An Examination of Certain Counterpart Accounts

James Milford

Christ Church
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Supervised by Professor Timothy Williamson

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Abstract

This work is closely tied to the metaphysical and formal work of David Lewis on modality. First, to Lewis' reduction of modality in terms of his modal realism and mature counterpart machinery. Second, to Lewis' 1968 formal account (and its subsequent developments) employing a notion of counterparthood, an account that is detachable from Lewis' full-blooded metaphysics. This work is primarily an examination of both of these accounts, as well as five other formal accounts that, inspired by Lewis' work, employ a notion of counterparthood.

The thesis divides into four chapters. The first chapter presents Lewis' reduction of modality in terms of his modal realism and mature counterpart machinery. It includes a presentation of an account that I take to be a straightforward regimentation of Lewis' sketch of joint possibilities in 1986.

The second chapter considers certain issues for this account. These concern existence, the combination of existence and iterated modality, and "contingent identity". The chapter ends with a discussion of alternatives to the machinery employed by the account.

The third chapter begins with a presentation and discussion of Lewis' 1968 formal account. That account employs formal machinery to deliver counterpart characterisations of natural language modal claims. First, Lewis presents a formal first-order theory, *counterpart theory*. Second, Lewis provides a method for translating natural language modal claims into sentences of the language of counterpart theory. This process has two steps. First, Lewis assumes a regimentation of natural language modal claims into sentences of "the language of quantified modal logic". Second, Lewis provides a translation scheme that maps sentences of this language of quantified modal logic to sentences of the language of counterpart theory.

This combination of (i) a theory and (ii) a translation scheme (that maps sentences of the language of quantified modal logic to sentences of the language of counterpart theory) generates a semantics for the language of quantified modal logic. Given this, I turn to a more foundational matter in the final part of the third chapter: comparing the usual direct model-theoretic

approach and this indirect approach (presenting a theory and translation scheme) to presenting a semantics for a formal language.

The fourth chapter raises issues for five formal accounts in the literature that in some way appeal to a notion of counterparthood: Hazen's 1977 stipulational semantics; Russell's 2013 counterpart semantics; Bacon's 2014 counterpart semantics; Graff Fara's 2008 relative-sameness semantics; and Meyer's 2013 counterpart account.

Chapter 1

Lewisian Modal Metaphysics

This chapter presents Lewis' reduction of modality in terms of his modal realism and mature counterpart machinery. It includes a presentation of an account that I take to be a straightforward regimentation of Lewis' sketch of joint possibilities in 1986.

1.1 Lewisian Modal Realism

Employing the terminology of Adams (1974: 224), let us distinguish a certain kind of metaphysical account which posits possible worlds: a *possibilist* account. According to possibilism, actual reality does not exhaust reality *simpliciter*. The *actual world* is actual reality. Other worlds are not part of actual reality.

Perhaps the most famous example of a possibilist account is Lewisian modal realism (LMR). An early partial characterisation of LMR is spelt out in section 4.1 of Lewis 1973. We have that:

(i) There is a certain entity, a world, that we each inhabit, namely the actual world (actuality).

(ii) There are other worlds, each of which is of the same ontological kind as our world.

(iii) The actuality of our world is nothing more than a relation between it and entities that inhabit it. The world we inhabit is actual relative to us in virtue of us inhabiting it. As a first pass, we might say that for any world w and entity x that inhabits w, w is actual relative to x.[1]

[1] As we will see, there is a complication with this first-pass if we posit *universals*. Put

Lewis subsequently makes clearer the nature of a world. As a preliminary, let us consider Lewis' background metaphysics. To introduce it, I propose the following framework:

Some entities are *located*, where to be located is to have a location in some *space*.[2]

For example, we might hold that the space we are located in is a four-dimensional space, *spacetime*, three dimensions of which are the usual spatial dimensions, and the fourth of which is time. The spacetime is fixed in the sense that there is never any change to the size of the spacetime, nor locations of entities in the spacetime.

Next, we turn to the nature of a space. Let S be a space, and consider the following (based on 1986: 76, f.n. 55):

(Substantivalism about S) There are S-regions.

On this position, there are distance relations between S-regions. Further, S-regions are straightforwardly S-located: S-region r is located at S-region r^+ iff r is r^+. There are two main versions of (Substantivalism about S).

According to the first version, the only S-located entities are S-regions. Let this be *super substantivalism* about S.

According to the second version, there are S-located non-S-region entities. Such entities are S-located in virtue of standing in occupation relations to S-regions. Let this be *dualist substantivalism* about S.

Dualist substantivalism about S itself divides into different versions depending on what S-located non-S-region entities are allowed. For one example, we might allow a wide variety of S-located non-S-region entities: donkeys, particles, and so on.

Standing in contrast to (Substantivalism about S) is:

(Relationalism about S) There are no S-regions.

On this position, S is just a system of distance relations between certain (non-S-region) entities.

Lewis does hold that the space we are located in is a four-dimensional space, *spacetime*, three dimensions of which are the usual spatial dimensions, and the

simply, if one posits universals, then one might also hold that for some universal u and distinct (from each other) worlds w_1 and w_2, u inhabits each of w_1 and w_2. In which case, each of w_1 and w_2 would be actual relative to u.

[2]Henceforth, by *located* I mean exactly located.

fourth of which is time. The spacetime is fixed in the sense that there is never any change to the size of the spacetime, nor locations of entities in the spacetime.

We distinguish between *tensed* temporal notions, and *tenseless* ones. Tensed temporal notions (past, present, future, now, has always been, will always be, ...) presuppose a certain vantage point "within time". Whereas tenseless temporal notions (before, after, earlier than, later than, ...) are relational, and do not presuppose such a vantage point. Following McTaggart (1908), let us call tensed and tenseless temporal notions *A-notions* and *B-notions*, respectively. Lewis' metaphysics is reductionist about every A-notion.

Lewis holds that (translated for our framework) there are other spaces. As with the space we are located in, each of these spaces is fixed. However, the nature of the spaces may vary in some respects. For example, it may be that for some space S, super substantivalism about S holds, whereas for some distinct space S^+, relationalism about S^+ holds (1986: 76, f.n. 55). For a further example, whereas our space is relativistic, there may be a four-dimensional space, three dimensions of which are the usual spatial dimensions, and the fourth of which is time, but which is Newtonian in nature (1986: 74).

Lewis holds that an issue arises here. Our usual notion of spatiotemporal relatedness concerns something that holds between entities located in the space we are located in. But given the varied nature of spaces, Lewis' concern (translated for our framework) is that for some space S, our usual notion of spatiotemporal relatedness does not concern anything which holds between any pair of entities located in S (1986: 74-75).

Lewis attempts to resolve this issue by instead talking of being "analogically spatiotemporally related" (1986: 76). On our framework, we have: entities b and c are AS related iff for some space S, each of b and c is located at some S-region. This last biconditional is provided only as an elucidation of the notion of being AS related (not as an analysis).

So much for the framework. One point to bear in mind when carving up Lewis' ontology is that he holds "that mereological composition is unrestricted: any old [plurality] of things has a mereological sum. Whenever there are some things, no matter how disparate and unrelated, there is something composed of just those things" (1986: 211). To emphasise the point, he adds: "I really do mean *absolutely* unrestricted - for instance, I see no bar to composition of sets with individuals, or particulars with universals, or cats with numbers." (1986: 212 f.n. 9). That is, for any plurality of any size of any entities, there is an entity that is the sum of exactly the members of the plurality.

Before we turn to Lewis' ontology, let us first turn to the particular-universal distinction. To introduce this distinction, consider: Alice is human. This (let

us suppose) is true. But ultimately in virtue of what features of reality is it true? According to one metaphysical position, as well as *particular* entities, entities that are not in the business of being instantiated, there are *universals*. Universals *are* in the business of being instantiated. Barring some potential degenerate cases, a universal has multiple instantiations. A monadic universal is instantiated by single entities. An n-ary universal ($n > 1$) is instantiated by n-sized collections of entities (perhaps ordered in a certain way).

According to the metaphysical position, the claim that Alice is human is true in virtue of certain entities (or groups of entities) instantiating certain universals. Here, versions of the position may differ on the universals that there are. On some versions, for example, there is a universal of humanness, in which case the claim is true in virtue of the particular Alice instantiating the universal humanness.

According to some versions of the position, some universals are instantiated by some universals. On such a version, we distinguish between (i) first-order universals, which are those universals only instantiated by particulars, and (ii) higher-order universals, which are those universals instantiated by some universals.[3] Further, some versions of the position analyse the notion of a particular in terms of the notion of a universal. For example, one might take a particular to be a "bundle of universals".[4]

On Lewis' usual *nominalist* ontology, there are no universals. That is, there are only particulars. We start with the particular *urelements* (the non-sets). Call them *concreta*. These are the non-derivatively located entities, plus their sums.

Next, we add to the ontology an iterative hierarchy of sets built out of the concreta. In the postscript to his 1968 paper printed in 1983b (1983b: 40), Lewis holds that the part-whole relation does not apply to sets. However, given the composition principle adopted in 1986, it is clear that Lewis there allows sets to stand in the parthood relation. In particular, he allows that they may be parts of certain bizarre fusions that are not themselves sets (for example, the fusion of a concretum with its singleton). In 1991, Lewis goes further, allowing for sets to have parts. In particular, the parts of a set are exactly all its subsets. Assuming at least the 1986 position, we now have that the nominalist ontology contains (i) concreta, (ii) sets, and (iii) those bizarre non-sets that have a set as a part.

In 1983a and section 1.5 of 1986, Lewis entertains adding universals into his ontology. He remains neutral between a certain theory of universals and other theories (each of which does not posit universals). The account of universals Lewis presents does not analyse the notion of a particular in terms of the notion of a universal, but it does hold:

[3] See Armstrong 1978b, Part 7 as an example.
[4] See Armstrong 1978a, Part 3 for a discussion.

(Mereological Immanence) For a given (monadic) universal, for each entity e that instantiates the universal, the universal is a part of e. Generally: for a given n-ary universal, for each n-many entities $e_1, ..., e_n$ that instantiate the universal, the universal is a part of the sum of $e_1, ..., e_n$.

Now, in the ontology we have (i) concreta, (ii) sets, (iii) universals, and (iv) a host of bizarre sums.

So much for the preliminaries. Given Lewis' comments in, *inter alia*, the postscript to his 1968 paper printed in 1983b and, in particular, section 1.6 of his 1986, we have that a world is a certain kind of concretum. Namely:

(W) Concretum b is a world iff both

(a) for all concrete parts p_1, p_2 of b, p_1 and p_2 are AS related, and

(b) for every concrete part p of b, every concretum AS related to p is a part of b.

Notice we have:

Lemma 1. No concretum is part of more than one world.

Proof. For *reductio*, let w and v be distinct worlds, and b a concretum that is a part of each of w and v. As (i) v is a part of v and (ii) w is a part of w, it follows by (W.a) that each of v and w are AS related to b. By (W.b), w is a part of v and v is a part of w. So, w and v are identical. $\perp$ ■

Now, we hold:

Entity b is *in* world w iff b is a part of w.

Entity b *inhabits* world w iff b is a part of w.

For every concretum b and world w, w is actual relative to b iff b is a part of w.[5]

The *possible* concreta are a subcollection of the concreta. We have:

Concretum b is possible iff for some world w, b is a part of w. Otherwise, b is impossible (1986: 211).

Concretum b is a *trans-world* concretum iff there is a pair of concrete parts p_1, p_2 of b which are not parts of the same world.

That is, the impossible concreta are exactly the transworld concreta.

[5]If we extend this to the universals, we have that relative to some universals u, multiple worlds are actual.

1.1.1 Common Sense and Reduction

LMR flies in the face of common sense. First, it posits a large number of spaces beyond the spacetime we are located in. Second, these other spaces are considered not part of actual reality (for us). Our actual reality is not reality *simpliciter*. Lewis (1986: section 2.8) views this as a serious cost, but holds that the theoretical virtues of LMR are worth it. Chief amongst these virtues is that LMR is supposedly reductionist about every modal notion. According to Lewis, no plausible rival account achieves this. Whether an acceptable reduction of modality can be given within the LMR framework is therefore of central importance. The first two chapters of this work concern this topic.

1.2 Counterparts

In section 1.2 of 1986, Lewis presents some simple instances of reducing modality (1986: 5-7):

> Possibly there are blue swans iff for some world w, at w, some swans are blue.
>
> Necessarily all swans are birds iff for every world w, at w, all swans are birds.

Here, Lewis introduces the sentential operator 'at world w' into the language of his metaphysics, which we are told mainly functions by restricting the domains of certain quantifiers in its scope. For example, in the above cases, the operator restricts the quantifiers 'some swans' and 'all swans' to the w-swans.

Generalising somewhat, let 'ϕ' be a sentence of our metaphysics that (i) is purely qualitative and (ii) features no modal locutions.[6] We hold:

> Possibly ϕ iff for some world w, at w, ϕ.
>
> Necessarily ϕ iff for every world w, at w, ϕ.

Notice that possibility and necessity are introduced *simpliciter* (not relative to a world). If we are concerned with a restricted flavour R of modality, we introduce the notion of a binary R-accessibility relation among the worlds, and hold (1986: 7):

> [At w, R-possibly ϕ] iff for some world w^+ R-accessible from w, at w^+, ϕ.
>
> [At w, R-necessarily ϕ] iff for every world w^+ R-accessible from w, at w^+, ϕ.

[6] Informally, a sentence is purely qualitative iff it does not mention a specific entity.

Reducing modality *de re* concerning concreta is more complex than the above reductions. Suppose Alice is blonde, and consider: Alice could have been brunette. Given (W), Alice is in exactly one world, namely our world, and there she is blonde. So, how do we reduce this modality?

To handle this, Lewis employs a *counterpart* relation, a binary relation which holds between concreta. Lewis analyses this relation in terms of qualitative similarity. Schematically: concretum b_1 in world w_1 is a counterpart of concretum b_2 in world w_2 iff b_1 is (i) *appropriately similar* to b_2, and (ii) at least as similar to b_2 as any other concretum in w_1 is (1968: 115-116). As a result, Lewis (ibid.) holds that (a) the counterpart relation is reflexive; (b) the counterpart relation need not be symmetric; (c) the counterpart relation need not be transitive; and (d) an entity need not have at most one counterpart per world.[7]

In 1968, Lewis holds that there is a single counterpart relation. Further, he takes it as axiomatic that a concretum has at most one counterpart in its own world, namely itself (1968: 114).

Given this, if we wanted to employ the 'at world w' machinery, we might try, as a first-pass: (i) [Alice could have been brunette] iff for some world w, at w, Alice is brunette; and (ii) [at world w, Alice is brunette] iff some w-counterpart of Alice is brunette.

1.2.1 Multiple Counterpart Relations

That there is a counterpart relation, and that modality *de re* concerning concreta reduces in terms of it, is seemingly required of Lewis, assuming we want to render *de re* claims like the above true. However, Lewis adds further complexities to the counterpart account.

First, Lewis (for example, 1971 and 1986) allows for multiple counterpart relations among the concreta. There seem to be three motivations for allowing multiple counterpart relations.

The first is that LMR seemingly requires it. This is because on Lewis' schematic analysis of a counterpart relation, we seemingly have a relation for

[7]Note that in 1968, Lewis introduces the counterpart relation independently of LMR. There, Lewis talks of the relation holding between *things*. However, the analysis of the relation there is seemingly adopted in 1986 (as holding between what I have called *concreta*). In this account of Lewis' modal metaphysics, I shall take a counterpart relation to *only* hold between concreta. This position is supported by Lewis' comments in his postscript to his 1968 paper published in 1983b, where he states that his 1968 account should only concern "individuals", and it is clear that in this postscript, Lewis is using 'individual' to denote concreta. That said, in an early discussion of counterparthood (1973: 39-40), Lewis holds, as a special case, that "the abstract entities that exist alike from the standpoint of all worlds, but inhabit none, are their own unique counterparts at all worlds".

each combination of (i) a kind of similarity and (ii) a standard of appropriate similarity. In order to have a single counterpart relation (which modality *de re* concerning concreta reduces in terms of), we would have to privilege one combination of a kind of similarity and a standard of appropriate similarity above the others. Without motivation for privileging a combination, it would be *ad hoc* and inelegant.

The second reason is that it can handle cases involving shifting intuitions about modality *de re* concerning concreta (Lewis 1986: section 4.5). In what follows, all contexts considered are taken to be of our world $w_@$.

Suppose we have a solid oak desk chair (call it 'Sturdy') and consider:

(1) Sturdy could have been made out of iron.

This is often taken to be a case of shifting intuitions: for some contexts c, it seems (1) (relative to c) is true, whereas for other contexts c, it seems (1) (relative to c) is false. Further, perhaps for some contexts c, we are unable to decide whether (1) (relative to c) is true or false. Having multiple counterpart relations allows us to accommodate this. First, we hold that context determines counterpart relation. Second, for some contexts c, for example, a counterpart relation concerning size and shape might be determined. In which case, (1) (relative to c) is rendered true in virtue of there being a world w_1 such that Sturdy has a size-and-shape w_1-counterpart made out of iron. For some other contexts c, a counterpart relation concerning what something is "made of" might be determined. In which case, (1) (relative to c) is rendered false in virtue of there not being a world w_1 such that Sturdy has a made-of w_1-counterpart made out of iron. We might also allow that some contexts underdetermine the relevant counterpart relation. For some of those contexts c, we might then hold that (1) (relative to c) is neither true nor false, or, alternatively, is both true and false.

Notice that although our metaphysical account is able to accommodate the variances in truth-value across contexts, it cannot causally explain the shifting intuitions.

The third reason for multiple counterpart relations, and a main motivation of Lewis in 1971, is that positing multiple counterpart relations allows us to accommodate cases of what we might hesitantly call "contingent identity". A common introduction to such kinds of cases is Gibbard's material constitution case (1975):

> Suppose that (i) at the exact time of its creation, a lump of clay, called 'Lumpl', is formed into a statue, called 'Goliath', and (ii) after a short while, the clay statue is vaporised. Here, the lump and the statue are (a) created at exactly the same time, and (b) destroyed at exactly the same time.

The lump and the statue occupy exactly the same time period, and are spatially coincident throughout, seemingly sharing, for each instant, exactly the same physical properties. Given this, some take it to be natural to identify the lump and the statue. However, it could have been that instead of being vaporised, the statue was instead squished into a ball. Here, it seems that the lump of clay survives the squashing but the statue does not. So, in this counterfactual scenario, it seems that we cannot identify the lump and the statue. Some thus hold the following:

(2) Lumpl and Goliath are identical but could have been distinct.[8]

We can accommodate this. To render (2) (relative to a given context c) true, we hold that the single concretum d of our world $w_@$ has two counterparts in a world w_2 (the squashing world), one the statue s, and one the lump of clay l, where $l \neq s$. However, there are two ways to spell this account out. One way is just to say that we have a single counterpart relation, and that each of s and l bears it to d. A natural alternative is to employ a multiple counterpart relation analysis. For example, we hold that d has exactly one lump w_2-counterpart, namely l, and exactly one statue w_2-counterpart, namely s. The value of 'Lumpl' (perhaps in conjunction with context) determines the lump counterpart relation, and the value of 'Goliath' (perhaps in conjunction with context) determines the statue counterpart relation.

Note that in our overall account we need not stop at context and names determining the relevant counterpart relation. For example, perhaps distinct definite descriptions with the same designatum outside a modal setting may determine distinct counterpart relations for the modal setting. And, perhaps, as Lewis writes: "in certain modal predications, the appropriate counterpart relation is selected not by the subject term but by a special clause" (1971: 210). For example, compare (i) 'I, regarded as the ϕ, could have been ψ', and (ii) 'I, regarded as the χ, could have been ψ'. The key is how a concretum is being considered.

Should we prefer the multiple counterpart relation approach over the single counterpart relation approach to account for the above "contingent identity" case? It seems that we should, as the single counterpart relation approach ostensibly faces counterexamples.

To see this, assume LMR and that there is a single counterpart relation. Now, suppose that Alice in $w_@$ is an only-child, and there is a world w_3 containing multiple Alice counterparts.

For one example, let w_3 be a world that has an initial segment qualitatively identical to the initial segment of $w_@$ up until a time shortly before Alice's

[8]The notion of distinctness I employ in the first two chapters requires that for x and y to be distinct, x and y must exist.

mother gives birth. In w_3, the zygote instead splits in the womb, and each of the resultant twins is a counterpart of Alice.

For another example, let w_3 be a world featuring so-called "two-way eternal recurrence". For a world featuring one-way eternal recurrence, "there is a beginning of time and there is a first epoch, a second epoch just like the first, and so *ad infinitum*" (Lewis 1986: 63). Each epoch is qualitatively identical to every other epoch, except with regards to the properties they satisfy concerning their place in the ordering of the epochs. For worlds featuring two-way eternal recurrence, "there is no last epoch and no first, the epochs are ordered like the integers rather than the natural numbers" (ibid.). Here, Lewis holds, each epoch is qualitatively identical to every other epoch. For this example, let us suppose that our world $w_@$ does not feature recurring epochs, and each epoch of w_3 is qualitatively identical to $w_@$ except with regards to properties the epoch satisfies concerning being one of a recurrence of epochs. Unless our counterpart relation is extreme, we presumably have infinitely-many w_3-counterparts of Alice (at least one from each epoch).[9]

Now, suppose that Alice is also known as 'A' by some of her peers, and there is a context c such that it is a mutually salient part of the conversational common-ground that the speaker associates the same descriptions with 'Alice' as they do with 'A'. That is, it is a mutually salient part of the conversational common-ground that the speaker thinks of the names in exactly the same way. Consider the following, which closely resembles the above (2):

(3) Alice and A are identical but could have been distinct.

We seemingly want (3) (relative to c) to come out as false. Crucially, it seems that this should come out as false because the speaker associates the same descriptions with the two coreferential names. It is rather like the following, which seemingly should also come out as false, at least relative to contexts where the context doesn't somehow enrich the pronoun with extra descriptive information:

(3^+) Alice and herself are identical but could have been distinct.

However, if there is a single counterpart relation, and what renders the above (2) (relative to a context) true is there being a world w^* such that concretum d in $w_@$ has multiple w^*-counterparts, then it seems we must have that (3) (relative to c) is true in virtue of there being a world w^* such that Alice has multiple w^*-counterparts.

[9] For an example of an extreme counterpart relation for which this does not hold, consider one for which something in w_3 fails to be appropriately similar to a concretum in $w_@$ in virtue of being situated in one of an eternal recurrence of epochs (or being a sum of concreta from different epochs, that is, a trans-epoch concretum). In which case, Alice would have no w_3-counterparts of this sort.

So, the multiple counterpart relation approach to our "contingent identity" case seems preferable to the single counterpart relation approach here. Crucially, the multiple counterpart relation approach is able to capture the phenomenon that (2) (relative to a relevant context) is true in virtue of the single concretum being considered in distinct ways. I return to this topic in the second chapter after additional machinery has been introduced.

As an addition to these three motivations, notice that a multiple counterpart relation analysis can also accommodate cases of shifting intuitions about such "contingent identity" cases. For example, for some contexts c, it seems (2) (relative to c) is true, whereas for other contexts c, it seems (2) (relative to c) is false. Further, perhaps for some contexts c, we are unable to decide whether (2) (relative to c) is true or false. We can account for this. First, we hold that for some contexts c, the names 'Lumpl' (together with c) and 'Goliath' (together with c) determine the same counterpart relation, and in virtue of this (2) (relative to c) is false. In addition, we hold that there are some contexts c where 'Lumpl' (together with c) determines a lump counterpart relation, 'Goliath' (together with c) determines a statue counterpart relation, and (2) (relative to c) is true in virtue of d having l as a lump w_2-counterpart and s as a statue w_2-counterpart. More generally, given the counterpart machinery alone, we presumably hold that for any context c, if 'Lumpl' (together with c) determines a distinct counterpart relation C_1 from the counterpart relation C_2 that 'Goliath' (relative to c) determines, then (2) (relative to c) is true if there is a world w, a C_1 w-counterpart d_1 of d, and a C_2 w-counterpart d_2 of d such that $d_1 \neq d_2$. Finally, there may be some contexts in which the relevant counterpart relations are underdetermined. For some of these contexts c, perhaps (2) (relative to c) is neither true nor false, or, alternatively, both true and false.

Before moving on, it is worth considering cases of what we might hesitantly call "contingent distinctness". Consider a variant of Gibbard's case:

> Suppose that (i) at the exact time of its creation, a lump of clay, called 'Lumpl', is formed into a statue, called 'Goliath', and (ii) after a short while, the clay statue is squashed into a ball.

Here it seems that the clay survives the squashing, but the statue does not. So, it seems that the clay and the statue are distinct. However, it could have been that instead of being squashed, the statue was instead vaporised. In which case, they would occupy exactly the same time period, and be spatially coincident throughout, seemingly sharing, for each instant, exactly the same physical properties. Some thus take it to be natural to identify the lump and the statue in this counterfactual scenario. And thus some hold:

(4) Lumpl and Goliath are distinct but could have been identical.

We can accommodate this. To render (4) (relative to a given context c) true, we hold that the distinct concretum l (the lump) and s (the statue) of our world $w_@$ share a counterpart d in a world w_4 (the vaporising world). Again, there are two ways to spell this account out. One way is just to say that we have a single counterpart relation, and d bears it to each of l and s. Again, a natural alternative is to employ a multiple counterpart relation analysis. For example, we hold that d is a lump counterpart of l, and a statue counterpart of s. 'Lumpl' (perhaps in conjunction with context) determines the lump counterpart relation, and 'Goliath' (perhaps in conjunction with context) determines the statue counterpart relation.

How should we handle the case? For the sake of systematicity, with the counterpart machinery alone, it seems we should allow that (relative to some contexts c), 'Lumpl' (together with c) determines a distinct counterpart relation C_1 from the relation C_2 that 'Goliath' (together with c) determines, and (4) (relative to c) is true in virtue of there being a world w and w-concretum that is both a C_1 counterpart of l and a C_2 counterpart of s.

However, it seems to me that the single counterpart relation approach for "contingent distinctness" does not have straightforward counterexamples in the same way that the single counterpart relation approach for "contingent identity" does. We may hold that (relative to some contexts c), 'Lump' (together with c) determines the same counterpart relation C as 'Goliath' (together with c) does, and (4) (relative to c) is true in virtue of there being a world w and w-concretum that is both a C counterpart of l and a C counterpart of s.

Interestingly, if all this is correct, it seems there is one way for the counterpart machinery to render "contingent identity" claims true, but two ways for it to render "contingent distinctness" claims true. Again, I return to this topic in the second chapter after additional machinery has been introduced.

1.2.2 Further Complexity

The added complexity does not end with multiple counterpart relations. Let us employ the term 'individual' to refer to a role that may be played by an entity of any kind. It is the role of subject of predication. Further, let an *individual-qualitative-role* be an exhaustive qualitative property. Individual-qualitative-roles are partitional. Let us say that according to a simple account of possibility/necessity *de re*, it is analysed simply in terms of existential/universal quantification over worlds, and world representation *de re* (representation of what individuals are playing what individual-qualitative-roles). Further, with regards to representation *de re*, for a world w and individual d, w represents d as a certain way iff it does not

represent d as not that way.[10] [11]

The LMR theorist may embrace or reject this account. In 1986, Lewis rejects it. First, Lewis formulates the doctrine of haecceitism explicitly in terms of worlds.[12] According to Lewis, there is a *haecceitistic difference* between two worlds iff those worlds (i) are qualitatively identical, but (ii) differ in their representation *de re* (representation of what individuals are playing what individual-qualitative-roles) (1986: 221). This definition can be straightforwardly extended so that it is compatible with more accounts of worlds: there is a haecceitistic difference between two worlds iff those worlds (i) have the same representation of the qualitative, but (ii) differ in their representation *de re*. We have (ibid.):

(Lewis Haecceitism) There are haecceitistic differences between worlds.

(Lewis Anti-Haecceitism) There are no haecceitistic differences between worlds.

Lewis continues by holding that according to anti-haecceitism, a world's representation *de re* holds (solely) in virtue of its qualitative character (1986: 223). Again, this definition can be straightforwardly extended so that it is compatible with more accounts of worlds:

(Lewis Anti-Haecceitism*) A world's representation *de re* holds (solely) in virtue of its representation of the qualitative.

However, notice that (Lewis Anti-Haecceitism) does not imply (Lewis Anti-Haecceitism*). It is consistent (though perhaps not very plausible) to hold that (i) there are no haecceitistic differences between worlds and yet (ii) world representation *de re* is not (solely) determined by world representation of the qualitative. For example, suppose that (a) no two distinct (from each other) worlds have the same representation of the qualitative [in which case, (Lewis Anti-Haecceitism) trivially holds], but (b) world representation *de re* is determined in part by something other than world representation of the qualitative. That said, (Lewis Anti-Haecceitism*) does imply (Lewis Anti-Haecceitism).

Corresponding to (Lewis Anti-Haecceitism*) is presumably:

(Lewis Haecceitism*) A world's representation *de re* does not hold (solely) in virtue of its representation of the qualitative.

[10] Here, I am thinking of failing to exist as a certain way. For every individual b and world w, w represents b as a certain way (it might just be that w represents b as failing to exist).

[11] This last condition rules out there being a property ϕ such that (i) w represents d as satisfying ϕ, and (ii) w represents d as not satisfying ϕ.

[12] The term 'haecceitism' was coined by Kaplan 1975.

(Lewis Haecceitism) implies (Lewis Haecceitism*), but (Lewis Haecceitism*) does not imply (Lewis Haecceitism).

Lewis holds (Lewis Anti-Haecceitism*). Even if there are qualitatively identical worlds w_1 and w_2 (something which Lewis does not rule out), their representation *de re* (of concreta) would be the same. This is because, as it were, for every world w, for every concretum b in w and counterpart relation C, w_1 and w_2 would provide the same C counterparts of b.

However, Lewis holds that there are intuitions ostensibly supporting (Lewis Haecceitism). For a simple example (based on 1986: 231), consider the following case. Suppose that Alice is an only child, but things could have been such that her mother instead gave birth to twins. Alice could have been the first-born twin, and Alice could have been the second-born twin. For these two distinct ways Alice could have been, there is no difference in the qualitative character of the setting in which she would be. How do we account for this? (Lewis Haecceitism) seems a natural fit: we hold that there are two distinct worlds which differ only in representation *de re*, one on which Alice plays the individual-qualitative-role of the first-born, and one on which Alice plays the individual-qualitative-role of the second-born.

However, according to LMR, there is a single world w with the twins (or multiple qualitatively identical worlds with the same representations). To deal with this, in short, Lewis holds: "possibilities are not always possible worlds" (1986: 230).

To start, Lewis (1986: 230-232) holds that a possibility for a given concretum b amounts to a counterpart of b (not necessarily a world). For our case, we now have that the two concreta in the world w, namely the first-born twin and second-born twin (each w-counterparts of Alice), are two distinct possibilities for Alice. The one world gives us one possibility for our qualitative setting as a whole, but two distinct possibilities for Alice.

The benefit of multiple counterpart relations applies here too: we can appeal to the multiplicity of counterpart relations in order to accommodate shifting intuitions concerning certain ostensibly-haecceitism-supporting cases. For example, suppose that Alice has exactly one sibling, Lorina. Could things have been qualitatively exactly as they actually are, but Alice plays the individual-qualitative-role that Lorina actually plays? Here, there seems to be a case of shifting intuitions again. The LMR theorist has the resources to accommodate this. The single world $w_@$, with Alice and Lorina as parts, is such that, for one "extraordinarily generous" counterpart relation, Alice has Lorina as a counterpart (1986: 232). In a context in which this counterpart relation is determined, the claim (relative to the context) is correct. In a context which determines a stricter counterpart relation (one for which a concretum has exactly one counterpart in its own world, namely itself), the

claim (relative to the context) is incorrect. Further, perhaps in some contexts, the counterpart relation is underdetermined. For some of those contexts c, we might then hold that the claim (relative to c) is neither true nor false, or, alternatively, is both true and false.

1.2.3 Tuple-Counterparts

Lewis goes further than just allowing for possibilities for single concreta (considered in single ways). He also briefly talks of there being *joint possibilities* for sequences of concreta that are all parts of the same world (1986: 232-233, 259, 52).[13] Joint possibilities for such sequences "are themselves sequences of appropriate length, with all their terms drawn from a single world" (1986: 232). In his sketch of joint possibilities, Lewis presents the following case. Suppose our world $w_@$ is a "spatially symmetric world. The entire history of one side is replicated on the other side" (1986: 232-233). There is also a world w which starts out the same as $w_@$, but at some time, one of the halves is destroyed. In $w_@$, there is lewis, l, and his twin concretum, t, in the other half of the world. In w, there is d_1 in the destroyed half, and d_2 in the other half. Lewis writes that "both [d_1 and d_2] are qualitative counterparts of both [l and t]; further, the relations between [d_1 and d_2] are like the relations between [l and t]; and so there is a natural sense in which both the pairs [$< d_1, d_2 >$ and $< d_2, d_1 >$] are qualitative counterparts of the pair [$< l, t >$]" (1986: 233). The pairs $< d_1, d_2 >$ and $< d_2, d_1 >$ are "two different joint possibilities" for $< l, t >$.

Although Lewis does not employ the term, I suggest we talk of *tuple-counterparthood*. We hold that for every $n \geq 1$, there is an n-tuple-counterpart relation R that holds between n-tuples of concreta (where each such tuple does not feature repeats, and all its members are from one world).

Lewis ostensibly adopts a two-part analysis of tuple-counterparthood: for $n \geq 1$, and worlds w and w^+, an n-tuple $< c_1, ..., c_n >$ of distinct (from each other) w^+-concreta is a tuple-counterpart of an n-tuple $< b_1, ..., b_n >$ of distinct (from each other) w-concreta iff (i) for every $1 \leq m \leq n$, c_m is a counterpart of b_m, and (ii) the relations between $c_1, ..., c_n$ are similar to the relations between $b_1, ..., b_n$.

Perhaps we can spell out this condition (ii) to more closely match Lewis' analysis of usual counterparthood. That is: (ii.a) the relations between

[13] Lewis states that his sketch of joint possibilities is based on Hazen 1977 (his dissertation) and Hazen 1979 (a paper that effectively reprints some of the dissertation material). Central to this work is Hazen's stipulational semantics. This semantics differs notably from the sketch of joint possibilities Lewis presents, but Hazen, as a heuristic to compare Lewis' 1968 counterpart account and his stipulational semantics, does briefly talk of a "double counterpart relation" which holds between pairs of individuals (1977: 122). I turn to stipulational semantics in detail later.

$c_1, ..., c_n$ are appropriately similar to the relations between $b_1, ..., b_n$, and (ii.b) the relations between $c_1, ..., c_n$ are as similar to the relations between $b_1, ..., b_n$ as the relations between any other distinct (from each other) w^+-concreta $d_1, ..., d_n$ are. For brevity, for condition (ii), I shall talk of $< c_1, ..., c_n >$ being *relationally similar* to $< b_1, ..., b_n >$.

Given the analysis of tuple-counterparthood, for $n \geq 1$, we ostensibly have a multiplicity of n-tuple-counterpart relations. Each n-tuple-counterpart relation R is associated with (i) a tuple $<C_1, ..., C_n>$ of counterpart relations, and (ii) a kind of relational similarity that holds between n-tuples of concreta. For $n \geq 1$, a tuple-counterpart relation R whose associated tuple of counterpart relations is $<C_1, ..., C_n>$ and whose associated kind of relational similarity is S, and worlds w and w^+, an n-tuple $< c_1, ..., c_n >$ of distinct (from each other) w^+-concreta is an R tuple-counterpart of an n-tuple $< b_1, ..., b_n >$ of distinct (from each other) w-concreta iff (i) for every $1 \leq m \leq n$, c_m is a C_m counterpart of b_m, and (ii) $< c_1, ..., c_n >$ is S relationally similar to $< b_1, ..., b_n >$.[14]

A possibility for n-many distinct (from each other) concreta $b_1, ..., b_n$ (all from one world) ($n \geq 1$) (considered in the way corresponding to tuple-counterpart relation R) amounts to an R tuple-counterpart of $< b_1, ..., b_n >$.

Why adopt the machinery of tuple-counterpart relations over the usual counterpart relations between concreta? One motivation is that tuple-counterpart relations present a way to avoid certain problem cases for Lewis's position up to this point.[15] For example, suppose Alice, a part of our world $w_@$, is an only child to her mother, Sarah, who is also an only child. However, there is a world w_5 that exhibits so-called two-way eternal recurrence. For this example, let us suppose that $w_@$ does not feature recurring epochs, and each epoch of w_5 is qualitatively identical to $w_@$, except with regards to properties each epoch of w_5 satisfies concerning being in a recurrence of epochs.

With the exception of a few extreme counterpart relations, for each counterpart relation, there seem to be infinitely-many w_5-counterparts of Sarah (at least one from each epoch), and infinitely-many w_5-counterparts of Alice (at least one from each epoch).[16] Indeed, for the sake of argument, let us say that a standard counterpart relation C is such that each of Sarah and Alice has exactly one C w_5-counterpart per epoch e, namely that concretum in

[14] Lewis alludes to this granularity in his sketch at 1986: 259.

[15] This problem was originally identified in Hazen's dissertation: 1977: 109-111 (reprinted in Hazen 1979). Hazen talks of Caesar and his death.

[16] Again, for an example of an extreme counterpart relation for which this does not hold, consider one for which something in w_5 fails to be appropriately similar to a concretum in $w_@$ in virtue of being situated in one of an eternal recurrence of epochs (or being a sum of concreta from different epochs, that is, a trans-epoch concretum). In which case, each of Sarah and Alice would have no w_5-counterparts of this sort.

e playing exactly the qualitative role they are playing, with the exception of qualitative properties concerning being situated in an eternal recurrence of epochs. Consider:

(5) It could have been that Sarah was not Alice's mother.

At the very least, it seems we want (5) (relative to *some* contexts where standard counterpart relations are determined) to not be rendered true by the presence of w_5. However, if we only have the machinery of usual counterpart relations between concreta, it seems difficult to present a systematic treatment of modality *de re* concerning concreta which provides this.

To see this, suppose that context c is such that 'Sarah' (together with c) determines standard counterpart relation C_1, and 'Alice' (together with c) determines standard counterpart relation C_2. So, there seem to be infinitely-many C_1 w_5-counterparts of Sarah (exactly one from each epoch), and infinitely-many C_2 w_5-counterparts of Alice (exactly one from each epoch). Let us fix on s_1 and a_1 of a single epoch (C_1 w_5-counterpart of Sarah, and C_2 w_5-counterpart of Alice, respectively), and s_2 and a_2 of a single epoch (C_1 w_5-counterpart of Sarah, and C_2 w_5-counterpart of Alice, respectively). Here, s_1 is the mother of a_1, and s_2 is the mother of a_2.

Employing just counterpart relations, it seems difficult to present a systematic treatment of modality *de re* concerning concreta such that (5) (relative to c) is not rendered true in virtue of the presence of w_5. This is because there is a C_1 w_5-counterpart of Sarah and a C_2 w_5-counterpart of Alice such that the former is not the mother of the latter (namely, s_1 and a_2).

By employing tuple-counterpart relations, we can have (5) (relative to c) at least not rendered true by the presence of w_5. For example, we might suppose that (5) (relative to c) determines a tuple-counterpart relation R whose associated tuple of counterpart relations is $<C_1, C_2>$, and whose associated kind of relational similarity is such that a pair $< x, y >$ of concreta is relationally similar to <Sarah, Alice> only if x and y are part of the same epoch. Now, (5) (relative to c) is not rendered true by the presence of w_5, as any R w_5-tuple-counterpart $< x, y >$ of <Sarah, Alice> is such that x is the mother of y (given the kind of relational similarity and the fact that C_1 and C_2 are standard).

The granularity of tuple-counterpart relations provides us flexibility. For example, one might think that in the Sarah and Alice case, there should be *some* contexts c in which standard counterpart relations are determined and (5) (relative to c) is rendered true by the presence of w_5. However, one also wants to hold, as before, that there are some contexts c in which standard counterpart relations are determined and (5) (relative to c) is not rendered true by the presence of w_5. This can be achieved, for example, by having some

context c that determines a tuple-counterpart relation R^+ whose associated tuple of counterpart relations contains only standard ones, and whose associated kind of relational similarity is more liberal, so that <Sarah, Alice> can have a pair $< x, y >$ of w_5-concreta as an R^+ tuple-counterpart even though x is not the mother of y. Then, as before, holding that some context c determines a tuple-counterpart relation R whose associated tuple of counterpart relations contains only standard ones, and whose associated kind of relational similarity is stricter, so that <Sarah, Alice> cannot have a pair $< x, y >$ of w_5-concreta as an R tuple-counterpart where x is not the mother of y.

1.2.4 Repetitions

It seems we must extend the above notion of tuple-counterparthood by allowing for tuple-counterparts that are tuples with repetitions.[17] One motivation for this extension is to ensure our reduction of modality *de re* concerning concreta is systematic. If some modality *de re* concerning concreta is to be reduced in terms of tuple-counterparts, it seems that all modality *de re* concerning concreta should be. Reconsider our reverse Gibbard case and (4):

(4) Lumpl and Goliath are distinct but could have been identical.

Recall that our strategy to render this true (relative to certain contexts) was to hold that the distinct lump l and statue s in $w_@$ share a counterpart d in a world w_4. However, if all modality *de re* concerning concreta should be reduced in terms of tuple-counterparts, we need to account for this case in terms of tuple-counterparts. Instead, then, we hold that (4) (relative to certain contexts) is rendered true in virtue of $< l, s >$ having $< d, d >$ as a relevant w_4-tuple-counterpart.

Going beyond this, it seems we must also extend the notion of tuple-counterparthood by allowing that tuples of concreta (all from one world) themselves with repetitions may be *borne* tuple-counterpart relations.[18] One motivation for this is again systematicity. If some modality *de re* concerning concreta is to be reduced in terms of tuple-counterpart relations, it seems we should reduce modality *de re* concerning a given concretum, but where that concretum is being considered in multiple ways, in terms of tuple-counterparts. For example, consider the original Gibbard case and (2):

(2) Lumpl and Goliath are identical but could have been distinct.

Our strategy to render this true (relative to certain contexts) was to hold that the single concretum d in $w_@$ has two distinct counterparts l and s in a world

[17] In his sketch, Lewis (1986: 233) presents an example where a tuple with repetitions serves as a possibility for a tuple of concreta.

[18] In his sketch, Lewis (1986: 259) presents an example where a tuple of concreta with repetitions has a possibility.

w_2 (where l is a lump counterpart of d, and s is a statue counterpart of d). However, for the sake of systematicity, we need to account for this case in terms of tuple-counterparts. Instead, then, we hold that (2) (relative to certain contexts) is rendered true in virtue of $< d, d >$ having $< l, s >$ as a tuple-counterpart (where the tuple-counterpart relation in play is such that its associated tuple of counterpart relations is $<\mathrm{C}_{\mathrm{lump}}, \mathrm{C}_{\mathrm{statue}} >$, and its associated kind of relational similarity allows a non-gappy non-identity pair to be relationally similar to an identity pair).

1.2.5 Gaps

Finally, Lewis also allows that a joint possibility for a tuple of concreta may be a tuple with "gaps" (1986: 233) (call them 'gappy tuples'). To capture this in our current tuple-counterparts framework, we extend the notion of tuple-counterparthood to allow gappy tuples of concreta to be tuple-counterparts. Lewis presents the following case (ibid.). Again, suppose our world $w_@$ is a "spatially symmetric world. The entire history of one side is replicated on the other side". There is also a world w with no replication, but which is qualitatively identical to each side of $w_@$ (except in regards to properties of being one-half of a spatially symmetric world). In $w_@$, there is Lewis, l, and his twin concretum, t, in the other half of the world. In w, there is concretum d, which is a w-counterpart of each of l and t. Here, Lewis is seemingly concerned with a single counterpart relation (let it be C), but presumably, for any counterpart relation, d is a w-counterpart of l of that kind iff d is a w-counterpart of t of that kind. After all, counterparthood is analysed in terms of qualitative similarity. Now, Lewis holds that the pair $< l, t >$ has three joint possibilities: $< d, * >, < *, d >$, and $< d, d >$.[19] Again, Lewis is seemingly only concerned with a single counterpart relation here. Indeed, Lewis is seemingly only concerned with a single pair-counterpart relation (let it be R^*) (whose associated pair of counterpart relations is <C, C>). We therefore seemingly have that $< l, t >$ has three R^* w-tuple-counterparts. These tuple-counterparts correspond to the following possibilities for l and t together (considered in the way corresponding to R^*): l exists, but t does not (set in the qualitative setting of w); t exists, but l does not (set in the qualitative setting of w); and both l and t exist (and play the same individual-qualitative-role) (set in the qualitative setting of w).

There is a question here of exactly what a gappy tuple is. A common position is to hold that for $n \geq 0$, a non-gappy n-tuple is a total function from $\{1, ..., n\}$. Equivalently, it is an indexed family whose index set is $\{1, ..., n\}$. Given this, one position is to hold that for $n \geq 1$, a gappy n-tuple is a partial function from $\{1, ..., n\}$. Equivalently, it is an indexed family whose index set is a proper subset of $\{1, ..., n\}$. This is Lewis' position (1986: 233, f.n. 25).

However, notice that a consequence for this account of gappy tuples is that the

[19] Following Lewis, I use '*' to mark a gap in the tuple.

entities serving as tuple-counterparts (hence as possibilities) do not have an arity built in. For example, a pair $< d_1, d_2 >$ of concreta (all from one world) can serve as an n-tuple-counterpart for any $n \geq 2$. Lewis himself is aware of this (ibid.).

Alternatively, we can ensure the entities serving as tuple-counterparts (hence as possibilities) have an arity built in. One way to achieve this is to hold that for every $n \geq 0$, an n-tuple is a total function from $\{1, ..., n\}$. Equivalently, it is an indexed family whose index set is $\{1, ..., n\}$. However, we say that the gap value is the empty set.

1.3 Conclusion

This is where Lewis' sketch of his counterpart account ends. I take it that the account I have developed so far is a straightforward regimentation of his sketch. Of course, even this account is only skeletal. For one deficiency, there is a requirement to provide a more general account of how modal statements involving contingent existence are to be reduced.

Chapter 2

Examining Lewisian Modal Metaphysics

In this chapter, I discuss certain issues for the counterpart account developed in the first chapter, an account that I take to be a straightforward regimentation of Lewis' sketch of joint possibilities in 1986. These issues concern existence, the combination of existence and iterated modality, and "contingent identity". Finally, I consider alternatives to the tuple-counterpart machinery itself.

2.1 Four Levels

There seem to me to be four levels in play in Lewis' modal metaphysics. At the zeroth level, we have qualitative features of concreta. At the first, we have counterparts and tuple-counterparts. At the second, we have possibilities. And at the third, we have the possible and the necessary.

The idea, it seems, is that on a comprehensive specification of the metaphysics, the levels are intimately related. Each level above the zeroth is analysed in terms of the level directly below it. Derivatively, for any two levels, there is an analysis of the higher in terms of the lower. Of central importance is the analysis of the third level (the possible and the necessary) in terms of the zeroth (qualitative features of concreta).

However, Lewis does not provide a comprehensive specification. To capture (within the four level framework) what he does specify, let us say that an *empty tuple* is a tuple that features only gaps. Further, let a possibility for concreta $b_1, ..., b_n$ (all from one world) together ($n \geq 1$) where not all of them exist be a *gappy possibility* for the concreta together. Finally, let a possibility for concreta $b_1, ..., b_n$ (all from one world) together ($n \geq 1$) where none of them exist be an *empty possibility* for the concreta together.

Lewis presents a gappy (but non-empty) possibility for given concreta together. On the relationship between the second and first levels, Lewis holds that possibilities for given concreta $b_1, ..., b_n$ (all from one world) together ($n \geq 1$) (non-gappy and gappy-but-non-empty) amount to n-tuples of concreta (non-gappy and gappy-but-non-empty, respectively) as tuple-counterparts of $< b_1, ..., b_n >$.

On the relationship between the first and zeroth levels, Lewis presents (i) an analysis of counterparthood in terms of qualitative similarity of concreta, and (ii) a two-part analysis of tuple-counterparthood in terms of counterparthood and relational similarity.

At the third level, we seemingly have the notion of the possible/necessary for concreta $b_1, ..., b_n$ (all from one world) together ($n \geq 1$). To capture what is presumably Lewis' position on the relationship between the third and second levels, let us distinguish between strong and weak senses of the possible/necessary for concreta $b_1, ..., b_n$ (all from one world) together ($n \geq 1$). The weak sense concerns what is possible/necessary for $b_1, ..., b_n$ when all of $b_1, ..., b_n$ exist. The strong sense concerns what is possible/necessary when some of $b_1, ..., b_n$ may fail to exist.

The strong sense itself divides into two senses. The moderately strong sense concerns what is possible/necessary for $b_1, ..., b_n$ when some (but not all) of $b_1, ..., b_n$ may fail to exist. The extremely strong sense concerns what is possible/necessary for $b_1, ..., b_n$ when some (perhaps all) of $b_1, ..., b_n$ may fail to exist.

Lewis' position is presumably that the possible/necessary for $b_1, ..., b_n$ together in the weak sense is analysed in terms of existential/universal quantification over non-gappy possibilities for $b_1, ..., b_n$ together. Further, the possible/necessary for $b_1, ..., b_n$ together in the moderately strong sense is analysed in terms of existential/universal quantification over all non-empty possibilities for $b_1, ..., b_n$ together. However, there is a lacuna concerning the extremely strong sense.

A qualification is in order: we ostensibly have a fine-grained account of modal flavour. To see this, let $n \geq 1$ and $b_1, ..., b_n$ be concreta (all from one world). On the first and zeroth levels, it seems we should say that there is an n-tuple-counterpart relation for every appropriate combination of (a) for every $1 \leq m \leq n$, (a.i) a kind of similarity between individual concreta, and (a.ii) a standard of appropriate similarity between individual concreta, and (b) a kind of relational similarity between n-tuples of concreta. On the second and first levels, it seems we should hold that there is a flavour of possibility for $b_1, ..., b_n$ together for every n-tuple-counterpart relation. And on the third and second levels, it seems we should hold that there is a flavour of the possible/necessary for $b_1, ..., b_n$ together for every flavour of possibility for $b_1, ..., b_n$ together.

Derivatively, then, there is a flavour of the possible/necessary for $b_1, ..., b_n$ together for every appropriate combination of (a) for every $1 \leq m \leq n$, (a.i) a kind of similarity between individual concreta, and (a.ii) a standard of appropriate similarity between individual concreta, and (b) a kind of relational similarity between n-tuples of concreta.[1]

So much for Lewis' specification. Clearly, one desideratum of the specification is that it can be extended to an acceptable comprehensive account. This chapter will consider issues with achieving such an extension.

2.2 Existence

2.2.1 Empty Possibilities

As noted earlier, in his 1986 sketch, Lewis presents examples of non-gappy and gappy-but-non-empty possibilities for given concreta together. A question immediately arises: should we include empty possibilities in our account of the second level? One motivation for their inclusion is to hold a more elegant relationship between the third and second levels. Ignoring modal flavour for simplicity, we have the following. With empty possibilities, we can analyse the extremely strong sense of the possible/necessary for concreta $b_1, ..., b_n$ (all from one world) together ($n \geq 1$) in terms of existential/universal quantification over all possibilities for $b_1, ..., b_n$ together. So, we have a more elegant relationship between the third and second levels: what is possible/necessary for $b_1, ..., b_n$ together is always analysed in terms of existential/universal quantification over possibilities for $b_1, ..., b_n$ together.

If we reject the notion of an empty possibility for given concreta together, then there is a particular awkwardness connecting the third and second levels. We cannot straightforwardly capture the extremely strong sense of the possible/necessary for given concreta together. For example, let 'ϕ' be a purely qualitative description, and consider: it could have been that ϕ but Alice failed to exist. This claim involves what is possible for Alice (in the extremely strong sense). However, if we reject empty possibilities for given concreta together, this cannot be analysed as there being an empty possibility for Alice whose qualitative setting is such that ϕ. Rather, it seems it must be analysed as there failing to be a certain possibility for Alice. The analysis connecting the third and second levels is non-uniform.

2.2.2 The Negative Approach

If we accept empty possibilities, we are faced with the question of how to analyse them. One option is the negative approach: to analyse an empty

[1] So far, the discussion has been simplified (following Lewis) in assuming that every total qualitative setting accesses every total qualitative setting. As a result, our modal flavours are restricted. I cover richer flavours in section 2.2.7.

possibility as something failing to hold. That is:

(Negative) For $n \geq 1$, concreta $b_1, ..., b_n$ (all from one world), world w, and n-tuple-counterpart relation R, the possibility for $b_1, ..., b_n$ together (considered in the way corresponding to R) that none of $b_1, ..., b_n$ exist (set in the qualitative setting of w) amounts to a lack of an R w-tuple-counterpart of $< b_1, ..., b_n >$.

There is a point in Lewis 1986 that suggests he has in mind a negative approach in the work. Before introducing the notion of a joint possibility, he states (1986: 11):

> "What I want to say, of course, is that Humphrey ... has the possibility of not existing iff at some world he lacks a counterpart"

However, an immediate issue with the negative approach is that it produces an inelegant non-uniform relationship between the second and first levels. We cannot hold that a possibility for concreta $b_1, ..., b_n$ (all from one world) ($n \geq 1$) always amounts to a tuple-counterpart. This mirrors the initial issue with rejecting empty possibilities in our account of the second level (that it produces an inelegant non-uniform relationship between the third and second levels).

2.2.3 The Third Level

A key issue for one who rejects empty possibilities, or accepts empty possibilities and (Negative), concerns the handling of the third level. To introduce the point, let us turn to a preliminary on the relationship between the third and first levels.

Let $n \geq 1$, $b_1, ..., b_n$ be concreta (all from one world), w a world, and R an n-tuple-counterpart relation.

Next, suppose we reject empty possibilities. Then presumably we have the following analysis: $b_1, ..., b_n$ (considered in the way corresponding to R) could have been such that none of $b_1, ..., b_n$ existed (set in the qualitative setting of w) iff there is a lack of a possibility for $b_1, ..., b_n$ (considered in the way corresponding to R) that is set in the qualitative setting of w. This in turn is then analysed as: there is a lack of an R w-tuple-counterpart of $< b_1, ..., b_n >$.

Alternatively, suppose we accept empty possibilities and (Negative). Then we have: $b_1, ..., b_n$ (considered in the way corresponding to R) could have been such that none of $b_1, ..., b_n$ existed (set in the qualitative setting of w) iff there is an empty possibility for $b_1, ..., b_n$ (considered in the way corresponding to R) (set in the qualitative setting of w). Given (Negative), this is turn is analysed as: there is a lack of an R w-tuple-counterpart of $< b_1, ..., b_n >$.

Importantly, regardless of whether we reject empty possibilities, or accept empty possibilities and (Negative), we seemingly have the following relationship between the third and first levels:

> $b_1, ..., b_n$ (considered in the way corresponding to R) could have been such that none of $b_1, ..., b_n$ existed (set in the qualitative setting of w) iff $< b_1, ..., b_n >$ lacks an R w-tuple-counterpart.

Independently, notice that regardless of whether we reject empty possibilities, or accept empty possibilities and (Negative), we also have:

> $b_1, ..., b_n$ (considered in the way corresponding to R) could have been such and such (involving some of them existing, and set in the qualitative setting of w) iff $< b_1, ..., b_n >$ has an R w-tuple-counterpart that is such and such.

The takeaway point is that regardless of whether we reject empty possibilities, or accept empty possibilities and (Negative), we end up with this same non-uniform relationship between the third and first levels. The difference in the two positions here is a matter of where the carpet bulges (of which adjacent layers are to have an inelegant non-uniform relationship).

So much for the preliminary. The issue is that this relationship seemingly generates an inappropriate account of the third level. To see this, take Lewis' case that I introduced at the start of section 1.2.5. There, we take $w_@$ to be a world of two replicate halves. There is also a world w with no replication, but which is qualitatively identical to each side of $w_@$ (except in regards to properties of being one-half of a spatially symmetric world). In $w_@$, there is Lewis, l, and his twin concretum, t, in the other half of the world. In w, there is concretum d, which is a C w-counterpart of each of l and t. $< l, t >$ has three R* w-tuple-counterparts: $< *, d >, < d, * >$ and $< d, d >$. These correspond to three possibilities for concreta l and t together (considered in the way corresponding to R*): one where t exists, but l does not (set in the qualitative setting of w); one where l exists, but t does not (set in the qualitative setting of w); and one where both exist (and play the same individual-qualitative-role) (set in the qualitative setting of w).

At the third level, we therefore have that (i) l and t (considered in the way corresponding to R*) could have been such that t existed and l did not (set in the qualitative setting of w), and (ii) l and t (considered in the way corresponding to R*) could have been such that t existed and l did not (set in the qualitative setting of w). Given (i) and (ii), it seems natural to hold that l and t (considered in the way corresponding to R*) could have been such that neither l nor t existed (set in the qualitative setting of w).

However, if we reject empty possibilities, or accept empty possibilities and (Negative), then given $< l, t >$ has some R* w-tuple-counterparts, we have

that l and t (considered in the way corresponding to R^*) could not have been such that neither l nor t existed (set in the qualitative setting of w).

To push the point, consider the following problem case. Suppose that instead of $w_@$ being a world of two replicate halves, it is a world of three replicate thirds. The entire history in each third is replicated in each of the other two thirds. In $w_@$, there is Lewis, l, his twin, t_1, in one of the thirds, and his other twin, t_2, in the other of the thirds. Again, there is the world w featuring no replication, but which is qualitatively identical to each third of $w_@$ (except in regards to properties of being one-third of a three-way-replication world).

In what follows, let us again focus on the counterpart relation C, and on the tuple-counterpart relation R^*. Further, let us focus on a triple-counterpart relation R^+. Let us say that R^+ is associated with the triple <C, C, C> of counterpart relations, and is associated with a kind of relational similarity very similar to the kind of relational similarity associated with R^*. The way of considering pairs of entities together corresponding to R^* is very similar to the way of considering triples of entities together corresponding to R^+.

We have that d is a C w-counterpart of each of l, t_1, and t_2. If we are staying loyal to Lewis' position, it seems that we should allow that $< l, t_1, t_2 >$ has $< *, *, d >$ as an R^+ w-tuple-counterpart. However, it also seems that we should allow that $< l, t_1 >$ has the following R^* w-tuple-counterparts: $< d, * >, < *, d >, < d, d >$. Importantly, we should allow that $< l, t_1 >$ has *some* R^* w-tuple-counterparts.

At the third level, we therefore have that (a) l, t_1, and t_2 (considered in the way corresponding to R^+) could have been such that l and t_1 failed to exist (set in the qualitative setting of w), but it is not the case that (b) l and t_1 (considered in the way corresponding to R^*) could have been such that l and t_1 failed to exist (set in the qualitative setting of w). This does not yet generate an outright contradiction, but it seems natural to hold that if (a), then (b).

So much for the exposition of the problem case. There seem to be two main approaches that one who rejects empty possibilities, or accepts empty possibilities and adopts (Negative), can take to handle this case. I turn to them now.

The first approach is to accept the case but reject that if (a), then (b). The idea would presumably be that R^+ and R^* are sufficiently different. However, this approach seems unattractive. The same single counterpart relation C is relevant for each position of each of the two tuple-counterpart relations, and we have supposed that the two kinds of relational similarity are very close. Further, even if we grant that R^+ and R^* are sufficiently different, it seems unattractive to impose a blanket ban on there being an analogue of my case

using a combination of a triple-counterpart relation and pair-counterpart relation for which the analogous conditional claim holds.

The second approach is to reject the initial description of the case. For example, we might introduce an additional condition on tuple-counterpart relations:

(Restrictive) For $n \geq 1$, a world w, counterparts relations C_1, ..., C_n, and a tuple-counterpart relation R associated with $<C_1, ..., C_n>$, a tuple $< c_1, ..., c_n >$ of w-concreta (which may feature repeats or gaps) is an R w-tuple-counterpart of a tuple $< b_1, ..., b_n >$ of concreta only if for every $1 \leq m \leq n$, if c_m is a gap, then b_m has no C_m w-counterparts.

Holding (Restrictive) constitutes a restriction of the non-empty possibilities for given concreta together that there are. On this position, for $n \geq 1$, a world w, counterparts relations C_1, ..., C_n, a tuple-counterpart relation R associated with $<C_1, ..., C_n>$, and concreta $b_1, ..., b_n$ (all from one world), for every $1 \leq m \leq n$, if b_m has a C_m w-counterpart, then there cannot be a (non-empty) possibility for $b_1, ..., b_n$ together (considered in the way corresponding to R) where b_m fails to exist (set in the qualitative setting of w).

Returning to the case, with (Restrictive) in place, we hold that $< l, t_1 >$ only has R^* w-tuple-counterpart $< d, d >$, and $< l, t_1, t_2 >$ only has R^+ w-tuple-counterpart $< d, d, d >$.

Perhaps there are counterpart relations C_1, C_2, and C_3, and there is a triple-counterpart relation R' associated with $<C_1, C_2, C_3>$ such that $< l, t_1, t_2 >$ has an R' w-tuple-counterpart $< *, *, x >$ (where x is not a gap). However, there we have, in virtue of (Restrictive), that there is no C_1 w-counterpart of l and no C_2 w-counterpart of t_1. Hence, there is no tuple-counterpart relation R" associated with $<C_1, C_2>$ such that $< l, t_1 >$ has an R" w-tuple-counterpart.

However, note that the negative approach combined with just (Restrictive) results in quite a strange position. Given the negative approach, for $n \geq 1$, we have two kinds of possibilities for concreta $b_1, ..., b_n$ (all from one world) together: non-empty ones and empty ones. Given (Negative), there is a very low requirement for there to be an empty possibility for $b_1, ..., b_n$ together. However, given (Restrictive), there is a very high requirement for there be to a gappy-but-non-empty possibility for $b_1, ..., b_n$ together.

Another worry for this response is that it currently appears quite *ad hoc*. The challenge for one interested in this response is to present a systematic analysis of tuple-counterparthood in terms of the zeroth level which generates the restriction.

2.2.4 The Positive Approach

To summarise my discussion of gaps so far, it seems that there are notable difficulties for one who rejects empty possibilities, or accepts empty possibilities but adopts (Negative).

A natural alternative to these positions seems to be to allow empty possibilities and adopt a positive approach. Put simply: allow empty tuples of concreta to serve as tuple-counterparts. Of course, an empty tuple is not relativised to a world, so we will have to somehow build world relativisation into our tuple-counterparts. A simple approach to achieving this is to hold that for $n \geq 1$, an n-tuple of concreta $< b_1, ..., b_n >$ (from a world w) relativised to a world is identified as the $(1 + n)$-tuple $< w, b_1, ..., b_n >$. Now, we hold that an n-tuple-counterpart relation holds between world-relativised n-tuples of concreta. The positive approach is then:

(Positive) For $n \geq 1$, concreta (of a world w) $b_1, ..., b_n$, world w^+, and n-tuple-counterpart relation R, the possibility for $b_1, ..., b_n$ together (considered in the way corresponding to R) that none of $b_1, ..., b_n$ exist (set in the qualitative setting of w^+) amounts to $< w^+, *_1, ..., *_n >$ as an R w^+-tuple-counterpart of $< w, b_1, ..., b_n >$.

This position results in a uniform relationship between the third and first levels. What is possible/necessary for concreta $b_1, ..., b_n$ (all from one world) together ($n \geq 1$) (considered in the way corresponding to n-tuple-counterpart relation R) is always analysed in terms of existential/universal quantification over R tuple-counterparts of $< w, b_1, ..., b_n >$.

Further, at least from the perspective of the first level and above, we can handle the case presented in 2.2.3. We hold that $< w_@, l, t_1, t_2 >$ has $< w, *, *, d >$ as an R^+ w-tuple-counterpart, and $< w_@, l, t_1 >$ has R^* w-tuple-counterparts $< w, d, d >, < w, d, * >, < w, *, d >$, and $< w, *, * >$.

2.2.5 Analysis

From the perspective of the first level and above, accepting empty possibilities and (Positive) seems attractive. However, there is an issue concerning the analysis of tuple-counterparthood.

First, we need to adjust our earlier analysis of tuple-counterparthood to allow gappy tuples to serve as tuple-counterparts. The natural approach is to hold: for $n \geq 1$, and worlds w and w^+, a world-relativised n-tuple $< w^+, c_1, ..., c_n >$ of distinct (from each other) w^+-concreta is a tuple-counterpart of a world-relativised n-tuple $< w, b_1, ..., b_n >$ of distinct (from each other) w-concreta iff (i) for every $1 \leq m \leq n$, c_m is a counterpart of b_m or it is a gap, and (ii) $< c_1, ..., c_n >$ is relationally similar to $< b_1, ..., b_n >$.

The issue is that we ostensibly must hold:

(Empty Tuple-Counterpart 1) For $n \geq 1$, worlds w and w^+, world-relativised n-tuple of w-concreta $< w, b_1, ..., b_n >$, and n-tuple-counterpart relation R, if $< w, b_1, ..., b_n >$ has no non-empty world-relativised n-tuple of w^+-concreta as an R w^+-tuple-counterpart, then $< w, b_1, ..., b_n >$ has $< w^+, *_1, ..., *_n >$ as an R w^+-tuple-counterpart.

To see this, suppose that $< w, b_1, ..., b_n >$ has no non-empty world-relativised n-tuple of w^+-concreta as an R w^+-tuple-counterpart. Then there is no possibility for $b_1, ..., b_n$ (considered in the way corresponding to R) set in the qualitative setting of w^+ on which some of $b_1, ..., b_n$ exist. That is, $b_1, ..., b_n$ (considered in the way corresponding to R) could not have been such that some of $b_1, ..., b_n$ existed (set in the qualitative setting of w^+). However, given w accesses w^+ (from the perspective of w, things could have been such that the qualitative setting of w^+ held), it seems we should have that $b_1, ..., b_n$ (considered in the way corresponding to R) could have been such that none of $b_1, ..., b_n$ existed (set in the qualitative setting of w^+). That is, there should just be the possibility for $b_1, ..., b_n$ (considered in the way corresponding to R) set in the qualitative setting of w^+ on which none of them exist. That is, $< w, b_1, ..., b_n >$ should have $< w^+, *_1, ..., *_n >$ as an R w^+-tuple-counterpart.[2]

The challenge now is to present an analysis of tuple-counterparthood that generates (Empty Tuple-Counterpart 1) in a principled way.

Perhaps one solution is to hold:

(Empty Tuple-Counterpart 2) For $n \geq 1$, worlds w and w^+, world-relativised n-tuple of w-concreta $< w, b_1, ..., b_n >$, and n-tuple-counterpart relation R, $< w, b_1, ..., b_n >$ has $< w^+, *_1, ..., *_n >$ as an R w^+-tuple-counterpart.

We arrive at this by holding that the world-relativised empty n-tuple of concreta $< w^+, *_1, ..., *_n >$ is treated like all other tuples, but it automatically satisfies the condition of relational similarity.

However, one issue with this approach is that it generates an account of the third level that some may find unpalatable. In particular, it allows a lot to be possible and little to be necessary.

2.2.6 Taking Stock

To summarise our discussion of existence, we have considered three extensions to Lewis' specification to handle existence: rejecting empty possibilities; accepting empty possibilities and (Negative); and accepting empty possibilities and (Positive). Each position faces difficulties.

[2]Notice that this issue does not arise for one who accepts empty possibilities and (Negative), or rejects empty possibilities.

If we reject empty possibilities, or accept empty possibilities and (Negative), then we end up with (i) an inelegant relationship between the third and first levels, and (ii) an ostensible mishandling of the third level.

If we accept empty possibilities and (Positive), then we end up with a challenge to present an analysis of tuple-counterparthood that generates (in a principled way) the condition (Empty Tuple-Counterpart 1). One might be tempted by my suggested solution, but some may find the conditions it imposes on the third level unacceptable.

2.2.7 Richer Flavour

Before moving on, I want to make a qualification concerning modal flavour. So far, the discussion has been simplified (following Lewis) by assuming that every total qualitative setting accesses every total qualitative setting. We said that there is a flavour of the possible/necessary (a flavour of possibility) for concreta $b_1, ..., b_n$ (all from one world) together ($n \geq 1$) for every appropriate combination of (a) for every $1 \leq m \leq n$, (a.i) a kind of similarity between individual concreta, and (a.ii) a standard of appropriate similarity between individual concreta, and (b) a kind of relational similarity between n-tuples of concreta.

However, our flavours should be richer than this: there should be a flavour of the possible/necessary (a flavour of possibility) for concreta $b_1, ..., b_n$ (all from one world) together ($n \geq 1$) for every appropriate combination of (a) for every $1 \leq m \leq n$, (a.i) a kind of similarity between individual concreta, and (a.ii) a standard of appropriate similarity between individual concreta, (b) a kind of relational similarity between n-tuples of concreta, and (c) a way of considering qualitative settings as a whole (that is, a kind of world accessibility).

In this short section, I want to note ways that the discussed positions would accommodate these richer flavours.

Suppose we rejected empty possibilities, or accepted empty possibilities and (Negative). We now say that for $n \geq 1$, concreta $b_1, ..., b_n$ (of a world w), world w^+, n-tuple-counterpart relation R, and world accessibility relation C:

> $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) could have been such and such (involving some of them existing, and set in the qualitative setting of w^+) iff there is a non-empty possibility for $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) on which $b_1, ..., b_n$ are such and such (set in the qualitative setting of w^+).
>
> There is a non-empty possibility for $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the

way corresponding to C) on which $b_1, ..., b_n$ are such and such (set in the qualitative setting of w^+) iff w^+ is C-accessible from w and there is an R w^+-tuple-counterpart of $< b_1, ..., b_n >$ that is such and such.

If we rejected empty possibilities, we now hold:

> $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) could have been such that none of them existed (set in the qualitative setting of w^+) iff there is no possibility for $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) set in the qualitative setting of w^+, yet w^+ is a possibility for w (when qualitative settings are considered in the way corresponding to C).
>
> There is no possibility for $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) set in the qualitative setting of w^+, yet w^+ is a possibility for w (when qualitative settings are considered in the way corresponding to C) iff w^+ is C-accessible from w but there is no R w^+-tuple-counterpart of $< b_1, ..., b_n >$.

If we accepted empty possibilities and (Negative), we now hold:

> $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) could have been such that none of them existed (set in the qualitative setting of w^+) iff there is an empty possibility for $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) set in the qualitative setting of w^+.
>
> There is an empty possibility for $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) set in the qualitative setting of w^+ iff w^+ is C-accessible from w but there is no R w^+-tuple-counterpart of $< b_1, ..., b_n >$.

Notably, both positions here still provide this same relationship between the third and first levels.

Separately, note that if we rejected empty possibilities, or accepted empty possibilities and (Negative), we cannot capture all modal flavour in our tuple-counterpart relations (by building world accessibility into the notion of tuple-counterparthood). This is because we need the granularity to allow that $< b_1, ..., b_n >$ has no R w^+-tuple-counterparts, but w^+ is nevertheless C-accessible from w.

Suppose we instead accepted empty possibilities and (Positive). On a simple approach on which world accessibility relations and tuple-counterpart relations

are independent, we hold: for $n \geq 1$, concreta $b_1, ..., b_n$ (all from one world w), world w^+, n-tuple-counterpart relation R, and world accessibility relation C, a possibility for $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) amounts to an R w^+-tuple-counterpart of $< w, b_1, ..., b_n >$ whilst w^+ is C-accessible from w.

On this position, we still require:

(Empty Tuple-Counterpart 1) For $n \geq 1$, worlds w and w^+, world-relativised n-tuple of w-concreta $< w, b_1, ..., b_n >$, and n-tuple-counterpart relation R, if $< w, b_1, ..., b_n >$ has no non-empty world-relativised n-tuple of w^+-concreta as an R w^+-tuple-counterpart, then $< w, b_1, ..., b_n >$ has $< w^+, *_1, ..., *_n >$ as an R w^+-tuple-counterpart.

The argument is similar to before. Suppose that $< w, b_1, ..., b_n >$ has no non-empty world-relativised n-tuple of w^+-concreta as an R w^+-tuple-counterpart. Further, consider the total world accessibility relation C. We therefore have that there is no possibility for $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) set in the qualitative setting of w^+ on which some of $b_1, ..., b_n$ exist. That is, $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) could not have been such that some of $b_1, ..., b_n$ existed (set in the qualitative setting of w^+). However, given w C-accesses w^+, it seems we should have that $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) could have been such that none of $b_1, ..., b_n$ existed (set in the qualitative setting of w^+). That is, there should just be the possibility for $b_1, ..., b_n$ (considered in the way corresponding to R, and where qualitative settings are considered in the way corresponding to C) set in the qualitative setting of w^+ on which none of them exist. That is, $< w, b_1, ..., b_n >$ should have $< w^+, *_1, ..., *_n >$ as an R w^+-tuple-counterpart.

So much for the simplest update for one who accepted empty possibilities and (Positive). As an aside, notice that we could instead update the position so that world accessibility is built into our tuple-counterpart relations. We say that for every $n \geq 0$, an n-tuple-counterpart relation holds between world-relativised n-tuples of concreta. Each such relation is associated with (i) an n-tuple of counterpart relations $<C_1, ..., C_n>$, (ii) a kind of relational similarity between n-tuples of concreta, and (iii) a world accessibility relation C. When $n = 0$, we have a relation between world-relativised 0-tuples of concreta, that is, between world one-tuples.

Now, our requirement on tuple-counterpart relations is slightly adjusted:

(Empty Tuple-Counterpart 1*) For $n \geq 1$, worlds w and w^+, world-relativised n-tuple of w-concreta $< w, b_1, ..., b_n >$, world accessibility relation C, and n-tuple-counterpart relation R (whose associated world accessibility relation is C), if w^+ is C-accessible from w, and $< w, b_1, ..., b_n >$ has no non-empty world-relativised n-tuple of w^+-concreta as an R w^+-tuple-counterpart, then $< w, b_1, ..., b_n >$ has $< w^+, *_1, ..., *_n >$ as an R w^+-tuple-counterpart.

My earlier suggestion is now:

(Empty Tuple-Counterpart 2*) For $n \geq 1$, worlds w and w^+, world-relativised n-tuple of w-concreta $< w, b_1, ..., b_n >$, world accessibility relation C, and n-tuple-counterpart relation R (whose associated world accessibility relation is C), if w^+ is C-accessible from w, then $< w, b_1, ..., b_n >$ has $< w^+, *_1, ..., *_n >$ as an R w^+-tuple-counterpart.

2.3 Existence and Iterated Modality

Let us now turn to an issue concerning iterated modality and existence. As a preliminary, I shall henceforth talk in terms of the old tuple-counterpart framework (without world relativisation). What is said is intended as neutral between the positions presented in the discussion on existence and can be translated into terms suitable for particular positions. Further, for simplicity, I shall assume the simpler modal flavours (I assume that every world accesses every world). Nothing turns on this.

As another preliminary on handling natural language sentences, note that there are two kinds of modal locution in natural language: sentential operators and auxiliary verbs. For example, compare: (i) 'it could have been that John was blond' and (ii) 'John could have been blond'. So far, I have been assuming that these sentences have (relative to the same contexts) the same meaning.

When we consider iterated modality, we have many combinations of sentential operators and auxiliary verbs. For example, consider: 'it could have been that it could have been that John was blond'; 'it could have been that John could have been blond'; 'John could have been such that things could have been such that John was blond'; and 'John could have been such that he could have been blond'. I shall also suppose that these sentences all have (relative to the same contexts) the same meaning. That is, I shall suppose that it does not matter to the meaning of a claim whether the modals that occur in the claim are sentential operators or auxiliary verbs.

That said, natural language sentences with multiple modal occurrences often feature scope ambiguities. Given this preliminary, it will sometimes be helpful to supplement a natural language sentence s (relative to a disambiguation of

s) with a translation s^+ in a first-order modal language L. To arrive at a translation of a natural language sentence s (relative to a disambiguation), we first translate s into a natural language sentence whose only modals are sentential operators, and then translate into an L-sentence s^+. The easily readable syntactic form of s^+ then elucidates the meaning of s.

2.3.1 Towards an Account

To begin, we restrict our attention to the weak sense of the possible for concreta (all from one world) together (what is possible for the concreta together when they all exist). Consider the following (where we suppose the sense of the possible involved is weak):

(6) It could have been that it could have been that John loved Mary.

(6^+) $\diamond\diamond Ljm$

Given our account so far, it seems we should say that (6) involves the possible for John and Mary together. In particular, it involves the possible for the possible. At the second level, it involves possibilities of possibilities of John and Mary together. At the first level, it involves tuple-counterparts of tuple-counterparts. As each modal occurrence may be concerned with different flavours of modality, the analysis is seemingly: there is a world w and non-gappy R w-tuple-counterpart $< x, y >$ of $< j, m >$ such that there is a world w^+ and non-gappy R^+ w^+-tuple-counterpart $< x_1, y_1 >$ of $< x, y >$ such that x_1 loves y_1 (where R and R^+ are the relevant tuple-counterpart relations in play).

Beyond this simple case, one complexity for the current overall account is that sometimes an *embedded* modal operator occurrence seemingly concerns a different length tuple of concreta from an *embedding* modal operator occurrence. Consider:

(7) It could have been that there was someone x whom John loved such that it could have been that John married x.

(7^+) $\diamond\exists x(Ljx \wedge \diamond Mjx)$

Given our account so far, it seems we should analyse (7) as follows: there is a world w, non-gappy R w-tuple-counterpart $< d_1 >$ of $< j >$, and concretum x in w such that d_1 loves x and there is a world w^+ and non-gappy R^+ w^+-tuple-counterpart $< d_2, d_3 >$ of $< d_1, x >$ such that d_2 is married to d_3.

For a slightly more complex case, consider:

(8) It could have been that there was someone x whom John loved such that it could have been that John was friends with Mary but married to x.

(8^+) $\Diamond\exists x(Ljx \wedge \Diamond(Fjm \wedge Mjx))$

Here, Mary is mentioned only *within* the scope of the embedded possibility operator occurrence (not outside of it). Nevertheless, in virtue of this mention also occurring within the scope of the outer possibility operator occurrence, the straightforward approach seems to be to unpack the first possibility operator occurrence in terms of quantification over tuple-counterparts of $< j, m >$.[3] That is, we analyse (8) as follows: there is a world w, non-gappy R w-tuple-counterpart $< d_1, d_2 >$ of $< j, m >$, and concretum x in w such that d_1 loves x and there is a world w^+ and non-gappy R^+ w^+-tuple-counterpart $< d_3, d_4, d_5 >$ of $< d_1, d_2, x >$ such that d_3 is friends with d_4 and d_3 is married to d_5.

This approach naturally extends to include the weak sense of the necessary for concreta (all from one world) together (what is necessary for the concreta together when they all exist). For example, consider the following (where we suppose the sense of the necessary involved is weak):

(9) It must be that it must be that John loves Mary.

(9^+) $\Box\Box Ljm$

(9) is analysed as follows: for every world w and non-gappy R w-tuple-counterpart $< x, y >$ of $< j, m >$, for every world w^+ and non-gappy R^+ w^+-tuple-counterpart $< x_1, y_1 >$ of $< x, y >$, x_1 loves y_1.

For a combined case, consider the following (where we suppose the senses of the possible and the necessary involved are weak):

(10) It could have been that there was someone x whom John loved such that from there, necessarily, John loves x.

(10^+) $\Diamond\exists x(Ljx \wedge \Box Ljx)$

(10) is analysed as follows: for some world w and non-gappy R w-tuple-counterpart $< d_1 >$ of $< j >$, for some concretum x in w, d_1 loves x and for every world w^+ and non-gappy R^+ w^+-tuple-counterpart $< d_2, d_3 >$ of $< d_1, x >$, d_2 loves d_3.

2.3.2 Strong Possibility and Necessity

The issue is extending the account to handle claims involving the strong sense of the possible/necessary for concreta (all from one world) together (what is possible/necessary for the concreta when some of them may fail to exist). Consider:

[3] After all, this phenomenon seemingly also occurs in (6), and we there unpacked the first possibility operator occurrence in terms of quantification over tuple-counterparts of $< j, m >$.

(11) It could have been that John failed to exist even though there was someone x such that from there it could have been that John loved x.

(11$^+$) $\diamond(\sim Ej \wedge \exists x \diamond Ljx)$

Suppose we begin to analyse (11) as before:

There is a world w and empty tuple R w-tuple-counterpart of $< j >$ (or, if one prefers, there is a lack of an R w-tuple-counterpart of $< j >$) ...

However, if we want to apply the simple tuple-counterparts of tuple-counterparts analysis of iterated modality and also have that (11) may be true, we need to continue the analysis in something like the following way:

... there is a concretum x in w such that there is a world w^+ and R$^+$ tuple-counterpart $< d_1, d_2 >$ of $< *, x >$ such that d_1 loves d_2.

It seems that to hold the simple tuple-counterparts of tuple-counterparts analysis of iterated modality and also have that (11) may be true, we would be required to allow gappy tuples to be *borne* tuple-counterpart relations. Further, connected to this observation is the crucial point: we still want the concretum d_1 in w^+ to be suitably connected to j. As it were, we want to consider w^+-tuple-counterparts of $< *, x >$ when it is understood that the gap is a j-gap. However, the machinery of tuple-counterparts alone cannot capture this.

2.3.3 Taking Stock

There seem to be two responses to this problem. The first response is to continue to hold the simple tuple-counterparts of tuple-counterparts analysis of iterated modality, but to just reject the kind of *de re* iterated modality that (11) claims there is. That is, we hold that (11) is in some sense contradictory. The idea I have in mind is that we continue to not allow gappy tuples to be *borne* tuple-counterpart relations. We hold that (11) states something contradictory. It first says that there is a world w and empty tuple R w-tuple-counterpart of $< j >$ (or, if one prefers, there is a lack of an R w-tuple-counterpart of $< j >$), then it says that the determined w-concretum that is a counterpart (for the counterpart relation in play) of j is such that it and x have a certain tuple-counterpart. However, no such w-concretum counterpart of j was determined.[4]

[4]The relative-sameness semantics of Graff-Fara 2008 is an example of a counterpart account on which contingent existence is allowed, but sentences like (11) cannot be rendered true. That said, Graff Fara has a different background metaphysics in play, and her counterpart relations (relative-sameness relations) are severely restricted in comparison to ours. Further, her account anyway fails to properly respect the combination of contingent existence and iterated modality. I turn to her semantics in detail in the fourth chapter.

The second response to the problem is to hold that (11) can be rendered true. It requires abandoning the simple tuple-counterparts of tuple-counterparts approach to iterated modality. However, this is a difficult challenge.[5]

The challenge itself has two parts. First, we need our machinery to be technically acceptable in the sense that it can render (11) true. Second, we need an acceptable analysis of the machinery.

Here is one point that highlights the difficulty of the first part of the challenge. A central desideratum of an account of iterated modality is that it should respect the iterative nature of iterated modality. Consider the following:

(12) Things could have been such that John was F and from there things could have been such that John was G.

(12^{+}) $\diamond(Fj \wedge \diamond Gj)$

(12) seems to state that John has a certain modal property. Namely, the modal property that he could have had the property [being F and being such that he could have been G]. It does not say that John himself has two separate modal properties: the modal property that he could have had the property [being F] and the modal property that he could have had the property [being G].

Of course, it may be that John himself does have these last two separate modal properties. However, the desideratum is that our account should not unpack (12) as saying this.

To motivate the point, for certain claims that involve iterated modality and multiple modal flavours, the iterative nature of the iterated modality must be respected to render the claim true. To see this, consider:

(13) [Strictly speaking, things could have been such that John was a bumble bee and from there, even merely physically speaking, things could have been such that John was an abnormally large bumble bee], but merely physically speaking, things could not have been such that John was an abnormally large bumble bee.

Here, the first possibility operator occurrence has a rather unrestricted flavour M_1, and the embedded possibility operator occurrence has a much more restricted flavour M_2. It may very well be that John could have (M_1 flavour) had the property [being a bumble bee and being such that he could have (M_2 flavour) been an abnormally large bumble bee]. However, it is presumably not the case that John could have (M_2 flavour) had the property [being an

[5] My (11) is similar to the (14) of Hazen 1977. Hazen introduces *fusion* machinery in his 1977 stipulational semantics to be able to render his (14) true. I turn to that semantics in detail in the fourth chapter.

abnormally large bumble bee]. Indeed, it is presumably not the case that John could have (M_2 flavour) had the property [being a bumble bee].

To render (13) true, our account must respect the iterative nature of the iterated modality in the claim. For the sake of systematicity, it therefore seems that our account should always respect the iterative nature of iterated modality. As a result, we cannot hope to account for claims like (11) with an approach that involves collapsing the iteration.

2.4 Contingent Identity

In this section, I raise an issue concerning the analysis of tuple-counterparthood. Reconsider (2), (3), and (3^+):

(2) Lumpl and Goliath are identical but could have been distinct.

(3) Alice and A are identical but could have been distinct.

(3^+) Alice and herself are identical but could have been distinct.

In section 1.2.1 (in the pre-tuple-counterpart framework) we held that sentences such as (2), (3), and (3^+) (relative to a context) should not be rendered true in virtue of the relevant single concretum b being such that for a certain counterpart relation C, for some world w, b has multiple C w-counterparts.

For (2), we adopted a multiple counterpart relation analysis to render it true (relative to certain contexts c). We held that the value of 'Lumpl' (perhaps in conjunction with c) determines the lump counterpart relation, and the value of 'Goliath' (perhaps in conjunction with c) determines the statue counterpart relation. (2) (relative to c) is true in virtue of d having a lump w_2-counterpart, and a distinct statue w_2-counterpart.

Crucially, the lesson (in the pre-tuple-counterpart framework) was that we wanted to allow (2) (relative to certain contexts) to come out as true because the single concretum is being individually considered in two distinct ways. In contrast, we wanted (3) to come out as false (relative to contexts where it is a mutually salient part of the conversational common-ground that the speaker thinks of the names 'Alice' and 'A' in exactly the same way) and (3^+) to come out as false (relative to contexts where the context doesn't somehow enrich the pronoun with extra descriptive information) because the single concretum is being individually considered in a single way.

Of course, the account of modal flavour with the tuple-counterpart machinery is more fine-grained than the account of modal flavour with counterpart relations alone. We consider not only how concreta are being individually

considered, but also how concreta together are considered. Importantly, on the tuple-counterpart account, we do not hold that for any context c, if 'Lumpl' (together with c) determines a distinct counterpart relation C_1 from the counterpart relation C_2 that 'Goliath' (relative to c) determines, then (2) (relative to c) is true if there is a world w, a C_1 w-counterpart d_1 of d, and a C_2 w-counterpart d_2 of d such that $d_1 \neq d_2$. This is because (2) (relative to a context c) may determine a tuple-counterpart R whose associated tuple of counterpart relations is $<C_1, C_2>$ (where $C_1 \neq C_2$), but whose associated kind of relational similarity is such that a non-gappy non-identity pair cannot be relationally similar to an identity pair, that is, the kind of relational similarity associated with R is such that when determining what is relationally similar to an identity pair, weight is placed on the identity.

In section 1.2.4, we held that (2) (relative to certain contexts) is rendered true in virtue of tuple $< d, d >$ having $< l, s >$ as a tuple-counterpart (where the tuple-counterpart relation in play is such that its associated tuple of counterpart relations is $<C_{\text{lump}}, C_{\text{statue}}>$, and its associated kind of relational similarity allows a non-gappy non-identity pair to be relationally similar to an identity pair).

However, one might wonder whether the negative lesson on "contingent identity" in the pre-tuple-counterpart framework even applies when we adopt the tuple-counterpart machinery. That is, should we still hold that (2), (3), and (3^+) are false relative to contexts where we have a single concretum b being individually considered in only one way?

To reject the lesson, we would hold that for one of (2), (3), and (3^+), there is a context c such that the sentence (relative to c) determines a tuple-counterpart relation R whose associated tuple of counterpart relations is an identity pair, and whose associated kind of relational similarity is such that a non-gappy non-identity pair can be relationally similar to an identity pair, that is, the kind of relational similarity associated with R is such that when determining what is relationally similar to an identity pair, weight is not placed on the identity. The sentence (relative to c) is true in virtue of the relevant identity pair of concreta having a non-gappy non-identity pair as an R tuple-counterpart.

As an aside, there is a question here of how far to take the approach. For example, can we hold that (14) is true (relative to a certain context c) in virtue of (i) (14) (relative to c) determining a tuple-counterpart relation R whose associated tuple of counterpart relations is an identity pair, and whose associated kind of relational similarity is such that a non-gappy non-identity pair can be relationally similar to an identity pair, and (ii) the relevant tuple $< b, b >$ having a non-gappy non-identity pair as an R tuple-counterpart?

(14) Alice and Alice are identical but could have been distinct.

To allow this to be true, we would presumably hold that in virtue of there being two occurrences of 'Alice', the sentence (relative to c) somehow invokes the possible for an identity pair, rather than a one-tuple. That is, we hold a certain position about the relationship between natural language sentences and the third level of the metaphysics.

So much for how one holding the tuple-counterpart machinery would reject the lesson. Although some may find the approach acceptable, it seems that adhering to the lesson is more in accord with pre-theoretic judgements about these claims.

If we want to adhere to the lesson, the natural position seems to be that we should restrict our tuple-counterpart relations: for a concretum b (of a world), counterpart relations C_1 and C_2, a tuple-counterpart relation R associated with $<C_1, C_2>$, and a non-gappy R tuple-counterpart $< x, y >$ of $< b, b >$, if C_1 and C_2 are identical, then x and y are identical.

It is helpful to consider the four level framework here. Given Lewis' account of the four levels, it seems natural to say that for each of (2), (3), and (3^+), and every context c, we have a concretum b such that the sentence (relative to c) involves the (weakly) possible for b and b (considered in a certain way). Hence, it involves existential quantification over (non-gappy) possibilities for b and b (considered in a certain way). Hence, it involves existential quantification over (non-gappy) tuple-counterparts of $< b, b >$ (where the tuple-counterpart relation in play is of a certain kind). For each sentence, for a context c, if we have a single concretum b being individually considered in a single way, then the sentence (relative to c) involves the (weakly) possible for b and b (individually considered in the same way). Hence, it involves existential quantification over (non-gappy) possibilities for b and b (individually considered in the same way). Hence, it involves existential quantification over (non-gappy) tuple-counterparts of $< b, b >$ where the tuple-counterpart relation in play stresses the same counterpart relation for the first and second positions of the tuples.

Given that, it seems adhering to the lesson should amount to imposing a certain restriction. This restriction impacts multiple levels. At the third level, we have the restriction that there is no concretum b such that b and b (individually considered in the same way) could have been such that b played two distinct individual-qualitative-roles (for the weak sense of the possible). At the second level, we have the restriction that there is no concretum b and (non-gappy) possibility for b and b (individually considered in the same way) on which b plays two distinct individual-qualitative-roles. At the first level, we have the restriction that there is no concretum b such that $< b, b >$ has a non-gappy non-identity tuple-counterpart (where the tuple-counterpart relation in play stresses the same counterpart relation for the first and second positions of the tuples).

One might respond by holding that in order to adhere to the lesson, we in fact don't need this restriction. One continues: although (2) (relative to certain contexts) is true in virtue of tuple $< d, d >$ having $< l, s >$ as a w_2-tuple-counterpart (for a certain kind of tuple-counterpart relation), it is not the case for each of (2), (3), and (3^+), for every context c, the analysis of the sentence (relative to c) is that a pair has a certain tuple-counterpart. Instead, for any of (2), (3), and (3^+), and any context c on which the sentence (relative to c) is such that we have a single concretum b being individually considered in a single way, the analysis of the sentence (relative to c) is just that the tuple $< b >$ has a certain tuple-counterpart. That is, for any of (2), (3), and (3^+), and any context c on which the sentence (relative to c) is such that we have a single concretum b being individually considered in a single way, we hold that the sentence just involves the possible for b (considered in a certain way) (not the possible for b and b). In which case, the sentence (relative to c) will not be rendered true (as a tuple-counterpart of $< b >$ will represent b playing at most one individual-qualitative-role). This approach involves holding a certain position about the relationship between natural language sentences and the third level of the metaphysics.

However, even if we grant that for some sentence of (2), (3), and (3^+), and some context c, the analysis of the sentence relative to c is that a one-tuple has a certain tuple-counterpart (rather than a pair having a certain tuple-counterpart), it still seems that we should impose the restriction given a desire to adhere to the lesson. This is because without the restriction, we would still have that some tuple of concreta $< b, b >$ has a non-gappy non-identity tuple as a tuple-counterpart (where the tuple-counterpart relation in play stresses the same counterpart relation for the first and second positions of the tuples). Hence, there is a (non-gappy) possibility for b and b (individually considered in the same way) on which b plays two distinct individual-qualitative-roles. Hence, b and b (individually considered in the same way) could have been such that b played two distinct individual-qualitative-roles (for the weak sense of the possible). And presumably one who wants to adhere to the lesson will find this third level feature unpalatable.

So much for motivating the restriction on tuple-counterpart relations given a desire to adhere to the lesson. The issue now is that if we want to impose this restriction, there is a challenge to present an analysis of tuple-counterparthood that generates the restriction in a principled way.

To spell out this challenge, suppose there is a concretum d in our world $w_@$, a world w, distinct (from each other) w-concreta d_1 and d_2, and counterpart relations C_1 and C_2 such that (i) d_1 is a C_1 counterpart of d, (ii) d_1 is a C_2 counterpart of d, and (iii) d_2 is a C_2 counterpart of d. Further, suppose there is a tuple-counterpart relation R^* whose associated tuple of counterpart relations is $<C_1, C_2>$, and which is such that $< d_1, d_2 >$ is an R^*

w-tuple-counterpart of $< d, d >$. Given our restriction on tuple-counterpart relations, note that there is no tuple-counterpart relation R with associated tuple $<C_2, C_2>$ of counterpart relations such that $< d_1, d_2 >$ is an R w-tuple-counterpart of $< d, d >$.

In order to account for this scenario, we cannot hold an analysis of tuple-counterparthood in which the two parts of our analysis, concerning counterparthood and relational similarity, are independent. That is, we cannot hold that for each $n \geq 1$, there is a collection of relational similarity relations S that hold between n-tuples of concreta such that for every combination of (a) an n-tuple of counterpart relations $<C_1, ..., C_n>$, and (b) a relation S in the collection, there is a corresponding n-tuple-counterpart relation R such that a (non-gappy) n-tuple of concreta $< c_1, ..., c_n >$ (from one world) bears R to an n-tuple of concreta $< b_1, ..., b_n >$ (from one world) iff (i) for every $1 \leq m \leq n$, c_m is a C_m counterpart of b_m, and (ii) $< c_1, ..., c_n >$ bears S to $< b_1, ..., b_n >$.

To see that this won't deliver our scenario, note that given $< d_1, d_2 >$ bears R^* to $< d, d >$, we have that $< d_1, d_2 >$ bears S^* to $< d, d >$, where S^* is the kind of relational similarity associated with R^*. Given the account of tuple-counterparthood under discussion, there is a tuple-counterpart relation R^+ that is associated with $<C_2, C_2>$, and associated with the relational similarity relation S^*. Given d_1 is a C_2 counterpart of d, d_2 is a C_2 counterpart of d, and $< d_1, d_2 >$ bears S^* to $< d, d >$, it follows that $< d_1, d_2 >$ bears R^+ to $< d, d >$, *contra* our scenario.

Instead, we must hold that for every pair-counterpart relation R, if R is associated with an identity pair of counterpart relations, then the kind of relational similarity associated with R does not allow a non-gappy non-identity pair to be relationally similar to an identity pair, that is, the kind of relational similarity associated with R is such that when determining what is relationally similar to an identity pair, it places weight on the identity. The challenge is to provide this restriction in a principled way.

If an analysis which delivers the restriction on tuple-counterpart relations in a principled way is not available, and we want to adhere to the lesson, it seems that our tuple-counterpart machinery is inappropriate.

2.5 Alternative Machinery

So far, we have considered issues concerning existence, the combination of existence and iterated modality, and "contingent identity". In this final section, I consider alternatives to the tuple-counterpart machinery itself. For simplicity, I begin by limiting the discussion to non-gappy possibilities.

For the tuple-counterpart machinery, when we are concerned with possibilities for concreta together, the entities that have possibilities are tuples of concreta.

Equivalently, they are indexed families of concreta each of whose index set is an initial segment of the positive integers. A possibility for such an indexed family of concreta (with index set I) is another indexed family of concreta with index set I.

At one point (1986: 233, f.n. 26), Lewis considers alternatively just taking the entities that have possibilities to be indexed families of concreta where each member of the family is indexed by itself. A possibility for such an indexed family of concreta (with index set I) is another indexed family of concreta with index set I.

However, this coarse-grained indexing is too crude to handle the rich possibilities that we have been considering (and indeed Lewis himself considers (1986: 259)). In particular, this coarse-grained indexing cannot allow for a possibility for a collection of concreta where one concretum of the collection is assigned to play multiple individual-qualitative-roles in the possibility. That is, we cannot account for those possibilities that have so far been involved in rendering true certain "contingent identity" claims.

Lewis' motivation for introducing this alternative machinery is to connect his sketch of joint possibilities with Hazen's stipulational semantics (1986: 233 f.n. 26). In particular, it seems Lewis there is interested in connecting his sketch of joint possibilities to the stipulations Hazen describes in his first-pass account of stipulational semantics. On this first-pass, world domains (of individuals) are disjoint. For worlds w and w^+, a stipulation from w to w^+ is a (perhaps partial) injection from the domain (of individuals) of w to the domain (of individuals) of w^+ (1977: 114-117). Hazen tells us that for a non-modal formula 'ϕ', we have '$\diamond\phi$' ('$\Box\phi$') is true iff for some (every) world w and stipulation f from $w_@$ to w, 'ϕ' is true at w when each individual variable occurring free in 'ϕ' and each individual constant in 'ϕ' has its original denotation d changed to $f(d)$ (1977: 115).

However, a central motivation for Hazen in presenting his stipulational semantics is to present a semantics that does not appeal to identity across worlds in the analysis of the modal operators, but which is such that a fairly standard deductive system (which includes T, 4, B, and statements of the necessity of identity and distinctness, and which is described in section II.A of Hazen 1977) is sound and complete with respect to it. For example, the stipulations of the first-pass are required to be functions to render statements of the necessity of identity valid, and they are required to be injective to render statements of the necessity of distinctness valid.[6]

Lewis and Hazen do not have the same background motivations. Unless we are willing to give up some of Lewis' motivations, our machinery must be more

[6]I turn to stipulational semantics in detail in the fourth chapter.

fine-grained than Hazen's. Nevertheless, the basic tuple-counterpart machinery does seem overly complex. For example, consider:

(15) Everyone here could have been rich.

According to the plural *de re* reading of the claim, a collection of concreta together has a certain modal property: the property that they together could have all been rich. The reading is ostensibly *not* that the collection (ordered in a certain way) has a certain modal property. We might be able to accommodate the claim in terms of the tuple-counterpart machinery as follows. First, we analyse the claim as: a certain (ordered) collection of concreta has a certain tuple-counterpart. Second, we hold that there is underdetermination of the relevant ordering (and perhaps also of the relevant tuple-counterpart relation). However, this is inelegant.

One therefore may hope to produce an alternative machinery that is more fine-grained than Hazen's, but which is not based so centrally on orderings.

One approach I want to present is to enrich our indices. Let us say that an indexed family of concreta (all from one world) $(\mathrm{D}_i)_{i\in\mathrm{I}}$ is *standard* iff (i) the index set I contains pairs $< d$, C$>$ (where $d \in$ D and C is a counterpart relation) such that for every $d \in$ D, for some counterpart relation C, $< d$, C$>\in$ I, and (ii) for every $< d$, C$>\in$ I, $< d$, C$>$ indexes d. We allow that for some $d \in$ D, there may be distinct counterpart relations C_1 and C_2 such that $< d$, $\mathrm{C}_1 >$ and $< d$, $\mathrm{C}_2 >\in$ I (in which case, two distinct $i \in$ I index the same $d \in$ D).

Next, we introduce the notion of *group-counterparthood*. For a standard indexed family of concreta (all from one world) $(\mathrm{D}_i)_{i\in\mathrm{I}}$, an indexed family of concreta (all from one world) $(\mathrm{U}_i)_{i\in\mathrm{I}}$ is a group-counterpart of $(\mathrm{D}_i)_{i\in\mathrm{I}}$ iff (i) for every $< d$, C$>\in$ I, the $u \in$ U that $< d$, C$>$ indexes is a C counterpart of d, and (ii) the relations that hold between the collection of indexed concreta $(\mathrm{U}_i)_{i\in\mathrm{I}}$ are like the relations that hold between the collection of indexed concreta $(\mathrm{D}_i)_{i\in\mathrm{I}}$, that is, $(\mathrm{U}_i)_{i\in\mathrm{I}}$ is relationally similar to $(\mathrm{D}_i)_{i\in\mathrm{I}}$.

With the group-counterpart machinery, because we already build counterpart relations into our indices, our notion of relational similarity ostensibly does not need to depend on the individual counterpart relations in play. We can employ a general notion of a collection of indexed concreta (all from one world) $(\mathrm{U}_i)_{i\in\mathrm{I}}$ being relationally similar to a collection of indexed concreta (all from one world) $(\mathrm{D}_i)_{i\in\mathrm{I}}$ (letting I be an arbitrary set, so long as $|\ \mathrm{I}\ | \geq |\ \mathrm{D}\ |$ and $|\ \mathrm{I}\ | \geq |\ \mathrm{U}\ |$). Further, there is no requirement to block certain kinds of relational similarity from generating a kind of group-counterparthood depending on the particular counterpart relations in play.

At the second level, we have the notion of a (non-gappy) possibility for a collection of concreta (all from one world) bb (where each $b \prec bb$ is considered

individually in certain ways, and the concreta together are considered in a certain way). This amounts to a group-counterpart of a certain standard indexed family of *bb* (for a certain kind of group-counterparthood). At the third level, we have the notion of the possible/necessary for *bb* together (where each $b \prec bb$ is considered individually in certain ways, and the concreta together are considered in a certain way).

So much for a presentation of the machinery. Notice that it automatically satisfies the analogue of the restriction on tuple-counterparts of the previous section. For any standard indexed family of concreta (all from one world) $(\mathrm{D}_i)_{i\in\mathrm{I}}$, for every indexed family of concreta (all from one world) $(\mathrm{U}_i)_{i\in\mathrm{I}}$, every $< d, \mathrm{C}> \in \mathrm{I}$ indexes at most one $u \in \mathrm{U}$. So, a (non-gappy) possibility for a collection of concreta (all of one world) *bb* (where each $b \prec bb$ is considered individually in certain ways, and the concreta together are considered in a certain way) cannot assign a single concretum (individually considered in a single way) multiple individual-qualitative-roles. So, *bb* (where each $b \prec bb$ is considered individually in certain ways, and the concreta together are considered in a certain way) could not have been such that a $b \prec bb$ (individually considered in a single way) plays multiple individual-qualitative-roles.

For each of (2), (3), and (3^+), for any context c, we hold that the sentence (relative to c) involves the (weakly) possible for a single-membered collection *bb* (where the member is considered in certain ways). That is, it involves existential quantification over (non-gappy) possibilities for *bb* (where the member is considered in certain ways). That is, it involves existential quantification over group-counterparts of a certain standard indexed family of concreta (for a certain kind of group-counterparthood).

We also allow that for some standard indexed family of concreta (all from one world) $(\mathrm{D}_i)_{i\in\mathrm{I}}$, there is an indexed family of concreta (all from one world) $(\mathrm{U}_i)_{i\in\mathrm{I}}$ such that for some $u \in \mathrm{U}$ and distinct (from each other) d_1 and $d_2 \in \mathrm{D}$, there are indices $< d_1, \mathrm{C}_1 >$ and $< d_2, \mathrm{C}_2 > \in \mathrm{I}$ that both index u. Here, C_1 and C_2 may be identical or they may be distinct. So, we allow for a possibility where two distinct concreta (considered individually in the same way) play the same individual-qualitative-role, and (ii) a possibility where two distinct concreta (considered individually in different ways from each other) play the same individual-qualitative-role.

So far, this alternative machinery seems preferable to the tuple-counterpart machinery. We have a more straightforward approach to handling the plural *de re*, and we straightforwardly adhere to the negative lesson of section 1.2.1 on "contingent identity".

However, the points of sections 2.2 and 2.3 concerning existence hold for the new machinery (suitably translated).

One point on iterated modality here is that because the new entities that stand in group-counterpart relations are more fine-grained than mere ordered collections of concreta (all from one world), the basic approach to iterated modality does not involve group-counterparts of group-counterparts. This is because (i) an entity can be borne the group-counterpart relation only if it is a standard indexed family of concreta (all from one world), and (ii) a group-counterpart need not be a standard indexed family of concreta (all from one world). Instead, at each stage when we iterate, we need to consider not the group-counterparts of the group-counterparts, but the group-counterparts of certain standard indexed families of concreta. For example, reconsider:

(6) It could have been that it could have been that John loved Mary.

(6^{+}) $\Diamond\Diamond Ljm$

Let D be the set containing exactly John and Mary. Then the basic group-counterpart machinery analysis here is that a certain standard indexed family of concreta $(\mathrm{D}_i)_{i\in\mathrm{I}}$ has a group-counterpart $(\mathrm{U}_i)_{i\in\mathrm{I}}$ (for a certain kind of group-counterparthood), and a certain standard indexed family of concreta $(\mathrm{U}_j)_{j\in\mathrm{J}}$ has a certain group-counterpart $(\mathrm{A}_j)_{j\in\mathrm{J}}$. Note that the change from $(\mathrm{U}_i)_{i\in\mathrm{I}}$ to $(\mathrm{U}_j)_{j\in\mathrm{J}}$ is a change in the index set, but not in the set of concreta.

2.6 Conclusion

In this chapter, I presented and discussed certain issues for the counterpart account developed in the first chapter. These issues concern existence, the combination of existence and iterated modality, and "contingent identity". Finally, I considered alternatives to the tuple-counterpart machinery itself, including the group-counterpart machinery. Although the group-counterpart machinery seems superior to the tuple-counterpart machinery in some respects, issues concerning existence that apply to the tuple-counterpart machinery apply to it.

Chapter 3

Formal Characterisation

Lewis 1986 only provides *ad hoc* counterpart characterisations of natural language *de re* modal claims. However, unless the semantics for our natural language modal talk is non-compositional, the defender of his later metaphysics should hope to eventually provide a systematic semantics for natural language modal talk that delivers, for each natural language modal claim ϕ, a characterisation of ϕ in terms of LMR and the mature counterpart machinery.

Lewis 1968 employs formal machinery to deliver counterpart characterisations of natural language modal claims. First, Lewis presents a formal first-order theory, *counterpart theory*. The theory itself consists of certain sentences which impose conditions on worlds, individuals, and the counterpart relation. It is detachable from the full-blooded metaphysics that Lewis later proposes. Second, Lewis provides a method for translating natural language modal claims into sentences of the language of counterpart theory. This process has two steps. First, Lewis assumes a regimentation of natural language modal claims into sentences of "the language of quantified modal logic". Second, Lewis provides a translation scheme that maps sentences of this language of quantified modal logic to sentences of the language of counterpart theory.

This combination of (i) a theory and (ii) a translation scheme (that maps sentences of the language of quantified modal logic to sentences of the language of counterpart theory) generates a semantics for the language of quantified modal logic.

Given this, the 1968 account is really a version of a common strategy to present a systematic semantics for a fragment F of natural language: presenting a systematic semantics for a formal language L, leaving implicit a regimentation of F-sentences to L-sentences. However, the standard version of this strategy is to present a semantics for the formal language L by presenting a direct model-theoretic account for L. The 1968 account is in part interesting because of the indirect nature of the semantics.

This chapter focusses on the 1968 account (and does not presuppose Lewis' mature metaphysics). After introducing some preliminary technical material, I present the 1968 account, issues with it, and developments Lewis makes to it. Finally, I turn to a more foundational matter: considering the indirect approach to semantics more generally.

3.1 Languages

A *vocabulary* is any set V, a V-*symbol* is any member of V, and a V-*expression* is any string of V-symbols. Now, we introduce the notion of a *language* L over a vocabulary V. At the heart of a language L over vocabulary V is a set of V-expressions: the L-*sentences*. The L-sentences are generated by a collection of syntactic rules over V (a syntactic grammar for V). So, we have a system of V-expressions, structured by a collection of syntactic rules, which culminates with the L-sentences.

For an example of a language, consider the following. The vocabulary of L^+ contains exactly the following symbols: denumerably many symbols S_1, S_2, ... (which we call the *sentence letters*), $\rightarrow$, $\sim$, and the parentheses) and (.[1] [2] The L^+-sentences are inductively defined immediately as follows:

(i) Every sentence letter P is an L^+-sentence.

(ii) For every L^+-sentence ϕ, [$\sim\phi$ is an L^+-sentence].

(iii) For all L^+-sentences ϕ, ψ, [$(\phi \rightarrow \psi)$ is an L^+-sentence].

(iv) Nothing else is an L^+-sentence.

For simplicity, we hold:

$$\mathrm{Atom} = \{P : P \text{ is a sentence letter}\}.$$

$$\mathrm{Sen(L)} = \{\phi : \phi \text{ is an L-sentence}\}.$$

3.1.1 A Model-Theoretic Account

A common approach to providing a semantics for a language is to provide a model-theoretic account for that language. A model-theoretic account for a language L begins by specifying a kind of mathematical entity to play the L-*structure* role. From here, we define the notion of *satisfaction* of an L-sentence at an L-structure.

For example, let us say that an L^+-structure is a unary function $M : \mathrm{Atom} \rightarrow \{1, 0\}$. And for an L^+-structure M, we recursively define a unary satisfaction function $v_M : \mathrm{Sen}(\mathrm{L}^+) \rightarrow \{1, 0\}$ as follows:

[1] We assume that no symbol is a finite sequence of other symbols.

[2] Unless otherwise noted, for simplicity, I shall omit quotation marks around object language expressions when they are mentioned in the meta-language.

(i) $v_M(P) = M(P)$.

(ii) $v_M(\sim\phi) = 1$ iff $v_M(\phi) = 0$.

(iii) $v_M((\phi \rightarrow \psi)) = 1$ iff $v_M(\phi) = 0$ or $v_M(\psi) = 1$.

Finally, the notion of *consequence* is defined. For our model-theoretic account of L^+, for a set Γ of L^+-sentences, and L^+-sentence ϕ,

> $\Gamma \models_S \phi$ iff for every L^+-structure M, if $[\forall\gamma \in \Gamma, v_M(\gamma) = 1]$, then $v_M(\phi) = 1$.[3]

3.2 Standard First-Order Languages

Let us say that the *standard* first-order (FO) languages are exactly all instances of the following schema for a language L.

When specifying the vocabulary V of L, we distinguish between two types of V-symbol: the *logical*, and the *extralogical*. The former belong to the vocabulary of every standard FO language; the latter do not.[4]

Logical: $\sim, \wedge, \vee, \rightarrow, \leftrightarrow, (,), \forall, \exists, =$, and denumerably-many symbols $v_1, v_2, ...$, the *individual variables*.

Extralogical: (i) 0 or more symbols c, the *individual constants* of L.

ii For every $n \geq 1$, 0 or more symbols P, the (extralogical) *n-ary predicate constants* of L.

A V-expression is an L-term iff it is (a) an individual constant of L or (b) an individual variable. The L-*formulas* are inductively defined:

(i) For every $n \geq 1$, for all L-terms $t_1, ..., t_n$, and every (extralogical) n-ary predicate constant P, $[Pt_1...t_n$ is an L-formula].

(ii) For all L-terms t_1, t_2, $[t_1 = t_2$ is an L-formula].

(iii) For every L-formula ϕ, $[\sim\phi$ is an L-formula].

(iv) For every $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$, for all L-formulas ϕ, ψ, $[(\phi * \psi)$ is an L-formula].

(v) For every $* \in \{\forall, \exists\}$, L-formula ϕ, and individual variable x, $[*x\phi$ is an L-formula].

(vi) Nothing else is an L-formula.

[3] S for Sentential semantics.

[4] We assume that each symbol is of exactly one of the following kinds, and that no symbol is a finite sequence of other symbols.

Let:

VAR = $\{x : x$ is an individual variable$\}$.

IC(L) = $\{c : c$ is an individual constant of L$\}$.

Term(L) = $\{t : t$ is an L-term$\}$.

PC(L) = $\{P : P$ is an extralogical predicate constant of L$\}$.

Form(L) = $\{\phi : \phi$ is an L-formula$\}$.

Next, we define what it is for an individual variable x to *occur free* in an L-formula ϕ. Officially, we define this as follows. We recursively define a unary function f: Form(L) $\rightarrow \mathcal{P}$(VAR):

(i) $f(Pt_1...t_n) = \{t_i : 1 \leq i \leq n$ and $t_i \in$ VAR$\}$.

(ii) $f(t_1 = t_2) = \{t_i : 1 \leq i \leq 2$ and $t_i \in$ VAR$\}$.

(iii) $f(\sim\phi) = f(\phi)$.

(iv) For every $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$, $f((\phi * \psi)) = f(\phi) \cup f(\psi)$.

(v) For every $* \in \{\exists, \forall\}$, $f(*x\phi) = f(\phi) - \{x\}$.

An individual variable x occurs free in an L-formula ϕ iff $x \in f(\phi)$.

Unofficially, we can think of this in terms of occurrences of V-expressions in V-expressions. Of particular importance to us are occurrences of individual variables in L-formulas. Here, however, there is a second, narrower notion: occurrence*. First, a quantifier is a two-symbol-long string whose first element is $\forall$ or $\exists$, and whose second element is an individual variable. For an individual variable x, an x-quantifier is a quantifier whose second element is x. An occurrence* of an individual variable is an occurrence of that variable that is not contained in an occurrence of a quantifier.

For an L-formula ϕ and individual variable x, the scope of an occurrence O of an x-quantifier in ϕ is the occurrence of the smallest formula ψ in ϕ containing O.

For an L-formula ϕ and individual variable x, an occurrence* o of x in ϕ is bound by an occurrence O of an x-quantifier in ϕ iff (i) o is in the scope of O, and (ii) there is no distinct occurrence O^+ of an x-quantifier in the scope of O such that o is in the scope of O^+.

An occurrence* o of an individual variable x in an L-formula ϕ is *free* iff there is no occurrence of an x-quantifier in ϕ which binds o. An individual variable x occurs free in an L-formula ϕ iff there is a free occurrence* of x in ϕ.

Finally, an L-formula ϕ is an L-sentence iff no individual variable occurs free in ϕ. Let Sen(L) = $\{\phi : \phi$ is an L-sentence$\}$.

3.2.1 A Model-Theoretic Account

In this section, I present a standard model-theoretic account for standard FO languages based on the work of Tarski in the early to mid twentieth century. In particular, see Tarski 1933, Tarski 1936a/1936b, and Tarski & Vaught 1956.[5]

For a standard FO language L, a *standard* L-structure M is a tuple <D, I> such that:

D is a non-empty set.

I is a function from IC(L) ∪ PC(L) such that:

(i) For every $c \in$ IC(L), I(c) ∈ D.

(ii) For every $n \geq 1$, for every n-ary $P \in$ PC(L), I(P) $\in \mathcal{P}(\text{D}^n)$.

These two elements represent, respectively: the domain of individuals, and an interpretation of the extralogical L-vocabulary.

For a standard FO language L, and standard L-structure M, a variable assignment s of M is a function: VAR → D. For all $d_1, ..., d_n \in$ D, and $x_1, ..., x_n \in$ VAR, let $s_{[d_1,...,d_n/x_1,...,x_n]}$ be the variable assignment that agrees with s, except that d_1 is assigned to x_1, ..., and d_n is assigned to x_n. For a standard L-structure M, and variable assignment s of M, we specify a denotation function $den_{M,s}$: Term(L) → D as follows:

$$den_{M,s}(t) = \begin{cases} \text{I}(t) \text{ if } t \in \text{IC(L)}. \\ s(t) \text{ if } t \in \text{VAR}. \end{cases}$$

For a standard FO language L, and standard L-structure M, we simultaneously define by recursion, for each variable assignment s of M, a unary satisfaction function $v_{M,s}$: Form(L) → {1, 0} as follows:

(i) $v_{M,s}(Pt_1...t_n) = 1$ iff $< den_{M,s}(t_1), ..., den_{M,s}(t_n) > \in \text{I}(P)$.

(ii) $v_{M,s}(t_1 = t_2) = 1$ iff $den_{M,s}(t_1) = den_{M,s}(t_2)$.

(iii) $v_{M,s}(\sim\phi) = 1$ iff $v_{M,s}(\phi) = 0$.

(iv) $v_{M,s}((\phi \wedge \psi)) = 1$ iff $v_{M,s}(\phi) = 1$ and $v_{M,s}(\psi) = 1$.

(v) $v_{M,s}((\phi \vee \psi)) = 1$ iff $v_{M,s}(\phi) = 1$ or $v_{M,s}(\psi) = 1$.

(vi) $v_{M,s}((\phi \rightarrow \psi)) = 1$ iff $v_{M,s}(\phi) = 0$ or $v_{M,s}(\psi) = 1$.

(vii) $v_{M,s}((\phi \leftrightarrow \psi)) = 1$ iff $v_{M,s}(\phi) = v_{M,s}(\psi)$.

[5] Tarski 1936a/1936b are Polish and German presentations, respectively, of the same lecture on consequence. Tarski 1936c is a German translation of Tarski 1933 (with an additional postscript). Tarski 1956 provides English translations of, *inter alia*, Tarski 1933 (1956: 152-278) and Tarski 1936a/1936b (1956: 409-420).

(viii) $v_{M,s}(\exists x\phi) = 1$ iff for some d ∈ D, $v_{M,s_{[d/x]}}(\phi) = 1$.

(ix) $v_{M,s}(\forall x\phi) = 1$ iff for every d ∈ D, $v_{M,s_{[d/x]}}(\phi) = 1$.

For a standard FO language L, and standard L-structure M, we also define the unary function v_M: Form(L)→ {1, 0, U} as follows:

$$v_M(\phi) = \begin{cases} 1 \text{ if } \forall s, v_{M,s}(\phi) = 1. \\ 0 \text{ if } \forall s, v_{M,s}(\phi) = 0. \\ \text{U otherwise.} \end{cases}$$

This is thought of as describing satisfaction of an L-formula at a standard L-structure *simpliciter*.[6]

For a standard FO language L, standard L-structure M, set Γ of L-formulas, and L-formula ϕ,

(a) M models ϕ iff $v_M(\phi) = 1$.

(b) M models Γ iff for every $\gamma \in \Gamma$, M models γ.

(c) Mod(Γ) is the class of standard L-structures that model Γ.

Finally, consequence is defined. For a standard FO language L, set Γ of L-formulas, and L-formula ϕ,

> $\Gamma \models_{\text{FO}} \phi$ iff for every standard L-structure M, and s of M, if [$\forall \gamma \in \Gamma$, $v_{M,s}(\gamma) = 1$], then $v_{M,s}(\phi) = 1$.[7]

3.3 Standard First-Order Modal Languages

Let us say that a *standard* FO modal (FOM) language L is an extension of a standard FO language with exactly the sentential operators $\diamond$, $\Box$, and **A**, glossed as 'possibly', 'necessarily', and 'actually'. We include the following clause in our inductive definition of an L-formula:

(vii) For every $* \in \{\diamond, \Box, \mathbf{A}\}$, and L-formula ϕ, [$*\phi$ is an L-formula].[8]

With regards to the notion of an individual variable x occurring free in a formula ϕ of a standard FOM language L, we officially include the following clause in our recursive definition of f: Form(L) → $\mathcal{P}$(VAR):

[6]The Agreement Lemma for our model-theoretic account states: for every standard FO language L, standard L-structure M, all variable assignments s, s' of M, and every L-formula ϕ, if s and s' agree on all variables that occur free in ϕ, then $v_{M,s}(\phi) = v_{M,s'}(\phi)$. So, notice that no L-sentence is assigned U by the function.

[7]FO for First-Order semantics.

[8]For the purposes of this work, for simplicity, we shall suppose that the following symbols serving as constants of certain standard FO languages cannot be constants of standard FOM languages: $W, A, I, C, D, R, @$. As a result, note that not every standard FO language extends to a standard FOM language. The motivation for this restriction will become clear later on.

(vi) For every $* \in \{\Diamond, \Box\ \mathbf{A}\}$, $f(*\phi) = f(\phi)$.

We can unofficially think of the notion in the same way as earlier.

3.3.1 Constant Domain Semantics

In this section, I present a simple model-theoretic account for standard FOM languages based on the model-theoretic approach developed by Kripke in the late 1950s and early 1960s. In particular, see Kripke 1963a and Kripke 1963b.

For a standard FOM language L, a *constant domain* L-structure M is a tuple <W, R, D, $w_@$, I> such that:

W and D are non-empty sets.

$\mathrm{R} \subseteq \mathrm{W}^2$.

$w_@ \in \mathrm{W}$.

I is a function from IC(L) $\cup$ PC(L) such that:

(i) For every $c \in$ IC(L), I(c) $\in$ D.

(ii) For every $n \geq 1$, every n-ary $P \in$ PC(L) is assigned a function that assigns to each $w \in$ W a subset of D^n.

These elements represent, respectively: a set of worlds, the accessibility relation between worlds, the domain of individuals, the actual world, and an interpretation of the extralogical L-vocabulary.

For a standard FOM language L, and constant domain L-structure M, a variable assignment s for M is a function: VAR $\rightarrow$ D. For a constant domain L-structure M, and variable assignment s of M, we specify a denotation function $den_{M,s}$: Term(L) $\rightarrow$ D as follows:

$$den_{M,s}(t) = \begin{cases} \mathrm{I}(t) \text{ if } t \in \mathrm{IC(L)}. \\ s(t) \text{ if } t \in \mathrm{VAR}. \end{cases}$$

For a standard FOM language L, and constant domain L-structure M, we simultaneously define by recursion, for each $w \in$ W, and variable assignment s of M, a unary satisfaction function $v_{M,w,s}$: Form(L) $\rightarrow \{1, 0\}$ as follows:

(i) $v_{M,w,s}(Pt_1...t_n) = 1$ iff $< den_{M,s}(t_1), ..., den_{M,s}(t_n) > \in \mathrm{I}(P)(w)$.

(ii) $v_{M,w,s}(t_1 = t_2) = 1$ iff $den_{M,s}(t_1) = den_{M,s}(t_2)$.

(iii) $v_{M,w,s}(\sim\phi) = 1$ iff $v_{M,w,s}(\phi) = 0$.

(iv) $v_{M,w,s}((\phi \wedge \psi)) = 1$ iff $v_{M,w,s}(\phi) = 1$ and $v_{M,w,s}(\psi) = 1$.

(v) $v_{M,w,s}((\phi \vee \psi)) = 1$ iff $v_{M,w,s}(\phi) = 1$ or $v_{M,w,s}(\psi) = 1$.

(vi) $v_{M,w,s}((\phi \to \psi)) = 1$ iff $v_{M,w,s}(\phi) = 0$ or $v_{M,w,s}(\psi) = 1$.

(vii) $v_{M,w,s}((\phi \leftrightarrow \psi)) = 1$ iff $v_{M,w,s}(\phi) = v_{M,w,s}(\psi)$.

(viii) $v_{M,w,s}(\exists x\phi) = 1$ iff for some $d \in$ D, $v_{M,w,s_{[d/x]}}(\phi) = 1$.

(ix) $v_{M,w,s}(\forall x\phi) = 1$ iff for every $d \in$ D, $v_{M,w,s_{[d/x]}}(\phi) = 1$.

(x) $v_{M,w,s}(\Diamond\phi) = 1$ iff for some $w^+ \in$ W such that Rww^+, $v_{M,w^+,s}(\phi) = 1$.

(xi) $v_{M,w,s}(\Box\phi) = 1$ iff for every $w^+ \in$ W such that Rww^+, $v_{M,w^+,s}(\phi) = 1$.

(xii) $v_{M,w,s}(\mathbf{A}\phi) = 1$ iff $v_{M,w_@,s}(\phi) = 1$.

For a constant domain L-structure M, and variable assignment s of M, we define the unary function $v_{M,s}$: Form(L) $\to \{1, 0\}$ as follows:

$$v_{M,s}(\phi) = v_{M,w_@,s}(\phi).$$

This is thought of as describing satisfaction of an L-formula at an L-structure relative to a variable assignment.

Finally, consequence is defined. For a standard FOM language L, class F of constant domain L-structures, set Γ of L-formulas, and L-formula ϕ,

> $\Gamma \models_{\text{DCD(F)}} \phi$ iff for every constant domain L-structure M in F, and s of M, if $[\forall\gamma \in \Gamma, v_{M,s}(\gamma) = 1]$, then $v_{M,s}(\phi) = 1$.[9]

So much for an exposition of the account. Notice that given the handling of the first-order quantifiers, they commute with the modal operators. As a result, the account generates controversial theorems. For example, consider formulas (NE), (N), (BF), and (CBF):

(NE) $\forall x\Box\exists y x = y$

(N) $\Box\forall x\Box\exists y x = y$

(BF) $(\forall x\Box Fx \to \Box\forall x Fx)$

(CBF) $(\Box\forall x Fx \to \forall x\Box Fx)$

(NE) is a statement of the necessity of existence, (N) a statement of the stronger *necessitism* (see Williamson 2013), and (BF) and (CBF) Barcan Marcus' Barcan Formula and Converse Barcan Formula, respectively (see Barcan 1946). For every standard FOM language L (containing F), class F of constant domain L-structures, and L-formula ϕ from the above list, $\models_{\text{DCD(F)}} \phi$.

[9] DCD for Direct Constant Domain semantics.

3.3.2 Varying Domain Semantics

Some find the above simple constant domain model-theoretic account philosophically acceptable. For example, see Linsky & Zalta 1994 and 1996, and Williamson 1998, 2002, and 2013. However, an alternative is available that (at least for certain classes of its structures) is able to block the theoremhood of each member of the above list. I turn to a version of this model-theoretic account now.

For a standard FOM language L, a *varying domain* L-structure M is a tuple <W, R, D, dom, $w_@$, I> such that:

W and D are non-empty sets.

$R \subseteq W^2$.

dom: $W \rightarrow \mathcal{P}(D)$.

$w_@ \in W$.

$dom(w_@) \neq \emptyset$.

I is a function from IC(L) $\cup$ PC(L) such that:

(i) For every $c \in$ IC(L), $I(c) \in dom(w_@)$.

(ii) For every $n \geq 1$, every n-ary $P \in$ PC(L) is assigned a function that assigns to each $w \in W$ a subset of D^n.

These elements represent, respectively: a set of worlds, the accessibility relation between worlds, the overall domain of individuals, the world domains of individuals, the actual world, and an interpretation of the extralogical L-vocabulary.

For a standard FOM language L, and varying domain L-structure M, a variable assignment s for M is a function: VAR $\rightarrow$ D. For a varying domain L-structure M, and variable assignment s of M, we specify a denotation function $den_{M,s}$: Term(L) $\rightarrow$ D as before:

$$den_{M,s}(t) = \begin{cases} I(t) \text{ if } t \in \text{IC(L)}. \\ s(t) \text{ if } t \in \text{VAR}. \end{cases}$$

For a standard FOM language L, and varying domain L-structure M, we simultaneously define by recursion, for each $w \in W$, and variable assignment s of M, a unary satisfaction function $v_{M,w,s}$: Form(L) $\rightarrow \{1, 0\}$. The clauses for this definition are the same ones used in constant domain semantics, with the exception of the $\exists$ and $\forall$ clauses, which become:

(viii*) $v_{M,w,s}(\exists x\phi) = 1$ iff for some $d \in dom(w)$, $v_{M,w,s_{[d/x]}}(\phi) = 1$.

(ix*) $v_{M,w,s}(\forall x\phi) = 1$ iff for every $d \in dom(w)$, $v_{M,w,s_{[d/x]}}(\phi) = 1$.

We define the satisfaction function $v_{M,s}$ as we did with constant domain semantics. Finally, consequence is defined. For a standard FOM language L, class F of varying domain L-structures M, set Γ of L-formulas, and L-formula ϕ,

> $\Gamma \models_{\text{DVD(F)}} \phi$ iff for every varying domain L-structure M in F, and s of M, if $[\forall \gamma \in \Gamma, v_{M,s}(\gamma) = 1]$, then $v_{M,s}(\phi) = 1$.[10]

So much for an exposition of the account. Crucially, in virtue of restricting the range of the first-order quantifiers to world domains, they do not commute with the modal operators. As such, for every standard FOM language L (containing F), and L-formula ϕ of our list, there is a varying domain L-structure M, and s of M such that $v_{M,s}(\phi) = 0$.

3.4 Lewis 1968

Lewis 1968 presents a theory of modality, *counterpart theory*. In line with Quinean methodology, the language of counterpart theory is a standard FO language without individual constants. Its signature contains four special predicate constants: two unary predicate constants A and W, and two binary predicate constants I and C. These are to be read as follows (1968: 113):

Wx	x is a world
Ixy	x is in y
Ax	x is actual
Cxy	x is a counterpart of y

The theory itself consists of certain sentences which impose conditions on worlds, individuals, and the counterpart relation.

To capture the counterpart theoretic analysis of our natural language modal talk, Lewis provides a method for translating natural language modal claims into sentences of the language of counterpart theory. This process has two steps. First, Lewis assumes a regimentation of natural language modal claims into sentences of "the language of quantified modal logic" (without individual constants). Second, Lewis provides a translation scheme that maps sentences of this language of quantified modal logic to sentences of the language of counterpart theory.

Here, I present the 1968 account. However, my presentation deviates from the original presentation in two (innocuous) ways. First, rather than considering a single language of quantified modal logic and a single language of counterpart theory, I shall consider a class of formal first-order modal languages and a corresponding class of counterpart theory languages. Second, I shall take a counterpart theory language to include exactly one individual constant @, which will be used to denote the actual world.

[10] DVD for Direct Varying Domain semantics.

3.4.1 Theories

Let the CT languages be a certain sub-class of the standard FO languages. A standard FO language L is a CT language iff it (i) contains exactly one individual constant @, and (ii) contains the unary predicate constants W and A, and the binary predicate constants I and C.

For a CT language L, let the L counterpart theory be the theory T containing exactly (P1)-(P8), (A), (B), and (C):

(P1) $\forall v_1 \forall v_2 (Iv_1v_2 \rightarrow Wv_2)$
Individuals are only in worlds.

(P2) $\forall v_1 \forall v_2 \forall v_3 ((Iv_1v_2 \wedge Iv_1v_3) \rightarrow v_2 = v_3)$
No individual is in more than one world.

(P3) $\forall v_1 \forall v_2 (Cv_1v_2 \rightarrow \exists v_3 Iv_1v_3)$
Any individual that is a counterpart is in a world.

(P4) $\forall v_1 \forall v_2 (Cv_1v_2 \rightarrow \exists v_3 Iv_2v_3)$
Any individual that has a counterpart is in a world.

(P5) $\forall v_1 \forall v_2 \forall v_3 (((Iv_1v_2 \wedge Iv_3v_2) \wedge Cv_1v_3) \rightarrow v_1 = v_3)$
An individual is a counterpart of no individual in its world other than itself.

(P6) $\forall v_1 \forall v_2 (Iv_1v_2 \rightarrow Cv_1v_1)$
Any individual in a world is a counterpart of itself.

(P7) $\exists v_1 (Wv_1 \wedge \forall v_2 (Iv_2v_1 \leftrightarrow Av_2))$
There is a world containing exactly all actual individuals.

(P8) $\exists v_1 Av_1$
Some individual is actual.

(A) $\forall v_1 (Wv_1 \vee \exists v_2 (Iv_1v_2))$
Each individual is a world or is in some world.

(B) $\forall v_1 (Wv_1 \rightarrow Iv_1v_1)$
Each world is in itself.

(C) $\forall v_1 (\forall v_2 (Iv_2v_1 \leftrightarrow Av_2) \leftrightarrow v_1 = @)$
@ is the actual world.

(P1)-(P8) are given by Lewis (1968: 114). I include (A) because Lewis notes that "the domain of quantification is to contain every possible world and everything in every world" (ibid.). I take it he here means contain exactly. I include (B) as a simplification that certainly a later Lewis (who conceives of worlds in mereological terms) would accept. I include (C) to characterise the primitive @.

For a CT language L, the CT L-structures are exactly the standard L-structures in Mod(T).

3.4.2 Translation

Our intermediary formal languages are standard FOM languages (without individual constants or **A**). For such a language L, there corresponds a CT language L^{CT} whose constants are exactly all the constants of L, plus $@, W, A, I, C$.[11]

Let L be a standard FOM language L (without individual constants or **A**). We define a *translation scheme* that maps an L-sentence to an L^{CT}-sentence. The translation of L-sentence ϕ is $\phi^{@}$, to be read as: ϕ holds at the actual world.

To define this, we simultaneously define by recursion, for every $\alpha \in$ [VAR $\cup\{@\}$], a unary function $(.)^{\alpha}$: Form(L) $\rightarrow$ Form(L^{CT}). Lewis tells us to read ϕ^{α} as: ϕ holds at world α.

To do this, we start with some notation. Associated with each L-formula ϕ is an n-ary sequence of distinct (from each other) individual variables $x_1, ..., x_n$, the (ordered) individual variables occurring free in ϕ. Sometimes, I will indicate this by attaching a subscript as follows:

$$\phi_{[x_1,...,x_n]}$$

Next, for an L-formula $\phi_{[x_1,...,x_n]}$, and individual variables $y_1, ..., y_n$, let $\phi(y_1, ..., y_n)$ be exactly the same as ϕ, except that every free occurrence of x_1 is replaced with y_1, ..., and every free occurrence of x_n is replaced with y_n.

Next, we hold:[12]

(i) $(Px_1...x_n)^{\alpha}$ is $Px_1...x_n$.

(ii) $(x_1 = x_2)^{\alpha}$ is $x_1 = x_2$.

(iii) $(\sim\phi)^{\alpha}$ is $\sim(\phi^{\alpha})$.

[11] Recall that $@, W, A, I, C$ cannot be constants of L.

[12] For simplicity, I will henceforth often omit some brackets. In which case, assume the written string is an abbreviation of a formula that is bracketed to the left. Sometimes, to stress scope, I shall include additional brackets that are not part of the object language formulas.

(iv) For every $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$, $(\phi * \psi)^\alpha$ is $(\phi^\alpha * \psi^\alpha)$.

(v) $(\exists x \phi)^\alpha$ is $\exists x(Ix\alpha \wedge \phi^\alpha)$.

(vi) $(\forall x \phi)^\alpha$ is $\forall x(Ix\alpha \rightarrow \phi^\alpha)$.

(vii) $(\diamond \phi_{[x_1,...,x_n]})^\alpha$ is

$\exists w \exists y_1 ... \exists y_n (Ww \wedge Iy_1 w \wedge Cy_1 x_1 \wedge ... \wedge Iy_n w \wedge Cy_n x_n \wedge$

$(\phi(y_1, ..., y_n))^w)$.

For some world w, w-counterpart y_1 of x_1, ..., and w-counterpart y_n of x_n, ϕ holds of $y_1, ..., y_n$ at w.

(viii) $(\Box \phi_{[x_1,...,x_n]})^\alpha$ is

$\forall w \forall y_1 ... \forall y_n ((Ww \wedge Iy_1 w \wedge Cy_1 x_1 \wedge ... \wedge Iy_n w \wedge Cy_n x_n) \rightarrow$

$(\phi(y_1, ..., y_n))^w)$.

For every world w, w-counterpart y_1 of x_1, ..., and w-counterpart y_n of x_n, ϕ holds of $y_1, ..., y_n$ at w.

As an aside, notice that this is not yet a recursive definition of the function $(.)^\alpha$. This is because for clauses (vii) and (viii), we have given a string containing the meta-variables $w, y_1, ..., y_n$ (which range over the individual variables of our object language), but we have not given a way to determine exactly which individual variables are contained in the output.

To specify the function $(.)^\alpha$, we must supplement clauses (vii) and (viii) so that an L-formula is mapped to a single L^{CT}-formula. The only desideratum here is that the resultant translation scheme does not deliver formulas with incorrectly bound occurrences of individual variables. There are two ways that we can arrive at an incorrectly bound occurrence of an individual variable. To see this, let us focus on clause (vii) (the discussion, suitably tweaked, holds for (viii) too). We have:

(a) If some β of $w, y_1, ..., y_n$ is equal to some x_i of $x_1, ..., x_n$, then the initial occurrence of $\exists \beta$ incorrectly binds an occurrence of x_i in the initial segment of the embedded long conjunction.

(b) If some β of $w, y_1, ..., y_n$ is such that there is an occurrence of a β-quantifier $Q\beta$ in ϕ, then an occurrence of $Q\beta$ in $(\phi(y_1, ..., y_n))^w$ may incorrectly bind an occurrence of β.

For example, consider:

$(\diamond \exists x Rxy)^\alpha$

$\exists w \exists x_1 (Ww \wedge Ix_1 w \wedge Cx_1 y \wedge \exists x(Ixw \wedge Rxx_1))$

On (a), if w is y, then the initial $\exists w$ occurrence incorrectly binds the occurrence of y in Cx_1y. And if x_1 is y, then the initial $\exists x_1$ occurrence incorrectly binds the occurrence of y in Cx_1y.

On (b), if w is x, then the later $\exists x$ occurrence incorrectly binds the occurrence of w in Ixw. And if x_1 is x, then the later $\exists x$ occurrence incorrectly binds the occurrence of x_1 in Rxx_1.

Put simply, in case (a), an initial quantifier occurrence binds too much. And in case (b), an initial quantifier occurrence binds too little.

Given all this, here is a simple way to determine individual variables. Recall that every individual variable of our object language is a v subscripted with a natural number $n \geq 1$. Given this, we specify the following for clauses (vii) and (viii). For an L-formula $O\phi_{[x_1,...,x_n]}$ (where O is $\diamond$ or $\Box$), w is v_i, y_1 is v_{i+1}, ..., and y_n is v_{i+n}, where:

> If no individual variables occur (either occur or occur*) in ϕ, $i = 1$.
>
> If some individual variables occur (either occur or occur*) in ϕ, $i = j + 1$, where (a) v_j occurs (either occurs or occurs*) in ϕ, and (b) $\forall k \in \mathbb{N}$ such that v_k occurs (either occurs or occurs*) in ϕ, $j \geq k$.

So much for a specification of object language variables. For simplicity, when discussing counterpart theoretic translation schemes, I shall henceforth leave the specification of object language variables implicit and talk in terms of strings involving meta-variables ranging over object language variables.[13]

For examples of the translation scheme:

$$\begin{array}{lll} (\forall xFx)^{@} & \text{is} & \forall x(Ix@ \rightarrow Fx) \\ (\Box\forall xFx)^{@} & \text{is} & \forall y(Wy \rightarrow \forall x(Ixy \rightarrow Fx)) \\ (\forall x\Box Fx)^{@} & \text{is} & \forall x(Ix@ \rightarrow \forall y\forall z((Wy \wedge Izy \wedge Czx) \rightarrow Fz)) \end{array}$$

The overall translation process now proceeds as follows. First, regiment a natural language claim ϕ (which does not feature names) as a sentence ψ of a standard FOM language L (without individual constants or $\mathbf{A}$). Second, apply the translation scheme to ψ to arrive at the counterpart theoretic translation $\psi^{@}$. For an example of the overall translation process:

> Some individual is handsome but could have been ugly.
>
> $\exists x(Hx \wedge \diamond Ux)$
>
> $\exists x(Ix@ \wedge Hx \wedge \exists w\exists y(Ww \wedge Iyw \wedge Cyx \wedge Uy))$

[13]Presenting a specification was required for certain proofs that appear later.

3.4.3 An Indirect Semantics

For a standard FOM language L (without individual constants or **A**), the combination of (i) an L^{CT} theory, and (ii) a translation scheme that maps an L-sentence to an L^{CT}-sentence, provides an indirect semantics for L.[14]

To capture this, for a given standard FOM language L (without individual constants or **A**) with its corresponding CT language L^{CT}, for a CT L^{CT}-structure M, we define a satisfaction function $v_M : \text{Sen(L)} \rightarrow \{1, 0\}$ as follows:

$v_M(\phi) = v_M(\phi^{@})$.

This characterises the notion of a CT L^{CT}-structure M indirectly interpreting L-sentences.

We can then define consequence. For a standard FOM language L (without individual constants or **A**), set Γ of L-sentences, and L-sentence ϕ,

> $\Gamma \models_{\text{ICT}} \phi$ iff for every CT L^{CT}-structure M, if $[\forall \gamma \in \Gamma, v_M(\gamma) = 1]$, then $v_M(\phi) = 1$.[15]

This raises an interesting foundational question: what is the technical significance of presenting a semantics for a language via this indirect approach as opposed to via a direct model-theoretic account? I turn to this question later in the chapter.

3.4.4 A Note on Machinery

For constant and varying domain semantics for a standard FOM language L, the notion of world representation is central to the semantics (and is captured by the notion of an L-formula being satisfied at a world relative to a variable assignment). Then, the modal operators are unpacked in terms of it. Broadly speaking, a world w represents that $\Diamond\phi$ ($\Box\phi$) iff some (every) w-accessible world represents that ϕ. The crucial point here is that this is not how the 1968 semantics operates.

To see this, let us start by considering the function $(.)^{\alpha}$. One might initially think that this function captures a notion of world representation. After all, Lewis tells us to read ϕ^{α} as: ϕ holds at world α. Further, we do have that for every standard FOM language L (without individual constants or **A**), and L-*sentence* ϕ, ϕ^{α} presents a world-relative condition. This is because ϕ will be built out of atomic formulas with just individual variables as terms, and individual variable occurrences are all bound by first-order quantifier

[14] Indeed, Lewis himself notes that the 1968 account effectively presents a semantics for the "standard language of modal logic" (1986: 9).

[15] ICT for Indirect Counterpart Theory semantics.

occurrences. Crucially, the first-order quantifier clauses for $(.)^\alpha$ effectively then unpack each first-order quantifier occurrence as ranging over the domain of a world. As a result, one can see $(.)^\alpha$ (restricted to the L-sentences) as characterising what holds at world α.[16] However, recall that the sentences we are currently concerned with feature no individual constants. They are purely qualitative.

When we expand our focus to include open L-formulas, we cannot see $(.)^\alpha$ as capturing a notion of world representation. For example, consider L-formula Fx. $(Fx)^\alpha$ is just L^{CT}-formula Fx, which is not a world-relative condition.

If we restricted our standard FOM language L (without individual constants or **A**) so that $\diamond$ and $\Box$ only attach to L-sentences, then we would have that (i) $(.)^\alpha$ (restricted to L-sentences) captures world representation, and (ii) the modal operator clauses of $(.)^\alpha$ are simply unpacked in terms of quantification over worlds and world representation. However, such a language only expresses modality *de dicto*. When L isn't restricted in this way, and $\diamond$ and $\Box$ can attach to open L-formulas, the modal operator clauses are much more complex.

The function $(.)^\alpha$ is instead set up to define $\phi^{@}$ (for an L-sentence ϕ).

3.4.5 Individual Constants

We have yet to cover how Lewis' overall translation process handles natural language modal claims with a name occurrence. To expand the overall translation process to handle such claims, we first expand our intermediary formal languages to all standard FOM languages (without **A**). Next, a natural language modal claim with a name occurrence is regimented to a standard FOM language (without **A**) sentence with an individual constant occurrence.

The question now is: how should we translate a standard FOM language (without **A**) sentence with an individual constant occurrence?

Put simply, Lewis suggests that for a standard FOM language L (without **A**), if an L-sentence contains an occurrence of an individual constant, we treat each such occurrence as a definite description, then eliminate the description by Russell's contextual definition (1968: 120). Then the 1968 translation scheme applies as usual.

Consider the following toy example which attempts to capture this idea. Let L be a standard FOM language (without **A**) with exactly one individual

[16] Suppose we had allowed 0-ary predicate constants in our standard FOM languages. Then for a 0-ary predicate constant P of standard FOM language L (without individual constants or **A**), P^α is just P. This is not a world-relative condition. For a CT L^{CT}-structure M, $\mathrm{I}(P) \subseteq \mathrm{D}^0 = \{\{\}\}$. That is, M just assigns it truth or falsity *simpliciter*. It is for this reason that I assume Lewis does not have 0-ary predicate constants in mind, and why I have excluded them in my overall presentation.

constant p. Corresponding to L is standard FOM language L^+ (without **A**) with the same vocabulary as L except that it omits p and includes unary predicate constant P (not in the vocabulary of L). We specify an individual-constant-elimination mapping which takes as input an L-formula ϕ and returns an L^+-formula ϕ^+ that is exactly the same as ϕ except that every occurrence of p has been understood as a definite description and contextually eliminated in the usual Russellian fashion employing our new predicate constant P. Next, we apply the original 1968 translation scheme to our outputs of the individual-constant-elimination mapping.

For our L, the corresponding counterpart language is now the CT language $\mathrm{L}^{+^{\mathrm{CT}}}$, the CT language whose constants are exactly all the predicate constants of L^+, plus $W, A, I, C, @$. In particular, P is a predicate constant of this counterpart language. Now, we need to specify our $\mathrm{L}^{+^{\mathrm{CT}}}$ theory T to determine the class Mod(T) of standard $\mathrm{L}^{+^{\mathrm{CT}}}$-structures that will indirectly interpret L-sentences.

Suppose we have specified T. Now, for an $M \in \mathrm{Mod(T)}$, we can define the satisfaction function $v_M : \mathrm{Sen(L)} \rightarrow \{1, 0\}$:

$$v_M(\phi) = v_M(\phi^{+^{@}})$$

Presumably, T is not just the set containing exactly (P1)-(P8), (A), (B), and (C). Rather, we must also include an axiom (or axioms) governing the predicate constant P. To see this, consider L-formula ϕ: $Fp \vee \sim Fp$. Suppose the individual-constant-elimination mapping maps ϕ to:

$$\phi^+ \;\; \exists x(Px \wedge \forall y(Py \rightarrow x = y) \wedge (Fx \vee \sim Fx))$$

By the original translation scheme, this maps to:

$$\phi^{+^{@}} \;\; \exists x(Ix@ \wedge Px \wedge \forall y(Iy@ \rightarrow (Py \rightarrow x = y)) \wedge (Fx \vee \sim Fx))$$

Given this, unless we include in T an axiom requiring that there is exactly one actual P, there will be an $M \in \mathrm{Mod(T)}$ such that $v_M(\phi^{+^{@}}) = 0$. Hence, an $M \in \mathrm{Mod(T)}$ such that $v_M(\phi) = 0$!

So much for Lewis' approach to individual constants. Notice that an alternative approach is just to augment the translation scheme to cover all standard FOM languages (without **A**). The idea I have in mind is as follows. Let a CT* language be a standard FO language that contains (i) the individual constant @, (ii) the unary predicate constants W and A, and (iii) the binary predicate constants I and C.

For a CT* language L, let the L counterpart theory* be the theory T containing exactly (P1)-(P8), (A), (B), (C), and every instance of schema (D):

(D) Ac

c is actual.

For a CT* language L, the CT* L-structures are exactly the standard L-structures in Mod(T).

Now, we turn to the translation scheme. For a standard FOM language L (without **A**), there corresponds a CT* language $\mathrm{L}^{\mathrm{CT}^*}$ whose constants are exactly all of the constants of L, plus $W, A, I, C, @$. Let L be a standard FOM language (without **A**), and $\mathrm{L}^{\mathrm{CT}^*}$ its corresponding CT* language.

We map an L-sentence to an $\mathrm{L}^{\mathrm{CT}^*}$-sentence. The translation of L-sentence ϕ is $\phi^{@}$.

To define this, we simultaneously define by recursion, for every $\alpha \in$ [VAR $\cup\{@\}$], a unary function $(.)^{\alpha}$: Form(L) $\rightarrow$ Form($\mathrm{L}^{\mathrm{CT}^*}$).

To do this, we need to update our notation. For simplicity, let us say that every occurrence of an individual constant of L is free. Now, associated with each L-formula ϕ is an n-ary sequence of distinct (from each other) L-terms $t_1, ..., t_n$, the (ordered) L-terms occurring free in ϕ. Sometimes, I will indicate this by attaching a subscript as follows:

$$\phi_{[t_1,...,t_n]}$$

Next, for an L-formula $\phi_{[t_1,...,t_n]}$, and L-terms $u_1, ..., u_n$, let $\phi(u_1, ..., u_n)$ be exactly the same as ϕ, except that every free occurrence of t_1 is replaced with u_1, ..., and every free occurrence of t_n is replaced with u_n.

The recursive definition of $(.)^{\alpha}$ is the same as before, except that clauses (i), (ii), (vii), and (viii) are adjusted to the following:

(i) $(Pt_1...t_n)^{\alpha}$ is $Pt_1...t_n$.

(ii) $(t_1 = t_2)^{\alpha}$ is $t_1 = t_2$.

(vii) $(\diamond\phi_{[t_1,...,t_n]})^{\alpha}$ is

$\exists w \exists y_1 ... \exists y_n (Ww \wedge Iy_1 w \wedge Cy_1 t_1 \wedge ... \wedge Iy_n w \wedge Cy_n t_n \wedge$

$(\phi(y_1, ..., y_n))^w)$.

(viii) $(\Box\phi_{[t_1,...,t_n]})^{\alpha}$ is

$\forall w \forall y_1 ... \forall y_n ((Ww \wedge Iy_1 w \wedge Cy_1 t_1 \wedge ... \wedge Iy_n w \wedge Cy_n t_n) \rightarrow$

$(\phi(y_1, ..., y_n))^w)$.

So much for the translation scheme. We would finally capture the idea of an indirect semantics by defining satisfaction and consequence in a similar fashion to the original indirect semantics.

For both simplicity and neutrality, when discussing the 1968 account, I shall henceforth ignore natural language modal claims featuring names, and will only consider standard FOM languages without individual constants.

3.4.6 Actuality

So much for the overall 1968 account. One of the central challenges made against the account is that it cannot handle certain natural language modal claims involving a notion of *actuality* (Hazen 1977 and 1979, and Fara & Williamson 2005).

The challenge has two parts. First, in order to handle certain natural language modal claims involving a notion of actuality, the initial formal modal language that natural language modal claims are regimented into must include something like an actuality operator **A**. Second, certain ways that Lewis' 1968 semantics might be extended to accommodate an actuality operator **A** are unacceptable.

The straightforward way to extend the 1968 semantics to handle **A** is as follows. First, we say that for a standard FOM language L (without individual constants), there corresponds a CT language L^{CT} whose constants are exactly all of the constants of L, plus $@, W, A, I, C$. Next, for a standard FOM language L (without individual constants), we define a mapping of L-sentences to L^{CT}-sentences in a similar fashion to our original mapping as follows. For a standard FOM language L (without individual constants), the translation of L-sentence ϕ is L^{CT}-sentence $\phi^{@}$. This is defined by simultaneously defining by recursion, for every $\alpha \in [\mathrm{VAR} \cup \{@\}]$, a unary function $(.)^{\alpha}$: Form(L) $\rightarrow$ Form(L^{CT}). In the recursive definition, we reuse the original 1968 clauses and add a clause for **A**. Given a mapping, we would finally capture the idea of an indirect semantics by defining satisfaction and consequence in a similar fashion to the original indirect semantics. For satisfaction: for a standard FOM language L (without individual constants), and CT L^{CT}-structure M, we define the satisfaction function v_M: Sen(L) $\rightarrow \{1, 0\}$:

$$v_M(\phi) = v_M(\phi^{@})$$

However, Hazen 1977 (reprinted 1979) argues that we cannot define $(.)^{\alpha}$ by combining Lewis' original 1968 clauses with either of the following clauses for **A** (1977: 111):

($\exists$-**A**) $(\mathbf{A}\phi_{[x_1,\ldots,x_n]})^{\alpha}$ is

$\exists y_1 \ldots \exists y_n (Iy_1@ \wedge Cy_1x_1 \wedge \ldots \wedge Iy_n@ \wedge Cy_nx_n \wedge$
$(\phi(y_1, \ldots, y_n))^{@})$.

For some actual counterpart y_1 of x_1, ..., and some actual counterpart y_n of x_n, ϕ holds of $y_1, ..., y_n$ at $w_@$.

($\forall$-**A**) $(\mathbf{A}\phi_{[x_1,...,x_n]})^\alpha$ is

$\forall y_1 ... \forall y_n((Iy_1@ \wedge Cy_1x_1 \wedge ... \wedge Iy_n@ \wedge Cy_nx_n) \rightarrow$

$(\phi(y_1, ..., y_n))^@)$.

For every actual counterpart y_1 of x_1, ..., and every actual counterpart y_n of x_n, ϕ holds of $y_1, ..., y_n$ at $w_@$.

If we adopt ($\exists$-**A**), then the supposedly intuitively unsatisfiable sentence (16) of basic FOM language L (without individual constants) is satisfiable (ibid.). That is, in the terminology of my framework, for some CT L^{CT}-structure M, $v_M((16)) = 1$.

(16) $\diamond\exists x(\mathbf{A}\exists yx = y \wedge \mathbf{A}Fx \wedge \mathbf{A}{\sim}Fx)$

Proof. $(16)^@$ is:

$\exists u(Wu \wedge \exists x(Ixu \wedge$

$\exists z(Iz@ \wedge Czx \wedge \exists y(Iy@ \wedge z = y)) \wedge$

$\exists z(Iz@ \wedge Czx \wedge Fz) \wedge$

$\exists z(Iz@ \wedge Czx \wedge {\sim}Fz))).$

Let M be a CT L^{CT}-structure such that:

$\mathrm{D} = \{w_@, w, d, d_1, d_2\}$.

$\mathrm{I}(W) = \{w_@, w\}$.

$\mathrm{I}(I) = \{< w_@, w_@ >, < w, w >, < d_1, w_@ >, < d_2, w_@ >, < d, w >\}$.

$\mathrm{I}(A) = \{w_@, d_1, d_2\}$.

$\mathrm{I}(C) = \{< w, w >, < w_@, w_@ >, < d_1, d_1 >, < d_2, d_2 >, < d, d >,$

$< d_1, d >, < d_2, d >\}$.

$\mathrm{I}(F) = \{d_1\}$.

Then for every variable assignment s for M, $v_{M,s}((16)^@) = 1$. So, $v_M((16)^@)$=1. So, $v_M((16))$=1. ■

The problem here is that a non-actual individual may have multiple actual counterparts, some of which are F, and some of which are not F.

If we adopt ($\forall$-**A**), then the supposedly intuitively unsatisfiable sentence (17) of basic FOM language L (without individual constants) is satisfiable (ibid.). That is, for some CT L^{CT}-structure M, $v_M((17)) = 1$.

(17) $\diamond\exists x(\mathbf{A}\exists y x = y \wedge \sim\mathbf{A}Fx \wedge \sim\mathbf{A}\sim Fx)$

Proof. $(17)^{@}$ is:

$\exists u(Wu \wedge \exists x(Ixu \wedge$

$\forall z((Iz@ \wedge Czx) \rightarrow \exists y(Iy@ \wedge z = y)) \wedge$

$\sim\forall z((Iz@ \wedge Czx) \rightarrow Fz) \wedge$

$\sim\forall z((Iz@ \wedge Czx) \rightarrow \sim Fz))).$

Let M be a CT L^{CT}-structure such that:

$\mathrm{D} = \{w_@, w, d, d_1, d_2\}.$

$\mathrm{I}(W) = \{w_@, w\}.$

$\mathrm{I}(I) = \{< w_@, w_@ >, < w, w >, < d_1, w_@ >, < d_2, w_@ >, < d, w >\}.$

$\mathrm{I}(A) = \{w_@, d_1, d_2\}.$

$\mathrm{I}(C) = \{< w, w >, < w_@, w_@ >, < d_1, d_1 >, < d_2, d_2 >, < d, d >,$
$< d_1, d >, < d_2, d >\}.$

$\mathrm{I}(F) = \{d_1\}.$

Then for every variable assignment s for M, $v_{M,s}((17)^{@}) = 1$. So, $v_M((17)^{@})$=1. So, $v_M((17))$=1. ■

Again, the problem is that a non-actual individual may have multiple actual counterparts, some of which are F, and some of which are not F.

Fara & Williamson 2005 extend the point. They show that in virtue of each of (a) the potential of a non-actual individual having multiple actual counterparts, and (b) the potential of a non-actual individual having no actual counterparts, Lewis' 1968 semantics cannot be extended to accommodate an actuality operator by, as it were, unpacking the actuality operator as a kind of logical quantifier over actual counterparts. I present this now in the terminology of my framework.

First, we consider standard FO languages enriched with binary quantification (BQFO languages). To each standard FO language L there corresponds a collection of BQFO languages L^* . Each L^* of this collection extends the vocabulary of L with exactly a set Q of quantifier symbols Q, as well as the colon symbol. To define the L^*-formulas, we include the following clause in our inductive definition:

> For every $Q \in \mathrm{Q}$, all L^*-formulas ϕ, ψ, and every individual variable x, $(Qx : \phi)\psi$ is an L^*-formula.

For every standard FO language L, and BQFO language L^* corresponding to L, the L^*-structures are exactly the standard L-structures.

For a BQFO language L^*, L^*-structure M, and variable assignment s for M, we simultaneously define by recursion, for every variable assignment s for M, the unary satisfaction function $v_{M,s}$: Form(L^*) $\rightarrow \{1, 0\}$. In our definition, we include a clause for each $Q \in \mathrm{Q}$. I turn to the schema of a clause now.

For every BQFO language L^*, and $Q \in \mathrm{Q}$, there corresponds a binary relation R_Q between cardinalities which determines Q's corresponding clause in the recursive definition. As a preliminary, some notation: for a BQFO language L^*, L^*-structure M, variable assignment s for M, L^*-formula ϕ, and individual variable x, let us hold:

$$\phi^{M,s,x} = \{d \in \mathrm{D} : v_{M,s_{[d/x]}}(\phi) = 1\}.$$

Then the schema of a clause is:

$$v_{M,s}((Qx : \phi)\psi) = 1 \text{ iff } |\phi^{M,s,x} \cap \psi^{M,s,x}| \ \mathrm{R}_Q \ |\phi^{M,s,x} \cap [\mathrm{D} - \psi^{M,s,x}]|.$$

Informally, $v_{M,s}((Qx : \phi)\psi) = 1$ iff the *amount* of satisfiers d of both ϕ (relative to $s_{[d/x]}$) and ψ (relative to $s_{[d/x]}$) bears the relation R_Q to the *amount* of satisfiers d of ϕ (relative to $s_{[d/x]}$) which do not satisfy ψ (relative to $s_{[d/x]}$).

Let a CT' language be a BQFO language that (i) contains exactly one individual constant @, (ii) contains the unary predicate constants W and A, and the binary predicate constants I and C, and (iii) contains the desired set Q of quantifier symbols. For a CT' language L, let the L counterpart theory' be the theory T containing exactly (P1)-(P8), (A), (B), and (C). The CT' L-structures are exactly the standard L-structures in Mod(T).

For a standard FOM language L (without individual constants), there corresponds a CT' language $\mathrm{L}^{\mathrm{CT'}}$ whose constants are exactly all of the constants of L, plus $@, W, A, I, C$. Translated for my framework, Fara & Williamson 2005 consider a more general version of Lewis' account: mappings that take sentences of a standard FOM language L (without individual constants) to sentences of the corresponding CT' language $\mathrm{L}^{\mathrm{CT'}}$.

One way to achieve such a mapping is with our now familiar process. For a standard FOM language L (without individual constants), the translation of L-sentence ϕ is $\mathrm{L}^{\mathrm{CT'}}$-sentence $\phi^{@}$. This is defined by simultaneously defining by recursion, for every $\alpha \in [\mathrm{VAR} \cup \{@\}]$, a unary function $(.)^{\alpha}$: Form(L) $\rightarrow$ Form($\mathrm{L}^{\mathrm{CT'}}$).

Given a mapping, we would finally capture the idea of an indirect semantics by defining satisfaction and consequence in the now familiar fashion. For

satisfaction: for a standard FOM language L (without individual constants), and CT' $\mathrm{L}^{\mathrm{CT'}}$-structure M, we define the satisfaction function v_M: Sen(L) $\rightarrow \{1, 0\}$:

$$v_M(\phi) = v_M(\phi^{@})$$

The main point of Fara & Williamson 2005 is that there is no acceptable version of this strategy when the definition of $(.)^{\alpha}$ combines Lewis' original 1968 clauses with an instance of the following schema for an **A** clause:

(Q-**A**) $(\mathbf{A}\phi_{[x_1,...,x_n]})^{\alpha}$ is

$(Qy_1 : Iy_1@ \wedge Cy_1x_1)...(Qy_n : Iyn@ \wedge Cy_nx_n)(\phi(y_1, ..., y_n))^{@}$.

For Q-many actual counterparts y_1 of x_1, ..., and Q-many actual counterparts y_n of x_n, ϕ holds of $y_1, ..., y_n$ at $w_{@}$.

If we adopt the 1968 clauses plus an instance of (Q-**A**), then the supposedly intuitively unsatisfiable sentence (18) of basic FOM language L (without individual constants) is satisfiable. That is, for some CT' $\mathrm{L}^{\mathrm{CT'}}$-structure M, $v_M((18)) = 1$.

(18) $\Diamond\exists x(\mathbf{A}Fx \leftrightarrow \mathbf{A}{\sim}Fx)$

Proof. $(18)^{@}$ is

$\exists u(Wu \wedge \exists x(Ixu \wedge ((Qy : Iy@ \wedge Cyx)Fy \leftrightarrow (Qy : Iy@ \wedge Cyx){\sim}Fy)))$

Let M be a CT' $\mathrm{L}^{\mathrm{CT'}}$-structure, and s a variable assignment for M. Then:

$v_{M,s}((18)^{@}) = 1$.

$\Updownarrow$

For some $< d, w > \in \mathrm{I}(I)$,

$v_{M,s_{[d,w/x,u]}}(((Qy : Iy@ \wedge Cyx)Fy \leftrightarrow (Qy : Iy@ \wedge Cyx){\sim}Fy)) = 1$.

$\Updownarrow$

For some $< d, w > \in \mathrm{I}(I)$,

$|(Iy@ \wedge Cyx)^{M,s_{[d,w/x,u]},y} \cap Fy^{M,s_{[d,w/x,u]},y}|$ R_Q

$|(Iy@ \wedge Cyx)^{M,s_{[d,w/x,u]},y} \cap [\mathrm{D} - Fy^{M,s_{[d,w/x,u]},y}]|$ iff

$|(Iy@ \wedge Cyx)^{M,s_{[d,w/x,u]},y} \cap (\sim Fy)^{M,s_{[d,w/x,u]},y}|$ R_Q

$|(Iy@ \wedge Cyx)^{M,s_{[d,w/x,u]},y} \cap [\mathrm{D} - (\sim Fy)^{M,s_{[d,w/x,u]},y}]|$.

$\Updownarrow$

For some $< d, w > \in \mathrm{I}(I)$,

$\kappa_1 \mathrm{R}_Q \kappa_2$ iff $\kappa_2 \mathrm{R}_Q \kappa_1$, where

$\kappa_1 = |(Iy@ \wedge Cyx)^{M,s_{[d,w/x,u]},y} \cap Fy^{M,s_{[d,w/x,u]},y}| =$

$|\{d^* \in \mathrm{D} :< d^*, \mathrm{I}(@) >\in \mathrm{I}(I) \wedge < d^*, d >\in \mathrm{I}(C) \wedge d^* \in \mathrm{I}(F)\}|$, and

$\kappa_2 = |(Iy@ \wedge Cyx)^{M,s_{[d,w/x,u]},y} \cap [\mathrm{D} - Fy^{M,s_{[d,w/x,u]},y}]| =$

$|\{d^* \in \mathrm{D} :< d^*, \mathrm{I}(@) >\in \mathrm{I}(I) \wedge < d^*, d >\in \mathrm{I}(C) \wedge d^* \notin \mathrm{I}(F)\}|$.

Given this, let M be such that for some $< d, w >\in \mathrm{I}(I)$, $\kappa_1 = \kappa_2$. Then for every variable assignment s for M, $v_{M,s}((18)^{@}) = 1$. So, $v_M((18)^{@}) = 1$. So, $v_M((18))$ =1. ■

Notice here that (i) if d has no actual counterparts, then $\kappa_1 = \kappa_2$, and (ii) if d has multiple actual counterparts, then it may be that $\kappa_1 = \kappa_2$. So, there are two ways in virtue of which (18) can be satisfied.

So much for the points of Hazen and Fara & Williamson. On actuality, it seems to me that we should hold the following. A possibility/necessity setting amounts to quantification over further possibilities (further iterations). An actuality setting seemingly differs in two respects. First, it does not involve further iterations, but, in some sense, amounts to a *return*. Second, it amounts to a return to a *single* situation (in contrast, typically, corresponding to a possibility/necessity setting is a multitude of possibilities).

It is therefore not surprising that an account which unpacks the actuality operator in terms of further actual counterparts (that is, which unpacks the actuality operator as a further iteration) should deliver some inappropriate truth-values.

3.4.7 Strong Possibility and Necessity

In virtue of the atomic and possibility/necessity operator clauses of the original recursive definition of ϕ^{α}, the 1968 semantics for standard FOM languages (without individual constants or **A**) ends up interpreting *de re* occurrences of $\diamond$ and $\Box$ as expressing weak *de re* possibility and necessity, respectively. That is, what is possible/necessary of individuals together when they all exist. As a result, a problem that the overall 1968 account faces is that it cannot render true certain claims concerning contingent existence. To see this, consider the following sentence of standard FOM language L (without individual constants or **A**):

(CE) $\exists x \diamond \sim \exists y x = y$

Lewis' 1968 semantics unpacks the possibility operator occurrence of (CE) as concerning weak *de re* possibility, and hence (CE) is analysed as a contradiction. That is, for every CT L^{CT}-structure M, $v_M((\mathrm{CE}))$=0.

Proof. $(\text{CE})^{@}$ is:

$\exists x(Ix@ \wedge \exists w \exists z(Ww \wedge Izw \wedge Czx \wedge \sim \exists y(Iyw \wedge z = y)))$

For *reductio*, let M be a CT L^{CT}-structure such that $v_M((\text{CE}))=1$.

$\Downarrow$

$v_M((\text{CE})^{@}) =1$.

$\Downarrow$

For some d such that $< d, w_@ > \in \text{I}(I)$, for some $< d_1, w_1 > \in \text{I}(I)$ such that $< d_1, d > \in \text{I}(C)$, $< d_1, w_1 > \notin \text{I}(I)$. $\perp$ ■

However, certainly on Lewis' metaphysical picture, (CE) (for certain flavours of strong possibility) should be satisfiable (and indeed satisfied at the intended structure). Even in 1968, Lewis notes that his account is in the frying pan for rendering $\forall x \Box \exists y x = y$ valid (1968: 119).

Lewis' 1968 account is therefore at the very least incomplete on this front too (at least given his metaphysics).

3.4.8 The Plural *De Re*

As I noted in section 1.2.3, a complexity arises when attempting to analyse plural *de re* modality in terms of a simple counterpart relation. This issue was first raised as a criticism by Hazen (1977: 109-111, 1979: 328-330) of Lewis' 1968 account. I briefly present this within my framework. A natural regimentation of natural language modal claim (19) into a sentence of a standard FOM language L (without individual constants or **A**) is (19*):

(19) For some x and y such that x is the mother of y, it could have been that x was not the mother of y.

(19*) $\exists x \exists y(Mxy \wedge \Diamond \sim Mxy)$

For certain flavours of modality, it seems (19) should be false. However, on the 1968 account, (19*) translates to the following L^{CT}-sentence:

(19**) $\exists x(Ix@ \wedge \exists y(Iy@ \wedge Mxy \wedge \exists w \exists x_1 \exists y_1(Ww \wedge Ix_1w \wedge Cx_1x \wedge Iy_1w \wedge Cy_1y \wedge \sim Mx_1y_1)))$

This is satisfied at a CT L^{CT}-structure at which for some actual individuals b and d such that b bears M to d, there is a world w, w-counterpart b_1 of b, and w-counterpart d_1 of d such that b_1 does not bear M to d_1.

Now, it seems the intended structure <D, I> should be such a structure (assuming M corresponds to motherhood). For example, consider the case from section 1.2.3. Suppose Alice, a part of our world $w_@$, is an only child to

her mother, Sarah, who is also an only child. However, there is a world w that exhibits so-called two-way eternal recurrence. For this example, let us suppose that $w_@$ does not feature recurring epochs, and each epoch of w is qualitatively identical to $w_@$, except with regards to properties each epoch of w satisfies concerning being in a recurrence of epochs.

The natural response to this problem seems to be to hold that there is an issue with the 1968 semantics for standard FOM languages (without individual constants or **A**). Lewis in fact responds to the problem in a different way in the postscript to his 1968 paper reprinted in Lewis 1983b (section D). He there attempts to retain his semantics by holding that natural language modal claims such as (19) should be read as concerning the singly *de re*. Put simply, (19) should be regimented as something like the following sentence of a standard FOM language L (without individual constants or **A**):

(19^{+}) $\exists x \exists y(Mxy \wedge \exists z(Fzxy \wedge \Diamond \exists x_1 \exists y_1(Fzx_1y_1 \wedge \sim Mx_1y_1)))$

Broadly speaking, z is meant to be an entity representing the plurality containing exactly x and y. Lewis (1983b: 44-45) talks informally in terms of pairs. If we take that seriously, F would stand for the relation of [being the pair of the two other items] and z would be the pair $< x, y >$. Then (19) is in fact treated as a claim concerning the modal profile of a pair. (19^{+}) translates to:

(19^{++}) $\exists x(Ix@ \wedge \exists y(Iy@ \wedge Mxy \wedge \exists z(Iz@ \wedge Fzxy \wedge \exists w \exists z_1(Ww \wedge Iz_1w \wedge Cz_1z \wedge \exists x_1(Ix_1w \wedge \exists y_1(Iy_1w \wedge Fz_1x_1y_1 \wedge \sim Mx_1y_1))))))$

The idea is then that this will not be satisfied at the intended CT L^{CT}-structure because the actual z has no such counterpart.

Notice that this strategy contains echoes of Lewis' 1986 sketch of joint possibilities, but at this point, he is trying to present the approach within the older 1968 framework.

There is one complication. In 1983b, Lewis holds that counterpart relations should not hold between abstract entities such as pairs (1983b: 40). Given this, he suggests that "in this easy case, we may take the pair simply as a mereological sum" (1983b: 44). In an attempt to adopt that position, we might say that F stands for the relation of [being the mereological sum of the two other *people*] and z would be the sum of x and y.[17]

[17]Note that we cannot just have that F stands for the relation of [being the mereological sum of the two other *items*]. If we did, it would be harder to prevent (19^{++}) being satisfied at the intended CT L^{CT}-structure, because there are presumably many pairs of individuals x_1 and y_1 (not related by the motherhood relation) of which z_1 is the mereological sum.

3.4.9 Beyond the 1968 Account

Until 1986, it seems that, broadly speaking, Lewis held both the theory and translation scheme of the 1968 account. Of course, he does allow for some augmentations to the theory: in 1971 and 1983b, he allows for multiple counterpart relations; and in 1983b, he allows that for some counterpart relations, the corresponding version of (P5) does not hold (the postulate that an individual is a counterpart of no individual in its world other than itself).

However, by 1986, it seems that although Lewis still wants to retain his theory (now perhaps enhanced), he has given up on the translation scheme (and providing a semantics for formal languages with $\diamond$ and $\Box$). He writes (1986: 12-13):

> "What is the correct counterpart-theoretic interpretation of the modal formulas of the standard language of quantified modal logic? Who cares? We can make them mean whatever we like. We are their master. We needn't be faithful to the meanings we learned at mother's knee - because we didn't. If the language of boxes and diamonds proves to be a clumsy instrument for talking about matters of essence and potentiality, let it go hang. Use the resources of modal realism *directly* to say what it would mean for Humphrey to be essentially human, or to exist contingently"

This is an interesting foundational position. In order to present an analysis of a suitable fragment F of our natural language modal talk, we need not first present a semantics for a standard FOM modal language (or some variant). Instead, we need only present a semantics for F itself.

The issue is that, again, a systematic semantics for a standard FOM language L is often employed as a systematic analysis of a fragment F of our natural language modal talk, by presupposing a regimentation of F-sentences to L-sentences. If Lewis rejects this approach, then unless he provides a systematic semantics for a fragment of natural language itself, he has provided no systematic analysis at all.

3.5 Direct Versus Indirect

Lewis 1968 effectively presents an indirect semantics for standard FOM languages (without individual constants or **A**). To summarise the indirect approach, for a given standard FOM language L (without individual constants or **A**), we specify: (i) a standard FO language L^+ (the translation language), (ii) a mapping of L-sentences ϕ to L^+-sentences ϕ^+, and (iii) an L^+-theory T. Next, satisfaction of an L-sentence ϕ at a standard L^+-structure $M \in$ Mod(T) is defined in terms of satisfaction of its translation ϕ^+ at M. Finally, consequence for L is defined in terms of satisfaction of L-sentences at members of Mod(T).

This section notes some technical points concerning this indirect approach. In the first half, I demonstrate that from a technical perspective, for the 1968 account, the indirect approach is unnecessary for the purpose of presenting a semantics. That is, for the 1968 indirect semantics for standard FOM languages (without individual constants or **A**), there is an equivalent (in the sense to be defined) direct semantics.

In the second half, I consider the indirect approach more generally. First, for each of constant and varying domain semantics for standard FOM languages (each restricted to the class of all its structures), I present an equivalent (in the sense to be defined) indirect semantics. Like the Lewis 1968 approach, these indirect semantics rely heavily on first-order logic. Second, I discuss the significance of the ability to present an indirect semantics of this sort.

3.5.1 1968 Semantics

For a standard FOM language L (without individual constants or **A**), a 1968 L-structure M is a tuple <D, W, C, dom, $w_@$, I> such that:

D and W are non-empty sets.

$\mathrm{W} \subseteq \mathrm{D}$.

dom: $\mathrm{W} \rightarrow \mathcal{P}(\mathrm{D})$.

For every $w \in \mathrm{W}$, $w \in \mathrm{dom}(w)$.

For all $w, v \in \mathrm{W}$, if $w \neq v$, then $[\mathrm{dom}(w) \cap \mathrm{dom}(v)] = \emptyset$.

For every $d \in \mathrm{D}$, for some $w \in \mathrm{W}$, $d \in \mathrm{dom}(w)$ $[\mathrm{D} = \bigcup_{w \in \mathrm{W}} \mathrm{dom}(w)]$.

$w_@ \in \mathrm{W}$.

$\mathrm{C} \subseteq \mathrm{D}^2$.

For every $d \in \mathrm{D}$, $< d, d > \in \mathrm{C}$.

For every $w \in \mathrm{W}$, for all $d_1, d_2 \in \mathrm{dom}(w)$, if $< d_1, d_2 > \in \mathrm{C}$, then $d_1 = d_2$.

I is a function from PC(L) such that:

(i) For every $n \geq 1$, and n-ary $P \in \mathrm{PC(L)}$, $\mathrm{I}(P) \in \mathcal{P}(\mathrm{D}^n)$.

Recall that a standard FO language L is a CT language iff it (i) contains exactly one individual constant @, and (ii) contains the unary predicate constants W and A, and the binary predicate constants I and C. Further, for a standard FOM language L (without individual constants or **A**), there corresponds a CT language L^{CT} whose constants are exactly all of the constants of L, plus $@, W, A, I, C$.

Also recall that for a CT language L, we let the L counterpart theory be the theory T containing exactly (P1)-(P8), (A), (B), and (C):

(P1) $\forall v_1 \forall v_2 (Iv_1v_2 \rightarrow Wv_2)$

(P2) $\forall v_1 \forall v_2 \forall v_3 ((Iv_1v_2 \wedge Iv_1v_3) \rightarrow v_2 = v_3)$

(P3) $\forall v_1 \forall v_2 (Cv_1v_2 \rightarrow \exists v_3 Iv_1v_3)$

(P4) $\forall v_1 \forall v_2 (Cv_1v_2 \rightarrow \exists v_3 Iv_2v_3)$

(P5) $\forall v_1 \forall v_2 \forall v_3 (((Iv_1v_2 \wedge Iv_3v_2) \wedge Cv_1v_3) \rightarrow v_1 = v_3)$

(P6) $\forall v_1 \forall v_2 (Iv_1v_2 \rightarrow Cv_1v_1)$

(P7) $\exists v_1 (Wv_1 \wedge \forall v_2 (Iv_2v_1 \leftrightarrow Av_2))$

(P8) $\exists v_1 Av_1$

(A) $\forall v_1 (Wv_1 \vee \exists v_2 (Iv_1v_2))$

(B) $\forall v_1 (Wv_1 \rightarrow Iv_1v_1)$

(C) $\forall v_1 (\forall v_2 (Iv_2v_1 \leftrightarrow Av_2) \leftrightarrow v_1 = @)$

For a CT language L, the CT L-structures are exactly the standard L-structures in Mod(T). Now, for a standard FOM language L (without individual constants or **A**), and its corresponding CT language L^{CT}, for a CT L^{CT}-structure $M^+ = <\mathrm{D}^+, \mathrm{I}^+>$, there corresponds a 1968 L-structure $M = <\mathrm{D}, \mathrm{W}, \mathrm{C}, \mathrm{dom}, w_@, \mathrm{I}>$ such that:

$\mathrm{D} = \mathrm{D}^+$.

$\mathrm{W} = \mathrm{I}^+(W)$.

$\mathrm{C} = \mathrm{I}^+(C)$.

For every $w \in \mathrm{W}$, $\mathrm{dom}(w) = \{d :< d, w >\in \mathrm{I}^+(I)\}$.

$w_@ = \mathrm{I}^+(@)$.

I is the restriction of I^+ to PC(L).

Theorem 1. For a standard FOM language L (without individual constants or **A**), and its corresponding CT language L^{CT}, there is a one-to-one correspondence between the 1968 L-structures and the CT L^{CT}-structures.

Proof. The above relation is a one-to-one correspondence. ■

Now, for a standard FOM language L (without individual constants or **A**), we must define satisfaction of an L-formula at a 1968 L-structure. Usually, when providing a model-theoretic account of a language L, the basic definition of satisfaction of an L-formula (relative to relevant indices) will be such that for a sentential operator $*$ of L, and L-formula ϕ, to determine the truth-value of $*\phi$ (relative to certain indices), we consider the truth-value of ϕ itself (relative to certain indices). Let us adhere to this approach.

For a standard FOM language L (without individual constants or **A**), and 1968 L-structure M, a variable assignment for M is a function s: VAR $\rightarrow$ D. As usual, for all $d_1, ..., d_n \in$ D, and $x_1, ..., x_n \in$ VAR, let $s_{[d_1,...,d_n/x_1,...,x_n]}$ be exactly the same as s, except that d_1 is assigned to x_1, ..., and d_n is assigned to x_n.

Now, for a standard FOM language L (without individual constants or **A**), and 1968 L-structure M, we simultaneously define by recursion, for each $w \in$ W, and s of M, a unary satisfaction function $v_{M,w,s}$: Form(L) $\rightarrow \{1, 0\}$ as follows:

(i) $v_{M,w,s}(Px_1...x_n) = 1$ iff $< s(x_1), ..., s(x_n) > \in$ I(P).

(ii) $v_{M,w,s}(x_1 = x_2) = 1$ iff $s(x_1) = s(x_2)$.

(iii) $v_{M,w,s}(\sim\phi) = 1$ iff $v_{M,w,s}(\phi) = 0$.

(iv) $v_{M,w,s}((\phi \wedge \psi)) = 1$ iff $v_{M,w,s}(\phi) = 1$ and $v_{M,w,s}(\psi) = 1$.

(v) $v_{M,w,s}((\phi \vee \psi)) = 1$ iff $v_{M,w,s}(\phi) = 1$ or $v_{M,w,s}(\psi) = 1$.

(vi) $v_{M,w,s}((\phi \rightarrow \psi)) = 1$ iff $v_{M,w,s}(\phi) = 0$ or $v_{M,w,s}(\psi) = 1$.

(vii) $v_{M,w,s}((\phi \leftrightarrow \psi)) = 1$ iff $v_{M,w,s}(\phi) = v_{M,w,s}(\psi)$.

(viii) $v_{M,w,s}(\exists x\phi) = 1$ iff for some $d \in$ dom(w), $v_{M,w,s_{[d/x]}}(\phi) = 1$.

(ix) $v_{M,w,s}(\forall x\phi) = 1$ iff for every $d \in$ dom(w), $v_{M,w,s_{[d/x]}}(\phi) = 1$.

(x) $v_{M,w,s}(\diamond\phi_{[x_1,...,x_n]}) = 1$ iff for some $w^* \in$ W,

for some $d_1, ..., d_n \in$ dom(w^*) such that $< d_1, s(x_1) >, ..., < d_n, s(x_n) > \in$ C,

$v_{M,w^*,s_{[d_1,...,d_n/x_1,...,x_n]}}(\phi) = 1$.

(xi) $v_{M,w,s}(\Box\phi_{[x_1,...,x_n]}) = 1$ iff for every $w^* \in$ W,

for all $d_1, ..., d_n \in$ dom(w^*) such that $< d_1, s(x_1) >, ..., < d_n, s(x_n) > \in$ C,

$v_{M,w^*,s_{[d_1,...,d_n/x_1,...,x_n]}}(\phi) = 1$.

For a 1968 L-structure M, and s of M, we also define the unary functions $v_{M,s}$: Form(L) $\rightarrow \{1, 0\}$ and v_M : Form(L) $\rightarrow \{1, 0\}$ as follows:

$v_{M,s}(\phi) = v_{M,w_@,s}(\phi)$.

$$v_M(\phi) = \begin{cases} 1 \text{ if } \forall s, v_{M,s}(\phi) = 1. \\ 0 \text{ if } \forall s, v_{M,s}(\phi) = 0. \\ \text{U otherwise.} \end{cases}$$

Finally, we define consequence. For a standard FOM language L (without individual constants or **A**), set Γ of L-sentences, and L-sentence ϕ,

$\Gamma \models_{\text{DCT}} \phi$ iff for every 1968 L-structure M, if $[\forall \gamma \in \Gamma, v_M(\gamma) = 1]$, then $v_M(\phi) = 1$.[18]

We have:[19]

Theorem 2. For every standard FOM language L (without individual constants or **A**), 1968 L-structure M (with its corresponding CT L^{CT}-structure M^+), and L-sentence ϕ,

$v_M(\phi) = v_{M^+}(\phi)$.

My direct 1968 semantics is equivalent to the indirect 1968 semantics in the following sense:

Theorem 3. For every standard FOM language L (without individual constants or **A**),

$$\models_{\text{DCT}} \;=\; \models_{\text{ICT}}$$

3.5.2 Indirect Semantics

I now consider the indirect approach to semantics more generally. First, I shall present, for each of constant and varying domain semantics for standard FOM languages (each restricted to the class of all its structures), an equivalent (in the sense to be defined) indirect semantics. For each indirect semantics, for each standard FOM language L, I specify: (i) a standard FO language L^+ (the translation language), (ii) a mapping of L-formulas ϕ to L^+-formulas ϕ^+, and (iii) an L^+-theory T. Next, satisfaction of an L-formula at an $M \in \text{Mod(T)}$ is defined in terms of satisfactions of its translation ϕ^+ at M. Finally, consequence for L is defined in terms of satisfaction of L-formulas at members of Mod(T).

After these presentations, I discuss the significance of the ability to present this kind of indirect semantics.

3.5.2.1 Constant Domain Semantics

A standard FO language L is a CD language iff it contains (i) the individual constant @, (ii) the unary predicate constants W and D, and the binary predicate constant R, and (iii) no other unary predicate constants.

For a CD language L, let the L constant domain theory be the theory T containing exactly (CD1)-(CD5), and every instance of each of the schemas (CD6) and (CD7):

(CD1) $\forall x(Dx \vee Wx)$

[18] DCT for Direct Counterpart Theory semantics.

[19] Unless otherwise noted, proofs of theorems are given in the appendix.

(CD2) $\exists x Dx$

(CD3) $\exists x Wx$

(CD4) $\forall x \forall y (Rxy \rightarrow (Wx \wedge Wy))$

(CD5) $W@$

(CD6) Dc [$c \in$ IC(L) distinct from @]

(CD7) $\forall y_1 ... \forall y_{n-1} \forall y_n (P^+ y_1 ... y_{n-1} y_n \rightarrow (Dy_1 \wedge ... \wedge Dy_{n-1} \wedge Wy_n))$
[$P^+ \in$ PC(L) distinct from W, D, R]

For a CD language L, the CD L-structures are exactly the standard L-structures in Mod(T).

Now, for a standard FOM language L, there is an injection mapping each $P \in$ PC(L) to a symbol P^+ (distinct from $W, D, R, @$) such that corresponding to L is a CD language L^{CD} such that (i) its individual constants are exactly all the individual constants of L, plus @, and (ii) its extralogical predicate constants are exactly the unary constants W and D, the binary constant R, and for every $n \geq 1$, and n-ary $P \in$ PC(L), the image P^+ of P under the mapping, which is an $n+1$-ary extralogical predicate constant of L^{CD}.

Next, for a standard FOM language L, and its corresponding CD language L^{CD}, for a constant domain L-structure $M = <\mathrm{W}, \mathrm{R}, \mathrm{D}, w_@, \mathrm{I}>$, there corresponds a CD L^{CD}-structure $M^+ = <\mathrm{D}^+, \mathrm{I}^+ >$ such that:

$\mathrm{D}^+ = \mathrm{D} \cup \mathrm{W}$.

$\mathrm{I}^+(D) = \mathrm{D}$.

$\mathrm{I}^+(W) = \mathrm{W}$.

$\mathrm{I}^+(R) = \mathrm{R}$.

$\mathrm{I}^+(@) = w_@$.

For every $c \in$ IC(L), $\mathrm{I}^+(c) = \mathrm{I}(c)$.

For every $n \geq 1$, for every n-ary $P \in$ PC(L),

$\mathrm{I}^+(P^+) =$

$\{< d_1, ..., d_n, w >: d_1, ..., d_n \in \mathrm{D}, w \in \mathrm{W}, \text{and} < d_1, ..., d_n > \in \mathrm{I}(P)(w)\}$.

Theorem 4. For a standard FOM language L, and its corresponding CD language L^{CD}, there is a one-to-one correspondence between the constant domain L-structures and the CD L^{CD}-structures.

Proof. The above relation is a one-to-one correspondence. ■

With regards to translation, for a standard FOM language L, an L-formula ϕ is translated to L^{CD}-formula $\phi^{@}$.

To define this, we simultaneously define by recursion, for every $\alpha \in [\mathrm{VAR} \cup \{@\}]$, a unary function $(.)^{\alpha}$: Form(L) $\rightarrow$ Form(L^{CD}) with the following:

(i) $(Pt_1...t_n)^{\alpha}$ is $P^{+}t_1...t_n\alpha$.

(ii) $(t_1 = t_2)^{\alpha}$ is $t_1 = t_2$.

(iii) $(\sim\phi)^{\alpha}$ is $\sim(\phi^{\alpha})$.

(iv) For every $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$, $(\phi * \psi)^{\alpha}$ is $(\phi^{\alpha} * \psi^{\alpha})$.

(v) $(\exists x\phi)^{\alpha}$ is $\exists x(Dx \wedge \phi^{\alpha})$.

(vi) $(\forall x\phi)^{\alpha}$ is $\forall x(Dx \rightarrow \phi^{\alpha})$.

(vii) $(\diamond\phi)^{\alpha}$ is $\exists w(Ww \wedge R\alpha w \wedge \phi^{w})$.

(viii) $(\Box\phi)^{\alpha}$ is $\forall w((Ww \wedge R\alpha w) \rightarrow \phi^{w})$.

(ix) $(\mathbf{A}\phi)^{\alpha}$ is $\phi^{@}$.

With regards to clauses (vii) and (viii), for an L-formula $O\phi$ (where O is $\diamond$ or $\Box$), w is v_i, where:

> If no individual variables occur (either occur or occur*) in ϕ or α, $i = 1$.
>
> If some individual variables occur (either occur or occur*) in ϕ or α, $i = j + 1$, where (a) v_j occurs (either occurs or occurs*) in ϕ or α, and (b) $\forall k \in \mathbb{N}$ such that v_k occurs (either occurs or occurs*) in ϕ or α, $j \geq k$.

Next, for a standard FOM language L, and its corresponding CD language L^{CD}, a CD L^{CD}-structure M, and variable assignment s for M, we define a satisfaction function $v_{M,s}$: Form(L) $\rightarrow$ {1, 0} as follows:

$v_{M,s}(\phi) = v_{M,s}(\phi^{@})$.[20]

[20] As an aside, note that the direct semantics also has the satisfaction function which returns a truth-value for an L-formula, world, and variable assignment. We can capture this quite easily with my indirect constant domain semantics. For a standard FOM language L, CD L^{CD}-structure $M = <\mathrm{D}, \mathrm{I}>$, $w \in \mathrm{I}(W)$, and variable assignment s for M, we define a unary satisfaction function $v_{M,w,s}$: Form(L) $\rightarrow \{1, 0\}$ as follows:

$$v_{M,w,s}(\phi) = \begin{cases} 1 \text{ if } \forall x \in \mathrm{VAR} \text{ not occurring in } \phi, v_{M,s_{[w/x]}}(\phi^{x}) = 1. \\ 0 \text{ if } \forall x \in \mathrm{VAR} \text{ not occurring in } \phi, v_{M,s_{[w/x]}}(\phi^{x}) = 0. \end{cases}$$

There is a complication with this satisfaction function. For a standard FOM language L, a CD L^{CD}-structure $M = <D, I>$ may be such that for some $d \in D$, $d \notin I(D)$. In which case, a variable assignment s for M may assign some $x \in VAR$ to an individual outside of $I(D)$. In which case, an L-formula ϕ may be strangely handled by M relative to s. For example, consider L-formula ϕ: $\forall x Fx \rightarrow Fy$. For any class F of constant domain L-structures M, we have that $\models_{DCD(F)} \phi$. However, the formula translates to L^{CD}-formula $\phi^{@}$: $\forall x(Dx \rightarrow F^{+}x@) \rightarrow F^{+}y@$. Now, let M be a CD L^{CD}-structure such that for every $b \in I(D)$, $< b, w_{@} > \in I(F^{+})$, let s be a variable assignment of M, and let $d \in D$ such that $d \notin I(D)$ and $s(y) = d$. Then $v_{M,s}(\phi^{@}) = 0$. So, $v_{M,s}(\phi) = 0$!

How should this peculiarity be handled when we come to define consequence? The first option is to restrict our focus to L-*sentences* which of course aren't affected by the peculiarity. The other option is to restrict our focus to a certain subset of variable assignments. Say that a variable assignment s of a CD L^{CD}-structure $M = <D, I>$ is *appropriate* iff the range of s is a subset of $I(D)$. Then for every standard FOM language L, set Γ of L-formulas, and L-formula ϕ,

> $\Gamma \models_{ICD} \phi$ iff for every CD L^{CD}-structure M, and appropriate s of M, if $[\forall \gamma \in \Gamma, v_{M,s}(\gamma) = 1]$, then $v_{M,s}(\phi) = 1$.[21]

We have:

Theorem 5. For every standard FOM language L, constant domain L-structure M (with its corresponding CD L^{CD}-structure M^{+}), variable assignment s of M, and L-formula ϕ,

$v_{M,s}(\phi) = v_{M^{+},s}(\phi)$.[22]

My indirect constant domain semantics is equivalent to direct constant domain semantics (restricted to the class of all its structures) in the following sense:

Theorem 6. For every standard FOM language L, for the class F of all constant domain L-structures,

$$\models_{DCD(F)} \; = \; \models_{ICD}$$

3.5.2.2 Varying Domain Semantics

We proceed in a similar fashion for an indirect varying domain semantics. A standard FO language L is a VD language iff it contains (i) the individual constant @, (ii) the unary predicate constants W and D, and the binary predicate constants I and R, and (iii) no other unary predicate constants.

[21] ICD for Indirect Constant Domain semantics.

[22] Notice that the variable assignments of M are exactly the appropriate variable assignments of M^{+}.

For a VD language L, let the L varying domain theory be the theory T containing exactly (VD1)-(VD6), and every instance of each of the schemas (VD7) and (VD8):

(VD1) $\forall x(Dx \vee Wx)$

(VD2) $\exists x Dx$

(VD3) $\exists x Wx$

(VD4) $\forall x \forall y(Rxy \rightarrow (Wx \wedge Wy))$

(VD5) $\forall x \forall y(Ixy \rightarrow (Dx \wedge Wy))$

(VD6) $\exists x Ix@$

(VD7) $Ic@$ [$c \in$ IC(L) distinct from @]

(VD8) $\forall y_1 ... \forall y_{n-1} \forall y_n(P^+ y_1 ... y_{n-1} y_n \rightarrow (Dy_1 \wedge ... \wedge Dy_{n-1} \wedge Wy_n))$
[$P^+ \in$ PC(L) distinct from W, D, I, R]

For a VD language L, the VD L-structures are exactly the standard L-structures in Mod(T).

Now, for a standard FOM language L, there is an injection mapping each $P \in$ PC(L) to a symbol P^+ (distinct from $W, D, R, I, @$) such that corresponding to L is a VD language L^{VD} such that (i) its individual constants are exactly all the individual constants of L, plus @, and (ii) its extralogical predicate constants are exactly the unary constants W and D, the binary constants I and R, and for every $n \geq 1$, and n-ary $P \in$ PC(L), the image P^+ of P under the mapping, which is an $n+1$-ary extralogical predicate constant of L^{VD}.

Next, for a standard FOM language L, and its corresponding VD language L^{VD}, for a varying domain L-structure $M = <\mathrm{W}, \mathrm{R}, \mathrm{D}, \mathrm{dom}, w_@, \mathrm{I}>$, there corresponds a VD L^{VD}-structure $M^+ = <\mathrm{D}^+, \mathrm{I}^+>$ such that:

$\mathrm{D}^+ = \mathrm{D} \cup \mathrm{W}$.

$\mathrm{I}^+(W) = \mathrm{W}$.

$\mathrm{I}^+(D) = \mathrm{D}$.

$\mathrm{I}^+(I) = \{< d, w >: d \in \mathrm{D}, w \in \mathrm{W}, \text{and } d \in \mathrm{dom}(w)\}$.

$\mathrm{I}^+(R) = \mathrm{R}$.

$\mathrm{I}^+(@) = w_@$.

For every $c \in$ IC(L), $\mathrm{I}^+(c) = \mathrm{I}(c)$.

For every $n \geq 1$, for every n-ary $P \in$ PC(L),

$\mathrm{I}^{+}(P^{+}) =$

$\{< d_1, ..., d_n, w >: d_1, ..., d_n \in \mathrm{D}, w \in \mathrm{W}, \text{and} < d_1, ..., d_n > \in \mathrm{I}(P)(w)\}$.

Theorem 7. For a standard FOM language L, and its corresponding VD language L^{VD}, there is a one-to-one correspondence between the varying domain L-structures and the VD L^{VD}-structures.

Proof. The above relation is a one-to-one correspondence. ■

With regards to translation, for a standard FOM language L, an L-formula ϕ is translated to L^{VD}-formula $\phi^{@}$.

To define this, we simultaneously define by recursion, for every $\alpha \in$ [VAR $\cup \{@\}$], a unary function $(.)^{\alpha}$: Form(L) $\rightarrow$ Form(L^{VD}) with the following:

(i) $(Pt_1...t_n)^{\alpha}$ is $P^{+}t_1...t_n\alpha$.

(ii) $(t_1 = t_2)^{\alpha}$ is $t_1 = t_2$.

(iii) $(\sim\phi)^{\alpha}$ is $\sim(\phi^{\alpha})$.

(iv) For every $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$, $(\phi * \psi)^{\alpha}$ is $(\phi^{\alpha} * \psi^{\alpha})$.

(v) $(\exists x\phi)^{\alpha}$ is $\exists x(Ix\alpha \wedge \phi^{\alpha})$.

(vi) $(\forall x\phi)^{\alpha}$ is $\forall x(Ix\alpha \rightarrow \phi^{\alpha})$.

(vii) $(\diamond\phi)^{\alpha}$ is $\exists w(Ww \wedge R\alpha w \wedge \phi^{w})$.

(viii) $(\Box\phi)^{\alpha}$ is $\forall w((Ww \wedge R\alpha w) \rightarrow \phi^{w})$.

(ix) $(\mathbf{A}\phi)^{\alpha}$ is $\phi^{@}$.

With regards to clauses (vii) and (viii), for an L-formula $O\phi$ (where O is $\diamond$ or $\Box$), w is v_i (the definition of i is the same as with indirect constant domain semantics).

Next, for a standard FOM language L, and its corresponding VD language L^{VD}, a VD L^{VD}-structure M, and variable assignment s for M, we define a satisfaction function $v_{M,s}$: Form(L) $\rightarrow$ {1, 0} as follows:

$v_{M,s}(\phi) = v_{M,s}(\phi^{@})$.[23]

[23]Again, as an aside, we can capture a satisfaction function which returns a truth-value for an L-formula, world, and variable assignment. For a standard FOM language L, VD L^{VD}-structure $M =$ <D, I>, $w \in \mathrm{I}(W)$, and variable assignment s for M, we define a unary satisfaction function $v_{M,w,s}$: Form(L) $\rightarrow \{1, 0\}$ as follows:

$$v_{M,w,s}(\phi) = \begin{cases} 1 \text{ if } \forall x \in \text{VAR not occurring in } \phi, v_{M,s_{[w/x]}}(\phi^{x}) = 1. \\ 0 \text{ if } \forall x \in \text{VAR not occurring in } \phi, v_{M,s_{[w/x]}}(\phi^{x}) = 0. \end{cases}$$

We define consequence in a similar way it was defined for indirect constant domain semantics. For every standard FOM language L, set Γ of L-formulas, and L-formula ϕ,

> $\Gamma \models_{\text{IVD}} \phi$ iff for every VD L^{VD}-structure M, and appropriate s of M, if $[\forall \gamma \in \Gamma, v_{M,s}(\gamma) = 1]$, then $v_{M,s}(\phi) = 1$.[24]

We have:

Theorem 8. For every standard FOM language L, varying domain L-structure M (with its corresponding VD L^{VD}-structure M^+), variable assignment s of M, and L-formula ϕ,

$v_{M,s}(\phi) = v_{M^+,s}(\phi)$.

My indirect varying domain semantics is equivalent to direct varying domain semantics (restricted to the class of all its structures) in the following sense:

Theorem 9. For every standard FOM language L, for the class F of all varying domain L-structures,

$$\models_{\text{DVD(F)}} \;=\; \models_{\text{IVD}}$$

3.5.2.3 The Significance of an Indirect Semantics

For each of my two indirect semantics, for each standard FOM language L, I specified: (i) a standard FO language L^+ (the translation language), (ii) a mapping of L-formulas ϕ to L^+-formulas ϕ^+, and (iii) an L^+-theory T. Next, satisfaction of an L-formula at an $M \in \text{Mod(T)}$ was defined in terms of satisfactions of its translation ϕ^+ at M. Finally, consequence for L was defined in terms of satisfaction of L-formulas at members of Mod(T).

Lewis' 1968 indirect semantics for standard FOM languages L (without individual constants or **A**) is the same, except that Lewis only focusses on a mapping of L-sentences to L^+-sentences.

What is the technical significance of this kind of indirect semantics? For a given direct semantics S, if there is an indirect semantics of the above kind that is equivalent to S (in the sense of generating the same consequence relation), then, as it were, S is capturable in first-order logic.

As a result, the direct semantics will exhibit certain metalogical features that first-order logic exhibits. For example, given that constant domain semantics for standard FOM languages (restricted to the class of all its structures) is equivalent to my indirect constant domain semantics, we can demonstrate that constant domain semantics for standard FOM languages (restricted to the

[24] IVD for Indirect Varying Domain semantics.

class of all its structures) is compact: for every standard FOM language L, and set Γ of L-sentences, if [for every finite $\Gamma_{\text{Fin}} \subseteq \Gamma$, some constant domain L-structure models Γ_{Fin}], then some constant domain L-structure models Γ.[25]

Proof. Let L be a standard FOM language, and Γ a set of L-sentences such that for every finite $\Gamma_{\text{Fin}} \subseteq \Gamma$, some constant domain L-structure models Γ_{Fin}.

Some notation: for any set Δ of L-sentences, let $\Delta^{@} = \{\phi^{@} : \phi \in \Delta\}$.

Consider [$\Gamma^{@} \cup$ T] (where T is the L^{CD} constant domain theory).

Let [$\Gamma^{@}_{\text{Fin}} \cup \text{T}_{\text{Fin}}$] be an arbitrary finite subset of $\Gamma^{@} \cup$ T. From our initial assumption, there is a constant domain L-structure M that models Γ_{Fin}. From this and theorem 5, there is a CD L^{CD}-structure M^{+} that models $\Gamma^{@}_{\text{Fin}}$. So, there is a standard L^{CD}-structure M^{+} that models $\Gamma^{@}_{\text{Fin}}$ and T. So, there is a standard L^{CD}-structure M^{+} that models [$\Gamma^{@}_{\text{Fin}} \cup \text{T}_{\text{Fin}}$].

So, for every finite subset of [$\Gamma^{@} \cup$ T], there is a standard L^{CD}-structure that models it. By the compactness of first-order logic, [$\Gamma^{@} \cup$ T] has a standard L^{CD}-structure that models it. So, there is CD L^{CD}-structure M^{+} that models $\Gamma^{@}$. By theorem 5, there is a constant domain L-structure M that models Γ. ■

By the same token, if a given direct semantics S for a language L does not exhibit certain metalogical features that first-order logic exhibits (such as compactness), then there is not an indirect semantics for L of our discussed kind that is equivalent to S.

It is worth briefly noting how familiar expressive limitations of first-order logic may limit our ability to present indirect semantics that are equivalent to certain direct semantics. Suppose we are trying to provide an indirect semantics for a language L (limiting our focus ultimately to L-sentences as Lewis 1968 does). There are two important areas where our (interpreted) translation language L^{+} is working in an indirect semantics of L:

(a) Specifying an L^{+}-theory T to determine the class Mod(T) of standard L^{+}-structures that will indirectly interpret L-sentences.

(b) Specifying our mapping from L-sentences to L^{+}-sentences. This (alongside the usual first-order definition of satisfaction) characterises satisfaction of L-sentences at the standard L^{+}-structures in Mod(T).

On (a), notice that we are here not limited by what conditions on L^{+}-structures single L^{+}-sentences can express, but rather, by what conditions on L^{+}-structures sets of L^{+}-sentences can express. For example, consider the

[25] The notion of a constant domain L-structure modelling an L-formula and a set of L-formulas is defined in the usual way.

compactness of first-order logic: for every standard FO language L^*, and set Γ of L^*-sentences, if every finite subset of Γ has a model, then Γ has a model. As is well known, given this, no single L^+-sentence can capture the condition that the domain of a structure (or, for example, the extension of a unary predicate) is infinitely large. However, this condition can be captured with an infinitely large set of L^+-sentences.

Whilst this holds, there are nevertheless certain conditions on standard L^+-structures that even a (perhaps infinitely large) set of L^+-sentences cannot capture. For example, as is well known, given compactness, we have that no set of L^+-sentences can capture the condition that the domain of a structure (or, for example, the extension of a unary predicate) is finite.

Proof. Let Γ be a set of L^+-sentences such that for every $n \in \mathbb{N}$, Γ has a model $M_n = <D, I>$ such that $|D| \geq n$. We show that Γ has a model $<D, I>$ such that $|D| \geq \aleph_0$.

For every $n \geq 1$, let α_n be the L^+-sentence that captures the condition that a structure $<D, I>$ is such that $|D| \geq n$. Let $\Gamma^* = \Gamma \cup \{\alpha_1, \alpha_2, ...\}$, let Δ^* be a finite subset of Γ^*, and let $m \in \mathbb{N}$ such that [(i) $\alpha_m \in \Delta^*$, and (ii) for every $i \in \mathbb{N}$ such that $\alpha_i \in \Delta^*, m \geq i$]. Given our assumption, there is a structure M_m that models Δ^*. By compactness, Γ^* has a model M. M also models Γ. ■

For another example, consider the Löwenheim-Skolem theorem: for every standard FO language L^* with signature σ, and standard L^*-structure $<D, I>$, if $|D| \geq \aleph_0$, then for every cardinality $\kappa \geq |\sigma|$, there is a standard L^*-structure $M^+ = <D^+, I^+>$ such that (i) $|D^+| = \kappa$, and (ii) M and M^+ are elementarily equivalent.[26] Given this, no set of L^+-sentences can capture the condition that the size of the domain of a structure is a particular infinite cardinality. Similarly, no such set can capture the condition that the domain of a structure is countable.

Our indirect semantics may therefore already be poorer than a direct semantics that, employing a rich meta-language, is able to more finely specify the direct L-structures.

On (b), in mapping an L-sentence ϕ to an L^+-sentence ϕ^+, we are trying to capture a condition on a standard L^+-structure (that models T). We are therefore still limited in this endeavour to capture a condition, as we are effectively just employing the set $T \cup \{\phi^+\}$ to capture a condition on standard L^+-structures.

[26] As we have omitted function constants in this presentation, $\sigma = [IC(L^*) \cup PC(L^*)]$.

Chapter 4

Other Counterpart Accounts

Lewis' 1968 account and his other writings on counterparthood have inspired the development of many formal accounts that appeal to a notion of counterparthood. In this chapter, I raise issues for five of them: the stipulational semantics of Hazen 1977; the counterpart semantics of Russell 2013; the counterpart semantics of Bacon 2014; the relative-sameness semantics of Graff Fara 2008; and the counterpart account of Meyer 2013.[1]

4.1 Hazen's Stipulational Semantics

As noted earlier, Lewis states that his 1986 sketch of joint possibilities is based on Hazen 1977 (his dissertation) and 1979 (a paper that effectively reprints some of the dissertation material). Central to this work is Hazen's *stipulational semantics* for standard FOM languages. This semantics differs notably from the sketch of joint possibilities Lewis presents, but Hazen, as a heuristic to compare Lewis' 1968 counterpart theory and his stipulational semantics, does briefly talk of a "double counterpart relation" which holds between pairs of individuals (1977: 122). Given this connection, I turn to Hazen's stipulational semantics first.

Hazen's central motivation in presenting stipulational semantics is to present a semantics that does not appeal to identity across worlds in the account of the modal operators, but which is such that a fairly standard deductive system (which includes T, 4, B, and statements of the necessity of identity and distinctness, and which is described in section II.A of Hazen 1977) is sound and complete with respect to it.

[1] Forbes (1982, 1985, 1990) and Ramachandran (1989, 1990a, 1990b) have also published counterpart accounts. See Fara & Williamson 2005 for criticisms. Further, Sider MS presents a counterpart account, but he there notes that the account mishandles certain sentences.

4.1.1 The Semantics

Hazen presents two versions of stipulational semantics in 1977: a first-pass account in section II.E, presented as a heuristic, and a rigorous account in section II.F. In this section, following Hazen's lead, I present both the heuristic and rigorous versions.[2]

First, the first-pass account of the semantics. We start with the notion of a world and a world's domain. The members of a world's domain are *individuals*. For our purposes, let us say that for a world w, the domain of w is dom(w). For the first-pass, we assume that the domains of worlds are disjoint (1977: 101-102).

Hazen takes a counterpart relation to be an equivalence relation. We "let an object's counterparts be all and only the objects of the same kind, where it is understood that an object is of one and only one kind" (1977: 113). I return to this in more detail later.

Next, for worlds w and w^+, a *stipulation* from w to w^+ is a (perhaps partial) function f: dom(w) $\rightarrow$ dom(w^+) such that (i) for every $x \in$ dom(w), $f(x)$ (if defined) is a counterpart of x, and (ii) f is injective (1977: 115-116).

Next, Hazen introduces the notion of a *fusion* of worlds. A fusion of two distinct (from each other) worlds w and w^+ is the pair $< \{w, w^+\}$, I$>$, where I is an equivalence relation in [dom(w) $\cup$ dom(w^+)] that does not relate distinct elements of the domain of a single world.

More generally, the fusion of n-many ($n \geq 2$) distinct (from each other) worlds $w_1, ..., w_n$ is the pair $< \{w_1, ..., w_n\}$, I$>$, where I is an equivalence relation in [dom (w_1) $\cup$ ... $\cup$ dom (w_n)] that does not relate distinct elements of the domain a single world.[3]

For a fusion W of worlds w and w^+, a stipulation from W to a world w^{++} (distinct from w and w^+) is a (perhaps partial) function g: [dom(w) $\cup$ dom(w^+)] $\rightarrow$ dom(w^{++}) such that (i) for every $x \in$ [dom(w) $\cup$ dom(w^+)], $g(x)$ (if defined) is a counterpart of x, (ii) for all x and $y \in$ [dom(w) $\cup$ dom(w^+)] such that Ixy, if $g(x)$ is defined, then $g(y)$ is defined and $g(x) = g(y)$, and (iii) $g \restriction$dom(w) and $g \restriction$dom(w^+) are each injective.

More generally, for a fusion W of n-many ($n \geq 2$) distinct (from each other) worlds $w_1, ..., w_n$, a stipulation from W to a world w_{n+1} (distinct from each of $w_1, ..., w_n$) is a (perhaps partial) function g: [dom (w_1) $\cup$ ... $\cup$ dom (w_n)] $\rightarrow$

[2] For ease of reading, when discussing Hazen's semantics, I include quotation marks around object language expressions when they are mentioned in the meta-language.

[3] Note the requirement that $w_1, .., w_n$ are distinct from each other. This restriction on the notion of a fusion only holds for the first-pass account.

dom(w_{n+1}) such that (i) for every $x \in$ [dom (w_1) $\cup$ ... $\cup$ dom (w_n)], $g(x)$ (if defined) is a counterpart of x, (ii) for all x and $y \in$ [dom (w_1) $\cup$ ... $\cup$ dom (w_n)] such that Ixy, if $g(x)$ is defined, then $g(y)$ is defined and $g(x) = g(y)$, and (iii) for every $1 \leq i \leq n$, $g \restriction$dom(w_i) is injective.

For a stipulation f from a world w to a distinct world w^+, the fusion of w and w^+ corresponding to f is the fusion $< \{w, w^+\}$, I$>$, where for every $x \in$ dom(w) and $y \in$ dom(w^+), Ixy iff $f(x) = y$.[4]

For a stipulation f from a fusion W $= < \{w_1, ..., w_n\}, \mathrm{I} >$ of n-many ($n \geq 2$) distinct (from each other) worlds to a world w_{n+1} (distinct from each of $w_1, ..., w_n$), the fusion corresponding to f is the fusion $< \{w_1, ..., w_n, w_{n+1}\}$, I$^+ >$ such that (i) I $\subseteq$ I$^+$, and (ii) for every $x \in$ [dom (w_1) $\cup$ ... $\cup$ dom (w_n)] and $y \in$ dom(w_{n+1}), Ixy iff $f(x) = y$.

So much for the machinery of the first-pass exposition. Worlds, stipulations, and fusions now play a central role in the account of modal formulas. We are told that for a non-modal formula 'ϕ', we have '$\Diamond\phi$' ('$\Box\phi$') is true iff for some (every) world w and stipulation f from $w_@$ to w, 'ϕ' is true at w when each individual variable occurring free in 'ϕ' and each individual constant in 'ϕ' has its original denotation d changed to $f(d)$ (1977: 115). With regards to iterated modality, Hazen presents the following example (1977: 117-119):

(20) $\Diamond(\sim\exists xj = x \wedge \exists x(Fx \wedge \Diamond Gjx))$[5]

According to the first-pass, (20) is to be handled as follows: there is a world w and stipulation f from $w_@$ to w such that the denotation of 'j' (which is in dom($w_@$)) is not defined for f, there is an $x \in$ dom(w) satisfying the property expressed by 'F', and there is a world w^+ and stipulation g from [the fusion W of $w_@$ and w corresponding to f] to w^+ such that g maps the denotation of 'j' to an individual d_1, g maps x to an individual d_2, and d_1 bears the relation expressed by 'G' to d_2.

I now turn to the rigorous account. First, a *fusion* is an ordered pair $<$(W$_k$)$_{k\in\mathrm{K}}$, I$>$. Here, the indexed family (W$_k$)$_{k\in\mathrm{K}}$ is such that (a) W is a set of worlds, (b) K is an initial segment of $\mathbb{N}$, and (c) $w_0 = w_@$. The domain of a fusion F $=$ $<$(W$_k$)$_{k\in\mathrm{K}}$, I$>$ is the set $\{< k, x >: k \in \mathrm{K} \wedge x \in \mathrm{dom}(w_k)\}$. Let this be dom(F). I is an equivalence relation in dom(F) such that for every $k \in$ K, for all $x, y \in$ dom(w_k), if $< k, x >$ and $< k, y >$ are in the same equivalence class of I, then $x = y$ (1977: 124-125).

[4]The first-pass definition of a fusion excludes a fusion of a world w with itself because there may be a stipulation f from w to w such that for some $x \in$ dom(w), $f(x)$ is defined, but $x \neq f(x)$. In which case, for the fusion of w and w corresponding to f, the relation I would need to relate distinct elements of the domain of a single world.

[5]His (14) with a slight adjustment to the constants used.

A *stipulation* from a fusion F = <$(W_k)_{k\in K}$, I> to a world w is a (perhaps partial) function f: dom(F)/I $\rightarrow$ dom(w).[6] That is, it assigns to an equivalence class of I a member of dom(w) (1977: 125).

Next, for fusions F = <$(W_k)_{k\in K}$, I> and F' = <$(U_j)_{j\in J}$, I'>, if K = J, then a bijection f: K $\rightarrow$ J is an *equivalence* of F and F' iff (i) $f(0) = 0$, (ii) for every $k \in$ K, w_k of [$(W_k)_{k\in K}$] is $u_{f(k)}$ [of $(U_j)_{j\in J}$], and (iii) for all $k, k^+ \in$ K, for every $x \in$ dom(w_k) [w_k of $(W_k)_{k\in K}$], and every $y \in$ dom(w_{k^+}) [w_{k^+} of $(W_k)_{k\in K}$], $< k, x >$I$< k^+, y >$ iff $< f(k), x >$I'$< f(k^+), y >$. Two fusions are *equivalent* iff there is an equivalence between them (ibid.).

For fusions F = <$(W_k)_{k\in K}$, I> and F' = <$(U_j)_{j\in J}$, I'>, equivalence f between them, and stipulation g from F to a world w, the *f-image* of g is the (perhaps partial) function h: dom(F')/I' $\rightarrow$ dom(w) such that for every $k \in$ K, every $x \in$ dom(w_k) [w_k of $(W_k)_{k\in K}$], and every $y \in$ dom(w), $g([< k, x >]_I) = y$ iff $h([< f(k), x >]_{I'}) = y$ (ibid.).[7]

For a fusion F = <$(W_k)_{k\in K}$, I>, world w, and stipulation f from F to w, a fusion F' = <$(U_j)_{j\in J}$, I'> is the *f-w-extension* of F iff (i) J extends K by exactly one member (let this member be n), (ii) for every $k \in$ K, w_k [of $(W_k)_{k\in K}$] is u_k [of $(U_j)_{j\in J}$], and w is u_n [of $(U_j)_{j\in J}$], and (iii) I' extends I by including the pairs $<< k, x >, < n, y >>$ and $<< n, y >, < k, x >>$ such that $k \in$ K, $x \in$ dom(w_k) [w_k of $(W_k)_{k\in K}$], $y \in$ dom(w), and $f([< k, x >]_I) = y$ (1977: 126).

A fusion F' is a *direct extension* of F (and F a *direct restriction* of F') iff there is a finite sequence of fusions F_0, ..., F_n such that (i) F_0 = F, (ii) F_n = F', and (iii) for every $0 \leq i \leq n$, F_i is the f-w-extension of $F_{(i-1)}$ for some w and f (ibid.). Finally, F' is an *extension* of F (and F a *restriction* of F') iff there are fusions F^+ and F^{++} such that F is equivalent to F^+, F' is equivalent to F^{++}, and F^{++} is a direct extension of F^+ (ibid.).

It is worth pausing to consider a toy example. Let F be a fusion <$(W_k)_{k\in K}$, I> such that W = $\{w, w^+, w^{++}\}$, K = $\{0, 1, 2\}$, w_0 [of $(W_k)_{k\in K}$] is w, w_1 [of $(W_k)_{k\in K}$] is w^+, and w_2 [of $(W_k)_{k\in K}$] is w^{++}. Suppose dom(w) = $\{a\}$, dom(w^+) = $\{b\}$, and dom(w^{++}) = $\{c\}$. Then dom(F) = $\{< 0, a >, < 1, b >, < 2, c >\}$.

[6] Notation-wise, for a set A and equivalence relation R in A, A/R is the quotient of A by R.

[7] The 1977 presentation seems somewhat loose here. Hazen introduces g as a stipulation from fusion <G, I> to world w, but then incorrectly goes on to talk of g containing pairs $<< i, x >, y >$ [which would require that g is in fact a function from dom(<G, I>) to dom(w), rather than a function from dom(<G, I>)/I to dom(w)]. Similarly, he introduces h as a function from dom(<G', I'>) to dom(w), but it is clear he has in mind that h is a stipulation here [in which case, we should have that h is a function from dom(<G', I'>)/I' to dom(w)].

Next, suppose relation I is such that $< 0, a >$I$< 1, b >$ and $< 1, b >$I$< 2, c >$. As I is an equivalence relation, we have that I = dom(F)^2 and hence dom(F)/I is the singleton {dom(F)}.

Next, let w^{+++} be a world such that dom(w^{+++}) = $\{d\}$, and f a stipulation from F to w^{+++} such that f: $[< 0, a >]_{\mathrm{I}} \mapsto d$ (it maps the single member of dom(F)/I, namely dom(F), to d). Further, let F' be a fusion <$(\mathrm{U}_j)_{j\in\mathrm{J}}$, I'> such that U = $\{w, w^+, w^{++}, w^{+++}\}$, J = $\{0, 1, 2, 3\}$, u_0 [of $(\mathrm{U}_j)_{j\in\mathrm{J}}$] is w, u_1 [of $(\mathrm{U}_j)_{j\in\mathrm{J}}$] is w^+, u_2 [of $(\mathrm{U}_j)_{j\in\mathrm{J}}$] is w^{++}, and u_3 [of $(\mathrm{U}_j)_{j\in\mathrm{J}}$] is w^{+++}. Then dom(F') = $\{< 0, a >, < 1, b >, < 2, c >, < 3, d >\}$. Let I' = dom(F')^2. Then F' is the f-w^{+++}-extension of F. So, F' is a direct extension of F.

Next, let F* be a fusion <$(\mathrm{V}_m)_{m\in\mathrm{M}}$, I* > such that V = $\{w, w^+, w^{++}\}$, M = $\{0, 1, 2\}$, v_0 [of $(\mathrm{V}_m)_{m\in\mathrm{M}}$] is w, v_1 [of $(\mathrm{V}_m)_{m\in\mathrm{M}}$] is w^{++}, and v_2 [of $(\mathrm{V}_m)_{m\in\mathrm{M}}$] is w^+. Then dom(F*) = $\{< 0, a >, < 1, c >, < 2, b >\}$. Suppose I* = dom(F*)^2. Then dom(F*)/I* is the singleton {dom(F*)}.

F and F* are equivalent. Further, F' is an extension (though not direct) of F*.

So much for the example. A *stipulational model structure* (SMS) for a standard FOM language L is an ordered quadruple <W, r, ST, FUS>, where W is a set of worlds "of the type corresponding to L" (to be explained), $r \in$ W, ST is a set of stipulations, and FUS is a set of fusions, and which meets the following conditions (1977: 127-128):

(i) For every initial segment K of $\mathbb{N}$, and U $\subseteq$ W such that $r \in$ U and |U| $\leq$ |K|, there is a fusion <$(\mathrm{U}_k)_{k\in\mathrm{K}}$, I> $\in$ FUS such that u_0 of $(\mathrm{U}_k)_{k\in\mathrm{K}}$ is r.

(ii) For every <$(\mathrm{U}_k)_{k\in\mathrm{K}}$, I> $\in$ FUS, u_0 of $(\mathrm{U}_k)_{k\in\mathrm{K}}$ is r.

(iii) For every F $\in$ FUS and $w \in$ W, there is a stipulation $f \in$ ST from F to w.

(iv) For every F $\in$ FUS, $w \in$ W, and $f \in$ ST from F to w, the f-w-extension of F is in FUS.

(v) For every F $\in$ FUS, every direct restriction of F is in FUS.

(vi) For every F $\in$ FUS, every fusion equivalent to F is in FUS.

(vii) For every <G, I> $\in$ FUS, $w \in$ W, $f \in$ ST from <G, I> to w, and direct extension <G', I'> of <G, I>, there is an $f' \in$ ST from <G', I'> to w such that f and f' "coincide on those equivalence classes of I' which have members in [dom(<G, I>)]".

I take it that for the last condition of (vii), Hazen has in mind: for every $< k, x >\in$ dom(<G, I>), $f([< k, x >]_{\mathrm{I}})$ is defined iff $f'([< k, x >]_{\mathrm{I'}})$ is, and if they are, $f([< k, x >]_{\mathrm{I}}) = f'([< k, x >]_{\mathrm{I'}})$.

(viii) For every <G, I> ∈ FUS, $w \in$ W, $f \in$ ST from <G, I> to w, and direct restriction <G', I'> of <G, I>, the "restriction of f to [dom(<G', I'>)]" is a stipulation in ST from <G', I'> to w.

Notice that we cannot take the restriction talk in (viii) literally, because dom(<G', I'>) $\not\subseteq$ the domain of f (remember that the domain of f is dom(<G, I>)/I). Presumably, Hazen has in mind the following. The "restriction of f to [dom(<G', I'>)]" is the function f': dom(<G', I'>)/I' $\rightarrow$ dom(w) such that for every $< k, x >\in$ dom(<G', I'>), $f'([< k, x >]_{\mathrm{I}'})$ is defined iff $f([< k, x >]_{\mathrm{I}})$ is, and if they are, $f'([< k, x >]_{\mathrm{I}'}) = f([< k, x >]_{\mathrm{I}})$.

(ix) For every F $\in$ FUS, $w \in$ W, $f \in$ ST from F to w, fusion F' equivalent to F, and equivalence g between F and F', the g-image of f is in ST.

(x) Every $f \in$ ST is injective (where defined).

(xi) For every F = $<(\mathrm{U}_k)_{k\in\mathrm{K}}$, I> ∈ FUS, and $k \in$ K, the stipulation f from F to u_k such that [for every equivalence class $c \in$ dom(F)/I and $x \in$ dom(u_k), $f(c) = x$ iff $< k, x >\in c$] is in ST.

Hazen initially restricts his focus to names which, if they denote anything, denote something in the actual world (1977: 128). A *stipulational model* for a standard FOM language L is an ordered pair <SMS, DEN>, where SMS is a stipulational model structure and DEN is a (perhaps partial) function from the individual constants of L to dom(r) (1977: 128).

For a stipulational model <SMS, DEN> for a standard FOM language L, and F ∈ FUS, an *assignment* a for F is a (perhaps partial) function from the variables of L to dom(F) (1977: 128-129).

For a standard FOM language L, stipulational model <SMS, DEN> for L, F ∈ FUS, L-term t, and $< k, x >\in$ dom(F), $< k, x >$ is assigned to t *primarily* by an assignment a for F iff either (i) t is a variable and $a(t) =< k, x >$, or (ii) t is a constant, $k = 0$, $x \in$ dom(r), and DEN(t) = x. $< k, x >$ is assigned to t *secondarily* by a for F = <G, I> iff for some $< j, y >$ primarily assigned to t by a, $< k, x >\in [< j, y >]_{\mathrm{I}}$ (1977: 129).

The most basic notion of truth-value employed by stipulational semantics is that of an L-formula being true (false) relative to an index of a fusion and assignment for that fusion. Truth (falsity) *simpliciter* is truth (falsity) relative to the fusion F whose only index is 0, for every assignment for F (1977: 129).

Earlier, I talked of the worlds "of the type corresponding to a language". In section II.D of 1977, Hazen specifies that the possible worlds involved in the semantics of a standard FOM language L are structures suited to verifying the sentences of the non-modal fragment of L. For a standard FOM language L,

we can take a world w to specify, for every $n \geq 1$ and n-ary predicate constant 'P' of L, a corresponding n-ary relation R $\subseteq$ dom$(w)^n$.

Finally, we turn to the clauses of the recursive definition of L-formula truth relative to an index of a fusion and assignment for that fusion.

An atomic formula '$Pt_1...t_n$' of L is true at index k of fusion $<(\mathrm{U}_k)_{k\in\mathrm{K}}, \mathrm{I}>$ on assignment a iff (i) for every $1 \leq i \leq n$, there is an $x \in$ dom(u_k) (let it be x_i) such that 't_i' is secondarily assigned $< k, x_i >$ by a, and (ii) $< x_1, ..., x_n >$ is a member of the relation, in the world u_k, corresponding to the predicate 'P'.

'$t_1 = t_2$' is true at index k of fusion $<(\mathrm{U}_k)_{k\in\mathrm{K}}, \mathrm{I}>$ on assignment a iff for some $x \in$ dom(u_k), $< k, x >$ is secondarily assigned by a to each of 't_1' and 't_2'.

The clauses for truth-functional connectives are the usual.

'$\exists x\phi$' is true at index k of fusion F $= <(\mathrm{U}_k)_{k\in\mathrm{K}}, \mathrm{I}>$ on assignment a iff for some $d \in$ dom(u_k), 'ϕ' is true at k of F on assignment a' (where a' is the same as a, except that it assigns $< k, d >$ to 'x').

'$\forall x\phi$' is true at index k of fusion F $= <(\mathrm{U}_k)_{k\in\mathrm{K}}, \mathrm{I}>$ on assignment a iff for every $d \in$ dom(u_k), 'ϕ' is true at k of F on assignment a' (where a' is the same as a, except that it assigns $< k, d >$ to 'x').

'$\diamond\phi$' is true at index k of fusion F on assignment a iff for some $w \in$ W and $f \in$ ST from F to w, 'ϕ' is true at index $n + 1$ of the f-w-extension of F on a (where n is the highest index of F).

'$\Box\phi$' is true at index k of fusion F on assignment a iff for every $w \in$ W and $f \in$ ST from F to w, 'ϕ' is true at index $n + 1$ of the f-w-extension of F on a (where n is the highest index of F).

'$\mathbf{A}\phi$' is true at index k of fusion F on assignment a iff 'ϕ' is true at index 0 of F on a.

For simplicity, let us introduce the following notation. For a standard FOM language L, fusion F, assignment a for F, variables 'x_1', ..., 'x_n', and $d_1, ..., d_n \in$ dom(F), let $a_{[d_1,...,d_n/\text{`}x_1\text{'},...,\text{`}x_n\text{'}]}$ be the same assignment as a, except that it assigns d_1 to 'x_1', ..., and d_n to 'x_n'.

For example, reconsider (20):

(20) $\diamond(\sim\exists xj = x \wedge \exists x(Fx \wedge \diamond Gjx))$

Let <SMS, DEN> be a stipulational model and F $\in$ FUS the fusion $<(\mathrm{U}_k)_{k\in\mathrm{K}}$, I> where U = $\{r\}$ and K = $\{0\}$. We have:

(20) is true (relative to <SMS, DEN>).

$\Updownarrow$

For every assignment a for F, there is a $w \in$ W and $f \in$ ST from F to w (hence f-w-extension F' = $<(\mathrm{V}_m)_{m\in\mathrm{M}}$, I'> of F) such that '$(\sim\exists xj = x \wedge \exists x(Fx \wedge \Diamond Gjx))$' is true at index 1 of F' on a.

$\Updownarrow$

For every assignment a for F, there is a $w \in$ W and $f \in$ ST from F to w (hence f-w-extension F' = $<(\mathrm{V}_m)_{m\in\mathrm{M}}$, I'> of F) such that (i) '$\exists xj = x$' is not true at 1 of F' on a, and (ii) '$\exists x(Fx \wedge \Diamond Gjx)$' is true at 1 of F' on a.

$\Updownarrow$

For every assignment a for F, there is a $w \in$ W and $f \in$ ST from F to w (hence f-w-extension F' = $<(\mathrm{V}_m)_{m\in\mathrm{M}}$, I'> of F) such that (i) it is not the case that for some $d \in \mathrm{dom}(w)$, '$j = x$' is true at 1 of F' on $a_{[<1,d>/\text{`}x\text{'}]}$, and (ii) for some $d^+ \in \mathrm{dom}(w)$, '$(Fx \wedge \Diamond Gjx)$' is true at 1 of F' on $a_{[<1,d^+>/\text{`}x\text{'}]}$.

$\Updownarrow$

For every assignment a for F, there is a $w \in$ W and $f \in$ ST from F to w (hence f-w-extension F' = $<(\mathrm{V}_m)_{m\in\mathrm{M}}$, I'> of F) such that (i) there is no $d \in \mathrm{dom}(w)$ such that $< 1, d > \in [< 0, \mathrm{DEN}(\text{`}j\text{'})>]_{\mathrm{I'}}$, (ii) for some $d^+ \in \mathrm{dom}(w)$, (ii.a) d^+ is a member of the property, in world w, corresponding to 'F', and (ii.b) there is a $w^+ \in$ W and $g \in$ ST from F' to w^+ (hence g-w^+-extension F" = $<(\mathrm{S}_j)_{j\in\mathrm{J}}$, I"> of F') such that '$Gjx$' is true at index 2 of F" on $a_{[<1,d^+>/\text{`}x\text{'}]}$.

$\Updownarrow$

There is a $w \in$ W and $f \in$ ST from F to w (hence f-w-extension F' = $<(\mathrm{V}_m)_{m\in\mathrm{M}}$, I'> of F) such that (i) there is no $d \in \mathrm{dom}(w)$ such that $< 1, d > \in [< 0, \mathrm{DEN}(\text{`}j\text{'})>]_{\mathrm{I'}}$, (ii) for some $d^+ \in \mathrm{dom}(w)$, (ii.a) d^+ is a member of the property, in world w, corresponding to 'F', and (ii.b) there is a $w^+ \in$ W and $g \in$ ST from F' to w^+ (hence g-w^+-extension F" = $<(\mathrm{S}_j)_{j\in\mathrm{J}}$, I"> of F') such that for some d^{++} and $d^{+++} \in \mathrm{dom}(w^+)$, $< 2, d^{++} > \in [< 0, \mathrm{DEN}(\text{`}j\text{'})>]_{\mathrm{I"}}$, $< 2, d^{+++} > \in [< 1, d^+ >]_{\mathrm{I"}}$, and $< d^{++}, d^{+++} >$ is a member of the relation, in world w^+, corresponding to the predicate 'G'.

So much for the example. It is worth noting that given the way that structures are constructed (and given the additional conditions Hazen imposes on structures) the semantics is quite restrictive. For example, for any stipulational model M, the T, B, and 4 schemas, as well as statements of the necessity of identity and the necessity of distinctness, are true at M. Indeed, recall that validating such sentences is part of the central purpose of presenting stipulational semantics: presenting a semantics that does not

appeal to identity across worlds in the handling of the modal operators, but which is such that a fairly standard deductive system (which includes T, 4, B, and statements of the necessity of identity and distinctness) is sound and complete with respect to it.

4.1.2 Flavours of Modality

So much for an exposition of the rigorous account. So far, we have remained silent on different flavours of modality. As a preliminary, Hazen initially talks of "sets of modal notions". Each of these sets "must contain one notion of necessity and one notion of possibility, and [possibility and necessity] must be interdefinable in the usual way" (1977: 6, f.n. 6). These different sets seem to correspond to different flavours of modality. Hazen later introduces a new term: "I will ... in what follows call what I have been referring to as "sets of modal notions" *applications of the modal concepts*" (1977: 7).

So far, all we have concerning counterpart relations is that we "let an object's counterparts be all and only the objects of the same kind, where it is understood that an object is of one and only one kind" (1977: 113). Hazen does expand on this:

> "To be sure, the question, "What kind of thing is that?" asked out of the blue will seldom if ever have a unique answer. In the context of a particular discussion, however, where asker and answerer share common interests and background information, it may. My suggestion is that a particular application of the modal concepts is similarly a context which brings one particular way of dividing things up into kinds to the fore. To specify an application of the modal concepts would, on this proposal, be to specify a class of possible worlds and also a partition of the possible objects in those worlds into kinds. When we say of an individual that it could or could not have had some property, our judgement is grounded in the properties it actually has. The class of counterparts of an individual corresponding to a particular application of the modal concepts is the class of those possible objects, in worlds possible for that application of the modal concepts, which have those of the individual's properties which are criterial, on that application of the modal concepts, for *de re* modal judgements." (1977: 113-114)

After introducing the notion of a fusion of worlds, Hazen adds:

> "To specify an application of the modal concepts in terms of stipulational semantics, one specifies a class of possible worlds, a class of fusions, and a class of stipulations." (1977: 119)

On the current account, for a given claim, we fix on an application of the modal concepts, and handle the claim with the corresponding class of worlds,

fusions, and stipulations. Of course, the current account cannot handle a claim involving multiple modal occurrences, with one occurrence of a different flavour from another (more on this soon). However, one might complain that the current account is anyway too coarse-grained to even handle all claims featuring a single modal occurrence. One continues: there are utterances of claims of the form '$\diamond Ft_1...t_n$' where the single flavour of the modality involved requires considering individuals $d_1, ..., d_n$ (the denotations of $t_1, ..., t_n$, respectively) individually in ways C_1, ..., C_n, respectively, where for some $1 \leq j \leq n$ and $1 \leq k \leq n$, $C_j \neq C_k$. Further, for some utterances of some claims of the above form, the flavour of the modality involved requires considering a single individual d individually in one way C, and individually in another way C^+. To accommodate this granularity of modal flavour, the account would need revision.

4.1.3 The Issue

So much for an exposition of the account. There seems to me an issue developing stipulational semantics so that it handles claims involving iterated modality (where an embedded modal operator occurrence has a different flavour from an embedding modal operator occurrence). To introduce the point, reconsider:

(12) Things could have been such that John was F and from there things could have been such that John was G.

(12^+) $\diamond(Fj \wedge \diamond Gj)$

(12^+) is true (relative to a given stipulational model) iff:

> For fusion F = $<(U_k)_{k\in K}$, I> (where U = $\{r\}$ and K = $\{0\}$),
>
> there is a $w \in$ W and $f \in$ ST from F to w,
>
> the f-w-extension F' = $<(V_m)_{m\in M}$, I'> of F is such that there is an $x \in$ dom(w) such that $< 1, x > \in [< 0, \text{DEN}(`j') >]_{I'}$ and x is a member of the property, in w, corresponding to 'F',
>
> there is a $w^+ \in$ W and $g \in$ ST from F' to w^+,
>
> the g-w^+-extension F'' = $<(S_j)_{j\in J}$, I''> of F' is such that there is a $y \in$ dom(w^+) such that $< 2, y > \in [< 0, \text{DEN}(`j') >]_{I''}$ and y is a member of the property, in w^+, corresponding to 'G'.

A key point is that the handling of the embedded modal claim involves holding that g assigns to the equivalence class $[< 0, \text{DEN}(`j') >]_{I'}$ a member of dom(w^+).

Next, consider an expanded version of (12), which involves multiple flavours of modality:

(21) Strictly speaking, things could have been such that John was a bumble bee and from there, even merely physically speaking, things could have been such that John was an abnormally large bumble bee.

Again, the first possibility operator occurrence has a rather unrestricted flavour M_1, and the embedded possibility operator occurrence has a much more restricted flavour M_2. It may very well be that John could have (M_1 flavour) had the property [being a bumble bee and being such that he could have (M_2 flavour) been an abnormally large bumble bee]. However, it is presumably not the case that John could have (M_2 flavour) had the property [being an abnormally large bumble bee]. Indeed, it is presumably not the case that John could have (M_2 flavour) had the property [being a bumble bee].

Of course, on Hazen's current account, we cannot handle a claim involving multiple flavours of modality. Currently, a claim is handled as being such that all of the modal occurrences in it are of the same flavour. However, my point is that given the way Hazen has handled iterated modality (in terms of fusions), there is an issue developing the account to handle iterated modality (where an embedded modal operator occurrence has a different flavour from an embedding modal operator occurrence).

To see this, note that informally, the natural way to handle (21) given Hazen's stipulational account is to posit two flavours of modality M_1 and M_2 and posit a class of stipulations for each.

Formally, let us now focus on the augmented FOM languages. For each standard FOM language L, there corresponds an augmented FOM language L^+ that is the same as L except that the single possibility operator is replaced with two operators $\diamond_1$ and $\diamond_2$, and the necessity operator is replaced with two operators $\Box_1$ and $\Box_2$. That is, we have a possibility and necessity operator pair for each of the flavours of modality M_1 and M_2. (21) is now regimented as:

(21^+) $\diamond_1(Bj \wedge \diamond_2 Lj)$

We develop stipulational semantics to interpret the augmented FOM languages by expanding a stipulational model structure so that for each $1 \leq i \leq 2$, there is a class of stipulations ST_i. That is, we have a class of stipulations for each of the flavours of modality M_1 and M_2. Next, we adjust the modal clauses of the recursive definition of truth relative to an index of a fusion and assignment for that fusion as follows:

> '$\diamond_i\phi$' ('$\Box_i\phi$') is true at index k of fusion F on assignment a iff for some (every) $w \in W$ and $f \in ST_i$ from F to w, 'ϕ' is true at index $n+1$ of the f-w-extension of F on a (where n is the highest index of F).

On the new proposal, (21^+) is true (relative to a given stipulational model) iff:

> For fusion $F = <(U_k)_{k\in K}, I>$ (where $U = \{r\}$ and $K = \{0\}$),

there is a $w \in$ W and $f \in \mathrm{ST}_1$ from F to w,

the f-w-extension F' = $<(\mathrm{V}_m)_{m\in\mathrm{M}}$, I'$>$ of F is such that there is an $x \in$ dom(w) such that $< 1, x > \in [< 0, \mathrm{DEN}(`j') >]_{\mathrm{I'}}$ and x is a member of the property, in w, corresponding to 'B',

there is a $w^+ \in$ W and $g \in \mathrm{ST}_2$ from F' to w^+,

the g-w^+-extension F" = $<(\mathrm{S}_j)_{j\in\mathrm{J}}$, I"$>$ of F' is such that there is a $y \in$ dom(w^+) such that $< 2, y > \in [< 0, \mathrm{DEN}(`j') >]_{\mathrm{I"}}$ and y is a member of the property, in w^+, corresponding to 'L'.

So much for the natural approach to handling (21). Notice that on the new proposal, (21^+) is true only if there is an M_2 stipulation to a world w^+ from a direct extension of F which assigns to the equivalence class that < 0, DEN('j')$>$ is a member of an individual that is a member of the property corresponding to 'L' in w^+.

The issue is that we seemingly want (21^+) to be true, but given M_2 is a much more restricted flavour of modality, it is perhaps inappropriate that there is an M_2 stipulation to a world w^+ from a direct extension of F which assigns to the equivalence class that < 0, DEN('j')$>$ is a member of an individual that is a member of the property corresponding to 'L' in w^+. Put simply, it is perhaps inappropriate that there is an M_2 stipulation which specifies as a representative for John an abnormally large bumble bee (or a bumble bee of any sort).

The issue here is ostensibly not a technical one. From a technical perspective, the developed semantics can render (21^+) true. It can even render (13^+) true:

(13) [Strictly speaking, things could have been such that John was a bumble bee and from there, even merely physically speaking, things could have been such that John was an abnormally large bumble bee], but merely physically speaking, things could not have been such that John was an abnormally large bumble bee.

(13^+) $\diamond_1(Bj \wedge \diamond_2 Lj) \wedge \sim \diamond_2 Lj$

We have that (13^+) is true (relative to a given stipulational model) iff:

For fusion F = $<(\mathrm{U}_k)_{k\in\mathrm{K}}$, I$>$ (where U = $\{r\}$ and K = $\{0\}$),

there is a $w \in$ W and $f \in \mathrm{ST}_1$ from F to w,

the f-w-extension F' = $<(\mathrm{V}_m)_{m\in\mathrm{M}}$, I'$>$ of F is such that there is an $x \in$ dom(w) such that $< 1, x > \in [< 0, \mathrm{DEN}(`j') >]_{\mathrm{I'}}$ and x is a member of the property, in w, corresponding to 'B',

there is a $w^+ \in$ W and $g \in \mathrm{ST}_2$ from F' to w^+,

the g-w^+-extension F" = <$(S_j)_{j\in J}$, I"> of F' is such that there is a $y \in$ dom(w^+) such that $< 2, y >\in [< 0, \text{DEN}(`j') >]_{I"}$ and y is a member of the property, in w^+, corresponding to 'L', and

there is no $w^* \in$ W and $f^* \in ST_2$ from F to w^* such that the f^*-w^*-extension F* = <$(R_l)_{l\in L}$, I* > of F is such that there is an $x \in$ dom(w^*) such that $< 1, x >\in$ [<0, DEN('j')>]$_{I^*}$ and x is a member of the property, in w^*, corresponding to 'L'.

The key point is that from a technical perspective, although $g \in ST_2$ from F' to w^+ assigns y to {<0, DEN('j')>, $< 1, x >$}, we can block there being an $f^* \in ST_2$ from F to w^+ that assigns y to {<0, DEN('j')>}. So, we can have that (13^+) is true as required.

To allow this is effectively to not impose a version of condition (viii) that Hazen imposed on stipulational model structures (which was imposed to guarantee the validity of schema 4).

The issue is whether we can provide an acceptable analysis of the developed account of stipulations. To push this point, Hazen employs the notion of a counterpart relation when analysing his first-pass notion of a stipulation (from the domain of a world to the domain of another world). For example:

> "A representative function maps objects into counterparts of themselves, but, to allow for internal relations, not every function mapping [objects] into counterparts will be a representative function" (1979: 334)

Given Hazen's approach, it seems we should hold that for each $1 \leq i \leq 2$, corresponding to our flavour of modality M_i is a counterpart relation that is employed in the analysis of M_i stipulations.

We might be tempted to hold that for every $1 \leq i \leq 2$, fusion F = <G, I>, $w \in$ W, and $f \in ST_i$ from F to w, f assigns to an equivalence class C in dom(F)/I an individual $d \in$ dom(w) only if for every $< k, x >\in$ C, d is an M_i counterpart of x. However, this seems unacceptable. Given this position, on the developed semantics, (25) is true only if John himself has an M_2 counterpart that is an abnormally large bumble bee (where M_2 is our stricter flavour). But it seems that John should have no such counterpart.

To render (25) true (as is seemingly required), we need to hold that for some fusion F = <G, I>, $w \in$ W, $f \in ST_2$ from F to w, equivalence class C in dom(F)/I, $< k, x >\in$ C, and $d \in$ dom(w), f assigns d to C, but d is not an M_2 counterpart of x. The challenge for Hazen's account is to provide an analysis of stipulations that accommodates this.

To expand on the challenge, it at least seems that for $1 \leq i \leq 2$, fusion F = <G, I> (whose highest index is n), $w \in$ W, $f \in ST_i$ from F to w, and

equivalence class C in dom(F)/I, if there is an item $< n, x >\in$ C, then f assigns a $d \in$ dom(w) to C only if d is an M_i counterpart of x.

One might think that a simple solution to the challenge is at hand: for $1 \leq i \leq 2$, fusion F = <G, I> (whose highest index is n), $w \in$ W, and $f \in ST_i$ from F to w, we simply hold that f is determined solely by factors concerning individuals in the domain of the n^{th} world of F and individuals in dom(w). For an equivalence class C in dom(F)/I, to determine the image of C under f, we do not need to consider those items $< k, x >\in$ C where $k < n$. However, Hazen seemingly cannot allow this. Given Hazen's approach to (20), it seems that a desideratum of his account is that the following variant of (20) is satisfiable:

(22) $\diamond_1(\sim\exists xj = x \wedge \exists x(Fx \wedge \diamond_1 Gjx))$

Let <SMS, DEN> be a stipulational model and F $\in$ FUS the fusion $<(U_k)_{k\in K}$, I> where U = $\{r\}$ and K = $\{0\}$. We have:

(22) is true (relative to <SMS, DEN>).

$\Updownarrow$

There is a $w \in$ W and $f \in ST_1$ from F to w (hence f-w-extension F' = $<(V_m)_{m\in M}$, I'> of F) such that (i) there is no $d \in$ dom(w) such that $< 1, d >\in [< 0,$ DEN('j')$>]_{I'}$, (ii) for some $d^+ \in$ dom(w), (ii.a) d^+ is a member of the property, in world w, corresponding to 'F', and (ii.b) there is a $w^+ \in$ W and $g \in ST_1$ from F' to w^+ (hence g-w^+-extension F" = $<(S_j)_{j\in J}$, I"> of F') such that for some d^{++} and $d^{+++} \in$ dom(w^+), $< 2, d^{++} >\in [< 0,$ DEN('j')$>]_{I"}$, $< 2, d^{+++} >\in [< 1, d^+ >]_{I"}$, and $< d^{++}, d^{+++} >$ is a member of the relation, in world w^+, corresponding to the predicate 'G'.

Here, we need the M_1 stipulation g to assign $d^{++} \in$ dom(w^+) to the equivalence class C = $[< 0,$ DEN('j')$>]_{I'}$ even though C has no element $< 1, x >$.

4.2 Russell's Counterpart Semantics

Russell 2013 presents a semantics for a certain formal first-order modal language that is close to the first-pass of Hazen's stipulational semantics. The aim is to demonstrate that a semantics for a first-order modal language based centrally on a notion of counterparthood can accommodate an actuality operator **A**.

4.2.1 The Semantics

Again, we start with the notion of a world and a world's domain. The members of a world's domain are individuals. The domains of worlds are disjoint (2013: 116). Central to Russell's account is the notion of a

counterpart-link. For worlds w and v, a counterpart-link from w to v is a relation $\mathrm{S} \subseteq [\mathrm{dom}(w) \times \mathrm{dom}(v)]$. In this work, Russell holds that a counterpart-link is "one-to-one", in the sense that for a relation $\mathrm{R} \subseteq \mathrm{X} \times \mathrm{Y}$, R is one-to-one iff for all $x_1, x_2 \in \mathrm{X}$ and $y_1, y_2 \in \mathrm{Y}$, if $x_1 \mathrm{R} y_1$ and $x_2 \mathrm{R} y_2$, then $x_1 = x_2$ iff $y_1 = y_2$ (2013: 99). Notice that a one-to-one relation $\mathrm{R} \subseteq \mathrm{X} \times \mathrm{Y}$ is just a (perhaps partial) injective function from X to Y. Further, as it is injective, its inverse R^{-1} is a (perhaps partial) injective function from Y to X.

Moving on, individuals $b_1, ..., b_n$ are *jointly-counterparts* of individuals $a_1, ..., a_n$ iff there is a counterpart-link S such that for every $1 \leq i \leq n$, $b_i \mathrm{S} a_i$.

To account for an actuality operator, Russell also introduces the notion of a representation relation. For a world w, a representation relation for w is a one-to-one relation $\mathrm{R} \subseteq [\mathrm{dom}(w) \times \mathrm{dom}(w_@)]$. For binary relations S and R, S$\circ$R is the binary relation: $x \mathrm{S} \circ \mathrm{R} z$ iff there is some y such that $x \mathrm{S} y$ and $y \mathrm{R} z$.

Russell presents his semantics for a standard FOM language L (without $\vee, \rightarrow, \leftrightarrow, \forall, \Box$, though these are allowed as meta-linguistic abbreviations defined in the usual way, and without individual constants) (2013: 115). A counterpart model M for L is an ordered tuple <W, $w_@$, D, dom, I, C> where the following holds (2013: 115-116):[8]

(i) W is a non-empty set.

(ii) $w_@ \in \mathrm{W}$.

(iii) dom: $\mathrm{W} \rightarrow \mathcal{P}(\mathrm{D})$ such that (a) $\mathrm{D} = \bigcup_{w \in \mathrm{W}} \mathrm{dom}(w)$, and (b) for all distinct (from each other) $w, v \in \mathrm{W}$, $\mathrm{dom}(w)$ and $\mathrm{dom}(v)$ are disjoint (if $[\mathrm{dom}(w) \cap \mathrm{dom}(v)] \neq \emptyset$, then $w = v$).

(iv) I is a function from PC(L) such that, for every $n \geq 1$ and n-ary $P \in \mathrm{PC(L)}$, $\mathrm{I}(P) \subseteq \mathrm{D}^n$ [for an n-ary $P \in \mathrm{PC(L)}$ and $w \in \mathrm{W}$, let $\mathrm{I}(P)_w = \mathrm{I}(P) \cap \mathrm{dom}(w)^n$].[9]

(v) C is a binary function which assigns to each pair of worlds w and v, a set of counterpart-links from w to v, with the condition that for every $w \in \mathrm{W}$, $\mathrm{Id}\restriction\mathrm{dom}(w) \in \mathrm{C}(w, w)$ [the identity relation restricted to $\mathrm{dom}(w)$ is in $\mathrm{C}(w, w)$].

For a counterpart model M, an **R**-point of M is a tuple $< v, \mathrm{R}, a_1, ..., a_n >$ where $v \in \mathrm{W}$, $n \geq 0$, for every $1 \leq i \leq n$, $a_i \in \mathrm{dom}(v)$, and R is a representation relation for v (2013: 117).

[8]Here, I make a slight adjustment to Russell's presentation to straightforwardly identify a counterpart model for L as a certain kind of ordered tuple. Nothing turns on this.

[9]Notice that on this treatment, we cannot have 0-ary predicate constants. If we did, then they would be assigned truth/falsity *simpliciter* (not relative to a world).

The most basic notion of truth-value is of an L-formula being true relative to an **R**-point of a model. Russell presents the following as a recursive definition of this notion (2013: 115-117). As a preliminary, we are told that there is an implicit "ordering of variables". It seems this ordering of variables combined with an **R**-point performs the job of a denotation function: for an **R**-point $< v$, R, $a_1, ..., a_n >$ and formula ϕ with at most n-many free variables, each free variable of ϕ corresponds to a position of $a_1, ..., a_n$, and the occupant of that position is the variable's denotation (relative to $< v$, R, $a_1, ..., a_n >$). It seems that for every $m \geq 0$, and L-formula ϕ with m-many free variables, we only consider ϕ relative to **R**-points with n-many places for individuals ($n \geq m$). Russell talks of a point having to be "long enough" for a formula ϕ "in the sense that it has enough objects for all of [ϕ's] free variables, with respect to the implicitly given ordering" (2013: 116).

(i) $< v$, R, $a_1, ..., a_n >\models Fx_{i_1}...x_{i_n}$ iff $< a_{i_1}, ..., a_{i_n} > \in \mathrm{I}(F)_v$.

(ii) $< v$, R, $a_1, ..., a_n >\models x_{i_1} = x_{i_n}$ iff $a_{i_1} = a_{i_n}$.

(iii) $< v$, R, $a_1, ..., a_n >\models \sim\phi$ iff $< v$, R, $a_1, ..., a_n > \not\models \phi$.[10]

(iv) $< v$, R, $a_1, ..., a_n >\models (\phi \wedge \psi)$ iff $< v$, R, $a_1, ..., a_n >\models \phi$ and $< v$, R, $a_1, ..., a_n >\models \psi$.

(v) $< v$, R, $a_1, ..., a_n >\models \exists x_{n+1}\phi$ iff for some $b \in \mathrm{dom}(v)$, $< v$, R, $a_1, ..., a_n, b >\models \phi$.

(vi) $< v$, R, $a_1, ..., a_n >\models \Diamond\phi$ iff there is an **R**-point $< w$, R', $b_1, ..., b_n >$ such that (i) $< v$, R, $a_1, ..., a_n > \overset{\Diamond}{\rightarrow} < w$, R', $b_1, ..., b_n >$ and (ii) $< w$, R', $b_1, ..., b_n >\models \phi$.

(vii) $< v$, R, $a_1, ..., a_n >\models \mathbf{A}\phi$ iff there is an **R**-point $< w$, R', $b_1, ..., b_n >$ such that (i) $< v$, R, $a_1, ..., a_n > \overset{\mathbf{A}}{\rightarrow} < w$, R', $b_1, ..., b_n >$ and (ii) $< w$, R', $b_1, ..., b_n >\models \phi$.

$< v$, R, $a_1, ..., a_n > \overset{\Diamond}{\rightarrow} < w$, R', $b_1, ..., b_n >$ iff for some S $\in$ C(w, v), (i) R' = S$\circ$R, and (ii) for every $1 \leq i \leq n$, $b_i \mathrm{S} a_i$ (2013: 117). Given this (and the definition of a counterpart-link), clause (vi) amounts to:

(vi*) $< v$, R, $a_1, ..., a_n >\models \Diamond\phi$ iff for some $w \in$ W, S $\in$ C(w, v), and b_1, ..., b_n $\in \mathrm{dom}(w)$ such that for every $1 \leq i \leq n$, $\mathrm{b}_i \mathrm{S} a_i$, $< w$, S$\circ$R, $b_1, ..., b_n >\models \phi$.

$< v$, R, $a_1, ..., a_n > \overset{\mathbf{A}}{\rightarrow} < w$, R', $b_1, ..., b_n >$ iff (i) $w = w_@$, (ii) R' = Id$\restriction$dom($w_@$), and (iii) for every $1 \leq i \leq n$, $a_i \mathrm{R} b_i$ (ibid.). Given this, clause (vii) amounts to:

[10] Russell includes the connective $\sim$ in L, but he in fact omits the clause corresponding to it in the recursive definition in his formal presentation (2013: 115-116). However, it seems clear that the one I present is the correct clause, given his earlier comments.

(vii*) $< v$, R, $a_1, ..., a_n >\models \mathbf{A}\phi$ iff $< w_@$, Id$\restriction$dom$(w_@)$, $b_1, ..., b_n >\models \phi$ (where for every $1 \leq i \leq n$, a_iRb_i).

Though $\forall$ and $\Box$ are not part of the vocabulary of L, they can be introduced as metalinguistic abbreviations in the usual way. Given this, we have:

(viii) $< v$, R, $a_1, ..., a_n >\models \forall x_{n+1}\phi$ iff for every $b \in$ dom(v), $< v$, R, $a_1, ..., a_n, b >\models \phi$.

(ix*) $< v$, R, $a_1, ..., a_n >\models \Box\phi$ iff for every $w \in$ W, S $\in$ C(w, v), and b_1, ..., $\mathrm{b}_n \in$ dom(w) such that for every $1 \leq i \leq n$, $\mathrm{b}_i\mathrm{S}a_i$, $< w$, S$\circ$R, $b_1, ..., b_n >\models \phi$.

An L-sentence ϕ is true in a counterpart model M iff $< w_@, Id \restriction \mathrm{dom}(w_@) > \models \phi$. That is, it is true relative to the **R**-point with the actual world $w_@$, a representation relation that is the identity relation restricted to dom$(w_@)$, and 0-many individuals .

4.2.2 Flavours of Modality

Like Hazen's stipulational semantics, Russell's counterpart semantics delivers a coarse-grained account of modal flavour. With regards to "contingent identity" and "contingent distinctness", note that (in virtue of a counterpart-link being a one-to-one relation) (NI), (ND), and (NID) are true at every **R**-point of every counterpart model:

(NI) $\forall x \forall y(x = y \rightarrow \Box x = y)$

(ND) $\forall x \forall y(\sim x = y \rightarrow \Box \sim x = y)$

(NID) $\Box \forall x \forall y(\diamond x = y \rightarrow \Box x = y)$

However, Russell does hold that his account is compatible with rendering certain "contingent identity" and "contingent distinctness" statements true. He continues (2013:99):

> "A plausible thing to say about statue–lump cases is that there are two different modalities involved: possibility-for-statues and possibility-for-lumps. We explain these in terms of different counterpart relations, (that is) different kinds of counterpart-links. What we want to say is that a single object could have been statuewise a different object than it would in that case have been lumpwise. But this claim is compatible with (NID) on any unequivocal interpretation. What (NID) rules out is that anything could have been distinct from what it then would have been, where 'could' and 'would' are used in *exactly the same sense*." (ibid.)

4.2.3 The Issues

Here, I note two issues for Russell's semantics. First, given that Russell's semantics handles the possibility/necessity operators as, broadly speaking, quantifying over tuples of individuals (tuples which may not feature gaps), we end up interpreting all *de re* occurrences of $\diamond$ and $\Box$ as weak *de re* possibility and necessity, respectively. That is, what is possible/necessary of individuals when they all exist.

To see this, consider the following:

(CE) $\exists x \diamond \sim \exists y x = y$

This is false at every **R**-point of every counterpart model.

Proof. Let M be a counterpart model and $< v, \mathrm{R}, a_1, ..., a_n >$ an **R**-point of M. For *reductio*, suppose:

$< v, \mathrm{R}, a_1, ..., a_n > \models \exists x \diamond \sim \exists y x = y.$

$\Downarrow$

For some $b \in \mathrm{dom}(v)$, $< v, \mathrm{R}, a_1, ..., a_n, b > \models \diamond \sim \exists y x = y.$

$\Downarrow$

For some $b \in \mathrm{dom}(v)$, for some $w \in \mathrm{W}$, $\mathrm{S} \in \mathrm{C}(w, v)$, and $c_1, ..., c_n, d \in \mathrm{dom}(w)$ such that $c_1 \mathrm{S} a_1$, ..., $c_n \mathrm{S} a_n$, and $d \mathrm{S} b$, $< w, \mathrm{S} \circ \mathrm{R}, b_1, ..., b_n, d > \models \sim \exists y x = y.$

$\Downarrow$

For some $b \in \mathrm{dom}(v)$, for some $w \in \mathrm{W}$, $\mathrm{S} \in \mathrm{C}(w, v)$, and $c_1, ..., c_n, d \in \mathrm{dom}(w)$ such that $c_1 \mathrm{S} a_1$, ..., $c_n \mathrm{S} a_n$, and $d \mathrm{S} b$, $< w, \mathrm{S} \circ \mathrm{R}, b_1, ..., b_n, d > \not\models \exists y x = y.$

$\Downarrow$

For some $b \in \mathrm{dom}(v)$, for some $w \in \mathrm{W}$, $\mathrm{S} \in \mathrm{C}(w, v)$, and $c_1, ..., c_n, d \in \mathrm{dom}(w)$ such that $c_1 \mathrm{S} a_1$, ..., $c_n \mathrm{S} a_n$, and $d \mathrm{S} b$, it is not the case that there is a $u \in \mathrm{dom}(w)$ such that $< w, \mathrm{S} \circ \mathrm{R}, b_1, ..., b_n, d, u > \models x = y.$

$\Downarrow$

For some $b \in \mathrm{dom}(v)$, for some $w \in \mathrm{W}$, $\mathrm{S} \in \mathrm{C}(w, v)$, and $c_1, ..., c_n, d \in \mathrm{dom}(w)$ such that $c_1 \mathrm{S} a_1$, ..., $c_n \mathrm{S} a_n$, and $d \mathrm{S} b$, it is not the case that there is a $u \in \mathrm{dom}(w)$ such that $d = u$. But $d \in \mathrm{dom}(w)$. $\perp$ ■

Likewise, consider:

(NE) $\forall x \Box \exists y x = y$

This is true at every **R**-point of every counterpart model (I leave this as an exercise for the reader).

On the current handling, *de re* occurrences of $\diamond$ and $\Box$ express weak *de re* possibility and necessity, respectively. Hence (CE) is always false, and (NE) always true.

One task now is to modify the account to handle strong *de re* possibility and necessity. Indeed, Russell himself states that when it comes to contingent existence, the approach "leaves much to be desired", and that future work is required (2013: 102, f.n. 13).

Now, the second point: the semantics seems to mishandle certain modal operator occurrences embedded in the scope of quantifier occurrences. Consider:

(23) $\forall x \diamond \exists y Fy$

Let M be a counterpart model. We have:

$\forall x \diamond \exists y Fy$ is true at M.

$\Updownarrow$

$< w_@, Id \restriction \mathrm{dom}(w_@) > \models \forall x \diamond \exists y Fy$.

$\Updownarrow$

For every $b \in \mathrm{dom}(w_@)$, $< w_@, Id \restriction \mathrm{dom}(w_@), b > \models \diamond \exists y Fy$.

$\Updownarrow$

For every $b \in \mathrm{dom}(w_@)$, for some $w \in \mathrm{W}$, $\mathrm{S} \in \mathrm{C}(w, w_@)$, and $c \in \mathrm{dom}(w)$ such that $c \mathrm{S} b$, $< w, \mathrm{S}, c > \models \exists y Fy$.

$\Updownarrow$

For every $b \in \mathrm{dom}(w_@)$, for some $w \in \mathrm{W}$, $\mathrm{S} \in \mathrm{C}(w, w_@)$, and $c \in \mathrm{dom}(w)$ such that $c \mathrm{S} b$, for some $d \in \mathrm{dom}(w)$, $d \in \mathrm{I}(F)_w$.

However, this seems to be a mishandling of (23). It seems to me that the correct handling of (23) should treat the initial quantification as vacuous: (23) states that for every actual individual, some *de dicto* modal feature obtains.[11] That is, the possibility operator occurrence is *de dicto*. (23) does not state that for every actual individual d, there is a possibility where d is such and such and some individual is F. (23) certainly does not state that for every actual individual d, there is a possibility where d exists and some individual is F. However, this is how (23) is handled on Russell's account.

[11] Let us suppose that F is purely qualitative.

The full mishandling of (23) is caused by two features: (i) individual d is taken as relevant to the handling of the possibility operator occurrence, and (ii) possibility/necessity operator occurrences are treated as quantifiers over tuples of individuals (tuples which may not feature gaps). As a result, the possibility operator occurrence in (23) is treated as a quantifier over non-gappy tuples of individuals which feature a position for d.

To demonstrate the point, let $M =<$W, $w_@$, D, dom, I, C$>$ be a counterpart model, $d \in$ D, and $w \in$ W such that $d \in \text{dom}(w)$ and $\text{I}(F) = \{d\}$. Further, suppose that for some $b \in \text{dom}(w_@)$, for every $\text{S} \in \text{C}(w, w_@)$, there is no $c \in \text{dom}(w)$ such that $< c, b > \in$ S. Then (23) is false at M according to Russell's semantics. However, it seems it should be true.

To push the point, consider:

(24) $\Diamond\exists yFy \rightarrow \forall x \Diamond \exists yFy$

This seems uncontroversially true. It seems to be an innocuous modal substitution instance of a theorem of classical first-order logic and inclusive first-order logic. However, it is false at M on Russell's semantics.

Russell does seem to be aware of the issue which causes the mishandling of sentences such as (23) and (24). He considers the following (2013: 129):

(Ugly) $\Diamond\forall yFy$

Russell informally presents an example of there being (i) an individual a which has no counterparts that are F, but (ii) a world at which every individual is F. On the current account, (Ugly) is false of a, despite (ii).

Here is a formal example in this vein: consider counterpart model $M =<$W, $w_@$, D, dom, I, C$>$, $a \in \text{dom}(w_@)$, and $w \in$ W such that (i) $\text{dom}(w) = \text{I}(F)$, but (ii) for every $\text{S} \in \text{C}(w, w_@)$, there is no $b \in \text{dom}(w)$ such that $< b, a > \in$ S. Then $< w_@, Id \restriction \text{dom}(w_@), a > \not\models$ (Ugly). Though, of course, $< w_@, Id \restriction \text{dom}(w_@) > \models$ (Ugly).

Russell notes:

> "(Ugly) is true, even though, considered as a trivial one-place predicate, (Ugly) is not true of a. For (Ugly) to be true of a, there would have to be an ugly world where a has a counterpart — and of course there is no such thing. This is a strange result: we are used to thinking that if a predicate is **syntactically trivial** in the sense that it has no free variables, it is also **semantically trivial** in the sense that it is true of anything if it is true at all. In our counterpart system, this is not so. It is not just terms that semantically 'depend on' particular things. Modal operators do too — since they require counterparts (and having a counterpart at a world is not trivial)." (2013: 129)

In response, it seems a pretty uncontroversial desideratum of an account that a sentence is semantically trivial in Russell's sense.

4.3 Bacon's Counterpart Semantics

Bacon 2014 presents a skeletal semantics for a certain formal first-order modal language. The semantics relies on the notion of a possibility (which is analysed in terms of a counterpart relation). The aim is to demonstrate that a semantics for a first-order modal language based centrally on a notion of counterparthood can accommodate an actuality operator **A**. The semantics is supervaluationist in order to accommodate **A**. The idea is that certain formulas with **A** occurrences can have an "indeterminate" truth-value at certain models of the semantics.

4.3.1 The Semantics

Bacon's semantics is for a standard FOM language L (without $\wedge, \rightarrow, \leftrightarrow, \forall, \Box$, though these are allowed as meta-linguistic abbreviations defined in the usual way, without $=$, and without individual constants).

A *counterpart structure* $\mathfrak{A}$ for L is a tuple <W, D, Ind, C, w^* >, where the following holds (2014: 100):

> W and D are non-empty sets.
>
> Ind: W $\rightarrow \mathcal{P}$(D).
>
> C $\subseteq$ D^2 (and is reflexive).
>
> $w^* \in$ W.
>
> Ind(w^*) $\neq \emptyset$.

These elements represent, respectively: a set of worlds, the overall domain of individuals, the world domains of individuals, a counterpart relation, and the actual world. Notice that world domains of individuals may overlap. Note that contrary to usual usage, on Bacon's account, Cxy is understood as "y is a counterpart of x".

A *counterpart model* M for L is a tuple <W, D, Ind, C, w^*, I>, where the following holds (ibid.):[12]

> <W, D, Ind, C, w^* > is a counterpart structure for L.
>
> I is a function that, for every $n \geq 1$, assigns to each n-ary predicate constant P a function that assigns to each $w \in$ W a subset of D^n.

[12]Here, I deviate slightly from Bacon's notation: I use the symbol 'I' rather than '[[ ]]'. Nothing turns on this.

A counterpart model <W, D, Ind, C, w^*, I> is *based* on the counterpart structure <W, D, Ind, C, w^* >.

A counterpart model <W, D, Ind, C, w^*, I> is *serious* iff for every $n \geq 1$, n-ary predicate constant P, and $w \in$ W, I(P)(w) $\subseteq$ Ind(w)^n. That is, an n-tuple of individuals $d_1, ..., d_n$ is in the extension of P at w only if each of $d_1, ..., d_n$ is in Ind(w). On his skeletal semantics, Bacon does not rule out non-serious models.

For a counterpart structure $\mathfrak{A}$ = <W, D, Ind, C, w^* > and $w \in$ W, a *counterpart function* σ for w at $\mathfrak{A}$ is a function from D to D such that $\sigma \subseteq$ C (ibid.).

For a counterpart structure $\mathfrak{A}$ = <W, D, Ind, C, w^* >, a *possibility* for $\mathfrak{A}$ is a tuple $< w, \sigma >$ such that (i) $w \in$ W, and (ii) σ is a counterpart function for w at $\mathfrak{A}$ (ibid.). Let S($\mathfrak{A}$) be the set of possibilities for $\mathfrak{A}$.

The idea here is that a possibility consists of a world plus a statement of which individuals are representing which individuals. Notice that the notion of a possibility is so far quite skeletal. For example, currently the condition for a function to be a counterpart function for w at $\mathfrak{A}$ has nothing to do with w itself. Bacon considers two further conditions on possibilities, but does not rule out possibilities which do not satisfy them (2014: 102):

(Exhaustion) $< w, \sigma >$ is such that for every $x \in$ Ind(w), for some $y \in$ D, $\sigma(y) = x$.

> Every individual in the domain of the world of the possibility represents some individual (according to the possibility).

(Identity) $< w, \sigma >$ is such that for every $x \in$ Ind(w), $\sigma(x) = x$.

> Every individual in the domain of the world of the possibility represents itself (according to the possibility).

Even with further conditions, it is worth noting that Bacon's possibilities are quite strange. For one point, we have that σ is required to be a total function from D to D. But why should it be that for every possibility $< w, \sigma >$, every individual is represented by some individual?[13]

Moving on, for a counterpart structure $\mathfrak{A}$ = <W, D, Ind, C, w^* >, a possibility $< w, \sigma >$ for $\mathfrak{A}$ is an *admissible actuality* iff (i) $w = w^*$, and (ii) for every $x \in$ Ind(w^*), $\sigma(x) = x$ (ibid.).

[13] σ being total is required for the version of the semantics Bacon presents: if we allow σ to be partial, then the recursive definition of satisfaction (to be introduced) is incomplete. However, after presenting his semantics (2014: 105, f.n. 12), Bacon does note in passing that we could introduce a notion of a *serious possibility* $< w, \sigma >$, where σ is a (perhaps partial) function σ: D $\rightarrow$ D that is a subset of C, and the range of σ is a subset of Ind(w). To accommodate this notion, the definition of satisfaction would have to be reworked.

The idea with the notion of an admissible actuality is that the way things actually are is represented by a possibility. However, there may be multiple possibilities which represent the ways things actually are. For example, for some x in D not in Ind(w^*), there may be two w^*-counterparts y_1 and y_2 of x. In which case, there is a possibility $< w^*, \sigma_1 >$ such that $\sigma_1(x) = y_1$ (whilst for every $z \in$ Ind(w^*), $\sigma_1(z) = z$), and a possibility $< w^*, \sigma_2 >$ such that $\sigma_2(x) = y_2$ (whilst for every $z \in$ Ind(w^*), $\sigma_2(z) = z$). According to Bacon, both of these represent the way things actually are; they are both admissible actualities.

Let an *assignment function* for a counterpart model $M =$ <W, D, Ind, C, w^*, I> be a function v: VAR $\rightarrow$ D. Further, for assignment v, individuals $d_1, ..., d_n \in$ D, and $x_1, ..., x_n \in$ VAR, let $v_{[d_1,...,d_n/x_1,...,x_n]}$ be the assignment that agrees with v, except that d_1 is assigned to x_1, ..., and d_n is assigned to x_n.[14]

The most basic notion of truth-value is of an L-formula ϕ being true (false) at a model M and admissible actuality $< w^*, \sigma^* >$ (for the structure $\mathfrak{A}$ that M is based on), with respect to a possibility $< w, \sigma >$ (for the structure $\mathfrak{A}$ that M is based on) and valuation v for M: $< M, < w^*, \sigma^* >>, < w, \sigma, v > \models \phi$. This is recursively defined as follows (reference to the model and admissible actuality is omitted for ease of reading) (2014: 103):

(i) $< w, \sigma, v > \models Px_1...x_n$ iff $< \sigma(v(x_1)), ..., \sigma(v(x_n)) > \in$ I(P)(w).

(ii) $< w, \sigma, v > \models \sim\phi$ iff $< w, \sigma, v > \not\models \phi$.

(iii) $< w, \sigma, v > \models (\phi \vee \psi)$ iff $< w, \sigma, v > \models \phi$ or $< w, \sigma, v > \models \psi$.[15]

(iv) $< w, \sigma, v > \models \exists x\phi$ iff for some $d \in$ Ind(w), $< w, \sigma, v_{[d/x]} > \models \phi$.[16]

(v) $< w, \sigma, v > \models \diamond\phi$ iff for some $< w^+, \sigma^+ > \in$ S($\mathfrak{A}$), $< w^+, \sigma^+, v > \models \phi$.

(vi) $< w, \sigma, v > \models \mathbf{A}\phi$ iff $< w^*, \sigma^*, v > \models \phi$.

Though $\forall$ and $\Box$ are not part of the vocabulary of L, they can be introduced as metalinguistic abbreviations in the usual way. Given this, we have:

(vii) $< w, \sigma, v > \models \forall x\phi$ iff for every $d \in$ Ind(w), $< w, \sigma, v_{[d/x]} > \models \phi$.

(viii) $< w, \sigma, v > \models \Box\phi$ iff for every $< w^+, \sigma^+ > \in$ S($\mathfrak{A}$), $< w^+, \sigma^+, v > \models \phi$.

[14]For simplicity, I deviate slightly from Bacon in the notation on valuations, and also in defining them as functions from VAR to D, rather than from $\mathbb{N}$ to D. Nothing turns on this.

[15]Strangely, Bacon explicitly includes $\vee$ in his object language L (and not $\wedge$), but in the recursive definition of satisfaction presents a clause for $\wedge$ (and not $\vee$). The clause I have given is straightforwardly what the $\vee$ one should be.

[16]In virtue of the slight deviations I made concerning valuations, the presentation of this clause slightly deviates from the presentation Bacon gives. Nothing turns on this.

An L-formula ϕ is true at a counterpart model M = <W, D, Ind, C, w^*, I> with respect to an admissible actuality $< w^*, \sigma^* >$ for <W, D, Ind, C, w^* > (the counterpart structure M is based on) and valuation v for M iff $< M, < w^*, \sigma^* >>, < w^*, \sigma^*, v > \models \phi$ (ibid.).

Corresponding to this: an L-formula ϕ is false at a counterpart model M = <W, D, Ind, C, w^*, I> with respect to an admissible actuality $< w^*, \sigma^* >$ for <W, D, Ind, C, w^* > and valuation v for M iff $< M, < w^*, \sigma^* >>, < w^*, \sigma^*, v > \not\models \phi$.

A motivating idea Bacon has is that for a given model M (and valuation v for M), the counterpart structure that M is based on may have multiple admissible actualities, and for some L-formulas ϕ, ϕ may be true at M with respect to one admissible actuality and v, and false at M with respect to another admissible actuality and v. Bacon holds that the correct way to handle this is to supervaluate; he talks of formulas being "determinately" true/false at a model, and "indeterminate" at a model (2014: 99). Although Bacon does not present the following definitions, they straightforwardly capture this motivating idea. For an L-formula ϕ, counterpart model M = <W, D, Ind, C, w^*, I>, and valuation v of M:

> ϕ is true at M with respect to v iff for every admissible actuality s of <W, D, Ind, C, w^* >, ϕ is true at M with respect to s and v.
>
> ϕ is false at M with respect to v iff for every admissible actuality s of <W, D, Ind, C, w^* >, ϕ is false at M with respect to s and v.
>
> Otherwise, ϕ lacks a truth-value at M with respect to v.

Notice that in the recursive definition of $< M, < w^*, \sigma^* >>, < w, \sigma, v > \models \phi$, the admissible actuality $< w^*, \sigma^* >$ is only relevant for the actuality operator clause. As a result, for every L-formula ϕ without an actuality operator occurrence, for every counterpart model M = <W, D, Ind, C, w^*, I>, and valuation v of M, either [for every admissible actuality s of <W, D, Ind, C, w^* >, ϕ is true at M with respect to s and v] or [for every admissible actuality s of <W, D, Ind, C, w^* >, ϕ is false at M with respect to s and v]. As a result, for every L-formula ϕ without an actuality operator occurrence, for every counterpart model M and valuation v of M, either [ϕ is true at M with respect to v] or [ϕ is false at M with respect to v]. Only L-formulas containing actuality operator occurrences may lack a truth-value at a counterpart model with respect to a valuation for the model.

To see the supervaluationist approach in action, let M = <W, D, Ind, C, w^*, I> be a model and v a valuation for M such that:

> W = $\{w, w^*\}$ ($w \neq w^*$).
>
> D = $\{d, d^*\}$ ($d \neq d^*$).

$\mathrm{Ind}(w^*) = \{d^*\}$.

$\mathrm{Ind}(w) = \{d\}$.

$\mathrm{V}(F)(w^*) = \{d^*\}$.

$v(x) = d$.

There are two admissible actualities for the structure M is based on:

$< w^*, \sigma_1 >$: $\sigma_1 = \{< d^*, d^* >, < d, d^* >\}$.

$< w^*, \sigma_2 >$: $\sigma_2 = \{< d^*, d^* >, < d, d >\}$.

We have:

(a) $\mathbf{A}Fx$ lacks a truth-value at M with respect to v.

(b) $\mathbf{A}{\sim}Fx$ lacks a truth-value at M with respect to v.

However:

(c) $\mathbf{A}(Fx \vee {\sim}Fx)$ is true at M with respect to v.

(d) ${\sim}\mathbf{A}(Fx \wedge {\sim}Fx)$ is true at M with respect to v.

Here, I shall just demonstrate (a) and (c).

Proof of (a). $\sigma_1(d) \in \mathrm{I}(F)(w^*)$, but $\sigma_2(d) \notin \mathrm{I}(F)(w^*)$.

$\Downarrow$

$< M, < w^*, \sigma_1 >>, < w^*, \sigma_1, v > \models Fx$, but

$< M, < w^*, \sigma_2 >>, < w^*, \sigma_2, v > \not\models Fx$.

$\Downarrow$

$< M, < w^*, \sigma_1 >>, < w^*, \sigma_1, v > \models \mathbf{A}Fx$, but

$< M, < w^*, \sigma_2 >>, < w^*, \sigma_2, v > \not\models \mathbf{A}Fx$.

$\Downarrow$

$\mathbf{A}Fx$ is true at M with respect to $< w^*, \sigma_1 >$ and v, but

$\mathbf{A}Fx$ is false at M with respect to $< w^*, \sigma_2 >$ and v.

$\Downarrow$

$\mathbf{A}Fx$ lacks a truth-value at M with respect to v. ■

Proof of (c). For *reductio*, let $< w^*, \sigma^* >$ be an admissible actuality of the structure M is based on such that $\mathbf{A}(Fx \vee \sim Fx)$ is false at M with respect to $< w^*, \sigma^* >$ and v.

$< w^*, \sigma^*, v > \not\models \mathbf{A}(Fx \vee \sim Fx)$.

$\Downarrow$

$< w^*, \sigma^*, v > \not\models (Fx \vee \sim Fx)$.

$\Downarrow$

$< w^*, \sigma^*, v > \not\models Fx$, and

$< w^*, \sigma^*, v > \not\models \sim Fx$.

$\Downarrow$

$\sigma^*(d) \notin \mathrm{V}(F)(w^*)$, and

$\sigma^*(d) \in \mathrm{V}(F)(w^*)$. $\perp$ ■

Again, a central motivation of Bacon in employing his supervaluationist approach is to handle certain sentences raised by Hazen 1977, 1979, and Fara & Williamson 2005. For example, reconsider the supposedly intuitively unsatisfiable (18):

(18) $\Diamond \exists x(\mathbf{A}Fx \leftrightarrow \mathbf{A}\sim Fx)$

On Bacon's semantics, (18) is a contradiction (as supposedly required). That is, for every model M and valuation v for M, (18) is false at M with respect to v.

Proof. For *reductio*, let $M =<$W, D, Ind, C, w^*, I$>$ be a model ($<$W, D, Ind, C, $w^* >= \mathfrak{A}$), v a valuation for M, and $< w^*, \sigma^* >$ an admissible actuality for $\mathfrak{A}$ such that (18) is true at M with respect to $< w^*, \sigma^* >$ and v.

$\Downarrow$

$< w^*, \sigma^*, v > \models \Diamond \exists x(\mathbf{A}Fx \leftrightarrow \mathbf{A}\sim Fx)$

$\Downarrow$

For some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$,

$< w, \sigma, v > \models \exists x(\mathbf{A}Fx \leftrightarrow \mathbf{A}\sim Fx)$

$\Downarrow$

For some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$, for some $d \in \mathrm{Ind}(w)$,

$< w, \sigma, v_{[d/x]} > \models \mathbf{A}Fx$ iff

$< w, \sigma, v_{[d/x]} > \models \mathbf{A}\sim Fx$.

$\Downarrow$

For some $< w, \sigma >\in$ S($\mathfrak{A}$), for some $d \in$ Ind(w),

$< w^*, \sigma^*, v_{[d/x]} > \models Fx$ iff

$< w^*, \sigma^*, v_{[d/x]} > \models \sim Fx$.

$\Downarrow$

For some $< w, \sigma >\in$ S($\mathfrak{A}$), for some $d \in$ Ind(w),

$\sigma^*(d) \in$ I(F)(w^*) iff $\sigma^*(d) \notin$ I(F)(w^*). $\bot$ ■

4.3.2 Iterated Modality

So much for an exposition of the account. In response, I want to raise two issues.

The first issue concerns iterated modality and is close to the main issue I raised against Hazen's stipulational semantics. To introduce the point, consider the following variant of (12):

(25) For some individual x, things could have been such that x was F and from there things could have been such that x was G.

(25$^+$) $\exists x \diamond (Fx \wedge \diamond Gx)$

(25) states that some individual x has a certain modal property. Namely, the modal property that it could have had the property [being F and being such that it could have been G]. It does not say that x itself has two separate modal properties: the modal property that it could have had the property [being F] and the modal property that it could have had the property [being G].

Indeed, one might think that there is a flavour of modality such that there are true utterances of (26) where all the modal locutions in (26) are concerned with that flavour.

(26) For some individual x, [things could have been such that x was F and from there things could have been such that x was G], but things could not have been such that x was G.

(26$^+$) $\exists x(\diamond(Fx \wedge \diamond Gx) \wedge \sim \diamond Gx)$

However, Bacon's semantics renders (26$^+$) unsatisfiable.

Proof. For *reductio*, let $M =$ <W, D, Ind, C, w^*, I> be a counterpart model ($\mathfrak{A} =$ <W, D, Ind, C, w^* >), v a valuation for M, and $< w^*, \sigma^* >$ an admissible actuality for $\mathfrak{A}$ such that (26$^+$) is true at M with respect to $< w^*, \sigma^* >$ and v.

$\Downarrow$

$< w^*, \sigma^*, v >\models \exists x(\diamond(Fx \wedge \diamond Gx) \wedge \sim \diamond Gx)$.

$\Downarrow$

For some $d \in \text{Ind}(w^*)$,

(a) $< w^*, \sigma^*, v_{[d/x]} >\models \diamond(Fx \wedge \diamond Gx)$, and

(b) $< w^*, \sigma^*, v_{[d/x]} >\models \sim \diamond Gx$.

$\Downarrow$

For some $d \in \text{Ind}(w^*)$,

(a) for some $< w, \sigma >\in \text{S}(\mathfrak{A})$,

$< w, \sigma, v_{[d/x]} >\models Fx \wedge \diamond Gx$, and

(b) for every $< w^+, \sigma^+ >\in \text{S}(\mathfrak{A})$, $< w^+, \sigma^+, v_{[d/x]} >\not\models Gx$.

$\Downarrow$

For some $d \in \text{Ind}(w^*)$,

(a) for some $< w, \sigma >\in \text{S}(\mathfrak{A})$, for some $< w^+, \sigma^+ >\in \text{S}(\mathfrak{A})$,

$< w^+, \sigma^+, v_{[d/x]} >\models Gx$, and

(b) for every $< w^+, \sigma^+ >\in \text{S}(\mathfrak{A})$, $< w^+, \sigma^+, v_{[d/x]} >\not\models Gx$. $\perp$ ■

The semantics validates T, 4, and B. It is appropriate only when we are concerned with an S5 flavour of modality, one that effectively collapses iterated modality.

Even if one does not think that there is a flavour of modality such that there are true utterances of (26) where all the modal locutions of (26) are concerned with that flavour of modality, we must be able to accommodate sentence utterances involving multiple flavours of modality. For example, consider:

(27) For some individual x, strictly speaking, things could have been such that x was a bumble bee and from there, even merely physically speaking, things could have been such that x was an abnormally large bumble bee.

Again, the first possibility operator occurrence has a rather unrestricted flavour M_1, and the embedded possibility operator occurrence has a much more restricted flavour M_2. It may very well be that x could have (M_1 flavour) had the property [being a bumble bee and being such that he could have (M_2 flavour) been an abnormally large bumble bee]. However, it is presumably not the case that x could have (M_2 flavour) had the property [being an abnormally large bumble bee]. Indeed, it is presumably not the case that x could have (M_2 flavour) had the property [being a bumble bee].

Indeed, it seems that (28) should be true:

(28) For some individual x, [strictly speaking, things could have been such that x was a bumble bee and from there, even merely physically speaking, things could have been such that x was an abnormally large bumble bee], but, physically speaking, x could not have been an abnormally large bumble bee.

In a similar fashion to our discussion of Hazen's semantics, to handle such sentences, we posit two flavours of modality. Our new language L^+ is the same as L, except that the single possibility operator $\diamond$ is replaced with two such operators $\diamond_1$ and $\diamond_2$. (27) and (28) are regimented as follows:

(27$^+$) $\exists x \diamond_1 (Bx \wedge \diamond_2 Lx)$

(28$^+$) $\exists x[\diamond_1(Bx \wedge \diamond_2 Lx) \wedge \sim \diamond_2 Lx]$

A desideratum of Bacon's semantics is that it can be adjusted to render (27$^+$) and (28$^+$) satisfiable. Perhaps a desideratum is also that it can be adjusted to render (26$^+$) satisfiable. However, the worry is that there is no acceptable adjustment. To support this point, I consider two approaches, raising an issue for each.

The first approach I will consider is motivated by the fact that a standard Kripke semantics for L whose L-structures omit accessibility relations (between worlds) also collapses iterated modality and only delivers appropriate truth-values when we are concerned with a flavour of modality that has an S5 character. Nevertheless, that semantics can capture the iterative nature of iterated modality by enriching an L-structure with an accessibility relation. Further, we can extend the semantics to handle L^+ by enriching our L-structures with two accessibility relations (one for each possibility operator).

Applying this idea to the counterpart semantics, we enrich a counterpart structure $\mathfrak{A}$ with an accessibility relation $\mathrm{R} \subseteq \mathrm{S}(\mathfrak{A})^2$, and adjust the possibility operator clause appropriately:

(v') $< w, \sigma, v > \models \diamond\phi$ iff for some $< w^+, \sigma^+ >$ such that
$< w, \sigma >$ R $< w^+, \sigma^+ >$,
$< w^+, \sigma^+, v > \models \phi$.

From a technical perspective, we can now render (26$^+$) satisfiable. Let $M =$ <W, D, Ind, C, w^*, I> be a counterpart model ($\mathfrak{A} =$ <W, D, Ind, C, w^* >), v a valuation for M, and $< w^*, \sigma^* >$ an admissible actuality for $\mathfrak{A}$. We have:

(26$^+$) is true at M with respect to $< w^*, \sigma^* >$ and v.

$\Updownarrow$

$< w^*, \sigma^*, v > \models \exists x(\diamond(Fx \wedge \diamond Gx) \wedge \sim \diamond Gx)$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$,

(a) for some $< w, \sigma >$ such that $< w^*, \sigma^* > \mathrm{R} < w, \sigma >$,

(a.i) $\sigma(d) \in \mathrm{I}(F)(w)$, and

(a.ii) for some $< w^+, \sigma^+ >$ such that $< w, \sigma > \mathrm{R} < w^+, \sigma^+ >$,

$\sigma^+(d) \in \mathrm{I}(G)(w^+)$, and

(b) for every $< w^+, \sigma^+ >$ such that $< w^*, \sigma^* > \mathrm{R} < w^+, \sigma^+ >$,

$\sigma^+(d) \notin \mathrm{I}(G)(w^+)$.

Informally, (26^+) is satisfied at a model with respect to an admissible actuality s iff for some actual individual d, there is (i) an s-accessible possibility p_1 according to which d is F, (ii) a p_1-accessible possibility p_2 according to which d is G, but (iii) no s-accessible possibility according to which d is G.

Turning to L^+, we include two accessibility relations R_1 and R_2 in our structures (one for each flavour of modality), and adjust the possibility operator clause appropriately:

(v") $< w, \sigma, v > \models \diamond_i \phi$ iff for some $< w^+, \sigma^+ >$ such that

$< w, \sigma > \mathrm{R}_i < w^+, \sigma^+ >$,

$< w^+, \sigma^+, v > \models \phi$.

From a technical perspective, we can now render (28^+) satisfiable. Let $M =$ <W, D, Ind, C, w^*, I> be a counterpart model ($\mathfrak{A} =$ <W, D, Ind, C, $w^* >$), v a valuation for M, and $< w^*, \sigma^* >$ an admissible actuality for $\mathfrak{A}$. We have:

(28^+) is true at M with respect to $< w^*, \sigma^* >$ and v.

$\Updownarrow$

$< w^*, \sigma^*, v > \models \exists x(\diamond_1(Bx \wedge \diamond_2 Lx) \wedge \sim \diamond_2 Lx)$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$,

(a) for some $< w, \sigma >$ such that $< w^*, \sigma^* > \mathrm{R}_1 < w, \sigma >$,

(a.i) $\sigma(d) \in \mathrm{I}(B)(w)$, and

(a.ii) for some $< w^+, \sigma^+ >$ such that $< w, \sigma > \mathrm{R}_2 < w^+, \sigma^+ >$,

$\sigma^+(d) \in \mathrm{I}(L)(w^+)$, and

(b) for every $< w^+, \sigma^+ >$ such that $< w^*, \sigma^* > \mathrm{R}_2 < w^+, \sigma^+ >$,

$\sigma^+(d) \notin \mathrm{I}(L)(w^+)$.

Informally, (28^+) is satisfiable at a model with respect to an admissible actuality s iff for some actual individual d, there is (i) a possibility p_1 (R_1 accessible from s) according to which d is B, (ii) a possibility p_2 (R_2 accessible from p_1) according to which d is L, but (iii) no possibility (R_2 accessible from s) according to which d is L .

So much for the candidate adjustment. The worry with this approach is that whilst ostensibly no technical problems arise, it is unclear whether there is an acceptable analysis of the machinery.

To begin, let us focus just on (26). A natural way to handle iterated modality in terms of counterpart machinery is to appeal to iterated counterparts. For example, (26) is true iff for some actual individual d, there is a counterpart d_1 of d that is F and a counterpart d_2 of d_1 that is G, but no counterpart of d itself is G.

Notably, this is not how Bacon's semantics (or the developed version) operates. Because of the clauses for atomics and the possibility operator in the the recursive definition of satisfaction, the semantics unpacks the possibility operator occurrence embedded within the scope of another possibility operator occurrence simply in terms of counterparts (representatives) of d itself. There is never any iteration of counterparts. The additional machinery that allows us to properly respect the iterative nature of iterated modality in (26^+) (on the developed version) is an accessibility relation between possibilities.

However, it is unclear how this additional accessibility relation is to be handled in a metaphysical account. In particular, it is unclear how to provide an adequate analysis of this accessibility relation between possibilities in non-modal terms. For example, in the case of (26^+), if we want to render it true, we need our actual individual d to have a counterpart that is G and for there to be a possibility p where (according to the possibility) d is G. However, in virtue of what is such a possibility accessible from some possibilities and not accessible from others?[17]

A similar point can be made on (28). A natural way to handle (28) in terms of counterparts is: for some actual individual d, there is a counterpart d_1 of d (of a sort corresponding to the first flavour of modality) that is B, and a counterpart d_2 of d_1 (of a sort corresponding to the second flavour of modality) that is L, but d itself has no counterparts (of the sort corresponding to the second flavour of modality) that are L.

Again, this is not how the developed semantics works. Instead, it unpacks the possibility operator occurrence embedded within the scope of another possibility operator occurrence simply in terms of counterparts

[17]Of course, one could hold the counterpart semantics without adopting the metaphysical requirement to take no modal notion as a primitive.

(representatives) of d itself. The developed semantics also employs a single counterpart relation. d is required to have both a counterpart that is B and a counterpart that is L. Then the additional machinery of multiple accessibility relations between possibilities is supposed to allow us to handle iterated modality involving multiple flavours of modality.

Again, it is unclear how to provide an adequate analysis of accessibility between possibilities in non-modal terms. One might also baulk at holding that a single counterpart relation is involved in the handling of sentences such as (28).

Perhaps we can handle this last point by positing multiple counterpart relations. A counterpart structure for L^+ has two counterpart relations C_1 and C_2. This in turn generates two kinds of possibility. For a counterpart structure $\mathfrak{A}$, a C_i-counterpart function σ for a $w \in \mathrm{W}$ at $\mathfrak{A}$ is a function from D to D such that $\sigma \subseteq \mathrm{C}_i$. Similarly, a C_i-possibility for $\mathfrak{A}$ is a pair $< w, \sigma >$ where $w \in \mathrm{W}$ and σ is a C_i-counterpart function. Each accessibility relation is now a binary relation in the set of all possibilities for $\mathfrak{A}$. We say that $< w, \sigma >$ $\mathrm{R}_i < w^+, \sigma^+ >$ only if $< w^+, \sigma^+ >$ is a C_i-possibility.[18]

However, even if we allow this, there is still the issue of providing an adequate analysis of accessibility between possibilities. Notice that we cannot have the opposite direction of the above claim. That is: $< w, \sigma > \mathrm{R}_i < w^+, \sigma^+ >$ if $< w^+, \sigma^+ >$ is a C_i-possibility. Such an account would collapse iterated modality and be unable to render (28^+) satisfiable.

So much for the first approach. To summarise the discussion: the approach is ostensibly acceptable from a technical perspective, but it is unclear whether there is an acceptable analysis of the machinery.

Let us turn to second approach. The motivating idea is that we can handle iterated modality in terms of iterated counterparts by altering the definition of satisfaction.

In particular, the approach is to simplify the atomic clause so that we are concerned directly with images under the relevant valuation function, and then build iteration into our possibility clause through an iterated valuation function:

(i^+) $< w, \sigma, v > \models Px_1...x_n$ iff $< v(x_1), ..., v(x_n) > \in \mathrm{I}(P)(w)$.

(v^+) $< w, \sigma, v > \models \Diamond\phi$ iff for some $< w^+, \sigma^+ > \in \mathrm{S}(\mathfrak{A})$,
$< w^+, \sigma^+, [\sigma^+ \circ v] > \models \phi$.[19]

[18] Here, it seems we should say that an admissible actuality for $\mathfrak{A}$ is any possibility $< w, \sigma >$ for $\mathfrak{A}$ such that $w = w^*$ and for every $x \in \mathrm{Ind}(w^*)$, $\sigma(x) = x$.

[19] $[\sigma^+ \circ v](x) = \sigma^+(v(x))$.

This now seemingly allows us to properly handle the iteration in (26^{+}).[20] To see this, let $M = <$W, D, Ind, C, w^{*}, I$>$ ($\mathfrak{A} = <$W, D, Ind, C, $w^{*}>$) be a counterpart model, v a valuation for M, and $< w^{*}, \sigma^{*} >$ an admissible actuality for $<$W, D, Ind, C, $w^{*}>$. Then we have:

(26^{+}) is true at M with respect to $< w^{*}, \sigma^{*} >$ and v.

$\Updownarrow$

$< w^{*}, \sigma^{*}, v > \models \exists x(\Diamond(Fx \wedge \Diamond Gx) \wedge \sim \Diamond\, Gx)$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^{*})$,

(a) $< w^{*}, \sigma^{*}, v_{[d/x]} > \models \Diamond(Fx \wedge \Diamond Gx)$, and

(b) $< w^{*}, \sigma^{*}, v_{[d/x]} > \models \sim \Diamond\, Gx$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^{*})$,

(a) for some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$,

$< w, \sigma, [\sigma \circ v_{[d/x]}] > \models Fx \wedge \Diamond Gx$, and

(b) for every $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$,

$< w, \sigma, [\sigma \circ v_{[d/x]}] > \not\models Gx$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^{*})$,

(a) for some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$,

(a.i) $< w, \sigma, [\sigma \circ v_{[d/x]}] > \models Fx$, and

(a.ii) for some $< w^{+}, \sigma^{+} > \in \mathrm{S}(\mathfrak{A})$,

$< w^{+}, \sigma^{+}, [\sigma^{+} \circ [\sigma \circ v_{[d/x]}]] > \models Gx$, and

(b) for every $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$,

$< w, \sigma, [\sigma \circ v_{[d/x]}] > \not\models Gx$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^{*})$,

(a) for some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$,

(a.i) $\sigma(d) \in \mathrm{I}(F)(w)$, and

(a.ii) for some $< w^{+}, \sigma^{+} > \in \mathrm{S}(\mathfrak{A})$,

[20] Though we now have a more complex handling of the modal operators (the operators are no longer unpacked in terms of quantification over possibilities and what holds at a possibility).

$\sigma^+(\sigma(d)) \in \mathrm{I}(G)(w^+)$, and

(b) for every $< w, \sigma >\in \mathrm{S}(\mathfrak{A})$,

$\sigma(d) \notin \mathrm{I}(G)(w)$.

This ostensibly handles the iterated modality in (26^+) in an appropriate way. However, with this new approach (iterating valuations and a simple atomic clause), an issue for the actuality operator arises. First, what should our clause for the actuality operator be? Suppose we kept the original:

(vi) $< w, \sigma, v >\models \mathbf{A}\phi$ iff $< w^*, \sigma^*, v >\models \phi$.

Given the current set-up, this is inappropriate. For a formula with an actuality operator occurrence embedded in a possibility operator occurrence, we would first handle the possibility operator occurrence by considering an iterated valuation function, and then handle the actuality operator occurrence by considering the same valuation function (not one suitably iterated to account for the admissible actuality). Given the simplification of the atomic clause, this results in a straightforward mishandling of certain formulas. For example, consider:

(29) $\exists x(Fx \wedge \Diamond\mathbf{A}{\sim}Fx)$

To see the mishandling of (29), let $M =$ <W, D, Ind, C, w^*, I> be a counterpart model ($\mathfrak{A} =$ <W, D, Ind, C, w^* >), v a valuation for M, and $< w^*, \sigma^* >$ an admissible actuality for <W, D, Ind, C, w^* >. Then on this approach we have:

(29) is true at M with respect to $< w^*, \sigma^* >$ and v.

$\Updownarrow$

$< w^*, \sigma^*, v >\models \exists x(Fx \wedge \Diamond\mathbf{A}{\sim}Fx)$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$,

(a) $d \in \mathrm{I}(F)(w^*)$, and

(b) $< w^*, \sigma^*, v_{[d/x]} >\models \Diamond\mathbf{A}{\sim}Fx$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$,

(a) $d \in \mathrm{I}(F)(w^*)$, and

(b) for some $< w, \sigma >\in \mathrm{S}(\mathfrak{A})$,

$< w, \sigma, [\sigma \circ v_{[d/x]}] >\models \mathbf{A}{\sim}Fx$.

$\Updownarrow$

For some $d \in \text{Ind}(w^*)$,

(a) $d \in \text{I}(F)(w^*)$, and

(b) for some $< w, \sigma > \in \text{S}(\mathfrak{A})$,

$< w^*, \sigma^*, [\sigma \circ v_{[d/x]}] > \models \sim Fx$.

$\Updownarrow$

For some $d \in \text{Ind}(w^*)$,

(a) $d \in \text{I}(F)(w^*)$, and

(b) for some $< w, \sigma > \in \text{S}(\mathfrak{A})$,

$\sigma(d) \notin \text{I}(F)(w^*)$.

On this bizarre handling, the actuality setting is unpacked in terms of what holds at w^* of d's representative according to possibility $< w, \sigma >$. We do not even unpack the actuality setting in terms of a representative according to the admissible actuality. As a result, for some counterpart models M, admissible actualities s for the counterpart structure M is based on, and valuations v for M, (29) is true at M with respect to s and v. Stronger still, for some counterpart models M and valuations v for M, (29) is true at M with respect to v.

Proof. Let $M = <\text{W, D, Ind, C, } w^*\text{, I}>$ be a counterpart model (based on counterpart structure $\mathfrak{A}$) such that:

$\text{W} = \{w^*, w\}$.

$\text{D} = \{d_1, d_2\}$.

$\text{Ind}(w^*) = \{d_1\}$.

$\text{Ind}(w) = \{d_2\}$.

$\text{C} = \{< d_1, d_1 >, < d_2, d_2 >, < d_1, d_2 >\}$.

$\text{I}(F)(w^*) = \{d_1\}$.

Then there is a single admissible actuality for $\mathfrak{A}$: $< w^*, \sigma_1 >$, where

$\sigma_1 = \{< d_1, d_1 >, < d_2, d_2 >\}$.

We also have $< w, \sigma_2 > \in \text{S}(\mathfrak{A})$, where

$\sigma_2 = \{< d_1, d_2 >, < d_2, d_2 >\}$.

Now, $d_1 \in \text{I}(F)(w^*)$, but $\sigma_2(d_1) \notin \text{I}(F)(w^*)$. So, for any valuation v for M, (29) is true at M with respect to $< w^*, \sigma_1 >$ and v. Hence, for any valuation v, (29) is true at M with respect to v. ■

However, it seems an uncontroversial desideratum that for any counterpart model M and valuation v for M, (29) should not be true at M with respect to v.

Given this, one might think to alter the actuality operator clause so that when we consider whether $\mathbf{A}\phi$ is true with respect to a possibility and valuation, we consider whether ϕ is true with respect to the admissible actuality s and a suitably iterated valuation that accounts for s:

(vi$^+$) $< w, \sigma, v > \models \mathbf{A}\phi$ iff $< w^*, \sigma^*, [\sigma^* \circ v] > \models \phi$.

However, because of this iteration, the resultant account also mishandles (29). To see this, let $M =$ <W, D, Ind, C, w^*, I> be a counterpart model ($\mathfrak{A} =$ <W, D, Ind, C, w^* >), v a valuation for M, and $< w^*, \sigma^* >$ an admissible actuality for <W, D, Ind, C, w^* >. Then we have:

(29) is true at M with respect to $< w^*, \sigma^* >$ and v.

$\Updownarrow$

$< w^*, \sigma^*, v > \models \exists x(Fx \wedge \Diamond \mathbf{A} \sim Fx)$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$,

(a) $d \in \mathrm{I}(F)(w^*)$, and

(b) for some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$,

$< w, \sigma, [\sigma \circ v_{[d/x]}] > \models \mathbf{A} \sim Fx$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$,

(a) $d \in \mathrm{I}(F)(w^*)$, and

(b) for some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$,

$< w^*, \sigma^*, [\sigma^* \circ [\sigma \circ v_{[d/x]}] > \models \sim Fx$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$,

(a) $d \in \mathrm{I}(F)(w^*)$, and

(b) for some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$,

$\sigma^*(\sigma(d)) \notin \mathrm{I}(F)(w^*)$.

The issue here is that $\sigma^*(\sigma(d))$ need not be d. As a result, for some counterpart models M, admissible actualities s for the counterpart structure M is based on, and valuations v for M, (29) is true at M with respect to s and v. Stronger still, for some counterpart models M and valuations v for M, (29) is true at M with respect to v.

Proof. Let $M = <$W, D, Ind, C, w^*, I$>$ be a counterpart model (based on counterpart structure $\mathfrak{A}$) such that:

$\mathrm{W} = \{w^*, w\}$.

$\mathrm{D} = \{d_1, d_2, d_3\}$.

$\mathrm{Ind}(w^*) = \{d_1, d_2\}$.

$\mathrm{Ind}(w) = \{d_3\}$.

$\mathrm{C} =$
$\{< d_1, d_1 >, < d_2, d_2 >, < d_3, d_3 >, < d_1, d_3 >, < d_2, d_3 >, < d_3, d_2 >\}$.

$\mathrm{I}(F)(w^*) = \{d_1\}$.

Then there are two admissible actualities for $\mathfrak{A}$: $< w^*, \sigma_1 >$ and $< w^*, \sigma_2 >$, where

$\sigma_1 = \{< d_1, d_1 >, < d_2, d_2 >, < d_3, d_3 >\}$, and

$\sigma_2 = \{< d_1, d_1 >, < d_2, d_2 >, < d_3, d_2 >\}$.

We have $< w, \sigma_3 > \in \mathrm{S}(\mathfrak{A})$, where

$\sigma_3 = \{< d_1, d_3 >, < d_2, d_3 >, < d_3, d_3 >\}$.

Now, $d_1 \in \mathrm{I}(F)(w^*)$, but (i) $\sigma_1(\sigma_3(d_1)) \notin \mathrm{I}(F)(w^*)$, and (ii) $\sigma_2(\sigma_3(d_1)) \notin \mathrm{I}(F)(w^*)$. For any valuation v for M, (i) (29) is true at M with respect to $< w^*, \sigma_1 >$ and v, and (ii) (29) is true at M with respect to $< w^*, \sigma_2 >$ and v. Hence, for any valuation v, (29) is true at M with respect to v. ■

However, again, it seems an uncontroversial desideratum that for any counterpart model M and valuation v for M, (29) should not be true at M with respect to v.

So much for the second approach (simplifying the atomic clause and building iteration into our modal operator clauses through an iterated valuation function). It seems that this fails from a technical perspective.

To summarise this section, a desideratum of Bacon's semantics is that it can be adjusted to render (27^+) and (28^+) satisfiable. Perhaps a desideratum is also that it can be adjusted to render (26^+) satisfiable. However, the worry is that no acceptable adjustment is available. To support this point, I considered two approaches, raising an issue for each.

4.3.3 Actuality

This brings me to my second overall issue with Bacon's initial semantics: it ostensibly inappropriately handles the actuality operator. Recall that it seems that we should hold the following. A possibility/necessity setting amounts to quantification over further possibilities (further iterations). In contrast, an actuality setting does not involve further iterations, but, in some sense, amounts to a *return*.

It seems to me that Bacon's account does not fully capture this. Although (a) an actuality operator occurrence is unpacked as a return to the admissible actuality, and then (b) we have the requirement that each actual individual represents itself (according to the admissible actuality), because of the atomic clause, we are nevertheless still concerned with further representatives (counterparts):

(i) $< w, \sigma, v > \models Px_1...x_n$ iff $< \sigma(v(x_1)), ..., \sigma(v(x_n)) > \in \mathrm{I}(P)(w)$.

(vi) $< w, \sigma, v > \models \mathbf{A}\phi$ iff $< w^*, \sigma^*, v > \models \phi$.

Given (a) and (b), the initial semantics avoids seriously mishandling certain simple L-formulas with an actuality operator occurrence. For example, for every counterpart model M and valuation v for M, (29) is false at M with respect to v.

Proof. For *reductio*, let $M =$ <W, D, Ind, C, w^*, I> be a counterpart model ($\mathfrak{A} =$ <W, D, Ind, C, w^* >), v a valuation for M, and $< w^*, \sigma^* >$ an admissible actuality for $\mathfrak{A}$ such that (29) is true at M with respect to $< w^*, \sigma^* >$ and v.

$\Downarrow$

$< w^*, \sigma^*, v > \models \exists x(Fx \wedge \diamond \mathbf{A} \sim Fx)$.

$\Downarrow$

For some $d \in \mathrm{Ind}(w^*)$, $d \in \mathrm{I}(F)(w^*)$, and

for some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$, $< w, \sigma, v_{[d/x]} > \models \mathbf{A} \sim Fx$.

$\Downarrow$

For some $d \in \mathrm{Ind}(w^*)$, $d \in \mathrm{I}(F)(w^*)$, and

for some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$, $< w^*, \sigma^*, v_{[d/x]} > \models \sim Fx$.

$\Downarrow$

For some $d \in \mathrm{Ind}(w^*)$, $d \in \mathrm{I}(F)(w^*)$, and

for some $< w, \sigma > \in \mathrm{S}(\mathfrak{A})$, $\sigma^*(d) \notin \mathrm{I}(F)(w^*)$.

$\Downarrow$

For some $d \in \mathrm{Ind}(w^*)$, $d \in \mathrm{I}(F)(w^*)$ and $d \notin \mathrm{I}(F)(w^*)$. $\perp$ ■

Given (a) and (b) of the initial semantics, it is ensured that when we unpack the actuality setting, we are here concerned with d itself.

However, issues with the initial semantics arise if we develop it to accommodate an extension of L with identity. Bacon considers a language L^+ that extends L by exactly including the predicates $=$ and $\simeq$, which he glosses as "strict" and "loose" identity, respectively. He develops his semantics to work for L^+ by providing two additional clauses for the recursive definition of satisfaction (2014: 108):

(vii) $< w, \sigma, v > \models x_1 = x_2$ iff $v(x_1) = v(x_2)$.

(viii) $< w, \sigma, v > \models x_1 \simeq x_2$ iff $\sigma(v(x_1)) = \sigma(v(x_2))$.

The first note is that it seems to me quite suspicious to even distinguish between two notions of identity here. It does not seem as if there is a separate notion of "loose" identity that features in natural language modal talk. Further, if we hold that two individuals d_1 and d_2 together satisfy a relation R expressible by an extralogical predicate constant at a possibility $< w, \sigma >$ iff $\sigma(d_1)$ and $\sigma(d_2)$ together satisfy R at w, then it seems natural to extend this treatment to identity. In which case, we would hold that d_1 and d_2 together satisfy the single relation [being identical] at $< w, \sigma >$ iff $\sigma(d_1)$ and $\sigma(d_2)$ satisfy [being identical] at w.

Consider:

(30) For some individual x, x is F and things could have been such that for some individual y, x and y are identical and actually y is not F.

Ideally, it seems to me that we should translate this with the standard identity predicate:

(30$^+$) $\exists x(Fx \wedge \Diamond \exists y(x = y \wedge \mathbf{A}{\sim}Fy))$

And, being uniform about the notion of something holding of individuals at a possibility, we hold:

(vii*) $< w, \sigma, v > \models x_1 = x_2$ iff $\sigma(v(x_1)) = \sigma(v(x_2))$.

If we did that, however, the fact that an actuality operator occurrence is unpacked in terms of further representatives (counterparts) causes trouble. To see this, let $M =$ <W, D, Ind, C, w^*, I> be a counterpart model ($\mathfrak{A} =$ <W, D, Ind, C, w^* >), v a valuation for M, and $< w^*, \sigma^* >$ an admissible actuality for $\mathfrak{A}$. Then we have:

(30$^+$) is true at M with respect to $< w^*, \sigma^* >$ and v.

$\Updownarrow$

$< w^*, \sigma^*, v >\models \exists x(Fx \wedge \Diamond\exists y(x = y \wedge \mathbf{A}{\sim}Fy))$

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$, $d \in \mathrm{I}(F)(w^*)$, and

for some $< w, \sigma >\in \mathrm{S}(\mathfrak{A})$,

$< w, \sigma, v_{[d/x]} >\models \exists y(x = y \wedge \mathbf{A}{\sim}Fy)$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$, $d \in \mathrm{I}(F)(w^*)$, and

for some $< w, \sigma >\in \mathrm{S}(\mathfrak{A})$, for some $d^+ \in \mathrm{Ind}(w)$,

$< w, \sigma, v_{[d,d^+/x,y]} >\models (x = y \wedge \mathbf{A}{\sim}Fy)$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$, $d \in \mathrm{I}(F)(w^*)$, and

for some $< w, \sigma >\in \mathrm{S}(\mathfrak{A})$, for some $d^+ \in \mathrm{Ind}(w)$,

(a) $\sigma(d) = \sigma(d^+)$, and

(b) $< w^*, \sigma^*, v_{[d,d^+/x,y]} >\models {\sim}Fy$.

$\Updownarrow$

For some $d \in \mathrm{Ind}(w^*)$, $d \in \mathrm{I}(F)(w^*)$, and

for some $< w, \sigma >\in \mathrm{S}(\mathfrak{A})$, for some $d^+ \in \mathrm{Ind}(w)$,

(a) $\sigma(d) = \sigma(d^+)$, and

(b) $\sigma^*(d^+) \notin \mathrm{I}(F)(w^*)$.

The issue here is that $\sigma^*(d^+)$ need not be d. As a result, for some counterpart models M, admissible actualities s for the counterpart structure M is based on, and valuations v for M, (30^+) is true at M with respect to s and v. Stronger still, for some counterpart models M and valuations v for M, (30^+) is true at M with respect to v.

Proof. Let $M =$ <W, D, Ind, C, w^*, I> be a counterpart model (based on counterpart structure $\mathfrak{A}$) such that:

W $= \{w^*, w\}$.

D $= \{d_1, d_2, d_3\}$.

$\mathrm{Ind}(w^*) = \{d_1, d_2\}$.

$\mathrm{Ind}(w) = \{d_3\}$.

C $=$
$\{< d_1, d_1 >, < d_2, d_2 >, < d_3, d_3 >, < d_1, d_3 >, < d_2, d_3 >, < d_3, d_2 >\}$.

$\mathrm{I}(F)(w^*) = \{d_1\}$.

Then there are two admissible actualities for $\mathfrak{A}$: $< w^*, \sigma_1 >$ and $< w^*, \sigma_2 >$, where

$\sigma_1 = \{< d_1, d_1 >, < d_2, d_2 >, < d_3, d_3 >\}$, and

$\sigma_2 = \{< d_1, d_1 >, < d_2, d_2 >, < d_3, d_2 >\}$.

Further, $< w, \sigma_3 > \in \mathrm{S}(\mathfrak{A})$, where

$\sigma_3 = \{< d_1, d_3 >, < d_2, d_3 >, < d_3, d_3 >\}$.

Now, $d_1 \in \mathrm{I}(F)(w^*)$, $\sigma_3(d_1) = \sigma_3(d_3)$, (i) $\sigma_1(d_3) \notin \mathrm{I}(F)(w^*)$, and (ii) $\sigma_2(d_3) \notin \mathrm{I}(F)(w^*)$. Given this, for any valuation v for M, (i) (30^+) is true at M with respect to $< w^*, \sigma_1 >$ and v, and (ii) (30^+) is true at M with respect to $< w^*, \sigma_2 >$ and v. Hence, for any valuation v for M, (30^+) is true at M with respect to v. ■

However, it seems to me that for any counterpart model M and valuation v for M, (30^+) should at least not be true at M with respect to v.[21]

To summarise this section, Bacon's semantics ostensibly inappropriately handles the actuality operator.

4.4 Graff Fara's Relative-Sameness Semantics

Graff Fara 2008 presents *relative-sameness* semantics, a semantics for a certain formal first-order modal and temporal language with the explicit aim of being able to render true certain contingent/temporary identity/distinctness claims (relative to certain contexts). For example:

(CI) $x = y \wedge \Diamond \sim x = y$

x and y are identical, but could have been distinct.

(TDF) $\sim x = y \wedge \mathbf{F}x = y$

x and y are distinct, but they will be identical.

Why adopt this aim? Like other accounts that allow for certain contingent identity statements to be true, such as Lewis 1971 and Gibbard 1975, the need for an account with such a feature comes from holding a certain identity thesis. In Graff Fara's case (2008: 168):

[21] Perhaps we can allow for it to lack a truth-value in certain circumstances.

> "I think that material objects are (actually now) identical to the matter that (actually now) constitutes them. And I believe that this holds no matter which world and time are the actual and current ones. People are identical to their bodies, which are in turn identical to the flesh, skin, and bones, etc., that make up those bodies."

The complication with such a thesis is that she also holds that the material constitution of a human body need be neither essential to it nor permanent. For example, (actually now) there is a human body and portion of matter such that: (i) (actually now) the body is constituted by the portion, and (ii) it could have been (now) that the body was not constituted by the portion.

Given Graff Fara's identity thesis, we seemingly have: (i) (actually now) the portion and body are identical, and (ii) it could have been (now) that the portion and body were distinct.

Let us assign proper names to our two objects in the spirit of Gibbard 1975: call the body 'Bod' and the portion 'Port'. Then we have a simple contingent identity statement that is seemingly true given Graff Fara's theses:

(31) Bod and Port are identical, but could have been distinct.

As we shall see, Graff Fara's semantics is able to render true certain contingent/temporary identity/distinctness claims (relative to certain contexts) by abandoning a straightforward handling of *de re* modal/temporal claims in terms of identity across worlds and times. Instead, such a claim is handled in terms of her *relative-sameness* relations (which need not be the identity relation), with the relevant relative-sameness relations in play determined by context and individual terms occurring in the claim.[22]

Unfortunately, there is a problem for the semantics concerning iterated modality/temporality, actuality/presentiality, and existence. Here, I present that problem, as well as a related feature that some may find problematic. I also note an issue with modal flavour.

4.4.1 The Semantics

Graff Fara's relative-sameness semantics is for a standard FOM language L (without $\vee, \rightarrow, \leftrightarrow, \exists, \Diamond$, though these are allowed as meta-linguistic

[22]The approach is extremely close to Lewis' 1971 developed version of his 1968 counterpart account, which is able to render true certain contingent identity/distinctness claims. For our more technical focus, the main differences between the accounts are: (i) Lewis' 1971 counterpart relations are not as restricted as Graff Fara's relative-sameness relations (as a result, the formal features of the accounts come apart); (ii) Lewis' account does not cover temporal operators; and (iii) Lewis' account does not explicitly cover an actuality operator. Graff Fara's approach is also close to Gibbard 1975.

abbreviations defined in the usual way, and without individual constants) enriched with the past, future, and now temporal operators **P**, **F**, and **N**, respectively.

A model M for L is a tuple <D, W, T, V>, where the following holds (2008: 170):

(i) D is a non-empty set.[23]

(ii) W is a non-empty set.

(iii) T $\subseteq \mathbb{R}$ and is non-empty.

(iv) V is a function that assigns to each $< w, t > \in$ W $\times$ T a function V_{wt} that, for every $n \geq 0$, assigns to each n-ary (extralogical) predicate constant a subset of D^n.

These four elements represent, respectively: the single domain of individuals, a set of worlds, a set of times, and an interpretation of the (extralogical) predicate constants.

Next, central to the semantics is the notion of a *relative-sameness* relation, a binary relation of sameness relativised to a sort of item. For example of such relations: [is the same person as] and [is the same statue as].[24] Graff Fara takes relative-sameness relations R to have the following properties (ibid.):

(a) R is a "weak equivalence relation" on D $\times$ W $\times$ T.[25]

(b) If $< d, w, t >$R$< d^*, w^*, t^* >$ and $< d, w, t >$R$< d^{**}, w^*, t^* >$, then $d^* = d^{**}$.

From (b) and the symmetry of R, we also have that if $< d^*, w^*, t^* >$R$< d, w, t >$ and $< d^{**}, w^*, t^* >$R$< d, w, t >$, then $d^* = d^{**}$. Given (b) and this, we have that, as it were, R is injectively functional on its first argument.

Crucially, Graff Fara holds that for an individual d, worlds w and w^*, times t and t^*, and sort S, if $w \neq w^*$ or $t \neq t^*$, then d (at wt) may not be the same S as d (at w^*t^*), despite d being identical with itself (2008: 168-169).

[23] And D $\notin$ D (2008: 172).

[24] Graff Fara in fact allows for another kind of relative-sameness relation: a binary relation of sameness relativised to a quality (2008: 169). For example of such relations: [is the same colour as] and [is the same height as]. However, she immediately restricts her attention to relative-sameness relations that are binary relations of sameness relativised to a sort of item. So, I shall follow suit.

[25] According to Graff Fara, a binary relation is a "weak equivalence relation" iff it is (i) symmetric, (ii) transitive, and (iii) "weakly reflexive", where a relation R is weakly reflexive iff every item that bears R to something bears R to itself (2008: 169). Notice that if a binary relation is symmetric and transitive, then it is a weak equivalence relation.

Next, a context c for a model M is a tuple $< w_c, t_c, \mathrm{I}_c >$, where the following holds (2008: 171):[26]

(i) $w_c \in$ W.

(ii) $t_c \in$ T.

(iii) I_c is a function that assigns to each variable x a *relative-sameness relation* R^x_c.

w_c and t_c are the world and time of c, respectively. From (iii), notice that a relative-sameness relation is determined by a variable and context together.

For a variable x and context c, there corresponds a three-place *rolemate function* f^x_c which takes as input a triple $< d, w, t > \in$ [D $\cup\{$D$\}$] $\times$ W $\times$ T, a world w^*, and a time t^*. The image under the function is the rolemate of $< d, w, t >$ at $< w^*, t^* >$ based on R^x_c. Let d_{wt} abbreviate $< d, w, t >$. Then we define the function as follows (2008: 171):

$$f^x_c(d_{wt}, w^*, t^*) = \begin{cases} d^* \text{ if } < d, w, t > \mathrm{R}^x_c < d^*, w^*, t^* > \\ \mathrm{D} \text{ if there is no such } d^*. \end{cases}$$

Here, D functions as a non-individual in the sense that it is not a member of D. To stress, Graff Fara allows that for a context c, variable x, $w, w^* \in$ W, and $t, t^* \in$ T, (assuming $w \neq w^*$ or $t \neq t^*$) it may that $f^x_c(d_{wt}, w^*, t^*) \notin$ D (it is not an individual). The motivating idea here is that it may be that at w^*t^*, no individual is the R^x_c same individual as d (at wt).[27]

Note that for every variable x, context c, all $w, w^* \in$ W, and $t, t^* \in$ T, $f^x_c(\mathrm{D}_{wt}, w^*, t^*)$ is defined and is D itself. As Graff Fara herself notes: "Because we need to allow for indefinite nestings of modal and tense operators, each f^x_c needs to be defined for $< d, w, t >$ [the first argument of the function] not only when $d \in$ D but also when $d =$ D. According to the definitions in D3, the value of the function is D in such cases" (2008: 172, f.n. 17).

Let VAR $= \{v_1, v_2, ...\}$. An *assignment* for a model $M =$ <D, W, T, V> is a total function g: VAR $\rightarrow$ D $\cup$ $\{$D$\}$ such for that for some $x \in$ VAR, $g(x) \in$ D. So, an assignment maps each variable to either an individual or the non-individual D.[28]

[26] Here, I make a slight adjustment to Graff Fara's presentation to straightforwardly identify a context as a triple. Nothing turns on this.

[27] An alternative technical approach that captures this idea would have been to restrict the range of f^x_c to D, but allow it to be partial. However, Graff Fara chooses her approach to allow for simpler clauses for the modal and temporal operators in the recursive definition of satisfaction (2008: 172).

[28] The technical approach of taking the range of an assignment to be (a subset of) D$\cup\{$D$\}$ is employed to straightforwardly accommodate the technical approach of taking the range of a rolemate function to be (a subset of) D$\cup\{$D$\}$. However, note the additional requirement:

Some notation: for assignment g, $d_1, ..., d_n \in$ D, and $x_1, ..., x_n \in$ VAR, $g_{[d_1,...,d_n/x_1,...,x_n]}$ is the assignment that agrees with g, except that d_1 is assigned to x_1, ..., and d_n is assigned to x_n (2008: 172).

The most basic notion of truth-value is of an L-formula ϕ being true (false) relative to a model M, context c, and assignment g, at a world w and time t: $\models^{Mcg}_{wt} \phi$ (ibid.).

We recursively define $\models^{Mcg}_{wt} \phi$ as follows (2008: 173-174):

(i) $\models^{Mcg}_{wt} x = y$ iff $g(x) = g(y)$.

(ii) $\models^{Mcg}_{wt} Px_1...x_n$ iff $< g(x_1), ..., g(x_n) > \in \mathrm{V}_{wt}(P)$.

(iii) $\models^{Mcg}_{wt} \sim\phi$ iff $\not\models^{Mcg}_{wt} \phi$.

(iv) $\models^{Mcg}_{wt} (\phi \wedge \psi)$ iff $\models^{Mcg}_{wt} \phi$ and $\models^{Mcg}_{wt} \psi$.

(v) $\models^{Mcg}_{wt} \forall x\phi$ iff for all $d \in$ D, $\models^{Mcg_{[d/x]}}_{wt} \phi$.

(vi) $\models^{Mcg}_{wt} \Box\phi$ iff for every $w^* \in$ W,
$\models^{Mcg_{[f_c^{x_1}(g(x_1)_{wt},w^*,t),...,f_c^{x_n}(g(x_n)_{wt},w^*,t)/x_1,...,x_n]}}_{w^*t} \phi$,
where $x_1, ..., x_n$ are exactly the distinct (from each other) variables occurring free in ϕ.

(vii) $\models^{Mcg}_{wt} \mathbf{A}\phi$ iff $\models^{Mcg_{[f_c^{x_1}(g(x_1)_{wt},w_c,t),...,f_c^{x_n}(g(x_n)_{wt},w_c,t)/x_1,...,x_n]}}_{wt} \phi$,
where $x_1, ..., x_n$ are exactly the distinct (from each other) variables occurring free in ϕ.

(viii) $\models^{Mcg}_{wt} \mathbf{P}\phi$ iff for some $t^* \in$ T: $t^* < t$,
$\models^{Mcg_{[f_c^{x_1}(g(x_1)_{wt},w,t^*),...,f_c^{x_n}(g(x_n)_{wt},w,t^*)/x_1,...,x_n]}}_{wt^*} \phi$,
where $x_1, ..., x_n$ are exactly the distinct (from each other) variables occurring free in ϕ.

(ix) $\models^{Mcg}_{wt} \mathbf{F}\phi$ iff for some $t^* \in$ T: $t^* > t$,
$\models^{Mcg_{[f_c^{x_1}(g(x_1)_{wt},w,t^*),...,f_c^{x_n}(g(x_n)_{wt},w,t^*)/x_1,...,x_n]}}_{wt^*} \phi$,
where $x_1, ..., x_n$ are exactly the distinct (from each other) variables occurring free in ϕ.

---

for every model M and assignment g, for some $x \in$ VAR, $g(x) \in$ D. What is the motivation for this requirement? Graff Fara states that when the requirement is not in place, "we have the following problems: $\forall x\phi \not\models \exists x\phi$ and $\not\models \exists x Ex$" (2008: 185). Here, E is understood as the usual metalinguistic abbreviation: $Ex =_{df} \exists y\, x = y$. However, this does not seem to be correct. Given Graff Fara's definition of satisfaction (to be shortly introduced) and the requirement that D is non-empty, both of these consequences hold regardless of whether the additional requirement on variable assignments holds.

(x) $\models^{Mcg}_{wt} \mathbf{N}\phi$ iff $\models^{Mcg_{[f_c^{x_1}(g(x_1)_{wt},w,t_c),\ldots,f_c^{x_n}(g(x_n)_{wt},w,t_c)/x_1,\ldots,x_n]}}_{wt_c} \phi$,

where $x_1, ..., x_n$ are exactly the distinct (from each other) variables occurring free in ϕ.

Though $\exists$ and $\diamond$ are not part of the vocabulary of L, they can be introduced as metalinguistic abbreviations in the usual way.[29] Given this, we have:

(xi) $\models^{Mcg}_{wt} \exists x\phi$ iff for some $d \in$ D, $\models^{Mcg_{[d/x]}}_{wt} \phi$.

(xii) $\models^{Mcg}_{wt} \diamond\phi$ iff for some $w^* \in$ W,
$\models^{Mcg_{[f_c^{x_1}(g(x_1)_{wt},w^*,t),\ldots,f_c^{x_n}(g(x_n)_{wt},w^*,t)/x_1,\ldots,x_n]}}_{w^*t} \phi$,

where $x_1, ..., x_n$ are exactly the distinct (from each other) variables occurring free in ϕ.

Finally, L-formula ϕ is true at a model M and context c for M *simpliciter* ($\models^{Mc} \phi$) iff for every assignment g for M, $\models^{Mcg}_{w_c t_c} \phi$.

So much for an exposition of the technical machinery.

Note that given clauses (i) and (ii), we have the notion of property ϕ holding of individuals $d_1, ..., d_n$ at a world-time point wt. This amounts to $d_1, ..., d_n$ (which are in the domain of wt) satisfying ϕ at wt. Given this, a straightforward approach to handling the modal operators would have been to employ this notion. That is:

(vi*) $\models^{Mcg}_{wt} \Box\phi$ iff for every $w^* \in$ W, $\models^{Mcg}_{w^*t} \phi$.

(vii*) $\models^{Mcg}_{wt} \mathbf{A}\phi$ iff $\models^{Mcg}_{w_c t} \phi$.

That is, an individual d is necessarily ϕ (at wt) iff for every world w^*, d itself is ϕ at w^*t. An individual d is actually ϕ (at wt) iff d itself is ϕ at $w_c t$.

Instead, Graff Fara chooses to employ her relative-sameness relations. An individual d (relative to sort S) is necessarily ϕ (at wt) iff for every world w^*, at w^*t, the same S as d_{wt} is ϕ. An individual d (relative to sort S) is actually ϕ (at wt) iff at $w_c t$, the same S as d_{wt} is ϕ.

Suppose that for any sort S, individuals d and d^*, and world-time points wt and w^*t^*, d_{wt} is the same S as $d^*_{w^*t^*}$ iff $d = d^*$. Then Graff Fara's position collapses into the more straightforward position. The richness of Graff Fara's semantics comes from abandoning this. The same point (suitably translated) holds for the temporal operators.

Again, the central motivation of Graff Fara to move away from the above straightforward position is to render certain contingent/temporary identity/distinctness claims true (2008: 168). For example, consider:

[29] Other temporal operators can also be introduced as metalinguistic abbreviations.

(CI) $x = y \wedge \Diamond\sim x = y$

First, suppose we adopt the straightforward position I described above, substituting (vi*) and (vii*) in for (vi) and (vii). Then for every model M, context c of M, assignment g for M, $w \in$ W, and $t \in$ T, $\not\models^{Mcg}_{wt}$ (CI).

Proof. Let M be a model, c a context for M, g an assignment for M, $w \in$ W, and $t \in$ T. For *reductio*, suppose that $\models^{Mcg}_{wt}$ (CI).

$\Downarrow$

(i) $\models^{Mcg}_{wt} x = y$ and (ii) $\models^{Mcg}_{wt} \Diamond\sim x = y$.

$\Downarrow$

(i) $g(x) = g(y)$ and (ii) for some $w^* \in$ W, $g(x) \neq g(y)$. $\perp$

■

Instead, suppose we adopt Graff Fara's semantics. Then (CI) can be rendered true. That is, for some model M, context c of M, assignment g for M, $w \in$ W, and $t \in$ T, $\models^{Mcg}_{wt}$ (CI).

Proof. Let M be a model, c a context for M, g an assignment for M, $w \in$ W, and $t \in$ T. Then we have:

$\models^{Mcg}_{wt}$ (CI).

$\Updownarrow$

(i) $\models^{Mcg}_{wt} x = y$ and (ii) $\models^{Mcg}_{wt} \Diamond\sim x = y$.

$\Updownarrow$

(i) $g(x) = g(y)$ and (ii) for some $w^* \in$ W,

$\models^{Mcg_{[f^x_c(g(x)_{wt},w^*,t),f^y_c(g(y)_{wt},w^*,t)/x,y]}}_{w^*t} \sim x = y$.

$\Updownarrow$

(i) $g(x) = g(y)$ and (ii) for some $w^* \in$ W,

$\not\models^{Mcg_{[f^x_c(g(x)_{wt},w^*,t),f^y_c(g(y)_{wt},w^*,t)/x,y]}}_{w^*t} x = y$.

$\Updownarrow$

(i) $g(x) = g(y)$ and (ii) for some $w^* \in$ W,

$f^x_c(g(x)_{wt}, w^*, t) \neq f^y_c(g(y)_{wt}, w^*, t)$.

Given this, suppose:

$d, d^* \in$ D $(d \neq d^*)$

$g(x) = g(y) = d$

$w^* \in$ W

$< d, w, t >$ R_c^x $< d^*, w^*, t >$

$< d, w, t >$ R_c^y $< d, w^*, t >$

Then $\models_{wt}^{Mcg}$ (CI). ■

Put simply, (CI) is true at wt relative to c iff the single individual d is such that for some world w*, at w^*t, the R_c^x same thing as d_{wt} is distinct from the R_c^y same thing as d_{wt}.

4.4.2 The Problem

Unfortunately, there is a problem for the semantics concerning (i) iterated modality/temporality and (ii) existence that renders it an inappropriate account of modal and temporal talk. There are two versions of the problem. One modal, one temporal. In this paper, I shall focus on the modal version, but everything said can be suitably translated for the temporal version.

In particular, the problem is a mishandling of certain claims involving an actuality operator occurrence within the scope of a possibility/necessity operator occurrence.

To introduce the problem, note that Graff Fara's semantics allows for certain contingent existence claims to be true. For example, for some models M and contexts c of M, $\models^{Mc}$ (CE):

(CE) $\exists x \diamond \sim \exists y x = y$

Proof. Let M be a model, and c a context for M. Then we have:

$\models^{Mc}$ (CE)

$\Updownarrow$

For every assignment g for M, $\models_{w_c t_c}^{Mcg}$ (CE).

$\Updownarrow$

For every assignment g for M, for some $d \in$ D, $\models_{w_c t_c}^{Mcg_{[d/x]}} \diamond \sim \exists y x = y$.

$\Updownarrow$

For every assignment g for M, for some $d \in$ D, for some $w \in$ W,

$\models_{wt_c}^{Mcg_{[f_c^x(d_{w_c t_c}, w, t_c)/x]}} \sim \exists y x = y$.

$\Updownarrow$

For every assignment g for M, for some $d \in$ D, for some $w \in$ W,

$\not\models^{Mcg_{[f_c^x(d_{w_c t_c}, w, t_c)/x]}}_{w t_c} \exists y x = y.$

$\Updownarrow$

For every assignment g for M, for some $d \in$ D, for some $w \in$ W,

no $d^* \in$ D is such that

$\models^{Mcg_{[f_c^x(d_{w_c t_c}, w, t_c), d^*/x, y]}}_{w t_c} x = y.$

$\Updownarrow$

For some $d \in$ D, for some $w \in$ W,

no $d^* \in$ D is such that $f_c^x(d_{w_c t_c}, w, t_c) = d^*$.

Given this, suppose:

> $d \in$ D.
>
> $w \in$ W ($w \neq w_c$).
>
> No $d^* \in$ D is such that $< d, w_c, t_c >$ R$_c^x$ $< d^*, w, t_c >$.

Then $\models^{Mc}$ (CE). ■

This is all fine. However, it seems that for any context at which (CE) is true, (CEA) should also be true.[30]

(CEA) $\exists x \diamond (\sim \exists y x = y \wedge \mathbf{A} \exists y x = y)$

The key issue is that on Graff Fara's semantics, for every model M, context c of M, assignment g for M, $w \in$ W, and $t \in$ T, $\not\models^{Mcg}_{wt}$ (CEA). That is, (CEA) is a contradiction.

Proof. Let M be a model, c a context for M, g an assignment for M, $w \in$ W, and $t \in$ T. For *reductio*, suppose $\models^{Mcg}_{wt}$ (CEA).

$\Downarrow$

For some $d \in$ D, $\models^{Mcg_{[d/x]}}_{wt} \diamond(\sim \exists y x = y \wedge \mathbf{A} \exists y x = y)$.

$\Downarrow$

For some $d \in$ D, for some $w^* \in$ W,

$\models^{Mcg_{[f_c^x(d_{wt}, w^*, t)/x]}}_{w^* t} \sim \exists y x = y \wedge \mathbf{A} \exists y x = y.$

$\Downarrow$

For some $d \in$ D, for some $w^* \in$ W,

(i) $\not\models^{Mcg_{[f_c^x(d_{wt}, w^*, t)/x]}}_{w^* t} \exists y x = y$, and

[30] If we allowed for truth-value gaps or truth-value gluts, then, at the very least, it seems (CEA) at such a context should not be (just) false.

(ii) $\models^{Mcg_{[f_c^x(d_{wt},w^*,t)/x]}}_{w^*t}$ $\mathbf{A}\exists yx = y$.

$\Downarrow$

For some $d \in$ D, for some $w^* \in$ W,

(i) no $d^* \in$ D is such that

$\models^{Mcg_{[f_c^x(d_{wt},w^*,t),d^*/x,y]}}_{w^*t}$ $x = y$, and

(ii) $\models^{Mcg_{[f_c^x(d_{wt},w^*,t)/x]}}_{w^*t}$ $\mathbf{A}\exists yx = y$.

$\Downarrow$

For some $d \in$ D, for some $w^* \in$ W,

(i) no $d^* \in$ D is such that $f_c^x(d_{wt}, w^*, t) = d^*$, and

(ii) $\models^{Mcg_{[f_c^x(d_{wt},w^*,t)/x]}}_{w^*t}$ $\mathbf{A}\exists yx = y$.

$\Downarrow$

For some $d \in$ D, for some $w^* \in$ W,

(i) $f_c^x(d_{wt}, w^*, t) =$ D, and

(ii) $\models^{Mcg_{[f_c^x(d_{wt},w^*,t)/x]}}_{w^*t}$ $\mathbf{A}\exists yx = y$.

$\Downarrow$

For some $d \in$ D, for some $w^* \in$ W,

(i) $f_c^x(d_{wt}, w^*, t) =$ D, and

(ii) $\models^{Mcg_{[f_c^x(f_c^x(d_{wt},w^*,t)_{w^*t},w_c,t)/x]}}_{w_ct}$ $\exists yx = y$.

$\Downarrow$

For some $d \in$ D, for some $w^* \in$ W,

(i) $f_c^x(d_{wt}, w^*, t) =$ D, and

(ii) for some $d^+ \in$ D,

$\models^{Mcg_{[f_c^x(f_c^x(d_{wt},w^*,t)_{w^*t},w_c,t),d^+/x,y]}}_{w_ct}$ $x = y$.

$\Downarrow$

For some $d \in$ D, for some $w^* \in$ W,

(i) $f_c^x(d_{wt}, w^*, t) =$ D, and

(ii) for some $d^+ \in$ D,

$f_c^x(f_c^x(d_{wt}, w^*, t)_{w^*t}, w_c, t) = d^+$.

$\Downarrow$

For some $d \in$ D, for some $w^* \in$ W,

(i) $f_c^x(d_{wt}, w^*, t) = \mathrm{D}$, and

(ii) $f_c^x(f_c^x(d_{wt}, w^*, t)_{w^*t}, w_c, t) \in \mathrm{D}$.

$\Downarrow$

For some $d \in \mathrm{D}$, for some $w^* \in \mathrm{W}$,

(i) $f_c^x(d_{wt}, w^*, t) = \mathrm{D}$, and

(ii) $f_c^x(\mathrm{D}_{w^*t}, w_c, t) \in \mathrm{D}$.

$\Downarrow$

For some $d \in \mathrm{D}$, for some $w^* \in \mathrm{W}$,

(i) $f_c^x(d_{wt}, w^*, t) = \mathrm{D}$, and

(ii) $f_c^x(\mathrm{D}_{w^*t}, w_c, t) \neq \mathrm{D}$.

However, $f_c^x(\mathrm{D}_{w^*t}, w_c, t) = \mathrm{D}$ (by the definition of the role-mate function).

$\perp$ ■

Given this, we of course have that for every model M and context c, $\not\models^{Mc}$ (CEA).

The mishandling of (CEA) is caused by the combination of two features. First, actuality settings are unpacked in terms of further role-mates (further iterations). Second, the simple role-mate function cannot map the non-individual D to an individual. As a result, the semantics cannot capture the nature of the actuality setting in (CEA) as a *return*.

4.4.3 A Similar Feature

At this point, it is worth noting a similar feature to the one just noted, but which concerns the semantics' handling of the combination of iterated modality (restricted to possibility and necessity) and existence.

Consider:

(32) $\exists x \diamond (\sim\exists y x = y \wedge \diamond\exists y x = y)$

(33) $\forall x \Box(\sim\exists y x = y \rightarrow \Box\sim\exists y x = y)$

On relative-sameness semantics, (32) is a contradiction, and (33) is valid. That is, for every model M, assignment g for M, context c of M, $w \in \mathrm{W}$, and $t \in \mathrm{T}$,

$\not\models_{wt}^{Mcg}$ (32)

$\models_{wt}^{Mcg}$ (33)

Notice that (32) is exactly the same as (CEA), except that the occurrence of **A** has been substituted out for an occurrence of $\diamond$. (32) and (33) are contradictories. Here, I shall just present a proof of the validity of (33):

Proof. For *reductio*, let M be a model, g an assignment for M, c a context of M, $w \in \mathrm{W}$, and $t \in \mathrm{T}$ such that $\not\models^{Mcg}_{wt}$ (33).

$\Downarrow$

For some $d \in \mathrm{D}$, $\not\models^{Mcg_{[d/x]}}_{wt} \Box(\sim\exists yx = y \rightarrow \Box\sim\exists yx = y)$

$\Downarrow$

For some $d \in \mathrm{D}$, for some $w^* \in \mathrm{W}$,

$\not\models^{Mcg_{[f^x_c(d_{wt},w^*,t)/x]}}_{w^*t} \sim\exists yx = y \rightarrow \Box\sim\exists yx = y$

$\Downarrow$

For some $d \in \mathrm{D}$, for some $w^* \in \mathrm{W}$,

(i) $\models^{Mcg_{[f^x_c(d_{wt},w^*,t)/x]}}_{w^*t} \sim\exists yx = y$, and

(ii) $\not\models^{Mcg_{[f^x_c(d_{wt},w^*,t)/x]}}_{w^*t} \Box\sim\exists yx = y$.

$\Downarrow$

For some $d \in \mathrm{D}$, for some $w^* \in \mathrm{W}$,

(i) $f^x_c(d_{wt}, w^*, t) \notin \mathrm{D}$, and

(ii) for some $w^{**} \in \mathrm{W}$, $\not\models^{Mcg_{[f^x_c(f^x_c(d_{wt},w^*,t)_{w^*t},w^{**},t)/x]}}_{w^{**}t} \sim\exists yx = y$.

$\Downarrow$

For some $d \in \mathrm{D}$, for some $w^* \in \mathrm{W}$,

(i) $f^x_c(d_{wt}, w^*, t) = \mathrm{D}$, and

(ii) for some $w^{**} \in \mathrm{W}$, $\models^{Mcg_{[f^x_c(f^x_c(d_{wt},w^*,t)_{w^*t},w^{**},t)/x]}}_{w^{**}t} \exists yx = y$.

$\Downarrow$

For some $d \in \mathrm{D}$, for some $w^* \in \mathrm{W}$,

(i) $f^x_c(d_{wt}, w^*, t) = \mathrm{D}$, and

(ii) $f^x_c(f^x_c(d_{wt}, w^*, t)_{w^*t}, w^{**}, t) \in \mathrm{D}$.

$\Downarrow$

For some $d \in \mathrm{D}$, for some $w^* \in \mathrm{W}$,

(i) $f^x_c(d_{wt}, w^*, t) = \mathrm{D}$, and

(ii) $f^x_c(\mathrm{D}_{w^*t}, w^{**}, t) \in \mathrm{D}$.

$\Downarrow$

For some $d \in \mathrm{D}$, for some $w^* \in \mathrm{W}$,

(i) $f^x_c(d_{wt}, w^*, t) = \mathrm{D}$, and

(ii) $f^x_c(\mathrm{D}_{w^*t}, w^{**}, t) \neq \mathrm{D}$.

However, $f^x_c(\mathrm{D}_{w^*t}, w^{**}, t) = \mathrm{D}$ (by the definition of the role-mate function).

$\bot$ ■

Of course, there are many who think that a semantics for a first-order modal language *should* render (32) a contradiction and (33) as valid. For example, take the defender of a standard constant domain Kripke semantics.

However, the crucial point here is that, on relative-sameness semantics, (32) is a contradiction and (33) is valid *whilst* for some model M and context c of M, $\models^{Mc}$ (CE).

(CE) $\exists x \diamond \sim \exists y x = y$

Like (CEA), the fact that (32) is a contradiction and (33) is valid is caused by the combination of two features. First, possibility/necessity settings are unpacked in terms of further role-mates (further iterations). Second, the simple role-mate function cannot map the non-individual D to an individual.

The question now is: is it acceptable that (32) is a contradiction *whilst* for some model M and context c of M, $\models^{Mc}$ (CE)? This does not seem as straightforwardly problematic as the feature I noted in the previous section. There are broadly two approaches: (A1) and (A2).

The first approach, (A1), is to hold that it is acceptable that (32) is a contradiction and (33) is valid, whilst for some M and c, $\models^{Mc}$ (CE). At least limited to these cases, the semantics does not inappropriately handle the combination of iterated modality (restricted to possibility and necessity) and existence.

The idea with (A1) is that one first recognises that a possibility/necessity setting amounts to quantification over further possibilities (further iterations). Next, one holds that Graff Fara's technical machinery has correctly captured this iterative nature (at least for (32) and (33)).

Given (A1), one might think that the original problem with the semantics is the result just of incorrect clauses for the indexical operators in the recursive definition of satisfaction. However, it seems that at the very least, additional machinery will need to be introduced to handle the indexical operators.

The second approach, (A2), is to hold that it is unacceptable that (32) is a contradiction and (33) is valid whilst for some M and c, $\models^{Mc}$ (CE). As such, relative-sameness semantics already inappropriately handles the combination of iterated modality (restricted to possibility and necessity) and existence, before we even turn to the actuality operator.[31]

Regardless of whether we adopt (A1) or (A2), the semantics requires revision if it is to provide an account of our modal and temporal talk.

[31]For an example of this kind of attitude, in his dissertation (Hazen 1977), Hazen requires that (i) certain contingent existence claims are satisfiable, (ii) (32) is not a contradiction and (33) is not valid, and (iii) modality *de re* is not handled simply in terms of transworld identity. His stipulational semantics is, in part, employed to achieve just that.

4.5 Meyer's Account

Meyer 2013 attempts to provide an alternative systematic method for generating counterpart theoretic characterisations of natural language modal claims (including those involving a notion of actuality) to the one provided in Lewis 1968. Following Lewis, Meyer's method has two steps: (i) regiment a natural language modal claim ϕ to a sentence ψ of a certain formal modal language, then (ii) translate ψ to a sentence of a slightly augmented language of counterpart theory.

4.5.1 The Eliminability of A

A central feature of Meyer's account is that the initial formal modal language that we regiment natural language modal claims into (including those involving a notion of actuality) does not contain an actuality operator **A**. Meyer holds that we can make do with other resources. He writes (2013: 33):

> "The actuality operator allows us to make claims about the actual world *inside* the scope of other modal operators. That is a nice feature, but describing the actual world is something we could do already, by using unmodalized sentences *outside* the scope of other operators. Offhand, one would therefore expect the actuality operator to make no difference to the expressive capacity of modal logic. Whenever a claim about the actual world is made within the scope of another modal operator, we should be able to move the claim outside that operator's scope, where we no longer need the actuality operator to talk about the actual world.
>
> However, this assumes that the rest of our logic is strong enough to permit this kind of transformation, and that is not always the case. The actuality operator does make a difference in expressive capacity when it is combined with a quantified modal logic that is too weak to permit its elimination. But there is no reason why counterpart theorists should measure the expressive capacity of their theory of modality against such a weak system, rather than against a stronger one from which [**A**] can be eliminated."

As Hazen 1978 shows, **A** is eliminable in propositional modal logic. We can capture this in model-theoretic terms. Let L^+ be the standard propositional modal language with **A** and assume it is interpreted with its usual Kripkean semantics. L^+-formulas ϕ and ψ are *equivalent* iff for every L^+-structure M, M satisfies ϕ (at the actual world of the structure) iff M satisfies ψ (at the actual world of the structure). The eliminability of **A** for this logic amounts to: for every L^+-formula ϕ with an occurrence of **A**, there is an equivalent L^+-formula ψ with no occurrence of **A**. Meyer himself (2009: 234) demonstrates this for the temporal analogue of modal propositional logic.[32]

[32] See Theorem 2 of Meyer 2009. Also see Kamp 1971.

As Hazen 1976 and 1978 note, **A** is not eliminable for standard FOM languages interpreted with varying domain semantics. Here, for a standard FOM language L, let L-formulas ϕ and ψ be equivalent iff for every varying domain L-structure M, and variable assignment s for M, $v_{M,s}(\phi) = v_{M,s}(\psi)$. Then the ineliminability of **A** for this logic amounts to: for some L-formula ϕ with an occurrence of **A**, there is no equivalent L-formula ψ with no occurrence of **A**. For example, consider $\Diamond\exists x{\sim}\mathbf{A}\exists y x = y$ (Hazen 1976: 40, 1978: 622). Meyer 2009 notes this for an analogous varying domain first-order temporal semantics.

However, Meyer 2013 makes an eliminability claim concerning **A** for a certain enriched constant domain modal semantics. This is close to the eliminability claim made in Meyer 2009 concerning the now operator **N** for a certain enriched constant domain temporal semantics.

The motivating idea with Meyer's enriched constant domain modal semantics is that we can enrich a first-order modal language with resources that allow us to talk about (i) "existence", and (ii) sets of n-tuples of members of our overall domain of individuals.[33]

Meyer's semantics can be captured as follows (Meyer 2013: 35-38, 2009: 242-245).[34] The languages of the semantics are certain enriched standard FOM languages. For a standard FOM language L, there corresponds an enriched FOM language L* whose vocabulary extends the vocabulary of L with exactly the following: E, $\in$, $<$, $>$, and for every $n \geq 0$, denumerably-many n-ary set variables $V_1^n, V_2^n, ...$ (let SVAR be the set of set variables). Let the L*-individual-terms be the L-terms. We include the following in our inductive definition of the L*-formulas:

> For every L*-individual-term t, [Et is an L*-formula].[35]
>
> For every $n \geq 0$, all L*-individual-terms $t_1, ..., t_n$, and every n-ary set variable X^n, [$< t_1, ..., t_n > \in X^n$ is an L*-formula].
>
> For every $* \in \{\exists, \forall\}$, set variable X, and L*-formula ϕ, [$*X\phi$ is an L*-formula].

[33] There are echoes of the third strategy of Bricker 1989 (section VI) in the approach.

[34] I have made an adjustment in this presentation of Meyer's semantics. In his presentations, Meyer talks just of "set variables $X, Y, Z, ...$" And he talks of a variable assignment of a structure mapping each set variable to a set in the hierarchy of sets built up from D (the hierarchy that takes the members of the overall domain D as urelements). Set variables range over sets in the hierarchy. However, the motivation of the enrichment of the language is to be able to talk about (i) "existence", and (ii) sets of n-tuples of members of our overall domain D. So, in my presentation, as we shall see, I have assigned an arity to each set variable and restricted the range of an n-ary set variable to $\mathcal{P}(\mathrm{D}^n)$. This is all we need for the eliminability thesis.

[35] Meyer (2013: 38) states that (i) one could allow formulas EX, where X is a set variable, and (ii) there are multiple ways in which the semantics might handle such a formula. He there proposes to ignore the issue, so I have excluded formulas of this kind here.

The L*-structures are the constant domain L-structures <W, R, D, $w_@$, I>, with the addition of a function dom: W $\rightarrow \mathcal{P}$(D) and the requirement on I that for every $n \geq 1$, for every n-ary $P \in$ PC(L), for every $w \in$ W, I(P)(w) $\subseteq$ dom(w)^n. A variable assignment s for an L*-structure M assigns to each individual variable a member of D, and for every $n \geq 0$, to each n-ary set variable a member of $\mathcal{P}$(D^n). A denotation function $den_{M,s}$ for an L*-structure M and variable assignment s is defined straightforwardly:

$$den_{M,s}(\alpha) = \begin{cases} \text{I}(\alpha) \text{ if } \alpha \in \text{IC(L)}. \\ s(\alpha) \text{ if } \alpha \in \text{VAR} \cup \text{SVAR}. \end{cases}$$

We include the following clauses in our recursive definition of $v_{M,w,s}$ to characterise our additional vocabulary:

$v_{M,w,s}(Et) = 1$ iff $den_{M,s}(t) \in$ dom(w).

$v_{M,w,s}(< t_1, ..., t_n > \in X^n) = 1$ iff

$< den_{M,s}(t_1), ..., den_{M,s}(t_n) > \in den_{M,s}(X^n)$.

$v_{M,w,s}(\exists X^n \phi) = 1$ iff for some S $\in \mathcal{P}$(D^n), $v_{M,w,s_{[\text{S}/X^n]}}(\phi) = 1$.

$v_{M,w,s}(\forall X^n \phi) = 1$ iff for every S $\in \mathcal{P}$(D^n), $v_{M,w,s_{[\text{S}/X^n]}}(\phi) = 1$.

So much for the semantics. Say that two L*-formulas ϕ and ψ are equivalent iff for every L*-structure M, and variable assignment s for M, $v_{M,s}(\phi) = v_{M,s}(\psi)$. Further, for an enriched language L*, let L$_\text{M}$ be the language that results from L* by removing **A**. The eliminability claim concerning **A** that Meyer 2013 makes is that every L-formula has an equivalent L$_\text{M}$-formula.[36]

Meyer 2009 covers the temporal analogue of this thesis (Theorem 4, 2009: 243). Meyer 2013 informally discusses why this result holds for the modal version. For our purposes, it is sufficient to consider two examples. First, consider:

(34) There could have been something that does not actually exist.

(35) It might have been that everyone who is in fact rich was poor.

For (35) (following the approach of Bricker 1989), we restrict our attention to the plural *de re* reading. Given Meyer's semantics, two formulas which supposedly capture the conditions of (34) and (35), respectively, are the following:

(34*) $\Diamond \exists x(Ex \wedge \mathbf{A} \sim Ex)$

(35*) $\Diamond \forall x(\mathbf{A}Gx \rightarrow Px)$

[36]Note that (i) every L-formula and every L$_\text{M}$-formula is an L*-formula, and (ii) every L-formula without an **A** occurrence is an L$_\text{M}$-formula.

Meyer states that the following are, respectively, equivalent:

(34**) $\exists x(\Diamond Ex \wedge \sim Ex)$

(35**) $\exists X(\forall x(< x >\in X \leftrightarrow Gx) \wedge \Diamond \forall x(< x >\in X \rightarrow Px))$

The idea with converting an L-formula ϕ to an L_M-formula ψ is that in ψ, we try to capture the relevant features about $w_@$ (that ϕ imposes) outside the scope of any modal operators which shift our evaluation world away from $w_@$.

For the simple case of (34*) and (34**), to capture (34*), we present a formula that, outside of any modal operators, captures the relevant feature about $w_@$, namely that some $d \in D$ in the domain of some world is not in $dom(w_@)$.

For the case of (35*) and (35**), in (35**) we antecedently introduce a set $S = \{d \in D : d \in I(G)(w_@)\}$. That is, the set containing exactly all individuals that are G at the actual world. Then, within the scope of the possibility operator occurrence, we can use individual membership of S to capture the property of an individual being G at the actual world.[37]

4.5.2 A Counterpart Account

Meyer states that we can employ this result when presenting a counterpart theoretic handling of natural language modal claims. He calls his approach the *antecedent elimination strategy*, and writes (2013: 33-34):

> "Instead of uniformly translating subformulae of type [**A**ϕ] into counterpart theory, I want to propose an antecedent elimination strategy that makes do without this operator altogether. We first eliminate all occurrences of [**A**] by translating the modal claims in question into a quantified modal logic in which this operator is redundant. After that, we apply a slightly modified version of Lewis's translation scheme to the resulting [**A**-free] sentences to generate substitutes in counterpart theory. This provides a systematic counterpart treatment of sentences involving [**A**]"

And (2013: 40):

> "We begin by regimenting all modal claims about physical objects in [a language L_M interpreted with Meyer's semantics]. By doing so, we eliminate all occurrences of the actuality operator [**A**]. After that, we apply our revised translation scheme to these [**A**]-free sentences to produce substitutes in counterpart theory."

[37] Notice that if we didn't restrict the range of set variables as I have suggested, (35**) would be handled slightly differently. In (35**) we would be antecedently introducing a set S whose intersection with D is $\{d \in D : d \in I(G)(w_@)\}$. That is, the intersection contains exactly all individuals that are G at the actual world. Then, within the scope of the possibility operator occurrence, we can use individual membership of S to capture the property of an individual being G at the actual world.

The idea, it seems, is that for a given natural language modal claim ϕ, we first regiment ϕ as an L_M-sentence ψ that (when interpreted with Meyer's semantics) supposedly captures ϕ. Next, we apply Meyer's translation scheme that maps an L_M-sentence ψ to a sentence of an enriched counterpart theory language corresponding to L_M.

Meyer restricts his attention to L_M languages with no individual constants. For an L_M language (with no individual constants), the corresponding counterpart theory language L_M^{CT} is an extension of the CT language L^{CT} (the standard FO language whose extralogical constants are exactly: @, W, A, I, C, and every extralogical predicate constant of L_M) with exactly: $\in$, $<$, $>$, and for every $n \geq 0$, denumerably-many n-ary set variables $V_1^n, V_2^n, ...$[38] The L_M^{CT}-formulas are inductively defined in the obvious way.

The L_M^{CT}-structures are the standard L^{CT}-structures. The definition of satisfaction is extended in the obvious way. Next, the L_M^{CT} counterpart theory itself seems to be our earlier counterpart theory. That is, the set containing exactly (P1)-(P8), (A), (B), and (C). Let it be CT.

Finally, for an L_M language (with no individual constants), the translation of L_M-sentence ϕ is $\phi^{@}$. To define this, we simultaneously define by recursion, for every $\alpha \in [VAR \cup \{@\}]$, the function ϕ^{α}: $Form(L_M) \rightarrow Form(L_M^{CT})$ (2013: 37-40):

(i) $(Px_1...x_n)^{\alpha}$ is $(Ix_1\alpha \wedge ... \wedge Ix_n\alpha \wedge Px_1...x_n)$.

(ii) $(x_1 = x_2)^{\alpha}$ is $(Ix_1\alpha \wedge Ix_2\alpha \wedge x_1 = x_2)$.

(iii) $(Ex)^{\alpha}$ is $Ix\alpha$.

(iv) $(< x_1, ..., x_n >\in X^n)^{\alpha}$ is $(Ix_1\alpha \wedge ... \wedge Ix_n\alpha \wedge < x_1, ..., x_n >\in X^n)$.

(v) $(\sim\phi)^{\alpha}$ is $\sim(\phi^{\alpha})$.

(vi) For every $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$, $(\phi * \psi)^{\alpha}$ is $(\phi^{\alpha} * \psi^{\alpha})$.

(vii) $(\exists x\phi)^{\alpha}$ is $\exists x(\phi^{\alpha})$.

(viii) $(\forall x\phi)^{\alpha}$ is $\forall x(\phi^{\alpha})$.

(ix) $(\exists X^n\phi)^{\alpha}$ is $\exists X^n(\phi^{\alpha})$.

(x) $(\forall X^n\phi)^{\alpha}$ is $\forall X^n(\phi^{\alpha})$.

(xi) $(\diamond\phi_{[x_1,...,x_n]})^{\alpha}$ is
$\exists w \exists y_1 ... \exists y_n (Ww \wedge Iy_1w \wedge Cy_1x_1 \wedge ... \wedge Iy_nw \wedge Cy_nx_n \wedge$
$(\phi(y_1, ..., y_n))^w)$.

[38] Strictly, it seems @ is not a primitive of Meyer's counterpart theory languages, but I shall include it for simplicity as I did with Lewis' 1968 account.

(xii) $(\Box\phi_{[x_1,...,x_n]})^\alpha$ is

$\forall w \forall y_1 ... \forall y_n((Ww \wedge Iy_1w \wedge Cy_1x_1 \wedge ... \wedge Iy_nw \wedge Cy_nx_n) \rightarrow$

$(\phi(y_1, ..., y_n))^w)$.

Notice that the clauses for the truth-functional connectives and modal operators remain the same as Lewis'. The differences are that (a) quantifiers are not translated as world-relative, and (b) the atomic clauses (i)-(iii) add world membership conditions. Meyer's idea with (i)-(iii) is that he is building back in world-relativisation of sentences removed from the first-order quantifier clauses.

For example, then, (34**) and (35**) are translated, respectively, as follows:

(34**) $\exists x(\diamond Ex \wedge \sim Ex)$

(34***) $\exists x(\exists w \exists y(Ww \wedge Iyw \wedge Cyx \wedge Iyw) \wedge \sim Ix@)$

(35**) $\exists X(\forall x(< x >\in X \leftrightarrow Gx) \wedge \diamond \forall x(< x >\in X \rightarrow Px))$

(35***) $\exists X(\forall x[(Ix@ \wedge < x >\in X) \leftrightarrow (Ix@ \wedge Gx)] \wedge$

$\exists w[Ww \wedge \forall x((Ixw \wedge < x >\in X) \rightarrow (Ixw \wedge Px))])$

So much for the account. In response, I have three points. First, we immediately run into trouble: Meyer's toy example (35)-(35**)-(35***) fails. Notice that (35***) just says that (a) there is a set S such that of the actual individuals $d_@$, [$d_@$ is in S iff $d_@$ is G], and (b) there is a world w such that of the w-individuals d_w, if d_w is in S, then d_w is P. The first conjunct $\forall x[(Ix@ \wedge < x >\in X) \leftrightarrow (Ix@ \wedge Gx)]$ that is supposed to define S in fact does not define S, but merely gives membership conditions for actual individuals. Given the first conjunct, it is open whether S contains non-actual individuals. The second conjunct $\exists w[Ww \wedge \forall x((Ixw \wedge < x >\in X) \rightarrow (Ixw \wedge Px))]$ then gives us a conditional for w-individuals. Namely, that if they're in S, then they are P. In no part of the statement is a counterpart relation mentioned. Clearly, the analysis fails to capture (35).

Second, given that Meyer's translation scheme reuses certain clauses from Lewis' original scheme, his account inherits some of the problems that Lewis' exhibits.

For example, Meyer reuses Lewis' clauses for the possibility and necessity operators. However, as we saw in section 3.4.7, given the way Lewis sets these up, the 1968 semantics interprets *de re* occurrences of the possibility and necessity operators as expressing weak *de re* possibility and necessity, respectively. That is, what is possible/necessary of individuals together when they all exist. As a result, a problem that the overall 1968 account faces is that it cannot render true certain claims concerning contingent existence. Consider:

(36) Some individual could have failed to exist.

On Lewis' 1968 account, this would be regimented as:

(36*) $\exists x \diamond \sim \exists y x = y$

Which is translated to the unsatisfiable:

(36**) $\exists x(Ix@ \wedge \exists w \exists z(Ww \wedge Izw \wedge Czx \wedge \sim \exists y(Iyw \wedge z = y)))$

However, certainly a later Lewis would want his counterpart theoretic analysis of (36) to allow it to be true.

Now, notice that Meyer inherits the problem of being unable to render true certain contingent existence claims. Presumably, on Meyer's account, (36) would be regimented as:

(36+) $\exists x(Ex \wedge \diamond \sim Ex)$

Which is translated to the unsatisfiable:

(36++) $\exists x(Ix@ \wedge \exists w \exists y(Iyw \wedge Cyx \wedge \sim Iyw))$

However, presumably Meyer does want his account to be able to render (36) true. After all, the regimentation (36+) of (36) is certainly satisfiable on Meyer's antecedent semantics.

This brings me to my third point, which concerns methodology. Why would we think that it is appropriate to arrive at the correct counterpart theoretic account of a natural language modal claim ϕ by first regimenting ϕ to an L_M-sentence ψ that supposedly captures ϕ when interpreted with an antecedently assumed semantics?

Recall that Lewis' overall 1968 strategy is effectively to provide an account of a natural language modal claim ϕ by (a) regimenting ϕ into a sentence of a formal modal language L^*, and (b) providing a semantics for L^*. Lewis' approach to step (b) is to present (i) a first-order theory, and then (ii) a translation scheme that maps L^*-sentences to sentences of the theory. Together, (i) and (ii) present an indirect semantics for L^*. Step (a) is meant to be a systematic regimentation of natural language modal claims to L^*-sentences *without a semantics interpreting* L^*.

Instead, Meyer antecedently assumes a semantics for a language L_M (without individual constants) which he relies on to regiment natural language modal claims into L_M-sentences. His counterpart theory and translation scheme combined then generate *another* semantics for L_M.

What is going on here? Meyer does tell us that his "solutions are of course based on the acceptance of [his antecedently assumed semantics]" (2013: 40).

Perhaps, then, the idea is that although we accept his antecedently assumed semantics for $\mathrm{L_M}$, we can present an equivalent counterpart theoretic semantics for $\mathrm{L_M}$ (equivalent in the sense of having the same consequence relation when restricted to $\mathrm{L_M}$-sentences). However, the account presented clearly does not achieve this. To see this, consider the following based on our observations concerning (35**) and (35***):

(37) $[(\exists x Gx \wedge \exists X(\forall x(x \in X \leftrightarrow Gx) \wedge \Diamond \forall x(x \in X \rightarrow Px)))] \rightarrow \exists x(Ex \wedge \Diamond Px)$

This is a theorem of Meyer's antecedent semantics. However, it translates to:

(37*) $[(\exists x(Ix@ \wedge Gx) \wedge$
$\exists X(\forall x[(Ix@ \wedge < x > \in X) \leftrightarrow (Ix@ \wedge Gx)] \wedge$
$\exists w[Ww \wedge \forall x((Ixw \wedge < x > \in X) \rightarrow (Ixw \wedge Px))])] \rightarrow$
$\exists x(Ix@ \wedge \exists w \exists y(Ww \wedge Iyw \wedge Cyx \wedge Py))$

And some $\mathrm{L_M^{CT}}$-structures that model CT do not satisfy (37*).

Proof. Let M be an $\mathrm{L_M^{CT}}$-structure that models CT such that (i) there is a single actual individual d, (ii) d is G, and (iii) d has no counterparts that are P. Then M satisfies the antecedent of the conditional (37*):

$[(\exists x(Ix@ \wedge Gx) \wedge$
$\exists X(\forall x[(Ix@ \wedge < x > \in X) \leftrightarrow (Ix@ \wedge Gx)] \wedge$
$\exists w[Ww \wedge \forall x((Ixw \wedge < x > \in X) \rightarrow (Ixw \wedge Px))])]$

This is because there is (i) an actual individual that is G (namely, d), and (ii) a set S such that (ii.i) for all actual individuals b, b is in S iff b is G, and (ii.ii) for some world w, every w-individual b in S is P (namely, S = $\{d\}$).

However, M does not satisfy the consequent of the conditional (37*):

$\exists x(Ix@ \wedge \exists w \exists y(Ww \wedge Iyw \wedge Cyx \wedge Py))$

This is because no actual individual has a counterpart that is P. ■

For another example, reconsider:

(36^{+}) $\exists x(Ex \wedge \Diamond \sim Ex)$

As we saw earlier, this is a contradiction on Meyer's counterpart theoretic semantics, but it is satisfiable on the antecedent semantics.

4.5.3 Revision

Meyer's current overall account fails to provide an acceptable counterpart theoretic account of our natural language modal talk.

Can it be salvaged? At the very least, it requires significant revision. However, if the counterpart theoretic semantics we generate for an L_M language (without individual constants) through a theory and translation scheme is meant to be equivalent (in the sense of having the same consequence relation restricted to sentences) to Meyer's antecedent semantics, then Meyer's approach anyway seemingly imposes certain limitations on our counterpart theoretic handling of natural language modal claims. For example, we seemingly will not be able to render true certain "contingent identity" statements. Consider:

(38) For some concretum x and some concretum y, x and y are identical but could have been distinct.

One might want to allow that (38) may be rendered true. However, it seems that any supposed capture of (38) as an L_M-sentence ϕ interpreted with Meyer's antecedent semantics will render ϕ unsatisfiable. Even if we extended the L_M languages with additional resources, it is unclear how (38) would be captured by a sentence ϕ of such a language (interpreted with a suitably enriched version of Meyer's antecedent semantics) that is satisfiable. As a result, if our counterpart theoretic semantics generated by a theory and translation scheme is meant to be equivalent to the antecedent semantics, it will render ϕ unsatisfiable. So, Meyer's overall strategy seemingly cannot allow for (38) to be rendered true.

Conclusion

In this work, I examined certain counterpart accounts. The first two chapters focussed on Lewis' reduction of modality in terms of his modal realism and mature counterpart machinery. I presented an account that I take to be a straightforward regimentation of his sketch of joint possibilities in 1986, then raised several issues for that account. Those issues concern existence, the combination of existence and iterated modality, and "contingent identity". Finally, I presented the group-counterpart machinery as an alternative. Although this seems superior to the tuple-counterpart machinery in some respects, issues concerning existence that apply to the tuple-counterpart machinery also apply to the group-counterpart machinery.

In the third chapter, I presented Lewis' formal 1968 counterpart account and several well-known issues with it. Those issues concern existence, the plural *de re*, and actuality. I also considered the indirect approach to presenting a semantics for a formal language. On this, I first demonstrated that from a technical perspective, for the 1968 account, the indirect approach is unnecessary for the purpose of presenting a semantics. That is, for the 1968 indirect semantics for standard FOM languages (without individual constants or $\mathbf{A}$), there is an equivalent (in the sense defined) direct semantics. Next, I considered the indirect approach more generally. First, for each of constant and varying domain semantics for standard FOM languages (each restricted to the class of all its structures), I presented an equivalent (in the sense defined) indirect semantics. Second, I discussed the significance of the ability to present an indirect semantics of this sort.

In the final chapter, I raised issues for five formal accounts that appeal to a notion of counterparthood. For Hazen's stipulational semantics, I noted that there seems to be an issue developing the semantics so that it handles claims involving iterated modality (where an embedded modal operator occurrence has a different flavour from an embedding modal operator occurrence). I considered a natural way that the semantics might be developed to handle such claims, one which seems acceptable from a technical perspective, but which faces a challenge to present an acceptable analysis of its machinery.

For Russell's counterpart semantics, I noted two issues. Those concern

existence and the handling of certain modal operator occurrences embedded in the scope of quantifier occurrences.

For Bacon's counterpart semantics, I noted that there seems to be an issue developing the semantics so that it handles claims involving iterated modality. I considered two natural ways that the semantics might be developed to handle such claims. The first way is seemingly acceptable from a technical perspective, but there is a challenge to present an acceptable analysis of the machinery involved. The second way is unacceptable from a technical perspective. I also argued that the actuality operator is inappropriately handled on Bacon's semantics.

For Graff Fara's relative-sameness semantics, I presented a problem for the semantics concerning iterated modality/temporality, actuality/presentiality, and existence. Finally, for Meyer's counterpart account, I demonstrated that his translation method is unacceptable for multiple reasons.

This work presents a gauntlet for one who wants to present an account of modality employing possible worlds, but without straightforwardly appealing to transworld identity to handle modality *de re*.

Such an account must instead employ a system of representatives to account for modality *de re*. On simple modal claims (those involving a single modal setting), such a system faces three central desiderata. First, a representative must be able to represent non-existence. If it cannot, then the account will interpret all *de re* occurrences of possibility/necessity operators as expressing weak *de re* possibility/necessity. That is, what is possible/necessary of individuals together when they all exist. So, the account will not be able to render true basic contingent existence claims. Second, the system of representatives must correctly handle the plural *de re*. Third, the system must deliver our desired account of modal flavour and "contingent identity and distinctness".

When we turn to more complex modal claims (those involving multiple modal settings), a system faces three further central desiderata. First, it must be able to handle actuality. Second, it must be able to handle complex contingent existence claims (involving iterated modality and actuality). And third, it must be able to handle claims in which the flavour of one modal setting is different to the flavour of another.

Finally, even if our machinery achieves technical acceptability, there is still a requirement to provide an acceptable analysis of it.

Appendix A

Theorem Proofs

This appendix contains proofs for certain theorems stated in section 3.5.

A.1 1968 Semantics

A.1.1 Lemmas

To begin, I help myself to:

Lemma 2 (FO Agreement)**.** For every standard FO language L, standard L-structure M, all variable assignments s, s^+ of M, and every L-formula ϕ, if s and s^+ agree on all individual variables free in ϕ, then $v_{M,s}(\phi) = v_{M,s^+}(\phi)$.

Next, we have:

Lemma 3 (CT Variables)**.** For every standard FOM language L (without individual constants or **A**), L-formula ϕ, $x \in$ VAR, and $\alpha \in$ VAR $\cup\{@\}$,

[(i) x is not free in ϕ, and (ii) $x \neq \alpha$] $\Rightarrow$ x is not free in ϕ^α.

Proof. Let L be a standard FOM language (without individual constants or **A**). We show by induction on the definition of an L-formula the stronger claim that for every L-formula $\phi_{[x_1,...,x_n]}$, $x \in$ VAR, distinct (from each other) $y_1, ..., y_n \in$ VAR substitutable for $x_1, ..., x_n$ in ϕ, and every $\alpha \in$ VAR $\cup\{@\}$, if [(i) x is not free in $\phi(y_1, ..., y_n)$, and (ii) $x \neq \alpha$], then x is not free in $(\phi(y_1, ..., y_n))^\alpha$. ■

Now, we have:

Lemma 4 (CT Agreement 1)**.** For every standard FOM language L (without individual constants or **A**), CT L^{CT}-structure M, s of M, L-formula ϕ, and $x \in$ VAR not in ϕ,

$v_{M,s}(\phi^{@}) = v_{M,s_{[w_@/x]}}(\phi^x)$.

Proof. Let L be a standard FOM language (without individual constants or **A**), and M a CT L^{CT}-structure. We show by induction on the definition of an L-formula that for every L-formula ϕ, assignment s of M, and $x \in$ VAR not in ϕ, $v_{M,s}(\phi^{@}) = v_{M,s_{[w_@/x]}}(\phi^x)$.

Base Cases

(i) ϕ is $Px_1...x_n$.

Let s be a variable assignment for M, and $x \in$ VAR not in $Px_1...x_n$.

$v_{M,s}((Px_1...x_n)^{@}) = 1.$

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s}(Px_1...x_n) = 1.$

$\Updownarrow$

$< den_{M,s}(x_1), ..., den_{M,s}(x_n) > \in \mathrm{I}(P).$

$\Updownarrow$ [$\forall 1 \leq i \leq n, x \neq x_i$, so $den_{M,s}(x_i) = den_{M,s_{[w_@/x]}}(x_i)$.]

$< den_{M,s_{[w_@/x]}}(x_1), ..., den_{M,s_{[w_@/x]}}(x_n) > \in \mathrm{I}(P).$

$\Updownarrow$

$v_{M,s_{[w_@/x]}}(Px_1...x_n) = 1.$

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s_{[w_@/x]}}((Px_1...x_n)^x) = 1.$

Clause (ii) for $=$ proceeds in a similar fashion to (i).

Inductive Step

IH: Let ψ, χ be L-formulas such that:

> For every variable assignment s for M, and $x \in$ VAR not in ψ, $v_{M,s}(\psi^{@}) = v_{M,s_{[w_@/x]}}(\psi^x)$.
>
> For every variable assignment s for M, and $x \in$ VAR not in χ, $v_{M,s}(\chi^{@}) = v_{M,s_{[w_@/x]}}(\chi^x)$.

(iii) ϕ is $\sim\psi$.

Let s be a variable assignment for M, and $x \in$ VAR not in $\sim\psi$.

$v_{M,s}((\sim\psi)^{@}) = 1.$

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s}(\sim(\psi^{@})) = 1.$

$\Updownarrow$

$v_{M,s}(\psi^{@}) = 0.$

$\Updownarrow$ [IH]

$v_{M,s_{[w_@,x]}}(\psi^x) = 0.$

$\Updownarrow$

$v_{M,s_{[w_@/x]}}(\sim(\psi^x)) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[w_@/x]}}((\sim\psi)^x) = 1$

Clauses (iv)-(vii) for $\wedge, \vee, \rightarrow, \leftrightarrow$ proceed in a similar fashion to (iii).

(viii) ϕ is $\exists y\psi$.

Let s be a variable assignment for M, and $x \in$ VAR not in $\exists y\psi$.

$v_{M,s}((\exists y\psi)^{@}) = 1.$

$\Updownarrow$

$v_{M,s}(\exists y(Iy@ \wedge \psi^{@})) = 1.$

$\Updownarrow$

For some $d \in$ D, $v_{M,s_{[d/y]}}((Iy@ \wedge \psi^{@})) = 1.$

$\Updownarrow$

For some $d :< d, w_@ >\in$ I(I), $v_{M,s_{[d/y]}}(\psi^{@}) = 1.$

$\Updownarrow$ [IH]

For some $d :< d, w_@ >\in$ I(I), $v_{M,s_{[d,w_@/y,x]}}(\psi^x) = 1.$

$\Updownarrow$

For some $d \in$ D, $v_{M,s_{[d,w_@/y,x]}}((Iyx \wedge \psi^x)) = 1.$

$\Updownarrow$

$v_{M,s_{[w_@/x]}}(\exists y(Iyx \wedge \psi^x)) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[w_@/x]}}((\exists y\psi)^x) = 1.$

Clause (ix) for $\forall$ proceeds in a similar fashion to (viii).

(x) ϕ is $\diamond\psi_{[x_1,\ldots,x_n]}$.

Let s be a variable assignment for M, and $x \in$ VAR not in $\diamond\psi$. Let $(\diamond\psi_{[x_1,\ldots,x_n]})^{@}$ and $(\diamond\psi_{[x_1,\ldots,x_n]})^x$ both be:

σ: $\exists u \exists y_1 ... \exists y_n (Wu \wedge Iy_1 u \wedge Cy_1 x_1 \wedge ... \wedge Iy_n u \wedge Cy_n x_n \wedge \psi(y_1, ..., y_n)^u)$

$v_{M,s}((\diamond\psi)^{@}) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s}(\sigma) = 1$.

$\Updownarrow$ [Either (a) x is one of $u, y_1, ..., y_n$ or (b) not. If (a), x is not free in σ (because of the initial x-quantifier occurrence). If (b), by (CT Variables), x is not free in $\psi(y_1, ..., y_n)^u$, so x is not free in σ. Then by (FO Agreement).]

$v_{M,s_{[w_@/x]}}(\sigma) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[w_@/x]}}((\diamond\psi)^x) = 1$.

Clause (xi) for $\Box$ proceeds in a similar fashion to (x). ■

Lemma 5 (CT Agreement 2)**.** For every standard FOM language L (without individual constants or **A**), 1968 L-structure M (with its corresponding CT L^{CT}-structure M^+), $w \in \mathrm{W}$, s of M, L-formula ϕ, and $x \in \mathrm{VAR}$ not in ϕ,

$v_{M,w,s}(\phi) = v_{M^+,s_{[w/x]}}(\phi^x)$.

Proof. Let L be a standard FOM language (without individual constants or **A**), and M a 1968 L-structure (with its corresponding CT L^{CT}-structure M^+). We show by induction on the definition of an L-formula that for every $w \in \mathrm{W}$, s for M, L-formula ϕ, and $x \in \mathrm{VAR}$ not in ϕ, $v_{M,w,s}(\phi) = v_{M^+,s_{[w/x]}}(\phi^x)$.

Base Cases

(i) ϕ is $Px_1...x_n$.

Let $w \in \mathrm{W}$, s be a variable assignment for M, and $x \in \mathrm{VAR}$ not in $Px_1...x_n$.

$v_{M,w,s}(Px_1...x_n) = 1$

$\Updownarrow$

$< s(x_1), ..., s(x_n) > \in \mathrm{I}(P)$.

$\Updownarrow$ [Defn M^+]

$< den_{M^+,s}(x_1), ..., den_{M^+,s}(x_n) > \in \mathrm{I}^+(P)$.

$\Updownarrow$ [$\forall 1 \leq i \leq n, x \neq x_i$, so $den_{M^+,s}(x_i) = den_{M^+,s_{[w/x]}}(x_i)$.]

$< den_{M^+,s_{[w/x]}}(x_1), ..., den_{M^+,s_{[w/x]}}(x_n) > \in \mathrm{I}^+(P)$.

$\Updownarrow$

$v_{M^+,s_{[w/x]}}(Px_1...x_n) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((Px_1...x_n)^x) = 1$.

(ii) ϕ is $x_1 = x_2$.

Let $w \in \mathrm{W}$, s be a variable assignment for M, and $x \in \mathrm{VAR}$ not in $x_1 = x_2$.

$v_{M,w,s}(x_1 = x_2) = 1$.

$\Updownarrow$

$s(x_1) = s(x_2)$.

$\Updownarrow$ [Defn M^+]

$den_{M^+,s}(x_1) = den_{M^+,s}(x_2)$.

$\Updownarrow$ [$\forall 1 \leq i \leq n, x \neq x_i$, so $den_{M^+,s}(x_i) = den_{M^+,s_{[w/x]}}(x_i)$.]

$den_{M^+,s_{[w/x]}}(x_1) = den_{M^+,s_{[w/x]}}(x_2)$.

$\Updownarrow$

$v_{M^+,s_{[w/x]}}(x_1 = x_2) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((x_1 = x_2)^x) = 1.$

Inductive Step

IH: Let ψ, χ be L-formulas such that:

> For every $w \in$ W, variable assignment s for M, and $x \in$ VAR not in ψ, $v_{M,w,s}(\psi) = v_{M^+,s_{[w/x]}}(\psi^x)$.
>
> For every $w \in$ W, variable assignment s for M, and $x \in$ VAR not in χ, $v_{M,w,s}(\chi) = v_{M^+,s_{[w/x]}}(\chi^x)$.

(iii) ϕ is $\sim\psi$.

Let $w \in$ W, s be a variable assignment for M, and $x \in$ VAR not in $\sim\psi$.

$v_{M,w,s}(\sim\psi) = 1.$

$\Updownarrow$

$v_{M,w,s}(\psi) = 0.$

$\Updownarrow$ [As x is not in ψ, and by IH.]

$v_{M^+,s_{[w/x]}}(\psi^x) = 0.$

$\Updownarrow$

$v_{M^+,s_{[w/x]}}(\sim(\psi^x)) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((\sim\psi)^x) = 1.$

(iv) ϕ is $(\psi \wedge \chi)$.

Let $w \in$ W, s be a variable assignment for M, and $x \in$ VAR not in $(\psi \wedge \chi)$.

$v_{M,w,s}((\psi \wedge \chi)) = 1.$

$\Updownarrow$

$v_{M,w,s}(\psi) = 1$ and $v_{M,w,s}(\chi) = 1.$

$\Updownarrow$ [As x is not in ψ or χ, and by IH.]

$v_{M^+,s_{[w/x]}}(\psi^x) = 1$ and $v_{M^+,s_{[w/x]}}(\chi^x) = 1$

$\Updownarrow$

$v_{M^+,s_{[w/x]}}((\psi^x \wedge \chi^x)) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((\psi \wedge \chi)^x) = 1.$

Clauses (v)-(vii) for $\vee, \rightarrow, \leftrightarrow$ proceed in a similar fashion to (iv).

(viii) ϕ is $\exists y\psi$.

Let $w \in$ W, s be a variable assignment for M, and $x \in$ VAR not in $\exists y\psi$.

$v_{M,w,s}(\exists y\psi) = 1.$

$\Updownarrow$

For some $d \in \mathrm{dom}(w)$, $v_{M,w,s_{[d/y]}}(\psi) = 1.$

$\Updownarrow$ [As x is not in ψ, and by IH.]

For some $d \in \mathrm{dom}(w)$, $v_{M^+,s_{[d,w/y,x]}}(\psi^x) = 1.$

$\Updownarrow$ [Defn M^+]

For some $d \in \mathrm{D}^+$, $< d, w > \in \mathrm{I}^+(I)$ and $v_{M^+,s_{[d,w/y,x]}}(\psi^x) = 1.$

$\Updownarrow$

For some $d \in \mathrm{D}^+$, $v_{M^+,s_{[d,w/y,x]}}((Iyx \wedge \psi^x)) = 1.$

$\Updownarrow$

$v_{M^+,s_{[w/x]}}(\exists y(Iyx \wedge \psi^x)) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((\exists y\psi)^x) = 1.$

Clause (ix) for $\forall$ proceeds in a similar fashion to (viii).

(x) ϕ is $\diamond\psi_{[x_1,...,x_n]}$.

Let $w \in$ W, s be a variable assignment for M, and $x \in$ VAR not in $\diamond\psi$. Let $(\diamond\psi_{[x_1,...,x_n]})^x$ be:

σ: $\exists u \exists y_1 ... \exists y_n (Wu \wedge Iy_1u \wedge Cy_1x_1 \wedge ... \wedge Iy_nu \wedge Cy_nx_n \wedge \psi(y_1, ..., y_n)^u)$

$v_{M,w,s}(\diamond\psi) = 1$

$\Updownarrow$

For some $w^* \in$ W, for some $d_1, ..., d_n \in \mathrm{dom}(w^*)$ such that
$< d_1, s(x_1) >, ..., < d_n, s(x_n) > \in$ C,

$v_{M,w^*,s_{[d_1,...,d_n/x_1,...,x_n]}}(\psi) = 1.$

$\Updownarrow$ [I appeal to the following lemma: for every 1968 L-structure M, assignment s of M, $w \in$ W, L-formula $\phi_{[x_1,...,x_n]}$, distinct (from each other) $y_1, ..., y_n \in$ VAR substitutable for $x_1, ..., x_n$ in ϕ, and $d_1, ..., d_n \in$ D,
$v_{M,w,s_{[d_1,...,d_n/x_1,...,x_n]}}(\phi) = v_{M,w,s_{[d_1,...,d_n/y_1,...,y_n]}}(\phi(y_1, ..., y_n))$.]

For some $w^* \in$ W, for some $d_1, ..., d_n \in \text{dom}(w^*)$ such that
$< d_1, s(x_1) >, ..., < d_n, s(x_n) > \in$ C,

$v_{M,w^*,s_{[d_1,...,d_n/y_1,...,y_n]}}(\psi(y_1, ..., y_n)) = 1.$

$\Updownarrow$ [By the defn $(.)^\alpha$, u is not in $\psi(y_1, ..., y_n)$. Then by IH.]

For some $w^* \in$ W, for some $d_1, ..., d_n \in \text{dom}(w^*)$ such that
$< d_1, s(x_1) >, ..., < d_n, s(x_n) > \in$ C,

$v_{M^+,s_{[d_1,...,d_n,w^*/y_1,...,y_n,u]}}(\psi(y_1, ..., y_n)^u) = 1.$

$\Updownarrow$

For some $w^*, d_1, ..., d_n \in \text{D}^+$, $w^* \in \text{I}^+(W)$, $< d_1, w^* >, ..., < d_n, w^* > \in \text{I}^+(I)$,
$< d_1, s(x_1) >, ..., < d_n, s(x_n) > \in \text{I}^+(C)$, and

$v_{M^+,s_{[d_1,...,d_n,w^*/y_1,...,y_n,u]}}(\psi(y_1, ..., y_n)^u) = 1.$

$\Updownarrow$

For some $w^*, d_1, ..., d_n \in \text{D}^+$,

$v_{M^+,s_{[d_1,...,d_n,w^*/y_1,...,y_n,u]}}((Wu \wedge Iy_1u \wedge Cy_1x_1 \wedge ... \wedge Iy_nu \wedge Cy_nx_n \wedge$
$\psi(y_1, ..., y_n)^u)) = 1.$

$\Updownarrow$

$v_{M^+,s}(\sigma) = 1.$

$\Updownarrow$ [Either (a) x is one of $u, y_1, ..., y_n$ or (b) not. If (a), x is not free in σ (because of the initial x-quantifier occurrence). If (b), by (CT Variables), x is not free in $\psi(y_1, ..., y_n)^u$, so x is not free in σ. Then by (FO Agreement).]

$v_{M^+,s_{[w/x]}}(\sigma) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((\Diamond\psi)^x) = 1.$

Clause (xi) for $\Box$ proceeds in a similar fashion to (x). ■

A.1.2 Theorems

Finally, we prove our theorems:

Theorem 2. For every standard FOM language L (without individual constants or **A**), 1968 L-structure M (with its corresponding CT L^{CT}-structure M^+), and L-sentence ϕ,

$v_M(\phi) = v_{M^+}(\phi)$.

Proof. Let L be a standard FOM language (without individual contants or **A**), M a 1968 L-structure (with its corresponding CT L^{CT}-structure M^+), and ϕ an L-sentence.

$v_M(\phi) = 1$

$\Updownarrow$

For every s of M, $v_{M,w_@,s}(\phi) = 1$

$\Updownarrow$ [(CT Agreement 2) and defn M^+]

For every s of M^+, for every $x \in$ VAR not in ϕ, $v_{M^+,s_{[w_@/x]}}(\phi^x) = 1$.

$\Updownarrow$ [(CT Agreement 1)]

For every s of M^+, $v_{M^+,s}(\phi^@) = 1$.

$\Updownarrow$

$v_{M^+}(\phi^@) = 1$.

$\Updownarrow$ [Defn v_{M^+}: Sen(L) $\rightarrow \{1, 0\}$]

$v_{M^+}(\phi) = 1$. ■

Theorem 3. For every standard FOM language L (without individual constants or **A**),

$$\models_{\mathrm{DCT}} \;=\; \models_{\mathrm{ICT}}$$

Proof. Follows straightforwardly from theorem 2. ■

A.2 Constant Domain Semantics

A.2.1 Lemmas

We have:

Lemma 6 (CD Variables)**.** For every standard FOM language L, L-formula ϕ, $x \in$ VAR, and $\alpha \in$ VAR $\cup\{@\}$,

[(i) x is not free in ϕ, and (ii) $x \neq \alpha$] $\Rightarrow$ x is not free in ϕ^α.

Proof. Let L be a standard FOM language. We straightforwardly show by induction on the definition of an L-formula that for every L-formula ϕ, $x \in$ VAR, and $\alpha \in$ VAR $\cup\{@\}$, [(i) x is not free in ϕ, and (ii) $x \neq \alpha$] $\Rightarrow$ x is not free in ϕ^α. ■

Next:

Lemma 7 (CD Agreement 1)**.** For every standard FOM language L, CD L^{CD}-structure M, $d \in$ D, variable assignment s of M, L-formula ϕ, and $x_1, x_2 \in$ VAR not in ϕ,

$v_{M,s_{[d/x_1]}}(\phi^{x_1}) = v_{M,s_{[d/x_2]}}(\phi^{x_2})$.

Proof. Let L be a standard FOM language, and M a CD L^{CD}-structure. We show by induction on the definition of an L-formula that for every $d \in$ D, variable assignment s for M, L-formula ϕ, and $x_1, x_2 \in$ VAR not in ϕ, $v_{M,s_{[d/x_1]}}(\phi^{x_1}) = v_{M,s_{[d/x_2]}}(\phi^{x_2})$.

Base Cases

(i) ϕ is $Pt_1...t_n$.

Let $d \in$ D, s be a variable assignment for M, and $x_1, x_2 \in$ VAR not in $Pt_1..t_n$.

$v_{M,s_{[d/x_1]}}((Pt_1...t_n)^{x_1}) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_1]}}(P^+t_1...t_nx_1) = 1$.

$\Updownarrow$

$< den_{M,s_{[d/x_1]}}(t_1), ..., den_{M,s_{[d/x_1]}}(t_n), den_{M,s_{[d/x_1]}}(x_1) > \in \mathrm{I}(P^+)$.

$\Updownarrow$ [$\forall 1 \leq i \leq n, x_1 \neq t_i$ and $x_2 \neq t_i$, so $den_{M,s_{[d/x_1]}}(t_i) = den_{M,s_{[d/x_2]}}(t_i)$.]

$< den_{M,s_{[d/x_2]}}(t_1), ..., den_{M,s_{[d/x_2]}}(t_n), den_{M,s_{[d/x_2]}}(x_2) > \in \mathrm{I}(P^+)$.

$\Updownarrow$

$v_{M,s_{[d/x_2]}}(P^+t_1...t_nx_2) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_2]}}((Pt_1...t_n)^{x_2}) = 1.$

(ii) ϕ is $t_1 = t_2$.

Let $d \in$ D, s be a variable assignment for M, and $x_1, x_2 \in$ VAR not in $t_1 = t_2$.

$v_{M,s_{[d/x_1]}}((t_1 = t_2)^{x_1}) = 1.$

$\Updownarrow$

$v_{M,s_{[d/x_1]}}(t_1 = t_2) = 1.$

$\Updownarrow$

$den_{M,s_{[d/x_1]}}(t_1) = den_{M,s_{[d/x_1]}}(t_2).$

$\Updownarrow$ [$\forall 1 \leq i \leq n, x_1 \neq t_i$ and $x_2 \neq t_i$, so $den_{M,s_{[d/x_1]}}(t_i) = den_{M,s_{[d/x_2]}}(t_i)$.]

$den_{M,s_{[d/x_2]}}(t_1) = den_{M,s_{[d/x_2]}}(t_2).$

$\Updownarrow$

$v_{M,s_{[d/x_2]}}(t_1 = t_2) = 1.$

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s_{[d/x_2]}}((t_1 = t_2)^{x_2}) = 1.$

Inductive Step

IH: Let ψ, χ be L-formulas such that:

> For every $d \in$ D, variable assignment s for M, and $x_1, x_2 \in$ VAR not in ψ, $v_{M,s_{[d/x_1]}}(\psi^{x_1}) = v_{M,s_{[d/x_2]}}(\psi^{x_2})$.
>
> For every $d \in$ D, variable assignment s for M, and $x_1, x_2 \in$ VAR not in χ, $v_{M,s_{[d/x_1]}}(\chi^{x_1}) = v_{M,s_{[d/x_2]}}(\chi^{x_2})$.

(iii) ϕ is $\sim\psi$.

Let $d \in$ D, s be a variable assignment for M, and $x_1, x_2 \in$ VAR not in $\sim\psi$.

$v_{M,s_{[d/x_1]}}((\sim\psi)^{x_1}) = 1.$

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s_{[d/x_1]}}(\sim(\psi^{x_1})) = 1.$

$\Updownarrow$

$v_{M,s_{[d/x_1]}}(\psi^{x_1}) = 0.$

$\Updownarrow$ [IH]

$v_{M,s_{[d/x_2]}}(\psi^{x_2}) = 0.$

$\Updownarrow$

$v_{M,s_{[d/x_2]}}(\sim(\psi^{x_2})) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_2]}}((\sim\psi)^{x_2}) = 1.$

(iv) ϕ is $(\psi \wedge \chi)$.

Let $d \in$ D, s be a variable assignment for M, and $x_1, x_2 \in$ VAR not in $(\psi \wedge \chi)$.

$v_{M,s_{[d/x_1]}}((\psi \wedge \chi)^{x_1}) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_1]}}((\psi^{x_1} \wedge \chi^{x_1})) = 1.$

$\Updownarrow$

$v_{M,s_{[d/x_1]}}(\psi^{x_1}) = v_{M,s_{[d/x_1]}}(\psi^{x_1}) = 1.$

$\Updownarrow$ [IH]

$v_{M,s_{[d/x_2]}}(\psi^{x_2}) = v_{M,s_{[d/x_2]}}(\psi^{x_2}) = 1.$

$\Updownarrow$

$v_{M,s_{[d/x_2]}}((\psi^{x_2} \wedge \chi^{x_2})) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_2]}}((\psi \wedge \chi)^{x_2}) = 1.$

Clauses (v)-(vii) proceed in a similar fashion to (iv).

(viii) ϕ is $\exists y\psi$.

Let $d \in$ D, s be a variable assignment for M, and $x_1, x_2 \in$ VAR not in $\exists y\psi$.

$v_{M,s_{[d/x_1]}}((\exists y\psi)^{x_1}) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_1]}}(\exists y(Dy \wedge \psi^{x_1})) = 1.$

$\Updownarrow$

For some $d^+ \in$ I(D), $v_{M,s_{[d,d^+/x_1,y]}}(\psi^{x_1}) = 1.$

$\Updownarrow$ [IH]

For some $d^+ \in$ I(D), $v_{M,s_{[d,d^+/x_2,y]}}(\psi^{x_2}) = 1.$

$\Updownarrow$

$v_{M,s_{[d/x_2]}}(\exists y(Dy \wedge \psi^{x_2})) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_2]}}((\exists y\psi)^{x_2}) = 1.$

Clause (ix) for $\forall$ proceeds in a similar fashion to (viii).

(x) ϕ is $\diamond\psi$.

Let $d \in$ D, s be a variable assignment for M, and $x_1, x_2 \in$ VAR not in $\diamond\psi$. Let:

$(\diamond\psi)^{x_1}$ be $\exists y_1(Wy_1 \wedge Rx_1y_1 \wedge \psi^{y_1})$.

$(\diamond\psi)^{x_2}$ be $\exists y_2(Wy_2 \wedge Rx_2y_2 \wedge \psi^{y_2})$.

$v_{M,s_{[d/x_1]}}((\diamond\psi)^{x_1}) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_1]}}(\exists y_1(Wy_1 \wedge Rx_1y_1 \wedge \psi^{y_1})) = 1$.

$\Updownarrow$

For some $w \in$ D, $v_{M,s_{[w,d/y_1,x_1]}}((Wy_1 \wedge Rx_1y_1 \wedge \psi^{y_1})) = 1$.

$\Updownarrow$

For some $w \in$ I(W) such that $< d, w > \in$ I(R), $v_{M,s_{[d,w/x_1,y_1]}}(\psi^{y_1}) = 1$.

$\Updownarrow$ [By defn $(.)^\alpha$, $x_1 \neq y_1$. As x_1 is also not in ψ, by (CD) variables, x_1 is not free in ψ^{y_1}. Then by (FO Agreement).]

For some $w \in$ I(W) such that $< d, w > \in$ I(R), $v_{M,s_{[w/y_1]}}(\psi^{y_1}) = 1$.

$\Updownarrow$ [IH]

For some $w \in$ I(W) such that $< d, w > \in$ I(R), $v_{M,s_{[w/y_2]}}(\psi^{y_2}) = 1$.

$\Updownarrow$ [By defn $(.)^\alpha$, $x_2 \neq y_2$. As x_2 is also not in ψ, by (CD) variables, x_2 is not free in ψ^{y_2}. Then by (FO Agreement).]

For some $w \in$ I(W) such that $< d, w > \in$ I(R), $v_{M,s_{[d,w/x_2,y_2]}}(\psi^{y_2}) = 1$.

$\Updownarrow$

For some $w \in$ D, $v_{M,s_{[w,d/y_2,x_2]}}((Wy_2 \wedge Rx_2y_2 \wedge \psi^{y_2})) = 1$.

$\Updownarrow$

$v_{M,s_{[d/x_2]}}(\exists y_2(Wy_2 \wedge Rx_2y_2 \wedge \psi^{y_2})) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_2]}}((\diamond\psi)^{x_2}) = 1$.

Clause (xi) for $\Box$ proceeds in a similar fashion to (x).

(xii) ϕ is $\mathbf{A}\psi$.

Let $d \in$ D, s be a variable assignment for M, and $x_1, x_2 \in$ VAR not in $\mathbf{A}\psi$.

$v_{M,s_{[d/x_1]}}((\mathbf{A}\psi)^{x_1}) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_1]}}(\psi^{@}) = 1$.

$\Updownarrow$ [$\forall 1 \leq i \leq 2, x_i \neq @$ and x_i is not in ψ. By (CD Variables), x_i is not free in $\psi^{@}$. Then by (FO Agreement).]

$v_{M,s_{[d/x_2]}}(\psi^{@}) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[d/x_2]}}((\mathbf{A}\psi)^{x_2}) = 1$. ■

Next:

Lemma 8 (CD Agreement 2)**.** For every standard FOM language L, CD L^{CD}-structure M, variable assignment s of M, L-formula ϕ, and $x \in$ VAR not in ϕ,

$v_{M,s}(\phi^{@}) = v_{M,s_{[w_{@}/x]}}(\phi^{x})$.

Proof. Let L be a standard FOM language, and M a CD L^{CD}-structure. We show by induction on the definition of an L-formula that for every variable assignment s of M, L-formula ϕ, and $x \in$ VAR not in ϕ,
$v_{M,s}(\phi^{@}) = v_{M,s_{[w_{@}/x]}}(\phi^{x})$.

Base Cases

(i) ϕ is $Pt_1...t_n$.

Let s be a variable assignment for M, and $x \in$ VAR not in $Pt_1...t_n$.

$v_{M,s}((Pt_1...t_n)^{@}) = 1$.

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s}(P^{+}t_1...t_n@) = 1$.

$\Updownarrow$

$< den_{M,s}(t_1), ..., den_{M,s}(t_n), den_{M,s}(@) > \in \mathrm{I}(P^{+})$.

$\Updownarrow$ [$\forall 1 \leq i \leq n, x \neq t_i$, so $den_{M,s}(t_i) = den_{M,s_{[w_{@}/x]}}(t_i)$.]

$< den_{M,s_{[w_{@}/x]}}(t_1), ..., den_{M,s_{[w_{@}/x]}}(t_n), den_{M,s_{[w_{@}/x]}}(x) > \in \mathrm{I}(P^{+})$.

$\Updownarrow$

$v_{M,s_{[w_{@}/x]}}(P^{+}t_1...t_nx) = 1$.

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s_{[w_{@}/x]}}((Pt_1...t_n)^{x}) = 1$.

Clause (ii) for $=$ proceeds in a similar fashion to (i).

Inductive Step

IH: Let ψ, χ be L-formulas such that:

> For every variable assignment s for M, and $x \in$ VAR not in ψ, $v_{M,s}(\psi^{@}) = v_{M,s_{[w_{@}/x]}}(\psi^{x})$.
>
> For every variable assignment s for M, and $x \in$ VAR not in χ, $v_{M,s}(\chi^{@}) = v_{M,s_{[w_{@}/x]}}(\chi^{x})$.

(iii) ϕ is $\sim\psi$.

Let s be a variable assignment for M, and $x \in$ VAR not in $\sim\psi$.

$v_{M,s}((\sim\psi)^{@}) = 1.$

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s}(\sim(\psi^{@})) = 1.$

$\Updownarrow$

$v_{M,s}(\psi^{@}) = 0.$

$\Updownarrow$ [IH]

$v_{M,s_{[w_{@},x]}}(\psi^{x}) = 0.$

$\Updownarrow$

$v_{M,s_{[w_{@}/x]}}(\sim(\psi^{x})) = 1.$

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s_{[w_{@}/x]}}((\sim\psi)^{x}) = 1$

Clauses (iv)-(vii) for $\wedge, \vee, \rightarrow, \leftrightarrow$ proceed in a similar fashion to (iii).

(viii) ϕ is $\exists y\psi$.

Let s be a variable assignment for M, and $x \in$ VAR not in $\exists y\psi$.

$v_{M,s}((\exists y\psi)^{@}) = 1.$

$\Updownarrow$

$v_{M,s}(\exists y(Dy \wedge \psi^{@})) = 1.$

$\Updownarrow$

For some $d \in \mathrm{I}(D)$, $v_{M,s_{[d/y]}}(\psi^{@}) = 1.$

$\Updownarrow$ [IH]

For some $d \in \mathrm{I}(D)$, $v_{M,s_{[d,w_{@}/y,x]}}(\psi^{x}) = 1.$

$\Updownarrow$

$v_{M,s_{[w_{@}/x]}}(\exists y(Dy \wedge \psi^{x})) = 1.$

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s_{[w_{@}/x]}}((\exists y\psi)^{x}) = 1.$

Clause (ix) for $\forall$ proceeds in a similar fashion to (viii).

(x) ϕ is $\diamond\psi$.

Let s be a variable assignment for M, and $x \in$ VAR not in $\diamond\psi$. Let:

$(\diamond\psi)^{@}$ be $\exists y_1(Wy_1 \wedge R@y_1 \wedge \psi^{y_1})$.

$(\diamond\psi)^{x}$ be $\exists y_2(Wy_2 \wedge Rxy_2 \wedge \psi^{y_2})$.

$v_{M,s}((\diamond\psi)^{@}) = 1$.

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s}(\exists y_1(Wy_1 \wedge R@y_1 \wedge \psi^{y_1})) = 1$.

$\Updownarrow$

For some $w \in$ D, $v_{M,s_{[w/y_1]}}((Wy_1 \wedge R@y_1 \wedge \psi^{y_1})) = 1$.

$\Updownarrow$

For some $w \in$ I(W) such that $< w_{@}, w >\in$ I(R), $v_{M,s_{[w/y_1]}}(\psi^{y_1}) = 1$.

$\Updownarrow$ [By defn $(.)^{\alpha}$, y_1 and y_2 are not in ψ. Then by (CD Agreement 1).]

For some $w \in$ I(W) such that $< w_{@}, w >\in$ I(R), $v_{M,s_{[w/y_2]}}(\psi^{y_2}) = 1$.

$\Updownarrow$ [By defn $(.)^{\alpha}$, $x \neq y_2$. As x is also not in ψ, by (CD Variables), x is not free in ψ^{y_2}. Then by (FO Agreement).]

For some $w \in$ I(W) such that $< w_{@}, w >\in$ I(R), $v_{M,s_{[w,w_{@}/y_2,x]}}(\psi^{y_2}) = 1$.

$\Updownarrow$

For some $w \in$ D, $v_{M,s_{[w,w_{@}/y_2,x]}}((Wy_2 \wedge Rxy_2 \wedge \psi^{y_2})) = 1$.

$\Updownarrow$

$v_{M,s_{[w_{@}/x]}}(\exists y_2(Wy_2 \wedge Rxy_2 \wedge \psi^{y_2})) = 1$.

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s_{[w_{@}/x]}}((\diamond\psi)^{x}) = 1$.

Clause (xi) for $\Box$ proceeds in a similar fashion to (x).

(xii) ϕ is $\mathbf{A}\psi$.

Let s be a variable assignment for M, and $x \in$ VAR not in $\mathbf{A}\psi$.

$v_{M,s}((\mathbf{A}\psi)^{@}) = 1$.

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s}(\psi^{@}) = 1$.

$\Updownarrow$ [As x is not in ψ, and $x \neq @$, by (CD Variables), x is not free in $\psi^{@}$. Then by (FO Agreement).]

$v_{M,s_{[w_{@}/x]}}(\psi^{@}) = 1$.

$\Updownarrow$ [Defn $(.)^{\alpha}$]

$v_{M,s_{[w_{@}/x]}}((\mathbf{A}\psi)^{x}) = 1$. ■

Next:

Lemma 9 (CD Agreement 3)**.** For every standard FOM language L, constant domain L-structure M and its corresponding CD L^{CD}-structure M^+, $w \in \mathrm{W}$, variable assignment s for M, L-formula ϕ, and $x \in \mathrm{VAR}$ not in ϕ,

$v_{M,w,s}(\phi) = v_{M^+,s_{[w/x]}}(\phi^x)$.

Proof. Let L be a standard FOM language, M a constant domain L-structure, and M^+ its corresponding CD L^{CD}-structure. We show by induction on the definition of an L-formula that for every $w \in \mathrm{W}$, variable assignment s for M, L-formula ϕ, and $x \in \mathrm{VAR}$ not in ϕ, $v_{M,w,s}(\phi) = v_{M^+,s_{[w/x]}}(\phi^x)$.

Base Cases

(i) ϕ is $Pt_1...t_n$.

Let $w \in \mathrm{W}$, s be a variable assignment for M, and $x \in \mathrm{VAR}$ not in $Pt_1...t_n$.

$v_{M,w,s}(Pt_1...t_n) = 1$

$\Updownarrow$

$< den_{M,s}(t_1), ..., den_{M,s}(t_n) > \in \mathrm{I}(P)(w)$.

$\Updownarrow$ [Defn M^+]

$< den_{M^+,s}(t_1), ..., den_{M^+,s}(t_n), w > \in \mathrm{I}^+(P^+)$.

$\Updownarrow$ [As x is not in $Pt_1...t_n$, $\forall 1 \leq i \leq n, t_i \neq x$.]

$< den_{M^+,s_{[w/x]}}(t_1), ..., den_{M^+,s_{[w/x]}}(t_n), den_{M^+,s_{[w/x]}}(x) > \in \mathrm{I}^+(P^+)$.

$\Updownarrow$

$v_{M^+,s_{[w/x]}}(P^+t_1...t_nx) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((Pt_1...t_n)^x) = 1$.

(ii) ϕ is $t_1 = t_2$.

Let $w \in \mathrm{W}$, s be a variable assignment for M, and $x \in \mathrm{VAR}$ not in $t_1 = t_2$.

$v_{M,w,s}(t_1 = t_2) = 1$.

$\Updownarrow$

$den_{M,s}(t_1) = den_{M,s}(t_2)$.

$\Updownarrow$ [Defn M^+]

$den_{M^+,s}(t_1) = den_{M^+,s}(t_2)$.

$\Updownarrow$ [As x is not in $t_1 = t_2$, $\forall 1 \leq i \leq n, t_i \neq x$.]

$den_{M^+,s_{[w/x]}}(t_1) = den_{M^+,s_{[w/x]}}(t_2)$.

$\Updownarrow$

$v_{M^+,s_{[w/x]}}(t_1 = t_2) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((t_1 = t_2)^x) = 1.$

Inductive Step

IH: Let ψ, χ be L-formulas such that:

> For every $w \in$ W, variable assignment s for M, and $x \in$ VAR not in ψ, $v_{M,w,s}(\psi) = v_{M^+,s_{[w/x]}}(\psi^x)$.
>
> For every $w \in$ W, variable assignment s for M, and $x \in$ VAR not in χ, $v_{M,w,s}(\chi) = v_{M^+,s_{[w/x]}}(\chi^x)$.

(iii) ϕ is $\sim\psi$.

Let $w \in$ W, s be a variable assignment for M, and $x \in$ VAR not in $\sim\psi$.

$v_{M,w,s}(\sim\psi) = 1.$

$\Updownarrow$

$v_{M,w,s}(\psi) = 0.$

$\Updownarrow$ [As x is not in ψ, and by IH.]

$v_{M^+,s_{[w/x]}}(\psi^x) = 0.$

$\Updownarrow$

$v_{M^+,s_{[w/x]}}(\sim(\psi^x)) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((\sim\psi)^x) = 1.$

(iv) ϕ is $(\psi \wedge \chi)$.

Let $w \in$ W, s be a variable assignment for M, and $x \in$ VAR not in $(\psi \wedge \chi)$.

$v_{M,w,s}((\psi \wedge \chi)) = 1.$

$\Updownarrow$

$v_{M,w,s}(\psi) = 1$ and $v_{M,w,s}(\chi) = 1.$

$\Updownarrow$ [As x is not in ψ or χ, and by IH.]

$v_{M^+,s_{[w/x]}}(\psi^x) = 1$ and $v_{M^+,s_{[w/x]}}(\chi^x) = 1$

$\Updownarrow$

$v_{M^+,s_{[w/x]}}((\psi^x \wedge \chi^x)) = 1.$

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+,s_{[w/x]}}((\psi \wedge \chi)^x) = 1$.

Clauses (v)-(vii) for $\vee, \rightarrow, \leftrightarrow$ proceed in a similar fashion to (iv).

(viii) ϕ is $\exists y\psi$.

Let $w \in$ W, s be a variable assignment for M, and $x \in$ VAR not in $\exists y\psi$.

$v_{M,w,s}(\exists y\psi) = 1$.

$\Updownarrow$

For some $d \in$ D, $v_{M,w,s_{[d/y]}}(\psi) = 1$.

$\Updownarrow$ [As x is not in ψ, and by IH.]

For some $d \in$ D, $v_{M^+,s_{[d,w/y,x]}}(\psi^x) = 1$.

$\Updownarrow$ [Defn M^+]

For some $d \in \mathrm{I}^+(D)$, $v_{M^+,s_{[d,w/y,x]}}(\psi^x) = 1$.

$\Updownarrow$

For some $d \in \mathrm{D}^+$, $v_{M^+,s_{[d,w/y,x]}}((Dy \wedge \psi^x)) = 1$.

$\Updownarrow$

$v_{M,s_{[w/x]}}(\exists y(Dy \wedge \psi^x)) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M,s_{[w/x]}}((\exists y\psi)^x) = 1$.

Clause (ix) for $\forall$ proceeds in a similar fashion to (viii).

(x) ϕ is $\diamond\psi$.

Let $w \in$ W, s be a variable assignment for M, and $x \in$ VAR not in $\diamond\psi$. Let:

$(\diamond\psi)^x$ be $\exists u(Wu \wedge Rxu \wedge \psi^u)$.

$v_{M,w,s}(\diamond\psi) = 1$

$\Updownarrow$

For some $w^* \in$ W such that Rww^*, $v_{M,w^*,s}(\psi) = 1$.

$\Updownarrow$ [By defn $(.)^\alpha$, u is not in ψ. Then by IH.]

For some $w^* \in$ W such that Rww^*, $v_{M^+,s_{[w^*/u]}}(\psi^u) = 1$.

$\Updownarrow$ [By defn $(.)^\alpha$, $u \neq x$. By assumption, x is not in ψ. So, by (CD Variables), x is not free in ψ^u. Then by (FO Agreement).]

For some $w^* \in$ W such that Rww^*, $v_{M^+,s_{[w^*,w/u,x]}}(\psi^u) = 1$.

$\Updownarrow$ [Defn M^+]

For some $w^* \in \mathrm{D}^+$, $v_{M^+, s_{[w^*, w/u, x]}}((Wu \wedge Rxu \wedge \psi^u)) = 1$.

$\Updownarrow$

$v_{M^+, s_{[w/x]}}(\exists u(Wu \wedge Rxu \wedge \psi^u)) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+, s_{[w/x]}}((\diamond\psi)^x) = 1$.

Clause (xi) for $\Box$ proceeds in a similar fashion to (x).

(xii) ϕ is $\mathbf{A}\psi$.

Let $w \in \mathrm{W}$, s be a variable assignment for M, and $x \in \mathrm{VAR}$ not in $\mathbf{A}\psi$.

$v_{M,w,s}(\mathbf{A}\psi) = 1$.

$\Updownarrow$

$v_{M,w_@,s}(\psi) = 1$.

$\Updownarrow$ [As x is not in ψ, by IH.]

$v_{M^+, s_{[w_@/x]}}(\psi^x) = 1$.

$\Updownarrow$ [(CT Agreement 2)]

$v_{M^+,s}(\psi^@) = 1$.

$\Updownarrow$ [As x is not in ψ and $x \neq @$, by (CD Variables), x not free in $\psi^@$. Then by (FO Agreement).]

$v_{M^+, s_{[w/x]}}(\psi^@) = 1$.

$\Updownarrow$ [Defn $(.)^\alpha$]

$v_{M^+, s_{[w/x]}}((\mathbf{A}\psi)^x) = 1$. ■

A.2.2 Theorems

Finally, we prove our theorems:

Theorem 5. For every standard FOM language L, constant domain L-structure M (with its corresponding CD L^{CD}-structure M^+), variable assignment s of M, and L-formula ϕ,

$v_{M,s}(\phi) = v_{M^+,s}(\phi)$.[1]

Proof. Let L be a standard FOM language, M a constant domain L-structure (with its corresponding CD L^{CD}-structure M^+), s a variable assignment, and ϕ an L-formula.

$v_{M,s}(\phi) = 1$

$\Updownarrow$ [Defn $v_{M,s}$]

$v_{M,w_@,s}(\phi) = 1$.

$\Updownarrow$ [(CD Agreement 3)]

For every $x \in$ VAR not in ϕ, $v_{M^+,s_{[w_@/x]}}(\phi^x) = 1$.

$\Updownarrow$ [(CD Agreement 2)]

$v_{M^+,s}(\phi^@) = 1$.

$\Updownarrow$ [Defn $v_{M^+,s}$: Form(L) $\rightarrow \{1, 0\}$]

$v_{M^+,s}(\phi) = 1$. ■

Theorem 6. For every standard FOM language L, for the class F of all constant domain L-structures,

$$\models_{\mathrm{DCD(F)}} \;=\; \models_{\mathrm{ICD}}$$

Proof. Follows straightforwardly from theorem 5. ■

A.3 Varying Domain Semantics

We straightforwardly proceed in a similar fashion to the way the constant domain theorems were proved.

[1] Notice that the variable assignments of M are exactly the appropriate variable assignments of M^+.

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