

Constructing Historical Euro-Zone Data

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Abstract

Existing methods of reconstructing historical Euro-zone data by aggregation of the individual countries' aggregate data raises numerous difficulties, especially due to past exchange rate changes. The approach proposed here is designed to avoid such distortions, and aggregate exactly when exchange rates are fixed. We first compute growth rates within states, aggregate these, then cumulate this Euro-zone growth rate to obtain the aggregated levels variables. The aggregate of the implicit-deflator price index coincides with the implicit deflator of our aggregate nominal and real data. We investigate the properties of this growth-rate method for aggregation, and construct Euro-zone measures for M3, GDP and prices over the previous two decades.

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JEL classifications:

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C82 (Methodology for Collecting, Estimating, and Organizing Macroeconomic Data)

1. Background

With the introduction on 31 December 1998 of the irrevocably-fixed Euro exchange rates, eleven countries of the European Union entered a new monetary union. From that date onwards, monetary policy for these countries is set by the newly-formed European Central Bank (ECB). The Governing Council of the ECB bases its monetary policy on 'medium-term assumptions regarding real GDP growth and the trend decline in the velocity of circulation of M3' (ECB (1999a), p.40). Therefore, the construction of historical data for aggregate M3 and GDP for the Euro zone is of practical relevance to policy

makers. In addition, any econometric model for Euro-zone countries requires such historical data. This paper describes an approach to doing so from the aggregate data of the individual member states. A number of previous re-constructions are noted, but as these have important drawbacks, an approach related to that in (Törnqvist 1936) is proposed and its properties are discussed.

Although there have been many important contributions to index-number theory (see section 4), few are directly relevant to the problem confronted here, where we wish to use the national aggregates from member states – not the individual-item data usually input into index calculations – to construct a Euro-zone aggregate. Moreover, the inherently non-stationary nature of those national aggregates, both from unit-roots inducing integrated time series (usually $I(1)$), and structural breaks, entail that any optimality properties of indexes will be hard to establish in this context. Specifically, data inaccuracies at the country level generally entail $I(1)$ errors, and this property will infect higher aggregates. Thus, we opt for the more limited objectives of developing an index that would aggregate correctly when a common currency always held, produces a unique aggregate price series (given the current emphasis on inflation targeting), and when any variable increases (or decreases) in every member state, then the aggregate should not move in the opposite direction. We comment on these in turn.

Approaches, such as simple summation of the historical data for each member country, which aggregate perfectly under a common currency, face numerous difficulties when there changes in exchange rates. Because precise purchasing-power parity (PPP) rates are neither known nor constant over the sample under consideration, the set of exchange rates used is of central concern for the aggregation process. Most choices of nominal or real exchange rates, when used to aggregate levels data, go awry for some scenarios. Secondly, a valuable consequence of the proposed method is that the aggregate of the implicit-deflator price index coincides with the implicit deflator of the aggregate nominal and real data. In most other methods, the aggregate of the individual deflators does not correspond to the deflator of the aggregates: indeed, the ECB advises against using their constructed deflator as the basis for an inflation measure. Finally, when large discrete exchange-rate changes occur, aggregators of levels can produce a series that falls even when (in own-currency terms) every member state experienced a rise. The resulting aggregate Euro-zone real GDP (say) could then appear to be unrelated to Euro-zone unemployment, even if real GDP was the main determinant within each member state. As shown below, these considerations restrict the admissible class of indexes to variable-weight aggregates of growth rates, namely: calculate within-country growth rates of each variable, aggregate them to a weighted-sum Euro-zone growth rate, then cumulate this Euro-zone growth rate to obtain the aggregate levels.

Even within a given country, it is well known that if nominal and real identities hold, but relative

price changes occur, then exact aggregation is impossible. Consider the simplest national income accounting identities, $Y_t \equiv C_t + I_t$ in nominal and $y_t \equiv c_t + i_t$ in real terms, where:

$$P_{y,t} = \frac{Y_t}{y_t}, \quad P_{c,t} = \frac{C_t}{c_t} \quad \text{and} \quad P_{i,t} = \frac{I_t}{i_t}.$$

Then:

$$y_t = \frac{Y_t}{P_{y,t}} = \frac{C_t + I_t}{P_{y,t}} = \frac{C_t}{P_{y,t}} + \frac{I_t}{P_{y,t}} = \frac{P_{c,t}}{P_{y,t}} c_t + \frac{P_{i,t}}{P_{y,t}} i_t.$$

In the first period, all the price indices can be set to unity, so $y_1 \equiv c_1 + i_1$, but when $P_{c,t}/P_{i,t}$ is not constant over time:

$$\frac{C_t}{P_{y,t}} = \frac{P_{c,t} c_t}{P_{y,t}} \neq c_t.$$

Thus, the implicit and direct deflators are unequal. The usual approach to reconciling the accounts is to deflate the sub-aggregates (C_t and I_t) and define aggregate real income as their sum, obtaining the implicit deflator $P_{y,t}$ as the resultant nominal divided by real. Below, we discuss the ‘conventions’ followed in this paper.

The Euro-zone data presented in (ECB 1999a) consists of aggregated M3 using the fixed, irrevocable Euro rates. All applications we have seen to date in the literature aggregate variables in levels, either using fixed weights (exchange rates or PPP rates) for nominal and real GDP, or occasionally fixed weight for real and variable weights for nominal GDP. (Winder 1997) discusses these methods, but does not consider the method proposed here. (Bayoumi and Kenen 1993) aggregate growth rates, but using fixed weights. (Fagan and Henry 1999) aggregate log-levels, and show that this is equivalent to aggregating growth rates with fixed weights.

The outline of the remainder of the paper is as follows. After discussing in §2 the choices to be confronted in constructing Euro-zone indexes, we provide a theoretical analysis in §3. We first discuss aggregation under fixed exchange rates (§3.1), followed by variable rates in §3.5. Some aspects of the constructed data are discussed in §5. We construct Euro-zone measures for M3 and GDP over the previous two decades. These differ from other currently available data owing to the different aggregation method. Detailed information on the data set is provided in the Appendix. The created data set is available for downloading.

2. Choices in constructing Euro-zone index numbers

It is essential that any method proposed for reconstructing ‘Euroland’ should also work correctly in a setting where the constituents use the same currency. Simple-sum aggregates work perfectly when

there is a common currency, but are inapplicable to ‘Euroland’ prior to the introduction of the Euro. Consequently, a large number of choices has to be made. Either levels or growth rates could be aggregated, in both cases using weights which are either constant or changing over time. Such weights could be based on nominal or real variables, be common across aggregates or differ for every series, and – if fixed – use base or final year values (or any date in between). The problems with fixed-weight aggregators are well known: both Paasche and Laspeyres indices can badly mis-measure when there are substantive changes to relative prices, and hence to the ‘budget’ shares. Thus, we do not consider any fixed weight aggregator to be a satisfactory solution to recreating ‘Euroland’ data when there have been considerable relative-price changes between member economies due to currency devaluations and revaluations.

Moreover, any proposal using variable weights must also be relatively immune to distortions induced by exchange-rate changes. As we show in §3.5, this precludes any aggregation of the levels using weights which vary with exchange rates. While fixed weights would avoid such a problem, they are prone to the criticism just noted, and even then, the choice of weight is crucial and can substantially affect the resulting time series.

Thus, we first demonstrate that, when aggregating across ‘countries’ that share a common currency, fixed weights are appropriate for either levels or differences, and that an almost-correct outcome results when aggregating either levels or growth rates with appropriate choices of variable weights. Next, we show that if the currency units altered, variable-weight levels aggregates fail, whereas variable-weight growth-rate aggregates would continue to be appropriate, leaving that as the only viable choice. Then we discuss the considerations determining the choice of weights for aggregating nominal and real GDP, prices and money.

3. Analysis

For simplicity, the analysis considers two countries to be aggregated with:

$$\begin{aligned} \text{nominal output:} & \quad Y_{i,t}, \\ \text{real output:} & \quad X_{i,t}, \\ \text{implicit deflator:} & \quad P_{i,t} \equiv Y_{i,t}/X_{i,t}, \end{aligned}$$

for countries $i = 1, 2$ and sample dates $t = 1, \dots, T$. Extensions to more countries follow directly, although other variables (such as monetary magnitudes, interest rates etc.) need separate consideration (see below). The corresponding growth rates of $Y_{i,t}$, $X_{i,t}$ and $P_{i,t}$ are denoted $\Delta y_{i,t}$, $\Delta x_{i,t}$ and $\Delta p_{i,t}$

where lower-case letters denote logs of corresponding capitals, so $\Delta y_{i,t} = y_{i,t} - y_{i,t-1} = \Delta \log Y_{i,t}$, and hence:

$$\Delta y_{i,t} \equiv \Delta x_{i,t} + \Delta p_{i,t}.$$

We prefer the ‘symmetric’ log difference for computing growth rates to the more conventional choice of:

$$g_{Y_{i,t}} = \frac{Y_{i,t} - Y_{i,t-1}}{Y_{i,t-1}}. \quad (1)$$

The two measures are close unless changes become large (i.e., in excess of 10% per period), and we approximate $\Delta \log Y_{i,t}$ by (1), and vice versa, below.

Possible aggregation methods include levels or growth aggregates, applied to groupings with and without exchange-rate changes. The possible combinations are summarized in the following table, using the *USA* to represent aggregation when there is a common currency, and *Euro* for variable exchange rates (*approx* denotes approximately correct):

		Possible aggregation combinations			
		aggregate levels		aggregate growth rates	
		constant weights	changing weights	constant weights	changing weights
exchange rate					
fixed	<i>USA</i>	✓	✓	<i>approx</i>	✓
variable	<i>Euro</i>	<i>approx</i>	<i>fail</i>	<i>approx</i>	✓

Of the 8 possible choices prior to deciding on the specification of the weighting function, only one delivers acceptable outcomes when applied to both fixed and varying exchange rates as we now show. We first examine fixed exchange rates, since if aggregators fail in that setting, they are generally inadmissible. Of course, simple sum must work, so the focus of the next section is to show that our proposal also aggregates correctly. In the process of doing so, we establish the uniqueness of the resulting price series, and show the need to use monetary weights to aggregate the money stock. The following section then considers the additional complications introduced by varying exchange rates, and establishes that the remaining potential methods of aggregation will fail in cases of practical importance, or have demanding information requirements.

3.1. Constant exchange rates

First consider the *USA* where the exchange rate is fixed at unity across all the ‘countries’. The aggregate is then given by variables without the subscript i :

$$\begin{aligned} Y_t &\equiv Y_{1,t} + Y_{2,t}, \\ X_t &\equiv X_{1,t} + X_{2,t}, \\ P_t &\equiv \frac{Y_t}{X_t}. \end{aligned}$$

These are clearly the correct aggregates, so aggregating both nominal and real GDP in levels with constant weights (of unity) is appropriate. However, some care is required when aggregating prices, since the aggregate price deflator is a variable-weighted average of the component deflators:

$$P_t \equiv \frac{Y_{1,t} + Y_{2,t}}{X_{1,t} + X_{2,t}} = \frac{Y_{1,t}}{X_{1,t}} \frac{X_{1,t}}{X_t} + \frac{Y_{2,t}}{X_{2,t}} \frac{X_{2,t}}{X_t} = w_{1,t} P_{1,t} + (1 - w_{1,t}) P_{2,t}, \quad (2)$$

where:

$$w_{1,t} = \frac{X_{1,t}}{X_t} = \frac{X_{1,t}}{X_{1,t} + X_{2,t}}. \quad (3)$$

The correct weights are therefore seen to be respective real GDP shares; on price indices more generally, see (Diewert 1976; Diewert 1988).

Alternatively, we can obtain approximately the same nominal and real GDP aggregates as follows. Although the logs of the individual data do not aggregate, other than $\Delta p_t \equiv \Delta y_t - \Delta x_t$, in terms of growth rates for real GDP, we have:

$$\begin{aligned} \Delta x_t &\simeq \frac{\Delta X_t}{X_{t-1}} \equiv \frac{\Delta X_{1,t} + \Delta X_{2,t}}{X_{t-1}} \\ &= \frac{\Delta X_{1,t}}{X_{1,t-1}} \frac{X_{1,t-1}}{X_{t-1}} + \frac{\Delta X_{2,t}}{X_{2,t-1}} \frac{X_{2,t-1}}{X_{t-1}} \\ &\simeq \Delta x_{1,t} w_{1,t-1} + \Delta x_{2,t} w_{2,t-1}. \end{aligned} \quad (4)$$

As in (2) we have $w_{1,t} + w_{2,t} \equiv 1$ for all t in the 2-country case. Both approximations in (4) of proportional changes by log differences are close for small changes, especially relative to the inherent data accuracy, and are partially offsetting. Similarly:

$$\begin{aligned} \Delta y_t &\simeq \frac{\Delta Y_{1,t}}{Y_{1,t-1}} \frac{Y_{1,t-1}}{Y_{t-1}} + \frac{\Delta Y_{2,t}}{Y_{2,t-1}} \frac{Y_{2,t-1}}{Y_{t-1}} \\ &= \frac{\Delta Y_{1,t}}{Y_{1,t-1}} \frac{P_{1,t-1} X_{1,t-1}}{P_{t-1} X_{t-1}} + \frac{\Delta Y_{2,t}}{Y_{2,t-1}} \frac{P_{2,t-1} X_{2,t-1}}{P_{t-1} X_{t-1}}, \\ &\simeq \Delta y_{1,t} w_{1,t-1} r_{1,t-1} + \Delta y_{2,t} w_{2,t-1} r_{2,t-1}, \end{aligned} \quad (5)$$

where $r_{i,t} = P_{i,t}/P_t$. From (2):

$$1 = w_{1,t} \frac{P_{1,t}}{P_t} + w_{2,t} \frac{P_{2,t}}{P_t} = w_{1,t} r_{1,t} + w_{2,t} r_{2,t}.$$

However, in place of (5), we shall use the same weights for nominal as for real aggregates, namely:

$$\Delta y_t \simeq \Delta y_{1,t} w_{1,t-1} + \Delta y_{2,t} w_{2,t-1}. \quad (6)$$

Although a closer aggregate results from nominal shares, this is at the cost of losing price-aggregation matching. Using real shares we find from (6) and (4):

$$\begin{aligned} \Delta p_t &\equiv \Delta y_t - \Delta x_t \\ &\simeq \Delta y_{1,t} w_{1,t-1} + \Delta y_{2,t} w_{2,t-1} - \Delta x_{1,t} w_{1,t-1} - \Delta x_{2,t} w_{2,t-1} \\ &= (\Delta y_{1,t} - \Delta x_{1,t}) w_{1,t-1} + (\Delta y_{2,t} - \Delta x_{2,t}) w_{2,t-1} \\ &= \Delta p_{1,t} w_{1,t-1} + \Delta p_{2,t} w_{2,t-1}, \end{aligned}$$

which is the appropriate weighted average of the component price indices. Thus, any resulting ‘distortions’ to the aggregate accounts affects the nominal GDP measure, which we suspect is the least relevant of the three for econometric studies and policy analyses. This is the procedure we adopt, although no issue of principle is involved in using nominal shares.

The unlogged levels magnitudes can be obtained by integration and exponentiation from a given first value, so for the nominal variable, given $\overline{y_0} = \log \overline{Y_0}$:

$$\begin{aligned} \hat{y}_0 &= \overline{y_0}, \\ \hat{y}_t &= \Delta y_t + y_{t-1} \text{ for } t = 1, \dots, \text{ and } \hat{Y}_t = \exp(\hat{y}_t). \end{aligned}$$

Writing out the cumulation process gives:

$$\hat{Y}_t = \exp(\overline{y_0} + \sum_{j=1}^t \Delta y_j) = \overline{Y_0} \exp(\sum_{j=1}^t \Delta y_j).$$

In practice, we prefer to match the constructed data with the most recent official observation $\overline{Y_T}$, so use:

$$\hat{Y}_t = \frac{\overline{Y_T}}{\exp(\sum_{j=1}^T \Delta y_j)} \exp(\sum_{j=1}^t \Delta y_j), \quad t = 1, \dots, T. \quad (7)$$

We checked the accuracy of the outcomes numerically from this approach for a small artificial data set (in a spreadsheet program), and found the results from (6), (4), and (7) to be quite close to the correct aggregates.

3.2. Sub-aggregates

A further requirement of a usable aggregator is that it is invariant to first aggregating to sub-groups, then aggregating these. We now show that our proposal also satisfies that property.

Two important classes of sub-aggregates need to aggregate consistently, namely ‘regional’ groups and ‘temporal’ groups. For the regional case, consider 4 ‘states’, first aggregated in pairs to East and West, then those are aggregated in turn to the total of the four. For real output, this delivers:

$$X_t \equiv \sum_{i=1}^4 X_{i,t} = \sum_{i=1}^2 X_{i,t} + \sum_{j=3}^4 X_{j,t} = X_{e,t} + X_{w,t}.$$

Extending (4):

$$\Delta x_t \simeq \sum_{i=1}^4 \Delta x_{i,t} w_{i,t-1}.$$

Let

$$\Delta x_{e,t} \simeq \sum_{i=1}^2 \Delta x_{i,t} w_{i,t-1}^e \quad \text{and} \quad \Delta x_{w,t} \simeq \sum_{j=3}^4 \Delta x_{j,t} w_{j,t-1}^w,$$

where the superscripted weights denote the denominator used, so $w_{1,t}^e = X_{1,t}/X_{e,t}$. Then we can reconstruct $X_{e,t}$ and $X_{w,t}$ using the analogue of (4) and (7), and hence (noting $w_{e,t} + w_{w,t} = 1$):

$$\begin{aligned} \Delta x_t &\simeq \Delta x_{e,t} w_{e,t-1} + \Delta x_{w,t} w_{w,t-1} \\ &= \left(\sum_{i=1}^2 \Delta x_{i,t} w_{i,t-1}^e \right) w_{e,t-1} + \left(\sum_{j=3}^4 \Delta x_{j,t} w_{j,t-1}^w \right) w_{w,t-1} \\ &= \sum_{i=1}^4 \Delta x_{i,t} w_{i,t-1} \end{aligned}$$

as required, because:

$$w_{1,t}^e w_{e,t} = \frac{X_{1,t}}{X_{e,t}} \frac{X_{e,t}}{X_t} = \frac{X_{1,t}}{X_t} = w_{1,t},$$

and similarly for the remaining weights.

A closely-related analysis applies for time aggregation: since the approach works for each frequency, correct monthly and quarterly magnitudes could be constructed for the ‘USA’ from the ‘state’ data. Appropriately aggregating the constructed series for stocks (end-of-period) and flows (integrals) then must match. Thus, aggregating the ‘within-state’ growth rates delivers acceptable aggregates under fixed rates.

3.3. Aggregating money stocks

For nominal money, however, monetary weights are needed if the outcome is to match aggregating an economy with a common currency:

$$\begin{aligned}\Delta m_t &\simeq \frac{\Delta M_t}{M_{t-1}} \equiv \frac{\Delta M_{1,t}}{M_{t-1}} + \frac{\Delta M_{2,t}}{M_{t-1}} \\ &= \frac{\Delta M_{1,t}}{M_{1,t-1}} \frac{M_{1,t-1}}{M_{t-1}} + \frac{\Delta M_{2,t}}{M_{2,t-1}} \frac{M_{2,t-1}}{M_{t-1}} \\ &\simeq \Delta m_{1,t} w_{1,t-1}^m + \Delta m_{2,t} (1 - w_{1,t-1}^m).\end{aligned}\tag{8}$$

If alternative weights are used, such as the GDP shares above, then the result becomes:

$$\Delta m_t^w \simeq \Delta m_{1,t} w_{1,t-1} + \Delta m_{2,t} (1 - w_{1,t-1}),\tag{9}$$

so:

$$\Delta m_t^w = \Delta m_t + (\Delta m_{1,t} - \Delta m_{2,t}) (w_{1,t-1} - w_{1,t-1}^m).$$

Consequently, if the monetary growth rates of the two ‘countries’ differ importantly, as will occur after financial innovation in either of them, the measured aggregate will not match the actual growth. For example, in an economy with 50% of the total money stock, but only 25% of GDP, where monetary growth is twice the average of the remaining country (20% p.a. rather than 10% say) then the aggregate growth is underestimated by 2.5% p.a. cumulatively, and increasingly so, since the weight of the first economy will be growing. Thus, (9) fails to deliver the correct measure in a common currency, whereas (8) provides a close approximation to the actual measured money stock when no exchange rate changes are involved.

3.4. Aggregating money sub-aggregates

Consider two sub-aggregates of money stocks, denoted by superscripts a and b , in states 1 and 2, when $M_{i,t} \equiv M_{i,t}^a + M_{i,t}^b$ within states. Let $M_t^a = M_{1,t}^a + M_{2,t}^a$ and $M_t^b = M_{1,t}^b + M_{2,t}^b$ denote the appropriate overall aggregates. Then the growth-rate aggregates satisfy the conditions that $M_t = M_{1,t} + M_{2,t} = M_t^a + M_t^b$, so do not depend on the order of aggregation (within states or money stocks).

We show this as follows, reversing the orders of the transformations for clarity. First, within states, $i = 1, 2$:

$$\Delta m_{i,t} \simeq \frac{\Delta M_{i,t}}{M_{i,t-1}} \equiv \frac{\Delta M_{i,t}^a}{M_{i,t-1}} + \frac{\Delta M_{i,t}^b}{M_{i,t-1}}$$

$$\begin{aligned}
&= \frac{\Delta M_{i,t}^a}{M_{i,t-1}^a} \frac{M_{i,t-1}^a}{M_{i,t-1}^a} + \frac{\Delta M_{i,t}^b}{M_{i,t-1}^b} \frac{M_{i,t-1}^b}{M_{i,t-1}^b} \\
&\simeq \Delta m_{i,t}^a w_{a,t-1}^i + \Delta m_{i,t}^b w_{b,t-1}^i,
\end{aligned} \tag{10}$$

so the growth-rate aggregates of different money measures are appropriate as usual. Similarly:

$$\begin{aligned}
\Delta m_t^k &\simeq \frac{\Delta M_t^k}{M_{t-1}^k} = \frac{\Delta M_{1,t}^k}{M_{1,t-1}^k} \frac{M_{1,t-1}^k}{M_{t-1}^k} + \frac{\Delta M_{2,t}^k}{M_{2,t-1}^k} \frac{M_{2,t-1}^k}{M_{t-1}^k} \\
&\simeq \Delta m_{1,t}^k w_{1,t-1}^k + \Delta m_{2,t}^k w_{2,t-1}^k
\end{aligned} \tag{11}$$

Since for $k = a, b$:

$$\frac{M_{i,t-1}^k}{M_{i,t-1}^k} \frac{M_{i,t-1}^k}{M_{t-1}^k} \equiv \frac{M_{i,t-1}^k}{M_{t-1}^k} \frac{M_{t-1}^k}{M_{t-1}^k},$$

then, in an obvious notation:

$$w_{k,t-1}^i w_{i,t-1}^m \equiv w_{i,t-1}^k w_{k,t-1}^m. \tag{12}$$

Thus:

$$\begin{aligned}
\Delta m_t &\simeq \frac{\Delta M_t}{M_{t-1}} \equiv \frac{\Delta M_{1,t}}{M_{t-1}} + \frac{\Delta M_{2,t}}{M_{t-1}} \\
&\simeq \Delta m_{1,t} w_{1,t-1}^m + \Delta m_{2,t} w_{2,t-1}^m \\
&\simeq (\Delta m_{1,t}^a w_{a,t-1}^1 + \Delta m_{1,t}^b w_{b,t-1}^1) w_{1,t-1}^m \\
&\quad + (\Delta m_{2,t}^a w_{a,t-1}^2 + \Delta m_{2,t}^b w_{b,t-1}^2) w_{2,t-1}^m,
\end{aligned} \tag{13}$$

and:

$$\begin{aligned}
\Delta m_t &\simeq \frac{\Delta M_t}{M_{t-1}} \equiv \frac{\Delta M_t^a}{M_{t-1}^a} + \frac{\Delta M_t^b}{M_{t-1}^b} \\
&\simeq \Delta m_t^a w_{a,t-1}^m + \Delta m_t^b w_{b,t-1}^m \\
&\simeq (\Delta m_{1,t}^a w_{1,t-1}^a + \Delta m_{2,t}^a w_{2,t-1}^a) w_{a,t-1}^m \\
&\quad + (\Delta m_{1,t}^b w_{1,t-1}^b + \Delta m_{2,t}^b w_{2,t-1}^b) w_{b,t-1}^m.
\end{aligned} \tag{14}$$

The final expressions in (13) and (14) are equal from (12). Generalizing to countries with different exchange rates involves the same additional approximations as for other variables discussed in section 3.5, noting that the same exchange rate is used for each money measure, and that weights like $w_{i,t-1}^m$ and $w_{k,t-1}^m$ have to be calculated recursively.

3.5. Variable exchange rates

The main impacts of variable exchange rates on the aggregation procedure proposed here is to alter the *weights* used in formulae such as (4): importantly, exchange rate changes do not alter the *magnitudes*

input into those formulae. Thus, only the shares in ‘Euroland’ nominal GDP alter as currency values change, since devaluations etc. do not affect the internally measured (own-country) growth rates. Clearly, weights must reflect the relative sizes of the components, and hence inherently involve both the ‘scale’ of each economy (measured domestically), and the exchange rates. A natural generalization of (3) in the two-country case is:

$$w_{1,t} = \frac{E_{1,t}X_{1,t}}{E_{1,t}X_{1,t} + E_{2,t}X_{2,t}} = \frac{X_{1,t}}{X_{1,t} + E_{2,t}X_{2,t}/E_{1,t}} \quad (15)$$

where $E_{1,t}$ and $E_{2,t}$ are the exchange rates for converting into a common currency (though only the relative rates matter). In ‘levels’ aggregation it can matter greatly whether nominal or ‘real’ exchange rates are used, but as we will show, that decision has a smaller impact on our approach.

First, consider any variable-weight aggregate of levels data, such as:

$$Z_t = \sum_{i=1}^n w_{i,t} Z_{i,t}. \quad (16)$$

When devaluations occur, the weights in (16) will change sharply and suddenly, unless very accurate PPP-based exchange rates are used – which do not, therefore, alter radically on devaluations. This dependence of the levels on the weights entails that such results become dependent on the particular numeraire selected, and hence on the frequency and magnitude of exchange-rate changes.

Next, fixed-weight aggregates are feasible, but even then the choice of weights is crucial (see e.g., Fagan and Henry (1999)). In any case, if fixed weights are used, based on the variable being aggregated, all in log-levels, then the outcome is equivalent to aggregating growth since if:

$$\log Z_t = \sum_{i=1}^n w_i \log Z_{i,t}$$

then:

$$\Delta \log Z_t = \sum_{i=1}^n w_i \Delta \log Z_{i,t}.$$

Consequently, such an approach is a special case of what we propose, obtained by selecting an arbitrary fixed weight, which could be interpreted as a specific function of the changing weights (such as the end period, or the average etc.). When the appropriate weights alter significantly over time, the results would seem inherently less useful, again because they do not deliver the correct result when there are no currency changes.

Finally, aggregating (non-log) levels with fixed weights, as in:

$$Z_t = \sum_{i=1}^n w_i Z_{i,t}$$

becomes highly problematic when the appropriate weights change over time, as has occurred on a substantive scale for several member states of the Euro zone, both because of differential real growth patterns (as measured in home currency), and from currency changes. Thus, when the weights are variable, as in (16), then changes in the weights can distort the outcome. This is most easily seen for $n = 2$, and aggregating real GDP:

$$X_t = w_{1,t} X_{1,t} + (1 - w_{1,t}) X_{2,t}.$$

Consider a counter example, where $X_{2,t} = \frac{1}{2} X_{1,t}$, so:

$$X_t = \frac{1}{2} (1 + w_{1,t}) X_{1,t},$$

when we use $w_{1,t}$ from (15). If a large devaluation occurred for the first country, so $w_{1,t}$ fell, measured aggregate output could fall even if *both* $X_{i,t}$ grew. Different problems affect the use of PPP-based exchange rates, namely the great difficulty of constructing accurate, and uncontroversial, measures, particularly when the resulting price series may be the basis for policy models. Thus, we exclude variable-weights with levels as a viable aggregator.

Consequently, the only remaining approach in Table 1 is to aggregate differences with variable weights as we propose. The aggregation formulae remain the same, e.g.:

$$\Delta x_t \simeq \Delta x_{1,t} w_{1,t-1} + \Delta x_{2,t} w_{2,t-1} \quad (17)$$

but the weights reflect exchange-rate changes as in (15). This is the approach implemented in section 5.

Mis-measuring the weights. It is important to note that the magnitudes of variables like $\Delta x_{i,t}$ are small, particularly for real growth rates, and relatively similar across countries as measured in their domestic currencies. Thus, errors in the weights have a ‘second-order’ impact on the aggregate. For example, let the measured weight be $w_{1,t-1}^o$, and the resulting measured aggregate be Δx_t^o , then in the 2-country case:

$$\begin{aligned} \Delta x_t - \Delta x_t^o &\simeq \Delta x_{1,t} (w_{1,t-1} - w_{1,t-1}^o) + \Delta x_{2,t} (1 - w_{1,t-1} - (1 - w_{1,t-1}^o)) \\ &= (\Delta x_{1,t} + \Delta x_{2,t}) (w_{1,t-1} - w_{1,t-1}^o). \end{aligned}$$

For example, when $\Delta x_{1,t} = 2\Delta x_{2,t} = 0.01$ (about 4% pa), and $w_{1,t-1} = 1.1w_{1,t-1}^o = 0.55$ (a 10% error in the dominant weight), then:

$$\Delta x_t - \Delta x_t^o \simeq 0.066\%,$$

when $\Delta x_t = 1\%$, so only a small relative error results.

More formally, the different effects of a change in, say, the exchange rate of country k at time s , $E_{k,s}$, on area-wide aggregates constructed by level aggregation or by growth rates can be seen by comparing the corresponding partial derivatives. For level aggregation:

$$Z_t^L = \sum_{i=1}^n E_i Z_{i,t}$$

and hence:

$$\frac{\partial Z_s^L}{\partial E_{k,s}} = Z_{k,s}. \quad (18)$$

For growth-rate aggregation:

$$Z_t^{BDH} = \exp \left(\sum_{r=1}^t \sum_{i=1}^n \frac{E_{i,r-1} X_{i,r-1}}{\sum_{j=1}^n E_{j,r-1} X_{j,r-1}} \Delta z_{i,r} \right),$$

the partial derivative with respect to a change in $E_{k,s}$ at $t = s + 1$ is therefore:

$$\begin{aligned} \frac{\partial Z_t^{BDH}}{\partial E_{k,s}} \Big|_{t=s+1} &= \frac{\partial \left(\sum_{r=1}^t \sum_{i=1}^n \frac{E_{i,r-1} X_{i,r-1}}{\sum_{j=1}^n E_{j,r-1} X_{j,r-1}} \Delta z_{i,r} \right)}{\partial E_{k,s}} \Big|_{t=s+1} Z_{s+1}^{BDH} \\ &= \frac{\partial \left(\sum_{i=1}^n \frac{E_{i,s} X_{i,s}}{\sum_{j=1}^n E_{j,s} X_{j,s}} \Delta z_{i,s+1} \right)}{\partial E_{k,s}} Z_{s+1}^{BDH} \\ &= \left(\Delta z_{k,s+1} - \frac{\sum_{i=1}^n E_{i,s} X_{i,s} \Delta z_{i,s+1}}{\sum_{j=1}^n E_{j,s} X_{j,s}} \right) \frac{X_{k,s}}{\sum_{j=1}^n E_{j,s} X_{j,s}} Z_{s+1}^{BDH} \\ &= (\Delta z_{k,s+1} - \Delta \bar{z}_{s+1}) Z_{k,s+1} \end{aligned} \quad (19)$$

where:

$$\Delta \bar{z}_{s+1} = \frac{\sum_{i=1}^n E_{i,s} X_{i,s} \Delta z_{i,s+1}}{\sum_{j=1}^n E_{j,s} X_{j,s}}$$

as:

$$\frac{X_{k,s}}{\sum_{j=1}^n E_{j,s} X_{j,s}} Z_{s+1}^{BDH} \simeq Z_{k,s+1}.$$

Thus, the effect of a change in the exchange rate in (19) is of a smaller order than in (18) unless $(\Delta z_{k,s+1} - \Delta \bar{z}_{s+1}) \simeq 1$. This, however, is unrealistic because it would require country k 's growth rate to be 100 times higher than the average growth rate across countries. Below, therefore, we have used nominal exchange rates as in (15).

4. Relation to index-number theory

The literature on index numbers is vast, and many approaches and criteria have been proposed: see (Diewert 1988). However, aggregating variables as described above yields indices which are close to those suggested by some aspects of index-number theory. In particular, our time-series indices are related to the Törnqvist approach (denoted TI): see (Törnqvist 1936). In a time-series context, TI is defined as a bilateral index for consecutive discrete points in time ($t = 0$ and 1, say) such that:

$$\log TI_{1,0} = \sum_{i=1}^n \frac{1}{2} (S_{1,i} + S_{0,i}) \log \left(\frac{Z_{1,i}}{Z_{0,i}} \right) = \sum_{i=1}^n \bar{S}_{1,i} \Delta z_{1,i} \quad (20)$$

where $Z_{i,t}$ is the scalar variable to be aggregated across n countries, $S_{t,i}$ are the weights and $\bar{S}_{1,i}$ is their average. A time series with more than two observations can be constructed by chain linking index numbers such as (20) over time.

The use of two-period average weights ‘smooths’ the series relative to the outcome from (4), but in practice does not greatly alter the calculated aggregates, as figure 1 shows. Consequently, we have retained the basic index form delivered by (4) in the computations reported in the next section.

The Törnqvist index is a ‘superlative’ index, namely, an exact index derived from a flexible aggregator function: see (Diewert 1976) and (Caves, Christensen, and Diewert 1982b; Caves, Christensen, and Diewert 1982a) for details and (Samuelson and Swamy 1974) for a technical survey of index-number theory. Time-series comparisons based on superlative bilateral chain-linked indexes have been shown to have desirable properties (see Caves, Christensen, and Diewert (1982b), for further discussion). Nevertheless, in the context of further aggregation of pre-aggregated national statistics, few of the usual justifications seem sustainable, whereas we have established the properties of the proposed approach above.

5. The constructed Euro-zone GDP and M3 data

Figure 2 shows the log-levels of the individual variables for the Euro-zone countries, after conversion to ECU at contemporaneous rates. Aggregation of levels at variable exchange rates is a simple addition of the lines in the graph, and it illustrates how the method is problematic in the light of exchange rate changes. Figure 3 shows the corresponding growth rates in the left three panels. The method proposed in this paper is a weighted addition of the growth rates. For GDP the weights are given by the share of real GDP (in current ECU) in the previous period, while money uses nominal money shares

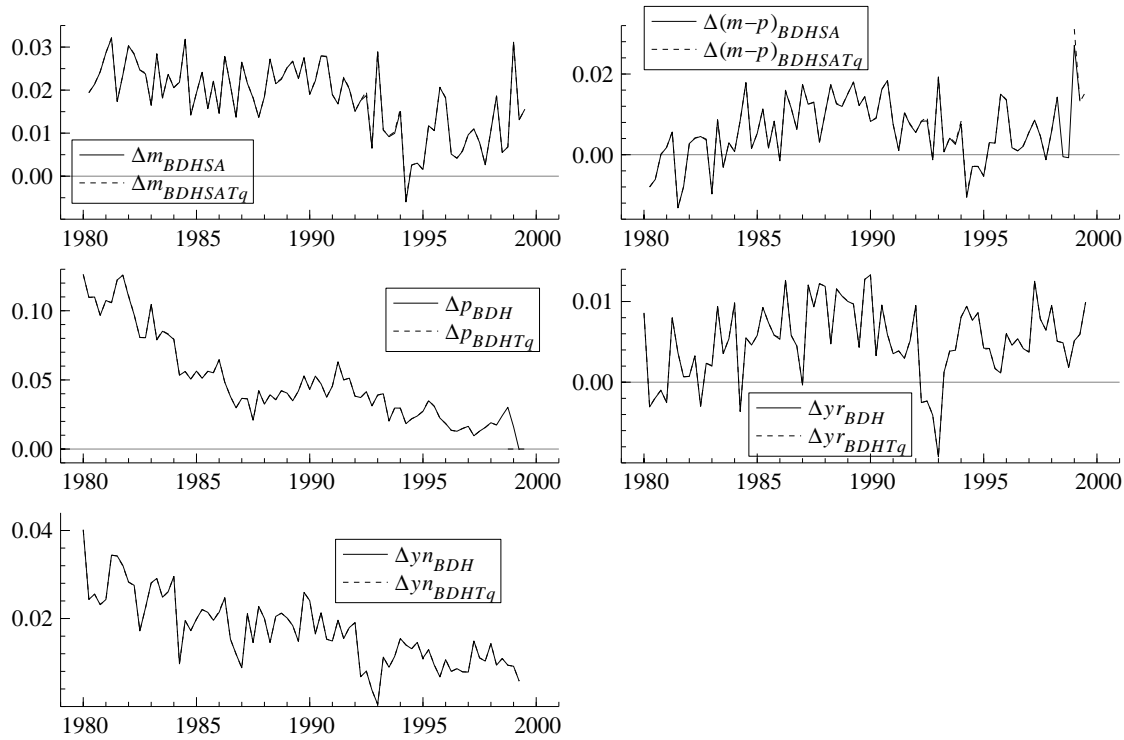


Figure 1. Growth rates of BDH E11 Variables, weighted by lagged shares and Törnqvist averages (Tq)

(again in current ECU). The resulting aggregate growth rates are shown in the right panels. As noted above, our method has the appealing feature that the aggregate of individual inflation is identical to the implied inflation of the aggregate. Seasonally-adjusted growth rates of M3 were constructed from seasonally-adjusted individual M3 (see the Appendix for details).

Figure 4 illustrates the drawbacks of some existing methods which aggregate levels. In the first graph, nominal GDP is aggregated with variable weights, and real GDP with fixed weights. The implied price levels are shown when using three numeraires: DE for German Marks, IT for Italian Liras, and ECU. Clearly, the outcome depends on the chosen units. This is the most problematic method, which has nonetheless been used in practice. In the second graph, the same method is used for both real and nominal GDP, once using fixed weights, and once using variable rates. The implied deflator is graphed for both cases, as well as the aggregate of the individual deflators (which uses GDP weights, with identical growth rates for nominal and real weights). The implied inflation rates are

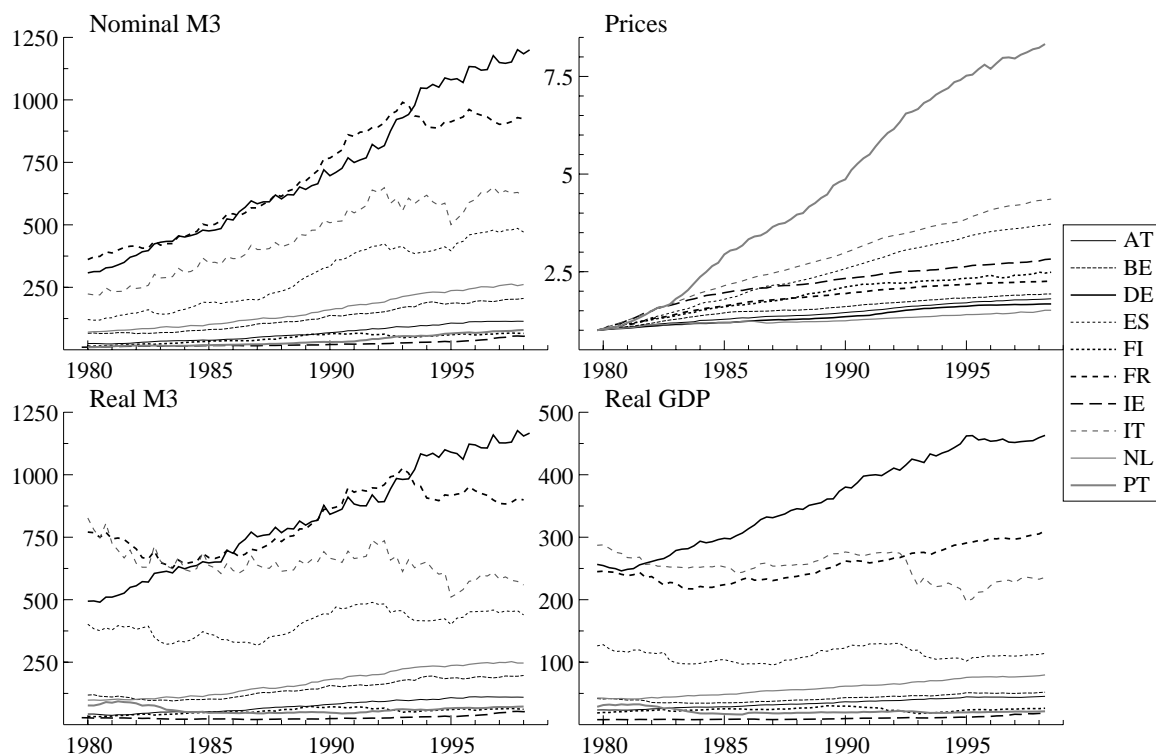


Figure 2. Levels of variables for individual Euro-zone countries (in current 10^9 ECU)

clearly different from the aggregated inflation rates.¹

The final graphs compare the constructed data with some published data sourced from the ECB and the OECD. Figure 5a compares the constructed $\log(M3)$ (with suffix .BDH), to that from (ECB 1999a). Figure 5b–c shows the logarithms of nominal and real GDP respectively, with the ECB figures from the statistical archive (dated November 1999); the published OECD figures use a fixed PPP US-dollar exchange rate. For comparison, the ECB and OECD data are scaled so that the observations for 1998Q2 are identical. The final graph gives the price level as an index which is unity in 1998Q2. The graphical evidence confirms the advantages of aggregating domestic growth rates with weights which reflect ‘common currency’ size.

¹This problem is illustrated by the warning in the explanatory notes in the ECB monthly bulletin for Table 5.1 (national accounts): ‘The data presented in Table 5.1 are, in particular, inappropriate as a means of deriving implicit deflators (see Table 4.2 for the ECB calculations of implicit deflators).’

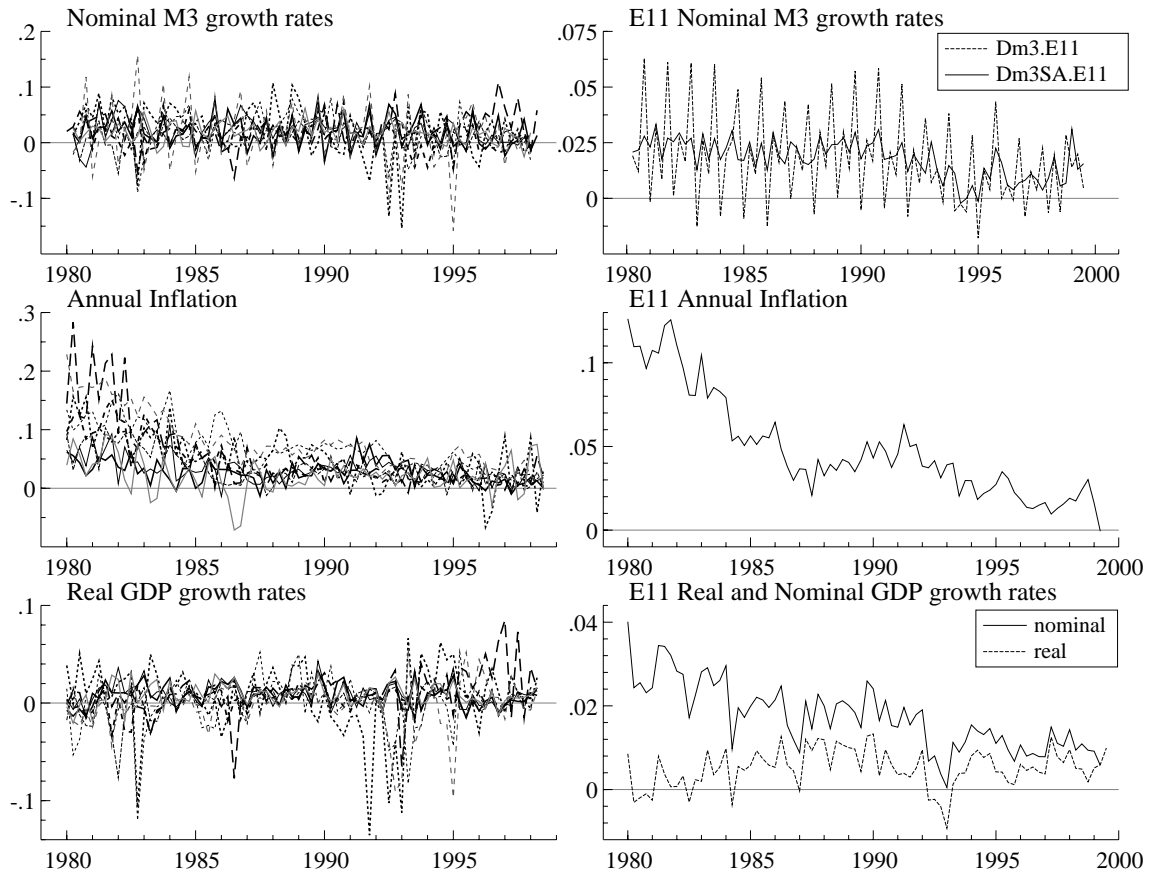


Figure 3. Growth rates for individual Euro-zone countries (left) and aggregate (right)

6. Conclusion

It is both feasible and valuable to reconstruct an aggregate dataset for the Euro-zone. The main problem that has vexed earlier investigators is handling exchange-rate changes between the member countries, involving many decisions on real or nominal rates, base year and whether PPP should be used. The method proposed in this paper instead aggregates growth rates, and shows that the choice of exchange-rate measure is less central to the resulting aggregates. Moreover, the method would aggregate essentially exactly if no currency changes had occurred.

The paper establishes the properties of the proposed aggregator, shows that sub-aggregation is unproblematic, whether temporally or spatially, and proves that the implicit deflator of the nominal

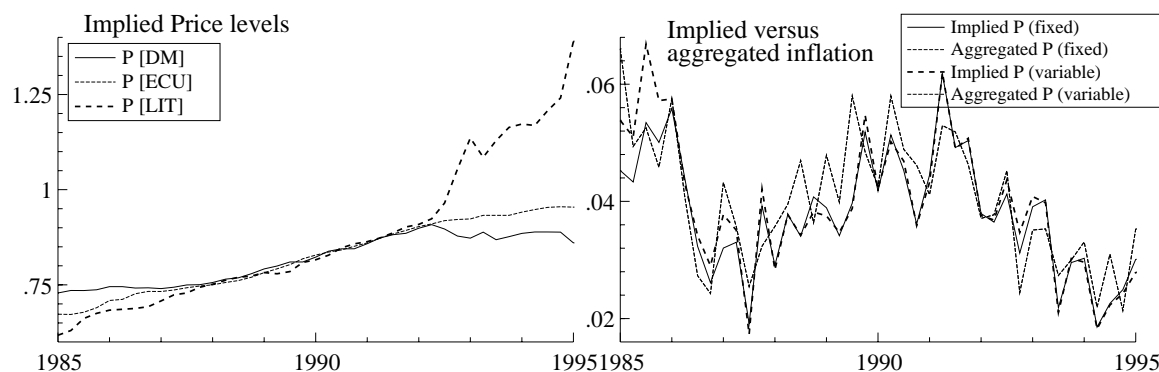


Figure 4. Problematic aspects of levels-aggregation

and real aggregates equals the aggregate implicit deflator, so a unique price measure results. It also addresses the aggregation of the money stock. The principles apply to other stock and flow variables.

The next stage is to econometrically analyze the resulting data to investigate the money-demand relations that might now confront the ECB with the adoption of a common currency.

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Appendix: Details of the Euro-zone data reconstruction

Using the procedure set out in section 3, we now reconstruct the data for the Euro-zone economy. The variables involved are:

$Y_{i,t}$	nominal GDP of country i at time t in local currency, seasonally adjusted,
$X_{i,t}$	real GDP in local currency, seasonally adjusted,
$P_{i,t}$	GDP deflator, $Y_{i,t}/X_{i,t}$,
$M_{i,t}$	nominal money stock M3 in local currency,
$M_{i,t}^{SA}$	nominal money stock M3 in local currency, seasonally adjusted,
$E_{i,t}$	exchange rate which converts the local currency into ECU.

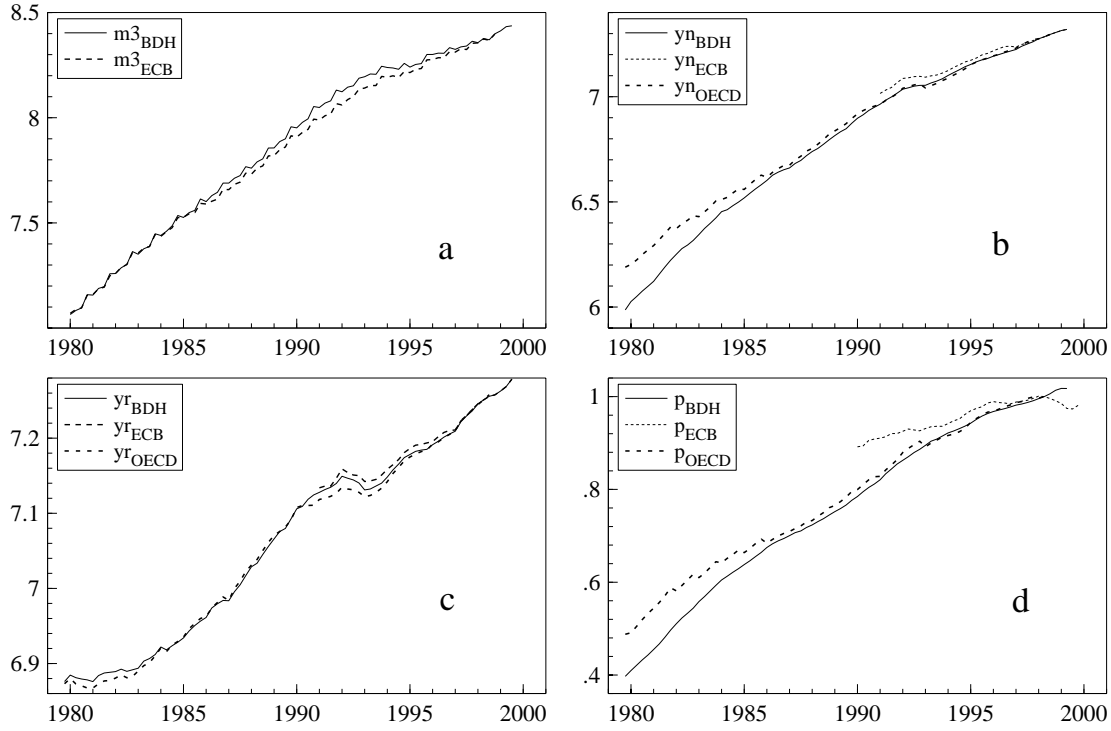


Figure 5. Comparison of constructed data with published data from ECB and OECD

The Euro zone consists of 11 countries, but we have no data for Luxembourg, leaving Austria, Belgium, Germany, Finland, France, Italy, Ireland, The Netherlands, Portugal, and Spain. For the Euroland aggregates we omit the subscript i . Belgium and Luxembourg have been in a monetary union over the period, and it is likely that Belgian M3 includes Luxembourg.

The weights are constructed as the share of real GDP expressed in ECU:

$$w_{i,t} = \frac{E_{i,t} X_{i,t}}{\sum_{i=1}^{10} E_{i,t} X_{i,t}}.$$

Because we use lagged weights, the weight for the initial period is missing. In that case, the weight for the second period is used: $w_{i,0} = w_{i,1}$.

To construct **nominal GDP**, we take the weighted sum of the country-specific growth rates:

$$\widehat{\Delta \log Y}_t = \sum_{i=1}^{10} w_{i,t-1} \Delta \log Y_{i,t}. \quad (21)$$

The created variable is then re-integrated using (7):

$$\hat{y}_t = \log \bar{Y}, \text{ for } t = 1998(2),$$

$$\begin{aligned}\widehat{y}_{t-1} &= y_t - \Delta y_t \text{ for } t = 1998(2), \dots, 1980(1), \\ \widehat{Y}_t &= \exp(y_t) \text{ for } t = 1979(4), \dots, 1998(2).\end{aligned}$$

Thus, the constructed aggregate is actualized with respect to the published figures for 1998(2).

To construct **real GDP**, the same procedure is used, with the same weights, but replacing Y s by X s.

The **GDP deflator** can be computed from Y_t/X_t , or, alternatively using the above procedure (substituting $P_{i,t}$ for $Y_{i,t}$ in the expressions). Both methods give identical results, as was shown in section 3.1. Consequently, one variable is redundant, and we did not include national real GDP data in our core database.

For the **money stock**, we use weights based on the nominal money shares (in ECU):

$$w_{i,t}^m = \frac{E_{i,t} M_{i,t}}{\sum_{i=1}^{10} E_{i,t} M_{i,t}}.$$

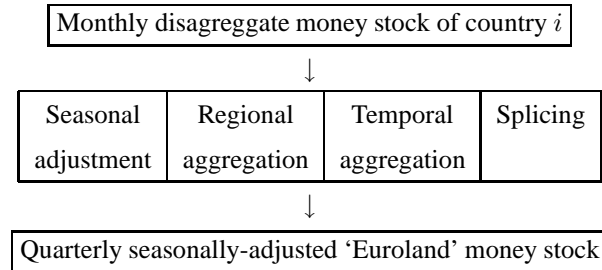
The money stock is available at a monthly frequency, but our principles apply:

$$\begin{aligned}\widehat{\Delta \log M}_t &= \sum_{i=1}^{10} w_{i,t-1}^m \Delta \log M_{i,t}, \\ \widehat{m}_t &= \log \overline{M}, \text{ for } t = 1998(5), \\ \widehat{m}_{t-1} &= m_t - \Delta m_t \text{ for } t = 1998(5), \dots, 1980(2), \\ \widehat{M}_t &= \exp(m_t) \text{ for } t = 1980(1), \dots, 1998(5).\end{aligned}$$

The same procedure is used for the seasonally-adjusted figures, with weights based on the seasonally-adjusted M3 data.

Related issues

Although monetary data is available at the monthly frequency, most other data are only available quarterly. The following table illustrates the steps involved in creating quarterly area-wide money stock from the raw monthly data:



We performed the steps in the order of the diagram. In particular, we performed seasonal adjustment first. Application of the adjustment after aggregation imposes the same filter on each country-specific series, and thus makes strong assumptions (see the comments by Findley (1996)). This conforms to the institutional practice in the US.

Data sources

variable	sample period	description	units	source
$X_{i,t}$	1979(Q4)–1998(Q2)	GDP at constant prices, $i \in E_{10}$	lc	OECD
X_t	1998(Q2)–1999(Q3)	base year is 1995	Euro	ECB
$P_{i,t}$	1979(Q4)–1998(Q2)	GDP deflator, $i \in E_{10}$	lc	OECD
P_t	1998(Q2)–1999(Q2)		Euro	Y_t/X_t
$Y_{i,t}$	1979(Q4)–1998(Q2)	nominal GDP, $i \in E_{10}$	lc	$P_{i,t}X_{i,t}$
Y_t	1998(Q2)–1999(Q2)		Euro	ECB
$M_{i,t}$	1980(M1)–1998(M5)	Harmonized M3, $i \in E_9$	ECU	NCBs
	1980(M1)–1998(M5)	M3, Ireland		OECD
M_t	1998(M6)–1999(M11)		Euro	ECB
$E_{i,t}$	1980(M1)–1998(M5)	exchange rates, $i \in E_{10}$	USD/ECU	OECD
			* ECU/lc	OECD
$E_9 =$	Austria (AT), Belgium (BE), Germany (DE), Finland (FI), France (FR), Italy (IT), The Netherlands (NL), Portugal (PT), Spain (ES).			
$E_{10} =$	$E_9 +$ Ireland (IE)			
lc	local currency			
NCBs	National Central banks.			

German unification

The effect of German unification for M3 was estimated by regressing the growth rates on lags 1, 2, 3, 6, 12, seasonal dummies, and a unification dummy variable for 1990(M6). The estimated coefficient on the unification dummy was 11.5% with a standard error of 0.94. Consequently, pre-unification M3 figures were multiplied by 1.115.

The effect for quarterly nominal GDP was estimated by regressing the growth rates on lags 1, 2, 3, and a unification dummy variable for 1991(Q1). The effect was estimated to be 9.2%, with a standard error of 0.78. The effect on real GDP was estimated as 9.5% (0.75).

Other adjustments

The Netherlands had M3 missing for 1980(M1) to 1982(M11). These observations were backcasted from a regression of the reversed growth rates on lags 1, 12 and seasonals.

Belgium had M3 missing for 1998(M1) to 1998(M5). These were forecasted from a regression of the growth rates on lags 1, 2, 3 and seasonals. Belgium had nominal GDP missing up to 1983(Q4). This was reconstructed from real GDP and the CPI index.

Harmonized M3 was not available for Ireland. We used the M3 index from the OECD up to 1990(M10), and levels afterwards. Irish nominal GDP after 1996(Q4) was extrapolated using the industrial production index (IIP).

Seasonal adjustment

Seasonal adjustment of M3 after the aforementioned adjustments is by X-12-ARIMA, developed by the US Census Bureau (see Findley, Monsell, Bell, Otto, and Chen (1998)), using X12arima for GiveWin. Multiplicative seasonal adjustment with Easter adjustment was applied to the levels of M3. The diagnostic ‘ Q (without $M2$)’ was always accepted, except for Ireland, which was ‘conditionally accepted’. For Germany, France and The Netherlands, none of the eleven M diagnostics were significant. For the other series there was some evidence of changed seasonality towards the end of the sample ($M10$, $M11$).

Note that the ECB also uses X-12-ARIMA for the published data, applying the multiplicative version (with trading day adjustments for some components of $M2$) to $M1$, $M2-M1$, and $M3-M2$; the seasonal adjustment is applied after aggregation (see e.g. ECB (1999b), p.48*).

Temporal aggregation

Quarterly Euro-zone M3 corresponds to the last month of each quarter.

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