Essays on Macroeconomic Policy



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Statement of Authorship The work in this thesis is based on my work at the University of Oxford between September 2018 and May 2023. Chapter 1 is single-authored work, whose earlier version was submitted in partial fulfillment of the requirements for the degree of Master of Philosophy in Economics at the Department of Economics, University of Oxford. Chapter 2 is co-authored with Ambrogio Cesa-Bianchi and Andrea Ferrero. Chapter 3 is co-authored with Daisuke Ikeda, Sophocles Mavroeidis, and Francesco Zanetti. Chapter 4 is single-authored work.

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This dissertation consists of four self-contained chapters which discuss topics related to challenges and the design for macroeconomic policy in the aftermath of the Global Financial Crisis 2007-2008. The first two chapters consider the implications of the global financial cycle (GFC) to monetary policy in two-country open market dynamic stochastic general equilibrium models where the dominant role of the reserve currency in trade and global financial transactions can account for the cross-border transmission of U.S. monetary policy shocks. Chapter 1 evaluates the gains from international monetary policy cooperation between the financial centre and periphery countries. The key finding is that although monetary policy cooperation may improve the global welfare, the asymmetric policy spill-overs through the global financial markets create winners and losers from cooperation at the individual country level. Chapter 2 discusses whether exchange rate regime matters in the presence of the GFC with a similar model estimated to match the empirical evidence from a panel vector autoregressive (VAR) model. We find that even with a GFC, a peg substantially increases macroeconomic volatility. Conversely, the introduction of an additional policy instrument to manage capital flows dampens economic fluctuations. A tax on domestic credit achieves nearly equivalent results. Chapter 3 examines the effectiveness of unconventional monetary policy (UMP). We develop a theoretical model that underpins the 'irrelevance hypothesis,' which states that UMP may make the effective lower bound (ELB) on the short-term interest rate irrelevant. We then test and firmly reject this hypothesis for Japan and the United States using a structural VAR model with the ELB. However, we find that UMP has had strong delayed effects. Chapter 4 analyses the factors that hinder the implementation of structural reforms using a macroeconomic political economy model. In the model, the effects of product market deregulation create its own resistance, and the resistance generates endogenous political costs to impair deregulation. In addition, adverse initial conditions, better risk-sharing among workers, and a lower share of temporary workers weaken opposition and help deregulation.

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Chapter 1

Global Financial Transmissions and International Monetary Policy Cooperation

Abstract This chapter evaluates the gains from international monetary policy cooperation between the financial centre and periphery countries in a two-country New Keynesian DSGE model with global financial transmissions. If the two countries cooperate to implement the optimal Ramsey cooperative equilibrium, the global welfare is generally improved over the welfare under optimal non-cooperative Nash equilibrium. However, at the individual country level, the optimal cooperative policy makes one country better off but the other country worse off. This pattern is robust even if both countries adopt implementable policy rules. Importantly, the significant welfare gains and losses for both countries are driven by the asymmetric policy spill-overs associated with the dominant role of the financial centre country's currency in the global financial markets. Given the other country's policy rule, responding to exchange rate movements dominates over purely inward-looking policy rules from both country's perspective.

1.1 Introduction

The international dimension of monetary policy remains one of the most extensively discussed topics in literature for decades. One of the central questions is whether central banks should coordinate their policy conducts with each other.[1] Most existing contributions generally find negligible gains from international monetary policy cooperation (e.g. Obstfeld and Rogoff (2002), Benigno and Benigno (2006)) The direct policy implication is that central banks should continue to follow Friedman (1953)'s conventional wisdom to act in their self-interest.

However, in the aftermath of the Great Financial Crisis, recent empirical literature such as Miranda-Agrippino and Rey (2020) documents that a monetary policy shock from a global financial centre country, such as the U.S., generates a global financial cycle that involves sizable policy spill-overs to emerging economies as the financial periphery. This poses new challenges to the policy trade-offs for central banks both in the financial centre and periphery countries and opens a debate on the role of monetary policy as discussed by, for example, Rey (2015) and Obstfeld (2021).

These new challenges to monetary policy motivates a reassessment of the gains from international monetary policy cooperation between the financial centre and periphery countries, which is the main aim of this paper. For this purpose, I build a two-country New Keynesian DSGE model where the monetary policy in the financial centre country can generate significant spill-overs to the financial periphery countries through the global financial transmissions, which is consistent with the global financial cycle as observed empirically in literature.

In my baseline analysis, the cooperative Ramsey equilibrium is chosen by a benevolent social planner to maximise the global joint welfare, and the non-cooperative Nash equilibrium is chosen by two self-oriented central banks to maximise their own national welfare, taking the other country's policy path as given. I find that although cooperative policy generally achieves higher global joint welfare, the distribution of the welfare gains for individual countries is highly uneven. Although one of the countries obtains large gains from policy cooperation, the other country suffers significant losses. Moreover, I show that this pattern is robust to the type and source of the shock, relative country size, and the instruments used.

I show that the key mechanism in the model that drives this results is that banks in the financial periphery country will finance their lending partly by borrowing from interna-

[1] Bordo and Schenk (2016) distinguish between policy cooperation and policy coordination. Although the scenarios discussed in this paper is closer their definition of policy coordination, the boundary between the two concepts is very ambiguous. Thus, I follow the convention of most literature and use the two terms interchangeably.

tional wholesale funds providers in the currency of the financial center countries, which creates a currency mismatch on their balance sheets.[2] This mismatch captures the dominant role of the currency of the financial centre country in the global financial markets, and it is also the key mechanism that enables the model to replicate the sizable policy spill-overs from the financial centre country to the financial periphery country.

In this case, the impact of a policy action in the financial centre country is twofold. Suppose the policy of the financial centre country leads to an appreciation of its currency, this impacts the balance-of-trade of the financial periphery country through the standard trade channel in literature. In addition, the net worth of its banking sector decreases due to higher real values of debt to serve if part of their debt is denominated in the foreign currency. This eventually leads to lower investment and output. By contrast, a similar policy action from the financial periphery only impacts the financial centre via the trade links. This asymmetry in policy spill-overs implies that one of the countries under non-cooperative equilibrium is potentially able to export some of the stabilisation costs to the other country without being affected by a matching spill-back from the other country's policy. Conversely, cooperation limits these strategic actions such that the other country benefits from smaller negative spill-overs of welfare costs.

As the financial periphery country borrows less in the foreign currency so that the degree of the currency mismatch becomes smaller, the asymmetry in policy spill-overs and trade-offs between the two countries is much less prominent. Consequently, I show that the cooperative gains become more evenly distributed. When the degree of mismatch is very low or absent in the model, the cooperative policy creates small positive gains for both countries compared to the non-cooperative policy.

As far as the implementability is concerned, I show that cooperation in either implementing the optimal Ramsey allocation or monetary policy rules may still fail to benefit both countries simultaneously compared to the non-cooperative equilibria with monetary policy rules. Nevertheless, within the monetary policy rules that I consider, countervailing the exchange rate movements with monetary policy rule dominates over purely inward-looking rules for the self-interested financial periphery country given any policy rules adopted by the financial centre country. Meanwhile, stabilising exchange rate movements is also beneficial for the financial centre country unless the financial periphery already responds to such movements strongly.

Literature Review This research is primarily related to two strands of literature. Firstly, my research question is inspired by the literature on the international dimension of monetary policy which has a long tradition and is comprehensively summarised by Corsetti

[2]The source of this friction is taken as exogenous in this study. Eichengreen and Hausmann (1999) Eichengreen et al. (2007), and Shin (2014) discuss the reasons for the mismatch in detail.

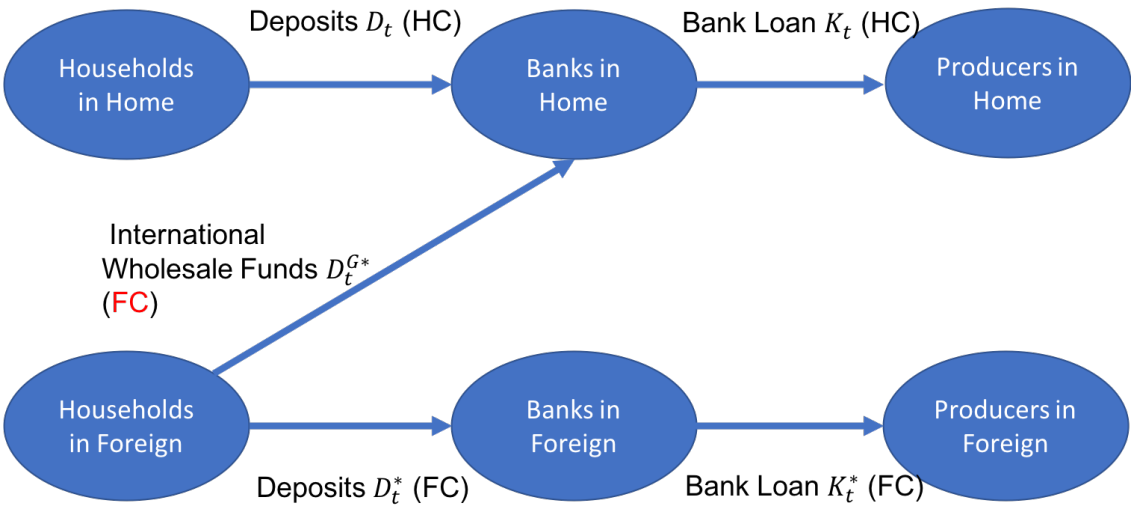
et al. (2010). More specifically, this chapter mainly evaluates gains from international monetary policy cooperation using modern macroeconomic models. Seminal contributions such as Obstfeld and Rogoff (2002) and Benigno and Benigno (2006) generally find that the cooperation gains are small in magnitude. Nevertheless, the size of the cooperative gain is sensitive to the parameterization of the model (Obstfeld and Rogoff, 2002), to the types of shocks (Benigno and Benigno, 2006), to frictions and policy trade-offs in the model (Tchakarov, 2004, Canzoneri et al., 2005, Coenen et al., 2007, Bergin and Corsetti, 2013), to whether central banks can commit (Cooley and Quadrini, 2003), and to what instruments central banks are using in the non-cooperative Nash game (Sims, 2007). My paper contributes to this literature by reassessing the cooperation gains within a model where the cross-border transmissions via the global banking sector play a prominent role, which is not much discussed in the previous literature.

Secondly, the key mechanism of currency mismatch that drives the asymmetric policy spill-overs in my model is built upon the contribution by Aoki et al. (2020). Different to them and Mimir and Sunel (2015) which examine the policy implications of the currency mismatch friction for a small open economy, I embed this mechanism in a two-country model for the purpose of evaluating the gains from monetary policy cooperation. Papers such as Akinci and Queralto (2019) also implement the currency mismatch friction in two-country models but the questions they discuss are different to mine.

Banerjee et al. (2016) also compares the optimal cooperative to the optimal non-cooperative Nash policy equilibrium in a two-country model with currency mismatch. My paper differs to theirs in 3 aspects. Firstly, the amount of foreign currency debt is endogenously chosen by the financial periphery country banks in my model. This implies policy trade-offs for central banks that are different to those in their model. Second, I approximate the model into the 2nd-order and explicitly calculate the welfare measures. Third, I consider the cooperation gains under a much wider range of shocks, calibrations, and instruments. All these lead to different policy implications to theirs.

The rest of the paper is organised as follows: Section 1.2 presents the model. Section 1.3 reports the calibration of the model and shows that the model is able to replicate the policy spill-overs of the financial centre to financial periphery as documented by the global financial cycle. Section 1.4 discusses the methodology and the main results. Section 1.5 extends the analyses to equilibria where countries adopt implementable monetary policy rules. Section 1.6 concludes.

Figure 1.1: The Financing Structure of the Model



Note: HC denotes the Home currency, FC denotes the Foreign currency.

1.2 The Model

The model assumes that there are two countries in the world economy, Home and Foreign, with population size m and $1-m$ respectively. The firms in each country specialise in one type of goods respectively, and each type of goods consists of a number of brands that are with equal measure of the population size. There is no barrier to trade so that the output of firms can be either used to satisfy domestic demand or exported.

Figure 1.1 shows the financing structure of the model. Assume Home to be the financial periphery country such as the emerging market economies (EMEs), whereas Foreign to be the financial centre country such as the US. The production of output in both countries is by combining labour and capital borrowed from banks in their own country. Home banks finance themselves by not only domestic household deposits, but also borrowing from international wholesale funds market in Foreign currency. Foreign banks finance their lending to firms by deposits from Foreign households only. The international wholesale funds and the local banks in Foreign can be regarded as two subsidiaries of a global bank that resides in the Foreign country. As documented by Cetorelli and Goldberg (2012) and Bruno and Shin (2015b), global banks transmit financial conditions across borders by channelling liquidity globally. Moreover, the US dollar plays a much more important role in the cross-border banking than any other major currencies in both asset and liability terms. The model set-up here captures these empirical aspects.

The behaviour of each sector of the economy is characterised below. All variables for the Foreign country are denoted by an asterisk.

1.2.1 Households

The representative household in each country consists of a continuum of bankers and workers. The bankers manage domestic banks. Each period, a banker will retire to become a worker with probability $1-\omega$, and will transfer the remaining net worth of her bank to the household. Meanwhile, an equal number of workers will become new bankers with entering endowments as a fraction ξ of the total assets of the bank in the current period. Workers supply labour to domestic firms each period.

The representative household in the Home country maximises the expected life-time utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [log(C_t)\epsilon_{c,t} - \frac{\zeta L_t^{1+\chi}}{1+\chi}], \tag{1.1}$$

where β is the discounting factor, χ is the inverse of the Frisch elasticity of labour supply, L_t is the total labour supply that the household supplies to firms, $\epsilon_{c,t}$ captures demand shocks. C_t is a CES aggregator of the household consumption over all brands of domestic and foreign goods:

$$C_t = [a^{\frac{1}{\phi}} C_{H,t}^{\frac{\phi-1}{\phi}} + (1-a)^{\frac{1}{\phi}} C_{F,t}^{\frac{\phi-1}{\phi}}]^{\frac{\phi}{\phi-1}},$$

where

$$C_{H,t} = ((\frac{1}{m})^{\frac{1}{\varphi}} \int_0^m C_t(h)^{\frac{\varphi-1}{\varphi}} dh)^{\frac{\varphi}{\varphi-1}}$$
$$C_{F,t} = ((\frac{1}{1-m})^{\frac{1}{\varphi}} \int_m^1 C_t(f)^{\frac{\varphi-1}{\varphi}} df)^{\frac{\varphi}{\varphi-1}}$$

are the aggregate consumption over domestic and foreign goods respectively, and $C_t(h)$ and $C_t(f)$ are the consumption of particular brands of goods. $a = 1-\nu(1-m)$ captures the home bias of Home residents over domestic goods, where ν is the degree of openness. ϕ is the elasticity of substitution between the H-type goods produced in Home and the F-type goods produced in Foreign, and φ is the elasticity of substitution among the brands within each type of goods.

The consumption aggregators above imply that the prices for aggregated bundles are:

$$P_{H,t} = (\frac{1}{m} \int_0^m P_t(h)^{1-\varphi} dh)^{\frac{1}{1-\varphi}}$$
$$P_{F,t} = (\frac{1}{1-m} \int_m^1 P_t(f)^{1-\varphi} df)^{\frac{1}{1-\varphi}}$$
$$P_t = [a{P_{H,t}}^{1-\phi} + (1-a){P_{F,t}}^{1-\phi}]^{\frac{1}{1-\phi}}.$$

In addition to purchasing consumption goods, the household may lend to banks. The budget constraint that the household faces in period t is:

$$P_t C_t + D_t = W_t L_t + (1 + i_{t-1}) D_{t-1} + P_t \Pi_t, \tag{1.2}$$

where D_t is the deposit to domestic banks, and i_t is the net nominal interest rate accrued to deposits.

In addition to the wage income from supplying labour, $W_t L_t$, and the interest payments from deposits, there are profits from owning various firms and financial intermediaries, as well as exogenous government transfers. This is characterised by Π_t:

$$\Pi_t = \frac{1}{m}\{[q_t - 1 - \frac{\kappa_I}{2}(\frac{I_t}{I_{t-1}} - 1)^2] I_t + \int_0^m [(\frac{\mathcal{P}_t(h)}{P_t} - mc_t) y_t(h) + (\frac{\mathcal{E}_t \mathcal{P}_t^*(h)}{P_t^*} - mc_t) y_t^*(h)] dh +$$
$$(1-\omega)[(R_t^K + (1-\delta) q_t) K_{t-1} - \widetilde{R}_t d_{t-1} - \varepsilon_t R_t^{G*} d_{t-1}^{G*}] - \xi (R_t^K + (1-\delta) q_t) K_{t-1} + TR_t\},$$

where the first component is the profit from owning investment goods production firms. The second component is the profit from owning output production firms. The third component is the profit brought by retiring bankers. The fourth component is the transfer made to new bankers. The last component is the government transfer. The parameters and variables in these components will be defined in sequence below in this section.

The first-order conditions that maximise the household utility (1.1) subject to the budget constraint (1.2) and a Transversality Condition are characterised below:

$$\zeta L_t^{\chi} C_t = \frac{W_t}{P_t} \epsilon_{c,t} = w_t \epsilon_{c,t} \tag{1.3}$$

$$\mathbb{E}_t[\beta (\frac{C_t \epsilon_{c,t+1}}{C_{t+1} \epsilon_{c,t}}) \frac{1 + i_t}{\pi_{t+1}}] = \mathbb{E}_t[\Lambda_{t,t+1} R_{t+1}] = 1, \tag{1.4}$$

where $\Lambda_{t,t+1} = E_t[\beta (\frac{C_t \epsilon_{c,t+1}}{C_{t+1} \epsilon_{c,t}})]$ is the stochastic discounting factor for the representative household in Home, $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross CPI inflation rate, R_t is the gross real returns for deposits.

The representative household in Foreign maximises a expected discounted utility function that is symmetrically defined subject to the budget constraint

$$P_t^* C_t^* + D_t^* + D_t^{G*} = W_t^* L_t^* + (1 + i_{t-1}^*)(D_{t-1}^* + D_{t-1}^{G*}) + P_t^* \Pi_t^*,$$

where D_t^{G*} is the deposit to international wholesale funds, and the rest of variables are defined similarly as those for Home. The utility maximisation problem yields the following

set of first-order conditions:

$$\zeta^* L_t^{*\chi} C_t^* = \frac{W_t^*}{P_t^*}\epsilon_{c,t}^* = w_t^* \epsilon_{c,t}^*$$
$$\mathbb{E}_t[\beta(\frac{C_t^* \epsilon_{c,t+1}^*}{C_{t+1}^* \epsilon_{c,t}^*})\frac{1+i_t^*}{\pi_{t+1}^*}] = \mathbb{E}_t[\Lambda_{t,t+1}^* R_{t+1}^*] = 1. \tag{1.5}$$

1.2.2 Financial Intermediaries

1.2.2.1 Banks in the Home Country

Table 1.1: The Balance Sheet of Banks in the Home Country

Loans to Firms $q_t K_t^i$	Deposits from Home Households d_t^i
	Borrowing from International Wholesale Funds $\varepsilon_t d_t^{Gi*}$
	Net Worth n_t^i

Table 1.1 describes the balance sheet of a typical bank in Home. The typical bank lends K_t^i to domestic production firms to obtain the rental and resale income. This is partly financed by deposits d_t^i from households and own net worth n_t^i. In addition, the bank borrows from international wholesale funds in the Foreign currency. These banks are however not large enough to hedge against this exchange rate risk.[3] The bankers manage the banks until their retirement. The retirement assumption assumes away the possibility that the bankers will be eventually unconstrained by the incentive compatibility constraint (described below) after having accumulated enough wealth.

The typical bank maximises the expected net worth

$$V_t^i = \mathbb{E}_t \sum_{j=1}^{\infty} \Lambda_{t,t+j} \omega^{j-1}(1-\omega) n_{t+j}^i,$$

where the net worth

$$n_t^i = (R_t^K + (1-\delta)q_t)K_{t-1}^i - \varepsilon_t R_t^{G*} d_{t-1}^{Gi*} - \widetilde{R}_t d_{t-1}^i,$$

where R_t^K is the real return for capital, δ is the depreciation rate, q_t is the resale value of capital, ε_t is the real exchange rate, R_t^{G*} is the cost of borrowing from international

[3] This assumption is in accordance with the empirical observation by Shin (2014) and Bruno and Shin (2015a) if we consider the balance sheet of banks and firms in the emerging markets jointly.

wholesale funds, $\widetilde{R}_t$ is the cost incurred by borrowing from households. Note that $\widetilde{R}_t - R_t = \tau_t^r > 0$, where τ_t^r is an exogenous premium incurred by banks when borrowing from domestic households.[4] The maximisation is subject to the balance sheet constraint

$$q_t K_t^i = n_t^i + d_t^i + \varepsilon_t d_t^{Gi*}, \tag{1.6}$$

and the incentive compatibility constraint (IC)

$$V_t^i \geq \vartheta(x_t^i) q_t K_t^i$$

as a limit for its ability to raise funds. The latter constraint is motivated by the moral hazard argument in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). The banker may choose to divert a fraction $\vartheta(x_t^i)$ of its total asset and transfer this amount back to the household. However, this activity cannot go through without being noticed. In the event, the creditors of the bank can force the bank into bankruptcy and recover the remaining $1 - \vartheta(x_t^i)$ fraction of the total asset, whilst the banker will lose the franchise to manage the bank and thus the expected net worth completely. Therefore, in equilibrium the contract between the creditors and the bank will involve the IC constraint above to eliminate the incentive to divert funds.

Specifically, the fraction of divertible funds is defined in the similar way as in Aoki et al. (2020):

$$\vartheta(x_t^i) = \vartheta(\frac{\kappa_b}{2}(x_t^i)^2 + 1),$$

where

$$x_t^i = \frac{\varepsilon_t d_t^{Gi*}}{q_t K_t^i}$$

is the ratio of the Foreign currency liability to total asset. This means that the fraction that the creditors recover in the event of the bank's bankruptcy, $1 - \vartheta(x_t^i)$, is lower if the bank finances its operation by a higher proportion of Foreign currency borrowing. This captures the idea that it is harder for foreign lenders to monitor and enforce contracts with the borrowing banks. In addition, as pointed out by Shin (2009), creditor banks such as wholesale funding providers are also subject to some external constraints. When they suddenly decrease lending due to its own prudential risk management, the resulting effect for the debtor banks can be devastating given the very short term nature of the inter-bank funding. Hence, creditors would impose stricter leverage constraint in response to a higher level of the bank's borrowing from wholesale funds banks.

[4] This ensures that borrowing from domestic households will be more expensive than from international wholesale funds in the steady state where households in both countries are equally patient. This provides a motive for Home banks to borrow internationally as in Akinci and Queralto (2019) and Aoki et al. (2020).

Define $\psi_t^i = \frac{V_t^i}{n_t^i}$ and substitute in the balance sheet constraint (1.6), we have:

$$\psi_t^i = \frac{V_t^i}{n_t^i} = \mu_{0,t}\Gamma_t^i + \mu_{1,t}\Gamma_t^i x_t^i + \mu_{2,t},$$

where $\Gamma_t^i = \frac{q_t K_t^i}{n_t^i}$ is the leverage ratio of the bank, and

$$\mu_{0,t} = \mathbb{E}_t\{\Omega_{t,t+1}(\frac{R_{t+1}^K + (1-\delta)q_{t+1}}{q_t} - \widetilde{R}_{t+1})\} \tag{1.7}$$

$$\mu_{1,t} = \mathbb{E}_t\{\Omega_{t,t+1}(\widetilde{R}_{t+1} - \frac{R_{t+1}^{G*}\varepsilon_{t+1}}{\varepsilon_t})\} \tag{1.8}$$

$$\mu_{2,t} = \mathbb{E}_t\{\Omega_{t,t+1}\widetilde{R}_{t+1}\} \tag{1.9}$$

$$\Omega_{t,t+1} = \mathbb{E}_t\{\Lambda_{t,t+1}(1-\omega+\omega\psi_{t+1})\}. \tag{1.10}$$

$\mu_{0,t}$ is the expected credit premium between the loan rate for firms and the deposit rate for households, and $\mu_{1,t}$ is the expected relative cost advantage of financing through the inter-bank lending over financing through domestic deposits, which can be regarded as a currency premium.

Then the maximisation problem of the bank can be rewritten as maximising ψ_t^i subject to the incentive compatibility constraint which can be rewritten as

$$\psi_t^i \geq \vartheta(x_t^i)\Gamma_t^i. \tag{1.11}$$

For the equilibrium where the IC always binds, we have

$$\Gamma_t^i = \frac{\mu_{2,t}}{\vartheta(x_t^i) - \mu_{0,t} - \mu_{1,t}x_t}, \tag{1.12}$$

so that the bank's leverage will be constrained by how profitable it is to extend loans measured by the credit premium $\mu_{0,t}$, and how strict the IC is which is measured by $\vartheta(x_t^i)$.

Combine the first-order conditions of this constrained maximisation problem with respect to Γ_t^i and x_t^i, we get

$$x_t^i = \frac{\sqrt{1 + \frac{2}{\kappa_b}(\mu_t)^2} - 1}{\mu_t}, \tag{1.13}$$

where

$$\mu_t = \frac{\mu_{1,t}}{\mu_{0,t}}. \tag{1.14}$$

It can be shown that x_t^i is increasing in μ_t. This means that the cheaper borrowing from international wholesale funds relative to borrowing from domestic households is, the higher proportion of interbank borrowing will there be in the bank's balance sheet.

Lastly, the aggregate net worth for banks in the Home country evolves according to

$$n_t = (\omega + \xi)(R_t^K + (1-\delta)q_t)K_{t-1} - \omega\varepsilon_t R_t^{G*} d_{t-1}^{G*} - \omega\widetilde{R}_t d_{t-1} \tag{1.15}$$

and the aggregate balance sheet constraint, the leverage ratio, and the proportion of the interbank lending are respectively:

$$q_t K_t = n_t + d_t + \varepsilon_t d_t^{G*}$$

$$\Gamma_t = \Gamma_t^i$$

$$x_t = x_t^i.$$

1.2.2.2 Banks in the Foreign Country

Table 1.2: The Balance Sheet of Banks in the Foreign Country

Loans to Firms $q_t^* K_t^{i*}$	Deposits from Foreign Households d_t^{i*}
	Net Worth n_t^{i*}

Banks in Foreign are standard Gertler and Karadi (2011) type banks. Table 1.2 describes the balance sheet of a typical foreign bank. The bank finances the lending of capital K_t^{i*} to domestic firms by deposits d_t^{i*} from households and own net worth n_t^{i*}.

The bank maximises the expected net worth:

$$V_t^{i*} = \mathbb{E}_t \sum_{j=1}^{\infty} \Lambda_{t,t+j}^* \omega^{j-1}(1-\omega) n_{t+j}^{i*},$$

where the net worth is

$$n_t^{i*} = (R_t^{K*} + (1-\delta)q_t^*)K_{t-1}^{i*} - R_t^* d_{t-1}^{i*}.$$

The maximisation is subject to the balance sheet constraint

$$q_t^* K_t^{i*} = n_t^{i*} + d_t^{i*}$$

and the incentive compatibility constraint (IC)

$$V_t^{i*} \geq \vartheta^* q_t^* K_t^{i*}.$$

Then the maximisation problem can be rewritten in a similar fashion as that for Home banks as maximising:

$$\psi_t^{i*} = \mu_{0,t}^* \Gamma_t^{i*} + \mu_{1,t}^*,$$

where

$$\begin{aligned}
\mu_{0,t}^* &= \mathbb{E}_t\{\Omega_{t,t+1}^*(\frac{R_{t+1}^{K*} + (1-\delta)q_{t+1}^*}{q_t^*} - R_{t+1}^*)\} \\
\mu_{1,t}^* &= \mathbb{E}_t\{\Omega_{t,t+1}^* R_{t+1}^*\} \\
\Omega_{t,t+1}^* &= \mathbb{E}_t\{\Lambda_{t,t+1}^*(1-\omega+\omega\psi_{t+1}^*)\},
\end{aligned} \tag{1.16}$$

subject to the IC constraint

$$\psi_t^{i*} \geq \vartheta^* \Gamma_t^{i*}. \tag{1.17}$$

In the equilibrium where the IC constraint always binds, we have

$$\Gamma_t^{i*} = \frac{\mu_{1,t}^*}{\vartheta^* - \mu_{0,t}^*}.$$

Lastly, the aggregate net worth for all banks in Foreign evolves according to

$$n_t^* = (\omega + \xi^*)(R_t^{K*} + (1-\delta)q_t^*)K_{t-1}^* - \omega R_t^* d_{t-1}^*.$$

The aggregate balance sheet constraint of the bank is given by

$$q_t^* K_t^* = n_t^* + d_t^*.$$

The leverage ratio is accordingly

$$\Gamma_t^* = \Gamma_t^{i*}.$$

1.2.2.3 International Wholesale Funds

International wholesale funds are owned by Foreign households and follow very simple rules to channel funds across banks and borders. They take deposits from Foreign households and promise the same payoff as domestic banks in the Foreign country. Then they

lend to banks in Home at the rate:

$$R_t^{G*} = R_t^*.^{5} \tag{1.18}$$

1.2.3 Monopolist Producers

The monopolistic producer of a particular brand of the H-type good produces output $Y_t(h)$ according to the production function

$$Y_t(h) = A_t(\frac{K_{t-1}(h)}{\alpha})^{\alpha}(\frac{L_t(h)}{1-\alpha})^{1-\alpha},$$

where $\alpha \in [0,1]$, $K_{t-1}(h)$ and $L_t(h)$ are the capital and labour inputs. A_t is the aggregate productivity which follows an exogenous process

$$A_t = A_{t-1}^{\rho_A} \exp(\epsilon_{a,t}).$$

The cost minimisation implies the following first-order conditions:

$$\begin{aligned} Y_t(h) &= A_t(\frac{K_{t-1}(h)}{\alpha})(\frac{R_t^K}{w_t})^{1-\alpha} \\ Y_t(h) &= A_t(\frac{L_t(h)}{1-\alpha})(\frac{w_t}{R_t^K})^{\alpha}. \end{aligned} \tag{1.19}$$

The corresponding real marginal cost is

$$mc_t(h) = \frac{1}{A_t}(R_t^K)^{\alpha} w_t^{1-\alpha} = mc_t, \tag{1.20}$$

which implies that the real marginal costs for producing different brands are equalised.

Every period, the producer can reset its prices in both countries in its domestic currency with probability $(1-\theta)$ as in Calvo (1983). They maximise the expected profit

$$\mathbb{E}_t\{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k}[(\frac{\mathcal{P}_t(h)}{P_{t+k}} - mc_{t+k})y_{t+k}(h) + (\frac{\mathcal{E}_t\mathcal{P}_t^*(h)}{P_{t+k}} - mc_{t+k})y_{t+k}^*(h)]\}$$

where $\mathcal{P}_t(h)$ and $\mathcal{P}_t^*(h)$ are the prices chosen by the firm, and $\mathcal{E}_t$ is the nominal exchange

[5] In practice, large banks that operate across borders are able to hedge away exchange rate risks incurred (Cesa-Bianchi et al., 2018). Therefore, I assume that Home banks are small local banks which cannot directly borrow from Foreign households. The cross-border lending activities are conducted by wholesale funds providers that operate internationally.

rate. The profit maximisation is subject to the demand for the particular brand from both countries

$$y_{t+k}(h) = (\frac{\mathcal{P}_t(h)}{P_{H,t+k}})^{-\varphi} y_{H,t+k} \qquad \text{and} \qquad y^*_{t+k}(h) = \frac{1-m}{m}(\frac{\mathcal{P}^*_t(h)}{P^*_{H,t+k}})^{-\varphi} y^*_{H,t+k}$$

where $y_{H,t}$ and $y^*_{H,t}$ are the total demand for the H-type goods in Home and Foreign respectively:

$$y_{H,t} = a(\frac{P_{H,t}}{P_t})^{-\phi}(C_t + I_t + G_t + \frac{\kappa_I}{2}(\frac{I_t}{I_{t-1}} - 1)^2 I_t)$$
$$y^*_{H,t} = a^*(\frac{P^*_{H,t}}{P^*_t})^{-\phi}[C^*_t + I^*_t + G^*_t + \frac{\kappa_I}{2}(\frac{I^*_t}{I^*_{t-1}} - 1)^2 I^*_t].$$

This gives the optimal domestic price as:

$$\mathcal{P}_t(h) = \mathbb{E}_t \frac{\sum_{k=0}^{\infty} \mu^P \theta^k \Lambda_{t,t+k} y_{H,t+k} P^{\varphi}_{H,t+k} m c_{t+k}}{\sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} y_{H,t+k} P^{\varphi}_{H,t+k} P^{-1}_{t+k}},$$

where $\mu^P = \frac{\varphi}{\varphi - 1}$.

Meanwhile, firms will optimally choose identical prices across borders, because the demand elasticities are constant and symmetric across borders. Thus

$$\mathcal{P}_t(h) = e_t \mathcal{P}^*_t(h)$$

holds for all brands of goods, where $\mathcal{P}_t(h)$ and $\mathcal{P}^*_t(h)$ are prices of good h in the two countries set by the producer. In other words, the Law of One Price holds for all brands of goods.

With this pricing behaviour, the aggregate price index of the H-type goods evolves according to:

$$P_{H,t} = (\theta P^{1-\varphi}_{H,t-1} + (1-\theta)\mathcal{P}_t(h)^{1-\varphi})^{\frac{1}{1-\varphi}},$$

which can be rewritten as

$$\theta \pi^{\varphi-1}_{H,t} + (1-\theta)(\frac{\mathcal{P}_t(h)}{P_{H,t}})^{1-\varphi} = 1.$$

Aggregate the demand for capital from (1.19):

$$K_{t-1} = \int_0^m K_{t-1}(h)dh = \frac{\alpha}{A_t}(\frac{R^K_t}{w_t})^{\alpha-1} Y_{H,t} s_t,$$

where $Y_{H,t} = y_{H,t} + y^*_{H,t}$ is the total output produced in Home, $s_t = \int_0^m (\frac{\mathcal{P}_t(h)}{P_{H,t}})^{-\varphi} dh$ is the price dispersion. The law of motion for s_t is

$$s_t = (1-\theta)(\frac{\mathcal{P}_t(h)}{P_{H,t}})^{-\varphi} + \theta \pi^{\varphi}_{H,t} s_{t-1}.$$

Similarly, the aggregate demand for labour is:

$$L_t = \frac{1-\alpha}{A_t}(\frac{R^K_t}{w_t})^{\alpha} Y_{H,t} s_t. \tag{1.22}$$

A symmetric set of equations hold for production firms in the Foreign country as well.

1.2.4 Capital Producers

Capital producers create a unit of capital good using a unit of final good, and pay a quadratic adjustment cost. They maximise

$$V_{Kt} = \left[q_t - 1 - \frac{\kappa_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t + \mathbb{E}_t[\Lambda_{t,t+1} V_{Kt+1}(I_t)].$$

This gives the first-order condition

$$q_t = 1 + \frac{\kappa_I}{2}(\frac{I_t}{I_{t-1}} - 1)^2 + \frac{I_t}{I_{t-1}} \kappa_I (\frac{I_t}{I_{t-1}} - 1) - \kappa_I \mathbb{E}_t[\Lambda_{t,t+1} (\frac{I_{t+1}}{I_t} - 1) \frac{I^2_{t+1}}{I^2_t}].$$

The aggregate capital accumulates according to:

$$K_t = (1-\delta) K_{t-1} + I_t.$$

A symmetric set of equations hold for the Foreign country as well.

1.2.5 Market Equilibrium

In both countries, the output produced by their domestic firms is either consumed, invested, exported, purchased by governments, or used to pay adjustment costs. The labour and capital markets also clear within each country.

The government budget constraints in both countries are respectively

$$G_t + TR_t = 0$$

$$G_t^* + TR_t^* = 0.$$

Lastly, by aggregating the Home household budget constraints, the net foreign liabilities evolve according to:

$$d_t^{G*} = R_t^{G*} d_{t-1}^{G*} + \frac{P_{F,t}^*}{P_t^*} y_{F,t} - \frac{P_{H,t}^*}{P_t^*} y_{H,t}^*. \tag{1.23}$$

1.3 Calibration and Empirical Relevance of the Model

For calibration, the Foreign economy is taken as a bloc of financial centre countries, such as the OECD group, most notably the US and the EU. The Home economy is taken as emerging market countries, such as Latin American and East European countries. The two country blocs are assumed to be equal in size so that $m = 0.5$. The model is calibrated around a steady state where there is no trend inflation in both blocs.

Table 1.3 reports the calibrated values of parameters. β is chosen so that the annualised real interest rates for both regions are 2%. The values of χ, δ, κ_I, and α standard in DSGE literature.[6] ϕ is set to be 1 so that its product with the intertemporal elasticity of substitution is 1. In a standard New Keynesian two country model, this setting implies that there is no motive for national benevolent planners to manipulate terms-of-trade in a non-cooperative Nash equilibrium that creates externalities to the other country.[7] This makes it easier to distinguish cooperation gains due to the financial transmissions. φ is set to 6 so that the steady state price mark-up is 20%.[8] θ is set so that the prices are not expected to change in a year. All above parameters are set symmetrically across the two regions. ν is set to 0.4 so that the home bias is 0.8 for both countries. This implies a steady state export-to-GDP ratio of 21% for Home, which compromises divergent values of various emerging markets.[9] ζ and ζ^* are chosen symmetrically, and their values imply that the steady state labour hour is $\frac{1}{3}$ in Home.

In terms of the parameters for the banking sector, I follow Aoki et al. (2020)'s calibrated value for ω and ω^* of 0.94, so that the annual dividend payout is $4(1-\omega) = 24\%$ of the net worth, which is reasonable if one includes the bonus payments to executives.

[6] e.g. Christiano et al. (2005), Coenen et al. (2007).
[7] As demonstrated in Obstfeld and Rogoff (2002).
[8] This is in line with estimates from Justiniano et al. (2010).
[9] See, for example, The World Bank (2018).

For Foreign banks, ϑ^* and ξ^* are pinned down by two targets: a credit spread of 150 basis points annually and a leverage multiple of 4. The latter is a compromised value between high leverage ratios in commercial and investment banks and low ratios in non-financial business.[10] For Home banks, I target a domestic credit spread of 150 basis points, a leverage multiple of 4, and a 25% dollar-debt-to-total-asset ratio in steady state. The exogenous premium for borrowing from domestic households, τ^r, is set to be 0.0025, so that the average currency premium for Home banks to borrow domestically over internationally is 100 basis points annually. These targets are broadly consistent with targets used in literature,[11] and they pin down the values of ϑ, ξ, and κ_b.

I assume 8 shocks in the model, 4 for each country. The productivity shocks in both countries are assumed to have a persistence of 0.95, a standard deviation of 0.01, and a correlation of 0.2, which is broadly consistent to the range of values estimated in literature.[12] The process for preference shocks are assumed to be the same as the productivity shocks.[13] Lastly, both the price mark-up shocks and net worth shocks in both coutries are assumed to be mutually independent one-off shocks with standard deviation of 0.01. The different types of shocks are assumed to be mutually independent as well.

In order to check that the calibration of the model produces reasonable qualitative and quantitative features, I close the model by assuming two symmetric benchmark Taylor rules as:[14]

$$\frac{1+i_t}{1+\bar{i}} = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\rho_m}\left(\frac{\pi_{H,t}}{\bar{\pi}_H}\right)^{\varphi_\pi(1-\rho_m)} \tag{1.24}$$

$$\frac{1+i^*_t}{1+\bar{i}^*} = \left(\frac{1+i_{t-1}}{1+\bar{i}^*}\right)^{\rho_m}\left(\frac{\pi^*_{F,t}}{\bar{\pi}^*_F}\right)^{\varphi_\pi(1-\rho_m)}, \tag{1.25}$$

where $\bar{i}$, $\bar{i}^*$, $\bar{\pi}_H$, and $\bar{\pi}^*_F$ are steady state levels of the corresponding variables, ρ_m is the smoothing parameter which is set to be 0.8, φ_π is the monetary policy response parameter with respect to the PPI inflation, which is set to be 1.5.

Table 1.4 shows the contributions of various shocks to fluctuations of selected macroeconomic variables. The variance decomposition under the baseline calibration is broadly consistent with Smets and Wouters (2007). The productivity shocks account for most variations in output and other main variables in both countries, although a large part of

[10] Here the combined balance sheet of the banking and the non-banking sectors is considered, which makes this value reasonable. See Gertler and Karadi (2011) and Aoki et al. (2020) for a discussion.

[11] See, for example, Akinci and Queralto (2019) and Mimir and Sunel (2015).

[12] e.g. Baxter (1995), Eichenbaum et al. (2021).

[13] as in Stockman and Tesar (1995) and Benigno (2009).

[14] The benchmark rules assume that central banks respond to domestic inflation only. Additional checks show that the results presented below are not significantly different if central banks respond to CPI inflation instead.

Table 1.3: Calibration of Parameters

Parameter	Value	Interpretation
Household, Firms, and Trade		
β, β^*	$1.02^{-0.25}$	discount factor for households
χ, χ^*	3	inverse of the Frisch elasticity of labour supply
ν	0.4	degree of openness
ζ, ζ^*	90	relative weight of labour in utility
ϕ, ϕ^*	1	elasticity of substitution between Foreign and Home goods
φ, φ^*	6	elasticity of substitution between brands of commodities within a single type of goods
δ, δ^*	0.025	depreciation rate
α, α^*	0.33	share of capital cost in production
κ_I, κ_I^*	1.2	adjustment cost for investment
θ, θ^*	0.75	price rigidity
Financial Intermediaries		
ω, ω^*	0.94	survival rate of bankers
ξ	0.0091	initial endowment for entry bankers
ξ^*	0.0103	initial endowment for entry bankers
ϑ	0.340	proportion of divertible funds
ϑ^*	0.329	proportion of divertible funds
κ_b	2.482	increase in the proportion of divertible funds with the increase in Foreign currency lending
Shocks		
ρ_A, ρ_A^*	0.95	persistence of the productivity shock
σ_A, σ_A^*	0.01	standard deviation of the productivity shock
σ_{AA^*}	0.2	correlation coefficient of the productivity shocks
ρ_c, ρ_c^*	0.95	persistence of preference shock
σ_c, σ_c^*	0.01	standard deviation of preference shock
σ_{cc^*}	0.2	correlation coefficient of preference shocks
σ_p, σ_p^*	0.01	standard deviation of mark-up shock
σ_n, σ_n^*	0.01	standard deviation of net worth shock

this can be driven by the large size and persistence of the shocks. The mark-up shocks are important drivers for inflation in respective countries. The preference shocks account for a fair amount of variations in consumption and labour, whilst the net worth shocks drive large amount of fluctuations in bank net worth and spreads.

Another interesting observation is that Foreign productivity shocks drive a significant portion of fluctuations in Home consumption, investment, capital price, and all bank variables. Nevertheless, productivity shocks in Home do not account for similar proportions of variations in Foreign variables. This suggests a potentially greater role of the currency mismatch frictions on the balance sheets of Home banks to propagate the cross-border spill-overs in the Home country than in the Foreign country.

Figure 1.2 illustrates more clearly the role of this currency mismatch in the banking sector in transmitting shocks from financial centre countries to financial periphery countries as documented empirically by, for example, Miranda-Agrippino and Rey (2020). The blue lines plot impulse responses of key variables in the model to a shock of 25 basis points increase in the Foreign (financial centre) interest rate. The shock induces an appreciation of the Foreign currency relative to the Home currency. Through the expenditure-switching channel, Home exports (Foreign imports) increase and imports from Foreign become more expensive. This contributes to the initial increase in output and CPI inflation. Meanwhile, the Foreign currency appreciation reduces the Home bank net worth due to a sudden rise in the cost of serving the Foreign currency denominated debt on their balance sheets. This leads to an increase in spreads and a decline in investment. The declining capital price further shrinks the bank net worth and further reduces investment. Ultimately, these effects put downward pressure on output and inflation.

The red dashed lines show the responses to the same shock if Home banks were able to raise debt from wholesale banks in the Home currency. In this counterfactual case, the wholesale funds providers still finance their lending to Home banks by deposits in the Foreign currency from Foreign households. Moreover, the wholesale funds providers as subsidiaries of internationally-operated large banks can hedge away any exchange rate mismatch on their balance sheets so that they are not subject to the exchange rate risks. The calibration and the steady state of the model is kept unchanged from the baseline model. The technical details of this model is presented in Appendix 1.A.[15] It is clear from the figure that the shock inflict much milder responses across all variables in Home when the currency mismatch is absent in the model. The banking sector is not significantly affected by the shock. Thus, the decrease in investment is much lower.

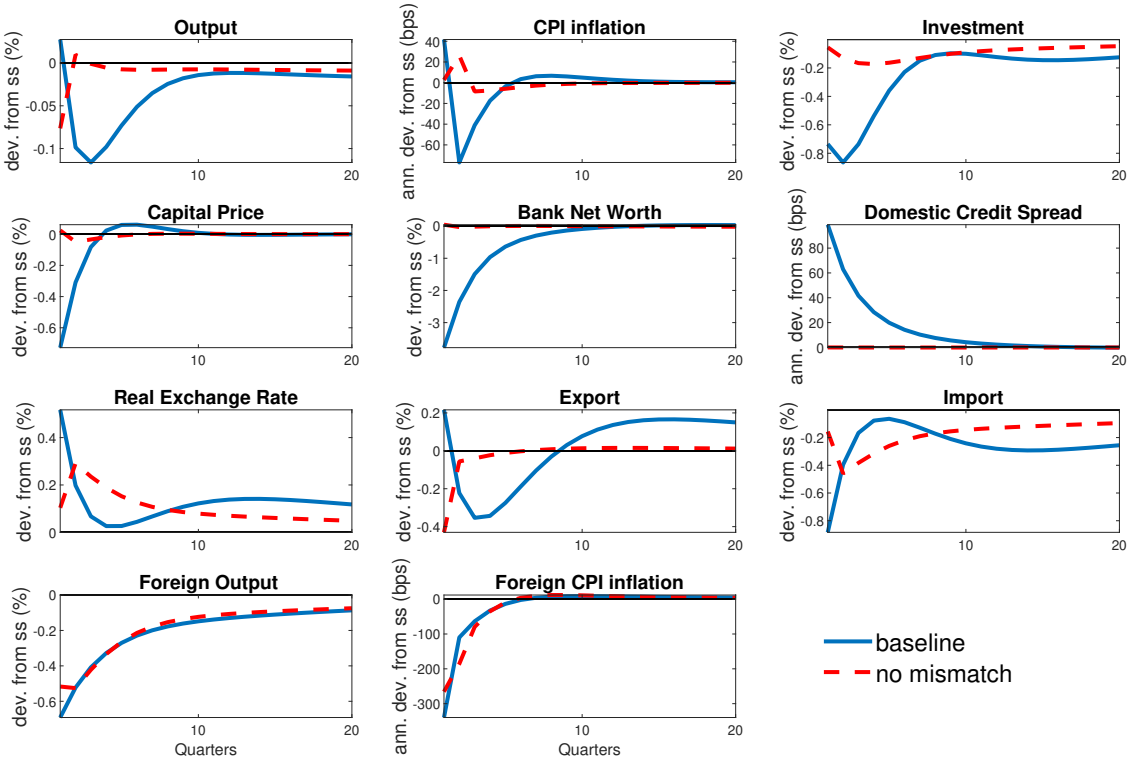
[15] Another counterfactual is to set $x_t = 0$. However, this assumption essentially means that Home banks cannot borrow internationally, not just that they cannot borrow in their own currency. Moreover, the steady state of such a model is different from the baseline model with different degrees of inefficiency. Therefore, the quantitative results from such a model for the optimal policy analyses that are discussed later in the chapter are not easily comparable to those of the baseline case. By contrast, the model described above is a comparable counterfactual to the baseline case with the same steady state and calibration.

Table 1.4: Variance Decomposition (in percent)

Variable	Productivity Home	Productivity Foreign	Net Worth Home	Net Worth Foreign	Preference Home	Preference Foreign	Mark-up Home	Mark-up Foreign
Home Variables								
Output	91.20	7.34	0.31	0.01	0.68	0.16	0.31	0.00
PPI Inflation	44.86	5.59	4.60	0.33	10.92	1.97	31.66	0.08
CPI Inflation	21.07	36.18	9.51	0.78	6.42	3.72	20.24	2.07
Consumption	60.97	26.77	0.14	0.08	11.67	0.17	0.18	0.02
Labour	28.37	10.73	2.81	0.23	41.31	5.79	10.64	0.11
Investment	66.89	19.90	8.12	0.27	1.90	1.48	1.42	0.01
Price of Capital	20.68	22.85	43.76	0.28	1.39	0.68	10.21	0.16
Bank Net Worth	19.13	18.45	53.89	0.05	2.93	0.74	4.78	0.04
Credit Spread	2.52	18.88	71.24	0.11	2.87	0.32	4.03	0.01
Currency Premium	2.36	19.83	70.20	0.12	3.06	0.43	3.99	0.02
Proportion of Foreign Currency Debt	6.77	27.10	4.07	0.37	50.97	10.31	0.29	0.14
Foreign Variables								
Output	0.48	98.01	0.06	0.38	0.09	0.61	0.00	0.35
PPI Inflation	1.68	33.48	2.33	11.83	1.43	14.02	0.11	35.12
CPI Inflation	24.42	28.43	14.31	6.84	4.52	7.37	0.59	13.52
Consumption	7.96	75.60	0.40	0.59	0.91	14.37	0.02	0.16
Labour	2.21	21.44	3.08	6.93	2.46	48.02	0.09	15.78
Investment	3.28	85.01	0.54	6.78	0.67	2.46	0.04	1.21
Price of Capital	0.36	58.24	1.54	30.26	0.28	0.21	0.06	9.05
Bank Net Worth	3.60	50.22	0.08	40.59	0.24	1.23	0.00	4.04
Credit Spread	3.16	39.73	0.12	53.29	0.10	0.23	0.01	3.34

In addition, the last two panels of the same figure indicate that output and inflation in the Foreign country decrease by more in the model with currency mismatch on the balance sheets of Home banks. This means that the currency mismatch friction not only increases the spill-over of the Foreign monetary policy shock to Home, but also the spill-back to the Foreign country itself. These spill-overs and spill-backs suggest potential room for policy cooperation.

Figure 1.2: Impulse Response Functions to a Foreign MP Shock



1.4 Gains from Monetary Policy Cooperation

This section examines the gains from international monetary policy cooperation. I firstly define the cooperative and non-cooperative equilibria. I also describe the methods for deriving policy rules and evaluating policy cooperation gains. Then, the quantitative results are reported. Lastly, I check the robustness of my results.

1.4.1 Derive and Evaluate Policy Cooperation Gains

The gains from policy cooperation is evaluated by comparing the world joint and individual country welfare measures under a cooperative optimal policy equilibrium against the same welfare measures under an open-loop Nash equilibrium.

In the optimal cooperative policy equilibrium, a benevolent social planner maximises the world joint welfare

$$\mathbb{W}_t = \upsilon U_t + (1-\upsilon)U_t^* + \beta^W \mathbb{E}_t \Delta \mathbb{W}_{t+1}$$

with respect to all endogenous variables, subject to the structural constraints of the economy, where

$$U_t = log(C_t)\epsilon_{c,t} - \frac{\zeta L_t^{1+\chi}}{1+\chi}$$

and U_t^* is the corresponding utility measure for the Foreign country. υ is the weight attached to the utility of Home residents. For the baseline case, I set $\upsilon = m = 0.5$. β^W is the discount parameter for the aggregate welfare, which is set to $1.02^{-0.25}$.

In the open-loop Nash equilibrium, there are two central banks as benevolent social planners, one for each country. They maximise the aggregate welfare of households in their own country with respect to all endogenous variables of the model except the policy instrument variable of the other country, subject to the structural constraints of the economy, taking the entire sequence of policy moves of the other central bank as given. In equilibrium, each central bank's policy is the best response to the other central bank's policy.

For the cooperative and the Nash equilibrium, the optimal policy paths are solved under the assumption of a timeless perspective. In addition, both national central banks have commitment devices. The optimal policy paths are derived by maximising the non-linear welfare functions subject to the non-linear structural constraints (i.e. the Lagragian approach). This is implemented in Dynare using the toolbox developed by Bodenstein et al. (2019).[16] For now, assume also that benevolent social planners use the producer-price-index (PPI) inflation as their operating instruments under both the cooperative and the non-cooperative regimes as in Benigno and Benigno (2006). The robustness of the results to the choice of instruments under the non-cooperative regime will be examined in section 1.4.3.

The model is approximated to the second-order when solving all equilibria considered. This ensures accurate measures for welfare evaluation as pointed out by Schmitt-Grohe and Uribe (2007). Let $\mathbb{W}_0^c$ be the welfare achieved by the optimal cooperative policy equilibrium, $\mathbb{W}_0^n$ be the welfare achieved by any non-cooperative policy equilibrium, and

[16] Another widely-used approach in literature for deriving the optimal policy is the linear-quadratic (LQ) approach, which derives the first-order conditions (FOCs) by maximising a welfare function approximated to the second-order, subject to structural constraints approximated to the first-order. An example is Benigno and Benigno (2006). For my medium-scale model here, I rely upon the algorithm implemented in Dynare using the Lagrangian approach, which significantly simplifies the derivation and facilitates the exploration of a wide range of sensitivity analyses as described below.

Table 1.5: Welfare Decompostion: Gains from Optimal Cooperative Equilibrium Relative to the Optimal Non-cooperative Nash Equilibrium

Shocks	Welfare Gains (Baseline)			Welfare Gains (No-Mismatch)		
	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$
All Shocks	-0.1288	0.1762	0.0236	0.0193	0.0206	0.0199
Productivity, Home	0.0874	-0.0758	0.0057	-0.0006	0.0050	0.0022
Productivity, Foreign	-0.1134	0.1140	0.0002	0.0020	0.0004	0.0012
Net Worth, Home	-0.1180	0.1188	0.0003	-0.0004	0.0013	0.0004
Net Worth, Foreign	0.0479	-0.0175	0.0152	0.0162	0.0056	0.0109
Mark-up, Home	0.0097	-0.0067	0.0015	-0.0008	0.0077	0.0035
Mark-up, Foreign	-0.0164	0.0189	0.0013	0.0025	0.0002	0.0013
Preference, Home	0.0064	-0.0053	0.0005	0.0001	0.0002	0.0001
Preference, Foreign	-0.0126	0.0129	0.0002	0.0001	0.0003	0.0002

λ^W be the unit of consumption that one would give up under the non-cooperative equilibrium to live under the optimal cooperative equilibrium. Then the welfare cost of not cooperating the monetary policy evaluated by the consumption-equivalent measure is

$$\lambda^W = [\exp((1-\beta^W)(\mathbb{W}_0^c - \mathbb{W}_0^n)) - 1] \times 100\%,$$

which is the measure reported in tables in the rest of this paper.

1.4.2 Quantitative Results

Table 1.5 displays the unconditional welfare gains for policy cooperation relative to the non-cooperative Nash equilibrium, as well as a decomposition of the welfare changes into contribution of each shock. The table also reports the same set of results for the model without the currency mismatch. The welfare gains for Home and Foreign are denoted by $\Delta\mathbb{W}_t^H$ and $\Delta\mathbb{W}_t^F$ respectively, and the gains for the joint world welfare is denoted by $\Delta\mathbb{W}_t$.

The table shows that the joint world welfare improves by 0.024% of the steady state consumption when central banks are cooperative compared to being non-cooperative conditional on all shocks in the baseline model. This magnitude is consistent with those found in other contributions in literature. However, the distribution of this cooperative gain is highly uneven despite the equal country size and largely symmetric parameter values chosen for both countries in my calibration. The Home country loses by 0.129% whilst the Foreign country gains by 0.176% of the steady state consumption from cooperating. The magnitude of these welfare gains and losses for individual countries are much higher than the joint welfare. In other words, cooperation is highly desirable for the Foreign country but undesirable for the Home country.

Meanwhile, the decomposition results suggest that the productivity and net worth shocks are the principal contributors to the welfare gains and losses. For the productivity shocks, this might be partly due to the large persistence and cross-country correlation of the shock.[17] Nevertheless, cooperation only benefits one country and hurt the other country even conditional on each individual shock.

By contrast, in the model without the currency mismatch friction, the cooperative gains for both countries become positive and much smaller in magnitude. This is the case not only when all shocks are considered jointly, but also for the majority of shocks when their respective contributions are considered. Even if for the cases where one country loses by cooperation, the magnitude of the welfare losses are negligible compared to those conditional on the same shock in the baseline model.

These results suggest that the cross-border transmission of shocks through the currency mismatch friction in the banking sector is crucial in the policy trade-off and design, especially for the Home country. Without cooperation, the self-interested Foreign central bank's policy generates spill-overs to Home via exchange rate fluctuations which is further propagated through the Home banks' balance sheet effects. Depending on the nature of the shock, this may either improve or compromise the Home central bank's policy trade-off. By contrast, the self-interested Home central bank's policy actions will not generate the same effects through the Foreign banking system.

This asymmetry in banking sectors implies that the policy spill-overs through exchange rate movements have higher influence over the inter-temporal efficiency of capital allocation in Home than in Foreign. For Home, the first-best inter-temporal allocation implies that:

$$\mathbb{E}_t\{\Lambda_{t,t+1}(R^K_{t+1}+1-\delta)\}=1.$$

A symmetric equation holds at the first-best allocation for Foreign as well. In the decentralised model, however, the inter-temporal allocation for Home is characterised by combining equations (1.4), (1.7), (1.8), (1.10), (1.11), and (1.18):

$$\mathbb{E}_t\{\Lambda_{t,t+1}\frac{R^K_{t+1}+(1-\delta)q_{t+1}}{q_t}\}=1+\Delta^{spread}_t+\Delta^{UIP}_t,$$

where

$$\Delta^{spread}_t=\frac{\mu_{0,t}+\mu_{1,t}-Cov(\Lambda_{t,t+1}(\frac{R^K_{t+1}+(1-\delta)q_{t+1}}{q_t}-\frac{R^{G*}_{t+1}\varepsilon_{t+1}}{\varepsilon_t}),1-\omega+\omega\vartheta(x_{t+1})\Gamma_{t+1})}{\mathbb{E}_t(1-\omega+\omega\vartheta(x_{t+1})\Gamma_{t+1})},$$

[17] However, note that the preference shock is calibrated in the same way as the productivity shocks and do not contribute much to the welfare changes.

and

$$\Delta_t^{UIP} = \mathbb{E}_t[\Lambda_{t,t+1}(\frac{R^*_{t+1}\varepsilon_{t+1}}{\varepsilon_t} - R_{t+1})] + \frac{Cov(\Lambda_{t,t+1}(\frac{R^{G*}_{t+1}\varepsilon_{t+1}}{\varepsilon_t}), 1-\omega+\omega\vartheta(x_{t+1})\Gamma_{t+1})}{\mathbb{E}_t(1-\omega+\omega\vartheta(x_{t+1})\Gamma_{t+1})}.$$

Similarly, by combining equations (1.5), (1.7), (1.16), and (1.17), the inter-temporal allocation for Foreign is:

$$\mathbb{E}_t\{\Lambda^*_{t,t+1}\frac{R^{K*}_{t+1}+(1-\delta^*)q^*_{t+1}}{q^*_t}\} = 1 + \Delta_t^{spread*},$$

where

$$\Delta_t^{spread*} = \frac{\mu^*_{0,t} - Cov(\Lambda^*_{t,t+1}(\frac{R^{K*}_{t+1}+(1-\delta^*)q^*_{t+1}}{q^*_t} - R^*_{t+1}), 1-\omega^*+\omega^*\vartheta^*\Gamma^*_{t+1})}{\mathbb{E}_t(1-\omega^*+\omega^*\vartheta^*\Gamma^*_{t+1})}.$$

Note that when there is no constraint to banks' leverage, the domestic credit and foreign currency spread terms, $\mu_{0,t}$, $\mu_{1,t}$, and $\mu^*_{0,t}$, are zero, and the covariance terms are zero as well. Thus the deviations from efficient inter-temporal allocation due to credit spreads, Δ_t^{spread} and $\Delta_t^{spread*}$, disappear. On the other hand, when there is no currency mismatch on Home banks' balance sheets, the inefficiency from deviations from the UIP condition, Δ_t^{UIP}, will become 0, and the change in exchange rate will not affect Δ_t^{spread}.

Therefore, as implied by the results in Table 1.5, self-interested central banks can take advantage of this asymmetry in the cross-border transmission of policy effects strategically under no-cooperation to help stabilise their domestic net worth shocks at the cost of the other country. However, they are constrained to do so under cooperation. By contrast, under productivity, mark-up, and preference shocks, leveraging this financial transmission channel becomes self-defeating for themselves and only helps the other country to contain the spill-overs. Thus, these shocks are better accommodated by cooperating with the other country.

However, in the model without the currency mismatch friction, this asymmetric transmission of policy effects does not exist. Self-interested central banks still have incentives to manipulate the exchange rate strategically due to other inefficiencies in the model. For example, in my model with capital accumulation and inefficient shocks, the manipulation may achieve better allocation of labour efforts even if the product of the inter-temporal elasticity and the intra-temporal elasticity is 1, which is argued by Benigno and Benigno (2006). Alternatively, the manipulation can reduce financial cost of liability or increase return for foreign assets given the non-zero initial foreign asset position in the model, as discussed by Benigno (2009). Nevertheless, these motives are more symmetric for both

countries. Hence, the welfare gains from cooperation become more evenly distributed and smaller compared to the baseline model.

Table 1.6 show the differences of unconditional means and standard deviations of a selection of variables between the cooperative and the non-cooperative equilibrium for the baseline model under all shocks.[18] Consumption and labour are included as the two arguments for welfare functions. Inflation and output are often the principal components in central banks' objective function in literature. Spreads are included as they affect intertemporal efficiency of capital allocation as established earlier. Differences in means for all variables except inflation are the percentage change relative to the mean under the non-cooperative equilibrium. Otherwise, they are simple differences.

From the table, the exchange rate is more volatile without cooperation, which may reflect more aggressive policy reactions of both countries under the Nash equilibrium. The cooperative gain for Foreign comes from an overall increase in consumption by 0.163% and save in labour hours by 0.013% compared to the non-cooperative equilibrium. Meanwhile, the volatility is lower for output, CPI inflation, and credit spread. For Home, the cooperative policy generates welfare costs by reducing consumption by 0.105% and raises labour input by 0.024%. Although policy cooperation helps reduce the volatilities of output and CPI inflation, this is at the cost of more fluctuations in the spreads. Overall, the self-interested Home central bank is able to export the stabilisation costs to the Foreign country by manipulating the exchange rates strategically without incurring a matching spill-back from the Foreign country. Policy cooperation, instead, limits the Home central bank to do so such that the welfare costs are redistributed among the two countries.

1.4.3 Sensitivity Analysis

As pointed out by Coenen et al. (2007), the magnitude of the cooperative gains is sensitive to the calibration of the model and the particular policy instruments used in the non-cooperative policy game. This section examines the robustness of the above results to different calibration and policy instruments used by central banks.

Strength of financial frictions The results in the previous section show that the currency mismatch friction is crucial in driving the large and uneven distribution of cooperation gains. Therefore, it is natural to investigate how cooperative gains and losses vary with degrees of this currency mismatch frictions. Table 1.7 lists cooperative gains under various degrees of currency mismatch. The financial friction parameters are re-calibrated

[18] More detailed tables that include the moments conditional on individual shocks are shown in Appendix 1.C.

Table 1.6: Differences in Unconditional Moments under All Shocks

Variable	Mean	STD
Home Variables		
Consumption	-0.1050	-0.0008
Labour	0.0238	0.0001
PPI Inflation	0.0000	0.0011
CPI Inflation	-0.0000	-0.0029
Output	0.0017	-0.0011
Credit Spread	0.6287	0.0009
Currency Premium	-4.2019	0.0005
Foreign Variables		
Consumption	0.1630	0.0003
Labour	-0.0133	-0.0001
PPI Inflation	0.0000	0.0013
CPI Inflation	-0.0000	-0.0024
Output	0.0792	-0.0027
Credit Spread	-5.9816	-0.0017
International Variables		
Real Exchange Rate	0.0333	-0.0009

Note: This table reports the differences in unconditional means of variables under the cooperative relative to the non-cooperative equilibrium. Differences in means for all variables except inflation are the percentage change relative to the mean under the non-cooperative equilibrium. The inflation measures are in simple differences.

accordingly. For comparison, the gains from the no-currency-mismatch model with the same calibration are reported as well.

In the baseline calibration, the foreign-currency-debt-to-total-asset ratio for Home banks, x, is a relatively high one, which corresponds to situations in countries such as Turkey. The alternative values of steady state x are in line with those suggested by Akinci and Queralto (2019), Chui et al. (2016), and Hahm et al. (2013) for various emerging economies.

The results in the table suggest that the cooperative gains are more evenly distributed with lower degree of currency mismatch from the top to the bottom row. This is because the spill-overs of policy actions become more symmetric as the Home banks' balance sheets are less sensitive to exchange rate fluctuations. This intuition is also confirmed by the observation that the cooperative gains are much more evenly distributed even if Home banks still borrow the same amount from abroad but in Home currency in the no-mismatch model. With high currency mismatch such as when $x = 0.3$ or 0.25, the asymmetry in policy trade-offs also generates significantly larger gains and losses compared to the no-mismatch model. By contrast, for low mismatch scenarios such as $x = 0.05$, the cooperation becomes preferable for both countries despite the currency mismatch. Nevertheless, the gains are more asymmetric than those in the no-mismatch model with the same proportions of foreign liabilities on the Home banks' balance sheets.[19]

Table 1.7: Cooperative Gains (in percent) with Different Strengths of Financial Frictions

Steady state foreign-currency debt ratio x	Welfare Gains (Baseline)			Welfare Gains (No-Mismatch)		
	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$
$x = 0.30$	-0.4221	0.4650	0.0205	0.0194	0.0216	0.0205
$x = 0.25$ (Baseline)	-0.1288	0.1762	0.0236	0.0193	0.0206	0.0199
$x = 0.18$	-0.0010	0.0493	0.0241	0.0191	0.0193	0.0192
$x = 0.05$	0.0160	0.0260	0.0205	0.0194	0.0216	0.0205

Investment adjustment costs and country size Table 1.8 reports how the baseline results change with alternative calibrations in other dimensions. In the baseline calibration, the investment adjustment cost parameters κ_I are 1.2 for both countries. If these parameters are higher, the propagation through the financial intermediation will be stronger. The policy spill-overs potentially becomes larger as a result which could strengthen my baseline results. The first two rows of the table confirm this intuition. If instead the size of the Home economy m is smaller, the cross-border spillovers through the trade channel increase for Home but decrease for Foreign. Consequently, the Foreign economy is more

[19] I also experimented with alternative steady state spreads and currency premium, the uneven distribution of cooperative gains and losses in the baseline is robust to all those cases.

insulated from the policy spillovers from Home. By contrast, the Home central bank has to offset the large externality of the Foreign central bank's policy. The Home is thus more likely to lose less and gain more by cooperating, and vice versa for the Foreign country.

Table 1.8: Cooperative Gains (in percent) with Alternative Calibrations

Calibrated Parameters	Welfare Gains (Baseline)			Welfare Gains (No-Mismatch)		
	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$
$\kappa_I = \kappa_I^* = 2.5$	-0.4164	0.4594	0.0206	0.0230	0.0267	0.0248
$\kappa_I = \kappa_I^* = 5$	-0.8231	0.8616	0.0157	0.0262	0.0318	0.0290
$m = 0.4$	-0.0120	0.0563	0.0290	NSE	NSE	NSE
$m = 0.25$	1.1181	-0.3162	0.0405	NSE	NSE	NSE

Note: NSE stands for "No Stable Equilibrium".

Policy instruments As pointed out by Corsetti et al. (2010) and Sims (2007), another important dimension that influences the magnitudes of cooperative gains is the instruments used by central banks. Table 1.9 reports the welfare gains from policy cooperation when policy makers in the model use different combinations of instruments. Four alternative instruments are considered for each country: PPI inflation ($\pi_{H,t}$, $\pi^*_{F,t}$), CPI inflation (π_t, π^*_t), growth of nominal GDP ($DY_t = \frac{P_{H,t}y_{H,t}}{P_{H,t-1}y_{H,t-1}}$, $DY^*_t = \frac{P^*_{F,t}y^*_{F,t}}{P^*_{F,t-1}y^*_{F,t-1}}$), and change in nominal exchange rate ($DNER_t$, $DNER^*_t$). This makes a total of 16 combinations.[20]

Among all these combinations, the Home country incur significant welfare loss and the Foreign country win significant welfare gains. However, both countries obtain more evenly distributed gains from cooperation in the no-mismatch model. The magnitude of cooperative gains varies greatly. Therefore, the key finding of the baseline results is robust to a wide range of non-cooperative instrument combinations.

[20]The nominal interest rates (i_t, i^*_t) are used by central banks in practice but are not considered here. This is because all those cases lead to equilibrium indeterminancy, which is consistent with findings in literature such as Coenen et al. (2007) and Bodenstein et al. (2019).

Table 1.9: Cooperative Gains (in percent) with Alternative Instruments

Instrument Combinations	Welfare Gains (Baseline)			Welfare Gains (No-Mismatch)		
(Home, Foreign)	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$
$\pi_{H,t}$, $\pi^*_{F,t}$ (baseline)	-0.1288	0.1762	0.0236	0.0193	0.0206	0.0199
$\pi_{H,t}$, π^*_t	-0.1188	0.1689	0.0249		NSE	
$\pi_{H,t}$, DY^*_t	-0.1333	0.1801	0.0233	0.0234	0.0279	0.0257
$\pi_{H,t}$, $DNER^*_t$	-0.1258	0.1744	0.0242		NSE	
π_t, $\pi^*_{F,t}$	-1.0976	1.2271	0.0580		NSE	
π_t, π^*_t	-1.1709	1.3468	0.0800	4.0730	10.2483	7.1161
π_t, DY^*_t	-1.0746	1.1941	0.0534		NSE	
π_t, $DNER^*_t$	-1.2186	1.4111	0.0876	2.4612	3.3110	2.8852
DY_t, $\pi^*_{F,t}$	-0.1423	0.1892	0.0233	0.0194	0.0212	0.0203
DY_t, π^*_t	-0.1333	0.1835	0.0250		NSE	
DY_t, DY^*_t	-0.1468	0.1931	0.0230	0.0235	0.0290	0.0263
DY_t, $DNER^*_t$	-0.1381	0.1867	0.0242		NSE	
$DNER_t$, $\pi^*_{F,t}$	-7.6707	13.6262	2.4257		NSE	
$DNER_t$, π^*_t	-4.4049	8.3182	1.7580		NSE	
$DNER_t$, DY^*_t	-11.7990	21.1817	3.3845		NSE	
$DNER_t$, $DNER^*_t$	-2.2829	4.6895	1.1433		NSE	

Note: NSE stands for "No Stable Equilibrium".

1.5 Implementable Monetary Policy Rules

The analyses of the optimal cooperative and non-cooperative policy rules above provide useful benchmarks to assess the desirability of policy cooperation. Nevertheless, the central bank practices are dominated by various simple implementable rules which do not necessarily replicate the optimal allocations. Although studying gains from cooperation when central banks implement simple rules is not the focus of this paper, it helps to get a sense of how my findings change when monetary policy is characterised by implementable rules in my model.

There are infinite numbers of alternative rules that can be considered. I initially restrict my focus on 3 alternative simple rules for each country: 1) complete stabilisation of the PPI inflation ($\pi_{H,t} = 1$ or $\pi^*_{F,t} = 1$ for all t), which replicates the optimal policy for a standard two-country New Keynesian model with complete markets (Benigno, 2009); 2) complete stabilisation of the CPI inflation ($\pi_t = 1$ or $\pi^*_t = 1$ for all t), which stabilises the CPI fluctuations that are directly relevant for welfare; 3) the standard Taylor rules (1.24) and (1.25) which typically feature in literature. The cooperative equilibrium is still defined in the same way as in section 1.4.1. In the non-cooperative equilibrium, each country uses one of the simple rules as described above. This sums up to the 9 scenarios

Table 1.10: Cooperative Gains (in percent) relative to Using Simple Rules

Simple Rule Combinations	Welfare Gains (Baseline)			Welfare Gains (No-Mismatch)		
(Home, Foreign)	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$	$\Delta\mathbb{W}_t^H$	$\Delta\mathbb{W}_t^F$	$\Delta\mathbb{W}_t$
$\pi_{H,t}=1$, $\pi^*_{F,t}=1$	1.1770	-0.7206	0.2237	0.1796	0.2029	0.1912
$\pi_{H,t}=1$, $\pi^*_t=1$	1.6413	-1.1310	0.2456	1.0116	1.5513	1.2811
$\pi_{H,t}=1$, Taylor	1.4721	-1.0257	0.2154	0.1159	0.2123	0.1641
$\pi_t=1$, $\pi^*_{F,t}=1$	1.5851	-0.7626	0.4044		NSE	
$\pi_t=1$, $\pi^*_t=1$	1.7746	-1.1336	0.3100		NSE	
$\pi_t=1$, Taylor	1.2847	-0.7621	0.2561		NSE	
Taylor, $\pi^*_{F,t}=1$	0.0751	0.1137	0.0944	1.0430	0.7567	0.8998
Taylor, $\pi^*_t=1$	0.5625	-0.3423	0.1090	5.1816	7.2391	6.2054
Taylor, Taylor	0.3707	-0.2030	0.0835	0.8404	0.7264	0.7834

Note: NSE stands for "No Stable Equilibrium".

in Table 1.10, which reports the unconditional welfare gains from cooperation relative to using the combination of simple rules in each scenario. In all cases but one, only the Home country gains from cooperation, whilst the Foreign country suffer significant losses from cooperation, even if the cooperative gains in joint welfare is sizable. Meanwhile, both countries gain from cooperation in the model where there is no currency mismatch if a stable equilibrium exists. The asymmetric policy spill-overs still create the highly uneven distribution of cooperative gains when simple rules are implemented in the non-cooperative equilibrium.

The results also imply that if the benevolent social planner under the cooperative equilibrium chooses combination of simple rules instead to maximise the joint world welfare, the cooperative policy dictates both countries to follow the standard Taylor rules (1.24) and (1.25). This simple rule cooperative equilibrium achieves lower joint welfare and Home welfare but higher Foreign welfare than the Ramsey cooperative equilibrium.

Table 1.11 reports gains from cooperating in implementable rules relative to scenarios where each country chooses one of the three simple rules independently. These gains are rearranged in the form of the pay-off matrix for each country and for the joint welfare. The 3 panels from the top to the bottom report welfare gains for Home, Foreign, and World joint respectively. The rows (columns) indicate the rules adopted by Home (Foreign). The table shows that cooperation still fails to improve both countries' welfare compared to any of the non-cooperative scenarios.

Moreover, if both national central banks behave strategically such that their policy rule is the optimal response to the rule adopted by the other country, there is a Nash equilibrium where the Home adopts the simple inward-looking Taylor rule, whilst the

Table 1.11: Welfare Gains from Cooperating in Implementable Rules (in percent)

For Home Welfare	$\pi^*_{F,t}=1$	$\pi^*_t=1$	Taylor
$\pi_{H,t}=1$	0.8033	1.2659	1.0973
$\pi_t=1$	1.2099	1.3987	0.9106
Taylor	-0.2945	<u>0.1910</u>	0.0000
For Foreign Welfare	$\pi^*_{F,t}=1$	$\pi^*_t=1$	Taylor
$\pi_{H,t}=1$	-0.5187	-0.9299	-0.8244
$\pi_t=1$	-0.5608	-0.9325	-0.5602
Taylor	0.3174	<u>-0.1396</u>	0.0000
For Joint Welfare	$\pi^*_{F,t}=1$	$\pi^*_t=1$	Taylor
$\pi_{H,t}=1$	0.1401	0.1620	0.1319
$\pi_t=1$	0.3207	0.2263	0.1725
Taylor	0.0110	<u>0.0256</u>	0.0000

Note: The 3 panels from the top to the bottom report welfare gains for Home, Foreign, and World joint respectively. The rows (columns) indicate the rules adopted by Home (Foreign). The Nash equilibrium is underlined.

Foreign stabilises CPI inflation completely.[21] This equilibrium is underlined in the table. Compared to this non-cooperative equilibrium, only Home benefits from the cooperation in simple rules by 0.191% of the steady state consumption. The Foreign loses 0.1396% of steady state consumption instead.

Another important question in monetary policy for open markets in literature is whether central banks should respond to the movements of external variables such as the nominal exchange rate. Hence, I expand the strategy space for simple rules in consideration by allowing central banks to implement the following augmented Taylor rules:

$$\frac{1+i_t}{1+\bar{i}} = \left(\frac{1+i_{t-1}}{1+\bar{i}}\right)^{\rho_m}\left[\left(\frac{\pi_{H,t}}{\bar{\pi}_H}\right)^{\varphi_\pi}\left(\frac{e_t}{e_{t-1}}\right)^{\varphi_e}\right]^{1-\rho_m} \tag{1.26}$$

$$\frac{1+i^*_t}{1+\bar{i}^*} = \left(\frac{1+i_{t-1}}{1+\bar{i}^*}\right)^{\rho_m}\left[\left(\frac{\pi^*_{F,t}}{\bar{\pi}^*_F}\right)^{\varphi_\pi}\left(\frac{e_t}{e_{t-1}}\right)^{\varphi^*_e}\right]^{1-\rho_m}, \tag{1.27}$$

where $\frac{e_t}{e_{t-1}}$ is the change in nominal exchange rate. The values of ρ_m and φ_π are kept the same as in (1.24) and (1.25). Then the value of φ_e is varied within the range $[0,5]$ and φ^*_e within the range $[-5,0]$.[22]

Within this expanded policy space, the optimal cooperative policy requires both central banks to adopt the augmented Taylor rule and respond to exchange rate movements with $\varphi_e = 0.2$, $\varphi^*_e = -0.1$. Compared to the optimal Ramsey cooperative equilibrium which is described in section 1.4.1, this cooperation in implementable rules achieves

[21] Only simple strategy Nash equilibrium is considered here.

[22] Intuitively, the interest rates should change to countervail changes in exchange rate. Thus, φ_e should be positive whilst φ^*_e should be negative.

0.077% lower joint welfare in consumption-equivalent measures. Home loses 0.3325% of consumption compared to the Ramsey policy equilibrium, whilst Foreign gains by 0.1778%. There is a simple strategy Nash equilibrium if both central banks adopt implementable rules non-cooperatively and strategically instead. This policy mix implies that the Home central bank uses an augmented Taylor rule with $\varphi_e = 0.7$, and that the Foreign central bank uses an augmented Taylor rule with $\varphi_e^* = -1.7$. Relative to this non-cooperative Nash equilibrium with implementable rules, the Home welfare increases by 0.281%, the Foreign welfare decreases by 0.119%, and the joint welfare increases by 0.081% of the steady state consumption if both central banks cooperate in implementable rules. Figures 1.4 - 1.5 in Appendix 1.C shows the cooperation gains relative to all possible combinations of simple rules considered in this paper. The uneven distribution of cooperation gains is robustly prevalent in all these combinations.

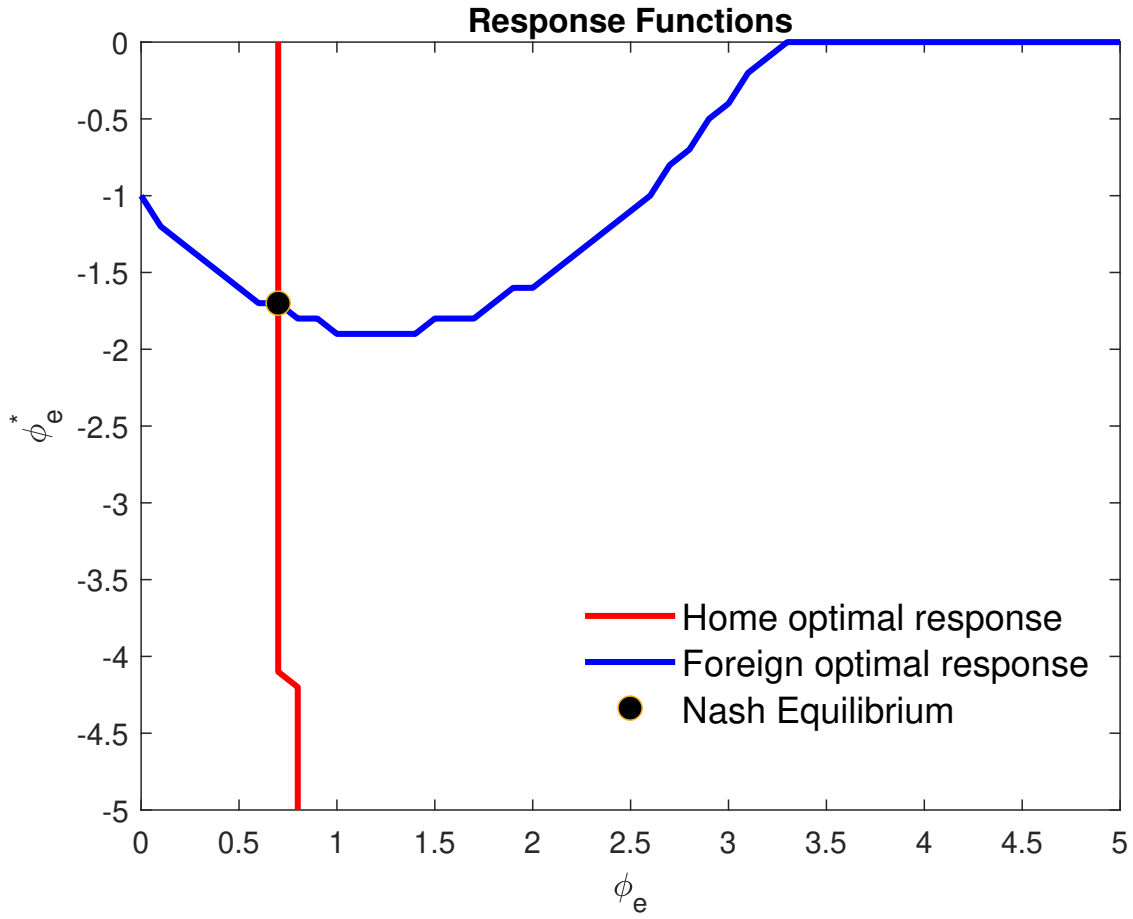
Another interesting observation is that in the Nash equilibrium, both countries respond to the exchange rate movements. Out of this Nash equilibrium, countervailing the exchange rate fluctuation still improves national welfare in most cases. Table 1.12 reports the optimal response of Home (Foreign) when Foreign (Home) adopts complete PPI or CPI stabilisation rules. If one of the countries adopts such a policy rule, it is always beneficial for the other country to moderately stabilising the exchange rate fluctuations. In addition, Figure 1.3 plots the optimal response functions for the two countries when they both implement the Taylor rules (1.26) and (1.27). Here, responding to exchange rate is a dominant strategy for Home as the financial periphery country. For Foreign which is the financial centre country, it is still worth reacting to exchange rates unless Home is already responding strongly with $\phi_e \geq 3.3$.

Table 1.12: The optimal responses for Stabilising Exchange Rate Movements I

Scenario	Optimal Response of the Other Country
Given $\pi^*_{F,t} = 1$ in Foreign	Taylor Rule (1.26) with $\varphi_e = 0.7$ in Home
Given $\pi^*_t = 1$ in Foreign	Taylor Rule (1.26) with $\varphi_e = 0.6$ in Home
Given $\pi_{H,t} = 1$ in Home	Taylor Rule (1.27) with $\varphi^*_e = -1.3$ in Foreign
Given $\pi_t = 1$ in Home	Taylor Rule (1.27) with $\varphi^*_e = -1.2$ in Foreign

Note: One of the countries adopts either complete PPI or CPI stabilisation.

Figure 1.3: The Optimal Responses for Stabilising Exchange Rate Movements II



Note: Both countries adopt the augmented Taylor rules (1.26) and (1.27).

1.6 Conclusion

This paper discusses the desirability for central banks of the financial centre and periphery countries to cooperate their monetary policy within a model with global financial transmissions. The dominance of the financial centre country's currency in the global banking system generates a currency mismatch on the balance sheets of banks in the financial periphery country. This leads to asymmetric transmissions of shocks and policy effects across borders. As a result, the distribution of the gains from monetary policy cooperation is highly uneven.

If the optimal allocations can be implemented by benevolent social planners in both countries under cooperative and non-cooperative Nash equilibria, one of the countries suffers significant losses from cooperation whilst the other country extract large welfare gains. By contrast, both countries obtain moderate gains if the currency mismatch friction is not present.

This finding still holds if both countries can only implement simple rules that characterise the monetary policy practice. Within the range of policy rules considered, counteracting the effects of exchange rate movements using the domestic monetary policy rules is beneficial from either country's perspective in response to given policy rules of the other country in most cases.

This study opens several avenues for future research. Firstly, it is worth calibrating this model with country-specific data to assess more precisely the magnitude of the welfare gain and loss from cooperation. Second, the policy trade-offs may be different when there are other frictions in the model. For example, it is interesting to see how our results change if firms in the emerging markets price their products abroad using the dominant currency of the financial centre country. Third, the range of implementable rules considered in this paper can be further extended. Simple rules that respond to credit spreads and the deviation from the UIP can potentially perform better than rules considered in this paper. Lastly, it is interesting to further explore how cooperative and non-cooperative macroprudential tools help stabilising the international transmission of shocks, and how these tools interact with monetary policy.

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Appendix

1.A The No-mismatch Model

In the no-mismatch model, we assume that Home banks now are able to raise debt d^G_{t-1} from the international wholesale funds providers in the Home currency in period $t-1$ and pay back $R_t d^G_{t-1}$ in period t. Therefore, the net worth of a typical Home bank becomes

$$n^i_t = (R^K_t + (1-\delta)q_t)K^i_{t-1} - R_t d^{Gi}_{t-1} - \widetilde{R}_t d^i_{t-1}.$$

The balance sheet constraint becomes

$$q_t K^i_t = n^i_t + d^i_t + d^{Gi}_t$$

and the incentive compatibility constraint remains the same.

The optimality conditions for leverage ratio Γ^i_t and foreign-currency debt ratio x^i_t remain unchanged. However, the definition for $\mu_{1,t}$ changes to

$$\mu_{1,t} = \mathbb{E}_t\{\Omega_{t,t+1}(\widetilde{R}_{t+1} - R_{t+1})\} = \mathbb{E}_t\{\Omega_{t,t+1}\tau^r_{t+1}\}.$$

The evolution of aggregate net worth and balance sheet constraint are thus

$$n_t = (\omega + \xi)(R^K_t + (1-\delta)q_t)K_{t-1} - \omega R_t d^G_{t-1} - \omega \widetilde{R}_t d_{t-1},$$

and

$$q_t K_t = n_t + d_t + d^G_t.$$

Meanwhile, the international wholesale funds providers raise d^{G*}_{t-1} in the Foreign currency from Foreign households in period $t-1$ and promise to pay back $R^*_t d^{G*}_{t-1}$ in period t. Then the wholesale banks can hedge away any exchange rate risks on their balance

sheets as in Cesa-Bianchi et al. (2018). In particular, I assume that the hedging is such that

$$d_t^{G*}\varepsilon_t = d_t^G.$$

Therefore, the profit for the wholesale bank is

$$\Pi_t^{wb} = (R_t - R_t^*)d_{t-1}^{G*} - \Phi,$$

where Φ is a constant hedging cost. Thus the balance of payments equation becomes:

$$d_t^{G*} = R_t d_{t-1}^{G*} + \frac{P_{F,t}^*}{P_t^*}y_{F,t} - \frac{P_{H,t}^*}{P_t^*}y_{H,t}^*.$$

Other equations of the model remain unchanged.

1.B Steady State

I approximate the model around a non-stochastic steady state with zero inflation in both countries. I also normalise all relative price terms to one so that the price dispersion terms $s = s^* = 1$. In addition, the nominal exchange rate is normalised to one which implies that the real exchange rate is also one.

I set the steady state Home labour hours $L = \frac{1}{3}$. From the inter-temporal Euler equations, we have $R = R^* = \frac{1}{\beta}$. Therefore, $i = i^* = \frac{1}{\beta} - 1$. The values of $\widetilde{R}$ and R^{G*} are determined by R, R^*, and the relevant targeted interest rate spreads for calibration. Moreover, the optimal pricing of capital implies $q = q^* = 1$.

From the optimal pricing equation of firms, $mc = \frac{\varphi-1}{\varphi}$. Similarly, $mc^* = \frac{\varphi^*-1}{\varphi^*}$. The value of R^K and R^{K^*} are pinned down by R, R^*, and the targeted credit spreads in steady state. Productivity is normalised to one in both countries.

From the first-order condition (FOC) with respect to labour for Home firms,

$$w = \left[\frac{mc}{(R^K)^\alpha}\right]^{\frac{1}{1-\alpha}}.$$

Substituting w back to the FOC for capital,

$$K = \frac{\alpha}{1-\alpha}\frac{wL}{R^K}.$$

Then from the capital accumulation equation, $I = \delta K$. By equation equation (1.20) and

(1.22), the total output produced by Home firms

$$Y_H = \frac{wL}{(1-\alpha)mc}.$$

Moreover, from equation (1.3), the steady state consumption

$$C = \frac{w}{\zeta L^{\chi}}.$$

Meanwhile, from the evolution of foreign assets (1.23), we obtain

$$Y_F^* = [\frac{(1-a)m}{(a-a^*)(1-m)}Y_H + (R^{G*}-1)d^{G*}]\frac{(a-a^*)}{a^*}.$$

Then from the Foreign counterpart equation of (1.22),

$$L^* = (1-\alpha)Y_F^*(\frac{R^{K*}}{w^*})^{\alpha}.$$

Then the values of K^*, I^*, and C^* are pinned down in a similar fashion as their counterparts for Home.

From the definitions of output Y_H and Y_F^* and demands for goods y_H, y_H^*, y_F, and y_F^*, the aggregate demand in the two countries are

$$C + I + G = \frac{(1-a^*)Y_H - a^*\frac{1-m}{m}Y_F^*}{a-a^*}$$
$$C^* + I^* + G^* = \frac{aY_F^* - (1-a)\frac{m}{1-m}Y_H}{a-a^*}.$$

Then the demands y_H, y_H^*, y_F, y_F^* and government spending G and G^* are determined accordingly.

Given K and the targeted leverage ratio Γ, the net worth of Home banks are given by $n = \frac{K}{\Gamma}$. Then given the targeted foreign-currency-debt-to-asset ratio x, the cross border lending $d^{G*} = xK$. From the aggregate balance sheet of banks, we can solve

$$d = K - n - d^{G*}.$$

By the evolution of the aggregate net worth (1.15), the value of ξ is

$$\xi = \frac{n + \omega R^{G*}d^{G*} + \omega\widetilde{R}d}{(R^K + 1 - \delta)K} - \omega.$$

Inserting the values of interest rates back to equations (1.7) $-$ (1.9) and (1.14), the corresponding values of the spread measures are determined. Then from equation (1.13),

$$\kappa_b = \frac{2\mu}{x^2\mu + 2x}.$$

By inserting these values back to (1.12), one can pin down the value of θ by

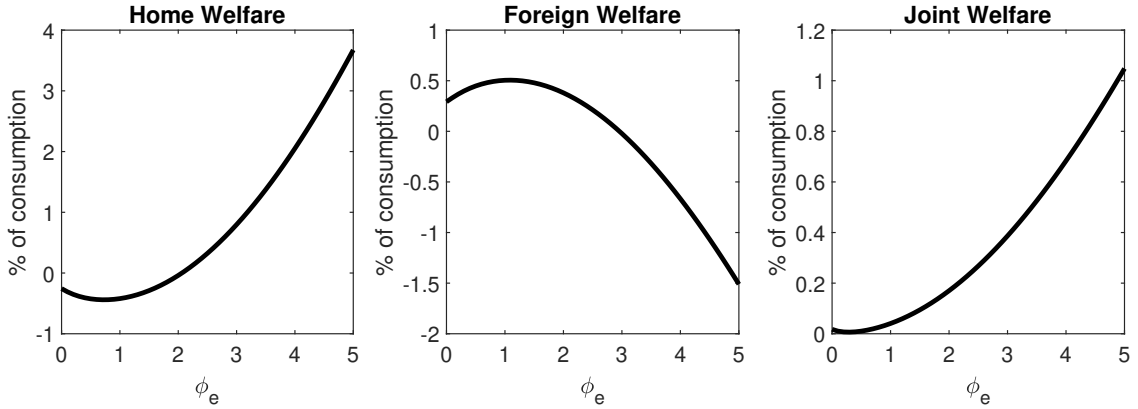
$$\theta = \frac{\beta(1-\omega)\widetilde{R} + \beta(1-\omega)\Gamma(R^K + 1 - \delta - \widetilde{R}) + \beta\Gamma(1-\omega)x(\widetilde{R} - R^{G*})}{\Gamma(\frac{\kappa_b}{2}x^2 + 1)[1 - \beta\omega\widetilde{R} - \beta\omega\Gamma(R^K + 1 - \delta - \widetilde{R}) - \beta\omega\Gamma x(\widetilde{R} - R^{G*})]}.$$

Given the values of Γ^* and K^*, n^* and d^* are accordingly determined in the similar fashion as those for Home. The values of μ_0^*, μ_1^*, and ψ^* can be obtained by substituting relevant variables into their respective definitions. Then the values of θ^* and ξ^* are given by

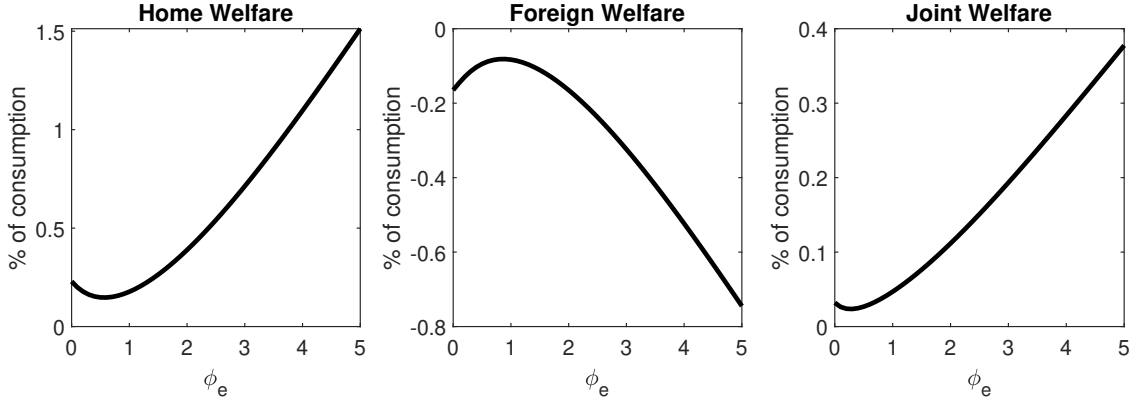
$$\theta^* = \frac{\beta(1-\omega)[(R^{K*} + 1 - \delta - R^*)\Gamma^* + R^*]}{\Gamma^* - \beta\omega(\Gamma^*)^2(R^{K*} + 1 - \delta - R^*) - \beta\omega\Gamma^* R^*}$$
$$\xi^* = \frac{n^* + \omega R^* d^*}{(R^{K*} + 1 - \delta)K^*} - \omega.$$

1.C Additional Figures and Tables

Figure 1.4: Welfare Gains from Cooperation in Implementable Rules I

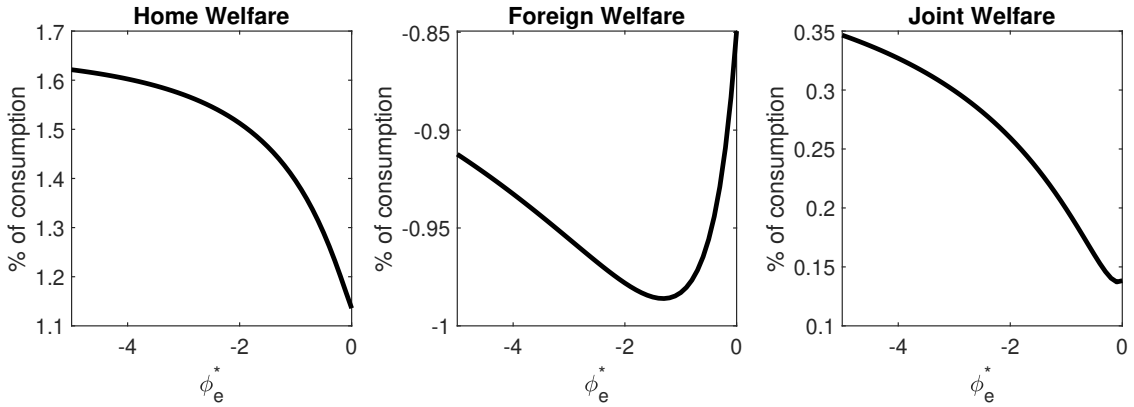


Note: The cooperative welfare is compared to equilibria with non-cooperative simple rules, when Home implements the augmented Taylor rule with varying ϕ_e and Foreign implements PPI targeting without cooperation.

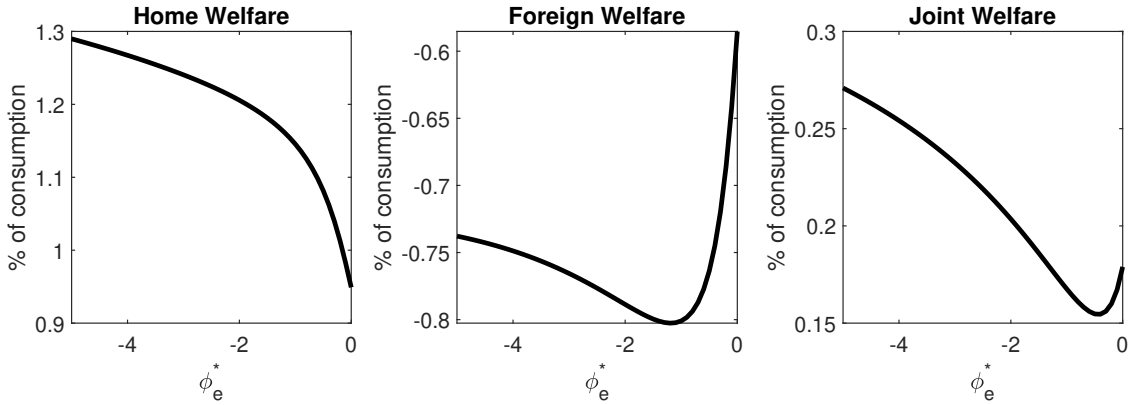


Note: The cooperative welfare is compared to equilibria with non-cooperative simple rules, when Home implements the augmented Taylor rule with varying ϕ_e and Foreign implements CPI targeting without cooperation.

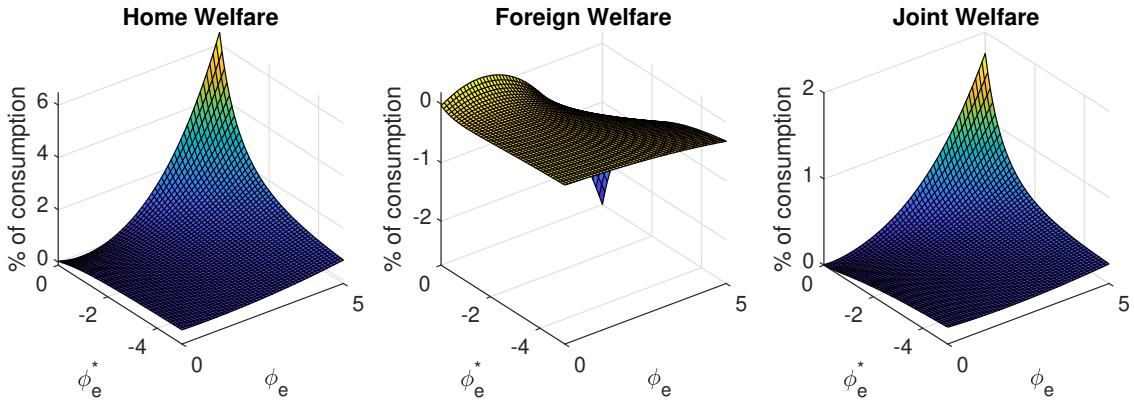
Figure 1.5: Welfare Gains from Cooperation in Implementable Rules II



Note: The cooperative welfare is compared to equilibria with non-cooperative simple rules, when Home implements PPI targeting and Foreign implements the augmented Taylor rule with varying ϕ_e^* without cooperation.



Note: The cooperative welfare is compared to equilibria with non-cooperative simple rules, when Home implements CPI targeting and Foreign implements the augmented Taylor rule with varying ϕ_e^* without cooperation.



Note: The cooperative welfare is compared to equilibria with non-cooperative simple rules, when the augmented Taylor rules (1.26) and (1.27) are adopted by both countries without cooperation.

Table 1.13: Differences in Unconditional Moments under Individual Shocks I

Variable	All Shocks		Productivity Home		Productivity Foreign		Net Worth Home		Net Worth Foreign	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD	Mean	STD
Home Variables										
Consumption	-0.1050	-0.0008	0.0722	-0.0003	-0.0963	-0.0013	-0.0962	0.0002	0.0420	0.0024
Labour	0.0238	0.0001	-0.0142	0.0003	0.0171	-0.0001	0.0193	0.0001	-0.0058	0.0001
PPI Inflation	0.0000	0.0011	0.0000	0.0011	0.0000	0.0004	0.0000	0.0005	0.0000	-0.0002
CPI Inflation	-0.0000	-0.0029	0.0000	0.0005	-0.0000	-0.0026	-0.0000	-0.0030	0.0000	0.0032
Output	0.0017	-0.0011	0.0047	-0.0012	-0.0110	-0.0000	0.0009	0.0010	0.0050	-0.0001
Credit Spread	0.6287	0.0009	-1.8671	-0.0034	0.8063	0.0000	3.4580	0.0031	-0.0005	0.0007
Currency Premium	-4.2019	0.0005	0.2822	-0.0023	-3.5224	-0.0000	-1.1041	0.0021	0.5113	0.0005
Foreign Variables										
Consumption	0.1630	0.0003	-0.0617	-0.0000	0.0969	0.0003	0.0985	-0.0008	-0.0042	0.0044
Labour	-0.0133	-0.0001	0.0131	0.0000	-0.0163	-0.0001	-0.0166	-0.0004	0.0081	0.0011
PPI Inflation	0.0000	0.0013	0.0000	0.0001	0.0000	0.0003	-0.0000	-0.0011	0.0000	0.0032
CPI Inflation	-0.0000	-0.0024	0.0000	0.0004	-0.0000	-0.0039	-0.0000	-0.0030	0.0000	0.0062
Output	0.0792	-0.0027	0.0057	0.0001	0.0110	-0.0028	0.0054	-0.0023	0.0493	0.0057
Credit Spread	-5.9816	-0.0017	-0.0175	-0.0001	-7.2652	-0.0063	-2.4856	-0.0038	6.3793	0.0096
International Variables										
Real Exchange Rate	0.0333	-0.0009	-0.0450	-0.0001	0.0426	-0.0011	0.0596	0.0001	-0.0398	0.0013

Note: : This table reports the differences in unconditional means of variables under the cooperative relative to the non-cooperative equilibrium. Differences in means for all variables except inflation are scaled as the percentage change relative to the mean under the non-cooperative regime. The inflation measures are in simple differences.

Table 1.14: Differences in Unconditional Moments under Individual Shocks II

Variable	Mark-up Home		Mark-up Foreign		Preference Home		Preference Foreign	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
Home Variables								
Consumption	0.0092	-0.0008	-0.0136	-0.0010	0.0056	0.0001	-0.0106	-0.0003
Labour	0.0002	-0.0002	0.0027	-0.0001	-0.0008	0.0001	0.0020	-0.0000
PPI Inflation	0.0000	0.0007	0.0000	-0.0001	0.0000	0.0005	-0.0000	-0.0002
CPI Inflation	0.0000	0.0003	-0.0000	-0.0015	0.0000	-0.0004	-0.0000	-0.0010
Output	0.0038	-0.0011	-0.0011	-0.0003	0.0008	0.0005	-0.0011	-0.0002
Credit Spread	-1.1565	-0.0026	-0.0636	-0.0012	-0.1421	-0.0006	0.0227	-0.0004
Currency Premium	-0.0997	-0.0018	-0.4402	-0.0009	0.0682	-0.0003	-0.4470	-0.0003
Foreign Variables								
Consumption	-0.0054	-0.0007	0.0175	-0.0014	-0.0042	0.0001	0.0118	-0.0005
Labour	0.0013	-0.0001	0.0010	-0.0004	0.0009	0.0000	-0.0016	0.0001
PPI Inflation	0.0000	-0.0003	0.0000	0.0014	-0.0000	0.0001	0.0000	-0.0001
CPI Inflation	0.0000	0.0001	0.0000	0.0024	-0.0000	0.0002	-0.0000	-0.0018
Output	0.0009	-0.0006	0.0071	-0.0025	0.0004	0.0001	0.0022	0.0008
Credit Spread	-0.0242	-0.0006	-2.2587	-0.0039	0.0023	0.0001	-0.7906	-0.0024
International Variables								
Real Exchange Rate	-0.0032	-0.0002	0.0048	-0.0009	-0.0028	0.0001	0.0048	-0.0003

Note: : This table reports the differences in unconditional means of variables under the cooperative relative to the non-cooperative equilibrium. Differences in means for all variables except inflation are scaled as the percentage change relative to the mean under the non-cooperative regime. The inflation measures are in simple differences.

Chapter 2

Capital Flows and Exchange Rates: A Quantitative Assessment of the Dilemma Hypothesis

Abstract In response to an unanticipated monetary policy tightening in the US, real GDP and exports of a typical small open economy fall, despite the depreciation of the local currency. The reason is that the financial channel of the transmission of monetary policy shocks across countries dominates over the traditional expenditure-switching effect. The dominant role of the reserve currency in trade and global financial transactions can account for the evidence in an otherwise standard two-country open economy model with nominal and real rigidities. Yet, even in the presence of a global financial cycle, the exchange rate regime matters. In particular, a peg substantially increases macroeconomic volatility. Conversely, the introduction of an additional policy instrument to manage capital flows dampens economic fluctuations. A tax on domestic credit achieves nearly equivalent results. Tax instruments can insulate the effects of foreign monetary policy shocks on real economic activity in a fixed exchange rate regime, but not on inflation.

2.1 Introduction

As central banks around the world started their tightening cycle following the recovery from the Covid crisis, issues related to exchange rate volatility and capital flows became once again front and center in the debate on the cross-country transmission of monetary policy.

The reference framework for this discussion often relies on the notion of 'Trilemma.' The Trilemma is a cornerstone proposition in international macroeconomics according to which a country can only attain two of the following three policy objectives: independent monetary policy, a fixed exchange rate, and free capital mobility.

Rey (2013) challenged this traditional view. Her critique contains two key points. The first is an empirical observation. Following a monetary policy shock in the US, asset prices and capital flows move together across countries, giving rise to a 'Global Financial Cycle' (GFC). Subsequent work (e.g. Miranda-Agrippino and Rey, 2020, di Giovanni et al., 2022) has confirmed and extended these findings. The second point focuses on the policy implications. Since the GFC holds regardless of the exchange rate regime, the key policy choice is not a Trilemma, but a 'Dilemma' between retaining monetary policy independence and free international capital mobility. A central bank can pursue inflation stability only if additional instruments, such as capital flows management and/or macroprudential tools, are available.

In this paper, we offer a quantitative evaluation of the Dilemma hypothesis. Our analysis proceeds in three steps. First, we show empirically that indeed a flexible exchange rate regime is not enough to insulate countries from the consequences of the GFC. In response to a contractionary monetary policy shock in the US, the nominal exchange rate of countries with a flexible exchange rate depreciates. Yet, exports and real GDP fall. The expenditure-switching channel (Engel, 2003) does not offset the negative consequences of lower global aggregate demand.

Second, we develop a two-country open economy model with real, nominal, and financial frictions that matches our empirical evidence. In the model, the GFC arises because of the cross-country links in the financial sector and the existence of a global reserve currency in trade and financial transactions. We then estimate the model by matching the impulse responses to a foreign monetary policy shock to those of the VAR. Several counterfactual experiments highlight the crucial interplay between financial frictions and trade pricing frictions. The former are necessary for the amplification of the shock. The latter allow the model to replicate the empirical response, in particular, of exports and inflation.

Third, endowed with the estimated model, we perform a number of policy experiments. In the model, GFC notwithstanding, abandoning a flexible exchange rate regime

in favor of a peg increases macroeconomic volatility because of the negative effects of higher interest rates on domestic demand. Conversely, the introduction of countercyclical tax instruments that respond to credit growth reduces macroeconomic volatility. Our experiments show that a capital flows management tool (a tax on borrowing in foreign currency) or a financial stability tool (a tax on domestic credit) have nearly equivalent effects. The adoption of one of these instruments in a country with a fixed exchange rate regime can approximately generate the same response of domestic output to a foreign monetary policy contraction as with flexible exchange rates, although inflation volatility remains higher.

Literature Review Initially, the empirical literature on the GFC has largely emphasized the response of financial variables to US monetary policy shocks. More recently, a number of contributions, such as Dedola et al. (2017), Han and Wei (2018), Degasperi et al. (2020), and Flaccadoro and Nispi Landi (2022), have extended the focus on to macro variables. While we share the interest of these papers in the joint dynamics of financial and macroeconomic variables, our empirical analysis relies solely on a sample of countries with a flexible exchange arrangement. Under the Trilemma paradigm, exchange rate flexibility should insulate countries from external shocks, including those driving the GFC. The fact that we observe the typical elements of the transmission of a US monetary policy shock even when we exclusively focus on countries with flexible exchange rates strikes a key point in favor of the GFC.

Our model is closely related to Aoki et al. (2020) and, especially, Akinci and Queralto (2019). The common element in those papers, as well as in ours, is the presence of a moral hazard friction between households and banks, as in Gertler and Karadi (2011). In open economy, the distinguishing feature is that the friction depends on the currency composition of the private sector's balance sheet, which gives rise to an endogenous wedge in the uncovered interest rate parity (UIP) condition. An international interbank market links domestic and foreign financial intermediaries, thus amplifying the transmission of foreign monetary policy shocks. On the trade side, two pricing frictions determine the propagation of exchange rate movements on to exports and inflation. In line with the recent evidence on the 'Dominant Currency Paradigm' (Gopinath et al., 2020), domestic firms price their exports in the reserve currency, allowing the model to match the decline of exports despite the depreciation of the domestic currency. The depreciation of the exchange rate has also a limited effect on domestic inflation because of imperfect pass-through (Monacelli, 2005). Overall, our framework thus features a dominant currency both in trade and finance, as in Gopinath and Stein (2018).

In our estimated model, as in Akinci and Queralto (2019), the volatility of output and inflation increases under a peg, while taxes on either foreign borrowing or domestic

credit reduce macroeconomic volatility.[1] This result is consistent with Farhi and Werning (2016), who find that this kind of taxes are generally desirable because of the interaction between nominal rigidities and imperfections in financial markets. In a small open economy with nominal rigidities and incomplete international financial markets, Farhi and Werning (2012) show that, under a peg, capital controls can restore domestic monetary policy independence, especially in response to risk-premium shocks.[2] In our setting, under a peg, we can design a countercyclical tax, either on foreign liabilities or domestic credit, that replicates the same response of output as under flexible exchange rates. The tax alleviates the negative consequences of a peg on real activity by reducing financial frictions. Nonetheless, inflation continues to fall because in a fixed exchange rate regime the domestic central bank must track the higher foreign nominal interest rate, thus lowering domestic demand.

Our paper is also closely related to the IMF Integrated Policy Framework. Basu et al. (2020) develop a three-period model with trade and financial frictions similar to those present in our setup. Adrian et al. (2020) add several quantitative features, including extending the model to an infinite horizon and allowing for behavioral features (a fraction of agents with adaptive expectations, and discounting in both aggregate demand and supply). Our estimation exercise complements this work and provides a solid empirical grounding to the literature on the GFC.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 reports estimation results for critical parameters of the model. Section 4 focuses on the model properties. Section 5 reports the quantitative analysis. Section 6 concludes. The appendix contains additional empirical results and details of the model.

2.2 Revisiting the GFC Evidence

In this section, we estimate a panel VAR (PVAR) on a sample of countries with a flexible exchange rate. The data cover 15 countries (Australia, Canada, Chile, Germany, Japan, Korea, Mexico, New Zealand, Norway, Singapore, South Africa, Sweden, Switzerland, Thailand and the United Kingdom) during the period 1997M1–2019M12. In this group of countries, the cross-sectional average of the classification of exchange rates by Ilzetzki et al. (2019) during the 1999–2019 period is 11.7, with a minimum value of 9.6 for Canada

[1] Rebucci and Ma (2020) survey the literature on the use of macroprudential instruments, and in particular capital controls, to address the inefficiencies arising from pecuniary externalities, which our model abstracts from.

[2] Schmitt-Grohé and Uribe (2016) reach a similar result in a model with downward wage rigidities.

(higher values correspond to a more flexible exchange rate regime).[3]

The empirical VAR specification for each country i is

$$x_{it} = a_i + b_i t + \sum_{\ell=1}^{L} F_{i\ell} x_{it-\ell} + u_{it},$$

where x_{it} is the vector of endogenous variables at time t, a_i is a vector of constants, t is a deterministic time trend, each $F_{i\ell}$ is a matrix of coefficients, and u_{it} is a vector of reduced form residuals with variance-covariance matrix Σ_{iu}. The vector x_{it} includes three US-specific variables and six country-specific variables. The US-specific variables are the monetary policy shock (ϵ_t^m), the excess bond premium (EBP_t^{US}) from Gilchrist and Zakrajsek (2012) and the log of real GDP (Y_t^{US}). The country-specific variables are the log of real GDP (Y_{it}), the (annualized) inflation rate (π_{it}), the log of exports in real terms (X_{it}), the level of the nominal interest rate (i_{it}), the depreciation of the nominal exchange rate (Δe_{it}), and the level of a measure of credit spreads (CS_{it}).[4]

To identify a US monetary policy shock, we employ the high-frequency monetary policy surprises constructed by Jarocinski and Karadi (2020) as an 'internal instrument' (Plagborg-Møller and Wolf, 2021).[5] Specifically, we include the series of monetary surprises as the first element of the vector of endogenous variables

$$x_{it} = \begin{bmatrix} \epsilon_t^m & EBP_t^{US} & Y_t^{US} & Y_{it} & \pi_{it} & X_{it} & \Delta e_{it} & i_{it} & CS_{it} \end{bmatrix}.$$

The introduction of the excess bond premium in the vector of endogenous variables is important for two reasons. First, given the small scale of our empirical model, this variable helps with the correct specification because of its strong predictive power for real economic activity, thus expanding the VAR information set (Caldara and Herbst, 2019). Second, the EBP can provide useful information about the quantitative strength of financial frictions in the transmission of monetary policy shocks (Gertler and Karadi, 2015).

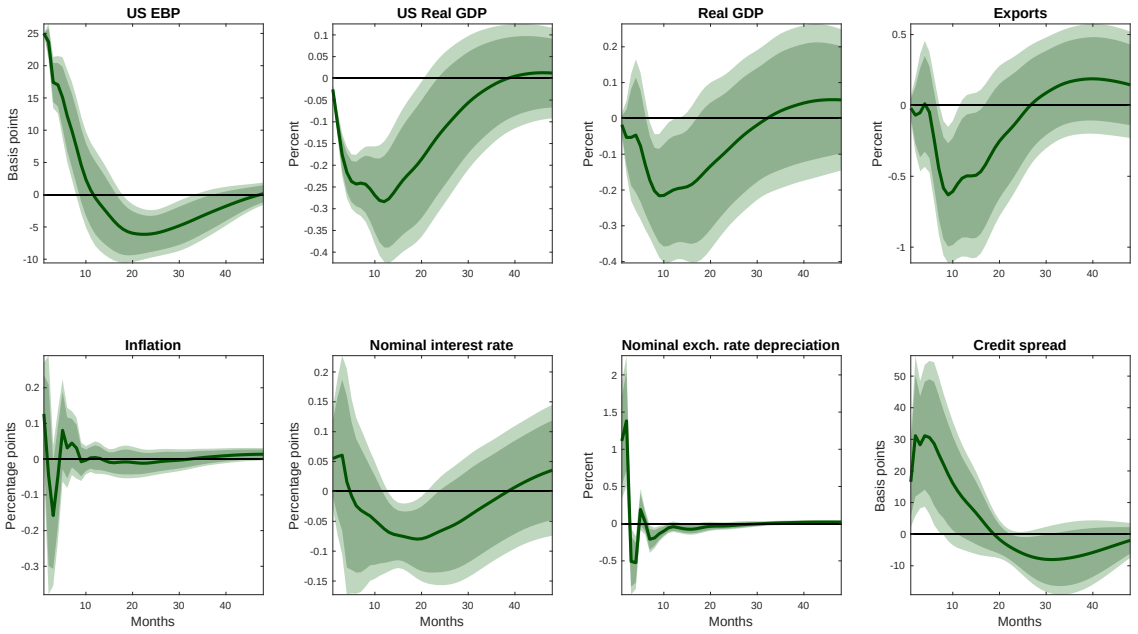
We obtain impulse responses from a Cholesky factorization of the reduced-form variance-covariance matrix of the VAR residuals. We estimate the VAR using the mean group estimator of Pesaran and Smith (1995) and Pesaran et al. (1996), as pooled estimators are not consistent in dynamic panel data with country-specific slope coefficients. The Bayesian

[3] For robustness, we also consider a larger panel of 24 countries, for which the average value of the exchange rate classification is 10.9, with a minimum value of 7.5 for India. See Appendix 2.A for details.

[4] Following Miranda-Agrippino and Rey (2020), we interpolate macroeconomic quantities (real GDP and exports) at monthly level. Appendix 2.A provides additional details on the definition and sources of the data.

[5] We downloaded the updated series of surprises from Marek Jarocinski's website (`https://marekjarocinski.github.io/`).

Figure 2.1: Impulse responses to a US monetary policy tightening.



Note: The figure displays the impulse responses of the EBP and real GDP in the US, and of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in a typical small open economy with a flexible exchange rate regime to a contractionary monetary policy shock in the US. The solid dark green line is the mean group estimate. The light and dark green shaded areas are the 95% and 99% confidence intervals, respectively. Inflation, the nominal interest rate, and spreads are in annualized terms. The shock is normalized to generate a 25 basis points increase in the EBP.

Information Criterion suggests 3 lags for the majority of countries (13 out of 15). For parsimony, we thus set $L = 3$ but the Appendix shows that the results are robust to a richer lag structure ($L = 6$).[6]

2.2.1 PVAR Baseline Estimation Results

Figure 2.1 reports the impulse responses of the EBP and real GDP in the US, and of macroeconomic and financial variables in a typical small open economy to an identified US monetary policy tightening.[7] We pick the size of the shock to generate a 25 basis points increase in the EBP, which roughly corresponds to a two-standard-deviation shock or, equivalently, to a 40 basis points increase in the 1-year rate in Gertler and Karadi (2015). The dark and light shaded areas represent the 95% and 99% confidence intervals computed scaling the variance of the country responses across countries, which provides a consistent estimate of the true cross-sectional mean impulse response (Pesaran et al.,

[6]The panel is unbalanced due to the data being available only starting after 1997:M1 for some countries. Nevertheless, we estimate the same empirical model for all countries to avoid introducing differences in country responses due to alternative specifications.

[7]Appendix 2.A reports the response of the high-frequency monetary policy surprises, which we omit from the figure for the sake of brevity.

1996).

The tightening of monetary policy in the US leads to an increase of the EBP and a persistent and hump-shaped decline of US real GDP, as in the closed economy analysis of Gertler and Karadi (2015) and Caldara and Herbst (2019). The remaining impulse responses show the average response of macroeconomic and financial variables in our sample of countries with flexible exchange rates. As expected, the monetary policy tightening in the US causes a short-lived depreciation of the local currency. A persistent contraction of credit conditions in the typical small open economy is a first indication of the GFC since domestic financial variables closely co-move with those in the US. The persistent fall of real GDP and exports is also in line with the idea of a GFC, in the sense that the financial channel of the transmission mechanism dominates over the expenditure-switching effect. In a Mundellian framework, the depreciation of the nominal exchange rate should boost exports and possibly insulate the small open economy from the foreign monetary policy shock. While this mechanism may be in place, the data highlight the importance of the transmission through financial variables. In particular, the negative effect of the monetary policy tightening on US real GDP explains the fall of exports in the small open economy because of lower foreign demand. The difference in the persistence of the response of the exchange rate compared to credit spreads is also likely to play a key role quantitatively in accounting for the relative strength of the two channels. Finally, in terms of the monetary policy variables, the nominal interest rate initially increases marginally to counteract the small inflationary blip, but quickly falls thereafter to stabilize output in the face of a contractionary shock.[8]

In summary, the results from our PVAR are consistent with the GFC hypothesis. In response to a contractionary monetary policy in the US, the nominal exchange rate of a typical small open economy that adopts a flexible exchange rate regime depreciates while credit spreads increase. The tightening of domestic financial conditions leads to a fall in real GDP and exports, despite the exchange rate depreciation. The central bank in the small open economy eases monetary policy over time to stabilize output and inflation. The next section develops and estimates a two-country DSGE model with financial frictions to account for the evidence from the PVAR.

2.3 A Model of the Dilemma Hypothesis

The model that we use is closely related to Akinci and Queralto (2019). The world consists of two countries, Home (of size n) and Foreign (of size $1-n$). Households in each

[8]The non-significant response of inflation among countries with flexible exchange rates is in line with the evidence in Flaccadoro and Nispi Landi (2022). Degasperi et al. (2020) find a statistically significant negative response of inflation, which rationalizes the monetary policy accommodation.

country consume Home and Foreign goods, supply labor, and can save via deposits in financial intermediaries (banks). Foreign banks operate both domestically and internationally, whereas Home banks only operate domestically. On the supply side, capital producers transform the final consumption good (a combination of diversified varieties) into investment goods subject to an adjustment cost function. Intermediate goods firms combine labor and capital to produce differentiated varieties. Retailers bring these products to consumers, and set prices on a staggered basis. In each country, a central bank sets monetary policy. In what follows, we describe the model from the perspective of the Home country. Where necessary, an asterisk denotes Foreign variables.

2.3.1 Households

In the Home country, the representative household consists of a continuum of members $i \in (0, n)$ who supply differentiated labor inputs $\ell_t(i)$. A representative union combines labor inputs into a homogeneous aggregate

$$\ell_t \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\nu}} \int_0^n \ell_t(i)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}},$$

where $\nu > 1$ is the elasticity of substitution among labor inputs. The demand for the i^{th} labor variety is

$$\ell_t(i) = \frac{1}{n} \left[\frac{W_t(i)}{W_t} \right]^{-\nu} \ell_t, \tag{2.1}$$

where $W_t(i)$ is the nominal wage specific to type$-i$ labor input and the aggregate wage index is

$$W_t = \left[\frac{1}{n} \int_0^n W_t(i)^{1-\nu} di \right]^{\frac{1}{1-\nu}}.$$

The representative household takes labor demand (2.1) as given, and sets wages on a staggered basis, where $\xi_w \in (0, 1)$ is the probability of keeping the wage fixed.

Because of perfect risk sharing among its members, the household chooses consumption c_t and savings in nominal deposits D_t on behalf of all its members to maximize

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left[\ln(c_{t+j} - h\bar{c}_{t+j-1}) - \frac{\chi}{1+\zeta} \int_0^n \ell_{t+j}(i)^{1+\zeta} di \right],$$

where $\bar{c}_{t-1}$ is a reference consumption level that the household takes as given, $h \in (0, 1)$ is the habits parameter, $\zeta > 0$ is the inverse Frisch elasticity of labor supply, and $\chi > 0$ is a parameter that pins down the steady state level of hours worked. The budget constraint

at time t is

$$P_t c_t + D_t = \int_0^n W_t(i)\ell_t(i)di + R_{t-1}D_{t-1} + T_t,$$

where P_t is the consumer price index (CPI), R_t is the nominal gross nominal interest rate on deposits, and T_t are profits from ownership of banks and firms net of lump-sum taxes.

The overall consumption bundle is a CES aggregator defined over goods produced in the Home and Foreign country

$$c_t \equiv \left[a^{\frac{1}{\epsilon}} c_{Ht}^{\frac{\epsilon-1}{\epsilon}} + (1-a)^{\frac{1}{\epsilon}} c_{Ft}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 0$ is the elasticity of substitution between Home and Foreign goods, and $a \in (n, 1)$ is the degree of home bias. Expenditure minimization implies that the consumer price index is

$$P_t = \left[aP_{Ht}^{1-\epsilon} + (1-a)P_{Ft}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \tag{2.2}$$

where P_{Ht} and P_{Ft} are the Home currency prices of goods produced in the Home and Foreign country, respectively. In turn, the consumption bundle for Home-produced goods consists of a continuum of varieties whose measure corresponds to the country size

$$c_{Ht} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varrho}} \int_0^n c_t(h)^{\frac{\varrho-1}{\varrho}} dh\right]^{\frac{\varrho}{\varrho-1}}$$

where $\varrho > 0$ is the elasticity of substitution among varieties. The implied price index for the Home goods bundle is

$$P_{Ht} = \left[\frac{1}{n}\int_0^n P_t(h)^{1-\varrho} dh\right]^{\frac{1}{1-\varrho}}.$$

Similarly, the consumption bundle for Foreign-produced goods is

$$c_{Ft} = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varrho}} \int_n^1 c_t(f)^{\frac{\varrho-1}{\varrho}} df\right]^{\frac{\varrho}{\varrho-1}},$$

and the corresponding price index is

$$P_{Ft} = \left[\frac{1}{1-n}\int_n^1 P_t(f)^{1-\varrho} df\right]^{\frac{1}{1-\varrho}}.$$

The structure for the Foreign country is symmetric (adjusting for country size), except that the degree of home bias may be different.

2.3.2 Financial Intermediation

We assume an asymmetric international financial structure. Foreign banks operate both domestically and internationally, whereas Home banks only operate domestically.[9] As a consequence of this asymmetric financial structure, we interpret the Home country as the typical small open economy and the Foreign country as a financial hegemon (e.g. the US).

2.3.2.1 Foreign Banks

Foreign banks fund their operations with domestic deposits and net worth, which in real terms we denote with n_t^*. Their assets consists of loans to both Foreign firms and Home banks, whose market value in real terms are $q_t^* z_t^*$ and $b_t^* \equiv B_t^*/P_t^*$, respectively.[10] International interbank borrowing and lending takes place in Foreign currency. The balance sheet of Foreign banks in real terms at the end of time t is

$$q_t^* z_t^* + b_t^* = d_t^* + n_t^*, \tag{2.3}$$

where $d_t^* \equiv D_t^*/P_t^*$. Net worth is the difference between the gross return on assets and liabilities

$$n_t^* = r_{kt}^* q_{t-1}^* z_{t-1}^* + \frac{R_{bt-1}^*}{\Pi_t^*} b_{t-1}^* - \frac{R_{t-1}^*}{\Pi_t^*} d_{t-1}^*, \tag{2.4}$$

where r_{Kt}^* is the real return on capital, R_{bt}^* is the gross nominal interest rate on loans to Home banks, and $\Pi_t^* \equiv P_t^*/P_{t-1}^*$ is the gross inflation rate.

As in Gertler and Karadi (2011), we assume that in each period banks continue their operations with probability ω. With the complementary probability, banks exit the industry, turn their net worth to households, and are replaced by an equal mass of new banks that start operating with a small transfer from households. The objective function of banks is the expected value of terminal wealth, which we can write in recursive form as

$$V(n_t^*) = \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1}^* [(1-\omega) n_{t+1}^* + \omega V(n_{t+1}^*)] \right\}, \tag{2.5}$$

where $\mathcal{M}_{t,t+1}^*$ is the stochastic discount factor of the Foreign representative household for real payoffs.

[9] Our international financial structure is slightly different from Akinci and Queralto (2019), who do not model banking frictions in the Foreign country. This different assumption has some consequences on the degree of amplification of financial frictions, as we discuss below.

[10] Formally, firms issue financial securities in exchange for the loans obtained from the bank.

In principle, bankers can divert a fraction $\theta^* \in (0,1)$ of their assets, in which case the bank gets liquidated. Without any constraint, banks would seek to expand their balance sheet indefinitely and maximize the benefit of diversion. Thus, the incentive compatibility constraint for banks must be such that the value of the bank is greater than the private benefit of diversion to the banker

$$V(n_t^*) \geq \theta^* \left(q_t^* z_t^* + b_t^*\right), \tag{2.6}$$

where $\theta^* > 0$.

The problem of a banker consists of maximizing (2.5) subject to the balance sheet constraint (2.3), the evolution of net worth (2.4) and the incentive compatibility constraint (2.6). In an equilibrium in which the incentive compatibility constraint binds, the solution to the bankers' problem implies

$$\lambda_t^* = \frac{\mu_{dt}^*}{\theta^* - \mu_{kt}^*}, \tag{2.7}$$

where $\lambda_t^* \equiv (q_t^* z_t^* + b_t^*)/n_t^*$ is the leverage ratio.[11] The numerator of (2.7) is the discounted expected real return on deposits

$$\mu_{dt}^* = \mathbb{E}_t \left(\mathcal{M}_{t,t+1}^* \Omega_{t,t+1}^* \frac{R_t^*}{\Pi_{t+1}^*} \right),$$

where $\Omega_{t,t+1}^* \equiv 1 - \omega + \omega \kappa_{t+1}^*$ is an additional discount rate that banks apply to future returns due to the probability of exiting, and $\kappa_t^* \equiv V(n_t^*)/n_t^* = \theta^* \lambda_t^*$ is the franchise value of the bank per unit of net worth. The denominator of (2.7) is instead the discounted expected excess return of capital over deposits

$$\mu_{kt}^* = \mathbb{E}_t \left[\mathcal{M}_{t,t+1}^* \Omega_{t,t+1}^* \left(r_{kt+1}^* - \frac{R_t^*}{\Pi_{t+1}^*} \right) \right].$$

As Gertler and Karadi (2011) stress, in the absence of financial frictions, μ_{kt} would be zero. In order to satisfy the incentive compatibility constraint (2.6), banks need to earn an excess return in expectation.

2.3.2.2 Home Banks

Home banks raise deposits from domestic households, borrow from Foreign banks, and use their net worth to lend to Home firms in domestic currency. Their balance sheet in

[11] Appendix 2.B reports the details of the derivations.

real terms is

$$q_t z_t = d_t + s_t b_t^* + n_t, \tag{2.8}$$

where $s_t \equiv \mathcal{E}_t P_t^*/P_t$ is the real exchange rate and $\mathcal{E}_t$ is the nominal exchange rate from the perspective of the Home country (units of Home currency to buy one unit of Foreign currency). Their net worth evolves according to

$$n_t = r_{kt} q_{t-1} z_{t-1} - \frac{R_{t-1}}{\Pi_t} d_{t-1} - \frac{R_{bt-1}^*}{\Pi_t^*} s_t b_{t-1}^*. \tag{2.9}$$

The presence of liabilities denominated in Foreign currency creates a balance sheet mismatch. For example, a depreciation of the Home currency for a given value of assets implies a reduction of the banks' net worth.[12]

Home banks are subject to a similar moral hazard problem as Foreign banks. The key difference is that, as in Akinci and Queralto (2019), we assume that the fraction of assets that Home banks can divert is an increasing function of the share of foreign currency liabilities

$$V(n_t) \geq \Theta(x_t) q_t z_t, \tag{2.10}$$

where

$$\Theta(x_t) \equiv \theta \left(1 + \frac{\gamma}{2} x_t^2\right),$$

and $x_t \equiv s_t b_t^*/(q_t z_t)$, with θ and $\gamma > 0$. The parameter θ governs the tightness of the financial friction, while γ determines the extent to which the financial friction varies with the share of foreign currency liabilities.

The problem of Home banks consists of maximizing

$$V(n_t) = \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1} [(1-\omega) n_{t+1} + \omega V(n_{t+1})] \right\},$$

subject to (2.8), (2.9) and (2.10). Similarly to Foreign banks, we can write the solution to this problem as

$$\lambda_t = \frac{\mu_{dt}}{\Theta(x_t) - (\mu_{kt} + \mu_{bt} x_t)}, \tag{2.11}$$

where $\lambda_t \equiv q_t z_t / n_t$ is the leverage ratio for Home banks. The definitions of the expected discounted real return on deposits μ_{Dt} and of the expected discounted excess return of

[12] In practice, financial intermediaries typically hedge the currency exposure on their balance sheet. In this sense, the balance sheet mismatch in the model better captures the overall exposure of the private sector in the small open economy.

capital over deposits μ_{Kt} correspond to their counterparts in the Foreign country

$$\mu_{dt} = \mathbb{E}_t \left(\mathcal{M}_{t,t+1} \Omega_{t,t+1} \frac{R_t}{\Pi_{t+1}} \right),$$
$$\mu_{kt} = \mathbb{E}_t \left[\mathcal{M}_{t,t+1} \Omega_{t,t+1} \left(r_{kt+1} - \frac{R_t}{\Pi_{t+1}} \right) \right],$$

where $\Omega_{t,t+1} \equiv 1 - \omega + \omega \kappa_{t+1}$ is the additional discount factor and $\kappa_t \equiv V(n_t)/n_t = \Theta(x_t)\lambda_t$ is the franchise value of Home banks. Home banks also arbitrage between domestic deposits and funds raised in foreign currency from the international interbank market. Financial frictions limit this arbitrage activity that banks perform and create an endogenous wedge in the uncovered interest rate parity condition, in line with a large body of empirical evidence since Fama (1984)

$$\mu_{bt} = \mathbb{E}_t \left[\mathcal{M}_{t,t+1} \Omega_{t,t+1} \left(\frac{R_{t+1}}{\Pi_{t+1}} - \frac{R^*_{bt}}{\Pi^*_{t+1}} \frac{s_{t+1}}{s_t} \right) \right].$$

Given the functional form assumed for $\Theta(\cdot)$, we can solve for optimal proportion of foreign currency debt

$$x_t = \frac{1}{\mu_t} \left(\sqrt{1 + \frac{2\mu_t^2}{\gamma}} - 1 \right), \tag{2.12}$$

where $\mu_t \equiv \mu_{bt}/\mu_{kt}$. Since interbank borrowing corresponds to the net external position, equation (2.12) in steady state pins down net foreign liabilities.

2.3.3 Firms

Four types of firms operate in the Home country: capital producers, importers, intermediate goods producers and final goods producers. This rich production structure allows us to introduce a number of key pricing frictions in the model.

As in Akinci and Queralto (2019), final goods producers in both countries set prices on a staggered basis (Calvo, 1983). In the Home country, these firms price their products in the currency of the market of destination. In a two-country setting, this assumption is consistent with the evidence on "dominant currency pricing" emphasized by Gopinath et al. (2020).

The presence of importers in the Home country distinguishes our framework from Akinci and Queralto (2019). Following Monacelli (2005), we assume that the law of one price for imported goods holds at the dock but importers adjust their price in domestic currency infrequently. This friction introduces imperfect exchange rate pass-through to

the Home economy and is a key ingredient for matching the empirical response of inflation to a Foreign monetary policy shocks in the model, as we later demonstrate.

Production in the Foreign country is symmetric except that we abstract from importers since Home final goods producers price their exports in Foreign currency.

2.3.3.1 Capital Producers

Capital producers transform final goods into capital goods. On top of the cost of acquiring one unit of final good, capital producers pay an adjustment cost, which we assume to be quadratic in the growth rate of investment. The problem of capital producers in the Home country is to maximize

$$\mathbb{E}_t \sum_{j=0}^{\infty} \mathcal{M}_{t,t+j} \left[q_{t+j} - 1 - \frac{\varphi_i}{2} \left(\frac{i_{t+j}}{i_{t+j-1}} - 1 \right)^2 \right] i_{t+j},$$

where $\varphi_i > 0$.

2.3.3.2 Intermediate Goods Producers

Competitive intermediate goods producers have access to a Cobb-Douglas technology in capital and labor

$$y_t = A_t k_{t-1}^{\alpha} \ell_t^{1-\alpha},$$

where $\alpha \in (0, 1)$ is the capital share. At the end of each period, firms issue securities to acquire capital for production in the subsequent period. After production takes place in a period, firms sell the undepreciated capital on the open market. Therefore, their balance sheet at time t is

$$q_t z_t = q_t k_t,$$

and their profits are

$$\mathcal{P}_t = p_{mt} y_t - w_t \ell_t - r_{kt} q_{t-1} z_{t-1} + (1 - \delta) q_t k_{t-1},$$

where p_{mt} is the relative price of intermediate goods and $w_t \equiv W_t / P_t$ is the real wage. The first order condition for labor is standard

$$w_t = (1 - \alpha) p_{mt} A_t k_{t-1}^{\alpha} \ell_t^{-\alpha}.$$

Plugging this expression back into the profits of intermediate goods producers, we obtain

$$\mathcal{P}_t = \alpha p_{mt} y_t - r_{kt} q_{t-1} k_{t-1} + (1-\delta) q_t k_{t-1}.$$

Since intermediate goods producers are perfectly competitive, zero profits implies

$$r_{kt} = \frac{\alpha p_{mt} A_t k_{t-1}^{\alpha-1} \ell_t^{1-\alpha} + (1-\delta) q_t}{q_{t-1}}.$$

2.3.3.3 Final Goods Producers

Final goods producers operate in monopolistic competition and set prices on a staggered basis (Calvo, 1983). In each period, the probability that the price remains unchanged is $\xi_p \in (0,1)$.

In the Home country, final goods producers set prices in the currency of the destination market (i.e., local currency pricing, or LCP). This assumption captures the idea that the Foreign country currency is "dominant" in goods markets, as we have also assumed for financial markets.

The problem for Home producers consists of choosing $\widetilde{P}_t(h)$ and $\widetilde{P}_t^*(h)$ to maximize

$$\mathbb{E}_t \sum_{j=0}^{\infty} \xi_p^j \mathcal{M}_{t,t+j} \left\{ \left[\frac{\widetilde{P}_t(h)}{P_{t+j}} - p_{mt+j} \right] y_{t,t+j}(h) + \left[\frac{\mathcal{E}_{t+j} \widetilde{P}_t^*(h)}{P_{t+j}} - p_{mt+j} \right] y_{t,t+j}^*(h) \right\},$$

where

$$y_{t,t+j}(h) = \left[\frac{\widetilde{P}_t(h)}{P_{Ht+j}} \right]^{-\varrho} y_{Ht+j} \qquad \text{and} \qquad y_{t,t+j}^*(h) = \left[\frac{\widetilde{P}_t^*(h)}{P_{Ht+j}^*} \right]^{-\varrho} y_{Ht+j}^*$$

and the demand for Home goods by Home and Foreign households, respectively, are

$$y_{Ht} = a \left(\frac{P_{Ht}}{P_t} \right)^{-\epsilon} \left[c_t + i_t + \frac{\varphi_i}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 i_t \right],$$

and

$$y_{Ht}^* = a^* \left(\frac{P_{Ht}^*}{P_t^*} \right)^{-\epsilon} \left[c_t^* + i_t^* + \frac{\varphi_i}{2} \left(\frac{i_t^*}{i_{t-1}^*} - 1 \right)^2 i_t^* \right].$$

The first-order conditions for $\widetilde{P}_t(h)$ and $\widetilde{P}_t^*(h)$ are standard and, together with the price indexes, determine two Phillips curves.

The problem for final goods producers in the Foreign country is similar, except that

those firms price in their domestic currency independently of the market of destination. As a consequence, the solution of their problem gives rise only to one Phillips curve.

2.3.3.4 Importers

The law of one price for imported goods holds at the dock, that is, $P_t^{im}(f) = \mathcal{E}_t P_t^*(f)$. However, in the Home country, the retailers that distribute imported goods each period change their prices with probability $\xi_{im} \in (0,1)$. Their problem is to choose the price $\widetilde{P}_t(f)$ to maximize

$$\mathbb{E}_t \sum_{j=0}^{\infty} \xi_{im}^j \mathcal{M}_{t,t+j} \left[\frac{\widetilde{P}_t(f)}{P_{t+j}} - \frac{P_{t+j}^{im}(f)}{P_{t+j}} \right] y_{t,t+j}(f),$$

where

$$y_{t,t+j}(f) = \left[\frac{\widetilde{P}_t(f)}{P_{Ft+j}} \right]^{-\varrho} y_{Ft+j},$$

and

$$y_{Ft} = (1-a) \left(\frac{P_{Ft}}{P_t} \right)^{-\epsilon} \left[c_t + i_t + \frac{\varphi_i}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 i_t \right].$$

The first-order condition for this problem is a Phillips curve that links the inflation rate of imported goods in domestic currency to the law of one price gap, that is, the difference between the price of imports in Foreign currency converted in domestic currency using the nominal exchange rate and the price of imports in domestic currency (Monacelli, 2005).

2.3.4 Monetary Policy

The baseline monetary policy configuration for the Home country assumes that the central bank sets the nominal interest rate according to a standard feedback rule (Taylor, 1993) with inertia

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left[\Pi_t^{\phi_\pi} \left(\frac{y_t}{y_{t-1}} \right)^{\phi_y} \right]^{1-\rho_R},$$

where $\rho_R \in (0,1)$ is the inertia parameter, $\phi_\pi > 1$ and $\phi_y > 0$ are the feedback coefficients on inflation and output, respectively, and $R = 1/\beta$ is the steady state nominal interest rate consistent with zero inflation (assumed to be the target for the central bank).

The monetary policy rule in the Foreign country is identical, except that we add a monetary policy innovation $\varepsilon_{Rt}^* \sim \mathcal{N}(0, \sigma_{R^*}^2)$, which is the focus of our empirical analysis.

2.3.5 Equilibrium

The labor and capital markets clear nationally within each country. From the perspective of the Home country, net foreign liabilities correspond to the amount of interbank borrowing. Their evolution, in real units of Foreign currency, is

$$b_t^* = \frac{R_{bt-1}^* b_{t-1}^*}{\Pi_t^*} + p_{Ft}^* y_{Ft} - \left(\frac{1-n}{n}\right) p_{Ht}^* y_{Ht}^*$$

Accordingly, the Home current account balance in real units of domestic currency is

$$ca_t \equiv -s_t(b_t^* - b_{t-1}^*).$$

2.4 Estimation and Counterfactuals

The quantitative assessment of the model relies on a mix of calibrated and estimated parameters. We calibrate all the parameters for the Foreign country, except those of the monetary policy rule, and some of the parameters of the Home country.

Table 2.1 reports the calibrated parameters. The values for the Foreign country are consistent with the estimated values for the US economy in Smets and Wouters (2007), and whenever possible we maintain symmetry across countries. The relative size of the Home country is equal to 0.1, which corresponds to the average size of the countries in our sample relative to the US. Given the calibrated individual discount factors, the steady state real interest rate in annualized terms is 3% in the Home country and 1% in the Foreign country. The home bias parameter implies that the export share of GDP for the Home country is about 10%, in line with the data for exports towards the US of major emerging markets in Asia and Latin America (Akinci and Queralto, 2019). The choice of parameters for the US banks gives a steady state leverage ratio of 5 and a credit spread of 150 basis points annualized, which are standard values in literature (see, e.g., Gertler and Karadi, 2011).

We estimate the remaining parameters by minimizing the distance between the model-implied impulse responses to a US monetary policy shock and the impulse responses of the PVAR presented in section 2.2 for the typical small open economy. Formally, the estimator $\hat{\eta}$ solves

$$\min_{\eta} \left\{ \left[\widehat{\Psi} - \Psi(\eta)\right]' \widehat{W}^{-1} \left[\widehat{\Psi} - \Psi(\eta)\right] \right\},$$

where η is the vector of parameters to be estimated, $\widehat{\Psi}$ is the vector of impulse responses from the PVAR, $\Psi(\gamma)$ is the vector containing of impulse responses from the DSGE model

Table 2.1: Calibrated parameters

Parameter		Home	Foreign
n	Relative size of country H	0.1	0.9
β	Individual discount factor	0.9926	0.9975
h	Habits in consumption	–	0.71
σ	Relative risk aversion	–	1.38
χ	Relative weight on disutility of labor	24	52
ζ	Inverse Frisch elasticity	1	1
ϱ	Elasticity of substitution among goods varieties	6	6
a	Home bias in consumption	0.91	0.99
ϵ	Elasticity of substitution between H and F goods	1.5	1.5
ν	Elasticity of substitution among labor varieties	6	6
ξ_w	Wage rigidity	0.66	0.66
ξ_p	Price rigidity	–	0.66
α	Capital share	0.33	0.33
δ	Depreciation rate	0.025	0.025
φ_i	Investment adjustment cost	–	5.74
ω	Bank survival rate	0.97	0.97
θ	proportion of divertible funds	–	0.51
ξ_b	bank transfer rate	–	0.002

Note: The missing parameters for the Home country are estimated.

and $\widehat{W}$ is a diagonal matrix collecting the estimated variances of each impulse response from the PVAR (i.e., the width of the error bands).

As in Christiano et al. (2011), we follow a Bayesian approach to estimate the parameters of the model with impulse response matching. Table 2.2 reports the list of estimated parameters (first column), their prior distributions (second to fourth columns), and their posteriors (fifth to eight column). We specify a prior directly on the steady state value of the leverage ratio of Home banks (λ) and on the steady state share of foreign currency debt (x). The data are particularly informative for the latter. At the posterior mode, the estimated values for these two variables imply a fraction of divertible funds θ equal to 0.516 (very close to the assumed value for θ^*), a degree of Home bias for bank funding γ equal to 2.125, and a bank transfer rate ξ_b equal to 0.0009, which is about half of the assumed value for Foreign banks.

The estimates of the other parameters are reasonably standard and the data are generally informative about their values.[13] The degree of price stickiness is fairly high, but very much comparable to recent estimates that include data after the financial crisis (Del Ne-

[13] Appendix 2.A reports a systematic comparison of prior and posterior distributions.

Table 2.2: Estimated parameters

Parameter	Prior			Posterior			
	Distribution	Mean	SD	Mode	Median	5%	95%
h	Beta	0.650	0.1	0.648	0.654	0.497	0.809
σ	Normal	1	0.375	0.703	0.839	0.517	1.258
λ	Normal	5	1	4.977	5.014	3.523	6.663
x	Normal	0.300	1	0.154	0.136	0.035	0.235
φ_i	Normal	2.850	1.5	0.587	0.799	0.182	1.862
ξ_p	Beta	0.660	0.1	0.914	0.882	0.786	0.962
ξ_{im}	Beta	0.660	0.1	0.702	0.688	0.525	0.841
ρ_R	Normal	0.750	0.1	0.666	0.689	0.543	0.839
ϕ_π	Normal	1.500	0.25	1.616	1.637	1.266	2.022
ϕ_y	Normal	0.125	0.05	0.155	0.147	0.065	0.231
ρ_R^*	Normal	0.750	0.1	0.731	0.702	0.564	0.819
ϕ_π^*	Normal	1.500	0.25	1.615	1.641	1.275	1.999
ϕ_y^*	Normal	0.125	0.05	0.126	0.126	0.129	0.208

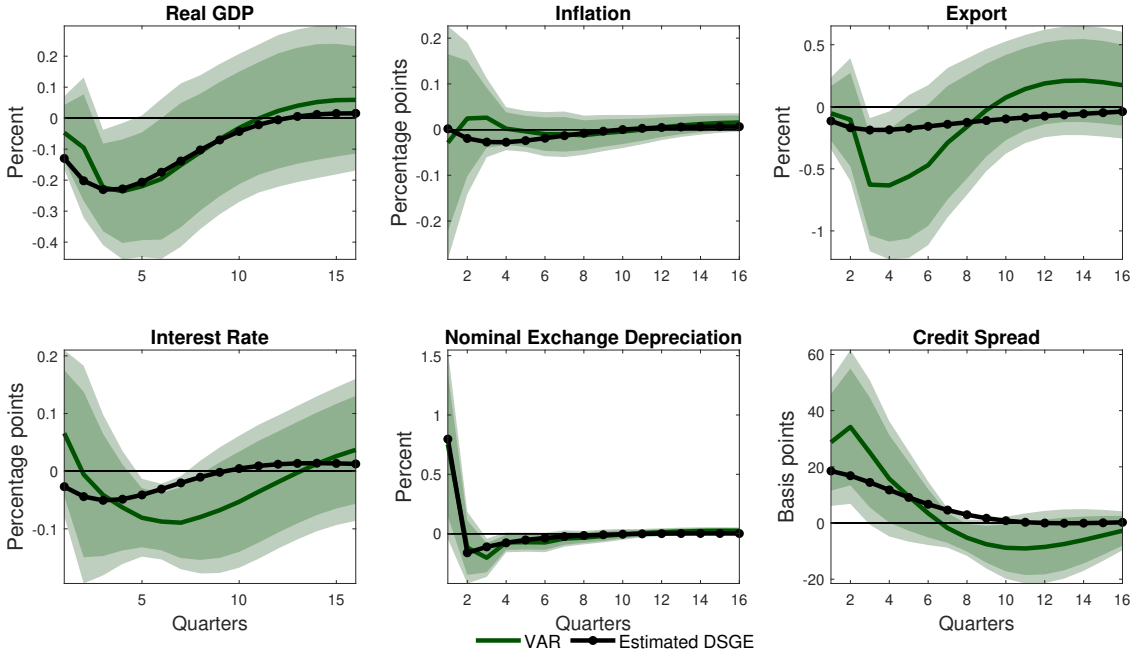
Note:

gro et al., 2015). Conversely, the degree of import price stickiness is significantly lower, implying an average contract duration between three and four quarters. The coefficients on inflation and output growth in the interest rate rule are very close across countries. Interest rate inertia is slightly higher in the Foreign country than in the Home country.

Figure 2.2 replicates the mean group estimate of the impulse responses for the typical small open economy to a contractionary monetary policy shocks in the US (solid green line) presented in Figure 2.1, as well as the 16-84 (dark shaded areas) and the 10-90 (lighter shaded areas) confidence bands. The solid black lines with markers in the figure correspond to the impulse responses to a Foreign monetary policy shock of the same variables in the model at the posterior mode. The model fits well the response of real GDP, inflation, the depreciation of the nominal exchange rate and credit spreads. Although quantitatively not as impressive, the response of exports and the nominal interest rate are also qualitatively in line with the data and lie at least within the 10-90 confidence bands throughout the simulation horizon.

The model, therefore, is able to replicate the dominant role of the financial channel in propagating Foreign monetary policy shocks to the domestic economy that we have highlighted in section 2.2. The next section investigates the key ingredients of transmission mechanism by way of two counterfactuals.

Figure 2.2: Impulse response matching.



Note: The figure displays the impulse responses of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in a typical small open economy to a contractionary monetary policy shock in the US in the data (solid green lines) and in the Foreign country in the model (solid black lines with markers). The light and dark grey shaded areas are the PVAR 16-84 and 10-90 confidence bands, respectively. Inflation, the nominal interest rate, and spreads are in annualized terms.

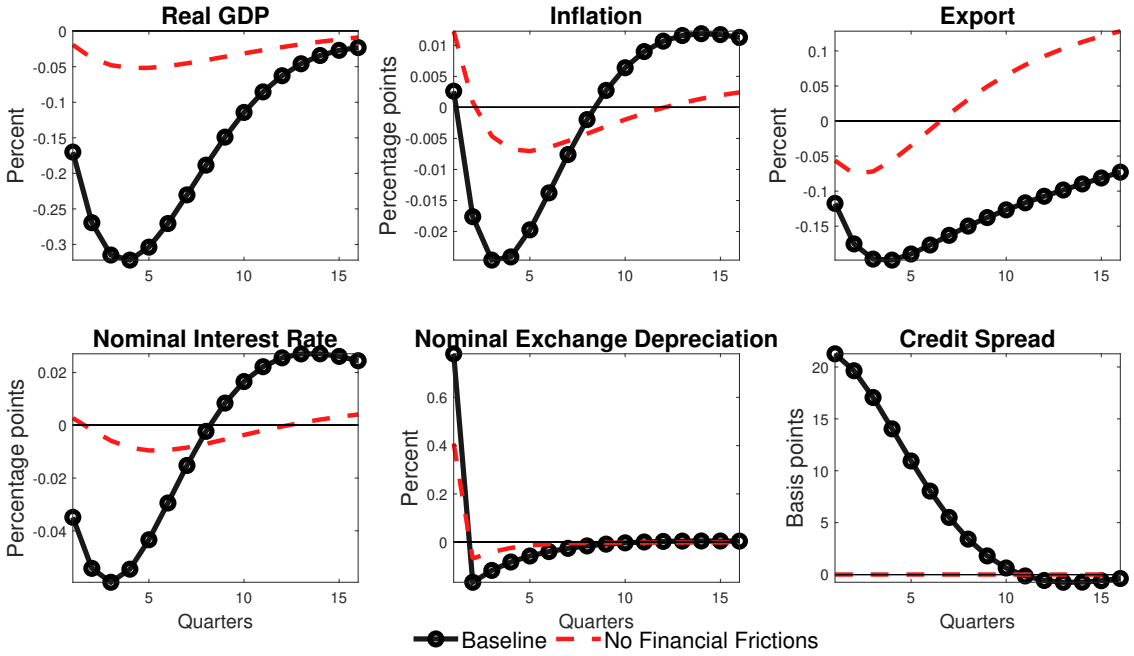
2.4.1 The Role of Financial and Trade Frictions

In the first counterfactual exercise, we shut down financial frictions in both countries. The difference with the baseline model is that households now invest directly in physical capital and in nominal bonds. Bonds denominated in Foreign currency are internationally traded, whereas bond denominated in Home currency only circulate domestically and are in zero net supply. These assumptions preserve the dominance of the Foreign currency (in practice the US dollar) in international financial markets even in the absence of global banks. In order to ensure stationarity of the net foreign asset position, we introduce a small portfolio-adjustment cost for trading bonds (Schmitt-Grohé and Uribe, 2003). The resulting model is thus a standard two-country open economy framework with incomplete international financial markets, similar to Baxter and Crucini (1995) and Kollmann (1996).

Figure 2.3 highlights the amplification effect of financial frictions on macroeconomic and financial outcomes. The dashed red line in the figure reports the response to a foreign monetary policy shocks without financial frictions, while the solid black line with markers is the estimated response from the baseline model. In the counterfactual, output drops by about 0.05% at the trough, compared to 0.3% with financial frictions.

At the same time, without financial frictions, the absence of an endogenous UIP wedge

Figure 2.3: The role of financial frictions.



Note: The figure displays the impulse responses of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in the Home country to a contractionary monetary policy shock in the Foreign country with (solid black line with markers) and without (dashed red line) financial frictions. Inflation, the nominal interest rate, and spreads are in annualized terms.

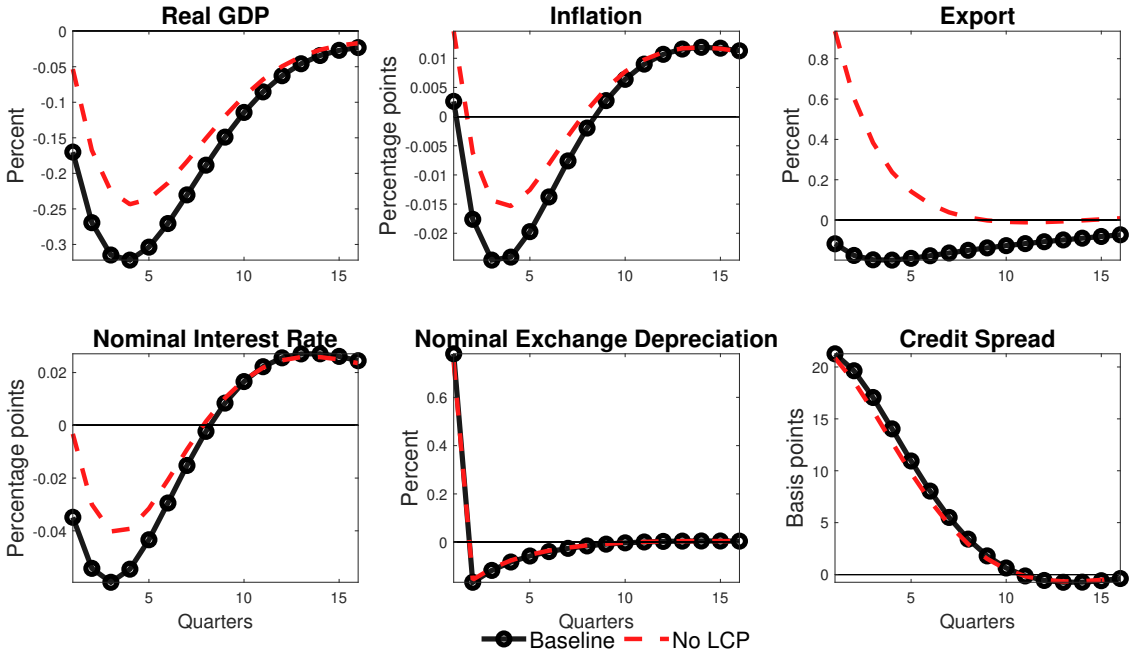
slightly reduces the depreciation of the Home currency. The first effect (smaller recession) quantitatively dominates over the second (smaller depreciation), implying somewhat higher inflation than in the baseline. As a consequence, the central bank keeps the nominal interest rate roughly unchanged to balance the tradeoff between output and inflation.

Financial frictions also significantly amplify the response of exports. In the frictionless model, the response is much less persistent than in the baseline case and turns positive after about eight quarters. The reason is that, in the absence of financial frictions, foreign demand drops less than in the baseline, which allows exports to recover more quickly.

The key element of the model that explains the sign of the response of exports is, however, currency pricing. Under our baseline assumption of LCP in the Home country aimed at mimicking the dominance of the US dollar in goods price invoicing, exports fall following a monetary policy contraction in the Foreign country.

In general, the response of exports is the result of two channels. On the one hand, the depreciation of the Home currency makes domestic goods more competitive and increases the demand for exports (expenditure-switching channel). On the other hand, the fall of Foreign demand reduces demand for Home exports (global-demand channel). Under LCP, exports are insensitive to the nominal exchange rate depreciation. Conversely, financial frictions make the global demand channel even stronger by amplifying the drop

Figure 2.4: Counterfactual with producer currency pricing (PCP).



Note: The figure displays the impulse responses of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in the Home country to a contractionary monetary policy shock in the Foreign country with (solid black line with markers) and without (dashed red line) LCP in the Home country. Inflation, the nominal interest rate, and spreads are in annualized terms.

of Foreign output.

Figure 2.4 illustrates the importance of the LCP assumption. The figure compares the baseline (solid black line with markers) with the impulse response functions under PCP (red dashed lines). In the counterfactual experiment, exports increase and return to steady state after about eight quarters. The positive effect of the depreciation on exports limits the decline of real GDP.[14] Furthermore, inflation is positive, at least on impact. As in the case without financial frictions, the response of the domestic interest rate on impact is muted to balance the two central bank mandates.

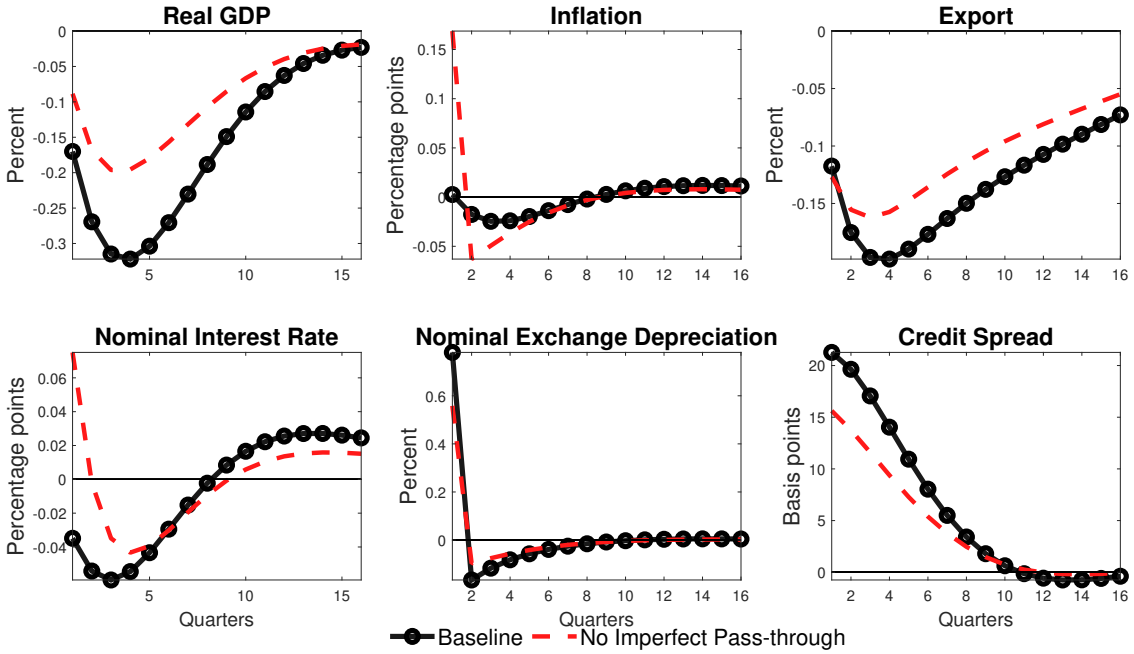
Overall, the two counterfactual exercises in this section clarify that the combination of financial and trade frictions is indeed crucial in accounting for the key empirical features of the GFC.

2.4.2 The Response of Inflation

From the perspective of monetary policy stabilization, the response of domestic inflation in a small open economy following a foreign monetary policy shock is an important dimension of the GFC and the dilemma debate. As Rey (2013) emphasizes, if a country

[14] In the absence of financial frictions (not shown), the positive response of exports pushes real GDP in positive territory, at least temporarily.

Figure 2.5: The role of imperfect pass-through.



Note: The figure displays the impulse responses of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in the Home country to a contractionary monetary policy shock in the Foreign country with (solid black line with markers) and without (dashed red line) price rigidities for importers. Inflation, the nominal interest rate, and spreads are in annualized terms.

with a flexible exchange rate regime does not manage to stabilize domestic inflation in the face of foreign shocks, the benefits of floating its currency may indeed prove elusive.

Our evidence suggests that the response of inflation to a contractionary foreign monetary policy shock is small and not significant. This result is consistent with the findings in Passari and Rey (2016) for the UK, in Rey (2016) for Sweden, and in Flaccadoro and Nispi Landi (2022) for their overall sample.[15]

Figure 2.5 shows that imperfect pass-through plays a key role in aligning the model with the evidence for the response of inflation, above and beyond the contribution of financial frictions discussed in the previous section. The dashed red line in the figure reports the response to a foreign monetary policy shocks when the law of one price holds at the consumer level. In this variant of the model, importers are perfectly competitive firms that deliver foreign goods to final consumers without pricing frictions ($\xi_{im} = 0$). The solid black line with markers is the estimated response from the baseline model.

The figure shows that the same depreciation of the nominal interest rate makes inflation jump in positive territory in the model without pricing frictions on importers. The exchange rate pass-through coefficient on impact is approximately 0.25, compared to es-

[15] In Rey (2016), the median response of inflation for Canada, the UK and New Zealand is positive but generally not significant. The response of inflation for floaters is negative and significant in Miranda-Agrippino and Rey (2020) and Degasperi et al. (2020), reinforcing the narrative that a flexible exchange rate regime provides little insulation against foreign monetary policy shocks.

sentially zero in the baseline. The inflation increase forces the central bank to raise the nominal interest rate, instead of providing accommodation as in the baseline formulation of the model and in the data. When the law of one price holds, the foreign monetary policy shock creates a tradeoff between inflation and output stabilization which is not present with imperfect pass-through.

Importantly, the response of the other variables does not display significant changes compared to the baseline, highlighting the complementarity of imperfect pass-through and financial frictions in determining the response of macroeconomic and financial variables to foreign monetary policy shocks.

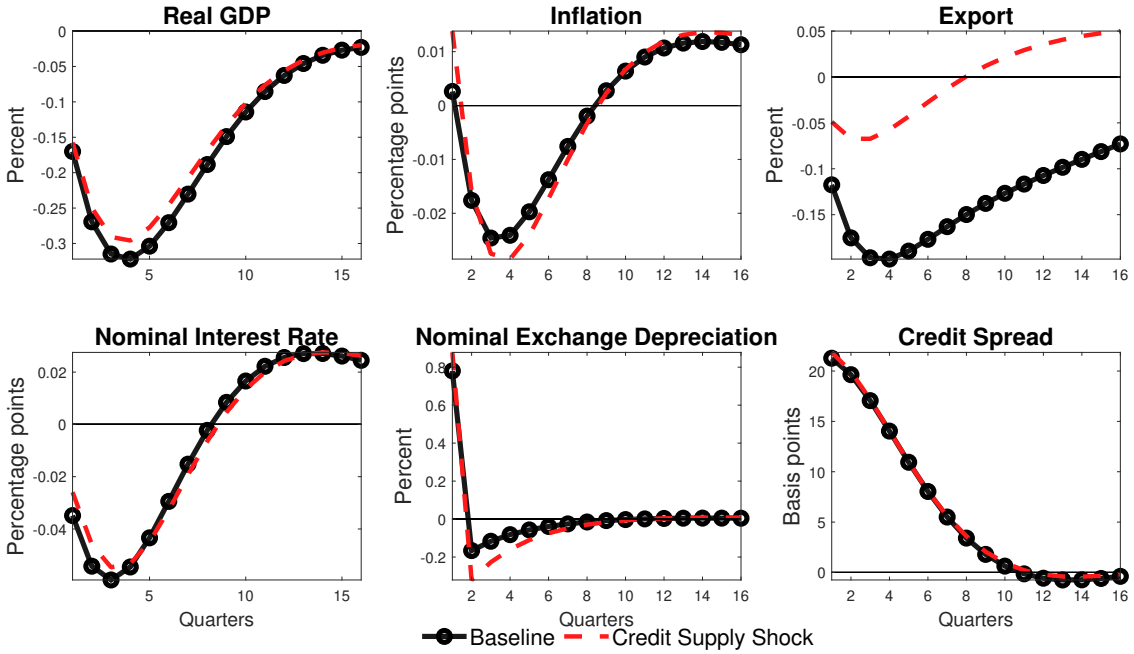
2.4.3 International Credit Supply Shocks

This section compares the responses of macroeconomic and financial variables in the Home country to the monetary policy shock considered so far with the responses to a contractionary credit supply shock for Foreign banks. For this experiment, we modify the baseline model by allowing the tightness of the financial constraint for Foreign banks to be time-varying. In particular, we assume that $\hat{\theta}_t^* \equiv \ln(\theta_t^*/\theta^*)$ follows a stationary process with persistence $\rho_{\theta^*} = 0.731$, which corresponds to the posterior mode of the inertia parameter in the monetary policy rule for Foreign country. The size of the credit supply shock is such that credit spreads in the Home economy increase by roughly 25 basis points on impact, as in the case of the monetary policy shock.

Figure 2.6 shows that the response of the Home country macroeconomic and financial variables is qualitatively and quantitatively very similar under monetary policy and credit supply shocks. Like a contractionary monetary policy shock, the negative credit supply shock in the Foreign country causes spreads to increase. Global banks tighten lending conditions in the international interbank market, giving rise to a similar transmission to macroeconomic and financial variables in the Home country.

Interestingly, the close correspondence between the response to monetary policy and credit supply shocks is consistent with the VAR evidence in Cesa-Bianchi et al. (2018). Their work demonstrates how shocks to the leverage of US broker-dealers are a complementary driver of the GFC, even when controlling for monetary policy and global demand shocks.

Figure 2.6: Monetary policy shock versus credit supply shock.



Note: The figure displays the impulse responses of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in the Home country to a contractionary monetary policy shock in the Foreign country (solid black line with markers) and to a negative shock to the US credit supply (dashed red line). The size of the credit supply shock is such that, under both shocks, credit spreads in the Home economy increase by roughly 25 basis points on impact. Inflation, the nominal interest rate, and spreads are in annualized terms.

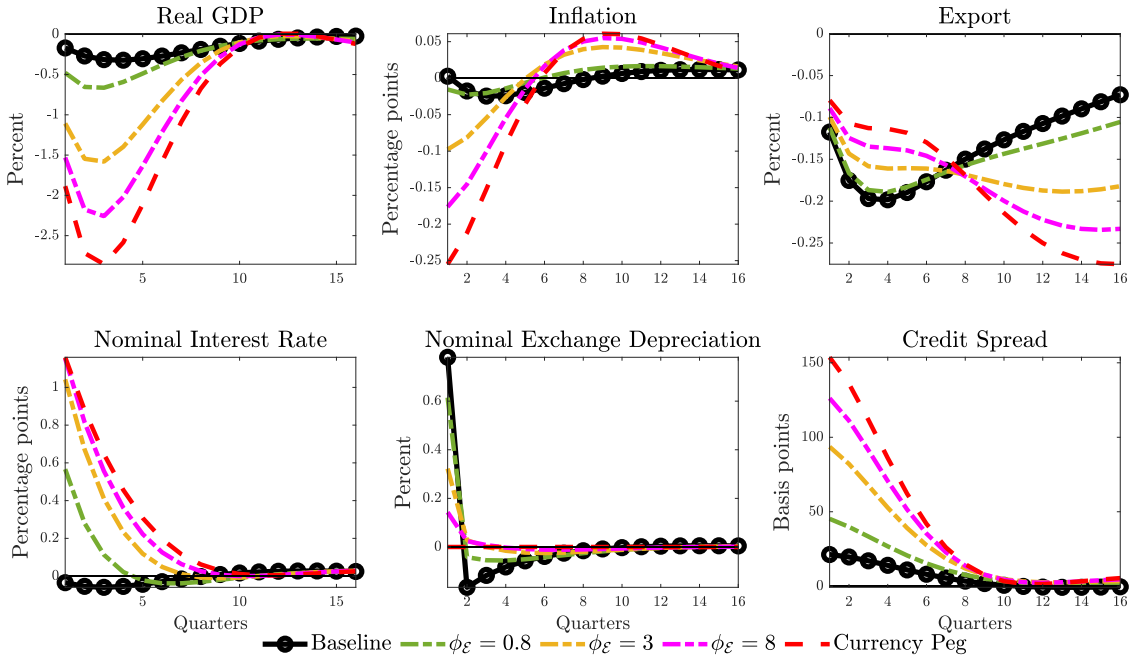
2.5 Policy Analysis

In this section, we study two questions. First, we revisit the benefits of flexible exchange rates in our estimated model. Second, we analyze the extent to which the introduction of additional policy instruments affects the volatility of macroeconomic and financial variables induced by foreign monetary policy shocks.

2.5.1 The Benefits of a Flexible Exchange Rate

The model developed in section 2.3 can match the empirical evidence that the financial channel dominates over the trade channel in the international transmission of monetary policy shocks. An interest rate increase in the Foreign country determines a tightening of domestic financial conditions. In spite of the depreciation of the domestic currency, exports decline and so does GDP. Thus, the flexible exchange rate regime does not appear to be providing any substantial insulation to the domestic economy. In fact, the depreciation of the exchange rate increases the cost of foreign funds for domestic financial intermediaries and thus contributes to exacerbate the negative impact of the shock.

Figure 2.7: Flexible versus fixed exchange rate.



Note: The figure displays the impulse responses of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in the Home country to a contractionary monetary policy shock in the Foreign country under a flexible exchange rate regime (solid black line with markers) and under a peg (dashed red line). The dashed-dotted green, yellow and magenta lines correspond to a managed float regime, in which the central bank of the Home country responds to the depreciation of the nominal exchange rate with increasingly higher coefficients ($\phi_{\mathcal{E}} = 0.8$, 3, and 8, respectively). Inflation, the nominal interest rate, and spreads are in annualized terms.

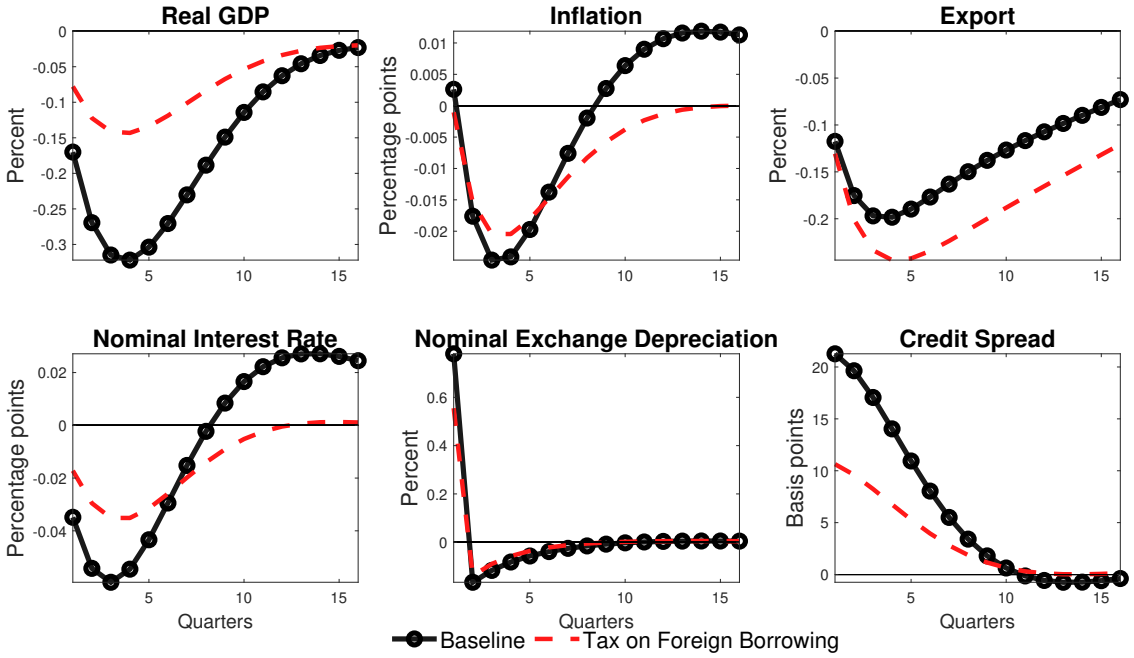
Figure 2.7 shows that a flexible exchange rate nevertheless limits macroeconomic volatility compared to a managed float or a peg for the Home country. The interest rate rule becomes

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\Pi_t^{\phi_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_y} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right)^{\phi_{\mathcal{E}}}\right]^{1-\rho_R},$$

with $\phi_{\mathcal{E}} \geq 0$.

In the extreme case when the nominal exchange rate is fixed (the limit for $\phi_{\mathcal{E}} \to \infty$, represented by the dashed red line in Figure 2.7), the central bank of the Home country must increase the nominal interest rate to track the monetary policy tightening abroad. Interestingly, in the model, the domestic nominal interest rate increases slightly more than one to one with the Foreign rate because of the endogenous wedge in the UIP condition. As a result, domestic financial conditions significantly worsen, with spreads increasing by a factor of six compared to the baseline. Real GDP falls more than 2.5 percentage points, compared to less the 0.5 percentage points under flexible exchange rates. The Home country also experiences a decline of inflation and a much more persistent fall in exports

Figure 2.8: A tax on foreign borrowing.



Note: The figure displays the impulse responses of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in the Home country to a contractionary monetary policy shock in the Foreign country with (dashed red line) and without (solid black line with markers) a tax on foreign borrowing. Inflation, the nominal interest rate, and spreads are in annualized terms.

compared with the baseline. As the Home central bank lowers the weight on the depreciation of the nominal exchange rate (dashed-dotted magenta, yellow and green lines), the response of macroeconomic and financial variables become progressively closer to the flexible exchange rate case ($\phi_{\mathcal{E}} = 0$, represented by the solid black line with markers).

Overall, the results in the model are consistent with the empirical evidence in Obstfeld et al. (2019). While a flexible exchange rate regime does not fully insulate a country from foreign monetary policy shocks, the transmission is not independent of the exchange rate regime, and a peg increases the volatility of both macroeconomic and financial variables.

2.5.2 A Tax on Foreign Borrowing

In addition to questioning the consensus on the advantages of flexible exchange rates, the recent literature on the GFC has also brought to the fore a discussion on the merits of a number of other policy instruments. In particular, Rey (2013) explicitly mentions the idea of actively managing the capital account as a way to curb excessive macroeconomic volatility due to global financial shocks.

In our setting, we can allow the government to actively manage the capital account by introducing a countercyclical tax on borrowing in foreign currency, denoted by τ_t^b. We model the tax as an additional cost (subsidy) that banks pay (receive) at maturity. The net

worth of financial intermediaries in the Home country thus becomes

$$n_t = r_{kt} q_{t-1} z_{t-1} - \frac{R_{t-1} d_{t-1}}{\Pi_t} - (1 + \tau_t^b) \frac{R_{bt-1}^*}{\Pi_t^*} s_t b_{t-1}^*, \tag{2.13}$$

while their balance sheet remains unchanged.

We assume that the government in the Home country sets the tax in response to deviations of the value of credit from its steady state value

$$\tau_t^b = \phi_b \ln \left(\frac{q_t z_t}{qz} \right),$$

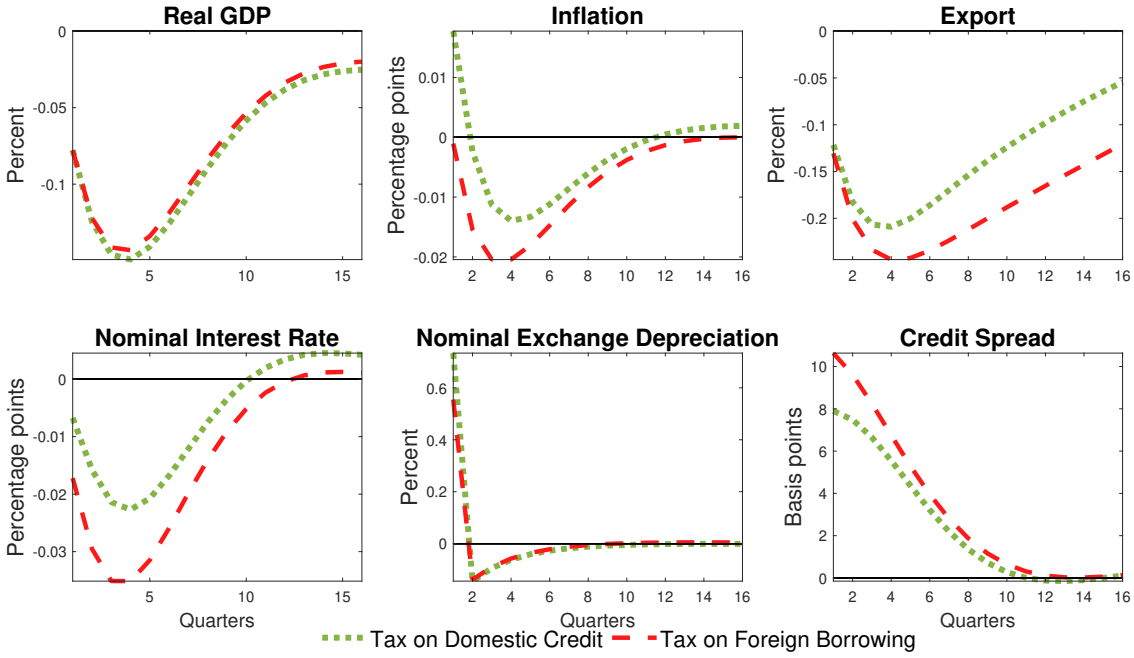
where $\phi_b > 0$.[16] With this formulation, the additional policy instrument directly targets the inefficiencies associated with fluctuations in the value of credit (Borio and Lowe, 2002). The proceedings of the tax are rebated lump-sum to households. The countercyclical nature of the tax reduces the UIP wedge, which now is

$$\mu_{bt} = \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1} \Omega_{t,t+1} \left[\frac{R_{t+1}}{\Pi_{t+1}} - (1 + \tau_{t+1}^b) \frac{R_{bt}^*}{\Pi_{t+1}^*} \frac{s_{t+1}}{s_t} \right] \right\}.$$

Figure 2.8 displays the effects of introducing the countercyclical tax on foreign borrowing by Home banks. The results clearly show that the policy is very effective in reducing the volatility of real GDP in response to the monetary tightening abroad. The main channel is the reduction of credit spreads, which roughly fall by a half relative to the baseline. Given the contractionary foreign monetary policy shock, the active management of the capital account actually requires the government to subsidize Foreign borrowing by reducing its cost. The subsidy narrows the UIP wedge, which in turn limits the leverage ratio of financial intermediaries (2.11) and reduces the cost of funding in foreign currency.

A side effect of a smaller UIP wedge is that the real exchange rate depreciates less than in the baseline case, which has a negative effect on exports. Nevertheless, the expansionary effect on domestic GDP associated with the easier financial conditions more than compensates the negative contribution of exports. The response of inflation and the nominal interest rate is quantitatively similar to the baseline case, albeit somewhat more persistent.

Figure 2.9: Tax on foreign borrowing versus tax on domestic credit.



Note: The figure displays the impulse responses of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in the Home country to a contractionary monetary policy shock in the Foreign country with a tax on foreign borrowing (dashed red line) and with a tax on domestic credit (dotted green line). Inflation, the nominal interest rate, and spreads are in annualized terms.

2.5.3 A Tax on Domestic Credit: A Near-Equivalence Result

Equation (2.13) highlights that a countercyclical tax on foreign borrowing stabilizes the net worth of Home banks. An instrument that has the potential to achieve a similar effect is a countercyclical tax on domestic credit, which we denote by τ_t^k. This tax reduces the effective return on loans that accrues to Home banks so that their balance sheet becomes

$$n_t = (1-\tau_t^k) r_{kt} q_{t-1} z_{t-1} - \frac{R_{t-1} d_{t-1}}{\Pi_t} - \frac{R^*_{bt-1}}{\Pi^*_t} s_t b^*_{t-1}.$$

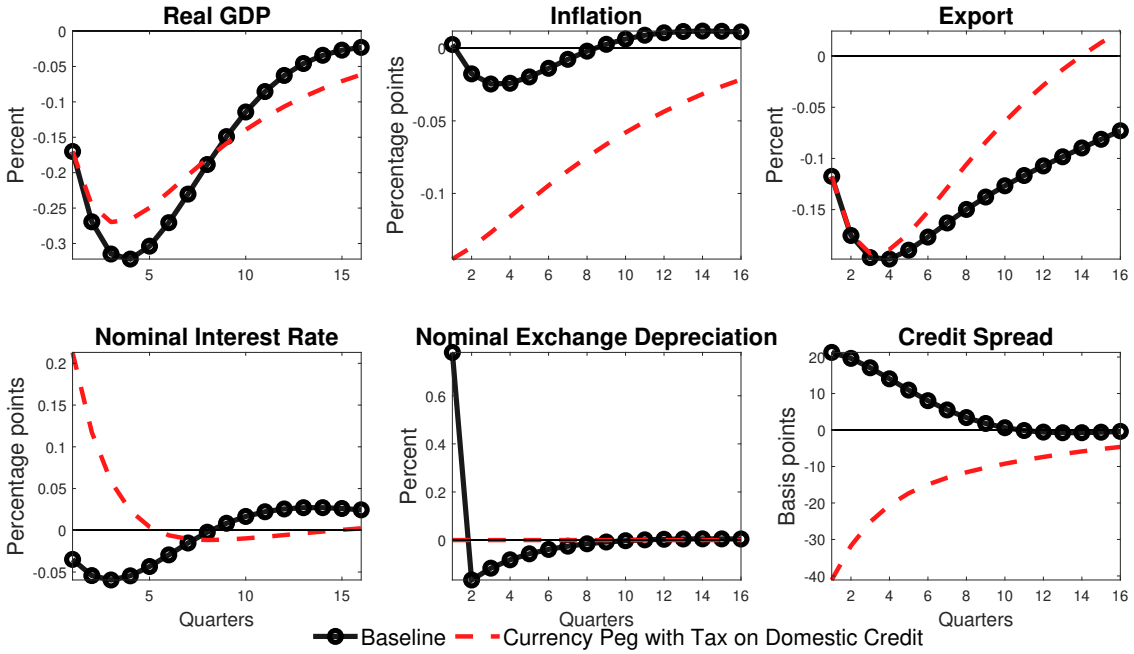
As in the case of a tax on foreign borrowing, we assume that the government sets the tax in response to deviations of the value of credit from steady state (like in rule 2.5.2) and rebates the revenues lump-sum to households.[17] The tax on domestic credit directly affects the credit spread

$$\mu_{kt} = \mathbb{E}_t \left\{ \mathcal{M}_{t,t+1} \Omega_{t,t+1} \left[(1-\tau^k_{t+1}) r_{kt+1} - \frac{R_t}{\Pi_{t+1}} \right] \right\}.$$

[16] For the exercise reported in Figure 2.8, we set ϕ_b so that the impact of the Foreign monetary policy shock on domestic spreads is halved.

[17] We calibrate the feedback coefficient so that the impact response on real GDP is the same as in the case of a tax on foreign borrowing.

Figure 2.10: A tax on domestic credit with a fixed exchange rate.



Note: The figure displays the impulse responses of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in the Home country to a contractionary monetary policy shock in the Foreign country with a tax on domestic credit under a fixed exchange rate regime (dashed red line) and with no taxes under a flexible exchange rate regime (solid black line with markers). Inflation, the nominal interest rate, and spreads are in annualized terms.

Figure 2.9 compares the effects of the tax on domestic credit (dotted green line) with those of the tax on foreign borrowing (dashed red line) discussed in the previous section. The effect on real GDP is almost identical, not only on impact (which happens by construction) but also over time. The tax on domestic credit reduces credit spreads even more than the tax on foreign borrowing. Conversely, the impact on the UIP wedge is smaller. Therefore, the domestic currency depreciates more in real terms and exports fall less, almost in line with the baseline without taxes. The combination of these two effects results in inflation actually rising above target, which requires relatively higher nominal interest rate. The differences in the response of inflation and the nominal interest rate remain nevertheless rather small. Overall, the two taxes have nearly equivalent consequences on macroeconomic volatility and can contribute to stabilize the domestic economy in the face of foreign monetary policy shocks.

2.5.4 A Tax on Domestic Credit with a Peg

The last policy experiment asks if a tax on domestic credit under a peg can approximate the macroeconomic outcomes under flexible exchange rates. We set the feedback coefficient on credit so that the impact effect on real GDP is the same in the two cases.

Figure 2.10 shows that the introduction of a tax on domestic credit reduces the volatility of real GDP under a peg for the first eight quarters. Exports are very close to the case of flexible exchange rates in the first year and then recover faster. The tax (indeed a subsidy in this case) substantially compresses domestic spreads (-40 basis points relative to steady state) and reduces the inefficiencies associated with financial frictions. Yet, as under a peg in the absence of the tax, the Home nominal interest rate continues to follow the Foreign one, which reduces aggregate demand and keeps inflation well below target. As a consequence, even in the presence of an additional instrument, inflation volatility remains higher under a peg than under flexible exchange rates.

2.6 Conclusions

Monetary policy shocks originating in large financial centers give rise to a Global Financial Cycle worldwide. Real GDP and inflation co-move across countries despite a fully flexible exchange rate regime. The expenditure-switching channel is not strong enough to fully offset the headwinds implied by higher spreads. Key to this transmission are the role of global banks in propagating the initial shock and the currency denomination of exports in the receiving country. Imperfect pass-through is crucial in accounting for the muted response of domestic inflation to the depreciation of the exchange rate.

While a flexible exchange rate regime does not insulate countries from foreign financial shocks, a fixed exchange rate increases domestic macroeconomic volatility. Capital flows management tools, in the form of a countercyclical tax on foreign borrowing, represent one policy instrument that can mitigate the transmission of foreign financial shocks. A tax on domestic credit has nearly-equivalent effects. With fixed exchange rates, either instrument can approximate the response of economic activity under flexible exchange rates. However, because the domestic nominal interest rate must still track the foreign interest rate, the combination of fixed exchange rates and a countercyclical tax on credit or foreign borrowing exerts disinflationary pressures that are largely absent under a flexible exchange rate regime.

As the world remains as financially interconnected as ever, the results in this paper are informative for the current monetary policy tightening cycle taking place in advanced economies as well as, more generally, for the design of appropriate policy frameworks that address the international transmission of financial shocks.

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Appendix

2.A Data and Empirics

2.A.1 Data Sources

The larger sample includes 24 countries: Argentina, Australia, Brazil, Canada, Chile, Colombia, Germany, India, Indonesia, Israel, Japan, Korea, Mexico, New Zealand, Norway, Philippines, Poland, Russia, Singapore, South Africa, Sweden, Switzerland, Thailand, United Kingdom. The choice of countries, in this case, depends on data availability:

1. **Exchange rate regime**: Countries need to have a relatively flexible exchange rates (floats and managed floats).
2. **Sample period**: Countries need to have a long enough sample period over which the exchange rate regime is classified as flexible.
3. **Size**: We ignore very small countries.

Data sources:

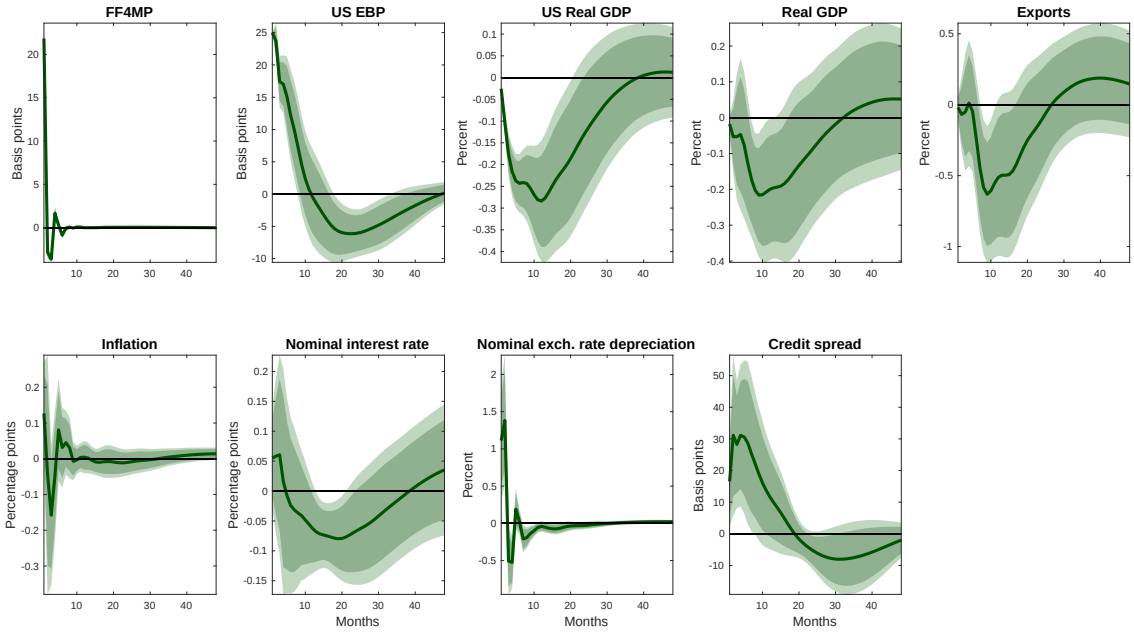
- **GDP** (real index): OECD, IMF IFS, Bloomberg. The quarterly level data are interpolated using a shape-preserving piecewise cubic interpolation (MatLab command: `y1 = interp1(t0,y0,t1,'pchip'))`.
- **Consumer prices** (CPI): OECD, IMF IFS, Bloomberg.
- **Short-term interest rates** (market rates): OECD, IMF, IFS, Bloomberg.
- **Nominal exchange rate** (units of local currency per US dollar, so that an increase corresponds to a depreciation of the local currency): Datastream.
- **Exports** (exports of goods and services, by expenditure in constant prices): OECD. The quarterly level data are interpolated using a shape-preserving piecewise cubic interpolation (MatLab command: `y1 = interp1(t0,y0,t1,'pchip'))`.

- **Credit spreads** (average of the option-adjusted spreads across non-financial firms in a country): ICE Bank of America Merrill Lynch Global Index.

2.A.2 Panel VAR

Figure 2.11 reports the impulse response that we obtain from a Cholesky factorization of the variance-covariance matrix of the estimated reduced-form residuals in a VAR that includes the monetary policy surprises directly in the VAR (ordered first in the vector of endogenous variables) (Plagborg-Møller and Wolf, 2021).

Figure 2.11: Impulse responses to a US monetary policy tightening

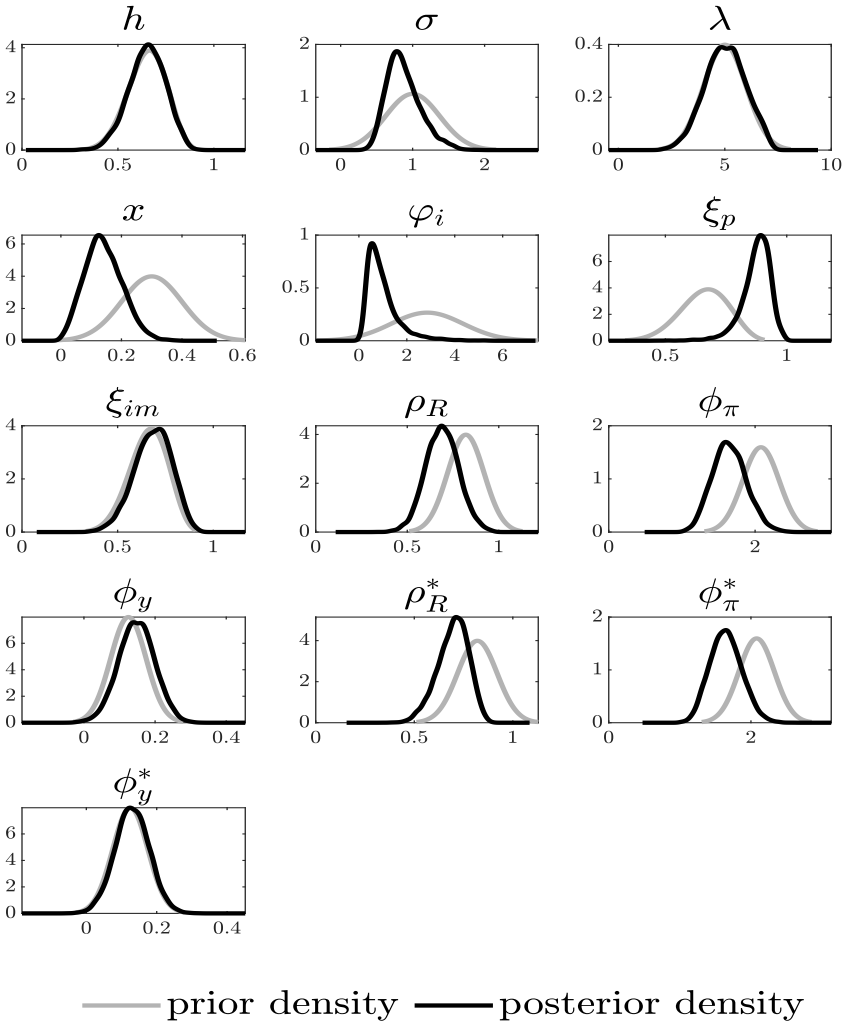


Note: The figure displays the impulse responses of the EBP and real GDP in the US, and of real GDP, CPI inflation, exports, interest rate, nominal exchange rate depreciation and credit spreads in a typical small open economy with a flexible exchange rate regime to a contractionary monetary policy shock in the US. The solid dark green line is the mean group estimate. The light and dark green shaded areas are the 95% and 99% confidence intervals, respectively. Inflation, the nominal interest rate, and spreads are in annualized terms. The shock is normalized to generate a 25 basis points increase in the EBP.

2.A.3 Prior and Posterior Densities

Figure 2.12 reports the prior and posterior densities of the estimation exercise.

Figure 2.12: Prior and posterior densities for impulse response matching exercise



Note: The figure displays the prior (light grey) and posterior (black) densities for the impulse response matching exercise for the habit parameter (h), the coefficient of relative risk aversion (σ), the leverage ratio (λ), the ratio of foreign-currency liabilities (x), the investment adjustment cost parameter (φ_i), the domestic price stickiness parameter (ξ_p), th import price stickiness parameter (ξ_{im}), and the parameters in the monetary policy rules of both countries (ρ_R, ϕ_π, ϕ_y, and their Foreign counterparts).

2.B Model Derivations

In this section, we report the first-order conditions for households, financial intermediaries, and firms, and characterize the steady state of the model.

2.B.1 Households

The first-order conditions for the Home household are

$$\begin{aligned}\Lambda_t &= \frac{1}{c_t - h\bar{c}_{t-1}} \\ \Lambda_t &= \beta \mathbb{E}_t \left(\Lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \right).\end{aligned}$$

The optimal wage-setting condition is

$$\widetilde{w}_t^{1+\nu\zeta} = \frac{\chi\nu}{\nu - 1} \frac{F_{1t}^W}{F_{2t}^W}$$

where $\widetilde{w}_t$ is the optimal real wage, and

$$\begin{aligned}F_{1t}^W &= \ell_t^{1+\zeta} w_t^{\nu\zeta} + \beta\xi_w \mathbb{E}_t \left[\left(\frac{w_{t+1}}{w_t} \right)^{\nu} \Pi_{t+1}^{\nu(1+\zeta)} F_{1t+1}^W \right] \\ F_{2t}^W &= \Lambda_t \ell_t + \beta\xi_w \mathbb{E}_t \left[\left(\frac{w_{t+1}}{w_t} \right)^{\nu} \Pi_{t+1}^{\nu-1} F_{2t+1}^W \right].\end{aligned}$$

The aggregate nominal wage level evolves according to

$$W_t = [(1 - \xi_w)\widetilde{W}_t^{1-\nu} + \xi_w W_{t-1}^{1-\nu}]^{\frac{1}{1-\nu}},$$

which we can rewrite in real terms as

$$w_t = \left[(1 - \xi_w)\widetilde{w}_t^{1-\nu} + \xi_w \left(\frac{w_{t-1}}{\Pi_t} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}}.$$

The first-order conditions for the Foreign representative household are symmetric.

2.B.2 Financial Intermediation

We define the leverage ratio of a typical bank in the Foreign country as $\lambda_t^* = (q_t^* z_t^* + b_t^*)/n_t^*$, where $b_t^* = B_t^*/P_t^*$ is the amount of lending to Home banks in real terms. The bank chooses an optimal leverage ratio to maximize the expected value of terminal wealth $V(n_t^*)$, subject to the balance sheet constraint and the incentive compatibility constraint. The fact that the bank can arbitrage between lending to firms and lending to Foreign banks implies the first order condition

$$\mathbb{E}_t(\mathcal{M}_{t,t+1}^* \Omega_{t,t+1}^* r_{kt+1}^*) = \mathbb{E}_t \left(\mathcal{M}_{t,t+1}^* \Omega_{t,t+1}^* \frac{R_{bt}^*}{\Pi_{t+1}^*} \right),$$

where

$$\Omega_{t,t+1}^* = \mathbb{E}_t(1 - \omega + \omega \kappa_{t+1}^*)$$

and

$$\kappa_t^* = \frac{V(n_t^*)}{n_t^*}.$$

In an equilibrium in which the incentive compatibility constraint binds, we have

$$\kappa_t^* = \theta^* \lambda_t^*,$$

which implies

$$\lambda_t^* = \frac{\mu_{dt}^*}{\theta^* - \mu_{kt}^*}$$

where

$$\mu_{kt}^* = \mathbb{E}_t \left[\mathcal{M}_{t,t+1}^* \Omega_{t,t+1}^* \left(r_{kt+1}^* - \frac{R_t^*}{\Pi_{t+1}^*} \right) \right]$$

and

$$\mu_{dt}^* = \mathbb{E}_t \left(\mathcal{M}_{t,t+1}^* \Omega_{t,t+1}^* \frac{R_t^*}{\Pi_{t+1}^*} \right).$$

The aggregate balance sheet of the banking sector is

$$q_t^* z_t^* + b_t^* = d_t^* + n_t^*.$$

The aggregate banking sector net worth evolves such that

$$n_t^* = (\omega + \xi_b^*) \left(r_{kt}^* q_{t-1}^* z_{t-1}^* + \frac{R_{bt-1}^* b_{t-1}^*}{\Pi_t^*} \right) - \omega \frac{R_{t-1}^* d_{t-1}^*}{\Pi_t^*}$$

where ξ_b^* is the proportion of total assets that the household transfers to new bankers.

The optimality conditions for Home banks are similar. The leverage ratio is $\lambda_t = q_t z_t / n_t$ and is determined by

$$\kappa_t = \underbrace{\theta\left(1 + \frac{\gamma}{2}x_t^2\right)}_{\equiv\Theta(x_t)}\lambda_t,$$

where $x_t = s_t b_t^* / (q_t z_t)$, which implies

$$\lambda_t = \frac{\mu_{dt}}{\Theta(x_t) - (\mu_{kt} + \mu_{bt}x_t)}$$

where

$$\begin{aligned}
\mu_{bt} &= \mathbb{E}_t\left[\mathcal{M}_{t,t+1}\Omega_{t,t+1}\left(\frac{R_t}{\Pi_{t+1}} - \frac{R_{bt}^*}{\Pi_{t+1}^*}\frac{s_{t+1}}{s_t}\right)\right] \\
\mu_{kt} &= \mathbb{E}_t\left[\mathcal{M}_{t,t+1}\Omega_{t,t+1}\left(r_{kt+1} - \frac{R_t}{\Pi_{t+1}}\right)\right] \\
\mu_{dt} &= \mathbb{E}_t\left(\mathcal{M}_{t,t+1}\Omega_{t,t+1}\frac{R_t}{\Pi_{t+1}}\right) \\
\Omega_{t,t+1} &= \mathbb{E}_t(1 - \omega + \omega\kappa_{t+1}) \\
\kappa_t &= \frac{V(n_t)}{n_t}.
\end{aligned}$$

The optimal proportion of foreign currency debt is given by

$$x_t = \frac{\sqrt{1 + \frac{2\mu_t^2}{\gamma}} - 1}{\mu_t},$$

where $\mu_t = \mu_{bt}/\mu_{kt}$.

The aggregate balance sheet is

$$q_t z_t = d_t + s_t b_t^* + n_t,$$

while net worth evolves according to

$$n_t = (\omega + \xi_b) r_{kt} q_{t-1} z_{t-1} - \omega\left(\frac{R_{t-1}d_{t-1}}{\Pi_t} + \frac{s_t R_{bt-1}^* b_{t-1}^*}{\Pi_t^*}\right).$$

2.B.3 Intermediate Goods Producers

The balance sheet of intermediate goods producer in the Home country is

$$q_t z_t = q_t k_t.$$

Profit maximization yields the first-order conditions with respect to ℓ_t and k_{t-1}, which are

$$w_t = (1-\alpha) p_{mt} k_{t-1}^{\alpha} \ell_t^{-\alpha}$$

and

$$r_{kt} = \frac{\alpha p_{mt} k_{t-1}^{\alpha-1} \ell_t^{1-\alpha} + (1-\delta) q_t}{q_{t-1}}.$$

The zero-profit condition implies that the price for intermediate goods is

$$p_{mt} = \frac{[r_{kt} q_{t-1} - (1-\delta) q_t]^{\alpha} w_t^{1-\alpha}}{A_t \alpha^{\alpha} (1-\alpha)^{1-\alpha}},$$

where $a_t \equiv \ln(A_t/A)$ follows an exogenous first-order autoregressive process

$$a_t = \rho_a a_{t-1} + e_{at},$$

with $\rho_a \in [0,1]$ and $e_{at} \sim \mathcal{N}(0, \sigma_a^2)$. The problem for Foreign intermediate goods producers and the resulting first-order conditions are symmetric.

2.B.4 Capital Producers

The first-order condition for capital producers in the Home country is

$$q_t = 1 + \frac{\varphi_i}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 + \varphi_i \left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}} - \varphi_i \mathbb{E}_t\left[\mathcal{M}_{t,t+1}\left(\frac{i_{t+1}}{i_t} - 1\right)\frac{i_{t+1}^2}{i_t^2}\right],$$

and the capital accumulatation equation is

$$k_t = i_t + (1-\delta) k_{t-1}.$$

The problem for capital producers in the Foreign country yields similar first-order conditions.

2.B.5 Final Goods Producers

Home final goods producers that serve the domestic market and can reset their price at time t choose

$$\frac{\widetilde{P}_t(h)}{P_{Ht}} = \frac{\varrho}{\varrho - 1}\frac{\mathcal{H}_{1t}}{\mathcal{H}_{2t}}$$

where

$$\mathcal{H}_{1t} = \Lambda_t p_{mt} y_{Ht} + \beta\xi_H \mathbb{E}_t(\Pi_{Ht+1}^{\varrho}\mathcal{H}_{1t+1})$$

$$\mathcal{H}_{2t} = \Lambda_t p_{Ht} y_{Ht} + \beta\xi_H \mathbb{E}_t(\Pi_{Ht+1}^{\varrho-1}\mathcal{H}_{2t+1}),$$

with $p_{Ht} \equiv P_{Ht}/P_t$ and

$$y_{Ht} = a p_{Ht}^{-\epsilon}\left[c_t + i_t + \frac{\varphi_i}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t\right].$$

The link between the optimal reset price and inflation is

$$\xi_H \Pi_{Ht}^{\varrho-1} + (1-\xi_H)\left[\frac{\widetilde{P}_t(h)}{P_{Ht}}\right]^{1-\varrho} = 1.$$

A similar set of conditions applies to all Foreign final goods producers.

Home final goods producers that serve the Foreign market and can reset their price at time t choose

$$\frac{S_t\widetilde{P}_t^*(h)}{P_{Ht}^*} = \frac{\varrho}{\varrho - 1}\frac{\mathcal{F}_{1t}}{\mathcal{F}_{2t}},$$

where

$$\mathcal{F}_{1t} = \left(\frac{1-n}{n}\right)\Lambda_t p_{mt} y_{Ht}^* + \beta\xi_H \mathbb{E}_t[(\Pi_{Ht+1}^*)^{\varrho}\mathcal{F}_{1t+1}]$$

$$\mathcal{F}_{2t} = \left(\frac{1-n}{n}\right)\Lambda_t p_{Ht}^* y_{Ht}^* + \beta\xi_H \mathbb{E}_t\left[\frac{S_{t+1}}{S_t}(\Pi_{Ht+1}^*)^{\varrho-1}\mathcal{F}_{2t+1}\right],$$

with

$$y_{Ht}^* = a^*(p_{Ht}^*)^{-\epsilon}\left[c_t^* + i_t^* + \frac{\varphi_i}{2}\left(\frac{i_t^*}{i_{t-1}^*} - 1\right)^2 i_t^*\right].$$

In this case, the link between the optimal reset price and inflation is

$$\xi_H(\Pi^*_{Ht})^{\varrho-1} + (1-\xi_H)\left[\frac{\widetilde{P}^*_t(h)}{P^*_{Ht}}\right]^{1-\varrho} = 1.$$

2.B.5.1 Importers

Importers who adjust their price at time t set

$$\frac{\widetilde{P}_t(f)}{P_{Ft}} = \frac{\varrho}{\varrho-1}\frac{\mathcal{I}_{1t}}{\mathcal{I}_{2t}},$$

where

$$\mathcal{I}_{1t} = \left(\frac{n}{1-n}\right)\Lambda_t p^{im}_t y_{Ft} + \beta\xi\mathbb{E}_t(\Pi^{\varrho}_{Ft+1}\mathcal{I}_{1t+1})$$
$$\mathcal{I}_{2Ft} = \left(\frac{n}{1-n}\right)\Lambda_t p_{Ft} y_{Ft} + \beta\xi\mathbb{E}_t(\Pi^{\varrho-1}_{Ft+1}\mathcal{I}_{2t+1}),$$

with

$$y_{Ft} = (1-a)p^{-\epsilon}_{Ft}\left[c_t + i_t + \frac{\varphi_i}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 i_t\right].$$

The link between the optimal reset price of Foreign goods and inflation is

$$\xi_F\Pi^{\varrho-1}_{Ft} + (1-\xi_F)\left[\frac{\widetilde{P}_t(f)}{P_{Ft}}\right]^{1-\varrho} = 1.$$

2.B.6 Relative Prices and Inflation

From the definition of the CPI index (2.2), we can link the relative price of Home goods p_{Ht} to the terms of trade according to

$$p^{\epsilon-1}_{Ht} = a + (1-a)\mathcal{T}_t^{1-\epsilon},$$

where $\mathcal{T}_t = P_{Ft}/P_{Ht}$.

Similarly, for the relative price of Foreign goods, we have

$$(p^*_{Ft})^{\epsilon-1} = a^*\left(\frac{\psi_t}{\psi^*_t\mathcal{T}_t}\right)^{1-\epsilon} + (1-a^*),$$

where $\psi_t = \mathcal{E}_t P^*_{Ht}/P_{Ht}$ and $\psi^*_t = \mathcal{E}_t P^*_{Ft}/P_{Ft}$ are the law-of-one-price gaps for Home and Foreign goods, respectively (the measure of price pass-through for tradable goods).

The link between the relative prices of the two goods in the Home country is

$$a p_{Ht}^{1-\epsilon} + (1-a) p_{Ft}^{1-\epsilon} = 1,$$

while the link between CPI inflation and domestic inflation is

$$\frac{\Pi_{Ht}}{\Pi_t} = \frac{p_{Ht}}{p_{Ht-1}}.$$

The last two equations have identical counterparts for the Foreign country.

Finally, the link between the terms of trade $\mathcal{T}_t$ and the real exchange rate is

$$S_t^{1-\epsilon} = \frac{a^* \psi_t^{1-\epsilon} + (1-a^*)(\psi_t^* \mathcal{T}_t)^{1-\epsilon}}{a + (1-a)\mathcal{T}_t^{1-\epsilon}}.$$

2.B.7 Monetary Policy

The central bank in the Home country follows the interest rate rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\Pi_t^{\phi_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_y} \left(\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}\right)^{\phi_\mathcal{E}}\right]^{1-\rho_R}.$$

In the baseline analysis, we set $\phi_\mathcal{E} = 0$, which instead we allow to be positive in some of the counterfactual experiments.

The interest rate rule in the Foreign country is

$$\frac{R^*_t}{R^*} = \left(\frac{R^*_{t-1}}{R^*}\right)^{\rho_R} \left[(\Pi^*_t)^{\phi_\pi} \left(\frac{y^*_t}{y^*_{t-1}}\right)^{\phi_y}\right]^{1-\rho_R} e^{\epsilon^*_{Rt}},$$

where $\epsilon^*_{Rt} \sim \mathcal{N}(0, \sigma^2_{R^*})$ is a monetary policy shock.

2.B.8 Market Clearing

In the Home country, the market clearing condition for intermediate goods requires

$$y_t = \Delta_{Ht} y_{Ht} + \left(\frac{1-n}{n}\right) \Delta^*_{Ht} y^*_{Ht},$$

where Δ_{Ht} and Δ^*_{Ht} are indexes of price dispersion defined as

$$\Delta_{Ht} \equiv \int_0^n \left[\frac{\widetilde{P}_t(h)}{P_{Ht}}\right]^{-\varrho} dh \qquad\qquad \Delta^*_{Ht} \equiv \int_n^1 \left[\frac{\widetilde{P}^*_t(h)}{P^*_{Ht}}\right]^{-\varrho} dh.$$

From the expression of the domestic price index, we can derive the law of motion of the two price dispersion variables as

$$\Delta_{Ht} = \xi_H \Pi^{\varrho}_{Ht} \Delta_{Ht-1} + (1-\xi_H)\left[\frac{\widetilde{P}_t(h)}{P_{Ht}}\right]^{-\varrho}$$

and

$$\Delta^*_{Ht} = \xi_H (\Pi^*_{Ht})^{\varrho} \Delta^*_{Ht-1} + (1-\xi_H)\left[\frac{\widetilde{P}^*_t(h)}{P^*_{Ht}}\right]^{-\varrho}.$$

The market clearing condition for the Foreign country is

$$y^*_t = \Delta^*_{Ft}\left(y^*_{Ft} + \frac{n}{1-n} y_{Ft}\right)$$

where

$$\Delta^*_{Ft} = \int_n^1 \left[\frac{\widetilde{P}^*_t(f)}{P^*_{Ft}}\right]^{-\varrho} df,$$

and the index of price dispersion evolves according to

$$\Delta^*_{Ft} = \xi^*_F (\Pi^*_{Ft})^{\varrho} \Delta^*_{Ft-1} + (1-\xi^*_F)\left[\frac{\widetilde{P}^*_t(f)}{P^*_{Ft}}\right]^{-\varrho}.$$

Lastly, the law of motion of net foreign debt for the Home country is

$$b^*_t = \frac{R^*_{bt-1} b^*_{t-1}}{\Pi^*_t} + p^*_{Ft} y_{Ft} - \left(\frac{1-n}{n}\right) p^*_{Ht} y^*_{Ht}.$$

2.C Steady State

We approximate the model around a non-stochastic steady state with zero inflation in both countries and relative prices normalized to one. As a consequence, steady state price dispersion is also equal to one.

In each country, the steady state real interest rate is equal to the inverse of the individ-

ual discount factor. We choose χ so that steady state hours ℓ are equal to one-third. Zero steady state investment adjustment costs imply that the relative price of capital q is equal to one. Moreover, since in steady state all firms can adjust their price, the relative price of intermediate goods is equal to the inverse of the steady state markup ($p_m = (\varrho - 1)/\varrho$). Given this value, the first order conditions for intermediate goods producers pin down w, r_k and k. In turn, from the production function, we obtain y, and from the law of motion of capital we can derive i.

In equilibrium, the quantity of securities that banks hold z corresponds to the capital stock k. Given the leverage ratio λ and the ratio of foreign currency liabilities x, banks' net worth of banks is $n = k/\lambda$ and the stock of foreign liabilities is $b^* = xk$.[18] The aggregate balance sheet of banks residually determines deposits ($d = (1 - 1/\lambda - x)k$). With these expressions, we can derive $V(n)$ and κ, which, together with the returns on deposits and capital, pin down μ_d, μ_k, and μ_b.

Finally, the demand equations and the market clearing conditions determine the quantities c_h, c_h^*, c_f, c_f^*, y_h, y_h^*, y_f, y_f^*, c and c^*.

[18] Our choice to directly estimating λ and x corresponds to choosing values for the parameters γ and θ consistent with the moral hazard constraint at equality and the first order condition for the optimal portfolio choice.

Chapter 3

Testing the effectiveness of unconventional monetary policy in Japan and the United States

Abstract Unconventional monetary policy (UMP) may make the effective lower bound (ELB) on the short-term interest rate irrelevant. We develop a theoretical model that underpins our empirical test of this 'irrelevance hypothesis,' based on the simple idea that under the hypothesis, the short rate can be excluded in any empirical model that accounts for alternative measures of monetary policy. We test the hypothesis for Japan and the United States using a structural vector autoregressive model with the ELB. We firmly reject the hypothesis but find that UMP has had strong delayed effects.

3.1 Introduction

Adjustments in the overnight nominal interest rate have been the primary tool for the implementation of monetary policy since the early 1980s. In recent years, however, the short-term nominal interest rate reached an effective lower bound (ELB) in several countries, making the standard policy tool *de facto* ineffective. Two prominent examples are Japan whose policy rate has been at the ELB for most of the past quarter century, and the United States that reached the ELB in the aftermath of the global financial crisis of 2007–2008. The central banks in these countries countervailed the inapplicability of the standard policy tool by embarking on unconventional monetary policy (UMP) that involves the purchases of long-term government bonds and the use of forward guidance to signal future policy action.[1]

The effectiveness of UMP is a central issue for policymakers. One view is that the ELB restricts the effectiveness of monetary policy, thus representing an important constraint on what monetary policy can achieve, as argued by Eggertsson and Woodford (2003), Gust et al. (2017) and Eberly et al. (2020). An alternative view is that UMP can affect long-term interest rates so significantly that UMP has been fully effective in circumventing the ELB constraint, as argued by Swanson and Williams (2014) and Debortoli et al. (2019). This latter view has been termed as the ELB 'irrelevance hypothesis.' The issue of the effectiveness of UMP has gained relevance since the ELB is likely to bind more often in the future with historically low levels of the longer-run natural rate of interest (Duarte et al., 2020).

This paper studies the irrelevance hypothesis of the ELB both theoretically and empirically. It develops a dynamic stochastic general equilibrium (DSGE) model with UMP. To the best of our knowledge, this is the first study that analytically characterizes the irrelevance hypothesis in a DSGE model. The model provides the theoretical underpinning to our novel empirical tests, and it shows that censored and kinked vector autoregressive models (VARs) are a suitable empirical framework to test the irrelevance hypothesis. Our empirical results show that the hypothesis is strongly rejected for both Japan and the U.S. Despite the rejection, the estimated impulse responses to a monetary policy shock indicate strong delayed effects of UMP in each country.

The idea of the irrelevance hypothesis is that observable properties of macroeconomic variables such as their dynamics and volatilities remain unchanged when the economy moves in and out of an ELB regime, for otherwise the ELB is empirically relevant. This idea can be applied to any theoretical model with the ELB. This insight allows us to test

[1] See Christensen and Rudebusch (2012), Liu et al. (2019), Campbell et al. (2020), and Carlson et al. (2020) for the U.S., and Ugai (2007), and Bank of Japan (2016) for Japan. Ueda (2012) provides a comparison of monetary policy between the U.S. and Japan.

the irrelevance hypothesis using reduced-form VAR models and therefore, our empirical results about this hypothesis hold for any theoretical model with the ELB as long as it has a VAR representation. The purpose of our DSGE model is to formalize this idea by showing that such a theoretical model actually exists, and to guide our empirical approach to identifying the effectiveness of UMP when the irrelevance hypothesis does not hold.

In our model, UMP consists of (i) quantitative easing (QE) implemented by long-term government bond purchases that directly affect long-term government bond yields, and (ii) forward guidance (FG) under which the central bank commits to keeping short-term interest rates low in the future. A key variable in the model is the 'shadow rate,' which we label i_t^* and it is defined as the short-term interest rate that the central bank would set if there were no ELB. The short rate i_t is given by $i_t = \max\{i_t^*, \underline{i}\}$, where $\underline{i}$ is the ELB. The shadow rate is equal to the short rate in a non-ELB regime, but it is unobservable in an ELB regime where the policy rate is constrained at the ELB. The shadow rate can be negative and interpreted as the indicator of the desired stance of monetary policy in terms of the short rate.

In the model the central bank resorts to UMP in an ELB regime by using the shadow rate for the guidance of its policy stance, as in the case of a non-ELB regime where the short rate is equal to the shadow rate. The model shows that UMP entails wide degrees of effectiveness, including the irrelevance of the ELB in which UMP retains the same effectiveness as the conventional policy that adjusts the short rate as if there were no ELB. We show that under the irrelevance of the ELB, the log-linearized DSGE model can be written in terms of inflation, the output-gap, the long-term interest rate or the shadow rate, and it retains the same VAR representation for both ELB and non-ELB regimes, thus providing the theoretical foundation to our empirical tests.

Motivated by our theoretical results, we use the censored and kinked structural VAR model developed by Mavroeidis (2021) to test the irrelevance hypothesis. Our DSGE model shows that a direct appraisal of the irrelevance hypothesis is to test whether a short-term interest rate, which is subject to the ELB, can be excluded from the VARs that include alternative measures of monetary policy that are not subject to the ELB, such as a long-term interest rate and the shadow rate. The exclusion of the short rate in VARs that include the long rate is a novel empirical test developed in this paper. The exclusion of the short rate in VARs that include the shadow rate, which is proposed by Mavroeidis (2021), is our second test for robustness.

Our theoretical model implies that under the irrelevance hypothesis there is no attenuation in the response of the long rate to shocks when short rates are at the ELB, a focal point for the assessment of the irrelevance hypothesis in Swanson and Williams (2014) and Gilchrist et al. (2015). We study the attenuation effect by focusing on the different impact of the monetary policy shock on the long rate in the non-ELB and ELB regimes.

Our monetary policy shock is a traditional monetary policy shock – a shock to the short rate – in a non-ELB regime while in an ELB regime it is a UMP shock – a shock to the shadow rate. Motivated by our theory, we use the VAR model to characterize analytically a formal test for the attenuation effect in the response of the long rate to a shock to monetary policy at the ELB.

We conduct the tests of the irrelevance hypothesis of the ELB on postwar data for Japan and the U.S. We consider several different VAR specifications, varying the lag order and the estimation sample (to account for structural change), and using alternative measures of monetary policy such as government bond yields with different maturities. In all cases, the tests overwhelmingly and consistently reject the hypothesis that the ELB has been empirically irrelevant for both economies. Our conclusion is therefore fairly robust: the ELB does represent a constraint on what monetary policy can achieve in those economies.[2] We also firmly reject the hypothesis of no attenuation in the response of the yield curve to monetary policy shocks during ELB regimes in both economies.

The rejection of the irrelevance hypothesis leaves open the question of the degree of effectiveness of UMP in an ELB regime compared to the conventional policy in a non-ELB regime. To address this question, we identify the dynamic effects of conventional and unconventional policies by combining the identifying power of the ELB with additional sign restrictions on impulse responses to a monetary policy shock à la Uhlig (2005). The ELB enables partial identification of impulse responses to a monetary policy shock, as shown in Mavroeidis (2021), because a change in the behaviour of the economy across ELB and non-ELB regimes is informative about the relative impact of conventional and unconventional policy. The identified set based only on the ELB turns out to be fairly wide, so we use the insights from our DSGE model to impose the theoretically-congruous sign restrictions that were used in Debortoli et al. (2019). The sign restrictions markedly sharpen the identified set of impulse responses.

We find that the effects of monetary policy on inflation and output on impact (i.e., within one quarter) declined when the economy entered an ELB regime: they dropped by more than 15 percent in the U.S., and more than 50 percent in Japan, relative to conventional policy. However, the cumulative effects of monetary policy exhibited the opposite pattern one and two years ahead: they appear to have been stronger during an ELB regime relative to a non-ELB regime, except for the response of output gap in the U.S., which remained weaker. Therefore, UMP seems to have had a delayed but stronger effect than conventional policy on inflation in the U.S., and on both inflation and output in Japan. Thus, we conclusively reject the hypothesis that the ELB has been empirically irrelevant

[2]This evidence corroborates Bernanke (2020) who claims that "it also seems unlikely that the new tools deployed during the Great Recession entirely compensated for the limits imposed by the lower bound,"and is consistent with the findings in Gust et al. (2017) and Del Negro et al. (2017), who attribute an important role to the ELB for the decline in output during the financial crisis.

in both countries, and find that responses of inflation and output to UMP have been different across time and across countries.

Literature Review Our analysis is closely related to two strands of research. The first pertains to theoretical studies that investigate the transmission mechanism of unconventional monetary policy. Among those, regarding QE, our theoretical model is close in spirit to Andrés et al. (2004), Chen et al. (2012), Harrison (2012), Gertler and Karadi (2013), Liu et al. (2019), and Sudo and Tanaka (2021). These studies introduce assets with different maturities and limit arbitrage across assets to break the irrelevance of QE that is shown by Eggertsson and Woodford (2003).[3] Regarding FG, our model follows Reifschneider and Williams (2000), and it considers this mechanism in a general equilibrium model that directly accounts for QE. Our main contribution to this first strand of literature is to develop a simple model of UMP, which incorporates the shadow rate and provides the theoretical underpinnings to our empirical analyses.

The second strand of literature pertains to empirical studies that assess the effectiveness of unconventional policy. In addition to Swanson and Williams (2014) who estimate the time-varying sensitivity of longer-maturity yields to macroeconomic news using high-frequency data, it includes Debortoli et al. (2019) who use a SVAR to investigate the (ir)relevance of the ELB constraint by comparing impulse responses to shocks between normal times and ELB episodes. Differing from our SVAR, their SVAR does not include short-term interest rates. Another related study by Inoue and Rossi (2019) uses an SVAR with shocks to the entire yield curve and finds evidence that UMP has been effective in the U.S. Our empirical methodology is closely related to Hayashi and Koeda's (2019), who propose an SVAR model for Japan that includes short rates and takes into account the ELB, and our empirical model for Japan relies heavily on the insights from their empirical analysis. The main difference of our methodology from Hayashi and Koeda's (2019) is that we use a shadow rate to model UMP, which nests QE as long-term government bond purchases and FG as a policy rule as in Reifschneider and Williams (2000), while Hayashi and Koeda (2019) use excess reserves to model QE and an inflation exit condition to model FG. Our methodology provides a simpler framework to test the irrelevance hypothesis of the ELB and to compare the effectiveness of UMP relative to conventional policy.

Finally, our empirical analysis uses the estimation methodology in Mavroeidis (2021), who also reports evidence against the irrelevance hypothesis for the U.S. using a three-equation VAR model. We have several differences from that study: we develop a novel theoretical DSGE model of UMP that provides the underpinnings for a new test of the

[3] For other possible channels of QE, see Krishnamurthy and Vissing-Jorgensen (2011). See also Sims and Wu (2021) for a recent discussion on the theoretical frameworks to study UMP.

irrelevance hypothesis based on the exclusion of short rates; we characterize analytically and obtain a new formal test of no attenuation of the effect of monetary policy on long rates at the ELB; we use sign restrictions motivated from our theoretical model to sharpen the identification of impulse responses; we estimate the dynamic effects of UMP and the shadow rates in each country; we study Japanese data and conduct several robustness checks.

The structure of the paper is as follows. Section 3.2 develops a simple New Keynesian DSGE model with UMP that provides theoretical underpinnings to our empirical model and the tests of the irrelevance hypothesis of the ELB. Section 3.3 introduces the econometric methodology and presents the tests of the irrelevance hypothesis from the reduced-form solution of the SVAR. Section 3.4 describes the data and reports our empirical results. Section 3.5 studies the effectiveness of UMP and its differences relative to conventional monetary policy. Section 3.6 concludes. An Appendix provides supporting material on the derivation of the DSGE model, additional empirical results, and the estimates of the shadow rates for Japan and the U.S.

3.2 A theoretical model of UMP

In this section, we develop a simple theoretical model of UMP and provide theoretical underpinnings to our empirical specifications and testing approaches to the irrelevance hypothesis. Section 3.2.1 presents the model with a focus on UMP. Section 3.2.2 studies the linear and non-linear VAR representations of the model that underpin our empirical analysis. Section 3.2.3 simulates the model and illustrates how UMP can make the ELB irrelevant. The details of the model, equation derivations, parameterization, and model simulations are reported in the Appendix 3.A.

3.2.1 Central equations

Overview The model is a New Keynesian model in which QE and FG are active under the ELB. The economy consists of households, firms, and a central bank. The firm sector is standard as in a typical New Keynesian model. The household sector comprises two types of households. Constrained households purchase long-term government bonds only, but unconstrained households can trade both short- and long-term government bonds subject to a trading cost. The trading cost captures bond market segmentation, as in the preferred habitat theory originally proposed by Modigliani and Sutch (1966), and it introduces imperfect substitutability between long- and short-term government bonds that

generates a spread between the yields of these bonds.[4] The trading cost depends on the amount of long-term government bonds circulated in the market. By purchasing long-term government bonds, the central bank can affect the spread and thereby the long-term yield.

Conventional monetary policy The central bank sets the short-term nominal interest rate i_t using a standard Taylor rule subject to the ELB. Let $\hat{y}_t$, $\hat{\pi}_t$, and $\hat{i}_t$ denote the deviation of output, inflation, and the short-term interest rate from the steady state in period t. Following conventional notation, the caret on a variable denotes the deviation of the variable from steady state. The short-term interest rate is set according to

$$\hat{i}_t = \max\{\hat{i}_t^*, \underline{\hat{i}}\}, \tag{3.1}$$

$$\hat{i}_t^* = -\alpha \hat{i}_t + (1+\alpha)\hat{i}_t^{\text{Taylor}}, \tag{3.2}$$

$$\hat{i}_t^{\text{Taylor}} = \rho_i \left((1-\lambda^*)\hat{i}_{t-1} + \lambda^* \hat{i}_{t-1}^* \right) + (1-\rho_i)\left(r_\pi \hat{\pi}_t + r_y \hat{y}_t\right) + \epsilon_t^i, \tag{3.3}$$

where $\alpha \geq 0$, $\lambda^* \geq 0$, $\rho_i \geq 0$, $r_\pi \geq 0$, $r_y \geq 0$, and ϵ_t^i is a monetary policy shock. Equation (3.1) encapsulates the ELB constraint, where $\underline{\hat{i}}$ is the ELB and $\hat{i}_t^*$ is the shadow rate.[5]

We use the term 'shadow rate' since $\hat{i}_t^*$ is unobserved under the ELB and therefore censored at the ELB, while it is observed and equal to $\hat{i}_t$ outside the ELB constraint. Our shadow rate represents the *desired* stance of monetary policy for the short-term interest rate, as opposed to the *effective* policy stance, e.g., in Wu and Xia (2016).

A monetary policy shock in our model is a shock to the shadow rate, which is identical to a shock to the short-term interest rate in the non-ELB regime. Equations (3.2) and (3.3) allow for FG to influence the system in the ELB regime, as we discuss below. In the non-ELB regime when $\hat{i}_t^*, \hat{i}_{t-1}^* \geq \underline{\hat{i}}$, equations (3.1)-(3.3) reduce to $\hat{i}_t = \hat{i}_t^* = \hat{i}_t^{\text{Taylor}}$ with the Taylor-rule rate $\hat{i}_t^{\text{Taylor}}$ being equal to

$$\hat{i}_t^{\text{Taylor}} = \rho_i \hat{i}_{t-1} + (1-\rho_i)\left(r_\pi \hat{\pi}_t + r_y \hat{y}_t\right) + \epsilon_t^i. \tag{3.4}$$

This equation is a standard Taylor rule that sets the current interest rate in response to the interest rate in period $t-1$, and current inflation and output.

[4] The preferred habitat model is the predominant modelling framework to study UMP. See among others Chen et al. (2012), Liu et al. (2019), and Sims and Wu (2021).

[5] For the interest rate, the deviation from steady state is expressed in terms of the gross interest rate. That is, $\hat{i}_t = (i_t - i)/(1+i)$, where i is the short-term net interest rate in steady state. Hence, since the ELB is equal to $\underline{i}$ and $i_t \geq \underline{i}$, the deviation of $\underline{i}$ from the steady-state interest rate can be written as: $\underline{\hat{i}} = (\underline{i} - i)/(1+i)$.

Forward guidance Equations (3.2) and (3.3) allow FG to maintain the short-term interest rate at a lower level than the rate implied by the standard Taylor rule (3.4). The intensity of FG is governed by the two parameters λ^* and α. To see the isolated effect of λ^*, we first consider the case of $\alpha = 0$ that implies $\hat{i}_t^* = \hat{i}_t^{\text{Taylor}}$ in equation (3.2). In the non-ELB regime equation (3.3) collapses to the standard Taylor rule (3.4). In the ELB regime of $\hat{i}_{t-1}^* < \hat{\underline{i}}$, if $\lambda^* = 0$, the lagged term of equation (3.3) is $\rho_i \hat{i}_{t-1} = \rho_i \hat{\underline{i}}$; if $\lambda^* > 0$, however, the lagged term becomes $\rho_i \hat{\underline{i}} + \rho_i \lambda^* (\hat{i}_{t-1}^* - \hat{\underline{i}}) < \rho_i \hat{\underline{i}}$, exerting additional downward pressures on $\hat{i}_t^{\text{Taylor}}$ in equation (3.3) and on i_t^* in equation (3.2) since $\hat{i}_t^* = \hat{i}_t^{\text{Taylor}}$. The decrease in $\hat{i}_t^*$ today keeps the interest rate $\hat{i}_t$ low from equation (3.1), and moreover it keeps the future interest rate low by reducing the shadow rate tomorrow, leading to the same effect of FG in Debortoli et al. (2019).

Next consider the case of $\alpha > 0$. Equation (3.2) implies that FG is additionally strengthened since the shadow rate is kept lower than the Taylor-rule rate, $\hat{i}_t^* = \hat{i}_t^{\text{Taylor}} + \alpha(\hat{i}_t^{\text{Taylor}} - \hat{\underline{i}}) < \hat{i}_t^{\text{Taylor}}$ under the ELB. Our implementation of FG is similar to Reifschneider and Williams (2000). For a given degree of interest rate smoothing ($\rho_i > 0$), the parameter $\lambda^* > 0$ generates downward pressure on the shadow rate in the ELB regime, and $\alpha > 0$ further magnifies the downward reduction of the shadow rate. Thus, λ^* primarily influences the persistence of the shadow rate, while α influences its level. Note that we also use λ^* below to measure the effectiveness of QE, i.e., it is not a free parameter in equation (3.3) but will be determined later by equation 3.13. This helps us simplify the theoretical model, and results in a parameter that jointly influences FG and QE policy, see Lemma 1 below.

Quantitative easing In the ELB regime, the short-term interest rate is fixed at the ELB, and the central bank starts QE by purchasing long-term government bonds (consol bonds). The long-term bond issued in period t pays μ^{j-1} dollars at time $t+j$. Let $P_{L,t}$ denote the price of the long-term bond, and let $R_{L,t+1}$ denote the return of holding it from period t to $t+1$. The price and the return conditional on period-t information are linked as

$$P_{L,t} = E_t \left(\frac{1 + \mu P_{L,t+1}}{R_{L,t+1}} \right). \tag{3.5}$$

where E_t is the expectation operator conditional on period-t information. The gross yield to maturity (or the long-term interest rate) at time t, $\bar{R}_{L,t}$, can be defined as

$$P_{L,t} = \frac{1}{\bar{R}_{L,t}} + \frac{\mu}{\left(\bar{R}_{L,t}\right)^2} + \frac{\mu^2}{\left(\bar{R}_{L,t}\right)^3} + \ldots$$

or

$$P_{L,t} = \frac{1}{\bar{R}_{L,t} - \mu}. \tag{3.6}$$

Log-linearizing equations (3.5) and (3.6) around steady state and combining them yields

$$\hat{\bar{R}}_{L,t} = \left(1 - \frac{\mu}{\bar{R}_L}\right) E_t \hat{R}_{L,t+1} + \frac{\mu}{\bar{R}_L} E_t \hat{\bar{R}}_{L,t+1}. \tag{3.7}$$

where $\bar{R}_L > \mu$ is the long-term interest rate in steady state.

Unrestricted households that trade both long- and short-term government bonds pay a unitary cost for trading long-term bonds. An arbitrage between holding short- and long-term bonds yields

$$E_t \hat{R}_{L,t+1} = \hat{i}_t + \frac{\zeta}{1+\zeta} \hat{\zeta}_t, \tag{3.8}$$

where $\hat{\zeta}_t$ and ζ are the trading cost in deviation from steady state and in steady state, respectively. The trading cost introduces a spread between the returns of holding long- and short-term bonds. The trading cost is assumed to be increasing in the real amount of long-term bonds circulated in the market, $\hat{b}_{L,t}$, and is given by

$$\hat{\zeta}_t = \rho_\zeta \hat{b}_{L,t}, \tag{3.9}$$

where $\rho_\zeta > 0$ represents the elasticity of the trading cost with respect to the amount of long-term bonds in the market.

The central bank conducts QE according to the rule:

$$\hat{b}_{L,t} = \min\left\{\gamma(\hat{i}^*_t - \underline{\hat{i}}), 0\right\}. \tag{3.10}$$

Parameter $\gamma \geq 0$ governs how aggressively the central bank purchases long-term bonds under the ELB, with $\gamma = 0$ corresponding to no purchase. In the ELB regime, where $\hat{i}^*_t \leq \underline{\hat{i}}$ holds, the central bank purchases and absorbs long-term bonds from the market, so that $\hat{b}_{L,t} \leq 0$. The central bank conducts QE by using the shadow rate $\hat{i}^*_t$ as policy guidance similar to the conventional monetary policy. For instance, assume $\alpha = 0$ in equation (3.2), such that $\hat{i}^*_t = \hat{i}^{\text{Taylor}}_t$. In the ELB regime, the more the shadow rate drops as a result of a decrease in inflation or output, the more the central bank purchases long-term government bonds. Thus, the central bank consistently aims at stabilizing inflation and output in both the non-ELB and ELB regimes.

Since constrained households hold long-term government bonds only while unconstrained households also hold short-term government bonds, the 'effective' interest rate relevant to output and inflation is the weighted sum of the returns of holding short- and

long-term government bonds, $\omega_u \hat{i}_t + (1-\omega_u) E_t \hat{R}_{L,t+1}$, where $\omega_u \in (0,1)$ is the population share of unconstrained households.[6] The Euler equation is then given by

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma}\left(\omega_u \hat{i}_t + (1-\omega_u) E_t \hat{R}_{L,t+1} - E_t \hat{\pi}_{t+1}\right) - \chi_b z_t^b, \tag{3.11}$$

where z_t^b is a demand (preference) shock, and $\sigma, \chi_b > 0$. Note that the expected return of holding the long-term bonds $E_t \hat{R}_{L,t+1}$ depends on the trading cost $\hat{\zeta}_t$ (equation 3.8), which depends on the real value of long-term bonds in the market $\hat{b}_{L,t}$ (equation 3.9), which in turn is controlled by QE that uses the shadow rate $\hat{i}_t^*$ as policy guidance (equation 3.10). Hence, $E_t \hat{R}_{L,t+1}$ can be written as a function of $\hat{i}_t^*$. Substituting equations (3.8)–(3.10) into the Euler equation (3.11) yields

$$\hat{y}_t = E_t \hat{y}_{t+1} - \frac{1}{\sigma}\left((1-\lambda^*)\hat{i}_t + \lambda^* \hat{i}_t^* - E_t \hat{\pi}_{t+1}\right) - \chi_b z_t^b, \tag{3.12}$$

where λ^* is given by

$$\lambda^* \equiv (1-\omega_u)\frac{\zeta}{1+\zeta}\rho_\zeta \gamma \tag{3.13}$$

The parameter λ^* determines the effectiveness of QE. If $\lambda^* = 0$, for instance as a result of $\rho_\zeta = 0$ or $\gamma = 0$, equation (3.12) reduces to the standard Euler equation that omits the shadow rate. If $\lambda^* = 1$, QE is 'fully effective' and the interest rate $\hat{i}_t$ becomes irrelevant to the Euler equation and the dynamics of the system, and consequently the ELB will be irrelevant for the dynamics of the economy. From equation (3.13), the effectiveness of QE increases in the share of restricted households, $1-\omega_u$, the trading cost in steady state, ζ, the elasticity of the trading cost with respect to the amount of long-term government bonds circulated in the market, ρ_ζ, and strength in the purchasing of long-term bonds by the central bank in response to a change in the shadow rate, γ.

Comparing equations (3.11) and (3.12) shows that the effective interest rate relevant to output and inflation is equal to the weighted interest rate, $(1-\lambda^*)\hat{i}_t + \lambda^* \hat{i}_t^*$. Thus the weighted interest rate can also be interpreted as the effective interest rate, and this appears as a lagged variable in the Taylor rule (3.3) that implements FG. The parameter λ^* depends on structural parameters pertaining to QE, as shown in equation (3.13). In this sense, the parameter λ^* encapsulates the effectiveness of UMP that reflects both QE and FG.

[6] The underlying assumption in deriving the effective interest rate is that the consumption of the two types of households is identical in steady state.

Long-term interest rates Combining equations (3.7)–(3.10) yields the long-term interest rate, given by

$$\hat{\bar{R}}_{L,t} = \begin{cases} \left(1 - \dfrac{\mu}{\bar{R}_L}\right) \hat{i}_t + \dfrac{\mu}{\bar{R}_L} E_t \hat{\bar{R}}_{L,t+1}, & \hat{i}^*_t > \hat{\underline{i}} \\ \left(1 - \dfrac{\mu}{\bar{R}_L}\right) \left(\dfrac{(1-\lambda^*)\hat{i}_t + \lambda^* \hat{i}^*_t - \omega_u \hat{i}_t}{1-\omega_u}\right) + \dfrac{\mu}{\bar{R}_L} E_t \hat{\bar{R}}_{L,t+1}, & \hat{i}^*_t \leq \hat{\underline{i}} \end{cases} \tag{3.14}$$

where $\hat{i}_t$ is set by equation (3.1). Equation (3.14) shows that the effects of the shadow rate and the degree of the effectiveness of UMP on the long-term interest rate are different between the two regimes. In the non-ELB regime, $\hat{i}_t = \hat{i}^*_t$ and the expectation hypothesis holds: the long rate is given by the weighted sum of the expected short rates today and in the future. In the ELB regime, the short rate is bounded at $\hat{i}_t = \hat{\underline{i}}$ and the long rate is affected by QE as the shadow rate appears in the lower equation in (3.14).

In the limiting case where the number of unconstrained households becomes infinitesimally small asymptotically, $\omega_u \to 0$, the difference between the two regimes in equation (3.14) vanishes and the long-term interest rate can be written in the form

$$\hat{\bar{R}}_{L,t} = \left(1 - \frac{\mu}{\bar{R}_L}\right) \left(\lambda^* \hat{i}^*_t + (1-\lambda^*)\hat{i}_t\right) + \frac{\mu}{\bar{R}_L} E_t \hat{\bar{R}}_{L,t+1}. \tag{3.15}$$

In this case, the expectation hypothesis holds with respect to the effective interest rate, $\lambda^* \hat{i}^*_t + (1-\lambda^*)\hat{i}_t$. In addition, equation (3.15) makes clear the presence of the attenuation bias of the long rate in the ELB regime and that it is related to the UMP effectiveness parameter λ^*. In the case of full effectiveness of UMP, i.e., $\lambda^* = 1$, the long rate is always given by the expected sum of the shadow rates and there will be no attenuation of the long rate in the ELB regime. However, in the case of less effective UMP, i.e., $\lambda^* < 1$, the long rate responds less to the shadow rate, giving rise to its attenuated response in the ELB regime.

System of equations We close the model by reporting the standard New Keynesian Phillips curve that relates the current inflation to expected inflation and output, given by

$$\hat{\pi}_t = \delta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t - \chi_a z^a_t, \tag{3.16}$$

where $\delta \geq 0$, $\kappa \geq 0$, and z^a_t is a supply (productivity) shock. The supply and demand shocks follow an AR(1) process. To summarize, our model of UMP comprises the six equations (3.1), (3.2), (3.3), (3.12), (3.14), and (3.16), the six endogenous variables $\{\hat{y}_t, \hat{\pi}_t, \hat{i}_t, \hat{i}^*_t, \hat{i}^{\text{Taylor}}_t, \hat{\bar{R}}_{L,t}\}$, and the three shocks $\{z^a_t, z^b_t, \epsilon^i_t\}$.

3.2.2 DSGE model and VAR representations

We use our DSGE model developed in Section 3.2.1 to derive the VAR representations that underpin our empirical models and tests to be introduced in Section 3.3.

The joint effect of QE and FG Before deriving VAR representations, it is useful to note that we cannot separate out the effects of the two parameters λ^* and α in the system, but the model allows us to identify their joint effect on the system, encapsulated by $\xi^* \equiv \lambda^*(1+\alpha)$. The next lemma states the result formally.

Lemma 1 *For any $\lambda^* \neq \lambda^{*\prime}$ and $\alpha \neq \alpha'$ that satisfy $\xi^* \equiv \lambda^*(1+\alpha) = \lambda^{*\prime}(1+\alpha')$, the model with λ^* and α is observationally equivalent to the model with $\lambda^{*\prime}$ and α'.*

The proof is straightforward. By using equation (3.2) to substitute out $\hat{i}_t^*$, the effective interest rate, $(1-\lambda^*)\hat{i}_t + \lambda^*\hat{i}_t^*$, can be replaced with $(1-\xi^*)\hat{i}_t + \xi^*\hat{i}_t^{\text{Taylor}}$. The parameter λ^* does not appear anywhere in the model except in $\xi^* = \lambda^*(1+\alpha)$. The parameter α appears only in equation (3.1) as $\hat{i}_t = \max\{\hat{i}_t^{\text{Taylor}} - \alpha(\hat{i}_t - \hat{i}_t^{\text{Taylor}}), \underline{\hat{i}}\}$. But this equation is observationally equivalent to $\hat{i}_t = \max\{\hat{i}_t^{\text{Taylor}}, \underline{\hat{i}}\}$ because only $\hat{i}_t$ is observable, and thus the joint effect of QE and FG is summarized by the parameter ξ^* that encompasses λ^* and α.[7]

As an illustrative example, consider a case in which QE is half as effective as the conventional monetary policy (i.e., $\lambda^* = 0.5$) but FG is active with $\alpha = 1$. In this scenario, the monetary policy shock ϵ_t^i is twice as large in the ELB regime as the same shock in the non-ELB regime (or, equivalently, at the ELB regime with $\lambda^* = 1$ and $\alpha = 0$). The impact of such a monetary policy shock at the ELB regime is of the same magnitude as the equivalent shock in the non-ELB regime. Thus, the effectiveness of UMP in the theoretical model depends on ξ^* that encapsulates the joint effect of λ^* and α, and encompasses the combination of QE and FG.

Irrelevance of the ELB and VAR representations Now we establish the VAR representations of the DSGE model under the irrelevance hypothesis of the ELB, where UMP is as effective as the conventional policy: $\xi^* = 1$. There will be different VAR representations, depending on what assumptions we impose on the model. Specifically, we begin by considering the following assumption.

[7] QE and FG are not separately identifiable here because they both operate only in the ELB regime during which the shadow rate is unobserved and only identified up to scale, see Mavroeidis (2021) for further discussion. It might be possible to disentangle the two policies if, for example, one of them operated also outside the ELB, as in Swanson (2021). This would require some additional assumptions to carefully model more than two endogenously-switching regimes.

Assumption 1 *The number of unrestricted households becomes infinitesimally small asymptotically:* $\omega_u \to 0$.

Under the irrelevance of the ELB, the solution to the model entails two VAR(1) representations, as formalized in the following proposition.

Proposition 1 *Consider the DSGE model in equations (3.1), (3.2), (3.3), (3.12), (3.14), and (3.16) under the irrelevance of the ELB:* $\xi^* = 1$. *Then,*

i) $[\hat{y}_t, \hat{\pi}_t, \hat{i}_t^*]$ *has a VAR(1) representation.*

ii) Under Assumption 1, $[\hat{y}_t, \hat{\pi}_t, \hat{\bar{R}}_{L,t}]$ *has a VAR(1) representation.*

Proof. Appendix 3.A.4. ■

Proposition 1(i) shows that under the irrelevance of the ELB, the short-term interest rate i_t is redundant for the dynamics of output and inflation once the shadow rate is included in the VAR. Whether the short rate is constrained by the ELB or not does not influence the dynamics of output and inflation, so that the ELB is irrelevant. What distinguishes our VAR from a standard linear VAR is that the shadow rate is censored at the ELB. In other words, for an econometrician, the shadow rate is observable, and equal to the nominal interest rate, only when it is above the ELB (equation 3.1).

Proposition 1(ii) shows that the long rate can be a sufficient indicator of monetary policy. Under Assumption 1, the shadow rate has no direct effect on the economy while having an indirect effect by affecting the long rate, and under the irrelevance of the ELB, the shadow rate and the long rate become interchangeable as implied by equation (3.15) with $\lambda^* = 1$ and $\alpha = 0$ without loss of generality from Lemma 1. Intuitively, in such a special case, the expectation hypothesis holds and the long rate is given by the expected sum of the shadow rates today and in the future, giving rise to a one-to-one relation between the long rate and the shadow rate. The corollary of Proposition 1(ii) is that under the irrelevance hypothesis there is no attenuation of the response of the long rate, i.e., the dynamics of the long rate are identical between the non-ELB and ELB regimes. We will use Proposition 1 to underpin our approach to testing the irrelevance hypothesis of the ELB and no attenuation effect in Section 3.3.

Assumption 1 plays a critical role in deriving the VAR representation with the long rate under the irrelevance of the ELB, i.e., the specific irrelevance hypothesis used in the literature (e.g., Debortoli et al., 2019). Without the assumption, equation (3.14) shows that even under the irrelevance of the ELB ($\xi^* = 1$), the long rate is kinked at the ELB: the first-term in equation (3.14) is smaller by $\omega_u/(1-\omega_u) \times \hat{\underline{i}}$ in the ELB regime than in the non-ELB regime. Why does the long rate have to decrease more in the ELB regime

to make the ELB irrelevant? The reason is straightforward. Unrestricted households hold short-term government bonds whose relevant interest rate is the short rate. To stimulate the aggregate consumption as if the economy were not at the ELB, consumption by the restricted households must be stimulated more, which requires a stronger decrease in the long rate under the ELB. Put differently, the decrease in the long rate needed to satisfy the irrelevance of monetary policy at the ELB regime becomes less pronounced with the reduction of unrestricted agents. Thus, Assumption 1 is important to validate the VAR representation with the long rate.

Relevance of the ELB and VAR representations Now consider the case of a less effective UMP than conventional policy (i.e., $\xi^* \leq 1$). In this general case, the model does not have a tractable solution under rational expectations and thus it would not have a VAR representation. However, the model admits a tractable solution if the formation of expectations slightly deviates from rational expectations by the following assumption.

Assumption 2 *In each period t, agents know the true $\xi^* \leq 1$ today and form expectations under the presumption of $\xi^* = 1$ from period $t+1$ onward.*

This assumption implies that agents entail behavioral expectations and believe that UMP will be as effective as the conventional policy from the next period $t+1$ onwards. Under this assumption, the VAR model has a piecewise linear representation, as stated in the following proposition.

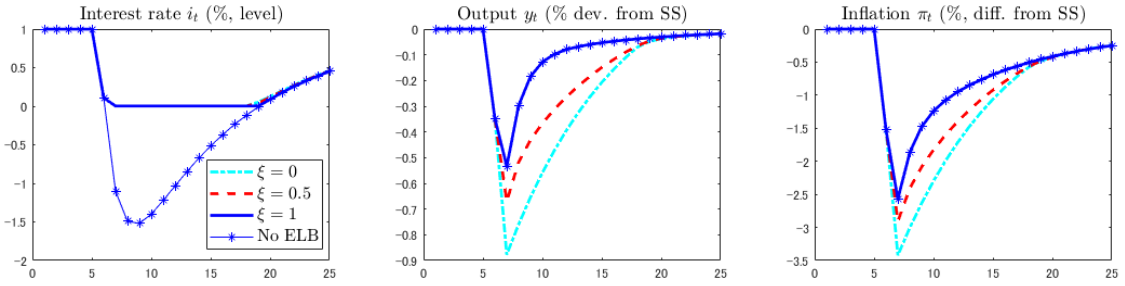
Proposition 2 *Under Assumption 2, the DSGE model in equations (3.1), (3.2), (3.3), (3.12), and (3.16) has a piecewise linear VAR representation with a kink at the ELB.*

Proof. Appendix 3.A.5 ■

Proposition 2 implies that in the case of $\xi^* < 1$ the dynamics of the economy differ between the non-ELB and ELB regimes as the dynamics in each regime is represented by a distinct VAR where a change in regimes occurs when the short-term interest rate crosses the ELB. This result echos with Aruoba et al. (2021), who argue that a piecewise linear solution to a DSGE model with an occasionally binding ELB constraint can be interpreted as describing the behavior of boundedly rational agents. Our specific assumption on the expectation formation in Proposition 2 represents a form of bounded rationality.

In our VAR representation in Proposition 2, equation (3.1) continues to hold, and the shadow rate is unobserved and censored in the ELB regime. By allowing for a kink in the dynamics, Proposition 2 shows that a censored and kinked VAR model provides a suitable empirical specification for testing the irrelevance hypothesis.

Figure 3.1: The effects of UMP



Note: The figure shows the dynamic path of the model under Proposition 2 where a severe demand shock hits the economy in periods $t = 6$ and $t = 7$. The dynamic path is computed by transforming the theoretical model under Proposition 2 into the reduced form equations (3.19b) and (3.19a) and calculating the response to the shocks.'No ELB' represents the model without the ELB, where the interest rate equation (3.1) is replaced by $i_t = i_t^*$. 'SS' denotes a steady state, 'dev.' denotes a deviation, and 'diff.' denotes a difference.

3.2.3 Simulations

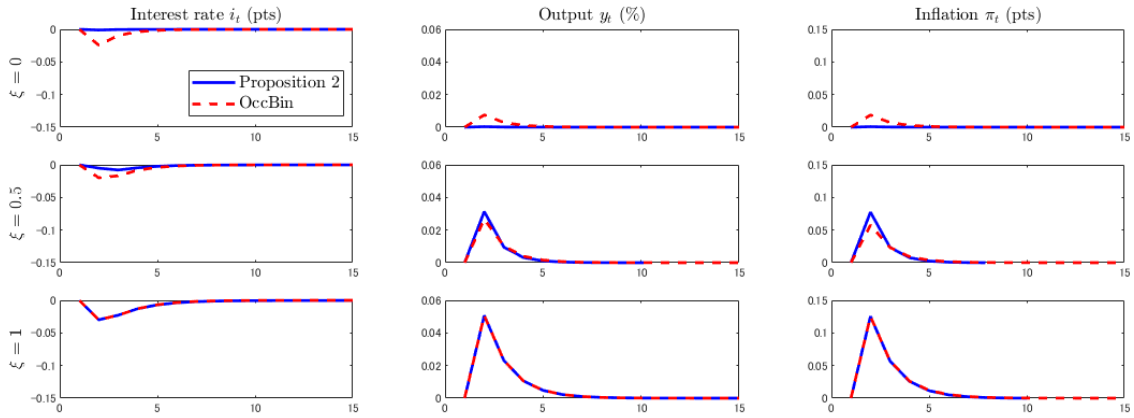
Before developing our empirical framework in the next section, we illustrate the effects of UMP and study impulse responses to a monetary policy shock in the ELB regime by solving the model under Proposition 2.[8] The analysis aims at illustrating the effects of UMP and it is not designed to draw quantitative implications. Appendix 3.A.3 reports the parameterization of the model.

No UMP The dash-dotted line in Figure 3.1 shows simulated paths for the theoretical model under Proposition 2 in the case of no UMP ($\xi^* = 0$). The economy starts from the steady state and large negative demand shocks hit in periods $t = 6$ and $t = 7$. The consecutive negative demand shocks bring the economy to the ELB and generate a severe recession by decreasing output and inflation sharply. At the ELB (dash-dotted line), the interest rate i_t cannot be lowered in response to the fall in inflation. This raises the real interest rate, decreases consumption and output, and puts further downward pressure on inflation through the Phillips curve (3.16). This negative feedback loop magnifies the falls in output and inflation compared to the hypothetical economy without the ELB (the star-marked line).

UMP UMP can offset the negative impact of the ELB. When UMP is partially effective ($\xi^* = 0.5$; the dashed line), the magnitude of the falls in output and inflation are mitigated relative to the case without UMP ($\xi^* = 0$; the dash-dotted line). When UMP is fully effective ($\xi^* = 1$; the solid line), although the interest rate i_t remains at the ELB, output and

[8] While our analysis focuses on monetary policy, Appendix 3.A.6 reports the responses of output and inflation to demand and supply shocks under the ELB to illustrate the role of the effectiveness of UMP for these shocks.

Figure 3.2: Impulse responses to a monetary policy shock at the ELB



Note: 'Proposition 2' denotes the theoretical model under Proposition 2 and 'OccBin' denotes the model solved by the algorithm developed by Guerrieri and Iacoviello (2015). For analyzing the impulse responses under the ELB, for each case of ξ^*, the initial condition is set as endogenous variables which are realized using OccBin in response to a severe negative demand shock.

inflation follow the same paths as in the case of no ELB (the star-marked line), as shown in Figure 3.1. In response to a decrease in the shadow rate i_t^*, the central bank increases the purchase of long-term government bonds and, by doing so, it lowers the long-term government bond yield by compressing its premium, which boosts consumption and output. When $\xi^* = 1$, UMP perfectly offsets the contractionary effect of the ELB. The interest rate i_t becomes irrelevant to the dynamics of the economy, which evolves as if there were no ELB.

Impulse responses to a monetary policy shock Figure 3.2 plots impulse responses to a 0.25 percentage points cut in the shadow rate under the ELB starting from period $t = 1$ for the theoretical model solved under Proposition 2 (the solid line) and the model solved by the OccBin algorithm (the dashed line), developed by Guerrieri and Iacoviello (2015), which has been a popular approach to solving DSGE models at the ELB (see Atkinson et al., 2020).[9] The OccBin solution assumes that agents in the model form expectations by treating the non-ELB regime as an absorbing state, i.e., by assuming that interest rates will remain positive once the economy exits the ELB regime. Thus, OccBin uses an alternative behavioural assumption on the formation of expectations than the one used in Proposition 2.

Overall the responses of the interest rate, output, and inflation are similar between the model solution under Proposition 2 and the OccBin solution, as shown in Figure 3.2. The responses of the interest rate are muted because the economy starts from the ELB triggered by a severe demand shock in period $t = 1$. Without the ELB, the interest rate

[9]The impulse responses are computed by using the same method employed in reporting our empirical results. For the details, see Section 3.5.1.

(left panels) would fall by about 0.15 percentage points (pts), reported in the figure as the lowest value on the y-axis.[10] In the case of no UMP ($\xi^* = 0$; top panels), the responses of output (central panels) and inflation (right panels) are muted for both the model solution under Proposition 2 and the OccBin solution. Because the economy is at the ELB, the monetary policy shock in period $t = 2$ does not have significant effects on the economy without UMP. In the case of partial UMP ($\xi^* = 0.5$; middle panels), QE is activated in response to a decrease in the shadow rate triggered by the monetary policy shock, and output and inflation increase. In the case of fully effective UMP ($\xi^* = 1$; bottom panels), the 'irrelevance hypothesis' holds and the responses of output and inflation coincide with those under the hypothetical economy with no ELB under both solution methods.

3.3 Empirical model

Our theoretical model in Section 3.2 highlights three important features to study the irrelevance hypothesis: i) the censoring of the shadow rate at the ELB, ii) a potential kink of the dynamics of the economy at the ELB, and iii) a parameter that encapsulates the effectiveness of UMP. We embed these features in our empirical model designed to test the irrelevance hypothesis using the flexible VAR approach of the literature on monetary policy. In this section, we present our empirical model, derive the tests of the irrelevance of the ELB and no attenuation hypotheses, and describe our approach to the identification of the effects of conventional and unconventional monetary policy.

3.3.1 Censored and kinked SVAR

The econometric model that we use is the censored and kinked SVAR (CKSVAR) developed by Mavroeidis (2021). Its structural form is given by:

$$i_t = \max\left\{i_t^*, \underline{i}_t\right\}, \tag{3.17a}$$

$$i_t^* = -\alpha i_t + (1+\alpha)\left(\gamma Y_{1t} + B_2 X_t + B_{22}^* X_{2t}^* + A_{22}^{*-1}\varepsilon_{2t}\right), \tag{3.17b}$$

$$Y_{1t} = \beta\left(\lambda i_t^* + (1-\lambda)\, i_t\right) + B_1 X_t + B_{12}^* X_{2t}^* + A_{11}^{-1}\varepsilon_{1t}, \tag{3.17c}$$

where i_t is the short-term interest rate that is subject to the observable lower bound of $\underline{i}_t$, i_t^* is the shadow rate, Y_{1t} is a vector of unconstrained endogenous variables such as inflation and output, X_t comprises exogenous and predetermined variables, including lags of Y_{1t}

[10] The responses of the interest rate are slightly negative because they are calculated relative to the expected interest rate conditional on no monetary policy shock in the initial period, which is slightly positive due to the realizations of shocks that bring the economy above the ELB.

and i_t, X_{2t}^* consists of lags of i_t^*, and ε_t are i.i.d. structural shocks with identity covariance matrix.

Equation (3.17a) represents the ELB constraint, and it corresponds to equation (3.1) in the theoretical model, except for the lower bound that is allowed to vary over time in equation (3.17a). Thus, the shadow rate is censored and unobservable under the ELB, as in the theoretical model.

Equation (3.17b) represents the short-term interest rate rule that nests the FG rule of Reifschneider and Williams (2000), and it corresponds to equations (3.2) and (3.3) in the theoretical model. The parameter α has the same interpretation as in the theoretical model.

Equation (3.17c) describes the dynamics of variables of interest such as inflation and output and the relationship between these variables and interest rates. The parameter λ is the equivalent of λ^* in the theoretical model and characterizes the effectiveness of UMP relative to conventional policy *on impact*. Specifically, from equation (3.17c) we see that above the ELB (i.e., when $i_t = i_t^* > \underline{i}_t$), the *contemporaneous* effect of a change in the short-term interest rate i_t by one unit on Y_{1t} is β, but the corresponding effect at the ELB, driven by a change in the shadow rate i_t^*, is $\lambda\beta$. When $\lambda = 1$, the two effects are equal, while $\lambda = 0$ corresponds to the case in which UMP has no contemporaneous effect on Y_{1t}.

Similar to the theoretical model, the parameter λ partially characterizes the impulse responses to a monetary policy shock at the ELB, since the response of the interest rate also depends on the degree of FG, and thus the joint effect of α and λ determines the response of endogenous variables to UMP. To see this in the context of the SVAR model, consider the impulse response to the monetary policy shock of $\varepsilon_{2t} = A_{22}^*$ ignoring non-linearities.[11] The effect on Y_{1t} is $\beta/(1-\gamma\beta)$ above the ELB, and $\xi\beta/(1-\xi\gamma\beta)$ at the ELB, where:

$$\xi = \lambda(1+\alpha). \tag{3.18}$$

So, it is, in fact, ξ, not λ, that measures the effectiveness of an UMP shock – a shock to the shadow rate below the ELB, see Mavroeidis (2021) for further discussion. The parameter ξ is the equivalent ξ^* in Lemma 1 of the theoretical model. In the proof of Proposition 2 (see Appendix 3.A.5), we show that the UMP parameter ξ^* in the theoretical model under Proposition 2 can be mapped in the system of equations (3.17a)-(3.17c), and the ξ^* in the theoretical model coincides with the corresponding parameter ξ in the empirical model.

Our discussion about the parameter ξ concerned the relative effectiveness of UMP *on impact*. The *dynamic* effects of UMP on Y_{1t} are governed by the coefficients on the lags of the shadow rate B_{12}^* in equation (3.17c). For example, the case of completely ineffective

[11] We will discuss the specification of the impulse response functions in Section 3.5, see equation (3.26).

UMP on Y_{1t} at all horizons can be represented by the joint restrictions $\xi = 0$ *and* $B^*_{12} = 0$. A more restrictive case is that UMP is also ineffective on the short-term interest rate i_t, in addition to having no effect on Y_{1t}, which can be implemented by $\xi = 0$, $B^*_{12} = 0$, *and* $B^*_{22} = 0$. This implies that the shadow rate has no contemporaneous and cumulative impact on the endogenous variables and completely drops out of the right-hand side of equations (3.17a)-(3.17c). We refer to this case as the *kinked* SVAR (KSVAR), using the same terminology in Mavroeidis (2021).

3.3.2 Reduced-form solution of the SVAR and identification

To implement our empirical tests and gain intuition on the identification and estimation of the CKSVAR model, we derive the reduced-form solution for Y_{1t} and i_t using equations (3.17a)-(3.17c). Mavroeidis (2021) develops the methodology for the identification and estimation of the CKSVAR, showing that the model is generally under-identified, but the parameter ξ, defined in equation (3.18), and the impulse responses to the monetary policy shock ε_{2t}, are partially identified in general. The reduced-form solution for Y_{1t} and i_t is:

$$i_t = \max\left\{C_{21}X_{1t} + C_{22}X_{2t} + C^*_{22}X^*_{2t} + u_{2t}, \underline{i}_t\right\} \tag{3.19a}$$

$$Y_{1t} = C_{11}X_{1t} + C_{12}X_{2t} + C^*_{12}X^*_{2t} + u_{1t} - \widetilde{\beta}D_t\left(C_2X_t + C^*_{22}X^*_{2t} + u_{2t} - \underline{i}_t\right) \tag{3.19b}$$

where $D_t \equiv \mathbb{1}_{\{i_t = \underline{i}_t\}}$ is the indicator of the ELB regime, $X_t \equiv (X'_{1t}, X'_{2t})'$, X_{2t} consists of the lags of i_t, the matrices C_{11}, C_{12}, C^*_{12}, $C_2 \equiv (C_{21}, C_{22})$, and C^*_{22} are reduced-form coefficients, $u_t \equiv (u'_{1t}, u'_{2t})'$ are reduced-form errors, and $\Omega \equiv var\left(u_t\right)$.

The reduced-form equations in (3.19) represent a censored and kinked VAR. Equation (3.19a) entails the censoring of the shadow rate, represented by the max operator. Equation (3.19b) allows for a kink at the ELB and the coefficients and variance can change across regimes. Therefore, our empirical model nests the theoretical model under Proposition 2, while imposing minimal structure and allowing for flexible coefficients in the VAR.

The coefficient of the kink $\widetilde{\beta}$ in equation (3.19b) is identified, together with the remaining reduced-form parameters. In other words, we can infer from the data whether the slope coefficients and the variance of Y_{1t} change across regimes by testing whether $\widetilde{\beta} = 0$. However, the parameter $\widetilde{\beta}$ does not have a structural interpretation and relates to the underlying structural parameters through the following equations:

$$\widetilde{\beta} = (1 - \xi)\left(I - \xi\beta\gamma\right)^{-1}\beta, \tag{3.20}$$

$$\gamma = \left(\Omega'_{12} - \Omega_{22}\beta'\right)\left(\Omega_{11} - \Omega_{12}\beta'\right)^{-1}. \tag{3.21}$$

As shown in Mavroeidis (2021), the structural parameters ξ, β and γ are partially identified, in the sense that there is a set of different combinations for values of the structural parameters that satisfy equations (3.20) and (3.21) and generate any given value of the reduced-form parameters $\widetilde{\beta}$ and Ω. Therefore, the impulse responses to a monetary policy shock are set-identified. In our empirical analysis below, we will sharpen the identified set by using sign restrictions on the impulse responses.

3.3.3 Hypothesis tests of the irrelevance of the ELB

We now develop our tests for the irrelevance hypothesis (IH) of the ELB from the reduced-form solution of the SVAR. A central implication of our theoretical framework was that the dynamics of the economy are independent from whether policy rates are at the ELB or not. We use this fundamental implication to formulate two testable hypotheses.

Irrelevance hypothesis 1 (IH_1) Our first approach to test the IH is motivated by Swanson (2018) and Debortoli et al. (2019), who argue that monetary policy remains similarly effective across ELB and non-ELB regimes and that long-term interest rates are a plausible indicator of the stance of monetary policy. Debortoli et al. (2019) use SVARs that include long-term, rather than short-term interest rates as indicators of monetary policy. They use such VARs to identify the impulse responses of the macroeconomic variables to monetary policy as well as the response of policy to economic conditions, and find that those responses are similar across ELB and non-ELB regimes in the U.S. The implicit and testable assumption that underlies their analysis is that the short-term interest rate can be excluded from the dynamics of all the other variables in the system, and the dynamics of the system do not change when the economy enters the ELB regime. In other words, under the IH, the dynamics of the economy can be represented by VARs with the long rate but without the short rate, which we formally showed in Proposition 1(ii).

The hypothesis can be tested as an exclusion restriction in an SVAR that includes both the short and long rates. Since the short rate is subject to the ELB constraint, the relevant framework is the CKSVAR (that allows for the shadow rate to affect the economy in the ELB regime) and the special case of KSVAR (that precludes the shadow rate to affect the economy in the ELB regime) introduced in Section 3.3.1. Specifically, looking at the reduced-form specification in equation (3.19b), the IH can be formulated as:

$$\text{IH}_1 : C_{12} = C_{12}^* = 0 \text{ and } \widetilde{\beta} = 0. \tag{3.22}$$

In words, $C_{12} = C_{12}^* = 0$ means that lags of the short rate (i_t) and the shadow rate (i_t^*) can be excluded from the equation (3.19b) that governs the unconstrained variables (Y_{1t})

in the VAR, and $\widetilde{\beta} = 0$ means that the slope coefficients and the variance of the errors of those equations (for Y_{1t}) remain the same when the economy moves across regimes.

Irrelevance hypothesis 2 (IH_2) The second test of the IH is motivated by Proposition 1(i), which shows that when UMP is fully effective, the dynamics of the economy can be adequately represented by a VAR that entails the pure censoring of i_t^* and no kink. Such a VAR is a special case of the reduced-form VAR (3.19) that arises when the following testable restrictions are imposed:

$$\text{IH}_2 : C_{12} = 0, C_{22} = 0 \text{ and } \widetilde{\beta} = 0. \tag{3.23}$$

In words, $C_{12} = 0$ and $C_{22} = 0$ means that the lags of the short rate (i_t) can be excluded from equation (3.19), and $\widetilde{\beta} = 0$ means that the slope coefficients and the variance of the errors of those equations (for Y_{1t}) remain the same when the economy moves across regimes. We call the structural form of the VAR under IH_2 as a purely *censored* SVAR (CSVAR) as the shadow rate is censored but affects the dynamics equally across regimes.

It is worth noting that both IH_1 and IH_2 do not rely on a specific model of UMP, such as the one in Section 3.2, as long as the long rate or the shadow rate represents the monetary policy stance, because any VAR that includes short rates must admit a CSVAR representation with constant parameters across regimes when the irrelevance hypothesis holds. Otherwise, the ELB would result in observable changes across regimes, violating the hypothesis that the ELB is empirically irrelevant.

3.3.4 Attenuation effect

Swanson and Williams (2014) and Gilchrist et al. (2015) argue that an implication of a binding ELB constraint is that the effect of shocks on long-term interest rates may be attenuated when the policy rate lies at the ELB. They investigate this phenomenon empirically using time-varying-parameter regressions of different maturity yields on news shocks at daily frequency.

We shall use the CKSVAR model to characterize analytically and obtain formal tests of the aforementioned attenuation effect. Specifically, we will show analytically how the model-implied impulse response of the long yields to a monetary policy shock on impact is attenuated during ELB-regimes relative to non-ELB regimes, and that this attenuation effect is state-dependent and thus time-varying.

Let $IR_{j,t}$ denote the impact response of variable j to an infinitesimal monetary policy shock derived from the CKSVAR model in Equation (3.17). Also, let $IR_{j,NA}$ denote the

same response under the assumption that there is no attenuation. Then, it can be shown that (see Appendix 3.B):

$$IR_{j,t} = (1 - a_{j,t})\, IR_{j,NA}, \qquad a_{j,t} := \frac{\widetilde{\beta}_j}{\beta_j} \Phi\left(\frac{\underline{i}_t - i^*_{t|t-1}}{\varpi}\right), \tag{3.24}$$

where β_j and $\widetilde{\beta}_j$ are the j th elements of the coefficient vectors that appear in structural and reduced-form equations, (3.17) and (3.19), respectively, $i^*_{t|t-1} := C_{21}X_{1t} + C_{22}X_{2t} + C^*_{22}X^*_{2t}$ is the predicted value of the shadow rate in period t, ϖ is the standard deviation of $i^*_t - i^*_{t|t-1}$ when the monetary policy shock ε_{2t} is zero, and $\Phi(\cdot)$ is the Standard Normal cumulative distribution function. The factor $a_{j,t}$ attenuates the impact of the monetary policy shock on $Y_{1t,j}$ as the economy approaches the ELB, provided $a_{j,t} \in [0, 1]$. This attenuation is notably time-varying and depends on the distance of the shadow rate from the ELB $\underline{i}_t$. When the shadow rate is far above the ELB, $\Phi\left(\frac{\underline{i}_t - i^*_{t|t-1}}{\varpi}\right)$ is approximately zero and there is no attenuation. The attenuation increases in $\underline{i}_t - i^*_{t|t-1}$, provided $\widetilde{\beta}_j/\beta_j > 0$.

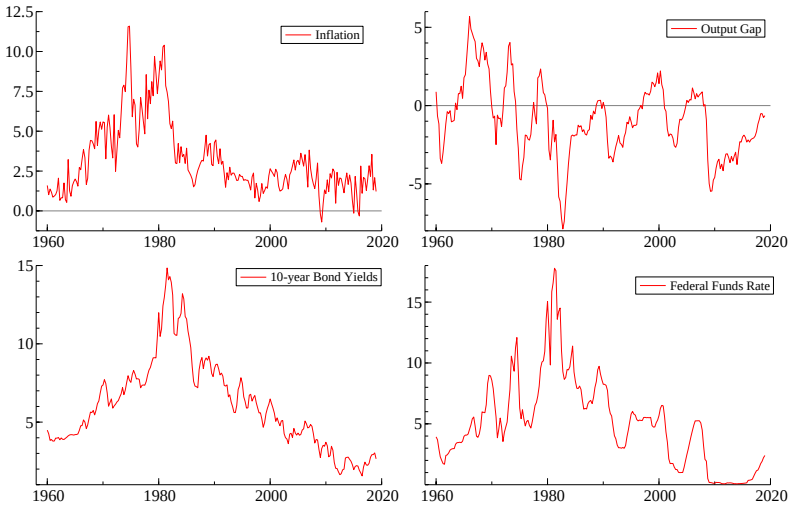
Equation (3.24) allows us to derive a formal test of the hypothesis that there is *no attenuation* based only on the *reduced-form* VAR specification (3.19). In other words, this test does not rely on any additional assumptions (such as sign restrictions) that one might use to identify the structural parameters β and the structural impulse responses. Specifically, suppose that the vector of unconstrained variables Y_{1t} in the empirical model (3.19) includes a long-term yield of a particular maturity, $\bar{R}_{L,t}$, and let $\widetilde{\beta}_L$ denote the coefficient of the kink in the reduced-form equation associated with $\bar{R}_{L,t}$ in equation (3.19b). Then, the hypothesis that there is no attenuation of the response of $\bar{R}_{L,t}$ to monetary policy shocks arising from the ELB is given by:

$$\mathrm{H}_{NA} : \widetilde{\beta}_L = 0. \tag{3.25}$$

In the theoretical model studied in Section 3.2, no attenuation effect is equivalent to the irrelevance of the ELB as implied by Proposition 1(ii). But the empirical model is less restrictive than the theoretical model, and H_{NA} is clearly a weaker hypothesis than the irrelevance hypotheses IH_1 (3.22) and IH_2 (3.23) defined above. Therefore, in our empirical framework, failing to reject H_{NA} does not necessarily imply that the ELB is empirically irrelevant.

We will test H_{NA} using long yields at various maturities in Section 3.4.3.

Figure 3.3: U.S. quarterly data



3.4 Empirical results

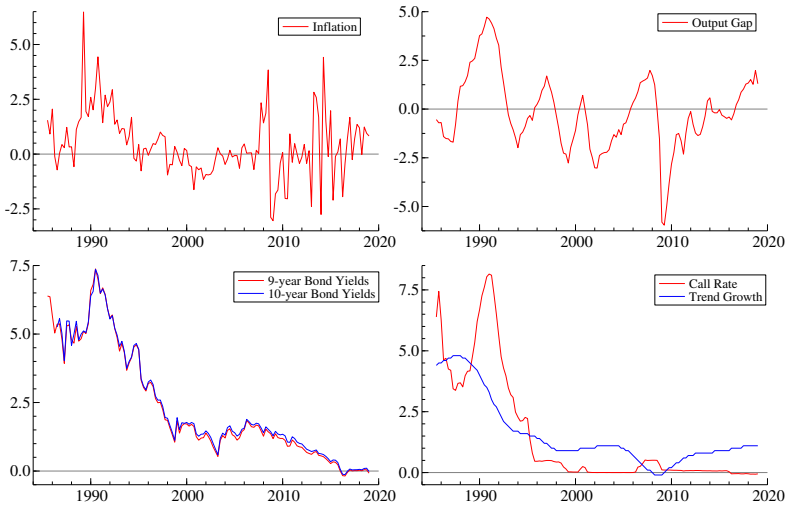
This section discusses the data for the U.S. and Japan, and tests the hypothesis that the ELB has been empirically irrelevant in each country using the hypothesis testing approach that is underpinned by Proposition 1 and established in Section 3.3.3. In addition, it tests the hypothesis of no attenuation effect in the response of long-term interest rates at the ELB, as formulated in Section 3.3.4, and it examines whether the short-term interest rate, including the shadow rate at the ELB, is a sufficient indicator of monetary policy including UMP, as implied by Proposition 2.

3.4.1 Data

Our empirical analysis focuses on the U.S. and Japan. We choose data series for the baseline specification of the SVAR model to maintain the closest specification possible to related studies and thereby include representative series for inflation, output, and measures for short- and long-term yields.

For the U.S., we use quarterly data for inflation based on the GDP deflator, a measure of the output gap from the Congressional Budget Office, the short-term interest rate from the Federal Funds Rate, and the 10-year government bond yield from the 10-year Treasury constant maturity rate. Figure 3.3 plots these series. We also consider the different measures of monetary aggregates listed in Appendix 3.C. The data are from the FRED database at the Federal Reserve Bank of St. Louis and the Center for Financial Stability databases. The estimation sample for the baseline specification is from 1960q1

Figure 3.4: Japanese quarterly data



to 2019q1.[12] We set the value of the effective lower bound on the Federal Funds Rate equal to 0.2, such that the short-term interest rate is at the ELB regime for 11 percent of the time, which is consistent with Bernanke and Reinhart (2004) who suggest that the effective lower bound on nominal interest rates may be above zero for institutional reasons.

For Japan, we use quarterly data for core CPI inflation, a measure of the output gap provided by the Bank of Japan, and the call rate. In addition, we use two alternative measures for long yields: the 9-year and the 10-year government bond yields, which are available for different sample periods. The data sources are the Bank of Japan for the output gap and the call rate, the Ministry of Finance for the 9-year and the 10-year government bond yields, and Statistics Bureau of Japan for core CPI inflation. The available sample is from 1985q3 to 2019q1 if we include the 9-year government bond yield in the VAR, which is our baseline case, and from 1987q4 to 2019q1 if we use the 10-year yield. Following Hayashi and Koeda (2019), we set the ELB to track the interest on reserves (IOR).[13] For the sample period 1985q3-2019q1, the call rate is at the ELB for 49 percent of the observations. Following Hayashi and Koeda (2019), we use a trend growth series to account for the declining equilibrium real interest rate in Japan during the 1990s.[14] Figure 3.4 plots these series.

[12] See Appendix 3.C for further details about the data. Alternative specifications with money are estimated over different time periods due to constraints on data availability.

[13] Specifically, ELB = IOR + 7bp, which is slightly higher than Hayashi and Koeda (2019) who use IOR+5bp, in order to treat 2016q1 as being at the ELB.

[14] Specifically, we use the annual average growth rate of potential GDP as an additional control in our model. See Hayashi and Koeda (2019, pp. 1081–1083) for an extended discussion of this issue and its implications.

Table 3.1: Test for excluding short rates from VAR that includes long rates

Panel A: KSVAR												
	United States						Japan					
p	loglik	pv-p	AIC	LR	df	p-val	loglik	pv-p	AIC	LR	df	p-val
5	-213.4	-	2.62	53.12	18	0.000	248.1	-	-2.18	27.82	18	0.065
4	-221.5	0.446	2.55	49.57	15	0.000	239.9	0.425	-2.30	28.10	15	0.021
3	-234.4	0.112	2.53	42.13	12	0.000	232.2	0.471	-2.42	28.58	12	0.004
2	-266.0	0.000	2.66	41.93	9	0.000	223.8	0.445	-2.53	25.71	9	0.002
1	-296.7	0.000	2.78	32.87	6	0.000	184.8	0.000	-2.19	32.32	6	0.000
Panel B: CKSVAR												
p	loglik	pv-p	AIC	LR	df	p-val	loglik	pv-p	AIC	LR	df	p-val
5	-191.3	-	2.60	82.43	33	0.000	284.7	-	-2.42	90.39	33	0.000
4	-202.7	0.290	2.53	72.15	27	0.000	277.1	0.766	-2.61	91.55	27	0.000
3	-223.0	0.011	2.53	51.93	21	0.000	258.1	0.081	-2.62	73.52	21	0.000
2	-256.3	0.000	2.64	49.44	15	0.000	242.1	0.018	-2.68	56.16	15	0.000
1	-290.3	0.000	2.76	37.12	9	0.000	204.8	0.000	-2.43	63.03	9	0.000

Note: Panel A reports results for a KSVAR(p) with inflation, output gap, long rate and policy rate. Panel B reports corresponding results for a CKSVAR(p) that includes shadow rates. Estimation sample is 1960q1-2019q1 for the U.S. and 1985q3-2019q1 for Japan. Long rates are 10-year government bond yields for the U.S. and 9-year yields for Japan. loglik is the value of the log-likelihood. pv-p is the asymptotic p-value of a LR test of (C)KSVAR(p) against (C)KSVAR($p+1$). AIC is the Akaike information criterion. LR is the test statistic for excluding short rates from equations for inflation, output gap and long rates. df is the number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

3.4.2 Testing the irrelevance hypothesis of the ELB

We now test the irrelevance hypothesis of the ELB for the U.S. and Japan using the two null hypotheses, IH_1 and IH_2, formalized by (3.22) and (3.23), respectively.

Testing IH_1 The basic idea of IH_1 is that a long-term interest rate is a sufficient indicator of monetary policy stance, and thereby a short-term interest rate becomes redundant and can be excluded from the VAR under the irrelevance of the ELB. The results of the likelihood ratio test of the null hypothesis IH_1 in (3.22) are reported in Table 3.1. Panel A reports results based on a KSVAR model, in which lags of the shadow rate i_t^* do not appear on the right-hand-side of equation (3.19). In this case, we test IH_1 against an alternative hypothesis that imposes $C_{12}^* = 0$, and this test only has power against the violation of $C_{12} = 0$. We do so because the KSVAR model is simpler to estimate, and a rejection of $C_{12} = 0$ would suffice to reject IH_1. Panel B reports the results using the general CKSVAR. The table reports results for specifications with different lag lengths of the VAR(p), where $p = 1, \ldots, 5$. Column pv-p reports the p-value of a test for selecting the number of lags in the model,which is an alternative approach to the Akaike Information Criterion (AIC), also reported in the Table. Both measures consistently select four lags

Table 3.2: Test for excluding short rates from VAR with long rates by maturity

Panel A: KSVAR										
	United States					Japan				
YTM	sample	p	LR	df	p-val	sample	p	LR	df	p-val
7	1970q4–2019q1	3	38.69	12	0.000	1985q3–2019q1	2	23.20	9	0.006
5	1963q2–2019q1	3	41.93	12	0.000	1985q3–2019q1	2	25.15	9	0.003
3	1963q2–2019q1	3	46.32	12	0.000	1985q3–2019q1	2	30.97	9	0.000
2	1977q4–2019q1	4	38.82	15	0.001	1985q3–2019q1	2	33.05	9	0.000
1	1963q2–2019q1	3	59.65	12	0.000	1985q3–2019q1	2	49.90	9	0.000
Panel B: CKSVAR										
YTM	sample	p	LR	df	p-val	sample	p	LR	df	p-val
7	1970q4–2019q1	3	51.03	21	0.000	1985q3–2019q1	2	58.32	15	0.000
5	1963q2–2019q1	3	56.33	21	0.000	1985q3–2019q1	2	58.50	15	0.000
3	1963q2–2019q1	3	58.54	21	0.000	1985q3–2019q1	2	62.04	15	0.000
2	1977q4–2019q1	4	66.84	27	0.000	1985q3–2019q1	2	63.77	15	0.000
1	1963q2–2019q1	3	73.06	21	0.000	1985q3–2019q1	2	72.41	15	0.000

Note: Panel A reports results for a KSVAR(p) with inflation, output gap, long rate, and policy rate. Panel B reports corresponding results for a CKSVAR(p) that includes shadow rates. Estimation sample varies for each long-term rate used for the U.S. and 1985q3–2019q1 for Japan. Long rates are government bond yields with year to maturity reported in the first column (YTM). p is the preferred lag length selected by AIC criteria. LR is the test statistic for excluding short rates from equations for inflation, output gap and long rates. df is the number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

for the U.S. and two lags for Japan. Column p-val reports the asymptotic p-value of our test of (3.22).

Table 3.1 shows that the data strongly reject the exclusion restrictions implied by IH_1 for both countries and in both the KSVAR and CKSVAR specifications. Hence, the short-term interest rate cannot be excluded from the VAR, and thus it makes the dynamics of the system differ between the non-ELB and ELB regimes even if the long-term interest rate – 10-year yields for the U.S. and 9-year yields for Japan – is added to the VAR. The result continues to hold for Japan with 10-year yields, which are available from 1987q3, and is reported in Table 3.14 in Appendix 3.D.

The result of the rejection of IH_1 continues to hold even if we use yields with shorter maturities. Table 3.2 shows the results for the null hypothesis IH_1 when we include yields with maturities in the range from 1 to 7 years in the VAR for the U.S. and Japan. The entries show results for the VAR specification with the preferred number of lags according to the AIC. The results show that the exclusion restrictions from IH_1 are strongly and consistently rejected across the whole range of maturities for both countries. Our results imply that a government bond yield with maturity within the 1- to 10-year range cannot replace the shadow rate as the indicator of monetary policy stance, and the dynamics of the economy differ between the non-ELB and ELB regimes.

Table 3.3: Testing CSVAR against CKSVAR with long rates by maturity

	United States					Japan				
YTM	sample	p	LR	df	p-val	sample	p	LR	df	p-val
10 or 9	1960q1–2019q1	4	34.42	19	0.016	1985q3-2019q1	2	51.02	11	0.000
7	1970q4–2019q1	3	26.80	15	0.030	1985q3–2019q1	2	50.86	11	0.000
5	1963q2–2019q1	3	29.26	15	0.015	1985q3–2019q1	2	49.27	11	0.000
3	1963q2–2019q1	3	31.83	15	0.007	1985q3–2019q1	2	53.09	11	0.000
2	1977q4–2019q1	4	45.49	19	0.001	1985q3–2019q1	2	56.03	11	0.000
1	1963q2–2019q1	3	51.04	15	0.000	1985q3–2019q1	2	70.62	11	0.000

Note: The estimated model is a CKSVAR(p) for the U.S. and Japan with inflation, output gap, policy rate, and a measure of long-term rate. Long rates are government bond yields with year to maturity reported in the first column (YTM). As baseline specifications, the 10-year yield is used for the U.S. and the 9-year yields is used for Japan. Sample availability varies for each long-term rate used. LR is the value of the LR test statistic. df is the number of exclusion restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

Testing IH$_2$ The idea of the hypothesis IH$_2$ in (3.23) is that once the shadow rate is included in the VAR, the short-term interest rate can be excluded from the VAR under the irrelevance hypothesis. We test IH$_2$ with the three core observables, inflation and output gap in Y_{1t}, and the short-term interest rate, and we also include the long-term interest rate in Y_{1t} for robustness. Under IH$_2$ the shadow rate is censored in the ELB regime but affects Y_{1t} equally in both regimes according to the reduced form equation (3.19) We test IH$_2$ using a likelihood ratio test as we tested IH$_1$.

Table 3.3 reports the results of the likelihood ratio test of IH$_2$ for the U.S. and Japan. The results for our baseline specifications are reported in the row starting with '10 or 9' in the first column. We include 4 lags for the U.S. and 2 lags for Japan, according to the AIC. The results show that the IH$_2$ is rejected for both economies at the 5 percent level of significance. The result continues to hold for Japan with 10-year yields and is reported in Table 3.15 in Appendix 3.D. Table 3.3 also shows that the result of the rejection of IH$_2$ continues to hold even if we include an alternative yield within the range of 1 to 7 years in Y_{1t}.

Robustness checks It is possible that the short rate is necessary to explain long rates even if the short rate is not necessary to characterize inflation and output dynamics. This could occur if real rates trend over time, so having both in the VAR picks up low-frequency movements. We address this concern by testing IH$_1$: $C_{12} = C^*_{12} = 0$ and $\tilde{\beta} = 0$ for the inflation and output equations only while allowing the short and shadow rates to affect the long rate. We find that this weaker version of IH$_1$ is also robustly rejected (Table 3.7 in Appendix 3.D).

Next we check the robustness of our results to possible omission of alternative channels of unconventional monetary policy, by adding money growth to the Y_{1t} variables of the VAR that we use to test the null hypothesis IH$_1$ in (3.22). Using several different

monetary aggregates for the U.S., we consistently reach the same conclusion: the IH_1 is firmly rejected (Table 3.8 in Appendix 3.D).

We also check the robustness of the U.S. results to the well-documented fall in macroeconomic volatility in the mid-1980s, known as the Great Moderation, as well as a possible change in monetary policy regime occurring at that time by performing the same tests over the subsample 1984q1–2019q1 (see Tables 3.9 and 3.10 in Appendix 3.D). In addition, Caldara and Herbst (2019) show that accounting for the endogenous reaction of financial conditions is critical to avoiding an attenuation bias in the responses of variables to monetary policy shocks. Gilchrist and Zakrajsek (2012) use credit spreads to internalize the central contribution of financial frictions during ELB episodes. Thus, we check the robustness of our results by including credit spreads in the VAR (see Tables 3.11-3.13 in Appendix 3.D). Our conclusion remains the same: the IH_1 and IH_2 are firmly rejected.

Finally, we check the power of our irrelevance tests IH_1 and IH_2 by generating simulated series from our theoretical model for values of ξ^* in the range $[0.7, 0.99]$. Since the theoretical model is solved under Proposition 2, ξ^* in the theoretical model coincides with ξ in the empirical model and the simulated series are equivalent between the two models. We find that the rejection rate of the tests declines as ξ^* approaches the value of one (i.e., the irrelevance hypothesis holds true), showing that our tests are powerful. The result is reported in Tables 3.16 and 3.17 in Appendix 3.D.

3.4.3 Testing the attenuation effect

We use our CKSVAR model to test the null hypothesis of no attenuation in the response of long-term interest rates to monetary policy shocks at the ELB, formalised as H_{NA} in 3.25 in Section 3.3.4. Table 3.4 reports the results of the test for the VAR model that includes long rates of different maturities ranging from 1 year to 10 years. The null is firmly and consistently rejected across different yields to maturity for both countries. The result implies that long rates of different maturities have responded differently to monetary policy shocks between the ELB and non-ELB regimes in both the U.S. and Japan.

Our approach to testing no attenuation effect differs from the one employed by Swanson and Williams (2014) who find that responses of 1- and 2-year U.S. bond yields to various macroeconomic news shocks, identified using high frequency data over the period 1990–2012, were not attenuated throughout 2008 to 2010 but became attenuated in late 2011. We instead focus on responses to a monetary policy shock – a shock to the shadow rate using quarterly data over a longer sample. We discuss the implications of different sample periods at the end of Appendix 3.D.

Table 3.4: Test for no attenuation

	United States			Japan		
YTM	p	LR	p-val	p	LR	p-val
10 or 9	4	15.84	0.000	2	16.63	0.000
7	3	10.33	0.001	2	14.95	0.000
5	3	13.19	0.000	2	15.13	0.000
3	3	17.15	0.000	2	27.70	0.000
2	4	16.81	0.000	2	33.17	0.000
1	3	35.90	0.000	2	46.06	0.000

Note: The table reports corresponding results for a CKSVAR(p) that includes shadow rates. Estimation sample is 1960q1–2019q1 for the U.S. and 1985q3–2019q1 for Japan. Long rates are government bond yields with year to maturity reported in the first column. As baseline specifications, the 10-year yield is used for the U.S. and the 9-year yield is used for Japan. p is the preferred lag length selected by AIC criteria. LR is the likelihood ratio test statistic for the hypothesis of no attenuation. p-value is the asymptotic p-value of the text.

Table 3.5: Test for excluding long rates from VAR by maturity

	United States					Japan				
YTM	sample	p	LR	df	p-val	sample	p	LR	df	p-val
10 or 9	1960q1–2019q1	3	2.43	6	0.876	1985q3–2019q1	2	6.32	4	0.177
7	1970q4–2019q1	3	5.15	6	0.524	1985q3–2019q1	2	10.38	4	0.035
5	1963q2–2019q1	3	4.99	6	0.545	1985q3–2019q1	2	7.00	4	0.136
3	1963q2–2019q1	3	7.05	6	0.316	1985q3–2019q1	2	4.99	4	0.288
2	1977q4–2019q1	4	11.53	8	0.174	1985q3–2019q1	2	3.40	4	0.493
1	1963q2–2019q1	3	11.70	6	0.069	1985q3-2019q1	2	5.19	4	0.268

Note: The estimated model is a CKSVAR(p) for the U.S. and Japan with inflation, output gap, policy rate and a different measure of long-term rates. Long rates are government bond yields with year to maturity reported in the first column (YTM). As baseline specifications, the 10-year yield is used for the U.S. and the 9-year yield is used for Japan. Sample availability varies for each long-term rate used. The null is that the lags of long rates can be excluded from the equations describing the dynamics of inflation and the output gap from the unrestricted model. p is chosen by AIC. LR is the value of the LR test statistic, df is number of exclusion restrictions, and p-val is the asymptotic χ^2_{df} p-value of the test.

3.4.4 Testing the (ir)relevance of long rates

Our statistical tests rejected the irrelevance hypothesis of the ELB and the possibility of excluding the short rate by controlling for the long rate. We now assess whether movements in the short rate, including the shadow rate during ELB regimes, are sufficient to encapsulate the effects of both conventional and unconventional monetary policies for inflation and output by testing the exclusion restriction on the long rate. Proposition 2 implies that the exclusion restriction on the long rate holds in the theoretical model.

We perform this test using the CKSVAR model that includes inflation, output gap, the long rate, and the short rate. The null hypothesis is that lags of the long rate can be excluded from the equations for inflation and output gap. In performing this test, we im-

pose the assumption that the short rate does not react to the long rate by imposing zero restrictions on the lags of long rates in the interest rate equation (3.19a). This is consistent with the standard Taylor rule (3.3) in the theoretical model, and our modelling of QE in equation (3.10). Table 3.5 reports the results using long rates of various maturities. It shows that the null hypothesis cannot be rejected and that the long rate can be excluded from the model at the 5 percent level of significance in both countries and at most maturities. The result implies that the short rate and the shadow rate may be sufficient indicators of monetary policy during the non-ELB and ELB regimes in both countries.

3.5 The effectiveness of UMP

Our testing results established that the dynamics of the economy are different across the ELB and non-ELB regimes for both the U.S. and Japan, leading us to conclude that the ELB has been empirically relevant. But the results are silent on the magnitude of the differences in the effects of monetary policy between the two regimes. Here we address this issue by estimating the (partially identified) impulse responses to a monetary policy shock from the CKSVAR models over time to gauge the effectiveness of UMP relative to conventional monetary policy. In doing so, we use inflation, output gap, the short rate, and the shadow rate for the CKSVAR models, as this specification is broadly in line with the test results obtained in Section 3.4.

3.5.1 State-dependent impulse responses

Since the empirical model is nonlinear, the impulse response functions (IRFs) are state-dependent. We will follow the approach in Koop et al. (1996), already used in Section 3.2.3, according to which the IRF to a monetary policy shock of magnitude ς is given by the difference in the expected path of the endogenous variables when the policy shock takes the value ς, versus the path when the shock is zero, conditional on the state of the economy prior to the shock. This approach is the most commonly used in the literature, see, e.g., Hayashi and Koeda (2019). In our model, there is an additional complication that lagged shadow rates are unobserved, so we evaluate the IRFs at the smoothed estimates of those latent variables. In the same notation of the empirical model in Section 3.3, let $Y_t \equiv (Y_{1t}', i_t)'$ denote the vector of endogenous variables, and $\overline{X}^*_{t,j}$ denote a state vector whose j-th component is given by $\min(i^*_{t-j} - \underline{i}_{t-j}, 0)$ for $j = 1, ..., p$, where p is the order of the VAR. Then, our IRFs starting from period t up to the horizon h are given by:

$$IRF_{h,t}\left(\varsigma, X_t, \widehat{\overline{X}}^*_t\right) = E\left(Y_{t+h}|\varepsilon_{2t} = \varsigma, X_t, \widehat{\overline{X}}^*_t\right) - E\left(Y_{t+h}|\varepsilon_{2t} = 0, X_t, \widehat{\overline{X}}^*_t\right), \quad (3.26)$$

where X_t consists of the lagged values of Y_{t-j} for $j = 1, ..., p$, and $\widehat{\overline{X}}_t^*$ is the smoothed estimate of the state vector $\overline{X}_t^*$ when it is unobserved.

3.5.2 The impact effects of UMP

As explained in Section 3.3, the IRFs are generally set-identified unless we assume there is no contemporaneous effect of UMP on Y_{1t}, which corresponds to setting $\xi = 0$ in the CKSVAR model. We will not be imposing such an assumption in our analysis. We proceed by first obtaining the identified set for ξ, β and γ by solving equations (3.20) and (3.21) at the estimated values of $\widetilde{\beta}$ and Ω, as explained in Section 3.3 (see the discussion following equations (3.20) and (3.21)), and then we simulate the paths of the empirical model at each of the values of the structural parameters in the identified set.

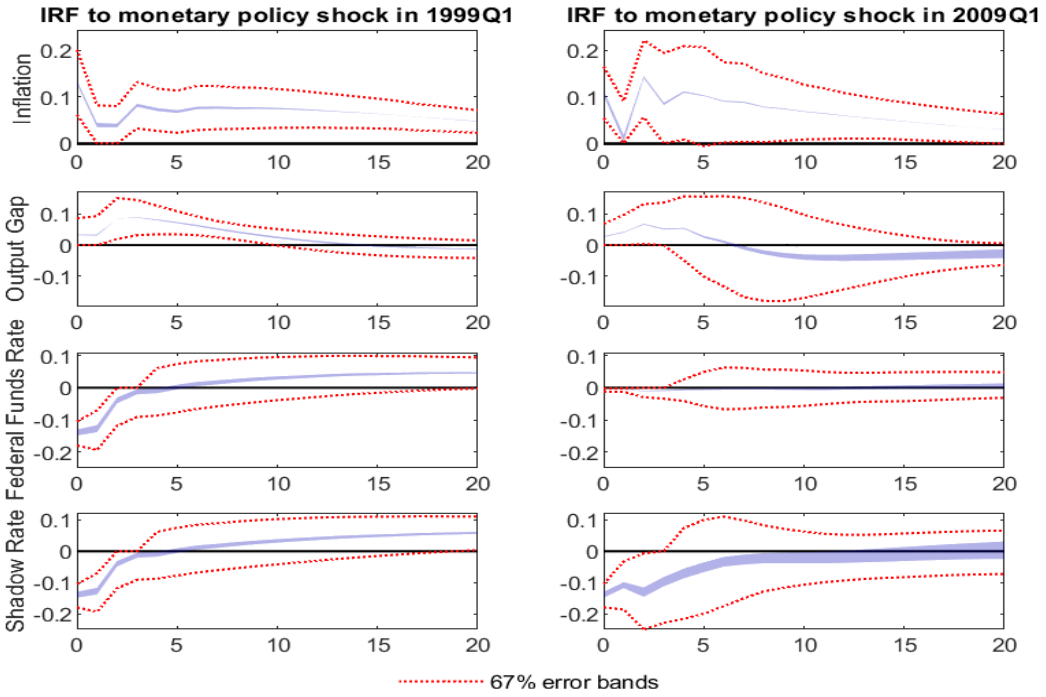
The estimation results for the parameter ξ are as follows. Recall that the parameter ξ determines the impact effect of UMP, where the two limiting cases of $\xi = 0$ and $\xi = 1$ correspond to UMP being completely ineffective on impact and as effective as conventional policy in non-ELB regimes on impact, respectively. When we restrict the range of ξ to $[0, 1]$ and impose no further identifying restrictions, the identified set for ξ is $[0, 0.75]$ for the U.S. and $[0, 0.34]$ for Japan.

We sharpen the identified sets by using sign restrictions. We follow Debortoli et al. (2019) and impose the restrictions that a negative monetary policy shock should have a nonnegative effect on inflation and output, and a nonpositive effect on interest rates at a one-year horizon.[15] These sign restrictions are consistent with the theoretical model studied in Section 3.2 (see Figure 3.2). With these sign restrictions, the identified set for the parameter ξ narrows down substantially for the U.S., from $[0, 0.75]$ to $[0.71, 0.73]$. For Japan the impact of the sign restrictions is more modest, from $[0, 0.34]$ to $[0, 0.26]$.

Appendix 3.E compares our identification based on sign restrictions against the standard Choleski identification that imposes restrictions on the impact response of the variables to the monetary policy shock. Consistent with the results in Gertler and Karadi (2015) and Kubota and Shintani (2022), we find that the Choleski identification presents several puzzling responses.

[15] Note that because IRFs are state-dependent, these sign restrictions need to be imposed for all values of the initial states. In principle, this means working out the worst cases over the support of the distribution of the variables. However, a very similar conservative estimate of the identified set can be obtained if we simply impose the sign restrictions in every period.

Figure 3.5: Impulse responses to a monetary policy shock in the U.S.



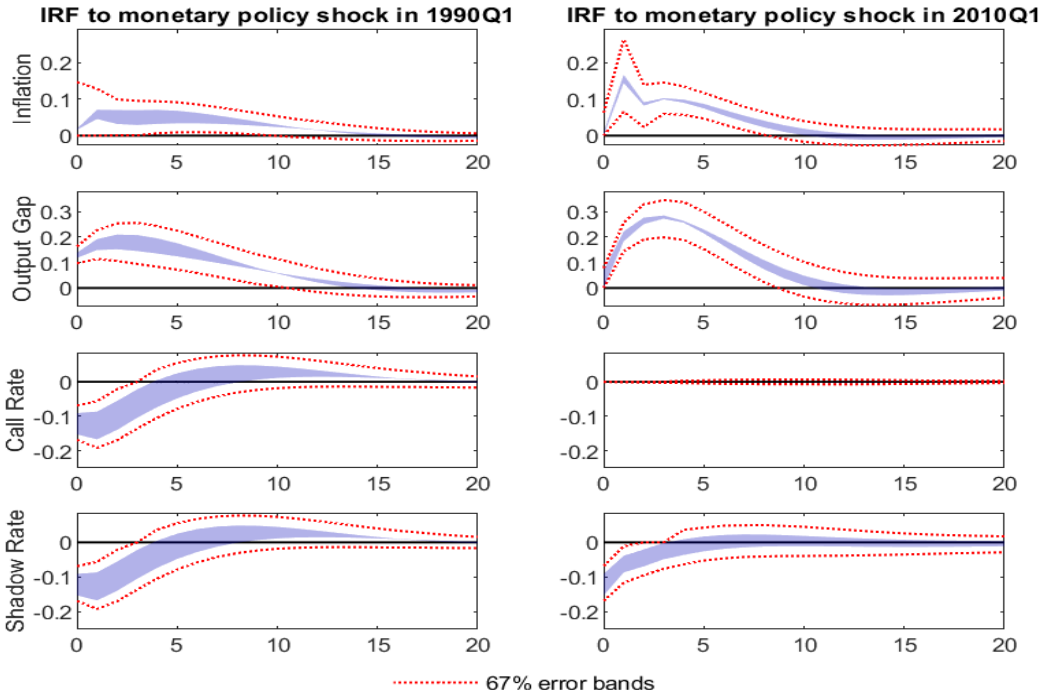
Note: Identified sets of IRFs in 1999q1 and 2009q1 to a -25bps monetary policy shock estimated from CKSVAR(3) model in inflation, output gap, the Federal Funds Rate, and the shadow rate for the U.S. over the period 1960q1-2019q1, identified by the sign restrictions that the shock has nonnegative effects on inflation and output and nonpositive effects on the federal funds rate and the shadow rate up to four quarters. Dotted lines give 67 percent error bands.

3.5.3 The dynamic effects of UMP

Now we study the dynamic effects of UMP by examining the IRFs for the identified sets of ξ for the U.S. and Japan. Figures 3.5 and 3.6 report the identified IRFs of inflation, the output gap, the policy rate, and the shadow rate to a -25 basis points monetary policy shock. The figure also reports asymptotic confidence intervals obtained using the method of Imbens and Manski (2004), where we also impose the sign restrictions on the confidence bands, as in Granziera et al. (2018). The IRFs are computed at two different dates: the left panels report IRFs at dates when interest rates are well above the ELB (1999q1 for the U.S. and 1990q1 for Japan) so that monetary policy is conventional; the right panels report IRFs at dates when interest rates are at the ELB (2009q1 for the U.S. and 2010q1 for Japan) so that monetary policy is unconventional.

The policy effects differ across the two periods. For both countries, the conventional monetary policy shock – the reduction in the short-term interest rate – has a larger *contemporaneous* effect on all variables in the pre-ELB dates than the corresponding unconventional policy shock – the reduction in the shadow rate – during the ELB dates, and the difference is larger in Japan than in the U.S. This is because ξ is estimated lower in Japan

Figure 3.6: Impulse responses to a monetary policy shock in Japan



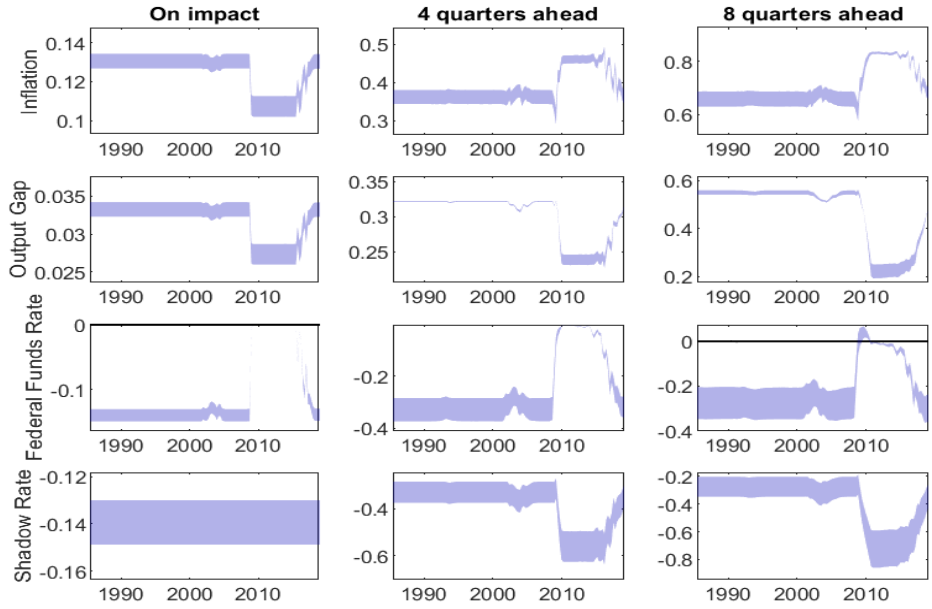
Note: Identified sets of IRFs in 1990q1 and 2010q1 to a -25bps point monetary policy shock estimated using a CKSVAR(2) model in inflation, output gap, the call rate, and the shadow rate for Japan over the period 1985q3-2019q1, identified by the sign restrictions that the shock has nonnegative effects on inflation and output and nonpositive effects on the call rate and the shadow rate up to four quarters. Dotted lines give 67 percent error bands.

than in the U.S. However, in Japan the impulse responses to the unconventional policy shock appear to be stronger a few quarters out.[16] In addition, the responses of short rates and shadow rates are identical when both rates are far above the ELB. By contrast, the shadow rates decrease whilst the responses of short rates are muted during the ELB dates. The reductions in shadow rates affect inflation and output through unconventional policy during the ELB regime.

To shed further light on the differences in the responses over time, and in light of the fact that the IRFs are time-varying, we look at the evolution of the impulse responses at given horizons, 0, 4 and 8 quarters, over time. The results are reported in Figures 3.7 and 3.8 for the U.S. and Japan, respectively. In each figure, the panels on the left column report the impact effects of a -25 basis point monetary policy shock at each quarter from 1985q3 to the end of our sample. The panels in the middle column show the cu-

[16]The reason why the delayed effects of UMP can be stronger than conventional policy even though $\xi < 1$ is because in the empirical model the coefficients on the lags of the shadow rate are completely unrestricted. This is more general than the theoretical model of Section 3.2 with the monetary policy rule (3.3), where the coefficient on the lagged shadow rate was restricted to be a constant fraction λ^* of the coefficient on the lagged policy rate above the ELB. The result of that restriction was that $\lambda^* < 1$ restricted UMP to have a uniformly weaker effect than conventional policy over all horizons. We did not need to impose this overidentifying restriction in the empirical analysis.

Figure 3.7: Responses to monetary policy shock in the U.S. over time



Note: Identified sets of (cumulative) impulse responses to a negative -25bps monetary policy shock at three different horizons, on impact, one year out and two years out, estimated using a CKSVAR(3) model in inflation, output gap, the Federal Funds Rate, and the shadow rate for the U.S. over the period 1960q1-2019q1, identified by the sign restrictions that the shock has nonnegative effects on inflation and output and nonpositive effects on the federal funds rate and the shadow rate up to four quarters.

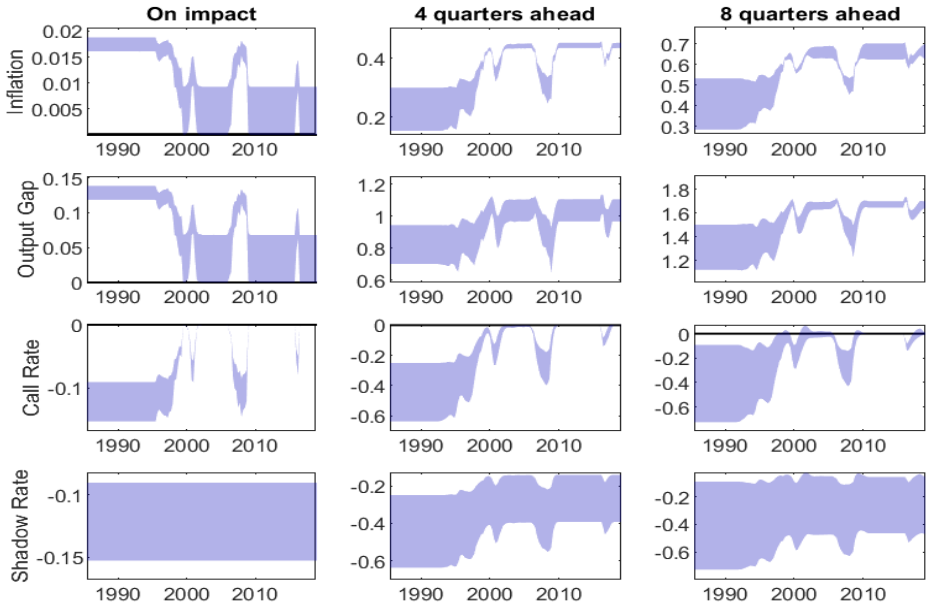
mulative impulse responses after a year, while the panels on the right column give the corresponding cumulative responses after two years.

Before discussing each country, it is worth noting that the impact effect of the shock on shadow rates is invariant and non-zero over time in both countries. This is because the shadow rates are unrestricted across both ELB and non-ELB regimes. By contrast, the responses of short rates vary according to the probability of entering the ELB regime over time. When the economy is far above the ELB, the effects on short rates are identical to those on shadow rates. As the short rates approach the ELB, the probability that the ELB constraint binds increases, and the average response of short rates diminishes.

Now we discuss each country in turn. In the U.S. (Figure 3.7), we see a drop in the impact effect of policy during the ELB period relative to the pre-ELB period. The relative difference in the effectiveness of policy on both inflation and output on impact is over 15 percent. For the output gap, this difference remains, and increases one and two years ahead. However, the effect on inflation is the reverse: the cumulative effect of UMP on inflation is much stronger one and two years ahead. Therefore, UMP in the U.S. seems to have had a delayed but strong effect on inflation, but has been persistently less effective on output than conventional policy.

In Japan (Figure 3.8), there is a clear drop in the contemporaneous effect of policy

Figure 3.8: Responses to monetary policy shock in Japan over time



Note: Identified sets of (cumulative) impulse responses to a -25bps monetary policy shock at three different horizons, on impact, one year out and two years out, estimated using a CKSVAR(2) model in inflation, output gap, the call rate, and the shadow rate for Japan over the period 1985q3-2019q1, identified by the sign restrictions that the shock has nonnegative effects on inflation and output and nonpositive effects on the call rate and the shadow rate up to four quarters.

on inflation and output during the ELB periods. There are three distinguishable ELB periods, 1999q2-2000q2, 2001q2-2006q2, and 2009q1 to the end of the sample. The contemporaneous policy effect on inflation is negligible, but the delayed effect one and two years later is stronger during the ELB periods than outside them. Like for the U.S., there is a stronger delayed effect of UMP on inflation in Japan. Turning to the policy effect on output, we see that UMP is more than 50 percent weaker on impact, but catches up within one year, and stays stronger two years out. So, unlike the U.S., where UMP has smaller effects on output at all horizons, in Japan, this is not the case.[17]

In sum, in the U.S. the response of inflation to UMP has been stronger than to conventional policy one or two years ahead, while it has been weaker for output over the same horizon. In Japan, UMP has had weaker effects than conventional policy on inflation and output on impact, but has had stronger delayed effects on these variables. In general, the findings show that the effects of UMP have been different across time, lending further support to our earlier finding that the ELB has been empirically relevant in both countries over that period.[18]

[17]The estimation results for Japan are based on a second-order VAR selected by the AIC and the sequential LR tests. Because these criteria are known to lead to overfitting and our sample for Japan is relatively small (34 years), we investigated the robustness of the above results to a first-order VAR, and found that our conclusions continue to hold. The results are available through our replication code.

[18]Appendix 3.F reports estimates of the shadow rate for Japan and the U.S. that can be interpreted as

3.5.4 Discussion

Our analysis of the Japanese data draws heavily on the seminal contribution of Hayashi and Koeda (2019). Specifically, the use of trend growth that they proposed to control for the decline in the short rate over our sample is essential to get a VAR that satisfies the sign restrictions on the IRFs. There are several apparent methodological differences between our papers. Hayashi and Koeda (2019) use monthly data over a shorter period 1992-2012, while we use quarterly data from 1985 to 2019. They model QE via excess reserves and FG via an exit condition on inflation, while we rely on the shadow rate to capture both forms of UMP and other forms of UMP such as the purchases of long-term government bonds, motivated by the theoretical model of Section 3.2. They use recursive identification as well as exclusion restrictions on the dynamics of the policy reaction function, assuming that inflation and output are predetermined, while we do not, and rely instead on sign restrictions and the changes in the dynamics and variances across regimes for identification. However, these differences are not as large as they appear. For example, their results show that inflation and output are not predetermined at quarterly frequency, which is consistent with our findings, since the identified impact effect of monetary policy easing on inflation and output is positive within the quarter. Both models have regime-dependent decision rules that are fairly similar when translated to quarterly frequency. Finally, even though they provide convincing documentation that an inflation exit condition fits better the narrative of Japanese monetary policy over their sample period, the use of a Reifschneider and Williams (2000) FG rule appears nearly observationally equivalent to an inflation exit condition because of the relative scarcity of movements in and out of the ELB regime over the sample. This explains why our conclusions are broadly consistent with theirs.[19]

Finally, our theoretical model abstracts from possible negative effects of UMP such as those of 'the reversal interest rate' (Brunnermeier and Koby, 2019), and our empirical analysis excludes those effects through sign restrictions. If we remove the sign restrictions during ELB regimes and maintain them during non-ELB regimes, we can allow ξ to be negative, which can capture policy reversals on impact. It turns out that removing the sign restrictions during the ELB periods does not affect the identified set for ξ for the impulse responses for the U.S., while the effect for Japan is limited.[20]

indicators of the desired monetary policy stance in terms of the short-term interest rate during ELB regimes.

[19]For example, the rejection of the irrelevance hypothesis (3.22) for Japan is due to both $\widetilde{\beta} \neq 0$ and $C_{12}^{*} \neq 0$. This accords with Hayashi and Koeda (2019), who report significant changes both in the constant term as well as the coefficient on the lag of the policy variable across regimes.

[20]The identified set for ξ for Japan becomes $[-0.08, 0.29]$, which includes negative responses of inflation and output to a decrease in the shadow rate under the ELB (an expansionary UMP shock) on impact. However, the delayed responses are not significantly affected, and thus any possible negative effects of UMP are short-lived.

3.6 Conclusion

The paper develops theoretical and empirical models to study the effectiveness of unconventional monetary policy in the U.S. and Japan. The theoretical model allows the degree of effectiveness of unconventional policy to range from being as effective as conventional policy to being completely ineffective, and it provides theoretical underpinnings to the empirical model and our approach to testing the irrelevance hypothesis of the ELB. The empirical analysis is based on an agnostic structural VAR model that accounts for the ELB on the policy rate and captures unconventional policy via the shadow rate. Our results provide strong evidence against the hypothesis that the ELB is empirically irrelevant, and support the view that the ELB has been an important constraint on monetary policy in both the U.S. and Japan. However, our results also reveal strong delayed effects and country-specific differences in the effect of unconventional policy relative to conventional policy.

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Appendix

3.A Theoretical Model

Appendix 3.A presents a simple New Keynesian model with an effective lower bound (ELB) and unconventional monetary policy (UMP). The model is a version of a preferred habitat model such as Chen et al. (2012), extended to incorporate two things: a policy rule for quantitative easing (QE) that is operated using a shadow rate as policy guidance and forward guidance (FG) in the spirit of Reifschneider and Williams (2000). To keep the analysis focused on the salient features of the transmission mechanisms of UMP, the model abstracts from capital accumulation and consumption habit formation. There are three shocks: a demand (preference) shock, a supply (productivity) shock, and a monetary policy shock.

3.A.1 Model building blocks

3.A.1.1 Long-term bonds

There is a long-term government bond (consol bond). The long-term bond issued at time t yields μ^{j-1} dollars at time $t+j$ over time. Let $R_{L,t+1}$ denote the gross nominal rate from time t to $t+1$. The period-t price of the bond issued at time t, $P_{L,t}$, is defined as

$$\begin{aligned} P_{L,t} &= E_t\left(\frac{1}{R_{L,t+1}} + \frac{\mu}{R_{L,t+1}R_{L,t+2}} + \frac{\mu^2}{R_{L,t+1}R_{L,t+2}R_{L,t+3}} + ...\right) \\ &= E_t\left(\frac{1}{R_{L,t+1}} + \frac{\mu}{R_{L,t+1}}P_{L,t+1}\right). \end{aligned} \tag{3.27}$$

The gross yield to maturity (or the long-term interest rate) at time t, $\bar{R}_{L,t}$ is defined as

$$E_t\left(\frac{1}{\bar{R}_{L,t}} + \frac{\mu}{\left(\bar{R}_{L,t}\right)^2} + \frac{\mu^2}{\left(\bar{R}_{L,t}\right)^3} + ...\right) = P_{L,t},$$

or

$$P_{L,t} = \frac{1}{\bar{R}_{L,t} - \mu}. \tag{3.28}$$

Let $B_{L,t|t-s}$ denote period-t holdings of bonds that were issued at time $t-s$. Suppose that a household has $B_{L,t|t-s}$ for $s = 1, 2, ...$ in the beginning of period t. The total amount of dividends the household receives in period t is

$$\sum_{s=1}^{\infty} \mu^{s-1} B_{L,t|t-s}.$$

Note that having one unit of $B_{L,t|t-s}$ is equivalent to having μ^{s-1} units of $B_{L,t|t-1}$ because they both yield μ^{s-1} dollars. The total amount of dividends then can be expressed in terms of $B_{L,t|t-1}$ as

$$\sum_{s=1}^{\infty} \mu^{s-1} B_{L,t|t-s} \equiv B_{L,t-1},$$

where $B_{L,t-1}$ denotes the amount of bonds in units of the bonds issued at time $t-1$, held by the household in the beginning of period t. Let $P_{L,t|t-s}$ denote the time-t price of the bond issued at time $t-s$. Then, the value of all bonds at time t is

$$\sum_{s=1}^{\infty} P_{L,t|t-s} B_{L,t|t-s}.$$

The bond price satisfies

$$\begin{aligned} P_{L,t|t-s} &= E_t \left(\frac{\mu^s}{R_{L,t+1}} + \frac{\mu^{s+1}}{R_{L,t+1} R_{L,t+2}} + \frac{\mu^{s+2}}{R_{L,t+1} R_{L,t+2} R_{L,t+3}} + ... \right) \\ &= \mu^s P_{L,t} \end{aligned}$$

Then the value of all bonds at time t is

$$\sum_{s=1}^{\infty} P_{L,t|t-s} B_{L,t|t-s} = P_{L,t} \mu \sum_{s=1}^{\infty} \mu^{s-1} B_{L,t|t-s} = \mu P_{L,t} B_{L,t-1}.$$

So the return of holding $B_{L,t-1}$ is given by the sum of dividends and the value of all bonds as:

$$B_{L,t-1} + \mu P_{L,t} B_{L,t-1} = (1 + \mu P_{L,t}) B_{L,t-1} = P_{L,t} \bar{R}_{L,t} B_{L,t-1} = \frac{\bar{R}_{L,t}}{\bar{R}_{L,t} - \mu} B_{L,t-1}.$$

3.A.1.2 Households

There are two types of households: unrestricted households (U-households) and restricted households (R-households). U-households, with population ω_u, can trade both short-term and long-term government bonds subject to a transaction cost ζ_t per unit of long-term bonds purchased. R-households, with population $\omega_r = 1 - \omega_u$, can trade only long-term government bonds. For $j = u, r$, each household chooses consumption c_t^j, hours worked h_t^j, the long-term government bond holdings $B_{L,t}^j$, and the short-term government bond holdings $B_{S,t}^j$ to maximize utility,

$$\sum_{t=0}^{\infty} \beta_j^t d_t \left[\frac{\left(c_t^j\right)^{1-\sigma}}{1-\sigma} - \psi \frac{\left(h_t^j\right)^{1+1/\nu}}{1+1/\nu} \right],$$

subject to: for a U-household,

$$P_t c_t^u + B_{S,t}^u + (1+\zeta_t) P_{L,t} B_{L,t}^u = (1+i_{t-1}) B_{t-1}^u + P_{L,t} \bar{R}_{L,t} B_{L,t-1}^u + W_t h_t^u - T_t^u + \Pi_t^u,$$

and for a R-household,

$$P_t c_t^r + P_{L,t} B_{L,t}^r = P_{L,t} \bar{R}_{L,t} B_{L,t-1}^r + W_t h_t^r - T_t^r + \Pi_t^r,$$

where P_t is the price level and i_t is the short-term interest rate. In addition $\bar{R}_{L,t}$ denotes the gross yield to maturity at time t on the long-term bond

$$\bar{R}_{L,t} = \frac{1}{P_{L,t}} + \mu, \quad 0 < \mu \leq 1.$$

The average duration of the bond is given by $\bar{R}_{L,t} / \left(\bar{R}_{L,t} - \mu\right)$. There is a shock d_t to the preference, and it is given by:

$$d_t = \begin{cases} e^{z_1^b} e^{z_2^b} ... e^{z_t^b} & \text{for } t \geq 1 \\ 1 & \text{for } t = 0 \end{cases},$$

where z_t^b is a preference (demand) shock, which is assumed to follow an AR(1) process

$$z_t^b = \rho_b z_{t-1}^b + \epsilon_t^b,$$

with $\epsilon_t^b \sim$ i.i.d. $N\left(0, \sigma_b^2\right)$.

We assume that the transaction cost of trading long-term bonds for the U-households is collected by financial firms and redistributed as a lump-sum profits to the U-households. Under the assumption, the transaction cost does not appear in the goods market clearing condition, which is given by:

$$y_t = \omega_u c_t^u + (1 - \omega_u) c_t^r. \tag{3.29}$$

Arranging the first-order conditions of the U-household's problem yields the following optimality conditions:

$$w_t = \psi (c_t^u)^\sigma (h_t^u)^{1/\nu}, \tag{3.30}$$

$$1 = E_t \beta_u e^{z_{t+1}^b} \left(\frac{c_{t+1}^u}{c_t^u} \right)^{-\sigma} \frac{1 + i_t}{\pi_{t+1}}, \tag{3.31}$$

$$1 + \zeta_t = E_t \beta_u e^{z_{t+1}^b} \left(\frac{c_{t+1}^u}{c_t^u} \right)^{-\sigma} \frac{R_{L,t+1}}{\pi_{t+1}}, \tag{3.32}$$

where $w_t \equiv W_t / P_t$ denotes the real wage, $\pi_t \equiv P_t / P_{t-1}$ denotes the inflation rate, and $R_{L,t+1}$ denotes the yield of the long-term bond between periods t and $t+1$, given by

$$R_{L,t+1} \equiv \frac{P_{L,t+1}}{P_{L,t}} \bar{R}_{L,t+1} = \frac{P_{L,t+1}}{P_{L,t}} \left(\frac{1}{P_{L,t+1}} + \mu \right) = \frac{1 + \mu P_{L,t+1}}{P_{L,t}}.$$

Similarly, arranging the first-order conditions of the R-household's problem yields

$$w_t = \psi (c_t^r)^\sigma (h_t^r)^{1/\nu}, \tag{3.33}$$

$$1 = E_t \beta_r e^{z_{t+1}^b} \left(\frac{c_{t+1}^r}{c_t^r} \right)^{-\sigma} \frac{R_{L,t+1}}{\pi_{t+1}}. \tag{3.34}$$

3.A.1.3 Firms

The firm sector consists of two types of firms: final-goods-producing firms and intermediate-goods-producing firms. The problems of these firms are standard except that the average discount rate between U-households and R-households is used in discounting the profits of these firms. The profits need to be derived explicitly because one of the two households' budget constraints constitutes an equilibrium condition as well as the goods market clearing condition.

Each final-goods-producing firm produces a unit of final goods y_t in a competitive

market by combining intermediate goods $\{y_t(l)\}_{l=0}^{1}$ according to

$$y_t = \left[\int_0^1 y_t(l)^{\frac{1}{\lambda_p}} dl\right]^{\lambda_p}, \ \lambda_p > 1.$$

The demand function for the l-th intermediate good is given by

$$y_t(l) = \left(\frac{P_t(l)}{P_t}\right)^{\frac{\lambda_p}{1-\lambda_p}} y_t.$$

Each intermediate-goods-producing firm uses labor and produce intermediate goods according to

$$y_t(l) = e^{z_t^a} h_t(l)^{\theta}, \ 0 < \theta \leq 1.$$

where z_t^a is a productivity shock, which is assumed to follow

$$z_t^a = \rho_a z_{t-1}^a + \epsilon_t^a,$$

with $\epsilon_t^a \sim$ i.i.d. $N(0, \sigma_a^2)$. Because there is no price dispersion in steady state, the aggregate output can be expressed up to the first-order approximation as:

$$\hat{y}_t = z_t^a + \theta \hat{h}_t, \tag{3.35}$$

where $\hat{y}_t$ and $\hat{h}_t$ denote the aggregate output and hours worked in terms of deviation from the steady state. The total cost of producing $y_t(l)$ is equal to

$$W_t h_t(l) = W_t \left(\frac{y_t(l)}{e^{z_t^a}}\right)^{\frac{1}{\theta}}.$$

In each period, intermediate-goods-producing firms can change their price with probability ξ identically and independently across firms and over time. For each l, the l-th intermediate-goods producing firm chooses the price, $\tilde{P}_t(l)$, to maximize the discounted sum of profits,

$$\max_{\tilde{P}_t(l)} E_t \sum_{s=0}^{\infty} (\xi\delta)^s \bar{\Lambda}_{t+s|t} \left[P_{t+s}(l) y_{t+s}(l) - W_{t+s}\left(\frac{y_{t+s}(l)}{e^{z_{t+s}^a}}\right)^{\frac{1}{\theta}}\right],$$

subject to the demand curve,

$$y_{t+s}(l) = \left(\frac{P_{t+s}(l)}{P_{t+s}} \right)^{\frac{\lambda_p}{1-\lambda_p}} y_{t+s},$$

where

$$\delta = \omega_u \beta_u + (1-\omega_u)\beta_r,$$

$$\bar{\Lambda}_{t+s|t} \equiv d_{t+s|t} \left(\omega_u \Lambda^u_{t+s|t} + (1-\omega_u) \Lambda^r_{t+s|t} \right),$$

$$\Lambda^j_{t+s|t} = \left(\frac{c^j_{t+s}}{c^j_t} \right)^{-\sigma} \frac{1}{P_{t+s}}, \quad d_{t+s|t} = \begin{cases} 1 & \text{if } s = 0 \\ e^{z^b_{t+1}} e^{z^b_{t+2}} ... e^{z^b_{t+s}} & \text{if } s = 1, 2, ... \end{cases}$$

$$P_{t+s}(l) = \tilde{P}_t(l) \Pi^p_{t,t+s},$$

$$\Pi^p_{t+s|t} = \begin{cases} 1 & \text{if } s = 0 \\ \prod_{k=1}^{s} (\pi_{t+k-1})^{\iota_p} (\pi)^{1-\iota_p} & \text{if } s = 1, 2, ... \end{cases}$$

The presence of $\Pi^p_{t+s|t}$ implies price indexation for firms that do not have a chance to change prices and $0 \leq \iota_p \leq 1$ governs the degree of indexation to the past inflation rates. Substituting the demand curve into the objective function yields

$$\max_{\tilde{P}_t(l)} E_t \sum_{s=0}^{\infty} (\xi\delta)^s \bar{\Lambda}_{t+s|t} \left[\tilde{P}_t(l) \Pi^p_{t+s|t} \left(\frac{\tilde{P}_t(l) \Pi^p_{t+s|t}}{P_{t+s}} \right)^{\frac{\lambda_p}{1-\lambda_p}} Y_{t+s} - W_{t+s} \left(\frac{\tilde{P}_t(l) \Pi^p_{t+s|t}}{P_{t+s}} \right)^{\frac{\lambda_p}{(1-\lambda_p)\theta}} \left(\frac{y_{t+s}}{e^{z^a_{t+s}}} \right)^{\frac{1}{\theta}} \right].$$

The first-order condition is

$$0 = E_t \sum_{s=0}^{\infty} (\xi\delta)^s \bar{\Lambda}_{t+s|t} \left[\frac{1}{1-\lambda_p} \Pi^p_{t+s|t} y_{t+s}(l) - W_{t+s} \frac{\lambda_p}{(1-\lambda_p)\theta} \left(\frac{y_{t+s}(l)}{e^{z^a_{t+s}}} \right)^{\frac{1}{\theta}} \frac{1}{\tilde{P}_t(l)} \right].$$

Since $\tilde{P}_t(l)$ does not depend on l, index l is omitted hereafter. Define $\tilde{p}_t \equiv \tilde{P}_t / P_t$ and

$$\tilde{\Pi}^p_{t+s|t} = \begin{cases} 1 & \text{if } s = 0 \\ \prod_{k=1}^{s} \frac{(\pi_{t+k-1})^{\iota_p} (\pi)^{1-\iota_p}}{\pi_{t+k}} & \text{if } s = 1, 2, ... \end{cases}$$

The first-order condition can be transformed as

$$0 = E_t \sum_{s=0}^{\infty} (\xi\delta)^s \bar{\Lambda}_{t+s} P_{t+s} \left[\frac{1}{1-\lambda_p} \frac{\Pi^p_{t+s|t}}{P_{t+s}} \left(\tilde{p}_t \tilde{\Pi}^p_{t+s|t} \right)^{\frac{\lambda_p}{1-\lambda_p}} y_{t+s} \right.$$
$$\left. - \frac{W_{t+s}}{P_{t+s}} \frac{\lambda_p}{(1-\lambda_p)\theta} \left(\tilde{p}_t \tilde{\Pi}^p_{t+s|t} \right)^{\frac{\lambda_p}{(1-\lambda_p)\theta}} \left(\frac{Y_{t+s}}{e^{z^a_{t+s}}} \right)^{\frac{1}{\theta}} \frac{1}{\tilde{P}_t} \right],$$

This equation can be written as:

$$\tilde{p}_t = \left(\frac{\lambda_p}{\theta} \frac{\omega_u K^u_{p,t} + (1-\omega_u) K^r_{p,t}}{\omega_u F^u_{p,t} + (1-\omega_u) F^r_{p,t}} \right)^{\frac{(1-\lambda_p)\theta}{\theta-\lambda_p}}, \tag{3.36}$$

where for $j \in \{r, u\}$

$$F^j_{p,t} = (c^j_t)^{-\sigma} y_t + \xi\delta E_t e^{z^b_{t+1}} (\tilde{\Pi}^p_{t+1|t})^{\frac{1}{1-\lambda_p}} F^j_{p,t+1}, \tag{3.37}$$

$$K^j_{p,t} = (c^j_t)^{-\sigma} \left(\frac{y_t}{e^{z^a_t}} \right)^{\frac{1}{\theta}} w_t + \xi\delta E_t e^{z^b_{t+1}} (\tilde{\Pi}^p_{t+1|t})^{\frac{\lambda_p}{(1-\lambda_p)\theta}} K^j_{p,t+1}. \tag{3.38}$$

The aggregate price level evolves following

$$P_t = \left[\xi [(\pi_{t-1})^{\iota_p} (\pi)^{1-\iota_p} P_{t-1}]^{\frac{1}{1-\lambda_p}} + (1-\xi) \tilde{P}_t^{\frac{1}{1-\lambda_p}} \right]^{1-\lambda_p},$$

which can be written as

$$\tilde{p}_t = \left[\frac{1 - \xi (\tilde{\Pi}^p_{t|t-1})^{\frac{1}{1-\lambda_p}}}{1-\xi} \right]^{1-\lambda_p}. \tag{3.39}$$

The conditions, (3.36)-(3.39), summarize the price setting behavior of intermediate-goods-producing firms.

The aggregate nominal profits earned by intermediate-goods-producing firms are given by:

$$\Pi^m_t = \int_0^1 \left(P_t(l) y_t(l) - W_t \left(\frac{y_t(l)}{e^{z^a_t}} \right)^{\frac{1}{\theta}} \right) dl = P_t y_t - W_t \left(\frac{y_t}{e^{z^a_t}} \right)^{\frac{1}{\theta}},$$

where the last equality holds up to the first-order approximation. Then, the aggregate real profits are given by $\pi^m_t = y_t - w_t \left(y_t / e^{z^a_t} \right)^{1/\theta}$.

3.A.1.4 Government

The government flow budget constraint is

$$(1+i_{t-1})\,B_{S,t-1}+(1+\mu P_{L,t})\,B_{L,t-1}=B_{S,t}+P_{L,t}B_{L,t}+T_t,$$

where $T_t=\omega_u T_t^u+(1-\omega_u)\,T_t^r$. We assume that the lump-sum tax is imposed on households equally so that $T_t^u=T_t^r=T_t$. To focus on the role of long-term government bonds, we assume that the amount of short-term bonds is constant at $b_{S,t}\equiv B_{S,t}/P_t=\bar{b}_S$.

3.A.1.5 Central bank

The nominal interest rate i_t set by the central bank is bounded below by the ELB as

$$i_t=\max\left\{i_t^*,\underline{i}\right\}, \tag{3.40}$$

where $\underline{i}$ is the ELB and i_t^* is a shadow rate – the short-term rate the central bank would set if there were no ELB. The shadow rate i_t^* is given by[21]

$$i_t^*=i_t^{\text{Taylor}}-\alpha\left(i_t-i_t^{\text{Taylor}}\right). \tag{3.41}$$

The shadow rate i_t^* consists of two parts: i_t^{Taylor} and $\alpha(i_t-i_t^{\text{Taylor}})$. First, i_t^{Taylor} is the Taylor-rule-based rate that responds to inflation π_t, output y_t, and the lagged 'effective' interest rate $(1-\lambda^*)i_{t-1}+\lambda^* i_{t-1}^*$:

$$i_t^{\text{Taylor}}-i=\rho_i\left((1-\lambda^*)i_{t-1}+\lambda^* i_{t-1}^*-i\right)+(1-\rho_i)\left[r_\pi\log\left(\pi_t/\pi\right)+r_y\log\left(y_t/y\right)\right]+\epsilon_t^i, \tag{3.42}$$

where ϵ_t^i is a monetary policy shock and variables without subscripts denote those in steady state. The parameter λ^* will be derived later in this appendix. Second, $\alpha(i_t-i_t^{\text{Taylor}})$ in equation (3.41) encapsulates the strength of FG. A positive value for α will maintain the target rate i_t^* below the Taylor rate i_t^{Taylor}. Under the ELB of $i_t=\underline{i}$, the more the

[21] Reifschneider and Williams (2000) use the following rule: $i_t^*=i_t^{\text{Taylor}}-\alpha Z_t$ and $Z_t=\rho_Z Z_{t-1}+(i_t-i_t^{\text{Taylor}})$ with $\rho_Z=1$.

central bank has missed to set the interest rate at its Taylor rate, the lower the central bank sets its target rate i_t^* through equation (3.41) as long as $\rho_i \lambda^* > 0$ in equation (3.42).[22]

The central bank activates QE when the economy hits the ELB. The central bank continues using the shadow rate as policy guidance in an ELB regime as in a non-ELB regime. Specifically, the amount of long-term bond purchases depends on the shadow rate, and as a result the amount of long-term government bonds, $b_{L,t} \equiv B_{L,t}/P_t$, held by the private agents is given by:

$$\hat{b}_{L,t} = \begin{cases} 0 & \text{if } i_t^* \geq \underline{i} \\ \gamma \frac{i_t^* - \underline{i}}{1+\underline{i}} & \text{if } i_t^* < \underline{i} \end{cases}, \tag{3.43}$$

where the caret on a variable denotes a deviation from the steady state. This QE rule implies that asset purchases by the central bank is zero (relative to the steady state) when the ELB is not binding (i.e. $i_t = i_t^* \geq \underline{i}$) and, given $\gamma > 0$, the purchases are positive (i.e., $\hat{b}_{L,t} < 0$) when the shadow rate goes below the ELB (i.e., $i_t^* < \underline{i}$).

3.A.1.6 Market clearing and equilibrium

As well as the goods market clearing condition (3.29), there are market clearing conditions for labor, long-term government bonds, and short-term government bonds:

$$\omega_u h_t^u + (1 - \omega_u) h_t^r = h_t, \tag{3.44}$$

$$\omega_u b_{L,t}^u + (1 - \omega_u) b_{L,t}^r = b_{L,t}, \tag{3.45}$$

$$\omega_u b_{S,t}^u = b_{S,t} \tag{3.46}$$

Also, either the U-household's budget constraint or the R-household's budget constraint should be added as an equilibrium condition. Here the latter budget constraint is added:

$$c_t^r + P_{L,t} b_{L,t}^r = \left(\bar{R}_{L,t}/\pi_t\right) P_{L,t} b_{L,t-1}^r + w_t h_t^r - T_t^r/P_t + \Pi_t^r/P_t, \tag{3.47}$$

[22] Debortoli et al. (2019) consider the case of $\alpha = 0$ and $\lambda^* = 1$ in equation (3.42) and interpret ρ_i – the coefficient of interest rate smoothing – as FG when i_t^* is below the ELB.

where

$$\frac{T_t^r}{P_t} = -\left(b_{S,t} + P_{L,t} b_{L,t}\right) + \frac{1 + i_{t-1}}{\pi_t} b_{S,t-1} + \frac{1 + \mu P_{L,t}}{\pi_t} b_{L,t-1},$$
$$\frac{\Pi_t^r}{P_t} = y_t - w_t h_t.$$

The cost of trading long-term bonds, ζ_t, is specified as

$$\frac{\zeta_t}{\zeta} = \left(\frac{b_{L,t}}{b_L}\right)^{\rho_\zeta}, \qquad \rho_\zeta > 0. \tag{3.48}$$

The trading cost is increasing in the amount of long-term bonds relative to its steady state value. The trading cost is ζ in steady state.

The system of equations for the economy consists of 19 equations, (3.29)-(3.47), with the following endogenous variables:

$$c_t^u, c_t^r, h_t^u, h_t^r, h_t, b_{L,t}^u, b_{L,t}^r, b_{L,t}, b_t^u, y_t, w_t, i_t, i_t^*, i_t^{\text{Taylor}}, R_{L,t}, \pi_t, \tilde{p}_t, F_{p,t}^j, K_{p,t}^j.$$

3.A.2 Log-linearized equations

We log-linearize the equilibrium conditions of the theoretical model presented in Appendix 3.A.1 around the steady state in which inflation is equal to the target rate of inflation set by the central bank. By doing so, we derive key equations in the system of equations (3.1)-(3.16) presented in Section 3.2 of the main text. We also derive a log-linearized equation for the long-term yield.

Euler equation. Log-linearizing equations (3.29), (3.31), (3.32), (3.34), and (3.48), we obtain[23]

$$\hat{y}_t = \frac{\omega_u c^u}{y} \hat{c}_t^u + \frac{(1 - \omega_u)\, c^r}{y} \hat{c}_t^r. \tag{3.49}$$

$$0 = E_t\left[-\sigma\left(\hat{c}_{t+1}^u - \hat{c}_t^u\right) + \hat{\imath}_t - \hat{\pi}_{t+1} + z_{t+1}^b\right], \tag{3.50}$$

$$\frac{\zeta}{1 + \zeta} \hat{\zeta}_t = E_t\left[-\sigma\left(\hat{c}_{t+1}^u - \hat{c}_t^u\right) + \hat{R}_{L,t+1} - \hat{\pi}_{t+1} + z_{t+1}^b\right], \tag{3.51}$$

[23] The variable $\hat{\imath}$ represents the deviation of the gross interest rate from the steady state.

$$0 = E_t\left[-\sigma\left(\hat{c}^r_{t+1} - \hat{c}^r_t\right) + \hat{R}_{L,t+1} - \hat{\pi}_{t+1} + z^b_{t+1}\right], \tag{3.52}$$

$$\hat{\zeta}_t = \rho_\zeta \hat{b}_{L,t}. \tag{3.53}$$

Equation (3.49) can be written as:

$$\hat{c}^u_t = \frac{y}{\omega_u c^u}\left\{\hat{y}_t - \frac{(1-\omega_u)\,c^r}{y}\hat{c}^r_t\right\}.$$

Subtracting $\hat{c}^u_{t+1}$ from $\hat{c}^u_t$ yields:

$$\begin{aligned}\hat{c}^u_{t+1} - \hat{c}^u_t &= \frac{y}{\omega_u c^u}\left\{\hat{y}_{t+1} - \hat{y}_t - \frac{(1-\omega_u)\,c^r}{y}\left(\hat{c}^r_{t+1} - \hat{c}^r_t\right)\right\}, \\ &= \frac{y}{\omega_u c^u}\left\{\hat{y}_{t+1} - \hat{y}_t - \frac{(1-\omega_u)\,c^r}{y}\frac{\left(\hat{R}_{L,t+1} - \hat{\pi}_{t+1} + z^b_{t+1}\right)}{\sigma}\right\},\end{aligned} \tag{3.54}$$

where equation (3.52) was used in the second equality. Substituting equation (3.54) into equation (3.50) yields:

$$\begin{aligned}0 &= E_t\left[-\sigma\left(\hat{c}^u_{t+1} - \hat{c}^u_t\right) + \hat{\imath}_t - \hat{\pi}_{t+1} + z^b_{t+1}\right], \\ &= E_t\left[-\frac{\sigma y}{\omega_u c^u}\left(\hat{y}_{t+1} - \hat{y}_t\right) + \frac{\sigma y}{\omega_u c^u}\frac{(1-\omega_u)\,c^r}{y}\frac{\left(\hat{R}_{L,t+1} - \hat{\pi}_{t+1} + z^b_{t+1}\right)}{\sigma}\right. \\ &\left. + \hat{\imath}_t - \hat{\pi}_{t+1} + z^b_{t+1}\right],\end{aligned}$$

or, by using equation (3.29) in steady state,

$$0 = E_t\left[-\sigma\left(\hat{y}_{t+1} - \hat{y}_t\right) + \frac{(1-\omega_u)\,c^r}{y}\hat{R}_{L,t+1} + \frac{\omega_u c^u}{y}\hat{\imath}_t - \hat{\pi}_{t+1} + z^b_{t+1}\right]. \tag{3.55}$$

Equation (3.55) shows that the interest rate relevant to the aggregate variables such as output and inflation is the weighted sum of the return of holding the long-term bonds $\hat{R}_{L,t+1}$ and the short-term interest rate $\hat{\imath}_t$. Also, substituting equation (3.54) into equation (3.51) yields:

$$\frac{\zeta}{1+\zeta}\hat{\zeta}_t = E_t\left[-\sigma\left(\hat{c}^u_{t+1} - \hat{c}^u_t\right) + \hat{R}_{L,t+1} - \hat{\pi}_{t+1}\right]$$

$$
\begin{aligned}
&= E_t\left[-\sigma\frac{y}{\omega_u c^u}\left\{\hat{y}_{t+1}-\hat{y}_t-\frac{(1-\omega_u)\,c^r}{y}\frac{\left(\hat{R}_{L,t+1}-\hat{\pi}_{t+1}+z^b_{t+1}\right)}{\sigma}\right\}\right.\\
&\left.+\hat{R}_{L,t+1}-\hat{\pi}_{t+1}+z^b_{t+1}\right],\\
&= E_t\left[-\frac{\sigma y}{\omega_u c^u}\left(\hat{y}_{t+1}-\hat{y}_t\right)+\frac{y}{\omega_u c^u}\left(\hat{R}_{L,t+1}-\hat{\pi}_{t+1}+z^b_{t+1}\right)\right],
\end{aligned}
$$

or, substituting out $\hat{\zeta}_t$ by using equation (3.53) yields

$$
E_t\left(\hat{R}_{L,t+1}-\hat{\pi}_{t+1}\right)=\sigma E_t\left(\hat{y}_{t+1}-\hat{y}_t\right)-E_t\left(z^b_{t+1}\right)+\frac{\omega_u c^u}{y}\frac{\zeta}{1+\zeta}\rho_\zeta\hat{b}_{L,t}. \tag{3.56}
$$

Combining equations (3.55) and (3.56) yields:

$$
\begin{aligned}
0 &= E_t\left[-\sigma\left(\hat{y}_{t+1}-\hat{y}_t\right)+\frac{(1-\omega_u)\,c^r}{y}\hat{R}_{L,t+1}+\frac{\omega_u c^u}{y}\hat{\imath}_t-\hat{\pi}_{t+1}+z^b_{t+1}\right]\\
&= E_t\left[-\sigma\left(\hat{y}_{t+1}-\hat{y}_t\right)+\frac{(1-\omega_u)\,c^r}{y}\left(\sigma\left(\hat{y}_{t+1}-\hat{y}_t\right)-z^b_{t+1}+\frac{\omega_u c^u}{y}\frac{\zeta}{1+\zeta}\rho_\zeta\hat{b}_{L,t}+\hat{\pi}_{t+1}\right)\right.\\
&\left.+\frac{\omega_u c^u}{y}\hat{\imath}_t-\hat{\pi}_{t+1}+z^b_{t+1}\right]\\
&= E_t\left[-\frac{\omega_u c^u\sigma}{y}\left(\hat{y}_{t+1}-\hat{y}_t\right)+\frac{\omega_u c^u}{y}\hat{\imath}_t-\frac{\omega_u c^u}{y}\left(\hat{\pi}_{t+1}-z^b_{t+1}\right)+\frac{(1-\omega_u)\,c^r}{y}\frac{\omega_u c^u}{y}\frac{\zeta}{1+\zeta}\rho_\zeta\hat{b}_{L,t}\right],
\end{aligned}
$$

or

$$
0 = E_t\left[-\sigma\left(\hat{y}_{t+1}-\hat{y}_t\right)+\hat{\imath}_t-\hat{\pi}_{t+1}+z^b_{t+1}+\frac{(1-\omega_u)\,c^r}{y}\frac{\zeta}{1+\zeta}\rho_\zeta\hat{b}\right],
$$

or

$$
\begin{aligned}
\hat{y}_t &= E_t\hat{y}_{t+1}-\frac{1}{\sigma}\left(\hat{\imath}_t-E_t\hat{\pi}_{t+1}+E_t z^b_{t+1}\right)-\frac{1}{\sigma}\frac{(1-\omega_u)\,c^r}{y}\frac{\zeta}{1+\zeta}\rho_\zeta\hat{b}_{L,t}\\
&= E_t\hat{y}_{t+1}-\frac{1}{\sigma}\left(\hat{\imath}_t-E_t\hat{\pi}_{t+1}\right)-\frac{1}{\sigma}\frac{(1-\omega_u)\,c^r}{y}\frac{\zeta}{1+\zeta}\rho_\zeta\hat{b}_{L,t}-\frac{\rho_b}{\sigma}z^b_t.
\end{aligned}
$$

This equation shows that the central bank's government bond purchase – a decrease in $\hat{b}_{L,t}$ – stimulates output, given $E_t\hat{y}_{t+1}$ and the real rate $\hat{\imath}_t-E_t\hat{\pi}_{t+1}$. Since $\hat{b}_{L,t}$ follows the simple rule (3.43), the equation can be written as equation (3.12) in the main text, which is reproduced here for convenience:

$$
\hat{y}_t = E_t\hat{y}_{t+1}-\frac{1}{\sigma}\left((1-\lambda^*)\hat{\imath}_t+\lambda^*\hat{\imath}^*_t-E_t\hat{\pi}_{t+1}\right)-\chi_b z^b_t \tag{3.57}
$$

where

$$\lambda^* = \frac{(1-\omega_u)\, c^r}{y} \frac{\zeta}{1+\zeta} \rho_\zeta \gamma$$
$$\chi_b = \frac{\rho_b}{\sigma} \tag{3.58}$$

The case of $\lambda^* = 1$ (and $\alpha = 0$) corresponds to the fully effective UMP, which makes the ELB irrelevant. Such a case can be achieved, e.g., when the central bank responds to the shadow rate aggressively enough to satisfy

$$\gamma = \left[\frac{(1-\omega_u)\, c^r}{y} \frac{\zeta}{1+\zeta} \rho_\zeta \right]^{-1}.$$

Phillips curve. The Phillips curve can be derived from equations (3.36)-(3.39). Log-linearizing equation (3.39) yields:

$$\widehat{\tilde{p}}_t = -\frac{\xi}{1-\xi} \widehat{\tilde{\Pi}}^p_{t|t-1}, \tag{3.59}$$

where

$$\widehat{\tilde{\Pi}}^p_{t|t-1} = (1-\nu_p)\, \hat{\pi}_{t-1} - \hat{\pi}_t.$$

Log-linearizing equation (3.36) yields:

$$\begin{aligned} \frac{\theta - \lambda_p}{(1-\lambda_p)\,\theta} \widehat{\tilde{p}}_t = & \frac{\omega_u K^u_p}{\omega_u K^u_p + (1-\omega_u)\, K^r_p} \hat{K}^u_{p,t} + \frac{(1-\omega_u)\, K^r_p}{\omega_u K^u_p + (1-\omega_u)\, K^r_p} \hat{K}^r_{p,t} \\ & - \frac{\omega_u F^u_p}{\omega_u F^u_p + (1-\omega_u)\, F^r_p} \hat{F}^u_{p,t} - \frac{(1-\omega_u)\, F^r_p}{\omega_u F^u_p + (1-\omega_u)\, F^r_p} \hat{F}^r_{p,t}. \end{aligned} \tag{3.60}$$

Combining equations (3.59) and (3.60) leads to:

$$\begin{aligned} -\frac{\xi}{1-\xi} \frac{\theta - \lambda_p}{(1-\lambda_p)\,\theta} \left[(1-\nu_p)\, \hat{\pi}_{t-1} - \hat{\pi}_t \right] = & \frac{\omega_u K^u_p}{\omega_u K^u_p + (1-\omega_u)\, K^r_p} \hat{K}^u_{p,t} + \frac{(1-\omega_u)\, K^r_p}{\omega_u K^u_p + (1-\omega_u)\, K^r_p} \hat{K}^r_{p,t} \\ & - \frac{\omega_u F^u_p}{\omega_u F^u_p + (1-\omega_u)\, F^r_p} \hat{F}^u_{p,t} - \frac{(1-\omega_u)\, F^r_p}{\omega_u F^u_p + (1-\omega_u)\, F^r_p} \hat{F}^r_{p,t}. \end{aligned} \tag{3.61}$$

Log-linearizing equation (3.37) and (3.38) yields:

$$\hat{F}_{p,t}^{j} = (1-\xi\delta)\left(-\sigma\hat{c}_t^j + \hat{y}_t\right) + \xi\delta E_t\left(z_{t+1}^b + \frac{1}{1-\lambda_p}\widehat{\widetilde{\Pi}}_{t+1|t}^{p} + \hat{F}_{p,t+1}^{j}\right),$$
$$\hat{K}_{p,t}^{j} = (1-\xi\delta)\left(-\sigma\hat{c}_t^j + \frac{1}{\theta}\hat{y}_t - \frac{1}{\theta}z_t^a + \hat{w}_t\right) + \xi\delta E_t\left(z_{t+1}^b + \frac{\lambda_p}{(1-\lambda_p)\theta}\widehat{\widetilde{\Pi}}_{t+1|t}^{p} + \hat{K}_{p,t+1}^{j}\right),$$

for $j \in \{r,u\}$. The term involving $\hat{F}_{p,t}^{u}$ and $\hat{F}_{p,t}^{r}$ in equation (3.61) is calculated as follows.

$$\begin{aligned}
&\frac{\omega_u F_p^u}{\omega_u F_p^u + (1-\omega_u)F_p^r}\hat{F}_{p,t}^{u} + \frac{(1-\omega_u)F_p^r}{\omega_u F_p^u + (1-\omega_u)F_p^r}\hat{F}_{p,t}^{r} \\
&= (1-\xi\delta)\left(-\sigma\frac{\omega_u F_p^u\hat{c}_t^u + (1-\omega_u)F_p^r\hat{c}_t^r}{\omega_u F_p^u + (1-\omega_u)F_p^r} + \hat{y}_t\right) \\
&+ \xi\delta E_t\left(z_{t+1}^b + \frac{1}{1-\lambda_p}\widehat{\widetilde{\Pi}}_{t+1|t}^{p} + \frac{\omega_u F_p^u}{\omega_u F_p^u + (1-\omega_u)F_p^r}\hat{F}_{p,t+1}^{u} + \frac{(1-\omega_u)F_p^r}{\omega_u F_p^u + (1-\omega_u)F_p^r}\hat{F}_{p,t+1}^{r}\right).
\end{aligned}$$

Similarly, the term involving $\hat{K}_{p,t}^{u}$ and $\hat{K}_{p,t}^{r}$ in equation (3.61) is calculated as:

$$\begin{aligned}
&\frac{\omega_u K_p^u}{\omega_u K_p^u + (1-\omega_u)K_p^r}\hat{K}_{p,t}^{u} + \frac{(1-\omega_u)K_p^r}{\omega_u K_p^u + (1-\omega_u)K_p^r}\hat{K}_{p,t}^{r} \\
&= (1-\xi\delta)\left(-\sigma\frac{\omega_u K_p^u\hat{c}_t^u + (1-\omega_u)K_p^r\hat{c}_t^r}{\omega_u K_p^u + (1-\omega_u)K_p^r} + \frac{1}{\theta}\hat{y}_t - \frac{1}{\theta}z_t^a + \hat{w}_t\right) \\
&+ \xi\delta E_t\left(z_{t+1}^b + \frac{\lambda_p}{(1-\lambda_p)\theta}\widehat{\widetilde{\Pi}}_{t+1|t}^{p} + \frac{\omega_u K_p^u}{\omega_u K_p^u + (1-\omega_u)K_p^r}\hat{K}_{p,t+1}^{u} + \frac{(1-\omega_u)K_p^r}{\omega_u K_p^u + (1-\omega_u)K_p^r}\hat{K}_{p,t+1}^{r}\right).
\end{aligned}$$

Let the right-hand-side of equation (3.61) be denoted as $\hat{X}_t$. Then, using the above relationships just derived, $\hat{X}_t$ can be written as:

$$\hat{X}_t = (1-\xi\delta)\left[\left(\frac{1}{\theta}-1\right)\hat{y}_t - \frac{1}{\theta}z_t^a + \hat{w}_t\right] + \xi\delta E_t\left(-\frac{\lambda_p-\theta}{(\lambda_p-1)\theta}\widehat{\widetilde{\Pi}}_{t+1|t}^{p} + \hat{X}_{t+1}\right)$$

Because $\hat{X}_t$ is the right-hand-side of equation (3.61), equation (3.61) can be written as:

$$\begin{aligned}
-\frac{\xi}{1-\xi}\frac{\lambda_p-\theta}{(\lambda_p-1)\theta}\left[(1-\nu_p)\hat{\pi}_{t-1} - \hat{\pi}_t\right] &= (1-\xi\delta)\left[\left(\frac{1}{\theta}-1\right)\hat{y}_t - \frac{1}{\theta}z_t^a + \hat{w}_t\right] \\
&+ \xi\delta E_t\left(-\frac{\lambda_p-\theta}{(\lambda_p-1)\theta}\widehat{\widetilde{\Pi}}_{t+1|t}^{p} - \frac{\xi}{1-\xi}\frac{\lambda_p-\theta}{(\lambda_p-1)\theta}\left[(1-\nu_p)\hat{\pi}_t - \hat{\pi}_{t+1}\right]\right),
\end{aligned}$$

or

$$\hat{\pi}_t = \frac{\xi(1-\nu_p)}{(\xi+1-\nu_p)}\hat{\pi}_{t-1} + \frac{(1-\xi\delta)(1-\xi)(\lambda_p-1)\theta}{(\lambda_p-\theta)(\xi+1-\nu_p)}\left[\left(\frac{1}{\theta}-1\right)\hat{y}_t - \frac{1}{\theta}z_t^a + \hat{w}_t\right] + \frac{\xi\delta}{(\xi+1-\nu_p)}E_t\hat{\pi}_{t+1}.$$

From equations (3.30) and (3.33), the wage $\hat{w}_t$ can be written as:

$$\begin{aligned}\hat{w}_t &= \omega_u \left(\sigma \hat{c}_t^u + \frac{1}{\nu} \hat{h}_t^u \right) + (1 - \omega_u) \left(\sigma \hat{c}_t^r + \frac{1}{\nu} \hat{h}_t^r \right), \\ &= \sigma \hat{y}_t + \frac{1}{\nu} \hat{h}_t = \left(\sigma + \frac{1}{\nu\theta} \right) \hat{y}_t - \frac{1}{\nu\theta} z_t^a,\end{aligned}$$

where the market clearing conditions (3.29) and (3.44) were used in the second equality and the production function (3.35) was used in the third equality. Since we assume $c^u = c^r$, the second equality holds. By using the expression for $\hat{w}_t$, the Phillips curve can be written as

$$\begin{aligned}\hat{\pi}_t &= \frac{\xi \left(1 - \nu_p\right)}{\left(\xi + 1 - \nu_p\right)} \hat{\pi}_{t-1} \\ &+ \frac{\left(1 - \xi\delta\right)\left(1 - \xi\right)\left(\lambda_p - 1\right)\theta}{\left(\lambda_p - \theta\right)\left(\xi + 1 - \nu_p\right)} \left[\frac{\nu + \nu\theta\left(\sigma - 1\right) + 1}{\nu\theta} \hat{y}_t - \frac{1 + \nu}{\nu\theta} z_t^a \right] + \frac{\xi\delta}{\left(\xi + 1 - \nu_p\right)} E_t \hat{\pi}_{t+1}.\end{aligned}$$

In the case of no price indexation to the past inflation rate and a linear production function, that is, in the case of $\nu_p = 1$ and $\theta = 1$, the Phillips curve collapses to the standard form:

$$\hat{\pi}_t = \frac{\left(1 - \xi\delta\right)\left(1 - \xi\right)}{\xi} \left(\sigma + \frac{1}{\nu} \right) \hat{y}_t + \delta E_t \hat{\pi}_{t+1} - \frac{\left(1 - \xi\delta\right)\left(1 - \xi\right)}{\xi} \frac{1 + \nu}{\nu} z_t^a.$$

This completes the derivation of equation (3.16) in the main text, where

$$\kappa = \frac{\left(1 - \xi\delta\right)\left(1 - \xi\right)}{\xi} \left(\sigma + \frac{1}{\nu} \right), \tag{3.62}$$

$$\chi_a = \frac{\left(1 - \xi\delta\right)\left(1 - \xi\right)}{\xi} \frac{1 + \nu}{\nu}. \tag{3.63}$$

Long-term yield. From equations (3.55) and (3.57), the interest rate relevant to the aggregate variables has the following equality:

$$\frac{(1 - \omega_u) c^r}{y} E_t \hat{R}_{L,t+1} + \frac{\omega_u c^u}{y} \hat{i}_t = (1 - \lambda^*) \hat{i}_t + \lambda^* \hat{i}_t^*. \tag{3.64}$$

This equation can be written as

$$E_t\hat{R}_{L,t+1} = \begin{cases} \hat{i}_t^* & \hat{i}_t^* \geq \hat{\underline{i}} \\ \frac{\lambda^* y}{(1-\omega_u)c^r}\hat{i}_t^* + \frac{(1-\lambda^*)y-\omega_u c^u}{(1-\omega_u)c^r}\hat{\underline{i}} & \hat{i}_t^* < \hat{\underline{i}} \end{cases} \tag{3.65}$$

By using $R_{L,t+1} = \bar{R}_{L,t+1}(\bar{R}_{L,t} - \mu)/(\bar{R}_{L,t+1} - \mu)$, which relates the return of holding long-term bonds $\hat{R}_{L,t+1}$ to the long-term yield $\hat{\bar{R}}_{L,t}$, the long-term yield can be written as

$$\hat{\bar{R}}_{L,t} = \frac{\bar{R}_L - \mu}{\bar{R}_L} E_t\hat{R}_{L,t+1} + \frac{\mu}{\bar{R}_L} E_t\hat{\bar{R}}_{L,t+1}, \tag{3.66}$$

where $\bar{R}_L > \mu$ in steady state. Substitution equation (3.65) into equation (3.66) yields

$$\hat{\bar{R}}_{L,t} = \begin{cases} \frac{\bar{R}_L-\mu}{\bar{R}_L}\hat{i}_t^* + \frac{\mu}{\bar{R}_L} E_t\hat{\bar{R}}_{L,t+1} & \hat{i}_t^* \geq \hat{\underline{i}} \\ \frac{\bar{R}_L-\mu}{\bar{R}_L}\left[\frac{y\lambda^*}{(1-\omega_u)c^r}\hat{i}_t^* + \frac{(1-\lambda^*)y-\omega_u c^u}{(1-\omega_u)c^r}\hat{\underline{i}}\right] + \frac{\mu}{\bar{R}_L} E_t\hat{\bar{R}}_{L,t+1} & \hat{i}_t^* < \hat{\underline{i}} \end{cases} \tag{3.67}$$

Equation (3.67) shows that the long-term yield is the discounted sum of the current and future short-term returns, where the short-term return is given by the shadow rate in the non-ELB regime and in the ELB regime it is given by the first two terms in the square brackets in (3.67).

3.A.3 Parameterization of the model

Instead of parameterizing the model presented in Appendix 3.A.1, we parameterize the system of log-linearized equations (3.1)-(3.16) in the main text. It is worth emphasizing that we use the parameterized model to illustrate the implications of the theoretical model, and not to study the quantitative implications, which would require a more complex system.

The relative risk aversion parameter is set at $\sigma = 2$. The discount factor is set close to unity at $\delta = 0.997$. The slope of the Phillips curve κ is set at $\kappa = 0.336$ using equation (3.62) with the Calvo parameter of $\xi = 0.75$ and the Frisch labor elasticity of $\nu = 0.5$. In the monetary policy rule, the persistence parameter is set at $\rho_i = 0.7$; the inflation coefficient is set at $r_\pi = 1.5$; the output coefficient is set at $r_y = 0.5$. The AR(1) coefficients

for the supply and demand shocks are set at $\rho_a = \rho_b = 0.9$, and the coefficients χ_b and χ_a are set according to equations (3.58) and (3.63), respectively. The term premium in steady state is set at $\zeta = 0.01/4$. We consider different values for the parameters λ^* and α (reported in the main text) to study the effects of UMP.

3.A.4 Proof of Proposition 1

Part (i) Because of the equivalence established in Lemma 1, without loss of generality, consider the case of $\lambda^* = 1$ and $\alpha = 0$ in the theoretical model. In this case, the variables $\hat{y}_t$, $\hat{\pi}_t$, and $\hat{i}_t^*$ have a closed system of equations, consisting of equation (3.12) with $\lambda^* = 1$, equation (3.16), and $\hat{i}_t^* = \hat{i}_t^{\text{Taylor}}$, where $\hat{i}_t^{\text{Taylor}}$ is given by equation (3.3).

In this case, the state of the economy in period t can be summarized by $\hat{i}_{t-1}^*$, ϵ_t^i, z_t^a, and z_t^b. Then decision rules for $\hat{y}_t$ and $\hat{\pi}_t$ have the following form:

$$\begin{aligned}\hat{y}_t &= d_{yi^*}\hat{i}_{t-1}^* + d_{yi}\epsilon_t^i + d_{ya}z_t^a + d_{yb}z_t^b,\\ \hat{\pi}_t &= d_{\pi i^*}\hat{i}_{t-1}^* + d_{\pi i}\epsilon_t^i + d_{\pi a}z_t^a + d_{\pi b}z_t^b,\end{aligned}$$

with coefficients $\{d_{yi^*}, d_{yi}, d_{ya}, d_{yb}, d_{\pi i^*}, d_{\pi i}, d_{\pi a}, d_{\pi b}\}$ uniquely determined under standard assumptions of the model (such as the Taylor principle). With these decision rules, the equation for $\hat{i}_t^*$ can be written as

$$\begin{aligned}\hat{i}_t^* =& \left[\rho_i + (1-\rho_i)\left(r_\pi d_{\pi i^*} + r_y d_{yi^*}\right)\right]\hat{i}_{t-1}^* + \left[(1-\rho_i)\left(r_\pi d_{\pi i} + r_y d_{yi}\right) + 1\right]\epsilon_t^i\\ &+(1-\rho_i)\left(r_\pi d_{\pi a} + r_y d_{ya}\right)z_t^a + (1-\rho_i)\left(r_\pi d_{\pi b} + r_y d_{yb}\right)z_t^b\\ =& d_{i^*i^*}\hat{i}_{t-1}^* + d_{i^*i}\epsilon_t^i + d_{i^*a}z_t^a + d_{i^*b}z_t^b. \qquad (3.68)\end{aligned}$$

Let $\mathbf{y}_t \equiv [\hat{y}_t, \hat{\pi}_t, \hat{i}_t^*]'$ denote the vector of endogenous variables. The decision rule implies

$$\mathbf{y}_t = \begin{bmatrix} d_{yi^*} & d_{yi} & d_{ya} & d_{yb}\\ d_{\pi i^*} & d_{\pi i} & d_{\pi a} & d_{\pi b}\\ d_{i^*i^*} & d_{i^*i} & d_{i^*a} & d_{i^*b}\end{bmatrix}\begin{bmatrix} i_{t-1}^*\\ \epsilon_t^i\\ \rho_a z_{t-1}^a + \epsilon_t^a\\ \rho_b z_{t-1}^b + \epsilon_t^b\end{bmatrix}$$

$$
\begin{aligned}
&= \begin{bmatrix} d_{yi^*} & \rho_a d_{ya} & \rho_b d_{yb} \\ d_{\pi i^*} & \rho_a d_{\pi a} & \rho_b d_{\pi b} \\ d_{i^* i^*} & \rho_a d_{i^* a} & \rho_b d_{i^* b} \end{bmatrix} \begin{bmatrix} i^*_{t-1} \\ z^a_{t-1} \\ z^b_{t-1} \end{bmatrix} + \begin{bmatrix} d_{yi} & d_{ya} & d_{yb} \\ d_{\pi i} & d_{\pi a} & d_{\pi b} \\ d_{i^* i} & d_{i^* a} & d_{i^* b} \end{bmatrix} \begin{bmatrix} \epsilon^i_t \\ \epsilon^a_t \\ \epsilon^b_t \end{bmatrix} \\
&= \mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}\epsilon_t. \qquad (3.69)
\end{aligned}
$$

The law of motion for $\mathbf{x}_t \equiv [\hat{i}^*_t, z^a_t, z^b_t]'$ is:

$$
\begin{aligned}
\mathbf{x}_t &= \begin{bmatrix} d_{i^* i^*} & \rho_a d_{i^* a} & \rho_b d_{i^* b} \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_b \end{bmatrix} \mathbf{x}_{t-1} + \begin{bmatrix} d_{i^* i} & d_{i^* a} & d_{i^* b} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \epsilon_t \\
&= \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}\epsilon_t. \qquad (3.70)
\end{aligned}
$$

Solving equation (3.69) for ϵ_t, and substituting the outcome in equation (3.70) yields:

$$
\mathbf{x}_t = \left(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\right) \mathbf{x}_{t-1} + \mathbf{B}\mathbf{D}^{-1}\mathbf{y}_t.
$$

If $\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C} = \mathbf{0}$, the vector of endogenous variables, $\mathbf{y}_t$, has a VAR(1) representation:

$$
\mathbf{y}_t = \mathbf{C}\mathbf{B}\mathbf{D}^{-1}\mathbf{y}_{t-1} + \mathbf{D}\epsilon_t.
$$

The rest of the proof shows $\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C} = \mathbf{0}$. Substituting the matrices $\mathbf{A}$ and $\mathbf{B}$ in equation (3.70) into this condition yields:

$$
\mathbf{D}^{-1}\mathbf{C} = \begin{bmatrix} d_{i^* i^*}/d_{i^* i} & 0 & 0 \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_b \end{bmatrix}.
$$

Further substituting the matrices $\mathbf{C}$ and $\mathbf{D}$ in equation (3.69) into this condition leads to: $\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C} = \mathbf{0}$ if and only if $d_{yi^*} = d_{yi}\left(d_{i^* i^*}/d_{i^* i}\right)$ and $d_{\pi i^*} = d_{\pi i}\left(d_{i^* i^*}/d_{i^* i}\right)$.

Substituting the decision rules into equation (3.12) yields:

$$\hat{y}_t = \left(d_{yi^*} - \frac{1}{\sigma} + \frac{d_{\pi i}}{\sigma}\right) d_{i^*i^*} \hat{i}^*_{t-1} + \left(d_{yi^*} - \frac{1}{\sigma} + \frac{d_{\pi i}}{\sigma}\right) d_{i^*i} \epsilon^i_t + ...,$$

where terms related to z^a_t and z^b_t are omitted. Matching coefficients on $\hat{i}^*_{t-1}$ and ϵ^i_t of both sides of the equation yields:

$$\begin{aligned} d_{yi^*} &= \left(d_{yi^*} - \frac{1}{\sigma} + \frac{d_{\pi i}}{\sigma}\right) d_{i^*i^*}, \\ d_{yi} &= \left(d_{yi^*} - \frac{1}{\sigma} + \frac{d_{\pi i}}{\sigma}\right) d_{i^*i}. \end{aligned}$$

These two equations imply $d_{yi^*} = d_{yi}\left(d_{i^*i^*}/d_{i^*i}\right)$. Next, substituting the decision rules into equation (3.16) yields:

$$\hat{\pi}_t = \left(\delta d_{\pi i^*} + \kappa d_{yi^*}\right) \hat{i}^*_{t-1} + \left(\delta d_{\pi i^*} d_{i^*i} + \kappa d_{yi}\right) \epsilon^i_t + ...,$$

where terms related to z^a_t and z^b_t are omitted. Matching coefficients on $\hat{i}^*_{t-1}$ and ϵ^i_t of both sides of the equation yields:

$$\begin{aligned} d_{\pi i^*} =& \delta d_{\pi i^*} + \kappa d_{yi} \left(\frac{d_{i^*i^*}}{d_{i^*i}}\right), \\ d_{\pi i} =& \delta d_{\pi i^*} d_{i^*i} + \kappa d_{yi}, \end{aligned}$$

where $d_{yi^*} = d_{yi}\left(d_{i^*i^*}/d_{i^*i}\right)$ is used in the first equation. Solving these two equations for $d_{\pi i^*}$ yields $d_{\pi i^*} = d_{\pi i}\left(d_{i^*i^*}/d_{i^*i}\right)$.

Part (ii) Again, without loss of generality, consider the case of $\lambda^* = 1$ and $\alpha = 0$. Under Assumption 1 and the irrelevance hypothesis, the long-term yield can be written as (3.15) with $\lambda^* = 1$ as

$$\hat{\bar{R}}_L = \frac{\bar{R}_L - \mu}{\bar{R}_L} \hat{i}^*_t + \frac{\mu}{\bar{R}_L} E_t \hat{\bar{R}}_{L,t+1}.$$

Solving this equation forward yields

$$\hat{\bar{R}}_{L,t} = \left(\frac{\bar{R}_L - \mu}{\bar{R}_L}\right) E_t \left[\hat{i}^*_t + \frac{\mu}{\bar{R}_L} \hat{i}^*_{t+1} + \left(\frac{\mu}{\bar{R}_L}\right)^2 \hat{i}^*_{t+2} + ...\right].$$

Because the right-hand-side of the equation depends on information in period t, which consist of $\hat{i}_t^*$, z_t^a, and z_t^b, the long-term interest rate can be written as:

$$\hat{\bar{R}}_{L,t} = f_{i^*}\hat{i}_t^* + f_a z_t^a + f_b z_t^b,$$

where f_{i^*}, f_a, and f_b are coefficients derived by using equation (3.68) as

$$\begin{aligned} f_{i^*} &= \frac{\bar{R}_L - \mu}{\bar{R}_L - d_{i^*i^*}\mu}, \\ f_a &= \frac{(\bar{R}_L - \mu)d_{i^*a}\rho_a\mu}{(\bar{R}_L - \rho_a\mu)(\bar{R}_L - d_{i^*i^*}\mu)}, \\ f_b &= \frac{(\bar{R}_L - \mu)d_{i^*b}\rho_b\mu}{(\bar{R}_L - \rho_b\mu)(\bar{R}_L - d_{i^*i^*}\mu)}. \end{aligned}$$

Again by using equation (3.68) the equation for the long-term yield can be written as:

$$\hat{\bar{R}}_{L,t} = f_{i^*}d_{i^*i^*}\hat{i}_{t-1}^* + f_{i^*}d_{i^*i}\epsilon_t^i + \left(f_{i^*}d_{i^*a} + f_a\right)z_t^a + \left(f_{i^*}d_{i^*b} + f_b\right)z_t^b.$$

Define $\mathbf{y}_t \equiv [\hat{y}_t, \hat{\pi}_t, \hat{\bar{R}}_{L,t}]'$, $\mathbf{x}_t \equiv [\hat{i}_{t-1}^*, z_t^a, z_t^b]'$, and $\epsilon_t = [\epsilon_t^i, \epsilon_t^a, \epsilon_t^b]'$. Then, the state space representation for $\mathbf{y}$ is

$$\begin{aligned} \mathbf{y}_t = &\begin{bmatrix} d_{yi^*} & \rho_a d_{ya} & \rho_b d_{yb} \\ d_{\pi i^*} & \rho_a d_{\pi a} & \rho_b d_{\pi b} \\ f_{i^*}d_{i^*i^*} & \rho_a(f_{i^*}d_{i^*a} + f_a) & \rho_b(f_{i^*}d_{i^*b} + f_b) \end{bmatrix}\mathbf{x}_t + \begin{bmatrix} d_{yi} & d_{ya} & d_{yb} \\ d_{\pi i} & d_{\pi a} & d_{\pi b} \\ f_{i^*}d_{i^*i} & f_{i^*}d_{i^*a} + f_a & f_{i^*}d_{i^*b} + f_b \end{bmatrix}\epsilon_t \\ =&\mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}\epsilon_t. \end{aligned}$$

and

$$\begin{aligned} \mathbf{x}_t = &\begin{bmatrix} d_{i^*i^*} & \rho_a d_{i^*a} & \rho_b d_{i^*b} \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_b \end{bmatrix}\mathbf{x}_{t-1} + \begin{bmatrix} d_{i^*i} & d_{i^*a} & d_{i^*b} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\epsilon_t \\ =&\mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}\epsilon_t. \end{aligned}$$

Similar to the part (i) in Proposition 1, a solution for $\mathbf{y}_t$ has a VAR(1) representation if

and only if $\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C} = \mathbf{0}$. This condition holds if and only if $d_{yi^*} = d_{yi}(d_{i^*i^*}/d_{i^*i})$ and $d_{\pi i^*} = d_{\pi i}(d_{i^*i^*}/d_{i^*i})$. The latter two conditions hold as shown in Part (i).

3.A.5 Proof of Proposition 2

We show that equations (3.1), (3.2), (3.3), (3.12), and (3.16) can be written in the empirical structural form of equations (3.17a), (3.17b), and (3.17c). This will prove the proposition since the structural form has a piecewise linear representation, as explained in the main text. It is straightforward to see that equations (3.1), (3.2), and (3.3) in the theoretical model can be written in the form of equations (3.17a) and (3.17b) in the empirical model. Below, we are going to show that equations (3.12) and (3.16) can be represented by the structural form equation (3.17c).

Without loss of generality, consider a case in which agents forming expectations assuming: $\lambda^* = 1$ and $\alpha = 0$. When forming expectations about variables in period $t+1$, the initial condition is given by $\tilde{\mathbf{x}}_t \equiv [(1-\lambda^*)\hat{i}_t + \lambda^*\hat{i}_t^*, z_t^a, z_t^b]'$. Under Assumption 2, the decision rule used for forming expectations about period $t+1$ variables is $\mathbf{y}_{t+1} = \mathbf{C}\tilde{\mathbf{x}}_t + \mathbf{D}\epsilon_{t+1}$, where $\mathbf{C}$ and $\mathbf{D}$ are those defined in equation (3.69). From period $t+s$ onward, for $s = 2, 3, ...$, time $t+s$ variables are expected *in period* t to follow $\mathbf{y}_{t+s} = \mathbf{C}\mathbf{x}_{t+s-1} + \mathbf{D}\epsilon_{t+s}$, where $\mathbf{x}_t \equiv [\hat{i}_t^*, z_t^a, z_t^b]'$. But, once the time proceeds and becomes period $t+1$, the initial condition is updated to $\tilde{\mathbf{x}}_{t+1}$ and this is used for forming expectations about $t+2$ variables as $E_{t+1}\mathbf{y}_{t+2} = \mathbf{C}\tilde{\mathbf{x}}_{t+1}$. Hence, under the assumption about expectations, the decision rule is given by $\mathbf{y}_{t+s} = \mathbf{C}\tilde{\mathbf{x}}_{t+s-1} + \mathbf{D}\epsilon_{t+s}$ for $s = 1, 2, ...$ In this system, in every period information is updated and $\tilde{\mathbf{x}}_{t+s-1}$ is used as an initial condition. The interest rate $\hat{i}_{t+s-1}$ in the initial condition is treated as if it were an exogenous variable.

By substituting the decision rule into the expected variables, equations (3.12) and (3.16) can be written as:

$$\hat{y}_t = \left(-\frac{1}{\sigma} + d_{yi^*} + \frac{d_{\pi i^*}}{\sigma}\right)\left((1-\lambda^*)\hat{i}_t + \lambda^*\hat{i}_t^*\right) + \left(\rho_a d_{ya} + \frac{\rho_a d_{\pi a}}{\sigma}\right) z_t^a + \left(\rho_b d_{yb} + \frac{\rho_b d_{\pi b}}{\sigma} - \chi_z\right) z_t^b, \tag{3.71}$$

$$-\kappa y_t + \hat{\pi}_t = \delta d_{\pi i^*}\left((1-\lambda^*)\hat{i}_t + \lambda^*\hat{i}_t^*\right) + (\delta\rho_a d_{\pi a} - \chi_a) z_t^a + \delta\rho_b d_{\pi b} z_t^b. \tag{3.72}$$

Since z_t^a and z_t^b follow AR(1) processes, equations (3.71) and (3.72) can be written in a matrix form as:

$$\mathbf{H}_1 \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix} = \mathbf{H}_2 \left((1-\lambda^*)\hat{i} + \lambda^* \hat{i}_t^* \right) + \mathbf{H}_3 \begin{bmatrix} z_{t-1}^a \\ z_{t-1}^b \end{bmatrix} + \mathbf{H}_4 \begin{bmatrix} \epsilon_t^a \\ \epsilon_t^b \end{bmatrix}.$$

or

$$\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \end{bmatrix} = \mathbf{H}_1^{-1}\mathbf{H}_2 \left((1-\lambda^*)\hat{i} + \lambda^* \hat{i}_t^* \right) + \mathbf{H}_1^{-1}\mathbf{H}_3 \begin{bmatrix} z_{t-1}^a \\ z_{t-1}^b \end{bmatrix} + \mathbf{H}_1^{-1}\mathbf{H}_4 \begin{bmatrix} \epsilon_t^a \\ \epsilon_t^b \end{bmatrix}. \tag{3.73}$$

Also, under Assumption 2, the expected values can be written as: $E_t\tilde{\mathbf{y}}_{t+1} = \mathbf{G}\tilde{\mathbf{y}}_t$, where $\tilde{\mathbf{y}}_t \equiv [\hat{y}_t, \hat{\pi}_t, (1-\lambda^*)\hat{i}_t + \lambda^*\hat{i}_t^*]'$ and $\mathbf{G} \equiv \mathbf{CBD}^{-1}$, as derived in the proof of Proposition 1. By using this equation, equations (3.12) and (3.16) can be written as:

$$\chi_z z_t^b = \left(g_{yy} + \frac{g_{\pi y}}{\sigma} - 1 \right) \hat{y}_t + \left(g_{y\pi} + \frac{g_{\pi\pi}}{\sigma} \right) \hat{\pi}_t + \left(g_{yi^*} + \frac{g_{\pi i^*}}{\sigma} - \frac{1}{\sigma} \right) \left((1-\lambda^*)\hat{i}_t + \lambda^* \hat{i}_t^* \right),$$

$$\chi_a z_t^a = (\delta g_{\pi y} + \kappa)\, \hat{y}_t + (\delta g_{\pi\pi} - 1)\, \hat{\pi}_t + \delta g_{\pi i^*} \left((1-\lambda^*)\hat{i}_t + \lambda^* \hat{i}_t^* \right),$$

where g_{ij}'s correspond to elements in the matrix $\mathbf{G}$. Then, the lagged shocks z_{t-1}^b and z_{t-1}^a in equation (3.73) can be represented by a function of $\tilde{\mathbf{y}}_{t-1} \equiv [\hat{y}_{t-1}, \hat{\pi}_{t-1}, (1-\lambda^*)\hat{i}_{t-1} + \lambda^*\hat{i}_{t-1}^*]'$. From this result, equation (3.73) is in the same form of equation (3.17c) in the structural form.

3.A.6 Impulse responses to demand and supply shocks

We study impulse responses to a demand shock and a supply shock, respectively, in an ELB regime, using the theoretical model presented in Section 3.2 of the main text. We show that the responses differ significantly depending on the effectiveness of UMP.

Figure 3.9 plots impulse responses of output and inflation in the theoretical model to the contractionary demand shock of $\epsilon_t^b = 0.25/400$ under the ELB. The responses are calculated exactly in the same way as those to a monetary policy shock, shown in Figure 3.2 in the main text. In the case of no UMP ($\xi = 0$), a negative demand shock causes the

Figure 3.9: Impulse responses to a demand shock at the ELB

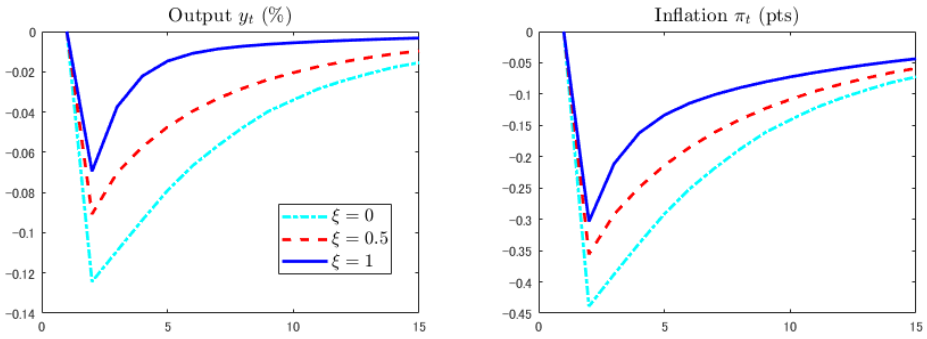
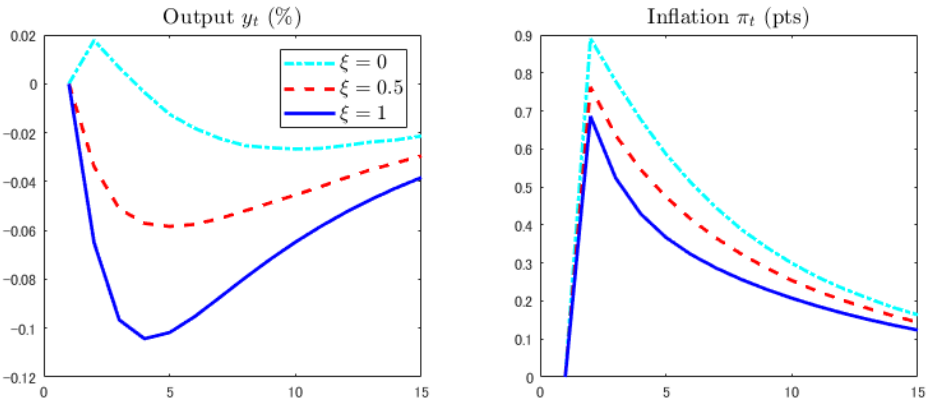


Figure 3.10: Impulse responses to a supply shock at the ELB



largest declines in output and inflation. As the effectiveness of UMP increases, i.e., as ξ increases, the negative responses of output and inflation become smaller.

Figure 3.10 plots impulse responses of output and inflation to the negative supply shock of $\epsilon_t^a = -0.25/100$ under the ELB. In the case of fully effective UMP ($\xi = 1$), output decreases and inflation increases in response to the negative supply shock, as in the responses in a non-ELB regime. However, as the effectiveness of UMP decreases, the degree of a decrease in output shrinks, and output even increases on impact in response to the negative supply shock in the case of no UMP ($\xi = 0$). This is driven by a stronger increase in inflation under the ELB. Such an increase in inflation mitigates the negative impact of the ELB on output. This effect dominates the direct effect of the negative supply shock, resulting in an increase in output on impact.

While we exclusively focus on monetary policy shocks in this paper, the same co-movement of the variables in response to supply and demand shocks when the economy approaches the ELB would pose a challenge for identifying responses to demand and supply shocks.

3.B Derivation of the attenuation effect (3.24)

Start from the definition of the IRF to the monetary policy shock $\bar{\varepsilon}_{2t} := A_{22}^{*-1}\varepsilon_{2t}$.[24] This is a function of the shock magnitude ς and horizon h:

$$IRF_{h,t}(\varsigma) = E(Y_{1t+h}|\bar{\varepsilon}_{2t} = \varsigma, x_t) - E(Y_{1t+h}|\bar{\varepsilon}_{2t} = 0, x_t), \tag{3.74}$$

where $x_t = (x_{1t}', x_{2t})'$, $x_{it} := C_i X_t + C_i^* X_t^*$ embodies all the relevant history of Y_t up to period $t-1$.[25] We only need to discuss the impact effects, so we set $h = 0$ in (3.74) and write $IRF_{0,t}(\varsigma) = g(\varsigma; x_t) - g(0; x_t)$, where

$$g(\varsigma; x_t) := E(Y_{1t}|\bar{\varepsilon}_{2t} = \varsigma, x_t). \tag{3.75}$$

Note that, despite the kink in the model, the fact that we are taking expectations with respect to the remaining shocks $\bar{\varepsilon}_{1t} := A_{11}^{-1}\varepsilon_{1t}$ implies that the function g is smooth in both arguments. So, we can consider infinitesimal interventions by computing $\lim_{\varsigma\to 0}\frac{g(\varsigma;x_t)-g(0;x_t)}{\varsigma} =: \frac{\partial g(\varsigma;x_t)}{\partial \varsigma}$.

Let $D_t = 1_{\{i_t^* < \underline{i}_t\}}$ denote the indicator that the interest rate is at the ELB. Then, using equation (3.19) in the main text, we obtain:

$$\begin{aligned}
g(\varsigma, x_t) &= C_1 X_t + C_{12}^* X_{2t}^* + E(u_{1t}|\bar{\varepsilon}_{2t} = \varsigma, x_t) \\
&\quad - \widetilde{\beta} E\left(D_t\left(C_2 X_t + C_{22}^* X_{2t}^* - \underline{i}_t + u_{2t}\right)|\bar{\varepsilon}_{2t} = \varsigma, x_t\right) \\
&= x_{1t} + (I_{k-1} - \beta\gamma)^{-1}\beta\varsigma - \widetilde{\beta} E(D_t|\bar{\varepsilon}_{2t} = \varsigma, x_t)\left(x_{2t} - \underline{i}_t + \frac{\varsigma}{1-\gamma\beta}\right) \\
&\quad - \widetilde{\beta} E\left(\frac{\gamma\bar{\varepsilon}_{1t}}{1-\gamma\beta} D_t \middle| \bar{\varepsilon}_{2t} = \varsigma, x_t\right) \\
&= x_{1t} + (I_{k-1} - \beta\gamma)^{-1}\beta\varsigma - \widetilde{\beta}\Phi\left(\frac{\underline{i}_t - x_{2t} - \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right)\left(x_{2t} - \underline{i}_t + \frac{\varsigma}{1-\gamma\beta}\right) \\
&\quad + \widetilde{\beta}\varpi\phi\left(\frac{\underline{i}_t - x_{2t} - \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right).
\end{aligned} \tag{3.76}$$

[24] Note that the derivation of (3.24) remains the same if we worked with perturbations to ε_{2t} instead of $\bar{\varepsilon}_{2t}$, but we choose the latter to avoid carrying A_{22}^* around in the derivation.

[25] x_t is the sufficient statistic for the entire history of Y_t in the conditional expectations, i.e., $E(Y_{t+h}|\bar{\varepsilon}_{2t}, Y_{t-1}, Y_{t-2}, ...) = E(Y_{t+h}|\bar{\varepsilon}_{2t}, x_t)$.

The second equality in equation (3.76) follows from the definitions of x_{it} and the fact that:

$$u_{1t} = (I_{k-1} - \beta\gamma)^{-1} (\bar{\varepsilon}_{1t} + \beta\bar{\varepsilon}_{2t}), \text{ and } u_{2t} := \frac{\gamma\bar{\varepsilon}_{1t} + \bar{\varepsilon}_{2t}}{1 - \gamma\beta},$$

see (Mavroeidis, 2021, equations 32 and 33), and the third equality in equation (3.76) follows from:

$$\begin{aligned} E(D_t|\bar{\varepsilon}_{2t} = \varsigma, x_t) &= \Pr(u_{2t} < \underline{i}_t - x_{2t}|\bar{\varepsilon}_{2t} = \varsigma, x_t) \\ &= \Pr\left(\frac{\gamma\bar{\varepsilon}_{1t}}{1 - \gamma\beta} < \underline{i}_t - x_{2t} - \frac{\varsigma}{1 - \gamma\beta}\right) \\ &= \Phi\left(\frac{b - x_{2t} - \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right), \quad \varpi^2 := var\left(\frac{\gamma\bar{\varepsilon}_{1t}}{1 - \gamma\beta}\right) \end{aligned}$$

and

$$\begin{aligned} E\left(\frac{\gamma\bar{\varepsilon}_{1t}}{1 - \gamma\beta} D_t \middle| \bar{\varepsilon}_{2t} = \varsigma, x_t\right) &= E\left(\frac{\gamma\bar{\varepsilon}_{1t}}{1 - \gamma\beta} \middle| \frac{\gamma\bar{\varepsilon}_{1t}}{1 - \gamma\beta} < \underline{i}_t - x_{2t} - \frac{\varsigma}{1 - \gamma\beta}, \bar{\varepsilon}_{2t} = \varsigma, x_t\right) \\ &\quad \times \Pr\left(\frac{\gamma\bar{\varepsilon}_{1t}}{1 - \gamma\beta} < \underline{i}_t - x_{2t} - \frac{\varsigma}{1 - \gamma\beta} \middle| \bar{\varepsilon}_{2t} = \varsigma, x_t\right) \\ &= -\varpi\phi\left(\frac{\underline{i}_t - x_{2t} - \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right), \end{aligned}$$

where the second equality in the last expression follows from the independence of $\bar{\varepsilon}_{1t}$ from $\bar{\varepsilon}_{2t}$ and x_t, and the properties of the truncated standard normal distribution, i.e., $E(z|z < a) = -\phi(a)/\Phi(a)$.

Differentiating equation (3.76) with respect to ς yields:

$$\begin{aligned} \frac{\partial g(\varsigma, x_t)}{\partial \varsigma} &= (I_{k-1} - \beta\gamma)^{-1} \beta - \frac{1}{1 - \gamma\beta}\widetilde{\beta}\Phi\left(\frac{\underline{i}_t - x_{2t} - \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right) \\ &\quad + \widetilde{\beta}\frac{1}{1 - \gamma\beta}\phi\left(\frac{\underline{i}_t - x_{2t} - \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right)\left(\frac{x_{2t} - \underline{i}_t + \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right) \\ &\quad + \widetilde{\beta}\frac{1}{1 - \gamma\beta}\left(\frac{\underline{i}_t - x_{2t} - \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right)\phi\left(\frac{\underline{i}_t - x_{2t} - \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right) \\ &= (I_{k-1} - \beta\gamma)^{-1} \beta - \frac{1}{1 - \gamma\beta}\widetilde{\beta}\Phi\left(\frac{\underline{i}_t - x_{2t} - \frac{\varsigma}{1-\gamma\beta}}{\varpi}\right), \end{aligned}$$

where the first equality follows from the fact that $\partial\phi(z)/\partial z = -z\phi(z)$. Evaluating the above expression at $\varsigma = 0$ yields the impact effect of a small monetary policy shock on

Y_{1t} in period t, which is a $k-1$ vector, namely,

$$IR_t := \left.\frac{\partial g(\varsigma, x_t)}{\partial \varsigma}\right|_{\varsigma=0} = (I_{k-1} - \beta\gamma)^{-1}\beta - \frac{1}{1-\gamma\beta}\widetilde{\beta}\Phi\left(\frac{i_t - x_{2t}}{\varpi}\right).$$

If there is no attenuation effect, the impact effect of the monetary policy shock $\bar{\varepsilon}_{2t}$ on Y_{1t} is common across regimes and is given by:

$$IR_{NA} = (I_{k-1} - \beta\gamma)^{-1}\beta = \frac{\beta}{1-\gamma\beta},$$

where the second equality follows from the fact that $(I_{k-1} - \beta\gamma)\beta = \beta(1-\gamma\beta)$. Therefore,

$$IR_t = IR_{NA} - \frac{1}{1-\gamma\beta}\widetilde{\beta}\Phi\left(\frac{i_t - x_{2t}}{\varpi}\right).$$

The jth element of the $k-1$ vector IR_t above can be written as

$$\begin{aligned} IR_{j,t} &= IR_{j,NA} - \frac{1}{1-\gamma\beta}\widetilde{\beta}_j\Phi\left(\frac{i_t - x_{2t}}{\varpi}\right) \\ &= \frac{\beta_j}{1-\gamma\beta} - \frac{\beta_j}{1-\gamma\beta}\frac{\widetilde{\beta}_j}{\beta_j}\Phi\left(\frac{i_t - x_{2t}}{\varpi}\right) \\ &= \left(1 - \frac{\widetilde{\beta}_j}{\beta_j}\Phi\left(\frac{i_t - x_{2t}}{\varpi}\right)\right) IR_{j,NA}. \end{aligned}$$

Renaming $x_{2t} = C_2 X_t + C_{22}^* X_{2t}^*$, the one-step ahead forecast of the reduced-form shadow rate, as $i_{t|t-1}^*$ yields (3.24) as required.

3.C Data description

We construct our quarterly data by taking averages of monthly series. For the U.S., the inflation rate is computed from the implicit price deflator (GDPDEF) as $\pi_t = 400 \times \log(P_t/P_{t-1})$, where P_t is the GDP deflator. The output gap is calculated as $100\% \times (GDPC1 - GDPPOT)/GDPPOT$, where GDPC1 is the series for the U.S. real GDP and GDPPOT is the U.S. real potential GDP. The long-term interest rate is from the 10-year Treasury constant maturity rate (GS10). All these series are from the FRED

database.[26] Money growth data for the U.S. are computed from 12 alternative indicators as listed in Table 3.6 as $m_t = 400 \times \log(M_t/M_{t-1})$, where M_t is the particular money supply considered. All M_t values are quarterly and computed by taking averages of their corresponding monthly values. The traditional monetary aggregates (MB, M1, M2, M2M, MZM), and securities held outright are from the FRED database. The Divisia monetary aggregates (DIVM1, DIVM2, DIVM2M, DIVMZM, DIVM4) are from the Center for Financial Stability Divisia database.

For Japan, the quarterly call rate, bond yields, and the core CPI are computed as the averages of their monthly counterparts. The quarterly inflation rate is computed from the core CPI (consumption tax changes adjusted) as $\pi_t = 400 \times (CPI_t - CPI_{t-1})/CPI_{t-1}$. The GDP gap is that published by the Bank of Japan. The trend growth is defined by the annualised growth rate of potential GDP from the previous quarter, which comes from the estimates of the Cabinet Office. The interest on reserves (IOR) is constructed from the interest rate that the Bank of Japan applies to the Complementary Deposit Facility.[27]

[26] The data can be retrieved from the following websites: GDP deflator https://fred.stlouisfed.org/series/GDPDEF; and series to construct the output gap: https://fred.stlouisfed.org/series/GDPC1 and https://fred.stlouisfed.org/series/GDPPOT; the Federal Funds Rate https://fred.stlouisfed.org/series/FEDFUNDS; and the long yield https://fred.stlouisfed.org/series/GS10. The data for the different monetary aggregates is available at: https://fred.stlouisfed.org/categories/24 and http://www.centerforfinancialstability.org/amfm_data.php.

[27] The data can be retrieved from the following websites: call rate (Bank of Japan): http://www.stat-search.boj.or.jp/index_en.html; 9-year and 10-year government bond yields (Ministry of Finance): https://www.mof.go.jp/jgbs/reference/interest_rate/data/jgbcm_all.csv; GDP gap (Bank of Japan): https://www.boj.or.jp/en/research/research_data/index.htm/; core CPI inflation (Statistics Bureau of Japan): https://www.e-stat.go.jp/stat-search/file-download?statInfId=000031431696&fileKind=1; trend growth rate (Cabinet Office): https://www5.cao.go.jp/keizai3/getsurei-e/index-e.html.

Table 3.6: Monetary Aggregates Data used in the Model

Monetary Aggregate (M_t)	Mnemonics in the Corresponding Database	Available Sample Periods
Monetary Base (MB)	MBSL	1948Q1-2019Q1
M1	M1SL	1959Q2-2019Q1
M2	M2SL	1959Q2-2019Q1
M2M	M2MSL	1959Q2-2019Q1
MZM	MZMSL	1959Q2-2019Q1
Securities Held Outright	WSECOUT	1989Q3-2019Q1
Divisia M1 (DIVM1)	Divisia M1	1967Q2-2019Q1
Divisia M2 (DIVM2)	Divisia M2	1967Q2-2019Q1
Divisia M2M (DIVM2M)	Divisia M2M	1967Q2-2019Q1
Divisia MZM (DIVMZM)	Divisia MZM	1967Q2-2019Q1
Divisia M4 (DIVM4)	DM4	1967Q2-2019Q1

3.D Additional empirical results

Weaker version of IH$_1$. Table 3.7 shows the results of the weaker version of IH$_1$, i.e., $C_{12} = C_{12}^* = \tilde{\beta} = 0$ for inflation and output equations only. As in the baseline CKSVAR specification reported in Table 3.1 in the main text, 4 lags are selected for the U.S. and 2 lags are selected for Japan. The p-values reported in Table 3.7 show that the weaker version of IH$_1$ is firmly rejected for both countries.

Adding alternative measures of monetary policy. Table 3.8 shows the results of tests for exclusion of the Federal Funds Rate from a SVAR that includes inflation, the output gap, the 10-year bond yield, and various alternative measures of the growth of monetary aggregates outlined in column (1). Column (3) shows the order of the VAR selected by the AIC, which varies between 3 and 4 lags, consistent with the benchmark model in Table 3.1 in the main text. Columns (4) and (6) report the likelihood ratio test statistics for the joint exclusion hypothesis and the corresponding asymptotic p-values, respectively. These results show that the data strongly and consistently reject the joint exclusion restrictions on the Federal Funds Rate across all the alternative specifications for all measures of money supply, which corroborates the findings in the baseline 4-equation model in Table 3.1 in the main text.

Table 3.7: Test for excluding short rates from VAR that includes long rates

Panel A: KSVAR												
	United States						Japan					
p	loglik	pv-p	AIC	LR	df	p-val	loglik	pv-p	AIC	LR	df	p-val
5	-213.4	-	2.62	36.08	12	0.000	248.1	-	-2.18	12.74	12	0.389
4	-221.5	0.446	2.55	33.42	10	0.000	239.9	0.425	-2.30	14.13	10	0.167
3	-234.4	0.112	2.53	27.12	8	0.001	232.2	0.471	-2.42	14.89	8	0.061
2	-266.0	0.000	2.66	28.29	6	0.000	223.8	0.445	-2.53	15.70	6	0.015
1	-296.7	0.000	2.78	24.62	4	0.000	184.8	0.000	-2.19	25.15	4	0.000
Panel B: CKSVAR												
p	loglik	pv-p	AIC	LR	df	p-val	loglik	pv-p	AIC	LR	df	p-val
5	-191.3	-	2.60	52.08	22	0.000	284.7	-	-2.42	48.79	22	0.001
4	-202.7	0.290	2.53	44.48	18	0.000	277.1	0.766	-2.61	52.42	18	0.000
3	-223.0	0.011	2.53	33.32	14	0.003	258.1	0.081	-2.62	42.61	14	0.000
2	-256.3	0.000	2.64	31.10	10	0.001	242.1	0.018	-2.68	28.23	10	0.002
1	-290.2	0.000	2.76	27.19	6	0.000	204.8	0.000	-2.43	43.99	6	0.000

Note: Panel A reports results for a KSVAR(p) with inflation, output gap, long rate, and policy rate. Panel B reports corresponding results for a CKSVAR(p) that includes shadow rates. The sample period is 1960q1-2019q1 for the U.S. and 1985q3-2019q1 for Japan. Long rates are 10-year government bond yields for the U.S. and 9-year yields for Japan. Under the null hypothesis, the short rate is excluded from the equations for inflation and output only. loglik is the value of the log-likelihood. pv-p is the p-value of the test for lag reduction. AIC is the Akaike information criterion. LR is the value of the LR test statistic for excluding short rates from equations for inflation and output gap. df is the number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

Table 3.8: Test for excluding short rates from VARs that include long rates and money

Mon. Aggr.	sample	p	LR	df	p-val
MB	1960q1–2019q1	3	55.05	16	0.0000
M1	1960q3-2019q1	3	55.50	16	0.0000
M2	1960q3-2019q1	3	54.77	16	0.0000
M2M	1960q3-2019q1	4	73.78	20	0.0000
MZM	1960q3-2019q1	4	79.65	20	0.0000
DIVM1	1968q3-2019q1	4	80.68	20	0.0000
DIVM2	1968q3-2019q1	4	111.50	20	0.0000
DIVM2M	1968q3-2019q1	4	110.88	20	0.0000
DIVMZM	1968q3-2019q1	4	107.10	20	0.0000
DIVM4	1968q3-2019q1	4	135.38	20	0.0000
SHO	1990q4-2019q1	3	94.38	16	0.0000

Note: The estimated model is a KSVAR(p) for the U.S. with inflation, output gap, the Federal Funds Rate, the 10-year government bond yield, and a different measure of money growth in each row. Sample availability varies for each monetary aggregate used. LR is the value of the LR test statistic for the testing that lags of the Federal Funds Rate can be excluded from all other equations in the model, df is the number of exclusion restrictions, and p-val is the asymptotic χ^2_{df} p-value of the test.

Robustness of test results for the U.S. to the Great Moderation. The test results of the IH over the full sample are subject to a possible misspecification arising from the 'Great Moderation', a drop in U.S. macroeconomic volatility in the mid-1980s. Therefore, we assess the robustness of our results by estimating the model and performing the above tests of the IH over the sub-sample which starts in 1984q1. Tables 3.9 and 3.10 report the results over this subsample, which correspond to the results reported in Tables 3.1 and 3.3 in the main text for the full sample, respectively. The results of the tests of the IH remain the same: the hypothesis is firmly rejected.

Table 3.9: Test for excluding short rates form VAR that includes long rates post-1984

	KSVAR(p)						CKSVAR(p)					
p	loglik	pv-p	AIC	LR	df	p-val	loglik	pv-p	AIC	LR	df	p-val
5	97.92	-	-0.01	25.63	18	0.11	122.37	-	-0.07	61.65	33	0.002
4	92.83	0.857	-0.17	28.01	15	0.022	119.99	1.000	-0.33	70.78	27	0.000
3	85.07	0.776	-0.28	22.31	12	0.034	103.32	0.556	-0.37	48.44	21	0.001
2	66.06	0.064	-0.24	22.99	9	0.006	77.86	0.009	-0.30	40.06	15	0.000
1	14.30	0.000	0.27	5.33	6	0.502	19.29	0.000	0.25	13.59	9	0.138

Note: The estimated model is a (C)KSVAR(p) for the U.S. with inflation, output gap, Federal Funds Rate, and the 10-year government bond yield. Estimation sample is 1984q1-2019q1. loglik is the value of the log-likelihood. pv-p is the p-value of the test for lag reduction. AIC is the Akaike information criterion. LR is the test statistic for excluding short rates from equations for inflation, output gap and long rates. df is the number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

Table 3.10: Testing CSVAR against CKSVAR post-1984

Country	p	LR	df	p-val
U.S.	3	31.17	15	0.008

Note: The unrestricted model is a CKSVAR(3) for the U.S. with inflation, output gap, 10-year government bond yields, and the Federal Funds Rate. Sample: 1984q1-2019q1. LR is the test statistics of the restrictions that the model reduces to CSVAR(3). Lag order is chosen by AIC. df is the number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

Robustness of results to the inclusion of credit spreads in the VAR. For the U.S., we use Moody's seasoned BAA corporate bond yield relative to 10-year treasury yield as the credit spreads, and the excess bond premium in Gilchrist and Zakrajsek (2012) for Japan. The test results show that our baseline results for IH_1 (Tables 3.11-3.12) and IH_2 (Table 3.13) are robust to the inclusion of credit spreads in the VAR.

Table 3.11: Test for excluding short rates from VAR that includes long rates and credit spreads

Panel A: KSVAR												
	United States						Japan					
p	loglik	pv-p	AIC	LR	df	p-val	loglik	pv-p	AIC	LR	df	p-val
5	298.6	-	-2.34	40.59	24	0.018	391.8	-	-3.52	45.64	24	0.005
4	286.3	0.486	-2.54	42.69	20	0.002	373.9	0.073	-3.63	38.10	20	0.009
3	266.5	0.086	-2.62	37.49	16	0.002	358.7	0.061	-3.77	34.18	16	0.005
2	232.1	0.000	-2.47	18.24	12	0.109	346.3	0.100	-3.96	31.84	12	0.001
1	178.4	0.000	-2.02	10.79	8	0.214	303.4	0.000	-3.69	44.30	8	0.000
Panel B: CKSVAR												
p	loglik	pv-p	AIC	LR	df	p-val	loglik	pv-p	AIC	LR	df	p-val
5	328.6	-	-2.42	87.26	44	0.000	441.6	-	-3.89	131.19	44	0.000
4	309.9	0.167	-2.59	83.99	36	0.000	412.4	0.001	-3.90	100.99	36	0.000
3	288.0	0.035	-2.72	73.60	28	0.000	376.7	0.000	-3.82	62.78	28	0.000
2	243.0	0.000	-2.48	37.62	20	0.010	356.6	0.000	-3.97	47.15	20	0.001
1	183.5	0.000	-2.10	20.41	12	0.060	316.2	0.000	-3.81	62.95	12	0.000

Note: Panel A reports results for a KSVAR(p) with inflation, output gap, long rate, credit spread, and policy rate. Panel B reports corresponding results for a CKSVAR(p) that includes shadow rates. Estimation sample is 1987q2-2019q1 for the U.S. and 1985q3-2019q1 for Japan. Long rates are 10-year government bond yields for the U.S. and 9-year yields for Japan. The credit spreads are Moody's seasoned BAA corporate bond yield relative to 10-year treasury yield for the U.S., and the excess bond premium introduced by Gilchrist and Zakrajsek (2012) for Japan. loglik is the value of the log-likelihood. pv-p is the p-value of the test for lag reduction. AIC is the Akaike information criterion. LR is the test statistic for excluding short rates from equations for inflation, output gap, credit spread, and long rates. df is the number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

Robustness of Japanese results to 10-year rates. Similarly, we test the robustness of our results for the Japanese data by using the 10-year yields instead. This shortens the available sample for estimation to 1987q4 to 2019q1. Tables 3.14 and 3.15 report test statistics for the two types of tests for the IH. From Tables 3.14 and 3.15, the IH is rejected across all lags. For the CKSVAR alternative, 2 lags are selected based on the AIC. Table 3.15 also suggests the rejection of the IH.

Power of the irrelevance tests IH_1 and IH_2. We use the theoretical model to generate 100 artificial time series under values for the parameter ξ in the range $[0.7, 0.99]$. Table 3.16 and 3.17 report the number of rejections for the tests of our irrelevance hypotheses IH_1 and IH_2, respectively. If our tests are powerful, we would expect the number of rejections to decline with ξ approaching the value of 1 for which the irrelevance hypothesis holds true in the simulated data.

Table 3.12: Test for excluding short rates from VAR that includes long rates and credit spreads

United States, with Excess Bond Premium												
	Panel A: KSVAR						Panel B: CKSVAR					
p	loglik	pv-p	AIC	LR	df	p-val	loglik	pv-p	AIC	LR	df	p-val
5	60.9	-	0.979	73.83	24	0.000	86.7	-	0.970	113.92	44	0.000
4	47.4	0.359	0.851	70.02	20	0.000	70.4	0.332	0.818	100.20	36	0.000
3	22.6	0.009	0.849	54.65	16	0.000	36.7	0.001	0.859	69.35	28	0.000
2	-13.8	0.000	0.975	33.35	12	0.001	0.4	0.000	0.929	49.91	20	0.000
1	-49.3	0.000	1.092	20.91	8	0.007	-36.6	0.000	1.006	37.12	12	0.000

Note: Panel A reports results for a KSVAR(p) with inflation, output gap, long rate, credit spread, and policy rate. Panel B reports corresponding results for a CKSVAR(p) that includes shadow rates. Estimation sample is 1974q2-2019q1. Credit spreads are the excess bond premium in Gilchrist and Zakrajsek (2012). loglik is the value of the log-likelihood. pv-p is the p-value of the test for lag reduction. AIC is the Akaike information criterion. LR is the test statistic for excluding short rates from equations for inflation, output gap, credit spread, and long rates. df is the number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

Table 3.13: Testing CSVAR against CKSVAR with credit spreads

Country	p	LR	df	p-val
U.S.(BAA)	3	49.58	19	0.000
U.S.(EBP)	4	53.56	24	0.000
Japan	2	40.62	14	0.000

Note: The unrestricted model is a CKSVAR(p) in inflation, output gap, long rate, credit spread, and policy rate. Long rate: 10-year government bond yield (U.S.), 9-year government bond yield (Japan). Policy rate: Federal Funds Rate (U.S.), call rate (Japan). Credit spread: Moody's seasoned BAA corporate bond yield relative to 10-year treasury yield (U.S.), the excess bond premium (U.S. and Japan). Sample: 1987q2-2019q1 (U.S. with BAA spread), 1974q2-2019q1 (U.S. with EBP), 1985q3-2019q1 (Japan). p chosen by AIC. LR is the test statistics of the restrictions that the model reduces to CSVAR(p). df is the number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

The tables show that the irrelevance tests are powerful. For instance, in the case of the KSVAR as the unrestricted model, the test rejects IH_1 at a 1 percent significance level with the rejection rate (frequency) of 99 percent when $\xi = 0.7$, while the rejection rate is 1 percent when $\xi = 0.99$ at the same significance level. Similar results hold for alternative significance levels (columns 2, 3), the CKSVAR as the unrestricted model (Table 3.16, Panel B), and the test for IH_2 (Table 3.17).

Testing no attenuation effect. We repeat our test of no attenuation in the response of long rates to monetary policy shocks for different sample periods for the U.S. Table 3.18 shows that no attenuation hypothesis is not rejected if the same sample period of 1990q1–

Table 3.14: Test for excluding short rates from VAR for Japan using 10-year bond yields

	KSVAR(p)						CKSVAR(p)					
p	loglik	pv-p	AIC	LR	df	p-val	loglik	pv-p	AIC	LR	df	p-val
5	285.1	-	-2.92	37.43	18	0.005	320.5	-	-3.17	99.85	33	0.000
4	275.1	0.217	-3.02	31.62	15	0.007	307.4	0.159	-3.28	86.95	27	0.000
3	270.5	0.605	-3.20	32.05	12	0.001	290.6	0.023	-3.33	62.07	21	0.000
2	256.2	0.155	-3.23	24.60	9	0.003	274.3	0.004	-3.39	50.90	15	0.000
1	196.4	0.000	-2.53	22.84	6	0.001	212.8	0.000	-2.73	38.81	9	0.000

Note: The estimated model is a (C)KSVAR(p) for Japan with inflation, output gap, 10-year government bond yields, and the call rate. Estimation sample is 1987q4-2019q1. loglik is the value of the log-likelihood. pv-p is the p-value of the test for lag reduction. AIC is the Akaike information criterion. LR is the test statistic for excluding short rates from equations for inflation, output gap and long rates. df is the number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

Table 3.15: Testing CSVAR against CKSVAR for Japan using 10-year bond yields

Country	p	LR	df	p-val
Japan	2	47.54	11	0.000

Note: The unrestricted model is a CKSVAR(2) for Japan with inflation, output gap, 10-year government bond yields, and the call rate. Estimation sample: 1987q4-2019q1. LR is the test statistics of the restrictions that the model reduces to CSVAR(2). Lag order is chosen by AIC. df is yje number of restrictions. p-val is the asymptotic χ^2_{df} p-value of the test.

2012q4 is adopted as in Swanson and Williams (2014). If the sample period is extended backwards (starting from 1960q1), the null is rejected at a 5 percent significance level. These results suggest that the responses of the long rate to a monetary policy shock may differ between non-ELB and ELB regimes, depending on the sample period.

Table 3.16: Test for excluding short rates from VAR that includes long rates, with simulated data

Panel A: KSVAR			
	(1)	(2)	(3)
ξ	$p \leq 0.01$	$p \leq 0.05$	$p \leq 0.1$
0.7	99	100	100
0.75	90	98	99
0.8	76	92	94
0.85	58	73	81
0.9	25	48	60
0.95	7	19	31
0.99	1	11	18
Panel B: CKSVAR			
	(1)	(2)	(3)
ξ	$p \leq 0.01$	$p \leq 0.05$	$p \leq 0.1$
0.7	99	100	100
0.75	93	99	100
0.8	85	92	94
0.85	69	84	88
0.9	42	62	71
0.95	20	33	45
0.99	13	25	32

Note: Panel A reports results for a KSVAR(1) with inflation, output gap, long rate, and policy rate. Panel B reports corresponding results for a CKSVAR(1) that includes shadow rates. Estimation sample is data simulated by the calibrated DSGE model for 237 quarters, which equals the length of the U.S. sample in section 3.4.2. For each value of ξ, we run 100 simulations. Columns 2-4 report how many times the irrelevant hypothesis is rejected with 1 percent, 5 percent, and 10 percent significance levels, respectively.

Table 3.17: Test CSVAR against CKSVAR, with simulated data

	(1)	(2)	(3)
ξ	$p \leq 0.01$	$p \leq 0.05$	$p \leq 0.1$
0.7	100	100	100
0.75	100	100	100
0.8	99	100	100
0.85	91	97	97
0.9	58	81	85
0.95	13	20	34
0.99	3	6	14

Note: This table reports results for a CKSVAR(1) with inflation, output gap, long rate, and policy rate. Estimation sample is data simulated by the calibrated DSGE model for 237 quarters, which equals the length of the U.S. sample in section 3.4.2. For each value of ξ, we run 100 simulations. Columns 2-4 report how many times the irrelevant hypothesis is rejected with 1 percent, 5 percent, and 10 percent significance levels, respectively.

Table 3.18: Test for no attenuation, various sample periods

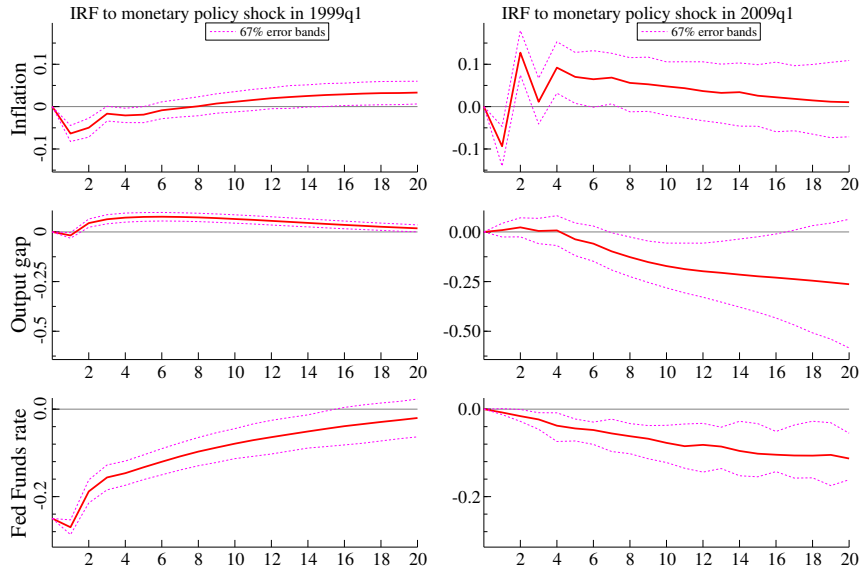
sample	p	LR	p-val
1990q1–2012q4	3	0.03	0.872
1960q1–2012q4	3	4.08	0.043

Note: The estimated model is a CKSVAR(p) for the U.S. with inflation, output gap, long-term rate, and policy rate. The long rate is the 10-year government bond yields. The hypothesis is tested with different sample periods, with the first one being consistent with Swanson and Williams. LR is the value of the likelihood ratio test statistic and asymptotic p-values are reported.

3.E Choleski identification

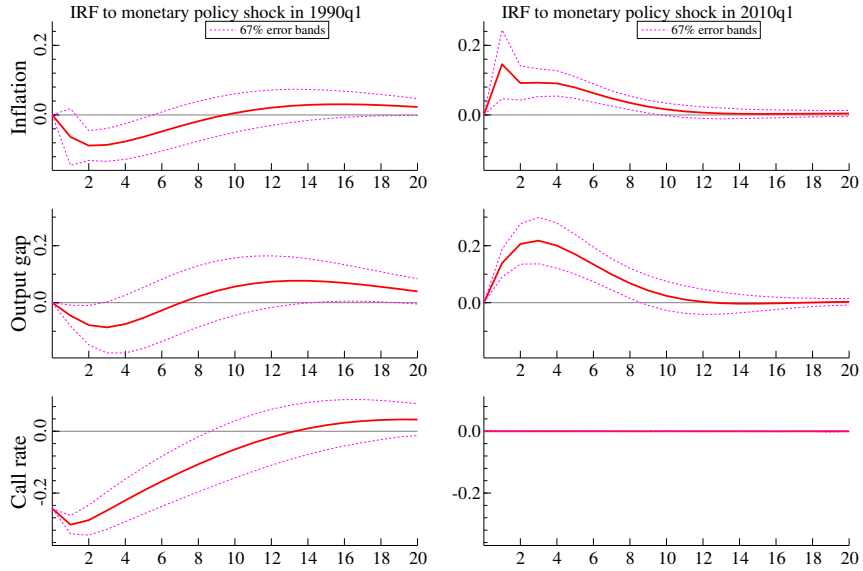
In our benchmark analysis we use the combination of the ELB identification developed by Mavroeidis (2021) and the sign restrictions similar to those employed by Debortoli et al. (2019) to estimate the UMP parameter ξ. This appendix shows the results from using the standard Choleski identification. Figures 3.11 and 3.12 reports results for the U.S. and Japan, respectively. They show that the Choleski identification generates several puzzling responses such as the instantaneous decreases in output and inflation in reaction to a negative monetary policy shock. These responses are consistent with the findings in Gertler and Karadi (2015) for the U.S. and Kubota and Shintani (2022) for Japan, who also show similar responses when using the Choleski identification. Thus, our analysis corroborates the results on the empirically-incongruous responses from the Choleski identification, while showing that the identification based on the combination of the ELB identification and sign restrictions provides plausible responses to monetary policy shocks for the U.S. and Japan when the economy is at the ELB. See Gortz et al. (2022) for a discussion of the issue and some additional corroborative evidence on U.K data.

Figure 3.11: Choleski identification: Impulse responses to a monetary policy shock in the U.S.



Note: Identified sets of IRFs in 1999q1 and 2009q1 to a -25bps monetary policy shock estimated from CKSVAR(4) model in inflation, output gap, and the Federal Funds Rate for the U.S. over the period 1960q1-2019q1, identified by the Choleski restrictions that the monetary policy shock has no contemporaneous effects on inflation, output, and the Federal Funds Rate. Dotted lines show the 67 percent asymptotic error bands.

Figure 3.12: Choleski identification: Impulse responses to a monetary policy shock in Japan



Note: Identified sets of IRFs in 1990q1 and 2010q1 to a -25bps point monetary policy shock estimated using a CKSVAR(2) model in inflation, output gap, long rate, and the call rate for Japan over the period 1985q3-2019q1, identified by the Choleski restrictions that the monetary policy shock has no contemporaneous effects on inflation, output, and the call rate. Dotted lines show the 67 percent asymptotic error bands.

3.F Shadow rates

Our analysis defines the shadow rate as the short-term interest rate that the central bank would set if there were no ELB. Thus defined, the shadow rate can be interpreted as an indicator of the desired monetary policy stance and we provide estimates of it for Japan and the U.S. Our estimates of the shadow rate do not impose the assumption that the model used to obtain them is constant across regimes, and therefore they explicitly account for the empirical relevance of the ELB over the estimation periods.

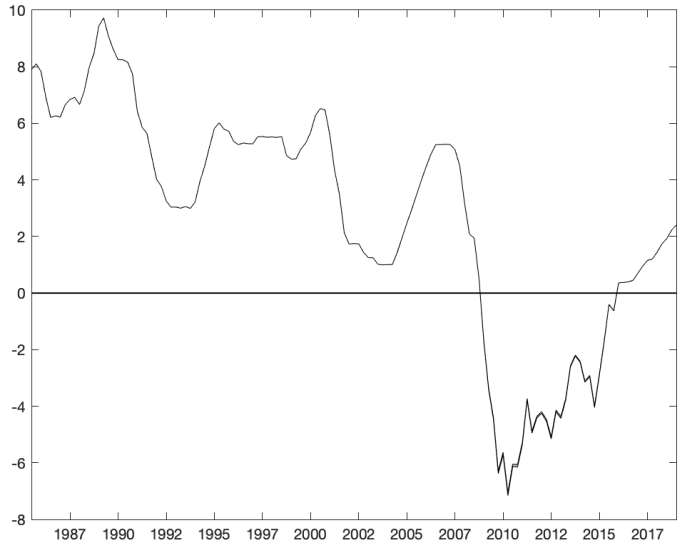
The important caveat is that the shadow rates are not identified under our present assumptions. As explained in Mavroeidis (2021), identifying the shadow rate i_t^* in the empirical model (3.17a)-(3.17c) in the main text requires knowledge of the parameter α, which scales the reaction function coefficients and policy shocks during the ELB regimes and is not identified without additional information. This parameter is needed *in addition* to the parameter ξ that measures the overall impact effect of UMP. In other words, to properly identify the shadow rate and interpret it as a measure of desired policy stance, we need to be able to isolate the effect of FG encapsulated by α. This exercise is beyond the scope of the present paper.

With the above caveat in mind, we report identified shadow rates under the assumption of $\alpha = 0$. The shadow rates are given in Figures 3.13 and 3.14 for the U.S. and Japan, respectively. Different values of α would scale those estimates by a factor $1 + \alpha$.[28] Note that, even with $\alpha = 0$, the shadow rate is only partially identified because it also depends on the parameter ξ that is partially identified. This uncertainty due to ξ is reflected in the shaded areas below the ELB in the figures.[29] In the case of the U.S., the shadow rate dropped sharply soon after the onset of the global financial crisis of 2007-2008. It reached its smallest value at the beginning of 2010 and gradually recovered until the exit from the ELB in 2016. In Japan, the behaviour of the shadow rate is different during the three ELB episodes. During the first episode, the shadow rate fell modestly. In the second episode,

[28] Results are available on request.

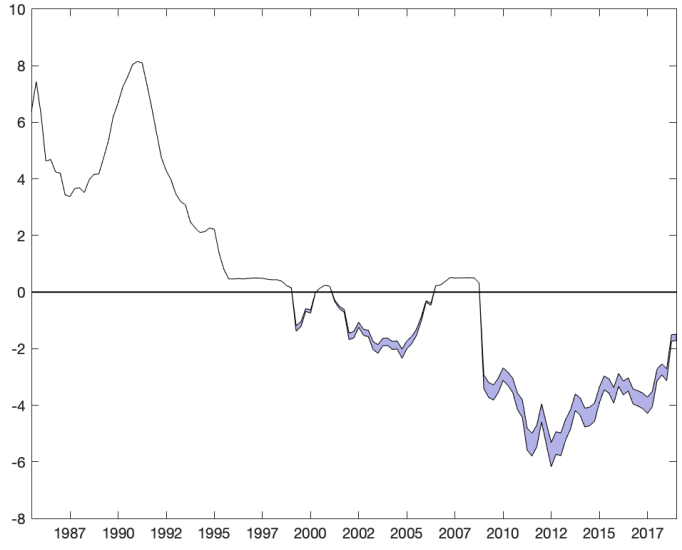
[29] The shadow rate is equal to the observed policy rate above the ELB, see equation (3.17a) in the main text. Below the ELB, it is given by the equation $Y_{2t}^* = \kappa \overline{Y}_{2t} + (1-\kappa) b_t$, where $\kappa = (1+\alpha)(1-\gamma\beta)/(1-\xi\gamma\beta)$ and $\overline{Y}_{2t}$ is a "reduced-form" shadow rate that can be filtered from the data using the likelihood, see Mavroeidis (2021).

Figure 3.13: Shadow policy rate for the U.S.



Note: Estimated using a CKSVAR(4) model in inflation, output gap, long rate, and the Federal Funds Rate for the U.S. over the period 1960q1-2019q1 (plotted over the sub-sample 1985q3-2019q1), identified by the sign restrictions that a -25bp monetary policy shock has nonnegative effects on inflation and output and nonpositive effects on the federal funds rate and the shadow rate up to four quarters.

Figure 3.14: Shadow policy rate for Japan



Note: Estimated using a CKSVAR(2) model in inflation, output gap, long rate, and the call rate for Japan over the period 1985q3-2019q1, identified by the sign restrictions that a -25bp monetary policy shock has nonnegative effects on inflation and output and nonpositive effects on the call rate and the shadow rate up to four quarters.

it exhibited a persistent decline until the beginning of 2005, followed by a quick reversal. In the third episode, which coincided with the ELB in the U.S., the decline was sharp, and followed by a second wave of declines that lasted until mid-2012. From that point on, the shadow rate exhibited a steady rise, but stayed far from zero even at the end of the sample, and remained near its trough in the second episode.

Chapter 4

What Hinders Structural Reforms

Abstract This chapter builds a macroeconomic political economy model to show that political costs associated with structural reforms hinder reform implementation. I first illustrate the main mechanism in a two-period model and then embed it into an infinite-horizon heterogeneous-agent model which is calibrated to Spain. In the model, the effects of product market deregulation create its own resistance, and the resistance generates endogenous political costs to impair deregulation. In addition, adverse initial conditions are associated with more reform progress, which is consistent with the empirical literature. A higher proportion of liquidity-constrained agents and a higher use of fixed-term contracts raise market regulation. By contrast, compensation schemes, labour market reform, and strong government leadership in negotiation help deregulation. Lastly, I use the model to discuss why product markets are more deregulated in some European countries than in others.

4.1 Introduction

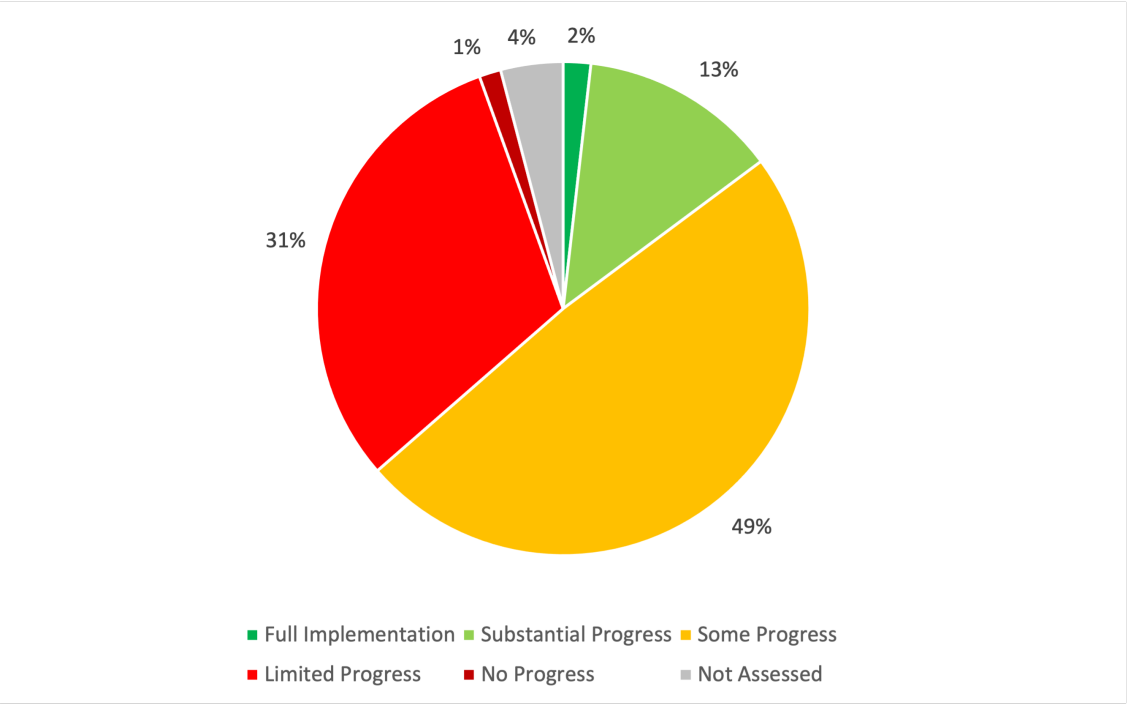
Structural reforms that deregulate product and labour markets frequently feature in policy recommendations by the European Central Bank (ECB), the World Bank, and the OECD as a promising mid- to long-term policy tool to promote growth and attain debt sustainability. Most recently, the former president of the ECB, Mario Draghi, has emphasised the importance of structural reforms for southern European countries hit by the debt crisis of 2010 (Draghi, 2017). The similar idea of deregulating market entry and promoting competition was also a central pillar of the Washington Consensus which guided the policy recommendation for many Latin American countries since the end of 1980s to restore macroeconomic stability and maintain fiscal discipline (Williamson, 2018).

Despite the widely-believed potential benefits, the implementation of structural reforms often makes weak progress in reality. Figure 4.1a shows the progress of all reform recommendations in the European Commission's annual country reports for all countries in the European Union throughout 2011-2020. Only 15% of reform measures recommended by the Commission are making substantial progress or fully implemented over the 10-year period. The rest are at best adopted by the legislation authorities of the respective countries but are implemented in very limited scales, which are assessed as making 'some progress' by the Commission. Among them, the progress in deregulating markets and encouraging competition is typically weak. No reform recommendation in this category is fully implemented, and only 8% of them are making substantial progress, as shown in Figure 4.1b.

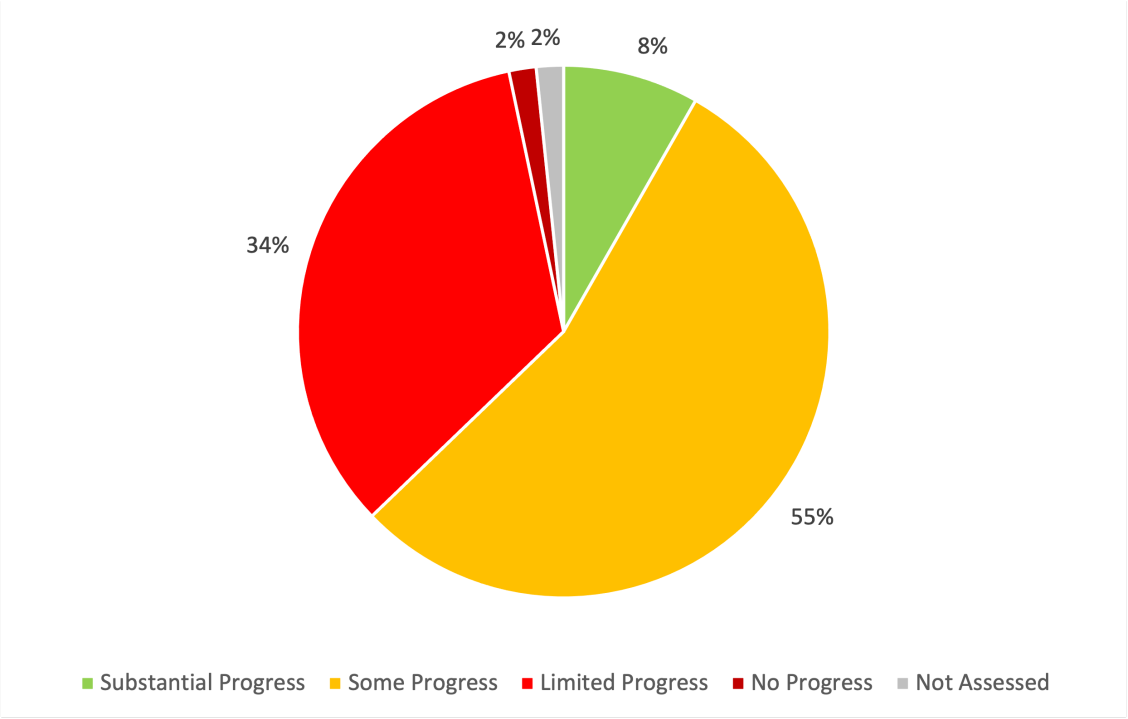
What explains this discrepancy between policy recommendation and implementation? This paper builds a political macroeconomic model to show that the political costs for the governments associated with reforms can hinder reform implementation. These political costs originates from the political actions participated by potential losers who expect to lose out if the government reforms. A key contribution of my paper is that I endogenise the political costs such that how much political costs that potential losers can impose on the government by participating in political actions depends upon how much they expect to lose from deregulation. The governments thus may not be able to reform, because this induces excessive political costs for them.

The arguments in my paper potentially apply to a range of structural reform measures. For concreteness, in my model, I focus on the deregulation of the product markets which increases the real income and creates more job opportunities for all workers on the aggregate level by making the market more competitive. However, on the individual level, the policy creates potential losers and winners among workers who influence the governments' decision on the regulation level to be implemented by imposing political costs for the governments.

Figure 4.1: Country-specific Reform Recommendation Implementation in European Union 2011-2020



(a) All Policy Areas



(b) Competition Policy

Note: The progress is assessed by the European Commission. Source: European Commission (2022) and the author's calculation.

I first illustrate the main mechanism in a two-period model with hand-to-mouth workers. In the model, the government can choose to deregulate the product market in period 1, which is captured by a cut in the entry cost for the product market. The policy is implemented in period 2 and encourages new entrants into the market. This results in a more competitive market where the average price mark-up charged by firms drops in the second period. For the outsider workers who are currently unemployed by the incumbent firms in the market, the lower price mark-up will raise their real income and the new entrant firms will offer them better employment opportunities in period 2. Thus, they are winners from the deregulation in expectation in period 1.

Meanwhile, the more competitive market after deregulation forces more incumbent firms to downsizing or closure. This means that the probability of losing jobs for for the insider workers who are currently employed by the incumbent firms increases if the market is deregulated in period 2. Once a worker is dismissed, she is subject to a significant income loss. In addition, there are some non-pecuniary costs associated with the more competitive market that hurt workers' welfare directly.[1] Therefore, the insiders potentially lose out from deregulation in expectation in period 1.

In period 1, both insider and outsider workers form a subjective view of the level of regulation that they think as fair to be implemented for the market in period 2 before the government decides and announces the actual level of regulation to be implemented. However, this subjective view is tainted by a self-serving bias. Given that the insiders expect themselves to lose out if the market becomes more deregulated, they oppose low regulation levels. Conversely, the outsiders support full deregulation.

The governments choose the regulation level to be implemented in period 2 by balancing between maximising aggregate workers' welfare and minimising the political costs to this regulation level as possible. The political costs comes from the political actions such as demonstrations, riots, lobbying, and negative media coverage that workers potentially take to impose costs to the government. Importantly, the likelihoods and the scales of these political actions depend not only on how well-organised the worker groups are politically, but also on how much the actual policy deviates from the level of regulation that the workers think as fair to be implemented. The larger this deviation is, the more aggrieved workers are towards the government. The more aggrieved workers are more likely to take political actions against the government, which imposes higher political costs. The government internalises the costs of these potential political reactions when deciding the regulation level. As a result, the governments have to keep an inefficiently high level of regulation in the market. If the government attempts to deregulate further

[1] This can be due to the distress toward higher wage inequality and having to exert more effort in work, as explained in more detail in section 4.2.2.

from this regulation level, aggregate workers' welfare may increase but the insiders expect to lose more. This increases their participation of political actions which induces higher costs for the government. The higher political cost will hinder the government to further deregulate.

Next, I extend the model to an infinite-horizon heterogeneous-agent political economy model (HAPEM). In this version of the model, workers have infinite horizon and a majority of them can use a risk-free asset to hedge against their idiosyncratic risk of losing jobs. The model shows that workers with assets expect higher real income following the deregulation but they no longer worry about the cost of increased job insecurity. Therefore, they support full deregulation of the market regardless of their current employment status and asset holding. However, the remaining minority of the workers are still liquidity-constrained and cannot hedge against the job-losing risk. This means that the liquidity-constrained insiders are still negatively affected by the increased job insecurity following deregulation. Moreover, the infinite-horizon liquidity-constrained outsiders will be more likely to be dismissed as well once they find a job in the deregulated market, despite that they are more likely to find a job immediately after the deregulation. Therefore, all types of workers who cannot hedge against the idiosyncratic job insecurity will resist the full deregulation. This is consistent with the observation made by Haggard and Kaufman (1989) that illiquid assets holders are more likely to oppose economic adjustments because they are unable to circumvent the adverse consequences. Even if they constitute only a small fraction of the entire population in the model calibrated to Spain, their opposition is strong enough to block full deregulation, which is the efficient policy implied by the model.

A key implication of the model is that economy with a higher initial regulation level faces weaker opposition to deregulation and is able to make more progress in deregulation. By contrast, a well-deregulated market may actually expect little further deregulation or even some reform reversals. This result resonates with the empirical finding in literature such as Da Silva et al. (2017) and Helbling et al. (2004) that adverse initial conditions and recessions are positively correlated with higher reform intensities, especially for the product market.

In addition, two novel insights emerge from the model. First, better risk-sharing among workers, which corresponds to a lower share of liquidity-constrained workers, contributes to a lower implemented regulation level. This is because the liquidity-constrained workers are unable to avoid the negative consequences of the reform, and a lower share of them endogenously weakens the political resistance against deregulation.

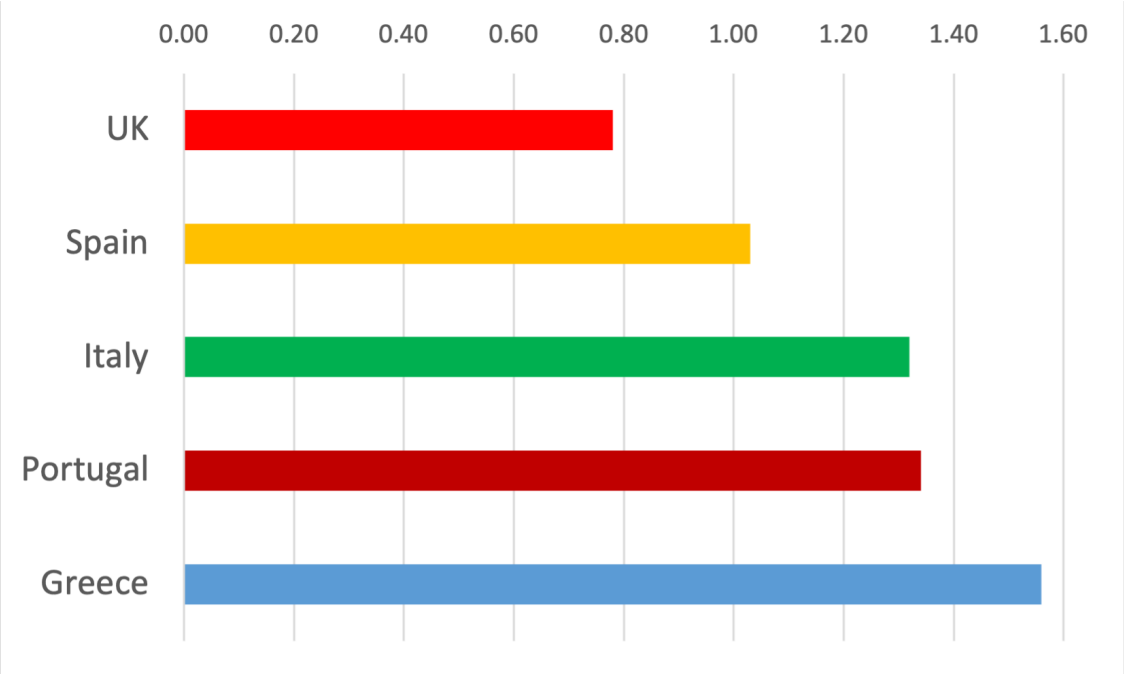
Second, the market is more regulated if the fixed-term contract is more prevalent in the labour market. The empirical work by Aparicio-Fenoll (2015) shows that temporary workers who are employed under this type of contract are subject to significant job

insecurity, and the risk of them being displaced from their current jobs is strongly and positively correlated with the market competition level. I extend the model so that a fraction of workers are employed under this type of contract. The model shows that the liquidity-constrained workers under this contract are mobilised to insist on a highly regulated market. Hence, the resistance to deregulation is stronger, which leads to a higher implemented regulation level.

In terms of policy, I show that several measures can be potentially useful in weakening the opposition to deregulation and promoting the reform. These include compensating those who lose out from the deregulation, labour market reform that reduces the use of the fixed-term contract, and strong government leadership in negotiation with the resisting group.

Lastly, I discuss how the key determinants for opposition to deregulation in the model offer potential explanations of why product markets in some European countries are more deregulated than in others as shown in Figure 4.2. These determinants include the share of liquidity-constrained agents, the use of fixed-term contract, the relative job insecurity, and the relative political influences of unions. The version of the model that account for the heterogeneity in these aspects is able to predict a ranking of the regulation level that is consistent to the ranking of the actual product market regulation indicators of these countries.

Figure 4.2: Product Market Regulation Indicators of Selected European countries in 2018



Source: OECD (2018).

Literature Review This paper is related to three strands of literature. The modelling of the effects of the product market deregulation is inspired by Blanchard and Giavazzi (2003). Recent DSGE literature models macroeconomic effects of different types of structural reforms in more detail, such as the contributions by Thomas and Zanetti (2009), Zanetti (2011), Eggertsson et al. (2014), and Cacciatore et al. (2016). My model focuses on the heterogeneous effects of product market deregulation on different types of workers instead, which is essential in considering the political support and resistance to structural reforms.

Second, the political economy component of my model is adapted from the recent contribution by Passarelli and Tabellini (2017). Their paper discusses the political economy motives for public debt accumulation, whereas this paper shows that a similar framework can be used to discuss the implementation issues for structural reforms. The idea that political economy concerns block reforms dates back to Olson (1965). More recent discussions can be found in Fernandez and Rodrik (1991), Alesina and Drazen (1991), Jain and Mukand (2003), Aghion and Schankerman (2004), Beetsma and Poplawski-Ribeiro (2008), Saint□Paul et al. (2016). The empirical contributions such as Da Silva et al. (2017) provide motivation and support the predictions of my model. This paper contributes to this literature in three ways. First, the political economy components in previous models assume either voting or lobbying mechanisms which implies that the political influences are exogenous. By contrast, my model enables the political pressure on the governments to vary endogenously to the structure of the economy and how much different types of agents expect to win or lose from the deregulation. Secondly, winners and losers from a given reform measure in previous models are assumed ad-hoc. In my paper, winners and losers endogenously emerge from the effects of deregulation in a calibrated macroeconomic model, which enriches the interaction between the effects of deregulation and the political reactions of workers. Third, my model endogenously pins down a level of regulation. Depending on the initial condition, partial deregulation and even reform reversals can occur. By contrast, deregulation is either fully implemented or blocked in previous models, which is at odds with the observed gradual reform implementation in reality.

Lastly, to the best of my knowledge, my HAPEM model is the first to embed a political economy mechanism other than the voting mechanism in a standard Bewley-Huggett-Aiyagari type heterogeneous-agent model(Bewley, 1977, Huggett, 1993, Aiyagari, 1994). Another difference of my model relative to the standard heterogeneous-agent model is that the Markov transition probabilities of income status are endogenous to the policy in my model which is the regulation level.

The rest of the paper is organised as follows. Section 4.2 illustrates the main mechanism in a simple two-period model. Section 4.3 extends the model to an infinite-horizon heterogeneous-agent model for additional insights. Section 4.4 considers the implication of the alternative measure for job insecurity associated with the fixed-term contracts to the model and the policy measures for promoting deregulation. Section 4.5 discusses how the insights from the model may help us understand the heterogeneity in regulation levels in European countries. Section 4.6 concludes.

4.2 Two-Period Model

I begin by illustrating the main mechanism in a simple 2-period model which consists of firms that employ workers to produce goods, workers that supply labour to firms and consume goods, and a government that set the regulation level in period 2 subject to the political pressure from the workers. For simplicity, inter-temporal savings are abstracted from the model so that all workers are hand-to-mouth.[2] With a numerical example, my model demonstrates how product market deregulation generates diverging views among workers on the optimal level of regulation, and how these views translate into political forces that shape the regulation that the governments eventually implement. Lastly, I discuss the impacts of initial conditions and ex-ante uncertainty on the implemented regulation level.

4.2.1 Producers

There are m_t incumbent producers in the economy,[3] each producing a different brand of goods Y_t^i according to the technology

$$Y_t^i = N_t^i.$$

The aggregate output Y_t is a composite of different brands of goods:

$$Y_t = [(m_t)^{-\frac{1}{\theta_t}} \sum_{i=1}^{m_t} (Y_t^i)^{\frac{\theta_t - 1}{\theta_t}}]^{\frac{\theta_t}{\theta_t - 1}},$$

[2] Section 4.3 examines the implications of infinite horizon and inter-temporal savings.

[3] For simplicity, I assume that these firms are owned by entrepreneurs who constitute an infinitely small proportion of the population and exert no political influence. Including these entrepreneurs in the analysis should not change the qualitative findings below. I leave the quantitative implication for including these entrepreneurs for future research.

where $\theta_t = g(m_t)$ is the elasticity of substitution between different brands of goods. I assume that $g'(\cdot) > 0$ so that θ_t is increasing with the number of brands in the economy. Thus the demand for each brand is

$$Y_t^i = \frac{1}{m_t}(\frac{P_t^i}{P_t})^{-\theta_t} Y_t,$$

where P_t^i is the price that the producer i charges for her goods. The corresponding aggregate price index P_t is

$$P_t = [\frac{1}{m_t}\sum_{i=1}^{m_t}(P_t^i)^{1-\theta_t}]^{\frac{1}{1-\theta_t}}.$$

Each period, the firm chooses the price and the number of employment to maximise the real profit

$$(\frac{P_t^i}{P_t} - w_t^i)N_t^i,$$

subject to the the firm's demand and production technology. This implies that the optimal price charged by the typical individual producer i is:

$$\frac{P_t^i}{P_t} = \frac{\theta_t(m_t)}{\theta_t(m_t) - 1} w_t^i = [1 + \mu_t(m_t)]w_t^i,$$

where $\mu(m_t)$ is the price mark-up that the firm charges over the marginal cost of production w_t^i. Note that the mark-up is the same across all firms as it negatively depends upon the total number of firms in the economy only. The higher the number of firms m_t, the more competitive the market is. This implies a higher elasticity of substitution between different brands θ_t. From the equation above, this means a lower price mark-up μ_t.

At the start of period 2, new firms can choose to enter the market by paying an entry cost ς. This cost captures the level of regulation in the product market. It is thus the policy variable of my interest. A deregulation corresponds to a cut in the entry cost ς which leads to more entry of new firms. In other words, the number of firms m_t decreases with the entry cost such that $m_t'(\varsigma) < 0$. This drives down profit per firm. For simplicity and without loss of generality, the entry cost ς is assumed to be proportional to the output. The entry of the market occurs until the expected profit is equal to the entry cost for individual firms:

$$(\frac{P_2^i}{P_2} - w_2^i)Y_2^i = \varsigma Y_2^i. \tag{4.1}$$

In an equilibrium where all firms are symmetric and make the same pricing decision, the relative price $\frac{P_t^i}{P_t} = 1$. Then the above can be rewritten as

$$w_2 = 1 - \varsigma,$$

which implies that the real wage in period 2 is equalised across all firms and decreases with the entry cost. The intuition is that a product market deregulation that cuts the entry cost ς increases market competition by encouraging new firm entry. This reduces the market-wide mark-up μ_t through an increase in the elasticity of substitution θ_t. As a result, the real wage paid to incumbent workers increases. This captures the positive effects on real income from deregulating the product markets.

4.2.2 Workers

There are workers with a total mass of 1 in the economy and all workers live for two periods. A proportion n_t^{in} of them are insider employees of the incumbent firms. The rest $1 - n_t^{in}$ of workers are outsiders of the market who are unemployed.

In order to capture the impact of deregulation on individual worker's job market prospects, I assume Markov transition probabilities for the workers' idiosyncratic employment status in each period. If a worker is an inside worker in period 1, the probability that she remains to be an insider in period 2 is

$$Pr(insider, t = 2|insider, t = 1) = q_i(\varsigma),$$

where I assume $\frac{\partial q_i(\varsigma)}{\partial \varsigma} > 0$. This captures the effect that deregulation in the product market increases the job insecurity of the insider workers. The reason is that deregulation that cuts the entry cost ς increases market competition. This typically pushes some incumbent firms out of the market. Meanwhile, surviving firms may have to downsize due to either a smaller market share or adopting more efficient operation procedures. These factors contribute to higher risk of job displacement for insider workers. This corroborates with the discussion in Blanchard and Giavazzi (2003). Aparicio-Fenoll (2015) and Anderton and Lupidio (2019) support this argument with empirical evidence using data from European countries. The probability of the insider workers experiencing a job displacement and becoming an outsider is thus $Pr(outsider, t = 2|insider, t = 1) = 1 - q_i(\varsigma)$.

Alternatively, if the worker is an outsider in period 1, the probability of her remaining to be an outsider in period 2 is

$$Pr(outsider, t = 2|outsider, t = 1) = q_o(\varsigma),$$

where I assume $\frac{\partial q_o(\varsigma)}{\partial \varsigma} > 0$. Accordingly, the probability of the outsider workers being employed in the industry is thus $Pr(insider, t = 2|outsider, t = 1) = 1 - q_o(\varsigma)$. These assumptions imply that the worker is more likely to be employed in the industry following a market deregulation. This is consistent with the empirical finding that product

market deregulation increases the overall employment rate and job finding rate (Bertrand and Kramarz, 2002, Nicoletti and Scarpetta, 2005). Therefore, the transition matrix of workers' employment status can be summarised as:

	insider in $t+1$	outsider in $t+1$
insider in t	$q_i(\varsigma)$	$1-q_i(\varsigma)$
outsider in t	$1-q_o(\varsigma)$	$q_o(\varsigma)$

In period 1, a particular worker j maximises her expected lifetime CRRA utility

$$V_1^j = \frac{(C_1^j/(C_1)^h)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t \frac{(C_2^j/(C_2)^h)^{1-\gamma}}{1-\gamma},$$

where C_1^j and C_2^j are consumption of the worker in period 1 and 2 respectively, and C_1 and C_2 are the average consumption of the economy in the two periods respectively. β is the discounting factor. γ measures the degree of risk-aversion.

h is the habit formation parameter in the "keeping up with the Joneses" fashion. It measures the extent to which the worker cares about the position of her consumption relative to the average level in the economy. This positional concern captures the non-pecuniary costs associated with product market deregulation. Following a deregulation, the average consumption of the economy will be higher. This decreases the utility of both types of worker's with different intuitions.

For the unemployed outsiders, their mental well-being is strongly positively correlated with the unemployment rate of the entire economy as a reference, as reported by Clark (2003). This is because workers want to deviate less from the social norm. In a more competitive market after deregulation, the overall unemployment rate is lower. Then the status of being unemployed hurts more when there is less of it around.

For insider workers, the non-pecuniary costs can come from various sources. For example, Colantone et al. (2019) demonstrates that a more competitive import sector is associated with higher levels of distress experienced by the workers in the sector. Guadalupe (2007) shows that wage inequality is higher with higher product market competition. Griffith (2001) documents that a more competitive market induces higher managerial efforts.

In period 2, the worker maximises her period 2 utility $V_2^j = \frac{(C_2^j/(C_2)^h)^{1-\gamma}}{1-\gamma}$ only. The

maximisation of utility in both periods are subject to the constraint

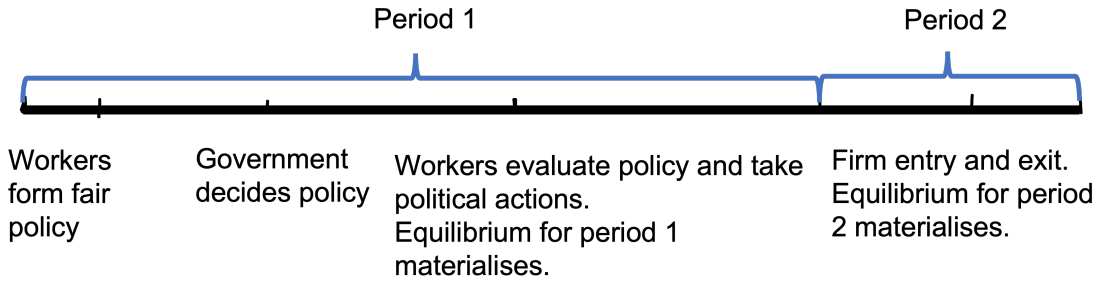
$$C_t^j = \begin{cases} w_t & \text{if insider} \\ 0 & \text{if just displaced} \\ b_t & \text{otherwise} \end{cases}$$

where w_t is the real wage given by firms in period t. $b_t = \nu w_t$ is the unemployment benefit. $\nu \in (0, 1)$ is the unemployment benefit replacement ratio. Each period, each insider worker provides 1 unit of labour only and earn the wage rate. Those who are just dismissed in the current period to become an outsider lose all their income. This captures the large losses from job displacement as documented by, for example, Burdett et al. (2020). Otherwise, if the worker has been an outsider since at least the previous period, she receives the unemployment benefit.

4.2.3 Fair Policy, Aggrievements, and Political Actions

In period 1, the government decides the entry cost for the market ς which comes into effect in period 2.[4] The policy decision on ς is shaped by a procedure of 3 stages, which is adapted from the political mechanism in Passarelli and Tabellini (2017). Figure 4.3 illustrates this sequence of interactions between the governments and the workers.

Figure 4.3: The Sequence of Interactions between the Governments and the Workers



In stage 1, each worker forms a subjective view of the regulation level that she considers as 'fair' to be implemented in the entire economy in the next period, before the governments choose the policy. The worker thus expects a certain level of entitled welfare achieved by this subjectively 'fair' policy as her reservation welfare. In stage 2, the government decides the actual regulation level to be implemented in the next period and announce the policy to the public. In stage 3, after observing the government policy announcement, workers will compare the expected welfare under the actual policy to her

[4] In this two-period model, any change to the entry cost in period 2 will not have any effect unless there is a period 3. Thus, I do not consider the government's policy decision in period 2.

reservation welfare. If her welfare realised by the governments' actual policy is lower than the welfare that she feels entitled to, the worker will feel aggrieved. The aggrieved workers will decide whether or not to take political actions that impose political costs on the government. The more her actual welfare deviates from her reservation welfare, the more likely she is to participate in political actions. These potential individual political reactions are aggregated and internalised by the government when it decides the regulation level in stage 2.

In the following, I describe how workers form their subjectively 'fair' policy in stage 1, and how they react to the actual policy in stage 3. The next subsection describes how the governments internalise workers' potential reactions when they make the actual policy decision in stage 2.

Since the 'fair policy' for each member of the same employment status is the same, I consider the subjective views made by each group $k \in \{in, out, disp\}$ for simpler notation, which corresponds to insider, outsider, and just displaced workers respectively. The 'fair' policy $\hat{\varsigma}^k$ is derived from maximising a modified aggregate social welfare

$$W_1^k = \sum_i \pi_{ik} V_1^i, i \in \{in, out, disp\} \tag{4.2}$$

in stage 1, where the weight that the particular group k attaches to its own welfare, $\pi_{kk} = \delta + (1-\delta)n_1^k$, whilst the other groups $i \neq k$ receive weights $\pi_{ik} = (1-\delta)n_1^i$. $\delta \in (0,1)$ captures the self-serving bias of the group. In other words, the members of each group think their welfare is more representative in the entire economy than those of others'. Hence, the weight attached to the welfare of their own group is larger than the share of population of the group. This bias generates the divergent subjective views among different groups of workers regarding the regulation level to be implemented in the next period.

The optimisation gives a subjective reservation utility for the particular group $R_1^k = V(\hat{\varsigma}^k)$ achieved by the group's subjective views. Note that this utility is not necessarily politically feasible, as workers disregards political pressure from the other groups that the government faces.

In stage 3, after the government's announcement of the actual entry cost to be implemented, workers feel aggrieved if and only if their individual welfare achieved by the actual policy falls short of the reservation utility R_1^k:

$$A_1^k(\varsigma) = \frac{\omega^k}{2} max[0, R_1^k - V_1^k(\varsigma)]^2,$$

where $\omega^k > 0$. A_1^k measures the degree of the worker group k's aggrievement, and it

increases with the positive gap between the reservation and actual utility.

If the worker is aggrieved toward the policy, the worker decides whether or not to participate in political actions by weighing the benefits against the costs of doing so. Here, political actions incorporate a wide range of activities from street demonstration, riots, and unrest that cause social harm, to lobbying party members and negative media coverage that put pressure on policy makers. Realised political actions impose costs for the governments.

The benefit from participating in political actions is an emotional gain that increases with aggrievements, $A_t^k(\varsigma)$. Moreover, the benefit increases with the number of people from the same group that participate in the action, $\mathcal{P}_1^k n_1^k$, where $\mathcal{P}_t^k$ is the participation probability for the worker type k. With increasing participation, workers feel their objection to the policy widely shared and anticipate higher chances of changing policy to their favour.[5]

The cost of participating in political actions comprises two components. There is a cost common to all workers, μ^c. This can be the potential legal punishment and repression following a street protest from workers, or monetary spending for lobbying and media coverage. The second component is an idiosyncratic cost, $\epsilon_{i,t}^k$, for each particular worker i. $\epsilon_{i,t}^k$ is assumed to be uniformly distributed with mean zero and density $\frac{1}{2\sigma^k}$. The standard deviation of this cost σ^k measures how organised a particular group is in initiating political actions. The lower σ^k is, the more organised is the worker type k.

Therefore, the worker will choose to take actions if the benefits of doing so are higher than the costs:

$$\mathcal{P}_1^k n_1^k A_1^k(\varsigma) - \mu^c - \epsilon_{i,t}^k \geq 0.$$

This implies that the participation rate

$$\mathcal{P}_1^k = Pr(\epsilon_{i,1}^k \leq \mathcal{P}_1^k n_1^k A_1^k(\varsigma) - \mu^c) = \frac{1}{2} + \frac{\mathcal{P}_1^k n_1^k A_1^k(\varsigma) - \mu^c}{2\sigma^k}.$$

Solving for $\mathcal{P}_1^k$ we get

$$\mathcal{P}_1^k = \frac{\sigma^k - \mu^c}{2\sigma^k - n_1^k A_1^k(\varsigma)} \equiv \mathcal{P}_1^k(\varsigma), \tag{4.3}$$

where I assume $\sigma^k > max\{n_1^k A_1^k(\varsigma) - \mu^c, \mu^c\}$, so that $0 < \mathcal{P}_1^k < 1$.

It is easy to show that $\mathcal{P}_{A,1}^k = \frac{\partial \mathcal{P}_1^k}{\partial A_1^k(\varsigma)} > 0$, $\mathcal{P}_{n,1}^k = \frac{\partial \mathcal{P}_1^k}{\partial n_1^k} > 0$, $\mathcal{P}_{\sigma,1}^k = \frac{\partial \mathcal{P}_1^k}{\partial \sigma^k} < 0$, so that the probability of participation is increasing if the aggrievement of the group, $A_1^k(\varsigma)$, or

[5] In equilibrium, the announced policy will not change despite that the protesting workers hope it will. This is because the governments already minimised the impacts of these political actions in stage 2 as detailed in the next subsection. An equivalent formulation of the process is that there is a fourth stage in period 1 that the government does reconsider the policy. However, the reconsideration always yields the same decision as in stage 2 in equilibrium because the trade-offs remain the same for the government.

the size of the group, n_1^k, is higher, and if the group is more organised (i.e. lower σ^k).

Moreover, $\frac{\partial \mathcal{P}_{A,1}^k}{\partial A_1^k(\varsigma)} > 0$, $\frac{\partial \mathcal{P}_{A,1}^k}{\partial n_t^k} > 0$, $\frac{\partial \mathcal{P}_{A,1}^k}{\partial \sigma^k} < 0$, so that the reactions of workers are more sensitive to aggrievement when the aggrievement is already higher and when the group is larger and more organised.

4.2.4 Government

In stage 2, the government trades off the social benefit of the policy against the political costs imposed by political actions of workers in response to the policy. Specifically, the government chooses the actual regulation level to be implemented in period 2 to maximise

$$W_1 = \sum_j (V_1^j - \kappa_j \mathcal{P}_1^j), \tag{4.4}$$

subject to the equilibrium conditions of the economy, where $\kappa_j \geq 0$ captures the cost for the government inflicted by political actions of a particular worker j. Given that the only heterogeneity in this model is workers' employment status, this government objective can be conveniently rewritten as

$$W_1 = \sum_k (V_1^k - \kappa_k \mathcal{P}_1^k) n_1^k, k \in \{in, out, disp\},$$

where $\mathcal{P}_1^k n_1^k$ measures the magnitude of political actions from the worker type k. Thus, the second component in the objective sums up to the aggregate political costs for the government from all workers.

4.2.5 Equilibrium

In equilibrium, the goods and labour markets clear:

$$Y_t = C_t = \int_0^1 C_t^j dj$$
$$n_t^{in} = \sum_{i=1}^{m_t} N_t^i.$$

An equilibrium consists of consumption, prices, labour, wages, the subjective views $\{\hat{\varsigma}^k\}$, the government policy $\{\varsigma^a\}$, and the political participation rates $\{\mathcal{P}_t^k\}$, such that

1. the individual utilities and firm profits are maximised;

2. the subjective views maximise the modified aggregate social welfare functions for each group (4.2);

3. the government policy maximises the objective function (4.4);

4. workers choose whether or not to participate in political actions given the government policy, their subjective views, and the participation of other group members.

4.2.6 Parameterisation

Now I illustrate the implications of the model on product market deregulation with a numerical example. The values of parameters are specified in Table 4.1. The discount factor β is chosen to be 0.96 for an annual interest rate of 4%. The risk aversion parameter γ is set to 0.9. The value of θ_1 implies an initial mark-up of 20%. All these parameter values are standard in literature. The unemployment benefit replacement ratio ν is set to 0.4, which is a reasonable value for European countries as in Thomas and Zanetti (2009).

In terms of the transition probability $q_i(\varsigma)$, Aparicio-Fenoll (2015) estimated a linear probability model for the relationship between the probability of becoming unemployed and the product market competition using the Spanish labour market flow data. Market competition in this paper is measured by the profit margin $\frac{P-MC}{P}$, where P is the price and MC is the marginal cost. The higher the profit margin, the less competitive the market is. From equation (4.1), ς in the model measures exactly the profit margin. Therefore, I define $q_i(\varsigma)$ as an increasing linear function with respect to ς:

$$q_i(\varsigma) = \beta_0^i + \beta_1^i \varsigma.$$

and use the corresponding slop estimates in Aparicio-Fenoll (2015) to set $\beta_1^i = 0.136$. β_0^i is chosen such that $q_i(\varsigma = 0.2) = 0.92$. In other words, with an average profit margin of 0.2, the probability of remaining employed is 0.92. These are within the range of the average mark-up and job transition probability estimates for Spain.[6]

As for $q_o(\varsigma)$, there is no existing direct empirical estimates of how it correlates with the profit margin or mark-up to the best of my knowledge. Hence, I calculate $q_o(\varsigma)$ by using the labour market flow equations in steady state:

$$q_o(\varsigma) = 1 - \frac{(1 - q_i(\varsigma))(1 - u_2(\varsigma))}{u_2(\varsigma)},$$

[6]From Soares (2020)'s estimates using data from 2004 to 2012, the aggregate profit margin in Spain is around 0.2. From Ward-Warmedinger and Macchiarelli (2013)'s estimates using data from 1998 to 2008, the probability of transiting from employment to either unemployment or out of labour force in Spain is 0.08.

where $u_2(\varsigma)$ is the unemployment rate in period 2 that satisfies

$$u_2(\varsigma) = 0.14 + 0.06 \times (\mu_2(\varsigma) - 0.2)$$

by the choice of $m_t(\varsigma)$. This implies that the period 2 equilibrium unemployment rate falls by 0.06 percentage points following a 1 percentage point reduction in the price mark-up, which is consistent with the finding by Bertinelli et al. (2013) from the simulation of a DSGE model calibrated to the European labour markets. In addition, the calibration implies that with an aggregate profit margin of 0.2, the unemployment rate is 0.14, which is consistent with the average unemployment rate in Spain over the period 2004-2012 (International Labour Organization, 2022). Appendix 4.A.1 presents the derivation of $q_o(\varsigma)$ and plots the transition probabilities against the profit margin.

The empirical estimate for the positional concern, h, can vary widely in literature across age and different goods types (Carlsson et al., 2007, Akay and Martinsson, 2012). It governs the strength of the non-pecuniary costs of deregulation. I set it to 0.3 in the baseline which is within the range estimated in literature and show the robustness of the results with respect to h.

δ measures the self-serving bias of workers when they form their subjectively views on policy. This is set to 0.7, which is in line with the estimates of altruism from experiments data in Andreoni and Miller (2002) and Fisman et al. (2007).

There is less guidance from the empirical side on how to calibrate the political participation parameters. I calibrate ω, μ^c, and σ symmetrically for the insiders and the outsiders. The choice of these parameters first have to satisfy the condition stated underneath Equation (4.3) to ensure that the participation probabilities are between 0 and 1. In addition, the baseline values imply that political participation rates of different worker types are within the range $[0.02, 0.03]$. This replicates the range of fractions of countries with social unrest within a year in Europe during the period 2010-2020 estimated by Barrett et al. (2022), which approximately measures the frequency of the social unrest in Europe. The values of κ are such that the weight attached to the loss from political actions in the government's objective function (4.4) is around 2.25 times of the weight attached to the aggregate welfare of all workers in the economy, which is a standard choice for government venality in lobby models such as in Bridgman et al. (2007) and Adamopoulos (2008).

4.2.7 Results: What Determines the Market Regulation

Table 4.2 presents the results from the numerical simulation of the model with the calibrated values discussed in the previous subsection. The first three rows reports the regulation levels that the insider, displaced, and outsider workers in period 1 think as fair to be

Table 4.1: Parameter Values of the Two-period Model

Parameter	Value	Interpretation
β	0.96	discount factor
γ	0.9	risk aversion
θ_1	5	elasticity of substitution in period 1
ν	0.4	unemployment benefit replacement ratio
β_0^i	0.893	parameter for probability q_i
β_1^i	0.136	parameter for probability q_i
h	0.3	positional concern
δ	0.7	self-serving bias
κ	1000	political influence
μ^c	19.2	constant cost of political actions
σ	20	standard deviation of idiosyncratic costs
ω	100	parameter for aggrievement

implemented in period 2 respectively. The fourth row is the efficient regulation level that a benevolent social planner would set to maximise the aggregate welfare of all workers in the economy. The last row is the actual regulation level implemented by the government that maximises the objective with political considerations (4.4). I convert the regulation levels measured by entry costs to their corresponding price mark-ups for a more intuitive interpretation.

Table 4.2: Preferred, Efficient, and Actual Regulation Level, Two-period Model

$\hat{\mu}^{in}$	96	mark-up preferred by the insiders
$\hat{\mu}^{disp}$	1	mark-up preferred by the displaced
$\hat{\mu}^{out}$	1	mark-up preferred by the outsiders
μ^i	66	efficient mark-up
μ^a	56	actual mark-up

Note: The table reports mark-ups (%) implied by corresponding entry costs ς from the numerical simulation of the two-period model.

From the table, the insiders advocate a high regulation level that corresponds to a high price mark-up. By contrast, the outsiders advocate full deregulation, which corresponds to 1% of price mark-up in my model, regardless of whether the worker is just dismissed or not. This suggests that the insiders prefer much more regulated markets than the displaced workers and the outsiders.

This divergence in subjective views between different workers is because that the insiders expect themselves to be losers whilst the outsiders expect themselves to be winners at low regulation levels. Consider a reform proposal that decreases the level of product

market entry cost ς from the level that is advocated by the insiders. This attracts more producers to enter the market, making the incumbent product market more competitive.

For insiders who are employed in the incumbent industry, the more competitive market reduces the price mark-up, which in turn increases their real wages. However, this pecuniary gain is compromised by two costs. Firstly, the deregulation decreases in the probability of remaining employed in the current job q_i. Once they are dismissed from the current job, they lose all current income. Secondly, deregulation increases the non-pecuniary cost to their utility. By contrast, although their real income is lower without deregulation, a higher regulation level increases the prospect for them to keep their jobs and reduces the non-pecuniary costs from higher competition. Hence, the insiders advocate more regulated markets when forming their subjective view as they attach higher weight to their own welfare (4.2).

For the unemployed outsiders, the deregulation increases the unemployment benefits as a pecuniary gain. Meanwhile, they are more likely to be employed by the incumbent firms, as reflected by the decrease in the probability of remaining an outsider q_o. Once they are employed in the incumbent industry, they will be able to earn even higher income. These gains are weighed against the higher welfare cost from potentially still remaining an outsider when there are lower unemployment in the economy after the deregulation. Nevertheless, the gains outweigh the loss for the outsiders. Thus, they advocate full deregulation.

In my model, though the total output is maximised in a fully deregulated market with the lowest entry cost, this is not pursued by the government that makes the policy decisions like a benevolent social planner without the political constraints (i.e. by setting $\kappa_{in} = \kappa_{out} = \kappa_{disp} = 0$). This implies that the reason for impeded product market deregulation might simply be that further deregulation does not make the market more efficient. This resonates the 'second-best' argument in Rodrik (2004) that fixing a subset of inefficiencies in an economy may actually reduce welfare. In my model, there are both price mark-up and involuntary unemployment as inefficiencies. In this case, too much competition leads to excessive welfare loss for the insider workers. These insiders constitute the majority of the population in this simple two-period model. Therefore, full deregulation does not maximise the aggregate social welfare function as it hurts the majority of the population excessively.

If the governments cares about the political costs, the actual implemented regulation level is different from the efficient level, even if the political participation parameters are the same for both groups. This is different to lobby models where the actual policy will not deviate from the socially optimal policy if the political powers of rival groups are the same. The reason is that the political pressure that a group exerts on the government depends upon not only the exogenous political participation parameters, but also how each

group endogenously reacts to the policy, which is captured by the political participation probabilities $\mathcal{P}_1^k$. As discussed, these probabilities are endogenously determined by the size of the group and how aggrieved different worker groups are towards a particular level of regulation. Under my baseline calibration, the efficient regulation level that implies 66% of mark-up disappoints the outsiders excessively because it is too far away from what they think is desirable (1% of mark-up). Therefore, the government sets a lower actual mark-up (56%) to prevent exorbitant political costs inflicted by political actions of the outsiders. Meanwhile, the government will not further deregulate the market to gratify the outsiders, because this provokes the insiders to impose political costs that outweighs the gain in outsiders' support.

The political participation parameters also influence the implemented regulation level. Table 4.3 shows four alternative scenarios where the insider workers possess better participation technology. In these circumstances, the actual implemented regulation level is higher than the efficient regulation level. Imagine that the incumbent workers belong to a union that can mobilise workers or has links to lobbying groups. This means that they may incur lower cost in organising protests (lower μ_{in}), that they are more organised (lower σ_{in}), that they are more able to incite workers to take political actions (higher ω_{in}), or that they exert higher political influence or social cost by their actions (higher κ_{in}). In these cases, the efficient regulation level that implies 66% of price mark-up will lead to disproportionate political backlash from the insiders. As a result, the government implements inefficiently high regulation levels. This conforms to the observation such as Haggard and Webb (1993) that in practice, oppositions to reforms often comes from sectors that are more incorporated into the political systems, smaller business, and organised labour force.

Table 4.3: Actual Regulation Level and Political Influence, Two-period Model

scenario	actual mark-up
baseline	56
lower participation cost for insiders ($\mu_{in} = 0.9\mu_{out}$)	84
more organised insiders ($\sigma_{in} = 0.5\sigma_{out}$)	80
more sensitive insiders ($\omega_{in} = 1.5\omega_{out}$)	70
more influential insiders ($\kappa_{in} = 2\kappa_{out}$)	73

Note: The table reports mark-ups (%) implied by corresponding entry costs ς from the numerical simulation of the two-period model.

In summary, this numerical example shows that deregulating product market can be hindered for two reasons. First, further deregulation may not be efficient because it reduces the aggregate social welfare. Second, deregulation may inflict too much political backlash from the insiders of the market. The opposition to deregulation from the insiders will be stronger if they are more politically integrated.

4.2.8 Initial Conditions and Ex-ante Uncertainty

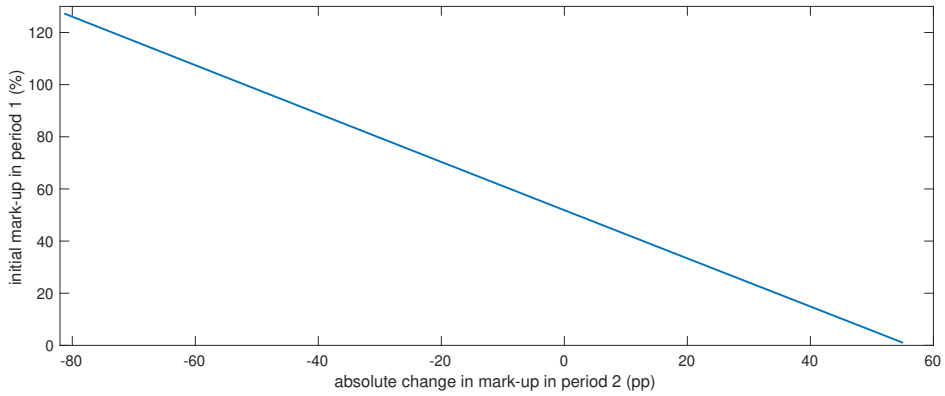
In addition to the factors discussed in the previous section, one of the most discussed aspects in literature that potentially influences the deregulation progress is the initial condition of the economy. Da Silva et al. (2017) and Helbling et al. (2004) demonstrate that weak initial conditions significantly promotes reform efforts. The explanation is that the opposition is weaker in the economy where the condition is further away from the best practice. Based upon the similar rationale, deep crises can foster more reform, which is supported empirically by Alesina et al. (2006) and Høj et al. (2006).

My model generates results that underpin this hypothesis. Figure 4.4a plots the initial mark-up of the economy in period 1 against the change that the government in the model makes in the actual level of mark-up in period 2 relative to the initial mark-up in the economy. The higher initial mark-up in period 1 represents a weaker initial condition. The figure shows that if the initial condition is weaker, more deregulation will be implemented in period 2. For example, with around 120% of initial mark-up in period 1, the government will be able to reduce the mark-up by around 76 percentage points in period 2. By contrast, if the economy starts with a low level of mark-up such as 20%, there will actually be a reform reversal in period 2 that increases the mark-up by around 38 percentage points. This negative relationship qualitatively replicates the experience of the OECD countries in Figure 4.4b.

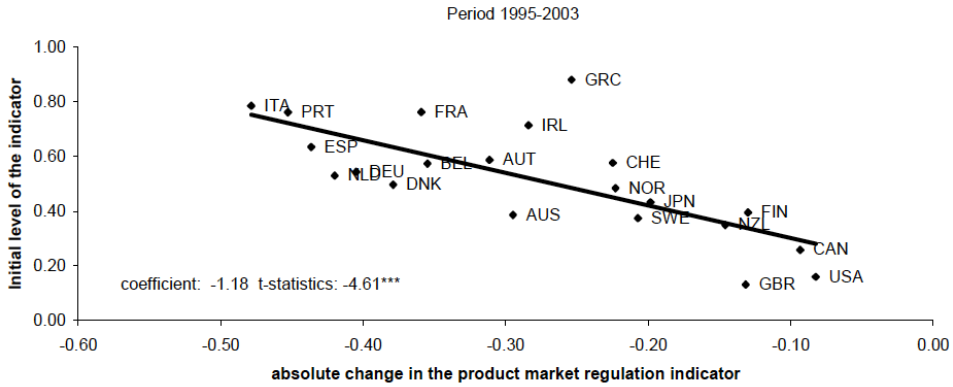
The intuition is that with a higher initial mark-up, the proportion of insiders in the economy is smaller as the unemployment rate is higher. This weakens the opposition to deregulation in three channels. First, workers' subjective views are more conductive to low regulation levels. This is because all workers attach smaller weight to the welfare of insiders when forming their subjective view by maximising (4.2), as long as their self-serving bias $\delta < 1$. This implies that the insider workers will be more sympathetic to the opinion of the outsiders and more tolerant if the government sets a lower regulation level. By contrast, the outsiders will be more insistent on full deregulation. Second, the government will attach smaller weight to the welfare and political influence of the insiders. The reason for the latter is that the insiders' political participation will be smaller given their overall smaller size, as implied by equation (4.3). Third, the opportunity cost of losing jobs, which is the wage rate, is smaller in a more regulated market. Therefore, the insiders are more willing to trade off more job security for potentially higher wage rate. Consequently, they will be less aggrieved if the government deregulates more. These three effects encourage the government to move the policy more to the outsiders' favour.

One of the reasons why all insider workers oppose low regulation levels in the model is that they are uncertain in period 1 about whether they will remain insiders or not in period

Figure 4.4: Initial Conditions and Deregulation, Two-period Model



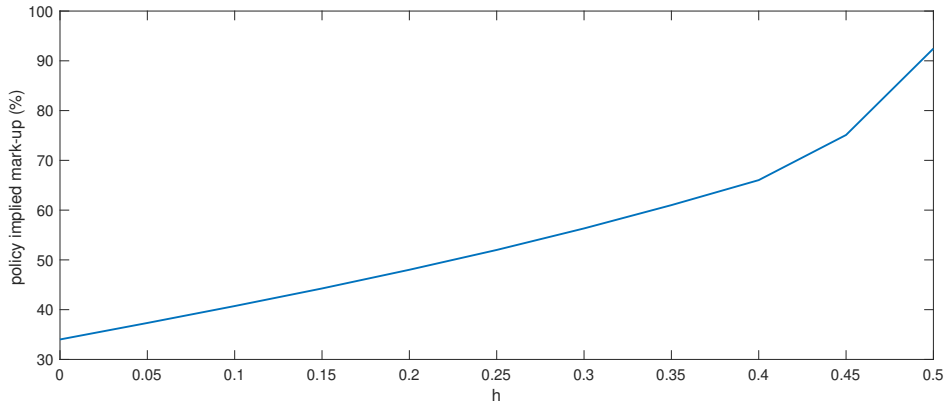
(a) Model



(b) Data from Høj et al. (2006), OECD countries, 1995-2003

2. If they remain to be insiders, they benefit from higher wages in a more deregulated market. Otherwise, they lose if they are dismissed and incur large displacement costs. This means that their expected income is not necessarily increasing with deregulation, which motivates their opposition. Fernandez and Rodrik (1991) argue that this ex-ante uncertainty creates a status-quo bias that hinders efficiency-enhancing reforms. Meanwhile, the other source of the insiders' unanimous opposition in my model comes from the direct non-pecuniary costs from deregulation which is captured by the positional concerns. To what extent do these factors drive the results? Figure 4.5 shows that the actual implemented mark-up increases if the workers are more sensitive to the non-pecuniary costs. However, even if $h = 0$ so that there is no non-pecuniary cost, the ex-ante uncertainty faced by insiders still mobilises strong enough opposition to block full deregulation.

Figure 4.5: Non-pecuniary Costs and Deregulation, Two-period Model



4.3 Infinite Horizon Heterogeneous-Agent Model

In this section, I extend my two-period model to an infinite-horizon heterogeneous-agent political economy model (HAPEM). In this version of the model, the majority of the workers can save via a risk-free asset. The motivation for these extensions is twofold. First, the deregulation has impacts on the workers' welfare beyond 2 periods so that the effects of higher job insecurity, which only the insiders care about in the 2-period model, becomes relevant for the outsider workers as well. Even if the current outsiders are more likely to find new jobs following deregulation, it is also more likely that they will be dismissed in a more competitive market once they become insiders. This potentially weakens their support for full deregulation. Similarly, the higher chances of getting re-employed becomes relevant in the trade-off of the insiders, which potentially weakens their opposition to deregulation. Second, the workers who are able to trade risk-free assets can allocate their income inter-temporally and hedge against their idiosyncratic job insecurity. As we will see, this eliminates the job insecurity concern for the insiders so that the opposition to deregulation is diluted.

4.3.1 Model Set-up

The set-up of the HAPEM model is based upon the Bewley-Huggett-Aiyagari framework and preserves the features of the 2-period model in section 4.2 in many aspects. In a small open economy, there are workers of a total mass of 1. Each of them maximises her expected lifetime CRRA utility

$$V_t^j = \frac{(C_t^j/(C_t)^h)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t[V_{t+1}^j(a_{t+1}^j, \ell_{t+1}^j, \ell_{t+1}^{dj})].$$

A proportion $1-\eta$ of them can trade a risk-free asset. For these workers, the utility maximisation is subject to the following constraints:

$$C_t^j + a_{t+1}^j = w_t \ell_t^j (1-\ell_t^{dj}) + b_t(1-\ell_t^j)(1-\ell_t^{dj}) + (1+r)a_t^j$$
$$a_{t+1}^j \geq -\bar{a}.$$

The first constraint is the budget constraint, where a_t^j is the holding of risk-free asset by the j-th worker in period t. r is the net world real interest rate paid on the savings. ℓ_t^j is an indicator variable which takes the value of 1 if the worker is an insider, and 0 otherwise. ℓ_t^{dj} is an indicator variable which takes the value of 1 if the worker is dismissed from the incumbent firms in period t, and 0 otherwise. The second constraint is a borrowing constraint, where $\bar{a} \geq 0$ is the upper bound that a worker can borrow.

The remaining η of workers are hand-to-mouth. Given that they cannot allocate consumption inter-temporally, their maximisation problem is identical to those faced by the workers in the two-period model, which is described in section 4.2.2. The Markov transition matrix for idiosyncratic employment statuses for all workers is also identical to that specified for the 2-period model in section 4.2.2.

The specification of producers is identical to those in section 4.2.1, except that the market entry condition becomes

$$\sum_{\tau=t}^{\infty} \beta^{\tau-t}\left(\frac{P_\tau^i}{P_t} - w_\tau^i\right) = \varsigma.$$

In period t, the government decides the regulation level that it will credibly implement from the next period onwards.[7] The political process in this period is identical to the process described in section 4.2.3. Thus, before the government decides and announces the actual regulation level, workers form their subjective view of the regulation level to be implemented from the next period onwards by maximising the modified aggregate social welfare

$$W_t^k(\varsigma, a_t^k, \ell_t^k, \ell_t^{dk}) = \sum_{i \in S} \pi_{ik,t} V_t^i(\varsigma, a_t^i, \ell_t^i, \ell_t^{di}), \tag{4.5}$$

where $i, k \in S$ and S is the set of all possible combination of states $\{a_t, \ell_t, \ell_t^d\}$. The weight $\pi_{kk,t} = \delta + (1-\delta)n_t^k$, whilst $\pi_{ik,t} = (1-\delta)n_t^i$ for $i \neq k$. The maximisation gives the the desired entry cost $\hat{\varsigma}_t^k(a_t^k, \ell_t^k, \ell_t^{dk})$ and the corresponding subjective reservation utility $R_t^k(a_t^k, \ell_t^k, \ell_t^{dk}) = V_t^k(\hat{\varsigma}_t^k(a_t^k, \ell_t^k, \ell_t^{dk}))$ for workers under every state $k \in S$.

[7] Here, I abstract from the possibility for the government to change the regulation level in the future. Adding this dimension is an interesting avenue for future research.

After the government announces the actual regulation level to be implemented, workers' aggrievements are also state-dependent:

$$A_t^k(\varsigma, a_t^k, \ell_t^k, \ell_t^{dk}) = \frac{\omega(a_t^k, \ell_t^k, \ell_t^{dk})}{2} max[0, R_t^k(a_t^k, \ell_t^k, \ell_t^{dk}) - V_t^k(\varsigma, a_t^k, \ell_t^k, \ell_t^{dk})]^2, k \in S.$$

Then workers participate in political actions if the benefits of doing so outweigh the costs:

$$\mathcal{P}_t^k(\varsigma, a_t^k, \ell_t^k, \ell_t^{dk}) n_t^k(a_t^k, \ell_t^k, \ell_t^{dk}) A_t^k(\varsigma, a_t^k, \ell_t^k, \ell_t^{dk}) - \mu^c(a_t^k, \ell_t^k, \ell_t^{dk}) - \epsilon_{k,t}(a_t^k, \ell_t^k, \ell_t^{dk}) \geq 0.$$

This implies a participation rate of political actions for workers in any particular state k

$$\mathcal{P}_t^k(\varsigma, a_t^k, \ell_t^k, \ell_t^{dk}) = \frac{\sigma(a_t^k, \ell_t^k, \ell_t^d) - \mu^c(a_t^k, \ell_t^k, \ell_t^{dk})}{2\sigma(a_t^k, \ell_t^k, \ell_t^{dk}) - n_t^k(a_t^k, \ell_t^k, \ell_t^{dk}) A_t^k(\varsigma, a_t^k, \ell_t^k, \ell_t^{dk})}.$$

Lastly, the government decides the actual regulation level to implement in the next period to maximise

$$W_t = \sum_{k \in S} [V_t^k(\varsigma, a_t^k, \ell_t^k, \ell_t^{dk}) - \kappa_k(a_t^k, \ell_t^k, \ell_t^{dk}) \mathcal{P}_t^k(\varsigma, a_t^k, \ell_t^k, \ell_t^{dk})]. \tag{4.6}$$

4.3.2 Stationary Equilibrium and Solution Algorithm

A stationary equilibrium consists of value functions $V : S \to R$, individual choices of consumption and asset holdings, prices, labour, wages, a measure Φ, the subjective views $\{\hat{\varsigma}^k\}$, the government policy $\{\varsigma^a\}$, and the political participation rates $\{\mathcal{P}_t^k\}$, such that

1. V satisfies the workers' Bellman equations, and a_{t+1}^j and C_t^j are the associated policy functions, given r, w_t;
2. the choices of P_t^i and N_t^i maximise the producers' profits;
3. markets for consumption goods and labour clear:

$$Y_t = C_t = \int_0^1 C_t^j d\Phi$$
$$n_t = \sum_{i=1}^{m_t} N_t^i$$

4. let Q be the transition function, for all $s \in S$,

$$\Phi(S) = \int Q(s_t, S) d\Phi;$$

5. subjective views maximise the modified aggregate social welfare functions for workers in each state (4.5);

6. the government policy maximises the objective (4.6);

7. the workers choose whether or not to participate in political actions given the government policy, their subjective views, and the participation of other group members.

The model is solved numerically. I firstly solve for the stationary equilibrium under each discretised grid of the entry cost ς using the value function iteration method. This gives the value functions and the stationary distribution of workers in all possible states given any value of ς. Then I aggregate these measures to obtain the individual modified social welfare for workers in each state. These welfare functions are then maximised to yield distributions of workers' subjective views on all possible values of the policy variable ς. I calculate the aggrievements and political participation rates based upon these views, which are in turn used to construct the government's objective function. Lastly, the actual implemented policy is obtained by maximising this government objective. The details of this solution algorithm is described in Appendix 4.B.

4.3.3 Market Regulation in the HAPEM

The model is calibrated in the same way as the two-period version of the model. All parameters are kept the same except for the political participation parameters. They are adjusted to match the same calibration targets for the two-period model, so that the political participation rate are within the range $[0.02, 0.03]$, and that the weight attached to the political costs in the government's loss function is around 2.25 times of the weight attached to the aggregate welfare of all workers in the economy. In addition, the world interest rate r is set to 3.95%, which is in line with the range of the EU government bond yields at the onset of the Greek debt crisis in 2010. The borrowing constraint $\bar{a}$ is set to 0. The proportion of hand-to-mouth workers η is set to 0.18, which is consistent with the estimate for Spain in Bracco et al. (2021). The calibrated parameters are reported in Table 4.4.

Figure 4.6 plots the mark-ups implied by the corresponding regulation levels that each type of workers think as fair to be implemented in the next period. The overlapped lines at

Table 4.4: Calibration of the HAPEM Model

Parameter	Value	Interpretation
β	0.96	discount factor
γ	0.9	risk aversion
ν	0.4	unemployment benefit replacement ratio
β_0^i	0.893	parameter for probability q_i
β_1^i	0.136	parameter for probability q_i
h	0.3	positional concern
δ	0.7	self-serving bias
κ	27000	political influence
μ^c	19.2	constant cost of political actions
σ	20	standard deviation of idiosyncratic costs
ω	5	parameter for aggrievement
r	0.0395	world interest rate
$\bar{a}$	0	borrowing constraint
η	0.18	proportion of hand-to-mouth workers

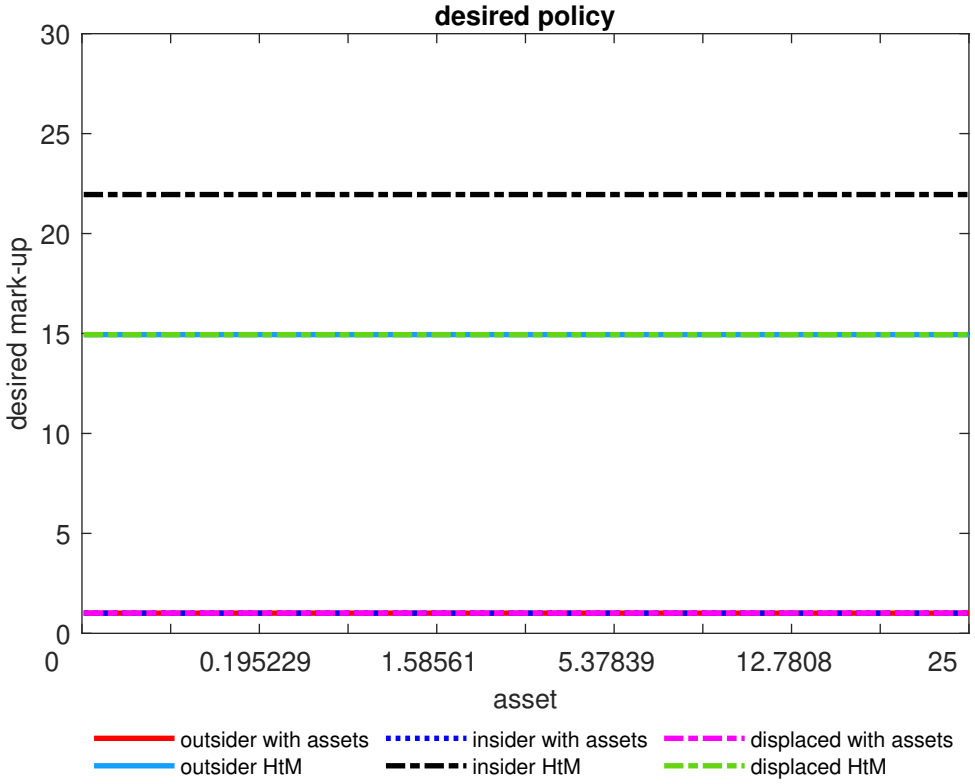
the bottom indicate that the workers who are not hand-to-mouth advocate full deregulation (i.e. mark-up of 1%) regardless their current employment status and asset levels. This is because they are able to hedge against their idiosyncratic future changes of employment status. In this case, the income effect from full deregulation dominates the job insecurity concerns weakened by hedging with assets and the non-pecuniary costs over all worker types.

By contrast, the dashed black line shows that the hand-to-mouth liquidity-constrained insiders advocate a regulation level that implies around 22% of price mark-up in the product market. This is because they are still preoccupied with the higher risk of losing jobs following the deregulation, but they cannot save to hedge against such risks. This motivates them to advocate high regulation levels as in the two-period model. Nevertheless, with a horizon beyond just two periods, they do realise that even if they are dismissed in a more competitive market, it will be easier for them to find another job as an outsider in periods after the next. This explains why these insiders advocate a lower regulation level than insiders in the two-period model.

By the same intuition, the hand-to-mouth outsiders and displaced workers will not advocate full deregulation as they do in the two-period model. Although full deregulation benefits them in the next period as in the two-period model, they now worry about the higher risk of losing their jobs in periods thereafter in a highly competitive market once they are employed as insiders. Since that they cannot hedge against this risk, they prefer some regulation rather than full deregulation.

Imagine the economy starts from a market regulation level which leads to around

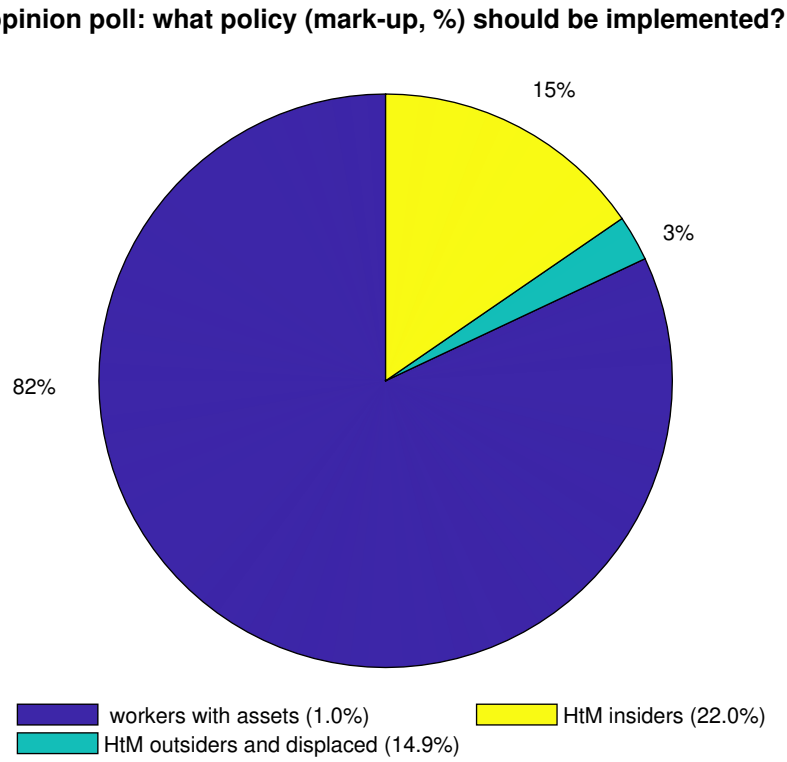
Figure 4.6: Desired level of Mark-ups (%) for All Agents, Baseline HAPEM



Note: The current market has a mark-up of 20%. Lines for HtM outsiders and displaced overlap in the middle, and lines for workers with assets overlap at the bottom.

20% price mark-up. Figure 4.7 reports the results of a opinion poll which asks each worker in the model the regulation level that they think should be implemented. This reflects the stationary distribution of the workers' subjective views of the fair policy. 82% of the workers advocate full deregulation (in dark blue). Consistent with the results in Figure 4.6, these are workers who can trade assets. The remaining workers are liquidity-constrained and they advocate higher mark-ups. The efficient policy that a benevolent social planner would implement is full deregulation in this case. However, the political pressure from the liquidity-constrained workers will force the government to implement an inefficiently high regulation level that implies a mark-up of 5% instead. Different to the two-period model where the majority of the population as insiders advocate high regulation levels, the anti-deregulation coalition here is formed by the minority of the population. This corroborates with the vested interest argument that dates back to Olson (1965). The cost of the policy concentrates on a small proportion of the economy whereas the benefit is spread among a large population. The group that incurs the loss, small yet organised, will act to impede further reform. Moreover, the fact that the liquidity-constrained workers form the anti-reform coalition resonates with the observation made by Haggard and Kaufman (1989) that illiquid assets holders are more likely to oppose

Figure 4.7: Opinion Poll of the Desired Mark-ups (%), Baseline HAPEM



Efficient mark-up: 1%. Actual implemented mark-up: 5%.

Note: The current market has a mark-up of 20%.

economic adjustments because they are unable to circumvent the adverse consequences.

Similar to the two-period model, if the insider workers can organise political actions more easily as captured by the more advantageous political participation parameters, the resistance to deregulation from the liquidity-constrained insiders are strengthened. This imposes higher political pressure on the government to increase market regulation. Consequently, the implemented mark-up is higher as shown in Table 4.7 in Appendix 4.E.

4.3.4 What Affects the Regulation Level

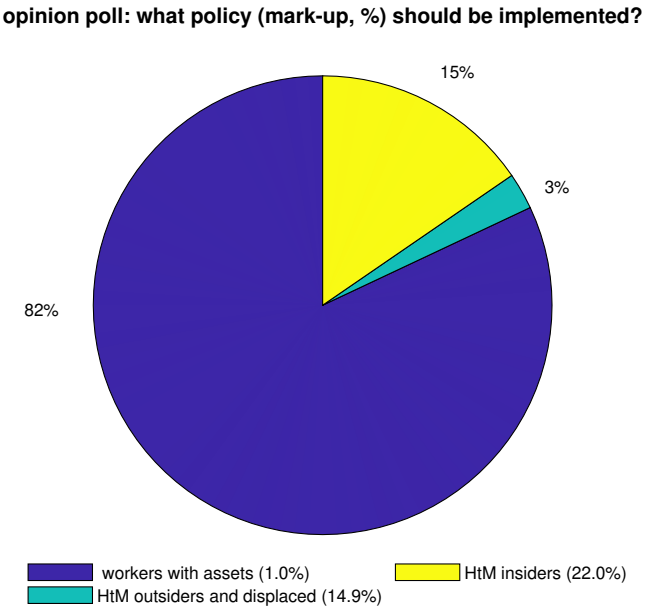
The results above suggest that a worker's support for and opposition to deregulation crucially depends upon whether the worker can trade assets to hedge against employment risks. Therefore, in this subsection, I first consider how the degree of risk-sharing among workers, which is measured by the proportion of liquidity-constrained workers, influences the implemented regulation level in my HAPEM model. Then I explore the impacts of initial conditions and non-pecuniary costs on the regulation level as in the two-period model.

Figure 4.8 compares the distribution of opinions on the regulation level to be implemented when the proportion of liquidity-constrained workers $\eta = 0.18$ and $\eta = 0.4$ respectively. The second case of high η corresponds to countries like Greece and Portugal as estimated by Bracco et al. (2021). When the proportion of liquidity-constrained workers is higher, the actual implemented mark-up increases from 5% in the baseline to 19%. In this case, the political opposition to low regulation levels is strengthened through the first two of the three channels discussed in section 4.2.8. First, all types of workers attach higher weights to the welfare of the liquidity-constrained workers in forming their subjective view on the regulation level to be implemented. Thus, workers with assets become more sympathetic to liquidity-constrained workers. Although this effect is not strong enough to persuade these workers to withdraw their support for full deregulation, they will be more tolerant if the implemented regulation level is higher. Meanwhile, the liquidity-constrained workers become more resistant to low regulation levels so that they advocate even higher regulation levels compared to the baseline. Second, the government realises that the liquidity-constrained workers become more influential not only in affecting aggregate social welfare, but also in inciting political actions with larger participation. Both channels work together to increase the costs for the government to maintain a low regulation level. Thus, the government set a higher regulation level to avoid potentially large political backlashes from the liquidity-constrained workers.

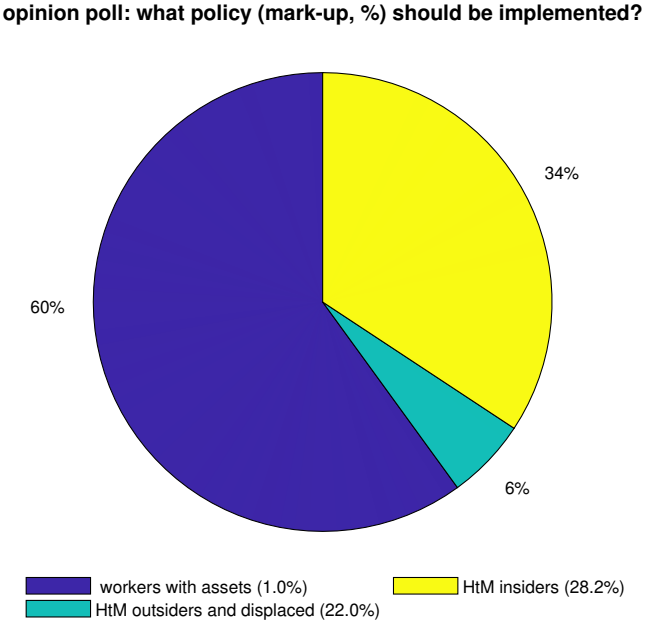
Meanwhile, a higher initial condition, which is captured by a higher initial mark-up implied by a higher initial regulation level in the economy, foreshadows larger progress in deregulation in the subsequent period. Figure 4.9a plots the initial mark-up of the economy against the absolute change in the implemented mark-up relative to the initial mark-up in the economy. Similar to the result of the two-period model in section 4.2.8, we observe a negative correlation between the two, which is consistent with the data. The higher initial regulation level leads to a less competitive market and smaller share of liquidity-constrained insiders among the workers. This weakens the opposition to deregulation in the same way as in the two-period model through the three channels that are discussed in section 4.2.8.

By contrast, higher non-pecuniary costs from deregulation should prompt higher implemented regulation levels. Figure 4.10 compares the opinion polls and the actual regulation levels when $h = 0$, $h = 0.3$, and $h = 0.5$. As expected, a higher h induces the liquidity-constrained workers to advocate higher regulation levels which strengthens the opposition to deregulation. When $h = 0.5$, the implemented regulation implies a mark-up of 27% which is higher than the baseline. By contrast, if there is no non-pecuniary cost (i.e. $h = 0$), all workers advocate full deregulation and this policy is implemented. Note that this unanimous view is achieved only when all workers are not purely self-biased in

Figure 4.8: Proportion of HtM workers and Regulation Level (%), Baseline HAPEM



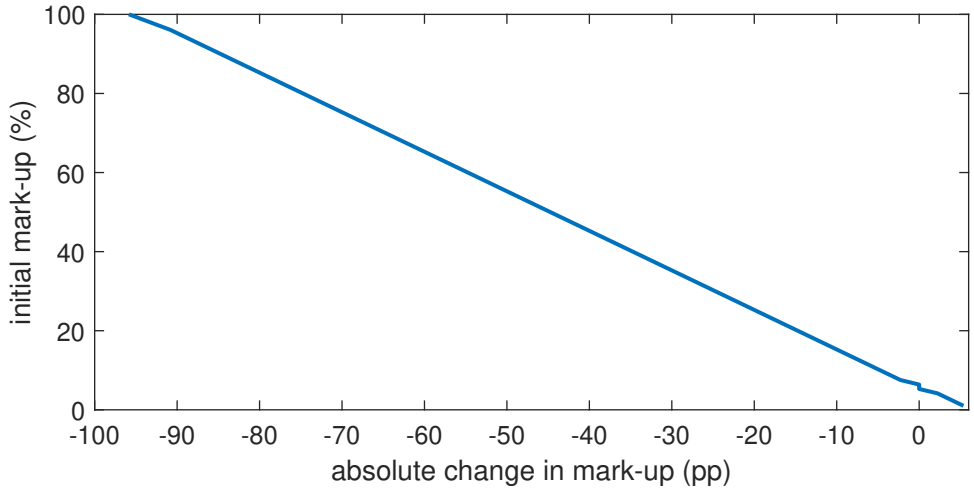
Efficient mark-up: 1%. Actual implemented mark-up: 5%.
(a) $\eta = 0.18$ (baseline)



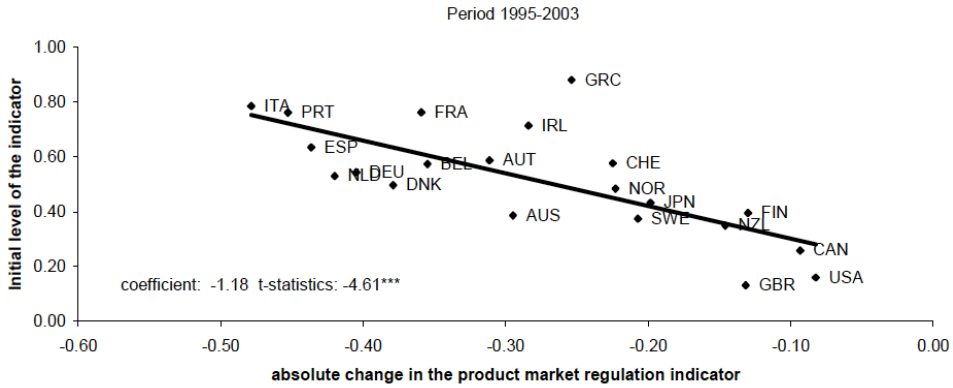
Efficient mark-up: 1%. Actual implemented mark-up: 19%.
(b) $\eta = 0.4$

Note: The current market has a mark-up of 20%.

Figure 4.9: Initial Conditions and Deregulation, Baseline HAPEM



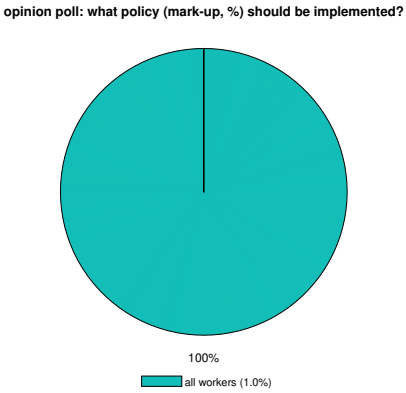
(a) Model



(b) Data from Høj et al. (2006), OECD countries, 1995-2003

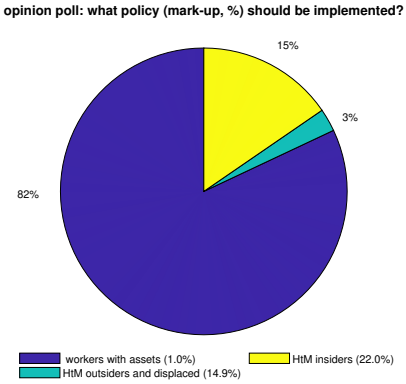
forming their subjectively views. In this case, the liquidity-constrained workers acknowledge that full deregulation is socially desirable and should be implemented, even though full deregulation is not the best policy for themselves. This is demonstrated by Figure 4.15 in Appendix 4.D. The figure shows that when these workers are purely self-biased, they will still oppose full deregulation. Different to the results of the two-period model in section 4.2.8, the increased risk of losing jobs after deregulation alone is not costly enough for workers to resist full deregulation in this case.

Figure 4.10: Strength of Non-pecuniary Costs and Regulation Levels, Baseline HAPEM



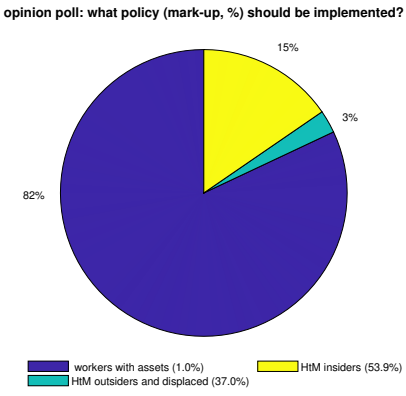
Efficient mark-up: 1%. Actual implemented mark-up: 1%.

(a) $h = 0$



Efficient mark-up: 1%. Actual implemented mark-up: 5%.

(b) $h = 0.3$ (baseline)



Efficient mark-up: 1%. Actual implemented mark-up: 27%.

(c) $h = 0.5$

Note: The current market has a mark-up of 20%.

4.4 Alternative Measure of Job Insecurity

In many European countries, many newly hired workers are temporarily employed under the fixed-term contracts that last no more than $3-5$ years. Aparicio-Fenoll (2015) finds that workers under the fixed-term contract are subject to a significant risk of failing to obtain an open-ended contract, and this risk is strongly and positively correlated with the market competition level. At the end of this contract, even if many of those who fail to secure an open-ended contract find another job immediately, the job transition process that involves job search and relocation can be still costly. This potentially explains the finding by Booth et al. (2002) that the lower job security offered by fixed-term contracts decreases workers' satisfaction. Therefore, interpreting the transition probability $q_i(\varsigma)$ as the probability of losing jobs may not able to fully capture the job insecurity experienced by temporary workers under the fixed-term contract. In this section, I show how incorporating the higher job insecurity induced by the fixed-term contract in my model strengthens the opposition to deregulation. Then I discuss the policy options for the government to promote deregulation in this case.

4.4.1 Set-up and Calibration

Suppose that a proportion η_F of workers in the model are temporary workers who will be offered a fixed-term contract whenever they get employed. At the end of each period, firms give one-period notices to some of their temporary employees to terminate their contracts. Assume that $1-q_i^f(\varsigma)$ is the probability that an insider worker in period t is informed that her contract will terminate by the end of $t+1$. Otherwise, with probability $q_i^f(\varsigma)$, she will be kept by the current employer in period $t+1$ without receiving any notice.

For an insider who has received the termination notice at the end of the previous period, she pays a transition cost that is equal to her wage and start job search while still on her current job in the current period. With probability $q_m(\varsigma)$, she fails to find another position immediately, so she becomes an unemployed outsider in period $t+1$. Otherwise, with probability $1-q_m(\varsigma)$, she finds another position that offers her a new fixed-term contract. For an outsider, with probability $q_o(\varsigma)$, he remains to be an outsider. Otherwise, with probability $1-q_o(\varsigma)$, he is offered an insider job. Thus, the transition matrix for the temporary workers is the following:

	insider without notice	contract not renewed	outsider
insider without notice	$q_i^f(\varsigma)$	$1 - q_i^f(\varsigma)$	0
contract not renewed	$1 - q_m(\varsigma)$	0	$q_m(\varsigma)$
outsider	$1 - q_o(\varsigma)$	0	$q_o(\varsigma)$

Note: Each row denotes the status in period t. Each column denotes the status in period $t+1$.

These workers' utility maximisation problem resembles what is defined in section 4.3.1, except that ℓ_t^{dj} is 1 if the worker receives the contract termination notice and 0 otherwise.

The rest $1 - \eta_F$ of workers are offered open-ended contracts whenever they get employed. Their transition of employment status is identical to that for workers in the baseline HAPEM model described in the previous section.[8] The key difference between the permanent and the temporary workers captured by my model is that the temporary workers are much less likely to remain employed by the same employer. For that reason, they are more likely to experience job displacements without necessarily becoming unemployed.

For simplicity, I assume that the permanent and temporary workers are equally likely to be liquidity-constrained. In other words, within workers of each contract type, η of them are hand-to-mouth. The rest of the model remains identical to the baseline HAPEM in section 4.3.

The calibration of this model is reported in Table 4.5. In terms of the transition probability $q_i^f(\varsigma)$, I calibrate a linear function that is similar to $q_i(\varsigma)$:

$$q_i^f(\varsigma) = \beta_0^f + \beta_1^f \varsigma.$$

and I choose β_1^f to match the estimates of Aparicio-Fenoll (2015) using the Spanish data. Meanwhile, the value of β_0^f implies that $q_i^f = 0.171$ when the price mark-up $\varsigma = 0.065$, which is consistent with the mean of the corresponding measures of the sample used in the same paper. Given the probability of outsiders remaining unemployed, $q_o(\varsigma)$, and the unemployment rate $u(\varsigma)$, the probability of a displaced temporary worker failing to find a new job is

$$q_m(\varsigma) = \frac{(1 - q_o(\varsigma))(2 - q_i^f(\varsigma))}{(1/u(\varsigma) - 1)(1 - q_i^f(\varsigma))}.$$

Appendix 4.A.2 shows the details of the derivation for q_m and plots the transition probabilities against the entry cost ς.

[8] It is legitimate to consider a model without segregation of contract types. This allows the temporary workers to become permanent employees, which captures the 'stepping stone' effect from fixed-term contracts as documented in Booth et al. (2002). I abstract from considering this effect explicitly here, because there is no reliable empirical guidance to calibrate all transition probabilities involved in such a model.

The proportion of the temporary workers, η_F, is chosen to be 0.26 as reported by Eurostat (2022a) and OECD (2022) for Spain in 2018. The strength of non-pecuniary cost, h, is set to 0 to highlight the role of job insecurity in this case. The political participation parameters are adjusted to match the same calibration targets as in the two-period model and the baseline HAPEM. The rest of parameters remain unchanged from the baseline HAPEM in the previous section.

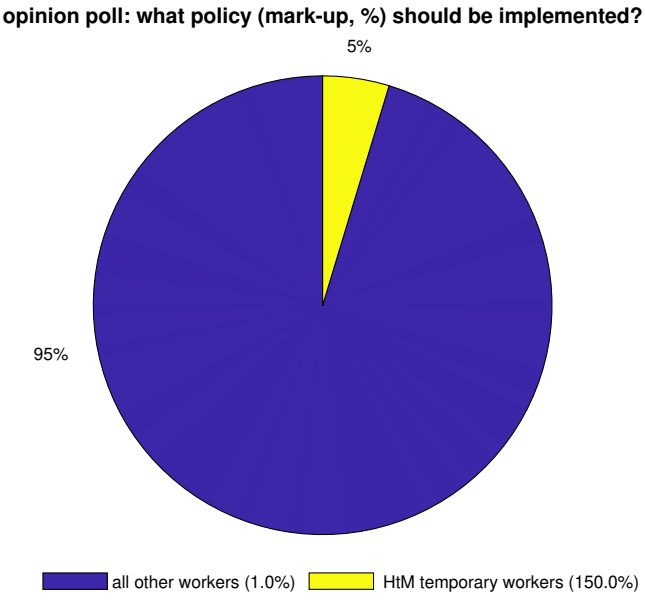
Table 4.5: Calibration of the HAPEM with Fixed-term Contracts

Parameter	Value	Interpretation
β	0.96	discount factor
γ	0.9	risk aversion
ν	0.4	unemployment benefit replacement ratio
β_0^i	0.893	parameter for probability q_i
β_1^i	0.136	parameter for probability q_i
β_0^f	0.831	parameter for probability q_i^f
β_1^f	1.352	parameter for probability q_i^f
h	0	positional concern
δ	0.7	self-serving bias
κ	25000	political influence
μ^c	38.4	constant cost of political actions
σ	40	standard deviation of idiosyncratic costs
ω	0.6	parameter for aggrievement
r	0.0395	world interest rate
$\bar{a}$	0	borrowing constraint
η	0.18	proportion of hand-to-mouth workers
η_F	0.26	proportion of fixed-term workers

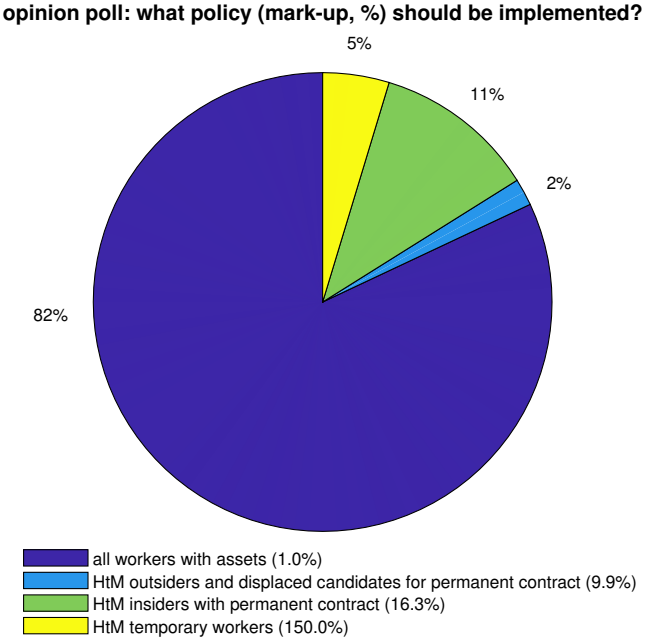
4.4.2 Market Regulation with High Job Insecurity

Figure 4.11a shows results of the opinion poll in the HAPEM model with temporary workers on fixed-term contracts. In this case, the liquidity-constrained temporary workers advocate extremely high regulation levels regardless of their current employment status. The intuition is that the risk of experiencing a job displacement for the temporary insiders is highly sensitive to market regulation levels. Following a deregulation, the temporary insiders are much less likely to stay in the same job. Meanwhile, this also implies a higher job insecurity for the displaced and outsider temporary workers if they become insiders in the future. Since these workers cannot trade assets to hedge against this high job insecurity, they are all motivated to advocate much less competitive markets.

Figure 4.11: Opinion Poll of Desired Mark-ups (%), HAPEM with Fixed-term Contracts



Efficient mark-up: 1%. Actual implemented mark-up: 41%.
(a) $\delta = 0.7$ (baseline)



Efficient mark-up: 1%. Actual implemented mark-up: 41%.
(b) $\delta = 1$

Note: The current market has a mark-up of 20%.

By contrast, workers with access to savings advocate full deregulation as in the baseline HAPEM model. In addition, the liquidity-constrained permanent workers, who advocate higher regulation levels in the baseline HAPEM model, now support full deregulation as well. This is because the workers are not purely self-biased when forming their subjective views on the regulation level to be implemented. Figure 4.11b shows that if workers are purely self-biased (i.e. $\delta = 1$), liquidity-constrained permanent workers will not support full deregulation. When they are more considerate of others instead, they acknowledge that full deregulation is welfare improving for the society as a whole, so that they are willing to lower their advocated regulation level.

In this case, full deregulation remains to be the efficient policy that a benevolent social planner would implement. However, the actual implemented mark-up is 41% despite that only 5% of the population wants a highly regulated market. This shows that even if the non-pecuniary cost of deregulation is absent, if a small proportion of workers are heavily affected by the job insecurity associated with the use of fixed-term contracts, they may form a powerful opposition to market deregulation.

Similar to the baseline HAPEM, the actual implemented regulation level also depends upon the relative political power of worker groups in this HAPEM model with temporary workers. Table 4.8 in Appendix 4.E shows that the market is even more regulated if the insider workers can be more easily mobilised with better political participation parameters than other groups. This result is driven by the strong political pressure from the political actions initiated by the liquidity-constrained temporary insiders.

In addition, Figure 4.16 in Appendix 4.D shows that initial conditions still negatively correlate with the deregulation progress in the next period. This means that economies with very high initial regulation levels are still able to make good progress in market deregulation.

4.4.3 Policy to Promote Deregulation

The analyses above suggest that the resistance to deregulation in my model comes from the liquidity-constrained workers who cannot hedge against the job-losing risk and the associated income loss following a deregulation. Moreover, the use of fixed-term contracts increases the job insecurity for workers and strengthens the opposition to deregulation. The intensified opposition eventually leads to high implemented regulation levels.

Therefore, the key for the government to promote product market deregulation is to weaken the opposition to deregulation. I consider three policy options below for the government to achieve this goal. The first option is to provide an insurance that compensate the insiders who lose their jobs after the deregulation. I simulate the effect of this policy

by a reduction in the proportion of liquidity-constrained agents η. This weakens the opposition by reducing the proportion of potential losers from deregulation. In my model, a 1 percentage point reduction in η pushes the implemented mark-up down to 32% from 41% in the baseline.[9]

Similarly, labour market reform that restricts the use of the fixed-term contract weakens the opposition by reducing the share of potential losers in the population. In my model, a 1 percentage point reduction in the proportion of temporary workers η_F lowers the implemented mark-up down to 33%.

Another popular strategy in practice is to conduct formal discussion between the governments and social partners to persuade the resisting groups to make concessions. I simulate the effect of this strategy as a reduction of the self-serving bias parameter δ in the model. This makes the workers' views more aligned with the benevolent social planner's objective, so that the implemented regulation level will be closer to full deregulation which is the efficient policy. My simulation suggests that the baseline implemented mark-up of 41% only starts to decline when δ is less than 0.18.[10] This implies that the government needs to persuade the workers to reduce their self-serving bias sufficiently from the baseline value of 0.7 to below 0.18 to weaken the resistance effectively. This potentially requires strong government leadership in terms of its readiness to act unilaterally or sanction non-cooperative parties, because this leadership plays a key role in the success of these negotiations as observed by Tompson and Dang (2010).

4.5 Application: Understanding the Heterogeneity in Regulation in Europe

The analyses so far identify some key factors that affect the regulation level implemented in the model, such as the share of liquidity-constrained workers η, the proportion of temporary workers with fixed-term contracts η_F, the relative political influence of different worker groups, and the impact of deregulation on job insecurity. In this section, I compare these factors among 5 European countries and illustrate how my model can potentially help to understand why some countries are more deregulated than others.

Table 4.6 reports the 4 factors mentioned above for Spain, the United Kingdom, Italy, Portugal, and Greece. The countries are ranked from top to bottom by their Product Market Regulation Index (PMR Index) composed by the OECD (2018) from low to high. Among them, Spain is the benchmark country that I calibrate the model with. The United

[9] Figure 4.17 in Appendix 4.D shows the corresponding poll.
[10] See Figure 4.18 in Appendix 4.D for the poll of opinions when $\delta = 0.18$.

Kingdom is a representative country with lower product market regulation compared to Spain. Italy, Portugal, and Greece are the three other Southern European countries with higher product market regulation levels. The proportion of liquidity-constrained agents η is estimated by Bracco et al. (2021). The proportion of temporary workers η_F comes from OECD (2022). The trade union density is reported by OECD (2020). This measures the proportion of employees who are union members, and is often used as an indicator for the strength of unions. Lastly, the probability of transiting to permanent contract from fixed-term contract is taken from Eurostat (2022b).

Table 4.6: Relevant Statistics in the Selected European Countries

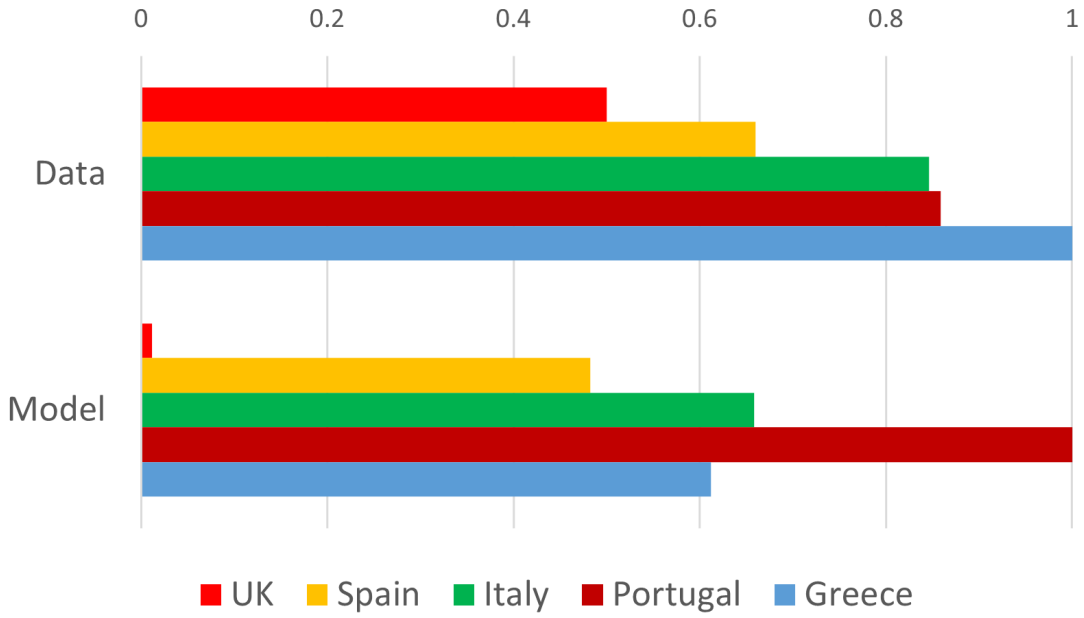
	Share of HtM η	Share of temporary workers η_F
UK	0.19	0.04
Spain	0.18	0.26
Italy	0.29	0.16
Portugal	0.43	0.20
Greece	0.44	0.11
	Trade union density	Probability of transiting to permanent contract from fixed-contract
UK	0.23	0.13
Spain	0.13	0.08
Italy	0.33	0.07
Portugal	0.15	0.19
Greece	0.19	0.02

Source: Bracco et al. (2021) for η, OECD (2022) for η_F, OECD (2020) for trade union density, Eurostat (2022b) for the transition probability.

The upper panel of Figure 4.12 shows the product market regulation levels of 5 European countries in 2018. I normalise these indices by the highest index value among the 5 countries, which is that of Greece. The lower panel plots the regulation levels of these countries that are predicted by the model by matching the values of η, η_F, and trade union density in the model to those reported in Table 4.6 for each country while keeping other parameters unchanged. The model-predicted regulation levels are normalised by that of Portugal which is the highest among the 5 countries. I account for the trade union density by setting the political influence parameter for the insiders in the model κ_{in} such that for any country i, the ratio $\kappa_{in}^{i}/\kappa_{in}^{Spain}$ is equal to the ratio of the union density of that country relative to the union density of Spain.

The figure demonstrates that the model is able to produce a ranking that is broadly consistent to the data. The UK is predicted to be the most deregulated market, followed by Spain, Italy, and Portugal. As reported in Table 4.6, Spain has the lowest share of liquidity-constrained workers and the highest share of temporary workers among the 5

Figure 4.12: Product Market Regulation, Data vs. Model



Note: The simulations match η, η_F and trade union density in the model to each country. Source: OECD (2018) and author's calculation.

countries. Compared to Spain, the UK has roughly the same proportion of liquidity-constrained agents, but a significantly lower share of temporary workers. This explains why it has the lowest regulation level. Italy has a higher share of liquidity-constrained workers but a lower share of temporary workers than Spain. Meanwhile, the union power in Italy is the strongest among all countries. This implies potentially higher political influence from insider workers in Italy to move the actual policy to the their favour. This results in a higher regulation level compared to Spain. Portugal instead features a much higher share of liquidity-constrained agents than the 3 countries above. Meanwhile, the share of temporary workers in Portugal is only lower than Spain. This explains its much higher regulation level compared to other countries.

Greece is the only outlier in this ranking comparison. Despite having highest share of liquidity-constrained workers among the 5 countries, Greece has a much lower share of temporary workers than Spain, Portugal, and Italy. Thus, the model predicts a regulation level in Greece that is higher than in the UK and Spain, but lower than Italy and Portugal. This inconsistency can potentially be resolved by factoring in the job insecurity faced by workers. The lower right panel of Table 4.6 shows that the probability of transiting from fixed-term contract to permanent contract is significantly lower in Greece than in other countries. This implies that the job displacement risk for temporary workers is potentially much higher in Greece. This should strengthen the opposition to reform in Greece and may lead to a more highly regulated market compared to other countries.[11]

[11] To the best of my knowledge, there is no empirical study that estimates how mark-ups affect the prob-

4.6 Conclusion

This paper identifies key factors that hinder structural reforms in a macroeconomic political economy model. In my model, the higher job insecurity and the associated income and welfare loss for workers following product market deregulation contribute to the opposition to the policy. The potentially losing workers impose political costs for governments through their political actions to maintain inefficiently high regulation levels.

However, the resistance is weaker if the initial regulation level of the market is high, if more workers can save to hedge against future employment risks, and if there is a lower share of temporary workers who are employed with fixed-term contracts. The policy implication is that compensating the potential losers, labour market reform that restricts the use of fixed-term contract, and strong government leadership in negotiations can help the government to promote deregulation. Lastly, I show that the factors identified by my model are useful for understanding the heterogeneous product market regulation levels in Europe.

There are several avenues for future research. First, the political consideration is an important dimension in the debate after the Greek debt crisis about whether structural reforms and fiscal consolidation are complementary. The framework in this paper can be used to explore this question and discuss the optimal design of such programs. Second, I consider only stationary equilibria in my HAPEM model. Studying the transitions between stationary equilibria in this model is potentially interesting. For example, the credibility of fully implementing and maintaining the reform proposal can be discussed in this context. Lastly, future empirical work can formally test the predictions of my model. Current empirical literature focuses on the role of political influence and initial conditions in affecting the deregulation progress. However, it is interesting to see how job insecurity, the risk-sharing among households, and the use of the fixed-term contracts affect the product market regulations empirically.

ability of transiting to permanent contract from fixed-term contract in other countries other than Spain. Therefore, I leave the exercise of matching this dimension quantitatively in my model to future research.

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Appendix

4.A Derivation of Transition Probabilities

4.A.1 Probability q_o

In the model, the labour market flows in steady state satisfy the following condition:

$$(1-q_i)n^{in} = (1-q_o)n^{disp} + (1-q_o)n^{out}, \tag{4.7}$$

where the left-hand side of the equation is the outflow from the incumbent industry, and the right-hand side is the inflow to the incumbent industry.

Meanwhile, the transition from being insiders to being the displaced implies that

$$n^{disp} = (1-q_i)n^{in},$$

and on aggregate

$$n^{in} + n^{disp} + n^{out} = 1.$$

Using these two conditions, we can rewrite equation (4.7) as

$$(1-q_i)n^{in} = (1-q_o)(1-q_i)n^{in} + (1-q_o)(1-n^{in}-(1-q_i)n^{in}).$$

Simplify to get

$$(1-q_i)n^{in} = (1-q_o)(1-n^{in}).$$

Recognise that the unemployment rate

$$u = n^{disp} + n^{out} = 1 - n^{in}.$$

Thus, the simplified equation can be further rewritten as

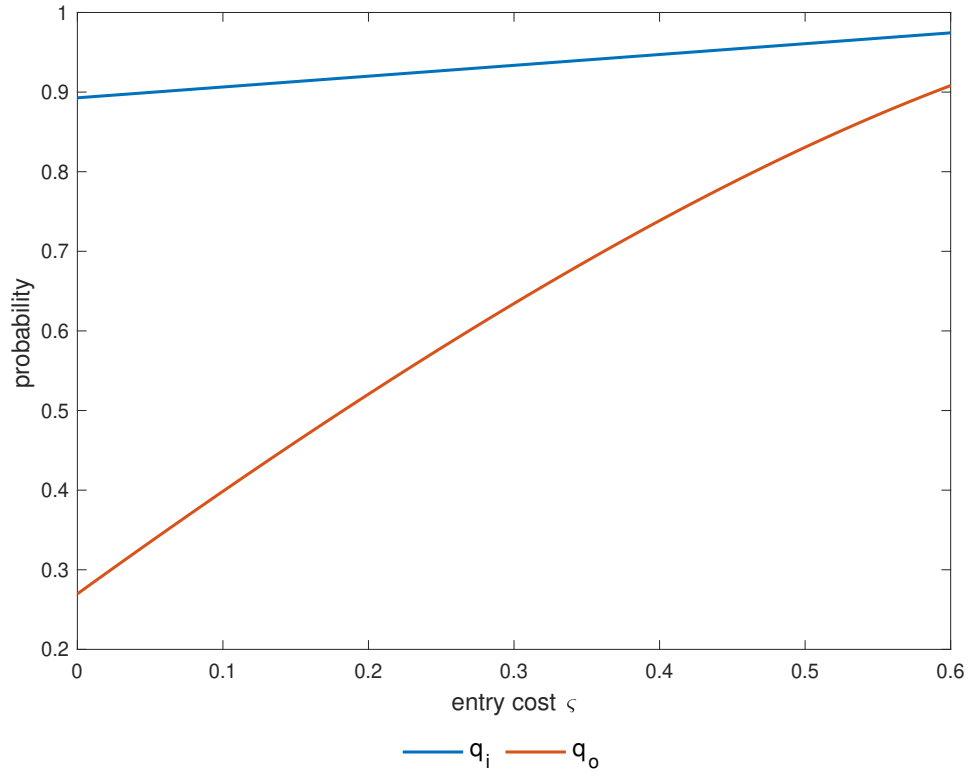
$$(1 - q_i)(1 - u) = (1 - q_o)u.$$

Therefore, we have

$$q_o = 1 - \frac{(1 - q_i)(1 - u)}{u}.$$

Figure 4.13 plots the transition probabilities q_i and q_o against the entry cost ς, which measures the profit margin of the product market, with the calibration specified in subsection 4.2.6.

Figure 4.13: Transition Probabilities of Employment Status



4.A.2 Probability q_m

In the HAPEM model with fixed-term contracts presented in section 4.4, the labour market flow in steady state for permanent workers is identical to what is described in the subsection above in this appendix. Thus, the probability of remaining an outsider, q_o, is the same as above

$$q_o = 1 - \frac{(1-q_i)(1-u)}{u}.$$

The labour market flow in steady state for temporary workers satisfies the following instead

$$(1-q_i^f)n_{temp}^{in} = (1-q_m)n_{temp}^{disp} + (1-q_o)n_{temp}^{out}, \tag{4.8}$$

where n_{temp}^{disp} denotes the workers who receive the notice of contract termination in the current period. In steady state, it thus also satisfies

$$n_{temp}^{disp} = (1-q_i^f)n_{temp}^{in}.$$

Therefore, equation (4.8) can be rewritten as

$$(1-q_i^f)n_{temp}^{in} = (1-q_m)(1-q_i^f)n_{temp}^{in} + (1-q_o)n_{temp}^{out}.$$

Recognise that $n_{temp}^{out} = \eta_F u$, the equation becomes

$$(1-q_i^f - (1-q_m)(1-q_i^f))n_{temp}^{in} = (1-q_o)\eta_F u.$$

Therefore,

$$\frac{n_{temp}^{in}}{u} = \frac{\eta_F(1-q_o)}{(1-q_i^f-(1-q_m)(1-q_i^f))} = \frac{\eta_F(1-q_o)}{q_m(1-q_i^f)}. \tag{4.9}$$

In addition, for temporary workers,

$$n_{temp}^{in} + n_{temp}^{disp} + n_{temp}^{out} = \eta_F,$$

which can be rewritten as

$$n_{temp}^{in} + (1-q_i^f)n_{temp}^{in} + \eta_F u = \eta_F.$$

Thus,

$$(2-q_i^f)\frac{n_{temp}^{in}}{u} = \eta_F(\frac{1}{u} - 1),$$

which means

$$\frac{n_{temp}^{in}}{u} = \eta_F \frac{1/u - 1}{2 - q_i^f}. \tag{4.10}$$

Then from equation (4.9) and (4.10), we can get

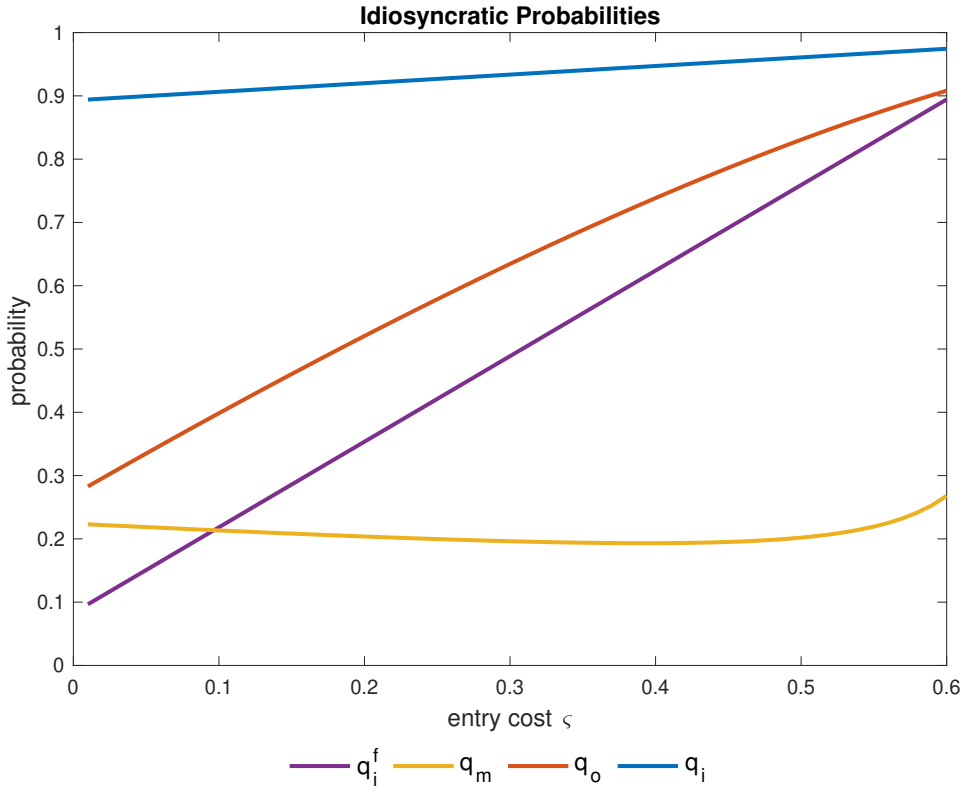
$$\frac{\eta_F(1 - q_o)}{q_m(1 - q_i^f)} = \eta_F \frac{1/u - 1}{2 - q_i^f}.$$

Solve this to get

$$q_m = \frac{(1 - q_o)(2 - q_i^f)}{(1/u - 1)(1 - q_i^f)}.$$

Figure 4.14 plots the transition probabilities q_i for the permanent insiders, q_i^f for the temporary insiders, q_m for the displaced temporary insiders, and q_o for the outsiders against the entry cost ς, which measures the profit margin of the product market, with the calibration described in subsection 4.4.1.

Figure 4.14: Transition Probabilities of Employment Status in the HAPEM with Fixed-term Contracts



4.B Algorithm for Solving the Infinite Horizon Heterogeneous-Agent Model

The algorithm follows the steps below:

1. Create grids for the assets a and the entry cost ς. Given each grid of ς, make a guess $\tilde{C}$ for the average consumption C.

2. Calculate elasticity θ, wage w, unemployment benefits b, and transition probabilities q_i and q_o.

3. Calculate the consumption and utility for each grid of the assets a_{t+1} held by the non-hand-to-mouth workers.

4. Solve for the policy functions of the non-hand-to-mouth workers $a_{t+1}(\varsigma, a_t, \ell_t, \ell_t^d)$ and $C_t(\varsigma, a_t, \ell_t, \ell_t^d)$ by the value function iteration method. Obtain the associated value functions $V_t(\varsigma, a_t, \ell_t, \ell_t^d)$.

5. Compute the stationary distribution using the policy functions, and then calculate the aggregate consumption C.

6. Check whether the aggregate consumption C converges to the initial guess $\tilde{C}$. If so, move to the next step. If not, go back to Step 1 to update the guess $\tilde{C}$.

7. Calculate the value functions for the hand-to-mouth workers.[12]

8. For each grid of ς, calculate the subjective weights π_{ik} using the stationary distributions, and aggregate the value functions using those weights to get the modified aggregate social welfare for workers under each state (a_t, ℓ_t, ℓ_t^d). Maximise these welfare functions to get the subjectively 'fair' policy $\hat{\varsigma}(a_t, \ell_t, \ell_t^d)$ and the associated reservation utility $R(a_t, \ell_t, \ell_t^d)$ for workers in any state.

9. Compute the functions for the aggrievements $A(\varsigma, a_t, \ell_t, \ell_t^d)$ and the associated participation rates of political actions $\mathcal{P}(\varsigma, a_t, \ell_t, \ell_t^d)$.

10. Compute the government's objective function. Maximise this objective function to get the implemented policy ς^a.

[12] The details are in Appendix 4.C.

4.C Value Functions for Hand-to-mouth Workers in HAPEM

For the hand-to-mouth workers in the baseline HAPEM model described in section 4.3, the value function for the insider, outsider, and displaced workers in steady state are respectively

$$V^{in} = U^{in} + \beta[q_i V^{in} + (1 - q_i)V^{disp}]$$
$$V^{out} = U^{out} + \beta[q_o V^{out} + (1 - q_o)V^{in}]$$
$$V^{disp} = U^{disp} + \beta[q_o V^{out} + (1 - q_o)V^{in}],$$

where V^{in}, V^{out}, and V^{disp} are values for being the insider, outsider, and displaced workers, and U^{in}, U^{out}, and U^{disp} are their corresponding per-period utilities respectively.

Solve these 3 equations for the values will yield the value for being an insider as

$$V^{in} = \frac{1 - \beta q_o}{1 - \beta q_i - \beta q_o - \beta^2(1 - q_i - q_o)}(U^{in} + \beta(1 - q_i)U^{disp} + \frac{\beta^2(1 - q_i)q_o}{1 - \beta q_o}U^{out}]).$$

Then the value for being an outsider can be calculated as

$$V^{out} = \frac{U^{out} + \beta(1 - q_o)V^{in}}{1 - \beta q_o}.$$

Finally, the value for being dismissed is

$$V^{disp} = U^{disp} + \beta[q_o V^{out} + (1 - q_o)V^{in}].$$

In the HAPEM model with fixed-term contracts in section 4.4, the values of being hand-to-mouth workers on permanent contracts remain the same as above. For the hand-to-mouth temporary workers, the value functions are

$$V^{in}_{temp} = U^{in}_{temp} + \beta[q^f_i V^{in}_{temp} + (1 - q^f_i)V^{disp}_{temp}],$$
$$V^{out}_{temp} = U^{out}_{temp} + \beta[q_o V^{out}_{temp} + (1 - q_o)V^{in}_{temp}],$$
$$V^{disp}_{temp} = U^{disp}_{temp} + \beta[q_m V^{out}_{temp} + (1 - q_m)V^{in}_{temp}],$$

where V^{disp}_{temp} and U^{disp}_{temp} correspond to the value and per-period utility of workers who

receive the contract termination notice. Solve this system to get the value for being a temporary insider

$$V_{temp}^{in} = \frac{(1-\beta q_o)[U_{temp}^{in} + \beta(1-q_i^f)U_{temp}^{disp} + \frac{\beta^2(1-q_i^f)q_m}{1-\beta q_o}U_{temp}^{out}]}{1-\beta q_i^f - \beta q_o - \beta^2(1-q_i^f-q_m) + \beta^2(q_o-q_m)[q_i^f + \beta(1-q_i^f)]}.$$

It follows that

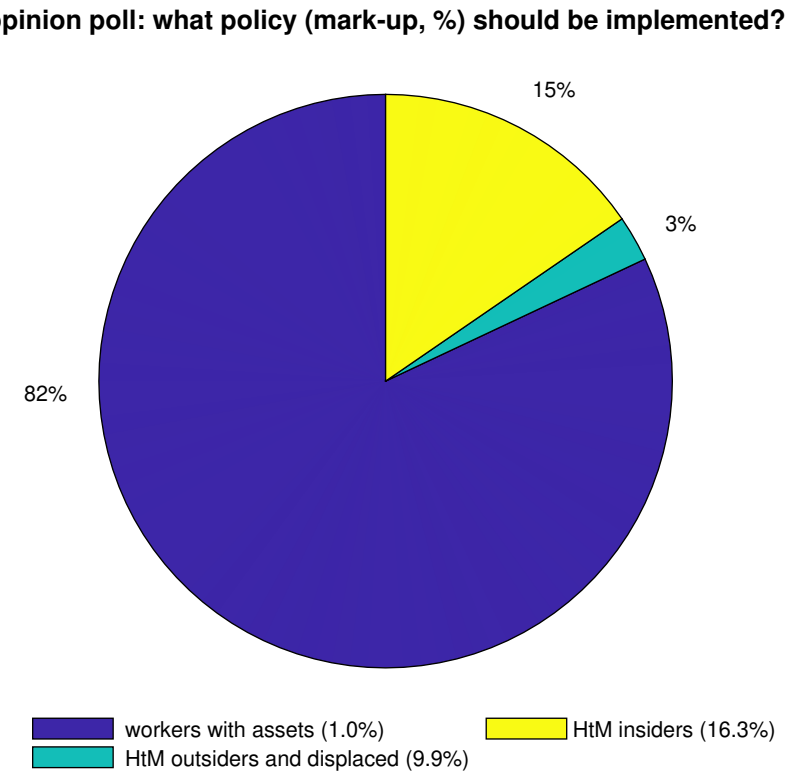
$$V_{temp}^{out} = \frac{U_{temp}^{out} + \beta(1-q_o)V_{temp}^{in}}{1-\beta q_o}$$

and

$$V_{temp}^{disp} = U_{temp}^{disp} + \beta[q_m V_{temp}^{out} + (1-q_m)V_{temp}^{in}].$$

4.D Additional Figures

Figure 4.15: Opinion Poll of Desired Mark-ups (%) with $h = 0$ and $\delta = 1$, Baseline HAPEM



Efficient mark-up: 1%. Actual implemented mark-up: 1%.

Note: The current market has a mark-up of 20%. The full deregulation is implemented because the opposition from the liquidity-constrained workers is not strong enough to impose high political costs for the government.

Figure 4.16: Initial Conditions and Deregulation, HAPEM with Fixed-term Contracts

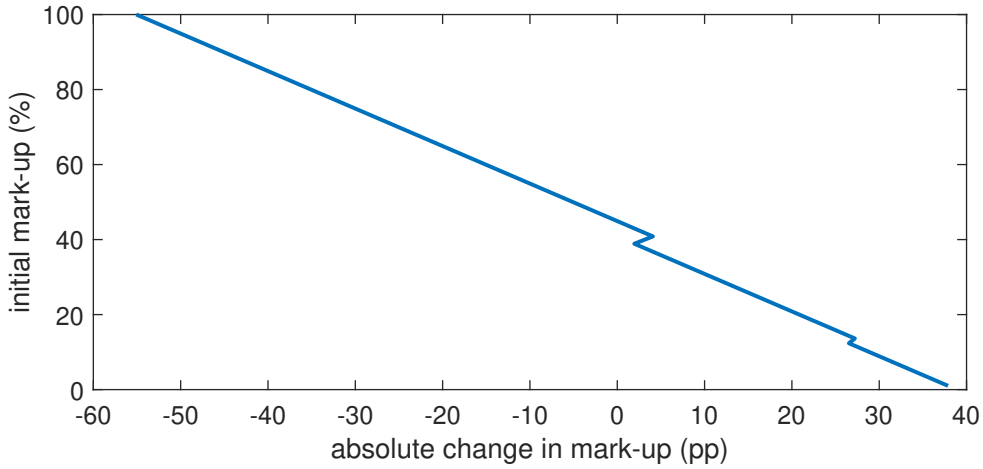
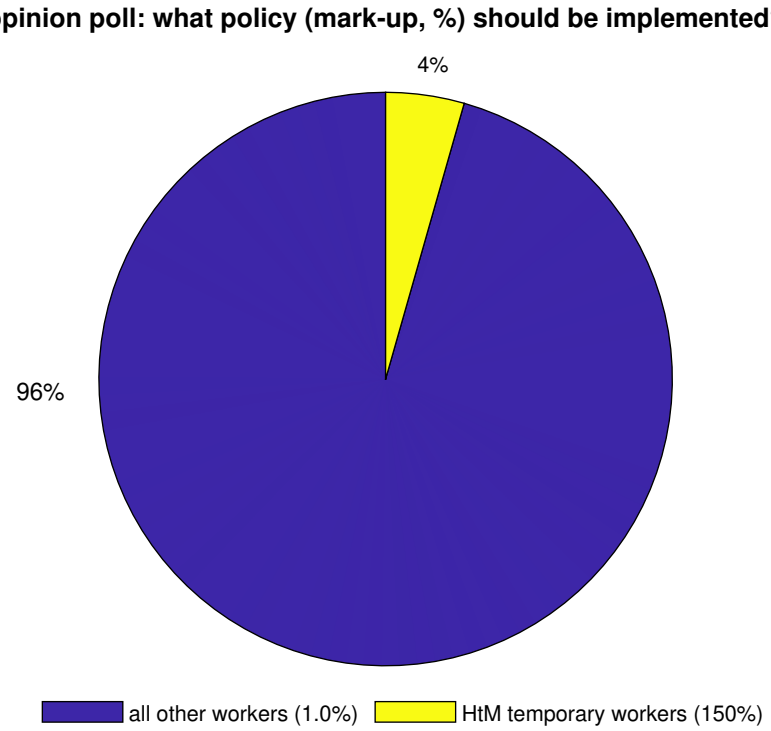


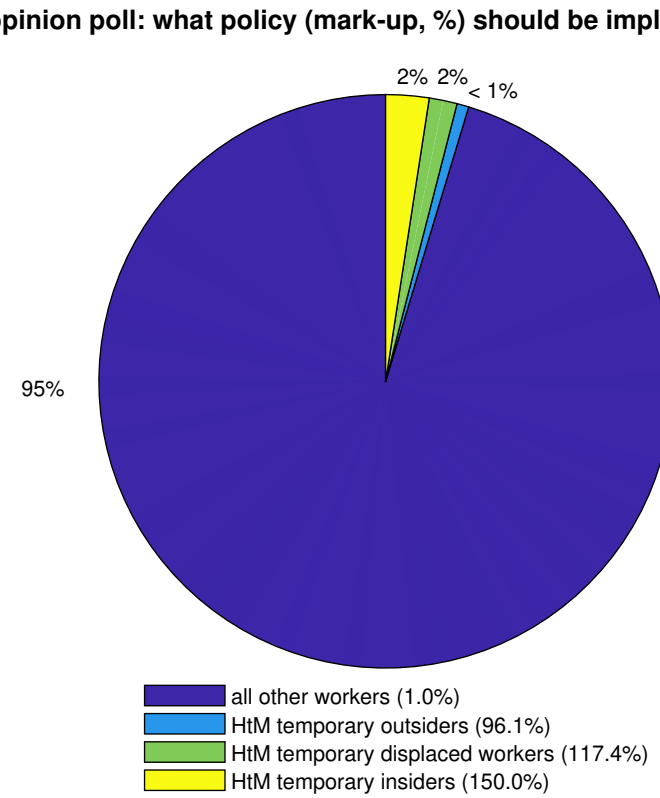
Figure 4.17: Opinion Poll of Desired Mark-ups (%) with Government Compensation, HAPEM with Fixed-term Contract



Efficient mark-up: 1%. Actual implemented mark-up: 33%.

Note: The government promises compensation for workers. The current market has a mark-up of 20%.

Figure 4.18: Opinion Poll of Desired Mark-ups (%) with $\delta = 0.18$, HAPEM with Fixed-term Contracts



Efficient mark-up: 1%. Actual implemented mark-up: 39%.

Note: The current market has a mark-up of 20%.

4.E Additional Tables

Table 4.7: Actual Policy (in mark-up, %) and Political Influence, baseline HAPEM.

scenario	actual mark-up
baseline	5
lower participation cost for insiders ($\mu_{in} = 0.9\mu_{out}$)	10
more organised insiders ($\sigma_{in} = 0.5\sigma_{out}$)	9
more sensitive insiders ($\omega_{in} = 5\omega_{out}$)	11
more influential insiders ($\kappa_{in} = 2\kappa_{out}$)	9

Table 4.8: Actual Policy (in mark-up, %) and Political Influence, HAPEM with Fixed-term Contracts

scenario	actual mark-up
baseline	41
lower participation cost for insiders ($\mu_{in} = 0.9\mu_{out}$)	67
more organised insiders ($\sigma_{in} = 0.5\sigma_{out}$)	61
more sensitive insiders ($\omega_{in} = 1.1\omega_{out}$)	45
more influential insiders ($\kappa_{in} = 2\kappa_{out}$)	54