

Fourier Methods for Turbomachinery

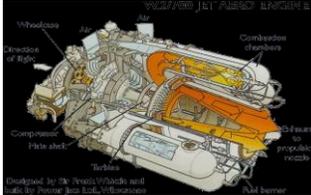
L. He

Department of Engineering Science
University of Oxford

'Turbomachinery' & 'CFD' in Turbomachinery CFD ?

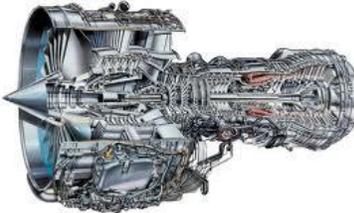
1941
Jet Engine

1940s
(Whittle's)
(No CFD!)



1975
Turbo CFD
(Denton's)

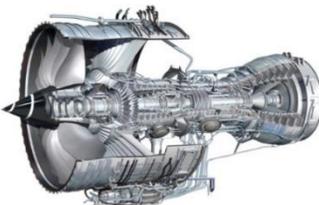
1970s
(bit CFD)



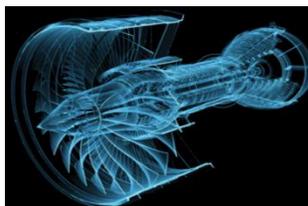
1990s
(more CFD)



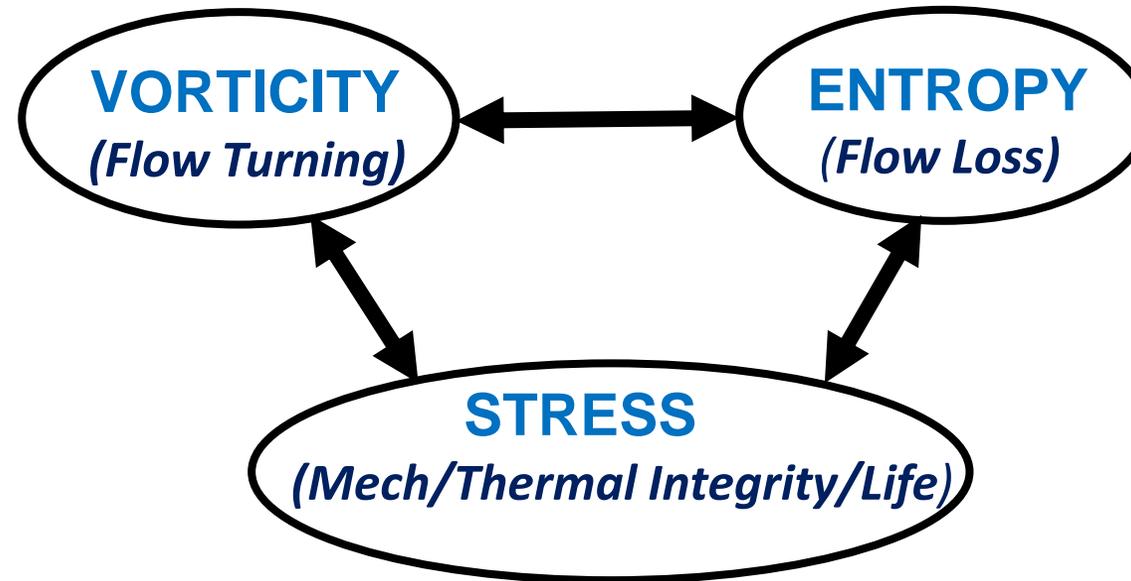
2000s
(lots CFD!)



2020s
(AI - CFD?)



- Basic Turbo Design Drivers: *What Make It Work & Live?*



'Turbo Δ '

- Multi-physics, M-disciplinary & M-component Interactive Settings
 - ➔ *Dictate how CFD methods are applied to Turbomachinery*
 - but also:
 - ➔ *Influence how Turbo-CFD methods may be developed.*

Outline

- **Background & Motivation**

- *Historical bits of 'Turbomachinery' in 'Turbomachinery CFD' (1975 - 1990)*
- *Motivating Context & Brief Account of 'Turbo Fourier Methods' (1990 -)*

- **Time-Domain Methods ('Spatial Truncation')**

- *Temporal Fourier Method: Multiple passages → Single-passage ('Phase-shift periodicity')*
- *Multi-disturbance Method: single-passage/row in multi-rows*
- *Spatial Fourier Method: Discrete Small Blocks for Whole Domain ('Block-spectral' model)*

- **Frequency-Domain Methods ('Temporal Truncation')**

- *Nonlinear Harmonic Model*
- *Time-spectral Model ('Harmonic Balance')*
- *Multi-stage (Rotor-Stator + Rotor-Rotor/Stator-Stator) Interactions*

Some Milestones in Turbomachinery CFD

- from Single Bladerow to Multi-rows/stages

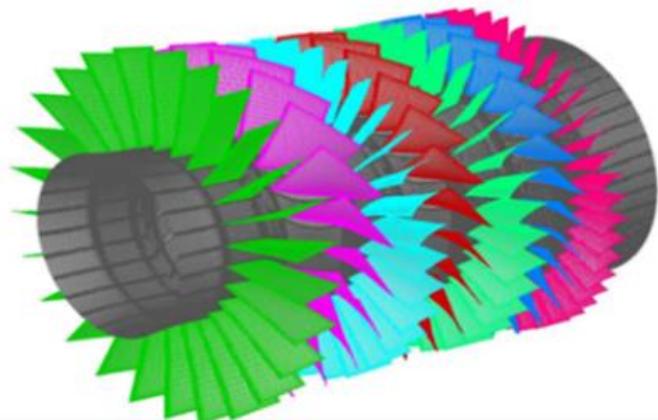
John Denton (1975) - marking the start of Turbo CFD

Full 3D Euler (Single blade-passage, Single blade-row, Steady Flow)

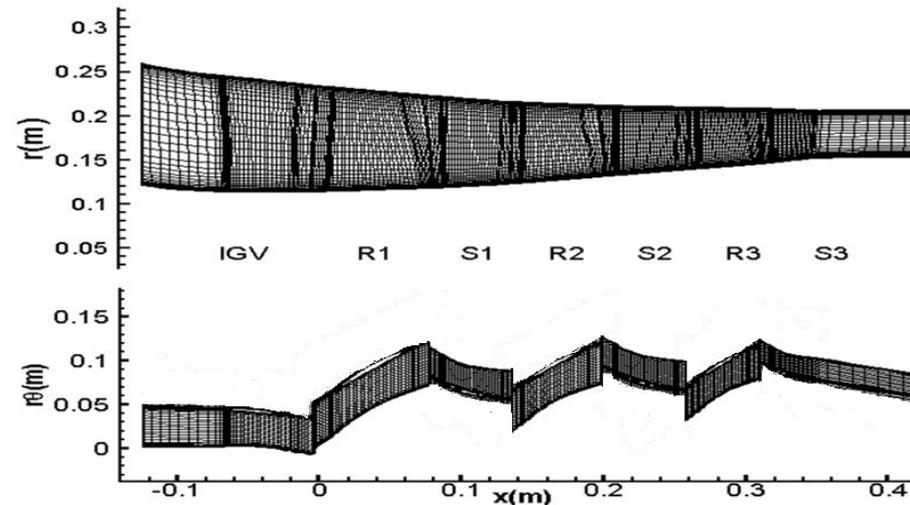
John Denton (1979)

'Mixing-plane' for Multi-Stage Interaction (Single-passage/row, Steady Flow)

-the WORKHORSE in designing all WORKING turbomachines (Air, Land & Sea)



Multistage Compressors



Multirow/stage Interaction:

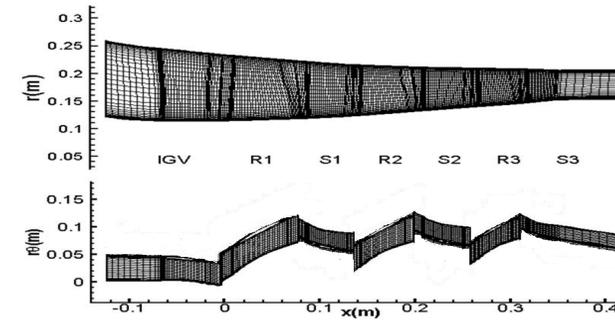
- 'Well designed/analysed' parts in isolation do not work well (when put together)!
- Power of the overall conservation (even with all missing/wrong details...!)

'Average Passage' Model: Informative Framework

John Adamczyk (NASA, 1985)

'Average-passage' for Rotor-Stator Coupling

- Single-passage/row,
- Averaging → An Open system needing Closure



• Averaging → Challenges and Opportunities

Reynolds-Averaging → 'Reynolds Stress' - *hard jobs for many turbulence modelers/profs...*

Passage-Averaging → 'Deterministic Stress' - *easier (?) jobs for a few turbo aerodynamists...*

→ Impetus for developing less-empirical & more efficient ways to close 'Deterministic Stresses'
-Closing Them by Computing Them (Efficiently) ?

'Shared' Challenge (& Motivation) for Both Aerodynamics and Aeroelasticity

- **More General Challenge:**

Aerodynamics: moving from RANS to URANS (just Time-marching in ' τ ' \rightarrow Time-marching in ' t ' ?)

Single Passage Steady \rightarrow Multi-passage/Whole annulus Unsteady

Extra cost $\sim O(10^3)$: $\sim O(10)$ from Steady to Unsteady; $\sim O(10^2)$ from 1 passage to whole annulus

Aeroelasticity: moving from Linear (Linearised based on RANS) to Nonlinear (URANS)

Single Passage 'Steady' (Harmonic) \rightarrow Multi-passage/Whole annulus Unsteady

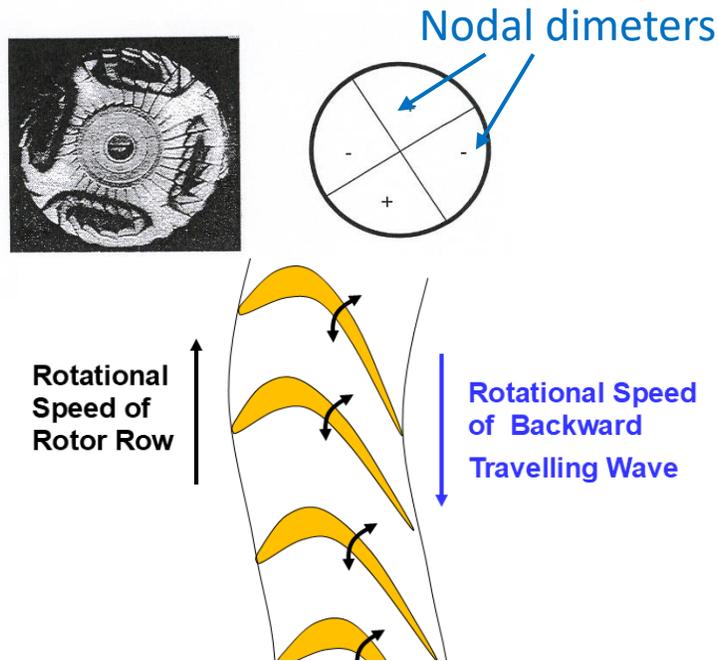
Extra cost $\sim O(10^3)$

- **More General Motivation for Fourier Methods**

- Much Faster (by $\sim O(10^2)$) Nonlinear Method than Conventional Multi-passage URANS.
- Primarily by significant circumferential truncation (to Single-passage or the like)

Some Shared Basics: 'Phase-Shift' Periodicity for Both Aerodynamics & Aeroelasticity

• Blade Flutter/Forced Response

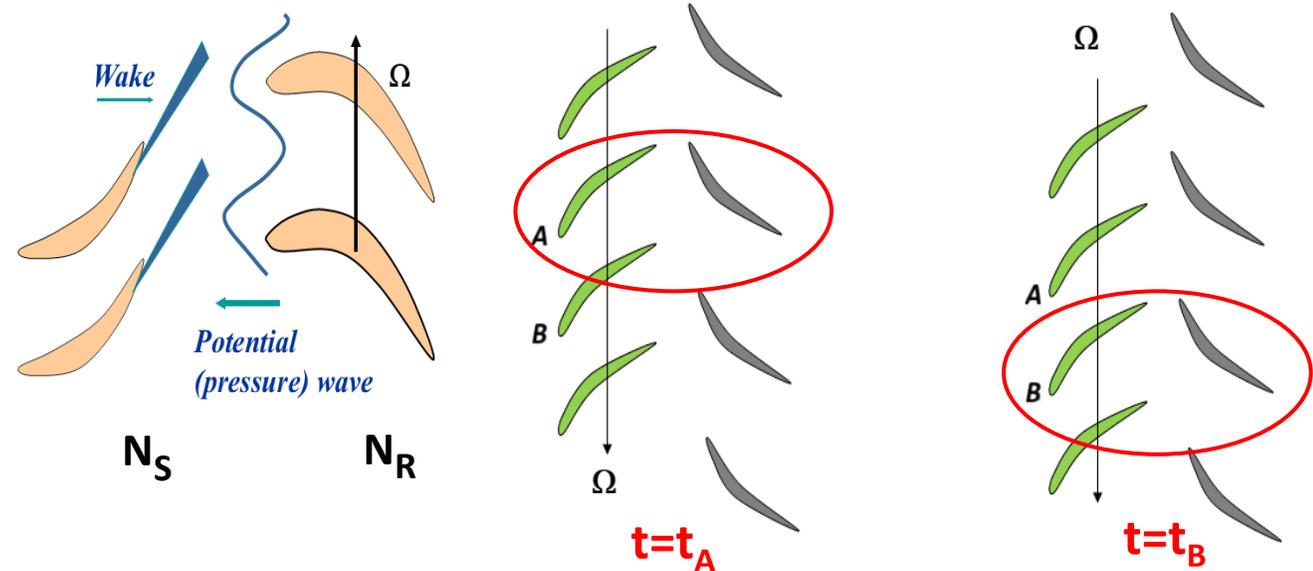


Inter-Blade Phase Angle (IBPA)

$$\sigma = \pm \frac{2\pi n}{N_b} \quad (n = 1, 2, \dots, N_b/2)$$

N_b – number of blades (blade row to be modelled)
 n – number of nodal diameters (structural mode)

• Stator-Rotor Interaction



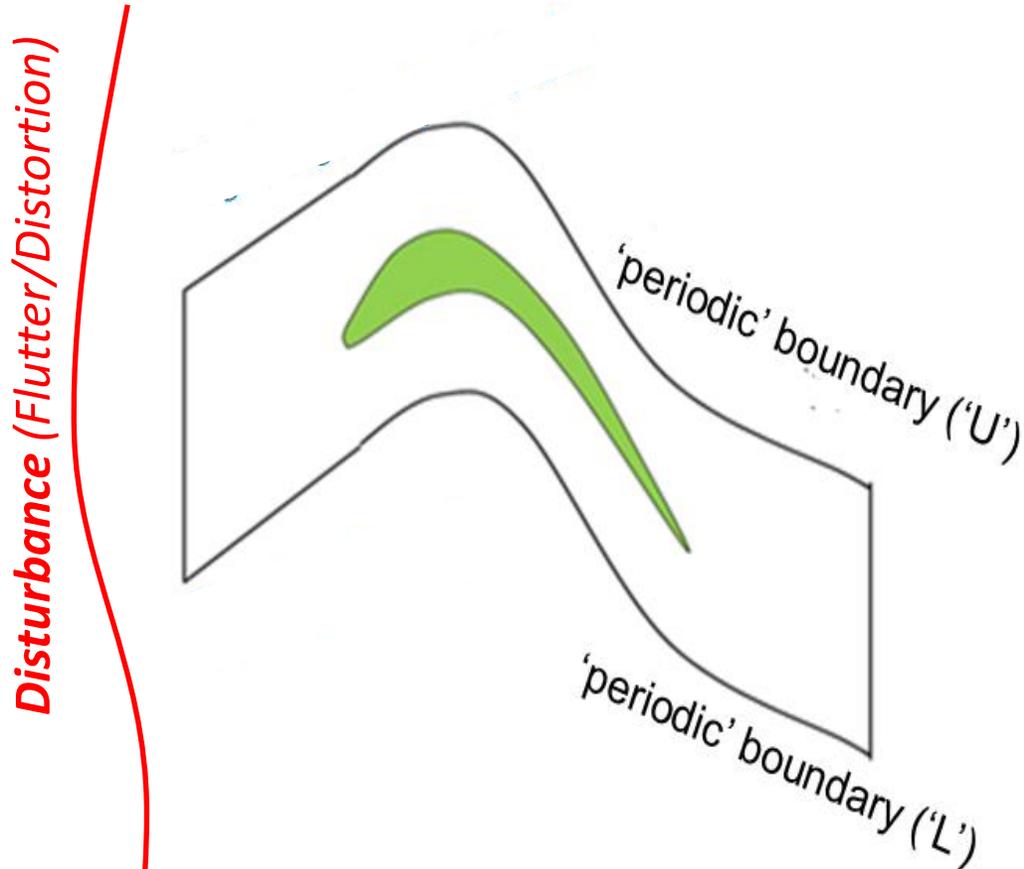
Inter-Blade Phase Angle (IBPA)

$$\sigma = \frac{2\pi N_S}{N_R}$$

N_R – number of rotor blades (bladerow to be modelled)
 N_S – number of stator blades (bladerow of disturbances)

Time-Domain Single-passage Solution: ‘Fourier Shape Correction’ (He,1990)

- Fourier Spectrum to enforce ‘Phase-shift’ Periodicity for Nonlinear Time-marching Solutions
(‘Tangible target’: can run low frequency flutter cases without memory limited by ‘Direct Store’, Erdos et al 1977)

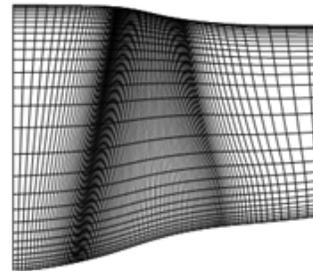
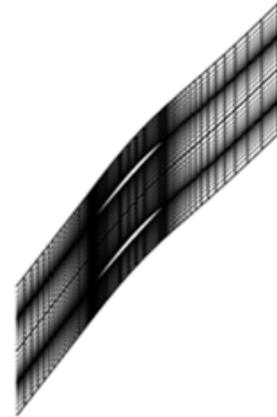
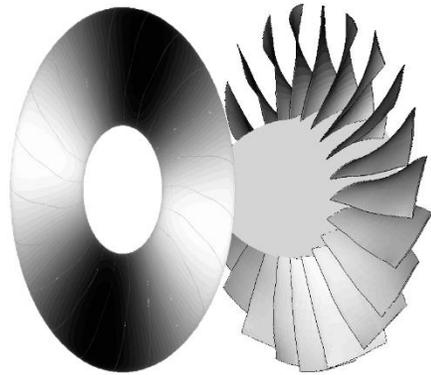


**Single-passage Domain
(Flutter/Forced Response)**

- **SHAPE** in time by Fourier Transfer at ‘periodic’ boundaries.
- **CORRECT** the current solution by updating Fourier coefficients accordingly.
- When periodically converged
(‘*Phase-shift Periodicity*’ satisfied):

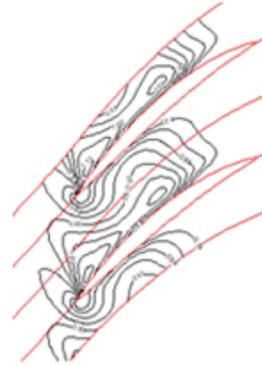
$$\bar{U}_U = \bar{U}_L, \quad A_U = A_L, \quad \varphi_U = \sigma \text{ (IBPA)}$$

NASA Rotor-67 under Inlet Distortion (Li & He 2002)

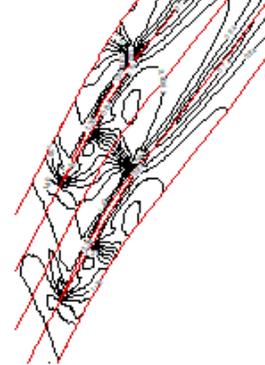


(a) Mesh

30% span



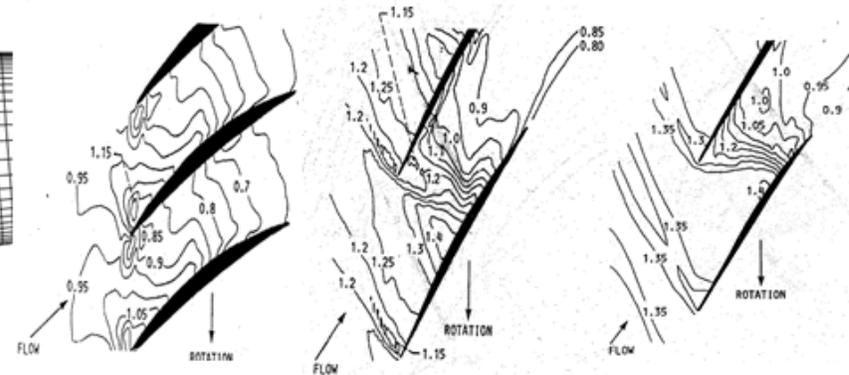
70% span



90% span



Computation



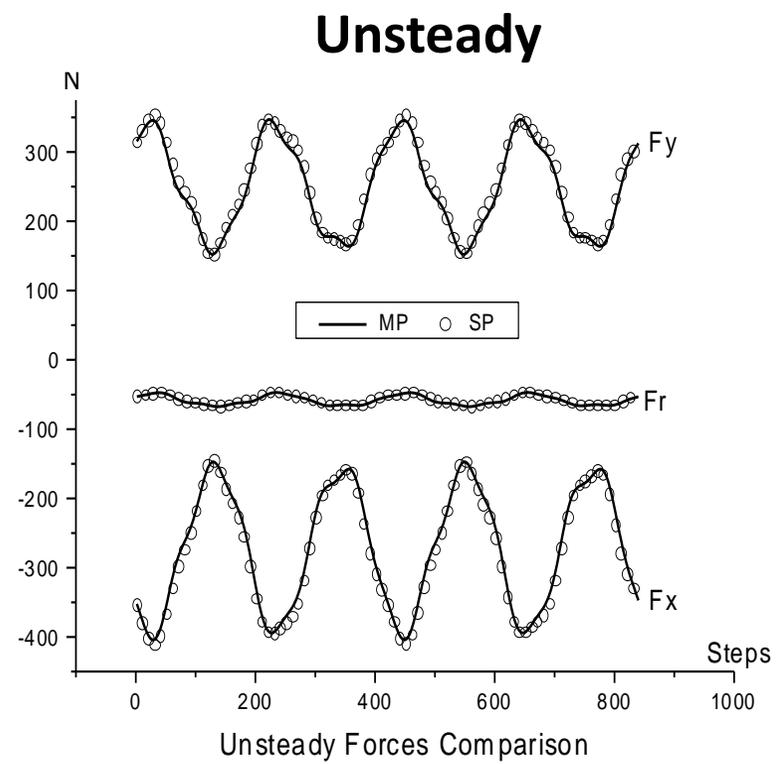
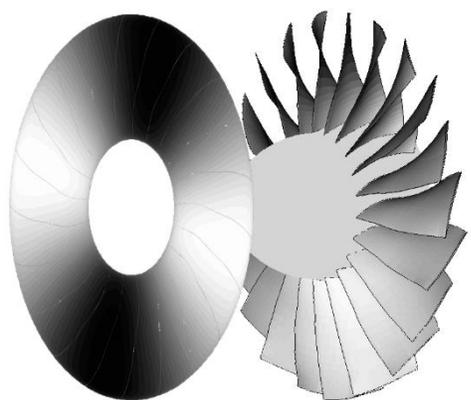
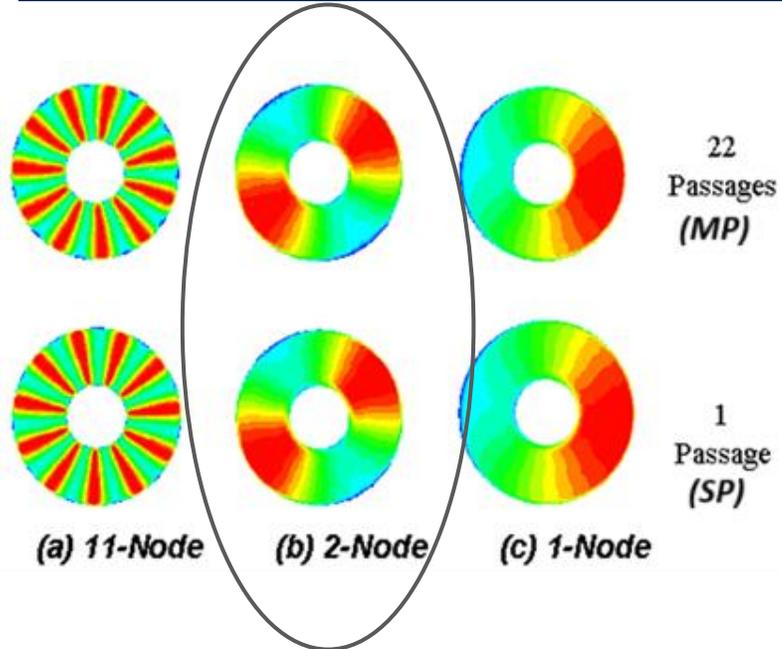
Experiment

(b) Relative Mach Number Compared with Exp Data

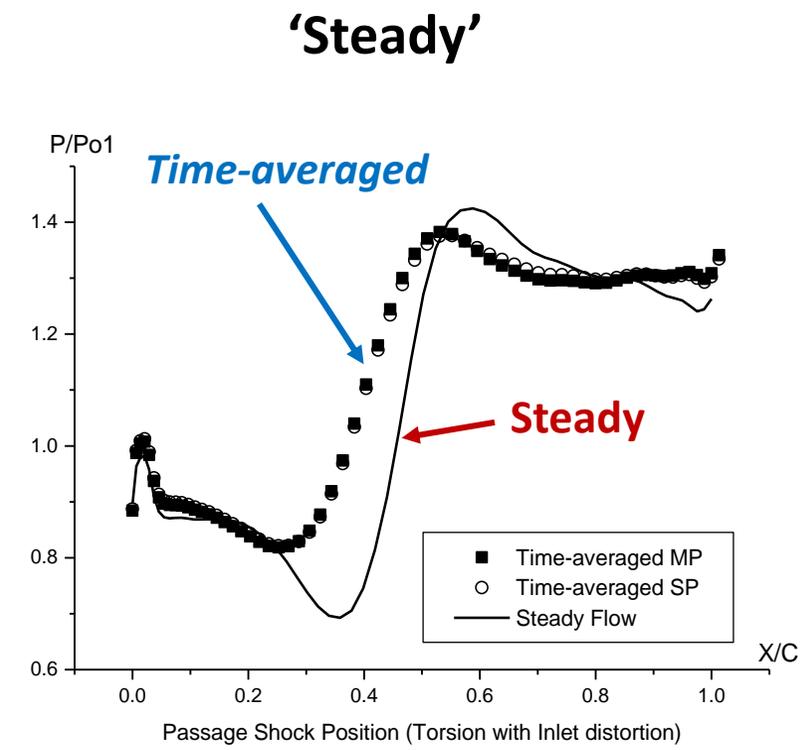
Steady Flow (Clean Inlet) Validation

NASA Rotor-67 under Two Nodes Distortion (Li & He 2002)

(Q: Fourier Transform is seemingly a linear operator, how can it model nonlinear flow?)

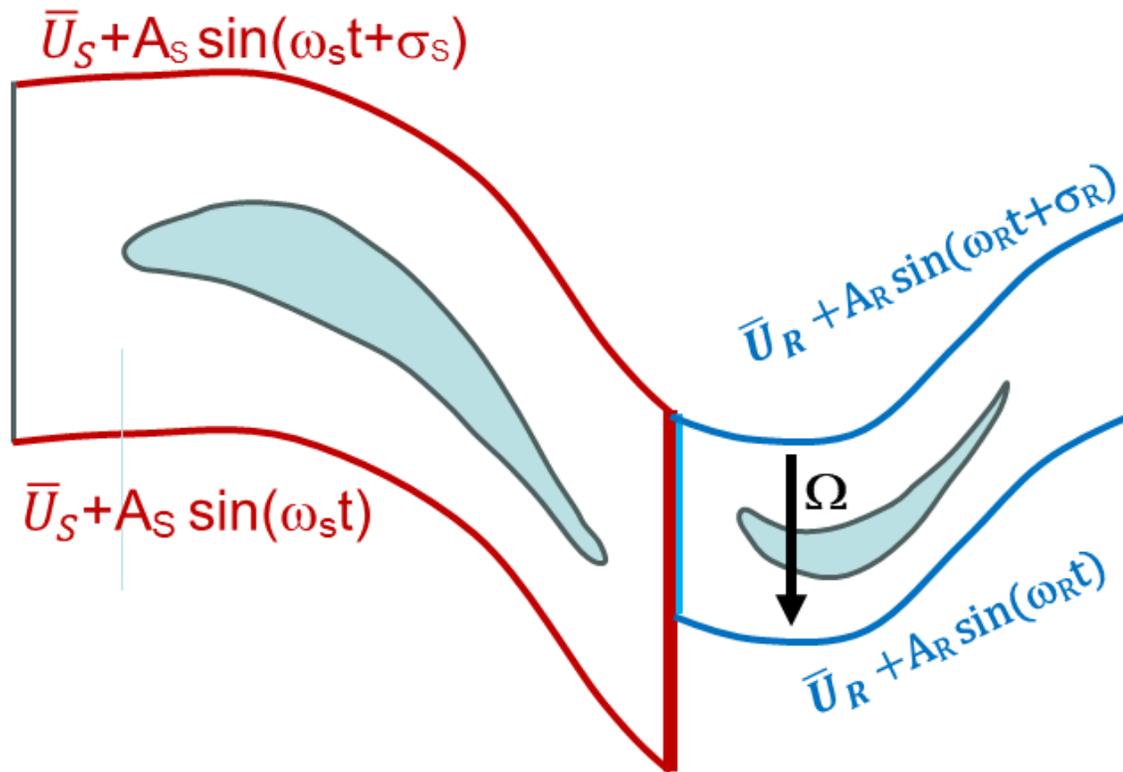


Time Traces of Blade Forces

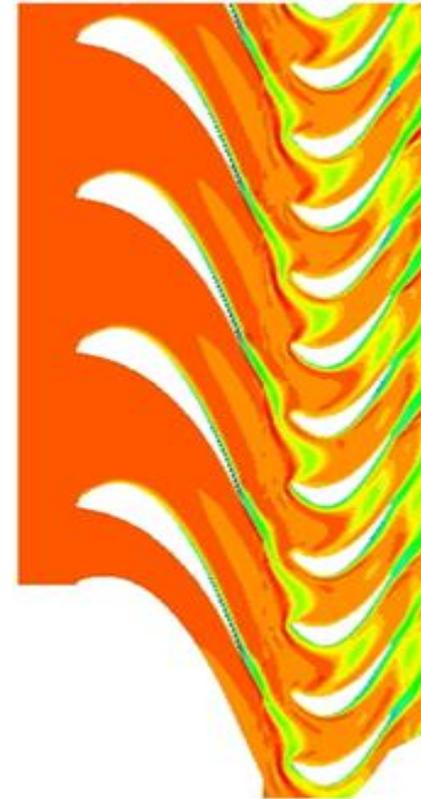


Time-Averaged Surface Pressures

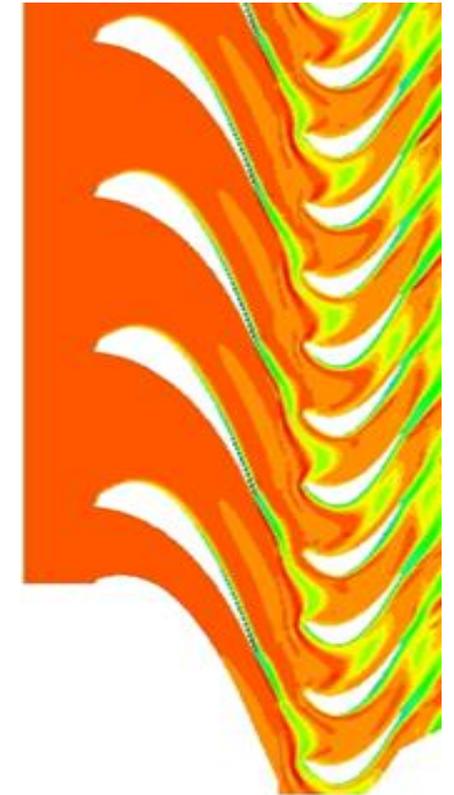
Stage Configurations *(Dewhurst & He 2000, Li and He 2002)*



Stator-Rotor (Bladerow Interaction)



Single-passage Solution
(‘Phase-shift’ reconstructed)

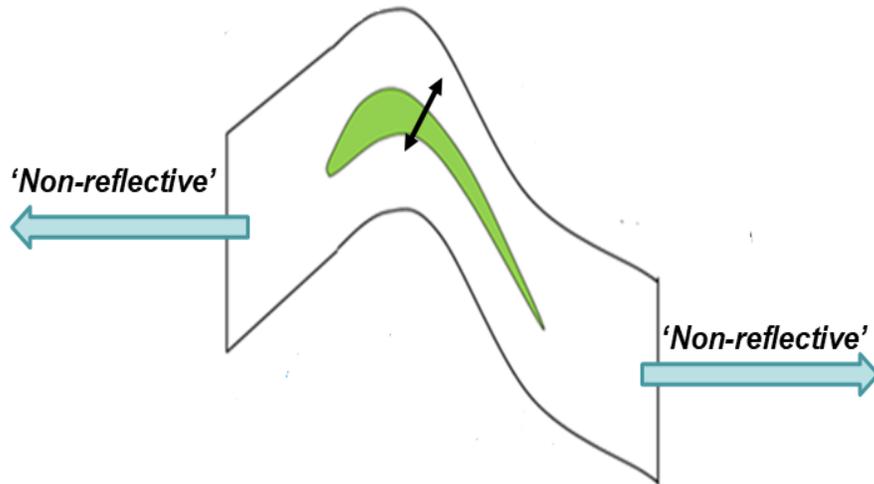


Multi-passage Solution
(Directly solved)

Towards Multiple Disturbances Settings: Accounting for 'Reflections'

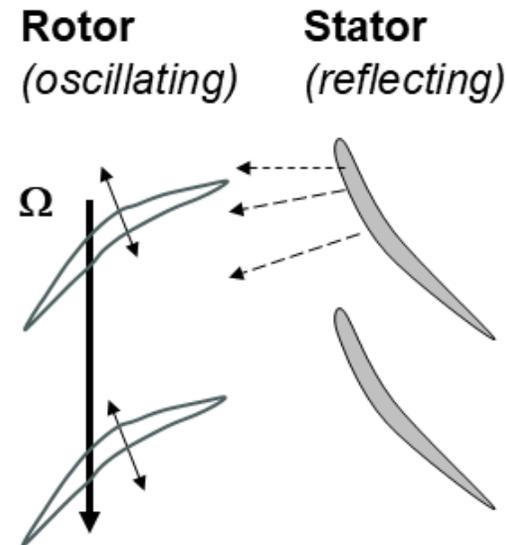
Idealised Oscillating Bladerow

('Tunned Cascade' in Infinite Long Duct)

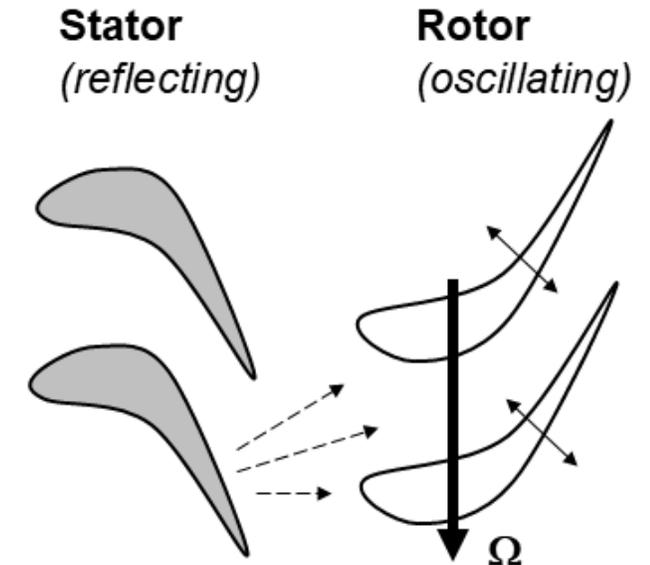


Oscillating Rotor Blades in Realistic Setting

(He, Cambridge Turbomachinery Course, 2008)



a) Frontal Fan/Compressor Stage



b) Rear Turbine Stage

➔ *Single-passage subject to multiples disturbances ?*

General Fourier Model for Multiple Disturbances

(*'Generalised Shape-Correction', He 1992*)

- General form: N_{dist} disturbances (each with a *primary frequency*):

$$U(\mathbf{x}, t) = \bar{U} + \sum_{i=1}^{N_{\text{dist}}} U_i(\mathbf{x}, t)$$

- Each with its own primary frequency ω_i ($i=1,2,\dots, N_{\text{dist}}$) modelled by its own Fourier spectrum:

$$U_i = \sum_{n=1}^{N_f} [A_{ni} \cos(n\omega_i t) + B_{ni} \sin(n\omega_i t)], \quad (i = 1, 2, \dots, N_{\text{dist}})$$

- Remarks on the basis:

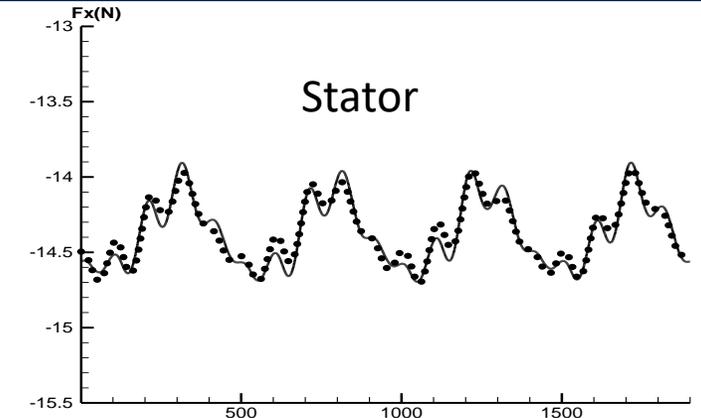
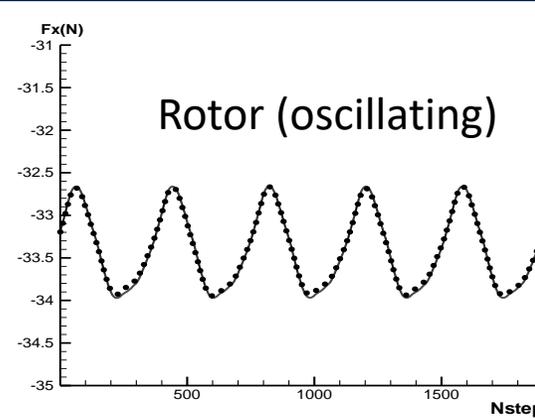
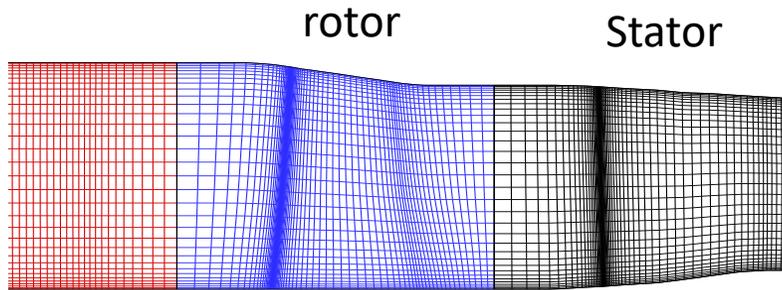
- i) All Disturbances are linearly superimposable ('orthogonal' like all Fourier modes).
- ii) Nonlinear cross-coupling among disturbances & the time-mean included (like all Fourier harmonics)

- Implementation:

- i) Straightforward Fourier transform needs all disturbances to beat –impractical (almost impossible for some)
- ii) An Effective Mitigation Enabler: '**Partial Substitution**' in updating Fourier-coefficients (*He, 1992*).

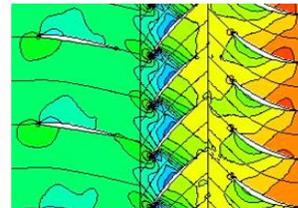
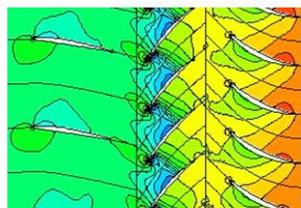
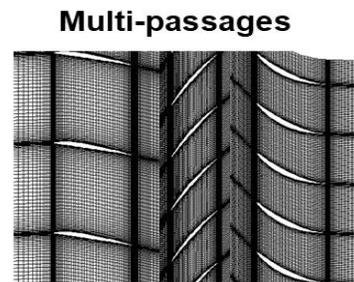
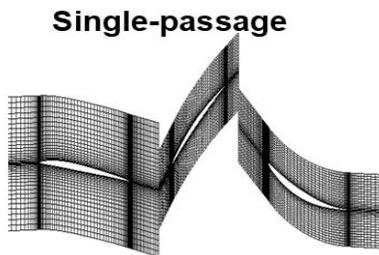
Blade Aeroelastic Stability of Embedded Rotor (Lyon Transonic Compressor)

-Blade numbers: 33 (IGV), 57 (rotor), 58 (stator), (*Li & He 2005*)



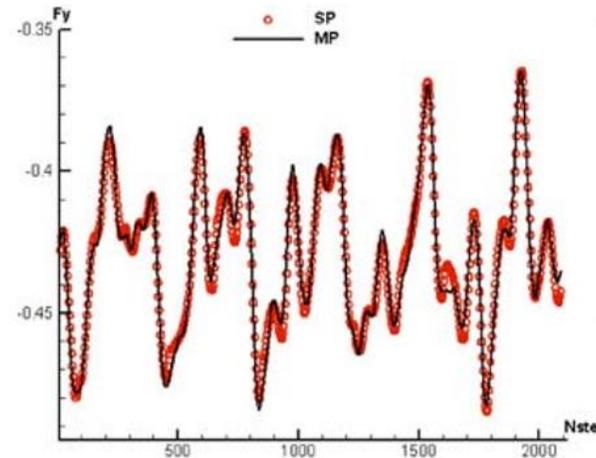
Unsteady Forces: **MP** vs **SP** (3D Stage: $N_{rotor}=19$, $N_{stator}=20$)

Gaping Analysis for 3-rows (*midspan section*)

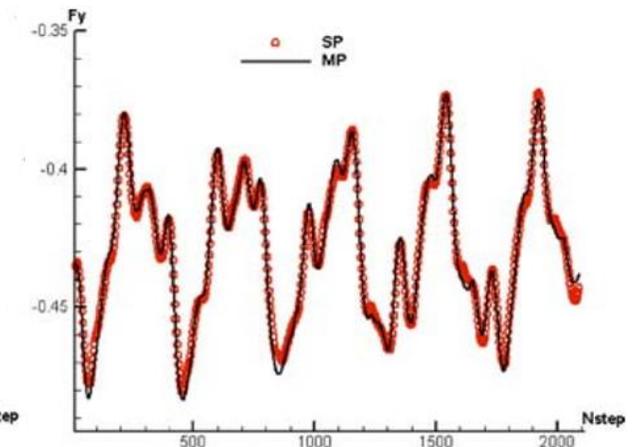


Pressure (*reconstructed*)

Pressure (*direct solution*)



(a) 30% Intra-row Gap

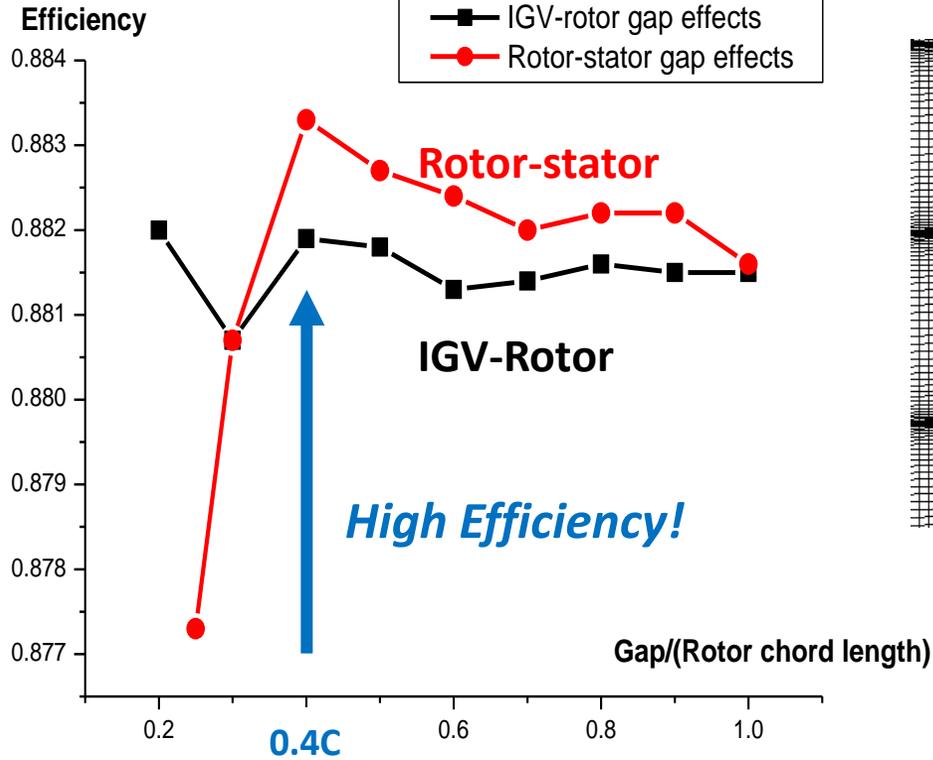


(b) 50% Intra-row Gap

Aeroelastic-Aerodynamic 'Balancing Act': Intra-row Gap Effect (3-row Lyon Compressor)

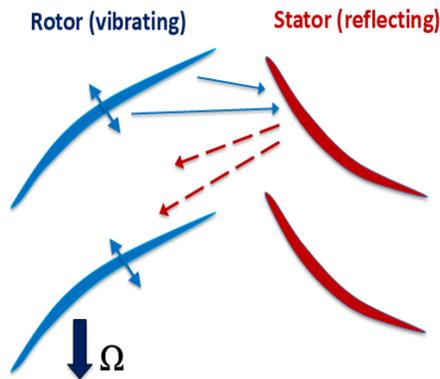
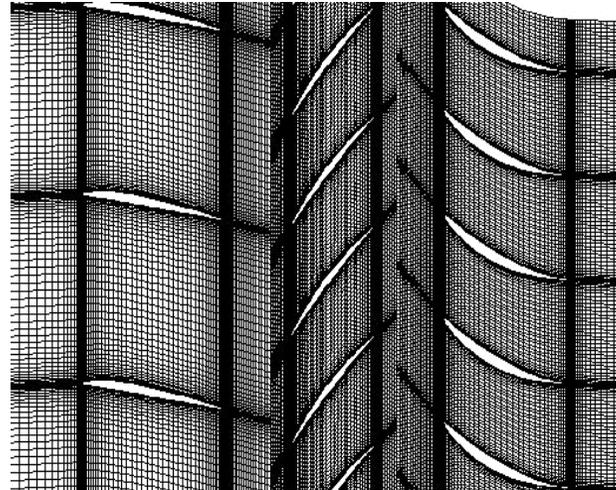
(Single-passage Time-Domain Multi-disturbance Fourier Solutions, Li & He 2005)

Efficiency (η_{is})

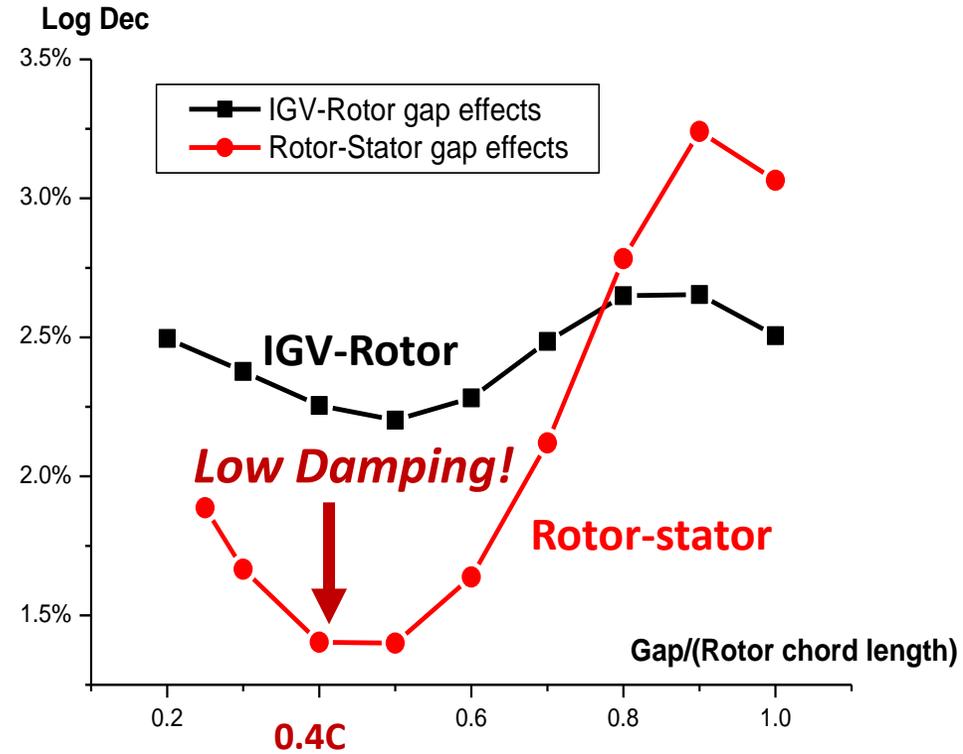


~0.7% η change
 (rotor-stator gap: 0.25C \rightarrow 0.4C)

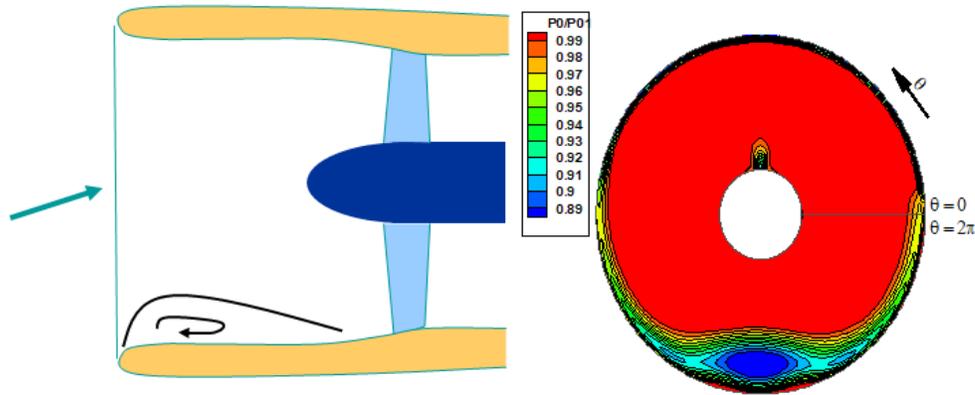
IGV Rotor Stator



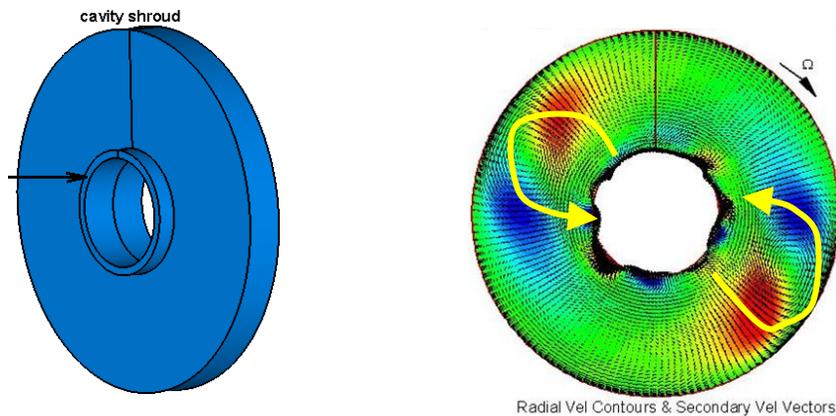
Rotor Aero-damping



>100% Aero-damping change
 (rotor-stator gap: 0.4C \rightarrow 0.9C)



Intake Lip Flow Separation at Angle of Attack

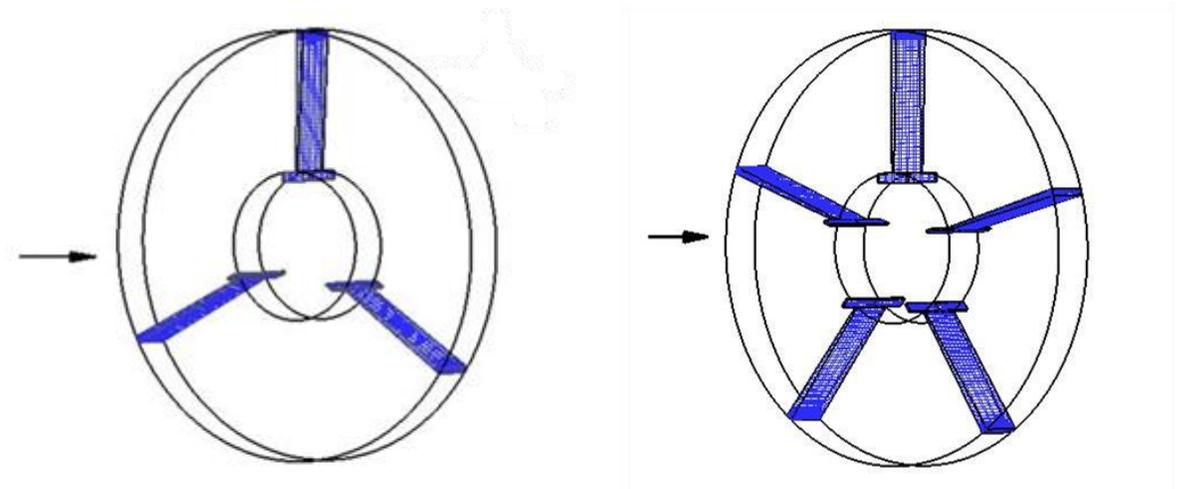


Self-excited Unsteady Cavity Flow

Circumferential (θ) Mesh-Pointwise Fourier Spectrum

$$U(x, r, \theta, t) = \bar{U}(x, r, t) + \sum_{n=1}^{N_f} [A_n(x, r, t) \cos(n\theta) + B_n(x, r, t) \sin(n\theta)]$$

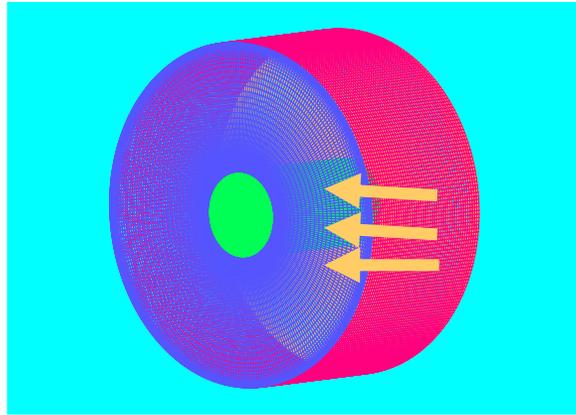
- Fully time-domain (no frequency/periodicity needed)
- Still $2N_f + 1$ unknowns for N_f harmonics



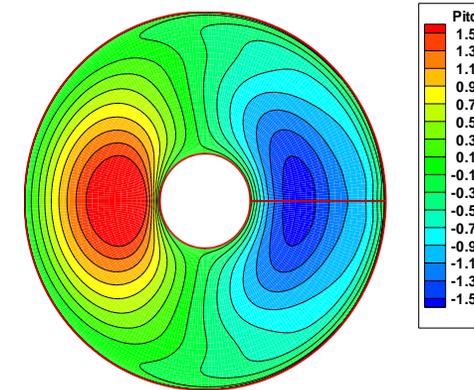
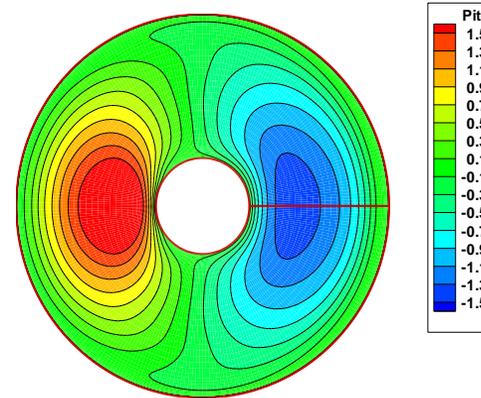
1 harmonic
(3 mesh cells/blocks)

2 harmonics
(5 mesh cells/blocks)

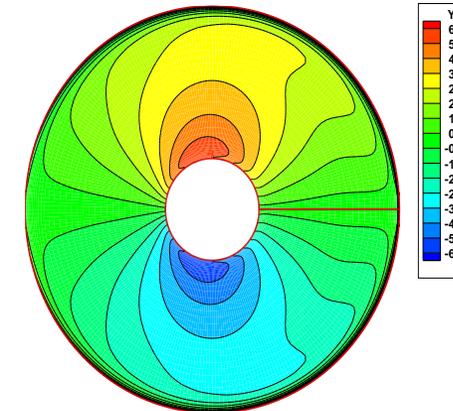
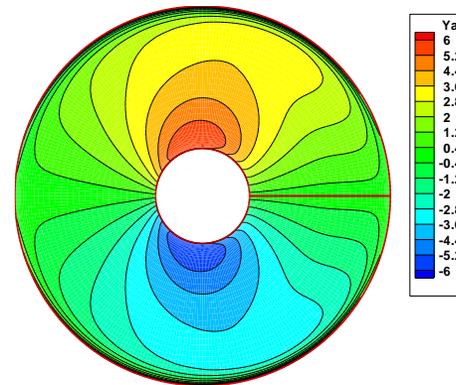
Duct subject to 10° crosswind (*He 2006*)



- **Circumferential Mesh Cells:**
Direct Solution (360°): **300**
Fourier Spectral (5 harmonics): **11**
- **Same Mesh Points in x, r directions**



Pitch Angle

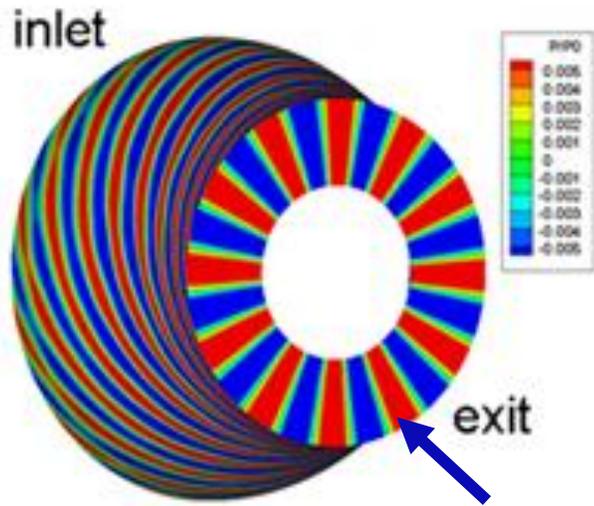


Yaw Angle

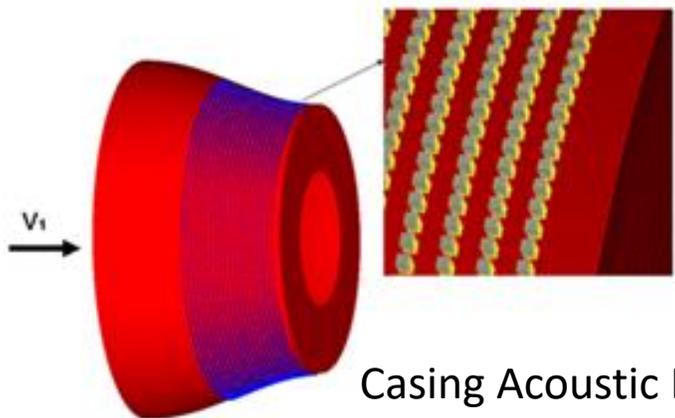
Fourier Solution (11 cells)
(Constructed Full Domain)

Direct Solution (300 cells)

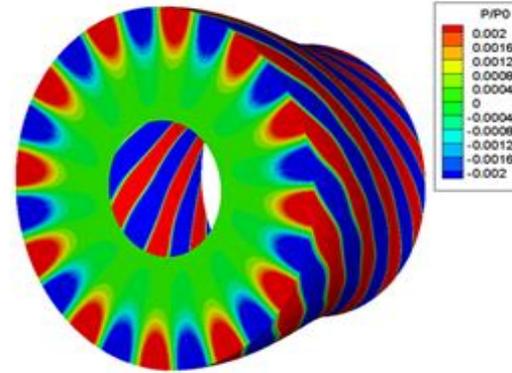
Fan Tone-Noise Propagation in Intake: *Effect of Casing Liner* (He 2013)



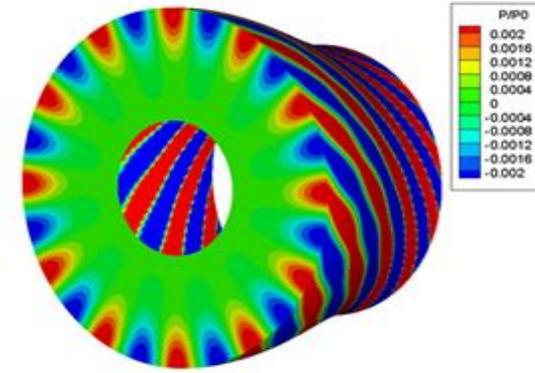
Exit Acoustic Disturbances from 12 Rotating Fan Blades



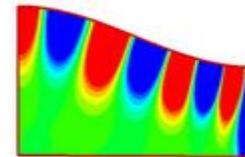
Casing Acoustic Liner
(500 Micro-cavities/30° Sector)
(fine-mesh blocks)



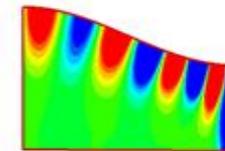
a) Smooth Casing



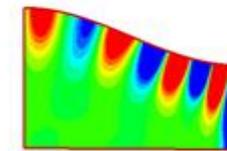
b) Lined Casing



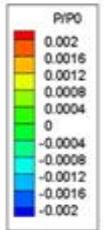
(c) Smooth Wall



(d) Lined Wall
(Direct Solution
solving 500 cavities)

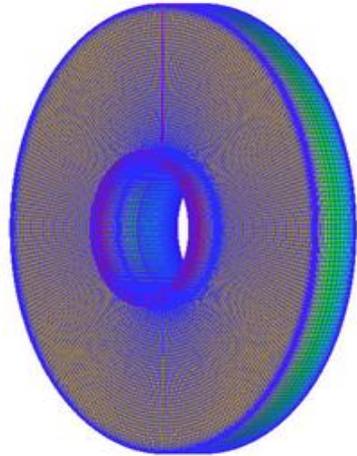


(e) Lined Wall
(Block-Spectral
solving 15 cavities)

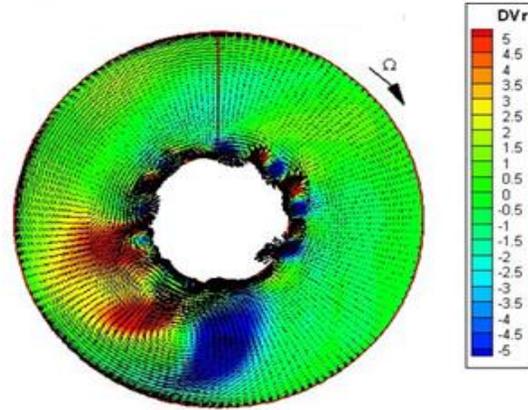


Cavity Flow & Heat Transfer (self-sustained: *Coriolis vs. Pressure Forces, He 2011*)

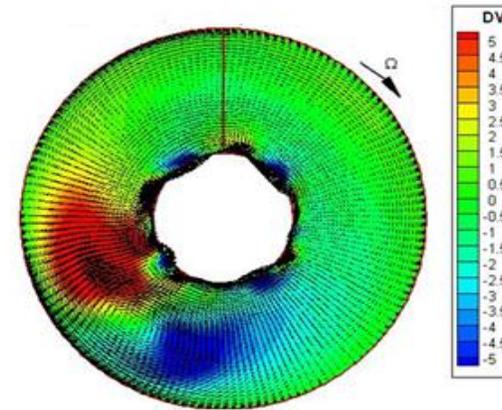
Instantaneous Radial Velocity Contours & Velocity Vectors



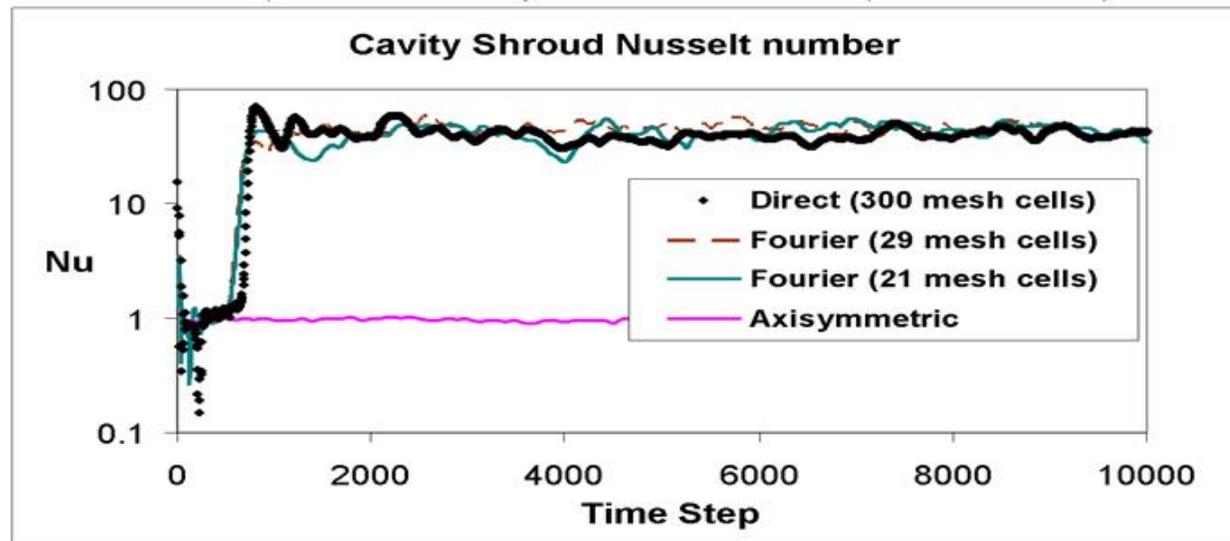
(a) Mesh (Direct Solution)



(b) Direct Solution
(300 mesh cells)



(c) Fourier Solution
(29 mesh cells)



(d) Nusselt Number Time Trace

- Self-excited unstable flow patterns well captured with $O(10)$ less mesh cells

- Statistic-mean changes due to instabilities well captured, far less sensitive than instantaneous flow field

'Harmonic Balance' - A Preliminary to Frequency-Domain Methods

- *The Label Says What It Does:*

1) **Nonlinear Flow Equation Set** (including $\partial U / \partial t$): $R(U) = 0$

2) **Fourier Spectrum of N_f harmonics:** $U = \bar{U} + \sum_{n=1}^{N_f} [A_n \cos(n\omega t) + B_n \sin(n\omega t)]$, **($2N_f + 1$ unknowns)**

3) **Substitute into Flow Equations and Expand:** $R(\bar{U} + \sum_{n=1}^{N_f} [A_n \cos(n\omega t) + B_n \sin(n\omega t)]) = 0$

4) **Group** coefficients for each **Harmonic** respectively for \bar{U} , $\cos(n\omega t)$ and $\sin(n\omega t) \rightarrow 2N_f + 1$ groups

5) **Balance** each **Harmonic** group respectively $\rightarrow 2N_f + 1$ equations for $2N_f + 1$ unknowns

• **Frequency-Domain Method: An Unsteady Problem \rightarrow A Set of Steady Problems**

Closing ‘Deterministic Stresses’ (Adamczyk, 1985) by Computing Them - A Preliminary to Nonlinear Harmonic Method

- ‘Deterministic’ – sum of periodic disturbances (primary and higher harmonics)
 - blade row interactions/distortion (frequencies linked to shaft speed)
 - blade aeromechanics (frequencies of structural modes)
- Cast Unsteady Disturbances equations in frequency domain with known primary frequency ω :
$$U' = \sum_{n=1}^{N_f} [A_n \cos(n\omega t) + B_n \sin(n\omega t)]$$
 - One unsteady problem $\rightarrow 2N_f$ “steady” problems (for a given time-average/steady base)
 - Cost benefit if N_f is small (e.g. $N_f=1$ for standard blade flutter analysis)
- Solve Harmonics Eq. (thus ‘Deterministic Stresses’) while coupling with the Base (Time-averaged Eq.)
 - More accurate than Empirical models
 - More cost-efficient than Multi-passage URANS

Time-averaged Base & Unsteady Disturbances: **Balancing Act**

Consider: 1-D Momentum for Compressible Inviscid Flow)

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} = - \frac{\partial P}{\partial x}$$

i) **Variable Decomposition** (based on **Time-averaging**):

Primary Variables: $\rho = \bar{\rho} + \rho'$

Secondary Variables: $P = \bar{P} + P'$

$$\rho u = \bar{\rho u} + (\rho u)'$$

$$u = \bar{u} + u' \quad (\bar{u}' = 0 !)$$

ii) **Equation Decomposition** (no assumption invoked):

$$\left\{ \frac{\partial}{\partial x} (\bar{\rho u} \bar{u} + (\rho u)' u') + \frac{\partial \bar{P}}{\partial x} \right\} + \left\{ \frac{\partial (u)'}{\partial t} + \frac{\partial}{\partial x} (\bar{\rho u} u' + \bar{u} (\rho u)') + \frac{\partial P'}{\partial x} \right\} = 0$$

Base State (Nonlinear)

Unsteady Disturbances ('quasi-linear')

iii) **Time-averaging for the Base** (still no assumption invoked):

$$\frac{\partial}{\partial x} (\bar{\rho u} \bar{u} + \overline{(\rho u)' u'}) + \frac{\partial \bar{P}}{\partial x} = 0$$

iv) **Balancing Unsteady Disturbances** (*quasi-linear on top of Base State*):

$$\frac{\partial (\rho u)'}{\partial t} + \frac{\partial}{\partial x} (\bar{\rho u} u' + \bar{u} (\rho u)') + \frac{\partial P'}{\partial x} = 0$$

Modelling Consistence for Time-Averaged Equation

Original Eq. $\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} = - \frac{\partial P}{\partial x}$

$\Rightarrow \underbrace{\left\{ \frac{\partial}{\partial x} (\overline{\rho u} \cdot \overline{u}) + (\rho u)' u' \right\} + \frac{\partial \overline{P}}{\partial x}}_{\text{Base State (Nonlinear)}} + \underbrace{\left\{ \frac{\partial (\rho u)'}{\partial t} + \frac{\partial}{\partial x} (\overline{\rho u} u' + \overline{u} (\rho u)') + \frac{\partial P'}{\partial x} \right\}}_{\text{Unsteady Disturbance ('quasi-linear')}} = 0$

$\overline{u'} = 0$

$\Rightarrow \boxed{\frac{\partial}{\partial x} (\overline{\rho u} \cdot \overline{u}) + \overline{(\rho u)' u'} + \frac{\partial \overline{P}}{\partial x} = 0}$

- **Requirement for Time-Averaged Eq:** unsteady disturbances terms $\rightarrow 0$ when time-averaged
 (*warranted as all disturbances are quasi-linear in time*)
- **Other averaging options (other than Time-averaging):** Time-averaged Eq. may be less consistent
 (*e.g. Favre-averaging: $u = \tilde{u} + u''$, $\tilde{u} = \overline{\rho u} / \bar{\rho} \rightarrow \overline{u''} \neq 0$*)

Nonlinear Harmonic Method (He 1996, Ning & He 1998)

1D Momentum Eq.:
$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} = -\frac{\partial P}{\partial x}$$

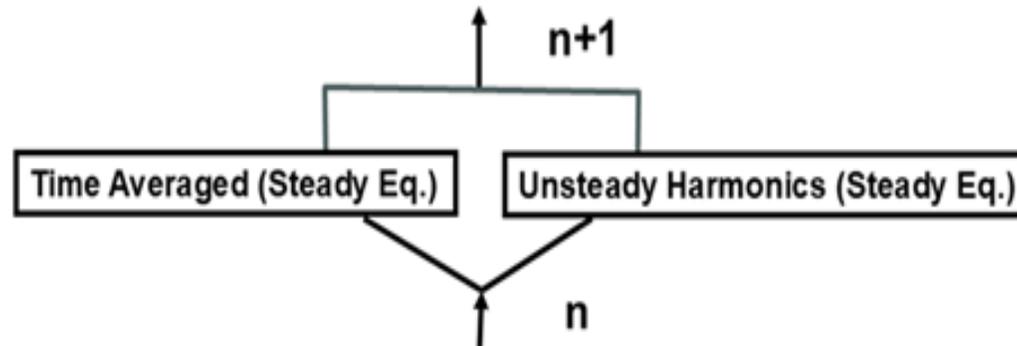
Fourier Spectrum of N_f harmonics:
$$\rho u = \bar{\rho u} + \sum_{n=1}^{N_f} [(A_{\rho u})_n \cos(n\omega t) + (B_{\rho u})_n \sin(n\omega t)] \quad (2N_f + 1 \text{ unknowns})$$

Harmonic Balance →
 $2N_f + 1$ 'Steady' Eqs:

$$\left\{ \begin{array}{l} \frac{\partial}{\partial x} (\bar{\rho u} \cdot \bar{u} + \overline{(\rho u)' u'}) + \frac{\partial \bar{P}}{\partial x} = 0 \quad (n=0!) \\ -n\omega (A_{\rho u})_n + \frac{\partial}{\partial x} [\bar{\rho u} (B_u)_n + \bar{u} (B_{\rho u})_n + (B_p)_n] = 0, \quad (n = 1, 2, \dots, N_f) \\ n\omega (B_{\rho u})_n + \frac{\partial}{\partial x} [\bar{\rho u} (A_u)_n + \bar{u} (A_{\rho u})_n + (A_p)_n] = 0, \quad (n = 1, 2, \dots, N_f) \end{array} \right.$$

Closure of 'Deterministic Stress':
$$\overline{(\rho u)' u'} = \frac{1}{2} \sum_{n=1}^{N_f} [(A_{\rho u})_n (A_u)_n + (B_{\rho u})_n (B_u)_n]$$

Strongly Coupled 'Steady' Solution:



Time spectral ('Harmonic Balance') Method (*Hall et al, 2002*)

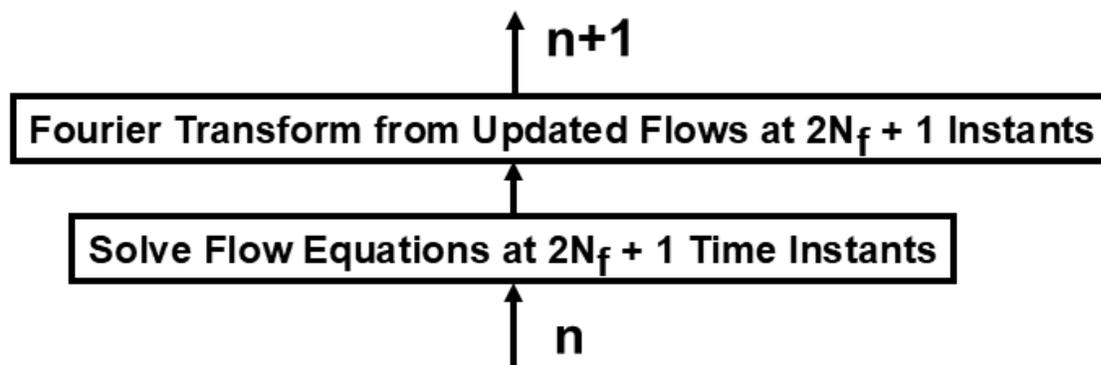
- Extra 'cross-coupling' terms may be included in $2N_f+1$ Harmonic Balance Eq. (at extra cost!)
- A different solution ('collocation time-spectral') method is chosen instead.

-Starting point: $\frac{\partial U}{\partial t} = R(U)$ and $U = \bar{U} + \sum_{n=1}^{N_f} [A_n \cos(n\omega t) + B_n \sin(n\omega t)]$ **($2N_f + 1$ Unknowns)**

- Equations hold at $2N_f+1$ time instants: $(\frac{\partial U}{\partial t})_i = R(U_i)$ ($i=1, 2, \dots, 2N_f+1$)

i.e. $\sum_{n=1}^{N_f} [n\omega(-A_n \sin(n\omega t_i) + B_n \cos(n\omega t_i))] = R(U_i)$, ($i=1, 2, \dots, 2N_f+1$)

-Solve $2N_f+1$ coupled eqs, & Update $2N_f+1$ knowns by Fourier-transform at each iteration



- Fully nonlinear including cross-coupling implicitly.
- Easy to implement including turbulence model eq.
- 'Harmonic Balance'? - neither used, nor needed.

- Take Time-domain Flow Equations at **3 phase angles** for single harmonic (He 2008):

$$\omega t_1 = 0^\circ, \quad \text{solution for } U_1 = \bar{U} + A,$$

$$\omega t_2 = 90^\circ \quad \text{solution for } U_2 = \bar{U} + B,$$

$$\omega t_3 = -90^\circ \quad \text{solution for } U_3 = \bar{U} - B,$$



\bar{U} , A, B directly without FT

- Similarly take Adjoint Equations (variable λ) at **3 phases** for single harmonic:

$$\omega t_1 = 0^\circ, \quad \text{solution for } \lambda_1,$$

$$\omega t_2 = 90^\circ \quad \text{solution for } \lambda_2,$$

$$\omega t_3 = -90^\circ \quad \text{solution for } \lambda_3,$$



Sensitivities of base flow \bar{U} and unsteady harmonic (A, B) to blade geometry perturbations.

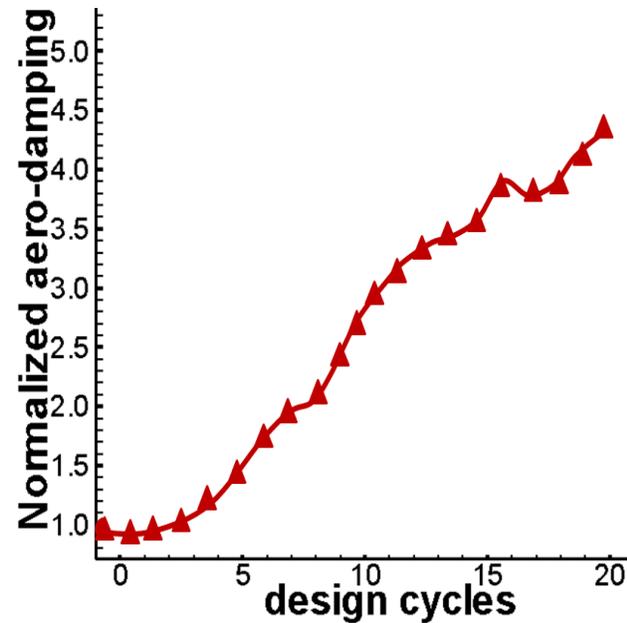


Concurrent Optimization to increase Aeroelastic Stability (Aero-damping \uparrow) without a compromise in Aerodynamic Performance (Aero-loss \downarrow).

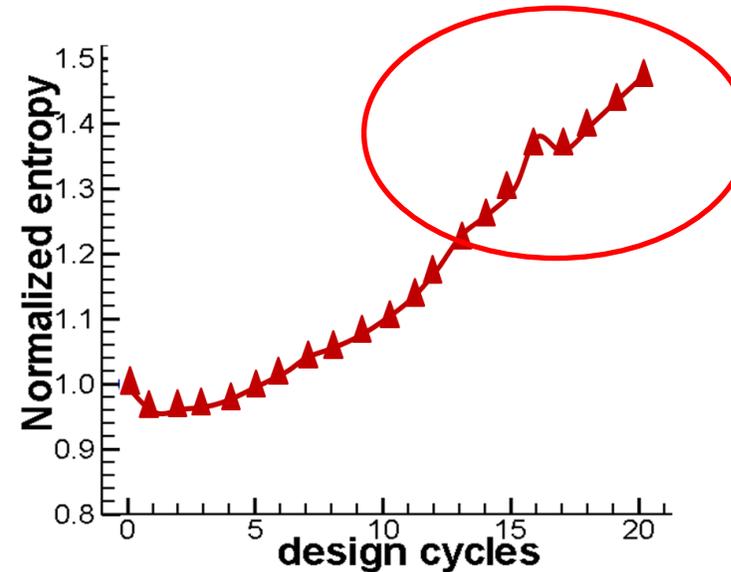
Why Concurrent D/O? - Limitation of Single-discipline D/D



*Task: shaping blade to increase stability/damping
(for a given high η aero-design)*



Aero-damping



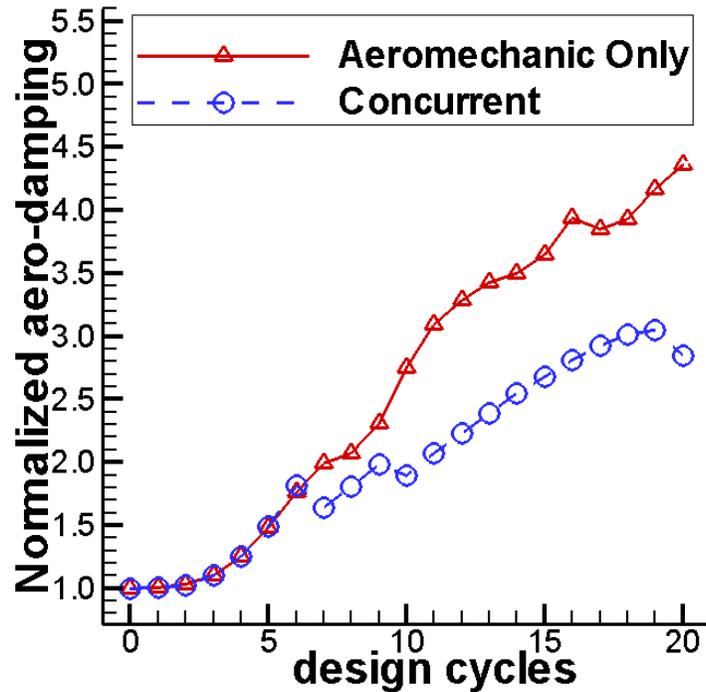
Aero-loss

- Aeroelastic-only design compromises aero-performance !

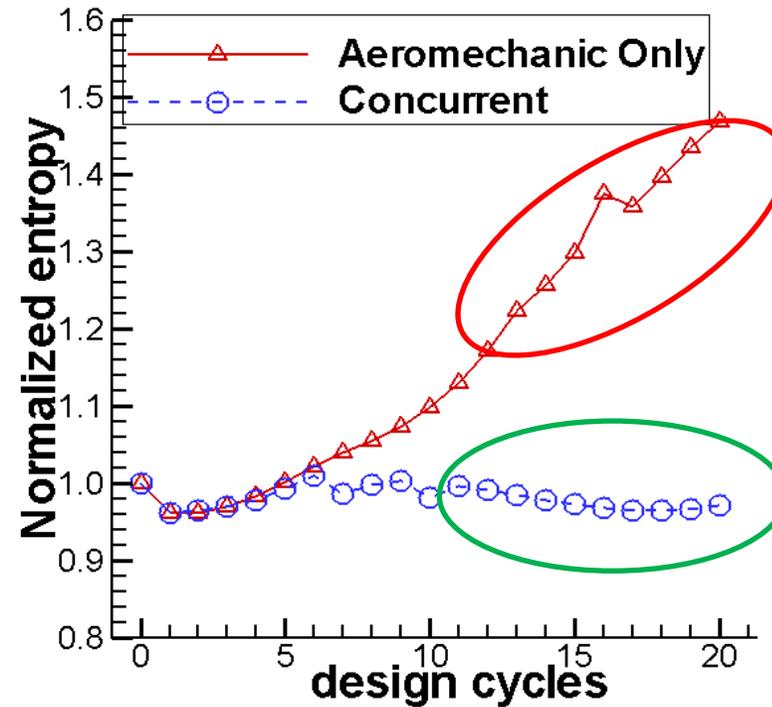
Benefit of Concurrent Design Optimization *(Wang and He, 2009)*

Concurrent D/O:

To ask for both Aerodynamic and Aeroelastic gains at the same time



Aero-damping

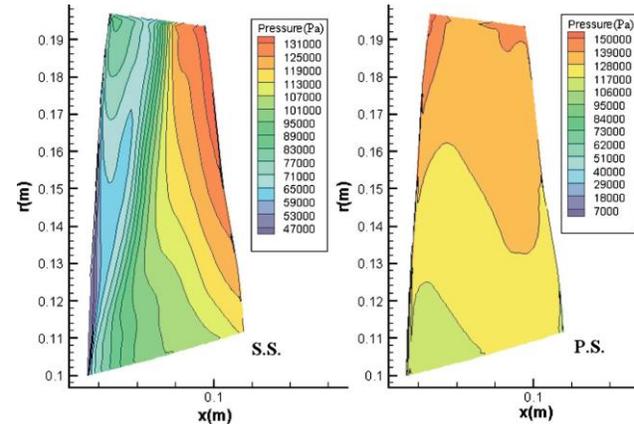
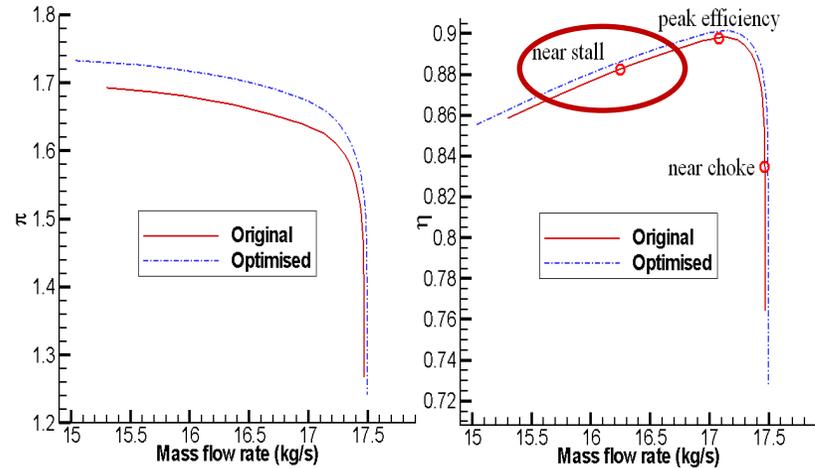


Aero-loss

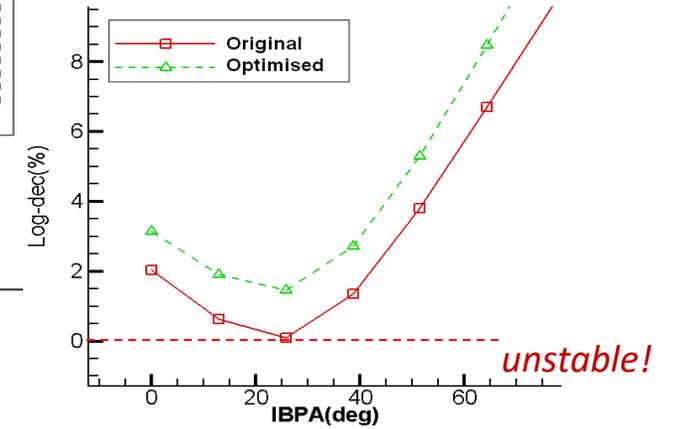
Multi-point Adjoint Aerodynamic-Aeroelastic D/O (DLR Transonic Rotor)

(Wang & He 2009, He & Wang 2011)

Aero-performance



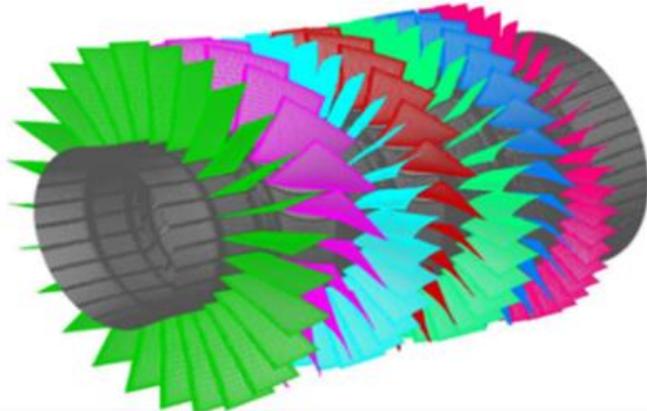
Aero-damping (near-stall)



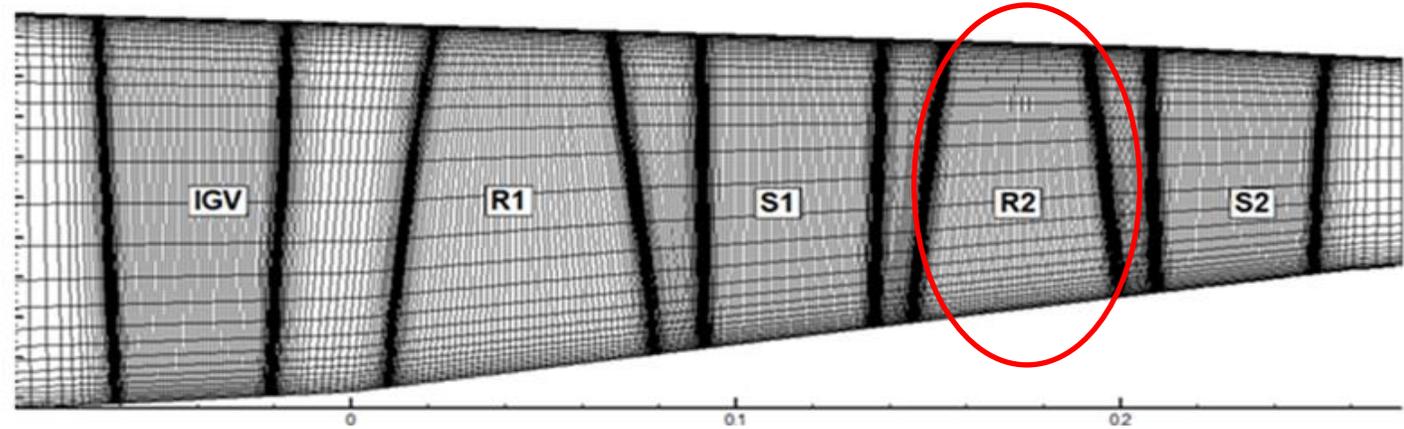
Optimized vs *Original*

Operating point	Pressure ratio	Efficiency	Damping (Log-dec %)
<i>Near stall</i>	1.716 (1.667)	89.57 (88.29)	1.6 (0.1)
<i>Peak efficiency</i>	1.661 (1.626)	89.76 (89.70)	3.2 (2.7)
<i>Near choke</i>	1.449 (1.421)	83.60 (83.47)	3.9 (3.4)

Nonlinear Harmonic Multi-stage Solution (*Rotor-Rotor/Sator-Stator Interactions*) (Closing Adamczyk “*Deterministic Stresses*” by computing them, efficiently)



Multistage Compressors



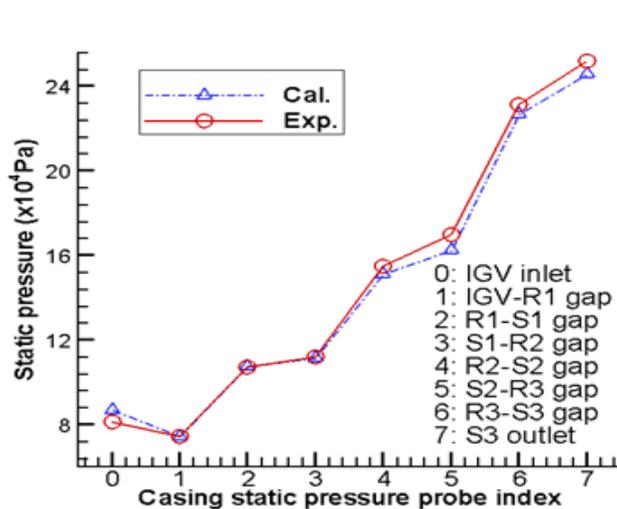
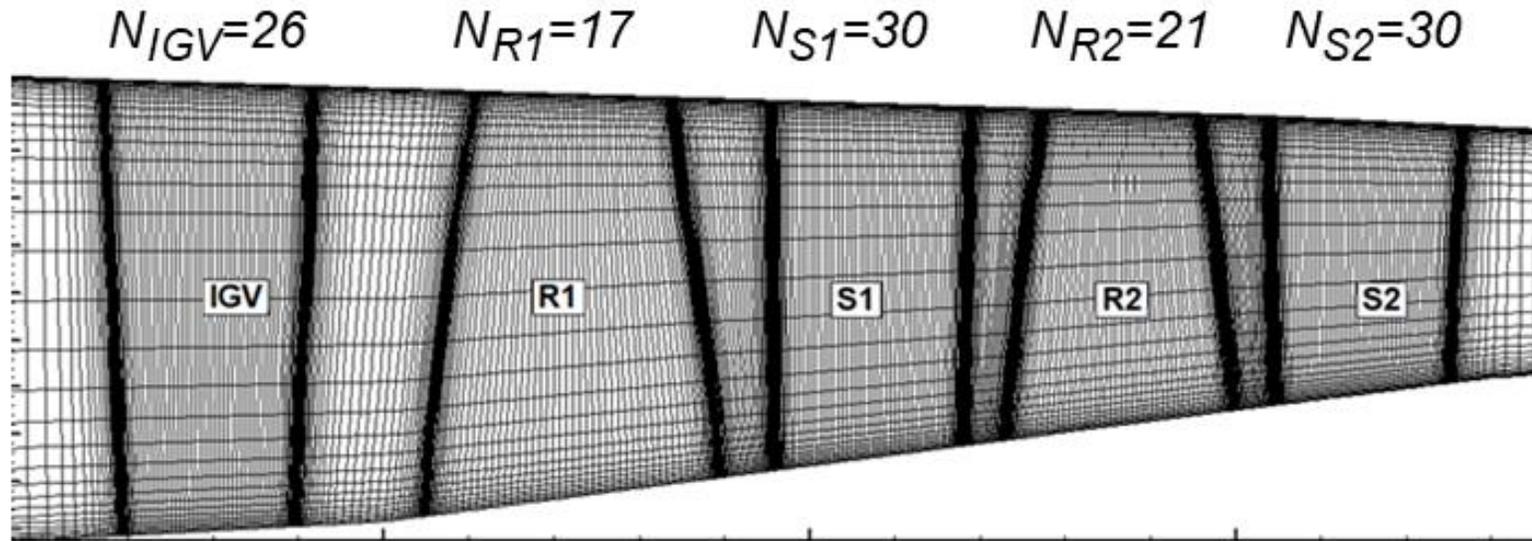
- **General Multi-disturbance Fourier Model** (He 1992):

$$U(\mathbf{x}, t)_{R2} = \bar{U}(\mathbf{x})_{R2} + U_{S1-R2}(\mathbf{x}, t) + U_{R1-R2}(\mathbf{x}, t)$$

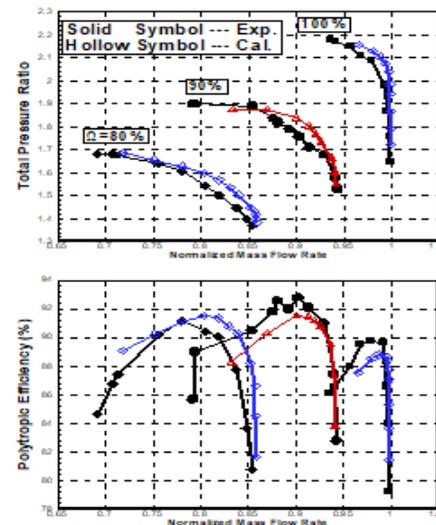
- **S1-R2 Interface:** *Rotating Harmonics from R1 in Stator (S1) → Stationary Harmonics from R1 ‘seen’ by R2*
- **R-R/S-S Interaction Effects** (‘Clocking’ / ‘Aperiodic Variation’): *Efficiently Reconstructed with Stationary Harmonics*

Coupled Unsteady Multi-stage Solution (Nonlinear Harmonic Method)

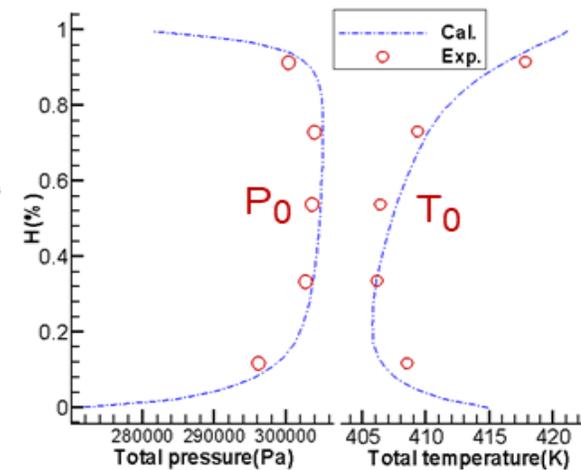
(He et al 2002)



(a) Casing Pressure Probe



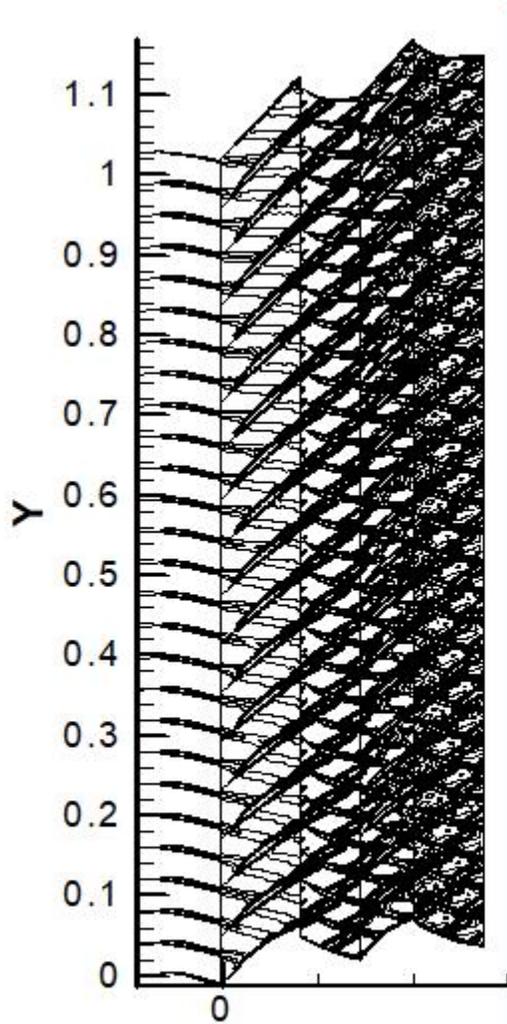
(b) 2-stage Performance



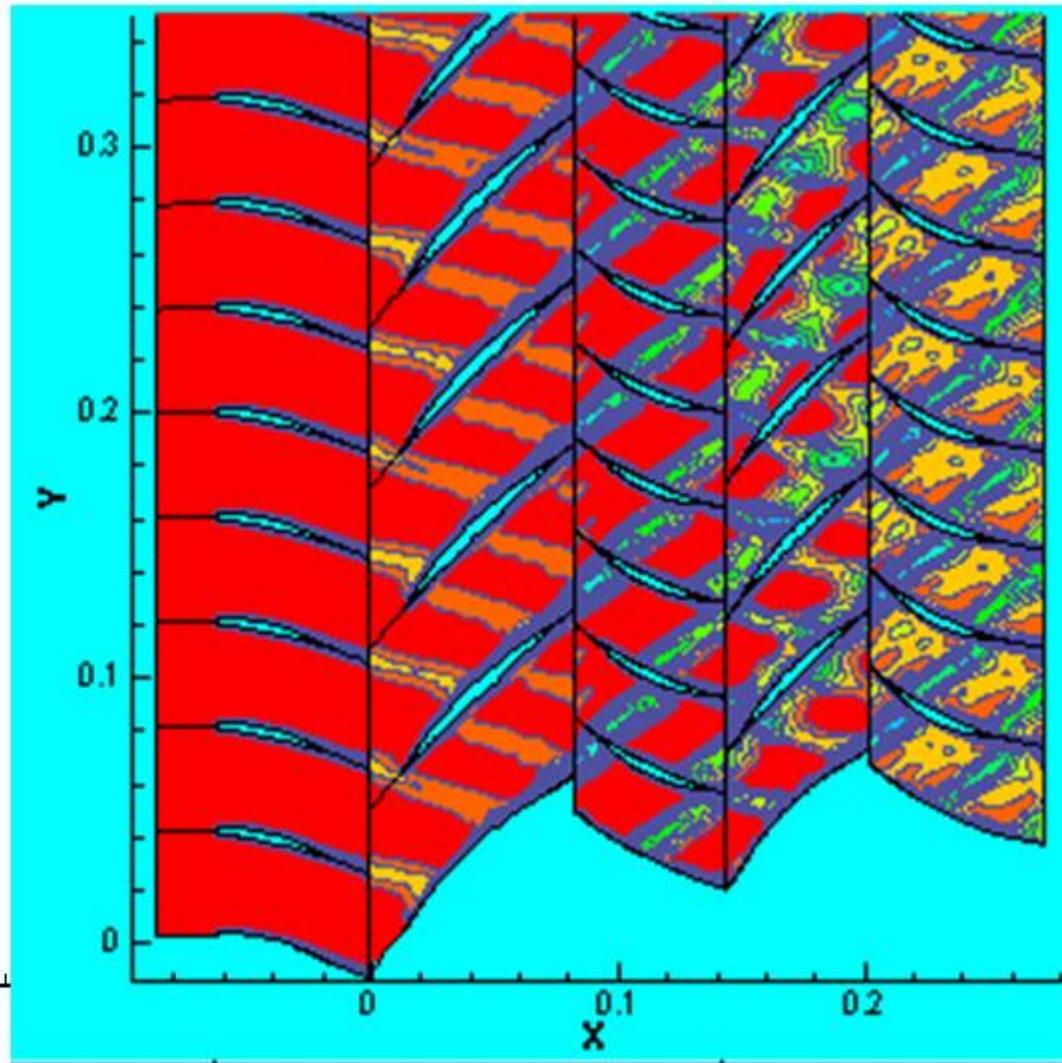
(c) Exit Spanwise Traverses

Rotor-Rotor/Stator-Stator Interferences (Nonlinear Harmonic Method)

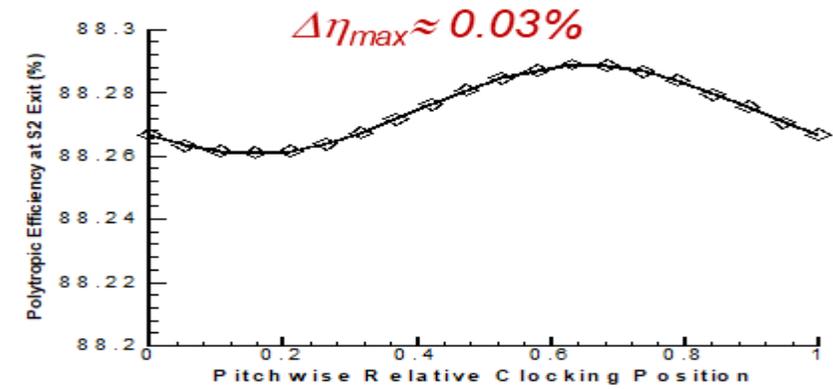
Stator-Stator 'Clocking' and Rotor-Rotor 'Aperiodicity' (He et al 2002)



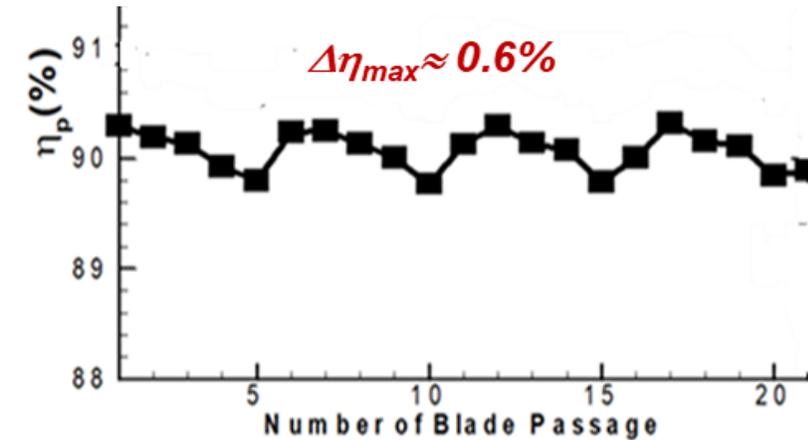
(a) Overall Domain



(b) Close-up



(a) Stator1 - Stator2 Clocking
($N_{S1} = N_{S2} = 30$)



(b) Rotor2 Passage Aperiodicity
($N_{R1} = 17, N_{R2} = 21$)

Closing Remarks

- **Multi-fidelity models are needed for Designs & Analyses, particularly in Multi-disciplinary Multi-component Interactive Settings (*Aerodynamics is Only Part of the Problem!*)**
- **The filtering (truncating) ability of a Fourier model → More efficient prediction & clearer understanding of what the related methods can (& cannot) do.**
- **Progress in past 30 years in the development of Fourier methods for turbomachinery → Part of Turbomachinery CFD Ecosystem.**
- **Continued development and applications of truncated/reduced methods aided by physical insight and computational modelling/method understanding are expected.**