Online appendix to

Controlling inflation with timid monetary-fiscal regime changes

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A.1. Parametrization

Table A1. Calibration

Parameter	Value	Description
β	0.99	Intertemporal discount factor
θ	11	Dixit-Stiglitz elasticity of substitution
α	0.75	Calvo probability not to optimise prices
N	0.333	Hours worked
$\bar{b}$	0.4	Debt-to-GDP ratio
$\bar{c}$	0.8	Consumption-to-GDP ratio

A.2. The model

In the paper we use a simple New Keynesian model with fiscal policy.

Households. The representative household maximises lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \mu N_t\right) \tag{A1}$$

under a sequence of budget constraints given by

$$P_t C_t + (1+i_t)^{-1} B_t \leq P_t w_t N_t + F_t - \tau_t + B_{t-1}. \tag{A2}$$

E_0 is the expectations operator conditional on time $t = 0$ information, C_t is real consumption, N_t is labour, w_t is the level of real wages, F_t are profits, τ_t are taxes, $R_t = 1 + i_t$ is the gross return on bonds purchased at date t (i.e., B_t). Maximization yields the first order conditions

$$1 = \beta \mathbb{E}_t \left[\frac{P_t}{P_{t+1}} (1 + i_t) \frac{C_t}{C_{t+1}} \right], \tag{A3}$$

$$w_t = \mu C_t. \tag{A4}$$

Final good producers. In each period, a final good Y_t is produced by perfectly competitive firms, using a continuum of intermediate inputs $Y_{i,t}$ indexed by $i \in [0, 1]$ and a standard CES production function $Y_t = \left(\int_0^1 Y_{i,t}^{(\theta-1)/\theta} di \right)^{\theta/(\theta-1)}$, with $\theta > 1$. Final good producers' demand schedules for intermediate good quantities are $Y_{i,t} = (P_{i,t}/P_t)^{-\theta} Y_t$, where the aggregate price index is defined as $P_t = \left(\int_0^1 P_{i,t}^{1-\theta} di \right)^{1/(1-\theta)}$.

Intermediate goods producers. There exists a continuum of intermediate goods produced by firms with constant returns to scale production function: $Y_{i,t} = N_{i,t}$. Intermediate goods producers compete monopolistically and set prices according to the usual Calvo mechanism. In each period each firm has a fixed probability $1 - \alpha$ to re-optimise its nominal price $P_{i,t}^*$ in order to maximise profits. With probability α, instead, the firm keeps its nominal price unchanged. Using the stochastic discount factor $Q_{t,t+j} = \beta^j \frac{P_t C_t}{P_{t+j} C_{t+j}}$, the first order condition of the firm's problem gives the optimal relative price

$$p_{i,t}^* \equiv \frac{P_{i,t}^*}{P_t} = \frac{\theta}{\theta - 1} \frac{\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} w_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta-1}}. \tag{A5}$$

As in Ascari and Ropele (2009), we introduce two auxiliary variables that allow to rewrite the last expression recursively

$$\psi_t \equiv \mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta} w_{t+j} = \frac{Y_t}{C_t} w_t + \alpha\beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta} \psi_{t+1} \right], \tag{A6}$$

$$\phi_t \equiv \mathbb{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \frac{Y_{t+j}}{C_{t+j}} \left(\frac{P_{t+j}}{P_t} \right)^{\theta-1} = \frac{Y_t}{C_t} + \alpha\beta \mathbb{E}_t \left[\Pi_{t+1}^{\theta-1} \phi_{t+1} \right], \tag{A7}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the aggregate gross rate of inflation. As all re-optimizing firms face the same problem and pick the same relative price, aggregate inflation evolves according to

(A8) $$1 = \alpha \Pi_t^{\theta-1} + (1-\alpha)\left(p_{i,t}^*\right)^{1-\theta}.$$

Individual firms demand labour according to the relation $N_{i,t} = \left(P_{i,t}/P_t\right)^{-\theta} Y_t$. Aggregating this expression yields $N_t = Y_t s_t$, where $N_t \equiv \int_0^1 N_{i,t} di$ and $s_t \equiv \int_0^1 \left(P_{i,t}/P_t\right)^{-\theta} di$. The variable s_t measures the dispersion of relative prices across intermediate firms. s_t is bounded below at one and it represents the resource costs (or inefficiency loss) due to relative price dispersion under the Calvo mechanism. s_t can be written recursively as

(A9) $$s_t = (1-\alpha)\left(p_{i,t}^*\right)^{-\theta} + \alpha \left(\Pi_t\right)^{\theta} s_{t-1}.$$

Fiscal and monetary policies. The government budget constraint is given by

(A10) $$\left(1+i_t\right)^{-1} b_t = \frac{b_{t-1}}{\Pi_t} + G - \tau_t,$$

where $b_t \equiv B_t/P_t$, G, and τ_t are the levels of government debt, expenditure, and taxes, all in real terms. Note that we assumed for simplicity that the government chooses a constant level of expenditure G. Taxes are set according to the fiscal policy rule

(A11) $$\tau_t = \tau \left(\frac{b_{t-1}}{b}\right)^{\gamma_{\tau,t}} e^{u_{\tau,t}},$$

while the central bank sets the interest rate following the simple Taylor rule

(A12) $$R_t = R \Pi_t^{\gamma_{\pi,t}} e^{u_{m,t}}.$$

Note that the parameters $\gamma_{\tau,t}$ and $\gamma_{\pi,t}$ are indexed with time as they can take different values according to the underlying Markov-switching process.

Complete nonlinear model. After imposing market clearing (with $Y_t = C_t + G$), the dynamics of aggregate variables is described by the following set of equations (here reproduced for convenience)

$$1 = \beta \mathbb{E}_t \left[\frac{R_t}{\Pi_{t+1}} \frac{Y_t - G}{Y_{t+1} - G}\right]$$

$$
\begin{aligned}
w_t &= \mu\left(Y_t - G\right) \\
1 &= \alpha \Pi_t^{\theta-1} + (1-\alpha)\left(p_{i,t}^{*}\right)^{1-\theta} \\
p_{i,t}^{*} &= \frac{\theta}{\theta-1}\frac{\psi_t}{\phi_t} \\
\psi_t &= \frac{Y_t}{Y_t - G} w_t + \alpha\beta\mathbb{E}_t\left[\Pi_{t+1}^{\theta}\psi_{t+1}\right] \\
\phi_t &= \frac{Y_t}{Y_t - G} + \alpha\beta\mathbb{E}_t\left[\Pi_{t+1}^{\theta-1}\phi_{t+1}\right] \\
s_t &= (1-\alpha)\ \left(p_{i,t}^{*}\right)^{-\theta} + \alpha\Pi_t^{\theta} s_{t-1} \\
N_t &= s_t Y_t \\
\frac{b_t}{R_t} &= \frac{b_{t-1}}{\Pi_t} + G - \tau_t \\
\tau_t &= \tau\left(\frac{b_{t-1}}{b}\right)^{\gamma_{\tau,t}} e^{\sigma_\tau u_{\tau,t}} \\
R_t &= R\Pi_t^{\gamma_{\pi,t}} e^{\sigma_m u_{m,t}}
\end{aligned}
$$

Zero-inflation steady state. If we switch off the exogenous processes $u_{\tau,t}$ and $u_{m,t}$, we can solve for the zero-inflation steady state

$$
\begin{aligned}
R &= \beta^{-1} \\
w &= \mu\bar{c}Y \\
p_i^{*} &= 1 \\
\psi &= \frac{1}{\bar{c}\left(1-\alpha\beta\right)} w \\
\phi &= \frac{1}{\bar{c}\left(1-\alpha\beta\right)} \\
s &= 1 \\
N &= sY = Y \\
\tau &= \left[(1-\bar{c}) + \bar{b}\left(1-\beta\right)\right] Y
\end{aligned}
$$

where we used the ratios $\bar{c} \equiv C/Y$ and $\bar{b} \equiv b/Y$. Further, note that the equation

$$
p_{i,t}^{*} = \frac{\theta}{\theta-1}\frac{\psi_t}{\phi_t}
$$

implies the following parameter restriction

$$1 = \frac{\theta}{\theta - 1}\frac{\psi}{\phi} = \frac{\theta}{\theta - 1}\mu Y \bar{c}. \tag{A13}$$

Log-linearised model. The nonlinear model can be log-linearised around the non-stochastic zero-inflation steady state. Standard computations lead to

$$\begin{aligned}
\frac{1}{\bar{c}}\hat{Y}_t &= \frac{1}{\bar{c}}\mathbb{E}_t\hat{Y}_{t+1} - \left(\hat{R}_t - \mathbb{E}_t\hat{\Pi}_{t+1}\right), \\
\hat{\Pi}_t &= \frac{\lambda}{\bar{c}}\hat{Y}_t + \beta\mathbb{E}_t\hat{\Pi}_{t+1}, \\
\hat{R}_t &= \gamma_{\pi,t}\hat{\Pi}_t + \sigma_m u_{m,t} \\
\hat{b}_t &= \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,t}\right)\hat{b}_{t-1} - \frac{1}{\beta}\hat{\Pi}_t + \hat{R}_t - \frac{1}{\beta}\frac{\tau}{b}\sigma_\tau u_{\tau,t},
\end{aligned}$$

with $\lambda \equiv (1-\alpha)(1-\alpha\beta)/\alpha$. These equations correspond to equations (12)-(15) in the main text.

A.3. The FRWZ solution method

The analysis of determinacy under Markov-switching coefficients can be performed by checking the existence of a unique stable MSV solution. In order to find all the MSV solutions, we adopt the perturbation method proposed by FRWZ. The method can be applied directly on the nonlinear version of the model. However, instead of considering the complete nonlinear model outlined above, it is convenient to manipulate the equations and reduce the dimensionality of the system. The smaller system turns out to be:

$$\begin{aligned}
1 &= E_t\left[\frac{\Pi_t^{\gamma_{\pi,t}}e^{u_{m,t}}}{\Pi_{t+1}}\frac{Y_t - G}{Y_{t+1} - G}\right], \\
\left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}}\phi_t &= \frac{\theta}{\theta-1}\mu Y_t + \alpha\beta E_t\left[\Pi_{t+1}^{\theta}\left(\frac{1-\alpha\Pi_{t+1}^{\theta-1}}{1-\alpha}\right)^{\frac{1}{1-\theta}}\phi_{t+1}\right], \\
\phi_t &= \frac{Y_t}{Y_t - G} + \alpha\beta E_t\left[\Pi_{t+1}^{\theta-1}\phi_{t+1}\right], \\
\frac{b_t}{R\Pi_t^{\gamma_{\pi,t}}e^{u_{m,t}}} &= \frac{b_{t-1}}{\Pi_t} + G - \tau\left(\frac{b_{t-1}}{b}\right)^{\gamma_{\tau,t}}e^{u_{\tau,t}}.
\end{aligned}$$

Using the notation of FRWZ, this system can be rewritten as

$$\mathbb{E}_t\boldsymbol{f}\left(\boldsymbol{y}_{t+1}, \boldsymbol{y}_t, b_t, b_{t-1}, \boldsymbol{\varepsilon}_{t+1}, \boldsymbol{\varepsilon}_t, \boldsymbol{\theta}_{t+1}, \boldsymbol{\theta}_t\right) = \boldsymbol{0},$$

where b_t is the only predetermined variable and the remaining non-predetermined variables are stacked in vector $\boldsymbol{y}_t' \equiv [Y_t, \Pi_t, \phi_t]$. The exogenous shocks appear in vector $\boldsymbol{\varepsilon}_t' \equiv [u_{m,t}, u_{\tau,t}]$, and $\boldsymbol{\theta}_t' \equiv [\gamma_{\pi,t}, \gamma_{\tau,t}]$ is the vector of Markov-switching parameters. We look for recursive solutions such as

$$b_t = h_t(b_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$$
$$\boldsymbol{y}_t = \boldsymbol{g}_t(b_{t-1}, \boldsymbol{\varepsilon}_t, \chi)$$

perturbed around the non-stochastic zero-inflation steady state $[b, \boldsymbol{y}']'$, where χ represents the perturbation parameter. Note that in our model the solutions are regime-dependent (h_t and $\boldsymbol{g}_t$ follow the latent Markov process too), while the steady state is not. The stability properties of each solution is governed by parameters of the first order expansion of the solutions, which reads as follows under regime i

$$b_t \approx b + h_{i,b}(b_{t-1} - b) + \boldsymbol{h}_{i,\varepsilon}\boldsymbol{\varepsilon}_t + h_{i,\chi}\chi, \tag{A14}$$

$$\boldsymbol{y}_t \approx \boldsymbol{y} + \boldsymbol{g}_{i,b}(b_{t-1} - b) + \boldsymbol{g}_{i,\varepsilon}\boldsymbol{\varepsilon}_t + \boldsymbol{g}_{i,\chi}\chi, \tag{A15}$$

for $i = 1, 2$. In these expressions we used a matrix notation for the partial derivatives: for example, $\boldsymbol{g}_{i,\varepsilon}$ is a (3×2) matrix whose first column is given by the partial derivative of $\boldsymbol{g}_i$ with respect to $u_{m,t}$, and so forth.

The elements in $h_{i,b}$, $\boldsymbol{h}_{i,\varepsilon}$, $h_{i,\chi}$, $\boldsymbol{g}_{i,b}$, $\boldsymbol{g}_{i,\varepsilon}$, $\boldsymbol{g}_{i,\chi}$ are unknown and can be found by exploiting the fact that the derivatives of $\mathbb{E}_t\boldsymbol{f}$ are equal to zero. Proposition 1 in FRWZ uses the chain rule to state that the coefficients in $h_{i,b}$ and $\boldsymbol{g}_{i,b}$ can be obtained by solving a system of quadratic polynomial equations that corresponds to equation (A4) in FRWZ. To derive such system, we need to compute the partial derivatives of $\boldsymbol{f}$

$$\begin{bmatrix} \boldsymbol{f}_{ij,\boldsymbol{y}_{t+1}} & \boldsymbol{f}_{ij,\boldsymbol{y}_t} \end{bmatrix} = \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 & -\frac{1}{\bar{c}Y} & -\gamma_{\pi,i} & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta & -\frac{\theta}{\theta-1}\mu & \frac{\alpha}{1-\alpha}\phi & 1 \\ 0 & \alpha\beta\left(1-\theta\right)\phi & -\alpha\beta & \frac{1-\bar{c}}{\bar{c}^2Y} & 0 & 1 \\ 0 & 0 & 0 & 0 & \left(1-\beta\gamma_{\pi,i}\right)b & 0 \end{bmatrix},$$

$$
\begin{bmatrix} \boldsymbol{f}_{ij,b_t} & \boldsymbol{f}_{ij,b_{t-1}} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \beta & \frac{\tau}{b}\gamma_{\tau,i} - 1 \end{bmatrix}.
$$

Note that the derivatives are indexed with ij to indicate that they must be evaluated at the steady state with $\boldsymbol{\theta}_t = i$ and $\boldsymbol{\theta}_{t+1} = j$ (refer to FRWZ for further details). With these derivatives in hand, we can apply formula (A4) of FRWZ and obtain a set of 8 equations in 8 unknowns

$$
\begin{aligned}
0 = & \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\tau}{b}\gamma_{\tau,1} - 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{\bar{c}Y} & -\gamma_{\pi,1} & 0 \\ -\frac{\theta}{\theta-1}\mu & \frac{\alpha}{1-\alpha}\phi & 1 \\ \frac{1-\bar{c}}{\bar{c}^2 Y} & 0 & 1 \\ 0 & (1-\beta\gamma_{\pi,1})\,b & 0 \end{bmatrix} \begin{bmatrix} g_{y,1} \\ g_{\pi,1} \\ g_{\phi,1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \beta \end{bmatrix} h_1 \\
& + p_{11} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta\,(1-\theta)\,\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,1} \\ g_{\pi,1} \\ g_{\phi,1} \end{bmatrix} h_1 + p_{12} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta\,(1-\theta)\,\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,2} \\ g_{\pi,2} \\ g_{\phi,2} \end{bmatrix} h_1, \\
0 = & \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\tau}{b}\gamma_{\tau,2} - 1 \end{bmatrix} + \begin{bmatrix} -\frac{1}{\bar{c}Y} & -\gamma_{\pi,2} & 0 \\ -\frac{\theta}{\theta-1}\mu & \frac{\alpha}{1-\alpha}\phi & 1 \\ \frac{1-\bar{c}}{\bar{c}^2 Y} & 0 & 1 \\ 0 & (1-\beta\gamma_{\pi,2})\,b & 0 \end{bmatrix} \begin{bmatrix} g_{y,2} \\ g_{\pi,2} \\ g_{\phi,2} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \beta \end{bmatrix} h_2 \\
& + p_{21} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta\,(1-\theta)\,\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,1} \\ g_{\pi,1} \\ g_{\phi,1} \end{bmatrix} h_2 + p_{22} \begin{bmatrix} \frac{1}{\bar{c}Y} & 1 & 0 \\ 0 & \alpha\beta\frac{\alpha\theta-\alpha-\theta}{1-\alpha}\phi & -\alpha\beta \\ 0 & \alpha\beta\,(1-\theta)\,\phi & -\alpha\beta \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{y,2} \\ g_{\pi,2} \\ g_{\phi,2} \end{bmatrix} h_2,
\end{aligned}
$$

where, with a slight abuse of notation, the h_i and $g_{x,i}$ coefficients are the elements of $h_{i,b}$ and $\boldsymbol{g}_{i,b}$ defined above. This system can be simplified by subtracting the third equation from the second, and the seventh from the sixth, to eliminate $g_{\phi,1}$ and $g_{\phi,2}$. We then arrive at

$$
\text{(A16)} \qquad 0 = \frac{1}{\bar{c}Y} g_{y,1} + \gamma_{\pi,1} g_{\pi,1} - h_1 \left[p_{11} \left(g_{\pi,1} + \frac{1}{\bar{c}Y} g_{y,1} \right) + p_{12} \left(g_{\pi,2} + \frac{1}{\bar{c}Y} g_{y,2} \right) \right],
$$

$$
\text{(A17)} \qquad 0 = g_{\pi,1} - \frac{\lambda}{\bar{c}Y} g_{y,1} - \beta h_1 \left(p_{11} g_{\pi,1} + p_{12} g_{\pi,2} \right),
$$

(A18) $$0 = \beta h_1 + b\left(1 - \beta\gamma_{\pi,1}\right) g_{\pi,1} + \frac{\tau}{b}\gamma_{\tau,1} - 1,$$

(A19) $$0 = \frac{1}{\bar{c}Y} g_{y,2} + \gamma_{\pi,2} g_{\pi,2} - h_2 \left[p_{21}\left(g_{\pi,1} + \frac{1}{\bar{c}Y} g_{y,1}\right) + p_{22}\left(g_{\pi,2} + \frac{1}{\bar{c}Y} g_{y,2}\right)\right],$$

(A20) $$0 = g_{\pi,2} - \frac{\lambda}{\bar{c}Y} g_{y,2} - \beta h_2 \left(p_{21} g_{\pi,1} + p_{22} g_{\pi,2}\right),$$

(A21) $$0 = \beta h_2 + b\left(1 - \beta\gamma_{\pi,2}\right) g_{\pi,2} + \frac{\tau}{b}\gamma_{\tau,2} - 1.$$

Note that the term λ appears after exploiting the restriction (A13). Finally, these equations can be further combined to obtain

$$\begin{aligned}
0 &= g_{\pi,1}\left[1 + \lambda\gamma_{\pi,1} - p_{11} h_1 \left(1 + \beta + \lambda\right) + p_{11}^2 \beta h_1^2\right] + \left(1 - p_{11}\right)\left(1 - p_{22}\right) \beta h_1 h_2 g_{\pi,1} \\
&\quad + \left(1 - p_{11}\right) h_1 g_{\pi,2} \left[p_{11}\beta h_1 + p_{22}\beta h_2 - \left(1 + \beta + \lambda\right)\right], \\
0 &= g_{\pi,2}\left[1 + \lambda\gamma_{\pi,2} - p_{22} h_2 \left(1 + \beta + \lambda\right) + p_{22}^2 \beta h_2^2\right] + \left(1 - p_{11}\right)\left(1 - p_{22}\right) \beta h_1 h_2 g_{\pi,2} \\
&\quad + \left(1 - p_{22}\right) h_2 g_{\pi,1} \left[p_{11}\beta h_1 + p_{22}\beta h_2 - \left(1 + \beta + \lambda\right)\right], \\
g_{\pi,1} &= \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) - h_1}{b\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)}, \\
g_{\pi,2} &= \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) - h_2}{b\left(\frac{1}{\beta} - \gamma_{\pi,2}\right)},
\end{aligned}$$

which correspond to equations (18)-(21) in the main text.

As this system cannot be solved using traditional approaches such as the generalised Schur decomposition, we follow FRWZ and adopt the Groebner basis algorithm to find all existing solutions, i.e., all the possible 8-tuples made by coefficients h_1, $g_{y,1}$, $g_{\pi,1}$, $g_{\phi,1}$, h_2, $g_{y,2}$, $g_{\pi,2}$, $g_{\phi,2}$ that satisfy the system of equations.

Note that to characterise the first order expansion of the MSV solution we still have to determine the other coefficients $\boldsymbol{h}_{i,\varepsilon}$, $h_{i,\chi}$, $\boldsymbol{g}_{i,\varepsilon}$, $\boldsymbol{g}_{i,\chi}$ that appear in equations (A14) and (A15). Fortunately, doing so is an easy task. Proposition 1 in FRWZ shows that one has to solve two separate systems of linear equations corresponding to their equations (A5) and (A6).

A.4. Mean square stability under regime switching

To asses the stability of the MSV solutions when some parameters are allowed to switch, FRWZ use the notion of mean square stability (MSS), which is discussed by Costa et al. (2005) and Farmer et al. (2009).

MSS requires the existence of

$$lim_{t\to\infty}\mathbb{E}_0\left(\begin{bmatrix} b_t \\ \boldsymbol{y}_t \end{bmatrix}\right), \quad \text{and} \quad lim_{t\to\infty}\mathbb{E}_0\left(\begin{bmatrix} b_t \\ \boldsymbol{y}_t \end{bmatrix}\begin{bmatrix} b_t \\ \boldsymbol{y}_t \end{bmatrix}'\right)$$

The MSS condition constrains the values of the autoregressive roots in the state variable policy function weighted by the probability of switching regimes. In our context with one state variable and two regimes, MSS formally states that one solution is stable if an only if all the eigenvalues of the matrix

$$\begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{bmatrix}\begin{bmatrix} h_1^2 & 0 \\ 0 & h_2^2 \end{bmatrix} = \begin{bmatrix} p_{11}h_1^2 & (1-p_{22})h_2^2 \\ (1-p_{11})h_1^2 & p_{22}h_2^2 \end{bmatrix}$$

are inside the unit circle. The characteristic polynomial of this matrix is

$$z^2 + a_1 z + a_0 = z^2 - \left(p_{11}h_1^2 + p_{22}h_2^2\right) z + \left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2 = 0.$$

As discussed in LaSalle (1986, p. 28), both eigenvalues are inside the unit circle if and only if both the following conditions hold

$$|a_0| < 1$$
$$|a1| < 1 + a_0,$$

which in our case give

$$\begin{aligned} \left|\left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2\right| &< 1, \\ p_{11}h_1^2 + p_{22}h_2^2 &< 1 + \left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2. \end{aligned}$$

If we assume $p_{11} + p_{22} \geq 1$, the two conditions can be be rewritten as

$$\begin{aligned} \left(p_{11} + p_{22} - 1\right) h_1^2 h_2^2 &< 1, \\ p_{11}h_1^2\left(1 - h_2^2\right) + p_{22}h_2^2\left(1 - h_1^2\right) + h_1^2 h_2^2 &< 1, \end{aligned}$$

which correspond to equations (10) and (11) in the main text.

A.5. Figure 3b: A numerical example for the timid fiscal deviations

Consider the case with $p_{11} = p_{22} = p < 1$ in Figure 3b in the main text. If regime 1 is AM/PF so that $g_{\pi,1} = g_{y,1} = 0$, from system (A16)-(A21) we can derive the equations

$$0 = \underbrace{\frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) - h_1}{b\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)}}_{g_{\pi,1}} \left[1 + \lambda\gamma_{\pi,1} - p_{11}h_1\left(1 + \beta + \lambda\right) + p_{11}^2\beta h_1^2\right], \tag{A22}$$

$$0 = \underbrace{\frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) - h_2}{b\left(\frac{1}{\beta} - \gamma_{\pi,2}\right)}}_{g_{\pi,2}} \left[1 + \lambda\gamma_{\pi,2} - p_{22}h_2\left(1 + \beta + \lambda\right) + p_{22}^2\beta h_2^2\right]. \tag{A23}$$

Call a solution stemming from $g_{\pi,i} = 0$, that therefore depends only on the fiscal coefficient $\gamma_{\tau,1}$, $\bar{h}_i(\gamma_{\tau,i})$. Take a passive fiscal policy in regime 1 with $\gamma_{\tau,1} = 0.2$ and $p = 0.95$. In this case the stable solution is $\bar{h}_1(\gamma_{\tau,1})$ that, under our calibration becomes $h_1 = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) = 0.9068$. In order to have MSS, conditions (10) and (11) must hold. Under this case the most stringent one of the two turns out to be equation (11) that becomes $0.822p\left(1 - h_2^2\right) + 0.178ph_2^2 - 1 + 0.822h_2^2 < 0$. Then, in order to have MSS the stable solution in regime 2 must satisfy the following: $-1.0209 < h_2 < 1.0209$.[1] If the stable solution in regime 2 is again $\bar{h}_2(\gamma_{\tau,2})$ then: $h_2 = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) \in (-1.0209, 1.0209)$. In this case we get a "timid" fiscal deviation for:

$$-0.02 < \gamma_{\tau,2} < 3.93$$

Substituting $h_2 = 1.0209$ in the square bracket in equation (A23) we get $\gamma_{\pi,2} = 0.955$ which is the lower bound of the correspondent "timid" monetary deviation:

$$0.955 < \gamma_{\pi,2} < 1$$

Note that these results hold for $\gamma_{\tau,1} = 0.2$; obviously, for every given $\gamma_{\tau,1}$ we could obtain different $\gamma_{\tau,2}$ and $\gamma_{\pi,2}$ coefficients.

A.6. The absorbing case

When regime 1 is absorbing ($p_{11} = 1$), MSS requires the eigenvalues of the following matrix to lie inside the unit circle:

[1] Condition (10) instead gives $-1.162 < h_2 < 1.162$.

$$\begin{bmatrix} h_1^2 & (1-p_{22})h_2^2 \\ 0 & p_{22}h_2^2 \end{bmatrix}$$

The two eigenvalues are equal to h_1^2 and $p_{22}h_2^2$, so that the conditions for MSS are

$$|h_1| < 1, \tag{A24}$$

$$|h_2| < \frac{1}{\sqrt{p_{22}}}. \tag{A25}$$

Moreover, by plugging $p_{11} = 1$ in equations (18) and (20) we obtain equation (23), that is

$$0 = \frac{\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) - h_1}{b\left(\frac{1}{\beta} - \gamma_{\pi,1}\right)} \left[1 + \lambda\gamma_{\pi,1} - h_1\left(1+\beta+\lambda\right) + \beta h_1^2\right].$$

This equation has three solutions for h_1: one $\bar{h}_1(\gamma_{\tau,1})$ that depends on the fiscal coefficient $\gamma_{\tau,1}$ (first term), the other two $\bar{h}_1(\gamma_{\pi,1})$ that depend on the monetary coefficient $\gamma_{\pi,1}$, (square brackets).

When regime 1 is AM/PF then $g_{\pi,1} = g_{y,1} = 0$, since debt has no impact on inflation and output. Using these restrictions, equations (19) and (21) yield equation (A23) for the non-absorbing regime.

A.6.1. Figure 3a

Consider an AM/PF regime 1. The stability condition for the absorbing state (A24) is the same as under fixed coefficients. As fiscal policy is passive, then

$$\left|\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right)\right| < 1,$$

that is $(1-\beta)\frac{b}{\tau} < \gamma_{\tau,1} < (1+\beta)\frac{b}{\tau}$. Employing our calibration, we have $\gamma_{\tau,1} \in (0.019, 3.892)$. The condition for not having another stable solution is that the two solutions in the square bracket of (23) should be outside the unit circle, that is, $\gamma_{\pi,1} > 1$: monetary policy needs to be active. We now analyse the MSS condition (A25) for regime 2, given that regime 1 is AM/PF. To do so, we have to solve the third order equation (A23) for h_2, and obtain one solution $\bar{h}_2(\gamma_{\tau,2})$ and two solutions $\bar{h}_2(\gamma_{\pi,2})$. Let us distinguish two cases according to the stability of the $\bar{h}_2(\gamma_{\tau,2})$ solution in regime 2.

A stable $\bar{h}_2(\gamma_{\tau,2})$ solution. In this case the solution must satisfy: $\left|\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right)\right| < \frac{1}{\sqrt{p_{22}}}$, that gives equation (24)

$$\frac{b}{\tau}\left(1 - \frac{\beta}{\sqrt{p_{22}}}\right) < \gamma_{\tau,2} < \frac{b}{\tau}\left(1 + \frac{\beta}{\sqrt{p_{22}}}\right)$$

which, employing our calibration, returns: $\gamma_{\tau,2} \in (-0.032, 3.952)$.

To have a unique stable $\bar{h}_2(\gamma_{\tau,2})$ solution the other (two $\bar{h}_2(\gamma_{\pi,2})$) solutions must be both unstable, which translates into equation (25)

$$\gamma_{\pi,2} > \sqrt{p_{22}} - \frac{\left(1-\beta\sqrt{p_{22}}\right)\left(1-\sqrt{p_{22}}\right)}{\lambda},$$

that is, $\gamma_{\pi,2} > 0.964$. This first case describes the upper-right zone in Figure 3a.

An unstable $\bar{h}_2(\gamma_{\tau,2})$ solution. Under this case $\left|\frac{1}{\beta}\left(1-\frac{\tau}{b}\gamma_{\tau,2}\right)\right| > \frac{1}{\sqrt{p_{22}}}$ which corresponds to equation (28). Under our calibration, we have $\gamma_{\tau,2} < -0.032, \gamma_{\tau,2} > 3.952$.

In order to have only one stable solution, the two $\bar{h}_2(\gamma_{\pi,2})$ solutions of (A23) must be one inside and the other outside the unit circle, which yields equation (29)

$$\gamma_{\pi,2} < \sqrt{p_{22}} - \frac{\left(1-\beta\sqrt{p_{22}}\right)\left(1-\sqrt{p_{22}}\right)}{\lambda}.$$

Employing our calibration, we get $\gamma_{\pi,2} < 0.964$. Again, monetary policy can be passive, and the more so, the lower p_{22}. This second case describes the lower-left zone in Figure 3a.

As the absorbing regime is AM/PF, we know that the only stable solution for this regime corresponds the fiscal backing one while, as the value of $\gamma_{\pi,1}$ is greater than 1, a stable $\bar{h}_1(\gamma_{\pi,1})$ solution does not exist. To have determinacy, there should only be one corresponding stable solution, h_2. The threshold values $\bar{\gamma}_{\pi,2}$ and $\bar{\gamma}_{\tau,2}$ for the timid changes in monetary and fiscal policies define the conditions for the existence of a stable solution in the second regime. In particular, starting from an AM/PF absorbing regime, determinacy in regime 2 admits either only timid deviations from AM/PF (upper-right zone, $\gamma_{\pi,2} > \bar{\gamma}_{\pi,2}$; $\gamma_{\tau,2} > \bar{\gamma}_{\tau,2}$) or large deviations in both monetary and fiscal policy, such that we are definitely in a PM/AF regime (lower-left zone, $\gamma_{\pi,2} < \bar{\gamma}_{\pi,2}$; $\gamma_{\tau,2} < \bar{\gamma}_{\tau,2}$). More precisely: (i) if fiscal policy is not too active (if $\gamma_{\tau,2} > \bar{\gamma}_{\tau,2}$), there exists a fiscal backing solution, where dynamics are Ricardian in both regimes; (ii) if monetary policy is sufficiently passive (if $\gamma_{\pi,2} < \bar{\gamma}_{\pi,2}$), there exists a fiscal unbacking solution, with wealth effect in the second regime.[2] Hence, to have only one solution, either (i) or (ii) should be satisfied. When both are satisfied, we have two solutions as in the white region; when neither is satisfied, we have no stable solutions as in the dark blue region.

[2]Obviously, there are no wealth effects in the first because it is absorbing, so that it does not admit spillovers.

A.6.2. Globally balanced policies: analytical results when regime 1 is absorbing

The fiscal frontier Consider the absorbing case and assume that monetary policy is always active ($\gamma_{\pi,1} = \gamma_{\pi,2} = 1.5$). The equation to be solved for h_1 is (23). If $\gamma_{\pi,1} > 1$, the roots of the second order equation in the square brackets are out of the unit circle. If in the first regime there is a PF policy, the equation for the second regime reduces to equation (A23). Given that we assumed an active monetary policy even in regime 2, we have global determinacy whenever fiscal policy is passive (or timidly active): $\frac{b}{\tau}\left(1 - \frac{\beta}{\sqrt{p_{22}}}\right) < \gamma_{\tau,2} < \frac{b}{\tau}\left(1 + \frac{\beta}{\sqrt{p_{22}}}\right)$, which corresponds to equation (25) in the text. So we have determinacy for the absorbing regime 1 when $\gamma_{\tau,1} > \frac{b}{\tau}(1-\beta)$ and for the non-absorbing regime 2 when $\gamma_{\tau,2} > \bar{\gamma}_{\tau,2}$. Figure A1 displays what we label *the fiscal frontier*: the fiscal policy combinations above this frontier admit only timid deviations into active fiscal behaviour in the second regime. The other fiscal policy combinations in Figure A1, by contrast, do not admit a mean square stable solution $\bar{h}_i(\gamma_{\tau,i})$. In these cases, if the $\bar{h}_1(\gamma_{\tau,1})$ solution in the first regime is outside the unit circle ($h_1 = \frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,1}\right) > 1$), that is if there is an AF policy, then all solutions are explosive, independently from what happens in the second regime.

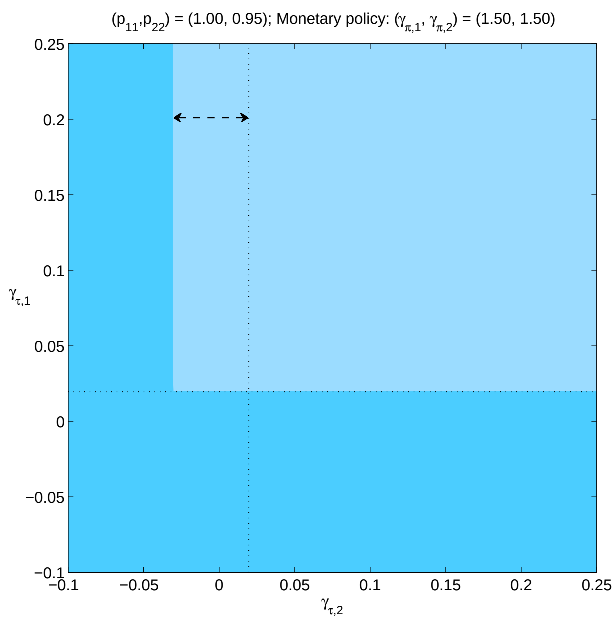

Figure A1: The fiscal policy frontier in the absorbing case.

The monetary frontier Figure A2 displays the monetary frontier in the absorbing case. Take an always passive fiscal policy, we know that a fiscal backing solution always exists under the two regimes, and thus, determinacy requires all solutions $\bar{h}_i(\gamma_{\pi,i})$ to be unstable. The condition for the absorbing state is (23). If in the absorbing regime 1 fiscal policy is passive, then the solution $\bar{h}_1(\gamma_{\tau,1})$ is inside the unit circle and $(1-\beta)\frac{b}{\tau} < \gamma_{1,\tau} < (1+\beta)\frac{b}{\tau}$. The condition for not having another stable solution is that the two $\bar{h}_1(\gamma_{\pi,1})$ solutions should be outside the unit circle, that boils down to the usual $\gamma_{\pi,1} > 1$: monetary policy needs to be active. As for the non-absorbing regime 2, consider again equation (A23). If fiscal policy is passive then to have a unique stable solution the other (two) $(A23)$ solutions must be both outside the unit circle, which gives (25): $\gamma_{\pi,2} > \sqrt{p_{22}} - \frac{(1-\beta\sqrt{p_{22}})(1-\sqrt{p_{22}})}{\lambda}$ that is, we have determinacy when $\gamma_{\pi,2} > \bar{\gamma}_{\pi,2}$.

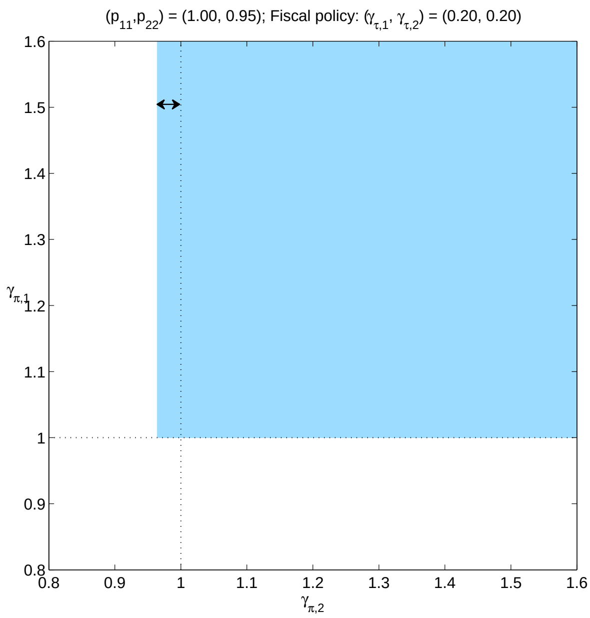

Figure A2: The monetary policy frontier in the absorbing case.

A.6.3. Figure 2b under an absorbing PM regime 1

Suppose now monetary policy is PM (with $\gamma_{\pi,1} = 0.9$) in the first (absorbing) regime and AM (with $\gamma_{\pi,2} = 1.5$) in the second regime. The condition for the absorbing state is, as usual, (23). That for the non-absorbing state is derived from (19) evaluated at $p_{11} = 1$ where, to simplify notation, we define $z = h_2\sqrt{p_{22}}$

$$
\begin{aligned}
0 &= g_{\pi,2}\left\{1 + \lambda\gamma_{\pi,2} - z\sqrt{p_{22}}\,(1+\beta+\lambda) + \beta z^2 p_{22}\right\} \\
&\quad + (1-p_{22})g_{\pi,1} z\left[\beta z - \frac{1}{\sqrt{p_{22}}}\,(1+\beta+\lambda-\beta h_1)\right]
\end{aligned}
\tag{A26}
$$

where $g_{\pi,1}$ and $g_{\pi,2}$ are given, as usual, by equations (20) and (21) in the text. We can re-write the condition for the non-absorbing state as

$$z^3 + b_2 z^2 + b_1 z + b_0 = 0$$

where

$$\begin{aligned}
b_2 &= -\frac{\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) p_{22} + (1+\beta+\lambda) + g_{\pi,1}(1-p_{22}) b \left(\frac{1}{\beta} - \gamma_{\pi,2}\right) \beta}{\beta\sqrt{p_{22}}}, \\
b_1 &= \frac{\sqrt{p_{22}}\,(1+\beta+\lambda)\frac{1}{\beta}\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right) + \frac{(1+\lambda\gamma_{\pi,2})}{\sqrt{p_{22}}} + g_{\pi,1}\frac{(1-p_{22})}{\sqrt{p_{22}}}\left(1+\beta+\lambda-\beta\bar{h}_1\right) b \left(\frac{1}{\beta} - \gamma_{\pi,2}\right)}{\beta\sqrt{p_{22}}}, \\
b_0 &= -\frac{\left(1 - \frac{\tau}{b}\gamma_{\tau,2}\right)(1+\lambda\gamma_{\pi,2})}{\beta^2\sqrt{p_{22}}}.
\end{aligned}$$

The necessary and sufficient condition for determinacy is that this cubic equation has exactly one solution inside the unit circle and the other two outside. By proposition C.2 in Woodford (2003), this is the case if and only if either of the following two cases is satisfied:

- Case I: $1 + b_2 + b_1 + b_0 < 0$ and $-1 + b_2 - b_1 + b_0 > 0$;
- Case II: $1 + b_2 + b_1 + b_0 > 0$, $-1 + b_2 - b_1 + b_0 < 0$, and $b_0^2 - b_0 b_2 + b_1 - 1 > 0$ or $|b_2| > 3$.

Let us study now how determinacy varies according to the fiscal policy undertaken in regime 1.

AF in the absorbing regime 1. Consider the case of an AF policy in the absorbing regime: regime 1 will have a PM/AF mix hence a unique stable solution. In this case the $\bar{h}_1(\gamma_{\pi,1})$ solutions in (23) should have a root inside and one outside the unit circle while the $\bar{h}_1(\gamma_{\tau,1})$ solution, being AF, is outside. The monetary coefficient $\gamma_{\pi,1} = 0.9$ generates a stable solution $h_1 = 0.94343$ (and an unstable one, that we discard, $h_1 = 1.1534$).

Studying the necessary and sufficient conditions for determinacy, we find that, under these parameters, Case I is never satisfied (since the second inequality never holds) while Case II is. In particular, the condition to have exactly one solution inside the unit circle for regime 2 (from the first condition in Case II) reads

(A27)

$$
\begin{aligned}
\gamma_{\tau,1} > & \frac{b}{\tau}(1-\beta h_1) \\
& - \frac{\left[1+\lambda\gamma_{\pi,2}-\sqrt{p_{22}}\,(1+\beta+\lambda)+\beta p_{22}\right]}{(1-p_{22})\left[\beta-\frac{1}{\sqrt{p_{22}}}\,(1+\beta+\lambda-\beta h_1)\right]\left(\frac{1}{\beta}-\gamma_{\pi,2}\right)}\left(\frac{1}{\beta}-\gamma_{\pi,1}\right)\frac{b}{\tau}\left[\beta\frac{1}{\sqrt{p_{22}}}-1+\frac{\tau}{b}\gamma_{\tau,2}\right],
\end{aligned}
$$

that is represented by a negative sloped line in the space $(\gamma_{\tau,1},\gamma_{\tau,2})$ and that depends, among other things, on h_1. Note that (A27) corresponds to (A26) for $z=1$.

Hence there is a unique stable solution when $\gamma_{\tau,1}$ is above this line for $h_1 = 0.94343$. As a result, when the first regime is PM/AF we have a global determinate equilibrium in the hatched area of Figure A3.

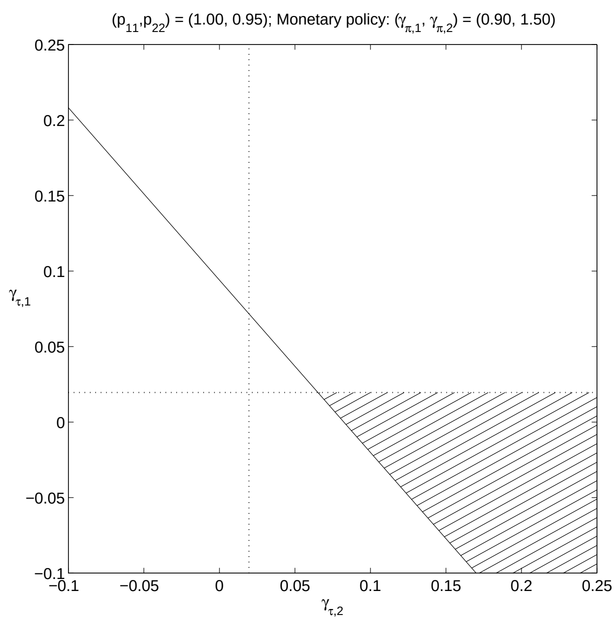

Figure A3: Stability for an absorbing PM/AF regime 1.

PF in the absorbing regime 1. When the first regime is PM/PF, the two $\bar{h}_1(\gamma_{\pi,1})$ solutions are one inside ($h_1 = 0.94343$) and the other outside while the $\bar{h}_1(\gamma_{\tau,1})$ solution is inside with $g_{\pi,1} = 0$.

Now we have to consider two areas, one for each $h_1 < 1$ in regime 1. The solution $h_1 = 0.94343$, returns the same negative sloped straight line as before. So, again, there is a unique stable solution for regime 2 when $\gamma_{\tau,1}$ is above this line. Hence, when the first regime is PM/PF we have a global determinate equilibrium in the hatched area of Figure A4.

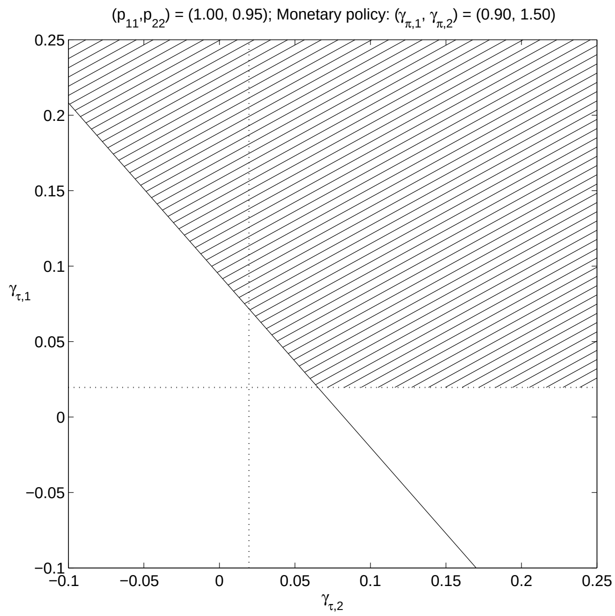

Figure A4: Stability for absorbing PM/PF regime 1: stable monetary solution case.

When we consider the other stable solution, with $g_{\pi,1} = 0$, equation (A26) reduces to

$$0 = g_{2,\pi,b} \left[1 + \lambda\gamma_{\pi,2} - z\sqrt{p_{22}} \left(1 + \beta + \lambda\right) + \beta z^2 p_{22} \right].$$

Hence, we have just one stable solution if and only if the regime 2 is AM/PF (in this case there are 2 $\bar{h}_2(\gamma_{\pi,2})$ solutions outside and 1 $\bar{h}_2(\gamma_{\tau,2})$ solution inside) and the area characterised by just one stable solution is the hatched area in Figure A5.

Overlapping these two figures (A4 and A5) we get the areas with just one stable solution when fiscal policy in regime 1 is passive: the two triangular areas in Figure A6.

Putting everything together. Putting together Figures A3 and A6, one obtains A7, which is the counterpart of Figure 2b for the case of absorbing PM regime 1.

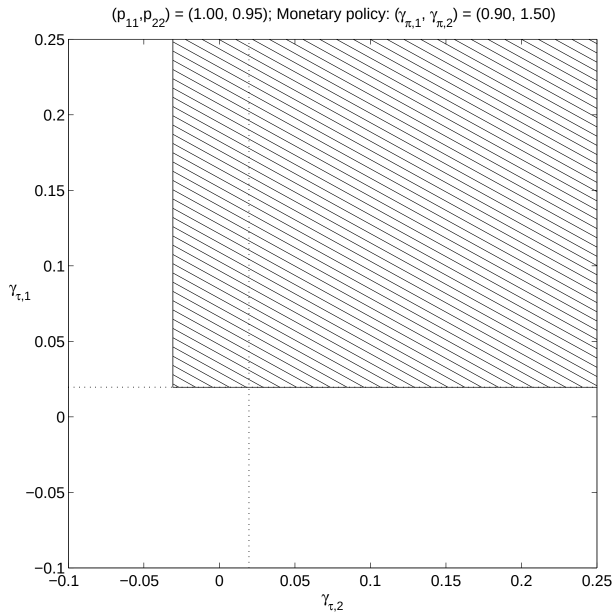

Figure A5: Stability for absorbing PM/PF regime 1: stable fiscal solution case.

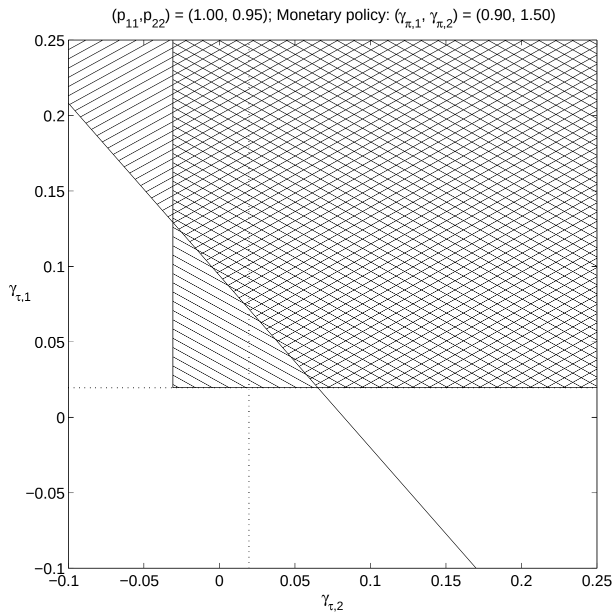

Figure A6: Stability for absorbing PM/PF regime 1: complete case.

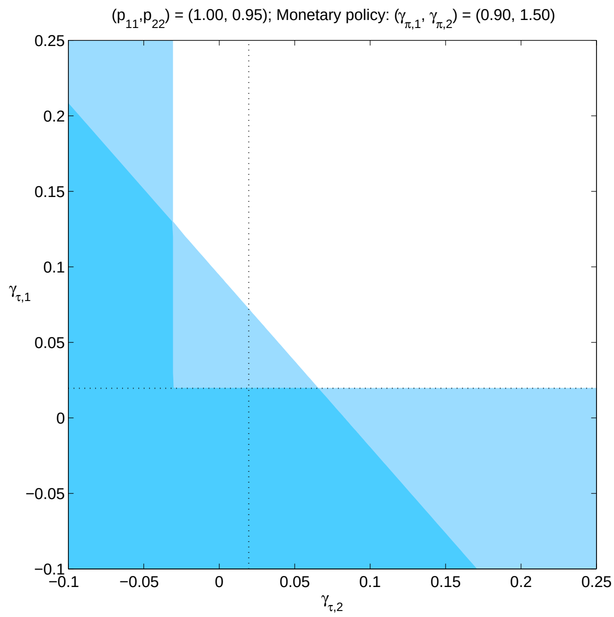

Figure A7: The fiscal policy frontier with substantial deviations in monetary policy and absorbing regime 1 with PM.

Notes: Light blue: unique solution; white: indeterminacy; dark blue: explosiveness.

A.7. Both regimes stable

In this section we want to show how our results change when the concept of both regimes stable (BRS) is used in place of the MSS concept that we used throughout the paper.[3] Note that MSS allows temporarily explosive paths while BRS does not. More specifically, a solution is BRS if all h_i lie inside the unit circle, for $i = 1, 2$.

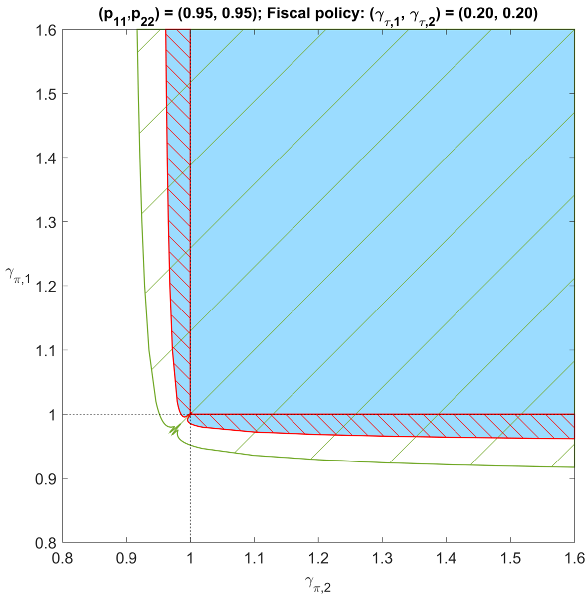

Figure A8: The monetary policy frontier under MSS and BRS.
Notes: Light blue: unique MSS solution; white: multiple MSS solutions; dark blue: no MSS solutions; green-hatching: unique BRS solution.

Figure A8 shows the monetary policy frontier (MPF): the figure reproduces Figure 1a in the main text, with the addition of sparse green hatch lines that indicate the area where a single BRS solution exists. The edge of the this area can be interpreted as the MPF under BRS. As you can see, the MPF shifts outwards if BRS is considered: the MPF under MSS, which encloses the policy mixes characterised by a unique stable (Ricardian) solution, lies inside the new MPF under BRS. In the area between the two frontiers, policy mixes are characterised by the co-existence of two MSS solutions: one solution is Ricardian, while the second solution is non-Ricardian and explosive in one regime (i.e., either h_1 or h_2 is greater than unity). Indeterminacy then arises under MSS but not under BRS as the second solution is not stable in both regimes. Hence this additional area is included in the determinacy region that therefore gets larger. Outside the green-hatched area, the policy mixes give rise to two stable solutions, under both stability concepts (one of these solutions is Ricardian). These

[3] Refer to Foerster (2016) for a discussion of BRS.

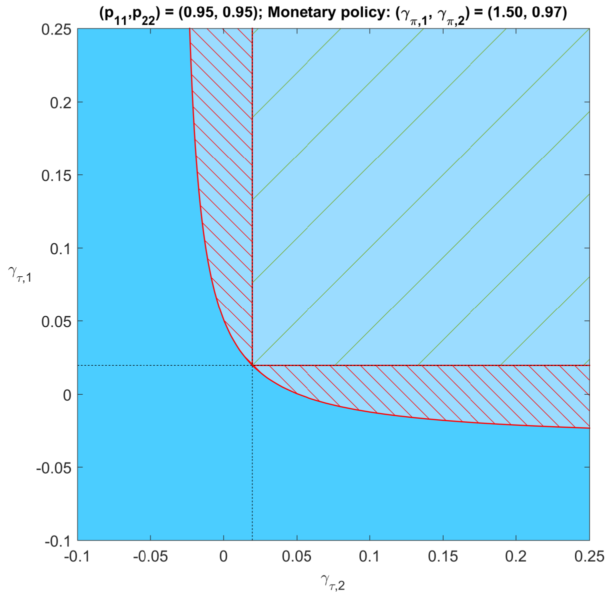

Figure A9: The fiscal policy frontier under MSS and BRS, for timid deviations in monetary policy.
Notes: Light blue: unique MSS solution; white: multiple MSS solutions; dark blue: no MSS solutions; green-hatching: unique BRS solution.

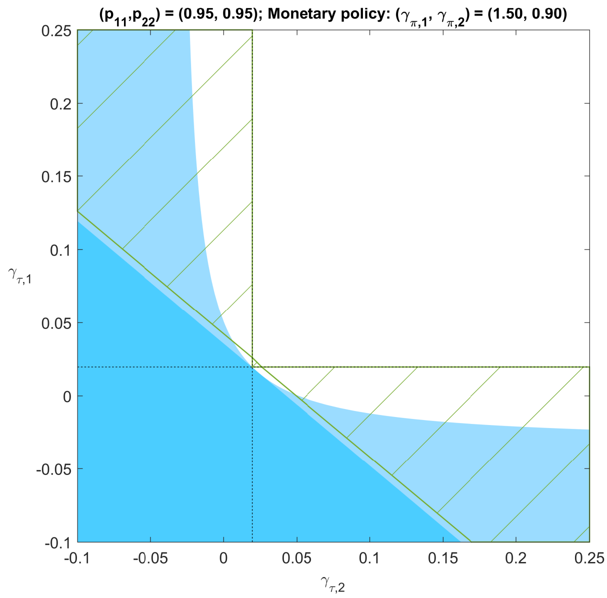

Figure A10: The fiscal policy frontier under MSS and BRS, for substantial deviations in monetary policy.
Notes: Light blue: unique MSS solution; white: multiple MSS solutions; dark blue: no MSS solutions; green-hatching: unique BRS solution.

parametrizations are therefore indeterminate.

Figure A9 depicts the fiscal policy frontier (FPF) for timid deviations into passive monetary policy. Policy mixes in the green-hatched area are characterised by single stable solution under both MSS and BRS. This solution, which is Ricardian, becomes explosive in one of the two regimes once fiscal policy deviates timidly into active fiscal territory (red-hatched area), but MSS is preserved. As no other stable solutions exist, we have determinacy under MSS but not under BRS. In other words, BRS rules out any flexibility in fiscal policy and the FPF coincides with the edges of the usual conditions for passive fiscal policy by Leeper (1991).

Figure A10 shows how Figure 2b, which depicts determinacy with substantial deviations in monetary policy, changes under BRS. Note that, even in this case, the FPF is not relevant anymore. A timid fiscal policy mix (e.g, consider the parametrization $\gamma_{\tau,1} = 0.2$, $\gamma_{\tau,2} = 0$) is characterised by one stable non-Ricardian solution and one MSS Ricardian solution that is explosive in one regime. The equilibrium is than indeterminate under MSS but determinate under BRS. In this case, therefore, employing this alternative stability concept, timid fiscal policy produces wealth effects (as in Chung et al., 2007) and the fiscal theory is always at work.

Starting from a AM/PF regime 1, a sufficient condition to have a unique stable solution and wealth effects under MSS is to have substantial variation in both monetary and fiscal policy in regime 2; under BRS just a substantial variation in monetary policy in regime 2 is needed while fiscal policy should be (even timidly) active.

A.8. An application to the ZLB

During the Great Recession, the Federal Reserve, the European Central Bank, the Bank of England and the Bank of Japan all brought their policy rates towards the ZLB with the aim of stimulating economic activity. This is a very special case of passive monetary policy, which can be described in our model by assuming $\gamma_\pi = 0$, i.e., the central bank avoids moving interest rates as inflation changes. In this section we want to conduct two exercises relative to the ZLB regime. First, imagine being in a crisis regime with interest rates stuck at the ZLB and agents expecting a return to business-as-usual regime—the traditional AM/PF regime. Then, how should fiscal policy be fixed in the current regime to guarantee the existence of a unique equilibrium? Which type of solution would that be? How long should the ZLB regime be in place? Figure A11 depicts determinacy results for different fiscal coefficients ($\gamma_{\tau,2}$) and durations (p_{22}) of crisis regime 2, when $\gamma_{\pi,2} = 0$ and an AM/PF regime 1 is expected with probability $p_{11} = 0.95$.

The first clear-cut result is that if the ZLB regime 2 is short-lasting ($p_{22} \ll 1$), then there is indeterminacy, whichever fiscal policy is adopted. Second, determinacy is unattainable when fiscal policy is passive, as $\gamma_{\tau,2}$ should be below the dotted line that represents the cutoff value for passive fiscal policy.

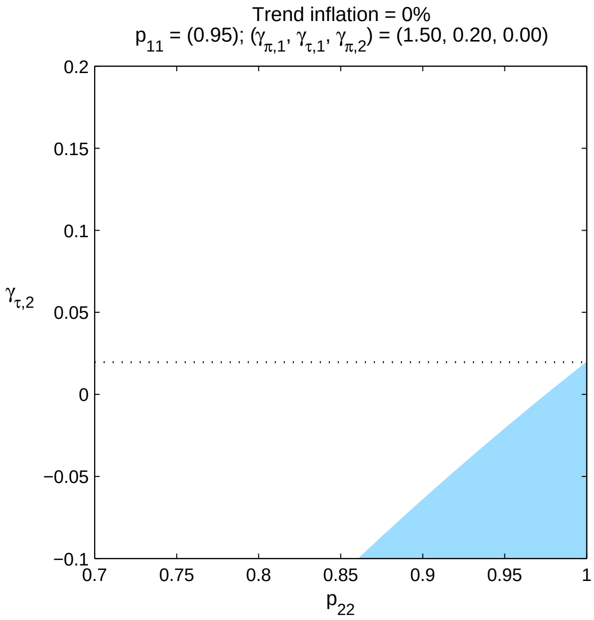

Figure A11: Determinacy and ZLB.

Notes: Light blue: unique solution; white: indeterminacy.

Third, the upper bound of the determinacy area is an upward-sloping line: the more short-lived the ZLB is, the more active fiscal policy must be. To have determinacy, agents must expect the ZLB to last for a long period of time and to be accompanied by an active fiscal policy. In the event that there were such a unique stable solution, it would be the result of an overall switching policy mix (a switching monetary policy combined with a switching fiscal policy), and hence, expectation effects would be operative. As in Figure 4b, the FTPL would apply, and an unbacked fiscal expansion would spur output, increase inflation and lower real debt: all desirable outcomes in a period of economic stagnation, below-target inflation and high indebtedness.

Two important points stem from these results. The first is the inadequacy of the conventional New Keynesian model, which does not consider active fiscal policies. The other is the importance of "forward guidance", both on the monetary and on the fiscal side. Even if agents expect to return in the future to the AM/PF regime, the question of how long the present policy is expected to last is key. To obtain determinacy, agents must be convinced of a long-lasting deviation from the AM/PF regime both through the promise, on the monetary side, of a long period of zero interest rates and, on the fiscal side, of a long period of no tax increases or spending cuts.

Conversely, in the presence of a passive fiscal policy, now and in the future, or of a short-lasting deviation from it, there would be multiple equilibria under a ZLB (the white area in the figure). These consist of two stable solutions: one characterised by Ricardian dynamics and the other by non-

Ricardian dynamics. In this case, inflation would become indeterminate. This could be the outcome for those countries, currently at their ZLB, where austerity imposes constraints on fiscal policy (the eurozone) or where fiscal policy is mainly passive (Japan).

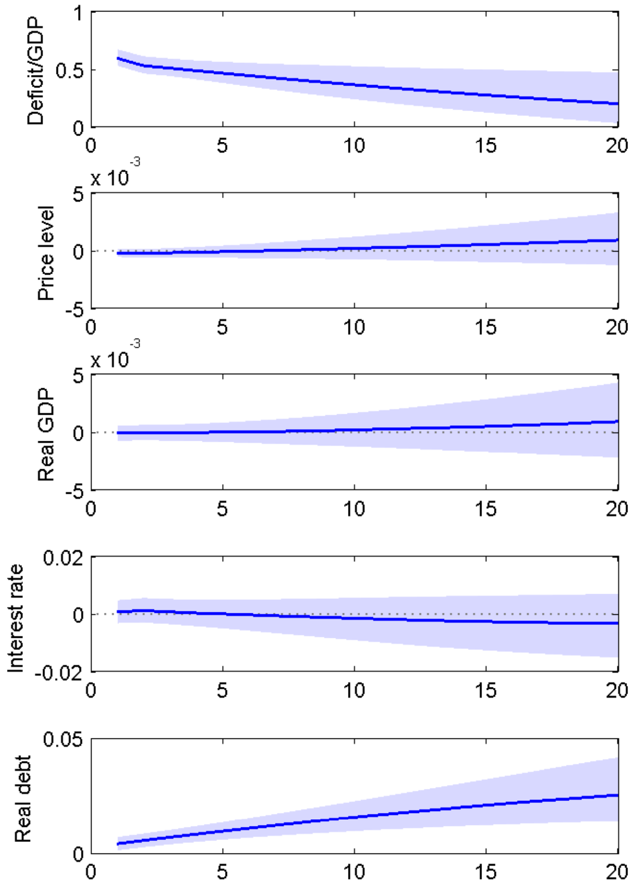

Figure A12: Impulse response functions from a BVAR

We now perform a second exercise to empirically investigate the policy mix in place in the U.S. during the ZLB period, through the lenses of our model. We examine the empirical impulse response functions computed using a Bayesian VAR fitted on quarterly data for the sample period 2008q4-2015q4. To assess the effect of a fiscal shock, we consider data for the primary deficit-to-GDP ratio, the federal funds rate, real GDP, the GDP deflator and real debt.[4] During this period, monetary

[4]The primary deficit-to-GDP ratio was constructed from NIPA data using the procedure illustrated in Bianchi and Melosi (2017), while real debt was computed by dividing the Market Value of U.S. Government Debt (FRED code: MVGFD027MNFRBDAL) by the GDP deflator. The remaining three variables were also taken from the FRED database. Real GDP, real debt and the GDP deflator are considered in log-levels. Our VAR includes two lags and a constant term, and we identify fiscal shocks by means of a recursive scheme in which the deficit-to-GDP ratio is ordered before all other variables. We use a Normal-Wishart prior augmented with two sets of dummy observations, i.e., both sums-of-coefficients

policy can be safely assumed to be passive, while there is more room for debate over the fiscal policy stance. According to our model, we can gain some insights into how agents perceive the prevalent policy mix by comparing the pattern of VAR-based impulse responses following a positive fiscal shock to the corresponding theoretical responses shown in Figure 4.

As Figure A12 shows, the impulse response functions are consistent with the PM/AF case of an overall AM/PF mix: while output and inflation do not move, there is a run-up in real debt. Assuming that agents expect to return to the same AM/PF equilibrium as in Figure A11, the only equilibrium consistent with these dynamics for the real debt is a Ricardian equilibrium due to timid AF policies in the indeterminacy (white) area in Figure A11.[5] Through the lens of our model, therefore, the data for the ZLB period suggest that the U.S. could well be in an indeterminate region and in a "timidity trap", where agents are coordinating on the Ricardian solution. This justifies the observed subdued path of inflation. Note that a more aggressive fiscal policy would eliminate one of the two equilibria and guarantee determinacy. Alternatively, an exogenous event, such as the election of a new government, could switch agents to coordinate on another possible equilibrium in the indeterminacy area, causing a spike in inflation, with the Fed being unable to do anything to avoid it.

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and dummy initial observations, to accommodate the presence of unit roots and cointegration in the data (see Sims and Zha, 1998).

[5] Impulse response functions undertaken when employing aggressive fiscal policies (either active or passive) return a decreasing path for the real debt, as in Figure 4b. These results are available from the authors upon request.

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