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Citation: *Appl. Phys. Lett.* **111**, 043505 (2017); doi: 10.1063/1.4996180

View online: <http://dx.doi.org/10.1063/1.4996180>

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Non-destructive measurement and monitoring of separation of charged particle micro-bunches

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(Received 24 March 2017; accepted 14 July 2017; published online 25 July 2017)

Micro-bunched particle beams are used for a wide range of research including wakefield-based particle acceleration and tunable sources of radiation. In all applications, accurate and non-destructive monitoring of the bunch-to-bunch separation is required. With the development of femtosecond lasers, the generation of micro-bunched beams directly from a photocathode becomes routine; however, non-destructive monitoring of the separation is still a challenge. We present the results of proof-of-principle experiments conducted at the Laser Undulator Compact X-ray accelerator measuring the distance between micro-bunches via the amplitude modulation analysis of a monochromatic radiation signal. Good agreement with theoretical predictions is shown; limitations and further improvements are discussed. © 2017 Author(s). All article content, except where otherwise noted, is licensed under a Creative Commons Attribution (CC BY) license (<http://creativecommons.org/licenses/by/4.0/>). [<http://dx.doi.org/10.1063/1.4996180>]

“Pre-bunched” or “micro-bunched” charged particle beams have attracted significant interest during the last decade.^{1–14} Potential applications of such beams include the development of the next generation of light sources^{1–5} and particle accelerators.^{6–12} These research activities are driven by the continuous interest and demand for both high-power source of terahertz (THz) radiation^{15–18} and compact particle accelerators.^{6–12,18} In wakefield accelerators, for example, the wakefield is generated by a “driver” beam and can achieve GV/m accelerating gradients.^{6–12} The following “witness” bunch harvests the wakefield energy. The separation of the “driver” and the “witness” bunches is comparable with their length and essential to optimize the efficiency of the energy transfer from the driver to the witness bunch. On the other hand, to enable the THz radiation source tunability and its stable operation,^{1–5} the accurate knowledge of the distance between bunches is also required, as the spacing between micro-bunches defines the spectrum of the radiation.^{18–24} In both examples and in other applications driven by such beams, the development of a non-invasive, a single-shot system capable of monitoring the distance between micro-bunches is important, and in many cases, non-destructive, single-shot evaluation of the bunch-to-bunch distance is still an unresolved challenge. At present, the most popular techniques used to study micro-bunch separations are via measurements of autocorrelation functions^{2,18,19} and use of a transverse deflection cavity. The first requires multi-shot measurements while the latter is a single-shot but the destructive method.

In this letter, we demonstrate that measurements of the coherent Smith-Purcell radiation (cSPR) signal amplitude variations, generated by micro-bunches propagating above the grating, can be used to monitor and evaluate the distance between the micro-bunches. Smith-Purcell radiation is emitted when a charged particle propagates in a vicinity of a periodic structure (grating). The particle excites a surface current on the surface which emits radiation when scatters on the

grating’s discontinuities.^{21,22} One notes that a pre-bunched electron beam can generate different types of coherent radiation,^{1–5,18–24} which can also be used for the micro-bunched beam monitoring. The use of cSPR was due to cSPR monitor compactness and convenience.^{21,22} The coherent radiation frequency spectrum from a single femtosecond bunch is broadband (up to tens of THz),^{21–24} and for a single electron bunch consisting of N_e electrons, the energy generated at frequency ω into a solid angle $d\Omega$ is given by^{21–24}

$$d^2I/(d\omega d\Omega) \propto (d^2I_e/(d\omega d\Omega)) (N_e - 1)N_e|F(\omega)|^2, \quad (1)$$

where I is the energy emitted by the bunch, I_e is the energy emitted by a single electron, and $F(\omega)$ is the normalized “form factor,” i.e., Fourier transform of the bunch temporal profile.^{21–24} If a pre-bunched beam is used, i.e., the beam consisting of M micro-bunches separated by interval Δt , the $F(\omega)$ is

$$|F(\omega)|^2 = |F_1(\omega)|^2 G_M(\omega, \Delta t), \quad (2)$$

where $F_1(\omega)$ is the form factor of a single micro-bunch. The form factor $|F(\omega)|$, which is measured during the experiments, is modulated by an oscillating function $G_M(\omega, \Delta t)$ depending on the micro-bunch spacing Δt . We also note that the behavior of the modulation function will depend on the charge distribution and individual bunch spacing. In particular, non-uniform charge distribution or micro-bunch separation will affect, for example, the amplitude and depth of modulation. Assuming M identical micro-bunches, equally separated by a time Δt , the oscillating function $G_M(\omega, \Delta t)$ has the form $G_M(\omega, \Delta t) = \frac{\sin^2(M\omega\Delta t/2)}{M^2 \sin^2(\omega\Delta t/2)}$. If the measurements are made at a single, fixed frequency of interest ω_i , any changes in the interval Δt between micro-bunches will lead to amplitude variation of G_M ; this will be referred throughout the paper as multi-bunch signal modulation (MBSM). Here,

we demonstrate that the MBSM function can be observed via the measurement of the cSPR intensity modulation to monitor the bunch-to-bunch distance in the train consisting of two micro-bunches. If the electron bunches have different charges, as it would happen in the case of witness and driver bunches in wake-field experiments, a “visibility” function $\Gamma = (I_{\max} - I_{\min}) / (I_{\max} + I_{\min})$ can be introduced to monitor the charge distribution, where $I_{\max, \min}$ are the maximal and minimal values of the signal intensity, respectively [Fig. 3(a)]. If $\Gamma = 1$, the bunches have equal charges, i.e., the charge ratio $\eta = 1$; otherwise, if the ratio of the micro-bunch charges is not 1, the visibility function will be proportional to $\Gamma \approx 1 - \frac{4\eta \sin^2(\frac{\omega \Delta t}{2})}{(1+\eta)^2}$.

The proposed monitor is independent of the micro-bunch production method and can be applied at any facility capable of generating such trains. At present, the micro-bunched beam can be generated by a beam in the following techniques: modulation in a plasma channel,^{6–8,10–12} “slicing,”^{2,3,18} seeded instability,^{4,13,14} and directly from a photocathode.^{5,19,25,26} In all of these techniques, the final spacing between micro-bunches can vary from shot to shot. Here (for reasons of clarity only), we

will focus on the generation of the micro-bunches directly from a photocathode. In this case, there are several phenomena that can affect the bunch spacing, e.g., space-charge effects, compression, or velocity bunching due to different initial phases of the micro-bunches, and the time jitter between laser pulses and RF acceleration potential. Any variation in the time delay between laser pulses and the RF acceleration phase (phase jitter) will lead to the acceleration of micro-bunches in different potentials [Fig. 1(a)] and thus to deviations of the bunch-to-bunch separation from its initial value. To illustrate the challenges, an analytical model²⁷ can be used whereby the distance between two micro-bunches is defined as the distance between two single electrons, injected at different RF phases. Using this model and taking into account the electron initial phase ϕ_0 with respect to the RF accelerating field [indicated as ϕ_{RF} on Fig. 1(a)], the electron phase ϕ at the cavity exit is given as

$$\phi = k \int_0^z \left(\frac{\gamma(z', \phi_0)}{\sqrt{\gamma(z', \phi_0)^2 - 1}} - 1 \right) dz' + \phi_0, \quad (3)$$

where k is the wavenumber, γ is the Lorentz factor, i.e., indicates the electron energy. Taking into account (3), the distance

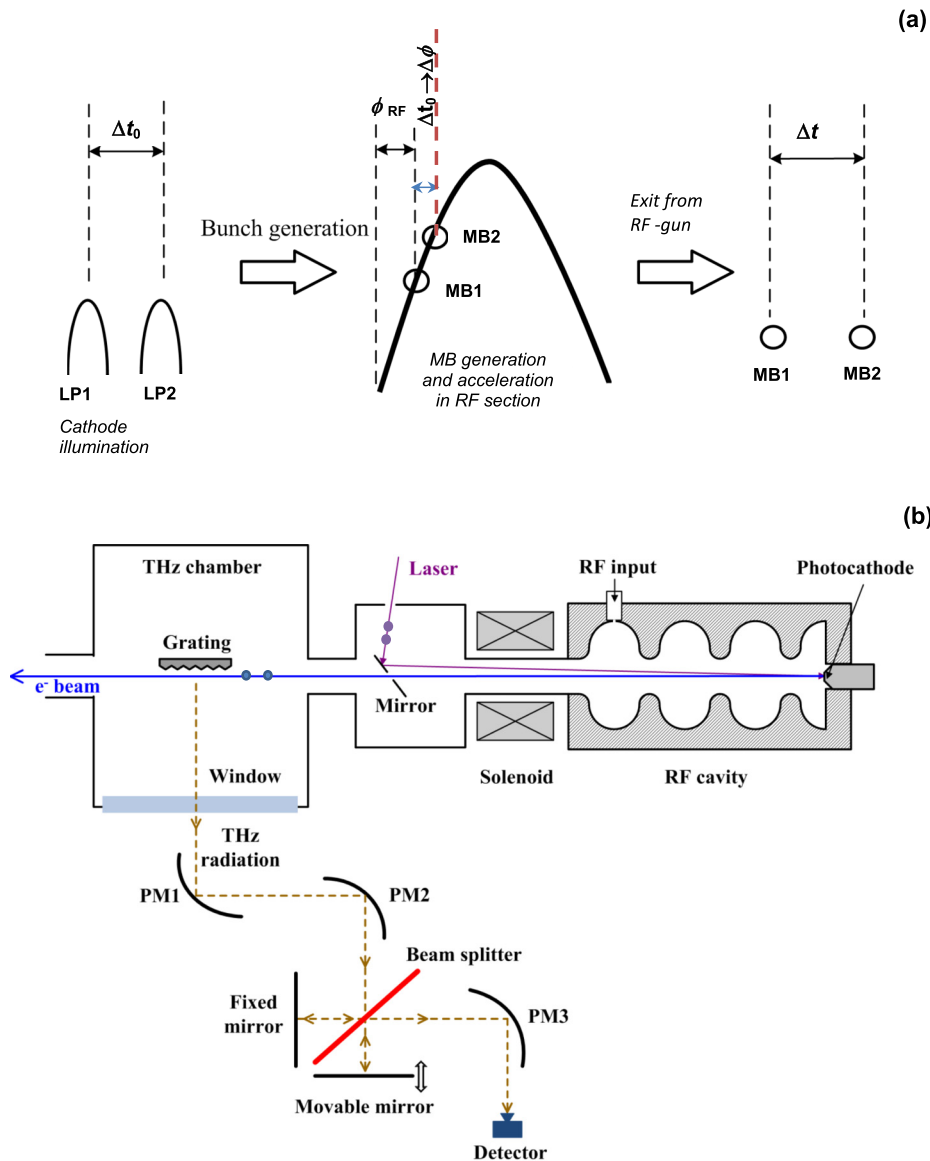


FIG. 1. (a) Schematic diagram of the generation and acceleration of two micro-bunches (MB1 and MB2) using two fs-long laser pulses (LP1 and LP2) separated by an initial time delay Δt_0 . (b) Schematic of the experimental set-up.

between micro-bunches at the exit from the photocathode can be calculated as $\Delta t = \Delta\phi/\omega_{rf}$ where ω_{rf} is the accelerator's operating frequency. The model assumes the knowledge of the initial phase ϕ_0 , which is an unknown parameter in the experiment and subject to the phase jitter, meaning that the only way to obtain the distance between the micro-bunches is to measure it.

First proof-of-principle experiments which demonstrate the monitor operation were performed at the Laser Undulator Compact X-ray source facility (LUCX) at KEK (High Energy Accelerator Research Organization, Japan).^{5,25,26} Two identical electron micro-bunches were generated directly from a Cs₂Te photocathode embedded in 3.6 cell RF cavity by illuminating it with a series of femtosecond laser pulses [see Fig. 1(b)]. The Gaussian-like micro-bunches (essentially identical) were emitted sequentially, and the initial distance between micro-bunches was determined by the laser pulse separation [Fig. 1(a)]. The micro-bunches' transverse dimensions were estimated to be $\sigma_{x,y} = 250 \mu\text{m}$ (well defined by the laser spot), the typical electron energy was 8 MeV, the single micro-bunch charge was around 30 pC (low space charge regime), the RF frequency was 2.856 GHz, and the accelerating field amplitude $E_0 = 80 \text{ MV/m}$. The beam parameters were measured using conventional beam diagnostics.^{25,26} There were no beam active dispersive/compressive elements between the gun and the interaction point, and by taking the distance between them to be around 1 m (drift space), a correction term to Δt , which accounts for the bunches' spatial dispersion due to energy variation, $\Delta t_{disp} = \frac{L}{c} \left(\frac{\Delta\beta}{\beta_1\beta_2} \right)$ (see Fig. 1), was added, where $\Delta\beta = \beta_2 - \beta_1$ and $\beta_{1,2} = v_{1,2}/c$ are the velocities of the first and second bunches, c is the speed of light. The experiments were carried out using a 1 mm period grating which was positioned 0.7 mm from the beam. The cSPr emerged from the vacuum chamber through the sapphire window. The signal was collected using a pair of off-axis parabolic mirrors, which were positioned to collect the radiation at 90° (normal to the grating). Initially, in order to confirm the cSPr generation, frequency measurements were carried out using a Michelson interferometer [Fig. 1(b)]; this was not used in subsequent measurements. A VDI WR2.2²⁸ zero bias detector was located at the focus of the third off-axis parabolic mirror. In Figs. 2 and 3, the measured signals were normalized to the square of the total charge in order to cancel shot-to-shot variations of the amplitudes due to variations of the beam charge. The radiation frequency was measured to be 300 GHz, as shown in Fig. 2. According to expression (2), the MBSM function G_2 for two identical micro-bunches is given by

$$G_2(\omega_i, \Delta t) = [(1 + \cos(\omega_i \Delta t))/2], \quad (4)$$

where ω_i is the measured frequency. To measure the signal modulation, the initial delay time Δt_0 [different from Δt in Eq. (4)] between the two laser pulses has been varied in the range from 0 ps to 20 ps for a set of RF accelerating phases ϕ_{RF} , and the intensity of the signal was measured. By changing Δt_0 , it was possible to determine the “reference point,” i.e., measuring a single bunch. In our case, $\Delta t_0 = 0$ was easy to realize, but any other reference point which can be independently confirmed can be used. The reference point is

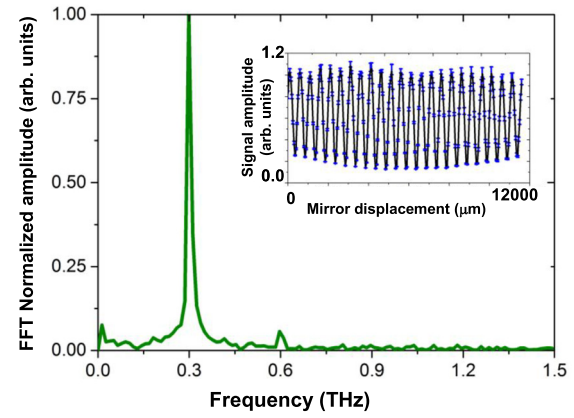


FIG. 2. The cSPr spectral line measured at 90° relative to the 1 mm period grating. The narrow line shown is very specific to the cSPr, and the inset to the figure shows the raw data interferogram observed using the interferometer shown in Fig. 1(b).

necessary for the evaluation of the bunch-to-bunch separation from the measured MBSM dependence [Fig. 3(a)]. Figure 3(a) shows the measured intensity (at $\omega_i = 300 \text{ GHz}$) as a function of the micro-bunch separation (dots with error bars) for four different RF phases ϕ_{RF} with the dashed lines showing prediction results [Eq. (4)]. At this stage, no assumption was made about how the bunches were generated and their dynamics prior to the measurements. Also, the experimental measurements were compared with the theoretical model Eq. (3) by taking into account the initial parameters. Both results, predictions (solid black lines) and measurements (dots with the error bars), are shown in Fig. 3(b). Figure 3(b) demonstrate the dependence of the distance between micro-bunches at the point of measurement on the initial interval Δt_0 . Each line in Fig. 3(b) corresponds to a specific phase ϕ_{RF} of the accelerating field, and an agreement between the results is clear.

In conclusion, we have used a non-destructive method of measuring the bunch-to-bunch separation in a two-bunch system and periodicity (i.e., Fourier harmonic) of a micro-bunch train by means of a single-frequency measurement of cSPr. Although the technique can determine the periodicity of the micro-bunch train, the measurement of the individual distances between the micro-bunches is a much more difficult issue, which would require an analysis of the full radiation spectrum, as described in Refs. 20–24. It is worth noting that in a single-frequency measurement, as described here, there is a limitation associated with the non-uniqueness of the oscillating MBSM function G_M which arises from the fact that a single value of this function corresponds to many values of the interval between micro-bunches [Fig. 3(a)]. This is a well-known problem,²⁹ which can be resolved by measuring G_M at two frequencies.^{29–31} Such a measurement at a second frequency can be performed using the same grating but with a second detector positioned at a different observation angle.^{21,22} The second measurement will provide a second curve, allowing the determination of the absolute bunch spacing over a larger unambiguous range as achieved in multi-wavelength interferometry systems.^{30,31} We note that in our experiment, the resolution limitations were linked to the detector properties and noise level. The maximum

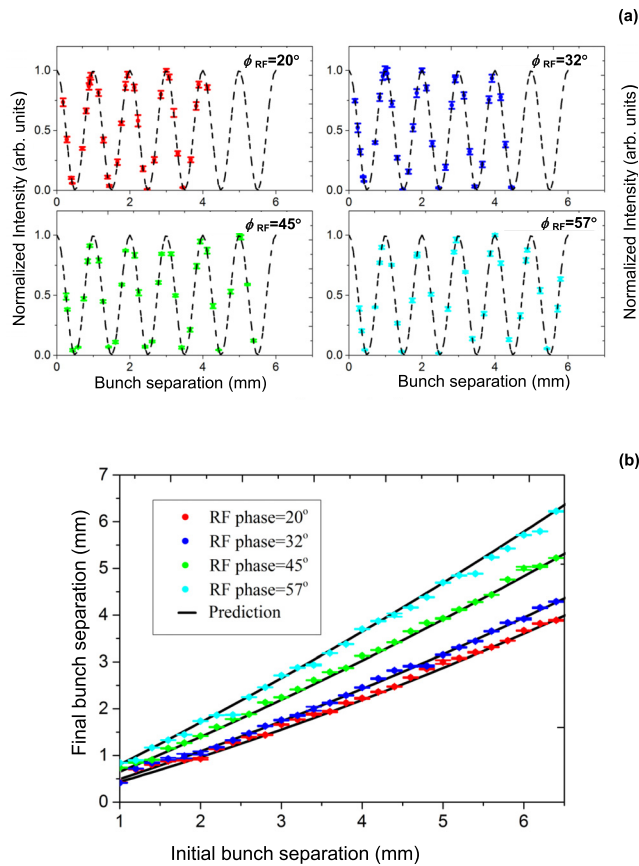


FIG. 3. (a) The measured (dots) spectral yield of coherent Smith-Purcell radiation as a function of the micro-bunch separation for the set of RF phases $\phi_{RF} = [20^\circ, 32^\circ, 45^\circ, 57^\circ]$; the results are superimposed on the predictions of Eq. (4) (dashed lines). (b) Graphs illustrating the dependence of the final bunch-bunch separation as measured (dots) and predicted using Eq. (3) (solid line) at the exit from the cavity on the initial the distance between the micro-bunches for $\phi_{RF} = [20^\circ, 32^\circ, 45^\circ, 57^\circ]$.

error of the normalized measured intensity was below 12%, leading to a maximum error of the separation measurements no larger than 5%. Using this method, the distance between the micro-bunches can be calculated during the experiment instantaneously from the measured intensity of the radiation in a single shot, non-destructive manner, assuming that other parameters are maintained the same. Although the results presented here were demonstrated for the case of two bunches generated directly from the photocathode, the same approach is applicable to any train of charged micro-bunches to evaluate the periodicity (i.e., main Fourier harmonic) of the bunch train. The proposed method can be used to provide feedback to control the sub-millimeter distances between the femtosecond long micro-bunches and, for example, to stabilize the operation of a THz oscillator or to position the witness bunch into the right phase of the wakefield.

The authors would like to acknowledge the partial support of the project from the STFC UK through PRD Grant No. ST/M003590/1, the Leverhulme Trust through the International Network Grant No. IN-2015-012 and the Photon and Quantum Basic Research Coordinated Development Program from the Ministry of Education, Culture, Sports, Science and Technology, Japan. H. Zhang would like to thank NUDT (China) for supporting his DPhil project.

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