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ORIGINAL RESEARCH

Are generics quantificational?

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Abstract
The standard view about generic generalizations is that they have a tripartite quantificational logical form involving a phonologically null quantificational expression called 'Gen'. However, proponents of the cognitive defaults theory of generics have forcefully rejected this view, instead arguing that generics express the default generalizations of our cognitive system, and, as such, they are different in kind from quantificational generalizations. While extant criticism of the cognitive defaults theory has focused on the extent to which it is supported by the empirical evidence, there has been little discussion of a neglected, albeit essential, theoretical argument in its defence, namely, that generics cannot be quantificational because they lack a central logical property of quantifiers: isomorphism invariance. This paper addresses this lacuna by considering and rejecting this argument. Consequently, an essential argument in favour of the cognitive default theory is found wanting.

Keywords Generics · Cognitive defaults · Logicality · Isomorphism invariance

1 Introduction

When we want to express generalizations in natural language, we usually do so in two distinct ways. On the one hand, we can use explicitly quantificational generalizations, such as those in (1):

(1) a. Most ravens are black.
b. Some lions have a mane.
c. Every whale is a mammal.
d. Birds usually fly.

These sentences express quantitative claims about how many members of a kind satisfy a certain property or statistical claims about how frequently they do so. For example, (1a) is true just in case the proportion of black ravens is greater than the proportion of non-black ravens; (1b) is true just in case the intersection of the set of lions and the set of things with a mane is non-empty; (1c) is true just in case the set of whales

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is a subset of the set of mammals; and (1d) is true just in case the probability that an arbitrary bird flies is greater than chance. Semantically speaking, such sentences have been productively analysed in terms of relations between sets or relative frequencies (Lewis, 1975; Barwise & Cooper, 1981; Cohen, 1999a).

On the other hand, we can use generic generalizations, such as those in (2):

(2) a. Ravens are black.
b. The lion has a mane.
c. A whale is a mammal.
d. Gold is valuable.

These sentences express generic claims about what is characteristic for a kind and its members, all without any explicit expression that is obviously responsible for generating this general content. For example, (2a) communicates a generalization about ravens – let us say, a generalization akin to *in general, ravens are black* – without an explicit expression, such as *in general*, to generate the general content. Similar remarks hold for (2b–d). But despite the lack of any such dedicated, phonologically articulated morpheme, the standard view amongst philosophers and linguists is that generic generalizations are essentially quantificational (cf. (Krifka et al., 1995; Mari et al., 2013)). More specifically, generic sentences are taken to have a tripartite logical structure consisting of a quantifier expression, a restrictor clause, and a matrix clause, akin to explicitly quantificational sentences like those in (1). To bridge the theoretical gap between generics and sentences containing explicit quantifier expressions, theorists postulate a phonologically null quantifier expression, which they call 'Gen', and provide it with a reductive semantic analysis in terms of quantification over suitable restricted individuals, worlds, situations, or histories (Heim, 1982; Schubert & Pelletier, 1989; Krifka et al., 1995; Asher & Morreau, 1995; Chierchia, 1995; Kratzer, 1995; Pelletier & Asher, 1997; Drewery, 1998; Cohen, 1999a, c; Eckardt, 2000; Greenberg, 2003, 2007; Nickel, 2009, 2016; Asher & Pelletier, 2013; Sterken, 2015b; van Rooij & Schulz, 2020; Kirkpatrick, 2023). Call this the *quantificational theory of generics*.[1]

Despite its popularity, some theorists have recently argued forcefully against the quantificational theory of generics and the reductive approach to their meaning within formal semantics. Instead, they have proposed an alternative theory according to which generics express cognitively primitive, default generalizations (Hollander et al., 2002; Leslie, 2007, 2008; Gelman, 2010; Leslie et al., 2011). According to this so-called 'cognitive defaults' theory, generics express the most basic means by which our conceptual system forms general judgements, and, as such, those judgements are not concerned with questions of quantity or *how many*. This categorical distinction

[1] The sentences in (2) are often called *characterising sentences* and are sometimes contrasted with so-called *reference-to-kind* genericity, another kind of phenomenon classified as generic that involves kind-referring noun phrases, as exhibited in the following sentences (cf. (Krifka et al., 1995), p. 2):

(i) a. The potato was cultivated in South America.
b. Potatoes were cultivated in South America.

It is debated whether such generics involve covert quantification, or whether they simply involve the direct predication of properties to the kinds to which they refer. Consequently, they are outside the scope of this paper.

between generic generalizations and explicitly quantificational generalizations is then linked to the dual process theory of cognition, as popularized by Kahneman and Frederick (2002). According to this theory, cognition involves two different systems, a fast, automatic, effortless lower-level system, and a slower, more effortful higher-level system. Generics are said to express cognitively primitive generalizations and are part of the intuitive system, while quantificational statements are concerned with quantity and are part of the reflective system (Leslie, 2007).

There are two kinds of arguments that have been levelled in support of the cognitive defaults theory. The first argument comes from experimental and developmental psychology. It has been argued that the cognitive defaults approach is best placed to explain numerous empirical observations, including the early acquisition of generics in child speech (Leslie, 2007, 2008), the tendency of humans to seemingly overgeneralize the truth of generics to *all*-generalizations (Khemlani et al., 2007, 2009, 2012; Leslie et al., 2011; Meyer et al., 2011; Leslie & Gelman, 2012), and our judgements about so-called striking property generics, such as *Mosquitos carry the West Nile Virus* and *Pit bulls attack people* (Leslie, 2007). Much critical attention has focused on assessing whether and to what extent the empirical evidence supports the cognitive defaults view (Lazaridou-Chatzigoga et al. 2015, (2019a), 9; Lazaridou-Chatzigoga et al. 2023; Sterken 2015a, c; Kirkpatrick 2024a).

But there is a second argument in favour of the cognitive defaults theory, one which is just as important as the empirical arguments, but which has so far received no critical attention. According to this argument, generic generalizations lack an important logical property commonly shared by quantifiers expressed by natural language: generic generalizations are not *isomorphism invariant*. Consequently, they cannot be quantificational.

This argument is an essential linchpin for the cognitive defaults theory, since it lays the grounds for the claim that generics and explicitly quantificational generalizations are different in kind. Without this distinction, we lose an important theoretical motivation for thinking that generics and explicitly quantificational generalizations express categorically different kinds of generalizations, and, as a result, an important argument for the cognitive defaults theory. Conversely, if there is a difference in kind between generics and explicitly quantificational statements, the implications for the proper semantic analysis of generics would be far-reaching: the quantificational theory, along with its myriad implementations, would be incorrect. Given the centrality of this argument for the cognitive defaults theory, the lack of any relevant critical discussion is surprising.

This paper aims to address this lacuna by critically evaluating the argument from isomorphism invariance that generics aren't quantificational. This paper is structured as follows. Section 2 expounds the quantificational analysis of generics together with its central motivations. Section 3 introduces the argument that generics are not quantificational on the grounds that they are not isomorphism invariant. Section 4 reevaluates the central examples to the claim that generics aren't isomorphism invariant and explores how they can be accommodated using extant resources. Section 5 outlines the notion of isomorphism invariance for two broad classes of models and shows that some prominent quantificational theories of generics are in fact invariant on these notions.

Section 6 concludes by reflecting upon the significance of these points for the cognitive defaults theory.

2 The quantificational theory of generics

This section outlines the quantificational theory of generics, beginning with some preliminary remarks about the general treatment of quantification in natural language on which it is built.

Natural language quantifier expressions are used to talk about quantities of things or amounts of stuff. There is a significant amount of syntactic variation between quantifier expressions, as demonstrated in (1). In particular, some quantifier expressions are determiners, like *most*, *some*, *every*, but quantification can also be expressed by adverbs of quantification, like *usually*, *generally*, *always*, and modal auxillaries, like *might* and *must*. There is also significant semantic variation between quantifier expressions. For example, the quantifier expressions in (1) quantify over things or stuff and they express binary relations between sets of things or stuff. Thus, for example, the quantified sentence in (1c) states that $every(W, M)$ holds, where W is the set of whales, M is the set of mammals, and $every$ is a relation between sets, namely, the subset relation $\subseteq$. But quantifier expressions vary considerably in their semantic type. For example, the quantifier expression *usually* in (3) presumably quantifies over stretches of time, not individuals, while in (4) time seems to be entirely irrelevant. Furthermore, the modal *must* in (5) seems to quantify over worlds or possibilities. And, finally, the quantifier expression in (6) seems to quantify over pairs of individuals, namely, pairs consisting of a person and a fireman who rescues that person; and it states that a majority of such pairs are such that the person is grateful.

(3) The fog usually lifts before noon here. (p. 3, (Lewis, 1975))
(4) A quadratic equation usually has two different solutions. (p. 5, (Lewis, 1975))
(5) All Maori children must learn the names of their ancestors. (p. 338, (Kratzer, 1977))
(6) People usually are grateful to firemen who rescue them.

The full range of natural language expressions of generality has lead theorists to think of the denotations of quantifier expressions not just as sets of sets of individuals, nor as relations between sets of individuals, but as relations between arbitrary relations (Mostowski, 1957; Lindström, 1966). To address the topic of natural language quantification, it is useful to avail ourselves of the notion of a *generalized quantifier*.

In this paper, we take a (generalized) *quantifier* to be any function Q assigning, to each universe E, a relation between arbitrary relations drawn from E.[2] We use the term *quantifier expression* for any natural language expression that can reasonably be taken to denote a quantifier. We shall also make use of the term of art *predicate expression* for expressions that can be taken to denote sets of individuals or relations between individuals. Here, and throughout, I will be diligent in distinguishing between

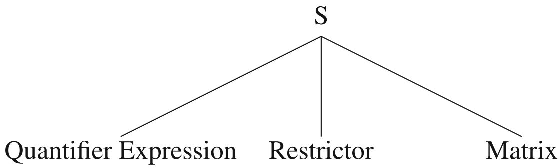
[2] Although, note that in Sect. 5.2, we consider a generalization of this notion of generalized quantifier to relational (Kripke) models for modal logic.

quantifier expressions, which are syntactic expressions, and quantifiers, which are the denotations or extensions of quantifier expressions.

By taking quantifiers to be relations between relations, we adopt a flatter representational structure, as exemplified in (7b), rather than the more familiar hierarchical form associated with a function-argument structure, as exemplified in (7a). In general, since we shall only be dealing with quantifiers that are binary relations between sets (or relations) of individuals, we shall have at some level of representation a tripartite structure consisting of a quantifier expression and two predicate expressions denoting sets of (or relations between) individuals as in (8).[3]

(7) a. Hierarchical structure: $[Q(A)](B)$
b. Relational structure: $Q(A; B)$

(8)



On this theory, the quantifier expression expresses a dyadic operator that relates the two predicate expressions, which are called the *restrictor clause* and the *matrix clause* (also known as the *nuclear scope*), respectively. The matrix clause makes the main assertion of the sentence, specifying the property attributed to the relevant members of the domain. The restrictor clause states the restricting cases relevant to the matrix. The quantifier expression, then, unselectively binds over any free variables in its scope, whether they be individual, situational, world, or eventual variables, and provides the quantificational force of the generalization.

This treatment unifies the observations made above. For example, Lewis (1975) argued that adverbs of quantification like *usually* denote quantifiers over 'cases', using examples like (4) with the posited logical form in (9b):

(9) a. (=(4)) A quadratic equation usually has two different solutions.
b. Usually x (x is a quadratic equation; x has two different solutions)

In (9), *usually* binds only one variable, and scopes over a restrictor clause and a matrix clause. But Lewis proposed that adverbs of quantification can unselectively bind any number of free variables in their scope, such as in (10):

(10) a. (=(6)) People usually are grateful to firemen who rescue them.
b. Usually x, y (x is a person and y is a fireman who rescues x; x is grateful)

Here, the quantifier expression *usually* binds pairs of objects. Then (10b) is true iff most pairs of things that satisfy the restrictor clause also satisfy the matrix clause.

[3] Note the flat, non-hierarchical structures are not intended to represent the linguistic structure of any of the examples. The notation is widely used without a commitment to its application to the grammar of any particular construction in natural language. It is easy to see how everything in this paper is made consistent with the assumption that all syntactic branching is binary, as well as functional type-theoretical implementations of natural language semantics, by recursively translating sets and relations into their characteristic functions and Schönfinkelizing/currying the resulting functions; see Heim and Kratzer (1998, Chapter 2) for details.

The general form of the tripartite structure applies to many different constructions (Partee, 1991). For example, Heim (1982) extended Lewis's treatment of adverbs of quantification to quantificational determiners like *most*, using examples like (11):

(11) a. Most quadratic equations have two different solutions.
  b. Most x (x is a quadratic equation; x has two different solutions)

But applications of Lewis's treatment of adverbs of quantification are not restricted to only extensional quantifiers. Kratzer (1977, 1981) and Heim (1982) have also extended Lewis's treatment of adverbs of quantification to modals and conditionals. For example, modal expressions like *must* are treated as quantifying over worlds, and so, they denote relations between sets of worlds (intuitively thought of as propositions). Thus, in (5), the sentence is analysed (very roughly) as in (12), where the restrictor clause is given by a free relative clause like *what is known*:

(12) a. (=(5)) All Maori children must learn the names of their ancestors.
  b. Must w (w is compatible with what is known; all Maori children learn the names of their ancestors in w).

Similarly, the word *if* can be taken to mark the restrictions on explicit quantification, as in (13) and (14), while in the case of 'bare' conditionals that have no overt indicator of modality in the *then*-clause, the *if*-clause is treated as restricting a covert modal expression like *must*, as in (15):

(13) a. Usually, if a man owns a donkey, he beats it.
  b. Usually x, y (x is a man and y is a donkey and x owns y; x beats y)

(14) a. If John enters the room, he might trip the switch.
  b. Might w (John enters the room in w; John trips the switch in w)

(15) a. If John enters the room, he will trip the switch.
  b. Must w (John enters the room in w; John trips the switch in w)

One might be cautious in taking modals and conditionals to be essentially quantificational simply on the grounds that they can be fruitfully analysed in terms of quantification, especially since sentences containing such expressions do not typically contain explicit expressions that denote the entities over which these expressions are claimed to quantify. But, since the work of Cresswell (1990), theorists have shown that natural language appears to make reference to times and worlds (e.g., through temporal and modal pronominal uses of *then*). In fact, it has been argued that the default assumption should be that natural language has expressions that quantify over these individual, world, and time variables (Partee, 1973; Stone, 1997; Percus, 2000; Kusumoto, 2005; Schlenker, 2006; Schaffer, 2018). Consequently, in this paper, I take the liberal view that modals and conditionals are essentially quantificational.

The standard quantificational approach to generics is build on top of this general theory of quantification in natural language. More specifically, it holds that generics have a tripartite logical form involving a phonologically null quantifier expression called 'Gen' (Heim, 1982; Krifka et al., 1995). For example, on a simple extensional analysis, the characterising sentence in (16) receives the (significantly simplified) logical form as in (16a):

(16) Ravens are black.

a. Gen x (x are ravens; x are black)

On this analysis of (16), the only kind of variable that *Gen* binds is an individual variable ranging over ravens. But since *Gen* is an unselective quantifier expression, theorists have proposed analyses whereby situation, event, and world variables are also present and bound, as in (17):

(17) Dogs bark.

a. Gen x, s (x are dogs in s; x bark in s)

More generally, the schematic logical form of generic sentences is as in (18) (cf. (Krifka et al., 1995), p. 26):

(18) Gen $x_1, \ldots, x_i$ (Restrictor($x_1, \ldots, x_i$); $\exists y_1, \ldots, y_j$ Matrix($\{x_1\}, \ldots, \{x_i\}, y_1, \ldots, y_j$)),

where $x_1, \ldots, x_i$ are variables of various sorts to be bound by *Gen*, $y_1, \ldots, y_j$ are variables to be bound existentially with scope just in the matrix, $\phi(\ldots x_m \ldots)$ is a formula where x_m occurs free, and $\phi(\ldots \{x_m\} \ldots)$ is a formula where x_m possibly occurs free. I will sometimes write $\ulcorner$Gen(ϕ; ψ)$\urcorner$ as shorthand, where 'ϕ' and 'ψ' are the restrictor and matrix clauses, respectively, and the variables bound by *Gen* are left implicit.

The standard quantificational approach to generics is primarily motivated by linguistic evidence that supports quantificational structure. The most influential argument is presented in Carlson (1989) and involves a systematic ambiguity in generic sentences between two readings, as witnessed by the following two interpretations of (19) (cf. Krifka et al. 1995, p. 26):

(19) Typhoons arise in this part of the Pacific.

a. Typhoons in general have a common origin in this part of the Pacific.
Gen x, y (x are typhoons; y is this part of the Pacific & x arise in y)
b. There arise typhoons in this part of the Pacific
Gen x (x is this part of the Pacific; $\exists y$[y are typhoons & y arise in x])

According to the first reading in (19a), typhoons are such that they generally arise in the part of the Pacific under discussion. This reading presumably requires at least a majority of typhoons to originate in that geographical region. On the second reading in (19b), the generalization says something about the part of the Pacific under discussion, namely, that generally typhoons arise there. This reading expresses a general property of a geographical region, rather than a general property of typhoons. So, unlike the first reading, the second reading is compatible with the existence of several geographical regions in which typhoons generally originate.

Other linguistic evidence in favour of the quantificational approach include: (i) the fact that generics exhibit embedding behaviour (Pelletier and Asher 1997, p. 1130), scope ambiguities under negation operators and quantifier expressions (Schubert & Pelletier, 1987; Krifka et al., 1995; Kirkpatrick, 2024b), and weak crossover effects (Leslie, 2015); (ii) the fact that there must be something that binds the free variables in generic sentences (Carlson, 1977; Nickel, 2013; Leslie, 2015; Sterken, 2016); and (iii) the fact that generics appear to be sensitive to focus and topic, just like explicitly quantified sentences (Partee, 1991; Krifka, 1995; Rooth, 1995). Quantificational analyses of generics have an easy time explaining these types of data, given the analogy to explicitly quantificational sentences in which all of these phenomena also arise.

There is a significant amount of flexibility built into the quantificational approach to generics. First, it is compatible with analysing *Gen* as an adverb of quantification (Heim, 1982; Krifka et al., 1995; Cohen, 1999a; Sterken, 2015b), as a quantificational determiner (Pelletier & Asher, 1997), or even as a modalized conditional (Asher & Morreau, 1995; Asher & Pelletier, 2013). Furthermore, the quantificational approach is also compatible with a variety of different semantic analyses in terms of quantification over individuals, worlds, situations, and histories. The important point is that, regardless of its syntactic category or semantic analysis, *Gen* essentially provides the 'conceptual glue' with which we express relations between restrictor and matrix clauses of generics.

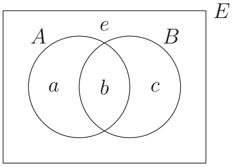
Second, the quantificational approach is compatible with numerous ways of partitioning the sentence material between the restrictor and matrix clauses (Kratzer, 1995; Chierchia, 1995; Krifka, 1995; Rooth, 1995; Cohen, 1999c; Asher & Pelletier, 2013; Kirkpatrick, 2023). The question of partitioning sentence material between the restrictor and matrix clauses is easy enough when the restriction comes from a conditional or a *when*-clause, but things are trickier in 'bare' cases and many proposals have been made. For example, Heim addresses this question with a sequence of syntactic operations (Heim 1982, Chapter 2). Rooth (1985, 1992, 1995), Krifka (1995), and Partee (1991) propose that focused material is mapped to the matrix, while the 'focus semantic value' of the sentence is mapped to the restrictor. Diesing (1990, 1992) proposes that the VP-external material at LF is mapped to the restrictor clause. Chierchia (1992) claims that topics are mapped to the restrictor clauses. Finally, von Fintel (2004) adopts a situation-semantic treatment of adverbial quantification, as pioneered by Berman (1987) and Heim (1990), and emphasizes the pragmatic determination of the restrictor. The result is a powerful and flexible theory of the expression of genericity in natural language.

3 The argument from isomorphism invariance

Despite the evidence adduced in favour of quantificational structure, the quantificational approach to generics has come under criticism due to concerns that generics do not share a certain property commonly held to be universal in natural language quantification: isomorphism invariance (Leslie, 2007). This section outlines a simplified notion of isomorphism invariance and then presents examples in support of this claim.

Quantification in natural language has a common logical structure for representing its meanings. This makes it possible to formulate general hypotheses about patterns and properties common across quantifiers. One recurrent constraint is the idea that quantifiers should be 'topic neutral' in the sense that quantifiers "should not allow us to distinguish between elements [of the domain of the model]" (Mostowski, 1957, p. 13). It is generally agreed amongst theorists that quantifiers are topic neutral in this way just in case they cannot distinguish between isomorphic structures (Tarski, 1986; Higginbotham & May, 1981; Sher, 1991; Larson & Segal, 1995; Peters &

Fig. 1 Isomorphism invariance



Westerståhl, 2006; Glanzberg, 2008). This condition, which we call *isomorphism invariance*, expresses the idea that whether a sentence expresses a quantifier should not be sensitive to individual traits of objects. In other words, the truth of sentences containing unembedded occurrences of an expression that denotes a quantifier depend only on the underlying logical structure of the model, and not on particular individuals, sets, or relations.

To make this idea formally precise, it is necessary to define a notion of isomorphism between structures or models, and then to state that isomorphism invariant quantifiers cannot distinguish between isomorphic structures. For simplicity, Leslie restricts her attention to quantifier expressions that bind only one variable, which allows herself utilize a simplified notion of isomorphism invariance in terms of the cardinalities of certain sets, forgoing any explicit definition of isomorphic structures in full generality. For exegetical purposes, I follow Leslie in this simplification, although I revisit this assumption in Sect. 5.

Let us begin with an informal notion of isomorphism invariance for binary quantifier expressions that bind only one variable:

Isomorphism Invariance (Informal). Let Q be any functor assigning, to each universe E, a binary relation Q_E between subsets of E. Then we say that Q is *isomorphism invariant* iff, for all sets E, E', all bijections f from E to E', and all $A, B \subseteq E$, $Q_E(A; B)$ iff $Q_{E'}(f(A); f(B))$.[4]

An immediate upshot of this definition is that such a quantifier Q is isomorphism invariant just in case whether $Q_E AB$ obtains depends only on the cardinalities a, b, c, and e, where $a = |A - B|$, $b = |A \cap B|$, $c = |B - A|$, and $e = |E - (A \cup B)|$ as representing in Fig. 1.

Observe that we can identify isomorphism invariant quantifiers using constraints on the quantity of individuals in different regions of E, such as the following:

(20) a. $all_E(A; B)$ iff $a = 0$
b. $some_E(A; B)$ iff $b \neq 0$
c. $no_E(A; B)$ iff $b = 0$
d. $all\ but\ one_E(A; B)$ iff $a = 1$
e. $most_E(A; B)$ iff $b > a$

The importance of isomorphism invariance as a constraint on quantifiers comes from the fact that the following claim seems true:

[4] A function $f : X \to Y$ is a *bijection* iff it is injective (i.e., distinct elements of X are mapped to distinct elements of Y) and surjective (i.e., every element of Y is the image of at least one element of X). Here $f(A) = \{f(d) : d \in A\}$.

> *All lexical quantifier expressions in natural languages denote isomorphism invariant quantifiers*.[5] (cf. Peters & Westerståhl, 2006, p. 330)

But does this claim about natural language quantification hold of the hypothesized generic quantifier expression *Gen*? Leslie argues not:

> The informal observation that quantifiers tell us *how much* or *how many* can be given a formal analog in terms of *isomorphism invariance*. […] For [operators that bind a single variable] this requirement reduces to the idea that, for it to be quantificational, it must be definable as a function from the cardinalities of sets to truth values […] I will argue that there is no such definition of Gen. (Leslie, 2007, pp. 391–392)

To illustrate her point, Leslie provides two arguments. The first argument involves the following sentence[6]

(21) Peacocks have fabulous blue-green tails.

Observe that (21) is true, even though only male peacocks have colourful tails; in actuality, female peacocks have frightfully dull brown tails. Now consider whether (21) would be true in a counterfactual possible world in which female peacocks have fabulous pink tails. Leslie argues not and this is a problem for the quantificational approach:

> As things stand, "peacocks have fabulous blue-green tails" is true. Were female peacocks to have fabulous pink tails in place of the brown stumps they in fact have, "peacocks have fabulous blue-green tails" would not be true; only the weaker "peacocks have fabulous blue-green or pink tails" would be true in those circumstances. This thought experiment does not involve altering any of the relevant cardinalities, however. The numbers of peacocks, blue-green-tailed individuals, blue-green-tailed peacocks, and non-blue-green-tailed non-peacocks all remained unaltered. That our intuitions about the truth of a generic can be altered

[5] The restriction to lexical quantifier expressions is essential for the validity of the claim, since there are some natural language expressions that may reasonably be taken to denote quantifiers (i.e., arbitrary relations between relations), but that are not themselves isomorphism invariant, such as proper names construed as denoting Montagovian individuals, restricted noun phrases (e.g., *every dog*, *some cat*), possessives (e.g., *John's book*), and exceptive quantifiers (e.g., *every professor except Susan*). However, the following claim does seem to be true of this class of expressions:

All non-isomorphism invariant quantifier expressions come from isomorphism invariant operations or relations by freezing some arguments (cf. Peters & Westerståhl, 2006, p. 330)

For an illustrative example, we can construct a quantificational expression *some cats* by combining the quantificational determiner *some* with the noun phrase *cats*. If Q is the quantifier denoted by *some*, we can *freeze* the restriction to the set A denoted by the noun *cats*, thereby obtaining a quantifier Q^A to serve as the interpretation of the quantificational expression *some cats* defined as follows: for all E and $B \subseteq E$, $(Q^{[A]})_E(B) \Leftrightarrow Q_{E \cup A}(A, B)$. Such a quantifier is not isomorphism invariant in the sense that it is definable in terms of cardinalities, but it is nevertheless built up out of an operation that is so definable. For a general discussion of freezing in full generality, see Peters & Westerståhl, (2006, p. 331).

[6] The common term for the two bird species in the genus *Pavo*: is 'peafowl'; male peafowl are referred to as peacocks and female peafowl are referred to as peahens. Leslie uses 'peacock' as the common term, irrespective of sex; e.g., "…nor do *female peacocks* sport blue-green tails" (Leslie, 2007, p. 376; my *emphasis*). Here and throughout, I defer to Leslie's usage.

> by such facts that do not pertain to cardinalities reflects the fact that generics depend on far more than cardinalities for their truth and falsity. (Leslie, 2007, p. 393)

The objection is this: the sentence *Peacocks have fabulously blue-green tails* is actually true; but if female peacocks had fabulous pink tails, rather than the brown stumps that they actually have, the sentence *Peacocks have fabulous blue-green tails* would not be true, even though the relevant cardinalities have not changed. Consequently, the truth of *Peacocks have fabulous blue-green tails* is not invariant under isomorphic structures, and so, generics cannot be quantificational.

The second argument that Leslie offers involves the following sentence:

(22) Mosquitos carry the West Nile virus.

Again, observe that (22) seems true, even though less than 1% of mosquitos carry the virus.[7] Now suppose that mosquitos are the only things that carry the West Nile virus, and all and only those mosquitos that carry the virus are blind. In other words, the set of WNV carriers is just the set of blind mosquitos:

$$\{x : x \text{ carries the West Nile virus}\} = \{x : x \text{ is a blind mosquito}\}$$

But then, Leslie argues, if generics were isomorphism invariant, the following sentence should also be true:

(23) Mosquitos are blind mosquitos.

But (23) is false. Here's what Leslie has to say:

> an operator that is sensitive only to the cardinalities of sets will not distinguish between [the set of West Nile carriers and the set of blind mosquitos]. If the truth of generics was definable in terms of set cardinalities, then "mosquitoes are blind mosquitoes" would be true in the circumstances described if "mosquitoes carry the West Nile virus" was true. Of course, we think the latter is true but the former false, which shows that Gen is not a function from cardinalities to truth values. (Leslie, 2007, p. 393)

Interestingly, these arguments do not depend essentially on counterfactual scenarios (Leslie, 2007, p. 394). For example, we can rigidify the predicates in (22) and (23) with *actually* or *in actuality*, and so assume that the predicates *in actuality carry the West Nile virus* and *in actuality are blind mosquitos* have the same extensions in every possible world that they actually have. Even then, we can observe that the sentence *Mosquitos in actuality carry the West Nile Virus* is true, but the sentence *Mosquitos in actuality are blind mosquitos* is false, even though their predicates have the same extensions in every possible world.[8]

[7] Note some theorists have disputed the claim that (22) is true on the relevant reading; see Sect. 4 for discussion.

[8] One confound to the discussion in Leslie is that generics seem to generally fail to exhibit *conservativity*. We say that a quantifier expression Q is *conservative* iff, for all sets $A, B \subseteq E$, $Q_E(A, B)$ is equivalent to

In summary, these arguments aim to establish that generics are not isomorphism invariant. If correct, they constitute a powerful argument against the quantificational approach to generics, since they show that generics cannot be quantificational, and an essential linchpin for the cognitive defaults view.

4 Variations in the restrictor

A closer look at the extant resources available to proponents of the quantificational approach makes it far from clear whether Leslie's examples are genuine counterexamples to the claim that generics are not isomorphism invariant. It is a familiar point that the logical forms of generics are not always reflected in their surface structures. Indeed, proponents of the quantificational approach typically avail themselves of various resources to partition the sentence material of generics between the restrictor and matrix clauses, as well as further ways to supplement the restrictor clause with additional contextually supplied material. In this section, I argue that these resources disarm Leslie's argument. While there are many ways of implementing this idea, I shall focus on the theory of generics in Cohen (1999a, b, c, 2004). I begin with a summary of his theory.

4.1 Cohen's theory

According to Cohen (1999a, b, c, 2004), generics express claims about probabilities. The basic idea is that a generic of the form $\ulcorner \text{Gen}(\phi; \psi) \urcorner$ is true, roughly speaking, iff the probability of an arbitrary ϕ being ψ is greater than 0.5, where probability is given a frequentist interpretation.[9] These relative probability judgements are interpreted in a Branching Time framework (Thomason, 1970, 1984), where we consider not only the sequence of events that we have actually observed, but also suitably smoothed out and suitably long possible continuations of that sequence into the future.[10]

Footnote 8 continued
$Q(A, A \cap B)$. For example, conservativity is exhibited by the equivalence between the sentences in (i) and (ii):

(i) a. All students run.
b. All students are students who run.
(ii) a. Some babies sleep.
b. Some babies are babies who sleep.

As an anonymous reviewer notes, the sentences in (iii) do not seem to convey the same thing.

(iii) a. Mosquitos are blind.
b. Mosquitos are mosquitos that are blind.

In particular, the sentence *Mosquitos are mosquitos that are blind* has a universal feel. For further discussion of the relation between generics and conservativity, see Cohen (1999c, 2001); Nickel (2018).

[9] Actually, Cohen's official theory is more complex than this, since he distinguishes absolute generics from relative generics. However, I shall focus on his simpler account for absolute generics, since nothing in my discussion hinges on the distinction.

[10] Here are the details of how Cohen interprets probability. Let a *history* be a linear course of time and say that history H *continues* history H' (in notation: $H' \sqsubset H$) iff H' forms an initial segment of H. Then a

An immediate issue is that this analysis appears to make the wrong predications for many intuitively true generics. For example, this proposal is not yet able to explain why (24) is true, at least not if its restrictor and matrix clauses are given as in (25):

(24) (=(21)) Peacocks have fabulous blue-green tails.
(25) Gen x (peacocks(x); $\exists y$(fabulous.blue.green.tails$(y) \wedge$ have(x, y)))

Since a strict majority of peacocks do not have fabulous blue-green tails, the probability of having a fabulous blue-green tail conditional on being a peacock is not greater than 0.5. Consequently, Cohen's basic analysis erroneously predicts that (24) is false.

To handle such generics, Cohen introduces the further idea that there is often more covert material in the restrictor clause than what one might have initially expected.[11] In particular, Cohen argues that the restrictor clause of a generic is further constrained by the set of contextually salient alternatives to the matrix clause. Consider the notion of the contextually salient alternatives to a predicate F, which we denote as $\textsc{alt}(F)$. This notion is intended to be natural and intuitive: the alternatives to *is walking* are other predicates of locomotion like *is cycling*, *is flying*, and so on; the alternatives to *lays eggs* are other predicates of reproduction like *births live young*, *reproduces via fission*, and so on; the alternatives to *is female* is *is male*. In each case, the predicate F is always included in the set $\textsc{alt}(F)$ and the alternatives need not be mutually inconsistent.

The idea of contextually salient alternatives helps us to accommodate the intuition that female peacocks are irrelevant to the truth of (24), as it allows us to assign to it the following logical form:

(26) Gen x (peacocks$(x) \wedge C(x)$; $\exists y$(fabulous.blue.green.tails$(y) \wedge$ have(x, y)))

where C is a contextually specified variable that is assigned the set of individuals satisfying any contextually salient alternative to the predicate *has fabulous blue-green tails*, namely:

$$\{x : \exists P(P \in \textsc{alt}(\text{fabulous.blue.green.tails}(z)) \wedge \exists y(P(y) \wedge \text{have}(x, y))\}$$

Before we calculate the truth-conditions for (26), we must augment the formal semantics with an explanation of what properties count as relevant alternatives to others. Again, there are numerous ways of making this precise, but I will again focus on the account given by Cohen (2004, p. 531), who places the following constraint on his truth conditions:

sequence of events can be understood as representing a possible history. Cohen then proposes the following definition of probability:

Definition 4.1 (Cohen probability) (Cohen, 1999a, p. 232)ss
$P(\phi|\psi) = l$ iff for every *admissible* history H and every $\epsilon > 0$, there is a history $H' \sqsubset H$, s.t. for every history H'', $H' \sqsubset H'' \sqsubset H$, the limiting relative frequency of ϕs among ψ will be within ϵ of l. Admissible histories are those histories that are sufficiently long so that "the relative frequency of ϕs among ψs in the admissible [history] will have "enough time" to come within ϵ of the probability" and that the histories are "extrapolated from the relevant part of the actual history" (Cohen, 1999a, p. 233).

[11] This idea enjoys wide application across quantificational theories of generics. For concreteness, I focus on Cohen's strategy for adding additional material to the restrictor, although other strategies work just as well; see, e.g., Asher and Pelletier (2013); Kirkpatrick (2023).

Homogeneity Constraint. The generic $\ulcorner \text{Gen}(\phi; \psi) \urcorner$ presupposes that exactly one of the following holds:
(i) for every psychologically salient partition Ω of ϕ, and for every $\phi' \in \Omega$, the probability of being a ψ conditional on being a ϕ' is high, or
(ii) for every psychologically salient partition Ω of ϕ, and for every $\phi' \in \Omega$, the probability of being a ψ conditional on being a ϕ' is low.

Cohen offers several mechanisms for what makes a partition salient, but what is important for present purposes is that the strikingness of colour may make for a salient partition. That is, the set of alternatives to having blue-green tails would be the psychologically salient alternative ways that sexual selection might visually mark a peacock's plumage. Since having a blue-green tail is a visually striking way in which sexual selection may mark one's plumage, the set of alternatives $\text{ALT}(\text{fabulous.blue.green.tails}(z))$ will include *fabulously blue-green tails*, as well as other predicates about strikingly coloured tails, including *red tails*, *orange tails*, or, as is most pertinent to the present case, *pink tails*. But, since having a brown tail is not a visually striking way in which sexual selection marks plumage, *brown tails* will not be a psychologically salient alternative.

With all of this machinery in place, let us see what Cohen's theory predicts about (24). According to Cohen's theory, (24) presupposes that *peacock with plumage visually marked by sexual selection* is homogeneous with respect to the property of having a blue-green tail; and, if defined, (24) is true just in case the probability of having a blue-green tail, conditional on being a peacock that has one's plumage visually marked by sexual selection, is greater than 0.5. Since the restrictor excludes brown-tailed peahens from consideration, the truth of (24) depends only on the male peacocks, the majority of which have blue-green tails. Thus, Cohen's theory predicts that (24) is true.

4.2 Peacocks have fabulous blue-green/pink tails

Let us now see how this strategy helps us to disarm Leslie's argument. Recall that the first problem Leslie raises is supposed to be that (24) is true, but we would judge it to be false if, in addition to blue-green tails enjoyed by male peacocks, female peacocks had pink tails. We have just seen that Cohen's theory predicts that (24) is actually true. But what does it predict for Leslie's counterfactual scenario?

Observe that if female peacocks had pink tails, the restrictor (24) would pick out a very different set of individuals than it actually does. In the counterfactual scenario, there are two ways peacocks have their plumages visually marked by sexual selection, namely, by having blue-green tails like the males do, or by having pink tails like the females do. As a result, the restrictor would pick out the set of male and female peacocks, and not just the set of male peacocks, since the set of alternatives for *having blue-green tails* includes *having pink tails*. Consequently, we can see that Cohen's theory does not predict that (24) is true in the counterfactual scenario: the probability of having a blue-green tail, conditional on being a peacock that has one's plumage visually marked by sexual selection, is not greater than 0.5.

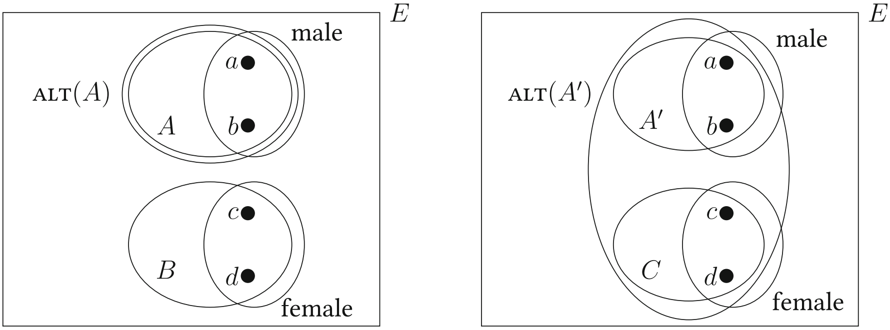


Fig. 2 Leslie's first example in full detail

I have just argued that Cohen's theory of generics can accommodate our truth-value judgements about Leslie's first example. But how does this show that Leslie's example is not a counterexample to the claim that generics are isomorphism invariant? The crucial observation is this: once we admit that the restrictor of (24) includes additional material — specifically, a predicate derived from the contextually salient set of alternatives of *having fabulous blue-green tails* — then it is clear that the cardinality of the restrictor denotation of (24) differs between its evaluation at the actual world and at the counterfactual scenario. Consequently, Leslie's example is simply no longer an example of an isomorphism invariant structure, and so, it is not a counterexample to the claim that *Gen* is isomorphism invariant.

To illustrate this point, consider the following toy model. Let E and E' be two simplified universes, each with only four peacocks, a, b, c, and d, two of each sex. Suppose further that a and b have blue-green tails in both E and E', while c and d have brown tails in E and pink tails in E'. We may then represent this toy model of Leslie's example as in Fig. 2, where A (A') is the property of having a fabulous blue-green tail in E (E'), B is the property of having a brown tail, C is the property of having a pink tail, and $\textsc{alt}(A)$ ($\textsc{alt}(A')$) is the property of having an alternative to a fabulous blue-green tail in E (E').

Simplifying Cohen's theory somewhat, suppose (24) is true iff the majority of individuals falling under the denotation of the restrictor (i.e., $\textsc{alt}(A)$ or $\textsc{alt}(A')$) also fall in the denotation of the matrix (i.e., A or A'). Then (24) is true in E, since all the individuals in $\textsc{alt}(A)$ are in A. But, (24) is not true in E'. Since the female peacocks have pink tails in E', they now fall in $\textsc{alt}(A')$, but since they do not also fall in A', the majority of individuals falling under the restrictor denotation do not also fall under the matrix denotation.

The toy model also illustrates why Leslie's example is not a counterexample to the claim that generics are isomorphism invariant. There are no bijections f from E to E', such that, for some $A, B \subseteq E$, $Gen_E(A; B)$ and not $Gen_{E'}(f(A); f(B))$, or vice versa. In particular, since $|\textsc{alt}(A)| \neq |\textsc{alt}(A')|$, there is no bijection f from E to E' such that $f(\textsc{alt}(A)) = \textsc{alt}(A')$. Furthermore, given the alternative-sensitive constraint imposed on the restrictor by the the matrix, there is no bijection

f from E to E' such that *Peacocks have fabulous blue-green tails* expresses $Gen_{E'}(f(\text{ALT}(A)); f(A))$ in E'.

The upshot: once we make clear how the restrictor is determined by the sentence material and contextually salient alternatives, any attempt to show that generics are not isomorphism invariant that changes the cardinality of the restrictor denotation across universes will fail.

4.3 Mosquitos carry the WNV/mosquitos are blind mosquitos

Similar remarks apply to Leslie's second supposed counterexample, repeated below for convenience, although details of the explanation differ.

(27) (=(22)) Mosquitos carry the West Nile virus.
(28) (=(23)) Mosquitos are blind mosquitos.

Recall that the problem is that (27) seems true, but if mosquitos were the only things that carry WNV, and all and only those mosquitos that carry the virus were blind, then the set $\{x : x$ carry the West Nile virus$\}$ would be the same as the set $\{x : x$ are blind mosquitos$\}$, but we would judge (28) to be false. How can proponents of the quantificational approach accommodate this datum?

A wrinkle. Leslie's second argument is complicated by the fact that so-called 'striking property generics' like (27) pose a *prima facie* problem for any quantificational theory of generics that takes their truth-conditions to be quasi-universal, such as Cohen's. The fact is that (27) seems true even though less than 1% of mosquitos carry WNV. This means that the probability of carrying WNV conditional on being a mosquito is less than 0.5, and so Cohen's theory predicts that (27) is false. Similar problems arise for other semantic analyses of generics that also require a contextually salient majority of the kind in question to have the predicated property.

Various solutions to this problem have been proposed. One approach is to treat striking property generics as ambiguous between a false generic reading and a true capacity reading (Asher & Pelletier, 2013; Nickel, 2016). The idea is that insofar as (27) has a true reading, it is equivalent to the claim that mosquitos have the capacity to carry WNV. Sterken (2015a, c) takes a less mollifying approach, arguing (convincingly to my mind) that such generics are just false, although we are apt to judge otherwise due to the alarming, dangerous, or striking nature of the predicated property.

A completely different approach takes seriously the idea that prosody affects the division of material between the matrix and restrictor, and, in particular, what is in focus goes in the matrix clause and the rest of the material is mapped to the restrictor. Then, by treating (27) as in (29), and appealing to Cohen's sets of alternatives, we would assign to it the logical form in (30)[12]:

(29) $[\text{Mosquitos}]_f$ carry the West Nile virus.
(30) Gen x (carries.the.WNV$(x) \wedge C(x)$; mosquitos(x)),

where C is assigned the set of individuals satisfying any contextually salient alternative to the predicate *mosquitos*, that is: $\{x : \exists P(P \in \text{ALT}(\text{mosquitos}) \wedge P(x))\}$. Then,

[12] Here, $[.]_f$ indicates focal stress.

according to Cohen's theory, (27) is true just in case the probability of being a mosquito, conditional on carrying WNV and satisfying some alternative property to being a mosquito (i.e., being some psychologically salient kind of insect), is greater than 0.5. Since the virus is primarily transmitted by mosquitos, (27) is predicted to be true, which is empirically adequate.

However, I want to stress that empirical questions about whether our judgements about these generics are veridical, or whether they involve an ambiguity or error, is entirely separable from Leslie's more theoretical argument that generics are not isomorphism invariant. Even if more than half of actual mosquitos carry WNV, we would still judge (27) to be true and (28) to be false. Consequently, I propose to abstract away from the question of how proponents of the quantificational approach to generics should handle striking property generics by stipulating that more than half of mosquitos carry WNV, while holding fixed Leslie's stipulation that mosquitos are the only things that carry WNV, and all and only those mosquitos that carry the virus are blind.

With this in mind, it is natural to suppose that the additional material Cohen's theory adds to the restrictor clauses of (27) and (28) results in them having the following denotations:

(31) a. $\{x : x \text{ are mosquitos}\} \cap \{x : \exists P(P \in \textsc{alt}(\text{carry.the.West.Nile.virus}(y)) \wedge P(x))\}$
b. $\{x : x \text{ are mosquitos}\} \cap \{x : \exists P(P \in \textsc{alt}(\text{are.blind.mosquitos}(y)) \wedge P(x)\}$

What are the psychologically salient alternatives to *carrying the West Nile virus* and *being a blind mosquito*? There are various ways to fill out this picture, but as far as I can see, proponents of the quantificational approach can appeal directly to Leslie's own remarks about positive and negative counterinstances to explain what is a psychologically salient alternative. In particular, consider the following remarks:

> Whether a counterinstance is positive or negative depends specifically on the generalization at hand—male birds are intolerable, positive counterinstances to "birds are female"', since they have the positive alternative property of *being male*, but are permissible, negative counterinstances to "birds lay eggs", as they simply fail to lay eggs, and do not have a positive alternative property in its place.…Now, the mosquitos that do not carry the M-virus do not possess a positive property that is an alternative to *carrying the M-virus*, but they *do* possess a positive alternative property to *being a mosquito that carries the M-virus* – namely, *being a mosquito that does not carry the M-virus*. This subtle shift turns merely negative counterinstances into positive ones, because they possess the positive, concrete property of *being a mosquito that is virus-free*. (Leslie, 2007, p. 400, ft. 7)

While Leslie is talking about the salient alternatives to the property of being a mosquito that carries the 'M-virus', her remarks apply equally to the property of being a blind mosquito. In other words, the salient alternative to being a *blind* mosquito is being a *non-blind* mosquito.

With this in mind, what does Cohen's theory predict about (27) and (28)? First, we can see that Cohen's theory correctly predicts that (27) is true, since the probability of carrying WNV, conditional on being a mosquito that has some alternative property to carrying WNV, is greater than 0.5. After all, in our imagined scenario, mosquitos are the only insects that carry WNV and more than 50% of them do.

Now, to see what Cohen's theory predicts about (28), we must make two observations. First, observe that, even though all and only those mosquitos that carry WNV are blind mosquitos, the restrictor of (28) picks out a very different set of individuals than that of (27), since being a non-blind mosquito is a salient alternative to being a blind mosquito, but not carrying WNV is *not* a salient alternative to carrying WNV. Consequently, the restrictor of (28) picks out the set of mosquitos that are either blind or not blind, that is, the set of mosquitos. Second, observe that, while the probability of being a blind mosquito, conditional on being either a blind or not blind mosquito, is still greater than 0.5, Cohen requires that the relevant probabilities must be homogeneous across every psychologically salient partition of the restrictor denotation. In particular, the relevant probabilities must be homogeneous across the set of blind mosquitos and the set of non-blind mosquitos. But since the probability of being a blind mosquito, conditional on being a non-blind mosquito, is trivially 0, Cohen's homogeneity constraint is not satisfied. Therefore, Cohen's theory does not predict that (28) is true. This accords with our intuitions.[13]

So far, we have built up a version of the quantificational approach to generics that makes the correct predications for Leslie's second example. All that remains is to show that, according to this approach, her example is not a counterexample to isomorphism invariance. The following toy model demonstrates this.

Let E be a simplified universe containing only four mosquitos, a, b, c, and d. Suppose a, b, c carry WNV, and hence are blind mosquitos. We may then represent this toy model of Leslie's example as in Fig. 3, where A is the property of carrying WNV, $\textsc{alt}(A)$ is the property of satisfying some alternative to A, B is the property of being a blind mosquito, and $\textsc{alt}(B)$ is the property of being either a blind or non-blind mosquito. It is easy to see that, even though $|A| = |B|$, since $|\textsc{alt}(A)| \neq |\textsc{alt}(B)|$, there is no bijection f from E to E, such that $Gen_E(\textsc{alt}(A); A)$, but not $Gen_E(f(\textsc{alt}(A)); f(A))$, or vice versa.

Futhermore, even if the above predicates were rigidified to preserve sameness of their intensions, as per (Leslie (2007), p. 394), the restrictors of the corresponding generics are not guaranteed to have identical cardinalities, since they would contain different ALT-sets. Consequently, we can explain why the sentence *Mosquitos in actuality carry the West Nile Virus* is true, but *Mosquitos in actuality are blind mosquitos* is false without rejecting the claim that generics are isomorphism invariant: the set of mosquitos in actuality that satisfy some alternative to carrying WNV differs in its cardinality from the set of mosquitos in actuality that satisfy some alternative to being a blind mosquito. The addition of different ALT-sets is enough to ensure that the cardinalities of the restrictors would be different.

[13] This manoeuvre also explains the sense in which *Mosquitos are mosquitos that are blind* seems universal in a way that *Mosquitos are blind* doesn't. As we can see in Fig. 3 below, the former sentence does express a generalization over all mosquitos, whereas the latter sentence doesn't. This manoeuvre also explains why such sentences are not enough to show that generics are not conservative: despite the seeming non-equivalence of *Mosquitos are blind* and *Mosquitos are mosquitos are not blind*, such sentences do not fit the schema of conservativity (i.e., $\mathrm{Gen}(A, B) \Leftrightarrow \mathrm{Gen}(A, A \cap B)$), since they differ in their restrictor.

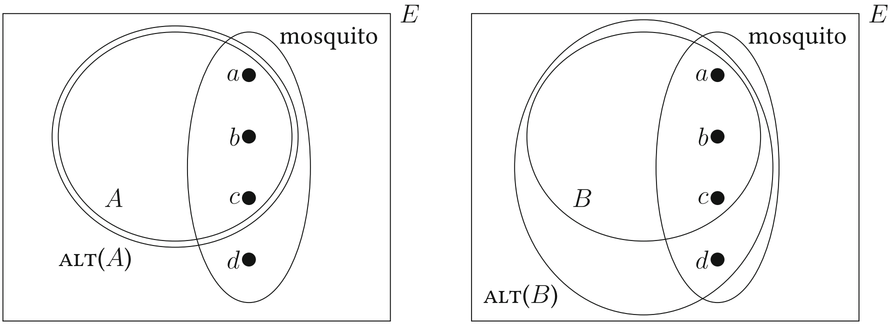


Fig. 3 Leslie's second example in full detail

4.4 Interim summary

To summarize, I have argued that proponents of the quantificational approach have extant resources to reject Leslie's examples as genuine counterexamples to the claim that generics are isomorphism invariant. While I have focused on Cohen's theory of generics for concreteness, the mechanisms by which sets of alternatives enter into the restrictor of generics are perfectly general and independently motivated. Consequently, proponents of other versions of the quantificational theory of generics can avail themselves of this general strategy and apply it to their favoured semantic analysis of *Gen*.

5 Variations on invariance

One restrictive feature of the kind of argument in Sect. 4 is that it only enjoys immediate application to the various sorts of generics with the same structure as those in Leslie's original examples. A general recipe for constructing Leslie-style examples is to take two generics that on the surface have the same subject term and two different predicates, but that differ in truth-value, even though the cardinalities of the subject terms and of the predicate denotations are the same. I argued such examples do not support the claim that generics aren't isomorphism invariant, since additional material may be added to the restrictor clause that is sensitive to the specific matrix clause, and so Leslie-style examples likely involve different restrictor clauses whose denotations differ in cardinality. But while this shows that Leslie's specific examples are ineffective for establishing that generics aren't isomorphism, it does not show *Gen in fact* denotes an isomorphism invariant quantifier.

Another restrictive feature of my argument is that it only immediately applies to theories of genericity that are structurally similar to the simplified version of Cohen's theory. But many implementations of the quantificational approach of genericity differ in important ways from this simplified theory. For example, some generics require *Gen* to bind multiple individual variables, such as in (32):

(32) a. A cat chases a mouse.

b. Gen x, y (x is a cat and y is a mouse and $C(x, y)$; x chases y)

The fact that *Gen* can bind multiple variables must be reflected in the strategy of accommodating additional restrictor material using a contextually specified variable. This is done by assigning to C the set of pairs of individuals satisfying some contextually salient alternative to the predicate *chases*, namely:

$$\{\langle x, y\rangle : \exists R(R \in \text{ALT}(\text{chases}(x, y)) \wedge R(x, y)\}$$

Then, on Cohen's simplified theory, (32) is true just in case the probability of an arbitrary pair $\langle x, y\rangle$ being such that x chases y, conditional on x being a cat, y being a mouse, and x and y being in some contextually salient alternative relation to x chasing y, is greater than 0.5. This is intuitively adequate. It is easy to see how the accommodation of additional restrictor material applies for generics involving greater numbers of individual variables.

A more pressing problem is that the informal cardinality-based version of isomorphism invariance explicitly discussed by Leslie is not immediately applicable to such examples, and so we cannot evaluate her argument with respect to generics that exhibit greater complexity than she allows. Issues compound when we consider intensional theories of generics according to which the generic operator *Gen* binds various sorts of variables over individuals, worlds, situations, or events, as in (33):

(33) a. Ravens are black.
b. Gen x, w (x is a raven in w and $C(x, w)$; x is black in w)

The problem is that Leslie's informal cardinality-based version of isomorphism invariance is not defined for intensional quantifiers, and so it is not immediately obvious how we can assess her argument against such versions of the quantificational approach.

This section resolves these problems by providing formally rigorous notions of isomorphism invariance for quantifiers in extensional and intensional structures. In turn, this allows us a setting general enough to explicitly define suitable denotations of *Gen* that are isomorphism invariant in the relevant ways.

5.1 First-order isomorphism invariance

Let us begin by generalizing Leslie's informal notion of isomorphism invariance in an extensional setting. To assess Leslie's argument against generics of greater complexity, we need to formulate a notion of isomorphism invariance for quantifiers of arbitrary type. We begin by introducing a first-order language and its corresponding models.

5.1.1 Basic first-order language

Let us begin with a basic first-order language $\mathcal{L} = \langle V, T(V), F(V)\rangle$ consisting of a set V of n-place relation symbols $P, Q, R, \ldots$, for each $n \geq 0$, a countably infinite set $T(V)$ of first-order variables $x, y, z, \ldots$, and the set of all formulas of vocabulary V, $F(V)$. For simplicity, we focus on a relational language without function symbols or constants; the only terms are the first-order variables. 0-place-predicate symbols correspond to propositional symbols. Formulas are defined inductively as follows:

$$\phi ::= P(x_1, \ldots, x_n) \mid x = y \mid \neg\phi \mid (\phi \wedge \phi) \mid \forall x \phi$$

where P is an n-place relation symbol, and $x_1, \ldots, x_n, x$, and y are variables. The notions of free and bound occurrences of variables are defined in the obvious way. Other propositional and first-order connectives are defined as usual.

We then define a model for $\mathcal{L}$ as follows:

Definition 5.1 A *model* for $\mathcal{L}$ is a tuple $\mathfrak{M} = \langle E, I \rangle$ consisting of:

(i) a non-empty set E;
(ii) a function I from each n-place relation symbol P of $\mathcal{L}$ to an n-place relation $I(P)$ on E.

Intuitively, E is *the domain of individuals*, and I is *the interpretation function* for non-logical vocabulary. As usual, we sloppily identity a set of one-tuples with the corresponding set of elements. There are two 0-ary relations $\varnothing$ and $\{\langle\rangle\}$; the first corresponds to falsity, the second to truth.

We shall sometimes write $\mathfrak{M} = \langle E, P_1^{\mathfrak{M}}, P_2^{\mathfrak{M}}, \ldots \rangle$ for a model for vocabulary $\{P_1, P_2, \ldots\}$. When it is clear that, say, vocabulary $\{P_1, P_2\}$ consists of 1-place relation symbols, we may simply write $\mathfrak{M} = \langle E, A, B \rangle$, where $A, B \subseteq E$; other variants are used.

Given a model $\mathfrak{M} = \langle E, I \rangle$, we say that an *assignment function* is a function that assigns to each variable an element of E. For any assignments g, g', $g' \overset{x}{\sim} g$ means that g' agrees with g on every variable except possibly x.

For each formula ϕ of $\mathcal{L}$ and assignment g to free variables of ϕ in $\mathfrak{M}$, we shall now define the conditions under which the relation $\mathfrak{M}, g \models \phi$ holds:

Definition 5.2 Given a model $\mathfrak{M}$ for $\mathcal{L}$, the relation $\mathfrak{M}, g \models \phi$ is defined by induction as follows:

$$\mathfrak{M}, g, \models P(x_1, \ldots, x_n) \text{ iff } \langle g(x_1), \ldots, g(x_n) \rangle \in I(P)$$
$$\mathfrak{M}, g \models x = y \text{ iff } g(x) = g(y)$$
$$\mathfrak{M}, g \models \neg\phi \text{ iff not } \mathfrak{M}, g \models \phi$$
$$\mathfrak{M}, g \models (\phi \wedge \psi) \text{ iff } \mathfrak{M}, g \models \phi \text{ and } \mathfrak{M}, g \models \psi$$
$$\mathfrak{M}, g \models \forall x\phi \text{ iff for any } g' \overset{x}{\sim} g, \mathfrak{M}, g' \models \phi$$

We say that a formula ϕ is *true in* $\mathfrak{M}$ *relative to an assignment* g if $\mathfrak{M}, g \models \phi$; *false in* $\mathfrak{M}$ *relative to an assignment* g, otherwise. And we say that a sentence ϕ is *true in* $\mathfrak{M}$ (in notation: $\mathfrak{M} \models \phi$) iff $\mathfrak{M}, g, \models \phi$ for every assignment g; *false in* $\mathfrak{M}$ otherwise.

5.1.2 Arbitrary generalized quantfiers

We shall now extend the basic first-order language with arbitrary generalized quantifiers. To do this, we first introduce the following notation:

Definition 5.3 For a formula $\phi = \phi(x_1, \ldots, x_n)$ consisting of at most the variables $x_1, \ldots, x_n$ free, let:

$$\phi(x_1, \ldots, x_n)^{\mathfrak{M},g} = \{\langle d_1, \ldots, d_n \rangle \in E^n : \mathfrak{M}, g' \models \phi(x_1, \ldots, x_n), \text{ for any } g' \overset{\overline{x}}{\sim} g\}$$

where $\overline{x} = x_1, \ldots, x_n$. Here $n \geq 1$; for $n = 0$ we stipulate:

$$\phi^{\mathfrak{M}} = \begin{cases} \{\langle\rangle\} & \text{if } \mathfrak{M} \models \phi \\ \varnothing & \text{otherwise} \end{cases}$$

Next, we define generalized quantifiers as follows.

Definition 5.4 A *type* is a finite sequence $\tau = \langle n_1, \ldots, n_k \rangle$ of positive integers ≥ 1.

A *generalized quantifier Q of type τ* associates with each universe E a k-ary second order relation Q_E over E, where the ith argument of Q_E is an n_i-ary relation on E. Let Q be a variable binding operator that denotes Q. Let the definition of a formula be extended to include $\mathrm{Q}\overline{x}_1, \ldots, \overline{x}_k(\phi_1; \ldots; \phi_k)$. Then the meaning of the quantified formula $\mathrm{Q}\overline{x}_1, \ldots, \overline{x}_k(\phi_1; \ldots; \phi_k)$, where each $\phi_i = \phi_i(x_{i_1}, \ldots, x_{i_{n_i}})$ is a formula with at most the distinct variables $\overline{x}_i = x_{i_1}, \ldots, x_{i_{n_i}}$ free, is given as follows:

$$\mathfrak{M}, g \models \mathrm{Q}\overline{x}_1, \ldots, \overline{x}_k(\phi_1; \ldots; \phi_k) \Leftrightarrow Q_E(\phi_1^{\mathfrak{M},g}; \ldots; \phi_k^{\mathfrak{M},g})$$

5.1.3 Isomorphism invariance

We shall now define the notion of isomorphic structure as follows:

Definition 5.5 If $\mathfrak{M} = \langle E, I \rangle$ and $\mathfrak{M}' = \langle E', I' \rangle$ are models for the same language $\mathcal{L}$, f is an *isomorphism* from $\mathfrak{M}$ to $\mathfrak{M}'$ iff:

(i) f is a bijection from E to E';
(ii) whenever P is an n-ary predicate symbol of $\mathcal{L}$ and $a_1, \ldots, a_n \in E$, $\langle a_1, \ldots, a_n \rangle \in I(P)$ iff $\langle f(a_1), \ldots, f(a_n) \rangle \in I'(P)$

We say that $\mathfrak{M}$ and $\mathfrak{M}'$ are *invariant under isomorphism*, or *isomorphic*, (in symbols: $\mathfrak{M} \cong \mathfrak{M}'$) iff there is an isomorphism from one model to the other.

The connection with the informal cardinality-based notion of isomorphism invariance for binary quantifiers binding a single variable in Sect. 3 can be stated as follows:

Fact 1 *If $\mathfrak{M} = \langle E, A, B \rangle$ and $\mathfrak{M}' = \langle E', A', B' \rangle$, then:*

$$\begin{aligned} \mathfrak{M} \cong \mathfrak{M}' \Leftrightarrow\ & |A - B| = |A' - B'|, |A \cap B| = |A' \cap B'|, |B - A| = |B' - A'|, \\ & \textit{and } |E - (A \cup B)| = |E' - (A' \cup B')| \end{aligned}$$

So, to say that a functor Q assigning, to each universe E, a binary relation Q_E between subsets of E is isomorphism invariant amounts to the condition that whenever $\mathfrak{M} \cong \mathfrak{M}'$, $Q_E(A; B) \Leftrightarrow Q_{E'}(A'; B')$.

We shall now state what isomorphism invariance amounts to for arbitrary quantifiers:

Definition 5.6 (Isomorphism Invariance for Arbitrary Quantifiers) ((Peters & Westerståhl, 2006), p. 99) A quantifier Q of type $\tau = \langle n_1, \ldots, n_k \rangle$ is isomorphism invariant iff whenever $\mathfrak{M} = \langle E, R_1, \ldots, R_k \rangle$ and $\mathfrak{M}' = \langle E', R'_1, \ldots, R'_k \rangle$ are isomorphic structures of type τ,

$$Q_E(R_1, \ldots, R_k) \Leftrightarrow Q_{E'}(R'_1, \ldots, R'_k)$$

Or, to put it slightly differently, Q is isomorphism invariant iff for every structure $\mathfrak{M} = \langle E, R_1, \ldots, R_k \rangle$ and every bijection f with domain E,

$$Q_E(R_1, \ldots, R_k) \Leftrightarrow Q_{f(E)}(f(R_1), \ldots, f(R_k))$$

where $f(R_i) = \{\langle f(a_1), \ldots, f(a_{n_i}) \rangle : \langle a_1, \ldots, a_{n_i} \rangle \in R_i\}$.

5.1.4 (Simplified) Cohen-semantics for generics

We are now in a position to say what it is for the generic quantifier expression *Gen* to express an isomorphism invariant generalized quantifier in an extensional setting. Restricting our attention to generics of the form $\ulcorner Gen(\phi; \psi) \urcorner$, where the restrictor clause ϕ and the matrix clause ψ denote arbitrary relations R_1, R_2 between individuals, the generic operator *Gen* expresses an isomorphism invariant quantifier Q of type $\langle n_1, n_2 \rangle$ just in case, for every structure $\mathfrak{M} = \langle E, R_1, R_2 \rangle$ and every bijection f with domain E:

$$Q_E(R_1; R_2) \Leftrightarrow Q_{f(E)}(f(R_1); f(R_2))$$

where $f(R_i) = \{\langle f(a_1), \ldots, f(a_{n_i}) \rangle : \langle a_1, \ldots, a_{n_i} \rangle \in R_i\}$.

In the simplified treatment of Cohen's theory in Sect. 4, we have essentially been treating *Gen* as expressing the strict majority quantifier *most*, as in (34).

(34) $most_E(A; B) \Leftrightarrow |A \cap B| > |A - B|$

To see that (34) is isomorphism invariant, suppose that there is a bijection f from E into E'. It is easy to see that $most_E(A; B)$ iff $most_{f(E)}(f(A); f(B))$, since $|A \cap B| = |f(A) \cap f(B)|$ and $|A - B| = |f(A) - f(B)|$.

We can now state the generalization of this idea as in (35):

(35) Let $\ulcorner Gen(\phi; \psi) \urcorner$ is a generic, such that ϕ, ψ denote the k-ary relations R_1, R_2, respectively. Then we say that $\ulcorner Gen(\phi; \psi) \urcorner$ is true (if defined) relative to a model $\mathfrak{M} = \langle E, R_1, R_2 \rangle$ just in case:

$$Res^k(most)_E(R_1; R_2) \Leftrightarrow |R_1 \cap R_2| > |R_1 - R_2|$$

where $Res^k(most)_E$ is the *resumption* of $most_E$ as defined as follows:

Definition 5.7 Let Q be monadic quantifier with n arguments, E a universe, and $R_1, \ldots, R_n \subseteq E^k$, for $k \geq 1$. We define the resumption operator as follows:

$$Res^k(Q)_E(R_1; \ldots; R_n) \Leftrightarrow Q_{E^k}(R_1; \ldots; R_n)$$

That is, $Res^k(Q)_E$ is just Q applied to the universe E^k, containing k-tuples.

It is easy to see that, since $most$ is isomorphism invariant, $Res^k(most)$ is as well. So the simplified version of Cohen's semantics is isomorphism invariant.

5.2 Modal isomorphism invariance

It is far from clear whether a first-order notion of isomorphism invariance as stated is sufficiently general to address the question of whether generics are quantificational. For many semantic analyses of generics treat *Gen* as an intensional quantifier expression that involves quantification over worlds or situations, as well as individuals. Consequently, we must formulate a notion of isomorphism invariance for such quantifiers. I begin by introducing basic first-order modal logic and its models. I then show how modal predicate logic can be linked to first-order predicate logic via the well-known standard translation. This allows us to see how extant intensional analyses of generics postulate isomorphism invariant quantifiers as the denotations of *Gen*.

5.2.1 Basic first-order modal logic

Let us begin by introducing the syntax of the basic first-order modal language. It is obtained by introducing a modal operator $\Box$ to the syntax of first-order predicate logic with identity previously introduced. Formulas are defined inductively as follows:

$$\phi ::= P(x_1, \ldots, x_n) \mid x = y \mid (\phi \wedge \phi) \mid \neg\phi \mid \Box\phi \mid \forall x\phi$$

where P is an n-place predicate symbol, $x_1, \ldots, x_n, x, y$ are variables. Other modal connectives are defined as usual.

We now define constant domain models for our language.[14]

Definition 5.8 A *constant domain model* is a tuple $\mathfrak{M} = \langle W, R, E, \{V_w\}_{w \in W}\rangle$, where

1. W is a non-empty set;
2. R is a binary relation on W;
3. E is a non-empty set;
4. for each w, V_w is a function that assigns to each n-place predicate symbol a subset of E^n.

A *pointed model* $\langle \mathfrak{M}, w\rangle$ is a pair consisting of a model $\mathfrak{M} = \langle W, R, E, \{V_w\}_{w \in W}\rangle$ and a point $w \in W$. It is common to omit the brackets and write $\mathfrak{M}, w$.

[14] For simplicity, we focus only constant domain semantics for first-order modal logic, since increasing and variable domain models introduce complications not relevant for present purposes.

Intuitively speaking, W is a set of *worlds*, R is an *accessibility relation*, E is *the domain of quantification*, and V_w is the *valuation at the world* w.

Given a constant domain model $\mathfrak{M} = \langle W, R, E, \{V_w\}_{w \in W} \rangle$, we say that an *assignment function* is a function that assigns to each variable an element of E. For any assignments g, g', $g' \overset{x}{\sim} g$ means that g' agrees with g on every variable except possibly x.

For each formula ϕ of the basic first-order modal language, assignment g to free variables of ϕ in $\mathfrak{M}$, and w in $\mathfrak{M}$, we now define the conditions under which the relation $\mathfrak{M}, g, w \models \phi$ holds:

Definition 5.9 Given a constant domain model $\mathfrak{M} = \langle W, R, E, \{V_w\}_{w \in W} \rangle$, the relation $\mathfrak{M}, g, w \models \phi$ is defined by induction as follows:

$$\mathfrak{M}, g, w \models P(x_1, \ldots, x_n) \text{ iff } \langle g(x_1), \ldots, g(x_n) \rangle \in V_w(P)$$
$$\mathfrak{M}, g, w \models x = y \text{ iff } g(x) = g(y)$$
$$\mathfrak{M}, g, w \models (\phi \wedge \psi) \text{ iff } \mathfrak{M}, g, w \models \phi \text{ and } \mathfrak{M}, g, w \models \psi$$
$$\mathfrak{M}, g, w \models \neg\phi \text{ iff not } \mathfrak{M}, g, w \models \phi$$
$$\mathfrak{M}, g, w \models \Box\phi \text{ iff for any } v \in W \text{ where } wRv, \mathfrak{M}, g, v \models \phi$$
$$\mathfrak{M}, g, w \models \forall x\phi \text{ iff for any } g' \overset{x}{\sim} g, \mathfrak{M}, g', v \models \phi$$

We say that a formula ϕ is a *true in* $\mathfrak{M}$ *at a world* w *relative to* g if $\mathfrak{M}, g, w \models \phi$; *false in* $\mathfrak{M}$ *at* w *relative to* g, otherwise.

5.2.2 Isomorphism invariance

We can now define the notion of isomorphism invariance between pointed constant domain models as follows:

Definition 5.10 Let $\langle \mathfrak{M}, w \rangle$ and $\langle \mathfrak{M}', w' \rangle$ be two pointed constant domain models, such that $\mathfrak{M} = \langle W, R, E, \{V_w\}_{w \in W} \rangle$ and $\mathfrak{M}' = \langle W', R', E', \{V'_w\}_{w \in W'} \rangle$ are constant domain models, and $w \in W$, $w' \in W'$. By a *strong homomorphism* of $\langle \mathfrak{M}, w \rangle$ into $\langle \mathfrak{M}', w' \rangle$, we mean a function f from W to W' and from E into E' such that $f(w) = w'$ with the following properties:

(i) For each predicate letter P, world $w \in W$, and sequence of objects $x_1, \ldots, x_n$, $\langle g(x_1), \ldots, g(x_n) \rangle \in V_w(P)$ iff $\langle f(g(x_1)), \ldots, f(g(x_n)) \rangle \in V'_{f(w)}(P)$.
(ii) For each $w, u \in W$, wRu iff $f(w)R'f(u)$.

An *isomorphism* is a bijective strong homomorphism. We say that $\langle \mathfrak{M}, w \rangle$ is *isomorphic* to $\langle \mathfrak{M}', w' \rangle$ (in symbols: $\langle \mathfrak{M}, w \rangle \cong \langle \mathfrak{M}', w' \rangle$) if there is an isomorphism from one to the other.

It is easy to see that if $\langle \mathfrak{M}, w \rangle \cong \langle \mathfrak{M}', w' \rangle$, then $\langle \mathfrak{M}, w \rangle$ and $\langle \mathfrak{M}', w' \rangle$ satisfy the same set of formulas via induction on the complexity of formulas.

5.2.3 Standard translation

The basic first-order modal language can be translated into a two-sorted first-order language with identity, where one sort is for individuals and the other sort is for worlds. This translation is truth-preserving for constant domain semantics. The two-sorted first-order language has a countably infinite set of first-order variables for worlds $a, b, c, \ldots$ and a countably infinite set of first-order variables for individuals x, y, z, disjoint from one another. Formulas of the language are defined inductively as follows:

$$\phi ::= P^*(a, x_1, \ldots, x_n) \mid R(a, b) \mid x = y \mid (\phi \wedge \phi) \mid \neg\phi \mid \forall a\phi \mid \forall x\phi$$

where for each n-place predicate P of the first-order modal language, there is a corresponding $(n+1)$-place predicate P^*. The $(n+1)$-place predicate is interpreted as relativising the corresponding n-place predicate to worlds. R is a designated predicate symbol used to interpret the accessibility relation.

We now give the standard translation. For a new first-order world variable a, the translation ST_a is defined by induction as follows:

$$\begin{aligned}
ST_a(P(x_1, \ldots, x_n)) &= P^*(a, x_1, \ldots, x_n) \\
ST_a(x = y) &= x = y \\
ST_a(\phi \wedge \psi) &= ST_a(\phi) \wedge ST_a(\psi) \\
ST_a(\neg\phi) &= \neg ST_a(\phi) \\
ST_a(\Box\phi) &= \forall b(R(a, b) \rightarrow ST_b(\phi)) \\
ST_a(\forall x\phi) &= \forall x ST_a(\phi)
\end{aligned}$$

To see that the translation is truth-preserving with respect to the constant domain semantics, we observe that a constant domain model for first-order modal logic can be considered as a model for two-sorted first-order logic and vice versa. First, given a constant domain model $\mathfrak{M} = \langle W, R, E, \{V_w\}_{w\in W}\rangle$ for first-order logic, let us define a model $\mathfrak{M}^* = \langle W, E, V^*\rangle$ for two-sorted first-order logic as follows:

i. $V^*(R) = R$
ii. $\langle w, d_1, \ldots, d_n\rangle \in V^*(P)$ iff $\langle d_1, \ldots, d_n\rangle \in V_w(P)$

It is straightforward to see that the map $(.)^*$ from $\mathfrak{M}$ to $\mathfrak{M}^*$ is bijective. Furthermore, if g is an assignment for first-order modal logic, then it can be extended to an assignment g^* that assigns a world to each first-order variable for worlds.

Next, given a model $\mathfrak{M}^*$ for two-sorted first-order logic, let us define the relation $\mathfrak{M}^*, g \models \phi$ by induction in the standard way. We say that ϕ is true if $\mathfrak{M}^*, g \models \phi$; false, otherwise.

We may now state that the translation is truth-preserving:

Proposition 5.1 *Let $\mathfrak{M}$ be a constant domain model. For any first-order modal formula ϕ and assignment g for $\mathfrak{M}$, $\mathfrak{M}, g, g^*(a) \models \phi$ iff $\mathfrak{M}^*, g^* \models ST_a(\phi)$, where g^* is any assignment extending g such that it assigns a world to each first-order variable for worlds.*

Proof By induction on complexity of ϕ.

Thus, we can see that first-order modal logic effectively becomes a fragment of the two-sorted first-order language. In turn, this makes it clear how pointed constant domain models can be used to interpret the array of intensional quantificational expressions in natural language surveyed in Sect. 2. This translation also allows us to directly show that first-order modal logic inherits a number of fundamental properties of full first-order logic.

5.2.4 Modal analyses of *Gen*

We are now in a position to check whether intensional analyses of *Gen* are isomorphism invariant. My aim here is not provide an exhaustive survey of every extant analysis of *Gen*, but rather to illustrate how to check whether a proposed semantics for *Gen* is isomorphism invariant. So, as a proof of concept, I focus on simplified toy theory based on (Krifka et al. (1995), p. 52).

The motivating idea behind this theory is that a generic of the form $\ulcorner$*Gen* $x_1, \ldots, x_n$ $(\phi; \psi)\urcorner$ is true relative to w just in case for every $x_1, \ldots, x_n$, and for every most normal world w' relative to w, if ϕ is true relative to w', then ψ is true relative to w', where $\phi = \phi(x_1, \ldots, x_n)$, $\psi = \psi(x_1, \ldots, x_n)$ consists of at most the variables $x_1, \ldots, x_n$ free.

To implement this idea in the present setting, we expand our basic first-order modal language with a generic operator GEN, with the following syntax:

$$\text{GEN}\ x_1, \ldots, x_n(\phi; \psi)$$

where ϕ and ψ are formulas with $x_1, \ldots, x_n$ free variables to be bound by GEN, and $y_1, \ldots, y_m$ free variables not bound by GEN.

Next, to capture the idea of 'every most normal world relative to w', we first define $R(w)$ as the set of accessible worlds relative to w using the accessibility relation R as follows:

$$R(w) = \{v \in W : wRv\}$$

Then we expand our constant domain models $\mathfrak{M} = \{W, R, D, \{V_w\}_{w \in W}\}$ to an extended constant domain model $\mathfrak{M}^+ = \{W, R, D, \{V_w\}_{w \in W}, \leq\}$, where $\leq$ is a ternary relation on W. Intuitively speaking, $w' \leq_w w''$ means that w' is at least as normal as w'' relative to w. We assume that $\leq_w$ is a *preorder* over W (i.e., reflexive, transitive).

Finally, we can extend the definition of satisfaction to our generic operator with the addition of the following clause:

(36) $\mathfrak{M}^+, g, w \models \text{GEN}\ x_1, \ldots, x_n(\phi; \psi)$ iff:

for any $g' \overset{\overline{x}}{\sim} g$, for all $w' \in R(w)$ such that $\mathfrak{M}^+, g', w' \models \phi$, there is a world $w'' \in R(w)$ such that $w'' \leq_w w'$, and for every $w''' \leq_w w''$, $\mathfrak{M}^+, g', w''' \models \psi$

where $\overline{x} = x_1, \ldots, x_n$.

To check whether our new generic operator is isomorphism invariant, we must first define the notion of isomorphic structures for our extended constant domain models.

Definition 5.11 Let $\langle \mathfrak{M}^+, w\rangle$ and $\langle \mathfrak{M}^{+'}, w'\rangle$ be extended pointed constant domain models, such that $\mathfrak{M}^+ = \langle W, R, E, \{V_w\}_{w\in W}, \leq\rangle$ and $\mathfrak{M}^{+'} = \langle W', R', E', \{V'_w\}_{w\in W'}, \leq'\rangle$ are extended constant domain models, and $w \in W$, $w' \in W'$. By a *strong homomorphism* of $\langle \mathfrak{M}^+, w\rangle$ into $\langle \mathfrak{M}^{+'}, w'\rangle$, we mean a function f from W to W' and E into E', such that $f(w) = w'$ with the following properties:

(i) For each predicate letter P, world $w \in W$, and sequence of objects $x_1, \ldots, x_n$, $\langle g(x_1), \ldots, g(x_n)\rangle \in V_w(P)$ iff $\langle f(g(x_1)), \ldots, f(g(x_n))\rangle \in V'_{f(w)}(P)$.
(ii) For each $w, u \in W$, wRu iff $f(w)R'f(u)$.
(iii) For each $w, u, v \in W$, $u \leq_w v$ iff $f(u) \leq'_{f(w)} f(v)$.

As before, an *isomorphism* is a bijective strong homomorphism. We say that $\langle \mathfrak{M}^+, w\rangle$ is *isomorphic* to $\langle \mathfrak{M}^{+'}, w'\rangle$ (in symbols: $\langle \mathfrak{M}^+, w'\rangle \cong \langle \mathfrak{M}^{+'}, w'\rangle$) if there is an isomorphism from $\langle \mathfrak{M}^+, w\rangle$ to $\langle \mathfrak{M}^{+'}, w'\rangle$.

Again, it is easy to see that isomorphic pointed extended constant domain models satisfy the same formulas, including $\textsc{Gen}\ x_1, \ldots, x_n\ (\phi; \psi)$.

Lastly, our extended first-order modal language can be translated into a two-sorted first-order language with identity by adding to the grammar of the latter the ternary predicate $a \leq_b c$, where a, b, c are variables over worlds, and extending the standard translation for $\textsc{Gen}\ x_1, \ldots, x_n(\phi; \psi)$ is given as follows:

(37) $ST_a(\textsc{Gen}\ x_1, \ldots, x_n(\phi; \psi)) =$
$\forall x_1, \ldots, x_n \forall b'(Rab' \wedge ST_{b'}(\phi) \rightarrow \exists b''(Rab'' \wedge b'' \leq_a b' \wedge \forall b'''(b''' \leq_a b'' \rightarrow ST_{b'''}\psi)))$

It should therefore be clear that this implementation of Krifka et al. (1995)'s modal analysis for *Gen* is both quantificational and isomorphism invariant.

More generally, it is straightforward to check whether other semantic analyses of *Gen* are isomorphism invariant by defining a suitable operator in the relevant language, constructing the appropriate models for its interpretation, defining a notion of isomorphism invariance between such models, and checking whether isomorphism invariant models entail the same generic sentences.

5.3 Summary

Many theorists have argued for intensional analyses of generics, and so appropriate notions of isomorphism invariance need to specified in order to evaluate whether such accounts predict generics to be isomorphism invariant. I have enriched Leslie's informal notion of isomorphism invariance by specifying suitable notions of invariance for a wide variety of quantificational theories of generics can be checked for isomorphism invariance. The result is positive: we can specify quantificational theories of generics that are isomorphism invariant.

6 Concluding remarks

This paper has argued that, despite initially compelling arguments to the contrary, generics are in fact isomorphism invariant. First, I argued that extant and independently-motivated resources allow proponents of the quantificational approach to account for examples that supposedly show generics are not isomorphism invariant. While this argument focuses on specific examples, the strategy generalizes: purported counterexamples that trade on pairs of generics differing in their predicates are unlikely to succeed, since there will also be predicate-induced variation in the restrictor clauses that undercut the force of these examples. Second, I formulated suitable notions of isomorphism invariance for different versions of the quantificational approach, and demonstrated that generics are isomorphism invariant on those theories. This is sufficient to show, as proof of concept, that many versions of the quantificational approach predict that generics are isomorphism invariant.

This argument has important implications for the recent debate between the quantificational approach to generics and the cognitive defaults view. Extant presentations of the cognitive defaults view rely heavily on a distinction in kind between generics and explicitly quantificational generalizations in order to underpin the apparent difference in their cognitive roles. In particular, it only makes sense to identify generic generalizations with System 1 judgements and explicitly quantificational generalizations with System 2 generalizations if a distinction in kind can be made between the respective generalizations. The essential linchpin of this distinction rests on the theoretical argument that generics are not isomorphism invariant. Consequently, the present reevaluation of this argument undercuts an essential piece of the cognitive defaults theory.

I should like to stress, however, that many of the important insights about generics revealed by the cognitive defaults approach can be reconciled with and adopted by the quantificational approach. For example, empirical evidence levied in favour of the competing cognitive defaults theory may well support the claim that some aspects of generic generalizations are specified or informed by default cognitive values and processes. Rather than taking this as evidence that generics express psychologically primitive generalizations distinct in kind from explicitly quantificational generalizations — an idea that I have been arguing against — we could instead interpret this as evidence that the domains over which generics generalize are partly specified or restricted by these default cognitive values and processes. This is perfectly compatible with quantificational theories of generics that are already committed to the idea that the domains of quantification are in part determined by features of the context (cf. Lazaridou-Chatzigoga et al., (2019b); 2023; 2024a). I hope the discussion in this paper encourages further work on how insights from the cognitive defaults approach may be incorporated into the quantificational approach.

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