

Filling a gap in the history of π : an exciting discovery

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The ratio of a circle's circumference to its diameter was calculated with steadily increasing precision during the 18th and 19th centuries: 100 decimal places in 1706 (John Machin), 140 places in 1789 (Jurij Vega), and 205 in 1844 (Leopold von Schulz Straszniaky). But one piece of the story has long been missing: an unknown resident of Philadelphia who calculated 154 decimal places in 1721 but never published them. This note reports on the unexpected and exciting rediscovery of the manuscript containing the calculations.

What Baron Zach saw

A persistent rumour says that someone calculated 154 digits of π in the 18th century but never published them. Wikipedia says so; Berggren and the Borweins' source book on π says so; the big recent survey article on the calculation of π by Agarwal, Agarwal and Sen says so. Much earlier, the great Victorian mathematician Augustus De Morgan said so in 1841, attributing the story to Jean-Étienne Montucla, the 18th-century author of a famous history of mathematics. And, sure enough, Montucla's *Histoire des mathématiques* tells the same story.

There the trail becomes colder. Montucla names Franz Xaver, Baron Zach as his source, and says he saw the calculation in a manuscript in Oxford. Zach was a relentlessly cosmopolitan astronomer, journal editor and retailer of scientific news in the 18th and early 19th centuries, whose eventful life took him from Budapest to Paris, and included visits to London and Oxford among many other places. He never

published anything about π .

I was intrigued, because this would have been the most accurate calculation of π ever made at that date. But tracking down an unnamed manuscript among the millions now held in Oxford's libraries seemed like looking for a needle in a haystack. According to Montucla, the manuscript was in 'la bibliothèque de Ratclif', the 'Ratclif' library, which doesn't exist. And it had last been seen in the 1780s, by Baron Zach. So, could I find it?

In fact, it tracked down surprisingly easily, through the now-digitized (and therefore searchable) manuscripts catalogue of the Bodleian Library. Wherever it was in the 18th century (probably the Radcliffe collection of scientific books), the manuscript that Zach saw is now in the Bodleian, the main library of the University of Oxford. It's a huge and impressive volume with over 500 pages, measuring nearly a foot wide. I had to find a special bookstand to look at it, and a wide-angle lens to photograph it. It gives a fascinating window into what could be done – and what *was* done – using pen and paper in the 18th century.

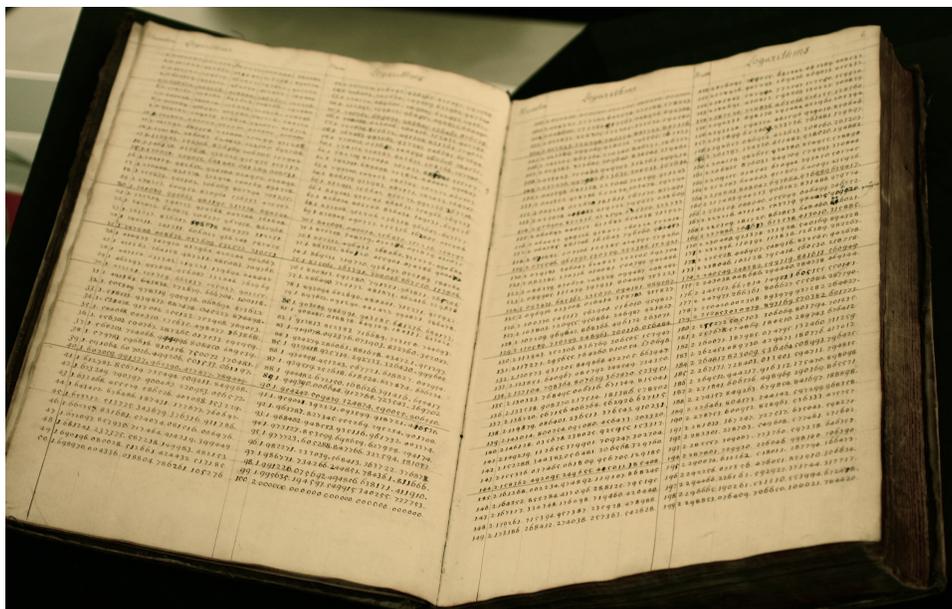


Figure 1: The 'Oxford' manuscript, open at one of the many pages of logarithm tables.

The first 200-odd pages are taken up with extensive tables of logarithms and sines, all calculated by hand to 30 decimal places. Astonishing stuff, and a testimony to

the author's powers of endurance. Towards the end comes the calculation of π : 'An easy Quadrature of the Circle', in the author's own phrase. The author would have known that squaring the circle was difficult geometrically, and that trying to do it was becoming the territory of eccentrics – though it hadn't yet been proved that it was impossible. Certainly, calculation was the only feasible approach that anyone knew.

The author of this manuscript calculated π in 1721, and checked it carefully a couple of times before finally making this neat copy in 1759. It was an obsession that lasted much of a lifetime, in other words.

The value given is

3.141592 653589 793238 462643 383279 502884 197164 399375 105820 974944 592307
816406 286208 998628 034825 342117 067982 148086 593242 306647 093844 609550 592231 725359 408128 4802.

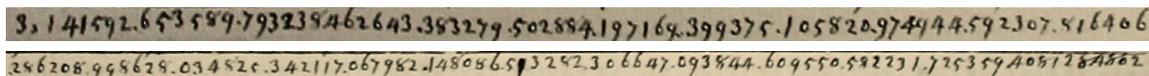


Figure 2: The value of π as it appears in the Oxford manuscript.

How was it done?

The calculation of π had seen a breakthrough in 1706, when the English FRS and Gresham Professor of Astronomy John Machin used the formula

$$\frac{\pi}{4} = 4 \arctan \frac{1}{5} - \arctan \frac{1}{239},$$

together with the series for the inverse tangent, to calculate and publish the first 100 digits of π : the best approximation up to that time.

But the author of this manuscript doesn't seem to have known it (or perhaps cared), and instead used an older and more primitive method based directly on the series for the inverse tangent. In modern terms it was to sum the series

$$\sqrt{12} - \frac{\sqrt{12}}{3.3} + \frac{\sqrt{12}}{9.5} - \frac{\sqrt{12}}{27.7} + \dots \pm \frac{\sqrt{12}}{3^n \cdot (2n + 1)} \dots$$

Edmond Halley had published this method of computing π in the 17th century. It works because $\tan(\frac{\pi}{6}) = \frac{1}{\sqrt{3}}$: the series is in fact the series for $\arctan(\frac{1}{\sqrt{3}})$, multiplied by 6.

The author of the manuscript calculated an amazing 314 terms of this series, using a value of $\sqrt{12}$ that was calculated first, to no fewer than 154 decimal places. An incredible labour: and the result was correct up to the 152nd place.

So the rumour turns out to be true, and this really was – as far as we know – the world’s most accurate value of π for over a century, from 1721 until 1844.

So, who discovered it? Unfortunately we don’t know much more about the author, who didn’t sign the manuscript. The only solid bit of information is that he or she was living in Philadelphia, Pennsylvania, when the π calculation was done.

Why the calculations were done is also a mystery. Others in the eighteenth century computed π for fame and glory, like Jurij Vega who calculated 144 digits in 1789 (using Euler’s new and better series for the inverse tangent) in the hope of being elected to the Petersburg Academy as a reward. The author of the Oxford manuscript didn’t publish it, and we’ve no record that he or she even tried to. The tables that make up most of the manuscript were a big undertaking – they took nearly 30 years to produce, on and off – and it’s hard to see what use they could ever have been. My own guess is that this was a hobby, a daily exercise: perhaps one that continued when an initial hope of publication had faded away.

And afterwards?

The manuscript somehow found its way to Oxford in time for Baron Zach to see it when he visited in the 1780s. I don’t know how it got there: there seems to be no paper trail. After Montucla’s report of Zach’s sighting, the ‘Oxford’ value of π was printed and commented on by quite a few people through the first half of the nineteenth century. But after it was superseded in 1844, it was forgotten. When the Bodleian Library’s manuscripts were catalogued, around 1905, this one was correctly described as the one Zach saw, but after that, for over a century, it didn’t appear in print, and dwindled into an unconfirmed rumour.

It’s a strange story and perhaps a melancholy one. The thought of a provincial schoolmaster – say – beavering away at mathematical tables and calculations for half a lifetime and never seeing them printed is a sobering one, even if it was done

as self-improvement, and not with the hope of publishing. The eighteenth century was an age of heroic by-hand calculations, and an age of conscious self-improvement through disciplined exercises like reading, walking or, perhaps, calculating. Who knows how many others computed π – or other things – and didn't publish?

This note is based on work first reported in Benjamin Wardhaugh, 'A "lost" chapter in the calculation of π : Baron Zach and MS Bodleian 949', *Historia mathematica* (forthcoming). I am grateful to Robin Wilson, Tomáš Pisanski and Marko Razpet for drawing my attention to this manuscript, and for Robin Wilson's help in drafting this note.