ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

COORDINATION AND CULTURE

Jean-Paul Carvalho

Number 489
June 2010

Manor Road Building, Oxford OX1 3UQ

Coordination *and* Culture

Jean-Paul Carvalho*

Department of Economics

University of Oxford

jean-paul.carvalho@economics.ox.ac.uk

Abstract

Culture constrains individual choice by making certain behaviours taboo. We propose an evolutionary model in which members of different groups attempt to coordinate over time. We show that cultural constraints can lead to a permanent break down in coordination between groups, even when coordination is attainable and Pareto-efficient. Hence restrictive cultures make coordination with out-group members more difficult. By limiting a person's options, however, highly restrictive cultures act as a strategic commitment, forcing out-group members to conform to in-group norms if they want to coordinate. In this way, cultural constraints on behaviour may lead to higher expected welfare. When people rationally choose their culture, we demonstrate that restrictive and permissive cultures can co-exist in the long run.

8 June 2010

JEL Classification: C72, C73, Z1

Keywords: Coordination Games, Culture, Taboos, Commitments, Cultural Evolution

*The author is heavily indebted to Peyton Young for his guidance and support. The paper has also benefitted from comments by Ken Binmore, Rob Boyd, Ryan Muldoon, David Myatt, Tom Norman and Chris Wallace. The usual caveat applies. Financial support from the General Sir John Monash Foundation and Commonwealth Bank Foundation is gratefully acknowledged.

1 Introduction

Culture constrains individual choice. In matters of diet, dress, language, manners and rules of social deference, people not only compare alternatives, but also rule out some options as culturally impermissible, i.e. taboo.[1] Vegetarians do not consider switching to eating meat as circumstances dictate. Orthodox Jews do not work on the Sabbath. Practicing Muslims are committed to refraining from alcohol consumption and gambling. The Amish are prohibited from using modern technology, even as it becomes more attractive to do so. Christians in the Middle Ages observed usury prohibitions. It was taboo to reveal the ankles in Victorian England. Japanese businessmen customarily bow and exchange business cards, even with foreign partners.

This paper addresses two questions: Why do people adopt cultures that restrict their choice? Can members of different cultural groups learn to coordinate with each other when they hold different cultural commitments? To study these issues, we develop an evolutionary model of coordination between cultural groups. Cultural commitments are modelled in a simple way. Let X be the global set of actions. A member of cultural group c_k is *committed* to taking actions in $X_k \subseteq X$. In this way, culture can serve as a strategic commitment and individuals may rationally choose restrictive cultures that limit their choice to gain a strategic advantage in social interactions.

We can immediately introduce the main ideas of the paper by analyzing a simple 3×3 coordination game, which resembles the Battle of the Sexes. Two friends, players 1 and 2, would like to meet. They can choose from attending venue A, B or C. The players receive a coordination payoff of one from going to the same venue and zero if they go it alone. In addition, each agent i receives idiosyncratic payoffs a_i, b_i and c_i when choosing venues A, B and C, respectively. Idiosyncratic payoffs are such that agents always prefer to coordinate, but they may prefer different venues to coordinate upon.[2]

The static game can be represented by the payoff matrix in figure 1. The three coordination (Nash) equilibria are underlined. Now let us impose cultural commitments. Suppose venues A, B and C correspond to Bar, $Cafe$ and $Church$. Player 1 is an atheist and rules out meeting at church on preconsequential grounds. Player 2 is a teetotaller and

[1] For example, Tetlock et al. (2000) find that experimental subjects express moral outrage at even contemplating certain taboo transactions, including buying and selling of human body parts for medical transplant operations, surrogate motherhood contracts, adoption rights for orphans, votes in elections for political office, the right to become a U.S. citizen and sexual favours (i.e. prostitution). People feel that contemplating taboo behaviour undermines their image as moral beings both to themselves and other members of society (Fiske & Tetlock 1997).

[2] Formally, agent i receives an idiosyncratic payoff of δ_{ix} from choosing venue x. To ensure that this is a coordination game, we assume that $\delta_{ix} - \delta_{ix'} < 1$ for both players $i \in \{1, 2\}$ and for all pairs of actions $(x, x') \in X^2$.

	A	B	C
A	$\underline{1+a_1}, \underline{1+a_2}$	a_1, b_2	a_1, c_2
B	b_1, a_2	$\underline{1+b_1}, \underline{1+b_2}$	b_1, c_2
C	c_1, a_2	c_1, b_2	$\underline{1+c_1}, \underline{1+c_2}$

Figure 1: Payoff matrix given a global action set $X = \{A, B, C\}$.

would not consider going to a bar. The restricted action sets yield the reduced game represented in figure 2.

	B	C
A	a_1, b_2	a_1, c_2
B	$\underline{1+b_1}, \underline{1+b_2}$	b_1, c_2

Figure 2: Reduced game given commitments $X_1 = \{A, B\}$ and $X_2 = \{B, C\}$.

In this paper, we emphasize the interpretation that cultural commitments are internalized through the process of *socialization*.[3] Notice, however, that the same restricted action set could be obtained by making C a strictly dominated strategy for player 1 and A a strictly dominated strategy for player 2, for example by introducing sanctions imposed by their respective communities. Our model is completely consistent with this alternative interpretation, though we will not emphasize it in the exposition.

After deleting inadmissible actions, only one coordination equilibrium (underlined) remains, i.e. (B, B) remains a Nash equilibrium. Now, however, the pair (A, C), which represents *miscoordination*, is also a Nash equilibrium if $a_1 > b_1$ and $c_2 > b_2$. This is not a

[3] A fundamental premise of economic theory is that individuals make choices by comparing the consequences of alternative actions. Yet evidence suggests that people tend to abide by moral commitments and taboos, rejecting certain choice options on preconsequential grounds. Tetlock et al. (2000) find that experimental subjects express moral outrage at even contemplating certain taboo transactions, including buying and selling of human body parts for medical transplant operations and surrogate motherhood contracts. People feel that contemplating taboo trade-offs undermines their image as moral beings, both to themselves and other members of society (Fiske & Tetlock 1997).

knife-edge phenomenon. In fact, the (A, C) miscoordination equilibrium is risk dominant under a range of parameter values.

Using techniques from stochastic evolutionary game theory (Foster & Young 1990, Kandori et al. 1993, Young 1993, Ellison 2000), we extend this basic insight to an evolutionary setting with n players, L actions and far greater payoff heterogeneity than analyzed in previous work. We demonstrate that common *conventions* (i.e. coordination equilibria) can emerge in intercultural interactions, even when agents choose from different action sets. However, social coordination can also *permanently* break down, even when coordination is Pareto-efficient. Indeed, there exist distributions of preferences for which recurrent miscoordination is the unique stochastically stable outcome. This phenomenon is clearly different to the selection of a Pareto-inefficient coordination equilibrium when agents use the same action set (Kandori, Mailath & Rob 1993, Young 1993). Here, play might not settle into any coordination equilibrium, even though every coordination equilibrium Pareto dominates miscoordination.

Therefore, restrictive cultures impose a cost upon adherents by inhibiting their ability to coordinate with out-group members. Why then do people choose to adopt cultures that restrict their choice? Adopting a restrictive culture can increase the welfare of adherents by committing them to taking actions that they most prefer in social interactions. The idea that precommitment can yield a strategic advantage is due to Schelling (1960) and is well established in game theory. It has found applications in industrial organization (e.g. Spence 1977, Dixit 1979), international relations and many other fields. However, the view of group membership as a strategic commitment has not featured in theories of culture. In our (cultural) evolutionary setting, restrictive cultural groups build up a reputation for choosing from within a small set of permissible actions. This means that if members of a more permissive group want to coordinate with members of a restrictive group in social interactions, they have to conform to the in-group norms of the restrictive group. In this way, cultural groups can improve the welfare of members by limiting their options.

The strategic commitment role of culture enables restrictive cultures to survive evolutionary selection. We extend our model to analyse the case in which agents are "exposed" to other cultures, and are able to adopt a new culture.[4] We demonstrate that in a society which begins with three cultural groups, at most two cultures survive in the long run: one restrictive culture and one permissive culture. Because the permissive culture can coordinate with the restrictive culture, coordination is achieved with probability one in every intercultural interaction. Therefore, surprisingly, the survival of restrictive cultures

[4]When an individual adopts a new culture it should be said that she is resocialized. Schaefer & Lamm (1992, p. 113) note that this is a frequent occurrence in the human life cycle.

need not inhibit coordination between groups, when we allow for cultural choice.

The remainder of the essay is structured as follows. Section 2.2 introduces our notion of culture as a strategic commitment and reviews related literature. Section 2.3 sets out the version of our model without cultural choice, and section 2.4 analyses the conditions under which members of different cultural groups can learn to coordinate with each other. In section 2.5, we extend the model to analyse which cultural configurations survive in the long run when individuals can adopt a new culture.

2 Modelling Culture

This section sets forth our view of culture as a strategic commitment.

2.1 Culture as Commitment

According to the simple notion of culture we introduce, an adherent to culture c_k is *committed* to taking actions in $X_k \subseteq X$. We suggest that this commitment arises from the internalization of culturally defined standards of behaviour through the process of *socialization*. According to Boyd & Richerson (2005, p. 3):

> *Culture is information that people acquire from others by teaching, imitation and other forms of social learning.* On a scale unknown in any other species, people acquire skills, beliefs, and values from the people around them, and these strongly affect behaviour. People living in human populations are heirs to a pool of socially transmitted information that affects how they make a living, how they communicate, and what they think is right and wrong. [*emphasis in original*]

One way in which culture might generate commitments is by influencing what an individual believes is right and wrong, as Boyd and Richerson have noted. During socialization, standards of "right" and "wrong" are internalized by the individual (e.g. Child 1943, Durkheim 1953, Merton 1957).[5] According to Coleman (1990), the internalization of behavioural norms means that individuals come to have an "internal sanctioning system" which provides punishment when they violate standards of acceptable behaviour [p. 293]. In other words, individuals incur a psychological cost from taking actions that are deemed to be wrong (e.g. Frank 1988, Elster 1989, Akerlof & Kranton 2000).[6] In addition, Frank

[5]Henrich et al. (2004) present evidence of the internalization of culturally defined standards of behaviour from their experiments in 15 small-scale societies spread over five continents.

[6]We do not explicitly model this psychological loss here. Of course, there may be other forces which preserve these rules of conduct, such as the imposition of sanctions by the community, which we do not model here. Nevertheless, as Rao & Walton (2004) claim regarding the ubiquitous incest taboo (e.g. Freud 1950), "[m]ost people would not consider breaking it, not just because of fear of social sanctions, but simply because the taboo is so deeply ingrained within their psyches" [p. 15].

(1988) argues that emotions such as shame, guilt, and anger enforce moral commitments, making credible the carrying out of threats and promises even when it is not in a person's material interest to do so.

Thus, culture might shape internal sanctions or cognitive processes which generate what Sen (1977) aptly calls "commitments", Harsanyi (1982) calls "moral values" and Rao & Walton (2004) call "constraining preferences." In effect, culture "limits choice sets" (Henrich et al. 2001, p. 357). In our model, individuals belonging to a particular cultural group must observe its commitments or taboos. Bowles (2001) points out that "[t]o explain why a person chose a point in a budget set, for example, one might make reference to her craving for the chosen goods, or to a religious prohibition against the excluded goods" [p. 157]. Clearly, individuals are not always compliant, and deviant behaviour is observed in real social settings. Nevertheless, individuals that reject social controls and experience no guilt or remorse when transgressing socially acceptable standards of behaviour constitute only 3-4 per cent of the male population and less than 1 per cent of the female population in the United States (Mealey 1995). These individuals are known as sociopaths (or psychopaths) in the literature.

2.2 Cultural Choice

The view of culture as commitment leads to a rationale for cultural choice. While this notion of culture has some support in the existing literature (though it has not been emphasized on the whole), the strategic role of cultural group membership that it implies has not been explored. In fact, culture has largely been treated as either a *fixed inheritance* (see Sen 2006) or as something that is *passively* acquired through social contact.

The formal analysis of cultural transmission was pioneered by Cavalli-Sforza & Feldman (1981) and Boyd & Richerson (1985, 2005). These social contagion models, including the agent-based model developed by Axelrod (1997), feature agents who do not engage in *intentional* choice, but are passive carriers of traits which are transmitted through social interaction like a virus (see also Coleman 1964, Bass 1969, Dodds & Watts 2005). We see an analogy between the acquisition of traits via social contact and socialization, which we believe is an important process in cultural evolution. On the other hand, intentional individual choice is an indispensable ingredient in models of social dynamics; agents are not really passive, even in the process of socialization.[7]

Therefore, we adopt a framework of *bounded rational choice*, and introduce to it a simple

[7]Dennis Wrong (1961) has argued persuasively against an "over-socialized" view of the individual, pointing out that while "socialization means transmission of culture, . . . this does not mean that [individuals] have been *completely molded* by the particular norms and values of their culture."

notion of culture. Agents adapt their behaviour in an intentional manner, but within culturally accepted bounds of behaviour which are internalized during socialization. Moreover, agents can rationally choose to adopt a different culture by switching group membership. In fact, due to the commitment role of culture, agents may strategically limit their options by joining restrictive cultural groups.

2.3 Related Research

Bisin & Verdier (2001) were possibly the first to develop a model of cultural transmission which combines socialization and rational choice. As in Cavalli-Sforza & Feldman (1981), agents are endowed with one of two cultural traits and each agent in the younger generation is exposed to a 'model' which is randomly selected from the older generation. Bisin & Verdier's innovation is to allow each parent to choose costly socialization effort τ (e.g. cultural education, segregation) in order to increase the chance that their offspring inherits their cultural trait. An agent retains their parent's trait with probability τ and acquires the trait borne by the model to whom they are exposed with the complementary probability. Bisin & Verdier show that the unique asymptotically stable distribution of traits is a polymorphic distribution, with positive weight on both traits. So by exerting greater socialization effort, minority cultures can survive the spread of the majority trait via social contagion. As with Bisin & Verdier, our model allows for intentionality in cultural choice. But whereas agents in Bisin & Verdier's model simply take a (non-strategic) decision based upon their cultural trait, we allow agents with different cultural traits to strategically interact by playing a coordination game. The evolution of play in the coordination game can make it profitable to switch cultural groups. In contrast, parents always prefer that their offspring inherit their cultural trait in Bisin & Verdier's formulation. The fact that multiple cultures can coexist in the long run in our model, is due to the benefits of cultural *commitment* on payoffs in the coordination game.[8]

As in this essay, Kuran & Sandholm (2008) develop a model in which social interactions take the form of a coordination game and agents have idiosyncratic preferences over actions. Agents in their model compromise between their ideal action and the action that yields the largest coordination payoff. However, culture in Kuran & Sandholm's model does not restrict an agent's choice set.[9] As such, there is no breakdown of Pareto-

[8]Axelrod (1997) develops an agent-based model in which multiple cultures can coexist in the long run. However, as he notes, this result is not robust to cultural mutations, in which players can randomly acquire mutant traits. In our model, multiple cultures can coexist even in the presence of mistakes and random experimentation on the part of agents. See also Bowles & Gintis (2004) on how ethnocentric networks can survive alongside an anonymous market offering unrestricted trading opportunities.

[9]Rather, culture is characterized by a distribution of preferences and actions among members of a cultural group.

efficient coordination. Another point of difference is that Kuran & Sandholm assume preferences evolve so that an agent's ideal action converges to their actual action over time. In our model, individual preferences remain fixed and it is agents' choice sets – or equivalently the composition of cultural groups – which evolve.

3 The Model

3.1 Social Interactions

Players. We analyse a recurrent game, in which there are n *roles* that may be filled by a changing cast of players (Jackson & Kalai 1997, Young 1998). Over time, we can think of players "dying" and their roles being filled by incoming players who inherit their predecessor's culture. This is analogous to the vertical transmission of culture from parent to child. For convenience, we speak of n players (rather than roles), where $n \geq 2$ and finite. The set of players is denoted by N, with members indexed by i.

Cultures. Each player belongs to one of K groups in any given period, denoted by $c \in C = \{c_1, c_2, ..., c_k, ..., c_K\}$. Each group has a different culture which is shared by its members. A player belonging to cultural group c_k is referred to as a c_k-member. The set of c_k-members is N_k, where $|N_k| = n_k$ and $\sum_k |N_k| = n$. A cultural group c_k is said to be *empty* if it has no members, i.e. $n_k = 0$. Each culture has its own visible *marker*, and individuals in a two-player interaction observe each others' cultural markers *prior* to selecting an action.

Actions. The set X, with cardinality L, is the set of all *pure* strategies that can potentially be taken in a two-player game. A player belonging to culture c_k must choose an action x from the set of pure strategies, X_k, which are acceptable to her culture, where $X_k \subseteq X$. This is the key assumption in our model:

Condition 1. *(Commitment)* A c_k-member in period t is *committed* to choosing from the set of actions, X_k, when interacting in period t.

Such a commitment might come about through the internalization of culturally accepted standards of behaviour during socialization, or as a result of sanctions imposed by the group, though we do not model this process here. Another interpretation, which is completely consistent with our model, is that this 'commitment' effect is induced by arbitrarily high costs to choosing an inadmissible action, due to sanctions imposed by the group. We shall not pursue this interpretation any further.

An agent's admissible choice set X_k is not assumed to be common knowledge. All that we require in this respect is that each player knows which actions they themselves are permitted to take in the current period.

The following definitions enable us to characterize cultures via the subset of actions they proscribe:[10]

RESTRICTIVENESS. A culture c_k is *permissive* if $|X_k| = L$.

COMPATIBILITY. Two cultures c_j and c_k are *radically opposed* if $X_j \cap X_k = \emptyset$.

Payoffs. Players in social interactions benefit from coordinating their actions. In addition, individuals have idiosyncratic preferences over possible actions, that are independent of the action taken by their partner in a social interaction. Formally, payoffs in social interactions are given by $u_i : X_j \times X_k \to \mathbb{R}$, for all $i \in N$ and $1 \leq j, k \leq K$. We assume u_i satisfies the usual von Neumann-Morgenstern axioms and takes the following form:

$$u_i(x, x_{-i}) = \mathscr{I} + \delta_{i,x}$$

Let $\mathscr{I}$ be player i's *interactive payoff*, where $\mathscr{I} = 1$ if $x = x_{-i}$ and zero otherwise. Therefore, the interactive payoff to each player is normalized to one for coordination, and zero for miscoordination. In addition, player i receives an *idiosyncratic payoff* equal to $\delta_{i,x}$ from playing action x. This idiosyncratic payoff represents player i's predisposition to taking certain actions.

We say that player i "most prefers" an action $\tilde{x}$, if $\tilde{x} \in argmax_{x \in X} \delta_{i,x}$. A player i most prefers an X_k action $\tilde{x}_k$, if $\tilde{x}_k \in argmax_{x \in X_k} \delta_{i,x}$. We treat an individual's idiosyncratic preferences as exogenous and private information to that individual. We impose the following non-degeneracy condition [ND] on idiosyncratic preferences:

Condition 2. *(Non-Degeneracy)* $\delta_{i,x} - \delta_{i,x'} < 1$ for all players i and for all actions $x, x' \in X$.

Due to players' idiosyncratic preferences, each social interaction is a different game. When ND is violated, not all social interactions take the form of a coordination game, since x is a strictly dominated strategy when $\delta_{i,x'} - \delta_{i,x} > 1$, for some $x \neq x'$. We are interested in social interactions that take the form of a coordination game, so we restrict our attention to preference distributions that satisfy ND. This mild condition is all we require on the distribution of preferences. When $\delta_{i,x} = 0$ for all players i and actions x, all social interactions take the form of a pure coordination game. When individual preferences are heterogeneous and condition ND is satisfied, then social interactions take the form of a

[10] More precisely, culture is information which shapes beliefs and mental structures, which in turn determine behaviour. For our purpose, we simply identify a culture by the particular behaviour through which it gains expression, and treat the intermediate stages in which beliefs and mental structures are formed as a black box. So in the interests of expositional convenience we at times speak loosely of culture as if it were an entity "proscribing actions" and occasionally equate culture with behaviour.

coordination game with conflicting interests, exemplified by the *Battle of the Sexes*.

3.2 Adaptive Choice

The structure of social interactions is based upon Young's (1993) *adaptive play*:

Timing. Time is discrete and denoted by $t = 1, 2, 3, \ldots$ Each period, two players are selected to engage in a social interaction. Each pair of players is selected with equal probability, regardless of the players' cultural affiliations.[11]

Information. A player selected to play a coordination game, forms an expectation of her partner's action in the social encounter, by looking back at the *history* of play. The player cannot isolate information on her partner's previous plays. However, she can observe her partner's cultural marker. Therefore, in our model players base their expectations on the prior plays of members of their partner's culture. We say that a *j-k interaction* occurs when a c_j-member and a c_k-member are paired to play a coordination game. We denote the history of play in a particular j-k interaction as follows:

HISTORIES. The j-k *history* in period t is denoted by $h^t_{j-k} = (\mathbf{x}^{t,-m}_{j-k}, \ldots, \mathbf{x}^{t,-1}_{j-k})$, which is the record of play in the m previous j-k interactions. Play in the most recent j-k interaction is captured by the action-tuple $\mathbf{x}^{t,-1}_{j-k} = (x^{t,-1}_{j,k}, x^{t,-1}_{k,j})$.

The individual action denoted by $x^{t,-1}_{j,k}$ is at time t the most recent action taken by a c_j-member in a j-k interaction, and $x^{t,-m}_{j,k}$ is the m^{th} most recent such action. In a k-k interaction, in which members of the same culture are paired, players are randomly allocated to the "first player" and "second player" positions, where the action taken by the first player is recorded as the first element of $\mathbf{x}^t_{k-k}$.[12] The history of play at time t, which is denoted by the vector $h^t = (h^t_{j-k})_{j \leq k}$, is a record of the previous m action-tuples played in every j-k interaction. This is not the same as in the canonical model (Young 1993), in which agents remember the actions played in the last m periods. If we were to retain the standard formulation, a particular j-k interaction could occur in m consecutive periods, leaving the j'-k' histories empty, for all $j' \neq j$ and $k' \neq k$.[13]

[11] Therefore, cultural membership does not define a local interactions structure in which individuals are more likely to interact with adherents to their own culture.

[12] By "random", we mean that the draw is from a distribution with full support on the relevant set.

[13] According to our formulation, play in a j-k interaction in period t' may be part of the period t history, while play in a j'-k' interaction in period $t'' > t'$ has been forgotten, if more j'-k' interactions occur in recent periods. We do not consider this to be implausible. For example, a person might remember her last encounter with an old friend which occurred years ago, while forgetting a more recent encounter with a friend she meets weekly. Our approach also ensures the independence of play in each j-k interaction.

When forming expectations, players in a j-k interaction obtain fragmented information on plays in prior j-k interactions by sampling the j-k history. In concrete terms, we can think of agents asking around about the experiences of players in earlier periods.[14] We formalize this as follows. In a j-k interaction which occurs in period t, the c_k-member draws a sample of size s (without replacement) from the j-k history h^t_{j-k}, and specifically from the last m actions taken by c_j-members in interactions with c_k-members. The c_j-member does the same, except that she samples from the actions taken by c_k-members in the m previous j-k interactions.[15] Apart from mistakes, both players independently choose (myopic) best replies to their resulting sample proportions, as illustrated below.

Social Behaviour. Agents are boundedly rational, in the sense that they *myopically* maximize their expected *current* period payoff when choosing an action in a social interaction. The c_k-member in a j-k interaction forms her expectation after drawing a sample of size s from the j-k history, h^t_{j-k}. This occurs with high probability $(1-\varepsilon)$. The agent then calculates the proportion, $\widehat{p}_{j,k}(x)$, of c_j-members playing action x in j-k interactions, for each $x \in X_j$. The c_k-member adopts this as a maximum likelihood estimate of the behavioural strategy used by a c_j-member in a j-k interaction. Therefore, the expected payoff to the c_k-member i from playing action x in a j-k interaction is $\widehat{p}_{j,k}(x)+\delta_{i,x}$. This in turn yields the set of pure strategies, $argmax_{x\in X_k}\{\widehat{p}_{j,k}(x)+\delta_{i,x}\}$, which maximize player i's expected current period payoff, given her sample information. When there are ties in best replies, each best reply is played with equal probability. With low probability ε, the c_k-member (resp. c_j-member) instead chooses an action in X_k (resp. X_j) at random, for reasons outside of the model.

Coordination and Miscoordination. A convention is a strict Nash equilibrium that has been played by the entire population for as long as anyone can remember (Young 1993). This means that any possible sample drawn from the history of play will be identical, and yield the same best reply, namely the conventional action itself. Let $\mathbf{x}^*_{j-k}$ be a strict Nash (coordination) equilibrium of a coordination game between c_j-members and c_k-members, where $\mathbf{x}^*_{j-k}$ indicates that $x_{j,k} = x_{k,j} \in X_j \cap X_k$.[16] We define a convention as follows:

Convention. A *convention* is a history h^*_{j-k} in which a strict Nash equilibrium $\mathbf{x}^*_{j-k}$ is played in m consecutive j-k interactions. We say the j-k *convention* is "x" when $h^*_{j-k} = (\mathbf{x}^*_{j-k}, \mathbf{x}^*_{j-k}, \ldots, \mathbf{x}^*_{j-k}) = ((x,x),(x,x),\ldots,(x,x))$, where $x \in X_j \cap X_k$.

[14] It should be clear, however, that we do not model this as an optimal search process, but rather the agent's information is considered to be a property of her environment.

[15] In a k-k interaction we assume that a player allocated to the first (resp. second) player position, randomly samples s of the m most recent plays by individuals occupying the second (resp. first) player position.

[16] Notice that even though idiosyncratic preferences mean each social interaction is a different game, condition ND ensures that the strict Nash equilibria of each game are the same for all interactions between c_j-members and c_k-members.

When cultures c_j and c_k are radically opposed, there are no *mutually admissible* actions that players can take, i.e. $X_j \cap X_k = \emptyset$, so a convention cannot arise, and players in a j-k interaction are doomed to perpetual miscoordination. We characterize such a state of miscoordination as follows:

State of Miscoordination. A *state of miscoordination* is a j-k history $h^{mc}_{j-k} = ((x^1_j, x^1_k), \ldots, (x^\ell_j, x^\ell_k), \ldots, (x^m_j, x^m_k))$ in which $x^\ell_j \notin X_k$ and $x^\ell_k \notin X_j$ for all $1 \leq \ell \leq m$.

3.3 Dynamical Process

As individuals interact recurrently, their adaptive behaviour gives rise to a particular dynamical process at the population level, which we will now characterize. The state in period t, $z^t = (h^t)$, specifies the history of play in period t. The associated state space is $Z = \prod_{j \leq k}(X_j \times X_k)^m$, where $(X)^m$ denotes the m-fold product of X. Clearly, Z is finite for finite K and m. We assume that the process begins in some initial state z^0 in which there are at least m plays in each j-k interaction, where the sequence of plays is otherwise arbitrary. There are well-defined, time-homogeneous transition probabilities between all pairs of states z, z', denoted by $P_{z,z'}$. Therefore, the adaptive process we have defined is a finite Markov chain, with a $|Z| \times |Z|$ transition probability matrix $P^{m,s,\varepsilon}$. For convenience, we will denote the unperturbed ($\varepsilon = 0$) process by P^0 and the perturbed process by P^ε.

P^ε is a regular perturbed Markov process (Young, 1993). When $\varepsilon > 0$, all pairs of states communicate, so the process P^ε is irreducible. This implies that the process has a unique recurrence class, the entire state space, so the Markov chain is ergodic, i.e. its limiting distribution is independent of the initial state z^0. Moreover, the Markov process P^ε has a unique stationary distribution μ^ε. The perturbed process is aperiodic, since there is a positive probability of remaining in any given state. This implies that not only does the relative frequency with which a state z is visited *up through* time t converge to the frequency given by the unique stationary distribution μ, but so does the probability of being in state z *at* time t, provided that t is sufficiently large. We rely on the following equilibrium concept due to Foster & Young (1990):

Stochastic Stability. A state is *stochastically stable* if it is in the support of $\mu = lim_{\varepsilon \to 0}\mu^\varepsilon$.

4 Coordination and Miscoordination

Can members of different cultural groups learn to coordinate with each other? In this section, we focus on the following case: (i) there are two cultures c_1 and c_2, and social interactions are always 1-2 interactions in which a c_1-member and a c_2-member are matched, (ii) there is no migration between cultural groups. In this case, we recover a process that is similar to Young's (1993) *adaptive play*, except that certain actions are proscribed by a player's culture.[17]

4.1 The Possibility of Long-Run Miscoordination

This subsection analyses the long-run behaviour of the *unperturbed* Markov process. We denote by P^0_{1-2} the 1-2 process under conditions (i) and (ii). The state of P^0_{1-2} in period t is fully characterized by the 1-2 history in that period, that is $z^t = h^t_{1-2}$. The process operates on the finite state space H_{1-2}, which is the set of all possible 1-2 histories defined as follows: $H_{1-2} = (X_1 \times X_2)^m$, where $(X)^m$ denotes the m-fold product of X.

By studying the behaviour of P^0_{1-2}, we can isolate the effect on the evolution of play of (a) cultural restrictions on individual behaviour, and (b) substantial heterogeneity in individual preferences. In Young's (1993) adaptive play, coordination is always achieved in the long run. In a moment, we will show that social conventions can emerge despite cultural restrictions and substantial heterogeneity in agents' preferences. However, the introduction of cultural restrictions yields a new long-run possibility, in which coordination permanently breaks down, even when cultures c_1 and c_2 are not radically opposed. It will be straightforward to generalize these results to a society with $K > 2$ cultures (see Section 2.5).

First, we need to introduce some definitions. We label the mutually admissible actions in $X_1 \cap X_2$ as $x_1, x_2, \dots x_\ell, \dots x_\mathcal{L}$, so that there are $\mathcal{L} \leq L = |X|$ possible conventions indexed by ℓ.

A c_k-member i maximizes her idiosyncratic payoff, without regard to coordination, if she simply chooses her *most preferred* X_k action $\tilde{x}_{i,k} \in argmax_{x \in X_k} \delta_{i,x}$. Let $\tilde{X}_k = \{\tilde{x} \mid \tilde{x} \in argmax_{x \in X_k} \delta_{i,x}$ for some $i \in N_k\} \subseteq X_k$. We can now define a further pairwise relation on the set of cultures:

[17] We will show (in a corollary) that our results apply to the case in which players from the same culture are matched in a social interaction. This yields one further departure from Young's approach. In our model, players in a k-k interaction are drawn from a single population, whereas in Young's version they are always drawn from separate populations.

WEAK OPPOSITION. Cultures c_j and c_k are *weakly opposed* if and only if $\tilde{X}_j \cap X_k = \emptyset$ and $\tilde{X}_k \cap X_j = \emptyset$.

If cultures c_j and c_k are radically opposed, i.e. $X_j \cap X_k = \emptyset$, then they are also weakly opposed, since $\tilde{X}_j \subseteq X_j$ and $\tilde{X}_k \subseteq X_k$. However, weak opposition unlike radical opposition depends on the distribution of idiosyncratic preferences in a cultural group.

Define the set $\mathcal{M} = (\tilde{X}_1 \times \tilde{X}_2)^m$. When c_1 and c_2 are weakly opposed, $\tilde{X}_1 \cap X_2 = \emptyset$ and $\tilde{X}_2 \cap X_1 = \emptyset$, so each state in $\mathcal{M}$ is a *state of miscoordination* (see section 2.3.2). Recall that a recurrence class of a Markov process is a set of states, each of which is accessible from any other state within the class, and for which no state outside the class is accessible from any state within the class.[18] In a moment, we will show that $\mathcal{M}$ is the *unique* recurrence class of P^0_{1-2}, when c_1 and c_2 are radically opposed. Since all states in $\mathcal{M}$ are states of miscoordination when the cultures are at least weakly opposed, and the process can become "locked into" $\mathcal{M}$, we refer to this long-run possibility as *recurrent miscoordination.* That recurrent miscoordination arises when cultures are radically opposed is not very surprising. Indeed, we will show that when cultures c_1 and c_2 are not radically opposed, social interactions are somewhat more conducive to coordination, in that the conventions are recurrence classes of P^0_{1-2}, despite cultural restrictions on behaviour. However, the striking result is that even when the cultures are only *weakly* opposed, recurrent miscoordination $\mathcal{M}$ remains a recurrence class. We have the following result:

Proposition 1 *Suppose $s/m \leq 1/2$. Then P^0_{1-2} converges almost surely to: (i) a convention h_1^*, h_2^*, … or h_L^*, if the cultures are not weakly opposed, (ii) a convention or recurrent miscoordination $\mathcal{M}$ if the cultures are weakly opposed, but not radically opposed, (iii) $\mathcal{M}$ if the cultures are radically opposed.*

The proof of Proposition 1 proceeds in two steps. First, we identify the conditions under which the conventions and the set $\mathcal{M}$ are recurrence classes of P^0_{1-2}. We then show that if $s/m \leq 1/2$, these are the only recurrence classes of the process.[19] Any finite Markov chain such as P^0_{1-2} converges almost surely to one of its recurrence classes. So this suffices to establish the Proposition.

[18] Also recall that a state z is accessible from state z', if there is a positive probability of moving from state z to z' in a finite number of periods.

[19] By making s/m sufficiently small, we ensure that there is enough randomness in the system to shake the process out of any other possible cycle. We do not claim that the bound $s/m \leq 1/2$ is the tightest possible bound for all preference distributions, only that it is sufficient to deliver the result without placing further restrictions on preferences.

Lemma 1 *(i) The conventions are recurrence classes of* P^0_{1-2} *if and only if the cultures are not radically opposed, (ii)* $\mathcal{M}$ *is a recurrence class if and only if the cultures are weakly opposed.*

Proof. To establish part (i), recall that a best reply for a c_1-member is an action $x^* \in argmax_{x \in X_1}\{\widehat{p}_{2,1}(x) + \delta_{i,x}\}$. A convention is a state of the form $((x,x),(x,x),\ldots(x,x))$, where $x \in X_1 \cap X_2$. So a convention can only arise if $X_1 \cap X_2 \neq \emptyset$, that is when c_1 and c_2 are not radically opposed. Without loss of generality, suppose an x_1 convention is in place. Then for all possible samples from h_1^* drawn by a c_1-member, the sample proportion of x_1 plays is $\widehat{p}_{2,1}(x_1) = 1$ and the sample proportion of x plays is $\widehat{p}_{2,1}(x) = 0$ for all $x \neq x_1$. So the expected payoff from playing action x_1 to a representative c_1-member i, for all possible samples when an x_1 convention is in place, is $\widehat{p}_{2,1}(x_1) + \delta_{i,x_1} = 1 + \delta_{i,x_1}$. The expected payoff from playing any action $x \neq x_1$ is $\delta_{i,x}$. According to condition ND, $|\delta_{i,x} - \delta_{i,x'}| < 1$ for all players i and for all actions $x, x' \in X$. This implies that $1+\delta_{i,x_1} > \delta_{i,x}$ for all i and x. Therefore, the unique best reply for any $i \in N$ is always x_1 when an x_1 convention is in place. In the unperturbed case ($\varepsilon = 0$) we are considering, there is a zero probability that a non-best reply is played. So no matter which players are drawn, the successor state to a convention $((x_1,x_1),(x_1,x_1),\ldots(x_1,x_1))$ is certainly $((x_1,x_1),(x_1,x_1),\ldots(x_1,x_1))$, and each convention is an absorbing state (i.e. a singleton recurrence class).

We shall now establish part (ii). When the cultures are weakly opposed, $\tilde{X}_1 \cap X_2 = \emptyset$ and $\tilde{X}_2 \cap X_1 = \emptyset$, so each state in $\mathcal{M} = (\tilde{X}_1 \times \tilde{X}_2)^m$ is a state of miscoordination. Now consider such a state in $\mathcal{M}$. For all possible samples drawn by c_1-member from a state of miscoordination, the sample proportion of action x is zero for all $x \in X_1$, since all plays by c_2-members in such a history are not in X_1. Therefore, the best reply for all c_1-members to a state of miscoordination is always to choose an action $\tilde{x}_{i,1} \in argmax_{x \in X_1}\delta_{i,x} \subseteq \tilde{X}_1$. Similarly, the best reply for all c_2-members to a state of miscoordination is always to choose an action $\tilde{x}_{i,2} \in argmax_{x \in X_2}\delta_{i,x} \subseteq \tilde{X}_2$. So once a state of miscoordination is reached, the action-tuple played in the next period is certainly in $\tilde{X}_1 \times \tilde{X}_2$, and can be any pair in $\tilde{X}_1 \times \tilde{X}_2$. This implies that no state outside $\mathcal{M} = (\tilde{X}_1 \times \tilde{X}_2)^m$ is accessible from any state in $\mathcal{M}$. Furthermore, every state in $\mathcal{M}$ is accessible from every other state in $\mathcal{M}$. Hence $\mathcal{M}$ is a recurrence class of P^0_{1-2} if c_1 and c_2 are weakly opposed.

Notice that if the cultures are not weakly opposed, then there exists an action $x' \in \tilde{X}_1 \cap X_2$. This implies that $((x',x),(x',x),\ldots(x',x))$ is in $\mathcal{M}$ for all $x \in \tilde{X}_2$. But all possible samples from such a state which can be drawn by a c_2-member, consist of s plays of $x' \in X_1 \cap X_2$. It follows from the proof of Lemma 2, case 1 (see the Appendix), that P^0_{1-2} transits from this point to the x' convention with positive probability in at most another m periods. By part (i) of Lemma 1, each convention is an absorbing state. This means that no other state in $\mathcal{M}$ is accessible from a convention. Therefore, if c_1 and

c_2 are not weakly opposed, the set $\mathcal{M}$ is not a recurrence class, unless it is simply the x' convention. □

To understand why recurrent miscoordination is a recurrence class when c_1 and c_2 are weakly opposed, suppose that the process is in a state of miscoordination. For all possible samples, both individuals selected to play expect that the probability of coordinating is zero. As far as they know, c_1-members have always taken actions that are inadmissible for c_2-members, and *vice versa*. Therefore, both individuals play one of their most preferred actions. If c_1 and c_2 are weakly opposed, then no most preferred action is in $X_1 \cap X_2$, so the process remains in a state of miscoordination. If the cultures are not weakly opposed, then at least one player will take a mutually admissible action in $X_1 \cap X_2$, so the unperturbed process can exit from a state of miscoordination, and a convention can emerge.

In conjunction with Lemma 1, the following suffices to establish Proposition 1:

Lemma 2 *If $s/m \leq 1/2$, then a set of states $\mathscr{X} \subset Z$ is a recurrence class of P^0_{1-2} only if it is a convention or $\mathcal{M}$.*

The proof is in the Appendix.

Proposition 1 fully characterizes the asymptotic behaviour of P^0_{1-2}. Firstly, members of different cultural groups can learn to coordinate with each other. Specifically, conventions can emerge in intercultural interactions despite cultural restrictions on behaviour, even when the cultures are weakly opposed. Secondly, we have derived the results with only a mild condition (ND) on the distribution of preferences. Therefore, our analysis incorporates a far greater degree of heterogeneity in individual preferences than does prior work. Young (1998) studies 2×2 coordination games in which preferences can vary between disjoint classes of players, but players within each disjoint class have the same preferences. Our analysis goes substantially further by allowing every individual to have different preferences. Thirdly, we have uncovered an interesting long-run possibility that has not been identified in previous work. Coordination between members of different cultural groups can *permanently* break down even where it need not, and even though coordination is *Pareto-efficient*.

The risk of "clashes" between members of disparate cultural communities has been emphasized by several influential commentators (e.g. Huntington 1993). We have shown how an analogous situation can arise within the framework of a (modified) coordination game.[20] In our model, permanent miscoordination can arise after a string of failed so-

[20] Since we partition society on the basis of culture alone, this result can also be viewed as support

cial interactions, which leads c_1-members to lose confidence that c_2-members will play a mutually admissible action, and *vice versa*. This can happen even when $X_1 \cap X_2 \neq \emptyset$, so that coordination can be achieved within cultural restrictions. Nevertheless, if initially all players are confident enough of achieving coordination, a convention might emerge which coordinates behaviour in social interactions.

The analysis so far has been conducted for interactions between members of two different cultures c_1 and c_2. We now derive the corresponding result for the evolution of play in interactions between adherents to the same culture.

Corollary 1 *Suppose $s/m \leq 1/2$. Then P^0_{k-k} converges almost surely to a convention.*

The proof is in the Appendix. Corollary 1 shows that a recurrent miscoordination cannot arise in interactions between adherents to the same culture, for *any* distribution of individual preferences. We attribute the fact that players share the same set of admissible actions to shared beliefs regarding acceptable behaviour. Therefore, this result captures the intuition that coordination in social interactions is more likely when individuals have the same cultural values.

4.2 Stochastically Stable Miscoordination

We have established that coordination between members of cultural groups that are weakly opposed can permanently break down. However, as long as the cultures are not radically opposed, the conventions remain recurrence classes of P^0_{1-2}. So we might expect that recurrent miscoordination is a tenuous phenomenon when cultures c_1 and c_2 are not radically opposed. In this section, we employ the stochastic stability framework to show that, on the contrary, there exist a range of distributions of individual preferences for which *recurrent miscoordination is the most likely outcome of play in the long run.*

For this purpose, we introduce the possibility that players make a mistake or engage in random experimentation with probability $\varepsilon > 0$. This gives rise to the perturbed process P^ε_{1-2}. Taking the limit $\varepsilon \to 0$ allows us to make sharp statements about the asymptotic behaviour of P^ε_{1-2}. We will also make use of two new conditions on the distribution of individual preferences that will jointly determine the stability of recurrent miscoordination.

First, let us define what we shall call the *weakest* idiosyncratic preference for miscoordination $\mathcal{M}$. This will determine the ease with which the process can exit the basin of

for Sen's (2006) claim that the "clash of civilizations" thesis follows naturally from a singular concept of identity.

attraction of $\mathcal{M}$. Recall that $X_1 \cap X_2 = \{x_1, \ldots x_\ell, \ldots x_\mathcal{L}\}$. Define:

$$\underline{\delta}_{1,\ell} = \min_{i \in N_1}(\max_{x' \in X_1 \setminus X_2} \delta_{i,x'} - \delta_{i,x_\ell})$$

Intuitively, $\underline{\delta}_{1,\ell}$ represents the *weakest* idiosyncratic preference for miscoordination over coordination on x_ℓ, among c_1-members. Similarly, we can define the corresponding variable for c_1-members, $\underline{\delta}_{2,\ell} = \min_{i \in N_2}(\max_{x' \in X_2 \setminus X_1} \delta_{i,x'} - \delta_{i,x_\ell})$. Let $\underline{\delta}_\ell = \min\{\underline{\delta}_{1,\ell}, \underline{\delta}_{2,\ell}\}$ represent the *weakest* idiosyncratic preference for miscoordination over coordination on x_ℓ, among all players. Then $\underline{\delta} = \min_{1 \leq \ell \leq \mathcal{L}} \underline{\delta}_\ell$ represents the *weakest* idiosyncratic preference for miscoordination. Notice that c_1 and c_2 are weakly opposed if and only if $\underline{\delta} > 0$. Suppose $\delta \leq 0$. Then for some player i, say in culture c_1, there exists a mutually admissible action x_ℓ such that $\delta_{i,x_\ell} \geq \delta_{i,x'}$ for all $x' \in X_1 \setminus X_2$. But that means that at least one of i's most preferred actions is a mutually admissible action, i.e. $\widetilde{X}_1 \cap X_2 \neq \emptyset$, so the cultures are not weakly opposed.

Second, let us define what we shall call the *strongest* idiosyncratic preference for miscoordination $\mathcal{M}$. This will determine the ease with which the process can enter the basin of attraction of $\mathcal{M}$. Let:

$$\overline{\delta}_{1,\ell} = \max_{i \in N_1}(\max_{x' \in X_1 \setminus X_2} \delta_{i,x'} - \delta_{i,x_\ell})$$

Define $\overline{\delta}_{2,\ell}$ similarly. Also, let $\overline{\delta}_\ell = \max\{\overline{\delta}_{1,\ell}, \overline{\delta}_{2,\ell}\}$. Then $\overline{\delta} = \min_{1 \leq \ell \leq \mathcal{L}} \overline{\delta}_\ell$ represents the *strongest* idiosyncratic preference for miscoordination. To see this suppose that $\underline{\delta}_{1,\ell} < \underline{\delta}_{2,\ell}$ and $\overline{\delta}_{1,\ell} > \overline{\delta}_{2,\ell}$ for all ℓ, $1 \leq \ell \leq \mathcal{L}$ (this simplifies the condition somewhat for inspection). In this case:

$$\underline{\delta} = \min_{x_\ell \in X_1 \cap X_2} \min_{i \in N_1}(\max_{x' \in X_1 \setminus X_2} \delta_{i,x'} - \delta_{i,x_\ell})$$

$$\overline{\delta} = \min_{x_\ell \in X_1 \cap X_2} \max_{i \in N_1}(\max_{x' \in X_1 \setminus X_2} \delta_{i,x'} - \delta_{i,x_\ell})$$

We now demonstrate that when the *weakest* and *strongest* idiosyncratic preference for miscoordination are sufficiently intense, then miscoordination is the most likely outcome of play:

Proposition 2 *Suppose c_1 and c_2 are weakly opposed and $s/m \leq 1/2$. If $\underline{\delta} + \overline{\delta} > 1$, then for s (and m) sufficiently large, recurrent miscoordination $\mathcal{M}$ is the unique stochastically stable class of P^ε_{1-2}.*

Proof. By Proposition 1, when c_1 and c_2 are radically opposed, $\mathcal{M}$ is the unique recurrence class of P^0_{1-2}. This directly implies that $\mathcal{M}$ is the unique stochastically stable class of P^{ε}_{1-2}, in this case. Now assume that c_1 and c_2 are not radically opposed. In this case, $X_1 \cap X_2 \neq \emptyset$ and we write $X_1 \cap X_2 = (x_1, \ldots x_\ell, \ldots x_\mathcal{L})$. In addition, assume that $s/m \leq 1/2$, and c_1 and c_2 are weakly opposed. By Proposition 1 part (ii), the only recurrence classes of P^0_{1-2} are the conventions $h^*_1, h^*_2, \ldots h^*_\mathcal{L}$ and the recurrent miscoordination $\mathcal{M}$. So the stochastically stable classes are among these (Young 1993). The perturbed process P^{ε}_{1-2} can transit from a state in one recurrence class to a state in another, with a sequence of erroneous plays. Define the *resistance* $r(z, \mathcal{M})$ of the transition $z \to \mathcal{M}$ as the minimum number of errors required for the process to transit from state z to a state in $\mathcal{M}$.

To prove the Proposition, we employ the *radius-coradius* technique of Ellison (2000). In our model, the *radius* of the *basin of attraction* of $\mathcal{M}$, $R(\mathcal{M})$, is the minimum resistance of a path between a state in $\mathcal{M}$ and a convention, or formally:

$$R(\mathcal{M}) = \min_{1 \leq \ell \leq \mathcal{L}} r(\mathcal{M}, h^*_\ell)$$

The *coradius* of the basin of attraction of $\mathcal{M}$, $CR(\mathcal{M})$, is the maximum resistance of a path between a convention and a state in $\mathcal{M}$:

$$CR(\mathcal{M}) = \max_{1 \leq \ell \leq \mathcal{L}} r(h^*_\ell, \mathcal{M})$$

We claim that $R(\mathcal{M}) > CR(\mathcal{M})$ if $\underline{\delta} + \overline{\delta} > 1$, for s (and m) are sufficiently large. By Ellison (2000) Proposition 1, if $R(\mathcal{M}) > CR(\mathcal{M})$ then $\mathcal{M}$ is the unique stochastically stable class of P^{ε}_{1-2}. So this would suffice to establish the Proposition.

To verify the claim, we first compute $R(\mathcal{M})$. Consider a transition between a state in $\mathcal{M}$ and h^*_ℓ. Suppose that in period t, P^{ε}_{1-2} is in a state in $\mathcal{M}$. Since c_1 and c_2 are weakly opposed, $\tilde{X}_1 \cap X_2 = \emptyset$ and $\tilde{X}_2 \cap X_1 = \emptyset$, so each state in $\mathcal{M} = (\tilde{X}_1 \times \tilde{X}_2)^m$ is a state of miscoordination. For all period t samples then, the sample proportion of any action $x \in X_1 \cap X_2$ is zero. Fix ℓ. Now suppose the c_2-member(s) drawn *erroneously* play action $x_\ell \in X_1 \cap X_2$ in the next b consecutive periods. Then in period $t + b + 1$, there is a positive probability that player $i \in N_1$ draws a sample of size s, which contains all b plays of x_ℓ. Based on this sample, i's expected payoff from playing action x_ℓ is $b/s + \delta_{i,x_\ell}$. Player i's expected payoff from an action $x' \in X_1 \backslash X_2$ is always $\delta_{i,x'}$. Since the sample proportion of any action $x \in (X_1 \cap X_2) \backslash x_\ell$ is zero, the expected payoff to i from any action $x \in (X_1 \cap X_2) \backslash x_\ell$, is $\delta_{i,x}$. Because c_1 and c_2 are weakly opposed, $argmax_{x \in X_1} \delta_{i,x}$ is contained in $X_1 \backslash X_2$ for all $i \in N_1$. Therefore, no action $x \in (X_1 \cap X_2) \backslash x_\ell$ is a best reply. Action x_ℓ is then a best reply for i, for a sample including all b erroneous plays, if $b/s + \delta_{i,x_\ell} \geq \delta_{i,x'}$ for all $x' \in X_1 \backslash X_2$, or equivalently if:

$$b \geq \max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell}) s \tag{1}$$

By (1), the minimum number of errors required for $x_\ell \in X_1 \cap X_2$ to be a best reply for i is:

$$b_i = \lceil \max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell})s \rceil \tag{2}$$

Therefore, given b_i erroneous plays by a c_2-member, there is a positive probability that $i \in N_1$ who is selected to play in period $t + b_i + 1$, samples all b_i errors, and plays a best reply $x_\ell \in X_1 \cap X_2$ to this sample. There is also a positive probability that i is selected to play in the next s consecutive periods, draws the same sample on each occasion (since $s/m \leq 1/2$), and plays the same best reply $x_\ell \in X_1 \cap X_2$ every time. It follows directly from the proof of Lemma 2, case 1 (see Appendix), that from this point the process transits to h_ℓ^* with no further errors.

The minimum number of errors required to transit from a state in $\mathcal{M}$ to h_ℓ^*, across all agents $i \in N_1$, is $b_{1,\ell}^* = \min_{i \in N_1} \lceil \max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell})s \rceil = \lceil \min_{i \in N_1} \max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell})s \rceil$. Similarly, the minimum number of errors for the transition $\mathcal{M} \rightarrow h_\ell^*$, across all $i \in N_2$, is $b_{2,\ell}^* = \lceil \min_{i \in N_2} \max_{x' \in X_2 \backslash X_1} (\delta_{i,x'} - \delta_{i,x_\ell})s \rceil$. Without loss of generality, assume $b_{1,\ell}^* < b_{2,\ell}^*$ for all ℓ, $1 \leq \ell \leq \mathcal{L}$, so that $b_{1,\ell}^*$ is the resistance of the transition $\mathcal{M} \rightarrow h_\ell^*$. $R(\mathcal{M})$ is the minimum such resistance over all ℓ:

$$\begin{aligned} R(\mathcal{M}) &= \min_{x_\ell \in X_1 \cap X_2} \lceil \min_{i \in N_1} \max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell})s \rceil \\ &= \lceil \min_{x_\ell \in X_1 \cap X_2} \min_{i \in N_1} (\max_{x' \in X_1 \backslash X_2} \delta_{i,x'} - \delta_{i,x_\ell})s \rceil \\ &= \lceil \underline{\delta} s \rceil \end{aligned} \tag{3}$$

We shall now establish an upper bound on $CR(\mathcal{M})$. Fix ℓ, and consider a transition between h_ℓ^* and a state in $\mathcal{M}$. Suppose that in period t, P_{1-2}^ε is in the x_ℓ convention $((x_\ell, x_\ell), \ldots (x_\ell, x_\ell))$. For all possible period t samples, the sample proportion of action x_ℓ is one. Now suppose the c_2-member(s) drawn erroneously plays an action $x'' \in X_2 \backslash X_1$ in the next $b \leq s$ consecutive periods. Then in period $t + b + 1$, there is a positive probability that player $i \in N_1$ draws a sample of size s, which contains all b plays of x''. Based on this sample, i's expected payoff from playing action x_ℓ is $(1 - b/s) + \delta_{i,x_\ell}$. Player i's expected payoff from an action $x' \in X_1 \backslash x_\ell$ is $\delta_{i,x'}$. So the only possible best replies for i in period $t + b + 1$ are x_ℓ or $\tilde{x} \in argmax_{x \in X_1} \delta_{i,x}$. [By hypothesis c_1 and c_2 are weakly opposed, so $\tilde{x} \notin X_1 \cap X_2$.] Action $\tilde{x}$ yields a weakly higher payoff for i than x_ℓ, for a sample including all b erroneous plays, if $\delta_{i,\tilde{x}} \geq (1 - b/s) + \delta_{i,x_\ell}$, or equivalently if:

$$b \geq (1 - (\delta_{i,\tilde{x}} - \delta_{i,x_\ell}))s \tag{4}$$

By (4), the minimum number of errors required for $\tilde{x} \in X_1 \backslash X_2$ to be a best reply for i is:

$$b_i = \lceil (1 - (\delta_{i,\tilde{x}} - \delta_{i,x_\ell}))s \rceil = \lceil (1 - \max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell}))s \rceil \tag{5}$$

The second equality follows from the definition of $\tilde{x}$. Therefore, given b_i or more erroneous plays by a c_2-member, there is a positive probability that $i \in N_1$ who is selected to play in period $t + b_i + 1$, samples all b_i errors, and plays a best reply $\tilde{x} \in X_1 \backslash X_2$ to this sample. There is also a positive probability that i is selected to play in the next s consecutive periods, draws the same sample on each occasion (since $s/m \leq 1/2$), and plays the same best reply $\tilde{x}$ every time. Since $\tilde{x} \notin X_1 \cap X_2$ and c_1 and c_2 are weakly opposed, we are in case 2(b) considered in the proof of Lemma 2. It follows that from this point, the unperturbed process P^0_{1-2} transits to a state in $\mathcal{M}$ with positive probability in at most $s + m$ additional periods. Therefore, P^ε_{1-2} can with b_i errors only, transit from h^*_ℓ to a state in $\mathcal{M}$.

The minimum number of errors across all $i \in N_1$ required to transit from h^*_ℓ to a state in $\mathcal{M}$ is at most $b^{**}_{1,\ell} = \min_{i \in N_1} \lceil (1 - \max_{x' \in X_1 \backslash X_2} (\delta_{i,x'} - \delta_{i,x_\ell}))s \rceil = \lceil (1 - \max_{i \in N_1} (\max_{x' \in X_1 \backslash X_2} \delta_{i,x'} - \delta_{i,x_\ell}))s \rceil$. Similarly, the minimum number of errors for the transition $h^*_\ell \to \mathcal{M}$, across all $i \in N_2$, is at most $b^{**}_{2,\ell} = \lceil (1 - \max_{i \in N_2} (\max_{x' \in X_2 \backslash X_1} \delta_{i,x'} - \delta_{i,x_\ell}))s \rceil$. Without loss of generality, assume $b^{**}_{1,\ell} < b^{**}_{2,\ell}$ for all ℓ, $1 \leq \ell \leq \mathcal{L}$, so that $b^{**}_{1,\ell}$ is an upper bound on the resistance of the transition $h^*_\ell \to \mathcal{M}$. $CR(\mathcal{M})$ is the maximum such resistance over all ℓ. Therefore:

$$\begin{aligned} CR(\mathcal{M}) &\leq \max_{x_\ell \in X_1 \cap X_2} \lceil (1 - \max_{i \in N_1} (\max_{x' \in X_1 \backslash X_2} \delta_{i,x'} - \delta_{i,x_\ell}))s \rceil \\ &= \lceil (1 - \min_{x_\ell \in X_1 \cap X_2} \max_{i \in N_1} (\max_{x' \in X_1 \backslash X_2} \delta_{i,x'} - \delta_{i,x_\ell}))s \rceil \\ &= \lceil (1 - \overline{\delta})s \rceil \end{aligned} \tag{6}$$

So far we have established that $R(\mathcal{M}) = \lceil \underline{\delta} s \rceil \geq \underline{\delta} s$, and that $CR(\mathcal{M}) \leq \lceil (1 - \overline{\delta})s \rceil \leq (1 - \overline{\delta})s + 1$. Therefore, $R(\mathcal{M}) > CR(\mathcal{M})$ if $\underline{\delta} s > (1 - \overline{\delta})s + 1$ or equivalently if:

$$\underline{\delta} + \overline{\delta} > 1 + \frac{1}{s} \tag{7}$$

By observation, if $\underline{\delta} + \overline{\delta} > 1$, there exists an integer $\overline{s}$ such that inequality (7) is satisfied for all $s \geq \overline{s}$. This establishes the claim, and concludes the proof. □

The intuition behind the result is as follows. It is more difficult for the process P^ε_{1-2} to exit the basin of attraction of recurrent miscoordination $\mathcal{M}$ when the weakest idiosyncratic preference for miscoordination, $\underline{\delta}$, is high. At the same time, it is relatively easy for P^{ε}_{1-2} to enter the basin of attraction of $\mathcal{M}$ when the strongest idiosyncratic preference for miscoordination, $\overline{\delta}$, is high. Therefore, when $\underline{\delta}$ and $\overline{\delta}$ are sufficiently high, it is difficult to exit the basin of attraction of $\mathcal{M}$ and easy to enter it. So as $\varepsilon \to 0$, P^{ε}_{1-2} spends virtually all of the time as $t \to \infty$ in $\mathcal{M}$, regardless of where the process begins. We have demonstrated that if c_1 and c_2 are weakly opposed, and s (and m) are sufficiently large, then $\underline{\delta} + \overline{\delta} > 1$ guarantees that individuals miscoordinate virtually all of the time. We remark that these conditions are not necessary for recurrent miscoordination to be stochastically stable. Nevertheless, they are stronger than weak opposition alone (i.e. $\underline{\delta} > 0$). The following "symmetric" example illustrates that the condition $\underline{\delta} + \overline{\delta} > 1$ can hold in plausible cases:

Example 1 *Consider the* Bar, Cafe, Church *example from the introduction with* $a_1 = c_2 = \delta_{\mathcal{M}}$ *and* $b_1 = b_2 = \delta_C$. *The payoffs in the reduced game are illustrated by figure 3.*

	B	C
A	$\delta_{\mathcal{M}}, \delta_C$	$\delta_{\mathcal{M}}, \delta_{\mathcal{M}}$
B	$\underline{1+\delta_C}, \underline{1+\delta_C}$	$\delta_C, \delta_{\mathcal{M}}$

Figure 3: Reduced game with 'symmetric' payoffs.

The players' cultures c_1 and c_2 are weakly opposed if $\delta_{\mathcal{M}} > \delta_C$. In this case, $\mathcal{M}$ is a recurrence class of P^0_{1-2}, by Proposition 1. The "symmetry" of preferences implies that the strongest and weakest idiosyncratic preferences for miscoordination are the same, that is $\underline{\delta} = \overline{\delta} = \delta_{\mathcal{M}} - \delta_C$. Therefore, by inequality (7), $\mathcal{M}$ is the unique stochastically stable class if:

$$\delta_{\mathcal{M}} - \delta_C > \frac{1}{2}\left(1 + \frac{1}{s}\right) \tag{8}$$

The right hand side of (8) goes to $\frac{1}{2}$ as $s \to \infty$. Therefore, for s sufficiently large, $\mathcal{M}$ is the unique stochastically stable class of P^{ε}_{1-2}, if the idiosyncratic preference for miscoordination, $\delta_{\mathcal{M}} - \delta_C$, is greater than half the coordination payoff of one. Therefore, there exists a range of preferences for which player 1 goes to the *Bar* and player 2 to goes to *Church*.

5 Cultural Evolution via Cultural Choice

When individuals make a reasoned choice of culture, which cultures survive in the long run, and which cultures die out? To answer this question, we extend our model to incorporate cultural choice. In this more general case, play in social interactions *coevolves* with the composition of cultural groups. Our focus here is on a salient case called a *tripartite society*, in which there are three cultures. We show that restrictive cultures can survive in the long run and coexist with permissive cultures.

5.1 Modelling Cultural Choice

Timing. At the beginning of each period, a player is "exposed" to a randomly selected (non-empty) culture with probability $0 < \theta < 1$. This occurs prior to players being matched in a social interaction. If selected, a player can choose to adopt the culture to which they are exposed or retain their existing culture. Equivalently, we say that the player can migrate to the new cultural group, and we assume that migration is costless.

Cultural Choice. A $c_{k'}$-member who is exposed to culture c_k, learns: (i) the admissible set of actions for that culture, X_k, (ii) the distribution of players across all cultures at the end of the previous period, denoted by $\nu^t = (n_1^t, n_2^t, \ldots, n_K^t)$, and (iii) a sample of size s drawn (without replacement) from the history $h^t = (h_{j-k}^t)_{j \leq k}$, of the last m actions taken by members of c_j when interacting with c_k-members, for each $j = 1, 2, ...K$. This final piece of information comprises a total of K samples of size s, which together capture how c_k-members have been treated in social interactions by adherents to other cultures, as well as fellow c_k-members. The player is assumed to possess the equivalent information, (i)-(iii), on her present culture. This enables her to calculate and compare expected payoffs between cultures, as follows.

The player's objective is to maximize her expected current period payoff, given that she is unaware who her partner will be in a forthcoming social interaction. She forms expectations based on her information on the history of play, h^t, and the distribution of players across cultures, ν^t. As in a social interaction, the c_k-member uses her sample from h_{j-k}^t to formulate the optimal pure strategy in a j-k interaction, denoted by $\widehat{x}_{k,j}^t(i) \in argmax_{x \in X_k}\{\widehat{p}_{j,k}^t(x) + \delta_{i,x}\}$. Now the player calculates the optimal strategy for each possible culture her partner could adhere to, i.e for all $j \in J = \{1, 2, \ldots K\}$. Since any two players are paired with equal probability, the likelihood of being paired with an adherent to any given culture can be inferred from ν^t. Suppose a player is currently a c_k'-member. Given the current history of play h^t and distribution of players across cultures ν^t, the expected payoff from becoming a c_k-member, before she learns who her partner is in the social interaction, is:

$$\widehat{\pi}^t_{i,k}(h^t,\nu^t) = \sum_{j\in J} \frac{n_j-\xi_j}{n}\{\widehat{p}^t_{j,k}(\widehat{x}^t_{j,k}(i)) + \delta_{i,\widehat{x}^t_{j,k}(i)}\} \tag{9}$$

where $\xi_j = 1$ if $j = k'$ and zero otherwise. If the sum in (9) is strictly greater than the expected payoff from membership in the player's existing cultural group $c_{k'}$, i.e. $\widehat{\pi}^t_{i,k} > \widehat{\pi}^t_{i,k'}$, then the player adopts culture c_k. In the case of a tie, the player chooses one of these two cultures, with uniform probability. Otherwise, she remains an adherent to $c_{k'}$.[21]

With cultural choice comes the problem of "ancient histories" which do not update if a culture has not been populated for some time. We deal with this issue by assuming that each period an empty culture, say k (i.e. for which $n^t_k = 0$), is selected with probability $\gamma > 0$. If k is selected, we assume that for one $j \in J$, the most recent action pair in a j-k interaction is randomly replaced by a pair (x, y) where $x \in X_j$ and $y \in X_k$. We call this a *renewal.* In effect, this allows dead cultural groups to be replaced by new cultural groups with the same cultural commitments, but possibly different expectations regarding their social behaviour.

Dynamical Process. With migration between cultural groups, the distribution of players across cultures, ν^t, is now relevant to individual choice. The state in period t, $z^t = (h^t, \nu^t)$, specifies the history of play in period t, and the distribution of players across cultures at the end of period $t-1$. The associated state space is $Z = \mathscr{N} \times \prod_{j\le k}(X_j \times X_k)^m$, where $\mathscr{N} = \{(n_1, \dots n_k, \dots n_K)|\ 0 \le n_k \le n, \forall k \in J, and \sum_{k\in J} n_k = n\}$ and $(X)^m$ denotes the m-fold product of X. Z is finite for finite K and m. Again, we assume that the process begins in some initial state z^0 in which there are at least m plays in each j-k interaction, where the sequence of plays is otherwise arbitrary. There are well-defined, time-homogeneous transition probabilities between all pairs of states z, z', denoted by $P_{z,z'}$. Therefore, the adaptive process we have defined is a finite Markov chain, with a $|Z| \times |Z|$ transition probability matrix $P^{m,s,\theta,\gamma,\varepsilon}$. The unperturbed process P^0 again involves $\varepsilon = 0$.

[21] So the choice of culture and its attending commitments are based *partly* on material interests, when interactive payoffs are interpreted as being materially based. There are numerous instances one can cite in which moral sentiments are shaped by material interests. For example, the Puritans who settled Providence Island off the coast of Nicaragua were lured by the sizable profits from trade in plantation crops to abandon their original ideals and become slave owners (Bowles 2004, p.4). Both the temporally dislocated Spartan and Venetian aristocracies took to polyandry in response to the taxation of separate households, which were legally deemed to be created upon marriage. So brothers saved on taxes by sharing a wife (Jones 2006).

5.2 Cultural Configurations in a Tripartite Society

In this section, we analyse the cultural configurations which emerge in the long run, when individuals make a reasoned choice of culture. Let us now denote the set of agents for which $\delta_{ix} > \delta_{ix'}$ for all $x' \in X$ by $\tilde{N}_x$. Call this the set of agents who most prefer action x.

Our analysis focusses on the following salient case, which we refer to as the *tripartite society*:

TRIPARTITE SOCIETY. *Each social interaction takes the form of a* 2×2 *coordination game, with pure strategies* $X = \{x_1, x_2\}$*. There are three cultures to which an individual may adhere:* c_1 *which permits only action* x_1*,* c_2 *which permits only action* x_2*, and a permissive culture* $c_{\mathcal{P}}$ *which allows both actions. In addition, all agents most prefer one of the actions, and for each action* x_1 *and* x_2*, there is at least one individual that most prefers that action.*

Define $\tilde{Z}_1 = \{z|N_1 = \emptyset, N_2 = \tilde{N}_2 \text{ and } N_{\mathcal{P}} = \tilde{N}_1\}$ as the set of states in which all agents who most prefer x_2 are members of c_2 and all agents who most prefer x_1 join the permissive culture $c_{\mathcal{P}}$. Similarly, define $\tilde{Z}_2 = \{z|N_1 = \tilde{N}_1, N_2 = \emptyset \text{ and } N_{\mathcal{P}} = \tilde{N}_2\}$ as the set of states in which all agents who most prefer x_1 are members of c_1 and all agents who most prefer x_2 join the permissive culture $c_{\mathcal{P}}$. In addition, let $\tilde{Z} = \tilde{Z}_1 \cup \tilde{Z}_2$.

Let us denote by $\mu^t(z|z^0)$ the relative frequency with which state z is visited by the process P^0 during the first $t > 0$ periods. As t goes to infinity, $\mu^t(z|z^0)$ converges almost surely to the probability distribution $\mu^\infty(z|z^0)$, called the *asymptotic frequency distribution* conditional on z^0. The process selects those states on which $\mu^\infty(z|z^0)$ puts positive probability. We can now state the following result:

Proposition 3 *Consider a tripartite society. If* $s/m \leq 1/2$*, then from any initial state* z^0*, the sequence* $\mu^t(z|z^0)$ *converges to zero, for all* $z \notin \tilde{Z}$*. In particular, the permissive culture* $c_{\mathcal{P}}$ *survives with probability one. In addition, only one restrictive culture survives. If an* x_1 *convention emerges in* $\mathcal{P}$-$\mathcal{P}$ *interactions, then with probability one* c_2 *survives and* c_1 *dies out. If an* x_2 *convention emerges in* $\mathcal{P}$-$\mathcal{P}$ *interactions, the opposite occurs.*

Proof. Assume $s/m \leq 1/2$. To establish the Proposition, it suffices to show that (i) from any state z, P^0 transits with positive probability in a finite number of periods to either a state in $\tilde{Z}_1$ in which the $\mathcal{P}$-$\mathcal{P}$ convention is x_1 or a state in $\tilde{Z}_2$ in which the $\mathcal{P}$-$\mathcal{P}$ convention is x_2 , and (ii) once P^0 enters one of these classes it does not leave.

Let P^0 reside in an arbitrary state z at the beginning of period t. Suppose that no player is selected to review her culture in $10s + 6m$ consecutive periods, beginning in period t. Recall that the probability with which a player is selected to review her culture in a given period is $\theta < 1$, so this event occurs with positive probability.

In a tripartite society there are six possible j-k interactions. Suppose first that both c_1 and c_2 are non-empty at the beginning of period $t+1$. There is a positive probability then that a c_1-member and a c_2-member are matched in a social interaction, in the next m periods. Since $X_1 = x_1$ and $X_2 = x_2$, the 1-2 history transits with probability one to the state of miscoordination $((x_1, x_2), (x_1, x_2), \ldots, (x_1, x_2))$ in that time. If one of these cultures is empty, then there is no possibility of a 1-2 interaction. Nevertheless, with positive probability γ^m, the 1-2 history transits to this state via m consecutive renewals.

Now suppose that c_1 and $c_{\mathcal{P}}$ are both non-empty at the beginning of period t, and thus remain non-empty at the beginning of period $t+m$. There is a positive probability then that a c_1-member and a $c_{\mathcal{P}}$-member are matched in a social interaction in the next $2s+m$ periods. Note that c_1 and $c_{\mathcal{P}}$ are not weakly opposed. Since by hypothesis there is no migration of players in this time interval, we can directly apply the argument used in the proof of Lemma 2, case 1 (see the Appendix), to show that there is a positive probability that the 1-$\mathcal{P}$ history converges to an x_1 convention in $2s+m$ periods. If instead one of these cultures is empty, then there is no possibility of a 1-$\mathcal{P}$ interaction. Nevertheless, with positive probability γ^m, the 1-$\mathcal{P}$ history can transit to an x_1 convention via m consecutive renewals. Therefore, the 1-$\mathcal{P}$ history transits to a convention with positive probability in at most $2s+m$ periods.

Each of the four remaining j-k interactions involves pairs of cultures that are not weakly opposed. Hence, by the same reasoning, the 1-1 history transits to an x_1 convention, the 2-2 history transits to an x_2 convention, the 2-$\mathcal{P}$ history transits to an x_2 convention and the $\mathcal{P}$-$\mathcal{P}$ transits to either an x_1 or x_2 convention with positive probability all in at most $4 \times (2s+m)$ periods.

The joint probability of all of this occurring in at most $m + 5 \times (2s+m) = 10s + 6m$ periods is positive.

We have already established that condition ND guarantees that no matter which individuals are selected to play, x_ℓ is a best reply to all samples from an x_ℓ convention. Therefore, whenever migration between cultural groups occurs, the conventions in 1-1, 2-2, 1-$\mathcal{P}$, 2-$\mathcal{P}$ and $\mathcal{P}$-$\mathcal{P}$ interactions remain in place. Since $X_1 \cap X_2 = \emptyset$, the 1-2 history remains in a state of miscoordination, no matter who is selected to play. The expected interactive payoff for any sample from such a 1-2 history is zero. Therefore, there are in effect two different types of overall histories; one in which an x_1 convention is in place in $\mathcal{P}$-$\mathcal{P}$ interactions and one in which an x_2 convention is in place.

Suppose that migration between cultural groups occurs in period $t' \equiv t + 10s + 6m$. A player selected to review her culture is randomly "exposed" to another culture, and can choose whether to adopt that culture. For convenience denote $n_k^{t'}$ by n_k, for the moment. Recall that each pair of players is drawn with equal probability, and $n = n_1 + n_2 + n_{\mathcal{P}}$. Consider a player i who is a c_1-member at the start of period t'. If i remains a c_1-member in period t' and she is matched in a social interaction, then i is paired with a c_1-member or a $c_{\mathcal{P}}$-member with probability $\frac{n_1-1+n_{\mathcal{P}}}{n-1}$. Recall that an x_1 convention is in place in 1-1 and 1-$\mathcal{P}$ interactions. So all c_1-members expect to receive an interactive payoff of one in 1-1 and 1-$\mathcal{P}$ interactions. Since 1-2 interactions are perpetually in a state of miscoordination, a c_1-member always expects to receive a zero interactive payoff in 1-2 interactions. Therefore, the expected payoff to i if she remains a c_1-member in period t' (and is thus restricted to playing x_1) is $\delta_{i,x_1} + \frac{n_1-1+n_{\mathcal{P}}}{n-1}$. Similarly, if player i is a c_2-member at the start of period t', her expected payoff from retaining c_2 is $\delta_{i,x_2} + \frac{n_2-1+n_{\mathcal{P}}}{n-1}$. Player i's expected payoff from membership in $c_{\mathcal{P}}$ is $(1 + \frac{n_1-(1-\xi)+\beta n_{\mathcal{P}}}{n-1}\delta_{i,x_1} + \frac{n_2-\xi+(1-\beta)n_{\mathcal{P}}}{n-1}\delta_{i,x_2})$, where $\beta = 1$ (resp. $\beta = 0$) indicates here that the $\mathcal{P}$-$\mathcal{P}$ convention is x_1 (resp. x_2), and $\xi = 1$ (resp. $\xi = 0$) indicates that player i adheres to c_2 (resp. c_1) at the beginning of period t'. As such, if i is a c_1-member at the beginning of period t', membership in $c_{\mathcal{P}}$ yields a (weakly) greater expected payoff than c_1, if and only if:

$$\delta_{i,x_1} + \frac{n_1 - 1 + n_{\mathcal{P}}}{n-1} \leq 1 + \frac{n_1 - 1 + \beta n_{\mathcal{P}}}{n-1}\delta_{i,x_1} + \frac{n_2 + (1-\beta)n_{\mathcal{P}}}{n-1}\delta_{i,x_2}. \tag{10}$$

If i is a c_2-member at the beginning of period t', membership in $c_{\mathcal{P}}$ yields a greater expected payoff than membership in c_2, if and only if:

$$\delta_{i,x_2} + \frac{n_2 - 1 + n_{\mathcal{P}}}{n-1} \leq 1 + \frac{n_1 + \beta n_{\mathcal{P}}}{n-1}\delta_{i,x_1} + \frac{n_2 - 1 + (1-\beta)n_{\mathcal{P}}}{n-1}\delta_{i,x_2}. \tag{11}$$

If $\beta = 1$, then (10) reduces to the following condition:

$$n_2(\delta_{i,x_1} - \delta_{i,x_2}) \leq n_2. \tag{12}$$

Under condition ND, $|\delta_{i,x_1} - \delta_{i,x_2}| < 1$ for all $i \in N$. Therefore, membership in $c_{\mathcal{P}}$ yields a (weakly) greater expected payoff than c_1 for all $i \in N$ (strictly if $n_2 > 0$). There is a positive probability that all players who are c_1-members at the start of period t' are exposed to culture $c_{\mathcal{P}}$ in $n_1^{t'}$ consecutive periods beginning in period t', and that each $i \in N_1$ chooses to adopt $c_{\mathcal{P}}$.

Then in period $t'' \equiv t' + n_1^{t'}$, all n agents in the population are members either of $c_{\mathcal{P}}$ or c_2. Setting $\beta = 1$ and rearranging (11), we find that a member of c_2 strictly prefers to join $c_{\mathcal{P}}$ if:

$$(n_1 + n_{\mathcal{P}})(\delta_{i,x_2} - \delta_{i,x_1}) < n_1. \tag{13}$$

Given that $n_1^{t''} = 0$, this reduces to:

$$\delta_{i,x_1} > \delta_{i,x_2}. \tag{14}$$

Therefore, there is a positive probability that all c_2-members who most prefer x_1 switch to $c_{\mathcal{P}}$. By the same reasoning, we can show that there is a positive probability that all $c_{\mathcal{P}}$-members who most prefer x_2 switch to c_2. All c_2-members who most prefer x_2 remain in c_2, and all $c_{\mathcal{P}}$-members who most prefer x_1 remain in $c_{\mathcal{P}}$. In particular, this means that there is a positive probability that in at most n periods, i.e. by period $t'' + n$, P^0 resides in a state in $\tilde{Z}_1$ in which the $\mathcal{P}$-$\mathcal{P}$ convention is x_1.

We shall now demonstrate that once P^0 reaches a state in $\tilde{Z}_1$, and the $\mathcal{P}$-$\mathcal{P}$ convention is x_1, P^0 never leaves this set. The only source of randomness in P^0 are renewals which can alter the histories involving the (only) empty culture c_1. Therefore, we need to show that there is no sequence of renewals which could induce players to join c_1. The payoff from joining c_1 is maximized when an x_1 convention is in place in 1-1 and 1-$\mathcal{P}$ interactions (the 1-2 interaction will always be in a state of miscoordination). When this is the case, a c_2-member will not switch to c_1 if:

$$\delta_{i,x_2} + \frac{n_2 - 1 + n_{\mathcal{P}}}{n-1} > \delta_{i,x_1} + \frac{n_1 + n_{\mathcal{P}}}{n-1}. \tag{15}$$

Since $n_1 = 0$ at the beginning of the period, this reduces to:

$$\delta_{i,x_2} + \frac{n_2 - 1}{n-1} > \delta_{i,x_1}, \tag{16}$$

which holds because all c_2-members at the beginning of the period most prefer x_2. Hence no c_2-member will switch.

When an x_1 convention is in place in 1-1 and 1-$\mathcal{P}$ interactions, a $c_{\mathcal{P}}$-member will not switch to c_1 if:

$$\delta_{i,x_1} + \frac{n_1 + n_{\mathcal{P}} - 1}{n-1} < 1 + \frac{n_1 + n_{\mathcal{P}} - 1}{n-1}\delta_{i,x_1} + \frac{n_2}{n-1}\delta_{i,x_2}. \tag{17}$$

This reduces to $n_2(\delta_{i,x_1} - \delta_{i,x_2}) < n_2$. By condition ND, this holds for $n_2 > 0$. Recall that for all states in $\tilde{Z}_1$, $n_2 > 0$, since by assumption at least one player most prefers x_2.

Therefore, even in the most attractive case in which an x_1 convention is in place in 1-1 and 1-$\mathcal{P}$ interactions, no player will switch to c_1. Hence, once P^0 reaches a state in $\tilde{Z}_1$ there is no sequence of renewals which can cause it to leave this set.

When the $\mathcal{P}$-$\mathcal{P}$ convention is x_2 (i.e. $\beta = 0$), the same reasoning can be applied to show that P^0 transits to a state in $\tilde{Z}_2$ and never leaves this set. □

Firstly, we have shown that it is possible for more than two cultural groups to learn to coordinate with each other, when players can migrate between cultural groups. Secondly, it is always the case that in a tripartite society with two strategies at most two cultures survive in the long run: one permissive and one restrictive culture. This result holds irrespective of the amount of heterogeneity in individual preferences. Restrictive cultures survive evolutionary selection due to the strategic commitment role of culture. Moreover, the survival of a restrictive culture need not hinder social coordination. Because the permissive culture always survives and can coordinate with the restrictive culture, coordination in a tripartite society is achieved in the long run with probability one in every intercultural interaction.

Appendix

Proof of Lemma 2

To establish the Lemma, it suffices to show that there is a positive probability of transiting from any state to a convention or a state in $\mathcal{M}$ in a finite number of periods. Consider the situation at some arbitrary time $t+1$. There is a positive probability that a particular player $i \in N_1$ is selected to play in s consecutive periods, and that i draws the same sample, which we denote by $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, on each occasion (since $m \geq 2s$). There is also a positive probability that i plays the same best reply, say x_ℓ, in the s consecutive periods from period $t+1$ to $t+s$. So in period $t+s+1$, the c_2-member sampling from the most recent m plays by a c_1-member up to period $t+s$, has a positive probability of drawing the sample $(x_\ell, x_\ell, \ldots x_\ell)$, comprised solely of s plays of x_ℓ. There are then two cases to consider.

Case 1. $x_\ell \in X_1 \cap X_2$. (By definition, if c_1 and c_2 are radically opposed, then $x_\ell \in X_1 \cap X_2$ is impossible.) Condition ND guarantees that x_ℓ is the unique best reply to the sample $(x_\ell, x_\ell, \ldots x_\ell)$ for all c_2-members. There is a positive probability that player $i \in N_1$ is again selected to play in period $t+s+1$, that she draws the same sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$ [since $m \geq 2s$], and that she plays the same best reply x_ℓ to this sample. Therefore, there is a positive probability that the action-tuple played in period $t+s+1$ will be (x_ℓ, x_ℓ). Since at the end of this period, the last s consecutive plays by the c_1-member are of x_ℓ, the c_2-member selected in period $t+s+2$ has a positive probability of again drawing a sample comprised of s plays of x_ℓ, to which the unique best reply is x_ℓ.

Now consider the c_1-member. There is a positive probability that player $i \in N_1$ is again selected to play in period $t+s+2$. Denote the history of the most recent m plays by c_2-members in period $t+s+1$ by $(x^1, x, \ldots)$. Once the c_2-member drawn in period $t+s+1$ plays action x_ℓ, the corresponding period $t+s+2$ history is $(x, \ldots x_\ell)$. The only change in the history is that x^1 is dropped and x_ℓ is added. If $x^1 \neq x^{-s}$, then player i can once again draw the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, and play the same best reply x_ℓ to this sample. If $x^1 = x^{-s}$, then i can draw the sample $(x^{-s+1}, \ldots x^{-1}, x_\ell)$, which contains at least as many plays of x_ℓ as the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$. Therefore, if x_ℓ is a best reply to the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, then it is also a best reply to $(x^{-s+1}, \ldots x^{-1}, x_\ell)$. As such, there is a positive probability that the action-tuple played in period $t+s+2$ is again (x_ℓ, x_ℓ). Iterating this argument, the process P^0_{1-2} transits with positive probability to the convention $h^*_\ell = ((x_\ell, x_\ell), \ldots (x_\ell, x_\ell))$ in at most $m-2$ additional periods. Notice that we have not relied on the cultures not being weakly opposed, at any stage. So this result applies whenever c_1 and c_2 are weakly, but not radically opposed.

Case 2(a). $x_\ell \notin X_1 \cap X_2$ and c_1 and c_2 are not weakly opposed. Then there exists an individual $i' \in N_2$, such that $\tilde{x} \in argmax_{x \in X_2} \delta_{i',x}$ is in $X_1 \cap X_2$. There is a positive probability that $i' \in N_2$ is selected in period $t+s+1$, and draws the sample $(x_\ell, x_\ell, \ldots x_\ell)$. For this sample, $\widehat{p}_{1,2}(x_\ell) = 1$. Since $x_\ell \in X_1$ (because x_ℓ was taken by a c_1-member), and $x_\ell \notin X_2$ (because $x_\ell \in X_1$ and $x_\ell \notin X_1 \cap X_2$), $\widehat{p}_{1,2}(x) = 0$ for all $x \in X_2$. Therefore, the expected payoff from any admissible action $x \in X_2$ for i' is $\widehat{p}_{1,2}(x) + \delta_{i',x} = \delta_{i',x}$. As such, the best reply to this sample for i' involves simply maximizing her idiosyncratic payoff by choosing an action in $argmax_{x \in X_2} \delta_{i',x}$. By assumption there exists such an action, say $\tilde{x}$, in $X_1 \cap X_2$. So there is a positive probability that i' plays action $\tilde{x}$ in period $t+s+1$. There is also a positive probability that i' is selected to play in the next s consecutive periods, that she draws the same sample $(x_\ell, x_\ell, \ldots x_\ell)$ in each of those periods (since $m \geq 2s$), and that she plays the same best reply $\tilde{x}$ on each occasion. Then in period $t+2s+1$, there is a positive probability that the c_1-member selected to play draws the sample $(\tilde{x}, \tilde{x}, \ldots \tilde{x})$. Since $\tilde{x} \in X_1 \cap X_2$, we are now in case 1, and there is a positive probability of transiting to the $\tilde{x}$ convention in at most another m periods.

Case 2(b). $x_\ell \notin X_1 \cap X_2$ and c_1 and c_2 are weakly opposed. Again, there is a positive probability that the c_2-member i' selected in period $t+s+1$ plays the best reply $\tilde{x} \in argmax_{x \in X_2} \delta_{i',x}$ to the sample $(x_\ell, x_\ell, \ldots x_\ell,)$. Only now $\tilde{x} \notin X_1 \cap X_2$. Consider the c_1-member. There is a positive probability that player $i \in N_1$ is again selected in period $t+s+1$, that i draws the same sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$ [since $m \geq 2s$], and that she again plays the best reply x_ℓ. Therefore, there is a positive probability that the action-tuple played in period $t+s+1$ is $(x_\ell, \tilde{x})$.

In period $t+s+2$, there is a positive probability that player $i' \in N_2$ is again selected to play. Since in this period, the last s consecutive plays by the c_1-member are of x_ℓ, there is a positive probability that i' again draws a sample comprised of s plays of x_ℓ, and that she again plays $\tilde{x}$ in response. There is also a positive probability that player $i \in N_1$ is again selected to play in period $t+s+2$. Denote the history of the most recent m plays by c_2-members in period $t+s+1$ by $(x^1, x, \ldots)$. Once the c_2-member drawn in period $t+s+1$ plays action $\tilde{x}$, the corresponding period $t+s+2$ history is $(x, \ldots \tilde{x})$. The only change in the history is that x^1 is dropped and $\tilde{x}$ is added. If $x^1 \neq x^{-s}$, then player i can once again draw the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, and play the same best reply x_ℓ to this sample. If $x^1 = x^{-s}$, then i can draw the sample $(x^{-s+1}, \ldots x^{-1}, \tilde{x})$. Since $\tilde{x} \notin X_1$, this implies that the sample $(x^{-s+1}, \ldots x^{-1}, \tilde{x})$ does not contain a greater frequency of any action $x \in X_1$ than does $(x^{-s}, x^{-s+1}, \ldots x^{-1})$. Since $x_\ell \notin X_2$ by hypothesis, the expected payoff from action x_ℓ is always δ_{i,x_ℓ}. Together with the preceding, this implies that if x_ℓ is a best reply for i to the sample $(x^{-s}, x^{-s+1}, \ldots x^{-1})$, then it is also a best reply to $(x^{-s+1}, \ldots x^{-1}, \tilde{x})$. As such, there is a positive probability that the action-tuple played in period $t+s+2$ is again $(x_\ell, \tilde{x})$. Iterating this argument, the process P^0_{1-2} transits with positive probability to the state $((x_\ell, \tilde{x}), \ldots (x_\ell, \tilde{x}))$ in at most another $s-2$ periods. Since $x_\ell \notin X_2$ and $\tilde{x} \notin X_1$, this is a state of miscoordination. All best replies to a state of miscoordination are in $\tilde{X}_1 \times \tilde{X}_2$. Therefore, P^0_{1-2} transits from this point to a state in $\mathcal{M}$ with probability one in at most m additional periods. $\square$

Proof of Corollary 1

Assume $s/m \leq 1/2$. Without loss of generality, fix $k = 1$ and consider an x_1 convention. We have already established in the proof of Lemma 1 part (i), that x_1 is the unique best reply for all possible samples from an x_1 convention, *no matter which players are selected to play.* So in a 1-1 interaction, each convention is an absorbing state and hence a distinct recurrence class of P^0_{1-1}. To establish the corollary then, it will (as before) suffice to show that P^0_{1-1} transits with positive probability from any state to a convention in a finite number of periods. Consider the event in which player $i \in N_1$ is selected to fill the first player position in $s+m$ consecutive periods and player $i' \in N_1$ is selected to fill the second player position in the same $s+m$ periods. This event occurs with positive probability, since in a 1-1 interaction each player in N_1 is drawn with positive probability and then randomly allocated to the first or second player position. Therefore, the pairing in this manner of i and i' in $s+m$ consecutive periods, is analogous to the case in which i and i' are drawn from two disjoint populations and belong to separate cultural groups c_1 and $c_{1'}$, respectively, with admissible action sets X_1 and $X_{1'} = X_1$. There is a positive probability that i draws the same sample in the first s periods, and plays the same best reply, say $x_\ell \in X_1$ on each occasion. Since $X_1 = X_{1'}$, x_ℓ is certainly in $X_1 \cap X_{1'}$. Therefore, we can directly apply the argument used in the proof of Lemma 2 (case 1) to show that from this point, the process converges with positive probability to the x_ℓ convention in at most m additional periods. $\square$

References

Akerlof, G. A. & Kranton, R. E. (2000), 'Economics and identity', *Quarterly Journal of Economics* **415**(3), 715–753.

Axelrod, R. (1997), 'The dissemination of culture: a model with local convergence and global polarization', *Journal of Conflict Resolution* **41**, 203–226.

Bass, F. (1969), 'A new product growth model for consumer durables', *Management Science* **15**, 215–227.

Bisin, A. & Verdier, T. (2001), 'The economics of cultural transmission and the dynamics of preferences', *Journal of Economic Theory* **97**, 298–319.

Bowles, S. (2001), Individual interactions, group conflicts, and the evolution of preferences, *in* S. N. Durlauf & H. P. Young, eds, 'Social Dynamics', Brookings Institution and MIT Press, Washington, D.C. and Cambridge, MA, pp. 156–190.

Bowles, S. (2004), *Microeconomics: Behavior, Institutions and Evolution*, Princeton University Press and Russell Sage, Princeton, NJ and New York, NY.

Bowles, S. & Gintis, H. (2004), 'Persistent parochialism: trust and exclusion in ethnic networks', *Journal of Economic Behavior & Organization* **55**, 1–23.

Boyd, R. & Richerson, P. (2005), *The Origin and Evolution of Cultures*, Oxford University Press, New York, NY.

Boyd, R. & Richerson, P. J. (1985), *Culture and the Evolutionary Process*, University of Chicago Press, Chicago, IL.

Cavalli-Sforza, L. L. & Feldman, M. W. (1981), *Cultural Transmission and Evolution: A Quantitative Approach*, Princeton University Press, Princeton, NJ.

Child, I. (1943), *Italian or American*, Yale University Press, New Haven, CT.

Coleman, J. S. (1964), *Introduction to Mathematical Sociology*, Glencoe, New York, NY.

Coleman, J. S. (1990), *Foundations of Social Theory*, Belknap Harvard, New York, NY.

Dixit, A. (1979), 'A model of duopoly suggesting a theory of entry barriers', *Bell Journal of Economics* **10**(1), 20–32.

Dodds, P. & Watts, D. J. (2005), 'A generalized model of social and biological contagion', *Journal of Theoretical Biology* **232**(4), 587–604.

Durkheim, E. (1953), *Sociology and Philosophy*, Free Press, New York, NY.

Ellison, G. (2000), 'Basins of attraction, long-run stochastic stability, and the speed of step-by-step evolution', *Review of Economic Studies* **67**, 17–45.

Elster, J. (1989), 'Social norms and economic theory', *Journal of Economic Perspectives* **4**, 99–117.

Fiske, A. P. & Tetlock, P. E. (1997), 'Taboo trade-offs: Reactions to transactions that transgress the spheres of justice', *Political Psychology* **18**(2), 255–297.

Foster, D. & Young, H. P. (1990), 'Stochastic evolutionary game dynamics', *Theoretical Population Biology* **38**, 219–232.

Frank, R. H. (1988), *Passions within Reason*, Norton, New York, NY.

Freud, S. (1950), *Totem and Taboo*, Routledge and Kegan Paul, London.

Harsanyi, J. (1982), Morality and the theory of rational behavior, *in* A. Sen & B. Williams, eds, 'Utilitarianism and Beyond', Cambridge University Press, Cambridge, pp. 39–62.

Henrich, J., Albers, R. W., Boyd, R., Gigerenzer, G., McCabe, K. A., Ockenfels, A. & Young, H. P. (2001), What is the role of culture in bounded rationality?, *in* G. Gigerenzer & R. Selten, eds, 'Bounded Rationality: The Adaptive Toolbox', MIT Press, Cambridge, MA.

Henrich, J., Boyd, R., Bowles, S., Camerer, C., Fehr, E. & Gintis, H., eds (2004), *Foundations of Human Sociality : Economic Experiments and Ethnographic Evidence from Fifteen Small-Scale Societies*, Oxford University Press, New York, NY.

Huntington, S. P. (1993), 'Clash of civilizations?', *Foreign Affairs* **72**(3), 22–49.

Jackson, M. O. & Kalai, E. (1997), 'Social learning in recurring games', *Games and Economic Behavior* **21**(1-2), 102–134.

Jones, E. L. (2006), *Cultures Merging: A Historical and Economic Critique of Culture*, Princeton University Press, Princeton, NJ.

Kandori, M., Mailath, G. & Rob, R. (1993), 'Learning, mutation, and long-run equilibria in games', *Econometrica* **61**(1), 29–56.

Kuran, T. & Sandholm, W. H. (2008), 'Cultural integration and its discontents', *Review of Economic Studies* **75**, 201–228.

Mealey, L. (1995), 'The sociobiology of sociopathy: an integrated evolutionary model', *Behavioral and Brain Sciences* **18**(3), 523–599.

Merton, R. K. (1957), *Social Theory and Social Structure*, Free Press, New York, NY.

Rao, V. & Walton, M. (2004), Culture and public action: relationality, equality of agency, and development, *in* V. Rao & M. Walton, eds, 'Culture and Public Action', Stanford University Press, Stanford, CA.

Schaefer, R. T. & Lamm, R. P. (1992), *Sociology. 4th ed.*, McGraw-Hill, New York, NY.

Schelling, T. (1960), *The Strategy of Conflict*, Harvard University Press, Cambridge, MA.

Sen, A. (1977), 'Rational fools: a critique of the behavioural foundations of economic theory', *Philosophy and Public Affairs* **6**, 317–344.

Sen, A. (2006), *Identity and Violence: The Illusion of Destiny*, W.W. Norton, New York, NY.

Spence, M. (1977), 'Entry, investment and oligopolistic pricing', *Bell Journal of Economics* **8**(2), 534–544.

Tetlock, P. E., Kristel, O. V., Elson, S. B., Green, M. C. & Lerner, J. S. (2000), 'The psychology of the unthinkable: Taboo trade-offs, forbidden base rates, and heretical counterfactuals', *Journal of Personality and Social Psychology* **78**(5), 853–870.

Wrong, D. (1961), 'The over-socialized conception of man in modern society', *American Sociological Review* **26**(3), 184–193.

Young, H. P. (1993), 'The evolution of conventions', *Econometrica* **61**(1), 57–84.

Young, H. P. (1998), *Individual Strategy and Social Structure*, Princeton University Press, Princeton, NJ.