





Global or Local Pure Condensible Atmospheres: Importance of Horizontal Latent Heat Transport

F. Ding¹  and R. T. Pierrehumbert² 

¹ Department of the Geophysical Sciences, University of Chicago, Chicago, IL 60637, USA; fding@uchicago.edu

² Department of Physics, University of Oxford, Oxford OX1 3PU, UK

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Abstract

The distribution of a pure condensible planetary atmosphere in equilibrium with a surface reservoir is revisited, employing the energy budget of the climate system and emphasizing the atmospheric horizontal latent heat transport. This configuration is applicable to icy solar system bodies such as Triton, as well as a range of possible exoplanet atmospheres, including water or CO₂ iceballs or ocean worlds, and lava planets with mineral vapor atmospheres. Climate regimes for slowly rotating planets with the hotspot near the substellar point and for rapidly rotating planets with a warm equatorial belt are both treated. A nondimensional parameter controlling the fractional variation of the surface pressure is derived; it measures whether the pure condensible atmosphere is global or localized. The global pure condensible atmosphere with the nondimensional parameter much less than order of unity is maintained by the strong horizontal latent heat transport associated with an “evaporation/sublimation-driven flow” from warm to cold places that compensates for the incoming differential radiative forcing. We show that the variation of surface temperature can be estimated in terms of this nondimensional parameter if it is not too large. In the case of a pure water-vapor atmosphere with an ice or liquid surface, we show that the atmosphere is thick enough to maintain nearly isothermal surface conditions even when the substellar surface temperature is around the freezing point. Finally, it is proposed that the evaporation/sublimation-driven flow regime for global atmospheres could be detected via its effect on the inhomogeneous distribution of minor noncondensable components in the atmosphere.

Key words: astrobiology – methods: numerical – planets and satellites: atmospheres – planets and satellites: terrestrial planets

1. Introduction

Condensation is ubiquitous in planetary atmospheres and forms various kinds of condensates that play an important role in the atmospheric radiative, chemical, and dynamical processes. The most familiar case is H₂O condensation, which is prevalent on Earth and the gas giants. Other examples includes CH₄ on Titan, NH₃ and NH₄SH on Jupiter and Saturn, and CH₄ on Uranus and Neptune. As is the case for water vapor in the Earth’s atmosphere, the condensible substances in these atmospheres are dilute in the sense that the condensible component makes up a small fraction of the atmosphere. These cases can therefore be treated using methods familiar from the extensively studied case of moist convection on the present Earth. The distinction between the dilute and nondilute behavior of atmospheres with a condensible component is discussed at length in Pierrehumbert & Ding (2016).

Observations of some of the icy satellites in the solar system suggest that the atmospheres are dominated by condensible components and most of the planetary surfaces covered by the condensate are in vapor equilibrium with the air overlying the surface (e.g., Io with condensing SO₂ atmosphere and Triton with condensing N₂ atmosphere). This kind of atmosphere, referred to as pure condensible atmospheres in this paper, has a completely different circulation regime compared to atmospheres with the dilute condensible component, as a consequence of the strong connection between the surface temperature and pressure via the Clausius–Clapeyron relation. Pure condensible atmospheres were also proposed to form under various scenarios with different kinds of volatiles, e.g., a dense condensing CO₂ atmosphere on early Mars

(Forget et al. 2013) and on planets near the outer edge of the habitable zone (Wordsworth et al. 2010), condensing N₂ atmosphere on Pluto (Forget et al. 2017) and on early Titan (Charnay et al. 2014), and the extreme hot rocky vapor atmosphere on close-in rocky exoplanets (Castan & Menou 2011). In all of these cases, the atmosphere is supplied by vaporization or sublimation of a volatile reservoir near the warmest part of the planet and flows to its sink where it condenses in the colder regions (e.g., the nightside of a tide-locked planet).

It has long been recognized that pure condensible atmospheres could be either global—with fractional surface pressure variation much less than order of unity as for N₂ on Triton—or local near the source regions as for SO₂ on Io. The thickness of the atmosphere has been implicated in determining whether the atmospheric circulation can carry enough heat to keep the surface pressure variations small (Trafton & Stern 1983; Trafton 1984). In this paper, we revisit this problem by means of the horizontal atmospheric energy transport equation with emphasis on the latent heat transport. A nondimensional parameter that determines the variation of surface pressure is derived from the energy budget of the pure condensible atmosphere. Idealized models are developed for two limiting cases: (1) rapid rotators for which the radiation balance can be zonally averaged, the dominant temperature gradient is from the hot equator to the cold pole and Coriolis forces dominate the dynamics, and (2) slow rotators for which the dominant temperature gradient is from the hot substellar point to the cold antistellar point. Tide-locked planets in sufficiently long period orbits are examples of the latter regime, but bodies with weak thermal inertia that have nonzero rotation relative to the

substellar point, such as Triton or Pluto, can also be in this regime. The simplified solution helps to broaden the understanding of the range of possible atmospheric configurations for pure condensable atmospheres on extrasolar rocky or icy planets, including the intriguing case of water-dominated atmospheres over an N_2 - and CO_2 -poor Snowball or Waterworld.

The thermodynamic and radiative assumptions on which our model rests are common to both classes of spin states and are summarized as follows:

1. The atmosphere consists of a single chemical substance, which is condensable in the range of temperatures encountered at the surface of the planet. It is assumed that the surface pressure is everywhere equal to the saturation vapor pressure corresponding to the local surface temperature. This assumption is met in the most straightforward way if there is a global surface reservoir, but that restriction could be relaxed in places where the atmosphere is cold enough to be condensing out onto the surface.
2. The atmosphere is in hydrostatic balance. This is important because in hydrostatic balance the surface pressure determines the mass contained in a column of the atmosphere.
3. Latent heat dominates the energy transport.
4. The infrared cooling to space (i.e., the outgoing longwave radiation, OLR) can be written as a function of the surface temperature.

The dynamic assumptions differ between the two spin cases considered. For slow rotators, it is assumed that all atmospheric properties are axisymmetric about the axis joining the substellar to the antistellar point and that Coriolis effects are negligible. It is also assumed that the vertically integrated mass transport can be estimated in terms of the surface wind. These assumptions are the same as those made in Ingersoll et al. (1985). In the case of fast rotators, the assumption is that the atmosphere is symmetric about the spin axis and that the mass and energy transports are dominated by the Ekman transport, without any significant contribution from transient eddy effects. The circumstances under which the various assumptions are justified will be discussed in the course of the exposition below.

2. Energy Budget on Planets with Pure Condensible Atmospheres

A pure condensable atmosphere is an atmosphere consisting of a single substance (e.g., N_2 or H_2O) that can condense over the range of temperatures encountered in the atmosphere. The key assumption in our theory is that the surface pressure is everywhere equal to the saturation vapor pressure of the atmospheric constituent corresponding to the local surface temperature. For a pure condensable atmosphere the partial pressure of the condensable substance is in fact the total pressure, so that the statement that a layer of the atmosphere is saturated is equivalent to the statement that the atmospheric pressure equals the saturation vapor pressure corresponding to the local atmospheric temperature; insofar as the temperature of the air in contact with the surface must equal the surface temperature, our assumption about surface pressure is equivalent to the statement that the atmosphere is saturated at the ground. More simply put, the atmosphere is assumed to be in gas/condensate equilibrium where it contacts the condensate

reservoir at the ground. Where the atmosphere is cold enough to be condensing, it will be in gas/condensate equilibrium with the condensate particles that form in situ (assuming a sufficient supply of condensation nuclei), and the requirement for a preexisting surface condensate reservoir can be relaxed.

Although for the most part our theory only requires the atmosphere to be saturated at the surface, some assumption about the vertical temperature structure must be made in order to compute OLR as a function of surface temperature. Assuming that the atmosphere is saturated throughout its depth determines the vertical temperature structure, and we will make use of this assumption in the OLR calculations carried out in the examples we provide below. The presence of a subsaturated layer in the high optically thin parts of the atmosphere would do little to change the results, but a deep subsaturated layer extending nearly to the surface—such as could be produced by strong near-infrared heating of the atmosphere owing to instellation by an M star—would require a more complex approach to computing the OLR on the dayside.

If the hydrostatic surface pressure due to the atmosphere were significantly lower than the saturation vapor pressure corresponding to the surface temperature, then a layer of gas would form just above the surface having pressure equal to the saturation vapor pressure. There would be a pressure discontinuity between this layer and the lower overlying pressure, driving an extremely strong nonhydrostatic vertical flow that would add mass to the atmosphere and simultaneously cool the surface through evaporation or sublimation, until the pressures are equalized. Where the surface vapor pressure becomes lower than that of the overlying atmosphere, the same process happens in reverse, with mass and heat flowing onto the surface. Condensation may also occur interior to the atmosphere and not just at the surface, owing to radiative cooling inside the atmosphere.

The saturation assumption needs to be modified for the lava planet case since mineral vapors in equilibrium with silicate melt are generally subsaturated relative to pure vapor condensation. Vapor pressures for various mineral vapors in equilibrium with silicate melt are given in Miguel et al. (2011), and at typical lava planet temperatures the vapor pressures for the dominant vapors (Na and SiO) are about an order of magnitude lower than the saturation vapor pressure in equilibrium with the respective pure liquids. The lava planet case will be briefly discussed in Section 3.

Since the surface temperature and pressure are so closely tied by the Clausius–Clapeyron relation for a saturated pure condensable atmosphere, the surface pressure gradient points from the cold region to the hot region in the pure condensable atmosphere. Therefore, the atmosphere circulation is characterized by a strong low-level flow from a warm region that carries both momentum and energy fluxes to cold places and is referred to as “sublimation-driven flow” in Ingersoll et al. (1985). In contrast, atmospheres with dilute condensable components are similar to dry atmospheres with surface pressure gradients pointing from the hot substellar or equatorial region to cooler regions, if the rotation effect of the planet is weak. Such atmospheres are characterized by an overturning circulation rising from the warm region and sinking in cool regions. The Hadley cells and Walker cell in the Earth’s tropics are examples of this type of atmospheric circulation. In terms of mass balance, in dilute atmospheres, the convergence of the low-level flow is balanced by the divergence of the high-level

return flow, while in pure condensible atmospheres, it is balanced by the local evaporation/sublimation minus precipitation flux through the surface. There is no pressure gradient that could support any return flow in the upper atmosphere.

We start from the steady-state energy balance at the top of the atmosphere (TOA). For any atmosphere the net radiative flux at TOA must be balanced by the divergence of horizontal energy transport in the climate system, including both in the atmosphere and the ocean (if present). Here, we shall neglect oceanic transport. The atmospheric energy transport consists of transport of moist static energy plus transport of kinetic energy. The former is composed of three terms: (1) potential energy flux, (2) sensible heat flux, and (3) latent heat flux (Peixoto & Oort 1992). The specific potential energy, gZ , is typically of the same order as, but somewhat smaller than, the sensible heat $c_p T$. This can be seen by using the scale height $H \equiv RT/g$ as the estimate for Z , whence $(gZ)/(c_p T) \approx R/c_p$, where R is the gas constant for the atmospheric composition. For a pure condensible atmosphere, the specific latent heat of the atmosphere is the latent heat L of the condensible gas, so the relative importance of sensible heat is given by the ratio $c_p T/L = T/T^*$, where $T^* \equiv L/c_p$. T^* is large for common condensibles. For example, based on thermodynamic properties near the triple point of the respective gases, it is 1350 K for H_2O , 484 K for CO_2 , 210 K for N_2 , and 3427 K for Na. All of these values are considerably supercritical in the sense of phase change thermodynamics, so that for any pure condensible atmosphere that operates in a substantially subcritical temperature range latent heat transport will dominate the sensible heat transport.

We will also neglect kinetic energy transport, which is the chief assumption by which our analysis departs from, and is much simpler than, the formulation of Ingersoll et al. (1985). If the speed of sound is used as a scale for the velocity, then the specific kinetic energy $v^2/2$ is equal to $\gamma RT/2$, which is of the same order as the specific sensible heat. Hence, when latent heat dominates the sensible heat, it will also dominate the kinetic energy transport, except where the atmosphere becomes substantially supersonic. Our approximation will therefore break down in supersonic regions of a very localized atmosphere with strong pressure gradients, but it nonetheless will suffice to discriminate between situations in which a local versus global atmosphere occur.

When the latent heat flux dominates the atmospheric energy transport, the vertically integrated energy budget of the pure condensible atmosphere can be written as

$$F_{\oplus}(1 - \alpha)\cos\zeta - \text{OLR} \simeq \nabla_h \cdot (LM), \quad (1)$$

where the left-hand side is the TOA radiative imbalance, and M the atmospheric horizontal mass transport vector. OLR is the outgoing longwave radiation, F_{\oplus} is the incident parallel-beam stellar flux, α the local planetary albedo, and ζ the local zenith angle. Since this is a vertically integrated equation, the divergence is in the horizontal direction alone. The problem will be closed by writing OLR as a function of surface temperature, but the statement remains valid regardless of how the OLR is determined.

In Ingersoll et al. (1985) for the case of Io and Castan & Menou (2011) for lava planets, the energy carried by latent heat is neglected. However, in both calculations, it is assumed the atmosphere does not carry enough heat to affect the surface temperature. The surface temperature is assumed to be the same

as it would be for a planet with no atmosphere, and the resulting temperature is used to drive the atmospheric mass flow. In this situation, heat transport is just a diagnostic, so errors in heat transport do not affect the results as long as the heat transport is indeed small. We will see shortly that even within our own framework, which includes latent heat transport, the conclusions of these two studies that the atmospheres are localized near the substellar point remain valid.

We will assume that the OLR can be written as a function of the surface temperature (T_s)³ that is related with the surface pressure (p_s) by the Clausius–Clapeyron relation. Then, the energy budget equation (Equation (1)) can be simplified as an ordinary differential equation on the spatial distribution of the surface pressure (p_s) because the surface momentum equation provides an additional relation between p_s and the horizontal mass transport. In the next two sections, we will consider two specific orbital configurations of the planet and further simplify the energy budget equation (Equation (1)) in the two different climate regimes to find how the surface pressure variation is controlled by the latent heat transport.

3. Slowly and Synchronously Rotating Planets

For slowly and synchronously rotating planets, it is convenient to use the tidally locked coordinate system (see Appendix B in Koll & Abbot 2015 for details) with tidally locked latitude $\theta_{\text{TL}} = 0$ at the terminator and $\theta_{\text{TL}} = \pi/2$ at the substellar point because slow planetary rotation leads to zonally symmetric flows in the tidally locked coordinate. Then, the energy budget equation and vertically averaged momentum equations are

$$\frac{1}{a \cos \theta_{\text{TL}}} \frac{d}{d\theta_{\text{TL}}} (\cos \theta_{\text{TL}} L v_s p_s / g) \simeq \begin{cases} F_a \sin \theta_{\text{TL}} - \text{OLR}(T_s), & \theta \in [0, \pi/2] \text{ (dayside)} \\ -\text{OLR}(T_s), & \theta \in [-\pi/2, 0] \text{ (nightside)} \end{cases} \quad (2)$$

$$\frac{1}{a} \frac{dp_s}{d\theta_{\text{TL}}} = -\frac{\rho_s C_d |v_s| v_s}{H}, \quad (3)$$

where a is the radius of the planet, g the surface gravity, $F_a \equiv (1 - \alpha)F_{\oplus}$ the absorbed stellar radiation at the substellar point, C_d the neutral drag coefficient, H the local isothermal scale height RT_s/g , T_s the surface temperature, p_s the surface pressure, ρ_s the surface air density, and v_s the velocity at the top of the friction layer. Following Ingersoll et al. (1985), we have assumed that the velocity is nearly independent of height over the depth that contains most of the mass of the atmosphere. The form of the vertically averaged momentum balance is approximate and assumes that the surface drag is distributed over the scale height. The planetary albedo is assumed uniform on the dayside and thus is included in F_a . Given the relation between T_s and p_s by the Clausius–Clapeyron relation, the spatial distribution of either surface temperature or pressure can be solved numerically using standard techniques for

³ This is a good assumption. If the atmosphere is optically thin, then the OLR is the blackbody emission from the surface. If not, the temperature jump between the surface and the surface air is small for a pure condensible atmosphere so that the OLR can still be expressed as a function of T_s . However, for very thin atmospheres, the surface pressure could differ greatly from the saturation vapor pressure corresponding to the surface temperature, owing to limits on the evaporation/sublimation rate imposed by the speed of sound.

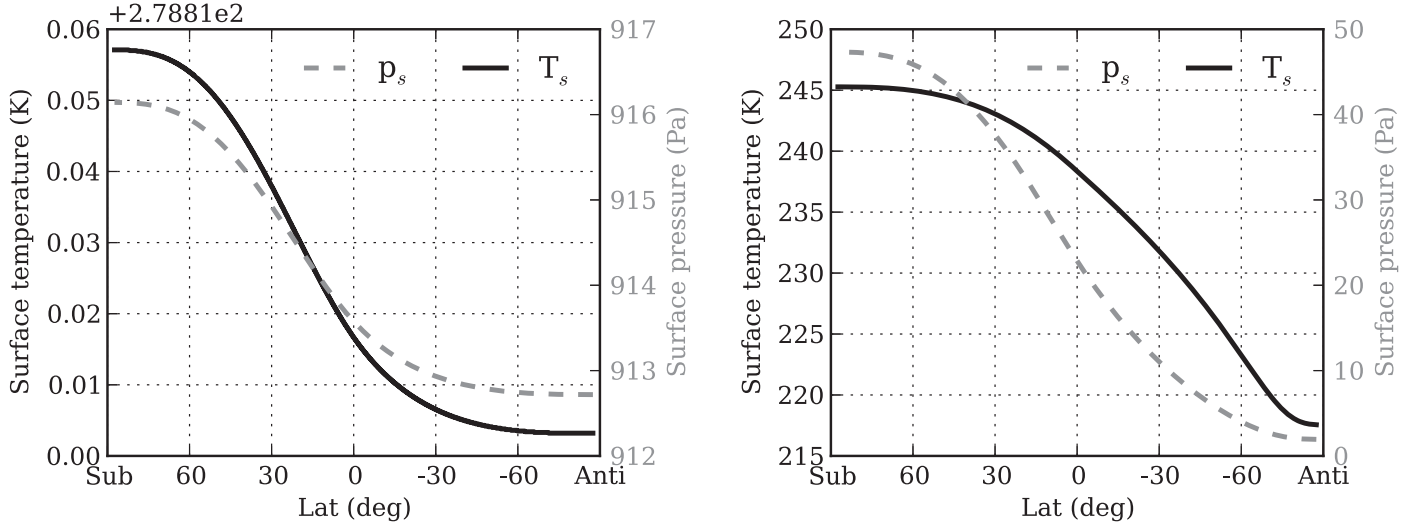


Figure 1. Meridional distribution of the surface temperature (solid black) and the surface pressure (gray dashed) in the tidally locked coordinate for the pure condensible H_2O atmosphere on a synchronously and slowly rotating planet, when (a) $F_a = 1112 \text{ W m}^{-2}$ and (b) $F_a = 712 \text{ W m}^{-2}$, respectively. Other parameters chosen to solve the 1D differential equation are $a = r_\oplus$, $g = g_\oplus$. The OLR of a pure condensible H_2O atmosphere is fit by a third-degree polynomial based on the realistic H_2O radiative transfer calculation: $\text{OLR}(T_s) = 4704.16 - 61.957T_s + 0.26912T_s^2 - 3.7244 \times 10^{-4}T_s^3$, when $200 \text{ K} < T_s < 280 \text{ K}$.

ordinary differential equations (we used the fourth-order Runge–Kutta method). Calculations reported here were carried out with $C_d = .01$, corresponding to a moderately rough surface with a roughness length of 20 cm (Pierrehumbert 2010, Equation (6.20)).

Figure 1 shows an example of the solutions for a pure condensible H_2O atmosphere. The solution in Figure 1(a) under a higher insolation represents a warm and global H_2O atmosphere in an ocean world with surface pressure variation less than 1%, while the one in Figure 1(b) represents a cold local atmosphere concentrated on the dayside of an icy planet with surface pressure variation of $\sim 95\%$. These two solutions confirm that the atmospheric thickness dominates the latent heat transport that compensates for the incoming differential radiative forcing and plays an important role in the distribution of pure condensible atmospheres. The heat redistribution is still quite efficient when the substellar point is at the freezing point of water. One consequence of this result is that, when the assumptions of our model are valid, Eyeball states with open water only near the substellar point such as discussed in Pierrehumbert (2011) can at best exist only in a very narrow range of instellations when water vapor is the only atmospheric constituent. Such worlds will tend to have surfaces that are either completely frozen over (in which case the atmosphere may be local if the system is cold enough) or completely liquid (in which case the atmosphere is global and the surface temperature is nearly uniform). The key difference with Pierrehumbert (2011) is that in the dilute-case strong nightside temperature inversions can form that allow substantial geographic variations of surface temperature even when the free tropospheric temperature is fairly uniform. Because of the control of total surface pressure by Clausius–Clapeyron, such inversions cannot occur in the pure condensible case, as they would be associated with extreme nonhydrostatic vertical pressure gradients that would drive a rapid flow to the surface, rectifying the imbalance quickly. Near the substellar point, the same processes operate in reverse and keep the surface temperature from greatly exceeding the free tropospheric

temperature at the top of the boundary layer. It remains to be seen how much noncondensable gas can be added to the atmosphere before strong nightside inversions and elevated substellar surface temperatures become possible again.

We will further explore the effect of atmospheric thickness by considering the limit of global atmosphere with weak temperature variations $\Delta T \ll T_s$. In this limit, the surface pressure gradient could still be large enough to maintain a global atmosphere due to the Clausius–Clapeyron relation

$$\frac{1}{p_s} \frac{dp_s}{d\theta_{\text{TL}}} = \frac{L}{RT_s} \frac{1}{T_s} \frac{dT_s}{d\theta_{\text{TL}}} \quad (4)$$

and the fact that $L/RT_s \gg 1$ for most condensing volatiles. Given the small surface temperature variation, $\text{OLR} \simeq F_a/4$. Note that this constraint, which is a consequence of planetary energy balance, means that the behavior will be independent of the strength of the atmosphere’s greenhouse effect, so long as our assumption that OLR can be written as a function of T_s is valid. We ignore the effect of clouds here that would alter the behavior both through their effect on OLR and on the spatial distribution of absorbed stellar radiation. However, unlike in the Earth’s tropics where clouds can be more variable than temperature, for pure condensible atmosphere, buoyancy generation in the interior of atmosphere is prohibited and convection works very differently, in fact, so differently it is unclear whether it should be called convection at all (Ding & Pierrehumbert 2016; Pierrehumbert & Ding 2016). The cloud effect needs to be evaluated consistently in 3D global climate models. The fractional variation of p_s then can be simplified as

$$\frac{1}{p_s} \frac{dp_s}{d\theta_{\text{TL}}} \simeq \begin{cases} \left(\mathcal{M} \frac{\sin \theta_{\text{TL}} + \cos 2\theta_{\text{TL}}}{\cos \theta_{\text{TL}}} \right)^2, & \theta \in [0, \pi/2] \text{ (dayside)} \\ \left(\mathcal{M} \frac{\sin \theta_{\text{TL}} + 1}{\cos \theta_{\text{TL}}} \right)^2, & \theta \in [-\pi/2, 0] \text{ (nightside)} \end{cases}, \quad (5)$$

where

$$\mathcal{M} = \frac{F_a g}{4L p_s} \frac{a \sqrt{ag C_d}}{RT_s}. \quad (6)$$

\mathcal{M} is a nondimensional parameter that can be used to distinguish whether the pure condensible atmosphere is global or local. Our assumption of a global atmosphere here requires that $\mathcal{M} \ll 1$. On the contrary, if $\mathcal{M} \geq O(1)$, the pure condensible atmosphere should exhibit large pressure variations and become local around the mass source. For example, consider the numerical solutions of the pure condensible H_2O atmosphere in Figure 1. For the global atmosphere solution in Figure 1(a), $\mathcal{M} = 0.046 \ll 1$; for the local atmosphere solution in Figure 1(b), $\mathcal{M} = 1.46$.

The nondimensional parameter \mathcal{M} can be understood by being rearranged into the ratio of two timescales: $\mathcal{M} = t_{\text{adv}}/t_{\text{evap}}$, where $t_{\text{evap}} = (p_s/g)/(F_a/4L)$ is the characteristic timescale to build up the condensible atmosphere by evaporation/sublimation under the global averaged insolation $F_a/4$, and $t_{\text{adv}} = a \sqrt{ag C_d}/(RT_s)$, the characteristic timescale to transport the air mass from the dayside to the nightside. Small values of \mathcal{M} means that the atmospheric transport is fast enough to redistribute air mass from the hot place to other regions, and thus results in a global atmosphere, and vice versa.

Ingersoll (1990) suggested a similar nondimensional parameter to distinguish the pure condensible atmosphere, $\mathcal{M}_T = (F_a g a / 4 p_s L) / c_s$, where the numerator is the velocity scale at the terminator estimated by assuming half of the evaporated mass on the dayside should be transported to the nightside in a global pure condensible atmosphere, and the denominator is the speed of sound corresponding to the global surface temperature. Our proposed nondimensional parameter can also be rearranged in a similar way, $\mathcal{M} = (F_a g a / 4 p_s L) / v_s$. The only difference is that the denominator here is the characteristic velocity scale if the atmosphere is local and the fractional variation of the surface pressure is $O(1)$ rather than the speed of sound, while the numerator is the same as the one defined in \mathcal{M}_T —the required mass transport to maintain a global pure condensible atmosphere.

The nondimensional parameter \mathcal{M} can also be used to estimate the surface temperature variation (ΔT_s) of the global pure condensible atmosphere by integrating Equation (5) from the substellar point to the antistellar point. The result is

$$\frac{T_{\text{sub}} - T_{\text{anti}}}{\bar{T}_s} \sim \mathcal{M}^2 (8 - 2\pi) \frac{RT_s}{L}. \quad (7)$$

We calculate the numerical solutions of the fractional change of the surface temperature $\Delta T/T_s$ in the pure condensible H_2O atmosphere under various values of insolation between 712 and 1112 W m^{-2} , shown in Figure 2. It is interesting that the expression in Equation (7) gives an accurate estimate of surface temperature variation not only for small values of \mathcal{M} but also when \mathcal{M} is order unity. We also show estimates of the right-hand side of Equation (7) for several planetary bodies with pure condensible atmospheres. The estimates are consistent with either observations or general circulation simulations published elsewhere: the condensing N_2 atmosphere on Triton, on Pluto (Forget et al. 2017), and possibly on early Titan (Charnay et al. 2014) should have small surface temperature variations; the condensing SO_2 atmosphere on Io should have large

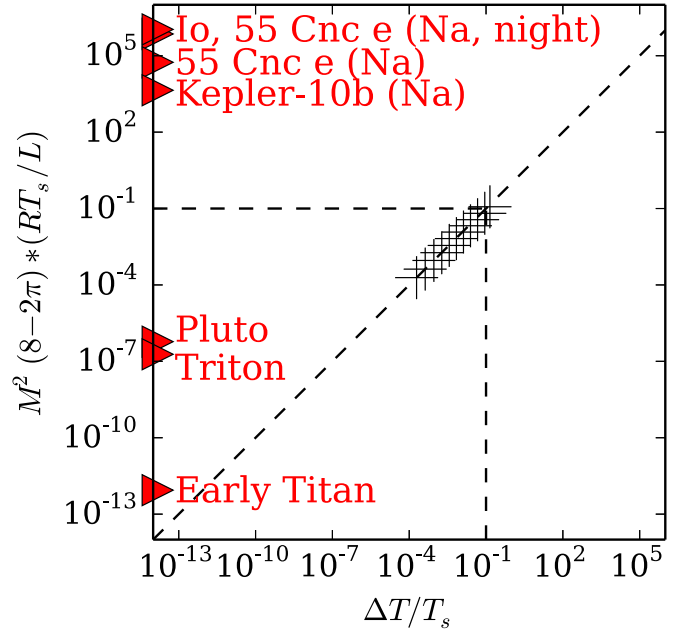


Figure 2. Theoretical fractional change of the surface temperature $\Delta T/T_s$ in a pure condensible atmosphere as a function of the expression $\mathcal{M}^2(8 - 2\pi) \frac{RT_s}{L}$. Numerical results for a pure H_2O atmosphere based on ΔT from the substellar point to the antistellar point under various values of insolation between 712 and 1112 W m^{-2} are shown as crosses. The nondimensional expression $\mathcal{M}^2(8 - 2\pi) \frac{RT_s}{L}$ of several planetary bodies with pure condensible atmospheres is estimated and marked on the vertical axis. For Io the atmosphere is assumed to be SO_2 ; for Pluto, Triton, and early Titan, N_2 ; and for the lava planets, Na. The scaling theory is only quantitatively valid for small values of \mathcal{M} , but large values are sufficient to indicate regimes in which the atmosphere is very localized to the dayside of the planet.

surface temperature variation as in Ingersoll et al. (1985). The magnitude of \mathcal{M} on these planetary bodies is primarily determined by that of p_s as discussed in Trafton & Stern (1983) and Trafton (1984), and therefore is ultimately determined by the absorbed stellar radiation. Other parameters in \mathcal{M} on these planetary bodies do not vary too much.

A complete treatment of the lava planet case would require some modifications to our formulation because vapor pressure of gases in equilibrium with a magma ocean are substantially subsaturated relative to the Clausius–Clapeyron relation for a pure gas in equilibrium with its liquid form (e.g., sodium vapor in equilibrium with a liquid sodium ocean). A crude application of the basic model to the lava planet case is nonetheless sufficient to confirm that the atmosphere will be localized to the dayside and cannot carry sufficient heat to extend the magma ocean to the nightside. We first do this by a form of *reductio ad absurdum*. Suppose that the magma ocean extends globally. In that case, we can do a Clausius–Clapeyron fit to the sodium vapor pressure curve over a silicate melt, based on the data in Miguel et al. (2011), which is valid wherever the vapor is in equilibrium with the silicate melt (hence, valid globally if the magma ocean is assumed global). With the modified effective Clausius–Clapeyron coefficients from this fit, our standard theory can be applied. The controlling nondimensional parameters assuming a global magma ocean for the cases of 55 Cancri-e and Kepler 10b are shown in Figure 2, and they show clearly that the atmospheres are in a similar regime to the localized atmosphere of Io, thus invalidating the assumption of a global magma ocean maintained by the evaporation-driven

atmosphere. If the magma ocean is localized to the dayside, then the sodium vapor atmosphere must travel some distance past the edge of the magma ocean before it cools down enough to condense. As a crude indication of the degree of localization in this regime, we also plot in Figure 2 the nondimensional parameter computed based on the Clausius–Clapeyron relation for sodium vapor, but using the temperature at which sodium at the vapor pressure at the edge of the magma ocean begins to condense (650 K). The modified calculation also puts the atmosphere in the same localized regime as Io’s SO_2 atmosphere. This calculation confirms that a thick noncondensable background atmosphere is required to account for the relatively high nightside temperature of 55 Cancr-e, as discussed in Hammond & Pierrehumbert (2017).

4. Fast-rotating Planets with Zonally Symmetric Stellar Forcing

On fast-rotating planets, Coriolis force becomes important in the atmospheric dynamics. We further simplify the problem by assuming that the surface reservoir has a thermal inertia large enough to damp out the diurnal cycle, so the atmospheric flow is zonally symmetric in the geographic coordinate system. Then, the meridional atmospheric transport that redistributes mass, momentum, and energy is characterized by the Ekman transport in the frictional boundary layer, as discussed in Pierrehumbert & Ding (2016). In the geographic coordinate system, the energy budget equation (Equation (1)) becomes

$$\frac{1}{a \cos \theta} \frac{d}{d\theta} (\cos \theta L M_E(\theta)) \simeq F_a \cos \theta / \pi - \text{OLR}(T_s(\theta)), \quad (8)$$

where θ is the geographic latitude, $M_E(\theta)$ the meridional Ekman mass transport, and F_a the absorbed stellar flux at the substellar point. Assuming that the frictional force in the Ekman layer is parameterized by a constant eddy viscosity A , the Ekman mass transport can be described by the product of the layer thickness and the zonal geostrophic wind speed (Vallis 2006, p. 112), and thus is related to the meridional surface pressure gradient

$$M_E \simeq -\frac{1}{4a\Omega} \sqrt{\frac{A}{\Omega}} (\sin^{-3/2} \theta) \frac{dp_s}{d\theta}, \quad (9)$$

where Ω is the angular spin rate of the planet. Similar to the discussion about the synchronously and slowly rotating planets, the meridional distribution of either the surface temperature or pressure can be solved numerically, and a nondimensional parameter can be derived to distinguish whether the pure condensable atmosphere is global or local, $\mathcal{M}_F = \sqrt{\Omega/A} (F_a a^2 \Omega) / (4Lp_s)$. Compared to the nondimensional parameter defined in the synchronously and slowly rotating case, \mathcal{M}_F has the same dependence on the insolation, the specific latent heat, and the surface pressure. However, \mathcal{M}_F here is independent of the gravity because the Ekman transport has no dependence on the gravity. Similarly, \mathcal{M}_F can also be rearranged as the ratio of the mass transport to maintain a global pure condensable atmosphere to the one if the atmosphere is local and the fractional variation of the surface pressure is $O(1)$.

Pierrehumbert & Ding (2016) developed a 3D general circulation model to study the basic features of condensable-rich atmospheres, including the case of a global pure condensable H_2O atmosphere. The meridional distribution of

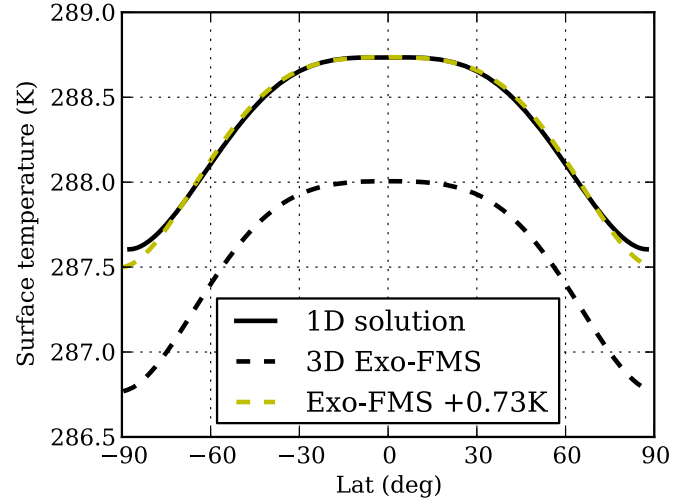


Figure 3. Meridional distribution of the surface temperature for a pure condensable H_2O atmosphere on an Earth-like fast-rotating planet. The black solid curve represents the solution of the 1D energy budget equation, and the black dashed curve represents the solution given by the 3D general circulation model in Pierrehumbert & Ding (2016). The 1D solution is generally 0.73 K higher than the 3D solution due to the energy imbalance of the GCM, which is marked by the yellow dashed curve. Parameters chosen to solve the 1D differential equation are $a = r_\oplus$, $g = g_\oplus$, $F_a = 1260 \text{ W m}^{-2}$, $A = 100 \text{ m}^2 \text{ s}^{-1}$. The OLR of the pure condensable H_2O atmosphere is given by a linear fit based on the gray radiation scheme in the 3D GCM: $\text{OLR}(T_s) = 17.53 + 1.032T_s$.

T_s from the 3D calculation is given by the black dashed curve in Figure 3. We solved the 1D energy budget equation using the same input parameters and a typical value of the boundary layer eddy viscosity $A = 100 \text{ m}^2 \text{ s}^{-1}$. The 1D solution (black solid curve in Figure 3) is generally 0.73 K higher than the 3D solution due to the energy imbalance of the GCM ($\sim 0.75 \text{ W m}^{-2}$), but has the same curvature as the 3D solution. This result confirms that for the rapidly rotating pure condensable case, the simple 1D solution of the energy budget equation reproduces the result given by more comprehensive 3D models, and the 1D energy budget equation is a generally useful tool to study pure condensable atmospheres.

5. Discussion

We have used pure water-vapor atmospheres in many of our examples, but the general principles discussed apply to any pure condensable atmosphere. The water-vapor atmospheres themselves are of potential interest for very water-rich planets, but it is unclear if such atmospheres can exist except as transients because the absence of cold-trapping generally leads to hydrogen escape and buildup of a noncondensable O_2 atmosphere (Wordsworth & Pierrehumbert 2014). However, it is possible that massive planets (inhibiting escape) with a strong oxygen sink at the surface could maintain water-dominated atmospheres. Given uncertainties about such sinks and also the possibility that a planet could be caught in a transient stage (including, perhaps, the early stages of a runaway greenhouse), it is worth knowing what the observational signatures of a water-dominated atmosphere would be.

The pure water case is relevant to the hypothetical water-dominated atmosphere considered by Turbet et al. (2016) in their series of simulations of the climate of Proxima Centauri b. Our analysis predicts that for substellar temperatures in the vicinity of the freezing point or warmer the surface temperature

should be very nearly uniform in a pure water case. The most water-dominated simulation of Turbet et al. (2016) is radically different from this prediction, exhibiting surface temperatures in excess of 320 K in a substellar ocean with the rest of the planet frozen over and nightside temperatures as low as 215 K, though the free tropospheric temperature is quite uniform globally. This case includes 0.01 bar of noncondensing N_2 , and the specific humidity at the substellar point shown in their Figure 7 is about 70% and independent of altitude. The resulting atmosphere is highly unsaturated at the surface, with relative humidity only 7% at the substellar point. This in itself violates one of the key assumptions of our analysis, and it would be tempting to conclude that the modest amount of noncondensable substance in the atmosphere has led to a radically different behavior. We do not believe this to be the case; instead, it appears that problems with the formulation of the evaporative flux in Turbet et al. (2016) have led to spurious subsaturation. Specifically, the saturation vapor pressure over water at 320 K is 0.1 bar and would lead to a layer of vapor immediately above the water surface with the same pressure. However, the hydrostatic pressure at the lowest model level (their Figure 7) is an order of magnitude lower, at only 0.01 bar. This should lead to an extremely strong evaporative flux of water vapor into the atmosphere—in fact, the upper layers of the ocean would literally boil into the atmosphere—rapidly saturating the lower atmosphere and increasing atmospheric mass. Evidently, this process is not properly represented in the LMD model employed in Turbet et al. (2016). The inconsistency we have identified depends only on the model surface pressure shown and the surface saturation vapor pressure and is in no way affected by the presence of N_2 in the atmosphere.

The question remains, however, as to how the addition of a moderate amount of noncondensable gas would affect our results. In addition, Proxima Centauri b has an orbital period of 11 days, so while Coriolis effects are weak, they are far from negligible. Both effects will require further study, most likely with the aid of 3D general circulation model simulations.

In our own solar system, the case of an evaporation/sublimation-driven pure water atmosphere may be relevant to one proposed scenario for the evolution of Pluto's crust. Sekine et al. (2017) proposed that the reddish deposits of Cthulhu Regio near Pluto's equator could have been formed in meltwater pools resulting from melting crustal ice by the giant impact that formed Charon. The scaling of temperature gradients given by Equations (6) and (7) and illustrated by the warm case in Figure 1, however, indicate that the resulting water-vapor atmosphere would likely become global and lead to global crustal melting, possibly diluting and globally redistributing organic material brought in by the impactor. There is much to be done to explore this scenario, as it depends much on the lifetime of the transient water-vapor atmosphere, but our calculations are sufficient to indicate that the possibility of a global ocean should be entertained.

The evaporation/sublimation-driven flow is a qualitatively different circulation regime as compared to the more familiar thermally direct overturning circulation. In the case of a local atmosphere, there would be a clear observational signal in the strong day/night contrast in conjunction with atmospheric spectral lines on the dayside detected by thermal emission. However, the case of a global pure condensable atmosphere would generate a flat phase curve that could be hard to


distinguish from that arising from other circulations with efficient horizontal energy transport. For example, a thick atmosphere with a moderate amount of water vapor that is dominated by an overturning circulation could also generate the same signal (Yang et al. 2013).

However, if a noncondensable radiatively active substance exists as a minor constituent in an otherwise pure condensable atmosphere, a unique observational signal could be detected. Suppose that the noncondensable gas has no sources or sinks, either at the surface or interior to the atmosphere. As a consequence of the evaporation/sublimation-driven flow in the pure condensable atmosphere, the noncondensable substance should accumulate near the antistellar point. The resulting dayside/nightside asymmetry in concentration of the noncondensable substance would affect the spectrally resolved thermal emission phase curve of the planet. Dayside emission before going into secondary eclipse would be lacking the absorption lines of the noncondensable gas. Insofar as the terminator region would also be depleted in the noncondensable gas, a similar absence could be discerned in transit-depth spectroscopy. Nightside thermal emission would on the other hand show a strong spectral signature of the noncondensing gas. Such measurements for small, cool exoplanets are probably beyond the reach of current technology, but it is to be hoped that future developments in hardware, as well as further refinements of such detection techniques as high-resolution spectroscopy, will make our suggested observational program feasible. The technique we suggest here could help to characterize the atmospheric circulation in condensable-dominated atmospheres by making use of the distribution of a noncondensable minor constituent. As an example of a related study that has been successful in the solar system, Sprague et al. (2007) studied the atmospheric argon (Ar) concentration in Mars's atmosphere from the gamma-ray spectrometer on the *Mars Odyssey* spacecraft, and they found a significant enhancement of Ar over the south polar latitudes occurring near the onset of the southern winter and rapid seasonal variations in Ar from 60°S to 90°S, owing to the concentration of Ar left behind when CO_2 condenses out onto the surface. This provides a valuable constraint on atmospheric transport mechanisms that are not directly observable. While the enhancement of noncondensable gas on exoplanets would be detected by infrared spectroscopy rather than gamma-ray spectroscopy, the general principles for interpreting the results are much the same.

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ORCID iDs

F. Ding  <https://orcid.org/0000-0001-7758-4110>

R. T. Pierrehumbert  <https://orcid.org/0000-0002-5887-1197>

References

- Castan, T., & Menou, K. 2011, *ApJ*, **743**, L36
- Charnay, B., Forget, F., Tobie, G., Sotin, C., & Wordsworth, R. 2014, *Icar*, **241**, 269
- Ding, F., & Pierrehumbert, R. T. 2016, *ApJ*, **822**, 24
- Forget, F., Bertrand, T., Vangichith, M., et al. 2017, *Icar*, **287**, 54
- Forget, F., Wordsworth, R., Millour, E., et al. 2013, *Icar*, **222**, 81

- Hammond, M., & Pierrehumbert, R. T. 2017, [ApJ](#), **849**, 152
- Ingersoll, A. P. 1990, [Natur](#), **344**, 315
- Ingersoll, A. P., Summers, M. E., & Schlipf, S. G. 1985, [Icar](#), **64**, 375
- Koll, D. D. B., & Abbot, D. S. 2015, [ApJ](#), **802**, 21
- Miguel, Y., Kaltenegger, L., Fegley, B., & Schaefer, L. 2011, [ApJL](#), **742**, L19
- Peixoto, J. P., & Oort, A. H. 1992, *Physics of Climate* (New York: AIP)
- Pierrehumbert, R. T. 2010, *Principles of Planetary Climate* (Cambridge: Cambridge Univ. Press)
- Pierrehumbert, R. T. 2011, [ApJL](#), **726**, L8
- Pierrehumbert, R. T., & Ding, F. 2016, [RSPSA](#), **472**, 20160107
- Sekine, Y., Genda, H., Kamata, S., & Funatsu, T. 2017, [NatAs](#), **1**, 0031
- Sprague, A. L., Boynton, W. V., Kerry, K. E., et al. 2007, [JGRE](#), **112**, e03S02
- Trafton, L. 1984, [Icar](#), **58**, 312
- Trafton, L., & Stern, S. A. 1983, [ApJ](#), **267**, 872
- Turbet, M., Leconte, J., Selsis, F., et al. 2016, [A&A](#), **596**, A112
- Vallis, G. K. 2006, *Atmospheric and Oceanic Fluid Dynamics: Fundamentals and Large-scale Circulation* (Cambridge: Cambridge Univ. Press)
- Wordsworth, R., & Pierrehumbert, R. 2014, [ApJL](#), **785**, L20
- Wordsworth, R. D., Forget, F., Selsis, F., et al. 2010, [A&A](#), **522**, A22
- Yang, J., Cowan, N. B., & Abbot, D. S. 2013, [ApJL](#), **771**, L45