Essays on Corporate Finance and Corporate Governance



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Declaration

I declare that this thesis is entirely my own work and, except where stated, describes my own research.

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Abstract

This thesis consists of three essays in corporate finance and corporate governance. Across the three chapters, I study how information, incentives, and governance structures shape corporate decision-making and firm value.

The first essay studies the feedback between financial markets and product markets. I develop a model in which consumers learn from financial markets while their purchasing decisions determine firm profitability. Firms strategically choose prices, taking into account how pricing affects financial market informativeness, which in turn influences consumer behaviour. The analysis identifies a feedback loop between financial markets and corporate decisions and shows how this interaction affects firm value, market informativeness, consumer welfare, and firms' incentives to invest in product quality.

The second essay studies director appointments and board decision-making. I develop a model in which shareholders appoint new directors in the presence of CEO moral hazard and directors' incentives to avoid opposing the CEO. Shareholders may rationally avoid appointing the best available candidate, preferring instead a lower-quality director whose presence deters value-destroying CEO proposals. The analysis highlights a tension between the advisory and monitoring roles of corporate boards and generates novel predictions on director appointments and board effectiveness.

The third essay studies the design of proxy access as a mechanism for expanding shareholder power. I develop a model in which shareholders can use proxy access to influence director appointments, but firms may resist adopting it when they anticipate shareholder

intervention. The analysis identifies an adoption-use trade-off: a lower ownership requirement encourages shareholder use and improves project choice, but can discourage adoption by increasing managerial resistance, while a higher requirement promotes adoption by reducing the expected threat of intervention. The model shows that a restrictive ownership requirement can therefore be welfare-maximizing even when proxy access is rarely used, and generates implications for the design of governance mechanisms and shareholders' incentives to revise existing proxy-access provisions.

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Chapter 1

Feedback Between the Financial Market and the Product Market

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Abstract

We study a model in which consumers learn from financial markets. Consumer learning creates a feedback effect; speculators trade on information about firm profitability, while consumers - who drive profitability - react to market information. The firm's pricing decision determines how informative the market becomes: high prices attract only high-valuation consumers, making speculator information about consumer valuations profitable; low prices induce all consumers to purchase, eliminating uncertainty and making speculator information worthless. Financial markets increase expected firm profits and can incentivize higher product quality. However, consumers may not benefit from financial market information, as it enables firms to extract consumer surplus.

Keywords: Feedback, Real effects of financial markets, Product market, Consumer learning

JEL Codes: D83, G12, G14

1.1 Introduction

Financial markets influence consumers. Initial Public Offerings (IPOs) often serve as advertisements to consumers (e.g., Stoughton et al. (2001), Pagano et al. (2002), and Demers and Lewellen (2003)). Signs of financial distress prompt consumers to reduce their demand (e.g., Hortaçsu et al. (2013), Custódio et al. (2023), and Antill and Hunter (2022)). Earnings announcements[1] and stock price movements[2] have also been shown to impact consumer behavior.

The relationship also runs in the other direction: changes in consumer demand, which directly impact firm profitability, affect financial markets. For example, in 2018, Facebook's share price fell 19% after the news broke that three million European Facebook users quit the platform in response to the Cambridge Analytica scandal.[3,4]

This interaction between the financial and product markets creates a feedback effect. We study how feedback from consumer learning affects the real economy. We build a model in which firm profits depend on consumer choice and financial markets care about firm profits. In the model, speculators may pay a cost to learn perfect information about the relevant product characteristic driving consumer choice. We label this characteristic as product quality. Speculators' information may come from surveys or superior information processing. The speculators trade on a financial market with liquidity traders and a market maker à la

[1]Graham et al. (2005) and Dichev et al. (2013) provide survey evidence that firms care about customer perceptions when reporting their earnings. Noh et al. (2021) find that consumer foot traffic (as measured by phone GPS data) to stores that sell durable goods increases with positive earnings announcements and decreases with negative ones.

[2]Sun (2017) finds that a firm's underperformance in the stock market leads to lower sales and lower shopping-related Google search volume. In a recent example, stock price declines for Credit Suisse prompted withdrawals of 10% of assets under management.The stock price declines were cited by posts on Twitter and Reddit as evidence that Credit Suisse was not in good financial health (see "How Switzerland's Oldest Bank Became a Meme Stock" by Maureen Farrell, Emily Flitter, and Joe Rennison, *New York Times*, October 18, 2022; the withdrawals figure is from "Credit Suisse forecasts $1.6bn loss as wealthy clients withdraw funds" by Owen Walker and William Langley, *Financial Times*, November 23, 2022).

[3]See "Over $119bn wiped off Facebook's market cap after growth shock" by Rupert Neate, *The Guardian*, July 26, 2018.

[4]In research tracing a direct connection between consumer demand and trading, Huang (2018) demonstrates that abnormal customer ratings on Amazon.com predict net purchases by hedge funds and abnormal returns. Several papers in the marketing literature (Sorescu and Sorescu (2016), Aksoy et al. (2008), and Anderson and Mansi (2009)) document abnormal returns from consumer satisfaction.

Kyle (1985). The firm observes the outcome of the financial market and maximizes its profits by setting a price for the product. Consumers receive private imperfect signals about the suitability of the product and observe the outcome of the financial market. They then decide whether to purchase or not.

A key insight of the model is that firm pricing determines the informativeness of financial markets. The firm may either set a high price to sell just to high valuation consumers, or a low price to sell to all consumers. If the firm sells to all consumers, it eliminates any uncertainty about firm profitability and erases the speculators' informational advantage over the market maker. If the firm sells just to high valuation consumers, speculators retain their informational advantage, as they know how profitable the firm will be (and the market maker does not). Speculators will, thus, consider the firm's pricing decision when deciding whether to acquire costly information. This creates an endogenous link between firm strategy, market informativeness, and consumer demand.

The presence of the financial market increases firm profits. When the financial market reveals that the product is high-quality, the firm is able to fully extract the surplus from all consumers. Of course, if the financial market reveals that the product is low-quality, the firm will not make any sales. We show that, in expectation, the benefit from surplus extraction outweighs the loss in sales. The marketing effect/ boost to pricing power offers an explanation of why firms enter public markets.

The feedback effect dampens the increase in firm profits - the dampening occurs when consumers act uniformly (with either all consumers purchasing or all consumers not purchasing), which reduces speculators' expected profits and, therefore, induces fewer speculators to acquire information, making the market less informative. However, we show that the dampening is a second-order effect.

While firms strictly benefit from the additional information coming from the financial market, consumers only weakly benefit from this information. While more information seems as though it would benefit them, it also can lead to the firm setting prices that extract their

surplus. We also demonstrate that as consumers' outside options increase in value, they are less likely to benefit from financial market information. This is because higher consumer outside options reduce firm profitability, reducing the incentives for speculators to acquire information.

Strategic complementarities between speculators arise when consumers' reservation utilities are very low or very high. When reservation utilities are very high (low), all consumers decide not to purchase (to purchase) when order flows reveal no information or some information that the product quality is low (high), eliminating the speculators' profits at those order flows. In these cases, an increase in the mass of speculators increases the range of the remaining order flows: those where some information is revealed that the product quality is high (low) and, thus, the speculators make positive profits. This link between the increase in the mass of speculators and increased speculator profits is a strategic complementarity. This could lead to multiple equilibria where either all speculators trade or none trade. However, given a large cost of information acquisition, only the equilibrium where no speculators trade remains.[5]

We extend the model to allow the firm to choose the product's quality. Higher-quality products both raise consumer valuations and allow speculators to benefit from information acquisition. These both provide incentives to the firm to increase product quality, with the second reason operating through the feedback effect.

Many product characteristics drive consumer choice, but for simplicity, there is only one in the model. The key assumption about this product characteristic is that consumers have the same preferences, but are uncertain about the characteristic's true value. Therefore, their information and the information provided by the market may foster heterogeneous responses where some consume, and some do not. There are many products that have

[5]Interestingly, strategic substitutability holds when the consumers' reservation utilities are intermediate - expected trading profitability is decreasing in the mass of informed speculators. As more speculators become informed and trade, the range of uninformative order flows shrinks, decreasing the expected trading profitability for these order flows - this is only partly compensated by an expanding range of partially revealing order flows and a corresponding increase in the expected trading profitability for these order flows.

vertical characteristics (e.g., quality) but must be experienced or researched in order to ascertain the value of the product to the consumer.[6]

Related literature. Our paper contributes to the literature on feedback effects (for a summary, see Bond et al. (2012) and Goldstein (2022)). A key strand of this literature studies a firm that learns from the market and makes an investment decision that affects its market value. Our paper studies learning by both consumers and the firm, with the firm using information to set prices (not make an investment) and consumers using the information to make purchase decisions. In the vast majority of papers in the firm feedback literature (e.g., Edmans et al. (2015), Goldstein and Guembel (2008), Goldstein and Yang (2019), Chen et al. (2021)), the firm's payoff from its investment choice depends on the state of the world.[7] Our result showing that pricing may induce all consumers to purchase implies a choice whereby the firm's payoff does not depend on the state of the world, leading to the erosion of the speculator's profits.[8]

In some papers, agents other than the firm learn from prices. For example, Bond and Goldstein (2015) examine financial market feedback with the government instead of a firm, motivated by the context of government interventions during the financial crisis. Nevertheless, the government learning in their model is similar to a firm learning in the papers cited above. Subrahmanyam and Titman (2001) model the feedback to non-financial stakeholders, including customers, employees, suppliers, etc. In their paper, however, non-financial stakeholders care about the firm valuation, making them similar to financial stakeholders

[6]Another example is that of sustainability, as (i) financial markets are racing to gather data on its implications; (ii) consumers are quite uncertain about the sustainability of given products; and (iii) all else equal, most consumers would prefer more-sustainable products. Interestingly, Serafeim and Yoon (2022) find that the ESG news that moves the financial market the most is news deemed material about customer related issues (product safety, quality, affordability, and access).

[7]Note that there is a recent literature where *investors'* payoffs may not entirely depend on firm profits due to sustainability concerns (Chen and Schneemeier (2025), Chen et al. (2025), and Goldstein et al. (2024)).

[8]There are two feedback papers that we are aware of in which the firm's payoff for some decision does not depend on the state. In Boleslavsky et al. (2017), the firm/regulator can take a costly action to eliminate the bad state, which reduces the speculator's profits and information in the market. In our model, we have a unit mass of decision makers, and the firm's pricing may make consumers act uniformly. In Dow et al. (2017), there is a low risk choice by the firm, which has the same payoff irrespective of the state. They model a firm learning from the financial market to inform its choice between a risky project and a riskless project, which differs from our environment.

and different from our consumers, who care only about their consumption decision. Huang and Xu (2023) examine the entry decision of a firm into the same product market as an incumbent, whose share price may reveal information about demand/profitability. Sun (2017) provides evidence that non-fundamental price drops due to mutual fund redemptions reduce customer demand, inducing a lower probability for the firm to launch a product.

There is a body of literature that relates firm market power to financial market informativeness. Stoughton et al. (2001) study the signaling choice of the firm to go public; this enables consumer learning about its product quality from the financial market and strengthens the firm's pricing power. In contrast, our main model endogenizes the informativeness of the financial market, which can be affected by strategic firm pricing in the product market. Like us, Perotti and Von Thadden (2003) endogenize information acquisition in the financial market, but unlike us, they focus on firm transparency and product market competition. Cong et al. (2024) also focus on product market competition, where firms learn from the financial market about how much they should produce. Tookes (2008) shows empirically that informed traders' incentives to trade indeed depend on the market power of firms under imperfect product market competition. This is consistent with our main model, in which the firm with monopoly power can affect the amount of information in the financial market through its strategic decision about price discrimination in the product market. Peress (2010) also finds that firm monopoly power improves the informativeness of the financial market, but with a very different mechanism from ours: in his model, monopoly power insulates firm profits, reduces risk, and incentivizes risk-averse traders to trade; in our model, price discrimination in the product market maintains residual uncertainty in firm profits and, thus, the information advantage of informed speculators.

1.2 The Model

The model has four agents who make active decisions: a unit mass of consumers, a firm, a unit mass of speculators and a market maker. In the financial market, there are also liquidity traders.

The firm produces a good that has a characteristic that is relevant to consumers' payoffs. Consumers learn about the characteristic from a private signal. Moreover, consumers may also learn about the characteristic from trade in the financial market.

The financial market (which consists of speculators, the market maker, and liquidity traders) takes place first and is followed by the product market (which consists of the firm and the consumers).

All agents in the model are rational and risk-neutral.

1.2.1 The Product Characteristic

The firm's product (or service) has one characteristic that influences consumers' payoffs. The characteristic is one that (i) consumers don't have perfect information about and (ii) the financial market has more information about than consumers have. This could be a vertical characteristic over which consumers have the same preferences. Consumers may have different information about the characteristic due to different experiences and information sources. To fix ideas, we will call this characteristic the quality of the product. The product is an experience good: consumers aren't fully aware of its quality until they have consumed it. Sophisticated investors, such as hedge funds, may gather information on quality through data collection, research, or consumer surveys.

The underlying quality θ can be high (H) or low (L) - i.e., $\theta \in \{H, L\}$. All consumers prefer high quality ($\theta = H$) over low quality ($\theta = L$). A consumer gets utility U from consumption of the product: $U = \omega > 0$ when quality is high and $U = 0$ when quality is low. In Section 1.9.1, we extend the model to allow the firm to choose the product quality.

All agents hold the prior:

$$\Pr(\theta = H) = \Pr(\theta = L) = \frac{1}{2}.$$

The product characteristic is revealed to all agents at the end of the game.

1.2.2 The Product Market

There are two types of agents in the product market: consumers and the firm.

Consumers have the same preferences but receive private signals about the product characteristic. There is a unit mass of consumers $I = [0, 1]$. Consumer $i \in I$ receives a private signal θ_i such that:

$$\Pr(\theta_i = H|\theta = H) = \Pr(\theta_i = L|\theta = L) = \mu,$$

where $\mu \in (\frac{1}{2}, 1)$. The private signals are iid across consumers for a given quality. Given the quality, receiving a correct private signal is the more likely outcome. The private signals create two types of consumers: those who receive an H signal (type H) and those who receive an L signal (type L). Define $q_i^{\theta_i}$ as the posterior probability of high quality given consumer i's information - i.e., q_i^H (q_i^L) represents the posterior belief of consumers with private signal $\theta_i = H$ ($\theta_i = L$). Consumers learn from the financial market in addition to their private signals.

Consumers maximize their expected utility by choosing whether to consume. Consumer i makes decision $d_i^{\theta_i} \in \{1, 0\}$, where 1 represents one unit of consumption and 0 represents no consumption. If consumer i does not consume (i.e., $d_i^{\theta_i} = 0$), then she will get her outside option and enjoy a reservation utility $U = \bar{U} \in [0, \omega)$.[9] For simplicity, we assume that all consumers have the same value for their outside option. We will also interpret $\bar{U}$ as a measure of market power: the higher the $\bar{U}$, the better are the alternatives to the product, and consumers will consume the firm's product only if they think that it is high-quality with

[9] In the uninteresting case of $\bar{U} \geq \omega$, all consumers will choose the outside option.

a sufficiently high probability (i.e., sufficiently high $q_i^{\theta_i}$) and/or has a low enough price.

Aggregate consumption of the product is given by $D = \int_{i\in[0,1]} d_i^{\theta_i} di$,[10] where, by definition, $D \in [0, 1]$.

The firm charges a price p_f that maximizes the firm's profits:

$$\pi_f = p_f D.$$

We assume for simplicity that the firm does not know the quality θ. This means that the firm does not know the match between the product and consumers, which could arise from limited/costly market research. This implies that the firm also learns from the financial market. It uses that information to form an expectation about consumer demand. In turn, the financial market payoffs are driven by the firm's profits, π_f.

Lastly, we define $D(\theta)$ and $\pi_f(\theta)$ as the aggregate consumption and the firm's profits, respectively, when the true quality is θ, where $\theta \in \{H, L\}$.

1.2.3 The Financial Market

There are three agents in this market: (i) a unit mass of speculators $J = [0, 1]$; (ii) liquidity traders who collectively trade $n \overset{iid}{\sim}$ Uniform$(-1, 1)$; and (iii) a market maker who sets the share price p_m (the subscript m represents the market) conditional on all public information to break even in expectation.[11]

We assume that with probability $Z \in (0, 1)$, speculators are absent from the financial market. This is similar to Edmans et al. (2015), which points out that since private information held by speculators is not public knowledge, the existence of speculators is also unlikely to be public knowledge. This will make the financial market less informative, as the market

[10] According to Uhlig (1996) (Theorem 2), we can write this integral as it is because, even if $d_i^{\theta_i}$ are pairwise uncorrelated (in the benchmark case where consumers only have iid private signals), their demands have the same common mean, and their variances are obviously bounded from above in this setup. Therefore, d is L2-Riemann integrable.

[11] Our financial market setup is similar to that in Schneemeier (2023).

maker will sometimes not know whether trades are coming from informed speculators, liquidity traders, or both. When there are speculators, each speculator j chooses whether or not to become perfectly informed about θ, at a cost c_j. The information acquisition cost of each speculator is drawn iid from the uniform distribution, i.e., $c_j \overset{iid}{\sim} \text{Uniform}(0, \bar{c})$. We denote the mass of informed traders $\alpha \in [0, 1]$. In equilibrium, speculators with information acquisition cost equal to or lower than $c^* = \alpha\bar{c}$ will become informed. We assume a prohibitively large $\bar{c}$ so that: (1) the speculator with the highest cost does not acquire information, and the solution for the mass of informed traders is always interior, i.e., $\alpha \in (0, 1)$; and (2) the model is tractable.[12] Note that the term α also is a measure of the informativeness of the financial market.

Speculator j's trading demand y_j is finite, and we impose $y_j \in [-1, 1]$ following the existing literature. Demand $y_j > 0$ represents a long position, while $y_j < 0$ represents a short position. An informed speculator's profit is denoted by π_s. He maximizes his expected *trading* profit:

$$\max_{y_j} \mathbb{E}[y_j(\pi_f - p_m)|\theta].$$

The informed speculators' total demand is denoted by $y = \int_0^\alpha y_j dj$. We will show below that y is either equal to α or $-\alpha$ because informed speculators optimally buy the share when they learn that $\theta = H$, and sell the share when they learn that $\theta = L$, and uninformed speculators do not trade.

As in Kyle (1985), the market maker observes the aggregate order flow $O = y + n$ from informed speculators and liquidity traders. The market maker sets the equilibrium share price equal to the expected firm value:

$$p_m(O) = \mathbb{E}[\pi_f|O].$$

[12]Discussions about how large $\bar{c}$ needs to be are in the relevant parts of the proofs.

We assume that the order flow is observable to all agents. Therefore, learning from the market happens through the aggregate order flow for the market maker, the firm, and consumers. Allowing agents to observe the order flow O simplifies our presentation, but is without loss of generality - our results will not change if agents learn from the share price.[13]

The firm's price can depend on the order flow, denoted $p_f(O)$; so does the decision of consumer i, $d_i^{\theta_i}(O)$, aggregate consumption, $D(O)$, and firm profits, $\pi_f(O)$. In addition, we denote aggregate consumption for the order flow O conditional on the true quality $\theta \in \{H, L\}$ as $D(O|\theta)$ and firm profits for the order flow O conditional on the true quality as $\pi_f(O|\theta)$.

1.2.4 Timeline

We now describe the timeline of the game:

1. Nature decides the quality of the product $\theta \in \{H, L\}$.
2. Consumers receive private signals θ_i about the characteristic. If speculators arrive, each speculator j chooses whether or not to learn the characteristic θ at a cost c_j.
3. Trading in the *financial market* occurs. The order flow O is public information.
4. The firm sets the product price p_f and consumers make purchases in the *product market*.
5. The true quality is revealed and payoffs realize.

We solve the game using the equilibrium concept of Perfect Bayesian Equilibrium and determine posterior beliefs for off-equilibrium order flows using the Intuitive Criterion. We note that all of the main proofs are in the appendix and the remaining proofs are in the online appendix.

[13]This is because, as we will see in the analysis later, there is a one-to-one mapping between different prices and different ranges of order flows in almost all cases. The only exception is when the share price is 0. Here there are two order flows consistent with the price: the order flow that partially reveals the quality is low, *and* the order flow that fully reveals the quality is low. Since a share price can correspond to two different ranges of order flows, the share price is a noisy signal of the order flow, which is, in turn, a noisy signal of the quality. Hence, assuming that the order flow is observable to all agents gives agents more information than learning from the price. If we were to assume that agents instead learn from the share price, consumer valuations will be weakly lower (than the case where they learn from the order flow) when the order flow partially reveals that the quality is low. With even lower consumer valuations, no consumer will consume and the share price will be 0. Therefore, there is no difference in equilibrium behavior between learning from the share price and learning from the order flow, even when the mapping between prices and ranges of order flows is not one-to-one.

Table 1.1: Consumer demand

	Consumer demand D			
	Both types willing to buy		Only H-type consumers willing to buy	Neither type willing to buy
	Firm sells to both types	Firm sells to only type H		
θ:				
H	1	μ	μ	0
L	1	1-μ	1-μ	0

1.3 Benchmark

We now look at a benchmark where the product market occurs before the financial market, which effectively shuts down the feedback channel. In the benchmark, the agents in the product market (consumers and the firm) do not learn from the financial market. We note that this flips stages 3 and 4 in the timing above. We assume that the quality of the product and firm profits/consumer purchases are not revealed when the financial market takes place. As in reality, firm performance may take time to be revealed - i.e., through financial reports and earnings announcements.[14]

Before we analyze the unique equilibrium of the model, it is useful to begin by discussing the key forces present in the model. In Table 1.1, we illustrate the possible realizations of consumer demand, given the true quality θ and the firm's pricing.

Consumer demand D is independent of the quality θ either when both types of consumers are willing to buy and the firm sells to both types, or when neither type is willing to buy. In these cases, speculators can't gain any information advantage by acquiring information. However, the firm's choice of price may break the independence of consumer demand from the quality when both types are willing to buy. The firm may profit by charging a very high price to the consumer type with the higher valuation, excluding the other consumer type. This monopoly distortion would make demand sensitive to the quality, allowing for informed

[14]That the true quality is not revealed until the end of the period maintains trading incentives in the financial market in this benchmark model. Some other feedback papers (e.g., Goldstein and Guembel (2008)) have a benchmark model without the feedback effect that assumes that agents have full information; however, such a benchmark is not interesting in our model given the nature of experience goods.

speculators to make more profits from trading.[15] Speculation and information in financial markets are, thus, responsive to firm pricing decisions.

When the product market takes place before the financial market, consumers update their beliefs about the product characteristic based only on their private signals. According to Bayes' rule, the consumers' posterior probabilities that the product is high-quality given signals H or L are $q_i^H = \mu$ and $q_i^L = 1 - \mu$, respectively, where $q_i^H > q_i^L$ given that $\mu > \frac{1}{2}$. A higher μ implies that consumers' private signals are more precise about the true quality.

The expected payoff of an H-type consumer from consuming ($d_i^H = 1$) is:

$$\mathbb{E}(U|\theta_i = H) = \omega q_i^H = \omega\mu.$$

The expected payoff of an L-type consumer from consuming ($d_i^L = 1$) is:

$$\mathbb{E}(U|\theta_i = L) = \omega q_i^L = \omega(1 - \mu).$$

For $\forall\mu \in (\frac{1}{2}, 1)$, $\mathbb{E}(U|\theta_i = H) > \mathbb{E}(U|\theta_i = L)$. Hence, there are two cases to discuss: $\bar{U} \in [\omega(1-\mu), \omega\mu)$ and $\bar{U} < \omega(1-\mu)$. Note that if $\bar{U} \geq \omega\mu$, no consumer would consume at any product price.

In the first case, when $\bar{U} \in [\omega(1-\mu), \omega\mu)$, only H-type consumers would consume if the price were lower than $\omega\mu - \bar{U}$. It is profit maximizing for the firm to sell to the H-type consumers at their willingness to pay, so $p_f = \omega\mu - \bar{U}$. In this case, the firm value is sensitive to the underlying quality θ because the mass of consumers who receive an H signal depends on the true quality.

We define the information sensitivity of firm value as:

$$\pi_f(H) - \pi_f(L).$$

[15] Note that, of course, in the model, the firm does not price this way for the benefit of speculators - the firm does this only to maximize its own profits.

When the information sensitivity is positive, the speculators are incentivized to become informed and trade in order to take advantage of the market maker, who will price the firm's share at its expected value. The larger this information sensitivity, the more valuable is the speculator's perfect information, because the gap between the firm's true profits and its expected profits widens. In the case of $\bar{U} \in [\omega(1-\mu), \omega\mu)$, the information sensitivity is positive, so some speculators acquire information about θ in equilibrium, i.e., $\alpha > 0$.

In the second case, when $\bar{U} < \omega(1-\mu)$, $\mathbb{E}(U|\theta_i = H) > \mathbb{E}(U|\theta_i = L) > \bar{U}$. Hence, all consumers would consume if the price were low. The firm chooses between selling to all consumers at a low price and selling only to H-type consumers at a high price. If the firm sells to all consumers, it will set the price $p_f = \omega(1-\mu) - \bar{U}$; if the firm sells only to H-type consumers, it will set the price $p_f = \omega\mu - \bar{U}$.

Hence, the firm will sell to all consumers if and only if:

$$\omega(1-\mu) - \bar{U} \geq \frac{1}{2}(\omega\mu - \bar{U}),$$

where $\frac{1}{2}$ represents the firm's expected demand from H-type consumers given the prior.

Therefore, the firm will sell to all consumers if and only if:

$$\bar{U} \leq \omega(2-3\mu).$$

When $\bar{U} \leq \omega(2-3\mu)$, the information sensitivity of firm value is 0.[16] When $\bar{U} > \omega(2-3\mu)$, the information sensitivity of firm value is positive and increasing in μ:

$$\pi_f(H) - \pi_f(L) = (\omega\mu - \bar{U})(2\mu - 1).$$

[16]Note that $\omega(2-3\mu) < 0$ if and only if $\mu \in (\frac{2}{3}, 1)$. In this case, $\bar{U} \leq \omega(2-3\mu)$ will not exist because $\bar{U}$ cannot be negative. As will be shown later, this will eliminate a case in which the financial market is not informative and will not affect our results - therefore, for expositional simplicity, we include the case in which $\bar{U} \leq \omega(2-3\mu)$ going forward.

Observing aggregate order flow O, the market maker sets the price:

$$p_m(O) = \Pr(H|O)\pi_f(H) + \Pr(L|O)\pi_f(L),$$

where she also uses Bayes' rule to update her beliefs where possible:

$$\Pr(H|O) = \frac{\Pr(O|H)\Pr(H)}{\sum_\theta \Pr(O|\theta)\Pr(\theta)}$$

$$\Pr(L|O) = \frac{\Pr(O|L)\Pr(L)}{\sum_\theta \Pr(O|\theta)\Pr(\theta)}$$

Summarizing the above analysis, we now characterize the unique equilibrium.

Proposition 1 *(Benchmark equilibrium)*

- *When $\omega(2-3\mu) < \bar{U} < \omega\mu$, there is a unique equilibrium E^{B} where only consumers with an H signal consume (at price $p_f = \omega\mu - \bar{U}$). Informed speculators choose to buy when $\theta = H$ and to sell when $\theta = L$. Uninformed speculators do not trade. The equilibrium mass of informed speculators is $\alpha_B = \frac{1}{\frac{2\bar{c}}{(\omega\mu-\bar{U})(2\mu-1)}+\frac{1}{Z+1}}$.*
- *When $\bar{U} \leq \omega(2-3\mu)$, there is a unique equilibrium E^{A} where all consumers consume (at price $p_f = \omega(1-\mu) - \bar{U}$). No speculator chooses to become informed, and uninformed speculators do not trade.*
- *When $\bar{U} \geq \omega\mu$, there is a unique equilibrium E^{N} where the product market collapses due to no consumption. No speculator chooses to become informed, and uninformed speculators do not trade.*

When the outside option is very high, consumers do not enter the product market and the firm doesn't produce, leading to no financial trading. When the outside option is very low, all consumers consume, but there is no value to trading for speculators, as information does not give them an advantage over the market maker. When the outside option is intermediate, only consumers with an H-signal consume, and the speculators are able to reap gains from

their information.

1.4 The Feedback Game

We now look at the main model: the financial market occurs before the product market. This creates feedback between the two markets: consumers observe the order flow and use their inferred information about the quality in combination with their private signal. The firm prices the product by rationally forecasting consumer behavior, incorporating the information from the public order flow signal. Consumer behavior and firm pricing determine the firm's value. The speculators and the market maker take this into account when trading.

1.4.1 Equilibrium

When there is feedback between the two markets, consumers' posterior beliefs are pinned down by Bayes' rule, now using two signals.

$$q_i^H(O) = \Pr(\theta = H|\theta_i = H, O) = \frac{\Pr(\theta_i = H|H)\Pr(O|H)\Pr(H)}{\sum_\theta \Pr(\theta_i = H|\theta)\Pr(O|\theta)\Pr(\theta)} = \frac{\mu \Pr(O|H)}{\sum_\theta \Pr(\theta_i = H|\theta)\Pr(O|\theta)}$$

$$q_i^L(O) = \Pr(\theta = H|\theta_i = L, O) = \frac{\Pr(\theta_i = L|H)\Pr(O|H)\Pr(H)}{\sum_\theta \Pr(\theta_i = L|\theta)\Pr(O|\theta)\Pr(\theta)} = \frac{(1-\mu) \Pr(O|H)}{\sum_\theta \Pr(\theta_i = L|\theta)\Pr(O|\theta)}$$

There is a unique equilibrium for the feedback game. The unique equilibrium will be one of the six possible configurations, determined by the parameters of the model. We begin by describing the configurations, which we denote by E^{I}, E^{II}, $\mathrm{E}^{\mathrm{III}}$, E^{IV}, E^{A}, and E^{N}. The configurations E^{A} and E^{N} are defined in the benchmark. In Table 1.2, we describe consumer choices and firm pricing in the other four cases. We will subsequently map these equilibria to the parameters in Propositions 2 and 3. In the table, there is no product price when no consumer buys because the product market collapses.

In E^{I}, only H-type consumers buy when the order flow is uninformative or partially reveals that the quality is low. When the order flow partially reveals that the quality is high, all

Table 1.2: List for equilibrium configurations E^{I}, E^{II}, E^{III}, and E^{IV} of: (i) which consumers purchase, and (ii) product prices.

Order Flow O	E^{I}	E^{II}	E^{III}	E^{IV}
$(-1-\alpha,-1)$	no consumers	no consumers	no consumers	no consumers
$[-1,-1+\alpha]$	H-type consumers $p_f=\omega\frac{\mu Z}{\mu Z+(1-\mu)-\bar{U}}$	H-type consumers $p_f=\omega\frac{\mu Z}{\mu Z+(1-\mu)}-\bar{U}$	no consumers	no consumers
$(-1+\alpha,1-\alpha)$	H-type consumers $p_f=\omega\mu-\bar{U}$	H-type consumers $p_f=\omega\mu-\bar{U}$	H-type consumers $p_f=\omega\mu-\bar{U}$	H-type consumers $p_f=\omega\mu-\bar{U}$
$[1-\alpha,1]$	all consumers $p_f=\omega\frac{1-\mu}{(1-\mu)+\mu Z}-\bar{U}$	H-type consumers $p_f=\omega\frac{\mu}{\mu+(1-\mu)Z}-\bar{U}$	H-type consumers $p_f=\omega\frac{\mu}{\mu+(1-\mu)Z}-\bar{U}$	all consumers $p_f=\omega\frac{1-\mu}{(1-\mu)+\mu Z}-\bar{U}$
$(1,1+\alpha)$	all consumers $p_f=\omega-\bar{U}$	all consumers $p_f=\omega-\bar{U}$	all consumers $p_f=\omega-\bar{U}$	all consumers $p_f=\omega-\bar{U}$

consumers buy. Hence, although the market maker does not know the true quality when the order flow partially reveals that the quality is high, there is no residual uncertainty about firm profits. In E^{II}, only H-type consumers buy when the order flows do not fully reveal the true quality, so there is residual uncertainty about firm profits for these order flows. In E^{III}, only H-type consumers buy when the order flow is uninformative or partially reveals that the quality is high. When the order flow partially reveals that the quality is low, no consumer buys and there is no residual uncertainty in the firm outcome. In E^{IV}, only H-type consumers buy when the order flow is uninformative about the true quality. When the order flow partially reveals that the quality is low, no consumer buys. When the order flow partially reveals that the quality is high, all consumers buy. Hence, when the order flow partially reveals either quality, there is no residual uncertainty in the firm's profits.

We now characterize the unique equilibrium of the feedback game for all parameters. The equilibrium depends on the probability that the speculators are absent from the financial market. We begin by looking at the case in which this probability is large (Z high).

Proposition 2 *(Equilibrium when the probability speculators are absent is large)*

Suppose that $Z>(\frac{1-\mu}{\mu})^2$. Given sufficiently large $\bar{c}$, the equilibrium is unique:

- *When $\omega(2-3\mu)<\bar{U}<\omega\mu$, informed speculators choose to buy when $\theta=H$ and to sell when $\theta=L$; uninformed speculators do not trade. Consumer and firm behavior*

are as described in Table 1.2 for the relevant equilibrium.

- *When* $\omega(2-3\mu) < \bar{U} \leq \omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu]$*, the unique equilibrium is* E^{I}*. The equilibrium mass of informed speculators is* α_I *(given by equation A.1 in the appendix).*
- *When* $\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu] < \bar{U} < \omega\frac{\mu Z}{\mu Z+(1-\mu)}$*, the unique equilibrium is* E^{II}*. The equilibrium mass of informed speculators is* α_{II} *(given by equation A.2 in the appendix).*
- *When* $\omega\frac{\mu Z}{\mu Z+(1-\mu)} \leq \bar{U} < \omega\mu$*, the unique equilibrium is* $\mathrm{E}^{\mathrm{III}}$*. The equilibrium mass of informed speculators is* α_{III} *(given by equation A.3 in the appendix).*

- *When* $\bar{U} \leq \omega(2-3\mu)$*, the unique equilibrium is* E^{A} *(where all consumers consume), as defined in the benchmark game.*
- *When* $\bar{U} \geq \omega\mu$*, the unique equilibrium is* E^{N} *(where no one consumes), as defined in the benchmark game.*

Consumers now learn from the financial market signal in addition to their private signal, so their valuation about the product can change with the order flow. Hence, the firm's pricing strategy will also be contingent on the order flow signal. We now examine how these results differ from the benchmark.

In equilibria E^{I}, E^{II} and $\mathrm{E}^{\mathrm{III}}$,[17] consumers learn the product quality perfectly when the order flow signal is fully revealing, whereas in the benchmark equilibrium E^B, they can never learn the quality perfectly via their private signals. If high quality is fully revealed ($O \in (1, 1+\alpha)$), all consumers value the product at ω, and the firm can sell to all consumers at their maximum willingness to pay $\omega - \bar{U}$; if low quality is fully revealed ($O \in (-1-\alpha, -1)$), no consumer would like to consume and the product market collapses.

Information from the order flow also may reveal enough information to make consumers act uniformly, eliminating speculator profits for those order flows. When the reservation

[17] In E^{IV}, consumers also learn the product quality perfectly when the order flow signal is fully revealing, but E^{IV} is not a possible equilibrium when $Z > (\frac{1-\mu}{\mu})^2$ (it will be when $Z \leq (\frac{1-\mu}{\mu})^2$).

utility is at the bottom of the range where the speculator trades ($\omega(2-3\mu) < \bar{U} \leq \omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu]$, where the unique equilibrium is E^{I}), the outside option is unattractive and all consumers buy at a relatively high product price for order flows that partially reveal that the quality is high. Hence, in this case, consumer demand in E^{I} eliminates residual uncertainty about firm profits; this does not occur in the benchmark E^{B}. When the reservation utility is at the top of the range where the speculator trades ($\omega\frac{\mu Z}{\mu Z+(1-\mu)} \leq \bar{U} < \omega\mu$, as in $\mathrm{E}^{\mathrm{III}}$), the outside option is attractive enough such that no consumer is willing to buy for order flows that partially reveal that the quality is low. Here, the lack of consumer demand in $\mathrm{E}^{\mathrm{III}}$ eliminates residual uncertainty about firm profits (which does not happen in the benchmark E^{B}).

When the order flow signal partially reveals the product's true quality, the shift in the distribution of consumer valuations relative to the benchmark depends on how noisy the order flow signal is. The amount of noise depends on whether speculators are likely to be absent. In Proposition 2, Z is large, so speculators are likely to be absent from the financial market. This implies that when the order flow signal partially reveals either quality, consumers may think it is noise and not change their valuations. Of course, when the order flow is uninformative, consumers do not change their valuations either. Therefore, when the order flow signal is not fully revealing, consumer and firm pricing choices remain similar to the benchmark.[18] Among the possible equilibria when Z is large, the distribution of consumer valuations shifts the least in E^{II} relative to the benchmark: when the order flow signal is not fully revealing, only H-type consumers consume, which is identical to the consumption pattern in the benchmark E^{B}.[19]

It is worth noticing that speculators acquire information and trade on the information if and only if the reservation utility is intermediate ($\omega(2-3\mu) < \bar{U} < \omega\mu$) - the same

[18] As Z approaches 1, speculators are likely to be completely absent from the financial market. In this limit case, consumer and firm pricing choices become identical to those in the benchmark E^{B}.

[19] In contrast, when Z is small, consumers are more likely to update their valuations upon partially revealing order flows. Consumer and firm pricing choices no longer remain similar to the benchmark: E^{II} will not exist when Z is small. We discuss this in Proposition 3.

range where speculators acquire information in the benchmark game. Speculators could have acquired information and traded in a wider range of reservation utility, but there are strategic complementarities in the feedback game - which are not present in the benchmark - that prevent speculators from doing so:

- First, when $\omega\mu \leq \bar{U} < \omega\frac{\mu}{\mu+(1-\mu)Z}$, the reservation utility is so high that no consumer consumes when the order flow is uninformative about the true quality, partially revealing that the quality is low, or fully revealing that the quality is low. This implies that speculators who are informed that the quality is low ($\theta = L$) cannot make trading profits by selling shares in the firm. Informed speculators can make trading profits when they are informed that the quality is high ($\theta = H$) and when the order flow partially reveals that the quality is high. However, in this case, a strategic complementarity arises due to the speculators' expected trading profits increasing in the mass of informed speculators. An increase in the number of informed speculators increases the number of informed trades - the range of order flows where high quality is partially revealed becomes bigger. This then increases the expected trading profits of all speculators from becoming informed. Such strategic complementarity coupled with our assumption that $\bar{c}$ is prohibitively large leads to unraveling: the speculator with the highest cost finds information acquisition unprofitable, which decreases the expected trading profits of the other speculators from becoming informed; then, the speculator with the second highest cost chooses not to become informed; and so on.[20]
- Second, when $\omega\frac{Z}{(1-\mu)Z+\mu}[\frac{(1-\mu)(Z+1)}{(1-\mu)Z+\mu} - \mu] < \bar{U} \leq \omega(2-3\mu)$, the reservation utility is so low that all consumers buy when the order flow is uninformative about the true quality, partially revealing that the quality is high, or fully revealing that the quality is high. This implies that speculators who are informed that the quality is high ($\theta = H$) cannot make trading profits by buying shares in the firm. Informed speculators could make trading profits when they are informed that the quality is low ($\theta = L$) and when the

[20]Note that there are no multiple equilibria here - the opposite effect of the 'unraveling' doesn't hold here. Because of unraveling, only the equilibrium where no speculator acquires information remains.

order flow partially reveals that the quality is low. The same strategic complementarity arises and leads to unraveling in speculators' information acquisition.[21]

Therefore, as in the benchmark, when the reservation utility is very high ($\bar{U} \geq \omega\mu$), all consumers choose the outside option regardless of their information about the firm's product; when the reservation utility is very low ($\bar{U} \leq \omega(2-3\mu)$), all consumers consume regardless of their information about the firm's product. In both cases, there is no information asymmetry about the firm's value between speculators and the market maker, so no speculator will acquire information and trade.

Next, we turn to the case in which the probability that speculators are absent is small (Z small).

Proposition 3 *(Equilibrium when the probability that speculators are absent is small)*

Suppose that $Z \leq (\frac{1-\mu}{\mu})^2$. *Given sufficiently large* $\bar{c}$, *there is a unique equilibrium:*

- *When* $\omega(2-3\mu) < \bar{U} < \omega\mu$, *informed speculators choose to buy when* $\theta = H$ *and to sell when* $\theta = L$; *uninformed speculators do not trade. Consumer and firm behavior are as described in Table 1.2 for the relevant equilibrium.*
  - *When* $\omega(2-3\mu) < \bar{U} < \max\{\omega\frac{\mu Z}{\mu Z+(1-\mu)}, \omega(2-3\mu)\}$, *the unique equilibrium is* $\mathrm{E^I}$. *The equilibrium mass of informed speculators is* α_I *(given by equation A.1 in the appendix).*
  - *When* $\max\{\omega\frac{\mu Z}{\mu Z+(1-\mu)}, \omega(2-3\mu)\} \leq \bar{U} \leq \min\{\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu], \omega\mu\}$, *the unique equilibrium is* $\mathrm{E^{IV}}$. *The equilibrium mass of informed speculators is* α_{IV} *(given by equation A.4 in the appendix).*
  - *When* $\min\{\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu], \omega\mu\} < \bar{U} < \omega\mu$, *the unique equilibrium is* $\mathrm{E^{III}}$. *The equilibrium mass of informed speculators is* α_{III} *(given by equation A.3*

[21] Strategic complementarity does not arise when $\omega(2-3\mu) < \bar{U} < \omega\mu$. When $\omega(2-3\mu) < \bar{U} < \omega\mu$, the informed speculators can make trading profits when the order flow signal is uninformative. Trading is more profitable when the financial market is uninformative than when the financial market is partially informative. Because the range of uninformative order flows decreases as the mass of informed speculators increases, the expected trading profits decrease as the mass of informed speculators increases. This eliminates the strategic complementarity.

in the appendix).

- *When $\bar{U} \leq \omega(2-3\mu)$, the unique equilibrium is* E^{A} *(where all consumers consume), as defined in the benchmark.*
- *When $\bar{U} \geq \omega\mu$, the unique equilibrium is* E^{N} *(where no one consumes), as defined in the benchmark.*

When Z is small, speculators are likely to be present in the financial market, and the financial market signal is more likely to be informative. When H-type (L-type) consumers observe a low (high) order flow that partially reveals that the quality is low (high), they may consider it to be informative and decrease (increase) their valuation of the product. Hence, the gap in product valuation between the two types of consumers becomes smaller when the order flow signal partially reveals either quality. Therefore, unlike Proposition 2, there does not exist an intermediate range of reservation utility where only H-type consumers consume when the order flow signal partially reveals either quality, as in E^{II}. Instead, when the reservation utility is in the intermediate range ($\max\{\omega\frac{\mu Z}{\mu Z+(1-\mu)}, \omega(2-3\mu)\} \leq \bar{U} \leq \min\{\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu], \omega\mu\}$, where the unique equilibrium is E^{IV}), the reservation utility is high enough such that no consumer is willing to buy for order flows that partially reveal that the quality is low and, at the same time, low enough such that the firm maximizes its profits by selling to all consumers at a relatively high price for order flows that partially reveal that the quality is high. Consequently, in equilibrium E^{IV}, informed speculators make trading profits only when the order flow is uninformative about the true quality. Equilibria E^{I}, $\mathrm{E}^{\mathrm{III}}$, E^{A}, and E^{N} arise in Proposition 3 due to forces analogous to those in Proposition 2.

In the following sections, we will look at the payoffs of different agents (consumers, the firm, and speculators). We measure their payoffs as follows: consumer surplus is denoted by CS, while the firm and speculators profits are given by π_f and π_s, respectively. All calculations for ex ante payoffs $\mathbb{E}[CS]$, $\mathbb{E}[\pi_f]$ and $\mathbb{E}[\pi_s]$ in each possible equilibrium are in the online appendix (OA.1). Unless specified otherwise, all the propositions henceforth follow directly from the payoff calculations.

1.5 Firm Profits

In comparison to the benchmark equilibrium, the firm's expected profits are at least as large in the feedback game (for any given $\bar{U}$). We demonstrate this in the following proposition.

Proposition 4 *For any given $\bar{U}$, the firm's expected profits in the feedback game are at least as large as in the benchmark. Expected firm profits are strictly higher if and only if $\bar{U}$ is intermediate, i.e., $\omega(2-3\mu) < \bar{U} < \omega\mu$.*

The proposition demonstrates that the information provided by financial markets benefits the firm. This can be thought of as a disciplining effect of markets. The intuition is that when the financial market conveys information, it is now possible for the quality to be revealed to consumers. In particular, when consumers learn that the product is high-quality, the firm is able to fully extract the rents from all consumers. However, when consumers learn that the product is low-quality, the firm is not able to generate any revenue. In expectation, the firm benefits from the pricing power it has over *all* consumers when high quality is revealed more than the loss of pricing power over *some* consumers (specifically, H-type consumers) when low quality is revealed.[22] The increase in pricing power is a benefit associated with firms listing in public markets.

There is a downside to the information provided by the financial market. Consumers, using this information, may act uniformly for some partially revealing order flows (while they would not act uniformly in the benchmark), and this feedback between the two markets

[22]To illustrate, we compare the benchmark equilibrium E^{B} and the feedback equilibrium E^{II}. Ex ante, the unconditional probability of high quality being fully revealed by the order flow in E^{II} is $\frac{\alpha_{II}}{2}\frac{1}{2}(1-Z)$. When high quality is revealed, relative to the benchmark, for each unit of sales, the firm gains pricing power of $\omega - \omega\mu$; moreover, the mass of consumers over which the firm exerts its pricing power expands from μ to 1. Similarly, ex ante, the unconditional probability of low quality being fully revealed by the order flow in E^{II} is $\frac{\alpha_{II}}{2}\frac{1}{2}(1-Z)$. When low quality is revealed, relative to the benchmark, for each unit of sales, the firm loses pricing power of $\omega\mu$; however, the mass of consumers over which the firm loses its pricing power is only $1-\mu$ (those consumers who receive high signals when $\theta = L$). Therefore, ex ante, the expected net gain in firm profits due to feedback is:

$$\frac{\alpha_{II}}{4}(1-Z)\omega[1-\mu^2-\mu(1-\mu)] = \frac{1}{4}(1-Z)\omega(1-\mu)\alpha_{II} > 0.$$

could lower speculator profits and, therefore, speculator entry. The negative effect on α can indirectly affect firm profits, namely by lowering the probability that the true quality is fully revealed and, thus, dampening the informational effect of markets described above. However, this downside to information is a second-order effect; the proof demonstrates that it does not dominate the effect of financial market information in any equilibrium of the feedback game.

Information from the financial market not only affects expected firm profits, but also makes firm profits more sensitive to the outside option.

Proposition 5 *When $\omega(2-3\mu) < \bar{U} < \omega\mu$, the firm's expected profits decrease in $\bar{U}$ in both the benchmark and the feedback game. In the feedback game, there is a larger decrease in the firm's expected profits for a unit increase in $\bar{U}$ than in the benchmark -, i.e., the firm's performance becomes more sensitive to the outside option due to the feedback effect.*

When the financial market is informative ($\omega(2-3\mu) < \bar{U} < \omega\mu$), the feedback between the two markets amplifies the sensitivity of the firm's expected profits to competitive forces in the product market. As shown in the proof of Proposition 4, the information effect increases the firm's expected profits, with this increase proportional to the number of informed speculators.[23] However, the feedback effect introduces an additional channel through which the outside option influences expected profits. As the outside option increases, there is (i) a direct effect: the firm's expected profits decrease due to more attractive alternatives; and (ii) an indirect effect: lower profits reduce the mass of informed speculators. A more appealing outside option constrains the firm's ability to charge higher prices, diminishing the value of the information asymmetry between informed speculators and the market maker, which weakens speculators' incentives to acquire information.

[23]To understand the proportionality, an increase in the mass of informed speculators increases the likelihood of the order flow revealing the underlying quality, and the firm benefits from such revelation in expectation.

1.6 Consumer Surplus

Given that firms benefit from learning from financial markets, it is natural to ask how consumers fare. We characterize their surplus in the following proposition.

Proposition 6 *When $\omega(2-3\mu) < \bar{U} < \omega\mu$, consumers are strictly better off in the feedback game than in the benchmark if and only if the feedback equilibrium is* E^{I} *or* E^{IV}*. When the feedback equilibrium is* E^{II} *or* $\mathrm{E}^{\mathrm{III}}$*, consumers receive the same payoff in the feedback game as in the benchmark.*

In both E^{II} and $\mathrm{E}^{\mathrm{III}}$, although the financial market signal is informative, consumers who consume always have their rents fully extracted by the firm, which is also the case in the benchmark equilibrium. Only when the feedback equilibrium is E^{I} or E^{IV} can the information effect make the firm switch from selling only to H-type consumers in the benchmark to selling to all consumers (for an order flow signal that partially reveals that the quality is high) in the feedback game. This gives H-type consumers a positive surplus.

Even though the information from the financial market may benefit consumers (in E^{I} or E^{IV}), it has a downside: because all consumers consume when the order flow signal partially reveals that the quality is high, there is no residual uncertainty about firm profits and, thus, the informed speculators do not profit from buying the share at that order flow. This reduces the incentive of speculators to acquire information. Lower α decreases the likelihood of an order flow that partially reveals that the quality is high, which, in turn, decreases the probability of H-type consumers having a positive surplus. Hence, the information benefit to consumer surplus is dampened by the feedback effect.

From Propositions 2 and 3, E^{I} or E^{IV} occurs (and, thus, consumers benefit from the information effect) only if $\omega(2-3\mu) < \bar{U} \leq \min\{\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu], \omega\mu\}$. In contrast, the firm benefits from the information effect when $\omega(2-3\mu) < \bar{U} < \omega\mu$. Because $\min\{\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu], \omega\mu\} \leq \omega\mu$, consumers benefit from the information effect

under more-restrictive conditions than the firm does.

Information from the financial market also alters how sensitive consumer welfare is to the outside option.

Proposition 7 *When the feedback equilibrium is* E^{I} *or* E^{IV}*, consumer surplus increases in* $\bar{U}$ *in both the benchmark and the feedback game. In the feedback game, there is a smaller increase in consumer surplus for a unit increase in* $\bar{U}$ *than in the benchmark - i.e., consumer surplus becomes less sensitive to outside options due to the feedback effect.*

When the equilibrium is E^{I} or E^{IV}, the feedback between the two markets diminishes the sensitivity of the consumer surplus to the outside option. As shown in the proof of Proposition 6, thanks to the information effect, consumer surplus is not entirely captured by the firm, and H-type consumers retain a positive surplus that is proportional to the number of informed speculators. However, the feedback effect introduces an additional channel through which the outside option influences consumer surplus. As the outside option increases, there is (i) a direct effect: consumer surplus increases because a higher outside option forces the firm to offer a more attractive price; and (ii) an indirect effect: a higher outside option reduces the product price that the firm can charge, reducing the value of the informed speculators' information and, therefore, the mass of informed speculators, which, in turn, reduces the likelihood that H-type consumers will receive positive surplus.

1.7 Market Power

Firm profits and consumer surplus are linked through the firm's market power. We now highlight how the feedback effect reduces the firm's market power relative to the benchmark. In line with the reduction in the firm's market power, consumers are weakly better off in the feedback game (Proposition 6).

Specifically, we define the firm's market power as the situation in which it fully extracts

Table 1.3: Parameters for which the firm uses its market power in the feedback game (Case 1, 2, or 3)

Order Flow O	Case 1	Case 2	Case 3
$[-1, -1+\alpha]$	$\omega\frac{(1-\mu)Z}{(1-\mu)Z+\mu} \leq \bar{U} < \omega\frac{\mu Z}{\mu Z+(1-\mu)}$	$\omega\frac{Z}{(1-\mu)Z+\mu}[\frac{(1-\mu)(Z+1)}{(1-\mu)Z+\mu} - \mu] < \bar{U} < \omega\frac{(1-\mu)Z}{(1-\mu)Z+\mu}$	NA
$(-1+\alpha, 1-\alpha)$	$\omega(1-\mu) \leq \bar{U} < \omega\mu$	$\omega(2-3\mu) < \bar{U} < \omega(1-\mu)$	NA
$[1-\alpha, 1]$	$\omega\frac{1-\mu}{(1-\mu)+\mu Z} \leq \bar{U} < \omega\frac{\mu}{\mu+(1-\mu)Z}$	$\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu] < \bar{U} < \omega\frac{1-\mu}{(1-\mu)+\mu Z}$	NA
$(1, 1+\alpha)$	NA	NA	$\bar{U} < \omega$

the surplus of the consumers it sells to. This includes three cases: the first case (Case 1) is when only H-type consumers are willing to consume and the firm fully extracts their surplus; the second case (Case 2) is when both H-type and L-type consumers are willing to consume, but the firm - through product pricing - excludes L-type consumers and fully extracts the surplus of H-type consumers; the third case (Case 3) is when the firm fully extracts the surplus of both H-type and L-type consumers (this happens only when high quality is fully revealed).

In the benchmark, the firm uses its market power to fully extract the surplus of H-type consumers when $\omega(1-\mu) \leq \bar{U} < \omega\mu$, because only H-type consumers are willing to consume. The firm also uses its market power to exclude L-type consumers by charging a product price higher than they are willing to pay when $\omega(2-3\mu) < \bar{U} < \omega(1-\mu)$.

In the feedback game, consumer valuations depend on the order flow signal. When the financial market signal fully reveals that quality is low, the firm does not use its market power because it cannot (no consumers are willing to purchase when low quality is fully revealed). Hence, it is possible for the firm to use its market power only when the order flow signal is uninformative, partially reveals the true quality, or fully reveals that the quality is high. We describe in Table 1.3 the (necessary) conditions that must be satisfied for the firm to use its market power at these order flows, and we differentiate the cases under which the firm uses its market power at these order flows ("NA" means there is no range of $\bar{U}$ where the type of market power the case specifies is exercised for the given order flow).

On the one hand, the outside option cannot be too attractive (i.e., $\bar{U}$ must be bounded

from above) so that H-type consumers are willing to purchase the firm's product given their private signals and the order flow signal. On the other hand, when H-type and L-type consumers have different valuations (i.e., the order flow signal is not fully revealing), if the outside option is too unattractive, the firm can charge a high price even to L-type consumers. This incentivizes the firm to sell at that price to all consumers to maximize profits (i.e., the firm does not use its market power to fully extract the surplus of the consumers it sells to). Hence, $\bar{U}$ is also bounded from below. In a given feedback equilibrium where the financial market is informative (E^{I}, E^{II}, E^{III}, or E^{IV}), the firm uses its market power if (i) the order flow does not fully reveal the quality is low; (ii) $\bar{U}$ satisfies the equilibrium conditions set out in Propositions 2 and 3; *and* (iii) $\bar{U}$ satisfies the conditions described in Table 1.3.

If the parameter conditions that satisfy (i), (ii) and (iii) are subsets of the corresponding parameter conditions in the benchmark equilibrium E^{B}, then the feedback effect reduces market power relative to the benchmark. In the benchmark equilibrium E^{B}, the firm uses its market power for any order flow as long as $\omega(2-3\mu) < \bar{U} < \omega\mu$. In contrast, in a given feedback equilibrium where the financial market is informative (E^{I}, E^{II}, E^{III}, or E^{IV}), the firm uses its market power only if the order flow does not fully reveal the low quality. Furthermore, because the equilibrium conditions of any feedback equilibrium (E^{I}, E^{II}, E^{III}, or E^{IV}) necessarily require $\omega(2-3\mu) < \bar{U} < \omega\mu$ (as set out in Propositions 2 and 3), the intersection of the equilibrium conditions and any of the conditions described in Table 1.3 must be a subset of $\omega(2-3\mu) < \bar{U} < \omega\mu$. Therefore, the feedback effect reduces market power relative to the benchmark. Because the firm does not use as much market power to extract consumer surplus in the feedback game as it does in the benchmark, consumers expect to have higher surplus in the feedback game.

Reduced market power/ higher consumer surplus results from both H-type and L-type consumers increasing their valuations compared to the benchmark when the order flow signal partially reveals that the quality is high. This can make the firm switch from selling only to H-type consumers in the benchmark (i.e., using market power under Case 2) to selling to

all consumers (and not extracting the full surplus). As set out in Proposition 6, this switch happens in the feedback equilibrium E^{I} or E^{IV} when the order flow signal partially reveals that the quality is high.

1.8 Speculators and Financial Market Informativeness

In the model, financial market informativeness is captured by the mass of informed speculators α. The term α also is a measure of the aggregate payoffs of speculators given that more entry implies that more speculators are profiting. We now examine more closely how α is affected by the feedback between the two markets.

Corollary 8 *Financial market informativeness α in the feedback game can be larger than, equal to, or lower than α in the benchmark. Specifically,*

- *When the feedback equilibrium is E^{I}, the financial market is strictly less informative than the benchmark equilibrium E^{B} - i.e., $\alpha_I < \alpha_B$.*
- *When the feedback equilibrium is E^{II}, the financial market is strictly more (less) informative than the benchmark equilibrium E^{B} - i.e., $\alpha_{II} > (<)\alpha_B$ - if and only if $Z > (<)\frac{3\mu-2}{2\mu-1}$.*
- *When the feedback equilibrium is $\mathrm{E}^{\mathrm{III}}$, the financial market is strictly more (less) informative than the benchmark equilibrium E^{B} - i.e., $\alpha_{III} > (<)\alpha_B$ - if and only if $\bar{U} > (<)\omega\mu\frac{(2\mu-1)+2(1-\mu)Z}{\mu+(1-\mu)Z}$.*
- *When the feedback equilibrium is E^{IV}, the financial market is strictly less informative than the benchmark equilibrium E^{B} - i.e., $\alpha_{IV} < \alpha_B$.*

The mass of informed speculators (α) is pinned down by the cost and the benefit of acquiring information. The cost is determined by the information acquisition technology. The benefit is determined by the markets (and the feedback therein), and is, thus, the focus of this corollary.

In both the benchmark and the feedback game, the expected trading profitability is decreasing in the mass of informed speculators.[24] As more speculators become informed and trade, the range of uninformative order flows shrinks, decreasing the expected trading profitability for these order flows - this is only partly compensated by an expanding range of partially revealing order flows and a corresponding increase in the expected trading profitability for these order flows.

The feedback between the two markets results in different trading profits at each order flow relative to the benchmark. This changes how much the expected trading profitability decreases for a unit increase in the mass of informed speculators. If the expected trading profitability decreases less (more) for a unit increase in the mass of informed speculators due to the feedback, in equilibrium, more (fewer) speculators will become informed and the financial market will be more (less) informative than in the benchmark.

The expected trading profitability decreases more for a unit increase in the mass of informed speculators in E^{I} or E^{IV} than in the benchmark equilibrium E^{B}. This means that the financial market is unambiguously less informative when the equilibrium is E^{I} or E^{IV}. Specifically, when the equilibrium is E^{I} (where consumer behavior is as given in Table 1.2), (i) for order flows that partially reveal high quality, the speculator makes zero profits; and (ii) for order flows that partially reveal low quality, trading profits are lower than in the benchmark because the product valuation of the marginal consumer (H-type) - the product price - is lower than in the benchmark. Similarly, when the feedback equilibrium is E^{IV} (where consumer behavior is as given in Table 1.2), for order flows that partially reveal either quality, the speculator makes zero profits. Therefore, the trading profits are unambiguously smaller in both E^{I} and E^{IV} than in the benchmark for order flows that partially reveal either quality. As the mass of informed speculators increases and the range of order flows expands

[24]Here, we refer to the parameter range $\omega(2-3\mu) < \bar{U} < \omega\mu$, where the financial market is informative in both the benchmark and the feedback game. Outside this parameter range, the financial market is uninformative in both the benchmark and the feedback game. Note that in Section 1.4, we point to a strategic complementarity where expected trading profitability is increasing in the mass of informed speculators: such strategic complementarity prevents the financial market from being informative, so it never occurs in equilibrium that expected trading profitability is increasing in the mass of informed speculators.

when either quality is partially revealed, the expected trading profits decrease more in both E^{I} and E^{IV} than in the benchmark. Consequently, fewer speculators will become informed.

In contrast, the expected trading profits can decrease more (or less) for a unit increase in the mass of informed speculators in E^{II} or E^{III} than in the benchmark equilibrium E^{B}. This means that the financial market will be less (or more) informative due to the feedback when the feedback equilibrium is E^{II} or E^{III}.

Specifically, when the feedback equilibrium is E^{II} (where consumer behavior is as given in Table 1.2), trading profits are larger than the benchmark for order flows that partially reveal high-quality because the product price is higher than in the benchmark; but trading profits are smaller than in the benchmark for order flows that partially reveal low-quality because the product price is lower than in the benchmark. Adding the two cases, the total difference in trading profits with respect to the benchmark is negative if and only if speculators are sufficiently likely to be present ($Z < \frac{3\mu-2}{2\mu-1}$). This is because when the presence of speculators is more likely, the marginal consumers (H-type) are more sensitive to partially revealing order flows, using them to update their beliefs. When H-type consumers update their beliefs, they update their valuation more when the order flow partially reveals that the quality is low than when it partially reveals that the quality is high.[25] When the total change in trading profits for partially revealing order flows relative to the benchmark is negative (positive), as the mass of informed speculators increases and the range of order flows where quality (high and low) is partially revealed expands, the expected trading profits across all order flows decrease more (less) in E^{II} than in the benchmark. Consequently, fewer (more) speculators will become informed.

[25] In the benchmark, H-type consumers value the product at $\mathbb{E}(U|\theta_i = H) = \omega\mu$. In the feedback game, with an order flow signal that partially reveals that quality is high ($O \in [1-\alpha, 1]$), H-type consumers' updated valuation is $\mathbb{E}(U|\theta_i = H, O) = \omega\frac{\mu}{\mu+(1-\mu)Z}$; with an order flow signal that partially reveals that quality is low ($O \in [-1, -1+\alpha]$), H-type consumers' updated valuation is $\mathbb{E}(U|\theta_i = H, O) = \omega\frac{\mu Z}{\mu Z+(1-\mu)}$. Hence, the positive change in H-type consumers' valuation with respect to the benchmark when the order flow partially reveals that quality is high, $\omega\frac{\mu(1-\mu)(1-Z)}{\mu+(1-\mu)Z}$, is smaller than the negative change in H-type consumers' valuation with respect to the benchmark when the order flow partially reveals that quality is low, $\omega\frac{\mu(1-\mu)(1-Z)}{(1-\mu)+\mu Z}$. Intuitively, repeating the same information has a smaller impact on the posterior belief than updating with different information.

When the feedback equilibrium is E^{III} (where consumer behavior is as given by Table 1.2), trading profits are larger than in the benchmark for order flows that partially reveal that the quality is high, but the speculator makes zero profits (thus smaller than the benchmark) for order flows that partially reveal that the quality is low. Adding the two cases, the total difference in trading profits with respect to the benchmark is positive if and only if the outside option is sufficiently large ($\bar{U} > \omega\mu\frac{(2\mu-1)+2(1-\mu)Z}{\mu+(1-\mu)Z}$). When the outside option is larger, the firm's pricing ability is more constrained, and the trading profits in the benchmark are smaller; since the speculator makes zero profits at order flows that partially reveal that the quality is low in the feedback game, the difference in trading profits with respect to the benchmark is, hence, smaller. In contrast, the increase in trading profits with respect to the benchmark for order flows that partially reveal that quality is high is unaffected by the outside option: when the outside option is larger, the firm's pricing ability is more constrained and the trading profits are smaller in both the benchmark and the feedback game, but the difference in trading profits remain the same. When the outside option is large enough, the increase in trading profits with respect to the benchmark for order flows that partially reveal that quality is high dominates the loss of trading profits with respect to the benchmark for order flows that partially reveal that quality is low. When the total difference in trading profits for partially revealing order flows relative to the benchmark is positive (negative), as the mass of informed speculators increases and the range of order flows where quality (high or low) is partially revealed expands, the expected trading profits (across all order flows) decrease less (more) in E^{III} than in the benchmark. Consequently, more (fewer) speculators will become informed.

1.9 Discussion

We now look at extensions of the model in two dimensions: allowing for ex ante investment by the firm in product quality and allowing for quality to be continuously distributed (rather

than binary).

1.9.1 Product Quality Choice

Keeping the setup for the product market and the financial market the same, we now assume that before both markets take place, the firm can choose the product quality ω at a cost $\frac{1}{2}k\omega^2$, where $k > 0$ is a constant.

It is clear from the previous propositions that in both the benchmark and the feedback game, ω determines which equilibrium occurs. Therefore, the firm can effectively choose its preferred equilibrium by choosing ω. To simplify, we assume that $k > \frac{(1-\mu)(2-3\mu)}{\bar{U}}$ so that when there is no financial market feedback, the firm's optimal choice of ω will never lead to the equilibrium E^{A}. This assumption improves tractability and allows us to focus on the more interesting equilibrium E^{B} (where the financial market is informative).

Specifically, in the benchmark, if $\mu > \frac{10-2\sqrt{3}}{11}$ and $k < \frac{\mu^2}{4\bar{U}}$, the firm's optimal choice of product quality will be $\omega_0 = \frac{\mu}{2k}$, resulting in the equilibrium E^{B}. Otherwise, the firm will optimally choose a product quality of zero ($\omega_0 = 0$) and the equilibrium will be E^{N}.

We now consider the choice of ω when there is financial market feedback. Here, the product quality ω and the mass of informed speculators α can only be jointly (and implicitly) solved for in any equilibrium where the financial market is informative (E^{I}, E^{II}, $\mathrm{E}^{\mathrm{III}}$, or E^{IV}). We prove the following result.

Proposition 9 *Assume $k > \frac{(1-\mu)(2-3\mu)}{\bar{U}}$. In the feedback game:*

- *If $\mu > \frac{10-2\sqrt{3}}{11}$ and $k < \frac{\mu^2}{4\bar{U}}$, there always exists a sufficiently small $\epsilon > 0$ such that the firm is better off choosing $\omega = \omega_0 + \epsilon$ than choosing $\omega_0 = \frac{\mu}{2k}$.*
- *Otherwise, the firm's optimal choice of product quality ω is weakly larger than $\omega_0 = 0$.*

The proof of Proposition 9 is in the online appendix OA.2.

The Proposition demonstrates that the feedback from financial markets can improve product quality. Product improvements happen for sure when consumer information is more pre-

cise ($\mu > \frac{10-2\sqrt{3}}{11}$) and investment is less costly ($k < \frac{\mu^2}{4\overline{U}}$). In the feedback game, the marginal benefit of investing in product quality is larger than in the benchmark. More specifically, increasing ω (i) raises consumer valuations without the presence of financial market information, allowing the firm to unconditionally charge a higher price; and (ii) makes informed trades more profitable and attracts more information acquisition by speculators - resulting in more informative financial markets and higher expected firm profits (see Proposition 4).

In contrast to the above, when consumer information is relatively imprecise ($\mu \leq \frac{10-2\sqrt{3}}{11}$) or when investment in product quality is sufficiently costly ($k \geq \frac{\mu^2}{4\overline{U}}$), the firm will choose a product quality of zero in the benchmark. We show that in some cases, the firm will switch to choosing positive quality (ω strictly greater than 0) once consumers learn about the product quality from the financial market; in other cases, the firm will stick to zero quality.

Generally, when the firm switches from choosing zero product quality in the benchmark to choosing positive quality in the feedback game, the increase will be discrete. This is because small product improvements don't benefit the firm at all. Specifically, investing in small product improvements will only result in E^{N}, where the firm incurs the product improvement costs to earn zero profits - the firm would rather not invest in product quality at all. If the firm wants to earn positive profits, some minimum product quality is required for the equilibrium to not be E^{N}.

1.9.2 Continuous Characteristic

Our model assumes that the product characteristic is binary (either high or low), but in reality, it may sit in a spectrum. In this case, more-varied learning/updating among consumers about product quality could lead to more-varied consumer behavior. It could, thus, be more difficult for the market maker to rationally predict consumer and firm behavior as precisely as informed speculators can. This could produce residual uncertainty in firm value for the speculators to trade on.[26] Recall that under the binary product characteristic setup, the

[26]We care about whether the financial market is informative or uninformative, as this is a key channel behind our key results, such as Proposition 4, Proposition 6, and Corollary 8.

financial market is not informative when consumers' private signals are not precise enough (i.e., $\mu \leq \max\{\frac{\bar{U}}{\omega}, \frac{1}{3}(2 - \frac{\bar{U}}{\omega})\}$).[27] We argue here that this finding - an informative financial market necessitates a minimum precision of consumers' private signals - may be robust to the alternative setup of continuous product characteristic, at least in a special case.

Suppose that the prior is now $\theta \sim \text{Uniform}[0, 1]$ and the truth is $\theta_T \in [0, 1]$. Without loss of generality, the unit mass of consumers I get independent and identically distributed private signals. Given the true quality θ_T, the private signal θ_i follows the probability distribution f_θ, which is a scaled normal distribution $N(\theta_T, \sigma_\theta^2)$ truncated at 0 and 1. For any consumer i, her marginal utility from one unit of the product is $U(\theta) = \omega\theta$ such that $\mathbb{E}(U) = \frac{1}{2}\omega$ under the prior. Other assumptions remain the same as in our main model.

When evaluating the utility from the product, consumer i still updates according to Bayes' rule:

$$\Pr(\theta|\theta_i, O) = \frac{\Pr(\theta_i|\theta)\Pr(O|\theta)\Pr(\theta)}{\int_{\theta\in[0,1]} \Pr(\theta_i|\theta)\Pr(O|\theta)\Pr(\theta)d\theta}.$$

Without fully solving this model, we show that at least in the limit case, where $\sigma_\theta \to \infty$ for $\forall\theta$ (similar to $\mu \to \frac{1}{2}$ in our main model), consumers' information set is no different from the prior, and their behavior is homogeneously driven by the public financial market signal since their posteriors degenerate to:

$$\Pr(\theta|\theta_i, O) = \frac{\Pr(O|\theta)\Pr(\theta)}{\int_{\theta\in[0,1]} \Pr(O|\theta)\Pr(\theta)d\theta}.$$

Hence, the market maker rationally knows consumer (and firm) behavior as precisely as informed speculators do, and as characterized in E^A and E^N, informed speculators do not know π_f any better than the market maker when the order flow signal turns out to be informative. Consequently, the financial market is uninformative when the consumers' private signals are very imprecise.

[27] Because $\max\{\frac{\bar{U}}{\omega}, \frac{1}{3}(2 - \frac{\bar{U}}{\omega})\} \geq \frac{1}{2}$ and $\mu \in (\frac{1}{2}, 1)$, there always exists a small enough μ such that the financial market is uninformative, except in the knife-edge case of $\frac{\bar{U}}{\omega} = \frac{1}{2}$.

1.10 Empirical Implications

We now examine the model's implications for (1) financial market informativeness, and (2) the real effects of financial markets, particularly on consumer behavior and firm pricing.

Financial market informativeness: Financial markets are most informative in the intermediate range of the outside option $\bar{U}$ ($\omega(2-3\mu) < \bar{U} < \omega\mu$). If we interpret the outside option as competitiveness, this implies that moderately competitive industries should have more-informed trading, while highly competitive and uncompetitive industries should have less-informed trading. The literature considers measures of financial market informativeness such as PIN, as in Easley et al. (2010), and ITI, as in Bogousslavsky et al. (2024). However, the relationship between financial market efficiency and consumer behavior is also subject to endogeneity concerns. To instrument for financial market price informativeness, Fang et al. (2009) use decimalization as an exogenous shock to stock liquidity, and, thus, price informativeness; Bharath et al. (2013) used financial crises as exogenous negative shocks to stock liquidity and price informativeness; and Jayaraman and Milbourn (2012) use stock splits to satisfy the exclusion restriction.

Real effects: In the model, a very positive order flow leads to high consumer demand and a high firm price; similarly, a very negative order flow suppresses consumer demand and, thus, the product price.

However, while more-positive order flows at least weakly increase consumer demand, they do not necessarily increase the firm's price because the signal from more-positive order flows makes more consumers willing to consume - the firm may react by lowering the product price to boost the quantity sold.

In the context of product launches, consumer demand and product prices/markups can be measured by microdata at the product level. Alternatively, consumer demand can be proxied by GPS tracking data on individual consumers (Noh et al. (2021)) or by online search volume related to a product (Sun (2017)). To overcome the endogeneity issue between the financial market order flow and real behavior, one can test the impact of order flow on consumer

demand or product price, with order flow instrumented by mutual fund redemptions of stocks, as in Edmans et al. (2012). Similarly, fire sales during financial crises that lead to exogenous underpricing, as in Hau and Lai (2013), can help identify very negative order flows. Another perspective one can take to test the prediction that financial markets affect consumer behavior is to compare consumer behavior variability shortly after product launches in a sample of public firms versus a sample of private firms, in the spirit of Asker et al. (2011) and Mortal and Reisel (2013).

1.11 Conclusion

Consumers are a key driver of firm profitability and, therefore, a major concern of financial markets. However, consumers may also learn from financial markets, creating a feedback loop. In our model, a firm benefits from this feedback loop. It may set a high price to extract surplus from consumers with high valuations. In this case, speculators in the financial market benefit from information acquisition, while consumers lose out from surplus extraction. However, the firm may set a low price and sell to all consumers. This eliminates speculators' information rents, as demand does not depend on quality. The low price benefits consumers with high valuations. It would be interesting to endogenize the competitive dynamics of firms in this context.

A Appendix

A.1 Proof of Proposition 1 (Derivation for the unique equilibrium in the benchmark)

It is clear that $p_m(O) \in [\pi_f(L), \pi_f(H)], \forall O$, so $\alpha \geq 0$.

When $\omega(2-3\mu) < \bar{U} < \omega\mu$, $\pi_f(L) < \pi_f(H)$. Since there will always be an uninformative order flow $\alpha - 1 < O < 1 - \alpha$, $p_m(\alpha - 1 < O < 1 - \alpha) \in (\pi_f(L), \pi_f(H))$. Then it must be true that in any equilibrium, the H-type (informed) speculators profit from buying on their private information and L-type (informed) speculators profit from selling on their private information. Other trading strategies of the informed speculators are strictly dominated. When (1) $\bar{U} \geq \omega\mu$ or (2) $\bar{U} \leq \omega(2-3\mu)$, $\pi_f(L) = \pi_f(H)$. For any order flow O in the financial market, there is no information asymmetry about the firm value between the speculators and the market maker because all consumers act uniformly. No speculator can make trading profits.

When $\omega(2-3\mu) < \bar{U} < \omega\mu$, consumers' purchase decisions in response to their private signals are $d_i^H = 1$ and $d_i^L = 0$. The product price $p_f = \omega\mu - \bar{U}$ in this game is not dependent on the public order flow signal. Hence, independent of the financial market, firm profits are:

$$\pi_f(H) = (\omega\mu - \bar{U})\mu$$

$$\pi_f(L) = (\omega\mu - \bar{U})(1-\mu)$$

Suppose that a mass $\alpha > 0$ of speculators have acquired perfect information about the quality θ. Because trading on their information is the dominant strategy of informed speculators, the market maker updates her beliefs about θ and prices the share according to Bayes' rule, as follows:

Note in Table 4 that when $\theta = H$ ($\theta = L$), $O \in (-1-\alpha, -1)$ ($O \in (1, 1+\alpha)$) is off the equilibrium path. We use the Intuitive Criterion to refine beliefs, which implies that the

Table 4: Market maker's strategy in E^{B}

O	$\Pr(H\|O)$	$\Pr(L\|O)$	$p_m(O)$
$O \in (-1-\alpha, -1)$	0	1	$\pi_f(L)$
$O \in [-1, -1+\alpha]$	$\frac{Z}{Z+1}$	$\frac{1}{Z+1}$	$\frac{Z}{Z+1}\pi_f(H) + \frac{1}{Z+1}\pi_f(L)$
$O \in (-1+\alpha, 1-\alpha)$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}[\pi_f(H) + \pi_f(L)]$
$O \in [1-\alpha, 1]$	$\frac{1}{Z+1}$	$\frac{Z}{Z+1}$	$\frac{1}{Z+1}\pi_f(H) + \frac{Z}{Z+1}\pi_f(L)$
$O \in (1, 1+\alpha)$	1	0	$\pi_f(H)$

quality must be L (H) given the observed order flow.

For uninformed speculators, according to the law of iterated expectations,

$$\mathbb{E}_O[\pi_f - p_m] = \frac{1}{2}[\pi_f(H) + \pi_f(L)] - \frac{1}{2}[\pi_f(H) + \pi_f(L)] = 0.$$

It follows that uninformed speculators cannot profitably trade without acquiring more information than their prior.

Hence, it is straightforward to see that given the market maker's strategy, informed speculators do not deviate from their dominant strategy - the informed trades are either α (when the $\theta = H$) or $-\alpha$ (when the true quality is $\theta = L$).

If $\theta = H$, the expected profits from buying the share are:

$$\begin{aligned}
\mathbb{E}[\pi_f - p_m|\theta = H] &= \frac{1}{2}\alpha\frac{Z}{Z+1}[\pi_f(H) - \pi_f(L)] + (1-\alpha)\frac{1}{2}[\pi_f(H) - \pi_f(L)] \\
&= [\frac{1}{2} - \frac{1}{2(Z+1)}\alpha](\omega\mu - \bar{U})(2\mu - 1) > 0.
\end{aligned}$$

If $\theta = L$, the expected profits from selling the share are:

$$\begin{aligned}
\mathbb{E}[p_m - \pi_f|\theta = L] &= \frac{1}{2}\alpha\frac{Z}{Z+1}[\pi_f(H) - \pi_f(L)] + (1-\alpha)\frac{1}{2}[\pi_f(H) - \pi_f(L)] \\
&= [\frac{1}{2} - \frac{1}{2(Z+1)}\alpha](\omega\mu - \bar{U})(2\mu - 1) > 0.
\end{aligned}$$

Therefore, the expected benefit to speculators becoming informed is:

$$\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = [\frac{1}{2} - \frac{1}{2(Z+1)}\alpha](\omega\mu - \bar{U})(2\mu - 1).$$

In equilibrium, the marginal informed speculator with an information acquisition cost of $c^* \in (0, \bar{c})$ is indifferent between being informed and uninformed; that is:

$$\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = c^*.$$

Because $c^* = \alpha\bar{c}$, the equilibrium mass of informed speculators is:

$$\alpha_B = \frac{1}{\frac{2\bar{c}}{(\omega\mu - \bar{U})(2\mu-1)} + \frac{1}{Z+1}}.$$

When $\bar{U} \leq \omega(2-3\mu)$, $d^H_{i,1} = d^L_{i,1} = 1$. When $\bar{U} \geq \omega\mu$, $d^H_{i,1} = d^L_{i,1} = 0$. In either case, $d^H_{i,1} = d^L_{i,1}$ and the firm value is not dependent on the quality. Hence, both the speculators and the market maker know the firm value. The speculators do not have any incentive to become informed. Uninformed traders do not trade.

The difference between these cases is that when $\bar{U} \leq \omega(2-3\mu)$, the product price will be $p_f = \omega(1-\mu) - \bar{U}$; all consumers consume; and the share price will be

$$p_m(O) = \pi_f(H) = \pi_f(L) = [\omega(1-\mu) - \bar{U}](1-\mu), \forall O,$$

whereas when $\bar{U} \geq \omega\mu$, no consumer consumes; the product price is irrelevant; and the share price will be $p_m(O) = 0, \forall O$.

A.2 Derivation of the unique equilibrium in the feedback game (Proposition 2 and Proposition 3)

This proof covers the derivation of the unique equilibrium in the feedback game, which is described in two Propositions (Proposition 2 and Proposition 3). We will proceed with the proof in two logically distinct parts.

Part 1: Equilibrium strategy of the speculators

In the first part of this section, we derive the equilibrium strategy of the speculators.

Because $q_i^H(O) \geq q_i^L(O)\ \forall O$, $\pi_f(O|H) \geq \pi_f(O|L)$ and $p_m(O) \in [\pi_f(O|L), \pi_f(O|H)]$ for $\forall O$. Hence, if a speculator has information that $\theta = H$, he will never sell; if a speculator has information that $\theta = L$, he will never buy. We first conjecture that informed speculators buy the firm's shares when $\theta = H$ and sell when $\theta = L$, and uninformed speculators do not trade. Then we reason through the consumer behavior, the firm behavior, and the market maker's pricing. Finally, we show that given the market maker's pricing, it is indeed optimal for speculators to follow this conjectured strategy when $\omega(2-3\mu) < \bar{U} < \omega\mu$.

The market maker updates her beliefs about θ as follows:

Table 5: Market maker's posterior beliefs about θ

O	$\Pr(H\|O)$	$\Pr(L\|O)$
$O \in (-1-\alpha, -1)$	0	1
$O \in [-1, -1+\alpha]$	$\frac{Z}{Z+1}$	$\frac{1}{Z+1}$
$O \in (-1+\alpha, 1-\alpha)$	$\frac{1}{2}$	$\frac{1}{2}$
$O \in [1-\alpha, 1]$	$\frac{1}{Z+1}$	$\frac{Z}{Z+1}$
$O \in (1, 1+\alpha)$	1	0

Note in Table 5 that when $\theta = H$ ($\theta = L$), $O \in (-1-\alpha, -1)$ ($O \in (1, 1+\alpha)$) is off the equilibrium path. We pin down beliefs using the Intuitive Criterion, which implies that the quality must be L (H) given the observed order flow.

Because the product quality is fully revealed when $O < -1$ or $O > 1$, the speculators cannot make trading profits. Hence, we examine next the speculators' trading profitability

when $O \in [-1, 1]$. We present the analysis in three steps, showing that: (1) trading profits would be positive when $\omega(2-3\mu) < \bar{U} < \omega\mu$, which incentivizes the speculators to trade; (2) no speculators trade when $\bar{U} \geq \omega\mu$; and (3) no speculators trade when $\bar{U} \leq \omega(2-3\mu)$.

Step 1: speculators trade when $\omega(2-3\mu) < \bar{U} < \omega\mu$

When $O \in [-1, 1]$, the order flow is not fully revealing and the market maker does not perfectly know the product quality. This information asymmetry between the informed speculators and the market maker about product quality becomes an information asymmetry about the firm's value, which incentivizes the speculators to trade, if and only if for a non-empty subset of $O \in [-1, 1]$, the firm sells only to H-type consumers.

Whether the firm sells only to H-type consumers at a given order flow depends on whether doing so maximizes its profits. Because the firm's profits are affected by consumers' outside option, $\bar{U}$, it follows that the information asymmetry between the informed speculators and the market maker about the firm's value is affected by $\bar{U}$.

- If $O \in [-1, -1+\alpha]$ (i.e., the order flow signal partially reveals that the quality is low), then $q_i^H(O) = \frac{\mu Z}{\mu Z+(1-\mu)}, \mathbb{E}(U|\theta_i = H, O) = \omega\frac{\mu Z}{\mu Z+(1-\mu)}$ and $q_i^L(O) = \frac{(1-\mu)Z}{(1-\mu)Z+\mu}, \mathbb{E}(U|\theta_i = L, O) = \omega\frac{(1-\mu)Z}{(1-\mu)Z+\mu}$. The firm will sell only to H-type consumers if and only if:
  - either $\bar{U} \in [\omega\frac{(1-\mu)Z}{(1-\mu)Z+\mu}, \omega\frac{\mu Z}{\mu Z+(1-\mu)})$; or
  - $\bar{U} < \omega\frac{(1-\mu)Z}{(1-\mu)Z+\mu}$ and $\omega\frac{(1-\mu)Z}{(1-\mu)Z+\mu} - \bar{U} < [\omega\frac{\mu Z}{\mu Z+(1-\mu)} - \bar{U}][\frac{Z}{Z+1}\mu + \frac{1}{Z+1}(1-\mu)]$. This can be simplified to $\omega\frac{Z}{(1-\mu)Z+\mu}[\frac{(1-\mu)(Z+1)}{(1-\mu)Z+\mu} - \mu] < \bar{U} < \omega\frac{(1-\mu)Z}{(1-\mu)Z+\mu}$.

  Hence, the firm will sell only to H-type consumers if and only if $\omega\frac{Z}{(1-\mu)Z+\mu}[\frac{(1-\mu)(Z+1)}{(1-\mu)Z+\mu} - \mu] < \bar{U} < \omega\frac{\mu Z}{\mu Z+(1-\mu)}$.
- If $O \in (-1+\alpha, 1-\alpha)$ (i.e., the order flow signal is uninformative about the underlying quality), then $q_i^H(O) = \mu, \mathbb{E}(U|\theta_i = H, O) = \omega\mu$ and $q_i^L(O) = 1-\mu, \mathbb{E}(U|\theta_i = L, O) = \omega(1-\mu)$. Similar to the analysis in the benchmark, the firm will sell only to H-type consumers if and only if $\omega(2-3\mu) < \bar{U} < \omega\mu$.
- If $O \in [1-\alpha, 1]$ (i.e., the order flow signal partially reveals that the quality is high), then $q_i^H(O) = \frac{\mu}{\mu+(1-\mu)Z}, \mathbb{E}(U|\theta_i = H, O) = \omega\frac{\mu}{\mu+(1-\mu)Z}$ and $q_i^L(O) = \frac{1-\mu}{(1-\mu)+\mu Z}, \mathbb{E}(U|\theta_i =$

$L, O) = \omega\frac{1-\mu}{(1-\mu)+\mu Z}$. The firm will sell only to H-type consumers if and only if:

- either $\bar{U} \in [\omega\frac{1-\mu}{(1-\mu)+\mu Z}, \omega\frac{\mu}{\mu+(1-\mu)Z}]$; or
- $\bar{U} < \omega\frac{1-\mu}{(1-\mu)+\mu Z}$ and $\omega\frac{1-\mu}{(1-\mu)+\mu Z} - \bar{U} < [\omega\frac{\mu}{\mu+(1-\mu)Z} - \bar{U}][\frac{1}{Z+1}\mu + \frac{Z}{Z+1}(1-\mu)]$. This can be simplified to $\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu] < \bar{U} < \omega\frac{1-\mu}{(1-\mu)+\mu Z}$.

Hence, the firm will sell only to H-type consumers if and only if $\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu] < \bar{U} < \omega\frac{\mu}{\mu+(1-\mu)Z}$.

Note that $\omega\frac{\mu}{\mu+(1-\mu)Z} > \omega\mu > \omega\frac{\mu Z}{\mu Z+(1-\mu)}$ and $\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu] > \omega(2-3\mu) > \omega\frac{Z}{(1-\mu)Z+\mu}[\frac{(1-\mu)(Z+1)}{(1-\mu)Z+\mu} - \mu]$.

When $\omega(2-3\mu) < \bar{U} < \omega\mu$, only H-type consumers consume for $O \in (-1+\alpha, 1-\alpha)$, implying residual uncertainty in consumer demand and firm profits. Because an order flow $O \in (-1+\alpha, 1-\alpha)$ is uninformative about the underlying quality whether or not informed speculators trade, $p_m(O) \in (\pi_f(O|L), \pi_f(O|H))$. Therefore, the dominant strategy of informed speculators is to trade on their information. Uninformed speculators do not trade.[28]

Step 2: no speculators trade when $\bar{U} \geq \omega\mu$

If (i) $\bar{U} \geq \omega\mu$, and (ii) $\bar{U} \geq \omega\frac{\mu}{\mu+(1-\mu)Z}$ or $\bar{U} \leq \omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu]$, it is clear from the above analysis that there is no residual uncertainty about firm profits for $O \in (-1-\alpha, 1+\alpha)$, implying that speculators cannot make trading profits at any order flow regardless of the true quality. Hence, no speculators have incentives to become informed, and we know that uninformed speculators do not trade.

If $\bar{U} \geq \omega\mu$ and $\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu] < \bar{U} < \omega\frac{\mu}{\mu+(1-\mu)Z}$, it is clear from the above analysis that no consumer consumes for $O \in (-1-\alpha, 1-\alpha)$, implying that speculators who

[28] At any given aggregate order flow O,

$$p_m(O) = \mathbb{E}[\pi_f|O].$$

Hence, for uninformed speculators whose information set is identical to that of the market maker, according to the law of iterated expectations,

$$\mathbb{E}_O[\pi_f - p_m] = \mathbb{E}_O[\mathbb{E}[\pi_f|O] - p_m(O)] = 0.$$

It follows that uninformed speculators cannot profitably trade without more information than the priors.

are informed about $\theta = L$ cannot make trading profits by selling the firm's shares and, thus, are indifferent between selling and not selling the firm's shares. This is true whether or not speculators who are informed about $\theta = H$ buy.[29]

Suppose that speculators who are informed about $\theta = L$ mix between selling and not selling the firm's shares, with probability $\delta_L \in [0, 1]$ of selling the firm's shares; suppose that speculators who are informed about $\theta = H$ buy the firm's shares. Table 6 shows the consumer demand $D(O)$, the product price $p_f(O)$, and the share price $p_m(O)$ in response to the conjectured speculator strategy ("NA" means the product market collapses, so there is no product price).

Table 6: Agent choices when $\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu] < \bar{U} < \omega\frac{\mu}{\mu+(1-\mu)Z}$ and $\bar{U} \geq \omega\mu$

O	$D(O)$	$p_f(O)$	$p_m(O)$
$O \in (-1-\alpha\delta_L, -1)$	0	NA	0
$O \in [-1, -1+\alpha]$	0	NA	0
$O \in (-1+\alpha, 1-\alpha\delta_L)$	0	NA	0
$O \in [1-\alpha\delta_L, 1]$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\frac{\mu}{\mu+(1-\mu)Z} - \bar{U}$	$\omega\frac{\mu}{Z+1} - \frac{\mu+(1-\mu)Z}{Z+1}\bar{U}$
$O \in (1, 1+\alpha)$	1	$\omega - \bar{U}$	$\omega - \bar{U}$

For the informed speculators, the expected profits from buying the share are:

$$\mathbb{E}[p_m - \pi_f|\theta = H] = [\omega\frac{\mu}{\mu+(1-\mu)Z} - \bar{U}]\frac{Z}{Z+1}(2\mu-1)\frac{\alpha\delta_L}{2}.$$

Clearly, $\mathbb{E}[p_m - \pi_f|\theta = H] > 0$ if and only if $\delta_L > 0$ and $\alpha > 0$.

Therefore, the expected benefit to speculators becoming informed is:

$$\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = [\omega\frac{\mu}{\mu+(1-\mu)Z} - \bar{U}]\frac{Z}{Z+1}(2\mu-1)\frac{\alpha\delta_L}{4}.$$

[29]To see this, when $\theta = L$ and informed speculators sell the firm's shares, whether or not the other agents believe that speculators who are informed about $\theta = H$ buy, the order flow can only be in one of the three situations: uninformative; partially revealing that the quality is low; and fully revealing that the quality is low. When $\bar{U} \geq \omega\mu$, none of the three situations will entail residual uncertainty in consumer demand and the firm profits.

Solving the equilibrium condition $\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = c^* = \alpha\bar{c}$,

$$\alpha = 0$$

for any $\delta_L \in [0,1]$. This is because we assume that $\bar{c}$ is sufficiently large, and the speculator with the highest cost does not acquire information: specifically in this case, we assume that $\bar{c} > \frac{1}{4}[\omega\frac{\mu}{\mu+(1-\mu)Z} - \bar{U}]\frac{Z}{Z+1}(2\mu-1)$.

The amount $\alpha = 0$ here reflects unraveling given a sufficiently large $\bar{c}$: because the expected trading profits are increasing in the number of informed speculators, there is a strategic complementarity between speculators to become informed and trade. However, the speculator with the highest information acquisition cost $\bar{c}$ finds acquiring information too costly, and the speculator with the second highest information acquisition cost follows suit, and so on. The amount $\alpha = 0$ means that no speculator chooses to become informed, and we know from the above analysis that uninformed speculators do not trade.

Step 3: no speculators trade when $\bar{U} \leq \omega(2-3\mu)$

If (i) $\bar{U} \leq \omega(2-3\mu)$ and (ii) $\bar{U} \leq \omega\frac{Z}{(1-\mu)Z+\mu}[\frac{(1-\mu)(Z+1)}{(1-\mu)Z+\mu} - \mu]$ or $\bar{U} \geq \omega\frac{\mu Z}{\mu Z+(1-\mu)}$, it is clear from the above analysis that there is no residual uncertainty about firm profits for $O \in (-1-\alpha, 1+\alpha)$, implying that speculators cannot make trading profits at any order flow regardless of the true quality. Hence, no speculators have incentives to become informed, and we know that uninformed speculators do not trade.

If $\bar{U} \leq \omega(2-3\mu)$ and $\omega\frac{Z}{(1-\mu)Z+\mu}[\frac{(1-\mu)(Z+1)}{(1-\mu)Z+\mu} - \mu] < \bar{U} < \omega\frac{\mu Z}{\mu Z+(1-\mu)}$, it is clear from the above analysis that the firm sells to all consumers and all consumers consume for $O \in (-1+\alpha, 1+\alpha)$, implying that speculators who are informed about $\theta = H$ cannot make trading profits by buying the firm's shares and, thus, are indifferent between buying and not buying the firm's shares. This is true whether or not speculators who are informed about $\theta = L$ sell.[30]

[30]To see this, when $\theta = H$ and informed speculators buy the firm's shares, whether the other agents believe speculators who are informed about $\theta = L$ sell or not, the order flow can only be in one of the three situations: uninformative; partially revealing the quality is high; or fully revealing that the quality is high. When $\bar{U} \leq \omega(2-3\mu)$, none of the three situations will entail residual uncertainty in consumer demand and

Suppose that speculators who are informed about $\theta = H$ mix between buying and not buying the firm's shares, with probability $\delta_H \in [0,1]$ of buying the firm's shares; suppose that speculators who are informed about $\theta = L$ sell the firm's shares. Hence, Table 7 shows consumer demand $D(O)$, the product price $p_f(O)$, and the share price $p_m(O)$ in response to this conjectured speculator strategy.

Table 7: Agent choices when $\omega\frac{Z}{(1-\mu)Z+\mu}[\frac{(1-\mu)(Z+1)}{(1-\mu)Z+\mu} - \mu] < \bar{U} < \omega\frac{\mu Z}{\mu Z+(1-\mu)}$ and $\bar{U} \leq \omega(2-3\mu)$

O	$D(O)$	$p_f(O)$	$p_m(O)$
$O \in (-1-\alpha, -1)$	0	NA	0
$O \in [-1, -1+\alpha\delta_H]$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\frac{\mu Z}{\mu Z+(1-\mu)} - \bar{U}$	$\omega\frac{\mu Z}{Z+1} - \frac{\mu Z+(1-\mu)}{Z+1}\bar{U}$
$O \in (-1+\alpha\delta_H, 1-\alpha)$	1	$\omega(1-\mu) - \bar{U}$	$\omega(1-\mu) - \bar{U}$
$O \in [1-\alpha, 1]$	1	$\omega\frac{1-\mu}{(1-\mu)+\mu Z} - \bar{U}$	$\omega\frac{1-\mu}{(1-\mu)+\mu Z} - \bar{U}$
$O \in (1, 1+\alpha\delta_H)$	1	$\omega - \bar{U}$	$\omega - \bar{U}$

For informed speculators, the expected profits from selling the share are:

$$\mathbb{E}[p_m - \pi_f | \theta = L] = [\omega\frac{\mu Z}{\mu Z + (1-\mu)} - \bar{U}]\frac{Z}{Z+1}(2\mu - 1)\frac{\alpha\delta_H}{2}.$$

Clearly, $\mathbb{E}[p_m - \pi_f | \theta = L] > 0$ if and only if $\delta_H > 0$ and $\alpha > 0$.

Therefore, the expected benefit to speculators becoming informed is:

$$\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = [\omega\frac{\mu Z}{\mu Z + (1-\mu)} - \bar{U}]\frac{Z}{Z+1}(2\mu - 1)\frac{\alpha\delta_H}{4}.$$

Solving the equilibrium condition $\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = c^* = \alpha\bar{c}$,

$$\alpha = 0.$$

for any $\delta_H \in [0,1]$. This is because we assume that $\bar{c}$ is sufficiently large and the speculator with the highest cost does not acquire information: specifically in this case, we assume that $\bar{c} > \frac{1}{4}[\omega\frac{\mu Z}{\mu Z+(1-\mu)} - \bar{U}]\frac{Z}{Z+1}(2\mu - 1)$.

the firm's profits.

To sum up the first part of this proof from the above analysis, we have shown that (i) the dominant strategy of informed speculators is to trade on their information when $\omega(2-3\mu) < \bar{U} < \omega\mu$; and (ii) all speculators are uninformed, and uninformed speculators do not trade when $\bar{U} \geq \omega\mu$ or $\bar{U} \leq \omega(2-3\mu)$.

Part 2: Equilibrium consumer behavior, firm behavior, and mass of informed speculators

In the second part of this section, given the equilibrium strategy of speculators derived in the first part, we detail the consumer and firm behavior and solve for the equilibrium mass of informed speculators in each possible equilibrium.

The market maker, the firm and consumers form beliefs about θ given the speculators' trading strategy, which is dependent on $\bar{U}$. When $\omega(2-3\mu) < \bar{U} < \omega\mu$, informed speculators trade on their information. They will certainly make trading profits when the order flow signal is uninformative about the true quality because only H-type consumers will consume, and the market maker faces residual uncertainty about how many H-type consumers there are. However, when the order flow partially reveals the true quality, we have not yet shown whether or not the informed speculators can make trading profits - this depends on how consumers and the firm will behave at these order flows.

In the following, we will discuss consumer and firm behavior - conditional on informative financial markets (i.e., $\omega(2-3\mu) < \bar{U} < \omega\mu$) - in two cases (corresponding to Proposition 2 and Proposition 3). The two cases are separated by a condition on the probability of speculator absence, Z, and differ in terms of the set of possible equilibria.

Case 1 Large Z $(Z > (\frac{1-\mu}{\mu})^2)$

When $\omega\frac{\mu Z}{\mu Z+(1-\mu)} > \omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu]$ - in other words, when $Z > (\frac{1-\mu}{\mu})^2$, consumer and firm behavior for partially revealing order flows depends on three possible ranges of $\bar{U}$: $\omega(2-3\mu) < \bar{U} \leq \omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu]$, $\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu] < \bar{U} < \omega\frac{\mu Z}{\mu Z+(1-\mu)}$, and $\omega\frac{\mu Z}{\mu Z+(1-\mu)} \leq \bar{U} < \omega\mu$.

•

- If $\omega(2-3\mu) < \bar{U} \leq \omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu]$, only H-type consumers will buy when the order flow is uninformative or partially reveals that the quality is low, but all consumers will buy when the order flow partially reveals that the quality is high. This implies that informed speculators are able to make trading profits only when the order flow is uninformative or partially reveals that the quality is low. Table 8 shows the consumer demand $D(O)$, the product price $p_f(O)$, and the share price $p_m(O)$.

Table 8: Equilibrium choices when only H-type consumers buy given $O \in [-1, 1-\alpha)$

O	$D(O)$	$p_f(O)$	$p_m(O)$
$O \in (-1-\alpha, -1)$	0	NA	0
$O \in [-1, -1+\alpha]$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\frac{\mu Z}{\mu Z+(1-\mu)} - \bar{U}$	$\omega\frac{\mu Z}{Z+1} - \frac{\mu Z+(1-\mu)}{Z+1}\bar{U}$
$O \in (-1+\alpha, 1-\alpha)$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\mu - \bar{U}$	$\frac{1}{2}(\omega\mu - \bar{U})$
$O \in [1-\alpha, 1]$	1	$\omega\frac{1-\mu}{(1-\mu)+\mu Z} - \bar{U}$	$\omega\frac{1-\mu}{(1-\mu)+\mu Z} - \bar{U}$
$O \in (1, 1+\alpha)$	1	$\omega - \bar{U}$	$\omega - \bar{U}$

Hence, the expected profits from buying the share are:

$$\mathbb{E}[\pi_f - p_m | \theta = H] = (\omega\mu - \bar{U})(\mu - \frac{1}{2})(1-\alpha).$$

The expected profits from selling the share are:

$$\begin{aligned}\mathbb{E}[\pi_f - p_m | \theta = L] &= [\omega\frac{\mu Z}{\mu Z+(1-\mu)} - \bar{U}]\frac{Z}{Z+1}(2\mu-1)\frac{\alpha}{2} + (\omega\mu - \bar{U})(\mu - \frac{1}{2})(1-\alpha) \\ &= \frac{1}{2}(2\mu-1)\Big\{(\omega\mu - \bar{U}) - \frac{1}{Z+1}\Big[\omega\mu\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)} - \bar{U}\Big]\alpha\Big\}.\end{aligned}$$

Therefore, the expected benefit to speculators becoming informed is:

$$\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = \frac{1}{2}(2\mu-1)\Big\{(\omega\mu-\bar{U}) - \Big[\frac{1}{2}(\omega\mu-\bar{U}) + \frac{1}{2(Z+1)}[\omega\mu\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)} - \bar{U}]\Big]\alpha\Big\}.$$

Solving the equilibrium condition $\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = c^* = \alpha\bar{c}$ yields:

$$\alpha_I = \frac{1}{\frac{2\bar{c}}{(\omega\mu-\bar{U})(2\mu-1)} + \frac{1}{2} + \frac{1}{2(Z+1)}\frac{\omega\mu\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)}-\bar{U}}{\omega\mu-\bar{U}}}. \tag{A.1}$$

Moreover, because $\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)} > 1$, it must be that $\alpha_I < \alpha_B$ - i.e., there are fewer speculators than in the benchmark.

- If $\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu] < \bar{U} < \omega\frac{\mu Z}{\mu Z+(1-\mu)}$, only H-type consumers will buy when the order flow is uninformative or partially reveals either quality. This implies that informed speculators are able to make trading profits as long as the order flow does not fully reveal the true quality. Table 9 shows the consumer demand $D(O)$, the product price $p_f(O)$, and the share price $p_m(O)$ for this case.

Table 9: Equilibrium choices when only H-type consumers buy given $O \in [-1, 1]$

O	$D(O)$	$p_f(O)$	$p_m(O)$
$O \in (-1-\alpha, -1)$	0	NA	0
$O \in [-1, -1+\alpha]$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\frac{\mu Z}{\mu Z+(1-\mu)} - \bar{U}$	$\omega\frac{\mu Z}{Z+1} - \frac{\mu Z+(1-\mu)}{Z+1}\bar{U}$
$O \in (-1+\alpha, 1-\alpha)$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\mu - \bar{U}$	$\frac{1}{2}(\omega\mu - \bar{U})$
$O \in [1-\alpha, 1]$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\frac{\mu}{\mu+(1-\mu)Z} - \bar{U}$	$\omega\frac{\mu}{Z+1} - \frac{\mu+(1-\mu)Z}{Z+1}\bar{U}$
$O \in (1, 1+\alpha)$	1	$\omega - \bar{U}$	$\omega - \bar{U}$

Hence, the expected profits from buying the share are:

$$\begin{aligned}\mathbb{E}[\pi_f - p_m|\theta = H] &= [\omega\frac{\mu}{\mu+(1-\mu)Z} - \bar{U}]\frac{Z}{Z+1}(2\mu-1)\frac{\alpha}{2} + (\omega\mu-\bar{U})(\mu-\frac{1}{2})(1-\alpha) \\ &= \frac{1}{2}(2\mu-1)\Big\{(\omega\mu-\bar{U}) - \frac{1}{Z+1}\Big[\omega\mu\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z} - \bar{U}\Big]\alpha\Big\}.\end{aligned}$$

The expected profits from selling the share are:

$$\begin{aligned}\mathbb{E}[\pi_f - p_m|\theta = L] &= [\omega\frac{\mu Z}{\mu Z+(1-\mu)} - \bar{U}]\frac{Z}{Z+1}(2\mu-1)\frac{\alpha}{2} + (\omega\mu-\bar{U})(\mu-\frac{1}{2})(1-\alpha) \\ &= \frac{1}{2}(2\mu-1)\Big\{(\omega\mu-\bar{U}) - \frac{1}{Z+1}\Big[\omega\mu\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)} - \bar{U}\Big]\alpha\Big\}.\end{aligned}$$

Therefore, the expected benefit to speculators becoming informed is:

$$\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = \frac{1}{2}(2\mu-1)\Big\{(\omega\mu-\bar{U})-\frac{1}{Z+1}\Big[\frac{1}{2}\omega\mu[\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z}+\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)}]-\bar{U}\Big]\alpha\Big\}.$$

Solving the equilibrium condition $\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = c^* = \alpha\bar{c}$ yields:

$$\alpha_{II} = \frac{1}{\frac{2\bar{c}}{(\omega\mu-\bar{U})(2\mu-1)} + \frac{1}{Z+1}\frac{\frac{1}{2}\omega\mu\left[\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z}+\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)}\right]-\bar{U}}{\omega\mu-\bar{U}}}. \tag{A.2}$$

Moreover, it can be shown that $\frac{1}{2}\left[\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z}+\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)}\right] < 1$ if and only if $Z > \frac{3\mu-2}{2\mu-1}$. Hence, $\alpha_{II} > \alpha_B$ iff $Z > \frac{3\mu-2}{2\mu-1}$.

- If $\omega\frac{\mu Z}{\mu Z+(1-\mu)} \leq \bar{U} < \omega\mu$, only H-type consumers will buy when the order flow is uninformative or partially reveals that the quality is high, but no consumer will buy when the order flow partially reveals that the quality is low. This implies that informed speculators are able to make trading profits only when the order flow is uninformative or partially reveals that the quality is high. Table 10 shows the consumer demand $D(O)$, the product price $p_f(O)$, and the share price $p_m(O)$ corresponding to this situation.

Table 10: Equilibrium choices when only H-type consumers buy given $O \in (-1+\alpha, 1]$

O	$D(O)$	$p_f(O)$	$p_m(O)$
$O \in (-1-\alpha, -1)$	0	NA	0
$O \in [-1, -1+\alpha]$	0	NA	0
$O \in (-1+\alpha, 1-\alpha)$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\mu - \bar{U}$	$\frac{1}{2}(\omega\mu - \bar{U})$
$O \in [1-\alpha, 1]$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\frac{\mu}{\mu+(1-\mu)Z} - \bar{U}$	$\omega\frac{\mu}{Z+1} - \frac{\mu+(1-\mu)Z}{Z+1}\bar{U}$
$O \in (1, 1+\alpha)$	1	$\omega - \bar{U}$	$\omega - \bar{U}$

Hence, the expected profits from buying the share are:

$$\begin{aligned}\mathbb{E}[\pi_f - p_m|\theta = H] &= [\omega\frac{\mu}{\mu+(1-\mu)Z} - \bar{U}]\frac{Z}{Z+1}(2\mu-1)\frac{\alpha}{2} + (\omega\mu-\bar{U})(\mu-\frac{1}{2})(1-\alpha) \\ &= \frac{1}{2}(2\mu-1)\Big\{(\omega\mu-\bar{U}) - \frac{1}{Z+1}\Big[\omega\mu\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z} - \bar{U}\Big]\alpha\Big\}.\end{aligned}$$

The expected profits from selling the share are:

$$\mathbb{E}[\pi_f - p_m|\theta = L] = (\omega\mu - \bar{U})(\mu - \frac{1}{2})(1-\alpha).$$

Therefore, the expected benefit to speculators becoming informed is:

$$\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = \frac{1}{2}(2\mu-1)\Big\{(\omega\mu-\bar{U}) - \Big[\frac{1}{2}(\omega\mu-\bar{U}) + \frac{1}{2(Z+1)}[\omega\mu\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z} - \bar{U}]\Big]\alpha\Big\}.$$

Solving the equilibrium condition $\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = c^* = \alpha\bar{c}$ yields:

$$\alpha_{III} = \frac{1}{\frac{2\bar{c}}{(\omega\mu-\bar{U})(2\mu-1)} + \frac{1}{2} + \frac{1}{2(Z+1)}\frac{\omega\mu\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z} - \bar{U}}{\omega\mu-\bar{U}}}. \tag{A.3}$$

Moreover, it can be shown that $\alpha_{III} > \alpha_B$ iff $\bar{U} > \omega\mu\frac{(2\mu-1)+2(1-\mu)Z}{\mu+(1-\mu)Z}$.[31]

Case 2 Small Z ($Z \leq (\frac{1-\mu}{\mu})^2$)

When $\omega\frac{\mu Z}{\mu Z+(1-\mu)} \leq \omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu]$ - in other words, when $Z \leq (\frac{1-\mu}{\mu})^2$, consumer and firm behavior for partially revealing order flows depends on three possible ranges of $\bar{U}$: $\omega(2-3\mu) < \bar{U} < \max\{\omega\frac{\mu Z}{\mu Z+(1-\mu)}, \omega(2-3\mu)\}$, $\max\{\omega\frac{\mu Z}{\mu Z+(1-\mu)}, \omega(2-3\mu)\} \leq \bar{U} \leq \min\{\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu], \omega\mu\}$, and $\min\{\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu], \omega\mu\} < \bar{U} < \omega\mu$.

- If $\omega(2-3\mu) < \bar{U} < \max\{\omega\frac{\mu Z}{\mu Z+(1-\mu)}, \omega(2-3\mu)\}$, consumer and firm behavior is identical to when $\omega(2-3\mu) < \bar{U} \leq \omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu]$ under Case 1. Hence, the equilibrium is E^{I}.
- If $\max\{\omega\frac{\mu Z}{\mu Z+(1-\mu)}, \omega(2-3\mu)\} \leq \bar{U} \leq \min\{\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu], \omega\mu\}$, only H-type consumers will buy only when the order flow is uninformative. This implies that informed speculators are able to make trading profits only when the order flow is uninformative. Table 11 shows the consumer demand $D(O)$, the product price $p_f(O)$, and the share price $p_m(O)$ in this case.

[31] Furthermore, it can be shown that $\omega\frac{\mu Z}{\mu Z+(1-\mu)} < \omega\mu\frac{(2\mu-1)+2(1-\mu)Z}{\mu+(1-\mu)Z} < \omega\mu$.

Table 11: Equilibrium choices when only H-type consumers buy given $O \in (-1+\alpha, 1-\alpha)$

O	$D(O)$	$p_f(O)$	$p_m(O)$
$O \in (-1-\alpha, -1)$	0	NA	0
$O \in [-1, -1+\alpha]$	0	NA	0
$O \in (-1+\alpha, 1-\alpha)$	μ if $\theta = H$; $1-\mu$ if $\theta = L$	$\omega\mu - \bar{U}$	$\frac{1}{2}(\omega\mu - \bar{U})$
$O \in [1-\alpha, 1]$	1	$\omega\frac{1-\mu}{(1-\mu)+\mu Z} - \bar{U}$	$\omega\frac{1-\mu}{(1-\mu)+\mu Z} - \bar{U}$
$O \in (1, 1+\alpha)$	1	$\omega - \bar{U}$	$\omega - \bar{U}$

Hence, the expected profits from buying the share are:

$$\mathbb{E}[\pi_f - p_m | \theta = H] = (\omega\mu - \bar{U})(\mu - \frac{1}{2})(1-\alpha).$$

The expected profits from selling the share are:

$$\mathbb{E}[\pi_f - p_m | \theta = L] = (\omega\mu - \bar{U})(\mu - \frac{1}{2})(1-\alpha).$$

Therefore, the expected benefit to speculators becoming informed is:

$$\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = (\omega\mu - \bar{U})(\mu - \frac{1}{2})(1-\alpha).$$

Solving the equilibrium condition $\mathbb{E}\Big[\mathbb{E}[\pi_s|\theta]\Big] = c^* = \alpha\bar{c}$ yields:

$$\alpha_{IV} = \frac{1}{\frac{2\bar{c}}{(\omega\mu-\bar{U})(2\mu-1)} + 1}. \tag{A.4}$$

Moreover, it is clear that $\alpha_{IV} < \alpha_B$ - i.e., there are fewer speculators than in the benchmark.

- If $\min\{\omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu], \omega\mu\} < \bar{U} < \omega\mu$, consumer and firm behavior is identical to when $\omega\frac{\mu Z}{\mu Z+(1-\mu)} \leq \bar{U} < \omega\mu$ under Case 1. Hence, the equilibrium is $\mathrm{E}^{\mathrm{III}}$.

Lastly, whether $Z > (\frac{1-\mu}{\mu})^2$ (Case 1) or $Z \leq (\frac{1-\mu}{\mu})^2$ (Case 2), no speculator will acquire information about the true quality, and, thus, no speculator will trade if and only if $\bar{U} \leq$

$\omega(2-3\mu)$ or $\bar{U} \geq \omega\mu$. In this case, $\alpha = 0$. Because no information will be impounded by speculators into the financial market, consumers and the firm will behave as in the benchmark: when $\bar{U} \leq \omega(2-3\mu)$, all consumers pay the product price $p_f = \omega(1-\mu) - \bar{U}$ to consume; when $\bar{U} \geq \omega\mu$, the product market collapses, as there is no consumption.

From the above analysis, for any given set of parameters $(Z, \mu, \bar{U}, \omega)$, only one of the aforementioned equilibria will realize, so the feedback equilibrium is unique.

Corollary 8 follows directly from the calculations in this appendix.

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Online Appendix for "Feedback Between the Financial Market and the Product Market"

By Ken Deng and Joel Shapiro

This Online Appendix contains the proofs that are skipped in the Appendix.

OA.1 Payoffs

We now detail the calculations for the payoffs of different agents in both the benchmark and the feedback game: consumer surplus CS, the firm's profits π_f and speculators profits π_s. We also include the comparative statics of these welfare measures with respect to the outside option $\bar{U}$. Propositions 4, 5, 6, and 7 all follow directly from the calculations in this appendix.

Benchmark

When $\omega(2-3\mu) < \bar{U} < \omega\mu$, the unique equilibrium is E^{B}.

Ex ante consumer surplus is measured by consumers' aggregate net-of-price utility:

$$\mathbb{E}[CS] = \bar{U}.$$

Ex ante firm profit is:

$$\mathbb{E}[\pi_f] = \frac{1}{2}p_f = \frac{1}{2}(\omega\mu - \bar{U}).$$

Ex ante aggregate trading profits, net of information acquisition costs, for speculators (if present) are:

$$\mathbb{E}[\pi_s] = \alpha_B(\alpha_B\bar{c}) - \int_0^{\alpha_B\bar{c}} \frac{1}{\bar{c}} j dj = \frac{1}{2}\bar{c}(\alpha_B)^2 = \frac{1}{2}\bar{c}\Big[\frac{1}{\frac{2\bar{c}}{(\omega\mu-\bar{U})(2\mu-1)} + \frac{1}{Z+1}}\Big]^2.$$

In E^{B}, consumers surplus increases in $\bar{U}$; firm profits decrease in $\bar{U}$; trading profits for

speculators decrease in $\bar{U}$ because α_B decreases in $\bar{U}$. Specifically,

$$\frac{d\mathbb{E}[CS]}{d\bar{U}} = 1 \tag{OA.1}$$

$$\frac{d\mathbb{E}[\pi_f]}{d\bar{U}} = -\frac{1}{2} \tag{OA.2}$$

We can later compare these comparative statics with those in the feedback game.

When $\bar{U} \leq \omega(2 - 3\mu)$, the unique equilibrium is E^{A}.

Ex ante consumer surplus is measured by consumers' aggregate net-of-price utility:

$$\mathbb{E}[CS] = \frac{1}{2}\bar{U} + \frac{1}{2}[\omega(2\mu - 1) + \bar{U}] = \bar{U} + \frac{1}{2}\omega(2\mu - 1).$$

Ex ante firm profit is:

$$\mathbb{E}[\pi_f] = p_f = \omega(1 - \mu) - \bar{U}.$$

Ex ante aggregate trading profits, net of information acquisition costs, for speculators (if present) are:

$$\mathbb{E}[\pi_s] = 0.$$

In E^{A}, speculators do not care about $\bar{U}$; firm profits decrease in $\bar{U}$; consumer surplus increases in $\bar{U}$.

When $\bar{U} \geq \omega\mu$, the unique equilibrium is E^{N}.

Ex ante consumer surplus is measured by consumers' aggregate net-of-price utility:

$$\mathbb{E}[CS] = \bar{U}.$$

Ex ante firm profit is:

$$\mathbb{E}[\pi_f] = 0.$$

Ex ante aggregate trading profits, net of information acquisition costs, for speculators (if

present) are:

$$\mathbb{E}[\pi_s] = 0.$$

Feedback game

When the unique equilibrium is E^{I}:

Ex ante consumer surplus is measured by consumers' aggregate net-of-price utility:

$$\mathbb{E}[CS] = \bar{U} + \frac{\omega}{4}\Big[\mu - (1-\mu)\frac{\mu + (1-\mu)Z}{(1-\mu)+\mu Z}\Big]\alpha_I > \bar{U}.$$

Ex ante firm profit is:

$$\mathbb{E}[\pi_f] = \frac{1}{2}(\omega\mu - \bar{U}) + \frac{1}{4}(1-Z)\omega(1-\mu)\alpha_I + \frac{1}{4}(1+Z)\Big[\omega\Big(\frac{1-\mu}{(1-\mu)+\mu Z} - \frac{\mu}{Z+1}\Big) - \bar{U}\frac{(1-\mu)+\mu Z}{Z+1}\Big]\alpha_I.$$

Ex ante aggregate trading profits, net of information acquisition costs, for speculators (if present) are:

$$\mathbb{E}[\pi_s] = \frac{1}{2}\bar{c}\alpha_I^2.$$

In order to see how these change with $\bar{U}$, we calculate that:

$$\frac{d\alpha_I}{d\bar{U}} = -\alpha_I^2\Big\{\frac{2(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}\bar{c} + \frac{\omega\mu}{2(Z+1)(\omega\mu-\bar{U})^2}\Big[\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)} - 1\Big]\Big\} < 0.$$

Hence, trading profits for speculators decrease in $\bar{U}$ because we have shown that α_I decreases in $\bar{U}$.

Then,

$$\frac{d\mathbb{E}[CS]}{d\bar{U}} = 1 + \frac{\omega}{4}\Big[\mu - (1-\mu)\frac{\mu+(1-\mu)Z}{(1-\mu)+\mu Z}\Big]\frac{d\alpha_I}{d\bar{U}} < 1.$$

Hence, comparing to equation OA.1, the consumer surplus is less sensitive to the outside option in E^{I} than in the benchmark equilibrium E^{B}.

Consider the change in firm profits with respect to $\bar{U}$:

$$\frac{d\mathbb{E}[\pi_f]}{d\bar{U}} = -\frac{1}{2} + \frac{1}{4}\Big\{(1-Z)\omega(1-\mu) + (1+Z)\Big[\omega\Big(\frac{1-\mu}{(1-\mu)+\mu Z} - \frac{\mu}{Z+1}\Big) - \bar{U}\frac{(1-\mu)+\mu Z}{Z+1}\Big]\Big\}\frac{d\alpha_I}{d\bar{U}}$$
$$-\frac{1}{4}[(1-\mu)+\mu Z]\alpha_I < -\frac{1}{2}.$$

Hence, comparing to equation OA.2, the firm's profits are more sensitive to $\bar{U}$ in E^{I} than in the benchmark equilibrium E^{B}.

When the unique equilibrium is E^{II}:

Ex ante consumer surplus is measured by consumers' aggregate net-of-price utility:

$$\mathbb{E}[CS] = \bar{U}.$$

Ex ante firm profit is:

$$\mathbb{E}[\pi_f] = \frac{1}{2}(\omega\mu - \bar{U}) + \frac{1}{4}(1-Z)\omega(1-\mu)\alpha_{II}.$$

Ex ante aggregate trading profits, net of information acquisition costs, for speculators (if present) are:

$$\mathbb{E}[\pi_s] = \frac{1}{2}\bar{c}\alpha_{II}^2.$$

In order to see how these change with $\bar{U}$, we calculate that:

$$\frac{d\alpha_{II}}{d\bar{U}} = -\alpha_{II}^2\Big\{\frac{2(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}\bar{c} - \frac{\omega\mu}{(Z+1)(\omega\mu-\bar{U})^2}\Big[1-\frac{1}{2}\Big(\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z} + \frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)}\Big)\Big]\Big\}.$$

Because $\bar{c}$ is prohibitively large - specifically in this case, we assume that $\bar{c} > \frac{\omega\mu(2\mu-1)}{2(1+Z)}\Big[1 - \frac{1}{2}\Big(\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z} + \frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)}\Big)\Big]$, it follows that $\frac{d\alpha_{II}}{d\bar{U}} < 0$. Hence, trading profits for speculators decrease in $\bar{U}$ because we have shown that α_{II} decreases in $\bar{U}$.

Consider how firm profits change with $\bar{U}$:

$$\frac{d\mathbb{E}[\pi_f]}{d\bar{U}} = -\frac{1}{2} + \frac{1}{4}(1-Z)\omega(1-\mu)\frac{d\alpha_{II}}{d\bar{U}} < -\frac{1}{2}.$$

Hence, comparing to equation OA.2, the firm's profits are more sensitive to $\bar{U}$ in E^{II} than in the benchmark equilibrium E^{B}.

When the unique equilibrium is $\mathrm{E}^{\mathrm{III}}$:

Ex ante consumer surplus is measured by consumers' aggregate net-of-price utility:

$$\mathbb{E}[CS] = \bar{U}.$$

Ex ante firm profit is:

$$\mathbb{E}[\pi_f] = \frac{1}{2}(\omega\mu - \bar{U}) + \frac{1}{4}(1-Z)\omega(1-\mu)\alpha_{III} - \frac{1}{4}(1+Z)\Big[\omega\frac{\mu Z}{Z+1} - \bar{U}\frac{\mu Z + (1-\mu)}{Z+1}\Big]\alpha_{III}.$$

Note that because $\bar{U} \geq \omega\frac{\mu Z}{\mu Z+(1-\mu)}$ in $\mathrm{E}^{\mathrm{III}}$, $\mathbb{E}[\pi_f] > \frac{1}{2}(\omega\mu - \bar{U})$.

Ex ante aggregate trading profits, net of information acquisition costs, for speculators (if present) are:

$$\mathbb{E}[\pi_s] = \frac{1}{2}\bar{c}\alpha_{III}^2.$$

Now consider the comparative statics with respect to $\bar{U}$. It is useful to calculate how α_{III} changes with $\bar{U}$:

$$\frac{d\alpha_{III}}{d\bar{U}} = -\alpha_{III}^2\Big\{\frac{2(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}\bar{c} - \frac{\omega\mu}{2(Z+1)(\omega\mu-\bar{U})^2}\Big[1 - \frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z}\Big]\Big\}.$$

Because $\bar{c}$ is prohibitively large - specifically in this case, we assume htat $\bar{c} > \frac{\omega\mu(2\mu-1)}{4(1+Z)}\Big[1 - \frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z}\Big]$, it follows that $\frac{d\alpha_{III}}{d\bar{U}} < 0$. Hence, trading profits for speculators decrease in $\bar{U}$ because we have shown that α_{III} decreases in $\bar{U}$.

Looking at how firm profits change with $\bar{U}$:

$$\frac{d\mathbb{E}[\pi_f]}{d\bar{U}} = -\frac{1}{2}+\frac{1}{4}\Big\{(1-Z)\omega(1-\mu)-(1+Z)\Big[\omega\frac{\mu Z}{Z+1}-\bar{U}\frac{\mu Z+(1-\mu)}{Z+1}\Big]\Big\}\frac{d\alpha_{III}}{d\bar{U}}+\frac{1}{4}[\mu Z+(1-\mu)]\alpha_{III}.$$

We can show that $\frac{1}{4}\Big\{(1-Z)\omega(1-\mu)-(1+Z)\Big[\omega\frac{\mu Z}{Z+1}-\bar{U}\frac{\mu Z+(1-\mu)}{Z+1}\Big]\Big\}\frac{d\alpha_{III}}{d\bar{U}}+\frac{1}{4}[\mu Z+(1-\mu)]\alpha_{III}<0$ when $\bar{c}$ is prohibitively large. Plugging in the expression of $\frac{d\alpha_{III}}{d\bar{U}}$, this is equivalent to:

$$\frac{1}{4}[\mu Z+(1-\mu)]\frac{1}{\alpha_{III}} <\frac{1}{4}\Big\{(1-Z)\omega(1-\mu)-(1+Z)\Big[\omega\frac{\mu Z}{Z+1}-\bar{U}\frac{\mu Z+(1-\mu)}{Z+1}\Big]\Big\}\times \Big\{\frac{2(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}\bar{c}-\frac{\omega\mu}{2(Z+1)(\omega\mu-\bar{U})^2}\Big[1-\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z}\Big]\Big\}.$$

Plugging in the expression of α_{III} (equation A.3) to the LHS, it is clear that both the LHS and the RHS of the above inequality are linear in $\bar{c}$. If the coefficient in front of $\bar{c}$ on the LHS is smaller than the coefficient in front of $\bar{c}$ on the RHS, it follows that when $\bar{c}$ is prohibitively large - specifically in this case, we assume that $\bar{c}$ is larger than a threshold at which the LHS is equal to the RHS, the above inequality holds. The comparison of coefficients in front of $\bar{c}$ shows that the LHS coefficient is indeed smaller than the RHS coefficient:

$$\frac{1}{4}[\mu Z+(1-\mu)]\frac{2}{(\omega\mu-\bar{U})(2\mu-1)} <\frac{1}{4}\Big\{(1-Z)\omega(1-\mu)-(1+Z)\Big[\omega\frac{\mu Z}{Z+1}-\bar{U}\frac{\mu Z+(1-\mu)}{Z+1}\Big]\Big\} \frac{2(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}.$$

To see this, the above inequality can be further simplified:

$$\begin{aligned} & \frac{1}{4}[\mu Z+(1-\mu)](\omega\mu-\bar{U}) < \frac{1}{4}\Big\{\omega(1-Z-\mu)+\bar{U}[\mu Z+(1-\mu)]\Big\} \\ \Leftrightarrow \quad & \frac{1}{4}\omega[-(1-Z)\mu^2+2\mu-(1-Z)] < \frac{1}{2}\bar{U}[\mu Z+(1-\mu)]. \end{aligned}$$

The last inequality clearly holds because in $\mathrm{E}^{\mathrm{III}}$, $\bar{U}\geq\omega\frac{\mu Z}{\mu Z+(1-\mu)}>\frac{1}{2}\omega\frac{-(1-Z)\mu^2+2\mu-(1-Z)}{\mu Z+(1-\mu)}$. Hence, $\frac{d\mathbb{E}[\pi_f]}{d\bar{U}}<-\frac{1}{2}$. Comparing to equation OA.2, the firm's profits are more sensitive to

$\bar{U}$ in $\mathrm{E}^{\mathrm{III}}$ than in the benchmark equilibrium E^{B}.

When the unique equilibrium is E^{IV}:

Ex ante consumer surplus is measured by consumers' aggregate net-of-price utility:

$$\mathbb{E}[CS] = \bar{U} + \frac{\omega}{4}\Big[\mu - (1-\mu)\frac{\mu + (1-\mu)Z}{(1-\mu)+\mu Z}\Big]\alpha_{IV} > \bar{U}.$$

Ex ante firm profit is:

$$\mathbb{E}[\pi_f] = \frac{1}{2}(\omega\mu - \bar{U}) + \frac{1}{4}(1-Z)\omega(1-\mu)\alpha_{IV} + \frac{1}{4}(1+Z)\omega\Big[\frac{1-\mu}{(1-\mu)+\mu Z} - \mu\Big]\alpha_{IV}.$$

Note that because $Z \leq (\frac{1-\mu}{\mu})^2$ in E^{IV}, $\mathbb{E}[\pi_f] > \frac{1}{2}(\omega\mu - \bar{U})$.

Ex ante aggregate trading profits, net of information acquisition costs, for speculators (if present) are:

$$\mathbb{E}[\pi_s] = \frac{1}{2}\bar{c}\alpha_{IV}^2.$$

Now consider the comparative statics with respect to $\bar{U}$. It is useful to calculate how α_{IV} changes with $\bar{U}$:

$$\frac{d\alpha_{IV}}{d\bar{U}} = -\alpha_{IV}^2 \frac{2(2\mu-1)}{[(\omega\mu - \bar{U})(2\mu-1)]^2}\bar{c} < 0.$$

Hence, trading profits for speculators decrease in $\bar{U}$.

Looking at consumer surplus, we find:

$$\frac{d\mathbb{E}[CS]}{d\bar{U}} = 1 + \frac{\omega}{4}\Big[\mu - (1-\mu)\frac{\mu + (1-\mu)Z}{(1-\mu)+\mu Z}\Big]\frac{d\alpha_{IV}}{d\bar{U}} < 1.$$

Hence, comparing to equation OA.1, the consumer surplus is less sensitive to the $\bar{U}$ in E^{IV} than in the benchmark equilibrium E^{B}.

The change in firm profits with respect to $\bar{U}$ is:

$$\frac{d\mathbb{E}[\pi_f]}{d\bar{U}} = -\frac{1}{2} + \frac{1}{4}\Big\{(1-Z)\omega(1-\mu) + (1+Z)\omega\Big[\frac{1-\mu}{(1-\mu)+\mu Z} - \mu\Big]\Big\}\frac{d\alpha_{IV}}{d\bar{U}} < -\frac{1}{2}.$$

Hence, comparing to equation OA.2, the firm's profits are more sensitive to $\bar{U}$ in E^{IV} than in the benchmark equilibrium E^{B}.

When the unique equilibrium is E^{A} or E^{N}, the consumer surplus, the firm profit, and the trading profits are identical to those in the benchmark.

OA.2 Proof of Proposition 9 (Endogenous product quality)

Quality choice in the benchmark equilibrium

We analyze two cases: $\frac{1}{2} < \mu < \frac{2}{3}$ and $\mu \geq \frac{2}{3}$.

Case 1: $\frac{1}{2} < \mu < \frac{2}{3}$.

If the firm chooses $\omega \leq \frac{\bar{U}}{\mu}$, the benchmark equilibrium will be E^{N} and the firm's profits will be zero. Hence, the local maximum of the firm's expected payoff (net of the cost) conditional on $\omega \leq \frac{\bar{U}}{\mu}$ is achieved at $\omega = 0$: the firm avoids any cost, and its expected payoff is zero.

If the firm chooses $\omega \geq \frac{\bar{U}}{2-3\mu}$, the benchmark equilibrium will be E^{A}. The firm maximizes $\mathbb{E}[\pi_f] - \frac{1}{2}k\omega^2 = \omega(1-\mu) - \bar{U} - \frac{1}{2}k\omega^2$. Hence, the first-order condition with respect to ω is:

$$(1-\mu) - k\omega = 0.$$

Because of the assumption $k > \frac{(1-\mu)(2-3\mu)}{\bar{U}}$ (i.e., $\frac{1-\mu}{k} < \frac{\bar{U}}{2-3\mu}$), the local maximum conditional on $\omega \geq \frac{\bar{U}}{2-3\mu}$ is achieved at $\omega = \frac{\bar{U}}{2-3\mu}$.

If the firm chooses $\frac{\bar{U}}{\mu} < \omega < \frac{\bar{U}}{2-3\mu}$, the benchmark equilibrium will be E^{B}. The firm maximizes $\mathbb{E}[\pi_f] - \frac{1}{2}k\omega^2 = \frac{1}{2}(\omega\mu - \bar{U}) - \frac{1}{2}k\omega^2$. Hence, the first-order condition with respect to ω is:

$$\frac{1}{2}\mu - k\omega = 0.$$

The assumption $k > \frac{(1-\mu)(2-3\mu)}{\bar{U}}$ implies that $k > \frac{\mu(2-3\mu)}{2\bar{U}}$ (i.e., $\frac{\mu}{2k} < \frac{\bar{U}}{2-3\mu}$). Hence, if $k < \frac{\mu^2}{2\bar{U}}$ (i.e., $\frac{\mu}{2k} > \frac{\bar{U}}{\mu}$), then the local maximum conditional on $\frac{\bar{U}}{\mu} < \omega < \frac{\bar{U}}{2-3\mu}$ is achieved at $\omega = \frac{\mu}{2k}$, which gives a higher expected payoff (net of the cost) than choosing $\omega = \frac{\bar{U}}{2-3\mu}$. If $k \geq \frac{\mu^2}{2\bar{U}}$,

then the local maximum conditional on $\frac{\bar{U}}{\mu} < \omega < \frac{\bar{U}}{2-3\mu}$ is achieved at $\omega = \frac{\bar{U}}{\mu}$, which renders a negative expected payoff (net of the cost).

Note that the firm's expected payoff is continuous in ω for its entire support ($\omega \geq 0$), despite the fact that the equilibrium may switch with ω. To see this:

- Equilibrium switches from E^{N} to E^{B} as ω crosses $\frac{\bar{U}}{\mu}$. Plugging $\omega = \frac{\bar{U}}{\mu}$ into the firm's expected payoff under E^{N}, we have $\mathbb{E}[\pi_f] - \frac{1}{2}k\omega^2 = 0 - \frac{1}{2}k\big(\frac{\bar{U}}{\mu}\big)^2 = -\frac{1}{2}k\big(\frac{\bar{U}}{\mu}\big)^2$. Plugging $\omega = \frac{\bar{U}}{\mu}$ into the firm's expected payoff under E^{B}, we have $\mathbb{E}[\pi_f] - \frac{1}{2}k\omega^2 = \frac{1}{2}(\omega\mu - \bar{U}) - \frac{1}{2}k\omega^2 = 0 - \frac{1}{2}k\big(\frac{\bar{U}}{\mu}\big)^2 = -\frac{1}{2}k\big(\frac{\bar{U}}{\mu}\big)^2$. Hence, there is no discontinuity in the firm's expected payoff at $\omega = \frac{\bar{U}}{\mu}$.
- Equilibrium switches from E^{B} to E^{A} as ω crosses $\frac{\bar{U}}{2-3\mu}$. Plugging $\omega = \frac{\bar{U}}{2-3\mu}$ into the firm's expected payoff under E^{B}, we have $\mathbb{E}[\pi_f] - \frac{1}{2}k\omega^2 = \frac{1}{2}(\omega\mu - \bar{U}) - \frac{1}{2}k\omega^2 = \frac{2\mu-1}{2-3\mu}\bar{U} - \frac{1}{2}k\big(\frac{\bar{U}}{2-3\mu}\big)^2$. Plugging $\omega = \frac{\bar{U}}{2-3\mu}$ into the firm's expected payoff under E^{A}, we have $\mathbb{E}[\pi_f] - \frac{1}{2}k\omega^2 = \omega(1-\mu) - \bar{U} - \frac{1}{2}k\omega^2 = \frac{2\mu-1}{2-3\mu}\bar{U} - \frac{1}{2}k\big(\frac{\bar{U}}{2-3\mu}\big)^2$. Hence, there is no discontinuity in the firm's expected payoff at $\omega = \frac{\bar{U}}{2-3\mu}$.

The firm's optimal choice, which achieves the global maximum, is pinned down by comparison among the local maxima described above. Notice that: (i) whether $k < \frac{\mu^2}{2\bar{U}}$ or $k \geq \frac{\mu^2}{2\bar{U}}$, choosing $\omega = \frac{\bar{U}}{2-3\mu}$ is dominated by choosing $\omega = \frac{\mu}{2k}$ or choosing $\omega = \frac{\bar{U}}{\mu}$; (ii) choosing $\omega = \frac{\bar{U}}{\mu}$ is dominated by choosing $\omega = 0$. Hence, the firm's optimal choice must be either $\omega = \frac{\mu}{2k}$ or $\omega = 0$. When the firm chooses $\omega = \frac{\mu}{2k}$, its expected payoff (net of the cost) is $\frac{\mu^2}{8k} - \frac{\bar{U}}{2}$, which is positive if and only if $k < \frac{\mu^2}{4\bar{U}}$. Note that $\frac{(1-\mu)(2-3\mu)}{\bar{U}} < \frac{\mu^2}{4\bar{U}}$ if and only if $\mu > \frac{10-2\sqrt{3}}{11}$.

Therefore, when $\frac{1}{2} < \mu \leq \frac{10-2\sqrt{3}}{11}$, the firm's optimal choice is $\omega_0 = 0$. When $\frac{10-2\sqrt{3}}{11} < \mu < \frac{2}{3}$, if $\frac{(1-\mu)(2-3\mu)}{\bar{U}} < k < \frac{\mu^2}{4\bar{U}}$, the firm's optimal choice is $\omega_0 = \frac{\mu}{2k}$; if $k \geq \frac{\mu^2}{4\bar{U}}$, the firm's optimal choice is $\omega_0 = 0$.

Case 2: $\mu \geq \frac{2}{3}$.

The analysis is similar to the above. If the firm chooses $\omega \leq \frac{\bar{U}}{\mu}$, the benchmark equilibrium will be E^{N}. If the firm chooses $\omega > \frac{\bar{U}}{\mu}$, the benchmark equilibrium will be E^{B}. In contrast to Case 1, there is no choice of ω such that the benchmark equilibrium will be E^{A}.

If the firm chooses $\omega \leq \frac{\bar{U}}{\mu}$, the local maximum of the firm's expected payoff (net of the cost) conditional on $\omega \leq \frac{\bar{U}}{\mu}$ is achieved at $\omega = 0$, and its expected payoff is zero.

If the firm chooses $\omega > \frac{\bar{U}}{\mu}$, the local maximum conditional on $\omega > \frac{\bar{U}}{\mu}$ is achieved at $\omega = \frac{\mu}{2k}$ if $k < \frac{\mu^2}{2\bar{U}}$; the local maximum conditional on $\omega > \frac{\bar{U}}{\mu}$ is achieved at $\omega = \frac{\bar{U}}{\mu}$ if $k \geq \frac{\mu^2}{2\bar{U}}$. Because choosing $\omega = \frac{\bar{U}}{\mu}$ renders a negative expected payoff (net of the cost), it is dominated by choosing $\omega = 0$.

The firm's expected payoff is continuous in ω for its entire support ($\omega \geq 0$), despite the fact that the equilibrium may switch from E^{N} to E^{B} as ω increases from below $\frac{\bar{U}}{\mu}$ to above $\frac{\bar{U}}{\mu}$. Plugging $\omega = \frac{\bar{U}}{\mu}$ into the firm's expected payoff under E^{N}, we have $\mathbb{E}[\pi_f] - \frac{1}{2}k\omega^2 = 0 - \frac{1}{2}k\left(\frac{\bar{U}}{\mu}\right)^2 = -\frac{1}{2}k\left(\frac{\bar{U}}{\mu}\right)^2$. Plugging $\omega = \frac{\bar{U}}{\mu}$ into the firm's expected payoff under E^{B}, we have $\mathbb{E}[\pi_f] - \frac{1}{2}k\omega^2 = \frac{1}{2}(\omega\mu - \bar{U}) - \frac{1}{2}k\omega^2 = 0 - \frac{1}{2}k\left(\frac{\bar{U}}{\mu}\right)^2 = -\frac{1}{2}k\left(\frac{\bar{U}}{\mu}\right)^2$. Hence, there is no discontinuity in the firm's expected payoff at $\omega = \frac{\bar{U}}{\mu}$.

Therefore, when $\mu \geq \frac{2}{3}$, if $k < \frac{\mu^2}{4\bar{U}}$, the firm's optimal choice is $\omega_0 = \frac{\mu}{2k}$; if $k \geq \frac{\mu^2}{4\bar{U}}$, the firm's optimal choice is $\omega_0 = 0$.

Combining Case 1 and Case 2, if $\mu > \frac{10-2\sqrt{3}}{11}$ and $k < \frac{\mu^2}{4\bar{U}}$, the firm's optimal choice of product quality is $\omega_0 = \frac{\mu}{2k}$, which will lead to the benchmark equilibrium E^{B}. If $\mu \leq \frac{10-2\sqrt{3}}{11}$ or $k \geq \frac{\mu^2}{4\bar{U}}$, the firm will optimally choose zero product quality and the benchmark equilibrium will be E^{N}.

Quality choice in the feedback equilibrium

In this section, instead of fully solving the optimal quality choice in the feedback equilibrium, we show that the firm's expected payoff in the feedback equilibrium can be higher than its expected payoff in the benchmark equilibrium if it chooses $\omega \geq \omega_0$, where ω_0 denotes the optimal quality choice in the benchmark equilibrium.

If $\mu \leq \frac{10-2\sqrt{3}}{11}$ or $k \geq \frac{\mu^2}{4\bar{U}}$, then $\omega_0 = 0$, and the firm's expected payoff in the benchmark equilibrium is 0. Because product quality (ω) cannot be negative, it is obvious that with the financial market feedback, the firm's optimal choice in the feedback equilibrium must be at least as large as its optimal choice in the benchmark equilibrium - i.e., $\omega \geq \omega_0 = 0$.

Indeed, in simulations, we show that both $\omega > 0$ and $\omega = 0$ can be optimal depending on the parameters.[32]

If $\mu > \frac{10-2\sqrt{3}}{11}$ and $k < \frac{\mu^2}{4\bar{U}}$, then $\omega_0 = \frac{\mu}{2k}$, and we have already shown above that under this choice, the firm's expected payoff in the benchmark equilibrium is positive. According to Proposition 4, the expected firm profits at $\omega = \omega_0 = \frac{\mu}{2k}$ in the feedback game are higher than the expected firm profits at $\omega_0 = \frac{\mu}{2k}$ in the benchmark, which means that the firm's expected payoff in the feedback equilibrium from choosing $\omega = \omega_0 = \frac{\mu}{2k}$ must also be positive. If the firm chooses $\omega = \omega_0 = \frac{\mu}{2k}$ in the feedback game, depending on the parameters, the feedback equilibrium will be E^{I}, E^{II}, $\mathrm{E}^{\mathrm{III}}$, or E^{IV}. Whichever occurs, we show below that the firm has incentives to increase product quality above the level that it would choose without the financial market feedback (i.e., ω_0), and would be better off. In other words, we show that in the feedback equilibrium,[33]

$$\frac{d\mathbb{E}[\pi_f]}{d\omega}\Big|_{\omega=\omega_0} - k\omega_0 > 0.$$

Hence, by choosing a slightly higher ω than ω_0, the firm gets a higher payoff in the feedback equilibrium than the positive payoff it could get in the benchmark equilibrium.[34]

If the feedback equilibrium is E^{I}, we can first calculate that:

$$\frac{d\alpha_I}{d\omega} = -\alpha_I^2\Big\{ - \frac{2\mu(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}\bar{c} - \frac{\bar{U}\mu}{2(Z+1)(\omega\mu-\bar{U})^2}\Big[\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)} - 1\Big]\Big\}.$$

[32] According to simulations, for parameters ($\mu = 0.55, Z = 0.5, \bar{U} = 1, \bar{c} = 10, k = 0.01$), the firm will optimally choose zero product quality ($\omega_0 = 0$) in the benchmark because $\mu < \frac{10-2\sqrt{3}}{11}$. However, in the feedback game, the firm will switch to choosing positive product quality ($\omega = 29.62$) and expect a positive payoff ($\mathbb{E}[\pi_f] - \frac{1}{2}k\omega^2 = 7.94$) in E^{A}. For another set of parameters ($\mu = 0.9, Z = 0.5, \bar{U} = 1, \bar{c} = 10, k = 1$), the firm will optimally choose zero product quality ($\omega_0 = 0$) in the benchmark because $k > \frac{\mu^2}{4\bar{U}}$. In the feedback game, the firm will stick to zero product quality ($\omega = \omega_0 = 0$) and expect zero payoffs in E^{N}.

[33] In the benchmark equilibrium, by the definition of optimality,

$$\frac{d\mathbb{E}[\pi_f]}{d\omega}\Big|_{\omega=\omega_0} - k\omega_0 = \frac{1}{2}\mu - k\frac{\mu}{2k} = 0.$$

[34] This does not, of course, mean that the optimal quality in the feedback game is above ω_0, as it does not demonstrate what the global maximum is.

Because $\frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)} - 1 > 0$,[35] it follows that $\frac{d\alpha_I}{d\omega} > 0$.

We can then calculate:

$$\frac{d\mathbb{E}[\pi_f]}{d\omega} = \frac{1}{2}\mu + \frac{1}{4}\Big\{(1-Z)\omega(1-\mu) + (1+Z)\Big[\omega\Big(\frac{1-\mu}{(1-\mu)+\mu Z} - \frac{\mu}{Z+1}\Big) - \bar{U}\frac{(1-\mu)+\mu Z}{Z+1}\Big]\Big\}\frac{d\alpha_I}{d\omega}$$
$$+ \frac{1}{4}\alpha_I\Big[(1-Z)(1-\mu) + (1+Z)\Big(\frac{1-\mu}{(1-\mu)+\mu Z} - \frac{\mu}{Z+1}\Big)\Big].$$

Because $\omega\Big(\frac{1-\mu}{(1-\mu)+\mu Z} - \frac{\mu}{Z+1}\Big) - \bar{U}\frac{(1-\mu)+\mu Z}{Z+1} > 0$ in E^{I},[36] it follows that $\frac{1-\mu}{(1-\mu)+\mu Z} - \frac{\mu}{Z+1} > 0$ and $\frac{d\mathbb{E}[\pi_f]}{d\omega} > \frac{1}{2}\mu$ for any ω.

Therefore, if the feedback equilibrium is E^{I} at $\omega = \omega_0 = \frac{\mu}{2k}$,

$$\frac{d\mathbb{E}[\pi_f]}{d\omega}\Big|_{\omega=\omega_0} - k\omega_0 > \frac{1}{2}\mu - \frac{1}{2}\mu = 0.$$

If the feedback equilibrium is E^{II}, we can first calculate that:

$$\frac{d\alpha_{II}}{d\omega} = -\alpha_{II}^2\Big\{-\frac{2\mu(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}\bar{c} + \frac{\bar{U}\mu}{(Z+1)(\omega\mu-\bar{U})^2}\Big[1 - \frac{1}{2}\Big(\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z} + \frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)}\Big)\Big]\Big\}$$

Because $\bar{c}$ is large - specifically in this case, we assume that $\bar{c} > \frac{\bar{U}(2\mu-1)}{2(1+Z)}\Big[1 - \frac{1}{2}\Big(\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z} + \frac{(1-\mu)(1-Z^2)+Z}{\mu Z+(1-\mu)}\Big)\Big]$ - it follows that $\frac{d\alpha_{II}}{d\omega} > 0$.

Hence, for any ω,

$$\frac{d\mathbb{E}[\pi_f]}{d\omega} = \frac{1}{2}\mu + \frac{1}{4}(1-Z)(1-\mu)\Big(\omega\frac{d\alpha_{II}}{d\omega} + \alpha_{II}\Big) > \frac{1}{2}\mu.$$

Therefore, if the feedback equilibrium is E^{II} at $\omega = \omega_0 = \frac{\mu}{2k}$,

$$\frac{d\mathbb{E}[\pi_f]}{d\omega}\Big|_{\omega=\omega_0} - k\omega_0 > \frac{1}{2}\mu - \frac{1}{2}\mu = 0.$$

[35] This is equivalent to showing $(1-\mu)Z > (1-\mu)Z^2$, which is obvious given $Z \in (0,1)$.

[36] This is equivalent to showing $\bar{U} < \omega\frac{1}{(1-\mu)+\mu Z}[\frac{(1-\mu)(Z+1)}{(1-\mu)+\mu Z} - \mu]$ in E^{I}, which we have already shown in the proof of feedback equilibrium (Appendix A.2).

If the feedback equilibrium is $\mathrm{E}^{\mathrm{III}}$, we can first calculate that:

$$\frac{d\alpha_{III}}{d\omega} = -\alpha_{III}^2\Big\{ - \frac{2\mu(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}\bar{c} + \frac{\bar{U}\mu}{2(Z+1)(\omega\mu-\bar{U})^2}\Big[1-\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z}\Big]\Big\}.$$

Because $\bar{c}$ is large - specifically in this case, we assume that $\bar{c} > \frac{\bar{U}(2\mu-1)}{4(1+Z)}\Big[1-\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z}\Big]$ - it follows that $\frac{d\alpha_{III}}{d\omega} > 0$.

Next, we can calculate that:

$$\frac{d\mathbb{E}[\pi_f]}{d\omega} = \frac{1}{2}\mu + \frac{1}{4}\Big\{(1-Z)\omega(1-\mu) - (1+Z)\Big[\omega\frac{\mu Z}{Z+1} - \bar{U}\frac{\mu Z+(1-\mu)}{Z+1}\Big]\Big\}\frac{d\alpha_{III}}{d\omega} + \frac{1}{4}\alpha_{III}[(1-Z)(1-\mu)-\mu Z].$$

We can show that $\frac{1}{4}\Big\{(1-Z)\omega(1-\mu) - (1+Z)\Big[\omega\frac{\mu Z}{Z+1} - \bar{U}\frac{\mu Z+(1-\mu)}{Z+1}\Big]\Big\}\frac{d\alpha_{III}}{d\omega} + \frac{1}{4}\alpha_{III}[(1-Z)(1-\mu)-\mu Z] > 0$ when $\bar{c}$ is very large. Plugging in the expression of $\frac{d\alpha_{III}}{d\omega}$, this is equivalent to:

$$\begin{aligned}\frac{1}{4}[Z-(1-\mu)]\frac{1}{\alpha_{III}} < &\frac{1}{4}\Big\{(1-Z)\omega(1-\mu) - (1+Z)\Big[\omega\frac{\mu Z}{Z+1} - \bar{U}\frac{\mu Z+(1-\mu)}{Z+1}\Big]\Big\} \\ &\Big\{\frac{2\mu(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}\bar{c} - \frac{\bar{U}\mu}{2(Z+1)(\omega\mu-\bar{U})^2}\Big[1-\frac{\mu+(1-\mu)Z^2}{\mu+(1-\mu)Z}\Big]\Big\}.\end{aligned}$$

Plugging in the expression of α_{III} (equation A.3) in the LHS, it is clear that both the LHS and the RHS of the above inequality are linear in $\bar{c}$. If the coefficient in front of $\bar{c}$ on the LHS is smaller than the coefficient in front of $\bar{c}$ on the RHS, then the LHS is smaller than the RHS when $\bar{c}$ is very large. Specifically, we assume that $\bar{c}$ is larger than a threshold at which the LHS is equal to the RHS.

Showing that the coefficient in front of $\bar{c}$ on the LHS is smaller than the coefficient in front of $\bar{c}$ on the RHS is equivalent to showing:

$$\begin{aligned}\frac{1}{4}[Z-(1-\mu)]\frac{2}{(\omega\mu-\bar{U})(2\mu-1)} < &\frac{1}{4}\Big\{(1-Z)\omega(1-\mu) - (1+Z)\Big[\omega\frac{\mu Z}{Z+1} - \bar{U}\frac{\mu Z+(1-\mu)}{Z+1}\Big]\Big\} \\ &\frac{2\mu(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}.\end{aligned}$$

Note that because $\bar{U} \geq \omega\frac{\mu Z}{\mu Z+(1-\mu)}$ in $\mathrm{E}^{\mathrm{III}}$, $\frac{1}{4}\Big\{(1-Z)\omega(1-\mu)-(1+Z)\Big[\omega\frac{\mu Z}{Z+1}-\bar{U}\frac{\mu Z+(1-\mu)}{Z+1}\Big]\Big\}$ is positive, and, thus, the RHS is positive. Hence, the above inequality clearly holds when $Z-(1-\mu)\leq 0$ (i.e., the LHS is negative), which means that we only need to show that the above inequality holds when $Z-(1-\mu)>0$.

To see this, the above inequality can be further simplified:[37]

$$\begin{aligned} & [Z-(1-\mu)](\omega\mu-\bar{U}) < \mu\Big\{\omega[(1-\mu)-Z]+\bar{U}[\mu Z+(1-\mu)]\Big\} \\ \Leftrightarrow \quad & \bar{U} > \omega\frac{2\mu[Z-(1-\mu)]}{\mu[\mu Z+(1-\mu)]+[Z-(1-\mu)]}. \end{aligned}$$

The above holds because in $\mathrm{E}^{\mathrm{III}}$, $\bar{U} \geq \omega\frac{\mu Z}{\mu Z+(1-\mu)} > \omega\frac{2\mu[Z-(1-\mu)]}{\mu[\mu Z+(1-\mu)]+[Z-(1-\mu)]}$. To see this,

$$\begin{aligned} & \omega\frac{\mu Z}{\mu Z+(1-\mu)} > \omega\frac{2\mu[Z-(1-\mu)]}{\mu[\mu Z+(1-\mu)]+[Z-(1-\mu)]} \\ \Leftrightarrow \quad & Z\Big\{\mu[\mu Z+(1-\mu)]+[Z-(1-\mu)]\Big\} > 2[Z-(1-\mu)][\mu Z+(1-\mu)] \\ \Leftrightarrow \quad & (1-\mu)(1-Z)[\mu Z+(1-\mu)] > [Z-(1-\mu)](1-\mu)(1-Z) \\ \Leftrightarrow \quad & \mu Z+(1-\mu) > Z-(1-\mu) \\ \Leftrightarrow \quad & 2 > Z. \end{aligned}$$

Hence, $\frac{d\mathbb{E}[\pi_f]}{d\omega} > \frac{1}{2}\mu$ for any ω.

Therefore, if the feedback equilibrium is $\mathrm{E}^{\mathrm{III}}$ at $\omega=\omega_0=\frac{\mu}{2k}$,

$$\frac{d\mathbb{E}[\pi_f]}{d\omega}\Big|_{\omega=\omega_0} - k\omega_0 > \frac{1}{2}\mu - \frac{1}{2}\mu = 0.$$

If the feedback equilibrium is E^{IV}, we can first calculate that:

$$\frac{d\alpha_{IV}}{d\omega} = \alpha_{IV}^2\frac{2\mu(2\mu-1)}{[(\omega\mu-\bar{U})(2\mu-1)]^2}\bar{c} > 0.$$

[37] Because $Z-(1-\mu)>0$, the expression $\mu[\mu Z+(1-\mu)]+[Z-(1-\mu)]$ in the denominators is positive, and we do not need to analyze the possibility of a flip of the inequality sign as we rearrange the algebra.

We can then calculate:

$$\begin{aligned}\frac{d\mathbb{E}[\pi_f]}{d\omega} =&\frac{1}{2}\mu+\frac{1}{4}\Big\{(1-Z)\omega(1-\mu)+(1+Z)\omega\Big[\frac{1-\mu}{(1-\mu)+\mu Z}-\mu\Big]\Big\}\frac{d\alpha_{IV}}{d\omega}\\&+\frac{1}{4}\alpha_{IV}\Big\{(1-Z)(1-\mu)+(1+Z)\Big[\frac{1-\mu}{(1-\mu)+\mu Z}-\mu\Big]\Big\}>\frac{1}{2}\mu.\end{aligned}$$

Because $Z \leq (\frac{1-\mu}{\mu})^2$ in E^{IV}, it follows that $\frac{1-\mu}{(1-\mu)+\mu Z}-\mu \geq 0$ and $\frac{d\mathbb{E}[\pi_f]}{d\omega} > \frac{1}{2}\mu$ for any ω. Therefore, if the feedback equilibrium is E^{IV} at $\omega = \omega_0 = \frac{\mu}{2k}$,

$$\frac{d\mathbb{E}[\pi_f]}{d\omega}\Big|_{\omega=\omega_0} - k\omega_0 > \frac{1}{2}\mu - \frac{1}{2}\mu = 0.$$

In summary, when $\omega_0 = \frac{\mu}{2k}$ is the optimal quality choice of the firm in the benchmark equilibrium (i.e., $\mu > \frac{10-2\sqrt{3}}{11}$ and $k < \frac{\mu^2}{4\bar{U}}$), whether the corresponding feedback equilibrium is E^{I}, E^{II}, $\mathrm{E}^{\mathrm{III}}$, or E^{IV}, the marginal payoff from increasing the quality at $\omega_0 = \frac{\mu}{2k}$ is positive. Therefore, there always exists a sufficiently small $\epsilon > 0$ such that the firm is better off choosing $\omega = \omega_0 + \epsilon$ than choosing $\omega_0 = \frac{\mu}{2k}$.

Chapter 2

Director Appointments and Board Dynamics

Ken Deng

Abstract

This paper develops a model of director appointments and their impact on firm value. A new appointee can bring new ideas to the incumbent board and change the direction of the firm; CEOs anticipate their arrival and set board agendas. The model features two frictions: CEO moral hazard and costs that directors face when opposing the CEO. Shareholders may rationally avoid appointing the best available candidate, as there is a trade-off between appointing this candidate, who adds the most value through new ideas, versus appointing a lower-quality director who prevents value destruction. These results provide novel predictions on director appointments and a positive benchmark for interpreting the effects of director appointments.

Keywords: Corporate Governance, Director Appointment, Moral Hazard, Collective Decision-Making, Voting Equilibrium

JEL Classification: G34, D82, D72, D23

2.1 Introduction

Corporate boards are central to firm governance: they shape oversight, strategic direction, and ultimately value creation. Director appointments are a key channel through which board composition evolves and determines how boards fulfill their governance role.

Director appointments can be described in multiple dimensions. Firms often prioritize high-caliber candidates - for example, 59% of new independent directors in S&P 500 firms brought CEO or financial expertise.[1] At the same time, public and media discussions often highlight demographic attributes such as gender and ethnicity - for example, women now hold approximately 43% of FTSE 350 board seats.[2] However, any selection criterion, experiential or demographic, must ultimately translate into substantive value for the firm.[3] In novel strategic contexts such as renewable energy transitions or AI adoption, legacy boards often struggle to formulate concrete plans for initiatives that fall outside of their established thinking, which is why new director appointments attract significant attention from shareholders and policymakers.[4] In these contexts, shareholders - especially activist ones - can play a more effective role in shaping director appointments than under the usual norm, where candidates are proposed by the nominating committee and formally ratified by shareholders.

This paper explores two central questions: (i) What types of directors will be appointed and (ii) How do these appointments influence firm value by shaping board behavior? Anchoring the analysis in *director quality* - defined as the value a director brings through

[1] Spencer Stuart US Board Index (2024)

[2] FTSE Women Leaders Review (2024)

[3] As Edmans (2021) points out, demographic diversity on the board may not always reflect diversity of thought, even if the two are correlated. It is the latter, rather than the former, that directly drives firm value.

[4] LGIM, a large shareholder of Exxon, stated that "Exxon's current board is full of incredibly accomplished CEOs but lacks people who understand oil and gas and the energy transition" (see "Investor Legal & General backs activist in Exxon proxy battle" by Jennifer Hiller, *Reuters*, May 11, 2021). The National Association of Corporate Directors (NACD) suggested that "the emergence of artificial intelligence means corporate boards will have to shake up their membership to better oversee the technology, bringing in tech-savvy recruits that could supplant some directors" (see "Board Shakeups Loom as AI Oversight Demands New Tech Expertise" by David Hood, *Bloomberg Law*, October 7, 2024). Consistently, when firm strategy shifts toward cutting-edge areas, VC directors raise innovation when they join mature-firm boards (Celikyurt et al., 2014).

thinking outside the box - this paper develops a microeconomic theory that connects board composition to corporate outcomes.

The model proceeds in three stages. First, shareholders appoint a new director to replace a departing one, focusing on whether the appointee can add value relative to the incumbent board. This captures a key dimension of director heterogeneity that motivates this paper, while abstracting from other attributes. Second, the CEO may propose a new project, but faces a moral hazard problem: incentives may diverge from shareholders', so the project can benefit the CEO at the expense of the firm. If the new director can think outside the box, they may put forward a counterproposal; otherwise, only the CEO's proposal is considered. Third, directors vote between the two options, but opposing the CEO comes at a cost. The model thus features two central frictions: a moral hazard problem that distorts the CEO's proposal and costs that make directors reluctant to oppose the CEO. Together, these frictions shape the optimal director appointment and determine whether a new appointee enhances firm value.

The model is grounded in institutional features of corporate governance, including the roles of key agents on boards and the frictions they face.

- **Shareholders and director appointment.** Shareholders formally elect directors, though in practice the process may be influenced by the CEO or other incumbent directors. Nevertheless, institutional activism and the market for corporate control can push appointments toward shareholder interests (Adams and Ferreira, 2007). The rise of proxy access since 2015 has arguably strengthened shareholder control (Gregory et al., 2019), and activist investors continue to shape board composition with growing effectiveness (DesJardine and Tetelbaum, 2025). In addition, Krüger et al. (2022) point out that the direct costs for blockholders to seek board representation are low relative to the size of their investments, underscoring the variety of channels through which shareholders can influence director appointments.[5]

[5]I will also analyze an extension in which the CEO, rather than shareholders, makes the appointment.

- **CEO agenda-setting.** CEOs typically initiate new issues and exert strong influence over board agendas (Adams, 2017).[6] Using board minutes, Schwartz-Ziv and Weisbach (2013) show that boards function as active monitors: although usually presented with a single option, directors can request updates or pursue further actions, creating scope for counterproposals. In this setting, where CEOs present and boards supervise, agency conflicts naturally arise (Jensen and Meckling, 1976).
- **Costs of opposing the CEO.** Directors may hesitate to oppose the CEO. In an influential study of group composition, Lau and Murnighan (1998) argue that "faultlines" - misalignment in individual characteristics - can silence disagreement and reduce board effectiveness. Veltrop et al. (2021) highlight the importance of psychological safety, as directors may fear punishment for dissent.[7] In a study on boards, Dewally and Peck (2010) find that director resignations often stem from conflict with the CEO, with public announcements of resignations driven by reputational concerns. Accordingly, my model captures the cost of opposing the CEO as a key friction, while allowing this cost to be negative to reflect that some directors may in fact be rewarded for dissent.

A key result of the paper is that shareholders may not always appoint the best available director, even when this candidate offers the greatest contribution to value. At first sight, this is counterintuitive: one would expect the best candidate to be the obvious choice. Yet the model shows that this candidate's expected value can be low when their counterproposal is unlikely to be supported by directors who are reluctant to bear the personal cost of opposing the CEO. In this case, the CEO proposes a value-destroying project. In contrast, a lower-quality director generates a counterproposal which offers limited improvement in firm value relative to the CEO's project. However, the counterproposal, if implemented, prevents the

[6] Boards do, however, initiate decisions on CEO compensation and dismissal, typically handled in executive sessions and outside the scope of this paper. More generally, directors rarely initiate new projects because they are often busy (Fich and Shivdasani, 2006) and may be stretched by serving on many boards (see "Overboarding: one investor looks for alternatives to numerical director thresholds" by Jessica Tasman-Jones, *The Financial Times*).

[7] Sonnenfeld (2002) describes a case where a newly appointed director was told not to speak during the first year.

CEO from obtaining private benefits and thus yields a negative payoff to the CEO. When such a counterproposal is likely to be implemented, the CEO refrains from proposing value-destroying projects so as not to provoke the counterproposal. In this case, the deterrence benefit of appointing a lower-quality director outweighs their limited direct contribution to firm value and the possible benefit of the best director.

Therefore, a director can enhance firm value even when their contribution is not directly observable. In general, directors may be appointed not only to generate valuable action but also to deter harmful action. Shareholders may therefore optimally appoint a lower-quality candidate whose role is not to propose alternatives but to prevent the CEO from advancing a value-destroying project. In such cases, no counterproposal is made and no new project is adopted, yet the director's presence preserves firm value. Empirically, however, this absence of visible action could be mistaken for ineffectiveness or tokenism, when, in fact, it reflects a hidden but important increase in value.

The model connects to two established views of boards: the agency perspective, which sees boards as monitors of executives on behalf of shareholders, and the resource dependence perspective, which views directors as providers of resources such as advice and external connections.[8] It is consistent with the agency perspective in that shareholders may appoint a new director primarily to deter the CEO from pursuing value-destroying projects. It is consistent with the resource dependence view in that directors can enhance value by improving on the CEO's proposal. At the same time, the model highlights a tension between the two: shareholders may prefer a lower-quality director who deters value-destroying proposals over a higher-quality director who improves new proposals. This trade-off arises only when there is CEO moral hazard. When moral hazard is absent, the contributions of the best candidate clearly dominate the status quo.

The paper also contributes theoretically by establishing uniqueness of the equilibrium in a majority-voting setting where multiplicity is typically the norm. The model delivers a

[8]For a comprehensive review of theories and evidence reflecting one or both perspectives, see Adams et al. (2010) and Ferreira (2010).

unique pure-strategy threshold equilibrium: directors oppose the CEO only if their private cost falls below a cutoff. Uniqueness arises from heterogeneity in directors' costs of opposing the CEO's proposal, which removes the strategic uncertainty that drives multiple equilibria in voting games.[9]

The analysis demonstrates how director appointments depend on the severity of moral hazard, the heterogeneity in opposition costs, and board size. A severe (mild) moral hazard problem makes the appointment of a lower-quality (best) candidate optimal. Counterproposals are less likely to be made and passed when opposition costs are more heterogeneous. And individual directors are less likely to oppose the CEO as board size increases.

A novel prediction of the model is that when the quality of the best available candidate is not sufficiently high, it can be optimal for firms to instead appoint a lower-quality candidate who may add little value through new ideas, but can effectively deter value-destroying projects.

The first extension of the model examines the case in which shareholders appoint the new director based on their cost of opposing the CEO rather than their quality. Shareholders may rationally appoint a candidate who incurs a cost rather than receives a reward for opposing the CEO, since both types can be equally effective in deterring value-destroying projects. A second extension considers the case where the CEO appoints the new director. CEO-led appointments may not align with shareholder interests, implying that activism to secure shareholder-backed candidates is valuable.

Related Literature. This paper contributes to the literature on board composition by examining which type of director, defined by idea quality, shareholders will appoint.

Earlier theories emphasize board independence. Hermalin and Weisbach (1998) model the

[9]The logic here is distinct from other settings where uniqueness has been obtained. In contest models such as the Tullock rent-seeking contest (Tullock et al., 1980), the agents' strategies are also threshold functions of private costs, but the context is very different: the outcome is continuous effort rather than binary vote. Likewise, in global games, uniqueness follows from players observing noisy signals of a common fundamental, as in the currency attack model of Morris and Shin (1998). In contrast, the setting here involves majority voting with a binary outcome, and the uniqueness comes directly from heterogeneity in directors' private costs of opposing the proposal.

monitoring role of boards as a function of independence, while Adams and Ferreira (2007) examine both monitoring and advisory roles. They highlight a conflict between the two, but the trade-off they study differs from mine: because both monitoring and advice rely on CEO communication, shareholders may prefer a less independent (friendlier) board to encourage such communication. In contrast, my results impose no restriction on director independence and instead focus on director quality, showing that even among equally independent candidates, shareholders may not choose the highest-quality one. Other key differences are: (i) in my model, the CEO may or may not initiate a project; (ii) director contributions can generate positive firm value (whereas in their model, the best outcome is eliminating moral hazard, yielding zero value); and (iii) directors vote during decision-making in my model.

More recent theories highlight the heterogeneity of directors, focusing on preferences and information. Donaldson et al. (2020) show in a dynamic voting model that preference diversity can be inefficient because it risks deadlock, whereas Ljungqvist and Raff (2023) argue that this risk can commit directors to ex ante information production, improving outcomes. Similarly, Malenko (2014) shows that under conformity biases, preference heterogeneity can encourage pre-meeting communication and lead to better board decisions. My paper differs in two respects. First, I focus on incremental changes in board composition rather than wholesale restructurings, highlighting how a new appointee can add value relative to the incumbent board without necessarily differing in preferences. Second, the trade-offs are distinct: the combination of CEO moral hazard and directors' disincentives to support better alternatives in collective decision-making - driven by the cost of opposing the CEO - can make it optimal not to appoint the highest-quality candidate. Relatedly, Drymiotes and Sivaramakrishnan (2021) find that a value-maximizing CEO may also prefer not to appoint the most able director, but for a different reason: to reduce the risk that other directors reject the CEO's good proposal because of uncertainty about the CEO's type.

Aghion and Tirole (1997) study the allocation and delegation of formal and real authority under asymmetric information, highlighting the trade-off between strengthening the agent's

initiative to acquire information and weakening the principal's control. Although this paper emphasizes different frictions and trade-offs, its results can be framed in their language: the board holds formal authority (the right to approve or oppose), while the CEO holds real authority (strong influence over board agendas). Because the board's formal authority is constrained by the costs of opposing the CEO, shareholders (the principal) may rationally avoid appointing the highest-quality director, as doing so does not fully counteract the moral hazard embedded in the CEO's real authority. In such cases, a limited improvement to the board's formal authority - through appointing a lower-quality director - may be optimal if it curbs the CEO's real authority and deters value-destroying proposals.[10]

2.2 The Model

The model unfolds in three stages: the director appointment stage, the proposal stage, and the decision-making stage. In the first stage, shareholders replace a departing director (e.g., a director whose term has ended) with a new one.[11] In the second stage, the CEO decides whether to propose a new project (e.g., a new business opportunity). I assume that the incumbent directors share a common strategic vision with the CEO about the firm's direction.[12] The key economic distinction is whether the new appointee can add value relative to the incumbent board, particularly by thinking outside the box in novel strategic contexts (e.g., renewable energy transitions or AI adoption). When the new director brings novel

[10] In a different context of human–technology integration, Zhong (2025) also studies multi-layer delegation. He shows that under the optimal rule, higher-quality technologies should be deployed at more downstream stages of the delegation chain. In a similar spirit, my paper suggests that improving downstream delegation (addressing the CEO's moral hazard) may create more firm value than improving upstream delegation (bringing new ideas to the board), when shareholders, the board, and the CEO are viewed as a delegation chain.

[11] The analysis is similar if shareholders add a new director instead. I focus on marginal changes in board composition because, in practice, such changes usually occur incrementally. Board turnover has been low for decades - for example, only 58% of S&P 500 boards appointed a new director in the 2024 proxy year, translating to less than one per board (see "The Boardroom's Refresh Dilemma: Why Turnover Lags And How to Fix It" by C.J. Prince, *Corporate Board Member*, 2024).

[12] Such alignment may arise for benign reasons, such as homophily and shared experiences within corporate boards (McPherson et al., 2001), or from CEO influence and "board capture" as CEOs gain power and entrenchment over time (Graham et al., 2020).

ideas, they may choose to counterpropose to the CEO, incurring a personal cost for doing so. I assume that any counterproposal strictly improves firm value relative to the CEO's initial proposal. If no counterproposal is made, the CEO's proposal is implemented without challenge. The final stage - the decision-making stage - occurs only if there are two options on the table: the CEO's proposal and the new director's counterproposal. The rest of the board then votes between the two. As with the new director's decision to counterpropose, each incumbent director incurs a cost when opposing the CEO (i.e., voting for the counterproposal). This cost could reflect the time and effort required to justify opposition or the fear and anxiety of jeopardizing one's position on the board. The cost can also be negative, meaning that a director might be privately rewarded for opposing the CEO. If a majority of directors oppose the CEO, the CEO's proposal is withdrawn and the counterproposal is implemented; otherwise, the CEO's proposal goes forward.[13]

Importantly, the incumbents' shared vision means they rarely challenge the CEO when no alternative is available,[14] but they may support an alternative when one is presented. This helps explain why activist shareholders (e.g., Elliott Investment Management) push for appointing directors with fresh perspectives and why policymakers advocate for bringing new ideas into boardrooms to broaden strategic vision and challenge entrenched thinking. The model formalizes these considerations.

In the director appointment stage, shareholders appoint a new director D. The board consists of $2N + 3$ seats ($N \in \mathbb{Z}^+$): the CEO, the new director, and $2N + 1$ incumbent directors (denoted as director $i \in \{1, ..., 2N + 1\}$).[15] The new director can be one of three types: a "similar candidate" (*Similar*) who shares the incumbents' strategic vision; the

[13]The results are robust to alternative decision rules - such as delegating authority to a single agent (e.g., Harris and Raviv (2008)) or using voting rules other than majority - in the sense that the qualitative conclusions remain unchanged: before a decision is made, shareholders and the CEO still face uncertainty about which project will be implemented.

[14]Shared strategic vision offers limited benefit when CEO moral hazard is present: Fahlenbrach et al. (2010) show that appointing CEOs from firms with similar strategies and governance structures to their boards is associated with little or even negative effects on operating performance.

[15]Because $N \in \mathbb{Z}^+$, I omit the uninteresting case where only one incumbent director votes during the decision-making stage.

"best candidate" ($Best$) who can think outside the box and generate a counterproposal *much* better than the CEO's proposal; or a "lower-quality candidate" (LrQ) who can think outside the box and generate a counterproposal *moderately* better than the CEO's proposal. Hence, shareholders choose $D \in \{Similar, LrQ, Best\}$, with the type observable to all agents based on past experiences and achievements. Because the type is observable and all shareholders maximize firm value without preference heterogeneity, I assume that they agree on the choice of D regardless of whether ownership is dispersed or concentrated. This abstraction allows the analysis to focus on board dynamics rather than shareholder conflicts. I also analyze an extension in which the CEO, rather than shareholders, makes the appointment, in which case the choice may diverge from shareholders' interests.

In the proposal stage, the CEO first considers whether to include a new firm project on the agenda of the next board meetings. They face a binary choice $\alpha \in \{0, 1\}$: if $\alpha = 0$, the CEO does not bring anything new to the attention of the board and the firm continues business as usual; if $\alpha = 1$, the CEO initiates discussions about the new project. When $\alpha = 1$, the CEO's proposal $\mathbb{P}$, if executed, will generate payoff $V \leq 0$ for the shareholders and the directors; in addition, $\mathbb{P}$ will generate private benefits $B > -V$ for only the CEO if it is executed. These private benefits could reflect control benefits or empire building.[16] This payoff assumption captures the classic moral hazard problem between the owners and the management: the CEO has incentive to propose a privately beneficial project at the expense of the firm owners. The parameters V and B are common knowledge.[17]

After the CEO's initial proposal ($\alpha = 1$), there may be a counterproposal. If shareholders have chosen $D = Similar$, the directors share a coherent vision and do not challenge the

[16]CEO private benefits at the expense of shareholders are well documented in several settings: in mergers and acquisitions (Hartzell et al., 2004), in dual-class firms where control rights exceed cash-flow rights (Masulis et al., 2009), and in CEOs' personal asset holdings (Décaire and Sosyura, 2020). Moreover, Cohen et al. (2012) find that when poorly governed firms appoint directors overly sympathetic to incumbents (i.e., similar directors), CEOs derive substantial private benefits following these appointments.

[17]V and B reflect the features of a business project that is on the CEO's radar at that time, so they are parameters and not choices of any director. This setup is similar to Donaldson et al. (2020), where an available firm project is characterized by how much value it creates for shareholders and how much it appeals to directors' biases (private benefits). The CEO has the choice of whether to propose this project to the board.

CEO. As a result, the CEO's proposal $\mathbb{P}$ will be executed. If shareholders have chosen $D = LrQ$, the new director can think of a counterproposal $\mathbb{P}^{LrQ}$. This counterproposal will generate payoff $V+\delta$ (where $\delta > 0$) for the shareholders and the directors if it is implemented, and there will be no private benefits for anyone.[18] The new director chooses whether to counterpropose $\mathbb{P}^{LrQ}$ against $\mathbb{P}$. Denote their choice as $\beta \in \{0, 1\}$: if $\beta = 0$, the new director does not counterpropose; if $\beta = 1$, the new director opposes the CEO by counterproposing $\mathbb{P}^{LrQ}$ and incurs a private cost (which I will define below). If shareholders have chosen $D = Best$, the new director can think of a counterproposal $\mathbb{P}^{Best}$. This counterproposal will generate payoff $V + \Delta$ (where $\Delta > \delta$) for the shareholders and the directors if it is implemented, and there will be no private benefits for anyone.[19] The new director chooses whether to counterpropose $\mathbb{P}^{Best}$ against $\mathbb{P}$ (i.e., chooses $\beta \in \{0, 1\}$) at a private cost.

The decision-making stage occurs if and only if there are a proposal ($\alpha = 1$) and a counterproposal ($\beta = 1$). Each of the $2N + 1$ incumbent directors chooses between the two proposals.[20] If the majority of the directors vote for the CEO, the CEO's proposal $\mathbb{P}$ will be executed. If the majority of directors oppose the CEO (i.e., vote for the counterproposal), the CEO will withdraw their proposal and the counterproposal will be executed.[21]

Opposing the CEO, by either counterproposing or voting, can be costly.[22] For the new

[18]Because shareholders and the directors get the same project payoff, the incumbent directors represent the interests of shareholders. This implies that inability to think outside the box need not come from misaligned interests.

[19]A counterproposal is characterized by the value it adds on top of the CEO's proposal (parameter δ or Δ). This allows flexibility in interpretation, as the added value (equivalently, the quality of the new director) can come from increasing expected return or reducing risk.

[20]It is natural to assume that as the proposing directors, the CEO and the new director are held to what they respectively propose (e.g., being perceived to renege on their proposals affects their credibility/reputation), so their votes would cancel out and the voting outcome depends on the $2N + 1$ incumbent directors.

[21]I assume away abstention in this paper because director abstentions/recusals are rare and typically stem from narrow, identifiable reasons such as conflicts of interest or procedural formality, rather than strategic reasons. As noted in Haas (2018), "Board votes to approve mergers with unaffiliated third parties are almost always unanimous. When the vote is not unanimous, it is usually attributed to a disclosed relationship the director has with the acquiror", underscoring that abstentions are generally explained by clear conflicts and do not attract judicial scrutiny as material information. This is in contrast to shareholder voting, in which shareholder abstention can be a strategic equilibrium move and is widely measured and shown to matter (e.g., Bar-Isaac and Shapiro (2020)).

[22]Chemmanur and Fedaseyeu (2018) also model a cost of dissent, but directors incur the cost of dissent only if they fail to oust the CEO in the context of forced CEO turnover. In addition, they do not have

director, the private cost of opposing the CEO (through counterproposing) is c_0. For director $i \in \{1, ..., 2N+1\}$, the private cost of opposing the CEO (through voting) is c_i. Both c_0 and c_i are private information, known only by the directors themselves.[23] Nevertheless, there is common knowledge that $c_i \overset{iid}{\sim} \text{Uniform}[-C, C], \forall i \in \{0, ..., 2N+1\}$. If a director's cost of opposing the CEO is positive, they will be reluctant to oppose the CEO. If a director's cost of opposing the CEO is negative, this is interpreted as the director being rewarded for their independence (e.g., in the form of reputation gain). In the model, the expected cost of opposing the CEO is zero for all directors.

Because it may be costly to oppose the CEO, new ideas may not emerge or be implemented even when the new director can think outside the box.[24] Faced with such uncertainty, shareholders need to weigh the firm value gains from the new director appointment against its impact on the CEO's initial proposal, without which the firm will continue business as usual.

Denote the implemented project as $\mathbb{P}^* \in \{\mathbb{P}, \mathbb{P}^{LrQ}, \mathbb{P}^{Best}\}$. The payoff for the shareholders (the firm value) will be:

$$\pi(\mathbb{P}^*) = V + \mathbb{1}(\mathbb{P}^* = \mathbb{P}^{LrQ}) \times \delta + \mathbb{1}(\mathbb{P}^* = \mathbb{P}^{Best}) \times \Delta \tag{2.2.1}$$

where $\mathbb{1}(.)$ is an indicator function that is equal to one if the condition in parentheses is true and zero if otherwise.

heterogeneity in the cost of dissent.

[23] I assume that directors cannot credibly convey their private cost of opposing the CEO, so the shareholders cannot select on this cost in the director appointment stage. Instead, shareholders select the quality of a director (e.g., whether they are the best candidate) based on their past experiences and achievements. I also analyze an extension in which shareholders appoint the new director based on their cost of opposing the CEO, and show that shareholders may rationally select a candidate who faces a positive cost of opposition.

[24] Giannetti and Zhao (2019) use directors' ancestral origins as a proxy for their ability to bring new ideas. They find that firms with greater ancestral diversity make less predictable decisions, indicating conflicts in the boardroom and inefficiency through greater uncertainty in the decision-making process. These findings are consistent with this paper.

The payoff for the CEO will be:

$$U_c(\mathbb{P}^*) = V + \mathbb{1}(\mathbb{P}^* = \mathbb{P}) \times B + \mathbb{1}(\mathbb{P}^* = \mathbb{P}^{LrQ}) \times \delta + \mathbb{1}(\mathbb{P}^* = \mathbb{P}^{Best}) \times \Delta \tag{2.2.2}$$

The payoff for the new director will be:

$$U_0(\mathbb{P}^*) = \pi(\mathbb{P}^*) - \mathbb{1}(\beta = 1) \times c_0 \tag{2.2.3}$$

The payoff for the incumbent director $i \in \{1, ..., 2N + 1\}$ will be:

$$U_i(\mathbb{P}^*) = \pi(\mathbb{P}^*) - \mathbb{1}(\text{voting against the CEO}) \times c_i \tag{2.2.4}$$

If the firm continues business as usual (i.e., $\alpha = 0$), there will be no new project. Hence, $\pi(\emptyset) = U_c(\emptyset) = U_0(\emptyset) = U_i(\emptyset) = 0, \forall i \in \{1, ..., 2N + 1\}$.[25]

2.2.1 Timeline

Now I summarize the timeline of the game.

1. Director appointment:
   - The shareholders choose the director appointment $D \in \{Similar, LrQ, Best\}$.
2. Proposal:
   - The CEO chooses $\alpha \in \{0, 1\}$, where $\alpha = 1$ means that they will propose a new firm project to the board. If $\alpha = 0$, the firm will continue business as usual.
   - If $\alpha = 1$ and $D \in \{LrQ, Best\}$, the new director chooses $\beta \in \{0, 1\}$, where $\beta = 1$ means that they will counterpropose against the CEO. If $\beta = 0$ or $D = Similar$, the CEO's proposal $\mathbb{P}$ will be implemented.[26]

[25] In principle, both shareholders and directors care about the value of a new project (if any). For simplicity, I assume they assign an equal weight of 1 to V, although this may not hold in reality. This simplification does not affect the qualitative nature of the results.

[26] The agents' trade-offs and the qualitative results of the model will not change if we modify the setup to one in which any director can choose to make the counterproposal after the new director has brought

3. Decision-making if and only if $\alpha = 1$ and $\beta = 1$ (only one of the following two scenarios will happen):
   - $D = LrQ$: If the majority of directors vote for the CEO, the CEO proposal $\mathbb{P}$ will be implemented. If the majority of directors oppose the CEO (i.e., vote for the counterproposal $\mathbb{P}^{LrQ}$), the counterproposal $\mathbb{P}^{LrQ}$ will be implemented.
   - $D = Best$: If the majority of directors vote for the CEO, the CEO proposal $\mathbb{P}$ will be implemented. If the majority of directors oppose the CEO (i.e., vote for the counterproposal $\mathbb{P}^{Best}$), the counterproposal $\mathbb{P}^{Best}$ will be implemented.

The equilibrium concept of the model is Perfect Bayesian Equilibrium (PBE).[27] The model will be solved by backward induction.

2.2.2 Parameter Assumptions

For exposition purposes, I also make the following assumptions.

Assumption 1: $\delta < -V < \Delta$.

Assumption 1 implies that by counterproposing $\mathbb{P}^{LrQ}$, a lower-quality candidate is not able to bring enough firm value to fully offset the damage of the moral hazard problem, while the best candidate can bring sufficient firm value, by counterproposing $\mathbb{P}^{Best}$, to reverse the effect of CEO moral hazard. Under this assumption, the analysis can focus on more interesting and realistic cases: in reality, the best director candidate in the labor market is more likely than a lower-quality director candidate to create value that fully compensates for CEO moral hazard.

Assumption 2: $C > \Delta$.

their new idea into the boardroom. The key point in both the model setup and this alternative setup is that appointing someone who can think outside the box makes it possible for the firm to pursue a different trajectory from the past.

[27] Despite the imperfect information about the directors' costs of opposing the CEO, there is no belief updating (e.g., signaling, learning) in the model.

Assumption 2 implies that the director with the highest possible cost of opposing the CEO will not oppose the CEO even if the counterproposal is made by the best director that a firm can find. Realistically, among the population distribution of directors, there always exist directors who find it prohibitively costly to oppose the CEO.

2.3 Analysis

In the model, directors face a private cost when opposing the CEO (or a private reward for showing independence if c_0 or c_i is negative). I show that such a cost creates disincentives to support counterproposals in the decision-making stage and thus reduces the expected benefit of the new director, which in turn affects the new director's decision about whether to counterpropose against the CEO, the CEO's decision about whether to propose a new project, and the shareholders' choice of director appointment.

2.3.1 Decision-Making

When there is a better counterproposal than the CEO's proposal, an incumbent director i who has a positive private cost of opposing the CEO (i.e., $c_i > 0$) is reluctant to oppose the CEO and prefers others to do so. However, in the case where N directors vote for the CEO while N directors oppose the CEO, director i is pivotal in determining which proposal gets implemented and director i has incentives to oppose the CEO if the marginal benefit (the difference in firm value between the counterproposal and the CEO's proposal) is greater than the marginal cost (c_i).

The following proposition summarizes the equilibrium in the decision-making stage when $D \in \{LrQ, Best\}$, $\alpha = 1$ and $\beta = 1$, in which any director with a private cost of opposing the CEO below (above) a threshold chooses to vote against (for) the CEO.

Proposition 1 *If $D = LrQ$, $\alpha = 1$ and $\beta = 1$, there is a unique $c^{LrQ} \in (0, \delta)$ (given by Equation A.1 in the appendix) such that any director i with $c_i \leq c^{LrQ}$ will oppose the CEO*

(i.e., vote for $\mathbb{P}^{LrQ}$), and any director j with $c_j > c^{LrQ}$ will vote for $\mathbb{P}$.

If $D = Best$, $\alpha = 1$ and $\beta = 1$, there is a unique $c^{Best} \in (c^{LrQ}, \Delta)$ (given by Equation A.2 in the appendix) such that any director i with $c_i \leq c^{Best}$ will oppose the CEO (i.e., vote for $\mathbb{P}^{Best}$), and any director j with $c_j > c^{Best}$ will vote for $\mathbb{P}$.

The detailed proofs are in Appendix A.1.

Proposition 1 shows that only directors with sufficiently low costs of opposing the CEO will vote for the counterproposal. Any director with a negative cost of opposing (i.e., one who receives a private reward for opposing the CEO) will always support the counterproposal. Conversely, any director who votes for the CEO's proposal must face a strictly positive cost of opposition. Moreover, any director whose cost exceeds the incremental value of the counterproposal relative to the CEO's proposal will always support the CEO. Because (i) both voting for and against the CEO can be dominant strategies, and (ii) the costs of opposing the CEO are heterogeneous and privately known, every director faces a positive probability of being pivotal.[28] Hence, no equilibrium exists in which all directors are expected to vote identically before the realization of their costs, although such an outcome may arise for particular realizations.

A marginal director - one who is indifferent between supporting the counterproposal and the CEO's proposal - is defined by the point where their expected marginal benefit from supporting the counterproposal equals their cost of opposing the CEO. This condition pins down the thresholds c^{LrQ} and c^{Best} in equilibrium. The marginal director's expected benefit depends on the probability of being pivotal (i.e., the probability that exactly N directors vote for and N against). As the marginal director's cost increases, it becomes less likely that other directors have higher costs, reducing the chance of an evenly split vote and thus lowering the expected benefit of voting for the counterproposal. This monotonicity ensures that the threshold cost at which a director is marginal is unique.

[28]This holds for any continuous distribution of opposition costs, provided that costs can be negative and may also be prohibitively large.

Furthermore, if the firm value improvement from a counterproposal relative to the CEO's proposal is larger, the expected marginal benefit of opposing increases for every director, increasing the threshold cost at which indifference occurs. Since $\delta < \Delta$, it follows that $c^{LrQ} < c^{Best}$.

Note that $c^{LrQ} < \delta$ and $c^{Best} < \Delta$. This reflects directors' disincentives to support better alternatives in collective decision-making: a director whose cost of opposing the CEO lies in $(c^{LrQ}, \delta]$ (or $(c^{Best}, \Delta]$) would support $\mathbb{P}^{LrQ}$ (or $\mathbb{P}^{Best}$) if they were the sole voter.

Since the information about the cost of opposing the CEO is private, we do not know if a counterproposal will be implemented before the decision-making stage. Based on the incumbent directors' voting behavior, the new director, the CEO, and the shareholders form an ex ante expectation about which project will be implemented.

Corollary 2 *If $D = LrQ$, $\alpha = 1$ and $\beta = 1$, $\mathbb{P}^{LrQ}$ will be implemented with probability $q^{LrQ} \in (0,1)$ and $\mathbb{P}$ will be implemented with probability $1 - q^{LrQ}$, where*

$$q^{LrQ} = \sum_{k=N+1}^{2N+1} \binom{2N+1}{k} \left(\frac{C - c^{LrQ}}{2C}\right)^{2N+1-k} \left(\frac{c^{LrQ} + C}{2C}\right)^{k} \tag{2.3.1}$$

If $D = Best$, $\alpha = 1$ and $\beta = 1$, $\mathbb{P}^{Best}$ will be implemented with probability $q^{Best} \in (0,1)$ and $\mathbb{P}$ will be implemented with probability $1 - q^{Best}$, where

$$q^{Best} = \sum_{k=N+1}^{2N+1} \binom{2N+1}{k} \left(\frac{C - c^{Best}}{2C}\right)^{2N+1-k} \left(\frac{c^{Best} + C}{2C}\right)^{k} \tag{2.3.2}$$

Corollary 2 shows that when $D \in \{LrQ, Best\}$, $\alpha = 1$ and $\beta = 1$, there is uncertainty about which project will be implemented in the decision-making stage. This uncertainty emerges because each director's cost of opposing the CEO is private information. If the majority of the directors have sufficiently high costs of opposing the CEO, the CEO's proposal will be implemented despite the fact that there is a better counterproposal.

Furthermore, a counterproposal is more likely to win the support of the board if it has

higher quality.

Lemma 3 $q^{Best} > q^{LrQ}$.

The detailed proofs are in Appendix A.2.

According to Proposition 1, a higher δ (Δ) raises the threshold cost c^{LrQ} (c^{Best}), making each director individually more likely to support the counterproposal. Consequently, a higher-quality counterproposal is more likely to win the support of the board.[29]

2.3.2 Proposal and counterproposal

When $D \in \{LrQ, Best\}$, if the new director chooses $\beta = 1$, they face uncertainty about whether their counterproposal will be implemented. This uncertainty affects their expected payoff from counterproposing against the CEO and in turn affects the choice of β. If the expected payoff from counterproposing is smaller than the private cost of opposing the CEO (c_0), the new director will not counterpropose.

Hence, when $D \in \{LrQ, Best\}$, if the CEO chooses to propose a new project in the first place ($\alpha = 1$), they expect that (i) there is a positive probability that the new director will not counterpropose; (ii) even if the new director makes a counterproposal, the counterproposal may not be voted through during the board decision-making.

The proposition below summarizes when the CEO will choose to initiate a proposal and how likely there will be a counterproposal in expectation.

Proposition 4 *If $D = Similar$, the CEO will choose $\alpha = 1$, and there will be no counterproposal.*

If $D = LrQ$, the CEO will choose $\alpha = 1$ if $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and choose $\alpha = 0$ if $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$. When the CEO chooses $\alpha = 1$, they

[29]The probability of a counterproposal passing depends not only on each director's individual likelihood of support but also on the joint distribution of votes across the board. As thresholds rise, unanimous support becomes more likely, but the probability of mixed-majority outcomes - where some directors support the counterproposal while others back the CEO - may fall. I show that any decrease in the probability of mixed-majority outcomes is dominated by an increase in the probability of the other majority outcomes.

expect the new director to counterpropose (i.e., $\beta = 1$) with probability $\frac{1}{2} + \frac{\delta}{2C} q^{LrQ}$.

If $D = Best$, the CEO will choose $\alpha = 1$, and they expect the new director to counterpropose (i.e., $\beta = 1$) with probability $\frac{1}{2} + \frac{\Delta}{2C} q^{Best}$.

The detailed proofs are in Appendix A.3.

When $D = Similar$, the CEO will always propose because their project $\mathbb{P}$ faces no counterproposal and benefits the CEO (albeit at the firm's expense).

When $D = LrQ$, uncertainty arises as to whether the implemented project will be $\mathbb{P}$ or $\mathbb{P}^{LrQ}$. While $\mathbb{P}$ benefits the CEO, $\mathbb{P}^{LrQ}$ yields neither sufficient firm value to compensate for CEO moral hazard nor any private benefits to the CEO. If the probability of $\mathbb{P}^{LrQ}$ being adopted is sufficiently low (i.e., $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$), the CEO will optimally choose $\alpha = 1$. Conditional on this, the new director counterproposes only if their expected payoff exceeds the private cost of opposition (c_0), which occurs with probability $\frac{1}{2} + \frac{\delta}{2C} q^{LrQ}$.

When $D = Best$, there is uncertainty between $\mathbb{P}$ and $\mathbb{P}^{Best}$, but here both projects benefit the CEO: $\mathbb{P}$ provides sufficient private benefits, and $\mathbb{P}^{Best}$ provides enough firm value to reverse the effect of CEO moral hazard. Thus, the CEO always proposes. The new director counterproposes only if their expected payoff exceeds the cost (c_0), with probability $\frac{1}{2} + \frac{\Delta}{2C} q^{Best}$.

2.3.3 Director Appointment

If the shareholders choose $D \in \{LrQ, Best\}$, their appointment can cause uncertainty about the implemented project $\mathbb{P}^*$. From the CEO's perspective, there is a positive probability that their proposal $\mathbb{P}$ with private benefits will not be implemented. From the shareholders' perspective, there is a positive probability that a better counterproposal (than the CEO's proposal) will not be implemented.

Conceptually, shareholders value the potential for new ideas to enhance firm value, but they also prefer to avoid the moral hazard problem from arising in the first place. When

appointing a director, they weigh the potential value that the appointment could generate against its effectiveness in deterring the CEO from pursuing a value-destroying proposal.

The following proposition summarizes the appointment of the equilibrium director.

Proposition 5 *The shareholders will choose $D = Similar$ under no circumstances.*

The shareholders will choose $D = LrQ$ if and only if $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $q^{Best} < \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$. In this case, the CEO will not propose a new firm project ($\alpha = 0$), there will be no counterproposal ($\beta = 0$), and there is no decision to be made.

The shareholders will choose $D = Best$ if and only if either of the following is true:

- $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ *and* $q^{Best} \geq \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$*; or*
- $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$

In either case, the CEO will propose a new firm project ($\alpha = 1$) and they expect the new director to counterpropose ($\beta = 1$) with probability $\frac{1}{2} + \frac{\Delta}{2C} q^{Best}$. When there is a counterproposal, $\mathbb{P}^{Best}$ will be implemented with probability q^{Best} (given by Equation 2.3.2) and $\mathbb{P}$ will be implemented with probability $1 - q^{Best}$.

The detailed proofs are in Appendix A.4.

Firstly, $D = Similar$ is strictly dominated by $D = Best$ and is never chosen by shareholders. While appointing $D = Best$ introduces uncertainty about the implemented project $\mathbb{P}^*$, the chance of implementing a higher-quality counterproposal is strictly positive. Moreover, if the CEO's proposal $\mathbb{P}$ is implemented, the firm value is the same as under $D = Similar$. Thus, the uncertainty does not eliminate the benefits of appointing the best candidate.

This uncertainty, however, creates a trade-off between appointing $D = LrQ$ and $D = Best$. Table 2.1 summarizes (i) the probability distribution of the implemented project conditional on a vote within each proposal–counterproposal pair, (ii) the corresponding payoffs for shareholders and the CEO under the implemented project, and (iii) the sign of each payoff (a minus sign indicates a negative payoff and a plus sign indicates a positive payoff).

Table 2.1: Uncertainty about the implemented project $\mathbb{P}^*$

$\mathbb{P}$ **vs** $\mathbb{P}^{Best}$	$\mathbb{P}$		$\mathbb{P}^{Best}$	
Probability	$1-q^{Best}$		q^{Best}	
Shareholder Payoff	V	[–]	$V+\Delta$	[+]
CEO Payoff	$V+B$	[+]	$V+\Delta$	[+]

$\mathbb{P}$ **vs** $\mathbb{P}^{LrQ}$	$\mathbb{P}$		$\mathbb{P}^{LrQ}$	
Probability	$1-q^{LrQ}$		q^{LrQ}	
Shareholder Payoff	V	[–]	$V+\delta$	[–]
CEO Payoff	$V+B$	[+]	$V+\delta$	[–]

In the model, uncertainty reduces the expected firm value of appointing the best candidate because the new director may not counterpropose, and even if they do, the board may not support it. When the probability of implementing $\mathbb{P}^{Best}$ is sufficiently small, the expected firm value from appointing the best candidate becomes negative. In such cases, shareholders may rationally prefer $D = LrQ$ because appointing a lower-quality candidate deters the CEO from proposing altogether, preserving the status quo $\pi(\emptyset) = 0$.

For $D = LrQ$ to result in $\alpha = 0$, the probability of implementing $\mathbb{P}$ must be small enough - equivalently, q^{LrQ} must be sufficiently large. Otherwise, the CEO will propose a new project ($\alpha = 1$) because the chance of capturing private benefits B from implementing $\mathbb{P}$ is high. Thus, shareholders will choose $D = LrQ$ if and only if q^{LrQ} is large ($q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$) and q^{Best} is small ($q^{Best} < \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$). Here, shareholders weigh the firm value gains from appointing the best candidate against the effectiveness of a lower-quality candidate in deterring a value-destroying proposal.[30]

[30]The deterrence effect arises from the quality of the counterproposal rather than from the new director, whose type is assumed to be synonymous with the quality of their counterproposal in the model. Although the best candidate could, in principle, make a lower-quality counterproposal, they will not do so in equilibrium. Once the CEO has proposed, the new director has no incentive to offer a counterproposal below their own quality. Hence, in equilibrium, the quality of the new director is mapped one-to-one with the quality of their counterproposal.

A key insight of Proposition 5 is that shareholders may rationally choose not to appoint the best candidate ($D = Best$). Even though the best candidate offers the highest potential value, this benefit may not materialize due to the cost of opposing the CEO and directors' disincentives to support better alternatives in collective decision-making. This implies that when the expected benefit of the best candidate is limited, shareholders may, if such candidates are available, bypass those who are only slightly worse and instead appoint a substantially lower-quality director.

A second insight is that a new appointee can preserve firm value even when their contribution is not directly observed. By appointing a lower-quality candidate ($D = LrQ$), shareholders may deter the CEO from initiating a value-destroying proposal. In such cases, the absence of visible action may be mistakenly interpreted as an ineffective or symbolic appointment (e.g., tokenism) in empirical analyses.

According to Proposition 5, whether the shareholders choose $D = LrQ$ or $D = Best$ depends on q^{LrQ} and q^{Best}. Because q^{LrQ} (q^{Best}) is affected by c^{LrQ} (c^{Best}), which is in turn affected by δ (Δ), the following proposition shows *sufficient* conditions in terms of δ and Δ for shareholders to optimally choose either $D = LrQ$ or $D = Best$. An interpretation of Proposition 6 is that we are able to attribute the optimal appointment of directors by shareholders to the quality of director candidates.

Proposition 6 *The shareholders will choose* $D = LrQ$ *if* $B < \frac{4}{3}|V|$, $\Delta < \frac{1}{2}(\sqrt{C(C-8V)} - C)$, *and* δ *approaches* 0.

The shareholders will choose $D = Best$ *if* $\Delta \geq \sqrt{C(C-8V)} - C$.

The detailed proofs are in Appendix A.5.

Proposition 6 examines how changes in δ and Δ affect the conditions under which shareholders optimally choose $D = LrQ$ or $D = Best$ (see Proposition 5). From the decision-making stage, a higher δ (Δ) raises the the probability that the counterproposal wins the majority support of the board, q^{LrQ} (q^{Best}). Hence, the likelihood of implementing a coun-

terproposal increases in director quality.

In the first part of Proposition 6, I show that shareholders will optimally appoint a lower-quality candidate ($D = LrQ$) if the CEO's private benefits from their proposal are small ($B < \frac{4}{3}|V|$), the best candidate's idea quality is limited ($\Delta < \frac{1}{2}(\sqrt{C(C-8V)}-C)$),[31] and the lower-quality candidate's idea adds little value (i.e., $\delta \to 0$). Intuitively, low private benefits make the CEO's proposal less attractive, and a lower-quality candidate of low quality also makes their counterproposal unattractive. In this environment, appointing a lower-quality candidate deters the CEO from proposing a value-destroying project, while appointing the best candidate yields a negative expected value due to limited quality. Thus, shareholders strictly prefer $D = LrQ$.

In the second part of Proposition 6, shareholders optimally appoint the best candidate ($D = Best$) if the quality of the best candidate is sufficiently high ($\Delta \geq \sqrt{C(C-8V)} - C$). High quality ensures that the expected firm value from appointing the best candidate is strictly positive, whereas appointing a lower-quality candidate yields at most zero firm value. Hence, $D = Best$ dominates $D = LrQ$ in this parameter region. This implies that policies or public pressure that push firms to tap into a larger talent pool, thereby raising the quality of the best candidate, can lead to the appointment of the best candidate and an expected increase in firm value.

Appendix B.1 provides simulations for parameter values both satisfying and not satisfying these sufficient conditions. The results show that when both δ and Δ are modest, shareholders are often more likely - but not always - to choose $D = LrQ$ over $D = Best$, consistent with the analytical results.

2.3.4 Welfare

Based on the previous analysis, I measure the ex ante payoffs of shareholders, the CEO, the new director, and the incumbent directors under the optimal appointment D^* in Proposition

[31] Note that $\frac{1}{2}(\sqrt{C(C-8V)} - C) > 0$ because $V \leq 0$.

5. Detailed payoff calculations are in Appendix A.6.

Corporate governance policies often argue that appointing the best candidate - one capable of generating the highest-value ideas - should improve firm performance. When opposing the CEO is costly, however, this intuition may fail. Even under optimal appointments, choosing the best candidate need not outperform the status quo (no new project). Proposition 5 shows that expected firm value under $D^* = Best$ can be negative when $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $q^{Best} < \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$. In this case, directors' disincentives to oppose the CEO in collective decision-making lower the chance that the superior counterproposal is implemented, eroding the benefits of appointing the best candidate - even though $\mathbb{P}^{Best}$ would increase value if adopted.

While the costs of opposing the CEO limit the effectiveness of board appointments, it is CEO moral hazard that ultimately leaves shareholders worse off. For policymakers, tackling this problem - for example, by better aligning the CEO's incentives with shareholders' interests or by making it easier to replace the CEO (e.g., through separation of the CEO and chair roles or caps on executive severance pay) - may be more effective than policies aimed at improving board appointments.

2.4 Role of Moral Hazard

This section examines how the main forces of the model evolve as the severity of CEO moral hazard changes. It also shows why moral hazard is essential for the results, using a benchmark without it as a point of contrast.

The model assumes a classic moral hazard problem between shareholders and the CEO: the CEO has incentives to propose a privately beneficial project at the expense of shareholder value. The severity of this problem is greater when the CEO's proposal $\mathbb{P}$ does more harm to firm value (i.e., V is more negative) and when the CEO's private benefits are larger (i.e., B is more positive).[32] Intuitively, larger $|V|$ (i.e., more negative V) or larger B correspond

[32]The assumption $B > -V$ ensures that the CEO has incentives to propose such a project.

to greater moral hazard severity, so comparative statics with respect to V and B can be interpreted as the impact of moral hazard severity on the equilibrium results.

The following proposition highlights that the moral hazard severity influences the choice of director appointment by the shareholders, who need to take into account the CEO's incentives to propose.

Proposition 7 *For any given parameters δ, Δ, C, N and B, if $V < \min\{V_1, V_2\}$ (V_1 is given by Equation A.3 in the appendix; V_2 is given by Equation A.4 in the appendix), shareholders will choose $D = LrQ$ and the firm will continue business as usual (i.e., $\alpha = 0$). Otherwise, if $V \geq \min\{V_1, V_2\}$, shareholders will choose $D = Best$ and the CEO will propose a new project (i.e., $\alpha = 1$).*

For any given parameters δ, Δ, C, N and V, if $B \leq B_1$ (B_1 is given by Equation A.5 in the appendix) or $B \geq B_2$ (B_2 is given by Equation A.6 in the appendix), shareholders will choose $D = Best$ and the CEO will propose a new project (i.e., $\alpha = 1$). If $B_1 < B < B_2$, the equilibrium director choice depends on V:

- *If $V < V_2$, shareholders will choose $D = LrQ$ and the firm will continue business as usual (i.e., $\alpha = 0$).*
- *If $V \geq V_2$, shareholders will choose $D = Best$ and the CEO will propose a new project (i.e., $\alpha = 1$).*

The detailed proofs are in Appendix A.7.

Intuitively, the severity of moral hazard determines which director shareholders prefer. When V is very negative, they appoint a lower-quality candidate, whose presence deters the CEO from proposing a new project. When B is large, the CEO will propose regardless of director type, since there is always a positive probability of obtaining private benefits; in this case, shareholders prefer the best candidate, who can offer the strongest counterproposal. In contrast, when moral hazard is mild - because either $|V|$ or B is small - shareholders again favor the best candidate, as their counterproposal is much better than the CEO's project.

The above interpretation allows us to attribute the optimal director appointment by shareholders to the severity of the moral hazard problem between the owners and the management.

While moral hazard is common in practice, there are situations where the CEO may instead propose a value-enhancing project ($V > 0$). In such cases, regardless of whether the CEO gains private benefits from the proposal, they do not benefit at the expense of the firm, and the moral hazard problem disappears. I now examine how this absence of moral hazard affects the director appointment.

Corollary 8 *(Benchmark Without Moral Hazard)*

Suppose $V > 0$. *The shareholders will choose* $D = Best$. *The CEO will propose a new firm project* ($\alpha = 1$) *and they expect the new director to counterpropose* ($\beta = 1$) *with probability* $\frac{1}{2} + \frac{\Delta}{2C} q^{Best}$. *When there is a counterproposal,* $\mathbb{P}^{Best}$ *will be implemented with probability* q^{Best} *(given by Equation 2.3.2) and* $\mathbb{P}$ *will be implemented with probability* $1 - q^{Best}$.

This corollary follows directly from Proposition 7 and represents a special case of that result. When the CEO's proposal increases firm value, they will always initiate the project ($\alpha = 1$), regardless of the director type. Without moral hazard, the monitoring benefit of appointing a lower-quality candidate - to deter the CEO from proposing a value-destroying project - no longer applies. Consequently, shareholders no longer face a trade-off between the advisory benefit of appointing the best candidate and the monitoring benefit of appointing a lower-quality candidate. The above highlights that moral hazard is necessary for the appointment of a lower-quality candidate to be valuable.

2.5 Comparative Statics and Implications

This section examines how the main forces of the model vary with key parameters beyond those capturing moral hazard, highlighting additional implications.

2.5.1 Heterogeneity in Costs of Opposing the CEO

The private costs of opposing the CEO - c_0 and $c_i, i \in \{1, ..., 2N+1\}$ - are known only by the directors themselves. It is common knowledge that $c_i \overset{iid}{\sim} \text{Uniform}[-C, C], \forall i \in \{0, ..., 2N+1\}$. A larger variance of this distribution is interpreted as greater heterogeneity in the directors' private costs of opposing the CEO. Because the variance of the uniform distribution is $\frac{1}{3}C^2$, which increases in C, the heterogeneity increases in C. Hence, the comparative statics of the equilibrium results with respect to C can be interpreted as the impact of heterogeneity in the directors' private costs of opposing the CEO on the equilibrium results.

Because the heterogeneity directly affects the identity of the marginal director during a vote (i.e., the cost of opposing the CEO of a director who expects the same payoff from voting for and against the CEO), the next proposition highlights that the heterogeneity affects the equilibrium results by affecting c^{LrQ} and c^{Best}.

Proposition 9 *Both c^{LrQ} and c^{Best} are increasing in C. Both q^{LrQ} and q^{Best} are decreasing in C.*

The detailed proofs are in Appendix A.8.

Suppose that as C increases, the identity of the marginal director in a vote does not change. For this director, a higher C lowers the probability that any other director votes for the counterproposal (i.e., lowers $\frac{c^{LrQ}+C}{2C}$ and $\frac{c^{Best}+C}{2C}$) and raises the probability that any other director votes for the CEO's proposal (i.e., raises $\frac{C-c^{LrQ}}{2C}$ and $\frac{C-c^{Best}}{2C}$), both moving closer to $\frac{1}{2}$. This makes a tie - and thus the marginal director being pivotal - more likely, increasing the expected benefit of supporting the counterproposal beyond their cost. The director is no longer marginal, and the new marginal director has a higher cost of opposing the CEO (i.e., higher c^{LrQ} and c^{Best}).

Because the expected benefit of opposing the CEO falls as the marginal cost rises,[33] c^{LrQ} and c^{Best} rise more slowly than C. As cost heterogeneity grows, the likelihood that any

[33]This follows from the uniqueness of c^{LrQ} and c^{Best} in Proposition 1.

director votes for (or against) the CEO converges toward $\frac{1}{2}$. As voting outcomes approximate random draws, the probability of a majority favoring the counterproposal (q^{LrQ} or q^{Best}) declines.

The key insight of Proposition 9 is that a greater heterogeneity in directors' private costs reduces the chance that a counterproposal is approved - and, in turn, reduces the ex ante probability that the new director will make a counterproposal at all.

Appendix B.2 provides simulations that illustrate these dynamics. When C is small, shareholders tend to prefer $D = LrQ$: a high q^{LrQ} deters the CEO from proposing a new project, making the lower-quality candidate optimal unless $D = Best$ offers strictly positive value. When C is large, q^{LrQ} is low, so the CEO proposes regardless of the appointment, and shareholders tend to favor $D = Best$ since the expected benefits of the best candidate outweigh those of a lower-quality one.

By this point, we have seen how the two central frictions of the model - moral hazard severity and heterogeneity in costs of opposition - each affects shareholders' optimal appointment individually. Appendix B.3 illustrates how the two frictions interact to shape the outcome. Simulations varying both moral hazard severity (V) and opposition costs (C) show that when the moral hazard problem is sufficiently severe, shareholders optimally choose $D = LrQ$, but as cost heterogeneity rises and the probability of board opposition falls, the optimal appointment may switch to $D = Best$.

2.5.2 Board Size

There are $2N+3$ director seats ($N \in \mathbb{Z}^+$). Hence, the comparative statics of the equilibrium results with respect to N can be interpreted as the impact of the size of the board on the equilibrium results.

The board size affects the equilibrium results by affecting the identity of the marginal director during a vote (i.e., c^{LrQ} and c^{Best}).

Proposition 10 *When $C \geq C_1$ (C_1 is given by Equation A.7 in the appendix), a unit*

increase in N will decrease both c^{LrQ} and c^{Best}.

The detailed proofs are in Appendix A.9.

From the intuition of Proposition 9, when C is sufficiently large, directors perceive each other's votes as nearly random between supporting the counterproposal or the CEO's proposal. In this environment, adding more voters (i.e., increasing N) reduces the likelihood that any individual director is pivotal (i.e., the other votes are equally split) in the final vote. As a result, the expected benefit of supporting the counterproposal falls, and the thresholds c^{LrQ} and c^{Best} (defining the marginal directors) decrease. This dynamic is likely relevant in practice, as there almost always exist some directors who face prohibitively high costs of opposing the CEO.

Appendix B.4 provides simulations to illustrate these patterns. For realistic board sizes,[34] both q^{LrQ} and q^{Best} decline slightly as N increases. However, the effect is generally modest, and equilibrium director appointments often remain unchanged across realistic board sizes.

2.6 Director Appointment Based on Cost of Opposing the CEO

I next consider a variation of the model in which shareholders observe candidates' costs of opposing the CEO and base their appointment decision on this information.

The setup is as follows: (i) all candidates have the same quality, so their counterproposals $\hat{\mathbb{P}}$ add value $\nu > 0$ to the CEO's proposal; (ii) shareholders can *coarsely* verify candidates' costs of opposing the CEO (e.g., through search interviews[35]) and appoint the new director based on this information; and (iii) shareholders cannot verify incumbent directors' costs of opposition. The proposal and decision-making stages, as well as the parameter assumptions,

[34]The simulation examines how q^{LrQ}, q^{Best}, and D change for $N \leq 10$. According to Jenter et al. (2023), typical board sizes average between 9 and 11, with larger boards (12-20 members) seen mainly in specific settings like German supervisory boards.

[35]Empirically, such costs are difficult to observe, making appointments based on them harder to test. In contrast, proxies for director quality are more readily available.

remain unchanged.

Formally, the new director can be one of two types: a *reluctant* director with a positive cost of opposing the CEO (i.e., $c_0 = c^+ \overset{iid}{\sim} \text{Uniform}[0, C]$), or a *conflictual* director with a negative cost (i.e., $c_0 = c^- \overset{iid}{\sim} \text{Uniform}[-C, 0]$). The sign of c_0 is publicly observable, but its exact value remains the director's private information. Thus, shareholders choose $D \in \{Reluctant, Conflictual\}$.

Since the decision-making stage is unchanged, if two proposals exist (i.e., $\alpha = 1$ and $\beta = 1$), any director whose private cost of opposing the CEO is below (above) a unique threshold $\hat{c} \in (0, \nu)$ ($\hat{c}$ is given by Equation A.8 in the appendix) will vote against (for) the CEO. The ex ante probability that the counterproposal is implemented is $\hat{q}$ ($\hat{q}$ is given by Equation A.9 in the appendix). Both $\hat{c}$ and $\hat{q}$ increase with ν.

The next proposition shows that shareholders can be indifferent between $D = Conflictual$ and $D = Reluctant$, so they may choose $D = Reluctant$ as a result.

Proposition 11 *The shareholders are indifferent between $D = Conflictual$ and $D = Reluctant$ if and only if $\nu < -V$ and $\hat{q} > \min\left\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\right\}$. In this case, regardless of the director type, the CEO will not propose a new firm project ($\alpha = 0$), there will be no counterproposal ($\beta = 0$), and there is no decision to be made.*

In all other cases, the shareholders will choose $D = Conflictual$:

- *If $\nu < -V$ and $\frac{V+B}{B-\nu} < \hat{q} \leq \min\left\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\right\}$, the CEO will not propose a new firm project ($\alpha = 0$).*
- *If $\nu \geq -V$ or $\hat{q} \leq \frac{V+B}{B-\nu}$, the CEO will propose a new project ($\alpha = 1$); the new director will counterpropose ($\beta = 1$); and the counterproposal $\hat{\mathbb{P}}$ will be implemented with probability $\hat{q}$.*

The detailed proofs are in Appendix A.10.

Intuitively, a conflictual director always counterproposes against the CEO, while a reluctant director may refrain if the cost of doing so is too high. Since both types have the same

quality, the expected advisory benefit of a conflictual director exceeds that of a reluctant director.

A conflictual director is also more effective at deterring value-destroying proposals when $\nu < -V$. In this case, if the counterproposal $\hat{\mathbb{P}}$ is implemented, it generates neither enough firm value to compensate for CEO moral hazard nor sufficient private benefits for the CEO. For any given increase in the probability that $\hat{\mathbb{P}}$ passes (i.e., an increase in $\hat{q}$), the CEO's expected payoff from proposing falls more sharply under $D = Conflictual$, because the expected advisory benefit is greater.

Shareholders are therefore indifferent between $D = Conflictual$ and $D = Reluctant$ if and only if both types deter value-destroying projects. In such cases, we may observe the appointment of a reluctant rather than a conflictual director.[36]

Since ν can be interpreted as the average quality of director candidates, Proposition 11 shows that quality still plays a central role in appointments based on the cost of opposing the CEO. For shareholders to optimally choose $D = Reluctant$, director quality must be intermediate. If ν is too low (approaching 0), a reluctant director will almost never counterpropose, leaving the CEO undeterred from proposing harmful projects. If ν is too high (i.e., $\nu \geq -V$), both $\mathbb{P}$ and $\hat{\mathbb{P}}$ benefit the CEO, so they will always propose regardless of director type. In either case, shareholders prefer $D = Conflictual$ for its greater advisory benefit. Thus, only when director quality is neither too low nor too high may shareholders optimally appoint a reluctant director.

The above interpretation helps explain shareholder activism on board composition. When activists push for board seats, they tend to choose more conflictual directors who are willing to challenge management and broaden the board's strategic vision, as illustrated by Elliott Investment Management's campaign at Phillips 66.[37] Gow et al. (2024) document that fol-

[36] There could be other unmodeled benefits of appointing a reluctant director that break this indifference. For example, à la Adams and Ferreira (2007), being less likely to oppose the CEO may be perceived as friendliness toward management, which in turn can incentivize the CEO to share more information with the board in other decisions where executive information is critical.

[37] "Activist hedge fund Elliott wins two seats on Phillips 66 board" by Oliver Barnes and Alexandra White, *The Financial Times*, May 21, 2025.

lowing the appointment of activist directors, firms undertake more strategic shifts consistent with activist directors' willingness to challenge management. Kang et al. (2022) find that the value gains from activism are more pronounced (i) when activist directors are appointed, relative to activism without board representation, and (ii) in contexts where challenging management is especially salient, such as entrenched boards.

2.7 The CEO Appoints the New Director

I next analyze a variation of the model in which the CEO, rather than shareholders, appoints the new director. This may occur for benign reasons, since appointing directors can be less costly for the CEO: the supply of directors is geographically limited, creating search costs for talent (Knyazeva et al., 2013), and incumbent boards - including CEOs - can often rely on personal connections to reduce these costs (Cai et al., 2022). It may also arise for less benign reasons, such as when boards become captive to CEOs.

The following result summarizes the CEO's preferred director choice given the existing model setup.

Proposition 12 *The CEO will choose $D = Similar$ if and only if $\Delta < B$. In this case, the CEO will propose a new firm project ($\alpha = 1$); since there will be no counterproposal, $\mathbb{P}$ is then implemented.*

The CEO will choose $D = Best$ if and only if $\Delta \geq B$. In this case, the CEO will propose a new firm project ($\alpha = 1$) and they expect the new director to counterpropose ($\beta = 1$) with probability $\frac{1}{2} + \frac{\Delta}{2C} q^{Best}$. When there is a counterproposal, $\mathbb{P}^{Best}$ will be implemented with probability q^{Best} (given by Equation 2.3.2) and $\mathbb{P}$ will be implemented with probability $1 - q^{Best}$.

The detailed proofs are in Appendix A.11.

Given $\delta < -V < B$, appointing a lower-quality candidate introduces a positive probability of a counterproposal, which reduces the CEO's payoff compared to a scenario without one.

Thus, the CEO would never choose $D = LrQ$. When the CEO makes the appointment, their choice D^{CEO} must be either $D^{CEO} = Similar$ or $D^{CEO} = Best$. In both cases, the CEO proposes a new project, so CEO-led appointments never provide the monitoring benefit of deterring value-destroying proposals.

Since $D^{CEO} = Similar$ is never optimal for shareholders, while $D^{CEO} = Best$ can also be optimal for them, alignment arises only when both parties prefer $D = Best$. In all other cases, where the CEO's choice diverges from the shareholders', the shareholders' appointment would generate higher firm value.

In this context, activism to secure shareholder-backed candidates can be profitable. Although activism is costly - Gantchev (2013) estimate that the expenses of influencing board composition (e.g., securing board seats) reduce activist returns by more than two-thirds - top-quartile activists still find it worthwhile, as their higher returns offset these costs. When activists step in to support their candidates, they face the same trade-off identified in the baseline model between the advisory benefit of appointing the best candidate and the monitoring benefit of appointing a lower-quality candidate.

2.8 Empirical Implications

This section summarizes the empirical implications of the model and offers new predictions.

As discussed in the welfare analysis, the effectiveness of director appointments is constrained by the costs of opposing the CEO. Ultimately, the CEO is undeterred from proposing value-destroying projects. This provides a benchmark for interpreting empirical evidence on director appointments: even when shareholders freely choose the best available director, firm value may rise or fall, making heterogeneous outcomes unsurprising. The model highlights one mechanism behind negative effects of director appointments: the cost of opposing the CEO can prevent novel ideas from being implemented, so the advisory benefit may not compensate for CEO moral hazard, leaving the firm worse off than under the status quo.

Consistent with this view, Hsu et al. (2024) show that increased job insecurity for directors reduces investment in long-term innovation.

Propositions 5 and 6 show that director supply shapes equilibrium appointments. The model provides a distinctive prediction: when the best candidate is not sufficiently strong, shareholders may optimally appoint a lower-quality director who adds limited value through new ideas yet effectively deters value-destroying projects. A recent governance trend is that both ESG advocates and opponents have sought leverage through corporate boards, suggesting that boards are increasingly arenas for political contestation.[38] When political views differ among government, shareholders, boards, and management, investors may appoint politically motivated directors to deter management from pursuing projects that risk government backlash and reduce firm value - even if these directors do not contribute the most innovative ideas. On the other hand, if corporate governance policies or heightened public attention to board composition drive firms to draw on a larger director supply and raise the quality of the best available candidate (e.g., Giannetti and Wang (2023), Ferreira et al. (2021)), then appointing the best candidate is optimal for shareholders.

To test the relationship between director supply and appointments, one must measure both director quality and supply empirically. On the one hand, consistent with the multidimensional nature of director appointments, director quality can be measured by the distinctiveness of a director's experiences, skills, and demographic characteristics, as these traits may translate into new ideas. Adams et al. (2018) illustrate this approach using examples from proxy statements that disclose the stated reasons for director appointments, which they use to capture director skills. On the other hand, because the supply of directors is geographically limited, one can construct a local measure of prospective director supply by identifying the pool of current executive officers or directors in nearby firms (Knyazeva et al., 2013).[39]

[38] "Corporate Boards Face ESG Pressure From Both the Left and Right" by Jenness E. Parker and Lauren N. Rosenello, *Skadden commentary*, March 2024.

[39] Although geography-based constraints on director participation have eased under virtual formats, current executives or directors at nearby firms may still represent the main pool of prospective directors. Appointing

More broadly, directors may be appointed not only to generate valuable action but also to deter harmful action. In the latter case, the absence of visible action can itself reflect an increase in firm value relative to the counterfactual, implying that measures of board effectiveness based solely on observable activity or firm performance may overlook this mechanism. Proposition 7 suggests that this measurement challenge is particularly acute in firms with severe CEO moral hazard, where shareholders may deliberately appoint lower-quality candidates to prevent harmful projects.

Beyond highlighting this measurement challenge, Proposition 7 also provides testable cross-sectional predictions: (i) when CEO moral hazard is mild, appointing the best candidate is optimal, leading to more new project approvals after board refreshment than under severe moral hazard; and (ii) when CEO moral hazard is severe, appointing a lower-quality candidate is optimal, resulting in little change in firm performance following board refreshment. The severity of CEO moral hazard can be proxied empirically by CEO ownership or pay structure (Masulis et al., 2009) or by entrenchment measures (Bebchuk et al., 2009).

2.9 Conclusion

This paper examines how director appointments affect firm value through their influence on board behavior. The model highlights two frictions - moral hazard in the CEO's proposals and the costs that discourage directors from opposing the proposals - and shows how these frictions can make it rational for shareholders to avoid appointing the best available candidate. Instead, shareholders may appoint a lower-quality candidate to deter value-destroying proposals. This choice reflects a tension between the advisory and monitoring roles of director appointments. The results provide a positive benchmark for interpreting empirical findings on board composition and generate novel predictions about director selection. Future research could extend the framework by exploring how these frictions interact with other

distant directors who serve remotely may have an ambiguous net effect on governance performance: while synchronous remote board meetings can improve monitoring relative to in-person meetings (Cai et al., 2023), greater geographic distance may raise directors' information-gathering costs (Alam et al., 2014).

governance challenges, such as informational frictions in boardroom communication.

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A Appendix: Proofs

A.1 Proof of Proposition 1

I detail the derivation for Proposition 1.

There are two cases to analyze: (i) $D = LrQ$, $\alpha = 1$, and $\beta = 1$; (ii) $D = Best$, $\alpha = 1$, and $\beta = 1$.

Case (i):

Firstly, note that if $c_i < 0$, director i will vote for $\mathbb{P}^{LrQ}$. When $c_i < 0$, whether director i is pivotal or not, the marginal benefit of voting for $\mathbb{P}^{LrQ}$ is always non-negative, which is greater than the marginal cost of opposing the CEO. Similarly, if $c_i \in (\delta, C]$, the director i will vote for $\mathbb{P}$: Whether the director i is pivotal or not, the marginal benefit of voting for $\mathbb{P}^{LrQ}$ is always less than the marginal cost of opposing the CEO. The above implies that neither of the following can be an equilibrium: (i) all directors vote for $\mathbb{P}^{LrQ}$ regardless of their cost of opposing the CEO and no director is pivotal; (ii) all directors vote for $\mathbb{P}$ regardless of their cost of opposing the CEO and no director is pivotal.

Next, I show that there is a unique pure-strategy threshold equilibrium in which any director i with $c_i \leq c^{LrQ}$ will oppose the CEO (i.e., vote for $\mathbb{P}^{LrQ}$) and any director j with $c_j > c^{LrQ}$ will vote for $\mathbb{P}$. We know from the above that $c^{LrQ} \in [0, \delta]$.

In a threshold equilibrium, a director i with $c_i = c^{LrQ}$ must expect equal payoff from voting for $\mathbb{P}^{LrQ}$ and from voting for $\mathbb{P}$, i.e., the marginal benefit of voting for the counterproposal must equal the marginal cost of opposing the CEO. Although the marginal cost is c^{LrQ}, the marginal benefit depends on the expectation of the director about the probability that they are pivotal. This is because the marginal benefit of voting for the counterproposal would be zero if they are not pivotal, whereas the marginal benefit would be δ (the difference in firm value between $\mathbb{P}^{LrQ}$ and $\mathbb{P}$) if they are pivotal.

Given the knowledge that for any director $j \neq i$, $c_j \overset{iid}{\sim} \text{Uniform}[-C, C]$, the probability that director i is pivotal is the probability that N directors vote for the CEO and N directors

opposing the CEO, i.e., $\binom{2N}{N}\left(\frac{C-c^{LrQ}}{2C}\right)^N\left(\frac{c^{LrQ}+C}{2C}\right)^N$, where $\binom{2N}{N}$ represents the number of combinations among $2N$ directors in which N directors vote for the CEO and N directors oppose the CEO. Hence, c^{LrQ} is pinned down by marginal benefit equal to marginal cost:

$$\binom{2N}{N}\left(\frac{C-c^{LrQ}}{2C}\right)^N\left(\frac{c^{LrQ}+C}{2C}\right)^N\delta = c^{LrQ}. \tag{A.1}$$

Setting $c^{LrQ}=0$, the LHS is greater than the RHS; setting $c_h=\delta$, the LHS is smaller than the RHS. For $c^{LrQ}\geq 0$, the LHS is continuous and strictly decreasing in c^{LrQ}, while the RHS is continuous and strictly increasing in c^{LrQ}. To show that the LHS is strictly decreasing in c^{LrQ}, it is equivalent to showing that the function $f(c^{LrQ})=\left(\frac{C-c^{LrQ}}{2C}\right)^N\left(\frac{c^{LrQ}+C}{2C}\right)^N$ is strictly decreasing in its argument. We can see that $\frac{df}{dc^{LrQ}}<0$ for $c^{LrQ}>0$ because

$$\frac{df}{dc^{LrQ}} = -\frac{N}{2C^2}\left(\frac{C-c^{LrQ}}{2C}\right)^{N-1}\left(\frac{c^{LrQ}+C}{2C}\right)^{N-1}c^{LrQ}<0.$$

Because the LHS is continuous and strictly decreasing in c^{LrQ} while the RHS is continuous and strictly increasing in c^{LrQ}, by the Intermediate Value Theorem, there exists a unique $c^{LrQ}\in(0,\delta)$.

Lastly, I rule out non-threshold equilibria by contradiction.

Suppose that directors with costs of opposing the CEO in some disjoint subsets of $[0,\delta]$ will vote for $\mathbb{P}^{LrQ}$ and directors with costs of opposing the CEO in the complement within $[0,\delta]$ will vote for $\mathbb{P}$. Given such a strategy, every director can have an expectation about any other director's probability of voting for $\mathbb{P}^{LrQ}$ - denote this probability by p. Hence, similar to the analysis of the threshold equilibrium, any director with a cost of opposing the CEO smaller (larger) than $\binom{2N}{N}(1-p)^Np^N\delta$ will vote for $\mathbb{P}^{LrQ}$ ($\mathbb{P}$). However, this is in contradiction with the disjoint subsets of $[0,\delta]$ in which the directors will vote for $\mathbb{P}^{LrQ}$.

Suppose that directors will follow mixed strategies (i.e., mix between voting for $\mathbb{P}^{LrQ}$ and voting for $\mathbb{P}$) if their costs of opposing the CEO fall in some subsets of $[0,\delta]$ with non-zero mass of probability. Again, given any mixed strategies, every director can have an

expectation about any other director's probability of voting for $\mathbb{P}^{LrQ}$ - denote this probability as p'. Hence, similar to the analysis of the threshold equilibrium, only the director(s) with a cost of opposing the CEO $c' = \binom{2N}{N}(1-p')^N(p')^N\delta$ will be indifferent between voting for $\mathbb{P}^{LrQ}$ and voting for $\mathbb{P}$. Because the realization of c' is a zero-probability event, this is in contradiction with the non-zero probability measure of some subsets of $[0, \delta]$ in which directors will follow mixed strategies.

Case (ii):

The analysis is similar to Case (i).

Firstly, note that if $c_i < 0$, director i will vote for $\mathbb{P}^{Best}$; if $c_i \in (\Delta, C]$, director i will vote for $\mathbb{P}$. This implies that neither of the following can be an equilibrium: (i) all directors vote for $\mathbb{P}^{Best}$ regardless of their cost of opposing the CEO and no director is pivotal; (ii) all directors vote for $\mathbb{P}$ regardless of their cost of opposing the CEO and no director is pivotal.

Next, I show that there is a unique pure-strategy threshold equilibrium where any director i with $c_i \leq c^{Best}$ will oppose the CEO (i.e. vote for $\mathbb{P}^{Best}$) and any director j with $c_j > c^{Best}$ will vote for $\mathbb{P}$. We already know $c^{Best} \in [0, \Delta]$.

In a threshold equilibrium, for a director i with $c_i = c^{Best}$, the marginal benefit of voting for the counterproposal must equal the marginal cost of opposing the CEO. Hence, c^{Best} is pinned down by marginal benefit equal to marginal cost:

$$\binom{2N}{N}\left(\frac{C - c^{Best}}{2C}\right)^N\left(\frac{c^{Best} + C}{2C}\right)^N\Delta = c^{Best}. \tag{A.2}$$

Setting $c^{Best} = 0$, the LHS is greater than the RHS; setting $c_H = \Delta$, the LHS is smaller than the RHS. Because for $c^{Best} \geq 0$, the LHS is continuous and strictly decreasing in c^{Best} whereas the RHS is continuous and strictly increasing in c^{Best}, by the Intermediate Value Theorem, there exists a unique $c^{Best} \in (0, \Delta)$.

Similar to Case (i), we can rule out by contradiction: (1) any non-threshold equilibria in which the directors with costs of opposing the CEO in some disjoint subsets of $[0, \Delta]$ will

vote for $\mathbb{P}^{Best}$ and the directors with costs of opposing the CEO in the complement within $[0, \Delta]$ will vote for $\mathbb{P}$; (2) any non-threshold equilibria in which directors will follow mixed strategies if their costs of opposing the CEO fall in some subsets of $[0, \Delta]$ with non-zero mass of probability.

Comparing Case (i) and Case (ii), I next show that $c^{LrQ} < c^{Best}$.

Suppose y is a function of x implicitly defined as follows:

$$\binom{2N}{N}\left(\frac{C-y}{2C}\right)^N\left(\frac{y+C}{2C}\right)^N x - y = 0$$

By the Implicit Function Theorem:

$$y'(x) = \frac{\binom{2N}{N}\left(\frac{C-y}{2C}\right)^N\left(\frac{y+C}{2C}\right)^N}{\frac{Nxy}{2C^2}\binom{2N}{N}\left(\frac{C-y}{2C}\right)^{N-1}\left(\frac{y+C}{2C}\right)^{N-1} + 1}.$$

We can see that $y'(x)$ is positive for $x > 0$ and $y \in (0, C)$.

Replacing (y, x) with (c^{LrQ}, δ) and (c^{Best}, Δ), because $\delta < \Delta$, it follows that $c^{LrQ} < c^{Best}$.

A.2 Proof of Lemma 3

I detail the derivation for Lemma 3.

According to Equations 2.3.1 and 2.3.2, the probability that a counterproposal wins the majority support of the board, q, can be written in the following general form:

$$q = \sum_{k=N+1}^{2N+1}\binom{2N+1}{k}(1-p)^{2N+1-k}p^k = 1 - \sum_{k=0}^{N}\binom{2N+1}{k}(1-p)^{2N+1-k}p^k$$

For $q = q^{LrQ}$, $p = p^{LrQ} \equiv \frac{c^{LrQ}+C}{2C}$; for $q = q^{Best}$, $p = p^{Best} \equiv \frac{c^{Best}+C}{2C}$. Note that $p^{Best} > p^{LrQ} > \frac{1}{2}$ because $c^{Best} > c^{LrQ} > 0$.

For each term in the sum, $f_k(p) = \binom{2N+1}{k}(1-p)^{2N+1-k}p^k$, we can see that $f_k(p)$ is

increasing (decreasing) in p if and only if $p < (>)\frac{k}{2N+1}$ because

$$\frac{df_k(p)}{dp} = \binom{2N+1}{k}(1-p)^{2N-k}p^{k-1}[k - p(2N+1)] > 0$$

if and only if $p < \frac{k}{2N+1}$.

For any $k \leq N$, $\frac{k}{2N+1} < \frac{1}{2} < p$. Hence, for any $k \leq N$, $f_k(p)$ decreases in p. It follows that $q = 1 - \sum_{k=0}^{N} f_k(p)$ increases in p.

A.3 Proof of Proposition 4

I detail the derivation for Proposition 4.

Regardless of the type of the new director, the CEO expects zero payoff from not proposing, i.e., $U_c(\emptyset) = 0$

If the shareholders have chosen $D = Similar$ and the CEO chooses to propose ($\alpha = 1$), there will be no counterproposal and the CEO's proposal $\mathbb{P}$ will be implemented. Hence,

$$\mathbb{E}[U_c(\alpha = 1|D = Similar)] = V + B > 0$$

If the shareholders have chosen $D = LrQ$ and the CEO chooses to propose ($\alpha = 1$), according to Corollary 2, $\mathbb{P}^{LrQ}$ will be implemented with probability q^{LrQ} and $\mathbb{P}$ will be implemented with probability $1 - q^{LrQ}$. Hence, the new director's expected payoff from counterproposing against the CEO is $q^{LrQ}\delta$ and they will not counterpropose ($\beta = 0$) if $c_0 > q^{LrQ}\delta$. Therefore, at the time of the proposal by the CEO, they expect that $\mathbb{P}^{LrQ}$ will be implemented with probability $\frac{q^{LrQ}\delta + C}{2C}q^{LrQ}$ and $\mathbb{P}$ will be implemented with probability $1 - \frac{q^{LrQ}\delta + C}{2C}q^{LrQ}$.

When $D = LrQ$, the CEO's expected payoff from proposing is

$$\mathbb{E}[U_c(\alpha = 1|D = LrQ)] = V + \big[\frac{q^{LrQ}\delta + C}{2C}q^{LrQ}\delta + (1 - \frac{q^{LrQ}\delta + C}{2C}q^{LrQ})B\big].$$

Because $\delta < -V < B$, $\mathbb{E}[U_c(\alpha = 1|D = LrQ)] \geq 0$ if and only if

$$\frac{q^{LrQ}\delta + C}{2C} q^{LrQ} \leq \frac{V + B}{B - \delta}.$$

Equivalently,

$$q^{LrQ} \leq \min\Big\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\Big\}.$$

If the shareholders have chosen $D = Best$ and the CEO chooses to propose ($\alpha = 1$), according to Corollary 2, $\mathbb{P}^{Best}$ will be implemented with probability q^{Best} and $\mathbb{P}$ will be implemented with probability $1 - q^{Best}$. Hence, the new director's expected payoff from counterproposing against the CEO is $q^{Best}\Delta$ and they will not counterpropose ($\beta = 0$) if $c_0 > q^{Best}\Delta$. Therefore, at the time of the proposal by the CEO, they expect that $\mathbb{P}^{Best}$ will be implemented with probability $\frac{q^{Best}\Delta + C}{2C} q^{Best}$ and $\mathbb{P}$ will be implemented with probability $1 - \frac{q^{Best}\Delta + C}{2C} q^{Best}$.

When $D = Best$, the CEO's expected payoff from proposing is

$$\mathbb{E}[U_c(\alpha = 1|D = Best)] = V + \big[\frac{q^{Best}\Delta + C}{2C} q^{Best}\Delta + (1 - \frac{q^{Best}\Delta + C}{2C} q^{Best})B\big].$$

Because $B > -V$ and $\Delta > -V$, it follows that $\mathbb{E}[U_c(\alpha = 1|D = Best)] > 0$.

To summarize the above, if $D \in \{Similar, Best\}$, the CEO will choose $\alpha = 1$; if $D = LrQ$, they will choose $\alpha = 1$ if $q^{LrQ} \leq \min\Big\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\Big\}$ and they will choose $\alpha = 0$ if $q^{LrQ} > \min\Big\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\Big\}$.

A.4 Proof of Proposition 5

I detail the derivation for Proposition 5.

Taking into account the voting behavior of the incumbent directors in the decision-making stage (Proposition 1) and the behavior of the CEO and the new director in the proposal stage (Proposition 4), the shareholders choose $D \in \{Similar, LrQ, Best\}$ in the director

appointment stage.

It is straightforward that if the shareholders choose $D = Similar$,

$$\mathbb{E}[\pi(Similar)] = V < 0$$

If the shareholders choose $D = LrQ$,

$$\mathbb{E}[\pi(LrQ)] = \begin{cases} V + (\frac{1}{2} + \frac{\delta}{2C} q^{LrQ}) q^{LrQ} \delta, & \text{if } q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\} \\ \pi(\emptyset) = 0, & \text{if } q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\} \end{cases}$$

Notice that when $q^{LrQ} = \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$, $\mathbb{E}[U_c(\alpha = 1|D = LrQ)] = V + \left[\frac{q^{LrQ}\delta + C}{2C} q^{LrQ}\delta + (1 - \frac{q^{LrQ}\delta + C}{2C} q^{LrQ})B\right] = 0$. Hence, when $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$, $\mathbb{E}[\pi(LrQ)] = V + (\frac{1}{2} + \frac{\delta}{2C} q^{LrQ}) q^{LrQ}\delta < 0$.

If the shareholders choose $D = Best$,

$$\mathbb{E}[\pi(Best)] = V + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) q^{Best} \Delta$$

Comparing the expected values of the firm dependent on the choice above, it is obvious that choosing $D = Similar$ is a dominated strategy for shareholders.

Between $D = LrQ$ and $D = Best$, shareholders will choose $D = LrQ$ if and only if either of the following is true:

- $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $V + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) q^{Best} \Delta < 0$; or
- $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $V + (\frac{1}{2} + \frac{\delta}{2C} q^{LrQ}) q^{LrQ}\delta > V + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) q^{Best}\Delta$.

The first case is equivalent to $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $q^{Best} < \min\left\{1, \frac{\sqrt{C(C-8V)} - C}{2\Delta}\right\}$.
The second case is equivalent to $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $q^{Best} < \frac{\delta}{\Delta} q^{LrQ}$.
Because $\Delta > \delta$ and $q^{Best} > q^{LrQ}$ (Lemma 3), the second case does not exist.

Shareholders will choose $D = Best$ if and only if either of the following is true:

- $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $V + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) q^{Best} \Delta \geq 0$; or

- $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $V + (\frac{1}{2} + \frac{\Delta}{2C}q^{Best})q^{Best}\Delta \geq V + (\frac{1}{2} + \frac{\delta}{2C}q^{LrQ})q^{LrQ}\delta$.

The first case is equivalent to $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $q^{Best} \geq \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$. The second case is equivalent to $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$.

A.5 Proof of Proposition 6

I detail the derivation for Proposition 6. Firstly, I prove that the shareholders will choose $D = LrQ$ if some sufficient conditions are satisfied. Then, I prove that the shareholders will choose $D = Best$ if a different set of sufficient conditions are satisfied.

Sufficient conditions for $D = LrQ$:

I show that if (i) $B < \frac{4}{3}|V|$; (ii) $\Delta < \frac{1}{2}(\sqrt{C(C-8V)} - C)$; (iii) δ approaches 0, then $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $q^{Best} < \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$. According to Proposition 5, the shareholders will choose $D = LrQ$.

- When $\Delta < \frac{1}{2}(\sqrt{C(C-8V)} - C)$, $\min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\} = 1$. Because $q^{Best} \in (0, 1)$, $q^{Best} < \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$.
- As $\delta \to 0$, according to Equation A.1, $c^{LrQ} \to 0$. According to Equation 2.3.1, $q^{LrQ} \to \sum_{k=N+1}^{2N+1} \binom{2N+1}{k} \left(\frac{1}{2}\right)^{2N+1}$. Because $\binom{2N+1}{k} = \binom{2N+1}{2N+1-k}$ for any $k \in \{N+1, ..., 2N+1\}$ and based on the binomial formula:

$$1 = (\frac{1}{2} + \frac{1}{2})^{2N+1} = \sum_{k=0}^{2N+1} \binom{2N+1}{k} (\frac{1}{2})^{2N+1},$$

we can see that $q^{LrQ} \to \frac{1}{2}$ as $\delta \to 0$. Furthermore, when $B < \frac{4}{3}|V| = -\frac{4}{3}V$, as $\delta \to 0$,

$$
\begin{aligned}
& & 8C(V+B) &< -\delta^2 + (B-2C)\delta + 2CB \\
\Leftrightarrow & & 8C(V+B) &< (\delta + 2C)(-\delta + B) \\
\Leftrightarrow & & \frac{2C(V+B)}{\delta(B-\delta)} &< \frac{\delta + 2C}{4\delta} \\
\Leftrightarrow & & \frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2} &< \big(\frac{1}{2} + \frac{C}{2\delta}\big)^2 \\
\Leftrightarrow & & \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta} &< \frac{1}{2}
\end{aligned}
$$

Hence, $q^{LrQ} > \min\Big\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\Big\}$ as δ approaches 0.

Sufficient conditions for $D = Best$:

I show that if $\Delta \geq \sqrt{C(C-8V)} - C$, then $q^{Best} \geq \min\Big\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\Big\}$ regardless of q^{LrQ}. According to Proposition 5, the shareholders will choose $D = Best$.

Because $\Delta \geq \sqrt{C(C-8V)} - C$, it follows that $\frac{\sqrt{C(C-8V)}-C}{2\Delta} \leq \frac{1}{2}$ and $\min\Big\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\Big\} = \frac{\sqrt{C(C-8V)}-C}{2\Delta}$.

Suppose $x \in (\frac{1}{2}, 1)$. According to the binomial formula:

$$1 = \big(x + 1 - x\big)^{2N+1} = \sum_{k=0}^{N} \binom{2N+1}{k} (1-x)^{2N+1-k} x^k + \sum_{k=N+1}^{2N+1} \binom{2N+1}{k} (1-x)^{2N+1-k} x^k$$

For any $k \in \{N+1, ..., 2N+1\}$,

- $\binom{2N+1}{k} = \binom{2N+1}{2N+1-k}$;
- $(1-x)^{2N+1-k} x^k > (1-x)^k x^{2N+1-k}$ given $x > \frac{1}{2}$.

Hence, $\sum_{k=N+1}^{2N+1} \binom{2N+1}{k} (1-x)^{2N+1-k} x^k > \sum_{k=0}^{N} \binom{2N+1}{k} (1-x)^{2N+1-k} x^k$ and $\sum_{k=N+1}^{2N+1} \binom{2N+1}{k} (1-x)^{2N+1-k} x^k > \frac{1}{2}$.

Replacing x with $\frac{c^{Best}+C}{2C}$, which has a value between $\frac{1}{2}$ and 1, we get $q^{Best} > \frac{1}{2}$. Hence, when $\Delta \geq \sqrt{C(C-8V)} - C$, $q^{Best} > \frac{1}{2} \geq \frac{\sqrt{C(C-8V)}-C}{2\Delta} = \min\Big\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\Big\}$.

A.6 Derivation for the ex ante payoffs of the shareholders and the directors

I detail the derivation for the ex ante payoffs of the shareholders, the CEO, the new director, and the incumbent directors from the perspective of a policymaker who does not know the private costs of opposing the CEO. Under optimal appointment D^* set out in Proposition 5, shareholder payoff is denoted $\mathbb{E}[\pi(D^*)]$; CEO payoff is denoted $\mathbb{E}[U_c(D^*)]$; new director payoff is denoted $\mathbb{E}[U_0(D^*)]$; incumbent director payoff is denoted $\mathbb{E}[U_i(D^*)]$ for any $i \in \{1, ..., 2N+1\}$.

When $D^* = LrQ$:

From Proposition 5, the firm continues business as usual, so $\mathbb{E}[\pi(D^*)] = 0$. In this case, it follows that $\mathbb{E}[U_c(D^*)] = \mathbb{E}[U_0(D^*)] = \mathbb{E}[U_i(D^*)] = 0, \forall i \in \{1, ..., 2N+1\}$.

When $D^* = Best$:

From Proposition 5, $D^* = Best$ if and only if either of the following is true:

- $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $q^{Best} \geq \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$; or
- $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$

In either case, from the proof of Proposition 5 (see Appendix A.4), $\mathbb{E}[\pi(D^*)] = V + (\frac{1}{2} + \frac{\Delta}{2C}q^{Best})q^{Best}\Delta$. Hence, $\mathbb{E}[\pi(D^*)] \geq 0$ if $q^{Best} \geq \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$ and $\mathbb{E}[\pi(D^*)] < 0$ if $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and $q^{Best} < \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$.

From the proof of Proposition 4 (see Appendix A.3), $\mathbb{E}[U_c(D^*)] \geq 0$ when $\alpha = 1$.

From a policymaker's perspective, directors' payoffs combine expected firm value and the expected (positive or negative) payoff from opposing the CEO.

For the new director,

$$
\begin{aligned}
&\mathbb{E}[U_0(D^* = Best)] \\
=&\mathbb{E}[\pi(D^*)] - \Big[\int_{-C}^{q^{Best}\Delta} \frac{1}{2C} x dx + \int_{q^{Best}\Delta}^{C} \frac{1}{2C} \times 0 \times dx\Big] \\
=&\mathbb{E}[\pi(D^*)] - \int_{-C}^{q^{Best}\Delta} \frac{1}{2C} x dx \\
=&\mathbb{E}[\pi(D^*)] + \frac{1}{4C}[C^2 - (q^{Best}\Delta)^2] \\
=&V + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) q^{Best}\Delta + \frac{1}{4C}[C^2 - (q^{Best}\Delta)^2] \\
=&V + \frac{1}{4C}(q^{Best}\Delta)^2 + \frac{1}{2} q^{Best}\Delta + \frac{C}{4}.
\end{aligned}
$$

Hence, $\mathbb{E}[U_0(D^* = Best)]$ increases in $q^{Best}\Delta$. It follows that $\mathbb{E}[U_0(D^*)] \geq 0$ if and only if $q^{Best} \geq \frac{1}{\Delta}(2\sqrt{-CV} - C)$.

Because the policymaker does not observe directors' private costs, they assume the new director is equally likely to have a positive or negative cost. If the new director always counterproposed regardless of cost, opposing the CEO would entail no expected payoff loss. However, the director only counterproposes when their cost is not too high, while always opposing when the cost is negative. Thus, the new director is expected to gain from opposing the CEO. As q^{Best} increases, both the expected firm value and the reward from opposition rise, raising their overall payoff. Hence, the new director is better off than under the status quo (i.e., $\mathbb{E}[U_0(D^*)] \geq 0$) if and only if q^{Best} is sufficiently large.

For any incumbent director $i \in \{1, ..., 2N+1\}$,

$$
\begin{aligned}
&\mathbb{E}[U_i(D^* = Best)] \\
=&\mathbb{E}[\pi(D^*)] - (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) \Big[\int_{-C}^{c^{Best}} \frac{1}{2C} x dx + \int_{c^{Best}}^{C} \frac{1}{2C} \times 0 \times dx \Big] \\
=&\mathbb{E}[\pi(D^*)] - (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) \int_{-C}^{c^{Best}} \frac{1}{2C} x dx \\
=&\mathbb{E}[\pi(D^*)] + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) \frac{1}{4C} [C^2 - (c^{Best})^2] \\
=&V + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) q^{Best} \Delta + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) \frac{1}{4C} [C^2 - (c^{Best})^2] \\
<&V + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) q^{Best} \Delta + (\frac{1}{2} + \frac{\Delta}{2C} q^{Best}) \frac{C}{4} \\
=&V + \frac{1}{2C} (q^{Best} \Delta)^2 + \frac{5}{8} q^{Best} \Delta + \frac{C}{8}.
\end{aligned}
$$

The last expression increases in $q^{Best}\Delta$. Hence, $\mathbb{E}[U_i(D^*)] < 0$ if $q^{Best} < \frac{1}{\Delta} \Big(\sqrt{-2CV + \frac{9}{64} C^2} - \frac{5}{8} C \Big)$.

The same logic that applies to the new director also applies to incumbent directors: they oppose the CEO only when their cost is sufficiently low, so they too expect rewards from opposition when the probability of a vote is positive. Since both the expected firm value and the likelihood of a vote increase with q^{Best}, their overall payoff becomes negative if q^{Best} is very low. However, when q^{Best} is above this threshold, directors are not necessarily better off than the status quo, because a high q^{Best} corresponds to a high c^{Best}. A higher threshold cost of opposing the CEO dilutes the expected reward averaged across all cost levels in $[-C, C]$.

A.7 Proof of Proposition 7

I detail the derivation for Proposition 7.

Based on Equations 2.3.1 and A.1, we see that q^{LrQ} is determined by δ, C, N. Based on Equations 2.3.2 and A.2, we see that q^{Best} is determined by Δ, C, N. Hence, when the parameters (δ, Δ, C, N) are fixed, q^{LrQ} and q^{Best} are also fixed.

Now, I examine how V changes the conditions for shareholders to optimally choose $D = LrQ$ or $D = Best$ (as defined in Proposition 5).

According to Proposition 5:

- If $\sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta} \geq q^{LrQ}$, then the shareholders will choose $D = Best$ and there will be a new project (i.e., $\alpha = 1$). We can rewrite $\sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta} \geq q^{LrQ}$ as $V \geq V_1$, where
$$V_1 \equiv \frac{\delta(B-\delta)}{2C} q^{LrQ}(q^{LrQ} + \frac{C}{\delta}) - B. \tag{A.3}$$
Note that $V_1 < 0$ because $q^{LrQ} < 1$.
- If $\sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta} < q^{LrQ}$ (i.e., $V < V_1$) and $\frac{\sqrt{C(C-8V)}-C}{2\Delta} \leq q^{Best}$, then the shareholders will choose $D = Best$ and there will be a new project (i.e., $\alpha = 1$). We can rewrite $\frac{\sqrt{C(C-8V)}-C}{2\Delta} \leq q^{Best}$ as $V \geq V_2$, where
$$V_2 \equiv -\frac{\Delta q^{Best}}{2}\big(\frac{\Delta q^{Best}}{C} + 1\big). \tag{A.4}$$
- If $\sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta} < q^{LrQ}$ (i.e., $V < V_1$) and $\frac{\sqrt{C(C-8V)}-C}{2\Delta} > q^{Best}$ (i.e., $V < V_2$), then the shareholders will choose $D = LrQ$ and the firm will continue business as usual (i.e., $\alpha = 0$).

We see that if $V < V_1$ and $V < V_2$ - in other words, V is sufficiently negative, then shareholders will choose $D = LrQ$ and the firm will continue business as usual (i.e., $\alpha = 0$).

Next, I show that if B is sufficiently small, then the shareholders will choose $D = Best$ and there will be a new project (i.e., $\alpha = 1$).

Given parameters $(\delta, \Delta, C, N, V)$, according to Proposition 5, if $\frac{\sqrt{C(C-8V)}-C}{2\Delta} \leq q^{Best}$, then the shareholders will choose $D = Best$.

Hence, it is sufficient for the shareholders to choose $D = Best$ if $\frac{\sqrt{C(C+8B)}-C}{2\Delta} \leq q^{Best}$.

We can rewrite $\frac{\sqrt{C(C+8B)}-C}{2\Delta} \leq q^{Best}$ as $B \leq B_1$, where
$$B_1 \equiv \frac{\Delta q^{Best}}{2}\big(\frac{\Delta q^{Best}}{C} + 1\big). \tag{A.5}$$

For completeness, I also list the choices of shareholders and CEO when $B > B_1$. In this case, the choices of shareholders and CEO may also depend on the level of V.

- If $\sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta} \geq q^{LrQ}$, then shareholders will choose $D = Best$ and there will be a new project (i.e., $\alpha = 1$). We can see that $\sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}$ is increasing in B because

$$\frac{d}{dB}\frac{V+B}{B-\delta} = \frac{-(V+\delta)}{(B-\delta)^2} > 0.$$

  Hence, we can rewrite $\sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta} \geq q^{LrQ}$ as $B \geq B_2$, where

$$B_2 \equiv \frac{-2CV - \delta q^{LrQ}(\delta q^{LrQ} + C)}{2C - q^{LrQ}(\delta q^{LrQ} + C)}. \tag{A.6}$$

  Note that $B_2 > -V$ because $\delta < -V$.
- If $\sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta} < q^{LrQ}$ (i.e., $B < B_2$) and $\frac{\sqrt{C(C-8V)}-C}{2\Delta} > q^{Best}$ (i.e., $V < V_2$), then shareholders will choose $D = LrQ$ and the firm will continue business as usual (i.e., $\alpha = 0$).
- If $\sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta} < q^{LrQ}$ (i.e., $B < B_2$) and $\frac{\sqrt{C(C-8V)}-C}{2\Delta} \leq q^{Best}$ (i.e., $V \geq V_2$), then shareholders will choose $D = Best$ and there will be a new project (i.e., $\alpha = 1$).

A.8 Proof of Proposition 9

I detail the derivation for Proposition 9.

To derive $\frac{dc^{LrQ}}{dC}$ and $\frac{dc^{Best}}{dC}$, I set up an auxiliary function. Suppose y is a function of x implicitly defined as follows:

$$\binom{2N}{N}\left(\frac{x-y}{2x}\right)^N\left(\frac{y+x}{2x}\right)^N z - y = 0,$$

where z is a positive constant.

By the Implicit Function Theorem:

$$y'(x) = \frac{\binom{2N}{N}\left(\frac{1}{2} - \frac{y}{2x}\right)^{N-1}\left(\frac{y}{2x} + \frac{1}{2}\right)^{N-1}\frac{Ny^2z}{2x^3}}{\binom{2N}{N}\left(\frac{1}{2} - \frac{y}{2x}\right)^{N-1}\left(\frac{y}{2x} + \frac{1}{2}\right)^{N-1}\frac{Nyz}{2x^2} + 1}.$$

We can see that $y'(x) \in (0, \frac{y}{x})$ for $x > y > 0$.

Replacing (x, y, z) with (C, c^{LrQ}, δ), we get $\frac{dc^{LrQ}}{dC} \in (0, \frac{c^{LrQ}}{C})$. Replacing (x, y, z) with (C, c^{Best}, Δ), we get $\frac{dc^{Best}}{dC} \in (0, \frac{c^{Best}}{C})$.

Next, I examine how q^{LrQ} and q^{Best} change with C. I analyze how q^{LrQ} changes with C in detail. The analysis of how q^{Best} changes with C will be similar.

According to Equation 2.3.1:

$$q^{LrQ} = \sum_{k=N+1}^{2N+1} \binom{2N+1}{k}(1-p)^{2N+1-k}p^k,$$

where $p \equiv \frac{c^{LrQ}(C)+C}{2C}$ and $c^{LrQ}(C)$ is written as a function of C based on Equation A.1. Note that $p > \frac{1}{2}$ because $c^{LrQ} > 0$.

How q^{LrQ} changes with C depends on how p changes with C. We can see that

$$\frac{dp}{dC} = \frac{1}{2C^2}\left(\frac{dc^{LrQ}}{dC}C - c^{LrQ}\right) < 0$$

because $\frac{dc^{LrQ}}{dC} \in (0, \frac{c^{LrQ}}{C})$.

From the proof of Lemma 3 (see Appendix A.2), q^{LrQ} increases in p, which decreases in C. Hence, q^{LrQ} decreases in C.

Following the same steps of analysis of how q^{LrQ} changes with C, one can show that q^{Best} decreases in C.

A.9 Proof of Proposition 10

I detail the derivation for Proposition 10.

Based on Equation A.1:

$$\binom{2N}{N}\Big[\frac{C-c^{LrQ}(N)}{2C}\Big]^N\Big[\frac{c^{LrQ}(N)+C}{2C}\Big]^N\delta = c^{LrQ}(N)$$
$$\binom{2N+2}{N+1}\Big[\frac{C-c^{LrQ}(N+1)}{2C}\Big]^{N+1}\Big[\frac{c^{LrQ}(N+1)+C}{2C}\Big]^{N+1}\delta = c^{LrQ}(N+1)$$

Hence,

$$\begin{aligned}\frac{c^{LrQ}(N+1)}{c^{LrQ}(N)} &= \frac{\binom{2N+2}{N+1}}{\binom{2N}{N}}\frac{\big(\frac{1}{2}\big)^{2N+2}}{\big(\frac{1}{2}\big)^{N}}\frac{\big\{1-\big[\frac{c^{LrQ}(N+1)}{C}\big]^2\big\}^{N+1}}{\big\{1-\big[\frac{c^{LrQ}(N)}{C}\big]^2\big\}^{N}}\\ &= \frac{(2N+2)(2N+1)}{4(N+1)^2}\big\{1-\big[\frac{c^{LrQ}(N)}{C}\big]^2\big\}^{-N}\big\{1-\big[\frac{c^{LrQ}(N+1)}{C}\big]^2\big\}^{N+1}\end{aligned}$$

Because $c^{LrQ}(N+1) < C$ and $\big\{1-\big[\frac{c^{LrQ}(N+1)}{C}\big]^2\big\}^{N+1} < 1$, if $\frac{(2N+2)(2N+1)}{4(N+1)^2}\big\{1-\big[\frac{c^{LrQ}(N)}{C}\big]^2\big\}^{-N} \leq 1$, then $\frac{c^{LrQ}(N+1)}{c^{LrQ}(N)} < 1$. We can rewrite $\frac{(2N+2)(2N+1)}{4(N+1)^2}\big\{1-\big[\frac{c^{LrQ}(N)}{C}\big]^2\big\}^{-N} \leq 1$ as follows:

$$\begin{aligned}&\frac{(2N+2)(2N+1)}{4(N+1)^2}\big\{1-\big[\frac{c^{LrQ}(N)}{C}\big]^2\big\}^{-N} \leq 1\\ \Leftrightarrow\quad &\big[\frac{(2N+2)(2N+1)}{4(N+1)^2}\big]^{\frac{1}{N}} \leq 1-\big[\frac{c^{LrQ}(N)}{C}\big]^2\\ \Leftrightarrow\quad &\frac{c^{LrQ}(N)}{C} \leq \big\{1-\big[\frac{(2N+2)(2N+1)}{4(N+1)^2}\big]^{\frac{1}{N}}\big\}^{\frac{1}{2}}\\ \Leftrightarrow\quad &c^{LrQ}(N) \leq MC,\end{aligned}$$

where $M \equiv \big\{1-\big[\frac{(2N+2)(2N+1)}{4(N+1)^2}\big]^{\frac{1}{N}}\big\}^{\frac{1}{2}}$.

According to Equation A.1, $c^{LrQ}(N) \leq MC$ if and only if

$$\binom{2N}{N}\big(\frac{1}{2}\big)^{2N}(1-M^2)^N\delta \leq MC.$$

Plugging in M, we can see that the above is equivalent to

$$C \geq \binom{2N}{N}\big(\frac{1}{2}\big)^{2N}\frac{(2N+2)(2N+1)}{4(N+1)^2}\delta\big\{1-\big[\frac{(2N+2)(2N+1)}{4(N+1)^2}\big]^{\frac{1}{N}}\big\}^{-\frac{1}{2}}$$

Therefore, if $C \geq \binom{2N}{N}\left(\frac{1}{2}\right)^{2N}\frac{(2N+2)(2N+1)}{4(N+1)^2}\delta\left\{1-\left[\frac{(2N+2)(2N+1)}{4(N+1)^2}\right]^{\frac{1}{N}}\right\}^{-\frac{1}{2}}$, then $c^{LrQ}(N+1) < c^{LrQ}(N)$.

Similarly, based on Equation A.2:

$$\frac{c^{Best}(N+1)}{c^{Best}(N)} = \frac{(2N+2)(2N+1)}{4(N+1)^2}\left\{1-\left[\frac{c^{LrQ}(N)}{C}\right]^2\right\}^{-N}\left\{1-\left[\frac{c^{LrQ}(N+1)}{C}\right]^2\right\}^{N+1}$$

Hence, if $\frac{(2N+2)(2N+1)}{4(N+1)^2}\left\{1-\left[\frac{c^{Best}(N)}{C}\right]^2\right\}^{-N} \leq 1$, then $c^{Best}(N+1) < c^{Best}(N)$.

We can rewrite $\frac{(2N+2)(2N+1)}{4(N+1)^2}\left\{1-\left[\frac{c^{Best}(N)}{C}\right]^2\right\}^{-N} \leq 1$ as $c^{Best}(N) \leq MC$. According to Equation A.2, $c^{Best}(N) \leq MC$ if and only if

$$\binom{2N}{N}\left(\frac{1}{2}\right)^{2N}(1-M^2)^N\Delta \leq MC.$$

Plugging in M, we can see that the above is equivalent to

$$C \geq \binom{2N}{N}\left(\frac{1}{2}\right)^{2N}\frac{(2N+2)(2N+1)}{4(N+1)^2}\Delta\left\{1-\left[\frac{(2N+2)(2N+1)}{4(N+1)^2}\right]^{\frac{1}{N}}\right\}^{-\frac{1}{2}}$$

In summary, both $c^{LrQ}(N+1) < c^{LrQ}(N)$ and $c^{Best}(N+1) < c^{Best}(N)$ if $C \geq C_1$, where

$$C_1 \equiv \binom{2N}{N}\left(\frac{1}{2}\right)^{2N}\frac{(2N+2)(2N+1)}{4(N+1)^2}\Delta\left\{1-\left[\frac{(2N+2)(2N+1)}{4(N+1)^2}\right]^{\frac{1}{N}}\right\}^{-\frac{1}{2}}. \tag{A.7}$$

A.10 Proof of Proposition 11

I detail the derivation for Proposition 11. The model is solved by backward induction.

Decision-making stage:

Following Proposition 1 and its proof in Appendix A.1, we know that when all director candidates have the same quality $\nu > 0$, there is a unique pure-strategy threshold equilibrium in the decision-making stage, in which any director i with $c_i \leq \hat{c}$ will oppose the CEO (i.e. vote for $\hat{\mathbb{P}}$) and any director j with $c_j > \hat{c}$ will vote for $\mathbb{P}$. In this equilibrium, for a director i with $c_i = \hat{c}$, the marginal benefit of voting for the counterproposal must equal the marginal

cost of opposing the CEO. Hence, $\hat{c}$ is pinned down by marginal benefit equal to marginal cost:

$$\binom{2N}{N}\left(\frac{C-\hat{c}}{2C}\right)^N\left(\frac{\hat{c}+C}{2C}\right)^N\nu=\hat{c}. \tag{A.8}$$

By the Intermediate Value Theorem, there exists a unique $\hat{c}\in(0,\nu)$. By the Implicit Function Theorem, $\hat{c}$ increases monotonically in ν.

Following Proposition 2, $\hat{\mathbb{P}}$ will be implemented with probability $\hat{q}\in(0,1)$ and $\mathbb{P}$ will be implemented with probability $1-\hat{q}$, where

$$\hat{q}=\sum_{k=N+1}^{2N+1}\binom{2N+1}{k}\left(\frac{C-\hat{c}}{2C}\right)^{2N+1-k}\left(\frac{\hat{c}+C}{2C}\right)^k. \tag{A.9}$$

Following Lemma 3 and its proof in Appendix A.2, $\hat{q}$ also increases monotonically in ν.

Proposal stage:

Based on the incumbent directors' voting behavior, the CEO and the shareholders face uncertainty about whether the counterproposal $\hat{\mathbb{P}}$ will be made and passed.

If the shareholders have chosen $D=Conflictual$ and the CEO chooses to propose ($\alpha=1$), the new director will counterpropose ($\beta=1$) with certainty because their expected payoff from counterproposing against the CEO is $\hat{q}\nu-c^->0$. Therefore, at the time of the proposal by the CEO, they expect that $\hat{\mathbb{P}}$ will be implemented with probability $\hat{q}$ and $\mathbb{P}$ will be implemented with probability $1-\hat{q}$.

When $D=Conflictual$, the CEO's expected payoff from proposing is

$$\mathbb{E}[U_c(\alpha=1|D=Conflictual)]=V+[\hat{q}\nu+(1-\hat{q})B].$$

If $\nu\geq -V$, then $\mathbb{E}[U_c(\alpha=1|D=Conflictual)]>0$. If $\nu<-V$, then $\mathbb{E}[U_c(\alpha=1|D=Conflictual)]\geq 0$ if and only if

$$\hat{q}\leq\frac{V+B}{B-\nu}.$$

If the shareholders have chosen $D=Reluctant$ and the CEO chooses to propose ($\alpha=1$),

the new director's expected payoff from counterproposing against the CEO is $\hat{q}\nu$ and they will not counterpropose ($\beta = 0$) if $c^+ > \hat{q}\nu$. Therefore, at the time of the proposal by the CEO, they expect that $\hat{\mathbb{P}}$ will be implemented with probability $\frac{\hat{q}^2\nu}{C}$ and $\mathbb{P}$ will be implemented with probability $1 - \frac{\hat{q}^2\nu}{C}$.

When $D = Reluctant$, the CEO's expected payoff from proposing is

$$\mathbb{E}[U_c(\alpha = 1|D = Reluctant)] = V + \big[\frac{\hat{q}^2\nu}{C}\nu + (1 - \frac{\hat{q}^2\nu}{C})B\big].$$

If $\nu \geq -V$, then $\mathbb{E}[U_c(\alpha = 1|D = Reluctant)] > 0$. If $\nu < -V$, then $\mathbb{E}[U_c(\alpha = 1|D = Reluctant)] \geq 0$ if and only if

$$\hat{q} \leq \min\Big\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\Big\}.$$

Director appointment stage:

Taking into account the voting behavior of the incumbent directors in the decision-making stage and the behavior of the CEO and the new director in the proposal stage, the shareholders choose $D \in \{Conflictual, Reluctant\}$ in the director appointment stage.

If the shareholders choose $D = Conflictual$,

$$\mathbb{E}[\pi(Conflictual)] = \begin{cases} V + \hat{q}\nu, & \text{if } \nu \geq -V \\ V + \hat{q}\nu, & \text{if } \nu < -V \text{ and } \hat{q} \leq \frac{V+B}{B-\nu} \\ 0, & \text{if } \nu < -V \text{ and } \hat{q} > \frac{V+B}{B-\nu} \end{cases}$$

Notice that when $\nu < -V$ and $\hat{q} = \frac{V+B}{B-\nu}$, $\mathbb{E}[U_c(\alpha = 1|D = Conflictual)] = V + [\hat{q}\nu + (1 - \hat{q})B] = 0$. Hence, when $\nu < -V$ and $\hat{q} \leq \frac{V+B}{B-\nu}$,, $\mathbb{E}[\pi(Conflictual)] = V + \hat{q}\nu < 0$.

If the shareholders choose $D = Reluctant$,

$$\mathbb{E}[\pi(Reluctant)] = \begin{cases} V + \frac{\hat{q}^2\nu^2}{C}, & \text{if } \nu \geq -V \\ V + \frac{\hat{q}^2\nu^2}{C}, & \text{if } \nu < -V \text{ and } \hat{q} \leq \min\left\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\right\} \\ 0, & \text{if } \nu < -V \text{ and } \hat{q} > \min\left\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\right\} \end{cases}$$

Notice that when $\nu < -V$ and $\hat{q} = \min\left\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\right\}$, $\mathbb{E}[U_c(\alpha = 1|D = Reluctant)] = V + \left[\frac{\hat{q}^2\nu}{C}\nu + (1 - \frac{\hat{q}^2\nu}{C})B\right] \leq 0$. Hence, when $\nu < -V$ and $\hat{q} \leq \min\left\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\right\}$,, $\mathbb{E}[\pi(Reluctant)] = V + \frac{\hat{q}^2\nu^2}{C} < 0$.

Comparing the expected values of the firm dependent on the choice above, shareholders will choose between $D = Reluctant$ and $D = Conflictual$.

- If $\nu \geq -V$, because $\hat{q}\nu > \frac{\hat{q}^2\nu^2}{C}$, then shareholders will choose $D = Conflictual$.
- If $\nu < -V$, then $\min\left\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\right\} > \frac{V+B}{B-\nu}$.[40] Because $\hat{q}\nu > \frac{\hat{q}^2\nu^2}{C}$, shareholders will choose $D = Conflictual$ if $\hat{q} \leq \min\left\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\right\}$ and shareholders will be indifferent between $D = Reluctant$ and $D = Conflictual$ if $\hat{q} > \min\left\{1, \sqrt{\frac{C(V+B)}{\nu(B-\nu)}}\right\}$.

A.11 Proof of Proposition 12

I detail the derivation for Proposition 12.

Firstly, $D = LrQ$ is dominated by $D = Similar$ and thus never chosen by the CEO. This is because whether $\alpha = 1$ or $\alpha = 0$, $\mathbb{E}[U_c(D = LrQ)] < \mathbb{E}[U_c(D = Similar)]$. To see this, if $\alpha = 1$, then $\mathbb{E}[U_c(\alpha = 1|D = LrQ)] = V + \left[\frac{q^{LrQ}\delta + C}{2C}q^{LrQ}\delta + (1 - \frac{q^{LrQ}\delta + C}{2C}q^{LrQ})B\right] < V + B = \mathbb{E}[U_c(D = Similar)]$; if $\alpha = 0$, then $\mathbb{E}[U_c(\alpha = 0|D = LrQ)] = 0 < V + B = \mathbb{E}[U_c(D = Similar)]$.

Next, between $D = Similar$ and $D = Best$, the CEO will choose $D = Best$ if and only if $\mathbb{E}[U_c(D = Best)] \geq \mathbb{E}[U_c(D = Similar)]$. We already know that whether $D = Similar$ or $D = Best$, the CEO will propose a new project (i.e., $\alpha = 1$), so the CEO will choose

[40] To see this, because $\nu < -V$ and $C > \nu$, $\frac{V+B}{B-\nu} < 1 < \frac{C}{\nu}$.

$D = Best$ if and only if

$$V + \big[\frac{q^{Best}\Delta + C}{2C} q^{Best}\Delta + (1 - \frac{q^{Best}\Delta + C}{2C} q^{Best})B\big] \geq V + B,$$

i.e., $\Delta \geq B$.

Comparing this outcome with the optimal appointment by shareholders (Proposition 5), the CEO and shareholders align in their director choices if $\Delta \geq B$ and

- either (i) $q^{LrQ} > \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ with $q^{Best} \geq \min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$,
- or (ii) $q^{LrQ} \leq \min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$.

In all other cases, their choices diverge.

B Appendix: Simulations

B.1 Simulation of equilibrium results with respect to director quality

This section complements Proposition 6 by using simulations to illustrate how equilibrium director appointments vary with δ and Δ under and outside the analytical sufficient conditions. Recall that $\delta < -V < \Delta$.

Firstly, I show that when the sufficient conditions set out in Proposition 6 are satisfied, the director appointment is indeed aligned with the prediction of the proposition.

In Figure 1, I show how D changes with $\delta \in (0, -V)$ and $\Delta > -V$ when $B < \frac{4}{3}|V|$. Consistent with Proposition 6, it is clear that (i) when $\Delta < \frac{1}{2}(\sqrt{C(C-8V)} - C)$ ($\frac{1}{2}(\sqrt{C(C-8V)} - C) \approx 13.72$ in this simulation) and as δ approaches 0, the shareholders will choose $D = LrQ$; (ii) when $\Delta \geq \sqrt{C(C-8V)} - C$ ($\sqrt{C(C-8V)} - C \approx 27.45$ in this simulation), the shareholders will choose $D = Best$ regardless of δ.

Next, I discuss the equilibrium director appointment when the parameters do not satisfy the sufficient conditions set out in Proposition 6. In Figure 1, we already see that (i)

shareholders can still choose $D = LrQ$ even when $\Delta \geq \frac{1}{2}(\sqrt{C(C-8V)} - C)$ or δ is not close to 0; (ii) shareholders can still choose $D = Best$ even when $\Delta < \sqrt{C(C-8V)} - C$.

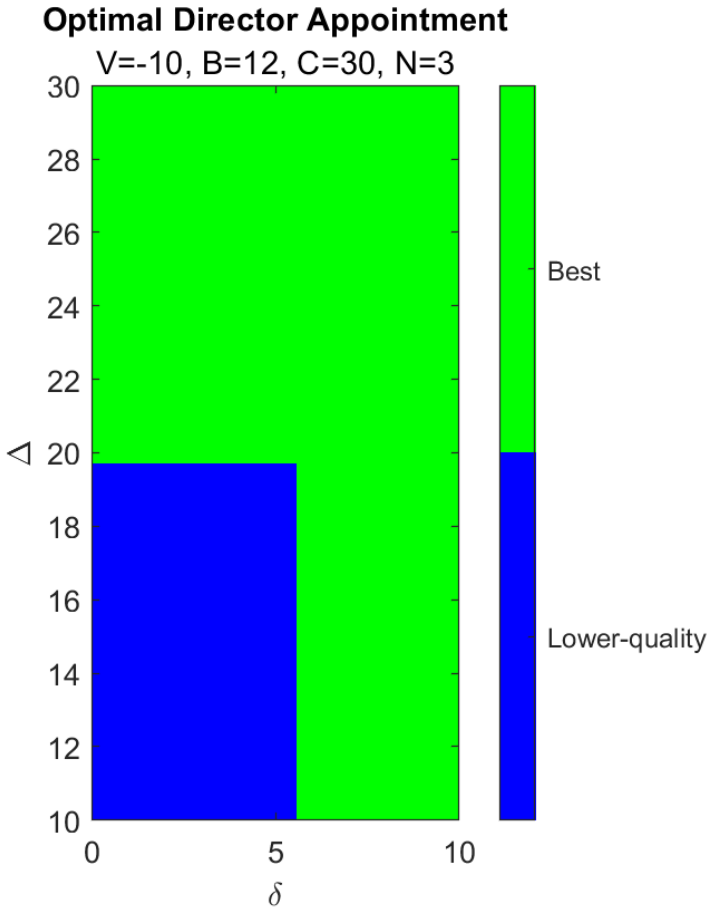


Figure 1: Equilibrium D when $V = -10, B = 12, C = 30, N = 3, \delta \in (0, 10), \Delta \in (10, 30)$

B.2 Simulation of equilibrium results with respect to C

This section complements Proposition 9 by using simulations to illustrate how q^{LrQ}, q^{Best}, and D change with C. Recall that $C > \Delta$.

In Figure 2, I set $V = -10, B = 12, \delta = 5, \Delta = 15, N = 3$. Consistent with Proposition 9, both q^{LrQ} and q^{Best} decrease in C.

In the first subplot, the red dashed line plots how the threshold of q^{LrQ} changes with C: the threshold of q^{LrQ} is $\min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ as set out in Proposition 5. In

the second subplot, the red dashed line plots how the threshold of q^{Best} changes with C: the threshold of q^{Best} is $\min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$ when q^{LrQ} is above its threshold; there is no threshold of q^{Best} when q^{LrQ} is below its threshold because $q^{Best}\Delta > q^{LrQ}\delta$ is always true - I plot $\frac{\delta}{\Delta}q^{LrQ}$ in place to reflect this.

We see that as C increases above a certain value, q^{LrQ} decreases below its threshold. When q^{LrQ} is above its threshold, q^{Best} is below its threshold; when q^{LrQ} is below its threshold, q^{Best} is above its threshold. According to Proposition 5, it follows that as C increases and q^{LrQ} decreases below its threshold, the equilibrium director appointment changes from $D = LrQ$ to $D = Best$.

In this simulation, greater heterogeneity in the directors' private costs of opposing the CEO makes it more likely for the shareholders to appoint the best candidate instead of a lower-quality candidate. Intuitively, greater heterogeneity implies a lower ex ante probability that the counterproposal $\mathbb{P}^{LrQ}$ is approved by the board, as well as a lower ex ante probability that the new director makes the counterproposal against the CEO. When C is sufficiently large, the CEO expects a low enough probability of $\mathbb{P}^{LrQ}$ being implemented and a high enough probability of their own proposal $\mathbb{P}$ being implemented, so they will initiate a new project (i.e., $\alpha = 1$) conditional on $D = LrQ$. Since the CEO will propose whether the shareholders appoint a lower-quality candidate or the best candidate, the shareholders will choose $D = Best$ over $D = LrQ$ because the expected contributions from the best candidate are greater than those from a lower-quality candidate.

B.3 Simulation of equilibrium results with respect to main frictions

The model features two frictions: CEO moral hazard and costs that directors face when opposing the CEO. I have shown analytically how equilibrium director appointments vary with the moral hazard severity (see Proposition 7). I have illustrated how equilibrium director appointments change with C using simulations (see Appendix B.2). This section comple-

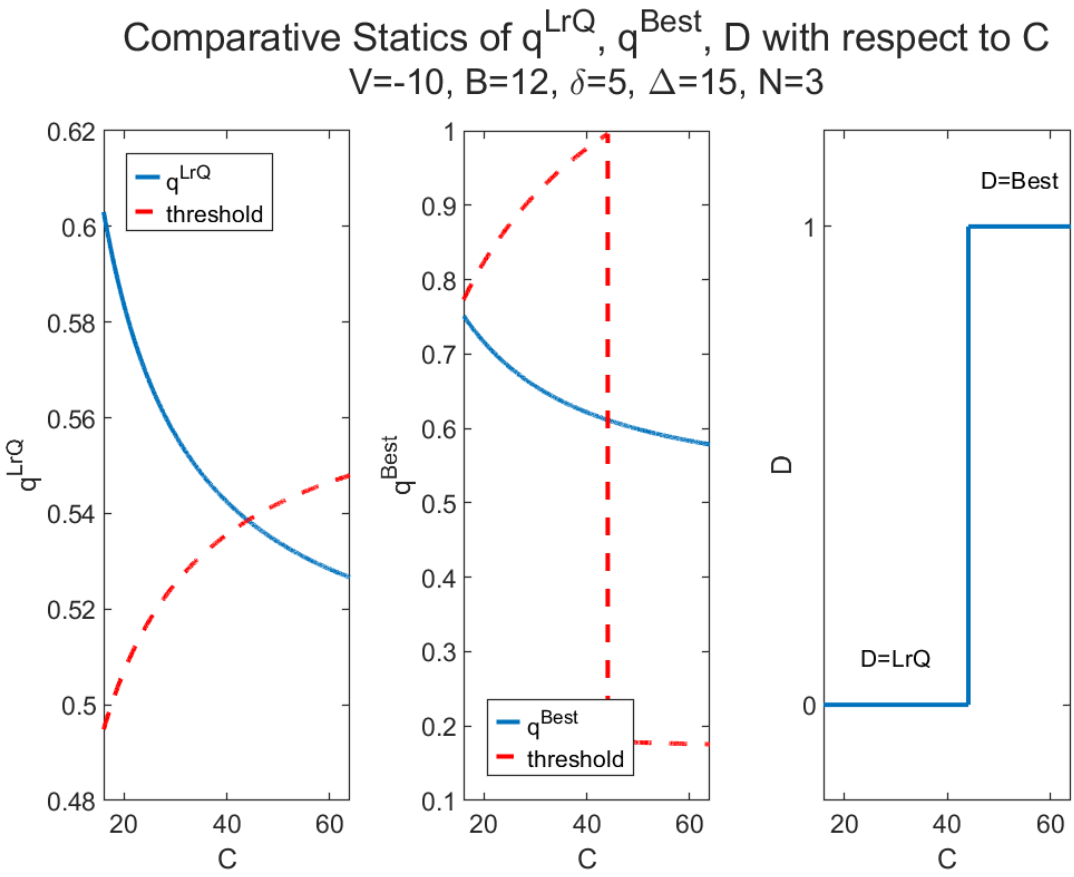


Figure 2: Equilibrium q^{LrQ}, q^{Best} and D when $V = -10, B = 12, \delta = 5, \Delta = 15, N = 3$

ments those results by using simulations to illustrate how equilibrium director appointments vary with V (a parameter that captures moral hazard) and C (a parameter that captures costs of opposition) at the same time. Recall that $|V| < \Delta$ and $C > \Delta$.

In Figure 3, shareholders choose $D = LrQ$ when the moral hazard problem is sufficiently severe (i.e., V is sufficiently negative), as shown in Proposition 7. Proposition 9 further shows that greater cost heterogeneity reduces the probability that a counterproposal passes the board vote. As q^{LrQ} and q^{Best} decline, Proposition 5 implies that shareholders' optimal appointment may switch from $D = LrQ$ to $D = Best$, since a lower-quality appointee may no longer deter the CEO from proposing value-destroying projects.

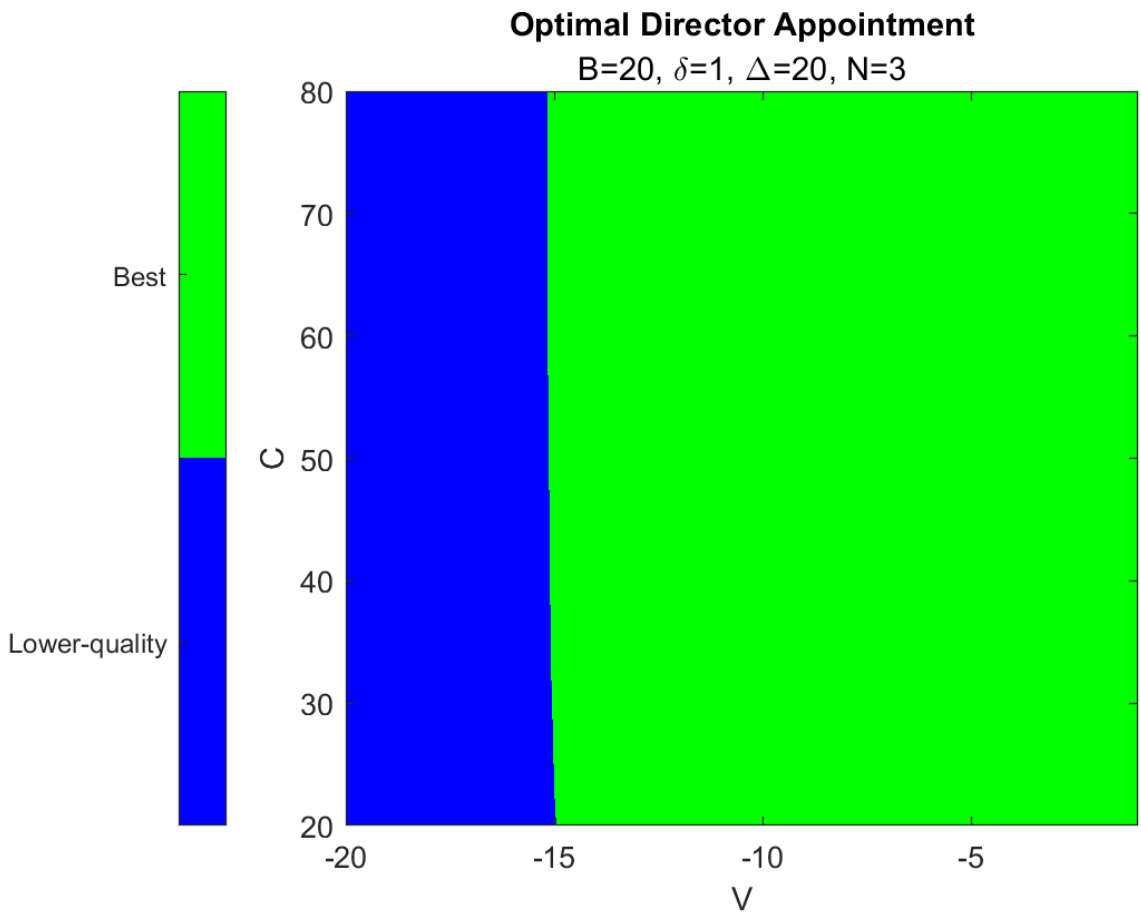


Figure 3: Equilibrium D when $B = 20, \delta = 1, \Delta = 20, N = 3, V \in (-20, -1), C \in (20, 40)$

B.4 Simulation of equilibrium results with respect to N

This section complements Proposition 10 by using simulations to illustrate how q^{LrQ}, q^{Best}, and D change with N.

In Figure 4, I set $V = -10, B = 12, C = 30, \delta = 5, \Delta = 15$. Consistent with Proposition 10, both q^{LrQ} and q^{Best} decrease in N when $C \geq C_1$ ($C_1 \leq C$ when $N \leq 6$ in this simulation). We also see that both q^{LrQ} and q^{Best} decrease in N even when $C < C_1$ (i.e., $N \geq 7$) in this simulation.

In the first subplot, the red dashed line plots the threshold of q^{LrQ}: the threshold of q^{LrQ} is $\min\left\{1, \sqrt{\frac{2C(V+B)}{\delta(B-\delta)} + \frac{C^2}{4\delta^2}} - \frac{C}{2\delta}\right\}$ and does not change with N. In the second subplot, the red dashed line plots the threshold of q^{Best}: the threshold of q^{Best} is $\min\left\{1, \frac{\sqrt{C(C-8V)}-C}{2\Delta}\right\}$,

which does not change with N, when q^{LrQ} is above its threshold; there is no threshold of q^{Best} when q^{LrQ} is below its threshold because $q^{Best}\Delta > q^{LrQ}\delta$ is always true.

We see that for any $N \leq 10$ (i.e., the number of board seats is at most 23), q^{LrQ} is above its threshold and q^{Best} is below its threshold in this simulation. According to Proposition 5, it follows that for any $N \leq 10$, the shareholders will choose $D = LrQ$. In this simulation, even though a larger board size implies a lower ex ante probability that the board will approve the counterproposal $\mathbb{P}^{LrQ}$ as well as a lower ex ante probability that the new director will counterpropose against the CEO, the CEO expects a high enough probability of $\mathbb{P}^{LrQ}$ being implemented for any $N \leq 10$. In other words, the CEO expects a low enough probability of their own proposal $\mathbb{P}$ being implemented for any $N \leq 10$, so they will not initiate a new project (i.e., $\alpha = 0$) conditional on $D = LrQ$. In contrast, the probability of $\mathbb{P}^{Best}$ being implemented is low enough for any $N \leq 10$, so the expected firm value conditional on $D = Best$ is negative for any $N \leq 10$. Therefore, shareholders will choose $D = LrQ$ to prevent the CEO from initiating a value-destroying project.

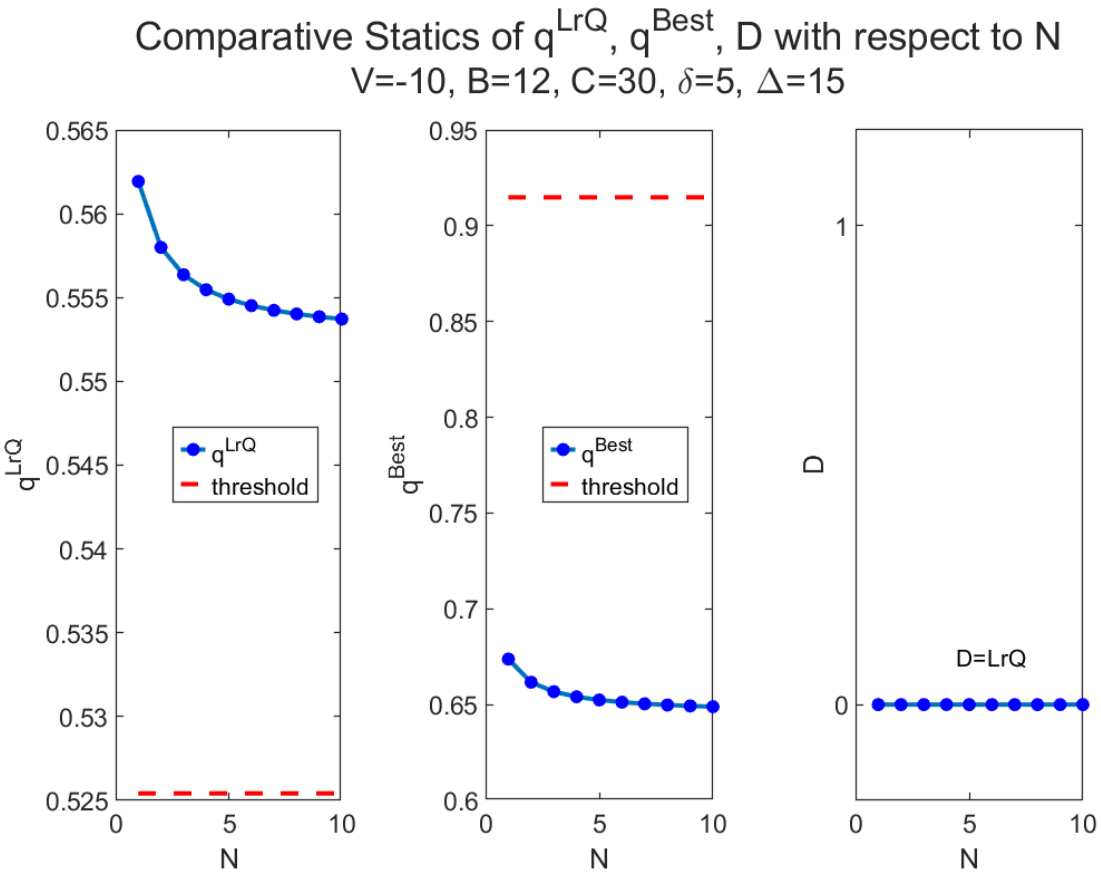


Figure 4: Equilibrium q^{LrQ}, q^{Best} and D when $V = -10, B = 12, C = 30, \delta = 5, \Delta = 15$

Chapter 3

Expanding Shareholder Power through Proxy Access

Ken Deng

Abstract

This paper develops a theory of shareholder power through proxy access, which allows eligible shareholders to nominate directors for inclusion in a company's proxy materials. Although proxy access has been widely adopted following shareholder proposals, it has rarely been used to nominate directors. The model rationalizes this disconnect through an adoption-use trade-off. A low ownership requirement encourages the use of proxy access and improves project choice, but also intensifies managerial resistance to adoption. By contrast, a high ownership requirement discourages use and thereby promotes adoption. In this setting, proxy access can create value not only through shareholder intervention itself but also through adoption-driven benefits such as reputational gains and capital-market effects. Consequently, a restrictive ownership requirement can be welfare-maximizing even when proxy access is rarely used in equilibrium. The model provides a new perspective on the SEC's vacated Rule 14a-11, generates empirical predictions regarding "fix-it" proposals, and yields implications for the optimal design of governance mechanisms that expand shareholder power.

Keywords: Proxy Access, Corporate Governance, Shareholder Power, Governance Regulation, Ownership Threshold

JEL Classification: G34, G38, K22, D72

3.1 Introduction

A central issue in corporate governance is the allocation of power between shareholders and management. Over time, shareholders have gained influence through a range of governance mechanisms, including board declassification, majority voting in director elections (and the elimination of supermajority provisions), and say-on-pay. More recently, proxy access has emerged as a prominent and controversial tool that allows eligible shareholders to nominate director candidates for inclusion in the company's proxy materials, without running a separate proxy contest.[1] Although the SEC's 2010 proxy access rule was vacated, proxy access has since spread through shareholder proposals, with a large majority of S&P 500 firms adopting bylaws with broadly similar terms. Despite its widespread adoption and the intense debate surrounding it, proxy access has rarely, if ever, been used by eligible shareholders to nominate directors.[2]

This paper develops a theoretical framework to rationalize this apparent disconnect between adoption and use. The model highlights that proxy access can generate value upon adoption through channels independent of its subsequent utilization, such as reputational gains and capital-market benefits associated with improved governance (e.g., fund inflows). These adoption-driven benefits coexist with, but are distinct from, the governance channel that operates through actual use.[3] The model treats the magnitude of these adoption-driven benefits as an economic characteristic of the environment and asks how the optimal design

[1] A more recent expansion in the ease of exercising shareholder power is the SEC's universal proxy rule (Rule 14a-19), which applies to contested director elections but, unlike proxy access, still requires dissidents to conduct a proxy solicitation.

[2] According to Gregory et al. (2020), approximately 76% of S&P 500 companies (and 50% of Russell 1000 companies) have adopted proxy access bylaws, representing a substantial increase from less than 1% in 2014. However, proxy access has been used only once to nominate a director in the United States (at The Joint Corp.), with a second attempt subsequently withdrawn.

[3] Adoption-driven benefits may reflect either perceived or real improvements in governance. On the perceived side, Kahan and Rock (2014) argue that if a governance change is largely symbolic, stock price increases following its adoption are plausibly driven by reputational signaling rather than expected director replacement. Consistent with a capital-market channel, empirical evidence shows that improvements in ESG ratings or classifications are associated with increased fund inflows (e.g., Hartzmark and Sussman, 2019; Gantchev et al., 2024). On the real side, the adoption of a governance change can enhance engagement by investors and proxy advisors (Dey et al., 2024), potentially generating valuation effects even if the adopted mechanism (e.g., proxy access) is rarely used.

of proxy access varies with it. As a result, firms may have incentives to adopt proxy access even when its use is not anticipated, helping to explain its broad diffusion in practice. The framework also provides a tractable tool for evaluating counterfactual scenarios, including the effects of a mandatory proxy-access regime and the incentives for shareholders to revise existing proxy-access terms.[4]

The model proceeds in three stages. First, a social planner chooses a standardized ownership requirement for proxy access before firm-specific managerial bias is publicly known. This captures the idea that regulators formulate governance rules at an aggregate level rather than tailoring them to individual firms. Second, in the baseline model, proxy access is not mandatory: firms individually decide whether to adopt it following shareholder proposals. The model features a manager-shareholder agency conflict: managers prefer projects that deviate from shareholder-value maximization, which creates demand for shareholder-backed directors in the first place. At the same time, managers can resist adoption when they expect proxy access to be used, but doing so is costly. Third, eligible shareholders decide whether to use proxy access, but activist shareholders may face trading frictions when building sufficiently large positions to satisfy the ownership requirement. The ownership requirement therefore governs not only whether shareholder intervention occurs, but also whether firms are willing to adopt proxy access in the first place. I later consider a mandatory proxy-access regime as a counterfactual.

The model is grounded in several institutional features of proxy access and shareholder voting. First, proxy-access proposals have converged toward a highly standardized set of terms across firms, particularly the SEC's "3% for 3 years" eligibility criterion. The model rationalizes this standardization by combining dispersed ownership and collective-action frictions with the fact that firm-specific managerial bias is not publicly observable when governance rules are formulated ex ante. Second, although proxy access is intended to increase shareholder influence over board composition, shareholder proposals are non-binding and

[4]Most proxy-access proposals are now "fix-it" proposals, which seek to amend existing bylaws but have not received the same level of support as proposals to adopt proxy access (Glass Lewis, 2021).

managers retain meaningful ability to resist adoption, either by influencing retail shareholder voting or by declining to implement proposals. Third, proxy access is not costless to use: activist shareholders may need to acquire additional shares to become eligible, and these financial frictions affect whether shareholder intervention is sufficiently credible to influence firms' adoption decisions.

A central result of the paper is that the disconnect between adoption and use can itself be welfare-maximizing. The model features an adoption-use trade-off. A low ownership requirement increases the set of firms that both adopt and use proxy access, thereby improving project choice. However, because managers anticipate a loss of control, they are more likely to resist adoption. By contrast, a high ownership requirement discourages use and thereby encourages adoption. Consequently, the optimal ownership requirement is typically located at one of two extremes: either sufficiently low to maximize use among adopting firms or sufficiently high to maximize adoption despite the absence of use.

This result offers a new perspective on the SEC's vacated Rule 14a-11. Although the mandatory proxy-access rule was successfully challenged, the model provides a welfare rationale for a relatively restrictive eligibility criterion of the type proposed by the SEC once subsequent voluntary adoption is taken into account. In particular, the model suggests that a relatively restrictive ownership requirement can optimally function as a focal benchmark around which firms coordinate when adoption is left to individual shareholder proposals. From this perspective, the empirical pattern of widespread adoption but little use is not necessarily evidence of policy failure; instead, it may reflect the welfare-maximizing outcome of the underlying adoption-use trade-off.

The model also generates several empirical and policy implications. First, because the ex-ante optimal ownership requirement need not remain optimal ex post, the model predicts heterogeneous proxy-access-related shareholder proposals across firms. In particular, when the equilibrium features widespread adoption but limited use, shareholders should have incentives to lower ownership requirements at firms that already adopted proxy access while

simultaneously supporting more restrictive thresholds at firms that have not yet adopted. This prediction is consistent with the growing importance of "fix-it" proposals that seek to amend existing proxy-access bylaws.

Second, adoption-driven benefits play a central role in determining the optimal ownership requirement. These benefits include reputational gains, improved governance ratings, increased investor engagement, and capital-market effects such as fund inflows. The model predicts that restrictive ownership requirements are most likely to emerge when adoption-driven benefits are neither too small nor too large. When such benefits are limited, welfare is maximized by encouraging use. When such benefits are sufficiently strong, many firms already adopt proxy access, and ensuring that these adopting firms use proxy access becomes relatively more valuable than further expanding adoption. More broadly, the model suggests that the optimal design of governance mechanisms should respond to the economic benefits generated by their adoption, which may vary across institutional environments and over time even when those benefits do not arise from subsequent use.

Third, while the ownership requirement is the most prominent and widely debated term of proxy access, policymakers may also influence how many directors shareholders can nominate through proxy access. Perhaps surprisingly, the model shows that increasing the effectiveness of proxy access can reduce welfare even though doing so increases the gains from shareholder intervention. The reason is that stronger shareholder intervention also intensifies managerial resistance, thereby reducing adoption and weakening adoption-driven benefits. Consequently, the regulator may optimally prefer proxy access to remain sufficiently limited in effectiveness so as to sustain adoption.

The model further highlights the importance of managerial resistance and financial market frictions in shaping equilibrium outcomes. Resistance by managers is central to generating heterogeneous adoption and use patterns: absent resistance, proxy access with lenient terms would be universally adopted and used. The model therefore sheds light on why mandatory proxy-access rules faced strong opposition from the business community. In addition,

financial market frictions that hinder activists from building sufficiently large positions can increase welfare when equilibrium welfare is driven primarily by adoption rather than use.

The paper also considers several extensions. First, I endogenize the adoption-driven capital-market benefit through an equilibrium in which proxy-access adoption improves an observable governance rating and attracts governance-sensitive investor demand. The resulting value effect depends on adoption itself rather than subsequent use, providing one economic foundation for the adoption-driven benefits in the baseline model. The second extension examines persistent information asymmetry, where managerial bias remains private information even after firms decide whether to adopt proxy access. In this setting, adoption decisions acquire a signaling role. However, the model shows that the adoption-use trade-off remains intact and that signaling may fail to generate separation in equilibrium: adoption can become completely uninformative when all firms adopt proxy access on the premise that it will not be used. The third extension allows activist shareholders to pursue biased objectives that diverge from shareholder-value maximization. In this setting, the relationship between activist bias and the optimal ownership requirement can become non-monotonic. When activist bias is either very low or very high, maximizing adoption without use can remain optimal, whereas intermediate activist bias can make inducing use relatively more valuable.

Related Literature. This paper contributes to the literature on the adoption and evolution of governance structures. As Bhandari et al. (2021) note, this literature broadly reflects two perspectives: some studies view observed governance structures as equilibrium outcomes shaped by optimizing market forces (e.g., Demsetz and Lehn, 1985; Hermalin and Weisbach, 1998), while others emphasize how suboptimal governance arrangements can emerge and persist because of principal-agent conflicts such as managerial bias (e.g., Bebchuk, 2003; Bebchuk, 2004; Schoar and Washington, 2011). This paper does not fit neatly into either perspective. Instead, it offers a framework that incorporates elements of both. While the adoption and use of proxy access are subject to frictions - including managerial bias, manage-

rial resistance, and costly shareholder intervention - and could in principle be improved by mitigating these frictions, the observed pattern of widespread adoption but limited use may nevertheless represent a constrained welfare-maximizing equilibrium from the perspective of a social planner operating under those frictions.

This paper also contributes to the literature specifically examining proxy access. Using event studies, Larcker et al. (2011) and Akyol et al. (2012) conclude that greater shareholder empowerment through proxy access is associated with negative market reactions. By contrast, Jochem (2012), Becker et al. (2013), and Cohn et al. (2016) find generally positive effects of increased shareholder control. The authors attribute these differences primarily to event selection and interpretation. For example, Larcker et al. (2011) interpret Delaware legislation facilitating voluntary proxy access adoption as reducing the likelihood of a mandatory federal rule, whereas Becker et al. (2013) and Cohn et al. (2016) focus on the SEC's October 4, 2010 announcement delaying implementation of proxy access and document negative abnormal returns. More broadly, these studies cannot reject the possibility that market beliefs about the value of shareholder control evolved over the periods they examine.

This paper contributes to this debate by reconsidering what the valuation effects surrounding early proxy-access events actually capture. Given the subsequent pattern of widespread adoption but limited use (Bhandari et al., 2021), positive market reactions to proxy access may reflect adoption-driven benefits through channels independent of actual utilization. From this perspective, the events studied in Larcker et al. (2011) - Delaware legislation facilitating voluntary adoption - may have reinforced expectations that a mandatory proxy-access regime would face strong resistance, leading the SEC to propose an ownership requirement that was ex-ante optimal once adoption was ultimately left to individual firms. In this interpretation, negative returns surrounding such events reflect the infeasibility of simultaneously achieving both widespread adoption and widespread use of proxy access. The seemingly contradictory findings in the literature may therefore be more reconcilable than previously recognized.

More broadly, the paper contributes to the literature documenting that shareholder proposals have become increasingly effective over time (e.g., Ertimur et al., 2011; Renneboog and Szilagyi, 2011; Buchanan et al., 2012). The model suggests that increasing shareholder power through proxy access can be value-enhancing even when adoption occurs through voluntary shareholder proposals rather than mandates. In this respect, the paper contrasts with the corporate-law traditionalist view that regulating shareholder control is unnecessary or potentially harmful because existing governance arrangements already reflect an optimal allocation of control (e.g., Bainbridge, 2005; Strine Jr, 2006).

3.2 The Model

The baseline model considers voluntary, firm-level adoption of proxy access through shareholder proposals. We consider a firm operated by a manager (equivalently, the incumbent board and management are aligned and treated as a single decision-maker). The firm faces a sequence of governance and investment decisions. First, proxy access may be adopted through shareholder proposal. If adopted, eligible shareholders may subsequently use proxy access to nominate directors. Finally, the firm undertakes a project $y \in \mathbb{R}$.

Firm value is

$$V + \mathbb{1}_{\text{adopt}} G - y^2,$$

where $V > 0$ is baseline firm value and $G \geq 0$ is a reduced-form measure of the net benefit that accrues upon adoption of proxy access independently of its subsequent use. Rather than representing a single primitive source of value, G serves as a sufficient statistic for adoption-driven benefits arising through channels such as reputation, capital markets, investor engagement, or expected savings from future proxy contests.[5] The analysis takes

[5]Reputational or capital-market benefits include, for example, expected savings from future proxy contests or increased fund inflows following improvements in governance ratings. These benefits are independent of the subsequent utilization of proxy access and of project choice. Consistent with this interpretation, Bhandari et al. (2021) document a positive market reaction to shareholder-driven proxy-access proposals, despite the fact that proxy access "has yet to be successfully utilized by an eligible shareholder" (Glass Lewis, 2021).

these underlying channels as given and asks how the optimal design of proxy access varies with the magnitude of the resulting adoption-driven benefit.

The quadratic loss normalizes the shareholder-value-maximizing project to $y = 0$ and provides a tractable representation of the value loss from deviations from this benchmark.

Throughout the paper, welfare refers to firm value (hereafter, welfare). Thus, the social planner maximizes value accruing to shareholders rather than the sum of agents' private utilities. Managerial private benefits and the private costs borne by the manager or activist affect welfare indirectly through their effects on equilibrium adoption, use, and project choice, but do not enter the planner's objective directly.

The manager is biased toward a project that deviates from the shareholder-value-maximizing choice. Let $b \geq 0$ denote the manager's bias. The manager's objective is

$$U_M = V + \mathbb{1}_{\text{adopt}} G - (y - b)^2 - \mathbb{1}_{\text{resist}}\, r,$$

where $r \geq 0$ is the manager's *cost of resistance*. The cost r is a reduced-form object that captures the effort, reputational consequences, or other *private* costs the manager incurs (i) when attempting to influence retail shareholders prior to a vote, or (ii) when declining to implement a majority-supported shareholder proposal.[6] For simplicity, the baseline model adopts the first interpretation and assumes non-strategic voting by retail shareholders.[7] Absent governance intervention, the manager chooses $y = b$; shareholders prefer $y = 0$. The parameter b measures the degree of managerial entrenchment.

Ownership is dispersed. An activist shareholder holds stake $\alpha_a \in (0, \frac{1}{2})$; a passive institutional investor ("indexer") holds $\alpha_p \in (0, \frac{1}{2})$ with $\alpha_a + \alpha_p < \frac{1}{2}$. We focus on the case in which both the activist and the indexer are minority shareholders. The remaining shares are held by a continuum of atomistic shareholders.

[6]The model abstracts from any direct firm-level resource costs associated with resistance. Such costs could be incorporated into firm value, but are distinct from the managerial incentive friction captured by r.

[7]One could model a richer voting mechanism with strategic shareholders. However, because shareholder proposals are non-binding, the manager ultimately retains discretion over implementation, so introducing such features would not qualitatively affect the model's implications.

The ownership stakes α_a and α_p are publicly observable when the social planner chooses the standardized ownership requirement. This assumption reflects that the holdings of large institutional and activist investors are generally more transparent than managerial preferences, in part because substantial ownership positions are subject to regulatory disclosure.[8] For tractability, the baseline abstracts from cross-sectional heterogeneity in ownership and instead focuses on heterogeneity in managerial bias.[9]

Shareholders vote on a proxy-access proposal. Votes are proportional to shareholdings, and each shareholder casts all votes either for or against the proposal (no abstention). If the manager chooses to resist - by swaying votes prior to the vote so that the proposal fails - she incurs the cost r as described above. Under this reduced-form voting setup, resistance prevents adoption with certainty, whereas absent resistance, value-maximizing shareholders support the proposal and proxy access is adopted. Thus, the observable adoption outcome reveals whether the manager resisted. Atomistic shareholders are non-strategic and, absent managerial influence, vote to maximize expected firm value.

Due to dispersed ownership and collective-action frictions, proxy-access proposals tend to adopt a common set of terms across firms. We therefore assume that proposals conform to a standardized set of terms advocated by a social planner (e.g., a regulator). The social planner standardizes the terms of proxy access but does not mandate its adoption; individual firms remain free to adopt or resist the proposal.[10]

The model unfolds over four dates. At $t = 0$, the managerial bias is drawn from

$$b \sim \text{Uniform}[0, B].$$

[8] Large ownership positions are often observable through regulatory filings and other ownership disclosures. The assumption is intended to capture the relative observability of ownership structure compared with managerial preferences, rather than to require that every shareholder position be perfectly observed in practice.

[9] Allowing observable ownership stakes to vary across firms would permit the planner to condition the ownership requirement on ownership structure, but would not alter the underlying adoption-use mechanism.

[10] Bhandari et al. (2021) document that proxy-access proposals initially featured heterogeneous terms across firms but subsequently converged to a nearly uniform set of provisions consistent with the ownership requirements of the vacated SEC rule.

The manager privately observes b at $t = 0$. At this stage, the social planner observes the ownership structure (α_a, α_p) but not firm-specific managerial bias and chooses the ownership requirement $\tau \in [\underline{T}, \bar{T}]$ that governs eligibility for proxy access.[11,12] The planner therefore cannot condition τ on b and maximizes expected firm value under the prior.

Before $t = 1$, the realization of b becomes publicly observable as firms are targeted by shareholder proposals and subject to scrutiny. Thus, at the time of voting and thereafter, b is common knowledge.

At $t = 1$, shareholders vote on the proxy-access proposal. If the proposal receives a majority of votes, proxy access is adopted into the firm's bylaws. If the manager instead resists by swaying votes prior to the vote so that the proposal fails, she incurs the cost r.

Proxy access permits any shareholder holding at least τ to nominate directors on the firm's proxy ballot. Both the activist and the indexer are eligible if their ownership stakes exceed τ.

At $t = 2$, eligible shareholders decide whether to use proxy access. The indexer does not trade (e.g., due to mandate) and therefore either meets τ initially or not; if eligible, the indexer can use proxy access at zero cost. The activist may acquire additional shares to meet τ. Let $x \geq 0$ denote the shares purchased from atomistic shareholders. Because there is no information asymmetry at the trading stage, shares are purchased at fair value. The purchase itself therefore generates neither gains nor losses for the activist; the incremental cost of building a larger position is captured by the convex trading cost $\frac{1}{2}\gamma x^2$, where $\gamma > 0$. This trading cost is privately borne by the activist and therefore affects her incentive to use proxy access, but does not enter firm value directly. Atomistic shareholders are infinitesimal and face no trading costs.

[11] According to Bhandari et al. (2021), the ownership requirement is the most prominent proposed term of access. Other terms include the number of years of ownership, whether or not a group of shareholders can join collectively to meet such thresholds, and the number of directors that can be nominated.

[12] When policymakers deliberate new governance rules, the feasible bounds of a proposed ownership threshold are likely to be anchored to existing thresholds associated with shareholder power. For example, $\underline{T}$ may capture the minimum ownership requirement for submitting shareholder proposals, while $\bar{T}$ (typically around 5%) corresponds to the threshold at which an investor is regarded as a substantial shareholder, potentially seeking influence or control, and becomes subject to mandatory disclosure requirements.

When both the indexer and the activist are eligible and willing to use proxy access, their intervention has the same effect on project choice. I therefore label the indexer as the user; this convention is also natural when the activist would otherwise need to incur trading costs to become eligible.

If an eligible shareholder uses proxy access to nominate directors, governance is partially shifted toward shareholders: with probability $s \in (0,1)$, shareholder intervention is effective and the project $y = 0$ is implemented; with probability $1 - s$, the manager retains control and chooses $y = b$. The parameter s captures the effectiveness of shareholder intervention through proxy access, rather than the mechanical probability that a shareholder nominee is elected. In practice, because proxy access typically permits only a minority of board seats to be filled (e.g., 20–25%), we take $s < 1$.[13] If proxy access is either not adopted or adopted but not used, the manager retains full control and chooses $y = b$ with probability one.

At $t = 3$ the project is implemented.

The activist's payoff is

$$U_A = \alpha_a\big(V + \mathbb{1}_{\text{adopt}} G - y^2\big) - \tfrac{1}{2}\gamma x^2,$$[14]

and the indexer's payoff is

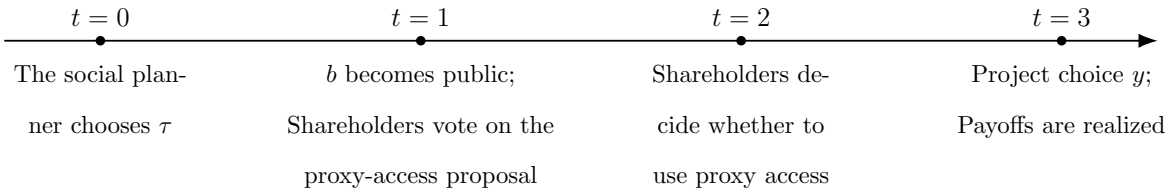
$$U_P = \alpha_p\big(V + \mathbb{1}_{\text{adopt}} G - y^2\big).$$

Atomistic shareholders have infinitesimal individual payoffs.

Now I summarize the timeline of the game.

[13] One can microfound the probabilistic outcome of board decision-making using the framework of Deng (2025), in which directors incur private costs to vote against the manager's preferred project and in favor of the value-maximizing alternative.

[14] The activist's payoff is expressed net of the fair-value purchase of additional shares: because these shares generate no trading surplus, only the value effect on the activist's initial stake and the cost of building the additional position enter the payoff.



The equilibrium concept is Perfect Bayesian Equilibrium (PBE).[15] An equilibrium specifies the social planner's choice of the ownership requirement, the proxy-access adoption decision, the activist's trading and use decisions, the indexer's use decision, and the resulting project choice, such that all agents maximize their expected payoffs given their beliefs and strategies. The model is solved by backward induction.

Finally, I impose parameter restrictions that ensure the model contains non-degenerate regions of both adoption and use. First, we assume $\alpha_p > \alpha_a > \underline{T}$, capturing that passive institutional investors typically hold larger stakes than activist investors. Second, we assume

$$B > \sqrt{\frac{\gamma}{2\alpha_a s}}(\alpha_p - \alpha_a),$$

so that eligibility and use decisions are non-degenerate, i.e., it is not always the case that either the indexer or the activist satisfies the incentive and ownership requirements to use proxy access. Third, we assume

$$B > \sqrt{\frac{G + r}{s}},$$

which ensures that the adoption decision is also non-degenerate. Together, these conditions guarantee interior regions in which both adoption and use decisions are economically meaningful.

[15] In the baseline model, managerial bias becomes publicly observable before adoption, so equilibrium beliefs play no substantive role after the planner's ex-ante choice. Belief updating becomes relevant in the extension with persistent information asymmetry.

3.3 Equilibrium Analysis

Under voluntary adoption, the manager's decision to resist a proxy-access proposal depends on her expectations about whether shareholders will subsequently use it. This strategic interaction generates a trade-off between the adoption and use of proxy access. The social planner, who chooses the ownership requirement - the most prominent term of proxy access - to maximize ex-ante welfare (i.e., firm value), internalizes this trade-off.

3.3.1 Trade-off Between Adoption and Use

When proxy access is adopted, the indexer uses it if and only if its initial stake satisfies the ownership requirement (i.e., $\tau \leq \alpha_p$), as the indexer does not trade due to mandate constraints. If the indexer is not eligible, the activist can acquire additional shares to meet τ and become eligible. The activist has incentives to build a larger position and use proxy access if and only if the expected benefit from using proxy access exceeds the associated trading costs:

$$\alpha_a[V + G - (1 - s)b^2] - \alpha_a(V + G - b^2) \geq \frac{1}{2}\gamma(\tau - \alpha_a)^2.$$

Hence, when proxy access is adopted, it is used if and only if

$$\tau \leq \max\left\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\, b\right\}.$$

If the ownership requirement is lenient, i.e.,

$$\tau \leq \max\left\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\, b\right\},$$

the manager expects that proxy access will be used once adopted. This prospect dilutes managerial control over project choice, and the manager may therefore sway retail share-

holders to vote against adoption at cost r. The manager blocks adoption if and only if the benefit of doing so outweighs the cost:

$$sb^2 \geq G + r.$$

The benefit of blocking adoption arises from retaining full control over project choice, while the cost consists of the vote-influencing cost (e.g., shareholder outreach or campaigns) and the foregone adoption benefit G.

Hence, under a lenient ownership requirement, only firms with sufficiently low managerial bias, i.e., $b < \sqrt{\frac{G+r}{s}}$, adopt proxy access and expect it to be used.

If the ownership requirement is restrictive, i.e.,

$$\tau > \max\left\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\, b\right\},$$

proxy access is not used despite adoption. Importantly, non-use is *credible* rather than merely anticipated: given the ownership requirement, neither the indexer nor the activist has an incentive to use proxy access. Because adoption itself generates benefits (e.g., reputational gains, improved governance ratings, or capital inflows), shareholders support the proposal and the manager has no incentive to sway votes. Hence, under a restrictive ownership requirement, firms with

$$b < \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$$

adopt proxy access and expect it *not* to be used.

The set of firms that adopt proxy access may be larger under a restrictive ownership requirement than under a lenient one. This reflects a trade-off between adoption and use: higher adoption comes at the cost that adopted firms may not use proxy access to improve project choice.

3.3.2 Optimal Ownership Requirement

Ex ante, the social planner does not observe firm-specific managerial bias when setting policy. For example, the SEC does not target particular firms when determining the terms of proxy access rules. In the model, the social planner chooses a standardized ownership requirement τ^* to maximize expected firm value:

$$\mathbb{E}\left(V + \mathbb{1}_{\text{adopt}} G - y^2\right).$$

Consistent with the welfare criterion defined above, the planner does not directly internalize the manager's private benefits, the manager's private cost of resistance, or the activist's trading costs. These objects nevertheless affect welfare indirectly by shaping equilibrium adoption, use, and project choice. Because the adoption of proxy access is not mandatory, the planner accounts for the fraction of firms that will adopt shareholder proposals under a given ownership requirement, in addition to the benefits from using proxy access to improve project choices at the firm level. Consequently, the social planner internalizes the trade-off between adoption and use.

The following proposition characterizes the optimal ownership requirement in light of this trade-off.

Proposition 1 *The optimal ownership requirement τ^* is characterized as follows:*

(i) If $\alpha_p \geq \bar{T}$, then the social planner is indifferent over all $\tau^* \in [\underline{T}, \bar{T}]$.

(ii) If $\alpha_p < \bar{T}$ and

$$\alpha_a + \left[\tfrac{1}{3}\left(4 + \tfrac{r}{G}\right)\right] \sqrt{\tfrac{2\alpha_a(G+r)}{\gamma}} \geq \bar{T},$$

then the social planner is indifferent over all $\tau^* \in [\underline{T}, \alpha_p]$.

(iii) If

$$\max\left\{\alpha_p,\ \alpha_a + \left[\tfrac{1}{3}\left(4 + \tfrac{r}{G}\right)\right] \sqrt{\tfrac{2\alpha_a(G+r)}{\gamma}}\right\} < \bar{T},$$

then the social planner chooses $\tau^* = \bar{T}$.

The proofs are provided in Appendix A.1.

Proposition 1 shows that the optimal ownership requirement may exhibit a corner solution. Case (i) is a benchmark in which passive institutional ownership is sufficiently large that the ownership requirement does not affect adoption or use. When passive institutional ownership is limited, Cases (ii) and (iii) characterize the two sides of the adoption-use trade-off. In Case (ii), the optimal ownership requirement is low, ensuring that proxy access is used whenever it is adopted. In Case (iii), the optimal ownership requirement is high, increasing the fraction of firms that adopt proxy access even though proxy access is not expected to be used.

The distinction between Cases (ii) and (iii) reflects how the ownership requirement shapes both adoption and use. A low ownership requirement ensures that proxy access is used whenever it is adopted, thereby maximizing the governance benefits from improved project choice. However, this comes at the cost of reduced adoption, as managers anticipate a loss of control and are more likely to resist. By contrast, a high ownership requirement eliminates the use of proxy access in equilibrium, which alleviates managerial resistance and increases the fraction of firms that adopt. In this case, welfare is driven primarily by adoption-driven benefits rather than improvements in project choice. The planner therefore chooses between maximizing use and maximizing adoption, producing the corner solution characterized in Cases (ii) and (iii).

Figure 3.1 illustrates the ownership-requirement ranges in the economically interesting case in which the activist's use cutoff exceeds the indexer's initial stake, i.e., the indexer becomes ineligible before the activist loses the incentive to build a sufficiently large position. The intermediate range is dominated, so the optimal ownership requirement is either low or high.

Therefore, Proposition 1 therefore provides a rationale for relatively restrictive proxy-access eligibility criteria of the type embodied in the SEC's "3% for 3 years" provision under

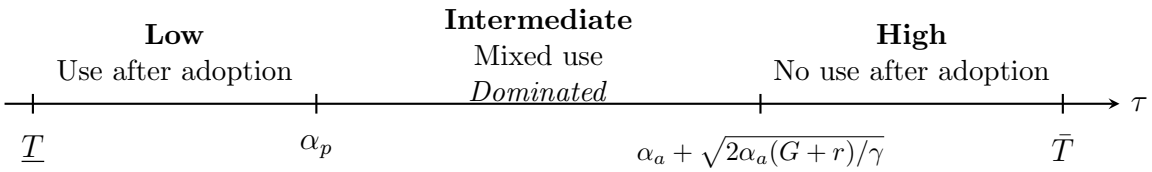


Figure 3.1: Ownership-requirement ranges when the activist's use cutoff exceeds the indexer's initial stake. The intermediate range is dominated, so the optimal ownership requirement is either low, sustaining use after adoption, or high, promoting adoption without use.

Exchange Act Rule 14a-11.[16] The model does not establish the optimality of the specific 3% threshold or three-year holding period; rather, it shows that a sufficiently restrictive eligibility criterion can maximize welfare by promoting adoption even when proxy access is not expected to be used. Although the proxy access mandate under Rule 14a-11 was vacated in 2011 following challenges by the Business Roundtable and the U.S. Chamber of Commerce, the "3% for 3 years" requirement nonetheless provided a focal, standardized term around which investors could coordinate in an environment characterized by dispersed ownership and collective-action frictions. As proxy access has subsequently been adopted voluntarily, the convergence of firm-level shareholder proposals toward these terms can be understood through the model as coordination around a benchmark consistent with the adoption side of the adoption-use trade-off.

Furthermore, perhaps surprisingly, the rationale underlying the optimal ownership requirement in Proposition 1 extends to settings in which the activist or the indexer, rather than the social planner, chooses the ownership requirement.

Corollary 2 *If the activist or the indexer chooses τ ex ante, they select the same τ^* as the social planner in Proposition 1.*

Intuitively, the indexer faces essentially the same problem as the social planner - maximizing expected firm value - because their preferences are aligned, up to a scaling by the

[16]The model abstracts from the holding-period dimension of proxy-access eligibility. Accordingly, the SEC's "3% for 3 years" provision should be interpreted as a composite eligibility criterion, of which τ captures the ownership-threshold dimension. A longer holding period, like a higher ownership threshold, makes eligibility more restrictive, but the model does not impose a quantitative mapping between the two.

indexer's ownership share. By contrast, the activist's private payoff also reflects the trading costs required to meet the ownership requirement and use proxy access. Despite this difference in incentives, the activist's preferred ownership requirement exhibits the same corner-solution structure.

The reason is that the activist faces a similar adoption-use trade-off, but with an additional cost of exercising proxy access. As discussed above, a low ownership requirement ($\tau \leq \alpha_p$) ensures that proxy access is used without requiring the activist to incur trading costs, as the indexer is eligible to act. By contrast, an intermediate ownership requirement - under which the indexer is ineligible but the activist may build a sufficiently large position to use proxy access - is dominated: it both reduces the likelihood of use and, when proxy access is used, requires the activist to incur trading costs. Finally, a high ownership requirement eliminates use altogether, thereby avoiding trading costs and shifting the trade-off toward adoption-driven benefits. Consequently, despite differing payoffs, the activist optimally chooses an ownership requirement at one of the extremes.

This model highlights a novel implication. Although the SEC's Rule 14a-11 was vacated, the model provides a rationale for the relatively restrictive eligibility criterion it proposed once subsequent voluntary adoption is taken into account. In particular, the 2011 amendment to Rule 14a-8, which facilitated shareholder proposals for proxy access, effectively transformed the original mandate into a benchmark around which firms coordinate. From this perspective, a relatively restrictive eligibility criterion can be welfare-enhancing ex ante, as it mitigates resistance and promotes adoption, even if proxy access is rarely used. This mechanism helps rationalize the empirical pattern of widespread adoption but limited use (Bhandari et al., 2021). As the next section shows, a mandatory regime would face stronger resistance, while voluntary adoption based on a standardized threshold naturally gives rise to incentives for ex post adjustments, consistent with the "fix-it" proposals observed in practice.

3.4 Implications

This section evaluates counterfactual policies, including the vacated proxy access mandate, and sheds light on ongoing shareholder initiatives related to proxy access. It also examines how incentives in the model vary with key parameters, yielding insights into policies that may enhance welfare.

3.4.1 Proxy Access Mandate

The baseline model considers voluntary adoption, under which individual firms can resist proxy-access proposals. I now consider the counterfactual in which proxy access is successfully mandated and firms can no longer resist adoption. This comparison sheds light on how the optimal ownership requirement changes when adoption is no longer endogenous and on why Rule 14a-11 faced strong opposition from the business community.

Because a successful mandate eliminates firms' ability to resist adoption, the next proposition characterizes the optimal ownership requirement under this counterfactual regime.

Proposition 3 *Suppose proxy access can be successfully mandated. The social planner optimally chooses a lenient ownership requirement, and is indifferent over all* $\tau^* \in [\underline{T}, \alpha_p]$. *Ex ante, the fraction of firm managers who are worse off under the mandate is* $1 - \frac{1}{B}\sqrt{\frac{G}{s}}$.

The detailed derivations are provided in Appendix A.2.

Intuitively, adoption is no longer endogenous under a mandate, and the adoption-use trade-off identified in Proposition 1 disappears. Absent this trade-off, the ownership requirement that maximizes welfare is the one that maximizes use - namely, a threshold that allows the indexer to use proxy access without incurring costs, i.e., $\tau^* \in [\underline{T}, \alpha_p]$.

While a firm manager is never worse off under voluntary adoption - since they can resist if adoption would reduce their payoff - the mandate makes managers with sufficiently large bias worse off. This allows the social planner to form ex ante expectations about the extent of opposition from the business community, captured by the fraction of managers who are

adversely affected. In particular, the mandate affects a larger set of firms than those that choose not to adopt proxy access under voluntary adoption. Consequently, mandating proxy access would face stronger opposition from businesses than a regime in which firms decide individually whether to adopt. This mechanism is consistent with the strong opposition to the SEC's Rule 14a-11 from the Business Roundtable and the U.S. Chamber of Commerce. More broadly, the result raises the possibility that anticipated resistance to expanding shareholder power may itself be relevant to the initial design of eligibility requirements, consistent with the model's broader implication that restrictive terms can mitigate managerial resistance.

3.4.2 "Fix-It" Proposals: Ex-Post Incentives to Change Ownership Requirement

The social planner may set a standardized ownership requirement for proxy access ex ante for several reasons. First, the planner may seek to mandate proxy access. Second, the proposed threshold can serve as a focal point that helps overcome coordination problems across firms and shareholders when adoption is left to individual firms. Third, it may be impractical or prohibitively costly to tailor ownership requirements to heterogeneous firms ex ante. Regardless of the specific motivation, the planner selects a common threshold that optimizes the adoption-use trade-off.

To study these ex-post incentives, I augment the baseline model with an additional revision opportunity. The social planner first chooses the standardized ownership requirement, denoted τ_0, and firms make their adoption decisions as in the baseline model. After managerial bias becomes publicly observable, shareholders can propose a firm-specific ownership requirement, denoted τ'. For a firm that has already adopted proxy access, this takes the form of a "fix-it" proposal to amend its existing bylaw; for a firm that has not adopted, it takes the form of a new proxy-access proposal with revised terms.

A standardized threshold that is optimal ex ante, before firm-specific managerial bias is known, need not remain optimal once b is publicly observable and the initial adoption

decisions have been made. Consistent with this idea, Glass Lewis (2021) report that all proxy-access shareholder proposals in 2020 - and most such proposals in recent years - are "fix-it" proposals that seek to amend existing proxy access bylaws, although they have received weaker support than proposals to adopt proxy access initially.

The next corollary characterizes the firm-specific ownership requirement τ' that value-maximizing shareholders would propose at the revision stage and compares it with the standardized ex-ante requirement τ_0 initially chosen by the social planner.

Proposition 4 *Suppose* $\alpha_p < \bar{T}$.

For $b < \sqrt{\frac{G+r}{s}}$, *the value-maximizing choice is* $\tau' \leq \alpha_p$. *For* $b \geq \sqrt{\frac{G+r}{s}}$, *the value-maximizing choice is* $\tau' > \max\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\, b\}$.

The ex-ante optimal choice τ_0 *is either low or high, as characterized in Proposition 1. If* $\tau_0 \leq \min\left\{\alpha_p,\ \alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\right\}$, *then it is ex-post optimal to target non-adopting firms by setting* $\tau' > \max\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\, b\}$. *If* $\tau_0 > \max\left\{\alpha_p,\ \alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\right\}$, *then it is ex-post optimal to set* $\tau' > \tau_0$ *for non-adopting firms and* $\tau' \leq \alpha_p < \tau_0$ *for adopting firms.*

More details are provided in Appendix A.3.

At the revision stage, the standardized ex-ante requirement can therefore be replaced by a requirement tailored to the now-observable managerial bias of each firm. Intuitively, if the ownership requirement could be tailored to individual firms (i.e., made dependent on managerial bias), it would be set sufficiently low for firms with low-bias managers to ensure both adoption and use of proxy access, and sufficiently high for firms with high-bias managers to maximize adoption by credibly discouraging its use - that is, by eliminating shareholders' incentives to use proxy access.

This implies that if τ_0 optimizes the adoption-use trade-off by prioritizing use among adopting firms, the ex-post optimal strategy is to target non-adopting firms with shareholder proposals that set sufficiently high ownership requirements to make adoption incentive-

compatible. By contrast, if τ_0 optimizes the trade-off by maximizing adoption without use, the ex-post optimal strategy is twofold: to target adopting firms with "fix-it" proposals that lower the ownership requirement and induce use, and to target non-adopting firms with shareholder proposals that set sufficiently high ownership requirements to make non-use credible and thereby make adoption incentive-compatible for managers.

Hence, Proposition 4 yields testable empirical predictions regarding shareholder proposals following proxy access adoption. To the extent that the data are consistent with the equilibrium in which the ownership requirement is high (i.e., $\tau_0 > \max\left\{\alpha_p,\ \alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\right\}$), such that proxy access is widely adopted but rarely used, we should observe heterogeneous proxy-access-related shareholder proposals across firms. In particular, ownership requirements are likely to be revised upward in proposals targeting non-adopting firms, while being revised downward in "fix-it" proposals targeting firms that have already adopted proxy access.

3.4.3 Effectiveness of Proxy Access and Board Representation

While the ownership requirement is the most prominent and widely debated term of proxy access, another important design feature is the number of directors that eligible shareholders can nominate.[17] Proxy-access bylaws typically cap shareholder nominations at around 20%-25% of the board.

In the model, the extent of board representation afforded by proxy access is captured by the parameter s: allowing shareholders to nominate a larger fraction of the board increases the effectiveness of shareholder intervention and therefore corresponds to a higher s.

Perhaps surprisingly, the next corollary shows that increasing the effectiveness of proxy access through greater board representation reduces welfare in equilibrium, even though greater effectiveness increases the gains from using proxy access.

[17] Other terms of proxy access include the required holding period (typically one to three years) for shareholders to qualify to nominate directors and whether shareholders can aggregate their holdings to meet the ownership requirement.

Corollary 5 *Given the optimal ownership requirement τ^* characterized in Proposition 1, welfare (the social planner's objective) is decreasing in s.*

The detailed derivations are provided in Appendix A.4.

To understand this result, recall from Proposition 1 that the optimal ownership requirement can be either low or high. When τ^* is low, adopting firms expect proxy access to be used. In this case, an increase in s has two opposing effects on welfare. On the one hand, it makes proxy access more effective and therefore improves project choice among firms that continue to adopt. On the other hand, greater effectiveness also makes the loss of managerial control more costly, inducing managers with relatively high bias to resist adoption and thereby shrinking the set of adopting firms. The latter effect dominates because firms induced to forgo adoption lose the adoption-driven benefit as well as the potential improvement in project choice, whereas the direct benefit of a higher s is only an incremental improvement in project choice among firms that continue to adopt. Hence, welfare decreases in s.

When τ^* is high, adopting firms do not expect proxy access to be used. In this case, a higher s makes the use of proxy access more attractive to the activist, which undermines the credibility of non-use and induces greater resistance by managers. As a result, the set of firms that adopt proxy access contracts, reducing adoption-driven benefits and lowering welfare. Therefore, regardless of whether proxy access is expected to be used under the optimal ownership requirement, welfare is decreasing in s.

While the optimal ownership requirement depends on the channel through which welfare is maximized - adoption-driven or use-driven - the social planner has no incentive to increase the effectiveness of proxy access by expanding board representation. In particular, when the ownership requirement is high and board representation is limited, proxy access remains sufficiently restrictive to sustain adoption while avoiding use.

3.4.4 Adoption-Driven Benefits

Adoption-driven benefits - arising through channels independent of the utilization of proxy access - play a central role in the model. Because G summarizes the value generated by these underlying channels, the model does not require a particular magnitude of adoption-driven benefits. Instead, this section characterizes how the optimal ownership requirement changes as their magnitude varies. In the absence of such benefits (i.e., when $G = 0$), the adoption-use trade-off disappears, and the social planner chooses a lenient ownership requirement that maximizes use. Consequently, the emergence of a high ownership requirement - under which proxy access is widely adopted but rarely used - depends critically on the presence and magnitude of G.

The next corollary characterizes this dependence and shows, in particular, that a high ownership requirement is optimal only when adoption-driven benefits take intermediate values.

Corollary 6 *The optimal ownership requirement is high ($\tau^* = \bar{T}$) if and only if (i) $\alpha_p < \bar{T}$, (ii) $\alpha_a < \bar{\alpha}_a$ (where $\bar{\alpha}_a$ is defined in Equation A.1 in the appendix), and (iii) $G_L < G < G_H$ (where G_L and G_H are defined in Equation A.2 in the appendix). Otherwise, the optimal ownership requirement is low ($\tau^* \leq \alpha_p$).*

The detailed derivations are provided in Appendix A.5.

Intuitively, when G is low ($G \leq G_L$), welfare gains from using proxy access dominate adoption-driven benefits. When G is high ($G \geq G_H$), a sufficiently large fraction of firms adopt proxy access, and ensuring that these firms use proxy access becomes more valuable than the marginal adoption benefits from expanding adoption further. In both cases, the optimal ownership requirement is therefore low. Consequently, the emergence of a high ownership requirement requires that G lies in an intermediate range.

Furthermore, Corollary 6 implies that a high ownership requirement that is initially optimal may cease to be so as adoption-driven benefits - such as reputational gains or capital

inflows - diminish over time. For example, late adopters may derive smaller benefits from adopting proxy access than early adopters. Even if managerial bias remains private information (i.e., shareholders cannot rely on "fix-it" proposals after managerial bias is observed), a social planner (or an indexer with aligned preferences) would optimally favor a lower ownership requirement as G declines. Conversely, policies that increase G - for example, by increasing the sensitivity of capital inflows to governance ratings - can also make a low ownership requirement optimal instead of a high one.

More generally, because G captures any adoption-driven benefits that do not arise from the actual use of proxy access, it can also be interpreted as the cost savings from avoiding expensive proxy contests, as discussed in Kahan and Rock (2011). From this perspective, the universal proxy cards mandated by the SEC under Rule 14a-19 may reduce the costs of proxy contests and therefore the cost savings embedded in G.[18] To the extent that universal proxy cards materially reduce these adoption-driven benefits, the optimal ownership requirement for proxy access should also adjust accordingly.

3.4.5 Managerial Resistance to Adoption

Managerial resistance to shareholder proposals is central to generating the heterogeneous adoption and use patterns observed in the data. Absent the ability to resist, proxy access with lenient terms would be universally adopted and used, and a regulatory mandate could have been implemented without difficulty. In practice, resistance may take the form of vote swaying, particularly through outreach or mail campaigns targeting retail shareholders, as modeled here. More generally, because shareholder proposals are non-binding, managers ultimately retain discretion over implementation and may simply choose not to adopt the

[18]SEC Rule 14a-19 mandates the use of a universal proxy card in contested elections for shareholder meetings held after August 31, 2022. Unlike proxy access, the universal proxy rules do not impose minimum ownership or holding period requirements; instead, a dissident must solicit holders of at least 67% of the voting power (see "Q&A: What Do Public Companies Need to Know About the Universal Proxy Rules" by Cynthia Angell and J.T. Ho, *Orrick*, December 15, 2022). Because universal proxy still requires running a proxy contest, proxy access remains a distinct governance mechanism with potential cost advantages (see "Universal Proxy Ballot vs. Proxy Access: Which Leads to Better Board Accountability?" by Matt Orsagh, *CFA Institute*, March 26, 2015).

proposed governance change.[19]

To the extent that the data are consistent with the equilibrium in which the ownership requirement is high and proxy access is widely adopted but rarely used, the cost for managers to resist proxy-access adoption cannot be too high (i.e., r must be bounded from above).[20] In this case, welfare is locally insensitive to r: marginal changes in the resistance cost do not affect equilibrium welfare. The reason is that when the optimal ownership requirement maximizes welfare by maximizing adoption without use, the marginal adopting firm is determined by the largest managerial bias that still discourages the activist from using proxy access. Firms with lower managerial bias adopt proxy access on the premise that it will not be used, without requiring managerial resistance. Consequently, the resistance cost does not affect the equilibrium fraction of adopting firms.[21]

Nevertheless, when the resistance cost is sufficiently high, the optimal ownership requirement becomes low.[22] In this case, firms that adopt proxy access also use it, while non-adopting firms are precisely those whose managers resist adoption. As the resistance cost increases, resistance becomes less attractive, leading to a larger fraction of firms adopting and using proxy access and thereby increasing welfare. Consequently, from the perspective of welfare maximization, policymakers would ideally prefer resistance costs to be sufficiently high so that all firms adopt and use proxy access.

To the extent that firm managers resist shareholder proposals by swaying retail shareholders, Brav et al. (2022) show that retail shareholder voting persistently differs from institutional shareholder voting, with retail shareholders tending to be more supportive of incumbent management due to self-selection into firms they favor. In light of this, the easier

[19]Levit and Malenko (2011) model non-binding voting through an endogenous threshold such that managers accept shareholder proposals in equilibrium if and only if favorable votes exceed this threshold. They show that non-binding voting generally fails to convey shareholder preferences when managerial and shareholder interests are not aligned.

[20]According to Proposition 1, $\tau^* = \bar{T}$ only if $\alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}} < \bar{T}$.

[21]According to Appendix A.1, when $\tau^* = \bar{T}$, the fraction of firms that adopt proxy access is $\sqrt{\frac{\gamma}{2\alpha_a s}}\frac{\bar{T}-\alpha_a}{B}$.

[22]According to Proposition 1, when r is sufficiently high such that $\alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}} \geq \bar{T}$, $\tau^* \leq \alpha_p$.

(harder) it is for firms to increase retail shareholder turnout, the lower (higher) the cost of resisting shareholder proposals. The growing use of digital platforms for proxy voting and shareholder communication may therefore affect managerial resistance costs by influencing retail shareholder turnout.[23] Consistent with this mechanism, Lee and Souther (2020) find that proxy delivery methods influence retail turnout: although electronic "notice-and-access" delivery lowers logistical costs, it also reduces retail participation relative to traditional mailed "full-set delivery." To the extent that lower retail turnout weakens managers' ability to mobilize management-friendly votes, digitalization may increase managerial resistance costs, which, according to the model, can improve welfare.

3.4.6 Financial Market Frictions

In the model, the parameter $\gamma > 0$ captures financial market frictions - such as transaction costs, illiquidity, and wealth constraints - in a manner similar to Levit et al. (2023). These frictions limit the activist's ability to build sufficiently large positions to become eligible to use proxy access. Consequently, financial market frictions affect both the optimal ownership requirement and welfare by shaping whether the use of proxy access is incentive-compatible, and thus credible, in equilibrium.

To the extent that the data are consistent with the equilibrium in which the ownership requirement is high and proxy access is widely adopted but rarely used, financial market frictions, captured by γ, must be sufficiently large.[24] In this case, perhaps surprisingly, welfare is locally increasing in γ: greater difficulty for the activist to become eligible to use proxy access - in the form of stronger financial market frictions - raises equilibrium welfare. The intuition is that when the optimal ownership requirement maximizes welfare by maximizing adoption without use, the marginal adopting firm is determined by the largest managerial

[23]For example, the Robinhood-owned Say Technologies launched a feature allowing public companies to communicate directly with retail investors. See "Robinhood-owned Say launches shareholder messaging" by Hope King, *Axios*, January 23, 2023. Trading platforms such as Robinhood, Public, and Webull have increasingly developed tools that facilitate direct communication between companies and retail shareholders.

[24]According to Proposition 1, $\tau^* = \bar{T}$ only if $\alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}} < \bar{T}$.

bias that still discourages the activist from using proxy access. As it becomes more difficult for the activist to build sufficiently large positions, fewer firms with relatively high managerial bias can credibly expect proxy access to be used. Consequently, a larger fraction of firms adopts proxy access on the premise that it will not be used, thereby increasing welfare.

Nevertheless, when financial market frictions are sufficiently low, the optimal ownership requirement becomes low.[25] In this case, firms that adopt proxy access also use it, and proxy access is implemented by the indexer at no cost. Hence, welfare is locally insensitive to γ, since the activist no longer needs to build large positions to induce use.

Consequently, from a welfare perspective, policymakers should not prioritize reducing financial market frictions, as welfare is either locally increasing in or locally insensitive to γ, unless the reduction is sufficiently large to shift the optimal ownership requirement from high to low. This implication is consistent with the evidence in Cohn et al. (2016), who find that firm value increases with perceived shareholder control through proxy access, particularly for firms in which acquiring a stake is relatively inexpensive.

3.5 Extensions

I next consider three extensions of the model. The first endogenizes the adoption-driven capital-market benefit through governance-sensitive investor demand. The second delays the resolution of information asymmetry about managerial bias until after project selection, while the third allows the activist to have biased preferences.

3.5.1 Capital-Market Benefits of Proxy-Access Adoption

In the baseline model, G is a reduced-form measure of the benefit that accrues upon the adoption of proxy access independently of its subsequent use. Such adoption-driven benefits can arise through multiple channels, and the baseline model deliberately abstracts from

[25] According to Proposition 1, when γ is sufficiently low such that $\alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}} \geq \bar{T}$, $\tau^* \leq \alpha_p$.

their specific origins. In this extension, I illustrate one particular capital-market channel by allowing proxy-access adoption to improve an observable governance rating and thereby affect investor demand. This channel is motivated by Hartzmark and Sussman (2019), who show that investors respond to sustainability ratings through fund flows, consistent with investor demand being sensitive to publicly observable assessments of firm practices. The governance-rating channel considered here is therefore intended as one illustration of how adoption-driven benefits can arise, rather than as the unique source of G.

The key distinction from the governance benefits generated by the subsequent use of proxy access is that the capital-market response in this extension is triggered by adoption itself. Proxy-access adoption changes the firm's observable governance rating, which attracts governance-sensitive investor demand and increases the firm's market valuation, regardless of whether proxy access is subsequently used. The resulting value effect therefore corresponds to the use-independent component captured by G in the baseline model.

To incorporate this channel without changing the subsequent proxy-access game, suppose there is a unit mass of atomistic *outside* investors who may purchase shares from the firm's existing atomistic shareholders. These outside investors are distinct from the activist and the indexer. Each outside investor remains infinitesimal after trading and therefore cannot individually satisfy the ownership requirement or use proxy access. Moreover, trades occur only within the atomistic portion of the firm's ownership, so the stakes α_a and α_p of the activist and the indexer remain unchanged. This setup allows governance-sensitive investor demand to affect the firm's market valuation while leaving the ownership structure relevant for subsequent proxy-access use unchanged.

The capital-market equilibrium occurs after the proxy-access adoption outcome is determined and before eligible shareholders decide whether to use proxy access. Let $z > 0$ denote the firm's publicly observable governance rating absent proxy-access adoption. If proxy access is adopted, the rating increases to $z + \Delta z$, where $\Delta z > 0$. The rating change is triggered by adoption itself and does not depend on subsequent proxy-access use, managerial bias,

or project choice. Although the capital-market equilibrium is realized following adoption, agents anticipate the resulting valuation effect when making earlier decisions.

Outside investors differ in their sensitivity to governance quality. Investor i has governance sensitivity q_i, where q_i is distributed according to a continuously differentiable CDF $F(q)$ with density $f(q) > 0$ over the relevant support. Given governance rating z and market price P, investor i purchases the firm's shares if

$$v + q_i z - P \geq 0,$$

where $v > 0$ denotes the component of outside investors' willingness to pay that is independent of the firm's governance rating.[26] Hence, the marginal investor satisfies

$$q^*(P, z) = \frac{P - v}{z},$$

and aggregate outside-investor demand is

$$D(P, z) = 1 - F\left(\frac{P - v}{z}\right).$$

Existing atomistic shareholders differ in their willingness to sell. Let $S(P)$ denote the aggregate supply of shares from these shareholders, where $S'(P) > 0$. An interior capital-market equilibrium satisfies

$$D(P, z) = S(P).$$

Let $P^*(z)$ denote the equilibrium market price.[27,28]

[26]The parameter v is distinct from the baseline firm-value parameter V: it captures only the governance-independent component of outside investors' willingness to pay in this auxiliary capital-market equilibrium.

[27]I focus on an interior equilibrium in which the marginal outside investor has strictly positive governance sensitivity, so that $q^*(P^*, z) > 0$, or equivalently $P^* > v$.

[28]The explicit market price in this extension is consistent with the treatment of activist trading in the baseline model. Here, the equilibrium price must be solved because the adoption-induced change in market valuation is the object that generates G^*. By the time the activist subsequently trades to satisfy the ownership requirement, the adoption outcome and its associated capital-market effect are public information and incorporated into the market price. Additional shares are therefore purchased at fair value, generating

Lemma 7 *Suppose the capital-market equilibrium is interior. A higher governance rating increases the equilibrium market price:*

$$\frac{\partial P^*(z)}{\partial z} > 0.$$

Consequently, proxy-access adoption generates a strictly positive adoption-driven capital-market benefit,

$$G^* \equiv P^*(z + \Delta z) - P^*(z) > 0.$$

The proof is provided in Appendix A.6.

The key feature of G^* is that it is generated entirely by adoption. Outside investors respond to the governance-rating change associated with adopting proxy access rather than to whether eligible shareholders subsequently exercise proxy access. Any value created through the actual use of proxy access continues to operate separately through project choice, exactly as in the baseline model.

The capital-market equilibrium is modular with respect to the remainder of the model. Conditional on the equilibrium adoption-driven benefit G^*, the subsequent proxy-access game is unchanged. Hence, all results from the baseline model continue to apply after replacing the reduced-form parameter G with G^*.

In particular, Corollary 6 implies that a high ownership requirement is optimal only when the endogenous capital-market benefit from adoption takes an intermediate value, $G_L < G^* < G_H$, together with the other conditions stated in the corollary. When governance-sensitive investor demand generates little adoption-driven value, the planner favors a low ownership requirement to promote use. When the capital-market benefit is intermediate, encouraging additional adoption can instead justify a high ownership requirement. When the benefit becomes sufficiently large, many firms are willing to adopt even when proxy access

no capital gain or loss for the activist; only the trading friction associated with building the position affects the activist's use decision.

is expected to be used, making it optimal again to prioritize use among adopting firms.

This extension illustrates one economic channel through which the adoption-driven benefit in the baseline model can arise without requiring that all such benefits operate through governance-sensitive investor demand. Other use-independent benefits, such as reputational effects or expected savings from future proxy contests, can continue to enter G. The relevant object for the baseline results is the aggregate value generated by adoption independently of subsequent use.

3.5.2 Information Revelation through Proxy-Access Adoption

In the baseline model, managerial bias b is privately observed by managers only when the social planner chooses a standardized ownership requirement ex ante. This assumption is natural because the social planner maximizes aggregate welfare rather than targeting a particular firm. By the time individual firms make adoption decisions, however, b becomes publicly observable - for example, information may be generated once a firm is targeted by a shareholder proposal for proxy access. In this extension, by contrast, managerial bias remains private information until after project selection at $t = 3$, so that only the manager precisely observes b throughout the adoption and use stages. This assumption represents the opposite extreme relative to the baseline timing of information revelation. In practice, the truth likely lies somewhere in between: information about managerial bias may be generated when firms are targeted by shareholder proposals, but such information is imperfect and managerial bias may only be revealed gradually over time. For expositional simplicity and to sharpen the contrast with the baseline model, I allow τ to lie outside the interval $[\underline{T}, \bar{T}]$. All other aspects of the model remain unchanged.

Under persistent information asymmetry, the manager's resistance decision can convey information about managerial bias. Because resistance prevents adoption with certainty in the model, the observable adoption outcome reveals whether the manager resisted and there-

fore carries the same information.[29] However, the next lemma shows that the information conveyed by the adoption outcome need not generate separation in equilibrium.

Lemma 8 *Under persistent information asymmetry, equilibrium adoption outcomes take one of two forms. Either (i) only firms with managerial bias* $b < \sqrt{\frac{G+r}{s}}$ *adopt proxy access, in which case adoption credibly signals low managerial bias and proxy access is expected to be used following adoption; or (ii) all firms adopt proxy access, in which case adoption is completely uninformative about managerial bias and proxy access is not expected to be used following adoption.*

The detailed derivations are provided in Appendix A.7.

Because resistance and adoption are perfectly linked in the model, conditioning shareholders' beliefs on the adoption outcome is equivalent to conditioning them on the manager's resistance decision. Intuitively, persistent information asymmetry implies that shareholders do not observe managerial bias b at the use stage and therefore cannot condition their decision to use proxy access on firm-specific bias. As a result, if proxy access is expected to be used for one adopting firm, it must be expected to be used for all adopting firms. However, managers privately observe their own biases. Consequently, any manager with sufficiently large bias, i.e., $b \geq \sqrt{\frac{G+r}{s}}$, would prefer to resist adoption if proxy access were expected to be used. Therefore, in an equilibrium with use following adoption, the best the social planner can achieve is to have all firms with $b < \sqrt{\frac{G+r}{s}}$ adopt and use proxy access. This outcome is identical to the corresponding use-driven equilibrium in the baseline model.

The same logic implies that, for firms with $b \geq \sqrt{\frac{G+r}{s}}$ to adopt proxy access, it must be credible that eligible shareholders will not use proxy access following adoption. Hence, in an equilibrium without use, the best the social planner can achieve is to have all firms with $b \in [0, B]$ adopt proxy access on the premise that it will not be used. This outcome resembles the adoption-driven equilibrium in the baseline model, except that persistent information

[29] The manager derives no direct benefit from signaling her type. Information revelation arises endogenously because the resistance decision depends on managerial bias and determines the observable adoption outcome.

asymmetry allows a larger set of firms to adopt without use because shareholders cannot distinguish among adopting firms.

Lemma 8 implies that the central trade-off in the baseline model is robust to both the persistence of information asymmetry and the potential signaling role of firms' adoption decisions. In the adoption-use trade-off, the social planner maximizes welfare either by maximizing the use of proxy access or by maximizing adoption. The conditions under which each outcome arises under persistent information asymmetry are characterized in the next proposition.

Proposition 9 *Under persistent information asymmetry, the optimal ownership requirement τ^* depends on the dispersion of managerial bias, captured by B.*

If

$$B > \frac{1}{3}\left(4 + \frac{r}{G}\right)\sqrt{\frac{G+r}{s}},$$

then the social planner chooses a sufficiently high ownership requirement such that

$$\tau^* > \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{3\gamma}}\,B\right\}.$$

If

$$B \leq \frac{1}{3}\left(4 + \frac{r}{G}\right)\sqrt{\frac{G+r}{s}},$$

then the social planner chooses a sufficiently low ownership requirement such that

$$\tau^* \leq \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{3\gamma}}\right\}.$$

The detailed derivations are provided in Appendix A.8.

Proposition 9 shows that, unlike in the baseline model, the optimal ownership requirement under persistent information asymmetry depends critically on the dispersion of managerial bias, captured by B. The reason is that persistent information asymmetry prevents share-

holders from conditioning proxy-access use on firm-specific managerial bias. As a result, adopting firms are pooled together from the perspective of shareholders at the use stage, and the social planner faces a sharper trade-off between inducing use and sustaining adoption.

When the dispersion of managerial bias is sufficiently large, high-bias firms account for a substantial fraction of firms in the environment. In this case, maximizing welfare through broad adoption becomes relatively more important, and the social planner optimally chooses a high ownership requirement that credibly discourages use and induces all firms to adopt proxy access. By contrast, when the dispersion of managerial bias is sufficiently limited, the gains from ensuring that adopting firms use proxy access dominate the additional adoption-driven benefits from inducing high-bias firms to adopt. The social planner therefore chooses a low ownership requirement that sustains both adoption and use among low-bias firms.

3.5.3 Biased Activist

In the baseline model, the activist's preferences over project choice are aligned with those of the other shareholders and the social planner. In practice, however, shareholder preferences may be heterogeneous. In particular, activist shareholders may pursue agendas that diverge from firm value maximization. Anabtawi and Stout (2008) provide a comprehensive discussion of private benefits and objectives that may motivate activist shareholders but are not shared by other shareholders, including short-termism, self-dealing involving other firms in activists' portfolios, labor-related objectives pursued by union fund managers, and the enhancement of political reputation through media exposure.

In this extension, I allow for a conflict of interest between the activist and the remaining shareholders in project preferences. The activist's payoff becomes

$$U_A = \alpha_a[V + \mathbb{1}_{\text{adopt}}G - (y - \beta)^2] - \frac{1}{2}\gamma x^2,$$

where $\beta > 0$ captures the activist's bias, so that the activist's preferred project choice is $y = \beta$.

For expositional simplicity, there is no indexer in this extension (i.e., $\alpha_p = 0$), and the activist's stake satisfies $\alpha_a < \frac{1}{2}$, so that the activist remains a minority shareholder. If the activist uses proxy access, shareholder intervention is effective with probability s, in which case the project $y = \beta$ is implemented. I also assume, for expositional simplicity, that the activist's bias is limited:

$$\beta < \sqrt{\frac{G+r}{s}},$$

which in turn implies $\beta < B$. All other aspects of the model remain unchanged.

The following proposition characterizes the optimal ownership requirement τ^* in a form analogous to Proposition 1 in the baseline model.

Proposition 10 *If*

$$\alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}(\beta + \sqrt{\frac{G+r}{s}})(-2\beta^2 + 2\beta\sqrt{\frac{G+r}{s}} + \frac{G+r}{s}) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}} \geq \bar{T},$$

then the social planner is indifferent over all $\tau^* \in [\underline{T}, \alpha_a]$.

If

$$\alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}(\beta + \sqrt{\frac{G+r}{s}})(-2\beta^2 + 2\beta\sqrt{\frac{G+r}{s}} + \frac{G+r}{s}) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}} < \bar{T},$$

then the social planner chooses $\tau^* = \bar{T}$.

The proofs are provided in Appendix A.9.

The main takeaway is that, as in the baseline model, the optimal ownership requirement in the presence of activist bias remains either low or high. Intuitively, although activist bias reduces the firm-value gains from using proxy access, it affects the payoff comparison only quantitatively and does not alter the underlying adoption-use trade-off. When the ownership

requirement is optimally low, welfare is maximized by ensuring that adopting firms use proxy access. By contrast, when the ownership requirement is optimally high, welfare is maximized by increasing the fraction of firms that adopt proxy access, even though adopting firms are not expected to use it.

The next corollary characterizes how the optimal ownership requirement depends on the magnitude of activist bias.

Corollary 11 *In the presence of activist bias, the optimal ownership requirement τ^* depends jointly on the upper bound of feasible ownership requirements, $\bar{T}$, and the magnitude of activist bias, β.*

If $\bar{T} \leq T_L$ (where T_L is defined in Equation A.3 in the appendix), then $\tau^* \leq \alpha_a$ regardless of β.

If $T_L < \bar{T} \leq T_M$ (where T_M is defined in Equation A.4 in the appendix), then $\tau^* = \bar{T}$ for $\beta < \beta_T$ and $\tau^* \leq \alpha_a$ for $\beta \geq \beta_T$, where β_T is defined in Equation A.6 in the appendix.

If $T_M < \bar{T} < T_H$ (where T_H is defined in Equation A.5 in the appendix), then $\tau^* = \bar{T}$ for $\beta < \beta_L$ or $\beta > \beta_H$, and $\tau^* \leq \alpha_a$ for $\beta \in [\beta_L, \beta_H]$, where β_L and β_H are defined in Equation A.7 in the appendix.

If $\bar{T} > T_H$, then $\tau^* = \bar{T}$ regardless of β.

The detailed derivations are provided in Appendix A.10.

Corollary 11 shows that, depending on the feasible upper bound of ownership requirements, the relationship between activist bias and the optimal ownership requirement can be non-monotonic. When activist bias is very low, the welfare-maximization problem resembles the baseline case without activist bias: although a low ownership requirement induces use and improves firm value among adopting firms, the fraction of adopting firms remains limited. In this case, setting the ownership requirement at the feasible upper bound - provided that $\bar{T}$ is not too low (see the case of $\bar{T} > T_L$) - can generate sufficiently large adoption-driven benefits to outweigh the gains from use.

When activist bias is very high, the benefits from using proxy access become minimal and may even be negative for some firms with $b < \beta$. Consequently, a high ownership requirement (see the case of $\bar{T} > T_M$) can again become optimal because maximizing adoption-driven benefits dominates the limited value of inducing use. By contrast, when activist bias is intermediate, the gains from setting a low ownership requirement are substantial, as a sizeable fraction of firms both adopts and uses proxy access. In this region, the feasible upper bound on ownership requirements may become binding (see the case of $T_M < \bar{T} < T_H$): even setting the ownership requirement at the highest feasible level may fail to generate sufficient adoption-driven benefits to outweigh the welfare gains from use, making a low ownership requirement optimal.

3.6 Conclusion

This paper develops a theory of proxy access that rationalizes the disconnect between widespread adoption and limited use. The model highlights an adoption-use trade-off: a low ownership requirement encourages the use of proxy access among adopting firms, while a high ownership requirement promotes adoption by reducing managers' incentives to resist. In this setting, proxy access can create value not only through shareholder intervention itself but also through adoption-driven benefits such as reputational gains and capital-market effects. Consequently, a restrictive ownership requirement can be welfare-maximizing even when proxy access is rarely used in equilibrium, providing a new perspective on the SEC's vacated Rule 14a-11 and the subsequent convergence of shareholder proposals toward standardized ownership requirements. The model also generates novel empirical implications regarding the ex-post incentives of shareholders to revise existing ownership requirements through "fix-it" proposals and shows how the optimal design of proxy access varies with the economic value generated by adoption, managerial resistance, financial frictions, and activist incentives. Future research could extend the framework by incorporating richer voting dy-

namics, such as strategic voting by institutional shareholders, to study how voting incentives interact with the design and adoption of governance mechanisms that expand shareholder power.

A Proofs

A.1 Proof of Proposition 1

The detailed derivation of Proposition 1 is provided below. The proof proceeds by comparing welfare across three ranges of the ownership requirement: low, intermediate, and high.

Low ownership requirement.

For $\tau \leq \alpha_p$, only firms with sufficiently low managerial bias, i.e., $b < \sqrt{\frac{G+r}{s}}$, adopt proxy access and expect it to be used.

$$\begin{aligned}
& \mathbb{E}(V + \mathbb{1}_{adopt} \times G - y^2) \\
= & \int_0^B (V + \mathbb{1}_{adopt} \times G - y^2) \frac{1}{B} db \\
= & \frac{1}{B} \int_0^{\sqrt{\frac{G+r}{s}}} [V + G - (1-s)b^2] db + \frac{1}{B} \int_{\sqrt{\frac{G+r}{s}}}^B (V - b^2) db \\
= & V + \frac{1}{B} \sqrt{\frac{G+r}{s}} G - \frac{1-s}{3B} (\frac{G+r}{s})^{\frac{3}{2}} - \frac{1}{3B} [B^3 - (\frac{G+r}{s})^{\frac{3}{2}}] \\
= & V + \frac{1}{B} \sqrt{\frac{G+r}{s}} G + \frac{s}{3B} (\frac{G+r}{s})^{\frac{3}{2}} - \frac{1}{3} B^2
\end{aligned}$$

Intermediate ownership requirement.

For $\tau > \alpha_p$, only the activist can acquire additional shares to meet the ownership requirement and become eligible.

When $\alpha_p < \tau \leq \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}$, i.e., $\sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a) \leq \sqrt{\frac{G+r}{s}}$, the activist has incentives to build a sufficiently large position to use proxy access for some firms. Despite anticipating this, the manager finds it cost-ineffective to block adoption. In this case, firms with intermediate

managerial bias, i.e., $\sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a) \leq b < \sqrt{\frac{G+r}{s}}$, adopt proxy access and expect it to be used. By contrast, firms with sufficiently low managerial bias, i.e., $b < \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$, adopt proxy access but do not expect it to be used. Finally, firms with high managerial bias, i.e., $b \geq \sqrt{\frac{G+r}{s}}$, do not adopt proxy access.

Hence, the intermediate ownership-requirement range is dominated by the low range and can be eliminated from consideration. For firms with sufficiently low managerial bias ($b < \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$), an ownership requirement in the range $\alpha_p < \tau \leq \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}$ leads to no use of proxy access, whereas $\tau \leq \alpha_p$ results in the use of proxy access and an improvement in project choice.

High ownership requirement.

When $\tau > \max\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\}$, i.e., $\sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a) > \sqrt{\frac{G+r}{s}}$, neither the indexer nor the activist has incentives to use proxy access. Anticipating this, firms with low managerial bias, i.e., $b < \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$, adopt proxy access but do not expect it to be used, while firms with high managerial bias, i.e., $b \geq \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$, do not adopt proxy access. In this case, no firm both adopts proxy access and expects it to be used.

When $\tau > \max\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\}$, the social planner expects the following welfare:

$$\begin{aligned}
& \mathbb{E}(V + \mathbb{1}_{adopt} \times G - y^2) \\
= & \int_0^B (V + \mathbb{1}_{adopt} \times G - y^2)\frac{1}{B}db \\
= & \frac{1}{B}\int_0^{\sqrt{\frac{\gamma}{2\alpha_a s}}(\tau-\alpha_a)} (V + G - b^2)db + \frac{1}{B}\int_{\sqrt{\frac{\gamma}{2\alpha_a s}}(\tau-\alpha_a)}^B (V - b^2)db \\
= & V + \frac{1}{B}\sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)G - \frac{1}{3}B^2
\end{aligned}$$

Because welfare is increasing in τ, welfare is higher for $\tau > \max\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\}$ than for $\tau \leq \alpha_p$ if and only if the ownership requirement is sufficiently restrictive, i.e.,

$$\tau > \alpha_a + \left[\tfrac{1}{3}\left(4 + \tfrac{r}{G}\right)\right]\sqrt{\tfrac{2\alpha_a(G+r)}{\gamma}}.$$

Thus, conditional on choosing the high ownership-requirement range, the planner optimally selects the highest feasible requirement, $\tau = \bar{T}$.

Since the intermediate range is dominated, the planner's problem reduces to comparing the optimal low ownership requirement with $\tau = \bar{T}$.

Therefore, if

$$\max\left\{\alpha_p,\ \alpha_a + \left[\tfrac{1}{3}\left(4 + \tfrac{r}{G}\right)\right]\sqrt{\tfrac{2\alpha_a(G+r)}{\gamma}}\right\} < \bar{T},$$

then the social planner maximizes welfare by choosing $\tau^* = \bar{T}$. If $\alpha_p < \bar{T}$ and $\alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}} \geq \bar{T}$, then the social planner maximizes welfare by choosing $\tau^* \leq \alpha_p$. Otherwise (i.e., if $\alpha_p \geq \bar{T}$), the choice of ownership requirement does not affect which firms adopt (and subsequently use) proxy access, and the social planner is indifferent over all $\tau^* \in [\underline{T}, \bar{T}]$.

A.2 Proof of Proposition 3

The proof of Proposition 3 is provided below.

Because all firms receive adoption-driven benefits G, the social planner maximizes welfare by maximizing improvements in project choice (i.e., maximizing the use of proxy access).

For $\tau > \alpha_p$, proxy access is used if and only if the activist has incentives to use it, i.e.,

$$b \geq \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a).$$

By contrast, for $\tau \leq \alpha_p$, the indexer uses proxy access, and project choices of all firms (i.e., for all b) are improved. Hence, $\tau \leq \alpha_p$ is the dominant choice for the social planner.

In this case, welfare is given by

$$\begin{aligned} & \mathbb{E}(V + \mathbb{1}_{\text{adopt}} G - y^2) \\ = & \int_0^B (V + G - y^2) \frac{1}{B}\, db \\ = & \frac{1}{B} \int_0^B [V + G - (1-s) b^2]\, db \\ = & V + G - \frac{1-s}{3} B^2. \end{aligned}$$

The social planner and shareholders (i.e., the indexer, the activist, and atomistic shareholders) are better off under the optimal mandate than under the optimal voluntary adoption described in Proposition 1. To see this, first note that

$$V + G - \frac{1-s}{3} B^2 > V + \frac{1}{B} \sqrt{\frac{G+r}{s}} G + \frac{s}{3B} \left(\frac{G+r}{s} \right)^{\frac{3}{2}} - \frac{1}{3} B^2.$$[30]

The right-hand side is the welfare when the optimal ownership requirement under the voluntary adoption regime is low. In addition,

$$V + G - \frac{1-s}{3} B^2 > V + \frac{1}{B} \sqrt{\frac{\gamma}{2\alpha_a s}} (\tau - \alpha_a) G - \frac{1}{3} B^2.$$[31]

The right-hand side is the welfare when the optimal ownership requirement under the voluntary adoption regime is high. Hence, welfare is higher under the optimal mandate than under optimal voluntary adoption, regardless of whether the optimal ownership requirement under voluntary adoption is low or high.

Finally, under the optimal mandate, the preferences of the social planner and shareholders are aligned, as the activist is not expected to build a large position to use proxy access. Therefore, both the social planner and shareholders are better off under the optimal mandate than under optimal voluntary adoption.

[30] This follows from $G > \frac{1}{B} \sqrt{\frac{G+r}{s}} G$ and $-\frac{1-s}{3} B^2 > \frac{s}{3B} \left(\frac{G+r}{s} \right)^{\frac{3}{2}} - \frac{1}{3} B^2$.

[31] This follows from $-\frac{1-s}{3} B^2 > -\frac{1}{3} B^2$ and the fact that the fraction of adopting firms is at most 1.

As for firm managers, they are better off under the proxy access mandate if and only if

$$V + G - sb^2 > V.$$

Thus, managers with sufficiently large bias, i.e., $b \geq \sqrt{\frac{G}{s}}$, are worse off under the mandate. Ex ante, the fraction of managers who are worse off is $1 - \frac{1}{B}\sqrt{\frac{G}{s}}$.

Recall from Appendix A.1 (the proof of Proposition 1) that under voluntary adoption, firms can resist proxy access if it would reduce their payoff, and hence are never worse off ex post. The fraction of firms that do not adopt proxy access under the optimal voluntary regime is at most $1 - \frac{1}{B}\sqrt{\frac{G+r}{s}}$.[32] Since $1 - \frac{1}{B}\sqrt{\frac{G}{s}} > 1 - \frac{1}{B}\sqrt{\frac{G+r}{s}}$, the mandate adversely affects a larger set of firms than those that choose not to adopt under voluntary adoption. Consequently, mandating proxy access would face stronger opposition from businesses than a regime in which firms decide individually whether to adopt proxy access.

A.3 Proof of Proposition 4

The proof of Proposition 4 is provided below.

Consider the ex-post revision stage, after the standardized requirement τ_0 has governed the initial adoption decision and managerial bias b is publicly observable. Shareholders can now propose a firm-specific requirement τ'. Since τ' can depend on b, there is no adoption-use trade-off for value maximization: adoption with use dominates mere adoption, which in turn dominates non-adoption. Hence, for $b < \sqrt{\frac{G+r}{s}}$, the firm manager has no incentive to resist adoption, and the optimal ownership requirement should allow the indexer to use proxy access costlessly, i.e., $\tau' \leq \alpha_p$. By contrast, for $b \geq \sqrt{\frac{G+r}{s}}$, the manager would resist adoption if proxy access were expected to be used. In this case, the optimal requirement should be sufficiently high to discourage use and thereby induce adoption, i.e., $\tau' > \max\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\, b\}$.

[32] If the optimal ownership requirement under voluntary adoption is high and prioritizes the fraction of adopting firms, the fraction of non-adopting firms is even smaller than $1 - \frac{1}{B}\sqrt{\frac{G+r}{s}}$.

From Proposition 1 and its proof (Appendix A.1), when $\tau_0 \leq \min\left\{\alpha_p,\ \alpha_a + \left[\frac{1}{3}\left(4+\frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\right\}$, firms with $b < \sqrt{\frac{G+r}{s}}$ adopt and use proxy access, which is already ex-post optimal, while firms with $b \geq \sqrt{\frac{G+r}{s}}$ do not adopt and therefore do not use proxy access. The latter outcome can be improved upon ex post. It is thus optimal to target these non-adopting firms with new shareholder proposals setting $\tau' > \max\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\, b\}$, so that they have incentives to adopt proxy access given that it will not be used.

When $\tau_0 > \max\left\{\alpha_p,\ \alpha_a + \left[\frac{1}{3}\left(4+\frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\right\}$, firms with $b < \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau_0 - \alpha_a)$ adopt but do not use proxy access, while firms with $b \geq \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau_0 - \alpha_a)$ do not adopt and therefore do not use proxy access. This outcome can be improved upon ex post in two ways:

- It is optimal to target adopting firms with "fix-it" proposals that set $\tau' \leq \alpha_p$. Since $\tau_0 > \alpha_p$, these proposals revise the ownership requirement downward. However, ex post, the manager blocks such revisions if and only if the benefit of doing so outweighs the cost, i.e., $sb^2 \geq r$. As a result, although it is optimal to propose revisions that induce use, only low-bias firms ($b < \sqrt{\frac{r}{s}}$) are likely to implement them, implying a low success rate of "fix-it" proposals among firms that have already adopted proxy access.
- It is optimal to target non-adopting firms with new shareholder proposals setting $\tau' > \max\{\alpha_p, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\, b\}$. Since non-adopting firms satisfy $b \geq \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau_0 - \alpha_a)$, the condition $\tau' > \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\, b$ implies $\tau' > \tau_0$. These proposals therefore revise the ownership requirement upward.

A.4 Derivation for Corollary 5

The detailed derivations supporting Corollary 5 are provided below.

If $\tau^* \leq \min\{\alpha_p, \alpha_a + [\frac{1}{3}(4+\frac{r}{G})]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\}$, then, according to Appendix A.1, welfare is given by

$$\mathbb{E}(V + \mathbb{1}_{\text{adopt}} G - y^2) = V + \frac{1}{B}\sqrt{\frac{G+r}{s}}\, G + \frac{s}{3B}\left(\frac{G+r}{s}\right)^{\frac{3}{2}} - \frac{1}{3}B^2.$$

Differentiating with respect to s,

$$\begin{aligned} & \frac{\partial}{\partial s}\mathbb{E}(V + \mathbb{1}_{\text{adopt}}G - y^2) \\ = \; & \frac{\partial}{\partial s}\left[\frac{G(G+r)^{\frac{1}{2}}}{B}s^{-\frac{1}{2}} + \frac{(G+r)^{\frac{3}{2}}}{3B}s^{-\frac{1}{2}}\right] \\ < \; & 0. \end{aligned}$$

Hence, welfare is decreasing in s.

If $\tau^* > \max\{\alpha_p, \alpha_a + [\frac{1}{3}(4 + \frac{r}{G})]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}}\}$, then, again using Appendix A.1, welfare is

$$\mathbb{E}(V + \mathbb{1}_{\text{adopt}}G - y^2) = V + \frac{1}{B}\sqrt{\frac{\gamma}{2\alpha_a s}}(\tau^* - \alpha_a)G - \frac{1}{3}B^2.$$

Differentiating with respect to s yields $\frac{\partial}{\partial s}\mathbb{E}(V+\mathbb{1}_{\text{adopt}}G-y^2) < 0$. Hence, welfare is decreasing in s.

A.5 Derivation for Corollary 6

The detailed derivations supporting Corollary 6 are provided below.

According to Proposition 1, the optimal ownership requirement is high ($\tau^* = \bar{T}$) if and only if $\alpha_p < \bar{T}$ and

$$\alpha_a + \left[\tfrac{1}{3}\left(4 + \tfrac{r}{G}\right)\right]\sqrt{\tfrac{2\alpha_a(G+r)}{\gamma}} < \bar{T}.$$

First, note that the left-hand side of the inequality diverges to infinity as $G \to 0$ or $G \to \infty$. In addition,

$$\begin{aligned} & \frac{\partial}{\partial G}\left[(4 + \tfrac{r}{G})\sqrt{G+r}\right] \\ = \; & -\frac{r}{G^2}\sqrt{G+r} + \left(2 + \tfrac{r}{2G}\right)\frac{1}{\sqrt{G+r}}. \end{aligned}$$

It follows that the left-hand side of the inequality is decreasing in G for $G \in \left[0, \frac{\sqrt{33}+1}{8}r\right)$ and increasing in G for $G > \frac{\sqrt{33}+1}{8}r$.

Therefore, the condition $\alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}} < \bar{T}$ holds only if the expression on the left-hand side, evaluated at $G = \frac{\sqrt{33}+1}{8}r$, is below $\bar{T}$ and $G \in (G_L, G_H)$, where G_L and G_H are the two roots satisfying $\alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}} = \bar{T}$.

Rewriting these conditions yields: (i) $\alpha_a < \bar{\alpha}_a$, where $\bar{\alpha}_a$ is implicitly defined by

$$\bar{\alpha}_a + \frac{\sqrt{33}+15}{24}\sqrt{\frac{(\sqrt{33}+9)r\,\bar{\alpha}_a}{\gamma}} = \bar{T}, \tag{A.1}$$

and (ii) $G \in (G_L, G_H)$, where G_L and G_H are the smaller and larger roots, respectively, of

$$\alpha_a + \left[\frac{1}{3}\left(4 + \frac{r}{G}\right)\right]\sqrt{\frac{2\alpha_a(G+r)}{\gamma}} = \bar{T}. \tag{A.2}$$

A.6 Proof of Lemma 7

The detailed derivation of Lemma 7 is provided below.

Given governance rating z and market price P, outside investor i purchases the firm's shares if

$$v + q_i z - P \geq 0.$$

Hence, the marginal investor is

$$q^*(P, z) = \frac{P - v}{z},$$

and aggregate outside-investor demand is

$$D(P, z) = 1 - F\left(\frac{P - v}{z}\right).$$

An interior capital-market equilibrium satisfies

$$1 - F\left(\frac{P - v}{z}\right) = S(P).$$

Define

$$H(P,z) \equiv 1 - F\left(\frac{P-v}{z}\right) - S(P).$$

The equilibrium price $P^*(z)$ is implicitly defined by $H(P^*(z),z) = 0$.

Because

$$\frac{\partial H}{\partial z} = f\left(\frac{P-v}{z}\right)\frac{P-v}{z^2} > 0$$

for an interior equilibrium with $P > v$, and

$$\frac{\partial H}{\partial P} = -\frac{1}{z}f\left(\frac{P-v}{z}\right) - S'(P) < 0,$$

the implicit function theorem implies

$$\begin{aligned}\frac{\partial P^*(z)}{\partial z} &= -\frac{\frac{\partial H}{\partial z}}{\frac{\partial H}{\partial P}} \\ &= \frac{f\left(\frac{P^*(z)-v}{z}\right)\frac{P^*(z)-v}{z^2}}{\frac{1}{z}f\left(\frac{P^*(z)-v}{z}\right) + S'(P^*(z))} \\ &> 0.\end{aligned}$$

Since proxy-access adoption increases the governance rating from z to $z + \Delta z$, where $\Delta z > 0$, it follows that

$$G^* = P^*(z + \Delta z) - P^*(z) > 0.$$

Importantly, the rating change Δz is generated by proxy-access adoption itself and does not depend on its subsequent use. Hence, G^* represents an adoption-driven capital-market benefit that is independent of proxy-access utilization. Conditional on $G = G^*$, the remainder of the model is identical to the baseline model.

A.7 Derivation for Lemma 8

The detailed derivations supporting Lemma 8 are provided below.

Unlike in the baseline model, shareholders do not observe managerial bias b at the use stage. The manager privately observes b and chooses whether to resist the proxy-access proposal. As in the baseline model, resistance prevents adoption with certainty, while absent resistance the proposal is adopted. Hence, observing adoption is equivalent to observing that the manager did not resist, and conditioning on the adoption outcome is equivalent to conditioning on the manager's resistance decision. Consequently, an eligible shareholder's decision to use proxy access cannot condition on firm-specific managerial bias. Let A ("Adopt") denote the set of managerial biases for which firms adopt proxy access, where $A \subseteq [0, B]$. Because shareholders cannot distinguish among adopting firms, managers of all firms with $b \in A$ share the same expectation regarding whether proxy access will be used following adoption.

As in the baseline model, if the manager expects proxy access to be used once adopted, she blocks adoption if and only if

$$b \geq \sqrt{\frac{G+r}{s}}.$$

In equilibrium, proxy access is used following adoption if and only if $\tau \leq \alpha_p$ or

$$\frac{1}{2}\gamma(\tau - \alpha_a)^2 \leq \alpha_a s \mathbb{E}(b^2 \mid b \in A).$$

Equivalently,

$$\tau \leq \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}\mathbb{E}(b^2 \mid b \in A)}\right\}.$$

Now suppose $A \supset [0, \sqrt{\frac{G+r}{s}})$ and $\tau \leq \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}\mathbb{E}(b^2 \mid b \in A)}\right\}$. Under these conditions, proxy access is expected to be used following adoption. However, any firm with $b \in A \setminus [0, \sqrt{\frac{G+r}{s}})$ would then have a manager who prefers to block adoption in the first place. This contradicts the assumption that such firms belong to A. Hence, $A \supset [0, \sqrt{\frac{G+r}{s}})$

and $\tau \leq \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}\mathbb{E}(b^2 \mid b \in A)}\right\}$ cannot simultaneously hold in equilibrium.

Therefore, in equilibrium, one of the following cases must hold: (i) $A \subseteq [0, \sqrt{\frac{G+r}{s}})$ and $\tau \leq \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}\mathbb{E}(b^2 \mid b \in A)}\right\}$; (ii) $A \supset [0, \sqrt{\frac{G+r}{s}})$ and $\tau > \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}\mathbb{E}(b^2 \mid b \in A)}\right\}$; or (iii) $A \subseteq [0, \sqrt{\frac{G+r}{s}})$ and $\tau > \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}\mathbb{E}(b^2 \mid b \in A)}\right\}$.

From the social planner's perspective, case (iii) is never optimal. If firms with $b \in A \subseteq [0, \sqrt{\frac{G+r}{s}})$ adopt proxy access but do not use it, welfare can be improved immediately by setting $\tau \leq \alpha_p$ to induce use.

By contrast, both cases (i) and (ii) can be optimal. In case (i), firms with $b \in A \subseteq [0, \sqrt{\frac{G+r}{s}})$ adopt and use proxy access. In this case, the social planner optimally sets the ownership requirement so that $A = [0, \sqrt{\frac{G+r}{s}})$, thereby maximizing the set of firms that both adopt and use proxy access. In case (ii), firms with $b \in A \supset [0, \sqrt{\frac{G+r}{s}})$ adopt proxy access but do not use it. The social planner then optimally sets the ownership requirement such that all firms adopt proxy access, i.e., $A = [0, B]$, in order to maximize adoption-driven benefits.

Therefore, in equilibrium, either adoption reveals that the firm has sufficiently low managerial bias $b \in [0, \sqrt{\frac{G+r}{s}})$, or adoption becomes uninformative about managerial bias because all firms adopt proxy access.

A.8 Proof of Proposition 9

The detailed derivation of Proposition 9 is provided below.

Based on Lemma 8, I derive the equilibrium conditions separately for two cases: (i) firms with $b < \sqrt{\frac{G+r}{s}}$ adopt and use proxy access; and (ii) all firms adopt proxy access but do not use it.

For case (i), proxy access is used if and only if $\tau \leq \alpha_p$ or

$$\frac{1}{2}\gamma(\tau - \alpha_a)^2 \leq \alpha_a s \mathbb{E}(b^2 \mid b < \sqrt{\tfrac{G+r}{s}}) = \frac{1}{3}\alpha_a s \left(\sqrt{\frac{G+r}{s}}\right)^2 = \frac{1}{3}\alpha_a (G+r).$$

Equivalently,

$$\tau \leq \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{3\gamma}}\right\}.$$

For $\tau \leq \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{3\gamma}}\right\}$,

$$\mathbb{E}(V + \mathbb{1}_{\text{adopt}}G - y^2) = V + \frac{1}{B}\sqrt{\frac{G+r}{s}}G + \frac{s}{3B}\left(\frac{G+r}{s}\right)^{\frac{3}{2}} - \frac{1}{3}B^2.$$

For case (ii), proxy access is not used if and only if $\tau > \alpha_p$ and

$$\frac{1}{2}\gamma(\tau - \alpha_a)^2 > \alpha_a s\mathbb{E}(b^2) = \frac{1}{3}\alpha_a s B^2.$$

Equivalently,

$$\tau > \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{3\gamma}}\,B\right\}.$$

For $\tau > \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{3\gamma}}\,B\right\}$,

$$\mathbb{E}(V + \mathbb{1}_{\text{adopt}}G - y^2) = V + G - \frac{1}{3}B^2.$$

Hence, welfare is higher under $\tau > \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{3\gamma}}\,B\right\}$ than under $\tau \leq \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{3\gamma}}\right\}$ if and only if

$$B > \frac{1}{3}\left(4 + \frac{r}{G}\right)\sqrt{\frac{G+r}{s}}.$$

Therefore, if

$$B > \frac{1}{3}\left(4 + \frac{r}{G}\right)\sqrt{\frac{G+r}{s}},$$

then the social planner maximizes welfare by choosing

$$\tau^* > \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a s}{3\gamma}}\,B\right\}.$$

Otherwise, the social planner maximizes welfare by choosing

$$\tau^* \leq \max\left\{\alpha_p,\ \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{3\gamma}}\right\}.$$

A.9 Proof of Proposition 10

The detailed derivation of Proposition 10 is provided below.

When proxy access is adopted, the activist has incentives to build a larger position and use proxy access if and only if the expected benefit from using proxy access exceeds the associated trading costs:

$$\alpha_a[V + G - (1-s)(b-\beta)^2] - \alpha_a[V + G - (b-\beta)^2] \geq \frac{1}{2}\gamma(\tau - \alpha_a)^2.$$

Hence, when proxy access is adopted, it is used if and only if

$$\tau \leq \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\,|b-\beta|.$$

If the ownership requirement is lenient, i.e., $\tau \leq \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\,|b-\beta|$, the manager blocks adoption if and only if

$$s(b-\beta)^2 \geq G + r.$$

Equivalently,

$$b \geq \beta + \sqrt{\frac{G+r}{s}},$$

since $\beta < \sqrt{\frac{G+r}{s}}$.

If the ownership requirement is restrictive, i.e., $\tau > \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\,|b-\beta|$, proxy access is not used despite adoption.

Next, we examine welfare across different ranges of τ. Specifically, we consider the following cases: (i) $\tau \leq \alpha_a$; (ii) $\alpha_a < \tau \leq \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\beta$; (iii) $\alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\beta < \tau < \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}$;

and (iv) $\tau \geq \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}$. By comparing welfare across these ranges, we can determine the ownership requirement τ^* that maximizes welfare.

For case (i), firms adopt and use proxy access if $b < \beta + \sqrt{\frac{G+r}{s}}$, while firms with $b \geq \beta + \sqrt{\frac{G+r}{s}}$ do not adopt proxy access.

$$\begin{aligned}
& \mathbb{E}(V + \mathbb{1}_{\text{adopt}} G - y^2) \\
= & \int_0^B (V + \mathbb{1}_{\text{adopt}} G - y^2) \frac{1}{B}\, db \\
= & \frac{1}{B} \int_0^{\beta + \sqrt{\frac{G+r}{s}}} [V + G - s\beta^2 - (1-s)b^2]\, db + \frac{1}{B} \int_{\beta + \sqrt{\frac{G+r}{s}}}^B (V - b^2)\, db \\
= & V + \frac{1}{B}\left(\beta + \sqrt{\frac{G+r}{s}}\right) G - \frac{s}{B}\beta^2 \left(\beta + \sqrt{\frac{G+r}{s}}\right) + \frac{s}{3B}\left(\beta + \sqrt{\frac{G+r}{s}}\right)^3 - \frac{1}{3}B^2.
\end{aligned}$$

This payoff is locally unaffected by τ when $\tau \leq \alpha_a$.

For case (ii), firms adopt and use proxy access if $\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a) \leq b < \beta + \sqrt{\frac{G+r}{s}}$ or $b \leq \beta - \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$. Firms adopt but do not use proxy access if $\beta - \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a) <$

$b < \beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$, while firms with $b \geq \beta + \sqrt{\frac{G+r}{s}}$ do not adopt proxy access.

$$
\begin{aligned}
& \mathbb{E}(V + \mathbb{1}_{\text{adopt}} G - y^2) \\
= & \int_0^B (V + \mathbb{1}_{\text{adopt}} G - y^2) \frac{1}{B}\, db \\
= & \frac{1}{B} \int_0^{\beta - \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)} [V + G - s\beta^2 - (1-s)b^2]\, db \\
& + \frac{1}{B} \int_{\beta - \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)}^{\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)} (V + G - b^2)\, db \\
& + \frac{1}{B} \int_{\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)}^{\beta + \sqrt{\frac{G+r}{s}}} [V + G - s\beta^2 - (1-s)b^2]\, db \\
& + \frac{1}{B} \int_{\beta + \sqrt{\frac{G+r}{s}}}^{B} (V - b^2)\, db \\
= & V + \frac{1}{B}\left(\beta + \sqrt{\frac{G+r}{s}}\right) G \\
& - \frac{s}{B}\beta^2 \left[\beta + \sqrt{\frac{G+r}{s}} - 2\sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)\right] \\
& + \frac{s}{3B}\left\{\left[\beta - \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)\right]^3 + \left(\beta + \sqrt{\frac{G+r}{s}}\right)^3 - \left[\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)\right]^3\right\} - \frac{1}{3}B^2.
\end{aligned}
$$

Because

$$
\begin{aligned}
& \frac{\partial}{\partial \tau} \mathbb{E}(V + \mathbb{1}_{\text{adopt}} G - y^2) \\
= & \frac{s}{B}\sqrt{\frac{\gamma}{2\alpha_a s}} \left\{\beta^2 - \left[\beta - \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)\right]^2\right\} \\
& + \frac{s}{B}\sqrt{\frac{\gamma}{2\alpha_a s}} \left\{\beta^2 - \left[\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)\right]^2\right\} \\
= & -\frac{2s}{B}\left(\sqrt{\frac{\gamma}{2\alpha_a s}}\right)^3 (\tau - \alpha_a)^2 < 0,
\end{aligned}
$$

this payoff is locally decreasing in $\tau \in (\alpha_a, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\beta)$. Hence, $\tau \in (\alpha_a, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\beta)$ is dominated by $\tau \leq \alpha_a$.

For case (iii), firms adopt but do not use proxy access if $b < \beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$, adopt and use proxy access if $\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a) \leq b < \beta + \sqrt{\frac{G+r}{s}}$, and do not adopt proxy access if $b \geq \beta + \sqrt{\frac{G+r}{s}}$.

$$
\begin{aligned}
& \mathbb{E}(V + \mathbb{1}_{\text{adopt}} G - y^2) \\
= & \int_0^B (V + \mathbb{1}_{\text{adopt}} G - y^2) \frac{1}{B}\, db \\
= & \frac{1}{B} \int_0^{\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)} (V + G - b^2)\, db \\
& + \frac{1}{B} \int_{\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)}^{\beta + \sqrt{\frac{G+r}{s}}} [V + G - s\beta^2 - (1-s)b^2]\, db \\
& + \frac{1}{B} \int_{\beta + \sqrt{\frac{G+r}{s}}}^{B} (V - b^2)\, db \\
= & V + \frac{1}{B} \left(\beta + \sqrt{\frac{G+r}{s}} \right) G \\
& - \frac{s}{B} \beta^2 \left[\sqrt{\frac{G+r}{s}} - \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a) \right] \\
& + \frac{s}{3B} \left\{ \left(\beta + \sqrt{\frac{G+r}{s}} \right)^3 - \left[\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a) \right]^3 \right\} - \frac{1}{3} B^2.
\end{aligned}
$$

Since $\frac{\partial}{\partial \tau} \mathbb{E}(V + \mathbb{1}_{\text{adopt}} G - y^2) < 0$, this payoff is locally decreasing in $\tau \in (\alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\beta, \alpha_a + \sqrt{\frac{2\alpha_a (G+r)}{\gamma}})$. Hence, this range is dominated by $\tau \in (\alpha_a, \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\beta)$, which is itself dominated by $\tau \leq \alpha_a$.

Finally, for case (iv), firms adopt but do not use proxy access if $b < \beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$,

while firms with $b \geq \beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)$ do not adopt proxy access.

$$\begin{aligned}
& \mathbb{E}(V + \mathbb{1}_{\text{adopt}} G - y^2) \\
= & \int_0^B (V + \mathbb{1}_{\text{adopt}} G - y^2) \frac{1}{B}\, db \\
= & \frac{1}{B} \int_0^{\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)} (V + G - b^2)\, db \\
& + \frac{1}{B} \int_{\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)}^B (V - b^2)\, db \\
= & V + \frac{1}{B}\left[\beta + \sqrt{\frac{\gamma}{2\alpha_a s}}(\tau - \alpha_a)\right] G - \frac{1}{3} B^2.
\end{aligned}$$

This payoff is locally increasing in $\tau \geq \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}$.

Hence, similar to the baseline model, the optimal ownership requirement τ^* is either low ($\tau^* \leq \alpha_a$) or high ($\tau^* \geq \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}$).

Therefore, welfare under $\tau \geq \alpha_a + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}$ exceeds welfare under $\tau \leq \alpha_a$ if and only if

$$\tau > \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}} \frac{s}{3G} (\beta + \sqrt{\frac{G+r}{s}}) \left(-2\beta^2 + 2\beta \sqrt{\frac{G+r}{s}} + \frac{G+r}{s} \right) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}.$$

If

$$\alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}} \frac{s}{3G} (\beta + \sqrt{\frac{G+r}{s}}) \left(-2\beta^2 + 2\beta \sqrt{\frac{G+r}{s}} + \frac{G+r}{s} \right) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}} < \bar{T},$$

then the social planner maximizes welfare by choosing $\tau^* = \bar{T}$. Otherwise, the social planner maximizes welfare by choosing $\tau^* \leq \alpha_a$.

A.10 Derivation for Corollary 11

The detailed derivations supporting Corollary 11 are provided below.

From Proposition 10, a high ownership requirement is optimal ($\tau^* = \bar{T}$) if and only if

$$\alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}\left(\beta + \sqrt{\frac{G+r}{s}}\right)\left(-2\beta^2 + 2\beta\sqrt{\frac{G+r}{s}} + \frac{G+r}{s}\right) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}} < \bar{T}.$$

Hence, the dependence of the optimal ownership requirement on activist bias β is determined by how the left-hand side of the above inequality varies with β.

Define

$$f(\beta) \equiv \left(\beta + \sqrt{\frac{G+r}{s}}\right)\left(-2\beta^2 + 2\beta\sqrt{\frac{G+r}{s}} + \frac{G+r}{s}\right).$$

The derivative of $f(\beta)$ with respect to β is

$$\begin{aligned} & \frac{\partial}{\partial\beta} f(\beta) \\ = \; & -2\beta^2 + 2\beta\sqrt{\frac{G+r}{s}} + \frac{G+r}{s} + \left(\beta + \sqrt{\frac{G+r}{s}}\right)\left(-4\beta + 2\sqrt{\frac{G+r}{s}}\right) \\ = \; & -6\beta^2 + \frac{3(G+r)}{s}. \end{aligned}$$

Therefore, $f(\beta)$ is increasing in β for $\beta \leq \sqrt{\frac{G+r}{2s}}$ and decreasing in β for $\beta \geq \sqrt{\frac{G+r}{2s}}$.

Evaluating $f(\beta)$ at several benchmark values,

$$f(0) = \left(\frac{G+r}{s}\right)^{\frac{3}{2}},$$

$$f\left(\sqrt{\frac{G+r}{2s}}\right) = (1+\sqrt{2})\left(\frac{G+r}{s}\right)^{\frac{3}{2}},$$

and

$$f\left(\sqrt{\frac{G+r}{s}}\right) = 2\left(\frac{G+r}{s}\right)^{\frac{3}{2}}.$$

Define

$$T_L \equiv \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}f(0) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}} = \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}\left(\frac{G+r}{s}\right)^{\frac{3}{2}} + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}, \tag{A.3}$$

$$T_M \equiv \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}f\left(\sqrt{\frac{G+r}{s}}\right) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}} = \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}2\left(\frac{G+r}{s}\right)^{\frac{3}{2}} + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}, \tag{A.4}$$

and

$$T_H \equiv \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}f\left(\sqrt{\frac{G+r}{2s}}\right) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}} = \alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}(1+\sqrt{2})\left(\frac{G+r}{s}\right)^{\frac{3}{2}} + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}}. \tag{A.5}$$

Therefore, if $\bar{T} > T_H$, then $\tau^* = \bar{T}$ regardless of β. If $\bar{T} \leq T_L$, then $\tau^* \leq \alpha_a$ regardless of β. If $T_L < \bar{T} \leq T_M$, then $\tau^* = \bar{T}$ for $\beta < \beta_T$ and $\tau^* \leq \alpha_a$ for $\beta \geq \beta_T$, where $\beta_T \in \left(0, \sqrt{\frac{G+r}{2s}}\right)$ is the unique root satisfying

$$\alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}\left(\beta_T + \sqrt{\frac{G+r}{s}}\right)\left(-2\beta_T^2 + 2\beta_T\sqrt{\frac{G+r}{s}} + \frac{G+r}{s}\right) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}} = \bar{T}. \tag{A.6}$$

Finally, if $T_M < \bar{T} < T_H$, then $\tau^* = \bar{T}$ for $\beta < \beta_L$ or $\beta > \beta_H$, and $\tau^* \leq \alpha_a$ for $\beta \in [\beta_L, \beta_H]$, where $\beta_L \in \left(0, \sqrt{\frac{G+r}{2s}}\right)$ and $\beta_H \in \left(\sqrt{\frac{G+r}{2s}}, \sqrt{\frac{G+r}{s}}\right)$ are the smaller and larger roots, respectively, of

$$\alpha_a + \sqrt{\frac{2\alpha_a s}{\gamma}}\frac{s}{3G}\left(\beta + \sqrt{\frac{G+r}{s}}\right)\left(-2\beta^2 + 2\beta\sqrt{\frac{G+r}{s}} + \frac{G+r}{s}\right) + \sqrt{\frac{2\alpha_a(G+r)}{\gamma}} = \bar{T}. \tag{A.7}$$

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