

ON THE PROPAGATION OF GALACTIC COSMIC RAYS

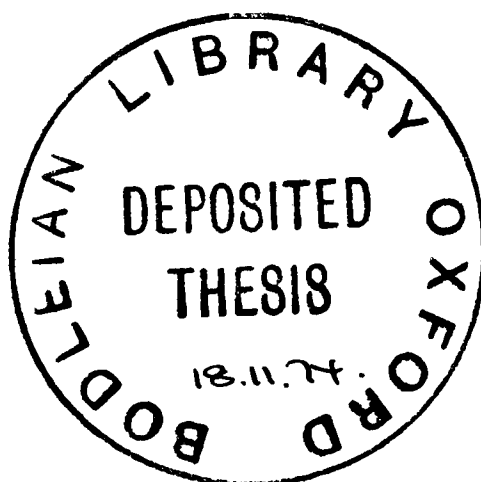
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ABSTRACT

Recent observations of cosmic ray composition imply that the average amount of interstellar matter traversed by the cosmic rays before escaping from the Galaxy decreases as their energy increases. These observations are reviewed in Chapter 1 together with other relevant observations of the cosmic ray composition, spectrum and anisotropy.

In Chapter 2 theories on the origin and propagation of galactic cosmic rays are reviewed in the light of the data presented in the first chapter. Models in which they are confined to the Galaxy by diffusion in the irregular galactic magnetic field experience difficulty in accounting for their slow leakage rate out of the Galaxy, in which observations indicate that they remain confined for an average time of about a million years. However, since the energy densities of the cosmic rays and the galactic magnetic field are similar, it is logical to expect that the field cannot influence the propagation of the cosmic rays without itself undergoing change. In Chapter 3 the effect of the cosmic rays on the field is discussed. A net streaming motion of the cosmic rays causes hydromagnetic waves to form in the field, which have a wavelength of the order of the cosmic ray gyro-radius. These waves in turn scatter the cosmic rays of corresponding gyro-radius, thereby reducing their streaming speed.

This process is used in Chapter 4 to explain the long residence time, the energy-dependent path length and the low

observed anisotropy of galactic cosmic rays. The hydromagnetic waves formed by the streaming cosmic rays have to compete against damping processes in the interstellar medium. The dominant linear damping process is that due to collisions between the charged particles moving with the wave and the neutral particles of the interstellar medium. The waves can only begin to form when the damping rate due to this process becomes smaller than their growth rate, that is at heights above the galactic plane where the neutral hydrogen density is low enough. Skilling's analysis of the scattering process is used to obtain an expression for the height at which waves resonating with cosmic rays of a particular rigidity begin to form. As the wave growth rate varies as the density of resonant cosmic rays, it decreases as the cosmic ray energy increases. Therefore higher energy cosmic rays can only encounter resonant waves at greater heights above the galactic plane, where the density of neutral hydrogen is smaller. The onset of the waves at their respective heights above the galactic plane produces a reflecting boundary which prevents free diffusion out of the Galaxy.

Once the waves have grown to a finite amplitude, non-linear damping processes become important. The waves which scatter the cosmic rays become degraded into other modes which are rapidly damped. Wentzel's analysis is used to investigate the effects of non-linear damping, after showing that it is equivalent to Skilling's analysis in the absence of non-linear damping. Under steady-state conditions, this process causes the net cosmic ray streaming speed through the waves to increase steeply

with energy. As a result, higher energy cosmic rays can escape from the Galaxy more readily than those of lower energy. The energy-dependence of their residence time in the Galaxy can account for the observed decrease in path length with increasing energy.

The effect of this confinement process on the spectrum of cosmic ray electrons is investigated in the first half of Chapter 5. The electrons experience the same decrease in residence time with energy as do the other cosmic ray species. The effect of this is to reduce the overall change in their spectral index which is expected when their residence time is similar to their timescale of energy loss due to synchrotron emission and the inverse Compton effect. This may be the reason why no change of order unity in the spectral index has been detected above 50 GeV .

The hydromagnetic waves which confine the cosmic rays to the Galaxy do not themselves produce compression of the interstellar medium. But the modes into which they decay by non-linear processes are compressive, and would therefore produce irregularities in the interstellar electron density with a length scale short compared to that which is expected from normal interstellar motions. The scattering of electromagnetic radiation from pulsars is discussed in the second half of Chapter 5, where it is suggested that these compressive modes may be responsible for the observed broadening of the pulses with increasing wavelength.

As a background for further study, the coupling between hydromagnetic wave modes is outlined in Chapter 6. The equations describing a MHD plasma are expanded to third order to obtain the wave-wave transition rates, and the results are interpreted in terms of collisions between plasmons.

ACKNOWLEDGEMENTS

I should like to thank Dr D. W. Sciama for first bringing this subject to my attention, for his supervision and helpful comments, and for directing my attention towards the connection between this confinement theory and other branches of astrophysics.

The calculations of nonlinear wave-wave interactions displayed in Chapter 6 were carried out in collaboration with Dr J. Skilling of DAMTP, Cambridge, whom I wish to thank for his permission to include them in this thesis and for the enlightening discussions we have had about cosmic rays and plasma waves.

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INTRODUCTION

Until the 1940's, almost all the information available to astronomers was obtained via the 'optical channel'. Despite the successes of astronomy, particularly in the last hundred years, the relative narrowness of this channel (from 0.3 microns in the ultraviolet region to a few tens of microns in the infrared) has severely limited the extent of the information available. The radio channel was opened up during the middle of this century, and during its 'maturity' in the last decade it has brought about a revolution in astronomical research. Other channels are beginning to be fruitfully exploited now. Until recently, X-ray and gamma-ray astronomy were at a similar stage to that which radio astronomy had reached twenty years ago. But thanks to the sophisticated technology of satellite observation these channels are now showing the promise of making an impact similar to that of radio astronomy.

Cosmic ray astronomy also shows this promise, but it has started with the handicap that, due to their nature as charged particles, the cosmic rays do not undergo rectilinear propagation. Consequently in order to investigate the sources of this radiation, we first have to understand the processes affecting its propagation. This disadvantage is partly offset by the additional information we can obtain from measurements of the chemical composition of the incoming radiation. Indeed, this information can tell us how long the radiation takes on average to travel between the sources and our detectors, and such knowledge gives us valuable

insight into its mode of propagation.

This thesis concentrates mainly on the problem of confining the radiation to the Galaxy for the estimated million years obtained from observations, and why the amount of interstellar matter traversed by the cosmic rays varies with their energy. In the first chapter the relevant observations are reviewed, and these are interpreted in the second chapter to provide information about the sources of the cosmic rays and their mode of propagation. The propagation theories reviewed in the second chapter, which involve only diffusion across irregularities in the galactic magnetic field, experience difficulty in explaining the observations. But as the energy densities of the cosmic rays and the galactic magnetic field are similar, it is logical to expect that the field cannot influence the cosmic rays without itself undergoing change. The influence which the cosmic rays have on the field, known as their 'collective effects', is discussed in Chapter 3, and the results are used in Chapter 4 to explain the long residence time and the energy-dependent path length of the galactic cosmic rays. In Chapter 5 we discuss the observable consequences of this confinement mechanism on the spectrum of cosmic ray electrons and on the scattering of radio waves emitted by pulsars. It is shown in Chapter 4 that nonlinear interactions between hydro-magnetic waves play an important part in the confinement of cosmic rays to the Galaxy, and these processes are outlined in Chapter 6. It is hoped that these results will act as a basis for future, more detailed, analysis of the phenomena outlined in this thesis.

CHAPTER 1

OBSERVATIONS OF THE COSMIC RADIATION

1. The Proton Spectrum.

Cosmic ray protons, by virtue of their energy density, are the dominant species of cosmic ray nuclei. Thus whenever the collective effects of the cosmic ray gas are expected to influence the medium through which they propagate, it is the proton component which controls the extent of the influence. We shall therefore review the measurements of the proton flux before turning to the other observations which can give us a deeper insight into the nature of their influence.

The galactic proton spectrum, for energies greater than a few GeV up to the highest measured energies of $\sim 10^{20}$ eV, can approximately be fitted to a single power law of the form

$$I(>E) \approx 1 \times 10^{16} E(\text{eV})^{-1.74 \pm 0.1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

(Lingenfelter 1973). It is recognized, however, that this fit is an oversimplification, since various experimenters have reported a number of bends in the spectrum, and parts of the spectrum have been fitted to power laws with integral exponents ranging from - 1.0 to - 2.2 . We shall therefore divide the spectrum into separate ranges of energies.

$E \lesssim 1$ GeV

In this region the differential spectrum $I(E) dE$ does not decrease monotonically with increasing energy, but rises to a maximum between 0.1 and 1 GeV. The position of the maximum and the extent of the reduction in the number of low-energy particles

in the vicinity of the Earth depend upon the phase of the solar cycle. During periods of minimum solar activity the reduction in cosmic ray flux is less pronounced and begins at lower energies, while when the Sun is most active the reduction is very distinct and begins at higher energies, (Figure 1). This indicates that the density of low-energy cosmic rays near the Earth is strongly influenced by the solar magnetic field. As well as undergoing convection and diffusion in the expanding magnetic field irregularities, the cosmic rays entering the solar system lose energy by adiabatic deceleration. Gleeson & Urch (1971) have shown that galactic cosmic rays of kinetic energy $\lesssim 80$ MeV/nucleon are virtually excluded from reaching the Earth, and that those cosmic rays of these energies which we do detect have in fact been decelerated from a range of higher energies. They also showed (Urch & Gleeson 1973) that the mean energy loss experienced by protons entering the solar system varies with solar conditions between 300 MeV and 800 MeV. They conclude that observational data on low-energy cosmic rays contains no information about galactic particles of similar energy.

The situation is complicated further by the fact that the Sun emits its own cosmic rays with energies below about 0.1 GeV. Several powerful 'bursts' of solar cosmic rays have been recorded during the last twenty years, the most intense of which occurred on the 23rd of February 1956 when the flux at the Earth's surface increased to several times its normal level.

Consequently, in order to investigate the spectrum of galactic cosmic rays, we must observe them at higher energies.

$$1 \text{ GeV} < E \lesssim 10^5 \text{ GeV}$$

This region is the most important part of the energy spectrum

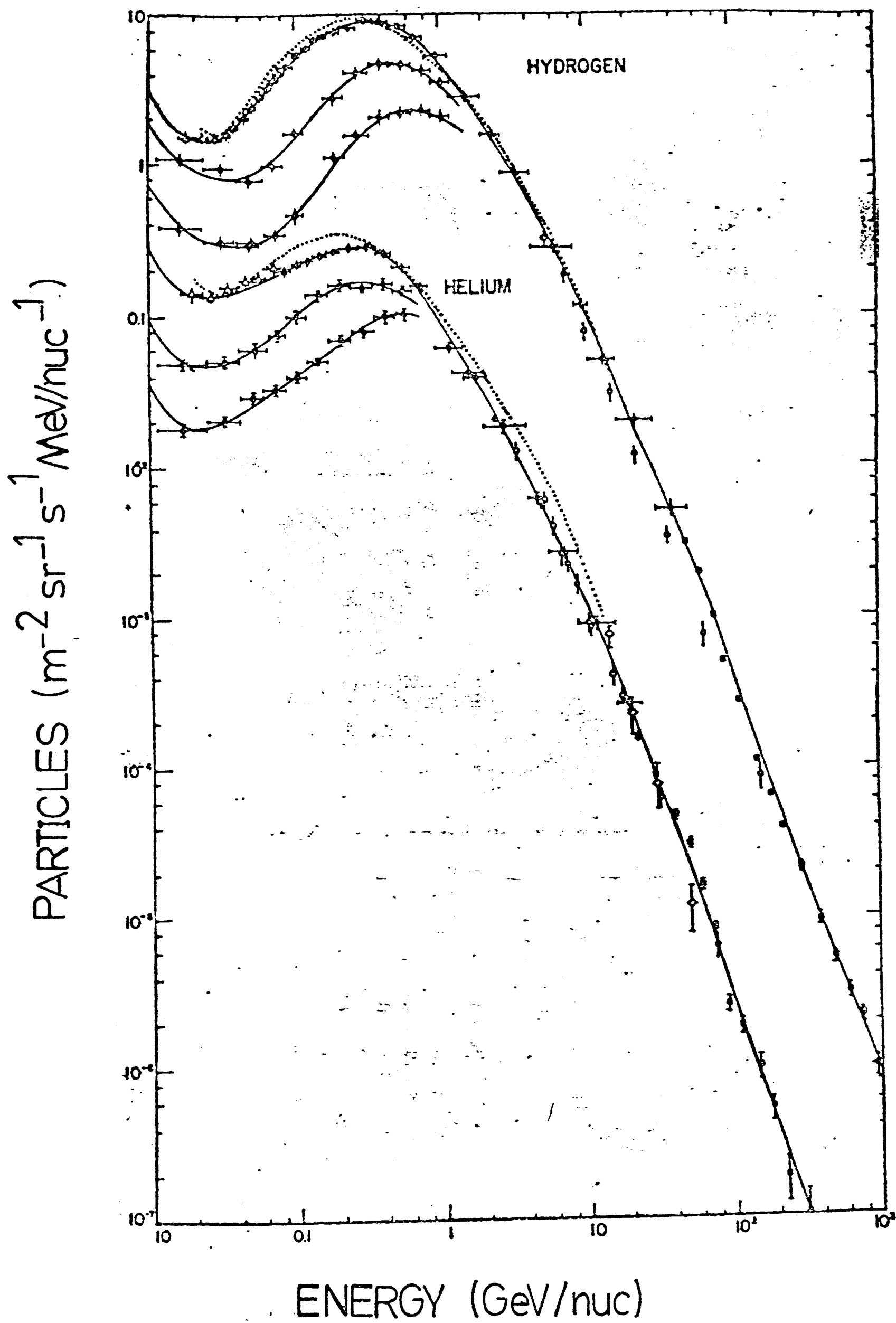


FIGURE 1. The spectrum of low-energy cosmic ray protons and helium (from Webber 1973).

as far as the collective effects of the cosmic ray gas in the Galaxy are concerned. The spectrum is not altered appreciably by the solar magnetic field above an energy of about 3 GeV, and at the lower end of this range the cosmic ray flux is strong enough to effect a dominating influence over its own mode of propagation in interstellar space. As the flux decreases at higher energies, the magnitude of this influence reduces until the cosmic rays become passive 'test particles' controlled entirely by the configuration of the galactic magnetic field. Whenever this influence is expected to have an observable effect on a certain parameter, such as cosmic ray residence time in the Galaxy, measurements of the energy dependence of the parameter, together with knowledge of the proton spectrum at these energies, can give us valuable information about the nature of the collective effects of the cosmic ray gas.

Fortunately this important energy region happens to be the one which is most readily observable. The particle flux is large enough to allow reliable statistical analysis of the measurements, and the detectors do not have to rely upon air shower methods with their inherent uncertainties of charge spectrum and scaling effects. The main methods of observation rely on the direct measurement of primary particles using ionization spectrometers, scintillation counters and spark chambers installed on satellites or balloons.

The results of various experiments are summarised in Figures 1 and 2. The existence of a bend in the spectrum at energies around 8 GeV should be noted. This bend also occurs at the same rigidity in the spectra of other cosmic ray nuclei (Figure 2).

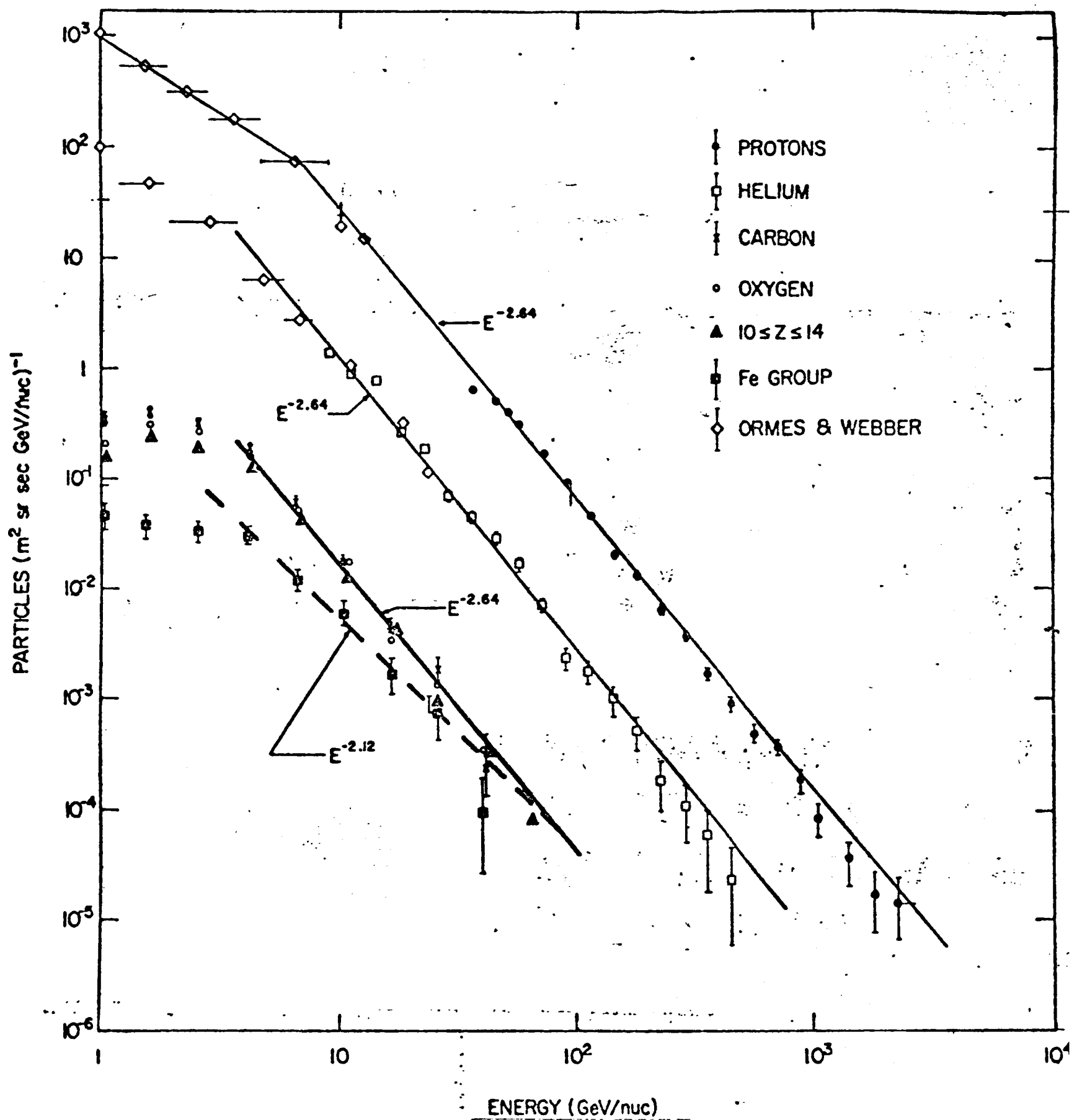


FIGURE 2. The differential spectra of medium-energy cosmic ray nuclei (from Ormes & Webber 1965 and Balasubrahmanyam & Ormes 1973).

It was first observed by Durney et al. (1964) in the spectrum of primary cosmic rays of atomic number $Z \geq 6$ (Figure 3). They originally suggested that this indicated an upper limit to the effects of the solar modulation process at the time of measurement, but subsequent observations during different phases of solar activity showed the spectrum in this region to be unchanged. It is therefore likely that this bend is a property of the galactic cosmic ray spectrum, and that the effects of solar modulation begin to occur at a slightly lower rigidity. In Chapter 4 it will be shown that this bend may be explained in terms of the onset of collective effects of the cosmic ray gas in our vicinity.

$$E \geq 10^5 \text{ GeV}$$

Cosmic rays with energies above about 10^5 GeV can only be studied using the method of extensive air showers. These are formed when a high-energy proton or nucleus entering the Earth's atmosphere collides with a N or O nucleus, producing a large number of K or π mesons, hyperons and nucleon-antinucleon pairs. These particles in turn collide with other nuclei or else decay. On reaching the surface of the Earth, the air shower consists mainly of electrons, positrons and neutrinos, due to the decay chain

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm + \nu \\ \mu^\pm &\rightarrow e^\pm + \nu + \bar{\nu} \end{aligned}$$

The area of the shower, which may cover several square kilometers, depends upon the energy of the primary particle. Arrays of detectors spread out over a large area and connected in coincidence

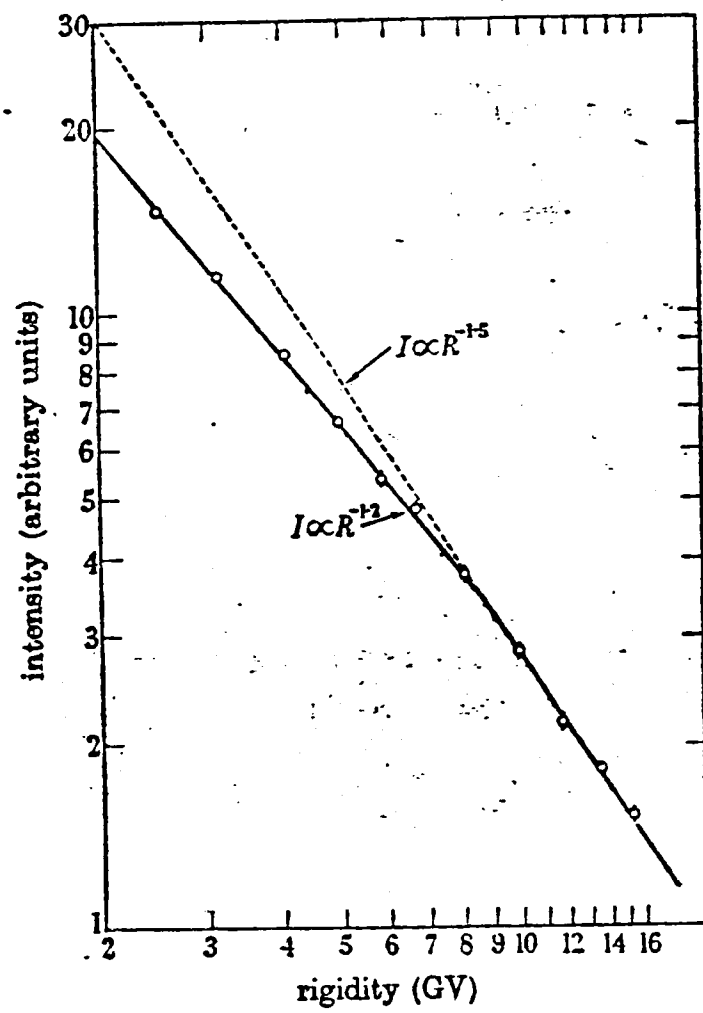


FIGURE 3. The integral spectrum of primary cosmic rays with $Z \leq 6$ (from Durney et al. 1964).

determine the energy and direction of incidence of the primary particle. These cannot be determined accurately due to uncertainties about the details of air shower development, and furthermore it is not possible to ascertain the chemical composition of the primary particle.

In general, results show a change in slope at energies of about 3×10^6 GeV. Below this energy the differential exponent is about -2.6, and above it the value is about -3.2. For example La Pointe et al. (1968) obtain an integral spectrum in the range $10^6 - 3 \times 10^8$ GeV of

$$I(>E) = (2.0 \pm 0.4) 10^{-10} (E(\text{eV})/10^{17})^{-2.2 \pm 0.15} \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

This compares well with the spectrum obtained by Edge et al. (1973) in the range $3 \times 10^8 - 10^{10}$ GeV. This was based on measurements in deep water Cerenkov detectors, chosen to have minimum sensitivity to the nature of the primary particles and to the details of air shower development. They obtained an integral spectrum of

$$I(>E) = (4.5 \pm 0.5) 10^{-10} (E(\text{eV})/10^{17})^{-2.17 \pm 0.03} \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$$

They further concluded that there was no evidence of any deviation from this form up to the highest observed energies of 10^{11} GeV. These and other measurements (Andrews et al. 1971, Bell et al. 1971) show no evidence for the reduction in the slope of the spectrum above 10^9 GeV which was reported during the last decade (Ginzburg & Syrovatskii 1964).

2. The Variation of Cosmic Ray Flux with Position in the Galactic Plane.

If cosmic rays are prevented from streaming freely away from their sources, either by collective effects of the cosmic ray gas or by diffusion in the irregular galactic magnetic field, then the variation of cosmic ray density with position in the galactic plane should show some relation to the distribution of their sources. Such estimates of cosmic ray density have recently been obtained from measurements of the diffuse γ -radiation from the galactic plane. When a cosmic ray particle undergoes a collision with a nucleus of the interstellar medium neutral pions are produced, which subsequently decay into γ photons. 70 per cent of the γ -radiation above 100 MeV is of pion-decay origin, and almost all of the π^0 mesons in the Galaxy are produced by cosmic rays in the energy range 0.5 - 3 GeV (Stecker 1973). Consequently observations of the diffuse γ -radiation in the Galaxy, together with knowledge of the density of the interstellar medium, should provide information about the density of cosmic rays in this energy range.

Results from the second Small Astronomy Satellite (SAS-2) (Kniffen et al. 1973) show that the high-energy γ emission is particularly intense in the region $315^\circ < l_{II} < 45^\circ$, concentrated in a narrow band along the galactic plane. In this interval there are two regions of maximum intensity situated about 20° either side of the galactic centre. From these results Stecker et al. (1974) obtain values for the flux of low-energy cosmic rays as a function of radial distance r from the galactic

centre. Beyond the Sun, at $r = 10$ kpc, the flux is approximately constant, but towards the galactic centre the flux increases to a maximum value, an order of magnitude higher than the local value, in a torroidal region with r between 4 and 5 kpc, (Figure 4). They suggest that this enhancement may be accounted for by Fermi acceleration caused by the expanding gas of the '3 kpc' arm. Puget and Stecker (1974) point out that the distribution of cosmic ray density with r is strikingly similar to that observed for ionized hydrogen. A possible explanation for this correlation will be discussed in Chapter 4.

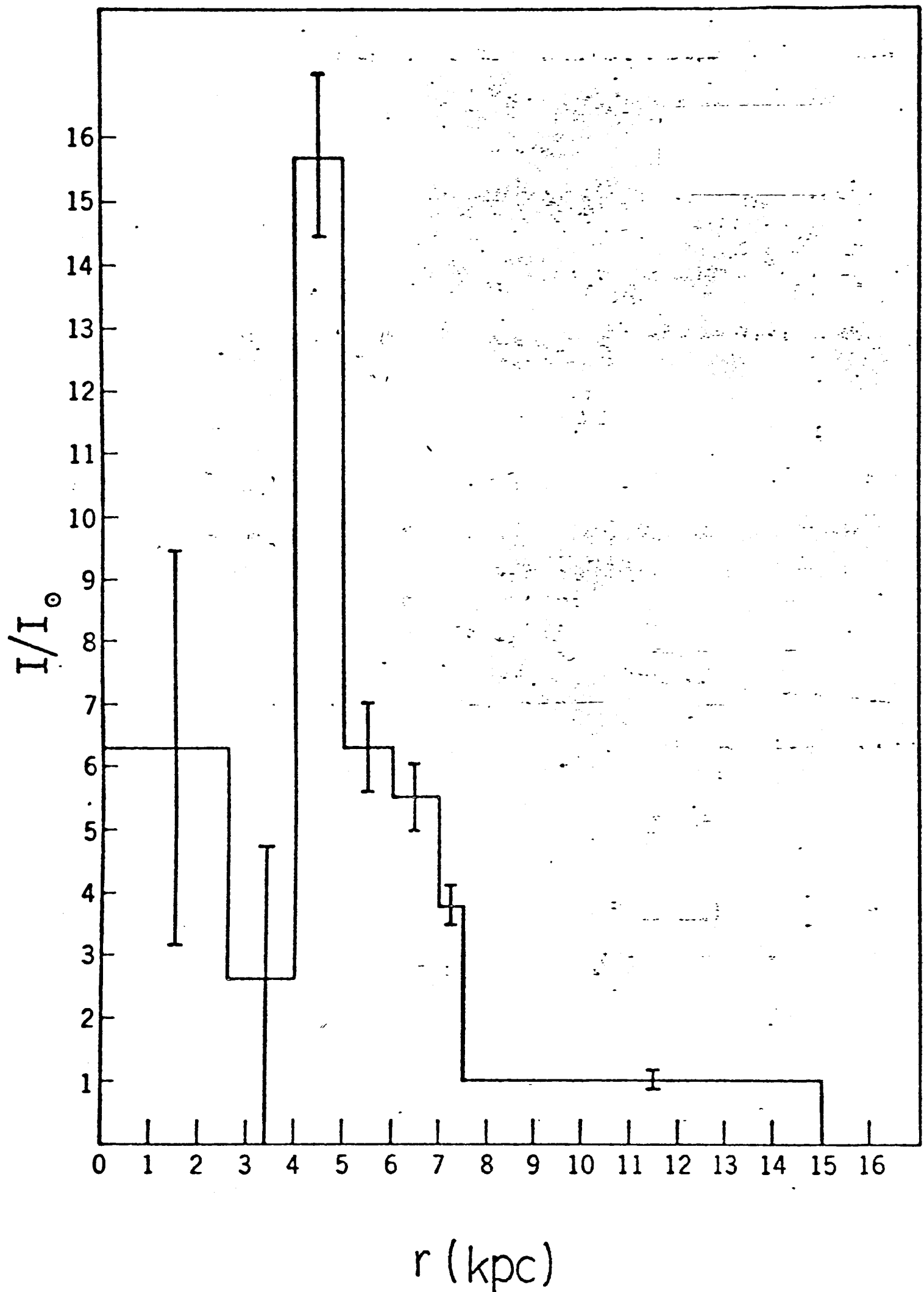


FIGURE 4. The intensity of the galactic cosmic ray flux as a function of position in the Galaxy (from Puget & Stecker 1974).

3. Cosmic Ray Anisotropy.

When large-scale variations in cosmic ray density exist there will be a tendency for the cosmic ray gas to stream down its density gradient. The streaming can be reduced by diffusion through an irregular magnetic field or by collective effects generated by the cosmic ray gas. Observations of the net streaming velocity can therefore provide us with information about the nature of the restrictions which prevent free flow.

The observed anisotropy is defined as

$$\delta = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Relativistic cosmic rays with anisotropy δ have a mean streaming velocity

$$V_s = \frac{1}{3} c \delta$$

relative to the Earth. Care must be taken however in transforming V_s to the velocity V of the frame in which the cosmic rays would appear isotropic, since in transforming between the two frames the distribution function is altered. The two velocities are related by a Compton-Getting factor C , $V_s = C V$.

When $\delta \ll 1$, the Compton-Getting factor can be expressed as

$$C = -\frac{1}{3} \frac{d \ln f}{d \ln r} = \frac{1}{3} \gamma$$

where the particle distribution function $f(r) \propto r^{-\gamma}$

(Forman 1970).

Because of the effects of the solar magnetic field, the galactic cosmic ray anisotropy cannot be measured for particles with energy below 100 GeV. The most stringent upper limit for δ

was obtained by Elliot et al. (1970), who measured a small but statistically significant sidereal variation in the muon intensity underground in London. After correcting for the known motion of the solar system relative to the local standard of rest, Speller et al. (1972) obtained an upper limit for the cosmic ray anisotropy relative to the galactic rotation frame of $\delta \leq 1.5 \times 10^{-4}$ (with the factor of $\frac{1}{2}$ in their definition ignored). This limits the streaming velocity relative to the galactic rotation frame to $V_s \leq 15 \text{ km s}^{-1}$. The actual value of this upper limit may be up to 2.5 times higher than this because of the effects of the solar magnetic field (Barnden & McCracken 1973), since their median primary cosmic ray energy was only 150 GeV.

Other observations at higher energies, mostly from air shower data, are plotted in Figure 5. The curve shows the estimate of Dickinson & Osborne (1974) of the upper limit to the anisotropy as a function of energy.

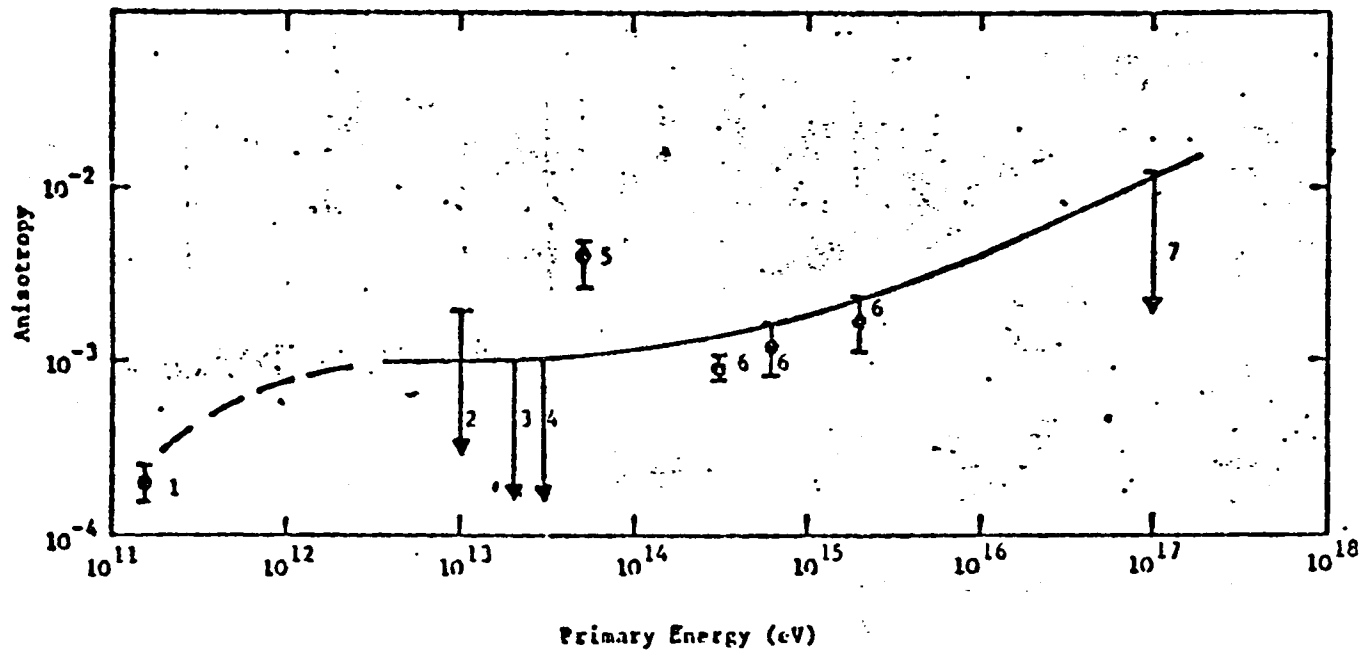


FIGURE 5. Experimental measurements of cosmic ray anisotropy, as reviewed by Dickinson & Osborne (1974). The curve represents their estimate of the upper limit as a function of energy.

4. The Chemical Composition of Galactic Cosmic Rays.

The most dramatic developments of the last two years in cosmic ray astronomy have taken place in observations of the chemical composition of the cosmic rays entering the Earth's atmosphere. These observations can provide us with information about the propagation parameters (such as the cosmic ray lifetime in the Galaxy and the amount of interstellar matter they traverse before escaping) and about the nature of the cosmic ray sources. During the last two years measurements have been published which show that the composition changes with particle energy, and the resulting variation of the propagation parameters provide an invaluable guideline for any theory of galactic cosmic ray confinement. Indeed it was these observations which first prompted the research into energy-dependent confinement models upon which this thesis is based. But it must be emphasized that a great deal of useful results had previously been obtained from integral measurements of the composition of particles within certain limited or unlimited energy ranges. We shall discuss the two types of observations, with their resulting conclusions, separately.

Integral Measurements

Nearly all of the early investigations into the composition of cosmic ray nuclei were carried out using nuclear research emulsion stacks suspended from stratospheric balloons. Allowance was made for interactions between the incoming particles and the residual atmosphere above the balloons, thereby obtaining the relative abundances of the different species of the galactic

cosmic ray particles. More recently, scintillation counters, Cerenkov detectors and spark chambers have been used.

The latest published measurements of the relative abundances of galactic cosmic rays above 5 GeV/nucleon are recorded in Table 1, normalized to carbon (Smith et al. 1973).

Before using the observed composition to ascertain the nature of the sources, allowance must be made for any changes in composition brought about by nuclear interactions in the interstellar medium. When a heavy cosmic ray nucleus collides with a nucleus of the interstellar medium, it breaks up into a collection of pions and lighter nuclei called 'spallation products'. We have already discussed how the γ -photons produced by the decay of these pions can give us information about the magnitude of the cosmic ray flux; observation of the abundance of the spallation products can also tell us how much interstellar material the cosmic ray nuclei traverse on average before leaving our part of the Galaxy. Fortunately we can identify some of the spallation products.

The 'light elements', Li, Be and B, are rapidly consumed in stellar interiors. Consequently their thermal abundance is very low, about 10^{-9} times that of hydrogen, as shown in Figure 6. There is no such disparity in their cosmic ray abundance, so if it is reasonable to assume that their source abundance is very low then they must be created by spallation reactions in the interstellar medium. In fact the light elements are products of spallation reactions between the 'medium elements', C, N and O, and interstellar H, He and D nuclei. The Li abundance is

<u>ELEMENT</u>	<u>ABUNDANCE</u>
H	24000 $\pm$ 2800
He	4200 $\pm$ 100
Li	18.1 $\pm$ 1.8
Be	4.2 $\pm$ 0.8
B	23.5 $\pm$ 1.3
C	100
N	24.8 $\pm$ 1.3
O	94.0 $\pm$ 2.3
F	2.1 $\pm$ 0.7
Ne	18.9 $\pm$ 1.3
Na	0.8 $\pm$ 0.5
Mg	23.6 $\pm$ 1.4
Al	1.8 $\pm$ 0.5
Si	19.6 $\pm$ 1.3
P - Cr	18.1 $\pm$ 1.4
Fe	12.0 $\pm$ 1.1

TABLE 1. The relative abundances of galactic cosmic rays above 5 GeV/nucleon , normalized to carbon (from Smith et al. 1973).

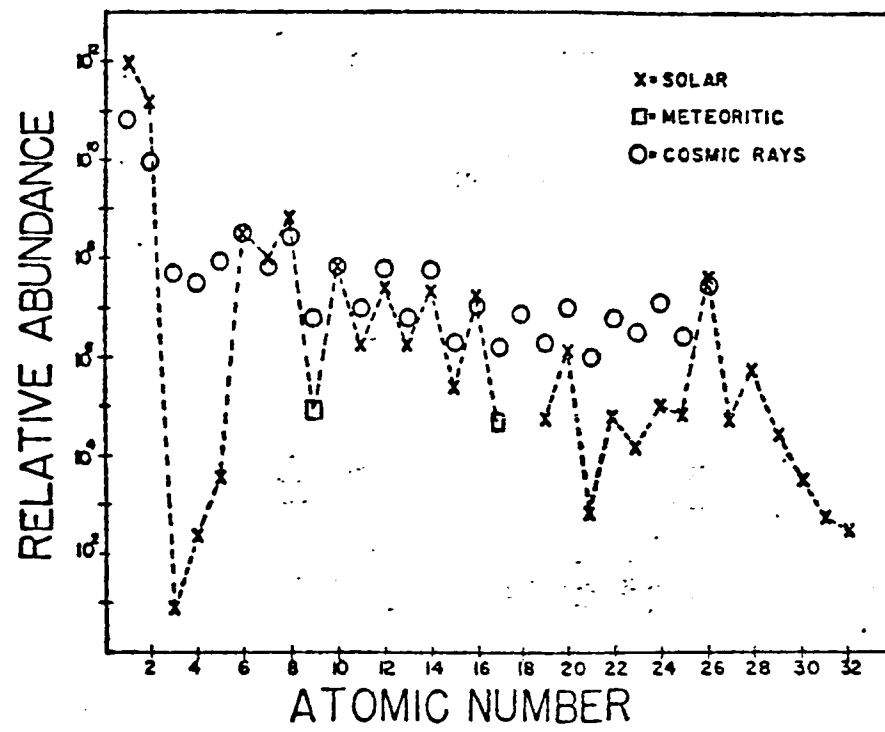


FIGURE 6. The abundances of the cosmic ray nuclei relative to solar abundances (from Shapiro & Silberberg 1970).

related to the medium element flux above 30 MeV/nucleon, and the B abundance to the flux above 5 MeV/nucleon. (Reeves et al. 1970). Measurements above 1 GeV/nucleon yield a L/M ratio

$$\frac{Li + Be + B}{C + N + O} = 0.25$$

(O'Dell et al. 1962, Durgaprasad et al. 1970, von Rosenvinge et al. 1969). When discussing cosmic ray composition we shall reserve the term 'secondary elements' for those cosmic rays which been created by spallation reactions in the interstellar medium. These are not to be confused with the 'atmospheric secondaries' created by spallation reactions in the Earth's atmosphere. Likewise the term 'primary' will apply to those elements created in the cosmic ray sources.

Using available cross sections for various spallation reactions, and allowing for the effects of decay in dilated time of the radioactive nuclides formed, Shapiro and Silberberg (1968) calculated the average amount of matter traversed by cosmic rays required to create the observed abundance of the light elements. They used a 'slab' approximation in which all the cosmic ray nuclei traversed the same amount of matter, and the value of the 'slab thickness' they obtained was

$$l = 4.0^{+1.0}_{-0.5} \text{ g cm}^{-2}.$$

This path length is sufficient to explain all the observed Li^6 , but not all the Li^7 . Meneguzzi et al. (1971) suggested that a substantial fraction of the Li^7 could come from the cool atmospheres of red giants.

Another estimate of the mean path length may be obtained from the abundance of He^3 , which like the light nuclei has a very low thermal abundance. It is a product of the breakup of cosmic ray He^4 , and measurement of its abundance between 100 and 400 MeV/nucleon gives

$$\frac{\text{He}^4}{\text{He}^3 + \text{He}^4} = 0.10 \pm 0.02$$

(Hildebrand et al. 1963). From this information Ramaty and Lingenfelter (1969) obtained a mean path length of 4.0 g cm^{-2} , which agrees with the results derived from the light nuclei, despite the difference in energy domains.

Of course, the assumption that all cosmic rays traverse the same amount of matter is an oversimplification. Davis (1959) calculated the distribution of path lengths resulting from different propagation models. A steady-state three-dimensional diffusion process from a point source in an infinite medium of constant density leads to a Gaussian-like function, while a steady-state diffusion inside a leaky box leads to an exponential distribution. Von Rosenvinge (1969) concluded that the exponential distribution fitted the He / H and He / M ratios best. Shapiro et al. (1969) obtained an exponential distribution

$$N(l) dl \propto e^{-(0.16 \pm 0.04) l} dl$$

(l in g cm^{-2}) as the best fit to reconcile the L / M and $(17 \leq Z \leq 23) / (24 \leq Z \leq 26)$ ratios. The 'sub-iron group' ($17 \leq Z \leq 23$) are known to be spallation products of the 'iron group' cosmic rays ($24 \leq Z \leq 26$).

If one is prepared to make assumptions about the region of propagation of cosmic rays, it is possible to obtain an estimate for their mean age. A particle confined to the galactic disk (of average density $\sim 1 \text{ atom cm}^{-3}$) requires a lifetime of 3×10^6 years in order to traverse 4 g cm^{-2} of matter. This estimate can be compared with that obtained from the abundance of Be^{10} , which acts as a 'radioactive clock'. If the cosmic ray light elements are separated into their respective isotopes, the amount of Be^{10} is found to be less than that predicted from a propagation model which satisfies the other isotopic abundances. The discrepancy is attributed to the radioactive decay of this isotope, which has a half-life of 1.5×10^6 years (Yiou & Raisbeck 1972). Webber et al. (1973a) conclude that the deficiency of Be^{10} in the energy range 100 - 200 MeV implies a mean cosmic ray lifetime of

$$\tau = \left(3.4 \begin{array}{c} +3.4 \\ -1.3 \end{array} \right) \times 10^6 \text{ yr}.$$

Differential Measurements

The first evidence of a variation in cosmic ray composition with energy came from Juliusson and Meyer (1972), who observed that the ratio of all the secondary nuclei to the primary nuclei decreased with increasing energy above about 20 GeV/nucleon. This measurement was subsequently confirmed and extended by several other groups. Smith et al. (1973) and Balasubrahmanyam and Ormes (1973) obtained spectra of the heavier elements (Figure 2), and found that the spectrum of the light nuclei Li, Be, B, is steeper than that of the medium nuclei C, N, O by a difference

of 0.14 ± 0.07 in their spectral indices. The L / M ratio is plotted as a function of energy in Figure 7.

The decreasing secondary / primary ratio is reflected in all of the secondary - primary pairs, e. g. $L / (C + O)$, $(17 \leq Z \leq 25) / Fe$ (Ormes & Balasubrahmanyam 1973, Webber et al. 1973). Above a few GeV/nucleon the cross sections for the spallation reactions do not vary appreciably with energy (Shapiro & Silberberg 1970, Kaufman et al. 1973). This implies that the higher-energy particles of each primary species must have passed through less material after acceleration than those of lower energy. Cesarsky and Audouze (1974) obtained an expression for the mean path length of cosmic rays in the Galaxy as a function of energy from analysis of the chemical composition data. This begins to decrease sharply above about 30 GeV/nucleon (Figure 8). They also confirm that the effects of a possible variation in the nuclear cross sections with energy would not alter the value of the path length by more than ten per cent.

Ormes and Balasubrahmanyam (1973) and Webber et al. (1973 b) also found another interesting effect. They discovered that the ratio of various pairs of primary nuclei groups, for example $(C + O) / Fe$, also decreases with energy. Balasubrahmanyam and Ormes (1973) calculated that this effect cannot be accounted for by the energy dependence of the path length as derived from the secondary / primary ratios, implying that some primary components must have different source spectra. Cesarsky and Audouze (1974) confirmed that the sources of high-energy cosmic rays must be richer in Fe relative to C and O than the

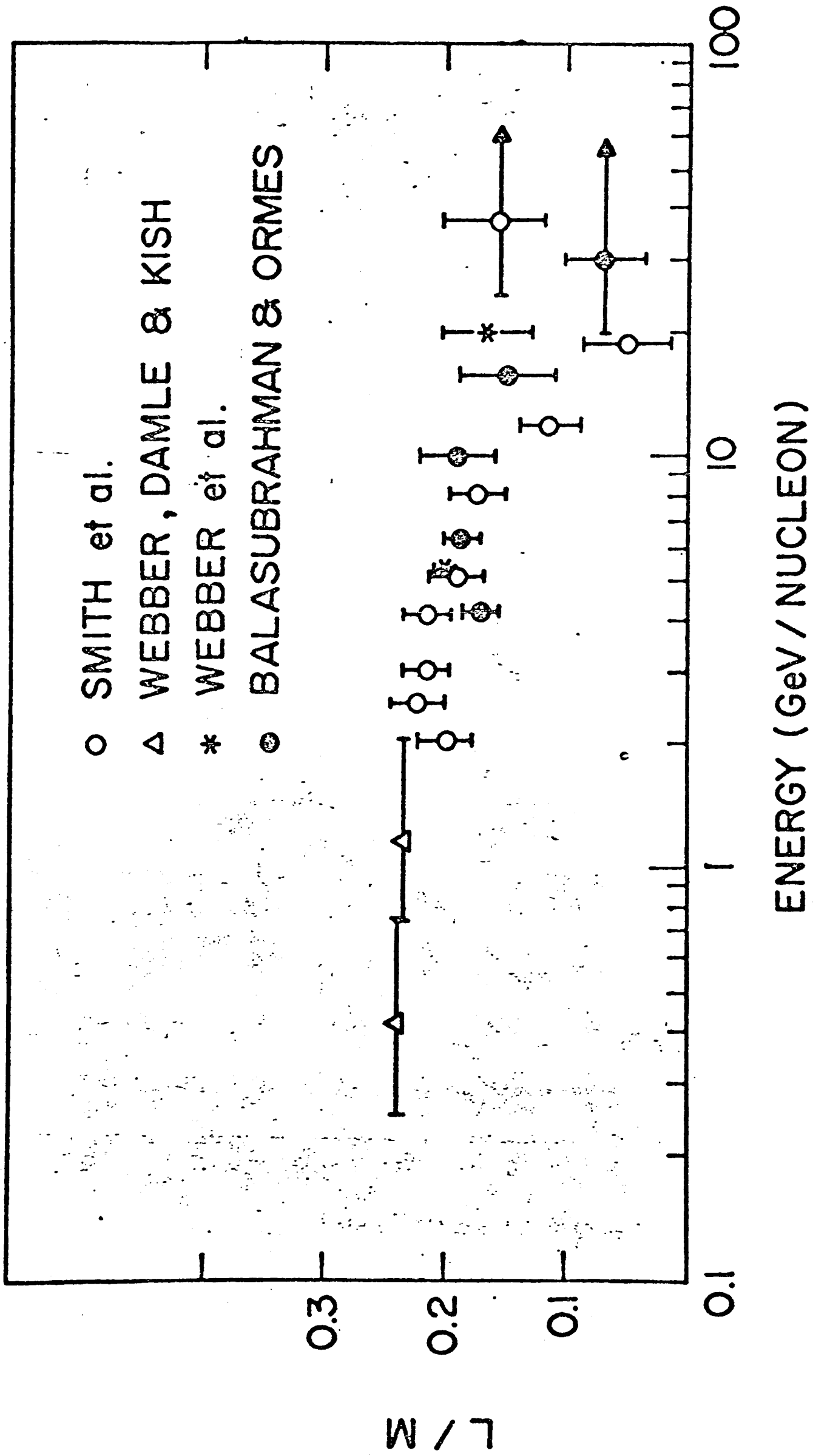


FIGURE 7. The ratio of light to medium nuclei in galactic cosmic rays as a function of energy per nucleon (the references

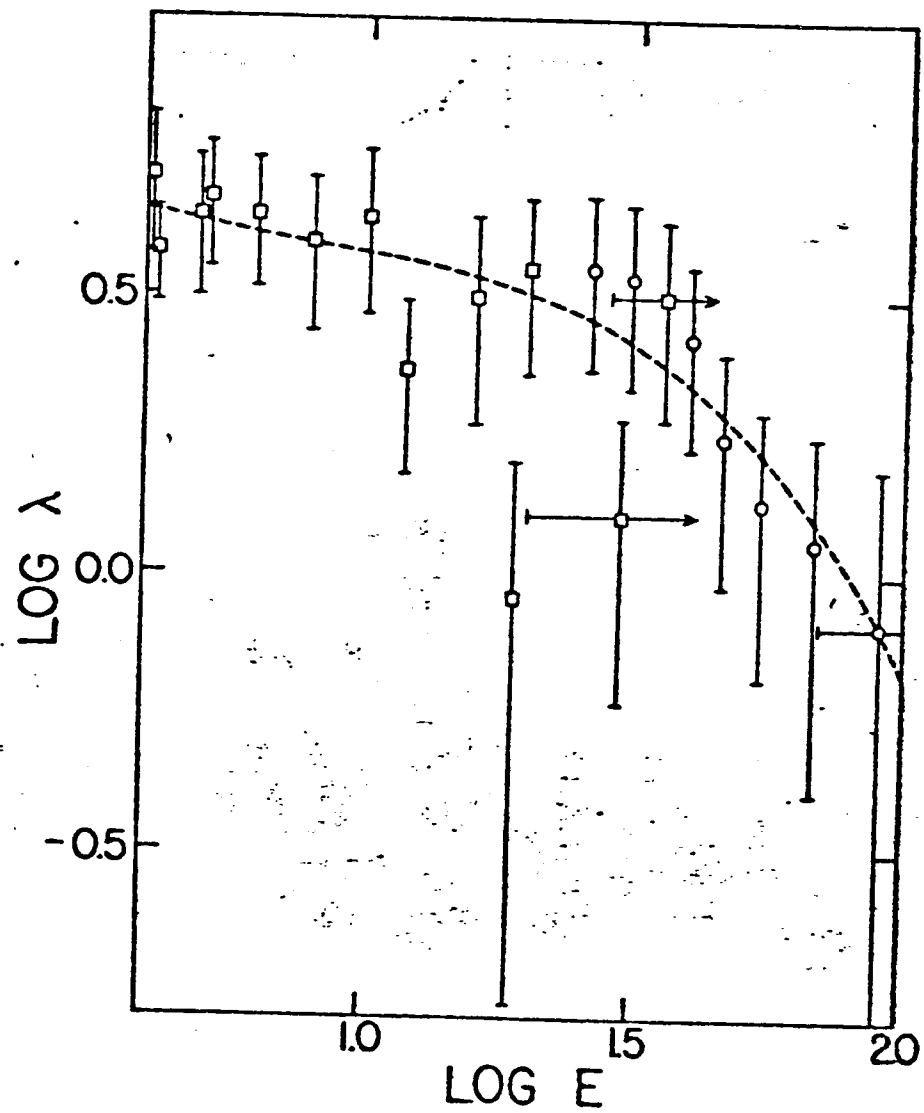


FIGURE 8. The mean path length of cosmic rays in the Galaxy as a function of energy per nucleon. λ is in gm cm^{-2} and E is in GeV/nucleon . The broken line is the best third-degree polynomial fit (from Cesarsky & Audouze 1974).

sources of lower-energy cosmic rays, while Ramaty et al. (1973) suggested that there may possibly exist two source mechanisms, one of which produces the iron (possibly from neutron stars) and the other is responsible for all the rest of the primary nuclei.

5. The Electron Spectrum.

Useful information about galactic cosmic ray propagation may be obtained from analysis of the relativistic electron spectrum. Since electrons are much lighter than protons, they suffer a proportionately greater energy loss, and this in turn produces characteristic changes in their spectrum. The theory behind this effect will be discussed in Chapter 5, so in this section we shall just summarize the observations.

Our main region of interest in the electron spectrum is between the energies of 10 and 1000 GeV, since some models of cosmic ray confinement predict a bend somewhere within this range. Unfortunately there is still too much scatter in the observed spectrum to ascertain precisely the presence of such a bend. The latest information from various groups, presented at the 13th International Conference on Cosmic Rays at Denver, 1973 (Webber 1973), is shown in Figure 9. The average spectral indices they obtain are listed in Table 2. There is no evidence for a change in the value of the spectral index ~ 1 in this region. The best single-power-law fit to these measurements is

$$I(E) = 200 E^{-3} \quad m^{-2} s^{-1} sr^{-1} GeV^{-1}$$

A certain fraction of these electrons may be of 'secondary' origin. The spallation of galactic cosmic rays produces both electrons and positrons, so observations of the relative flux of the positrons should indicate what fraction of the electrons are secondary. If all the electrons were of secondary origin then the fraction $e^+ / (e^+ + e^-)$ should approximately equal 0.6 (Webber 1973). Measurements by Buffington (1973) above

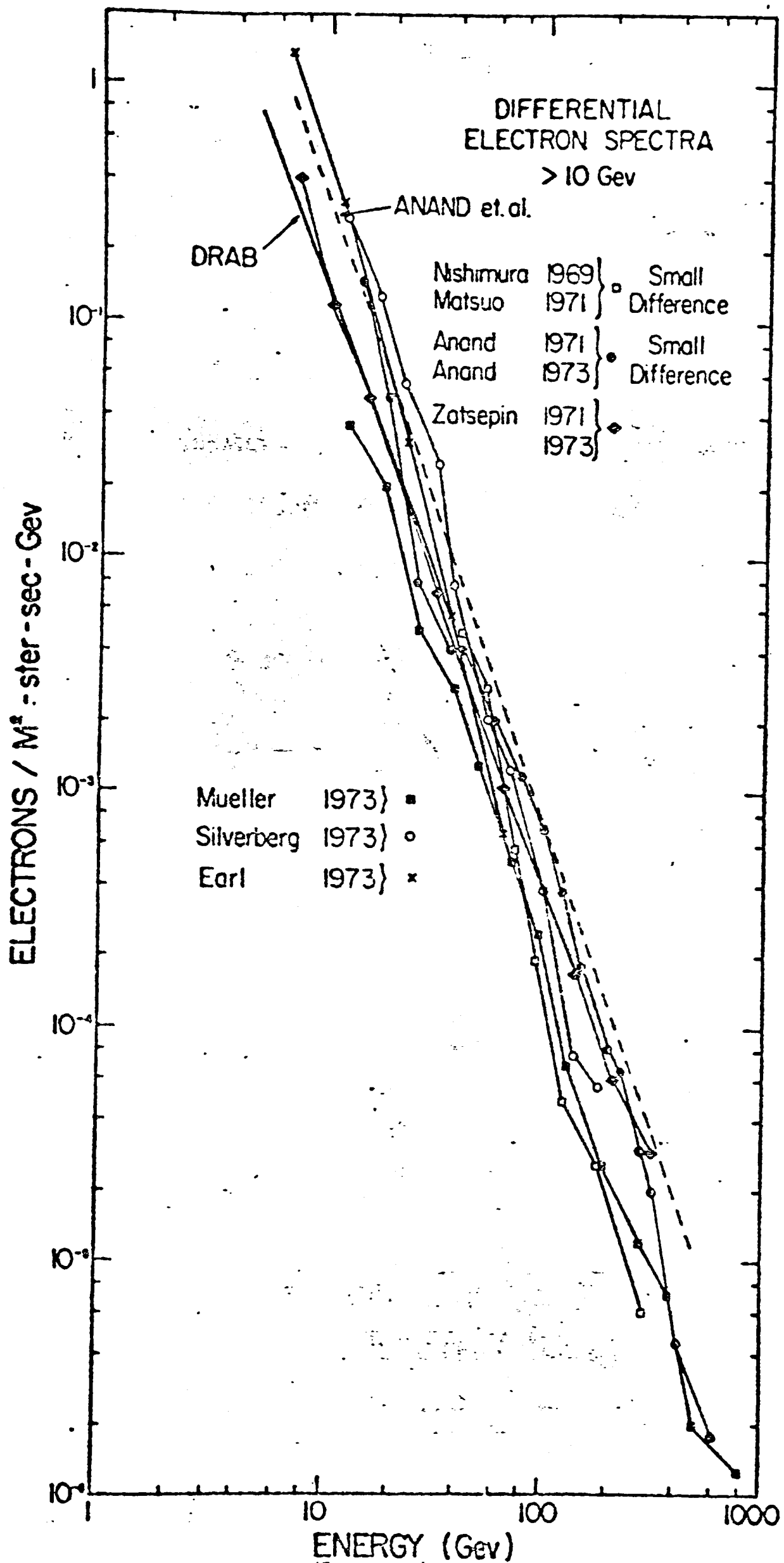


FIGURE 9. The differential spectrum of cosmic ray electrons
(from Webber 1973).

	<u>10 - 100 GeV</u>	<u>> 100 GeV</u>
Mueller	- 2.72	- 3.0
Meegan & Earl	- 3.5	
Silberberg et al.	- 3.2	- 3.2
Nishimura et al.		- 3.2
Anand et al.	- 2.62	- 3.1
Zatsepin et al.	- 2.7	

TABLE 2. The slope of the cosmic ray electron spectrum,
from measurements presented at the 13th International Conference
on Cosmic Rays, Denver 1973 (from Webber 1973).

5 GeV suggest a ratio of ~ 0.1 . The flux of secondary electrons and positrons which this implies lies a factor of 2 - 3 below that calculated by Ramaty and Lingenfelter (1968) on the assumption that the average amount of matter traversed by cosmic rays is 4 g cm^{-2} . This discrepancy is also confirmed by Dilworth et al. (1974), who conclude that it can be reconciled with an energy-dependence in the average amount of interstellar matter encountered by cosmic rays in the Galaxy. When energy-dependent values of the observed electron / positron ratio are available, it would be interesting to compare the path length derived from them with that derived from the primary / secondary cosmic ray ratio.

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CHAPTER 2

ORIGIN AND PROPAGATION THEORIES

Since the existence of cosmic radiation was established about fifty years ago, the two major problems it has posed concern (i) the nature of the sources, and (ii) the acceleration mechanism. These questions still remain unanswered, although several viable theories have emerged from the wealth of observational material collected in the past decade. But as well as bringing us closer to an answer to these two questions, the data obtained in the last decade has posed additional questions. The most important problem posed by the observations concerns the propagation of the cosmic rays through the Galaxy. If most of the cosmic ray sources are situated in the Galaxy, as several proposed answers to question (i) indicate, then we would expect to be able to detect the cosmic rays streaming from the source region out into the halo. But measurements of their anisotropy indicate that they must be streaming with a velocity less than about 30 km s^{-1} , which suggests that they are prevented from streaming freely. Consequently another question must be posed: (iii) what is their mode of propagation through the Galaxy? In this chapter we will discuss the answers which have been suggested for the three questions.

1. The Source Composition.

The first clues about the composition of cosmic ray sources

were offered by Bradt and Peters (1950), who discovered that cosmic radiation entering the Earth's atmosphere consisted of nuclei up to Si as well as protons. They deduced from the charge spectrum that the ratio of heavy elements to light elements was greater in the accelerated particle flux than in the solar system abundances, and that the flux of Li, Be and B could be produced by the spallation of the heavy elements. This coincided with Hoyle's work on nucleosynthesis (Hoyle 1949), in which he proposed that most of the heavy nuclei in the Galaxy were formed in the interior of stars and distributed throughout space when the stars exploded. It was recognized that the elemental abundances of the galactic primary cosmic rays would reflect the chemical composition of their sources, and would perhaps offer some clues about their acceleration mechanism. But calculation of the primary abundances involves detailed knowledge of the spallation processes in interstellar space and of the composition of the radiation incident on the Earth, which was at that time incomplete. When more extensive cross-section data became available, coinciding with better charge resolution of the incident radiation, Beck and Yiou (1968) calculated the primary abundances of some of the heavy elements, employing a slab approximation in which all the particles traversed 5.4 g cm^{-2} of interstellar material.

A more detailed calculation of primary composition was carried out by Shapiro et al. (1970). They compiled abundance data from experiments with good charge resolution, and applied the exponential path length distribution discussed in the previous

chapter. Starting with a trial primary composition similar to the accepted universal composition, they calculated the resulting composition after propagation. The primary composition was adjusted in successive approximations until the calculated abundances matched those of the elements C to Fe observed at the top of the atmosphere. The calculated primary abundances were found to be insensitive to the initial trial values. The resulting source composition is shown in Figure 10, together with information about whether each element is mainly of primary or secondary origin.

If there are no preferential acceleration effects, the primary abundances should match the source composition. For some time it has been thought that supernovae could be the sources of galactic cosmic rays (Ginzburg & Syrovatskii 1964), since they are enriched in heavy elements and can provide enough energy to accelerate the cosmic rays. But the large $(C + O) / Fe$ ratio eliminates supernovae of mass less than about 8 solar masses, as such supernovae burn their C and O to the Fe region (Arnett & Schramm 1973). Arnett and Schramm suggest that instead, massive supernovae of $M > 8 M_{\odot}$ are likely candidates, since they do not burn up their C and O explosively. The event rate of these supernovae is about the same as that for supernovae with $4 M_{\odot} \lesssim M \lesssim 8 M_{\odot}$, which is adequate to supply the observed cosmic ray flux. Furthermore these supernovae leave behind a small, dense iron-enriched remnant of mass $\sim 1.4 M_{\odot}$, which could become a pulsar, providing an electromagnetic acceleration mechanism and an additional source of Fe.

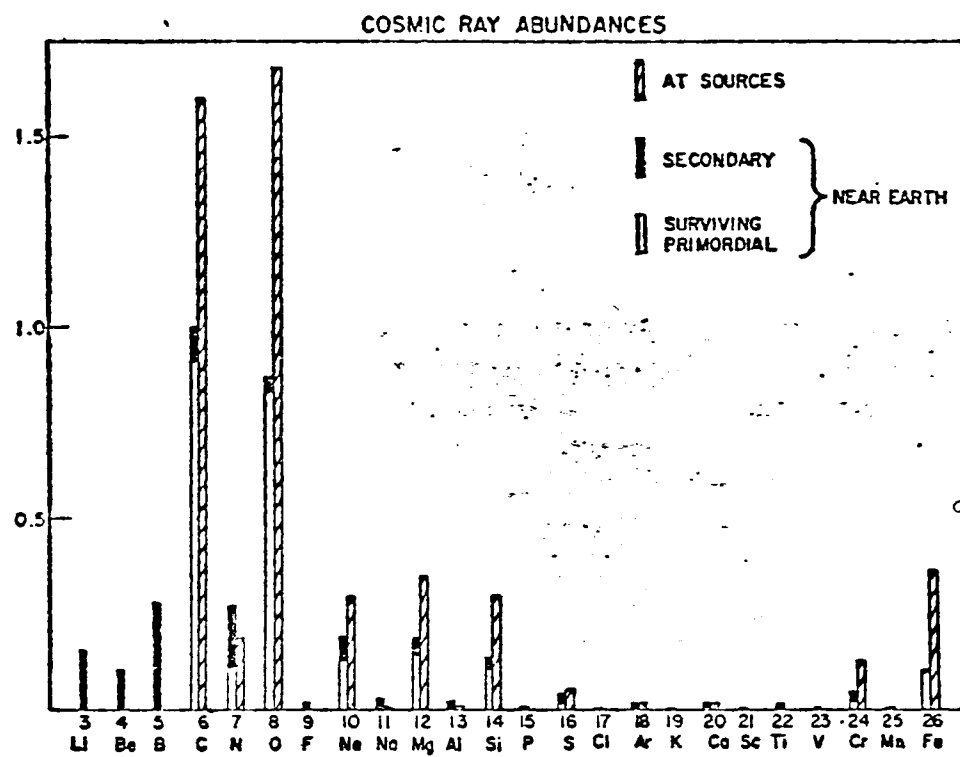


FIGURE 10. The composition of primary and secondary galactic cosmic rays (from Shapiro & Silberberg 1970).

Unfortunately the estimate of the primary Fe / C ratio obtained from the observed composition of galactic cosmic rays is rather sensitive to the path length distribution used. The exponential distribution employed by Shapiro et al. gives a ratio of $0.23 \pm .05$, while a slab model of thickness 4 g cm^{-2} gives a ratio of $0.35 \pm .06$. But in all practical distributions the Fe / C ratio is small enough to rule out the carbon-detonation supernova as a possible source. Furthermore recent observations suggest that this ratio increases as the cosmic ray energy increases (see Chapter 1), which may be due to the acceleration of Fe from a separate source. It is quite possible that the pulsar left behind by a massive supernova explosion could be the source of the high-energy Fe.

Finally let us examine how the conclusions of the last two paragraphs are modified if we allow for preferential acceleration effects. The existence of a preferential acceleration process seems quite likely, as analysis by Cassé and Goret (1973) and Havnes (1973) shows that the ratio between the cosmic ray abundance and the universal abundance of each element exhibits a fair degree of correlation with the first ionization potential of the element (Figure 11). If this correlation is real, it implies that elements with small first ionization potentials, which are the most easily ionized, are accelerated preferentially to those with larger first ionization potentials. This suggests that the acceleration of cosmic rays is electromagnetic rather than explosive, and that the source region is originally composed of neutral atoms which become ionized by some violent event.

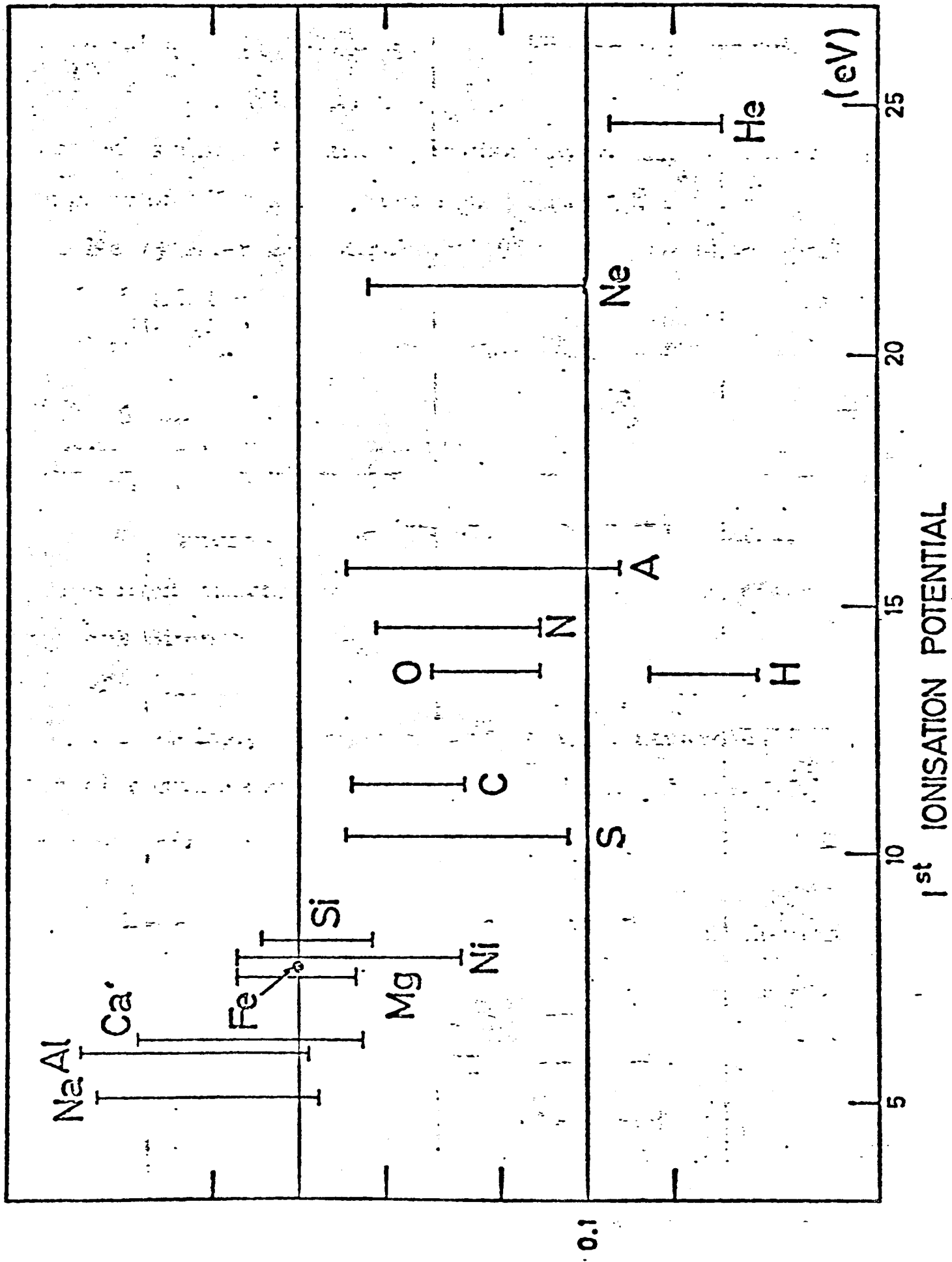


FIGURE 11. The correlation between the first ionization potential of cosmic ray nuclei and their over-abundance (from Reeves 1973).

Non-explosive acceleration is advocated by Reeves (1973) who explains the difference between the cosmic ray and universal abundances in terms of a two-component theory. He states that the primary composition is a mixture of one part of supernova material with three parts of interstellar medium (Table 3). These calculations do not take any preferential acceleration into account, but whether it occurs or not it seems highly likely that there is a period between a supernova explosion and cosmic ray acceleration when mixing is allowed to take place between the supernova ejecta and the interstellar medium.

It therefore seems that three major difficulties have to be overcome in order to determine the composition of the cosmic ray sources : (i) detailed knowledge of the path length distribution and spallation reaction cross sections are required in order to calculate the primary abundances from the observed abundances, (ii) the composition of the source-medium mixture must then be determined, taking into account any preferential acceleration mechanisms that may exist, (iii) finally the degree of mixing between the source material and the interstellar medium must be known in order to determine the true source composition. Nuclear astrophysicists will not run out of problems for many years hence, if indeed they ever do.

	<u>X</u>	<u>Y</u>	<u>Z_{CNO}</u>	<u>Z_{NeSi}</u>	<u>Z_{Fe}</u>
1 part of supernova	.60	.10	.15	.09	.06
+					
3 parts of interstellar medium	.73	.25	.013	.003	.001
=					
cosmic ray composition	.70	.21	.046	.025	.016

TABLE 3. The mixing of supernova material with the interstellar medium to produce cosmic ray abundances (from Reeves 1973).

2. The Acceleration Mechanism.

Having discussed the type of object which may produce the galactic cosmic rays, let us now investigate the means by which they attain their relativistic energies. The magnitude of the energy involved in the acceleration process may be estimated from the energy density and lifetime of cosmic rays in the Galaxy. The energy density of the cosmic ray flux near the Earth is $\sim 1 \text{ eV cm}^{-3}$. Taking this to be the average energy density of galactic cosmic rays, which are assumed to be confined in a disk of radius 15 kpc and thickness 1 kpc, the total energy of cosmic rays in the Galaxy is $\sim 2 \times 10^{67} \text{ eV}$. If they escape from this region after $3 \times 10^6 \text{ yr}$, the total power required by their sources to maintain this density is $\sim 10^{49} \text{ erg yr}^{-1}$. This criterion does not rule out the possibility that the energy required to accelerate the cosmic rays comes from supernovae explosions. If an average rate of 1 supernova every 30 years in the Galaxy can be assumed, then $\sim 3 \times 10^{50} \text{ erg}$ from each explosion must be used in the acceleration process. This is quite possible, since a type II supernova releases on average $10^{51} - 10^{52} \text{ erg}$.

The means by which supernovae accelerate the cosmic rays is still uncertain, although it seems likely that they could be accelerated electromagnetically in the expanding shell produced by the explosion. Colgate and White (1966) carried out hydrodynamic analyses of supernova explosions to investigate the possibility of direct acceleration of cosmic rays by the shock wave produced in these events. By assuming a particular stellar model they predicted a cosmic ray energy spectrum which was in good agreement

with observations. But the efficiency of this process is very low ($\sim 10^{-3}$), as most of the energy of the explosion goes into mass motions of the gas with sub-relativistic velocities. This is too low, as the arguments in the previous paragraph demand an efficiency of $10^{-2} - 10^{-1}$. Gull (1973) considered the evolution of the expanding supernova remnant, and showed that this is a more plausible location for the acceleration process. The young remnant slows down due to the 'snowplow effect' as it collects an increasing quantity of interstellar matter in front of it. The denser material of the supernova remnant penetrates the layer of interstellar matter by means of the Rayleigh-Taylor instability, and the resulting turbulence amplifies the magnetic field in the shell, thereby accelerating the cosmic rays. The efficiency of this process is $\sim 10^{-2}$, which is high enough to account for the observed cosmic ray flux; and furthermore the interpenetration between the layers of supernova remnant and interstellar medium allows the required degree of mixing discussed in the previous section to take place. This process can account for the acceleration of particles up to energies of about 10^{18} eV (Gurevich & Rumyantsev 1973), and is therefore eminently suitable as a model of galactic cosmic ray origin.

Another source of cosmic ray particles (possibly some of the Fe component of primary cosmic rays) could be pulsars. Goldreich and Julian (1969) pointed out that the electric field induced by the rotating magnetic field is strong enough to pull charged particles from the surface of the neutron star, leading to the formation of a dense magnetosphere. Unfortunately the

presence of a magnetosphere renders calculation of the electromagnetic fields very difficult, as they can no longer be described by the vacuum-dipole solution. But if it can be assumed that the field at large distances from the pulsar, where the density of the magnetosphere is negligible, is that of a rotating dipole, then it is possible to calculate the trajectory of a test particle in this vicinity. Gunn and Ostriker (1969) investigated the interaction between test particles and the low-frequency electromagnetic waves that are produced by oblique rotators, and concluded that the test particles can be accelerated up to energies of about 10^{21} eV, which is greater than that of the most energetic cosmic ray particle which has been observed. By a suitable choice of parameters, the spectrum of accelerated particles can be made to match the observed cosmic ray spectrum. Karakula et al. (1974) suggested that pulsars could be the dominant source of cosmic rays in the energy range $10^{14} - 10^{16}$ eV. The change in slope of the cosmic ray spectrum at 3×10^{15} eV may be explained in terms of the process by which pulsars lose their energy. In their early stage the energy loss is mainly due to gravitational effects, while later on magnetic losses dominate. Each stage gives rise to its characteristic energy spectrum of accelerated particles, and the superposition of the two spectra produces a kink in the total energy spectrum. By a suitable choice of pulsar parameters, the kink can be made to occur at $\sim 10^{15}$ eV.

It is possible that some cosmic rays are accelerated by the Fermi process (Fermi 1949, 1954). A constriction in the interstellar

magnetic field, where the field increases from $\bar{B}$ to $B_{\max}$, will reflect all incident particles with pitch angle θ such that

$$\sin^2 \theta > \bar{B} / B_{\max} .$$

When a particle travelling with velocity $\underline{v}$ is reflected from a mirror moving with velocity $\underline{u}$ ($u \ll v$), the particle experiences a gain in energy

$$\Delta E = - 2 E (\underline{u} \cdot \underline{v}) / c^2 .$$

Since the probability of an approaching collision is greater than that of an overtaking one, it follows that there will be a net gain in energy of particles propagating in a region of randomly-moving magnetic irregularities. Fermi showed that such a situation could produce a power-law spectrum with integral exponent $-t c^2 / \tau V_A^2$, where t is the mean time between collisions, τ is the cosmic ray lifetime in the accelerating region, c is the velocity of light, and V_A is the Alfvén velocity. If the cosmic rays were accelerated by this process in the interstellar medium, where the Alfvén velocity is $\sim 10^6$ cm s⁻¹ and their mean lifetime is $\sim 3 \times 10^6$ yr, the observed exponent of ~ 1.7 requires the mean time between collisions to be $\sim 6 \times 10^{-3}$ yr. At relativistic velocities, this corresponds to a collision mean free path of 2×10^{-3} pc. This is several orders of magnitude less than the accepted length-scale of irregularities in the galactic magnetic field due to random motions, which is thought to be about 10 - 30 pc. Furthermore this process is unable to accelerate particles to energies above $\sim 10^{12}$ eV, where their gyro-radii are greater than their mean free path. It is therefore unlikely that Fermi acceleration

in the interstellar medium is the dominant method of generating cosmic rays, although it may be important in smaller-scale regions of turbulence such as supernova envelopes.

3. Cosmic Ray Propagation in the Galaxy.

The answer to the third question posed at the beginning of this chapter, namely the mode of cosmic ray propagation in the Galaxy, must be consistent with: (a) the stringent limit of $\sim 30 \text{ km s}^{-1}$ for their maximum streaming velocity in the Galactic plane, (b) a containment lifetime of $\sim 3 \times 10^6 \text{ yr}$ before escaping from the Galaxy, and (c) a mean path length which decreases with increasing energy. The most obvious mechanism of restraining charged particles from streaming freely in the Galaxy is the process of scattering in a turbulent magnetic field. Use of the diffusion equation with a 3-dimensional diffusion coefficient $D = \frac{1}{3} c L$, where L is the diffusion mean free path, yields an expression for the density $N(r, z, t)$ of particles in a cylindrically-symmetric diffusing region of dimensions $-h < z < h$ and $0 < r < a$. With an initially uniform density N_0 , allowing free escape across the boundaries of the diffusing region,

$$N = N_0 \Psi(z, t) \chi(r, t)$$

$$\text{where } \Psi(z, t) = \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} e^{-D(2n+1)^2 \pi^2 t / 4h^2} \cos \frac{(2n+1)\pi z}{2h}$$

$$\text{and } \chi(r, t) = \frac{2}{a} \sum_{n=1}^{\infty} e^{-D\alpha_n^2 t} \frac{J_0(r\alpha_n)}{\alpha_n J_1(a\alpha_n)},$$

α_n are the roots of $J_0(a\alpha) = 0$ (see Carslaw & Jaeger, 1959).

Because $h \ll a$, most of the particles are lost across the plane faces of the disk, and only the term $\Psi(z, t)$ need be considered when calculating their mean lifetime, which is found to be

$$\tau = 0.4 h^2 / D \quad \text{at } z = 0. \quad \text{A mean free path of } L \sim 10 \text{ pc}$$

gives a diffusion coefficient of $L \sim 1 \text{ pc}^2 \text{ yr}^{-1}$, so the mean lifetime in a disk of thickness 200 pc would be only $\sim 10^4 \text{ yr}$. This is $2\frac{1}{2}$ orders of magnitude less than the measured lifetime, so one or more of the following conclusions must be true:

(i) the geometry of the system is incorrect, (ii) the propagation is not governed by 3-dimensional diffusion with a mean free path of 10-30 pc, (iii) the particles do not escape freely across the boundaries of the diffusing region. We shall discuss each of these conclusions in order.

(i) The disk model only begins to produce an acceptable lifetime when its thickness exceeds 2 kpc. This would extend into the halo, where the magnetic field is weaker. Similar objections apply to diffusion models with confinement in spiral arms. The average lifetime of particles in a long cylinder of radius r , when they are free to escape across the curved surface, is $\tau = r^2 / 4 D$, and a spiral arm of radius 200 pc gives a lifetime of 10^4 yr .

The difficulty in resolving lifetime and anisotropy measurements with galactic diffusion theories gives support to the metagalactic origin hypothesis (Brecher & Burbidge 1972), in which the cosmic rays pervade the local supercluster. Indeed any galactic confinement theorem runs into difficulties with particles of energy greater than 10^{17} eV , whose trajectories would not be affected significantly by the galactic magnetic field. Karakula et al. (1972) examined the trajectories of high-energy cosmic rays in various models of the galactic magnetic field, and concluded that data on the arrival directions of extensive air

showers are inconsistent with the galactic origin of cosmic rays with energy greater than 10^{18} eV . A similar conclusion was drawn (Osborne et al. 1973) when irregularities in the galactic magnetic field were taken into account. The metagalactic particle spectrum should steepen at energies around 10^{20} eV due to energy losses which become important in this region, but because of the rarity of detected arrivals of these high-energy particles, it is not yet possible to say conclusively whether the spectrum does or does not steepen here.

Arguments for a metagalactic origin are less conclusive for cosmic rays with energy below 10^{17} eV . The hypothesis implies a high universal energy density in cosmic rays, and it has already been shown that these particles can be made in our own Galaxy by supernovae or pulsars. A viable galactic propagation theory which predicts a reasonable lifetime and anisotropy could remove the need of a metagalactic origin hypothesis for cosmic rays of energy below 10^{17} eV .

(ii) To counter the second objection to the diffusion hypothesis it is necessary to examine the motion of a charged particle in a magnetic field. A relativistic particle of charge Ze propagating in a uniform magnetic field spirals with a Larmor radius

$$R_L = 10^{-6} \frac{E \text{ (GeV)}}{|Z|B(\mu\text{G})} \text{ pc}$$

around the field line along which it is travelling. Its instantaneous direction of motion maintains a constant pitch angle α with the direction of the field line, so the particle's velocity along the field is $v_{\parallel} = c \cos \alpha = c \mu$. The magnetic moment

of the particle, proportional to $v_{\perp}^2 / B$, is a constant of motion. So any change in the field strength B will produce a proportional change in $\sin^2 \alpha$. An increase in field strength from B_0 to B_1 will reflect all particles having an original pitch angle α_0 such that $\sin^2 \alpha_0 > B_0 / B_1$. Jokipii and Parker (1969) concluded that, although cosmic rays may be tightly bound to the magnetic field lines, which on average run approximately parallel to the galactic plane, each individual field line will be expected to random-walk to the surface of the disk due to the turbulent nature of the galactic magnetic field. The particles reach the surface of the disk after travelling ~ 1 kpc, where the pressure of the cosmic ray gas causes a 'bubble' to inflate, which allows them to disengage from the galactic field. Lingenfelter et al. (1971) studied the process of 'compound diffusion', in which the cosmic rays execute 1-dimensional diffusion along the field lines, scattering from minor irregularities in the field, which themselves experience a 3-dimensional random walk. They concluded that reasonable values of the mean free path and step length give an overall anisotropy consistent with observations. But Allan (1972) pointed out that if cosmic rays are tightly bound to one field line, the anisotropy observed at the Earth is dependent upon conditions relevant to that field line alone, and could therefore be considerably greater than the overall value calculated by Lingenfelter et al. Jones (1971) calculated the anisotropy at points along a randomly wandering field line on to which cosmic rays are injected continuously and uniformly, but the low anisotropy observed can only occur in this model if

by chance the Earth is situated near the mid-point of its field line. Dickinson and Osborne (1974) investigated this model using a more realistic source distribution, namely point sources distributed randomly along the field line, in which the anisotropy fluctuates with time. But this model also gives a very small probability (< 0.05) for the anisotropy to be as low as is observed.

It therefore seems that the compound diffusion model alone cannot satisfy the observational conditions on the cosmic ray lifetime and anisotropy. But the assumption that cosmic rays remain tightly bound to their individual field lines needs further consideration, because irregularities in the field cause the particles to drift across the field lines. In a field which has uniform strength but which is changing in direction with a radius of curvature R , particles drift normal to the plane of the curve with a drift velocity $v_D = v_{\parallel}^2 / R \omega_c$, where $\omega_c = Z e B / m$ is the cyclotron frequency (see e. g. Morrison 1961). An additional drift $v_D = \frac{1}{2} \frac{v_{\perp}^2}{\omega_c} \frac{\nabla_{\perp} |B|}{|B|}$ is caused by a field gradient perpendicular to the field lines. For negligible current flow $\nabla \times \underline{B} \approx 0$, then $\nabla_{\perp} |B| / |B| \approx 1/R$. So the total drift velocity is $v_D = (v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2) / R \omega_c$. It is therefore likely that, when a cosmic ray particle is reflected from a magnetic mirror, it will also tend to drift across the field lines, and so will not return along the same field line as the one on which it approached the mirror. Skilling (1970) examined the motion of particles across field lines, and concluded that cosmic rays can diffuse a few thousand Larmor

radii across the field lines in their lifetime in the Galaxy. Furthermore, when a particle is reflected by a magnetic irregularity, it moves on average a Larmor radius across the field; and due to the turbulence of the field, the new field line to which the particle becomes attached will rapidly diverge away from the particle's former field line. (Skilling et al. 1974). If the scattering mean free path is L , then the field lines separate by L after travelling a distance L .

The above argument suggests that the propagation of cosmic rays in the irregular galactic magnetic field is more akin to a 3-dimensional random walk than compound diffusion. If the average field strength is $\bar{B}$, then irregularities of strength $B_{\max}$ will act as mirrors for particles whose initial pitch angles obey $\sin^2 \alpha > \bar{B} / B_{\max}$. These particles constitute a fraction $\sqrt{1 - \bar{B} / B_{\max}}$ of the total cosmic ray flux, assuming an isotropic distribution of pitch angles. Taking $\bar{B} / B_{\max} \sim \frac{1}{2}$, it follows that about 70 per cent of the particles undergo a 3-dimensional random walk between scattering centres a distance $L \sim 10 - 30$ pc apart. The remainder of the particles would be free to follow the field lines out of the disk in the absence of any other obstruction.

Our arguments have thus turned a full circle, finally concluding that galactic cosmic rays undergo a 3-dimensional random walk between locations of enhanced magnetic field strength, such as interstellar clouds. In reality, of course, the clouds will not all be of the same size and strength. Bell et al. (1974) have analysed the process of diffusion in a region of varying

cloud size and field strength. The model predicts a change in spectral index near the energy of 10^{16} eV , above which the particles' Larmor radii are greater than the average cloud size so that the scattering effect is reduced. But the predicted change in slope is significantly larger than the measured bend. The model also experiences difficulties in explaining the residence time of 3×10^6 yr and the small measured anisotropy. It therefore appears that the concept of a galactic containment mechanism for cosmic rays can only be saved by invoking statement (iii) , that the particles do not escape freely across the boundaries of the diffusing region. In fact this point is the basis of this Thesis, and the nature of the mechanism which prevents free escape, due to the onset of collective effects of the cosmic ray plasma, will be discussed in the following chapters.

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CHAPTER 3

THE ROLE OF PLASMA WAVES IN COSMIC RAY CONFINEMENT

An effective galactic cosmic ray confinement mechanism must give rise to a process which inhibits the escape of cosmic rays across the boundaries of the diffusing region. This must be so in order to reconcile the observed lifetime and anisotropy with diffusion in the disk. A 'leaky-box' model of steady-state diffusion inside partially-transmitting boundaries was proposed in Chapter 1 as the best way of obtaining various chemical abundance ratios measured in the cosmic ray flux entering our atmosphere. In this chapter we propose that the walls of the leaky box are formed by Alfvén waves in the ionized component of the interstellar gas, which can exist at certain distances above and below the central plane of the Galaxy.

Since the energy densities of the galactic magnetic field and the cosmic ray gas are similar, $\sim 1 \text{ eV cm}^{-3}$, if the field influences the propagation of the cosmic rays then it follows that the cosmic rays must exert an influence on the field. The effects of the cosmic ray gas on the field must be investigated by treating the cosmic rays as a plasma rather than a collection of individual particles. To this end, the ionized component of the interstellar medium and the cosmic ray gas are treated as collisionless plasmas, and their interaction is investigated by use of the Vlasov equation.

Lerche (1967) and Wentzel (1968) investigated the growth

rate of magnetohydrodynamic (MHD) waves in the interstellar plasma which are formed when the cosmic ray plasma streams through it. They found that MHD waves are generated in a wide range of θ (the angle between the wave vector and the magnetic field) whenever the streaming velocity of the cosmic rays along the field exceeds $\sim V_A / \cos \theta$, where $V_A = B / \sqrt{4\pi\varrho_i}$ is the Alfvén velocity, and ϱ_i is the density of the ionized component of the interstellar medium. The growth rate of these waves is greater than the rate at which the cosmic rays escape from the Galaxy, so that the cosmic rays can be effectively scattered by the waves which they themselves create (Wentzel 1969, Kulsrud & Pearce 1969). Wentzel showed that, whenever the cosmic rays are able to produce MHD waves, they will be scattered by the waves so that their streaming motion is reduced. In conditions of equilibrium, the streaming rate of the cosmic rays, as determined by the wave spectrum, will be of the correct magnitude required to form the waves. Any enhancement of the streaming rate will produce a stronger wave spectrum, which will reduce the streaming rate to the original value within a few hundred years.

The wave growth rate may be investigated as follows. Since the pressure of the ionized component of the interstellar medium is very small compared to the magnetic pressure (a factor of $\sim 10^{-5}$), the medium can be treated as a cold plasma. In the regime of low wave frequencies, where $\omega \ll$ the ion gyro-frequency $e B / M c$, MHD waves can propagate. In the cold-plasma approximation the MHD waves can be separated into three simple modes; magnetosonic

waves which propagate with velocity V_A in all directions, Alfvén waves which propagate with velocity $V_A \cos \Theta$, and sound waves which propagate with velocity $\sqrt{\frac{3P}{\rho}} \cos \Theta$, where P is the pressure of the interstellar plasma. A plasma distribution function $f_0 = n_i \delta(\underline{p}) + F(\underline{p})$ is substituted into the relativistic Vlasov equation, where $F(\underline{p})$ represents the cosmic ray momentum distribution function and $n_i \delta(\underline{p})$ represents the background plasma at zero temperature, of number density n_i .

Solving for first-order wave-like perturbations of the form

$e^{i(\underline{k} \cdot \underline{r} - \omega t)}$ yields a relation for the complex wave frequency

$\omega(k) = \omega_k + i\Gamma_k$. The real frequencies of the magnetosonic and Alfvén modes of wave-number k are (Kulsrud & Pearce 1969)

$$\omega_k = \begin{cases} k V_A \\ k V_A \cos \Theta \end{cases}$$

and their growth rates are

$$\Gamma_k = \sum q^2 \left(\frac{V_A}{c}\right)^2 \sum_{n=-\infty}^{\infty} \int d^3p \, U_{\perp}^2 \delta(\omega_k - k_{\parallel} U_{\parallel} - n\Omega) \left\{ \frac{J_n'^2(x)}{n^2 J_n^2(x)} \right\} \left[\frac{\partial F}{\partial E} + \frac{k_{\parallel}}{\omega_k p} \frac{\partial F}{\partial p} \right]$$

The first sum is over all cosmic ray species of charge q . The second sum is over the Bessel functions J_n , the upper term applying to magnetosonic waves and the lower one to Alfvén waves. E is the cosmic ray energy, Ω is the cosmic ray gyro-frequency, $U_{\parallel}$ and $U_{\perp}$ are their velocities parallel to and perpendicular to the magnetic field, and $x = k_{\perp} U_{\perp} / \Omega$.

The above result is valid only when $V_A \ll c$, which is true in the interstellar medium, and when $\Gamma_k \ll \omega_k$.

The significance of the Dirac delta term $\delta(\omega_k - k_{\parallel} U_{\parallel} - n\Omega)$ is that the wave growth is a resonant interaction. When $V_A \ll c$, ω_k can be neglected and the resonance condition is $k_{\parallel} U_{\parallel} + n\Omega = 0$.

Only the cases $n = \pm 1, 0$ need be considered (Tademaru 1969).

The first order resonance occurs for waves propagating parallel to the field with wavelength

$$\lambda = 2\pi v_{\parallel} / \Omega$$

$$\text{i.e. } \lambda = 7 \times 10^{-6} \mu E(G) / |z| B(\mu G) \text{ pc},$$

which is of the order of the cosmic ray gyro-radius. This is also the condition for the resonant scattering of cosmic rays by the waves, since when a particle performs one gyration around the field while travelling a distance λ , the Lorentz force due to the wave has a steady component $\underline{v}_{\perp} \times \underline{B}_1$ in the direction of $\underline{B}$, where $\underline{B}_1$ is the wave amplitude, and changes the pitch angle $\propto$ systematically with time. The energy-dependence of the resonant growth and scattering condition necessitates that each energy range of cosmic rays independently creates its own waves to modify its own streaming motion.

The growth rate of Alfvén waves is only appreciable for $\theta \lesssim 30^\circ$ (Tademaru 1969), whereas the magnetosonic mode can be formed at all angles to the field. But in the cold plasma approximation, the possibility has been neglected that collisionless (Landau) damping might occur. In fact the magnetosonic mode is heavily damped when $v_A \lesssim v_e$ the electron thermal velocity. Since this is so in the interstellar medium ($10^6 \text{ cm s}^{-1} < 3 \times 10^7 \text{ cm s}^{-1}$), it seems that self-generated magnetosonic waves cannot play a part in confining cosmic rays to the Galaxy. The collisionless damping of the Alfvén mode however is negligible. So in the absence of any other damping process, cosmic rays would be constrained to stream at about the Alfvén velocity, which would

explain their low anisotropy.

However, Alfvén waves do experience other forms of damping. The dominant linear damping process in interstellar space is due to collisions between the charged particles moving with the waves and the neutral atoms of the interstellar medium (Kulsrud & Pearce 1969). When the damping rate Γ_D is much less than the wave frequency ω , the ion-neutral collision damping rate is

$$\Gamma_D = \begin{cases} \nu_{in}/2 & \text{if } \omega \gg \nu_{in} \\ \omega^2/2\nu_{in} & \text{if } \omega \ll \nu_{in} n_i/n_H \end{cases}$$

where $\nu_{in} = n_H \langle \sigma v \rangle$ is the ion-neutral collision frequency, n_H is the neutral particle number density, n_i is the ion and electron number densities, and $\langle \sigma v \rangle$ is the ion-neutral interaction cross section averaged over the thermal velocities.

In order that the waves may grow, the linear damping rate must be less than the growth rate, which for a power-law cosmic ray distribution function $F(p) \propto p^{-\gamma}$ may be approximated to

$$\Gamma_G = \frac{\pi}{4} \frac{\gamma-1}{\gamma} \Omega_H \frac{N(>E)}{n_i} \left(\frac{V_s}{V_A} - 1 \right)$$

where $\Omega_H = e B / M_H c$, the ion gyro-frequency of the interstellar plasma, $N(>E)$ is the integral cosmic ray density, and V_s is the cosmic ray streaming velocity. Using a density spectrum

$$N(>E) = 2.5 \times 10^{-10} E^{-1.5} \text{ cm}^{-3},$$

where E is in GeV, and an ion density $n_i = 0.025 \text{ cm}^{-3}$ in the intercloud medium of the galactic plane, we obtain

$$\Gamma_G = 1.8 \times 10^{-10} \left(\frac{V_s}{V_A} - 1 \right) E^{-1.5} \text{ s}^{-1}$$

If $n_H \sim 0.1 \text{ cm}^{-3}$ in the intercloud medium, then the damping rate Γ_D is also of the order of 10^{-10} s^{-1} , so that cosmic

rays of energy greater than a few GeV cannot generate the waves necessary to reduce their streaming rate to $\sim V_A$. This point was emphasized by Kulsrud and Cesarsky (1971), who concluded that if cosmic rays of energy greater than about 100 GeV are confined to the Galactic plane by Alfvén waves, then the waves must be generated by sources more powerful than the cosmic rays themselves.

The existence of more powerful wave sources to confine the higher-energy cosmic rays to the Galaxy is not essential, as n_i and n_H decrease with height above the Galactic plane, so that Γ_c increases and Γ_D decreases. Consequently cosmic rays of energy greater than a few GeV need only to travel a certain distance out of the plane before they encounter waves which can scatter them. This explanation was suggested by Skilling (1971). He calculated the distribution function $f(\underline{p}, \underline{x}, t)$ of cosmic rays in the presence of Alfvén waves by transforming to a frame of reference moving with the waves and expanding f in inverse powers of the particle-wave scattering frequency. He obtained the scattering frequency from the wave energy, which he determined by equating the growth rate of the waves with their damping rate. The waves which are the most important in cosmic ray confinement are linearly-polarized Alfvén waves propagating nearly parallel to the background magnetic field. The resulting expression for f is

$$\left(\frac{\partial}{\partial t} + \underline{w} \cdot \nabla \right) f = \frac{1}{j} (\nabla \cdot \underline{w}) \rho \frac{\partial f}{\partial \rho} - \frac{1}{\rho^3} \nabla \cdot \left[\left(\frac{\Gamma_D \beta_0}{4\pi^3 \mathcal{M}_H \mathcal{R}_H V_A} \right) \beta_0 \hat{n} \right]$$

where $\underline{w}$ is the wave velocity, $\beta_0 \hat{n}$ represents the background

magnetic field, and p is the cosmic ray momentum. The first term on the right hand side represents a cosmic ray distribution that convects with the waves, since their number density

$$N(p) = \int f(p) 4 \pi p^2 dp \quad \text{obeys the convection equation}$$

$$\left(\frac{\partial}{\partial t} + \underline{w} \cdot \nabla \right) N + N \nabla \cdot \underline{w} = 0$$

if the second term is disregarded. The second term describes the streaming of the cosmic rays through the wave frame. The expression is valid when $V_A \ll c$ and the cosmic ray gyro-radius R_L is much less than the scale of irregularities in the background magnetic field.

Whenever $\Gamma_D > \Gamma_c$ the waves cannot exist; the cosmic rays will be able to stream freely along the field between irregularities. Hence in regions of high damping (designated the 'free zone') $\hat{n} \cdot \nabla f = 0$. The onset of the waves can be localized by determining the conditions in which the particle distribution function in the wave zone, obtained from the above equation, satisfies $\hat{n} \cdot \nabla f = 0$. In the next chapter we carry out this analysis in an axially-symmetric model of the Galaxy, to determine the height at which the waves begin to grow, as a function of the rigidity of the cosmic rays which they scatter. The results are then used to predict a residence time and path length for galactic cosmic rays of given rigidity.

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CHAPTER 4

AN ENERGY-DEPENDENT CONFINEMENT MODEL

We begin with the equation for the distribution function $f(\underline{p}, \underline{x}, t)$ of cosmic ray protons in the wave zone,

$$\left(\frac{\partial}{\partial t} + \underline{\omega} \cdot \nabla \right) f = \frac{1}{3} (\nabla \cdot \underline{\omega}) p \frac{\partial f}{\partial p} - \frac{1}{p^3} \nabla \cdot \left[\left(\frac{\Gamma_0 B_0}{4\pi^3 M_H \Omega_H V_A} \right) \beta_0 \hat{n} \right] \quad (1)$$

We use a cylindrical polar coordinate system with origin at the centre of the Galaxy. The approximation is made that the Galaxy is cylindrically-symmetric and in a steady state, so that all quantities depend only upon the distance from the centre of the Galaxy r and the height above the galactic plane z . The velocity of the waves in this system is the sum of the Alfvén velocity and the velocity of the background plasma $\underline{v}_0$,

$$\underline{\omega} = \underline{V}_A + \underline{v}_0$$

$\underline{v}_0$ is taken to be the (azimuthal) galactic rotation velocity, and since this does not vary in the azimuthal direction, equation (1) reduces to

$$\underline{V}_A \cdot \nabla f = \frac{1}{3} (\nabla \cdot \underline{V}_A) p \frac{\partial f}{\partial p} - \frac{1}{p^3} \nabla \cdot \left(\frac{\Gamma_0 B_0 \hat{n}}{4\pi^3 M_H \Omega_H V_A} \right) \quad (2)$$

The damping rate in the intercloud medium is proportional to the density of neutral hydrogen, $\Gamma_D \approx G n_H$

Substituting $V_A = B_0 (4\pi M_H n_i)^{-\frac{1}{2}}$ into equation (2), and remembering $\nabla \cdot \beta_0 = 0$, we obtain

$$\begin{aligned}
 \frac{\partial f}{\partial z} + \frac{B_r}{B_z} \frac{\partial f}{\partial r} = & - \frac{1}{6} p \frac{\partial f}{\partial p} \left(\frac{1}{n_i} \frac{\partial n_i}{\partial z} + \frac{B_r}{B_z} \frac{1}{n_i} \frac{\partial n_i}{\partial r} \right) \\
 & - g \frac{1}{p^3} \frac{n_i n_n}{B_0} \left[\frac{1}{n_H} \frac{\partial n_H}{\partial z} + \frac{1}{2 n_i} \frac{\partial n_i}{\partial z} - \frac{1}{B_0} \frac{\partial B_0}{\partial z} \right. \\
 & \left. + \frac{B_r}{B_z} \left(\frac{1}{n_H} \frac{\partial n_H}{\partial r} + \frac{1}{2 n_i} \frac{\partial n_i}{\partial r} - \frac{1}{B_0} \frac{\partial B_0}{\partial r} \right) \right] \quad (3)
 \end{aligned}$$

where $g = \frac{G \epsilon M_H}{\pi^2 e}$, and B_r and B_z are the r and z components of $B_0 \hat{n}$.

Let the density of the interstellar medium and the strength of the galactic magnetic field vary exponentially with height above the plane,

$$\begin{aligned}
 n_H &= n_H(r) e^{-z/z_H} \\
 n_i &= n_i(r) e^{-z/z_i} = n_i(r) e^{-x z/z_H} \\
 B_0 &= B_0(r) e^{-z/z_B} = B_0(r) e^{-w z/z_H}
 \end{aligned}$$

where $x = z_H / z_i$ and $w = z_H / z_B$. The direction of the galactic magnetic field is predominantly azimuthal (Manchester 1972) in the local vicinity. This would be the configuration of the galactic field if it originated from a metagalactic field which remained frozen into the gas as the Galaxy condensed, and was subsequently amplified by differential rotation (Piddington 1973). If the galactic field remains connected to the universal field then the cosmic rays would flow out of the Galaxy along the field lines, their rate of escape being regulated by the Alfvén waves. But Parker (1973) proposes that the galactic field is disconnected from the universal field due to turbulent

diffusion. In this case the cosmic rays would be transported up to the galactic halo where they would overwhelm the field and escape by the process which Parker (1965) suggested.

Unless $B_r \gg B_z$, we can neglect the terms involving radial gradients of B_0 , n_i and n_H . Assuming $\frac{\partial f}{\partial z} \gg \frac{\partial f}{\partial r}$, which will be justified later in this chapter, equation (3) approximates to

$$Z_H \frac{\partial f}{\partial z} - \frac{x}{6} p \frac{\partial f}{\partial p} = g(1 + \frac{x}{2} - \omega) \frac{1}{p^3} b(r) e^{-(1+x-\omega)z/Z_H} \quad (4)$$

where $b(r) = n_i(r) n_H(r) / B_0(r)$. The homogeneous equation can be solved by the method of separation of variables to yield

$$f = f_0 p^{6\alpha} e^{x\alpha z/Z_H}$$

where α is the separation constant. The particular integral is

$$- g b(r) p^{-3} e^{-(1+x-\omega)z/Z_H}$$

Hence the solution to equation (4) is

$$f(r, z, p) = f_0(r) p^{-\delta} e^{\frac{2x}{6}\delta z/Z_H} - g b(r) p^{-3} e^{-(1+x-\omega)z/Z_H} \quad (5)$$

where we have substituted δ for -6α .

We shall now apply the condition that at the boundary between the wave zone and the free zone, at $Z = Z(r, p)$, the distribution function is equal to that in the free zone. In the free zone the cosmic rays undergo 3-dimensional diffusion with mean free path $L \sim 10$ pc. We shall show later on that the variation of cosmic ray density with height is negligible in the free zone, so let the distribution function here be

$f = F(r) p^{-\gamma}$. Setting $\frac{\partial f}{\partial z} = 0$ at $Z = Z(r, p)$ in equation (5)

we obtain

$$g b(r) p^{-3} e^{-(1+x-w)z(r,p)/2H} = f_0(r) p^{-\delta} \frac{\delta x/6}{1+x-w} e^{-\frac{x}{6}\delta z(r,p)/2H} \quad (6)$$

Substituting this relation back into equation (5), we find that the distribution function at the boundary is

$$f(r, z(r,p), p) = f_0(r) \left(\frac{1+x-w - \frac{\delta x}{6}}{1+x-w} \right) p^{-\delta} \left[\frac{1+x-w}{\delta x/6} p^{\delta-3} \frac{g b(r)}{f_0(r)} \right]^{-\frac{x\delta/6}{1+x-w - \frac{\delta x}{6}}} \quad (7)$$

This is a power-law in p , which can be equated directly with the free zone distribution function $F(r) p^{-\gamma}$ to give a relation between γ and δ ,

$$\left. \begin{aligned} \gamma - \delta &= \frac{\delta(\delta-3)x}{6(1+x-w) - \delta x} \\ \text{or } \delta &= \frac{\gamma}{1 + \frac{(\gamma-3)x}{6(1+x-w)}} \end{aligned} \right\} \quad (8)$$

The boundary height $z(r,p)$ may be obtained from equation (6),

$$e^{\left\{ \left[(1+x-w) - \frac{x}{6}\delta \right] z(r,p)/2H \right\}} = \frac{1+x-w}{\delta x/6} \frac{g b(r)}{f_0(r)} p^{\delta-3} \quad (9)$$

Let us normalize the damping coefficient g so that protons of momentum p encounter a boundary in the vicinity of the Earth, at $r = 10$ pc and $z = 0$. Then equation (9) yields

$$\frac{1+x-w}{\delta x/6} g = \frac{f_0(r=10)}{b(r=10)} p^{-(\delta-3)} \quad (10)$$

and hence

$$e^{z(r,p)/2H} = \left[\frac{b(r)}{b(r=10)} \frac{f_0(r=10)}{f_0(r)} \left(\frac{p}{p_0} \right)^{\delta-3} \right]^{\frac{1}{1+x-w - \frac{x\delta}{6}}} \quad (11)$$

Equation (10) can be used in conjunction with equation (5) to obtain an expression for the cosmic ray distribution function in the wave zone,

$$f(r, z, p) = f_0(r) p^{-\delta} e^{-\frac{x\delta}{6} z/2H} \left[1 - \frac{\delta x/6}{1+x-w} \frac{f_0(r=10)}{f_0(r)} \frac{b(r)}{b(r=10)} \left(\frac{p}{p_0} \right)^{\delta-3} e^{-(1+x-w - \frac{x\delta}{6}) z/2H} \right] \quad (12)$$

This may be simplified by use of equation (11),

$$f(r, z, p) = f_0(r) p^{-\delta} e^{-\frac{\alpha\delta}{6} \frac{z}{z_H}} \left[1 - \frac{\delta\alpha/6}{1+\alpha-\omega} e^{-(1+\alpha-\omega - \frac{\alpha\delta}{6})(z - z(r, r))/z_H} \right] \quad (13)$$

This represents the distribution function of cosmic rays in the wave zone, at $z \gg z(r, r)$. The free zone distribution function will be equal to the above value at the boundary $z(r, r)$,

$$F(r) p^{-\delta} = f(r, z(r, r), p) = f_0(r) p^{-\delta} e^{-\frac{\alpha\delta}{6} z(r, r)/z_H} \left(\frac{\delta-3}{\delta-3} \right) \quad (14)$$

It is now time to review the results obtained so far and to explain their importance. To this end we present a summary of this chapter's discussion so far.

Summary.

(i) Cosmic rays scatter themselves on the Alfvén waves which they generate in the interstellar magnetic field, but those of energy above a few GeV are too few in number to generate the waves at a sufficient rate to overcome damping in the galactic plane. These waves can begin to grow at a height $z(r, r)$ above the plane, where the damping rate is smaller. Above this height the cosmic rays are strongly scattered by the waves, and are constrained to stream at about the wave speed V_A . This region is termed the 'wave zone'. Below this height, in the 'free zone', they are free to diffuse between irregularities in the magnetic field, with a length-scale $L \sim 10 - 30$ pc. The position of this boundary is energy-dependent, since the density of the cosmic rays, and hence their wave generating power, decreases with energy. The cosmic rays of higher energy can only form their waves when the damping rate is correspondingly smaller, at

greater heights above the galactic plane.

(ii) The cosmic ray proton distribution function in the wave zone is given by equation (13), correct to order V_A / c . Their density $N(r, z, p) dp$ is related to the distribution function by $N(r, z, p) dp = 4 \pi p^2 f(r, z, p) dp$. It can be seen that the differential density spectrum will have a gradient $-(\delta - 2)$ when p is sufficiently less than $p_0(r, z)$, or z is sufficiently greater than $z(r, p)$, to render the second term in the brackets of equation (13) negligible compared to unity ($p_0(r, z)$ is defined as the momentum of those protons which encounter a boundary at coordinates (r, z)). In the free zone, when $z < z(r, p)$ or $p > p_0(r, z)$, the spectral gradient is $-(\gamma - 2)$. Equation (8) shows that the spectrum in the wave zone will be flatter than that in the free zone by an amount

$$\gamma - \delta = \frac{\delta(\delta - 3)x}{6(1+x-w) - \epsilon^2}.$$

This relation has been derived independently by McIvor and Skilling (1974) for the case when $x = 1$, $w = 0$. It was noted in Chapter 1 that such a bend has been observed in the spectrum around a rigidity of about 8 GV. If we take the gradients measured by Durney et al. (1964), (Figure 3), $\gamma = 4.5$, $\delta = 4.2$, then equation (8) requires that

$$5x + 2w = 2.$$

An important constraint on the proposed model is that in the wave zone V_A must increase with z , otherwise the cosmic rays would be streaming up their own density gradient, which is

clearly absurd (Skillling 1971). Therefore this model will be valid only if $z_B > 2 z_i$, that is if $w < \frac{1}{2} x$. The constraint that $0 < w < \frac{1}{2} x$ then requires that

$$1/3 < x \leq 2/5$$

$$0 < w < 1/6 .$$

Observations of pulsar dispersion measures and 21-cm hydrogen emission suggest that z_i may be greater than z_H by a factor of 2 or 3 (Bridle & Venugopal 1969), and Falgarone and Lequeux (1973) state that this factor is at least 2. The above condition on x is not in conflict with the observational inferences that $x \leq \frac{1}{2}$. If the neutral particle scale height is 130 pc, then the condition on w requires that $z_B > 780$ pc. In the absence of any clear knowledge about the z -distribution of the galactic magnetic field, this is difficult to verify or dispute. Radio observations suggest a region of enhanced magnetic field strength in the form of a disk of thickness 750 pc and radius ~ 8 kpc (Piddington 1970), and that the Sun lies outside this region in one of lower field strength. This outer region, which Piddington calls the 'halo region', has a radius of ~ 15 kpc and a field strength of $2 - 3 \mu G$. Ilovaisky and Lequeux (1972) suggest the thickness of this region is ~ 2 kpc. In the light of such data, it can be stated that observations do not conflict with the assumption that the scale height of the galactic magnetic field in the vicinity of the Sun is greater than about 800 pc.

The results shown in Figure 2 display a similar behaviour of the spectrum around a rigidity of 8 GV, although the change

in spectral index is much greater. Their values of $\gamma = 4.64$ and $\delta = 3.35$ require that

$$0.825 < x \leq 1.4$$

$$0 \leq w < 0.41$$

which conflict with the observations that $x < 0.5$. However, Figure 2 is a combination of two sets of data obtained by different groups. The values above 50 GeV/nucleon were obtained by Ryan et al. (1972), while Ormes and Webber (1965) supplied the values below 10 GeV/nucleon from measurements taken on flights below the top of the atmosphere. The latter measurements were subject to a large error in energy calibration. Closer examination of their results shows their values to be consistent with a proton spectral gradient of -2.4 in the energy range 3 - 8 GeV which does not vary with solar conditions.

(iii) The presence of a bend in the proton spectrum at 8 GeV suggests that protons above this energy experience a free zone in the region $r = 10$ kpc, $z = 0$. Using this assumption to normalize the damping rate in equation (10), the height of the boundary can be obtained from equation (11),

$$e^{\gamma(r,r)/2H} = \left[\frac{b(r)}{b(r=10)} \frac{f_0(r=10)}{f_0(r)} \left(\frac{p_{\text{G2V/C}}}{\delta} \right)^{\delta-3} \right]^{\frac{1}{1+x-w-x\delta/6}} \quad (15)$$

A negative value of $\gamma(r,r)$ implies that waves which resonate with protons of momentum p exist in the galactic plane at that particular value of r . Using equation (8) to simplify the exponent, equation (15) may be written

$$e^{z(r,r)/z_H} = \left(\frac{p}{p(r,z=0)} \right)^{\frac{6(\gamma-\delta)}{x\delta}} \quad (16)$$

where $p(r,z=0) = p \left[\frac{f_0(r)}{f_0(r=10)} \frac{b(r=10)}{b(r)} \right]^{\frac{1}{\delta-3}} \quad G=V/c$

$p(r,z=0)$ is the maximum momentum of the protons which encounter waves in the galactic plane. In general,

$$p(r,z) = p(r,z=0) e^{\frac{x\delta}{6(\gamma-\delta)} \frac{z}{z_H}} \quad (17)$$

(iv) The cosmic ray density in the free zone is obtained from equation (14), eliminating $z(r,r)$ by use of equation (16),

$$N(r,z,p) = N_0(r) \left(\frac{p(r,z=0)}{p} \right)^{\gamma-\delta} p^{-(\gamma-2)} \quad (18)$$

when $z \leq z(r,r)$, $p \geq p(r,z)$,

where $N_0(r=10) p^{-(\gamma-2)}$ is the density at the Sun. We choose

$$N_0(r=10) = 3.8 \times 10^{-10} \text{ cm}^{-3}$$

and $\gamma = 4.5$

which will produce a flux of 3×10^{-3} particles $\text{cm}^{-2} \text{ s}^{-1} \text{ ster}^{-1} \text{ GeV/c}^{-1}$ at $p = 10 \text{ GeV/c}$.

In a similar way, the cosmic ray density in the wave zone may be derived from equation (13),

$$N(r,z,p) = \frac{N_0(r)}{p^{\gamma-6}} \left(\frac{\delta-3}{\delta-3} \right) p^{-(\delta-2)} e^{-\frac{x\delta}{6} \frac{z}{z_H}} \left[1 - \frac{\delta-5}{\delta-3} \left(\frac{p}{p(r,z)} \right)^{\delta-3} \right] \quad (19)$$

when $z \geq z(r,r)$, $p \leq p(r,z)$.

It is evident from equation (19) that the cosmic ray density in the wave zone decreases exponentially with height, with a scale height of $6 z_H / x\delta \sim 500 \text{ pc}$. As this is small compared to distances over which the density changes by a factor e in the galactic plane, we are justified in neglecting the radial

density gradients as we did at the beginning of this chapter.

Figure 12 shows schematically the behaviour of the cosmic ray spectrum at different heights above the galactic plane. For purposes of clarity the difference between γ and δ has been greatly exaggerated. In the galactic plane, the spectrum bends at momentum $\mathcal{P}(r, z=0)$, which is given by equation (16) and normalized so that $\mathcal{P}(r=10, z=0) = 8 \text{ GeV}/c$. At a height z above the plane, the bend occurs at a higher momentum $\mathcal{P}(r, z)$ which is related to $\mathcal{P}(r, z=0)$ by equation (17). The height at which cosmic ray protons of momentum p first encounter waves, at $\mathcal{Z}(r, p)$, is given by equation (16). In the next section we use this relation to determine the leakage rate of cosmic rays from the Galaxy.

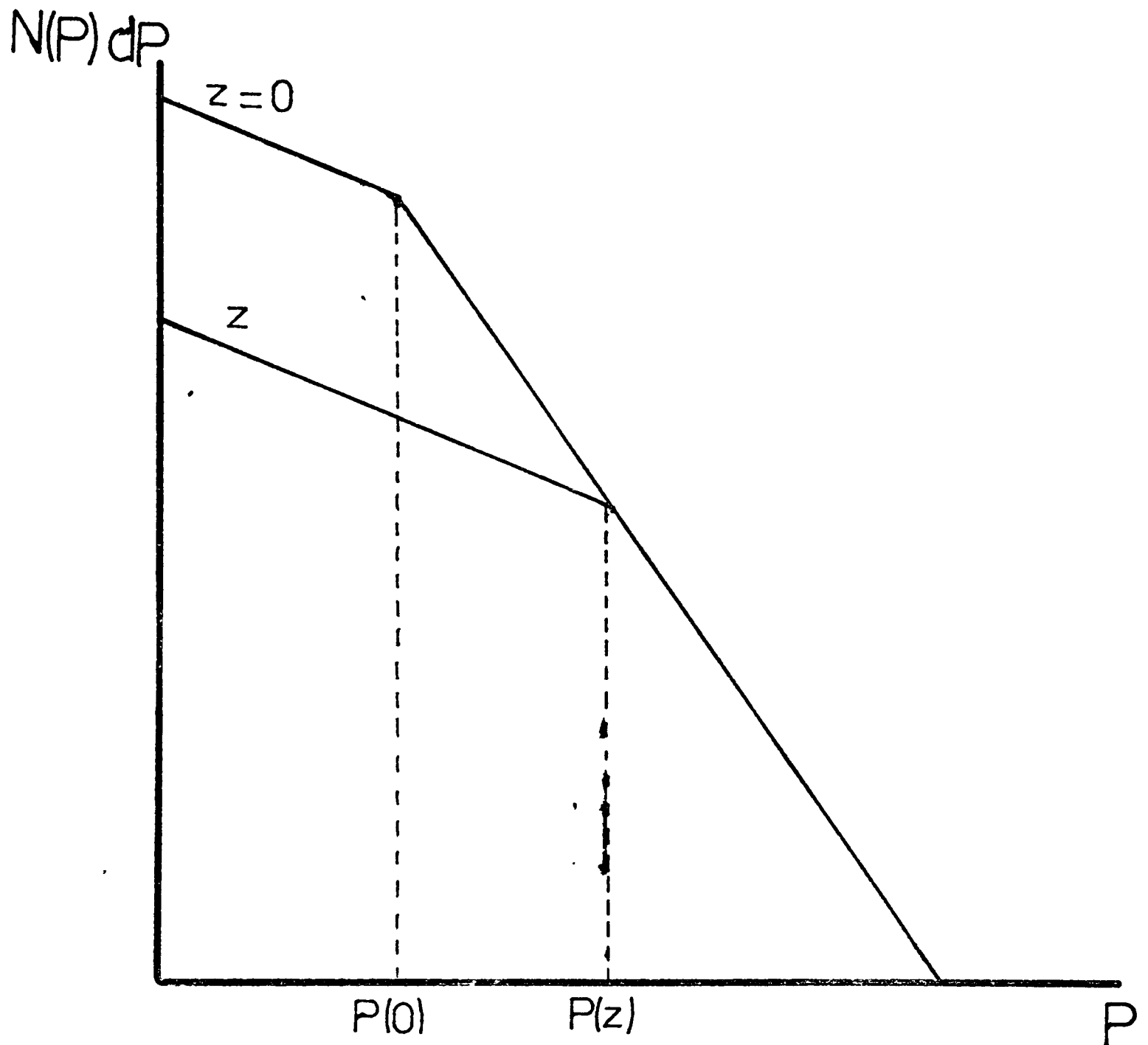


FIGURE 12. A schematic representation of the behaviour of the cosmic ray spectrum at different heights above the galactic plane. For purposes of clarity, the difference between the spectral indices in the wave zone and the free zone has been exaggerated.

2. The Leakage Mechanism.

At the boundary of the wave zone, the cosmic rays stream through the waves with a velocity of $V_A / 3$ (Skilling 1971), so that they will be isotropic in a frame moving along the field with velocity $4/3 V_A$. An observer at rest on the boundary would therefore see them stream with velocity $4/3 V_A \approx 2 V_A$. Due to the decrease in n_i with height above the galactic plane, the Alfvén velocity depends upon $\mathcal{J}(r, p)$. Since protons are the dominant species of cosmic ray, it is their flux which determines the position of the boundary. But all particles of equal rigidity resonate with the same waves, so particles of other cosmic ray species will encounter the same boundary that protons of corresponding rigidity encounter. We shall therefore generalize the above treatment to apply to all species of cosmic ray by replacing momentum p with rigidity R .

The leakage rate of cosmic rays across the boundary is

$$\phi = 2 V_z(r, R) N(r, R) \text{ particles cm}^{-2} \text{ s}^{-1} \quad (20)$$

where V_z is the component of the Alfvén velocity normal to the boundary at that position, and N is the cosmic ray density in the free zone. Each particle will therefore have a probability $\sim V_A / c$ of escaping from the free zone at every encounter with the boundary. In the free zone the cosmic rays diffuse with a mean free path $L \sim 10 - 30$ pc. In their lifetime τ they will have diffused a distance $(v \tau L)^{1/2}$ in the galactic plane, where v is their average velocity along the field lines, $\frac{1}{2} c$. With $\tau = 3 \times 10^6$ yr, $L \sim 30$ pc, this distance is about 3 kpc. So if we can assume that the boundary height

does not vary appreciably over an area of $\pi(3 \text{ kpc})^2$ centered at the Sun, then it is possible to determine the lifetime of the cosmic rays which we detect from knowledge of $\mathcal{J}(r=10, R)$ obtained from equation (16). Before we do so, however, we shall discuss the validity of our assumption that the cosmic ray density in the free zone does not vary appreciably with z .

3. Cosmic Ray Density in the Free Zone.

If we assume that conditions in the galactic plane are uniform within ~ 3 kpc of the Sun, the cosmic rays diffuse from their sources in the z -direction until they reach the boundary at height z . There they are transported out of the diffusing region with a net velocity of $2 V_z$ (Figure 13). In a steady state the density in the free zone obeys the diffusion equation when $z \gg L$,

$$\frac{d^2 N}{dz^2} = - \frac{S}{D} \quad (21)$$

S represents the source density, which is taken to vary exponentially with height,

$$S = S_0 e^{-z/z_s}$$

The solution to equation (21) is obtained subject to the boundary conditions

$$(i) \quad \text{at } z = 0, \quad N = N_0$$

$$(ii) \quad \text{at } z = z, \quad -D \frac{dN}{dz} = 2 V_z N$$

The source coefficient is eliminated by equating the loss rate per unit area across the boundary to $\int_0^z S dz$.

The result is

$$\frac{N(z)}{N_0} = 1 - \frac{z/z_s - (1 - e^{-z/z_s})}{z/z_s - (1 - \frac{D}{2V_z z_s})(1 - e^{-z/z_s})}$$

It is evident that the approximation $N(z) \approx N_0$ is valid if $\frac{2 V_z z}{D} \ll 1$. Since $z \gg L$, this condition is more stringent than $V_z \ll c$.

On the assumption that B_z does not vary with z , the z -component of the Alfvén velocity varies as

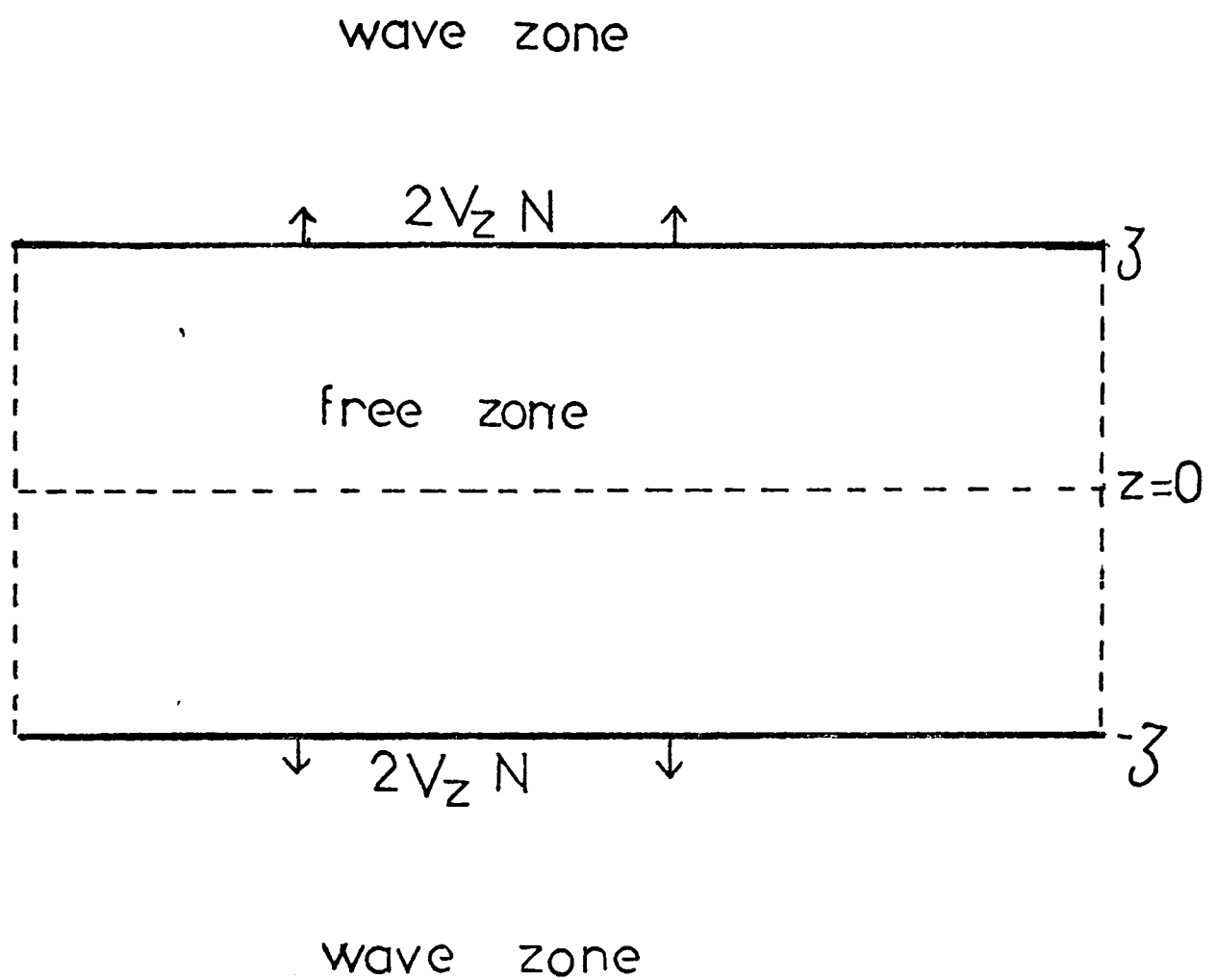


FIGURE 13. Assuming uniform conditions in the galactic plane within 3 kpc of the Sun, cosmic rays diffuse in the free zone out towards the boundary at height $\mathfrak{z}$. There they are transported into the wave zone with a streaming velocity $2 V_z$.

$$V_z = V_0 e^{\frac{1}{2} z/z_H}$$

where $V_0 = B_z / (4 \pi M_H n_i)^{\frac{1}{2}}$ is its value at $z = 0$.

Taking $B_z = 10^{-6}$ G and $n_i = 0.025 \text{ cm}^{-3}$, $V_0 = 1.4 \times 10^6 \text{ cm s}^{-1}$.

By application of equation (16) we find that the approximation

is invalid when
$$\left(\frac{R_{cv}}{\rho} \right)^{\frac{3(\gamma-1)}{\epsilon}} \gtrsim \frac{\alpha \delta}{6(\gamma-1)} \frac{10^4}{5.6 (n(R/\rho))} \frac{L}{z_H}$$

As $L / z_H \sim 10^{-1}$, it is evident that diffusion effects are unimportant up to a rigidity of $\sim 10^8$ GV, corresponding to $\beta / z_H \sim 20$. But this model ceases to be valid at rigidities where the particle gyro-radius approaches the length scale of the magnetic field, at $R \sim 10^6$ GV.

In reality, of course, the validity of this model is limited to lower rigidities by our lack of knowledge of galactic structure at large heights above the plane. At an undefined value of z the disk merges into the halo, where $n_i \sim 10^{-3} \text{ cm}^{-3}$. In our model this value is reached at $\beta / z_H \approx 3.2$, corresponding to a rigidity of $R \sim 1.5 \times 10^4$ GV. We take this value as the high-rigidity limit of validity. The low-rigidity limit is ~ 3 GV, where the galactic spectrum is thought to begin to depart significantly from a power-law, and the relativistic approximations cease to be applicable.

Having justified our method of approach for a certain range of rigidities, in the next section we shall apply the model to calculate the average residence time and path length of cosmic rays in the free zone. Our range of validity fortunately coincides with the region of interest, where observations have implied a path length which decreases with increasing rigidity.

4. Residence Time and Path Length.

As long as $\mathcal{J}(r, R)$ does not vary appreciably over a distance of ~ 3 kpc in the vicinity of the Sun, the residence time of cosmic rays in the free zone in our part of the Galaxy may be determined from their rate of escape across the boundary,

$$-\frac{dN}{dt} = \frac{2V_z N}{\mathcal{J}(r=10, R)}$$

which gives a lifetime of

$$\tau(R) = \mathcal{J}(r=10, R) / 2V_z(R) \quad , \quad (23)$$

On application of equation (16), we find

$$\tau(R) = 5 \times 10^6 \ln\left(\frac{R_{cr}}{\rho}\right) \left(\frac{R_{cr}}{\rho}\right)^{-0.214} \text{ yr} \quad (24)$$

This expression becomes invalid when the variations in $\mathcal{J}(r, R)$ within 3 kpc of the Sun are of the order of $\mathcal{J}(r=10, R)$, as R approaches 8 GV. We must determine τ for this rigidity by tracing the boundary which passes through the Sun. For this purpose we take $n_H(r)$ and $n_i(r)$ as the surface density of atomic hydrogen and the density of giant HII regions respectively, from Lequeux (1973) (Figure 14a). We normalize these values at $r = 10$ kpc to the accepted densities in the local inter-cloud medium, $n_H = 0.16 \text{ cm}^{-3}$ and $n_i = 0.025 \text{ cm}^{-3}$, and we take the cosmic ray density to be proportional to the distribution obtained from diffuse γ -radiation measurements shown in Figure 4. The strength of the galactic magnetic field is assumed to vary from $3 \mu\text{G}$ at $r = 10$ to a value of $7 \mu\text{G}$ in the region $r < 7$ kpc. The resulting boundary heights for selected rigidities, obtained from equation (16), are shown in Figure 14b.

The boundary for $R = 8$ GV, which passes through $r = 10$, $z = 0$, cuts the plane again at $r = 12$ kpc. This is due to

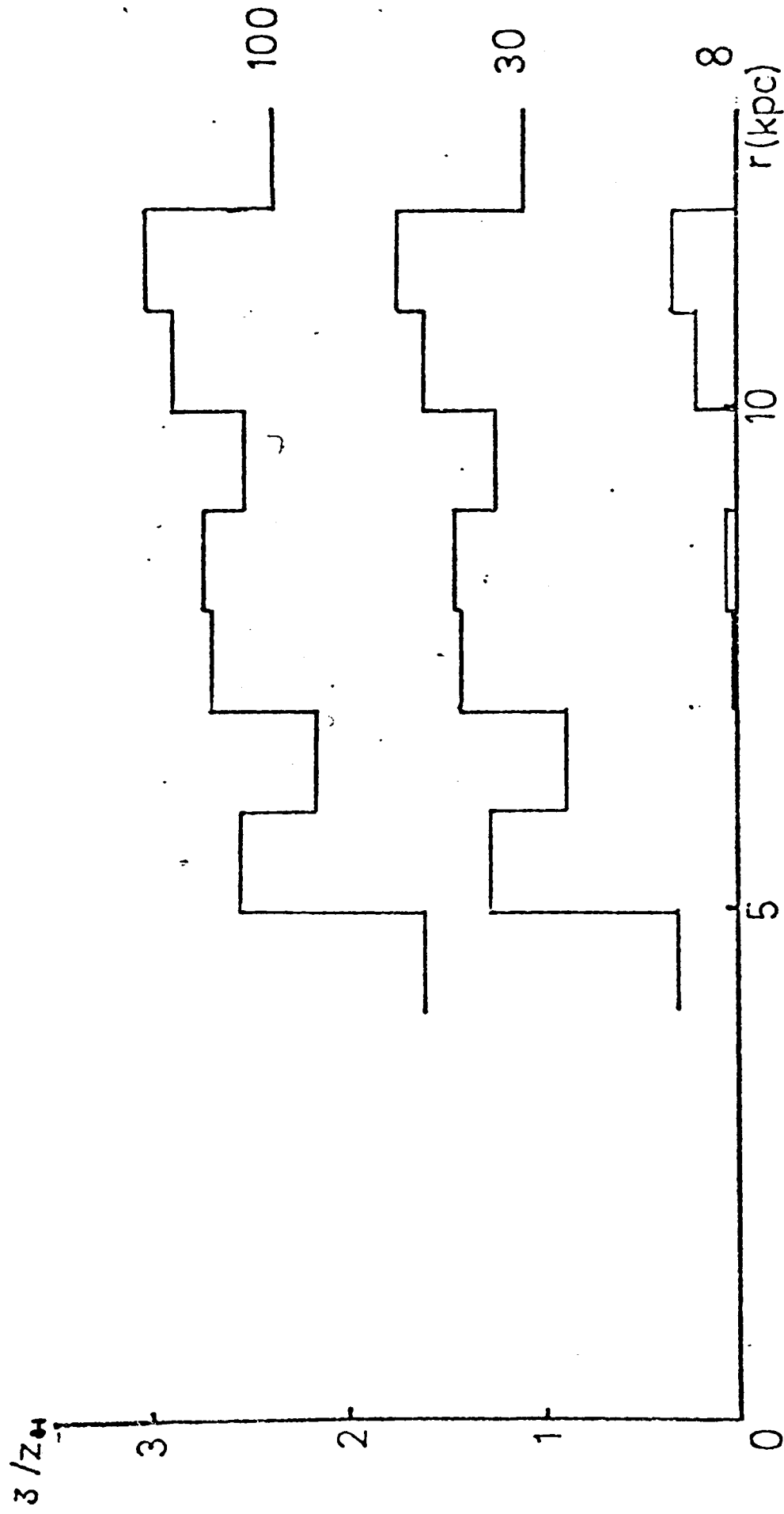


FIGURE 14 b. The location of wave zone boundaries for particles of rigidity 8 GV , 30 GV , and 100 GV . The 8 GV boundary passes by the Sun.

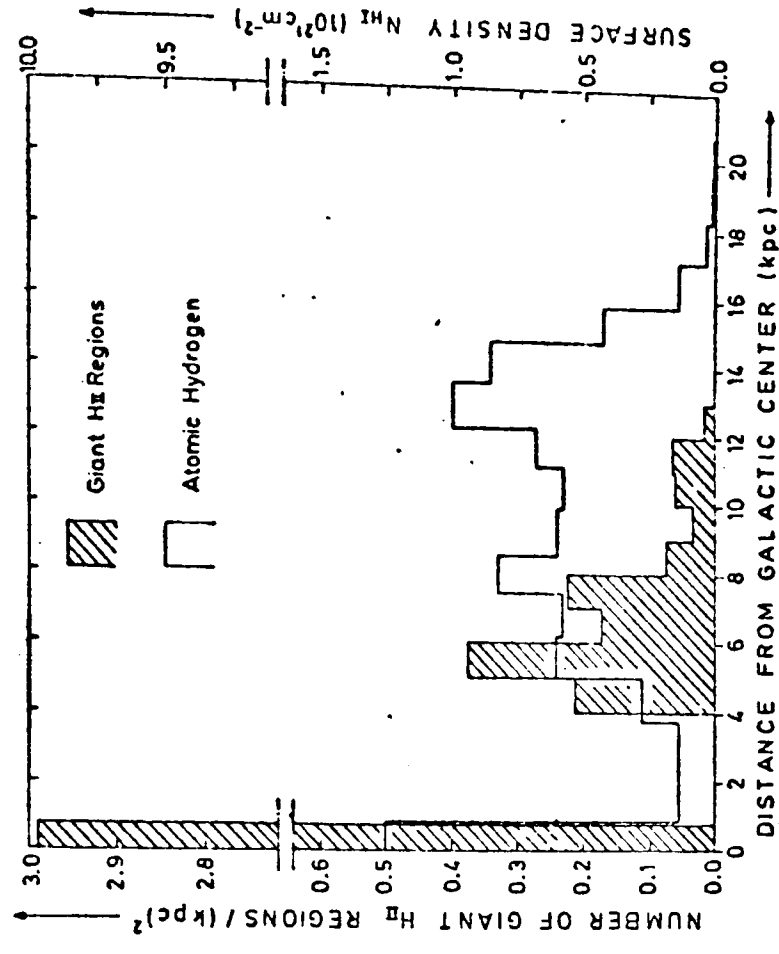


FIGURE 14 a. The distribution of atomic and ionized hydrogen in the Galaxy (from Lequeux 1973).

the supposed reduction in n_i at that location, which increases the growth rate of the waves, allowing them to form in the plane again. The average lifetime of cosmic rays inside this region is 1.1×10^6 yr . The overall variation of the average cosmic ray residence time with rigidity obtained from the above analysis is displayed as curve (i) in Figure 15 a .

It can be seen from Figure 14 that the cosmic rays which are responsible for the background γ -radiation, those with $R < 10$ GV , experience a wave zone in the galactic plane at all values of radius r . As a result, they are constrained to convect away from their sources at the Alfvén velocity. For a given rate of production by the sources, the cosmic ray density will be higher in regions of smaller Alfvén velocity. This could be the reason for the observed correlation of cosmic ray density with ionized hydrogen concentration mentioned in Chapter 1 (Puget & Stecker 1974). In regions of higher HII concentration the Alfvén velocity will be smaller, so that the density of the cosmic rays as they convect away from their sources will be higher. This correlation will, of course, be affected by variations in magnetic field strength and source density, but it is unlikely that an increase in source density alone could account for the substantial increase in cosmic ray density at $r = 5$ kpc .

The cosmic ray path length in the free zone is obtained by multiplying the residence time with the average density of interstellar matter in the free zone,

$$l = \tau \bar{c} \bar{\rho}$$

As the free zone becomes larger at higher rigidities, the average

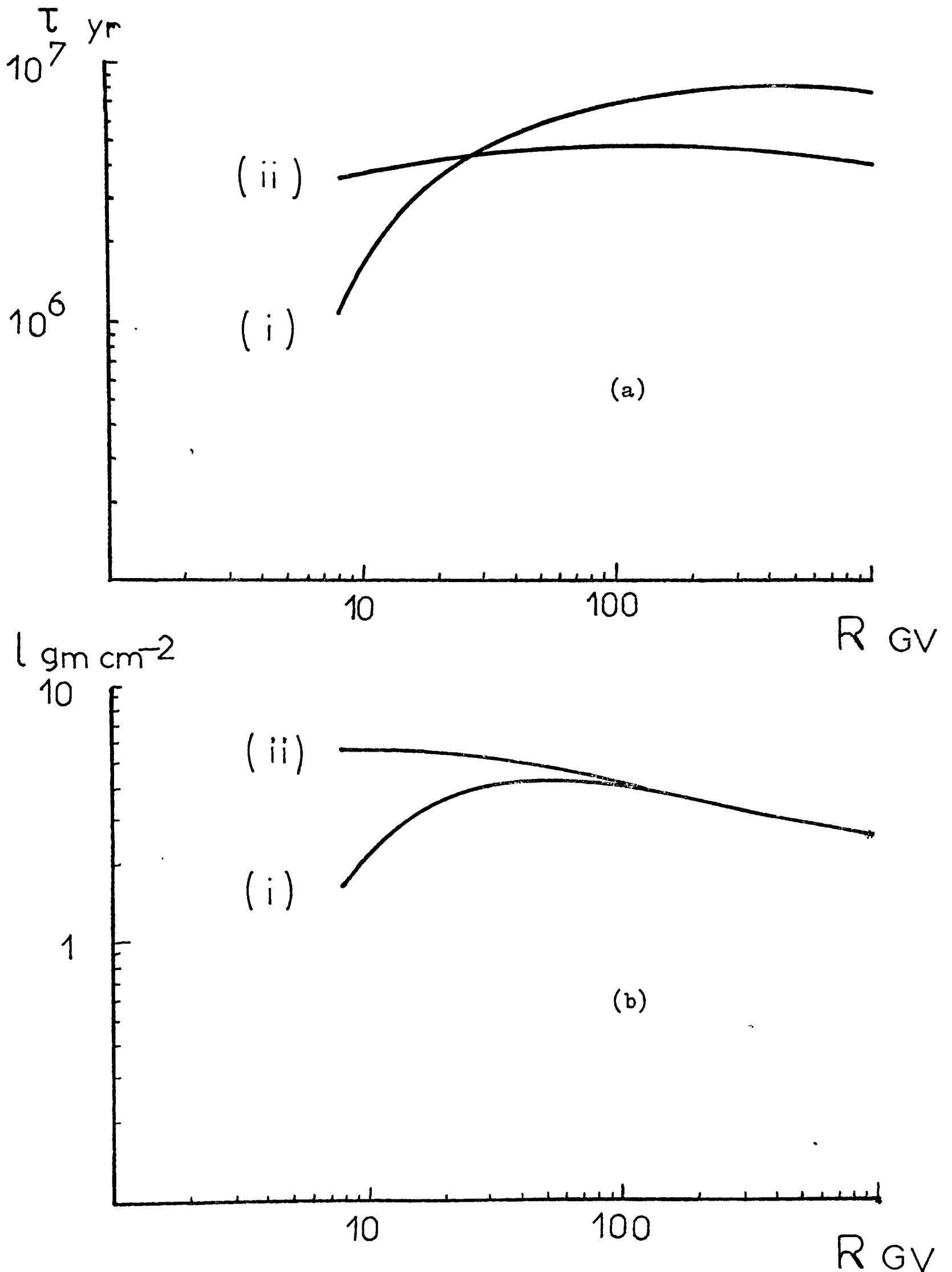


FIGURE 15. (a) the residence time and (b) the path length of cosmic rays in the free zone, obtained from the previous analysis. Curve (i) assumes an exponential variation of density with height, while curve (ii) assumes a Gaussian one.

density of interstellar matter it encloses becomes smaller. This can explain the observed decrease in path length as the rigidity increases above ~ 30 GV. Assuming the average density of the interstellar medium to vary exponentially with a scale height of 130 pc, with a value of 1 atom cm^{-3} at $z = 0$, the path length can be expressed as

$$l(R) = 7.2 \left[1 - \left(\frac{R_{cv}}{\rho} \right)^{-1.07} \right] \left(\frac{R_{cv}}{\rho} \right)^{-0.214} \text{ gm cm}^{-2}, \quad (25)$$

when $R \gtrsim 30$ GV.

At $R = 8$ GV, the path length may be determined directly from the lifetime, $l = 1.7 \text{ gm cm}^{-2}$. $l(R)$ is displayed as curve (i) in Figure 15 b.

$l(R)$ and $\tau(R)$ are directly proportional to z_H and inversely proportional to B_z . As these two quantities are not known accurately, the lifetime and path length may be multiplied by a numerical factor (probably of order unity) to normalize them to the observations. The shape of these functions offers a crucial test to the theory, not their absolute values.

The lifetime and path length resulting from low boundary heights, such as for $R = 8$ GV where z / z_H averaged over the free zone ≈ 0.3 , are strongly influenced by uncertainties about local galactic structure. If we allow for the possibility that the Sun is situated 10 pc above the galactic plane, then another 10 pc must be added to z , increasing $\tau(8)$ and $l(8)$ by about 30 per cent. Furthermore, an appreciable difference in this region of low rigidity is obtained from applying a Gaussian z -distribution in the place of the exponential

one. As long as the z -gradients are larger than the radial gradients, the results of the previous analysis of boundary height carry over on replacing z/z_H by $(z/z_G)^2$, where z_G is the Gaussian scale height which produces the same total amount of matter as that of the exponential distribution, that is $z_G = 2 z_H / \sqrt{\pi} \sim 150$ pc . At a height of 10 pc above the plane, $n / \frac{\delta n}{\delta z} \sim 2$ kpc . So as long as conditions do not vary appreciably over 2 kpc in the galactic plane, and assuming the Sun is located 10 pc above the plane, the boundary height will be given by

$$\left(\frac{z}{z_G}\right)^2 = \left(\frac{10}{z_G}\right)^2 + \frac{6(\gamma-1)}{x\delta} \ln\left(\frac{R_{GW}}{\rho}\right) \quad (26)$$

when $z \gg 10$ pc , where $x = (z_G/z_i)^2$.

The resulting residence time and path length are represented as curve (ii) in Figure 15 .

Above $R \sim 30$ GV , $l(R)$ can be represented as a power-law $l(R) \propto R^{-0.214}$. This is too shallow to be fitted to the results of Cesarsky and Audouze (1974) (Figure 8). But we have in fact neglected another important damping mechanism, which we shall discuss in the next section.

5. The Non-linear Damping of Alfven Waves.

Up to now, we have assumed that the dominant damping mechanism of Alfvén waves is the process of ion-neutral collisions, which decreases in proportion to the neutral particle density, that is exponentially with height. If we allow for the effect on this damping rate of an increase in kinetic temperature, the path length still does not fall off with rigidity as rapidly as observations require. Assuming an increase from 10^4 °K to 10^6 °K between the disk and the halo, the damping coefficient g in equation (3) will increase by an order of magnitude, so that from equations (10) and (11) we should expect $e^{3/2 h}$ at large R to be a factor $10^{\frac{6(r-\delta)}{8(1-\beta)^2}} \approx 8$ larger than as previously calculated. Consequently the path length, which varies as $e^{-\frac{x}{2} \frac{1}{2h}}$ at large R , would decrease by a factor 1.5. This plainly does not increase the fall-off in path length with rigidity enough to fit the observations.

The answer lies in the fact that it is wrong to assume that ion-neutral damping remains dominant when n_H becomes small. From arguments supplied in the appendix of Kulsrud & Pearce (1969), it is evident that ion-neutral damping remains dominant over viscosity damping and resistive damping, since the strength of the latter two processes decreases in proportion to the square of the wave-number, and at large heights above the galactic plane it is the long-wavelength waves which influence the confinement of high-rigidity cosmic rays. In the plane, the viscous and resistive damping rates are several orders of magnitude less than the ion-neutral damping rate.

The Alfvén wave energy, however, can also be degraded by non-linear wave-wave interactions. Wentzel (1969) considered the transformation of an Alfvén wave of wave-number k into two other modes of wave-numbers k' and k'' . The selection rules, analagous to the conservation of energy and momentum, require that

$$\begin{aligned}\omega_k &= \omega_{k'} + \omega_{k''} \\ \underline{k} &= \underline{k'} + \underline{k''}\end{aligned}$$

and consequently

$$\begin{aligned}k_z &= \pm k'_z + \omega_{k''}/V_A \\ k_z &= k'_z + k''_z\end{aligned}$$

If we take the positive sign for k'_z and add, the selection rules are satisfied if k'' is an Alfvén wave. But this 3-Alfvén wave transition has a vanishing coupling coefficient, since the Alfvén wave dispersion relation is an exact solution of the MHD equations. If we take the minus sign and subtract, the selection rules are satisfied if $\omega_{k''} < k''_z V_A$, which implies that k'' must be a sound wave, which has a speed $\sim 3 \times 10^{-3} V_A$ in the interstellar medium. A forward-travelling Alfvén wave can therefore transform into a backward-travelling Alfvén wave and a sound wave. The sound wave is Landau-damped, while the backward-travelling Alfvén wave will interact with another Alfvén wave k travelling forwards. The result of this interaction must be a wave k'' which obeys

$$\begin{aligned}\omega_k + \omega_{k'} &= \omega_{k''} \\ \underline{k} + \underline{k'} &= \underline{k''}\end{aligned}$$

and consequently

$$\begin{aligned}k_z - k'_z &= \omega_{k''}/V_A \\ k_z + k'_z &= k''_z\end{aligned}$$

After subtracting, it can be seen that these selection rules require $\omega_{tr}'' > V_A k_z''$, which is satisfied by an off-axis magnetosonic wave. The non-linear decay of Alfvén waves therefore takes place in two stages: (i) the wave decays into a sound wave and a backward-travelling Alfvén wave, (ii) the backward-travelling Alfvén wave combines with a forward-travelling one to produce a magnetosonic wave. The sound wave and the magnetosonic wave are subsequently Landau-damped.

Wentzel (1969) calculated the non-linear damping rate of Alfvén waves. After correcting for a factor $\frac{1}{2}$ error in the transition rate, the damping rate is found to be

$$\Gamma_{w-w} = 16 \pi (m-1) \frac{k^2 M_k}{\rho_i V_A} \quad (27)$$

where M_k is the wave energy, proportional to k^{-m} .

Since Γ_{w-w} depends upon the wave energy, the method we have adopted so far to obtain the boundary position is unable to include this form of damping. Skilling's approach, namely that either (i) waves exist and cosmic rays stream at V_A , or (ii) the waves are damped out and the cosmic rays are free to stream at c , does not require knowledge of the wave amplitude. This approach is criticized by Wentzel, who states that Skilling's expansion of the scattering equation in inverse powers of the scattering frequency ignores the regime of weak scattering. In order to accommodate the effects of non-linear damping we shall adopt the approach of Wentzel, as outlined in his paper of 1969 and reviewed in his article of 1974. But before proceeding in this way, we shall first show that these two methods of approach are equivalent in that they both yield the same results in the absence of non-linear damping.

6. Wentzel's Approach ; a Comparison with Skilling's.

From the method outlined in Chapter 3 , Wentzel obtained a relation between the streaming velocity of cosmic rays V_s and the growth rate of the Alfvén waves which they excite,

$$\Gamma_G = \frac{1}{M_H} \frac{\partial M_H}{\partial t} = \frac{e}{e_i c} N(>p) (V_s - \frac{2}{3} V_A) \quad (28)$$

As before, we shall call the damping rate due to ion-neutral collisions $\Gamma_D = G n_H$.

Letting $n_i = 0.025 e^{-xz/z_H}$

$$n_H = 0.16 e^{-z/z_H}$$

$$B_z = 10^{-6} G$$

$$N(>p) = 2.5 \times 10^{-10} p^{-1.5},$$

the streaming speed in the z-direction is obtained by equating Γ_D and Γ_G ,

$$\frac{V_s}{c} = 7 \times 10^{-5} e^{\frac{x}{2} z/z_H} + G 2.73 \times 10^5 p^{1.5} e^{-(1+x/2)z/z_H} \quad (29)$$

Clearly, the streaming speed out of the galactic plane is governed by the minimum streaming speed along the field lines. Calling the height at which this occurs $z(r)$, the waves cannot exist where $z < z(r)$, otherwise the cosmic rays would be streaming up their own density gradient. Consequently the region $z < z(r)$ corresponds to the free zone in Skilling's approach. Minimizing equation (29),

we obtain
$$e^{z/z_H} = \left(\frac{p}{p_0} \right)^{\frac{1.5}{1+x}} \quad (30)$$

where $p_0 = \left[\frac{x/2}{1+x/2} \frac{7 \times 10^{-5}}{2.73 \times 10^5} \frac{1}{G} \right]^{\frac{1}{1.5}}$ is the momentum of the

cosmic ray protons which experience their minimum streaming

velocity at $z = 0$. By application of equation (8) it can be

seen that equation (30) is identical to equation (16) which we

obtained using Skilling's approach.

The rate at which the protons stream across the boundary at $z = \zeta(p)$ is obtained by substituting equation (30) into equation (29),

$$\frac{V_s}{c} \Big|_3 = \left(1 + \frac{x/2}{1+x/2} \right) \frac{x}{3} \frac{V_A}{c} \Big|_3 = \left(1 + \frac{x/2}{1+x/2} \right) 7 \times 10^{-5} e^{\frac{x}{2} \tau/2\pi} \quad (31)$$

For $0 \leq x \leq 1$, V_s lies between $\frac{x}{3} V_A \Big|_3$ and $\frac{4}{3} \frac{x}{3} V_A \Big|_3$, which agrees with Skilling's statement that the drift velocity through the wave frame at the boundary is $V_A / 3$.

Now that we have reconciled the approaches of Wentzel and Skilling, we shall use Wentzel's method to include the effects of non-linear damping.

7. The Effect of Non-linear Damping on Cosmic Ray Confinement.

Wentzel showed that the growth rate Γ_G of equation (28) can be balanced with the damping rate Γ_w of equation (27) to yield a steady-state solution for the streaming speed

$$V_s - \frac{\delta}{3} V_A \propto \left[\frac{B}{L N(r)} \right]^{\frac{1}{2}} \quad (32)$$

and the wave energy

$$M_K \propto r^2 \left[\frac{N(r)}{L B} \right]^{\frac{1}{2}} \quad (33)$$

where L is the cosmic ray scale height in the presence of the waves. When non-linear damping dominates over ion-neutral collisions, he showed that, for $B = 3 \times 10^{-6}$ G ,

$$V_s = \frac{\delta}{3} V_A + 56 (100 r_c / L)^{\frac{1}{2}} (P/M_c)^{0.75} \text{ km s}^{-1}$$

When we include the ion-neutral damping term, we obtain an expression for the streaming velocity in the z -direction, in a uniform z -field of 10^{-6} G ,

$$\frac{V_s}{c} \Big|_z = 7 \times 10^{-5} e^{\frac{x}{2} z_{H}} + 6 \cdot 273 \times 10^{-5} p^{1.5} e^{-(1+\frac{x}{2}) z_{H}} + 2.9 \times 10^{-5} p^{0.75} \quad (34)$$

where we have used a scale height $L = 6 z_H / (4.2 \times 0.4) \approx 460$ pc as suggested by equation (19).

Since the non-linear damping term is independent of gas density, and therefore of height, the height at which V_s is minimum is not altered by its inclusion. Therefore the boundary height remains as described by equation (30) . The streaming rate across the boundary is now

$$\frac{V_s}{c} \Big|_z = \frac{1+x}{1+x/2} 7 \times 10^{-5} e^{\frac{x}{2} z_{H}} + 2.9 \times 10^{-5} p^{0.75}$$

which becomes after substitution for $e^{\frac{x}{2} z_{H}}$

$$\frac{V_s}{c} \Big|_z = 8.2 \times 10^{-5} \left(\frac{p}{p_0} \right)^{0.214} + 2.9 \times 10^{-5} p^{0.75} \quad (35)$$

having set $x = 0.4$. This equation suggests that non-linear damping effects are appreciable in the confinement of cosmic rays of rigidity $R \gtrsim 12 R_0^{-0.4} \text{ GV}$, where R_0 is the rigidity of the cosmic rays which begin to encounter resonant waves at the Sun. If we let $R_0 = 8 \text{ GV}$, as in the previous analysis, non-linear damping is appreciable when $R \gtrsim 5 \text{ GV}$, that is for all rigidities of interest in this model. The effect of non-linear damping is to increase the streaming speed across the boundary, and it will therefore affect the lifetime and path length of galactic cosmic rays.

The lifetime and path length of cosmic rays in the free zone of height $z(R)$, from which the streaming velocity is $V_z/2$, are plotted as functions of rigidity in Figures 16a and b respectively. Curve (i) assumes an exponential density distribution, while curve (ii) assumes a Gaussian one. The Sun is located at $z = 10 \text{ pc}$ for this purpose. The path length has been normalized to fit the results of Cesarsky and Audouze (1974), and the lifetime subsequently normalized to produce these path lengths in a medium of density $n_H = 1 \text{ atom cm}^{-3}$ at $z = 0$. The Gaussian density distribution produces a better fit to the data than the exponential one.

At rigidities below 8 GV , the cosmic rays convect along the field lines at velocities just greater than the Alfvén velocity. The path length and residence time are determined by the average distance along the field from the nearby sources. They will not vary much with energy in this region, especially below 5 GV when the slippage through the wave frame due to non-linear wave damping becomes negligible.

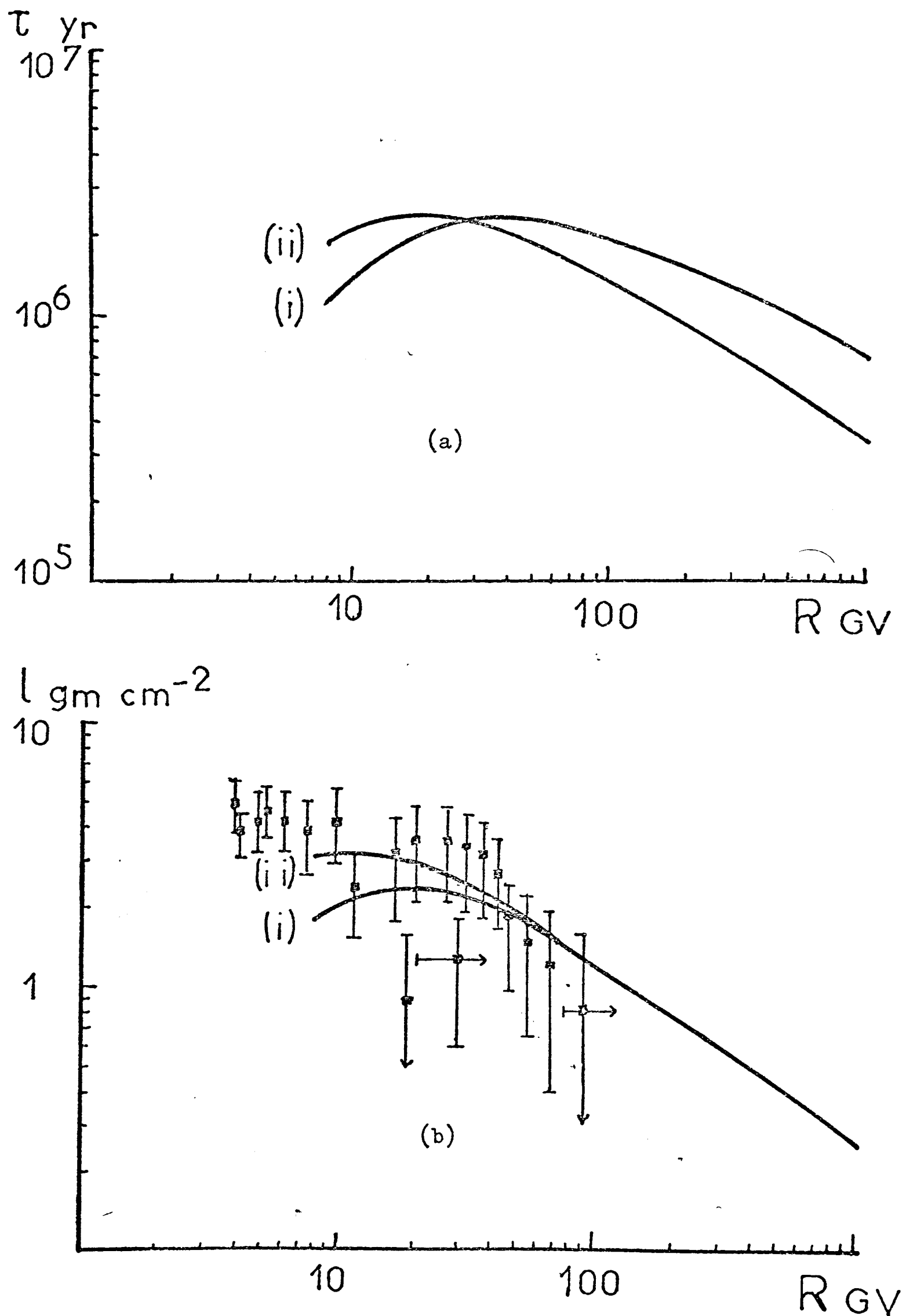


FIGURE 16. (a) the residence time and (b) the path length of cosmic rays in the free zone, taking into account nonlinear wave damping. The path length is normalized to fit the results of Cesarsky and Audouze (1974). Curve (i) assumes an exponential density distribution, while curve (ii) assumes a Gaussian one.

8. Cosmic Ray Anisotropy.

The low observed anisotropy of the galactic cosmic rays may be explained in terms of this leaky-box model, since each cosmic ray particle has only a small probability, $\sim V_s/c$, of free passage across the boundary at each encounter. In this section we shall discuss the factors which determine the anisotropy, in the light of Skilling's conclusion (1974) that the individual field lines diverge rapidly away from each other due to turbulence in the galactic magnetic field and that at each reflection a cosmic ray particle moves across the field through a distance of the order of its gyro-radius.

The cosmic rays which we detect are those particles which spiral along field lines passing within a gyro-radius of the Sun. Particles with small pitch angles are free to travel along the field across the free zone until they are reflected at the boundary. If there are no cosmic ray sources feeding particles directly on to these lines, the particles must have transferred to these lines during reflection at the boundary from other field lines which reach the boundary within a gyro-radius. Their anisotropy would therefore depend upon the difference between the Alfvén velocities at the boundaries at either end of the field lines. As the lines passing close to the Sun have diverged by a distance of about $\mathcal{L}$ by the time they reach the boundary, the Alfvén velocities must be averaged over an area $\sim \pi \mathcal{L}^2$ of either side of the galactic disk.

Cosmic rays with larger pitch angles are scattered by mirrors separated by distances of 10 - 30 pc, the mean separation

of irregularities in the galactic magnetic field. Their local anisotropy is determined by the distribution of the mirrors from which they were last reflected. The rapid divergence of field lines away from the Sun ensures that the cosmic rays which pass within a gyro-radius of the Sun encounter a selection of different mirrors. As a result, two particles of similar pitch angle on neighbouring field lines at the Sun could have experienced their previous reflection from two different mirrors, each with its own strength and turbulent velocity. Furthermore, the cosmic rays on any particular field line would not all be reflected at the same mirror, as their pitch angles determine the minimum strength a mirror requires in order to reflect them, the particles of smaller pitch angle in general having to travel further in order to find a stronger mirror. Particles with pitch angles smaller than $\sin^{-1}(\bar{B}/B_{\max})^{\frac{1}{2}}$ would travel along their field lines until they reach the boundary, where they would be reflected on to other field lines.

Consequently the anisotropy of cosmic rays in the free zone is determined by the field lines along which they are travelling, and hence their position in the Galaxy, and their pitch angles relative to them. The low observed streaming velocities discussed in Chapter 1 imply that the average motion of the nearby clouds relative to the local standard of rest is less than $\sim 30 \text{ km s}^{-1}$, since the majority of the cosmic rays we observe will be reflected from them. This is, of course, to be expected, since their r.m.s. velocity in the line of sight is only $\sim 9 \text{ km s}^{-1}$. Measurement of the anisotropy

of cosmic rays with small pitch angles would provide useful information about the streaming velocity at the boundaries, but unfortunately this is not feasible as yet.

Now that we have explained the energy-dependent path length and low observed anisotropy of galactic cosmic rays in terms of our confinement model, let us examine the effects this model will have on other observable parameters, as more observational tests are necessary in order to verify this theory.

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CHAPTER 5

OBSERVATIONAL CONSEQUENCES OF THE CONFINEMENT MODEL

1. The Electron Spectrum.

Melrose and Wentzel (1970) have shown that the motion of the cosmic ray electrons is dominated by that of the protons. In regions where protons can generate hydromagnetic waves, electrons of the same rigidity (and hence the same relativistic energy) will be scattered by the waves, and any anisotropy caused by synchrotron emission or streaming from sources will be reduced. Consequently the leakage rate of electrons from the Galaxy is governed by that of the protons, and therefore the lifetime of cosmic ray electrons will be the same as that of protons of the same energy. But unlike the protons, electrons lose energy continually by synchrotron emission and the inverse Compton effect on microwave photons. This energy loss produces characteristic changes in the electron spectrum.

In a steady state, the electron number density $N(E) dE$ is related to their lifetime $\tau(E)$ and source rate $Q(E) dE$ by

$$\frac{\partial}{\partial E} \left(N(E) \frac{dE}{dt} \right) + \frac{N(E)}{\tau(E)} = Q(E) \quad (1)$$

(see for example Daniel & Stevens 1970). The energy loss by synchrotron emission and the inverse Compton effect is

$$-\frac{dE}{dt} = b E^2 \quad (2)$$

where $b = 4 \times 10^{-6} B_{\perp}^2 + 4 \times 10^{-19} T_{bb}^4 \text{ GeV}^{-1} \text{ s}^{-1}$.

$B_{\perp}$ is the average perpendicular component of the magnetic

field in Gauss and T_{bb} is the temperature of the background black body radiation in $^{\circ}\text{K}$. For an energy-independent lifetime, this equation predicts a change of 1 in the spectral index of the electrons centered on the energy at which the energy-loss lifetime $E / \frac{dE}{dt}$ is equal to the containment lifetime τ , when $E b \tau = 1$. When the energy losses during the containment time are negligible, i.e. when $E b \tau \ll 1$, the solution to equation (1), with a power-law source term $Q(E) dE = A E^{-\beta} dE$, is

$$N(E) = \tau A E^{-\beta} \quad (3a)$$

When energy losses dominate (when $E b \tau \gg 1$), the solution is

$$N(E) = \frac{A}{b(\beta-1)} E^{-(\beta+1)} \quad (3b)$$

If we take $B = 3 \times 10^{-6}$, $T_{bb} = 2.7$, and $\tau = 10^6$ yr, then in the case of a constant lifetime we should expect a bend in the electron spectrum at an energy of ~ 500 GeV.

However, the situation is altered significantly if the containment lifetime changes with energy. If $\tau(E)$ decreases faster than E^{-1} , the spectrum would have a constant index, since the escape time decreases with increasing energy faster than the characteristic energy-loss time ($1 / b E$). A decrease in $\tau(E)$ slower than E^{-1} would produce an overall change in index of less than 1. Silverberg and Ramaty (1973) considered the effect on the electron spectrum of a lifetime which decreases as a power-law, $\tau(E) = \tau_0 (E_0 / E)^{\delta}$. They showed that the overall change in spectral index between $E \ll 1/b\tau$ to $E \rightarrow \infty$ is $1 - \delta$ when $0 \leq \delta < 1$, and 0 when $\delta \geq 1$. Furthermore the energy range over which the index changes appreciably becomes

wider as δ increases. From Figure 16 we see that above an energy of about 100 GeV, the lifetime decreases approximately as a power-law with index $\sim 0.5 - 0.6$. From the results of Silverberg and Ramaty it is evident that the change in index of the electron spectrum up to 10^3 GeV will only be $\lesssim 0.2$ in this case.

Our confinement theory therefore implies that the expected bend in the electron spectrum will be considerably reduced in magnitude. The observations reviewed in Chapter 1 are not consistent with a change in index of 1 around 500 GeV, although the scatter is still too great to obtain an estimate of the overall extent of any bend. Future measurements of the electron spectrum in this region will be very useful as a means of determining the rate of decrease of lifetime with energy.

2. Pulsar Scintillation.

Pulses received from distant pulsars are observed to be considerably broadened at low frequencies (Ables et al. 1970, Lang 1971). Their width is approximately proportional to the fourth power of the wavelength at which they are detected. This effect has been explained in terms of fluctuations in the electron density in the line of sight which scatter the radiation, producing a distribution in optical path lengths between the pulsar and the Earth.

The refractive index for waves of angular frequency ω in a medium where there are n_e free electrons cm^{-3} is $(1 - 4\pi n_e e^2/m_e \omega^2)^{1/2}$. When a plane wave with wavelength λ passes through a fluctuation Δn_e of linear dimension a , it experiences a phase change $\Delta\phi = \Delta n_e r_e \lambda a$, where $r_e = e^2/m_e c^2 = 2.8 \times 10^{-13}$ cm is the classical radius of the electron. If the scattering is produced at a thin screen half way between the Earth and the pulsar, the radiation will be scattered through a range of angles with root mean square value

$$\Theta_{\text{rms}} \approx 4 \times 10^7 (D/a)^{1/2} \langle \Delta n_e^2 \rangle^{1/2} / \nu^2 \quad (4)$$

where D is the distance of the pulsar (Lang 1971). The difference in path length between the direct and scattered rays is $1/4 D \Theta^2$, and hence the r.m.s. time delay is $D \Theta_{\text{rms}}^2 / 4c$, which is proportional to λ^4 .

The observations require a r.m.s. density fluctuation $\langle \Delta n_e^2 \rangle^{1/2} \sim 10^{-4} \text{ cm}^{-3}$ over a length scale $a \sim 10^{11} \text{ cm}$.

Wentzel (1969) suggested that this indicates the existence of

hydromagnetic waves generated by streaming cosmic rays, although these waves would be resonant only with low-energy cosmic rays ($E \lesssim 1$ MeV). The waves relevant to our confinement model have wavelengths greater than about 3×10^{14} cm. These waves cannot be observed by this method because of Fresnel filtering, which renders irregularities closer to the source or observer than a/θ ineffective in scattering radiation. So in order to observe fluctuations half way between the pulsar and Earth,

$$\frac{1}{2} D > a/\theta_{rms} . \text{ Using equation (4) , this condition becomes}$$

$$a < 7 \times 10^4 D \left[\langle \Delta n_e^2 \rangle^{\frac{1}{2}} \right]^{\frac{2}{3}} \nu^{-\frac{4}{3}} . \quad (5)$$

With $D \sim 1$ kpc , $\langle \Delta n_e^2 \rangle^{\frac{1}{2}} \sim 10^{-4} \text{ cm}^{-3}$, and $\nu \sim 10^9 \text{ Hz}$, we see that pulsar scintillation will only be observed if $a < 10^{12} \text{ cm}$.

Furthermore, Alfvén waves do not compress the interstellar plasma to the first order in wave amplitude. It is therefore evident that the Alfvén waves which cause the confinement of relativistic cosmic rays are not the fluctuations which produce pulsar scintillation. However, we have seen that these waves decay into magnetosonic and sound waves, both of which produce compression of the interstellar plasma. Hence the fluctuations of size $\sim 10^{11} \text{ cm}$ may be evidence of confinement of low-energy cosmic rays by hydromagnetic waves. A useful problem for future research would be to investigate the equilibrium spectrum of hydromagnetic waves, with an input of Alfvén and magnetosonic waves due to cosmic ray streaming, and a transfer of energy to sound and magnetosonic waves which are subsequently damped. The growth rate of small-wavelength Alfvén waves cannot be

found at the moment due to the difficulty of obtaining the galactic low-energy cosmic ray spectrum from solar demodulation calculations. But although density fluctuations of length scale $a \gg 10^{11}$ cm do not cause scintillation, they can manifest themselves through the effect of refraction. Shishov (1973) showed that a combination of two irregularity length scales, with one being much greater than the other, can produce a variation of scintillation effects with frequency, over and above that predicted by equation (4). He concluded that the scintillation of CP 0328 (distant 300 pc) indicates the existence of irregularities with a length scale $10^{14} - 10^{15}$ cm, which would be produced in the confinement of 10 GeV cosmic rays.

Further support for the theory that pulsar scintillation is produced by hydromagnetic waves comes from the conclusions of Williamson (1974 and earlier references therein). The scattering of electromagnetic radiation from a pulsar is not of course expected to take place just in a thin screen between Earth and the pulsar, so Williamson analysed the pulse shapes expected from various configurations of the scattering region. He concluded that scattering must take place in regions of finite size, but which do not occupy the whole of the line of sight. This can be interpreted in terms of waves forming in regions of enhanced ionization, where the linear damping rate is smaller. Such a situation would produce a correlation between the amount of scattering and the pulsar dispersion measure, which has indeed been observed. He also mentions that there is no apparent relationship between the amount of scattering and the degree

of HI absorption. In fact four pulsars which exhibit scintillation effects show no HI absorption. This suggests that the scattering does not take place in HI clouds, which is in agreement with the consideration that an enhanced concentration of neutral hydrogen would damp the hydromagnetic waves.

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CHAPTER 6

THE NONLINEAR COUPLING OF HYDROMAGNETIC WAVES

IN A LOSS-FREE PLASMA

In section 7 of Chapter 4 we showed that the nonlinear decay of Alfvén waves plays an important role in determining the confinement time of cosmic rays in the Galaxy. The nonlinear damping rate in equation (27) was calculated by Wentzel (1969) using the approximation of a negligible flux of sound waves, as these would be rapidly Landau-damped. Clearly there exists opportunity for a more thorough examination of mode-coupling between hydromagnetic waves, in which energy is supplied to the Alfvén and magnetosonic modes by streaming cosmic rays, and the magnetosonic and sound modes lose their energy by collisionless damping. With this in mind, we present in this chapter the results of an investigation into the process of mode-coupling, carried out in collaboration with Dr J. Skilling of DAMTP, Cambridge, to whom I am greatly indebted for his permission to reproduce the results in this thesis.

In a turbulent plasma, energy is distributed over a spectrum of excited waves. When the wave energy is small compared to the total energy of the plasma, the theory of weak turbulence can be applied to investigate the passage of energy between wave modes. In this regime, arbitrary perturbations can be expressed as a superposition of eigenmodes, and the nonlinearity of the equations which represent the plasma provides a weak interaction between the modes. Here we investigate the effects of wave-wave interactions

in a MHD plasma where the wave-particle interactions can be neglected. The turbulence is considered weak enough for wave-wave interactions of order higher than two to be neglected. We apply the results to loss-free double-adiabatic and single-adiabatic plasmas, in which the magnetic pressure is much greater than the thermal pressure, and show that the coupling coefficients for a double-adiabatic plasma are the same as those for a single-adiabatic plasma with a specific heat ratio of 3 . Finally, the interactions are interpreted as collisions in a plasmon gas, which stimulate the decay of plasmons into other plasmons of different types.

1. General Theory.

Hydromagnetic waves can be expressed in terms of state vectors Ψ^j whose components represent the variables describing the wave, i.e. components of magnetic field, of particle velocity, and number density. In the weak turbulence regime, the state of the plasma can be written as a perturbation expansion in successive orders of Ψ^j . We denote the state vector of the n th order perturbation as Ψ_n^j . The zeroth order state vector Ψ_0^j describes a homogeneous plasma in a uniform magnetic field.

A closed set of hydromagnetic equations may be expanded into successive orders of perturbation Ψ_r^j . Expanding as far as the third order, using the summation convention, the successive orders may be written as

$$\begin{aligned} \xi^{ij}(-i\nabla, i\frac{\partial}{\partial t}) \Psi_1^j &= 0 \\ \xi^{ij}(-i\nabla, i\frac{\partial}{\partial t}) \Psi_2^j &= \Psi_1^j A^{ijr}(-i\nabla, i\frac{\partial}{\partial t}) \Psi_1^r \\ \xi^{ij}(-i\nabla, i\frac{\partial}{\partial t}) \Psi_3^j &= \Psi_1^j A^{ijr}(-i\nabla, i\frac{\partial}{\partial t}) \Psi_2^r \\ &+ \Psi_2^j A^{ijr}(-i\nabla, i\frac{\partial}{\partial t}) \Psi_1^r \\ &+ \Psi_1^j \Psi_1^r B^{ijrq}(-i\nabla, i\frac{\partial}{\partial t}) \Psi_1^q \end{aligned} \quad (1)$$

The arrays ξ, A, B have elements which depend upon the background plasma parameters Ψ_0^j and the derivative operators of space and time. ξ is the dielectric tensor, A measures the second order interaction, and B is included

in the third order expansion.

Equations (1) are Fourier analysed, with normalization

$$\xi(\underline{x}) = \int \frac{d^3 k}{(2\pi)^3} e^{i \underline{k} \cdot \underline{x}} \xi_{\underline{k}}$$

to give the dependence on wavevector $\underline{k}$. The dispersion relation

$$\det \xi(\underline{k}, \omega) = 0$$

has roots $\omega_{\underline{k}}^{\sigma}$, which are the frequencies of oscillation of each mode σ . The first order equation in (1) shows that the elements of the state vector describing the first order mode σ oscillate with frequency $\omega_{\underline{k}}^{\sigma}$. The net first order disturbance is the sum over modes,

$$\psi_{1,\underline{k}}^{\sigma,j}(t) = \sum_{\sigma} \psi_{1,\underline{k}}^{\sigma,j}(0) e^{-i \omega_{\underline{k}}^{\sigma} t},$$

where $\psi_{1,\underline{k}}^{\sigma,j}$ stands for the j th component of the first order state vector describing a wave of mode σ with wavevector $\underline{k}$.

Fourier analysing the second and third of equations (1) and substituting in them the above expression for the first order disturbance enables us to express the second and third order perturbations in terms of the first order eigenmodes,

$$\begin{aligned} \psi_{2,\underline{k}}^{\sigma,j}(t) = & \sum_{\sigma_1} \sum_{\sigma_2} \int \frac{d^3 l}{(2\pi)^3} \psi_{1,\underline{k}-\underline{l}}^{\sigma_1,a}(t) \psi_{1,\underline{l}}^{\sigma_2,b}(t) \\ & \times (\epsilon^{-1})^{ji}(\underline{k}, \omega_{\underline{k}-\underline{l}}^{\sigma_1} + \omega_{\underline{l}}^{\sigma_2}) A^{iab}(\underline{l}, \omega_{\underline{l}}^{\sigma_2}) \\ & \times \int_{\underline{k}}^{\sigma} \left(\begin{matrix} \sigma_1 & \sigma_2 \\ \underline{k}-\underline{l} & \underline{l} \end{matrix} \right) \end{aligned} \quad (2a)$$

$$\begin{aligned} \Psi_{3, \underline{k}}^{\epsilon, j}(t) = & \sum_{\epsilon_1} \sum_{\epsilon_2} \sum_{\epsilon_3} \iint \frac{d^3 l}{(2\pi)^3} \frac{d^3 l'}{(2\pi)^3} \Psi_{1, \underline{k}-\underline{l}}^{\epsilon_1, a}(t) \Psi_{1, \underline{l}-\underline{l}'}^{\epsilon_2, b}(t) \\ & \times \Psi_{1, \underline{l}'}^{\epsilon_3, c}(t) (\xi^{-1})^{ji}(\underline{k}, \omega_{\underline{k}-\underline{l}}^{\epsilon_1} + \omega_{\underline{l}-\underline{l}'}^{\epsilon_2} + \omega_{\underline{l}'}^{\epsilon_3}) \\ & \times \left\{ [A^{iad}(\underline{l}, \omega_{\underline{l}-\underline{l}'}^{\epsilon_2} + \omega_{\underline{l}'}^{\epsilon_3}) + A^{ida}(\underline{k}-\underline{l}, \omega_{\underline{k}-\underline{l}}^{\epsilon_1})] (\xi^{-1})^{de}(\underline{l}, \omega_{\underline{l}-\underline{l}'}^{\epsilon_2} + \omega_{\underline{l}'}^{\epsilon_3}) \right. \\ & \left. \times A^{ebc}(\underline{l}', \omega_{\underline{l}'}^{\epsilon_3}) + B^{ibc}(\underline{l}', \omega_{\underline{l}'}^{\epsilon_3}) \right\} \int_{\underline{k}}^{\epsilon} \left(\begin{matrix} \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \underline{k}-\underline{l} & \underline{l}-\underline{l}' & \underline{l}' \end{matrix} \right) . \end{aligned}$$

(2 b)

The term $(\xi^{-1})^{ji}(\underline{k}, \omega_{\underline{k}-\underline{l}}^{\epsilon_1} + \omega_{\underline{l}}^{\epsilon_2})$ represents the element j, i of the inverse matrix of $\xi(-i\nabla, i\frac{\partial}{\partial t})$, with ∇ replaced by $i\underline{k}$ and $\frac{\partial}{\partial t}$ replaced by $-i(\omega_{\underline{k}-\underline{l}}^{\epsilon_1} + \omega_{\underline{l}}^{\epsilon_2})$. Similar definitions hold for the other arrays. The $\int_{\underline{k}}^{\epsilon}$ terms are selectors, used to assign combinations of first order disturbances of different modes to a 'net' mode ϵ . Thus $\int_{\underline{k}}^{\epsilon} \left(\begin{matrix} \epsilon_1 & \epsilon_2 \\ \underline{k}-\underline{l} & \underline{l} \end{matrix} \right) = 1$ if $\omega_{\underline{k}}^{\epsilon}$ has the same sign as $\omega_{\underline{k}-\underline{l}}^{\epsilon_1} + \omega_{\underline{l}}^{\epsilon_2}$, and $= 0$ otherwise, with a similar definition for $\int_{\underline{k}}^{\epsilon} \left(\begin{matrix} \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \underline{k}-\underline{l} & \underline{l}-\underline{l}' & \underline{l}' \end{matrix} \right)$.

To make further progress we must now ensemble average (denoted by angular brackets), so that we may use the random phase approximation

$$\langle \psi^{(1)} \psi^{(2)} \psi^{(3)} \psi^{(4)} \rangle = \langle \psi^{(1)} \psi^{(2)} \rangle \langle \psi^{(3)} \psi^{(4)} \rangle + \langle \psi^{(1)} \psi^{(3)} \rangle \langle \psi^{(2)} \psi^{(4)} \rangle + \langle \psi^{(1)} \psi^{(4)} \rangle \langle \psi^{(2)} \psi^{(3)} \rangle .$$

The equation governing the rate of change of wave intensity is obtained by ensemble averaging the expression for

$$\left(\frac{1}{2} \frac{d}{dt} - \gamma_{\underline{k}}^{\epsilon} \right) (\Psi_{\underline{k}}^{\epsilon, j*} \Psi_{\underline{k}}^{\epsilon, j}) \text{ expanded in terms of } \Psi_1^{\epsilon, j} + \Psi_2^{\epsilon, j} + \Psi_3^{\epsilon, j} .$$

$\gamma_{\underline{k}}^{\sigma}$ is the linear growth rate of a wave σ with frequency $\omega_{\underline{k}}^{\sigma}$. An asterisk denotes the complex conjugate. Neglecting terms of order higher than 4,

$$\left. \begin{aligned} \frac{1}{2} \frac{d}{dt} \langle \psi_{\underline{k}}^{\sigma, j*} \psi_{\underline{k}}^{\sigma, j} \rangle &= \gamma_{\underline{k}}^{\sigma} \langle \psi_{\underline{k}}^{\sigma, j*} \psi_{\underline{k}}^{\sigma, j} \rangle & (I) \\ &+ \left(\frac{1}{2} \frac{d}{dt} - \gamma_{\underline{k}}^{\sigma} \right) \langle \psi_{2, \underline{k}}^{\sigma, j*} \psi_{2, \underline{k}}^{\sigma, j} \rangle & (II) \\ &+ \text{Re} \langle \psi_{1, \underline{k}}^{\sigma, j*} \left(\frac{d}{dt} + i\omega_{\underline{k}}^{\sigma} \right) \psi_{3, \underline{k}}^{\sigma, j} \rangle & (III) \end{aligned} \right\} (3)$$

Term (I) on the right hand side represents the linear growth rate, while (II) and (III) are wave-wave interaction terms.

It is now convenient to introduce wave intensities $I_{\underline{k}}^{\sigma}$ and normalized eigenvectors $\Psi_{\underline{k}}^{\sigma, j}$, defined by

$$\langle \psi_{1, \underline{k}}^{\sigma, j*} \psi_{1, \underline{k}}^{\sigma, j} \rangle = \Psi_{\underline{k}}^{\sigma, j*} \Psi_{\underline{k}}^{\sigma, j} I_{\underline{k}}^{\sigma}$$

for each component j . The expressions for ψ_2 and ψ_3 from equations (2a) and (2b) are substituted into terms (II) and (III) in equation (3), and the random phase approximation is used to reduce the ensemble average of four first-order waves to a sum of products of wave intensities. Two first-order disturbances only correlate if they define the same wave, so that

$$\langle \psi_{1, \underline{k}}^{\sigma_1, j} \psi_{1, \underline{k}}^{\sigma_2, j} \rangle = \delta_{\sigma_1, \sigma_2} \delta_{\underline{k}, -\underline{k}} \Psi_{\underline{k}}^{\sigma_1, j} \Psi_{-\underline{k}}^{\sigma_1, j} I_{\underline{k}}^{\sigma_1}.$$

For a loss-free medium, the elements of the array β^{iabc} are purely imaginary, so there is no functional dependence on β^{iabc} in term (III) after taking its real part. Some algebra and complex analysis then yields, for a loss-free medium:

$$\begin{aligned}
 & \text{term (II)} \\
 & = \frac{\pi}{2V} \sum_{\sigma_1} \sum_{\sigma_2} \int \frac{d^3 l}{(2\pi)^3} I_{\underline{k}-\underline{l}}^{\sigma_1} I_{\underline{l}}^{\sigma_2} \delta(\omega_{\underline{k}-\underline{l}}^{\sigma_1} + \omega_{\underline{l}}^{\sigma_2} - \omega_{\underline{k}}^{\sigma}) |V(\underline{k} \quad \underline{k}-\underline{l} \quad \underline{l})|^2 \\
 & \text{term (III)} \\
 & = -\frac{\pi}{V} \sum_{\sigma_1} \sum_{\sigma_2} \int \frac{d^3 l}{(2\pi)^3} I_{\underline{k}}^{\sigma} I_{\underline{k}-\underline{l}}^{\sigma_1} \delta(\omega_{\underline{k}-\underline{l}}^{\sigma_1} + \omega_{\underline{l}}^{\sigma_2} - \omega_{\underline{k}}^{\sigma}) V(\underline{k} \quad \underline{k}-\underline{l} \quad \underline{l}) V(\underline{l} \quad \underline{l}-\underline{k} \quad \underline{k})
 \end{aligned} \quad (4)$$

The normalization volume V enters when changing a Kronecker

$\delta_{\underline{k}, \underline{l}}$ into a Dirac $\delta(\underline{k}-\underline{l})$ in the 2-wave averages. The 'interaction strength' is

$$V(\underline{k} \quad \underline{k}-\underline{l} \quad \underline{l}) = -i U^c(\underline{k}, \omega_{\underline{k}}^{\sigma}) [A^{ca}(\underline{l}, \omega_{\underline{l}}^{\sigma_1}) + A^{ba}(\underline{k}-\underline{l}, \omega_{\underline{k}-\underline{l}}^{\sigma_2})] \Psi_{\underline{k}-\underline{l}}^{\sigma_1, a} \Psi_{\underline{l}}^{\sigma_2, b}$$

and U^c is given by

$$\Psi_{\underline{k}}^{\sigma, d} U^c(\underline{k}, \omega_{\underline{k}}^{\sigma}) = (\xi^{-1})^{dc}(\underline{k}, \omega_{\underline{k}}^{\sigma}) / \frac{\partial(\ln \det \xi)}{\partial \omega_{\underline{k}}^{\sigma}}$$

We now define

$$W(\underline{k} \quad \underline{k}-\underline{l} \quad \underline{l}) \equiv \frac{|V(\underline{k} \quad \underline{k}-\underline{l} \quad \underline{l})|^2}{\rho \pi^2 V (\omega_{\underline{k}}^{\sigma})^2}$$

By considering an isolated wave-wave interaction and comparing

the total gain in wave energy from term (II) with the total

loss in wave energy from term (III) in equations (4), it can

be seen that $W(\underline{k} \quad \underline{k}' \quad \underline{k}'')$ is invariant under the

interchange of any subscripts in all cases satisfying the

selection rules $\underline{k} + \underline{k}' + \underline{k}'' = 0$ and $\omega_{\underline{k}}^{\sigma} + \omega_{\underline{k}'}^{\sigma_1} + \omega_{\underline{k}''}^{\sigma_2} = 0$.

In particular

$$V(\underline{k} \quad \underline{k}-\underline{l} \quad \underline{l}) / \omega_{\underline{k}}^{\sigma} = V(\underline{l} \quad \underline{l}-\underline{k} \quad \underline{k}) / \omega_{\underline{l}}^{\sigma_2}$$

So terms (II) and (III) in equations (4) can be combined

with term (I) in equation (3) to give the total rate of change

of intensity of a wave due to linear growth and non-linear wave-wave interaction;

$$\frac{d}{dt} (I_k^\epsilon) = 2\gamma_k^\epsilon I_k^\epsilon + \sum_{\sigma_1} \sum_{\sigma_2} \int d^3l \left[(\omega_k^\epsilon)^2 I_{k-l}^{\sigma_1} I_l^{\sigma_2} - 2\omega_k^\epsilon \omega_l^{\sigma_2} I_k^\epsilon I_{k-l}^{\sigma_1} \right] \times \delta(\omega_{k-l}^{\sigma_1} + \omega_l^{\sigma_2} - \omega_k^\epsilon) W \left(\begin{smallmatrix} \epsilon & \sigma_1 & \sigma_2 \\ k & k-l & -l \end{smallmatrix} \right) \quad (5)$$

In this there is an implicit approximation that losses are unimportant ($\gamma_k^\epsilon \ll \omega_k^\epsilon$) because a finite γ broadens the resonance conditions and alters the form of the interaction strength.

Equation (5) is a general formula for the evolution of a spectrum of waves. We now proceed to evaluate W in special cases.

2. Application to a Double-adiabatic Plasma.

The double-adiabatic plasma equations, which allow for anisotropy in the pressure due to the magnetic field, are:

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B}) \quad (6)$$

$$\frac{\partial N}{\partial t} = -\nabla \cdot (N \underline{u}) \quad (7)$$

$$Nm \left(\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right) = -\nabla \left(P_{\perp} + \frac{B^2}{2\mu_0} \right) + \underline{B} \cdot \nabla \left[\left(\frac{1}{\mu_0} + \frac{P_{\perp} - P_{\parallel}}{B^2} \right) \underline{B} \right] \quad (8)$$

$$\left(\frac{\partial}{\partial t} + \underline{u} \cdot \nabla \right) \frac{P_{\parallel} B^2}{N^3} = 0 \quad (9)$$

$$\left(\frac{\partial}{\partial t} + \underline{u} \cdot \nabla \right) \frac{P_{\perp}}{NB} = 0 \quad (10)$$

(Chew et al. 1956).

$P_{\parallel}$ is the pressure along the field and $P_{\perp}$ is the pressure perpendicular to it. The zero-order magnetic field $\underline{B}_0$ is taken to be constant and parallel to the z-axis. In the frame in which the background plasma is at rest, $\underline{u}_0 = 0$.

N_0 is the density of the background plasma, and we let

$$P_{\parallel 0} = P_{\perp 0} = P_0.$$

Equations (9) and (10) are used to eliminate $P_{\parallel}$ and $P_{\perp}$ from equation (8) to give

$$Nm \left(\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right) = -\nabla \left(\frac{P_0}{N_0 B_0} NB + \frac{B^2}{2\mu_0} \right) + \underline{B} \cdot \nabla \left[\left(\frac{1}{\mu_0} + \frac{P_0}{N_0 B_0} \frac{N}{B} - \frac{P_0 B_0^2}{N_0^3} \frac{N^3}{B^4} \right) \underline{B} \right] \quad (11)$$

Equations (6), (7) and (11) form a complete set, which can be expanded in terms of 7-component state vectors:

$$\Psi_n^j = \left(B_{xn}, B_{yn}, B_{zn}, u_{xn}, u_{yn}, u_{zn}, N_n \right).$$

The elements of the arrays ε^{ij} and A^{ijr} can then be determined. The dispersion relation $\det \varepsilon = 0$ becomes

$$\omega (\omega^2 - V_A^2 k_z^2) \left[\omega^4 - \{ (2V_s'^2 + V_A^2) k^2 + V_s'^2 k_z^2 \} \omega^2 + 3V_s'^2 V_A^2 k_z^2 k^2 + 5V_s'^4 k_x^2 k_z^2 \right] = 0$$

where $V_A^2 = B_0^2 / (\mu_0 N_0 m)$, $V_s'^2 = P_0 / (N_0 m)$.

Letting $B_0^2 / 2\mu_0 \gg P_0$, the six non-zero roots of the dispersion relation can be approximated to :

$$\omega_{\underline{k}}^{\pm A} = \pm V_A k_z \quad \text{Alfvén wave}$$

$$\omega_{\underline{k}}^{\pm M} = \pm V_A |\underline{k}| \quad \text{Magnetosonic wave}$$

$$\omega_{\underline{k}}^{\pm S} = \pm V_s k_z \quad \text{Sound wave}$$

$$\text{where } V_s = \sqrt{3} V_s' = \sqrt{\frac{3 P_0}{N_0 m}} .$$

Their respective eigenvectors are:

$$\Psi_{1, \underline{k}}^A = \left(B_0 k_y k_z , -B_0 k_x k_z , 0 , -\omega k_y , \omega k_x , 0 \right)$$

$$\Psi_{1, \underline{k}}^M = \left(-B_0 k_x k_z , -B_0 k_y k_z , B_0 (k_x^2 + k_y^2) , \omega k_x , \omega k_y , N_0 (k_x^2 + k_y^2) \right)$$

$$\Psi_{1, \underline{k}}^S = \left(0 , 0 , 0 , 0 , 0 , N_0 \right) .$$

The eigenvectors are normalized by requiring that $I_{\underline{k}}^{\epsilon}$ represent the total (i.e. kinetic and magnetic) energy density of the waves of type ϵ and wavevector $\underline{k}$. The total energy density of all the waves is then $\sum_{\epsilon} \int_{\infty} d^3k I_{\underline{k}}^{\epsilon}$.

The values of $W\left(\begin{smallmatrix} \epsilon, & \epsilon, & \epsilon \\ \underline{k} & \underline{k}' & \underline{k}'' \end{smallmatrix}\right)$ obtained by this method, to be inserted into equation (5), are displayed in Table 4.

When terms of order V_S^2/V_A^2 are neglected compared to unity, it is evident that the frequency selection rule for interactions between three sound waves is automatically satisfied for all cases obeying the wavevector selection rule. However, this is only true to zeroth order. A more exact solution of the dispersion relation shows that the δ -function argument in equation (5) has a frequency offset of order $(V_S^2/V_A^2)\omega_{\underline{k}}^s$. Now the effect of a finite growth or damping rate $\gamma_{\underline{k}}^s$ is to broaden the resonance condition by order $\gamma_{\underline{k}}^s$. So to determine the validity of the δ -function it is necessary to compare the frequency offset with the width of the resonance. If $\gamma_{\underline{k}}^s < (V_S^2/V_A^2)\omega_{\underline{k}}^s$; the δ -function will be valid for frequencies determined to the first order of V_S^2/V_A^2 and will not be automatically satisfied for all cases obeying the wavevector selection rule. In this case it will be

$$\delta\left(\frac{V_S^2}{V_A^2}\left[\frac{l_x^2 + l_y^2}{|\underline{l}|^2} l_z + \frac{(k_x - l_x)^2 + (k_y - l_y)^2}{|\underline{k} - \underline{l}|^2} (k_z - l_z) - \frac{k_x^2 + k_y^2}{|\underline{k}|^2} k_z\right]\right).$$

If $\gamma_{\underline{k}}^s > (V_S^2/V_A^2)\omega_{\underline{k}}^s$, equation (5) is invalid. However, for a significant damping rate, it could be argued that $I_{\underline{k}}^s$ will be small. Then the sound-wave sound-wave interaction, being quadratic in I^s , may be negligible compared to those involving just one sound wave.

3. Comparison With a Single-adiabatic Plasma.

The method described in section 2 was also used on the equations which govern a plasma with scalar pressure:

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{u} \times \underline{B}) \quad (12)$$

$$\frac{\partial N}{\partial t} = -\nabla \cdot (N \underline{u}) \quad (13)$$

$$Nm \left(\frac{\partial \underline{u}}{\partial t} + \underline{u} \cdot \nabla \underline{u} \right) = \frac{1}{\mu_0} (\nabla \times \underline{B}) \times \underline{B} - \nabla P \quad (14)$$

$$P = (P_0 N_0^{-\gamma}) N^\gamma \quad (15)$$

The expressions concerning the Alfvén and magnetosonic waves remain unchanged, while the velocity of the sound wave becomes

$$V_s = \sqrt{\frac{\gamma P_0}{N_0 m}}$$

The only expressions for W to be changed are those involving one or more sound waves. These are displayed in Table 5. In the regime where $B_0^2 / \mu_0 \gg P_0$, the effects of pressure are negligible in interactions involving only Alfvén and magnetosonic waves. For interactions involving sound waves, the values of W in Table 4 are identical to those in Table 5 with $\gamma = 3$. This is reasonable since the double-adiabatic equations effectively describe a plasma with $\gamma = 3$ for longitudinal sound waves along the magnetic field.

4. Interpretation.

If a 'plasmon' is defined such that the density of plasmons of type $\begin{pmatrix} \sigma \\ k \end{pmatrix}$ is $n_k^\sigma = I_k^\sigma / \hbar |\omega_k^\sigma|$, equation (5) can be written

$$\begin{aligned} \frac{d}{dt} n_k^\sigma = & 2 \gamma_k^\sigma n_k^\sigma + \sum_{\sigma_1} \sum_{\sigma_2} \int d^3l \delta(\omega_k^\sigma - \omega_{k-l}^{\sigma_1} - \omega_l^{\sigma_2}) \hbar \\ & \times W \begin{pmatrix} \sigma & \sigma_1 & \sigma_2 \\ k & k-l & l \end{pmatrix} |\omega_k^\sigma \omega_{k-l}^{\sigma_1} \omega_l^{\sigma_2}| \left[n_{k-l}^{\sigma_1} n_l^{\sigma_2} \right. \\ & \left. - n_k^\sigma n_{k-l}^{\sigma_1} \text{sign}(\omega_k^\sigma \omega_l^{\sigma_2}) - n_k^\sigma n_l^{\sigma_2} \text{sign}(\omega_k^\sigma \omega_{k-l}^{\sigma_1}) \right]. \end{aligned} \quad (16)$$

The wave-wave interaction terms are the same as in the formula (I-53) of Sagdeev and Galeev (1969). The term $n_{k-l}^{\sigma_1} n_l^{\sigma_2}$ represents the gain in $\begin{pmatrix} \sigma \\ k \end{pmatrix}$ plasmons from interactions between $\begin{pmatrix} \sigma_1 \\ k-l \end{pmatrix}$ and $\begin{pmatrix} \sigma_2 \\ l \end{pmatrix}$ plasmons,

$$\begin{pmatrix} \sigma_1 \\ k-l \end{pmatrix} + \begin{pmatrix} \sigma_2 \\ l \end{pmatrix} \rightarrow \begin{pmatrix} \sigma \\ k \end{pmatrix}$$

The term $-n_k^\sigma n_{k-l}^{\sigma_1} \text{sign}(\omega_k^\sigma \omega_l^{\sigma_2})$ represents the effect on the number of $\begin{pmatrix} \sigma \\ k \end{pmatrix}$ plasmons due to interactions with $\begin{pmatrix} \sigma_1 \\ k-l \end{pmatrix}$ plasmons. Keeping $\omega_k^\sigma > 0$, if $\omega_l^{\sigma_2} > 0$ then $\omega_k^\sigma > \omega_{k-l}^{\sigma_1}$ and the $\begin{pmatrix} \sigma \\ k \end{pmatrix}$ plasmons have more energy than the $\begin{pmatrix} \sigma_1 \\ k-l \end{pmatrix}$ plasmons. So there is a loss of $\begin{pmatrix} \sigma \\ k \end{pmatrix}$ plasmons due to the stimulated decay reaction

$$\begin{pmatrix} \sigma \\ k \end{pmatrix} + \begin{pmatrix} \sigma_1 \\ k-l \end{pmatrix} \rightarrow \begin{pmatrix} \sigma_1 \\ k-l \end{pmatrix} + \begin{pmatrix} \sigma_2 \\ l \end{pmatrix} + \begin{pmatrix} \sigma_1 \\ k-l \end{pmatrix}$$

If $\omega_l^{\sigma_2} < 0$ then $\omega_k^\sigma < \omega_{k-l}^{\sigma_1}$ and the $\begin{pmatrix} \sigma \\ k \end{pmatrix}$ plasmons stimulate the $\begin{pmatrix} \sigma_1 \\ k-l \end{pmatrix}$ plasmons to decay,

$$\begin{pmatrix} \sigma \\ k \end{pmatrix} + \begin{pmatrix} \sigma_1 \\ k-l \end{pmatrix} \rightarrow \begin{pmatrix} \sigma \\ k \end{pmatrix} + \begin{pmatrix} \sigma_2 \\ l \end{pmatrix} + \begin{pmatrix} \sigma \\ k \end{pmatrix}$$

resulting in a net gain in N_k^ϵ . As can be seen from formula (16), the gain in number of a given type of plasmon due to a certain reaction is proportional to the densities of the plasmons that collide to initiate the reaction.

Interpreting I_k^ϵ as the energy of the wave gives $\hbar |\omega_k^\epsilon|$ the characteristics of energy per plasmon. The $\delta(\omega_k^\epsilon - \omega_{k-l}^\epsilon - \omega_l^\epsilon)$ term thus ensures conservation of total plasmon energy in a loss-free medium. Similar considerations of wave-particle interactions give $\hbar k \text{sign}(\omega_k^\epsilon)$ the characteristics of momentum per plasmon, and wave-wave interaction conserves total plasmon momentum.

References

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TABLE 4. Expressions for interaction strengths W in a double-adiabatic plasma.

$$W(\underline{A} \underline{A} \underline{A}) = 0$$

$$W(\underline{M} \underline{M} \underline{M}) = \frac{9\pi}{4} \frac{\mu_0}{B_0^2} \frac{k_x^2 + k_y^2}{k^2}$$

$$W(\underline{S} \underline{S} \underline{S}) = \frac{4\pi}{3} \frac{1}{P_0}$$

$$W(\underline{M} \underline{A} \underline{A}) = 4\pi \frac{\mu_0}{B_0^2} \frac{[(k'_x k''_x + k'_y k''_y)k'_z k''_z - (k'_x k''_y - k''_x k'_y)^2]^2}{k^2 (k_x'^2 + k_y'^2) (k_x''^2 + k_y''^2)}$$

$$W(\underline{A} \underline{M} \underline{M}) = \frac{\pi}{4} \frac{\mu_0}{B_0^2} \frac{(k''_x k'_y - k'_x k''_y)^2 [k''_z k'_z (k_x'^2 + k_y'^2) + k'_z k''_z (k_x''^2 + k_y''^2) + (k''_z k'_z + k'_z k''_z) (k_x^2 + k_y^2)]^2}{k_x'^2 k_y''^2 k_z^2 (k_x^2 + k_y^2) (k_x'^2 + k_y'^2) (k_x''^2 + k_y''^2)}$$

$$W(\underline{S} \underline{A} \underline{A}) = \frac{\pi}{12} \frac{1}{P_0} \frac{(k'_x k''_x + k'_y k''_y)^2}{(k_x'^2 + k_y'^2) (k_x''^2 + k_y''^2)}$$

$$W(\underline{S} \underline{M} \underline{M}) = \frac{\pi}{12} \frac{1}{P_0} \frac{(k'_x k''_x + k'_y k''_y)^2}{(k_x'^2 + k_y'^2) (k_x''^2 + k_y''^2)}$$

$$W(\underline{S} \underline{A} \underline{M}) = \frac{\pi}{12} \frac{1}{P_0} \frac{(k''_x k'_y - k'_x k''_y)^2}{(k_x'^2 + k_y'^2) (k_x''^2 + k_y''^2)}$$

$W(\underline{S} \underline{S} \underline{M})$ and $W(\underline{S} \underline{S} \underline{A})$ are not included as these transitions are forbidden by the selection rules.

TABLE 5. Expressions for interaction strengths W in a single-adiabatic plasma.

$$W(\underline{S} \underline{S} \underline{S}) = \frac{(\gamma+1)^2}{4\gamma} \frac{\pi}{P_0}$$

$$W(\underline{S} \underline{A} \underline{A}) = \frac{1}{4\gamma} \frac{\pi}{P_0} \frac{(k'_x k''_x + k'_y k''_y)^2}{(k_x'^2 + k_y'^2) (k_x''^2 + k_y''^2)}$$

$$W(\underline{S} \underline{M} \underline{M}) = \frac{1}{4\gamma} \frac{\pi}{P_0} \frac{(k'_x k''_x + k'_y k''_y)^2}{(k_x'^2 + k_y'^2) (k_x''^2 + k_y''^2)}$$

$$W(\underline{S} \underline{A} \underline{M}) = \frac{1}{4\gamma} \frac{\pi}{P_0} \frac{(k''_x k'_y - k'_x k''_y)^2}{(k_x'^2 + k_y'^2) (k_x''^2 + k_y''^2)}$$

CONCLUSIONS

Models in which galactic cosmic rays are free to undergo diffusion across irregularities in the galactic magnetic field, which have a length scale of $10 - 30$ pc, cannot account for the slow leakage of cosmic rays out of the Galaxy. But since the energy densities of the cosmic ray gas and the galactic magnetic field are of similar magnitude, it is logical to expect that, whenever the field influences the propagation of the cosmic rays, the cosmic ray gas must also affect the configuration of the field. When the cosmic rays are treated collectively, it has been found that a net streaming motion causes hydromagnetic waves to form in the magnetic field, which have a wavelength of the order of the cosmic ray gyro-radius. These waves in turn scatter the cosmic rays of corresponding gyro-radius, thereby reducing their streaming speed.

The hydromagnetic waves have to compete against damping by various processes in the interstellar medium. The dominant linear damping process is that of collisions between the charged and the neutral particles of the interstellar medium. The waves can only begin to form when the damping rate due to this process becomes smaller than their growth rate due to cosmic ray streaming, that is at heights above the galactic plane where the neutral hydrogen density is low enough. As the wave growth rate is proportional to the density of the resonant cosmic rays, it decreases as the cosmic ray energy increases. Therefore higher energy cosmic rays can only encounter resonant waves at greater

distances from the galactic plane, where the density of neutral hydrogen is smaller. The onset of the waves at their respective heights above the galactic plane produces a reflecting boundary which prevents free diffusion out of the Galaxy.

Once the waves have grown to a finite amplitude, non-linear damping becomes important. The waves which scatter the cosmic rays become degraded into other modes which are rapidly damped. Under steady-state conditions, this causes the net cosmic ray streaming speed through the waves to increase steeply with energy. Therefore higher energy cosmic rays can escape from the Galaxy more readily than those of lower energy. The resulting energy dependence of their residence time in the Galaxy can account for the observed decrease in path length with increasing energy.

The steep decrease in residence time with energy also applies to cosmic ray electrons. This has the effect of reducing the overall change in their spectral index expected when their residence time is similar to their characteristic energy-loss time. This may be the reason why no change of order unity in the spectral index above 50 GeV has been detected.

The hydromagnetic waves which confine the cosmic rays to the Galaxy do not themselves produce compression of the interstellar medium. But the modes into which they decay by non-linear processes are compressive, and would therefore produce irregularities in the interstellar electron density of length scale short compared to that which is expected from normal interstellar motions. This may be the mechanism responsible for scattering

the radiation from pulsars, which causes the duration of each pulse to increase with the wavelength at which they are detected.

The physical processes outlined in this thesis can therefore offer explanations for a number of different phenomena. The principles mentioned here can, I believe, offer a base for future, more detailed, research into these processes. A steady-state analysis of cosmic ray streaming in a spectrum of the three hydromagnetic wave modes, subject to collisional damping, Landau damping, and non-linear mode-coupling, would be of particular interest.