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ORIGINAL ARTICLE

The "ins and outs" of student engagement in mathematics: shifts in engagement factors among high and low achievers

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Abstract

Student engagement in mathematics in the early secondary years can be fragile. Engagement in learning fluctuates in response to students' mathematics experiences and is underpinned by numerous adaptive and maladaptive factors. Thirty-seven 11–12 year old students (grades 6–7) responded twice to a questionnaire to measure shifts in their engagement and motivation over a 1-year period as they transitioned from primary to secondary school. When plotted on spider graphs, the results of specific adaptive and maladaptive factors visually demonstrate "in and out" movements as students' engagement levels shifted from time 1 to time 2. Subsequent semistructured interviews complemented questionnaire data by eliciting student beliefs about their achievement, feelings and behaviours towards mathematics. Interview data shed light on the reasons for individual student shifts in motivation and engagement during the transition. Together, data reveal four unique engagement/achievement characteristics. Significantly, students who were more alike in terms of their engagement reported similar factor patterns regardless of their achievement level. Findings draw attention to the importance of addressing mathematics engagement for students of all achievement levels.

Keywords Engagement · Disengagement · Motivation · Mathematics · Achievement · Secondary school

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Introduction

Student engagement in mathematics is of major concern with the transition from primary to secondary school identified as a time when engagement levels can significantly decrease (Skilling et al., 2015). Mathematics is not perceived as a popular subject among young people (Council of Australian Governments [COAG] 2008), with only 53% of 15-year-old students in the Organisation for Economic Co-operation and Development (OECD) countries reporting that they were interested in learning mathematics (OECD 2013). In this OECD report, it was acknowledged that "achievement is in fact only one of the factors that can help countries maximize the economic value of investments in education" and that "students need to be engaged, motivated, willing to learn new things", emphasising the importance of participation and effort to succeed (V3, p. 30). This statement reflects contemporary engagement and motivation research that has consistently highlighted the multifaceted nature of learning and the importance of motivational, emotional and social elements that influence learning outcomes and achievement in mathematics classrooms (Reschly and Christenson 2012).

While student engagement in learning is acknowledged as an important predictor of general academic achievement (Fredricks et al. 2004; Lutz et al., 2010), there is evidence that not all students who are highly engaged in mathematics experience similar levels of achievement in the subject (Skilling et al., 2015). Similarly, not all high achieving students are motivated to continue studying mathematics beyond school or to pursue mathematics-related pathways (Sullivan et al., 2006). Clearly there are differences between the engagement levels of individual students that are not explained by variations in achievement. Moreover, research by Martin et al., (2011) identified that individual differences in mathematics engagement are more salient than contextual factors and differences between environments. Further evidence exists confirming differential effects residing at the individual student level at major transition points in schooling, such as the move from primary to secondary school (Skilling et al. 2015; Sullivan et al. 2006). As students transition to secondary school, they are also likely to experience adolescence and changes in their physical, cognitive, social and psychological characteristics (Dinham and Rowe 2007). We acknowledge that these changes and prior mathematics experiences may potentially threaten student engagement. However, the significance attributed to individual-level student characteristics in multiple studies over the past two decades without any qualitative investigation into the nature of those characteristics provided the stimulus for the current study's exploration of the reasons for individual student shifts in motivation and engagement during their transition from primary to secondary school.

To better understand the reasons for variations and shifts over time in students' engagement in mathematics, we focussed on eliciting individual student's beliefs about their academic achievement in mathematics and their behaviours and feelings towards the subject. Beliefs are defined as the "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Richardson 1996, p. 103). Our interest is in understanding the phenomena of engagement from the perspective of the individual participants and therefore our study rests in an interpretive paradigm. Here reality is viewed as subjective and mediated by the individual as a way of explaining differences in value-laden constructs such as beliefs and understandings

(Skilling and Stylianides 2019). Therefore, this research investigates beliefs that reside within individual students and the research questions for this study include:

(1) What beliefs do high and low engaging students hold about their achievement and engagement in mathematics?
(2) What individual characteristics differentiate high and low engaging students; and how do they influence students' responses to learning in mathematics classrooms?

In this study, we drew upon quantitative data to investigate the shifts in engagement and disengagement of 37 students during their first year of secondary school. These data were complemented by semistructured interviews that sought to elicit students' beliefs concerning possible reasons for these shifts. Initial findings of the collective responses of student interviews were presented in a short conference paper (Skilling et al. 2015). The present paper reports additional findings by: (a) reporting individual student responses to the quantitative survey tool used to measure shifts in engagement; and motivation and (b) detailing the in-depth qualitative responses of four students who are presented as exemplar cases with different engagement/achievement characteristics.

Prior theory and research

Engagement research has typically been concerned with psychological perspectives of how students are engaged in learning settings through identifying influential factors from a range of motivational theories that are manifested in how students act, feel and think (e.g. Ryan and Deci 2000; Skinner 2016; Wigfield and Eccles 2002). It is important to distinguish between engagement and motivation as each offers a different perspective; as articulated by Reeve, "those who study motivation are interested in engagement mostly as an outcome of motivational processes, whereas those who study engagement are interested mostly in motivation as a source of engagement" (2012, p. 151).

Middleton and Spanias (1999) identified the relevance and importance of using motivational theories for understanding why students are not motivated or achieving in mathematics. Although motivational factors play more than an ancillary role in learning settings, they are not easy for teachers to identify (Walter and Hart 2009). Therefore, the current study aims to contribute to the field by conceptualising the relationship between multiple motivational factors and types of engagement. It does this by drawing on two complementary frameworks: the first focuses on behavioural, emotional and cognitive engagement (Fredricks et al. 2004); the second presents a multidimensional motivation and engagement framework (Martin 2003, 2007). The combined frameworks provide a unique theoretical basis for supporting the findings and to inform teachers and educators about the complexities of motivational factors and their relationship to student engagement.

Engagement—definitions, conceptions and theoretical framework

As engagement research from educational and psychological traditions have converged, clearer definitions and coherency for how engagement is conceptualised have been sought (Reschly and Christenson 2012; Fredricks et al. 2004). The widely acknowledged multidimensional framework proposed by Fredricks et al. (2004) distinguishes three types of engagement—behavioural, emotional and cognitive. Behavioural engagement refers to a student's participation and involvement in academic, social or extra-curricular activities associated with school. Emotional engagement is concerned with students' positive and negative affective reactions to teachers, schoolwork, peers and school and reflects emotions and other non-cognitive aspects such as interest, values and attitudes. Cognitive engagement is linked to improving student learning (Harris 2011) and draws from two perspectives: psychological investment in learning, which emphasises the efforts students make; and practices used to enhance learning and instruction, such as self-regulation strategies and metacognitive processes (Fredricks et al. 2004; Fredricks et al. 2016).

Conceptually, engagement operates dynamically and is likely to fluctuate due to individual factors and changing contexts. Hence, in school settings such as the classroom environment, the teacher's approach to instruction and inter-personal relationships, in addition to individual factors (such as motivations), are accommodated. Not only are students likely to be responsive to a range of motivational, emotional and social factors, it is expected that intensities of engagement for individual students will vary depending on activity type and cognitive demands.

Engagement—relationship to motivation

Although the terms motivation and engagement are frequently used interchangeably, they are distinct. Motivation is concerned with the psychological processes of an individual that are the sources or drivers for taking action and subsequently displayed as visible engagement characteristics (Skinner and Pitzer 2012). It is generally agreed that the motivation and engagement relationship is reciprocal, with motivation occurring before and during activities leading to engagement as well as prior engagement explaining "significant variance in subsequent motivation" (Durksenet al. 2017). It is also agreed that engagement mediates and explains the motivation to achievement relationship (Martin et al. 2011; Skinner 2016). This relationship is crucial and draws attention to an important aspect of engagement—that it is malleable and reactive to instructional environments (Fredricks et al. 2004). Shernoff (2013) stresses the importance of the instructional environment for learners, acting as compensation for bioecological influences such as gender, age, socio-economic and cultural differences, reinforcing the need to understand more clearly factors influencing student engagement.

Distinguishing specific motivational factors that contribute to the engagement characteristics of individual students has compelled theorising about the association of engagement and motivation helping to clarify their association (Durksen et al. 2017; Martin 2007; Skinner 2016). Importantly, this includes discussions about negative notions of engagement, which have variably been referred to as disaffection, disengagement, low engagement or an absence of engagement. For some, disaffection refers to students who are compliant, non-disruptive, 'invisible' and often underachieving in mathematics (Nardi and Steward 2003) resulting in negative influences on learning (Skinner 2016). In motivational terms, disengagement refers to students who actively impede their progress or have abandoned their efforts in school (Skilling et al. 2015). References to negative forms of engagement imply that dis/engagement presents as a continuum. However, as Durksen et al. (2017) notes, despite correlations between engagement and disengagement, there also exists unique variances between them, suggesting that engagement and disengagement share both complementary and distinct factors.

Martin's motivation and engagement wheel

To reflect the wide range of factors influencing engagement, Martin (2003, 2007) combines major motivation theories integrating adaptive (positive) and maladaptive (negative) constructs on a unifying framework. This is in contrast to researchers who conceive engagement from specific perspectives (e.g. Reeve prefers a self-determination perspective; Eccles and Wigfield use an expectancy-value model).

Martin's (2003, 2007) framework, known as the 'Motivation and Engagement Wheel' (MEW), includes four higher order factors under which 11 first order factors (sub-scales) are clustered (see Fig. 1). The two higher order adaptive clusters include: *adaptive motivation* comprised of self-efficacy, learning focus and valuing (and constructs of cognitive engagement) and *adaptive engagement* comprising persistence, planning and task management (and constructs of behavioural engagement). The higher order maladaptive clusters include *maladaptive motivation* factors such as anxiety, failure avoidance and uncertain control (and constructs of emotional and cognitive engagement) and *maladaptive engagement* includes self-handicapping and disengagement (and constructs of behavioural engagement). Martin's framework is used as the second theoretical framework for this study because its multidimensional nature helps unpack and describe the factors contributing to an individual's overall behavioural, cognitive and/or emotional engagement. Measures of these specific contributing factors

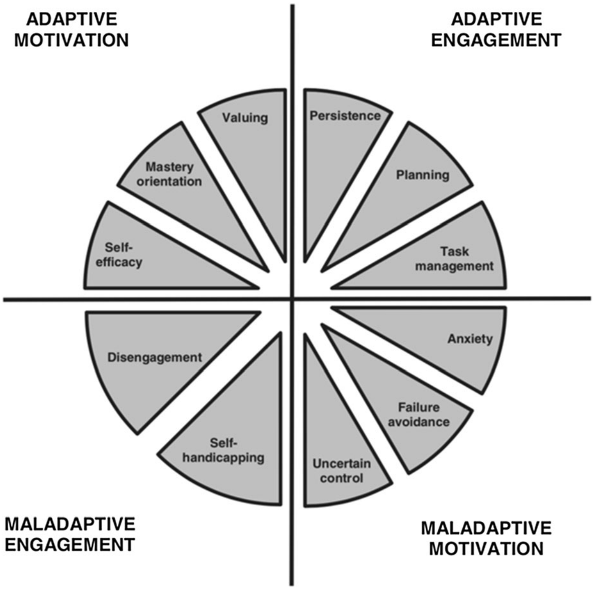

Fig. 1 Motivation and engagement wheel (reproduced with permission from Martin 2003 and Lifelong Achievement Group, www.lifelongachievement.com)

can be obtained at different points in time and thus indicate shifts in engagement or disengagement.

Influences of motivation on student engagement in mathematics learning

It is in the early secondary years that student motivation and enjoyment or dislike for mathematics tends to crystallise (Middleton and Spanias 1999). At this time, it can be particularly challenging to meet students' cognitive needs when they do not enjoy mathematics, lack confidence or fail to see its relevance (Attard 2013; Nardi and Steward 2003; Sullivan et al. 2006).

As students progress from primary to secondary school, they meet greater cognitive challenges, encounter more complex mathematics content and are expected to adopt "new kinds of classification, new kinds of perceptions and interpretation, and new representational tools" (Watson 2010, p. 137). Students also encounter more frequent exposure to problem solving tasks which are cognitively demanding, require more effort, pose the risk of failure and activate affective factors including beliefs and emotions (Stylianides and Stylianides 2014). Mathematics education researchers have found teacher practices that encourage risk taking, promote autonomy and values, nurture feelings of competency to foster conceptual understanding, and also positively affect student motivation (Reeve 2009; Stipek et al. 1998). Many of these supporting practices overlap with factors identified by motivational theorists. For example, self-efficacy, which is the extent to which individuals believe they are capable of successfully completing/performing a task (Bandura 1977), influences student learning and achievement and is a factor included in Martin's MEW (2003).

Similarly, expectancy-value theorists link student beliefs to achievement, persistence and task choice. Eccles and Wigfield's (2002) expectancy-value model describes two sets of beliefs, reflected in Martin's MEW as the relative value of a task and the probability of successfully completing it. Linked to both self-efficacy and expectancy-value is the notion of perceived control; students who believe they can control their achievement outcomes fulfil their need for competency. However, students who perceive low control or are uncertain of being successful (represented on the MEW as maladaptive motivation) tend to have their motivation undermined, reflecting disaffected engagement (Connell and Welborn 1991).

Fulfilling ones' need for competency is also affected by how students attribute their outcomes of success and failure (Skinner 2016; Skinner and Pitzer 2012) reflecting both adaptive and negative motivations on Martin's MEW. For example, learners attributing their performance to their ability are likely to have stable and strong feelings of control, whereas students who attribute their performance to luck are likely to feel unstable, low control and lower expectations of success (Weiner 1985). A student's locus of control is strongly linked to affective reactions, so that students who experience failure are likely to feel shame or anger about poor achievement.

Causal directions concerning mathematics anxiety and performance also hold important implications for student engagement and subsequent learning. Carey et al. (2016) propose that mathematics anxiety which influences performance can occur at various stages of learning, leading to responses such as avoiding mathematic situations, compromising working memory function and preferences for choosing less effective strategies—all of which are likely to lead to worsening performance. However, they also identify studies where individuals with high mathematics anxiety have been shown to suppress their negative emotional responses, resulting in a reduced performance deficit (Carey et al. 2016). This highlights the need for attending to emotional and cognitive factors that influence mathematics anxiety when addressing student engagement.

In learning environments where students feel inadequate or uncertain about their academic competency, they may impair their chances of success by 'not trying' or procrastinating to protect their self-worth (Covington 2000). Feeling competent is identified as one of three innate human psychological needs, the others being need for autonomy (exerting choice) and social relatedness, which are central constructs of self-determination theory (SDT) (Ryan and Deci 2000) and also reflected on Martin's MEW (2003).

In this section, we have endeavoured to illustrate how adaptive and maladaptive motivational factors relate to the constructs of student behavioural, emotional and cognitive engagements described by Fredricks et al. (2004). In doing so, we seek to alert teachers and educators to motivational factors that may signal shifts in student's behavioural emotional and cognitive engagement and lead to supportive classroom practices. For example, it has been identified that classroom environments that encourage competency, self-determination, autonomy and belonging also promote students' intrinsic motivation for learning and facilitate engagement (Turner et al. 2011). Other practices noted for sustaining student engagement include identifying clear content goals, promoting self-regulation of strategy use, cognitive processes for storing and retrieving information and monitoring learning (Cleary and Zimmerman 2012).

Methodology

This study was situated within a larger longitudinal mixed methods project that researched student motivation and engagement in mathematics (Skilling et al. 2016). Following ethical approval, the first phase of the project collected survey data from secondary school students across 37 schools using the Motivation and Engagement Scale (MES) (Martin 2007). The schools belonged to a large school system in a metropolitan region of Australia. The second phase of the project involved interviewing grade 7 students from 10 of the schools who evinced significant upward and downward shifts in engagement over a 12-month period. All grade 7 students were from mixed ability classes and represented a range of social and economic levels.

To understand the beliefs and influencing factors of students with different engagement and achievement characteristics, it was important to distinguish between their engagement and achievement levels. To do this, we took measures from the MES and a mathematics achievement instrument between time 1 (in the month of May in the last year of primary school) and time 2 (in the month of May in the first year of secondary school). Student-level data obtained from these instruments enabled us to determine high and low achieving students who evinced the greatest upward and downward shifts in engagement between the two time points. We were then able to invite students (approximately 11–12 years of age) from this purposefully selected sample to an interview.

Instruments and data collection procedure

The MES comprises 44 items relating to the 11 factors in the motivation and engagement wheel described earlier (see Fig. 1) against which the students rated themselves from a scale of 1 (strongly disagree) to 7 (strongly agree). For this study, the MES was adapted to the subject of 'mathematics' and four additional items were included to capture students' *enjoyment of mathematics* to reflect a measure for emotional engagement (e.g. When I'm doing maths, I feel pretty happy). Reliability, descriptive and distributional properties and confirmatory factor analytic results of the questionnaire component have been published elsewhere (Martin et al. 2011).

From these data, measures for individual students were obtained factor-by-factor and according to factor clusters. This allowed for comparisons between students in the same class, year, with similar or different levels of achievement and the same student over time. The MES is one of the few multidimensional survey instruments to reflect all three aspects of engagement and because it includes adaptive and maladaptive factors, it is useful for identifying students whose engagement levels are rising and declining (Fredricks and McColsky 2012).

A mathematics assessment instrument including age relevant numerical and algebraic questions was adapted by experienced Australian teacher educators from the Wide Range Achievement Test 3 (WRAT) (Wilkinson 1983) to measure student achievement. The adapted instrument was successfully used by Martin et al. (2012) in their study of middle years students' engagement and achievement in mathematics. The results of time 1 generated data regarding students' engagement and achievement in mathematics and established a baseline for monitoring shifts (both positive and negative) in student engagement and achievement in the following year. Using a unique individual code, it was possible to track students from grades 6 to 7 ($n = 378$) who completed the MES in time 1 and time 2.

Identifying specific students and purposeful groupings

In keeping with the aims of the study, it was important to consider engagement and achievement separate from each other for the grades 6 to 7 cohort. Analysis from the quantitative phase included multilevel modelling to analyse hierarchically structured data. One of these models focused on the transition between grades. Analysis of the factors measured by the MES showed significant ($p < .001$) declines in most of the adaptive factors and increases in disengagement over the transition period between grade 6 and grade 7 (see Martin et al. 2012 for details of quantitative data analysis and an explanation of these findings). Results of the mathematics assessment showed significant ($p < .001$) increases in average scores from 14.45 to 16.12 out of 30—in line with expectations that students' achievement levels would be higher with an additional year of schooling. It was from this group of 378 students (grades 6 to 7) that parameters for inviting students for interview were set.

Using MES data for both time 1 and time 2, individual student scores for each adaptive (six) and maladaptive (five) factor were obtained and the average for each factor calculated by applying the scoring procedures advised by Martin et al. (2011). We aggregated the adaptive and maladaptive factor scores for all students (grades 6 to 7) and calculated the net shift to group them as either becoming more *engaging* or becoming more *disengaging* (although the measures were calculated from data collected from two points in time, we view engagement as fluctuating rather than fixed, and therefore, we refer to students as engaging and disengaging rather than engaged and disengaged). Using each student's mathematical achievement score at time 2, which ranged between 6 and 26 out of a possible 30, we refined the groups using a tripartite split: scores of 18 and over determined high achievers; scores of 13 and below determined low achievers. After applying the achievement parameters to the engaging and disengaging student groups, 93 grade 7 students were identified as potential participants for interview.

With knowledge of their engagement/disengagement and high/low achievement characteristics, the 93 students were placed into one of four categories: high achieving + engaging ('HAE'), low achieving + engaging ('LAE'), high achieving + disengaging ('HAD') and low achieving + disengaging ('LAD'). For each student, the overall net shifts in engagement (i.e. where adaptive shifts increased and maladaptive decreased) and in disengagement (i.e. where adaptive shifts decreased and maladaptive increased) were calculated. Students who experienced the most significant shifts (between 1.00 and 5.00 points) in each group were identified and invited to interview. A total of 56 students were invited to interview, 37 of whom provided consent and participated in the interview. The final number of interviewees fell into each of the four categories as depicted in Table 1.

Interview protocol and collection

The semistructured interviews elicited student beliefs about their mathematical capabilities, reports on their participation and feelings towards mathematics—and of the possible reasons how changes to these or other aspects might have affected their engagement and achievement in mathematics. Regarding mathematics, the students discussed their achievement (e.g. How do think you achieve in mathematics and what tells you this? Was this the same in primary school?), described their behaviours in lessons, their feelings (e.g. Do you like mathematics and what do you like about it?), interactions with the teacher (e.g. Do you talk to your teacher about your mathematics?), homework completion (e.g. What are your mathematics homework routines?) and help seeking behaviours (e.g. Who else do you ask for help with your mathematics?). All interviews took place in the final term of the school year and took between 20 and 40 min. Interviews were audio recorded and later transcribed. Field notes taken by the interviewer included information about each student's favourite subject, homework habits and possible career choices.

Table 1 Engaging and disengaging student categories

Engaging students ($N = 19$)		Disengaging students ($N = 18$)	
High achieving (HAE) $n = 9$	6 (female) 3 (male)	High achieving (HAD) $n = 8$	4 (female) 4 (male)
Low achieving (LAE) $n = 10$	6 (female) 4 (male)	Low achieving (LAD) $n = 10$	7 (female) 3 (male)

Approach to data analysis

Analysis of interview data was mainly deductive, guided by the theoretical framework of Fredricks et al. (2004). The NVivo software programme (QSR-International 2008) was used to assist with the management of data, run queries and draw reports. The researcher began by coding comments that reflected behavioural, emotional and cognitive aspects of mathematics learning as follows: involvement and participation in mathematics, enjoyment or not of mathematics and views about mathematics achievement and strategies for learning mathematics. For example, when asked to comment on their mathematics ability, one student responded: "I'm pretty good. I go very well in tests. I study [for] maths. It's not my best subject, but I go really good in tests when I try hard" (HAD3). This multifaceted response was coded as follows: behaviours that referred to strategies ("I study [for] math") and effort ("when I try hard"), perceptions of achievement ("I'm pretty good. I go very well in tests") and comparisons to other school subjects ("It's not my best subject").

As coding continued, repeated aspects were attributed to categories, and those about similar phenomenon were established as themes. Three themes reflected the three types of engagement identified by Fredericks et al. (2004) and include the following: *Behaviours Towards Mathematics* (*behavioural engagement*), *Students' Emotional Responses Towards Mathematics* (*emotional engagement*) and *Students' Beliefs about their Mathematics Achievement* (*cognitive engagement*). Inter-rater reliability was established following recommendations by Krippendorf (2004), whereby a second member of the research team independently coded a 13.5% sample of the student interviews. Coding reliability ranged between 92 and 100% accuracy with an average of 95.8%, therefore satisfying Krippendorf's suggested level of agreement between two or more scholars (i.e. $\alpha \geq .80$).

Findings

It will be recalled that students were categorised into one of four groups: HAE, LAE, HAD and LAD. These four groupings enabled the researcher to investigate reasons for shifts in engagement or disengagement in mathematics in two distinctive ways: by type of engagement or by level of achievement. For this paper, we report the findings firstly by types of engagement and then high and low achievement. Findings are reported at both the group and individual level. At the group level, overall engagement features between the engaging/disengaging and low/high achieving students are discussed according to the themes: *behaviours towards mathematics*, *students' emotional responses towards mathematics* and *students' beliefs about their mathematics achievement.*

Then, the measures of the four individual students' adaptive and maladaptive factors, are reported to illustrate—the "ins and outs"—of specific motivational factors and their influence on overall engagement, as described in 'Martin's 'Motivation and Engagement Wheel''. Here measures of student engagement, motivation and achievement collected through the survey and quiz in the quantitative phase (previously reported by Martin et al. 2012) are presented to complement the interview data. Spider graphs (or radar charts) are used to graphically illustrate shifts in student measures on the MES from time 1 and time 2 (see Figs. 2, 3, 4, and 5). The six adaptive factor variables on the MES (self-efficacy, mastery orientation, valuing, persistence, planning and task management) plus "enjoyment" are reflected in the upper semicircle of the graphs and the five maladaptive factor variables (anxiety, failure avoidance, uncertain control, self-handicapping and disengagement) in the lower semicircle. The scores for each factor range from 0 (centre of the graph) to 7 (circumference of the graph) and when plotted, a line resembling a spider's web is created. An engaged profile would typically result in an outward line (toward the circumference of the graph) on the upper semicircle (representing high adaptive measures) and inward line (toward the centre of the graph) on the lower semicircle (representing low maladaptive measures). The opposite case indicates shifts towards disengagement: where an outward line in the lower semicircle indicates high maladaptive factors and an inward line on the upper semicircle reflects lower adaptive factor measures. The survey data results for four students are represented in Figs. 2, 3, 4 and 5, with the darker lines indicating time 1 results and the lighter line shows time 2 results. Using graphics to display the survey results visually highlights the increase and decrease in specific adaptive and maladaptive factors over 1 year—evident from the inward and outward line shifts in the graphs. This is helpful for drawing attention to factors that motivation theorists identify as significantly influencing student engagement. To illustrate the ins and outs of individual student engagement profiles, the interview and survey data of one student from each of the HAE, LAE, HAD and LAD groups are then presented as four cases: Heather, Leo, Hayden and Lydia respectively.

Engaging students

Overall, engaging students reported relatively high levels of adaptive measures and low maladaptive measures. These measures are pronounced for the high achieving engaged students (HAE) who report very high adaptive measures and a relative absence of maladaptive measures (see Fig. 2 as an example). While it might be anticipated that high achieving students would also be highly engaged, the details reported here provide a basis for comparing this group of students to unexpected student characteristics. This includes students who are as equally high achieving but disengaged (discussed in the next section). The results of the HAE group can also be compared to students who are

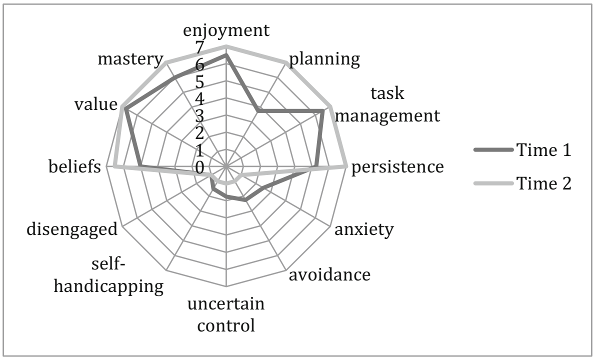

Fig. 2 High achieving engaging student HAE1

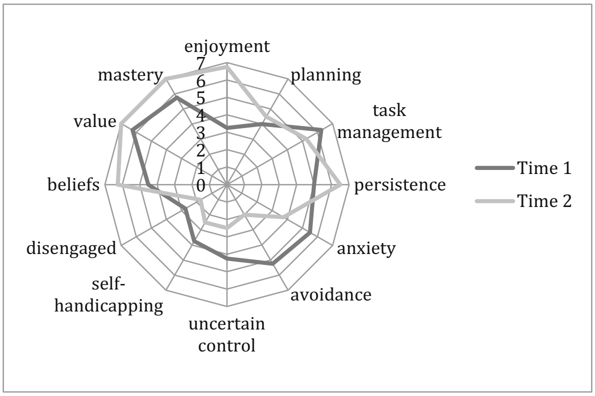

Fig. 3 Low achieving engaging student LAE7

engaged but are low achieving (LAE) (see Fig. 3 as an example). By identifying the characteristics of each group, we are able to 'notice' and discuss the similarities and differences of students with the following: (a) similar achievement levels and (b) similar engagement levels, assisting with addressing the research questions.

High achieving engaging

Students' beliefs about their mathematics achievement Most HAE students believed they were positioned in the higher range of the class and gauged this by assessment results, which were typically in the range 66 to 90%. For example, HAE8 considered that he was "pretty smart and pretty good at maths…on tests and stuff I get good marks… [I am] one of the higher people in the class". Students in the HAE group reported monitoring their learning and knew when they understood concepts when "it makes sense…and I can explain to someone else" (HAE1).

Behaviours towards mathematics Overall, HAE students reported proactive study behaviours such as asking questions of the teacher and independently seeking help to enhance their learning. If HAE students encountered problems, they would take action

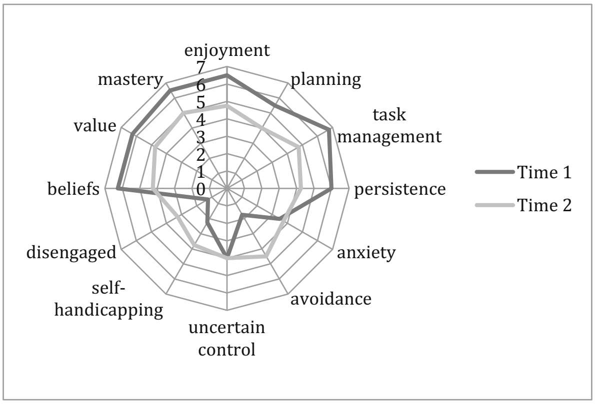

Fig. 4 High achieving disengaging student (HAD1)

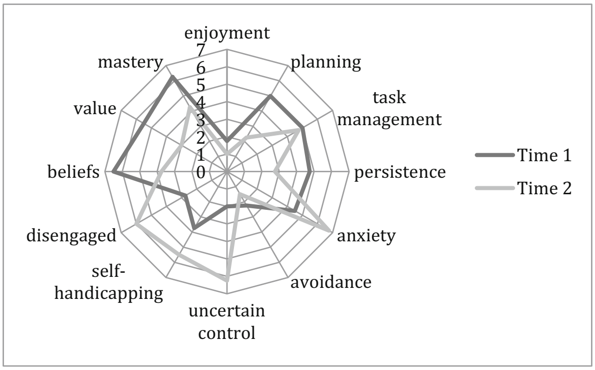

Fig. 5 Low achieving disengaging student (LAD2)

themselves, "I usually understand but if I don't, I ask" the teacher (HAE3) or ask a "smart friend in class" (HAE8) or "if I don't get things in class I go home and practice" (HAE1). Several students anticipated studying mathematics throughout school and use in their potential careers (e.g. actuary (HAE2), engineer (HAE9), mathematics teacher (HAE4) and scientist (HAE7).

Students' emotional responses towards mathematics HAE students described various reasons why they enjoyed mathematics. In some cases, students' enjoyment of mathematics was linked to their level of competency and being "easy" to understand (HAE2, HAE3, HAE4, HAE9). Three students mentioned that they liked learning new things and problem solving (HAE1, HAE2 and HAE6), because it is "more interesting and more challenging" (HAE2). Four HAE students reported that mathematics was their favourite subject and recalled previous successful experiences throughout school. Students also commented on the value of mathematics and expected future use, for example:

> I like the subject a lot, especially when I learn new things, I like learning even more because everyone says that you are going to need it after in life, so why would not you want to like that subject now, you are going to need it later on. (HAE6).

The case of Heather Heather attended a mixed-ability class in an all-girls' school. She rated herself as "pretty high" in this class, explaining, "in my last test I came first". She intended to study mathematics beyond the compulsory years for her final school qualification.

Heather reported that "maths is really interesting …and when I am older I would like a career in maths…I want to be an accountant". Heather explained that she "always loved" mathematics and it had been her favourite subject since the start of school: "I always exceled in maths…whenever I came home, I'd tell her [mum] how much fun I had in class with maths". She reported that she still discusses mathematics with her parents and that their attitude had been influential, saying, "they always tell me to try my best and to look on the positive side".

Heather normally sat at the "front of the teacher's desk. It is easier when I have questions if he's [the teacher] right there". Outside of class, Heather reported that she always completed homework and if she "doesn't get things in class" she practiced at home and then her strategy was to ask her teacher to "explain it one more time". Heather gauged her mathematics understanding when "it makes sense to me and if someone asks me, I can just tell them."

Heather spoke positively about mathematics in secondary school saying that "Year 7 work is a bit more challenging" and "we get better textbooks… to refer to questions". The part of mathematics Heather liked least was "marking and going through [workings] again", and if the teacher was teaching something she already knew, she found things "a bit boring". However, in these situations, Heather was proactive and would "see if there is an easier way to do it".

In summary, there was a consistency between the motivational factors, behaviours (proactive) and emotions (positive) Heather reported about mathematics. Survey data indicated high levels of self-efficacy, valuing of maths, enjoyment and an orientation towards understanding mathematics over performance. Interview data supports these measures. Heather believed that her efforts affected her achievement and behaved in ways that supported her interests and portrayed intrinsic motivation to do well. She monitored and regulated her learning and used strategic approaches at school and for study at home. Heather did not report feelings of uncertainty or anxiety towards mathematics nor did she avoid or act to self-handicap her learning. These personal attributes were evidenced by the combination of very low maladaptive and very high adaptive scores (illustrated in Fig. 2), reflecting high levels of all types of engagement as described by Fredricks et al. (2004).

Low achieving engaging

Students' beliefs about their mathematics achievement LAE students tended to be less certain about their mathematics performance relative to other students in the class, describing their ability as "average" or "around the middle of the class". This uncertainty may be one of the reasons why the self-efficacy scores (from the survey data) for the LAE students ranged from 3 to 6 compared to a range of 5–7 for the HAE students at time 2.

Approaches to exam study varied between LAE students, with some knowing what they needed to study and planning for it. However, the study approaches of other LAE students were often focused on practicing procedures rather than more sophisticated strategies for enhancing understating. For example, one student's approach was to "go over the work that I've learned, I memorise it and I keep doing it" (LAE3). Other students revealed that their teachers helped them prepare for the exams by telling them what to study using online resources (LAE6, LAE8).

Behaviours towards mathematics Several LAE students were aware of their lack of understanding and actively sought help when needed. For example, LAE4 explained: "I'll start whatever it is and then I'll find it difficult" so I will "ask the teacher to explain it again" and then "I know when I've got it because I find it easier to do and I get the hang of doing it". Other LAE students mentioned particular concepts in mathematics were "hard to understand" (LAE2, LAE6). However, these students also demonstrated a desire to persist and seek help. Despite LAE students struggling to perform well in mathematics, they employed behaviours that were conducive to enhancing their learning, such as paying attention in class: "You have to listen so you can understand…don't muck around when [the teacher] is speaking" (LAE10).

Students' emotional responses towards mathematics Despite not achieving well, several of the LAE students indicated they enjoyed the challenges of mathematics in secondary school, for example: "It's a little harder than primary school, but I'm learning more" (LAE4). Students also liked "learning new things" (LAE8), enjoyed the challenge of "solving different problems" (LAE9), and "hard things where you can do it by yourself" (LAE7).

The case of Leo The results of Leo (LAE7) are represented in Fig. 3. His time 2 results revealed outward shifts in most adaptive measures and inward shifts of all maladaptive measures.

Leo attended an all-boys' school and was in a mixed ability mathematics class. When asked about his favourite subjects, he reported liking sport and English, and considered "Maths is alright. I don't really mind maths" and perceived these subjects were important for his future. Leo recalled differences between teaching approaches for mathematics, explaining that primary school involved copying from books and worksheets, but in secondary school "our teacher explains it on the board and then we understand it more". Leo believed that high school maths teachers were better because they provided "more examples and they teach you more than at primary".

Leo described a shift in his beliefs regarding his current mathematics ability: "I wasn't that smart…in primary school, but when I got to high school, I got a bit smarter". In terms of understanding mathematics content, Leo reported inconsistencies, for example: "[I] get it, but when it comes to tests, I don't get it that good". When asked about what he enjoyed in mathematics, Leo said that although he needed help with algebra, he "enjoys learning it" because "I like hard things when you can do it by yourself". Leo's feelings of self-efficacy and enjoyment were supported by high self-report measures evidenced in Fig. 3.

At home, Leo used an online mathematics programme to watch lessons and do practice questions. He studied with his "mum a bit" and listened to the online lessons. Leo reported that his teacher gave notice of upcoming tests so he knew what to study, however he sometimes left study until "the last day".

Leo described proactive classroom behaviours, such as asking to move away from a talkative student so he could concentrate on his work. Leo also copied examples into his notebook so he could ask questions if he did not understand. He claimed that using the textbook was helpful because "if you don't get something…you could do homework by yourself. You could do a test…at the back of the book you have the answers and you could mark it and figure it out". Leo valued mathematics and demonstrated high levels of persistence although his strategies were low level and relatively unsophisticated. Even when faced with difficulties, he reported a positive attitude and emphasised his efforts several times during his interview. For example:

"Mostly I think it is difficult, but I don't really have feelings that I hate it…I know it is difficult, but I have to get on with it… I know I have to do it. So I sit down and have a positive feeling about it and I do it" (LAE7)

Overall, although Leo's time 2 MES measures showed high levels of valuing, mastery orientation and persistence, although measures for planning and task management were not particularly high (Fig. 3) and he did not use specific strategies to help plan and monitor his learning needs. He did not report feeling anxious, uncertainty or avoiding doing work. When asked to explain his strategies for understanding a particular concept, he replied that "before asking the teacher I will try and do it" and only if really "stuck" would he ask the teacher for help. Time 2 results in Fig. 3 show reduced measures for all maladaptive factors, including anxiety, uncertain control, avoidance, self-handicapping and disengagement.

Disengaging students

The profiles of high achieving and low achieving disengaged students present quite differently. High achieving students showed numerous robust adaptive measures in time 1, all of which were lower and shifted inward at time 2. The relative absence of maladaptive factors in time 1 gave way to notable increases in all five maladaptive factors in time 2 (see Fig. 4 as an example). Although a similar pattern of decreasing adaptive and increasing maladaptive measures were noted for the LAD group of students, the size of the shifts here were more pronounced and irregular (see Fig. 5 as an example).

High achieving disengaging

Students' beliefs about their mathematics achievement Similar to engaging students, HAD students commented on their grades obtained in class assessments to explain their achievement level in mathematics. Several students indicated low self-efficacy and feelings of uncertainty, believing their performance in mathematics had fallen during the first year of secondary school leading to feelings of uncertainty about their competency. For example, "I was one of the smarter kids in primary school but in high school, I don't know, I just lost it" (HAD6) and not feeling "as confident as last year" (HAD4).

Behaviours towards mathematics Some of the HAD students reported completing extra work at home and using interactive computer programs for revision (HAD3, HAD4, HAD7). While HAD students did not make many specific comments about study strategies, five reported asking their parents (mostly fathers) for help with mathematics at home. At school, some disengaging students reported that they asked their teacher for help in class, but none reported seeking a teacher's help outside of class time.

Students' emotional responses towards mathematics HAD students reported a range of feelings towards mathematics. Four students found repetition of concepts and lack of challenge in mathematics as "kind of boring" particularly when the teacher "explains stuff…we already know" (HAD4) or asked questions for which they already knew the answers (HAD8). Others who had experienced success at mathematics in primary school now encountered challenges and difficulties. Students had to apply more effort to master new concepts (HAD6) and because of this "I haven't really enjoyed it…as much as I used to" (HAD7). Two students reported liking the challenges of new material but not enjoying struggling with it: "I like it more because we get to learn new stuff, but I like it less because it is hard sometimes" (HAD2). These changes resulted in HAD students reported feeling uncertain about their mathematics competency and to what they should attribute this to.

The case of Hayden The results of Hayden (HAD1) are represented in Fig. 4. Her time 2 results reveal reduced adaptive measures (inwards shift) and increased maladaptive measures (outwards shifts). Hayden attended a mixed-ability class in an all-girls' school.

In interview, Hayden reported that mathematics was her favourite subject because she found it challenging and enjoyable. She also considered herself to be "better at maths" than other subjects and "I like my teachers and I like the work". She considered herself to be in the top to middle range of the class and reported liking "algebra, fractions, decimals… but I am not really good at negative numbers…". She also reported herself to be engaged in mathematics because "I focus and concentrate". Despite feeling competent and enjoying mathematics, many of her responses were indicative of a performance orientation rather than for understanding content for example, "it doesn't really matter if you enjoy it, you are just there to learn". Hayden's interview responses resonate with the reduced measures of self-efficacy, valuing, and mastery between times 1 and 2 in Fig. 4.

Hayden wanted to be a "lawyer or barrister", and although she was not sure if she needed mathematics for her career, she intended to study mathematics throughout school, believing "most jobs would need maths". Hayden reported ongoing support for mathematics from her parents at home and she worked with a mathematics tutor whom she had seen weekly since year 4. She also expressed feeling some pressure to do well in mathematics, which may explain increases in several maladaptive factors at time 2, most notably for avoidance, self-handicapping, and anxiety. In combination, the dips in adaptive factors and rise in several maladaptive factors indicate that despite her current achievement, her engagement in mathematics is waning.

Low achieving disengaging

Students' beliefs about their mathematics achievement LAD students reported a general lack of understanding and this often led them to "hope for the best" (LAD9) when completing class tests. Although the students recognised their lack of knowledge, they were reluctant to seek help from teachers and were not sure about how to help themselves. Overall, the students portrayed negative perceptions as mathematics learners, for example, "I don't think I am very good at it" (LAD3) and "Last year I felt I am doing alright but then this year of my God I am doing badly" (LAD5).

Behaviours towards mathematics A number of the LAD students reported their preference for working in collaborative ways in the mathematics classrooms, such as oral warm-up activities (LAD10), and using mini-whiteboards for sharing whole class responses. Further, working in groups was more appealing than working alone, where "everyone gets to talk and then we all understand each other" (LAD6).

Students' emotional responses towards mathematics Many students in the LAD group conveyed feelings of helplessness, frustration and anxiety, and these seemed to impede their engagement in learning and in many cases led to situations where students viewed success as unattainable. For example, "I can't seem to figure out some answers…and even if [the teacher] helps…I just keep getting lost in it" (LAD7).

The case of Lydia The results of Lydia (LAD2) are represented in Fig. 5. Time 2 results reveal declines in all adaptive measures, including self-efficacy beliefs, valuing, mastery, planning and persistence. Worryingly, Lydia's survey results also show significant increases in anxiety, uncertain control, self-handicapping and disengagement.

Lydia (LAD2) was in a mixed ability class at an all-girls' school. She explained that "I don't really like" mathematics because "I think it's boring and I don't really get it". She reported that when the teacher "tries to explain something, I don't understand it". Lydia rarely asked the teacher for help, preferring to ask her friend "because she gets good grades" or she looks at examples in the textbook. Regarding her mathematics achievement, she explained that "I don't really think I do good … I get bad marks" and described herself as "near the bottom" of the class. Although Lydia considered that she continued "to try", she confided that she was "not engaged or motivated" by mathematics and was not intending to continue studying it "…because I don't find it very interesting". Lydia found mathematics "confusing", leading to her feeling "angry… because I want to get it". The reports from Lydia reflected negative experiences and feelings of confusion and dislike as well as indicating low self-efficacy about her competency. Lydia's reports correspond to her time 2 survey results that show significant declines in measures of self-efficacy, valuing, mastery and enjoyment, as well as increases in anxiety, uncertainty, self-handicapping, and disengagement.

Lydia did report seeking help, but that help was not always forthcoming, stating that she would put her "hand up for 15 minutes and he [the teacher] won't pick you. So, I put it down and ask someone else". She reported attempting homework "most of the time" but when she did not "get it" she would "give up" rather than persist or seek further help. This resulted in her rarely completing homework and she found that the maths textbook unhelpful and "doesn't really explain it properly". Lydia recalled talking to her father about mathematics, but not "very often" and although the school offered an online mathematics program for practice at home, she did not use it. She believed that her achievement was lower than in primary school, reporting, "last year I did average…but this year I have gone down", causing her to feel that mathematics is now a "bit harder". Lydia's reports suggest that she is very aware of her low achievement and although for now she continues to make efforts at school to complete work, multiple references to low self-belief, lack of enjoyment and uncertainty about future success place her at risk of giving up. Supporting Lydia with mathematics skills, in conjunction with motivation support, would be helpful for capitalising on any future successes and gains in learning to promote her self-efficacy beliefs and persistence.

Discussion

Using Fredricks et al.'s (2004) types of engagement framework and Martin's (2003, 2007) MEW, we were able to identify and connect a range of specific adaptive and maladaptive motivational factors and how they influence student engagement. To address the research questions, students with varying engagement and achievement levels were asked to report their beliefs about their mathematics competency, their behaviours in class and at home to support their mathematics learning as well as their emotional responses towards mathematics. Students' responses to interview questions not only reflected the measures that helped categorise them as engaging or disengaging but also provided additional information and examples to enrich our understanding of the nuanced profiles of students with varying engagement/achievement characteristics. The profiles helped identify important factors that reside within students with high/low engagement and high/low achievement. Further, the reporting of individual student cases unveiled the personal nature of mathematics experiences and how they influence and inform students' current and future decisions about mathematics learning.

Engaging students

Irrespective of achievement levels, engaging students believed that mathematics was an important subject to study at school and valuable for their future. Most students in both high and low achieving groups stated an intention to study mathematics beyond compulsory requirements and related this to their potential career. Engaging students reported activities indicating high levels of participation and involvement for mathematics learning—they paid attention in class, listened to instructions and explanations by their teachers, completed homework, planned and managed their study and sought help. Overall, survey data indicated high levels of self-efficacy (Bandura 1977), valuing of mathematics (Eccles and Wigfield 2002), enjoyment, and an orientation towards understanding mathematics over performance (Dweck 2000; Middleton and Spanias 1999) evidenced by high measures in time 2.

The HAE students' survey results consistently reflected outward shifts for all adaptive factors between times 1 and 2. Additionally, of all the student groupings, HAE students were the only ones that reported a high capacity and orientation for articulating their approaches to mathematics learning and self-regulating their actions to master mathematics work. These characteristics are in keeping with research proposing that students who see themselves as being capable of doing mathematics tend to value it and employ strategies to help themselves be successful (Bandura 1977; Eccles and Wigfield 2002; Middleton and Spanias 1999; Sullivan et al. 2006).

LAE students also displayed relatively high adaptive factors but not consistently. Their self-efficacy beliefs were not as strong as high achieving students; however, they reported being as persistent and portrayed positive feelings and learning orientations towards their mathematics work. As noted by Middleton and Spanias (1999), effort can mediate ability, and failure can be seen as an acceptable part of mathematics learning which may explain why this group of students maintained a relatively strong sense of confidence for future successes. We also noted reductions in maladaptive factors for this group of students, with lower measures of anxiety, avoidance and uncertainty over the year. As suggested by Carey et al. (2016), it is possible that this group of students were able to suppress or alleviate negative emotional responses and use higher cognitive functions to mitigate the effect of mathematics anxiety.

However, while LAE students were aware when they needed to seek help with their mathematics learning, their strategies for doing so were restricted to clarifying and rehearsing mathematics work rather than using sequential strategic processes involving forethought, performance control and self-reflection as suggested by Cleary and Zimmerman (2012). Increased cognitive awareness and strategy use may be particularly helpful for improving low achieving students cognitive investment and meeting their learning needs. For example, Sullivan et al. (2006) point out that by understanding the source of the mathematics problems encountered and identification of what can help overcome their difficulties would help students to monitor their learning and meet learning goals.

Disengaging students

Overall, profiles of the students in the disengaging groups were more diverse than those of students in the engaging groups, with several factors distinctive of this group. While both high and low disengaging students portrayed decreases in adaptive and increases in maladaptive measures, the time 2 results of many HAD students' shifts were more pronounced and evidenced in outward shifts measures of anxiety, uncertainty and self-handicapping.

Although many of the HAD students believed that they achieved reasonably well at mathematics, they mainly cited their test marks and academic class rank as evidence, rather than discussing their mathematical understandings. These reports indicate a focus on performance indicators rather than a mastery orientation to learning, where the learner is more focused on the judgement of their competency rather than becoming more competent as explained by Dweck (2000). These students were also concerned about achievement levels that were lower than in the past and several worried about coping with new and challenging mathematical material, creating some confusion about their competency.

The results of the LAD students showed the most 'marked' shifts between time 1 and time 2 with many believing that they were not very "good" at mathematics and concerned with their lack of understanding. They indicated low self-efficacy beliefs about their competency, reported confusion, negative emotions and experiences to the point where some students expected failure. Students attributed their failures to internal or external causes resulting in respectively, feelings of hopelessness or anger and frustration as described by Weiner (1985). Unlike their high achieving peers the LAD students made less effort and were less proactive in seeking help, however, they did value the support of their teachers. For example, these students reported incidents of teacher encouragement and group learning settings as helpful which, as discussed by Ryan and Deci (2000), can foster feelings of belonging and act to counter feelings of being isolated within mathematics classrooms.

However, encouragement and amenable learning environments may not always enough to cultivate the students' belief of success and need for competence, as students in the LAD group reported greater shifts towards disengagement. Many of the LAD students indicated the importance of mathematics and study yet also reported activities such as failing to study in a timely manner, or grew distracted by other events, thereby reducing their chances of success. As suggested by Covington (2000), it appears that it was important for these students to maintain their self-worth although they were engaged in self-handicapping behaviours.

Factors reflecting cognitive engagement, such as persistence, planning of work and task management, were also low or absent for the majority of this group of students. For students who have experiences of repeated failure at mathematics and attribute this to their ability, they may see little value in expending effort to improve. If, as Middleton and Spanias (1999) suggest, students learn to dislike mathematics and this becomes part of their mathematical self-concept, further failure or perceived failure exacerbates their dislike and inhibits future engagement. However, being introduced to methods for managing negative emotions (Carey et al. 2016) and strategies for planning, organising and regulating their learning (Cleary and Zimmerman 2012) offer effective interventions for addressing disengagement.

Using Martin's framework (2003, 2007), a comprehensive range of both adaptive and maladaptive motivational factors were considered and connected to the three types of engagement identified by Fredricks et al. (2004), providing clarity about the interactive relationship between motivation and learning. However, there are some limitations to this study, for example, the sample size is relatively small and the students are drawn from one school system. We also used the highest and lowest measures of the mathematics quiz and MES levels, and we acknowledge that other students are also likely to have fluctuating levels of engagement that may not be as pronounced but are nevertheless important. We also note that the mathematics quiz used only numerical and algebraic questions and therefore not reflective of all mathematic curriculum content. However, the questions were of increasing complexity so that the quiz could be completed by students of different ages/grade levels and therefore had a low floor/high ceiling capacity for assessing student numerical/algebraic processes and it had been previously used and validated (Martin et al. 2012). We also note that other physical, cognitive, social and psychological characteristics may influence a lack of engagement in early secondary years, yet our concerns about more pronounced engagement in mathematics warranted this investigation of individual student beliefs.

Conclusion

In this paper, we have presented findings of high and low achieving students who are both engaging and disengaging in mathematics in early secondary years. Drawing on longitudinal quantitative and qualitative data, we were able to identify and group students who evinced the greatest upward and downward shifts in dis/engagement and conduct in-depth interviews with them. Although the number of participants in each group was relatively small, the parameters used to group the students established unique engagement/achievement groupings from which we elicited detailed student beliefs about their thinking, emotions and behaviours towards mathematics.

Grouping students according to their engagement/achievement characteristics provided a basis for comparing factors of students with similar types of engagement but different levels of achievement. The groupings also provided a basis for comparing students with similar achievement levels and distinct engagement characteristics. Through these comparisons, we could identify both patterns and significant factors that students in different categories reported as being critical for their mathematics learning. As the factors represent important motivational constructs, this information, in conjunction with interview data, is helpful for illuminating the complexity of motivations, their fluctuating nature and helps explain some of the less obvious and observable factors that influence students' mathematics experiences (Shernoff 2013; Walter and Hart 2009).

There are several implications of our findings. First, individual student engagement and motivational needs in mathematics may not necessarily be met if teachers/schools focus solely on achievement levels. For example, in this study, high achieving students who valued performance over mastery reported feelings of uncertainty and anxiety about maintaining their achievement and their future mathematics performance. Such students are likely to be at risk of disengaging with mathematics over time if not appropriately supported. Low achieving students with different engagement profiles were also identified. For example, we reported about students who have difficulties with mathematics yet still see themselves as capable of doing well in mathematics and report valuing, persisting and actively seeking help in the belief that they can augment their learning. The group of students most at risk of giving up in mathematics are the low achieving students with low self-efficacy and expectancy for success in mathematics. These students also do not use strategies to regulate their learning and tend to feel overwhelmed by uncertainty and anxiety about their mathematics capabilities often leading to failure avoidance and self-handicapping behaviours.

This research also contributes to the field by using a comprehensive motivational framework, representing multiple motivational theories and related constructs, that include important adaptive and maladaptive factors. We used survey and interview instruments specific to mathematics to identify students with high and low engagement/achievement characteristics to observed patterns of their engagement over 1 year, and through in-depth interviews, we were able to elicit their beliefs concerning possible reasons for these shifts.

The present study is innovative because it links a wide range of motivational factors to engagement by using a Martin's multidimensional MEW, which includes adaptive and maladaptive factors, and connecting these more clearly to Fredricks et al. (2004) types of engagement. By using qualitative methods, we were able to elaborate on and explain the quantitative data results. In this way, the influence of motivational factors and how students adapt to particular situations can be more clearly understood. Further, greater knowledge about the nuance of factors influencing student engagement in mathematics is revealed, reinforcing that promoting engagement is as important for low achieving students as for high achieving ones. The credibility of our inquiry is strengthened by the choice of complementary theoretical frameworks, and alignment between the methodology, methods and research questions.

This study aimed to deepen our understanding of the engagement and achievement characteristics of individual students by exploring through qualitative methods, the influence of underlying motivational factors that shape student responses to experiences with mathematics. However, we acknowledge, as Middleton and Spanias (1999) have before us, that teachers may not have sufficient background knowledge of motivational factors or their students' motivational perspectives to enable them to adequately differentiate or address their students' needs. We envisage that by discussing individual student self-reports of their engagement and achievement characteristics in the context of key motivational theories, we can raise teacher awareness about the importance of student motivation and engagement and the language to discuss this in relation to mathematics learning. In this way, teachers will be better positioned to shape their instructional approaches to better promote engagement (and dissuade disengagement) in mathematics.

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