Essays in Macroeconomics



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To all my teachers.

Statement of Originality

I certify that the intellectual content of this thesis is the product of my own work and that all assistance received in preparing this thesis and relevant sources have been acknowledged. This thesis has not been submitted for any other degree or purposes.

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Abstract

This thesis consists of three self-contained chapters.

In the first chapter, I present analytical expressions for fiscal multipliers under the Fiscal Theory of the Price Level. In the associated 'fiscal regime', taxation multipliers turn positive, while the government spending multiplier has the same functional form as its counterpart in the 'monetary regime', augmented by a nominal wealth effect. As a result, fiscal multipliers tend to be larger in the fiscal regime, with the degree of price stickiness being a key determinant of their exact sizes. I also analyze the effectiveness of money-financed fiscal stimulus. In the fiscal regime, money-financed stimulus is equivalent to a particular form of debt-financed stimulus. The effectiveness of money-financed stimulus in raising output (relative to inflation) decreases as monetary policy becomes more responsive to inflation.

In the second chapter, I analytically study the effects on economic activity from expected changes to future fiscal variables. I document four transmission mechanisms, the sizes of which depend crucially on the level of nominal rigidities. In general, news shocks to fiscal policy produce larger effects when the policy rate is unresponsive to inflation and when the fiscal authority has the power to issue unbacked nominal debt. Within this framework, I show that 'forward guidance' of fiscal policy can expand economic activity without ever increasing the real value of government debt.

In the third chapter, I use the falling relative price of investment to explain the growing divergence between gross and net asset positions in the U.S. A lower investment price leads shadow banks to increase their borrowing and lending (i.e. gross

balance sheet) to profit from better investment returns. Although more investment increases the capital stock, a lower investment price also lowers the replacement cost and, by extension, the per-unit market value of capital. Net asset positions remain, as a result, relatively unchanged. I present a stylized model that accounts for these facts.

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Introduction

This thesis consists of two parts.

Part I includes two self-contained chapters, in which I study the impact of fiscal policy under the Fiscal Theory of the Price Level ('FTPL', 'fiscal theory' or 'fiscal regime'). The fiscal theory has attracted much attention over recent years. Drastic changes in central bank operations have raised questions about previously standard approaches to explaining the determination of the price level. Above all, why has inflation remained stable at near-zero interest rates (for almost a decade in the euro area, the U.S., and the U.K., and for more than two decades in Japan), despite an unprecedented growth in interest-paying reserves? The fiscal theory has emerged as a common answer that is consistent with recent events.

In Chapter 1 (published in the *European Economic Review*; see Beck-Friis and Willems, 2017), we derive and present analytical expressions for fiscal multipliers on output and inflation under the FTPL. In the associated fiscal regime, taxation multipliers turn positive, while the government spending multiplier has the same functional form as its counterpart in the 'monetary regime', augmented by a nominal wealth effect. As a result, fiscal multipliers tend to be larger in the fiscal regime,

with the degree of price stickiness being a key determinant of their exact sizes. In particular, multipliers go to infinity as prices become fully fixed, and collapse to their equivalents in the monetary regime as prices become fully flexible. Standard calibrations suggest that the fiscal regime hosts a government spending multiplier on output that lies between 1 and 3. We also analyze the effectiveness of money-financed fiscal stimulus. In the fiscal regime, money-financed stimulus is equivalent to a particular form of debt-financed stimulus. The effectiveness of money-financed stimulus in raising output (relative to inflation) decreases as monetary policy becomes more responsive to inflation.

In Chapter 2, I analytically study the effects on economic activity from expected changes to future fiscal variables, in both the monetary and fiscal regime. I document four transmission mechanisms, the sizes of which depend crucially on the level of nominal rigidities. In general, news shocks to fiscal policy produce larger effects when the policy rate is unresponsive to inflation and when the fiscal authority has the power to issue unbacked nominal debt. Within this framework, I show that 'forward guidance' of fiscal policy can expand economic activity without ever increasing the real value of government debt.

The model that underpins the analysis in Chapter 1 and 2 is set in a simple New-Keynesian environment without capital and distortionary taxes. I present the linearized model in the body of the chapters, but I relegate the full description of the model, derivation and solution to Appendices 2.A - 2.C.

Part II consists of Chapter 3, in which I use the falling relative price of investment to explain the growing divergence between gross and net asset positions in the U.S. A

lower relative price of investment leads shadow banks to increase their borrowing and lending (i.e. gross balance sheet) to profit from better investment returns. Although more investment increases the capital stock, a lower relative price of investment also lowers the replacement cost and, by extension, the per-unit market value of capital. Net asset positions remain, as a result, relatively unchanged. This result suggests that the large disparity between gross and net positions is here to stay, and will continue to grow as long as the relative price of investment continues to fall. Large gross positions are a result of technological advances and better investment opportunities, rather than of, as commonly argued, lenient credit regulation, financial deepening, or excessive risk taking in the financial system. I present a stylized model that accounts for these facts.

Part I

CHAPTER 1

Dissecting Fiscal Multipliers under the Fiscal Theory of the Price Level

Chapter Abstract

We derive analytical expressions for fiscal multipliers on output and inflation under the Fiscal Theory of the Price Level. In the associated 'fiscal regime', taxation multipliers turn positive, while the government spending multiplier has the same functional form as its counterpart in the 'monetary regime', augmented by a nominal wealth effect. As a result, fiscal multipliers tend to be larger in the fiscal regime, with the degree of price stickiness being a key determinant of their exact sizes. In particular, multipliers go to infinity as prices become fully fixed, and collapse to their equivalents in the monetary regime as prices become fully flexible. Standard calibrations suggest that the fiscal regime hosts a government spending multiplier on output that lies between 1 and 3. We also analyze the effectiveness of money-

financed fiscal stimulus. In the fiscal regime, money-financed stimulus is equivalent to a particular form of debt-financed stimulus. The effectiveness of money-financed stimulus in raising output (relative to inflation) decreases as monetary policy becomes more responsive to inflation.

1.1 Introduction

In this paper, we derive analytical expressions for fiscal multipliers on output and inflation under the Fiscal Theory of the Price Level ('FTPL') and the associated 'fiscal regime'. We show that these are closely linked to their equivalents in the more-standard 'monetary regime', but that fiscal policy is typically more potent in the fiscal regime.

In an environment where governments issue nominal debt, the monetary and fiscal regime differ in their treatment of the government budget equation (which equates the real value of government debt to the present-discounted value of all future real primary surpluses). While the monetary regime treats this equation as a constraint (implying that policy needs to adjust when the present-discounted value of future surpluses differs from the real value of debt), the fiscal regime views it as an equilibrium condition.[1] In the latter case, there is 'fiscal dominance' and a shock to the current primary surplus does not have to lead to changes in future primary surpluses. Instead, the price level can adjust to restore equality. Leeper (1991) has dubbed the underlying, autonomous form of fiscal policy 'active', while the associated mechanism has led Woodford to characterize the FTPL as 'non-Ricardian': since an increase in government spending under the FTPL is not automatically associated with higher future primary surpluses, agents perceive the government bonds that are placed into their hands as net wealth. As a result, they demand more consumption

[1] See Begg and Hague (1984), Auernheimer and Contreras (1990), Leeper (1991), Sims (1994), Woodford (1995) and Cochrane (1998; 2005) for early contributions to this theory. Canzoneri et al. (2011) and Leeper and Leith (2016) offer overviews of the associated literature.

goods and the price level rises.

The FTPL seems to have particular relevance for developing countries (to the extent that they issue domestic currency debt[2]): they often lack the fiscal capacity to collect the necessary real tax revenues (giving rise to an 'active' fiscal authority; cf. Baldini and Poplawski-Ribeiro, 2008), while concerns for capital flows imply that monetary policy tends to be 'passive'.[3] Others have moreover argued that the FTPL has applied at times in the U.S.,[4] with Cochrane (2011, 2014) forcefully making the case that the FTPL provides a useful framework to think about the U.S. economy post-financial crisis.

Although there is a large literature analyzing fiscal multipliers in the monetary regime,[5] there is no such focused analysis for fiscal multipliers in the fiscal regime. Fiscal multipliers play a big role in FTPL-based papers like Kim (2003), Davig and Leeper (2011), and Leeper, Traum, and Walker (2016), but these papers typically have a different focus and report only a limited set of analytical results. Bhattarai,

[2]For a sample of 18 emerging markets, Ottonello and Perez (2016) document that about one-third of all debt is issued in local currency. It ranges from as low as 0% in Bulgaria, to as high as 68% in Thailand.

[3]Central banks in developing economies are often reluctant to increase the interest rate when the economy is booming, as that would attract further capital inflows—throwing oil on fire. The reverse worry implies that they are less keen to cut the interest rate when the economy is in recession. Federico, Vegh and Vuletin (2014) document the passivity of monetary policy for developing economies empirically.

[4]See Davig and Leeper (2006), Bianchi (2012), Bianchi and Ilut (2014) and Chen, Leeper and Leith (2015).

[5]Cf. Aiyagari, Christiano and Eichenbaum (1992) and Baxter and King (1992), both of which analyze the size of the multiplier in a neoclassical model; Christiano, Eichenbaum and Rebelo (2011), who look at the size of the multiplier when nominal interest rates are at the zero lower bound; Woodford (2011), who derives analytical expressions for the multiplier in various incarnations of the New Keynesian model; Galí (2014), who compares multipliers depending on whether the underlying government spending is money-financed or debt-financed; and Rendahl (2015), who shows that the existence of equilibrium unemployment can increase the size of the government spending multiplier.

Lee, and Park (2014) obtain a broad set of analytical results for the fiscal regime, but only focus on monetary policy.

With the present paper, we fill this gap by obtaining analytical expressions for fiscal multipliers under the FTPL. We uncover that fiscal multipliers in the monetary and fiscal regime are intimately linked. In particular, the functional form of the government spending multiplier in the fiscal regime is equal to the government spending multiplier in the monetary regime plus the taxation multiplier in the fiscal regime, minus a 'fiscal inflation tax'. This additive relationship arises because debt-financed government spending in the fiscal regime produces the same real effects as in the monetary one, but also adds a nominal wealth effect, resulting from unbacked bonds being placed in the private sector.

The analytical expressions reveal three novel facts. First, the degree of price stickiness is key in determining the size of the nominal wealth effect and hence the fiscal multipliers. In particular, the multipliers on output in the fiscal regime explode to infinity as prices become fully fixed, and collapse to their counterparts in the monetary regime as prices become fully flexible. Standard calibrations suggest that the fiscal regime hosts a government spending multiplier on output that lies between 1 and 3. In the monetary regime, the standard three-equation New Keynesian model only attributes a marginal role to price stickiness in determining the size of fiscal multipliers.

Second, in contrast to the monetary regime, multipliers in the fiscal regime depend on the government's debt-to-output ratio. In particular, more indebted countries have smaller multipliers, in line with the empirical evidence documented by Leeper,

Traum, and Walker (2015) and Huidrom et al. (2016). The literature has suggested two transmission channels to account for this empirical relationship: first, a Ricardian channel, whereby households expect fiscal stimulus by more indebted governments to be offset by tax increases sooner; and second, an interest rate channel, whereby fiscal stimulus by more indebted governments increases sovereign credit risk, thereby crowding out private consumption through higher interest rates. In this paper, we add a third channel, operating through the nominal wealth of the private sector (similar to Leeper, Traum, and Walker, 2015): in the fiscal regime, bonds issued by a more-indebted government represent a smaller nominal wealth increase (relative to the size of the existing debt stock), thereby leading to a smaller nominal wealth effect of fiscal stimulus on output and inflation.

Third, as the fiscal regime adds a wealth stock to the model, fiscal multipliers start displaying persistence, even when the underlying shock is purely transitory. This result is consistent with empirical evidence presented in Leeper, Traum, and Walker (2015), who find large persistence in their estimates of the government spending multiplier in the fiscal regime. In the monetary regime, the standard three-equation New Keynesian model displays no persistence in multipliers unless the underlying shock itself is persistent.

Finally, we use our model to analyze the effects of money-financed fiscal stimulus in the fiscal regime. Our analysis shows that the effects of money-financed stimulus can be replicated by debt-financed stimulus if the fiscal authority is willing to take an 'active' stance, with the monetary authority accepting a 'passive' role. Our results indicate that the effectiveness of money-financed stimulus in raising output relative

to inflation decreases as monetary policy becomes more responsive to inflation.

1.2 Model

To 'dissect' the multipliers, we start by considering a standard New Keynesian model. It is set in a discrete-time, infinite horizon environment, lacks capital, but allows for non-zero, wasteful government spending. The representative household has per-period preferences that are of the CRRA-form:

$$U(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\xi}}{1+\xi},$$

where c_t is consumption, n_t is labor supply, σ is the coefficient of relative risk aversion and ξ is the inverse of the elasticity of labor supply. Firms set prices according to the Calvo-mechanism, which allows them to reset their price in each period with probability $(1-\theta)$. Throughout this paper, variables without a time-subscript denote steady-state values, variables with a hat indicate log-deviations from this steady state (so $\hat{x}_t = (x_t - x)/x$), and variables with a tilde express steady-state deviations as a fraction of steady-state output (i.e. $\tilde{z}_t = (z_t - z)/y$).

As the model is well-documented, we relegate a complete description of the model

and its solution to Appendices 2.A - 2.C and proceed by presenting its linear form:

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} + \tilde{g}_t - \mathbb{E}_t \tilde{g}_{t+1} - \left(\frac{1 - g/y}{\sigma}\right)[\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}], \tag{1.1}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa \left[\left(\xi + \frac{\sigma}{1 - g/y}\right)\hat{y}_t - \left(\frac{\sigma}{1 - g/y}\right)\tilde{g}_t\right], \tag{1.2}$$

$$\hat{i}_t = \phi \hat{\pi}_t, \tag{1.3}$$

$$\tilde{\tau}_t = \psi \tilde{b}_{t-1} + \varepsilon_t^{\tau}, \tag{1.4}$$

$$\tilde{g}_t = \rho \tilde{g}_{t-1} + \varepsilon_t^{g}, \tag{1.5}$$

$$\beta \tilde{b}_t = \tilde{b}_{t-1} - \tilde{s}_t - \left(\frac{b}{y}\right)\hat{\pi}_t + \left(\frac{b}{y}\right)\beta \hat{i}_t. \tag{1.6}$$

Here, y_t refers to output, g_t to real government spending, i_t to the nominal interest rate, π_t to inflation, τ_t to real lump-sum taxation, b_t to real government debt, and s_t to the real primary surplus $(\tau_t - g_t)$. The variables ε_t^{τ} and ε_t^{g} are mean zero i.i.d taxation and government spending shocks, respectively. $\beta \in [0, 1)$ is the household's subjective discount factor. $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta > 0$ is the standard Calvo-measure of price stickiness (with $\kappa \to \infty$ as prices become fully flexible).

Equations (1.1) and (1.2) are the standard Euler equation and the New Keynesian Phillips Curve (accounting for the impact of government spending), respectively. Equation (1.3) describes the monetary policy rule, according to which the central bank reacts to current-period inflation with strength parameter $\phi \geq 0$. Equation (1.4) captures the fiscal policy rule as in Leeper (1991), according to which the fiscal authority adjusts lump-sum taxes to the deviation of lagged real debt with strength parameter $\psi \geq 0$. Equation (1.5) describes the evolution of government spending,

with the parameter $\rho < 1$ governing its persistence. Finally, equation (1.6) is the government's flow budget identity. The government issues nominal debt and collects nominal taxes to finance its nominal spending and interest cost on its existing debt stock.

We follow Leeper (1991) in calling monetary policy 'passive' when $\phi < 1$ and 'active' when $\phi > 1$. Similarly, we call fiscal policy 'active' when $\psi < 1 - \beta$ and 'passive' when $\psi \in (1 - \beta, 1 + \beta]$.

As is well-established in the literature, the system is overdetermined when both monetary and fiscal policy are active. In that case, there are no bounded solutions. When both policies are passive, the system is indeterminate and there is an infinite number of bounded solutions. Only when one of the policies is active and the other one is passive is there a unique, bounded solution for inflation, output and real debt.

We refer to the case with an active monetary and passive fiscal policy as the 'monetary regime', M. Similarly, we refer to the case with an active fiscal and passive monetary policy as the 'fiscal regime', F.

1.3 Fiscal multipliers

In this section, we derive expressions for the taxation multiplier ('TM') and the government spending multiplier ('GSM'). We calculate multipliers at horizon $j \geq 0$

for inflation (π), output (y) and real debt (b) as follows:[6]

$$
\begin{aligned}
TM^{y}(j) &\equiv -\frac{\partial y_{t+j}}{\partial \tau_t} = -\frac{\partial \hat{y}_{t+j}}{\partial \varepsilon_t^{\tau}}, \\
TM^{\pi}(j) &\equiv -\frac{\partial \pi_{t+j}}{\partial \tilde{\tau}_t} = -\frac{\partial \hat{\pi}_{t+j}}{\partial \varepsilon_t^{\tau}}, \\
TM^{b}(j) &\equiv -\frac{\partial b_{t+j}}{\partial \tau_t} = -\frac{\partial \tilde{b}_{t+j}}{\partial \varepsilon_t^{\tau}}, \\
GSM^{y}(j) &\equiv \frac{\partial y_{t+j}}{\partial g_t} = \frac{\partial \hat{y}_{t+j}}{\partial \varepsilon_t^{g}}, \\
GSM^{\pi}(j) &\equiv \frac{\partial \pi_{t+j}}{\partial \tilde{g}_t} = \frac{\partial \hat{\pi}_{t+j}}{\partial \varepsilon_t^{g}},
\end{aligned}
$$

In the remainder of this section we derive expressions for these objects under both the monetary and the fiscal regime.

1.3.1 Monetary regime

The monetary regime, characterized by active monetary policy ($\phi > 1$) and passive fiscal policy ($\psi \in (1-\beta, 1+\beta]$), is the conventional regime that has been the focus of many earlier analyses (versions of all results in this sub-section can for example be found in Woodford, 2011). It is well-established that in this regime taxation

[6] We define taxation multipliers with a minus-sign up front, so that a positive multiplier implies that a tax cut boosts output and inflation. We also define the multipliers on inflation as the inflationary response to a fiscal shock as a fraction of steady state output. This ensures that the multipliers on inflation do not depend on the model's steady state level of output.

multipliers on output and inflation are zero at all horizons $j \geq 0$:

$$TM_M^y(j) = 0, \tag{1.7}$$

$$TM_M^\pi(j) = 0. \tag{1.8}$$

The reason lies in the well-known Ricardian Equivalence theorem (Barro, 1974). As the government adjusts taxes aggressively to deviations of lagged real debt, the newly-issued government bonds used to finance the tax cut will eventually be soaked up by higher real taxes. As a result, the household does not perceive the bonds as wealth and does not modify its consumption behavior in response. Output, inflation, and the real interest rate (defined as $i_t - \mathbb{E}_t\hat{\pi}_{t+1}$) remain unaffected in response to changes in taxation.

The real value of government debt initially increases, but converges back to its steady state over time through higher real taxes:

$$TM_M^b(j) = \Big(\frac{1-\psi}{\beta}\Big)^j \frac{1}{\beta}. \tag{1.9}$$

When $\psi \in (1-\beta, 1]$, real debt remains (weakly) above its steady state level throughout the episode as the government increases real taxes less than one-for-one with the deviation of lagged real debt. When $\psi \in (1, 1+\beta)$, debt oscillates around its steady state as real taxes adjust by more than one-for-one with the deviation of lagged real debt. In both cases, both the real *and* nominal value of government debt eventually return to their pre-tax-cut levels (as there is no change in the price level; cf. equation (1.8)).

Government spending multipliers in the monetary regime are positive for both output and inflation:[7]

$$GSM_M^y(j) = \rho^j \times \frac{(1-\rho)(1-\beta\rho) + \kappa(\phi-\rho)}{(1-\rho)(1-\beta\rho) + \kappa(\phi-\rho)\{1+\sigma^{-1}\xi[1-(g/y)]\}} \leq 1, \quad (1.10)$$

$$GSM_M^\pi(j) = \rho^j \times \frac{(1-\rho)\kappa\xi}{(1-\rho)(1-\beta\rho) + \kappa(\phi-\rho)\{1+\sigma^{-1}\xi[1-(g/y)]\}}. \quad (1.11)$$

Expressions (1.10)-(1.11) are positive for two reasons. The first reason lies in the household's optimality condition for labor (equation (2.27) in Appendix 2.A.1), which equates the marginal rate of substitution between labor and real consumption to the real wage. Since government spending is wasteful, increasing g_t effectively amounts to destroying some real output. If hours remained constant, goods market clearing implies that the household would consume less, which in turn would increase its marginal utility of consumption. At the given real wage, the household would then choose to work more to finance further consumption, up until its marginal rate of substitution between labor and consumption is brought back to equal the real wage. This is equivalent to an outward shift in labor supply: for any given real wage, the household chooses to supply more hours. Barro and King (1984) refer to this as the 'wealth effect on labor supply'.

Next to this outward shift in labor supply, labor demand increases too. As long as there is some price stickiness, firms cannot fully raise prices in response to the higher demand. Instead, they lower their mark-up and increase production, thereby raising labor demand for any given real wage. If prices are fully flexible, firms merely

[7]The corresponding multiplier for debt is a complicated expression that does not yield any additional insights and is omitted as a result.

adjust their prices, leaving labor demand unchanged.

Four facts emerge from the analytical expressions. First, the stronger the central bank's response to inflation (i.e. the higher ϕ), the smaller is the size of the demand increase following increased government spending, and thus the smaller the impact of government spending on output and inflation. The impact on the real interest rate, $\partial\left(i_{t+j} - \mathbb{E}_t\hat{\pi}_{t+j+1}\right)/\partial\varepsilon_t^g = (\phi - \rho) \times GSM_M^{\pi}(j)$, is unambiguously positive at all times (as $\phi > 1$ while $\rho < 1$). Through this channel, the central bank cools down the economy after a fiscal stimulus.

Second, as long as taxation responds sufficiently strongly to lagged real government debt (i.e. as long as $\psi > 1 - \beta$), the household does not perceive government liabilities as wealth and multipliers are independent of ψ.

Third, note that ρ, the parameter governing the persistence of government spending (cf. equation (1.5)), also governs the persistence in multipliers. When $\rho = 0$, multipliers are only positive on impact; for any period $j > 0$, they are zero. So without any exogenous persistence in government spending, there are no dynamic effects in the monetary regime. As we show in Section 1.3.2, this result no longer holds true in the fiscal regime.

Fourth, note that the multipliers are an increasing function of the steady state ratio of government spending to output (g/y). Under CRRA-preferences, a higher steady state ratio of government spending implies a smaller steady state level of private consumption, and thus a higher steady state marginal utility of consumption. Any additional increase in government spending therefore increases the supply of labor by more for larger governments, as the disutility from lower private consump-

tion (and thus the change in the marginal rate of substitution between labor and consumption) is larger.

Crucially, however, the multipliers are independent of the steady state ratio of government debt to output (b/y).[8] Consequently, in the monetary regime, more indebted economies display the same magnitude of fiscal multipliers as less indebted ones. As we highlight in the next section, this result does not translate to the fiscal regime.

1.3.2 Fiscal regime

Let us now turn to the fiscal regime, which is the focus of this paper. In the fiscal regime, monetary policy is passive ($\phi < 1$) while fiscal policy is active ($\psi < 1 - \beta$), reflecting a situation of fiscal dominance.

To understand some of the results that are to follow, one should keep in mind that the fiscal regime has unconventional implications for monetary policy: in this regime, raising the interest rate *increases* inflation. This is the so-called neo-Fisherian effect, discussed at length in Cochrane (2016), and forms a stark contrast with the monetary regime. The reason is that, in the fiscal regime, a higher interest rate increases the return the household receives on its unbacked government bond holdings, without changing current and future real primary surpluses. The household feels wealthier as a result and increases demand, thereby creating inflation (Kim, 2003; Bhattarai, Lee, and Park, 2014). As we illustrate in Section 1.4, the inflation multiplier in the fiscal regime inherits this effect, leading to markedly different comparative statics in

[8]Incorporating distortionary taxation may change this result.

the two regimes.

Turning to the actual multipliers, we have that:

Result 1.1. *In the fiscal regime, taxation multipliers on output and inflation are positive:*[9]

$$TM_F^y(j) = \lambda^j \times \left(\frac{y}{b}\right) \times \frac{(1-\beta\lambda)(1-\beta\lambda-\psi)}{\kappa\left(\xi + \frac{\sigma}{1-g/y}\right)(1-\beta\phi)(1-\psi)} > 0, \tag{1.12}$$

$$TM_F^\pi(j) = \lambda^j \times \left(\frac{y}{b}\right) \times \frac{(1-\beta\lambda-\psi)}{(1-\beta\phi)(1-\psi)} > 0, \tag{1.13}$$

where

$$\lambda = \frac{1}{2\beta}\Big[1+\beta+\kappa\left(\xi\frac{1-g/y}{\sigma}+1\right) - \sqrt{\left(1+\beta+\kappa\left(\xi\frac{1-g/y}{\sigma}+1\right)\right)^2 - 4\beta\left(1+\phi\kappa\left(\xi\frac{1-g/y}{\sigma}+1\right)\right)}\Big] \in (0,1).$$

Taxation multipliers are positive (implying that a tax cut boosts output and inflation) due to the break-down of Ricardian Equivalence associated with the FTPL. In this regime, a tax cut is not associated with sufficiently large future real taxes to repay the debt. So the household perceives the newly-issued government bonds as an increase to its nominal wealth, leading to increased demand and giving rise to positive taxation multipliers on output and inflation. Ricardian Equivalence breaks down not because the household is irrational or myopic, but rather because the government is never expected to adjust real primary surpluses in response to the newly-issued debt.

[9]Since $\lambda < 1$ and $\psi < 1-\beta$ in the fiscal regime, we have that $1-\beta\lambda-\psi > 1-\beta-(1-\beta) = 0$. This result ensures positivity of both equation (1.12) and (1.13).

A tax cut in the fiscal regime is a *nominal* wealth effect, because the newly-issued bonds are a *nominal* asset for the household. The labor supply curve remains intact, independent of preferences. So output expands only if real labor demand increases, i.e. only if firms are unable to adjust prices to meet the extra nominal demand for their goods. As a result, as prices become fully flexible ($\kappa \to \infty$), firms merely adjust their prices in response to this extra demand, leaving output unchanged ($TM_F^y \to 0$). On the other hand, as prices become fully fixed ($\kappa \to 0$), firms have to produce and sell an infinite amount of goods to satisfy the higher nominal demand ($TM_F^y \to \infty$).[10] We thus conclude that:

Result 1.2. *The degree of price stickiness is key in determining the size of the taxation multiplier on output.*

Equations (1.12)-(1.13) reveal two additional results. First, in contrast to the monetary regime, the multipliers in the fiscal regime depend on the steady state ratio of government debt to output. In particular:

Result 1.3. *Taxation multipliers on output and inflation are smaller for more indebted governments.*

The more debt outstanding, the smaller the relative size of the newly-issued government bonds (as a proportion of the household's existing bond holdings), and thus the smaller the nominal wealth effect of the tax cut on output and inflation.

In addition, the analytical expressions show that:

Result 1.4. *Even though there is no persistence in the underlying tax shock, its*

[10]When prices are fully fixed, nominal debt becomes real debt and an equilibrium with active fiscal policy no longer exists.

effect on output and inflation continues to be positive after the impact period.

The reason is that the fiscal regime hosts a wealth stock in the form of government bonds. A tax cut, albeit transitory, represents a permanent increase to this wealth, and thus a permanent increase in nominal demand. With sticky prices, firms are only able to adjust their prices gradually to meet the permanently higher nominal demand, until real private wealth eventually returns to its steady state level. This result contrasts with the monetary regime, where a transitory increase in government spending only represents a temporary increase in nominal demand.

The factor determining the persistence in output and inflation, λ, is governed by the monetary authority's response of the interest rate to inflation, ϕ. Consider the flexible price limit ($\kappa \to \infty$) where $\lambda \to \phi$. In this limit, output is not affected by tax shocks, as firms merely adjust their prices to offset the change in nominal demand. Yet inflation continues to display persistence, by a factor of ϕ, as higher (lower) nominal interest rates call for higher (lower) expected inflation via the neo-Fisherian effect.

Just like in the monetary regime, the real value of government debt initially increases, but converges back to its steady state over time:

$$TM_F^b(j) = \lambda^j \times \Big(\frac{1}{1-\psi}\Big). \tag{1.14}$$

As $\lambda \in (0,1)$ does not depend on the fiscal strength parameter ψ, real debt converges back to its steady state level even if the government leaves future real taxes unchanged, i.e. if $\psi = 0$. The government instead remains fiscally solvent through

higher *nominal* tax revenues. Put differently, the deficit is financed by higher future inflation, not by future tax hikes or spending cuts. As a result, and in contrast to the monetary regime (cf. equation (1.9), nominal debt eventually settles at a permanently higher level.

It is also interesting to analyze the behavior of the real interest rate in response to changes in taxation. In the monetary regime, taxation has no impact on the real rate. In the fiscal regime, on the other hand, the real interest rate is affected by tax shocks:

Result 1.5. *When prices are sticky, the real interest rate falls (when $\phi < 1$) or remains unaffected (when $\phi = 1$) following a tax shock. That is: $\partial \left(i_{t+j} - \mathbb{E}_t \hat{\pi}_{t+j+1} \right) / \partial \varepsilon_t^{\tau} = (\phi - \lambda) \times TM_M^{\pi}(j) \leq 0$.*[11]

This result shows that the knife-edge case where $\phi = 1$ is indeed a special one: in that case, tax shocks have the exact same effect on the real interest rate in both the monetary and fiscal regime (namely a zero response).

[11] To see this, note that $\partial\lambda/\partial\phi \in (0, 1]$. As $\lambda(0) > 0$ and $\lambda(1) = 1$ this implies that $\phi - \lambda(\phi) \leq 0$ for all $\phi \in [0, 1]$.

Government spending multipliers in the fiscal regime are given by:

$$GSM_F^y(j) = \rho^j \times \frac{(1-\rho)(1-\rho\beta)+\kappa(\phi-\rho)}{(1-\rho)(1-\rho\beta)+\kappa\left(\xi\frac{1-g/y}{\sigma}+1\right)(\phi-\rho)} - \frac{\lambda^j(1-\beta\lambda)[1-\beta\lambda-\psi]}{\kappa\left(\xi+\frac{\sigma}{1-g/y}\right)(1-\beta\phi)[1-\psi-\beta\rho]}\times \left[\frac{(1-\rho)\kappa\xi}{(1-\beta\rho)(1-\rho)+\kappa\left(\xi\frac{1-g/y}{\sigma}+1\right)(\phi-\rho)} - \frac{y}{b}\frac{1}{(1-\beta\phi)}\right], \tag{1.15}$$

$$GSM_F^\pi(j) = \rho^j \times \frac{(1-\rho)\kappa\xi}{(1-\beta\rho)(1-\rho)+\kappa\left(\xi\frac{1-g/y}{\sigma}+1\right)(\phi-\rho)} - \frac{\lambda^j[1-\beta\lambda-\psi]}{1-\psi-\beta\rho}\times \left[\frac{(1-\rho)\kappa\xi}{(1-\beta\rho)(1-\rho)+\kappa\left(\xi\frac{1-g/y}{\sigma}+1\right)(\phi-\rho)} - \frac{y}{b}\frac{1}{(1-\beta\phi)}\right]. \tag{1.16}$$

These expressions for government spending multipliers in the fiscal regime may not seem insightful, but they are closely linked to their equivalents in the monetary regime.

To see this, first consider the case with no persistence in government spending, i.e. $\rho = 0$. Rewriting expressions (1.15)-(1.16) then brings us to the following conclusion:

Result 1.6. *Fiscal multipliers in the monetary and fiscal regime are related in their functional form. In particular:*

$$GSM_F^y(j) = \underbrace{GSM_M^y(j)}_{\text{Keynesian effect}} + \underbrace{TM_F^y(j) - \frac{b}{y}(1-\beta\phi)GSM_M^\pi(0)\times TM_F^y(j)}_{\text{Nominal wealth effect}}, \tag{1.17}$$

$$GSM_F^\pi(j) = \underbrace{GSM_M^\pi(j)}_{\text{Keynesian effect}} + \underbrace{TM_F^\pi(j) - \frac{b}{y}(1-\beta\phi)GSM_M^\pi(0)\times TM_F^\pi(j)}_{\text{Nominal wealth effect}}. \tag{1.18}$$

The government spending multiplier in the fiscal regime has the same functional form as the government spending multiplier in the monetary regime, plus a nominal wealth effect.[12] The first term is standard. Government spending in the fiscal regime still brings about labor supply and labor demand effects as in the monetary regime. Real labor demand shifts out due to firms being unable to fully adjust prices to meet the extra nominal demand. Real labor supply shifts out to equate the marginal rate of substitution between labor and consumption to the real wage. We refer to these effects as 'Keynesian'.

On top of these, higher government spending in the fiscal regime also increases the nominal wealth of the private sector, since it places unbacked government bonds in the hands of the household. The effect of this operation is equivalent to that of a debt-financed tax cut in the fiscal regime, which explains the presence of the second term, $TM_F^{\{y,\pi\}}(j)$, in expressions (1.17)-(1.18).

The third term represents an interaction effect. It features because government spending in the fiscal regime still brings about 'Keynsian inflation' (cf. equation (1.11)). The interaction term comes in two parts, one which is contractionary and one which is expansionary. First, inflation depresses the real value of the outstanding government bonds. In this respect, inflation acts like a tax on the household's wealth and thus lowers demand. Second, inflation increases the nominal interest rate (provided that $\phi > 0$), which increases demand through the neo-Fisherian effect. The net impact of the Keynesian inflation on the household's wealth, $-(b/y)(1-\beta\phi)GSM_M^{\pi} < 0$,

[12]The relationship between multipliers in the fiscal and monetary regime is robust to using any type of $U(c,n)$-preferences. It might break down when taxation is distortionary.

is then equivalent to that of a tax hike of the same size.[13] This is a 'fiscal inflation tax' and can be seen as the FTPL-equivalent to Phelps' seigniorage-driven inflation tax under the monetary regime (Phelps, 1973).

Due to this fiscal inflation tax, the nominal size of the wealth increase brought about by a tax cut is larger than the nominal size of the wealth increase brought about by an increase in government spending. As a result, $GSM_F^{\{y,\pi\}}$ is smaller than the sum of $GSM_M^{\{y,\pi\}}$ and $TM_F^{\{y,\pi\}}$. The total 'nominal wealth effect' – deriving its name from the fact that this term goes to zero (infinity) as prices become fully flexible (fixed) – at time j is equal to $[1-(b/y)(1-\beta\phi)GSM_M^{\pi}(0)] \times TM_F^{\{y,\pi\}}(j)$.

A few results are worth commenting upon. First, as spending multipliers are functionally related to taxation multipliers, several of our previous results carry over:

Result 1.7. *Government spending in the fiscal regime creates dynamic effects even when the driving process is purely transitory, i.e. when* $\rho = 0$.

By placing unbacked bonds in private hands, a transitory increase in government spending creates a permanent increase in nominal demand. This forms a stark contrast with the monetary regime, where multipliers are only positive on impact (and collapse to zero afterwards; unless government spending itself is persistent, $\rho > 0$) as transitory government spending only creates a temporary increase in nominal demand.

Second, the contribution of the nominal wealth effect also implies that:

Result 1.8. *The government spending multiplier on output can exceed unity in*

[13] Another way to see where the coefficient $(b/y)(1-\beta\phi)$ comes from, is to substitute the monetary and fiscal policy rules (1.3) and (1.4) into the government's flow budget identity (1.6). This yields $\partial\tilde{b}_t/\partial\hat{\pi}_t = (b/y)(1-\beta\phi)\partial\tilde{b}_t/\partial\varepsilon_t^{\tau}$.

the fiscal regime, even when preferences are separable between labor and consumption.

This forms an important contrast with the monetary regime, where government spending crowds out private consumption (recall equation (1.10)). In the fiscal regime, the nominal wealth effect goes to infinity (zero) as prices become fully fixed (flexible). As a result, the spending multiplier on output in the fiscal regime explodes to infinity as $\kappa \to 0$ (the fixed-price limit).

Third, from substituting the taxation multipliers (1.12)-(1.13) into the government spending multipliers (1.17)-(1.18) it follows that:

Result 1.9. *The government spending multiplier on output is smaller for more indebted governments.*

This result is in line with the empirical evidence presented in Leeper, Traum, and Walker (2015) and Huidrom et al. (2016). As with taxation multipliers, the more debt outstanding, the smaller is the relative size of the newly-issued government bonds.

To compare the sizes of the multipliers between the two regimes, consider first the limit as $\phi \to 1$. Arguably, this limit is the only point where a fair comparison between the two regimes is possible, as it is the point where the monetary ($\phi > 1$) and the fiscal regime ($\phi < 1$) meet.[14] In this limit, the fiscal inflation tax becomes negligibly small, as $(b/y)(1-\beta) \approx 0$ for reasonable debt-to-output ratios. The net contribution of the nominal wealth effect to the government spending multiplier is therefore positive and multipliers in the fiscal regime are larger than their equivalents

[14]When we move further into the respective regimes, any resulting differences are also attributable to the differing inflation-responses of the monetary authority.

in the monetary regime for all reasonable calibrations.

As ϕ decreases in the fiscal regime, the Keynesian effects strengthen (it is possible to show that $\partial GSM_M^{\{y,\pi\}}(j)/\partial\phi < 0$), while taxation multipliers continue to be positive. However, due to the differing behavior of the central bank, this does not imply that fiscal multipliers are unambiguously larger in the fiscal regime. To see this, recall that a lower ϕ also increases the fiscal inflation tax, via two effects. First, higher Keynesian inflation further depresses the real value of the outstanding debt, and second, a smaller interest rate hike by the central bank produces a smaller expansionary neo-Fisherian effect. For a sufficiently small ϕ, the nominal wealth effect can become negative, i.e. $[1-(b/y)(1-\beta\phi)GSM_M^{\pi}(0)] < 0$. This occurs when the fiscal inflation tax more-than-offsets the positive wealth effect that the newly-issued unbacked bonds represent. In this case, with $\rho = 0$, fiscal multipliers in the monetary regime exceed their equivalents in the fiscal regime for any period $j \in \{1, 2, 3, ...\}$.

Turning to the case of persistent government spending ($\rho > 0$), equations (1.17)-(1.18) generalize to:

$$GSM_F^y(j) = GSM_M^y(j) + \underbrace{\sum_{i=0}^{\infty}(\beta')^i\left[\rho^i - \frac{b}{y}(1-\beta\phi)\,GSM_M^{\pi}(i)\right]}_{\text{Time-0 PDV of Nominal Wealth Increase}} \times TM_F^y(j), \quad (1.19)$$

$$GSM_F^{\pi}(j) = GSM_M^{\pi}(j) + \underbrace{\sum_{i=0}^{\infty}(\beta')^i\left[\rho^i - \frac{b}{y}(1-\beta\phi)\,GSM_M^{\pi}(i)\right]}_{\text{Time-0 PDV of Nominal Wealth Increase}} \times TM_F^{\pi}(j), \quad (1.20)$$

with $\beta' \equiv \frac{\beta}{1-\psi}$.

With persistence in government spending, the total nominal wealth increase is

the sum of all discounted future nominal wealth increases. On impact, the household anticipates that higher future spending implies issuance of more government debt in period i by a factor of ρ^i, which it views as wealth in the fiscal regime. It moreover anticipates the inflation tax in period i to be $(b/y)(1-\beta\phi) \times GSM_M^{\pi}(i)$. The household discounts all nominal wealth increases into present-value terms at time zero.

1.4 Numerical results for the New Keynesian model

To quantify the sizes of the various effects at play, we calibrate the standard New Keynesian model at a quarterly frequency and use the model to analyze government spending multipliers. Most of the baseline calibration choices are uncontroversial. We set the discount factor (β) to 0.99; the coefficient of relative risk aversion (σ) to 1; the inverse elasticity of labor supply (ξ) to 1; and the steady state fraction of government spending to output (g/y) to 0.3. Changing the values of any of these parameters brings only marginal changes to the quantitative results.

Four parameters play a crucial role in determining the size of the nominal wealth effect: the degree of price flexibility (θ), the degree of persistence in government spending (ρ), the steady state ratio of government debt to output (b/y), and the strength parameter of the monetary authority (ϕ). In our baseline calibration, we set θ to 0.75 (implying an average duration of prices of four quarters), ρ to 0.5 (following Galí (2014)), and (b/y) to 1. We keep $\phi = 1$, thereby considering the point where the monetary and fiscal regime meet.[15]

[15]This is a knife-edge point, only of theoretical interest. As ϕ decreases to values smaller than

Figures 1.1 and 1.2 depict the size of the (impact) government spending multipliers on output and (annualized) inflation, respectively, under varying degrees of fiscal policy strength ψ. For low values of ψ, the newly-issued government debt generates a positive nominal wealth effect. The larger ψ, the smaller the effect on both output and inflation (since a larger ψ is associated with larger future real tax increases). Once ψ becomes larger than $1-\beta$ (at $\psi = 0.01$) the economy switches from the fiscal to the monetary regime. At that point, the nominal wealth effect disappears and the strength of the taxation rule parameter becomes redundant: for all $\psi > 1-\beta$, the household does not perceive government debt as net wealth.

Under the baseline calibration of θ, (b/y) and ρ, the nominal wealth effect on output is small, with most of the wealth increase translating into inflation instead. Consider a 1% increase in government spending as a fraction of steady state output. With $\psi = 0$, the nominal wealth effect then only generates a 0.1% increase in output, but an increase in annual inflation that lies close to 8%.

1, output multipliers rise, while inflation multipliers fall (giving rise to a more favorable trade-off ratio) – cf. Figures 1.3 and 1.4, as well as Section .

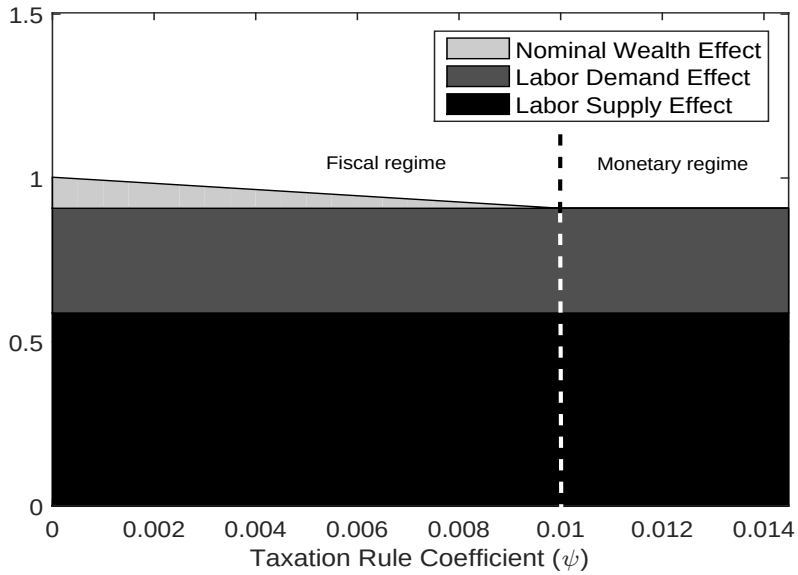


Figure 1.1: $GSM_F^y(0)$ as a function of ψ (for $\phi = 1$) in the New Keynesian Model

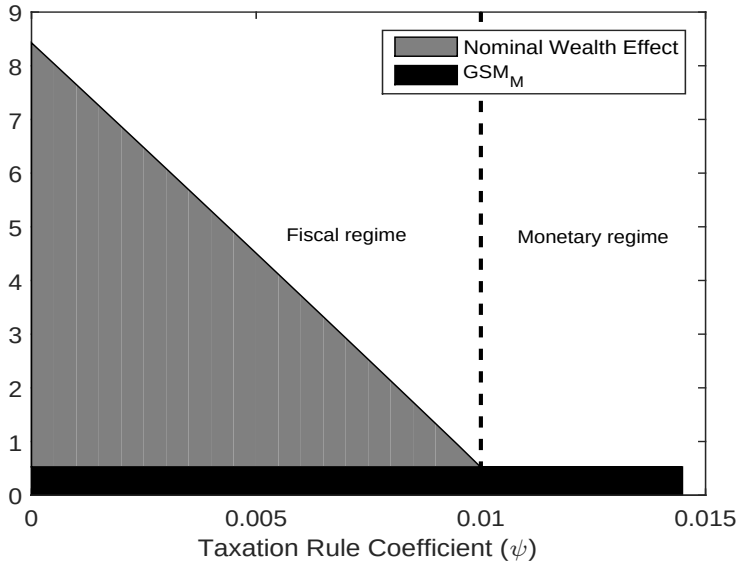


Figure 1.2: $GSM_F^{\pi}(0)$ as a function of ψ (for $\phi = 1$) in the New Keynesian Model

Figure 1.3 plots the size of the (impact) government spending multiplier on output, varying the Calvo probability θ. From this point onward, we fix ψ at 0.005 which ensures an active fiscal policy. At low degrees of price stickiness, the nominal wealth effect is tiny and the difference between multipliers in the two regimes is small. A large number of firms are then able to adjust their prices in response to the increase in nominal demand, leaving real demand for labor rather intact. But as the degree of price stickiness increases, more firms are forced to lower their markup and increase production. Consequently, the nominal wealth effect gains strength. In the limit where $\theta \to 1$, prices become completely fixed, and the goods market would fail to clear. At that stage, any increase in nominal wealth would translate into an infinitely large output multiplier. In the monetary regime, the output multiplier is increasing in θ as well, but remains less than one for *all* degrees of price stickiness. Figure 1.3 also illustrates the sensitivity with respect to ϕ, by displaying results for $\phi = 0.5$ and $\phi = 10$. As the figure shows, the sensitivity is higher when ϕ goes below one, relative to when ϕ exceeds one. Generally, the government spending multiplier on output is larger when ϕ is lower. The reason is that a central bank with higher ϕ is more willing to sacrifice output in order to fight inflation.

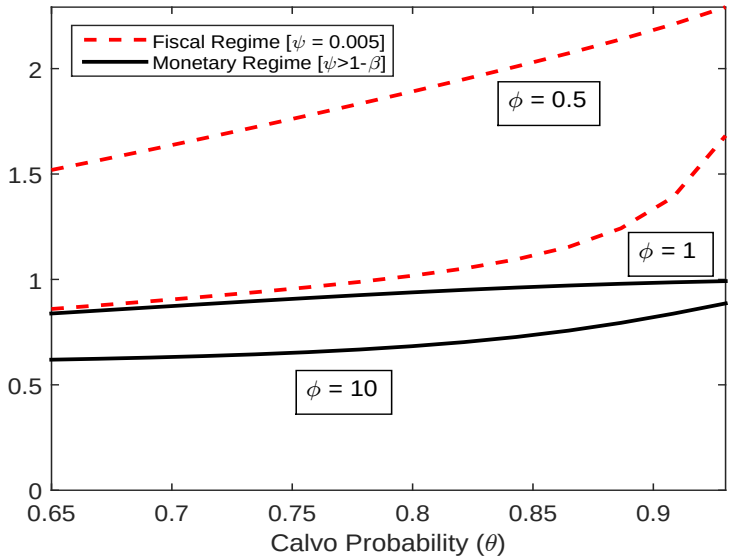


Figure 1.3: $GSM_F^y(0)$ as a function of θ in the New Keynesian Model

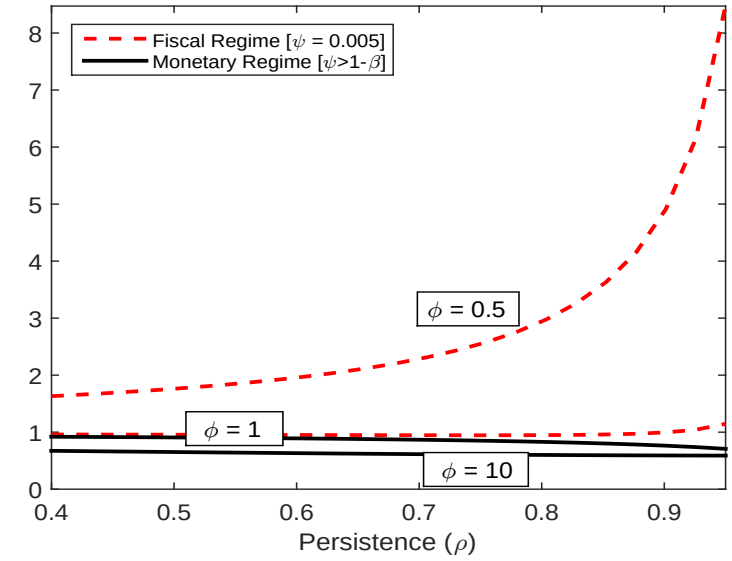


Figure 1.4: $GSM_F^y(0)$ as a function of ρ in the New Keynesian Model

Figure 1.4 plots the size of the (impact) government spending multiplier on output, varying the degree of persistence in government spending. The larger ρ, the larger the size of the present discounted value of the future nominal wealth increase, and the larger the output multiplier on impact. Note, in contrast, that the government spending multiplier in the monetary regime is a *decreasing* function of level of persistence.[16] The reason lies in the active stance of monetary policy: because a more persistent increase in government spending creates more inflation upfront (due to the forward-looking behavior of price setters) it is met by an interest rate hike, which depresses output. In the fiscal regime, monetary policy is passive and the positive relation between ρ and the size of the nominal wealth effect dominates. This implies that higher persistence translates into larger output multipliers for all reasonable calibrations.[17]

Figures 1.5 depicts the size of the (impact) government spending multiplier on output, varying the steady state ratio of government debt to output (b/y). In the monetary regime, the multiplier is independent of the level of indebtedness. In the fiscal regime, it is not, with a larger indebtedness translating into a smaller nominal wealth effect, as the newly-issued bonds represent a smaller proportion of the existing debt stock. As the debt-to-output ratio approaches zero, the multiplier explodes to infinity; as the ratio approaches infinity, the nominal wealth effect goes to zero. These results hold for the multiplier on inflation, too.

[16]Differentiating (1.16) tells us that $sgn(\partial GSM^{y}_{M,CRRA}(0)/\partial\rho) = sgn([(2\rho-1)\beta-1-\beta\rho^2/\phi]\phi+1)$. One can show that this expression is negative over the entire domain of the monetary regime ($\phi > 1$) , which establishes that $\partial GSM^{y}_{M,CRRA}(0)/\partial\rho < 0$.

[17]The negative relationship can still arise, but only as $\theta \to 0$ (i.e. for extremely high levels of price flexibility, at which stage the nominal wealth effect gets eroded).

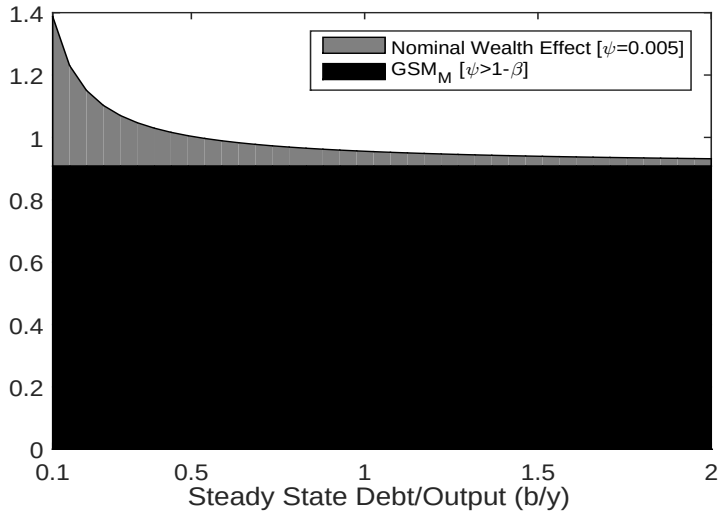


Figure 1.5: $GSM_F^y(0)$ as a function of b/y in the New Keynesian Model

Lastly, Figures 1.6 and 1.7 show the *dynamic* responses of output and (annualized) inflation under the baseline calibration, setting $\phi = 0.5$ in the fiscal regime and $\phi = 1.5$ in the monetary regime.[18] Given that persistence in government spending ρ is small (0.5), the multipliers in the monetary regimes weaken rapidly as time passes. The multipliers in the fiscal regime, however, show a large degree of persistence as the fiscal regime adds a wealth stock to the model.

[18]For $\phi \to 1$, the eigenvalue $\lambda \to 1$. As that happens, the nominal wealth effect ceases to decrease over time and the figure would no longer show any decay.

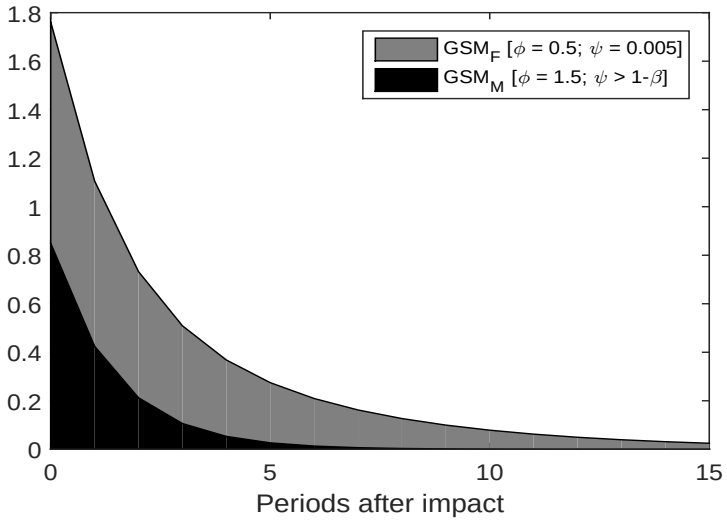


Figure 1.6: Dynamic representation of $GSM_F^y(j)$ in the New Keynesian Model

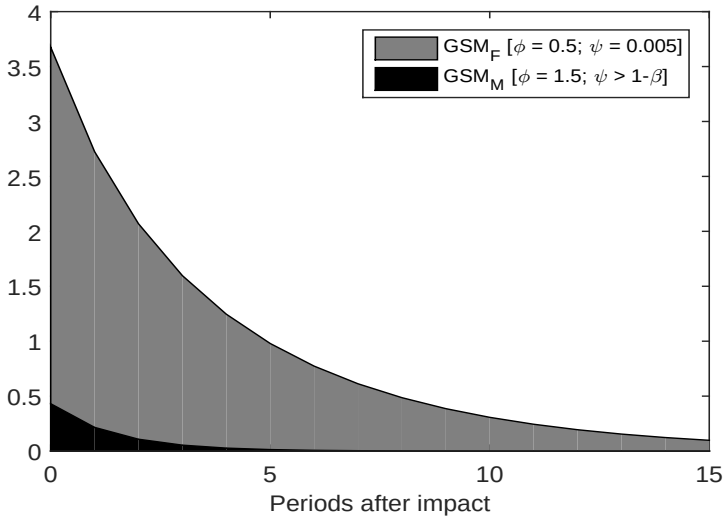


Figure 1.7: Dynamic representation of $GSM_F^\pi(j)$ in the New Keynesian Model

Another way to the capture the dynamic effects in the two regimes is to set $\rho = 0$ and consider the dynamic multiplier on output, as defined by:

$$\text{Dynamic Multiplier } (k) \ = \frac{\partial \mathbb{E}_t \sum_{j=1}^{k+1} \left[\prod_{i=1}^{j-1} (1 + \hat{r}_{t+i})^{-1} \right] \hat{y}_{t+j-1}}{\partial \varepsilon_t^g},$$

where $\hat{r}_{t+i}$ is the real interest rate at time $t+i$ (in log-deviations from steady state). This expression measures the additional output produced, in present-value terms, over a k-period horizon from a one-off exogenous change in government spending. When $k = 0$, the dynamic multiplier is equal to the impact multiplier.

In the monetary regime, the dynamic multiplier is always equal to the impact multiplier as there is no dynamic effect on output without any underlying persistence in government spending (see equation (1.10)). Under the baseline calibration with $\phi = 1.5$ and $\rho = 0$, both the dynamic and static multiplier equal 0.93.

In the fiscal regime (now having $\phi = 0.5$), the impulse-response function displays persistence and the dynamic multiplier grows bigger as we consider longer time-horizons k. Eventually (at $k \approx 20$), the dynamic multiplier settles at a value of around 2.85 (while the original impact multiplier is only 1.37).

Summarizing, the calibrations considered in this section suggest that the impact government spending multiplier on output lies somewhere in between 1 and 3 for the standard New Keynesian model in the fiscal regime (with the output multiplier being larger when prices are sticker, spending is more persistent, the government is less indebted, and monetary policy is less active). The same calibration suggests that the government spending multiplier in the monetary regime lies somewhere in

between 0.8 and 1. When taking a dynamic perspective, these differences are even more pronounced as multipliers in the fiscal regime display endogenous persistence.

1.5 Numerical results for the Smets-Wouters model

Our analysis has so far focused on the effects in a simple New Keynesian model. While this analysis produces insightful closed-form solutions, one is left wondering to what extent our results generalize to more involved models and what such models would suggest as to the size of the various multipliers and underlying effects.

We therefore extend our analysis to the DSGE-model of Smets and Wouters (2007). That model includes capital and features a rich set of frictions, including: wage and price stickiness, wage and price indexation, habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. We refer to their paper for a complete description of the model.

Crucially, we modify the Smets-Wouters model in two ways. First, we change the reaction function of the monetary authority to match the simple inflation-feedback rule described in (1.3). Second, we include a fiscal block, adding the tax policy rule (1.4) alongside the flow budget identity (1.6). Numerical analysis confirms that the monetary and fiscal regimes then follow from the same determinacy regions on ϕ and ψ as in Section 1.2.

The higher complexity of the Smets-Wouters model makes it impossible to derive closed-form solutions. We resort to a numerical analysis instead, which suggests that the relationship between the fiscal multipliers under the monetary and fiscal

regime (captured in equations (1.19) and (1.20)) generalizes to the Smets-Wouters environment.[19] This result is not surprising: the intuition underpinning the relationship between the multipliers in the two regimes is independent of the structure of the underlying model. While the size of the nominal wealth effect depends on the characteristics of the model, its existence is merely a function of the regime in which the model solution lies.

For our numerical exercise we calibrate the model using the posterior mode estimates in Smets and Wouters (2007). To maintain comparability with Section 1.4, we stick to Galí's (2014) calibration of $\rho = 0.5$ and maintain $\phi = 1$ (to ensure that any differences between the monetary and fiscal regime are not due to different responses by the central bank).[20] Figures 1.8-1.14 correspond to our Figures 1.1-1.7, but now for the Smets-Wouters model. From these figures it emerges that the Smets-Wouters government spending multiplier on output tends to be bigger than in the baseline New Keynesian model, while the multiplier on inflation is smaller (Figures 1.8 and 1.9). But differences are small and most calibrations still suggest a government spending multiplier on output in the fiscal regime that lies in between 1 and 3 on impact (with the same calibrations suggesting that the multiplier lies in between 1 and 1.5 in the monetary regime). Apart from these quantitative differences, the underlying qualitative story remains intact. First, the output response continues to be crucially reliant upon the level of nominal rigidities, with stickier prices translating

[19] As the monetary regime ceases to exist when $\phi < 1$, one cannot obtain numerical values for $GSM_M^{\{y,\pi\}}(j)$ when $\phi < 1$. As a result, we can only show that the relationship holds in the limit as $\phi \to 1$, where multipliers can be evaluated numerically in both regimes.

[20] Smets and Wouters (2007) estimate $\rho = 0.97$. When using that value for ρ in our calibration, results are similar but suggest that the government spending multiplier on output is somewhat higher (in between 2 and 4).

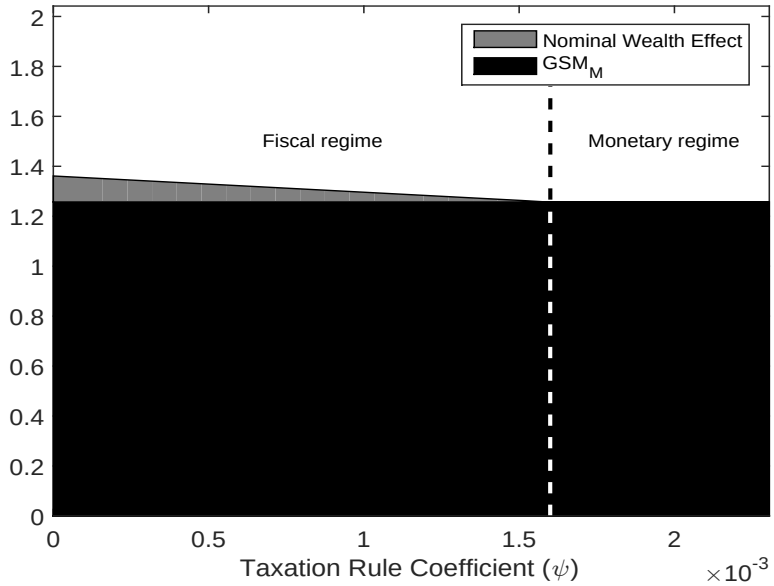


Figure 1.8: $GSM_F^y(0)$ as a function of ψ (for $\phi = 1$) in the Smets-Wouters Model

into a larger output-effect (Figure 1.10). As the Calvo price (or wage) probability approaches unity, the government spending multiplier in the fiscal regime explodes to infinity (just as in the New Keynesian model; cf. Section 1.4). Second, adding more persistence in government spending generates a larger nominal wealth effect in the fiscal regime, leading to larger fiscal multipliers (Figure 1.11). Third, the government spending multiplier on output is a decreasing function of the government's level of indebtedness (Figure 1.12), again in line with the baseline New Keynesian model.

Finally, Figures 1.13 and 1.14 show the dynamic responses of output and (annualized) inflation for the Smets-Wouters model. In this case, the introduction of frictions (habit formation in particular) renders the impulse-respones hump-shaped, but as in the standard New Keynesian model, multipliers in the fiscal regime are more persistent than their monetary equivalents.

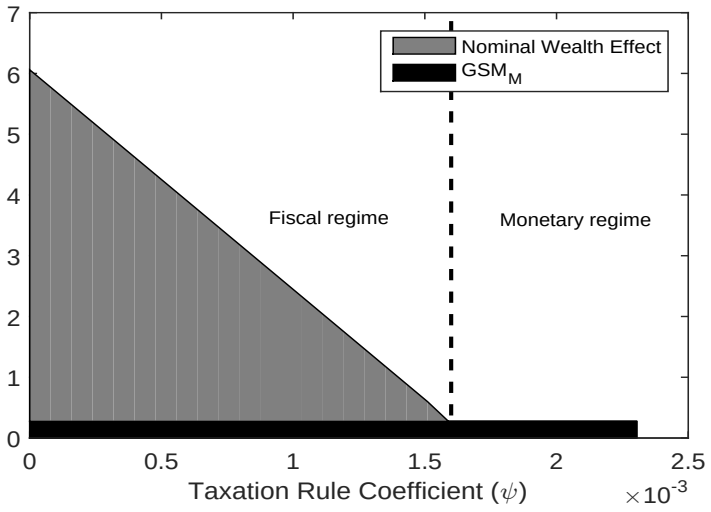


Figure 1.9: $GSM_F^{\pi}(0)$ as a function of ψ (for $\phi = 1$) in the Smets-Wouters Model

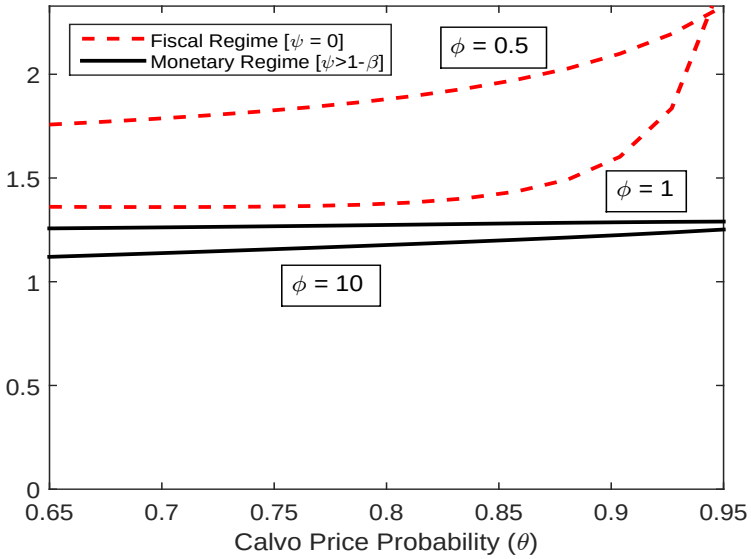


Figure 1.10: $GSM_F^{y}(0)$ as a function of θ in the Smets-Wouters Model

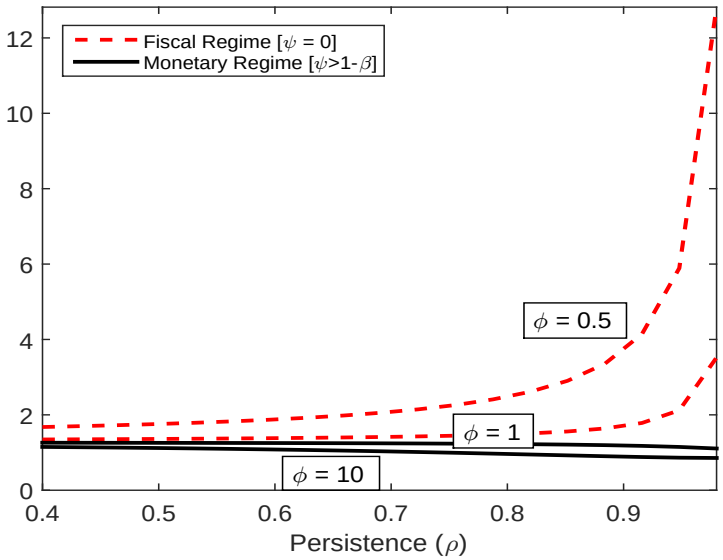


Figure 1.11: $GSM_F^y(0)$ as a function of ρ in the Smets-Wouters Model

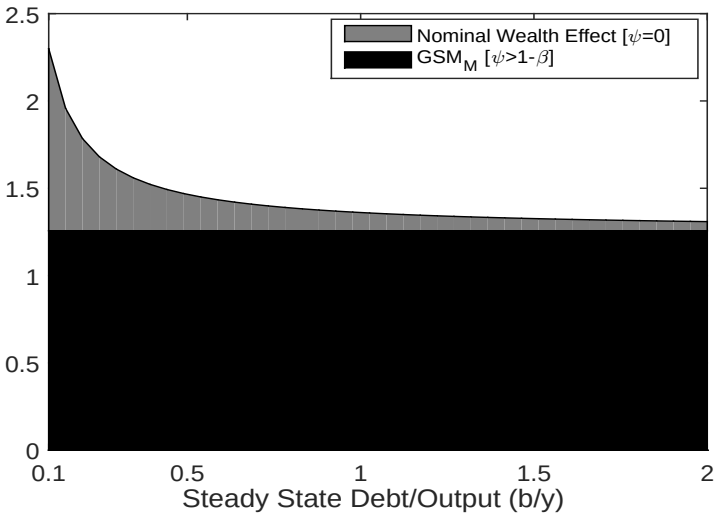


Figure 1.12: $GSM_F^y(0)$ as a function of b/y in the Smets-Wouters Model

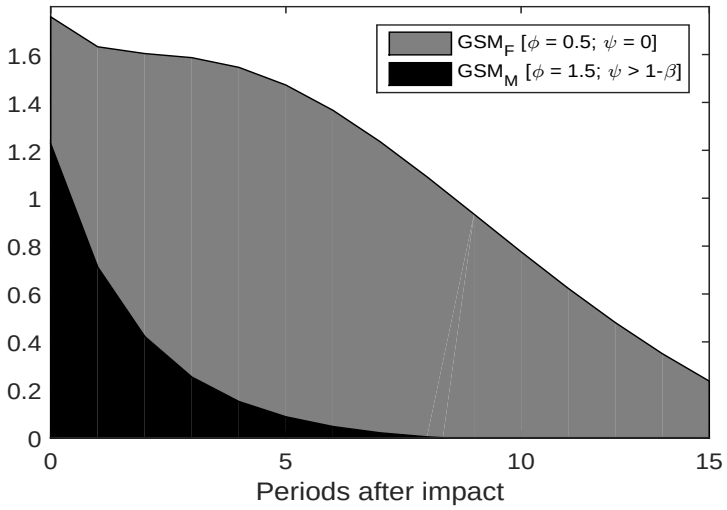


Figure 1.13: Dynamic representation of $GSM_F^y(j)$ in the Smets-Wouters model

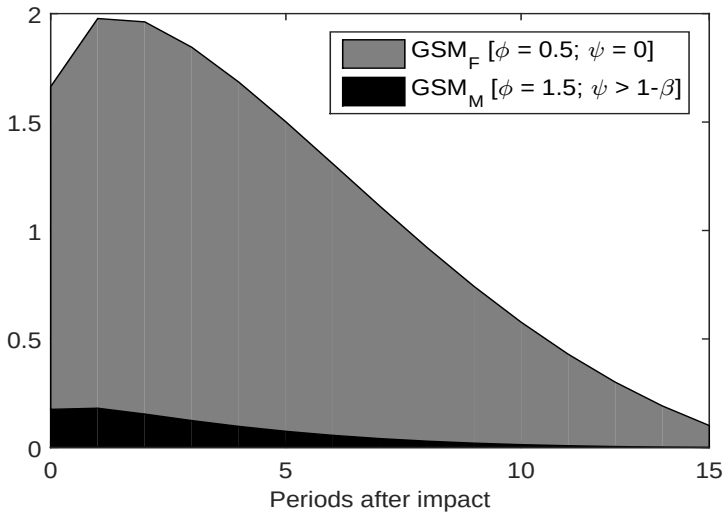


Figure 1.14: Dynamic representation of $GSM_F^\pi(j)$ in the Smets-Wouters model

1.6 Money-financed fiscal stimulus

With interest rates currently constrained by the zero lower bound and inflation rates below-target in many countries, various authors have recently called for policy makers to resort to money-financed fiscal stimulus (also known as 'overt monetary financing'). Under this policy, government stimulus (either in the form of additional spending or a tax cut) is financed by a permanent increase in the monetary base. Since this permanently increases the stock of nominal wealth in the hands of the private sector, it is thought to provide an answer to today's problems of subpar inflation and weak output growth (without a need for further decreases in the interest rate). Recent proponents of such a policy include Buiter (2014), Galí (2014), and Turner (2015), but earlier notable advocates include Lerner (1943), Friedman (1969), and Bernanke (2003).

By analyzing a New Keynesian model, Galí (2014: 1) concludes that "a money-financed fiscal stimulus [has] very strong effects on economic activity, with relatively mild inflationary consequences". His analysis is however limited to the monetary regime. In this section, we analyze the effects of money-financed fiscal stimulus in an economy that operates under the fiscal regime.

We believe that the fiscal regime is more relevant to countries in which interest rates are stuck at the zero lower bound (also see Cochrane 2011; Cochrane, 2014; and Leeper and Leith, 2016). In a liquidity trap, monetary policy loses its ability to respond in an 'active' manner, while the dire economic situation may induce the fiscal authority to start acting in an 'active' way. Moreover, Cochrane (2014)

demonstrates that, in the case of permanent regimes, the fiscal regime is the only regime that ensures determinacy when interest rates are stuck at zero. Typically, countries at the zero lower bound are also the countries in which calls for monetary financing are the loudest.

Analyzing the effects of money-financed fiscal stimulus is easily done in the fiscal regime. As for example set out in Buiter (2014), a key distinguishing feature of fiat money (and other components of the monetary base) is its 'irredeemable' nature: while money is seen as an asset to its holder, it is not considered to be a liability by its issuer.[21] This holds for government bonds B in our framework when ψ (the coefficient in the taxation policy rule; cf. equation (1.4)) is equal to zero. Then, an increase in government debt is never met by a current or future increase in the primary surplus and B turns into an asset-class that is seen as an asset by its holder, but not as a liability by its issuer. That is, it has become base money (also see Kocherlakota (2016) on the equivalence between money and bonds under these circumstances). Consequently, one can analyze the effectiveness of money-financed fiscal stimulus in the fiscal regime, by computing the effects of increases in B in this regime when $\psi = 0$.

This insight immediately implies that the effects of money-financed stimulus in the fiscal regime are identical to those of debt-financed stimulus when the increase in debt is permanent. The effects from money-financed stimulus can thus be replicated via fiscal policy if the fiscal authority is willing to adopt an active stance, with

[21] As Buiter (2014: 16) explains: "The owner of a $20 Federal Reserve Note [...] has no claim on the Federal Reserve, now or ever, other than for an amount of Federal Reserve Notes adding up to $20 in value."

monetary policy turning passive (which is the only way of guaranteeing determinacy at the zero-lower bound; cf. Cochrane (2014)). When $\psi \in (0, 1-\beta)$, fiscal policy can *approximate* the effects of money-financed stimulus (with the approximation getting closer as ψ approaches 0).

To be clear, money can still exist in our model even though we have not introduced an explicit money demand function. First, if base money pays interest, money and government bonds become perfect substitutes: both are interest-paying government liabilities (Cochrane, 2014). In that case, money and government bonds provide the private sector with the same exact savings mechanism to transform current nominal consumption to future nominal consumption. In the model outlined in Section 1.2, the real government debt b_t then encompasses the real value of all government liabilities, money and bonds.

Second, if base money does not pay interest, the private sector may still demand money to the extent it is the only government liability that is accepted as payment for goods and services. But even then, Cochrane (2005) demonstrates that we do not have to introduce an explicit money demand function to model this mechanism. In particular, in the model outlined in Section 1.2, we can add a cash-in-advance constraint ($Pc_t \leq M^d$) to ensure that the household needs money to pay for its nominal consumption. But if we then re-open the securities market at the end each period, the household can return and trade money for interest-paying bonds. Put differently, the household can use money during the day without holding it overnight: the end-of-period demand for money is zero, although the intra-period demand is

positive. The cash-in-advance constraint is then redundant.[22]

Let us consider money-financed stimulus (thus setting $\psi = 0$) in the standard New Keynesian setting, calibrated as in Section 1.4. In line with many countries in practice (U.S., U.K., Eurozone), we model the economy as operating under an 'interest on reserves' regime, such that interest rates can be set independently from the monetary base (Borio and Disyatat, 2009; Cochrane, 2014), and so that money and bonds become perfect substitutes.

Figure 1.15 shows the size of the taxation multiplier on impact in response to a money-financed tax cut (also known as 'helicopter money') as a function of ϕ, while Figure 1.16 shows the response to a money-financed increase in government spending.[23] A couple of observations are in order. Firstly, money-financed government spending has a more favorable trade-off ratio (i.e. it moves output more relative to inflation) than a money-financed tax cut. This is yet another manifestation of our finding that government spending multipliers tend to be larger than taxation multipliers (provided that taxation is non-distortionary). Setting $\psi = 0$ does not change that. Secondly, money-financed stimulus can give rise to output multipliers that are larger than one (with the effect again gaining strength as prices become stickier). Most reasonable calibrations suggest an output multiplier that lies in between 1 and 3 on impact.[24]

[22]Note, however, that if money differed from government bonds along other dimensions (e.g through a money-in-the-utility function or through differing liquidity), a money-financed fiscal expansion would have a different effect on economic activity than an unbacked debt-financed one.

[23]Since we model a tax cut as a one-off event in our model, results for money-financed government spending are also generated with $\rho = 0$. This ensures comparability between Figures 1.15 and 1.16.

[24]Since money-financed stimulus is very similar to debt-financed stimulus in the fiscal regime, it is no surprise that the resulting multipliers are alike.

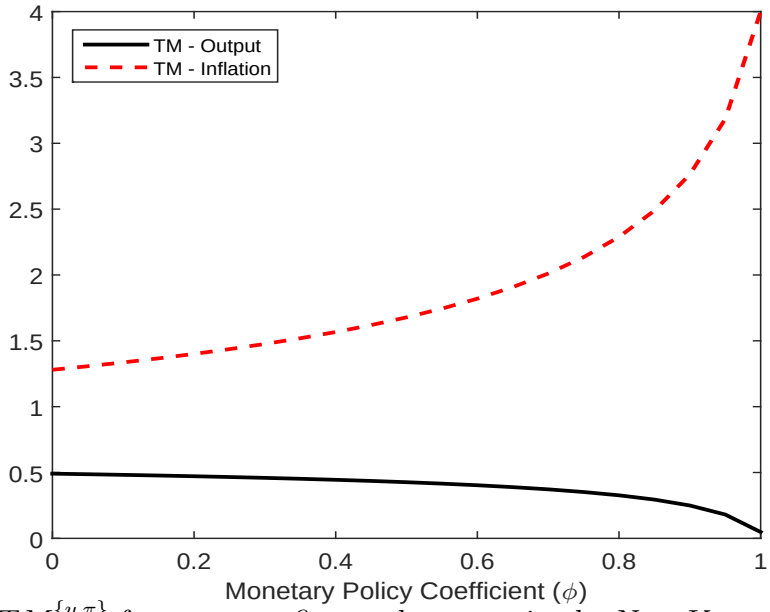


Figure 1.15: $TM_F^{\{y,\pi\}}$ for a money-financed tax cut in the New Keynesian Model as a function of ϕ

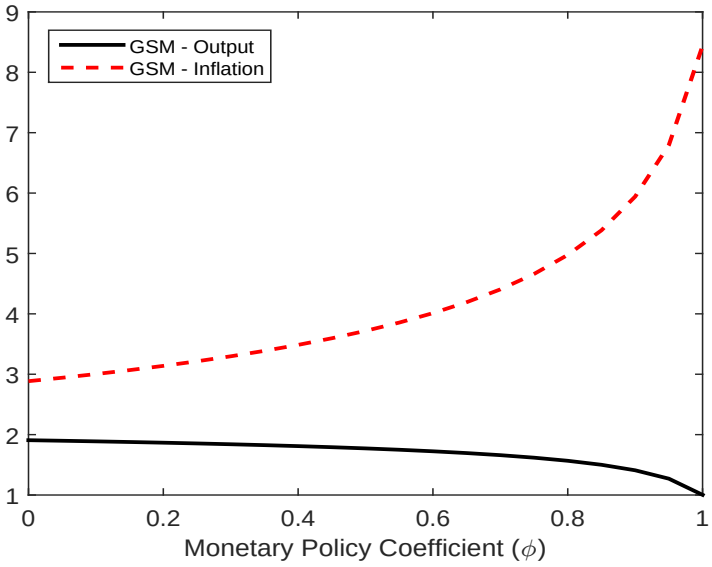


Figure 1.16: $GSM_F^{\{y,\pi\}}$ for money-financed government spending in the New Keynesian Model as a function of ϕ

Finally, note that the size of the inflation multiplier is increasing in the monetary policy coefficient ϕ. This is a direct consequence of the 'neo-Fisherian effect' that underlies the fiscal regime and has the unconventional implication that, in order to increase inflation, the central bank has to *increase* the interest rate. That however comes at a significant cost, as it will have a contractionary effect on output. Figures 1.15 and 1.16 suggest that money-financed stimulus offers a less-painful way out of a de-/low-flationary environment: such an operation *pushes up both output and inflation*, thereby avoiding the difficult trade-off between a positive inflation-response and a negative output-response that comes with an interest-rate increase. If monetary policy is too responsive (ϕ is too close to 1), the trade-off ratio associated with money-financed fiscal stimulus can, however, turn pretty unfavorable: while the inflation multiplier quickly increases with ϕ, the output multiplier falls as ϕ rises. Then, the operation might generate a lot of inflation without bringing much stimulus to output. Consequently, and somewhat counterintuitively, money-financed fiscal stimulus in the fiscal regime is only effective in raising output with mild inflationary consequences if the central bank is willing to adopt a very passive stance (operating with $\phi \ll 1$).

The above observations generalize to the Smets-Wouters environment (giving similar numerical results, available upon request).

1.7 Concluding remarks

In this paper we have analyzed the effects of fiscal policy under the Fiscal Theory of the Price Level, which characterizes a situation of fiscal dominance. We have shown that, in the associated fiscal regime, taxation multipliers turn positive, while government spending multipliers have the same functional form as their equivalents in the monetary regime, augmented by taxation multipliers and corrected for the 'fiscal inflation tax'. The reason is that the household in the fiscal regime perceives government bonds as nominal wealth. Consequently, the accumulation of government debt makes the household feel wealthier. As a result, fiscal multipliers in the fiscal regime tend to be larger than those in the monetary regime. Moreover, even if the underlying stimulus process is purely transitory, the wealth effect generates fiscal multipliers that are persistently positive.

The framework can also be used to analyze the effects of money-financed stimulus in the fiscal regime. Our analysis shows that all effects generated by money-financed fiscal stimulus can be replicated through fiscal policy when the fiscal authority takes an active stance, with the monetary authority adopting a passive one. We believe that this fiscal regime is particularly relevant to economies in which the interest rate is stuck at the zero lower bound. Such countries are typically the ones in which calls for monetary financing are loudest. Interestingly, our analysis illustrates that, for exactly those countries, monetary financing is just a particular form of fiscal policy. This implies that possible beneficial effects of monetary financing do not necessarily require the central bank to get involved with this (controversial) practice. What

is essential is that the authorities create irredeemable nominal wealth. Whether this is done by the fiscal authority or by the central bank is irrelevant. Our results furthermore suggest that money-financed stimulus (or debt-financed stimulus, given the aforementioned equivalence) is able to push countries out of a low-/deflationary environment without engineering a recession. In this way, policy makers are able to avoid the neo-Fisherian conundrum which suggests that inflation can only be lifted by raising the interest rate (which would depress output). Our analysis finally indicates that money-financed stimulus in the fiscal regime is only effective in raising output with mild inflationary consequences if the central bank is willing to adopt a very passive stance.

We end by stressing that our analysis takes place in a simple environment with no distortionary taxes, no long-term government debt, and no dynamic switching between the two regimes. We leave these extensions for future research.

CHAPTER 2

Forward Guidance of Fiscal Policy

Chapter Abstract

I analytically study the effects on economic activity from expected changes to future fiscal variables. I document four transmission mechanisms, the sizes of which depend crucially on the level of nominal rigidities. In general, news shocks to fiscal policy produce larger effects when the policy rate is unresponsive to inflation and when the fiscal authority has the power to issue unbacked nominal debt. Within this framework, I show that 'forward guidance' of fiscal policy can expand economic activity without ever increasing the real value of government debt.

2.1 Introduction

The policy response after the Great Recession can be broadly described in the following sequential steps. First, central banks cut the short-term policy rate to zero. Second, once short-term rates were constrained by the zero lower bound, central banks turned to 'forward guidance' of the future course of monetary policy. Third, central banks reinforced pressure on long-term rates by large scale asset purchases. Fourth, once long-term rates, too, were approaching the zero lower bound, policy discussion turned to fiscal policy. But fiscal policy quickly turned impotent, with political pressures and debt ceilings limiting the scope for fiscal deficits. The situation seemed to be most gridlocked in the Eurozone where explicit limits on debt and deficits instead pushed many countries to introduce austerity measures.

The natural progression along these lines would be to consider 'forward guidance' of the future course of fiscal policy. Yet as an explicit policy tool, it has not been discussed in policy circles. As I shall show in this paper, just like forward guidance of monetary policy allows the central bank to stimulate economic activity when conventional tools are constrained by the zero lower bound, forward guidance of fiscal policy allows the fiscal authority to stimulate economic activity when conventional tools are constrained by debt or deficit limits.

The issue is topical. Some observers (e.g. Bernanke, 2017; and Brainard, 2017) argue that, since the Trump administration took office in January 2017, inflation and interest rates in the U.S. have increased on the expectation of more expansionary fiscal policies in the future (including large tax cuts, more military spending,

and substantial infrastructure investments). But the recent experience is not new. Governments frequently signal their intentions of future fiscal policies, with immediate effects on economic activity. Indeed, all changes in fiscal policy come with an implementation lag and are thus pre-announced. Schmitt-Grohe and Uribe (2012) estimate that news shocks to government spending account for roughly 7% of the variance of U.S. output growth. Born, Peter and Pfeifer (2013) find that 8% to 12% of the variance of inflation is due to anticipated capital tax shocks.

This paper analytically examines the transmission mechanisms of forward guidance of fiscal policy — that is, the channels through which future changes in fiscal variables propagate to current economic activity. I do so using the standard three-equation New Keynesian model, in both the 'monetary' and 'fiscal' regime. I show that forward guidance can have strong and lasting effects on economic activity. The effects are more pronounced when the policy rate is unresponsive to inflation (e.g. when it is stuck at the zero lower bound) and when the fiscal authority has the power to issue unbacked nominal debt.

I document four transmission mechanisms. First, higher expected future inflation lowers the ex-ante real interest rate and thus increases aggregate demand. Second, if government spending crowds out (in) consumption, lower (higher) future private consumption decreases (increases) aggregate demand via the consumption smoothing motive. Third, higher future inflation produces a stagflationary effect, much like a cost-push shock, via the New Keynesian Phillips curve, as firms raise their prices in anticipation of higher future demand. And fourth, higher future deficits generate immediate wealth effects if the deficits are expected to be unbacked by primary

surpluses.

In all transmission mechanisms, economic activity expands or contracts only through changes in aggregate demand: labor supply remains unchanged (independent of preferences), as fiscal policy only removes resources from private consumption when spending is higher, not in the periods preceding it. The level of price stickiness therefore plays a key role in determining the effect on output. With flexible prices, firms merely increase (decrease) their prices to offset the higher (lower) demand for their goods and services, leaving labor demand and output unchanged. With sticky prices, however, changes in aggregate demand translate into changes in labor demand, as firms as forced to lower (raise) their markup to meet the higher (lower) demand for their goods and services.

Standard calibrations suggest that news shocks to government spending produce a small stagflationary effect in the monetary regime. A 1% increase in government spending (as a fraction of steady state output) announced one year in advance contracts output and expands inflation by roughly -0.1% and 0.1%, respectively. The fiscal regime, however, displays larger and more persistent effects, with the same announcment expanding output and inflation by roughly 0.75% and 1%, respectively.

Forward guidance of fiscal policy resembles conventional fiscal policy, but differs in one important aspect. Conventional fiscal policy is typically constrained by debt or deficit limits; forward guidance of fiscal policy needs not to be. On the contrary, in the periods preceding the fiscal shock, forward guidance can create fiscal space by eroding the real value of outstanding debt through higher prices. If the announcement is made sufficiently early (to allow for a sufficient price increase in the periods preceding the

fiscal shock), the real value of government debt needs *never* to increase above its pre-announcement level, even after the fiscal shock is implemented. Fiscal forward guidance, if credibly believed by the private sector, is therefore not restricted by any (potentially binding) debt ceiling.

I take the analysis one step further. When the policy rate is unresponsive to inflation, I show that by permanently lowering lump-sum taxes, economic activity expands while the real value of debt permanently *decreases*. Lower taxes increase the wealth of the private sector, leading to higher output and inflation through an increase in aggregate demand. Higher prices, in turn, permanently erode the real value of debt. On the flip side, by the reverse logic, increasing taxes at the zero lower bound (when the policy rate is de facto unresponsive to inflation) may be self defeating, as it increases the real value of debt through a fall in aggregate demand. This result is consistent with the empirical findings in Cherif and Hasanov (2012), De Grauwe and Ji (2013), and Holland and Portes (2012).

This paper is related to two strands of the literature. First, the literature has identified that news shocks to fiscal policy account for a large share of the business cycle volatility (e.g. see Schmitt-Grohe and Uribe, 2012; and Born, Peter and Pfeifer, 2013). Other authors (e.g. House and Shapiro, 2006; and Parker, 1999), use single tax events to demonstrate the importance of fiscal foresight. Although the empirical evidence makes clear that fiscal foresight matters for economic conditions, little is known about the mechanisms whereby fiscal news shocks propagate to economic activity. This study fills that gap. Second, this paper adds to the large literature on fiscal multipliers (e.g. see Aiyagari et al., 1992; Baxter and King, 1992; Christiano

et al., 2011; Woodford, 2011; and Leeper et al., 2016). In particular, I borrow from and extend the work by Beck-Friis and Willems (2017), who analytically derive and link the fiscal multipliers in the 'monetary' and 'fiscal' regime.

This paper proceeds as follows. Section 2.2 outlines the model. Section 2.3 derives the 'announcement multipliers' in the two regimes and discusses the channels through which fiscal news shocks propagate to economic activity. Section 2.4 discusses the effects of committing to a path of fiscal variables. Section 2.5 concludes.

2.2 Model

I consider a standard New Keynesian model. It is set in a discrete-time, infinite horizon environment, lacks capital, but allows for non-zero, wasteful government spending. Firms set prices according to the Calvo-mechanism, which allows them to reset their price in each period with probability $(1-\theta)$. Throughout this paper, variables without a time-subscript denote steady-state values, variables with a hat indicate log-deviations from this steady state (i.e. $\hat{x}_t = (x_t - x)/x$), and variables with a tilde express steady-state deviations as a fraction of steady-state output (i.e. $\tilde{x}_t = (x_t - x)/y$).

As the model is well-documented, I relegate a complete description of the model

and its solution to Appendices 2.A - 2.C and proceed by presenting its linear form:

$$\hat{y}_t - \alpha_1 \tilde{g}_t = \mathbb{E}_t \hat{y}_{t+1} - \alpha_1 \mathbb{E}_t \tilde{g}_{t+1} - \alpha_2 [\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}], \tag{2.1}$$

$$\hat{\pi}_t = \beta \mathbb{E}_t [\hat{\pi}_{t+1}] + \kappa \alpha_3 \hat{y}_t - \kappa \alpha_4 \tilde{g}_t, \tag{2.2}$$

$$\hat{i}_t = \phi \hat{\pi}_t, \tag{2.3}$$

$$\tilde{\tau}_t = \psi \tilde{b}_{t-1} + \varepsilon_t^{\tau}, \tag{2.4}$$

$$\tilde{g}_t = \rho \tilde{g}_{t-1} + \varepsilon_t^{g}, \tag{2.5}$$

$$\tilde{b}_t = \frac{1}{\beta} \tilde{b}_{t-1} - \frac{1}{\beta} (\tilde{\tau}_t - \tilde{g}_t) - \frac{1}{\beta} \frac{b}{y} \hat{\pi}_t + \frac{b}{y} \hat{i}_t, \tag{2.6}$$

Here, y_t refers to output, g_t to real government spending, i_t to the nominal interest rate, π_t to inflation, τ_t to real lump-sum taxation, and b_t to real government debt. The variables ε_t^{τ} and ε_t^{g} are mean zero i.i.d taxation and government spending shocks, respectively. $\beta \in [0, 1)$ is the household's subjective discount factor. $\kappa \equiv (1-\theta)(1-\beta\theta)/\theta$ is the standard Calvo-measure of price stickiness (with $\kappa \to \infty$ as prices become fully flexible).

Equations (2.1) and (2.2) are the standard Euler equation and the New Keynesian Phillips Curve (accounting for the impact of government spending), respectively. Equation (2.3) describes the monetary policy rule, according to which the central bank reacts to current-period inflation with strength parameter $\phi \geq 0$. Equation (2.4) captures the fiscal policy rule as in Leeper (1991), according to which the fiscal authority adjusts lump-sum taxes to the deviation of lagged real debt with strength parameter $\psi \geq 0$. Equation (2.5) describes the evolution of government spending, with the parameter $\rho \in [0, 1)$ governing its persistence. Finally, equation (2.6) is the

government's flow budget identity. The government issues nominal debt and collects nominal taxes to finance its nominal spending and interest cost on its existing debt stock.

The parameters $\{\alpha_i\}_{i=1}^{4}$ are a function of the household's period utility function $U(c_t, n_t)$ (with U_c denoting the steady-state value of the partial derivative $\partial U/\partial c$ and so forth):

$$\begin{aligned}
\alpha_1 &= \frac{U_{cc}}{U_{cc}+U_{cn}} \geq 0, \\
\alpha_2 &= -\frac{U_c}{y(U_{cc}+U_{cn})} \geq 0, \\
\alpha_3 &= y\Big[\frac{U_{nc}+U_{nn}}{U_n} - \frac{U_{cc}+U_{cn}}{U_c}\Big] \geq 0, \\
\alpha_4 &= y\Big[\frac{U_{nc}}{U_n} - \frac{U_{cc}}{U_c}\Big] \geq 0,
\end{aligned}$$

with $U_c, -U_n > 0$ and $U_{cc}, U_{cn}, U_{nn} \leq 0$. Preferences are convex. I shall consider CRRA-preferences in the numerical illustrations, but I keep this general form as the analytical results extend to any types of preferences.

As is well-established in the literature (e.g. see Leeper, 1991), with static policy parameters, the system has a unique, bounded solution under two regimes. In the 'monetary regime', the central bank 'actively' adjusts the policy rate to inflation, $\phi > 1$, while fiscal policy 'passively' adjusts taxes to deviation of lagged real debt, $\psi > (1-\beta)$. In the 'fiscal regime', the central bank 'passively' adjusts the policy rate to inflation, $\phi < 1$, while fiscal policy 'actively' adjusts taxes to deviation of lagged real debt, $\psi < (1-\beta)$.

2.3 Announcement multipliers

I define the government spending multiplier ('GSM') and taxation multiplier ('TM') on output (y) and inflation (π) at horizon $j, k \geq 0$ as:[1]

$$GSM^y(j,k) \equiv \mathbb{E}_t \frac{\partial \hat{y}_{t+k}}{\partial \varepsilon^g_{t+j}}, \tag{2.7}$$

$$GSM^\pi(j,k) \equiv \mathbb{E}_t \frac{\partial \hat{\pi}_{t+k}}{\partial \varepsilon^g_{t+j}}, \tag{2.8}$$

$$TM^y(j,k) \equiv \mathbb{E}_t \frac{\partial \hat{y}_{t+k}}{\partial \varepsilon^\tau_{t+j}}, \tag{2.9}$$

$$TM^\pi(j,k) \equiv \mathbb{E}_t \frac{\partial \hat{\pi}_{t+k}}{\partial \varepsilon^\tau_{t+j}}. \tag{2.10}$$

That is, the multipliers measure the expected effect on economic activity in period $t+k$ from a fiscal shock announced in period t but implemented in period $t+j$. When $j = 0$, the expressions collapse to the standard multipliers in the literature. I shall refer to these expressions as the *impact multipliers* when $j = k = 0$, and the *tail multipliers* when $j = 0$ and $k > 0$. When $j > 0$ and $k = 0$, the expressions capture the immediate effects on economic activity in the period in which the future fiscal shock is announced (or suddenly expected). I shall refer to these expressions as the *announcement multipliers* for any $j > 1$.

In this remainder of this section, I derive expressions for these objects under both the monetary ('M') and fiscal ('F') regime.

[1] I define taxation multipliers with a minus-sign up front, so that a positive multiplier implies that a tax cut boosts output and inflation. I also define the multipliers on inflation as the inflationary response to a fiscal shock as a fraction of steady state output. This ensures that the multipliers on inflation do not depend on the model's steady state level of output.

2.3.1 Monetary regime

Taxation

In the monetary regime, the taxation multipliers are zero across the entire episode:

$$TM_M^y(j,k) = 0,$$
$$TM_M^\pi(j,k) = 0,$$

for any $j, k \geq 0$. The reason lies in the well-known Ricardian Equivalence (Barro, 1974). As the government adjusts taxes aggressively to deviation of lagged real debt, the bonds issued to finance a tax cut, now or in the future, will eventually be soaked up by higher real taxes. Bonds in the monetary regime are, put differently, 'backed' by future primary surpluses. As a result, the life-time wealth of the household is independent of the timing of lump-sum taxes, leaving aggregate demand, output and inflation unchanged throughout time.

Government spending

In the monetary regime, the impact multipliers of government spending are positive:

$$GSM_M^y(0,0) = \frac{\alpha_1(1-\rho)(1-\beta\rho) + \kappa\alpha_2\alpha_4(\phi-\rho)}{(1-\rho)(1-\beta\rho) + \kappa\alpha_2\alpha_3(\phi-\rho)} > 0, \tag{2.11}$$
$$GSM_M^\pi(0,0) = \frac{(1-\rho)\kappa(\alpha_1\alpha_3 - \alpha_4)}{(1-\rho)(1-\beta\rho) + \kappa\alpha_2\alpha_3(\phi-\rho)} > 0, \tag{2.12}$$

with the tail multipliers decaying by a factor of ρ:

$$GSM_M^{\{y,\pi\}}(0,k) = \rho^k \times GSM_M^{\{y,\pi\}}(0,0). \tag{2.13}$$

The multipliers on output are positive for two reasons, both of which I shall refer to as the 'Keynesian' effects (following Beck-Friis and Willems, 2017). First, labor supply shifts out as government spending removes resources from private consumption.[2] To make up for the lost consumption, the household chooses to work more for any given real wage.[3] Second, if prices are sticky, labor demand shifts out too, as firms are forced to lower their markup to meet the higher demand for their goods. If prices are fully flexible, however, firms merely adjust their prices, leaving labor demand unchanged.

The monetary policy coefficient ϕ affects output through changes in aggregate demand. A higher ϕ depresses aggregate demand as the ex-ante real interest rate increases. Lower aggregate demand, in turn, translates into lower prices and, to the extent prices are sticky, lower labor demand.[4] The shift in labor supply, however, is unrelated to monetary policy. To see this, note that in the flexible-price limit where

[2]In a general equilibrium model, as all markets must clear, a *shift* in the labor supply can be defined in multiple ways. In particular, by Walras' Law, one has to specify which market clears by construction. Throughout this paper, I shall refer to a shift in the labor supply in the following way. Start with the relationship that equates the marginal rate of substitution of private consumption and labor to the real wage (2.27). Substitute out private consumption by using the aggregate resource constraint (2.B) and production function (2.B). The remaining relationship equates the real wage to a function of labor and government spending. I refer to shifts in labor supply as shifts in this curve (in the two-dimensional space with real wage on the y-axis and labor on the x-axis).

[3]This holds true for most preferences, and for all preferences that are separable in consumption and leisure. In the case of GHH-preferences (Greenwood, Hercowitz, and Huffman, 1988), the labor supply curve stays still; and in the case of KPR-preferences (King, Plosser, and Rebelo, 2002), the labor supply curve shifts down.

[4]The derivative of the multipliers in (2.11) and (2.12) with respect to ϕ is negative.

$\kappa \to \infty$, the multiplier on output, (α_4/α_3), is independent of ϕ.

Before turning to the analytical expressions of the announcement multipliers, it is helpful to first present the following relation:

$$GSM_M^{\{y,\pi\}}(j,k) = GSM_M^{\{y,\pi\}}(j+m,k+m), \text{ for any } m \geq 0. \tag{2.14}$$

That is, the expected effect on economic activity in period $t+k$ is independent of when the announcement of the spending shock was made. Put differently, the multipliers are history-independent: economic activity today does not depend on economic activity yesterday. For $k < j$, what matters is only the distance $j-k$ to the actual implementation of the shock. In particular,

$$GSM_M^{\{y,\pi\}}(j,j) = GSM_M^{\{y,\pi\}}(0,0). \tag{2.15}$$

The effect on economic activity in the period in which the spending shock is implemented is independent of when in the past the spending shock was announced. This result is not surprising. The only backward-looking equations in this economy are the government's taxation rule (2.4) and flow budget identity (2.6). But in the monetary regime, the path of government debt is irrelevant for economic activity, because all debt is backed by primary surpluses. So the economy is instead completely dictated by the forward-looking Euler equation (2.1), Phillips curve (2.2) and Taylor rule (2.3).

To understand the transmission mechanisms of news shocks to government spend-

ing, consider first the case with flexible price. In the limit $\kappa \to \infty$, we get:

$$\lim_{\kappa \to \infty} GSM_M^y(j,k) = 0, \text{ for any } k < j. \tag{2.16}$$

Under flexible prices, an announcement of higher future spending leaves output unchanged in the periods preceding the spending shock. Neither labor supply nor labor demand adjusts. Labor supply remains unchanged as government spending only removes resources from private consumption when government spending is higher, not in the periods preceding it. Labor demand remains unchanged as firms merely adjust their prices to offset any changes in aggregate demand.

Inflation follows a linear combination of the impact multipliers:

$$\lim_{\kappa \to \infty} GSM_M^\pi(j,0) = \frac{1}{\phi^j} \lim_{\kappa \to \infty} GSM_M^\pi(0,0) + \frac{1}{\phi^j \alpha_2} \Big[\lim_{\kappa \to \infty} GSM_M^y(0,0) - \alpha_1 \Big]. \tag{2.17}$$

Two effects are in operation, both via the Euler equation (2.1). First, higher future inflation (cf equation (2.12)) lowers the ex-ante real interest rate, which boosts aggregate demand and prices. Second, changes in future private consumption (cf equation (2.11)) affect aggregate demand through the consumption smoothing motive. If government spending decreases the future marginal utility of private consumption, i.e. $GSM_M^y(0,0) > \alpha_1$ (see Appendix 2.B.1 for derivation), the household increases consumption demand when the announcement is made. In this case, the net inflationary response is unambiguously positive. If government spending instead increases the future marginal utility of private consumption, i.e. $GSM_M^y(0,0) < \alpha_1$, consumption demand decreases, and the net inflationary response depends on the

relative strengths of the two effects.[5,6]

One can alternatively derive the inflationary response from the Fisher equation:

$$\hat{i}_t = \hat{r}_t + \mathbb{E}_t \hat{\pi}_{t+1}, \tag{2.18}$$

where $\hat{r}_t$ is the linearized real interest rate. By Euler equation (2.1), given next-period inflation, the nominal interest rate has to adjust to equate the ex-post real interest rate to the growth in the marginal utility of private consumption. The nominal interest rate, in turn, determines inflation through the Taylor rule. To see a simple example of this relationship, consider the case in which the impact multiplier on output is α_1 (=1, if the utility function is separable in consumption and leisure), so that the marginal utility of private consumption, and thus the real interest rate, is unchanged throughout the episode. Substituting $\hat{r}_t = 0$ and the Taylor rule (2.3) in the Fisher equation (2.18) tells us that inflation grows by a factor of ϕ in the periods leading up to the spending shock. This result is, of course, also consistent with equation (2.17), where the last term drops out when $GSM_M^y(0,0) = \alpha_1$.

Let us now turn on price stickiness. The effects of news shocks still depend

[5]If the utility function is separable in consumption and leisure, preferences satisfy $U_{cn} = 0$ and $\alpha_1 = 1$. Consumption demand then increases (decreases) if government spending crowds in (out) private consumption.

[6]As the economy is history-independent, the sign of the inflationary response, $GSM_M^\pi(j,k)$, depends only on the sign of the inflationary response one period before the spending shock is implemented, $GSM_M^\pi(j,j-1)$. To see why, recall from (2.16) that $GSM_M^y(j,k) = 0$ for any $k < j$ under flexible prices. There is thus no demand effect in period $j-2$ from the consumption smoothing motive. If $GSM_M^\pi(j,j-1)$ is positive (negative), the linearized ex-ante real interest rate in period $j-2$ is negative (positive), which therefore increases (decreases) aggregate demand and prices in period $j-2$. By (2.14), we can extend this logic by backward induction: if $GSM_M^\pi(j,j-1) > 0$, it must be that $GSM_M^\pi(j,k) > 0$ for any $k < j$, and vice versa.

linearly on the impact multipliers:

$$GSM_M^y(j,0) = p_{1,2}(j)GSM_M^\pi(0,0) + p_{1,1}(j)[GSM_M^y(0,0) - \alpha_1], \tag{2.19}$$

$$GSM_M^\pi(j,0) = p_{2,2}(j)GSM_M^\pi(0,0) + p_{2,1}(j)[GSM_M^y(0,0) - \alpha_1], \tag{2.20}$$

where the p's are given by (2.39) - (2.42) in Appendix 2.C.1, with $p \to 0$ as $j \to \infty$.

Consider first the effect on output and inflation in the period preceding the spending shock. Three effects come into play. The first two appear via the Euler equation (2.1) and are the same as above with flexible prices. First, higher next-period inflation (cf equation (2.12)) lowers the ex-ante real interest rate, which increases aggregate demand. Second, higher (lower) future marginal utility of private consumption (cf equation (2.11)) depresses (increases) aggregate demand through the consumption smoothing motive. With sluggish price adjustment, the net demand of these effects translates into a change in output, as firms are unable to fully adjust their prices.

Sticky prices introduces a third transmission channel. Higher next-period inflation raises current inflation through the Phillips curve (2.2), as firms increase their prices in anticipation of a higher future demand. Higher prices, in turn, depresses output along an unchanged aggregate demand curve. The stagflationary nature of this transmission mechanism resembles the effects of a more standard cost-push shock, and grows stronger the stickier are prices.[7]

As the system is completely forward-looking, the effects in period $t + k$ depend

[7]The level of price stickiness also affects the impact multipliers, and thus, by extension, the announcement multipliers. The stickier are prices, the larger is the impact multiplier on output and smaller is the impact multiplier on inflation (cf equations (2.11) and (2.12)).

only on the economic conditions in period $t+k+1$. We can therefore extend the above intuition by backward induction to each period preceding the spending shock.

I shall refer to these three channels as the 'Keynesian transmission mechanisms'. Note that if the impact multipliers on output and inflation are one and zero, respectively, these transmission mechanisms become impotent. In that specific case, as future inflation and private consumption remain unaffected by the spending shock, there is neither any incentive to change private consumption demand nor any reason for firms to smooth any price increase over time.

Quantitative Illustration

To quantity the results, I consider CRRA-preferences:

$$U(c_t, n_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\xi}}{1+\xi},$$

where σ is the coefficient of relative risk aversion and ξ is the inverse of the elasticity of labor supply. As is well-documented in the literature, the impact multiplier on output is smaller than one with CRRA-preferences (for all calibrations).

Unless otherwise noted, the simulations throughout the paper assume the following baseline calibration at the quarterly frequency. I set the discount factor (β) to 0.99; the coefficient of relative risk aversion (σ) to 1; the inverse elasticity of labor supply (ξ) to 2; the steady state fraction of government spending to output (g/y) to 1/3; the steady state fraction of government debt to output (b/y) to 1; and the parameter governing the persistence in government spending (ρ) to 0.5. In the monetary regime, I set the monetary policy parameter (ϕ) to 1.2 and fiscal policy

parameter (ψ) to 0.8. In the fiscal regime, I set the same policy parameters to 0.5 and 0, respectively.

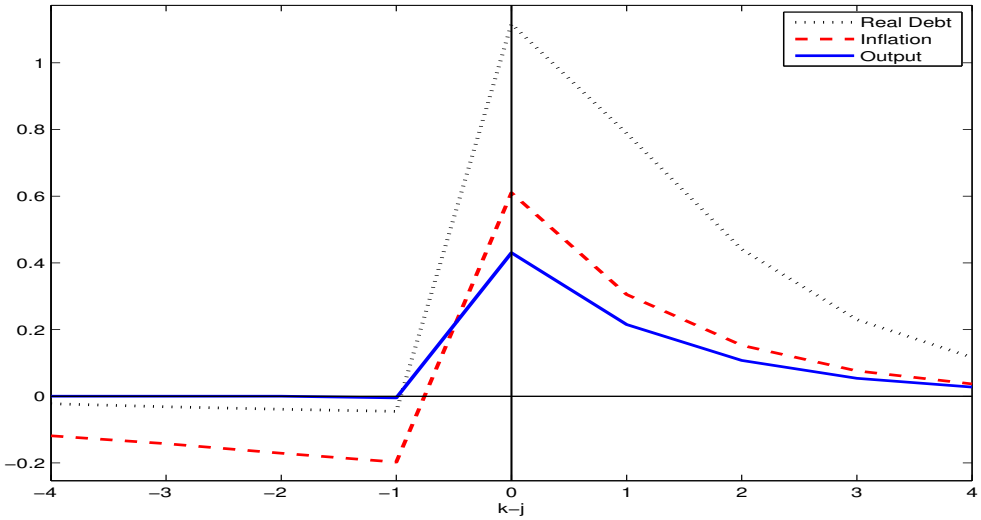
Figure 2.1(a) plots the dynamic effects under flexible prices. I fix $j = 4$ (so that the announcement is made one year prior to the spending shock) and let the x-axis measure $k - j$, the number of periods before and after the spending shock is implemented. With CRRA-preferences, as government spending crowds out private consumption, there are two transmission mechanisms with opposite effects: (1) higher future inflation lowers the ex-ante real interest rate and thus increases aggregate demand; and (2) lower future consumption decreases aggregate demand through the consumption smoothing motive. Here, the second effect dominates the first, leading to deflation in all periods leading up to the spending shock. More generally, this result holds true for all calibrations and for all preferences that are separable in consumption and labor (see equation (2.43) in Appendix 2.C.1 for a proof). Lastly, output remains unchanged until the spending shock is implemented.

Figures 2.1(b) plots the same graph but with sticky prices, setting $\theta = 0.75$ (implying an average price duration of four quarters). With sluggish price adjustment, the net negative demand effect translates into lower labor demand and output. Yet despite the negative shift in demand, inflation increases in the periods preceding the spending shock, as firms raise their prices in anticipation of higher future inflation. Higher prices, in turn, depress output further.

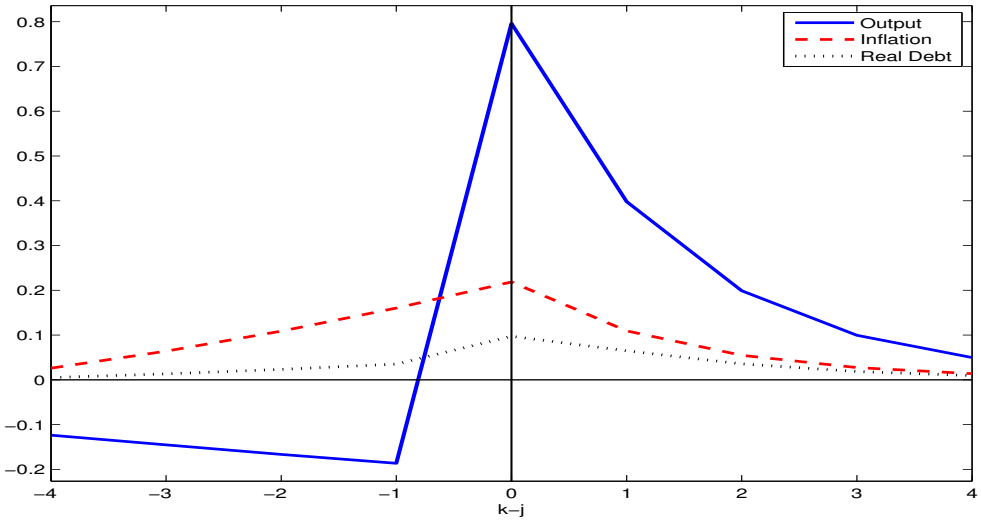
Two additional points emerge from the graphs. First, the real value of debt increases (decreases) in response to inflation (deflation). It is true that inflation erodes the real value of outstanding nominal debt. But higher inflation also calls for

Figure 2.1: Announcement Multipliers of Spending in the Monetary Regime

(a) Flexible Prices ($\theta = 0$)



(b) Sticky Prices ($\theta = 0.75$)



Notes: The graphs plot the impulse response functions of economic activity in response to a news shock to government spending in the monetary regime. The shock is announced four ($j = 4$) quarters before it is implemented. The x-axis measures the number of periods before (-) and after (+) the spending shock is implemented. The y-axis measures the size of the multipliers, with the multiplier on debt being defined analogously as in (2.7) - (2.10).

a higher nominal interest rate (via the Taylor rule), leading to a larger fiscal deficit and more issuance of nominal debt. If $\phi\beta > 1$, the second effect dominates the first, so that the real value of debt increases in response to inflation.[8] So, in the monetary regime with a sufficiently aggressive central bank, forward guidance of fiscal policy, to the extent it is inflationary, is just as constrained by binding debt or deficit limits as conventional fiscal policy.

Second, the multipliers on the positive x-axis resemble the standard impulse response functions of government spending, with economic activity decaying by a factor of $\rho = 0.5$ in each period. This result follows from the monetary regime being history-independent.

2.3.2 Fiscal regime

Taxation

As documented in Beck-Friis and Willems (2017), the impact multipliers of taxation in the fiscal regime are positive:

$$TM_F^y(0,0) = \left(\frac{y}{b}\right) \times \frac{(1-\beta\lambda)(1-\beta\lambda-\psi)}{\kappa\alpha_3(1-\beta\phi)(1-\psi)} \geq 0, \tag{2.21}$$

$$TM_F^\pi(0,0) = \left(\frac{y}{b}\right) \times \frac{(1-\beta\lambda-\psi)}{(1-\beta\phi)(1-\psi)} \geq 0, \tag{2.22}$$

[8] To see this, substitute the Taylor rule (2.3) in the government's budget identity (2.6) and differentiate $\tilde{b}_t$ with respect to $\hat{\pi}_t$.

with the tail multipliers decaying by a factor of λ:

$$TM_F^{\{y,\pi\}}(0,k) = \lambda^k \times TM_F^{\{y,\pi\}}(0,0), \tag{2.23}$$

where,

$$\lambda = \frac{1}{2\beta}\Big[1+\beta+\kappa\alpha_2\alpha_3 - \sqrt{(1+\beta+\kappa\alpha_2\alpha_3)^2 - 4\beta(1+\phi\kappa\alpha_2\alpha_3)}\Big] < 1.$$

The taxation multipliers are positive for reasons well-documented in the literature. As a tax cut is not associated with sufficiently large future real taxes to repay the debt, the household perceives the newly-issued government bonds as an increase to its wealth. Consumption demand increases as a result, giving rise to positive multipliers on output and inflation. Crucially, although labor demand increases (if prices are sticky), labor supply remains fixed, as the government does not remove resources from private consumption.

For $j > 0$, the multipliers generalize to:

$$TM_F^{y}(j,k) = \Big(\frac{\beta}{1-\psi}\Big)^j TM_F^{y}(0,k) \geq 0, \tag{2.24}$$

$$TM_F^{\pi}(j,k) = \Big(\frac{\beta}{1-\psi}\Big)^j TM_F^{\pi}(0,k) \geq 0. \tag{2.25}$$

The announcement multipliers are identical to the impact and tail multipliers, but now discounted by a factor of $\beta/(1-\psi) < 1$ for each period preceding the taxation shock. Economic activity expands for similar reasons as above. In the announcement period, the household anticipates that lower future taxes imply future issuance

of unbacked government debt. This debt adds to the life-time wealth of the household. The household discounts this wealth increase into present-value terms in the announcement period, and increases consumption demand accordingly. Prices increase, while output expands through an increase in labor demand (if prices are sticky).

The discount factor, $\beta/(1-\psi)$, is *increasing* in the fiscal strength parameter ψ. At first sight, this result may seem surprising. After all, the impact and tail multipliers in (2.21)-(2.23) are all decreasing in ψ: the real value of debt increases when the tax cut is implemented, so a more aggressive fiscal policy response implies higher future taxes, suppressing household wealth and demand. But as news of a tax cut is inflationary, the real value of debt *decreases* in the periods preceding the tax cut.[9] A larger ψ therefore produces larger endogenous tax cuts in the periods preceding the taxation shock. On announcement, the household incorporates these tax cuts in its wealth calculation through the discount factor. When $\psi = 0$, so that there are no changes in taxes in response to debt, the discount factor collapses to the household's subjective discount factor β.

The total derivative of the announcement multipliers with respect to ψ depends on the relative strengths of the two effects. A larger ψ increases the discount factor but lowers the sizes of the impact multipliers. For large j, so that there are many endogenous tax cuts preceding the tax shock, the former effect outweighs the latter.

The earlier the tax cut is announced (i.e. the larger is j), the smaller is the present-discounted value of the wealth increase, and thus the smaller is its effect on

[9]See footnote [8], and recall that $\phi < \beta^{-1}$ in the fiscal regime

output and inflation. As $j \to \infty$, the announcement multipliers tend to zero.[10] But the speed of convergence is slow. The discount factor applied in the wealth calculation is between β and one, so whether the announcement is made one quarter, one year, or two years in advance has only marginal effects on the response of economic activity.

The announcement multipliers resemble the impact and tail multipliers in (2.21)-(2.23) in two important ways. First, the effect on economic activity is strongest in the announcement period, and then decays over time by a factor of λ. Second, the announcement multipliers of taxation are smaller for more indebted governments. The more debt outstanding, the smaller is the relative size of the newly-issued government bonds (as a proportion of the household's existing bond holdings), and thus the smaller is the wealth effect of the tax cut on economic activity. News of a tax cut is thus less expansionary when government debt is high (and thus when convention fiscal policy space is limited), which is when, perhaps, calls for forward guidance of fiscal policy would be the loudest.

Further note that

$$TM_F^{\{y,\pi\}}(j,k) \neq TM_F^{\{y,\pi\}}(j+m,k+m) \text{ for } m > 0.$$

Unlike the monetary regime, the fiscal regime is history-dependent. As debt is an important state variable, determining wealth, the backward- and forward-looking

[10]Note, therefore, that the cumulative effect on economic activity over time

$$\sum_{k=0}^{\infty} TM_F^{\{y,\pi\}}(j,k)$$

is a strictly decreasing function of j, and converges to zero as $j \to \infty$.

nature of the government's budget identity (2.6) plays an important role in shaping demand. In particular,

$$TM_F^{\{y,\pi\}}(j,j) \neq TM_F^{\{y,\pi\}}(0,0).$$

The effect on economic activity in the period in which the taxation shock is implemented depends on when in the past it was announced. Output and inflation continue to smoothly decay by a factor of λ even when the taxation shock is implemented.

Quantitative Illustration

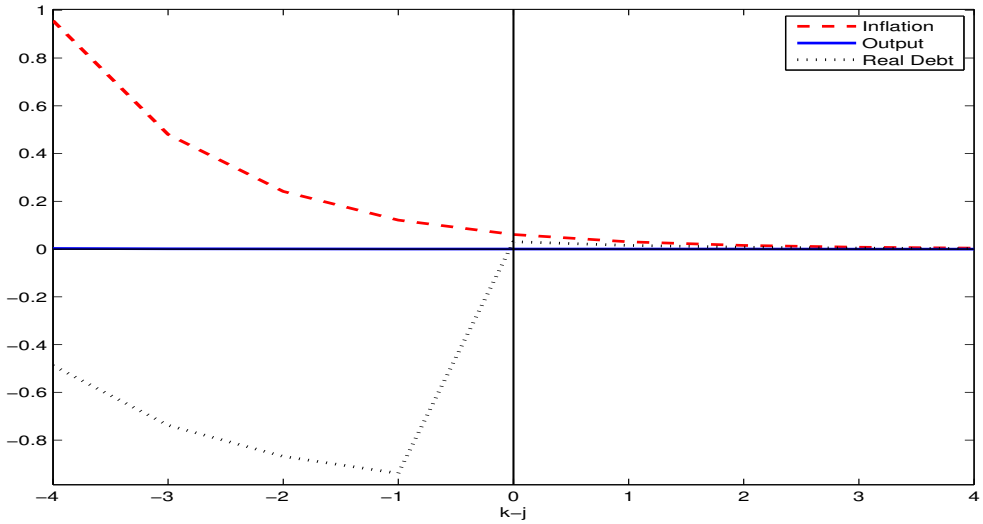
Figure 2.2(a) and 2.2(b) plot the announcement multipliers of taxation with flexible and sticky prices, respectively. I maintain the baseline calibration, now with $\phi = 0.5$ and $\psi = 0$ in the fiscal regime. Five points are worth highlighting. First, under flexible prices, the taxation multiplier on output is zero throughout the entire episode ($TM_F^y(j,k) \to 0$ as $\kappa \to \infty$). As a tax cut represents an increase to *nominal* wealth, firms merely adjust their prices to offset the increase in demand. Labor supply and demand remain unchanged.

Second, inflation increases in the announcement period as firms raise their prices to meet the higher nominal demand. Subsequently, under flexible prices, inflation decays by a factor of $\phi = 0.5$.[11] The reason lies in the Fisher equation (2.18). As real consumption, and thus the real interest rate, is unchanged throughout the episode, the positive nominal interest rate, in response to higher current inflation, must translate into positive next-period inflation. This is the well-documented Neo-

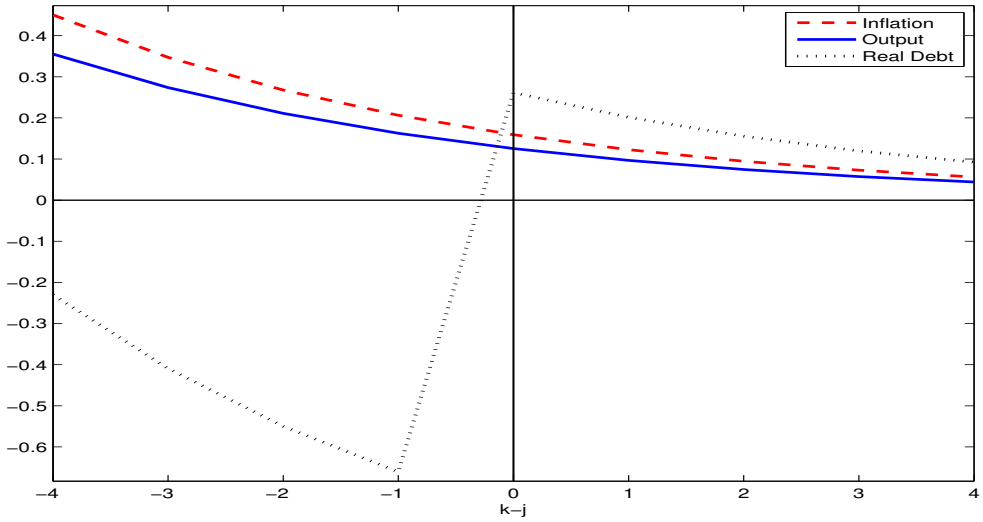
[11] As $\kappa \to \infty$, it can be shown that $\lambda \to \phi$.

Figure 2.2: Announcement Multipliers of Taxation in the Fiscal Regime

(a) Flexible Prices ($\theta = 0$)



(b) Sticky Prices ($\theta = 0.75$)



Notes: The graphs plot the impulse response functions of economic activity in response to a news shock to taxation in the fiscal regime. The shock is announced four ($j = 4$) quarters before it is implemented. The x-axis measures the number of periods before (-) and after (+) the spending shock is implemented. The y-axis measures the size of the multipliers, with the multiplier on debt being defined analogously as in (2.7) - (2.10).

Fisherian effect (e.g. see Cochrane (2016)): in the fiscal regime, a higher nominal interest rate results in higher growth of unbacked fiscal deficits, leading to persistent growth in nominal demand and prices. If $\phi = 0$, both the nominal and real interest rates are unchanged throughout the episode, leading to no persistence in inflation.

Third, with sluggish price adjustment, the increase in aggregate demand translates into higher output, as firms are forced to lower their mark-up. The inflationary dynamics follow a similar pattern as under flexible prices, but with a smaller magnitude and more persistence. As prices become infinitely sticky ($\kappa \to 0$), the goods market fails to clear, as firms have to produce and sell an infinite amount of goods to satisfy the higher nominal demand ($TM_F^y(j,k) \to \infty$).

Fourth, inflation continues its downward trend when the taxation shock hits the economy (at $k - j = 0$). The real value of debt jumps back to its saddle-path.

Fifth, in the fiscal regime, higher inflation decreases the real value of debt. Unlike in the monetary regime, the interest rate hike (or lack thereof if $\phi = 0$) is insufficient to offset the erosion of real debt though higher prices. Forward guidance in the fiscal regime is therefore not constrained by binding debt or deficit limits as conventional fiscal policy. On the contrary, it *creates* fiscal space by lowering the real value of debt. Moreover, if the announcement is made sufficiently early (i.e. if j is large), so that both output and inflation are back to their steady state values in the period in which the taxation shock is implemented, real debt is never above its steady state. In that case, forward guidance of fiscal policy allows the fiscal authority to stimulate economic activity without ever increasing the real value of its debt.

Government spending

As documented in Beck-Friis and Willems (2017), when $\rho = 0$, the impact and tail multipliers of government spending in the fiscal regime are:

$$GSM_F^y(0,k) = GSM_M^y(0,k) + TM_F^y(0,k) - \Big(\frac{b}{y}\Big)(1-\beta\phi)GSM_M^\pi(0,0)TM_F^y(0,k),$$

$$GSM_F^\pi(0,k) = \underbrace{GSM_M^\pi(0,k)}_{\text{Keynesian Effect}} + \underbrace{TM_F^\pi(0,k) - \Big(\frac{b}{y}\Big)(1-\beta\phi)GSM_M^\pi(0,0)TM_F^\pi(0,k)}_{\text{Nominal Wealth Effect}}.$$

The multipliers come in two parts. First, there are the standard Keynesian demand and supply effects present in the monetary regime. Labor demand shifts out because firms are unable to fully adjust their prices to meet the higher demand for their goods. Labor supply shifts out as government spending removes resources from private consumption.

Second, there is a 'nominal wealth effect' resulting from changes to the wealth of the private sector. The first nominal wealth term captures the direct wealth increase that the newly-issued, unbacked bonds represent. The second nominal wealth term captures a 'fiscal inflation tax' that results from Keynesian inflation revaluing the real value of the outstanding nominal debt (for a detailed discussion, see Beck-Friis and Willems, 2017).

Let us now turn to the announcement multipliers. The analytical expressions are complicated objects and are omitted as a result.[12] But the intuition follows the same

[12] As news shocks to government spending create Keynesian inflation in the periods preceding the spending shock (see Section 2.3.1), the 'fiscal inflation tax' becomes a complicated analytical expression.

logic as in the preceding sections. In particular, the timings of the Keynesian effects and the nominal wealth effect differ. The Keynesian effects occur when spending is higher. The wealth effect occurs immediately upon the announcement. As government spending in the fiscal regime produces a combination of the two effects, the impulse response functions display two bursts of activity across time.

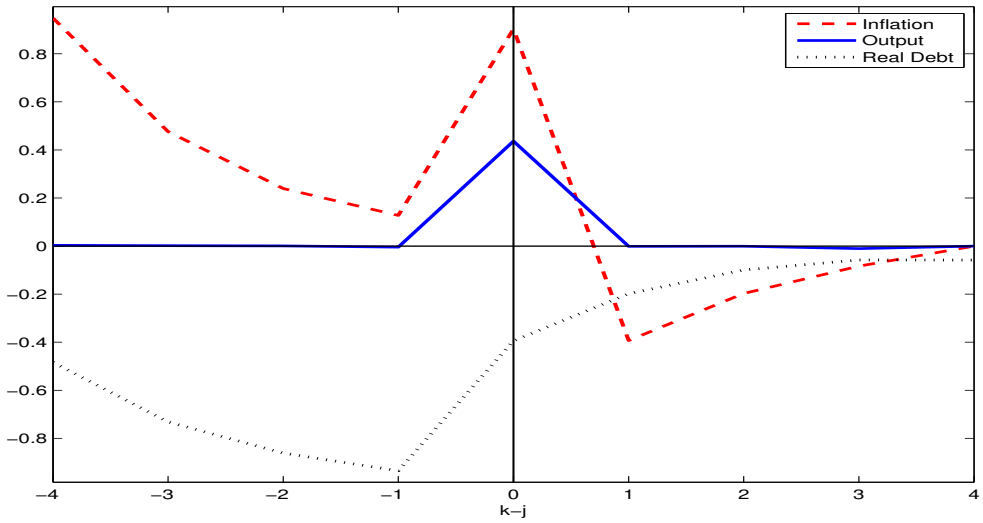
Figure 2.3(a) plots the announcement multipliers under flexible prices, setting $\phi = 0.5$ and $\rho = 0$. Output remains unchanged until the spending shock is implemented. It thereafter expands (as labor supply increases through the Keynesian effect), and then displays no persistence, as there is no persistence in the underlying spending shock. Inflation increases in the announcement period, as more nominal wealth translates into higher consumption demand, and then decays by a factor of $\phi = 0.5$, because of the Neo-Fisherian effect. Inflation increases again on spending impact, as firms raise their prices to meet the higher Keynesian demand for their goods, and then immediately turns negative to satisfy the Fisher equation.[13] Subsequently, inflation reverts back to steady state by a factor of $\phi = 0.5$ in each period.

Figure 2.3(b) plots the same graph, but with sticky prices and with persistence in government spending ($\rho = 0.5$). Here, I set $j = 8$ (so that the announcement is made two years prior to the spending shock) to more visibly disentangle the two effects. The underlying qualitative story remains the same as under flexible prices. But with sluggish price adjustments, increased consumption demand now translates into higher labor demand. Output therefore expands in the announcement period, similar to the taxation multiplier on output in Figure 2.2(b).

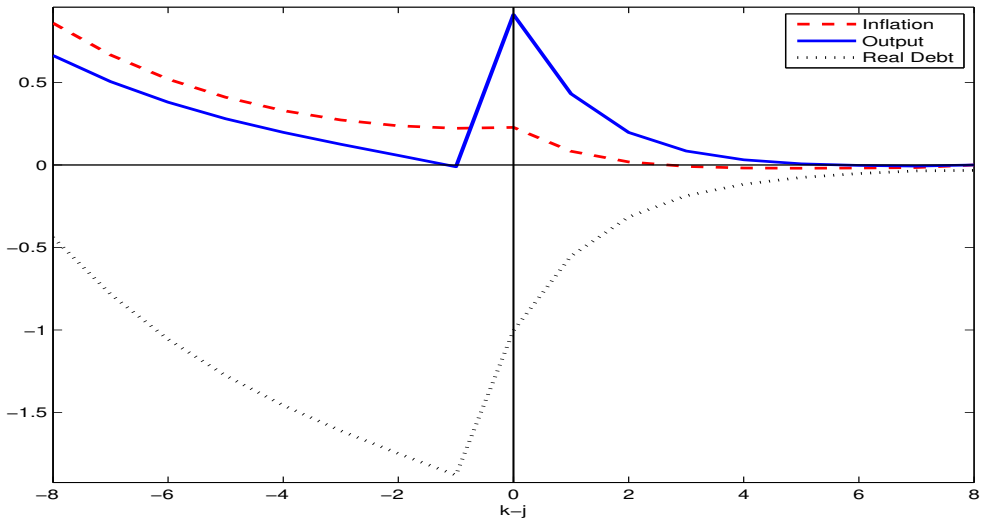
[13]The real interest rate increases by more than the nominal interest rate. Next-period inflation must therefore turn negative by the Fisher equation.

Figure 2.3: Announcement Multipliers of Spending in the Fiscal Regime

(a) Flexible Prices ($\theta = 0$) with No Persistence ($\rho = 0$)



(b) Sticky Prices ($\theta = 0.75$) with Persistence ($\rho = 0.5$)



Notes: The graphs plot the impulse response functions of economic activity in response to a news shock to government spending in the fiscal regime. The shock is announced four ($j = 4$) quarters before it is implemented in panel (a), and eight ($j = 8$) quarters before it is implemented in panel (b). The x-axis measures the number of periods before (-) and after (+) the spending shock is implemented. The y-axis measures the size of the multipliers, with the multiplier on debt being defined analogously as in (2.7) - (2.10).

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Note in Figure 2.3(a) and 2.3(b) that a news shock to government spending expands economic activity *without ever increasing the real value of government debt.*

2.3.3 Sensitivity analysis

How do the simulated effects depend on the calibration of parameters? In this section, I consider variations in six parameters—the coefficient of relative risk aversion (σ), the inverse of the elasticity of labor supply (ξ), the level of price stickiness (θ), the announcement period (j), the monetary policy coefficient (ϕ) and the persistence in government spending (ρ)—from the baseline calibration. Table 2.1 and 2.2 summarize the values of the spending multipliers in the monetary and fiscal regime, respectively.

Inverse Elasticity of Labor Supply & Coefficient of Relative Risk Aversion

Panels A and B give results for three values of the inverse elasticity of labor supply ($\xi = 0, 2, 5$) and three values of the coefficient of relative risk aversion ($\sigma = 1, 5, 10$).

Consider first the case $\xi = 0$, which corresponds to Hansen's (1985) model of indivisible labor. In this case, government spending leaves private consumption and inflation unchanged in the monetary regime.[14] No future shifts in private consumption and inflation imply, in turn, no change in the ex-ante real interest rate. So, in the monetary regime, both output and inflation remain unaffected by the fiscal news shock (cf. equations (2.19) and (3.29)). In the fiscal regime, however, wealth effects

[14]That is, when $\xi = 0$, the impact multipliers on output and inflation in the monetary regime are one and zero, respectively.

are present, giving rise to positive multipliers on output and inflation, even when labor is indivisible.

As the inverse elasticity of labor supply and the coefficient of relative risk aversion increase: (1) the impact multiplier on output decreases; and (2) the household becomes less tolerant to large variations in consumption over time. Both effects lead to a greater reduction in aggregate demand through the consumption smoothing motive. To see this, note with flexible prices ($\theta = 0$) that the inflationary response decreases as both parameters increase. With sticky prices, however, the direction of the multipliers depends on complex interaction effects between the two parameters.

Price Stickiness

Panels A and B vary the level of price stickiness across five values ($\theta = 0, 0.5, 0.75, 0.9, 1$). With flexible prices ($\theta = 0$), news shocks to government spending leave output unchanged. Labor supply remains intact, as fiscal policy only removes resources from private consumption when spending is higher, not in the periods preceding it. Labor demand remains unchanged, as firms merely adjust their prices to offset any changes in aggregate demand.

With sticky prices, the results vary across the two regimes. In the monetary regime, output contracts in all the calibrations (for $j = 4$). Aggregate demand falls as the household saves for the future drop in private consumption. Inflation decreases for low levels of price stickiness (as firms lower their prices to offset the lower demand for their goods) but increases for high levels of price stickiness (as firms raise their prices in anticipation of a higher future output gap). Moreover, as the level of price

stickiness increases, the impact multipliers on output and inflation converge to one and zero, respectively, thereby weakening the Keynesian transmission mechanisms (cf. equations (2.19) and (3.29)). As a result, in the sticky-price limit $\theta \to 1$, news shocks to government spending have no effect on economic activity in the periods preceding the spending shock.

In the fiscal regime, both output and inflation expand in all the calibrations. That is, the increase in private sector wealth more than offsets the contractionary effect of the Keynesian transmission mechanisms. As prices become more sticky, the increase in aggregate demand translates into higher output and lower inflation. As prices become infinitely sticky ($\theta \to 1$), the goods market fails to clear, as the firms have to produce and sell an infinite amount of goods to satisfy the higher nominal demand ($TM_F^y(j,k) \to \infty$).

Monetary Policy Strength

Panels C and D give results for six values of the monetary policy coefficient — three in the monetary regime ($\phi = 1.2, 5, 10$) and three in the fiscal regime ($\phi = 0, 0.5, 0.9$). A stronger central bank weakens the Keynesian transmission mechanisms (see equation (2.17)), but it also lowers the impact multipliers on output and inflation. As ϕ increases in the monetary regime, the net effect on output and inflation depends on the relative strengths of the two effects.

In the fiscal regime, a larger monetary policy coefficient reduces output in all the calibrations. Inflation, however, increases because of the Neo-Fisherian effect.

Table 2.1: Sensitivity Analysis: Government Spending in Monetary Regime

Panel A: Output ($j = 4$)						
		$\theta = 0$ (flex)	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.9$	$\theta \to 1$
$\sigma = 1$	$\xi = 0$	0	0	0	0	0
	$\xi = 2$	0	-0.01	-0.12	-0.06	0
	$\xi = 5$	0	-0.01	-0.12	-0.12	0
$\sigma = 5$	$\xi = 0$	0	0	0	0	0
	$\xi = 2$	0	-0.02	-0.04	-0.01	0
	$\xi = 5$	0	-0.02	-0.09	-0.03	0
$\sigma = 10$	$\xi = 0$	0	0	0	0	0
	$\xi = 2$	0	-0.01	-0.02	-0.01	0
	$\xi = 5$	0	-0.02	-0.05	-0.02	0

Panel B: Inflation ($j = 4$)						
		$\theta = 0$ (flex)	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.9$	$\theta \to 1$
$\sigma = 1$	$\xi = 0$	0	0	0	0	0
	$\xi = 2$	-0.12	-0.13	0.03	0.03	0
	$\xi = 5$	-0.16	-0.18	-0.05	0.07	0
$\sigma = 5$	$\xi = 0$	0	0	0	0	0
	$\xi = 2$	-0.21	-0.18	0.10	0.04	0
	$\xi = 5$	-0.42	-0.42	0.16	0.09	0
$\sigma = 10$	$\xi = 0$	0	0	0	0	0
	$\xi = 2$	-0.24	-0.18	0.11	0.04	0
	$\xi = 5$	-0.52	-0.45	0.23	0.10	0

Panel C: Output						
		$j = 0$ (impact)	$j = 1$	$j = 2$	$j = 4$	$j = 8$
$\phi = 1.2$	$\rho = 0.25$	0.86	-0.13	-0.12	-0.09	-0.04
	$\rho = 0.50$	0.80	-0.19	-0.17	-0.12	-0.05
	$\rho = 0.75$	0.67	-0.30	-0.26	-0.18	-0.05
$\phi = 2$	$\rho = 0.25$	0.78	-0.22	-0.19	-0.09	0.05
	$\rho = 0.50$	0.69	-0.29	-0.24	-0.09	0.07
	$\rho = 0.75$	0.55	-0.39	-0.29	-0.07	0.11
$\phi = 10$	$\rho = 0.25$	0.56	-0.25	0.04	0.05	-0.01
	$\rho = 0.50$	0.49	-0.25	-0.01	0.06	-0.01
	$\rho = 0.75$	0.44	-0.23	0.02	0.06	-0.01

Panel D: Inflation						
		$j = 0$ (impact)	$j = 1$	$j = 2$	$j = 4$	$j = 8$
$\phi = 1.2$	$\rho = 0.25$	0.17	0.13	0.09	0.03	-0.04
	$\rho = 0.50$	0.22	0.16	0.11	0.03	-0.06
	$\rho = 0.75$	0.28	0.19	0.11	-0.01	-0.13
$\phi = 2$	$\rho = 0.25$	0.14	0.07	0.01	-0.06	-0.04
	$\rho = 0.50$	0.16	0.07	-0.01	-0.09	-0.04
	$\rho = 0.75$	0.14	0.02	-0.07	-0.14	-0.04
$\phi = 10$	$\rho = 0.25$	0.05	-0.02	-0.03	0.00	0.00
	$\rho = 0.50$	0.04	-0.03	-0.04	0.00	0.00
	$\rho = 0.75$	0.02	-0.05	-0.04	0.00	0.00

Table 2.2: Sensitivity Analysis: Government Spending in Fiscal Regime

		Panel A: Output ($j = 4$)				
		$\theta = 0$ (flex)	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.9$	$\theta \to 1$
$\sigma = 1$	$\xi = 0$	0	0.49	0.88	1.23	$+\infty$
	$\xi = 2$	0	0.30	0.67	1.07	$+\infty$
	$\xi = 5$	0	0.20	0.52	0.94	$+\infty$
$\sigma = 5$	$\xi = 0$	0	0.10	0.18	0.25	$+\infty$
	$\xi = 2$	0	0.08	0.15	0.23	$+\infty$
	$\xi = 5$	0	0.07	0.13	0.22	$+\infty$
$\sigma = 10$	$\xi = 0$	0	0.05	0.09	0.12	$+\infty$
	$\xi = 2$	0	0.04	0.08	0.12	$+\infty$
	$\xi = 5$	0	0.03	0.07	0.11	$+\infty$
		Panel B: Inflation ($j = 4$)				
		$\theta = 0$ (flex)	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.9$	$\theta \to 1$
$\sigma = 1$	$\xi = 0$	1.90	1.19	0.65	0.29	0
	$\xi = 2$	1.89	1.44	0.93	0.44	0
	$\xi = 5$	1.88	1.59	1.12	0.58	0
$\sigma = 5$	$\xi = 0$	1.90	1.19	0.65	0.29	0
	$\xi = 2$	1.87	1.29	0.78	0.35	0
	$\xi = 5$	1.85	1.37	0.92	0.42	0
$\sigma = 10$	$\xi = 0$	1.90	1.19	0.65	0.29	0
	$\xi = 2$	1.86	1.26	0.76	0.33	0
	$\xi = 5$	1.83	1.33	0.89	0.39	0

		Panel C: Output				
		$j = 0$ (impact)	$j = 1$	$j = 2$	$j = 4$	$j = 8$
$\phi = 0$	$\rho = 0.25$	1.49	0.52	0.53	0.54	0.53
	$\rho = 0.50$	1.75	0.78	0.80	0.81	0.79
	$\rho = 0.75$	2.55	1.58	1.59	1.59	1.55
$\phi = 0.5$	$\rho = 0.25$	1.39	0.42	0.43	0.45	0.44
	$\rho = 0.50$	1.61	0.64	0.65	0.67	0.66
	$\rho = 0.75$	2.26	1.30	1.32	1.33	1.30
$\phi = 0.9$	$\rho = 0.25$	1.15	0.17	0.19	0.21	0.23
	$\rho = 0.50$	1.24	0.27	0.29	0.32	0.34
	$\rho = 0.75$	1.55	0.59	0.62	0.66	0.68
		Panel D: Inflation				
		$j = 0$ (impact)	$j = 1$	$j = 2$	$j = 4$	$j = 8$
$\phi = 0$	$\rho = 0.25$	0.61	0.56	0.52	0.48	0.44
	$\rho = 0.50$	0.87	0.81	0.76	0.71	0.67
	$\rho = 0.75$	1.61	1.52	1.45	1.38	1.30
$\phi = 0.5$	$\rho = 0.25$	0.77	0.71	0.68	0.63	0.58
	$\rho = 0.50$	1.11	1.04	0.99	0.93	0.86
	$\rho = 0.75$	2.06	1.97	1.89	1.79	1.68
$\phi = 0.9$	$\rho = 0.25$	1.15	1.10	1.06	1.00	0.92
	$\rho = 0.50$	1.68	1.62	1.56	1.48	1.37
	$\rho = 0.75$	3.18	3.08	2.99	2.86	2.67

Announcement Period

Panels C and D consider five values of the announcement date ($j = 0, 1, 2, 4, 8$). Not surprisingly, the earlier the announcement is made (i.e. the larger is j), the smaller is the effect in both regimes. In the limit $j \to \infty$, the multipliers tend to zero.

Note, however, that the multipliers converge to zero at vastly different speeds in the two regimes. In the monetary regime, an announcement made two years ($j = 8$) prior to the spending shock generates negligible effects. The same announcement in the fiscal regime, however, produces large movements in economic activity. The reason is that the household discounts the future wealth increase into present-value terms by a factor of $\beta = 0.99$ (when $\psi = 0$, see equations (2.24)-(2.25)). As a result, the effects are only marginally affected by how far in advance the announcement is made.

Persistence in Government Spending

Lastly, panels C and D give results for three values of persistence in government spending ($\rho = 0.25, 0.5, 0.75$). The results address the following question: does the length of the fiscal stimulus matter for economic conditions at the time of the announcement? The question is topical because, at the time of writing, there is uncertainty about the the length of the Trump administration's future fiscal policies.

In the monetary regime, the answer is: yes, but only marginally. The p's in (2.19) and (3.29) are independent of the persistence in government spending, ρ. So the length of expected fiscal stimulus only affects economic conditions at the time of the announcement through its effect on the impact multipliers. In general, a

larger persistence lowers the impact multiplayer on output, which in turn lowers the announcement multiplier on output through the consumption smoothing motive.

In the fiscal regime, however, the length of expected fiscal stimulus has a significant impact on economic activity in the announcement period. Every period of a fiscal deficit adds to private sector wealth through issuance of unbacked government debt.[15] On announcement, the household discounts all wealth increases into present-value terms. A higher persistence in government spending therefore translates into higher multipliers on announcement. In the limit $\rho \to 1$, the announcement multipliers on output and inflation explode to infinity.

2.4 Committing to a path of fiscal variables

In the preceding analysis, I considered news shocks to fiscal variables. In response to these shocks, the paths of fiscal variables, both before and after the fiscal shock, adjusted (endogenously) according to the fiscal rule. But forward guidance of monetary policy, as implemented by central banks in response to the Great Recession, looked different. Instead of announcing a shock to future monetary policy, central banks committed to a path of the nominal interest rate.

In this section, I consider the analogous policy experiment for the fiscal authority. That is, instead of announcing a shock to future fiscal policy, I let the fiscal authority

[15] Too see this result mathematically, suppose $\psi = 0$ and $\varepsilon^{\tau}_{t+i} < 0$ for $i \in \{0, 1, 2, ..., n-1\}$, where n is the number of taxation shocks. Suppose all the shocks are announced in period t. Then the aggregate effect on economic activity in period t is:

$$TM_F^{\{y,\pi\}}(0,0) + \beta TM_F^{\{y,\pi\}}(0,0) + ... + \beta^{n-1} TM_F^{\{y,\pi\}}(0,0) = \frac{1-\beta^n}{1-\beta} TM_F^{\{y,\pi\}}(0,0)$$

commit to a path of future fiscal variables.[16]

2.4.1 Can the path of taxes stimulate economic activity?

Changing the path of taxes affects economic activity only under certain conditions. To understand why, I start by stating the following result:

Result 2.1. *Suppose the economy reverts permanently to the monetary regime in period $T < \infty$. Then there is a unique, bounded solution for the system, for any (possibly switching) fiscal and monetary rules in periods $t < T$. In this solution, fiscal policy does not generate wealth effects.*

That is, debt cannot be 'temporarily' unbacked. If fiscal policy is Ricardian in the long run, it cannot be non-Ricardian in the short run. This result is not surprising. Fiscal solvency requires that the real value of government debt equates the present value of future primary surpluses. But the precise path of primary surpluses is irrelevant. Even if the fiscal authority puts real debt on an explosive path in the short run, the fact that it will raise taxes aggressively in the future ensures that the fiscal rule is Ricardian.[17]

As a result, neither time-contingent forward guidance of lump-sum taxation (in which the fiscal authority keeps taxes low for a given time period) nor state-contingent forward guidance of lump-sum taxation (in which the fiscal authority

[16] As government spending creates difficult inter-temporal Keynesian effects over time, I limit the analysis to specifying paths for taxation and/or debt, and thus omit any analysis on government spending.

[17] I shall refer to changes to the path of taxes as 'Ricardian' if the changes preserve the present value of primary surpluses. Any other changes to the path of taxes are 'non-Ricardian', and require the price level to adjust to restore fiscal solvency.

links the path of taxes to economic conditions) has any effect on economic activity if the private sector expects fiscal policy to revert to being passive in the long run. For taxation to generate wealth effects, the private sector has to assign a non-zero probability that the fiscal authority will credibly let nominal debt explode. See Chung et al. (2007) and Ascari et al. (2017) for a discussion on wealth effects under regime switching policy mixes.

2.4.2 Fiscal consolidation

The government may also manage expectations of permanent changes to fiscal variables. A topical example includes fiscal consolidation plans, in which the government commits to a path of primary surpluses or a path of debt, with the aim of permanently lowering the debt-to-gdp ratio. Many European countries (notably Greece, Italy, and the UK) announced such plans at the height of the sovereign debt crisis. Indeed, the bulk of political manifestos since 2011, in both the developed and developing world, have included debt reduction plans to reverse the worsened fiscal outlook in the aftermath of the Great Recession.

What are the effects of permanently reducing the debt-to-gdp ratio? First recall that, with lump-sum taxes, the steady state level of debt has no effect on the steady state level of economic activity (in either regime). It does, however, affect the steady state level of taxes. In particular, a lower level of debt implies a lower primary surplus needed to cover the interest cost of servicing that debt.[18] So a commitment to permanently lowering the debt level must also imply a commitment to permanently

[18]In steady state, the real value of debt, b, equals the steady state present value of primary surpluses, $(\tau - g)/(1 - \beta)$.

lowering long-run taxes.

The government may reduce the real value of debt through one of two policy instruments: either (1) it commits to an exogenous path of taxes, and lets debt endogenously adjust to the new target; or (2) it commits to an exogenous path of debt. As we shall see, the two are not necessarily equivalent.

Committing to a path of taxes

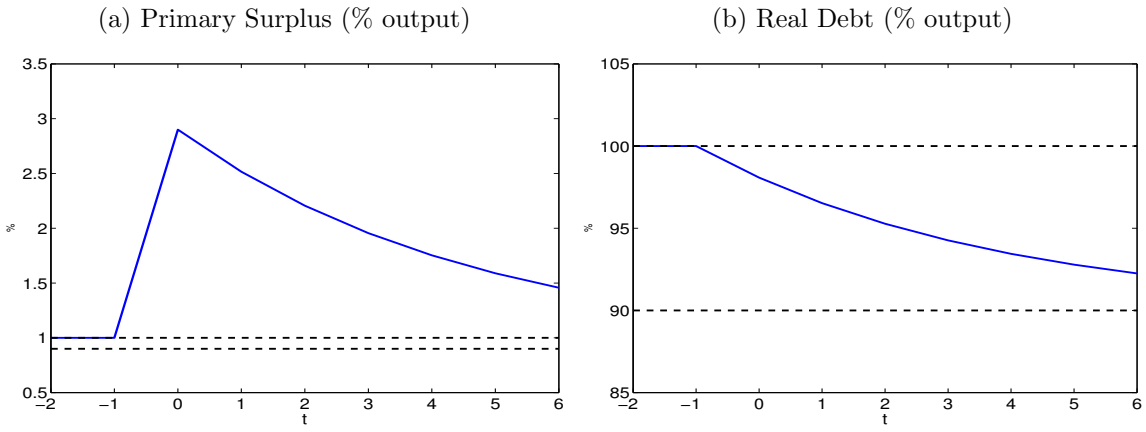
Let us begin by studying the case in which the government commits to a Ricardian path of taxes.

Result 2.2. *A permanent reduction in government debt through committing to a Ricardian path of taxes yields a unique equilibrium with no effect on economic activity. This result is independent of whether monetary policy is active ($\phi > 1$) or passive ($\phi < 1$).*

That is, the government can reduce the real value of debt through changing the path of taxes without taking the response of the central bank into account. Any such Ricardian path involves cutting taxes in the long run (to ensure that the steady state level of primary surpluses is consistent with a lower steady state level of debt), but raising taxes in the short run (to ensure that the path preserves the present value of primary surpluses).

Figure 2.4 depicts these dynamics with the following simulation. I use the baseline calibration, and let the government in period $t = 0$ announce a new steady state debt-to-output ratio (b/y) from 100% to 90%. The government implements this fiscal

Figure 2.4: Example of Debt Reduction in Monetary Regime



Notes: The top and bottom dashed lines denote the old and new steady state levels, respectively.

consolidation through committing to a Ricardian path of taxes.[19] The effects are trivial. Primary surpluses immediately increase, and then converge to a lower steady state consistent with the lower steady state level of debt. Real debt falls throughout the episode and ultimately settles at its new target. Output and inflation remain unaffected by Ricardian Equivalence: higher short-run taxes are offset by lower long-run taxes to ensure that the life-time wealth of the private sector is unchanged.

Let us now turn to non-Ricardian paths of taxes.

Result 2.3. *A permanent reduction in government debt through committing to a non-Ricardian path of taxes yields a unique equilibrium only if monetary policy is passive.*

That is, the fiscal authority can inflate away its debt only by taking the response

[19]I use the following path of taxes. I linearize the fiscal rule (2.4) and the government's flow budget identity (2.6) around the new steady state, and back out the Ricardian path of taxes that is consistent with an equilibrium of no inflation. Note that as the steady state level of economic activity is independent of the steady state level of debt, the announcement does not change the linearized Euler equation (2.1), Phillips Curve (2.2) or Taylor rule (2.3).

of the central bank into account. There is, of course, nothing surprising in this result. An active fiscal policy has to be paired with a passive monetary policy to yield a unique bounded solution. If monetary policy is active, a non-Ricardian change in the path of taxes produces explosive dynamics.

Figure 2.5 depicts the effects of reducing debt by committing to a non-Ricardian path of taxes under a passive monetary policy ($\phi = 0.5$). In period $t = 0$, the government commits to pegging taxes at a lower level forever. Lower taxes increase the wealth of the private sector, leading to higher inflation and output through an increase in aggregate demand. Higher prices, in turn, ensures that real debt falls throughout the episode and ultimately settles at its new target.

The reverse results holds true if we flip the signs in the example above:

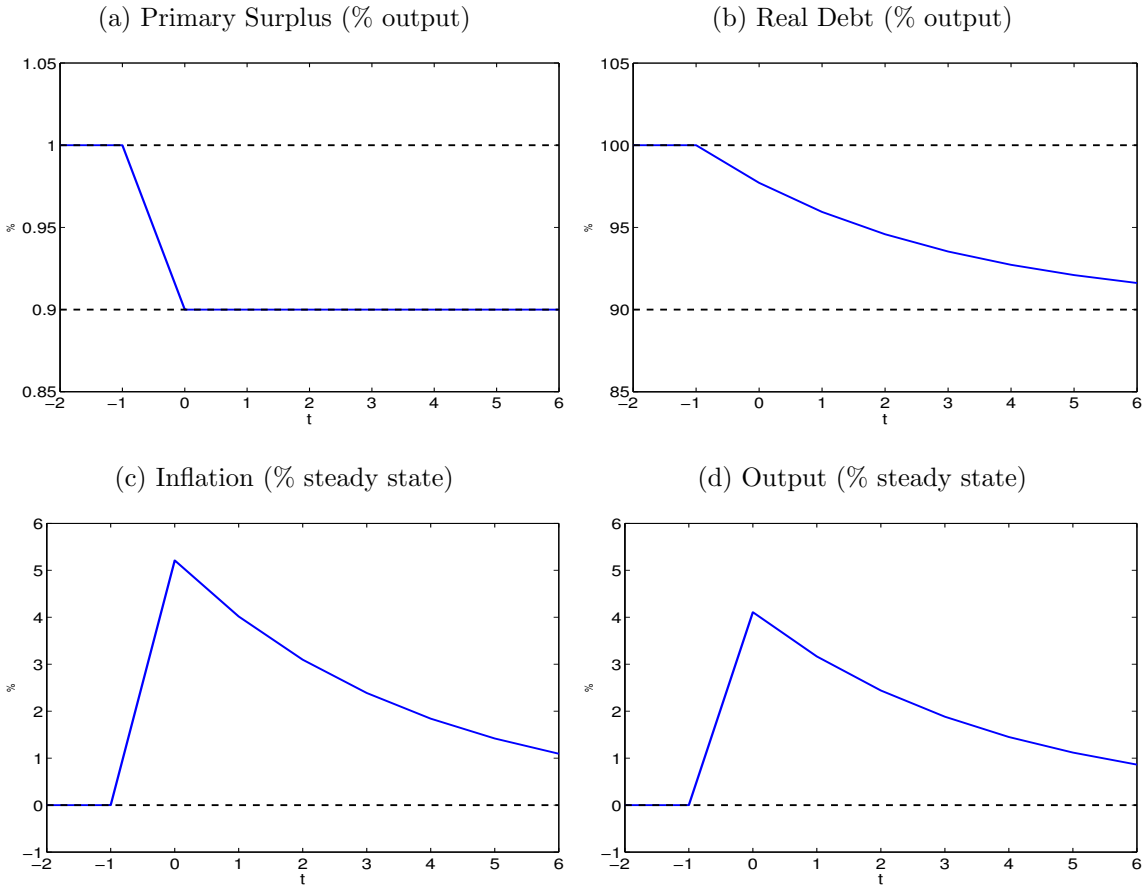
Result 2.4. *If monetary policy is passive, a permanent increase in taxes increases the real value of debt.*

Increasing taxes at the zero lower bound (when the policy rate is de facto unresponsive to inflation) may thus be self defeating: it increases the real value of debt through a fall in aggregate demand and prices. This result is consistent with the empirical findings in Cherif and Hasanov (2012), De GrauI and Ji (2013), and Holland and Portes (2012). To reduce debt, the fiscal authority must crucially commit to a path of lower, not higher, long-run taxes.

Committing to a path of debt

Instead of committing to an exogenous path of taxes, the government may commit to an exogenous path of real debt. But this policy experiment yields determinacy

Figure 2.5: Example of Debt Reduction in Fiscal Regime



Notes: For panels (a) and (b): the top and bottom dashed lines denote the old and new steady state levels, respectively.

under only certain conditions:

Result 2.5. *A permanent reduction in government debt through committing to a path of government debt yields a uniquely bounded solution only if monetary policy is active ($\phi > 1$). If monetary policy is passive ($\phi < 1$), committing to a path of government debt leads to indeterminacy.*

The logic runs as follows. If monetary policy is active, the fiscal authority cannot inflate away its debt. All changes in debt have to be implemented via changes in primary surpluses. Committing to a path of debt therefore implies committing to a path of primary surpluses. But if monetary policy is passive, the government may use both inflation and primary surpluses to reduce its debt. To yield a unique equilibrium, it must therefore specify *how* it will reduce the debt. So with passive monetary policy, announcing a reduction in debt is insufficient; the government also needs to commit to a path of primary surpluses.[20]

Figure 2.6 illustrates this result with the following simulation. In period $t = 0$, the government commits to reducing the debt-to-output ratio by 1% per quarter for five consecutive quarters. It subsequently pegs debt at 95% of output. Figure 2.6(a) plots this path of debt. Crucially, the government does not commit to any particular path of taxes.

The blue line plots the unique solution if monetary policy is active. All changes

[20]Mathematically, an active monetary policy pins down a unique path of inflation. In particular, in the absence of government spending shocks, inflation has to be zero in all periods to satisfy the Euler equation (2.1) and Phillips curve (2.2). For a given path of inflation, there is then a one-to-one mapping between changes in real debt and changes in taxes through the government's flow budget identity (2.6). If monetary policy is passive, however, there is an infinite number of bounded paths of inflation that satisfy the Euler equation (2.1) and Phillips curve (2.2).

in debt are implemented through changes in primary surpluses. Primary surpluses rise for five quarters, and then settle at a lower steady state level. Inflation and output remain unchanged throughout the episode.

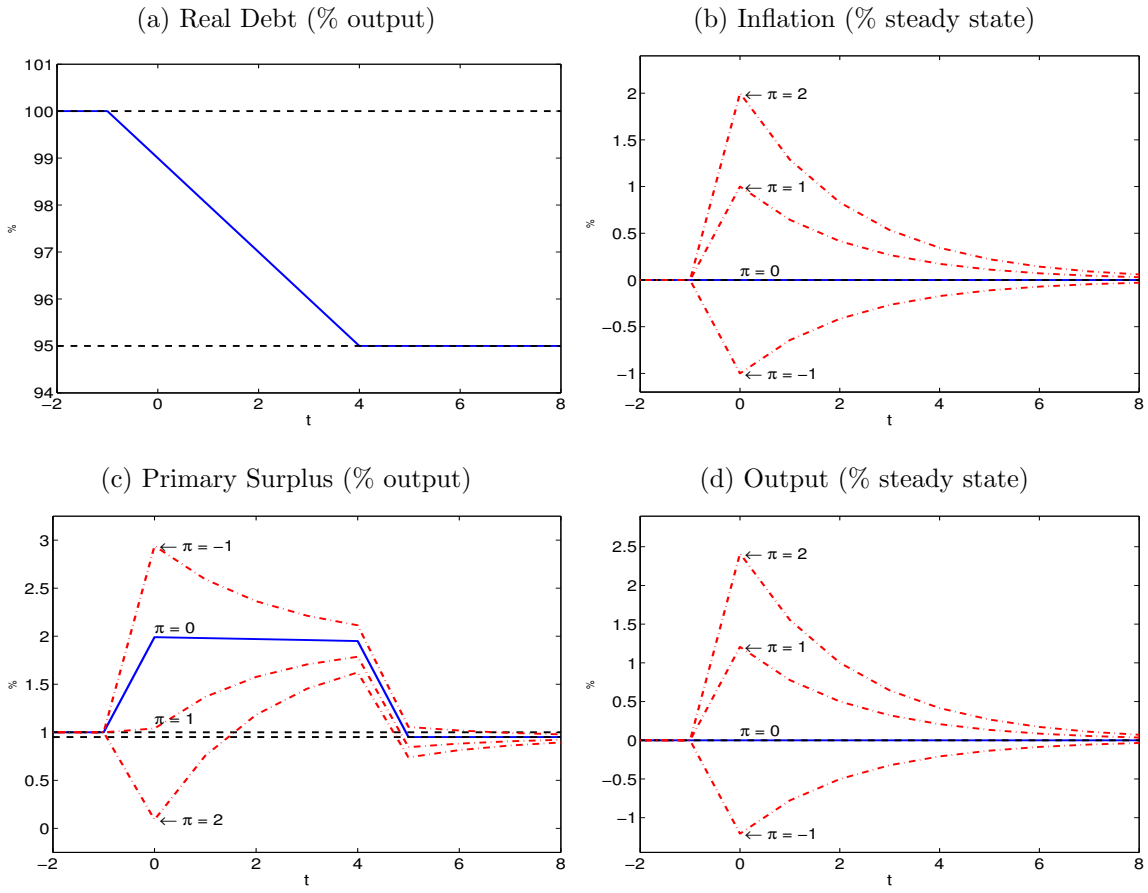
The red lines plot three sample solutions when monetary policy is passive (I set $\phi = 0$). I categorize them by their initial jump in inflation ($\hat{\pi}_0 = 2\%, \hat{\pi}_0 = 1\%$, and $\hat{\pi}_0 = -1\%$). In all these solutions, the path of taxes adjusts to ensure that the debt ratio falls in line with the government's commitment and with the path of inflation. Note that there is an infinite number of bounded solutions—here I plot only three—all of which are consistent with the fall in the debt ratio plotted in Figure 2.6(a). To pick a unique solution, the fiscal authority has to specify the initial value of taxes (or, equivalently, the initial value of inflation).

2.5 Concluding remarks

In this paper, I have studied the effects on economic activity from expected changes to future fiscal variables. I documented four transmission mechanisms, the timings of which vary, and the sizes of which crucially depend on the level of price stickiness. In general, forward guidance of fiscal policy produces larger effects in the fiscal regime, as expected changes to the path of taxes lead to immediate wealth effects on aggregate demand. Through these wealth effects, announcing a future tax shock, or committing to a path of future tax cuts, can expand economic activity without ever increasing the real value of government debt.

Forward guidance of fiscal policy is subject to the same issues as forward guidance

Figure 2.6: Fiscal Consolidation Through Committing to a Path of Debt



Notes: In this simulation, the government commits to the path of debt plotted in panel (a). The blue line plots the unique equilibrium if monetary policy is active. The red lines plot three sample solutions when monetary policy is passive. For panels (a) and (c): the top and bottom dashed lines denote the old and new steady state levels, respectively.

of monetary policy. Managing expectations is hard. Time inconsistencies may impair credibility. And without credibility, announcements to change the future path of fiscal variables may just be seen as empty promises. Arguably, fiscal authorities may be less credible than monetary authorities given the political time horizon. I have not addressed these issues in this paper. Nor have I discussed the effects of reneging on a promise. Instead, I have tied the hands of the fiscal authority to keep past promises, much like the central bank in 'Odyssean' forward guidance. What would happen, then, if the government reneged on a previous (and credible) promise? In the monetary regime, the output gap and inflation would immediately revert back to zero. In this regime, economic activity is dictated by forward-looking jump variables, so reneging on a past promise merely brings economic activity back to its steady state. In the fiscal regime, however, debt is an important state variable, determining wealth, so any unexpected changes to the path of debt changes aggregate demand. Reneging on a previous promise to cut taxes, for instance, lowers the wealth of the private sector, reducing economic activity through a fall in aggregate demand.

I end by stressing that the analysis takes place in a simple environment with no capital, no distortionary taxes, and no long-term government debt. Changing any of these assumptions would change the results. The monetary regime would no longer be history-independent. The steady state level of debt would affect steady state economic activity. And with long-term debt, fiscal news shocks may translate into future rather than current inflation (see Cochrane, 2001). I leave these extensions for future research.

Appendix

In this appendix, I spell out the full model and describe the solution-technique employed to derive the closed-form solution presented in the body of the Chapter 1 and 2.

Chapter 1 assumes per-period CRRA-preferences of the form:

$$U(c_t, n_t) = \frac{1}{1-\sigma} c_t^{1-\sigma} - \frac{1}{1+\xi} n_t^{1+\xi},$$

where c_t is time-t consumption, n_t is time-t labor suppy, σ is the coefficient of relative risk aversion, and ξ is the inverse of the elasticity of labor supply.

Chapter 2 employs a more generalized version of the model, without specifying the functional form of preferences. I therefore proceed by presenting the model and its solution for Chapter 2. The solution for Chapter 1 then follows trivially by substituting CRRA-preferences as utility function in the results.

2.A Model

2.A.1 Households

There is an infinitely-lived representative household whose preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t), \tag{2.26}$$

where $U(c_t, n_t)$ is the per-period utility function, c_t is time-t consumption, n_t is time-t labor suppy.

The household maximizes (2.26) subject to a sequence of period-budget constraints of the form:

$$P_t c_t + P_t \tau_t + Q_t B_t = W_t n_t + B_{t-1} + \Omega_t,$$

where P_t is the price level in period t, τ_t is real lump-sum taxation, and B_t denotes nominal, one-period, risk-less government bonds. Markets are complete. The bonds are purchased in period t and mature in period $t+1$. Each bond pays one unit at maturity and costs Q_t ($\equiv 1/i_t$) in period t. Finally, W_t is the nominal wage rate in period t, while Ω_t denotes nominal profits that are rebated from the firm to the household.

This problem yields the two familiar intra- and inter-temporal optimality condi-

tions:

$$\frac{W_t}{P_t} = -\frac{U_n(c_t, n_t)}{U_c(c_t, n_t)}, \tag{2.27}$$

$$\frac{1}{i_t} = \beta \mathbb{E}_t \left[\frac{U_c(c_{t+1}, n_{t+1})}{U_c(c_t, n_t)} \frac{1}{\pi_{t+1}} \right], \tag{2.28}$$

where $\pi_{t+1} \equiv P_{t+1}/P_t$ is inflation in period $t+1$.

2.A.2 Firms

The economy is populated by a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated intermediary good by using the same linear production technology given by:

$$y_t(i) = n_t(i).$$

A representative and perfectly competitive final goods producer bundles the intermediary goods into a final one that can be consumed. It does so with a constant returns technology:

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1.$$

The aggregate production function is therefore equal to:

$$y_t = n_t.$$

Each intermediate firm is subject to the familiar Calvo-price setting friction, implying that it can reset its price with probability $(1-\theta)$ in any given period. This

structure gives rise to the following standard first-order condition for a re-optimizing firm:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ M_{t,t+k} y_{t+k|t} \left(P_t^* - \frac{\epsilon}{\epsilon - 1} W_t \right) \right\} = 0,$$

where $M_{t,t+k}$ is the stochastic discount factor applied to nominal payoffs, P_t^* is the price that maximizes the value of the firm, and $y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t.

2.A.3 Government

The fiscal authority adheres to a taxation rule, which states that real taxes are adjusted in proportion to steady-state deviations of debt:

$$\left(\frac{\tau_t}{\tau}\right)^{(\tau/y)} = \left(\frac{b_{t-1}}{b}\right)^{\psi(b/y)} \exp(\varepsilon_t^\tau),\ \psi \geq 0,$$

where $b_t \equiv B_t/P_t$ is the real debt stock and ε_t^τ is an exogenous i.i.d. taxation shock.

Real government spending g_t follows:

$$\left(\frac{g_t}{g}\right)^{(g/y)} = \left(\frac{g_{t-1}}{g}\right)^{\rho(g/y)} \exp(\varepsilon_t^g),\ \phi \geq 0,$$

where ε_t^g is an exogenous i.i.d. government spending shock.

The monetary authority follows a simple interest rate rule of the Taylor-type:

$$\frac{i_t}{i} = \pi_t^\phi.$$

The government budget identity is given by:

$$Q_t B_t + P_t s_t = B_{t-1},$$

where $s_t \equiv \tau_t - g_t$ is the real primary surplus.

Finally, the aggregate resource constraint is:

$$y_t = c_t + g_t.$$

2.B Linearized model

Let variables without subscript denote steady state variables. Let 'hat' denote deviation from steady state, and 'tilde' deviation from steady state output (y), i.e.

$$\hat{x}_t = \frac{x_t - x}{x},$$
$$\hat{x}_t = \frac{x_t - x}{y}.$$

Then the model reduces to the following linearized form:

Linearized aggregate resource constraint:

$$\hat{y}_t = \left(\frac{c}{y}\right)\hat{c}_t + \tilde{g}_t.$$

Linearized production function:

$$\hat{y}_t = \hat{n}_t.$$

Linearized Euler equation:

$$-U_c \cdot [\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}] = U_{cc} \cdot c[\hat{c}_{t+1} - \hat{c}_t] + U_{cn} \cdot n[\hat{n}_{t+1} - \hat{n}_t],$$

where U_c, U_{cc} and U_{cn} denote the steady state values of the partial derivatives. Combining the Euler equation with the resource constraint and production function yields

$$\hat{y}_t - \alpha_1 \tilde{g}_t = \mathbb{E}_t \hat{y}_{t+1} - \alpha_1 \mathbb{E}_t \tilde{g}_{t+1} - \alpha_2 [\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}],$$

where

$$\alpha_1 = \frac{U_{cc}}{U_{cc} + U_{cn}} \in \mathbb{R},$$
$$\alpha_2 = -\frac{U_c}{y(U_{cc} + U_{cn})} \in \mathbb{R}.$$

The optimal intra-temporal consumption/labour choice:

$$\hat{w}_t - \hat{p}_t = c\Big[\frac{U_{nc}}{U_n} - \frac{U_{cc}}{U_c}\Big]\hat{c}_t + n\Big[\frac{U_{nn}}{U_n} - \frac{U_{cn}}{U_c}\Big]\hat{n}_t.$$

Combining with the resource constraint and production function gives

$$\hat{w}_t - \hat{p}_t = \alpha_3 \hat{y}_t - \alpha_4 \tilde{g}_t,$$

where

$$\alpha_3 = y\Big[\frac{U_{nc}+U_{nn}}{U_n} - \frac{U_{cc}+U_{cn}}{U_c}\Big] \in \mathbb{R}$$
$$\alpha_4 = y\Big[\frac{U_{nc}}{U_n} - \frac{U_{cc}}{U_c}\Big] \in \mathbb{R}.$$

Phillips curve:

$$\hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa\hat{mc}_t,$$

where κ is the standard measure of price rigidity. With a linear production function, real marginal cost equals the real wage. we can therefore rewrite the Phillips curve as

$$\hat{\pi}_t = \beta\mathbb{E}_t[\hat{\pi}_{t+1}] + \kappa\alpha_3\hat{y}_t - \kappa\alpha_4\tilde{g}_t.$$

Interest rate rule:

$$\hat{i}_t = \phi\hat{\pi}_t.$$

Taxation rule:

$$\tilde{\tau}_t = \psi\tilde{b}_{t-1} + \varepsilon_t^{\tau}.$$

Government spending:

$$\tilde{g}_t = \rho\tilde{g}_{t-1} + \varepsilon_t^{g}.$$

Government budget identity:

$$\tilde{b}_t = \frac{1}{\beta}\tilde{b}_{t-1} - \frac{1}{\beta}(\tilde{\tau}_t - \tilde{g}_t) - \frac{1}{\beta}\frac{b}{y}\hat{\pi}_t + \frac{b}{y}\hat{i}_t.$$

2.B.1 Linearized marginal utility of consumption

By Taylor expansion of the marginal utility of consumption around the steady state, we get:

$$\begin{aligned}
\frac{\partial U_c(c_t, n_t)}{\partial \tilde{g}_t} &= \frac{\partial}{\partial \tilde{g}_t}\Big[U_c + U_{cc}(c_t - c) + U_{cn}(n_t - n)\Big], \\
&= \frac{\partial}{\partial \tilde{g}_t}\Big[U_c + y\tilde{c}_t U_{cc} + y\tilde{y}_t U_{cn}\Big], \\
&= \frac{\partial}{\partial \tilde{g}_t}\Big[U_c + y(\tilde{y}_t - \tilde{g}_t)U_{cc} + y\tilde{y}_t U_{cn}\Big], \\
&= y(U_{cc} + U_{cn})GSM_M^y(0,0) - yU_{cc}, \\
&= y(U_{cc} + U_{cn})\Big[GSM_M^y(0,0) - \alpha_1\Big],
\end{aligned}$$

where $\alpha_1 = U_{cc}/(U_{cc} + U_{cn})$. The marginal utility of consumption therefore increases in response to government spending if $GSM_M^y(0,0) > \alpha_1$. This addresses the statement raised in Section 2.3.1: Government Spending.

2.C Model solution

The economy is characterized by the following linearized equations

$$\begin{aligned}
\hat{y}_t - \alpha_1 \tilde{g}_t &= \mathbb{E}_t \hat{y}_{t+1} - \alpha_1 \mathbb{E}_t \tilde{g}_{t+1} - \alpha_2 [\hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1}], \\
\hat{\pi}_t &= \beta \mathbb{E}_t [\hat{\pi}_{t+1}] + \kappa \alpha_3 \hat{y}_t - \kappa \alpha_4 \tilde{g}_t, \\
\hat{i}_t &= \phi \hat{\pi}_t, \\
\tilde{b}_t &= \frac{1}{\beta} \tilde{b}_{t-1} - \frac{1}{\beta} (\tilde{\tau}_t - \tilde{g}_t) - \frac{1}{\beta} \frac{b}{y} \pi_t + \frac{b}{y} \hat{i}_t, \\
\tilde{\tau}_t &= \psi \tilde{b}_{t-1} + \varepsilon_t^{\tau} \\
\tilde{g}_t &= \rho \tilde{g}_{t-1} + \varepsilon_t^{g},
\end{aligned}$$

which in vector form can be written as

$$\begin{bmatrix} 1 & \alpha_2 & 0 & 0 & -\alpha_1 \\ 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t \hat{y}_{t+1} \\ \mathbb{E}_t \hat{\pi}_{t+1} \\ \tilde{b}_t \\ \tilde{\tau}_{t+1} \\ \tilde{g}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & \alpha_2 \phi & 0 & 0 & -\alpha_1 \\ -\kappa \alpha_3 & 1 & 0 & 0 & \kappa \alpha_4 \\ 0 & -\frac{1}{\beta} \frac{b}{y} (1 - \beta \phi) & \frac{1}{\beta} & -\frac{1}{\beta} & \frac{1}{\beta} \\ 0 & -\psi \frac{b}{y} \frac{1}{\beta} (1 - \beta \phi) & \psi \frac{1}{\beta} & -\psi \frac{1}{\beta} & \psi \frac{1}{\beta} \\ 0 & 0 & 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \tilde{b}_{t-1} \\ \tilde{\tau}_t \\ \tilde{g}_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \varepsilon_{t+1}^{\tau} \\ \varepsilon_{t+1}^{g} \end{bmatrix}$$

In period t, suppose that the government announces that it will implement a non-zero government spending shock and a non-zero taxation shock in period $t+j$. That is, let $\mathbb{E}_t \varepsilon_{t+i}^{g} = \mathbb{E}_t \varepsilon_{t+i}^{\tau} = 0$ for $i \in \{0, 1, ..., j-1, j+1, j+2, ...\}$ and $\mathbb{E}_t \varepsilon_{t+j}^{g}, \mathbb{E}_t \varepsilon_{t+j}^{\tau} \neq 0$.

How do we find a solution to this system? Any innovations $\delta_{t+1}^{y}, \delta_{t+1}^{\pi}$ such that

$\mathbb{E}_t\delta^y_{t+1} = \mathbb{E}_t\delta^\pi_{t+1} = 0$ satisfying the system

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{\pi}_{t+1} \\ \tilde{b}_t \\ \tilde{\tau}_{t+1} \\ \tilde{g}_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{\alpha_2}{\beta} & 0 & 0 & \alpha_1 \\ 0 & \frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \alpha_2\phi & 0 & 0 & -\alpha_1 \\ -\kappa\alpha_3 & 1 & 0 & 0 & \kappa\alpha_4 \\ 0 & -\frac{1}{\beta}\frac{b}{y}(1-\beta\phi) & \frac{1}{\beta} & -\frac{1}{\beta} & \frac{1}{\beta} \\ 0 & -\psi\frac{b}{y}\frac{1}{\beta}(1-\beta\phi) & \psi\frac{1}{\beta} & -\psi\frac{1}{\beta} & \psi\frac{1}{\beta} \\ 0 & 0 & 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \tilde{b}_{t-1} \\ \tilde{\tau}_t \\ \tilde{g}_t \end{bmatrix} + \begin{bmatrix} \alpha_1\varepsilon^g_{t+1} + \delta^y_{t+1} \\ \delta^\pi_{t+1} \\ 0 \\ \varepsilon^\tau_{t+1} \\ \varepsilon^g_{t+1} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\beta}(\beta + \alpha_2\kappa\alpha_3) & -\frac{\alpha_2}{\beta}(1-\phi\beta) & 0 & 0 & -\frac{1}{\beta}[\kappa\alpha_2\alpha_4 + \beta\alpha_1(1-\rho)] \\ -\frac{\kappa\alpha_3}{\beta} & \frac{1}{\beta} & 0 & 0 & \frac{\kappa\alpha_4}{\beta} \\ 0 & -\frac{1}{\beta}\frac{b}{y}(1-\beta\phi) & \frac{1}{\beta} & -\frac{1}{\beta} & \frac{1}{\beta} \\ 0 & -\psi\frac{b}{y}\frac{1}{\beta}(1-\beta\phi) & \psi\frac{1}{\beta} & -\psi\frac{1}{\beta} & \psi\frac{1}{\beta} \\ 0 & 0 & 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \tilde{b}_{t-1} \\ \tilde{\tau}_t \\ \tilde{g}_t \end{bmatrix}$$

$$+ \begin{bmatrix} \alpha_1\varepsilon^g_{t+1} + \delta^y_{t+1} \\ \delta^\pi_{t+1} \\ 0 \\ \varepsilon^\tau_{t+1} \\ \varepsilon^g_{t+1} \end{bmatrix}$$

$$X_{t+1} = AX_t + \delta_{t+1},$$

forms a solution. The eigenvalues and corresponding eigenvectors of A are

$$\lambda_1 = 0, \qquad \leftrightarrow q_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda_2 = \rho, \qquad \leftrightarrow q_2 = \begin{bmatrix} q_{1,2} \\ q_{2,2} \\ q_{3,2} \\ q_{4,2} \\ q_{5,2} \end{bmatrix}$$

$$\lambda_3 = \frac{1}{2\beta}\Big[1+\beta+\kappa\alpha_2\alpha_3 - \sqrt{(1+\beta+\kappa\alpha_2\alpha_3)^2 - 4\beta(1+\phi\kappa\alpha_2\alpha_3)}\Big], \quad \leftrightarrow q_3 = \begin{bmatrix} (1-\beta\lambda_3)[1-\beta\lambda_3-\psi] \\ \kappa\alpha_3[1-\beta\lambda_3-\psi] \\ \kappa\alpha_3(b/y)(1-\beta\phi) \\ \psi\kappa\alpha_3(b/y)(1-\beta\phi) \\ 0 \end{bmatrix}$$

$$\lambda_4 = \frac{1}{2\beta}\Big[1+\beta+\kappa\alpha_2\alpha_3 + \sqrt{(1+\beta+\kappa\alpha_2\alpha_3)^2 - 4\beta(1+\phi\kappa\alpha_2\alpha_3)}\Big], \quad \leftrightarrow q_4 = \begin{bmatrix} (1-\beta\lambda_4)[1-\beta\lambda_4-\psi] \\ \kappa\alpha_3[1-\beta\lambda_4-\psi] \\ \kappa\alpha_3(b/y)(1-\beta\phi) \\ \psi\kappa\alpha_3(b/y)(1-\beta\phi) \\ 0 \end{bmatrix}$$

$$\lambda_5 = \frac{1}{\beta}(1-\psi), \qquad \leftrightarrow q_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \psi \\ 0 \end{bmatrix}$$

where

$$
\begin{aligned}
q_{1,2} &\equiv \left[\alpha_1(1-\rho) + \kappa\alpha_2\alpha_4\frac{(\phi-\rho)}{(1-\rho\beta)}\right], \\
q_{2,2} &\equiv \frac{(1-\rho)\kappa(\alpha_1\alpha_3 - \alpha_4)}{1-\beta\rho}, \\
q_{5,2} &\equiv \left[1-\rho + \kappa\alpha_2\alpha_3\frac{(\phi-\rho)}{(1-\rho\beta)}\right], \\
q_{3,2} &\equiv \frac{(1-\beta\phi)(b/y)q_{2,2} - q_{5,2}}{[1-\psi-\beta\rho]}, \\
q_{4,2} &\equiv \psi q_{3,2}.
\end{aligned}
$$

Define

$$
Q \equiv [q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5], \quad \Lambda \equiv \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \end{bmatrix},
$$

and let $q_{i,h}$ denote the element of Q at row i and column h.

By eigendecomposition of A, we then get

$$\begin{aligned}
X_t &= AX_{t-1} + \delta_t, \\
X_t &= Q\Lambda Q^{-1} X_{t-1} + \delta_t, \\
Q^{-1}X_t &= \Lambda Q^{-1} X_{t-1} + Q^{-1}\delta_t, \\
Z_t &= \Lambda Z_{t-1} + V_t, && \text{defining } Z_t \equiv Q^{-1}X_t \text{ and } V_t \equiv Q^{-1}\delta_t, \\
z_{i,t} &= \lambda_j z_{i,t-1} + v_{i,t}, && \text{for } i \in \{1,2,3,4,5\}.
\end{aligned}$$

To rule out explosive solutions, for any eigenvalue λ_i that is outside the unit circle, we then need

$$\begin{aligned}
&\mathbb{E}_t[z_{i,t+j}] = 0, \\
&\mathbb{E}_t[\lambda_i z_{i,t+j-1} + v_{i,t+j}] = 0, \\
&\mathbb{E}_t[\lambda_i^2 z_{i,t+j-2} + \lambda_i v_{i,t+j-1} + v_{i,t+j}] = 0, \\
&... \\
&\mathbb{E}_t\Big[\lambda_i^{j+1} z_{i,t-1} + \sum_{k=0}^{j} \lambda_i^k v_{i,t+j-k}\Big] = 0, \\
&\lambda_i^j v_{i,t} + \mathbb{E}_t v_{i,t+j} = 0, \qquad (2.29)
\end{aligned}$$

where the last line follows from $\mathbb{E}_t v_{i,t+k} = 0$ for $k \in \{1,2,...,j-1\}$, and that $z_{i,t-1} = 0$ as we start from steady state.

Note that $\lambda_i^j v_{i,t} + \mathbb{E}_t v_{i,t+j}$ is the i's element of column vector $\Lambda^j V_t + \mathbb{E}_t V_{t+j}$. So

to find the solution, write

$$\Lambda^j V_t + \mathbb{E}_t V_{t+j} = \Lambda^j Q^{-1}\delta_t + Q^{-1}\mathbb{E}_t\delta_{t+j} \tag{2.30}$$

$$Q[\Lambda^j V_t + \mathbb{E}_t V_{t+j}] = \underbrace{Q\Lambda^j Q^{-1}}_{A^j}\delta_t + \mathbb{E}_t\delta_{t+j} \tag{2.31}$$

where the first equality follows by definition, and the second line follows from pre-multiplying both sides by Q. Combining (2.29) with (2.31) gives a system of five equations with five unknowns, two of which are δ_t^y and δ_t^y. We can then obtain the response of output and inflation by recalling that $\hat{y}_t = \delta_t^y$ and $\hat{\pi}_t = \delta_t^\pi$.

Before deriving the five equations, we first need to find an expression for $A^j = Q\Lambda^j Q^{-1}$. Inverting Q gives

$$Q^{-1}(:, 1-3) = \begin{bmatrix} 0 & 0 & -\psi/(1-\psi) \\ 0 & 0 & 0 \\ q_{2,4}/(q_{1,3}q_{2,4} - q_{2,3}q_{1,4}) & -q_{1,4}/(q_{1,3}q_{2,4} - q_{2,3}q_{1,4}) & 0 \\ -q_{2,3}/(q_{1,3}q_{2,4} - q_{2,3}q_{1,4}) & q_{1,3}/(q_{1,3}q_{2,4} - q_{2,3}q_{1,4}) & 0 \\ q_{3,4}(q_{2,3} - q_{2,4})/(q_{1,3}q_{2,4} - q_{2,3}q_{1,4}) & q_{3,4}(q_{1,4} - q_{1,3})/(q_{1,3}q_{2,4} - q_{2,3}q_{1,4}) & 1/(1-\psi) \end{bmatrix}$$

$$Q^{-1}(:, 4-5) = \begin{bmatrix} 1/(1-\psi) & 0 \\ 0 & 1/q_{5,2} \\ 0 & (q_{1,4}q_{2,2} - q_{2,4}q_{1,2})/[q_{5,2}(q_{1,3}q_{2,4} - q_{2,3}q_{1,4})] \\ 0 & (q_{2,3}q_{5,2} - q_{1,3}q_{2,2})/[q_{5,2}(q_{1,3}q_{2,4} - q_{2,3}q_{1,4})] \\ -1/(1-\psi) & -q_{3,2}/q_{5,2} - \Big[q_{3,4}[q_{1,2}(q_{2,3} - q_{2,4}) + q_{2,2}(q_{1,4} - q_{1,3})]\Big]/(q_{1,3}q_{2,4} - q_{2,3}q_{1,4}) \end{bmatrix}$$

which, after some algebra, gives (with $\cdot$ denoting unknown variable)

$$Q\Lambda^j Q^{-1} = \begin{bmatrix} \omega_{1,1}(j) & \omega_{1,2}(j) & 0 & 0 & \cdot \\ \omega_{2,1}(j) & \omega_{2,2}(j) & 0 & 0 & \cdot \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & 0 & 0 & 0 & \rho^j \end{bmatrix} \tag{2.32}$$

where,

$$\begin{aligned} \omega_{1,1}(j) &= \frac{\lambda_3^j q_{1,3} q_{2,4} - \lambda_4^j q_{1,4} q_{2,3}}{q_{1,3} q_{2,4} - q_{2,3} q_{1,4}} \\ \omega_{1,2}(j) &= \frac{q_{1,3} q_{1,4} (\lambda_4^j - \lambda_3^j)}{q_{1,3} q_{2,4} - q_{2,3} q_{1,4}} \\ \omega_{2,1}(j) &= \frac{q_{2,3} q_{2,4} (\lambda_3^j - \lambda_4^j)}{q_{1,3} q_{2,4} - q_{2,3} q_{1,4}} \\ \omega_{2,2}(j) &= \frac{-\lambda_3^j q_{2,3} q_{1,4} + \lambda_4^j q_{2,4} q_{1,3}}{q_{1,3} q_{2,4} - q_{2,3} q_{1,4}} \end{aligned}$$

Substituting the expressions for q's from the eigenvectors gives

$$\omega_{1,1}(j) = \frac{\lambda_3^j (1 - \beta\lambda_3) - \lambda_4^j (1 - \beta\lambda_4)}{\beta(\lambda_4 - \lambda_3)} \tag{2.33}$$

$$\omega_{1,2}(j) = \frac{(1 - \beta\lambda_3)(1 - \beta\lambda_4)(\lambda_4^j - \lambda_3^j)}{\kappa\alpha_3\beta(\lambda_4 - \lambda_3)} \tag{2.34}$$

$$\omega_{2,1}(j) = \frac{\kappa\alpha_3 (\lambda_3^j - \lambda_4^j)}{\beta(\lambda_4 - \lambda_3)} \tag{2.35}$$

$$\omega_{2,2}(j) = \frac{\lambda_4^j (1 - \beta\lambda_3) - \lambda_3^j (1 - \beta\lambda_4)}{\beta(\lambda_4 - \lambda_3)} \tag{2.36}$$

We are now in a position to find the multipliers in the two regimes.

2.C.1 Monetary regime

In the monetary regime, two eigenvalues λ_3 and λ_4 are outside the unit circle. From (2.29), we then need $\lambda_i^j v_{i,t} + \mathbb{E}_t v_{i,t+j} = 0$ for $i \in \{3, 4\}$.

Impact and tail multipliers

Let us start with the impact and tail multipliers, i.e. $j = 0$, so that $v_{3,t} = v_{4,t} = 0$. We then have

$$\delta_t = QV_t,$$

$$\begin{bmatrix} \alpha_1 \varepsilon_t^g + \delta_t^y \\ \delta_t^\pi \\ 0 \\ \varepsilon_t^\tau \\ \varepsilon_t^g \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 \end{bmatrix} \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \\ v_{5,t} \end{bmatrix},$$

$$= \begin{bmatrix} 0 & q_{1,2} & 0 \\ 0 & q_{2,2} & 0 \\ 1 & q_{3,2} & 1 \\ 1 & q_{4,2} & \psi \\ 0 & q_{5,2} & 0 \end{bmatrix} \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{5,t} \end{bmatrix}, \qquad \text{as } v_{3,t} = v_{4,t} = 0,$$

which implies that

$$
\begin{aligned}
v_{1,t} &= \frac{1}{1-\psi}\varepsilon_t^{\tau}, \\
v_{2,t} &= \frac{1}{q_{5,2}}\varepsilon_t^{g}, \\
v_{5,t} &= -\frac{1}{1-\psi}\varepsilon_t^{\tau} - \frac{q_{3,2}}{q_{5,2}}\varepsilon_t^{g}.
\end{aligned}
$$

To find the impact and tail multipliers, write

$$
\begin{aligned}
X_t &= QZ_t, \\
\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \tilde{b}_{t-1} \\ \tilde{\tau}_t \\ \tilde{g}_t \end{bmatrix} &= \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \\ z_{4,t} \\ z_{5,t} \end{bmatrix}, \\
&= \begin{bmatrix} 0 & q_{1,2} & 0 \\ 0 & q_{2,2} & 0 \\ 1 & q_{3,2} & 1 \\ 1 & q_{4,2} & \psi \\ 0 & q_{5,2} & 0 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{5,t} \end{bmatrix}, \qquad \text{as } z_{3,t} = z_{4,t} = 0,
\end{aligned}
$$

where

$$
\begin{aligned}
z_{1,t} &= v_{1,t} = \frac{1}{1-\psi}\varepsilon_t^{\tau}, \\
z_{2,t} &= v_{2,t} = \frac{1}{q_{5,2}}\varepsilon_t^{g}, \\
z_{5,t} &= v_{5,t} = -\frac{1}{1-\psi}\varepsilon_t^{\tau} - \frac{q_{3,2}}{q_{5,2}}\varepsilon_t^{g}.
\end{aligned}
$$

and for $k \in \{1, 2, ...\}$

$$
\begin{aligned}
z_{1,t+k} &= \lambda_1 z_{1,t+k-1} = 0, \\
z_{2,t+k} &= \lambda_2 z_{2,t+k-1} = \rho z_{2,t+k-1}, \\
z_{5,t+k} &= \lambda_5 z_{5,t+k-1}.
\end{aligned}
$$

The impact and tail multipliers of taxation in the monetary regime are therefore

$$
\begin{aligned}
TM_M^{y}(0,k) &\equiv \frac{\partial \hat{y}_{t+k}}{\partial \tilde{\tau}_t} = 0, \\
TM_M^{\pi}(0,k) &\equiv \frac{\partial \hat{\pi}_{t+k}}{\partial \tilde{\tau}_t} = 0,
\end{aligned}
$$

and the impact and tail multipliers of government spending in the monetary regime are

$$
GSM_M^{y}(0,k) \equiv \frac{\partial \hat{y}_{t+k}}{\partial \tilde{g}_t} = \frac{\partial \hat{y}_{t+k}}{\partial \varepsilon_t^{g}} = \rho^k \frac{q_{1,2}}{q_{5,2}}, \tag{2.37}
$$

$$
GSM_M^{\pi}(0,k) \equiv \frac{\partial \hat{\pi}_{t+k}}{\partial \tilde{g}_t} = \frac{\partial \hat{\pi}_{t+k}}{\partial \varepsilon_t^{g}} = \rho^k \frac{q_{2,2}}{q_{5,2}}. \tag{2.38}
$$

Announcement multipliers

I now let $j \neq 0$, so that

$$\delta_t = [\delta_t^y, \delta_t^\pi, 0, 0, 0]',$$
$$\mathbb{E}_t \delta_{t+j} = [\alpha_1 \varepsilon_{t+j}^g, 0, 0, \varepsilon_{t+j}^\tau, \varepsilon_{t+j}^g]'.$$

From (2.31), we then have

$$Q[\Lambda^j V_t + \mathbb{E}_t V_{t+j}] = Q\Lambda^j Q^{-1}\delta_t + \mathbb{E}_t \delta_{t+j}$$
$$\begin{bmatrix} 0 & q_{1,2} & 0 \\ 0 & q_{2,2} & 0 \\ 1 & q_{3,2} & 1 \\ 1 & q_{4,2} & \psi \\ 0 & q_{5,2} & 0 \end{bmatrix} \begin{bmatrix} \lambda_1^j v_{1,t} + \mathbb{E}_t v_{1,t+j} \\ \lambda_2^j v_{2,t} + \mathbb{E}_t v_{2,t+j} \\ \lambda_5^j v_{5,t} + \mathbb{E}_t v_{5,t+j} \end{bmatrix} = \begin{bmatrix} \omega_{1,1}(j) & \omega_{1,2}(j) \\ \omega_{2,1}(j) & \omega_{2,2}(j) \\ \cdot & \cdot \\ \cdot & \cdot \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_t^y \\ \delta_t^\pi \end{bmatrix} + \begin{bmatrix} \alpha_1 \varepsilon_{t+j}^g \\ 0 \\ 0 \\ \varepsilon_{t+j}^\tau \\ \varepsilon_{t+j}^g \end{bmatrix}$$

This is a system of five equations with five unknown. The last equation in this system gives

$$\lambda_2^j v_{2,t} + \mathbb{E}_t v_{2,t+j} = \frac{1}{q_{5,2}} \varepsilon_{t+j}^g,$$

so that the system reduces to

$$\begin{bmatrix} \omega_{1,1}(j) & \omega_{1,2}(j) \\ \omega_{2,1}(j) & \omega_{2,2}(j) \end{bmatrix} \begin{bmatrix} \delta_t^y \\ \delta_t^\pi \end{bmatrix} = \begin{bmatrix} q_{1,2}/q_{5,2} - \alpha_1 \\ q_{2,2}/q_{5,2} \end{bmatrix} \varepsilon_{t+j}^g$$

with solution

$$\begin{aligned}
\begin{bmatrix} \delta_t^y \\ \delta_t^\pi \end{bmatrix} &= \frac{1}{\omega_{1,1}(j)\omega_{2,2}(j) - \omega_{1,2}(j)\omega_{2,1}(j)} \begin{bmatrix} \omega_{2,2}(j) & -\omega_{1,2}(j) \\ -\omega_{2,1}(j) & \omega_{1,1}(j) \end{bmatrix} \begin{bmatrix} q_{1,2}/q_{5,2} - \alpha_1 \\ q_{2,2}/q_{5,2} \end{bmatrix} \varepsilon_{t+j}^g \\
&= \frac{1}{(\lambda_3\lambda_4)^j} \begin{bmatrix} \omega_{2,2}(j) & -\omega_{1,2}(j) \\ -\omega_{2,1}(j) & \omega_{1,1}(j) \end{bmatrix} \begin{bmatrix} q_{1,2}/q_{5,2} - \alpha_1 \\ q_{2,2}/q_{5,2} \end{bmatrix} \varepsilon_{t+j}^g \\
&= \frac{1}{(\lambda_3\lambda_4)^j} \begin{bmatrix} \omega_{2,2}(j) & -\omega_{1,2}(j) \\ -\omega_{2,1}(j) & \omega_{1,1}(j) \end{bmatrix} \begin{bmatrix} GSM_M^y(0,0) - \alpha_1 \\ GSM_M^\pi(0,0) \end{bmatrix} \varepsilon_{t+j}^g
\end{aligned}$$

We thus get

$$\begin{bmatrix} GSM_M^y(j,0) \\ GSM_M^\pi(j,0) \end{bmatrix} = \begin{bmatrix} p_{1,1}(j) & p_{1,2}(j) \\ p_{2,1}(j) & p_{2,2}(j) \end{bmatrix} \begin{bmatrix} GSM_M^y(0,0) - \alpha_1 \\ GSM_M^\pi(0,0) \end{bmatrix},$$

where,

$$p_{1,1}(j) = \frac{\lambda_4^j(1-\beta\lambda_3) - \lambda_3^j(1-\beta\lambda_4)}{(\lambda_3\lambda_4)^j\beta(\lambda_4-\lambda_3)}, \tag{2.39}$$

$$p_{1,2}(j) = -\frac{(1-\beta\lambda_3)(1-\beta\lambda_4)(\lambda_4^j - \lambda_3^j)}{(\lambda_3\lambda_4)^j\kappa\alpha_3\beta(\lambda_4-\lambda_3)}, \tag{2.40}$$

$$p_{2,1}(j) = -\frac{\kappa\alpha_3(\lambda_3^j - \lambda_4^j)}{(\lambda_3\lambda_4)^j\beta(\lambda_4-\lambda_3)}, \tag{2.41}$$

$$p_{2,2}(j) = \frac{\lambda_3^j(1-\beta\lambda_3) - \lambda_4^j(1-\beta\lambda_4)}{(\lambda_3\lambda_4)^j\beta(\lambda_4-\lambda_3)}, \tag{2.42}$$

using the expressions for ω's in (2.33) - (2.36).

In the flexible price limit ($\kappa \to \infty$), one can show that

$$\begin{aligned} p_{1,1}(j) &\to 0, \\ p_{1,2}(j) &\to 0, \\ p_{2,1}(j) &\to \frac{1}{\phi^j \alpha_2}, \\ p_{2,2}(j) &\to \frac{1}{\phi^j}. \end{aligned}$$

These expressions then give equations (2.16) and (2.17) in the body of Chapter 2.

I lastly show that, under flexible prices, inflation decreases in the periods leading up to the spending shock for all utility functions that are separable in consumption and labor ($U_{cn} = 0$). Such preferences imply that $\alpha_1 = 1$ and $\alpha_3 > \alpha_4$. Using the expressions for the impact multipliers in (2.37) - (2.38), and letting $\kappa \to \infty$, we get

$$\begin{aligned} GSM_M^{\pi}(j,0) &= \frac{1}{\phi^j \alpha_2}\left[\frac{\alpha_4}{\alpha_3} - 1\right] + \frac{1}{\phi^j}\frac{(1-\rho)(\alpha_3 - \alpha_4)}{\alpha_2 \alpha_3 (\phi - \rho)}, \\ &= \frac{(\alpha_3 - \alpha_4)(1-\phi)}{\phi^j \alpha_2 \alpha_3} < 0, \end{aligned} \tag{2.43}$$

as desired.

2.C.2 Fiscal regime

In the fiscal regime, two eigenvalues λ_4 and λ_5 are outside the unit circle. From (2.29), we then need $\lambda_i^j v_{i,t} + \mathbb{E}_t v_{i,t+j} = 0$ for $i \in \{4, 5\}$.

Impact and tail multipliers

Let us start with the impact and tail multipliers, i.e. $j = 0$, so that $v_{4,t} = v_{5,t} = 0$. We have

$$\delta_t = QV_t,$$

$$\begin{bmatrix} \alpha_1 \varepsilon_t^g + \delta_t^y \\ \delta_t^\pi \\ 0 \\ \varepsilon_t^\tau \\ \varepsilon_t^g \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 \end{bmatrix} \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \\ v_{4,t} \\ v_{5,t} \end{bmatrix},$$

$$= \begin{bmatrix} 0 & q_{1,2} & q_{1,3} \\ 0 & q_{2,2} & q_{2,3} \\ 1 & q_{3,2} & q_{3,3} \\ 1 & \psi q_{3,2} & \psi q_{3,3} \\ 0 & q_{5,2} & 0 \end{bmatrix} \begin{bmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \end{bmatrix}$$

which implies that

$$\begin{aligned} v_{1,t} &= \frac{1}{1-\psi} \varepsilon_t^\tau, \\ v_{2,t} &= \frac{1}{q_{5,2}} \varepsilon_t^g, \\ v_{3,t} &= -\frac{1}{q_{3,3}} \left[\frac{1}{1-\psi} \varepsilon_t^\tau + \frac{q_{3,2}}{q_{5,2}} \varepsilon_t^g \right]. \end{aligned}$$

To find the impact and tail multipliers, write

$$X_t = QZ_t,$$

$$\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \tilde{b}_{t-1} \\ \tilde{\tau}_t \\ \tilde{g}_t \end{bmatrix} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \\ z_{4,t} \\ z_{5,t} \end{bmatrix},$$

$$= \begin{bmatrix} 0 & q_{1,2} & q_{1,3} \\ 0 & q_{2,2} & q_{2,3} \\ 1 & q_{3,2} & q_{3,3} \\ 1 & q_{4,2} & q_{4,3} \\ 0 & q_{5,2} & 0 \end{bmatrix} \begin{bmatrix} z_{1,t} \\ z_{2,t} \\ z_{3,t} \end{bmatrix}$$

where

$$z_{1,t} = v_{1,t} = \frac{1}{1-\psi}\varepsilon_t^{\tau},$$
$$z_{2,t} = v_{2,t} = \frac{1}{q_{5,2}}\varepsilon_t^{g},$$
$$z_{3,t} = v_{3,t} = -\frac{1}{q_{3,3}}\left[\frac{1}{1-\psi}\varepsilon_t^{\tau} + \frac{q_{3,2}}{q_{5,2}}\varepsilon_t^{g}\right].$$

and for $k \in \{1, 2, ...\}$

$$z_{1,t+k} = \lambda_1 z_{1,t+k-1} = 0,$$

$$z_{2,t+k} = \lambda_2 z_{2,t+k-1} = \rho z_{2,t+k-1},$$

$$z_{3,t+k} = \lambda_3 z_{3,t+k-1}.$$

The taxation multipliers in the fiscal regime are therefore

$$TM_F^y(0,k) \equiv -\frac{\partial \hat{y}_{t+k}}{\partial \tilde{\tau}_t} = -\frac{\partial \hat{y}_{t+k}}{\partial \varepsilon_t^\tau} = \lambda_3^k \frac{q_{1,3}}{q_{3,3}(1-\psi)}, \tag{2.44}$$

$$TM_F^\pi(0,k) \equiv -\frac{\partial \hat{\pi}_{t+k}}{\partial \tilde{\tau}_t} = -\frac{\partial \hat{\pi}_{t+k}}{\partial \varepsilon_t^\tau} = \lambda_3^k \frac{q_{2,3}}{q_{3,3}(1-\psi)}. \tag{2.45}$$

The government spending multipliers ('GSM') in the Fiscal ('F') regime are

$$
\begin{aligned}
GSM_F^y(0,k) \equiv \frac{\partial \hat{y}_{t+k}}{\partial \tilde{g}_t} &= \frac{\partial \hat{y}_{t+k}}{\partial \varepsilon_t^g} \\
&= \rho^k \frac{q_{1,2}}{q_{5,2}} - \lambda_3^k \frac{q_{1,3}}{\kappa\alpha_3(b/y)(1-\beta\phi)} \frac{q_{3,2}}{q_{5,2}}, \\
&= \rho^k \frac{q_{1,2}}{q_{5,2}} - \lambda_3^k \frac{q_{1,3}}{\kappa\alpha_3(b/y)(1-\beta\phi)[1-\psi-\beta\rho]} \left[(1-\beta\phi)(b/y)\frac{q_{2,2}}{q_{5,2}} - 1 \right], \\
&= GSM_M^y(0,k) + \frac{[1-\psi]}{[1-\psi-\beta\rho]} \left[TM_F^y(0,k) - \lambda_3^k \frac{q_{1,3}(1-\beta\phi)}{\kappa\alpha_3(1-\beta\phi)[1-\psi]} \frac{q_{2,2}}{q_{5,2}} \right], \\
&= GSM_M^y(0,k) + \frac{[1-\psi]}{[1-\psi-\beta\rho]} \left[TM_F^y(0,k) - (b/y)(1-\beta\phi) \times TM_F^y(0,k) \times GSM_M^\pi(0,0) \right] \\
&= GSM_M^y(0,k) + \frac{1}{1-\beta'\rho} \left[1 - \left(\frac{b}{y}\right)(1-\beta\phi) \times GSM_M^\pi(0,0) \right] TM_F^y(0,k), \\
&= GSM_M^y(0,k) + TM_F^y(0,k) \sum_{i=0}^{\infty} (\beta'\rho)^i \left[1 - \left(\frac{b}{y}\right)(1-\beta\phi) \times GSM_M^\pi(0,0) \right], \\
&= \underbrace{GSM_M^y(0,k)}_{\text{Keynesian Effects}} + TM_F^y(0,k) \underbrace{\sum_{i=0}^{\infty} (\beta')^i \left[\rho^i - \left(\frac{b}{y}\right)(1-\beta\phi) \times GSM_M^\pi(0,i) \right]}_{\text{Time 0 PDV of Nominal Wealth Increase}},
\end{aligned}
$$

where,

$$\beta' \equiv \frac{\beta}{1-\psi}.$$

For $\rho = 0$, the term collapses to expression (1.17) in Chapter 1, where $\lambda = \lambda_3$.

Similarly,

$$
\begin{aligned}
GSM_F^{\pi}(0,k) \equiv \frac{\partial \hat{\pi}_{t+k}}{\partial \tilde{g}_t} &= \frac{\partial \hat{\pi}_{t+k}}{\partial \varepsilon_t^g} \\
&= \rho^k \frac{q_{2,2}}{q_{5,2}} - \lambda_3^k \frac{q_{2,3}}{\kappa\alpha_3(b/y)(1-\beta\phi)} \frac{q_{3,2}}{q_{5,2}}, \\
&= \rho^k \frac{q_{2,2}}{q_{5,2}} - \lambda_3^k \frac{q_{2,3}}{\kappa\alpha_3(b/y)(1-\beta\phi)[1-\psi-\beta\rho]} \left[(1-\beta\phi)(b/y)\frac{q_{2,2}}{q_{5,2}} - 1 \right], \\
&= GSM_M^{\pi}(0,k) + \frac{[1-\psi]}{[1-\psi-\beta\rho]} \left[TM_F^{\pi}(0,k) - \lambda_3^k \frac{q_{2,3}(1-\beta\phi)}{\kappa\alpha_3(1-\beta\phi)[1-\psi]} \frac{q_{2,2}}{q_{5,2}} \right], \\
&= GSM_M^{\pi}(0,k) + \frac{[1-\psi]}{[1-\psi-\beta\rho]} \left[TM_F^{\pi}(0,k) - (b/y)(1-\beta\phi) \times TM_F^{\pi}(0,k) \times GSM_M^{\pi}(0,0) \right] \\
&= GSM_M^{\pi}(0,k) + \frac{1}{1-\beta'\rho} \left[1 - \left(\frac{b}{y}\right)(1-\beta\phi) \times GSM_M^{\pi}(0,0) \right] TM_F^{\pi}(0,k), \\
&= GSM_M^{\pi}(0,k) + TM_F^{\pi}(0,k) \sum_{i=0}^{\infty} (\beta'\rho)^i \left[1 - \left(\frac{b}{y}\right)(1-\beta\phi) \times GSM_M^{\pi}(0,0) \right], \\
&= \underbrace{GSM_M^{\pi}(0,k)}_{\text{Keynesian Effects}} + TM_F^{\pi}(0,k) \underbrace{\sum_{i=0}^{\infty} (\beta')^i \left[\rho^i - \left(\frac{b}{y}\right)(1-\beta\phi) \times GSM_M^{\pi}(0,i) \right]}_{\text{Time 0 PDV of Nominal Wealth Increase}}.
\end{aligned}
$$

Announcement multipliers

Now let $j \neq 0$. I only consider a news shock to taxation, so that

$$
\begin{aligned}
\delta_t &= [\delta_t^y, \delta_t^{\pi}, 0, 0, 0]', \\
\mathbb{E}_t \delta_{t+j} &= [0, 0, 0, \varepsilon_{t+j}^{\tau}, 0]'.
\end{aligned}
$$

To obtain the announcement multipliers, instead of using (2.31), I will use (2.30).

The last two rows of (2.30) give

$$\Lambda^j V_t + \mathbb{E}_t V_{t+j} = \Lambda^j Q^{-1}\delta_t + Q^{-1}\mathbb{E}_t\delta_{t+j}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \frac{1}{q_{1,3}q_{2,4} - q_{2,3}q_{1,4}} \begin{bmatrix} -\lambda_4^j q_{2,3} & \lambda_4^j q_{1,3} \\ \lambda_5^j q_{3,4}(q_{2,3} - q_{2,4}) & \lambda_5^j q_{3,4}(q_{1,4} - q_{1,3}) \end{bmatrix} \begin{bmatrix} \delta_t^y \\ \delta_t^\pi \end{bmatrix} + \begin{bmatrix} 0 \\ -1/(1-\psi)\varepsilon_{t+j}^\tau \end{bmatrix}$$

We therefore get,

$$\begin{bmatrix} \delta_t^y \\ \delta_t^\pi \end{bmatrix} = -\frac{q_{1,3}q_{2,4} - q_{2,3}q_{1,4}}{(\lambda_4\lambda_5)^j[q_{2,3}q_{3,4}(q_{1,4} - q_{1,3}) + q_{1,3}q_{3,4}(q_{2,3} - q_{2,4})]} \begin{bmatrix} \lambda_5^j q_{3,4}(q_{1,4} - q_{1,3}) & -\lambda_4^j q_{1,3} \\ -\lambda_5^j q_{3,4}(q_{2,3} - q_{2,4}) & -\lambda_4^j q_{2,3} \end{bmatrix} \begin{bmatrix} 0 \\ \varepsilon_{t+j}^\tau/(1-\psi \end{bmatrix}$$

$$= -\frac{1}{\lambda_5^j} \begin{bmatrix} q_{1,3}/[q_{3,4}(1-\psi)] \\ q_{2,3}/[q_{3,4}(1-\psi)] \end{bmatrix} \varepsilon_{t+j}^\tau$$

$$= \frac{1}{\lambda_5^j} \begin{bmatrix} TM_F^y(0,0) \\ TM_F^\pi(0,0) \end{bmatrix},$$

which means that

$$\begin{bmatrix} TM_F^y(j,0) \\ TM_F^\pi(j,0) \end{bmatrix} = \Big(\frac{\beta}{1-\psi}\Big)^j \begin{bmatrix} TM_F^y(0,0) \\ TM_F^\pi(0,0) \end{bmatrix}.$$

Next, we know that the system evolves according to:

$$\mathbb{E}_t X_{t+k} = \begin{cases} A^k X_t, & \text{for } 0 \leq k \leq j-1 \\ A^j X_t + \mathbb{E}_t \delta_{t+j}, & \text{for } k = j \\ A^k X_t + A^{k-j} \mathbb{E}_t \delta_{t+j}, & \text{for } k > j \end{cases}$$

Recalling that $A^k = Q\Lambda^k Q^{-1}$, from (2.32) we then get

$$\begin{bmatrix} TM_F^y(j,k) \\ TM_F^\pi(j,k) \end{bmatrix} = \begin{bmatrix} \omega_{1,1}(k) & \omega_{1,2}(k) \\ \omega_{2,1}(k) & \omega_{2,2}(k) \end{bmatrix} \begin{bmatrix} TM_F^y(0,0) \\ TM_F^\pi(0,0) \end{bmatrix}.$$

Lastly, plug the expressions for ω's (2.33) - (2.36) and for the impact multipliers (2.44) - (2.45) into the equations above to get

$$\begin{bmatrix} TM_F^y(j,k) \\ TM_F^\pi(j,k) \end{bmatrix} = \Big(\frac{\beta}{1-\psi}\Big)^j \begin{bmatrix} TM_F^y(0,k) \\ TM_F^\pi(0,k) \end{bmatrix}.$$

Part II

CHAPTER 3

Gross Financial Flows and the Relative Price of Investment

Chapter Abstract

Gross asset positions have rocketed to record levels over the past four decades. Yet *net* asset positions have remained rather unchanged. I explain these trends with the falling relative price of investment. A lower relative price of investment leads shadow banks to increase their borrowing and lending (i.e. gross balance sheet) to profit from better investment returns. Although more investment increases the capital stock, a lower relative price of investment also lowers the replacement cost and, by extension, the per-unit market value of capital. Net asset positions remain, as a result, relatively unchanged. This result suggests that the large disparity between gross and net positions is here to stay, and will continue to grow as long as the relative price of investment continues to fall. Large gross positions are a result of technological

advances and better investment opportunities, rather than of, as commonly argued, lenient credit regulation, financial deepening, or excessive risk taking in the financial system. I present a stylized model that accounts for these facts.

3.1 Introduction

Gross asset positions have rocketed to record levels over the past four decades. Yet its causes remain poorly understood. Much of the policy debate has instead focused on *net* positions. Net positions are important because they reflect the economy's saving and investment decisions in the goods market. But net positions alone are silent about the underlying gross exposures in the asset market, i.e. the volume and direction of capital flows. Countries, banks, and corporates typically hold large gross positions vis-à-vis each other that dwarf their corresponding net positions.

As shown by a wave of recent empirical studies, the size of gross asset positions has important implications for the functioning and fragility of the monetary system (e.g. see Forbes and Warnock, 2012; Obstfeld, 2012a, 2012b; Shin, 2012; Broner et. al., 2013; Rey, 2013; and Borio and Disyatat, 2015). Rapid growth in gross lending and borrowing is a dominant predictor of financial crises. These episodes are often preceded by a build-up of gross positions, and followed by an abrupt retrenchment in it. Borio and Disyatat (2011), Gourinchas and Obstfeld (2012), and Hume and Sentance (2009), among other observers, link the origin of the global financial crisis of 2007-10 to the gradual build-up of gross positions.

What has triggered the build-up in gross positions? Four arguments have been put forward in the literature: (1) lenient credit regulation (or 'excess financial elasticity' as coined by Borio and Disyatat, 2011) that has systematically encouraged banks to extend more loans; (2) increased risk tolerance by banks as a result of deposit insurance and lender of last resort (moral hazard argument, e.g. see Taylor,

2012); (3) financial deepening, through the creation of new financial products; and (4) financial globalization, through falling barriers to international asset trade (e.g. see Obstfeld, 2011).

All these arguments have merits. But they are not fully borne out by the data. First, the growth in gross assets started in 1980s, well before the liberalization and proliferation of the U.S. banking industry in the mid-1990s, and well after the introduction of deposit insurance and other safety net measures. Second, although cross-border gross positions have increased, the bulk of the increase has taken place within domestic sectors. And third, more fundamentally, none of these explanations give reasons for why net positions have not increased in tandem with the growing gross positions.

This paper adds a fourth explanation that accounts for these facts: the widespread decline in the price of investment goods (e.g. computers, cars, and machinery) relative to consumption goods. Capital goods prices have been steadily falling over the last four decades, dropping by more than a third since the early 1980s. The literature (see e.g. Karabarbounis and Neiman, 2014; and Eichengreen, 2015) typically attributes the fall to advances in technology, R&D, and the computer age. In this paper, I connect these trends, arguing that the decline in the relative price of investment has caused the build-up of gross positions (while leaving net positions relatively unchanged), and may thus, by extension, have been a source of increased financial fragility.

The mechanism runs as follows. All else equal, a lower relative price of investment raises the return on investing in capital goods. Facing better investment re-

turns, shadow banks invest more. To fund their investment, they issue more debt in the wholesale market, as a source of leveraged financing. Private equity firms, for instance, borrow from broker dealers to perform leveraged buyouts. Hedge funds, to give another example, use margin loans from from money market funds to finance their investments. But for diversification purposes, banks not only increase their capital investment, they increase their lending in the wholesale market too, to spread out risk. That is, banks increase their lending to and borrowing from the wholesale market. Larger bilateral (two-way) gross positions arise.

Viewed through this lens, gross flows arise from 'debt collateralizing debt'. In the first place, shadow banks issue debt to fund their investments. In the second place, other financial intermediaries, in turn, use that newly-issued debt as collateral to borrow from other sources, and so on. One debt contract can thus magnify total gross borrowing and lending by a large multiple. The financial system is, in this way, intertwined by debt securing debt, establishing large gross positions between market participants.

Although a falling relative price of investment increases borrowing and lending (i.e. gross balance sheet), it leaves the value of net positions rather unchanged. The economy's net balance sheet, after cancelling all offsetting gross positions, equals the value of all real assets (e.g. capital stock and housing) plus equity instruments. And while a lower relative price of investment increases investment and thus the capital stock, each unit of capital is now valued at a lower market price: with a long-run Tobin's q of unity, a lower relative price of investment lowers the replacement cost and, by extension, the market value of capital.

This result suggests that large disparity between gross and net positions is here to stay, and will continue to grow as long as the relative price of investment continues to fall. Large gross positions are a result of technological advances, better investment opportunities and risk sharing, rather than of, as commonly argued, lenient credit regulation, financial deepening, or excessive risk taking in the financial system.

A lower relative price of investment increases the steady state level of economic activity. But it presents risks too. In particular, the economy becomes more vulnerability to drops in asset prices, through two channels. First, a lower relative price of investment lowers the per-unit market value of capital that if necessary could be liquidated to repay maturing debt. So for a given level of borrowing, banks have a smaller buffer to cushion a fall in the value of their assets. And second, because lending is restricted by the value of existing collateral, a fall in assets can cut investment and output by a large multiple. Consider an unexpected fall in the capital stock. As debt liabilities are fixed, the net equity of banks falls by a multiple of the fall in the capital stock. The higher is leverage, the larger is the fall in net equity. Less equity, in turn, leads to less collateral to borrow. The problem now is that the savers (which get all of their deposits back, assuming that banks stay solvent) have most of the funds that the economy needs for investments to rebuild the capital stock. But the financial sector cannot efficiently channel these funds to the firm because the banks have insufficient collateral to borrow and make new investments. Moore (2011) refers to a similar amplification mechanism as the 'collateral-value multiplier', through which higher interest rates lower collateral values, which in turn limits borrowing and increases interest rates even further.

I embed these ideas in a simple general equilibrium framework with a financial sector intermediating savings from households to firms. The financial sector consists of a set of heterogeneous banks. Think of these banks as big financial powerhouses (akin to 'The Big Four' in the U.S.) that raise deposits from households, but also perform shadow banking services such as private equity and venture capital investments. In each period, the banks can invest in capital at the relative price of investment. The investment price is randomly distributed across banks and time, so in every period some banks face better investment opportunities than others.

The banks can borrow from and lend to each other in the over-night wholesale interbank market. A bank therefore faces two investment opportunities: invest in firm capital, or lend in the wholesale market. In equilibrium, the banks invest in both. Why? The banks value capital at its replacement cost. Capital is hence a risky investment—the effective (one-period) return on capital is conditional on the next period's random investment price—even in the face of no aggregate uncertainty. Short-term interbank bonds, however, are free from such idiosyncratic risk. The risk-averse banks thus diversify their assets across both risky capital and risk-free wholesale bonds.

The banks can also raise deposits, at a cheaper rate, from households. Crucially, household and interbank lending are subject to a collateral requirement. The banks accept any form of assets as collateral. Households, however, only accept interbank bonds, lacking the expertise to evaluate the quality of capital as collateral. This asymmetry in collateral requirements, first introduced by Moore (2011), ensures that banks cannot directly borrow from households to finance their venture capital

investments, and is motivated by the real-world regulatory framework that prohibits shadow banks from raising deposits.

To maximize the return on their investment portfolio, the banks, then, borrow from the wholesale market to amplify the return on their capital investment, and borrow from the household to amplify the return on their wholesale bond investments. The banks thus simultaneously (intra-period) borrow from and lend to the wholesale market, even at the same interest rate, without netting out their mutually gross positions. Gross bilateral positions arise.

I calibrate the model and show a near-perfect relationship between the decline in the relative price of investment and the build-up of gross positions. A lower investment price increases the return of capital investments, prompting banks to invest more. Borrowing increases too, both from households and the interbank market, as a source of leveraging up their investments. As the investment good price falls, the economy, and in particular the financial sector, increases its gross balance sheet. Consistent with data, however, the economy's value of net positions remains rather unchanged: the capital stock is higher, but is now valued at a lower per-unit replacement cost.

This paper belongs to a growing literature using the decline in the relative price of investment to explain trends. Karabarbounis and Neiman (2014) explain the global decline of the labor share by the fall in investment prices. Cheaper investments, they argue, encourage firms to shift away from labor and toward capital. Thwaites (2015) builds on this analysis to explain the widespread decline in real interest rates. In his model, less investment spending chasing the same savings results in lower

real interest rates through an excess of desired saving over desired investment. This mechanism emerges, and plays a key role, in this paper too.

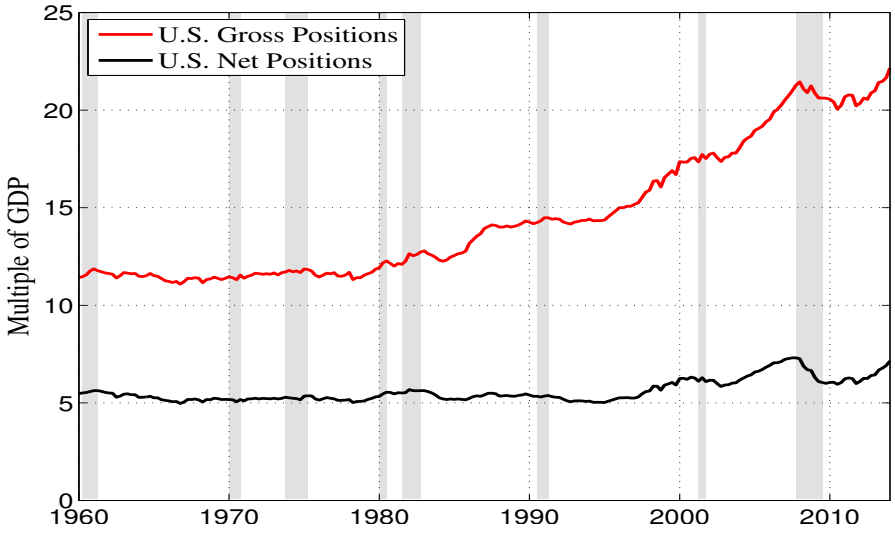
Two papers are particularly close to the framework I offer. First, I base the model on the interbank equilibrium sketched in Moore (2011). Moore introduces an asymmetry in the collateral requirement between banks and households and shows why banks, as a result, borrow from each other to maximize the return on their investment portfolios. And second, I use the solution technique in Kiyotaki and Moore (2012) to aggregate the financial sector among a set of heterogeneous banks.

The remainder of the paper is structured as follows. Section 3.2 sets out the key stylized facts that the model aims to connect. Section 3.3 sets out a simple example of how gross bilateral positions arise in the financial sector. Section 3.4 outlines the model. Section 3.5 reviews its equilibrium. Section 3.6 discusses the calibration and model results. Section 3.7 relates the model to the recent financial crisis. Section 3.8 concludes. I relegate the proofs and derivations to Appendices 3.D - 3.G.

3.2 Motivating facts

Figure 3.1 plots the evolution of the U.S. gross balance sheet (assets plus liabilities, red line) versus net balance sheet (assets minus liabilities, black line). Before the 1980s, the two appeared to be on a stationary trend relative to output. Ever since, gross asset positions have nearly doubled relative to output, and continue to grow. Net positions, however, have remained relatively stable. That is, the U.S. has increased its borrowing from and lending to itself, for the same value of net positions.

Figure 3.1: U.S. Balance Sheet



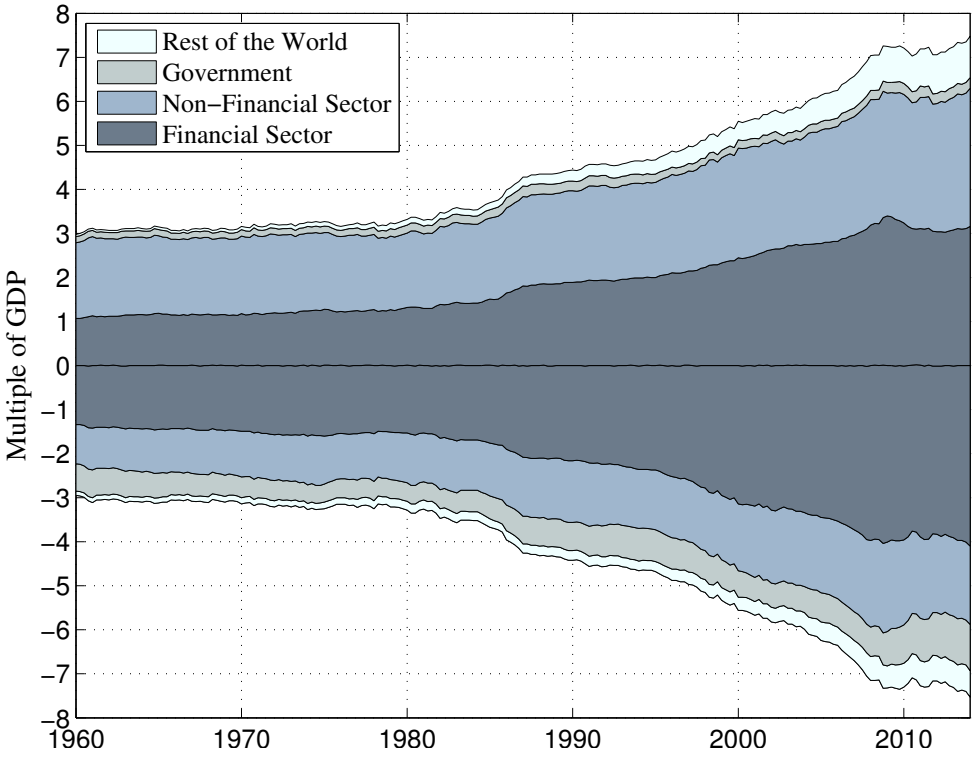
Note: Author's own calculation. See Appendix 3.B for data source and calculations. Shaded areas indicate NBER recessions.

Taylor (2012) has coined this observation 'The Great Leveraging'.

The bulk of this expansion has taken place within the financial sector, with 'banks' lending to and borrowing from each other. Figure 3.2 depicts the breakdown of U.S. gross positions by sector. Although all sectors have experienced an increase in their gross balance sheets, the growth in the domestic banking sector has been unprecedented: its gross liabilities relative to output have trebled over the last four decades; its assets have more than doubled.

I use the term 'bank' in a broad sense. As documented by Bhatia and Bayoumi (2012), the growth in the financial sector has largely been due to an expanding 'shadow banking' market. Shadow banks perform traditional banking operations (e.g. maturity and liquidity transformation), but do not fund their activities by deposits. Instead, they raise money in the wholesale interbank market, i.e. from other

Figure 3.2: U.S. Financial Balance Sheet by Sector



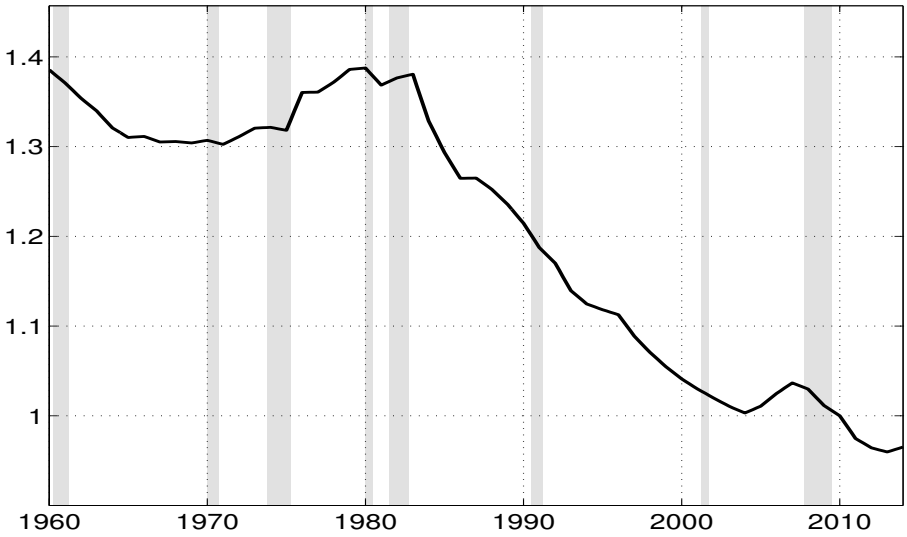
Note: Size of U.S. gross financial balance sheet by sector, excluding equity instruments. Positive y-axis measures financial assets. Negative y-axis measures liabilities. Author's own calculation. See Appendix 3.B for data source and calculations.

financial intermediaries and institutional investors, often through collateralized loans (e.g. repos), to fund their investments. Examples include broker dealers, investment banks, private equity funds, hedge funds, and other asset managers. These 'banks' cannot raise money from depositors either for regulatory reasons, or because depositors lack the knowledge to evaluate the risk of the investments.

Yet the line between shadow banks and traditional banks is still blurred. A large number of banks perform both. 'The Big Four' in the U.S. (J.P. Morgan, Bank of America, Citigroup, Wells Fargo) are good examples of that. These banks raise deposits from households to fund their low-risk investments, e.g. in mortgages and lending to the wholesale market. They also raise money from the wholesale market to finance their more risky investments, e.g. in venture capital and private equity. Through this mechanism, these banks are *simultaneously* borrowers and lenders in the wholesale market, creating large bilateral gross positions, though their corresponding net positions may be small.

The literature has proposed a number of reasons for the rise in gross positions. Many point to lenient credit regulation (e.g. see Borio and Disyatat, 2011), financial deepening (e.g. see Swiston, 2005) and financial globalization (e.g. see Obstfeld, 2011). Others have focused on a growing risk appetite by banks resulting from deposit insurance and other financial safety net measures (e.g. see Taylor, 2012). But Figures 3.1 and 3.2 reveal that these arguments are, at best, imperfect explanation. First, gross positions started to rise in the early 1980s, well before the deregulation and proliferation of the U.S. banking industry in the mid-1990s (Figure 3.1). For instance, the McFadden Act (which prohibited banks from opening branches

Figure 3.3: Relative Price of U.S. Investment Goods



Note: The relative price of investment to consumption goods. Data is taken from Eichengreen (2015). Shaded areas indicate NBER recessions.

in multiple states), the Glass-Steagall act (which separated the core businesses of commercial and investment banking) and the Bank Holding Act (which prohibited bank holding companies from owning other financial institutions) were repealed in 1994, 1999 and 1999, respectively. Second, the U.S. introduced deposit insurance and lender of last resort long before the 1980s. Third, it is true that cross-border flows have increased from falling barriers to international asset trade. Gross international assets plus liabilities have soared more than tenfold relative to output (Figure 3.2, light blue). But the expansion of gross flows in the domestic sectors dwarf that of cross-border flows. And fourth, none of these explanations account for why net positions have remained relatively unchanged (Figure 3.1).

The divergence between gross and net positions has near-perfectly coincided with a fall in the relative price of investment goods. Machinery, equipment, mobile phones

and vehicles are all in real terms cheaper today. Figure 3.3 plots this price, revealing a clear structural break in the early 1980s, in line with when gross positions started to rise. Cumulatively, the investment price has dropped more than a third over the last four decades. The literature (see e.g. Karabarbounis and Neiman, 2014; and Eichengreen, 2015) typically attributes the fall to advances in technology, R&D, and the computer age. I connect these observed trends in Section 3.4.

3.3 Schematic example

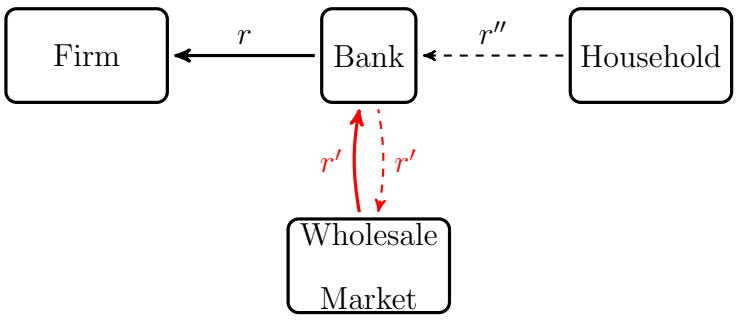
Before turning to the model, it is helpful to start by giving a simple schematic example of how gross bilateral flows arise in the wholesale interbank market. This mechanism, whereby debt collateralizes debt, lies at the core of the analysis to come. I sketch the example in Diagram 3.1.

Consider a bank that invests in (risky) venture capital that returns an expected per-period interest rate r. It has two sources of funding: (1) either it borrows from other banks in the wholesale interbank market at (cheaper) interest rate $r' < r$; or (2) it raises deposits from households at (even cheaper) interest rate $r'' < r'$. Both types of borrowing require collateral. Crucially, the household does not accept venture capital as collateral, thereby preventing the bank from raising deposits to fund its risky investment (recall that shadow banks cannot raise deposits). But other banks accept venture capital as collateral. So to maximize the return on its venture capital investment, the bank uses its venture capital as collateral to borrow from the wholesale interbank market: it borrows at r' to invest in r. The solid arrows show

the direction of these credit flows.

To spread out risk, the bank also lends in the wholesale market at interest rate r'. To maximize the return on its lending in the wholesale market, the bank uses the interbank bonds as collateral to borrow from the household: it borrows at r'' to invest in r'. The dashed arrows show the direction of these credit flows.

Diagram 3.1.



Note the bilateral gross flows (in red). The bank simultaneously lends to and borrows from the wholesale market, *even* at the same interest r'. It does not net out the bilateral positions, because it uses the gross assets as collateral to borrow at a cheaper rate from the household. Instead, for a given leverage constraint, the bank maximizes the size of its gross flows vis-à-vis the wholesale market. It uses debt to collateralize debt. In this example, net flows between the bank and the wholesale market may be zero, although the corresponding gross flows could be substantial.

3.4 Baseline model

The model is set in an infinite-horizon, discrete-time real business cycle environment with no aggregate uncertainty. There are three sets of agents: a representative household, a representative firm, and a financial sector intermediating savings from the household to the firm. The firm and household problems are standard. The financial sector consists of a set of banks that can borrow and invest subject to a collateral and leverage constraint.

3.4.1 Representative firm

At time t, the representative firm chooses capital K_t and labor N_t to maximize its profit

$$Y_t - r_t K_t - w_t N_t,$$

subject to the Cobb-Douglas production function $Y_t = K_t^{\alpha} N_t^{1-\alpha}$, where $\alpha \in (0, 1)$ is the output elasticity of capital, r_t is the competitive real interest rate, and w_t is the competitive real wage. The output price is normalized to unity. The firm rents capital for one period. Production is completed in period t, after which the capital depreciates at factor $\lambda \in (0, 1)$. In the competitive equilibrium, the representative firm makes zero profit.

3.4.2 Representative household

The representative household consumes C_t^h, works N_t, and saves in bank deposits B_{t+1}^*. At time t, it maximizes

$$\sum_{j=t}^{\infty}(\beta')^{j-t}\left[\log C_j^h - \frac{1}{1+\nu}N_j^{1+\nu}\right], \quad \beta' \in (0,1),$$
$$\text{s.t. } C_t^h + q_t^* B_{t+1}^* = w_t N_t + B_t^*,$$

where $\nu > 0$ is the inverse Frisch elasticity of labor supply, $q_t^* < 1$ is the competitive real deposit bond price, and β' is the household's subjective discount factor.

3.4.3 Banks

There is a continuum of heterogeneous banks of unit mass. Each bank maximizes time-t expected discounted utility

$$\mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} \log c_j, \quad 0 < \beta < \beta', \tag{3.1}$$

of real consumption path $\{c_t, c_{t+1}, c_{t+2}, ...\}$. The discount factor, β, is below that of the representative household to ensure the banks borrow in equilibrium. The banks may invest in capital goods to the representative firm. Investment is completed in period t, although newly-produced capital does not become available as an input to the production of output until the next period $t+1$.

Let i_t denote a bank's capital investment in period t. The law of motion for the

bank's capital is then given by

$$k_{t+1} = \lambda k_t + i_t. \tag{3.2}$$

Investment, i_t, may be positive or negative. If it is negative, the bank is liquidating parts of its existing capital stock into consumption goods.

Although every bank may invest in capital, the banks each face an idiosyncratic, stochastic investment price. In each period, a positive investment opportunity arrives with probability $\pi \in (0, 1)$, whereby the bank pays a low real price of investment η_t^l; and a negative investment opportunity arrives with probability $1 - \pi$, whereby the bank pays a high real price of investment $\eta_t^n > \eta_t^l$.

The arrival of the investment price is independently distributed across banks and through time. Therefore, by the law of large numbers, in every period, a fraction π of the banks face a positive investment opportunity (henceforth *lead* banks), with the remaining fraction $1 - \pi$ facing a negative one (henceforth *non-lead* banks). As we shall see, this ex-post heterogeneity across banks creates an interbank market in which non-lead banks channel funds to lead banks.

A bank has two sources of financing. Either it issues one-period deposit bonds to households at competitive real price $q_t^* < 1$. Or it issues one-period interbank bonds at competitive real price $q_t < 1$. Both types of borrowing are subject to a collateral and leverage constraint a la' Kiyotaki and Moore (2012). In particular, at time t, a bank can maximally pledge a fraction $\theta \in (0, 1)$ of its next-period assets.

The banks accept any form of assets as collateral. But, crucially, households only accept interbank bonds as collateral, lacking the expertise to evaluate the quality of

capital as collateral. At time t, a bank can therefore issue a maximum of

$$\theta k_{t+1} = \theta\lambda k_t + \theta i_t \tag{3.3}$$

interbank bonds, and a maximum of

$$\theta b_{t+1}$$

deposit bonds, where b_{t+1} denotes the bank's interbank bond holdings at time $t+1$. The first term on the right-hand side (RHS) of (3.3) is the bank's ability to roll over its debt. As its existing short-term debt matures, it can partially refinance its capital by issuing new interbank bonds using its depreciated capital, λk_t, as collateral. The second term is the cap on the number of interbank bonds a bank can issue to finance new investments. Let $\theta_t, \theta_t^* \in [0, \theta]$ denote the bank's time-t interbank and deposit leverage ratios, respectively.

Table 3.1 summarizes the balance sheet of a typical bank at the start of time t.

Table 3.1: The Balance Sheet of a Bank

Assets	**Liabilities**
k_t [Capital]	$\theta_t k_t$ [Interbank bonds]
b_t [Interbank bonds]	$\theta_t^* b_t$ [Deposit bonds]
	$k_t(1-\theta_t) + b_t(1-\theta_t^*)$ [Own equity]

The bank may have two types of assets: (i) capital, bought at some time in the

past; and (ii) interbank bonds, bought in period $t-1$. Similarly, the bank may have two types of liabilities: (i) interbank bonds, issued in period $t-1$; and (ii) deposit bonds, issued in period $t-1$. The difference is the bank's own equity.

At time t, a bank of type $h \in \{l, n\}$ (l = lead-bank; n = non-lead bank), with initial capital k_t, interbank bond positions b_t, and leverage ratios θ_t and θ_t^* face the following flow-of-funds constraint:

$$c_t^h + \eta_t^h i_t^h + q_t b_{t+1}^h = r_t k_t + b_t - \theta_t k_t - \theta_t^* b_t + q_t \theta_{t+1}^h k_{t+1}^h + q_t^* \theta_{t+1}^{*,h} b_{t+1}^h. \tag{3.4}$$

The left-hand side (LHS) is the bank's liquidity needs: it can allocate expenditure on consumption, capital investment, or interbank bond investment. The RHS is the total liquidity supplied: the first four terms reflect the flows generated from the time-t assets and liabilities, and the final two terms are the interbank and deposit bonds issued at price q_t and q_t^*, respectively.

The bank's problem is to maximize (3.1) subject to the flow-of-funds constraint (3.4), the law of motion for capital (3.2), the leverage constraints $\theta_{t+1}^h, \theta_{t+1}^{*,h} \in [0, \theta]$, non-negativity constraints on c_t^h, k_{t+1}^h and b_{t+1}^h, and the Markov process describing the arrival distributions of the relative price of investment.

3.5 Description of equilibrium

I restrict the parameter space to ensure that the competitive equilibrium features two components.

First, given the non-linearity of the flow-of-funds constraint (3.4), there are com-

plicated dynamic interactions between the distribution of leverage across banks and their corresponding asset holdings. Aggregation is thus a non-trivial task. To overcome this problem, I only consider equilibria in which both types of banks maximize leverage.

Proposition 3.1. *Suppose the market clearing bond prices satisfy:*

$$q_t \in (\theta q_t^*, q_t^*). \tag{3.5}$$

Then both types of banks maximize leverage, i.e. $\theta_{t+1}^h = \theta_{t+1}^{*,h} = \theta$, *for* $h \in \{l, n\}$.

Appendix 3.D.1 contains the proof, but the underlying reasoning runs as follows. For a given level of downpayment, the banks maximize deposit leverage, $\theta_{t+1}^{*,h}$, as interbank bonds earn a higher rate of return than the cost of deposit borrowing. The banks also maximize interbank leverage, θ_{t+1}^h, as they can use the additional liquidity to increase the downpayment on which their interbank bond investments are based.

Second, I am interested in describing an equilibrium in which gross bilateral interbank flows arise. Consider an equilibrium in which

$$b_{t+1}^n, k_{t+1}^n > 0, \tag{3.6}$$

so that non-lead banks invest a strictly positive amount $q_t b_{t+1}^n$ in interbank bonds, while concurrently (intra-period) issuing a strictly positive amount $q_t \theta k_{t+1}^n$ of interbank bonds. In this equilibrium, gross interbank flows exceed net flows: non-lead banks are both borrowers and lenders in the interbank market, even at the same

interest rate. Further assume that lead banks allocate their entire investment expenditure on capital, so that

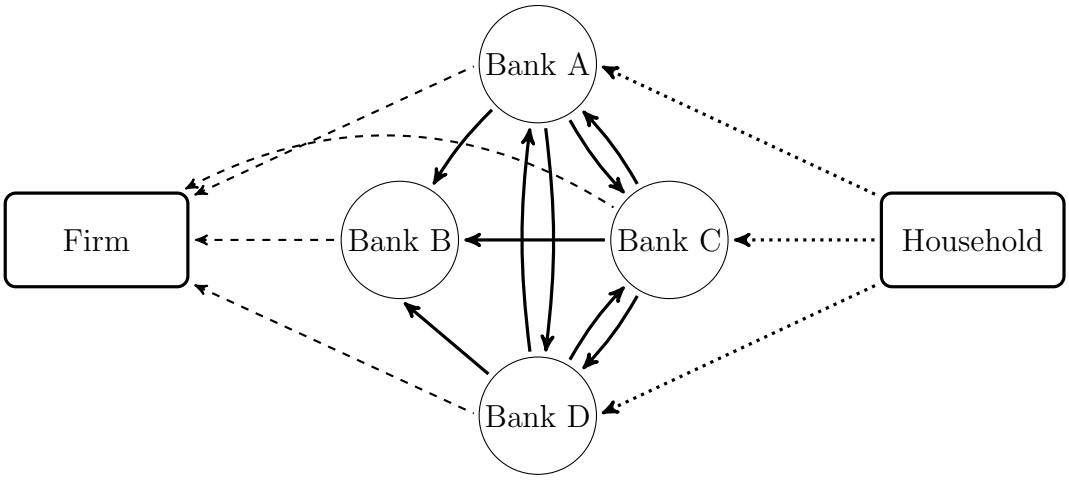
$$b^l_{t+1} = 0, i^l_t > 0. \tag{3.7}$$

The last restriction (3.7) is not strictly necessary. But it helps isolate the area of interest by making the distribution of interbank assets across banks simple.

Claim 3.1. *There exists a non-empty parameter space in which the competitive equilibrium satisfies (3.5), (3.6) and (3.7).*

Diagram 3.2 depicts the capital flows in the steady state (no growth in aggregate variables) of this equilibrium. Bank B represents a lead bank. Bank A, C, and D all represent non-lead banks. The dotted arrows show the direction of deposits from households; the dashed arrows show the direction of capital to firms; and the solid arrows show the direction of interbank flows. All banks allocate investment expenditure on capital goods, levered by issuing interbank bonds. Non-lead banks also invest in interbank bonds, levered by issuing deposit bonds. Crucially, non-lead banks do not net out their mutually gross interbank positions vis-a-vis each other, although they lend and borrow at the same interest rate $1/q_t$. As we shall see in the next section, non-lead banks borrow from the interbank market to finance their capital investment, and they invest in interbank bonds to diversify their investment portfolio. In this steady state, gross interbank flows exceed net ones.

Diagram 3.2. Credit Flows in Steady State



3.5.1 Equilibrium conditions

I derive the equilibrium conditions in Appendix 3.D by using a guess-and-verify approach akin to the one in Kiyotaki and Moore (2012). The optimality conditions for the firm and household are standard. To understand why bilateral gross flows arise, this subsection presents the optimality conditions for the aggregate banking sector.

By substituting the law of motion for capital (3.2) into (3.4), the bank's flow-of-funds constraint collapses to

$$c_t^h + (\eta_t^h - q_t\theta)k_{t+1}^h + (q_t - \theta q_t^*)b_{t+1}^h = (r_t - \theta + \eta_t^h\lambda)k_t + (1-\theta)b_t, \tag{3.8}$$

for $h \in \{l, n\}$. The RHS is the bank's start-of-period net worth: the liquidity supplied from its time-t assets and liabilities, plus its depreciated capital, λk_t, valued at its

time-t replacement cost of η_t^h. The second and third terms of the LHS are the internally financed downpayments of investing k_{t+1}^n in capital goods and θb_{t+1}^n in interbank bonds. Given logarithmic preferences, the bank consumes a fraction $(1-\beta)$ of its net worth

$$c_t^h = (1-\beta)\Big[(r_t - \theta + \eta_t^h \lambda)k_t + (1-\theta)b_t\Big], \tag{3.9}$$

and partitions the residual net worth as downpayments for an investment portfolio of capital and interbank bonds

$$(\eta_t^h - q_t\theta)k_{t+1}^h + (q_t - q_t^*\theta)b_{t+1}^h = \beta\Big[(r_t - \theta + \eta_t^h \lambda)k_t + (1-\theta)b_t\Big]. \tag{3.10}$$

In equilibrium, by construction, $b_{t+1}^l = 0$, so that lead banks only invest in capital goods. Note that lead banks consume less than non-lead banks with the same amount of start-of-period capital and interbank bond positions. This result is not surprising. As cheaper investment opportunities appear, the banks employ more funds for investment purposes, knowing that they may face worse investment costs in the future.

There is a spectrum of interest rates in the economy: the marginal return on capital, the return on interbank bonds, and the return on deposits. All these returns are return on assets (ROA). The banks, though, only care about their return on equity (ROE), i.e. the return on the downpayment of investing in these assets. Let R_{t+1}^b be the ROE of interbank bonds from time t to $t+1$, and $R_{t+1}^{hh'}$ be the ROE of capital from time t to $t+1$ when the bank is type h at time t and h' at time $t+1$.

The flow-of-funds condition gives

$$R^b_{t+1} \equiv \frac{1-\theta}{q_t - q^*_t\theta}, \quad R^{hh'}_{t+1} \equiv \frac{r_{t+1} - \theta + \eta^{h'}_{t+1}\lambda}{\eta^h_t - q_t\theta}. \tag{3.11}$$

Crucially, then, as the banks value capital at its replacement cost, the ROE of capital is conditional on the next period's investment price. Capital is hence a *risky* investment, even in the face of no aggregate uncertainty. Interbank bond investments are free from such idiosyncratic risk.

Proposition 3.2. *The following interest rate ordering holds in the competitive equilibrium:*

$$\frac{1}{q^*_t} < \frac{1}{q_t} < R^b_{t+1}, \tag{3.12}$$

$$R^{nl}_{t+1} < R^b_{t+1} < R^{nn}_{t+1} < R^{ll}_{t+1} < R^{ln}_{t+1}. \tag{3.13}$$

These relationships hold by construction of the equilibrium. First, relationship (3.12) follows trivially from (3.5). Second, relationship (3.13) follows from (3.6) and (3.7). Lead banks do not allocate investment expenditure on interbank bonds as the ROE of interbank bonds is smaller than the ROE of capital in both next-period states, $R^b_{t+1} < R^{ll}_{t+1}, R^{ln}_{t+1}$. Non-lead banks, however, face a trade-off between risk and return, since $R^{nl}_{t+1} < R^b_{t+1} < R^{nn}_{t+1}$, and therefore diversify their portfolios across risky capital investment and risk-less interbank bonds. The concavity of the logarithmic preferences specifies the level of risk aversion.

The optimal investment portfolio for non-lead banks is determined where the expected marginal utility from investing one more unit of downpayment in inter-

bank bonds equals the expected marginal utility from investing one more unit of downpayment in capital, so that no free gain can be made:

$$\pi R_{t+1}^{nl}\beta\frac{1}{c_{t+1}^{nl}}+(1-\pi)R_{t+1}^{nn}\beta\frac{1}{c_{t+1}^{nn}}=R_{t+1}^{b}\cdot\left[\pi\beta\frac{1}{c_{t+1}^{nl}}+(1-\pi)\beta\frac{1}{c_{t+1}^{nn}}\right] \tag{3.14}$$

Here, $c_{t+1}^{hh'}$ denotes the optimal consumption at time $t+1$ when the bank is type h at time t and h' at time $t+1$. The LHS is the expected gain from investing one more unit of downpayment in capital. If the bank is a lead bank in the next period with probability π, the downpayment increases utility by R_{t+1}^{nl} times the discounted marginal utility $1/c_{t+1}^{nl}$. If the bank is a non-lead bank with probability $(1-\pi)$, the downpayment increases utility by R_{t+1}^{nn} times the discounted marginal utility $1/c_{t+1}^{nn}$. The RHS is the expected gain from investing one more unit of downpayment in interbank bonds, which risklessly yield R_{t+1}^{b}.

By iterating time-t optimal consumption (3.9) forward, rewrite (3.14) as

$$\pi\cdot\left[\frac{R_{t+1}^{b}-R_{t+1}^{nl}}{f_tR_{t+1}^{nl}+(1-f_t)R_{t+1}^{b}}\right]=(1-\pi)\cdot\left[\frac{R_{t+1}^{nn}-R_{t+1}^{b}}{f_tR_{t+1}^{nn}+(1-f_t)R_{t+1}^{b}}\right], \tag{3.15}$$

where

$$f_t\equiv\frac{(\eta_t^n-q_t\theta)k_{t+1}^n}{(\eta_t^n-q_t\theta)k_{t+1}^n+(q_t-q_t^*\theta)b_{t+1}^n}\in(0,1) \tag{3.16}$$

is the share of capital in the portfolio for the non-lead bank. By Proposition 3.2., the ratios on both sides are strictly positive and less than one in the neighborhood of the steady state.

Note that the optimal investment portfolio (3.15) is the same for *all* non-lead

banks—it is independent of initial time-t capital and interbank holdings. All non-lead banks therefore choose the same fraction of capital, f_t, in their investment portfolios. The size of their investment portfolios is idiosyncratic, though, as dictated by the individual bank's initial time-t capital and interbank holdings in (3.10).

Now consider the aggregate financial sector. Index lead banks by $i \in [0, \pi)$, non-lead banks by $i \in [\pi, 1]$, and let capital letters denote aggregate variables

$$\begin{aligned} X_t^l &= \int_0^\pi x_t^l(i)\, di, \\ X_t^n &= \int_\pi^1 x_t^n(i)\, di, \\ X_t &= X_t^l + X_t^n \end{aligned}$$

for $(X_t, x_t) \in \{(C_t, c_t), (K_t, k_t), (B_t, b_t), (I_t, i_t)\}$. By construction of the model, aggregation is easy—the choice variables in (3.9), (3.10), and (3.15) are all linear in start-of-period capital k_t and interbank bond position b_t. Using the law of large numbers, at the start of time t, lead banks thus hold a fraction π of aggregate capital and interbank bonds:

$$K_t^l = \int_0^\pi k_t^l(i)\, di = \pi K_t, \tag{3.17}$$

$$B_t^l = \int_0^\pi b_t^l(i)\, di = \pi B_t. \tag{3.18}$$

In equilibrium, the agents maximize their objectives subject to their constraints. The market-clearing conditions for composite labor and capital are standard. The

interbank market clears at the equilibrium price q_t:

$$\theta K_{t+1} = B_{t+1}, \tag{3.19}$$

where the LHS is the total supply of interbank bonds, and the RHS is the total demand for interbank bonds. The deposit market clears at the equilibrium price q_t^*:

$$\theta B_{t+1} = B_{t+1}^*, \tag{3.20}$$

where the LHS is the total supply of deposit bonds, and the RHS is the total demand for deposit bonds. Finally, the goods market

$$Y_t = C_t^h + C_t^n + C_t^l + \eta_t^l I_t^l + \eta_t^n I_t^n$$

clears by Walras' Law. Appendix 3.E lists the full set of equilibrium conditions.

3.5.2 Balance sheets

Tables 3.2 and 3.3 summarize the aggregate balance sheet for the financial and non-financial sector, respectively, at the end of period t.

The financial sector's balance sheet in Table 3.2 is the aggregate equivalent to the individual bank's balance sheet in Table 3.1. The financial sector holds two types of assets: (i) capital, bought by both types of banks at some time in the past, loaned to the representative firm; and (ii) interbank bonds, bought by non-lead banks in period $t-1$. Similarly, the financial sector has two types of liabilities: (i) interbank

Table 3.2: Aggregate Balance Sheet of Financial Sector

Assets	**Liabilities**
K_t [Capital]	$\theta_t K_t$ [Interbank bonds]
B_t [Interbank bonds]	$\theta_t^* B_t$ [Deposit bonds]
	$K_t(1-\theta_t) + B_t(1-\theta_t^*)$ [Own equity]

Table 3.3: Aggregate Balance Sheet of Non-Financial Sector

Assets	**Liabilities**
$[\pi\eta_t^l + (1-\pi)\eta_t^n]K_t$ [Capital]	K_t [Capital loan]
B_t^* [Deposit bonds]	$[\pi\eta_t^l + (1-\pi)\eta_t^n - 1]K_t + B_t^*$ [Own equity]

bonds, issued by both types of banks in period $t-1$; and (ii) deposit bonds, issued by non-lead banks in period $t-1$. The difference is the sector's net own equity.

The non-financial sector in Table 3.3 consists of the representative firm and household. The firm borrows capital from the banks, recorded as a financial liability and real asset. The aggregate capital stock is proportionally valued at its replacement cost. The household holds deposit bonds as a financial asset and has no debt.

In equilibrium, by clearing conditions (3.19) and (3.20), the economy's net financial wealth is zero—all financial assets are offset by financial liabilities. Deposits are household assets, but bank liabilities. Capital loans are bank assets, but liabilities for the firm. And interbank bonds are offsetting assets and liabilities within the financial sector. The economy's net wealth is therefore equivalent to its capital stock, proportionally valued at its replacement cost.

3.6 Calibration and results

I calibrate the model at quarterly frequency. Appendix 3.C summarizes the parameter values and details the data and calculations underpinning the calibration.

Most of the calibration choices are uncontroversial. I set the household's discount factor, β', to 0.99; depreciation factor, λ, to 0.975; output elasticity of capital, α, to 1/4; and inverse Frisch elasticity of labor supply, ν, to 1. Leverage, θ, is set to 0.74, corresponding to the sample average of U.S. financial leverage from 1951Q4 to 2015Q3 (values ranging from 0.65 to 0.82). And I set the fraction of lead banks, π, to 0.5 to impose symmetry on the arrival distribution of the relative price of investment.

Next, I let η_t be the model's mean relative price of investment and σ_η be the deviation of the investment price from this average:

$$\eta_t \equiv \frac{\eta_t^l + \eta_t^n}{2}, \quad \sigma_\eta \equiv |\eta_t^h - \eta_t|, \text{ for } h \in \{l, n\}. \tag{3.21}$$

The parameter σ_η captures the dispersion of the Markov process describing the arrival distributions of the relative price of investment. I set it to 0.00745, corresponding to the standard error of an AR(1) regression of the relative price of investment from 1959Q4 to 2013Q4.

Finally, I calibrate the banks' discount factor, β, to match the steady-state size of the model's gross balance sheet to that of the U.S. economy in 1960 (i.e. using steady state $\eta = \eta_{1960}$). The value of β pins down the steady state return on the banks' investments, the capital stock, and thus, ultimately, the size of their gross balance

sheets. As desired, the calibrated value, 0.905, results in a higher impatience in banks relative to that of the representative household. The steady state values of the calibrated equilibrium satisfy conditions (3.5) - (3.7).

3.6.1 Stochastic dynamics

This subsection illustrates the dynamic response of gross and net interbank flows to a temporary drop in the relative price of investment. To do so, I temporarily deviate from the baseline model by adding aggregate uncertainty. In particular, I let the average investment price follow an AR(1) process with mean η:

$$(\eta_t - \eta) = \rho(\eta_{t-1} - \eta) + \varepsilon_t,$$

where ε_t is a mean-zero i.i.d. innovation to the investment price, and $\rho \in (0, 1)$ is the parameter governing the persistence of the process. With aggregate uncertainty present, the banks neither know which type $h \in \{l, n\}$ they will be in the next period, nor the values of the future investment prices, η^h_{t+1} $(h \in \{l, n\})$.[1]

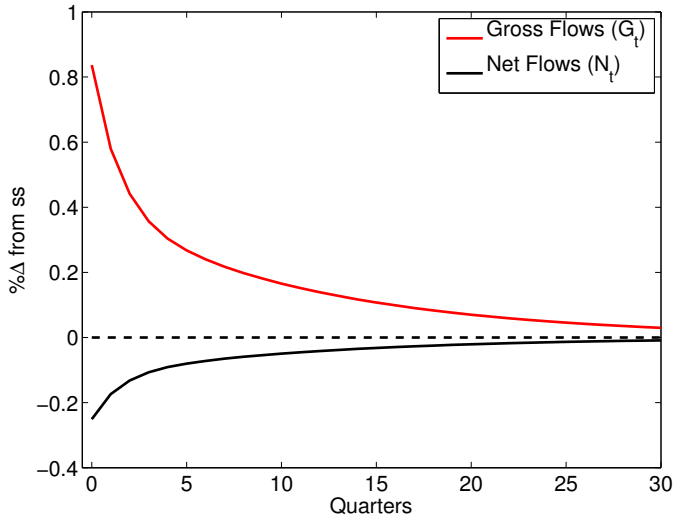
Two variables are of interest. I define them as follows:

$$N_{t+1} \equiv B_{t+1} - \theta K^n_{t+1} > 0,$$
$$G_{t+1} \equiv B_{t+1} + \theta K^n_{t+1}.$$

N_{t+1} measures net interbank lending from non-lead banks to lead banks in period

[1]The equilibrium conditions are the same as before, but now augmented with the expectation operator present in the forward-looking equations. The solution requires linearization of the equilibrium around the model's steady state.

Figure 3.4: Responses to Investment Price Shock



t. It is strictly positive because of market clearing (3.19) and aggregation (3.17). G_{t+1} measures gross interbank lending between non-lead banks to lead banks in period t. Note that, if the banks netted out their bilateral gross positions, net flows, N_{t+1}, would equal gross flows, G_{t+1}. But as non-lead banks choose to simultaneously borrow from and lend to the interbank market, the two differ.

Figure 3.4 plots the evolution of N_{t+1} and G_{t+1} in response to a 1% unexpected fall in the size of the investment innovation ε_t, setting $\rho = 0.5$. Think of this shock as a sudden improvement in the investment opportunities of the economy. These impulse response functions are not meant as a detailed estimation of the magnitude in movements, but rather as a representation of some of the key dynamic features of the model.

At time zero, a lower replacement cost, η_t^h, leads to a deterioration in the banks' net worth, the RHS of (3.9). The banks therefore choose to consume less to instead

allocate more liquidity to their investment portfolio. Both capital investment and interbank investment increase. Why? A lower η_t^n (cf equation (3.11)) makes capital a more attractive investment class, encouraging banks to invest more in capital. But for diversification purposes, non-lead banks also increase their interbank bond investments. This is a 'wealth' effect: non-lead banks expand their gross balance sheets by fueling both their total interbank borrowing (to finance their increased capital investment) and total interbank bond investing. In short, gross flows increase as banks increase their borrowing and lending to profit from better investment returns and to diversify risk.

N_{t+1} is, in contrast, dictated by the corresponding 'substitution' effect. A lower η_t^n induces non-lead banks to shift their investment portfolio into a larger proportion f_t (cf. equation (3.16)) of capital. This translates into proportionally lower interbank investment, B_{t+1}, and higher interbank borrowing, θK_{t+1}^n (to finance the increased capital investment). Net interbank lending decreases, as more funds flow out of the banking system.

In the subsequent periods, as capital becomes a less attractive asset class, non-lead banks gradually modify the composition of their investment portfolios to include a larger share of interbank bonds. Net and gross lending eventually converge back to their steady state values.

3.6.2 Trend dynamics

The decline in the relative price of investment is unlikely to be a temporary shock. Its underlying drivers, such as advances in technology and computing power, imply

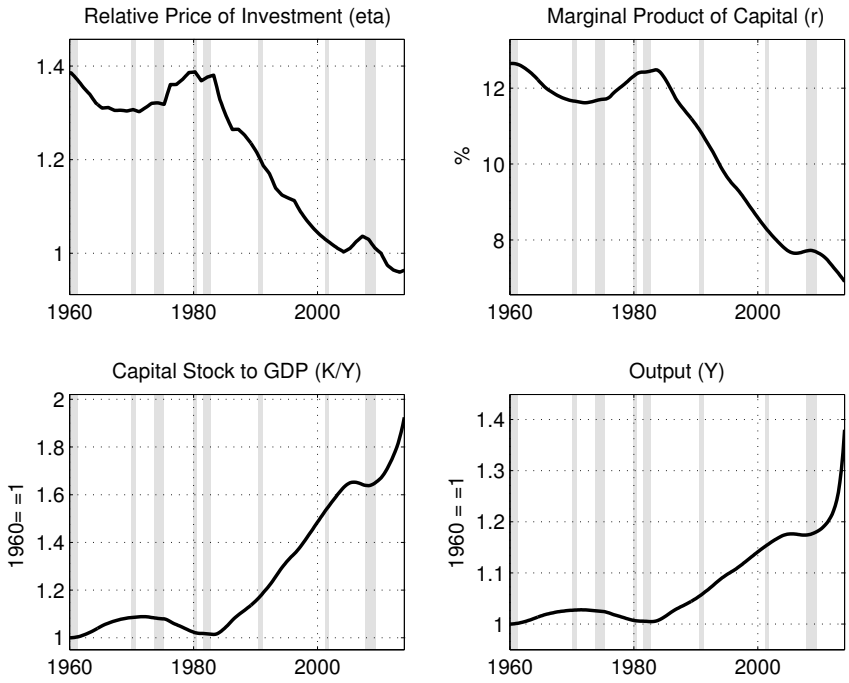
a permanent reduction in the cost of producing one unit of capital. As such, the economy's steady state quantities, flows, and relative prices should move in response.

To capture this permanent shift, this section studies the economy's response to the observed decline in the relative price of investment. In particular, I set η_t to the investment price series (1959Q4—2013Q4) in Eichengreen (2015) and study the perfect foresight solution of the calibrated equilibrium. That is, the model starts at its steady state using the investment price in 1959Q4, and then evolves with full information, perfect foresight, and no aggregate uncertainty around the subsequent trajectory of future investment prices. The banks therefore know the values of η_t^h ($h \in \{l, n\}$) for all t, although they do not know which type h they will be, which is instead dictated by the two-state idiosyncratic Markov process describing the arrival distribution of the bank type h. The paths of each variable are fully known and the solution does not require linearization.

Figure 3.5 plots the evolution of the real interest rate (marginal product of capital), the capital stock to output ratio, and output. All signs are consistent with observed trends in U.S. data since 1980 (when the relative price of investment started to fall). First, the real interest rate falls. Other things equal, a lower price of investment raises the return on the banks' investment portfolios. But in steady state, the banks' expected return on its investment portfolio is pinned down by their subjective discount factor, and the steady state cost of borrowing from deposits is pinned down by the household's subjective discount factor. So the marginal product of capital must eventually fall to offset the lower investment price.[2] Thwaites (2015) obtains a

[2]This result is independent of the type of production function.

Figure 3.5: Perfect Foresight Solution



Note: Perfect foresight dynamics of model variables in response to changes in the relative price of investment in the top-left panel.

similar result. Second, the capital stock to GDP ratio increases, reflecting its inverse relation to the marginal product of capital in the firm's demand for capital. Third, output increases too.

Figure 3.6 plots the calibrated model's equivalent to the U.S. balance sheet in Figure 3.1. The top panel plots the sizes of the model's gross (assets plus liabilities) and net (assets minus liabilities) balance sheets, as a multiple of output. By construction of the calibrated value of β, the gross balance sheet coincides with that of the U.S. economy in 1960. The subsequent growth in both the gross and net balance sheet is strikingly similar to the U.S. data. To see this connection, in the bottom

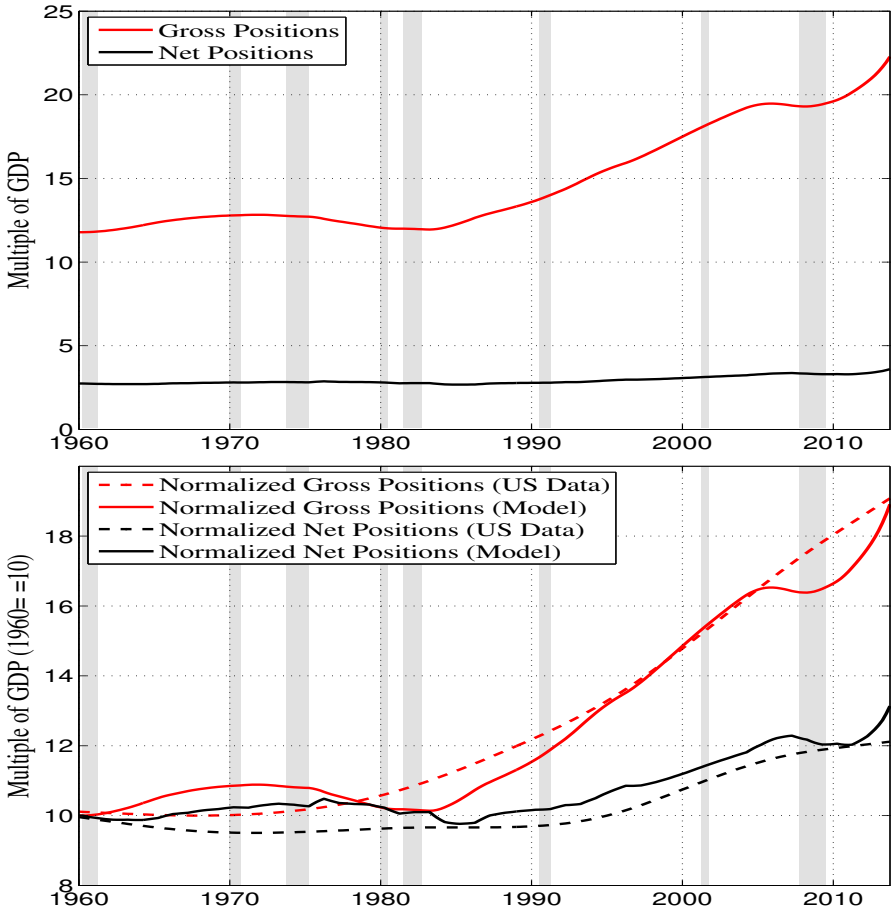
panel, I normalize both series (solid lines) to index 10 in year 1980, and compare them to their de-trended equivalents (dotted lines) in U.S. data (see Figure 3.1). The expansion in the model's balance sheet tracks the data near-perfectly.

Why do gross positions significantly outpace net ones? By clearing conditions (3.19) and (3.20), the economy's total financial claims depend only on the leverage ratio, θ, and (linearly) on the aggregate capital stock, K_t. The growth in the ratio of total financial claims to output therefore mirrors the growth in the capital stock to output ratio, K_t/Y_t. The economy's net equity, $\eta_t K_t$, is a linear function of the aggregate capital stock too. But its growth is partially offset by a lower per-unit valuation (replacement cost) η_t.

Figure 3.7 plots the calibrated model's equivalent to the U.S. sectoral balances in Figure 3.2. The positive y-axis measures financial assets; the negative y-axis measures liabilities. By accounting identity, net financial wealth is zero—total financial assets exactly offset all financial liabilities. Consistent with data, most of the growth in financial claims take place *within* the financial sector. Banks increase their investments. Borrowing increases too, both from households and the interbank market, as a source of leveraging up their investments. But the growth in deposits (assets for the non-financial sector) is negligible. Funds are instead taken from within the financial system.

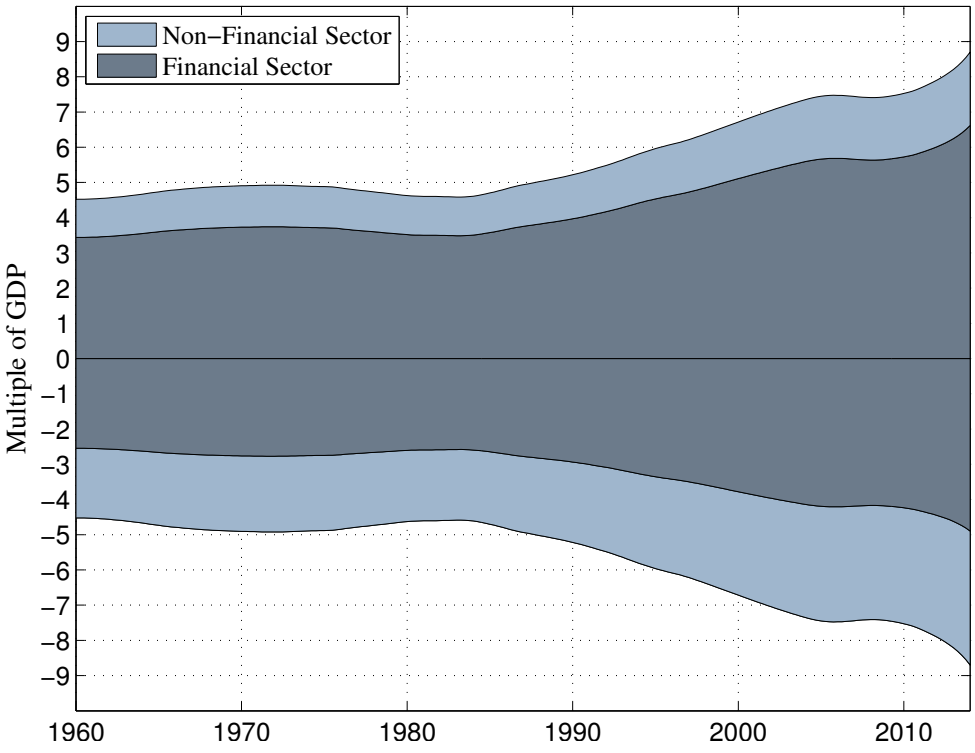
With a permanently lower relative price of investment, the gross asset positions remain elevated forever. A higher steady state capital stock implies a higher steady state investment rate, which in turn implies a higher rate of lending and borrowing within the financial sector. Large gross asset positions (and small net asset positions)

Figure 3.6: Balance Sheet of Calibrated Model



Note: Solid lines are the perfect foresight solution of gross and net balance sheets as a multiple of GDP of calibrated model. In the bottom panel, series are normalised to index 10 in year 1960. The dotted lines are the normalized, de-trended (HP-filter: smoothing parameter 100,000) series from *U.S. data* (see Figure 3.1).

Figure 3.7: Gross Balance Sheet by Sector in Calibrated Model



Note: Size of the gross financial balance sheet by sector in the perfect foresight solution, in response to changes in the relative price of investment in Figure 3.3. Positive y-axis measures financial assets. Negative y-axis measures liabilities.

are thus the new normal, even if the relative price of capital goods stops falling.

Comparative statics

The preceding subsection assumed that agents ex-ante perfectly anticipated the observed decline in the relative price of investment. This assumption may seem unrealistic. Few financiers knew in the mid-1980s that computers, machinery, and cars would almost half in price over the following three decades. Even so, a permanent shift in the investment price, anticipated or not, will eventually have to push the economy to a new steady state. As such, it is helpful to also consider the effects of varying the relative price of investment on the steady state of the model at the baseline parameter values.

The change in the steady state values of the variables is similar to that of the perfect foresight solution above. A 30% fall in the relative price of investment halves the steady state real interest rate. It almost doubles the steady state size of the gross balance sheets of banks. And the net balance sheet increases by merely a third. Given the similarities between the two exercises in this section, I relegate the figures to Appendix 3.A.

3.7 Leverage and collateral value multiplier

Is the diverging trend between gross and net positions a source of financial fragility? Policy makers are convinced so. To strengthen the resilience of the financial sector, regulators have since the Great Recession imposed tighter leverage constraints on

banks. Basel III serves as a recent example. Such regulation is thought to present a trade-off. One the one hand, a lower leverage ratio decreases financial vulnerability. All else equal, less indebted banks have a larger buffer to cushion a fall in the value of their assets. One the other hand, a lower leverage ratio allows funds to less efficiently flow from the households to the firms, as banks can borrow less to invest in productive capital.

This trade-off emerges in this model too. A higher leverage ratio, θ, increases the steady state size of the economy. For a given level of downpayment, a higher leverage ratio allows banks to issue more interbank bonds. More interbank bonds, in turn, lead to more collateral that banks can use to increase their deposit borrowing. Put differently, debt collateralizes even more debt. Figure 3.8 plots the steady state level of output as a function of the leverage ratio. Steady state economic activity expands and contracts by more than one-for-one with percentage point changes in the leverage ratio.

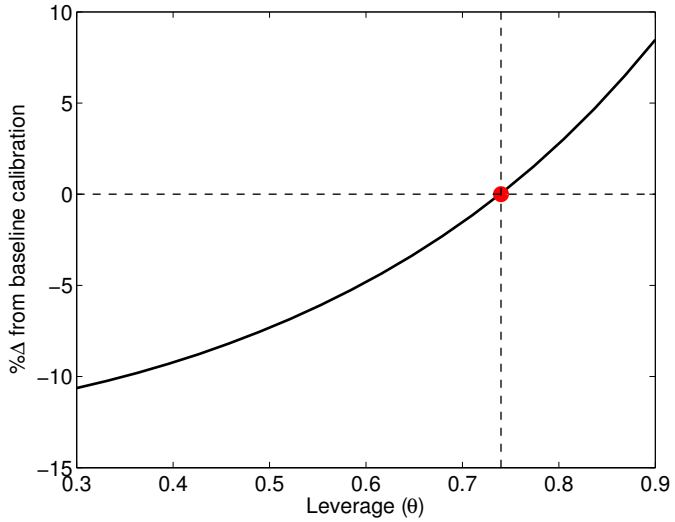
On the flip side, a higher leverage ratio, θ, increases financial fragility. The most vulnerable banks are lead banks that were lead banks in the previous period too. They hold no cushion of interbank bonds. And they can if necessary only liquidate their capital at a low price (relative to non-lead banks). Formally, their balance sheets fall under water if the capital stock unexpectedly declines by a factor of less than $\theta/(r_t + \eta_t^l \lambda) < 1$.[3] This cut-off value is increasing in θ and decreasing in η_t^l. A

[3]From (3.8), recall that a non-lead bank's net worth is:

$$\underbrace{(r_t + \eta_t^l \lambda)k_t + b_t}_{\text{Assets}} - \underbrace{(\theta k_t + \theta b_t)}_{\text{Fixed Liabilities}}$$

Set $b_t = 0$ (as the bank was a non-lead bank in the previous period) and rearrange to obtain the

Figure 3.8: Comparative Statics of Steady State Output



higher leverage ratio decreases the net worth as a proportion of total assets. And a lower relative price of investment lowers the per-unit market value of capital that if necessary could be liquidated to repay maturing debt.

The model uncovers an additional vulnerability. Because lending is restricted by the value of existing collateral, a fall in assets can cut investment and output by a large multiple. Consider an unexpected fall in the capital stock. As debt liabilities are fixed, the net equity of banks falls by a multiple of the fall in the capital stock. Less equity, in turn, leads to less collateral to borrow. The problem now is that the households (which get all of their deposits back, assuming that banks stay solvent) have most of the funds that the economy needs for investments to rebuild the capital stock. But the financial sector cannot efficiently channel these funds to the firm because the banks have insufficient collateral to borrow and make new investments.

cut-off value.

Moore (2011) refers to a similar amplification mechanism as the 'collateral-value multiplier', through which higher interest rates lower collateral values, which in turn limits borrowing and increases interest rates even further.

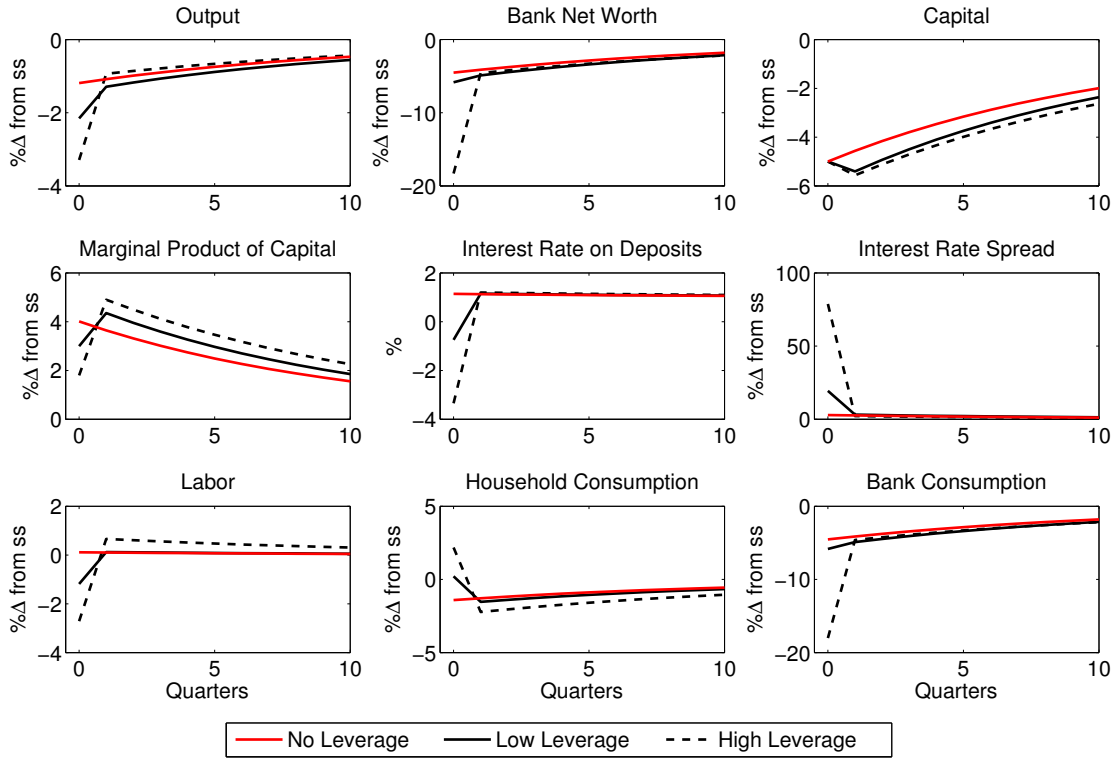
To illustrate this effect in the model, consider the following crisis experiment. In period $t = 0$, the existing capital stock, K_0, unexpectedly falls by 5%. I consider three scenarios: 'high leverage' ($\theta = 0.95$), 'low leverage' ($\theta = 0.74$, baseline calibration), and 'no leverage'. The first two assume fixed liabilities, i.e. unchanged B_0 and B_0^*. The third simulation ('no leverage') assumes that the existing liabilities, B_0 and B_0^*, adjust in proportion to the fall in the capital stock. This case is equivalent to all lending being in the form of equity rather than debt, as the face value of all claims depend on the value of the banks' assets. Figure 3.9 shows the response of the model economy.

In the model with equity instruments ('no leverage', red line), the shock produces only a modest decline in output. The decline in the banks' net worth is of a similar magnitude as the decline in the capital stock.[4] The household, too, faces a proportional decline in its net worth, with the value of deposits falling. Both the banks and the household cut consumption and increase investment, incentivized by a higher return on capital. Labor increases (marginally) as the household cuts consumption, limiting the fall in output. Funds channel efficiently from the household to the firm to rebuild the capital stock.

In the model with fixed liabilities ('low' and 'high leverage', black lines), the shock produces a larger drop in output. The net worth of the banks declines by a

[4]The rise in the interest rate partially offsets the fall. See the RHS of (3.8).

Figure 3.9: Responses to Capital Shock



multiple of the fall in the capital stock. A lower net worth, in turn, decreases the collateral available to borrow. Investment drops, leading to a second-round decline in capital. With a fall in the demand for next-period deposits, the real deposit rate turns negative. A lower deposit rate and a higher return on capital increases the interest rate spread (that is, the return on assets minus the cost of liabilities). But the banks are unable to profit from the higher spread as they has insufficient collateral to increase their borrowing. The household, which gets all its deposits back, increases its consumption in light of the lower interest rate on deposits.[5] Labor decreases as a result, which amplifies the fall in output.

The higher is leverage, the larger is the fall in bank collateral, and thus the larger is the drop in labor and output. But the recovery is quicker. A higher leverage deepens the crisis but allows banks to more quickly rebuild their net worth, as they can more rapidly borrow to exploit the spread between the funding costs and investment returns.

Note the source of the friction. With equity instruments or low leverage, a fall in asset prices hit all agents in equal proportions. With high leverage and fixed liabilities, shocks hit agents disproportionally hard. Debtors get their assets back, while the net worth of equity owners falls by a multiple of the initial fall in asset prices.

[5]Household consumption, C_t^h, increases, but total consumption, $C_t^h + C_t^n + C_t^l$, decreases as banks cut their consumption.

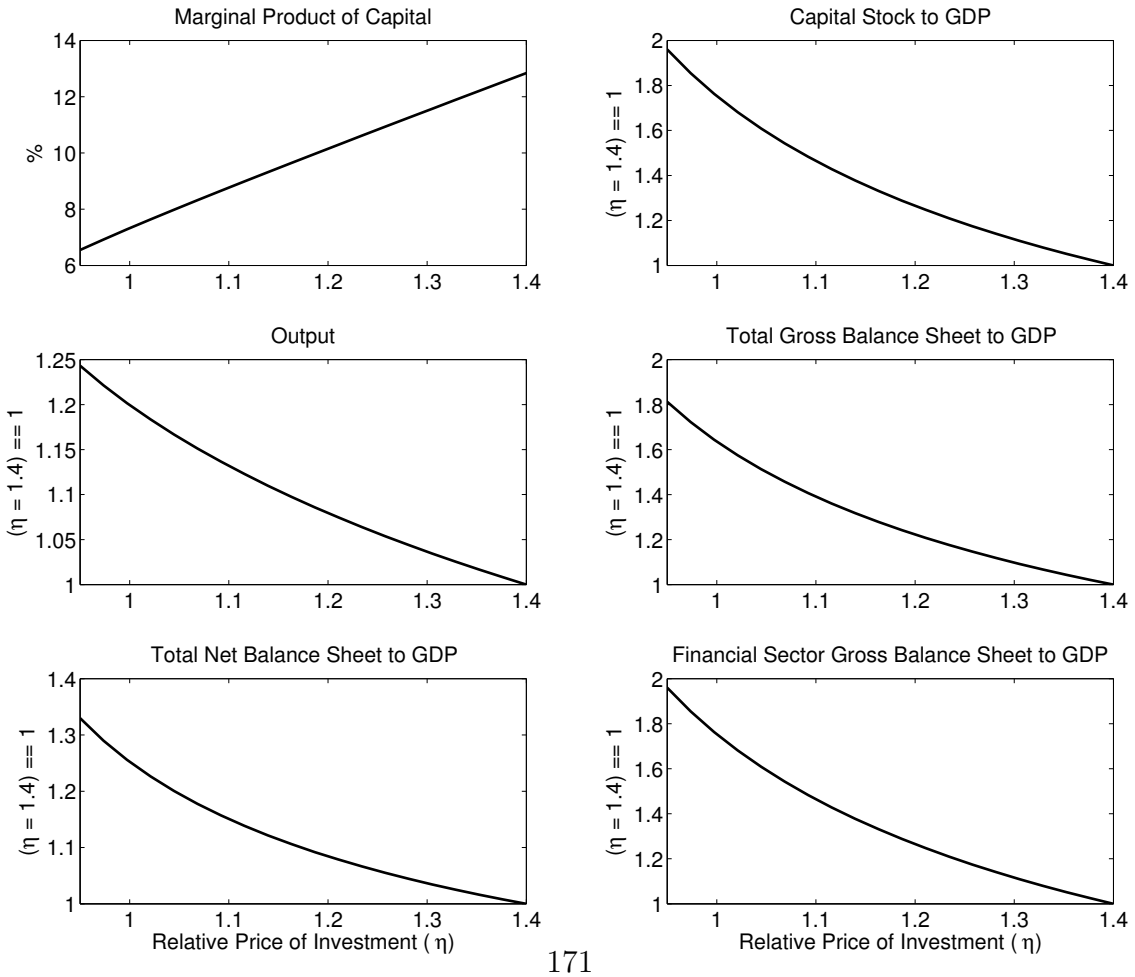
3.8 Concluding remarks

Why do bilateral gross flows arise? And why have they soared? Despite a renewed interest in the literature to examine gross flows, there is no formal framework that addresses these questions. In simple neoclassical theory, any two agents would never simultaneously lend to each other: there is no inherent value in borrowing from someone for the sole purpose of lending it out to the same person. Although net flows matter in these models, the size of bilateral gross flows is thus both irrelevant and indeterminate. Viewed through this lens, Obstfeld (2010, p.1) concludes that: "*economists' knowledge of the determinants of gross flows...is exceedingly slim*".

This paper has presented a framework in which banks simultaneously lend and borrow in the wholesale interbank market. The model challenges the conventional belief that bilateral gross positions are irrelevant. In evaluating the dynamic features of the model, I have showed that gross positions grow at a faster rate than net positions in response to a reduction in the relative price of investment. This finding differs from other arguments put forward in the literature (e.g. lenient regulation, excessive risk taking, or financial globalization) triggered the expansion of gross balance sheets in the run-up to the Great Recession. The dynamic simulations predict that the disparity between gross and net positions will stay elevated, even if the relative price of capital goods stops falling.

Appendix

3.A Comparative statics of steady state variables



3.B Data source and calculations

Figure 1: Data source from Z.1. Financial Accounts of the U.S. Total net assets is the sum of net equity of: household and nonprofit (B.101, line 41); nonfinancial corporate business (B.103, line 38); nonfinancial noncorporate business (B.104, line 35); financial business (L.108, line 1 + S.6.a, line 100 - L.108, line 28); state and local governments (L.107, line 1 + S.8.a, line 76 - L.107, line 18); federal government (L.106, line 1 + S.7.a, line 96 - L.106, line 16); and rest of the world (L.132, line 1 - L.132, line 21). Financial assets and liabilities are reported on a quarterly basis. Real assets are linearly interpolated on a quarterly basis from annual data. The gross balance sheet is the sum of total assets and total liabilities. See Figure 3.2 for calculations. All variables are divided by seasonally adjusted annual nominal GDP (source: Federal Reserve Economic Data). The data sample is from end-of-period 1959Q4 to 2013Q4.

Figure 2: Data source from Z.1. Financial Accounts of the U.S. Sectors are defined as: Non-Financial Sector (L.100 - L.105); Financial Sector (L.108); Government (L.105); and Rest of the World (L.132). Gross assets are defined as 'Total Financial Assets' minus 'Corporate Equities' minus 'Equity in Non-Corporate Business' minus 'Miscellaneous Assets'. Liabilities are defined as 'Total Liabilities' minus 'Miscellaneous Liabilities'. All variables are divided by seasonally adjusted annual nominal GDP (source: Federal Reserve Economic Data). The data sample is from end-of-period 1951Q4 to 2015Q3.

3.C Calibrated parameter values

Table 3.C.1: Calibrated parameter values

	Calibrated Parameters	Value	Reference
β'	Discount Factor Household	0.99	RBC Literature
λ	Depreciation Factor	0.975	RBC Literature
α	Capital Share	1/4	RBC Literature
ν	Inverse Frisch Elasticity	1	RBC Literature
θ	Leverage	0.74	U.S. Financial Leverage[1]
π	Fraction of Lead Banks	0.5	Symmetric Distribution
σ_η	Dispersion of Investment Price	0.00745	SE of AR(1)[2]
β	Discount Factor Banks	0.905	Match U.S. Gross Balance Sheet 1960

[1] Average financial leverage from 1951Q4 to 2015Q3. Data source from Z.1. Financial Accounts of the U.S. I define financial leverage as total liabilities divided by total financial assets. Total financial assets is the sum of total financial assets of: domestic non-financial sectors (L.100, line 1); domestic financial sectors (L.108, line 1); and rest of the world (L.132, line 1). Total liabilities is the sum of total liabilities of: domestic non-financial sector (L.100, line 27); domestic financial sectors (L.108, line 28.); and rest of the world (L.132, line 1).

[2] Standard error of AR(1) regression of the relative price of investment. Annual data is taken from Eichengreen (2015). The data sample is from 1959 to 2013. The standard error is divided by $\sqrt{4}$ to convert into quarterly frequency.

3.D Optimality conditions

3.D.1 Banks

For notational convenience, I henceforth use h to denote any element in the set $\{l, n\}$.

Insert the law of motion for capital (3.2) into the flow-of-funds constraints (3.4) to obtain

$$c_t^h = \Big(r_t - \theta_t + \eta_t^h \lambda\Big) k_t + \Big(1 - \theta_t^*\Big) b_t - \Big(\eta_t^h - q_t \theta_{t+1}^h\Big) k_{t+1}^h - \Big(q_t - q_t^* \theta_{t+1}^{*,h}\Big) b_{t+1}^h, . \tag{3.22}$$

Let $V(k_t, b_t, \theta_t, \theta_t^*)$ denote the value function of a bank with capital k_t, interbank bond positions b_t, and leverage θ_t and θ_t^* at the beginning of the period t before facing the stochastic investment price. The Bellman equation is then

$$\begin{aligned} V(k_t, b_t, \theta_t, \theta_t^*) = \pi \cdot \max_{\substack{c_t^l, k_{t+1}^l, b_{t+1}^l, \theta_{t+1}^l, \theta_{t+1}^{*,l} \\ s.t.(3.22)}} \Big\{ & \log c_t^l + \beta V(k_{t+1}^l, b_{t+1}^l, \theta_{t+1}^l, \theta_{t+1}^{*,l}) \\ & + \mu_t^l (k_{t+1}^l - \lambda k_t) + \nu_t^l b_{t+1}^l - \omega_t^l (\theta_{t+1}^l - \theta) - \omega_t^{*,l} (\theta_{t+1}^{*,l} - \theta) \Big\} \\ + (1 - \pi) \cdot \max_{\substack{c_t^n, k_{t+1}^n, b_{t+1}^n, \theta_{t+1}^n, \theta_{t+1}^{*,n} \\ s.t.(3.22)}} \Big\{ & \log c_t^n + \beta V(k_{t+1}^n, b_{t+1}^n, \theta_{t+1}^n, \theta_{t+1}^{*,n}) \\ & + \mu_t^n (k_{t+1}^n - \lambda k_t) + \nu_t^n b_{t+1}^n - \omega_t^n (\theta_{t+1}^n - \theta) - \omega_t^{*,n} (\theta_{t+1}^{*,n} - \theta) \Big\}, \end{aligned}$$

where $\mu_t^h, \nu_t^h, \omega_t^h, \omega_t^{*,h}$ denote the time-t Lagrange multipliers for the constraints $k_{t+1}^h \geq \lambda k_t, b_{t+1}^h \geq 0, \theta_{t+1}^h \leq \theta, \theta_{t+1}^{*,h} \leq \theta$, respectively. I am looking for a corner solution in

which: lead banks invest all their funds in capital goods; non-lead banks split their investment across interbank bonds and capital goods; and both types of banks maximize leverage. Put differently, I want $\nu_t^l, \omega_t^h, \omega_t^{*,h} > 0$ and $\nu_t^n, \mu_{t+1}^h = 0$ to hold at the optimum. Substituting (3.22) into the Bellman equation and differentiating yields the following first order conditions:

$$k_{t+1}^h: \quad -\frac{1}{c_t^h}(\eta_t^h - q_t\theta_{t+1}^h) + \beta V_k(k_{t+1}^h, b_{t+1}^h, \theta_{t+1}^h, \theta_{t+1}^{*,h}) = -\mu_t^h = 0, \tag{3.23}$$

$$b_{t+1}^h: \quad -\frac{1}{c_t^h}(q_t - q_t^*\theta_{t+1}^{*,h}) + \beta V_b(k_{t+1}^h, b_{t+1}^h, \theta_{t+1}^h, \theta_{t+1}^{*,h}) = -\nu_t^h \leq 0, \tag{3.24}$$

$$\theta_{t+1}^h: \quad \frac{1}{c_t^h} \cdot q_t k_{t+1}^h + \beta V_\theta(k_{t+1}^h, b_{t+1}^h, \theta_{t+1}^h, \theta_{t+1}^{*,h}) = \omega_t^h > 0, \tag{3.25}$$

$$\theta_{t+1}^{*,h}: \quad \frac{1}{c_t^h} \cdot q_t^* b_{t+1}^h + \beta V_{\theta^*}(k_{t+1}^h, b_{t+1}^h, \theta_{t+1}^h, \theta_{t+1}^{*,h}) = \omega_t^{*,h} > 0, \tag{3.26}$$

where I use $V_k, V_b, V_\theta, V_{\theta^*}$ to denote the partial derivatives of the value function $V(k, b, \theta, \theta^*)$, and where equality of (3.24) holds if and only if $h = n$. The envelope theorem gives

$$\begin{aligned}
V_k(k_t, b_t, \theta_t, \theta_t^*) &= \pi\Big[\frac{1}{c_t^l}(r_t - \theta_t + \eta_t^l\lambda) - \mu_t^l\lambda\Big] + (1-\pi)\Big[\frac{1}{c_t^n}(r_t - \theta_t + \eta_t^n\lambda) - \mu_t^n\lambda\Big], \\
V_b(k_t, b_t, \theta_t, \theta_t^*) &= \pi\frac{1}{c_t^l}(1 - \theta_t^*) + (1-\pi)\frac{1}{c_t^n}(1 - \theta_t^*), \\
V_\theta(k_t, b_t, \theta_t, \theta_t^*) &= -\pi\frac{1}{c_t^l}k_t - (1-\pi)\frac{1}{c_t^n}k_t, \\
V_{\theta^*}(k_t, b_t, \theta_t, \theta_t^*) &= -\pi\frac{1}{c_t^l}b_t - (1-\pi)\frac{1}{c_t^n}b_t.
\end{aligned}$$

Iterating forward, and recalling that $\mu^h_{t+1} = 0$, yields

$$V_k(k^h_{t+1}, b^h_{t+1}, \theta^h_{t+1}, \theta^{*,h}_{t+1}) = \pi \frac{1}{c^{hl}_{t+1}}(r_{t+1} - \theta^h_{t+1} + \eta^l_{t+1}\lambda) + (1-\pi)\frac{1}{c^{hn}_{t+1}}(r_{t+1} - \theta^h_{t+1} + \eta^n_{t+1}\lambda), \tag{3.27}$$

$$V_b(k^h_{t+1}, b^h_{t+1}, \theta^h_{t+1}, \theta^{*,h}_{t+1}) = \pi \frac{1}{c^{hl}_{t+1}}(1 - \theta^{*,h}_{t+1}) + (1-\pi)\frac{1}{c^{hn}_{t+1}}(1 - \theta^{*,h}_{t+1}), \tag{3.28}$$

$$V_\theta(k^h_{t+1}, b^h_{t+1}, \theta^h_{t+1}, \theta^{*,h}_{t+1}) = -\pi \frac{1}{c^{hl}_{t+1}}k^h_{t+1} - (1-\pi)\frac{1}{c^{hn}_{t+1}}k^h_{t+1}, \tag{3.29}$$

$$V_{\theta^*}(k^h_{t+1}, b^h_{t+1}, \theta^h_{t+1}, \theta^{*,h}_{t+1}) = -\pi \frac{1}{c^{hl}_{t+1}}b^h_{t+1} - (1-\pi)\frac{1}{c^{hn}_{t+1}}b^h_{t+1}, \tag{3.30}$$

where $c^{kk'}_{t+1}$ is date $t+1$ consumption when the bank is of type k at date t and k' at date $t+1$. Then, insert equations (3.28) - (3.30) into the first order conditions (3.24) - (3.26) to rewrite the Euler equations for leverage and interbank bond positions as

$$\pi \frac{\beta c^h_t}{c^{hl}_{t+1}} + (1-\pi)\frac{\beta c^h_t}{c^{hn}_{t+1}} \leq \frac{(q_t - q^*_t\theta^{*,h}_{t+1})}{(1-\theta^{*,h}_{t+1})}, \quad \text{(interbank bonds)} \tag{3.31}$$

$$\pi \frac{\beta c^h_t}{c^{hl}_{t+1}} + (1-\pi)\frac{\beta c^h_t}{c^{hn}_{t+1}} < q_t, \quad \text{(interbank leverage)} \tag{3.32}$$

$$\pi \frac{\beta c^h_t}{c^{hl}_{t+1}} + (1-\pi)\frac{\beta c^h_t}{c^{hn}_{t+1}} < q^*_t, \quad \text{(deposit leverage)}, \tag{3.33}$$

where equality of (3.31) holds if and only $h = n$. Now suppose $q_t \in (\theta q^*_t, q^*_t)$. As $\theta^{*,h}_{t+1} \leq \theta$, it follows that

$$q_t \in (\theta q^*_t, q^*_t) \implies q_t \in (\theta^{*,h}_{t+1} q^*_t, q^*_t) \implies \frac{(q_t - q^*_t\theta^{*,h}_{t+1})}{(1-\theta^{*,h}_{t+1})} < q_t < q^*_t.$$

As there is no upper bound on interbank bond holdings b^h_{t+1}, an optimizing bank

would never choose a consumption policy such that the LHS:s of (3.31) − (3.33) are strictly bigger than then the RHS of (3.31). Therefore, if (3.5) holds, we then know that (3.32) and (3.33) always hold, i.e. banks always maximize leverage. This result proves Proposition 3.1. Let me now focus on equilibria in the neighborhood of which $q_t \in (\theta q_t^*, q_t^*)$, so that $\theta_{t+1}^h = \theta_{t+1}^{*,h} = \theta$.

Substituting (3.27) - (3.28) into the first order conditions (3.23) - (3.24) yields the final Euler equations for capital and bond holdings

$$1 = \pi \frac{\beta c_t^l}{c_{t+1}^{ll}} R_{t+1}^{ll} + (1-\pi) \frac{\beta c_t^l}{c_{t+1}^{ln}} R_{t+1}^{ln}, \tag{3.34}$$

$$1 = \pi \frac{\beta c_t^n}{c_{t+1}^{nl}} R_{t+1}^{nl} + (1-\pi) \frac{\beta c_t^n}{c_{t+1}^{nn}} R_{t+1}^{nn}, \tag{3.35}$$

$$1 > \pi \frac{\beta c_t^l}{c_{t+1}^{ll}} R_{t+1}^{b} + (1-\pi) \frac{\beta c_t^l}{c_{t+1}^{ln}} R_{t+1}^{b}, \tag{3.36}$$

$$1 = \pi \frac{\beta c_t^n}{c_{t+1}^{nl}} R_{t+1}^{b} + (1-\pi) \frac{\beta c_t^n}{c_{t+1}^{nn}} R_{t+1}^{b}. \tag{3.37}$$

Note that (3.36) and (3.37) and just special cases of (3.31) when $\theta_{t+1}^{*,h} = \theta$. It remains to show there exists a corner solution satisfying the above constraints, and which ultimately yields an interbank bond price satisfying (3.5). Let me guess the

solution takes the following form:

$$
\begin{aligned}
c_t^l &= (1-\beta)\Big[(r_t - \theta + \eta_t^l\lambda)k_t + (1-\theta)b_t\Big], \\
b_{t+1}^l &= 0, \\
k_{t+1}^l &= \frac{1}{(\eta_t^l - q_t\theta)}\beta\Big[(r_t - \theta + \eta_t^l\lambda)k_t + (1-\theta)b_t\Big], \\
c_t^n &= (1-\beta)\Big[(r_t - \theta + \eta_t^n\lambda)k_t + (1-\theta)b_t\Big], \\
b_{t+1}^n &= \frac{1}{(q_t - q_t^*\theta)}(1-f_t)\beta\Big[(r_t - \theta + \eta_t^n\lambda)k_t + (1-\theta)b_t\Big], \\
k_{t+1}^n &= \frac{1}{(\eta_t^n - q_t\theta)}f_t\beta\Big[(r_t - \theta + \eta_t^n\lambda)k_t + (1-\theta)b_t\Big].
\end{aligned}
$$

where f_t is the fraction of capital in the portfolio for the non-lead banks, as defined in (3.16). Under this guess, iterating forward gives

$$
c_{t+1}^{hh'} = (1-\beta)\Big[(r_{t+1} - \theta + \eta_{t+1}^{h'}\lambda)k_{t+1}^h + (1-\theta)b_{t+1}^h\Big],
$$

when the bank is of type h at date t and h' at date $t+1$. Dividing by time-t consumption gives

$$
\frac{c_{t+1}^{ll}}{c_t^l} = \beta R_{t+1}^{ll}, \quad \frac{c_{t+1}^{ln}}{c_t^l} = \beta R_{t+1}^{ln},
$$

which means that (3.34) is always satisfied. Condition (3.36) can then be rewritten as

$$
1 > \pi\frac{R_{t+1}^b}{R_{t+1}^{ll}} + (1-\pi)\frac{R_{t+1}^b}{R_{t+1}^{ln}}, \tag{3.38}
$$

which is also satisfied in the neighborhood of our steady state where $R_{t+1}^b < R_{t+1}^{ll} <$

R^{ln}_{t+1}, by Proposition 3.2. (proved in Appendix 3.G). Similarly, write

$$\begin{aligned}\frac{c^{nl}_{t+1}}{c^{n}_{t}} &= \beta\Big[f_t R^{nl}_{t+1} + (1-f_t)R^{b}_{t+1}\Big],\\ \frac{c^{nn}_{t+1}}{c^{n}_{t}} &= \beta\Big[f_t R^{nn}_{t+1} + (1-f_t)R^{b}_{t+1}\Big],\end{aligned}$$

so that (3.37) and (3.35) can be rewritten as

$$1 = \pi\frac{R^{b}_{t+1}}{f_t R^{nl}_{t+1} + (1-f_t)R^{b}_{t+1}} + (1-\pi)\frac{R^{b}_{t+1}}{f_t R^{nn}_{t+1} + (1-f_t)R^{b}_{t+1}}, \tag{3.39}$$

$$1 = \pi\frac{R^{nl}_{t+1}}{f_t R^{nl}_{t+1} + (1-f_t)R^{b}_{t+1}} + (1-\pi)\frac{R^{nn}_{t+1}}{f_t R^{nn}_{t+1} + (1-f_t)R^{b}_{t+1}}. \tag{3.40}$$

Note that multiplying the RHS of (3.40) by f_t and the RHS of (3.39) by $1-f_t$, and adding them together always equals $1 = f_t + (1-f_t)$. Thus, the above expressions are not mutually independent: if one of them holds, then so does the other. Combing the two equations into one conveys the same amount of information as both of them separately. Subtracting (3.39) by (3.40) and rearranging yields

$$\pi \cdot \left[\frac{R^{b}_{t+1} - R^{nl}_{t+1}}{f_t R^{nl}_{t+1} + (1-f_t)R^{b}_{t+1}}\right] = (1-\pi)\cdot\left[\frac{R^{nn}_{t+1} - R^{b}_{t+1}}{f_t R^{nn}_{t+1} + (1-f_t)R^{b}_{t+1}}\right], \tag{3.41}$$

which fully characterizes the optimal portfolio construction for the non-lead bank. This equation is the same as (3.15) in the text. Note that, by Proposition 3.2., we have $R^{nl}_{t+1} < R^{b}_{t+1} < R^{nn}_{t+1}$ so that both sides in (3.41) are strictly positive.

Aggregating the idiosyncratic solutions across all banks yields the following set of optimality conditions for the aggregate banking system:

Bank total consumption:

$$C_t^n = (1-\pi)\cdot(1-\beta)\Big[(r_t - \theta + \eta_t^n\lambda)K_t + (1-\theta)B_t\Big], \tag{3.42}$$

$$C_t^l = \pi\cdot(1-\beta)\Big[(r_t - \theta + \eta_t^l\lambda)K_t + (1-\theta)B_t\Big]. \tag{3.43}$$

Bank total investment:

$$(q_t - \theta q_t^*)B_{t+1}^n + (\eta_t^n - q_t\theta)K_{t+1}^n = (1-\pi)\cdot\beta\Big[(r_t - \theta + \eta_t^n\lambda)K_t + (1-\theta)B_t\Big], \tag{3.44}$$

$$(\eta_t^l - q_t\theta)K_{t+1}^l = \pi\cdot\beta\Big[(r_t - \theta + \eta_t^l\lambda)K_t + (1-\theta)B_t\Big]. \tag{3.45}$$

Total non-lead bank investment portfolio:

$$\pi\cdot\left[\frac{R_{t+1}^b - R_{t+1}^{nl}}{F_tR_{t+1}^{nl} + (1-F_t)R_{t+1}^b}\right] = (1-\pi)\cdot\left[\frac{R_{t+1}^{nn} - R_{t+1}^b}{F_tR_{t+1}^{nn} + (1-F_t)R_{t+1}^b}\right], \tag{3.46}$$

where

$$f_t = F_t \equiv \frac{(\eta_t^n - q_t\theta)K_{t+1}^n}{(\eta_t^n - q_t\theta)K_{t+1}^n + (q_t - q_t^*\theta)B_{t+1}^n} \in (0,1).$$

Law of motion for capital:

$$K_{t+1}^l = \pi\lambda K_t + I_t^l, \tag{3.47}$$

$$K_{t+1}^n = (1-\pi)\lambda K_t + I_t^n. \tag{3.48}$$

Capital and interbank bond aggregation:

$$K_{t+1} = K^l_{t+1} + K^n_{t+1}, \tag{3.49}$$

$$B_{t+1} = B^n_{t+1}. \tag{3.50}$$

3.D.2 Representative firm

Aggregate production, and the standard capital and labor demand equations:

$$Y_t = K_t^\alpha N_t^{1-\alpha}, \tag{3.51}$$

$$r_t = \alpha(Y_t/K_t), \tag{3.52}$$

$$w_t = (1-\alpha)(Y_t/N_t). \tag{3.53}$$

3.D.3 Representative household

The standard Euler and labor supply equations:

$$q^*_t = \beta \mathbb{E}_t[C^h_t/C^h_{t+1}], \tag{3.54}$$

$$w_t = N^\nu_t C^h_t. \tag{3.55}$$

3.E Market clearing and equilibrium

Interbank market clearing:

$$\theta K_{t+1} = B_{t+1}, \tag{3.56}$$

Goods market clearing:

$$Y_t = C_t^h + C_t^n + C_t^l + \eta_t^l I_t^l + \eta_t^n I_t^n. \tag{3.57}$$

Twelve endogenous quantities

$$(Y_t, C_t^h, C_t^l, C_t^n, K_{t+1}, K_{t+1}^l, K_{t+1}^n, I_t^l, I_t^n, B_{t+1}, B_{t+1}^n, N_t)$$

and four prices

$$(r_t, w_t, q_t, q_t^*),$$

then, define the recursive competitive equilibrium as a function of the state variables (K_t, B_t) which satisfies the sixteen equilibrium conditions (3.42) - (3.57). Once all the market clearing conditions are satisfied, the deposit market $\theta B_{t+1} = B_{t+1}^*$ clears by Walras' Law.

3.F Steady state calculations

I consider a steady-state economy in which there are no changes in prices or in any of the aggregate endogenous quantities. In what follows, $\bar{X}$ denotes the steady state value of quantity X as a fraction of steady state capital K. Once K is recovered, the steady state values of the absolute quantities trivially follow.

First, the household's discount factor β pins down the steady state deposit price q^* in (3.54).

Next, incorporate interbank market clearing (3.56) and aggregation (3.49)-(3.50)

in equations (3.44)-(3.46) to get steady state conditions

$$(q - \theta q^*)\theta + (\eta_t^n - q\theta)\bar{K}^n = (1-\pi)\cdot\beta\cdot(r + \eta_t^n\lambda - \theta^2), \tag{3.58}$$

$$(\eta_t^l - q\theta)(1-\bar{K}^n) = \pi\cdot\beta\cdot(r + \eta_t^l\lambda - \theta^2), \tag{3.59}$$

$$\pi\cdot\left[\frac{R^b - R^{nl}}{(r-\theta+\eta_t^l\lambda)\bar{K}^n + (1-\theta)\theta}\right] = (1-\pi)\cdot\left[\frac{R^{nn} - R^b}{(r-\theta+\eta_t^n\lambda)\bar{K}^n + (1-\theta)\theta}\right], \tag{3.60}$$

where R^b, R^{nl}, and R^{nn} are the steady state ROE as functions of q and q^*. Equations (3.59) - (3.60) form a system of three equations in three unknowns: q, r, and $\bar{K}^n$. Note, crucially, that capital demand does not feature in determining steady state marginal return to capital r.

Steady state investment then follows from (3.47) - (3.49)

$$\bar{K}^l = 1 - \bar{K}^n,$$

$$\bar{I}^l = \bar{K}^l - \pi\lambda,$$

$$\bar{I}^n = \bar{K}^n - (1-\pi)\lambda,$$

and steady state consumption follows from (3.42) - (3.43)

$$\bar{C}^n = (1-\pi)\cdot(1-\beta)\cdot(r + \eta_t^n\lambda - \theta^2),$$

$$\bar{C}^l = \pi\cdot(1-\beta)\cdot(r + \eta_t^l\lambda - \theta^2).$$

Capital demand (3.52) at steady state implies

$$\bar{Y} = r/\alpha.$$

Steady state household consumption $\bar{C}^h$ then follows immediately from goods market clearing (3.57)

$$\bar{C}^h = \bar{Y} - \bar{C}^n - \bar{C}^l - \eta_t^l \bar{I}^l - \eta_t^n \bar{I}^n.$$

Next, equate labor demand (3.53) and supply (3.55), and divide by K, to get steady state absolute level of labor

$$N = \left[(1-\alpha) \frac{\bar{Y}}{\bar{C}^h} \right]^{\frac{1}{\nu+1}}.$$

Finally, the steady state level of capital then follows from production function (3.51)

$$K = N \bar{Y}^{\frac{1}{\alpha-1}}.$$

3.G Proof of proposition 3.2.

As a first step, it is easy to see that $R_{t+1}^{nl} < R_{t+1}^{nn} < R_{t+1}^{ln}$ and $R_{t+1}^{nl} < R_{t+1}^{ll} < R_{t+1}^{ln}$ as $\eta_t^l < \eta_t^n$. As the denominators of the LHS and RSH of (3.15) are both strictly positive, it must be that

$$R_{t+1}^{nl} < R_{t+1}^{b} < R_{t+1}^{nn}, \tag{3.61}$$

to ensure that the numerators are also of the same sign. Note that when $q_t \in (\theta q_t^*, q_t^*)$, we have

$$R_{t+1}^b \equiv \frac{1-\theta}{q_t - q_t^*\theta} > \frac{1-\theta}{q_t - q_t\theta} > 1, \tag{3.62}$$

so that, by (3.61), it follows that

$$1 < R_{t+1}^{nn}. \tag{3.63}$$

To complete the proof, it remains to show that $R_{t+1}^{nn} < R_{t+1}^{ll}$.

Consider the real-valued function $g : \mathbb{R} \to \mathbb{R}$ mapping $x \mapsto \frac{r_{t+1}-\theta+\lambda x}{x-q_t\theta}$ so that $g(\eta_t^n)$ and $g(\eta_t^l)$ are R_{t+1}^{nn} and R_{t+1}^{ll}, respectively. Consider only $\eta_t^l > q_t\theta$ so that the denominators in $g(\eta_t^n)$ and $g(\eta_t^l)$ are strictly positive. g is then a continuous and differentiable function on $[\eta_t^l, \infty)$ with derivative satisfying

$$g'(x) = \frac{1}{x - q_t\theta}[\lambda - g(x)] \implies g'(\eta_t^l) = \frac{1}{\eta_t^l - q_t\theta}[\lambda - g(\eta_t^l)]. \tag{3.64}$$

Further note that for $x \in [\eta_t^l, \infty)$,

$$g'(x) = 0 \iff g(x) \equiv \frac{r_{t+1} - \theta + \lambda x}{x - q_t\theta} = \lambda \iff r_{t+1} = \theta(1 - q_t\lambda). \tag{3.65}$$

We are now in a position to show that $g(\eta_t^n) < g(\eta_t^l)$. First, suppose that $r_{t+1} = \theta(1 - q_t\lambda)$ so that $g(x) = \lambda, \forall x \in \mathbb{R}$. In particular, $g(\eta_t^n) = \lambda < 1$, which contradicts (3.63). From (3.65), we therefore know that $\nexists\, x \in [\eta_t^l, \infty)$ s.t. $g'(x) = 0$, or equivalently, by (3.64), s.t. $g(x) = \lambda$.

Next, suppose that $g(\eta_t^l) < \lambda$. As $g(x)$ is continuous on $[\eta_t^l, \infty)$ with no x satisfying

$g(x) = \lambda$, it follows that $g(x) < \lambda, \forall x \in [\eta_t^l, \infty)$. In particular, we have $g(\eta_t^n) < \lambda < 1$. This is a contradiction to (3.63).

It must therefore be the case that $g(\eta_t^l) > \lambda$. By (3.64), we learn that $g'(\eta_t^l) < 0$. As $g'(x)$ is continuous on $[\eta_t^l, \infty)$ with $g'(x) \neq 0$ on the same domain, we know that $g'(x) < 0$ for all $x \in [\eta_t^l, \infty)$. Then

$$g(\eta_t^n) = g(\eta_t^l) + \int_{\eta_t^l}^{\eta_t^n} g'(x)dx < g(\eta_t^l), \tag{3.66}$$

which proves the desired result. ■

References

Aiyagari, S, Rao, Lawrence J. Christiano and Martin Eichenbaum (1992): "The Output, Employment, and Interest Rate Effects of Government Consumption", *Journal of Monetary Economics*, 30 (1), pp. 73-86.

Ascari, Guido, Anna Florio, and Alessandro Gobbi (2017): "Controlling Inflation with Switching Monetary and Fiscal Policies: Expectations, Fiscal Guidance and Timid Regime Changes", mimeo, University of Oxford.

Auernheimer, Leonardo and Benjamin Contreras (1990): "Control of the Interest Rate with a Government Budget Constraint: Determinacy of the Price Level and Other Results", mimeo: Texas A&M University.

Baldini, Alfredo and Marcos Poplawski-Ribeiro (2008): "Fiscal and Monetary Anchors for Price Stability: Evidence from Sub-Saharan Africa", IMF Working Paper No. 08/121, Washington, DC.

Barro, Robert J. (1974): "Are Government Bonds Net Wealth?", *Journal of Political Economy*, 82 (6), pp. 1095-1117.

Barro, Robert J. and Robert G. King (1984): "Time-Separable Prefer-

ences and Intertemporal-Substitution Models of Business Cycles", *Quarterly Journal of Economics*, 99 (4), pp. 817-839.

Baxter, Marianne and Robert G. King (1992): "Fiscal Policy in General Equilibrium", *American Economic Review*, 83 (3), pp. 315-334.

Beck-Friis, Peder and Tim Willems (2017): "Dissecting Fiscal Multipliers under the Fiscal Theory of the Price Level", *European Economic Review*, 95, pp. 62-83.

Begg, David K.H. and Badrul Hague (1984): "A Nominal Interest Rate Rule and Price Level Indeterminacy Reconsidered", *Greek Economic Review,* 6 (1), pp. 31-46.

Bernanke, Ben (2003): "Some Thoughts on Monetary Policy in Japan" , Speech, Tokyo, May, available at: http://www.federalreserve.gov/BoardDocs/Speeches/2003/20030531/.

_____ (2017): "The Fed and Fiscal Policy", Blog Post (January 13), available at: https://www.brookings.edu/blog/ben-bernanke/2017/01/13/the-fed-and-fiscal-policy.

Bhatia, Ashok and Tamim Bayoumi (2012): "Leverage? What Leverage? A Deep Dive into the U.S. Flow of Funds in Search of Clues to the Global Crisis" *IMF Working Paper*, 12/162.

Bhattarai, Saroj, Jae Won Lee,and Woong Yong Park (2014): "Inflation Dynamics: The Role of Public Debt and Policy Regimes", *Journal of Monetary Economics*, 67, pp. 93-108.

Bianchi, Francesco (2012): "Evolving Monetary/Fiscal Policy Mix in the United States", *American Economic Review Papers & Proceedings*, 101 (3), pp. 167-172.

Bianchi, Francesco and Cosmin Ilut (2014): "Monetary/Fiscal Policy Mix and Agents' Beliefs", NBER Working Paper No. 20194.

Borio, Claudio and Piti Disyatat (2009): "Unconventional Monetary Policies: An Appraisal", BIS Working Paper No. 292.

_____ (2011): "Global imbalances and the financial crisis: Link or no link?" BIS Working Paper No. 346, May.

_____ (2015): "Capital flows and the current account: Taking financing (more) seriously" BIS Working Paper No. 525, October.

Born, Benjamin, Alexandra Peter, and Johannes Pfeifer (2013): "Fiscal News and Macroeconomic Volatility", *Journal of Economic Dynamics and Control*, 37(12), pp. 2582-2601.

Brainard, Lael (2017): "Transitions in the Outlook and Monetary Policy", Federal Reserve Official Speech (March 1), available at: https://www.federalreserve.gov/newsevents/speech/brainard20170301a.htm.

Broner, Fernando, Tatiana Didier, Aitor Erce and Sergio Schmukler (2013): "Gross Capital Flows: Dynamics and Crises" *Journal of Monetary Economics*, 60 (1), pp. 113-133.

Buiter, Willem H. (2014): "The Simple Analytics of Helicopter Money: Why

it Works – Always", *Economics*, 8, pp. 1-53.

Canzoneri, Matthew, Robert Cumby, and Behzad Diba (2011): "The Interaction Between Monetary and Fiscal Policy", in: Benjamin M. Friedman and Michael Woodford (eds.), *Handbook of Monetary Economics*, Volume 3B, The Netherlands: North-Holland, pp. 935-999.

Chen, Xiaoshan, Eric M. Leeper, and Campbell Leith (2015): "U.S. Monetary and Fiscal Policy: Conflict or Cooperation?", mimeo, University of Glasgow.

Cherif, Reda and Fuad Hasanov (2012): "Public Debt Dynamics: The Effects of Austerity, Inflation, and Growth Shocks", IMF working paper, 12/230.

Christiano, Lawrence J., Martin Eichenbaum, and Sergio Rebelo (2011): "When is the Government Spending Multiplier Large?", *Journal of Political Economy*, 119 (1), pp. 78-121.

Chung, Hess, Troy Davig and Eric Leeper (2007): "Monetary and Fiscal Policy Switching", *Journal of Money, Credit and Banking* 39 (4), pp. 809-842.

Cochrane, John H. (1998): "A Frictionless View of U.S. Inflation", in: Ben S. Bernanke and Julio J. Rotemberg (eds.), *NBER Macroeconomics Annual*, Volume 14, Cambridge, MA: MIT Press, pp. 323-384.

_____ (2001): "Long-term Debt and the Optimal Policy in the Fiscal Theory of the Price Level", *Econometrica* 69 (1), pp. 69-116.

_____ (2005): "Money as Stock", Journal of Monetary Economics, 52 (3), pp.

501-528.

_____ (2011): "Understanding Policy in the Great Recession: Some Unpleasant Fiscal Arithmetic", *Journal of Economic Dynamics and Control*, 55, pp. 2-30.

_____ (2014): "Monetary Policy with Interest on Reserves", *Journal of Economic Dynamics and Control*, 49, pp. 74-108.

_____ (2016): "Do Higher Interest Rates Raise or Lower Inflation?", mimeo, Hoover Institution.

De Grauwe, Paul and Yuemei Ji (2013): "Panic-driven austerity in the Eurozone and its implications", VoxEU.org, 21 February, available at: http://www.voxeu.org/article/panic-driven-austerity-eurozone-and-its-implications.

Davig, Troy and Eric M. Leeper (2006): "Fluctuating Macro Policies and the Fiscal Theory", in: Daron Acemoglu, Kenneth Rogoff and Michael Woodford (eds.), *NBER Macroeconomics Annual*, Volume 21, Cambridge, MA: MIT Press, pp. 247-298.

_____ (2011): "Monetary-Fiscal Policy Interactions and Fiscal Stimulus", *European Economic Review*, 55 (2), pp. 211-227.

Eichengreen, Barry (2015): "Secular stagnation: The long view." National Bureau of Economic Research, Working Paper 20836, January.

Federico, Pablo, Carlos A. Vegh, and Guillermo Vuletin (2014): "Reserve Requirement Policy over the Business Cycle", NBER Working Paper No. 20612.

Forbes, Kristin J. and Francis E. Warnock (2012): "Capital Flow Waves: Surges, Stops, Flight and Retrenchment." *Journal of International Economics* 88 (2), pp. 235-251.

Friedman, Milton (1969): "The Optimum Quantity of Money", in: Milton Friedman, *The Optimum Quantity of Money and Other Essays*, Chicago, IL: Adline Publishing Company.

Galí, Jordi (2014): "The Effects of a Money-Financed Fiscal Stimulus", mimeo, UPF.

Gertler, Mark and Nobuhiro Kiyotaki (2010): "Financial Intermediation and Credit Policy in Business Cycle Analysis." *Handbook of Monetary Economics* 3 (11), pp. 547-599.

Gourinchas, Pierre-Oliver and Maurice Obstfeld (2012): "Stories of the Twentieth Century for the Twenty-First" *American Economic Journal: Macroeconomics* 4 (January), pp. 226-265.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory Huffman (1988): "Investment, Capacity Utilization, and the Real Business Cycles", *American Economic Review* 78, pp. 402-417.

Hansen, Gary (1985): "Indivisible Labor and the Business Cycle", *Journal of Monetary Economics*, 16(3), pp. 309-327.

Holland, Dawn and Jonathan Portes (2012): "Self-defeating austerity?", *National Institute Economic Review*, 222.

House, Christopher, and Matthew Shapiro (2006): "Phased-in Tax Cuts and Economic Activity", *American Economic Review*, 96(5), pp. 1835-1849.

Huidrom, Raju, M. Ayhan Kose, Jamus J. Lim, and Franziska L. Ohnsorge (2016): "Do Fiscal Multipliers Depend on Fiscal Positions?", World Bank Policy Research Working Paper No. 7724.

Hume, Michael and Andrew Sentance (2009): "The global credit boom: challenges for macroeconomics and policy." *Journal of International Money and Finance* 28 (8), pp.1426-1461.

Karabarbounis, Loukas and Brent Neiman (2014): "The global decline of the labor share." *The Quarterly Journal of Economics* 129 (1), pp. 61-103.

King, Robert, Charles Plosser, and Sergio Rebelo (2002): "Production, Growth and Business Cycles: Technical Appendix" *Computational Economics* 20 (1?2), pp. 87?116.

Kim, Soyoung (2003): "Structural Shocks and the Fiscal Theory of the Price Level in the Stick Price Model", *Macroeconomic Dynamics*, 7 (5), pp. 759-782.

Kiyotaki, Nobuhiro and John Moore (2012): "Liquidity, Monetary Policy and Business Cycles." NBER Working Paper No. 17934, March.

Kocherlakota, Narayana (2016): "'Helicopter Money' Won't Provide Much Extra Lift", available at http://www.bloombergview.com/articles/2016-03-24/-helicopter-money-won-t-provide-much-extra-lift.

Leeper, Eric M. (1991): "Equilibria under Active and Passive Monetary and

Fiscal Policies", *Journal of Monetary Economics*, 27 (1), pp. 129-147.

Leeper, Eric M. and Tack Yun (2006): "Monetary-Fiscal Policy Interactions and the Price Level: Background and Beyond", *International Tax and Public Finance*, 13 (4), pp. 373-409.

Leeper, Eric M. and Campbell Leith (2016): "Understanding Inflation as a Joint Monetary-Fiscal Phenomenon", *Handbook of Macroeconomics,* Volume 2, forthcoming.

Leeper, Eric M. Nora Traum, and Todd B. Walker (2016): "Clearing Up the Fiscal Multiplier Morass", mimeo, Indiana University.

Lerner, Abba P. (1943): "Functional Finance and the Federal Debt", *Social Research*, 10 (1), pp. 38-51.

Moore, John (2011): "Leverage Stacks and the Financial System." Econometric Society, Presidential Address Lecture, June.

Obstfeld, Maurice (2010): "Expanding Gross Asset Positions and the International Monetary System" Remarks at the Federal Reserve Bank of Kansas City symposium on ?Macroeconomic Challenges:The Decade Ahead,? Jackson Hole, Wyoming, August 26-28.

_____ (2012a): "Does the Current Account Still Matter?" *American Economic Review* 102 (3), pp. 1-23.

_____ (2012b): "Financial Flows, Financial Crises, and Global Imbalances." *Journal of International Money and Finance* 31, pp. 469-480.

Ottonello, Pablo and Diego J. Perez (2016): "The Currency Composition of Sovereign Debt", mimeo, New York University.

Parker, Jonathan (1999): "The Reaction of Household Consumption to Predictable Changes in Social Security Taxes", *American Economic Review*, 89 (4), pp. 959-973.

Phelps, Edmund S. (1973): "Inflation in the Theory of Public Finance", *Swedish Journal of Economics*, 75 (1), pp. 67-82.

Rendahl, Pontus (2015): "Fiscal Policy in an Unemployment Crisis", *Review of Economic Studies*, forthcoming.

Rey, Helene (2013): "Dilemma not Trilemma: The Global Financial Cycle and Monetary Policy Independence." Paper presented at the 25th Jackson Hole symposium, The Federal Reserve Bank of Kansas City Wyoming, August.

Schmitt-Grohe, Stephanie and Martin Uribe (2012): "What's News in Business Cycles", *Econometrica*, 80 (6), pp. 2733-2764.

Shin, Hyun Song (2012): "Global Banking Glut and Loan Risk Premium." *IMF Economic Review* 60 (2), pp. 155-192.

Sims, Christopher A. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy", *Economic Theory*, 4 (3), pp. 381-399.

Smets, Frank and Rafael Wouters (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", *American Economic Review*, 97 (3),

pp. 586-606.

Swiston, Andrew (2005): "A Global View of the U.S. Investment Position" *IMF Working Paper*, 06/181

Taylor, Alan (2012): "The Great Leveraging" Centre for Economic Policy Research, Discussion Paper 9082.

Thwaites, Gregory (2015): "Why are real interest rates so low? Secular stagnation and the relative price of investment goods." Bank of England Staff Working Paper No. 564, November.

Turner, Adair (2015): "The Case for Monetary Finance – An Essentially Political Issue", Paper Presented at the 16th IMF Jacques Polak Research Conference.

Woodford, Michael (1995): "Price-Level Determinacy Without Control of a Monetary Aggregate", *Carnegie-Rochester Conference Series on Public Policy*, 43, pp. 1-46.

_____ (2011): "Simple Analytics of the Government Spending Multiplier", *American Economic Journal: Macroeconomics*, 3 (1), pp. 1-35.