USE OF DERIVED FACT STRATEGIES BY CHILDREN WITH MATHEMATICAL DIFFICULTIES

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Abstract

339 children aged 6 and 7 at Oxford primary schools took part in a study of arithmetic. 204 of the children had been selected by their teachers as having mathematical difficulties and the other 135 children were unselected. They were assigned to an addition performance level on the basis of a calculation pretest, and then given Dowker's (1998) test of derived fact strategies in addition, involving strategies based on the Identity, Commutativity, Addend + 1, Addend – 1, and addition/subtraction Inverse principles. The exact arithmetic problems given varied according to the child's previously assessed calculation level of the child, and were selected to be just a little too difficult for the child to solve unaided.. The technique was used of giving children the answer to a problem and then asking them to solve another problem that could be solved quickly by using this answer, together with the principle under consideration. The children were also given the WISC Arithmetic subtest and the British Abilities Scales Basic Number Skills Subtest. Analyses of variance showed that performance on the standardized arithmetic tests was independently affected by both addition performance level and group membership (unselected children versus those with mathematical difficulties). Derived fact strategy use was affected by addition performance level, but there was no independent effect of group membership.

Keywords: Mathematical difficulties; Young children; Addition; Arithmetical development; Derived fact strategies.

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One of the most crucial aspects of arithmetical reasoning is the ability to derive and predict unknown arithmetical facts from known facts, by using arithmetical principles such as commutativity, associativity and the addition/ subtraction inverse principle. For example, if we know that 44 + 23 = 67, then we can use the commutativity principle to derive the fact that 23 + 44 must also be 67. Without this ability, children will depend entirely on the facts that they have already learned, and will be unable to go beyond them independently. Also, the use of derived fact strategies is an indication of the extent to which children have an explicit understanding of the connections and interrelationships between individual number facts.

Many studies indicate that young primary school children often do use derived fact strategies, often without direct teaching (Baroody, Ginsburg and Waxman, 1983; Canobi, 2004; Canobi, Reeve & Pattison, 1998, 2003; Carpenter & Moser, 1984; Cowan & Renton, 1995; Dowker, 1998, 2005; Gilmore & Bryant, 2008). One of the earliest to emerge is the 'counting-on-from-larger' or 'min' concrete addition strategy, whereby the child adds two numbers (e.g. 2 + 6), by representing the larger number (e.g. with fingers) first, and then 'counting-on' the smaller number: "6, 7, 8 - it's 8!" This involves implicit use, with or without an explicit knowledge, of the commutativity principle (Baroody & Gannon, 1984; Cowan & Renton, 1995). By contrast, there are many sophisticated strategies involving the use of decomposition and decomposition for multi-digit arithmetic, that appear late and appear to characterise unusually skilled mental calculators (Hope and Sherrill, 1987).

A key issue for consideration is whether the use of derived fact strategies is significantly worse in children with known mathematical difficulties than in unselected children. On the one hand, there is certainly evidence from some studies that children with mathematical difficulties often rely on counting strategies to the exclusion of both retrieval and derived fact strategies (Gray & Tall, 1994; Jordan, Hanich & Kaplan, 2003; Ostad, 1997, 1998).. For example, Geary, Hoard, Byrd-Craven & DeSoto (2004) found that elementary school children with mathematical disabilities relied more on counting, made more counting errors and made less frequent use of derived fact strategies, from the 'min' strategy to decomposition strategies, less frequently than children without such difficulties.

Such relative infrequency of derived fact strategies may be in part due to problems with working memory, which makes it harder for children to keep track of several steps of a problem in memory. Since the use of derived fact strategies involves keeping a known fact in memory while carrying out the strategy necessary to derive the new fact, it is likely to be impaired by working memory difficulties. Moreover, children with mathematical disabilities know fewer facts to start with. This could work in two directions: knowing relatively few facts by heart may make it more necessary to use derived fact strategies, as direct retrieval is less often possible. On the other hand, if extremely few facts are known, then there may be insufficient basis for deriving new facts.

Some studies suggest that conceptual limitations could also be a factor. Geary, Hoard, Nugent & Byrd-Craven (2007) suggest that young children with mathematical disabilities sometimes have persistent difficulties in understanding counting concepts, and that this may affect their progression from counting-based strategies to more sophisticated ones.. Canobi et al (1998, 2003) suggest that understanding part-whole relations may be a prerequisite to the use of most derived-fact strategies.

Despite the findings of associations between mathematical disabilities and restrictions of strategy use, there is also much evidence that some people are much better at derived fact strategy use than at calculation. This is seen particularly strikingly in the case of some patients with dyscalculia following brain damage. Some such patients have developed severe impairments in their ability to retrieve arithmetical facts, but have been able to compensate to varying degrees by using derived fact strategies (Cappelletti, Kopelman, Morton & Butterworth, 2005; Hittmair-Delazer, Semenza & Denes, 1994; Sokol & McCloskey, 1991; Warrington, 1982; Zamarian, Lopez-Rolon & Delazer, 2007). Dissociations and discrepancies between factual and procedural knowledge and derived fact strategy use have also been noted in children with mathematical disabilities, especially those associated with dyslexia (Macaruso & Sokol, 1998; Pritchard, Miles, Chinn & Taggart, 1989; Russell & Ginsburg, 1984; Steeves 1983) and in typically developing children (Ginsburg, 1977; Dowker, 1998, 2005).

The present study investigates the use of derived fact strategies for addition by 6-and 7-year-old children with and without known mathematical difficulties, in relation to some other tests of mathematical performance.

The derived fact strategy test used was the same as that used by Dowker (1998), except that the N X 10 principle was omitted, because it was so rarely used by children in this age range. The principles were selected for their apparent importance, combined with their applicability across a fairly wide range of difficulty. Some derived fact strategies, such as most counting-based strategies or those based on the use of doubles, are mainly applicable to single-digit arithmetic (Carpenter& Moser, 1984); others, including certain decomposition strategies (; Beishuizen, Van Putten and Van Mulken, 1997; Carpenter, Franke, Jacobs, Fennema and Empson, 1997; Fuson & Burghardt, 2003) are mainly applicable to multi-digit arithmetic. The strategies investigated in the study to be described were restricted to those that may be used for both single-and multi-digit arithmetic.

Method:

Participants:

The participants included 339 children in Years 2 and 3 at Oxford primary schools. 204 of the children had been selected by their teachers as having mathematical difficulties in and recommended to undergo intervention in the Numeracy Recovery project described by Dowker (2001, 2008). These children came from six state schools in Oxford, but two schools furnished the majority. These are referred to as the ‘children with mathematical difficulties. The other 135 children were children taken from the same classes, at similar times, in the two schools that contributed the largest number of children in the group with mathematical difficulties. These are described simply as the ‘unselected’ children, as they had not been formally assessed as *not* having mathematical difficulties. However, as they had not been put forward for intervention, it may be assumed that they were not, as a group, considered to have difficulties at the same level as those in the first group.

The children with mathematical difficulties included 107 boys and 97 girls. The unselected children included 80 boys and 55 girls. Although the proportion of boys was a little higher in the unselected group, a Fisher’s exact test showed no significant difference between the groups (p = 0.19).

The mean age of the children with mathematical difficulties was 81.5 months (s.d. 6.47). The mean age of the unselected children was 82.6 months (s.d. 11.56). An univariate ANOVA with Group (Children with mathematical difficulties versus Unselected children) as the factor and Age in months as the dependent variable showed no significant group difference (F(1,338) = 1.27; p n.s.),

<u>Procedure</u>:

The task described here is Dowker’s (1998, 2005) test of use of arithmetical principles in derived fact strategies.

In order to evaluate the children’s competence in addition calculations, each child was given the mental addition test previously devised to assess children’s arithmetical performance prior to an estimation task (Dowker, 1997). It consisted of a list of 20 addition sums graduated in difficulty from 4 + 5, 7 + 1, etc. to 235 + 349. These sums were simultaneously presented orally and visually in a horizontal format. The children’s answers were oral.

The sums were as follows:

(1)	6 + 3	(11)	31 + 57
(2)	4 + 5	(12)	68 + 21
(3)	8 + 2	(13)	52 + 39
(4)	7 + 1	(14)	45 + 28
(5)	4 + 9	(15)	33 + 49
(6)	7 + 5	(16)	26 + 67
(7)	8 + 6	(17)	235 + 142
(8)	9 + 8	(18)	613 + 324
(9)	26 + 72	(19)	523 + 168
(10)	23 + 44	(20)	349 + 234

Testing continued with each child until (s)he had failed to give a correct response to six successive items.

The children were then assigned to an addition performance level according to their performance on the mental calculation task. Though there are six potential levels, only the first four are relevant to the present study:

(1) The *Beginning Arithmetic* level. Children at this level can usually count to 10 or higher, and do basic concrete arithmetic up to at least 5; but they cannot answer more than 1 of the first 4 items in the calculation test.

(2) The *Facts to 10* level. Children at this level can solve addition sums that add up to no more than 10, but are not reliable with larger addends. They can answer at least 3 of the first 4 items in the task, but no more than 1 of items 5 to 8

(3) The *Facts to 25* level. Children at this level can solve single-digit addition sums, even if the digits add up to more than 10, and sometimes sums that involve slightly higher numbers such as 11 + 12. However, they cannot yet deal effectively with tens and units, and rarely respond correctly to sums that add up to more than 25. They can answer at least 3 of items 5 to 8, but no more than 1 of items 9 to 12.

(4) The *2-digit Addition (No Carry)* level. Children at this level can solve 2 –digit addition sums, so long as carrying is not required. They can answer at least 3 of items 9 to 12, but no more than 1 of items 13 to 16.

They were then given an arithmetical reasoning test involving *use of arithmetical principles in derived fact strategies*. The technique was used of giving children the answer to a problem and then asking them to solve another problem that could be solved quickly by using this answer, together with the principle under consideration. Problems preceded by answers to numerically unrelated problems were given as controls. The exact arithmetic problems given varied according to the child's previously assessed calculation level of the child, and were selected to be just a little too difficult for the child to solve unaided. Such a set of problems is referred to here, as in earlier studies (Dowker, 1997), the child's *base corresponding set*).

Each child was shown the arithmetic problems, while the experimenter simultaneously read them to him/her. Children were asked to respond orally. The children received three arithmetical problems per principle: on rare occasions, when there was serious ambiguity about the interpretation of their responses, they received a fourth problem.

The principles investigated were as follows:

(1) The *Identity principle* (e.g. if one is told that 8 + 6 = 14, then one can automatically give the answer “14", without calculating, if asked “What is 8 + 6?”). N.b. This is the most basic of arithmetical principles: that if an arithmetical operation produces a given result, then the repetition of the same arithmetical principle will produce the same result. Its use in predicting the result of an arithmetical operation is properly speaking not a 'derived-fact strategy' but a 'same-fact' strategy. Thus, its inclusion in the study is intended to investigate whether children possess the concept of using the result of an operation to predict the result of another at all, over and above the particular principles that they are able to use in such predictions.

(2) The *Commutativity principle* (e.g., if 9 + 4 = 13, 4 + 9 must also be 13).

(3) The *Addend + 1 principle* (e.g., if 23 + 44 = 67, 23 + 45 must be 68).

(4) The *Addend – 1 principle* (e.g., if 9 + 8 = 17, 9 + 7 must be 17 – 1 or 16).

(5) The *addition/subtraction Inverse principle* (e.g., if 46 + 27 = 73, then 73 – 27 must be 46).

A child was deemed to be able to use a principle if (s)he could explain it and/or used it to derive at least 2 out of 3 unknown arithmetical facts, while being unable to calculate *any* sums of similar difficulty when there was no opportunity to use the principle.

Results

Table 1 shows the main characteristics of the children with and without mathematical difficulties at the different addition performance levels, and overall.

Table 1 about here

Not surprisingly, there is an association between group and addition performance level. Most children at the two lower addition performance levels (Beginning Arithmetic and Facts to 10) belonged to the group with mathematical difficulties; most at the highest level (2-Digit Addition - No Carrying) belonged to the unselected group.

However, as can be seen, there was overlap at all levels.

A multivariate analysis of variance was carried out with Age in months, WISC Arithmetic Scaled Score, BAS Basic Number Skills Standard Score, and Number of Derived Fact Strategies Used as the dependent variables. The grouping factors were Group (Children with Mathematical Difficulties versus Unselected Children) and Addition Performance Level (Beginning Arithmetic versus Facts to 10 versus Facts to 25 versus 2-DigitAddition (No Carrying).

Addition Performance Level had a significant effect on Age (($F(3, 324) = 7.2$; $p < 0.01$). Tamhane T2 post-hoc tests showed that children at the Beginning Arithmetic level were significantly younger than those at the Facts to 25 and 2-digit Addition (No Carry) levels, but no other age difference between performance levels were significant. Group did not have a significant effect ($F(1,324) = 1.39$; p n.s) and there was no significant interaction between Addition Performance Level and Group ($F(3, 324) = 1.99$; p n.s.)

WISC Arithmetic Scaled Score was significantly affected by both Group ($F(1,324) = 11.06$; $p < 0.01$) and by Addition Performance Level ($F(3,324) = 16.7$; $p < 0.01$). Tamhane T2 post-hoc tests showed that all paired comparisons were significant, except that between the two highest levels of Facts to 25 and 2-Digit Addition (No Carrying). There was a significant interaction between Group and Addition Performance Level (F(3,324 = 3.92; $p < 0.01$), indicating a general increase in group differences with increasing performance level, and a slight reversal of the difference at the lowest level.

BAS Basic Number Skills Standard Score was significantly affected by both Group ($F(1,324) = 15.94$; $p < 0.01$) and by Addition Performance Level ($F(3,324) = 28.51$; $p < 0.01$). Tamhane T2 post-hoc tests showed that all paired comparisons were significant. There was a significant interaction between Group and Addition Performance Level (F(3,324 = 4.67; $p < 0.01$), indicating a strong progressive increase in group differences with increasing performance level.

Number of Derived Fact Strategies Used was significantly affected by Addition Performance Level ($F(3,324) = 10.43$; $p < 0.01$) but not by Group ($F(1,324) = 0.23$; p n.s). Tamhane T2 post-hoc tests showed that all paired comparisons were significant, except that between the two intermediate levels of Facts to 10 and Facts to 25. There was no significant interaction between Group and Addition Performance Level (F(3,324 = 0.56).

Tamhane post-hoc tests indicated that each of the addition levels differed significantly from all the others.

A similar ANOVA was carried out with the same dependent variables and Gender as the grouping factor. Gender had no significant effect on any variable, and will not be discussed further.

A univariate analysis of covariance was then carried out with Number of Derived Fact Strategies Used as the dependent variable. The grouping factors were Group (Children with Mathematical Difficulties versus Unselected Children) and Addition Performance Level (Beginning Arithmetic versus Facts to 10 versus Facts to 25 versus Addition (No Carrying). The coveriates were Age in months, Scaled Score on the WISC Arithmetic subtest, and Standard Score on the BAS Basic Number Skills subtest. Addition Performance Level still had a highly significant effect on number of derived fact strategies used ($F(3, 324) = 4.65$; $p < 0.01$). Tamhane post-hoc tests indicated that each of the addition levels differed significantly from all the others. There was no significant effect of Group (F(1,324(= 0.11; p n.s) or any significant interaction between Group and Addition Performance Level ($F(3,324) = 0.99$; p n.s.). BAS Basic Number Skills was a significant covariate ($F(1,327) = 6.59$; $p < 0.05$) but WISC Arithmetic was not ($F(1,324) = 0.49$; p n.s); nor was Age (($F(1,324) = 1.15$; p n.s).

Table 2 shows the proportions of children in the Unselected group and those with mathematical difficulties who used each of the specific derived fact strategies. Fisher exact tests showed highly significant group differences for each of the strategies.

Table 2 about here

Table 3 shows the proportions of children at each addition performance level who used each of the specific derived fact strategies. Fisher exact tests showed highly significant differences between the performance levels for each of the strategies.

Table3 about here

Discussion

There are clear and strong differences between children with mathematical difficulties and unselected children. Children identified by their teachers as having problems with mathematics do indeed show weaknesses in comparison with other children in measures of numerical competence (BAS Number Skills), word problem solving and arithmetical reasoning (WISC Arithmetic) and derived fact strategy use.

The greater use of derived fact strategies by unselected children than those with mathematical difficulties was not specific to any one strategy. As found in previous studies (Dowker, 1998, 2005), some strategies were generally easier than others. The order of difficulty, as demonstrated by frequency of use, appeared to be: Identity (easiest); Commutativity; Addend + 1; Addend – 1; and Inverse (by far the most difficult). However, each strategy was used with significantly greater frequency by unselected children than by those with mathematical difficulties (Table 2). Similarly, each strategy was used with greater frequency by children at higher addition performance levels than by those at lower addition performance levels (Table 2).

Is the likelihood of using derived fact strategies mainly a matter of absolute level of competence in addition; or is there a difference between children who do and do not have *difficulties* in arithmetic, even where their absolute level of competence is similar? Although there is unsurprisingly a strong relationship between addition performance level and whether or not a child has mathematical difficulties, the two do not go together in a rigid fashion. This is particularly noticeable at the two intermediate levels: there are substantial numbers of children with and without mathematical difficulties at the Facts to 10 and Facts to 25 levels. There are relatively few children without mathematical difficulties at the Beginning Arithmetic level (as is to be expected given that the children were already in at least their second year of formal instruction), or children with mathematical difficulties at the 2-digit Addition (No Carry) level; but exceptions existed in both groups. Thus, it was possible to tease apart the effects of absolute performance level in addition and the presence or absence of mathematical difficulties. This gave some interesting results.

Both addition performance level and group membership had independent effects on the standardized tests of arithmetical performance. Children at higher addition performance levels performed better on standardized tests than did children at lower addition performance levels. Children with mathematical difficulties performed better than those without mathematical difficulties. Each effect continued to be significant even after controlling for the other.

By contrast, only addition performance level had a significant independent effect on derived fact strategy use. As has been previously found (Dowker, 1998), children at higher addition performance levels tended to use more derived fact strategies. This may be either because knowledge of arithmetic facts and procedures leads to greater access to arithmetical strategies, or because derived fact strategy use contributes to calculation, or most likely both. However, once addition performance level was taken into account, there ceased to be an influence of group membership. Children with mathematical difficulties used just as many derived fact strategies as children without such difficulties at the same addition performance level.

Why is there this difference between standardized test performance and use of derived fact strategies? Performance on standardized tests – both of written arithmetic (British Abilities Scales) and oral word problem solving (WISC Arithmetic) seem to be related to addition calculation ability, but additionally, independently to the existence teacher-assessed mathematical difficulties; derived fact strategy use is independently related only to the former. This reflects indicates that mathematical difficulties, at least as normally defined by schools, are not predominantly characterized by specific problems with derived fact strategy use. They evidently do involve a number of other characteristics, that lead to poor performance in these standardized tests. This finding may be related to the fact that British schools are currently expected to make extensive use of formal testing ('school assessment tests') to monitor children's performance. Although these are not the same tests as used in this study, teachers will therefore be accustomed to assessing children's mathematical ability through tests, and this may mean that identification of mathematical difficulties is particularly influenced by characteristics measured on tests, and less associated with derived fact strategy use. It should however be remembered that in Britain use of derived fact strategies is explicitly emphasized as part of the curriculum, so that one cannot assume the sharp distinction between ‘derived fact strategies’ and ‘school-taught strategies’ as was the case in some earlier studies (e.g. Baroody, Ginsburg & Waxman, 1983).

It is not entirely clear whether this increased formal focus on derived fact strategy use in Britain has in fact strongly influenced children’s use of such strategies. The Primary Numeracy Strategy, which emphasizes use of derived fact strategies, was instituted in 1998-1999. It may therefore be of interest to compare the use of derived fact strategies by unselected children in the present sample with those in the sample reported by Dowker (1998), who were studied shortly before this strategy was instituted. The mean numbers of derived fact strategies used at each level by children in that study were 0.72 at the Beginning Arithmetic level (0.4 in the current study); 1.82 at the Facts to 10 level (1.5 in the current study); 3.09 at the Facts to 25 level (2.25 in the current study) and 4.0 at the 2-Digit Addition – No Carrying level (3.18 in the current study). Thus, the unselected children in the current study seem to make slightly less use of derived fact strategies than those in the earlier study. This may be because more older children were included in the earlier study. In any case, it suggests that the increased explicit focus on derived fact strategies in primary school mathematics teaching may not be leading to their increased use. It may however be that schools were already beginning to place greater emphasis on derived fact strategies before the formal implementation of the Primary Numeracy Strategy, and that this already affected the earlier findings.

Intriguingly, the analysis of covariance suggested some independent relationship between derived fact strategy use and a test of written calculation (BAS Basic Number Skills), but not between derived fact strategy use and a test of oral arithmetical reasoning and word problem solving (WISC Arithmetic). Further research with other written and oral calculation and reasoning tasks might help to elucidate the reasons for this distinction.

In any case, the results do indicate at least partial functional independence of derived fact strategy use from some other mathematical skills. In this respect they support earlier studies, which suggest that derived fact strategy use can dissociate from factual and conceptual knowledge both in children (e.g. Russell & Ginsburg, 1984) and in dyscalculic patients (e.g. Zamarian et al, 2007), and give further evidence for the componential nature of arithmetical ability (Dowker, 2005). At the same time, the confirmation of a relationship between addition performance level and derived fact strategy use does indicate that the dissociation is not complete, and that although derived fact strategy use is not totally dependent on each other. As Zamarian et al point out (2007, pp. 256-257), despite the dissociability of different types of arithmetical knowledge, 'the different types of knowledge benefit from each other. For example, good conceptual knowledge allows the targeted selection and efficient application of calculation procedures, and a rich store of stored facts supports insight into the relations of different number combinations and leads to a better conceptual understanding of the number domain'.

The relationship between calculation performance level and derived fact strategy use could indicate that a certain level of arithmetical knowledge is a prerequisite for the use of such strategies. Alternatively, the derived fact strategies may develop first, and contribute to an improvement in calculation performance. These alternative possibilities have parallels with the 'some principles first' (Gelman & Gallistel, 1977) and 'skills first' (Briars & Siegler, 1984; Fuson, 1988) theories of the relationship between counting principles and procedures. Those studies (Cowan, Dowker, Bailey & Christakis, 1996; Fluck & Henderson, 1996) that have simultaneously assessed children's counting proficiency and their use of principles have suggested that the two are closely related, and that discrepancies can nonetheless occur between the two *in either direction*. Such findings are difficult to reconcile with the strongest versions of either a 'principles-first' or a 'skills first' theory, and suggest some degree of 'mutual development' between the two (Baroody, 1992). In the present study, similar findings of a strong relationship between calculation and derived fact strategy use, combined with discrepancies occurring in both directions, suggest a similar mutual development.

Ultimately, longitudinal studies, preferably incorporating training experiments, would be desirable to establish any direction(s) of causation between calculation performance level and derived fact strategy use. There have already been some studies that suggest that it is possible, at least in the short term, to train children successfully to use derived fact strategies (Adetula, 1996; Markowits & Sowder, 1994; Steinberg, 1985; Tournaki, 2003).

Further research should also investigate the relationships between derived fact strategy use and other mathematical skills over a wider age and ability range. It may for example be that with age, and with increasingly complex mathematical material at later stages of schooling, other mathematical abilities will become increasingly dependent on derived fact strategy use, and thus a greater independent relationship may be found between derived fact strategy use and the presence or absence of mathematical difficulties at later ages

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