

# LES-CHT for Smooth and Micro-structured Walls

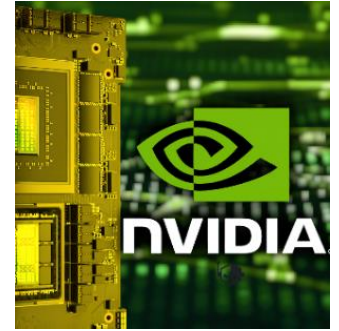
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# 'Machinable Surface': What Do We Want (& How)?

2017 - a 'disruptive' year.

- **World sees the foundational work on the LLMs (thus the current 'AI Storm')**
  - Vaswani et al (2017) "**Attention is All You Need**" (~190,000 cites) → 'Transformer'
  - **Nvidia** (market value \$3 Trillions, ↑100 times, 2017-2024)



- **Additive Manufactured HP Turbine: AM made & tested at 13,000 rpm; 1523 K**

- Siemens Press Release: Feb 2017;
- ASME Emerging Tech Award: Dec 2017



- **'Role Reversal': Manufacture Engineer vs. Aerodynamic Designer:**

- **Past (20-30 years ago)**

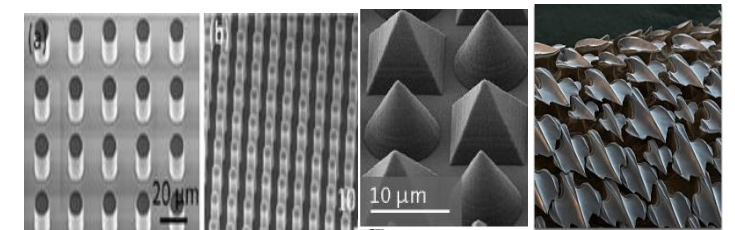
Aero-Designer: 'here is my design'.

Manufacturer: 'very nice, but we cannot make it';

- **Present (& Near Future?)**

AM Manufacturer: 'what surface finish would you like?'

Aero-Designer: 'oh..., how can we design it in the first place?'



Chu et al (2012)    KIT (2015)    Nature (...?!)

→ **"Surface Design" (both Shape and Finish) ??**

# Outline (*'Multiscale Solution for Multiscale Problem'*)

- **Multi-scale Issues of Interest (Motivations & Challenges):**

- Spatial Scale Disparity (Macro vs. Micro): Random/'Regular' Surface Roughness.*
- Temporal Scale Disparity (Fluid vs. Solid): Conjugate Heat Transfer.*

- **Two-scale Solution (*in Time*)**

- Long-scale Slow Transient: 'Dual Time-Marching'*
- Short-scale Turbulence: Fluid-Solid 'Temperature Harmonic Transfer Function'*

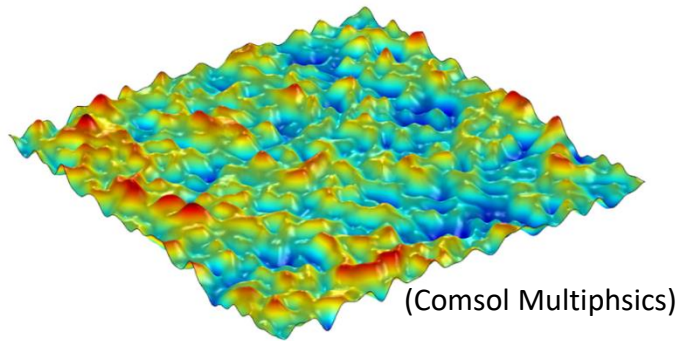
- **Two-scale Solution (*in Space*)**

- Applied to Fluid-domain with Micro-structured Wall.*
- Extended to Solid-domain (Source-term for Coarse-mesh (ML or not): **Reynolds Stresses?!**)*

- **Case Examples**

- CHT validation (VKI ribbed channel)*
- LES for micro-structured wall (Fluid-domain only)*
- Conduction for micro-structured wall (Solid-domain only)**
- Coupled CHT for micro-structured wall (Fluid-Solid domain).*

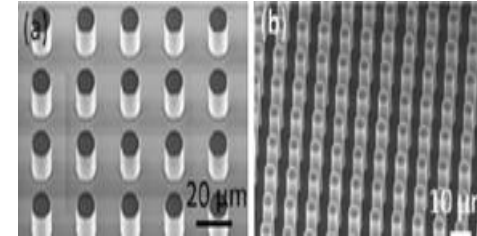
## Stochastic Roughness



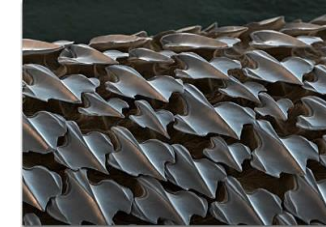
- **Conventional Modelling Path** (Nikuradse, 1933):  
- same BL structure as smooth with velocity offset by 'Roughness Function' ( $\Delta U^+$ )
- **Validity in Question** (e.g. for large roughness),  
 $\Delta U^+$  depending on detailed geometry (Flack, 2019)  
→ Need for Local Resolving Capability

## 'Regular Roughness'

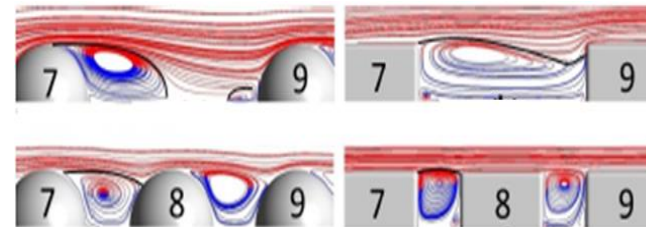
(New Impetus: Additive Manufacture, AM)



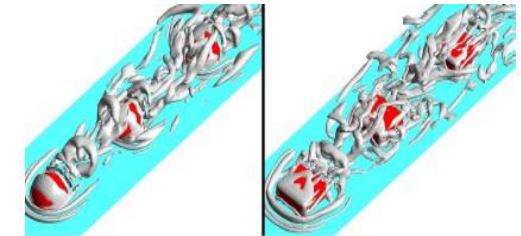
Chu et al (2012)



Nature (!)



(Kapsis & He, 2018)



- Roughness Shapes & Patterns Do Matter.  
→ Need for Local Resolving Capability

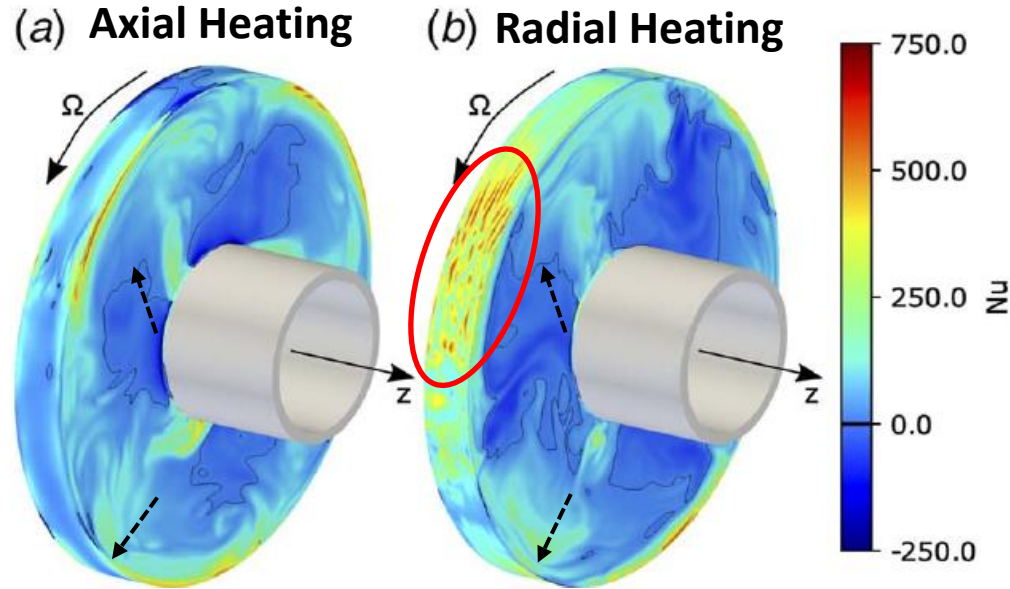
*Issues of Interest: Interplay between roughness & bulk flow/surface shape.  
Long-term Prospect: towards 'Surface Design' (both Shape & Finish)*

# Why LES & CHT? – ‘Feedback’ of Heat Transfer to Turbulence (*smooth wall*)

Rotating Cavity at Different Thermal BC (*Hickling & He, 2023*)

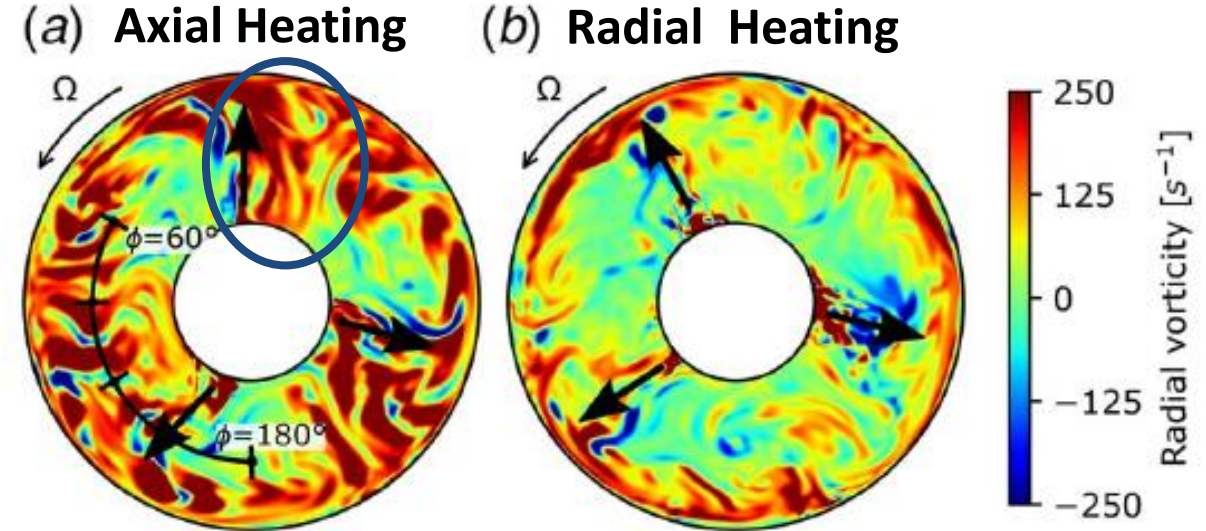
## Heat Transfer

(on shroud and disk surfaces)



## Flow

(on axial cut-plane at  $2\delta_{Ek}$ )



**Radial Vorticity** (arrows: outflow ‘radial arms’)

- Disk heating  $\rightarrow$  Imbalance ( $\Delta P$  vs.  $F_{Coriolis}$ ) (thus, unstable vortices / large-scale turbulence)

- Wall Heat Transfer (Temperature  $T_w$ ) does affect Nearwall Turbulence Generation

# Why LES & CHT?—‘Feedback’ of Heat Transfer to Transition (*micro-structured wall*)

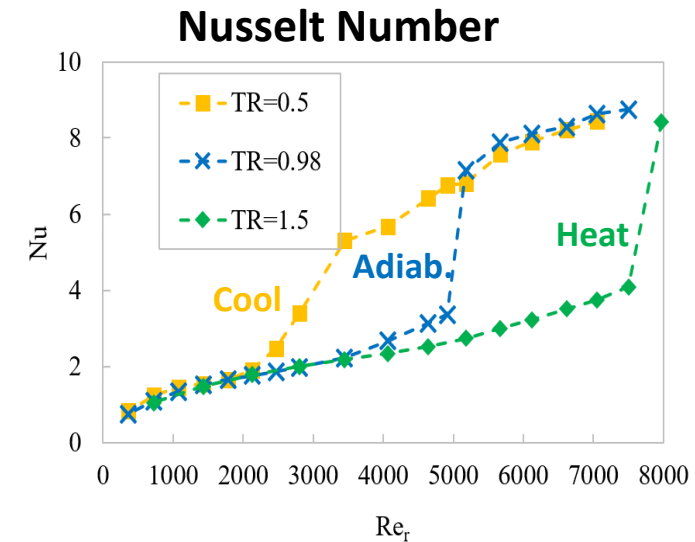
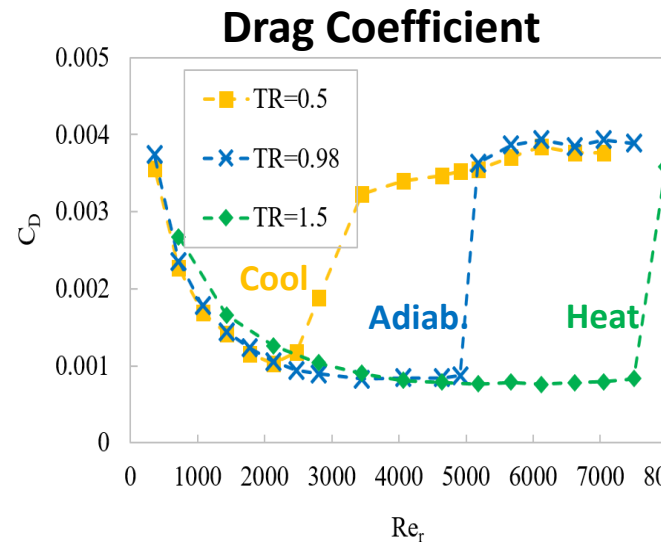
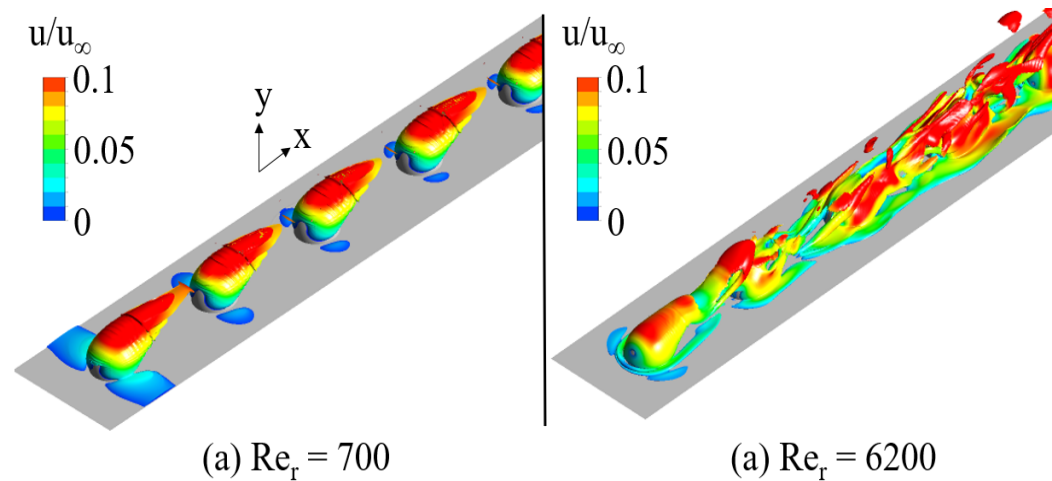
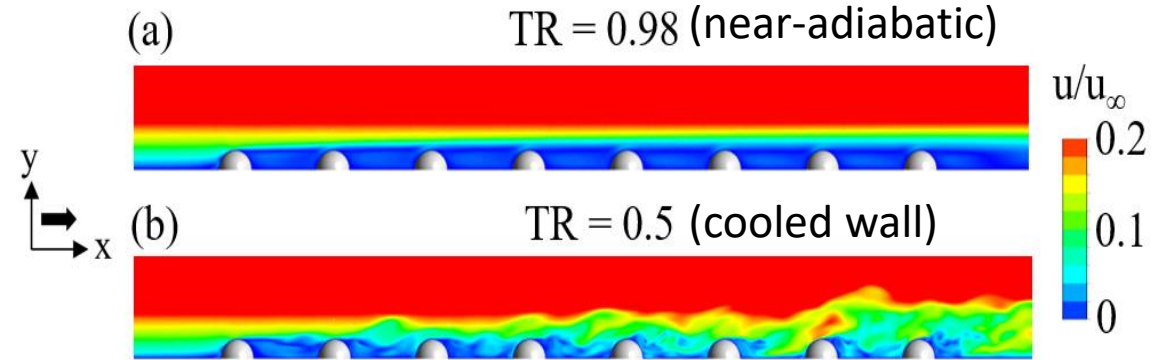
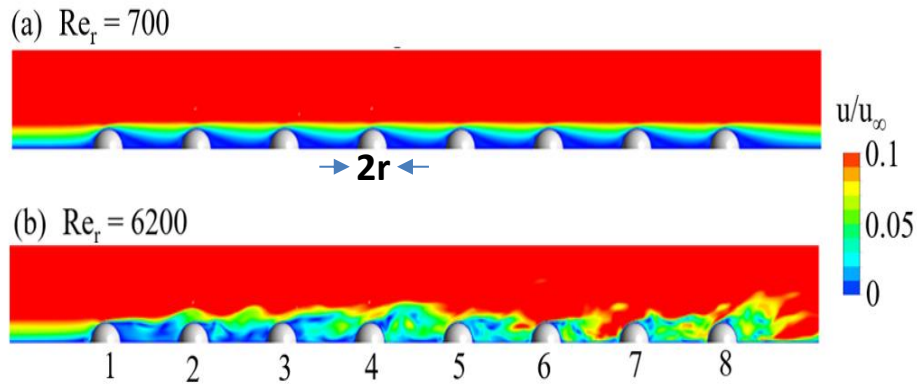
(Campanaro & He, 2023)

## Adiabatic BC at Two Reynolds Numbers

( $Re_r = \rho_\infty V_\infty r / \mu_\infty$  --- *bulk flow based Re*)

## Diabatic at Same Reynolds Number ( $Re_r = 4000$ )

(Temperature Ratio:  $TR = T_w / T_{01}$ , with specified  $T_w$ )

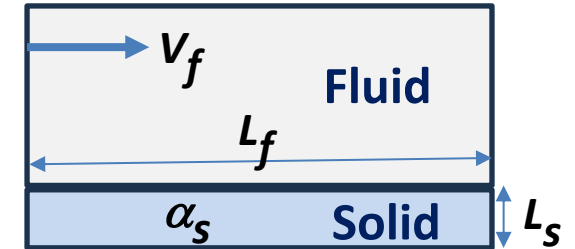


• Wall cooling  $\rightarrow$  increase of Local Reynolds number

# Time-Scale Disparity: Fluid vs. Solid

**Fluid Convection:**  $\tau_f = L_f / V_f$ ;    **Solid Diffusion**  $\tau_s = L_s^2 / \alpha_s$

$$\Rightarrow \frac{\tau_s}{\tau_f} = \left(\frac{c p_s}{c p_f}\right) \left(\frac{\rho_s}{\rho_f}\right) \left(\frac{k_f}{k_s}\right) \left(\frac{L_s}{L_f}\right)^2 (Pr_f) (Re_{L_f}) \quad (\text{He, 2023})$$



**Take:** Stainless steel & Air,  $L_s = 0.1 L_f$  and  $Re_{L_f} \sim 10^5$

$$\Rightarrow \frac{\tau_s}{\tau_f} \sim 10^4 \quad \left(\text{cf. } \frac{\Delta t_s}{\Delta t_f} \sim 10^4 \text{ - numerical stability requirement (He \& Oldfield, 2011)}\right)$$

## Dual Requirement for Unsteady CHT:

**Time-accuracy:**  $\Delta t_{\text{CHT}} = \Delta t_f$     (*much smaller  $\Delta t$  required for LES*)

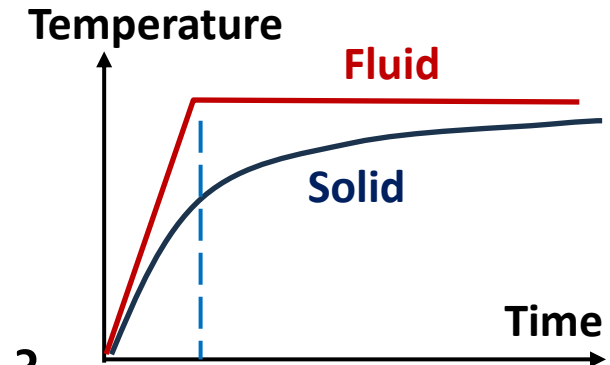
**Time-consistency:**  $\tau_{\text{CHT}} = \tau_s$     (*otherwise initial transients still remain in domain*)

**Time steps for unsteady CHT ( $N_{\text{CHT}}$ ) compared to that for a flow-only solution ( $N_f$ ):**

$N_{\text{CHT}} \sim 10^4 N_f$     - *Direct time-accurate and consistent CHT: prohibitive!*  
 - *'Fixes' (different materials/initial conditioning): erroneous.*

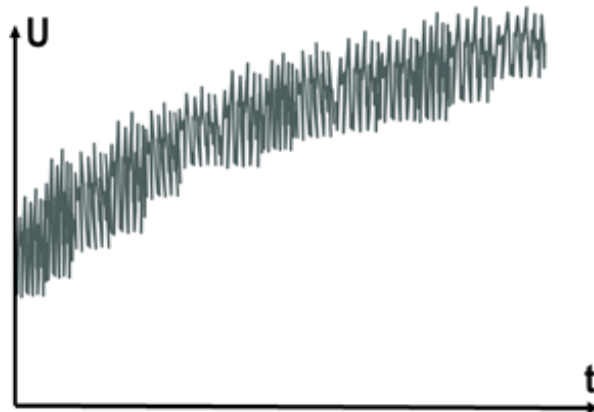
# Long Time-scale Transient Process: ‘Quasi-Steady’ Flow Model?

- Applications: Engine startup/shutdown, Flexible operations
- Common ‘Quasi-steady’ CHT: Unsteady Solid + ‘Steady Flow’ Each Solid Step
- Quasi-steady assumption may be challenged in part of an overall long process
- Can we do without the quasi-steady assumption for the fluid side (*at what cost*) ?



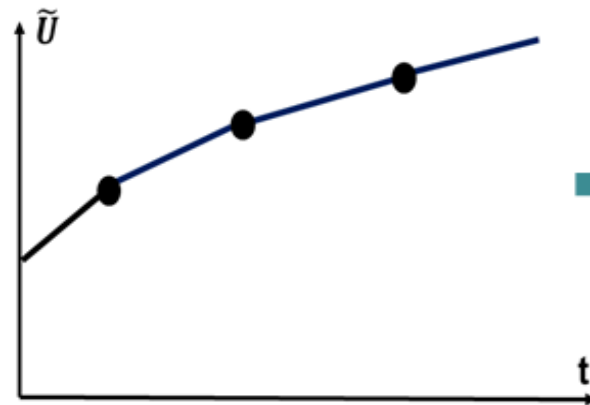
Conventional Scale-split:

$$U(x, t) = \tilde{U}(x, t) + U'(x, t)$$



(a) Full Unsteady Variable

=



(b) Filtered Base Part (Large Step Sampling)

+



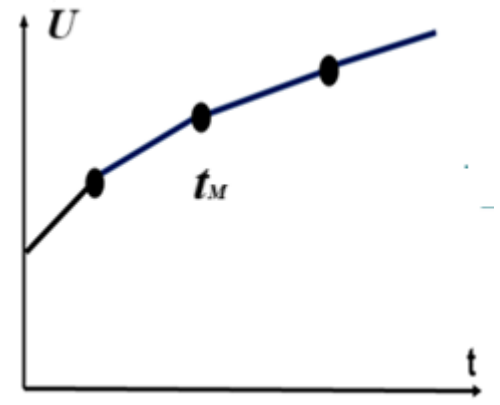
(c) Fluctuation Part (small step sampling)

Very Small Flow Time-steps  
Very Long Solid Time Scale

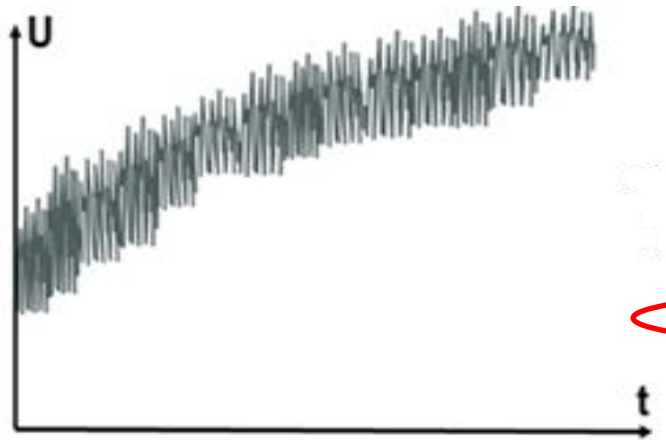
# 'Dual Time-Marching' for Long Transient CHT (*He and Fadl, 2017*)

- Dual Time-stepping for URANS (*Jameson 1991*)

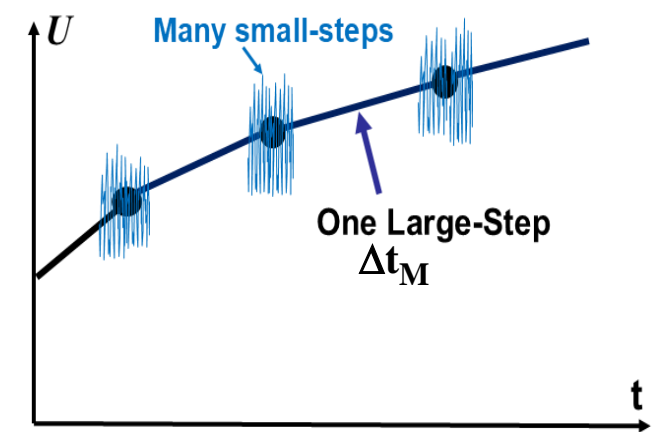
$$\frac{\partial U}{\partial t} + R(U) = 0 \quad \xrightarrow{\text{Sub Iteration at } t_M} \quad \left(\frac{\partial U}{\partial \tau}\right)_k + R(U)_k = -\left(\frac{\partial U}{\partial t}\right)_{t_M}$$

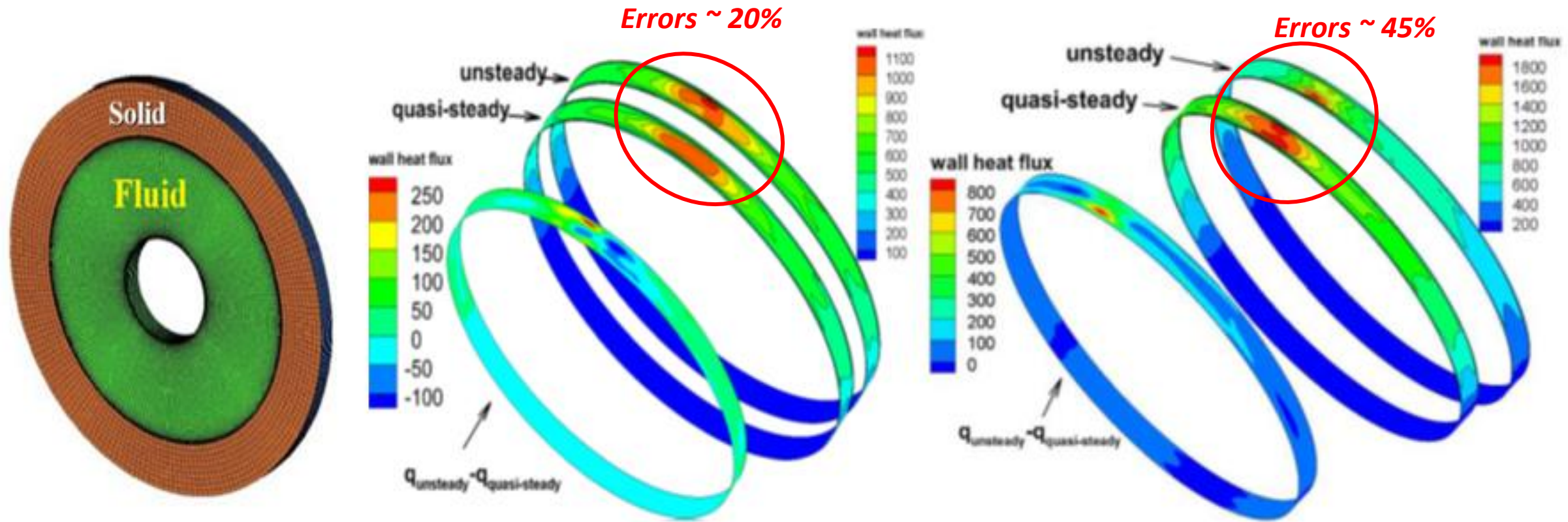


- Dual Time-marching  $\xrightarrow{\text{'Sub Time-marching' at } t_M} \left(\frac{\partial U}{\partial t}\right)_n + R(U)_n = -\left(\frac{\partial \tilde{U}}{\partial t}\right)_{t_M}$



$\xrightarrow{\text{Local Small-step Time-marching/integration conditioned by Large-step Time Gradient}}$





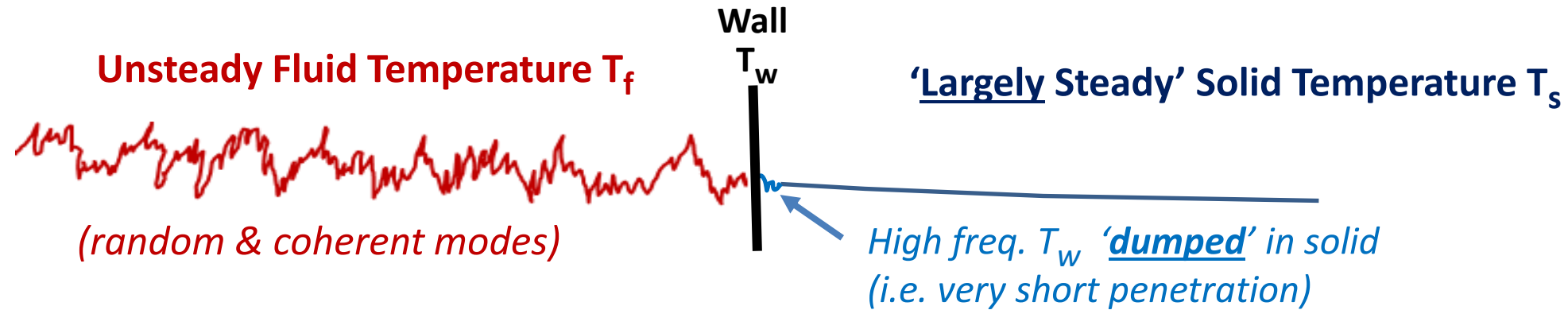
(a) Configuration

(b) Slow Transient ( $100K/30s$ )

(c) Fast Transient ( $100K/1s$ )

- Quasi-steady Model: Unsafe for local high-gradient changing part in an otherwise slow process. (& some history effects on later parts of the process as well)

# Short Time-scale Turbulence: *Why & Where Should We Bother?*



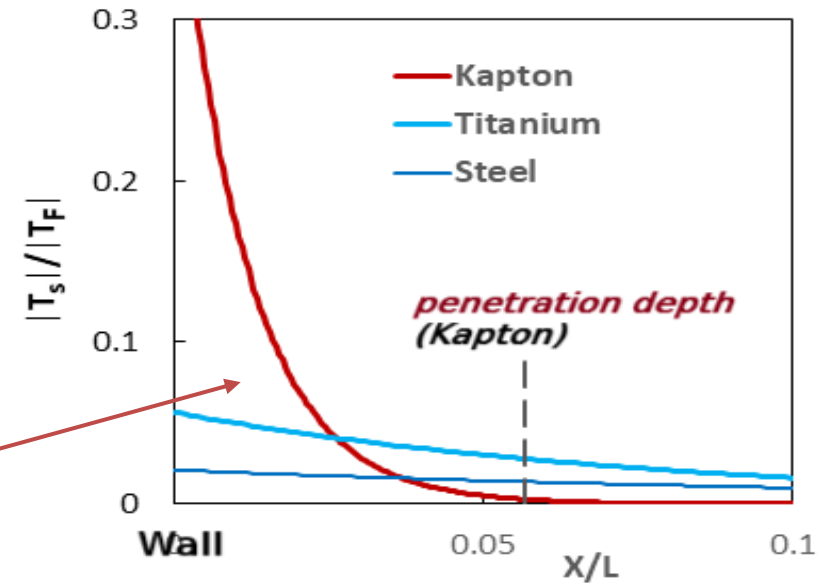
• **The conventional direct time-domain CHT:**

- 1) Time-consistency: Number of time-steps has to be up by  $\sim O(10^4)$
- 2) Truncat. err/strong solid mesh-dependency

**Penetration Depth:**  $\delta_p \sim \sqrt{\alpha_s / f}$  *(material & frequency-dependent)*

**e.g. Low conductivity 'TBC' at high frequency: small penetration depth**

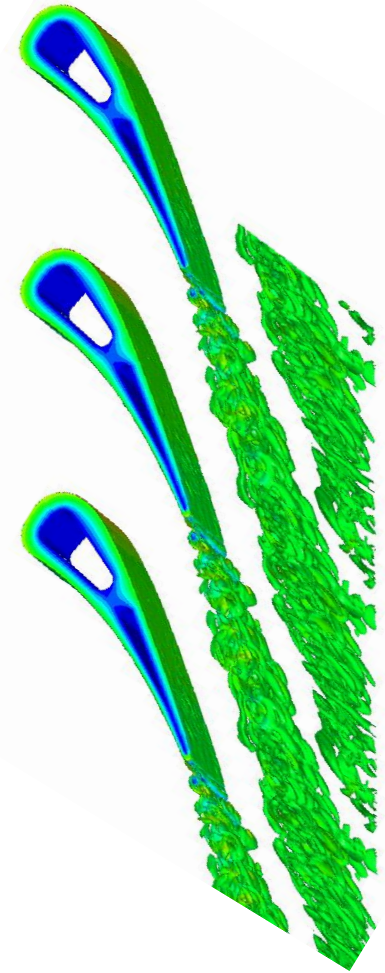
- i) High  $T_w$  fluctuations (physical)*
- ii) very High mesh-dependency (numerical)*



Penetration of Unsteadiness in Solid  
*(He, CHT chapter 2023)*

## Target Framework for LES-CHT: Pay When & Where Needed

- ‘Deconstruct’ One Full Time-domain CHT into Two Parts
  - (i) **Baseline ‘Steady’ Part:**
    - *coupling time-averaged fluid & solid conduction solution*
    - (Cost  $\sim$  Steady CHT).
  - (ii) **‘Add-on’ Unsteady CHT:**
    - *capturing  $T_w$  unsteadiness over a range of frequencies.*
    - (Cost  $\leq$  Baseline Steady-like CHT)



# Primary Intent: 'Time Realignment' at Fluid-Solid Interface

- **Fourier Spectrum for Temperatures**

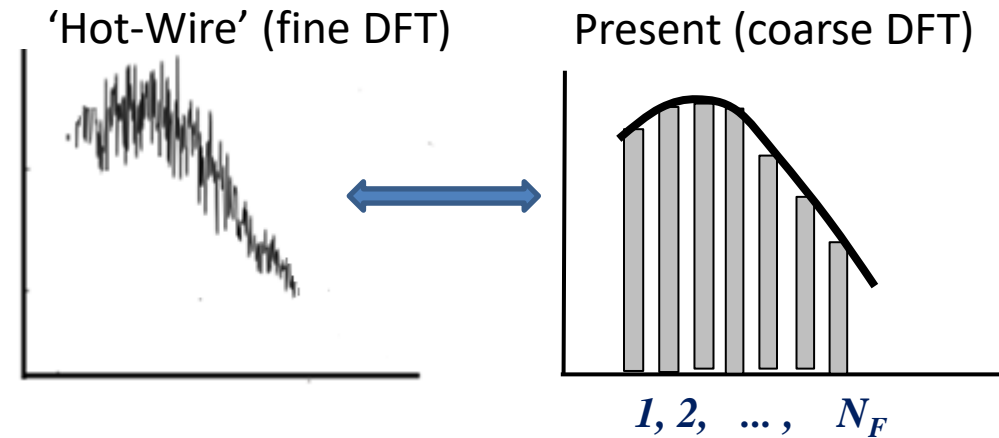
$$T(x, t) = \bar{T}(x) + T'(x, t)$$

$$T'(x, t) = \sum \hat{T}(x) e^{in\omega t}$$

- Unsteady Interface in Time-domain

→ A set of harmonic ('steady') interfaces

(0th harmonic: Time-average 'Steady' Baseline CHT!)

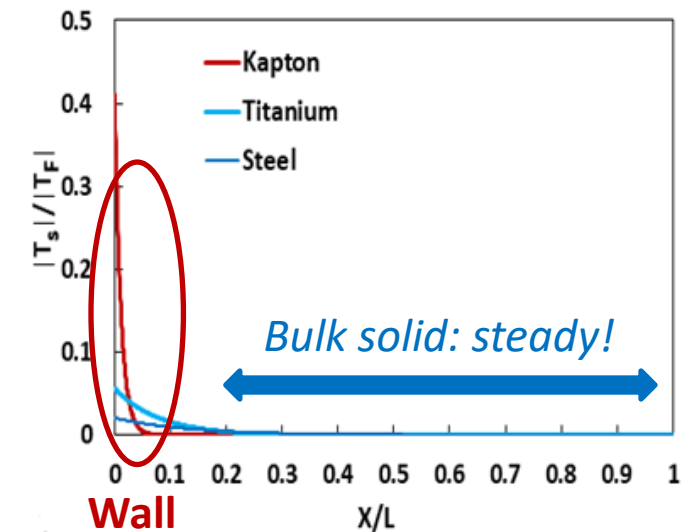


**However,**

- Solving  $N_F$  harmonics (for Solid Conduction): Not appealing (if  $N_F \sim 10^2!$ )
- Unsteady conduction for bulk of solid domain (>penetration depth) is hardly relevant.

→ Only small region near/on WALL (not bulk solid!) is where we need/want to know!

Can we get Unsteady  $T_w$  without solving Unsteady Conduction in Solid Domain?



(a) Full Domain View (Wall at  $x/L=0$ )

# Harmonic Transfer Function *Unsteady* CHT Interface (*'Learning CFD from Experiment'*)

(He & Oldfield 2011, He 2019)

- Analytical harmonic transfer function (*Unsteady 1-D Conduction for Solid*) heat flux ( $q_w$ ) vs. wall temperature ( $T_w$ ), (Schulz & Jones, 1973)

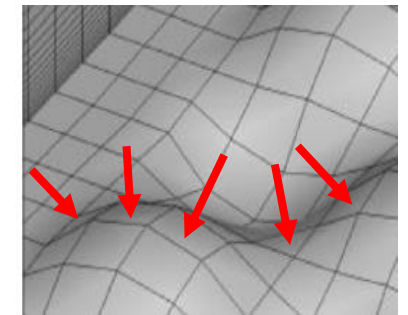
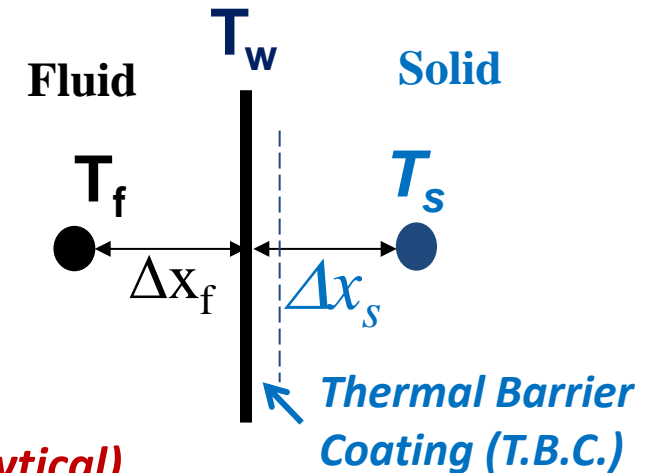
$$\hat{q}_w = TF_{Tq} \hat{T}_w \quad (TF_{Tq} - \text{"}T_w\text{-}q \text{ Transfer Function"} - \text{Analytical})$$

- Linking to Fluid side by Harmonic Balance ( $n = 0, 1, 2, \dots, N_F$ ):

$$(\hat{T}_w)_n = (TF_w)_n \hat{T}_f \quad (TF_w - \text{"}T_f\text{-}T_w \text{ Harmonic Transfer Function"} - \text{Semi-Analytical})$$

## → Very Efficient Framework for Unsteady CHT

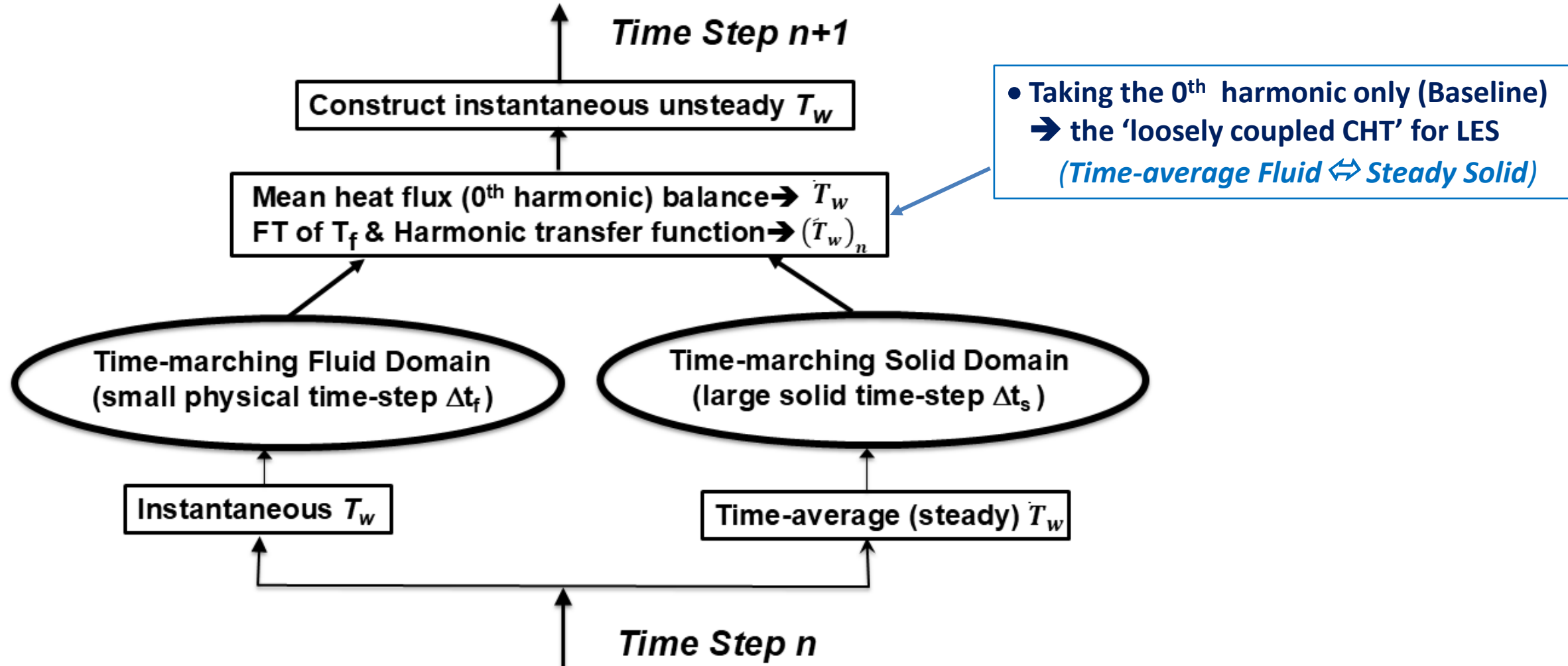
- **Fluid:** Time-domain scale-resolving URANS/LES (*subject to unsteady  $T_w$* )
- **Interface:** Frequency-domain (input FT on-the fly for  $T_f$ )  
the baseline  $0^{th}$  harmonic ( $\bar{T}_w$ ): time-mean (steady) CHT  
other harmonics  $(\hat{T}_w)_n$ : **updated analytically** by  $(\hat{T}_f)_n, (TF_{Tq})_n$
- **Solid:** Steady Conduction (thus a much faster solution with  $\sim 10^4$  larger  $\Delta t_s$ ).



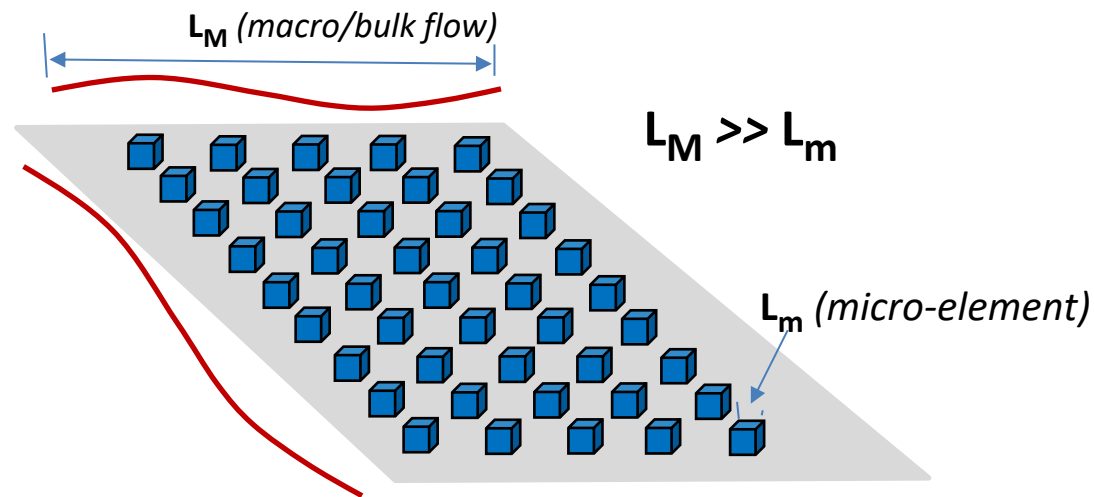
'Local 1-D' (each mesh cell)

- **Unsteady CHT at largely the cost as Steady CHT** (No solid temperature harmonics needed!)  
-the fluid-side DFT ('moving-average'): only  $\sim 20\%$  overhead to Baseline 'Steady' CHT!

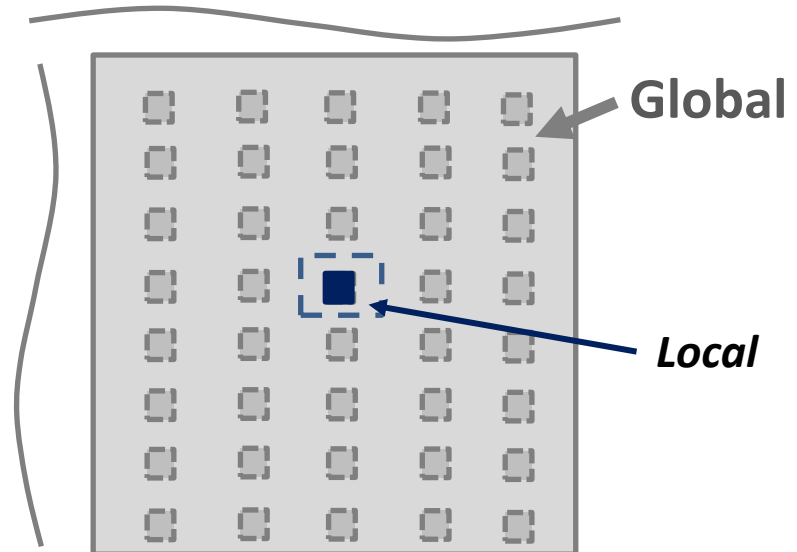
# Flow Chart of Time-Frequency Domain Unsteady CHT



## Micro-structured Surface (with slow-varying bulk flow)



## Macro-scale (Global) vs. Micro-scale (Local)



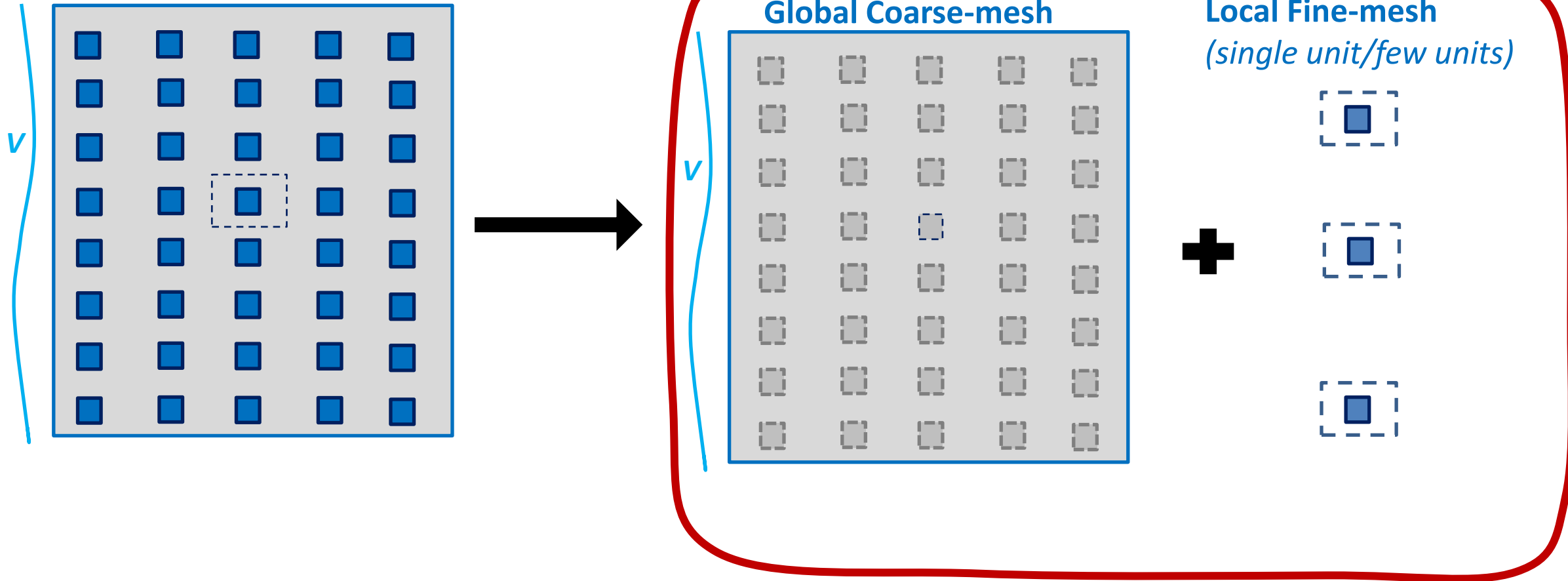
'Scale-dependent Solvability':

- Local Micro-scale: High gradient within a Unit
- Global Macro-scale: Smooth Unit-Unit variation

# Basic Conflict: Local Resolution vs. Global Conditioning

Full Domain (fine-mesh everywhere)

*'Problem Decomposition'*



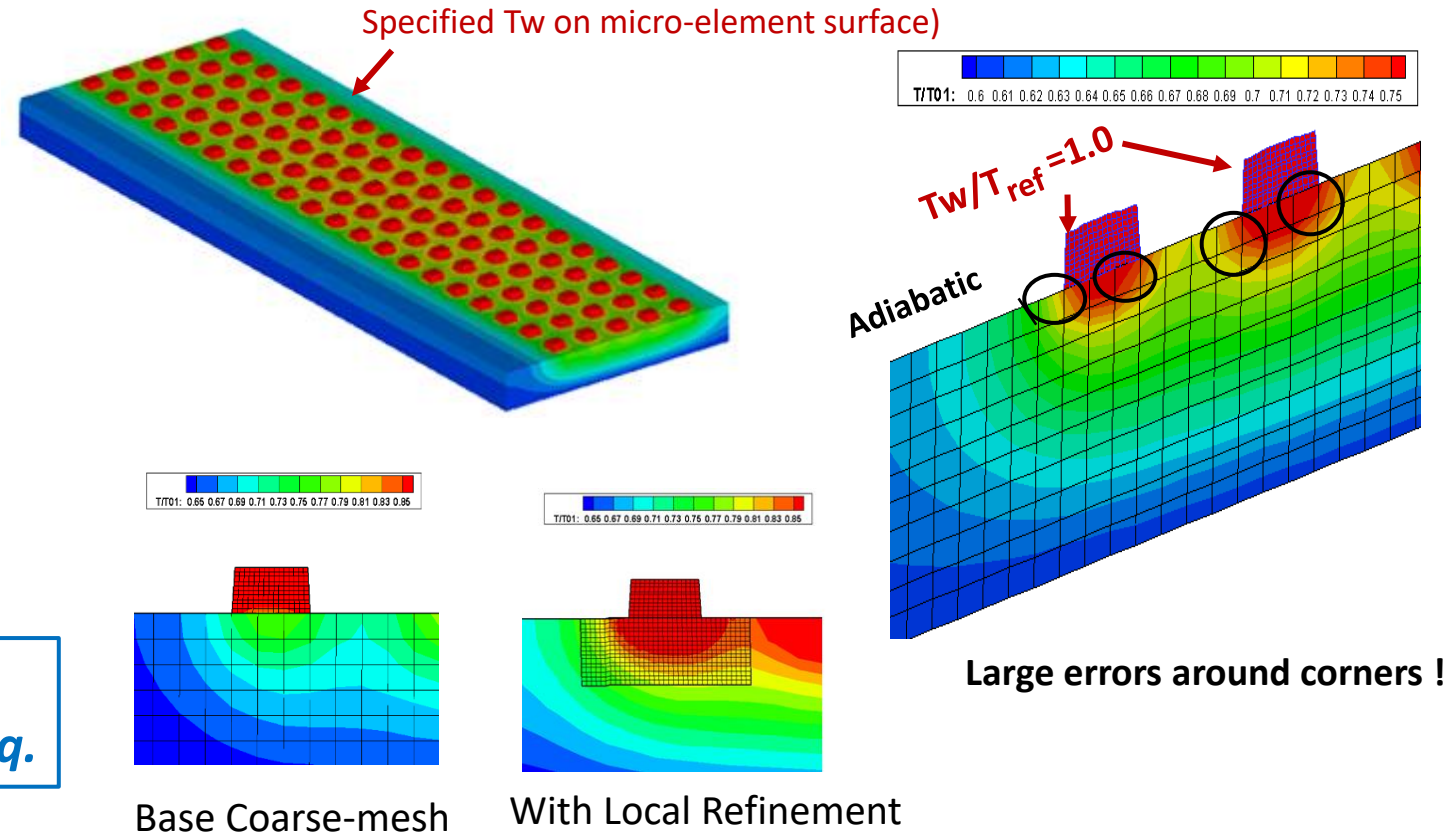
*Pathway to Solution: to couple two sub-systems to correct each other.*

# Solving Solid Temperature with a Unified ('Monolithic') CFD Method

## The Same Method for both Fluid & Solid

- **Conduction Equation:** *Fluid energy equation with zero velocities and solid properties.*
- **Discretization:** *Finite-Volume on multi-block meshes (with local embedding)*
- **Solution:** *Time-marching (Multi-step Runge-Kutta) for a steady temperature field in solid*

- *The two-scale method also extended to solid domain for solving the conduction eq.*



# Space-Averaging Two-scale Method for Steady Conduction in Solid

- Dual meshing (the same as flow)
- Local spatial-averaging (same as flow domain) : volumetric average of fine-mesh results as the corresponding coarse-mesh cell target:

$$\tilde{U}_f = \frac{1}{V} \sum U_f \Delta v$$

- No Time-averaging here : we are only after steady temperatures in solid

- Coupled System of Two parts/domains (as for flow domain):

- Local fine-mesh 'f' ('Inverse Mode'):

$$\mathbf{S}_s = \mathbf{R}_c(\tilde{U}_f)$$

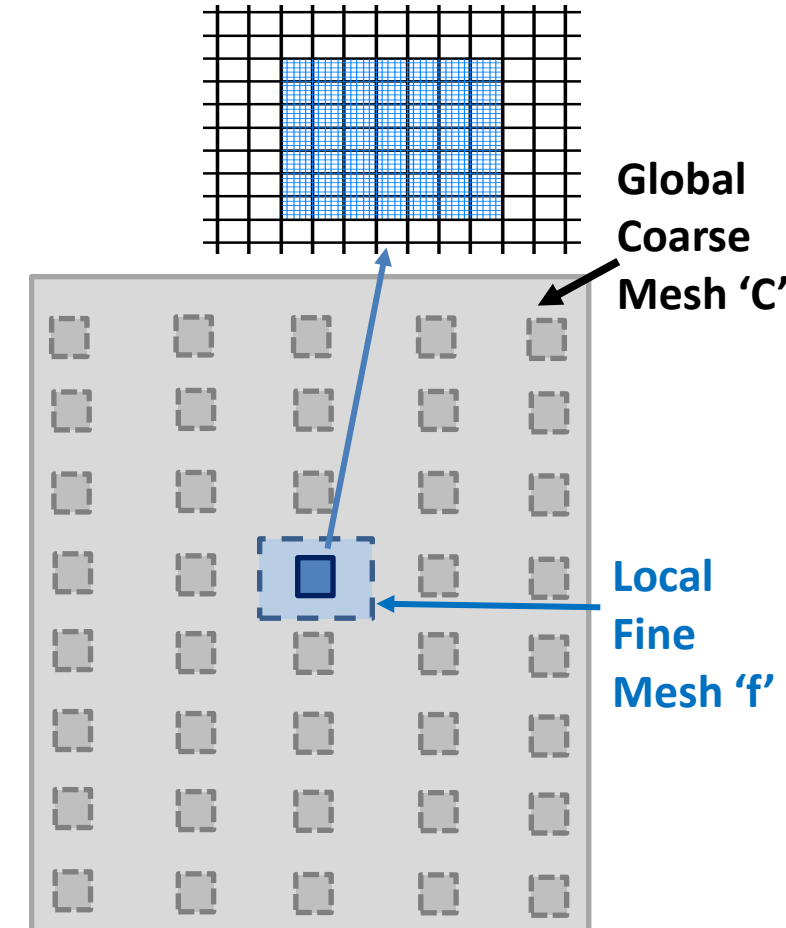
- Global coarse-mesh 'C' ('Direct Mode'):

$$\frac{\partial U_c}{\partial \tau} + \mathbf{R}_c(U_c) = \mathbf{S}_s$$

(Source term  $\mathbf{S}_s$  drives the coarse-mesh solution to the target:  $U_c \Rightarrow \tilde{U}_f$ )

- Converged Solution (same as flow domain):

- Local fine-mesh: *much improved conditioning*
- Global coarse-mesh: *much improved resolution*



## Attention to Source Term for Solid Conduction: Why/How Should It Work ?

### Contrast (in Equations):

- Flow domain:

Space-Time averaging mimics Reynolds-averaging (source terms originate from **Nonlinearity** of flow equations ).

**BUT:**

- Solid domain:

**Linear** conduction equation (for given material property/conductivity). Where does the source originate ?

### Similarity (Balancing the Averaged Equations)

- Flow Domain:

$$\overline{R(\mathbf{U})} = 0 \quad \text{but} \quad R(\bar{\mathbf{U}}) \neq 0$$

→ Need lumped **Reynolds stress terms** to balance the time-averaged equation, to produce the target  $\bar{\mathbf{U}}$ .

- Solid Conduction:

$$R_f(\widetilde{\mathbf{U}}_f) = 0 \quad \text{but} \quad R_c(\widetilde{\mathbf{U}}_f) \neq 0$$

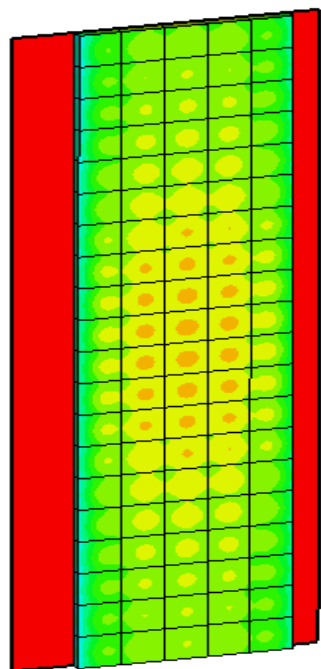
Target solution does not hold on coarse-mesh, due to **'mesh-dependent' discretization errors.**

→ Need **'mesh-informed' source term** to balance the coarse-mesh equation, to produce the target  $\widetilde{\mathbf{U}}_f$ .

# Global Fine-mesh vs. Local Embedded Fine-mesh (*Solid-domain only, He 2023*)

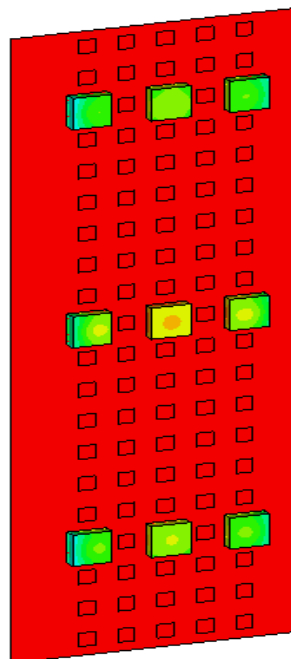
(Coarse-mesh Solution : no Reynolds Stresses to Close!)

**Direct Solution**  
(100 fine-mesh blocks)

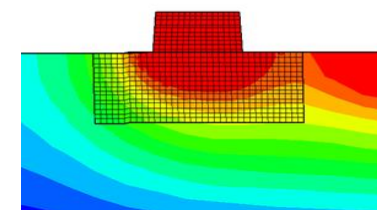
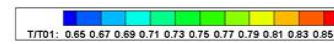
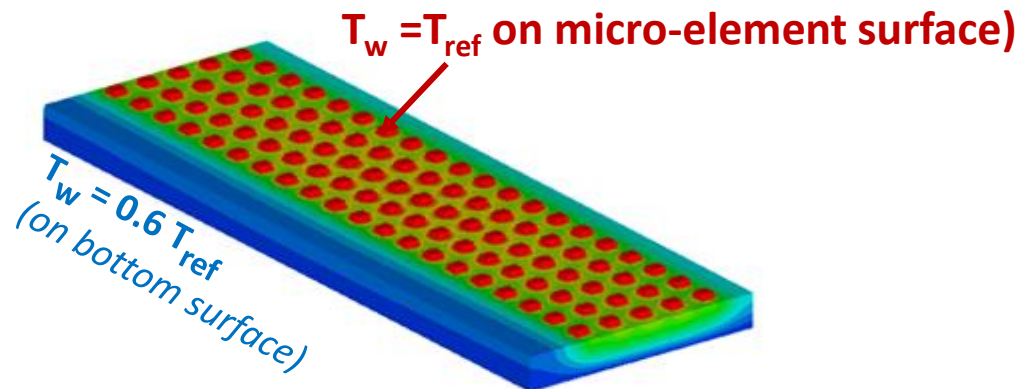


Heated Surface

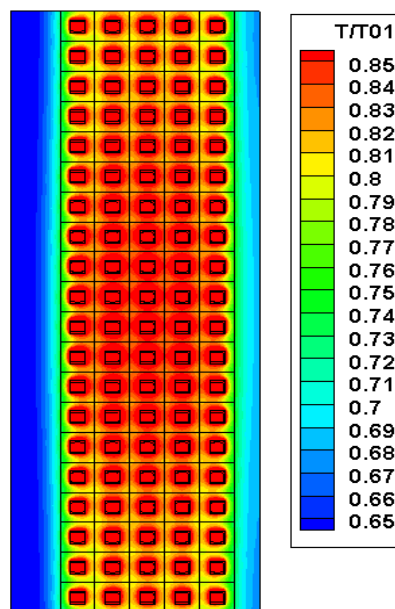
**Local Fine-mesh Blocks**  
(only 9 fine-mesh blocks)



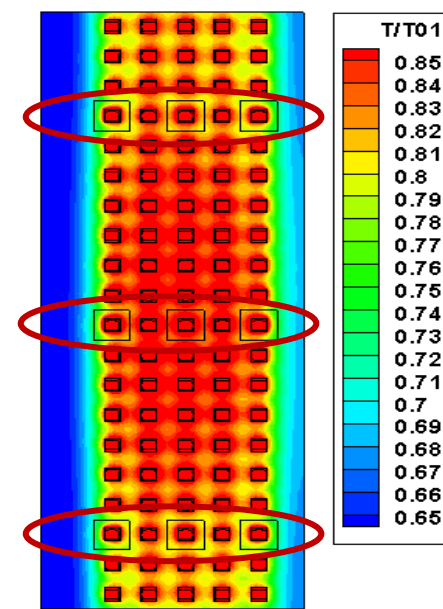
Back View (only fine-mesh blocks & top surface shown)



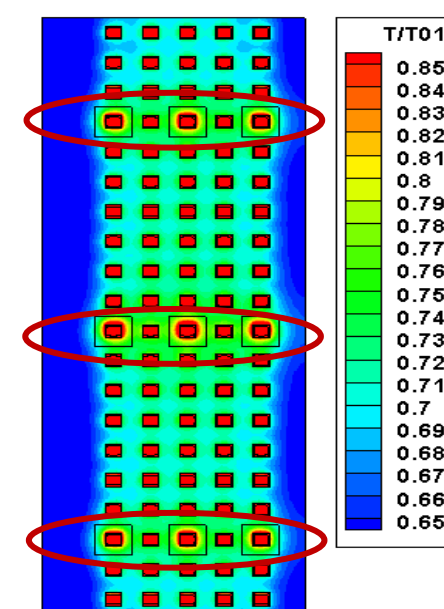
Locally Embedded Fine-mesh (5x5x5)



**Full Fine-mesh Solution**  
(100 fine-mesh blocks)



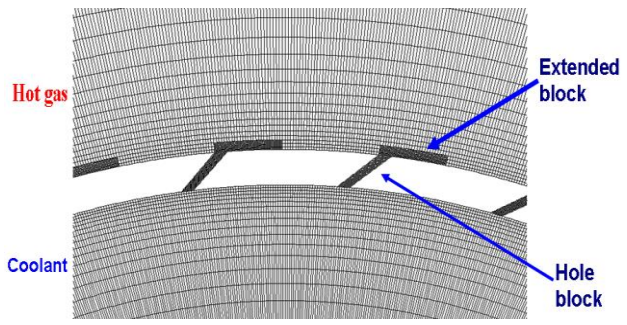
**Two-scale Coupling**  
(9 fine-mesh blocks)



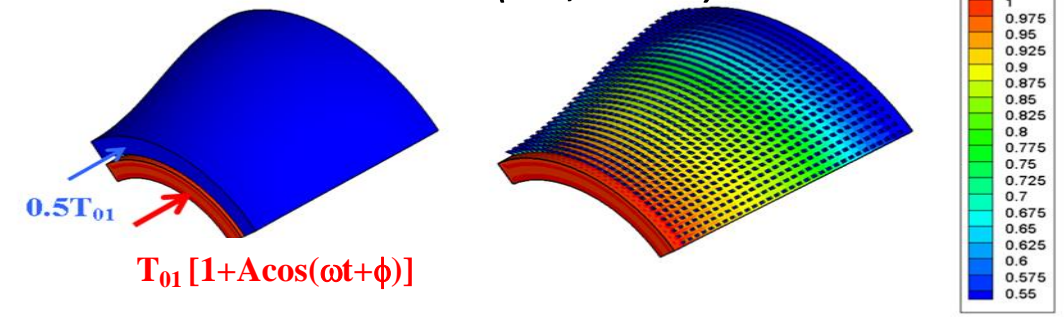
**'One-scale' (no coupling)**  
(9 fine-mesh blocks)

# Two-scale Tests on Cooling: Multi-hole Film/Effusion Cooling (RANS/URANS)

RANS (He, 2012)

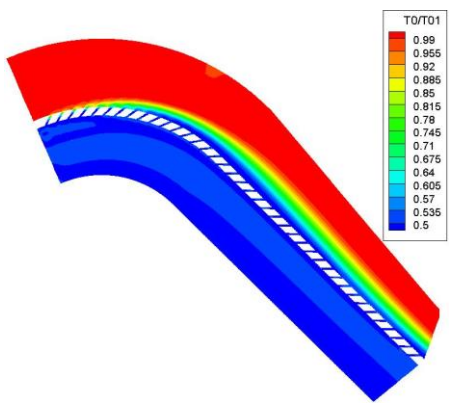


URANS (He, 2013)

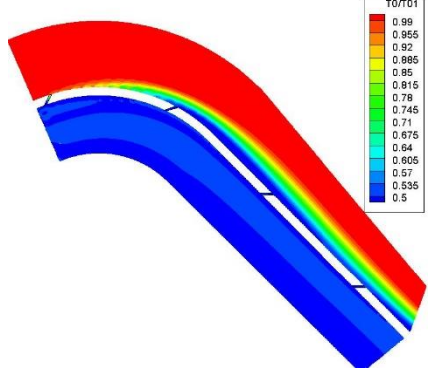


(a) Sector Domains and inflow temperatures

(b) Effusion cooling holes and steady wall temperatures

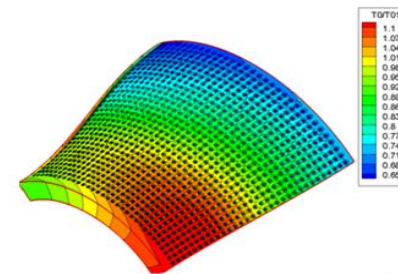
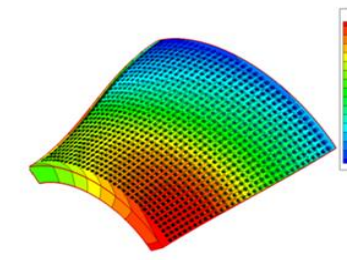


Direct Global Solution  
(resolving 39 holes)

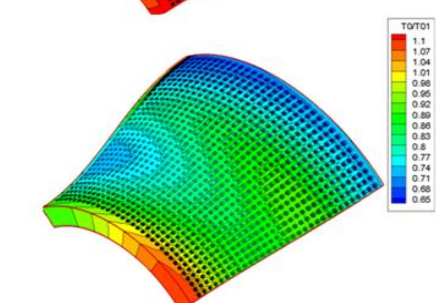
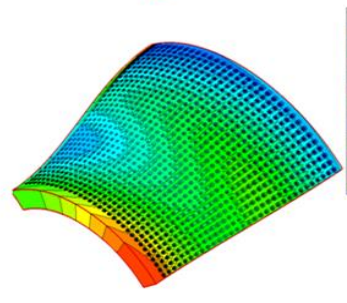


Two-scale solution  
(resolving 4 holes)

$t = 1/4T$



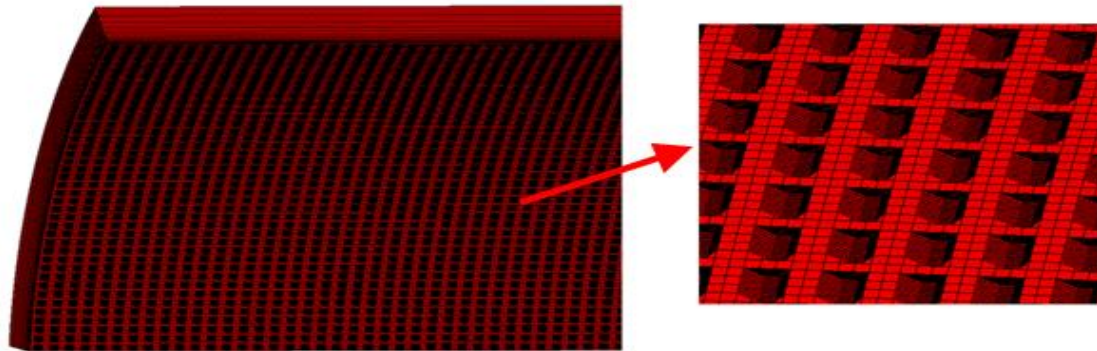
$t = 3/4T$



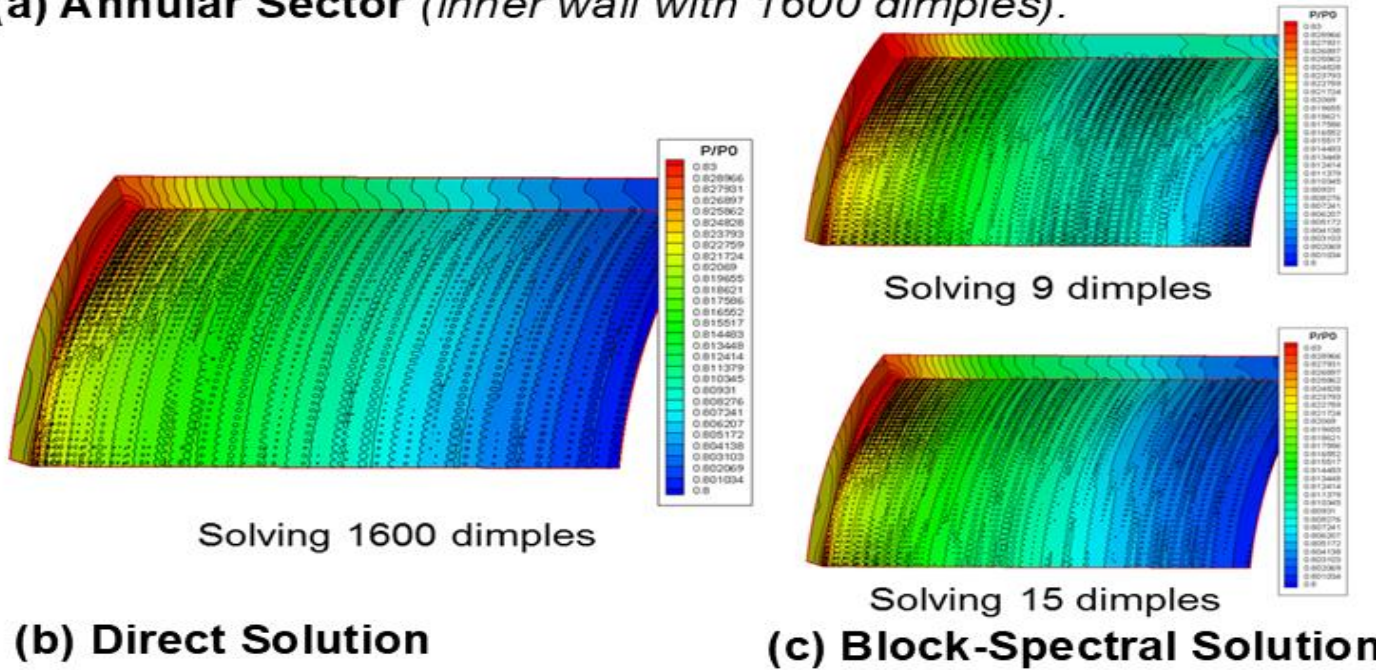
Direct Global Solution  
(Fine-mesh for 992 holes)

Two-scale Solution  
(Fine-mesh for 8 holes)

# Two-scale Tests for Surface Micro-structures (*RANS, He, 2013*)

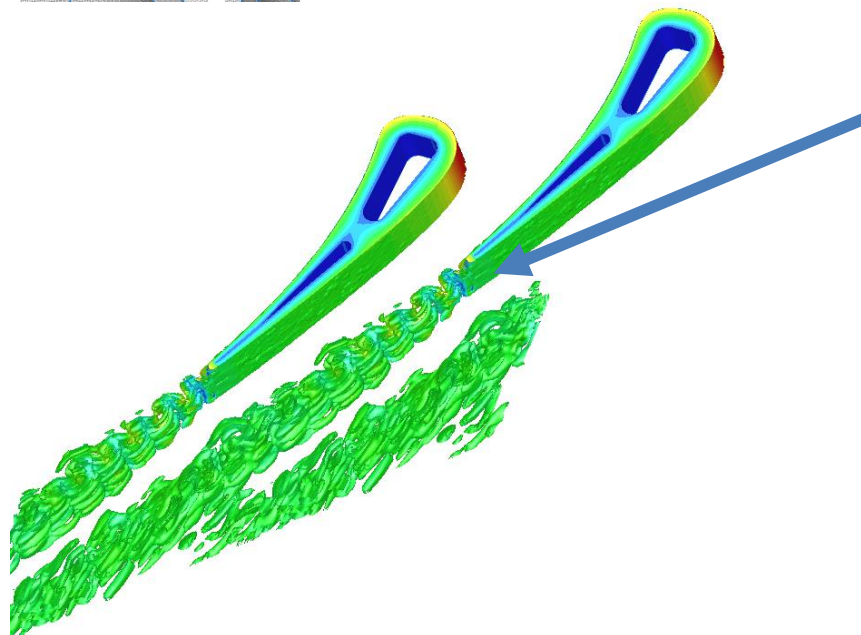
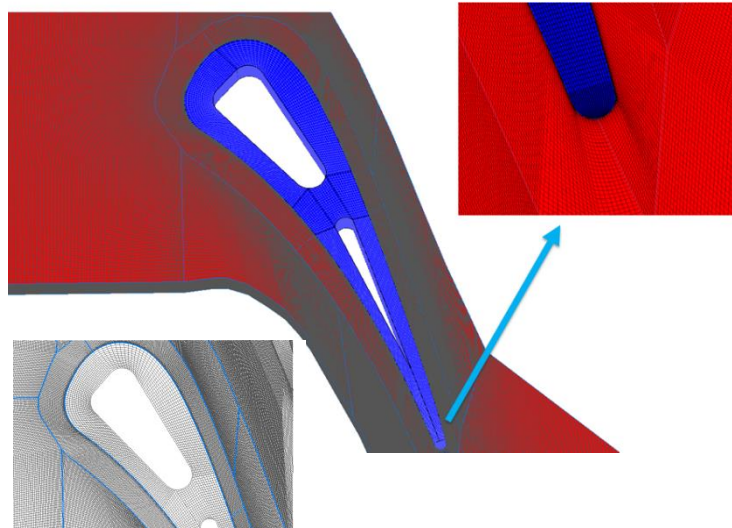


(a) Annular Sector (inner wall with 1600 dimples).

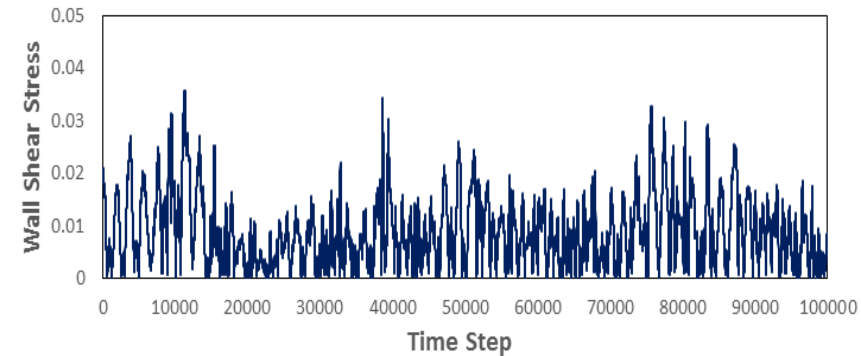


# Impact of Heat Transfer on Near-wall Flow (smooth wall)

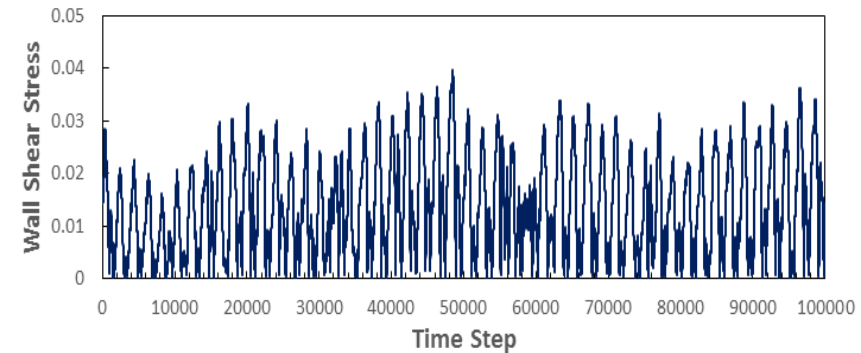
(LES-CHT for internally cooled blade passage, He 2019)



## Time traces of Shear Stress at Trailing-Edge



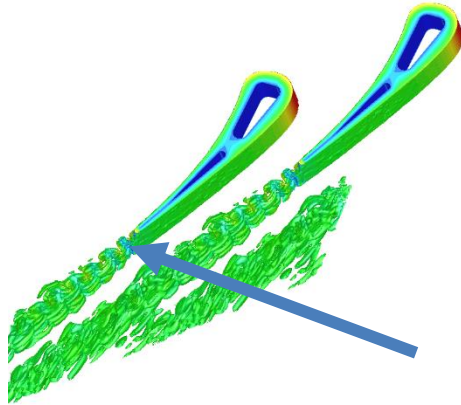
a) Adiabatic Wall Condition



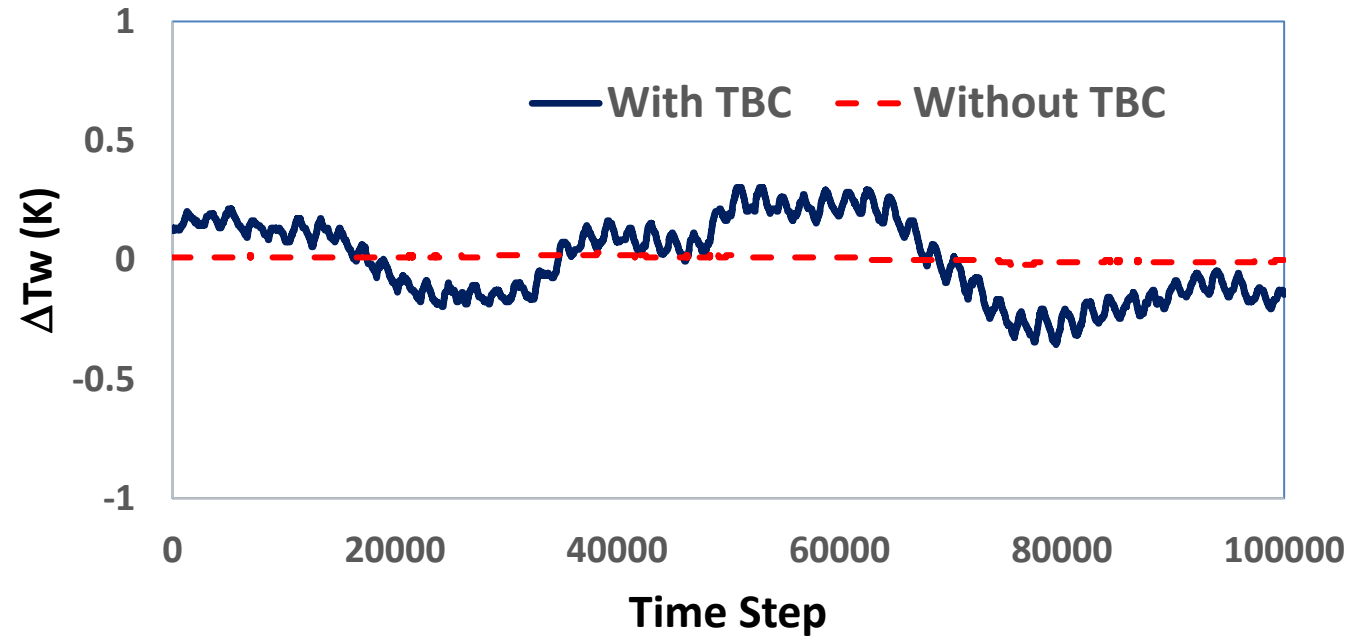
b) Conjugate Heat Transfer

- Time-mean: *changed by more than 50%*
- Unsteady: *markedly different characteristics*

# Impact of Solid Material Property (with a thin TBC layer of 100 $\mu\text{m}$ )

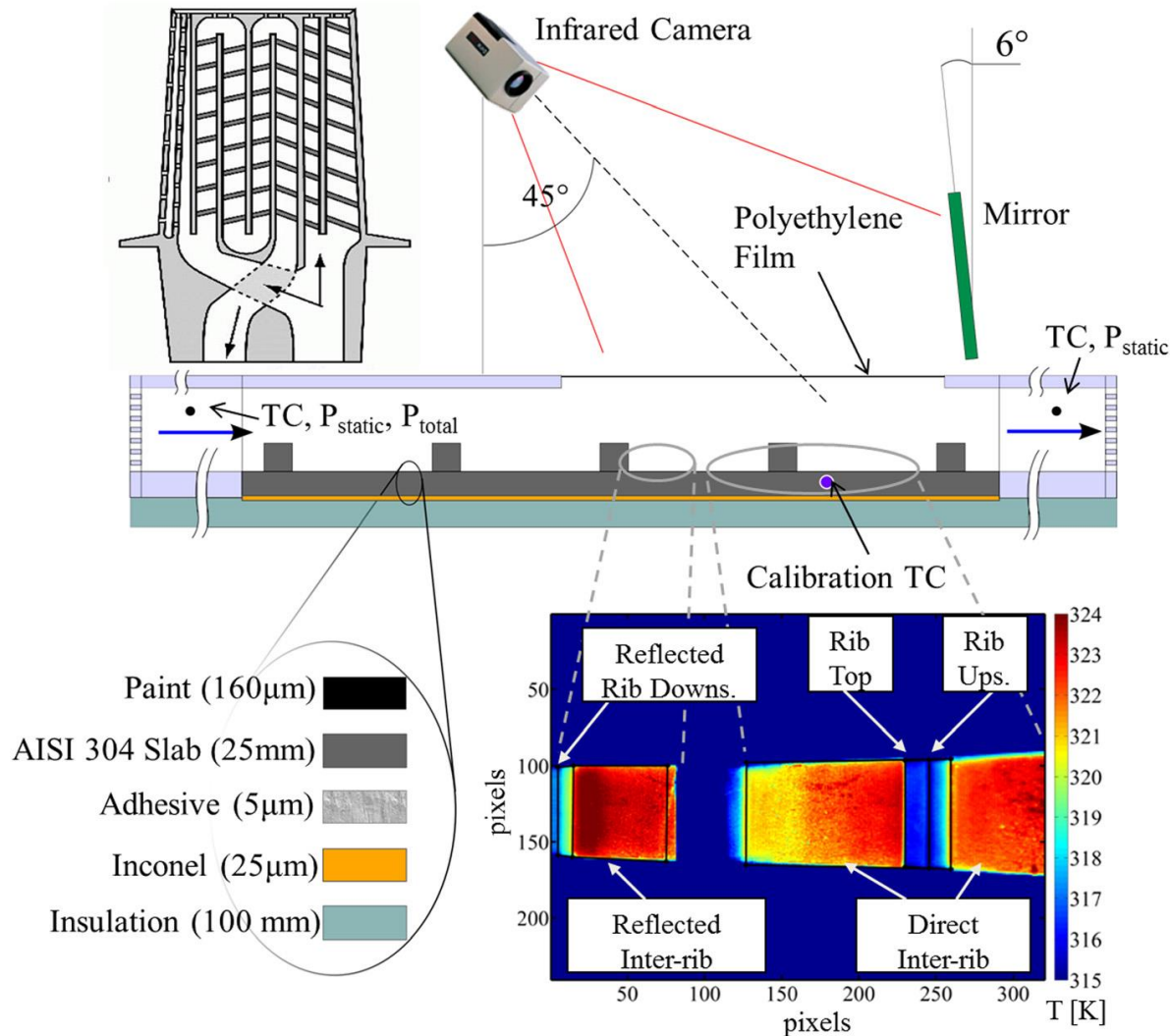


## Wall Metal Temperature in time



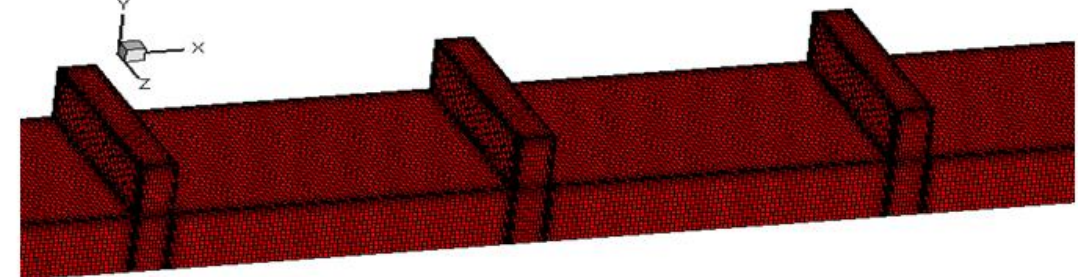
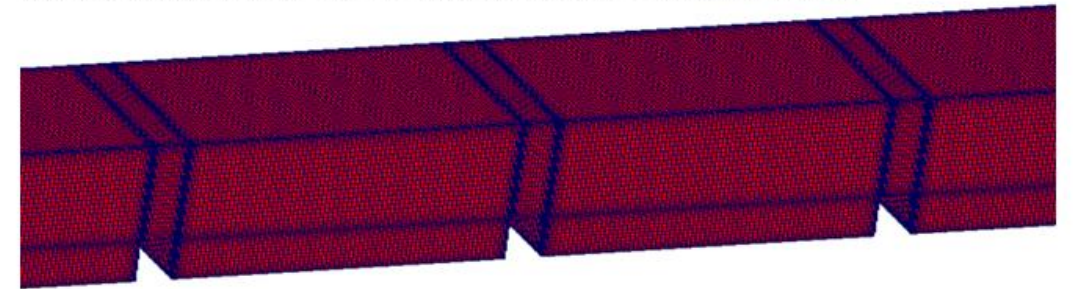
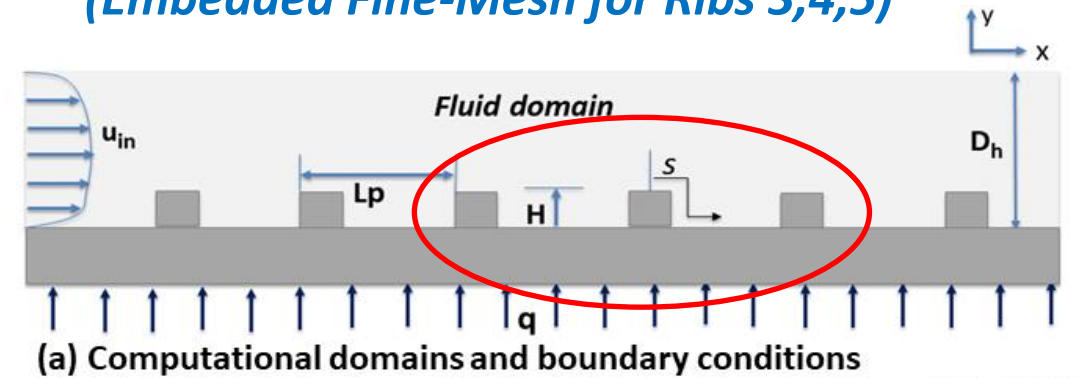
- **High Conductivity Solid:** *largely constant wall temperature (~isothermal)*
- **Low Conductivity TBC :** *much higher wall temperature fluctuations.*

## Experimental Setup



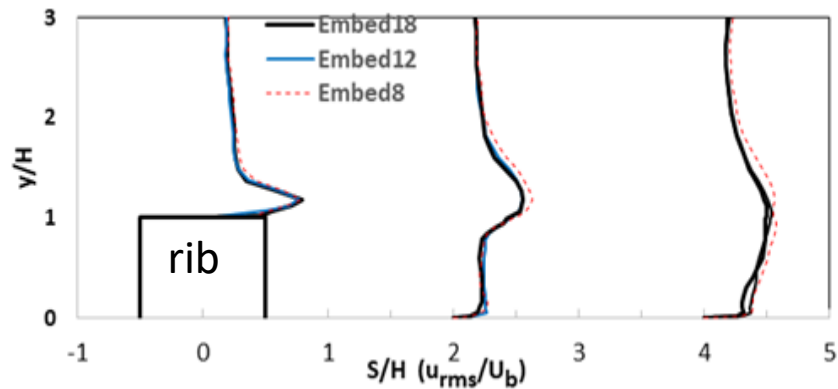
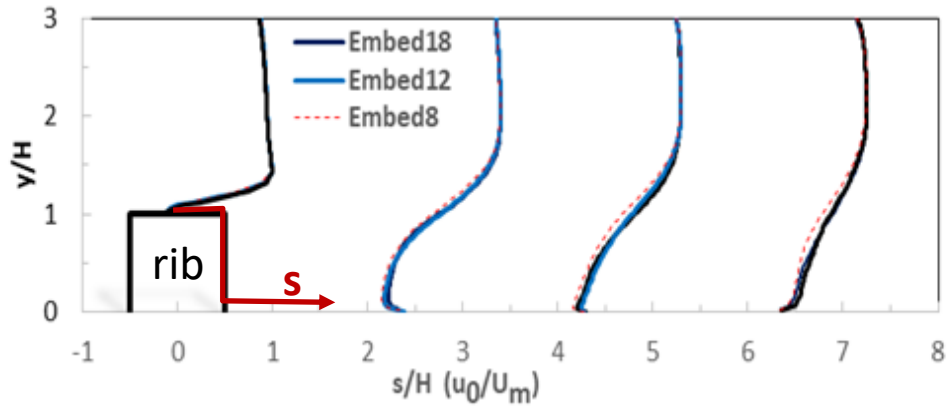
## Computational Base Mesh (He, 2023)

*(Embedded Fine-Mesh for Ribs 3,4,5)*



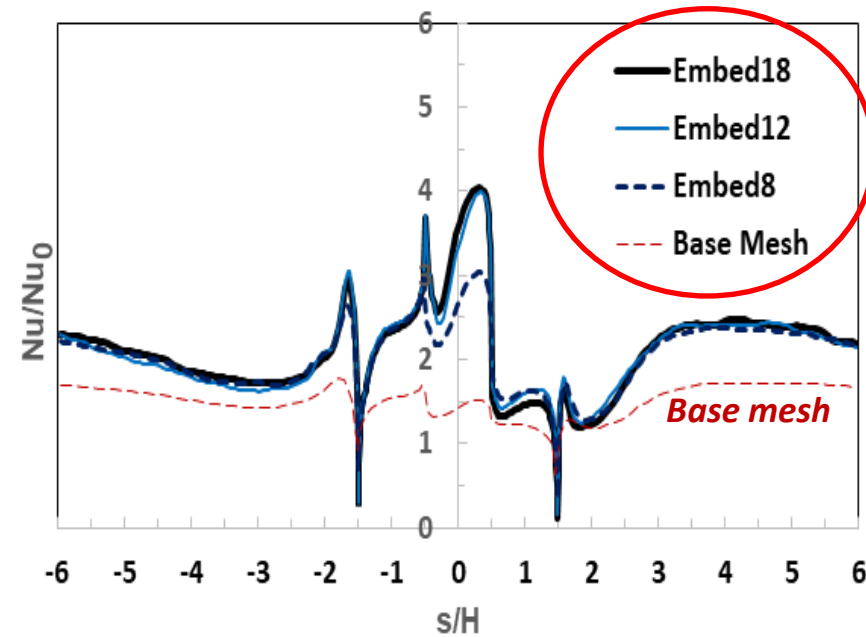
# Mesh Sensitivity (distance: 's' measured along rib surface)

## Aerodynamics (Velocity Traverses)



## Heat Transfer ( $Nu/Nu_0$ )

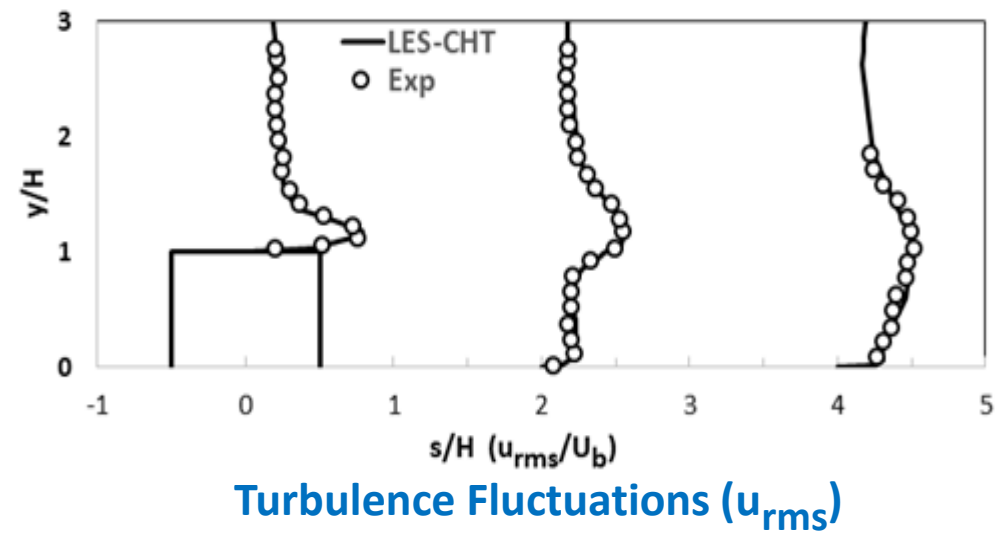
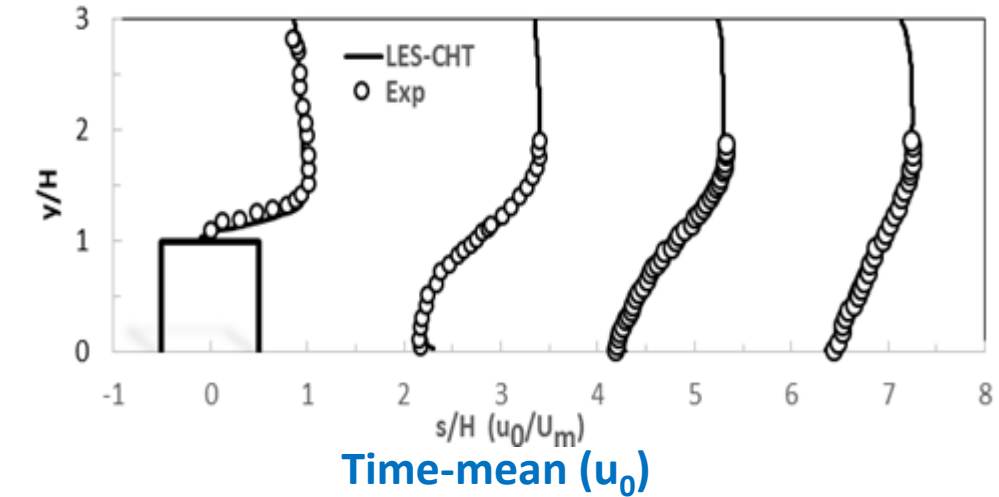
( $Nu_0$  – smooth (no rib) channel)



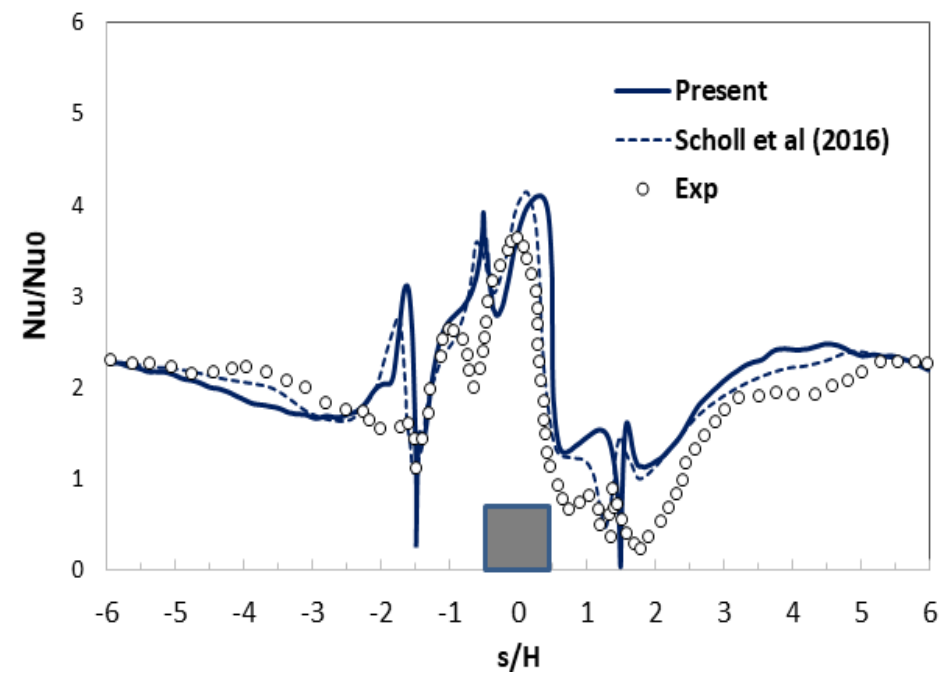
• Heat Transfer more sensitive than Flow!

# Comparison between Present LES-CHT and VKI Exp Data

## Aerodynamics (Velocity Traverses)

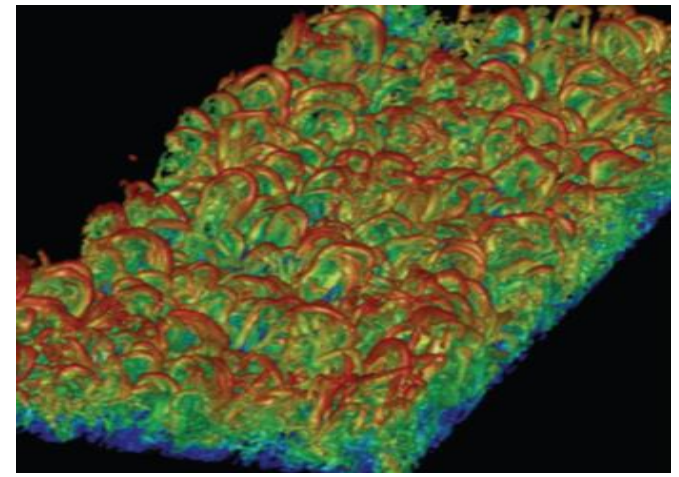
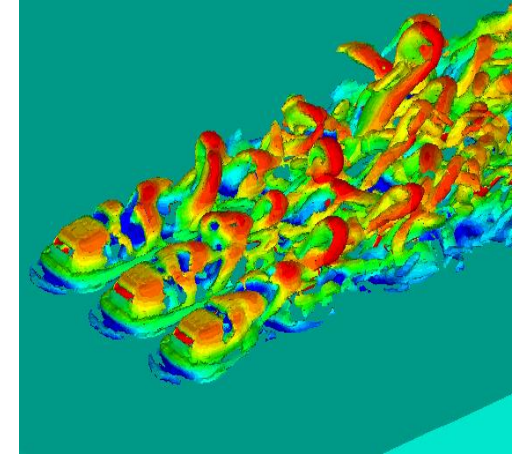
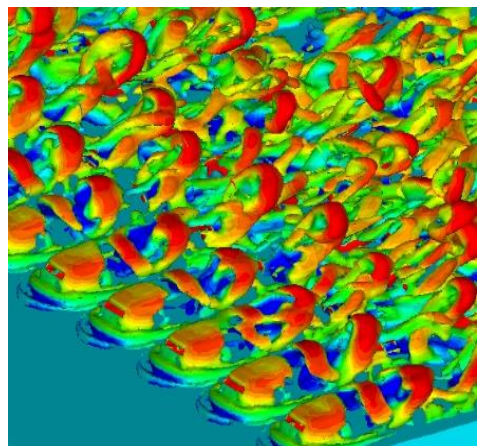
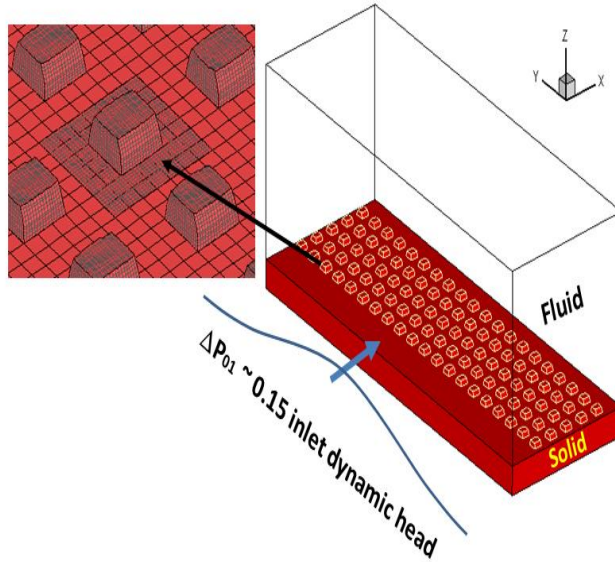


## Heat Transfer ( $Nu/Nu_0$ )



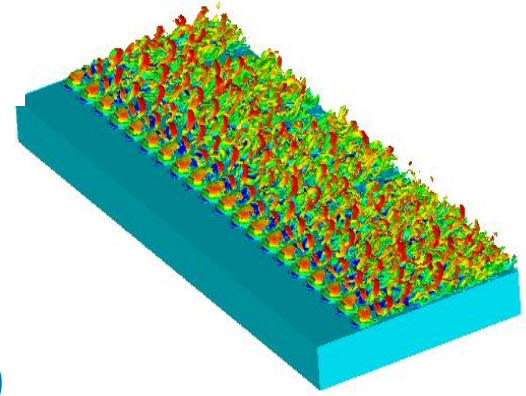
**Heat Transfer more sensitive than Flow!**

Micro-structured Wall: 100 micro-elements (*Fluid-domain with Adiabatic wall*)  
 (Direct Full-Domain Solution vs. Local Embedded Two-scale Solution (He, IJNFM 2023))

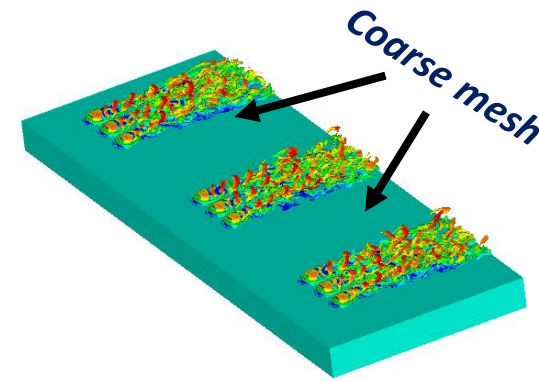


- Base coarse mesh ~0.6 million
- Direct fine-mesh: ~10 million  
 (100 micro-elements with local fine-mesh embedding: 5x5x5)

• Local 3D refinement by  $O(10^2)$ !



**Direct Solution**  
 (20 fine-mesh arrays)

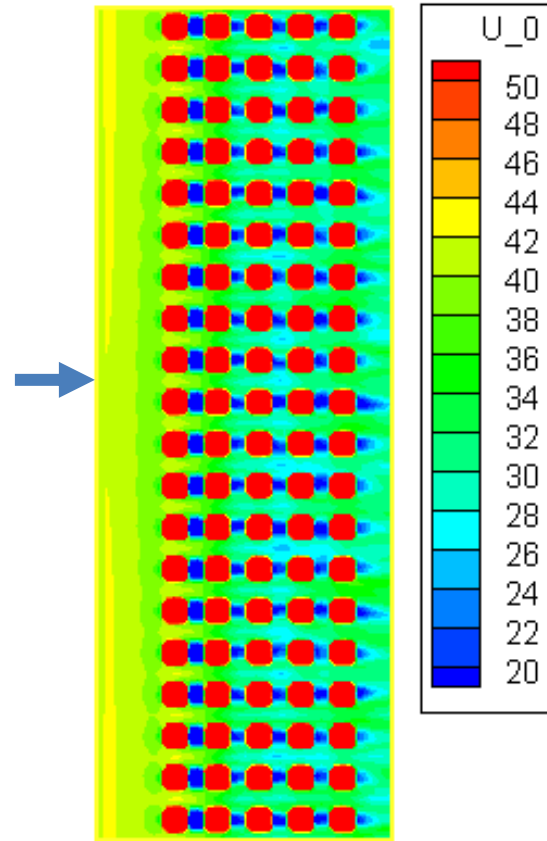


**Two-scale Solution**  
 (3 fine-mesh blocks)

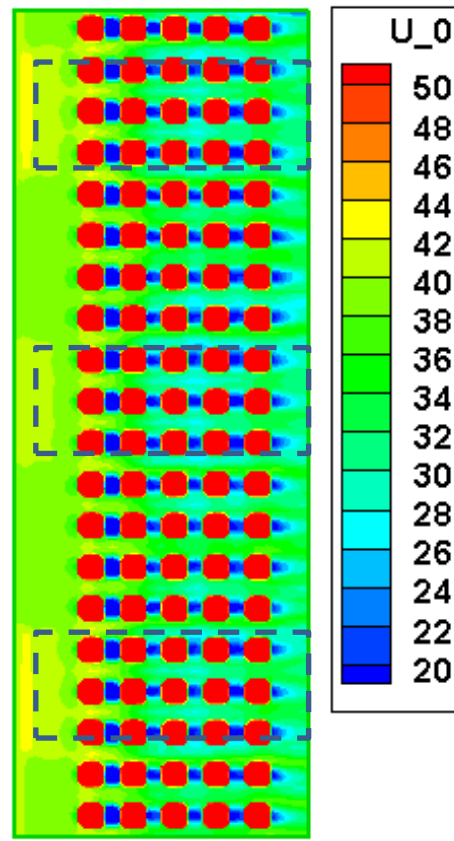
**Perspective of Hairpin Vortices (DNS)**  
 (Marusic & Monty, Ann Rev Fluid, 2019)

# Direct Solution vs. Block spectral Solution (Fluid-domain only)

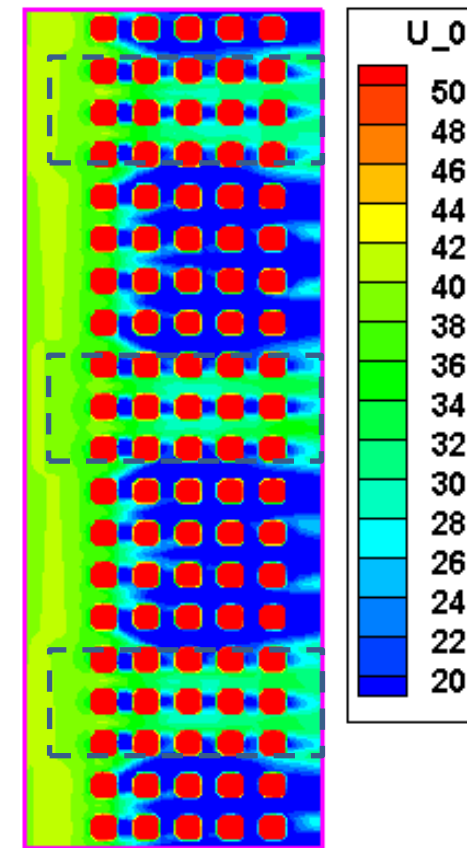
( $V_{x0}$  Contours on Plane cut at top of micro-elements)



**Full fine-mesh  
Solution**



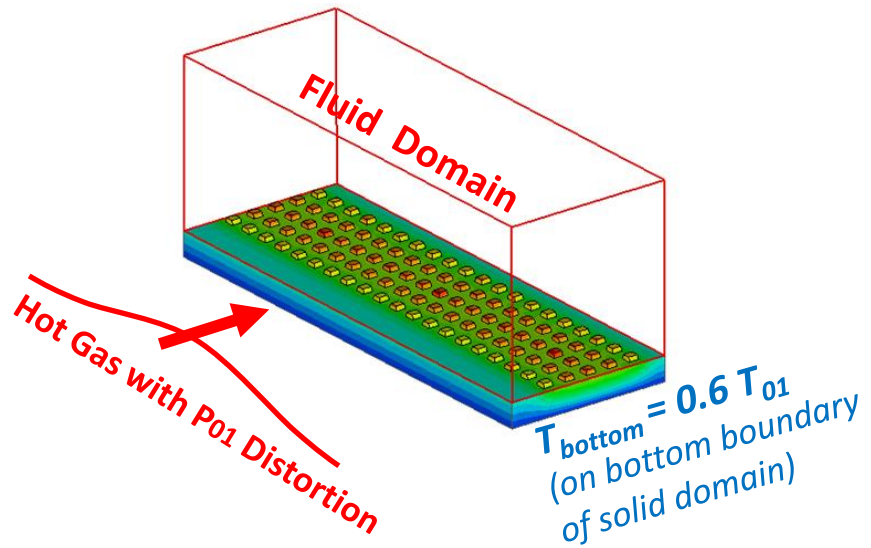
**Two-scale**  
(Local fine-mesh;  
*With Coupling*)



**'One-scale'**  
(Local fine-mesh;  
*No Coupling*)

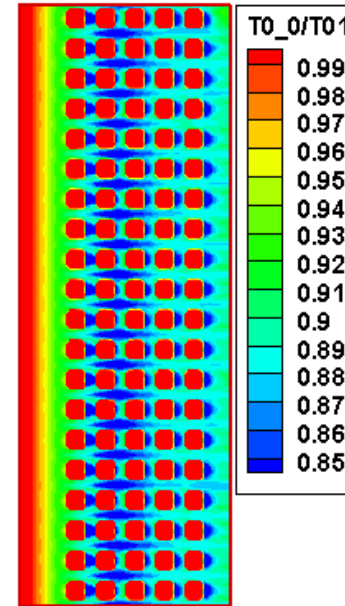
# Fluid-Solid coupled Conjugate Heat Transfer (CHT) Solution

(Time-mean Fluid Temperature  $T_0$ )

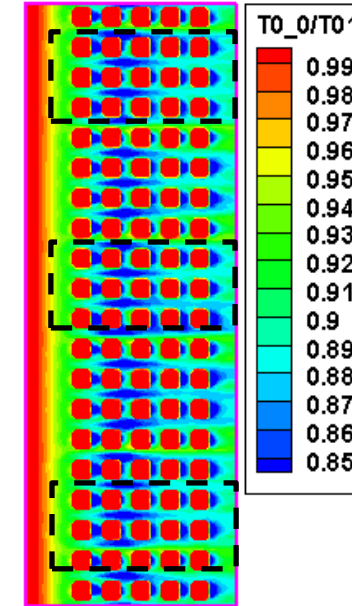


## Fluid Stagnation Temperature on Near-wall Plane

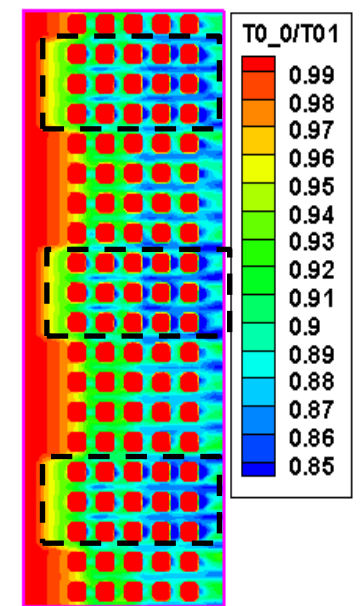
Direct Full Solution



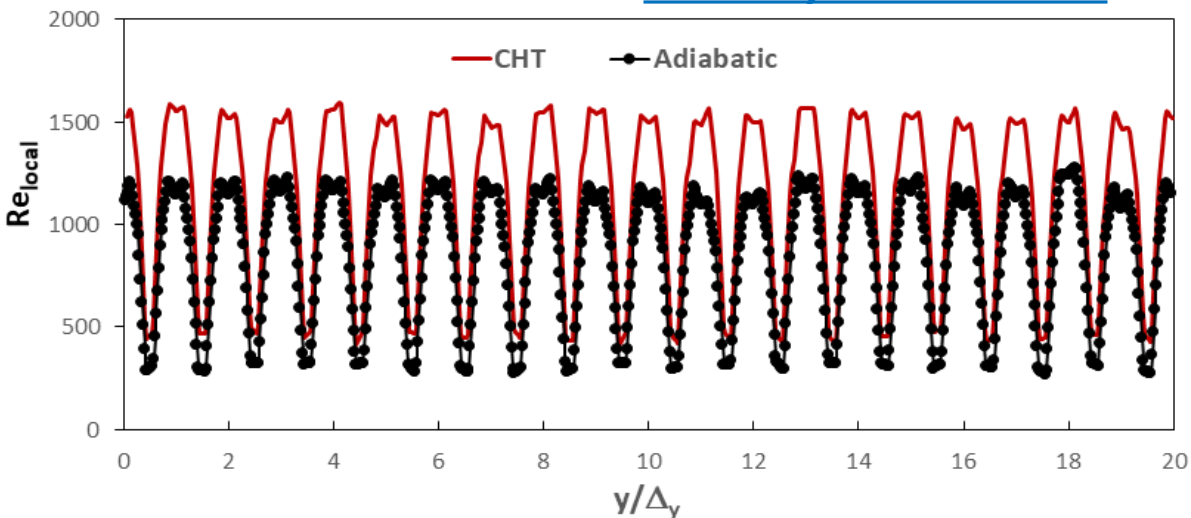
Two-scale Solution



'One-scale' Solution



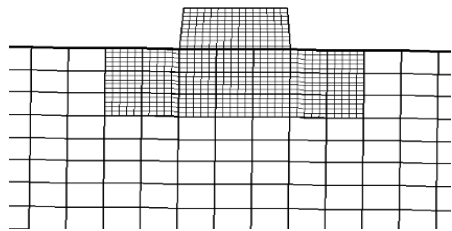
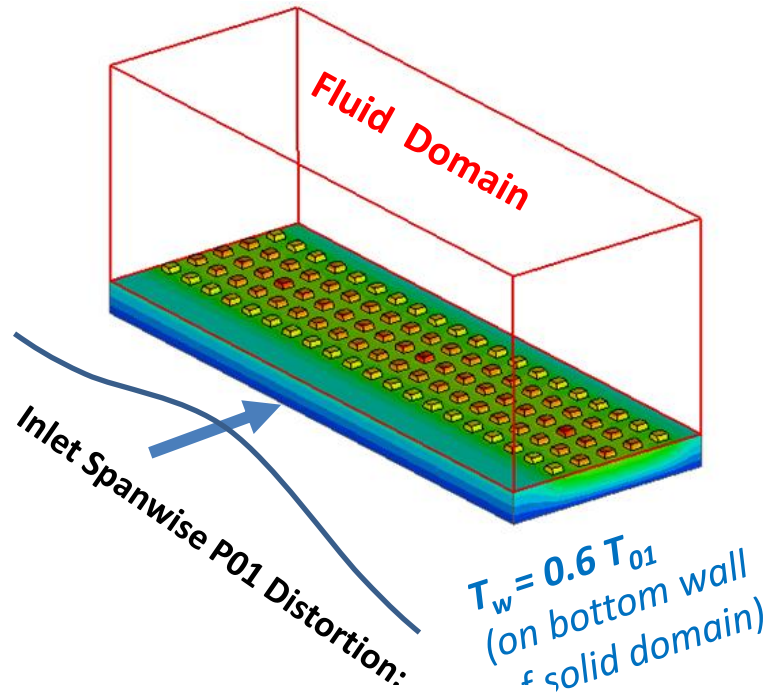
## Downstream Traverse of Local Reynolds Number



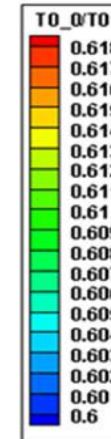
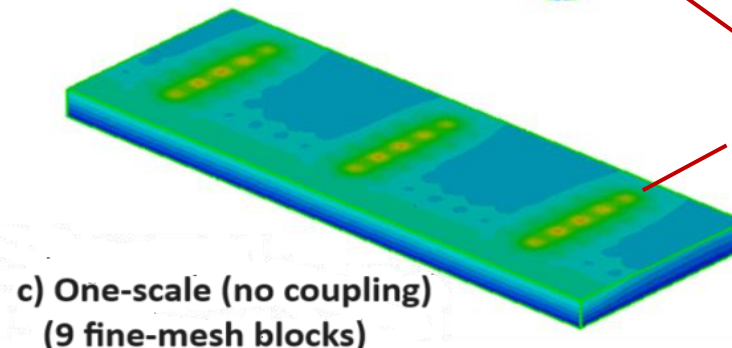
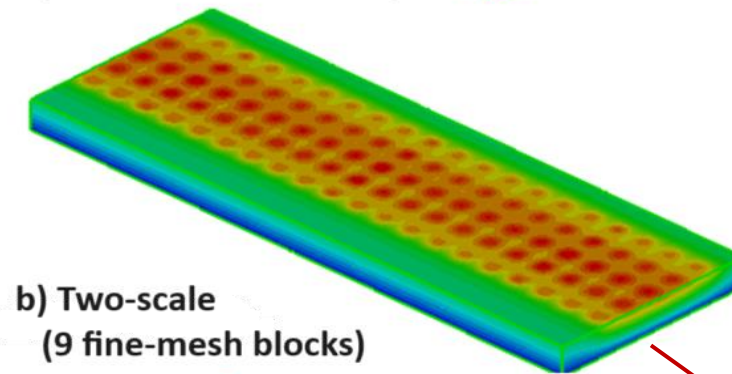
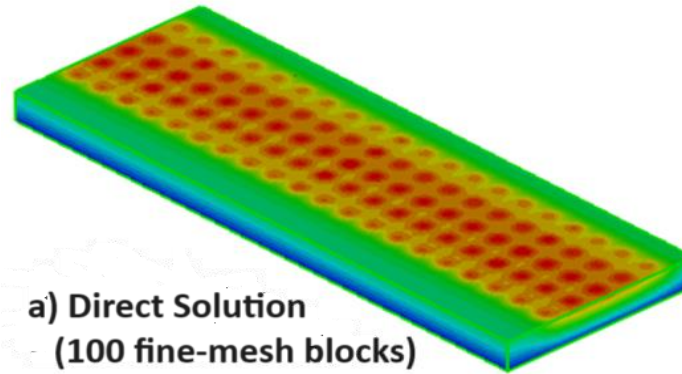
*Wall Cooling raises the Local Reynolds number - more usefully indicative than bulk flow based  $Re$ !*

# Conjugate Heat Transfer (CHT) Solution (Fluid-Solid Coupled Domain) (Solid Surface Temperatures)

## Solid Temperature on Top Wall



Local mesh for micro-structure  
( $10^2$  refinement: 5x5x5 fine/coarse)

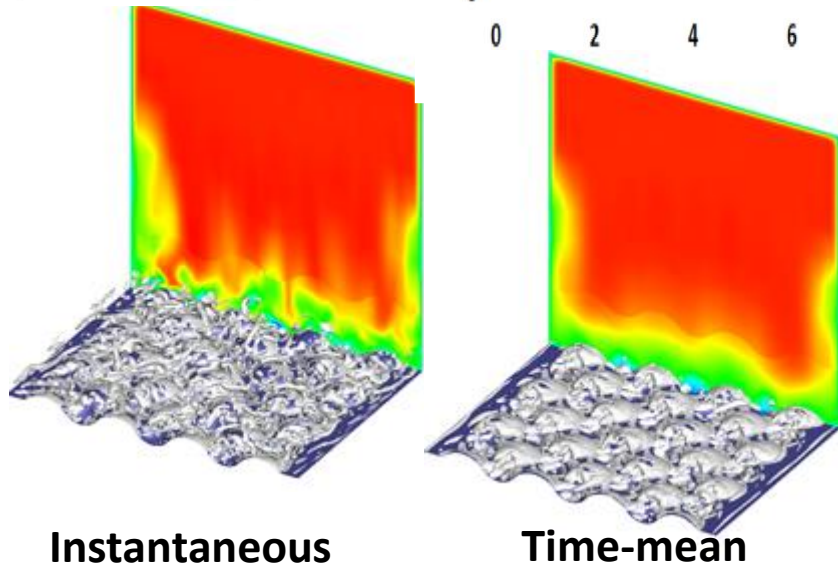
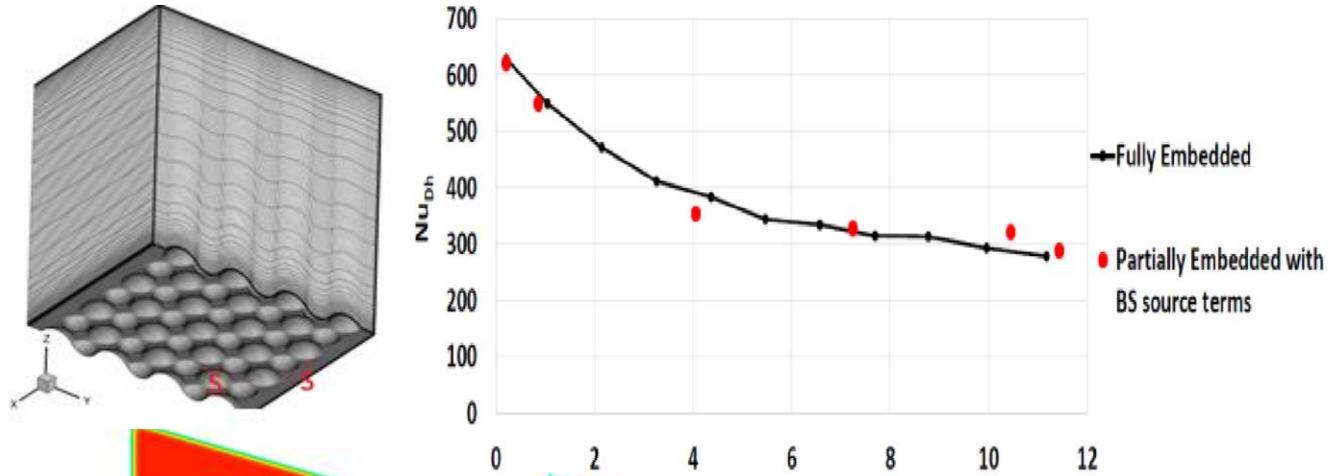


*Same mesh for both cases: underlining the impact of Two-scale coupling*

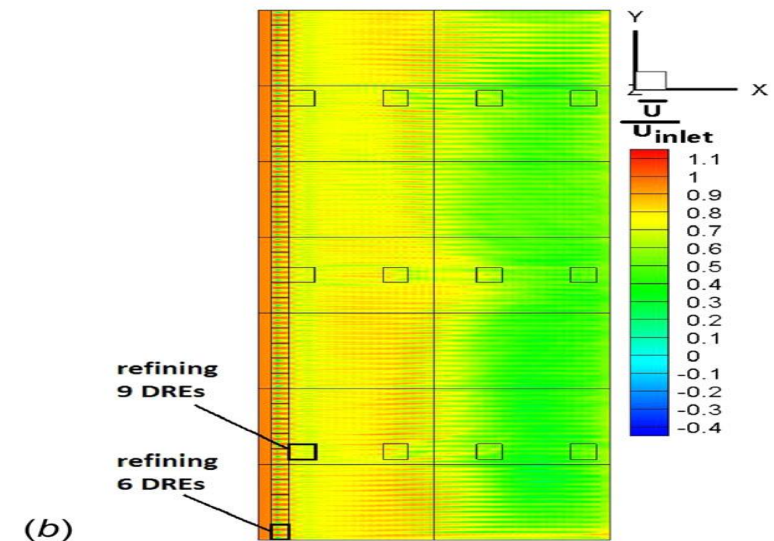
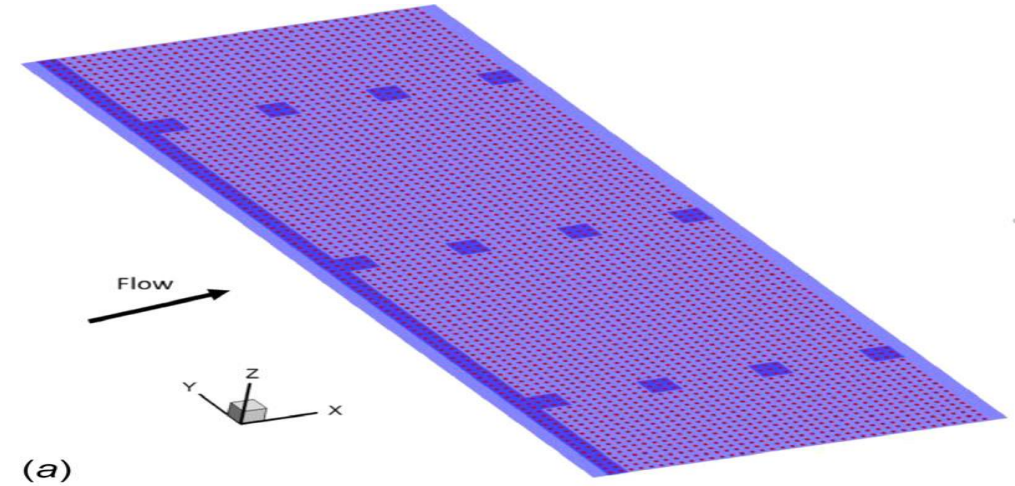
# Validation & Demo for Dimpled Surface and Large-scale Problems

(Kapsis et al, 2020)

## Pimple-Dimple Surface (Two-scale LES)



## 4000 Microstructures on Wall Surface (5x5x5 embedding)



## Closing Remarks

- Wall Microstructure (Random/Regular Roughness, Micro-Cooling): *Multi-scale Challenges.*
- Prospect of '*Surface-Design*' (both *SHAPE* and *FINISH*): *Multi-scale Opportunities.*
- Two-scale Approach (*in Space, Time, Fluid, Solid*): *Multi-scale Source-term based Method.*
  - Aimed at correcting 'Mesh-dependent' discretization errors in coarse-mesh (adopted by choice);
  - 'Mesh-informed' source terms generated in situ to drive coarse-mesh solution to the target;
  - Inclusive of closing Reynolds-Stresses (part of the problem in Fluid/not a problem at all in Solid!).

**Reynolds Stresses: Only Part of the Problem for Coarse-mesh WMLES – Attention to Mesh Errors..**