Quantificational Subordination as Anaphora to a Function*

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Abstract. In [11], a semantics for cross-sentential and donkey anaphora is presented that is inspired by approaches using dependent types but couched in simple type theory with parametric polymorphism. In this paper, the approach is extended to cover quantificational subordination. It is argued that the approach enjoys advantages over existing accounts in type-theoretical semantics.

Keywords: quantificational subordination · telescoping · anaphora · polymorphism.

1 Introduction

The history of dynamic semantic theories can be seen as a series of generalizations of what sentence meaning is taken to be, in order to account for the range of constructions out of which and into which it turns out that a pronoun can be bound. The pioneering work of [12, 14, 17] was developed in order to account for cases like (1). Covariation between donkeys and the interpretation if *it* cannot straightforwardly[1] be accounted for in a static semantic system according to which the interpretation of a sentence is its truth conditions, or its set of verifying assignments, as in classical logic.

(1) Every farmer who owns a donkey beats it. [10]

However, via a generalization to the interpretation of a sentence in terms of a relation between assignments, this covariation can be accounted for. By way of example, the interpretation of (1) in the system described by [12], assuming the translation into logical formalism shown in (2), is shown in (3).

(2) $$\forall x((\mathsf{farmer}(x) \wedge \exists y(\mathsf{donkey}(y) \wedge \mathsf{own}(x,y))) \rightarrow \mathsf{beat}(x,y))$$

(3) $$\left\{ \langle g,g \rangle \mid \begin{array}{l} \left\{ \begin{array}{l} h \mid g \approx_{x,y} h \;\&\; h(x) \in [\![\mathit{farmer}]\!] \;\&\; h(y) \in [\![\mathit{donkey}]\!] \\ \quad \& \; \langle h(x), h(y) \rangle \in [\![\mathit{own}]\!] \end{array} \right\} \\ \subseteq \{ h \mid \langle h(x), h(y) \rangle \in [\![\mathit{beat}]\!] \} \end{array} \right\}$$

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[1] 'Straightforwardly', here, means by treating pronouns basically as variables. Static semantic systems can account for cases like this by treating pronouns as something else, for example concealed descriptions [7, 9].

In turn, however, the first generation of dynamic semantic theories are challenged by examples like (4)–(6). This has led to a further generalization of theories such that now, sentential semantic values are taken to be relations between sets of assignments [4, 5] or something more complex altogether [19].

(4) If you give every child a present, some child will open it. [25]
(5) Every woman bought a book. Most of them read it immediately. [19]
(6) Every player chooses a pawn. He puts it on square one. [12]

What examples like these have in common is that the anaphoric relationships that we have to capture seem to depend on functional relationships established in the first clause: between children and presents they are given in (4), between women and books they bought in (5), and between players and pieces they chose in (6). So, second-generation dynamic semantic theories look for ways of preserving those relationships, either by making any output set of assignments for the first clause such that it guarantees that the relationships are preserved [4, 5], or by allowing the update that the first sentence engenders to be stored so that it can be reintroduced at the appropriate point in the second sentence [19].

There is an alternative approach, though, which is to see the semantic values of sentences as producing the very functions themselves, not mediated via variable assignments. This is the approach taken in type-theoretical semantics (TTS).

2 Type-Theoretical Semantics

Type-theoretical semantics is a proof-theoretic variety of logical semantics in which the language of types, rather than terms, is the meaning representation language. That is to say, instead of the meaning of a sentence (say) being represented by a term (of a type), which in turn is interpreted in a model, in TTS (in the simplest case) the meaning of a sentence is a type T, where the inhabitants of T are the (intuitionistic) proofs of T. Building on previous work by [29] showing the application to sentences like (1), [25] used the intuitionistic type theory (ITT) of [23] to give a detailed analysis of many natural language phenomena, which has been followed up in recent years by several authors, summarized in [6]. Formation, introduction and elimination rules for the crucial ITT type constructors Σ and Π are shown in Fig. 1. Σ is a generalization of $\times$, from pairs to dependent pairs, and Π is a generalization of $\rightarrow$, from functions to dependent functions.

A possible TTS representation for (1) is shown in (7).[2] The sentence is taken to be true if and only if there is something of the type shown (the type is inhabited), i.e. a function mapping every tuple $\langle a, \langle b, \langle \langle c, d \rangle, e \rangle \rangle \rangle$, where b is a

[2] Following the approach of [2] rather than that of [29] or [25], i.e. treating the interpretation of e.g. *donkey* not as the type DONKEY (inhabited by donkeys), but rather as the dependent type DONKEY(x) (for any $x : e$), inhabited by proofs that x is a donkey.

$$
\begin{array}{lcc}
 & \Sigma & \Pi \\[1em]
\text{Formation} & \dfrac{x : A \quad \begin{array}{c}[x:A]^n \\ \vdots \\ B : \mathbf{type}\end{array}}{(\Sigma x : A)B : \mathbf{type}}\; n & \dfrac{x : A \quad \begin{array}{c}[x:A]^n \\ \vdots \\ B : \mathbf{type}\end{array}}{(\Pi x : A)B : \mathbf{type}}\; n \\[2em]
\text{Introduction} & \dfrac{a : A \quad b : B[a/x]}{(a, b) : (\Sigma x : A)B} & \dfrac{\begin{array}{c}[x:A]^n \\ \vdots \\ b : B\end{array}}{\lambda x.b : (\Pi x : A)B}\; n \\[2em]
\text{Elimination} & \dfrac{u : (\Sigma x : A)B}{\pi_1(u) : A} \quad \dfrac{u : (\Sigma x : A)B}{\pi_2(u) : B[\pi_1(u)/x]} & \dfrac{a : A \quad f : (\Pi x : A)B}{f(a) : B[a/x]}
\end{array}
$$

Fig. 1. Formation, introduction and elimination rules for Σ and Π types

proof that a is a farmer, d is a proof that c is a donkey, and e is a proof that a owns c, to a proof that a beats c.

(7)
$$
\begin{aligned}
&\big(\Pi u : (\Sigma x : e)(\textsc{farmer}(x) \times (\Sigma v : (\Sigma z : e)\,\textsc{donkey}(z))\,\textsc{own}(x, \pi_1(v)))\big) \\
&\qquad \textsc{beat}(\pi_1(u), \pi_1(\pi_1(\pi_2(\pi_2(u)))))
\end{aligned}
$$

As [25, Sect. 3.7] points out, the fact that in TTS a universal statement expresses a function means that the functional dependency in cases like (4) can easily be expressed, as shown in (8).

(8)
$$
\begin{aligned}
&\big(\Pi f : \big(\Pi u : (\Sigma x : e)\,\textsc{child}(x)\big) \\
&\qquad\qquad (\Sigma v : (\Sigma y : e)\,\textsc{present}(y))\,\textsc{give}\big((\mathit{you}, \pi_1(v), \pi_1(u))\big) \\
&\qquad (\Sigma w : (\Sigma z : e)\,\textsc{child}(z))\,\textsc{open}(\pi_1(w), \pi_1(\pi_1(f(w))))
\end{aligned}
$$

Given any function f mapping every pair $\langle a, b\rangle$, where b is a proof that a is a child, to a tuple $\langle\langle c, d\rangle, e\rangle$, where d is a proof that c is a present and e is a proof that you give c to a, (8) expresses the existence of a function mapping f to a tuple $\langle\langle g, h\rangle, i\rangle$ where h is a proof that g is a child and i is a proof that g opens $\pi_1(\pi_1(f(g, h))$. So TTS automatically has the ability to capture antecedent-pronoun relationships that first-generation dynamic semantic theories struggle with. This point has been futher explored in [31, 32].

Nevertheless, the direct applicability of this antecedence-to-a-function strategy is limited. In the case of (4), it was actually crucial that an appropriate argument for the function (*some child*) was present in the second sentence. In a discussion of (6), [25, p. 73] acknowledges as much in saying that 'the only way to interpret the text [...] is by treating the pronoun *he* as an abbreviation of *every player*'. Obviously, this 'abbreviation' strategy is unsatisfactory. Below, I will show how it can be improved by taking a different tack.

3 The Proposal

In [11], an implementation of the ideas behind TTS is given in simple type theory with polymorphism, once again treating the language of terms as the meaning representation language. Taking the perspective outlined in [11], equivalents for (7) and (8) are shown in (9) and (10), respectively. The reader can verify that these represent the same meanings as given in (7) and (8) according to the glosses provided, on the understanding that, following a suggestion by [25, Sect. 2.26], in turn picked up by [8], we allow eventualities (type v) to serve as 'proof objects'.[3] N.B. in what follows, in order to save space, application will be written as fa rather than $f(a)$ and left and right projections will be written as a_0 and a_1 rather than $\pi_1(a)$ and $\pi_2(a)$, respectively. In type annotations, $\times$ binds more tightly than $\to$, and both associate to the right.

(9) $\exists f^{e\times e\times v\to v}.\forall u^{e\times e\times v}.\mathsf{farmer}\,u_0 \wedge \mathsf{donkey}(u_1)_0 \wedge \mathsf{own}\,u \to \mathsf{beat}(u_0,(u_1)_0, fu)$

(10) $$\begin{aligned}&\exists f^{\tau\to e\times v}.\forall g^{\tau}.\big(\forall x^{e}.\mathsf{child}\,x \to (\mathsf{present}(gx)_0 \wedge \mathsf{give}(\mathsf{you},(gx)_0,x,(gx)_1))\big)\\ &\qquad\to \big(\mathsf{child}(fg)_0 \wedge \mathsf{open}((fg)_0,(g(fg)_0)_0,(fg)_1)\big)\end{aligned}$$

where $\tau := e \to e\times v$

Like TTS, the system described in [11] does not straightforwardly have the means to account for the examples (5)–(6). In this section an extension is described that does so, on the basis of the lexicon shown in Fig. 2.

3.1 Syntax and Semantics

The syntactic theory assumed here is a modified version of Combinatory Categorial Grammar (CCG, [27]) according to which syntactic categories are potentially parameterized by types. Lexical entries are pairs $M : C$ of meaning M and category C such that the type of M is $\mathrm{Ty}(C)$, where Ty is as defined in (11). The combinatory rules for the fragment are shown in Fig. 3. Note that we have adopted the G rule from [15] for passing pronominal dependencies (without adopting the theory of pronouns described in [15]), to which we have added a rule X of exchange.[4]

(11) For any types α, β and any categories A, B:
- $\mathrm{Ty}(\mathrm{S}_{\alpha,\beta}) = \mathrm{Ty}(\mathrm{N}_{\alpha,\beta}) = \alpha \to \beta \to t$
- $\mathrm{Ty}(\mathrm{S}) = t$
- $\mathrm{Ty}(\mathrm{NP}_{\alpha}) = \alpha \to e$
- $\mathrm{Ty}(\mathrm{NP}) = e$

[3] I only claim that eventualities (states or Davidsonian events) can be operationalized in this way, not that this is the only way to make sense of proof objects in a model-theoretic perspective. Other options worth considering would be situations [21] or even, as one reviewer suggests, to fill the proof object slot with a dummy object of the unit type (as I have done for common nouns).

[4] In the G and X rules, | could be / or \.

$$
\begin{aligned}
\textit{if} \rightsquigarrow\ & \lambda p^{\alpha\to\beta\to t}.\lambda q^{\alpha\times\beta\to\gamma\to t}.\lambda i^{\alpha}.\lambda f^{\beta\to\gamma}.\mathrm{dom}f = (\lambda b^{\beta}.pib) \\
& \qquad\qquad\qquad\qquad\qquad\qquad \wedge \forall b^{\beta}.\mathrm{dom}fb \to q(i,b)(fb) \\
& : (\mathrm{S}_{\alpha,\beta\to\gamma}/\mathrm{S}_{\alpha\times\beta,\gamma})/\mathrm{S}_{\alpha,\beta} \\
\textit{and,;} \rightsquigarrow\ & \lambda p^{\alpha\to\beta\to t}.\lambda q^{\alpha\times\beta\to\gamma\to t}.\lambda i^{\alpha}.\lambda o^{\beta\times\gamma}.pio_0 \wedge q(i,o_0)o_1 \\
& : (\mathrm{S}_{\alpha,\beta\times\gamma}/\mathrm{S}_{\alpha\times\beta,\gamma})\backslash\mathrm{S}_{\alpha,\beta} \\
\textit{a} \rightsquigarrow\ & \lambda P^{\alpha\to e\times\beta\to t}.\lambda V^{e\to\alpha\times e\times\beta\to\gamma\to t}.\lambda i^{\alpha}.\lambda u^{(e\times\beta)\times\gamma}.Piu_0 \wedge V(u_0)_0(i,u_0)u_1 \\
& : (\mathrm{S}_{\alpha,(e\times\beta)\times\gamma}/(\mathrm{S}_{\alpha\times e\times\beta,\gamma}\backslash\mathrm{NP}))/\mathrm{N}_{\alpha,e\times\beta} \\
\textit{child} \rightsquigarrow\ & \lambda i^{\alpha}.\lambda u^{e\times 1}.\mathsf{child}\,u_0 : \mathrm{N}_{\alpha,e\times 1} \\
\textit{bought} \rightsquigarrow\ & \lambda D^{\tau''\to\beta\to\gamma\to t}.\lambda x^{e}.D(\lambda y^{e}.\lambda i^{\alpha}.\lambda e^{v}.\mathsf{buy}(x,y,e)) \\
& : (\mathrm{S}_{\beta,\gamma}\backslash\mathrm{NP})/(\mathrm{S}_{\beta,\gamma}/(\mathrm{S}_{\alpha,v}\backslash\mathrm{NP})) \\
\textit{who} \rightsquigarrow\ & \lambda V^{\tau'}.\lambda P^{\tau}.\lambda i^{\alpha}.\lambda o^{e\times\beta\times\gamma}.Pi(o_0,(o_1)_0) \wedge Vo_0(i,(o_1)_0)(o_1)_1 \\
& : (\mathrm{N}_{\alpha,e\times\beta\times\gamma}\backslash\mathrm{N}_{\alpha,e\times\beta})/(\mathrm{S}_{\alpha\times e\times\beta,\gamma}\backslash\mathrm{NP}) \\
\textit{he,it} \rightsquigarrow\ & \lambda g^{\alpha\to e}.\lambda V^{e\to\alpha\to\beta\to t}.\lambda i^{\alpha}.V(gi)i : (\mathrm{S}_{\alpha,\beta}/(\mathrm{S}_{\alpha,\beta}\backslash\mathrm{NP}))^{\mathrm{NP}\alpha} \\
\textit{they} \rightsquigarrow\ & \lambda G^{\alpha\to e\to t}.\lambda V^{e\to\alpha\to\beta\to t}.\lambda i^{\alpha}.\lambda f^{e\to\beta}.\mathrm{dom}f = (\lambda y^{e}.Giy) \\
& \qquad\qquad\qquad\qquad\qquad\qquad \wedge \forall x^{e}.\mathrm{dom}fx \to Vxi(fx) \\
& : (\mathrm{S}_{\alpha,e\to\beta}/(\mathrm{S}_{\alpha,\beta}\backslash\mathrm{NP}))^{\mathrm{NPL}_{\alpha}} \\
\textit{of them} \rightsquigarrow\ & \lambda G^{\alpha\to e\to t}.\lambda i^{\alpha}.\lambda u^{e\times 1}.Giu_0 : (\mathrm{N}_{\alpha,e\times 1})^{\mathrm{NPL}_{\alpha}} \\
\textit{det}_{\text{weak}} \rightsquigarrow\ & \lambda P^{\tau}.\lambda V^{\tau'}.\lambda i^{\alpha}.\lambda f^{e\times\beta\to\gamma}.\mathrm{dom}f \subseteq (\lambda v^{e\times\beta}.Piv) \\
& \qquad \wedge \mathsf{det}(\lambda x^{e}.\exists b^{\beta}.Pi(x,b))(\lambda x^{e}.\exists b^{\beta}.\mathrm{dom}f(x,b)) \\
& \qquad \wedge \big(\forall a^{e\times\beta}.\mathrm{dom}fa \to Va_0(i,a)(fa)\big) \\
& \qquad \wedge \big(\neg\exists Y^{e\times\beta\to t}.\mathrm{dom}f \subsetneq Y \wedge Y \subseteq (\lambda v^{e\times\beta}.Piv) \\
& \qquad\qquad \wedge \forall a^{e\times\beta}.Ya \to \exists c^{\gamma}.Va_0(i,a)c\big) \\
\textit{det}_{\text{strong}} \rightsquigarrow\ & \lambda P^{\tau}.\lambda V^{\tau'}.\lambda i^{\alpha}.\lambda f^{e\times\beta\to\gamma}.\mathrm{dom}f \subseteq (\lambda v^{e\times\beta}.Piv) \\
& \qquad \wedge \mathsf{det}(\lambda x^{e}.\exists b^{\beta}.Pi(x,b))(\lambda x^{e}.\exists b^{\beta}.\mathrm{dom}f(x,b)) \\
& \qquad \wedge \big(\forall a^{e\times\beta}.\mathrm{dom}fa \to Va_0(i,a)(fa)\big) \\
& \qquad \wedge \big(\forall x^{e}.\forall b^{\beta}.(Pi(x,b) \wedge \exists c^{\beta}.\mathrm{dom}f(x,c)) \\
& \qquad\qquad\qquad \to \mathrm{dom}f(x,b)\big) \\
& \qquad \wedge \big(\neg\exists Y^{e\times\beta\to t}.\mathrm{dom}f \subsetneq Y \wedge Y \subseteq (\lambda v^{e\times\beta}.Piv) \\
& \qquad\qquad \wedge \forall a^{e\times\beta}.Ya \to \exists c^{\gamma}.Va_0(i,a)c\big) \\
& : (\mathrm{S}_{\alpha,e\times\beta\to\gamma}/(\mathrm{S}_{\alpha\times e\times\beta,\gamma}\backslash\mathrm{NP}))/\mathrm{N}_{\alpha,e\times\beta}
\end{aligned}
$$

$$[\text{close}] := \lambda p^{1\to\alpha\to t}.\exists a^{\alpha}.p * a : \mathrm{S}/\mathrm{S}_{1,\alpha}$$

where $\tau := \alpha \to e\times\beta \to t, \tau' := e \to \alpha\times e\times\beta \to \gamma \to t$ and $\tau'' := e\to\alpha\to v\to t$

Fig. 2. The lexicon

- $\text{Ty}(\text{NPL}_\alpha) = \alpha \to e \to t$
- $\text{Ty}(\text{NPL}) = e \to t$
- $\text{Ty}(A\backslash B) = \text{Ty}(A/B) = \text{Ty}(A^B) = \text{Ty}(B) \to \text{Ty}(A)$

$$\frac{f : B/A \quad a : A}{fa : B} > \qquad \frac{f : A \mid B}{\lambda g^{\text{Ty}(C)\to\text{Ty}(B)}.\lambda c^{\text{Ty}(C)}.f(gc) : A^C \mid B^C} G$$

$$\frac{a : A \quad f : B\backslash A}{fa : B} < \qquad \frac{f : (A \mid B)^C}{\lambda b^{\text{Ty}(B)}.\lambda c^{\text{Ty}(C)}.fcb : A^C \mid B} X$$

Fig. 3. CCG rules used

On the semantic side, lexical entries are also parameterized by type, so we can see Fig. 2 as in effect giving us schemata over lexical entries. The base types are e (entities), v (eventualities), 1 (unit) and t (booleans), and the type (meta)variables range over the closure of this set under the type constructors $\times$ and $\to$. Furthermore, we are assuming a partial theory of types, as described for example in [13, §4]. For each base type τ the undefined object of type τ, $\star^\tau$, is stipulated,[5] and then for complex types undefined objects are as specified in (12).[6]

(12) For any types α, β:
- $\star^{\alpha\times\beta} := \left(\star^\alpha, \star^\beta\right)$
- $\star^{\alpha\to\beta} :=$ the unique $f :: \alpha \to \beta$ such that for all $a :: \alpha$, $fa = \star^\beta$

This notion of definedness then features in the definition of dom, shown in (13) and used in the analysis of quantificational subordination (dom is mnemonic for 'domain').

(13) For any types α, β and term $f :: \alpha \to \beta, \text{dom} f := \lambda a^\alpha.fa \neq \star^\beta$.

Finally, note the general lexical entries for (strong and weak) determiners, which represents an advance on the ad-hoc entries given in [11]. In these entries, det is the meaning of the determiner understood as a relation between sets, in terms of generalized quantifier theory. In those terms, the meanings given are roughly equivalent to saying that a sentence [[D N] VP], assuming that $[\![\text{D}]\!]$ is a relation between sets of entities, $[\![\text{N}]\!]$ is a set of entities and $[\![\text{VP}]\!]$ is a relation between entities and events, expresses the existence of a function f with domain X, where X is a witness set of $[\![\text{D}]\!]$ $([\![\text{N}]\!])$,[7] such that f maps every $x \in X$ to some

[5] Space precludes proper discussion of various technical questions here; suffice to say that we do have a third truth value but we do not have a second element of the unit type.

[6] $\star$ must not be confused with $*$, which is the unique object of the unit type.

[7] That is, a witness set in the sense of [1, Sect. 4.9], i.e. a set S such that $S \subseteq [\![\text{N}]\!]$ and $\langle [\![\text{N}]\!], S\rangle \in [\![\text{D}]\!]$.

e such that $\langle x, e\rangle \in [\![\mathrm{VP}]\!]$, and there is no other set Y such that $X \subsetneq Y \subseteq [\![\mathrm{N}]\!]$ and $Y \subseteq \{y \mid \text{there is an } e \text{ such that } \langle y, e\rangle \in [\![\mathrm{VP}]\!]\}$. So, for example, *two boys jump* expresses a function f with domain X, where X is a set of two boys, such that f maps every $x \in X$ to an event of x jumping, and there is no proper superset Y of X such that Y is a set of boys and, for every $y \in Y$, there is an event of y jumping.

3.2 Derivations

We now are in a position to show derivations for some examples. First, the simple donkey sentence (1), the derivation of which is given in (14). $\mathrm{GQ}_{\alpha,\beta,\gamma,\delta}$ is an abbreviation for $\mathrm{S}_{\gamma,\delta}/(\mathrm{S}_{\alpha,\beta}\backslash\mathrm{NP})$ and $\mathrm{GQ}_{\alpha,\beta*2}$ is an abbreviation for $\mathrm{GQ}_{\alpha,\beta,\alpha,\beta}$.

(14) Derivation of (1). Let $\epsilon := e \times 1, \sigma := e \times 1 \times \epsilon$ and $\tau := e \times 1 \times e \times 1 \times v$. Then:

$$\dfrac{\overset{\textit{owns}}{(\mathrm{S}_{1\times\epsilon,e\times1\times v}\backslash\mathrm{NP})/\mathrm{GQ}_{1\times\sigma,v,1\times\epsilon,e\times1\times v}} \quad \dfrac{\overset{\textit{a}}{\mathrm{GQ}_{1\times\sigma,v,1\times\epsilon,e\times1\times v}/\mathrm{N}_{1\times\epsilon,\epsilon}} \quad \overset{\textit{donkey}}{\mathrm{N}_{1\times\epsilon,\epsilon}}}{\mathrm{GQ}_{1\times\sigma,v,1\times\epsilon,e\times1\times v}}}{\mathrm{S}_{1\times\epsilon,e\times1\times v}\backslash\mathrm{NP}}>$$

$$\dfrac{\dfrac{\overset{\textit{every}}{\mathrm{GQ}_{1\times\tau,v,1,\tau\to v}/\mathrm{N}_{1,\tau}} \quad \dfrac{\overset{\textit{farmer}}{\mathrm{N}_{1,\epsilon}} \quad \dfrac{\overset{\textit{who}}{(\mathrm{N}_{1,\tau}\backslash\mathrm{N}_{1,\epsilon})/(\mathrm{S}_{1\times\epsilon,e\times1\times v}\backslash\mathrm{NP})} \quad \overset{\textit{owns a donkey}}{\mathrm{S}_{1\times\epsilon,e\times1\times v}\backslash\mathrm{NP}}}{\mathrm{N}_{1,\tau}\backslash\mathrm{N}_{1,\epsilon}}}{\mathrm{N}_{1,\tau}}<}{\mathrm{S}_{1,\tau\to v}/(\mathrm{S}_{1\times\tau,v}\backslash\mathrm{NP})}>}{(\mathrm{S}_{1,\tau\to v})^{\mathrm{NP}_{1\times\tau}}/(\mathrm{S}_{1\times\tau,v}\backslash\mathrm{NP})^{\mathrm{NP}_{1\times\tau}}}G$$

$$\dfrac{\dfrac{\overset{\textit{beats}}{(\mathrm{S}_{1\times\tau,v}\backslash\mathrm{NP})/\mathrm{GQ}_{1\times\tau,v*2}}}{(\mathrm{S}_{1\times\tau,v}\backslash\mathrm{NP})^{\mathrm{NP}_{1\times\tau}}/(\mathrm{GQ}_{1\times\tau,v*2})^{\mathrm{NP}_{1\times\tau}}}G \quad \overset{\textit{it}}{(\mathrm{GQ}_{1\times\tau,v*2})^{\mathrm{NP}_{1\times\tau}}}}{(\mathrm{S}_{1\times\tau,v}\backslash\mathrm{NP})^{\mathrm{NP}_{1\times\tau}}}>$$

$$\dfrac{\dfrac{\overset{\text{[close]}}{\mathrm{S}/(\mathrm{S}_{1,\tau\to v})}}{\mathrm{S}^{\mathrm{NP}_{1\times\tau}}/(\mathrm{S}_{1,\tau\to v})^{\mathrm{NP}_{1\times\tau}}}G \quad \dfrac{\overset{\textit{every farmer who owns a donkey}}{(\mathrm{S}_{1,\tau\to v})^{\mathrm{NP}_{1\times\tau}}/(\mathrm{S}_{1\times\tau,v}\backslash\mathrm{NP})^{\mathrm{NP}_{1\times\tau}}} \quad \overset{\textit{beats it}}{(\mathrm{S}_{1\times\tau,v}\backslash\mathrm{NP})^{\mathrm{NP}_{1\times\tau}}}}{(\mathrm{S}_{1,\tau\to v})^{\mathrm{NP}_{1\times\tau}}}>}{\mathrm{S}^{\mathrm{NP}_{1\times\tau}}}>$$

Weak interpretation:

$$\begin{aligned}
\lambda g^{1\times\tau\to e}.\exists f^{\tau\to v}.\mathrm{dom}f \subseteq (\lambda u^{\tau}.&\mathsf{farmer}\, u_0 \wedge \mathsf{donkey}((u_1)_1)_0 \\
&\wedge \mathsf{own}(u_0, ((u_1)_1)_0, ((u_1)_1)_1)) \\
&\wedge \mathsf{every}(\lambda x^e.\mathsf{farmer}\, x \wedge \exists z^e.\mathsf{donkey}\, z \wedge \exists e^v.\mathsf{own}(x, z, e)) \\
&\quad (\lambda x^e.\exists o^{1\times e\times 1\times v}.\mathrm{dom}f(x, o)) \\
&\wedge \forall a^{\tau}.\mathrm{dom}fa \to \mathsf{beat}(a_0, (g(*, a)), (fa))
\end{aligned}$$

Strong interpretation:

$$\begin{aligned}\lambda g^{1\times\tau\to e}.\exists f^{\tau\to v}.\mathrm{dom}f \subseteq (\lambda u^{\tau}.\mathsf{farmer}\,u_0 \wedge \mathsf{donkey}((u_1)_1)_0 \\ \wedge\ \mathsf{own}(u_0, ((u_1)_1)_0, ((u_1)_1)_1)) \\ \wedge\ \mathsf{every}(\lambda x^e.\mathsf{farmer}\,x \wedge \exists z^e.\mathsf{donkey}\,z \wedge \exists e^v.\mathsf{own}(x,z,e)) \\ (\lambda x^e.\exists o^{1\times e\times 1\times v}.\mathrm{dom}f(x,o)) \\ \wedge\ (\forall a^{\tau}.\mathrm{dom}f a \to \mathsf{beat}(a_0, (g(*,a)), (fa))) \\ \wedge\ \forall x^e.\forall z^e.\forall e^v.(\mathsf{farmer}\,x \wedge \mathsf{donkey}\,z \wedge \mathsf{own}(x,z,e) \\ \wedge\ \exists v^{e\times v}.\mathrm{dom}f(x,v)) \to \mathrm{dom}f(x,z,e)\end{aligned}$$

The open abstraction λg represents anaphoric resolution, and so is resolved contextually. In this case, the function that gets the right resolution is shown in (15).

(15) $\lambda i^{1\times e\times 1\times e\times 1\times v}.(((i_1)_1)_1)_0$

The strong interpretation differs from the weak in requiring that, if some farmer x and donkey z that x owns are in the domain of f, then so are x and y for every donkey y that x owns. With the resolution shown in (15) applied, the strong interpretation shown above is equivalent to the one shown in (9). In neither case is the final clause of the definition of a determiner from Fig. 2 shown, because this maximality clause only makes a truth-conditional difference for non-monotone-increasing determiners.

We will come back to the issue of appropriate anaphoric resolution after considering our next example, (5), in (16).

(16) Derivation of (5). Let $\epsilon := e \times 1, \sigma := \epsilon \to \epsilon \times v$ and $\tau := 1 \times \sigma \times \epsilon$. Then:

$$\dfrac{\dfrac{\textit{bought}}{(\mathrm{S}_{1\times\epsilon,\epsilon\times v}\mathrm{NP})/\mathrm{GQ}_{(1\times\epsilon)\times\epsilon,v,1\times\epsilon,\epsilon\times v}} \quad \dfrac{\dfrac{\textit{a}}{\mathrm{GQ}_{(1\times\epsilon)\times\epsilon,v,1\times\epsilon,\epsilon\times v}/\mathrm{N}_{1\times\epsilon,\epsilon}} \quad \dfrac{\textit{book}}{\mathrm{N}_{1\times\epsilon,\epsilon}}}{\mathrm{GQ}_{(1\times\epsilon)\times\epsilon,v,1\times\epsilon,\epsilon\times v}}>}{\mathrm{S}_{1\times\epsilon,\epsilon\times v}\mathrm{NP}}>$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\textit{every}}{\mathrm{GQ}_{1\times\epsilon,\epsilon\times v,1,\sigma}/\mathrm{N}_{1,\epsilon}} \quad \dfrac{\textit{woman}}{\mathrm{N}_{1,\epsilon}}}{\mathrm{S}_{1,\sigma}/(\mathrm{S}_{1\times\epsilon,\epsilon\times v}\backslash\mathrm{NP})}> \quad \dfrac{\textit{bought a book}}{\mathrm{S}_{1\times\epsilon,\epsilon\times v}\mathrm{NP}}}{\mathrm{S}_{1,\sigma}}> \quad \dfrac{;}{(\mathrm{S}_{1,\sigma\times(\epsilon\to v)}/\mathrm{S}_{1\times\sigma,\epsilon\to v})\backslash\mathrm{S}_{1,\sigma}}}{\dfrac{\dfrac{\mathrm{S}_{1,\sigma\times(\epsilon\to v)}/\mathrm{S}_{1\times\sigma,\epsilon\to v}}{(\mathrm{S}_{1,\sigma\times(\epsilon\to v)})^{\mathrm{NPL}_{1\times\sigma}}/(\mathrm{S}_{1\times\sigma,\epsilon\to v})^{\mathrm{NPL}_{1\times\sigma}}}G}{((\mathrm{S}_{1,\sigma\times(\epsilon\to v)})^{\mathrm{NPL}_{1\times\sigma}})^{\mathrm{NP}_\tau}/((\mathrm{S}_{1\times\sigma,\epsilon\to v})^{\mathrm{NPL}_{1\times\sigma}})^{\mathrm{NP}_\tau}}G}<$$

$$\dfrac{\dfrac{\dfrac{\textit{read}}{(\mathrm{S}_{\tau,v}\backslash\mathrm{NP})/\mathrm{GQ}_{\tau,v*2}}}{(\mathrm{S}_{\tau,v}\backslash\mathrm{NP})^{\mathrm{NP}_\tau}/(\mathrm{GQ}_{\tau,v*2})^{\mathrm{NP}_\tau}}G \quad \dfrac{\textit{it}}{(\mathrm{GQ}_{\tau,v*2})^{\mathrm{NP}_\tau}}}{(\mathrm{S}_{\tau,v}\backslash\mathrm{NP})^{\mathrm{NP}_\tau}}>$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\overset{\textit{most}}{\mathrm{GQ}_{\tau,v,1\times\sigma,\epsilon\to v}/\mathrm{N}_{1\times\sigma,\epsilon}}}{(\mathrm{GQ}_{\tau,v,1\times\sigma,\epsilon\to v})^{\mathrm{NPL}_{1\times\sigma}}/(\mathrm{N}_{1\times\sigma,\epsilon})^{\mathrm{NPL}_{1\times\sigma}}}\,G \quad \overset{\textit{of them}}{(\mathrm{N}_{1\times\sigma,\epsilon})^{\mathrm{NPL}_{1\times\sigma}}}}{(\mathrm{S}_{1\times\sigma,\epsilon\to v}/(\mathrm{S}_{\tau,v}\backslash\mathrm{NP}))^{\mathrm{NPL}_{1\times\sigma}}}>}{(\mathrm{S}_{1\times\sigma,\epsilon\to v})^{\mathrm{NPL}_{1\times\sigma}}/(\mathrm{S}_{\tau,v}\backslash\mathrm{NP})}\,X}{((\mathrm{S}_{1\times\sigma,\epsilon\to v})^{\mathrm{NPL}_{1\times\sigma}})^{\mathrm{NP}_\tau}/(\mathrm{S}_{\tau,v}\backslash\mathrm{NP})^{\mathrm{NP}_\tau}}\,G \quad \overset{\textit{read it}}{(\mathrm{S}_{\tau,v}\backslash\mathrm{NP})^{\mathrm{NP}_\tau}}}{((\mathrm{S}_{1\times\sigma,\epsilon\to v})^{\mathrm{NPL}_{1\times\sigma}})^{\mathrm{NP}_\tau}}>$$

$$\dfrac{\overset{\textit{every woman bought a book;}}{((\mathrm{S}_{1,\sigma\times(\epsilon\to v)})^{\mathrm{NPL}_{1\times\sigma}})^{\mathrm{NP}_\tau}/((\mathrm{S}_{1\times\sigma,\epsilon\to v})^{\mathrm{NPL}_{1\times\sigma}})^{\mathrm{NP}_\tau}} \quad \overset{\textit{most of them read it}}{((\mathrm{S}_{1\times\sigma,\epsilon\to v})^{\mathrm{NPL}_{1\times\sigma}})^{\mathrm{NP}_\tau}}}{\dfrac{((\mathrm{S}_{1,\sigma\times(\epsilon\to v)})^{\mathrm{NPL}_{1\times\sigma}})^{\mathrm{NP}_\tau}}{\vdots\ [\mathrm{close}] \\ (\mathrm{S}^{\mathrm{NPL}_{1\times\sigma}})^{\mathrm{NP}_\tau}}}>$$

Weak interpretation:

$$\begin{aligned}
&\lambda g^{\tau\to e}.\lambda G^{1\times\sigma\to e\to t}.\exists W^{\sigma\times(e\times 1\to v)}.\mathrm{dom}W_0 = (\lambda o^{e\times 1}.\mathsf{woman}\, o_0)\\
&\qquad\wedge (\forall u^{e\times 1}.\mathrm{dom}W_0 u \to (\mathsf{book}((W_0 u)_0)_0 \wedge \mathsf{read}(u_0,((W_0 u)_0)_0,(W_0 u)_1)))\\
&\qquad\wedge \mathrm{dom}W_1 \subseteq (\lambda v^{e\times 1}.G(*,W_0)v_0)\\
&\qquad\wedge \mathsf{most}(\lambda x^e.G(*,W_0)x)(\lambda x^e.\mathrm{dom}W_1(x,*))\\
&\qquad\wedge \forall a^{e\times 1}.\mathrm{dom}W_1 a \to \mathsf{read}(a_0, g(*,W_0,a),(W_1 a))
\end{aligned}$$

In this case there are two open abstractions to resolve, for *it* and *of them* respectively. The appropriate resolutions are shown in (17) and (18) respectively.

(17) $\lambda n^{1\times(e\times 1\to(e\times 1)\times v)\times e\times 1}.\big(((n_1)_0\,(n_1)_1)_0\big)_0$

(18) $\lambda m^{1\times(e\times 1\to(e\times 1)\times v)}.\lambda x^e.\exists i^1.\mathrm{dom}\, m_1(x,i)$
$\equiv \lambda m^{1\times(e\times 1\to(e\times 1)\times v)}.\lambda x^e.\mathrm{dom}\, m_1(x,*)$

With those resolutions in place, the (weak) interpretation of the sentence is:

$$\begin{aligned}
&\exists W^{\sigma\times(e\times 1\to v)}.\mathrm{dom}W_0 = (\lambda o^{e\times 1}.\mathsf{woman}\, o_0)\\
&\qquad\wedge (\forall u^{e\times 1}.\mathrm{dom}W_0 u \to (\mathsf{book}((W_0 u)_0)_0 \wedge \mathsf{read}(u_0,((W_0 u)_0)_0,(W_0 u)_1)))\\
&\qquad\wedge \mathrm{dom}W_1 \subseteq \mathrm{dom}W_0 \wedge \mathsf{most}(\lambda x^e.\mathrm{dom}W_0(x,*))(\lambda x^e.\mathrm{dom}W_1(x,*))\\
&\qquad\wedge \forall a^{e\times 1}.\mathrm{dom}W_1 a \to \mathsf{read}(a_0,(W_0 a)_0,(W_1 a))
\end{aligned}$$

This expresses the existence of a pair of functions, the first f mapping every woman to a book she bought and the second mapping most things x in the domain of f (i.e. most women) to an event of x reading fx.

The anaphoric resolution functions (15), (17) and (18) are all natural resolution functions (NRFs) according to the definition shown in (19).

(19) The set of NRFs is the smallest set such that, for any types α, β and γ and any terms $F :: \alpha \to \beta \to \gamma, G :: \beta \to \gamma$ and $H :: \alpha \to \beta$:
- $\lambda a^\alpha.a$ is an NRF

- $\lambda A^{\alpha\times\beta}.A_0$ is an NRF
- $\lambda A^{\alpha\times\beta}.A_1$ is an NRF
- $\lambda X^{\alpha\times\beta\to t}.\lambda a^{\alpha}.\exists b^{\beta}.X(a,b)$ is an NRF
- $\lambda X^{\alpha\times\beta\to t}.\lambda b^{\alpha}.\exists a^{\beta}.X(a,b)$ is an NRF
- $\lambda f^{\alpha\to\beta}.\mathrm{dom}f$ is an NRF
- $\lambda a^{\alpha}.G(Ha)$ is an NRF if G and H are NRFs
- $\lambda a^{\alpha}.Fa(Ha)$ is an NRF if F and H are NRFs

Of course, this definition has been formulated post-hoc. Nevertheless, there is a naturalness to it: a resolution function can select projections, sets of projections, and the domain of a function, and can apply one thing it selects to another when the types match.

4 Telescoping

To deal with example (6), however (repeated below), additional machinery is required.

(6) Every player selects a pawn. He puts it on square one.

The reason that examples like (6), sometimes called 'telescoping' [26], pose a particular challenge is that there is nothing in the second sentence that is explicitly anaphoric on the function expressed by the first sentence. In contrast, the seemingly equivalent sentence (20) could be dealt with in basically the same way as (5), by making *they* anaphoric to the domain of the function mapping every player to the pawn he selects, i.e. the set of players.

(20) Every player selects a pawn. They put it on square one.

4.1 Covert Subordination

In order to deal with examples like (6), [26] posits the existence of a covert adverbial at the start of the second sentence, meaning something like 'in every case'. We will adopt essentially the same strategy. In (21) we postulate a silent subordinating operator that, when applied to the usual sentential conjunction shown in Fig. 2, gives (22) as an alternative, subordinating, sentential conjunction.

(21) $$\lambda C^{(\alpha\to(\beta\to\gamma)\to t)\to(\alpha\times(\beta\to\delta)\to(\beta\to\delta)\to t)\to\alpha\to(\beta\to\gamma)\times(\beta\to\delta)\to t}.\\ \lambda p^{\alpha\to(\beta\to\gamma)\to t}.\lambda q^{\alpha\times\beta\times(\beta\to\gamma)\to\delta\to t}.Cp\big(\lambda o^{\alpha\times(\beta\to\gamma)}.\lambda f^{\beta\to\delta}.\mathrm{dom}o_1=\mathrm{dom}f\\ \wedge\forall b^{\beta}.\mathrm{dom}fb\to q(o_0,b,o_1)(fb)\big)$$

(22) $$;_{\mathrm{sub}}\rightsquigarrow\lambda p^{\alpha\to(\beta\to\gamma)\to t}.\lambda q^{\alpha\times\beta\times(\beta\to\gamma)\to\delta\to t}.\lambda i^{\alpha}.\lambda o^{(\beta\to\gamma)\times(\beta\to\delta)}.pio_0\\ \wedge\mathrm{dom}o_0=\mathrm{dom}o_1\wedge\forall b^{\beta}.\mathrm{dom}o_1b\to q(i,b,o_0)(o_1b)\\ :(\mathrm{s}_{\alpha,(\beta\to\gamma)\times(\beta\to\delta)}/\mathrm{s}_{\alpha\times\beta\times(\beta\to\gamma),\delta})\backslash\mathrm{s}_{\alpha,\beta\to\gamma}$$

We can now give a (summarized) derivation for (6), in (23).

(23) Derivation of (6). Let $\epsilon := e \times 1, \sigma := \epsilon \times v, \tau := \epsilon \to \sigma$ and $\omega := 1 \times \epsilon \times \tau$. Then:

every player selects a pawn
$\vdots$
$\mathrm{S}_{1,\tau}$ $\quad$ $\vdots$ sub $\quad (\mathrm{S}_{1,\tau\times(\epsilon\to v)}/\mathrm{S}_{\omega,v})\backslash \mathrm{S}_{1,\tau}$

$$\frac{\dfrac{\dfrac{\dfrac{\mathrm{S}_{1,\tau} \quad (\mathrm{S}_{1,\tau\times(\epsilon\to v)}/\mathrm{S}_{\omega,v})\backslash \mathrm{S}_{1,\tau}}{\mathrm{S}_{1,\tau\times(\epsilon\to v)}/\mathrm{S}_{\omega,v}}<}{(\mathrm{S}_{1,\tau\times(\epsilon\to v)})^{\mathrm{NP}_\omega}/(\mathrm{S}_{\omega,v})^{\mathrm{NP}_\omega}}G}{((\mathrm{S}_{1,\tau\times(\epsilon\to v)})^{\mathrm{NP}_\omega})^{\mathrm{NP}_\omega}/((\mathrm{S}_{\omega,v})^{\mathrm{NP}_\omega})^{\mathrm{NP}_\omega}}G \qquad \overset{\textit{he puts it on square one}}{\vdots}\;((\mathrm{S}_{\omega,v})^{\mathrm{NP}_\omega})^{\mathrm{NP}_\omega}}{((\mathrm{S}_{1,\tau\times(\epsilon\to v)})^{\mathrm{NP}_\omega})^{\mathrm{NP}_\omega}}>$$

$\vdots$ [close]
$(\mathrm{S}^{\mathrm{NP}_\omega})^{\mathrm{NP}_\omega}$

Interpretation:

$$\lambda g^{\omega\to e}.\lambda h^{\omega\to e}.\exists F^{\tau\times(\epsilon\to v)}.\mathrm{dom}F_0 = (\lambda o^{e\times 1}.\mathsf{player}\, o_0)$$
$$\wedge\, (\forall u^{\epsilon}.\mathrm{dom}F_0 u \to (\mathsf{pawn}((F_0u)_0)_0 \wedge \mathsf{select}(u_0, ((F_0u)_0)_0, (F_0u)_1)))$$
$$\wedge\, \mathrm{dom}F_0 = \mathrm{dom}F_1 \wedge \forall a^{\epsilon}.\mathrm{dom}F_1 a \to \mathsf{put}(h(i,a,F_0), g(i,a,F_0), \mathsf{onsq1}, (F_1a))$$

We need to apply this formula to the resolutions for *it* and *he* respectively. The appropriate resolutions are shown in (24) and (25) respectively. They are both NRFs as defined in (19).

(24) $\lambda n^{1\times(e\times 1)\times((e\times 1)\to(e\times 1)\times v)}.\big(\big((n_1)_1\,(n_1)_0\big)_0\big)_0$
(25) $\lambda n^{1\times(e\times 1)\times((e\times 1)\to(e\times 1)\times v)}.((n_1)_0)_0$

With those resolutions in place, the sentence is interpreted as shown below.[8]

$$\exists F^{\tau\times(\epsilon\to v)}.\mathrm{dom}F_0 = (\lambda o^{e\times 1}.\mathsf{player}\, o_0)$$
$$\wedge\, (\forall u^{\epsilon}.\mathrm{dom}F_0 u \to (\mathsf{pawn}((F_0u)_0)_0 \wedge \mathsf{select}(u_0, ((F_0u)_0)_0, (F_0u)_1)))$$
$$\wedge\, \mathrm{dom}F_0 = \mathrm{dom}F_1 \wedge \forall a^{\epsilon}.\mathrm{dom}F_1 a \to \mathsf{put}(a_0, ((F_0a)_0)_0, \mathsf{onsq1}, (F_1a))$$

This expresses the existence of a pair of functions, the first f of which maps every player to a pawn he chooses, and the second of which maps every player x to an event of x putting fx on square one.

The subordinating operator defined in (21), which is hypothesized to apply covertly in cases like (6), can also apparently be overt, as for example in (27).

[8] As a result of the semantics assumed for *every*, the part of this formula corresponding to *every player selects a pawn* can be seen as a Skolemized version of (26).

(26) $\forall u^{e\times 1}.\mathsf{player}\, u_0 \to \exists v^{(e\times 1)\times v}.\mathsf{pawn}(v_0)_0 \wedge \mathsf{select}(u_0, (v_0)_0, v_1)$

But since we have the Skolem function F_0, in the next conjunct the pronoun can be represented by $((F_0a)_0)_0$, achieving the desired binding. A reviewer points out that this gives the analysis presented a certain resemblance to approaches that use epsilon terms to model indefinites and donkey pronouns (e.g. [18, 24, 28]), an observation for which I'm grateful.

(27) Every player chooses a pawn. He always puts it on square one.

And in fact, *always* in this sense seems to be just a special case of a variety of possible subordinators, as we can see from (28)–(29).

(28) Every player chooses a pawn. He usually puts it on square one.
(29) Every player chooses a pawn. He rarely puts it on square one.

The subordinating sentential conjunctions that apply in each of these cases are special instances of (30), where, as we have seen, det can at least be every (for *always*), most (for *usually*) or few (for *rarely*).

(30)
$$\lambda p^{\alpha\to(\beta\to\gamma)\to t}.\lambda q^{\alpha\times\beta\times(\beta\to\gamma)\to\delta\to t}.\lambda i^{\alpha}.\lambda o^{(\beta\to\gamma)\times(\beta\to\delta)}.pio_0 \\ \wedge \mathrm{dom}o_1 \subseteq \mathrm{dom}o_0 \wedge \mathsf{det}(\mathrm{dom}o_0)(\mathrm{dom}o_1) \\ \wedge \forall b^{\beta}.\mathrm{dom}o_1 b \to q(i,b,o_0)(o_1 b)$$

4.2 Constraints

Understood as a covert operator, the subordinator defined in (21) will cause vast overgeneration if allowed to apply too freely. It would, for example, allow the interpretation of *he* to covary with players in (31), but this is surely undesirable.

(31) ? Every player chooses a pawn. He has brown hair.

Empirical evidence is presented in [34] to show that a major constraint on this kind of quantificational subordination is the discourse relation that holds between the two sentences, where discourse relations are defined as in the framework of segmented discourse representation theory (SDRT, [22]). In that framework, the relation is taken to be *Narration* in the case of (6), but *Background* in the case of (31), for example. We can adopt this insight and take the discourse relation holding between sentences S_1 and S_2 as as a constraint on the applicability of a covert subordinator to the sentential conjunction coming between S_1 and S_2. I leave open the question of precisely the level at which this constraint should be stated.

5 Comparison with TTS

Existing discussion of quantificational subordination in TTS [31, 32] has focused almost exclusively on examples like (4). Now, the intended interpretations of (5) and (6) can certainly be *represented* in TTS, as shown in (32) and (33) respectively.[9]

(32)
$$(\Sigma f : (\Pi v : (\Sigma x : e)\textsc{woman}(x))) \\ (\Sigma u : (\Sigma y : e)\textsc{book}(y))\textsc{choose}(\pi_1(v), \pi_1(\pi_1(u))) \\ \mathit{Most}(\lambda x.\textsc{woman}(x))(\lambda x.\textsc{woman}(x) \times \textsc{read}(x, \pi_1(\pi_1(f(x)))))$$

[9] See [30, 33] for discussion of generalized quantifiers like `Most` in TTS.

(33) $$(\Sigma f : (\Pi v : (\Sigma x : e)\textsc{player}(x))) \\ (\Sigma u : (\Sigma y : e)\textsc{pawn}(y))\textsc{select}(\pi_1(v), \pi_1(\pi_1(u))) \\ (\Pi v : (\Sigma x : e)\textsc{player}(x))\textsc{put}(\pi_1(v), \pi_1(\pi_1(f(v))), \textsc{onsq1})$$

But the question is, how easily can those representations be derived compositionally? In the variety of TTS that has dealt most fully with the issues of compositionality and anaphora resolution, Dependent Type Semantics (DTS, [2, 3, 20, 31, 32]), pronouns are represented by @ terms, as defined in (34).

(34) $$\frac{A : \textbf{type} \quad A \text{ true}}{(@ : A) : A}\,@$$

A term $@ : A$ is well-typed in a context, then, iff it is provable that there is some term of type A in that context. Anaphoric resolution then amounts to replacing the @ term with some $a : A$ at the point of type checking. In the version of DTS presented in [2, 3], pronouns express @ terms that are functions from left contexts to entities, much like the system presented in this paper. An example is given in (35).[10]

(35) $$\textit{it} \rightsquigarrow \lambda P^{e\to\alpha\to\textbf{type}}.\lambda c^{\alpha}.P((@_i : \alpha \to e)(c))(c) : \textsc{s}/(\textsc{s}\backslash\textsc{np})$$

So much for *it*, what about *they* or *of them*? A clue is given as to how this would work is given in [31, p. 133], where it is stated that 'the type annotation of the @-term associated with *they* requires a predicate and a proof term of the cardinality condition'. However, no type annotation is actually given, so it is difficult to judge this claim. *Most* requires its first argument to be a predicate, i.e. (in DTS) something of type $e \to \textbf{type}$. It is reasonable to assume, then, that the @-term associated with *of them* should encode a function from left contexts to predicates. In the case of (5) the relevant @ term would therefore be as shown in (36).

(36) $$@_j : \begin{pmatrix} (\Pi v : (\Sigma x : e)\textsc{woman}(x))) \\ (\Sigma u : (\Sigma y : e)\textsc{book}(y)) \\ \textsc{choose}(\pi_1(v), \pi_1(\pi_1(u))) \end{pmatrix} \to e \to \textbf{type}$$

Without some equivalent of dom as discussed above, there is no way to get the right predicate, $\lambda x.\textsc{woman}(x)$, out of the left context in (36).

Alternatively, one could eschew the functions-from-a-left-context approach to pronouns (as in [31, 32]) and instead adopt a simpler perspective according to which *of them* would (presumably) be translated as $@_k : e \to \textbf{type}$. But then, in order to encode a subordinating conjunction to deal with cases like (6), something like dom is still needed. Standard sentential conjunction in this version of DTS is shown in (37); (38) shows an attempt to formulate a subordinating conjunction, but we don't know what type to put in in place of the question mark.

[10] Each @ term bears a unique index, which in the following examples are (arbitrarily) chosen as i, j, k.

(37) $\lambda p.\lambda q.(\Sigma u : p)q$
(38) $\lambda p.\lambda q.(\Sigma u : p)(\Pi v : ?)q$

Furthermore, the system presented in this paper benefits from a general definition of determiner meanings, as shown in Fig. 2 and discussed in Sect. 3.1. In contrast, while there has been some work on constructive generalized quantifiers appropriate for TTS [30, 33], this is still at the ad-hoc, case-by-case stage, and there has been no discussion of monotone-decreasing determiners, for example.

6 Discussion and Future Work

At face value, many examples of anaphoric dependencies look like they depend on functional relationships established in discourse. We have shown that quite some progress in capturing those anaphoric dependencies can be made by taking that impression seriously, i.e. by having sentences denote functions and allowing those functions to serve as pronominal antecedents. We hope to have shown that this is a viable alternative to placeholders like sets of assignment functions, from which those functions have to be extracted.

One obvious next place to look for applications of this approach is in the treatment of 'paycheck' pronouns; for example, the interpretation of (39) according to which the second sentence is interpreted as equivalent to *every fourth grade boy hates his (own) mother.*

(39) Every third grade boy loves his mother. But every fourth grade boy hates her. [16]

Once again, the interpretation of the second sentence gives the impression of depending on a functional relationship established in the first, namely between people and their mothers. And, in fact, many accounts of paycheck pronouns do in fact take that approach, either by saying that the relevant function is contextually salient [7] or, in a recent TTS analysis [32], that it is introduced by the presupposition of the possessive pronoun in the first sentence.

Another obvious avenue for extension is the phenomenon of modal subordination, as exemplified by (40).

(40) A wolf might come in. It would eat you first. [26]

Of course, this would require an account of modality, which has not been offered yet.

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