Essays in climate policy and exhaustible resource economics



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Abstract

Owners of exhaustible resources will respond to climate policies, and these policies have to take such responses into account. This thesis considers three separate instances in which market power and exhaustible resources interact with climate policy.

Chapter 2 considers research and development (R&D) into green substitutes to oil as a climate policy instrument. Oil exporters will respond to such R&D efforts in ways which reduce the effectiveness of the policy. Making substitute technologies competitive against current oil prices is not sufficient. R&D efforts will only force higher oil supplies, aggravating short-term pollution. Eventually, the oil age will end as the substitutes become competitive against the marginal cost of producing oil. This motive encourages an R&D push to leave more oil underground. Strategic gaming between the importers and exporters may reduce both oil supply and R&D efforts.

Chapter 3 considers fixed costs into opening a deposit of an exhaustible resource. Counterintuitively, a monopolist may invest too early, into too much capacity. I then apply this model to an unconventional exhaustible resource: empty space underground, in which to store captured carbon emissions. I focus on the case of storing European emissions under the North Sea. Monopolistic storage is only a concern if storage space is sufficiently abundant. In this case, the monopolist will not invest enough, to cut back the cumulative storage capacity. Duopolistic storage may involve tacit collusion.

Chapter 4 considers an unconventional climate policy instrument: capital income taxes imposed on oil exporters. Such taxes can motivate conservation of polluting resources and allow oil importers to appropriate some oil wealth. These benefits come at the cost of inducing productive distortions, which diminish overall economic output.

This thesis contains approximately 70,000 words (using page 14 as a representative page).

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I now appear to have come to some milestone. The latest leg has been long and, often, hard going. I have benefited to no end from the advice and encouragement of my supervisors, Rick van der Ploeg and Cameron Hepburn. I am deeply grateful and indebted to them both.

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Chapter 1

Introduction

Climate change is inextricably linked to exhaustible resources. The modern economy is fundamentally based on the extraction of carbon-based fossil fuels from the Earth's crust. Burned in order to extract energy, the waste product—carbon dioxide (CO_2)—is then vented into the atmosphere. A part of these emissions is rapidly taken up by the uppermost layer of the oceans and the biosphere. However, removal of carbon from the system composed of the atmosphere, surface waters and vegetation is much slower. A substantial part of the CO_2 disposed of in the atmosphere persists, for economic purposes, forever. Climate change is best understood as resulting from this transfer of carbon from the subsoil into the atmosphere-ocean surface-biosphere subsystem (the 'surface subsystem').

The Earth as a whole tends toward a radiative balance, in which the energy coming into the system equals the energy flowing out. The Sun, of course, acts as the source of incoming energy. Outgoing energy results from the infrared radiation the Earth emits. All bodies emit radiation: the higher the temperature of the body, the more energy the radiation

carries, and the shorter the wavelength. As the Sun is much hotter than the Earth, solar radiation has a much shorter wavelength than the infrared radiation emitted by the Earth. Greenhouse gases, such as carbon dioxide, let this incoming shortwave radiation in, but partially block the outgoing longwave radiation. This 'insulation' means that the temperature of the lower atmosphere increases, and so does the amount of energy the Earth radiates. This process continues until the system is again in radiative balance. The resulting changes in the atmospheric energy balance affect the air circulation and water cycling patterns in complex ways which are hard to predict. This phenomenon is known as anthropogenic (human-caused) climate change. Existing human societies are adapted to the existing climate, and may thus be maladapted to a changed climate. This will have economic costs in terms of productive losses and impacts on human welfare.

Mitigating climate change—reducing the degree of change—requires the carbon concentration in the atmosphere to fall, relative to some baseline. The most common idea is to reduce emissions by extracting and burning less carbon. Another method is to keep burning carbon, but prevent the waste product from entering the surface subsystem, instead storing it underground. This is known as carbon capture and storage (CCS). A third possibility is the idea of negative emissions: removing carbon which has already been released into the atmosphere and locking it away. One way to do this would be by burning biomass in a conventional power plant and capturing the waste emissions (bioenergy with carbon capture and storage, or BeCCS). As the CO_2 generated in biomass combustion comes from the surface subsystem, capturing and storing it also reduces concentrations in the atmosphere. Another possibility would be to capture carbon directly

from the atmosphere by some chemical process (air capture).

Instead of mitigation, changes offsetting the effect of higher carbon concentrations might be considered. This is known as geoengineering. A popular idea is to spray reflective particles into the upper atmosphere. These particles will reflect a fraction of incoming solar radiation into space, and so achieve radiative balance by reducing the amount of shortwave radiation which reaches the surface. However, this 'solar radiation management' does not perfectly offset the changes resulting from adding carbon to the atmosphere, and may have unforeseen side effects. Other geoengineering options seek to accelerate the removal of carbon from surface subsystem into the deep ocean.

This thesis focuses on mitigation. Chapters 2 and 4 consider difficulties, related to the supply of fossil fuels under imperfect competition, in slowing the rate of fossil fuel extraction. Chapter 3 focuses on CCS, which can be used to reduce emissions or to generate negative emissions. I will not consider air capture *per se* or geoengineering.[1]

Fossil fuels—underground carbon—are finite, or exhaustible, resources. The total stock of such resources is, ultimately, limited. In some cases, extraction may be limited by physical exhaustibility (Hotelling, 1931). A more realistic case is one in which the resource becomes progressively more expensive to extract as cheaper deposits are exhausted first (Heal, 1976). The resulting scarcity of the resource encourages conservation. Retaining stocks has value, in terms of saving more of the resource, or more deposits which are cheap to exploit, for the future.[2]

[1]For some economics of air capture and geoengineering, see Barrett (2008); Pielke Jr (2009); Keith et al. (2006).

[2]The assumption on scarcity has been questioned by some, most notably Morris Adelman (Adelman, 1990; Adelman et al., 1991; Adelman, 1997). Periodic warnings

The canonical Hotelling framework for modelling exhaustible resources has been criticised both for not being microfounded on the engineering and geological facts (see Adelman, 1990, for discussion in the context of oil) and for being disproven empirically (Livernois, 2009). Despite this, I resort to the standard framework, for several reasons. Firstly, the lack of empirical validity in the past does not rule out the relevance of a model in the future (Hamilton, 2009). Particularly in the context of oil markets, observational evidence is hampered by changes in market structure, technology, and intentional misdirection.[3] Scarcity issues are likely to gain in importance; thus, the exhaustible resource framework may provide useful intuition about these markets in the future. Secondly, the lack of microfoundations is likely to be less important when studying these markets at a more aggregated level and over the long run. The models in Chapters 2 and 4 should be interpreted as describing exactly such long-run phenomena. Finally, these models remain normatively sound; if the assumptions hold, they will explain how rational agents might or should behave in resource markets. This applies particularly to the market for CO_2 storage space considered in Chapter 3.

Out of the three main fossil fuels—coal, oil and natural gas—coal is the most polluting, per unit of energy delivered, in terms of climate change. Coal is also the most abundant of the three: coal reserves generally exceed oil and gas reserves by an order of magnitude (Rogner, 1997). Kharecha and Hansen (2008) report that proven coal reserves could contribute more than

regarding resource exhaustion have been proved wrong by improvements in technology and new exploration. Currently, exactly such improvements are increasing proven reserves of shale gas and tight oil. However, there will be some limits on technological progress and so ultimate economic exhaustibility will eventually occur (but see Stürmer and Schwerhoff, 2012).

[3]On asymmetric information, see Sauré (2009); Gerlagh and Liski (2012b).

500 parts per million (ppm) to atmospheric concentrations, versus little more than 200 ppm for all proven oil and gas reserves taken together. For comparison, since the Industrial Revolution, atmospheric concentrations have grown by roughly 110 ppm, to around 390 ppm today. Coal can be considered effectively inexhaustible for the foreseeable future.

Exhaustibility plays a bigger role in oil and natural gas markets, although the recent shale gas and tight oil revolutions have greatly increased projected recoverable reserves. The abundance of coal does not mean that oil and gas should not be regulated. The source of carbon does not matter in terms of its marginal effect on climate. If a tonne of CO_2 emitted by a coal-fired power station has a high impact—maybe because existing concentrations are very high—then so will a tonne of CO_2 emitted by burning petrol as a transport fuel. Thus, the shadow price of these emissions, per a tonne of carbon, should be the same. Were the marginal climate impacts caused by a tonne of carbon to increase with background concentrations, abundant coal reserves would act as a lever to make both oil and gas more damaging as well.

Owners of large stocks of exhaustible resources, such as fossil fuels, have an incentive to form a cartel; in the oil markets, they have done so (Mason and Polasky, 2005). The Organization of the Petroleum-Exporting Countries (OPEC) controls 70% of proven reserves and currently supplies some 38% of liquid fuel consumption; this share is predicted to increase to 45% over the next 20 years as other suppliers' stocks dwindle (BP, 2012). Recent technological advances, despite reducing OPEC's market power in the medium term, leave its long-term importance in the oil market unchanged (IEA, 2013). There is some controversy regarding the exact

extent to which OPEC has been able to exploit its market power (Smith, 2005). However, OPEC certainly has been an important actor since its inception in 1961, and may well continue to be one in the future.

Any market dominance leads to strategic behaviour: the dominant actors recognise they are able to affect markets and/or other parties' behaviour. With respect to the present thesis, oil-dependent rentier economies have an interest in preventing tough climate policy from being implemented. A successful policy, intended to reduce demand for fossil fuels, would reduce the value of their assets. Divergent interests of the countries involved have led to the well-known failure of climate negotiations. Thus climate policy must take into account strategic behaviour by various parties, including OPEC. The challenge is to incentivise cooperation.

Any first-best climate agreement would involve carbon pricing. Recent studies have recommended prices, per tonne of CO_2, in the region of \$10–\$40.[4] Political difficulties, compounded by the economic crisis, have led to a price collapse in the sole operational regional trading scheme—the EU Emissions Trading Scheme—to \$6/tCO_2.[5] More generally, the political failure to implement such policies has led some to look for alternative policy instruments. In particular, funding research into developing cleaner technologies is often touted as a more workable policy instrument (Lomborg, 2012; Acemoglu et al., 2012). However, resource owners' responses may dull such policies: in the extreme case, if cumulative emissions are perfectly inelastic, climate policy by definition cannot affect them. This is

[4]Nordhaus (2010); Golosov et al. (2011); Gerlagh and Liski (2012a).

[5]On the other hand, the ongoing battle over U.S. fiscal policy has led to some conservative voices, traditionally opposed to carbon taxes, to soften their stance.

the idea known as the 'Green Paradox' (Sinn, 2008). The problem of inelastic supply can only be solved by either surprising resource owners, for example, by unexpected technological innovation; or by coercing or bribing the resource owners into leaving a fraction of the stock unexploited. As an example, Ecuador has already offered to leave the billion-barrel Yasuní oil reserve unexploited for $3.6bn. Were we confident of the security of property rights, it would be possible to solve the problem by buying out the marginal deposits of carbon (Harstad, 2012).

In Chapters 2 and 4 I study two unconventional climate policies, and the oil exporters' strategic responses to them. The specific policies are R&D subsidies (Chapter 2) and capital income taxation intended to alter the incentives of resource owners to conserve their assets, including oil (Chapter 4). The latter policy also allows the resource importers to appropriate some of the accumulated resource rents. To focus sharply on the key issues, and recognising that the dominance of OPEC is set to increase in the future, I will ignore interaction between dominant and 'fringe' producers (Karp and Newbery, 1993). I also ignore questions of bribery, coercion or surprise: in particular, I assume that the major research, development and deployment investments required to substitute for oil at a large scale are not sudden, stochastic events, but rather gradual processes to which resource owners can adjust. On capital income taxes, I show that such taxes are feasible, but that they have the undesirable side effect of distorting productive efficiency.

Both of these instruments are clearly second-best policies. I model them under the assumption that political considerations rule out the first-best of carbon pricing. I do not consider the effect of very patient 'Stern' time preferences, even though these could be of course implemented straightfor-

wardly. I also ignore the possibility that the various agents in the economy might have different time preferences, or that these might depart from the social planner's time preference. It might be reasonable to suppose that resource owners discount the future more heavily, for example because of perceived political risk (Long, 1975); however, the capital-intensive nature of these industries may in fact imply the opposite (Bohn and Deacon, 2000). I do not consider catastrophic risk (Weitzman, 2009), but rather focus on gradual climate change. Finally, I do not consider the stability of the coalitions implied in the model (Barrett, 2006b). To simplify the problem, I assume that both OPEC and the coalition conducting climate policy are stable and act cohesively.

In Chapter 3, I focus on carbon capture and storage. The idea that we may be able to use conventional power generation technologies, yet capture and lock away the resulting emissions, has the potential to reduce emissions drastically. CCS is developing rapidly and has generated substantial policy interest. As an example, the United Kingdom has established a £1bn CCS commercialisation fund.[6]

Until such demonstration projects yield more information, our understanding of the ultimate geological storage capacity remains limited. Some of the extracted underground carbon—oil and gas—has left behind empty reservoirs suitable for storing CO_2. Coal mines, however, will yield little (if any) storage capacity. Hence we will need additional space to store a substantial fraction of our future emissions. Saline aquifers are the main candidate, but their ultimate suitability is yet unknown. The potential certainly exists for CCS to play a major role in the transition to renewable

[6]On the other hand, the EU's recent competition for €1.5bn of funding for clean energy projects failed to attract any satisfactory CCS bids.

energy. CO_2 emissions from the European power sector are likely to be in the region of 2 $GtCO_2$ per annum by 2030, under a business-as-usual scenario (IEA, 2008). A major assessment of European storage capacities concluded total capacity, onshore and offshore, could be as high as 115 $GtCO_2$, sufficient to store all emissions from power generation for over 50 years. However, 95 Gt of this capacity is provided by saline aquifers. A major downward revision regarding total aquifer capacity would reduce the overall potential of CCS. Similarly, an increase in electricity demand, due to e.g. an electrified transport system, would put pressure on overall capacity. Of course, this would not reduce the potential of CCS at the margin. It would require higher mitigation efforts by other means.

In studying CCS, I move fossil resources into the background and instead consider empty storage capacity as the exhaustible resource. The economics of exhaustible resources can then be brought to bear on the question of how to deploy CCS optimally. In particular, the Hotelling model—often criticised as empirically invalidated—still retains substantial normative power. I offer insight into how the technology should be deployed: if ultimate storage capacity is scarce, the storage operations should be postponed in order to put off paying the high costs related to capturing and storing carbon. Of course, this result might be tempered were learning-by-doing taken into account. The relative maturity of the component technologies means there may not be scope for much learning, however.[7]

[7]Transport and injection technologies have been operational for several decades at commercial scale, as part of enhanced oil recovery (EOR) operations. CO_2 capture based on aqueous amine absorption is similarly considered a mature technology; however, alternative capture technologies may provide scope for cost reductions (Jones, 2011). See footnote 5, Chapter 3 for further details.

Most assessments of the economics of CCS assume the technology will be deployed in a socially optimal fashion. I also consider what happens should an owner of storage capacity—such as Norway in the European context—have market power and exploit it non-cooperatively. With high fixed investments, the resource monopolist can create more market power by investing—counterintuitively—too early, and too much. However, this result is unlikely to be important in the context of CCS. In fact, if storage capacity is very scarce, then the laissez-faire outcome may be quite acceptable, despite market power. Much more important is the conventional possibility that the monopolist cuts back supply, by not selling the entire stock of capacity. In an exhaustible resource context, this occurs mainly in a dynamic sense, by curtailing investment (Gaudet and Lasserre, 1988). Coordination of policy is much more important if the higher projections of overall capacity are realised. I also show that duopolistic competition, for example between Norway and Scotland, may quite naturally lead to tacit collusion to hold back supply of capacity.

A few words on methodology. All of the papers employ dynamic optimisation methods, including in a strategic framework. Often, analytical study of such models requires very particular assumptions to retain tractability. In this thesis, I resort to numerical methods in all three chapters. Computational methods are powerful and allow a wider range of models to be studied. In addition to providing quantitative solutions to the model, they can also provide additional insight to further the analytical efforts (Judd, 1997). In Chapter 2, I contribute to the literature on numerical methods by employing a novel algorithm to solve for the Markov Perfect Nash Equilibrium to a differential game. Chapter 3 takes a workhorse model of

preemptive capacity investment (Gilbert and Harris, 1984) and develops it further by including scale economies in capacity build-up.

The problem of climate change will only truly be solved once economies stop transferring carbon from the Earth's crust into the atmosphere. This will require alternative, non-exhaustible energy sources. The technologies to exploit these sources have to become very cheap. Both market power and exhaustibility imply that fossil fuel prices do not reflect marginal costs. Thus, the correct target for clean R&D is not the price of fossil fuels, but rather the marginal cost. As long as the price is above marginal cost, clean technologies will temporarily increase pollution, whether the resource is abundant or not, until finally driving the resource out of the market.

CCS could be seen as a bridging technology. Moreover, it would maintain the value of coal resources should climate policy become unavoidable, thus reducing political opposition to such policy. However, it is relatively costly. As long as even modest carbon pricing remains out of political reach, it is difficult to see how CCS could play any significant role. Under this pessimistic scenario, it may be that the best bet for unilateral climate policy would be to push for R&D, push very hard—and hope that the technological process for once goes where it was intended to go.

Chapter 2

Green technologies and the protracted end to the age of oil: A strategic analysis

Abstract

This paper considers competition between an oil exporter depleting and selling an exhaustible resource, and an oil importer able to gradually lower the cost of substitutes. R&D into clean fuels begins before the substitutes are competitive, in order to reduce overall development costs. The substitute constrains the oil exporter's market power: after an initial Hotelling-type stage, oil pricing becomes constrained by the ever-cheaper substitute technology. Supply is thus non-monotonic, initially falling, then forced up by competition from the substitute. Climate change slows down substitute development: rapid R&D forces the exporter to extract oil faster, aggravating near-term environmental impacts. If oil extraction becomes more expensive as supplies are depleted, the importer switches into clean fuels once these price oil out of the market; technological development will eventually be hastened to leave more of the oil locked underground. Novel numerical methods for solving PDEs are introduced into a differential game context.

2.1 Introduction

Developed economies have many reasons to worry about their dependence on oil. The Organization of the Petroleum Exporting Countries (OPEC), a cartel, is a powerful player in the market for the resource, accounting for some 40% of current production. The cartel is likely to become even more dominant going forward, as it controls more than 70% of global proven reserves (Central Intelligence Agency, 2011).[1] Firstly, the market power of the suppliers means oil importers feel they are getting a bad deal. Despite slow growth, oil prices have remained around levels seen in the run-up to the 2008 financial crisis. Secondly, as proven reserves are largely concentrated in politically volatile countries, this dependence has geopolitical and security ramifications. These have become painfully obvious over the past decade. At the time of writing, there is some light at the end of the tunnel regarding deadlock over the Iranian nuclear program, but few people would confidently predict an era of increased political stability in the Middle East. Thirdly, some worry that the resource will run out relatively suddenly, leading to a severe economic shock (known as 'peak oil').[2] The infrastructure in developed economies is, at present, fundamentally built around the assumption of abundant oil. This infrastructure is very long-lived, and a

[1] It should be noted that recent developments may curtail this market power. Technological developments in the extraction of 'tight oil' may increase both production and reserves of the non-OPEC 'fringe' producers (Maugeri, 2012). However, currently this boom is expected to run out of steam by 2030, again increasing the power of OPEC (BP, 2013). A second factor, reducing the ability of OPEC to hold substitute energy sources at bay, are the higher fiscal requirements needed to pacify a population restive in the wake of the 'Arab Spring' (Blas, 2012; Griffin, 1985). The present paper focuses on the long term: I will be focusing on a perfectly monopolistic OPEC. This simplification is chosen for model tractability, as well as to sharpen the intuition. Of course, a more detailed model would consider e.g. cartel-fringe interactions (Newbery, 1981; Groot et al., 2003).

[2] A relatively rapid cessation of oil supply is on the extreme fringe of 'peak oil' concerns. There also exist more nuanced versions of this argument.

sudden contraction in oil availability could prove to be very costly. Finally, as oil is a fossil fuel, there are concerns over its environmental impacts, particularly climate change.

How does an economy kick the cheap oil habit? For some, the answer is 'drill, baby, drill'—increasing domestic supply. In the United States, production of so-called 'tight oil' is projected to increase by up to 3.5 bn barrels per day, or almost 20% of domestic consumption, in the next decade (Maugeri, 2012). However, domestic production will not insulate an oil consumer from commodity markets: the Middle East would retain its geopolitical significance. Oil production outside the Middle East also tends to be expensive. Of course, domestically produced oil still contributes to climate change; it can also cause other environmental damages during production. Domestic production can delay the day of reckoning in terms of peak oil, but eventually these reserves too will dwindle.

An alternative option would be to rely on unconventional oil, such as tar sands, or to start converting coal to liquid fuels (CTL). These solutions would kick the exhaustibility can much further down the road. However, such fuels are expensive to produce, although profitable given that oil prices remain at the relatively high level seen since 2006. Moreover, they are extremely polluting: CTL fuels emit twice as much CO_2 as conventional oil (EPA, 2007). Ranking countries by CO_2 emissions, the Secunda CTL plant in South Africa would feature at around number 50.

In the wake of the failure of the climate summit in Copenhagen in 2009, R&D subsidies into clean technologies have been proposed as a form of climate policy in financial newspapers, think tank publications, and the US

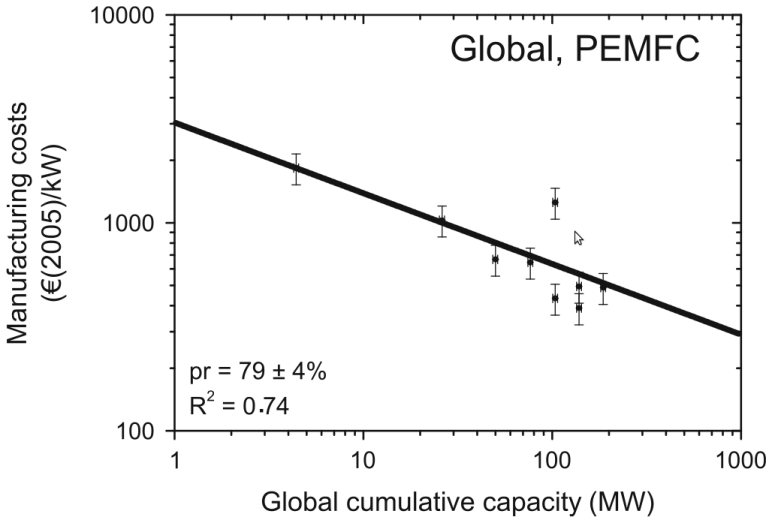


Figure 2.1: Manufacturing costs for proton exchange membrane fuel cells, as function of cumulative produced capacity. Reprinted from Energy Policy, Vol 38, Schoots, K., G.J. Kramer and B.C.C. van der Zwaan, "Technology learning for fuel cells: An assessment of past and potential cost reductions", 2887–2897, Copyright (2010), with permission from Elsevier.

Congress.[3] Substitutes to oil could include third-generation biofuels, or a hydrogen- or electrity-based transport system fuelled by clean electricity, e.g. wind or solar power, or CCS-equipped coal power. Such green technologies would solve the security issues and the environmental question. The problem is cost: most green technologies are much more costly than their dirty counterparts.

As an example, consider hydrogen fuel cells for transport. In fuel cells, hydrogen is oxidized to generate electricity, which can be used to drive an electric motor. The hydrogen acts as a storage medium for energy. For the technology to be 'clean', the hydrogen must be generated by low-emission technologies, such as renewable electricity, nuclear power or CCS-fitted coal generation. The cost of producing fuel cells has fallen with produced ca-

[3]Lomborg (2011); Greenstone (2010); Nordhaus and Shellenberger (2012).

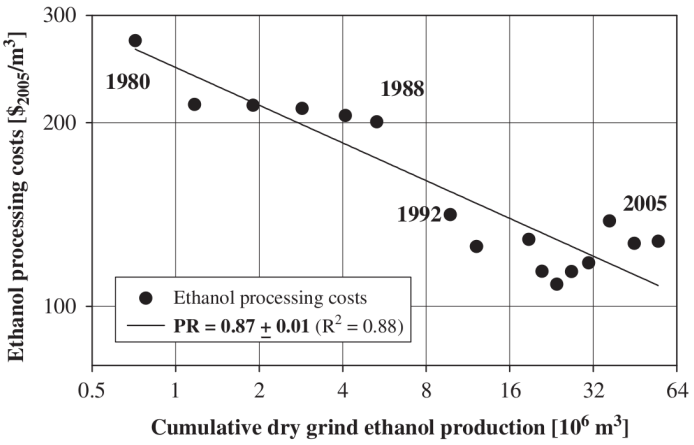


Figure 2.2: Ethanol processing costs, as function of cumulative production. Reprinted from Energy Policy, Vol 37, Hettinga, W.G., H.M. Junginger, S.C. Dekker, M. Hoogwijk, A.J. McAloon and K.B. Hicks, "Understanding the reductions in US corn ethanol production costs: An experience curve approach", 190–203, Copyright (2009), with permission from Elsevier.

pacity, due to learning-by-doing effects (Figure 2.1; Schoots et al., 2010). Note that around 200 megawatts of capacity has been manufactured, even though these technologies are not expected to be competitive against the internal combustion engine in the foreseeable future. Schoots et al. argue that these costs could potentially fall by up to a further factor of ten. Extrapolating, they estimate this to involve a learning investment of €40 bn. Should the learning effects not be appropriable, this development work would have to be subsidised for hydrogen fuel cells to be the solution to oil dependence. Moreover, if fuel cells are clean yet carbon taxes are politically impossible, private incentives to develop the technology would be nonexistent until oil started becoming truly scarce. In this case, public funding would be required to make a hydrogen-based transport system a reality.

Alternatively, a clean transport system could remain based on the in-

ternal combustion engine, but use renewable fuels. Such fuels, generated from plant matter, are called biofuels (the two main fuel candidates are ethanol and biodiesel). Hettinga et al. (2009) document learning-by-doing related to corn-based biofuel production in the US (Figure 2.2). They document a halving of unit production cost in the two decades after 1983, and argue that this results from learning-by-doing.[4] Learning-by-doing involves improvements due to experience in production, as well as research intended to develop more efficient production techniques. This chapter focuses explicitly on research, ignoring learning-from-experience. Lynd et al. (2008) assess the potential of purely research-driven cost reductions in biofuel production. They note that most of this potential lies in research into cheaper ways to convert low-value feedstocks (cellulosic biomass, such as switchgrass, which does not compete with food crops) into usable forms; with potential cost reductions of more than 50%, making cellulosic ethanol competitive against oil at plausible prices.

Oil exporters are worried about such prospects: substitutes threaten to destroy the value of what is, for many of these exporters, their sole asset. To deter R&D efforts, or development of unconventional reserves, exporters have to convince their customers that their worries are overblown. For example, they would want to reassure their customer that a sudden 'peak oil' event does not lie just around the corner, but that resources remain plentiful (Sauré, 2009; Gerlagh and Liski, 2012b). Customers would also have to be convinced that oil prices will remain at reasonably low levels. Indeed:

[4]Of course, the caveats related to private incentives to conducting R&D apply here as well.

> OPEC is working hard to bring down oil prices that jumped towards $130 a barrel earlier this year (...) and is pumping much more than its official target even as exports from cartel-member Iran dwindle.
>
> "We are not happy with prices at this level because there will be destruction as far as demand is concerned," OPEC Secretary General Abdullah al-Badri told an energy conference. "We're working hard to bring down the price. We're not comfortable." (*Reuters*, May 3rd, 2012)

In this paper, I consider two questions. I first focus on the strategic competition between a monopolistic oil exporter and an importing country (or a group a cooperating countries) able to gradually reduce the cost of a substitute technology. I focus on abundant substitutes, so that exhaustibility of the substitute is not an issue. The substitute might, thus, be thought of as a transport system fueled by renewable technologies, or by utilising abundant coal reserves.

In the second half of the paper, I focus explicitly on non-polluting substitutes, to consider how an environmental externality linked to oil use affects optimal R&D into clean technologies. I strip away meaningful strategic interaction to deliver a simple message.

The key points are as follows. The presence of substitutes curtails the oil cartel's market power: eventually, substitutes will impose a price ceiling on oil. Substitute development starts immediately, before the substitute is even close to being competitive, as the importer seeks to spread the cost of research over time. The supply of oil may initially be decreasing. Eventually the oil exporter will be constrained by the competing technologies, and will set prices so as to just keep these out of the market. The substitute will only be used once oil is exhausted.

Thus, eventually, the price of the substitute technology will effectively determine the price and supply of oil. A cheaper alternative fuel forces oil prices down; the exporter is forced to ramp up production, to keep the substitute at bay. When oil is polluting but the substitute is clean, cheaper substitutes will just lead to more oil extraction, and more pollution. Climate concerns induce the importer to slow down R&D efforts (assuming carbon pricing is not feasible, say, for political reasons). If oil extraction stops because of increasing marginal costs of extraction, rather than due to the entire physical stock being depleted, this result changes. In this more realistic case, the importer will eventually speed up substitute development, in order to shut a greater fraction of the oil reserves permanently out of the market (so preventing the embodied carbon from entering the atmosphere). If oil stocks will last for a long time, and near-term climate damages are substantial, the climate problem is still best tackled by initially conducting less research.

The existing literature on substitute development has tended to focus on cases in which the R&D investments are indivisible, making the R&D decision an optimal stopping problem. Gerlagh and Liski (2011) model a deterministic game in which the importer can trigger a process which ends with the introduction of the substitute. The delay between the decision to develop the substitute, and the arrival of the technology, acts as a commitment device: supposing the decision has not been made by a given period, the resource importer is committed to consuming the resource for at least an interval of length equal to the delay. The less resource remains, the more costly will this interval be, and the resource owner is forced to 'bribe' the importer into not switching—by increasing supply of the resource as

stocks fall.[5]

Earlier papers look at a similar situation without the adoption delay, explicitly modelling a *backstop technology*—that is, a technology which can perfectly substitute for the resource at constant marginal cost (Nordhaus et al., 1973). These authors also consider indivisible investments, with various assumptions on the ability to commit and the timing of moves (e.g. Dasgupta et al., 1983; Gallini et al., 1983; Olsen, 1993).

Harris and Vickers (1995) model a probabilistic R&D process in which a new innovation, once it arrives, makes the resource obsolete overnight. Thus, R&D produces discrete results, even though efforts takes place continuously. A particularly simple modification of the Hotelling rule characterises the resource owner's extraction rate, incorporating the strategic effect resource extraction has on the R&D efforts of the importer.

Incremental development of the backstop technology has been considered by Tsur and Zemel (2003). In their paper, cumulative investment reduces the unit price of producing the backstop. With linear R&D costs, socially optimal R&D implies immediate technological development at maximal rate, until reaching a steady state at which the benefits of investment just balance the costs (including the depreciation of accumulated knowledge).

Wirl (1991) considers the case in which the quality of the backstop is given, but investment is required in order to ramp up production capacity. A competitive backstop sector will invest to gain a profit, and to allow for these, the resource price must temporarily rise above the backstop production cost (while approaching it in the long run). However, the model

[5]See also Michielsen (2012).

ignores strategic investment into the backstop capacity.[6]

Gerlagh (2011) and Van der Ploeg and Withagen (2012) note that if resource extraction costs increase with cumulative extraction, lowering the cost of a substitute may lead to more of the resource being left unused. This result is obtained for some exogenous change in the backstop price.

The present paper extends the Hoel (1978) model of a limit-pricing monopolist into a dynamic game, with the R&D process involving convex (per period) costs. Hoel shows that a resource monopolist, faced with a perfect substitute that can be produced at constant, fixed marginal cost, will eventually limit price. That is, the monopolist will set prices to just keep the substitute out of the market for a prolonged period of time. With elastic demand and high initial resource endowments, this period is preceded by a Hotelling-type stage with increasing resource prices.[7]

Unlike in most of the existing literature, I assume that the substitute technology cannot be made competitive overnight.[8] Investments into clean energy technologies are not indivisible, but have to be built up gradually, over time. The present paper seeks to model such a more gradual R&D process, in which the accumulated stock of knowledge determines how competitive the substitute is. With convex R&D costs, it will be optimal to spread development work over time, even if the substitute will not initially be competitive against oil. In other words, research into substitutes takes place at all times, certainly before the substitutes are used, and even before they are competitive against the resource.

As R&D efforts only bear fruit in the future, the importer is effectively

[6]Tsur and Zemel (2011) consider a similar model, but without exhaustibility.

[7]This result has largely been absent from the theoretical literature; but see Van der Ploeg and Withagen (2012).

[8]The exceptions (Wirl, 1991; Tsur and Zemel, 2003) do not consider strategic issues.

dependent on foreign oil in the short term. The resource owner has the ability and the incentives to keep resource prices high. In addition to higher revenues, in the absence of commitment a strategic effect exists: high oil prices reduce current demand, ensuring more oil remains for the future, so weakening the importer's incentives to conduct research.[9] The importer, on the other hand, can induce lower oil prices today by credibly committing to higher prices once the price ceiling becomes binding. That is, slowing down R&D efforts yields the strategic benefit of cheaper oil in the near-term. This counterintuituve result agrees with the comparative static results of Hoel (1978) and Gilbert and Goldman (1978), who show that a lower entry price of a substitute, or the threat of entry itself, leads to higher prices charged by a resource monopolist prior to entry.

Thus, the paper relates to the literature on entry deterrence with natural resources. In my model, a resource monopolist tries to keep a competitive fringe at bay. The import cartel is able to subsidise the production technology of the fringe. This has two effects. A preference for low post-entry prices encourage subsidising the entrants' technology. The import cartel very likely employs inefficiently high R&D subsidies, as a response to the monopolist holding back production. The fringe costs become relevant as soon as the threat of entry starts constraining the monopolist, so that post-entry prices matter more for the importer than in the efficient case. However, dynamic strategic concerns partially offset this effect: the

[9]Note, however, that for most functional specifications equilibrium R&D effort is certainly likely to be above the *efficient* level, exactly because of high oil prices. I show that this holds for e.g. isoelastic utility in the commitment (open-loop) case. The commitment equilibrium is used as the point of reference for the strategic effects outlined in the text: without the ability to commit, the importer will conduct more R&D the less of the resource remains. The exporter can thus induce lower R&D effort tomorrow by lowering current extraction today, i.e. by keeping prices high.

import cartel recognises that the expectation of a higher-quality substitute leads the monopolist to hike up short-run resource prices (as in Hoel, 1978; Gilbert and Goldman, 1978).

This result is specific to the case of a privately-owned resource: with a common access (renewable) resource, an incumbent producer will lower prices, to deter entry by reducing the exploitable resource stock (Mason and Polasky, 1994). Common access harvesting imposes a negative externality on other firms, so that entry also has undesirable welfare effects, offsetting some of the desirable effect of reduced market power. In the second half of this paper, I allow resource extraction to cause a negative externality on the importers in terms of climate change. In the present paper, there are no common access resources *per se*. However, entry will force the resource monopolist to speed up extraction, as in the limit pricing stage a cheaper substitute leads to more oil and more emissions. Hence, the threat of entry causes a temporary negative effect, in terms of accelerated climate change; this is an undesirable consequence of the threat of entry. If extraction costs increase as the resource stock diminishes, then lower backstop costs will also leave oil unprofitable to extract while higher stocks remain. These carbon stocks, kept out of the atmosphere forever, yield a long-run benefit. Thus, the present paper duplicates the result of Gerlagh (2011) and Van der Ploeg and Withagen (2012), but with an endogenous, optimal R&D process.[10]

[10]The 'Green Paradox' of Sinn (2008) refers to supply-side effects of 'green' policies in exhaustible resource markets. Specifically, any policy which tends to depress future demand *relative* to current demand will lead to resource suppliers reoptimising to extract their resources faster, hence expediting emissions and exacerbating the environmental problem. Thus the correct policy should aim to depress current demand more, for example by an ad valorem tax decreasing over time.

In the present paper, a supply-side effect is shown to imply that the environmental problem should lead to less intensive development of substitutes. However, the cause of this effect is not exhaustibility of fossil fuels. Consider a reproducible good, produced at constant marginal cost c and associated with a negative externality also of magnitude

Note that a substantial literature exists on the wider issue of technological change, the environment and natural resources. Acemoglu et al. (2012) show that temporary R&D subsidies are sufficient to allow 'green' technologies to catch up with polluting ones, after which the economy will of itself develop in a green direction. While their rich model allows for exhaustible resources, it does not tackle strategic issues to do with supply of these. Popp (2008) finds that R&D subsidies complement, but cannot substitute for, carbon taxes by internalising the spillovers from innovation. In the present paper, I assume that spillovers are always internalised, but rule out carbon pricing. I also implicitly assume that there is perfect cooperation on 'green' R&D, an optimistic assumption (Barrett, 2006a). One of the purposes of the present paper is to assess whether supply-side effects (in fossil fuel markets) can blunt R&D-based climate policies even when these can otherwise overcome the problem of global cooperation.

To obtain the Markov-perfect solution to the game, I employ and extend numerical techniques which have not received much attention in resource economics. In a continuous-time framework, I use policy function iteration to obtain the solution as a system of two coupled, non-linear PDEs. This implicit system of functional equations is then solved using polynomial approximation methods. Such methods of solving differential equations have

c, with a linear demand curve. Now, the monopoly quantity q_M^* is efficient. Call the associated price $p_M^* = 2c$. Suppose there exists a perfect substitute, the initial production cost of which is given by $s > p_M^*$. The cost s can be driven down gradually, with some convex costs $C(-\dot{s})$. Let $\min s(t) \equiv \underline{s} < c$: the substitute is potentially superior, by virtue of being cheaper to produce when fully developed. Then it may be profitable to gradually drive down s to $\underline{s}$, in order to benefit from the cheaper technology. However, as s changes gradually, as long as $s(t) \in (c, p_M^*)$, the monopolist will still produce, keeping the substitute at bay, but the produced quantity $q_M > q_M^*$. This causes more external damages until, when $s(t) < c$, the monopolist is finally driven out. Were the externality absent, s would be optimally be driven down faster. This effect, not related to exhaustibility, is what I demonstrate in the present paper.

been introduced to economics by Judd (1998), Dangl and Wirl (2004) and Caporale and Cerrato (2010); see Balikcioglu et al. (2011) for an application in an environmental context. All of these papers use optimal stopping problems as examples; none tackle dynamic games or systems of PDEs. The method solves the present model relatively quickly and accurately.

The paper is structured as follows. I will first develop the basic model with physical exhaustion and absent the externality. This will serve to illustrate the basic structure of the problem, as well as reminding the reader of the model of Hoel (1978). Section 2.2 sets up the model and the social optimum is solved as a benchmark. Section 2.3 develops the non-cooperative equilibrium of this model. Section 2.4 extends the model to include a stock pollutant and extraction costs. Section 2.5 concludes.

2.2 The social optimum

An economy uses a natural resource, the flow of which is denoted by $q_F(t)$. To fix ideas, I will call this resource 'oil'. Oil is exhaustible, with remaining stock denoted by $S(t)$, and the (given) initial stock by S_0. Resource extraction is costless (I will modify this assumption in Section 2.4).

There is a perfect substitute for the resource—for example, biofuels, solar energy or coal-fired power with carbon capture and storage—called the backstop technology.[11,12,13] This substitute is produced perfectly com-

[11] As oil is a transport fuel, the latter two options would have to be combined with an electrified transport infrastructure.

[12] The assumption of perfect substitutability is extreme, but an often-used assumption in the literature. I will discuss it in Section 2.5.

[13] Biofuels are direct, and nearly perfect, substitutes to petrol; the blending of ethanol with petrol is currently mandated in many European countries. The latter technologies are substitutes on an economy-wide scale, once the (long-lived) transport infrastructure is allowed to adjust. The model thus represents these technologies only in a very reduced-form sense.

petitively at a unit cost x, and the production rate is denoted $q_B(t)$. In fact, the backstop production cost depends on the accumulated knowledge of technologies used in backstop production:

Assumption 1. Backstop technology. The backstop production cost x is a function of accumulated knowledge $K(t)$: $x = x(K(t))$, $x' < 0$, $x'' \geq 0$. The knowledge stock is normalised so that $K(0) = 0$, and the initial price is denoted $\overline{x} \equiv x(0)$. There exists a strictly positive lower bound to the backstop price: $\lim_{K\to\infty} x(K) = \underline{x} > 0$. If this bound is attained at $\overline{K}$, then $x'(K) = 0$ for $K > \overline{K}$.

Assumption 2. R&D process. R&D investment reduces the price of the resource incrementally. The rate of this research is denoted $d(t)$ and it builds up the knowledge stock according to $\dot{K} = d$. There are strictly convex monetary costs to conducting research[14] $c(d)$: $c \geq 0$, $c' \geq 0$, $c'' > 0, c(0) = 0$, $c'(0) = 0$. Knowledge does not depreciate.[15]

Assumption 3. Climate change. There are no externalities related to the use of the resource.[16]

The representative consumer has a quasilinear felicity function $v(q_F, q_B, M) = u(q_F + q_B) + M$, with $u' > 0$, $u'' < 0$. I assume that using the backstop resource is always preferable to zero resource use: $\lim_{q\to 0} u'(q) > \overline{x}$. M denotes money, normalised so that the exogenously given money income is zero. This yields the inverse oil demand curve:

$$p(q_F, K) = \min\{u'(q_F), x(K)\} \tag{2.1}$$

[14]Relaxing the assumption of zero marginal cost at $d = 0$ is straightforward but yields no further intuition.

[15]This incremental research effort could perhaps be thought of as more like development and deployment investment. I will refer to it, for brevity, as 'research' or 'R&D'.

[16]This assumption will be relaxed in Section 2.4.

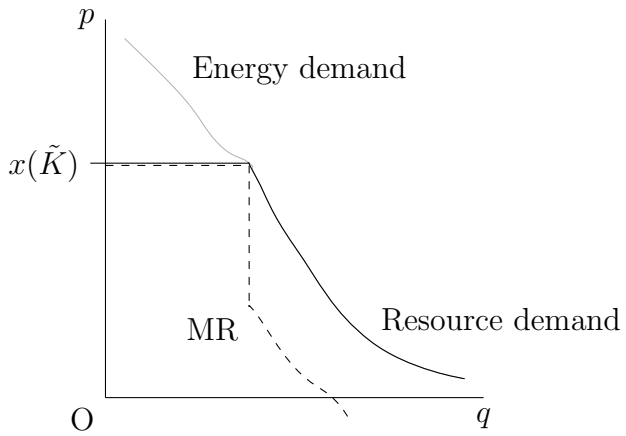


Figure 2.3: Inverse demand curve (gray); inverse demand for the exhaustible resource, given a backstop price $x(\tilde{K})$ (solid); and the corresponding marginal revenue curve (dashed).

Inverse demand is depicted in Figure 2.3. The backstop is supplied to satisfy the balance of the demand:

$$q_B(K) = \max\{0, u'^{-1}(p) - q_F\} \tag{2.2}$$

I assume the utility function is such that either $u'''(q)q + 2u''(q) < 0$, or $u''(q)q + u'(q) < 0$, for all q. In words, either revenue is concave, or marginal revenue is always negative, assuring the existence of a unique optimum to the monopolist's problem later on.

All agents in the economy live forever and discount the future at the common rate ρ. I omit notation to indicate the dependence of all variables on time.

Consider a social planner who weights the per capita welfare of all

people identically.[17] The planner's problem is

$$\begin{aligned} \max_{q_F,q_B,d} \int_0^\infty e^{-\rho t} \left(u(q_F+q_B) - x(K)q_B - c(d)\right) \mathrm{d}t \\ \text{s.t. } \dot{S} = -q_F, \quad S(0) = S_0, \quad S \geq 0 \\ \dot{K} = d, \quad K(0) = 0 \end{aligned} \tag{2.3}$$

The problem is solved using Pontryagin's maximum principle:

Lemma 1. Suppose an optimum to problem 2.3 exists.[18] Denoting the costate variables on the resource stock and the knowledge stock, respectively, by λ_S and λ_K, the necessary conditions for an optimum are

$$u'(q_F+q_B) \leq \lambda_S, \qquad q_F \geq 0, \quad \text{C.S.} \tag{2.4a}$$

$$u'(q_F+q_B) \leq x(K), \qquad q_B \geq 0, \quad \text{C.S.} \tag{2.4b}$$

$$c'(d) \leq \lambda_K, \qquad d \geq 0, \quad \text{C.S.} \tag{2.4c}$$

$$\dot{\lambda}_S = \rho\lambda_S \tag{2.4d}$$

$$\dot{\lambda}_K = \rho\lambda_K + q_B x'(K) \tag{2.4e}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_S(t)S(t) = 0 \tag{2.4f}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = 0 \tag{2.4g}$$

Proof. In Appendix 2.A.[19] □

These conditions are easily interpreted. The marginal utility of con-

[17] There are efficient outcomes in which the social planner gives preferential treatment of citizens of either country. As utility is quasilinear, these involve all numeraire consumption being allocated to the preferred country. Zero numeraire consumption pushes the assumption of quasilinear utility too far, and so these outcomes—while efficient given the model—seem to be beyond reasonable bounds of applicability for the model. For this reason, and for expositional clarity, I relegate the full discussion to the proof of Lemma 1.

[18] All later propositions assume the existence of an optimum.

[19] All proofs are in the Appendix.

suming an energy resource must be equal to its marginal cost, in the case of the fossil resource the scarcity rent ((2.4a) and (2.4b)). The marginal cost of research into the backstop technology has to equal the marginal benefit: the value of the marginal unit of knowledge ((2.4c)). As there are no extraction costs, the scarcity rent of the resource is constant in present value terms ((2.4d). The marginal value of the knowledge stock rises at the rate of interest plus capital gains ((2.4e)). The transversality conditions (2.4f) and (2.4g) indicate that the stocks of the resource and knowledge have to be used or built up so that the stock value, as $t \to \infty$, is zero in present value terms.

Definition 1. The *terminal path* refers to the optimal R&D process when the exhaustible resource is not used. It is the trajectory of R&D intensity $d(t)$, the R&D stock $K(t)$ and the associated costate variable $\lambda_K(t)$ which solve the social planner's problem for $S(0) = 0$. This solution is unique (see Proposition 2). As $K(t)$ is weakly monotonic and the optimisation problem is autonomous (not dependent on the starting date), I can denote the terminal path as the triplet $\{K, d^\infty(K), \lambda_K^\infty(K)\}$.[20]

Lemma 2. The terminal path is unique and satisfies

$$\lim_{K \to \overline{K}} \lambda_K^\infty(K) = \lim_{K \to \overline{K}} d^\infty(K) = 0.$$

Proof. In Appendix 2.A. □

The terminal path (Figure 2.4) describes the optimal R&D process once resource use stops, as a function of K. Even though defined here as the socially optimal path, it will appear also in the non-cooperative models: in

[20] If $\overline{K}$ is finite, then $\forall K \geq \overline{K}$, $d^\infty(K) = \lambda_K^\infty(K) = 0$.

the stage following exhaustion, the oil importer optimally conducts R&D according to the terminal path. This provides the terminal condition for the optimisation problem solved in the resource-using stage. Note that the R&D intensity may behave non-monotonically. The marginal benefit of knowledge λ_S is just the present value of the stream of future cost reductions it yields: integrating (2.4e) in the interval $[t, \infty)$,

$$\lambda_K(t) = -\int_t^\infty e^{-\rho(s-t)} q_B(s) x'(K(s))\,\mathrm{d}s \tag{2.5}$$

At any moment, the total reduction in the cost flow is the marginal reduction in backstop cost, multiplied by the quantity of the substitute consumed. Thus, capital gains may be low if the backstop is consumed in small amounts, *or* if a marginal unit of knowledge only reduces the costs a little. When capital gains are low, the shadow value mostly represents future benefits and will fall more slowly, or even rise. The precise behaviour depends on the interaction of the demand for the resource and the effectiveness with which cumulative R&D effort reduces the backstop cost.[21]

Proposition 1. The social optimum is characterised by two stages:

Stage I. $t \in [0, t^*)$, $t^* > 0$. Initially, only the exhaustible resource is used, with rate of extraction decreasing monotonically. The resource is

[21] With the present assumptions, little more can be said. If $x(K)$ is linearly decreasing, then the terminal path will be single-peaked (with the peak possibly occurring at $K = 0$). To see this, note that $\frac{\mathrm{d}\lambda_K}{\mathrm{d}K} = \frac{\dot\lambda_K}{\dot K} = 0$ only if $\dot\lambda_K = 0$. In this case, $\ddot\lambda_K$ has the same sign as $q_B'(x)x'(K)^2 + q_B(x)x''(K)$; this is negative if $x(K)$ is linear. A single-peaked terminal path can be analytically demonstrated by assuming, in addition a linear $x(K)$, linearity of demand and a quadratic R&D cost. The system $\begin{pmatrix}\dot K & \dot\lambda_K\end{pmatrix}'$ is then linear in $\begin{pmatrix}K & \lambda_K\end{pmatrix}'$, and so the solution paths are sums of competing exponentials. The numerical specification in Section 2.3.2 also gives rise to a single-peaked terminal path (Figure 2.8). I have not been able to rule out more complicated forms of non-monotonic behaviour, but neither have I been able to categorically rule out such behaviour. More complicated forms of non-monotonicity would seem to require regions of high curvature of the demand curve and/or of $x(K)$.

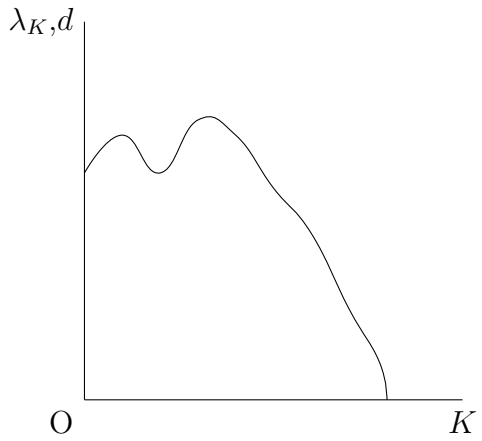


Figure 2.4: A generic example of the terminal path in (K, λ_K^∞)-space. d^∞ increases monotonically with λ_K^∞. The economy moves to the right along the path at a rate increasing with λ_K.

fully used up by the switching date t^*. R&D intensity is strictly positive and increases monotonically.

Stage II. $t \in [t^*, \infty)$. In the second stage, the economy uses only the substitute and moves along the terminal path. Substitute use increases monotonically as the unit costs falls, until the date t^{**} (if finite) when the lower bound on the backstop cost is attained. Research effort is strictly positive until this date. Ultimately R&D effort falls to zero: $\lim_{t\to\infty} d(t) = 0$.

Proof. In Appendix 2.A. □

Thus, in the social optimum, initial resource use is sufficiently high so that, by the time the marginal utility of resource use (denoted p_F) rises to the backstop price, exhaustible resource use stops as the stock is fully depleted. This will not hold in the non-cooperative equilibrium. Figure 2.5 illustrates the social optimum for a case in which the lower bound on the backstop cost is attained in finite time.

I will now consider some comparative statics of the social optimum.

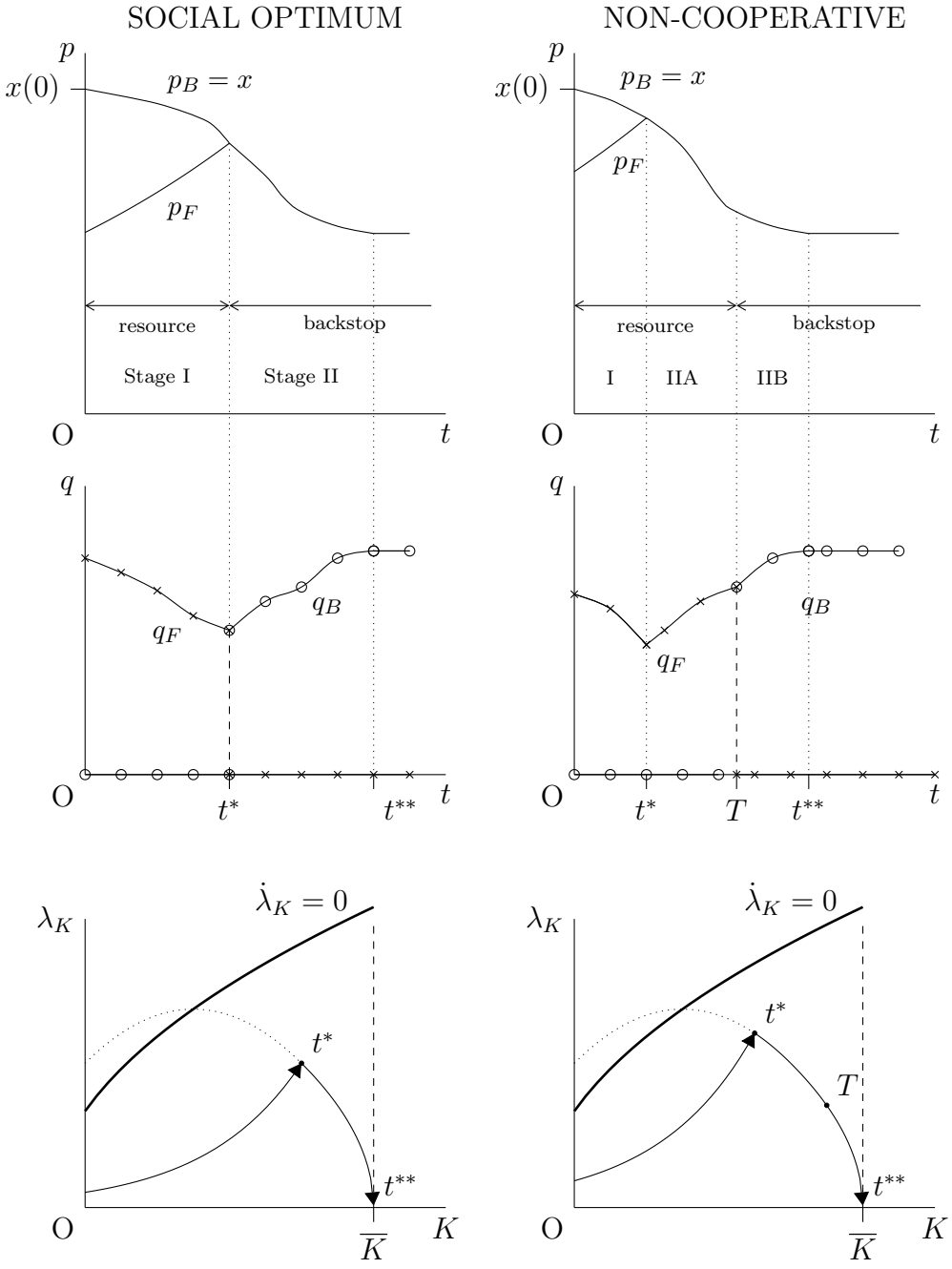


Figure 2.5: (*top*) Time paths of the backstop and resource prices (p_B and p_F, respectively) under the social optimum (*left*) and the non-cooperative equilibrium (*right*); (*middle*) Quantities consumed of the exhaustible resource (crosses) and the backstop resource (dots); (*bottom*) Trajectories in (K, λ_K)-space (solid line) and the terminal path (dotted line).

It might be conjectured that a less patient planner would consume the resource more quickly, and invest less for the future. This intuition is partly true:

Proposition 2. For the social optimum, an increase in impatience (a rise in the discount rate ρ) implies the backstop price at the moment of the switch will be higher: $\frac{\mathrm{d}x(t^*)}{\mathrm{d}\rho} > 0$. Either the initial extraction rate will rise, or the initial R&D intensity fall, or both. Effect on the timing of the switch is ambiguous: an earlier switch implies that the initial resource extraction rate rises, but the effect on initial R&D intensity is ambiguous; while a later switch implies that the initial R&D rate falls:

$$\frac{\mathrm{d}t^*}{\mathrm{d}\rho} < 0 \Rightarrow \frac{\mathrm{d}q_F(0)}{\mathrm{d}\rho} > 0$$
$$\frac{\mathrm{d}t^*}{\mathrm{d}\rho} > 0 \Rightarrow \frac{\mathrm{d}d(0)}{\mathrm{d}\rho} < 0.$$

Proof. In Appendix 2.A. □

Proposition 2 says that an increase in impatience will lead to at least one type of asset falling in marginal valuation. Two assets exist in the economy: the exhaustible resource and knowledge. An increase in impatience will either increase the depletion of the former, or slow the accumulation of the latter, or both.[22] However, the assets are linked by the optimality condition that the marginal utility of energy consumption never jump. If the consumption of the exhaustible resource increases sufficiently, this dominates the desire to invest less in the substitute: the R&D efforts must intensify for the backstop to be available at the optimal price by the time

[22]Numerical examples confirm that the rise in ρ can yield more initial R&D effort; the crucial factor is a low R&D investment cost. However, no examples have been found in which the resource extraction rate falls with a rise in ρ.

the resource runs out. The switch will occur earlier if the incentives to conserve the resource are more responsive to time preference than the incentives to accumulate knowledge.

Proposition 3. An increase in the initial resource stock S_0 implies a higher initial extraction rate, a lower initial R&D rate, and a delay in introducing renewables: $\frac{\mathrm{d}q_F(0)}{\mathrm{d}S_0} > 0$, $\frac{\mathrm{d}d(0)}{\mathrm{d}S_0} < 0$, $\frac{\mathrm{d}t^*}{\mathrm{d}S_0} > 0$. An increase in the initial knowledge stock implies a higher initial extraction rate.

Proof. In Appendix 2.A. □

Thus, a higher resource stock makes the problem of substitute development less pressing and will allow the social planner to share the benefits between higher resource consumption and being able to develop the substitute at a more leisurely pace. This would also apply to technological developments which increase the resource base. A more advanced technological state (initial knowledge stock) will also allow higher resource consumption. The effect on the R&D programme cannot be signed under general functional forms as it depends on the R&D profile along the terminal path.

2.3 The non-cooperative equilibrium

2.3.1 Equilibrium with commitment

Consider now setting up the above problem as a non-cooperative differential game, in which one agent (the exporter, indexed by E) owns the resource stock; and a second agent (the importer, indexed by I) buys the resource for consumption, and strategically develops and deploys the backstop technology. R&D is not conducted here by firms, but by the importing

government. This might be because the government wants to coordinate R&D spending, or because the benefits due to R&D are not appropriable and hence R&D has to be funded by the government.[23]

For purposes of intuition, I will first consider an equilibrium in the case in which commitment is possible; i.e. an equilibrium in open-loop strategies. Open-loop strategies are entire time paths of the choice variables. Hence the exporter is optimising extraction given a path for $d(t)$, and thus for the substitute cost $x(t)$. The importer, on the other hand, is trying to optimise $d(t)$, and so $x(t)$, given a time path of the extraction rate.

The open-loop equilibrium is intended to illustrate the qualitative features of the Markov-perfect (closed-loop) Nash equilibrium (MPNE). For some initial states, a MPNE exists which coincides with the open-loop equilibrium. In the next section, I will show numerically that the two equilibria are very similar both qualitatively and quantitatively.

The limit-pricing argument discovered by Hoel (1978) is at the heart of the strategic equilibrium. Consider a monopolist supplying a resource, for which there exists a competitively supplied perfect substitute with a fixed, constant price. The substitute imposes a price ceiling, with strictly positive demand at the ceiling price, and a discontinuity in the marginal revenue function (Figure 2.3).

The monopolist will eventually start selling the resource at a price only just undercutting the marginal cost of the substitute, satisfying the entire demand at this price. Initially, the resource may be optimally priced below the backstop price. If resource demand is elastic, the resource owner has to

[23]Popp (2002) finds that federal R&D spending in the United States is a costly way of inducing new patents. The government R&D spending here could be interpreted as directed R&D subsidies and deployment support.

choose between selling the marginal unit of the stock immediately, possibly depressing revenue earned for the inframarginal units, or at the time of exhaustion at the backstop price.[24] If the initial resource stock is large, exhaustion may occur a long time in the future and immediate sale is preferred. It is straightforward to show that the same result holds for a given decreasing backstop price path (see Proposition 4 below).

The exporter maximises the discounted revenue stream

$$\begin{aligned} &\max_{q_F} \int_0^\infty e^{-\rho t} R(q_F; K)\, \mathrm{d}t \\ &\quad \text{s.t. } \dot{S} = -q_F, \quad S(0) = S_0, \quad S \geq 0 \end{aligned} \tag{2.6}$$

where $R(q; K) \equiv p(q; K)q$ denotes revenue, with inverse demand given by (2.1). I will from now on omit the dependence on K. The problem is solved subject to the path of R&D spending $d(t)$, and the (decreasing) path of the substitute price $x(t)$, both taken as given.

Proposition 4. Consider a monopolist solving (2.6), subject to a backstop price path $x(t)$, with $x(t)$ continuous and weakly decreasing. Then the monopolist will eventually limit price, from date t^* until date of exhaustion T:

$$\exists t^*, T : \left\{0 \leq t^* < T; \forall t \in [t^*, T),\ q_F^*(t) = p^{-1}(x(t))\right\}$$

with $T \equiv \arg\inf_t \{S(t) = 0\}$. If energy demand is elastic, and given sufficiently high S_0, the monopolist will initially price substantially below the backstop price, so that marginal revenue R' satisfies

$$\frac{\mathrm{d}R'(q_F)}{\mathrm{d}t} = \rho R'(q_F),\ t \in [0, t^*).$$

[24] Of course, if resource demand is everywhere inelastic, the monopolist will always raise the price as high as she can—to the price of the backstop.

Proof. In Appendix 2.B. □

The importer maximises the discounted stream of utility of the representative consumer, i.e. utility from resource consumption less spending on purchasing the exhaustible resource and R&D activities:

$$\begin{aligned} &\max_{d} \int_0^{\infty} e^{-\rho t} \left(u(q_F + q_B) - p(q_F)q_F - x(K)q_B - c(d)\right) \mathrm{d}t \\ &\quad \text{s.t. } \dot{K} = d, \quad K(0) = 0 \end{aligned} \tag{2.7}$$

subject to the exhaustible resource supply path $q_F(t)$, taken as given; and assuming that the representative consumer maximises utility, taking prices as given. I omit any tax or tariff instruments, to focus solely on the effect of technological development.[25] Note that, once the resource is exhausted, $q_F = 0$ and the objective function coincides with that of the social planner in the previous section.

The equilibrium is solved in detail in Appendix 2.B. The perfect substitutability affects the exporter's problem as in Hoel (1978). Once limit pricing begins, however, the importer recognises he is in control of the energy price. Without externalities, he is indifferent between consuming the substitute and consuming the resource at the price of the substitute. The importer will thus conduct R&D as in the case in which no resource exists, i.e. according to the terminal path. In the next section I will show that this limit-pricing stage is also subgame perfect.

Proposition 5. The open-loop equilibrium features three stages:

[25] This assumption is for analytical convenience. It could be justified by the observed political difficulty of agreeing to a globally binding agreement on carbon pricing. The failure of climate policy has also recently been used as an argument to focus efforts solely on policies to promote clean substitute technologies.

Stage IA. $t \in [0, t^*)$, $t^* \geq 0$. Initially, only the exhaustible resource is used ($q_F > 0$), with rate of extraction decreasing monotonically ($\dot{q}_F < 0$), and marginal revenue rising, according to a monopolist's Hotelling Rule, at the discount rate ($\frac{\mathrm{d}R'(q_F)}{\mathrm{d}t} = \rho R'(q_F)$). A strictly positive quantity of the resource is left at the date t^* ($q_F(t^*) > 0$). Resource price is strictly below the unit cost of the backstop ($p_F < x(K)$) and the backstop is not used ($q_B = 0$). R&D intensity is strictly positive ($d > 0$) and increases monotonically ($\dot{d} > 0$), with the marginal cost increasing at the discount rate ($\frac{\mathrm{d}c'(d)}{\mathrm{d}t} = \rho c'(d)$).

Stage IB. $t \in [t^*, T]$. Only the exhaustible resource is used ($q_B = 0$, $q_F > 0$), with the monopolist limit pricing at the backstop price ($p_F = x(K)$). Resource use increases monotonically ($\dot{q}_F > 0$), and T is determined by the date at which the stock is fully exhausted ($S(T) = 0$). R&D intensity is initially strictly positive ($d > 0$ for some $t > t^*$) and may behave non-monotonically. The R&D process follows the terminal path, with marginal cost satisfying

$$\frac{\frac{\mathrm{d}c'(d)}{\mathrm{d}t}}{c'(d)} = \rho + \frac{q_F x'(K)}{c'(d)}$$

If the date t^{**} at which the lower bound on the backstop cost is attained is less than T, then R&D intensity is zero following this date and resource use is constant.

Stage II. $t \in [T, \infty)$. In the final stage, the economy uses only the substitute and follows the terminal path.

Proof. In Appendix 2.B. □

The equilibrium is illustrated in Figure 2.5. Assuming that Stage IA is not degenerate ($t^* > 0$), it features non-monotonic extraction. This is not

'peak oil', but its inverse: supply of oil first falls, as the monopolist follows her Hotelling Rule, but once limit pricing begins, oil supply has to increase in order to fend off the ever cheaper substitute. R&D begins before the substitute becomes properly competitive, in order to spread the (convex) investment costs over time. Once the resource is exhausted, the economy switches to the substitute and the importer will follow the terminal path.

Intuition suggests that the non-cooperative equilibrium would feature excessively low extraction and excessive R&D effort, compared to the socially optimal $q_{F,\mathrm{SP}}$, d_{SP}, as the exporter seeks to push up revenues and the importer tries to force the exporter to sell the resource faster. At the present level of generality, it is difficult to confirm this. However, if the elasticity of resource demand $\epsilon(q) \equiv \left|\frac{p(q)}{qp'(q)}\right|$ is weakly monotonic with respect to quantity, it is straightforward to verify the following:

Proposition 6. If $\epsilon'(q) \geq 0$, the open-loop equilibrium will feature inefficiently high initial R&D effort: $d > d_{\mathrm{SP}}$ for $t \leq t^*$. If $\epsilon'(q) \leq 0$, then there is excessive resource conservation: $q_F < q_{F,\mathrm{SP}}$ for $t \leq t^*$. With isoelastic utility, $\epsilon'(q) = 0$, both hold and the substitute becomes (nearly) competitive inefficiently early: $t^* < t^*_{\mathrm{SP}}$.

Proof. In Appendix 2.B. □

Thus, under the assumption of isoelastic utility, the open-loop equilibrium will indeed imply excessively low initial resource extraction rates, as the monopolist cuts extraction from the socially optimal level, and excessively high R&D rates, as the importer starts benefiting from low backstop costs earlier, at the time limit pricing begins. Both of these effects imply that the resource price meets the backstop price too soon, compared to

what would be socially optimal. [26]

Proposition 7. With isoelastic demand, an increase in the initial resource stock S_0 increases initial equilibrium oil supply $q_F(0)$, lowers initial R&D efforts $d(0)$ and leads to a delay in the substitute becoming competitive (t^* rises).

Proof. In Appendix 2.B. □

Hence, having more of the exhaustible resource has similar effects as in the socially optimal case: loosening the resource constraint of course yields higher resource supply, but also a reduced need to conduct costly R&D, and thus a delay in the substitute becoming competitive.[27]

2.3.2 Non-cooperative case without commitment

I will now turn to the equilibrium in the absence of commitment, limiting myself to Markovian strategies—strategies which are functions of the current state of the system only—and thus to the Markov-perfect Nash equilibrium concept (MPNE). In the previous sections, I have resorted to Pontryagin's Maximum Principle in order to solve the social optimum and the open-loop equilibrium. In the present section, I will reformulate the problem in terms of dynamic programming. This change is motivated by

[26] It should be noted that, for the special case of isoelastic utility but with a backstop technology, the outcomes under monopolistic and competitive resource extraction do not coincide, as would happen in the absence of a backstop technology (Stiglitz, 1976). The reason is that, with a backstop technology, there exists a price at which energy demand remains strictly positive, but oil demand goes to zero because of the backstop. This results in the limit-pricing outcome studied in the present paper. Inefficient resource extraction then also leads to an inefficient R&D process.

[27] It is more difficult to sign the effects of a higher initial knowledge stock.

the computational methods I use to solve the equilibrium outcome: specifically, I numerically approximate the two value functions.[28]

The exporter will, at each moment, solve the Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} \rho V^E(K,S) = \max_{q_F} \left\{ R(q_F) + \tilde{d}(K,S) V_K^E(K,S) - q_F V_S^E(K,S) \right\} \\ \text{s.t. } q_F \in \mathbb{R}^+, \quad S = 0 \Rightarrow q_F = 0 \end{aligned} \tag{2.8}$$

in which $\tilde{d}(K,S)$ denotes the belief over the importer's strategy. Crucially, the exporter recognises that its choice of extraction rate will affect the future state of the economy, and hence the importer's future R&D rate.

The importer's problem is similarly

$$\rho V^I(K,S) = \max_{d} \Big\{ u(\tilde{q}_F(K,S) + q_B) - R(\tilde{q}_F(K,S)) - x(K) q_B - c(d) \tag{2.9}$$

$$+ d V_K^I(K,S) - \tilde{q}_F(K,S) V_S^I(K,S) \Big\} \tag{2.10}$$

$$\text{s.t. } d \in \mathbb{R}^+ \tag{2.11}$$

where p_F and q_B are given by (2.1) and (2.2). The importer recognises the effect of its R&D rate on the future state, and thus the future extraction

[28] Optimal control methods offer better analytical tractability for the socially optimal and open-loop equilibrium outcomes, as they do not require solving partial differential equations. This is often the case (Liberzon, 2012). Dynamic programming could be seen as more naturally suited to Markov perfect equilibria, as the solution method implies solving for the feedback control rule (Başar and Olsder, 1999). However, either of the two methods can in principle be used (Starr and Ho, 1969). As an example, dynamic programming methods are convenient for open-loop analysis if, for example, the model specification is linear-quadratic (as in, e.g., Wirl, 1994; List and Mason, 2001), or if qualitative, phase-portrait analysis suffices (Wirl, 2007). Similarly, the Maximum Principle can also be used to analyse Markov perfect equilibria (e.g. Karp, 1992; Karp and Livernois, 1992; Clemhout and Wan, 1985).

rate chosen by the exporter.

I will first obtain a result pertaining to open-loop equilibria in which the initial knowledge stock, denoted by $K_0 \geq 0$, is now allowed to vary. I will index the equilibria by their initial state (K_0, S_0).

Definition 2. Denote by Φ the set of open-loop equilibria such that limit-pricing begins immediately, i.e.

$$\Phi = \{(S_0, K_0) : MR(p^{-1}(x(K_0))) \leq e^{-\rho(T-t^*)} x(K(T))\}$$

where $MR(\cdot)$ denotes marginal revenue, $p^{-1}(x(K))$ is inverse demand at the backstop price, and $T = T(K_0, S_0)$. The upper boundary of this set is given by $S_0 = \phi(K_0)$, along which the above holds as an equality.

Proposition 8. In the set Φ, the open-loop equilibrium coincides with a Markov-perfect Nash equilibrium.

Proof. In Appendix 2.C. □

Thus, following the open-loop strategies with $\dot{\lambda}_K = \rho\lambda_K + q_F x'(K)$ (synthesised as functions of the state variables) is subgame-perfect once limit-pricing has started. Under these strategies, the importer's strategy is not a function of the resource stock. Thus the exporter cannot influence the importer's future actions, and will optimally follow the open-loop strategy; in other words, she will always limit price. This, on the other hand, implies that it is indeed optimal for the importer to develop the substitute technology as if the resource did not exist.

I will now focus on a particular MPNE: that which indeed coincides with the open-loop equilibrium in the set Φ. There could potentially be a

large set of equilibria which satisfy this condition. I will proceed to find one which is continuously differentiable in terms of the value functions outside the set Φ. In other words, I am ruling out equilibria which feature coordinated jumps in strategies in the non-limit pricing stage.

Proposition 9. Suppose there exist continuously differentiable functions $V^I(K,S)$, $V^E(K,S)$, defined in the set Φ, which satisfy the HJB equations (2.8) and (2.9), and which, for any $K \in [0, \overline{K}]$, satisfy

$$\lim_{S \downarrow \phi(K)} V^i(K,S) = V^{\mathrm{OL},i}(K, \phi(K))$$

where $V^{\mathrm{OL},i}$ is the value for the open-loop solution for player i. Then the HJB equations yield equilibrium strategies to the MPNE.

Proof. By assumption, both players are following Markovian strategies. The problem then conforms to Theorem 5.3 in Başar and Olsder (1999) and the result follows immediately. □

I do not claim that the numerical method below will find a unique equilibrium. It is well-known that, even in relatively well-understood and simple linear-quadratic differential games, a multiplicity of equilibria may exist (see Tsutsui and Mino, 1990; Rowat, 2007). Existing approaches to show uniqueness, by using a 'natural boundary condition' to pin down the equilibrium, rely on uniqueness results for ODEs (Karp, 1996). This applies to models in which the state variable is a scalar. Applicable uniqueness results to systems of PDEs, which would be required to use the same approach in the present model, are not readily available. I have sought alternative equilibria by varying the initial guess and have not found any; however, at best this is only weakly suggestive of there not being any nearby

equilibria.[29] Equilibrium uniqueness in differential games is in general an unsolved problem, and I thus only present one of potentially many possible equilibria to the game. This approach is often taken in the literature: see Salo and Tahvonen (2001) and Harris et al. (2010) for applications to resource games.[30]

A Chebyshev collocation method to solve for the MPNE

I will numerically obtain the value functions outside the set Φ.[31] Instead of discretising with respect to time, I will approximate the continuous-time value functions. This is more satisfactory in a model with a finite, endogenous date of exhaustion. Note also that the open-loop case has been solved in continuous time.

The first-order conditions to the problems (2.8) and (2.9), with limit pricing not binding, are

$$d^* \equiv d^*(V_K^I) = c'^{-1}(V_K^I)$$
$$q_F^* \equiv q_F^*(V_S^E) = MR^{-1}(V_S^E)$$

which are uniquely defined, given the assumptions made, for any V_K^I and V_S^E. Substituting these into the HJB equations, the optimal value functions

[29] Of course, the rootfinding algorithm might, because of some feature of the problem, tend to converge to the equilibrium I have found even were other nearby equilibria to exist.

[30] Klein et al. (2008), in a very different model, cite the fact that their algorithm finds only one equilibrium as circumstantial evidence that this is unique.

[31] The equilibrium values are not twice differentiable at the regime boundary locus $S = \Phi(K)$. For this reason, standard Chebyshev collocation on the entire state space does not work and splines provide an inaccurate solution.

satisfy

$$\begin{aligned} \rho V^I &= u(q_F^*) - q_F^* p(q_F^*) - c(d^*) + V_K^I d^* - V_S^I q_F^* \\ \rho V^E &= q_F^* p(q_F^*) + V_K^E d^* - V_S^E q_F^* \end{aligned} \tag{2.12}$$

where I have omitted the dependence of q_F^* and d^* on V_S^E and V_K^I, respectively. I will thus have to solve a system of two nonlinear, first-order partial differential equations. The unknowns in these equations are the functions V^I and V^E. The boundary conditions will be given by the continuity of the value functions at the upper boundary of the set Φ.

I solve the system using the Chebyshev collocation method for solving partial differential equations: that is, I find k-dimensional approximations $\tilde{V}^I$, $\tilde{V}^E$ which satisfy the above system at k points. This approach to PDEs is briefly described by Judd (1998). It has been developed in more detail by Dangl and Wirl (2004), Caporale and Cerrato (2010) and Mosiño (2012); the approach has been applied in an environmental context by Balikcioglu et al. (2011). All of these papers focus on solving optimal stopping problems with one unknown function and a single (partial or ordinary) differential equation. In the present paper, I apply the method to solve a pair of coupled PDEs, with continuous actions and two unknown functions.

By choosing Chebyshev polynomials, I am imposing differentiability of an arbitrary degree in the set $\Phi^{-1} \equiv [0, \overline{K}] \times [0, \overline{S}] \backslash \Phi$ (choosing $\overline{S} > \phi(\overline{K})$, unless $\phi(K) \to \infty$ for some $K < \overline{K}$). This implies that the strategies to be found will be smooth in Φ^{-1}, in particular ruling out coordinated switches of strategies in the interior of this set.

The Chebyshev collocation method is well-understood and, by choosing the interpolation nodes appropriately, will yield small interpolation errors

(Judd, 1998). Chebyshev nodes require the approximation domain to be rectangular, so I transform the set Φ^{-1} into a rectangular set in the space (K, s), by using

$$s \equiv \frac{S - \phi(K)}{\overline{S} - \phi(K)} \tag{2.13}$$

implying $s \in [0, 1]$. I choose n basis functions in the K-dimension and m basis functions in the s-dimension.

I will thus approximate transformed value functions $v^I(K, s)$, $v^E(K, s)$, the partial derivatives of which satisfy, for $i \in \{I, E\}$,

$$\begin{aligned} v^i_K &= V^i_K(K, S) + V^i_S(K, S)(1 - s)\phi'(K) \\ v^i_s &= V^i_S(K, S)(\overline{S} - \phi(K)) \end{aligned} \tag{2.14}$$

The function approximations will be of the form

$$\tilde{v}^i(K, s) = V^{\phi,i}(K) + sA^i g(s, K) \tag{2.15}$$

where $V^\phi(K)$ is the relevant value function at the limit-pricing boundary $S = \phi(K)$, A^i is a coefficient matrix with dimensions (nm, nm), and $g(\cdot)$ is a (nm) vector of Chebyshev polynomials. Note that the boundary condition will be satisfied by construction.[32]

I choose a 400-degree Chebyshev approximation, with 20 basis functions in each dimension, utilising the off-the-shelf routines in the COMPECON package developed by Miranda and Fackler (2002). This yields a system with 800 equations and unknowns. I obtain the coefficients for the

[32]Because of the formulation of the function approximation (2.15), the meaning of the minimax property of Chebyshev approximation in terms of the actual value functions V^i is not clear. However, the errors of the HJB equations seem to behave well even near the boundary $S = \phi(K)$.

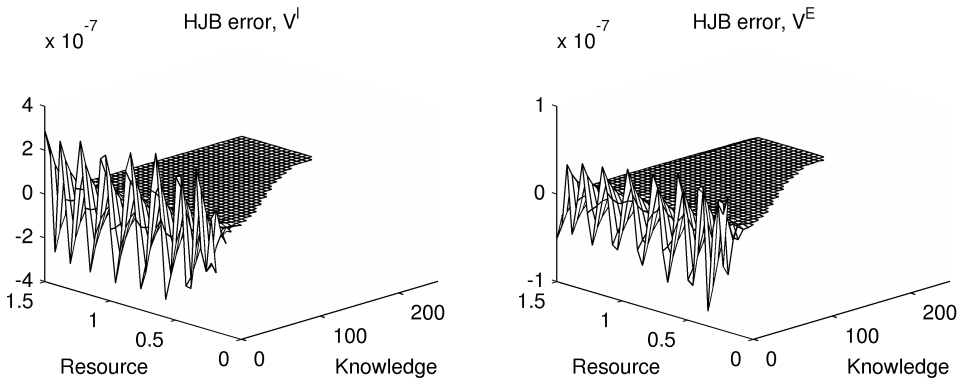


Figure 2.6: HJB equation errors for the two players, outside the collocation nodes, relative to the HJB equation LHS. The relative errors are smaller than 10^{-7}.

open-loop solution to use as my initial guess. The functional forms and parameters are as follows: let utility be of the standard isoelactic form, $u(q) = \frac{q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}$. Let the backstop cost be given by $x(K) = \underline{x} + \frac{\gamma}{2}(\overline{K} - K)^2$. Let R&D costs be quadratic also: $c(d) = \frac{\xi}{2}d^2$. To illustrate the qualitative results, I parameterise arbitrarily with $\eta = 2$, $\xi = .01$, $\gamma = 1.6(-4)$.

The system is solved rapidly by a standard non-linear rootfinding algorithm in Matlab, probably largely due to the good initial guess. For initial guesses 'near' the open-loop equilibrium values, the system converges to effectively identical results; for very different initial guesses, convergence does not occur. HJB equation errors are small, of the order of 10^{-6} relative to the HJB equation values (Figure 2.6).[33]

Results

The results are displayed in Figures 2.7 and 2.8. As the system evolves, the economy travels towards the bottom right in the state space. In the limit pricing regime, importer value (Figure 2.7) of course does not depend

[33]One initial guess converged to a solution which was ruled out based on very large HJB equation errors.

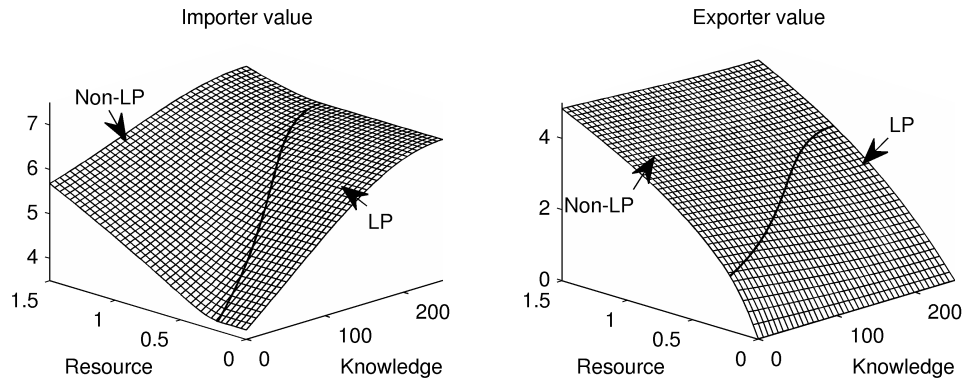


Figure 2.7: (*left*) Resource importer value increases with knowledge, up to $K = 250$ at which substitute cost achieves its minimum value. Under limit pricing, stocks of the exhaustible resource make the importer no better off. With oil stocks high relative to knowledge stocks, the resource is initially priced strictly below substitute and the importer value increases as more oil is supplied. (*right*) Resource exporter value increases with oil stocks. More competitive substitute (more knowledge) decreases value, up to the level after which substitute cost no longer falls (here $K = 250$).

on the resource stock, but only on the knowledge stock. Maximum value is attained here at $\overline{K} = 250$, corresponding to a permanent stream of constant resource use. In the non-limit pricing regime, a higher initial resource stock implies higher value: the exporter sells some of the plentiful resource early on, and the importer captures a part of the surplus.

Exporter value increases with the resource stock, being zero when no resource exists. Higher knowledge stocks reduce value, up until the lower bound on backstop cost (with the parameterisation used, this effect is hard to distinguish in the figure).

Optimal actions, as functions of the state, are shown in Figure 2.8. Under the limit pricing regime, R&D intensity by construction coincides with the terminal path. It is also constant with respect to the resource stock. In the non-limit pricing regime, R&D intensity falls with the resource stock. Under the limit-pricing regime, the quantity of the resource sold is

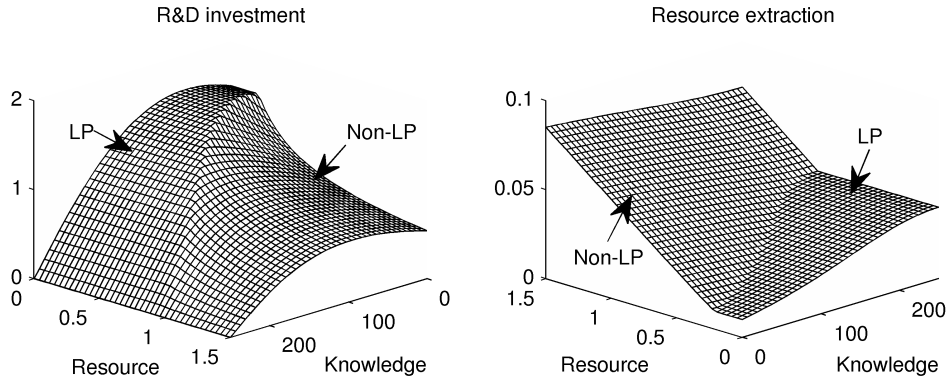


Figure 2.8: (*left*) R&D intensity. Note that the axes have been reversed. Under limit pricing, the importer conducts R&D as per the terminal path (the concave part). For high oil stocks relative to knowledge stocks, oil is initially priced strictly below the substitute and the importer relaxes R&D efforts. (*right*) Exhaustible resource sales. Under limit pricing, extraction is determined by the substitute cost. For high oil stocks, relative to knowledge, exporter initially sells strictly more than the limit-pricing quantity.

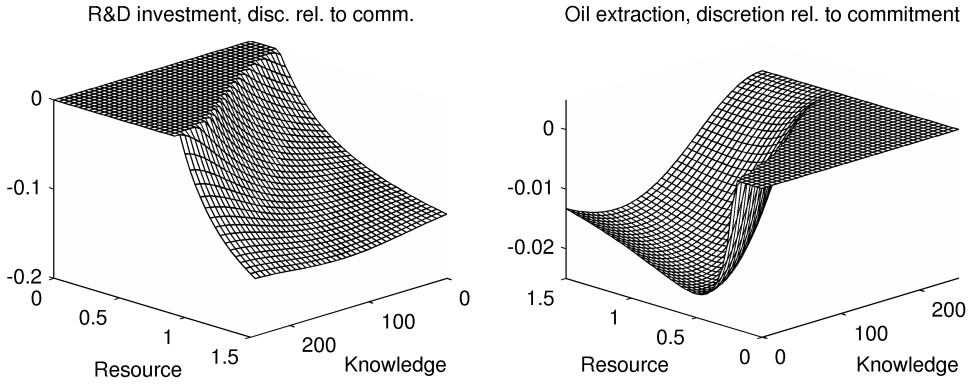


Figure 2.9: Excess R&D (*left*, note reversed axes) and oil extraction (*right*) rates in the MPNE, relative to the open-loop case. R&D rates are up to some 14% lower when commitment is not possible, and oil extraction rates are up to 2.2% lower.

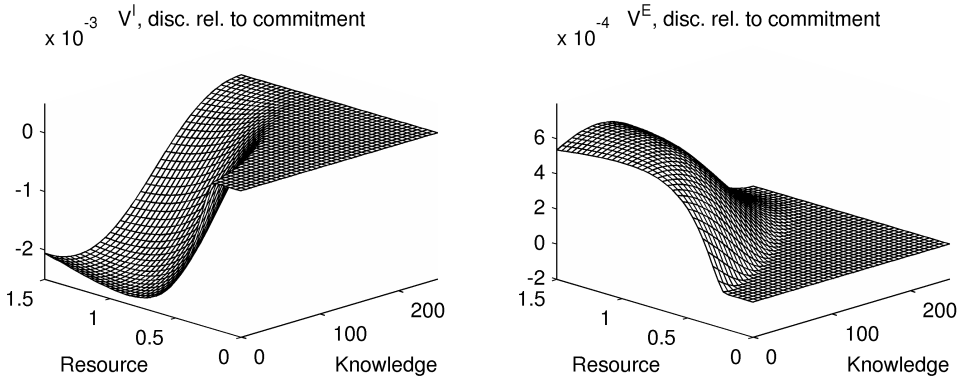


Figure 2.10: The importer loses when commitment is not possible (*left*), with the change in the value up to .24%. (*right*) The exporter gains by up to .06%.

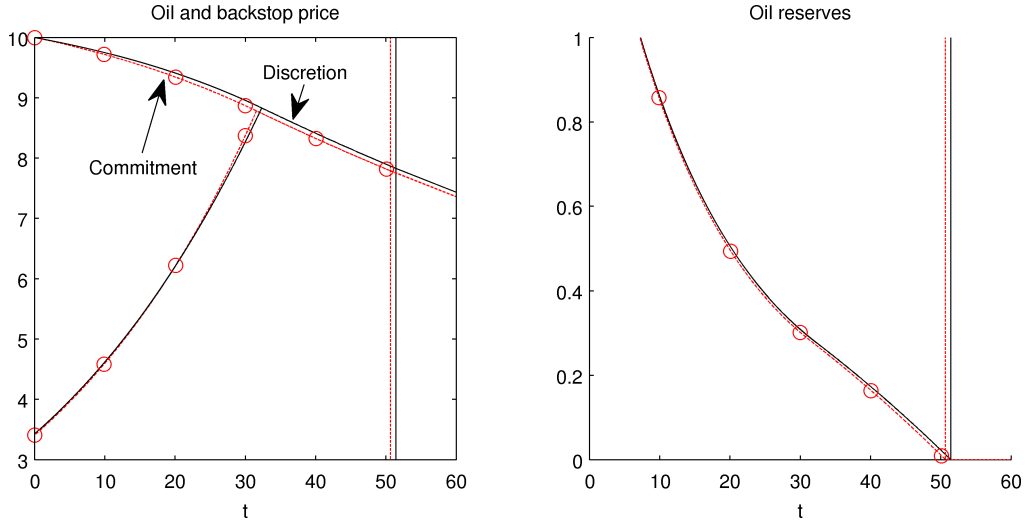


Figure 2.11: (*left*) Price paths of the backstop resource (always decreasing) and the exhaustible resource (initially increases), for the commitment (open-loop) outcome (*dashed red*) and the discretionary (closed-loop) outcome (*solid black*). Lack of commitment implies lower initial oil extraction and lower R&D effort. Times of exhaustion are very close (*vertical lines*). (*right*) Resource stocks under both solutions.

of course a function only of the knowledge stock. In the non-limit pricing regime, resource sales are higher.

The open-loop case looks qualitatively similar to the MPNE. Excess R&D and oil extraction rates, relative to the open-loop case, are shown in Figure 2.9. It is apparent that, with the chosen functional specification, the inability to commit leads the importer to conduct less R&D (by up to 14%), and the exporter to slow down extraction (by up to 2.2%). Without the ability to commit to future actions, the players are forced to use present actions, and their effect on the state of the economy, as commitment devices.

In particular, the exporter will sell less oil in order to leave more oil for the future: the promise of plentiful oil will make substitute development seem less urgent to the importer. The importer will invest less in the substitute. To understand this effect, it is worth discussing a result by Hoel (1978). Recall that in this model the backstop price is just a given, constant parameter. In the case with isoelastic demand and no extraction costs, as in the present case, a fall in the backstop price induces the resource monopolist to raise her initial price (provided she does not limit-price from the starting date onwards). In other words, a more competitive backstop will not induce the exporter to sell more oil, but instead less. Similarly, Gilbert and Goldman (1978) show that a monopolist constrained by potential entry will charge higher prices in the short run than an unconstrained one.

The present numerical solutions confirm that this counterintuitive result survives in a dynamic game with an endogenous backstop price. The importer wants to encourage the exporter to sell more oil in the non-limit

pricing regime. This can be managed by the importer 'tying its hands' to ensure the substitute price is relatively high when the economy shifts into the limit pricing regime. When it is not possible to commit to future actions, underinvestment can act as a commitment device: due to the convex investment costs, is would be prohibitively expensive to try and undo today's underinvestment in a rapid R&D push later. Thus, the exporter accepts that the backstop price will be higher at the start of the limit pricing regime, and increases her resource extraction today.

Note also that, under the present simulations, the impossibility of commitment hurts the importer and benefits the exporter, assuming the economy starts in the non-limit pricing regime (Figure 2.10). However, these effects are fairly insubstantial with the current parameterisation (corresponding roughly to much less than 1% of the present discounted surplus, in monetary terms, of oil and backstop consumption or oil revenues, respectively, until infinity). The paths in the open-loop equilibrium and the MPNE are shown in Figure 2.11. In the MPNE, there is less R&D, just slightly less oil extraction, limit pricing begins at a higher backstop price, and exhaustion occurs later.

Finally, it is worth noting that the present model implicitly rules out storage of the resource, and thus arbitrage opportunities by agents in the importing economy are not relevant. However, with the above functional specification, the MPNE is robust to a resale market existing, assuming there is no initial storage of the resource. The resource price changes at the rate $\frac{\dot{p}_F}{p_F} < \rho$. In this case, arbitrageurs would like to draw contracts to supply more oil today and less in the future. However, unless they hold initial positive stocks, they cannot access the physical resource as the

exporter controls the amount available in the economy.[34]

2.4 Stock pollution and economic exhaustion

I will now extend the model to take into account economic, rather than physical, exhaustion of the resource (Heal, 1976). Economic exhaustion occurs when resource extraction stops due to increasing extraction costs, rather than total depletion of physical reserves. This substantially increases the realism of the present model. I will also introduce a stock externality related to the cumulative use of the resource; the obvious motivation is climate change, resulting from the use of fossil fuels. Jointly, the two assumptions introduce interesting new dynamics to the model.

I will illustrate the basic dynamics by solving the special case in which limit pricing begins immediately at the start of the game. This could result, for example, from resource demand being inelastic. To avoid technicalities and focus on the mechanism in action, I will consider a situation in which the importer recognises he is effectively in control of the resource price.[35] In other words, I consider a degenerate MPNE in which the exporter will only ever limit price. This way, I can focus on the importer's problem only.

Assumption 4. Resource extraction costs. The marginal cost of increasing the extraction rate at any moment is $C(S)$, $C' < 0$.

This is a standard assumption in which the extraction costs depend

[34]Were the Hotelling Rule broken in the opposite direction, then costless storage would impose a constraint on the monopolist. This could potentially happen with alternative demand specifications.

[35]Solving the proper open-loop equilibrium is more involved, as the Hamiltonian of the importer is discontinuous. In any case, I am primarily interested in the MPNE.

solely on the remaining stock of the resource, such that the first units of the stock are relatively cheap to extract, but extraction costs rise as the stock diminishes. I assume that, if extraction costs are present, then exhaustion is always economic: $C(0) > x(0)$.

Assumption 3'. Climate change. Climate change impacts are an increasing function $Z(G)$ $(Z' > 0)$ of the cumulative emissions $G(t) \equiv S_0 - S(t)$. The impacts enter the importer's welfare additively, but do not affect the exporter.

Two issues regarding Assumption 3' require discussion. Firstly, the climate change impacts are assumed to depend on the cumulative carbon emissions. The conventional way to model climate impacts is as a function of the temperature deviation from the preindustrial era (e.g. Nordhaus, 2009).[36] It turns out that cumulative emissions may be a very good proxy for this deviation. Matthews et al. (2009) show that warming closely follows cumulative carbon emissions to date. As atmospheric concentrations increase, the radiative window (the spectral band) in which CO_2 has its greatest effect becomes saturated, leading to less radiative 'kick' per unit of CO_2. However, this effect is largely offset by similar saturation in terrestrial carbon sinks, with a higher fraction of emitted CO_2 remaining in the atmosphere. The net effect is that temperature may be close to linear with respect to cumulative emissions.

After emissions stop, the atmospheric CO_2 starts gradually decaying. The temperature response is more persistent. Solomon et al. (2010) consider a scenario of increasing emissions to 2050, followed by zero emissions.

[36] Analytical models often consider damages to be a function of the atmospheric pollutant stock, considering it a good proxy for the temperature difference and related climatic shifts; see e.g. Withagen (1994); Hoel and Kverndokk (1996).

The resulting temperature path is nearly flat for some 50 years after cessation of emissions, and then starts decreasing very gradually (see also Solomon et al., 2009; Allen et al., 2009).[37] Thus, given that the rest of the model is highly stylised, to proxy climate change by cumulative emissions seems to be a reasonable first-order approximation.[38]

The second issue—the assumption that the exporter is unaffected by the climate change impacts—is crucial. One can justify asymmetric damages by appealing to asymmetries in the size of the two countries: if the exporting country's population is very small, it bears a very small burden of overall damages, but receives all the revenues (List and Mason, 2001). For expositional clarity, I focus here on the extreme case in which the exporter suffers no damages due to climate change.[39]

[37]Roe and Baker (2007) and Zickfeld et al. (2011) argue that uncertainty in the strength of positive feedbacks leads to a fat upper tail in terms of climate sensitivity. Their analysis, to date, is limited to an equilibrium analysis and it is unclear as to what the implications are for the transient temperature response. In the present model, uncertainties in the strength of the climate response imply a need for a robust sensitivity analysis with respect to the magnitude of climate change impact damages.

[38]Note that it seems plausible to argue that impact damages might be a function also of the rate of change of temperature, instead of only the level: over a very long time period, society can potentially adapt to even extreme climate states. Were damages a function only of the rate of change, the first-order approximation to the result of Matthews et al. (2009) would be to treat carbon emissions as a flow pollutant.

[39]To sketch this in a simple static model: suppose there is an inexhaustible good, produced at quantity q at zero cost but giving rise to a per-capita externality $Z(q)$. There are two countries: a passive importer, with share of population γ and quasilinear per-capita utility, with demand for the good $p(q)$; and an exporter who produces the good and gets linear utility from the numeraire good. A social planner treating all individuals equally would set $\gamma p(q) = Z'(q)$. A monopolistic exporter, instead, would set $\gamma q p'(q) = (1-\gamma)Z'(q)$. It is easy to see that, as $\gamma \to 1$, the social planner sets marginal benefit of per-capita consumption to marginal per-capita damage; while the monopolist completely ignores the marginal damage.

The importer's problem is now[40]

$$\max_{d(t)} \int_0^\infty e^{-\rho t} \left(u(q_F + q_B) - p(q_F)q_F - x(K)q_B - c(d) - Z(G)\right) \mathrm{d}t \quad (2.16)$$

$$\dot{G} = q_F, G(0) = G_0 \quad (2.17)$$

$$\dot{K} = d, K(0) = K_0 \quad (2.18)$$

subject to the resource supply being

$$q_F = \begin{cases} p^{-1}(x(K)) & \text{if } x(K) \leq C(S) \\ 0 & \text{otherwise.} \end{cases}$$

I will use the equilibrium in the absence on climate change as a benchmark case:

Definition 3. Given some instance of the model, the *reference equilibrium* is the equilibrium of the same instance absent the externality: $Z(G) \equiv 0$. The R&D process in the reference equilibrium will follow the terminal path at all dates.

The equilibrium features limit pricing at all times, and it is obvious that—in the absence of climate change impacts—the importer will develop the substitute as if the resource weren't available at all.

Proposition 10. With zero extraction costs ($C(S) \equiv 0$) and $t^* = 0$, given any level of knowledge, taking the externality into account reduces the optimal R&D rate relative to the reference equilibrium.

Proof. In Appendix 2.D. □

[40]Note that I am again using the more general function specifications.

If the resource is supplied by a monopolist who always limit prices, it is optimal to slow down the development of substitutes to the polluting resource. The monopolist will just bring down the price of the resource in lockstep with the substitute, always undercutting to keep the substitute out of the market. The pollutant thus introduces an extra cost to investing in substitutes: a fall in the substitute price forces the monopolist to supply larger amounts of the polluting resource while the stock remains positive. This brings emissions forward and raises the near-term damages due to the externality.

Consider now the case with extraction costs. The choice of R&D intensity has a third effect: it also influences the ultimate fraction of the resource extracted. A marginal unit of knowledge will imply exhaustion at a lower level of cumulative extraction, as it lowers the unit cost of producing the backstop, relative to the unit extraction cost of the exhaustible resource. This effect will encourage faster development of the backstop. However, the prospect of higher near-term pollution will still tend to deter it. The overall effect on the R&D process will depend on the balance of these effects.

Economic exhaustion occurs when extraction becomes unprofitable, i.e. the unit extraction cost equals the price:

$$C(S(T)) = x(K(T)) \tag{2.19}$$

Following exhaustion, the stock externality presents a constant burden on welfare but does not affect incentives to conduct R&D. The economy will thus follow the terminal path. Hence, the importer's problem is to

solve

$$\max_{d(t),T} \int_0^T e^{-\rho t} \left(u(p^{-1}(x)) - xp^{-1}(x) - c(d) - Z(G)\right) \mathrm{d}t + e^{-\rho T} \left(\pi^{\infty}(K(T)) - \frac{Z(G(T))}{\rho}\right)$$

where $\pi^{\infty}(K)$ denotes the welfare obtained from backstop use following the terminal path after exhaustion. The choice of T is constrained by equation (2.19). I will focus on cases in which exhaustion is, indeed, economic: $S(t)$ is always strictly positive. Denoting the optimal values by an asterisk:

Proposition 11. Initial R&D intensity may be higher or lower in the equilibrium with the externality and extraction costs, compared to the reference equilibrium: $d^*(0) \lesseqgtr d^{\infty}(K_0)$. Immediately preceding exhaustion, R&D intensity is higher than in the reference equilibrium for the same level of knowledge: $\lim_{t \uparrow T^*} d^*(K^*(t)) > d^{\infty}(K^*(T^*))$. If initial intensity is lower than in the reference equilibrium, it will equal the reference equilibrium rate for a unique level of the knowledge stock, being lower (higher) for lower (higher) knowledge levels. As soon as exhaustion occurs, the R&D rate jumps discretely down to the terminal path.

Proof. In Appendix 2.D. □

In words, in the run-up to exhaustion, the importer will always race to drive the polluting resource out of the market: R&D effort will be higher, for the level of the knowledge stock, that it would be in the absence of climate change (Figure 2.12). This way, the importer avoids the marginal damages due to long-term pollution (suffered in perpetuity). Of course, R&D also makes energy cheaper as in the case without the externality. Once oil is rendered uncompetitive, R&D intensity falls discretely to the

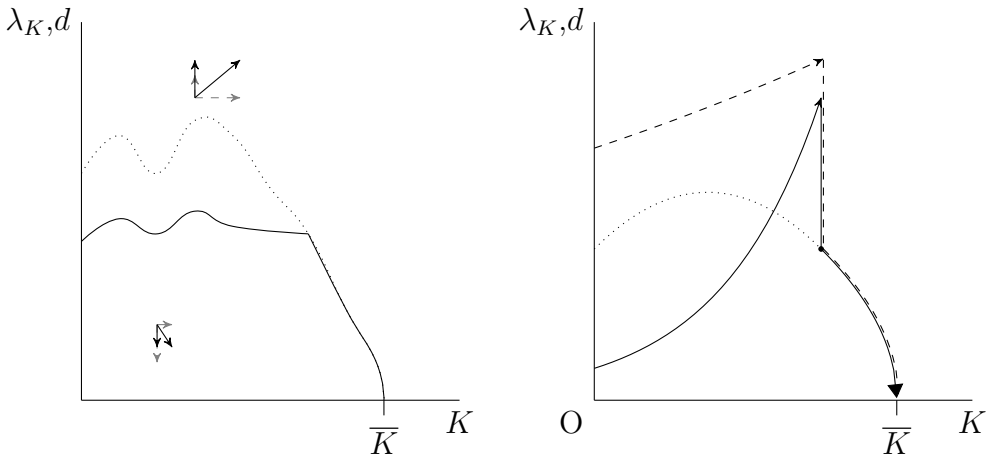


Figure 2.12: Alternative trajectories in (K, λ_K)-space when monopolist always limit prices. Note that R&D intensity is monotonic with respect to λ_K. (*left*) Physical exhaustion. The inclusion of the climate impacts will bend phase arrows upward. As the economy has to end up on the terminal path, the optimum must lie below this in the absence of climate change. (*right*) Economic exhaustion. If the concern for long-term damages outweighs the near-term pollution impacts (dashed line), initial R&D intensity is higher than in the case without the externality (terminal path; dotted line). High near-term damages can lead to initial R&D intensity being lower than without the externality (solid line), although eventually it will become optimal to start intensive R&D to halt resource use. Both trajectories feature a discontinuous fall in R&D intensity as soon as economic exhaustion occurs.

terminal path: the additional marginal value to R&D, associated with the prospect of shutting out the polluting resource, has already been realised.

If marginal damages at low levels of pollution are fairly significant, relative to the long-term damages, and the resource is plentiful, then early R&D efforts may be below the reference rate (with respect to the accumulated knowledge): the importer wants to delay short-term damages by delaying R&D (thus keeping short-term extraction rates low). In this case, there will come a unique point in time at which the importer starts to focus more on long-term concerns, beginning the crash programme; after this moment, R&D rates exceed the corresponding rates without the pollution problem, until the resource is exhausted.

2.4.1 A calibrated example

Before concluding, I will conduct a back-of-the-envelope calibration to test whether the effects discovered above are important. I use the model with climate change and extraction costs, augmented with exogenous non-oil carbon emissions. I consider the case in which limit pricing starts immediately in isolation, as this allows me to focus on the effects identified in Section 2.4 independently of the terms-of-trade effects studied in the previous numerical examples. All examples presented in this chapter should be regarded as purely illustrative. In principle, tying the two models together should be straightforward. This would, however, confound the different effects.[41]

The common discount rate $\rho = .03$. I measure quantities in billions

[41]The model at present lacks realism due to the assumption that the backstop sector is able to kick in effectively overnight. I thus omit a more careful calibration. Future work will consider a model with a third state variable, backstop production capacity. Such a model should be readily applicable to data.

of barrels of oil (equivalent), and prices in 2011 dollars. As the felicity function is quasilinear, I can ignore economic growth. The utility function is $u(q) = \alpha \frac{q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}$, with the elasticity of oil demand $\eta = .6$ and the coefficient $\alpha = 1.74e4$ chosen so that an oil price of \$60/bbl yields an annual demand of 30 billion barrels.

The initial price of the backstop is $\overline{x} = \$300$/bbl, and the minimum price is $\underline{x} = \$70$/bbl. The knowledge stock is normalised so that $\overline{K} = 1000$. The backstop price is given by $x(K) = \underline{x} + \frac{\gamma}{2}(\overline{K} - K)^2$, with $\gamma = 4.6e(-4)$. R&D costs are quadratic: $c(d) = \frac{\xi}{2}d^2$, with $\xi = 1$. This implies that a constant R&D rate sufficient to bring the substitute price to \$100/bbl over 15 years would cost roughly \$0.9tn annually, i.e. some 1.3% of 2011 gross world product.

Initial oil reserves owned by the monopolist are 10^{12} barrels, which is of the order of magnitude estimated by the IEA for conventional oil reserves owned by OPEC. Extraction costs are hyperbolic: $C(S) = \frac{\kappa}{S}$, with $\kappa = 10000$. This implies that initial extraction cost is \$10/bbl, and the extraction cost when 100 billion barrels remain is \$100/bbl.

Climate change damages are $Z(G) = \frac{\zeta}{2}G^2$, with $\zeta = 6.32e(-5)$ and $G(0) = 0$—that is, I assume that the flow of impacts at 2011 concentrations is zero. Greenhouse gases are measured in the carbon-equivalent of billions of barrels of oil. Exogenous emissions are constant ar 12.5 GtC/year for 100 years, then zero. Absent oil use, these imply a long-term concentration of 790 ppm (assuming the carbon cycle weakens, with the airborne fraction constant at .65).[42] Damages are calibrated such that the exogenous emis-

[42]Present airborne fraction is roughly .55 (Solomon et al., 2007). Some studies indicate this might change fairly rapidly as a result of climate feedbacks on the carbon cycle (Schmittner et al., 2008).

sions yield damages of roughly 5% of 2011 gross world product. OPEC oil contributes, at most, some 37 ppm on top of this.

With this parameterisation, limit-pricing would of course start immediately as demand is inelastic. The backstop price and reserves are illustrated in Figure 2.13. Backstop prices start very high, but reach \$100/bbl in less than 15 years. Oil demand would be below the currently observed 30 bn barrels per year, starting below 12 bn bbl/year and ending at 27 bn bbl/year. Exhaustion occurs at 2043, with just under 300 bn bbl of conventional oil left underground. The value of the oil trade in the first year would be some \$3.4tn. Importantly, recognising climate change has a very modest effect on R&D effort; climate concerns reduce R&D in the first year by roughly 0.1%. It seems that climate worries do indeed, optimally, lead to less R&D effort initially, but that this effect is not very noticeable.

Clearly the simple calibration does not explain the real world particularly well. Worries over 'demand destruction' imply that the demand elasticity, over the very long run, is higher. Further, it could be argued that OPEC countries may in fact have a higher discount rate than the consumer countries do. Very high oil prices might increase the risk of expropriation. Finally, it is questionable as to whether OPEC in fact currently behaves as a cohesive cartel. All of these factors would tend to lower the oil price below the backstop price.

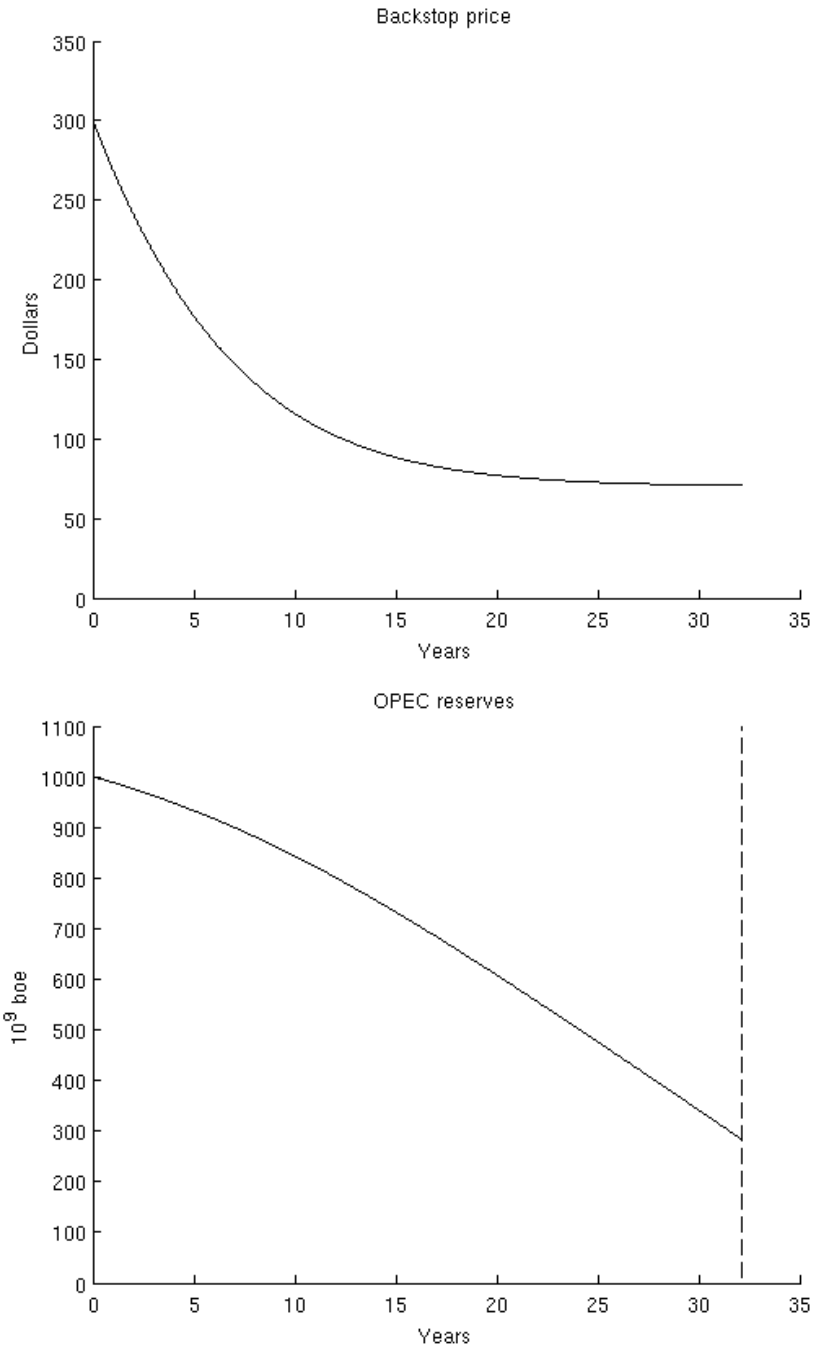


Figure 2.13: Backstop price (*top*) and oil reserves (*bottom*) until exhaustion. Exhaustion occurs when the backstop price is close to its minimum (at \$70.65/bbl). This implies that 283 bn barrels of oil are left unextracted.

2.5 Conclusions

2.5.1 A discussion of assumptions and possible extensions

The model presented in this paper is certainly highly stylized, and I wish to discuss some of the assumptions here.

The Hotelling exhaustible resource framework, while still the canonical model in resource economics, can be seen as controversial. This is primarily the result of its empirical failings (Livernois, 2009; Hart and Spiro, 2011). However, lack of past empirical success is not necessarily sufficient to rule out a model in terms of its future validity (Hamilton, 2009). The model is probably more relevant over the very long run: complications such as lumpy investments and uncertainty may become less important over a timescale of decades. Over long time periods, the model is likely to tell us something about the intertemporal considerations at play.

The assumption of a single, perfect substitute to oil is a drastic simplification. In reality, oil is used for many purposes in the economy; while substitutes are available for all of these, none will be perfect, and they will be priced differently. There may also be capacity constraints related to scaling up production of these substitutes (a point made clearly by Wirl, 1991). The assumption is made primarily for reasons of tractability, and in order to bring the main message of the paper into sharper focus.

Two alternative ways to model substitute development would be possible. One would be to assume that a fraction of demand would be satisfied by the substitutes, so that the demand curve, instead of being clipped off by a perfect substitute, would instead shift left as R&D investment accu-

mulates. The problem with this formulation is that a clearer specification of *how* the residual demand curve would shift would have to be imposed: there is only one way for a price of the perfect substitute to fall, but many ways in which residual demand could shift left. One would also have to specify how exactly the substitutes are supplied to make welfare evaluations.

A second alternative R&D model would be to consider many substitutes, each with its own production cost, elasticity of substitution and R&D process. This model would be a extension of the present model towards the model employed by Hoel (1984). Such a model would require numerical solutions. With several perfect substitutes, there would be several stages of limit-pricing. R&D would always tend to be most intensive for the substitute which was setting the limit price at any given moment: the substitutes with the higher costs could be developed more slowly, until the oil exporter decided to stop competing against the marginal substitute, hiking up prices. This approach could also be used to consider several importing countries, with each deploying their own substitute technologies. Some countries would free ride on the cheaper oil on offer because of other countries' R&D efforts. However, at some point the oil exporter would no longer wish to compete with the advanced technologies, hiking prices up and only selling to the technological laggards. The marginal country's research efforts would then have to pick up. Substitutability (for any purpose oil were used for) would tend to eliminate limit-pricing behaviour, and the resource price would follow the relevant version of the Hotelling Rule.

Another obvious extension would be to consider a market-driven R&D process. This would require further assumptions on the model, as at present

the assumption of perfect competition, perfect substitutability (and possibly the continuous entry of competitors with cheaper costs) would give firms no profits, and hence no incentives to invest in R&D.

Finally, the model could be extended to the case in which the substitute is not clean. Substitutes cleaner than oil, but still polluting, would imply qualitatively similar effects as in the present paper. Were the substitutes even more dirty—such as petroleum products developed from tar sands—then R&D efforts would be slowed down even more, to prolong the era of relatively clean oil and to deter the entry of the very dirty substitutes.

2.5.2 Conclusions

I have analysed strategic competition between a resource exporter, selling an exhaustible resource, and a resource-consuming country, able to gradually improve, with convex per-period costs, a perfect substitute to this. Per-period convex costs imply that the cost of developing the resource is optimally spread out across time. Unlike most other models of resource extraction and substitute development, the present model explains why R&D is undertaken even when the substitutes are far from being competitive against the resource. With incremental technological progress, the non-cooperative outcome features three stages. Initially, the resource is priced substantially below the substitute cost, with decreasing resource use (thus increasing resource price) over time. R&D efforts are already undertaken, with a view to the future value of the substitute. After the substitute becomes competitive, the resource exporter will price oil just below the substitute, in order to keep the substitute off the market. As technological progress keeps making substitutes cheaper, the resource exporter is forced

to supply increasing quantities. The path of resource extraction is thus non-monotonic and V-shaped. Finally, once the resource is depleted, the importer switches to the backstop technology.

Importantly, strategic considerations tend to increase the current price of oil: low demand today can convince the importer that future scarcity is less pressing. On the other hand, the importer may similarly lower R&D efforts to induce the exporter to sell more oil today.

When use of the exhaustible resource results in a stock pollution externality—as climate change follows from consumption of a fossil fuel such as oil—limit-pricing behaviour implies that, in the absence of carbon prices, it will be optimal to slow down research. The importer effectively controls oil supply; aggressive R&D programs will just result in the oil stock being depleted faster, leading to greater emissions. With oil extraction costs increasing as supplies dwindle, there is a third effect: R&D can make oil obsolete, actively bringing the oil age to a close with a part of the resource remaining unused. I have shown that this effect will always eventually dominate. As exhaustion looms close, the importer will race to drive the polluting resource out of the market.

These findings are important, as they inform the public debate over whether technological programs would prove to be a workable climate policy instrument, if carbon pricing remains politically difficult. Aggressive R&D subsidies can be used to wean economies off oil, provided that the moment of (economic) exhaustion is relatively close. However, if oil can be expected to remain competitive with the substitutes for a long time, more aggressive R&D may only result in greater near-term emissions, possibly aggravating climate change. Hence, the optimal response may still be to initially slow

down R&D efforts. A very rough calibration, however, indicates that this effect is not very large in magnitude.

These results are necessarily indicative only, due to the simplicity of the model (Hart and Spiro, 2011). Furthermore, it could be argued that conventional oil reserves are not the crucial issue when tackling climate change, as the potential pollution embodied in coal and unconventional oil reserves are much more substantial. Nevertheless, the present paper gives partial intuition to a particular outcome of climate policy which has not been considered previously. The results would be more likely to hold in a situation in which a coalition of countries tries to develop a clean substitute to liquid fuels, forgoing the use of any unconventional oil reserves its members might possess; with the oil exporting countries acting as a cohesive cartel and potentially supplying conventional and non-conventional oil. The model in the present paper might also shed some light on other resource markets which may feature market power, such markets in rare earth elements, supposing appropriate backstops exist.

Appendix 2.A Proofs for Section 2.2

Proof of Lemma 1. I will develop the set of all Pareto optima to the model, with the case discussed in the main text (with equal Pareto weights) arising as a particular case. Consider a social planner who has Pareto weights θ_I and θ_E for the citizens of Country I and Country E, respectively, with $\theta_I + \theta_E = 1$.[43] Denoting the populations by L_I and L_E, the planner's

[43] I ignore the optima in which the social planner discriminates between citizens *within* either country.

problem is

$$\max_{q_F,q_B,d,F} \int_0^\infty e^{-\rho t} \left\{ \theta_I L_I \left(u\left(\frac{q_F+q_B}{L_I}\right) - \frac{q_B x(K)+c(d)+F}{L_I} \right) + \theta_E L_E \left(\frac{F}{L_E}\right) \right\} \mathrm{d}t$$

$$\text{s.t. } M_I \geq q_B x(K) + c(d) + F$$

$$M_E \geq -F$$

in which per-capita welfare of a Country I citizen is quasilinear, with $u(\cdot)$ the per-capita utility of individual energy consumption, and the linear term referring the per-capita numeraire consumption. Country E does not have the technology to consume energy, and obtains welfare linearly from per-capita numeraire consumption. F denotes the aggregate numeraire transfer from Country I to Country E, and q_F and q_B refer to aggregate energy consumption. The two constraints specify that the most that can be transferred from one country to the other is given by the exogenous numeraire income (less any expenditures in Country I's case).

Suppose an optimum exists. Forming the Hamiltonian and augmenting it by the constraints, the Lagrangean is

$$\mathcal{L} = \theta_I L_I \left(u\left(\frac{q_F+q_B}{L_I}\right) - \frac{q_B x(K)+c(d)+F}{L_I} \right) + \theta_E L_E \left(\frac{F}{L_E}\right)$$
$$+\lambda_K d - \lambda_S q_F - \delta_I(-M_I + q_B x(K) + c(d) + F) + \delta_E(M_E + F)$$

in which λ_K and λ_S are the costates on the knowledge stock and the resource stock, respectively; and δ_I and δ_E, respectively, are the shadow prices on the importer's and exporter's budget constraints.

From the Maximum Principle, the first-order conditions are

$$\theta_I u'\left(\frac{q_F+q_B}{L_I}\right) \leq \lambda_S, \qquad q_F \geq 0, \quad \text{C.S.} \tag{2.20a}$$

$$\theta_I u'\left(\frac{q_F+q_B}{L_I}\right) \leq (\theta_I+\delta_I)x(K), \qquad q_B \geq 0, \quad \text{C.S.} \tag{2.20b}$$

$$(\theta_I+\delta_I)c'(d) \geq \lambda_K, \qquad d \geq 0, \quad \text{C.S.} \tag{2.20c}$$

$$\dot{\lambda}_S = \rho\lambda_S \tag{2.20d}$$

$$\dot{\lambda}_K = \rho\lambda_K + (\theta_I+\delta_I)q_B x'(K) \tag{2.20e}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_S(t)S(t) = 0 \tag{2.20f}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = 0 \tag{2.20g}$$

as well as the complementary slackness conditions on the constraints

$$\delta_I \geq 0, \qquad M_I \geq q_B x(K) + c(d) + F, \qquad \text{C.S.} \tag{2.21a}$$

$$\delta_E \geq 0, \qquad M_E \geq -F, \qquad \text{C.S.} \tag{2.21b}$$

Finally, the derivative of the Lagrangian is linear with respect to F:

$$\frac{\mathrm{d}\mathcal{L}}{\mathrm{d}F} = -\theta_I + \theta_E - \delta_I + \delta_E$$

Thus, the necessary condition has to be separated into three cases:

Case 1: $\frac{d\mathcal{L}}{dF} > 0$. This implies $F = M_I - q_B x(K) - c(d)$, $\delta_E = 0$, and $\delta_I \geq 0$. Thus $\theta_E - \theta_I > \delta_I \geq 0$, i.e. the case occurs only if $\theta_E > \theta_I$.

Case 2: $\frac{d\mathcal{L}}{dF} < 0$. This implies $F = -M_E$, $\delta_I = 0$, and $\delta_E \geq 0$. Thus $\theta_E - \theta_I < -\delta_E \leq 0$, i.e. the case occurs only if $\theta_E < \theta_I$.

Case 3: $\frac{d\mathcal{L}}{dF} = 0$. This implies $\theta_E - \theta_I = \delta_I - \delta_E$. Suppose $\theta_E > \theta_I$. Then we must have $\delta_I > 0$ and $F = M_I - q_B x(K) - c(d)$. Similarly $\theta_E < \theta_I$

implies $F = -M_E$. The interesting case is $\theta_E = \theta_I = \frac{1}{2}$, which implies $\delta_I = \delta_E = 0$, and F indeterminate.

Normalise $L_I = 1$. Note that the 'importer dictator' solution ($\theta_I = 1$, $\theta_E = 0$) yields $\delta_I = 0$, and thus the system (2.4) given in the main text.

Define now $\dot{\tilde{\lambda}}_K = \frac{\lambda_K}{\theta_I + \delta_I}$. Then (2.20e) becomes $\dot{\tilde{\lambda}}_K = \rho\dot{\tilde{\lambda}} + q_B x'(K)$, (2.20c) becomes $c'(d) \geq \dot{\tilde{\lambda}}$, and (2.20b) becomes $\theta_I u'(q_F + q_B) \leq (\theta_I + \delta_I)x(K)$. Note that if $\theta_I \geq \theta_E$, $\delta_I = 0$ and the above system in $(d, K, \tilde{\lambda}_K)$ is equivalent to system in (d, K, λ_K) in (2.4), as is the transversality condition. Thus, the R&D process and backstop consumption follow the same process as in the 'importer dictator' case if $\theta_I \geq \theta_E$. As to the numeraire transfer, $F = -M_E$ if $\theta_I > \theta_E$; otherwise F is indeterminate.

If $\theta_I < \theta_E$, then $\delta_I \neq 0$ and the (2.20b) is no longer equivalent to (2.4b): backstop consumption is lower for any backstop price, as the social planner would prefer to transfer the funds instead to Country E. This is reflected in the shadow value of knowledge $\tilde{\lambda}_K$. In the 'exporter dictator' case ($\theta_I = 0$, $\theta_E = 1$), no R&D is undertaken or backstop ever consumed, but instead all Country I income is transferred to Country E. Resource consumption is indeterminate.

Note that the assumption of quasilinear utility is not well suited to modelling cases in which consumption of the numeraire departs substantially from some benchmark level. The actual welfare from numeraire consumption is better thought of as concave, with the quasilinear form representing a linear approximation when consumption levels do not vary a lot. Thus, the quasilinear model is not well-suited to cases with $\theta_I \neq \theta_E$, and hence the main text focuses on the case $\theta_I = \theta_E = \frac{1}{2}$. □

Proof of Lemma 2. Existence of the optimal solution seems obvious. As

neither the Hamiltonian nor the maximised Hamiltonian are concave in (K, d), given the assumptions on functional forms, sufficient conditions cannot be used. Instead I prove existence using Theorem 10, Chapter 7 in Seierstad and Sydsæter (1987), and then show that the necessary conditions have a unique solution.

I assume the control set D has some arbitrarily high but finite upper bound: $d \in [0, \overline{d}]$. Clearly the objective function and equation of motion for K are both continuous. The assumption of the set N in the Seierstad and Sydsæter (1987) is equivalent to the function $\tilde{u}(d) \equiv \max_d u(d, q_B^*)$ being concave for any given K, with q_B^* chosen optimally. This is clearly the case, as $\tilde{u}(d) = f(K) - c(d)$. The function $y(t)$ in the theorem plays no role as I have assumed a bounded control set. I can ignore condition (3.189), as no state constraints are required. Setting $b_T = \overline{d}$, the theorem applies and existence of a maximum is guaranteed.

Necessary conditions are given in the main text, with $q_F = 0$ always. I will show that the solution will satisfy $K \to \overline{K}$, $\lambda_K \to 0$. Phase diagrams in (K, λ_K)-space, for the different cases below, are illustrated in Figure 2.14.

Case 1: $\overline{K}$ is finite, $x'(\overline{K}) < 0$. All points on the K-axis are stationary, and of course trajectories never cross as the system is autonomous. Suppose the optimal path satisfies $K(\tilde{t}) = \overline{K}$, with $\lambda_K(\tilde{t}) > 0$. Then for any $\epsilon > 0$, $d(\tilde{t} + \epsilon) > 0$ and $K \to \infty$. As $x'(K) = 0$ for $K > \overline{K}$, this breaks the transversality condition. Suppose instead that $\lim_{t\to\infty} K(t) < \overline{K}$. In this case, welfare could be increased by a marginal unit of R&D, as $c'(0) = 0$ but this marginal unit would yield a benefit of $-\int_0^\infty e^{-\rho t} x'(K) q_B \, \mathrm{d}t > 0$. Thus the optimal trajectory has to approach $(\overline{K}, 0)$.

Case 2: $\overline{K}$ is finite, $x'(\overline{K}) = 0$. The system approaches, but never

reaches, $(\overline{K}, 0)$: all other paths are ruled out as above. By linearising around the candidate steady state, the system is found to be saddlepath-stable; thus the solution must be unique.

Case 3: $\overline{K} = \infty$. The system does not reach a steady state; instead, R&D continues forever: $d(t) > 0$, for all t. It has to be shown that only one path is consistent with the transversality condition (2.4g). Suppose such a path exists. A necessary condition is $\lim_{t\to\infty} e^{-\rho t}\lambda_K = 0$. Consider (2.5). The assumptions on $x(\cdot)$ and (2.4c) imply that $K \to \infty$, and $\lim_{t\to\infty} x'(K(t)) = 0$. Denoting the quantity of backstop resource consumed at the minimum price by $\overline{q}_B$, $\lambda(t) < \int_t^\infty e^{-\rho(s-t)}\overline{q}x'(K(t))\,\mathrm{d}s \to 0$. The assumptions on $c(\cdot)$ then dictate that also $d \to 0$.

$\dot{K} > 0$ for all $\lambda_K > 0$, with loci $\dot{K} = 0$ located at $\lambda_K = 0$. The loci of points $\dot{\lambda}_K = 0$ is illustrated; it is decreasing and approaches the K-axis asymptotically. The optimal path has to be sandwiched between the two loci. This path is unique. Suppose it weren't; then there would exist two paths $\lambda_K^1(K)$ and $\lambda_K^2(K)$, both asymptotically converging to the K-axis. Suppose $\lambda_K^1(\tilde{K}) > \lambda_K^2(\tilde{K})$, for some large $\tilde{K}$. As K increases, the vertical distance between the two paths would have to decrease. However, at $\tilde{K}$

$$\begin{aligned}\frac{\mathrm{d}(\lambda_K^1 - \lambda_K^2)}{\mathrm{d}K} &= \frac{\dot{\lambda}_K^1}{\dot{K}^1} - \frac{\dot{\lambda}_K^2}{\dot{K}^2} \\ &= \frac{\dot{\lambda}_K^1}{r^1} - \frac{\dot{\lambda}_K^2}{r^2} > 0\end{aligned}$$

as both terms are negative, and decreasing in absolute value with λ_K. Hence the paths would diverge, while converging towards zero—a contradiction. □

Proof of Proposition 1. Existence of the optimum is assumed.[44] The backstop will always be used eventually as $u'(0) > x(0)$. Note that this implies that $\lambda_K(0) > 0$; otherwise the costate variable will become negative and the transversality condition (2.4g) is not met (note that $x'(\cdot) < 0$). For the same reason, $\lambda_K(t) = 0$ is only possible for $t \geq t^{**}$ (in fact, integrating (2.4e), one confirms that, if t^{**} is finite, then $\lambda(t) = 0$ for $t \geq t^{**}$). But then, due to the assumptions on $c(\cdot)$, research takes place at all times until the attainment of the lower bound (if ever): $d > 0$ for all $t < t^{**}$.

Suppose there is an interval of time of non-zero length such that both resources are used simultaneously. Then, from the first-order conditions, during this period $\lambda_S = x$. Taking time derivatives and using (2.4d), $0 \leq \rho\lambda_S = x'(K)d < 0$ which is a contradiction. Hence, there cannot exist an interval during which both resources are used.

That the exhaustible resource will be used up entirely is immediately implied (2.4a) and (2.4f). Marginal utility of resource consumption increases in stage one; in stage two, as the backstop cost decreases, marginal utility decreases. This yields the monotonicity properties of resource use over time. Prior to the time of switch t^*, $\dot{\lambda}_K > 0$ (as $q_B = 0$). This yields the monotonicity of R&D intensity prior to the switch.

In (K, λ_K)-space, following exhaustion, we have

$$\left.\frac{\mathrm{d}\lambda_K}{\mathrm{d}K}\right|_{q_B>0} = \frac{\dot{\lambda}_K}{\dot{K}} = \frac{\rho\lambda_K + p^{-1}(x(K))x'(K)}{(c')^{-1}(\lambda_K)} \leq \frac{\rho\lambda_K}{(c')^{-1}(\lambda_K)} = \left.\frac{\mathrm{d}\lambda_K}{\mathrm{d}K}\right|_{q_B=0}$$

and so the path will lie below the terminal path in (K, λ_K)-space (Figure 2.14).

[44] The method used in Lemma 2 cannot be used, as including the resource adds a non-negativity constraint.

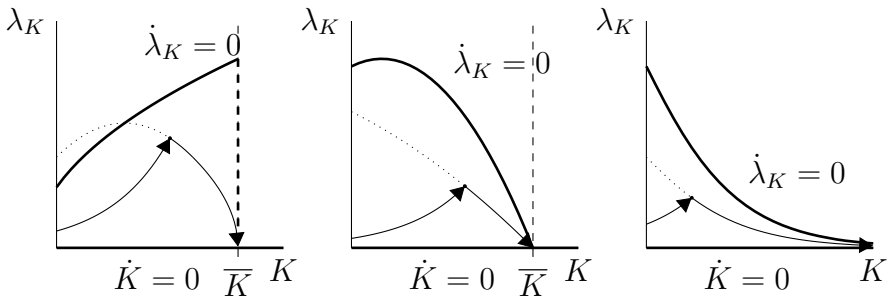


Figure 2.14: Behaviour of the economy in (K, λ_K)-space, for the cases with finite $\overline{K}$, $x'(\overline{K}) < 0$ (*left*); finite $\overline{K}$, $x'(\overline{K}) = 0$ (*middle*); and $\overline{K} = \infty$ (*right*). The terminal path is given by the dotted line and the solid continuation. The locus $\dot{K} = 0$ lies along the K-axis, $\dot{\lambda}_K = 0$ given by the thick line.
The socially optimal trajectory, with strictly positive resource stocks, coincides with the terminal path after exhaustion (*thin solid line*) and approaches $(\overline{K}, 0)$. Before this, the economy has reached the terminal path at some finite date, prior to which it lies on a path below the terminal path. A higher knowledge stock at the switching date $K(t^*)$ implies a lower R&D intensity path before the switch. The knowledge stock at the switching date is determined by the resource constraint.

□

Proof of Proposition 2. Note first that, for any given K, $\lambda_K > 0$, an increase in ρ increases the slope of the phase arrows in (K, λ_K)-space: $\dot{K}$ remains unchanged, but $\dot{\lambda}_K$ strictly increases (Figure 2.15a). This further implies that the new terminal path will lie strictly below the old terminal path. Both have to end at $(\overline{K}, 0)$. Suppose the new terminal path would, somewhere, lie (weakly) above the old one. Then it would be impossible for the terminal path to arrive at the required point.

Take optimal paths for economies A and B such that $\rho_A < \rho_B$. Suppose $K_B(t_B^*) \geq K_A(t_A^*)$. Prior to exhaustion, path B must lie always below path A, implying $d_B(t) < d_A(t)$ for $t \leq t_A^*$, and so that $t_A^* < t_B^*$. Now note that the marginal utility ('price') of consuming fossil fuels also has

to rise at a higher rate, and terminate at $x_B(t_B^*) \leq x_A(t_A^*)$. This implies that the marginal utility will always be lower along B than along A, that is extraction rates have to be always higher for $t \leq t_A^*$. This will break the resource constraint. Hence, $K_B(t_B^*) < K_A(t_A^*)$.

Suppose $\frac{\mathrm{d}t^*}{\mathrm{d}\rho} < 0$. Then $\lambda_S(0)$ has to fall with ρ. Suppose it doesn't; as the price of oil increases at a higher rate, there will be less resource extraction at all times, *and* for a shorter period of time. This implies not all of the resource is not used up which cannot be optimal.

Suppose $\frac{\mathrm{d}t^*}{\mathrm{d}\rho} > 0$. Now, from the phase diagram, it is obvious that if $\lambda_K(0)$ were to rise with ρ, path B would lie above path A until exhaustion, so that $d(t)$ would be greater for all $t \leq t_B^*$, and the terminal path would be hit more quickly—a contradiction.

Thus at least one of the capital stocks must fall in terms of the initial shadow values; in fact, both may do so. This means that either the initial R&D rate or the initial extraction rate (or both) have to fall. It is straightforward to find numerical examples with $\frac{\mathrm{d}d(0)}{\mathrm{d}\rho} > 0$; for example, $u(q) = \frac{q^{1-\frac{1}{\eta}}}{1-\frac{1}{\eta}}$, $c(d) = \frac{\xi}{2}d^2$, $x(K) = \underline{x} + \frac{\gamma}{2}(\overline{K} - K)^2$; with $\eta = 2$, $\xi = 10^{-4}$, $\underline{x} = .5$, $\overline{x} = 10$, $\gamma = .0304$, $\overline{K} = 25$, $S_0 = 5$, $\rho = .03$. Numerical examples of the case in which $\frac{\mathrm{d}q_F(0)}{\mathrm{d}\rho} < 0$ have not been discovered. □

Proof of Proposition 3. By arguments employed in the proof of Proposition 2, for two equilibria A and B which do not vary in the terminal path (i.e. which have identical discount rates, R&D cost functions and backstop technologies)

$$K_A(t_A^*) \geq K_B(t_B^*) \Leftrightarrow t_A^* \geq t_B^*, \quad q_A(t^*) \geq q_B(t_B^*), d_A(0) < d_B(0)$$

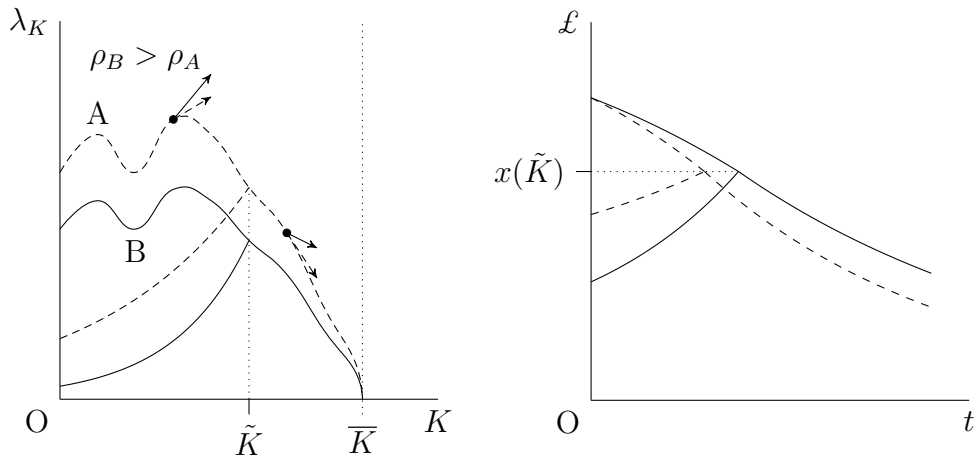


Figure 2.15: An increase in ρ leads to the terminal path contracting down and the phase arrows all skewing up (2.15a). A (weakly) lower knowledge stock at switching time would imply higher t^* and higher extraction (lower marginal utility) at all moments, breaking resource constraint (2.15a).

Suppose A and B vary only in terms of initial resource stock: $S_A(0) > S_B(0)$, $d_A(0) \geq d_B(0)$. Then the path $q_A(t) < q_B(t)$ for all $t \in [0, t_A^*]$ and the resource constraint is broken. Hence it must be that $d_A(0) < d_B(0)$ and $q_A(0) > q_B(0)$.

Suppose instead that A and B vary only in terms of the initial knowledge stock: $K_A(0) \geq K_B(0)$. Then if $q_A(0) \leq q_B(0)$, the price of the exhaustible resource will hit the backstop price earlier: $t_A^* < t_B^*$, and again not all of the resource is used up in A. Hence $q_A(0) \geq q_B(0)$. Similar claims are not applicable for the R&D process. □

Appendix 2.B Proofs for Section 2.3

Proof of Proposition 4. I follow the proof in Hoel (1978), making a minor adjustment to account for a given, decreasing price path instead of a

constant backstop price. The exporter facing a given downward-sloping price path solves

$$\max_{q_F} \int_0^\infty e^{-\rho t} q_F(t) p(q_F(t); x)\, \mathrm{d}t$$

$$\text{s.t. } p(q_F; x) = \begin{cases} p(q_F) & \text{if } p(q_F) \leq x \\ x & \text{otherwise} \end{cases}$$

$$\dot{S} = -q_F,\ S \geq 0,\ S(0) = S_0$$

where $p(\cdot; x)$ is the demand curve the monopolist faces, incorporating the price ceiling, and (slightly abusing notation) $p(\cdot)$ denoting the underlying energy demand curve. Note that $p'(q; x) = 0$ for $q < p^{-1}(x)$, $p'(q; x) = p'(q)$ for $q > p^{-1}(x)$.

Due to positive discounting, it can never be optimal for the monopolist to satisfy less than the full demand, at the price $x(t)$, for any period; she could then increase her profits by rearranging her sales within this period so that $q_F = p^{-1}(x)$ in the first part of the period, and $q_F = 0$ in the second part. Similarly, it can never be optimal to set $q_F(t) = 0$ for $t \in (t^1 - \delta, t^1 + \delta)$ and $q_F(t^2) > 0$ for for $t \in (t^2 - \delta, t^2 + \delta)$, for any small δ and $t^2 > t^1$. In other words, the monopolist satisfies full demand when limit pricing and it is never optimal to stop extraction and then start again. The problem can be rewritten as a maximising revenues up to the date of exhaustion T,

with a control constraint $p(q_F(t)) \leq x(t)$ and a free terminal time T:

$$\begin{aligned} &\max_{q_F,T} \int_0^T e^{-\rho t} q_F(t) p(q_F(t); x)\, \mathrm{d}t \\ \text{s.t.} \quad &p(q_F) \leq x(t) \\ &\dot{S} = -q_F,\ S \geq 0,\ S(0) = S_0 \end{aligned}$$

Augmenting the Hamiltonian associated with the exporter's problem with the constraint, the Lagrangean is

$$\mathcal{L} = q_F p(q_F; K) - \lambda_S q_F - \mu \left(p^{-1}(x(t)) - q_F \right)$$

where $\mu(t)$ is the Lagrange multiplier associated with the price ceiling at all points in time.

The (necessary) first-order conditions are

$$\begin{aligned} MR(q_F; x) \equiv p(q_F; x) + q_F p'(q_F; x) &= \lambda_S - \mu \\ q_F \geq p^{-1}(x(t)),\ \mu \geq 0,\ &\text{C.S.} \\ \dot{\lambda}_S &= \rho\lambda \\ \lim_{t\to\infty} e^{-\rho t} \lambda_S(t) S(t) &= 0 \\ \mathcal{L}(T) &= 0 \end{aligned}$$

Assume that the resource is truly scarce, i.e. that $\lambda > 0$, $\forall t$. It is straightforward to see that the monopolist, once limit pricing, will never price below the substitute cost again. Likewise, when limit pricing, she will never set $q_F < p^{-1}(x)$; were she to do this, any barrels unsold due to the full demand not being satisfied could be sold only later, at a weakly lower

current price (as $\dot{x} < 0$), which would furthermore be discounted. Thus profits would be increased by satisfying full demand earlier instead.

The last condition yields the optimal T, and from the Kuhn-Tucker conditions it is immediate that $\lambda_S(T) = p(q_F(T))$. The constraint must bind at some point in time. Suppose it doesn't; then $\mu(t) = 0$ always, and we would have $0 < -q_F(T)p'(q_F(T)) = p(q_F(T)) - \lambda(T) = 0$, a contradiction. Thus we must have $\lambda(T) = x(T)$, $q_F(T) = p^{-1}(x(T))$ and $\mu(T) = -q_F(T)p'(q_F(T) > 0$. Assuming the inverse demand function is twice differentiable, by continuity limit pricing goes on for a period of time before T of non-zero measure. The moment at which limit-pricing begins is given by $p(q_F(t^*)) + q_F(t^*)p'(q_F(t^*; x(t^*)) = \lambda(t^*) = e^{-\rho(T-t^*)}x(T)$. With strictly concave revenues, this is uniquely given for any T and $x(T)$, as the RHS is strictly decreasing in t^*. Thus $t^* < T$. A negative t^* is impossible and instead implies that limit pricing begins immediately, i.e. $t^* = 0$.

For any given T, the resource constraint may not be satisfied. However, the cumulative extraction $\int_0^T q_F(t)\,\mathrm{d}t$ is increasing in T. To see this, consider two candidate exhaustion dates $T^1, T^2, T^1 < T^2$. Then $x(T^1) > x(T^2)$. Denote the resulting two paths of q_F, λ and other variables by superscripts. Then $\lambda^2(T_1) = e^{-\rho(T^2-T^1)}x(T^2) < x(T^1) = \lambda^1(T^1)$, and clearly $\lambda^2(t) < \lambda^1(t) \forall t \in [0, T^1]$. This implies that $MR(q_F(t^{*,1})) \leq MR(q_F(t^{*,2})) = \lambda^2(t^{*,2}) < \lambda^1(t^{*,2})$; where the first inequality captures the possibility that path might not involve limit pricing at $t^{*,2}$ and the following equality results from the definition of t^* for path 2. But this means that $\mu^1(t^{*,2}) = MR(q_F(t^{*,1})) - \lambda^1(t^{*,2}) < 0$, i.e. that $t^{*,2} > t^{*,1}$. Clearly, then, $q_F^2(t) > q_F^1(t), \forall t \in [0, t^{*,2}]$. But this, together with the fact that $T^1 < T^2$, implies that the cumulative quantity extracted along path 2 is

strictly higher than that along path 1. Thus, there is only one value of T which exactly satisfies the resource constraint.

If demand is inelastic, marginal revenue is negative for all $q_F > p^{-1}(x(t))$, but positive for all q_F below this limit, and clearly the exporter limit prices always. If demand is elastic and $MR(p^{-1}(x(0))) > 0$, so that marginal revenue at the limit price is positive at least for small t, then the limit pricing stage always exists provided S_0 is high enough. □

Proof of Proposition 5. All costate variables are denoted by the same symbols as for the social planner's problem, but now represent the marginal value of the stocks to their respective 'owners': λ_K is the shadow price of knowledge for the importer, and λ_S the shadow price of the resource to the exporter. For either player, the shadow price of the other player's asset plays no role; under commitment, this asset's path is effectively taken as given.

Given a path for R&D, the backstop price path is determined, and by Proposition 4 the necessary conditions for a solution to the exporter's problem are

$$R'(q_F) \leq \lambda_S, \qquad q_F \geq \max\{0, q^{-1}(x)\}, \quad \text{C.S.} \tag{2.22a}$$

$$\dot{\lambda}_S = \rho\lambda_S \tag{2.22b}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_S(t)S(t) = 0 \tag{2.22c}$$

$$\lambda_S(T) = x(K(T)) \tag{2.22d}$$

where T denotes the time at which the resource is exhausted. Equation (2.22a) is just the Hotelling Rule for the monopolist: marginal revenue $R'(q_F) = p'(q_F)q_F + p(q_F)$ has to equal the scarcity rent. From (2.22b),

this increases at the discount rate. From (2.22b) and (2.22c), it is clear that the entire stock has to be exhausted eventually. The optimal date of exhaustion is given by (2.22d) (following from the condition that the Hamiltonian equal zero at the date at which the resource is used up): the scarcity rent is pinned down by the substitute price at the time of ehaustion. In other words, the marginal revenue for the stock is given by the price received for the very last barrel sold, at the end of the limit-pricing stage.

The resulting solution has three stages. For $t \in [0, t^*)$, the price of the resource is strictly below the backstop cost and only the exhaustible resource is consumed. If resource stocks are low, or if resource demand is inelastic for the relevant range, this stage is degenerate with $t^* = 0$ (Hoel, 1978). In the second stage, for $t \in [t^*, T)$, the resource price equals the backstop cost but only the exhaustible resource is consumed. The costate trajectory is continuous and so $p(q_F(t^*)) = x(K(t^*))$. Finally, from $t = T$, the exhaustible resource has been used up and only the backstop resource is consumed. The exporter has nothing further to do as the extraction rate is constrained to zero.

Turn now to the importer's problem. Taking q_F as a given, the backstop demand

$$q_B = \begin{cases} 0 & \text{if } q_F \geq u'^{-1}(x(K)) \\ u'^{-1}(x(K)) - q_F & \text{if } q_F < u'^{-1}(x(K)) \end{cases}$$

is continuous, but not differentiable in the state K:

$$\frac{\mathrm{d}q_B}{\mathrm{d}K} = \begin{cases} 0 & \text{if } q_F > u'^{-1}(x(K)) \\ (u'^{-1})'(x(K))x'(K) & \text{if } q_F < u'^{-1}(x(K)) \end{cases}$$

This discontinuity poses a problem: namely, that the objective function

in (2.7) is not differentiable with respect to K at $x(K) = p^{-1}(q_F)$. Hence the Maximum Principle must be modified, as by Hartl and Sethi (1984), to obtain necessary conditions:

$$c'(d) \leq \lambda_K, \qquad d \geq 0, \quad \text{C.S.} \tag{2.23a}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_K(t)K(t) = 0 \tag{2.23b}$$

$$\dot{\lambda}_K = \rho\lambda_K + q_B x'(K), \qquad p(q_F) \neq x(K), q_F > 0 \tag{2.23c}$$

$$\dot{\lambda}_K \in [\rho\lambda_K + q_F x'(K), \rho\lambda_K], \qquad p(q_F) = x(K) \tag{2.23d}$$

Again, the marginal cost of R&D effort has to equal the shadow price of knowledge (2.23a); and the present value of the knowledge stock has to equal zero in the limit $t \to \infty$. But now, when the monopolist is limit pricing, then the time derivative of the costate variable can take any value in an interval (2.23d).

At the time of exhaustion, the equilibrium has to be on the terminal path. If there exists a period in the limit-pricing stage in which $\dot{\lambda}_K > \rho\lambda_K + q_F x'(K)$, it is clear from the phase diagram that in this period λ_K will be lower than on the terminal path; otherwise the trajectory would 'overshoot' the terminal path and never be able to return to it. This implies that such an equilibrium would feature less R&D than along the terminal path. Were the importer to deviate to the terminal path, he would drive the energy price to below the backstop price, which would be welfare-improving. Thus any strategy of the importer which would feature non-terminal path R&D in the limit-pricing stage is not credible.

Following the exhaustion of the resource at time T, the exporter ceases to play a role in the game and the importer behaves as the social planner.

The final stage can be analysed as for the social optimum, and it is of course optimal to follow the terminal path from T onwards. The stages are tied together by continuity of K and λ_K at times t^* and T. □

Proof of Proposition 6. It is well-known that if $\epsilon'(q) \leq 0$ (note that I have defined $\epsilon(q)$ to be positive), the rate of increase of the resource price is greater than ρ. Suppose that the open-loop equilibrium path hits the terminal curve at a higher K than the socially optimal path: $K_S(t_S^*) < K_{OL}(t_{OL}^*)$. Then, by arguments used in the previous proposition, $t_S^* < t_{OL}^*$; further, $q_F^S(t_S^*) < q_F^{OL}(t_{OL}^*) < q_F^{OL}(t_S^*)$ and the extraction path $q_F^{OL}(t)$ lies above $q_F^S(t)$. But the social optimum exhausts the entire stock by t_S^*, in which case the along open-loop trajectory exhaustion occurs before t_{OL}^*. Thus $K_{OL}(t_{OL}^*) < K_S(t_S^*)$, implying $d_{OL}(0) > d_S(0)$.

Suppose $\epsilon'(q) \geq 0$, so that $\dot{p}_F^{OL} < \rho p_F^{OL}$. If $p_F^{OL}(0) \leq p_F^S(0)$, then $q_F^{OL}(t) > q_F^S(t)$ for $t > 0$, and again the resource stock is exhausted before t_{OL}^*. Hence $q_F^{OL}(0) < q_F^S(0)$. □

Proof of Proposition 7. With isoelastic demand $q = p^{-\frac{1}{\sigma}}$, and extending the argument used by Hoel (1978), as

$$q_F = \max\left\{\left(\frac{\lambda_S(t)}{1-\sigma}\right)^{-\frac{1}{\sigma}}, x(K(t))^{-\frac{1}{\sigma}}\right\}$$

(where the max operator captures the limit pricing behaviour), and as

$$\lambda_S(t) = e^{-\rho(T-t)}x(K(T)) \tag{2.24}$$

it follows that $e^{-\rho(T-t^*)} = (1-\sigma)\frac{x(K(t^*))}{x(K(T))}$. Suppose S_0 increases but $q_F(0)$ falls (weakly). Then t^* falls weakly; further, limit pricing begins at a lower

$K(t^*)$, with $S(t^*)$ higher and T higher. This implies that the RHS of (2.24) (less than one) will be greater, and so $T - t^*$ is lower—a contradiction. Hence $q_F(0)$, $K(t^*)$, and $d(0)$ all increase. □

Appendix 2.C Proofs for Section 2.3.2

Proof of Proposition 8. Suppose that, in the set Φ, the importer conducts R&D as according to the terminal path: $d(K, S) = d^{\infty}(K)$. Clearly, the exporter's choice of q_F will not affect the R&D rate and, clearly, it is optimal for the exporter to always limit-price as in the open-loop equilibrium. Suppose that, in the set Φ, the exporter limit prices: $q_F = p^{-1}(x(K))$. Then the importer recognises that, whatever its R&D rate, it will always have to pay the backstop price for energy. Then, clearly the optimal strategy is to follow the terminal path. □

Appendix 2.D Proofs for Section 2.4

Proof of Proposition 10. The Hamiltonian for the importer's problem is

$$\mathcal{H} = u(q_F + q_B) - x(K)(q_F + q_B) - c(d) - Z(G) + \lambda_K d - \lambda_S q_F + \lambda_G q_F \quad (2.25)$$

with $q_F = p^{-1}(x(K))$ and backstop demand given as before. The first-order conditions are as in 2.B for the limit-pricing stage, but

$$\dot{\lambda}_K = \rho\lambda_K + q_F x'(K) + (\lambda_S - \lambda_G)\frac{\mathrm{d}q_F}{\mathrm{d}K} \tag{2.26a}$$

$$\dot{\lambda}_S = \rho\lambda_S \tag{2.26b}$$

$$\dot{\lambda}_G = \rho\lambda_G + Z'(G) \tag{2.26c}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_S(t)S(t) = 0 \tag{2.26d}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_G(t)G(t) = 0 \tag{2.26e}$$

where λ_S signifies the shadow price of the resource, and λ_G the shadow price of a unit of greenhouse gases. λ_S is not pinned down by the conditions; however, $\lambda_S = \frac{\mathrm{d}(V(K,S,G)}{\mathrm{d}S}$; by assumption the exporter always limit prices, so $\lambda_S = 0$. The transversality condition is only satisfied if $\lambda_G = -\frac{Z'(G)}{\rho}$ for $t \geq T$, implying $\lambda_G < 0$ for all t.

Thus, $\dot{\lambda}_K = \dot{\lambda}_K^{\infty} - \lambda_G\frac{\mathrm{d}q_F}{\mathrm{d}K} > \dot{\lambda}_K^{\infty}$. In (K, λ_K)-space, the phase arrows for any given point bend upwards. Thus, if the economy is ever on the terminal path before exhaustion, if will immediately move above the terminal path and will never return to it. However, the optimal solution implies the economy must be on the terminal path for $t \geq T$. Thus, the optimal solution must imply the economy is below the terminal path for $t < T$; that is, for any K, $\lambda_K < \lambda_K^{\infty}$ and $d < d^{\infty}$. □

Proof of Proposition 11. The Hamiltonian for this problem is

$$\mathcal{H} = u(p^{-1}(x)) - x(K)p^{-1}(x) - c(d) + \lambda_K d - (\lambda_S - \lambda_G)p^{-1}(x)$$

assuming limit-pricing begins immediately. The necessary conditions are,

from Note 2, Chapter 2.2 in Seierstad and Sydsæter (1987):

$$c'(d) = \lambda_K \tag{2.27a}$$

$$\dot{\lambda}_K = \rho\lambda_K + x'(K)\left(p^{-1}(x) + (\lambda_S - \lambda_K)(p^{-1})'(x)\right) \tag{2.27b}$$

$$\dot{\lambda}_S = \rho\lambda_S \tag{2.27c}$$

$$\dot{\lambda}_G = \rho\lambda_G + Z'(G) \tag{2.27d}$$

$$\lambda_K(T) = \lambda_K^{\infty}(K(T)) - \mu x'(K) \tag{2.27e}$$

$$\lambda_S(T) = \mu C'(S(T)) \tag{2.27f}$$

$$\lambda_G(T) = -\frac{Z'(G)}{\rho} \tag{2.27g}$$

$$\mathcal{H}(T) = \rho\left(\pi^{\infty}(K(T)) - \frac{Z(G)}{\rho}\right) \tag{2.27h}$$

where μ is a shadow value related to the constraint $C(S(T)) = x(K(T))$. Equation (2.27h) yields the optimal stopping time T. Note that for the terminal path, this holds for all K with $\lambda_S = \lambda_G = 0$, and $\lambda_K = \lambda_K^{\infty}(K)$, $d = d^{\infty}(K)$. Hence

$$\begin{aligned}\lambda_K(T)d(T) - c(d(T)) - (\lambda_K^{\infty}(K(T))d^{\infty}(K(T)) - c(d^{\infty}(K(T))))& \\ = (\lambda_S(T) - \lambda_G(T))\, p^{-1}(x(K(T)))&\end{aligned} \tag{2.28}$$

Using the first-order condition on d, the function $\Phi(d) \equiv c'(d)d - c(d) = \lambda_K d - c(d) =$ is increasing in d. The above statement thus relates the difference between $\Phi(d(T))$ and $\Phi(d^{\infty}(K(T))$ to the reduction in welfare caused by pollution at the moment of exhaustion. Note that $\lambda_G(T) < 0$.

I will now argue that $d(T) > d^{\infty}(K(T))$. This follows if $\lambda_S(T) < 0$, so that $\mu > 0$: then $\lambda_K(T) - \lambda_K^{\infty}(K(T)) = -\mu x'(K) > 0$, which yields the result. Then, from (2.28), $\lambda_S(T) - \lambda_G(T) > 0$, i.e. $\lambda_G(T)$ is more negative

than $\lambda_S(T)$. This makes intuitive sense: the welfare impact of having more of the exhaustible resource lies in the fact that a part of it will eventually become a pollutant—but only part, and only eventually.

To complete the argument, suppose that $\lambda_S(T) \geq 0$, so that $\mu \leq 0$. Then, for sure, the LHS of (2.28) is positive, so that $d(T) > d^{\infty}(K(T))$; but from the transversality condition on $\lambda_K(T)$, the opposite must hold—a contradiction.

I will finally establish the property that the optimal trajectory crosses the terminal path once at most; and, so, that there are two distinct phases of R&D, the first (if it exists) with R&D lower, and the latter with R&D higher, than in the reference equilibrium. Note that $\lambda_S - \lambda_G$ has a positive sign. Now, suppose there exists a point in time when the optimal trajectory coincides with the terminal path. Then, along the optimal path, $\dot{\lambda}_K$ must be higher (comparing the equations of motion for λ_K) while clearly the R&D rate, and hence $\dot{K}$, is equal in both cases. Thus the optimal trajectory will cross to above the terminal path and stay there until exhaustion. $\square$

Chapter 3

Monopolistic sequestration of European carbon emissions

Abstract

Mitigating climate change by carbon capture and storage (CCS) will require vast infrastructure investments. These investments include pipeline networks for transporting carbon dioxide (CO_2) from industrial sites ('sources') to the storage sites ('sinks'). This paper considers the decentralised formation of trunk-line networks when geological storage space is exhaustible and demand is increasing. Monopolistic control of an exhaustible resource may lead to overinvestment and/or excessively early investment, as these allow the monopolist to increase her market power. The model is applied to CCS pipeline network formation in northwestern Europe. The features identified above are found to play a minor role. Should storage capacity be effectively inexhaustible, underinvestment due to the inability of the monopolist to capture the entire social surplus is likely to have substantial welfare impacts. Multilateral bargaining to coordinate international CCS policies is particularly important if storage capacity is plentiful. A duopolistic case may feature tacit collusion to cut supply of storage.

3.1 Introduction

> Scotland-based companies and the government at Holyrood hope proximity to the rapidly-depleting oil and gas fields of the North Sea will put the country at the forefront of a potentially lucrative new industry storing carbon dioxide. (...) The development of CCS in Scotland including power stations and storage networks has the potential to support 10,000 jobs. (*Financial Times*, 16th Aug 2010)[1]

> [Norwegian] Statoil has dedicated a group of geologists to mapping the undersea region with the aim of one day providing carbon storage for power plants and manufacturers across Europe. "We want to build a business at Statoil as a carbon dioxide storage provider," says Kristofer Hetland, a Statoil executive. (*Financial Times*, 2nd Sep 2010)

Carbon capture and storage (CCS) is a technology intended to mitigate climate change.[2] CCS involves the capture of carbon dioxide (CO_2) from large point sources: typically coal- or gas-fired power plants or industrial installations. The gaseous pollutant is separated from other flue gases and compressed into either a liquid or a 'supercritical' ('dense') phase. Once transported to an appropriate location, the pollutant is then injected underground. Storage is conventionally considered to occur in suitable geological formations: depleted oil and gas reservoirs, or saline aquifers.[3] CO_2 will fill any vacant pore space in the rock, potentially displacing water or existing

[1] A policy document from the Scottish government concurs:

> We want the North Sea to be seen as Europes principal CO2 storage hub (...) bringing new investment and a long term future for our offshore industries as hydrocarbon production eventually declines. (Scottish Government and Scottish Enterprise, 2010)

[2] IPCC (2005), although dated, remains the 'bible' on carbon capture and storage; see also Kheshgi et al. (2012).

[3] Storage in the deep ocean has also been suggested. See Lontzek and Rickels (2008) for an economic analysis of ocean sequestration.

oil or gas in the formation.[4] Once underground, the storage site would be sealed (if necessary), with the carbon dioxide trapped by low-permeability layers above the formation, slowly dissolving into the formation water and eventually mineralising into carbonate rock.

As the quotes above illustrate, the expectation of climate policies can transform subsurface storage capacity into a valuable asset. Two questions naturally arise: how *should* such assets be employed, and how *will* they be employed? The answer to the latter will depend on the regime of property rights and the relevant regulatory environment—that is, who controls the assets, and how they are allowed to utilise them. The two answers might well be different: under some regulatory environments, self-interested actors might utilise subsurface storage capacity in ways which are suboptimal from society's point of view. For example, a country such as Norway might want to ration storage capacity, in order to maximise the fees it can charge for storing the carbon emitted by German power plants. Such rationing would push up the costs of generating electricity in Germany; but such costs would not fall on Norway, and would hence not be taken into consideration.

As economy-scale CCS would almost certainly require storage in reservoirs which are not yet well understood, in particular saline aquifers, total available storage capacity remains an unknown quantity.[5] Storage in de-

[4]With large-scale CCS, large quantities of brine may be produced and would need to be disposed of adequately, e.g. by pumping it underground into a different formation or by desalination (Surdam et al., 2011; Bourcier et al., 2011). Not producing brine would imply a reduced overall storage capacity or higher pressures in the formation. The latter would increase the risk of fracturing the rock 'seal' overlying the storage formation, leading to CO_2 leakage or groundwater contamination; or of inducing seismic activity (Buscheck et al., 2011).

[5]Many of the component technologies of CCS are mature and have been commercially available for a long time. Transport and injection technologies have been used for decades in enhanced oil recovery operations. Capture based on aqueous amine absorption is also considered mature; however, alternative post-combustion capture technologies have scope for improvement (Jones, 2011). These include technologies based

pleted oil and gas fields is fairly well understood; these could provide total capacity of up to 900 $GtCO_2$ worldwide. However, many such fields will not be located close to where the emissions are being generated; in Europe, such capacity is estimated to be only 20 $GtCO_2$ (Vangkilde-Pedersen et al., 2009b). Note that the potential emissions would well exceed this capacity: using projections by the International Energy Agency, European CO_2 emissions could be around 2-3 $GtCO_2$ per annum.[6] Storage in saline aquifers would at least double this capacity worldwide; in Europe, saline aquifers could provide 100 $GtCO_2$ additional capacity.

Several factors might limit the actual available storage capacities. Firstly, the expected abundance of storage capacity in saline aquifers might have been overestimated. The statistical methods used to assess regional storage capacities have been criticised as ignoring crucial geological and physico-chemical details (such as the migration of the CO_2 plume in the reservoir or the pressure and temperature profiles which affect CO_2 density). Detailed studies have shown that ignoring such detail may result in reported storage volumes exceeding actual potential by one or two orders of magnitude (Spencer et al., 2011).

Furthermore, restrictions on onshore storage may prove to be serious.

on CO_2-selective membranes, on adsorption of CO_2 by solids, or on other methods. Technologies which remove CO_2 before combustion, such as integrated gasification and combined cycle combustion, also have major potential (Figueroa et al., 2008). Oxyfuel combustion, in which the concentration of CO_2 in the flue gas is increased by high-oxygen combustion may also play a part. Furthermore, to have a substantial impact of greenhouse gas concentrations, these technologies would have to be deployed at a vastly greater scale than that experienced to date. For example, some 50 $MtCO_2$ is currently injected underground each year as part of enhanced oil recovery operations (ITFCCS, 2010). Integrated assessment models project storage rates of 10-20 $GtCO_2$ by 2100; an increase or two or three orders of magnitude.

[6]Note that while the original fossil fuel content partially corresponds to available capacity, CCS is *not* about 'putting back in' whatever was taken out: the CO_2 injected would be mostly sourced from coal-burning facilities, not oil or gas combustion.

At present, these seem most likely to stem from public health and safety concerns.[7] While fears over very large risks seem to be unfounded, they could severely constrict overall storage capacity (due to expected liability exposure and/or public opposition to storage and transport schemes) and increase the overall cost of CCS. Onshore storage could also prove unattractive to storage operators if expected liabilities are much greater onshore than offshore, due to e.g. risks to public health or to groundwater resources (Damen et al., 2006).[8] Hence, at this point, it seems prudent to take into account scenarios in which onshore storage capacity is rather scarce.

Economy-scale CCS implies major investments: plants to separate CO_2 from a flue gas stream and to compress it, transport infrastructure (pipelines and/or ships and related facilities) to transport the pollutant to the injection site, and the injection plants. As an example, In Europe these investments could run into the tens and hundreds of billions of euros. The capture costs dominate overall costs. However, in the case of offshore storage, the cost of developing the transport infrastructure would gain more weight. Such investments are costly: a recent study estimated the cost of long offshore pipelines to be in excess of €2m/km (ZEP, 2011d), with fixed costs related to transportation and injection making up almost

[7]People may be worried about a catastrophic release, such as the notorious incident of a release of naturally accumulated CO_2 in Lake Nyos, Cameroon, which led to thousand of fatalities by suffocation, or of induced earthquakes (Bradbury, 2012). However, these are seen to be extremely unlikely in the geological contexts appropriate for CCS (DOE, 2006).

[8]The risks depend on prudent site selection. Trabucchi et al. (2012) consider a 'best-practice' storage operation and estimate the liability cost to be around $0.34/ton of CO_2 sequestered. However, this study is based on a storage operation in Texas; a similar assessment of European onshore storage, with a much higher average population density, might turn out very differently. Note that the long-term liability for the climate changes induced by gradual leakage of stored carbon would not differ between onshore and offshore storage, at least under a sensible regulatory regime.

a third of the overall costs of CCS.[9] Major backbone infrastructure would require many pipelines of this capacity.[10]

This paper studies CCS under decentralised infrastructure investment and potentially exhaustible storage. Both features are novel in the study of optimal CCS infrastructure. I will use an exhaustible resources framework: the 'resource' being underground storage capacity, one can think of CO_2 injections as 'extraction' of storage capacity.

The main theoretical innovation of the paper is to consider the effect of set-up costs and market power in exhaustible resource markets. I will show that, under particular demand structures, a resource monopolist may build too much infrastructure, too early, compared to what is socially efficient. The conditions required for this to occur are rapidly increasing demand, an anticipated collapse in demand in the future (for example, due to technological innovation introducing a substitute for the resource) and the ability of the monopolist to appropriate an increasing fraction of the social value of resource use by cutting back demand.

With rapidly increasing demand, resource consumption would optimally be loaded towards the future, peaking at maximum capacity in the run-up to the termination of demand, when the excess value of consumption (net of any extraction costs) is the highest. This creates an incentive to postpone investment into opening the deposit. A monopolist will invest

[9] According to ZEP (2011b) and the detailed sister reports, capturing a tonne of CO_2 post-combustion at a hard coal power plant, shipping it 1500 km offshore and injecting into a depleted gas field would cost roughly €53. Of this cost, the capture costs make up 59%, divided evenly between capital and operating costs. Pipeline and storage capital costs make up another 31%. The operating costs related to injection make up 8%, and the remaining 2% is due to transport operating costs.

[10] The 40" offshore pipeline used for the estimates can transport 20 Mt CO_2 per annum. Projected transport requirements run into the hundreds of megatonnes per annum.

too early, and in too much capacity. The intuition is that the distorted investment decisions create market power for the monopolist. Investing early prolongs the remaining period until the technology revolution makes the resource worthless. High extraction capacity permits larger sales in the run-up to the technology revolution. Both factors allow the monopolist to increase prices immediately after investment, so capturing a larger share of the surplus due to the use of the resource, while still selling the entire stock.

These theoretical results may have implications for many capital-intensive resource markets. For example, the deposits of some heavy rare earth elements—necessary for many high-tech applications, e.g. as permanent magnets used in computer hard drives, wind turbines and the engines of hybrid cars—are well-known to be concentrated in China.[11] The exhaustibility of known deposits has been said to be a serious issue. The industry is very capital intensive, due to the complicated supply chain required to extract and refine these metals.[12]

In the second half of the paper, I apply the model to the European CCS market under decentralised pipeline network formation. I find that the above questions of overinvestment or early investment are not very important from a welfare perspective: if the exhaustibility of storage capacity truly bites, then the monopolist is likely to behave in ways which come

[11]Reports indicate that non-Chinese reserves of terbium and yttrium, in particular, are very scarce. There has been substantial activity recently to develop alternative sources of these elements.

[12]Other particularly capital-intensive resource industries, such as supply of liquefied natural gas or exploitation of unconventional oil reserves, may feature a lower degree of market power on part of the owner of the capital-intensive deposit. In oil markets, the market power is instead held by OPEC, who sell mainly conventional oil; although even extraction of conventional oil is rather capital intensive. In LNG markets, on the other hand, market power is less likely to be an issue in the near future due to the rapidly increasing shale gas supply from the United States, and potentially elsewhere.

close to the efficient outcome. However, the more important case is that in which storage capacity is plentiful. In this case, while the potential social benefits of CCS are higher, so are the potential losses due to lack of coordination between the countries which emit CO_2 and the countries with the possibility to store these emissions.

The last section will consider duopolistic supply of storage. As I want to focus on the fixed costs of increasing capacity, I extend a workhorse model of preemptive capacity expansion. I find that this setup may well enable tacit collusion. Both suppliers cut back on their capacity expansion, instead allowing the competitor to the market. The intuition is that allowing the competitor to build some capacity makes her less hungry to expand in the future. On the other hand, preemptive outcomes, in which cutthroat investment competition eats away all duopoly rents, can also occur.

The existing literature on CCS economics is not very extensive.[13,14] One strand of literature uses stylised, long-run, macro-scale analytical models to consider the optimal timing of CO_2 sequestration. Lafforgue et al. (2008) consider the efficient timing of CCS when multiple storage sinks exist, under the constraint of atmospheric CO_2 concentrations not exceeding a given ceiling. The sinks are assumed to have finite capacity. They are only used once the atmospheric ceiling becomes binding. Before this is the case, 'storage in the atmosphere' is preferred as this is not only cheaper, but also has the added benefit of natural decay proportional to the stock.[15] Once

[13]I am omitting the voluminous engineering-economic literature considering the costs of various detailed processes in the CCS chain.

[14]I will cover the theoretical literature on set-up costs in Section 3.2.

[15]Exponential decay of the entire excess carbon stock is a common assumption made in analytical models of climate change; early contribution are Nordhaus (1991); Withagen (1994); Hoel and Kverndokk (1996); more recent works include, among many others, Newell and Pizer (2003); Eyckmans and Tulkens (2003); Leach (2007). This is, strictly speaking, unrealistic. Anthropogenic carbon emissions are gradually removed from the

the economy hits the ceiling, resource use continues over and above the maximum rate of natural decay rate at the ceiling, with the excess emissions stored in sinks, and the cheapest sinks used first. Amigues et al. (2010) and Coulomb and Henriet (2011) point out that if noncapturable emissions, such as the diffuse emissions resulting from transport fuel consumption, are very large, then the entire flow of capturable emissions might be stored while at the ceiling. With late capture constrained and so unable to fully substitute for early capture, CCS begins before the atmospheric ceiling is reached. Amigues et al. (2012) consolidate and extend this line of inquiry further by studying the optimal timing of CCS when the costs are affected by the cumulative capture (due to either learning-by-doing or scarcity of storage capacity) or purely by the flow rate of capture.

Grimaud et al. (2008) consider an endogenous growth model, with an exhaustible resource, the use of which produces a stock pollutant. They allow for a carbon capture process which uses labour to remove a fraction of the carbon emissions. It is found that the optimal carbon tax, while increasing, can be interpreted as a decreasing ad valorem tax on resource use. This is result consistent with the 'Green Paradox' literature (Sinclair, 1994): resource owners seek to postpone extraction in response to a lower ad valorem tax rate in the future.[16] Importantly, the level of the tax becomes

atmosphere, in a complex interaction of climate change and biogeochemical processes (Archer et al., 2009). For the first millennium or so, the drawdown is primarily a result of the mixing of the CO_2-saturated surface waters into the deep ocean. Following this, the oceanic carbon is gradually depleted by chemical reactions with minerals. Following equilibration of the ocean, some 20-35% of the anthropogenic emissions will remain in the atmosphere for 10,000 years or longer. Farzin and Tahvonen (1996) model optimal carbon taxes using an analytical model with multiple boxes, based on Maier-Reimer and Hasselmann (1987). Archer et al. (1997) present more recent estimates of a reduced-form multiple-box model of the carbon cycle. See Section 2.4 for a discussion of modeling climate impacts based on cumulative emissions.

[16]The 'Green Paradox' states that any policy constricting future demand, as perceived by the resource owner, relative to current demand will tend to accelerate extraction (and,

important (alongside the rate of change) as this incentivises optimal CCS efforts.

None of the above papers take into account the 'energy penalty' related to CCS. Part of the cost of CCS results from need to burn more fuel to drive the energy-intensive capture process. Thus, to avoid a given amount of emissions, a larger amount has to be captured; in addition to the avoided emissions, the CO_2 emitted during production of energy used in capture itself has to be captured. This of course increases the demand for fuel, in particular coal. Hoel and Jensen (2012) point out that, if CCS is feasible, an expected carbon tax in the future may in fact *increase* future demand for coal, because of this energy penalty. This effect may eliminate the Green Paradox, as a tightening climate policy leads to increasing demand for coal, thus incentivising conservation of the resource.

At the other end of the spatial scale, Leach et al. (2011) study how a profit-maximising firm optimally schedules oil extraction by CO_2-enhanced oil recovery (EOR), thereby sequestering carbon in the process. Their model departs from commonly used models of resource extraction by being founded more closely on models employed by reservoir engineers and oil industry practitioners. These models typically feature falling oil extraction due to falling pressure in the reservoir (the 'production decline curve') (Adelman, 1990; Cairns and Davis, 2001; Mason and vant Veld, 2013). CO_2 injections are found to decrease over time, driven by exhaustibility of storage capacity and a fall in the marginal product of injected CO_2 (in terms of produced oil) as reservoir pressure falls. Carbon sequestration is found to be much more sensitive to the oil price than to the carbon

presumably, pollution) as resource owners reoptimise their extraction schedules.

price. Other authors have pointed out that the timing of EOR-related CO_2 demand, among other factors, imply that while enhanced oil recovery is likely to be important in terms of higher oil production, it may play a rather small role in sequestering meaningful amounts of carbon (Davidson et al., 2011; Dooley et al., 2010).

Recently, a number of studies (some of these funded or conducted by the European Commission) have studied optimal CO_2 pipeline networks using geographically detailed economic models (Middleton and Bielicki, 2009; Morbee et al., 2012; Neele et al., 2010). These studies, all of which have an operations research flavour, consider minimisation of overall system-wide costs required to meet exogenous goals; such as sequestering a given amount of carbon per year, or complying with a carbon tax. The implicit assumption is that there exists a regulator with the incentives and legal powers to mandate the construction of the cost-minimising network. The studies are either static or use a relatively coarse temporal resolution.

All of the above papers (except Leach et al., 2011) thus focus on the centrally planned, socially optimal or social cost-minimising outcome. Such central planning may not be feasible. There may not exist the political will or desire to regulate the industry (this could well be the case in the United States). Alternatively, a regulator with sovereignty over the various actors might not exist; say, were CO_2 pipelines to cross international borders, a situation seen as unavoidable in the EU.[17] In the present chapter, I focus first on monopolistic supply of CO_2 storage capacity. In the last section, I also consider duopolistic storage, for which game-theoretic methods are

[17]Heitmann et al. (2012) argue that failures to subordinate local and/or national decisionmakers to EU-wide CCS policies may hinder the development of European CCS projects.

required.

The application in the present paper falls somewhere between the models considering the optimal global timing of CCS and those focusing on infrastructure network formation. I strip down the geographical detail in order to focus on the large-scale features of transport networks; in particular, the timing of 'backbone' pipelines transporting large quantities of CO_2. I do this in order to focus on the dynamics of sequestration when coordination fails, in the sense that the backbone is unilaterally constructed by the owner of a major storage site, in order to profit from CCS storage.

The present paper does not consider issues such as regulation of monopolies or network industries (Armstrong et al., 1994). These would be natural questions to consider when focusing on appropriate incentives for a pipeline network located under a single jurisdiction. By assumption, in the present paper, no regulator with the required powers exists. As I only focus on a major 'backbone' component of the pipeline network, the problem is really one of fixed costs for a single investment: network externalities do not play any role in the model.[18] I also abstract from questions of long-term leakage rates of stored carbon (Ha-Duong and Keith, 2003). The question of optimal carbon taxes is ignored; this paper focuses on regional mitigation efforts so that the carbon tax can be taken as exogenous. Similarly, the availability and optimal use of fossil fuel resources is ignored; coal reserves are, in any case, estimated to be plentiful for the foreseeable future.

The paper is divided into three parts. Section 3.2 introduces a simple model of set-up costs with exhaustible resources, focusing on monopolistic extraction and investment timing. Section 3.3 applies the model to CCS,

[18]But see the discussion in Section 3.3.

taking storage capacity as the resource. Section 3.4 consider a preemptive duopoly investment model to evaluate strategic competition between two storage suppliers. Section 3.5 summarises the findings, considers future research directions and concludes.

3.2 Exhaustible resources and set-up costs

Opening a deposit of an exhaustible resource may involve set-up costs to develop the associated infrastructure. The interaction between such costs and market power has received surprisingly little attention. Stiglitz (1976) showed that, faced with isoelastic demand and a given period of extraction, a monopolistic resource owner is unable to use market power to her advantage. An attempt to hike up prices in one period will require more of the resource to be sold in other periods, depressing prices, for the resource stock to be fully used up. This increases overall profits if prices are raised precisely when demand is less elastic, and decreased when demand is more elastic. With isoelastic demand profits cannot be increased.

Hartwick et al. (1986) show that this result breaks down when set-up costs are introduced: a monopolist who owns two deposits plans to open the second deposit too late, delaying the infrastructure investment. Fischer and Laxminarayan (2005), on the other hand, show that there are two different effects at play. The monopolist, deciding when to open a second (and final) deposit, is faced with a choice between hiking up prices, thus stretching out the period of extraction from the penultimate deposit; or depressing prices and opening the terminal deposit sooner. In the case of isoelastic demand, the latter effect outweighs the former and the monopolist will always invest

excessively early.

Gaudet and Lasserre (1988) model upfront exploration investment, which determines the size of the initial resoure stock. They find that, under isoelastic demand, the monopolist will not invest enough in exploration. This is because the monopolist does not capture the entire surplus from consuming the resource. Both the planner and the monopolist will set the marginal cost of developing a resource deposit to the marginal benefit, i.e. the scarcity rent; this will be lower for the latter, as the marginal social value of the stock exceeds the marginal private value. The monopolist cuts back *cumulative* supply, analogously to the static case.[19]

I will below present related results. Suppose that demand is such that lowering the quantity sold increases the share of revenue in gross surplus.[20] If demand is increasing over time, until some date T at which a substitute enters and makes the resource obsolete, the monopolist will tend to invest too early into opening a deposit. Furthermore, if investment costs are not fixed but associated with an expansion of maximum capacity, the monopolist will also have an incentive to invest in too much capacity.

These results are, in some ways, the mirror image of those in Gaudet and Lasserre (1988). In their paper, the period of extraction and the capacity were fixed (indeed, with capacity not binding), while stock was endogenous. In the present paper, the stock is fixed, but both the period of extraction and the capacity are endogenous. These quantities determine the degree of market power the monopolist has. The date of investment determines

[19]Without extraction costs, the scarcity rent for the social planner is just the resource price at the time of opening the deposit, while for the monopolist it is the marginal revenue.

[20]Linear demand satisfies this, while for isoelastic demand, of course, the share is constant.

the effective length of the period of extraction. As the quantity of the resource is fixed, a longer period implies a lower average rate of extraction. Further, increasing demand causes extraction to be postponed, so that any capacity constraints are hit at the very end of the extraction period. Higher capacities allow more to be extracted late in the deposit's lifetime, hence allowing extraction to be constrained early on.

For ease of exposition, I will focus on the special case of time-varying linear demand and a finite and given terminal date. This choice is motivated by the application to CO_2 storage in Section 3.3. I will also briefly comment on the isoelastic and infinite horizon cases, as well as on uncertainty with respect to either climate policy or the substitute entry date.

3.2.1 Single exhaustible deposit and set-up costs

Suppose that, at any moment $t \in [0, T]$, demand for an exhaustible resource is linear:[21]

$$p(q, t) = \overline{p}(t) - \xi q$$

This yields gross consumer surplus (the area under the demand curve)

$$CS(q) = \overline{p}q - \frac{\xi}{2}q^2 \tag{3.1}$$

Let the choke price $\overline{p}(t)$ grow at some rate $\gamma \in \mathbb{R}$ until the given terminal date T, following which it is zero forever:

$$\overline{p}(t) = \begin{cases} \overline{p}(0)e^{\gamma t} & \text{for } t \leq T \\ 0 & \text{for } t > T \end{cases}$$

[21] The results up to the characterisation of the optimal investment date (equation (3.6)) in fact hold for any downward-sloping demand curve.

The justification for this effectively finite time horizon is the arrival of some cheaper substitute which makes the resource unnecessary. This drastic assumption is made for simplicity. The results would not change qualitatively if the demand were assumed to die off e.g. smoothly yet rapidly; a peaking demand (over time) is the essential feature.[22] The increase in the choke price might reflect exogenous economic growth driving up demand.[23]

The decisionmaker (either the social planner or a monopolist) initially has a fixed stock of the resource S(0), which can be extracted costlessly. Before extraction begins, a fixed investment cost I has to be paid. For now, this investment allows the extraction and sale of the resource at any rate.[24] The decisionmaker discounts the future at rate ρ.

Social planner

The social planner maximises the discounted stream of total surplus; as resource extraction is costless, this equals gross consumer's surplus less investment costs:[25]

$$\max_{q(t),t^*} \int_{t^*}^{T} e^{-\rho t} \left(CS\left(q(t)\right)\right) \, \mathrm{d}t - e^{-\rho t^*} I$$

subject to $\dot{S} = -q$, $S \geq 0$; $q(t) = 0, \forall t < t^*$.

The problem is solved in two stages: by first optimising for a given investment date t^*; second, by choosing the investment date t^* which max-

[22] Of course, technology revolutions are very uncertain *a priori*. It might be more satisfying to interpret T as reflecting the resource owner's subjective expectation of such a revolution. I tackle uncertainty later.

[23] In the application in Section 3.3, the choke price will reflect an exogenously increasing carbon price.

[24] Capacity constraints are introduced in 3.2.5.

[25] In other words, firms do not play a role in the social optimum; the resource is simply allocated to consumers.

imises profits. The plan has to yield a positive net surplus for the investment to be undertaken at all.

Given t^*, the Hamiltonian for this problem is

$$\mathcal{H} = \overline{p}(t)q(t) - \frac{\xi}{2}q(t)^2 - \lambda_S(t)q(t)$$

and an optimal path has to satisfy the necessary conditions

$$\begin{aligned} \lambda(t) &= \overline{p}(t) - \xi q(t) \\ \lambda(t) &= \lambda(t^*)e^{\rho(t-t^*)} \\ \lambda(T)S(T) &= 0 \end{aligned} \tag{3.2}$$

with $\lambda(t)$ denoting the scarcity rent of the resource. The interpretation of the above conditions is straightforward. The first states that the marginal benefit of selling the resource has to, at all times, equal the scarcity cost of not being able to sell the same unit at some other moment. The second implies that the present value of the scarcity rent is constant, so that profits cannot be increased by shifting extraction between any two points in time. The final condition just requires that either the resource be fully used by up by the terminal date; or, if otherwise, that the resource is not scarce to begin with, so that demand will be fully satisfied at all times following t^*.

I will, for now, focus on situations in which scarcity bites, i.e. $\lambda(t) > 0$, $\forall t$. Denoting the final date on which extraction takes place by $\tilde{T} \leq T$, this implies

$$\int_{t^*}^{\tilde{T}} q(s)\,\mathrm{d}s = S(0) \tag{3.3}$$

It should be noted that, differentiating this with respect to the investment

date t^*, one obtains

$$q(t^*) = \int_{t^*}^{\tilde{T}} \frac{\partial q(s)}{\partial t^*} \, \mathrm{d}t + q(\tilde{T}) \frac{\partial \tilde{T}}{\partial t^*} \tag{3.4}$$

This just implies that a small delay in investment date requires the amount extracted in the initial period to be reallocated along the extraction schedule, with the final date of extraction potentially adjusted. Clearly any delay in investment can only be optimal if $\gamma > 0$. If demand is not increasing, the social planner can do no better by delaying, but in fact will end up worse off: any profits are delayed and restrictions imposed by the terminal date (if binding) will get worse.

From the necessary conditions, following investment the optimal extraction path is given by

$$q(t) = \xi^{-1} \left(\overline{p} e^{\gamma(t-t^*)} - \lambda(t^*) e^{\rho(t-t^*)} \right) \tag{3.5}$$

with $q(t) \geq 0$ required. For a given investment date t^*, the optimal extraction path can be obtained from (3.3) and (3.5). The path is continuous; if extraction ever falls to zero, it will stop forever. Hence, paths such that the time horizon does not bind ($\tilde{T} < T$) are characterised by $q(\tilde{T}) = 0$. This implies that the last term in (3.4) drops out, as either the extraction flow at $t = \tilde{T}$ is zero, or the terminal date is fixed ($\tilde{T} = T$).

The optimal date of investment is obtained by differentiating profits, given optimal extraction, with respect to t^*:

$$\begin{aligned} \frac{\mathrm{d}\pi(t^*, q(t))}{\mathrm{d}t^*} = \int_{t^*}^{\tilde{T}} e^{-\rho t} \frac{\partial CS(t, q(t))}{\partial q(t)} \frac{\partial q(t)}{\partial t^*} \, \mathrm{d}t + e^{-\rho \tilde{T}} CS(\tilde{T}, q(\tilde{T})) \frac{\mathrm{d}\tilde{T}}{\mathrm{d}t^*} \\ - e^{-\rho t^*} CS(t^*, q(t^*)) + \rho e^{-\rho t^*} I = 0 \end{aligned}$$

By using (3.4), the fact that the first derivative of consumer's surplus yields $\frac{\partial CS(t,q(t))}{\partial q(t)} = \lambda(t)$, and the fact that either $q(\tilde{T}) = 0$ or $\frac{\mathrm{d}\tilde{T}}{\mathrm{d}t^*} = 0$, this gives the first-order condition for investment:

$$q(t^*)\left(\frac{CS(t^*, q(t^*))}{q(t^*)} - \lambda(t^*)\right) \geq \rho I, \quad t^* \geq 0, \quad \text{C.S.} \tag{3.6}$$

The second-order condition becomes, after some manipulation,

$$\xi \int_{t^*}^{\tilde{T}} e^{-\rho(t-t^*)} \left(\frac{\partial q(t)}{\partial t^*}\right)^2 \mathrm{d}t \geq -\rho\lambda(t^*)q(t^*)$$

which is clearly always satisfied.

The first-order condition (3.6) is very similar to the result obtained by Fischer and Laxminarayan (2005) in a slightly different setting. The right-hand side is the benefit obtained by a short delay in investment: the avoided opportunity cost of the investment outlay. The left-hand side is the cost of delaying. The small amount of resource not extracted during this delay has to be reallocated along the planned extraction schedule. Doing this, the per-unit return is just the marginal surplus $\lambda(t)$; or, discounted back to t^*, $\lambda(t^*)$. Had these units been extracted in the initial period, they would have instead earned the average surplus $\frac{CS(q(t^*))}{q(t^*)}$. Of course, the foregone average surplus exceeds the marginal surplus, and so delaying has a cost.

For the linear demand curve, the optimal investment date is characterised by

$$q(t^*) = \sqrt{\frac{2\rho I}{\xi}} \tag{3.7}$$

Thus, the 'day-one' extraction rate, assuming an interior solution for the

investment date, does not depend on the level or rate of increase of the demand curve (Figure 3.1).[26] An increase in ρI raises the opportunity cost of investment, that is, the benefit of delaying. Hence investment occurs later; as there is a shorter period of extraction, the extraction rate has to also increase. An increase in ξ raises, for any $q(t^*)$, the difference between average surplus and price—thus increasing the marginal cost of delaying. Investment occurs earlier and extraction rates fall.

Proposition 12. Suppose resource depletion continues until the terminal date: $\tilde{T} = T$ and that the investment date $t_S \in (0, T)$. The social planner's optimum (with linear demand) is given by the unique $(t_S, \lambda_S(t_S))$ which satisfy

$$\overline{p}(t_S)\frac{e^{\gamma(T-t_S)}-1}{\xi\gamma} - \lambda_S(t_S)\frac{e^{\rho(T-t_S)}-1}{\xi\rho} = S(0) \tag{3.8a}$$

$$\frac{\overline{p}(t_S)-\lambda_S(t_S)}{\xi} = \sqrt{\frac{2I\rho}{\xi}} \tag{3.8b}$$

where the first equation is the resource constraint, and the second the first-order condition for investment.

Proof. In the text. Uniqueness of interior solution proven in Appendix 3.A. □

I will now consider some comparative statics of the optimal investment date.

Proposition 13. Supposing the optimal date of investment is given by an interior solution, it satisfies $\frac{\partial t^*}{\partial S_0} < 0$, $\frac{\partial t^*}{\partial I} > 0$. An increase in the discount

[26] For the linear demand curve, the difference between the price and (gross) consumer's surplus does not depend of the level of the demand curve, only on the slope (Figure 3.1).

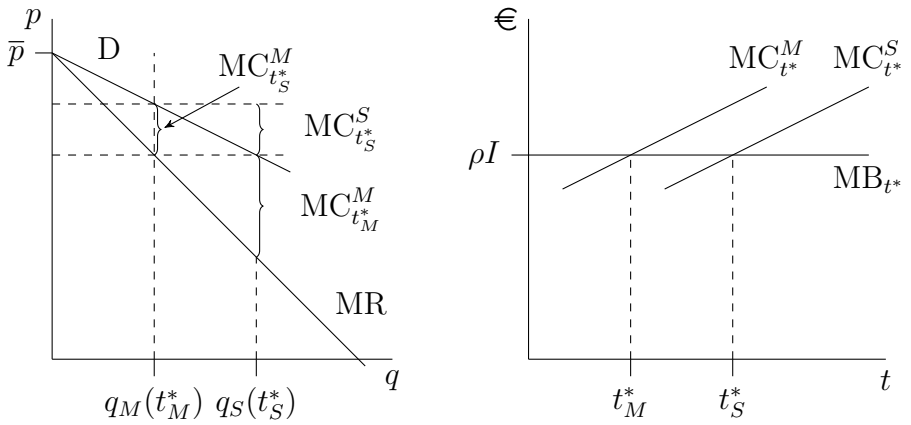


Figure 3.1: (*left*) The cost of a marginal delay in investment for the social planner is average surplus less price: $\mathrm{MC}^S_{t^*} = \frac{CS(q(t^*))}{q(t^*)} - p(q^*)$. With linear demand, this depends only on the slope of demand, not the level $\overline{p}$. The cost for the monopolist is price less marginal revenue: $\mathrm{MC}^M_{t^*} = p(q^*) - \mathrm{MR}(q^*) = 2\mathrm{MC}_{t^*}$. As the marginal benefit of delaying is the same for both, the monopolist's 'day one' extraction rate will be half that of the social planner. (*right*) For any given t, the monopolist's marginal cost of delay is higher than the social planner's; the marginal benefit is, for both, the opportunity cost of funds ρI.

rate has an ambiguous effect on the investment date: $\frac{\partial t^*}{\partial \rho} \gtreqless 0$.

Proof. In Appendix 3.A. □

The first two effects are quite obvious. A higher resource stock increases the resource available post-investment. With no extension of the period of sales, the momentary extraction rate would increase, at all times. In particular, the day-one extraction rate would rise, thus also raising the opportunity cost of delaying investment—and thus investment is optimally brought forward. Similarly, an increase in the investment cost increases the benefit of marginally delaying investment.

An increase in the discount rate, on the other hand, affects both the marginal cost and marginal benefit of delaying investment. It obviously increases the marginal benefit, as the opportunity cost of funds increases

with the discount rate. This implies that the optimal 'day-one' extraction rate rises. Other things given, this would imply that investment is optimally delayed (as otherwise the resource constraint is broken). However, post-investment, a higher discount rate also makes the resource extraction schedule more front-loaded: it is optimal to extract the resource faster. The faster decrease of the extraction rate may partially or fully offset higher day-one extraction, in terms of cumulative extraction, and may even more than offset it. Thus, for the resource constraint to be satisfied, the date of investment may have to be delayed or brought forward.

To see this more clearly, note that, following investment, the extraction rate changes according to

$$\begin{aligned}\dot{q} &= \frac{1}{\xi}\left(\gamma\overline{p} - \rho\lambda\right) \\ &= \gamma q - \frac{\rho - \gamma}{\xi}\lambda\end{aligned}$$

For $\rho = \gamma$, clearly $\dot{q} = \rho q$: this characterises the Hotelling extraction path, absent the twist due to the divergence between the rate of increase of the price and the discount rate. The second term characterises the twist: for $\rho > \gamma$, $\dot{q} < \gamma q$. An increase in ρ raises $q(t^*)$, and hence the Hotelling extraction path absent the 'twist'. However, it also increases the downward twist, undoing the effect of ρ on increasing $q(t)$—but only gradually.

It is challenging to obtain clear analytical results, but a combination of analytical and numerical work indicates that the net effect depends non-linearly on the resource stock. For very low stock levels, conditional on investment still yielding positive overall profits, the resource is extracted over a very short period. Optimal investment requires higher day-one ex-

traction. The higher ρ gradually twists the extraction rate down. However, this effect operates only on $\dot{q}$ and needs time to have an appreciable effect on q. With a low stock, the time horizon is short and the first effect dominates: investment is delayed in order to satisfy the resource constraint. For a medium resource stock, the time horizon is longer and there is more time for the front-loading 'twist' to operate, with extraction falling below the comparison case (i.e. the case with lower ρ) for an extended period of time. This means that the entire stock would not be exhausted unless the investment is brought forward. However, for very large stock levels, the scarcity rent may be very low; this weakens the twist (see the second term above) and, again, it may be optimal to delay investment.[27]

Numerical work also indicates that the effect of an increasing discount rate on investment timing may be non-monotonic in the discount rate itself. For example, for very high stock levels, an increase in the discount rate may initially bring investment forward. Further increases may, however, reverse this effect.[28,29]

3.2.2 Monopolistic extraction

Consider now the case in which a monopolist owns the stock of the resource. The monopolist maximises the stream of discounted revenues, less

[27] An example of this nonlinearity is given by the solution for $S_0 \in \{20, 200, 800\}$, with $I = 75, \rho = .0456, \gamma = .0306, \xi = .1, \overline{p}_0 = 1, T = 50$.

[28] Take the parameter value in the previous footnote, but $S_0 = 700$, $\rho \in \{.0306, .0406, .0506\}$.

[29] In the next subsection, I obtain the optimal investment date t_M for a monopolistic supplier. Analytical comparisons of effect of an increase in ρ on t_M, relative to t_S, are difficult, but numerical experiments indicate that the difference $t_S - t_M$ is decreasing in ρ; that is, as the discount rate increases, the difference between the investment dates diminishes and eventually reverses.

the investment cost:

$$\max_{q(t),t^*} \int_{t^*}^{T} e^{-\rho t} \left(p(q(t),t)q(t)\right) \, \mathrm{d}t - e^{-\rho t^*} I$$

subject to the same conditions as the social planner. The solution method proceeds exactly as above; I will only state the differences here. The FOC for maximising the Hamiltonian becomes

$$\lambda(t) = \overline{p}(t) - 2\xi q(t)$$

so that the optimal extraction path is

$$q(t) = (2\xi)^{-1} \left(p(t^*)e^{\gamma(t-t^*)} - \lambda(t^*)e^{\rho(t-t^*)}\right) \tag{3.9}$$

The first-order condition for investment becomes

$$q(t^*) \left(p(q(t^*)) - \lambda(t^*)\right) \geq \rho I \tag{3.10}$$

where the intuition is as before: delaying investment leads the need to extract at a higher rate following investment. For a monopolist, this depresses prices on the inframarginal units at all future dates; the discounted scarcity rent $\lambda(t^*)$ just equals marginal revenue, which is of course below the price $p(t^*)$. The monopolist also has a constant 'day one' extraction rate, lower than the social planner's:

$$q(t^*) = \sqrt{\frac{\rho I}{\xi}} \tag{3.11}$$

The intuition for the relationship between the two day-one extraction rates is shown in Figure 3.1 (left panel).

For extraction continuing until the terminal date, the monopolist's optimal solution is implicitly given by

$$\overline{p}(t_M)\frac{e^{\gamma(T-t_{\mathrm{M}})}-1}{\xi\gamma}-\lambda_M(t)\frac{e^{\rho(T-t_{\mathrm{M}})}-1}{\xi\rho}=2S(0) \tag{3.12a}$$

$$\frac{\overline{p}(t_M)-\lambda_M(t_M)}{\xi}=2\sqrt{\frac{I\rho}{\xi}} \tag{3.12b}$$

Note that the entire plan in conditional on the monopolist actually achieving positive profits following it. Otherwise the monopolist will simply remain inactive and make zero profits.

3.2.3 Comparison of monopolistic and optimal outcomes

If the choke price rises at the discount rate ρ, the elasticity of demand is constant along the optimal path (although clearly not, in general, for the demand curve) for both social planner and monopolist. Hence:

Lemma 3. For $\gamma = \rho$, both the monopolist and the social planner will follow the same Hotelling Rule:

$$\frac{\dot{p}_M}{p_M}=\frac{u'(q_S)\dot{q}_S}{u'(q_S)}=\rho,\forall t>t_i$$

for $i \in \{S, M\}$. Hence, for a given investment date $t_S^* = t_M^* = t^*$, the time profiles of extraction will be identical: $q_S(t) = q_M(t), \forall t > t^*$.

Proof. Immediate from (3.5) and (3.9), and from combining these with the resource constraint (3.3). □

Proposition 14. As long as the choke price grows at some rate sufficiently close to ρ, the monopolist will invest earlier than socially optimal:

$$\exists \epsilon > 0 : \text{for } \gamma \in (\rho - \epsilon, \rho], t_M^* < t_S^*$$

Proof. For $\gamma = \rho$, obtained by explicitly solving (3.8) and (3.12). As the systems defining the two equilibria are continuous, this will also hold for some $\gamma < \rho$. □

Under the given demand curve, as long as demand increases sufficiently quickly, the monopolist will find it profitable to invest excessively early. The motivation for this is to increase the length of the period over which the resource is extracted, in order to push up the resource price. If there is a terminal date which is binding for the social planner, the monopolist can only extend the extraction interval by investing too early.[30]

Consider the case $\gamma = \rho$.[31] Constant elasticity of demand along the optimal path means that the monopolist does not 'frontload' or 'backload' extraction, relative to the social planner. The optimal investment date is the point at which the cost, to the decisionmaker, of deferring investment (delaying getting the surplus, plus any decreases in the current value surplus) just equals the benefit (delaying payment of the set-up costs) (Figure 3.1, right panel).

[30]Suppose that demand is linear but the time horizon is not binding or infinite. This case requires $\gamma < \rho$, as otherwise it is optimal to spread extraction over an infinite interval of time. This complicates the solution (see footnote 31) and analytical comparisons of the two investment dates have failed to yield clear results. Numerical solutions to the model, over a large region of the parameter space, suggest that the monopolist will unambiguously invest later than the social planner.

[31]For lower γ, the intuition is complicated by the fact that demand is no longer isoelastic along the optimal paths, and so the monopolist will load extraction, over time, differently compared to the social planner.

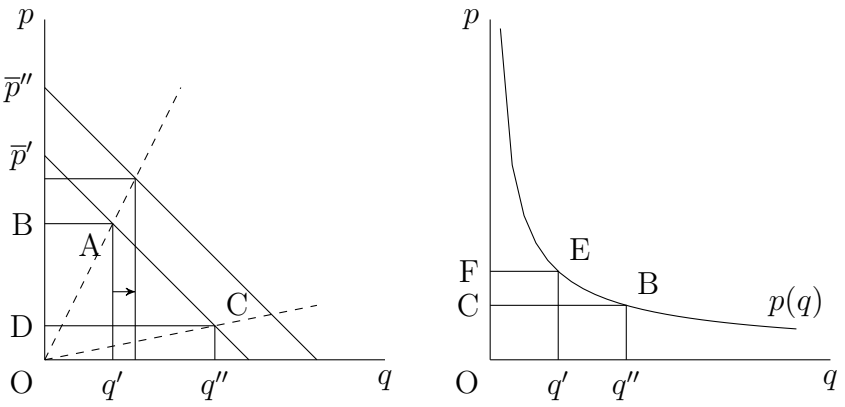


Figure 3.2: Surplus, revenue streams. (*left*) Linear demand, with $\gamma = \rho$. The extraction rate and price increase proportionally along a ray (through A) from the origin. An optimal plan just exhausts the stock by time T. The ratio of revenues to total surplus $\frac{\text{Oq'AB}}{\text{Oq'A}\overline{p}}$ remains constant, for a given initial rate of extraction. Delaying investment, however, increases initial $q(t^*)$ and so reduces this ratio to $\frac{\text{Oq"CD}}{\text{Oq"C}\overline{p}'}$. (*right*) With isoelastic demand, the monopolist's share of total surplus is unaffected by $q(t^*)$: $\frac{Oq''BC}{\int_O^{q''} p(q)} = \frac{Oq'EF}{\int_O^{q'} p(q)}$.

The monopolist captures a share $\frac{pq}{CS}$ of the total surplus as revenues. For the particular case we are considering, as both the choke price and extraction rate increase at the rate ρ, this is constant given an investment date:

$$\frac{pq}{CS} = \frac{\overline{p}_0 - \xi q_0}{\overline{p}_0 - \frac{\xi}{2} q_0}$$

where it is understood that q_0 only indexes the level of the extraction schedule; of course extraction is only positive after the investment date. Notice that the fraction of surplus captured decreases with q_0. Earlier investment implies a lower extraction schedule, and thus a higher fraction of total surplus captured (Figure 3.2, left panel).

For the planner, the cost of a short delay is given by

$$\begin{aligned}\mathrm{COST}^{\mathrm{SP}}(t^*) &= -\frac{\mathrm{d}}{\mathrm{d}t^*}\left(\int_{t^*}^{T} e^{-\rho t} CS\,\mathrm{d}t\right) \\ &= -\int_{t^*}^{T} e^{-\rho t}\frac{\mathrm{d}CS}{\mathrm{d}q(t)}\frac{\mathrm{d}q(t)}{\mathrm{d}t^*}\,\mathrm{d}t + e^{-\rho t^*}\,CS|_{t=t^*}\end{aligned}$$

which is positive for the particular demand curve we are considering.

With a constant $\frac{pq}{CS}$, the same cost for the monopolist can be written[32]

$$\begin{aligned}\mathrm{COST}^{\mathrm{MON}}(t^*) &= -\frac{\mathrm{d}}{\mathrm{d}t^*}\left(\int_{t^*}^{T} e^{-\rho t}\frac{pq}{CS} CS\,\mathrm{d}t\right) \\ &= \frac{pq}{CS}\mathrm{COST}^{\mathrm{SP}}(t^*) - \int_{t^*}^{T} e^{-\rho t}\frac{\mathrm{d}(pq/CS)}{\mathrm{d}q_0}\frac{\mathrm{d}q_0}{\mathrm{d}t^*} CS\,\mathrm{d}t\end{aligned} \tag{3.13}$$

for any given t^*. Suppose the monopolist considers investing at the socially optimal date t^*_{SP}. The cost of delay is smaller due to the monopolist capturing a smaller fraction of the total surplus (first term). This effect is very similar to that investigated by Gaudet and Lasserre (1988). However, it is larger due to the delay leading to this fraction falling (second term). With linear demand, the latter effect outweighs the former and the monopolist faces a higher cost of delaying than the social planner does. The benefit of delay, the opportunity cost of investment funds, is the same for both the social planner and the monopolist. The monopolist will hence want to bring investment forward.

Robustness to alternative assumptions

The result of a monopolist investing excessively early is counterintuitive. I now consider the robustness of this result to the assumptions made.

[32] Write revenues $pq = \frac{pq}{CS}CS$, differentiate, and use the expression for $\mathrm{COST}^{\mathrm{SP}}$.

Non-linear demand. It is clear that the result above follows from the monopolist being able to capture a higher share of total surplus by cutting back supply. Consider briefly the case of isoelastic demand with a finite terminal date (Figure 3.2, right panel). In this case, for a given investment date, the monopolist and planner again follow the very same extraction paths. Further, $\frac{pq}{CS}$ is again constant; it is constant along the entire demand curve! Equation (3.13) still holds, but the second term of the right-hand side is zero. In this case, the cost of a marginal delay, for a given day one extraction rate, is smaller for the monopolist than for the social planner. The monopolist thus chooses to invest later. These two polar cases illustrate the mechanism leading to the monopolist investing early in the case of rising linear demand. For other types of demand, any demand schedule in which $\frac{pq}{CS}$ decreases with quantity supplied would involve some degree of the effect incentivising early investment for the monopolist.

Exhaustibility. Next, consider the case in which the resource is plentiful: $\lambda_S = \lambda_M = 0$. It is straightforward to observe from (3.7) and (3.11) that this implies $\overline{p}(t_S^*) < \overline{p}(t_M^*)$, i.e. $t_S^* < t_M^*$. This is obvious: when scarcity does not play a role, a change in the investment date does not affect the day one extraction rate nor, hence, the share of surplus the monopolist appropriates. The only effect at play is the monopolist's appropriating a lower share of the rents, and thus the monopolist optimally delays investment. Thus, exhaustibility of the resource stock is crucial to the result.

Policy uncertainty. I now consider the effect of uncertainty regarding the persistence of climate policy on optimal CCS investment. Suppose that there is a random event which, on arrival, kills off the carbon price immediately. This might occur because of a breakdown of the international

policy regime. The uncertainty might also reflect a possibility that the technology revolution may occur before the terminal date T. For brevity, I will only refer to 'policy uncertainty'. I assume that the event arrives as a Poisson process, with an arrival rate of π.

Lemma 4. Under policy uncertainty, given a small enough $\pi > 0$, the optimal investment dates are given by (3.8) (social planner) and (3.12), with the discount rate ρ replaced by $\tilde{\rho} \equiv \rho + \pi$.

Proof. In Appendix 3.A. □

In other words, the possibility of storage capacity becoming worthless leads the decisionmaker to utilise a higher effective discount rate. This leads to a frontloading of extraction, compared to the case $\pi = 0$: the expected future value of the resource falls because of the uncertainty, and the seller wants to sell it faster. Uncertainty also affects the investment choice. The marginal cost of delay is modified due to the day-one extraction rate changing. The marginal benefit of delaying is also higher: in addition to saving the opportunity cost of funds ρI, the decisionmaker also gets the option value of the funds πI. The latter term reflects the marginal benefit from the recognition that the policy breakdown may in fact occur during the marginal delay, thus allowing the decisionmaker to avoid making the costly investment at all.

Proposition 15. A marginal increase in the degree of policy uncertainty π reduces the welfare of the planner and the profits of the monopolist. Conditional on investment still being profitable, the effect on investment timing is ambiguous: $\frac{\partial t_S}{\partial \pi}, \frac{\partial t_M}{\partial \pi} \gtreqless 0$.

Proof. Consider a marginal increase in uncertainty from π to $\pi + \mathrm{d}\pi$. Denote the corresponding values by V^{π} and $V^{\pi+\mathrm{d}\pi}$. Suppose there is path of controls which yields $V^{\pi+\mathrm{d}\pi} > V^{\pi}$. Then this control path will yield the same flow of current value profits under π, but these would be discounted by less (as the risk of policy breakdown is lower under π). Hence V^{π} cannot have been the optimal value under π. The second part of the proof follows directly from Lemma 4 and Proposition 13. □

An increase in policy uncertainty will increase the front-loading of the extraction schedule, increasing $q(t^*)$ for any given investment date. This increases the difference between the average and marginal welfare (profit for the monopolist), increasing the cost of delay for any t^*; but the benefit of delaying, i.e. the opportunity cost of funds, is similarly increased, with the net effect ambiguous (as explained in Proposition 13 and the discussion following it). The effect of uncertainty may also be non-monotonic, with small levels of uncertainty bringing investment forward, and further increases delaying it. Of course, an increase in uncertainty may lower expected revenue below the investment cost, in which case investment is no longer profitable. Also, a high degree of uncertainty may imply that the resource is exhausted before the terminal date.

Terminal date. Finally, suppose that demand is linear, choke price rising at rate ρ, but that demand does not vanish immediately at the terminal date T. Instead, the technology revolution which makes the resource obsolete arrives as a Poisson process at date $\tilde{T} \geq T$: that is, as in the policy uncertainty case, again with arrival rate π, but only after some date

T. The demand function is

$$\overline{p}(t) = \begin{cases} \overline{p}(0)e^{\gamma t} & \text{for } t < \tilde{T} \\ 0 & \text{for } t \geq \tilde{T} \end{cases} \tag{3.14}$$

The monopolist may still invest inefficiently early:

Proposition 16. Let the choke price rise at the discount rate ρ, and let demand follow (3.14). Assume that, for $t \geq T$, the stochastic technology revolution arrives as a Poisson process with arrival rate π. Then $t_M^* < t_S^*$, provided that π is sufficiently high.

Proof. In Appendix 3.A. □

In other words, the results above do not depend on the hard, deterministic boundary of the terminal date. A stochastic boundary may suffice to yield similar results, and I conjecture that a deterministic but smooth contraction of demand will give similar results, provided the contraction is sufficiently rapid.

3.2.4 Optimal policy with commitment

I will now consider the ability of the regulator to achieve the socially optimal outcome by committing to taxes on resource sales and on set-up investment. An ad valorem tax on the resource, $\theta(t)$, has to motivate an efficient extraction path following investment.[33] A unit tax on investment, τ_I, can be used to ensure that the resource stock is opened up at an optimal date. Negative taxes imply subsidies.

[33]The solution is different if the regulator sets a time-varying unit tax on resource sales.

Following investment, the monopolist now faces the same inverse demand curve but only gets a share $1 - \theta(t)$ of the revenue flow. The cost of investment inclusive of taxes is $I + \tau_I$. The taxes can be characterised further:

Proposition 17. If the social optimum satisfies $p(t^*)S_0 \geq 2I$, the regulator can obtain the efficient outcome with the monopolist's profits being at any weakly positive level. If the condition does not hold, full nationalisation is the only way to obtain the efficient outcome.

If the rate of change of the choke price is strictly less than (equal to) the discount rate ρ, the efficiency-inducing ad valorem resource tax rises (does not change) over time:

$$\gamma < \rho \Rightarrow \dot{\theta} > 0$$
$$\gamma = \rho \Rightarrow \dot{\theta} = 0$$

The level of this tax can be freely chosen, as long as $\theta(t) \leq 1$ for all t. The optimal investment tax depends on this level, and is characterised by

$$\tau_I = \big(1 - 2\theta(t^*)\big)I$$

Proof. In Appendix 3.A. □

If resource sales are not taxed, the tax on investment should just equal the investment cost again; this increases the opportunity cost of the funds required for investment, and encourages the monopolist to delay investment. If the initial resource tax is at 50%, investment should be neither subsidised nor taxed. As θ tends to one, i.e. as the regulator appropriates almost all of the potential resource revenues, investment should be

subsidised at a rate approaching 100%; this would be equivalent to a 'nationalisation' of the sector. These tax (or subsidy) instruments allow any distributional outcomes to be satisfied; the regulator can effectively choose any level of (weakly positive) profits it allows the monopolist to obtain.

3.2.5 Set-up costs with capacity constraints

In the previous section, set-up costs have been assumed independent of the quantity of the resource sold. I will now relax this assumption and consider the case in which an investment gives the resource owner the ability to extract and sell the resource up to a chosen capacity constraint. The resource owner thus has an extra choice variable. Furthermore, there may be an incentive to make several investments: if there is increasing demand for the resource, it may be optimal to invest first in a small amount of capacity and then increment this later with additional investments. These investments could describe investment into rigs and infrastructure, or, as in the next section, into the large transport pipelines which would be required to transport the pollutant to storage fields in the North Sea.

For simplicity, consider the case $\gamma = \rho$ and linear demand. Suppose the resource owner makes n investments. These are described by the set of investment times and capacities $\{(t_1^*, \overline{q}_1), \ldots, (t_n^*, \overline{q}_n)\}$. The capacity constraint at time t is now given by

$$q(t) \leq Q(t) \equiv \sum_{\{j:t_j^* \leq t\}} \overline{q}_j$$

To focus on interesting cases, I will assume that there are some fixed costs

to investment.[34] I will also assume costs are weakly convex in capacity. Thus, in current value terms, the investment cost is given by $C(\overline{q}), C(0) > 0, C' > 0, C'' \geq 0$. Thus, the social planner's problem is to solve

$$\max_{q(t),\{(t_i^*,\overline{q}_i)\}} \int_{t^*}^{T} e^{-\rho t} \left(CS\left(q(t)\right)\right) \, \mathrm{d}t - \sum_i e^{-\rho t_i^*} C(\overline{q}_i)$$

subject to the resource and capacity constraints.

For a given investment schedule, the social planner's first-order condition for extraction now becomes

$$p(q) \geq \lambda, \quad q \leq \overline{q}, \quad \text{C.S.} \tag{3.15}$$

coupled, of course, with the associated non-negativity constraint (given my assumptions, these will never hold). Other first-order conditions and the resource constraint remain unchanged. The resulting optimal extraction path will have a discontinuous shape, with capacity caps clipping off a part of the curve (Figure 3.3).

Proposition 18. For ease of notation, denote $t_{n+1}^* \equiv T$. Starting with $t = t_1^*$, the optimal path consists of $2n$ stages, divided by the dates of investment, and in between these dates, the dates at which the current quantity gap starts binding $\{\tilde{t}_i\} \equiv \{\min t \, : \, \xi^{-1}(\overline{p}(t) - \lambda(t)) = Q(t_i^*)\}$. These latter dates satisfy $\tilde{t}_i \in [t_i^*, t_{i+1}^*)$.

[34] Otherwise it would be optimal to make an infinite number of infinitely small investments.

The social planner's optimal investment schedule is characterised by

$$\Delta_{t_i^*}\left[CS(q) - p(t_i^*)q\right] = \rho C(\overline{q}_i), \quad t_i^* \geq 0, \quad \text{C.S.} \tag{3.16a}$$

$$\sum_{j\in[i,n]} \int_{\tilde{t}_j}^{t_{j+1}^*} e^{-\rho t}\left(p(Q(t)) - \lambda(t)\right)\,\mathrm{d}t = e^{-\rho t_i^*} C'(\overline{q}_i) \tag{3.16b}$$

in which $\Delta_t[X(s)] \equiv \lim_{s\downarrow t} X(s) - \lim_{s\uparrow t} X(s)$, i.e. the upward discontinuous jump in variable X at time t. The optimal number of investments $n \in \{0, 1, 2 \dots\}$ is chosen so that welfare is maximised.

Proof. In Appendix 3.A. □

In words, investment at t_i^* is followed by two stages: first, the new quantity cap may not be binding, but from $\tilde{t}_i$, it will. The first stage may be degenerate, but the second always exists: the next investment will not be made until the capacity constraint becomes binding—otherwise the investment could have been profitably delayed. The intuition for the optimal dates is as before: a marginal delay in investing has the benefit of saving the opportunity cost of investment, but extends the period of capped sales, which has a cost in terms of revenues (net of the scarcity rent, i.e. taking into account that the resource can be sold in other periods; equation (3.16a)). As the quantity will only ever jump up, this cost will be positive (Figure 3.4). The marginal benefit of investment quantity is just given by the cumulative marginal welfare due to being able to sell more resource in all future periods when the quantity cap binds, net of the scarcity rent (as these units can be reallocated to be sold in periods in which the cap is not binding) (equation (3.16b)). This marginal benefit is always positive (by (3.15)), and has to equal the marginal cost of capacity.

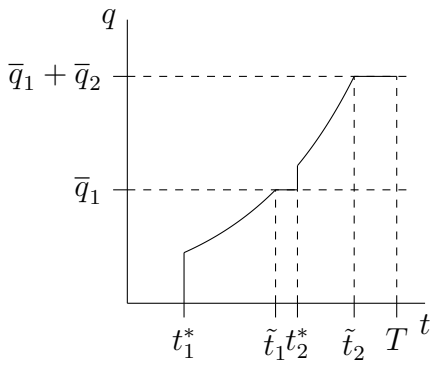


Figure 3.3: The optimal path consists of a series of investments. After an investment, the quantity sold may be below the cap; it will reach the cap before the next investment.

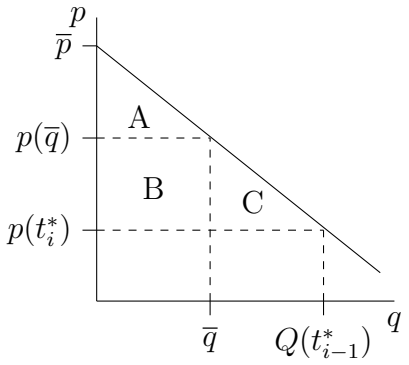


Figure 3.4: The gross marginal cost of extending the next investment is ABC, but A can be recouped by reallocating sales. Net marginal cost is BC > 0.

For the monopolist, the problem changes in an analogous manner. The outcome is similar, with the FOC (3.15) changing to

$$MR(q) \geq \lambda, \quad q \leq \overline{q}, \quad \text{C.S.} \tag{3.17}$$

The FOCs for investment date and quantity become, respectively,

$$\Delta_{t_i^*}\Big[(p(q) - \lambda(t_i^*))q\Big] = \rho C(\overline{q}_i) \tag{3.18a}$$

$$\sum_{j\in[i,n]} \int_{\tilde{t}_j}^{t_{j+1}^*} e^{-\rho t}\left(MR(Q(t)) - \lambda(t)\right) \mathrm{d}t = e^{-\rho t_i^*} C'(\overline{q}_i) \tag{3.18b}$$

where the LHS are positive, by positivity of marginal revenue and equation (3.17), respectively.

Proposition 19. Suppose demand is linear, investment costs are weakly convex in capacity ($C'' \geq 0$), that a single investment is optimal for both the monopolist and the social planner ($n_M = n_S = 1$), and that the resource is scarce for both ($\lambda_S, \lambda_M > 0$). Then the monopolist either invests too

early ($t^*_{1,\mathrm{M}} < t^*_{1,\mathrm{M}}$), or invests in too much capacity ($\overline{q}_{1,\mathrm{M}} > \overline{q}_{1,\mathrm{S}}$), or both.

Proof. In Appendix 3.A. □

Thus, the counterintuitive result obtained in the absence of capacity constraints may still hold when such constraints are included. A further counterintuitive result is added: with linear demand, there is a tendency for the monopolist to also invest too much. At least one of these effects will always be observed. The intuition for the latter effect is that, when only a single investment is made, higher capacity allows the monopolist to sell more when the cap is reached. This occurs close to T, allowing the monopolist to raise prices early in the extraction period, yet still sell the entire stock. As in the case with unlimited capacity, the monopolist's share of early surplus increases . With multiple investments, the entire extraction path changes and it is difficult to make claims at this level of generality. In the next section, I will show numerically that, with multiple investments, the monopolist will tend to overinvest; and will tend to make at least some investments excessively early.

3.3 Investment in CCS infrastructure

I will now sketch out an application of the above model to carbon capture and storage. I will consider the particular scenario of transporting CO_2 emissions from northwestern Europe to the North Sea by large pipelines. Such a scenario has been widely discussed in the policy literature on CCS (Haszeldine, 2009; Neele et al., 2010; Morbee et al., 2012). The dominant players would be Norway and the UK, particularly were the storage led by EOR projects; in the long term, should projections of potential stor-

age in saline aquifers be borne true, Norway in particular might hold 25% of European storage capacity, and a much larger share were onshore storage ruled out for whatever reason (Neele et al., 2011b; Vangkilde-Pedersen et al., 2009b).[35]

There is a potential need to construct large trunk line networks out to the North Sea, in particular if onshore storage is not usable. The detailed routing of such networks varies between existing studies; Haszeldine (2009) and Neele et al. (2010) envision a connected network for the entire region, while Morbee et al. (2012) find the optimal trunk lines to be more fragmented. It is apparent the debate over the spatial structure of such a backbone network is not settled, and that the existence of a single connected network for the entire region cannot be ruled out.

I will focus on a very simplified spatial structure in order to consider decentralised network formation and the dynamic effects of market power. Quite clearly, the various actors (the storage sites and the emitting firms) are under separate, sovereign jurisdictions. Under a single jurisdiction, a regulator could implement the first-best solution. Here, I assume there is no central coordination among the emitting firms, i.e. that the emitters behave competitively. I will first extensively consider monopolistic supply of storage capacity. In the last section, I will also consider duopolistic storage.[36] Moreover, as the emissions are produced by a large number of firms, the assumption of ruling out bilateral contracts is also appropriate. Of course, the results are contingent on the assumption that cooperation between various countries fails. The only other study to consider imperfect

[35]However, the Netherlands might play a non-negligible role given its share of depleted gas fields in the southern North Sea.

[36]Of course, the CO_2-exporting countries could also try to cartelise CO_2 supply. I leave strategic behaviour on the polluter side for future research.

coordination is Morbee (2012), who adopts a cooperative game theoretic approach in a static framework.

At the heart of the climate change problem lies a market failure—the absence of markets for the public bad of climate change. Hence, in the decentralised problem, appropriate incentives (e.g. carbon taxes or a cap-and-trade scheme) are required for the agents to undertake CCS at all. I take climate policy as exogenous as northwestern Europe cannot substantially affect the path of atmospheric CO_2 concentrations.

The assumption of exogenous climate policy implies that there exists a regulator with the desire to implement such incentives, and the power to enforce them. This contrasts with the absence of a regulator for CCS policy. Such asymmetry is plausible, however. There might exist sufficient political pressure to get an overall climate policy regime in place, but not enough to implement detailed regulation on how it should be complied with (for which the regulator would also require better information). Climate policy also turns subsurface storage sites into valuable assets, and there could be organised lobbying against the regulator interfering with the management of such assets. I assume full compliance with the climate policy, and will not consider whether the agents might want to withdraw from such a policy regime.

The North Sea has major storage potential, with UK and Norwegian depleted oil and gas fields providing capacity for up to 10.5 $GtCO_2$ and large saline aquifers potentially multiplying this capacity several times over (Vangkilde-Pedersen et al., 2009a).[37] Some of the oil and gas fields have

[37]Some CCS experts have pointed out that commonly used statistical methods to assess regional storage capacities tend to overestimate actual, final capacities by one or two orders of magnitude (Spencer et al., 2011). These issues affect storage in saline aquifers; the geology of the hydrocarbon fields has been mapped in detail as part of

legacy infrastructure in place; others are still producing, and CO_2 injections could begin as enhanced oil recovery operations.[38] The geology of the oil and gas fields is also already well understood.

Major clusters of emission sources lie relatively close, both in the UK and in the Rhine valley. Onshore storage of these emissions would be cheaper, assuming saline aquifer storage turns out to be feasible. Were this assumption to fail, onshore geological storage capacity would fall substantially. Moreover, public concerns over health and safety, possibly the most important obstacle to CCS (House of Commons Science and Technology Committee, 2012), may rule out onshore storage. As an example, a major pilot storage project in Barendrecht in the Netherlands was cancelled in 2010 due to public outcry (Feenstra et al., 2010), and an exploration project in Beeskow is Germany has met similar public opposition (Dütschke, 2011).

I will now map the model of the previous section into a scenario of carbon storage in the North Sea. It should be emphasised that the assumptions (no onshore storage, and no saline aquifer storage capacity offshore) tend to err on the pessimistic side, and the exercise below is entirely conditional on these assumptions holding. Offshore storage in saline aquifers would substantially reduce the threat of scarcity of storage, with the conclusions reported below for cases without scarce storage holding. Were onshore saline aquifer storage feasible, the resulting network structure, and the decentralised pipeline investment problem, would be very different: in particular, the onshore network structure would be more fragmented, con-

the oil and gas extraction operations, and estimates of overall storage capacity can be estimated fairly accurately.

[38] Dooley et al. (2010) argue that while EOR projects may help advance the relevant technologies, in the US they are unlikely to be important in terms of actually storing meaningful quantities of carbon.

sisting of many individual, small pipelines. This alternative problem would have to be analysed using a much more complicated model.

Spatial structure. Consider an industrial cluster emitting CO_2 ('Source') and a site suitable for geological storage of the pollutant ('Sink'). Considering the Sink to be Norway and the Source to be the industrial agglomeration in northwestern Europe, I will assume the sites are separated by a distance of roughly 1200 km. This corresponds to the pipeline from the Ruhr to the Utsira formation in Haszeldine (2009).

Timing. I take the base year to be 2020. Emissions will continue for $T = 50$ years, after which there is an overnight transition to clean energy, resulting in capturable carbon emissions falling to zero. An alternative case considers a longer time horizon with $T = 80$. All agents discount the future at the common rate $\rho = .03$.

Carbon pricing. An exogenous climate policy governs both nodes, mandating a carbon tax $\overline{p}(t)$ per unit of carbon emitted. The carbon price rises at the rate $\gamma = \rho$. The fact that the rate of increase equals the discount rate implies that the cumulative amount of carbon emitted is capped, as suggested, for example, by Allen et al. (2009). The carbon price is not paid for stored emissions as these do not contribute to climate change.

This policy is assumed to be optimal, in the sense that the carbon tax corresponds to the social cost of carbon emissions, allowing direct comparisons of social welfare versus private profits: attention can then be focused on the efficiency of the CCS process itself. I consider two cases: one with a low initial (2020) carbon price (€73/tCO_2) and one with a high initial carbon price (€146/tCO_2).[39]

[39]By the year 2030, the prices are fairly close to the projections in the two scenarios in IEA (2008). The initial values may seen implausibly high for 2020.

CO_2 capture. Instead of being emitted to the atmosphere, carbon emissions can be captured, transported and buried underground up to the capacity of the pipeline linking the Source to the Sink.[40] Capture costs are incurred in this process. Provided the carbon price is higher than the marginal capture costs, capturing and storing carbon is (weakly) welfare-improving (after any pipeline costs have been sunk).

The Source is a representative polluter, composed of a large number of small actors, all faced with different capture cost curves. The aggregate capture cost curve $C\left(q\left(t\right)\right)$ is strictly convex, and for convenience specified quadratic: $C(q) = \frac{\xi}{2}q^2$. The representative firm faces a choice between emitting the marginal unit and paying the carbon tax $\overline{p}(t)$, or capturing the marginal unit at marginal cost $\xi q(t)$ and paying the Sink the asking price $p(t)$ to dispose of it. Equalising these for optimality yields the inverse demand curve for carbon storage:

$$p(t) = \overline{p}(t) - \xi q(t)$$

The key assumption here is that of increasing marginal cost. This could result from a combination of marginal costs varying across firms and marginal costs varying within firms. Variation across firms exists, particularly if industrial plants are included as sources alongside power-generating facilities.[41] It is more difficult to ascertain the degree to which individual installations are able to adjust at the intensive margin (the capture rate), and how costs would vary with respect to this. Most engineering-economic studies assume a fixed capture rate and report costs given this. A few stud-

[40] I abstract from questions of pipeline and reservoir leakage.

[41] See e.g. Damen et al. (2009), who provide an aggregate CO_2 supply curve for the Netherlands.

ies do consider variation in the capture rate, either in terms of switching off capture and instead venting the carbon into the atmosphere (Chalmers et al., 2009), or in terms of altering the fraction of CO_2 captured (Wiley et al., 2011; Ziaii et al., 2009; Brunetti et al., 2010). These studies indicate that there does, indeed, exist an intensive margin for captured CO_2, even if they provide no information on costs or elasticities of supply. As the present paper is concerned with margins aggregated at the level of a large industrial cluster, I will assume that the linear marginal cost curve is a reasonable first step.[42]

I implicitly assume that there is no fixed investment into capture equipment: instead, these costs are subsumed into the operating costs. This also includes the costs of any feeder pipelines to connect to the backbone. In this sense, the detailed infrastructute investment is assumed to be 'below the resolution' of the model. This is a major assumption, made in order to simplify the model. In particular, the (related) issues of network externalities and investment hold-up arise here. I will briefly discuss these issues here.

The point of this chapter is to consider economies of scale with respect to pipeline capacity. Such scale economies also apply to small-scale networks, such as the local feeder pipelines connecting CO_2 sources to the backbone. These local networks are likely to consist of smaller trunk lines

[42]Note that I have not explicitly considered the energy penalty. This is because the storage demand curve aggregates the intensive and extensive margins in a stylised fashion. Were all adjustment to happen at the intensive margin, the penalty could be interpreted as follows. Suppose that the additional energy required is linear in the captured quantity q, so that emissions are $E(q) = E_0 + \nu q$, where E_0 are the emissions when fully vented (CCS switched off). Suppose also that emissions are linear in fuel consumption, and that the fuel price c is rising at the discount rate. Then the model holds as is, but we must interpret $\overline{p} = p_{CO_2}(1 - \nu) - c$; that is, the carbon price is expressed net of the energy penalty and fuel price.

leading to industrial agglomerations, with feeder pipelines connecting to individual plants producing CO_2. The presence of a local trunk line would of course make it much cheaper for an individual firm to get access to the large backbone, provided the trunk line has spare capacity; this constitutes a positive externality. There is thus scope for coordination between the individual installations to coordinate their investments. Firstly, local networks should be constructed so as to exploit any scale economies. Secondly, any externalities related to the construction of network arms should be internalised. To the extent that plants fail to achieve such coordination, the costs of capturing and transporting CO_2 to the backbone would be higher, so that the CO_2 storage demand curve would be lower. Interpreting network effects in this simplified sense, the qualitative results should be unchanged. In any case, coordination should be possible, either by joint ventures, or by regulation at local or national level.[43]

Incorporating capture investment would introduce investment hold-up issues, as a marginal plants might face the risk of not recouping investment costs. Dealing with such issues explicitly would require including individual polluting firms in the model. These firms might be heterogenous with respect to e.g. their variable cost of capture, or their investment costs. The marginal plant would have to be able to capture sufficient surplus to just cover their investment cost. This could occur, for example, if the marginal (variable) capture cost curve were sufficiently steep. This rising marginal cost might arise on the level of the individual plant, but more likely across several plants, in which case coordination might be again required.[44]

[43]The assumption that the sector producing CO_2 behaves non-strategically implies that the sector cannot be fully organised or regulated at the market level.

[44]With a constant unit capture cost, varying across plants, the CO_2 storage supplier would always seek to extract all surplus from the marginal firm, i.e. the one with the

Calibrating the cost curve 'bottom-up' is a challenging exercise, to put it mildly, involving projections of economic growth, the change in the composition of energy infrastructure, and so on. I will take the shortcut of using projections from IEA (2008). In particular, they provide projections of CCS-equipped infrastructure in the OECD power-generating sector in 2030 at two different carbon price levels: 78 GW (all coal-fired) at a carbon price of \$90/tCO_2, and 170 GW (120 GW coal-fired, the rest gas turbines) at a carbon price of \$180/tCO_2. To obtain the equivalent figure for the region in question, I assume the generating capacity is divided between countries in proportion to their 2010 electricity generation (OECD, 2012). I pick the countries in the region; for UK, Germany and France I scale emissions in proportion to the total quantity of CO_2 emissions originating broadly in the relevant region (using the European Pollutant Release and Transfer Register database). 96% of German, 84% of UK and 55% of French power generation CO_2 emissions fall in the relevant region; with northwestern Europe accounting for roughly 14% of total OECD electricity generation. This implies that, at the lower carbon price, the region would contain 14 hard coal power plants with CCS (800 MW each). At the higher carbon price, there would be an additional 7 hard coal power plants and 14 combined-cycle gas turbines (500 MW each). I calculate the carbon captured for both price levels (ZEP, 2011a)[45] to obtain $\xi = 1.6$ and I assume CCS demand is stationary at this level for the entire time period.

CO_2 transport and storage. The sink's upfront capital costs are

highest capture cost. Thus, the marginal firm would never invest and the equilibrium would unravel.

[45]Note that the carbon emissions avoided needs to be transformed into carbon captured; due to the energy penalty imposed by capturing carbon, i.e. the need to burn additional fuel to power the capture process, these two quantities are not equal.

just the pipeline capital costs. Pipeline investment cost is given by the formula $C(\overline{q}) = \alpha_1 + \alpha_2\overline{q}$ per kilometer, a linear relationship as estimated from a number of previous studies by Morbee et al. (2012) (with $\alpha_1 = 1.066$, $\alpha_2 = .038$ after adjusting by the terrain factor for offshore) This implies that a 20 Mtpa pipeline would cost €1.9m per kilometer. This is slightly higher than the cost estimated by ZEP (2011d). The capital costs of storage facilities (\$6 per tCO_2) are calculated for the volume stored between two consecutive pipeline investment dates. ZEP (2011c) uses a central assumption of 66 Mt capacity per field, with a 5 $MtCO_2$ per annum injection rate. This would imply field lifetime of 13 years, which is mostly of similar order of magnitude as the optimal investment intervals given in the present model.

I include constant, fixed operating costs incorporating the operating costs of transport and storage (\$1 and \$4 per tCO_2, respectively). Note that I am implicitly assuming a very long pipeline lifetime, as there is no depreciation of installed capacity. This assumption might be contested in the context of the longer time horizon.[46]

It should be noted that operational considerations impose minimum and maximum bounds on the volume transported through a pipeline at a given moment: sufficient pressure is required to force CO_2 into the supercritical phase for transport. Hence, the underutilisation of capacity may

[46]ZEP (2011d) assumes capital costs are depreciated over a 40-year period, and includes maintenance costs in its overall cost estimates. Supercritical CO_2 can be highly corrosive in the presence of impurities and water. Careful purification and drying of the gas stream prior to transport is sufficient to eliminate the problem of carbon steel corrosion in low-pressure pipelines; some EOR-related pipelines have been in operation for nearly 30 years without corrosion-related problems. ZEP (2011d) assumes purity rates stated to be sufficient to avoid corrosion issues; these fall between the requirements for the Canyon Reef EOR pipeline (IPCC, 2005) and those adopted by Kinder Morgan, the largest U.S. pipeline operator (Cole et al., 2011). Were the purity assumptions relaxed, high-pressure CO_2 pipelines could face more severe problems (Cole et al., 2011).

imply that CO_2 is temporarily stored at source until sufficient a quantity has been accumulated for transportation. Alternatively, as the results imply transport links of magnitude which cannot be constructed as a single pipeline, but rather as several lines running side by side, it may be that initially some of these component pipelines are not used at all times.

Storage capacity. The sink initially has a fixed stock of storage space $S_0 = 3200$ $GtCO_2$, corresponding to the depleted oil and gas fields in the Norwegian sector in the North Sea (Vangkilde-Pedersen et al., 2009a). As an alternative scenario, I consider storage by the UK, with a larger capacity $S_0 = 7300$.

3.3.1 Results

I will compare outcomes under monopolistic supply of CCS by each of the countries considered, and the socially optimal case.[47] In the base case (Figure 3.5, top panel), a monopolistically behaving Sink does not find scarcity biting for either level of initial capacity; hence there is no difference between the two countries. Investment occurs immediately, with three further capacity expansions. The social planner invests earlier and in more capacity; the Norwegian monopolist's initial capacity is 20% lower than efficient, and the UK monopolist's 45%. The welfare costs are substantial, leading to present value monetary losses of 14% and 24% of the total value of CCS operations, respectively. These differences in investment are driven by the inability of the monopolist to capture the entire social surplus. In other words, the monopolist cuts back cumulative supply, analogously to monopolistic behaviour in a static model. Scarcity bites for the social

[47] See the next section for a treatment of the duopoly case.

planner if storage occurs in Norway, but not if it occurs in UK. Note that the fixed cost of laying pipelines induces both the social planner and the monopolist to practice 'oversizing' (as reported by Morbee et al., 2012): the constraints on the capacity are never binding immediately after investment. In fact, they only bind for roughly half of the entire time horizon in the base case, and even less under the alternative scenarios.

Suppose now that the technology revolution is perceived to occur much later, in 2100 instead of 2070. With storage in saline aquifers proving to be infeasible, capacity becomes scarce for all agents. A Norwegian monopolist would now invest inefficiently early, building the first pipeline (of five) in 2023 whereas efficient investment would be postponed until 2036 (Figure 3.5, bottom panel). Following investment, as both types of agent will want to fill the entire capacity, the social planner must capture slightly more. The differences are relatively small, and the welfare impact of monopolistic storage is only .3% of total CCS value. In the case of the UK, both the monopolist and the planner follow very similar paths. The larger storage capacity requires more transport capacity, construction of which is spread over seven investments. The paths are nearly identical and welfare differences are essentially zero. These results indicate that, if storage capacity is scarce even for the monopolist, then market power is not worth worrying about. However, note that in the base case, when the monopolist finds it optimal to cut back supply and leave some storage unutilised, the welfare impacts are substantial.

These are very basic points. Consider a static context, with a quantity cap which binds even for a monopolist. Then the monopolist will sell as much as she can, as will the social planner. This result holds also in

the dynamic context examined here, with the cap being analogous to the cumulative quantity of resource available.

Note that either outcome results in substantially lower total sequestration rates than projections consistent with the stated long-term climate goals of the European Union. Neele et al. (2010), as an example, project total sequestration rates of up to 750 $MtCO_2$ per year by 2050, should onshore storage be ruled out. These compare with 20–100 Mt per year (depending on the case considered) by the same date in the present model.[48] This partly reflects the assumption of much more limited storage capacity: Neele et al. assume offshore saline aquifer storage to be feasible.

High carbon prices (starting at \$146/tCO_2 in 2020) motivate more storage, ensuring that even with a short time horizon the entire capacity is used up except by a monopolistic UK (Figure 3.6). A Norwegian monopolist overinvests but by very little; the welfare impacts are vanishing. In the case of the UK, the inability to capture the entire surplus leads to underinvestment by the monopolist, with substantial effects on welfare (18% of the total present value of CCS operations).

The model projects cost shares which are consistent with those obtained from engineering-economic studies, despite explicit optimisation of investment timing and capture rates and the rather abstract calibration of the storage demand curve. The share of total discounted capture costs is between 65% and 82% of total costs, with higher costs resulting from higher carbon prices or from having more storage capacity and a longer time horizon. Of course, both factors lead to higher overall stocks of sequestered carbon. These compare with engineering-economic benchmarks of 59% (for

[48]It should be mentioned that Neele et al. (2011a) also consider such large volumes 'unrealistic'.

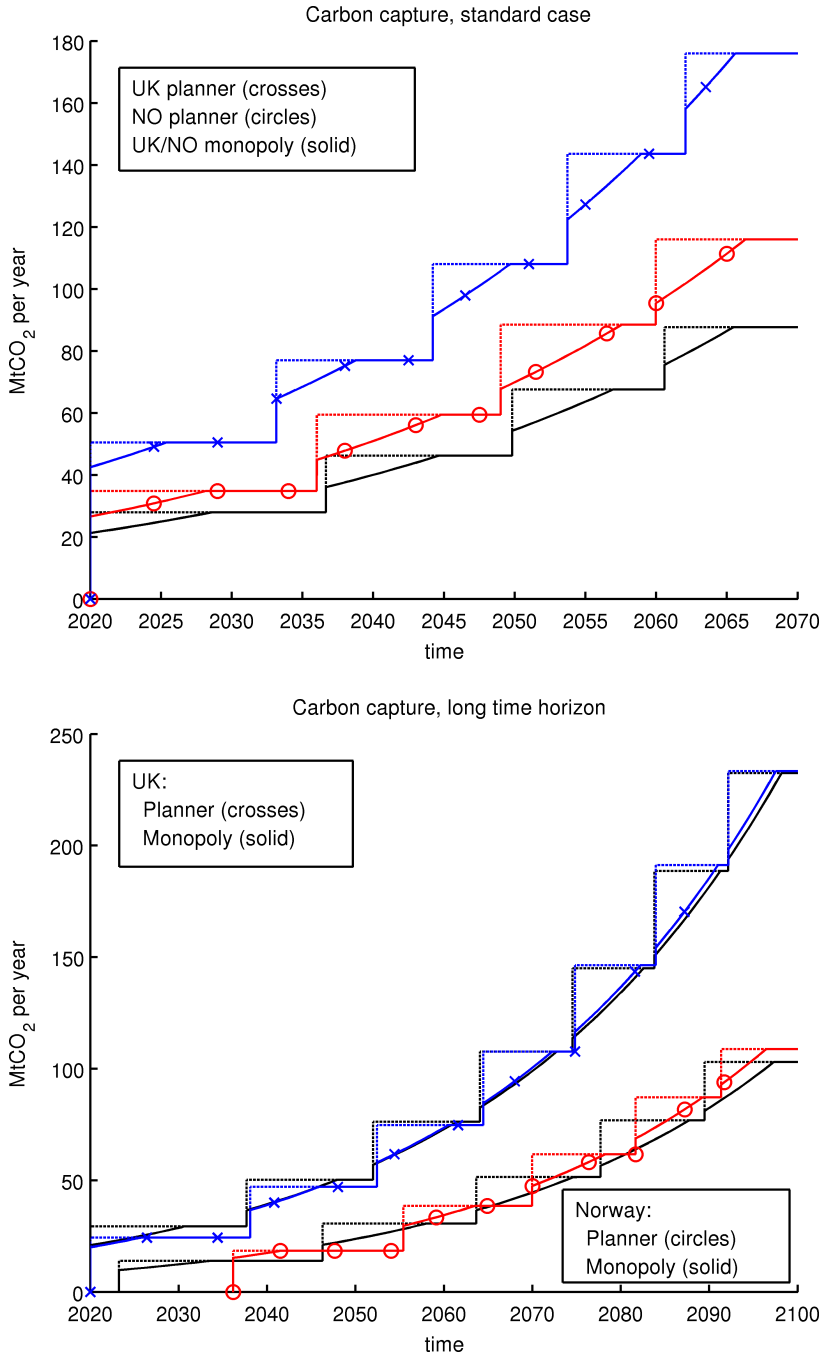


Figure 3.5: (*top*) Comparison of the socially optimal schedule (*red with cirles*: Norway / low capacity, *blue with crosses*: UK / high capacity), with the monopolistic outcome, identical for both capacities (*solid*). (*bottom*) The same comparison when clean energy arrives in 2100. Now scarcity bites in all cases. To utilise the entire UK capacity, a larger volume must be transported and it is efficient to build up to total capacity gradually. Note that in the Norwegian case the monopolist would invest 13 years earlier than efficient.

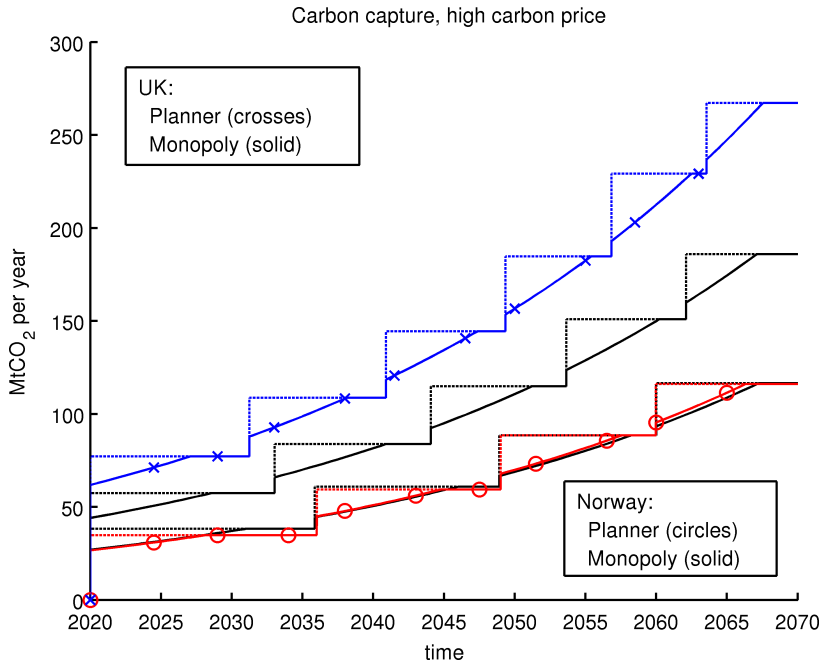


Figure 3.6: (*top*) Comparison of the socially optimal schedule (*red with cirles*: Norway / low capacity, *blue with crosses*: UK / high capacity), with the monopolistic outcome (*solid*), when the initial carbon price is high. Higher capture rates make capacity investment more profitable, so that the resource is scarce for all except the UK monopolist. Overinvestment by the monopolist is barely visible in the Norwegian case, and has negligible welfare impacts. The inability of the monopolist to capture the entire surplus leads to substantial underinvestment, with a welfare cost of 17%.

a hard coal post-combustion capture plant) and 84% (for natural gas combined cycle plant with post-combustion capture) (ZEP, 2011b).[49] Pipeline and storage capital costs are projected to comprise between 11% and 25% of total costs; the actual costs are correlated with capture costs, but the latter respond more so that the cost shares are inversely correlated with capture costs. These compare with benchmark shares of 26% and 12% for the hard coal and natural gas combustion, respectively.

3.4 Strategic pipeline investment

So far, I have investigated monopolistic supply of carbon storage capacity. The calibration of the previous section interpreted the identity of the supplier as either Norway or the United Kingdom, the two parties with the largest stocks of undersea storage capacity in Europe. What if both these suppliers want to benefit from the CO_2 storage market, behaving noncooperatively with respect to each other? This closing section investigates such strategic pipeline investment.

As the key focus of this chapter has been on fixed consecutive investments into pipelines, the natural framework for considering strategic behaviour is that of cumulative capacity investment developed by Gilbert and Harris (1984). They derive very stark results for a duopolistic preemptive equilibrium. In particular, market structure, in terms of the distribution of existing capacity between the two firms, plays no role in the equilibrium outcome: the future development of the sector, in terms of new investments and supply of the product, only depends on existing aggregate capacity, i.e.

[49] Assuming a 1500 km offshore spine with storage in depleted oil and gas fields without legacy technology; in the present study, the pipeline is 300 km shorter.

not on whether the industry consists of, for example, one large incumbent and one potential entrant, as opposed to consisting of two firms of similar size. A second notable feature of this model is that, in equilibrium, capacity is built up as a sequence of investments, all incrementing capacity by the smallest possible (discrete) investment quantity, with no two investments made at the same time. Thirdly, in this model, all individual plants make zero profits, as do both firms.[50]

Gilbert and Harris (1984) admit that their model may admit a multiplicity of equilibria, including some involving threats. This is correct: it is straightforward to show that the results they obtain are crucially dependent on the particular equilibrium they choose to investigate. This equilibrium is constructed so that the strategies employed are extremely aggressive. In particular, the equilibrium features 'purely self-defensive' investment (in the sense of Fudenberg and Tirole, 1985): situations in which both players are happy to invest only because the other one also intends to invest, even though both players would prefer to have no further investment made by either firm. Such equilibria may not seem very reasonable, and the stark results are not very surprising.[51] The key issue is the willingness of the smaller firm to invest: an entrant with no capacity is very hungry and wants to enter the market, while a small firm with some capacity is less keen to expand as it has inframarginal revenues to protect.

Further, as discussed in Section 3.3, it is apparent that construction of

[50]Non-zero profits for the first plant built, and for individual firms, are possible in the case in which the initial investment occurs immediately at the beginning of the game. In such a case, the first investment may also involve multiple plants.

[51]Gilbert and Harris (1984) also impose a small, exogenous timing advantage on one of the firms, in order to break ties in cases in which both firms want to invest at a given time, conditional on the other not doing so. The firm with the advantage ends up making all investments. However, this is not crucial for their key results.

pipeline capacity involves economies of scale. In particular, there are fixed costs involved with laying down any undersea pipeline at all: ships have to be commissioned, plans drawn up, and so on. Total costs consist of these fixed investment costs plus the (integrated) marginal costs of increasing pipeline capacity.

In this section, I will amend the model of Gilbert and Harris (1984). Instead of a simultaneous move order, I employ a sequential move order, which eliminates the highly competitive equilibrium. I also include the possibility that there are economies of scale to pipeline investment. The focus on more 'reasonable' equilibria alters the results. In particular, typically an incumbent will want to allow an entrant into the market.[52] In particular: a) market structure (the distribution of existing capacity) becomes very relevant to the future capacity expansion path, so much so that an incumbent often wants to allow an entrant into the market; b) at least some capacity investments may well be large, rather than small; c) clustering of investments is possible, i.e. at times the two firms may choose to increment capacity at the same moment; d) it is possible that some individual plants make profits, while others make losses; and e) the firms may make aggregate profits, with rent equalisation not necessarily holding.[53,54]

To develop a model which is even remotely tractable, I will need to alter

[52]Boyer et al. (2012) investigate a related model; in their model incumbents also let other firms in, although for a different reason. They thus obtain results similar to mine.

[53]Rent equalisation will hold if the first investment involves a first-mover advantage, as then preemption would apply. If the first investment involves a second-mover advantage, rent equalisation does not, in general, hold.

[54]Mills (1990) develops a similar model as the one outlined below. However, the purported equilibrium in that paper is not an equilibrium. To see this, note that there is a profitable deviation to the equilibrium given for Example 7 in that paper: namely, following the first investment in a small plant, for the now-incumbent to build another small plant at time $t = 4.07 - \epsilon$, for some small ϵ. Mills' claim (in his footnote 6) that the existing capacity distribution is immaterial is incorrect, and following the implications through results in the present model.

some assumptions employed in the previous sections. Three substantive assumptions are required to simplify the model. A technical assumption on timing is also needed to derive a clean solution; I defer discussion of this until later.

The first substantive change is that I now assume that ultimate storage scarcity is close to inconsequential. According to Vangkilde-Pedersen et al. (2009a), the inclusion of North Sea saline aquifers owned by the UK and Norway increase total available capacity to over 40 $GtCO_2$, or 6 times the capacity considered in the more generous case in the previous section. Note that some of the previous simulations yielded non-binding capacity constraints. With saline aquifer storage, it seems plausible that CCS operations may not be limited by ultimate storage capacity, assuming that the technology revolution eliminating the need for CCS is not very far in the future. Alternatively, the assumption could be motivated by considering injection as involving stock-dependent extraction costs, with storage ending due to economic exhaustion. Provided that costs increase very slowly for most of the reservoir's early lifetime, this implies that the scarcity rent is very low; I implicitly approximate this rent to be zero.[55]

Secondly, I will assume that demand is isoelastic at any given point in time, with an elasticity larger than unity. This makes the supply choice at

[55]Economic exhaustion implies that the carbon price, less any capture and transportation costs, must equal the injection cost on the date of exhaustion. As the carbon price implicitly increases over time, injection costs must eventually rise. Denoting injections cost by $C(S)$, with S denoting storage capacity and $C'(S) < 0$, the scarcity rent is equal to the future stream of cost reductions from not injecting the previous unit: $\lambda(t) = -\int_t^T e^{-\rho(\tau-t)} C'(S(\tau)) q(\tau)\,\mathrm{d}\tau$. If there are any periods in which the injection cost increases rapidly with respect to storage capacities, and for which injection rates are not very small, these must lie far enough in the future so that the decisionmaker discounts any benefits from retaining a marginal unit of stock. Close to the exhaustion date, this approximation may be less accurate. For economic exhaustion to occur, the demand for storage would have to eventually stop growing.

any given point in time very simple: both firms supply as much storage as they can, i.e. up to their existing capacity. With linear demand, as in the previous sections, one or both firms would in some states (and times) sell at less than full capacity, leading to a much more complicated equilibrium structure which, even were it tractable, would only mask the key messages. I feel the assumption of isoelastic demand is, a priori, neither more nor less plausible than that of linear demand, provided that any calibrated examples are checked for plausibility.[56]

The third substantive assumption is that the set of investment quantities is discrete, unlike in previous sections. I will work with linear investment costs as in Section 3.3.

The model is effectively a sequence of complicated games of timing, with endogenously arising asymmetry. Many equilibrium situations are driven by preemption concerns: both players would prefer to delay investment, but are forced to act by the threat of the opponent moving first. Some outcomes also have features which resemble a war of attrition, with one or both players wanting the other player to move first (Hendricks et al., 1988). The model analysis uses and extends the methods developed by Fudenberg and Tirole (1985) and Katz and Shapiro (1987).

[56]Isoelastic demand does not have a choke price, nor a saturation quantity. For robustness, any calibrated examples should not involve very high prices for stored carbon, nor very large quantities of stored carbon. The numerical illustrations below satisfy these requirements.

3.4.1 Game set-up

Demand and capacity technology

Suppose the model conforms to that in Section 3.2.5, with the following amendments. Let there be two firms, indexed by $k \in \{1, 2\}$, with supply by firm k given by $q^k \leq Q^k$, and aggregate supply denoted by $q = \sum_k q^k$. Let the resource be inexhaustible, so that the resource constraint can be ignored, and let the marginal cost of extraction be zero. It will become clear below that both firms will always supply up to capacity, and I will hence dispose of the variable Q^k and instead denote the vector of output/capacities and investment quantities, respectively, by $\boldsymbol{q} \equiv (q^1, q^2)$ and $\overline{\boldsymbol{q}} \equiv (\overline{q}^1, \overline{q}^2)$.

Let demand be given by

$$p(q,t) = \begin{cases} p_0(q)e^{\gamma t} & \text{if } t \leq T \\ 0 & \text{otherwise.} \end{cases}$$

with $p_0(q) \equiv Aq^{-\frac{1}{\sigma}}$, $\sigma > 1$, $\gamma < \rho$. The scaling parameter A is, in principle, redundant and could be eliminated by a convenient choice of units; I retain it here with a view to the numerical examples later. Demand is thus isoelastic, with elasticity greater than unity, with the level of demand growing at rate γ until T, after which the market disappears as before. Note that this implies marginal revenue is strictly positive at any given moment in time $t \leq T$, for each firm, irrespective of the other firm's supply. As claimed above, the firms will both supply up to capacity as there are no costs (monetary or opportunity) of supply.

Let the cost structure be given by the linear specification as in Section

3.3; however, for convenience of notation, I will also explicitly define zero cost for no investment:

$$c(\overline{q}) = \begin{cases} \alpha_1 + \alpha_2 \overline{q} & \text{if } \overline{q} > 0; \\ 0 & \text{if } \overline{q} = 0. \end{cases}$$

Let the investment quantity be chosen from a discrete set: $\overline{q} \in \{0, \delta, 2\delta, \ldots, n\delta\}$.[57] I assume that $n\delta$ is sufficiently large to cover all investment quantities the players could desire in equilibrium, so that a player never wants to make two consecutive investments at the same moment, but would rather prefer one larger investment to save on the fixed costs.

Observe that, as demand dies off at time T, there will be only a finite number of investments. In particular, an upper bound to aggregate capacity is given by

$$q^{\text{MAX}} \equiv \min \left\{ q : \int_{t'}^{T} e^{-\rho(\tau - t')} p(q+1, t) \, \mathrm{d}t - \frac{c(\tilde{q})}{\tilde{q}} \leq 0, \forall t' \in [0, T] \right\}$$

in which $\tilde{q}$ denotes the investment quantity which minimises the average investment cost; with linear costs, of course, $\tilde{q} = n\delta$. The bound on capacity is thus derived as the maximal capacity for which the increment of one further unit of capacity, at the lowest possible average cost and with no further investment, will yield a negative profit for an entrant with no existing capacity, irrespective of the investment date. No firm could ever make a profit making such an investment.

[57] I could easily use some other discrete set, with arbitrary capacity increments. The key assumption is one of a discrete, rather than continuous, capacity choice set: the latter presents substantial difficulties in terms of obtaining the appropriate first- and second-order conditions, as the equilibrium often involves corner solutions. Furthermore, the full model has to be solved numerically and a discrete choice set makes this much easier.

Timing assumptions

I will assume the timing of the game is as follows: time flows continuously, but the firms get to make choices at discrete intervals of length κ. At each moment of time, firms choose their actions sequentially. This could reflect a vanishing observational advantage as in Gilbert and Harris (1984). The identity of the firm with the observational advantage is randomised at the beginning of each period, with both firms having equal probability of moving first.[58] When either player invests, time is immediately incremented by the decision interval, the state changes, and the game continues with the randomisation of the player with the advantage.

I allow the length of the decision interval become arbitrarily small ($\kappa \to 0$), to capture the fact that investment dates are in reality chosen from a continuous set.[59] The game could alternatively be set up in continuous time, as a sequence of stopping games (as in Murto and Välimäki, 2013). Due to numerous open-set issues, particular restrictions on strategies and a carefully selected tie-breaking rule (for situations in which both players try to invest at the same time) would be required to ensure the

[58]The timing assumptions are close to those made by Gerlagh and Liski (2011), except for my per-period randomisation of the move order; Katz and Shapiro (1987) use similar assumptions but with simultaneous moves. Alternative assumptions include having a fixed move order; or randomising e.g. after each investment. These choices would in most cases not make a difference, but would change the equilibrium with some parameter combinations. A move order fixed from period to period implies that firms know with certainty which firm has a very marginal advantage in the future and can plan accordingly, and/or that there might be complex correlations in terms of who has the advantage across time. I elaborate on this below. While fixing the move order would be a more standard approach, I find it less plausible. For this reason, and with the added benefit of a simplified computational algorithm, I choose per-period randomisation instead.

[59]I do not work with a purely continuous-time model as defined by Simon and Stinchcombe (1989); see Hoppe and Lehmann-Grube (2005) and Argenziano and Schmidt-Dengler (forthcoming) for applications of this framework to preemption games. Instead, I show that the continuous-time formulation is an arbitrarily precise approximation to the discrete-decision-interval game as the period length vanishes.

existence of equilibrium. Furthermore, a multiplicity of equilibria would still exist, although the equilibria would be very closely related to the ones presented here. I have chosen the discrete decision interval framework as the assumptions required are more transparent.

Strategies

I assume the players condition their actions on the history of the game. However, generically, this is equivalent to the players conditioning their actions only on the current state $\boldsymbol{q}$, the calendar date t and the identity of the player moving first in period in question; in other words, the players play Markov-perfect strategies (in the sense of Maskin and Tirole, 2001).[60] I thus focus on the case in which strategies are a function ϕ only of the calendar date, the existing distribution of capacity and the binary variable indicating move order: $q^{k,*} = \phi^k(\boldsymbol{q}, t, \mathbf{1}(k \text{ moves first}))$. From the structure of the game it is apparent that the equilibrium will feature symmetric strategies.

Given any initial state $(\boldsymbol{q}_0, t_0)$, I will denote the equilibrium capacity, investment quantity and investment date sequences (for $j \geq 1$) by

$$\begin{aligned} \boldsymbol{q}^* &\equiv \{\boldsymbol{q}_j^*\} \equiv \{(q_j^{1,*}, q_j^{2,*})\} \\ \overline{\boldsymbol{q}}^* &\equiv \{\overline{\boldsymbol{q}}_j^*\} \equiv \{(\overline{q}_j^{1,*}, \overline{q}_j^{2,*})\} \\ t^* &\equiv \{t_j^*\} \end{aligned}$$

respectively, so that

$$q_{j+1}^{k,*} = q_j^{k,*} + \overline{q}_{j+1}^{k,*}.$$

[60] I discuss this in footnote 66, after first setting up the structure of the game.

If player i makes the jth investment, then $\overline{q}_j^{-i,*} \equiv 0$. As before, aggregate equilibrium capacity is given by $q_j^* \equiv \sum_k q_j^{k,*}$.

For convenience of notation, also define

$$t_0^* \equiv t_0$$

$$t_{|\overline{\boldsymbol{q}}^*|+1}^* \equiv T$$

$$\overline{q}_{|\overline{\boldsymbol{q}}^*|+1}^{k,*} \equiv 0$$

with $|\overline{\boldsymbol{q}}^*|$ denoting the number of investments in equilibrium.

Value functions

I can now state the value[61] of the equilibrium to player k:

$$V^k(\boldsymbol{q}_0, t_0) = \mathbb{E}\left\{\sum_{j=0}^{|\overline{\boldsymbol{q}}^*|} \int_{t_j^*}^{t_{j+1}^*} e^{-\rho(t-t_0)} p\left(q_j^*, t\right) q_j^{k,*}\, \mathrm{d}t - e^{-\rho(t_{j+1}^*-t_0)} c\left(\overline{q}_{j+1}^{k,*}\right)\right\}$$

This is just the discounted stream of revenues from selling at full capacity, less any investment costs. The expectation is taken with respect to the randomisation of the first mover at the beginning of each period, which is the only source of uncertainty. Observe that by the definition of the cost function and the investment vectors, if equilibrium investment j is made by player i, the cost for player $-i$ is (of course) zero.

Slightly abusing notation, I will also denote the next equilibrium investment, given any initial state $(\boldsymbol{q}_0, t_0)$ by $\overline{\boldsymbol{q}}_{\boldsymbol{q}_0,t_0}^*$, $t_{\boldsymbol{q}_0,t_0}^*$. I can then express

[61] Recall that this is an approximation of the value function as the time interval between decisions κ goes to zero. As time is defined to run continuously, the integrals are exact; approximation errors arise only from the investment dates being forced to lie on the grid of decision points. As $\kappa \to 0$, these approximation errors go linearly to zero (see Appendix 3.A.1).

the value function recursively:

$$V^k(\boldsymbol{q}_0, t_0) = \mathbb{E}\left\{\int_{t^0}^{t^*_{\boldsymbol{q}_0,t_0}} e^{-\rho(t-t_0)} p\left(q_0^*, t\right) q_0^k \,\mathrm{d}t - e^{-\rho(t^*_{\boldsymbol{q}_0,t_0}-t_0)} c\left(\overline{q}^{k,*}_{\boldsymbol{q}_0,t_0}\right) + e^{-\rho(t^*_{\boldsymbol{q}_0,t_0}-t_0)} V^k(\boldsymbol{q}_0 + \overline{\boldsymbol{q}}^*_{\boldsymbol{q}_0,t_0}, t^*_{\boldsymbol{q}_0,t_0})\right\}$$

This recursive formulation allows the analysis of the game using the framework developed by Fudenberg and Tirole (1985) and Katz and Shapiro (1987), and thus yields clearer insight to the equilibrium than a simple brute-force numerical approach.

Recursive equilibrium investment

I will now construct the equilibrium outcome and the value functions using the recursive structure above. Given initial state $(\boldsymbol{q}_0, t_0)$, the profit function for player k, as a function of the next investment date t' and investment quantity vector $\overline{\boldsymbol{q}}'$, is

$$\begin{aligned}
&\pi^k_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t') \\
&\quad = \int_{t_0}^{t'} e^{-\rho(t-t_0)} p(q_0, t) q_0^k \,\mathrm{d}t + \int_{t'}^{t^*_{\boldsymbol{q}',t'}} e^{-\rho(t-t_0)} p(q', t) q'^k \,\mathrm{d}t \\
&\qquad - e^{-\rho(t'-t_0)} c(\overline{q}'^k) \\
&\qquad + e^{-\rho(t^*_{\boldsymbol{q}',t'}-t_0)} \mathbb{E}\left(V^k(\boldsymbol{q}' + \overline{\boldsymbol{q}}^*_{\boldsymbol{q}',t'}, t^*_{\boldsymbol{q}',t'}) - c(\overline{q}^{k,*}_{\boldsymbol{q}',t'})\right)
\end{aligned} \tag{3.19}$$

with $\boldsymbol{q}' \equiv \boldsymbol{q}_0 + \overline{\boldsymbol{q}}'$. I will denote the profit function if player k leads (i.e. with $\overline{q}'^k \neq 0$, $\overline{q}'^{-k} = 0$) by $\pi^{k,L}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t')$. Similarly, if player k follows (with $\overline{q}'^k = 0$, $\overline{q}'^{-k} \neq 0$), I will denote the resulting profit function by $\pi^{k,F}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t')$. I will from now on suppress the subscripts indexing the values to a given state.

Assume, in what follows, that the equilibrium value has been determined for all $\tilde{\boldsymbol{q}} \geq \boldsymbol{q}_0$, i.e. all states with weakly higher capacity (strictly for at least one firm) and for all $\tilde{t} \in [t_0, T]$. To determine the equilibrium for state $(\boldsymbol{q}_0, t_0)$, I will need to construct the profits for either player following and leading at any t'. I can then construct the equilibrium outcome for this particular state by backward induction.

I will first show how to construct the profits conditional on firm i leading on the next investment.[62] Fix a scalar $\overline{q}'$ and let this be the ith component of $\overline{\boldsymbol{q}}'$ (with the other component zero). Then, for any t', the subsequent investment date $t^*_{\boldsymbol{q}',t'}$ and the corresponding value are known by assumption. For now, consider only cases such that $t^*_{\boldsymbol{q}',t'} > t'$, i.e. there is an interval of strictly positive length separating the investment under consideration from the subsequent one. Then, by subgame perfection, $\frac{\mathrm{d}t^*_{\boldsymbol{q}',t'}}{\mathrm{d}t'} = 0$, i.e. a small delay in investment will not affect the subsequent investment date.[63]

It is now straightforward to show that the profit from leading is quasi-concave in t'. To see this, note that

$$\frac{\partial \pi^{L,i}}{\partial t'} = e^{-\rho(t'-t_0)} \left(p(q_0, t') q_0 \frac{q_0^i}{q_0} - p(q', t') q' \frac{q'^i}{q'} + \rho c \left(\overline{q}'^i \right) \right) \tag{3.20}$$

$$\frac{\partial^2 \pi^{L,i}}{\partial t'^2} = -\rho \frac{\partial \pi^{L,i}}{\partial t'} + e^{-\rho(t'-t)} \left(p_t(q_0, t') q_0^i - p_t(q', t') q'^i \right) \tag{3.21}$$

Suppose $\frac{\partial \pi^{L,i}}{\partial t'} = 0$, $\frac{\partial^2 \pi^{L,i}}{\partial t'^2} \geq 0$; then clearly $p_t(q_0, t') q_0^i - p_t(q', t') q'^i \geq 0$. But, by the assumed separability of $p(q, t)$, this implies $p_0(q_0) q_0^i - p_0(q') q'^i \geq 0$, so the bracketed term in (3.20) has to also be positive, and thus $\frac{\partial \pi^{L,i}}{\partial t'} > 0$,

[62] I refer by the index i to the investing firm, by $-i$ to the non-investing firm, and by k to any firm.

[63] The time derivatives of various quantities have to be defined so that $\mathrm{d}t$ and the grid size κ approach zero in lockstep. Note that I use the derivatives to characterise the equilibrium, but the agents in the model do not need to calculate them, so approximation errors in the derivatives do not affect the equilibrium outcome.

contrary to the initial assumption. This yields quasiconcavity. To repeat, the π^L-curves given here are defined only for t' satisfying $t^*_{x',t'} > t'$.

The choice of investment quantity $\overline{q}'$ is simple to determine. Given that player i gets to lead and given t', she will choose $\overline{q}'$ which yields the highest possible profit. In other words, from the perspective of moment t_0, as long as player i leads on the next investment, the optimal value is given by the upper envelope, with respect to $\overline{q}'$, of the curves $\pi^{L,i}(t', \overline{\boldsymbol{q}}')$ (Figure 3.7). Three features are worth observing. Firstly, as the profit curves are differentiable, any local maximum of the upper envelope will also be a critical point, i.e. there will exist no 'kinked' maxima. Secondly, the curves' concavity is increasing in $\overline{q}'$, in the sense that for $\overline{q}' > \overline{q}''$,

$$\left.\frac{\partial \pi^{L,i}}{\partial t'}\right|_{\overline{q}'} = \left.\frac{\partial \pi^{L,i}}{\partial t'}\right|_{\overline{q}''} \Rightarrow \left.\frac{\partial^2 \pi^{L,i}}{\partial t'^2}\right|_{\overline{q}'} < \left.\frac{\partial^2 \pi^{L,i}}{\partial t'^2}\right|_{\overline{q}''}$$

implying that any two profit curves cross at most twice. Finally, note also that, as $\overline{q}'$ increases, for any t', the first integral term in (3.19) is unchanged; the second integral term increases; the investment cost becomes larger; and the continuation value may either increase or decrease. Hence, in general, it is not possible to order π_L with respect to $\overline{q}'$. In particular, the continuation values may change non-monotonically with respect to $\overline{q}'$, as the continuation equilibrium path may vary non-trivially with the continuation state.

On the other hand, the profits player $-i$ gets for following when player

i invests $\overline{q}'$ at time t' are strictly increasing:

$$\frac{\partial \pi^{F,-i}}{\partial t'} = e^{-\rho(t'-t_0)} \left(p(q_0,t') - p(q',t')\right) q_0^{-i} > 0 \tag{3.22}$$

$$\frac{\partial^2 \pi^{F,-i}}{\partial t'^2} = -\rho \frac{\partial \pi^{F,-i}}{\partial t'} + e^{-\rho(t'-t_0)} \left(p_t(q_0,t') - p_t(q',t')\right) q_0^{-i} \tag{3.23}$$

where $q'^{-i} = q_0^{-i}$ as player $-i$ does not invest; $p_t(q_0,t') = q_0^{-\frac{1}{\sigma}} \gamma e^{\gamma t'} = \gamma p(q_0,t')$, so that $\frac{\partial^2 \pi^{F,i}}{\partial t'^2} \leq 0$ if $\frac{p_t}{p} = \gamma \leq r$, which holds by assumption. In words, given that the other player is next to invest, the later this occurs, the better: the non-investing player prefers the price drop associated with new capacity to be delayed.[64] I also illustrate some π^F-curves in Figure 3.7. Note that $\lim_{t' \to T} \pi^F$ is independent of $\overline{q}'$ and equal to the profits obtained if no further investment is undertaken.

The solution is similarly straightforward in the case of simultaneous investment, i.e. if the investment subsequent to the next is immediate, or $t^*_{\boldsymbol{q}',t'} = t' + \kappa$ (recall that after any investment, the time period advances by one). As $\kappa \to 0$, then, $t^*_{\boldsymbol{q}',t'} \to t'$; I will henceforth only consider this limit. The sequence of investments at a given moment results in state $(\boldsymbol{q}'', t')$. Thereafter the following investment occurs a strictly positive interval of time later: $t^*_{\boldsymbol{q}'',t'} > t'$.

For simultaneous investments, the profit curve π^S is given by (3.19), with $\boldsymbol{q}'$ replaced by $\boldsymbol{q}''$, and the investment costs for both players given by the sum of the costs of their respective investments in the sequence.[65]

[64] The model thus departs from the framework of Hoppe and Lehmann-Grube (2005), who only consider cases such that profits for following decrease over time.

[65] One might think that simultaneous investments always feature exactly one investment by each player, as this would minimise total costs given the total capacity increment by each player. Due to the sequential timing assumptions, this is difficult to demonstrate analytically. However, numerical experiments have not revealed outcomes in which multiple investments are made by one player at a given moment in time.

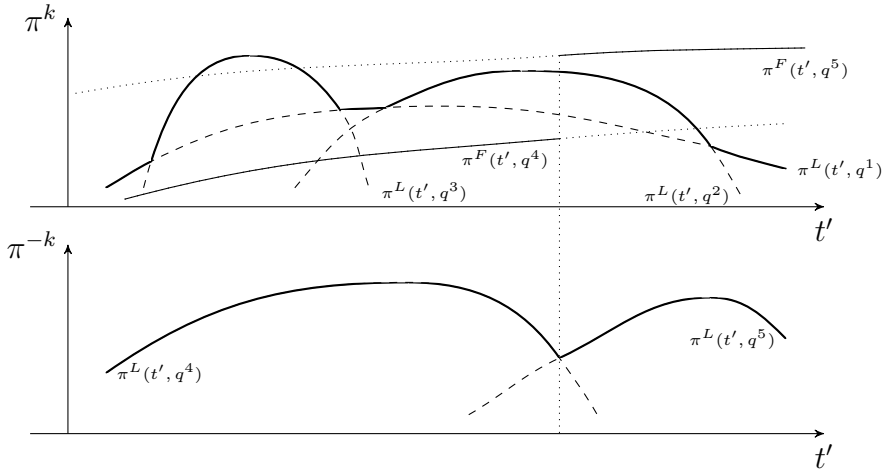


Figure 3.7: The profits for leading, as function of time, are given by the upper envelope (*thick line*) of the π^L-curves (*dashed lines*) for different investment quantities. For the top player, curves displaying profits for different lead quantities chosen by the other player (*dotted lines*) are also shown. The actual π^F-curve (*solid line*) may be discontinuous at points at which the other player's lead quantity changes.

Quasiconcavity holds by the same arguments used previously. Finally, fix any $\overline{\boldsymbol{q}}'$, and denote by $\tilde{t}$ the point such that, in a neighbourhood of $\tilde{t}$, $t^*_{\boldsymbol{q}',t} > t$ for $t < \tilde{t}$, but $t^*_{\boldsymbol{q}',t} = t$ for $t > \tilde{t}$. That is, $\tilde{t}$ is a point at which, given an investment quantity, the outcome switches from the subsequent investment being delayed (unilateral investment) to immediate subsequent investment (simultaneous investment). Then

$$\lim_{t\uparrow\tilde{t}} \pi^{L,i}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t) = \lim_{t\downarrow\tilde{t}} \pi^{S,i}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t)$$
$$\lim_{t\uparrow\tilde{t}} \frac{\partial \pi^{L,i}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t)}{\partial t'} < \lim_{t\downarrow\tilde{t}} \frac{\partial \pi^{S,i}_{\boldsymbol{q}_0,t_0}(\overline{\boldsymbol{q}}', t)}{\partial t'}.$$

In words, at the moment at which investment becomes simultaneous, the π^L- and π^S-curves join up but there is an upward kink.

3.4.2 Subgame-perfect equilibrium

I will now describe the subgame-perfect equilibrium to the game. Existence of such an equilibrium holds trivially, by Zermelo's Theorem (Fudenberg and Tirole, 1991). The equilibrium also seems to be generically unique.[66]

Note that the timing structure I utilise rules out e.g. self-defensive equilibria, in which both parties are willing to invest only because the other one is, even though both would prefer delaying, unlike various continuous-time or simultaneous-move formulations (Gilbert and Harris, 1984; Fudenberg and Tirole, 1985; Katz and Shapiro, 1987). The timing assumptions in the present paper—in particular, the sequential move order—ensure that Pareto-dominated equilibria to any subgame are never played (in the limit as $\kappa \to 0$).[67]

The construction of the subgame perfect equilibrium is not difficult, if a little tedious. Equilibrium investments can be characterised and interpreted in a relatively tidy fashion. I will here only enumerate and intuitively describe the various types of equilibrium investment; the formal character-

[66]At each decision node, a player will have exactly two choices: investing at the optimal quantity, or letting the game continue, with the continuation payoff obtained by backward induction. The optimal quantity is uniquely determined. Firm i investing on any given date t' will choose to invest at a quantity which maximises $\pi^{L,i}(\overline{\boldsymbol{q}}', t')$, i.e. its profits for leading. Two or more values of $\overline{q}'^i$ yield the same profits, by definition, at a crossing of the respective $\pi^{L,i}$-curves. As decision moments are discrete, the date t' coincides with such an intersection only by chance. Such a coincidence would not be robust to a small perturbation in any key parameter, e.g. the period length.

This heuristic argument is based on the failure to find any *a priori* mechanisms systematically causing such equalities to hold, or any numerical examples in which they do hold. Should the argument fail, uniqueness could be ensured by adding a very small stochastic perturbation to e.g. the cost functions. The magnitude of such a perturbation would have to fall sufficiently rapidly with the period length to ensure the perturbation would never outweigh any approximation errors, so that the continuous-time approximation would still hold.

[67]Of course, in a simultaneous-move game, coordination on a self-defensive Pareto-dominated equilibrium might be used as a threat strategy; I do not imply that the equilibrium picked out by my assumptions might be Pareto-dominant for the full game.

isation is relegated to Appendix 3.A.1.

Unilateral investment

Given the state $(\boldsymbol{q}_0, t_0)$, consider unilateral equilibrium investment, that is, investment such that the subsequent investment takes place only after a strictly positive interval of time: $t_2^* > t_1^*$. Any such outcome can be classified as belonging to one of seven different types. These types are illustrated in Figure 3.8, in terms of the two players' respective profit curves $\pi^{k,L}$ and $\pi^{k,F}$. These points are equilibrium *candidates* only; the actual equilibrium outcome is determined from the exact sequence of such points by backward induction.

The first three cases consider equilibrium investments in situations in which both players' optimal actions are continuous, that is, neither player's optimal lead quantity is about to change. The next three cases describe equilibrium investment driven by a change in this optimal lead quantity. The last case completes the list. I will refer to the two players as 'top' and 'bottom', with reference to Figure 3.8.

(i) Preemption. (Figure 3.8, top left.) This is most basic case: bottom (weakly) prefers to lead, while top is indifferent between following and leading. Note that any potential equilibrium in which investment were to occur a short time later would unravel by the desire of both players to preempt the other. If top strictly prefers to lead, she will get to invest. If both players are indifferent between following and leading, the player who moves first invests at the first moment following the crossing of the two curves.

(ii) Unilateral investment without preemption. (Figure 3.8, top

right.) In this case top prefers leading to following, while bottom prefers following to leading. Further, top's π_L-curve has a local maximum, that is, top can choose an interior optimum, with constraints imposed by preemption not binding.

(iii) Forced investment. (Figure 3.8, middle left.) Bottom prefers following to leading or continuation, and wants to force top to invest; top would prefer to continue, but would rather lead than follow. Bottom can force top to invest at the point at which bottom's profits from leading are just about to fall below the value she obtains from continuing the game. At this point, bottom's profits from leading have to be decreasing; otherwise bottom could leave the threat until a while later and get even higher profits for following. Investment is determined by the fact that, a moment later, the threat to force investment would no longer be credible.

(iv) Symmetric forced investment. (Figure 3.8, middle right.) This case occurs, generically, only when the players both have equal capacity, at a moment at which the optimal lead quantity changes.[68] Both players would prefer the other player to lead with the quantity optimal running up to the investment point. Following this point, both prefer leading to following, and preemption forces immediate investment. Thus, the player who moves last just before the crossing is forced to invest; the first mover has the first option to decline to invest. Note that the π^L-curves might also be decreasing.

(v) Preemption with discontinuity. (Figure 3.8, bottom left.) This case is the standard preemption case, except that bottom's π^F-curve

[68] With unequal capacities, the case in which both players' optimal actions change at the same moment is not robust to a small perturbation of the model parameters.

crosses her π^L-curve discontinuously, due to top's optimal investment quantity changing. The threat of impending preemption means top invests at the last possible moment before the curves cross.

(vi) Forced investment with discontinuity. (Figure 3.8, bottom right.) This case is like the forced investment case; top strictly prefers following to leading, and leading to continuing. However, in the previous case, investment is forced by the expiration of the threat's credibility. Here, the threat itself expires; a moment later, bottom would rather follow than lead, and can so no longer be threatened into investing.

(vii) Immediate investment. Given the date t_0, i.e. the start of the subgame, it is always possible that the optimal investment occurs immediately.

I will make a few comments about the potential equilibrium investment dates. Types (i) and (ii) were investigated by Katz and Shapiro (1987). For type (iii), their assumptions on timing (simultaneous moves) and tie-breaking (a coin flip) led to non-existence of equilibrium; in the present paper, with sequential moves, the equilibrium exists. The discontinuous cases have not, to my knowledge, been previously considered in the literature.

Case (iv) may seem odd: the firms are symmetric, but both would rather follow than lead. In this, the equilibrium resembles a war of attrition (Hendricks et al., 1988). Numerical examples demonstrate that such cases are not impossible. In such a case, both firms want someone to invest, as continuation would eventually (or immediately) lead to higher ultimate capacity. However, both would still strictly prefer the other firm to be

the first investor, knowing they will be allowed into the market later in the subsequent equilibrium, possibly with higher capacity and saving the opportunity cost of investment funds for the time being.

Per-period randomisation of the move order is important here. With a fixed move order, the firm moving first would have a first-mover advantage in any symmetric pre-emptive investment outcomes, but its opponent would hold a second-mover advantage in a symmetric forced investment case. Such persistence of the infinitesimal advantage would alter the continuation payoffs, and thus lead to a very different equilibrium. I feel it is unrealistic to assume one of the firms is able to consistently hold on to a very small advantage.

Simultaneous investment

In the case of simultaneous investment, the profits from leading and following can be calculated as outlined above. The equilibrium types are thus as above, except that the profit curves for the other player leading first (equivalent to 'following') may now also decrease. One particular special case deserves highlighting.

Suppose there is some time t' at which the players optimally make one investment each simultaneously. Denote the optimal investment quantities by players 1 and 2, respectively, by $\overline{q}'$ and $\overline{q}''$, the corresponding unilateral investment vectors by $\overline{\boldsymbol{q}}' \equiv (\overline{q}', 0)$ and $\overline{\boldsymbol{q}}'' \equiv (0, \overline{q}'')$, and the corresponding next states by $\boldsymbol{q}' \equiv \boldsymbol{q} + \overline{\boldsymbol{q}}'$ and $\boldsymbol{q}'' \equiv \boldsymbol{q} + \overline{\boldsymbol{q}}''$. Suppose now that the optimal responses to these investments are $\overline{\boldsymbol{q}}^*_{\boldsymbol{q}',t'} = \overline{\boldsymbol{q}}''$ and $\overline{\boldsymbol{q}}^*_{\boldsymbol{q}'',t'} = \overline{\boldsymbol{q}}'$, respectively. Then, irrespective of who invests first, the continuation state is going to be $\boldsymbol{q} + \overline{\boldsymbol{q}}' + \overline{\boldsymbol{q}}''$, and both players get the same profits irrespective of whether

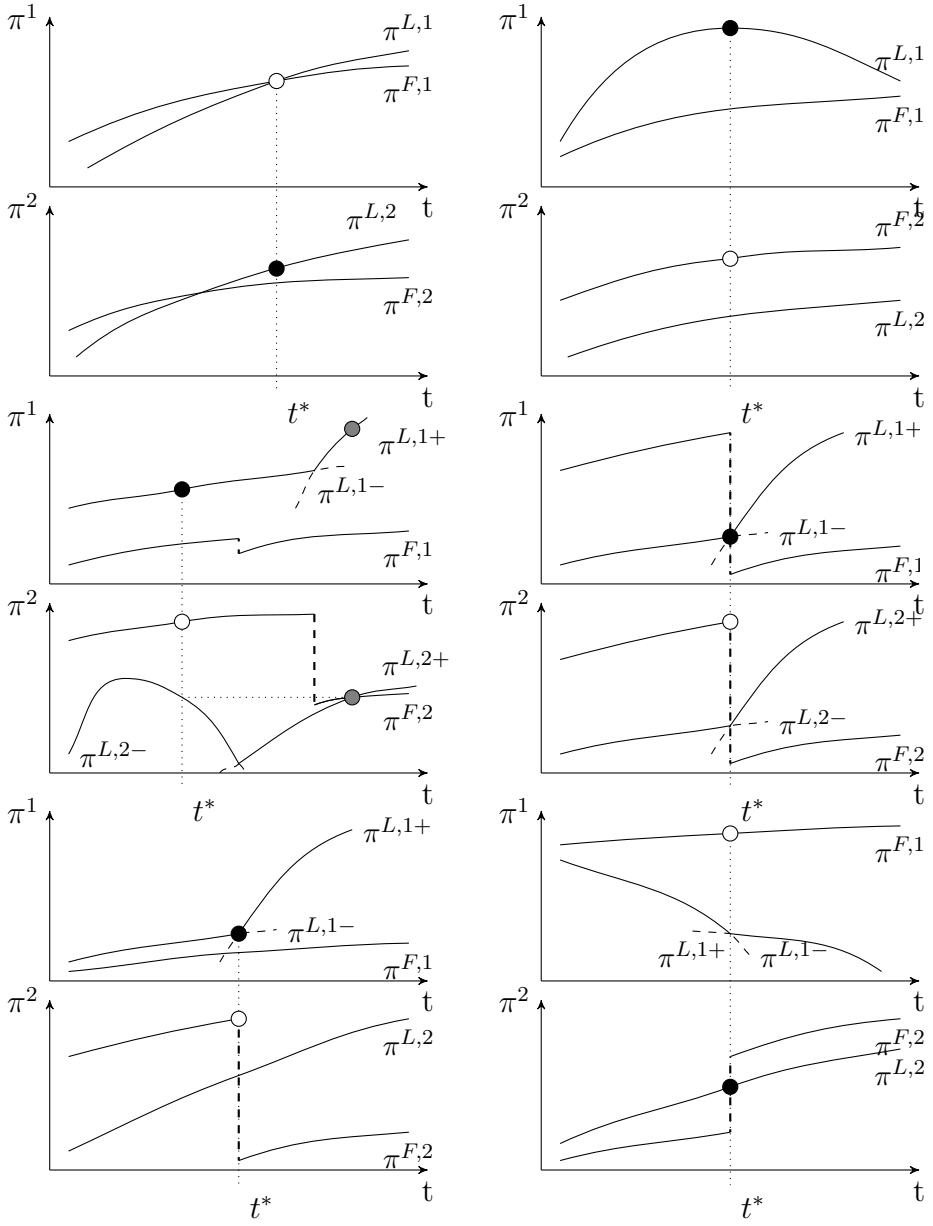


Figure 3.8: Equilibrium candidates. Black (white/grey) dot indicates equilibrium profit for leading (following/continuation). (*top left*) Preemptive investment. (*top right*) Investment without preemption. (*center left*) Forced investment due to a credible threat. (*center right*) Symmetric forced investment. (*bottom left*) Preemption with an asymmetric discontinuity. (*bottom right*) Forced investment with an asymmetric discontinuity.

they follow or lead.

Over an interval on which this holds, both players' profits from simultaneous investment, leading, coincide with her profits from simultaneous investment, following. On such an interval, equilibrium investment takes place at the earliest moment on which the slope of either player's profit curve is (weakly) negative.

In other words, it may be that under simultaneous investment, the players both delay investment, as in the joint adoption outcome described by Fudenberg and Tirole (1985). Note that the presence of such an outcome—both players tacitly delaying and then investing at the same time—depends crucially on the incentives for any active players to let its competitor enter or expand capacity. Without such incentives, such a tacitly collusive outcome would not exist; either player would have an incentive to preempt, by building $\overline{q}' + \overline{q}''$ units just before the equilibrium investment date.[69]

3.4.3 Numerical results

The computational algorithm

The model can be solved using a straightforward computational algorithm. Take any $\boldsymbol{q}$ such that the equilibrium is known for all $\tilde{\boldsymbol{q}}$ with higher aggregate capacity (i.e. $\tilde{q} > q$). I partition the timeline $[0, T]$ into a collection $\mathcal{T}$ of disjoint intervals, with the elements separated:

[69]Fudenberg and Tirole (1985), by assumption, restrict cumulative investment to one unit each for both players as they consider technology adoption rather than capacity build-up. This is why their model can have simultaneous, or clustered, investments. Argenziano and Schmidt-Dengler (forthcoming) show that clustering can occur for an alternative mechanism for three or more players. Mills (1990) also obtains clustering for some cases, but only because the maximum investment size in his model is capped at 2. The mechanism presented here, also demonstrated by Boyer et al. (2012) in a subtly different context, is different to any of these.

- at points at which optimal lead quantities change;
- at critical points of the π^L-curves; and
- at all the points at which the continuation outcome changes (either changing from delayed investment to immediate subsequent investment, or, with immediate subsequent investment, changing from one quantity to another).

The backward induction algorithm will further keep partitioning the time intervals as the continuation value changes; this is explained below. The functional assumptions made allow the number of crossings of any two π^k-curves, or of π^k and a constant, to be determined analytically, with some crossing points solved in closed-form and others numerically.[70]

In this way, I obtain a sequence of disjoint time intervals, ordered in time. Each of these elements of $\mathcal{T}$ can be classified in terms of: a) the ordering of $\pi^{k,L}$, $\pi^{k,F}$ and the continuation value $V^{k,C}$; and b) the slope of $\pi^{k,L}$. The optimal strategies and equilibrium outcomes on all of these intervals are straightforward to classify (see Appendix 3.A.1).

Close to the end of the game, investment is no longer profitable, so the default candidate equilibrium from which to start iterating is $t^C = T$, $\overline{\boldsymbol{q}}^C = (0, 0)$. Take the last element of $\mathcal{T}$, and denote by $\underline{t}$ the starting point of this element.

Then:

1. Based on the ordering of π_L^k, π_F^k and $V^{k,C}$, determine the equilibrium strategies for the players in this interval. If the equilibrium outcome is continuation, go to step 4.

[70] As decisions are taken at discrete intervals, it is immaterial to which element the cutting point is assigned.

2. Update the candidate investment time t^C and values $V^{k,C}$.

3. For both players, project $V^{k,C}$ backwards from t^C to determine the point at which it crosses the $\pi^{k,L}$-curve, or $\max\{t \in [0, \underline{t}] : \forall \tau \in [t, \underline{t}), \pi^{k,L} \leq V^{k,C}\}$. Partition the corresponding element at this point.

4. If $\underline{t} > 0$, move to the next (earlier) interval element, updating $\underline{t}$. Otherwise stop.

The outer nest of the algorithm will simply backward induct with respect to aggregate capacity. I have shown above that there exists a cap q^{MAX} to aggregate capacity. I can then solve for the equilibrium for all $\boldsymbol{q} \times [0, T]$ such that $q = q^{\text{MAX}} - 1$, and so work backward all the way to $q = 0$.

Results

For arbitrary specifications of the model, the equilibria can become quite complex. All of the different equilibrium investment types characterised above can be observed. Typical equilibrium outcomes involve tacit collusion, so that the first firm to enter will later allow the competitor into the market. Clustering of investments—both firms investing at the same moment—is common. In many cases, the equilibrium outcome is perfectly symmetric so that both firms make equal investments at the same moment.

As the demand specification differs from that used in previous sections, the model is more difficult to calibrate. I thus offer only two illustrative numerical experiments, under somewhat *ad hoc* assumptions regarding CO_2 storage demand. All the parameters are as before, except for resource demand which is now parameterised with $\sigma = 1.3$ (1.5 in the alternative

experiment), $A = 15$ (50 similarly), $\gamma = .29$, $T = 80$.

These experiments indicate that an isoelastic demand specification makes the fixed costs of investment (using the same specification and calibration as in the previous section) rather important. More specifically, the typical investments made, whether under a social planner, a duopoly, or a monopoly, tend to be much larger than under linear demand, for comparable levels of ultimate investment; and the outcomes between a social planner and a monopolist diverge much more than in the case with linear demand.

The equilibrium paths of the experiments are shown in Figure 3.9. I use pipeline increments of 20 $MtCO_2$ per annum. In the top case, elasticity of demand is low ($\sigma = 1.3$). The equilibrium features preemption, with both firms waiting until $t = 4.5$, at which point both want to invest into a 100 $MtCO_2$ pipeline. Only one of these investments takes place; no further investments are made, and both firms make zero profits. The socially optimal outcome is to immediately build an 80 $MtCO_2$ pipeline, followed by a further 180 $MtCO_2$ of capacity at time $t = 38$. A monopolist would build a very small, 40 $MtCO_2$ pipeline immediately and refrain from further investment. Thus, in this case, preemption holds; all profits are zero, and only one firm is active in the market. Nevertheless, total capacity is held back compared to the efficient case as the duopolists do not consider consumers' surplus in their decisions.

With more elastic demand ($\sigma = 1.5$), and also a higher implicit carbon price, so that a social planner's cumulative investment would reach 580 $MtCO_2$, the equilibrium outcome features tacit collusion with a second-mover advantage. That is, both firms want a 100 $MtCO_2$ pipeline to be

built at time $t = 11$, but moreover both prefer this pipeline to be built by the other firm. The firm which follows on the first investment then enters by building 140 $MtCO_2$ of capacity at time $t = 34$. The firm forced to make the first investment is the one who moves second at the last instant before the investment date. Immediately following this date, it would be optimal to lead by building a much larger capacity pipeline, with no further investments in equilibrium. Neither firm wants to take their chances under this preemptive outcome as they worry they will be the one left outside the market. Both firms make positive profits; in particular, the firms' roles in the market are reversed along the equilibrium path, with the initial incumbent ending up smaller than its competitor, and also making lower profits. Thus, rent equalisation does not hold. Following the first investment, it will not make sense for the incumbent to preempt the second entrant as a very large pipeline is required to keep the other firm permanently out of the market, with ultimate capacities rising to $q(T) = 360$, instead of $q(T) = 220$ as in equilibrium.

3.5 Conclusions

Poor infrastructure investments may drive up the total, lifetime, system-wide costs of large-scale CCS. Studies to date have considered CO_2 pipelines from a social planner's perspective, seeking to minimise CCS system costs. However, these studies assume the existence of a regulator with the powers to mandate or incentivise the construction of the optimal network. These studies also ignore the possible exhaustibility of geological storage capacity.

I have studied the interaction between set-up costs and market power

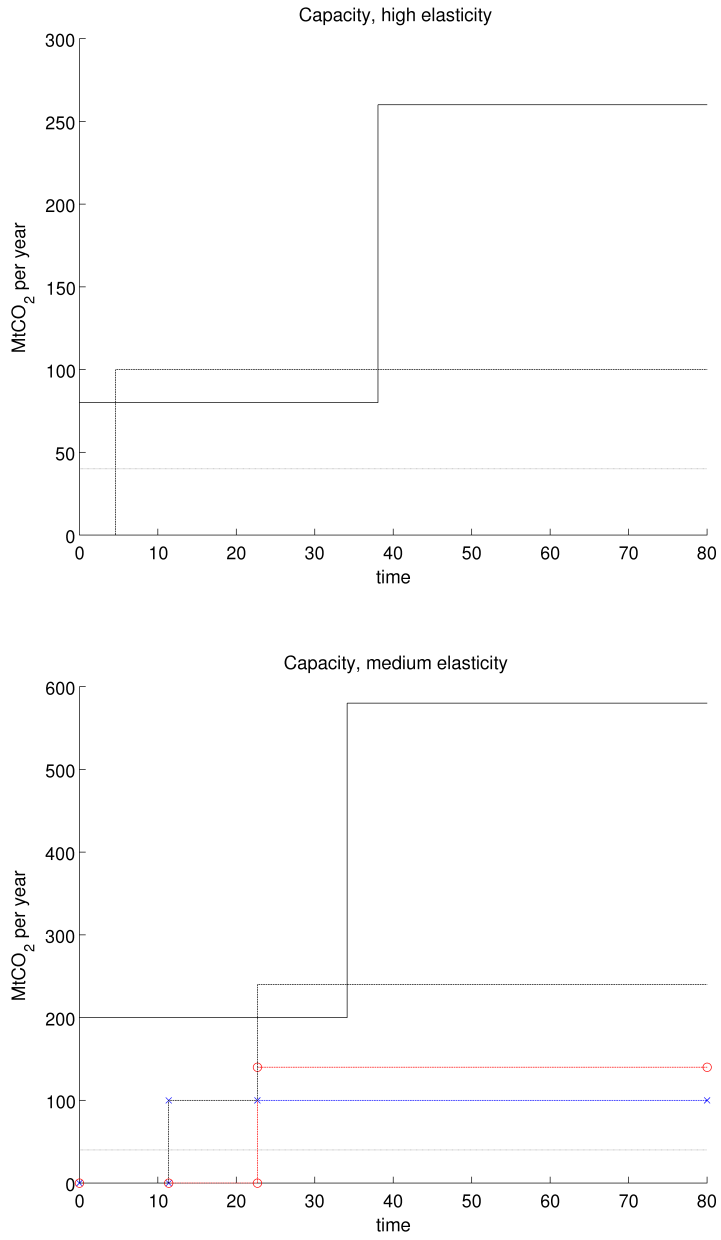


Figure 3.9: Comparison of the preemptive investment schedule (*dashed*: *blue with crosses, red with circles*: individual firms, *black*: aggregare), with the socially optimal (*solid*) and monopolistic (*dotted*) outcomes. The two graphs differ with respect to demand elasticity and the implicit carbon price: the bottom case involves more elastic demand, but also a higher carbon price, in the sense of leading to higher cumulative investment. As the top example features preemption under duopoly, only aggregate capacity is shown.

in a market for an exhaustible resource. A well-known result is that, facing isoelastic demand, a monopolist is unable to use market power. Wanting to sell the entire resource stock, profits cannot be increased by reordering extraction over time. This changes when the monopolist gets to choose the date and magnitude of investment, and when demand is such that the monopolist is able to affect the share of overall surplus she captures by her extraction choices. A monopolist can then create market power by extending the period over which she extracts the resource, or by committing to high future extraction by overinvesting. I have shown this effect to hold for linear demand rising sufficiently quickly, but the feature will hold for other types of 'similar' demand functions.

I have applied the above model to CCS pipeline investment for the case of a single sink, single source. In principle, a monopolistic carbon sink may invest excessively early into CCS infrastructure, or invest in too much infrastructure. These results hold when storage capacity is effectively exhaustible, as it might be if, for example, the overall capacity of saline aquifers happened to be systematically overestimated. However, I have shown that these effects are not very substantial.

A much more important concern arises when storage capacity is plentiful, as would be the case were the massive saline aquifers under the North Sea brought online for CCS. While this would of course be good news overall, it increases the potential losses due to unilateral action by monopolistic suppliers. Specifically, there is a risk that the monopolist will underinvest as she is unable to capture the entire social surplus due to CCS. Such concerns may have very large welfare effects and it seems appropriate that the nations concerned coordinate their policies.

I have also considered the case of duopolistic supply of carbon storage services. Dynamic competition in terms of pipeline investment may lead to various qualitatively different outcomes. One possible outcome is preemptive competition, with the two suppliers engaged in such a tough race to be the first mover that they squander any rents due to market power in the process. However, a tacitly collusive outcome is also possible. Such tacit collusion does not require any threats of punishment, but arises naturally as the firms recognise that allowing a competitor to build up some capacity makes the competitor less hungry in the future. In any case, the ultimate capacity is likely to lie somewhere between the socially optimal and the monopolistic outcome, as one would expect.

At present, the model remains highly stylised. In particular, I have ignored the effect of uncertainty on CCS deployment. This has been in order to retain analytical tractability and to focus on the effect of market power and set-up costs, in the specific case of exhaustible storage capacity. As uncertainty is a key element of the environment in which these long-term investment decisions are made, I will briefly discuss how it might affect the results of the paper.

An important source of uncertainty for CCS investment is policy uncertainty. In the present paper, I have assumed carbon pricing to be exogenous. In reality, the persistence of carbon pricing could not be taken for granted. Furthermore, the trajectory of carbon prices might be stochastic: changes in policy regimes, or new information regarding the severity of climate change, might cause unpredictable movements in the carbon price. A real option model would be required for modelling such scenarios. Intuition would suggest that the optimal solutions would involve the deci-

sionmakers delaying investment until the carbon price was sufficiently high, so as to be satisfied of making an expected profit. This would imply higher storage rates than in the present paper, to exhaust capacity before the technology revolution. Assuming carbon price shocks due to either policy changes or new information would be persistent, shocks themselves would not necessarily have much effect on the storage path following investment: a positive shock would raise the value of storage both today and in the future (Aleksandrov et al., 2013). Temporary shocks would affect storage rates immediately.

A second source of uncertainty would be with respect to the characteristics of technology, such as capture costs. From the point of view of the storage firm, capture costs affect the slope of the storage demand curve: lower capture costs imply higher demand and increase the value of delaying investment. This option value would form an extra part of the optimal trade-off in the decision of when to invest. Plausibly, however, capture costs would not be purely random, but rather something the firms would learn as they started CCS operations. This would call for government subsidies on pilot projects, in order to discover the true capture costs. As the focus of the present paper is on the region-level infrastructure investments, such pilot projects—on the scale of one or two full-scale plants—could be argued to have little effect on the present results.

Third, the decisionmakers might be uncertain about the terminal date. Using the simple framework introduced in Section 3.2.5, this could translate to the (subjective) arrival rate of the technology revolution π being a stochastic variable. An increase in the arrival rate would imply bringing forward investment.

The key question is the interaction between uncertainty and the effects of market power, as discussed in the present paper. 'Bad news' with respect to the value of storage capacity might, of course, make investment not profitable to begin with—especially if the option value had induced the decisionmaker to delay investment initially. Assuming investment remains profitable, then the social planner and the monopolist will both face the same incentives as in the deterministic case, together with the new incentives related to the option value of delaying. Thus, in principle, the incentives to invest too early and too much remain, although their effect might be muted even further. Based on the calibrated results, in the CCS setting it seems safe to ignore them.

These considerations emphasise the need to draw up contracts which ensure the entire surplus due to carbon storage is captured and shared among the parties. These parties should include all countries which either have major storage reservoirs and/or large domestic emissions. Countries without North Sea coast should be included, as several previous studies have shown that, were onshore storage not possible, even long-distance transport of captured CO_2 would be welfare-improving. The obvious question is how the surplus should be divided among the various parties. Morbee (2012) is a first comment on this question. A more detailed, dynamic treatment, taking into account the capacities of the various available reservoirs, would be a more satisfying way to assess how the surplus from CCS would be shared. However, this would be a substantially more difficult problem to tackle.

Multilateral bargaining between the countries responsible for emitting CO_2 and the providers of storage capacity is essential to ensure the entire

value of potential CO_2 storage capacity is used. This paper thus presents further arguments for multilateral coordination of CCS policies (Heitmann et al., 2012). On the other hand, if there are grounds to believe that storage owners behave as if storage capacity were exhaustible, then the potential losses arising from lack of coordination may not be very high.

Appendix 3.A Proofs

Proof of Proposition 12. Rewrite equations (3.8) as function $\lambda_S(t_S)$ to get

$$\lambda_S^R(t_S) = \overline{p}_0 \frac{\rho}{\gamma} \frac{e^{\gamma T} - e^{\gamma t_S}}{e^{\rho(T-t_S)} - 1} - \frac{\rho \xi S_0}{e^{\rho(T-t_S)} - 1}$$
$$\lambda_S^I(t_S) = \overline{p}_0 e^{\gamma t_S} - \sqrt{2\rho I \xi}$$

with the superscripts R and I referring to the resource constraint and the investment FOC, respectively. Any interior solution will have these functions crossing. Differentiating both,

$$\frac{\partial \lambda_S^R}{\partial t_S} = -\frac{\rho}{e^{\rho(T-t_S)} - 1} \left(\overline{p}(t_S) - \lambda_S(t_S) e^{\rho(T-t_S)} \right)$$
$$\frac{\partial \lambda_S^I}{\partial t_S} = \gamma \overline{p}(t_S)$$

To see that $\lambda_S^I(t_S)$ crosses $\lambda_S^R(t_S)$ from below, evaluate the following at the intersection:

$$\begin{aligned}
\frac{\partial \lambda_S^I}{\partial t_S} - \frac{\partial \lambda_S^R}{\partial t_S} &= \left(\gamma + \frac{\rho}{e^{\rho(T-t_S)}-1}\right)\overline{p}(t_S) - \frac{\rho}{e^{\rho(T-t_S)}-1}\lambda_S(t_S) \\
&= \left(\gamma + \frac{\rho}{e^{\rho(T-t_S)}-1} - \frac{\rho}{e^{\rho(T-t_S)}-1}\frac{\rho}{\gamma}\frac{e^{\gamma(T-t_S)}-1}{e^{\rho(T-t_S)}-1}\right)\overline{p}(t_S) \\
&\quad + \frac{\rho}{e^{\rho(T-t_S)}-1}\frac{\rho\xi S_0}{e^{\rho(T-t_S)}-1} \\
&= \left(\gamma + \frac{\rho}{e^{\rho(T-t_S)}-1}\left(1 - \frac{\rho}{\gamma}\frac{e^{\gamma(T-t_S)}-1}{e^{\rho(T-t_S)}-1}\right)\right)\overline{p}(t_S) \\
&\quad + \frac{\rho}{e^{\rho(T-t_S)}-1}\frac{\rho\xi S_0}{e^{\rho(T-t_S)}-1} \\
&> 0
\end{aligned}$$

as all terms are positive; note that $\left(1 - \frac{\rho}{\gamma}\frac{e^{\gamma(T-t_S)}-1}{e^{\rho(T-t_S)}-1}\right) \in (0,1)$ as $\frac{e^{\alpha x}-1}{\alpha}$ is increasing in α, and I have assume $\rho > \gamma$. Hence the two curves can cross only once. □

Proof of Proposition 13. From (3.8b) it is clear that S_0 does not affect the day-one extraction rate $q(t^*)$. Note that we can write $\dot{q} = \rho q - \frac{\rho-\gamma}{\xi}\overline{p}$. Clearly, given $q(t^*)$, the evolution of q as a function of time elapsed since investment is independent of S_0. Hence, if $\frac{\partial t_S^*}{\partial S_0} \geq 0$, the resource constraint will not be satisfied, as $\int_{t^*}^{T} q(t)\,\mathrm{d}t \leq S_0$. Hence it must be that $\frac{\partial t_S^*}{\partial S_0} < 0$.

Similarly, an increase in I raises $q(t^*)$; from $\dot{q}$ above, it is apparent that the entire extraction path must rise at all times. This clearly implies that t^* must rise, as otherwise the resource constraint is broken: $\frac{\partial t_S^*}{\partial I} > 0$.

To analyse the sign of $\frac{\partial t_S^*}{\partial \rho} \geq 0$, solve (3.8b) for $\lambda_S(t_S^*)$ and substitute into (3.8a), to get

$$\overline{p}_0 e^{\gamma t_S} - \sqrt{2I\rho\xi} - \overline{p}_0\frac{\rho}{\gamma}\frac{e^{\gamma T} - e^{\gamma t_S}}{e^{\rho(T-t_S)}-1} + \frac{\rho\xi S_0}{e^{\rho(T-t_S)}-1} = 0$$

which implicitly defines t_S as a function of the parameters. Implicitly differentiating and after some algebra, I obtain

$$\frac{\mathrm{d}t_S}{\mathrm{d}\rho} = \frac{\frac{1}{2\rho}\sqrt{2I\rho\xi} + \lambda_S(t_S)\left[\frac{1}{\rho} - \frac{(T-t_S)e^{\rho(T-t_S)}}{e^{\rho(T-t_S)}-1}\right]}{\overline{p}_0 e^{\gamma t_S}\left[\gamma + \frac{\rho}{e^{\rho(T-t_S)}-1}\right] - \lambda_S(t_S)\frac{\rho e^{\rho(T-t_S)}}{e^{\rho(T-t_S)}-1}} \tag{3.24}$$

The denominator of (3.24) is just $\frac{\partial \lambda_S^I}{\partial t_S} - \frac{\partial \lambda_S^R}{\partial t_S}$ (see proof of Proposition 12), and thus positive.

The sign of the numerator is positive if

$$\frac{\rho(T-t_S)e^{\rho(T-t_S)}}{e^{\rho(T-t_S)}-1} \leq \frac{\overline{p}_0 e^{\gamma t_S} - \frac{1}{2}\sqrt{2I\rho\xi}}{\overline{p}_0 e^{\gamma t_S} - \sqrt{2I\rho\xi}} \tag{3.25}$$

The right hand side is clearly strictly greater than 1, decreases with t_S, and increases with I. Note that for $\alpha > 0$, $x \geq 0$

$$\begin{aligned}
\lim_{x\to 0} \frac{\alpha x e^{\alpha x}}{e^{\alpha x}-1} &= \lim_{x\to 0} \frac{\alpha e^{\alpha x}(1+\alpha x)}{\alpha e^{\alpha x}} = 1 \\
\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\alpha x e^{\alpha x}}{e^{\alpha x}-1}\right] &= \frac{\alpha e^{\alpha x}}{e^{\alpha x}-1}\left[1 - \frac{\alpha x}{e^{\alpha x}-1}\right] \geq 0 \\
\lim_{x\to\infty} \frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\alpha x e^{\alpha x}}{e^{\alpha x}-1}\right] &= \rho
\end{aligned}$$

where I have used L'Hôpital's Rule. Thus, clearly the LHS of (3.25) is decreasing in t_S and approaches 1 as $t_S \to T$. Hence, for t_S close to T (i.e. if resource stock is low), $\frac{\partial t_S}{\partial \rho} > 0$ (assuming positive profits). Furthermore, suppose this is the case and consider decreasing I. The RHS of (3.25), as a function of t_S, will decrease. Thus, assuming the time horizon T is sufficiently long, eventually there will be a point such that the functions given by the two sides of (3.25) will cross. For such an I, it is possible to find an optimal solution (with high enough S_0) such that $\frac{\partial t_S}{\partial \rho} < 0$.

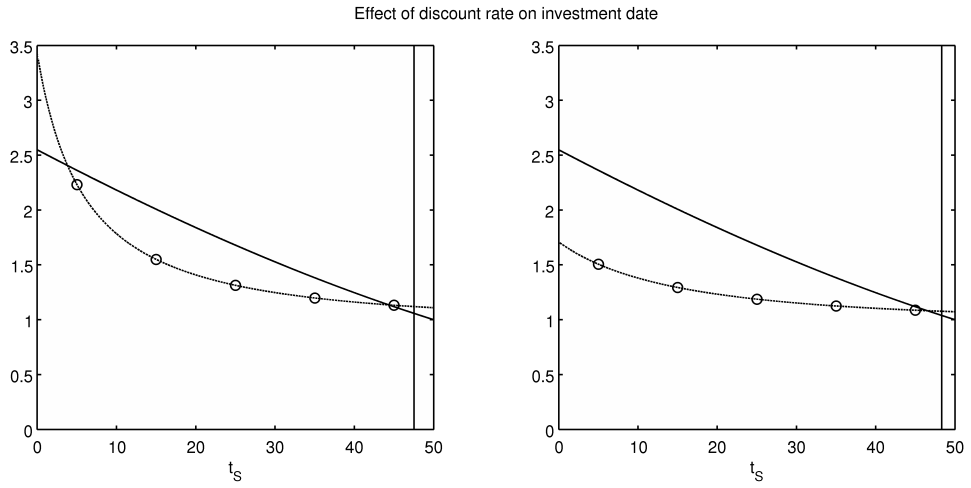


Figure 3.10: The effect of the discount rate on socially optimal investment. The curves illustrate the two sides of equation (3.25), as a function of optimal t_S. $\frac{\partial t_S}{\partial \rho} > 0$ if the dotted line with circles lies above the solid line. For high investment cost (*left*), an increase in the discount rate delays optimal investment is t_S is close to T (low resource stock) or close to 0 (high resource stock). For intermediate values, the effect is reversed. Investment is not profitable for resource stocks which imply t_S to the right of the vertical line. For lower investment cost (*right*), the regime for high initial stock may not be reversed.

Note also that the RHS of (3.25) approaches infinity as $\overline{p}(t_S) \to \sqrt{2I\rho\xi}$ from above. As the LHS definitely takes a finite value at such t_S, it is clearly possible that the inequality holds for a sufficiently low t_S (i.e. as long as the resource stock is sufficiently large). Thus, for a large resource stock, it may again be that $\frac{\partial t_S}{\partial \rho} > 0$. This requires that the time horizon is long enough.

In other words, the effect of a marginal increase in ρ on t_S depends on the parameters and can be non-monotonic in the resource stock, being positive for low or high stock levels and negative for intermediate levels. It is not difficult to find parameter combinations which yield the above non-monotonicity and I illustrate this with a numerical example in Figure 3.10. □

Proof of Lemma 4. Following investment, the stochastic event kills off the profit stream for good. It is well known that, with the event arriving as a Poisson process with arrival rate π, the optimal extraction plan coincides with the optimal extraction plan in a deterministic setting with a discount rate $\rho + \pi$. I will adapt the more general proof from (Dixit and Pindyck, 1994).

In a short interval of time Δt, the equation of transition for the price ceiling is $\Delta\overline{p} = \rho\overline{p}\Delta t - \overline{p}\Delta x$, with Δx a random variable: $P(\Delta x = 0) = \pi\Delta t$, $P(\Delta x = 1) = 1 - \pi\Delta t$. It is obvious that if the event arrives, the choke price $\overline{p}$ becomes zero for all time. The resource stock evolves deterministically: $\Delta S = -q\Delta t$.

Suppose that the decisionmaker has to commit to a given extraction rate q for a time interval $\mathrm{d}t$. The Bellman equation for this interval is

$$V(S, \overline{p}, t) = \max_{q} \left\{ v(S, \overline{p}, q)\Delta t + (1 + \rho\Delta t)^{-1} \mathbb{E}\left[V + \Delta V\right] \right\}$$

in which $v(\cdot)$ indicates the decisionmaker's flow utility. Because of the finite time horizon, the problem is not stationary and thus t has to be included as an argument on V.

Clearly $V(S, 0, t) = 0$ for all S, t. Hence

$$\begin{aligned} \mathbb{E}\Delta V &= -V(S, \overline{p}, t)\pi\Delta t + \left(V(S - q\Delta t, \overline{p} + \rho\overline{p}\Delta t, t + \Delta t) - V\right)(1 - \pi\Delta t) \\ &\equiv -V(S, \overline{p}, t)\pi\Delta t + \Delta V^{\mathrm{c}}(1 - \pi\Delta t) \end{aligned}$$

in which I denote the evolution of the value function, given the policy breakdown does not occur, by ΔV^{c}. In words, if the event arrives, the value is killed off; otherwise it evolves deterministically.

Multiplying by the inverse discount factor $(1+\rho\Delta t)$ and rearranging gives

$$(\rho+\pi)\Delta t V = \max_q \left\{ v(S,\overline{p},q)\Delta t(1+\rho\Delta t) + \Delta V^{\mathrm{c}}(1-\pi\Delta t) \right\}$$

Dividing by Δt, and letting the time interval go to zero, $\frac{\Delta V^{\mathrm{c}}}{\Delta t} \to \frac{\mathrm{d}V}{\mathrm{d}t}$, and $\rho\Delta t$ vanishes, giving

$$\begin{aligned}(\rho+\pi)V &= \max_q \left\{ v(S,\overline{p},q) + \frac{\mathrm{d}V}{\mathrm{d}t} \right\} \\ &= \max_q \left\{ v(S,\overline{p},q) - \frac{\partial V}{\partial S}q + \frac{\partial V}{\partial \overline{p}}\rho\overline{p} + \frac{\partial V}{\partial t} \right\}.\end{aligned} \tag{3.26}$$

Following investment, the decisionmaker's first-order condition is to set $\frac{\partial v}{\partial q} = \frac{\partial V}{\partial S}$, i.e. to set the marginal benefit equal to the scarcity rent. Furthermore, by using the envelope theorem and observing that for neither the social planner nor the monopolist does the immediate payoff flow depend on the resource stock, $(\rho+\pi)V_S = \frac{\mathrm{d}^2 V}{\mathrm{d}t\,\mathrm{d}S}$. This means the scarcity rent V_S increases at the rate $\rho+\pi$. Thus, given investment has taken place at t^*, the decisionmaker's expected value is given by the deterministic solution using the discount rate $\rho+\pi$. For a small enough π, the terminal date T remains binding. The value function can thus be solved in closed form for region of the state space following investment.

I will denote the value function when the investment has not been made by $F(S,\overline{p},t)$. Before investment, (3.26) holds for the option value $F(\cdot)$, with $q=0$ and, hence, $v(S,\overline{p},0)=0$. The two stages are tied together by the

value-matching condition, and the smooth pasting condition, respectively:

$$V(S_0, \overline{p}(t^*), t^*) = F(S_0, \overline{p}(t^*), t^*) + I \tag{3.27}$$

$$\frac{\partial V(S_0, \overline{p}(t^*), t^*)}{\partial \overline{p}} = \frac{\partial F(S_0, \overline{p}(t^*), t^*)}{\partial \overline{p}} \tag{3.28}$$

Subtracting the two HJB equations from one another, and evaluating at $t = t^*$:

$$\begin{aligned}(\rho + \pi)I &= \tilde{\rho}(V - F) \\ &= q\left(\frac{v}{q} - V_S\right) + V_t - F_t\end{aligned} \tag{3.29}$$

I can consider the system in $(t^*, \overline{p}(t^*))$-space. Consider the values and option values along the locus of points at which optimal investment occurs. Any two points on this locus satisfy $\mathrm{d}V^* = V^*_{\overline{p}}\,\mathrm{d}\overline{p}(t^*) + V^*_S\,\mathrm{d}S + V^*_t\,\mathrm{d}t^*$, given $\mathrm{d}t^*$, $\mathrm{d}\overline{p}(t^*)$ sufficiently small. Note that all points on the locus satisfy $S = S_0$, so $\mathrm{d}S = 0$. Thus $\mathrm{d}V^* - \mathrm{d}F^* = (V^*_{\overline{p}} - F^*_{\overline{p}})\,\mathrm{d}\overline{p}(t^*) + (V^*_t - F^*_t)\,\mathrm{d}t^*$. Using the value matching ($V = F + I$ at all points, so $\mathrm{d}V = \mathrm{d}F$) and smooth pasting conditions, $V_t - F_t = 0$ along the investment locus. Optimal investment is thus determined by

$$\tilde{\rho}I = q\left(\frac{v}{q} - V_S\right) \tag{3.30}$$

which coincides with the deterministic case, but using the discount rate $\tilde{\rho}$. This completes the proof. □

Proof of Proposition 16. Consider a dynamic optimisation problem which is deterministic *except* for a stochastic, one-off event, which kills off all profits for good and is governed by a Poisson process with arrival

rate π. As shown in the proof of Lemma 4, the solution to this problem coincides with the solution to the same problem *absent* the Poisson process and with the discount rate augmented by π. From Dixit and Pindyck (1994), the stochastic Hamilton-Jacobi-Bellman equation for the problem of maximising resource revenues[71] is

$$\rho V(S;d) = \begin{cases} \max_q \left\{ p(q)q + \frac{1}{\mathrm{d}t}\mathbb{E}\left(\mathrm{d}V\right) \right\} & \text{if } d = 0 \\ 0 & \text{if } d = 1 \end{cases}$$

where d indicates whether the technological revolution has occurred or not. The expected change in value, as $\mathrm{d}t \to 0$, is given by

$$\mathbb{E}\left(\mathrm{d}V\right) = \pi\,\mathrm{d}t[V(S;1) - V(S;0)] + (1 - \pi\,\mathrm{d}t)\,\mathrm{d}V(S;0)$$

Here, in a short interval there is a probability $\pi\,\mathrm{d}t$ that the resource loses all value (i.e. if the technology revolution occurs); otherwise the optimisation continues as normal. Substituting into the HJB equation,

$$\begin{aligned} \rho V(S;0) &= \max_q \left\{ p(q)q - \pi V(S;0) - (1 - \pi\,\mathrm{d}t)V'(S)q \right\} \\ &= p(q^*)q^* - \pi V(S;0) - V'(S)q^* \end{aligned}$$

where the optimal extraction q^* is given by $p(q^*) + p'(q^*)q^* = V'(S)$, and clearly $(1 - \pi\,\mathrm{d}t) \to 1$ as $\mathrm{d}t \to 0$. Rearranging,

$$(\rho + \pi)V(S;0) = p(q^*)q^* - V'(S;0)q^*.$$

[71] The proof is amended for the planner case in the obvious manner.

Differentiating this with respect to S and using the envelope theorem,

$$(\rho + \pi)V'(S;0) = -V''(S;0)q^*.$$

or, writing $\lambda_S \equiv V'(S;0)$, $\dot{\lambda}_S = (\rho + \pi)\lambda_S$. This is the equation of motion for the costate variable, and it replaces the corresponding condition derived using the Maximum Principle.

The economic decisionmaker now faces a two-stage optimisation problem:

$$V(S_0) = \max_{q(t)} \int_0^T e^{-\rho t} p(q) q \, \mathrm{d}t + e^{-\rho T} \tilde{V}(S(T))$$

where $\tilde{V}(S(T))$ is the value from the second-stage problem:

$$\tilde{V}(S(T)) = \max_{q(t)} \mathbb{E} \int_T^\infty e^{-\rho t} p(q) q \, \mathrm{d}t$$

Both problem are subject to the usual resource constraints $\dot{S} = -q$, $S \geq 0$. In addition, the second-stage problem involves the stochastic arrival of the technological revolution.

The first-stage problem is solved exactly as before, but with the additional transversality condition $\lambda(T) = \tilde{V}'(S(T))$ which yields the optimal quantity of stock to retain into the second stage.

The necessary conditions for the second stage are as in equation 3.2, except that I will denote the second-stage costate by $\tilde{\lambda}$, with the equation of motion for the costate

$$\dot{\tilde{\lambda}} = (\rho + \pi)\tilde{\lambda}.$$

Now note that $q = \frac{\overline{p} - \tilde{\lambda}}{\xi}$ and the infinite horizon imply that exhaustion occurs at $\tilde{T}$ given by $\overline{p}_0 e^{\rho \tilde{T}} = \tilde{\lambda}_0 e^{(\rho+\pi)t}$, or $\tilde{T} = \frac{1}{\rho+\pi} \log\left(\frac{\overline{p}_0}{\tilde{\lambda}_0}\right)$. Substituting

this into the expression for $\tilde{V}(S(T))$ above, and using Leibniz's rule, I can differentiate the value function:

$$\begin{aligned}\tilde{V}'(S) &= \int_T^{\tilde{T}} e^{-(\rho+\pi)(t-T)} \left(p'(q)q + p(q)\right) \frac{\mathrm{d}q(t)}{\mathrm{d}S(T)}\,\mathrm{d}s \\ &= e^{(\rho+\pi)T} \int_T^{\tilde{T}} \tilde{\lambda}_0 \left(-\frac{e^{(\rho+\pi)t}}{\xi}\right) \frac{\mathrm{d}\tilde{\lambda}_0}{\mathrm{d}S(T)}\,\mathrm{d}s \\ &= \tilde{\lambda}(T) \left(-\frac{1}{\xi}\frac{\mathrm{d}\tilde{\lambda}(T)}{\mathrm{d}S(T)}\frac{1}{\rho+\pi}\left(e^{(\rho+\pi)(\tilde{T}-T)} - 1\right)\right)\end{aligned}$$

where I have immediately removed the term $e^{-(\rho+\pi)(\tilde{T}-T)}p(q(\tilde{T}))q(\tilde{T})\frac{\mathrm{d}\tilde{T}}{\mathrm{d}S(T)}$ as $q(\tilde{T}) = 0$; used the FOC on q; and integrated.

I now write the resource constraint as a function of $\tilde{\lambda}_0$ and $S(T)$:

$$\int_T^{\tilde{T}(\tilde{\lambda}_0)} \frac{\overline{p}_0 e^{\rho t} - \tilde{\lambda}_0 e^{(\rho+\pi)t}}{\xi}\,\mathrm{d}t - S(T) = 0$$

and use the implicit function theorem to obtain

$$\frac{\mathrm{d}\tilde{\lambda}(T)}{\mathrm{d}S(T)} = -\frac{\xi(\rho+\pi)}{e^{(\rho+\pi)(\tilde{T}-T)} - 1} < 0.$$

Then clearly $\tilde{V}'(S(T)) = \tilde{\lambda}(T)$, i.e. the costate variable is continuous at T. As $S(T) \to 0$, $\tilde{T} \to T$, and $\tilde{\lambda}(T) \to \overline{p}(T)$. As $S(T) \to \infty$, $\tilde{T} \to \infty$ and $\tilde{\lambda}(T) \to 0$. Thus, $\tilde{\lambda}(T)$, as a function of $S(T)$, decreases monotonically from $\overline{p}(T)$ to 0. On the other hand, $S(T) = S_0 - \int_0^T q(t)\,\mathrm{d}t$ is an increasing function of $\lambda(T)$. Thus, there exists a unique level λ_0 (equal to $\tilde{\lambda}_0$) which gives the optimal extraction schedule. Note that the lower limit for λ_0 is that level which implies full extraction by T.

Similarly, using the implicit function theorem on the resource constraint

again, I can obtain

$$\frac{\mathrm{d}\tilde{\lambda}(T)}{\mathrm{d}\pi} = -\frac{(\rho+\pi)\int_T^{\tilde{T}} \tilde{\lambda}_0 t e^{(\rho+\pi)t}\,\mathrm{d}t}{e^{(\rho+\pi)(\tilde{T}-T)}-1} < 0.$$

In other words, an increase in the hazard rate π will lower $V'(S(T))$ for any $S(T)$, leading to a lower optimal $S(T)$. As $\pi \to \infty$, $\lambda_0 \to \lambda_0|_{T \text{ terminal}}$, i.e. the solution approaches that with a fixed terminal date T. As all the variables are continuous, a sufficiently high π thus implies that $t_M^* < t_S^*$ as in the main text. □

Proof of Proposition 17. The socially optimal Hotelling Rule is

$$\dot{q} - \rho q = \frac{\gamma - \rho}{\xi}\overline{p} \tag{3.31}$$

For a given capture tax path, the Hotelling Rule the monopolist follows is given by

$$\dot{q} - \rho q = \frac{1}{2\xi}\left((\gamma-\rho)\overline{p} - \frac{\dot{\theta}}{(1-\theta)^2}\lambda\right) \tag{3.32}$$

where λ is determined by the resource constraint $\int_{t^*}^{T} q(t)\,\mathrm{d}t \leq S(0)$ and the monopolist's first-order condition

$$q(t) = \frac{\overline{p} - \frac{1}{1-\theta(t)}\lambda}{2\xi}$$

For any $\lambda(t) = \lambda(0)e^{\rho t} \geq 0$, equating (3.31) and (3.32) implies that the socially optimal capture tax path satisfies

$$-\frac{\dot{\theta}}{(1-\theta)^2} = \frac{\overline{p}(0)}{\lambda(0)}(\gamma-\rho)e^{(\gamma-\rho)t} \leq 0$$

which establishes the sign of $\dot{\theta}$. This can also be solved to get

$$1-\theta = \frac{\lambda}{\overline{p}+K\lambda}, \qquad K>0$$

with K an arbitrary constant.

For any given path $\theta(t)$, the monopolist's profits are

$$\pi^M = \int_{t^*}^{T} e^{-\rho t}(1-\theta)pq\,\mathrm{d}t - e^{-\rho t^*}(I+\tau_I) \tag{3.33}$$

and optimal investment date is given by

$$\begin{aligned}
\frac{\mathrm{d}\pi^M}{\mathrm{d}t^*} &= \int_{t^*}^{T} e^{-\rho t}(1-\theta)(\overline{p}-2\xi q)\frac{\mathrm{d}q(t)}{\mathrm{d}t^*}\,\mathrm{d}t \\
&\qquad - \big(1-\theta(t^*)\big)e^{-\rho t^*}\,pq|_{t=t^*} + \rho e^{-\rho t^*}(I+\tau_I) \\
&= e^{-\rho t^*}\left(\lambda(t^*)\int_{t^*}^{T}\frac{\mathrm{d}q(t)}{\mathrm{d}t^*}\,\mathrm{d}t - \big(1-\theta(t^*)\big)\,pq|_{t=t^*} + \rho(I+\tau_I)\right) \\
&= e^{-\rho t^*}\left(-\xi q(t^*)^2\big(1-\theta(t^*)\big) + \rho(I+\tau_I)\right) \\
&= 0
\end{aligned}$$

assuming an interior solution, and where I have used the monopolist's FOC and the derivative (with respect to t^*) of the resource constraint. This yields the $q(t^*)$; setting this to be equal to that chosen by the social planner determines the optimal tax on investment.

Finally, using the socially optimal values in (3.33), the optimised profits are

$$\pi^M_{\mathrm{OPT}} = (1-\theta)(\overline{p}S_0 - 2I)$$

which is positive as long as $\theta \leq 0$ and the bracketed term is positive. Provided this is the case, any positive profits can be given to the monopolist

by setting θ sufficiently low. □

Proof of Proposition 18. With n investments, there are at most $2n$ stages: following each investment there may be a stage in which the cap does not bind, after which there is a stage in which the cap is binding. Given an investment schedule $\{(t_i^*, \overline{q}_i\}$, from (3.15) and the equation of motion $\dot{\lambda} = \rho\lambda$ it is apparent that, given a series of investments, the extraction rate increases monotonically until it hits the cap; and, after hitting the cap, it will remain constant until the next investment (after the last investment, the terminal time T). Clearly the next investment will never be made until the current cap has become binding; were this to be the case, profits could be increased by postponing the investment in question.

Denote the times at which the cap becomes binding by $\{\tilde{t}_i\}$ and let $t_{n+1}^* \equiv T$. With $q^*(t)$ denoting the optimal extraction quantity given the sequence of investments, I can write the profits as

$$W(\{t_i^*, \overline{q}_i\}) = \sum_{j\in\{1,\dots,n\}} \int_{t_j^*}^{\tilde{t}_j} e^{-\rho t} p(q^*(t)) q^*(t)\,\mathrm{d}t + \int_{\tilde{t}_j}^{t_{j+1}^*} e^{-\rho t} p(Q(t)) Q(t)\,\mathrm{d}t \tag{3.34}$$

and the resource constraint as

$$\sum_{j\in\{1,\dots,n\}} \int_{t_j^*}^{\tilde{t}_j} q^*(t)\,\mathrm{d}t + \int_{\tilde{t}_j}^{t_{j+1}^*} Q(t)\,\mathrm{d}t \leq S_0 \tag{3.35}$$

Differentiating the latter with respect to t_i^* ad using Leibniz's Rule, we get

$$\sum_{j\in\{1,\dots,n\}} \int_{t_j^*}^{\tilde{t}_j} \frac{\mathrm{d}q(t)}{\mathrm{d}t_i^*}\,\mathrm{d}t = \Delta_{t_i^*}\left[q(t)\right]$$

as the quantity sold in the capped periods does not change, as the quantity

path is continuous at times $\tilde{t}_j$, and as for $j \neq i$, $\frac{\mathrm{d}t_j^*}{\mathrm{d}t_i^*} = 0$. In words, a marginal delay in investment means the units not sold during this delay (proportional to the upward jump in quantity) have to be reallocated to the periods in which extraction is not capped. On the other hand, differentiating the resource constraint with respect to $\overline{q}_i$ gives

$$\sum_{j\in\{1,\ldots,n\}} \int_{t_j^*}^{\tilde{t}_j} \frac{\mathrm{d}q(t)}{\mathrm{d}t_i^*}\,\mathrm{d}t = -\sum_{j\in\{i,\ldots,n\}} \int_{\tilde{t}_j}^{t_{j+1}^*} 1\,\mathrm{d}t$$

The RHS shows how a marginal increase in the magnitude of investment i increases the extraction rate during all future periods in which extraction is capped. For the resource constraint to hold, the extraction rate has to fall during all time periods when extraction is capped.

Differentiating (3.34) with respect to t_i^*, and understanding that $q(t)$ refers to the optimal value $q^*\left(t;\left\{(t_j^*,\overline{q}_j)\right\}\right)$ and that the sums are taken over $j \in \{1,\ldots,n\}$,

$$\begin{aligned}
\frac{\mathrm{d}W}{\mathrm{d}t_i^*} &= \sum_j \int_{t_j^*}^{\tilde{t}_j} e^{-\rho t_i^*} \frac{\mathrm{d}CS(t)}{\mathrm{d}q(t)} \frac{\mathrm{d}q(t)}{\mathrm{d}t_i^*}\,\mathrm{d}t - \Delta_{t_i^*} e^{-\rho t} CS(t) + \rho e^{-\rho t_i^*} C(\overline{q}_i) \\
&= e^{-\rho t_i^*}\left(p(t_i^*)\sum_j \int_{t_j^*}^{\tilde{t}_j} \frac{\mathrm{d}q(t)}{\mathrm{d}t_i^*}\,\mathrm{d}t - \overline{p}(t_i^*)\Delta_{t_i^*}\left[q(t)\right] + \frac{\xi}{2}\Delta_{t_i^*}\left[q(t)^2\right] + \rho C(\overline{q}_i)\right) \\
&= e^{-\rho t_i^*}\left(p(t_i^*)\Delta_{t_i^*}\left[q(t)\right] - \overline{p}(t_i^*)\Delta_{t_i^*}\left[q(t)\right] + \frac{\xi}{2}\Delta_{t_i^*}\left[q(t)^2\right] + \rho C(\overline{q}_i)\right) \\
&= e^{-\rho t_i^*}\left(-\frac{\xi}{2}\left(\Delta_{t_i^*}\left[q(t)\right]\right)^2 + C(\overline{q}_i)\right) \\
&= e^{-\rho t_i^*}\left(-\Delta_{t_i^*}\left[CS(q(t)) - p(q(t_i^*)q(t)\right] + C(\overline{q}_i)\right)
\end{aligned}$$

where I have used the fact that, given the assumptions, the resource price grows at rate ρ; the derivative of the resource constraint with respect to t_i^*; the last line is straightforward to verify by calculation. Setting this equal

to zero, for optimality, yields (3.16a). To check second-order conditions, differentiate again and use the FOC to get

$$\begin{aligned}
\frac{\mathrm{d}^2 W}{\mathrm{d}t_i^{*2}} &= -e^{-\rho t_i^*} \frac{\mathrm{d}}{\mathrm{d}t_i^*} \left(\Delta_{t_i^*} \left[CS(q(t)) - p(t_i^*) q(t) \right] \right) \\
&= -e^{-\rho t_i^*} \frac{\mathrm{d}}{\mathrm{d}t_i^*} \left(\frac{\xi}{2} \left(\Delta_{t_i^*} \left[q(t) \right]^2 \right) \right) \\
&= -e^{-\rho t_i^*} \frac{\xi}{2} 2 \Delta_{t_i^*} \left[q(t) \right] \frac{\mathrm{d}}{\mathrm{d}t_i^*} \Delta_{t_i^*} \left[q(t) \right] \\
&< 0
\end{aligned}$$

as delaying investment will increase the uncapped extraction rate, and holding the cap constant, thus increase the upward jump in the extraction rate.

To obtain the FOC for $\overline{q}_i$, note that

$$\begin{aligned}
\frac{\mathrm{d}W}{\mathrm{d}\overline{q}_i} &= \sum_j \int_{t_j^*}^{\tilde{t}_j} e^{-\rho t} \frac{\mathrm{d}CS(t)}{\mathrm{d}\overline{q}_i} \, \mathrm{d}t + \sum_{j \geq i} \int_{\tilde{t}_j}^{t_{j+1}^*} e^{-\rho t} \frac{\mathrm{d}CS(Q(t))}{\mathrm{d}Q} \, \mathrm{d}t - e^{-\rho t_i^*} C'(\overline{q}_i) \\
&= \lambda(0) \sum_j \int_{t_j^*}^{\tilde{t}_j} \frac{\mathrm{d}q(t)}{\mathrm{d}\overline{q}_i} \, \mathrm{d}t + \sum_{j \geq i} \int_{\tilde{t}_j}^{t_{j+1}^*} e^{-\rho t} p(Q(t)) \, \mathrm{d}t - e^{-\rho t_i^*} C'(\overline{q}_i) \\
&= \sum_{j \geq i} \int_{\tilde{t}_j}^{t_{j+1}^*} e^{-\rho t} \left(p(Q(t)) - \lambda(t) \right) \, \mathrm{d}t - e^{-\rho t_i^*} C'(\overline{q}_i)
\end{aligned}$$

where $\lambda(t) = p(t)$ at all times such that resource sales are positive and the cap is non-binding; and using the derivative of the resource constraint with respect to $\overline{q}_i$. Setting the last line equal to zero yields (3.16b). Differenti-

ating again to get the second-order condition,

$$\begin{aligned}\frac{\mathrm{d}W^2}{\mathrm{d}^2\overline{q}_i} &= \sum_{j\geq i}\int_{t_j^*}^{\tilde{t}_j} e^{-\rho t}\left(p'(Q)-\frac{\mathrm{d}\lambda}{\mathrm{d}\overline{q}_i}\right)\mathrm{d}t + e^{-\rho t_{j+1}^*}(p(Q)-\lambda(t_{j+1}^*))\frac{\mathrm{d}t_{j+1}^*}{\mathrm{d}\overline{q}_i} \\ &\quad - e^{-\rho\tilde{t}_j}(p(Q)-\lambda(\tilde{t}_j))\frac{\mathrm{d}\tilde{t}_j}{\mathrm{d}\overline{q}_i} - e^{-\rho t_i^*}C''(\overline{q}_i) \\ &= \sum_{j\geq i}\int_{t_j^*}^{\tilde{t}_j} e^{-\rho t}\left(-\xi Q-\frac{\mathrm{d}\lambda}{\mathrm{d}\overline{q}_i}\right)\mathrm{d}t - e^{-\rho t_i^*}C''(\overline{q}_i) \\ &< 0\end{aligned}$$

where the second equality follows from the fact that either $\frac{\mathrm{d}t_j^*}{\mathrm{d}\overline{q}_i}=0$ and that $p(Q(\tilde{t}_j))=\lambda(\tilde{t}_j))$. Increasing $\overline{q}_i$ increases extraction at all times $t\geq t_i^*$ such that demand is capped; to meet the resource constraint, extraction has to fall when demand is not capped, i.e. $\frac{\mathrm{d}\lambda(t)}{\mathrm{d}\overline{q}_i}>0$.

The formulae for the monopolist are derived similarly. □

Proof of Proposition 19. From the analysis of second-order conditions in the previous proof it becomes apparent that, in present value terms, the marginal benefit of delaying investment is constant with respect to t^*, while the marginal cost is increasing (Figure 3.11). Assuming $\overline{q}_M=\overline{q}_S$, it is straightforward to show that the monopolist's marginal cost is higher for any t_* and she will thus invest earlier. Suppose the monopolist invests less than the planner: $\overline{q}_M<\overline{q}_S$. We have

$$\begin{aligned}\frac{\partial \mathrm{MB}}{\partial\overline{q}} &= \rho C'(\overline{q})>0 \\ \frac{\partial \mathrm{MC}_M}{\partial\overline{q}} &= -\frac{\mathrm{d}\lambda(t^*)}{\mathrm{d}\overline{q}}q^*<0\end{aligned}$$

as the term in the brackets is zero; the cap will not be binding by assumption, as $\lambda_S>0$. This is illustrated graphically in Figure 3.11 and it is

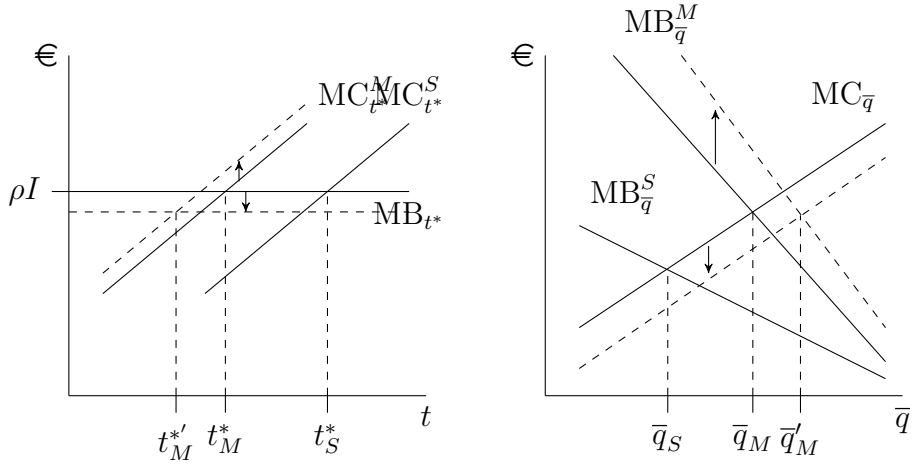


Figure 3.11: (*left*) The marginal costs and benefits of delaying investment. Given $\overline{q}$, the monopolist will invest earlier: $t_M^* < t_S^*$. Supposing the monopolist invests in less capacity, these curves shift (*dashed lines*) so that the result holds *a fortiori* ($t_M^{*'}$). (*right*) Marginal benefit and cost of pipeline capacity (*solid lines*). Supposing the planner and the monopolist invest at the same time, the monopolist invests more: $\overline{q}_M > \overline{q}_S$. Supposing the monopolist invests earlier, the marginal costs and benefits shift (*dashed lines*) and the monopolist invests in even more capacity $\overline{q}_M'$.

apparent that the monopolist will certainly invest even earlier.

A similar argument holds shows that, were the monopolist to invest later than the planner, then she would for sure invest in higher capacity. Firstly, for the same t^*, the monopolist's marginal benefit of capacity is higher than the planner's. It is straightforward to show that the marginal cost of capacity falls at t^* increases. Above we showed that the marginal cost of delay decreases with capacity, i.e. that the cross-partial derivative of the monopolist's operating profit is positive. Hence, the marginal benefit of capacity increases as t^* increases.

Thus the monopolist will always invest earlier and/or in more capacity than the social planner. □

3.A.1 Technical details of the CCS game

Continuous-time approximation

In the main text, I claim that the discrete-time equilibrium can, as the time period becomes very small, be characterised by the continuous-time description of the various equilibrium points. More precisely, I claim that the equilibrium investment times and quantities are given, in the limit, by the crossing points of the various continuous-time profit curves.

Lemma 5. In the approximation of the discrete-time framework by the continuous-time formulae, the approximation errors in $\pi^k_{\boldsymbol{q_0},t_0}(\overline{\boldsymbol{q}}', t')$ are linear in the period length κ (as the period length becomes sufficiently small).

Proof. Take the continuous-time approximation of the π^k-curves as the benchmark; for any κ, the values at the decision points do not lie exactly on this curve. I want to establish a bound on the difference ϵ^π between the values in the discrete-time formulation and the continuous-time formulation, and show that this value goes to zero as $\kappa \to 0$.

Take any initial state $(\boldsymbol{q}_0, t_0)$. Assume that, as we change κ, the sequence of states in the rest of the game does not change. I will later show how to ensure this is the case.

As time has been defined to run continuously, any integrals between two dates $\tilde{t}_1$ and $\tilde{t}_2$ hold exactly. Thus, there are two sources of approximation error in (3.19). The first is the continuation value term $e^{-\rho(t^*_{\boldsymbol{q}',t'}-t_0)}V^k(\boldsymbol{q}', t^*_{\boldsymbol{q}',t'})$ (here described before the subsequent investment), in which the value of the subsequent state is approximate, and the subsequent investment date $t^*_{\boldsymbol{q}',t'}$ is not constrained to lie on the grid of decision points (as it is in the true, discrete-time model). Call the total error ϵ^{V+}.

The second source is the error in the second revenue term; the upper limit of integration is given by $t^*_{\boldsymbol{q}',t'}$, which carries an error of magnitude ϵ^{t+}. Call the second error ϵ^R. In reality, the two terms offset each other, but I conservatively add them up when obtaining error bounds. Both error terms are a function of κ, but, given κ, constant with respect to t'. Thus, this error implies that, for any κ,

$$\pi^k_{\text{TRUE}}(\overline{\boldsymbol{q}}', t') = \pi^k(\overline{\boldsymbol{q}}', t') + \epsilon^{V+}(\kappa) + \epsilon^R(\kappa)$$

i.e. the true profits are given by a sequence of points which has the shape of the continuous-time approximation, but has been shifted up or down. As I have argued in the text that the continuous-time curves have at most two crossings, so the shifted curves also cross at most twice—and thus the true values also.

These vertical shifts induce an error on the crossing point. Furthermore, there is an additional source of error: the true discrete-time formulation requires the equilibrium to lie on the grid of decision points. In fact, the equilibrium point itself may not be in the node immediately next to the crossing, but in the next one, that is, a distance of up to 2κ from the true crossing point. Hence, the approximation error in the investment timing ϵ^t, for small κ, is bounded by

$$|\epsilon^t| < 2\kappa + \frac{\left|\epsilon^{V+}(\overline{\boldsymbol{q}}^1;\kappa)\right| + \left|\epsilon^{V+}(\overline{\boldsymbol{q}}^2;\kappa)\right|}{\left|\frac{\mathrm{d}\pi^k(\overline{\boldsymbol{q}}^1)}{\mathrm{d}t'}\right| + \left|\frac{\mathrm{d}\pi^k(\overline{\boldsymbol{q}}^2)}{\mathrm{d}t'}\right|}$$

in which the continuation error terms refer to the two curves whose crossing we are considering (these could be from following, leading or simultaneous investment). The denominator contains the slopes of the curves at the

crossing; with small enough κ, the curves are close to linear and the above establishes an upper bound on the error term.[72]

Note that if the terms ϵ^{V+} are linear in κ, so is the bound on the timing error ϵ^t. Suppose that this is the case, and also that the approximation error ϵ^{t+} is similarly linear. With the initial state still $(\boldsymbol{q}_0, t_0)$, denote by $\overline{t}_j^* \equiv \max(t_j^*, t_{j,\text{TRUE}}^*)$ the latter of the continuous-time crossing date and the true investment date for the jth equilibrium investment, and, correspondingly, by $\underline{t}_j^*$ the earlier. Then the error in the optimal value for a state, ϵ^V, is also bounded and this bound is linear in κ:

$$\begin{aligned}
\left|\pi^k - \pi^k_{\text{TRUE}}\right| \leq & \left| \int_{\underline{t}_1^*}^{\overline{t}_1^*} e^{-\rho(t-t_0)} \left(p(q,t)q^k - p(q',t)q'^k\right) \, \mathrm{d}t \right| \\
& + \left| \int_{\underline{t}_2^*}^{\overline{t}_1^*} e^{-\rho(t-t_0)} \left(p(q',t)q'^k\right) \, \mathrm{d}t \right| \\
& + \left(e^{-\rho\left(\underline{t}_1^* - t_0\right)} - e^{-\rho\left(\overline{t}_1^* - t_0\right)} \right) c(\overline{\boldsymbol{q}}'^k) + \epsilon^{V+} \\
\leq & \, e^{-\rho\left(\underline{t}_1^* - t_0\right)} \left| p(q, \overline{t}_1^*)q^k - p(q', \overline{t}_1^*)q'^k - c(q_1^*) \right| \epsilon^t \\
& + \epsilon^{V+} + e^{-\rho\left(\underline{t}_2^* - t_0\right)} p(q', \overline{t}_2^*)q'^k \epsilon^{t+}
\end{aligned}$$

The final step is to observe that, for any state and investment quantity such that the following state yields no further investment in equilibrium, the errors in the continuation value, continuation timing and current timing are $|\epsilon^{V+}| = 0$, $|\epsilon^{t+}| = 0$, $|\epsilon^t| < 2\kappa$. By induction on the state space, all approximation errors are then linear in κ.

The sequence of states following any given $(\boldsymbol{q}_0, t_0)$ given by the continuous-time approximation coincides with the discrete-time approximation, provided the period length is small enough so that no potential

[72]The case in which both slopes were zero would not be a crossing of interest, as the π_L-envelope would not change at this point.

equilibrium investment point is 'jumped over' and that the approximation errors are made sufficiently small. It is straightforward to establish this argument formally by induction. □

Based on the above Lemma, it is apparent that the continuous-time equilibrium gives an arbitrarily good approximation to the discrete-time equilibrium as the period length becomes infinitesimal. The characterisation of the equilibrium below makes it clear that equilibrium points are determined by intersections of the various π^k-curves, and the equilibrium strategies hinge on various inequalities between π^L, π^F and the continuation value V^C. These quantities will be correctly ordered provided that κ is small enough. The only potential problem in the limit would be if the continuous-time approximations of two of these quantities would be exactly equal: then approximation errors could result in the discrete-time equilibrium not converging as $\kappa \to 0$. However, such a case would be a knife-edge case, not robust to a small perturbation of e.g. the cost parameters.

Characterisation of candidate equilibrium investments

Observe first that any point t' at which the π^L-curves are continuous for both players can only be an equilibrium if $\pi^{k,F} > \pi^{k,L}$ for at least one k; otherwise preemption unravels the equilibrium (unless $t' = t_0$, the starting moment of the game under consideration). Investment will occur only for cases (i)-(iii); if the conditions described do not hold, then the investing player always has an incentive to either bring investment forward or to delay it.

The only other moments at which equilibrium investment can take place involve discontinuities in one player's optimal lead quantity. I do not con-

sider cases in which both players have discontinuous lead quantities at the same moment as these are not robust to a perturbation in model parameters, except in the case in which the players both have equal capacity.

To characterise potential equilibrium investments at a point of discontinuity, I classify possible equilibrium candidates by: the continuation value $V^{k,C}$, relative to $\pi^{k,L}$, for both k; the slopes of the $\pi^{k,L}$-curves for both players; the ordering of $\pi^{-i,L}$, $\lim_{t\uparrow t'} \pi^{k,F}$ and $\lim_{t\downarrow t'} \pi^{k,F}$ for the noninvesting player $-i$. I go through these candidates one by one to rule out all scenarios which cannot be an equilibrium. Many of these are easy to rule out. For example, no equilibrium investment can (obviously) take place at which both players get a higher payoff by continuation to the next candidate. Similarly, no investment can take place where the identity of the next investor is known with certainty, and that player's π^L-curve is decreasing.

The remaining candidates have to be worked through one by one. As an example, cases (v) and (vi) are illustrated in Figure 3.12. The lines are given by the continuous-time approximations of the profit curves, which are very close to the true values. I show a few decision moments around the continuous-time 'equilibrium point'. Each period, the black dot gives the value at the beginning of the period if top moves first; the circle gives the value if bottom moves first. These are easy to confirm by constructing the decision tree in both cases. The diamond illustrates the expected continuation value in the previous period, which is just the mean between the two outcomes. It is straightforward to work through these examples to confirm that the discrete-time equilibrium investment takes place at the earliest depicted timestep (in the upper case, top invests just before the crossing; in the lower case, bottom does so but at the second step before

the crossing).

This process results in a set of conditions for equilibrium candidates (the candidates have been more intuitively described in the main text). Take some candidate moment for equilibrium investment $\tilde{t}^*$. Denote the limiting optimal lead quantities in the neighbourhood of $\tilde{t}^*$ by $\overline{q}'^{k-} \equiv \lim_{t\uparrow\tilde{t}^*} \arg\max_{\overline{q}'^k;\overline{q}'^{-k}=0} \pi^{k,L}(t,\overline{\boldsymbol{q}}')$, $\overline{q}'^{k+} \equiv \lim_{t\downarrow\tilde{t}^*} \arg\max_{\overline{q}'^k;\overline{q}'^{-k}=0} \pi^{k,L}(t,\overline{\boldsymbol{q}}')$. Denoting the investing player with i, and evaluating all profit functions at the optimal lead quantities, equilibrium candidates then satisfy one of the following conditions:

(i) $\tilde{t}^* > t_0$, $\overline{q}'^{k-} = \overline{q}'^{k+}$, $\forall k \in \{1,2\}$: a) $\frac{\partial \pi^{i,L}}{\partial t'} \geq 0$, b) $\pi^{-i,L} - \pi^{-i,F} = 0$, and c) $\frac{\partial \pi^{-i,L}}{\partial t'} \geq 0$;

(ii) $\tilde{t}^* > t_0$, $\overline{q}'^{k-} = \overline{q}'^{k+}$, $\forall k \in \{1,2\}$: a) $\frac{\partial \pi^{i,L}}{\partial t'} = 0$, b) $\pi^{-i,L} - \pi^{-i,F} \leq 0$, and c) $\frac{\partial \pi^{-i,L}}{\partial t'} \geq 0$ if (i-b) holds with equality;

(iii) $\tilde{t}^* > t_0$, $\overline{q}'^{k-} = \overline{q}'^{k+}$, $\forall k \in \{1,2\}$: a)[73] $\pi^{i,L} - \pi^{i,F} > 0$ and $\frac{\partial \pi^{i,L}}{\partial t'} \geq 0$, b) $\frac{\partial \pi^{-i,L}}{\partial t'} \leq 0$ and $\pi^{-i,L} - \pi^{-i,F} < 0$, and c) $\pi^{-i,L} = \lim_{\tau\downarrow\tilde{t}^*} V^{-i,C}(x,\tau)$;

(iv) $\tilde{t}^* > t$, $q_0^k = q_0^{-k}$, $\overline{q}'^* = \overline{q}'^{k-} \neq \overline{q}'^{k+}$, for $i,k \in \{1,2\}$: a) $\lim_{t\uparrow\tilde{t}^*} \pi^{k,L} - \pi^{k,F} < 0$ and $\lim_{t\downarrow\tilde{t}^*} \pi^{k,L} - \pi^{k,F} > 0$, and b) $\lim_{t\downarrow\tilde{t}^*} V^{k,C}(\boldsymbol{q}_0,t) < \pi^{k,L}$;

(v) $\tilde{t}^* > t$, $\overline{q}'^* = \overline{q}'^{k-} \neq \overline{q}'^{k+}$: a) $\lim_{t\uparrow\tilde{t}^*} \frac{\partial \pi^{i,L}}{\partial t'} \geq 0$, b) $\pi^{i,L} - \pi^{i,F} > 0$, $\lim_{t\uparrow\tilde{t}^*} \pi^{-i,L} - \pi^{-i,F} < 0$ and $\lim_{t\downarrow\tilde{t}^*_{x,t}} \pi^{-i,L} - \pi^{-i,F} > 0$, and c) $\lim_{t\downarrow\tilde{t}^*} V^{k,C}(\boldsymbol{q}_0,t) < \pi^{k,L}$ for $k \in \{1,2\}$;

(vi) $\tilde{t}^* > t$, $\overline{q}'^* = \overline{q}'^{k-} \neq \overline{q}'^{k+}$: a) $\lim_{t\uparrow\tilde{t}^*} \frac{\partial \pi^{i,L}}{\partial t'} \geq 0$ and $\lim_{t\uparrow\tilde{t}^*} \frac{\partial \pi^{-i,L}}{\partial t'} \leq 0$, b) $\lim_{t\uparrow\tilde{t}^*} \pi^{i,L} - \pi^{i,F} > 0$, $\lim_{t\downarrow\tilde{t}^*} \pi^{i,L} - \pi^{i,F} < 0$, and $\pi^{-i,L} - \pi^{-i,F} < 0$; c) $\lim_{t\downarrow\tilde{t}^*} V^{i,C}(\boldsymbol{q}_0,t) > \pi^{i,L}$ and $\lim_{t\downarrow\tilde{t}^*} V^{-i,C}(\boldsymbol{q}_0,t) < \pi^{-i,L}$; or

[73] I ignore, as non-generic, the case $\pi^{i,L} = \pi^{i,F}$.

<table>
<tr><th></th><th colspan="2">$\pi^L > V^C > \pi^F$ | $\pi^L > \pi^F > V^C$</th><th>$\pi^F > \pi^L > V^C$</th></tr>
<tr><td>$\pi^L > V^C > \pi^F$
$\pi^L > \pi^F > V^C$
$V^C > \pi^L > \pi^F$</td><td colspan="2">*Row* / *Col* (equal prob), $\underline{t}$.</td><td>*Col*, t_{MAX}.</td></tr>
<tr><td>$V^C > \pi^F > \pi^L$
$\pi^F > V^C > \pi^L$
$\pi^F > \pi^L > V^C$</td><td colspan="2">*Row*, t_{MAX}.</td><td>*Col*, t_{MAX}.
Row/*Col* (equal prob), $\bar{t}$</td></tr>
</table>

Table 3.1: Equilibrium investor identity and investment date, given the rankings of lead profits π^L, follower profits π^F and continuation value V^C for both players. If both players have $V^C > \pi^L$ then no investment takes place, so these cases are omitted. t_{MAX} indicates the moment at which the investing player's π^L is maximised along the interval; as intervals are also broken with respect to critical points of π^L, this will be either the beginning or the end of the interval.

(vii) $\tilde{t}^* = t_0$.

Note that these conditions simply summarise the formal conditions which are discussed more intuitively in the main text.

Numerical equilibrium types

The numerical algorithm considers intervals $[\underline{t}, \bar{t}]$ along which, for each player: a) the ordering of $\pi^{k,L}$, $\pi^{k,F}$ and $V^{k,C}$ are constant, where the continuation value refers to continuation beyond the interval in question; b) the slope $\frac{\mathrm{d}\pi^{k,L}}{\mathrm{d}t'}$ does not change sign; c) the optimal lead quantity $\arg\max_{\overline{q}'} \pi^{k,L}(\overline{\boldsymbol{q}}', t')$ is constant; d) the equilibrium strategies for the continuation state are constant.

Note that if $V^{k,C} > \pi^{k,L}$ for both k, then the equilibrium outcome for each player is to continue. Thus, at least one player must have the opposite hold for investment to take place. I characterise the equilibrium outcomes in Table 3.1 below.

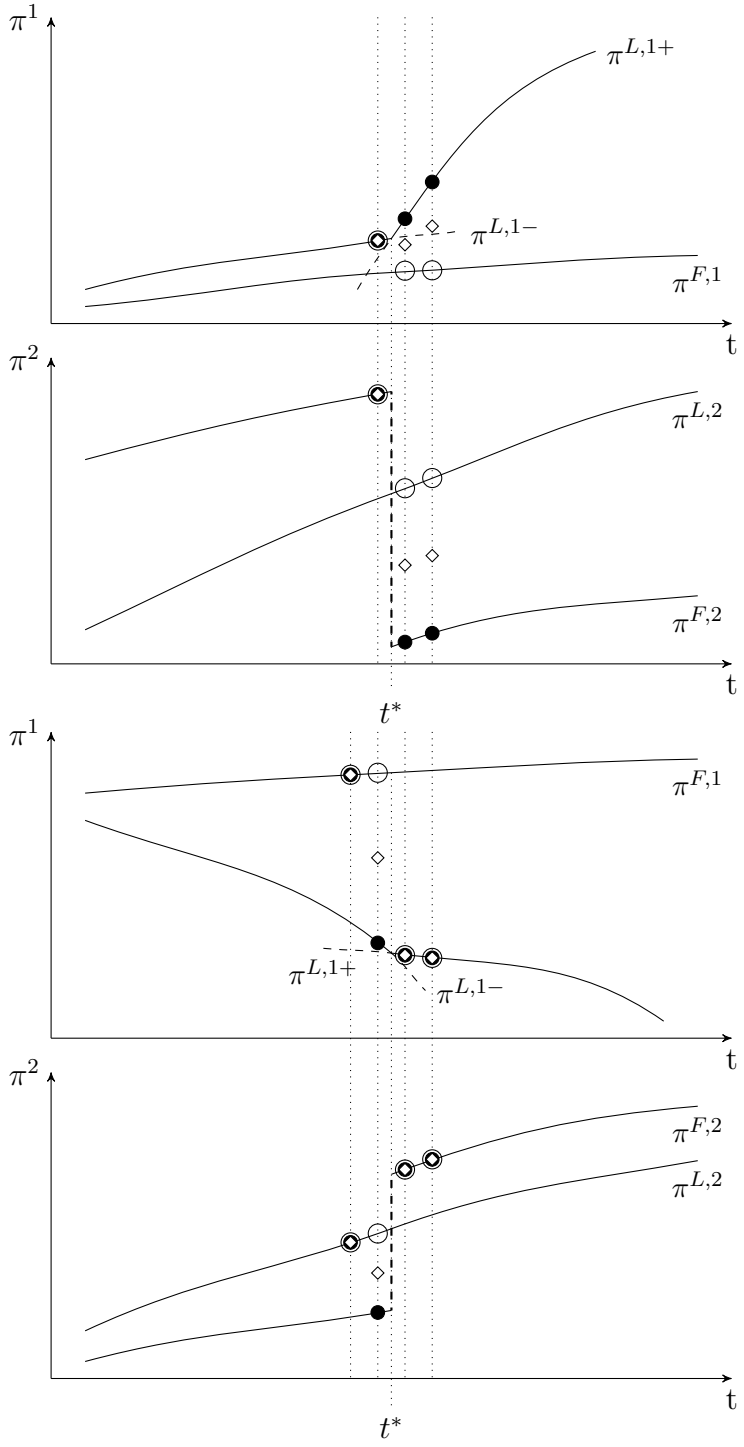


Figure 3.12: Example of the determination of the discrete-time equilibrium. The black dots (white circles) denote the outcome for both players when top (bottom) leads in a given period. The diamonds denote the expected continuation payoffs in the previous period.

Chapter 4

Can we save the planet by taxing OPEC capital income?

Abstract

Capital income taxes on resource owners' wealth alter their savings decisions, including saving in the resource asset. Such taxes can be used as an unconventional climate policy instrument in a two-country Ramsey growth model with an essential resource input. If the resource owner has no productive technologies, the aggregate efficient solution can be attained. However, the instruments distort saving decisions, leading the exporter to bring consumption forward and the importer to delay consumption. The latter also captures some of the exporter's assets. The possibility of the resource owner developing a domestic production sector limits the effectiveness of policy based on capital income taxes, as capital is allocated inefficiently. In an equilibrium with commitment, in the long run either capital income taxes are zero or the importer becomes a net investor in the exporting economy.

4.1 Introduction

The use of fossil fuels is associated with the accumulation of greenhouse gases in the atmosphere. These gases trap heat and cause the Earth's climate to shift, damaging both human welfare and economic productivity. One of these fossil fuels, oil, is predominantly supplied by a cartel: The Organization of the Petroleum-Exporting Countries (OPEC). This cartel presently supplies 38% of all oil, and this fraction will only increase as other suppliers gradually run down their reserves (BP, 2012). Selling oil is a lucrative business, for some. OPEC is considered to be able to extract the marginal barrel for less than $10, possibly as low as $3 (Adelman and Watkins, 2008), while oil prices at the time of writing are in the neighbourhood of $90 per barrel. The resulting profits make OPEC a rentier economy, producing little else but oil. As a result, these countries are reliant on Western asset markets for their saving: according to the SWF Institute, OPEC countries have some $1.9tn deposited in sovereign wealth funds.[1]

Some have considered whether carbon taxes could be used by the consuming countries to appropriate a part of the excess profits, or resource rents, earned by oil-exporting countries (Liski and Tahvonen, 2004). In other words, the oil-consuming countries, reliant on foreign suppliers, might want to use environmental policy to also improve their terms of trade in the oil market. This paper asks the opposite question: whether the oil-consuming countries, acting as the main supplier of financial assets to OPEC countries, could use capital income taxes—potentially useful for appropriating resource rents after these have been deposited into bank ac-

[1] www.swfinstitute.org.

counts with Western banks—to also do a bit of climate policy?

Conventionally, economists prescribe demand-side policies—carbon pricing by taxes or tradable emission permits—for tackling the pollution externalities associated with exhaustible fossil resources. Such first-best solutions have proven difficult to implement. There have been attempts to impose taxes on carbon, but such schemes have been hobbled by exemptions given to vulnerable (energy-intensive) industries, or relaxed after popular opposition. The EU Emissions Trading Scheme has imposed a cap on EU emissions, but the level of the cap has been set so high that this has had little real impact. Negotiations to apply worldwide carbon pricing or other emission targets have floundered following the failure of the Copenhagen Summit in 2009.

According to the 'Green Paradox' (Sinn, 2008, 2012), supply-side responses may render these conventional policies ineffective. Worse, poorly designed policies may perversely accelerate the depletion of such resources, hence aggravating the pollution problem. A rational owner of a stock of an exhaustible resource will have planned a depletion schedule such that profits cannot be increased by shifting extraction across time: along an optimal schedule, the marginal net revenues gained from selling the resource must be constant, in present value terms, at all moments. Policies which affect demand, and so the marginal net revenues, differently across time will result in the resource owner changing her depletion plans accordingly. In particular, policies perceived to cut future demand, relative to present demand, will increase present depletion rates. Such policies include rapidly rising carbon taxes, or R&D efforts to develop future energy technologies. The resource owner foresees that the resource will fetch a lower price in the

future, and seeks to sell more of the resource now. Such unintended outcomes matter: they would both bring climate impacts forward, and make them more expensive as rapid changes in the Earth's climate are likely to require more expensive adaptation than gradual changes.

The counterpart of Western reliance on OPEC oil, that is, OPEC reliance on Western asset markets, suggests a second-best policy instrument. OPEC's intertemporal optimisation hinges not only on demand for the resource, but also on the available investment opportunities, represented by the interest rate. According to the well-known Hotelling Rule, a resource owner should be indifferent between storing her wealth as oil in the ground, and as 'oil in the bank'. Recognising this, Sinn (2008) suggests that—instead of trying to adjust demand for oil—one might motivate more efficient depletion of oil reserves by adjusting the interest rate oil-exporting countries face. Financial isolation of the oil-producing countries and consequent taxation of the returns on e.g. Saudi-Arabian sovereign wealth investments could work as a form of climate policy.[2]

The present paper studies this suggestion in detail. I develop a simple differential game model with two countries: a resource exporter which owns a stock of a polluting exhaustible resource essential in production, and a resource importer who has a comparative advantage in the production of manufactured goods. In the extreme case in which the exporter has no domestic manufacturing sector, capital income taxes can indeed be used to obtain the globally efficient aggregate outcome. This is done by taxing the

[2]Of course the OPEC countries could invest in less wealthy countries, but they may be deterred from doing so by the reasons which deter investment in poor countries in general (Robert E. Lucas, 1990). Despite substantial home bias, two-thirds of all SWF investments target OECD-headquartered companies (Bortolotti et al., 2009; Dyck and Morse, 2011).

resource owners' capital income more heavily in the short-to-medium term, and loosening the taxes in the long term. Hence, in the short term, the resource owner adjusts her portfolio to include more oil in the ground and less financial wealth (as in Van Wijnbergen, 1985). This will lead to aggregate underinvestment, which must be offset by subsidising saving in the importing country. This outcome has substantial distributional outcomes, with the exporting country suffering from both some appropriation of its wealth as well as from fairly large intertemporal distortions in consumption and saving.

I then consider a more nuanced question, tackling Sinn's main argument head on: could the importing bloc conduct climate policy without caring for the intertemporal distortions suffered by the exporting countries? In other words, what is the equilibrium outcome in a game between two purely self-interested governments, the importer taxing the capital income of the the exporter only, and the exporter trying to maximise the value of its oil wealth? In this scenario, a capital income tax still expropriates some of the returns accruing to the resource owner's capital. However, it will also drive capital back to Country E, where it will be less productive. These distortions also reduce aggregate resource use: thus, the importing country has a distorting lever by which to affect resource extraction schedules. I show that, in the long run, the optimal capital income tax is weakly negative. Zero long-run taxes imply no net foreign investment in either country, with the long-run solution tending to an efficient outcome. Negative long-run taxes are possible if the importing country becomes a net investor in the exporting economies; in this case, the resource-importing country uses these taxes to push up the rate of return it obtains on its foreign investments.

The unconventional policy studied in this chapter involves discrimination based on the origin of the taxable funds. The legality of such a system, under World Trade Organization rules, is easily questioned: after all, discriminatory taxes imposed by, say, the United States on a Saudi-owned firm would comprise a limitation of market access. Sinn (2012) would respond to this question as follows: industrialised countries already have in place a system of residence-based capital income taxes. All that is required is a switch into a source-based system of capital taxation. Under the assumption that the industrialised countries can harmonise such capital income tax rates amongst themselves, the net effect on FDI between rich countries is nil. However, resource rents tend to be invested via tax havens, currently paying essentially no capital income taxes. A system of source-based capital income taxation would bring these revenues also under taxation. Thus, shutting down tax havens would have the added benefit of improving the resource owners' supply decisions.

This counterargument is correct. However, depending on the level of optimal capital income taxation, a tax authority instructed to use capital income taxes to effect supply-side improvements in fossil resource allocation would still have to differentiate between investors, based on whether they owned resources or not. This brings differentiation, and potential legal challenges to it, back into the picture. Secondly, this argument is confounding two issues: inefficiencies in the current system of capital income taxation and the use of capital income taxation to correct for an intertemporal climate externality. In this paper, I assume away any inefficiency of capital income taxes in general, in order to focus sharply on the use of capital income taxes as an instrument of climate policy. It should be noted

that the political costs of turning to such an instrument may be substantial. These costs should be borne in mind as additional arguments against such policies, even though they are not explicitly considered in what follows.[3]

The long-term model presented in this paper is a version of the so-called Dasgupta-Heal-Solow-Stiglitz (DHSS) economy, and fits into a tradition of modelling economic growth when an essential factor input is exhaustible. The global economy is divided into two countries, a resource exporter which owns the entire resource stock, and a producer which has to import the resource in order to produce goods and services. This model is augmented with a stock externality tied to the use of the exhaustible resource, to consider how such an externality affects what should be considered 'optimal' growth. I focus on a second-best case in which carbon pricing is ruled out, to bring the implications of capital taxation into sharp focus.

The line of literature on sustainable resource use and capital accumulation began with the seminal papers by Stiglitz (1974), Dasgupta and Heal (1974) and Solow (1974). Dasgupta and Heal show that, with essential exhaustible resources and fixed factors, accumulation of reproducible capital cannot compensate for the diminishing exhaustible input and that consumption must necessarily fall asymptotically to zero. Other authors have since furthered the analysis of the centralised model, including Pezzey and Withagen (1998), with Benchekroun and Withagen (2011) providing a full analytical solution to the model. A decentralised differential game solution was developed by Chiarella (1980), who divided the DHSS economy into two countries, one of which owns the resource, the other the pro-

[3]Of course, shutting down tax havens in general may be politically difficult. Such a policy would also affect domestic investors, who would resist the policy. Some of these investors may be wealthy and wield corresponding political influence. The welfare effects of closing tax havens are complicated (see Hines, 2010, for a review).

ductive technology. Saving occurs via international capital markets. The equilibrium under commitment is efficient provided both countries have equal discount factors and population growth rates; if these diverge, the long-run equilibrium is dominated by the more patient country. None of these models have included a pollution externality related to the use of the exhaustible resource; this is one of the innovations of the present study.

Concerning capital income taxation, Van Wijnbergen (1985) shows that taxing capital income will induce a resource monopolist to shift saving towards the alternative asset class—the resource—by conserving more of it for the future. The present paper shows a similar effect, with the obvious corollary that if it is socially optimal to delay extraction of the resource because of pollution, then this response in fact is desirable. Groth and Schou (2007) develop an endogenous growth model with an exhaustible resource and growth driven by positive externalities to investment. They find that, along a balanced growth path, taxing interest income (earned by reproducible capital) does not affect long-run growth rates, as these taxes are fully offset by changes in capital demand and the consequent changes in the marginal product of capital.

Similarly, Daubanes and Grimaud (2010) consider taxation of a polluting resource in a two-region endogenous growth model. They point out that, for productive efficiency, environmental taxes should be internationally coordinated; but that resource importers have incentives to set higher taxes, as these also help capture some resource rents, leading to distortions in production. This effect shows up in the present paper as well, except that the instrument considered is a capital income tax.

More realistic models show that the Green Paradox, with respect to

backstop technologies becoming less costly to produce, does not hold. Gerlagh (2011) and Van der Ploeg and Withagen (2012) together show that economic exhaustibility takes away the bite of the paradox. If the entire resource stock is not exhausted, cheaper substitutes imply lower cumulative extraction. Similarly, if alternative technologies can substitute imperfectly for fossil resources, a fall in the cost of producing the substitutes tends to lower extraction rates.

4.1.1 A note on assumptions

The present paper uses a model which has as few moving parts as possible while being still useful in analysing the Sinn (2008) suggestion. It has a number of assumptions which may be questioned, and I want to discuss these upfront. Some are inherited from Sinn's original suggestion itself.

Firstly, the model features Hotelling-type intertemporally optimising behaviour by the resource owners. This model often comes under criticism for its lack of empirical verification (Livernois, 2009; Hart and Spiro, 2011), while still being employed widely in the theoretical literature on resource economics. The Hotelling model is primarily employed due to its being used by Sinn. Despite its empirical problems, the model may still offer useful insights into future behaviour, should oil scarcity begin to bite (Hamilton, 2009).[4] It is, of course, possible to develop alternative models based on e.g. real option theory (Dixit and Pindyck, 1994; Adelman, 1990; Mason, 2001) and to consider the effects of capital income taxation in these settings.

Second, as stated above, the model assumes that climate policy is con-

[4]It should be noted that there have been public statements from Gulf countries indicating some type of intertemporal optimisation, with respect to resource depletion, is already taking place (Hamilton, 2009).

ducted only using a set of capital income taxes. This assumption is used to focus clearly on Sinn's suggestion, so as to see the implications in an extreme case. With an additional carbon tax instrument, a part of the 'climate policy burden' could be shifted off the capital income tax, allowing this instrument to focus more on appropriating OPEC financial wealth. The carbon tax would then be used partially for climate policy, and partially to appropriate oil rents (as in Liski and Tahvonen, 2004).

Third, the model assumes that the two blocs of countries operate cohesively, one as an oil cartel and the other as an importing bloc. OPEC's cohesion has often been questioned, but it is likely to increase in the future as conventional oil reserves will increasingly be concentrated in the hands of a few Persian Gulf states. More importantly, is it realistic to assume that all the relevant economies would adhere to a capital income taxation regime? An individual member of the importing bloc will always have an incentive to act as the cartel's money launderer, offering preferential interest rates while taking a cut for themselves. These types of defections might be difficult to police; although, with large sovereign wealth investments, the actual capital stocks would accumulate somewhere and might raise questions. This assumption does constitute a weak part of the model. However, such assumptions are required for Sinn's suggestion to work in the first place: without coordinated policies, the oil-exporting countries would simply shift their capital into a non-taxing state. The model here allows for some oil-importing countries to be allied with OPEC, but coalition membership is taken as given.

The paper is divided into 5 parts. In Section 4.2, I set up the basic model and consider the benchmark case of the social optimum. In Section

4.3, I set up the market institutions for this economy and consider efficient policies in a simple special case. In Section 4.4, I develop the open-loop Nash equilibrium for the economy. Section 4.5 concludes.

4.2 Social optimum

Consider an economy which consists of two countries. One of these countries, denoted E, is an exporter of an exhaustible resource; the other country, denoted I, imports the resource. The resource is essential in production.[5] Both countries have access to a technology for producing a homogeneous good:[6]

Assumption 5. Production technology. Country i has a constant-returns-to-scale production technology given by

$$F_i(\Omega_i, K_i, R_i, L_i) = \Omega_i K_i^{\alpha} R_i^{\beta} L_i^{(1-\alpha-\beta)} \tag{4.1}$$

with K_i denoting capital, L_i labour, R_i the flow of the exhaustible resource, and $\alpha, \beta > 0$ parameters. $\Omega_i \geq 0$ is a Hicks-neutral total factor productivity term (with strict inequality holding for at least one country). Aggregate output is denoted $F = F_I + F_E$.

Assumption 6. Investment. Output is either consumed or invested, so that aggregate capital stock $K \equiv K_I + K_E$ evolves according to[7]

$$\dot{K} = F - C \tag{4.2}$$

[5] In the sense of Dasgupta and Heal (1979).

[6] All variables are functions of time; this dependence is omitted, for notational clarity, where possible.

[7] Note that capital does not depreciate. It would be straightforward to incorporate a constant depreciation rate.

where $C \equiv C_I + C_E$ denotes aggregate consumption.

Assumption 7. Resource extraction. The stock of the exhaustible resource (owned by Country E) is denoted $S(t)$, with the initial stock given: $S(0) = S_0$. The flow of the resoure is denoted by $R \equiv R_I + R_E$:

$$\dot{S} = -R \tag{4.3}$$

The resource can be extracted costlessly.

Assumption 8. Economic growth. Both economies grow at a constant and identical rate:[8]

$$\hat{\Omega}_I = \hat{\Omega}_I = g \tag{4.4}$$

with $g < \frac{(1-\alpha)\rho\sigma}{\sigma-1}$ if $\sigma > 1$.

The limitation on the growth rate is required to ensure that transversality conditions are satisfied. Otherwise, an optimum does not exist: fast economic growth implies there is no need to invest for the future, so that it is always welfare-improving to consume a little bit more.

Both countries contain a mass of consumers, normalised so that the total population equals one: $L_I + L_E = 1$. Population is constant over time. I will assume perfectly inelastic labour supply throughout. The welfare of these consumers flows from per-capita consumption and the impacts of climate change:

Assumption 9. Utility. A consumer in country i derives a momentary

[8] I use the standard hat notation to denote growth rates: $\hat{X} \equiv \frac{\dot{X}}{X}$.

flow of utility given by the isoelastic utility function:

$$u\left(\frac{C_i}{L_i}\right) = \begin{cases} \frac{\left(\frac{C_i}{L_i}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} & \text{for } \sigma \neq 1, \\ \log\left(\frac{C_i}{L_i}\right) & \text{for } \sigma = 1 \end{cases} \tag{4.5}$$

Assumption 10. Climate change. Climate change impacts, in country i, cause a per-capita disutility flow of magnitude $D_i(G)$, with $D_i' \geq 0$, $D_i'' \geq 0$. Greenhouse gas concentrations G evolve according to

$$\dot{G} \equiv R + \dot{\tilde{G}}, \tag{4.6}$$

with $G(0) = 0$ and $\tilde{G}$ a given time profile of concentrations resulting from emissions outside the model.

All agents in the economy discount the future at the common rate ρ.

This economy is effectively a Dasgupta-Heal-Solow-Stiglitz economy, augmented with a stock externality and with the production technology split into two (Dasgupta and Heal, 1974; Stiglitz, 1974). It is worth noting that as the resource is essential in production, and as the use of the exhaustible resource must eventually approach zero, if growth rates are low the economy will feature production and consumption decreasing asymptotically to zero as $t \to \infty$.

Suppose there exists a social planner wishing to implement a Pareto-optimal allocation for the economy. Take any 'welfare weight' for the importer $\lambda_I \in [0, 1]$, with the corresponding term for the exporter defined as $\lambda_E \equiv 1 - \lambda_I$. Then, the problem is to solve

$$\max_{\substack{C_I, C_E, K_I, \\ K_E, R_I, R_E}} \int_0^\infty e^{-\rho t} \sum_{i \in \{I,E\}} \lambda_i \left(L_i u\left(\frac{C_i}{L_i}\right) - L_i D_i(G) \right) \mathrm{d}t \tag{4.7}$$

subject to equations (4.1), (4.2), (4.3), (4.4), (4.5), (4.6), and the identities for aggregate capital stock and aggregate output.

I assume an equilibrium exists. The necessary conditions can be obtained by using Pontryagin's Maximum Principle. Augmenting the Hamiltonian with the equation for aggregate capital stock, the Lagrangian for the social planner's problem is

$$\begin{aligned}\mathcal{L}_{\text{SP}} = \sum_{i\in\{I,E\}} \Bigg(\lambda_i \left(L_i u\left(\frac{C_i}{L_i}\right) - L_i D_i(G)\right) + \mu_K(F_i(K_i,R_i) - C_i) \\ - (\mu_S - \mu_G)R_i \Bigg) - \nu(K_I + K_E - K)\end{aligned} \tag{4.8}$$

in which μ_K, μ_S and μ_G denote the costate variables of the corresponding state variables, and ν denotes the Lagrange multiplier associated with the capital stock. Note that μ_S and μ_G are interpreted as the shadow value of adding a unit of carbon to the economy, either underground or in the atmosphere. Holding the total amount of carbon constant, the shadow value of keeping the marginal unit of carbon in the ground, rather than emitting it, is given by $\mu_S - \mu_G$.

The necessary conditions for this problem are, for $i \in \{I, E\}$,

$$\lambda_i u'(\frac{C_i}{L_i}) = \mu_K \qquad \text{if } \lambda_i > 0; C_i = 0 \text{ otherwise} \tag{4.9a}$$

$$\mu_K \alpha \frac{F_i}{K_i} = \nu_K \qquad \text{if } \Omega_i > 0; K_i = 0 \text{ otherwise} \tag{4.9b}$$

$$\mu_K \beta \frac{F_i}{R_i} = \mu_S - \mu_G \qquad \text{if } \Omega_i > 0; R_i = 0 \text{ otherwise} \tag{4.9c}$$

$$\dot{\mu}_S = \rho \mu_S \tag{4.9d}$$

$$\dot{\mu}_G = \rho \mu_G + \lambda_I L_I D'_I(G) + \lambda_E L_E D'_E(G) \tag{4.9e}$$

$$\dot{\mu}_K = \rho \mu_K - \nu_K \tag{4.9f}$$

as well as the usual transversality conditions. These are all easy to interpret. The welfare gain from the marginal unit of consumption has to equal the shadow value (in welfare terms) of capital (4.9a). Note that this welfare gain is just the marginal utility of consumption, times the weight the respective country receives in the social welfare function. Thus, if country i receives no weight, it is allocated no consumption; if both countries' welfares are equally weighted, per capita consumption is equalised across countries.

The marginal product of capital in either country has to equal the marginal product in aggregate; the marginal product of the resource has to equal the marginal cost, i.e. the scarcity rent ((4.9b) and (4.9c)). In the absence of stock-dependent extraction costs, the scarcity rent rises at the discount rate (4.9d). The shadow value of the greenhouse gas stock is just the sum of its future contributions to welfare, again adjusted by the

welfare weights, as can be seen by integrating (4.9e):

$$\mu_G(t) = -\int_t^\infty e^{-\rho t}(\lambda_I L_I D_I'(G) + \lambda_E L_E D_E'(G))\,\mathrm{d}t \leq 0$$

Similarly, the value of the marginal unit of the capital stock is just the integral of the marginal product of the capital stock from now to infinity (from (4.9f) and (4.9b)).

Foreshadowing the market outcome, I will denote the marginal product of the resource in Country I by $p_i \equiv \beta\frac{F_i}{R_i}$. I also define $\Phi(L_I, L_E, \lambda_I, G) \equiv \lambda_I L_I D_I'(G) + \lambda_E L_E D_E'(G) \geq 0$. The following two propositions characterise the social optimum.

Proposition 20. The optimal outcome is characterised as follows: per capita consumption is divided in a constant proportion, given by

$$\frac{C_I/L_I}{C_E/L_E} = \left(\frac{\lambda_I}{\lambda_E}\right)^\sigma$$

The marginal products of both capital and the resource are equated for productive efficiency: $r_I = r_E$, $p_I = p_E$. These values are thus denoted r and p, respectively. The actual allocations of both capital and the resource depend on the relative endowments of labour and the relative technology levels. Consumption grows according to the Ramsey Rule

$$\hat{C}_I = \hat{C}_E = \hat{C} = \sigma(r - \rho) \tag{4.10}$$

while resource use follows a modified Hotelling rule:

$$\hat{p} = r - \frac{\Phi}{\mu_S - \mu_G} \tag{4.11}$$

where the second term captures the effect of the environmental externality on the optimal extraction path; the term is bounded so that $\frac{\Phi}{\mu_S - \mu_G} \in [0, \rho]$. Finally, the resource constraint will be satisfied:

$$\int_0^\infty R(t)\,\mathrm{d}t = S_0.$$

Proof. In Appendix 4.A.[9] □

The properties are self-explanatory, except for (4.11). This states that, at the margin, the two assets must yield equal returns. For oil in the ground, the rate of return is just the rate of appreciation of the price $\hat{p}$. Sold and invested, it yield the interest rate r. The term $-\frac{\Phi}{\mu_S - \mu_G}$ is the 'rate of return' on the stock of carbon in the atmosphere, due to the flow of damages it causes.

Following the work of previous authors (Stiglitz, 1974; Chiarella, 1980; Pezzey and Withagen, 1998), I will define the consumption-capital ratio and the output-capital ratio:

$$x \equiv \frac{C}{K} \tag{4.12a}$$

$$y \equiv \frac{F}{K} \tag{4.12b}$$

and will proceed to solve the system in terms of these variables. The respective equations of motion are

$$\hat{x} = x - (1 - \sigma\alpha)y - \rho\sigma \tag{4.13a}$$

$$\hat{y} = \frac{1}{1-\beta}\left(g + (1 - \alpha - \beta)x - (1-\alpha)(1-\beta)y + \beta\frac{\Phi}{\mu_S - \mu_G}\right) \tag{4.13b}$$

[9] All proofs have been relegated to the appendices.

I now characterise the steady state of the economy:[10]

Proposition 21. As $t \to \infty$, the social optimum will converge to

$$x^{\infty} = \frac{(1-\beta)(1-\alpha)\rho - (\alpha - \frac{1}{\sigma})g}{\alpha\big(1-\alpha-\beta(1-\frac{1}{\sigma})\big)} \tag{4.14a}$$

$$y^{\infty} = \frac{(1-\alpha-\beta)\rho + \frac{1}{\sigma}g}{\alpha\big(1-\alpha-\beta(1-\frac{1}{\sigma})\big)} \tag{4.14b}$$

The economy is saddle-path stable. The long-run growth rates of capital stock, production and consumption are

$$\hat{K}^{\infty} = \hat{C}^{\infty} = \hat{F}^{\infty} = \frac{\sigma(g-\beta\rho)}{\sigma(1-\alpha-\beta)+\beta}$$

Proof. In Appendix 4.A. □

The steady state is independent of the externality: in the very long run, as extraction of the resource falls to zero and its marginal product rises without bound, the damages due to climate change impacts (being bounded) become immaterial (i.e. the term $\frac{\Phi}{\mu_S - \mu_G}$ in (4.13b) goes to zero as $\mu_S \gg \mu_G$). The rate of economic growth has an unambiguous positive effect on the long-run output-capital ratio. If capital is very important in production (α is higher than the elasticity of intertemporal substitution), the long-run consumption-capital ratio is decreasing in the rate of TFP growth: the faster productivity rises, the more it makes sense to invest for the future. The economy experiences long-run growth if the growth rate is high, the resource is not very important in production and/or the social planner is very patient ($g > \beta\rho$); otherwise the economy decays, with capital, consumption and production all approaching zero in the long run.

[10] I use the notation $X^{\infty} \equiv \lim_{r\to\infty} X(t)$, for any variable X.

From the equality of the marginal products above, we can obtain

$$\frac{K_I}{K_E} = \frac{R_I}{R_E} = \left(\frac{\Omega_I}{\Omega_E}\right)^{\frac{1}{1-\alpha-\beta}} \frac{L_I}{L_E} \tag{4.15}$$

and, using this in the aggregate production identity, it is straightforward to confirm the well-known result that $F(K, R, L) = QK^{\alpha}R^{\beta}L^{1-\alpha-\beta}$, with $Q = Q(\Omega_I, \Omega_E, L_I, L_E)$. In other words, the aggregate production function still has constant returns to scale and a Cobb-Douglas functional form. In the absence of climate change damages, or if the social welfare function does not take into account the welfare of those suffering damages ($\lambda_I D'_I = \lambda_E D'_E = 0$, i.e. $\Psi = 0$, $\forall G$), the aggregate economy is of the Dasgupta-Heal-Solow-Stiglitz type. This has been solved by Pezzey and Withagen (1998) (except for the TFP growth term) whose graphical construction I will use to show how the climate externality affects the optimal solution. I will restrict myself to the case $\alpha\sigma < 1$; the other cases are straightforward to solve.

The economy is illustrated in Figure 4.1. The long-run steady state is shown by the intersection of the loci $\hat{x} = 0$ and $\hat{y} = 0$. In the case $\Psi = 0$, the optimal solution will feature initial R and C chosen so that the economy starts on the saddle path. The exact point is determined by the relative abundance of factors K_0, S_0. Given any K_0, a higher S_0 implies the economy will have higher output-capital and consumption-capital ratios. To see this, consider any optimal path and then marginally increase S_0. Were the initial point not adjusted, the economy would have a strictly positive amount of the resource left unused forever. Hence, it will be optimal to increase $R(0)$ marginally, thus increasing $F(0)$ and $y(0)$. But this implies $x(0)$ will also have to increase. Part of the increased output made available

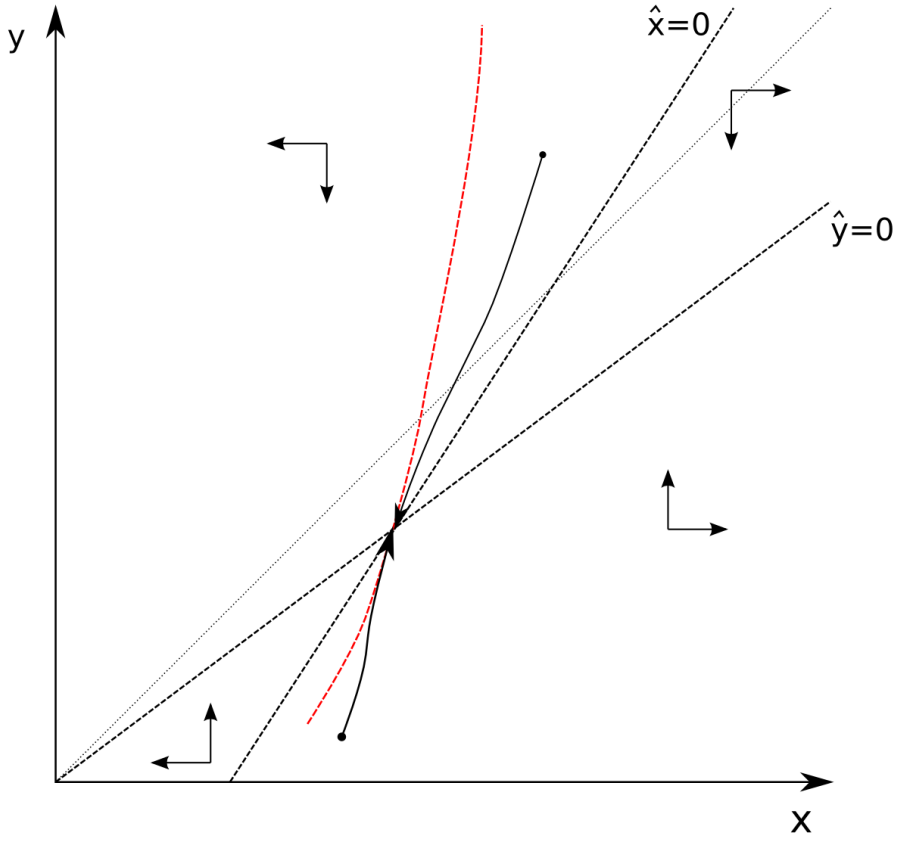


Figure 4.1: Phase diagram for the global optimum. The steady-state loci for $\hat{x} = 0$, $\hat{y} = 0$ are shown. When an externality is present, the loci of points such that $\hat{y} = 0$ will initially rise and become steeper, until eventually falling down again. Without an externality, the economy approaches the steady state along a saddle path (*red dashed line*). With climate change, the optimal saddle path (*solid line*) will start from a point to the right of the no-externality saddle-path but approach the same steady state.

by a more plentiful resource is consumed, part invested.

Now assume $\Phi > 0$ for some t. From (4.13), it is apparent that for any (x, y), this change will not affect $\hat{x}$ and it will increase $\hat{y}$. In other words, the phase arrows will 'bend upwards'. Supposing the economy now were to start anywhere on the saddlepath, or to the northwest of it, clearly the economy would diverge off the saddlepath and never be able to reach the steady state. Hence, taking climate change into account will shift the

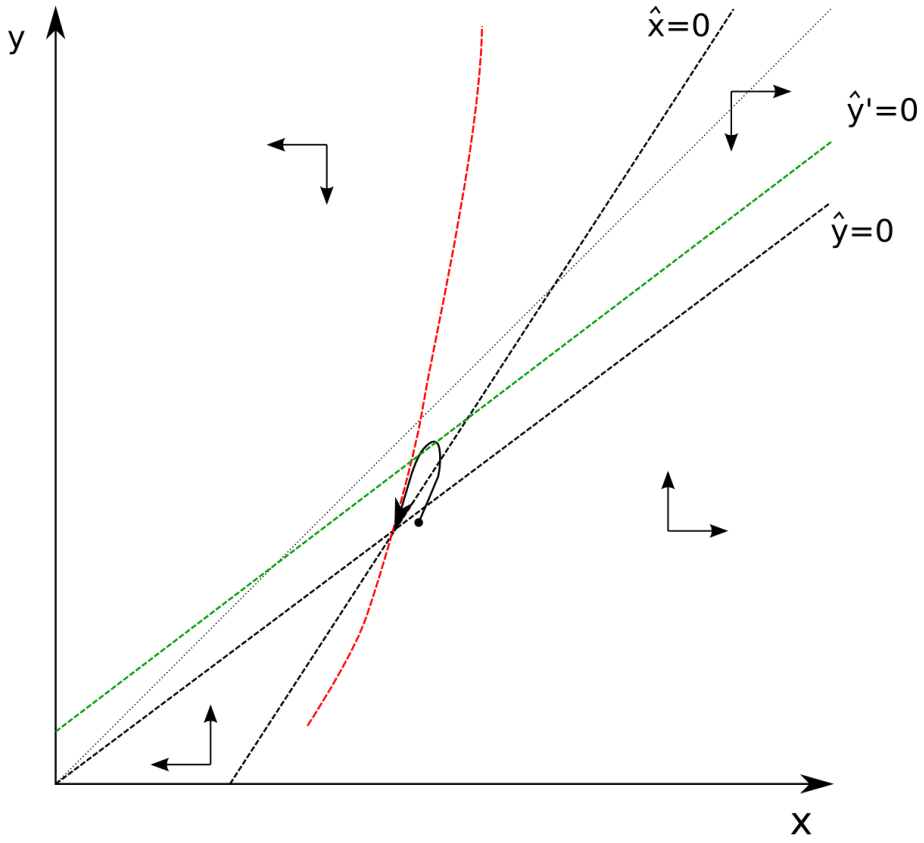


Figure 4.2: Phase diagram for the global optimum. It is possible for the economy to follow a non-monotonic path in x and y when the externality is present. The loci of points $\hat{y} = 0$ shifts gradually up, then down, with $\frac{\Phi}{\mu_S - \mu_G}$ (*green dashed line*); as $t \to \infty$, the line converges to the corresponding one for the case of decentralised equilibrium without taxes.

saddlepath right. The optimal path may start from below the steady-state capital-output ratio, 'overshoot' and then approach it from above (Figure 4.2).

To appreciate how much taking climate change into account makes the economy diverge from the optimal path in the absence of climate change, suppose the two damage function are proportional to each other, and hold aggregate damages constant, so that $D_I(G) = \eta_I D(G)$, $D_E(G) = \eta_E D(G)$, with $\eta_I + \eta_E = 1$. Then it is straightforward to see that

$$\begin{aligned} \Phi &= (\eta_I L_I + \eta_E L_E) D'(G) \\ \frac{\mathrm{d}\Phi}{\mathrm{d}\lambda_I} &= (\eta_I L_I - \eta_E L_E) D'(G) \end{aligned}$$

implying that the divergence is stronger as aggregate damage rises; and that increasing concern for a given country increases the divergence if total marginal damage, i.e. the marginal per capita damage multiplied by the affected population, is higher in that country.

Further results may be obtained by numerical methods. The parameterisation is summarised in Table 4.1. Due to the simple model structure—in particular, the absence of substitutes to the resource—this has to be regarded as a rough, back-of-the-envelope calibration and the model results should be considered as indicative only. Given this caveat, I choose parameter values which could be considered plausible.

Country I and Country E have populations of 2.5bn and 0.5bn people, respectively. Country I represents the wealthy OECD nations, Chile, South Korea and China; and Country E the OPEC countries. Country I GDP is roughly $52tn (using World Bank data for both population and GDP, with base year 2011).

Table 4.1: Parameterisation of the model (default values italicised).

Population (bn)	Country I	2.5
	Country E	.5
Initial capital stocks ($ tn)	K(0)	450
Country E's share of initial capital stock	z_0	.1
Share of capital in capital-labour composite	$\frac{\alpha}{1-\beta}$	.4
Share of the resource in production	β	.05
Total factor productivity	$\Omega_I(0)$	2.57
	$\Omega_E(0)$	1.93
TFP growth rate	g	.01
Rate of pure time preference	ρ	.03
Inverse elasticity of intertemporal substitution	σ	2
Initial resource stocks (bn bbl)	S_0	1200
Exogenous carbon emissions ($t \in [0, 150]$)	$\dot{\bar{G}}_0$	116
Convexity of climate damages	θ	2
Level of climate damages	ξ_I	1.2(-8)
	ξ_E	0

The share of oil is .05, chosen based on the share of global oil trade out of world GDP. I assume that capital accounts for 40% of the capital-labour aggregate, implying $\alpha = .38$. Based on oil demand of some 33 bn bbl per year, assuming an oil price of $90 per barrel, and with a world interest rate of .05, I pin down world capital stocks at $450 tn. I assume this is held entirely by Country I and Country E citizens; this ignores the wealth held by most low-to-middle income countries. Given these values and Country I GDP, I can pin down the TFP for Country I at 2.57. Country E TFP represents the long-term technological possibilities; I assume these possibilities are roughly similar as in non-resource rich middle-income countries.[11] I set Country E TFP level at 75% of Country I TFP, broadly consistent with Hall and Jones (1997).[12] Both countries' TFP grows exogenously at

[11]The data for the resource rich countries includes resource revenues in GDP and TFP estimates are thus not representative of those relevant to production of goods and services.

[12]I use the working paper version, instead of Hall and Jones (1999), as the former

rate .01.

The resource stock represents OPEC oil reserves of 1,200 billion barrels. Non-oil carbon emissions are exogenously given at 116 billion barrels' worth of carbon per year, for 150 years. This represents global non-oil emissions of roughly 13.6 GtC per year, in line with the SRES A1 scenarios (Nakicenovic and Swart, 2000). Following 2160, exogenous emissions are taken to be zero. Assuming the airborne fraction (the proportion of emissions retained in the atmosphere in the short term) is .65, this implies an ultimate atmospheric concentration of 1065 ppm.[13]

Climate change impacts welfare. I use a quadratic damage function: $Z(G) = \frac{\xi_i}{2}G^2$. The level of these damages is parameterised so that, assuming a temperature increase of 5 °C over preindustrial by 2200, climate change would impose a welfare hit on Country E equivalent to losing 7% of consumption; that is, the willingness-to-pay to avoid climate change altogether is 7% of consumption. I assume a climate sensitivity of 3 °C (Solomon et al., 2007), with no delays in the temperature response. This is somewhat arbitrarily chosen based on the damage parameterisation used in the DICE model, which yields a 7% hit to output for the same temperature change (Nordhaus, 2009). Country E is assumed to be immune to climate change.

I assume the inverse elasticity of intertemporal substitution to be 2, and the rate of pure time preference to be relatively high at .03. These assumptions together imply a total growth rate of 1.4% in steady state, as

uses a Hicks-neutral TFP specification as in the present model.

[13]This value is fairly well understood, and according to the IPCC Fourth Assessment Report, the current value is .55 (Solomon et al., 2007). Climate-carbon cycle feedbacks may lead to dramatic increases even in the relatively short term (Schmittner et al., 2008).

expansion of the capital stock complements TFP growth.

The optimum is illustrated in Figure 4.3, together with a counterfactual 'business-as-usual' case without climate change. The economy experiences long-run growth despite the exhaustibility of an essential input. Initial extraction rate is 28 bn bbl per year, interestingly fairly close to the actual present rate of 33 bbl per year.

Taking climate change into account implies an 8% drop in initial resource use, with extraction rates overtaking the BAU case at $t = 38$ (with time measured in years). The polluting, exhaustible resource is thus conserved. Of course, asymptotically, the stock is fully depleted. As more resource remains for the future, capital stocks only just overtake the BAU case around $t = 56$—to better utilise the higher resource inputs—despite a slowing down in initial investment. Consumption behaves similarly; for both, the differences are very minor at less that 1%.

Note that the effect of the exogenous emissions is very important, as they push up the background carbon concentrations, and so the marginal damage caused by a tonne of carbon. Even though oil by itself contributes relatively little to total concentrations, it is the marginal damage which determines how tightly a pollutant should be controlled. In other words, even though oil consumption contributes only some 30 ppm of the eventual atmospheric concentrations, at 1065 ppm (the ultimate concentration in the present model) those 30 ppm actually count for rather a lot.

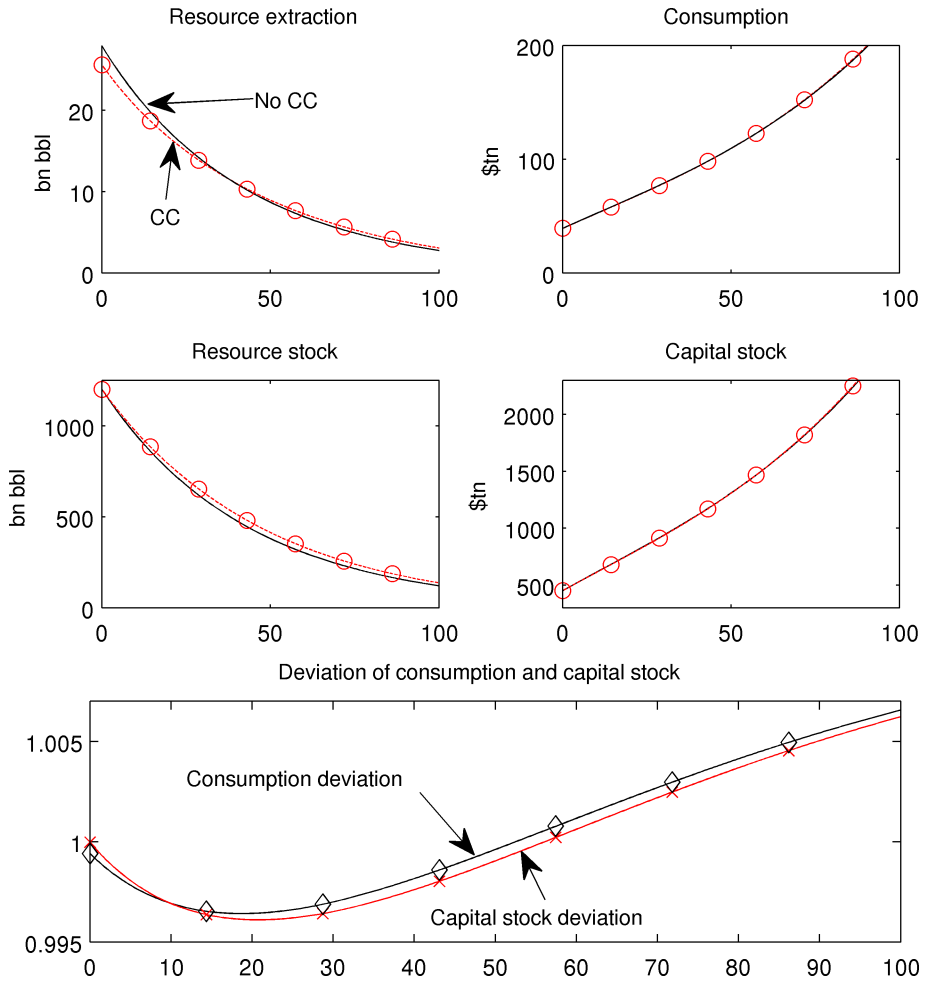


Figure 4.3: Aggregate solution for the global optimum in the business-as-usual case without climate change (*black line*), and with climate change (*red dashed line with circles*). Social welfare weights are in proportion to the relative populations. Taking climate change into account implies an immediate reduction in oil consumption of 8% over BAU; the extraction rate overtakes BAU extraction around $t = 38$. Capital accumulation and consumption slow down very little compared to BAU case, but later overtake it, as more resource is left for the far future. (*bottom*) Efficient extraction leads to a fall in early consumption and capital stocks, relative to BAU. Later, with higher resource stocks, both consumption and capital stocks exceed BAU values.

4.3 Decentralised equilibrium

Now consider decentralising the above economy. There will be three types of agents in each country: a mass of competitive firms, a mass of forward-looking but price-taking consumers, and a government. Due to reasons of tractability, I will only consider commitment (open-loop) equilibria. The resource-owning government chooses the price of the resource p; the resource-importing government chooses taxes on capital income paid to foreign investors $\tau_E \in \mathbb{R}$, and on domestic investors' income due to foreign and domestic investments $\tau_{I,E}, \tau_{I,I} \in \mathbb{R}$. The tax rates are expressed as percentage points, so that the tax rate on foreign investors (for example) is given by $\frac{\tau_E}{r_I}$, with an after-tax rate of return of $r_I - \tau_E$.

Assumption 11. No carbon taxes. There is no carbon pricing.

I make the above assumption to focus clearly on the instrument of capital income taxation in climate policy. The assumption could be justified by, for example, noting the major political difficulties in implementing carbon pricing. Thus, I focus on second-best policy.

Consider now what happens in the production sector at any moment, given that all taxes and the oil price have been set. I assume labour is immobile, but that there exist secondary markets for both capital and the resource.

Assumption 12. Instantaneous market clearing. All markets (for goods, labour in each country, capital and the resource) clear instantaneously at all times.

This assumption is natural as the model is intended to represent very long-term mechanisms. Recall that the price of goods is used as the nu-

meraire. With inelastic labour supply, wages adjust until labour is paid its share: $w_i L_i = (1 - \alpha - \beta)F_i$. The international secondary market for the resource implies that there is a uniform global resource price: $p_I = p_E = p$, and that this equals the marginal product of the resource: $p = \beta \frac{F_i}{R_i}$. Finally, in each country the rental rate for capital must equal the marginal product: $r_i = \alpha \frac{F_i}{K_i}$.

I will denote assets held by Country E and Country I consumers by A and B, respectively. Initial assets are given: $A(0) = A_0, B(0) = B_0$. The total stock of assets must equal the total amount of capital: $A + B = K$.

Assumption 13. No limits on borrowing. The representative consumers' assets may take any positive or negative value: $A, B \in \mathbb{R}$. Capital income taxes do not depend on the net position of the investor.

In other words, a tax on capital income implies a subsidy on debt interest payments. It will below become clear that taxing the capital income earned by Country E residents in Country I will drive capital to Country E. An alternative modelling assumption would be to e.g. suppose that capital income earned by foreign investors is taxed, but borrowing by foreign parties (equivalently, foreign investments of domestic investors) is not subsidised. This alternative assumption would imply that, once this process was complete, such that all capital owned by Country E residents was invested in Country E ($A = K_E$), an interest rate differential would be created and the tax would lose its bite: in terms of capital, the two economies would effectively operate autarkically. The tax would also have no further effect on resource extraction choices. The above assumption is chosen so as to analyse the Sinn suggestion in a more favourable light—in the case in which capital income taxes have the most effect on resource extraction.

Assumption 14. Balanced government budgets. The importing government balances its budget each period, collecting any necessary taxes (or refunding collected capital income taxes) lump-sum.

Note that the resource exporting government effectively has no budget. The assumption of lump-sum taxation for Country I is clearly unrealistic, but made here to eliminate features unnecessary to make the simple points I wish to make in this chapter. A more realistic model would assume the importing country had some (flow) revenue requirement, and absent capital income taxes would finance this by way of distortionary taxation of e.g. labour. A simple reduced-form way to model this would be to impose a marginal cost of public funds on such distortionary taxes (Browning, 1976). Alternatively, I could explicitly model labour supply instead of taking it as fixed.

The effects of relaxing Assumption 14 on the results which follow are fairly simple. The ability to use the proceeds of capital income taxes to offset distortionary taxation would increase the welfare benefits of the tax policy when the resource exporter owned net assets in the importing country. However, should the solution involve the exporting country going into net debt, the welfare gains from more efficient resource extraction due to capital income taxes would be offset by the additional distortions from taxes required to subsidise Country E's investment returns. With endogenous labour supply, the ability to cut income taxes in periods with high capital income taxes would increase labour supply, thus raising the marginal product of capital and the resource price. The former effect would induce a shift towards financial assets, thus accelerating resource depletion and offsetting some of the climate benefits. The latter effect would similarly encourage

oil sales in periods when labour input was higher. Provided Country E net financial assets were positive, oil extraction would be reallocated from periods with low capital income taxes to periods with high capital income taxes. A negative asset position by Country E would imply reduced labour supply when capital income taxes were high, thus reversing both effects. Of course, capital accumulation would also be affected, leading to knock-on effects on prices, so that the overall effects are difficult to gauge.

4.3.1 Consumers' problem

Consider now the problem facing a price-taking representative consumer in Country E:

$$\begin{aligned} \max_{C_E} & \int_0^\infty e^{-\rho t} L_E u(C_E/L_E) - L_E D_E(G)\,\mathrm{d}t \\ \text{s.t. } & \dot{A} = r_E A_E + (r_I - \tau_E) A_I + w_E L_E + pR(p) - C_E \end{aligned} \tag{4.16}$$

where A_E denotes assets invested domestically, A_I assets invested abroad, and $A = A_E + A_I$. I will denote Country E's share of total assets $z \equiv \frac{A}{K}$. As Country E does not impose capital income taxation, domestic assets must just yield the rental rate r_E. I assume that assets can be negative as well as positive. The solution to this problem then requires that $r_E = r_I - \tau_E$, as otherwise a money pump would exist: investors would rush to pull out their assets in the economy which yielded a lower rate of return, and furthermore would be willing to take unlimited amounts of debt to invest in the other economy.

On the other hand, considering the same problem for Country I consumers, the marginal products must satisfy $r_I - \tau_{I,I} = r_E - \tau_{I,E}$; the after-

tax rates of return on home and foreign investments must be equal. Together, these arbitrage conditions imply

$$\tau_{I,E} = r_E - r_I + \tau_{I,I} = \tau_{I,I} - \tau_E$$

In other words, to prevent money pumps, the tax rates on domestic investors have to be consistent with the tax rate on foreigners' capital income. Suppose domestic returns are untaxed ($\tau_{I,I} = 0$); then a tax τ_E on foreigners' capital income will push down the marginal product of capital abroad. Unless home investors' investments abroad are subsidised at the same rate (or interest payments on foreign debt are taxed), there would be an incentive to borrow money at cheap rates abroad, bring it home and invest domestically. I assume that $\tau_{I,E} = \tau_{I,I} - \tau_E$ always and denote, for simplicity, $\tau_I \equiv \tau_{I,I}$.

The first-order conditions yield the Ramsey Rules:

$$\begin{aligned} \hat{C}_E &= \sigma(r_E - \rho) \\ \hat{C}_I &= \sigma(r_I - \rho) \end{aligned} \tag{4.17}$$

These characterise the behaviour of aggregate consumption. Denoting Country I's share of aggregate consumption by $s_C \equiv \frac{C_I}{C}$,

$$\begin{aligned} \hat{C}_W &= s_C \hat{C}_I + (1 - s_C)\hat{C}_E \\ &= \sigma(r_I - \rho - s_C \tau_I - (1 - s_C)\tau_E) \end{aligned} \tag{4.18}$$

4.3.2 Momentary equilibrium

I will denote the demands for the resource and for capital in a momentary equilibrium by $\overline{R}_i$ and $\overline{K}_i$, and all other quantities in momentary equilibrium similarly. These are functions of the resource price and the tax rates τ_E, τ_I, $\tau_{I,E} = \tau_I - \tau_E$, as well as the aggregate capital stock K:

$$\overline{R}_i = \overline{R}_i(p, \tau_E, \tau_I, K)$$
$$\overline{K}_i = \overline{K}_i(p, \tau_E, \tau_I, K)$$

Proposition 22. $\overline{R}_i$ and $\overline{K}_i$ are uniquely defined for all $\{p, \tau_E, \tau_I\}$, $i \in \{I, E\}$. Resource demand satisfies

$$\overline{R}_i = \left(\frac{\Omega_i \beta \overline{K}_i^{\alpha} L_i^{1-\alpha-\beta}}{p}\right)^{\frac{1}{1-\beta}} \tag{4.19}$$

while capital stocks satisfy $\overline{K}_I + \overline{K}_E = K$. The shares of capital, resource and output are uniquely determined and given by

$$\frac{\overline{K}_I}{\overline{K}_E} = \left(\frac{\overline{r}_I - \tau_E}{\overline{r}_I}\right)^{\frac{1-\beta}{1-\alpha-\beta}} \left(\frac{\Omega_I}{\Omega_E}\right)^{\frac{1}{1-\alpha-\beta}} \frac{L_I}{L_E} \tag{4.20}$$

$$\frac{\overline{R}_I}{\overline{R}_E} = \frac{\overline{r}_I}{\overline{r}_I - \tau_E} \frac{\overline{K}_I}{\overline{K}_E} \tag{4.21}$$

$$\frac{\overline{F}_I}{\overline{F}_E} = \left(\frac{\overline{r}_I - \tau_E}{\overline{r}_I}\right)^{\frac{\alpha}{1-\alpha-\beta}} \left(\frac{\Omega_I}{\Omega_E}\right)^{\frac{1}{1-\alpha-\beta}} \frac{L_I}{L_E} \tag{4.22}$$

The following comparative statics hold:

$$\frac{\partial \overline{K}_I}{\partial \tau_E} < 0, \quad \tau_E \frac{\partial \overline{R}}{\partial \tau_E} \leq 0, \quad \tau_E \frac{\partial \overline{F}}{\partial \tau_E} \leq 0, \quad \frac{\partial \overline{r}_I}{\partial \tau_E} > 0, \quad \frac{\partial \overline{r}_E}{\partial \tau_E} < 0$$

$$\frac{\partial \overline{K}_I}{\partial \tau_I} = \frac{\partial \overline{R}}{\partial \tau_I} = \frac{\partial \overline{F}}{\partial \tau_I} = \frac{\partial \overline{r}_I}{\partial \tau_I} = \frac{\partial \overline{r}_E}{\partial \tau_I} = 0$$

$$\tau_E \frac{\partial \overline{K}_I}{\partial p} < 0, \quad \frac{\partial \overline{R}}{\partial p} < 0, \quad \frac{\partial \overline{F}}{\partial p} < 0, \quad \frac{\partial \overline{r}_I}{\partial p} = \frac{\partial \overline{r}_E}{\partial p} < 0$$

As $|\tau_E|$ grows arbitrarily large,

$$\lim_{\tau_E \to -\infty} r_I = \alpha \frac{F_I(K, R_I(K)}{K}, \quad \lim_{\tau_E \to -\infty} r_E = \infty$$

$$\lim_{\tau_E \to \infty} r_I = \infty, \quad \lim_{\tau_E \to -\infty} r_E = \alpha \frac{F_E(K, R_E(K)}{K}$$

Proof. In Appendix 4.B. □

In the absence of capital income taxes, as the technology is homothetic, $r_I = r_E$ and so the capital and resource are employed in the same proportions in both countries. Resource demand is then isoelastic at both country and aggregate level, with $\epsilon \equiv \left| \frac{\mathrm{d}R}{\mathrm{d}p} \frac{p}{R} \right| = \frac{1}{1-\beta}$.

Introducing capital income taxes drives a distortionary wedge between the prices of capital in the two countries as $r_E = r_I - \tau_E$. A capital income tax on Country E earnings will drive capital, resource use and production towards Country E. This will, of course, reduce aggregate output (and thus aggregate resource use) over the laissez-fair equilibrium. As the capital income tax rises without bound, it will shift an arbitrarily high fraction of capital to Country E; the rental rate in Country E is bounded below by the optimal rental rate using the entire capital stock and given the oil price p. The rental rate in Country I rises without bound. Of course, a subsidy will have the opposite effect: capital and output are driven to Country I, with the tax wedge reducing aggregate output and resource use.

Given my assumption that the tax on domestic investors' income from

Country E investments is set to prevent capital flight, capital income taxes on Country I investors do not affect the allocation of capital and hence are neutral in momentary equilibrium. Of course they do affect the incentives to consume and save, and hence will have dynamic effects.

Increasing the oil price will of course curb oil demand and hence reduce output. This will reduce the marginal product of capital. For any non-zero tax τ_E, a fall in r_I (equivalently, r_E) will increase the distorting effect of the capital income tax and thus accentuate any shift in capital away from the laissez-faire allocation (equation (4.20)). This will have a further downward impact on aggregate output and resource use.

It should now be clear that capital income taxes have effects beyond the intended outcome of motivating a more desirable resource extraction schedule. If both countries have access to a production technology, they cause inefficiencies in production, reducing the total amount of output available to the economy over time. On the other hand, these inefficiencies reduce resource demand at any moment. Finally, they distort the dynamic consumption-investment choices.

4.3.3 Globally efficient taxes and distributional issues

I will now illustrate a simple case in which the importing country is able to achieve the globally efficient outcome; that is, the outcome from which the social optimum is achievable by a given set of lump-sum transfers to reallocate consumption. It will become apparent that, assuming these transfers are not feasible, this allocation imposes a substantial cost on the exporting country, distorting intertemporal incentives to save and consume. The importing country can benefit from both a more benign climate as well as

from capturing some of the resource owner's wealth.

Suppose that the exporter has no production technology, i.e that $\Omega_E = 0$, and so $r_E = 0$, for all K_E. Now the exporter is completely reliant on Country I for saving in reproducible capital. I will thus denote $K_I = K$. Suppose further that the exporting country government does not observe the impact of its oil extraction decisions on Country I interest rate r_I, nor on the accumulation of Country I assets B. Finally, assume that $D_E(G) = 0, \forall G$. This is a reasonable approximation: assuming that the impacts of climate change, or the costs of adaptation, are small compared to resource revenues, Country E would not sacrifice its resource revenues.

Now, the exporting government's Hamiltonian is

$$\mathcal{H}_{\text{GE}} = L_E u\left(\frac{C_E}{L_E}\right) + \mu_A^E\left(r_E A + pR(p) - C_E\right) - \mu_S^E R(p)$$

where the costate variables μ_S^E and μ_A^E refer to the costates as perceived by Country E. As resource demand is isoelastic, the first order condition yields, after simplification, $\beta p = \frac{\mu_S}{\mu_A}$. As is known from Stiglitz (1976), this implies the standard Hotelling Rule:

$$\hat{p} = r_E = r_I - \tau_E \tag{4.23}$$

Proposition 23. Denoting Country I's share of consumption by $s_C \equiv \frac{C_I}{C}$, the capital income taxes to obtain the aggregate efficient outcome are

$$\tau_E = \frac{\Phi}{\mu_S - \mu_G} \in [0, \rho], \quad \tau_I = -\frac{1 - s_C}{s_C}\tau_E \tag{4.24}$$

where the costate variables refer to the social planner's costate variables

along the Pareto-optimal path for the required λ_I. Denoting Country E's share of assets by $z \equiv \frac{A}{K}$,

$$z^\infty = \frac{(1 - s_C^\infty)x^\infty - \beta y^\infty}{x^\infty - (1-\alpha)y^\infty} \tag{4.25}$$

where s_C^∞ is determined by

$$A_0 + p(0)S_0 = \int_0^\infty \exp\left(\int_0^t -r_E(s)\,\mathrm{d}s\right)(1 - s_C(t))C(t)\,\mathrm{d}t \tag{4.26}$$

and the equation of motion for s_C:[14]

$$\dot{s}_C = \sigma(1 - s_C)\frac{\Phi}{\mu_S - \mu_G} > 0 \tag{4.27}$$

Proof. In Appendix 4.B. □

Remark 1. In the absence of climate change, the laissez-faire equilibrium ($\tau_E = \tau_I = 0, \forall t$) is efficient.[15]

To induce slower extraction of the polluting resource, the resource importer must impose a positive capital income tax on the exporter. However, this drives a wedge between the two interest rates, distorting the incentives to save. Other distortions have been eliminated by the absence of Country E production technology: there is nowhere for capital to flee. Taxing the capital income of Country E leads to undersaving at the aggregate level

[14]Note that as $s_C \in (0,1)$ and as the term $\frac{\Phi}{\mu_S - \mu_G}$ is just a given function of time, (4.27) implies a unique path given any initial s_C. This implies that the total value of consumption in (4.26) is monotonic in the initial consumption share; it is straightforward to solve for the optimal initial consumption share.

[15]This follows from two observations: all agents except the resource exporter behave non-strategically; and the resource demand, given that the exporter only considers its effect on the oil price, is isoelastic. Then, by Stiglitz (1976), the optimal extraction path will coincide with the competitive path. Given this, by the Fundamental Theorem of Welfare Economics, the solution is otherwise efficient.

(see (4.18)). To compensate, an offsetting subsidy must be imposed on Country I capital income. These two taxes will obtain the aggregate efficient outcome. Note that if Country E accounts for a very small share of total consumption (maybe because it has a very small population), the countervailing distortion does not have to be very large.

Provided foreign investors have positive net positions in domestic assets, at least some of the revenues required to subsidise domestic investment can be collected from the tax on foreign sovereign wealth investments. This would reduce the need to collect extra revenues in a lump-sum fashion. No lump-sum taxes at all need to be collected when $z(t) > 1 - s_C(t)$; i.e. when Country E's share of total global asset stock exceeds their share of aggregate consumption.

The numerical solution is shown in Figure 4.4 (the aggregate solution is as in Figure 4.3). Up to .2 percentage points, or around 5% of the resource owner's capital income, is taxed away. The taxes are heavier in the short-to-medium term to induce conservation. As the resource becomes exhausted, the externality becomes less important relative to the productive value of the resource, and the capital income taxes gradually fall to zero. Thus, the last drops of oil are extracted almost efficiently.

The capital income subsidy offered to Country I residents is roughly an order of magnitude lower, or some .02 percentage points. This is because Country I has much larger weight in terms of population and assets, and so a fairly minor subsidy can correct for the investment distortion caused by reduced saving by the resource owners.

The resulting distortions on intertemporal consumption patterns are similarly more pronounced in Country E, with the introduction of the tax

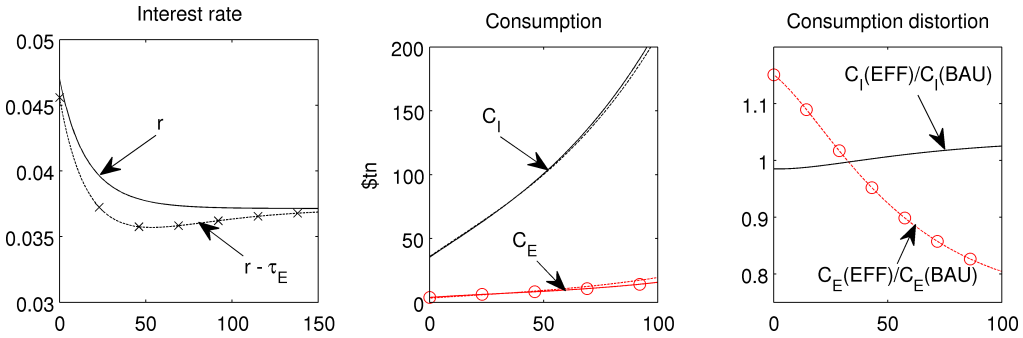


Figure 4.4: The aggregate efficient solution is achieved by taxing away a fraction of the resource exporter's capital income in the short-to-medium term (*left*). The interest rate faced by the exporter (dashed with crosses) lies below the marginal product of capital (solid). The importer faces a small countervailing subsidy (not shown). (*center*) The capital income taxes induce an intertemporal distortion in consumption (*black solid line*: Country I consumption, *red solid line with circles*: Country E consumption, *dashed lines*: respective BAU consumption). (*right*) Consumption with efficient taxes, relative to BAU (*black solid line*: Country I, *red dashed with circles*: Country E). The exporter consumes more than in the BAU case in the short run and less in the future. The importer delays consumption; however, the effects are relatively weaker as the countervailing capital income subsidy can be set at a fairly low level.

boosting consumption immediately by ∼15%, but long-run consumption being ∼25% below the business-as-usual benchmark. The subsidies encourage excess saving in Country I, with consumption falling immediately by 1.5%, and being 3.5% higher in the long run.

Welfare effects are shown in Table 4.2. All effects are measured in terms of a proportional change over the permanent consumption stream in the

Table 4.2: Welfare effects as a permanent proportional change in consumption in laissez-faire.

	Importer	Exporter
No climate change	4.9%	0.0%
Capital income taxes	0.3%	-2.3%
Revenue-neutral carbon taxes	-0.5%	6.0%

laissez-faire case. Climate damages on Country I are worth 4.9% of laissez-faire consumption, in terms of willingness-to-pay to avoid all damages; Country E is by design immune to climate change. It can be seen that implementing climate policy by the efficient capital income taxes improves Country I welfare by .3%; this incorporates the net effect of less climate change, some appropriation of Country E assets and the intertemporal distortion in consumption. Country E's consumption distortions, being more severe, are equivalent to a 2.3% consumption loss. Finally, were the efficient allocation implemented by carbon taxation, refunded lump-sum to the monopolist, the resulting cut in consumption would exceed the climate benefits, with the net effect equivalent to a .5% drop in consumption. As the carbon taxes hike up the oil price, the exporting country gains in terms of assets (see equation (4.26)), equivalent to a 6% increase in consumption.

Thus, capital income taxation as climate policy does have some benefits in terms of allowing the importing country to appropriate some of the exporter's assets, as well as moving the economy on the aggregate efficient path. The exporting country is hit quite severely, in terms of both losing some of its wealth as well as suffering from the substantial intertemporal distortions illustrated in Figure 4.4. Note that were the importing country to be the one setting carbon taxes, thus capturing some resource rents, it could do better than in the case in which the exporter collects the carbon taxes.

4.4 Equilibrium taxes and prices

I will now tackle the main question posed by Sinn (2008): should a capital income tax be imposed on foreign resource owners only, in order to motivate conservation of the polluting resource? I will do this by having both governments maximise their own citizens' welfare by using their respective tax and price instruments. I will impose $\tau_I = 0$ to avoid intertemporal distortions related to distorting the savings decisions of the large population in Country I. The capital income taxes will now also be used by the importer to capture a larger share of the pie—the stream of aggregate consumption into the future. On the other hand, as the exporter has access to production technology, but has lower productivity and a smaller quantity of labour available, the tax will drive production towards the less efficient country. This will diminish the size of the pie. In particular, this rules out attempts to appropriate fully Country E's assets employed in Country I by using taxes on capital income: very high taxes on capital income cause very severe distortions in production while encouraging Country E to take a net debt position with respect to Country I, changing the taxes into subsidies on borrowing.

To maintain tractability, I will assume both countries ignore the effects of their actions on the other's asset holdings. In other words, Country E does not recognise that its choice of p affects the investment of Country I and vice versa. This approach is not fully satisfactory. However, the alternatives are to either model strategic behaviour as a Stackelberg game, as the representative consumers behave non-strategically; or to assume governments are able to control consumers' savings decisions. The former approach has proven to be intractable to date, and would further-

more likely pose issues of time-inconsistency under the open-loop equilibrium concept. A closed-loop Stackelberg approach would not be analytically tractable.[16] Alternatively, one could assume the governments are able to control consumption-savings allocations (possibly using some tax instruments). This would not make the model any more plausible; furthermore, the analysis would still be complicated by several additional terms in the Ramsey Rules, reflecting general equilibrium and strategic concerns on savings decisions. I proceed with the slightly unsatisfactory approach of limited strategic interaction, in order to obtain at least some interesting results below.[17]

The government of Country E now solves

$$\begin{aligned} &\max_{p} \int_0^\infty e^{-\rho t} L_E u(C_E/L_E)\,\mathrm{d}t \\ &\quad \text{s.t. } \dot{A} = r_E A_E + (r_I - \tau_E)A_I + w_E L_E + pR(p) - C_E \\ &\qquad \dot{S} = -R(p) \end{aligned} \tag{4.28}$$

taking as given the time path of τ_E, B, C_I and C_E. However, the effect of the resource price on all other components of $\dot{A}$ is now taken into account when optimising: Country E observes that its choices will affect the interest rate and domestic wages, as well as shifting capital around. I have again assumed Country E to be immune to climate change. Note that $r_E = r_I - \tau_E$ and that labour expenditure share is $1 - \alpha - \beta$, which implies $\dot{A} = r_E(A - K_E) + F_E + \beta F_I - C_E$. From the derivatives (with respect to price) in Proposition 22, it is apparent that marginal revenue is always

[16] Numerical closed-loop solutions would also present difficulties as the derivatives of the value functions go to infinity near the steady state.

[17] Further work along these lines may require a fundamental rethink regarding model assumptions, in particular the potential introduction of a backstop substitute (as in Chapter 2). This is a line of inquiry for future research.

positive; while the marginal cost is the scarcity rent.

The importer solves

$$\begin{aligned} \max_{\tau_E} & \int_0^\infty e^{-\rho t} L_I u(C_I/L_I) - L_I D_I(G)\,\mathrm{d}t \\ \text{s.t. } & \dot{B} = r_I B_I + (r_E + \tau_E) B_E + w_I L_I + \tau_E(A - K_E) - C_I \\ & \dot{S} = -R(p) \end{aligned} \tag{4.29}$$

taking as given the time path of p, A, C_I and C_E. Note that domestic investments abroad are subsidised, so that $r_I = r_E + \tau_E$; and that collected tax revenues are returned lump-sum for any net inward investment (alternatively, funds for subsidies are raised by lump-sum taxes). Hence, $\dot{B} = r_E(K_E - A) + F_I - \beta F_I - C_I$. Country I receives as income its output less what it spends on oil, less any net factor payments for foreign capital or plus any income from positive net outward investment. Note that raising τ_E, assuming this is positive (negative), will tend to lower (increase) domestic output, lower the Country E rental rate, increase net outward investment $(K_E - A)$ and lower (increase) resource demand.

As in Chiarella (1980), the dynamics of the system are challenging to analyse analytically. However, the steady state is amenable to characterisation. The long-run behaviour of the economy is given by

Proposition 24. In the long run, the optimal tax satisfies $\tau_E^\infty \in (-\rho, 0]$, with the importer being a net investor in the exporting economy:

$\lim_{t\to\infty} B - K_I \geq 0$. The economy will converge to the steady state

$$y^\infty = \frac{1}{\alpha}\frac{1-\alpha-\beta}{(1-\alpha-\beta)\sigma\frac{s_F^\infty}{s_K^\infty}+\beta\frac{1-s_F^\infty}{1-s_K^\infty}}\sigma(\rho+\tau^\infty(1-s_C^\infty))) \tag{4.30}$$

$$x^\infty = \frac{1}{\alpha}\frac{1-\alpha-\beta+\alpha\beta\frac{1-s_F^\infty}{1-s_K^\infty}}{(1-\alpha-\beta)\sigma\frac{s_F^\infty}{s_K^\infty}+\beta\frac{1-s_F^\infty}{1-s_K^\infty}}\sigma(\rho+\tau^\infty(1-s_C^\infty))) \tag{4.31}$$

with the ratios s_K, s_F determined by the tax and the resulting long-term interest rates. If the long-run tax is zero, this coincides with the socially optimal steady state. The Hotelling Rule holds:

$$\hat{p} = r_E^\infty = \frac{1-s_F}{1-s_K}\alpha y^\infty$$

The importer's long-run consumption share s_C^∞ is determined by the intertemporal budget constraints. The asset shares z, $1-z$ converge to constants given by

$$z^\infty = -\frac{(1-s_C^\infty)x^\infty - \beta y^\infty - (1-\alpha-\beta)(1-s_F^\infty)y^\infty}{(1-\alpha\frac{1-s_F}{1-s_K})y^\infty - x^\infty}$$

while the consumption share converges to

$$s_C^\infty = \begin{cases} 0 & \text{if } \tau^\infty < 0 \\ \frac{\alpha}{1-\beta}(1-z^\infty) + \frac{(1-\alpha-\beta)^2}{(1-\alpha)(1-\beta)}s_F^\infty & \text{if } \tau^\infty = 0 \end{cases}$$

Proof. In Appendix 4.C. □

It should be noted that, for $\tau^\infty = 0$, $s_F^\infty = s_K^\infty$. Thus, in the long run, the economy can tend to the efficient steady state, with taxes falling to zero and the exporter gradually decreasing their foreign investments to exactly zero. Alternatively, the economy reaches a steady state with lower output-

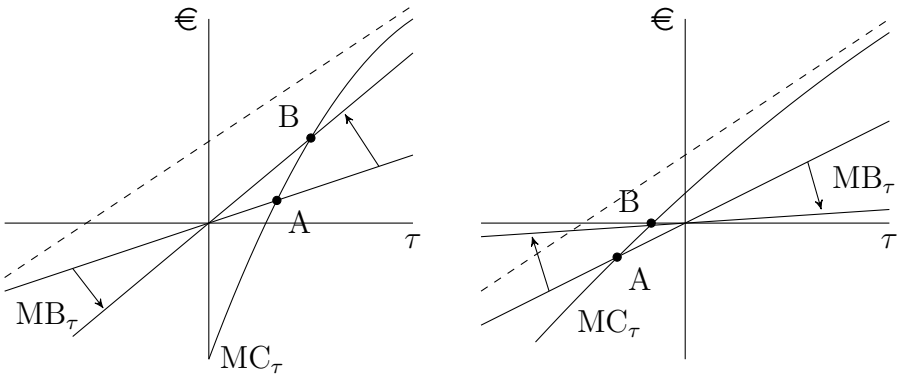


Figure 4.5: (*left*) Marginal benefit of raising capital income taxes is linear and passes through origin. Marginal cost, were all capital invested in Country E, is linear (*dashed line*). As $\tau \to \infty$, more and more capital is driven to Country E. The actual MC curve is concave and tends asymptotically to the dashed line. Suppose steady state is at A, with $\text{MC}_\tau = \text{MB}_\tau$. Then the value of the resource to the importer rises (MB rotates anticlockwise). To reduce resource use, optimal tax rates increase to B, and keep increasing; A cannot be a steady state. (*right*) Negative tax rates are possible in the long run. With a negative tax rate, the MB curve over time rotates clockwise about the origin, tending to the x-axis. Thus, a negative tax rate is feasible with Country I holding net assets in Country E (point B).

capital and consumption-capital ratios, the importing country holding net investments in the less productive country and taxing domestic investors' foreign returns.

If long-run taxes are strictly negative, then from the Ramsey Rules it is easy to see that, in the limit, $\hat{C}_I < \hat{C}_E$. This implies that, asymptotically, the exporting country consumes the entire output that is not invested. The expression for the long-run consumption share of the importer when $\tau^\infty = 0$ is derived from the budget constraints as, in the limit, all variables change at constant rates. This share is increasing in the importing country's asset share $(1-z^\infty)$ as well as the importing country's advantages in productivity and labour endowment, summarised by s_F^∞.

I will now explain why taxes cannot be strictly positive in the long run.

The importing country values the resource stock too. In the long term, climate change is insignificant compared to the value of the resource in production; in the absence of a backstop technology, the marginal product of the resource goes to infinity as resource input diminishes to zero (Dasgupta and Heal, 1974). However, the importing country captures some of this value in terms of wages. Thus, a higher resource stock still guarantees higher future wages. The importer also has an instrument by which to control this stock: it can use the distortions introduced by the capital income tax to affect aggregate resource use in the economy.

A marginal increase in the capital income tax will increase these distortions (or lower pre-existing distortions, were the tax rate negative). Thus, when taxes are positive, a marginal increase has positive value, as it conserves some more resource for the future. When taxes are negative, a marginal increase involves a cost: reducing distortions increases resource use, conserving less for the future. In fact, the marginal benefit of the tax rate is proportional to $\frac{\partial \overline{K}_I}{\partial \tau}$ times τ. Thus, at zero taxes, marginal tax increases do not really distort the economy, and thus do not affect resource extraction. As taxes go up, the marginal effect increases; but with high enough taxes, capital stock is mostly located in the exporting country, and the marginal tax increases shift very extra little capital.

On the other hand, there is also a marginal cost of raising the tax rate. This cost results from lower current wages, a lower return on any capital invested abroad, and a higher fraction of capital shifting abroad. This marginal cost is also proportional to $\frac{\partial \overline{K}_I}{\partial \tau}$, times a function linear in τ. The marginal benefit must equal marginal cost, both per unit of capital shifted,

for the capital income tax to be at an optimal level:

$$\frac{\mu_{S,I}/\mu_B}{p}\frac{\beta}{1-\alpha-\beta}\tau = \left(\frac{K_E - A}{K_E}r_E + \frac{1-\beta}{1-\alpha-\beta}\tau\right)$$

where both benefits (LHS) and costs (RHS) are measured in money terms (Figure 4.5).

Note that the marginal costs (RHS) are linear for a given (importer's) net foreign investment position $\frac{K_E - A}{K_E}$. Thus, if the importer holds net capital assets in the exporting country when the taxes are zero ($K_E > A$ for $\tau = 0$), a marginal increase in the tax rate will have a strictly positive marginal cost, as the rate of return for these assets falls. Increasing τ further will also increase the net investment position, as more capital is driven to the exporting country.

To understand why a long-run steady state cannot have strictly positive taxes, suppose this were the case. In this steady state, the marginal costs are all constant: the asset share $z = \frac{A}{K}$ is constant, the tax $\tau^\infty > 0$ is too and thus so is the share of capital employed in Country E $\frac{K_E}{K} = 1 - s_K$. Now the marginal benefit curve must be increasing, i.e. the resource is becoming more important to the importer. This is because the value of the resource to the importer, $\frac{\mu_{S,I}}{\mu_B}$ is rising at the interest rate:

$$\hat{\mu}_{S,I} - \hat{\mu}_B = r_I$$

This is the importer's Hotelling Rule. The importer can also influence resource extraction—in particular, it can kill demand by introducing severe distortions in the economy. The exporter's Hotelling Rule involves a different interest rate: $\hat{p} = r_E = r_I - \tau < r_I$. Thus, the shadow value of the

resource stock to the importer, relative to the resource price, is growing over time. This would imply increasing tax rates in order to conserve ever more of the resource (Figure 4.5).

A negative steady state tax rate, on the other hand, is feasible. In this case, the resource will become less and less valuable, over time, to the importer, so that the marginal benefit of τ (i.e. the marginal cost of *decreasing* the tax rate) will become very small; the resource-conserving effect of distorting the economy matters less and less. However, with a positive net foreign investment position, a small reduction in the tax rate (rather, taxing domestic investors' foreign returns) is beneficial, as it raises returns abroad, and also brings some capital back home, driving up wages.

It is difficult to characterise the transition dynamics further by using analytical methods. Furthermore, numerical methods have proven challenging. The model involves some variables which decrease asymptotically to zero (the capital and resource stocks, as well as the tax rate) and some which grow arbitrarily large (the resource price and marginal utility). As the first-order conditions cannot be solved explicitly, the optimal tax rate and resource price both have to be obtained numerically. The latter becomes very high even a moderate distance from the steady state, leading to great inaccuracies in the first-order conditions. Thus attempts to find the equilibrium path by reverse shooting methods have failed. Both numerical and analytical work would be made easier by the incorporation of a backstop substitute to the resource, bounding both marginal utility and the resource price. This would turn the model into one of limit pricing (as in Chapter 2). Such a change would likely partially nullify the effectiveness of capital income taxation as a climate policy instrument, as the

Hotelling Rule would hold only before the limit pricing stage. This remains a potential line of future research.

4.5 Conclusions

The use of capital income taxation has been proposed as a potential second-best (or complementary) solution to the Green Paradox (Sinn, 2008). The present paper has provided the first detailed analysis of this proposal. In principle, taxes on financial returns earned by sovereign wealth funds of resource-rich countries, together with domestic capital income subsidies, can be used as an instrument of climate policy. I have shown that climate policy based on such instruments can achieve the efficient aggregate consumption and resource depletion schedules. Such a policy can also appropriate some of the accumulated resource wealth the oil exporters have invested into Western asset markets. The costs of such a policy would be primarily borne by future generations in resource-exporting countries. The tax instruments would discourage aggregate saving in the resource-exporting economies today, leaving lower overall assets for tomorrow.

To the extent that the oil-exporting countries have domestic productive opportunities, they may respond to taxation by shifting investment to domestic assets. If these assets suffer from low productivity or low labour endowments, this will be inefficient and lower the stream of aggregate output. Because of this, even a 'selfish' climate regulator, one that acts solely in the interests of the importing countries, will not seek to tax resource owners' assets maximally. One of the motives for the importer to set higher taxes is to reduce resource extraction by imposing international

productive distortions. Presently, resource-exporting economies have much higher resource intensities than the importers. Thus, until technological development irons out such disparities, this mechanism would not work for positive taxes: driving production into exporting economies might, in fact, increase resource demand.

Resource-rich countries are often thought to be resource-dependent to a harmful degree (Van der Ploeg, 2011). Increased domestic output and investment, especially if the latter were appropriately focused, could help increase domestic productivity. In a richer model, capital income taxation might in fact reduce such dependence and have overall beneficial effects on the exporters' welfare. On the other hand, if oil-rich countries choose their extraction levels based on a need to satisfy the government's budget constraint, subject to consumption habits among a restive population, any decrease in income from financial assets might be offset by higher resource extraction in order to satisfy revenue requirements (Griffin, 1985; Ramcharran, 2001, 2002).

An obvious question is how the present model would change if a backstop technology, able to substitute for oil, were to exist. With a perfect substitute to oil, limit pricing behaviour would arise. This problem could be tackled as in Chapter 2.

Like any other climate policy mechanism, the success of the proposed tax instruments relies on the cooperation among the countries seeking to mitigate climate change. The tax instruments clearly give individual countries incentives to defect, by taxing investments at less than the agreed rate, attracting a disproportionate share of investment and the resulting tax revenues and income for the fixed factor. Such deviations might be difficult

to observe and thus reduce the usefulness of the proposed instrument.

Whether capital income taxes would be helpful in tackling the Green Paradox turns on the question of whether the paradox itself is a relevant phenomenon. While this appears to be so in simple models, more elaborate models tend to imply the issue is less severe. Capital income taxes might, however, play some part as a complementary climate policy mechanism, or as a threat to induce more cooperative behaviour from oil exporters.

Appendix 4.A Proofs for Section 4.2

Proof of Proposition 20. Equality of marginal welfare and the marginal products is immediate from the first-order conditions. As consumption is shared in constant proportions, clearly the growth rates are both equal. The Ramsey Rule follows from differentiating (4.9a) woth respect to time and using (4.9b) and (4.9f). The Hotelling Rule follows from differentiating (4.9c) with respect to time and using (4.9d) and (4.9e). The transversality condition on S is

$$\lim_{t\to\infty} e^{-\rho t}\mu_S(t)S(t) = 0$$

and from (4.9d) it is obvious that this implies $S(t) \to 0$. □

Proof of Proposition 21. Note that $\hat{y} = \hat{F} - \hat{K}$ and $\hat{x} = \hat{C} - \hat{K}$. From the production function, $\hat{F} = g + \alpha\hat{K} + \beta\hat{R}$. From (4.2), $\hat{K} = y - x$. From (4.9c), (4.9d), (4.9e) and (4.9f), (4.13b) can be derived. The Ramsey Rule allows (4.13a) to be derived. Setting both equal to zero at (x^∞, y^∞) yields the steady state. On a phase diagram, it is clear that either the economy approaches this steady state, or both $x, y \to \infty$, or $x, y \to 0$.

Any trajectories with $x \to \infty$ are clearly not feasible, as along any such

path there exists t' such that, for all $t > t'$,

$$\frac{C_W}{F} = \frac{C_W/K}{F/K} = \frac{x}{y} > \frac{1-\alpha-\beta+\alpha\beta}{1-\alpha-\beta} > 1$$

Clearly, it is not feasible that consumption exceeds production by a non-infinitesimal amount for an unbounded time interval, as production decays to zero and the entire capital stock would eventually be used up.

Any paths converging to $(0,0)$ will break the resource constraint. For the resource constraint to hold, it is required that $R \to 0$; I will show that this does not hold when the system converges to the origin in (x,y)-space. As $\hat{R} = -\frac{\alpha}{1-\beta}x$,

$$R(t) = R(0)\exp\big(-\frac{\alpha}{1-\beta}\int_0^t x(s)\,\mathrm{d}s\big)$$

Taking the limit as $t \to \infty$, for any positive $R(0)$, $R(t)$ will tend to zero only if the integral does not converge, instead tending to (positive) infinity. However, if the system tends to $x = y = 0$, then $\hat{x}$ tends to $-\sigma\rho < 0$. Thus there will exist a time t' after which x decays exponentially, at some rate $\sigma\rho - \epsilon$ or faster (for arbitrarily small ϵ). Breaking up the integral,

$$\lim_{t\to\infty}\int_0^t x(s)\,\mathrm{d}s = \int_0^{t'} x(s)\,\mathrm{d}s + \lim_{t\to\infty}\int_{t'}^t x(s)\,\mathrm{d}s$$

we note that the second integrand decays exponentially, and so the integral converges. Furthermore, as $x(t)$ is well behaved, in particular exhibiting no singularities, the first term also takes a finite value. Hence, the whole integral converges to a finite value. But this implies $R(t)$ tends to a strictly positive value, breaking the resource constraint.

Hence, the only feasible path is the one converging to (x_∞, y_∞). $\square$

Appendix 4.B Proofs for Section 4.3

Proof of Proposition 22. The derivative of K_I with respect to τ_E can be obtained by implicitly differentiating the equality $r_I(\overline{K}_I, \overline{R}_I(\overline{K}_I)) - \tau_E = r_E(K - K_I, R_E(K - K_I))$. As capital stock is fixed at any moment, this also gives the derivative of K_E:

$$\frac{\mathrm{d}K_I}{\mathrm{d}\tau_E} = -\frac{\mathrm{d}K_E}{\mathrm{d}\tau_E} = -\frac{1-\beta}{1-\alpha-\beta}\left(\frac{r_I}{K_I} + \frac{r_E}{K_E}\right)^{-1} < 0$$

Note that the derivatives are monotonic and $\lim_{\tau\to\infty} K_E(\tau, p) = K$, $\lim_{\tau\to-\infty} K_E(\tau, p) = 0$. Thus, for any tax, the equilibrium quantities are uniquely given. Clearly this holds irrespective of the value of p (as long as $p > 0$). The quantity of resource employed and hence of output produced in each country are entirely determined by the quantity of capital employed and similarly unique. The remaining derivatives are obtained simply using the Chain Rule.

$$\frac{\mathrm{d}R}{\mathrm{d}\tau_E} = \frac{\beta}{1-\beta}\frac{\tau_E}{p}\frac{\mathrm{d}K_I}{\mathrm{d}\tau_E}$$

$$\frac{\mathrm{d}F}{\mathrm{d}\tau_E} = \frac{1}{1-\beta}\tau_E\frac{\mathrm{d}K_I}{\mathrm{d}\tau_E}$$

$$\frac{\mathrm{d}r_I}{\mathrm{d}\tau_E} = \frac{r_I}{K_I}\left(\frac{r_I}{K_I} + \frac{r_E}{K_E}\right)^{-1}$$

$$\frac{\mathrm{d}r_E}{\mathrm{d}\tau_E} = \frac{\mathrm{d}r_I}{\mathrm{d}\tau_I} - 1 = -\frac{r_E}{K_E}\left(\frac{r_I}{K_I} + \frac{r_E}{K_E}\right)^{-1}$$

The derivatives with respect to p are straightforward, with a change in p having a direct effect and an indirect effect due to its effect on K_I and

K_E.

$$\frac{\mathrm{d}K_I}{\mathrm{d}p} = -\frac{\beta}{1-\alpha-\beta}\frac{\tau_E}{p}\left(\frac{r_I}{K_I}+\frac{r_E}{K_E}\right)^{-1}$$

i.e. this has the opposite sign of τ_E; given any τ_E, increasing the price of oil reduces the rental rate r_I and thus makes the tax more effective.

$$\frac{\mathrm{d}r_I}{\mathrm{d}p} = \frac{\mathrm{d}r_E}{\mathrm{d}p} = -\frac{\beta}{1-\beta}\frac{r_I r_E}{p}\left(\frac{r_I}{K_I}+\frac{r_E}{K_E}\right)^{-1}\left(\frac{1}{K_I}+\frac{1}{K_E}\right) < 0$$

The equality follows as, for any given τ_E, if one rental rate changes the other must too.

$$\frac{\mathrm{d}F_I}{\mathrm{d}p} = -\frac{\beta}{\alpha}\frac{r_I}{p}\left(\frac{r_I}{K_I}+\frac{r_E}{K_E}\right)^{-1}\left(\frac{1}{1-\beta}r_E\left(1+\frac{K_I}{K_E}\right)+\frac{1}{1-\alpha-\beta}\tau_E\right)$$

$$\frac{\mathrm{d}F_E}{\mathrm{d}p} = -\frac{\beta}{\alpha}\frac{r_E}{p}\left(\frac{r_I}{K_I}+\frac{r_E}{K_E}\right)^{-1}\left(\frac{1}{1-\beta}r_I\left(1+\frac{K_E}{K_I}\right)-\frac{1}{1-\alpha-\beta}\tau_E\right)$$

$$\frac{\mathrm{d}F}{\mathrm{d}p} = -\frac{\beta}{\alpha}\frac{1}{p}\left(\frac{r_I}{K_I}+\frac{r_E}{K_E}\right)^{-1}\left(\frac{1}{1-\beta}r_E r_I\left(\frac{K}{K_I}+\frac{K}{K_E}\right)+\frac{1}{1-\alpha-\beta}\tau_E^2\right) < 0$$

$$\begin{aligned}\frac{\mathrm{d}R_I}{\mathrm{d}p} &= R_I\left(\frac{\alpha}{1-\beta}\frac{\frac{\mathrm{d}K_I}{\mathrm{d}p}}{K_I}-\frac{1}{1-\beta}\frac{1}{p}\right)\\ &= \frac{R_I}{(1-\beta)p}\left(-\frac{\alpha\beta}{1-\alpha-\beta}\tau_E\left(r_I+\frac{K_I}{K_E}r_E\right)^{-1}-1\right) < 0\end{aligned}$$

$$\frac{\mathrm{d}R_E}{\mathrm{d}p} = \frac{R_E}{(1-\beta)p}\left(\frac{\alpha\beta}{1-\alpha-\beta}\tau_E\left(\frac{K_E}{K_I}r_I+r_E\right)^{-1}-1\right)$$

$$\begin{aligned}\frac{\mathrm{d}R}{\mathrm{d}p} &= -\frac{R}{(1-\beta)p}-\frac{\alpha\beta}{(1-\beta)(1-\alpha-\beta)}\frac{\tau_E}{p}\left(\frac{r_I}{K_I}+\frac{r_E}{K_E}\right)^{-1}\left(\frac{R_I}{K_I}-\frac{R_E}{K_E}\right)\\ &= -\frac{R}{(1-\beta)p}-\frac{\alpha\beta}{(1-\beta)(1-\alpha-\beta)}\frac{\tau_E}{p}\left(\frac{r_I}{K_I}+\frac{r_E}{K_E}\right)^{-1}\frac{R_I}{K_I}\frac{\tau_E}{r_I}\\ &= -\frac{R}{(1-\beta)p}+\frac{\beta}{(1-\beta)p}\tau_E\frac{\mathrm{d}K_I}{\mathrm{d}p} < 0\end{aligned}$$

Finally, given that the tax rates on home residents' invesments at home

and abroad are tied together, it follows that

$$\frac{\mathrm{d}K_I}{\mathrm{d}\tau_{I,I}} = 0.$$

□

Proof of Proposition 23. The efficient taxes are obtained immediately by equating the socially optimal and decentralised Hotelling Rules (4.11) and (4.23) and aggregate Ramsey Rules (4.10) and (4.18). Equation (4.26) is obtained by integrating Country E's budget constraint. Equation (4.27) comes from

$$\begin{aligned}
\dot{s}_C &= \hat{s}_C s_C \\
&= s_C \left(\hat{C}_I - \hat{C} \right) \\
&= \sigma(-\tau_I + s_C \tau_I + (1 - s_C)\tau_E)
\end{aligned}$$

where I have used the Country I and aggregate Ramsey Rules and the optimal τ_I.

To obtain z^∞, note that

$$\begin{aligned}
\hat{z} &= \hat{A} - \hat{K} \\
&= r_E + \frac{1}{z}(\beta y - (1 - s_C)x) - (y - x) \\
&\to -(1 - \alpha)y^\infty + x^\infty + \frac{1}{z^\infty}(\beta y^\infty - (1 - s_C^\infty)x^\infty)
\end{aligned}$$

assuming all the limits exist. We will have either $z \to z^\infty \in \mathbb{R}$ or $z \to \pm\infty$. In the former case, z^∞ is immediately obtained. To rule out the latter case, consider the transversality condition for A: $\lim_{t\to\infty} e^{-\rho t}\mu_A(t)A(t) = 0$.

Note that

$$\frac{\frac{\mathrm{d}e^{-\rho t}\mu_A A}{\mathrm{d}t}}{e^{-\rho t}\mu_A A} = \frac{\beta y - (1-s_C)x}{z}$$

and, integrating, we must have

$$e^{-\rho t_2}\mu_A(t_2)A(t_2) = e^{-\rho t_1}\mu_A(t_1)A(t_1)e^{\int_{t_1}^{t_2}\frac{\beta y-(1-s_C)x}{z}\,\mathrm{d}t}$$

For the transversality condition to hold, this requires that the LHS goes to zero as $t_2 \to \infty$. This, on the other hand, will never happen if the integral in the exponent converges, as $e^{-\rho t_1}\mu_A(t_1)A(t_1)$ will be finite for any t_1. But the rate of change of the integrand tends to $-\hat{z}$. If z explodes, this is certainly negative so that the integral converges, and the transversality condition will not be satisfied. □

Appendix 4.C Proofs for Section 4.4

Proof of Proposition 24. I will retain the aggregate variables $x \equiv \frac{C}{K}$, $y \equiv \frac{F}{K}$; then $r_I = \alpha\frac{s_F}{s_K}y$ where s_X denotes the share of Country I: $s_X \equiv \frac{X_I}{X}$, for some variable X (and symmetrically for Country E). Recall that I have set $g = 0$. From the consumers' Ramsey rules, as $r_E = r_I - \tau_E$, I obtain

$$\begin{aligned}\hat{x} &= \hat{C}_I s_K + \hat{C}_E(1-s_K) \\ &= -\left(1-\sigma\alpha\frac{s_F}{s_K}\right)y + x - \sigma\left(\rho + \tau_E(1-s_C)\right)\end{aligned} \tag{4.32}$$

which reduces to the socially optimal $\hat{x}$ if $\tau_E = 0$. From the production function,

$$\begin{aligned}\hat{y} &= -(1-\alpha)\hat{K} + \beta\hat{R} = -(1-\alpha)(y-x) + \beta\hat{R} \\ &= -(1-\alpha)(y-x) + \frac{\beta}{1-\beta}\left(\alpha\hat{K} - \hat{p}\right)\end{aligned} \tag{4.33}$$

where I have used the momentary resource demand function (4.19) to get the last line.

Step 1: $\tau_E \to \tau_E^\infty \in \mathbb{R}, r_I \to r_I^\infty \in \mathbb{R}$. Consider first the long-run behaviour of τ_E. It must hold that $\lim_{t\to\infty} \dot{\tau}_E = 0$. Otherwise $\tau_E \to \pm\infty$, which implies that one of r_I, r_E and, by the Ramsey Rule, also of $\hat{C}_I, \hat{C}_E$ will tend to infinity. This is clearly not feasible. For the same reason, $\dot{r}_I \to 0$.

Thus it must be that $\frac{r_I - \tau_E}{r_I}$ tends to a constant in the long run, implying $\frac{K_I}{K_E} \to \frac{s_K^\infty}{1-s_K^\infty} \in \mathbb{R}^+$, $\frac{R_I}{R_E} \to \frac{s_R^\infty}{1-s_R^\infty} \in \mathbb{R}^+$ and $\frac{F_I}{F_E} \to \frac{s_F^\infty}{1-s_F^\infty} \in \mathbb{R}^+$. Also $\hat{K}_I^\infty = \hat{K}_E^\infty = \hat{K}^\infty$, and similarly for R and F.

Step 2: With constant τ^∞, derive steady state. Note that we must have $x \to x^\infty \in \mathbb{R}^+$, $y \to y^\infty \in \mathbb{R}^+$. Were either to tend to infinity, then either the consumption-capital ratio would become infinite, eventually consuming all capital and output; or the output-capital ratio would become infinite, requiring increasing amounts of the resource which would not be sustainable.

To obtain the equivalent of the Hotelling Rule, I will consider the exporter's problem. The Hamiltonian for this is given by

$$\begin{aligned}\mathcal{H}_E =& L_E\left(u\left(\frac{C_E}{L_E}\right) - D_E(G)\right) \\ &+ \mu_B^E\left(r_E(A-K_E) + F_E + \beta F_I - C_E\right) - \mu_S^E R + \mu_G^E(R)\end{aligned}$$

Differentiating this with respect to p to obtain the FOC, and using the derivatives given in the proof of Proposition 22, optimality requires

$$\begin{aligned}
&\frac{\mu_A}{\mu_{S,E}}\beta r_I r_E \left(\frac{\alpha(z-(1-s_K))+(1-s_K)+\beta s_K}{\alpha(1-\beta)} \right. \\
&\quad \left. + \frac{\tau_E}{1-\alpha-\beta} s_K(1-s_K) \left(\frac{-(1-\alpha)r_E+\beta r_I}{\alpha r_I r_E} \right) \right) \\
&\quad = \frac{1}{p}\frac{\beta}{1-\beta} \left(y(r_I - \tau s_K) + \frac{\beta}{1-\alpha-\beta}\tau^2 s_K(1-s_K) \right)
\end{aligned}$$

As $\hat{\tau} \to 0$, the second term in the first brackets and the term in the last brackets tend to constants. The first term in the first brackets, in the limit, changes at rate $\max(0, \hat{z})$. Note that, by arguments employed in the social planner case, in the long run μ_S^E dominates μ_G^E. Thus the Hotelling Rule, in the long term, is

$$\hat{p}^\infty = r_E^\infty - \max(0, \hat{z}^\infty)$$

I will show below that the cases with $\hat{z}^\infty \neq 0$ are not interesting and will thus only report the steady state for the relevant case. Setting (4.32) and (4.33) to zero and solving:

$$y^\infty = \frac{1}{\alpha}\frac{1-\alpha-\beta}{(1-\alpha-\beta)\sigma\frac{s_F^\infty}{s_K^\infty} + \beta\frac{1-s_F^\infty}{1-s_K^\infty}}\sigma(\rho + \tau^\infty(1-s_C^\infty))) \tag{4.34}$$

$$x^\infty = \frac{1}{\alpha}\frac{1-\alpha-\beta+\alpha\beta\frac{1-s_F^\infty}{1-s_K^\infty}}{(1-\alpha-\beta)\sigma\frac{s_F^\infty}{s_K^\infty} + \beta\frac{1-s_F^\infty}{1-s_K^\infty}}\sigma(\rho + \tau^\infty(1-s_C^\infty))) \tag{4.35}$$

which exists only if $\tau^\infty \geq -\rho$. Note that if $\tau^\infty = 0$, $s_F^\infty = s_K^\infty$ and the steady state is the same in the social optimum.

From the phase diagrams, it is apparent that either (x, y) tends to either (x^∞, y^∞) or to $(0, 0)$. If $\tau^\infty \leq -\rho$, no steady state exists; thus any equilibrium outcome must satisfy $\tau^\infty > -\rho$.

Step 3: $(x, y) \to (0, 0)$ *cannot be optimal.* Suppose a candidate solution converges to $(0, 0)$. By calculating $\hat{y} - \hat{x} = \sigma(\rho + \tau(1 - s_C)) > 0$, it is found that x decays at a faster rate than y in the limit. This implies that the economy approaches the steady state $(0, 0)$ almost vertically, with for either country $x_i \equiv \frac{C_i}{K} \approx 0$, $\hat{C}_i \to -\sigma\rho$ but the capital stock increasing as $\hat{K} = y - x > 0$. This cannot be optimal, as too much output is being saved. At least one of the countries would be better off by stopping all production and consumption and instead just consuming the asset stock as a cake, increasing their consumption immediately to a fraction $\frac{1}{\rho\sigma}$ of the asset stock and with consumption decreasing at rate $\rho\sigma$ thereafter. Thus any optimal path must tend to a non-zero steady state.

Step 4: $z \equiv \frac{A}{K} \to z^\infty \in \mathbb{R}$. In other words, the share of Country E's assets out of total capital stock tends to a finite constant. Clearly either $\dot{z} \to 0$ and $z \to z^\infty$, or $z \to \pm\infty$.

Suppose $z \to \pm\infty$. From the budget constraint,

$$\begin{aligned}\hat{A} &= r_E - \frac{1}{z}\left((1 - s_K)r_E - \frac{\beta}{\alpha}s_K r_I - \frac{(1 - s_K)r_E}{\alpha} + (1 - s_C)x_E\right) \\ &= r_E - \frac{1}{z}\left((1 - s_F)\alpha y - s_F\beta y - (1 - s_F)y + (1 - s_C)x\right)\end{aligned}$$

which tends to r_E, and so, using the Ramsey Rule,

$$\frac{\frac{\mathrm{d}e^{-\rho t}\mu_A^E A}{\mathrm{d}t}}{e^{-\rho t}\mu_A^E A} = -\frac{1}{z}\left((1 - s_K)r_E\left(1 - \frac{1}{\alpha}\right) - \frac{\beta}{\alpha}s_K r_I + (1 - s_K)x_E\right) \quad (4.36)$$

which approaches zero as the bracketed term tends to a constant. Integrating,

$$e^{-\rho t_2}\mu_A^E(t_2)A(t_2) = e^{-\rho t_1}\mu_A^E(t_1)A(t_1)\exp\int_{t_1}^{t_2}\frac{\frac{\mathrm{d}e^{-\rho t}\mu_A^E A}{\mathrm{d}t}}{e^{-\rho t}\mu_A^E A}\,\mathrm{d}t$$

The transversality condition requires that, as $t_2 \to \infty$, the LHS goes to zero. This happens only if the integral term explodes to $-\infty$. From (4.36), the rate of change of the integrand is $-\hat{z}$. For $z \to \infty$, it must be that $\hat{z}^\infty \geq 0$. If this holds with inequality, then $-\hat{z} < 0$, that is, the integrand decays at a finite rate and the integral converges, breaking the transversality constraint. The other alternative is that $\hat{z}^\infty = 0$. Then, as $z \to \infty$,

$$\begin{aligned}\hat{z}^\infty &\equiv \hat{A} - \hat{K} \\ &= -\left(1 - \alpha\frac{s_F^\infty}{s_K^\infty}\right) y^\infty + x^\infty \\ &= 0\end{aligned}$$

and, using the steady states, this implies

$$\alpha\frac{s_F^\infty}{s_K^\infty} = -\frac{\alpha\beta(1-s_F^\infty)}{(1-\alpha-\beta)(1-s_K^\infty)} < 0$$

which clearly cannot be the case.

Step 5: $\tau_E^\infty \in (-\rho, 0]$. The importer's Hamiltonian is

$$\begin{aligned}\mathcal{H}_I =& L_I\left(u\left(\frac{C_I}{L_I}\right) - D_I(G)\right) \\ &+ \mu_B^I\left(r_E(K_E - A) + F_I - \beta F_I - C_I\right) - \mu_S^I R + \mu_G^I R\end{aligned}$$

and the first-order condition with respect to τ yields

$$\tau = \left(1 - \frac{1-\beta}{\beta}\frac{\mu_B p}{\mu_{S,I}}\right)^{-1} \frac{1-\alpha-\beta}{\beta}\frac{\mu_B p}{\mu_{S,I}}\frac{1-s_K-z}{1-s_K} r_E \tag{4.37}$$

Using the Hotelling and Ramsey rules, and the fact that $\hat{z}^{\infty} = 0$,

$$\begin{aligned} \hat{\mu}_B + \hat{p} - \hat{\mu}_S &= \rho - r_I + r_E - \rho \\ &= -\tau \end{aligned} \tag{4.38}$$

Now suppose $\tau \to \tau^{\infty} > 0$. Then (4.37) and (4.38) imply together that $\tau \to \infty$, which is not feasible. Furthermore, it cannot be always welfare-improving to keep increasing τ; this would mean that Country I would have very low consumption for a very long period of time, followed by consuming a large fraction of a very low aggregate consumption stream. Thus we must have $\tau^{\infty} \leq 0$.

□

Appendix 4.D The numerical method

The social optimum. As the system is saddle-path stable, these are conducted by the method of reverse shooting: starting from the saddlepath close to the steady state, and trying to shoot backwards in time so as to hit the (known) initial state. I use a quasi-Newton algorithm (the Broyden Method from the COMPECON package by Miranda and Fackler (2002)) to find the $K(T), T$ for the socially optimal case. These variables pin down $C(T)$, $\mu_K(T)$, $F(T)$ and so $R(T)$. $S(T)$ is given by assuming that, for $t \geq T$, $\hat{R} \approx \hat{R}^{\infty}$ which is constant. Then

$$S(T) = \frac{R(T)}{\hat{R}^{\infty}}.$$

With climate damages switched on, the steady state moves over time

until settling in the long-run steady state. This implies that the system often exhibits very high curvature in terms of the functions to be minimised $K(0) - K_0$, $S(0) - S_0$. Hence the algorithm requires a careful search for an appropriate starting point for the quasi-Newton algorithm to converge.

Efficient taxes. The aggregate solution for the efficient taxes case is the social optimum. To determine the optimal assets, I solve (4.26) jointly with (4.27), augmented with the value of assets at time T to ensure the approximation of the steady state has no effect:

$$\begin{aligned} A_0 + p(0)S_0 = &\int_0^\infty \exp\Big(\int_0^t -r_E(s)\,\mathrm{d}s\Big)(1 - s_C(t))C(t)\,\mathrm{d}t \\ &+ \exp\left(\int_0^T -r(s)\,\mathrm{d}s\right)\left(z^\infty + \beta y^\infty\right)K(T) \end{aligned}$$

substituting in z^∞ from (4.25).

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