Mix-and-Match Compatibility and Asymmetric Costs

Hunter Monroe

Balliol College

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ABSTRACT

This thesis analyzes how the ability of consumers to buy components of a system from different firms affects prices, profit margins, R&D effort, and welfare. It also examines firms' incentives to make their products compatible, that is, to allow consumers to mix-and-match different brands of components into systems.

Chapter 1 reviews the economic literature on product compatibility with motivating material drawn from the personal computer industry. Three strands of the literature study compatibility using definitions based on the ability of consumers to mix-and-match components, to capture externalities arising from networks, and to switch brands costlessly. The mix-and-match literature has found that compatibility raises prices compared with those under incompatibility in a variety of settings. In practice, however, compatible computers appear to be less expensive than incompatible computers, and computer buyers have promoted standardization.

Chapter 2 develops models of mix-and-match compatibility which make predictions that are the opposite of the literature's. If many Bertrand competitors draw their component costs, qualities, or characteristics from independent random distributions, then expected prices and profit margins are lower under compatibility than under incompatibility, while expected consumer surplus is higher. In addition, the chapter examines the incentives of firms to form coalitions around competing standards. It is found that a subset of firms may become compatible with each other to attract customers away from other firms, creating excess incentives for firms to become compatible from the perspective of industry profits. However, compatibility raises welfare if it is costless and components are homogeneous, because incompatibility is a restriction on the technology for combining components into systems.

Chapter 3 shows that shifting from incompatibility to compatibility has an ambiguous impact on R&D effort to reduce costs. In an industry with sufficiently many firms that faces elastic demand, compatibility lowers prices and raises output, and therefore leads to greater R&D incentives. If effort lowers costs without changing the shape of the cost distribution function, compatibility induces firms to choose R&D effort levels that are closer together than under incompatibility.

Chapter 4 relaxes the assumption that consumers combine components in fixed proportions. With variable coefficients, compatibility does not necessarily raise the profits of duopolists. For instance, compatibility prevents a dominant firm from setting the price of either component above its competitor's cost. On the other hand, when two "mirror-image" firms each have the lowest cost in one component and demand is symmetric across components, the firms prefer compatibility, as they did in the fixed coefficients case. When sufficiently many firms draw their costs from discrete random distributions, this ambiguity disappears, and expected profits are higher under incompatibility.

Variable coefficients also allow analysis of quantity competition, by eliminating the problem of unmatched components when there are asymmetric quantity choices. In this case, firms with mirror-image costs prefer compatibility to incompatibility because they can specialize in their low-cost component. However, when each firm has the same cost across components, firms are indifferent between the two regimes.

To my parents

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Notation

Chapter 1: Mix-and-Match Model

A,B Components labelled A and B
1,2 Firms labelled 1 and 2
(a,b) Location of a consumer whose ideal component characteristics are (a,b)
(a_i,b_i) Location of firm i with a component A of type a_i and component B of type b_i
R Reservation price of a consumer with inelastic demand for one system
p Firm 1's system price
p_i^j Price of firm i for component j
p_i System price of firm i. With fixed proportions, $p_i=p_i^A+p_i^B$.
d_i Distance between a consumer and firm i
q Demand of a firm, i.e. a firm's market area
d Distance between two firms
l Length of the margin between two firms
$-q'$ Number of consumers on the margin between two firms, $-q'=l/2d$
π_i^C Firm i's profit under compatibility
π_i^I Firm i's profit under incompatibility
n Number of firms

Chapter 1: Network Externalities Model

R_0 Consumers' reservation prices are uniformly distributed over $(-\infty, R_0]$
R Consumer's reservation price ignoring network benefits
q_i^e Expected sales of firm i
y_i^e Expected size of firm i's network.
$v(y_i^e)$ Benefits of belonging to a network of expected size y_i^e
q Industry output
q_i Firm i's output

Chapter 1: Switching Costs Model

R Consumer's reservation price for one unit, each period
ϵ Small price cut
δ Discount rate
$\Gamma_1(p)$ Mixed-pricing strategy of the firm that sold last period
$\Gamma_0(p)$ Mixed-pricing strategy of the firm that did not sell last period
V_1 Continuation value of the game for a firm that sold last period
V_0 Continuation value of the game for a firm that did not sell last period

Chapter 2: Mix-and-Match Compatibility in Oligopoly

a_i Firm i's component cost in component A
b_i Firm i's component cost in component B
c_i Unit system cost of firm i. In Chapter 2, $c_i=a_i+b_i$

p	Industry system price, the price paid by the consumer
c	Industry system cost, the cost of the system bought by the consumer
$\approx$	Approximately equal
iff	If and only if
Δp	$\Delta p = p^C - p^I$, effect of compatibility on price
$\Delta\pi$	$\Delta\pi = \pi^C - \pi^I$, effect of compatibility on profit
π_{klm}	Profit of a firm in a group of size k when there are other groups of size l and m
k	Number of components in a system
c_1^n	Lowest of n draws from the random variable c
c_2^n	Second-lowest of n draws from the random variable c
$E[.]$	Expectation operator
$F(.)$	Random distribution describing the unit cost in a component
$G(.)$	Random distribution describing the average of two draws from $F(.)$
e	Probability that a firm has cost zero in a component rather than cost one
$U(.)$	Utility function
$P(.)$	Probability of an event
c_1	Lowest cost in a market, firm 1's cost
c_2	Second-lowest cost in a market, firm 2's cost
$c^C(n)$	Industry's expected costs under compatibility with n firms
$c^I(n)$	Industry's costs under incompatibility with n firms
$\pi^C(n)$	Firm's expected profit under compatibility with n firms
$\pi^I(n)$	Firm's expected profit under incompatibility with n firms
$f \succ g$	f is of higher order than g ($\lim f/g = \infty$)
$f \prec g$	f is of smaller order than g ($\lim g/f = \infty$)
$f \asymp g$	Functions f and g are of the same order
$\mathbb{C}$	Set of all partitions of size n
a_i	Characteristic (cost) of firm i for component A
b_i	Characteristic (cost) of firm i for component B

Chapter 3: Mix-and-Match Compatibility and Research Effort

$F(c,e)$	Distribution function for costs in a component with effort e
C'	Marginal cost of effort in a component
e_i^j	Effort level of firm i in component j
e_i	Effort level by firm i
$\Delta\pi'$	$\Delta\pi' = \pi^{C\prime} - \pi^{I\prime}$, effect of compatibility on the marginal return to R&D effort

Chapter 4: Mix-and-Match Compatibility with Variable Coefficients

$v(.)$	Indirect utility function of the representative consumer
U_2	Utility per system that firm 2 can offer
q_i^j	Quantity of firm i for component j
q^C	Industry output under compatibility
q^I	Industry output under incompatibility
ρ	Parameter of constant elasticity of substitution demand function
r	Parameter of CES function, $r = \rho/(\rho-1)$
ϵ	Elasticity of substitution, $\epsilon = 1/(1-\rho)$

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Introduction

This thesis develops theoretical explanations for why compatible products, such as IBM PC compatible computers, might have lower prices than incompatible products, although some existing economic literature on product compatibility predicts that the opposite would be the case. Furthermore, it considers why firms might pursue strategies of compatibility although compatibility appears to lower profit margins. This is primarily a theoretical work, but the personal computer industry provides motivation and examples.

Personal computer users deal constantly with questions of compatibility and incompatibility. Consider three questions that a computer user might face. Can his computer work with software not made by the same supplier? Can his computer use the same electronic mail network as a friend's computer? Will software and skills acquired for his present computer be useful with his next computer?

Computer firms also must make important decisions about compatibility. Should a firm develop its own standard or adopt the same standard as other firms? Should it adopt now or later? Should it license or give away its technology to attract allies to its standard? These decisions form an important part of a computer manufacturer's strategy.

Although technical considerations determine whether products are compatible, firms make decisions about compatibility in an economic context. As a working definition, let us say that two products are *compatible* if their manufacturers have decided to make them complementary goods. Three strands of the economic literature define compatibility based on mix-and-match, network externalities, and switching costs.

In the mix-and-match literature, a consumer simultaneously buys the two potentially complementary products, which are components of a system. When two firms' components are compatible, a consumer can combine them into a system. When their components are incompatible, a consumer must buy all of a system's components from the same firm. In the network externalities literature, different consumers buy products, such as telephones, that are potentially complementary across consumers. Compatible products belong to the same network whereas incompatible products do not, and the utility derived from a network grows with its size. In the switching costs literature, potentially complementary products are bought sequentially by the same consumer. If two firms' products are compatible, then the consumer can switch from one firm to the other without cost, and is not locked into repeated purchases from the same supplier.

Models based on the first two definitions predict that compatibility raises prices and profit margins. However, the IBM PC compatible industry appears to have low prices and profit margins. Although this thesis does not attempt to show that compatibility has indeed lowered prices in this industry, it does ask why compatibility might lower prices in theory. One explanation is that compatibility lowers consumers' cost of switching firms, and thereby increases competition. This thesis develops explanations based on mix-and-match compatibility.

In the computer industry, coalitions of firms often back competing standards. A standard in this context is a set of compatible products. For instance, the IBM PC standard is a set of computers that can use the same software and accessories. The thesis analyzes a game in which firms join coalitions through their choice of standard, and finds that all firms may choose the same standard although industry profits are higher when they choose different standards.

One must be very careful to distinguish compatibility standards from other types of standards, which are not addressed here. For instance, some standards are simply descriptive. Quality grading standards for tobacco fall into this category. Governments also mandate standards for health, safety, and environmental protection. A descriptive standard may serve as a point of reference in government regulations, and may also have force when written into private contracts or when referred to in liability suits. Finally, a firm may standardize internally, say by using interchangeable spare parts, but the emphasis here is on standardization across firms.

The plan of the thesis is as follows. Chapter 1 reviews the literature and presents stylized facts about compatibility in the personal computer industry to motivate theoretical models. Chapter 2 shows that the effect of mix-and-match compatibility on prices and profit margins depends on whether there are two firms or many firms. It also looks at the coalitional behavior of firms in choosing standards, and illustrates how their individually rational decisions can lower industry profits. Chapter 3 finds that compatibility has an ambiguous impact on R&D effort in a variety of settings. Chapter 4 relaxes the assumption that consumers combine components in fixed proportions. With variable coefficients, compatibility can lower profits by constraining the price of a dominant firm. It also presents a mix-and-match model with Cournot competition.

Chapter 1: Compatibility in Theory and Practice

This chapter reviews the economic literature on product compatibility with motivation drawn from the personal computer industry. More comprehensive reviews of the literature can be found in the *Journal of Industrial Economics* (March 1992) and *Journal of Economic Perspectives* (forthcoming) symposia on compatibility and in David and Greenstein (1990).

Section 1 presents stylized facts about compatibility drawn from an examination of the personal computer industry contained in an appendix. Section 2 reviews the literature on mix-and-match compatibility, and Section 3 discusses the choice of standards by firms. Sections 4 and 5 present two other strands of the literature using definitions of "compatibility" based on network externalities and the lack of switching costs. Section 6 locates the thesis in relation to the literature. The appendices follow.

1 Stylized Facts About the Personal Computer Industry

Participants in the computer industry uses the term "compatibility" to mean one of several things. For instance, hardware and software are "compatible" if they work together. The term can also mean that computers can work together in a network. Finally, "compatibility" may also mean that buyers can switch suppliers easily. We postpone a more careful definition for the moment.

The following table summarizes stylized facts about compatibility in the computer industry. An appendix presents evidence supporting each point.

Stylized Facts About Compatibility in the Personal Computer Industry

Computer buyers prefer compatibility. Buyers have shifted their purchases from large incompatible computers to small, compatible computers. Computer users have aggressively promoted a movement towards open systems (compatibility) for larger computers. Buyers who prefer incompatible Macintosh computers have nevertheless benefitted from price competition and innovation in the IBM PC compatible industry.

Profit margins and prices are lower in the PC compatible industry than in other parts of the computer industry, with some notable exceptions. Some industry observers believe that compatibility is the cause. Profit margins and prices for PC compatible computers are lower than for larger incompatible computers, Macintosh computers, and for incompatible PCs in Japan. IBM has abandoned the highly competitive PC market that it created in favor of its proprietary PS/2 standard. Intel and Microsoft have become highly profitable as suppliers of essential components for PC compatible computers primarily through high output rather than high prices.

Until the 1980s, most computer firms tried to prevent other firms from adopting their proprietary standards. However, almost all companies now pursue strategies of compatibility to one degree or another, sometimes by forming coalitions around competing standards or by giving away their technology to competitors. In the early computer industry IBM usually avoided compatibility with other firms, and in 1984 the European Commission felt it necessary to require IBM to publish its interface specifications. Today almost all computer suppliers make compatibility a selling point, and many give away valuable technology to persuade other firms to adopt their standard, as IBM did with its PC. Coalitions of firms have supported competing standards in many cases.

Increased compatibility in the personal computer industry has been accompanied by increased specialization, variety, and R&D. Before the 1980s, when most computers were incompatible, a single company made most of the components of the typical personal computer system. Now few companies make all components. The range of off-the-shelf software, printers, and other accessories for PC compatibles exceeds that available for Macintoshes or for Japanese PCs.

The level and growth rate of sales in markets with compatibility appear greater than in markets with incompatibility. Japan's PC market is smaller than the PC market outside Japan, and has grown less quickly. New markets with competing standards have not grown as quickly as those with a single standard.

These "facts" should not be interpreted as causal claims, for instance, that compatibility lowers profits, although some industry observers do hold this view. Even a careful, in-depth case study of the personal computer industry might not be able to distinguish causal relation from coincidence. In addition, compatibility is not exogenous, but results from the design decisions of firms. With these caveats, however, the stylized facts should help motivate a review of the literature in this chapter and the theoretical models contained in chapters that follow.

As explained above, economic theory has taken three approaches to analyzing product compatibility. The next sections compare the different approaches' theoretical predictions about the effects of compatibility, in the context of stylized facts about the personal computer industry. A table at the end of this chapter (on page 56) summarizes the comparison. As will be seen, the mix-and-match and network externality literatures predict that compatibility raises prices and profit margins. By contrast, later chapters develop mix-and-match models predicting that compatibility lowers profits and prices. The goal of later chapters is not to develop a working model of the PC industry, but explore theoretical explanations for why compatibility might have effects other than those predicted in the existing literature.

2 Mix-and-Match Compatibility

Compatibility in the personal computer industry most commonly refers to the ability to combine hardware and software made by different companies. The literature calls this mix-and-match compatibility, because consumers can mix-and-match compatible components made by different firms into systems. Mix-and-match compatibility is also an issue with other multicomponent products, such as video-cassette recorders (VCRs) and videocassettes, cameras and lenses, video games and cartridges, and razors and razor blades. In a geographical context, the location of buyers and suppliers in the same place can promote mixing-and-matching in their relationships (see Krugman (1992)).

Section 2.1 presents simple models of mix-and-match compatibility drawn from the literature that agree in their prediction that compatibility raises prices and profits. Section 2.2 discusses other issues touched upon in the literature on mix-and-match compatibility.

2.1 Some Simple Models of Mix-and-Match Compatibility

The mix-and-match literature concludes that the prices and profits of symmetric firms are higher under compatibility than under incompatibility. The next chapter develops mix-and-match models with predictions that are the opposite of those in the literature. To put the following chapters into context, the next sections present simple models of mix-and-match with vertical and horizontal differentiation.

2.1.1 Vertical Differentiation in Duopoly

An example based on Einhorn's (1992) model, in which components vary in quality, quickly captures the results of the mix-and-match literature. To illustrate, suppose that a representative consumer has inelastic demand for one system consisting of exactly one unit each of components A and B. Duopolists with zero marginal costs compete in prices. Suppose that the consumer has reservation price one for firm 1's component A and reservation price zero for firm 1's component B, written (1,0). The consumer's reservation prices for firm 2's components are (0,1). Components and systems are otherwise homogeneous.

In the regime of *incompatibility*, a consumer must buy both components from the same firm. Then, firms essentially compete in selling a single product—systems. Under incompatibility, the consumer has reservation price one for each firm's system. Because the two systems are identical, each firm sets a price of one and makes zero profit.

In the regime of *compatibility*, the consumer can combine components made by different firms into a system. For instance, buyers of stereo systems can readily combine stereos and speakers made by different companies. Under compatibility, firms compete component-by-component. Firm 1's high quality component A competes against firm 2's low quality component A. Even if firm 2 sets a price of zero for component A, firm 1 can raise its as high as one and still offer more surplus to the consumer, thereby making a profit of one. Similarly, firm 2 makes a profit of one in component B. Profits and prices are higher in the regime of compatibility than in the regime of incompatibility, or for short, compatibility raises profits and prices. Consumer surplus is zero in both regimes, so compatibility raises welfare as well.[1] The prediction that compatibility increases welfare is not surprising, given that incompatibility is a restriction on the technology for combining components into systems.

Einhorn also looks at the case in which one firm has the lowest cost in both components, and finds that compatibility has no effect on prices or profits. Suppose that the consumer has reservation prices (1,1) for firm 1's components and reservation prices

[1] This thesis measures of welfare by summing consumer surplus and industry profits.

(0,0) for firm 2's components. Under incompatibility, firm 1 has profit $(1+1)-(0+0)=2$; under compatibility, it has profit $(1-0)+(1-0)=2$.

Einhorn's prediction that firms specialize in making one component under compatibility captures a important feature of the personal computer industry. It is obvious that firms cannot sell more of one component than another under incompatibility, because a single component is without value by assumption. However, specialization does not occur elsewhere in the literature, which assumes that firms are symmetric as described below. Chapter 4 presents a model in which Cournot duopolists specialize partially under incompatibility, but specialize even more under compatibility.

The example above has the analytically interesting feature that the market for each component under compatibility can be solved separately. The examples below are chosen to share this feature for simplicity.

2.1.2 Horizontal Differentiation in Duopoly

More complicated mix-and-match models make the same predictions, that prices and profits are higher under compatibility than under incompatibility. Matutes and Regibeau (1988) began the mix-and-match literature with a duopoly model of horizontally differentiated components. A computer system consists of exactly one unit of component A and one unit of component B; think of a computer and software.[2] A single characteristic describes each component, and consumers have horizontally

[2] Note that Matutes and Regibeau (1988) refer to firms A and B and components 1 and 2. Later models will have more than two firms, so firms and not components are referred to by number in this thesis. We also adopt the convention of referring to consumers as "he" and firms as "it."

differentiated tastes over characteristics, following Hotelling (1929). In other words, each consumer has a preferred variety of component A, denoted by a point a on the unit interval. Similarly, the consumer's preferred component B is a point b on the unit interval. Thus, the ordered pair (a,b) on the unit square describes a consumer's tastes in systems. Consumers are uniformly distributed over the unit square. Each has inelastic demand with reservation price R for his ideal system, and R is the same across consumers.

The two firms each sell both components and have zero production costs. Firm 1 makes the system that is ideal for the consumer at (0,0), and firm 2 makes the system that is ideal for the consumer at (1,1) (see Figure 1 below). When a consumer buys a system other than his ideal system, he pays a transport cost equal to the square of his Euclidean distance from that system.[3] If the consumer buys from a firm i which makes a system with characteristics (a_i,b_i), the transport cost is

$$d_i = (a_i-a)^2+(b_i-b)^2. \tag{1}$$

For instance, a consumer at (½,½) pays a transportation cost when he buys from firm 1 of $(\frac{1}{2})^2+(\frac{1}{2})^2=\frac{1}{2}$. A consumer buys the system offering the largest surplus net of transportation costs and system price. More specifically, when the firms have system prices p_1 and p_2, a consumer at (a,b) prefers firm 1's system to firm 2's system if

$$R-[p_1+a^2+b^2] > R-[p_2+(1-a)^2+(1-b)^2]. \tag{2}$$

[3] Matutes and Regibeau (1988) assume linear transportation costs and the block metric. However, later we extend their result to many firms, and it is necessary to avoid Hotelling-type existence problems that arise with linear transportation costs. Also, the margin is only piece-wise linear under the block metric, and in some cases, has positive area.

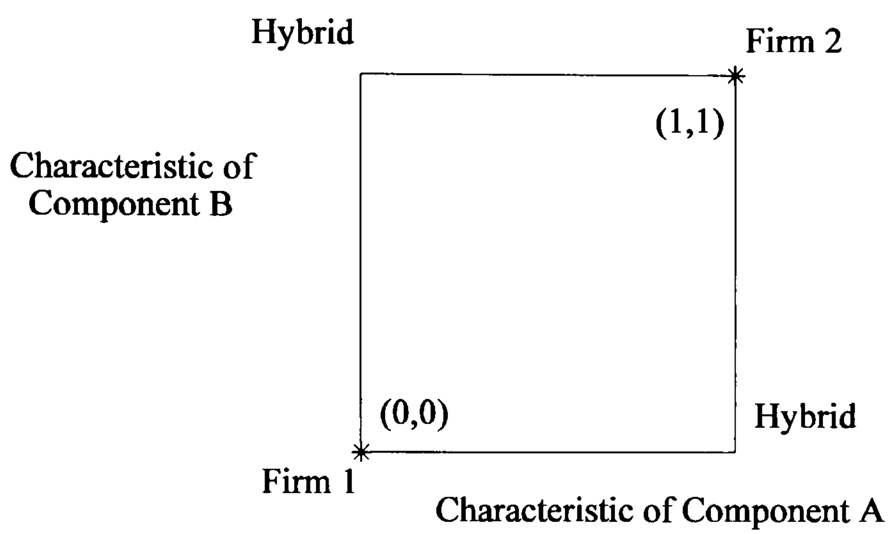

Figure 1: Firms' Locations in Product Space

When the firms are incompatible, only two systems are available: firm 1's system at (0,0) and firm 2's system at (1,1). When the firms are compatible, consumers can buy two new *hybrid* systems as well, consisting of components made by different firms (see Figure 1 above). Consumers can combine firm 1's computer and firm 2's software, creating a hybrid system at (0,1) or combine firm 2's computer and firm 1's software to create a hybrid system at (1,0). In the first stage of the model, firms decide whether to become compatible or not. In the second stage, firms compete in prices given their compatibility decisions in the first stage.

With these assumptions, Matutes and Regibeau show that prices and profits are higher under compatibility than under incompatibility. There are two reasons. First, the variety of products increases under compatibility from two systems to four systems. Second, firms face less elastic demand under compatibility. Under incompatibility, a firm that raises the price of component A loses sales of components A and B in equal numbers, because consumers combine components in fixed, one-to-one proportions. Under compatibility, raising the price of component A loses proportionately fewer sales of component B, because firm 2 sells some of the matching components.

This second effect is clearest when the consumers' reservation price R is so high that market coverage is complete in equilibrium in both regimes. Then, no consumer is on the margin between buying and not buying at all. This condition simplifies firms' maximization problems under compatibility, because it implies that the market for each component can be solved separately. In that case, we can solve two simple problems in which each firm sets one component price, rather than one complicated problem in which each firm sets two component prices. Then, raising the price of a component under compatibility loses sales only of that component, while a price increase under incompatibility loses sales of both components.

A formal solution when there is complete market coverage follows. In either regime, firm 1 maximizes revenue $p_1q(p_1,p_2)$ holding p_2 constant, where $q(p_1,p_2)$ is firm 1's demand. Abbreviating p_1 as p and $q(p_1,p_2)$ as q, firm 1 sets $q=-pq'$. The left-hand side, q, is the area of firm 1's market, because consumers are distributed with uniform density and have inelastic demand for one system. When market coverage is complete, no consumer is on the margin between buying and not buying. Consumers are however on the margin between buying from firm 1 and firm 2.

First, we show that when transportation costs are the square of Euclidean distance, the margin between two firms with locations (a_1,b_1) and (a_2,b_2) is a line perpendicular to the line between the firms. A consumer is on the margin if both firms offer him the same surplus. A consumer's surplus when he buys from firm i is

$R-(p_i+d_i)$, where p_i is the price of firm i and d_i is the transport cost between the consumer and firm i. Therefore, $p_1+d_1=p_2+d_2$ for a marginal consumer. If a marginal consumer is at (a,b) and firm i is at (a_i,b_i), then

$$p_1+d_1 = p_1+(a_1-a)^2+(b_1-b)^2 = p_2+(a_2-a)^2+(b_2-b)^2 = p_2+d_2. \tag{3}$$

The squared terms in a and b drop out, so the margin is a line with slope $-(a_2-a_1)/(b_2-b_1)$. The line between the two firms has slope $(b_2-b_1)/(a_2-a_1)$, so the two lines are perpendicular. If firms set the same prices, the margin is the perpendicular bisector. Now we calculate the effect of a marginal increase in price on the location of the margin. For the purpose of this calculation, suppose for the moment that $b_1=b_2$. From (3) it follows that the marginal consumer at a satisfies

$$a = \frac{(p_2-p_1)+a_2^2-a_1^2}{2(a_2-a_1)}, \qquad \frac{da}{dp_1} = \frac{-1}{2d} \tag{4}$$

where d is the distance between the two firms. The margin shifts by a similar amount when $b_1 \neq b_2$. Therefore, if l is the length of the margin between two firms, and d is the distance between the firms, then the marginal effect of a price increase on demand is

$$-q' = \frac{l}{2d}. \tag{5}$$

Under incompatibility in a symmetric price equilibrium, the margin between the two firms at (0,0) and (1,1) is the perpendicular bisector of the line segment between them, that is, the NW-SE diagonal. Firm 1 sells in the lower left half of the square and firm 2 in the upper right half (see Figure 2). Each firm has a market area of $q=½$, and the boundary between their markets lies along the NW-SE diagonal. Suppose that firm 1 raises its price slightly. Firm 1's customers pay more, but the higher price causes some

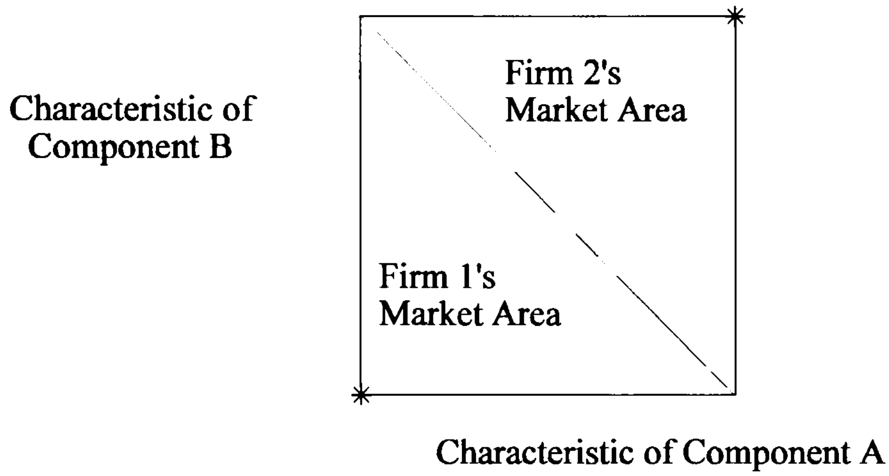

Figure 2: Market Areas under Incompatibility

customers to switch from firm 1's system to firm 2's system. These marginal consumers lie along the boundary between the two firms' market areas, along the NW-SE diagonal. The length of the margin is $l=\sqrt{2}$, and the distance between the two firms is $d=\sqrt{2}$. By (5) the margin is $-q'=½$, so each firm's system price is $p=-q/q'=1$ and each firm's profit is $\pi^J=½$.

Under compatibility, suppose that there is a symmetric price equilibrium in component A. Each firm's market area in component A is a rectangle of size ½ (see Figure 3). The margin between firm 1's component A and firm 2's component A is a vertical line splitting the square down the middle. This line has length $l=1$, and the distance between the two firms' component A characteristics is $d=1$ in the horizontal

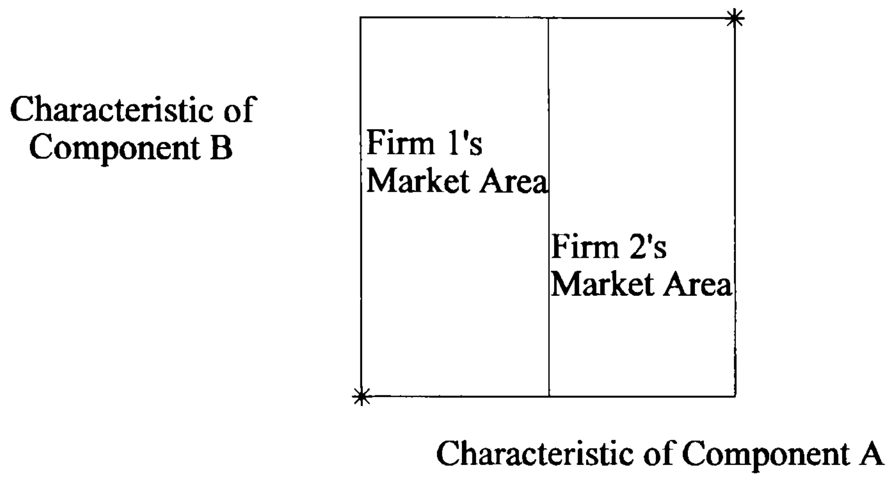

Figure 3: Market Areas in Component *A* under Compatibility

direction. Therefore, $-q'=½$ and each firm's price for component A is $p^A=-q/q'=1$.[4] Similarly, $p^B=1$, and each firm's system price is $p=p^A+p^B=2$. Each firm's profit from both components is $\pi^C=pq=1$. Therefore, compatibility raises profits and prices. The reason is that there are as many consumers on the margin in each component under compatibility as there are consumers on the margin in systems under incompatibility. Each component's price under compatibility is equal to the system price under incompatibility, so prices are twice as high under incompatibility. Consumer surplus falls, because the system price increases by one, but no consumer saves that much on transportation costs.

[4] The subscript indicating that the price is for firm 1 is suppressed, because firms charge the same price in equilibrium.

More generally, compatibility has an ambiguous effect on welfare in Matutes and Regibeau's model. Consumers who buy the same system in both regimes are worse off under compatibility, because prices are higher. However, the surplus of a consumer who switches to a hybrid system under compatibility can go up, because the system is closer to his ideal system.

Economides (1991) finds a similar result when compatibility is not an all-or-nothing decision. In his specification, an adapter or converter is required to create hybrid systems from components not originally designed to work together. He interprets the cost of the adapter as a measure of partial compatibility. If the cost is prohibitively high, then the firms are incompatible in the sense used above, while if the cost is zero, then firms are compatible. Economides assumes that firms can control the cost of competitively-produced adapters through the design of their components. The demand function for each of the four possible systems is linear in the four component prices, and is not derived from a distribution of consumers over the unit square.

He finds that if demand is symmetric for hybrid and single-producer systems, firms will make the adapter cost as low as possible. Thus, he reproduces Matutes and Regibeau's conclusion when compatibility is a matter of degree. His intuition for this result is that the adapter is complementary with components, and the elasticity of demand for components rises with the price of the adapter. Firms prefer inelastic demand, and therefore a low adapter cost. This result holds as well in the case of more than two firms that can control the cost of adapters that are specific to each pair of firms. The analysis does not address effects on industry profits when there are more than two firms.

However, Economides (1991) shows that the conclusion that prices and profits are higher under compatibility is reversed if the demand for hybrid systems is sufficiently low. If demand for hybrid systems is much lower than demand for single-producer systems, compatibility increases competition without an offsetting advantage. Then, both firms prefer incompatibility. Alternatively, if demand for one producer's system is high, while demand for hybrids and the other producer's system is low, then the high-demand producer prefers incompatibility, while the low-demand producer prefers compatibility.

2.1.3 Mix-and-Match Compatibility with Many Firms

Several authors have attempted to extend Matutes and Regibeau (1988) to the case of many firms, with varied success. This research has attempted to create models with symmetric firms to avoid solving n distinct first-order conditions for n prices. However, when consumers are distributed over the unit square, a firm near the corner of the square will usually not have the same first-order condition as one near the edge or in the middle. It is helpful to think about the simpler case in which consumers lie on a unit interval as in Hotelling's (1929) model. Three or more firms cannot be symmetric because there is always a left-most firm and a right-most firm. Salop (1979) restores symmetry among firms by assuming that consumers lie along the circumference of a unit circle. Using a circle rather than an interval eliminates boundary effects.

In the same manner, Economides (1989) attempts to restore symmetry when there are two dimensions of differentiation by assuming that consumers lie on the surface of a unit sphere. He concludes that compatibility raises prices and profits if there are n firms. Unfortunately, the analysis presumes that firms have square market areas, and it is impossible to divide the surface of a sphere into a grid of squares of equal area. For instance, lines of latitude and longitude divide a globe into trapezoids and triangles.

Matutes and Regibeau (1991) note this flaw in Economides' model, and consider alternative approaches.[5] They solve a model in which firms lie along the diagonal of the unit square (see Figure 4). Because of boundary effects, firms are not symmetric. Therefore, the authors solve the n first-order conditions numerically when all firms are compatible and when all firms are incompatible. Firms near the center of the square have large market shares under incompatibility, and tend to oppose compatibility. By contrast, firms near the corners have small market shares under incompatibility and tend to prefer compatibility.

However, an appendix to this chapter shows that it is possible to solve a symmetric mix-and-match model with n firms located along the diagonal of the unit square. This model employs a generalization of Salop's (1979) approach when there are two dimensions of product differentiation. There, it is found that profits and prices are higher under compatibility than incompatibility for any n. Thus, Economides' (1989) conclusion is correct, despite the flaws in the model noted above.

This section has shown that in a variety of settings, prices and profits of symmetric firms are higher under compatibility than incompatibility if demand for hybrid products is not too low. However, this prediction is unsatisfactory theoretically and empirically. The appendix on the personal computer industry argues that increased compatibility in the personal computer industry has been accompanied by lower prices and profits, and

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[5] These authors also note that Economides (1989) ignores the effect of compatibility on the distance between firms. In addition, Economides incorrectly states that the margin is the perpendicular bisector for the block metric, which is not true when transport costs are linear.

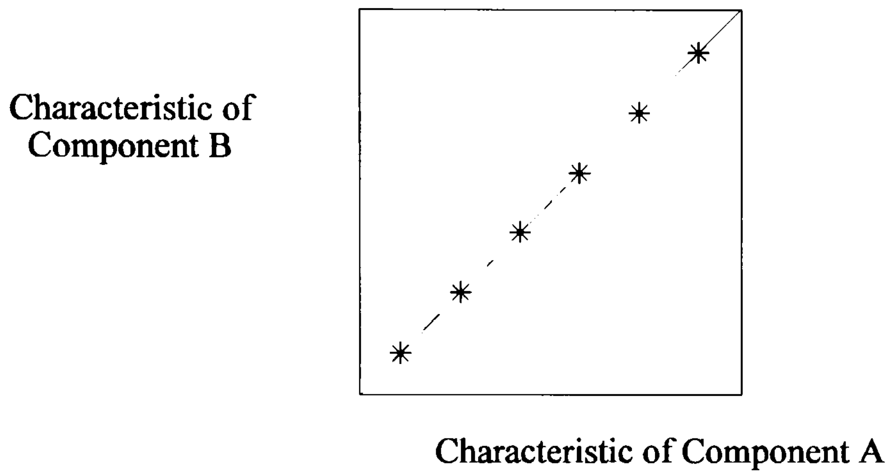

Figure 4: Locations of the *n* Firms

some industry observers believe that compatibility has caused this change. Furthermore, most computer buyers build hybrid systems, with hardware, software, and printers made by different companies. By one account, "the availability of clone and compatible personal computer hardware not only lowers prices and expands the market for PCs, but is demanded by the user community."[6] Although other factors may have lowered prices and profits, the personal computer industry does not offer ready confirmation of the literature's predictions.

From a theoretical perspective, Economides' (1989) conclusion requires the *ad hoc* assumption that firms lie along the diagonal. By contrast, suppose that four firms

[6] *Datamation*, "When Copyright Goes Wrong," **38**, 15 August 1992, p. 110.

lie not along the diagonal, but at the four corners of the unit square.[7] Under compatibility, there are two identical versions of each component, so Bertrand competition drives prices and profits down to zero. Under incompatibility, $l=1$, $d=1$, $q=\frac{1}{4}$, so $p=\frac{1}{2}$ and each firm's profit is $\pi^I=\frac{1}{8}$. Economides' result that prices and profits are higher therefore depends on the perfect correlation of firms' characteristics.

Also, Economides' (1991) model, in which consumers need adapters to mix-and-match components, does not address how compatibility affects industry profits when there are many symmetric firms. The article does show that increased compatibility between two of many firms, as a result of a lower adapter cost, raises those two firms' profits. However, increased compatibility also lowers the other $n-2$ firms' profits, as follows. Assuming that demand for hybrids equals demand for single-producer systems, let D_{ij} be the demand for a system with component A from firm i and component B from firm j. Demand is linear in the $2n$ component prices. In a symmetric price equilibrium, all systems have price p excluding adapter costs, so industry profits are

$$\Pi = p\sum D_{ij}. \tag{8}$$

Let R_{12} be the cost of an adapter between firm 1's components and firm 2's components. The price to a consumer of buying a hybrid system from firms 1 and 2 is $p+R_{12}$, so lowering R_{12} increases demand for the two hybrid systems made by those firms, but reduces demand for the other n^2-2 systems. Therefore, the effect of a small reduction in R_{12} on industry profits, using the envelope theorem, is

$$\frac{\partial\Pi}{\partial(-R_{12})} = p\frac{\partial(D_{12}+D_{21})}{\partial(-R_{12})} + (n^2-2)p\frac{\partial D_{ij}}{\partial(-R_{12})}. \tag{9}$$

[7] This example does not assume that tastes wrap around, as defined in an appendix.

The first term is positive and the second term is negative. If raising n does not alter the ratio of the own-price and cross-price elasticities, then for n large enough, the second term predominates. Then, increasing compatibility reduces industry profits.

Chapter 2 makes the same prediction. It shows that symmetric firms with random and independent component characteristics also have lower expected profits and prices under compatibility than incompatibility, when there are sufficiently many firms. There, firms also wish to increase compatibility individually, sometimes with negative effects on industry profits.

2.2 Other Issues with Mix-and-Match

Many other important issues arise with mix-and-match compatibility. These include tying, entry deterrence, economies of scale, bundling, specialization versus integration, and product line decisions.

2.2.1 Tying

Tying is a decision by a firm 1, which makes two components A and B, to prevent its customers from buying component B from firm 2, which does not make component A. We can interpret tying as mix-and-match incompatibility imposed by firm 1. Economists have long debated whether tying can improve the profit of firm 1, particularly in a socially detrimental way that anti-trust authorities should regulate.

The Chicago school holds that firm 1 cannot raise its profit at the expense of firm 2 and of social welfare by becoming incompatible. Commentators such as Bork (1978) and Posner (1976) argue that there is only one monopoly profit for firm 1 to extract. Under compatibility, firm 1 can make up any losses on component B by raising the price of component A.

Whinston (1990) argues that the model implicitly used by the Chicago school is not appropriate to the task of assessing claims that tying can raise profits. Whinston shows that the Chicago argument depends on implicit assumptions of constant returns-to-scale (CRS) in component B, a fixed market structure, homogeneous preferences for the component A, and fixed proportions in combining components. He shows that relaxing these assumptions can make tying a profitable strategy. For instance, given firm 2's price, firm 1 may wish to tie in order to price discriminate. In a dynamic setting, a commitment to tie can induce firm 2 to exit. Also, if the component A is essential to all uses of component B, tying can reduce competition in the component B market.

The mix-and-match models above assume a fixed market structure, CRS production, and fixed proportions, just as the Chicago model does. Therefore, the mix-and-match models are potentially subject to similar criticisms. Relaxing the Chicago-style assumptions in a mix-and-match model means that duopolists sometimes prefer incompatibility to compatibility. Section 2.2.2 relaxes the assumption of fixed market structure, and considers entry deterrence strategies. Section 2.2.3 discusses economies of scale. Finally, Chapter 4 relaxes that assumption that consumers combine components in fixed proportions.

Matutes and Regibeau (1988) and Economides (1989) differ from the tying literature in that they assume that firms are symmetric, and in particular, that each firm makes both components. Einhorn (1992), on the other hand, is nearly identical to the Chicago model when firm 2 has zero quality in component B.

2.2.2 Entry Deterrence

When market structure is not fixed, Tirole (1988) interprets Whinston to show that an incumbent can sometimes raise its profit by becoming incompatible with a potential entrant. Although incompatibility lowers profits given that entry occurs, as described above, lower profits may deter entry.

On the other hand, standardization can deter entry as well. For instance, in Matutes and Regibeau (1989) standardization forces an incumbent firm to charge the same price in two markets. They argue that the incumbent might standardize to encourage an entrant to enter in one rather than both markets. Then, entry in both markets would provoke a price war, while entry in one market does not lead the incumbent to cut price as much, because of sales lost in the other market. Here, compatibility reduces entry, while in Tirole's example, incompatibility deters entry.[8]

2.2.3 Compatibility and Economies of Scale

There are parallels between some issues in the theory of international trade and in the study of mix-and-match compatibility. Trade theory shows that gains from trade can arise when countries specialize by their comparative advantage. Similarly, compatibility improves welfare in Einhorn (1992) when each firm has an absolute advantage in one component. International trade can also improve welfare by increasing

[8] Ordover and Willig (1981) also discuss predatory incompatibility.

product variety.[9] In Matutes and Regibeau (1988), compatibility increases the variety of systems available and sometimes improves welfare.

Trade theory also studies gains from trade that arise from increasing returns to scale and greater competition, but the mix-and-match literature has not closely examined these phenomena. Compatibility can allow firms to achieve economies of a scale in a component, such as software. On the other hand, the lack of PC standards in Japan has hampered software development there by comparison to the rest of the world, because each software package works with only one brand.

Compatibility can create dominant firms in components produced with increasing returns. The U.S. government has investigated allegations that Microsoft and Intel have used their dominant positions in the markets for operating systems and microprocessors to the disadvantage of firms making other personal computer components.

Church and Gandal (1992) analyze one of the few mix-and-match models in which increasing returns play a role. These authors model independent software producers' choices between two differentiated hardware standards. Software is produced with increasing returns to scale. Free entry reduces profits to zero, so the welfare comparison depends on consumer surplus. They find that consumer surplus is higher when producers make software for only one type of hardware if consumers have a weak preference for variety in hardware.

[9] See, for instance, Dixit and Norman (1980), p. 273.

2.2.4 Bundling

Matutes and Regibeau (1992) model bundling under compatibility, that is, the practice of charging a different price to a consumer who buys a single-producer system than the price charged to a consumer who buys a hybrid system. Bundling differs from tying in that a bundling firm may still sell components separately. In practice, computer manufacturers often bundle software with hardware. PCs commonly come with DOS operating system software, Windows, and perhaps other software as well.

Bundling models typically focus on the monopoly case.[10] Matutes and Regibeau (1992) are unusual for considering bundling in duopoly. In this context, a strategy of pure bundling, that is, setting a price only for systems, is equivalent to a decision to be incompatible. Therefore, the analysis focuses on mixed bundling, that is, setting a price for systems in addition to setting prices for components sold separately. In an extension of Matutes and Regibeau (1988), they show that choosing to bundle is a prisoner's dilemma for duopolists if consumers' reservation prices are not too high. The firms' profits are lower if both bundle, but each prefers to bundle given its competitor's decision about whether to bundle. However, compatibility with bundling still gives higher profits than incompatibility.

2.2.5 Specialization versus Integration

The literature offers contrary views on whether a specialized or integrated industry structure is associated with higher welfare. Economides and Salop (1992)

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[10] See, for instance, McAfee, McMillan, and Whinston (1989).

examine how ownership of component production affects the prices of systems assuming that all firms are compatible. There are two types each of two components, and demand is symmetric. Economides and Salop compare equilibrium system prices under several types of ownership structure. The structures, ranked in order of increasing price, are: composite goods competition, in which there are four firms selling each possible combination of components; parallel vertical integration, with two firms selling systems that differ in both components; independent ownership, in which there are four firms each selling one of the four components; and finally joint ownership, in which one firm owns all four components. The results follow from which of various price externalities are internalized by the ownership structure.

In Einhorn's (1992) framework, specialization is associated with higher welfare. However, in Economides and Salop, parallel vertical integration gives the lowest prices and therefore the highest welfare. Einhorn's result arises from increased productive efficiency, because compatibility ensures that the high-quality (or low-cost) producer of each component is the one that sells. Economides and Salop's conclusions derive from the internalization of pricing externalities. Einhorn corresponds more closely to the specialized nature of the PC compatible industry, where integration occurs more often in incompatible products, such as the Macintosh or Japanese PCs. In Chapter 2's model, when there are many firms, pricing externalities become unimportant and the efficiency effect predominates.

2.2.6 Product Line Decisions

Klemperer (1992) models firms' incentives to differentiate their products, which we call components, when consumers vary in their willingness to buy components from more than one firm. The literature, such as Shaked and Sutton (1982), has traditionally argued that profit maximization leads firms to differentiate their products. However, when consumer have "shopping costs" that discourage them from buying components from different firms, Klemperer shows that differentiation can intensify competition. The reason is that differentiation implies that consumers buy components from more than one firm, which increases the price sensitivity of demand. By contrast, when firms choose similar characteristics for their components, consumers buy only from one firm, and demand is less inelastic. Therefore, firms prefer "head-to-head" competition, in which their components have similar characteristics.

3 Choice of Standards

Decisions about standards are made by markets, by committees, and by governments. In the first case, consumers choose among the standards that firms offer, and firms' market shares decide which standards are the most widely used. In the second case, committees made up of firms and consumers decide among competing standards. In the final case, governments mandate a standard, for instance, for connecting a telephone to the telephone network.

Farrell and Saloner (1988) contrast standard-setting by committees with *de facto* standard-setting by markets. They find that the threat firms will adopt a standard outside the committee process encourages committees to set standards more quickly. This section focuses on the first case, in which the market sets the standards.

The appendix on the personal computer industry notes that firms often form coalitions supporting competing standards. Product compatibility models typically include a first stage in which firms choose standards, where firms that choose the same standard are compatible. From a theoretical perspective, the game of standard choice is an example of a more general class of games in which agents partition themselves into groups. Other examples include the formation of customs unions, currency areas, and research consortia, as well the locational decisions of firms. Although there has been no general treatment of this topic, many articles look at special cases.

Matutes and Padilla (1989) consider the incentives of banks to share their networks of Automated Teller Machines (ATMs). They distinguish two effects of sharing. First, sharing reduces the differentiation in services between the sharing banks. Second, if a subset of banks shares, these banks take business away from those that do not. For instance, two banks will share networks if they steal enough business from a third incompatible bank. However, if all banks share their network, there is no potential to steal business. If banks can be excluded from a network, intermediate compatibility[11] is the result. With three banks, two would share while excluding a third.

By contrast, Economides (1991) argues that each of n symmetric banks wishes to increase compatibility (reduce the adapter cost) with each other bank. However, as argued above, when n is large, the negative externality of increased compatibility between two banks outweighs the increased profits for those two banks, so industry

[11] We use the term *intermediate* compatibility to describe a situation in which a only a subset of firms are compatible with each other, and *partial* compatibility a situation in which compatibility between two products is a matter of degree.

profits fall. Then, firms' choice of compatibility is a prisoner's dilemma—firms maximize compatibility although this lowers industry profits. Chapter 2 presents another model with similar coalitional effects.

Yi (1992) examines the endogenous formation of coalitions in a variety of settings. In one case, coalitions are open to all comers, while in another, they are not. Firms choose which coalition to join simultaneously in one setting, and sequentially in another. Yi provides conditions for existence and uniqueness of equilibrium in each of these cases. Although this work is a step towards a general theory of coalition formation, he places restrictions on the payoff function to ensure uniqueness and existence, and these restrictions are at odds with the behavior of the computer industry. In Yi's framework, if a subset of firms forms a coalition with each other, other firms are better off. In other words, coalition formation has a positive externality on outsiders. On the other hand, when a subset of computer firms becomes compatible with each other, the computer industry sees this as an aggressive action that hurts competitors. In Chapter 2's model below, coalition formation is an aggressive move.

Zachau (1987) and Farrell and Scotchmer (1988) analyze the question of who joins whom in cooperative games, and they obtain similar results to each other. Zachau assumes that the payoff to a member of a coalition depends only on the abilities of its members. In the core's unique outcome, all members receive the same payoff, and all agents of similar abilities join the same coalition.

The theory of clubs in public economics examines how individuals cooperate to provide excludable public goods, such as a swimming pool. In this literature, the payoff of one club does not depend on the number or size of other clubs, nor are the memberships of clubs necessarily mutually exclusive (see Cornes and Sandler (1986)).

By contrast, observers of the computer industry view compatibility between a subset of computer companies as an aggressive move that hurts the profits of other companies.

4 Network Externalities

Another strand of the literature studies *network externalities*. Typically, when someone buys a product that connects to a network, such as a telephone, this purchase produces an external benefit for other network users. The external effect is called a network externality. For instance, a consumer's decision to buy a telephone has a positive externality on other people who want to call him, or on people who benefit from his calls to them. Being part of a bigger network is more valuable than being part of a small network. Some authors have written that network externalities generate benefits to "going along with the crowd." However, if consumers have varying tastes, it is more accurate to say that a consumer would like the crowd to join the network that he prefers rather than joining the crowd on another network. Other examples of networks include electronic mail networks and fax machines. In this literature, two products are *compatible* if they are part of the same network. For instance, any two telephones that can communicate with one another are compatible, while telephones and walkie-talkies are *incompatible*, because they cannot communicate with one another in normal circumstances.

Network externalities occur in other settings as well. The quality and variety of software for one consumer's personal computer increases with the number of other consumers who buy computers that can use similar software. The larger the installed base of consumers using similar computers, the greater are economies of scale in producing and distributing software. Given that the network and mix-and-match definitions both apply to hardware and software, how then do the two definitions differ? Under the mix-and-match definition the two components, hardware and software, are explicitly part of the model. However, most models of network externalities focus explicitly on one component, hardware, and mention software only to motivate the existence of externalities among hardware products. Another difference is that many mix-and-match models ignore the possibility of increasing returns to focus on other issues, while network externalities imply the existence of demand-side increasing returns by definition.

Farrell and Saloner (1986b) and Gilbert (1992) argue that there is a tradeoff between the respective advantages of standardization and variety in the presence of network externalities. In the extreme, this trade-off implies that products connected to the same network are undifferentiated. For instance, suppose that some people prefer communicating by talking on the telephone, while others prefer communicating using fax machines, but a person cannot afford both a fax machine and a telephone. Then, society faces a choice between standardizing on one form of communication, which achieves large network benefits, or allowing people to use their preferred method of communication. This tradeoff is similar to the standard tradeoff between variety and economies of scale, as discussed by Dixit and Stiglitz (1977).

Several factors can relax the tradeoff between variety and network benefits. First, networks can become compatible *ex post* through converters, gateways, and adapters as discussed by David and Bunn (1988). For instance, most electronic mail services now connect to each other through gateways, although initially many networks could not communicate with each other. Another *ex post* response to incompatibility is for a person to join several networks, say by owning both a telephone and a fax machine (see de Palma *et al* (1992)). However, these options are often imperfect and costly, as discussed in the section on converters below. Second, significant and enduring tradeoffs between variety and networks benefits seem scarce. Often agents acquire a taste for variety through sunk costs in one standard, so the real issue is the durability and specificity of these sunk costs. Finally, the philosophy of computer networking has shifted towards a client-server architecture, which unbundles the way in which a user communicates with a network from the functions the network provides. Then the network appears differently to different users depending upon their tastes, while providing each of them the same underlying functions. For instance, a wide variety of telephones connect to the same network. This approach divides a network service into a component with variety and a component without variety, which can then be mixed-and-matched.

4.1 A Simple Model of Network Externalities

The following simple model of network externalities is based on Katz and Shapiro (1985). Duopolists make a product for which there is a network externality. In the regime of incompatibility, a consumer's valuation for the product increases in the number of other consumers buying the same brand. In the regime of compatibility, his valuation increases in the number of consumers buying either brand. The firms' products are otherwise homogeneous. A consumer's preference among products depends on his expectations about how many people buy which brand. Competition takes place in three stages: (1) consumers form expectations about the sales of each firm,

(2) the firms choose quantities, taking expectations as given, and (3) consumers choose the product offering the largest surplus, that is, network benefits minus price.

A consumer's reservation price for firm 1's product is a function $R+v(y_1^e)$, where R is the consumer's hedonic reservation price exclusive of network benefits, y_1^e is the expected size of firm 1's network, and $v(.)$ describes the benefits of belonging to a network of size y_1^e. Under incompatibility, the expected size of firm 1's network is $y_1^e=q_1^e$, where q_1^e is firm 1's expected sales. If firms 1 and 2 are compatible, firm 1's customers also receive a network benefit from purchases of firm 2's product, so $y_1^e=q_1^e+q_2^e$. Assume that $v(.)$ is an increasing concave function passing through the origin such that

$$\lim_{q\to\infty} v'(q) = 0. \tag{10}$$

Consumers' hedonic reservation prices R are uniformly distributed between R_0 and minus infinity with density one.

Suppose that firms choose an industry output of $q=q_1+q_2$. Because consumers are distributed with density one, the hedonic reservation price of the marginal consumer is R_0-q. Each firm must offer a surplus $p_i-v(y_1^e)$ equal to the hedonic reservation price of the marginal consumer, so firm 1's prices must satisfy

$$p_1 = R_0+v(y_1^e)-q. \tag{11}$$

Firm 1's profit, if marginal costs are zero, is

$$\pi_1 = q_1(R_0+v(y_1^e)-q). \tag{12}$$

A firm chooses its quantity taking consumer expectations and the other firm's output as given.[12] Differentiating with respect to q_1 gives the first-order condition

$$q_1^* = R_0 + v(y_1^e) - q, \tag{13}$$

which happens to equal p_1. Solving the first-order condition gives the familiar-looking solution

$$q_1^* = \frac{R_0 + 2v(y_1^e) - v(y_2^e)}{3}, \tag{14}$$

although there may be corner solutions as well. Cournot equilibrium exists for any output expectations, but outcomes must fulfill consumers' output expectations. In a symmetric equilibrium, (11) and (13) imply that firm 1's price is $p_1 = q/2$, which increases in the equilibrium industry output. Consumer surplus is $q^2/2$, firm 1's profit is q_1^2, and industry profits are $q^2/2$.

Under compatibility, fulfilled expectations imply that $y_1^e = q_1^* + q_2^*$. Both firms have the same equilibrium output

$$q_1^* = \frac{R_0 + v(2q_1^*)}{3}. \tag{15}$$

The assumptions on $v(.)$ above imply a unique solution.

Under complete incompatibility, firm 1's first-order condition gives

[12] Katz and Shapiro examine a variant of their model in which firms do not take consumers' expectations as given, but assume that expected output equals actual choices of output, and find similar results.

$$q_2 = R_0 + v(q_1^*) - 2q_1^* \tag{16}$$

which Katz and Shapiro call the firm's equilibrium reaction correspondence. It is not a standard reaction function, because this correspondence picks out only reactions that fulfill consumer expectations.

There are three types of equilibria: symmetric, in which all firms are active; asymmetric, in which the two firms have positive but different output levels; and monopolistic, in which only one firm has a positive output. Figure 5 illustrates the three types of equilibria. In the asymmetric case, one firm has a larger market share because consumers expect it to do so, though firms are otherwise symmetric. If the larger firm's output falls in a shift to compatibility, then its profit does as well.

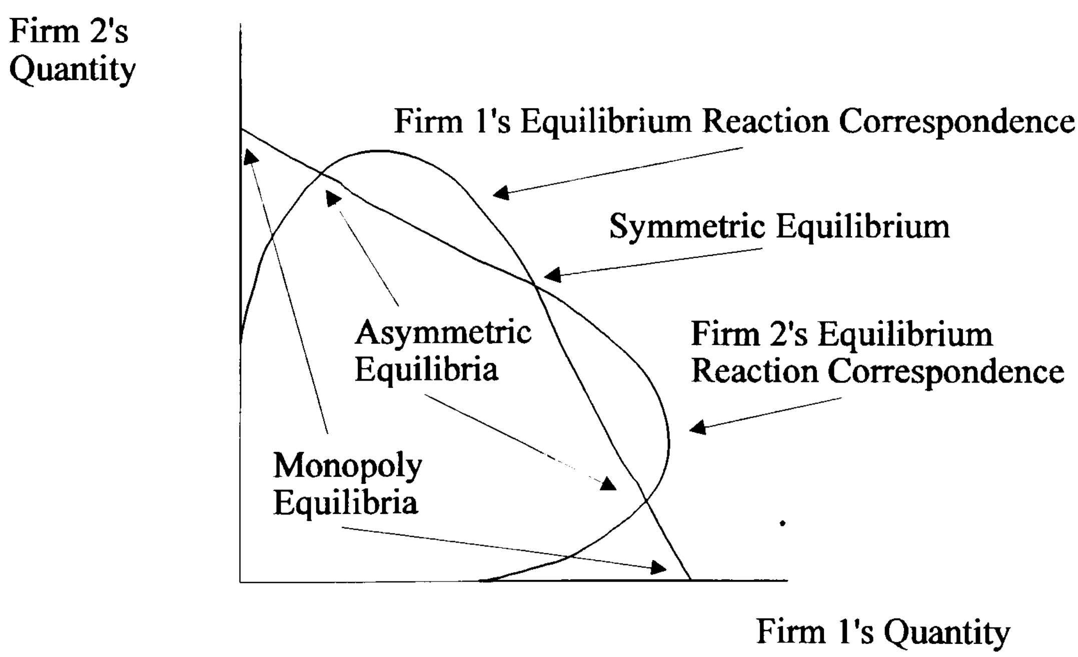

Figure 5: Equilibria in Katz and Shapiro (1985)

Katz and Shapiro show that industry output q is greater when all firms are compatible than in any equilibrium with less than complete compatibility. The above formulae imply prices and consumer surplus rise in a shift to complete compatibility as well. If the equilibrium under incompatibility is symmetric with all firms active, profits rise with compatibility implying that welfare is higher as well.

The model above readily extends to the case of n firms. Then, comparing two equilibria in which some but not all firms are compatible is more difficult. If two groups combine and their output beforehand is less than R_0, then their combined output as a group rises, the output of any other firm falls, and industry output rises. Higher industry output means that prices and consumer surplus rise. Firms with higher output have higher profits, and those with lower output have lower profits, but higher industry output does not necessarily imply higher industry profits. When the original output is greater than R_0, the number of active firms can drop after two groups combine, causing industry output to fall. Firms choices of standards do not uniquely determine profits, so Katz and Shapiro do not explicitly solve the game in which firms choose standards.

4.2 Other Network Issues

Network externalities raise many other important issues, some of them dynamic. These include the timing of adoption, network momentum, product preannouncements, converters, divestiture, and inducements to enter.

4.2.1 Adoption and Technological Progress

Present buyers of network products care about future sales, because these sales bring network benefits. Therefore, consumers also care about technological progress in their network even if they make no future purchases. Katz and Shapiro (1986) examine the interaction between consumers' expectations and differing rates of exogenous technical advance for competing products. One might expect the product that is initially superior to develop a large network in the first period, and therefore to dominate in the second period even if it is no longer the better product. However, in Katz and Shapiro's model, the opposite may occur, because first-period consumers care about future growth in their network. A product that is superior in the future may experience more growth in its network, and attract purchases now as well.

4.2.2 Network Momentum and Inertia

The timing of the adoption of standards can be an important strategic variable. Firms often have a strong incentive to adopt preemptively, for instance, to introduce a network product and begin building installed base before a competitor can do so.

Katz and Shapiro (1992) examine the timing of the introduction of new network products. The conventional wisdom has it that the first network product to reach the market has excess inertia from a social perspective once it has developed a large installed base, that is, there is insufficient incentive to introduce new products. However, their analysis shows that there may also be an excess incentive to introduce new networks, because new consumers do not consider the negative effect on old consumers of choosing the new network.

Farrell and Saloner (1985) show that firms may have trouble coordinating a switch to a new network because of imperfect information about each others' preferences over networks. Sometimes all firms adopt the strategy "switch if others do," even if everyone agrees that the new network is better. However, a switch to a new network can also have excess momentum.

4.2.3 Predatory Product Preannouncements

Farrell and Saloner (1986a) explore the potential for product preannouncements to be predatory. A U.S. antitrust investigation against IBM investigated charges that the company allegedly used product preannouncements in an anticompetitive manner. However, some analysts argued that preannouncements only help~~s~~ buyers to make more informed decisions. Farrell and Saloner provide support for the former view, by showing that a preannouncement can cut short growth in a new competing network by persuading consumers to wait for another product.

Their analysis fails to explore sufficiently the credibility of preannouncements. Computer companies have repeatedly failed to introduce products as announced. Credibility is such a common problem that the computer trade press has a word for software that does not appear as announced: "vaporware."[13] Firms have imperfect means to convince a market that a product will be released as predicted. Preannouncements are more likely to be credible if the dominant firm is known to have a technological lead, and can respond to the introduction of a competing network product with something superior. Thus, an incumbent might innovate to deter entry, by implicitly threatening to undermine the entrant's network.

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[13] *Computerworld*, "Users Say PC Promises Wearing Thin," **22**:14, 4 April 1988, pp. 1,6.

4.2.4 Converters

Farrell and Saloner (1992) examine the economics of converters, which allow network benefits to extend from one incompatible network to another. They find that converters can reduce the welfare cost of incompatibility. However, converters also lower the private cost of incompatibility, which can encourage too many consumers to buy incompatible products with converters.

Their model focuses on the tradeoff between standardization and variety. Consumer tastes for technologies are distributed over a unit interval, and there are two technologies, one at each end of the interval. Because of network externalities, each consumer wants other consumers to use the same technology or a technology made compatible through a converter. The authors distinguish one-way and two-way converters. A one-way converter allows a consumer to gain network benefits from users of other technologies, without providing any in return. A business that provides a one-way converter can only raise the value of its product. Consequently, their analysis focuses on two-way converters.

Farrell and Saloner also examine three different market structures: (1) monopoly, in which one firm controls both technologies, (2) duopoly, in which the two technologies are separately owned, and (3) perfect competition, in which both technologies are nonproprietary. Under perfect competition, one might expect consumers to buy too few converters, because they convey external benefits. However, the opposite is the case. Because converters are imperfect, buying the dominant technology conveys a larger externality on a larger group, than does buying the alternative technology and a converter. Given consumers' choices of technology, the social benefit of buying a converter exceeds the private benefit. However, converters encourage consumers to buy the "wrong" technology.

Although a monopolist can internalize the problem, it will also encourage too much reliance on converters. A monopolist wants to maximize the price extracted from the marginal consumer, which he can do by providing two technologies instead of one. The increase in variety raises the price that can be charged to the marginal consumer, while lowering the network contribution to the surplus of inframarginal consumers. Converters get used for more parameter values than under competition, and in greater quantities.

Under duopoly, prices are higher and consumers use even more converters. Because the firm with the dominant technology prices above cost, the alternative firm is better able to entice consumers to switch and use converters.[14]

4.2.5 Divestiture

Economides and Woroch (1992) compare social and private incentives to make networks compatible under several ownership structures. Compatibility can raise profits if it creates new services and sometimes if new services compete with existing ones. When two networks provide components of a service, divestiture lowers consumer and industry surplus because of double marginalization. The presence of network

[14] These observations about converters have analogues in the literature on economic geography (see Krugman (1992)). Cities sometimes exert enormous effort to connect themselves well by transport, in the hopes that these connections will attract manufacturing investments. However, sometimes exactly the opposite has occurred, because the high quality of transport allows manufacturers to serve the city's markets from other locations. Similarly, the sponsor of a large network may find that connection to other networks encourages consumers to buy from those competing networks.

externalities increases the benefits of internalizing the externalities through integration. The authors assume a fixed market structure and do not, for instance, consider the possibility that divestiture allows new entry by specialists.

4.2.6 Inducements to Enter

Economides (1992) explores the possibility that an incumbent firm might encourage entry by firms using the same standard. The appendix on the personal computer industry notes many examples of technology giveaways, which encourage entry by compatible firms. Economides shows that a quantity-setting incumbent that is a Stackelberg leader will license its network technology without charge to entrants if the network externality is sufficiently strong, to build up the size of its network. If the network externality is very strong, the incumbent may subsidize entrants.

This argument has parallels in the literature on second-sourcing. In this literature, an incumbent licenses its technology to an alternative supplier to lessen consumers' concerns that they will become locked into a single supplier. This brings us to a third definition of compatibility, which is discussed below.

5 Lock-in and Switching Costs

Under a third definition, two products are *compatible* if consumers can switch between their suppliers easily. Compatibility means that consumers are not locked into repeated purchases from a single supplier. For instance, two typewriters with conventional QWERTY keyboards are compatible in this sense, because typing skills acquired with one typewriter on this standard are useful with any other brand. However, a typewriter with the alternative DVORAK keyboard layout is incompatible with a QWERTY typewriter, because a buyer switching from one to the other would have to exert considerable effort learning to type again.[15] Similarly, spreadsheet packages are sometimes said to be Lotus 123-compatible because they have the same user interface and keystroke sequences. When switching costs are prohibitively high, a consumer becomes completely locked into a single technology or a supplier. Then, the product is *incompatible* with other products.

A common cause of switching costs is the need to acquire skills to use a product. Firms can lower switching costs either by making a product easy to use, or by designing it to require the same skills as another product. Switching costs can also arise when a consumer makes a sunk investment in hardware and must later buy the corresponding software from the same supplier. On the other hand, a company locating near several suppliers can reduce its cost of switching suppliers in the future.

The switching cost definition of compatibility ties in closely with the previous two definitions. The combination of switching costs and increasing returns to scale can lead to network externalities, as pointed out by Beggs (1992). Suppose that consumers buy a product in the first period that sometimes needs spare parts in the second period, and firms produce spare parts with increasing returns. Spare parts will be cheaper in the second period the more people buy the product in the first period, so a network externality occurs.

[15] David (1985) and Liebowitz and Margolis (1990) give contrasting views on whether the continued dominance of QWERTY over DVORAK represents a market failure.

Network externalities can produce switching costs as well. Given that most consumers have chosen one network, it is costly for a single consumer to switch to a new network if others do not follow. The cost of switching networks disappears if all consumers switch to the new network. Farrell and Saloner (1985) explore problems of coordination in switching from one network to another. In addition, dynamic models of networks often assume that consumers are locked into using the first network they choose to ensure that installed base carries over from one period to the next.

Finally, switching cost compatibility and mix-and-match compatibility are both issues for multicomponent products. A personal computer user may have a sunk investment in computer software that creates a cost of switching to incompatible hardware. Thus, the difference between mix-and-match and switching cost compatibility is whether products are bought sequentially or simultaneously.

5.1 A Simple Model of Switching Costs

Padilla (1991) illustrates how the inability of consumers to switch suppliers can relax competition, in a model drawing upon Farrell and Shapiro (1988) and Beggs and Klemperer (1992). We interpret his model as describing competition under incompatibility, and later compute the equilibrium under compatibility, that is, when consumers can freely switch suppliers. Padilla's infinite-horizon model employs overlapping generations of consumers, each of whom lives two periods. In each period consumers have inelastic demand with reservation price R for one unit of a good which is homogeneous before their first purchase. However, a consumer's purchase in the next period must come from the same supplier as his first purchase, so the good becomes differentiated *ex post*. Duopolists set prices in each period simultaneously and independently, by contrast with Farrell and Shapiro's (1988) model, in which firms alternate in setting prices. In each period, a firm must balance setting a high price to exploit its locked-in customers against setting a low price to attract new consumers. Firms cannot price discriminate between old and new consumers.

New consumers buy from the firm with the lowest price, or if the firms' prices are equal, the smaller firm. Thus, a firm has a customer base of size zero or one at the beginning of any period. No pure-strategy equilibrium in prices exists by the following argument. If both firms charge the same price, the firm that sold in the previous period will undercut by a small amount ϵ. If both firms charge a price of zero, the firm that sold last period will deviate to price R. Finally, if the firms charge different prices, the lower-price firm will raise its price by ϵ.

Padilla finds a unique symmetric Markovian perfect equilibrium in mixed strategies. If the discount rate is δ, the probability distribution giving the pricing strategy of the large firm is

$$\Gamma_1(p) = 1 - \frac{R(1+\frac{\delta}{2+\delta})}{2(p+\frac{R\delta}{2+\delta})}, \qquad p \in [\frac{R}{2+\delta}, R] \tag{17}$$

with an atom of ½ at R.[16] The small firm uses the strategy

$$\Gamma_0 = 2\Gamma_1, \qquad p \in [\frac{R}{2+\delta}, R] \tag{18}$$

but with no atom at R.

[16] An atom is defined as a positive probability weight on a single point; with a continuous distribution function, no single point receives positive probability weight.

In equilibrium, prices and profits are above the competitive level of zero with probability one. Market share is valuable, because the value of the game for the large firm (V_1) is greater than the value of the game for the small firm (V_0)

$$V_1 = \frac{2R}{2 - \delta(1+\delta)} > V_0 = \frac{R(1+\delta)}{2 - \delta(1+\delta)}. \tag{19}$$

As described above, we interpret Padilla's model as describing incompatibility in the sense that buyers cannot switch suppliers. Compatibility eliminates lock-in and allows consumers to switch suppliers without cost. Then, prices and profits fall to zero (assuming firms do not collude). The large firm had higher profit under incompatibility, so it has more to lose from compatibility.

The results for this infinite horizon model contrast with those of two-period models such as Klemperer (1987). In two-period models, switching costs relax competition in the second-period, but intensify competition to build a customer base in the first period, with ambiguous total effects on profits. Beggs and Klemperer (1992) argue the relaxation effect predominates in multiperiod models because discounting encourages firms to take profits immediately, and higher prices now soften the behavior of the competitor tomorrow.

Padilla (1992) shows that the same basic results apply to the oligopolistic case. His analysis does not explore the possibility that a subset of firms might reduce switching costs among their products to attract customers from other firms.

5.2 The Formation of Switching Costs

The literature has not focussed on the formation of switching costs. Models such as the one above suggest that firms will raise switching costs to achieve higher profits, say by designing products to be difficult to use or to require spare parts available only from the original supplier. In practice, firms often do the opposite. For instance, many auto parts have become standardized across firms.

Below, we suggest several explanations of the puzzle of why firms might lower switching costs. First, consider the case in which switching costs are symmetric, that is, it costs the same to switch to a product as it does to switch away from that product. One explanation of the puzzle is that a forward-looking consumer will choose products without switching costs, say by choosing a product with standard spare parts over one with nonstandard parts. That is, two firms whose products use the same spare parts can attract customers away from a third firm with nonstandard parts. If the group of firms eliminating all switching costs among themselves cannot exclude members, this behavior may lead firms to eliminate switching costs among themselves. This argument implicitly requires three firms, but the literature, excepting Padilla (1992), builds models with two firms. Also, a model in which some but not all firms are compatible may not be tractable because of the implied asymmetry between firms.

The following example illustrates coalitional effects in oligopoly. Suppose that firms choose their location in the first period, a consumer with inelastic demand chooses its location in the second period, and the consumer makes a single purchase in the third period from a firm in its location. Although the consumer does not make two sequential purchases as is usual in the switching cost literature, he does make a decision that locks him into a purchase in the next period. The consumer does not learn firms' prices until the third period. Therefore, he will choose a location with several competing firms rather than one with a single firm, in the expectation that a location with many competing firms will have lower prices.[17] Industry profits are highest when all firms choose different locations, because the price is highest. However, if two of many firms choose the same location, they can attract the consumer. They will find this profitable if profits do not fall too quickly with the number of firms in the same location. Then, firms will locate together in an attempt to attract the consumer, with detrimental effects on industry profits. Chapter 2 finds a similar result in a model of mix-and-match compatibility; all firms become compatible even if incompatibility maximizes industry profits.

Another version of this argument is that a consumer might prefer a product with spare parts that are competitively supplied. Knowing this, a firm might encourage other firms to make its spare parts, though this increases competition in the future. The appendix on the personal computer industry notes that companies often invite other firms to become compatible, perhaps by giving away a technology. They may invite competition from new entrants, or cooperate with incumbents. Farrell and Gallini (1988) model the incentives of a monopolist to create a second source for its technology. In their model, a monopolist may invite competition to make consumers more willing to incur setup costs of purchasing the monopolist's product. Although this model focuses on the monopoly case, the argument is stronger in oligopoly, because a firm that second-sources can also attract customers away from other firms.

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[17] One form of competition with this property is the uncertain costs model of Chapter 2.

The literature focuses on the analysis of symmetric switching costs to ensure analytical tractability. However, asymmetric switching costs are possibly more important in practice. For instance, products require varying degrees of effort to learn how to use them. For that reason, it can cost more to switch from product A to B than from B to A. A firm has nothing to lose by making it easy for old consumers already locked into another supplier to switch to its product. On the other hand, new, uncommitted consumers may decide to try another product first.

In practice, firms usually try to make their products easier to use, not harder to use. The main selling point of the Macintosh is its ease of use. Also, word processors, spreadsheets and databases are often designed so they can use the same keystrokes or files as an established package, which lowers the cost of switching to a product. Lotus and Apple have sued several other companies for copying the "look-and-feel" of their software.[18] Not all companies protect their user interface so zealously; the SQL database query language was originally part of an IBM application, but is now becoming a standard.

Firms can also make it difficult for consumers to switch away from their product. Long-term contracts, as discussed by Aghion and Bolton (1987), are one method. However, consumers may avoid those firms that make it difficult to switch away if there are other firms that do not. While long-term contracts may successfully deter entry, the strategy is less effective when there are other incumbents who may offer shorter term contracts. Thus, an individual firm may have more to gain by reducing the cost of switching to its product than by increasing the cost of switching away from its product.

[18] Congressional Research Service. "Telecommunications and Information-Systems Standardization—Is America Ready?" by David Hack, U.S. Congress, 21 May 1987, p. 36.

As a result, competition in the choice of switching costs may tend to reduce switching costs rather than raise them; asymmetric switching cost formation appears to be a prisoner's dilemma.

6 Aims of the Thesis

This chapter has attempted to set the stage for the chapters that follow. It presents models based on three definitions of compatibility: mix-and-match, network externalities, and the lack of switching costs. These models reach differing conclusions about the effects of compatibility, which are summed up in the following table.

Effects of Compatibility	"Facts"	Mix-and-match	Network externalities	Switching costs
Literature	Appendix on the PC industry	Matutes & Regibeau, Economides, Einhorn	Katz & Shapiro, Economides	Padilla, Beggs & Klemperer, Farrell & Shapiro
Prices	–	+	+	–
Profit margins	–	+	+	–
Industry profit	+/–	+	+	–
Cons. surplus	+	+/–	+	+
Welfare	+	+/–	+	+
Output	+	+/–	+	+
Costs	–	–	?	?
Variety	+	+	–	?
Specialization	+	+	?	?

The column headed "Facts" summarizes material presented in the appendix below on the personal computer industry. A quick survey of anecdotal evidence suggests that increased compatibility in this industry has been accompanied by lower prices, profits margins, and costs, but increased output, variety, and specialization. The information in this column should not be interpret as causal claims backed by a rigorous case study. However, most industry observers say that compatibility lowers prices and profit margins. Also, the fact that computer buyers have aggressively promoted increased standardization is taken as an indication that compatibility increases consumer surplus. Many observers would also claim that standardization in the computer industry has been socially beneficial.

The next column lists the predictions of the mix-and-match literature. Matutes and Regibeau (1988), Economides (1989), and Einhorn (1992) find that compatibility raises the prices and profit margins of symmetric firms when the demand for hybrids is not too low. In Matutes and Regibeau (1988) compatibility increases variety, while in Einhorn compatibility increases quality (lowers costs), and increases specialization. Effects on consumer surplus, welfare, and output are ambiguous.

In the network externalities literature, compatibility also increases prices and profits, but for different reasons. It raises consumer surplus, welfare, and output as well. Standardization reduces variety. The literature has not focussed on issues of specialization and costs.

The switching costs literature can be interpreted as describing a type of incompatibility that makes it costly for consumers to switch suppliers. Under this interpretation, compatibility eliminates switching costs, which has the effect of lowering prices and profit margins, but increases consumer surplus, welfare, and output. Thus, these models, which are not always considered to be part of the product compatibility literature, offer predictions that are more in line with stylized facts about compatibility.

This thesis develops mix-and-match models with predictions that are the opposite of the mix-and-match and network externalities literatures. Chapter 2 shows that mix-and-match compatibility lowers profit margins and prices on average when there are sufficiently many firms, if their component characteristics, qualities, or costs are independent random variables. This chapter also examines coalitional behavior in the choice of standards, motivated by examples given in the appendix on the personal computer industry. When all standards are nonproprietary, complete compatibility is the unique equilibrium for noncooperative choice of standards although complete incompatibility sometimes gives higher industry profits.

Chapter 3 examines the role of R&D effort in a mix-and-match model. When there are sufficiently many firms, compatibility lowers expected prices relative to incompatibility, and higher sales then increase the return to R&D. Under compatibility, firms tend to choose effort levels that are closer together than under incompatibility, because then their costs are more likely to be complementary.

Chapter 4 shows that duopoly profits can fall under compatibility when components are combined in variable proportions. Compatibility constrains a dominant firm from setting its price in either component above the other firm's cost. This constraint does not alter profits when consumers combine components in fixed proportions, but does when consumers combine components in variable proportions. Relaxing the constraint of fixed coefficients also permits examination of Cournot competition, and finds results that are similar to Bertrand duopoly. Although firms specialize partially in the low-cost components under incompatibility, they specialize even more under compatibility.

7 Appendix: Compatibility in the Personal Computer Industry

This appendix presents stylized facts about compatibility in the personal computer industry.[19] Its purpose is to motivate a review of the theoretical literature in this chapter, and the development of theoretical models in later chapters. This material is not intended to form a thorough case study of the industry or to assess causal relationships.

It will be helpful to begin with a description of the elements that make up a personal computer, assuming some familiarity with personal computers by the reader. The central processing unit (CPU) processes series of very simple instructions, which are called software. IBM PC compatible computers use Intel CPUs or imitations of Intel chips, while Apple Macintoshes use Motorola CPUs.[20] Computers have two types of memory. Random access memory (RAM) holds information only when the computer remains turned on, and floppy disks and hard disks store information for longer periods of time. Users communicate with the computer with a keyboard, screen, and perhaps a mouse; optional accessories include a printer, or a modem for connection with other computers by telephone. These memory and input/output devices are all known as peripherals.

[19] Information is up-to-date as of June 1993.

[20] As described below, IBM no longer makes "IBM compatible" computers, so the term "PC compatible" or "PC" is sometimes used instead.

Hardware by itself can accomplish nothing without software, which is a set of instructions to the CPU. An operating system is software that manages the basic functions of the different pieces of hardware: reading keys pressed on the keyboard, sending information to the screen and printer, reading and writing information on the disc drives, and scheduling of the CPU's work. Common operating systems include DOS and Windows for PCs, System 7 for the Macintosh, and Unix for larger computers. The additional software needed for word-processing, spreadsheets and databases are called applications packages.

7.1 Before the Personal Computer

Most early computers were incompatible with one another. Software and peripherals developed for one computer could not work with another computer, perhaps even ones produced by the same manufacturer. A buyer typically bought all parts of a system, including service and supplies, from the same manufacturer. Manufacturers did not always make new systems "backwardly compatible" with existing systems and often sold several incompatible systems simultaneously. Each practical application of computers required a machine specifically designed for that purpose.

An important early step towards compatibility occurred within the IBM product line. The IBM 360 product line broke new ground as a range of general-purpose, compatible and modular machines of different sizes, all using standard interfaces.[21]

[21] Franklin M. Fisher, John J. McGowan, and Joen E. Greenwood, *Folded, Spindled, and Mutilated: Economic Analysis and U.S. v. IBM.* (Cambridge, Mass.: MIT Press, 1983), pp. 8, 59. See also Kenneth Flamm, *Creating the Computer*, (Washington, DC: The Brookings Institution, 1988), p. 97.

The same software and peripherals worked with machines of different sizes used for a variety of purposes.[22] Before long, "plug-compatible" manufacturers (PCMs) took advantage of IBM's standard interface to sell competitively-priced printers and disc drives that worked with IBM 360 computers.

Although IBM pursued a strategy of compatibility within its product line, it attempted to prevent other companies from becoming compatible with it. The PCMs accused IBM of manipulating interfaces to make their equipment unusable with IBM equipment, and of invalidating the maintenance contracts of IBM computers connected to PCM peripherals. IBM also opposed some government efforts at standardization. For instance, the U.S. government developed the ASCII code as a standard way of encoding information for computers, to allow computers to share information. IBM refused to adopt this standard until the 1980s, and used its own proprietary method, EBCDIC.

The computer industry took another step towards compatibility when IBM also unbundled (sold separately) its hardware and software under pressure from a U.S. government antitrust suit. Unbundling made it easier for companies to enter by selling only one component. Fujitsu and Hitachi began making IBM-compatible CPUs following this decision, because their customers could buy the IBM 360 operating system software without having to buy IBM hardware.[23] Unbundling also led to the creation

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[22] In economic history, the development of interchangeable spare parts was analogous.

[23] Marie Anchordoguy, *Computers, Inc.: Japan's Challenge to IBM* (Cambridge, Mass.: Harvard University Press, 1989), p. 109.

of the independent software and services industry.[24] Independent software developers accounted for 30-50% of systems software developed for the IBM 360's generation of computers.[25]

After IBM unbundled its systems, competitors continued to claim that IBM kept interface information secret or made changes in interface specifications to discourage competition from PCMs. After an investigation by the European Commission, IBM agreed in 1984 to make available the interface specifications for new mainframe computers within four months of announcing the products.[26]

IBM successfully fought off almost all of the many legal challenges to its ability to set standards. Of the many actions against IBM in U.S. courts, only one private suit, by CDC, was successful; an appeals court overturned a victory by Telex. IBM's consent degrees in the 1950s and 1960s were not important constraints on its behavior. Also, the U.S. Justice Department dropped its antitrust efforts in 1981.[27] Of course, the mere existence of government suits had some effect on IBM behavior.

Despite the U.S. government's failure to win a court judgment against IBM, one analyst of the computer industry wrote in 1975 that "most of the progress that has taken

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[24] John Cavanagh, *Alien's Guide to the Computer Industry* (London: Reed Business Publications for Computer Weekly, 1988), p. 8.

[25] Fisher *et al*, p. 211.

[26] Kenneth Flamm, *Targeting the Computer: Government Support and International Competition*, Washington, DC: The Brooking Institution, 1987, p. 113.

[27] Flamm, *Targeting the Computer*, p. 113.

place in computer standards has been because of government pressure."[28] The advent of the personal computer would later make such a assessment unlikely.

7.2 The Early Personal Computer Industry

In the early personal computer industry, consumers also had to buy the computer, software, printer, disc or tape drive, screen, and keyboard all from the same manufacturer. Brands included Apple, Commodore, Kaypro, Osborne, and the BBC. Independent manufacturers supplied some components and software. However, designing products that could work with more than one brand was difficult, and third-party suppliers became dependent on the success of particular brands. Some companies attempted to standardize around the CP/M operating system, but software was often not truly portable across brands of hardware.

7.3 The IBM PC

IBM introduced its PC in 1981, and in doing so departed radically from its traditional practice. It designed the PC around off-the-shelf components and published the system's specifications to encourage the introduction of compatible products. It sold printers made by another company, Epson, with IBM nameplates. Microsoft developed

[28] Gerald Brock, *The U.S. Computer Industry: A Study of Market Power*. (Cambridge, Mass.: Balinger Publishing Co., 1975), p. 144.

the MS DOS operating system for the Intel 8088/8086 processor chips, and Microsoft and Intel retained the right to sell these products to IBM's competitors.

The PC was an immediate success. IBM employees had ordered 30,000 on the day it was announced.[29] Producers of software, printers, and other add-ons rushed to develop compatible products. The availability of Lotus 123, a spreadsheet program, helped to sell the computer to businesses. Other companies tried with partial success to make computers that used the same software and peripherals as IBM. Eventually, the manufacturers of "clones," led by Compaq, achieved 100% compatibility. Compaq then outdid IBM by introducing a portable computer sooner and with more success. IBM began losing share in the PC market, but regained its position with the IBM AT, which used the Intel 286 CPU. The clone-makers soon managed to imitate the IBM AT as well, and eventually the competition became so intense that IBM stopped making PCs. Instead, IBM developed the PS line of computers based around a proprietary standard, Micro Channel Architecture, which was initially not available to other firms. Buyers did not respond favorably to the idea of going back to proprietary IBM standards, having experienced the benefits of compatibility, and IBM began licensing the technology to other firms. The IBM PC standard went on without IBM, developing into 386, 486, and now Pentium-based computers. These computers are now called "PC compatibles," rather than IBM PC compatibles. The term "PC" refers only to PC compatibles, and not to other types of personal computer, such as the Macintosh.

Spurred on by the success of the PC standard, buyers began asking for more compatibility between larger computers. A variety of organizations began promoting

[29] David Mercer, *IBM: How the World's Most Successful Corporation is Managed* (London: Kogan Page, 1987), p. 92.

"open systems," with some disagreement about what the word "open" meant. One definition was that an open system used nonproprietary standards. In Europe, open systems meant use of the Unix operating system, which a group of companies own jointly. Another definition says that "openness" implies portability, scaleability, and interoperability. Portability means that software works on different brands of machines, while scaleability means that software works on different sizes of computers. Interoperability allows buyers to mix-and-match hardware components into systems and computers into networks.

Compatibility has proved much harder to achieve for large computers than for personal computers, in part because of their greater complexity. For that reason, the Corporation for Open Systems spends large sums of money testing conformance to standards. In addition, the IBM PC served as the benchmark against which clone manufacturers measured compatibility. No company wants to give another manufacturer of large computers the advantage of having its computer define the standard. Almost all computer manufacturers claim in their advertisements to make open systems, but it is not clear that this means much.

The shift to compatibility appears to have had a significant impact on the international competitiveness of U.S.-based computer production, and on IBM as well.[30] The U.S. trade surplus in computers at the beginning of the 1980s amounted to 20% of world trade in computers, but the surplus had vanished by the end of the decade. The

[30] Allen Lenz, Hunter K. Monroe, and Bruce Parsell, *Narrowing the U.S. Current Account Deficit: A Sectoral Assessment*. (Washington, DC: Institute for International Economics, 1992).

increased importance of high-volume, low-margin, off-brand production appears to have played to the strengths of Asian producers.[31]

The move towards open systems and smaller computers appears to have caught IBM off guard. The price of its stock dropped sharply in 1992 and the company has had to lay off many workers. One analyst argues that IBM made mistakes with the IBM 360, which in retrospect seem similar to the mistakes it appears to have made with the IBM PC. By this account, IBM could have eliminated the plug-compatible industry with an early price cut.[32] In addition, in responding to the plug-compatible manufacturers, IBM probably paid insufficient attention to the 360 compared with the 370.[33] In any case, the IBM PC launched a movement towards compatibility that is changing the shape of IBM and the computer industry as a whole.

With this background, the next few sections present evidence on the economic effects of compatibility.

7.4 The Effect of Compatibility on Prices and Profit Margins

The next few sections compare prices and profit margins for PC compatibles with those of less compatible computers. They compare PCs with larger computers, with the Apple Macintosh, and with Japanese PCs.

[31] See also Charles Ferguson and Charles Morris, *Computer Wars* (New York: Times Books, 1993).

[32] Brock, p. 112.

[33] Brock, p. 370.

7.4.1 Mainframes and Minicomputers

It is a common view among observers of the computer industry that compatibility lowers profit margins. The *Financial Times* says that "almost every mainframe or minicomputer manufacturer in the United States and Europe is demonstrating declining profitability, if not actual losses."[34] It names open systems (compatibility) as the culprit, though it also gives credit to falling costs, the growing importance of service suppliers and system integrators, and falling demand. More recently, it states that "Open systems have already forced down the price of hardware."[35] Similarly, *The Economist* notes that personal computers have become commodities, and that larger computers are moving the same way.[36] For instance, margins on mainframe computers often exceed 70%, but can be lower than 30% on personal computers. Again, this analysis points to user-driven standardization, but also claims that the trend towards computers-on-a-chip limits the potential for computer makers to add value and differentiate their products. In addition, increasingly sophisticated users are demanding fewer post-purchase services. Niche companies that do a few things well are profitable, but big companies that do many things and offer customers "solutions" rather than computers are not.[37] Finally,

[34] *Financial Times*, "On the brink of disaster," 23 April 1991, supplement on The Computer Industry, p. I.

[35] *Financial Times*, "Big Change in Users' Expectations," 23 September 1993, supplement on Using Computers in Business, p. I.

[36] *The Economist*, "Pile 'em high, sell 'em cheap," 2 November 1991, p. 93.

[37] Intel and Microsoft are presumably examples of the first, IBM the second.

another account states that "the computer industry's adoption of open systems provides greater value for a lower price."[38]

A rigorous case study would be necessary to assess the claim that open systems lower prices. Also, while greater compatibility might explain why profit margins for PCs are lower than for larger computers, there are many other potential explanations. For instance, the minimum scale necessary for entry into the personal computer industry is undoubtedly smaller than for larger computers. Therefore, comparisons of similarly-sized computers might give a better indication of the effects of compatibility on prices and profit margins.

7.4.2 Macintosh versus PC Prices

The Macintosh computer is made only by Apple. The Macintosh is incompatible with all other brands of computer, and requires different software and printers than a PC. It therefore offers a potential benchmark for measuring the effect of compatibility.

The following compares Macintosh and PC prices. Mail order prices are used to control for differential distribution costs. A Macintosh IIci with a Motorola 25 MHz 68030 processor and built-in coprocessor, 8Mb of RAM, and 210 MB hard disk cost £2299 without a screen from one mail-order supplier.[39] On the other hand, a Viglen

[38] *Computing Canada*, "Open Systems: A Good Return on Investment," May 1992, p. 6.

[39] *MacUser*, Camelot advertisement, 8:12, 12 June 1992, p. 54.

386DX with a coprocessor, 8Mb of RAM, and 200MB hard disk cost £1723. Viglen is a large mail-order supplier of PCs in the United Kingdom.[40]

A potentially clearer comparison is software, because nearly identical packages are available for PCs and Macintoshes. WordPerfect for Windows cost £152 for the PC by mail order,[41] while WordPerfect 2.0 for the Macintosh cost £199.[42] Other comparisons are: Microsoft Word 5, £123 and £215 respectively; Grammatik 2.0, £31 and £79; Ventura Publisher, £284 and £595; Microsoft Excel 3.0, £171 and £225; Microsoft Project £243 versus £365. The spread for games is smaller: the top ten games in the United Kingdom for the PC cost an average of £35 versus £38 for the Macintosh.[43] One explanation for higher prices for Macintosh software is that the Macintosh operating system places greater demands on programmers. Also, because Apple has only about one-fifth of the personal computer market, Macintosh software developers spread their fixed costs over fewer sales.

A comparison of printer prices avoids these problems, because only minor redesign is necessary to convert printers from one computer to another. An HP Deskjet 500 cost £289 for the PC while the identical printer for the Macintosh, a Deskwriter, cost £319 from the same supplier.[44] An HP PaintJet XL costs £1048 for the PC versus £1269 for the PaintWriter XL for the Macintosh.[45]

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[40] *Personal Computer World*, Viglen advertisement, **15**:5, May 1992, p. 206.

[41] *Personal Computer World*, SkyWay Data advertisement, **15**:5, May 1992, p. 280.

[42] *MacUser*, MacLine advertisement, **8**:12, 12 June 1992, p. 6.

[43] *Personal Computer World*, "PC Chart, Macintosh Chart," **15**:5, May 1992, p. 373.

[44] *Personal Computer World*, **15**:5, May 1992, p. 330.

[45] *Personal Computer World*, **15**:5, May 1992, p. 309.

The above comparison shows that buyers pay a premium for the Macintosh and related products. Although the data only cover one country at one point in time, the existence of the premium is widely acknowledged.[46] However, it would be difficult to attribute the premium to the incompatibility of the Macintosh alone. First, the Macintosh and the PC are clearly differentiated goods. Macintosh buyers claim that the computer ease of use justifies paying a higher price. On the other hand, some PC users claim that a PC with the Windows operating system is as easy to use as a Macintosh. Second, to assess the impact of compatibility on prices in the same way as the literature, it would be more useful to compare an industry in which all firms are compatible with a similar industry in which all companies are incompatible. Third, entry costs into the PC industry are low, while Apple has successfully prevented entry into the "Macintosh industry." Finally, consumers can buy PCs through a variety of distribution channels, but buy Macintoshes mostly from authorized dealers.

There is some evidence that price cuts in the PC market lead price cuts in the Macintosh market.[47] A series of price wars among PC manufacturers have lowered prices by as much as 40% per year in the last few years.[48] Mail-order firms, such as Dell, have put downward pressure on prices, and even IBM has entered the mail order business. The latest round of price cuts began in June 1992, when Compaq introduced a new range of low-cost computers. Because of these cuts, Çompaq's prices are only 15

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[46] *Marketing*, "It's the sound and vision thing," 11 February 1993, pp. S12-S13.

[47] *Marketing*, "Is Apple Losing Its Byte?" 22 October 1992, pp. 23-28. See also *Computerworld*, "Apple Confident in War with Windows PCs," **26**, 10 August 1992, p. 12.

[48] *Financial Times*, Survey on Personal and Portable Computers, 19 February 1993, p. I.

percent higher than those of clones with little brand recognition. Articles about later Macintosh price cuts present them as a response to falling PC prices.[49]

Price wars have taken place in the PC software industry as well.[50] The best-known software applications, including database, spreadsheet, and word-processing packages, had been selling in the range $400-800 through 1992. However, Microsoft introduced its new Windows database software for $99; the popular rival Paradox, made by Borland, had been selling for $795. In February, Borland introduced the Windows version of Paradox for only $140, after a dramatic series of layoffs. The president of Borland, Philippe Kahn, expects most PC software to sell for around $100-150 in the future.[51] By contrast, prices for Macintosh software have not come down as quickly.[52]

7.4.3 Japanese versus Rest-of-World PC Market

The comparisons of PCs, Macintoshes, and mainframes above are problematic. Fortunately, the Japanese PC market has recently provided a clearer example of the effects of compatibility.

Until recently, different brands of Japanese PCs were incompatible because each manufacturer developed its own proprietary hardware for entering Japanese characters. As a result, each brand of PC required its own software. NEC dominated Japan's PC

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[49] *Electronic Business*, "How Windows Cuts into the Macintosh Market," 19, March 1993, pp. 113-115.

[50] *BusinessWeek*, "The PC Price Wars are Sweeping into Software," 13 July 1992, p. 132.

[51] *The Economist*, "Borland International: Kahn unable," 20 March 1993, p. 103.

[52] *Info Canada*, "Apple Offers a Splash of Color," 18, March 1993, pp. 1-2.

market with roughly a 50% share.[53] The personal computer market grew slowly there because of a complete lack of compatibility, though some analysts blamed small desks and the importance of face-to-face relationships.

This situation is now changing. IBM, which had a market share of less than 10%, persuaded 25 PC manufacturers to join forces against NEC by becoming compatible with each other. They agreed to use DOS/V, a version of DOS that can handle Japanese characters, but works with the same hardware sold in other countries.[54] A base of software has grown quickly.[55] Compaq has introduced computers with prices half those of NEC's, and IBM has introduced range of low-priced computers as well.[56] Because of DOS/V, "The Japanese personal computer market is starting to witness something new—tough price and product competition."[57]

In part, Japanese producers' high hardware prices reflected the cost of developing software for each PC. Because Compaq computers will use existing software, the company does not bear similar expenses. There is a widely acknowledged shortage of software programmers in Japan for computers of all sizes, in large part because they must reinvent the wheel for each new computer, each application, or even each buyer.

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[53] *Washington Post*, "IBM, Japanese Join Forces on Standard: Move an Effort to Dethrone NEC," 13 March 1991, p. C1.

[54] *Financial Times*, "Engineers Aim For Ever Smaller Goal," Supplement on Portable Computers, 8 February 1991, p. III.

[55] *Personal Computer World*, "Japanese DOS wins converts," 14:10, October 1991, p. 108.

[56] *Financial Times*, "U.S. Price Warriors Spark Revolution in Desktop Japan," 2 March 1993, p. II. See also *Mitsubishi Bank Review*, "Trade and Investment Briefs: Computers," 24 January 1993, pp. 1347-1348.

[57] *Far Eastern Economic Review*, "Battle of the Desktops: Japanese PC Buyers Finally Get a Price Break," 5 November 1992, pp. 52-53.

Most computers sold in Japan do not use the same software as those sold elsewhere.[58] Off-the-shelf packages account for 60% of software purchases in the United States, compared with 10% in Japan. Thus, lower prices for DOS/V PCs could result from lower entry barriers. On the other hand, Dataquest, a market research firm, predicts that the market shares of existing Japanese firms other than NEC, not of new entrants, will increase.[59]

7.4.4 Monopolized Components

Some observers argue that compatibility raises profits by allowing a firm to obtain a dominant position in a component. The more components there are in a system, the more possibilities there are for one firm to dominate. In the PC industry, two suppliers have had dominant positions in the markets for essential components: Intel, which makes CPU chips, and Microsoft, which created the DOS and Windows operating systems and is the world's largest supplier of software. The U.S. Federal Trade Commission has investigated both of them.

In developing the Windows operating system, competitors accuse Microsoft of informing its inhouse applications programmers about changes in design and strategy before competitors. They also accuse Microsoft of spreading false rumors that competing applications would not work well with Windows. The two top-selling

[58] *Financial Times*, "Big plans are afoot for computer systems segment," 8 June 1990, p. 17.

[59] *Wall Street Journal*, "U.S.-Style PCs May Raise Share in Japan Market," 6 July 1993, p. B10.

applications for Windows are made by Microsoft.[60] Also, Microsoft and IBM encouraged software developers to write for the OS/2 operating system; IBM and the developers were very unhappy when Microsoft began pushing Windows as an alternative.[61] Finally, PC manufacturers pay Microsoft for DOS and Windows based on the number of computer they sell, and producers of competing operating systems such as DR DOS say that this locks them out of this market. Windows has been very successful, and has already sold millions of copies, though there remain around fifty million copies of DOS in use. Similar tensions exist between independent software producers and computer manufacturers who also provide software.[62] The FTC has closed its investigation of Microsoft, but the U.S. Justice Department has begun its own investigation.

One charge against Intel is that it induces buyers of its processors to buy other Intel chips as well. It has allegedly threatened to withhold scarce chips from buyers who use competitors' products. For instance, Cyrix, which makes Intel-compatible math coprocessors, claims that Intel has intimidated PC manufacturers who buy from Cyrix with the threat that it will withhold Intel CPUs. The FTC investigation overlapped with a private suit by AMD, a competitor of Intel.[63] By an agreement with Intel, AMD was

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[60] *Financial Times*, "Not just a pretty face," Supplement on Software at Work, 27 September 1991, p. 6.

[61] *PC Week*, "FTC Probe Incites Microsoft Criticism," 18 March 1991, p. 1. *Washington Post*, "Software Publisher Microsoft Says It Faces FTC Investigation," 13 March 1991, p. C1. *Wall Street Journal*, "Microsoft Corp. is the Target of FTC Probe," 12 March 1991, p. A3.

[62] Cavanagh, p. 10.

[63] *Financial Times*, "Front runners fast losing ground," Supplement on Personal Computers and Software, 17 September 1991, p. IV.

a second source for the 286 microprocessor, which was the basis for the IBM AT standard. However, Intel has made it difficult for AMD to act as a second source for the 386 as originally agreed. For instance, in times of shortage, Intel grants spaces in its order queue based on purchases of the previous generation, which hampers sales by AMD of 386s.[64] In 1983-4 there was a processor shortage when demand surged for PCs.[65] In any case, the FTC is not expected to act against Intel.[66]

Observers agree that the Intel-Microsoft combination is facing significant challenges.[67] Apple and IBM have formed several joint venture which will compete with Intel and Microsoft, as discussed below. Microsoft faces increasing competition from the Unix operating system, as the distinction blurs between the markets for Unix-based workstations and PCs. Also, Sun has developed software that will run software written for Windows on Unix machines.[68] Unix and workstations are discussed further below.

Intel also faces serious challenges to its dominant position in the market for CPUs. As mentioned above, workstations, which use different types of CPUs, are providing increasing competition to Intel-based PCs. Also, AMD and Cyrix have "cloned" Intel's CPU chips, and IBM has announced its attention to do so as well.

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[64] *PC Week*, "AMD Chip Debut Poses Little Threat to Intel," 18 March 1991, p. 139.

[65] Jordan's *Britain's Computer Industry 1988*, (Bristol, Jordan's & Sons, 1988), p. 15.

[66] *Financial Times*, "The Hottest Act in Town," March 1993, p. 15.

[67] *Financial Times*, "Waiting for the Explosion," Supplement on Personal Computers and Software, 17 September 1991, p. I.

[68] *Financial Times*, "Sun Attacks Microsoft's Dominance in Software," 7 May 1993, p. 24.

Compatibility has probably raised the total profits and the unit profit margins of Intel and Microsoft. However, their products do not appear to have noticeably high prices. In particular, the Microsoft Windows operating system is fairly inexpensive, at £35. It appears that their profits derive primary from high volume, rather than high prices compared to competitors.

7.5 Consumer Surplus

Computer buyers like compatibility. Buyers have shifted their purchases to smaller, compatible computers and manufacturers use compatibility as a selling point. An account of the development of local-bus standards for internal PC communications gives an explanation. It states that "a local-bus standard ... could increase competition and lower prices for users. It would also give users more flexibility and upgrade-ability."[69]

Some standardization results from demands by large buyers. The U.S. government, especially the Department of Defense (DoD), promotes standards by writing them into its procurement specifications. For instance, most DoD contracts require use of the ADA computer language. Also, GM has promoted a Manufacturing Automation Protocol (MAP) for the interconnection automated factory equipment. Boeing also developed the related Technical Office Protocol (TOP) which serves a

[69] *InfoWorld*, "Local Bus: 1 Need, 2 Specification," 14, 29 June 1992, p. 27.

similar role for office equipment. However, MAP has not caught on as quickly as hoped, and GM is behind schedule in making MAP a specification in all purchasing.[70]

Several user groups have been prominent in promoting open systems. The user group Standards and Open Systems (SOS) is strongly in favor of the Open Software Foundation standards. Many SOS members are drawn from another group, the User Alliance for Open Systems, which has harshly criticized suppliers of incompatible computers and their buyers. Other user groups promoting compatibility are the North American MAP/TOP Users Group, the North American ISDN User Forum, and the OSI Network Management Forum. Many supporters of the Corporation for Open Systems, which tests conformance to standards, are users.

However, some analysts dispute the "conventional wisdom" that nonproprietary standards best serve the needs of users. They argue the nonproprietary standards are set by committees, and represent "lowest-common denominator, compromise solutions."[71]

7.6 Coalitional Behavior and Technology Giveaways

Coalitions of firms back competing standards, perhaps based on technology given away by one of them. This section presents a number of examples of this behavior.

[70] *Scientific American*, "Off the MAP: Has GM's factory automation network lost its way," August 1991, p. 85.

[71] Charles R. Morris and Charles H. Ferguson, "How Architecture Wins Technology Wars," *Harvard Business Review*, **71**, p. 89.

Of course, some standards are not disputed. For instance, manufacturers unanimously agreed upon a standard for laptop memory cards.[72] Also, manufacturers of the next generation of 2.88M 3.5 inch floppy disk drives agreed upon a common standard.[73] IBM has set many PC-related standards by fiat. The Sony-Philips standard for music CDs mentioned below is similar in this respect.

Firms often join forces against the dominant firm in a market by agreeing upon an alternative standard. Some observers say that IBM made its PC specifications public to potential competitors to attract allies against Apple. In addition, IBM may have wanted to draw software developers and add-on manufacturers away from Apple, in an attempt to gain control over standards in a new market.

The formation of one coalition may stimulate the formation of others (see Saloner (1989)). When the threat of a competing standard arises, a dominant firm may share its standard, as IBM did with MCA. When IBM's ability to set standards has been challenged, it has repeatedly sought allies to support a different standard, as it did with the IBM PC. Recent challengers, as described below, have included: the Gang of Nine (EISA), Microsoft (Windows and multimedia), AT&T (Unix), ACE and Sun (workstations), and NEC (DOS/V). Other companies have pursued this strategy as well.

Sometimes firms imitate a dominant company's standard rather than forming a coalition around a competing one. Once the Hayes modem established a large lead,

[72] *Wall Street Journal*, "Computer Makers Agree on a Standard for New Memory Cards for Laptops," 22 May 1990.

[73] *PC Week*, "High-Capacity Floppy Drives Emerge: Market Waiting for IBM Approval of De Facto 2.88M-Bye Standard," 25 March 1991, p. 101.

competitors began offering Hayes-compatible modems that emulate the Hayes modem.[74] These modems could then work with communications software designed around the Hayes modem. Similarly, Epson printers for PCs sold well, many with IBM nameplates, and competitors began making Epson-compatible printers that could work with the existing base of word processing programs. The Hewlett-Packard LaserJet printer serves the same role for laser printers. Companies do not always discourage imitation—Hayes, Epson, and Hewlett-Packard receive free advertising of their status as market leader, and in some cases alternative protocols could easily be designed.

A number of examples of coalitions of firms supporting competing standards now follows, not all of which are from the personal computer industry.

IBM developed its Micro Channel Architecture (MCA) as a proprietary standard for its PS/2 series of computers. In response, Compaq organized the largest clone makers into the "Gang of Nine", which developed the competing Extended Industry Standard Architecture (EISA), which is a backwardly compatible with the IBM PC. IBM, faced with slow adoption and the possibility of competition from EISA, offered the MCA technology to other manufacturers. They must pay royalties to IBM as result of patents inherent in MCA, unlike manufacturers of IBM PC clones. Because of uncertainty about which standard would succeed, many board makers waited before

[74] A modem is a device that allows a computer to communicate with other computers over a telephone line.

introducing new products.[75] Compaq bought MCA licenses from IBM, although it was promoting a competing standard.[76]

Three standards are competing in the workstation market. Workstations rank just above personal computers in power, and have fast Reduced Instruction Set Computing (RISC) processors and high resolution graphics for scientific applications. Their growing commercial use and falling costs may make the fast-growing workstation market as important as the PC market is today. Sun Microsystems' Sparc chip currently has the largest market share, but an alliance of IBM, Apple, and Motorola and another led by Compaq has announced the intention to offer competing standards.

The first group, Sparc International, is a consortium of 130 companies pledged to support Sparc, the RISC microprocessor that Sun uses in its workstations. Sun has given away the specifications for Sparc, and is aggressively promoting it as a standard.[77] Japanese companies have taken up the offer, raising concerns within the U.S. Commerce Department about the implications for U.S. competitiveness. Sun has also given away the source code for its Macintosh-like Open Look user interface.[78] Sun's Unix-based Solaris operating system will work on both RISC-based and Intel-based computers.

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[75] *Electronic Business*, "Board Makers Hedge Their Bets in EISA-MCA Bus Wars," **16**:3, 5 February 1990, p. 73.

[76] Jordan's *Britain's Personal Computer Industry*, (Bristol, Jordan's & Sons, 1990), p. A11.

[77] *Financial Times*, "An open architecture route to fame," 22 October 1991, supplement, p. VI.

[78] *Infoworld*, "Sun offers developers open windows code," 26 November 1990, p. 27.

Compaq created the second group, known as ACE, "to develop workstation standards in a direct challenge to market leaders."[79] The consortium included MIPS, which will offer a RISC-based processor as an alternative to Sun's Sparc technology.[80] Other members were Microsoft, Olivetti, Wang, NEC, and Silicon Graphics, with twenty-one in total.[81] They would have supported Microsoft's operating system for high-performance computers, Windows New Technology,[82] as well as a version of the Unix operating system supplied by the Santa Cruz Operation. Compaq dropped out of ACE in 1992 once it became satisfied with Intel's ability to develop a satisfactory next-generation RISC chip.[83]

Finally, the third group arose when IBM and Apple agreed to develop an operating system for IBM's RISC workstations and Apple's next-generation PCs.[84] Thus, IBM is considering a future apart from its old allies, Intel and Microsoft, but with its old rival Apple. This group will use IBM's RISC processor, Power PC, which will be made by Motorola.[85] IBM and Apple will form a company to sell the operating system

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[79] *Wall Street Journal*, "Compaq to Announce Workstation Consortium," 4 April 1991, p. 10A.

[80] *Infoworld*, "Compaq Spurns Sparc, Taps MIPS' RISC Chip," 11 February 1991, p. 1.

[81] *Financial Times*, "Gibraltar's peace treaty," 5 April 1991, p. 10. See also *Personal Computer World*, "When Two Worlds Collide," October 1991, 14:10, p. 400.

[82] *Financial Times*, "A veteran after four years," 22 October 1991, p. VI.

[83] *Infoworld*, "Compaq Emerges from ACE with Commitment to Intel," 14, May 1992, p. 110.

[84] *Financial Times*, "Sun operating system shines in two spheres," 12 September 1991, p. 33.

[85] *Financial Times*, "Apple Uses Chip Developed with IBM, Motorola," 11 May 1993, p. 23.

to other vendors, and Motorola will also be free to sell its chip to competing manufacturers.[86] As mentioned above, Apple is also cooperating with IBM in multimedia, in competition with Microsoft.

A group led by Microsoft, AT&T, and Tandy announced in November 1990 its intention to develop standards for multimedia, which store linked text, sound and graphics on CD-ROMs. For example, the entry under Beethoven in a multimedia encyclopedia might include a biography, a video of a performance with CD quality sound, musical scores linked to the soundtrack, and pictures of the composer. IBM and Apple formed a rival group in March 1991 with 170 members. Some say the large size of the group will slow development.[87] Both standards will be based around personal computers, and packages from Microsoft and Apple (Multimedia PC and QuickTime) are now on the market.[88] In addition, Sony, Philips, and Matsushita are developing a CD-based standard not requiring a PC, which is oriented towards the consumer market.[89] They have lined up 180 companies in support of their standard, known as CD-I (for interactive), which can show interactive movies.[90] These membership numbers must be taken with a grain of salt, since many companies join groups promoting

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[86] *Personal Computer World*, "Sculley tells of RISCy strategy behind Apple-IBM partnership," October 1991, **14**:10, p. 99.

[87] *Wall Street Journal*, "IBM, Apple Join Microsoft Rivals Over Standards," 18 March 1991, p. B3.

[88] *Personal Computer World*, "When Two Worlds Collide," October 1991, **14**:10, p. 400.

[89] Congressional Budget Office. *Using R&D Consortia for Commercial Innovation: SEMATECH, X-ray Lithography, and High-Resolution Systems*. (Washington, DC: GPO, July 1990), p. 63.

[90] *Wall Street Journal*, "Philips, Sony, Matsushita Form Interactive CD Group," 22 March 1991, p. 5A. See also the periodical *CD-I News*.

competing standards.[91] A more informative measure of commitment to a standard would be the actual introduction of products, but this information is harder to obtain.

Two groups have until recently been fighting for control of the Unix operating system. Each hoped to create a standard version of Unix to play the role for larger computers that DOS has played for PCs. Unix is already widely used, but computer manufacturers and computer installations often make enhancements to it, which then limit the portability of software between computers. Nevertheless, competition among Unix computers is increasingly quickly. For instance, having the fastest computer on the market does not give a manufacturer the same advantage as it did in the past, because buyers expect a faster one to be introduced quickly.[92]

The first group, Unix International, consists of AT&T, which originally developed Unix, along with Sun Microsystems, and over 240 other companies. IBM and DEC formed the second group, the Open Software Foundation (OSF), out of a concern that AT&T and Sun exerted too much control over Unix.[93] Both groups released versions of Unix, but the OSF version has not been selling as well.[94] In another rivalry, OSF supports Hewlett-Packard's Motif standard for a Macintosh-like graphical user interface, while Unix International supports Sun's competing product Open Look.

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[91] *Financial Times*, "Too lucrative to let go," Supplement on Personal Computers and Software, 17 September 1991, p. II.

[92] *Wall Street Journal*, "Data General to Unveil Fast Computer Using Unix That Serves Network of 500," 4 March 1991, p. B2.

[93] *Wall Street Journal*, "IBM, Microsoft Develop Pacts on PC Software," 18 September 1990.

[94] *Financial Times*, "AT&T to tie up deals for shared ownership of Unix," 21 December 1990, p. 23. *Financial Times*, "The age of the answer," 22 October 1991, supplement, p. I. *Financial Times*, "Those who set standards hope to call the tunes," 22 October 1991, supplement, p. X.

The two groups decided to cooperate when they realized that the confusion created by their competition was hurting sales. AT&T sold 20% of its Unix software business to eleven other companies.[95] Also, OSF's Distributed Computing Environment, which allows computers with different operating systems to work together, has been included in Unix International's Atlas system.[96]

Rival versions of pen-based operating systems, which allow users to communicate with a computer through pen and pad rather than through a keyboard or mouse, are competing with each other. Go Corp. has lined up thirty hardware and software vendors in support of its system.[97] However, Microsoft has twenty-one companies supporting its pen-based extension of Windows.[98] In 1993, Apple introduced the Newton, which it describes as a personal digital assistant, with a pen-based operating system.

Competition between standards also takes place outside the computer industry. Sony's Betamax standard for videocassette records (VCRs) was by some accounts superior to JVC's VHS standard. However, VHS has almost completely displaced Betamax. Unlike Sony, JVC was more liberal in giving away its technology. Philips, based in the Netherlands, also offered a VCR standard. It ran into competition from VHS, because JVC signed agreements in 1977 with three other European companies, Thorn-EMI, Telefunken, and Thomson. By the time Philips and its partner Grundig

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[95] *Wall Street Journal*, "AT&T to Disclose It has Sold 20% of Its Unix Unit," 2 April 1991, p. B5.

[96] *Financial Times*, "Truce to boost sales ends Unix wars," 22 October 1991, p. II.

[97] *PC Week*, "Developers Get the Go-Ahead For PenPoint OS," 28 January 1991, p. 1.

[98] *Wall Street Journal*, "Microsoft Increases Pressure on Rival in Pen Software."

began looking for allies in 1981, many companies were already committed to VHS.[99] Sony and Philips also viewed VCRs strictly as recording machines and Philips provided better prerecorded material for its video disc system. Like Philips, Sony was hesitant to allow other companies to sell its VCRs under their brand names.[100] Some analysts attribute the failure of the Betamax standard to the wider availability of VHS technology.[101] Betamax failed when video rental stores decided they could not keep inventories in both standards. Similarly, after sales of music CDs reached a critical mass, record stores withdrew LPs from their shelves. In each of these cases, software distribution, not production, was the bottleneck; most movies and albums were released in both media. Rivalry between Sony and JVC continues with camcorders; Sony leads a group of companies using 8mm tape, and JVC a group using VHS.

Another standards competition is taking place between the next-generation music cassette tape formats. Digital Audio Tape (DAT) systems were kept off the market until recently by music companies concerned about the ease of copying with digital technologies. After some conflict, manufacturers agreed to add a copy protection device, but sales of DAT have not been spectacular. Some speculate that delay introduction meant that DAT lost its novelty to the technology enthusiasts that are usually early buyers of new gadgets. Also, Philips has introduced the competing Digital Compact Cassette (DCC) technology, which is backwardly compatible with existing cassette

[99] Alan Cawson, Kevin Morgan, Douglas Weber, Peter Holmes, and Anne Stevens, *Hostile Brothers*, (Oxford: Clarendon Press, 1990), p. 226.

[100] Cawson *et al*, p. 371.

[101] *The Economist*, "Survey: Consumer Electronics," 13 April 1991, p. 13.

recorders.[102] The Sony MiniDisc, a recordable CD, is also coming on the market. The problems with copy protection may have caused DAT to miss the opportunity to establish a user base without a competing technology. Sony and Philips have swapped rights to each other's standards.[103]

Governments also compete in setting standards. Japanese companies developed the first high-definition television (HDTV) standard, called MUSE. The Japanese government has approved it and transmissions have already begun. Japan and the United States proposed this standard for international approval in 1986, but the European Community (EC) objected. These objections were originally on technical grounds, but the Japanese managed to satisfy these objections. Thus, many observers think that the EC rejected the Japanese HDTV standard to prevent Japan from having an advantage in selling TVs and related equipment.[104] The EC has subsidized development of the HD-MAC standard by Philips and Thomson, which is incompatible with the Japanese standard, but is backwardly compatible with existing European standards. Thus, a coalition of countries is joining forces against a competitor, but in this case the governments have the power to mandate conformance with the standard. The EC has also tried to force satellite broadcasters, such as Sky Television, to use HDTV standards, rather than the existing PAL standard. The EC was concerned that new TVs that enhanced normal PAL broadcasts with advanced signal processing would

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[102] *The Economist*, "Survey: Consumer Electronics," 13 April 1991, p. 8.

[103] *The Economist*, "Philips vs. Sony: Sounding Off," 7 November 1992, p. 90.

[104] U.S. Congress. General Accounting Office. *High Definition Television: The Effects of Standards on U.S. Entertainment Industries*. GAO/IMTEC-90-33. (Washington, DC: GPO, March 1990), p. 18.

undercut broadcasts using HDTV standards.[105] The HD-MAC effort collapsed when Philips announced that it would not produce HD-MAC TVs.

In the United States, the Federal Communications Commission (FCC) has invited proposals for an HDTV standard.[106] The standard must be backwardly compatible with existing TVs, which rules out the Japanese standard. The developer of the standard may win royalties for its compression techniques and chip designs.[107] In 1993, the FCC persuaded the three remaining contenders to consolidate their proposals into one.

Even if HDTV is successful, producers of films and TV programs may continue to produce and exchange programs using 35mm film for three reasons: (1) an international HDTV standard is unlikely, (2) 35mm film allows higher production quality, and (3) producers prefer film as a creative medium.[108] Therefore, the main impact of incompatibility in HDTV transmission standards may be to limit economies of scale in producing TV sets and transmission equipment. On the other hand, there is some potential to build "open architecture receivers" that can switch standards when a different software cartridge is inserted. Some argue that maneuvering among governments over HDTV standards could turn out to be misguided in several ways. First, any advantage to choosing a standard because it is not Japanese is likely to be fleeting, while the costs of incompatibility are not. Second, the business computer and video game markets, which drive innovation in digital graphics, may set the

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[105] Cawson *et al*, p. 319.

[106] *The Economist*, "The world at war," 4 August 1990, p. 58.

[107] *Financial Times*, "AT&T and Zenith team up to pursue HDTV standard," 18 December 1990, p. 21.

[108] General Accounting Office, p. 4.

technologically and commercially important standards. Finally, consumers may prefer to take advantage of the compression technology developed for HDTV by having four times as many normal channels rather having one HDTV channel with four times as much information.[109]

The histories of HDTV standards and existing TV standards are similar. American companies developed the first TV standard, NTSC, which was in production before other countries chose their standard. Most of Europe adopted the PAL standard, while France, Greece, and the Eastern bloc chose the SECAM standard.[110] European governments initially denied PAL licenses to Japanese producers, until the companies threatened to sell a very low quality, low cost standard.[111] The French goal in developing SECAM was not to isolate itself, but to promote a French standard worldwide.[112] In any event, SECAM has not protected French companies from Japanese competition, and has limited their export potential.[113]

Despite the popularity of standardized products, sufficiently differentiated proprietary products can still be successful. Apple has taken a proprietary approach with its Macintosh, and survived many predictions of failure, most recently with the success of the Mac Classic computer. The company has successfully guarded the Macintosh against cloning, so far, and continues to sell its computers based on their ease of use. Nevertheless, this proprietary approach is under challenge. Nutek Computers, Inc. has

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[109] See Farrell and Shapiro (1992) for further discussion of HDTV standard-setting.

[110] Cawson *et al*, p. 223.

[111] Cawson *et al*, p. 224.

[112] Cawson *et al*, p. 274.

[113] Cawson *et al*, p. 275.

announced that it will be selling a chip set and operating system for use in making Macintosh clones. They claim that because the technology was "clean-roomed" that it will not infringe on Apple's patents or copyrights.[114] Another challenge comes from competing graphical user interfaces for other machines, such as Microsoft Windows, Hewlett Packard's Motif, and Sun's Open Look, which provide the same features that used to be unique to the Macintosh: windows, icons, menus, and a mouse pointer. Already, most new software now appears on Windows first rather than on the Macintosh.[115] Apple has sued Hewlett-Packard and Microsoft for copying the look-and-feel of the Macintosh, though Xerox originally developed the underlying ideas.[116] Courts have dismissed the suit against Hewlett-Packard.[117]

There are indications that Apple will make more compatible products in the future. Apple and IBM are developing a hardware-independent operating system that will allow Macintosh applications to work on hardware not made by Apple.[118] Apple might have prevented the PC standard from becoming dominant by licensing the Macintosh operating system software more freely.[119]

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[114] *Infoworld*, "Borrowing From the Mac Can be a Risky Proposition," 11 February 1991, p. 40. See also, *Macworld*, "Clone Alone," June 1993, pp. 25-26.

[115] *Electronic Business*, "How Windows Cuts into the Macintosh Market," **19**, March 1993, pp. 113-115.

[116] Congressional Research Service, p. 36.

[117] *Financial Times*, "Apple's Claim on Copyright Dismissed," 3 June 1993, p. 28.

[118] *PC Week*, "Apple Considering Non-Macintosh OS," 25 March 1991, p. 6. *PC Week*, "From Courtroom to Courtship: Apple's About-Face on Windows," 25 February 1991, p. 126. *The Economist*, "Apple Computer: Falling to Earth," 28 August 1993, p. 57. *Financial Times*, "Apple moves towards open software," 12 May 1993, p. 26.

[119] See, for instance, Charles R. Morris and Charles H. Ferguson, "How Architecture Wins Technology Wars," *Harvard Business Review*, **71**, p. 89.

7.7 Market Growth, R&D Effort, and Compatibility

Competition between different standards can impede the adoption of any standard by consumers and providers of third party products and services, as described in the Unix, MCA versus EISA, and 1K VGA graphics examples above. When confusion about standards slows sales, a war of attrition can occur, with supporters of each standard waiting for the other to back down.

On the other hand, once manufacturers agree upon standards, markets grow more quickly. For instance, the market for local area networks only began growing quickly with the arrival of the Novell Netware standard, which now has a 60% share.[120] An early agreement between Philips and Sony on music CD standards allowed that market to grow quickly. The two companies apparently wanted to avoid competition between standards, having lost the competition to set VCR standards.[121]

De facto market standards often use inferior technology. For instance, the Betamax VCR standard was considered superior to VHS by some. Similarly, the IBM PC did not use the best-available technology when it was introduced; Intel acknowledges that its microprocessor for the original IBM PC was inferior to Motorola's competing product at the time.

Standards give-aways are important means of diffusing innovations. Companies may give away standards to undermine the development of a competing technology; Sun is doing this with its Sparc processor chip, as described above.

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[120] *Financial Times*, "A trend to attachment," 22 October 1991, supplement, p. V.

[121] Cawson *et al*, p. 372.

Innovation in one component can have large spillover benefits for other components. For instance, a major stimulus to the acceptance of the Apple II computer was a spreadsheet package, Visicalc, not made by Apple. Similarly, the IBM PC caught on with businesses largely because of the Lotus 123 spreadsheet and the WordStar word processing programs. The quality of software for a computer system can affect its value more than the design of its hardware.

Spillovers were important in the growth of the market for portable PCs. The first portable computers were large, heavy, and could not use batteries. Once light weight liquid crystal displays (LCDs) were developed, hard disk drives were the greatest obstacle to portability, because of their size and power requirements. Eventually, smaller hard drives and larger capacity memory chips made laptops even lighter. These combined improvements have increased the market for laptops sufficiently that Intel has now developed a new processor just for laptops, which cuts itself off to save power and needs fewer external chips. Notebook-size computers now rival desktops in functionality, and increasingly in price.[122]

7.8 Summary

This appendix has brought together stylized facts about compatibility in the personal computer industry, which are summarized on page 12. It has described how increased compatibility in this industry has been accompanied by lower profits and prices but increased specialization, variety and R&D. It has also noted that some industry

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[122] *Financial Times*, "The dream has become a reality," Supplement on Portable Computers, 8 February 1991, p. VI.

observers believe that compatibility has caused these changes. Nonetheless most firms now pursue strategies of compatibility to one degree or another, often by joining coalitions promoting competing standards. It was also noted that computer buyers have aggressively promoted greater standardization among computer manufacturers.

This appendix has not attempted to assess claims that increased compatibility has caused prices and profits to fall in the PC industry. Its purpose is to motivate the literature review in this chapter and the theoretical models developed in later chapters.

8 Appendix: Mix-and-Match Compatibility with n Firms

This appendix solves a simple model of mix-and-match compatibility with horizontal differentiation when there are n symmetric firms. Economides' (1989) conclusion that n firms prefer compatibility is correct in this model, in spite of flaws in his analysis pointed out by Matutes and Regibeau (1991). The model below corrects these problems by generalizing Salop (1979), who solves a model with n symmetric firms when there is only one dimension of product differentiation.

In the one-dimensional case, we interpret Salop's use of the unit circle as allowing consumers to jump without cost from one end of Hotelling's unit interval to the other. If a describes a consumer's location on the unit interval, a_i describes a firm's location on the unit interval, and transportation costs are quadratic,[123] then the consumer pays a transportation cost given by the metric

$$d = \min[(a - a_i)^2, 1 - (a - a_i)^2]. \tag{20}$$

A similar stratagem works in the two-dimensional case. Consider a consumer who is travelling horizontally to the right across the unit square along the unit interval where the characteristic of component B satisfies $b = ½$. When he reaches the right edge, suppose that he jumps from the point (1,½) to the point (0,½) (see Figure 6). Then, consumers at (0,½) and (1,½) have the same tastes. The line segment $b = ½$ becomes a circle, just as Hotelling's unit interval becomes Salop's unit circle.

More generally, suppose that a consumer can travel at no cost from any point $(1,b)$ on the right edge of the square to the corresponding point $(0,b)$ on the left edge and vice versa. Then, equation (20) measures distance in the horizontal direction. Similarly, let a consumer travel at no cost from any point $(a,0)$ on the bottom edge to the corresponding point $(a,1)$ on the top edge, and vice versa. When consumers can jump from edge to edge in this way, the implied distance function, by generalization of (20), is

$$d = \min[(a - a_i)^2, 1 - (a - a_i)^2] + \min[(b - b_i)^2, 1 - (b - b_i)^2]. \tag{21}$$

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[123] With linear transport costs, d'Aspremont, Gabszewicz and Thisse (1979) show that pure-strategy equilibrium in prices may not exist in Hotelling's model of two firms on a unit interval. The nonexistence problem arises because there exist prices such that all of the customers in the other firm's "back yard" are indifferent between the two firms. A small price cut will steal all of those consumers. An additional source of nonexistence problems arises when there are two dimensions of product differentiation with a block metric and linear transportation costs. If firm 2 lies northeast of firm 1, there are prices such that all consumers north of firm 2 and west of firm 1 are indifferent. Again, a small price cut steals all of these consumers.

Because our goal in discussing Economides is to show that the assumption of perfectly correlated tastes drives the result, we do not attempt to show that the conclusion is correct for general metrics and transportation cost functions.

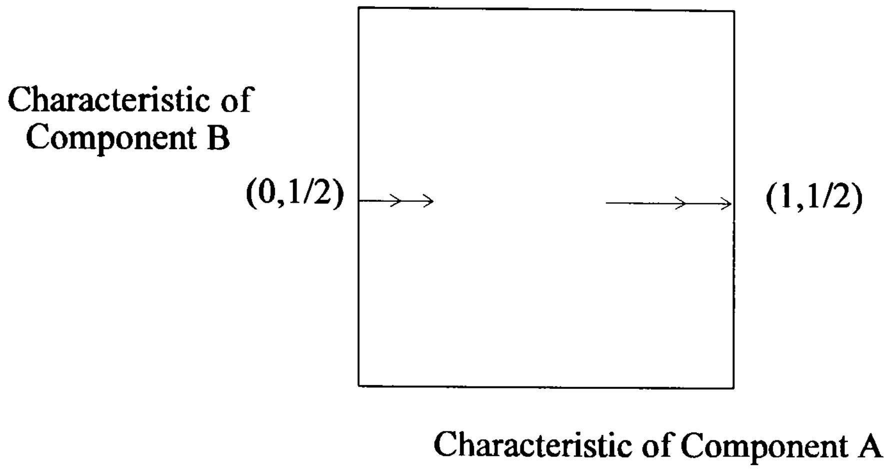

Figure 6: Horizontal Wrap Around

Allowing consumers to jump from one edge to another eliminates boundary effects on those edges. When this metric describes consumers' tastes, we will say that consumer tastes *wrap around.*

It is possible to solve a symmetric model of mix-and-match compatibility with n firms when consumer tastes wrap around. Suppose that consumers combine components with fixed coefficients, and that the reservation price R is sufficiently high that market coverage is complete in either regime. Suppose that consumers are distributed with density one over the unit square. To obtain a symmetric solution, space the firms evenly along the diagonal (again, see Figure 4). In other words, locate the i^{th} of n firms at the point $(\frac{2i-1}{2n},\frac{2i-1}{2n})$. Transport costs are given by (21).

Under compatibility, firm 1's market area for component A when firms have symmetric prices is a rectangle with width $1/n$ and two sides of length one (see Figure 7).

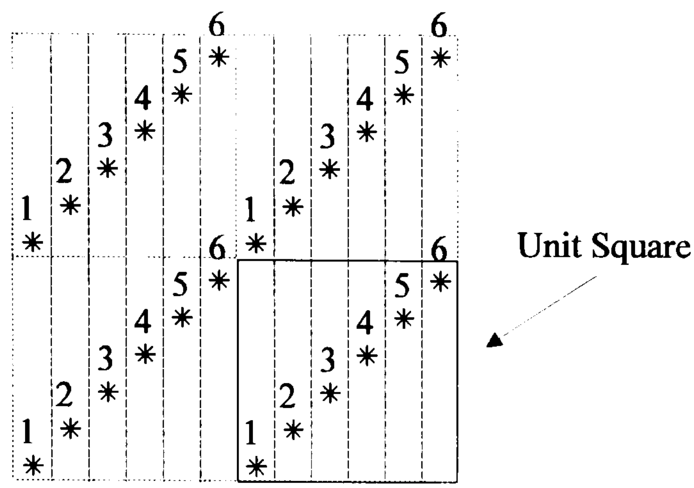

Figure 7: Market Areas under Compatibility with Wrap Around and n=6

Figure 7 shows several copies of the unit square to illustrate that firm 1's market area touches firm 6's market area. Consumers along the left and right edges of each rectangle are on the margin, so l=2 and q=1/n. The distance in the horizontal direction between two neighboring firms' component A characteristics is d=1/n, giving $-q'=n$. Firm 1's price for component A is therefore $p^A=-q/q'=1/n^2$. Firm 1's combined profit in both components is $2/n^3$.

Under incompatibility, firms have rectangular market areas of size 1/n when prices are equal (see Figure 8). Figure 8 illustrates that the market areas are rectangular, and that consumers are on the margin on all four sides of the rectangle. By contrast, consumers are on the margin on only two sides of a firm's rectangle under compatibility. Under incompatibility, a firm's rectangle has two sides of length $l=\frac{\sqrt{2}}{2}$ with $d=\frac{\sqrt{2}}{n}$ and two sides of length $l=\frac{\sqrt{2}}{n}$ with $d=\frac{\sqrt{2}}{2}$ The combined size of the firm's margin is

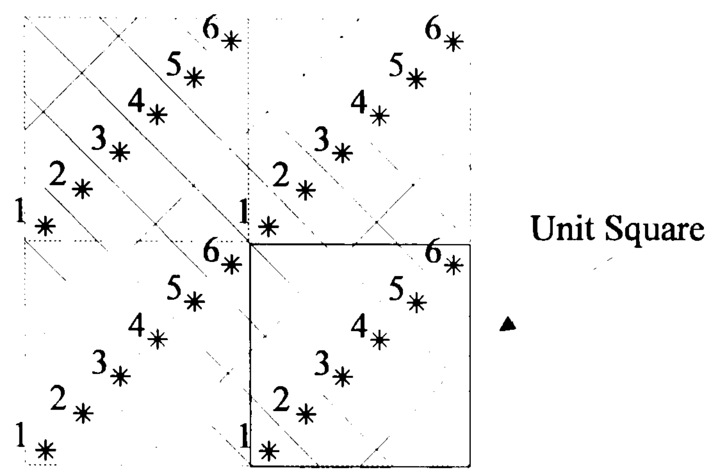

Figure 8: Market Areas under Incompatibility with Wrap Around and n=6

therefore $-q' = 2\left(\frac{n}{4}+\frac{1}{2n}\right) = \frac{n^2+2}{2n}$. The system price is $\frac{2}{n^2+2}$. Multiplying the system price times the market area of $1/n$, profit is

$$\pi^J = \frac{2}{n^3+2n}. \tag{29}$$

Prices and profits are higher under compatibility than incompatibility for any n, when consumers' tastes wrap around. In this sense, Economides' (1989) conclusion is correct although the reasoning appears not to be. Compatibility has several effects on profits. Compatibility shortens l by a factor of $\sqrt{2}$, but it increases d by the same amount, and the two effects cancel out. The remaining effect is that the length of a firm's margin is shorter under compatibility, because a firm faces competition on all four sides under incompatibility.

In sum, the assumption of wrap around allows us to solve a mix-and-match model with n symmetric firms. The solution reveals that the n-firm case gives the same prediction as the duopoly case, that compatibility raises prices and profits.

Chapter 2: Mix-and-Match Compatibility in Oligopoly

1 Introduction

Sometimes consumers can mix-and-match different brands of components into a system, and sometimes they cannot. For instance, most stereos and speakers are compatible in this way, but not all computers and software are. Other multicomponent products for which compatibility is an issue include cameras and film, cameras and lenses, VCRs and videocassettes, razors and blades, and video game systems and game cartridges. In the rest of this thesis, the term compatibility refers to mix-and-match compatibility, and not to compatibility in the sense of network externalities or the lack of switching costs.

Chapter 1 sets the stage for this chapter. Models of mix-and-match compatibility by Matutes and Regibeau (1988), Economides (1989), and Einhorn (1992) find that prices and profits are higher under compatibility in a variety of settings. However, some observers of the computer industry believe that compatibility lowers profit margins and prices. Yet many computer manufacturers pursue strategies of compatibility. Also, computer buyers have been demanding increased compatibility, but the models above predict that compatibility has an ambiguous or negative impact on consumer surplus. This chapter explores some theoretical explanations for why prices and profit margins might be lower and consumer surplus higher when products are compatible, and why producers nonetheless pursue strategies of compatibility.

We find that price competition among sufficiently many firms yields expected profits and prices that are lower under incompatibility than under compatibility when firms have independent and random costs, qualities, or characteristics for components. An intuition when the distribution function for component characteristics is discrete is that the main determinant of a firm's profit under price competition as n grows large is the probability that it draws unique characteristics. If two firms draw identical characteristics, they sell homogeneous goods and have zero profits. Under incompatibility, firms compete in a market for systems made of two components, so they have two opportunities to draw a unique component characteristic. Under compatibility, firms compete in markets for each component, and they have only one opportunity to draw a unique characteristic in each market. Therefore, firms have higher expected profits under incompatibility than compatibility because they are more likely to have a unique system than to have a unique component.

Models in the literature do not incorporate this effect for two reasons. First, the literature has, with one exception, focussed on the duopoly case, and duopolists prefer compatibility to incompatibility in most settings. Second, Economides (1989) does consider the case of n firms, but imposes symmetry to avoid the problem of solving n different first-order conditions for n prices. This chapter's result requires that the assumptions of duopoly and symmetry be relaxed simultaneously. Finally, Economides (1991) allows a subset of firms to increase compatibility, but does not assess effects on industry profits.

Although industry profits are sometimes higher when all firms are incompatible than when all firms are compatible, firms choose to be compatible in the unique equilibrium of the game in which firms choose among nonproprietary standards. For instance, suppose that firm 1 becomes compatible with firm 2 but not with firm 3. Firms 1 and 2 thereby attract customers away from firm 3 and lower its profit. Then, firm 3 wishes to become compatible with firms 1 and 2 rather than remain the only incompatible firm, although this lowers those firms' profits. Although industry profits are higher under incompatibility when there are sufficiently many firms, firms independently decide to become compatible. When a group of compatible firms can prevent other firms from becoming compatible, there may be additional equilibria in which coalitions of firms back competing standards.

This model has several interpretations. First, we can interpret it to describe not only industries with multicomponent products, but also industries in which production takes place in several stages potentially performed by different companies.[124] One component corresponds to the upstream industry, and the other corresponds to the downstream industry. An industry with incompatibility is similar to an industry in which each upstream firm sells to only one downstream firms, and each downstream firm buys only from that upstream firm. On the other hand, a compatible industry is formally similar to an industry in which a downstream firm can purchase from any upstream firm, and vice versa. By this interpretation, the results of this chapter imply that duopolists prefer not to have exclusive relationships, while industries with sufficiently many firms have higher profits when all firms have exclusive relationships.

A second interpretation is that an industry with risk-neutral firms prefers to have riskier costs exactly when industry profits are higher under compatibility than incompatibility. Finally, Bertrand competition among homogeneous goods is equivalent to a

[124] This interpretation is not the independent work of the author, but is drawn from Farrell, Monroe, and Saloner (1993), which includes some results from this chapter. This chapter is the author's independent work except where footnoted.

second-price auction; in both cases the low bidder wins and receives a payment equal to the second-lowest bid. The Revenue Equivalence Theorem therefore implies that results proved for Bertrand competition hold for a broader class of auction mechanisms.

Section 2 specifies the model's assumptions. Section 3 compares industry profits under complete compatibility and complete incompatibility. Section 4 considers firms' choices of standards. Section 5 shows that firms prefer compatibility when they prefer riskier costs. Section 6 analyzes the case of elastic demand. Section 7 extends the results about many firms to models with horizontal or vertical differentiation. Section 8 applies the results to auctions using the Revenue Equivalence Theorem. Section 9 considers the effects of compatibility on entry and welfare. Section 10 interprets the results in terms of multimarket contact and also the exclusiveness of vertical relationships.

2 The Given Costs Model

The *given costs model* assumes the following. Suppose that a representative consumer has inelastic demand with reservation price R for one unit of a system. A system consists of two components A and B, such as a computer and a screen, combined in fixed proportions.[125] Measure quantities of components so that a system requires exactly one component of each type. A component has no value except in combination with exactly one component of the other type. Each of the n firms makes both components at constant unit costs which may vary across firms and components. If two

[125] Although some propositions generalize to any number of components, systems have two components unless otherwise specified.

firms choose the same standard, then their components are compatible and a consumer can mix-and-match their components into a system. Components are homogeneous, excepting considerations of compatibility. Firms decide which standard to adopt in stage one of the game, and in stage two, they compete in prices.[126] The game is solved for the sub-game perfect Nash equilibrium. Information is perfect, except in Section 8.

In stage one, firms choose from a set of homogeneous, costless standards. If two firms choose the same standard, then they are compatible, and consumers can mix-and-match their components into systems. By implication, compatibility is an equivalence relation. More precisely, let "$i \approx j$" denote the statement "firm i's component A is compatible with firm j's component B." Then, the relation "$\approx$" is an equivalence relation if for any firms i, j, and k, (1) $i \approx i$, (2) $i \approx j$ implies $j \approx i$, and (3) $i \approx j$ and $j \approx k$ implies $i \approx k$. A *standard* is an equivalence class generated by the equivalence relation "$\approx$." *Complete compatibility* describes a situation in which all firms choose the same standard, and *complete incompatibility* one in which no two firms choose the same standard. Otherwise, the industry structure is one of *intermediate compatibility*. Firms cannot resell components to an incompatible firm.

In stage two, firms compete in prices given their choices of standards in stage one. Components are homogeneous, excepting considerations of compatibility. When different firms charge the same price, assume, for analytical convenience, that all sales go to the

[126] Quantity competition raises questions about what happens when more of one component is produced than the other, given that a component is useless by itself. One possibility is that the price of the surplus component falls to zero. Also, if compatibility does not affect the response of prices to quantities chosen, then any symmetric equilibrium under compatibility will also be an equilibrium under incompatibility, because the set of possible deviations from equilibrium under incompatibility is a subset of those possible under compatibility. Chapter 4 shows that quantity competition makes more sense when consumers combine components in variable proportions.

lower-cost firm.[127] Let a_i and b_i be the constant unit costs of firm i for components A and B, and let c_i be its system cost, where $c_i=a_i+b_i$ by the assumption of fixed coefficients. Let c denote the *industry cost*, that is, the cost of the system that the consumer buys. Similarly, let p_i^A and p_i^B be the component prices of firm i, and p_i be firm i's system price, where $p_i=p_i^A+p_i^B$. Let p be the *industry price*, that is, the price of the system that the consumer buys. Without loss of generality, let firm 1 be the firm with the lowest system cost under incompatibility: $a_1+b_1 \leq a_i+b_i$. Below, we will abbreviate the phrase "a shift from the regime of complete incompatibility to the regime of complete compatibility has effect x" with the phrase "compatibility has the effect x." Welfare is the sum of consumer surplus and industry profits.

3 Price Competition in the Second Stage

This section solves the price game in the second period taking as given firms' choices of standards in the first period. In duopoly, there are only two possibilities: the firms are compatible, or they are incompatible. We begin with this case. Section 3.1 compares equilibria when duopolists are compatible and incompatible. Section 3.2 compares equilibria in oligopoly when all firms are compatible and when all firms are incompatible. Section 3.3 develops a method for computing expected profits when some, but not all firms are compatible.

[127] It can be argued that sales must go the lower-cost firm in equilibrium, or that firm would lowers its price otherwise.

3.1 Price Competition in Duopoly

The following example suggests that duopoly profits and prices are higher and costs are lower under compatibility.

Example. Firm 1 has a unit cost of zero for component A and a unit cost of one for component B, written (0,1). Firm 2's costs are the mirror-image of firm 1's costs, that is, (1,0). Each firm has the lowest cost in one component, that is, they have *complementary costs*. A single consumer has inelastic demand with reservation price R for one system where $R>2$. Under incompatibility, firms compete in selling systems. Each firm's system cost is one. Because systems are homogeneous, if either firm sets a price above one, the other firm can profitably set a lower price. In Nash equilibrium, each firm sets a system price of one and has zero profit. Under compatibility, firms compete separately in selling each component. Firm 1's component A costs zero while firm 2's component A costs one. Firm 1 sets its price for component A equal to firm 2's cost, and makes a profit on that component of $1-0=1$. Similarly, firm 2 has a lower cost in component B, and sets its component B price to one and makes a profit of one. A shift from incompatibility to compatibility raises the industry system price from one to two, raises industry profits from zero to two. Also, compatibility lowers industry costs from one to zero, and lowers consumer surplus from $R-1$ to $R-2$.

These results occur more generally in duopoly, as stated below in Proposition 1 below. The proof goes into some detail which will be useful in later analysis of competing standards and of variable coefficients in combining components.

Proposition 1. In the given costs model, compatibility among duopolists weakly raises prices, profits, and welfare, and it weakly lowers costs and consumer surplus.

Proof. The following table gives firms' equilibrium prices and profits under incompatibility and compatibility in various cases when there are two components.

In duopoly, the assumption above that firm 1 has the lower system cost implies that it has a lower cost in at least one component. Without loss of generality, let this be component A, so that $a_1 \leq a_2$. The effect of compatibility on the industry price is $\Delta p = p^C - p^I$, where p^j is the price of the system the consumer buys in regime j for $j = C, I$. Similarly, the effect of compatibility on firm i's profit is $\Delta\pi_i = \pi_i^C - \pi_i^I$, where π_i^j is the profit of firm i in regime j. In Cases I and II in the table, the consumer's reservation price R is so high that it does not constrain firms' prices. In the other cases, R is sufficiently low to be a binding constraint on prices. In Cases I and III, firm 1 has the lowest cost in component B; recall that it has the lowest cost in component A and the lowest system cost by assumption. In the other cases, each firm has the lowest cost in one component. Cases IV, V, and VI differ in how low the consumer's reservation price is.

The equilibria in the table are computed as follows. In Case I, firm 1 has the lowest costs in both components. Under incompatibility, its system price is $a_1 + b_1$, and firm 2's system price is $a_2 + b_2$. If firm 1 sets a system price greater than firm 2's system cost, then firm 2 will undercut firm 1 with a lower system price. The consumer's reservation price is high enough that it is not a constraint on firm 1's price. Therefore, firm 1 sets a system price of $p_1 = a_2 + b_2$, and makes a profit of $\pi_1^I = (a_2 + b_2) - (a_1 + b_1)$. Under compatibility, in Case I, firm 1 has the lowest cost in component A. Firm 1 sets

Profits and prices with two firms	Incompatibility	Compatibility	Change
Case I. $b_1 \leq b_2$ $a_2+b_2 \leq R$	$p_1^A+p_1^B=a_2+b_2$ $\pi_1^I=a_2+b_2\text{-}(a_1+b_1)$ $p_2^A+p_2^B=a_2+b_2$ $\pi_2^I=0$	$p_1^A=a_2,\ p_1^B=b_2$ $\pi_1^C=(a_2\text{-}a_1)+(b_2\text{-}b_1)$ $p_2^A=a_2,\ p_2^B=b_2$ $\pi_2^C=0$	$\Delta p=0$ $\Delta\pi_1=0$ $\Delta\pi_2=0$
Case II. $b_1>b_2$ $a_2+b_1 \leq R$	$p_1^A+p_1^B=a_2+b_2$ $\pi_1^I=a_2+b_2\text{-}(a_1+b_1)$ $p_2^A+p_2^B=a_2+b_2$ $\pi_2^I=0$	$p_1^A=a_2,\ p_1^B=b_1$ $\pi_1^C=a_2\text{-}a_1$ $p_2^A=a_2,\ p_2^B=b_1$ $\pi_2^C=b_1\text{-}b_2$	$\Delta p=b_1\text{-}b_2$ $\Delta\pi_1=b_1\text{-}b_2$ $\Delta\pi_2=b_1\text{-}b_2$
Case III. $b_1 \leq b_2$ $a_1+b_1<R<a_2+b_2$	$p_1^A+p_1^B=R$ $\pi_1^I=R\text{-}(a_1+b_1)$ $p_2^A+p_2^B \geq R$ $\pi_2^I=0$	$p_1^A+p_1^B=R,\ a_1 \leq p_1^A \leq a_2,$ $b_1 \leq p_1^B \leq b_2,$ $\pi_1^C=R\text{-}(a_1+b_1)$ $p_2^B \geq p_1^B,\ p_2^A \geq p_1^A$ $\pi_2^C=0$	$\Delta p=0$ $\Delta\pi_1=0$ $\Delta\pi_2=0$
Case IV. $b_1>b_2$ $a_1+b_2<R \leq a_1+b_1$	$p_1^A+p_1^B \geq R$ $\pi_1^I=0$ $p_2^A+p_2^B \geq R$ $\pi_2^I=0$	$a_1 \leq p_1^A \leq R\text{-}b_2,\ p_1^B \geq p_2^B$ $\pi_1^C=p_1^A\text{-}a_1$ $p_2^B=R\text{-}p_1^A,\ p_2^A \geq p_1^A$ $\pi_2^C=p_2^B\text{-}b_2$	$\Delta p=0$ $\Delta\pi_1=p_1^A\text{-}a_1$ $\Delta\pi_2=p_2^B\text{-}b_2$
Case V. $b_1>b_2$ $a_1+b_1<R \leq a_2+b_2$	$p_1^A+p_1^B=R$ $\pi_1^I=R\text{-}(a_1+b_1)$ $p_2^A+p_2^B \geq$ $\pi_2^I=0$	$R\text{-}b_1 \leq p_1^A \leq R\text{-}b_2,\ p_1^B \geq p_2^B$ $\pi_1^C=p_1^A\text{-}a_1$ $p_2^B=R\text{-}p_1^A,\ p_2^A \geq p_1^A$ $\pi_2^C=p_2^B\text{-}b_2$	$\Delta p=0$ $\Delta\pi_1=$ $p_1^A\text{-}(R\text{-}b_1)$ $\Delta\pi_2=p_2^B\text{-}b_2$
Case VI. $b_1>b_2$ $a_2+b_2<R<a_2+b_1$	$p_1^A+p_1^B=a_2+b_2$ $\pi_1^I=a_2+b_2\text{-}(a_1+b_1)$ $p_2^A+p_2^B=a_2+b_2$ $\pi_2^I=0$	$R\text{-}b_1 \leq p_1^A \leq a_2,\ p_1^B \geq p_2^B$ $\pi_1^C=p_1^A\text{-}a_1$ $p_2^B=R\text{-}p_1^A,\ p_2^A \geq p_1^A$ $\pi_2^C=p_2^B\text{-}b_2$	$\Delta p=$ $R\text{-}(a_2+b_2)$ $\Delta\pi_1=p_1^A+b_1\text{-}$ (a_2+b_2) $\Delta\pi_2=p_2^B\text{-}b_2$

a component A price of a_2, and makes a profit of a_2-a_1 in that component. Its profit or in both components is $\pi_1^C=(a_2-a_1)+(b_2-b_1)$. The equilibria in the other cases are computed similarly.

As indicated in the table, compatibility weakly raises the price of the system that the consumer buys and the profit of each firm, in all cases. Because prices weakly rise and demand is inelastic, consumer surplus weakly falls. Compatibility weakly lowers the industry cost by ensuring that the low-cost producer of each component sells. Welfare weakly rises, because with inelastic demand welfare depends only on costs. *QED*.[128]

Remarks. There is a shorter proof that profits rise under compatibility as follows. Firm 2 can only do better under compatibility, since it has no profit under incompatibility. Thus, it is sufficient to show that firm 1's profit do not fall under compatibility. Suppose that firm 1 sets component prices under incompatibility so that it takes all of its profit in component A, i.e. $p_1^A - a_1 = \pi_1^I$ and $p_1^B = \min[R, a_2 + b_2] - p_1^A$. Firm 1 can continue to set these prices under compatibility, since firm 2 does not set its price in component A below cost in equilibrium. Thus, firm 1 makes at least as much profit under compatibility as it did under incompatibility.

In Case I, firm 1 has the lowest cost in each component. The firm does not care whether it takes its profit component-by-component under compatibility or on the sale of a single system under incompatibility. Although compatibility appears to have no effect here, it does tie down the prices of individual components. This fact is significant in Chapter 4 when consumers combine components with variable coefficients. Case II is the most interesting, and corresponds to the example above.

In Cases III-VI, the consumer's reservation price R is sufficiently low that it constrains firms' prices. Case III is similar to Case II in that firm 1 has the lowest cost in both components. Compatibility again places a tighter constraint on component prices. However, now there is a continuum of price equilibria under compatibility. Industry profits and the system price are the same in each of these equilibria, and are

[128] This proposition and most of those that follow readily generalize to the case of more than two components.

unchanged from incompatibility. Each equilibrium corresponds to a different division of profits between components but the same division of profits between firms.

The same is not true in Cases IV-VI. Again there is a continuum of price equilibria under compatibility with the same levels of industry profits and system price, which are higher than under incompatibility. Now, each equilibrium corresponds to a different division of industry profits between the two firms. Firm 1 achieves higher profit under compatibility even in the equilibrium that gives it the smallest share of industry profits.

Compatibility raises both firms' profits by the same amount in Cases I-III. By contrast, in Cases IV-VI the strength of a firm's preference for compatibility depends on which price equilibria results, that is, how profits are divided.

Case IV differs from the others in that output rises under compatibility from zero to one. When the firms are incompatible, each of them has a system cost above the consumer's reservation price. However, compatibility allows firms to produce a system at a cost less than the reservation price.

Prices strictly rise in Cases II and VI. In these cases, firm 2's system cost is the binding constraint on firm 1's system price under incompatibility. Compatibility relaxes this constraint.

In Cases I and II, the model has the analytically convenient property that the price for one component under compatibility does not affect the demand for the other component. As a result, we can solve for equilibrium prices in each component market separately.[129] This property is sensitive to the assumptions of the given costs model. For instance, it fails to hold in Cases III-VI when R is low. Also, if some, but not all

[129] Einhorn (1992) uses the term *disjoint markets* to refer to this type of situation.

firms are compatible with each other, the low-cost incompatible system is similar to a low R, as discussed in Section 3.3. The property can also fail to hold with elastic demand, discussed in Section 6, with horizontal differentiation and incomplete market coverage, discussed in Section 7, and with variable coefficients, discussed in Chapter 4. Below, we focus on cases in which the equilibrium price for each component can be computed separately from the price for the other component. To that end, the phrase "the consumer's reservation price is high" will denote the condition $R>\max[a_i]+\max[b_i]$.

3.2 Price Competition in Oligopoly

The result that duopoly profits and prices rise under compatibility does not generalize to oligopoly.

Example. When there are many firms, compatibility can weakly lower all firms' profits and prices. For instance, suppose that three firms have costs (1,1), (10,1) and (1,10) respectively and R is high. When all three firms are incompatible, firm 1 makes a profit of $(10+1)-(1+1)=9$, and firms 2 and 3 have zero profits. However, when all three firms are compatible, each firm has zero profit. In each component, there are two firms with the lowest cost of one, so neither can raise its price above cost without being undercut. Firm 1 faces the toughest competitor component-by-component under compatibility. Thus, a firm that has the lowest system cost but does not have the lowest cost in either component prefers incompatibility. On the other hand, specialist firms that do not have the lowest system cost make no profit under incompatibility and can only do better under compatibility. Another difference from the duopoly case is the price falls here, from eleven to two.

Profits and prices do not always fall under compatibility when there are many firms. For instance, if three firms have costs (0,1), (1,0), and (10,10), the results are the similar to the duopoly example above. Industry profits rise from zero to two and the industry price from one to two.

These two examples show that compatibility has an ambiguous effect on prices and profits when there are many firms. Compatibility relaxes competition among duopolists with complementary costs. However, compatibility toughens an oligopolist's competitors when the competitors' costs are complementary with each other.

Which effect predominates on average? To answer this question, define the *uncertain costs model*, in which firms draw their costs in each component from random and independent distributions before second-period price competition. Firms and consumers are risk-neutral firms. Firms draw their costs from the same distributions, so they are symmetric *ex ante*. Although the effect of compatibility on profits depends on the realization of costs, we can compare expected profits in the two regimes. In the following example, compatibility lowers expected profits, by contrast with the duopoly case.

Example. For each of three firms in each component, let the probability of having cost zero be 0.8 and the probability of having cost one be 0.2. When a firm has cost zero, say that it "succeeds" and otherwise that it "fails." Under compatibility, a firm's expected profit in one component is the probability that it succeeds times the probability that the other two fail, which is $(0.8)(0.2)^2$. The expected profit in both components is thus $\pi^C(3)=0.064$. Under incompatibility, a firm makes a profit in three cases. First, the firm makes profit two if it succeeds in both components and the other two firms each fail in both, which has probability $(0.8)^2(0.2)^4$. Second, the firm makes profit one if it succeeds in both components and at least one other firm succeeds in exactly one but no more, which has probability $(0.8)^2((1-0.8^2)^2-(0.2)^4)$. Third, the firm makes profit one it succeeds in exactly one component and the other firms fail in both, which has probability $2(0.8)(0.2)(0.2)^4$. The sum of these is approximately $\pi^I(3)\approx 0.084$, so a firm's expected profit is higher under incompatibility than under compatibility.

However, compatibility does not lower expected profits for all cost distributions. To show this, define the *two-point cost* distribution with a probability e of cost zero and the probability $(1-e)$ of cost one. Assume that $R>2$, so that markets can be solved separately under compatibility. A firm makes a positive profit in a component when it succeeds only if all other firms fail; the expected profit of a firm under compatibility in both components when there are n firms is $\pi^C(n)=2e(1-e)^{n-1}$. Under incompatibility, there are three cases in which a firm makes positive profit. First, a firm makes profit two if it succeeds in both components and all other firms fail in both, giving expected profit $2e^2(1-e)^{2n-2}$. Second, a firm makes profit one if it succeeds in both components, no other firm succeeds in both components, and not all other firms fail in both components, giving $e^2[(1-e^2)^{n-1}-(1-e)^{2n-2}]$. Third, a firm makes profit one if it succeeds in exactly one component, and all other firms fail in both, giving expected profit $2e(1-e)(1-e)^{2n-2}$. Combining these results, the expected profit of a firm under incompatibility when there are n firms is

$$\begin{aligned}\pi^I(n) &= 2e^2(1-e)^{2n-2}+e^2[(1-e^2)^{n-1}-(1-e)^{2n-2}]+2e(1-e)(1-e)^{2n-2} \\ &= 2e(1-e)^{2n-1}+e^2(1-e^2)^{n-1}.\end{aligned} \tag{31}$$

Expected profits are not higher under incompatibility for all distributions. Suppose that the probability of having cost zero is $e=0.2$ and the probability of cost one is e=0.8. Using the above formulae, the expected profit under compatibility is $\pi^C(3)=0.256$. However, $\pi^I(3)\approx 0.167$. By contrast to the case when e=0.8, profits are higher under compatibility for this distribution. More generally, the comparison is ambiguous for any given n.

Proposition 2. For any given n, there is a two-point cost distribution for which expected profits are higher under complete compatibility than complete incompatibility, and vice versa.

Proof. The ratio of a firm's profits in the two regimes is

$$\frac{\pi^I(n)}{\pi^C(n)} = (1-e)^n + \frac{e(1+e)^{n-1}}{2}. \tag{32}$$

For a given n, the limit of this ratio as e goes to one is greater than one for n greater than two. Then, expected profit is higher under complete incompatibility. The limit of the ratio as e goes to zero approaches one from below. For e close to zero, expected profit is higher under complete compatibility. *QED.*

Remarks. Proposition 2 shows that for any fixed n, the effect of compatibility on profits is ambiguous. Nonetheless, it is possible to get unambiguous results by fixing the cost distribution and letting n grow large. The following shorthand will be useful for asymptotic results. If $f(n)$ and $g(n)$ are positive functions of n, define $f \succ g$ such that

$$f \succ g \quad \text{iff} \quad \lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty. \tag{33}$$

Read $f \succ g$ as "f is of higher order than g." Note that this is stronger than an alternative definition that f/g is not bounded above, which does not require the limit to exist. Write

$g \prec f$ if and only if (abbreviated iff) $f \succ g$. Write $f \asymp g$ iff f/g and g/f are both bounded above. The following properties are easy to verify. If $f \succ g$, then $f+g \asymp f$. In particular, if x, y, v, and w are positive real numbers with $x>y$, then

$$vx^{n}+wy^{n} \asymp x^{n} \quad \text{and} \quad vx^{n} \succ wy^{n}. \tag{34}$$

The proofs below use these properties repeatedly. Write $g \preceq f$ iff $g \prec f$ or $g \asymp f$.

If we fix e and let n grow large, then the ratio of a firm's expected profits in the two regimes for the two-point distribution, using (34) and $1+e>1-e$, is

$$\begin{aligned}\frac{\pi^{I}(n)}{\pi^{C}(n)} &= \left[(1-e)^{n}+\frac{e(1+e)^{n-1}}{2}\right] \\ &\asymp \frac{e(1+e)^{n-1}}{2}.\end{aligned} \tag{35}$$

Not only is it the case that $\pi^{I}>\pi^{C}$ as n grows large, but the ratio π^{I}/π^{C} has limit infinity, i.e. $\pi^{I} \succ \pi^{C}$. The ratio is exponential in n. As a result, n does not have to be very large for incompatibility to give higher expected profits than compatibility. An intuition for why expected profits are higher under incompatibility is that a firm makes a profit when n is large only if it is uniquely successful in getting the lowest cost. It is easier to get the lowest cost in a component under compatibility than it is to get the lowest cost in both components under incompatibility. When there are many firms, it is easier to be uniquely successful under incompatibility because it is less likely that any other firm succeeds.

We can make a similar comparison for expected costs. Under compatibility, expected industry costs per component $c^{C}(n)$ are

$$c^{C}(n) = (1-e)^{n} \tag{36}$$

while under incompatibility expected industry costs per component $c^{I}(n)$ are

$$c^I(n) = \tfrac{1}{2}[(1-e^2)^n - (1-e)^{2n}] + \tfrac{1}{2}2(1-e)^{2n}. \tag{37}$$

The ratio of costs as n grows large is

$$\begin{aligned} \frac{c^I(n)}{c^C(n)} &\approx \frac{\tfrac{1}{2}(1-e^2)^n}{(1-e)^n} \\ &\approx \frac{(1+e)^n}{2} \end{aligned} \tag{38}$$

so $c^I > c^C$. Because $p = \pi + c$, it immediately follows that $p^I > p^C$. Prices and costs are higher in ratio under incompatibility than under compatibility for large n.

To investigate whether these results generalize to other distributions, we turn to the theory of order statistics. Industry profit in a market is the difference between the second-lowest and the lowest cost drawn in that market. Denote these as c_2 and c_1. The lowest of n draws is known as the first order statistic, and the second-lowest of n draws is known as the second order statistic. Here, the word "order" refers to the rank order.

The cost in a market is equal to the first order statistic on n draws, that is, the lowest cost. Suppose that firms make independent and identical draws for their component costs from a distribution F with support $[\underline{c},\overline{c}]$ and $R \geq 2\overline{c}$. The density function for the lowest of n draws, which can be found in most statistics textbooks, is $n(1-F(c_1))^{n-1}f(c_1)$. That is, there are n firms that might have the lowest cost, and for each one the probability that the other $n-1$ have higher costs than c_1 is $(1-F(c_1))^{n-1}$, and the probability of cost c_1 is $f(c_1)$. Therefore, expected industry costs per component under compatibility are[130]

[130] This calculation (and those that follow) is valid for distributions with atoms as well as continuous distributions. Let all integrals be Lebesgue integrals, replacing dc_1 with $d\alpha(c_1)$ in the final step. If F is continuous, $\alpha(c_1) = c_1$. If F is not continuous, suppose there are a finite number of discontinuities at j_i, $i = 1,\ldots,J$. Let $\alpha'(c_1) = 1$ at all other points. For each j_i create an upward jump in α of j_{i+1}-j_i. For a countable number of jumps, add in the largest J jumps, and define the integral as the limit as J goes to

$$\frac{c^C(n)}{2} = \int nc_1(1-F)^{n-1}dF$$
$$= \underline{c} + \int (1-F)^n dc_1 \tag{41}$$

integrating by parts with $u=c_1$ and $dv=n(1-F)^{n-1}dF$.

The price in a market is equal to the second-lowest of the n draws. The joint density function for the lowest and second-lowest of n draws is, according to the theory of order statistics, $n(n-1)(1-F(c_2))^{n-2}f(c_2)f(c_1)$. There are $n(n-1)$ way of choosing the lowest and second-lowest cost firms, the probability that the other $n-2$ firms have higher costs is $(1-F(c_2))^{n-2}$, and the probability that c_1 and c_2 are drawn is $f(c_2)f(c_1)$. Let $\pi^C(n)$ be the expected profit of a single firm under compatibility in both components. Expected industry profits under compatibility in a single component are therefore

$$\frac{n\pi^C(n)}{2} = E[c_2-c_1] = \int_{\underline{c}}^{\bar{c}} \int_{c_1}^{\bar{c}} n(n-1)(c_2-c_1)(1-F(c_2))^{n-2}dF(c_2)dF(c_1). \tag{42}$$

This formula gives the expected value of the difference between the lowest and second-lowest of n draws from a distribution F. Integrating by parts simplifies this formula considerably. With

$$u = \int_{c_1}^{\bar{c}} n(n-1)(c_2-c_1)(1-F(c_2))^{n-2}dF(c_2), \quad \text{and} \quad dv=dF(c_1) \tag{43}$$

then

$$\frac{\pi^C(n)}{2} = \int (1-F)^{n-1}Fdc. \tag{44}$$

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infinity. The number of jumps is countable since they can be ordered in size and have a finite sum.

All integrals are over the support of F unless otherwise stated.

This equation gives profit per component under incompatibility as well, substituting G for F, where G is the distribution function for the average of two draws from F.

The result (44) for order statistics is originally due to Pearson (1902).[131] If c_i^n is the i^{th} lowest of n independent draws from the same distribution, then Pearson's result implies

$$\frac{E[c_2^n - c_1^n]}{n} = \int (1-F)^{n-1} F dc = \int [(1-F)^{n-1} - (1-F)^n] dc = E[c_1^{n-1} - c_1^n]. \tag{45}$$

This equation has a natural economic interpretation.[132] The left-hand side of this equation is the expected profit of a firm in a market, while the right-hand side is the effect on expected industry costs of entry by the nth firm. Thus, an entrant's profit is the amount by which it lowers industry costs: $\pi(n)=c(n-1)-c(n)$. The same is true *ex post* for any given costs. Although entry lowers the profits of existing firms, consumer surplus rises by exactly the amount that incumbents' profits fall.

These results for order statistics allow us to generalize the results for the two-point distribution to any distribution for component costs.

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[131] Pyke (1965) reviews the theory of spacings, that is, the differences between consecutive order statistics.

[132] I am indebted to Joe Farrell for pointing out the relevance of this result. I independently proved Proposition 3 in the case of discrete distributions F. The observation that $\pi(n)=c(n-1)-c(n)$ allows the proof to be adapted to cover any distribution.

continuous

Proposition 3. In the uncertain costs model, suppose that n firms draw their costs from nondegenerate distributions F_A and F_B such that draws are independent across firms and components. Suppose that the F_j have support bounded below by $\underline{c}$, where the consumer's reservation price satisfies $R>2\underline{c}$. Then, if there are sufficiently many firms, expected profits under complete incompatibility are strictly higher than under complete compatibility. Compatibility strictly lowers expected prices, profits, and costs, and strictly raises expected welfare and consumer surplus.

Proof. Consider first the case in which $F=F_A=F_B$, and the consumers' reservation price is high in the sense that $R\geq 2\bar{c}$. Then, the markets for each component can be solved separately under compatibility.

Using the formulas developed above,

$$\begin{aligned}\frac{\pi^C(n)}{2} &= \int (1-F)^{n-1}F\,dc \\ &= \int [(1-F)^{n-1}-(1-F)^n]\,dc \\ &= c^C(n-1)-c^C(n) \\ &= \int_n^{n-1} \frac{dc^C(m)}{dm}\,dm \\ &= \int_n^{n-1}\int_{\underline{c}}^{\bar{c}} (1-F)^m \log(1-F)\,dc\,dm.\end{aligned} \tag{47}$$

Let G be the distribution function for the average of two draws from F. Let $\pi^I(n)$ be the expected profit of a single firm under incompatibility in both components. A single firm's expected profit under incompatibility per component is

$$\begin{aligned}\frac{\pi^I(n)}{2} &= \int (1-G)^{n-1} G\,dc \\ &= \int [(1-G)^{n-1} - (1-G)^n]dc \\ &= c^I(n-1) - c^I(n) \\ &= \int_n^{n-1} \frac{dc^I(m)}{dm} dm \\ &= \int_n^{n-1} \int_{\underline{c}}^{\bar{c}} (1-G)^m \log(1-G)\,dc\,dm.\end{aligned} \tag{48}$$

The goal is to show that $\pi^I \succ \pi^C$. Using the fact that $(1-F)$ is decreasing and (34), for any ϵ

$$\int_{\underline{c}}^{\underline{c}+\epsilon} (1-F)^m \log(1-F)dc \succ \int_{\underline{c}+\epsilon}^{\bar{c}} (1-F)^m \log(1-F)dc. \tag{49}$$

The fact that G describes the average of two draws from F implies that $1-G(c)>1-F(c)$ for all c is a neighborhood of $\underline{c}$. For instance, Figure 9 compares F and G when F is a uniform distribution over the unit interval. If $F(\underline{c})>0$, then $G(\underline{c})=F(\underline{c})^2<F(\underline{c})$, and for ϵ sufficiently small, $G(c)<F(c)$ for c in the half-open interval $[\underline{c},\underline{c}+\epsilon)$. Alternatively, if $F(\underline{c})=0$, $G(c)$ is computed by integrating $f(a)f(b)$ over the triangular region $(a+b)/2<c$ which has probability weight of the second-order for ϵ sufficiently small. On the other hand, F is computed by integrating $f(a)f(b)$ over the rectangular region $a<c$, which has first-order probability weight. In either case, $1-G(c)>1-F(c)$ for c in the half-open interval $[\underline{c},\underline{c}+\epsilon)$. Therefore, by (34), we have the result

$$\begin{aligned}\frac{\pi^C(n)}{2} &\asymp \int_n^{n-1} \int_{\underline{c}}^{\underline{c}+\epsilon} (1-F)^m \log(1-F)\,dc\,dm \\ &\prec \int_n^{n-1} \int_{\underline{c}}^{\underline{c}+\epsilon} (1-G)^m \log(1-G)\,dc\,dm \\ &\asymp \frac{\pi^I(n)}{2}.\end{aligned} \tag{50}$$

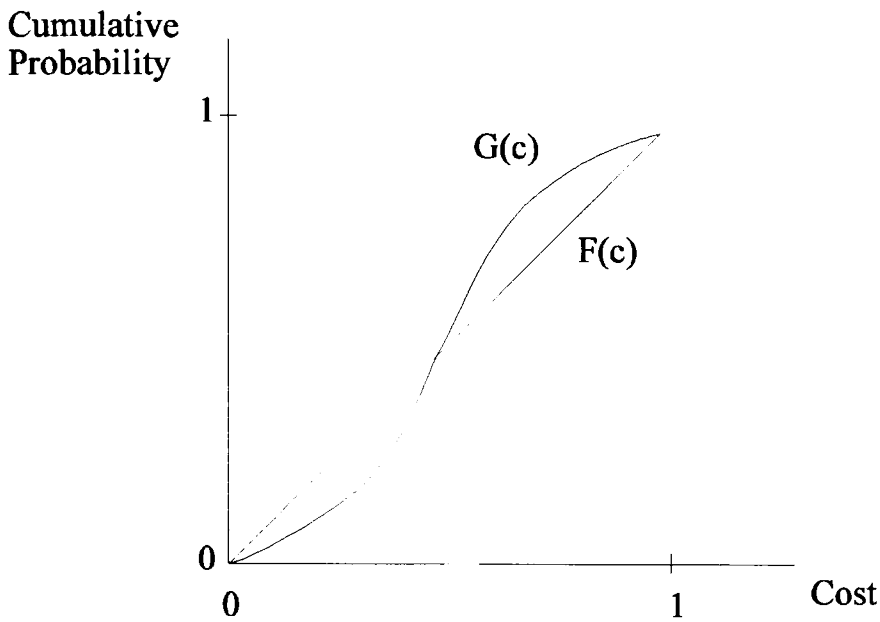

Figure 9: *F* versus *G* for a Uniform Distribution

Using the same methods as above,

$$
\begin{aligned}
c^{C}(n) - \underline{c} &\approx \int_{\underline{c}}^{\underline{c}+\epsilon} (1-F)^{n} dc \\
&\prec \int_{\underline{c}}^{\underline{c}+\epsilon} (1-G)^{n} dc \qquad (51) \\
&\approx c^{I}(n) - \underline{c}.
\end{aligned}
$$

Because $p=\pi+c$, we know immediately

$$p^{C}(n) - \underline{c} \prec p^{I}(n) - \underline{c}. \qquad (52)$$

Let W^{j} be welfare in regime $j=C, I$. Welfare is $W=R-c$, so

$$W^{C}(n) - (R - \underline{c}) \succ W^{I}(n) - (R - \underline{c}). \qquad (53)$$

The results extend to the case in which there are different distributions F_A and F_B for the costs of components A and B. The expected profit of a firm in both components under compatibility is

$$\begin{aligned}\pi^C(n) &= \int_{\underline{c}}^{\bar{c}}(1-F_A)^{n-1}F_A dc + \int_{\underline{c}}^{\bar{c}}(1-F_B)^{n-1}F_B dc \\ &= \int_n^{n-1}\int_{\underline{c}}^{\bar{c}}[(1-F_A)^m\log(1-F_A)+(1-F_B)^m\log(1-F_B)]dcdm.\end{aligned} \tag{54}$$

Let G be the distribution function for the sum of a draw from F_A and a draw from F_B. (Note that G described the average of two draws above.) Let $\pi^I(n)$ be the expected profit of a single firm under incompatibility in both components, which is given by

$$\begin{aligned}\pi^I(n) &= \int(1-G)^{n-1}G dc \\ &= \int_n^{n-1}\int_{\underline{c}}^{\bar{c}}(1-G)^m\log(1-G)dcdm.\end{aligned} \tag{55}$$

The fact that G describes the sum of a draw from F_A and a draw from F_B implies that $G(c)<\min[F_A(c),F_B(c)]$ for all c in a neighborhood of $\underline{c}$ as follows. If $F_A(\underline{c})>0$ and $F_B(\underline{c})>0$, then

$$G(\underline{c}) = F_A(\underline{c})F_B(\underline{c}) < \min[F_A(\underline{c}),F_B(\underline{c})] \tag{56}$$

and for ϵ sufficiently small, $G(c)<\min[F_A(c),F_B(c)]$ for c in the half-open interval $[\underline{c},\underline{c}+\epsilon)$. Alternatively, if $F_A(c)=0$ or $F_B(c)=0$, $G(c)$ is computed by integrating $f_A(a)f_B(b)$ over the triangular region $(a+b)/2<c$ which has probability weight of the second-order for ϵ sufficiently small. On the other hand, $F_A(c)$ is computed by integrating $f_A(a)$ over the rectangular region $a<c$, which has first-order probability weight and similarly for $F_B(c)$. In either case, $1-G(c)>1-\max[F_A(c),F_B(c)]$ for c in the half-open interval $[\underline{c},\underline{c}+\epsilon)$. Then,

$$\begin{aligned}\pi^C(n) &\propto \int_n^{n-1} \int_{\underline{c}}^{\underline{c}+\epsilon} [(1-F_A)^m \log(1-F_A) + (1-F_B)^m \log(1-F_B)] dc\, dm \\ &\prec \int_n^{n-1} \int_{\underline{c}}^{\underline{c}+\epsilon} (1-G)^m \log(1-G) dc\, dm \\ &\propto \pi^I(n).\end{aligned} \tag{57}$$

Now consider the case in which the consumer's reservation price is low enough sometimes to be a binding constraint on firms' prices. The computation of profit above does not allow for this possibility. Take any rule for choosing among multiple price equilibria when cases IV-VI of Proposition 1 occur. For ϵ sufficiently small, these cases occur with arbitrarily small probability, and profits can be computed as before with probability arbitrarily close to one. Therefore, the proof remains valid for $R>2\underline{c}$. *QED.*

Remarks. For any distribution, profits under compatibility are of higher order than profits under incompatibility. If $\underline{c}=0$, compatibility also has order effects on costs and prices, but not on welfare or consumer surplus.

Examples. The following examples show how Proposition 3 can fail to hold when its assumptions are violated. First, suppose that costs are correlated across components, but are independent across firms, such that each firm has costs (0,1) or (1,0) with equal probability. Under incompatibility, all firms have a system cost of one with certainty, so each firm's profit is zero. Under compatibility, each firm has a profit of $(\frac{1}{2})^{n-1}$. Then, expected profit are higher under compatibility than incompatibility for any number of firms.

Second, suppose that costs for each type of component are correlated across firms such that in each outcome, exactly one firm has cost zero in a component and all other firms have cost one. Then, a firm's expected profit under compatibility is $2/n$ (a $1/n$

chance of getting profit one in each component). A firm's expected profit of incompatibility is $2/n^2$ (a $1/n^2$ chance of getting profit two), because the firm must get the lowest cost in both components to make a profit. Otherwise, two firms will have a system cost of one, and all others will have a cost of two, and no firm makes a profit.

Finally, suppose that the distribution function for the costs of one component is degenerate, that is, all firms have the same cost in that component with certainty. For any realization, costs correspond to Case I in the proof of Proposition 1, and compatibility does not affect profits.

In one respect, the assumptions of Proposition 3 are stronger than necessary. Let $F(a,b)$ be the joint distribution of each firm's component costs, let F_A and F_B be the marginal distributions of F, and define G as the distribution for the average of draws a and b. The proof needs only $G<F_i$ in a neighborhood of the lower bound of the distributions' respective supports. For instance, the proof above continues to be valid for any $F(a,b)$ with rectangular support, since G measures probability in a neighborhood of the lower left corner, which has second-order weight. Thus, the result is robust to some violations of the independence assumptions.

3.3 Price Competition with Intermediate Compatibility

When some, but not all firms are compatible with each other, price competition does not uniquely determine each firm's profit for some cost realizations. The lowest-cost incompatible system can act as a constraint on the prices of the two firms, just as the consumer's reservation price R constrained prices in Cases III-VI of Proposition 1 above.

Example. Suppose that two firms have costs (0,1) and (1,0), and the lowest-cost incompatible system costs one. Under compatibility, the two firms jointly make a profit of one. Any nonnegative pair of prices such that $p_1^A + p_2^B = 1$ is an equilibrium, and each equilibrium has a different division of profits.

The multiplicity of equilibria means that firms' choices of standards do not uniquely determine their profits for some cost realizations. However, because firms are symmetric *ex ante*, any rule for choosing among the equilibria that does not depend on the identity of the firms will lead to the same expected profit for each firm. Call such a rule a *symmetric sharing rule*. Possible rules include dividing the profits evenly, or choosing the equilibrium that gives the highest profit to the firm with the lowest cost in component A. Henceforth, when we consider situations of intermediate compatibility, we assume that firms draw their costs from the same distributions, and divide profits according to a symmetric sharing rule.

4 Equilibrium Standard Choice in the Uncertain Costs Model

In the first stage, firms choose among a set of homogeneous standards, and firms that choose the same standard are compatible. The previous section compares the profits of complete compatibility, when all firms choose the same standard, and complete incompatibility, when all firms choose different standards. However, we have not yet looked at the incentive for a subset of firms to choose the same standard when other firms choose different standards. In duopoly, this possibility does not arise. When $n>2$, however, it found that the decision of a subset of firms to choose the same standard hurts other firms. As a result, individually rational compatibility decisions need not maximize industry profits.

In the uncertain costs model, firms' choices of standards in the first period uniquely determine their expected profits in second period price competition with a symmetric sharing rule, as described above. Section 4.1 examines the properties of the implied reduced form payoff function. Section 4.2 solves the game in which firms simultaneously choose from among nonproprietary standards. In Section 4.3 firms choose standards sequentially. Section 4.4 considers the case in which standards are proprietary, that is, when a group of firms using the same standard can prevent other firms from using that standard.

4.1 Properties of the Payoff Function

Proposition 3 above does not address what happens to industry profits when some, but not all firms choose the same standard. Nor does it address how a firm chooses between being incompatible with all other firms and becoming compatible with one of several groups of firms. The answers to these questions can be derived from the reduced form payoff function.

Firms' choices of standards partition the set of firms into mutually exclusive groups of compatible firms. Because firms are symmetric, the number of firms in each group completely describes firms' choices. Let $\mathbb{C}$ be the set of partitions of n. For $n=3$, $\mathbb{C}=\{\{3\},\{2,1\},\{1,1,1\}\}$. Call each partition a *coalition structure*. The second-period payoff of firm i is a mapping from the set of coalition structures into expected profit

$$\pi_i\colon\ \mathbb{C}\rightarrow\mathbb{R}^+. \tag{58}$$

This payoff function has the property that private and social incentives for entry sometimes coincide. When welfare is measured by the sum of consumer surplus and industry profits, Mankiw and Whinston (1986) identify two ways in which the private and social benefits of entry may differ. First, incentives for entry are too low from a social perspective to the extent that entry increases welfare in a way that is not appropriated by the entrant. Second, incentives for entry are too high when entry lowers the profits of other firms, that is, when the entrant steals business from other firms. Private and social incentives for entry coincide when the nonappropriability effect are equals the business stealing effect. In the uncertain costs model, the condition is simpler. Entry incentives are socially optimal when the entrant's profit is equal to the amount by which it lowers the industry cost, which in turn measures the social benefit of entry because demand is inelastic.

To describe when private entry incentives are socially optimal, let the term *compatible entry* describe entry in which the entrant is compatible with at least one other firm. Let *incompatible entry* describe entry by a firm that stands alone after entry, that is, does not become compatible with any other firm. A firm that is incompatible with all other firms is said to *stand alone*.[133]

Proposition 4. In the uncertain costs model, the private benefits of incompatible entry are equal to the social benefits. When all incumbents are compatible with one another and $R>2\bar{c}$, the private incentives for compatible entry are also socially optimal. When some but not all firms are compatible or if the consumer's reservation price R is low, private incentives for compatible entry are suboptimal.

[133] This phrase is taken from Yi (1992).

Proof. Incompatible entry is profitable if and only if it lowers industry costs. If it does lower industry costs, the entrant sets a price equal to the industry cost before entry, so its profit is the difference between industry costs before entry and after entry.

The same computation describes the profit of a compatible entrant that has the lowest cost in both components. Suppose therefore that the entrant has the lowest cost only in component A. If the price following entry is not constrained by the cost of an incompatible firm (or the consumer's reservation price), the entrant's profit and the social benefit of entry are both equal to the amount by which the entrant lowers the cost of component A. Therefore, the incentives for compatible entry when all firms are compatible and R is high are socially optimal.

The same is not true when a compatible entrant lowers the cost only of component A and its post-entry price is constrained by the cost of an incompatible system or a low reservation price. Let c^I be the cost of the lowest-cost incompatible system, let (a_2,b_2) be the lowest costs in the group that the entrant joins, and let the entrant's cost in component A be a_1. Consider two cases. In the first case, $c^I<a_2+b_2$. Costs following entry are a_1+b_2. Entry lowers industry costs by $c^I-(a_1+b_2)$. There is a continuum of price equilibria, each corresponding to a different division of profits. At best, the entrant's profit is $c^I-(a_1+b_2)$. In a second case, let $c^I\geq a_2+b_2$. Costs following entry are again a_1+b_2, and entry lowers industry costs by a_2-a_1. The entrant's profit is at best a_2-a_1. Thus, in both cases an entrant's profit is at best the amount by which it lowers industry costs. Profits are shared by some sharing rule that picks one of the price equilibria. Suppose that the rule is to split the difference by choosing the equilibrium in the middle of the continuum of equilibria. In every outcome, the entrant's profit is less than the amount by which it lowers industry costs. Any symmetric sharing rule gives

the same expected profits. Therefore, the private benefit of compatible entry is less than the social benefit when the cost of an incompatible firm (or a low reservation price) constrains the post-entry price with positive probability. *QED*.

Remarks. When there is intermediate compatibility, the private incentive for incompatible entry is socially optimal, but the payoff to compatible entry is suboptimal. Thus, the entrant has a bias towards choosing incompatible entry over compatible entry. However, we will see below that the private benefits of compatible entry exceed those of incompatible entry.

The following proposition describes the payoff function when the total number of firms is fixed. The following terminology will be useful in stating its properties. Say that a distribution function G *first-order stochastically dominates* a distribution H if $G(c) \geq H(c)$ for all c, with strict inequality for some c. Then, say that costs are "FOSD-lower" with G than H.

Proposition 5. The expected profit function in the uncertain costs model has the following properties:

(i) no firm wants to stand alone, that is, remain incompatible with all other firms;

(ii) the coalition structure that maximizes industry profits can involve complete compatibility, incomplete compatibility, or intermediate compatibility, depending on the cost distribution;

(iii) with a given cost distribution, a firm sometimes but not necessarily always wants other firms to join its group;

(iv) if two groups of compatible firms join into one, other firms are worse off and industry costs are FOSD-lower;

(v) if a firm moves to a larger group, industry costs are FOSD-lower and are FOSD-lowest with the largest group;

(vi) a firm prefers to join a larger group, the benefits of switching to a larger group increase in the size of the group joined, and firms not in the group joined are worse off;

(vii) in a given coalition structure, a firm in a larger coalition has higher profit than a firm in a smaller coalition;

(viii) a firm cannot do worse than to be incompatible when all other firms are compatible with each other;[134] and

(ix) an incompatible firm's private incentive to become compatible when there are multiple standards is socially suboptimal.

Proof. (i) Take any cost realization. Proposition 1 above shows that in duopoly a firm is weakly better off when it becomes compatible with another firm. The same result implies that a stand-alone firm is weakly better off joining a firm or group of firms, letting R represent the price after the firm joins the group, and letting (a_2,b_2) be the least-cost components in the group beforehand. Because this property is true with certain costs, it is also true for uncertain costs.

[134] Proposition 5 (viii), which says that a firm can do no worse than to be incompatible when all other firms are compatible may have some bearing on the outlook for the Apple Macintosh computer. The Macintosh is incompatible with all other brands of personal computers, most of which are compatible with each other. Until recently, the Macintosh's ease of use has differentiated it from PCs. However, Microsoft's Windows operating system for PCs has made PCs and Macintoshes closer substitutes. Apple has responded to increased competition from PCs with an effort to make its operating system compatible. See *The Economist*, "Apple Computer: Falling to Earth," 28 August 1993, p. 57.

(ii) With $n=3$ and a two-point cost distribution, industry profits are maximized under complete incompatibility for $e=0.2$, under complete compatibility for $e=0.8$, and under intermediate compatibility (two compatible firms and one incompatible firm) for $e=2/3$.

(iii) A duopolist always wants the other duopolist to join its group, by Proposition 1. However, when there are sufficiently many firms, complete incompatibility gives higher profits than complete compatibility. Then, a firm does not want all other firms to join it.

(iv) When two groups of compatible firms join into one, the least-cost incompatible system that other firms face has a FOSD-lower cost. Other firms' costs do not change, so industry costs are FOSD-lower.

(v) Suppose for the moment that it is possible to make the firm's component A compatible with the larger group while leaving the component B with the smaller group. The larger group's lowest component B cost is FOSD-lower than the smaller group's, because the larger group makes more draws. Therefore, moving the component A to the larger group FOSD-lowers industry costs, because the component A is matched with an FOSD-lower component B. By the same argument, moving the firm's component B to the larger group also FOSD-lowers the industry cost.

(vi) If the firm moving to a larger group has the lowest costs in both components, its profit is the same whichever group it joins. Suppose therefore that it has the lowest cost only in component A. The firm is more likely to be part of the winning system with a larger group, because that group's lowest component B cost is FOSD-lower than that of a smaller group. Given that the firm makes part of the winning system, consider its profit in two cases. On the one hand, suppose that the price of the winning system is not constrained by the cost of an incompatible system. Then, its profit is equal to the amount by which it lowers industry costs, and these costs are FOSD-lower with the larger group, by (v). On the other hand, suppose that the cost of an incompatible system constrains the price of the winning system. This is less likely to happen with the larger group, and the constraint is less binding with the larger group, because the larger group has FOSD-lower costs than a smaller one. With a constrained system price, the firm's profit is less than the amount by which it lowers industry costs. Therefore, with a smaller group, a firm is more likely to get a partial share of a smaller pie than with a larger group. On all counts, its profit is FOSD-higher with a larger group. The benefits of joining a larger group increase with the size of the group joined, since each of these effects increases in the size of the group joined. Firms not in the group joined have FOSD-higher costs, and, they face competitors with FOSD-lower costs as a group. Therefore, firms not in the group joined are worse off.

(vii) Same as (vi).

(viii) Follows directly from (vii); its competitors' costs are FOSD-lowest.

(ix) In Proposition 4 above, the incentives for incompatible entry are socially optimal, but the incentives for compatible entry when there are multiple standards are suboptimal. Therefore, the incentive to become compatible given that entry has occurred is suboptimal. *QED*.

Remarks. The ranking of coalition structures by industry profits is highly sensitive to the cost distribution. With a two-point cost distribution and $n=4$, any partition of firms maximizes industry profits for some e, except the one with two compatible firms and two incompatible firms, {2,1,1}. With $n=3$ or $n=4$, the ranking of coalition structures by industry profits reverses completely between low e and high e.

The game of standard choice in the uncertain costs model is one of a class of games in which agents form mutually exclusive groups. The set of outcomes in such a game is a partition of the set of agents. Other members of this class are games in which countries form competing trading blocks or currency unions, or firms join R&D consortia. The role of coalitions in such games should not be confused with the role of coalitions in equilibrium concepts such as the core. The coalitions used in identifying the core exist only notionally to test for deviations from equilibrium, and the games in question may have any type of outcome. By contrast, in games of group formation, the groups themselves are the outcomes, and the equilibrium concept may be noncooperative or otherwise. For instance, firms can choose their standards noncooperatively, and the groups would consist of firms who choose the same standard.

A familiar game of endogenous group formation, in which Cournot firms choose whether to join cartels, has payoffs that differ sharply from those in the first stage of the uncertain costs model. Salant, Switzer, and Reynolds (1983) note that if a subset of Cournot firms form a cartel, firms not in the cartel are better off. By contrast, if a subset of firms become compatible with each other in the uncertain costs model, other firms are worse off.[135] The following table compares the properties of the payoff function in the Cournot cartels game and in the uncertain costs model, drawing upon Yi's (1992) analysis of the cartels game.

[135] Yi (1992) proves the existence and uniqueness of equilibrium in the Cournot cartels game.

Cournot cartels: Salant, Switzer, and Reynolds (1983), Yi (1992)	Uncertain costs model
A cartel with the membership of all firms maximizes industry profits	Complete compatibility does not maximize industry profits for large n
Smaller cartels have higher profits than larger cartels in a given cartel structure	Larger coalitions have higher profits in a given coalition structure (vii)
A firm is better off when other firms create a cartel	A firm is worse off when other firms become compatible with each other (iv)
When a firm moves to a larger cartel, firms left behind are better off	When a firm moves to a larger group, firms left behind are worse off (viii)
A member of the largest cartel strictly prefers joining the smallest cartel	A firm prefers to join the largest group (vi)

Having established the properties of the payoff function, it is now possible to solve the game in which firms choose standards.

4.2 Simultaneous Choice of Nonproprietary Standards

Suppose that decisions about compatibility are made in the following manner: each firm chooses a standard from a set of homogeneous, costless standards. Firms that choose the same standard are compatible. Suppose also that standards are *nonproprietary*, that is, a firm cannot prevent other firms from adopting the same standard, for instance, through patent protection.

Consider first games with general payoff functions of the form (58) that do not necessarily have the properties described in Proposition 5. At least one Nash equilibrium exists, because the players have finite action spaces. However, the only equilibria may involve mixed strategies. For instance, a pure-strategy equilibrium will not exist if one of two firms prefers compatibility and the other prefers incompatibility.

In the uncertain costs model, duopolists cannot have such preferences, by Proposition 1, or more simply, by the assumption that firms are symmetric. However, the following example shows that when $n>2$, symmetry does not guarantee the existence of pure-strategy equilibrium.

Example.[136] Let $\pi_{klm...}$ stand for the profit of a firm in a group of size k, when there are other groups of size l, m, and so on. With five firms, pure-strategy equilibrium does not exist if $\pi_{221}>\pi_{311}$, $\pi_{32}>\pi_{122}$, and $\pi_{113}>\pi_{23}$, and all other payoffs are lower than $\min[\pi_{221},\pi_{311},\pi_{32},\pi_{122},\pi_{113},\pi_{23}]$. Existence is not a problem with fewer than five symmetric firms. These payoffs are inconsistent with the uncertain costs model, because the third inequality violates Proposition 5 (i), which states that a firm prefers becoming compatible with any other firm to standing alone.

Existence of pure-strategy equilibrium is not a problem in the uncertain costs model.[137]

Proposition 6. For any given costs, a firm that is incompatible with all other firms weakly does better by becoming compatible with any other firm or group of firms. Therefore, complete compatibility is a Nash equilibrium in the given costs model and in the uncertain costs model. Complete incompatibility is never an equilibrium.

[136] Yi (1992) independently describes the same example, in a general analysis of the existence and uniqueness of equilibrium in games of endogenous group formation. The material in this thesis is my independent work except where noted.

[137] Mixed-strategy equilibria exist, for instance, if each duopolist flips a coin to choose its standard. We focus on pure-strategy equilibria as being more relevant in practice.

Proof. The proposition follows immediately from Proposition 5 (i), which states that a firm does not want to stand alone. *QED*.

Remarks. Although firms' compatibility decisions do not always maximize industry profits, the socially optimal outcome, complete compatibility, is always an equilibrium. An intuition for this result is that compatibility can raise the profit of a single incompatible firm only when it improves on the industry's low-cost system, which is precisely when compatibility would raise welfare. Starting from complete incompatibility, each firm raises its profit by joining the same group one by one. With $n=4$, Proposition 5 (i) implies that $\pi_{211} \geq \pi_{1111}$, $\pi_{31} \geq \pi_{112}$, and $\pi_4 \geq \pi_{13}$ for any costs. However, for some cost distributions, $\pi_{1111} > \pi_4$, by Proposition 3.

Proposition 5 (i) does not address whether a firm with profit π_{112} will prefer joining the two firms instead of the other single firm, i.e. it does not show that $\pi_{31} \geq \pi_{22}$. Thus, Proposition 6 does not rule out two pairs of compatible firms as an equilibrium. However, intermediate compatibility is never an equilibrium, by Proposition 5.

Proposition 7. In the uncertain costs model, a firm will always do better switching to a larger group. Complete compatibility is the unique equilibrium with nonproprietary standards.

Proof. See Proposition 5 (vi). *QED*.

Remarks. This result shows that even with intermediate compatibility, firms make their compatibility decisions in a socially optimal manner if standards are costless, though they do not capture the full social benefits. A social planner would like to move firms to the largest group, and each firm would like to move to the largest group as well. However, if some standards cost more than others, this fortunate coincidence can break down, because Proposition 5 (ix) shows that the benefit to a stand-alone firm of joining one of several groups of compatible firms is socially suboptimal.

4.3 Sequential Choice of Nonproprietary Standards

The equilibrium coalition structure depends upon the cost distribution if firms choose standards sequentially. Suppose that firms choose standards in a given order, and solve the game for the subgame perfect Nash equilibrium. Let firm i be the i^{th} firm to move. For any cost distribution, the last firm to move, firm n, will always choose to join the largest group of firms given the choices of the other $n-1$ firms. The strategy of firm $n-1$ depends on the cost distribution. Since all standards are homogeneous *ex ante*, firm 1's choice of standard does not affect the outcome. The following example shows that the equilibrium coalition structure depends upon the cost distribution.

Example. Suppose that $\pi_3 < \pi_{111}$, as happens when firms draw costs from a two-point cost distribution with $e = 0.8$. By Proposition 6 above, $\pi_{111} < \pi_{21}$. Transitivity implies that $\pi_3 < \pi_{21}$. Firm 3's strategy is to join the largest group, if there is one. If firm 2 does not choose the same standard as firm 1, then firm 3 is indifferent between joining firm 1 and joining firm 2. Suppose that firm 3 flips a coin to decide. Then firm 2 will join firm 1 if $\frac{1}{2}(\pi_{21} + \pi_{12}) > \pi_3$, and the unique equilibrium will be to have two firms compatible and one incompatible. On the other hand, if $\frac{1}{2}(\pi_{21} + \pi_{12}) < \pi_3$, firm 2 will join firm 1 and the unique equilibrium will involve complete compatibility. Thus, this example shows that the outcome of sequential standard choice depends upon the distribution of costs.

4.4 Simultaneous Choice of Excludable Standards

Suppose now that a coalition of firms can prevent other firms from adopting its standard if that would lower their profits, i.e. standards are *excludable*. Complete compatibility remains an equilibrium in this game, because there is no provision for a group to eject members. Proposition 5 (iii) above shows that a group of firms sometimes does not want to be joined. Therefore, there are other equilibria for some cost distributions, because a group of firms sometimes wants to say no.

For instance, suppose that $\pi_3 < \pi_{111}$. By Proposition 6 above, $\pi_{111} < \pi_{21}$. This implies that $\pi_3 < \pi_{21}$, so the two compatible firms will reject the single firm's request to join. Therefore, the outcome in which two of three firms choose the same standard is an equilibrium.

Compatibility is welfare optimal in the uncertain costs model. Therefore, excludability creates new equilibria, which have undesirable welfare properties. Matutes and Padilla (1992) provide another example of this phenomenon, in the case of Automatic Teller Machines. In their model, two banks join their ATM networks together in part to attract business away from a third bank. Although joining all three ATM networks would be socially optimal, this would eliminate product differentiation and create too much head-to-head competition from the perspective of firms.

A more compelling solution to this game might exclude complete compatibility as an equilibrium. One method of doing this would be to allow groups to eject members. Another way would be to allow group deviations. Yi (1992) uses a refinement of Nash equilibrium that does this, coalition-proof Nash equilibrium. Further research could apply that solution concept here.

5 Compatibility and Risk

This section shows that the results above can be reinterpreted as describing when firms, which are risk-neutral with respect to profits, prefer to have riskier costs. Below, it is found that compatibility induces a mean-preserving spread in the distribution of a firm's costs per component. As a result, an industry's preference for or against compatibility can be interpreted as a preference for or against cost risk. Although firms are risk-neutral, riskier costs for all firms raise expected industry profits when the industry prefers complete compatibility to complete incompatibility, and vice versa.

Section 5.1 reviews basic results on risk preference. Section 5.2 shows that compatibility induces a mean-preserving spread of costs in the given costs and uncertain costs models. Section 5.3 shows that firms, despite being risk-neutral, prefer an increase in the riskiness of all firms' costs in duopoly, but prefer a decrease in the riskiness of all firms' costs, when there are sufficiently many firms.

5.1 Basic Results on Risk Preference

This section reviews basic results about risk preference used below. Variance is an inappropriate measure of risk; it is possible to lower the variance of a lottery faced by a risk-averse individual while holding the mean constant and make him worse off. Rothschild and Stiglitz (1970) prove that the following three definitions of increasing risk are equivalent: (i) adding noise to a distribution, (ii) moving probability towards the tails of the distribution without changing its means (a mean-preserving spread), and (iii) changing the distribution such that all risk-averse agents are made worse off, without changing its mean. A mean-preserving spread of a distribution is said to *second-order stochastically dominate* the original distribution.

Jensen's inequality will be used extensively below. If $U(.)$ is a concave function, Y is a random variable, and $E[]$ is the expectation operator, then it states that

$$E[U(Y)] \leq U(E[Y]). \tag{60}$$

In words, a risk-averse individual prefers to receive the expected value of a gamble with certainty to the gamble itself. A related result is that if Y is a mean-preserving spread of random variable X, then

$$E[U(Y)] \leq E[U(X)] \tag{61}$$

that is, increasing risk makes a risk-averse individual worse off. If $U(.)$ is a convex function, each of the above inequalities reverses. If the convexity or concavity is strict, the inequalities are also strict. Finally, if $U(.)$ is linear, the individual is risk-neutral and the inequalities become equalities.

5.2 Compatibility as a Mean-Preserving Spread of Costs

Compatibility induces a mean-preserving spread in the distribution of costs in both the given costs and uncertain costs models.

Example. When duopolists become compatible in the given costs model, the low-cost system becomes less costly, and the high cost system becomes more costly. For instance, with costs (1,2) and (2,1), under incompatibility the industry's low-cost system costs three and the high-cost systems costs three, while these cost two and four respectively under compatibility. It appears that costs are more spread out under compatibility. More precisely, let G be the distribution function for system costs under incompatibility, where $G(c)$ is the proportion of firms with systems costing $c/2$ or less, and let g be the associated density function. Here, $g(3)=1$. Under compatibility, define a set of n costs c_i, where c_i is the average of the i^{th} lowest cost for component A and the i^{th} lowest cost for component B. Industry profits per component under compatibility are c_2-c_1. Let F be the distribution function for the costs c_i and let f be the associated density function. In this example, $f(2)=½$ and $f(4)=½$. Then, F is a mean-preserving spread of G.

This property holds for any given costs.

Proposition 8. For any given component costs, with F and G defined as above, F is a mean-preserving spread of G.

Proof. The two distributions have the same mean, because they sum over the same set of costs. The following proof by induction will show that F is a spread of G. Start out with all firms incompatible and make the two firms with the highest system costs compatible. By Cases I and II, this causes a spread in costs. The cost of the low-cost system weakly goes down and the system price, equal to the cost of the second-lowest cost system, weakly goes up. Take as induction hypothesis that $k-1$ firms with the highest system costs are compatible, and that the distribution of costs is a spread of G. Label the costs of these $k-1$ systems (a_i,b_i), where $a_{i+1}\geq a_i$ and $b_{i+1}\geq b_i$. Now make the k^{th} highest-cost firm compatible. It has the lowest cost in at least one component, so without loss of generality call it component A. Making this firm compatible only affects the cost of the i^{th} system if $b_i<b_k$. Let j be the highest-cost system that is affected. The new lowest-cost system has costs (a_k,b_1), which are weakly lower than before. The j^{th}

system's costs rise to (a_j,b_k). The other $j-1$ systems' costs rise from (a_i,b_i) to (a_i,b_{i+1}). Because the cost of the lowest-cost system falls, and the cost of all other systems rises, making the k^{th} firm compatible is a spread of costs with $k-1$ systems compatible. A combination of spreads is also a spread, so making the k highest-cost systems compatible is a spread of the cost distribution with all firms incompatible. *QED.*

Remarks. A similar result is true in the uncertain costs model. Suppose that a firm draws its two component costs from the same distribution. Under compatibility a firm competes in a component market with one draw from the distribution, while under incompatibility it competes with the sum of two draws in the market for systems. Thus, a firm's cost per component under incompatibility is the average of two draws rather than with a single draw. It follows immediately that costs are riskier under compatibility, by the following standard result.

Lemma. If G is the distribution function for the average of two draws from a distribution F, then F is a mean-preserving spread of G.

Proof. The average of two draws a and b has the same mean as a single draw. Any risk-averse individual prefers the average of two draws to a single draw as follows. Make two draws from F. Let choice one be the average of the two draws. Let choice two be a lottery that gives either of the two draws with equal probability. For any outcome of the draws, a risk-averse individual prefers receiving the average with certainty to the lottery over the draws because his utility is concave in income. Rothschild and Stiglitz (1970) show that if all risk-averse individuals prefer one distribution to another with the same mean, then the second is a mean-preserving spread of the first. Because choice two is equivalent to making one draw, the distribution of one draw is a mean-preserving spread of the average of two draws. *QED*.

Remarks. This result is reminiscent of the Law of Large Numbers, which says that the probability weight in the distribution of the average of n numbers converges in probability to the mean. However, the lemma above applies to two draws, not many draws.

5.3 Preference for Compatibility as Risk Preference

Propositions 1 and 3 show that an industry with two firms prefers compatibility to incompatibility, while an industry with sufficiently many firms prefers incompatibility. The previous section shows that compatibility increases the riskiness of costs. This proposition shows that risk-neutral duopolists' preference for compatibility can be interpreted as a preference towards riskier costs for all firms. Similarly, the preference of many firms for incompatibility can be interpreted as an aversion to riskier costs for all firms.

First, say that F is a *simple mean-preserving spread* of G, then G crosses F once from below. In other words, there is some cost c_0 such that $1-G(c) \geq 1-F(c)$ for $c<c_0$, $1-G(c) \leq 1-F(c)$ for $c>c_0$, and, if F and G are continuous at c_0, $1-G(c_0)=1-F(c_0)$. Intuitively, probability to the left of c_0 does not shift to the right and probability to the right of c_0 does not shift to the left. An increase in risk is *simple* if it is the composition of a finite number of simple mean-preserving spreads.

Proposition 9. In homogeneous Bertrand competition among risk-neutral firms with uncertain costs, an increase in the riskiness of all firms' costs raises expected profits in duopoly. However, a given simple increase in risk lowers industry profits with sufficiently many firms. For any number of firms, riskier costs for any firm raise expected welfare.

Proof. Let F be any mean-preserving spread of G. In duopoly, firm 1's profit is convex in its own cost, holding firm 2's cost constant (see Figure 10).

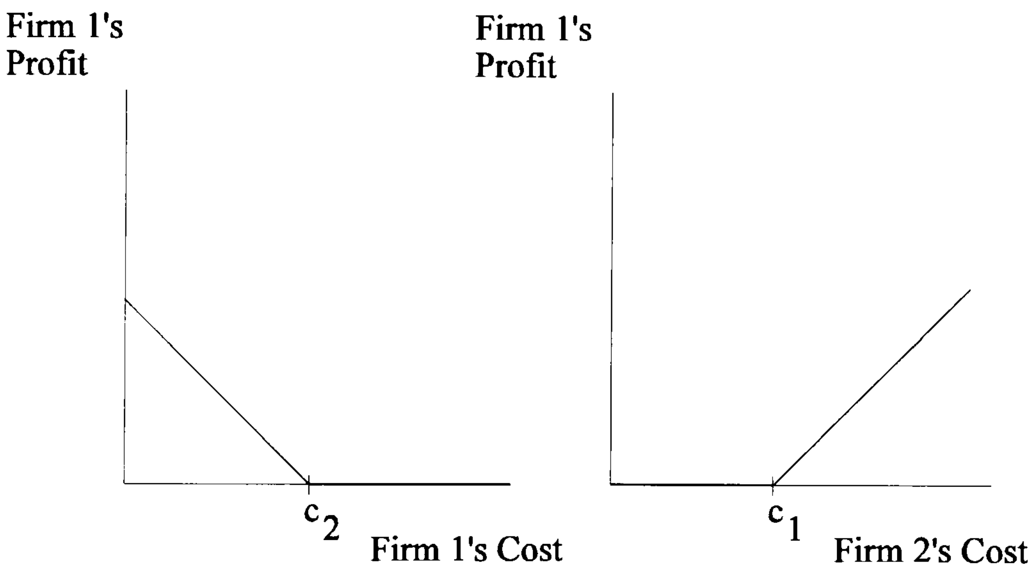

Figure 10: Convexity of a Duopolist's Profit in Costs

In addition, firm 1's profit is are convex in firm 2's cost, holding its own cost constant. Therefore, a shift from G to F raises expected profit by two applications of Jensen's inequality

$$\frac{\pi_1^C}{2} = E[\max[c_2^F - c_1^F, 0]] \geq E[\max[c_2^G - c_1^F, 0]] \geq E[\max[c_2^G - c_1^G, 0]] = \frac{\pi_1^I}{2}.$$

When there are many firms, firm 1's cost is neither convex nor concave in firm 2's cost, if a third firm has a cost higher than firm 1 (see Figure 11), so Jensen's inequality does not apply. For given F and G, where G is a simple increase in risk over F, let c_0 be the first point at which G crosses F from below.[138] The proof of Proposition 3 above shows that expected profit with F is less than expected profit with G, letting $\epsilon = c_0$.

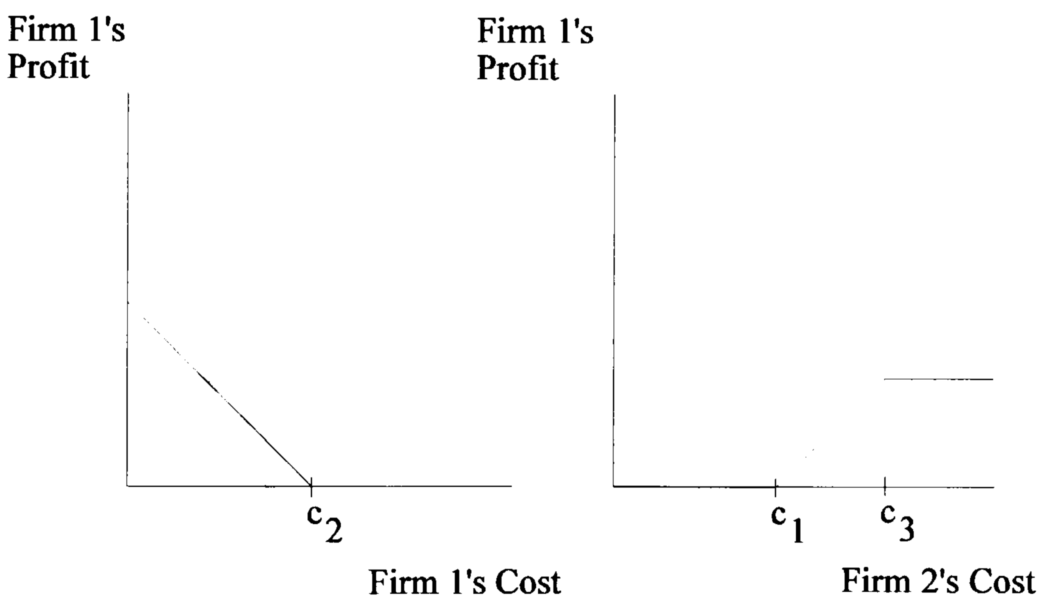

Figure 11: Nonconvexities in Oligopoly Profits

Welfare, which equals $R - \min[a_i, b_i]$, is convex in each of its arguments. Therefore, a mean-preserving spread of any firm's component cost distribution raises expected welfare. *QED.*

[138] If the increase in risk is not simple, F may cross G an infinite number of times, and a lowest crossing point may not exist.

Remarks. The result above is sensitive to the assumption of Bertrand competition. For instance, in Dasgupta and Maskin (1987) duopolists draw their product qualities from random distributions, and the firm with higher quality gets a payoff equal to his quality. Unlike Bertrand competition, the winning firm does not care about its margin of victory over the losing firm. It is shown that a single firm prefers an increase in the riskiness of its own quality, and that risk has a negative externality on the other firm if the density function for quality has an elasticity with respect to quality greater than minus one, that is, if the density function does not fall too quickly. This condition implies that more is at stake when quality is high than when quality is low. Expected industry profits are higher when any firm's quality becomes riskier, since the expected winning quality is higher. For the same reason, expected welfare increases with risk. Dasgupta and Maskin do not consider the case of many firms. However, preferences for or against risk would then be even stronger, because yet higher qualities are necessary to win.

Klette and de Meza (1986) analyze a game of timing in which the first firm to complete a particular R&D project receives a fixed prize. In such a game, discounting convexifies payoffs with respect to time and induces a private and social preference towards increasing risk in the distribution of payoffs over time. The benefits of advancing an early discovery are greater than the costs of delaying a late discovery because of discounting. For the same reason, riskier outcomes for one firm lowers the expected payoff of another firm. Even without discounting, the presence of many firms creates a individual preference towards risk, because with many firms it takes a very early discovery to win. Firms benefit more from increasing the chance of early discovery than they do from decreasing the chance of late discovery. This line of reasoning is extremely similar to the proof of Proposition 3. However, in Klette and de Meza, many firms are necessary to create a individual preference towards risk, while individual firms prefer risk in the uncertain costs model in an industry with any number of firms. In Klette and de Meza's model, there is no industry or social preference towards risk since the industry payoff and social payoff are equal to the fixed prize with certainty. However, if there were a chance that discovery did not occur at all (at $t=\infty$), then an increase in risk that raises the probability of no discovery would be socially undesirable.

The uncertain costs model differs from these two settings in that a firm cares only about its margin of victory, and not how well it does in absolute terms. A firm might care about its margin of victory in R&D competition if success by a second firm reduced or eliminated profits. Without discounting, payoffs would depend only upon the length of time between first and second discovery.

On the other hand, the uncertain costs model, Dasgupta and Maskin's model, and Klette and de Meza's model share the feature that the presence many firms focuses their attention on extreme outcomes (with low costs, high quality, or early discovery). Thus, the presence of many firms encourages a individual preference towards risk, and means that higher risk for one firm has a negative externality on other firms.

6 Elastic Demand

The results so far have assumed inelastic demand. Below we show that with elastic demand, duopolists continue to prefer compatibility to incompatibility for all outcomes. Also, the higher-cost firm in duopoly has a stronger preference for compatibility. Compatibility remains an equilibrium for firms' choices of standards when standards are nonproprietary. Even if demand is elastic, sufficiently many firms with uncertain costs again have lower expected prices and profits. In all cases, complete compatibility remains welfare optimal, and continues to be the equilibrium outcome for compatibility decisions with nonproprietary standards.

Section 6.1 examines the duopoly case and the strength of firms' preferences for compatibility. Section 6.2 shows that firms prefer incompatibility to compatibility when the cost distribution is discrete and there are sufficiently many firms.

6.1 Elastic Demand in Duopoly

Proposition 1 above shows that compatibility weakly raises prices when demand is inelastic. However, if demand is price sensitive, then higher prices will reduce demand. Despite this difference, the duopoly case with elastic demand is similar to the duopoly case with inelastic demand.

Let demand for systems with price p be a strictly decreasing function $q(p)$. We focus on the case in which the low-cost firm's price in each market is equal to its competitor's cost, implying that prices are the same as they were with inelastic demand. When this condition holds, say that *costs constrain prices*, as a shorthand for the statement "the second-lowest cost for a system (component) is a binding constraint on the price of the lowest-cost system (component)." The following table shows equilibrium prices and profits when costs constrain prices. Recall that $a_1 \leq a_2$ by assumption.

One case in which costs constrain prices is as follows. Let $p^m(c)$ be the price a monopolist would charge when facing demand $q(p)$, and suppose that the monopolist's profit as a function of price, $(p-c)q(p)$ is single-peaked. Then costs constrain prices if

Profits and prices with two firms	Incompatibility	Compatibility
Case I. $b_1 \leq b_2$	$p_1^A+p_1^B=a_2+b_2$ $\pi_1^I=[(a_2+b_2)-(a_1+b_1)]q(a_2+b_2)$ $p_2^A+p_2^B=a_2+b_2$ $\pi_2^I=0$	$p_1^A=a_2,\ p_1^B=b_2$ $\pi_1^C=[(a_2-a_1)+(b_2-b_1)]q(a_2+b_2)$ $p_2^A=a_2,\ p_2^B=b_2$ $\pi_2^C=0$
Case II. $b_1>b_2$	$p_1^A+p_1^B=a_2+b_2$ $\pi_1^I=[(a_2+b_2)-(a_1+b_1)]q(a_2+b_2)$ $p_2^A+p_2^B=a_2+b_2$ $\pi_2^I=0$	$p_1^A=a_2,\ p_1^B=b_1$ $\pi_1^C=[a_2-a_1]q(a_2+b_1)$ $p_2^A=a_2,\ p_2^B=b_1$ $\pi_2^C=[b_1-b_2]q(a_2+b_1)$

$$a_2+\max[b_1,b_2] < p^m(a_1+\min[b_1,b_2]) \qquad \textbf{(63)}$$

that is, when the high-cost system (calculated under compatibility) costs less than the price a monopolist would charge if it made the low-cost system (calculated under compatibility).

When costs constrain prices, many of the results that are true when demand is inelastic remain true when demand is elastic.

Proposition 10. Suppose that duopolists have given costs, and demand is elastic such that costs constrain prices. Then, each firm weakly prefers compatibility to incompatibility, and the preference for compatibility of the firm with the higher system cost is stronger than that of the lower-cost firm.

Proof. If one firm has the lowest costs in both components, compatibility has no effect on prices or profits. Otherwise, $c^I=a_1+b_1$, $p^I=a_2+b_2$, $c^C=a_1+b_2$, and $p^C=a_2+b_1$ because costs constrain prices. Welfare is the sum of consumer surplus and industry profits, so if $p(q)$ is the inverse demand curve, welfare under compatibility is

$$W^C = \int_0^{q(p^C)} [p(q)-c^C]dq \qquad \textbf{(64)}$$

and welfare under incompatibility. Then,

$$W^I = \int_0^{q(p^I)} [p(q) - c^I]dq \tag{65}$$

so the difference between welfare under compatibility and under incompatibility is

$$W^C - W^I = (c^I - c^C)q(p^C) - \int_{q(p^C)}^{q(p^I)} [p(q) - c^I]dq. \tag{66}$$

By the assumption that costs constrain prices, a monopolist with cost c^C has higher profit with price p^C than with price p^I

$$(p^C - c^C)q(p^C) > (p^I - c^C)q(p^I). \tag{67}$$

Rearranging (67), and using the fact that $p^I - c^C = p^C - c^I$ gives

$$(c^I - c^C)q(p^C) - (p^C - c^I)[q(p^I) - q(p^C)] > 0. \tag{68}$$

Because $p^C > p(q)$ for q in the interval $[q(p^C), q(p^I)]$, then the second term in (68) is greater in absolute value than the second term in (66). Therefore, (68) implies that (66) is positive as well, so welfare is higher under compatibility.

When firm 1 has the lowest costs in both components, each firm's profit is the same in each regime. Otherwise, when the firms have complementary costs, firm 2's profit is zero under incompatibility, so its profit can only be higher under incompatibility. Firm 1's profit per unit under compatibility is $a_2 - a_1$ which happens to equal $p^C - c^I$. Therefore, firm 1's profit is higher under compatibility than under incompatibility, because deducting $(c^I - c^C)q(p^C)$ from both sides of (67) shows that

$$\pi_1^C = (p^C - c^I)q(p^C) > (p^I - c^I)q(p^I) = \pi_1^I. \tag{69}$$

using $p^I \leq p^C$

To show that firm 2 has a stronger preference for compatibility, note that in regime j, for $j = C, I$, firm 1's profit exceeds firm 2's profit by $(c_2 - c_1)q(p^j)$. Because $p^C \geq p^I$, output is weakly higher under incompatibility than under compatibility, so firm 1's profit exceeds firm 2's profit by more under incompatibility. This implies that firm 2 has a weakly stronger preference for compatibility than firm 1. *QED.*

6.2 Elastic Demand in Oligopoly

When demand is inelastic, it was found that the effect of compatibility on a firm's profit depends on the cost realization. The same is true when demand is elastic. As in the inelastic case, firms' views on compatibility depends upon the costs they have, when $n>2$.

Example. Suppose that demand for systems is $p=1/q$, so demand is unit elastic and costs constrain prices. First, suppose that firms have costs (1,1), (10,1) and (1,10). Under incompatibility the price is eleven, quantity is 1/11, and the profit per system is $11-2=9$, so profit for firm 1 is 9/11. The other firms have zero profits. Under compatibility, no firm makes a profit, because in each component there are two firms with the same cost. Therefore, firm 1 prefers incompatibility. Second, suppose that costs are (1,2), (2,1), and (10,10). Under incompatibility, two firms have the lowest cost of three, and no firm makes a profit. Under compatibility, firm 1's profit is

$$\pi_1^A = \frac{p_1^A - 1}{p_1^A + p_2^B} = 1 - \frac{p_2^B + 1}{p_1^A + p_2^B}. \tag{70}$$

Firm 1's profit is increasing in price, so it charges the highest possible price, which is two. Firm 2 also charges price two for component B, so firm 1's profit is ¼. For these costs, firms 1 and 2 prefer compatibility. Thus, firms may prefer either compatibility or incompatibility, depending upon their costs.

Remarks. When demand is inelastic, Proposition 3 proves that compatibility unambiguously lowers profits on average when there are sufficiently many firms. It is possible to extend this result to the case of elastic demand when the distribution of costs is discrete.

Proposition 11. For any discrete cost distribution F which is independent across firms and components and any demand function for systems $q(p)$, profits are higher under complete incompatibility than complete compatibility when there are sufficiently many firms.

Proof. As is usual, let G be the distribution function for the average of two draws from F, and let g be its density function. Under incompatibility, suppose that firm 1 draws cost per component of c, which has probability $g(c)$. It makes a positive profit only when all other firms have higher costs, which has probability $[1-G(c)]^{n-1}$. Let $\pi^{\min}(c)$ be the least profit per component that the firm can make with cost c, given that other firms have higher costs. Then, firm 1's expected profit per component under incompatibility is at least

$$\frac{\pi_1^I}{2} \geq \sum_c \pi^{\min}(c)[1-G(c)]^{n-1}g(c) \tag{71}$$

where the summation is over the support of G. As n grows large, (34) implies that only the term for which $[1-G(c)]$ is highest matters. This is the term where $c=\underline{c}$, where $\underline{c}$ is the lower bound of the support of F (and G). Therefore

$$\pi_1^I \succeq [1-G(\underline{c})]^{n-1}. \tag{72}$$

Under compatibility, if firm 1 draws cost c in a component, it makes a profit in that component if all other firms have higher costs, which has probability $[1-F(c)]^{n-1}$.

Let $\pi^{\max}(c)$ be the most profit that the firm can make with cost c in a component, given that other firms have higher costs. Then, firm 1 has expected profit in a component of at most

$$\frac{\pi_1^C}{2} \leq \sum_c \pi^{\max}(c)[1-F(c)]^{n-1}f(c) \tag{73}$$

where the summation is over the support of F. As n grows large, again the only term that matters is the one for which $[1-F(c)]$ is highest, so

$$\pi_1^C \precsim [1-F(\underline{c})]^{n-1}. \tag{74}$$

For a discrete distribution F, $F(\underline{c})=f(\underline{c})$ and $G(\underline{c})=f(\underline{c})^2$, so $F(\underline{c})\geq G(\underline{c})$. Therefore, by (34),

$$\pi_1^I \succsim [1-G(\underline{c})]^{n-1} \succ [1-F(\underline{c})]^{n-1} \succsim \pi_1^C. \tag{75}$$

So, profits are higher under incompatibility for large enough n. *QED.*

Remarks. This method of proof does not apply when F is continuous, because a firm gets the lowest possible cost with probability zero. Elastic demand does not alter other conclusions about welfare and standard choice that obtain when demand is inelastic.

Proposition 12. If costs constrain prices, welfare is weakly higher under compatibility than under incompatibility for any number of firms. If profits are in increasing in price, complete compatibility is an equilibrium for standard choice when standards are nonproprietary.

Proof. Welfare is higher under compatibility when there are n firms as follows. If one firm has the lowest cost in both components, then $c^C=c^I$ and $p^C\leq p^I$, which imply that welfare is higher under compatibility. Otherwise, let firms 1 and 2 be the firms with the lowest cost in components A and B respectively. Let c_3 be the firm with the lowest system cost among the other $n-2$ firms. If $c_3>a_2+b_1$, then prices and costs are the same as in the duopoly case in both regimes, and the additional $n-2$ firms do not matter. If $a_2+b_1 \geq c_3 > a_2+b_2$, then the price is lower under compatibility than it was in the duopoly case, but price and cost under incompatibility are unchanged, so welfare is even higher under compatibility than under incompatibility than it was in the duopoly case. If $a_2+b_2 \geq c_3 > a_1+b_1$, then $p^C \leq p^I = c_3$ implies that welfare is higher under compatibility. If $a_1+b_1 \geq c_3 > a_1+b_2$, then $p^C \leq c_3 < p^I = a_1+b_1$, and again welfare is higher under compatibility.

Consider the decision of a firm i to join a group rather than stand alone. If it does not have the lowest cost system when it stands alone, it can do no worse than to join a group. Suppose therefore that it does have the lowest cost system when it stands alone. Let a_L and b_L be the group's lowest cost in each component (excluding firm i), let $c_L=a_L+b_L$, and let c_3 be the lowest-cost incompatible system not including the group or firm i. If $c_3 \leq c_L$ then firm i's profit is c_3-c_i whether it joins the group or not. Suppose therefore that $c_3>c_L$. Then, the firm's profit when it stands alone is $(c_L-c_i)q(c_L)$. Its profit when it joins the group is the same if firm i has the lowest cost in the group for each component. Suppose that it has the lowest cost only in component A. If b_2 is the second-lowest cost in the group (including firm i), then firm i's profit with the group is $(a_L-a_i)q(a_L+b_2)$ if $a_L+b_2<c_3$. The assumption that profits are increasing in price implies

$$[(a_L+b_2)-c_i]q(a_L+b_2) > (c_L-c_i)q(c_L). \tag{76}$$

The right-hand side is equal to firm i's profit under incompatibility, and the left-hand side is less than firm i's profit under compatibility, because

$$a_L-a_i = (a_L+b_i)-c_i \geq (a_L+b_2)-c_i. \tag{77}$$

Therefore, firm i's profit is weakly higher when it joins a group. In the final case $a_L+b_2 \geq c_3$. Then, firm i's profit with the group is at worst $(c_3-a_i)q(c_3)$. The assumption that profits are increasing in price implies that

$$(c_3-c_i)q(c_3) > (c_L-c_i)q(c_L). \tag{78}$$

The right-hand side is equal to firm i's profit under incompatibility, and the left-hand side is less than firm i's profit under compatibility, because

$$a_L-a_i \geq (a_L+b_2)-c_i \geq c_3-c_i. \tag{79}$$

Therefore, in all cases, firm i weakly prefers joining the group to standing alone. As before, complete compatibility is therefore an equilibrium for standard choice. *QED.*

The remainder of this chapter reverts to the assumption of inelastic demand except where noted.

7 Parallels with Differentiation Models

The mix-and-match literature concludes that complete compatibility raises prices and profits compared with incompatibility, in several different settings. In particular, Economides (1989)[139] shows that many firms prefer compatibility. The uncertain costs model reaches the opposite conclusion, apparently because it focuses on costs rather horizontal differentiation.

This section shows, however, that in most settings, many firms with random, independent characteristics prefer incompatibility to compatibility. In particular, Einhorn's (1992) results for vertical product differentiation depend upon the assumption

[139] See page 24 of Chapter 1 for a discussion of problems in this article.

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of duopoly. Economides' (1989) conclusion is also reversed when firms' component characteristics are not perfectly correlated.

Section 7.1 shows that the uncertain costs model can be reinterpreted as a model with uncertain quality. Section 7.2 shows in horizontal differentiation models that many firms with random characteristics prefer incompatibility, and identifies some formal similarities with the uncertain costs model. The remainder of the chapter assumes that demand is inelastic, with the exception of Proposition 16.

7.1 Random Quality with Many Firms

Einhorn (1992) shows in a model with vertical differentiation that duopolists prefer compatibility, but does not address the case of many firms. The results above apply to this case because the uncertain costs model can be reinterpreted as one with uncertain quality. The following example, repeated from the previous chapter, illustrates the similarity between the two models.

Example. Suppose that a consumer has reservation prices (1,0) and (0,1) respectively for the components of firms 1 and 2. Firms have zero unit costs in each component. Under incompatibility, the reservation price for each firm's system is one. If either firm sets a price above zero, the other firm will undercut it. Profits are therefore zero. Under compatibility, the consumer has a higher reservation price for firm 1's component A than for firm 2's component A. Even if firm 2 sets a price of zero, firm 1 can set a price as high as one and still make the sale. Firm 1 has a profit of one in component A, and similarly firm 2 has a profit of one in component B.

These results are almost identical to the example in the given costs model with costs (0,1) and (1,0). Both firms have zero profits under incompatibility while each firm has a profit of one under compatibility. In both cases, what matters is which firm can offer the consumer a higher surplus while making a given level of profit. Whether the source of the firm's high surplus is a high quality or a low cost does not affect its competitive position.

Using this intuition, it is possible to reinterpret the uncertain costs model as one of uncertain quality. Define the *vertical differentiation* model as follows. In the uncertain costs model, assume that components have random qualities rather than random costs. That is, when firms draw c according to F in component j, for $j=A$, B, the consumer has reservation price $R^j = \bar{c} - c$ in that component, where R^j is a measure of product quality. Firms have zero unit production costs. When two firms offer the same surplus, for analytical convenience give the sale to the firm with higher reservation price. This model has a single consumer, while by contrast Einhorn has a continuum of consumers with different reservation prices.

In both the vertical differentiation and uncertain costs models, profits depend on firms' draws from a distribution F. Say that two such models are *ex post revenue equivalent* if for any realization, a firm's profit is the same in each model.[140]

Proposition 13. The vertical differentiation model is *ex post* revenue equivalent to the uncertain costs model with high reservation price. The same propositions apply to both models, although compatibility now raises product quality rather than lowering costs.

[140] *Ex post* revenue equivalence is stronger than *ex ante* revenue equivalence, because the former requires the profits be equivalent for all outcomes, not just in expectation. Auction theory uses the latter definition, as discussed below.

Proof. In the vertical differentiation model, consumers choose the product that offers the largest surplus, $R-p$, and only one firm makes a sale for any prices. Let firm 1 be the firm that draws the highest reservation price R_1 in a market, and firm 2 the one that draws the second-highest reservation price R_2. The only firm whose price cannot be undercut is firm 1. Firm 2 is willing to undercut firm 1 until its price is zero, in which case firm 2 offers surplus R_2. Firm 1 must offer the same surplus, so it cannot set a price higher than R_1-R_2. The profit it earns with this price is the same as the profit it would have earned in the uncertain costs model, because $R_1-R_2=c_2-c_1$. *QED.*

Remarks. Einhorn (1992) found that prices and profits are higher under compatibility than incompatibility in duopoly. Proposition 13 shows that the assumption of duopoly is not innocuous. When there are sufficiently many firms in a model with uncertain quality in this model, profits and prices are higher under incompatibility. Thus, the duopoly case does not generalize.

7.2 Horizontal Differentiation with Many Firms

The literature on mix-and-match compatibility began with models with horizontally differentiated components, which are discussed in Chapter 1. Matutes and Regibeau (1988) has the same prediction as the uncertain costs model, that duopolists prefer compatibility. By contrast, Economides (1989) finds that when there are many firms, they prefer compatibility, while Proposition 3 shows the opposite is true in the uncertain costs model. In Economides' model, firms have differentiated components and identical costs, while in the uncertain costs model, firms have homogeneous components and differing costs. These contrasting results appear to arise from different specifications. In fact, this section shows that when firms' components have random, independent characteristics, profits and prices are higher under compatibility.

Define the *horizontal differentiation model* as follows. Each component comes in several varieties, and consumers differ in their preferred variety. A given component has a characteristic corresponding to a point on the unit interval. A consumer has a preferred characteristic of a for component A and b for component B. A consumer's ideal system is therefore described by his location on a unit square, (a,b). Each consumer has inelastic demand with reservation price R for a system consisting of exactly one of each component.[141]

A firm makes exactly one variety of each component, so its two component characteristics, (a_i,b_i), completely describe its system. Firms have zero unit production costs. A consumer pays a transport cost of buying components different from his ideal components. This transport cost is a function of the distance between his ideal system (a,b) and the system bought (a_i,b_i), measured according to some metric.

One convenient metric for measuring distance is the block or "Manhattan" metric, which adds up coordinate differences component-by-component. This metric measures distance from the perspective of a pedestrian walking along horizontal and vertical streets. If (a,b) are the characteristics of the consumer's ideal system, and (a_i,b_i) is the system he buys, his distance from his ideal system is .

$$d = \|a_i - a\| + \|b_i - b\|. \tag{81}$$

The unit transport cost is one, so the transport cost that the consumer pays is equal to the distance to his ideal system. The analysis below will assume unit transportation costs

[141] No assumption is made here that consumer tastes wrap around, as discussed in the previous chapter.

of one whenever the block metric is used. By contrast, Chapter 1 used the Euclidean metric and quadratic transportation costs to avoid Hotelling-type existence problems.

Economides (1989) concludes that profits and prices are higher under complete compatibility than complete incompatibility in a model with n symmetric firms. An appendix to Chapter 1 shows that this claim is correct when firms are evenly spaced along the diagonal of the unit square. However, the assumption that firms lie on the diagonal is a crucial one, as seen in the following example.

Example. Suppose that the four firms lie not along the diagonal, but at the four corners of the unit square. Under incompatibility, profits are positive. Under compatibility, there are two identical versions of each component, so homogeneous Bertrand competition drives prices and profits down to zero. Economides' conclusion that prices and profits are higher therefore depends on the perfect correlation of firms' characteristics.

Remarks. The example shows that the effect of compatibility on prices and profits when there are many firms depends on the particular characteristics firms happen to have. The same feature was found in the uncertain costs model. There, it was possible to get unambiguous results by examining expected prices and profit margins when many firms have random costs. The following proposition does the same by giving firms random characteristics.

Proposition 14. The uncertain costs model with costs drawn from the unit interval is *ex post* revenue equivalent to a model in which firms draw their component characteristics from the same distribution functions, a single consumer with reservation price R is at the point (0,0), and the consumer has tastes described by the block metric and a unit transportation cost of one.

Proof. Let c_1 be the draw closest to the origin under the block metric, and let c_2 be the second-closest draw. In the uncertain costs model, the firm with draw c_1 has the lowest costs. Firm 1 is the only firm to make a profit, and that profit is $\pi = c_2 - c_1$, with price c_2 and cost c_1. In the characteristics model, only firm 1 makes a profit. If it sets a price higher than $c_2 - c_1$, firm 2 will undercut it. Therefore, the winning firm has profit $c_2 - c_1$ and costs zero. The same equivalence holds under compatibility as well by a similar argument. *QED.*

Remarks. Having established this result, it is easy to show that the firms do not have characteristics that are in the first quadrant for revenue equivalence to hold.

Proposition 15. In the horizontal differentiation model, suppose that firms draw their component characteristics from a distribution with support $[-\bar{c}, \bar{c}]$ and a single consumer with inelastic demand and high reservation price is at the point (0,0), with tastes described by the block metric and a unit transportation cost of one. This model is *ex post* revenue equivalent to the uncertain costs model, if a firm that draws characteristic c from the same distribution has a cost $\|c\|$, the absolute value of a draw.

Proof. In both models and in both regimes, profits are $c_2 - c_1$, as defined in the previous proof. *QED.*

Remarks. The horizontal differentiation model of Proposition 15 is similar to that of Matutes and Regibeau (1988) in the following way. One can interpret their model as having a single consumer with random tastes described by a uniform distribution over the unit square. Firms choose their prices before the uncertainty about the consumer's tastes resolves. By contrast, in Proposition 15, firms choose their prices after they learn the consumer's tastes.[142]

[142] Joe Farrell suggested this interpretation.

Although Propositions 14 and 15 demonstrate an interesting relationship between the uncertain costs and horizontal differentiation models, the differentiation model has neither a variety of consumers nor a love of variety by the single consumer. A model in which many consumers have varying tastes would also avoid the unattractive feature that only one firm makes positive sales in a given market.

The following proposition has more consumers than firms, but revenue equivalence with the uncertain costs model does not hold. It allows for a general distribution of consumer tastes and elastic demand, but it requires that the distribution of firms' characteristics be discrete. Essentially, the proposition below argues that it is easier for a firm to draw unique system characteristics than to draw a unique component characteristic, and uniqueness is all that matters when n is large and the distribution of characteristics is discrete.

Proposition 16. Suppose that consumers with any demand function $q(p)$ are distributed over the unit square according to any joint distribution function, and that firms independently draw their characteristics in each component from discrete distribution F. For any metric and transportation cost function, such that equilibrium exists in any outcome, expected profits are lower under compatibility when there are sufficiently many firms.

Proof. There are a finite number of possible draws for firms' characteristics (a,b), which form a grid by the assumption of independence. Consider the profit of firm 1 when it draws characteristics (a_1,b_1), which has probability $f(a_1)f(b_1)$. Under incompatibility, firm 1 makes a profit only if there are not other firm at that grid point. The probability that firm 1 is alone at that point is $f(a_1)f(b_1)[1-f(a_1)f(b_1)]^{n-1}$. Consider firm 1's expected profit in two cases, conditional on firm 1 being alone at (a_1,b_1). In the first case, all grid points but (a_1,b_1) are occupied by at least two firms. Let $P^I(n)$ be the probability that this is the case, conditional on firm 1 being alone at (a_1,b_1). At the other grid points with two or more firms, the firms at that point set a price of zero, because they sell homogeneous products. This is true for any metric and transportation cost function. When all other firms set a price of zero and firm 1 is alone, write firm 1's profit as γ. Pulling all the terms together, firm 1's expected profit from being alone at (a_1,b_1) when all other grid points are occupied by at least two firms is $\gamma P^I(n)f(a_1)f(b_1)[1-f(a_1)f(b_1)]^{n-1}$.

In the other case, at least one other grid point is occupied by less than two firms. Let the greatest profit firm 1 can earn in this case be γ_0. Firm 1's expected profit from this case is at most $\gamma_0[1-P^I(n)]f(a_1)f(b_1)[1-f(a_1)f(b_1)]^{n-1}$. As n grows large, the probability that all other grid points have at least two firms goes to one

$$\lim_{n\to\infty} p^I(n) = 1. \tag{83}$$

Because γ and γ_0 do not depend on n, profit in the second case are of lower order than profit in the first case. Therefore, firm 1's expected profit under incompatibility is of the order of the expression for expected profit in the first case, that is,

$$\pi_1^I \times \gamma P^I(n)f(a_1)f(b_1)[1-f(a_1)f(b_1)]^{n-1}. \tag{84}$$

Under compatibility, firm 1 makes a profit in component A only if no other firm draws characteristic a_1. The probability that firm 1 uniquely draws a_1 is $f(a_1)[1-f(a_1)]^{n-1}$. As before, consider profit in two cases. In the first case, at least two firms draw any other component A characteristic. Let $P^C(n)$ be the probability that this happens, conditional on firm 1 uniquely drawing a_1. Let ι be the greatest profit that firm 1 can make in this case. Then, firm 1's expected profit in this case is $\iota P^C(n)f(a_1)[1-f(a_1)]^{n-1}$. In a second case, at least one other grid point has less than two firms. Let the greatest profit that firm 1 can earn in this case be ι_0. Firm 1's expected profit in this case is at most $\iota_0[1-P^C(n)]f(a_1)[1-f(a_1)]^{n-1}$. As n grows large, $P^C(n)$ goes to one, so profit in the first case are of higher order than profits in the second case. Firm 1's expected profit is therefore of order of the expression for expected profit in the first case. Without loss of generality, let A be the component which produces greater profit. Then, expected profit in both components is of order of at most twice the expected profits in the first case, that is

$$\pi_1^C \times 2\iota P^C(n)f(a_1)[1-f(a_1)]^{n-1}. \tag{85}$$

The ratio of firm 1's expected profits in the two regimes is thus

$$\frac{\pi_1^I}{\pi_1^C} \times \frac{\gamma P^I(n)f(a_1)f(b_1)[1-f(a_1)f(b_1)]^{n-1}}{2\iota P^C(n)f(a_1)[1-f(a_1)]^{n-1}} \times \frac{P^I(n)[1-f(a_1)f(b_1)]^{n-1}}{P^C(n)[1-f(a_1)]^{n-1}}. \tag{86}$$

The last expression in (86) cancels out terms which are not functions of n, by (34). As n grows large,

$$\frac{P^I(n)}{P^C(n)} \times 1. \tag{87}$$

Therefore,

$$\frac{\pi_1^I}{\pi_1^C} \times \frac{[1-f(a_1)f(b_1)]^{n-1}}{[1-f(a_1)]^{n-1}}. \tag{88}$$

Because $f(a_1)f(b_1)<f(a_1)$, expected profit under incompatibility is of higher order than expected profit under compatibility. *QED.*

Remarks. The argument in this proof works with price competition in many settings. In short, it is easier for a firm to be unique under incompatibility, and being unique is all that matters when there are many firms.

In the uncertain costs model, the result that many firms prefer incompatibility to compatibility depended on the assumption of independence between component costs. The theorems of this section have assumed that a firm's two component characteristics are not correlated as well. By contrast, Economides (1989) implicitly assumes in effect that firms' characteristics are perfectly correlated, in placing them along the diagonal. Correlation in component characteristics in the horizontal differentiation model has a similar effect to correlation in component costs in the uncertain costs model. In both models, correlation of the right kind reverses the result that many firms prefer incompatibility to compatibility.

This section has shown that many firms prefer incompatibility with various combinations of distributions of consumers and of firms. It is an open question whether this result remains valid with any distributions of firms and consumers. Such a model is inherently more difficult to solve. The models in this section have been nonstrategic in the sense that at most one firm makes positive sales in the outcomes of interest, and any other firm sets a price of zero.

8 Bertrand Competition and Auctions

The uncertain costs model makes the strong assumption that firms knows each others' costs exactly. Firms rarely know each other's costs exactly in practice, so relaxing that assumption would make the model more realistic. At first glance, relaxing the assumption that firms know each others costs perfectly complicates the analysis of the model considerably. However, results from the theory of auctions allow us to introduce imperfect information into the uncertain costs model with little further work, when demand is inelastic.

In the uncertain costs model under complete incompatibility, Bertrand firms compete in selling homogeneous systems to a single consumer with inelastic demand for one unit. The firm with the lowest system cost is the only one to make a sale, and its payoff is the difference between its cost and the second-lowest cost. These payoffs are identical to those in a *second-price auction* in which firms know each other's bids. In such an auction, the n firms simultaneously announce the price at which they can supply a system. The consumer buys from the firm with the lowest bid. However, in a second-price auction, the price the consumer pays that firm is the second-lowest bid, and not the lowest bid.

It might appear that this can only hurt the consumer, which is true if he knows suppliers' costs. Suppose, however, that he does not, but the firms know their own and each others' costs. Then, it is a dominant strategy for a firm to make a bid that is equal to its cost. In other words, no matter what bidding strategy other firms choose, a firm will make a bid equal to its cost, by the following argument. A firm's bid does not affect its profit when it has the winning bid, but only affects whether it wins. A bid above its cost would prevent it from winning sometimes even when it would make profit from doing so. On the other hand, a bid below its cost would only cause it to win sometimes when it might be worse off as a result. Therefore, a bid different from its true cost can only make it worse off. If firms bid their costs, then the lowest cost firm wins, and makes a profit equal to the difference between the second-lowest cost and the lowest cost.

Thus, Bertrand competition and a second-price auction under complete incompatibility yield profits and prices that are identical. This is true for any given costs, so the two mechanisms are *ex post* revenue equivalent.

The two mechanisms are also revenue equivalent under complete compatibility, if the consumer's reservation price is high. Define a second-price auction under compatibility as two separate second-price auctions, one in each component. Then, the argument made above about the market for systems under incompatibility applies to the market for each component under complete compatibility, because the assumption of inelastic demand and a high reservation price guarantees that the component markets can be solved separately.

Proposition 17. In the uncertain costs model, suppose that demand is inelastic, the consumer's reservation price is high, and firms' costs are common knowledge. Under complete incompatibility, Bertrand competition is *ex post* revenue equivalent to a second-price auction. Under complete compatibility, Bertrand competition in each component is *ex post* revenue equivalent to a second-price auction in each component.

Proof. As presented above. *QED.*

Remarks. Although Bertrand competition and a second-price auction are *ex post* revenue equivalent, they are not identical games. One difference is that Bertrand competitors do not have dominant strategies. For instance, if the second-lowest cost firm raises its price above cost, the lowest-cost firm will raise its price in response. However, *ex post* revenue equivalence means that the two settings have the same equilibria, so the results of this chapter apply to a second-price auction as well as Bertrand competition.

The revenue equivalence of Bertrand competition to a second-price auction is significant, because the Revenue Equivalence Theorem (RET) from auction theory shows that many types of auctions yield the same expected revenue as the second-price auction in the right conditions. Say that two models are *ex ante revenue equivalent* if they give the consumer and each firm the same expected surplus before the resolution of uncertainty. *Ex post* revenue equivalence is a stronger condition, because it holds for all outcomes, while *ex ante* revenue equivalence holds only in expectation. The Revenue Equivalence Theorem proves that auctions with a certain type of imperfect information are *ex ante* revenue equivalent.

In these auctions, firms independently draw their costs from a random distribution G, which is common knowledge. A firm knows its own cost, but it cannot directly observe its competitors' costs. Such an auction is known as an *independent, private values* auction. It is straightforward to solve a second-price auction with independent and private values, even if a firm cannot observe the other firms' bids, i.e. bids are sealed. Because firms' have dominant strategies in a second-price auction, their bidding strategies are unaltered if they do not know other firms' costs or bids. Then, in a second-price sealed bids auction with independent, private values, firms' bids and therefore their expected profits are the same as in a second-price auction with perfect information.

Revenue Equivalence Theorem (RET). Suppose that n risk-neutral symmetric firms compete to supply a consumer who has inelastic demand for one unit of a good, and that the firm make independent draws for their costs from a continuous distribution function G, which is common knowledge. Consider any auction mechanism, such that (1) the mechanism is efficient in equilibrium, that is, it always awards the sale to the firm with the lowest cost, (2) a firm drawing the highest possible cost has zero profit, (3) firms' payoffs depend only upon their bids, and (4) firms have private values, that is, they known their own cost but only the distribution G of other firms' costs. Then, the expected profits of each firm and the expected surplus of the consumer are the same under any such mechanism.

Proof. Milgrom (1989) proves the theorem as follows. A firm with cost c and bid p has profit $\pi(c)=(p-c)P(p)$, where $P(p)$ is the probability that it wins. If the firm chooses the price that it receives and the probability that it wins to maximize its profit, then by the envelope theorem, $\pi'=-P(p)$. Because the mechanism is efficient, $P(p)=[1-G(c)]^{n-1}$. The firm's expected profit is

$$\begin{aligned} E\pi(c) &= \int \pi dG \\ &= G\pi\Big|_{\underline{c}}^{\bar{c}} - \int G\pi' dc \\ &= \int (1-G)^{n-1} G dc \end{aligned} \tag{89}$$

integrating by parts and using the fact that $\pi(\bar{c})=0$ by assumption. Any auction mechanism satisfying the assumptions gives expected profit as described by this formula. The consumer's expected surplus, which is its reservation price minus the expected industry surplus, is therefore also the same for any such auction mechanism. Thus, auctions satisfying the assumptions are *ex ante* revenue equivalent. *QED*.

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Remarks. To summarize the results so far, under complete incompatibility, perfect information Bertrand competition is *ex post* revenue equivalent to a second-price auction with perfect information, which is *ex post* revenue equivalent a second-price auction with private values. Furthermore, an independent, private values second-price auction satisfies the assumptions of the RET since it always awards the sale to the lowest bidder and gives a high-cost firm zero profit. Therefore, perfect information Bertrand competition under complete incompatibility is *ex ante* revenue equivalent to any auction satisfying the assumptions of the RET. Under completely compatibility, this equivalence between Bertrand competition and auctions satisfying the assumptions of the RET holds in the market for each component, when the consumer's reservation price is high.[143]

The literature has shown that the following auctions satisfy the assumptions of the RET, when values are private and independent (see MacAfee and McMillan (1987)). In an English auction, bidders successively lower the price until one bidder remains. In a Dutch auction, the auctioneer raises the price from a low level until one bidder accepts the price. In a first-price sealed bid auction, bidders simultaneously announce their bids, the lowest bidder receives the price that it bids, and bidders cannot observe each other's bids. The first-price and second-price sealed bids auctions differ only in whether the lowest bidders pays its own bid or the second-lowest bid.

A central question in auction theory is to specify which auction mechanism maximizes the expected surplus of the buyer, on the assumption that the buyer can impose the mechanism he chooses. Similarly, a buyer conducting an auction might be able to impose compatibility or incompatibility on firms. For instance, if the buyer only takes bids on systems, and does not allow firms to cooperate in making bids, the firms are completely incompatible. On the other hand, if the buyer only accepts bids for

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[143] When there is intermediate compatibility or a low consumer reservation price, several problems arise. First, the market for each component cannot be solved separately, as discussed previously. Second, when there is intermediate compatibility with groups of different sizes, then firms are asymmetric. Myerson (1981) shows that auctions with asymmetric bidders are not necessarily efficient. An inefficient auction does not satisfy the assumptions of the RET.

components which can be mixed-and-matched across firms, then the firms are completely compatible.

The uncertain costs model combined with the equivalences developed in this section have implications for whether the buyer should impose compatibility or incompatibility. By Proposition 1, imposing incompatibility when there are two bidders lowers the expected winning bid. On the other hand, Proposition 3 implies that, when there are sufficiently many bidders, imposing compatibility lowers the expected winning bid. Because welfare is higher under compatibility, the buyer has a bias towards incompatibility when the number of bidders is small.

These recommendations do not apply only when a buyer is acquiring systems made up of components. The "components" A and B could be completely unrelated products which happen to be supplied by the same bidders. Then, the above recommendations are valid if each bidder can supply both products, and the buyer's reservation price for each product exceeds the highest cost that any firm might have.

The Revenue Equivalence Theorem is sensitive to the assumption of inelastic demand. Hansen (1988) shows that when the buyer has an elastic demand function and there is imperfect information about costs, the first-price auction gives lower expected bids than the second-price auction. The reason is that bidders in the first-price auction have the additional incentive to lower their bid in order to increase demand. This effect is not present in a second-price auction, because another firm's bid determines the price.

9 Mix-and-Match and Entry

Analyses of the computer industry often suggest that more firms enter under compatibility, because a "niche" entrant needs only to make one component, and imply that higher entry is better. For instance, Fisher *et al.* (1983) argue that niche entry in the computer industry is more likely than entry with a full system. Similarly, Brock (1975) states that it is harder to make a full computer system than to make one component of a system.

Although compatibility may lower the cost of entry, Proposition 3 shows that it lowers the benefits of entry as well when there are sufficiently many firms. As a result, the effect of a shift from complete incompatibility to complete compatibility on entry is ambiguous.

Example. Suppose that each firm has a probability of success of $e=0.8$ with a two-point distribution and inelastic demand with a high reservation price. In the cases of $n=2$ and $n=3$, profits are $\pi^C(2)=0.32$, $\pi^I(2)\approx0.27$, $\pi^C(3)=0.064$, and $\pi^I(3)\approx0.084$, using the formulae developed on page 111 above. If the fixed cost of entry lies between 0.27 and 0.32, then two firms enter under compatibility and one firm enters under incompatibility. On the other hand, if the entry cost lies between 0.064 and 0.084, three firms enter under incompatibility and two firms enter under compatibility. Thus, the effect of compatibility on entry is ambiguous.

When the entry cost is sufficiently low, then entry is higher under incompatibility than under compatibility, because profits are higher under incompatibility for given n. A question for further research is whether incompatibility can attract enough additional firms that prices and costs are lower than under compatibility.

10 Vertical Relationships and Multimarket Contact

The section presents two more interpretations of the uncertain costs model, in terms of the exclusivity of vertical relationships and the effect of multimarket contact.

It is possible to interpret components A and B as representing successive stages in the production of a final good.[144] Then, incompatibility implies that each upstream firm sell only to one downstream firm, and the downstream firm buys only from that upstream firm. However, under compatibility, a downstream firm can make purchases from any upstream firm, and relationships are said to be "open." Under this interpretation, Proposition 1 says that duopolists prefer open relationships to exclusive relationships. On the other hand, Proposition 3 says that an industry with sufficiently many firms prefers exclusive relationships. However, Proposition 5 (iv) shows that the benefits of joining a group of firms with open relationships grow with the size of that group. Thus, there can be a bandwagon towards open relationships.

By contrast, in Hart and Tirole's (1990) model of vertical integration, integration by one firm can lead to vertical integration by another. However, vertical integration between an upstream and downstream firm does not necessarily imply that they have an exclusive relationship, or vice versa.

Compatibility also implies that firms interact in many component markets rather than one market for systems. Bernheim and Whinston (1990) show that contact between two firms in many possibly unrelated markets can facilitate collusive behavior. By contrast, Proposition 1 above shows that multimarket contact (compatibility) among

[144] This interpretation is due to Joe Farrell.

duopolists raises their profits, given that they do not collude. On the other hand, when there are sufficiently many firms, Proposition 3 shows that multimarket contact lowers their profits, given that they do not collude. In both cases, multimarket contact increases total surplus, and therefore increases the potential benefits from collusion.

11 Robustness of the Results

The literature has shown that an industry with symmetric firms prefer compatibility to incompatibility in a variety of settings, if the demand for hybrids is not too low. This result holds in duopoly with horizontal differentiation (Matutes and Regibeau (1988)) or with vertical differentiation (Einhorn (1992)), and in oligopoly with horizontal differentiation (Economides (1989)). By contrast, this chapter has shown that sufficiently many firms which are asymmetric *ex post* as a result of uncertainty prefer incompatibility *ex ante*. This result is valid in the same variety of settings, that is, if firms draw their component costs, qualities, or characteristics from discrete, independent distributions.

The conclusion that compatibility raises industry profits appears to arise from assumption of either duopoly or symmetry. This chapter reaches a different conclusion by relaxing both assumptions simultaneously. Economides (1991) relaxes only the assumption of symmetry, and shows that a duopolist with higher demand than its competitor sometimes prefers incompatibility.

Chapter 3 introduces a role for cost-reducing effort into the uncertain costs model. Again, it is found in one case that a duopolist with a lower cost of effort sometimes prefers incompatibility. Chapter 4 identifies another exception to the conclusion that duopolists prefer compatibility, when consumers combine components into systems with variable coefficients. There, a dominant firm's component prices are more tightly constrained under compatibility, so its profits are lower. Thus, it appears that the result that many firms prefer incompatibility is more robust than the result that duopolists prefer compatibility.

Chapter 3: Mix-and-Match Compatibility and R&D Effort

1 Introduction

This chapter introduces a role for cost-reducing effort into the uncertain costs model of Chapter 2. It asks how a shift from a regime of incompatibility to a regime of compatibility affects the private return to R&D effort. It also compares private and social R&D incentives in both regimes.

Many of the results below follow directly from those of the previous chapter. First, it is shown there that the private and social returns to entry coincide in some conditions, namely when the entrant is incompatible with all other firms, or when all firms are compatible and the consumer's reservation price is high. In the same conditions, the private and social return to R&D coincide as well.

Second, the previous chapter establishes that duopolists' profits are higher under compatibility than under incompatibility when their costs are complementary, that is, when neither firm has the lowest cost in both components. Similarly, we find below that compatibility increases the return to R&D effort precisely when effort increases the probability that costs are complementary. As a result, duopolists choose levels of cost-reducing effort that are closer together under compatibility than under incompatibility to increase the chances of having complementary costs.

Third, the previous chapter's showed that in the uncertain costs model, expected prices are lower under compatibility when there are sufficiently many firms. If demand is elastic, lower prices increase sales, and therefore increase the return to lowering unit costs as well.

On the other hand, because compatibility itself lowers costs, it may serve as a substitute for R&D, if it is harder to reduce costs when costs are already low. This effect reduces R&D effort under compatibility, but costs are still lower under compatibility. In addition, compatibility allows firms to specialize completely in making one component. Then, because of double marginalization, prices are too high and output is too low from an industry perspective, which implies that R&D incentives are lower as well.

Section 2 specifies the model's assumptions. Section 3 shows that private and social returns to R&D coincide in certain conditions. Section 4 analyzes the case in which the relationship between effort and cost is deterministic, and Section 5 the case in which costs are uncertain given the effort level.

2 The Model

The model is identical to the uncertain costs model of Chapter 2, but with the additional feature that firms can alter their costs by exerting R&D effort. We present the assumptions again for convenience. Suppose that a representative consumer has inelastic demand with reservation price R for one unit of a system. At some points the assumption of inelastic demand is relaxed, but demand is assumed to be inelastic unless otherwise stated. A system consists of two components A and B, such as a computer and a screen, combined in fixed proportions. Measure quantities of components so that a system requires exactly one component of each type. A component has no value except in combination with exactly one component of the other type. Each of the n firms makes both components at constant unit costs, which may vary across firms and components. If two firms are compatible, then the consumer can mix-and-match their components into a system. Components are homogeneous, excepting considerations of compatibility. Compatibility is an equivalence relation.

Let a_i and b_i be the unit costs of firm i for components A and B, let c_i be its system cost, where $c_i=a_i+b_i$. Let c be the industry cost, that is, the cost of the system that the consumer buys. Similarly, let p_i^A and p_i^B be the component prices of firm i, let p_i be the system price, $p_i=p_i^A+p_i^B$, and let p be the industry price.

The model three stages. In stage one, firms decide which standards to adopt. In the second stage, firms choose their R&D effort. In the third and final stage, firms compete in prices. The solution concept for the game is sub-game perfect Nash equilibrium.

To find sub-game perfect equilibria, the game is solved backwards. Price competition in the third stage is solved given all possible choices of standards and effort levels that might have been chosen in the first and second stages. When different firms charge the same price, assume that all sales go to the lower-cost firm.

Standard choice in the first stage takes place much as in the previous chapter, but receive less attention in this chapter.

In the second stage, firms choose their R&D effort based on the profits they expect to earn in the third period, taking as given their compatibility decisions made in the first period. Unlike the previous chapter, firms here may be asymmetric unless otherwise stated. More specifically, firms may be asymmetric even when both choose zero effort, and they also may face different R&D opportunities. A firm choosing effort level e has costs in each component governed by the distribution function $F(c,e)$ which may vary across firms and components. Any uncertainty about costs resolves after firms choose their effort and before price competition in the third period. In Section 4, the F distributions are degenerate and the relationship between cost and effort is deterministic. In Section 5, the F distributions are not degenerate, and the same F applies to all firms and components. The analysis below specifies various ways in which effort e affects the distribution of costs F. In either case, we normalize so that additional effort e_0 lowers a firm's expected cost by e_0.

The cost of effort is independent across firms and components. The cost of effort is specified as necessary below, but for the most part the analysis focusses on benefits of effort rather than the cost of effort. Statements about profits and the marginal benefit of additional effort refer to benefits exclusive of R&D costs.

In the third stage, firms take their effort levels and compatibility decisions as given, so the results of the previous chapter apply. Firms and consumers are risk-neutral.

3 Social Optimality of R&D Incentives

The previous chapter shows that in some settings, an entrant's profit is equal to the amount by which it lowers the industry cost.[145] Then, entry incentives are socially optimal. In those cases, firms' R&D incentives are socially optimal as well. This is true

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[145] Recall that the *industry cost* is the cost of the system which the consumer buys.

whether or not firms are symmetric, and whether or not the relationship between effort and cost is deterministic or not.

Proposition 18. An incompatible firm's R&D incentives are socially optimal. Under completely compatibility, if the consumer's reservation price is sufficiently high, then each firm's R&D incentives are socially optimal. Otherwise, private R&D incentives are suboptimal in the uncertain costs model when there is intermediate compatibility and when there is complete compatibility and *R* is low.

Proof. For given costs, an incompatible firm's benefit from R&D is equal to the amount by which it lowers industry costs,[146] so its incentives are optimal. Under completely compatibility with a sufficiently high reservation price, markets can be solved separately, and in each market a firm's benefit from R&D is equal to the amount by which it lowers industry costs. The same result carries over into the uncertain costs model. With intermediate compatibility or a low reservation price, the private benefit of R&D is at best optimal, and on average is therefore suboptimal, as argued in the proof of Proposition 4. *QED.*

Remarks. It may seem surprising that R&D incentives are socially optimal, given that R&D by a firm can hurt its competitors. However, in the conditions outlined by the proposition, consumer surplus increases exactly by the amount that competitors' profits fall. In the terminology of Mankiw and Whinston (1986), the nonappropriability and business-stealing effects are equal.

The remainder of the chapter assumes that *R* is high and that firms are either completely compatible or completely incompatible. Otherwise, the price game does not have a unique equilibrium, and the effect of R&D on profits depends upon the choice

[146] Recall that the industry cost is the cost of the system which the consumer buys.

of equilibrium. Assuming that firms are symmetric and applying a symmetric sharing rule does not solve this problem as it did in the last chapter, because we must compute profits when firms make asymmetric effort choices.

With the above assumptions, additional effort by a firm does not alter the sum of consumer surplus plus other firms' profits, so the quantity

$$CS + \sum_{i \neq j} \pi_j \tag{91}$$

is constant, if demand is inelastic.

4 Deterministic R&D Effort

This section examines R&D effort when the relationship between R&D effort and cost is deterministic, that is, there is no uncertainty in a firm's costs given its choice of effort. This can be seen as a special case of the uncertain costs model in which firms draw their costs from a distribution with all probability weight concentrated at a single point.

Section 4.1 shows that symmetric firms may specialize in their R&D efforts under compatibility, that is, a firm may choose a high R&D effort in one component and a low effort in the other component. Section 4.2 finds that R&D incentives may be higher or lower under compatibility than under incompatibility. Section 4.3 analyzes the case with elastic demand.

4.1 Specialization

In the previous chapter, compatibility sometimes leads firms to specialize in production, that is, produce more of one component than the other. There, initially symmetric firms can become asymmetric because of randomness in their costs. In the example below, compatibility can lead firms to exert more research effort in one component than the other. Here, the relation between effort and cost is deterministic, and firms specialize by choice rather than by chance.

Example. Suppose that symmetric Bertrand duopolists start out with identical costs (1,1) when they make no effort. By exerting a unit of effort at a small cost ϵ, a firm can lower its costs in any component from one to zero. Under incompatibility, exactly one firm will exert positive effort in both components; the other firm has nothing to gain by lowering costs to equal its competitors'. There are two pure-strategy equilibria, in which one firm exerts effort and the other does not.[147]

Under compatibility, exactly one firm exerts effort in a market as before, but there is now one market for each component. There are four pure-strategy equilibria, two in which one firm does all the research, and two in which each firm does research in one component. In this example, compatibility creates new equilibria with specialization. This result occurs because equilibria in any one market are asymmetric;

[147] There is also a solution in mixed strategies in this example. Suppose firm i exerts effort one in both components with probability p_i and effort zero with probability $1-p_i$. Then, firm 1's payoff in both components is $2p_1(1-p_2\text{-}\epsilon)$. Then, the only interior solution to the first-order conditions occur when $p_1=p_2=1-\epsilon$. However, the expected payoff of each firm is zero in this equilibrium, so we focus on pure-strategy equilibria in the remainder of this section.

one firm exerts more effort than the other. If equilibria were symmetric, there would be no specialization.

We now show that equilibria for effort more generally are similar to this example. First, note that at most one firm exerts positive effort in a market in a pure-strategy equilibrium. In price competition, at most one firm can earn a profit in the market for systems under incompatibility or in the market for a component under compatibility.

Describe a pure-strategy effort equilibrium in duopoly by the vector $(e_1^A, e_1^B, e_2^A, e_2^B)$.

Proposition 19. Suppose that the relationship between effort and cost is deterministic, firms are symmetric prior to choice of effort, the outcomes of R&D projects in different components are independent, and R is high. Then, there are two pure-strategy equilibria of the form $(e^A, e^B, 0, 0)$ and $(0, 0, e^A, e^B)$ under incompatibility, and there are two additional pure-strategy equilibria $(e^A, 0, 0, e^B)$ and $(0, e^A, e^B, 0)$ under compatibility. In particular, compatibility and specialization do not affect the total amount of R&D performed.

Proof. Effort in component B has two potential effects on effort in component A. First, it can directly affect costs in component A. Second, it can affect the payoff to a given outcome in component A. The assumption of independent R&D projects eliminates the first effect. The assumption that R is high means that markets can be solved separately, which eliminates the second effect. *QED.*

Remarks. In the example above, specialization does not affect the total amount of R&D performed. The reason for this is that the marginal benefit of effort in one component does not depend on effort exerted in the other component. This phenomenon recalls the market separation property discussed in the previous chapter, which described the feature that the price games in the two components could be solved separately. The two features are closely related.

So far, we have assumed inelastic demand with a high reservation price, which means that markets can be solved separately under compatibility. When demand is elastic, the price in one market affects demand in the other. When firms specialize, double marginalization can raises prices, and lower output and industry profits. Then, the return to effort is lower when the result is specialization, as seen in the following example.

Example. Take the previous example, but suppose that demand is $q=2-p$. If firm 1 exerts effort in both components, then it has costs (0,0) while firm 2 has costs (1,1). Firm 1 maximizes profit $p(2-p)$, so it sets a system price of one and has profit of $1-2\epsilon$. Now suppose that each firm exerts R&D effort in one component, so costs for firms 1 and 2 are (0,1) and (1,0). In the price game, firm 1 maximizes profit $p_1^A(2-p_1^A-p_2^B)$. The first-order condition for firm 1 is $2-2p_1^A-p_2^B=0$, so in a symmetric equilibrium each firm sets a price of ⅔ in its low-cost component. The system price is 4/3, and industry profits are $8/9-2\epsilon$, so the firms fail to maximize joint profits. Component markets are no longer independent, and a specialized firm ignores the negative externality on the other firm of raising its price. Because output is lower, the private return to reducing unit costs is lower when firms specialize.

Remarks. The previous proposition shows that symmetric firms may choose to specialize. Of course, asymmetric firms will have even more reason to specialize by devoting their R&D effort to the component in which they have a lower cost of effort.

Example. Suppose that Bertrand duopolists have costs (2,2) when they make no effort, and that demand is again inelastic. By exerting effort ϵ in component A, firm 1 can lower its cost in that component from two to zero. By exerting effort ϵ in component B, firm 1 can lower its cost in that component from two to one. Thus, firm 1 is more effective at R&D in component A. Firm 2 is more effective at R&D in component B; effort in that component lowers its cost from two to zero, while effort in component A lowers costs from two to one.

Under incompatibility, exactly one firm exerts positive effort in both components. Its cost is one and its profit is $(4-1)-2\epsilon$. Under compatibility, in the unique equilibrium, firm 1 exerts effort in component A and firm 2 in component B, and each firm has a profit of $(2-0)-\epsilon$. The industry cost, zero, is lower under compatibility, and industry profits, $4-2\epsilon$, are higher. However, the firm that exerts effort under incompatibility has lower profit under compatibility.

Remarks. Compatibility also affects the direction in which firms wish to move their costs slightly, as shown in Figure 12. Under incompatibility the low-cost firm wishes to move its costs down and to the left. The same is true of a dominant firm under compatibility. However, if neither firm is dominant, a firm only does research in the component in which it has the lowest cost, unless it can catch up and surpass the low-cost firm. Compatibility allows firms to specialize in R&D, so they develop complementary costs, which in turn encourages them to remain compatible. In this way, a regime of compatibility might be self-perpetuating. •

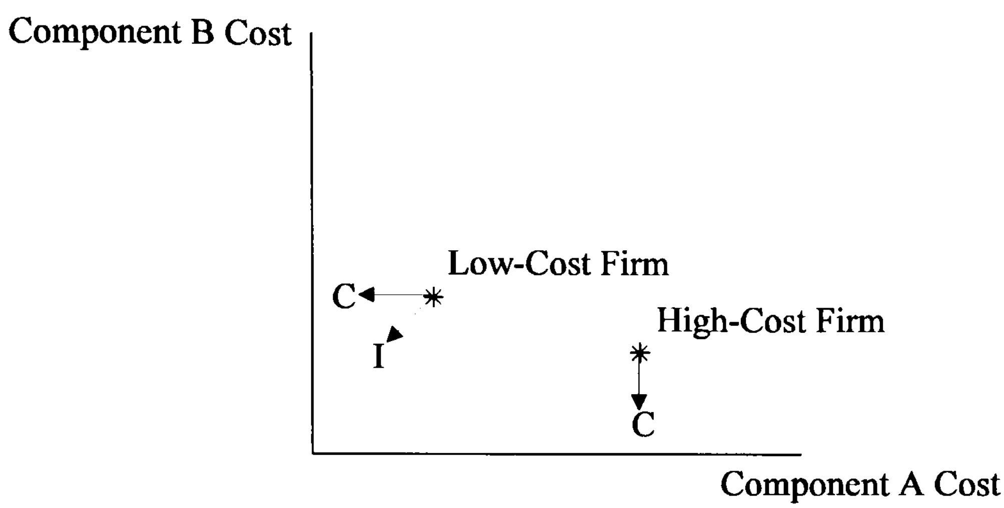

Figure 12: Firms' Preferred Directions for Cost Reduction

4.2 Compatibility and the Level of R&D

Compatibility and R&D both lower costs. If it is harder to reduce a low cost than to reduce a high cost, then compatibility and R&D are substitutes, in that more of one leads to less of the other.

Example. Let a firm's cost of effort in a component when it starts with cost c be $C(e,c)$, such that $C_e \geq 0$, $C_{ee} > 0$, and because it is harder to reduce a low cost, $C_{ec} < 0$. Suppose firms 1 and 2 have complementary costs (a_1,b_1) and (a_2,b_2), and that the low cost firm is the only one to perform R&D. Assuming the first-order approach is valid, firm 1 sets $\pi'(e_1^A) = C_e(e_1^A, a_1)$ in either regime, where $\pi'(e_1^A) = 1$ by the normalization of effort units. Under incompatibility, firm 1 sets $1 = C_e(e_1^B, b_1)$. Under compatibility, firm 2 has the lower cost in component B, so it sets $1 = C_e(e_2^B, b_2)$. Because $b_2 < b_1$ by the assumption of complementary costs, then $e_2^B > e_1^B$. Thus, R&D effort is higher in component B under incompatibility.

The effect of compatibility on R&D incentives is ambiguous when effort to lower one component's cost affects the other component's cost.

Example. Suppose that firm j has costs (1,3) for any level of effort it chooses. Suppose that firm i has three possible effort levels, labelled I, II, and III, which produce costs (3,3), (2,2) and (1,1) respectively. Under compatibility, firm i has profits zero, one, and two for effort levels I, II, and III. Under incompatibility, firm i has profits zero, zero, and two for effort levels I, II, and III. Firm i's marginal incentive to lower costs from I to II is higher under compatibility (one versus zero). However, its marginal incentive to lower costs from II to III is higher under incompatibility (two versus one).

Remarks. The explanation of these results is straightforward. When firm i chooses effort II, compatibility raises its profit because the costs of the two firms are complementary. On the other hand, compatibility does not affect profits at effort levels I and III. Increasing effort from I to II makes costs complementary, so R&D incentives are higher under compatibility. After an increase in effort from II to III, costs are no longer complementary, so R&D incentives are lower under compatibility.

4.3 R&D Effort with Elastic Demand

Now, relax the assumption the demand is inelastic for a moment. Suppose that the demand curve for systems, $q(p)$, is differentiable and decreasing, $q' \geq 0$, and suppose that costs constrain prices.

We begin with the duopoly case. Recall that firm 1 has lower costs under incompatibility by assumption. Under incompatibility, firm 1's profit is $[(a_2+b_2)-(a_1+b_1)]q(a_2+b_2)$ and firm 2's profit is zero. The marginal benefit to lowering costs for firm 1 is $q(a_2+b_2)$ in each component. Firm 2 has nothing to gain from a marginal reduction in its costs, although a large reduction might yield a positive return if firm 2 can "leapfrog" firm 1, that is, achieve a lower cost than firm 1.

If firm 1's costs are lower in both components, then outcome of the price game under compatibility is the same as under incompatibility. Suppose that firm 1's costs are lower only in component A. The hybrid system with costs (a_1,b_2) will be the only one sold under compatibility, and firms will charge the highest prices possible, that is, (a_2,b_1), by the assumption that costs constrain prices. Under compatibility, firm 1's profit will rise to $(a_2-a_1)q(a_2+b_1)$ and firm 2's profit becomes $(b_1-b_2)q(a_2+b_1)$. Firm 1's marginal benefit of lowering a_1 is $q(a_2+b_1)$, which equals the social benefit of lowering a_1. However, the private benefit of R&D is less than under incompatibility at the original costs, because the price is now higher and quantity is lower: $q(a_2+b_1)<q(a_2+b_2)$.

Firm 1 also has benefit of $(a_2-a_1)(-q'(a_2+b_1))$ to lowering the cost b_1 of its high-cost component. Lowering b_1 reduces the price that firm 2 can charge for its component B, which increases demand for firm 1's component A. Firm 1's marginal benefit to lowering b_1 under compatibility could be higher or lower than under incompatibility, depending on whether the elasticity of demand, pq'/q, is large or small. Firm 2 had no incentive to lower costs under incompatibility, but under compatibility it faces similar incentives to firm 1. The marginal benefit to lowering b_2 is $q(a_2+b_1)$ and the benefit to lowering a_2 is $(b_1-b_2)(-q'(a_2+b_1))$. In first case, effort increases unit profits on existing sales, and in the second case, effort increases sales holding unit profits constant.

Thus, the effect of compatibility on R&D incentives is ambiguous. The marginal return to R&D for firm 2 is clearly higher under compatibility, because it was zero under incompatibility. Firm 1's R&D incentives are unchanged if it has the lowest cost in both components. If it does not, and the elasticity of demand is large, then compatibility leads to higher effort by both firms. On the other hand, if the elasticity of demand is small, firm 1 has smaller R&D incentives in component B under compatibility.

When there are many firms, the results are not ambiguous in one case. Suppose that n firms draw their component costs from a random distribution F, and then choose effort levels with a deterministic relation between effort and cost.[148] The previous chapter showed that expected prices are then lower under compatibility. As a result, the effect of compatibility on R&D incentives is clearer.

Proposition 20. Suppose that the demand function for systems, $q(p)$, is decreasing, and that costs constrain prices. Suppose that firms draw their component costs from a random distribution which is independent across firms and components, and then choose their R&D effort, with a deterministic relation between effort and costs. Then, when there are sufficiently many firms, the expected marginal benefit to R&D effort by low-cost firms is higher under complete compatibility than under complete incompatibility.

Proof. Under incompatibility, call the firm with the lowest system cost firm i. If the second-lowest cost firm has cost p^I, firm i's marginal benefit to additional R&D effort in component A is $E[q(p^I)]$, as it was in the duopoly case. Under compatibility, there are two cases. In the first, the winning firm has the lowest-cost in component A. If the second-lowest cost system has cost p^C, the winning firm's expected marginal benefit to reducing its component A cost is $E[q(p_C)]$.

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[148] In Section 5 below, firms choose their effort and then draw their costs.

For n sufficiently large, p^C is FOSD-lower than p^I in a neighborhood of $\underline{c}$, as follows. The cost under compatibility c^C is FOSD-lower than the cost of c^I. By the proof of Proposition 3, the unit profit under compatibility, p^C-c^C, is FOSD-lower than the unit profit under incompatibility, p^I-c^I, in a neighborhood of $\underline{c}$. Therefore, p^C is FOSD-lower than p^I in a neighborhood of $\underline{c}$. Because q is decreasing, and p^C is FOSD-lower than p^I in the outcomes that matter, then $E[q(p_C)]>E[q(p_I)]$ for n sufficiently large.

This comparison ignores the incentive of a firm that has the lowest in component A and the second-lowest cost in component B to lower the cost of component B. However, this case occurs with probability $1/n$, so this term is of smaller order as n grows large. *QED.*

The following restores that assumption that demand is inelastic.

5 R&D Effort with Uncertain Outcomes

In practice, firms do not know the results of their R&D effort with certainty. In this section, a firm's component costs are uncertain given its effort level. Section 4 found that when the relation between effort and cost is deterministic, compatibility has an ambiguous effect on the level of R&D effort. Not surprisingly, the same is true when costs are uncertain, and as seen in the following example. •The section again ignores the possibility that effort levels vary across components, as in Section 4.1, to focus on other issues.

Example. Suppose that each of two firms draws its component costs from a two-point cost distribution such that its probability of having component cost zero instead of one is equal to its effort e_i.[149] If a firm draws a cost of zero in a component, say that it "succeeds." The cost distributions are independent across components. Let $\Delta\pi$, be the effect of compatibility on firm i's expected profits, that is, $\Delta\pi = \pi^C - \pi^I$.[150] When firm i chooses $e_i = 0$, it has the highest cost in each component with certainty, so $\Delta\pi = 0$. Similarly, when firm i chooses $e_i = 1$, it has the lowest cost in each component with certainty, and again $\Delta\pi = 0$. However, compatibility raises the expected profit of intermediate effort, because the firms have complementary costs with positive probability. Thus, without going any further, it is clear that compatibility will have an ambiguous effect on the marginal benefit of additional effort. More formally, firm i's expected profit under compatibility in both components is $\pi^C = 2e_i(1-e_j)$, and the marginal benefit of additional effort by firm i is $\pi^{C\prime} = 2(1-e_j)$.[151] Under incompatibility, firm i's expected profit is

$$\pi^I = 2e_i^2(1-e_j)^2 + e_i^2 2e_j(1-e_j) + 2e_i(1-e_i)(1-e_j)^2 \tag{92}$$

and the marginal benefit of additional effort for firm i is $\pi^{I\prime} = (1-e_j) - (1-2e_i)e_j(1-e_j)$. Then, the difference in firm i's expected profits between the two regimes is

$$\Delta\pi = \pi^C - \pi^I = 2e_i(1-e_i)e_j(1-e_j). \tag{93}$$

The right-hand side is the probability that the firms have complementary costs, i.e. the probability that firm i succeeds only in one component and firm j succeeds only in the

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149 In this example, a firm's expected cost is $1-e_i$. The derivative with respect to e_i is -1, so effort e reduces the firm's expected cost by e, so the normalization of effort units is satisfied.

150 We suppress the superscript indicating that the profits are for firm i for economy of notation.

151 Profits are measured exclusive of R&D costs.

other component. Therefore, the marginal benefit of additional effort will be higher under compatibility than incompatibility if and only if additional effort increases the probability that firms have complementary costs. Let $\Delta\pi' = \pi^{C\prime} - \pi^{I\prime}$ measure the effect of compatibility on marginal R&D incentives. Then $\Delta\pi' = 2(1-2e_i)e_j(1-e_j)$. The marginal benefit of additional effort by firm i is higher under compatibility if $e_i > ½$, and is lower if $e_i > ½$.

Based on this example, the following sections examine how various types of effort affect own profit, competitors' profits, and industry profits. They also consider how the number of firms and regime of compatibility alters the findings.

Section 5.1 considers the case in which a firm's R&D effort lower expected costs without changing the shape of its cost distribution function. Section 5.2 considers the case in which a firm's R&D effort does change the shape of its cost distribution.

5.1 Cost-Reducing R&D Effort

The example above suggested that the effect of compatibility on the level of R&D depends on whether effort increases the chances that cost are complementary. We now attempt to formalize this intuition.

Let there be a family of distributions $F(c,e)$ such that a firm choosing effort e draws its component costs from the distribution $F(c,e)$. Suppose that e is an index of first-order stochastic dominance (FOSD). In other words, $F(c,e) \geq F(c,e_0)$ for all c if $e > e_0$, so the probability of drawing a cost below any given level increases with effort. Assume that F_e exists, in which case $F_e > 0$. As a short-hand, write that "effort FOSD-reduces cost." In the two-point distribution example above, effort FOSD-reduces costs.

Units of effort are normalized such that additional effort e reduces a firm's expected costs by e, which implies

$$\frac{dE[c]}{de} = -\int F_e(c,e)dc = -1. \tag{94}$$

With these preliminaries, we can establish the following properties, which hold in either regime. No assumptions about symmetry among firms are made, but effort is the same across components by assumption.[152]

Proposition 21. In either regime, when effort FOSD-reduces costs, then the marginal benefit of effort to a firm is nonnegative (ignoring R&D costs). Effort has a negative externality on any other firm and raises expected consumer surplus and welfare. Choices of effort by duopolists are strategic substitutes.

Proof. Firm i's expected profit per component under compatibility, when markets can be solved separately and when other firms choose effort levels e_j, is[153]

$$\frac{\pi_i}{2} = \int \prod_{j \neq i} [1 - F(c,e_j)] F(c,e_i) dc. \tag{95}$$

To obtain the corresponding formula for incompatibility, substitute G for F, where G is the average of two draws from F. The marginal benefit of additional effort per component for firm i is

152 See Section 4.1 above for analysis of the case in which effort varies across components.

153 Note that $\prod$ represents iterated multiplication.

$$\tfrac{1}{2}\frac{\partial \pi_i}{\partial e_i} = \int \prod_{j \neq i} [1 - F(c,e_j)] F_e(c,e_i)\, dc \geq 0. \tag{96}$$

The expected profit per component of another firm, labelled firm j, is

$$\frac{\pi_j}{2} = \int \prod_{j \neq i} [1 - F(c,e_i)] F(c,e_j)\, dc, \tag{97}$$

and the effect of firm i's additional effort on firm j (per component) is

$$\tfrac{1}{2}\frac{\partial \pi_j}{\partial e_i} = \int -F_e(c,e_i) \prod_{k \neq i,j} [1 - F(c,e_k)] F(c,e_j)\, dc \leq 0. \tag{98}$$

In duopoly, the cross partial is

$$\frac{\partial^2 \pi_i}{\partial e_i \partial e_j} = 2\int -F_e(c,e_j) F_e(c,e_i)\, dc \leq 0 \tag{99}$$

so effort choices are strategic substitutes. By (91), increasing effort raises consumer surplus because it lowers other firms' expected profits. Increased effort by firm i raises its expected profit, so increasing effort raises welfare as well by Proposition 18. *QED.*

To get further results, it is necessary to restrict the way in which effort affects costs. In the following example, additional effort FOSD-reduces costs without changing the shape of the distribution.

Example. Suppose that a firm with effort e has a uniform distribution of costs on the interval $[1-e, 2-e]$. Suppose that firm 1 exerts greater effort than firm 2. When $e_1 - e_2 > 1$, firm 1 has lower costs in both components with certainty, so its profits are the same in both regimes. Suppose that $e_1 - e_2 < 1$, so the two firms have complementary costs with positive probability. Then, the supports of their cost distributions overlap, and the length of the overlap is $L=1-(e_1-e_2)$. By Proposition 1, firms' profits are higher under compatibility than under incompatibility precisely when they are complementary. It seems plausible that the marginal benefit of additional effort will also be higher under compatibility than incompatibility precisely when additional effort increases the probability that costs are complementary. Notice that higher effort by firm 2 increases L, and therefore increases the probability that the firms have complementary costs. However, higher effort by firm 1 reduces L, and lowers the probability that costs are complementary. If this intuition is correct, then compatibility increases the marginal benefit of additional effort by firm 2 and lowers the marginal benefit of additional effort by firm 1.

To formalize this intuition, consider a special type of FOSD, in which $F(c,e)$ has the form $F(c+e)$. Then, increasing effort does not change the shape of F. This formulation obeys the normalization of units of effort described in (94), because $F_e=F_c$.

Proposition 22. If $F(c,e)$ has the form $F(c+e)$, and there are n firms, then a firm's marginal benefit of additional effort for a component is the probability that the component is part of the system the consumer buys. In particular, for any cost such distribution in either regime, the marginal benefit of additional effort when symmetric firms choose the same effort levels is $1/n$.

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Proof. Firm i's expected profit in each component under compatibility is

$$\frac{\pi_i^C}{2} = \int \prod_{i \neq j} [1 - F(c,e_j)] F(c,e_i) dc. \tag{100}$$

Since $F_e = F_c$, firm i's marginal benefit of additional effort in component A under compatibility is

$$\begin{aligned} ½\frac{\partial \pi_i^C}{\partial e_i} &= \int \prod_{i \neq j} [1 - F(c,e_j)] F_e(c,e_i) dc \\ &= \int \prod_{i \neq j} [1 - F(c,e_j)] dF(c,e_i) \\ &= P(a_i \leq \min_{i \neq j}[a_j]). \end{aligned} \tag{101}$$

Therefore, firm i's marginal benefit of effort in a component is the probability that the component is the probability firm i has the lowest cost in that component.When firms choose the same effort levels, each has the lowest cost with probability $1/n$.

Under incompatibility, let G be the distribution function for the average of two draws from F. Then $G_e = G_c$, and firm i's marginal benefit of additional effort per component under incompatibility is again the probability that firm i has the lower cost.

$$\begin{aligned} ½\frac{\partial \pi_i^I}{\partial e_i} &= \int \prod_{i \neq j} [1 - G(c,e_j)] G_e(c,e_i) dc \\ &= \int \prod_{i \neq j} [1 - G(c,e_j)] dG(c,e_i) \\ &= P(c_i \leq \min_{i \neq j}[c_j]). \end{aligned} \tag{102}$$

When firms choose the same effort levels, the marginal benefit of additional effort is once again $1/n$. *QED.*

Remarks. Intuitively, additional effort has two effects when it does not change the shape of the cost distribution. First, additional effort ϵ by firm i increases its expected profit by ϵ when it wins. Second, additional effort increases the probability that firm i wins by $\epsilon f(c,e_i)$ yielding additional profit of ϵ, so the marginal benefit of additional effort in this case, $\epsilon^2 f(c,e_i)$, is of the second order. Therefore, firm i's marginal benefit of additional effort in a component is the probability that firm i has the lower cost in that component.

By the definition of FOSD, additional effort increases the probability that a firm wins. The previous proposition shows that the marginal benefit of additional effort, π', is equal to the probability that a firm wins. Therefore, $\pi''>0$, i.e. profit is convex in effort. As a result, second-order conditions will not hold and the first-order approach is invalid, unless C is sufficiently concave.

This result shows that the marginal benefit of R&D, $1/n$, is lower the more firms there are. By contrast, Arrow (1962) argues that R&D effort is higher in a competitive industry, because R&D by a monopolistic incumbent in part replaces existing profit. The settings for these two results are very different. In Proposition 22, demand is inelastic, which precludes the Arrow effect. Also, research by a firm's competitors reduces the chance that its research will be fruitful, because a firm must have the lowest cost to make any profit. The context of Proposition 20, in which uncertainty about costs resolves before firms choose their R&D effort and demand is elastic, give results more similar to Arrow. There, increasing the number of firms FOSD-reduces price, increases output, therefore increases the marginal benefit to reducing the unit cost.

When $F(c,e)$ has the form $F(c+e)$, it is also possible to get results when one firm chooses higher effort than another. Suppose that firm 1 chooses higher effort because its cost of effort function C lies below that of firm 2. Demand is inelastic as usual.

Proposition 23. Suppose that $F(c,e)$ has the form $F(c+e)$. If $e_1>e_2$, firm 1 has a higher marginal benefit of additional effort than firm 2 in either regime. Firms choose effort levels that are closer together under compatibility than under incompatibility. Additional effort by firm 2 reduces expected industry profits on the margin and additional effort by firm 1 increases expected industry profits, and compatibility moderates these effects for given effort levels. Firm 2 prefers compatibility to incompatibility, but firm 1's preferences are ambiguous, as is the effect of compatibility on expected industry profits.

Proof. When $e_1>e_2$, firm 1's costs are FOSD-lower than firm 2's. Firm 1's marginal benefit of additional effort is higher under incompatibility than compatibility because the probability that it wins is higher under incompatibility, as follows. Compatibility affects the probability that firm 1 wins only when the firms have complementary costs. In those cases, firm 1 wins exactly half the time in a given component under compatibility, since it has the low cost in exactly one component by assumption. Under incompatibility, firm 1 wins more than half the time conditional on costs being complementary, since its costs are FOSD-lower. Firm 1 has a lower cost less often under compatibility than incompatibility, so its marginal benefit of additional effort is lower. By the same argument, firm 2 has a higher marginal benefit of additional effort under compatibility, since it wins more often than under incompatibility.

Expected industry profit per component is

$$\begin{aligned}\frac{\pi_1^I + \pi_2^I}{2} &= \int[(1-G(c,e_2))G(c,e_1) + (1-G(c,e_1))G(c,e_2)]dc \\ &= \int[G(c,e_1) + G(c,e_2) - 2G(c,e_1)G(c,e_2)]dc.\end{aligned} \tag{103}$$

The marginal effect of additional effort by firm 2 on expected industry profit per component is

$$\frac{1}{2}\frac{\partial(\pi_1^I+\pi_2^I)}{\partial e_2} = 1-2\int G(c,e_1)dG(c,e_2)$$
$$= 1-2P(c_2>c_1) \tag{104}$$
$$< 0.$$

Firm 2 has a higher cost than firm 1 more than half the time, so increasing its effort lowers expected industry profits. Similarly, firm 1 has a lower cost more than half the time, so increasing its effort raises expected industry profits. Equation (104) also gives the marginal effect of additional effort under compatibility, substituting F for G and component costs for system costs. Firm 2 wins more often and firm 1 loses more often under compatibility than under incompatibility, so these effects are smaller under compatibility.

Firm 2 always prefers compatibility to incompatibility, because compatibility raises expected profits for given effort levels by the duopoly result in Proposition 1, and because firm 1 chooses lower effort under compatibility. Firm 1's views about compatibility depend on whether the increased profits of compatibility for given effort levels outweighs negative effect of greater effort by firm 2. The result is ambiguous, because compatibility can have a large or small effect on firm 2's effort depending on the cost function. The effect of compatibility on expected industry profits is ambiguous for the same reasons. *QED.*

Example. The conclusions of Proposition 22 and 23 depend on the assumption that $F(c,e)$ has the form $F(c+e)$. Suppose that a firm with effort e has cost zero with probability e and cost one with probability $1-e$. As computed above, the effect of compatibility on firm 1's marginal R&D incentives, which is $\Delta\pi'=2(1-2e_1)e_2(1-e_2)$ as computed above, is positive if $e_1<\frac{1}{2}$. In this case, in which additional effort changes the shape of the cost distribution, compatibility raises firm 1's marginal return to R&D effort whether $e_1 > e_2$ or not. Here, complementary costs are not necessarily more likely if firms choose effort levels that are close together.

5.2 R&D Effort and Risk

The previous section studied effort that does not change the shape of the cost distribution, but changed its mean. This section studies effort that changes the shape of the distribution by inducing a mean-preserving spread, that is, e is an index of second-order stochastic dominance in the family of distributions $F(c,e)$. As discussed in the previous chapter, a mean-preserving spread in the distribution of costs increasing risk by definition. The normalization (94) does not apply here, because effort does not alter a firm's expected cost.

5.2.1 Duopoly versus Oligopoly

Recall from the previous chapter that F is a *simple mean-preserving spread* of G, then G crosses F once from below. In other words, there is some cost c_0 such that $1-G(c) \geq 1-F(c)$ for $c<c_0$, $1-G(c) \leq 1-F(c)$ for $c>c_0$, and, if F and G are continuous at c_0, $1-G(c_0)=1-F(c_0)$. Intuitively, probability to the left of c_0 does not shift to the right and probability to the right of c_0 does not shift to the left. An increase in risk is *simple* if it is the composition of a finite number of simple mean-preserving spreads.

The following proposition describes the effect of effort in either regime.

Proposition 24. Let e be an index of second-order stochastic dominance, such that higher effort corresponds to a mean-preserving spread of costs. In duopoly the marginal effect of additional effort on own profit and the competitor's profit is positive, but increasing effort lowers expected consumer surplus. With any number of firms, increasing effort raises expected welfare. For a given e, if additional effort induces a simple increase in risk, then when there are sufficiently many firms, the marginal effect of additional effort on other firms and on prices is negative and the effect on expected consumer surplus is positive.

Proof. The previous chapter showed that profits are convex in own cost and in the competitor's cost in duopoly. Therefore, a mean-preserving spread of a firm's costs raises the expected profits of the firm and its competitor. Welfare, $R-\min[c_i]$, is a convex function for any number of firms, so expected welfare is increasing in effort. The price in duopoly, $\max[c_1,c_2]$, is convex in costs, so additional effort raises the expected price and lowers expected consumer surplus.

In oligopoly, fix e for firm i. Suppose that additional effort ϵ causes a simple increase in risk from $F(c,e)$ to $F(c,e+\epsilon)$. Then, there is some cost c_0 such that $1-F(c,e)\geq 1-F(c,e+\epsilon)$ for $c<c_0$, that is, $F(c,e+\epsilon)$ is FOSD-lower than $F(c,e)$ for $c<c_0$. Intuitively, c_0 is the first point at which the distributions cross. By implication, $F_e>0$ for costs less than c_0. If other firms choose effort levels e_j, the marginal effect of effort by firm i on another firm is by (98)

$$\frac{\partial \pi_j}{\partial e_i} = \int_{\underline{c}}^{\bar{c}} -F_e(c,e_i)F(c,e_j)\prod_{k\neq i,j}[1-F(c,e_k)]dc. \tag{105}$$

As in the proof of Proposition 3, for a large enough n, probability greater than c_0 is irrelevant, because $1-F(c,e_j)$ is decreasing in c. Therefore,

$$\frac{\partial \pi_j}{\partial e_i} \approx \int_{\underline{c}}^{c} -F_e(c,e_i)F(c,e_j)\prod_{k\neq i,j}[1-F(c,e_k)]dc < 0. \tag{106}$$

For costs less than c_0, $F_e>0$, so increasing effort has a negative externality. *QED.*

Remarks. This result suggests an intuition for Proposition 3. When there are many firms, riskier costs for other firms lower firm *i*'s expected profit, and the size of this effect increases in the number of firms. This effect outweighs the benefit to firm *i* of an increase in the riskiness of its own costs which, for a given risk, declines in the number of other firms. Therefore, riskier costs for all firms lower firm *i*'s expected profit.

5.2.2 Compatibility versus Incompatibility

R&D that does not affect expected costs may seem like a theoretical curiosity. However, we can decompose the effects of additional effort on the cost distribution into (1) its role in lowering expected costs given the shape of the distribution, and (2) its role in changing the shape of the distribution given its mean. Moreover, in comparing the marginal benefit of effort under compatibility and incompatibility, Proposition 22 showed that when effort equilibria are symmetric and increasing effort does not alter the distribution's shape, the marginal benefit of additional effort is the same in both regimes. Therefore, in assessing how compatibility affects R&D effort in symmetric equilibria, we can focus on effort that induces mean-preserving changes in the shape of the distribution.

In the following example, the effect of effort in spreading risk is weaker under incompatibility than under compatibility. Then, firms will undertake more R&D effort that increases risk under compatibility.

Example. Suppose that the distribution when no effort is made, F, gives costs 2 and 4 with equal probability, and the distribution after effort, F_0, gives 1 and 5 with equal probability. Clearly, effort induces a mean-preserving spread in the cost distribution. The average of two draws from the first distribution, G, gives outcomes 2, 3, and 4 with probabilities ¼, ½, and ¼ respectively. The average of two draws from the second distribution, G_0, gives outcomes 1, 3, and 5 with probabilities ¼, ½, and ¼ respectively. The spread on one draw moves probability weight of ½ from 2 to 1 and from 4 to 5, while the spread on the average two draws moves only a probability weight of ¼ from 2 to 1 and 4 to 5. Effort under incompatibility moves less probability than under compatibility.

The reason that less probability is spread under incompatibility is that averaging causes the upper and lower parts of the spread to cancel each other out half the time. There are four possible outcomes for two draws: (2,2), (2,4), (4,2) and (4,4). In the second case, the spread changes outcome (2,4) to outcome (1,5), but under incompatibility the average is 3 before and 3 after with certainty. No probability is spread in the distribution of the average. The same is true of outcome (4,2) in the third case. Only in the first and fourth cases does probability get spread in the average.

Because more probability gets spread under compatibility, expected profit increases by more under compatibility. Effort raises expected profit from one to two under compatibility, and from 3/8 to 6/8 under incompatibility. The benefit of effort is higher under compatibility, because incompatibility averages costs and dilutes the effect of effort in spreading probability.

This example does not readily generalize to any mean-preserving spread. For instance, change the example so that effort results a distribution F_0 in which costs 0 and

5 have probabilities of 2/5 and 3/5 respectively. Effort still induces a mean-preserving spread. Under incompatibility, the distribution G_0 after effort has costs per component 0, 2.5, and 5 with probabilities of 4/25, 12/25, and 9/25 respectively. Before effort, the distribution again gives costs 2, 3, and 4 with probabilities ¼, ½, and ¼. Then, the argument that effort moves less probability does not apply. Under incompatibility, effort moves probability from 3 to 2.5, something that does not take place under compatibility. It is an open question whether the marginal benefit of risk-increasing effort is always higher under compatibility.

Example. When firms are asymmetric, the marginal benefit of additional effort that increases risk can be lower under compatibility than incompatibility. For instance, suppose that firm j has costs (4,1) with certainty. Firm i has costs 2 or 3 in each component with equal probability if it makes no R&D effort, and costs 1 or 4 with equal probability when it does exert R&D effort. It is trivial to verify that the R&D effort causes a mean-preserving spread. Under compatibility, effort does not affect expected profit. Firm i's costs in component B are always too high to compete. Its expected profit on component A remains the same after effort is exerted (3 or 0 with equal probability) as they were before (2 or 1 with equal probability). The spread lies along linear segments of the profit function, so the firm exhibits risk neutrality for this risk.

Under incompatibility, the spread does occur across a convex portion of the profit function, so firm i exhibits a preference for risk. With no effort, profit is positive only when costs are (2,2), so expected profit is ¼(1)=¼. With effort, profit is positive only if costs are (1,1), so expected profit is ¾(1)=¾. The crucial difference in this example is whether the spread moves probability weight across the convex bend in the profit function or not.

6 Conclusions

This chapter has addressed firms' choices of R&D effort in regimes of complete compatibility and complete incompatibility. It has shown that firm's R&D incentives are socially optimal in certain settings in which the negative effect of R&D on a firm's competitors is exactly offset by the positive effect on the consumer (or vice versa). Symmetric firms sometimes choose asymmetric levels, and under compatibility this asymmetry may take the form of each firm exerting higher effort in one component than the other. It is found that when compatibility and R&D effort both reduce costs, and when it is harder to reduce a low cost than a high cost, that compatibility may reduce R&D effort. On the other hand, when demand is elastic and firms draw their costs before exerting effort, then when there are sufficiently many firms, compatibility lowers prices, increases output, and as a result, produces greater R&D incentives.

When costs are uncertain, and effort lowers costs without changing the shape of the cost distribution, asymmetric duopolists choose effort levels that are closer together under compatibility than incompatibility. When effort increases the riskiness of costs, effort by a firm has a positive externality on another firm in duopoly, but has a negative externality on other firms when there are sufficiently many firms.

The next chapter moves back to the setting in which firms cannot alter their costs through R&D effort. It examines the effects of relaxing Chapter 2's assumption that consumers combine components in fixed proportions. This change in specification will also permit an examination mix-and-match compatibility when there is Cournot competition.

Chapter 4: Mix-and-Match Compatibility with Variable Coefficients

In practice, consumers buy components in varying proportions; some of them buy more software than hardware, and others buy more hardware than software. For many products consumers buy one base unit (a video game system), but vary in the number of complements they purchase (video game cartridges). The mix-and-match literature has, up to this point, assumed fixed coefficients—components are combined in one-to-one proportions. For instance, most personal computer owners have exactly one system unit, one screen, and one keyboard. These components are perfect complements; demand for one component is the same as demand for the others. However, if consumers vary in the degree to which they care about quality in one component versus the other, then once again there is some substitutability.

This chapter relaxes the assumption of perfect complementarity. Thus, the proportions in which a consumer combines components depend upon his tastes and the prices he faces. At some prices a photographer might buy two cameras and a lense; for others he will buy two lenses and one camera.

It is found that when components are not perfect complements, compatibility has an ambiguous effect on profits and welfare. By contrast to Matutes and Regibeau (1988) and Einhorn (1992), a duopolist with the lowest cost in both components is weakly worse off under compatibility than under incompatibility because it faces the additional constraint that it cannot set the price in any component above its competitor's cost. When neither firm dominates, but one firm almost does, a similar result obtains.

These results arise under price competition. Quantity competition can be modelled when coefficients are variable, unlike the fixed coefficients case.[154] We find that when each firm has the same cost across components, and demand is also symmetric across components, compatibility does not alter equilibrium outcomes. However, mirror-image duopolists, which each have the low cost in exactly one component, prefer compatibility.

Section 1 considers price competition with general demand in duopoly and oligopoly. Section 2 considers the special case of price competition with constant elasticity of substitution (CES) demand. Section 3 presents a Cournot model of mix-and-match compatibility with CES demand.

1 Price Competition with General Demand

Below, we examine the case of price competition with general demand. It found that variable coefficients can change the preference of duopolists for compatibility that obtains when coefficients are fixed. However, the finding that when there are sufficiently many firms, once again they prefer incompatibility.

Section 1.1 examines price competition in duopoly, and finds that the effect of compatibility in industry profits depends on whether or not one firm has the lowest costs in both components. Section 1.2 shows that an industry with sufficiently many firms that have random costs prefers incompatibility, as was the case in Chapter 2.

[154] See footnote 126 above.

1.1 Price Competition in Duopoly

Relaxing the assumption of fixed coefficients overturns the result that compatibility raises profits and welfare in duopoly. Under incompatibility, it is necessary to change the meaning of the assumption that "costs constrain prices." Suppose that the representative consumer has a quasiconcave indirect utility function $v(p^A,p^B)$. This is not an indirect utility function, in that income and outside goods are not arguments. Without loss of generality, assume $v(a_1,b_1) \geq v(a_2,b_2)$. Only firm 1 has positive sales under incompatibility, because it can always undercut firm 2 and offer the consumer more utility. Say that costs constrain prices here when the constrain that firm 1 offers more utility than firm 2, $v(p_1^A,p_1^B) \geq v(a_2,b_2)$, is binding.

Proposition 25. Suppose that components and systems are homogenous, that a representative consumer has a quasiconcave indirect utility function $v(p^A,p^B)$ and that costs constrain prices. If one duopolist has lower costs in both components, profits and welfare are weakly lower under compatibility than incompatibility. Compatibility weakly raises the price of one component and weakly lowers the price of the other.

Proof. The highest utility that firm 2 can offer without losing money is $U_2 = v(a_2,b_2)$. Firm 1 sets prices subject to the constraint $v(p_1^A,p_1^B) \geq v(a_2,b_2) = U_2$, which is binding by assumption (see Figure 13). In the figure, the constraint on firm 1's price is denoted U_2. Section 2 shows that with CES demand, firm 1 sets an equal markup in each component, so the dotted line through the origin picks out the point on U_2 corresponding the firm 1's prices in that case.

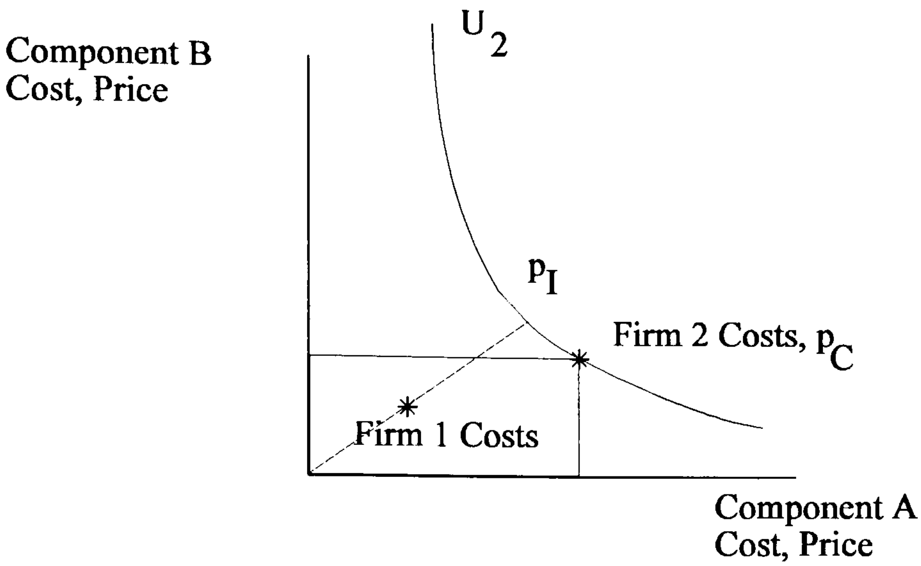

Figure 13: Price Constraints under Compatibility and Incompatibility

Under compatibility, firms compete component-by-component. Firm 1 has the lowest cost in each component by assumption, so it is the only firm to sell either component. It cannot set a price above firm 2's cost in either component, or firm 2 would undercut it. Therefore, firm 1 sets prices subject to the constraints $p_1^A \leq a_2$ and $p_1^B \leq b_2$, which are binding by assumption.

Under incompatibility, firm 1 can set prices corresponding to any point on U_2, but under compatibility, it must choose the point (a_2,b_2) (see Figure 13). Under incompatibility, firm 1 can set the price of one component above firm 2's costs; if it strictly prefers doing so, its profit is strictly lower under compatibility. Utility is U_2 is the same in either regime by the assumption that costs constrain prices, and industry profits are weakly lower under compatibility. Therefore, welfare is weakly lower under compatibility. *QED*.

Remarks. When coefficients are fixed in Chapter 2, compatibility does not affect a dominant firm's profit. There, the locus of points offering utility U_2 is a straight line, and firm 1 faces the constraint $p_1^A+p_1^B=a_2+b_2$, and each point on that line gives firm 1 the same profit. Then, when R is high, compatibility forces firm 1 to set components prices equal to firm 2's costs, $p_1^A=a_2$ and $p_1^B=b_2$, but this constraint has no effect on firm 1's profit. By contrast, U_2 is concave to the origin when coefficients are variable, and different points on U_2 may give firm 1 different profits (see Figure 13).

Matutes and Regibeau (1988) note that in their mix-and-match model with horizontal differentiation, duopolists prefer compatibility if components are sufficiently strong complements. However, Proposition 25 holds even if components are nearly perfect complements, that is, if U_2 is only slightly concave to the origin.

When neither firm has the lower cost in both components, there is one case in which is easy to show that duopolists prefer compatibility. Say that duopolists have *mirror-image costs* when firm 1 has costs (c,c_0) and firm 2 has costs (c_0,c).

Proposition 26. Suppose that firm 1 has cost (c,c_0) and firm 2 has costs (c_0,c). Suppose that utility is symmetric across components, so that $v(p^A,p^B)=v(p^B,p^A)$. Then, firms have zero profits under incompatibility and positive profits under compatibility, and compatibility raises prices.

Proof. Under incompatibility, both firms offer the same utility when they set price at cost, because $v(c,c_0)=v(c_0,c)$. Because systems are homogenous, firms undercut each other in price until each sets price at cost, and neither makes a profit. Under compatibility, each firm has a cost advantage in one component, so it can make a positive profit in that component. Each firm raises the price of its low-cost component and does not change the price of its high-cost component. *QED.*

When neither firm dominates and the assumptions of Proposition 26 do not apply, then general results are difficult to obtain. However, Section 2 below solves the case with CES demand for general costs. The CES case below suggests the following intuition for Propositions 25 and 26. The industry has constant revenue of one, so industry profits depend on how efficiently the industry delivers utility. Compatibility has two opposing effects on efficiency. First, it ensures that the consumer buys from the low-cost firm in each component. Second, it means that the ratio of component prices under compatibility, a_2/b_1, bears no relation to the ratio of costs, a_1/b_2. In Proposition 25, the first effect is not present, because firm 1 already has the lowest cost in both components. In Proposition 26, the second effect is not a factor, because the ratio of prices, b/b, is equal to the ratio of costs, a/a.

1.2 Price Competition with Many Firms

The previous section shows that when coefficients are variable, the effect of compatibility on profits is ambiguous. However, in the same manner as Chapter 2, it is possible to get unambiguous results when many firms draw their costs from independent random distributions. The proof of the following proposition is very similar to that of Proposition 11, where demand is elastic. Intuitively, a firm is more likely to be the only firm to draw the lowest system cost under incompatibility than to be the only firm to draw the lowest cost in a component under compatibility, and this is all that matters when n is large.

Proposition 27. Suppose the representative consumer has utility $v(p_i^A, p_i^B)$ of buying from firm i, and that Bertrand firms draw their component costs from the discrete distribution F, which is independent across firms and components. Then, when there are sufficiently many firms, expected profits are lower under compatibility.

Proof. Under compatibility, $\pi^{\max}$ in the proof of Proposition 11 takes different values than before, but the conclusion in Equation (74) that

$$\pi_1^C \preceq [1 - F(\underline{c})]^{n-1} \tag{107}$$

is unchanged. Under incompatibility, let G be the distribution of the random variable $-v(a,b)$, where a and b are drawn from F. Suppose that firm 1 draws costs (a,b) which has probability $g(-v(a,b))$. It makes a positive profit only when all other firms have higher $-v(a,b)$, which has probability $[1-G(-v(a,b))]^{n-1}$. Let $\pi^{\min}(a,b)$ be the least profit that the firm can make with costs (a,b), given that other firms have higher $-v(a,b)$. Then, firm 1's expected total profit under incompatibility is at least

$$\pi_1^I \geq \sum_a \sum_b \pi^{\min}(a,b)[1 - G(-v(a,b))]^{n-1} g(-v(a,b)) \tag{108}$$

where the summation is over the support of G. As n grows large, (34) implies that only the term for which $[1-G(-v(a,b))]$ is highest matters. This is the term where $a=b=\underline{c}$, where $\underline{c}$ is the lower bound of the support of F. Therefore

$$\pi_1^I \succeq [1 \quad G(-v(\underline{c},\underline{c}))]^{n-1}. \tag{109}$$

For a discrete distribution F, $F(\underline{c})=f(\underline{c})$ and $G(-v(\underline{c},\underline{c}))=f(\underline{c})^2$, so $F(\underline{c}) \geq G(-v(\underline{c},\underline{c}))$. Therefore, by (34),

$$\pi_1^I \succeq [1 - G(-v(\underline{c},\underline{c}))]^{n-1} \succ [1 - F(\underline{c})]^{n-1} \succeq \pi_1^C. \tag{110}$$

So, profits are higher under incompatibility for large enough n. *QED.*

2 Price Competition with a Constant Elasticity of Substitution (CES)

This section solves the special case when the representative consumer has a constant elasticity of substitution (CES) between components. This case will illustrate the general results derived above, and give some insight into the case in which neither firm has the lowest cost in both components. The constant elasticity of substitution demand function is a natural specification, because it includes Leontief demand, in which coefficients are fixed, as a special case. Despite its name, this function can describe complements or substitutes. The degree to which components are substitutes or complements is a parameter. This formulation does not capture the case in which each consumer buys exactly one of a component, because the quantities bought of each component vary with prices.

Consumers combine components into systems according to

$$q = ((q^A)^\rho + (q^B)^\rho)^{1/\rho} \tag{111}$$

where q is the industry output of systems, q^j is the number of components used, and ρ is a measure of the substitutability of components. The elasticity of substitution is $\epsilon = 1/(1-\rho)$. Under compatibility, industry output is $q^j = q_1^j + q_2^j$ for $j = A, B$. Under incompatibility, the consumer buys only from a single firm i, so $q^j = q_i^j$. Utility is assumed to be $U(q) = \ln q + y$, where y is expenditure on outside goods. Therefore, expenditure on systems is constant, without loss of generality, the inverse demand function for systems is $p = 1/q$. Indirect utility is[155]

[155] See for example Varian (1985), p. 130, although his definition of r appears to have the wrong sign.

$$v(p^A,p^B) = [(p^A)^r + (p^B)^r]^{-\frac{1}{r}}, \qquad r = \frac{\rho}{\rho - 1}. \tag{112}$$

2.1 Price Competition under Incompatibility

Under incompatibility, firms will cut prices until at most one makes a profit. By (112), consumers prefer the firm for which

$$(p_i^A)^r + (p_i^B)^r \tag{113}$$

is lowest. Notice that in the Leontief case, for which $r=1$, consumers prefer the firm for which the sum of component prices is lowest. The firm for which (113) is lowest when it sets prices equal to costs cannot be undercut, and is therefore the firm that makes positive sales. Without loss of generality, let that firm be firm 1, and say that it is the "low-cost" firm. If it has the lowest cost in only one component, let that be component A. Let U_2 be the level of utility offered by the second-ranked firm when it sets prices equal to costs. Firm 1 chooses prices subject to the constraint that

$$[(p_1^A)^r + (p_1^B)^r]^{-\frac{1}{r}} = U_2. \tag{114}$$

Demand for component A is

$$q_1^A = \frac{(p_1^A)^{r-1}}{(p_1^A)^r + (p_1^B)^r} \tag{115}$$

and profit under incompatibility is

$$\pi_1^I = (p_1^A - a_1)q_1^A + (p_1^B - b_1)q_1^B = \frac{\frac{p_1^A - a_1}{p_1^A}(p_1^A)^r + \frac{p_1^B - b_1}{p_1^B}(p_1^B)^r}{(p_1^A)^r + (p_1^B)^r}. \tag{116}$$

Thus, profit is the weighted average of the inverses of the markups. Because expenditure on systems is constant, profit is increasing in prices and the constraint (114) is binding. Eliminating the quantity variables from (116) using (115), and maximizing profit subject to the constraint (114), gives

$$\frac{p_1^A}{a_1} = \frac{p_1^B}{b_1}. \tag{117}$$

By implication, firms set the same markup in each component.

With equal markups in each component, firm 1's profit in both components reduces to

$$\pi_1^I = \frac{p_1^A - a_1}{p_1^A}. \tag{118}$$

Profit is increasing in price, so the lowest cost firm sets the markup so that a consumer has utility U_2 in equilibrium. Therefore, firm 1's price-to-cost ratio equals the ratio of the utilities provided by the two firms

$$\frac{p_1^A}{a_1} = \left[\frac{a_1^r + b_1^r}{a_2^r + b_2^r}\right]^{-\frac{1}{r}} \tag{119}$$

and profit is

$$\pi^I = \frac{(a_2^r + b_2^r)^{\frac{1}{r}} - (a_1^r + b_1^r)^{\frac{1}{r}}}{(a_2^r + b_2^r)^{\frac{1}{r}}}. \tag{120}$$

2.2 Price Competition under Compatibility

Now consider competition under compatibility. A consumer will buy each component from the firm offering the lowest price in that component. Suppose that neither firm has the lowest cost in both components. Costs constrain prices again, i.e. $p_1^A = a_2$ and $p_2^B = b_1$. Firm 1 has a lower cost in component A with profit in that component of

$$\pi_1^C = \frac{p_1^A - a_1}{p_1^A} \frac{(p_1^A)^r}{(p_1^A)^r + (p_2^B)^r}. \tag{121}$$

The first term, the markup, is always increasing in price, but the second term is only increasing in price if $r>0$, i.e. when the component are complements. When profit is increasing in price, firm 1 sets price equal to firm 2's cost in component A:

$$p_1^A = a_2. \tag{122}$$

Comparing to equation (119), the effect of compatibility on price is ambiguous, and depending on cost and the substitutability parameter r. Profit is

$$\pi_1^C = \frac{a_2 - a_1}{a_2} \frac{a_2^r}{a_2^r + a_1^r}. \tag{123}$$

Industry profits are

$$\Pi^C = \pi_1^C + \pi_2^C = \frac{\frac{a_2 - a_1}{a_2} a_2^r + \frac{b_1 - b_2}{b_1} b_1^r}{a_2^r + b_1^r} \tag{124}$$

so the industry markup is a again weighted average of the firm's inverse markups in each component.

Compatibility has two opposing effects on efficiency. On the one hand, it ensures that a consumer buys from the low-cost producer of each component. On the other hand, component prices no longer reflect costs, so the consumer does not buy components in the "correct" proportions. Under incompatibility, $p^A/p^B = a_1/b_1$, by (117). Under compatibility, $p^A/p^B = a_2/b_1$, so the price ratio is unrelated to the cost ratio, a_1/b_2. When one firm is dominant, the first effect does not matter. Therefore, only the second effect, which reduces efficiency, is present, with the result that welfare is lower when there is a dominant firm.

We can now examine how the parameter r affects the comparison between of industry profits under compatibility and incompatibility. By the assumptions made about costs,

$$a_1^r + b_1^r > a_2^r + b_2^r, \qquad a_1 \leq a_2, \qquad b_2 \leq b_1. \tag{125}$$

Consider three cases, which are summarized in the table below:

1. Leontief technology ($r=1$): This is the fixed coefficients case dealt with in Chapter 2, with unit elastic demand for systems. Under compatibility, costs are lower, and industry profits are higher than under incompatibility. Each firm's profit is higher too, and welfare rises despite an increase in prices.

2. Cobb-Douglas ($r=0$): Under compatibility

$$\pi_1^C = \frac{1}{2} - \frac{a_1}{2a_2}, \quad \Pi^C = 1 - \frac{\frac{a_1}{a_2} + \frac{b_2}{b_1}}{2}, \quad p_1^A = a_2, \quad p_2^B = b_1, \quad U^C = \sqrt{\frac{1}{a_2 b_1}}.$$

Under incompatibility,

$$\pi_1^I = \Pi^I = 1 - \sqrt{\frac{a_1}{a_2}\frac{b_1}{b_2}}, \quad p_1^A = \sqrt{\frac{a_1 a_2 b_2}{b_1}}, \quad U^I = \sqrt{\frac{1}{a_2 b_2}}. \tag{127}$$

The effect of compatibility on industry profit is ambiguous. When firms have identical advantages in their low-cost component, i.e. $a_1 = b_2$ and $a_2 = b_1$, then profits under incompatibility are zero, and they are positive under compatibility. On the other hand, when $a_2/a_1 > b_1/b_2$, incompatibility produces higher industry profits. Because firm 2's profit can only rise from zero, a fall in industry profits means firm 1's profit must have fallen. This case is similar to the dominant firm case discussed in Proposition 25 above. Prices are higher under compatibility, because

$$\frac{(p_1^A)^C}{(p_1^A)^I} = \frac{(p_2^B)^C}{(p_2^B)^I} = \sqrt{\frac{b_1 a_2}{a_1 b_2}} > 1 \tag{128}$$

by the assumptions about costs. Utility is lower under compatibility.

3. Perfect substitutes ($r = -\infty$): Components are perfect substitutes for one another; under compatibility and incompatibility, each firm sells only the component with the lowest unit cost. These components compete head-to-head, whether they are type A or B. Industry profits are

$$\Pi^C = \Pi^I = 1 - \frac{a_1}{\min[a_2, b_2]}. \tag{129}$$

The following table summarizes the effects of shifting from incompatibility to compatibility when neither firm has the lowest costs in both components in the above case, and the case in which one firm dominates in costs.

Technology	r	ρ	ϵ	Profits			Prices	Utility	Welfare
				Π	π_1	π_2	(A, B)		
Leontief	1	$-\infty$	0	+	+	+	+	–	+
Cobb-Douglas	0	0	1	?	?	+	+	–	?
Perfect substitutes	$-\infty$	1	∞	0	0	0	0	0	0
Dominant Firm ($1 > r > -\infty$)				–	–	0	(–,+)	0	–

3 Quantity Competition with CES Demand

Variable coefficients allow an examination of quantity competition, which the mix-and-match literature has not yet explored. Under incompatibility, firms compete in systems, and choose their outputs of quantities to minimize their unit system cost. By a standard result, the minimum unit cost is

$$c(q_1) = [a_1^r + b_1^r]^{\frac{1}{r}}. \tag{130}$$

With inverse demand $p = 1/q$, quantity competition in systems yields quantities

$$q_1 = \frac{c_2}{(c_1 + c_2)^2} \tag{131}$$

and profit

$$\pi_1^I = \frac{c_2^2}{(c_1 + c_2)^2}. \tag{132}$$

Under compatibility, if both firms sell a component, they must receive the same price for that component. The consumer's demand for component A as a function of industry prices is, by (115),

$$q^A = \frac{(p^A)^{r-1}}{(p^A)^r + (p^B)^r}. \tag{133}$$

Therefore

$$\frac{q^A}{q^B} = \left(\frac{p^A}{p^B}\right)^{r-1}, \quad \frac{p^A}{p^B} = \left(\frac{q^A}{q^B}\right)^{\frac{1}{r-1}}. \tag{134}$$

By the assumption of unit expenditure on systems

$$1 = p^A q^A + p^B q^B = p^A q^A + p^A \left(\frac{q^B}{q^A}\right)^{\frac{1}{r-1}} q^B \tag{135}$$

so

$$p^A = \frac{1}{q^A + \left(\frac{q^B}{q^A}\right)^{\frac{1}{r-1}} q^B}. \tag{136}$$

Firm 1's total profit is

$$\pi_1^C = (p^A - a_1) q_1^A + (p^B - b_1) q_1^B. \tag{137}$$

The first-order condition with respect to q_A^1 as derived in Mathematica is

$$a_1 = \frac{1}{q_1^A + q_2^A + \frac{(q_1^B + q_2^B)^{\frac{r}{r-1}}}{(q_1^A + q_2^A)^{\frac{1}{r-1}}}} - \frac{q_1^B (q_1^A + q_2^A)^{\frac{1}{r-1}} \left(\frac{r}{r-1}\right)}{(q_1^B + q_2^B)^{\frac{1}{r-1}} \left(q_1^B + q_2^B + \frac{(q_1^A + q_2^A)^{\frac{r}{r-1}}}{(q_1^B + q_2^B)^{\frac{1}{r-1}}}\right)^2}$$

$$- \frac{q_1^A \left(1 - \frac{(q_1^A + q_2^A)^{-\frac{r}{r-1}} (q_1^B + q_2^B)^{\frac{r}{r-1}}}{r-1}\right)}{\left(q_1^A + q_2^A + \frac{(q_1^B + q_2^B)^{\frac{r}{r-1}}}{(q_1^A + q_2^A)^{\frac{1}{r-1}}}\right)^2}.$$

We could not solve the first-order conditions except by focussing on two special cases. First, let firm 1 have costs (c,c) and let firm 2 have costs (c_0,c_0). Call this case *symmetric dominance*. Under incompatibility, cost minimization implies that a firm will choose the same quantity across components in equilibrium. Profits and prices are the same in both regimes when $q_i^A = q_i^B$, because the CES utility function is homogeneous of degree one in its two arguments. Therefore, a symmetric equilibrium under compatibility must also be a symmetric equilibrium under incompatibility, because a firm has the same benefits of deviating while maintaining $q_i^A = q_i^B$.

This argument applies with more general costs. Suppose that costs are (kc, c) and (kc_0, c_0) for some constant $k>0$. Component quantities are not equal but have the ratio

$$\frac{q_i^A}{q_i^B} = k^{r-1} \tag{139}$$

which is implied by (130) and cost minimization.

Now, consider a different case in which firms 1 and 2 have mirror-image costs, (c, c_0) and (c_0, c), where $c_0 > c$, so firm 1 has the lower cost in component A. Under incompatibility, each firm has the same unit system cost, so (132) implies that each firm has profit of ¼ and industry profits are ½.

Under compatibility, setting the derivative of (137) to zero, and then letting $q_2^B = q_1^A$, $q_1^B = q_2^A$, and $q^A = q_1^A + q_2^A$, yields the first-order condition

$$c = \frac{1}{4q^A}\left(1 + \frac{2q_1^A - q^A}{q^A(r-1)}\right)$$

and by symmetry

$$c_0 = \frac{1}{4q^A}\left(1 + \frac{2q_2^A - q^A}{q^A(r-1)}\right). \tag{141}$$

Computation in Mathematica indicates that the second-order condition holds when

$$\frac{q_1^A}{q_1^B} < 4 - r, \tag{142}$$

that is, when a firm does not choose a much higher output in its lower-cost component than in its higher-cost component. Adding (140) and (141) shows that industry output of component A is

$$q^A = \frac{1}{2(c+c_0)}. \tag{143}$$

Normalize quantity units so that $c+c_0=1$, and (140) gives

$$q_1^A = \frac{2c_0(1-r)+r}{4}. \tag{144}$$

Computation in Mathematica shows that the second-order condition (142) holds for the quantities implied by (144), so profit is maximized at least locally. Industry costs are

$$2(q_1^A c + q_2^A c_0) = 2[c_0(1-c_0)(1-r) + \frac{r}{4}] \leq ½ \tag{145}$$

so costs are lower under compatibility when $r>0$, that is, when components are complements. Industry revenue is constant at one, so industry profit under compatibility is greater than ½, and each firm's profit is greater than ¼ by symmetry. Therefore, profits are higher under compatibility than under incompatibility.

Now, compare the degree to which firms specialize in producing their low-cost component in the two regimes, as measured by the ratio of their component quantities. These ratios are

$$\left.\frac{q_1^A}{q_1^B}\right|_C = \frac{2(1-c)(1-r)+r}{2c(1-r)+r} \geq \left(\frac{1-c}{c}\right)^{1-r} = \left.\frac{q_1^A}{q_1^B}\right|_I \tag{146}$$

where $c<\frac{1}{2}$ by the normalization. Therefore, specialization is greater under compatibility than under incompatibility.

Output of systems falls under compatibility. Output in the two regimes is

$$q^I = \frac{1}{2(c^r + c_0^r)^{1/r}} \geq \frac{1}{2^{1/r}} = q^C. \tag{147}$$

Output is greater under incompatibility, particularly when c is near zero and $r=\frac{1}{2}$.

The first-order approach may be invalid under compatibility when firms specialize, because the second-order condition (142) may fail to hold. Then, a corner solution may occur, in which output of the higher-cost component is zero, $q_1^B=q_2^A=0$, and firms are completely specialized. However, then, $q_2^B=q_1^A=-r/[4c(1-r)]$, which is negative when components are complements ($r>0$). An equilibrium with complete specialization can occur only when components are substitutes, $r<0$. Although the price competition case in Chapter 2 suggests that firms specialize completely when components are perfect complements ($r=1$), they do not specialize at all, by (146).

Thus, quantity competition among duopolists gives similar results to price competition with fixed or variable coefficients when the first-order approach is valid. Output falls, prices rises, and firms increase the share of their production in their low-cost component. The mirror-image case is more difficult to generalize than the symmetric dominance case. More broadly, it is an open question whether profits are weakly higher under compatibility than incompatibility for any costs.

These results for quantity competition are not general. However, they suggest a line for further research. One approach would be to assume inverse demand functions for each component that are linear in component prices.

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