

Probing BEC phase fluctuations with atomic quantum dots

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Abstract. We consider the dephasing of two internal states $|0\rangle$ and $|1\rangle$ of a trapped impurity atom, a so-called atomic quantum dot (AQD), where only state $|1\rangle$ couples to a Bose–Einstein condensate (BEC). A direct relation between the dephasing of the internal states of the AQD and the temporal phase fluctuations of the BEC is established. Based on this relation we suggest a scheme to probe BEC phase fluctuations non-destructively via dephasing measurements of the AQD. In particular, the scheme allows to trace the dependence of the phase fluctuations on the trapping geometry of the BEC.

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1. Introduction

The coherence properties of Bose–Einstein condensates (BECs) have attracted considerable theoretical and experimental interest since the first experimental realization of BECs in trapped ultracold clouds of alkali atoms [1, 2]. Part of this interest is due to the importance of coherence effects for the conceptual understanding of BECs and their use as a source of coherent matter waves. In particular, the absence of spatial coherence in low dimensional BECs, which exhibit strong spatial and temporal phase fluctuations, has been investigated theoretically [3]–[5] and demonstrated for one-dimensional (1D) condensates [6, 7]. Moreover, temporal first- and second-order phase coherence of an atom laser beam extracted from a BEC has been observed [8, 9]. However, to our knowledge, temporal phase fluctuations in a BEC have not yet been measured directly, despite the fact that they ultimately limit the coherence time of an atom laser beam.

In this paper, we propose a scheme to measure temporal phase fluctuations of the BEC based on a single trapped impurity atom coupled to the BEC, hereafter called an *atomic quantum dot* (AQD) [10]. More specifically, we consider an AQD with two internal states $|0\rangle$ and $|1\rangle$, where we assume for simplicity that only state $|1\rangle$ undergoes collisional (s-wave scattering) interactions with the BEC. This set-up could be implemented using spin-dependent optical potentials [11], where the impurity atom, trapped separately [12]–[14], and the BEC atoms correspond to different internal atomic states. The collisional properties of the AQD can, to a good approximation, be engineered by means of optical Feshbach resonances [15], which allow to change the atomic scattering length *locally* over a wide range and are available even when no magnetic Feshbach resonance exists.

By identifying the combined system of the AQD and the BEC with an exactly solvable *independent boson model* [16]–[18], we show that the dephasing of the internal states due to the asymmetric interaction with the BEC is directly related to the temporal phase fluctuations. This dephasing can be detected under reasonable experimental conditions, for example, in a Ramsey-type experiment [19], and hence it is possible to use the AQD to probe BEC phase fluctuations. Since the phase fluctuations depend strongly on the temperature and the density of states of the BEC, determined by the trapping geometry, the proposed scheme allows us to measure the BEC temperature and to observe the crossover between different effective BEC dimensions, notably transitions from 3D to lower dimensions. Our scheme is non-destructive and hence it is possible, in principle, to investigate the dependence of phase fluctuations on the BEC dimension and temperature for a single copy of a BEC.

Probing a BEC with an AQD was proposed recently in [10] and in a different context in [20]. However, in [10], two states corresponding to the presence of a single atom in the trap or its absence were considered, as opposed to internal atom states. More importantly, in addition to the collisional interactions, the impurity atom was coupled to the BEC via a Raman transition, allowing the realization of an independent boson model with *tunable* coupling. In particular, Recati *et al* [10] proposed to measure the Luttinger liquid parameter K by observing the dynamics of the independent boson model for different coupling strengths.

Other schemes based on interactions between impurity atoms and BECs have been proposed recently. In [21], for example, an impurity was used to implement a single-atom transistor, whereas in [22] a quantum gate exploiting phonon-mediated interactions between two impurities was investigated. Single-atom cooling in a BEC was considered in [23] where some aspects of dephasing of a two-state system (qubit) due to BEC fluctuations were addressed. Whereas their treatment focused on a 3D BEC and was based on a master equation approach, in our work

we use an analytical approach similar to dephasing calculations for semiconductor quantum dots [24, 25].

The paper is organized as follows. In section 2, we introduce our model for the AQD coupled to the BEC and identify it with an exactly solvable independent boson model, which enables us to establish a relation between the temporal phase fluctuations of the BEC and the dephasing of the AQD. We first consider an effective D -dimensional BEC, which is strongly confined in $(3 - D)$ directions and, as an ideal case, assumed to be homogeneous along the loosely confined directions. Later, we discuss corrections to this zero potential approximation due to a shallow trap potential. In section 3, we show by means of a concrete example how dephasing depends on the interaction time τ between the AQD and the BEC, the condensate temperature T and the effective condensate dimension D . Section 4 addresses corrections to our results due to an imperfect measurement of the AQD state. We conclude in section 5.

2. The model

We consider the system of an AQD coupled to a BEC in thermal equilibrium with temperature T . The BEC is confined in a harmonic trapping potential

$$V_{\text{trap}}(\mathbf{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2), \quad (1)$$

with $\mathbf{r} = (x, y, z)$ the position vector and m the mass of the condensate atoms. The trap frequencies ω_x , ω_y and ω_z assume the value(s) ω_\perp or ω , where $\omega_\perp \gg \omega$ is the trap frequency in the strongly confined (transverse) direction(s) and ω the trap frequency in the loosely confined direction(s). Depending on the number of strongly confined directions—none, one or two—the BEC is either 3D or assumes effective 2D or 1D character, provided that thermal excitations are suppressed, i.e. $k_B T \ll \hbar\omega_\perp$, and that the interaction energy of the weakly interacting BEC does not exceed the transverse energy, i.e. $mc^2 \ll \hbar\omega_\perp$, with k_B Boltzmann's constant and c the speed of sound. In directions that are loosely confined, the extension of the trap potential is assumed to be much larger than the length scale σ set by the AQD size, so that it is justified to approximate the potential by zero, as discussed at the end of this section. We consider the case where the spectrum of the BEC excitations, as compared to the impurity spectrum, is practically continuous.

The AQD consists of an impurity atom in the ground state of a harmonic trap potential centred at \mathbf{r}_0 and is described by the wavefunction $\psi_\sigma(\mathbf{r} - \mathbf{r}_0)$. We assume that $\psi_\sigma(\mathbf{r} - \mathbf{r}_0)$ takes the form of the BEC density profile in the strongly confined direction(s) and has ground state size σ in the loosely confined direction(s). For example, in case of a 1D condensate,

$$\psi_\sigma(\mathbf{r} - \mathbf{r}_0) \propto \frac{1}{\sqrt{a_\perp^2} \sigma} \exp \left[- \left(\frac{x - x_0}{\sqrt{2} a_\perp} \right)^2 - \left(\frac{y - y_0}{\sqrt{2} a_\perp} \right)^2 - \left(\frac{z - z_0}{\sqrt{2} \sigma} \right)^2 \right], \quad (2)$$

with the harmonic oscillator length $a_\perp \ll \sigma$. We further assume that the impurity atom has two internal states $|0\rangle$ and $|1\rangle$, and undergoes s-wave scattering interactions with the BEC atoms only in state $|1\rangle$.

2.1. Dephasing in the zero potential approximation

The total Hamiltonian of the system can be written as

$$H_{\text{tot}} = H_A + H_B + H_I, \quad (3)$$

where $H_A = \hbar\Omega|1\rangle\langle 1|$, with level splitting $\hbar\Omega$, is the Hamiltonian for the AQD, H_B is the Hamiltonian for the BEC, and H_I describes the interaction between the AQD and the BEC. Provided that $\sigma/l \gg 1$, with l the average interparticle distance in the BEC, we can represent the D -dimensional (quasi)condensate in terms of the phase operator $\hat{\phi}(\mathbf{x})$ and number density operator $\hat{n}_{\text{tot}}(\mathbf{x}) = n_0 + \hat{n}(\mathbf{x})$. Here, $n_0 = l^{-D}$ is the equilibrium density of the BEC, $\hat{n}(\mathbf{x})$ is the density fluctuation operator and \mathbf{x} is a D -dimensional vector describing the position in the loosely confined direction(s). We describe the dynamics of the BEC by a low-energy effective Hamiltonian [26, 27]

$$H_B = \frac{1}{2} \int d^D \mathbf{x} \left(\frac{\hbar^2}{m} n_0 (\nabla \hat{\phi})^2(\mathbf{x}) + g \hat{n}^2(\mathbf{x}) \right), \quad (4)$$

where the interaction coupling constant is defined² by $g \equiv mc^2/n_0$. We shall see that the use of this model is fully justified, even though it exhibits only Bogoliubov excitations with a linear dispersion relation $\omega_k = ck$, henceforth called phonons. The canonically conjugate field operators $\hat{\phi}(\mathbf{x})$ and $\hat{n}(\mathbf{x})$ can be expanded as plane waves

$$\hat{\phi}(\mathbf{x}) = \frac{1}{\sqrt{2L^D}} \sum_{\mathbf{k}} A_k^{-1} [\hat{b}_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{x}) + \text{h.c.}], \quad (5)$$

$$\hat{n}(\mathbf{x}) = \frac{i}{\sqrt{2L^D}} \sum_{\mathbf{k}} A_k [\hat{b}_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{x}) - \text{h.c.}], \quad (6)$$

where $\hat{b}_{\mathbf{k}}^\dagger$ and $\hat{b}_{\mathbf{k}}$ are bosonic phonon creation and annihilation operators and L^D is the sample size. With the amplitudes $A_k = \sqrt{\hbar\omega_k/g}$, the Hamiltonian (4) takes the familiar form

$$H_B = \sum_{\mathbf{k}} \hbar\omega_k \left(\hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \frac{1}{2} \right). \quad (7)$$

The coupling between AQD and the BEC occurs in the form of a density–density interaction

$$H_I = \kappa |1\rangle\langle 1| \int d^D \mathbf{x} |\bar{\psi}_\sigma(\mathbf{x} - \mathbf{x}_0)|^2 \hat{n}_{\text{tot}}(\mathbf{x}), \quad (8)$$

where $\bar{\psi}_\sigma(\mathbf{x} - \mathbf{x}_0)$ is the AQD wavefunction integrated over the transverse direction(s), and κ is the coupling constant, which can be positive or negative. To avoid notable deviations of the BEC density from n_0 in the vicinity of the AQD, we require $|\kappa| \sim g$ [29, 30]. The interaction Hamiltonian H_I acts on the relative phase of the two internal states, but does not change their population. This effect is customarily called pure dephasing.

² We neglect the weak dependence of c on the dimensionality of the BEC [28] for simplicity.

The total Hamiltonian can be identified with an independent boson model, which is known to have an exact analytic solution [16]–[18]. Inserting the explicit expression (6) for $\hat{n}(\mathbf{x})$ into equation (8), we can rewrite the total Hamiltonian as

$$H_{\text{tot}} = (\kappa n_0 + \hbar\Omega)|1\rangle\langle 1| + \sum_{\mathbf{k}} (g_{\mathbf{k}}\hat{b}_{\mathbf{k}}^{\dagger} + g_{\mathbf{k}}^*\hat{b}_{\mathbf{k}})|1\rangle\langle 1| + \sum_{\mathbf{k}} \hbar\omega_{\mathbf{k}}(\hat{b}_{\mathbf{k}}^{\dagger}\hat{b}_{\mathbf{k}} + \frac{1}{2}), \quad (9)$$

where κn_0 is the mean field shift. The coupling coefficients $g_{\mathbf{k}}$, which contain the specific characteristics of the system, are given by

$$g_{\mathbf{k}} = -\frac{i\kappa}{\sqrt{2L^D}} A_{\mathbf{k}} f_{\mathbf{k}}, \quad (10)$$

with the Fourier transform of the AQD density

$$f_{\mathbf{k}} = \int d^D\mathbf{x} |\bar{\psi}_{\sigma}(\mathbf{x} - \mathbf{x}_0)|^2 \exp(-i\mathbf{k} \cdot \mathbf{x}). \quad (11)$$

As can be seen from expression (9), the effect of the AQD in state $|1\rangle$ is to give rise to a displaced harmonic oscillator Hamiltonian for each phonon mode. This problem is identical with the problem of a charged harmonic oscillator in a uniform electric field and can be solved by shifting each phonon mode to its new equilibrium position [18]. As a consequence, the AQD does not change the phonon frequencies and hence the phase fluctuations are the same in the presence of the AQD as in its absence.

The state of the system is described by the density matrix $\rho_{\text{tot}}(t)$. We assume that the AQD is in state $|0\rangle$ for $t < 0$ and in a superposition of $|0\rangle$ and $|1\rangle$ at $t = 0$, which can be achieved, for example, by applying a short laser pulse. Hence the density matrix $\rho_{\text{tot}}(t)$ at time $t = 0$ is of the form

$$\rho_{\text{tot}}(0) = \rho(0) \otimes \rho_{\text{B}}, \quad \rho_{\text{B}} = \frac{1}{Z_{\text{B}}} \exp[-H_{\text{B}}/(k_{\text{B}}T)], \quad (12)$$

where $\rho(0)$ is the density matrix for the AQD and ρ_{B} the density matrix of the BEC in thermal equilibrium, with Z_{B} the BEC partition function. After a change to the interaction picture, the time evolution operator of the total system takes the form [17]

$$U(t) = \exp \left[|1\rangle\langle 1| \sum_{\mathbf{k}} (\beta_{\mathbf{k}}\hat{b}_{\mathbf{k}}^{\dagger} - \beta_{\mathbf{k}}^*\hat{b}_{\mathbf{k}}) \right], \quad (13)$$

with $\beta_{\mathbf{k}} = g_{\mathbf{k}}[1 - \exp(i\omega_{\mathbf{k}}t)]/(\hbar\omega_{\mathbf{k}})$. The reduced density matrix of the AQD can be determined from the relation $\rho(t) = \text{Tr}_{\text{B}} [U(t)\rho_{\text{tot}}(0)U^{-1}(t)]$, where Tr_{B} is the trace over the BEC. The coherence properties of the AQD are governed by the off-diagonal matrix elements

$$\rho_{10}(t) = \rho_{01}^*(t) = \rho_{10}(0)e^{-\gamma(t)}, \quad (14)$$

with the dephasing function

$$\gamma(t) = -\ln \left\langle \exp \left[\sum_{\mathbf{k}} (\beta_{\mathbf{k}}\hat{b}_{\mathbf{k}}^{\dagger} - \beta_{\mathbf{k}}^*\hat{b}_{\mathbf{k}}) \right] \right\rangle, \quad (15)$$

where angular brackets denote the expectation value with respect to the thermal distribution ρ_B .

The dephasing function $\gamma(t)$ can be expressed in terms of the phase operator in the interaction picture $\hat{\phi}(\mathbf{x}, t) = \exp(iH_B t/\hbar) \hat{\phi}(\mathbf{x}) \exp(-iH_B t/\hbar)$. We introduce the coarse-grained phase operator averaged over the AQD size σ

$$\hat{\phi}_\sigma(\mathbf{x}_0, t) \equiv \int d^D \mathbf{x} |\bar{\psi}_\sigma(\mathbf{x} - \mathbf{x}_0)|^2 \hat{\phi}(\mathbf{x}, t), \quad (16)$$

and the phase difference $\delta\hat{\phi}_\sigma(\mathbf{x}_0, t) \equiv \hat{\phi}_\sigma(\mathbf{x}_0, t) - \hat{\phi}_\sigma(\mathbf{x}_0, 0)$ to rewrite the phase coherence as

$$e^{-\gamma(t)} = \left\langle \exp \left[i \frac{\kappa}{g} \delta\hat{\phi}_\sigma(\mathbf{x}_0, t) \right] \right\rangle \quad (17)$$

$$= \exp \left[-\frac{1}{2} \left(\frac{\kappa}{g} \right)^2 \langle (\delta\hat{\phi}_\sigma)^2(\mathbf{x}_0, t) \rangle \right], \quad (18)$$

where the second equality can be proven by direct expansion of the exponentials [18]. Thus we have established a direct relation between the dephasing of the AQD and the temporal phase fluctuations of the BEC averaged over the AQD size σ . Moreover, the phase coherence (18) is closely related to the correlation function $\langle \Psi^\dagger(\mathbf{x}, t) \Psi(\mathbf{x}', t') \rangle$, with $\Psi(\mathbf{x}, t)$ the bosonic field operator describing the BEC in the Heisenberg representation. In the long-wave approximation, where the density fluctuations are highly suppressed, we have [26, 27]

$$\langle \Psi^\dagger(\mathbf{x}_0, t) \Psi(\mathbf{x}_0, 0) \rangle \approx n_0 \exp[-\frac{1}{2} \langle (\delta\hat{\phi})^2(\mathbf{x}_0, t) \rangle]. \quad (19)$$

Therefore expression (18) can be interpreted as a smoothed out temporal correlation function of the BEC field operator $\Psi(\mathbf{x}, t)$ provided that $\kappa = g$.

To further investigate the effect of the BEC phase fluctuations on the AQD, we require an explicit expression for the dephasing function $\gamma(t)$ depending on the parameters of our system. For this purpose, we express the dephasing function $\gamma(t)$ in terms of the coupling coefficients $g_{\mathbf{k}}$ [17]

$$\gamma(t) = \sum_{\mathbf{k}} |g_{\mathbf{k}}|^2 \coth \left(\frac{\hbar \omega_{\mathbf{k}}}{2k_B T} \right) \frac{1 - \cos(\omega_{\mathbf{k}} t)}{(\hbar \omega_{\mathbf{k}})^2} \quad (20)$$

and take the thermodynamic limit of $\gamma(t)$, which amounts to the replacement $\sum_{\mathbf{k}} \rightarrow \int_0^\infty dk g(k)$, with the density of states $g(k) = S_D L^D k^{D-1}$, $S_D \equiv D/[2^D \pi^{D/2} \Gamma(D/2 + 1)]$ and $\Gamma(x)$ the gamma function. Substituting expression (10) for $g_{\mathbf{k}}$, we find

$$\gamma(t) = \frac{S_D \kappa^2}{2g} \int_0^\infty dk k^{D-1} |f_{\mathbf{k}}|^2 \coth \left(\frac{\hbar \omega_{\mathbf{k}}}{2k_B T} \right) \frac{1 - \cos(\omega_{\mathbf{k}} t)}{\hbar \omega_{\mathbf{k}}}. \quad (21)$$

The integral is well defined due to the factor $|f_{\mathbf{k}}|^2 = \exp[-\sigma^2 k^2/2]$, which provides a natural upper cut-off. Since for typical experimental parameters $\xi/l \sim 1$, with $\xi \sim \hbar/(mc)$ the healing length, the condition $\sigma/l \gg 1$ implies that $\sigma \gg \xi$. Thus the upper cut-off at $k \sim 1/\sigma$ is still in the phonon regime, which justifies the use of the effective Hamiltonian (4).

2.2. Corrections due to a shallow potential

We now give the conditions under which the effects of a shallow trap on result (21) are negligible in the thermodynamic limit. This limit is taken such that $\omega \propto 1/R$, with R the radius of the BEC, to ensure that the BEC density at the centre of the trap remains finite. To determine the trap-induced corrections to the density of states and the phonon wavefunctions, which affect $f_{\mathbf{k}}$ defined by (11), we use a semi-classical approach based on the classical Hamiltonian [31]

$$H(\mathbf{p}, \mathbf{x}) = c(\mathbf{x})|\mathbf{p}| + V(\mathbf{x}), \quad (22)$$

with $c(\mathbf{x}) = c(1 - \mathbf{x}^2/R^2)^{1/2}$ the position-dependent speed of sound, \mathbf{p} the phonon momentum and $V(\mathbf{x}) = m\omega^2\mathbf{x}^2/2$ the shallow trap potential. The range of phonon energies ε relevant for the AQD dephasing is $\hbar\omega/2 < \varepsilon < \hbar c/\sigma$, where the lower bound goes to zero in the thermodynamic limit.

Given the semi-classical phonon wavefunctions, we find that corrections to $f_{\mathbf{k}}$ are negligible if $\sigma \ll L_\varepsilon$, with $L_\varepsilon = \sqrt{2\varepsilon/(m\omega^2)}$ the classical harmonic oscillator amplitude. For the density of states $g(\varepsilon)$, we use the semi-classical expression

$$g(\varepsilon) = \frac{1}{(2\pi\hbar)^D} \int d^D\mathbf{x} \int d^D\mathbf{p} \delta[\varepsilon - H(\mathbf{p}, \mathbf{x})] \quad (23)$$

to obtain $g(\varepsilon) \propto L_\varepsilon^D \varepsilon^{D-1} [1 + \mathcal{O}(L_\varepsilon/R)]$. Thus in the regime where $\sigma \ll L_\varepsilon \ll R$ and after the substitutions $L \rightarrow L_\varepsilon$ and $\hbar\omega_k \rightarrow \varepsilon$ we have

$$\gamma(t) \propto \frac{\kappa^2}{g} \int_0^\infty d\varepsilon \varepsilon^{D-1} |f_\varepsilon|^2 \coth\left(\frac{\varepsilon}{2k_B T}\right) \frac{1 - \cos(\varepsilon t/\hbar)}{\varepsilon}, \quad (24)$$

which up to numerical constants is identical to expression (21). The conditions on L_ε , together with the bounds of the phonon spectrum, imply that the dephasing in a shallow trap does not differ from the homogeneous case if $\xi \ll \sigma \ll a_\omega$, with $a_\omega = \sqrt{\hbar/(m\omega)}$ the harmonic oscillator length.

3. Application

In this section, we discuss by means of a concrete example³ how the AQD dephasing depends on the interaction time τ , the condensate temperature T and the effective condensate dimension D .⁴ To find the phase coherence $e^{-\gamma(\tau)}$, we have evaluated the integral (21) numerically (shown in figures 1–3) and analytically in the high-temperature regime $k_B T \gg \hbar c/\sigma$.

Figure 1 shows the phase coherence $e^{-\gamma(\tau)}$ as function of the interaction time τ . The evolution of the phase coherence is split into two regimes separated by the typical dephasing time $\sigma/c \sim 10^{-3}$ s, which can be viewed as the time it takes a phonon to pass through the AQD.

³ For the numerical values of the system parameters, see the caption of figure 1.

⁴ A discussion of a similar model in the context of quantum information processing can be found in [17].

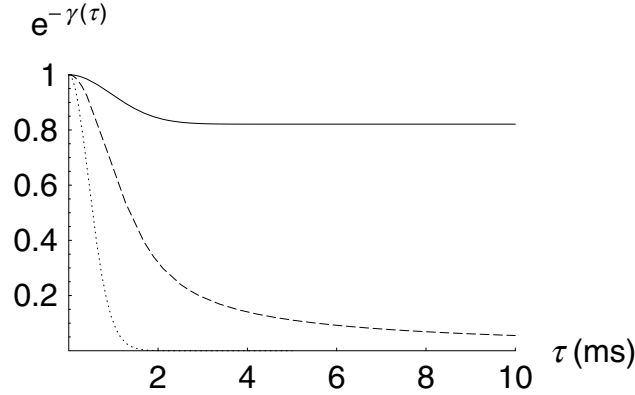


Figure 1. The phase coherence $e^{-\gamma(\tau)}$ as a function of the interaction time τ . In the regime $\tau \gg \sigma/c$, the phase coherence approaches a non-zero value in 3D (solid line), falls off as a power law in 2D (dashed line) and shows exponential decay in 1D (dotted line). The system parameters were set to $m = 10^{-25}$ kg, $l = 5 \times 10^{-7}$ m, $c = 10^{-3}$ ms $^{-1}$, $T = 2 \times 10^{-7}$ K, $\sigma = 10^{-6}$ m and $\kappa = g$.

It follows from the analytic calculations that the asymptotic behaviour of $e^{-\gamma(\tau)}$ for times $\tau \gg \sigma/c$ is

$$e^{-\gamma(\tau)} = \begin{cases} C_T \exp \left[- \left(\frac{\kappa}{g} \right)^2 \frac{mck_B T}{2\hbar^2 n_0} \tau \right] & \text{for } D = 1, \\ C'_T \left(\frac{\sigma}{c\tau} \right)^\nu & \text{for } D = 2, \\ \exp \left[- \left(\frac{\kappa}{g} \right)^2 \frac{mk_B T}{(2\pi)^{3/2} \hbar^2 n_0 \sigma} \right] & \text{for } D = 3, \end{cases} \quad (25)$$

where $\nu = (\kappa/g)^2 mk_B T / (2\pi\hbar^2 n_0)$ and C_T , C'_T are temperature-dependent constants. Thus the phase coherence tends asymptotically to a nonzero value in 3D, falls off as a power law in 2D and shows exponential decay in 1D. This result reflects the fact that the physics of 1D and 2D condensates differs significantly from that of a 3D condensate. Given the relation between the phase coherence and the BEC correlations, discussed in subsection 2.1, we note that the results for the phase coherence are consistent with findings for temporal and spatial coherence properties of the BEC [26, 27], where in the latter case the distance $|\mathbf{x}|$ has to be identified with $c\tau$.

In particular in 3D the dephasing of the AQD is incomplete under reasonable experimental conditions due to the suppressed influence of low-frequency fluctuations, which has been noted in the context of quantum information processing [17, 23]. However, the residual coherence of the AQD will eventually disappear due to processes as, for example, inelastic phonon scattering [23], which are not taken into account in our model. We also point out that, in contrast to the remnant coherence of the AQD, the typical dephasing time does not depend on the coupling constant κ .

The fact that phase fluctuations depend strongly on the density of states, or equivalently the dimension of the BEC, allows us to observe the crossover between different effective dimensions, especially transitions from 3D to lower dimensions. Figure 2 shows the phase coherence $e^{-\gamma(\tau)}$

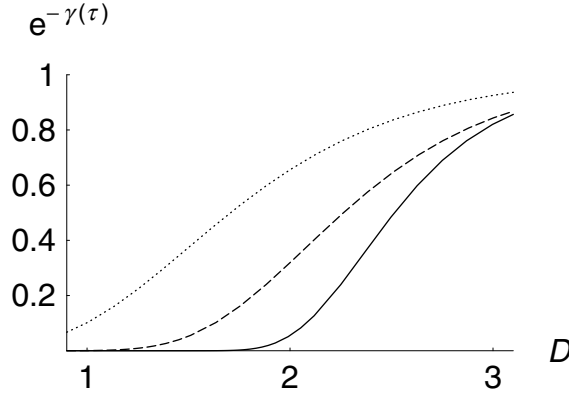


Figure 2. The phase coherence $e^{-\gamma(\tau)}$ as a function of the effective condensate dimension D for the interaction times $\tau_1 = \sigma/c$ (dotted line), $\tau_2 = 2\sigma/c$ (dashed line) and $\tau_3 = 10\sigma/c$ (solid line). The phase coherence drops significantly while the BEC excitations in the strongly confined directions are frozen out. The plot was produced with the same set of parameters as in figure 1.

as a function of the dimension D for the interaction times $\tau_1 = \sigma/c$, $\tau_2 = 2\sigma/c$ and $\tau_3 = 10\sigma/c$. The phase coherence $e^{-\gamma(\tau)}$ drops significantly as the BEC excitations in the strongly confined direction(s) are frozen out. Therefore we expect that the change of phase fluctuations depending on the effective dimension should be experimentally observable.

In addition, the AQD can be used to measure the BEC temperature since the dephasing function $\gamma(\tau)$ is approximately proportional to the temperature T according to equation (25). Figure 3 shows the relation between the phase coherence $e^{-\gamma(\tau)}$ and the BEC temperature T in all three dimensions for the interaction times $\tau_1 = \sigma/c$, $\tau_2 = 2\sigma/c$ and $\tau_3 = 10\sigma/c$. In 1D and 2D, the interaction time τ can be chosen to assure that the AQD dephasing changes significantly with temperature, whereas in 3D the interaction time τ has little influence on the thermal sensitivity of the AQD.

In the high-temperature regime $k_B T \gg \hbar c/\sigma$, the phase coherence is reduced mainly due to thermal phonons. However, even at zero temperature coherence is lost due to purely quantum fluctuations. This loss of coherence takes place on the same time scale as in the high-temperature regime, but is incomplete in both 3D and 2D, which can be seen by evaluating the integral (21) in the limit $t \gg \sigma/c$ with $T = 0$.

4. Measurement of the internal states

The AQD dephasing can be detected in a Ramsey-type experiment [19]: The AQD is prepared in state $|0\rangle$ and hence initially decoupled from the BEC. A first $\pi/2$ -pulse at $t = 0$ changes the state to a superposition $(|0\rangle + |1\rangle)/\sqrt{2}$ and a spin-echo-type π -pulse at $t = \tau/2$ neutralizes the mean field shift. After a second $\pi/2$ -pulse at $t = \tau$, the AQD is found in state $|1\rangle$ with probability

$$P(|1\rangle) = \frac{1}{2}(1 - e^{-\gamma(\tau)}). \quad (26)$$

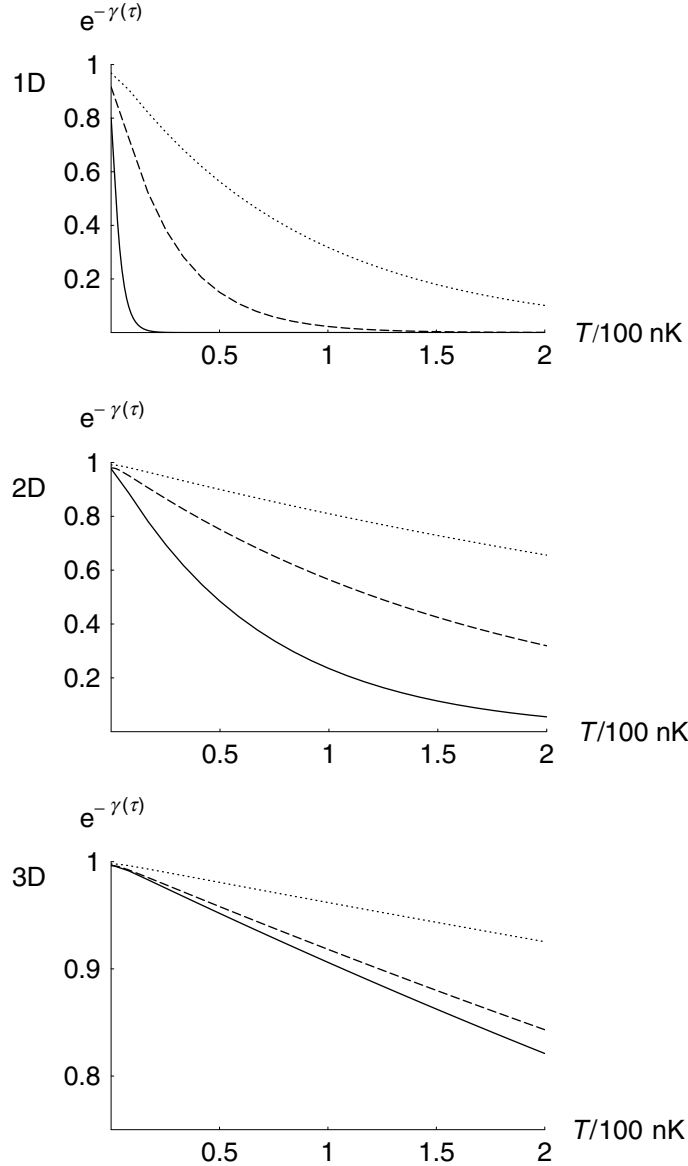


Figure 3. The phase coherence $e^{-\gamma(\tau)}$ as a function of temperature T for interaction times $\tau_1 = \sigma/c$ (dotted line), $\tau_2 = 2\sigma/c$ (dashed line) and $\tau_3 = 10\sigma/c$ (solid line). In 1D and 2D, the interaction time τ can be chosen to assure that dephasing changes significantly with temperature, whereas in 3D dephasing is independent of τ for $\tau \gg \sigma/c$. The plot was produced with the same set of parameters as in figure 1, except for the temperature.

However, this result is altered by decay of state $|1\rangle$ into state $|0\rangle$, atom loss, imperfect (noisy) detection of state $|1\rangle$, and additional dephasing due to environmental noise. We subsume these processes into three phenomenological constants, namely the detection probability P_d , the probability of a spurious detection P_s and the dephasing rate γ_d , which can all be determined experimentally. Taking these effects into account, we find an effective probability \tilde{P} to detect state $|1\rangle$

$$\tilde{P}(|1\rangle) = \frac{1}{2} P_d (1 - e^{-\gamma(\tau) - \gamma_d \tau}) + P_s, \quad (27)$$

and the visibility $V \equiv (\tilde{P}_{\max} - \tilde{P}_{\min})/(\tilde{P}_{\max} + \tilde{P}_{\min})$ in the limit $\gamma_d \tau \ll 1$ is given by

$$V = \frac{1 - \gamma_d \tau}{1 + \gamma_d \tau + 4P_s/P_d}, \quad (28)$$

where \tilde{P}_{\max} and \tilde{P}_{\min} correspond respectively to $\tilde{P}(|1\rangle)$ in the case of complete dephasing and no dephasing due to phase fluctuations. Kuhr *et al* [32] recently demonstrated state-selective preparation and detection of the atomic hyperfine state for single caesium atoms stored in a red-detuned dipole trap. They showed that dephasing times of 146 ms and ratios P_s/P_d of the order of 5×10^{-2} are achievable. If we choose $\gamma_d = 10 \text{ s}^{-1}$, $\tau = 10 \text{ ms}$ and $P_s/P_d = 5 \times 10^{-2}$, we find a visibility of $V = 69\%$, which shows that our scheme is feasible even in the presence of additional dephasing due to environmental noise.

5. Conclusion

We have shown that the dephasing of the internal states of an AQD coupled to a BEC is directly related to the temporal phase fluctuations of the BEC. Based on this relation, we have suggested a scheme to probe BEC phase fluctuations non-destructively via measurements of the AQD coherences, for example, in a Ramsey-type experiment. It was shown that the scheme works for a BEC with reasonable experimental parameters even in the presence of additional dephasing due to environmental noise. In particular, the scheme allows us to trace the dependence of the phase fluctuations on the trapping geometry of the BEC and to measure the BEC temperature.

Our scheme is applicable even if the BEC is trapped in not strongly confined directions provided that the trapping potential is sufficiently shallow. We expect that the observed dephasing will be qualitatively different from our results only if the AQD size σ is comparable to the classical harmonic oscillator amplitude $L_\varepsilon \sim 1/\omega$. In addition, our findings indicate that the use of AQDs for quantum information processing, proposed recently in [22, 23], may be constrained because of the unfavourable coherence properties of low-dimensional BECs.

The results in [17] suggest a natural extension of our work to entangled states between several AQDs, which might lead to a probe with higher sensitivity. However, the experimental requirements for the state-selective preparation and detection of entangled states are considerably higher than for a single AQD, which has to be considered in the analysis of an extended scheme.

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