

***Nova methodus investigandi. On the Concept of Analysis in
John Wallis's Mathematical Writings*¹**

By

PHILIP BEELEY (OXFORD)

As most academic authors learn from experience, it is generally unwise to promise one's readers too much too soon. The Savilian professor of geometry in the University of Oxford, John Wallis, made this mistake at the beginning of his long career which would see him make a major contribution to methods of quadrature and perhaps more significantly still to the modernization of the dissemination of mathematical knowledge. In the final paragraph of *Mathesis universalis* (1657), one of his earliest publications, which the author describes as being "a complete arithmetical work, presented both philologically and mathematically, encompassing both the numerical and the specious or symbolic, or geometric calculus"², we find the decisive passage. Wallis explains that he has had to leave out the doctrine of analysis on account of the sheer volume of material he has gathered together. And thus "the analysis of powers", or as it is also called, "the extraction of roots", to which "the doctrine of surd roots" is conjoined, could not as he had originally intended be included in the complete transmission of arithmetic he had set out to achieve³. The final line of *Mathesis universalis* did not auger well: "which I hope will all some time, if God should

1 All dates are given old style, i. e. according to the Julian calendar, except where otherwise stated.

2 The complete title of the work, which was published with separate pagination as part of the first collected edition of John Wallis's works, *Operum mathematicorum pars prima*, Oxford 1657, is *Mathesis universalis: sive, arithmeticum opus integrum, tum philologicæ, tum mathematicæ traditum; arithmetica tum numerosa, tum speciosa sive symbolica complectens, sive calculum geometricum; tum etiam rationum proportionumve traditionem; logarithmorum item doctrinam; aliaque, quæ capitulum syllabus indicabit* (hereafter: *Mathesis universalis*). The *Mathesis universalis* was reprinted in J. Wallis: *Johannis Wallis [...] opera mathematica*, 3 vols, Oxford 1693-1699 (hereafter: *Opera mathematica*), vol. I, pp. 11-228.

3 Wallis: *Mathesis universalis*, p. 398; *Opera mathematica*, vol. I, p. 228: "Superest adhuc ad perfectam Arithmetici negotii traditionem, *Analyticae* doctrina. Quam quidem statueram praesenti operi adjunxisse. Verum cum videam jam ultra quam speraveram molem crevisse; nec interim posse totum illud negotium ita in compendium redigi, ut huic commode subungi possit: Placuit potius rem illam totam hic intactam praeterire; adeoque & Potestatum Analysis, sive (ut loquuntur) Radicum extractionem, quaeque huic conjuncta est de Radicibus Surdis doctrinam".

want it thus way, be completed in its own special volume”⁴. Less on account of the vagaries of mathematical book production in early modern England⁵ than Wallis’s other pre-occupations, the promised volume did not appear until almost thirty years later under the title *A Treatise of Algebra, both historical and practical* (London 1685), by which time, as we shall see, the concept of analysis had been considerably extended if not to say revised⁶.

1. Universality and Abstraction

Wallis’s remarks at the end of *Mathesis universalis* were less prescient than they were programmatic. That work, which he preferred to call his “Opus arithmeticum”, had after all sought to elevate arithmetic to the status of universal mathematics by contending that results in geometry could be achieved more evidently and naturally by the use of arithmetical arguments. Geometry, Wallis claimed, took its principles ultimately from arithmetic, because the principles of arithmetic are presupposed in any geometrical demonstration⁷. This had direct consequences for his fundamental views on algebra, which he seeks explicitly, as in the case of conic sections, to disentangle from its geometric underpinnings⁸. Although geometric terms such as root, square, and cube appear in algebra, Wallis rejects the conclusion that algebra is somehow based on geometry. On the contrary, in founding algebra on arithmetic, he effectively proclaimed algebra’s independence from geometry. Thus he writes in *Mathesis universalis*:

- 4 Wallis: *Mathesis universalis*, p. 398; *Opera mathematica*, vol. I, p. 228: “Quae omnia aliquando forsan, modo Deus ita voluerit, peculiari seorsim volumine absolvenda spero”.
- 5 See Mordechai Feingold: “Isaac Barrow: Divine, Scholar, Mathematician”, in: *Before Newton: the Life and Times of Isaac Barrow*, ed. by M. Feingold, Cambridge 1990, pp. 1-104, pp. 71-79; Philip Beeley: “The Progress of Mathematic Learning: John Wallis as Historian of Mathematics”, in: *The History of the History of Mathematics*, ed. by B. Wardhaugh, Frankfurt – New York 2012, pp. 9-30, especially pp. 11-14. It is a reflection of these problems that Wallis’s mathematical writings first appeared in a two volume edition of his works, printed in Oxford 1656-1657. The printing of such specialized tracts as *Arithmetica infinitorum* or *Mathesis universalis* individually would scarcely have been a sound financial proposition for the Oxford printers and booksellers.
- 6 In the preface, Wallis explicitly refers to the promise of a treatise on algebra or analytics made in 1657. As he explains, his preoccupation with other matters (“many incident Diversions”) prevented him from fulfilling the promise earlier, although the body of the text had been with London printers since 1676 and many of the sections were substantially older than that. See J. Wallis: *A Treatise of Algebra, both Historical and Practical*, London 1685 (hereafter: *Treatise of Algebra*), sig. A a2^r; a Latin translation and substantially augmented version is in *Opera mathematica*, vol. II, pp. 1-482.
- 7 Wallis: *Mathesis universalis*, p. 11; *Opera mathematica*, vol. I, p. 53. See also Douglas M. Jesseph: *Squaring the Circle: The War Between Hobbes and Wallis*, Chicago – London 1999, p. 38.
- 8 Wallis: *Treatise of Algebra*, p. 291: “This I have shewed [...] by taking them out of the Cone, and considering them abstractly as Figures in *plano*, without the embranglings of the Cone” (*Opera mathematica*, vol. II, p. 317).

“Indeed many geometric things can be discovered or elucidated by algebraic principles, and yet it does not follow that algebra is geometrical, or even that it is based on geometric principles (as some would seem to think). This close affinity of arithmetic and geometry comes about, rather, because geometry is, as it were, subordinate to arithmetic, and applies universal principles of arithmetic to its special objects”⁹.

The determination of algebraic formulae for conic sections was for Wallis an important stage on the path to finding a general method of the quadrature or cubature of curved spaces, as set out in his important work *Arithmetica infinitorum* (1656). General algebra, which he describes thereby as being “truly arithmetical, not geometrical”¹⁰ and also as an “art of invention”¹¹, could serve this function precisely because it allows lines and figures to be considered in abstraction. Nor were such views restricted to his published work. When he briefly introduces the modern algebra or analysis of Viète in his 1651 Oxford lectures on Euclid, he not only contrasts it with classical algebra but also emphasizes the enormity of its usefulness¹². An important part of this utility is in Wallis’s view the universality which the employment of algebra affords and which, he suggests, can only be achieved through abstraction. He reflects on this fundamental approach to mathematical objects years later in his *Treatise of Algebra* and confirms that it was a motivating concern for him when writing the *Arithmetica infinitorum* and had continued to be so ever since. While some contemporaries prefer to look at geometrical lines and figures, he chooses rather

“[...] to demonstrate universally from the nature of Proportions, and regular Progressions; because such Arithmetical Demonstrations are more Abstract, and therefore more universally applicable to particular occasions. Which is one main design that I aimed at in this Arithmetick of Infinites”¹³.

But such forcefulness was not without cost. By championing the primacy of arithmetic over geometry, Wallis brought himself into conflict with two noteworthy, and in one case rather troublesome contemporaries: the philosopher Thomas Hobbes and the Cambridge mathematician Isaac Barrow. Both of these men held opposite views to Wallis on the role of algebra and analytic geometry, insist-

- 9 Wallis: *Mathesis universalis*, p. 73; *Opera mathematica*, vol. I, p. 56: “Quamvis enim Geometrica multa Algebricis principiis vel inveniantur vel elucidentur, non tamen inde sequitur Algebram esse Geometricam, aut quidem Geometricis principiis nixam, (prout, qui sic procedunt, videntur imaginari,) sed ob intimam Arithmeticae Geometriaeque affinitatem illud evenit, aut potius quoniam Geometria sit Arithmeticae quasi subordinata, adeoque universalis Arithmetices effata rebus suis specialiter applicet”. The translation is in Jesseph: *Squaring the Circle* (see note 7), p. 38. See also *Treatise of Algebra*, p. 291; *Opera mathematica*, vol. II, p. 317.
- 10 Wallis: *Mathesis universalis*, p. 73; *Opera mathematica*, vol. I, p. 56: “[...] universa Algebra est vere Arithmetica, non Geometrica”.
- 11 Wallis: *Treatise of Algebra*, p. 290: “[...] without some such Art of Invention, as what we now call Algebra”; *Opera mathematica*, vol. II, p. 316.
- 12 Oxford, Bodleian Library MS Don. d. 45, 1: “Neque enim Arithmetica solummodo quam vocant Vulgarem, in numeris tam Integris quam Fractis tradidimus, sed Speciosam illam, quae (post Vietam) apud omnes nunc dierum Mathematicos, maxima quidem cum utilitate passim obtinet”. This was part of the text of a lecture originally given on 15 October 1651.
- 13 Wallis: *Treatise of Algebra*, p. 292; *Opera mathematica*, vol. II, p. 318.

ing instead on the supremacy of geometry in mathematics and of the synthetic method over methods of analysis. It is to them that we now turn.

2. Hobbes on Algebra and Symbols

Hobbes's rejection of Wallis's position has a number of sources, which it is useful to recall. Fundamentally, he was hostile to the analytic procedures set out in Descartes's *Géométrie* on account of what he considered to be the excessive use of symbols in analytic geometry. This hostility had a strong epistemological component. Hobbes opposed the view shared by Descartes and Wallis that the employment of symbols leads to new knowledge rather than simply serving to abbreviate truths known already by other means. By implication Descartes is also the target, when Hobbes contrasts Wallis's approach with that of Viète in the epistle dedicatory to his *Six Lessons to the Professors of the Mathematiques* (London 1656), his response to Wallis's *Elenchus geometriae Hobbiana* (Oxford 1655):

"And I verily believe that since the beginning of the world there has not been, nor ever shall be so much absurdity written in Geometry, as is to be found in those books of his [sc. Wallis]; [...] The cause whereof I imagine to be this, that he mistook the study of *Symboles* for the study of *Geometry*, and thought *Symbolicall* writing to be a new kinde of Method, and other mens Demonstrations set down in *Symboles* new Demonstrations. The way of *Analysis* by Squares, Cubes, &c. is very antient, and usefull for the finding out whatsoever is contained in the nature and generation of rectangled Plains (which also may be found without it) and was at the highest in *Vieta*; but I never saw any thing added thereby to the Science of Geometry, as being a way wherein men go round from the Equality of rectangled Plains to the Equality of Proportion, and thence again to the Equality of rectangled Plains; wherein the *Symboles* serve only to make men go faster about, as greater Winde to a Winde-mill"¹⁴.

Hobbes was not being entirely consistent in his evaluation of analysis here, even in respect of Viète. To the extent that contemporary algebra was considered by Wallis to be an arithmetic of species (*arithmetica speciosa*), which could be applied to different kinds of quantity, Hobbes rejected it on account of the asserted diversity of its application. As an essentially arithmetical method, algebra should not on his view be applied beyond arithmetic to geometry. In addition, Hobbes was only prepared to concede presentational significance to algebra: it could at most enable conciseness of expression in presenting a demonstration, but it could not serve any cognitive function, whether in the sense of abbreviating thought or of leading to new knowledge¹⁵. Methodological questions opened up an epistemic divide. In contrast to Wallis, Hobbes fundamentally denied that algebra was in any way to be considered a method of discovery¹⁶.

This methodological position is crucial to his distinction between algebra and analysis and explains the lengths to which Hobbes goes in praising Viète at

14 Thomas Hobbes: *Six Lessons to the Professors of the Mathematiques, One of Geometry, the Other of Astronomy*, London 1656 (hereafter: *Six Lessons*), sig. A3^v.

15 Jesseph: *Squaring the Circle* (see note 7), p. 242.

16 Thomas Hobbes: *Examinatio et emendatio mathematicae hodiernae*, London 1660 (hereafter: *Examinatio et emendatio*), p. 3: "Falsum item Algebram methodum esse inveniendi".

the expense of Wallis. Just as Hobbes rejects Wallis's identification of algebra and analysis, arguing that Pappus certainly had an analytics without algebra¹⁷, so, too, he rejects the Savilian professor's view that algebra is a method of discovery. Indeed, going even further, he also rejects the notion that it is in any meaningful sense a method at all.

The main reason for this stance is to be found in the systematic importance ascribed to deductive reasoning in Hobbes's philosophy. Geometry, proceeding deductively from well-chosen definitions represents for him the model of rigour to which other areas of rational knowledge should aspire. Conversely, knowledge based on induction and experimentation does not constitute philosophical knowledge, because it is not achieved through rational deduction¹⁸. In this respect, Wallis's position is diametrically opposed to that of Hobbes¹⁹. Moreover, it is because of these fundamental epistemological considerations that Hobbes rejects what he finds to be the reliance of analytic geometry on hypothetical procedures²⁰. In his view right reasoning (*recta ratio*) must proceed from causes to effects, and that is to say, to be demonstrative knowledge grounded on the synthetic exposition of the properties of geometric objects²¹. To ignore such a deductive approach is accordingly for Hobbes quite simply unscientific, while employing algebraic symbols for what is already known synthetically makes algebra effectively superfluous.

3. Barrow and the Objects of Mathematical Thought

Attitudes to algebra and analysis in early modern England hinged in many ways on perceptions of the scientific heritage of classical antiquity and particularly of authors such as Pappus and Diophantus. But epistemological considerations were also important in so far as these often determined what was seen to constitute the true object of mathematical thought. For Barrow geometry was the superior mathematical science, because of its association with sensible objects,

17 Hobbes: *Six Lessons*, p. 23. With explicit reference to Oughtred, Hobbes writes in *Examinatio et emendatio*, p. 3: "Falsum etiam est (quod ille innuit) Algebram & Analyticam eandem esse rem". See Helena M. Pycior: *Symbols, Impossible Numbers, and Geometric Entanglements: British Algebra Through the Commentaries on Newton's Universal Arithmetick*, Cambridge 1997, pp. 144-145.

18 Thomas Hobbes: *Elementorum philosophia sectio prima de corpore*, London 1655, p. 2; Thomas Hobbes: *Elements of Philosophy, the First Section, Concerning Body*, London 1656, pp. 2-3.

19 See Wallis: *Treatise of Algebra*, p. 306: "[...] I look upon *Induction* as a very good Method of *Investigation*; as that which doth very often lead us to the easy discovery of a General Rule"; *Opera mathematica*, vol. II, p. 331.

20 See Hobbes: *Examinatio et emendatio*, pp. 6, 125-127.

21 Jesseph: *Squaring the Circle* (see note 7), p. 243.

from which magnitude and quantity were abstracted²². The synthetic method of geometry, which he describes in his Cambridge lectures as “the art of proving theorems”²³, was considered by him to have a sound and ancient heritage. At the same time, he denied that the analytic method was the inverse of synthesis. Algebra or analysis, whose rules he claims can be known completely and distinctly from geometry and arithmetic, no more belong to mathematics than they do to physics or ethics. Consequently, algebra for Barrow is not a part of mathematics, indeed it is not even a scientific discipline²⁴, but instead, as he sets out, “a certain kind of logic, or a certain way of using reason in order to solve a question or search or prove a conclusion, such as is not rarely done in all other sciences”²⁵.

Barrow’s arguments, which are directed explicitly at Wallis, rest on his conviction that numbers themselves are mere symbols whose content derives essentially from the division of continuous geometric extension. He is thus able to move from a denial that numbers have a proper existence apart from the magnitudes they denominate to the assertion that algebra is no more than an artifice or a set of rules for manipulating symbols²⁶. It is not a part or kind of mathematics, he argues, but rather serves as an instrument of mathematics. And, as we have seen, this means for him that it is not unlike logic in respect of reasoning in general.

- 22 See Michael S. Mahoney: “Barrow’s Mathematics: Between Ancients and Moderns”, in: *Before Newton: the Life and Times of Isaac Barrow*, ed. by M. Feingold, Cambridge 1990, pp. 179-249, 185-186, 189; Pycior: *Symbols, Impossible Numbers, and Geometric Entanglements* (see note 17), pp. 156-157.
- 23 Isaac Barrow: *Lectiones mathematicae XXIII; in quibus principia matheseos generalia exponuntur*, London 1683, in: *The Mathematical Works of Isaac Barrow*, 2 vols., Cambridge 1860, vol. 1 (hereafter: *Lectiones mathematicae XXIII*), p. 45: “Analysis [...] non est pars, aut species, sed instrumentum potius Mathematicae subministrans; ut neque Synthesis, quae modus est theoremata demonstrandi analysi contradistinctus, inversusque”.
- 24 Barrow: *Lectiones mathematicae XXIII*, p. 59. See Mahoney: “Barrow’s Mathematics” (see note 22), p. 189; Pycior: *Symbols, Impossible Numbers, and Geometric Entanglements* (see note 17), p. 162.
- 25 Barrow: *Lectiones mathematicae XXIII*, p. 45: “Quia nimirum Analysis (eatenus intellecta, quatenus a Geometriae vel Arithmeticae pronunciatis et regulis distincti quid innuit) non magis ad Mathematicam, quam ad Physicam, aut Ethicam, aut aliam quamvis scientiam videtur spectare. Est enim duntaxat pars quaedam aut species Logicae, seu modus quidam utendi ratione circa quaestionum solutionem, inventionemque vel probationem conclusionum, qualis in aliis omnibus scientiis exercetur haud raro”.
- 26 Ibid., p. 46: “[...] haec nullum habet distinctum, et sibi proprium objectum, sed artificium solummodo quoddam tradat, in Geometria (vel Arithmetica) fundatum, qua magnitudines et numeros certis notis, vel symbolis designandi, qua summas ipsorum et differentias colligendi ac comparandi. Unde minime constituit partem aliquam Matheseos, a Geometria, vel Arithmetica distinctam, sed in ipsis omnino continetur”.

4. Specious Arithmetic

Both Hobbes and Barrow rejected the incorporation of algebraic techniques into geometry, as had been promoted by Viète, Descartes, and the English mathematician William Oughtred – Wallis would later also emphasize the importance of Thomas Harriot in this context. In the dedicatory epistle which prefaced *De sectionibus conicis* (1655) and which was addressed to Seth Ward and Lawrence Rooke, Wallis named precisely the three figures Viète, Descartes, and Oughtred as providing the model for his own application of symbols²⁷. It was of course Viète who had first introduced symbolic notation for unknowns and coefficients and who had identified algebra in his *In artem analyticam Isagoge* (1591) as the analytical tool by which classical theories had first been discovered. It was principally on account of Viète that algebra became known as the analytic art or simply as analysis for much of the seventeenth century. In the *Isagoge* Viète follows Pappus and distinguishes two classical forms of analysis: the zetetic, by which is found the relationship or equation which an unknown quantity must satisfy, and the poristic, by which the truth of a proposed theorem is tested by means of an equation or proportion. To these two kinds he adds a third, namely, rhetoric or exegetic analysis, which is the process of determining the sought magnitude in a proposed equation or proportion²⁸. Not without reason, Viète was convinced of the enormous power of this analytical tool. Having distinguished the three forms, he describes the whole analytical art as being “the doctrine of correct discovery in mathematics”²⁹, and he concludes the work famously by claiming for the analytical art “that it leaves no problem unsolved”³⁰.

Wallis endorsed Viète’s analytic method and ascribed to it the status of a general mathematical technique. In *Mathesis universalis* he refers to the “universal art of algebra or analytics”³¹ and to the “algebraic or analytic method”³², and similar expressions are to be found in other works. As his lecture notes on Euclid show, Wallis had imbued Viète’s *Isagoge* as well as Oughtred’s *Clavis mathematicae* (1631) early in his mathematical career and already in his inaugural

27 John Wallis: *De sectionibus conicis*, Oxford 1655, sig. i2^v-i3^r; *Opera mathematica*, vol. I, p. 294.

28 François Viète: *In artem analyticam isagoge*, Tour 1591; reprinted in: *Francisci Vietae opera mathematica*, ed. by F. van Schooten, Leiden 1646 (hereafter: *In artem analyticam isagoge*), pp. 1-12, p. 1. See Michael S. Mahoney: *The Mathematical Career of Pierre de Fermat (1601-1665)*, second edition, Princeton, NJ 1994, pp. 28-48.

29 Viète: *In artem analyticam isagoge*, p. 1: “Atque adeo tota ars Analytica triplex illud sibi vendicans officium definiatur, Doctrina bene inveniendi in Mathematicis”.

30 Ibid., p. 12: “Denique fastuosum problema problematum ars Analytica, triplicem Zeteticas, Poristicas & Exegeticas formam tandem induta, jure sibi adrogat, Quod est, Nullum non problema solvere”. See Pycior: *Symbols, Impossible Numbers, and Geometric Entanglements* (see note 17), pp. 34-35.

31 Wallis: *Mathesis universalis*, p. 68: “monendum duco, Universam Artem Algebrae sive Analyticae”; *Opera mathematica*, vol. I, p. 53.

32 Wallis: *Mathesis universalis*, p. 80: “ut Methodo Algebraica seu Analytica procedatur”; *Opera mathematica*, vol. I, p. 59.

lecture, delivered in the University of Oxford on 31 October 1649, he talks of many theorems lately discovered in mathematics and whose discovery was greatly facilitated by the mode of discovery itself, “that is to say, algebra, or analytical practices, beyond those known to the ancients, now becoming known”³³.

But Wallis differs in his approach to algebra or analytics considerably from Viète. Effectively, he ignored what can be considered to be Viète’s most important innovation, the application of algebra to geometry, and saw algebra instead as primarily a generalized method of arithmetic³⁴. In place of Viète’s specious logistics (*logistica speciosa*) he sets his own concept of specious arithmetic (*arithmetica speciosa*) which he nevertheless tentatively ascribes to the French mathematician and indeed traces to his legal training: “The name of *Specious Arithmetick*, is given to it (I presume) with respect to a sense wherein the *Civilians* use the word *Species*”³⁵. As Wallis explains, common lawyers are accustomed to denote unknown parties in their legal cases by general names. He continues

“And to this way of *Specious Arithmetick*, Vieta doth accommodate, not only the ordinary Operations of common Arithmetick; but the Rules of *Algebra* before invented by former *Algebrists*; differing from them herein, not so much in the substance of the Rules themselves (which are the same in the Old and New Algebra,) as in a new Notation or Designation of those former Rules”³⁶.

As a specious arithmetic, the new algebra was conceived by Wallis to be endowed with near unlimited possibilities for application. All the basic arithmetic operations extended now to algebra. Conversely, the scope of arithmetic was extended by the inclusion of specious arithmetic to cover whatsoever was capable of being expressed in terms of proportion³⁷.

5. James Gregory’s Concept of Analysis

The task which Wallis sets out to achieve in the *Treatise of Algebra* is no small one. It is outlined in the preface as being to provide “an Account of the Original, Progress, and Advancement of (what we now call) *Algebra*”³⁸. On the face of it, his motivation was similar to that of Viète, because as he emphasizes not only in the *Treatise of Algebra* itself but also in a number of other places his aim was to reveal the true art of discovery or method of investigation, “which the

33 John Wallis: *Oratio inauguralis*, Oxford 1657, f. c3v: “Algebrae nempe, sive Analytices usue, ultra quam veteribus innotuit, jam innotescit”; *Opera mathematica*, vol. I, p. 8.

34 See Jacqueline A. Stedall: *A Discourse Concerning Algebra: English Algebra to 1685*, Oxford 2002, p. 54.

35 Wallis: *Treatise of Algebra*, p. 66; *Opera mathematica*, vol. II, p. 70.

36 Ibid. See Pycior: *Symbols, Impossible Numbers, and Geometric Entanglements* (see note 17), pp. 112–113.

37 John Wallis: *A Defense of the Treatise of the Angle of Contact*, London 1684, p. 92; *Opera mathematica*, vol. II, p. 649.

38 Wallis: *Treatise of Algebra*, f. A^r; *Opera mathematica*, vol. II, f. A^r.

Ancients were wont (as great secrets) to conceal from us”³⁹. In a broader sense he presents algebra, suitably configured in its arithmetical dress, as the analytics which classical mathematicians hid from posterity. And indeed when Wallis uses the term “analysis” it is meant almost exclusively in this way to signify what he calls “arithmetica speciosa”. Admittedly, sometimes when talking of analytical operations, he distinguishes subtraction, division and the extraction of roots from their synthetic counterparts of addition, multiplication, and the composition of powers. In this narrower sense he also occasionally talks of “analytical operations”, for example in his exposition of Oughtred’s work on algebra⁴⁰. But this distinction was of no substantial consequence for his overall position, which was remarkably consistent.

This consistency contributed to making Wallis an ideal adjudicator when, following the publication of James Gregory’s *Vera circuli et hyperbolae quadratura* (1667), an acrimonious conflict broke out between the Scottish mathematician and Christiaan Huygens over one of his major propositions⁴¹. Gregory sought to employ a new method for determining the magnitude of the area of a circle in relation to its radius, which did not rely simply on the five basic operations addition, subtraction, multiplication, division, and root extraction, but also used convergent sequences. This procedure was suggested to him by the geometric construction in which the area of the circle is approached by means of circumscribed polygons⁴². It led him to a convergent double sequence whose limit was held to represent the sought area. In *Vera circuli et hyperbolae quadratura*, Gregory investigated the limits of such sequences, which on his terms did not always need to be an “analytical” expression. This requires explanation. Under “analytical” Gregory understood a magnitude which could be composed from magnitudes which are commensurable to each other using the five basic operations. Now, since it was shown that the limit of the sequence used to determine

39 Wallis: *Treatise of Algebra*, p. 306; *Opera mathematica*, vol. II, p. 331. See also Wallis to Digby, 21 November 1657, in: John Wallis: *Commercium epistolicum de quaestionibus quibusdam mathematicis nuper habitum*, Oxford 1658, p. 43: “Sed nec Archimedes solus, verum & veterum plerique omnes, Analyticen suam, (quam habuisse, extra dubium est,) eousque celarunt posteros, ut Recentioribus facilius jam fuerit novam suo Marte comminisci (quod praesenti praeteritoque seculo factum est) quam indagasse veterem”; *Correspondence of John Wallis (1616-1703)*, ed. by P. Beeley and C. J. Scriba, 1-(4) vols., Oxford 2003ff, vol. I, p. 330. See also Christoph J. Scriba: “The Autobiography of John Wallis, F.R.S.”, in: *Notes and Records of the Royal Society of London* 25 (1970), pp. 17-46, p. 41.

40 See Wallis: *Treatise of Algebra*, pp. 2, 97, 254; *Opera mathematica*, vol. II, pp. 2, 110-112, 272.

41 Huygen’s criticism of Gregory’s approach was published in *Journal des Sçavans*, 2 July 1668 (new style), pp. 52-56. Gregory’s response, “Mr. Gregoryes Answer to the Animadversions of Mr. Hugenius upon his Book, De vera circuli & hyperbolae quadratura”, appeared in *Philosophical Transactions* 37 (13 July 1668), pp. 732-735. Huygens responded in turn in *Journal des Sçavans*, 12 November 1668 (new style), pp. 109-112.

42 See Christoph J. Scriba: “Gregory’s Converging Double Sequence. A New Look at the Controversy between Huygens and Gregory over the ‘Analytical’ Quadrature of the Circle”, in: *Historia Mathematica* 10 (1983), pp. 274-285, pp. 275-276.

the area of a circle could not readily be closed, Gregory sought to prove that this limit is not in his sense analytical to the radius. In modern terms he sought to prove the transcendence of π and was indeed the first to do so.

Huygens set out to show that Gregory was wrong, arguing that it was still an open question as to whether the area of a circle could be “analytically” derived from its radius. Called upon to adjudicate in a potentially harmful dispute between two members of the Royal Society, Wallis soon arrived at the conclusion that Gregory’s argumentation was wrong⁴³. But rather than engage with the fundamentals of his mathematical procedure, which no doubt would have had the undesirable effect of prolonging the dispute still further, the Savilian professor switched uniquely in his mathematical writings to logical considerations:

“The summe of his Demonstration (since he will needs defend what I would have waved for him) depends upon these two Syllogisms. If the Sector, indefinitely taken, can be, in such manner as he speakes, Analytically compounded; Then the Circle can be Analytically squared: But the Sector cannot bee so compounded: Therefore the Circle cannot be so squared. Which syllogism is manifestly peccant in form. The Minor of that, hee proves by another in the same form. If there can be a quantity in the same manner compounded of the two first, & of the two second Terms; Then the sector can be Analytically compounded: But there cannot be any such quantity: Therefore the Sector cannot be so compounded. [...] Both are false Syllogisms; and can conclude nothing, be the premises never so true, unless not onely the Consequences of the Majors, but also the Converse of those consequences be demonstrated. (Which any Logician, though no Mathematician, will easily discern)”⁴⁴.

Wallis does not challenge Gregory’s employment of the term “analytical” in this context, but instead leaves it standing in precisely the sense in which the author intended its use. In doing so, it would seem that the Savilian professor made a clear attempt to avoid ambiguity in the use of crucial terms, just as we find him later doing in response to the employment of the term analysis in yet another sense in Leibniz’s writings on his infinitesimal calculus.

6. Malebranche and Prestet

The concept of analysis played a role in another controversy with which Wallis was more directly involved. When the anonymously published first edition of Jean Prestet’s *Elemens des mathematiques* (1675) appeared⁴⁵, Wallis like contemporaries on both sides of the English Channel attributed the work to Nicolas Malebranche. This ascription was not entirely ill-founded, because the author had indeed lived for a time as a young man in the house of the eminent

43 Wallis’s account of the controversy took the form of a letter to the president of the Royal Society. See Wallis – Brouncker, 4 November 1668, in: *Correspondence of John Wallis (1616-1703)* (see note 39), vol. III, pp. 25-37.

44 Wallis to Oldenburg, 14 November 1668, in: *Correspondence of John Wallis (1616-1703)* (see note 39), vol. III, pp. 46-48, 46-47.

45 [Prestet]: *Elemens des mathematiques ou principes generaux de toutes les sciences*, Paris 1675. This work was reviewed in *Philosophical Transactions* 126 (20 June 1676), pp. 638-642.

philosopher⁴⁶. Moreover there is evidence that the *Elemens* not only had considerable impact on Malebranche's *Recherche de la vérité*, which appeared at the same time, but also that there was reciprocal influence of that work on the *Elemens*⁴⁷. Wallis, who at the time was working on the first draft of his *Treatise of Algebra* and who was keen to promote the algebraic endeavours of Harriot to the disadvantage of Descartes⁴⁸ – he would later be supported in this by Leibniz⁴⁹ – noticed above all in Prestet's compendious publication that only the names of Viète and Descartes appeared:

“Monsieur Malbranche, hath lately published (but without putting his Name to it,) his *Elemens des Mathematiques*; which is a Collection out of all or most of the Writers of this nature; especially from *Vieta*'s time downwards. But for the most part, without troubling his Reader with the Names of the Authors where he found those things by him Collected, (except his two Country-men, *Vieta*, and *Des Cartes*;) And without adding any great matter of his own, to what was before taught by others”⁵⁰.

Not surprisingly, this suggestion of partisanship and the attribution of the work to Malebranche did not go down well with Prestet, who in later years distanced himself considerably from his former friend and patron⁵¹. Correspondents including Leibniz⁵² and the sometime vice-president of Magdalen College, Oxford, Thomas Smith⁵³ corrected the Savilian professor on his mistake concerning the authorship, and in the long-awaited second edition of the *Treatise of Algebra*, which was published in Latin as volume II of Wallis's monumental *Opera mathematica* in 1693, the offending references to Prestet were removed. But this was too late to prevent the French mathematician from using the preface of the second edition of his *Nouveaux elemens des mathematiques* (1689) to reject vehemently

46 See André Robinet: “Jean Prestet ou la bonne foi cartésienne (1648-1691)”, in: *Revue d'histoire des sciences et de leurs application* 13 (1960), pp. 95-104, p. 97.

47 See André Robinet: *Malebranche et Leibniz: relations personnelles*, Paris 1955, p. 27.

48 See Stedall: *A Discourse Concerning Algebra* (see note 34), pp. 117-125.

49 See Philip Beeley and Christoph J. Scriba: “Wallis, Leibniz und der Fall von Harriot und Descartes. Zur Geschichte eines vermeintlichen Plagiats im 17. Jahrhundert”, in: *Acta Historica Leopoldina* 45 (2005), pp. 115-129.

50 Wallis: *Treatise of Algebra*, p. 214. As explained below, this paragraph is no longer present in the second (Latin) edition of the *Treatise of Algebra*.

51 On the consequences of this development for Prestet's thought see Catherine Goldstein: “On a Seventeenth Century Version of the ‘Fundamental Theorem of Arithmetic’”, in: *Historia Mathematica* 19 (1992), pp. 177-187, pp. 180-182.

52 See also Leibniz's review of Wallis's *Treatise of Algebra*, in: *Acta eruditorum*, June 1686, pp. 283-289, p. 286, and his review of Wallis's *Opera mathematica*, vols. I and II, in: *Acta eruditorum*, June 1696, pp. 249-59, p. 256.

53 See Smith to Wallis, 30 June 1692, Oxford, Bodleian Library MS Smith 66, f. 9^r: “The Author, M. Prestet, who formerly lived with M. Malbranch, a Preist of the Orators, I presume is extremly concerned for the honour of Vieta & Des-Cartes; as if to these two cheifly the great impurements, that have been made in Geometry, are to bee ascribed, and seemes angry with you for your reflecting on the latter, as if hee had been beholden to our learned Countryman M. Harriot, and on himselfe also, as if hee had professed severall things exstant in the first volume of his elements out of your writings”.

Wallis's suggestion that he had set out deliberately to suppress the contributions of other, non-French authors when he had originally written the work:

"One might not be surprised that I do not undertake here to deliver a history of algebra and of analysis. It appears to me to be without any legitimate foundation that Mr Wallis attempts to quibble and start a dispute with me over this, when he suggests in his grand publication on algebra historical and practical that my first *Elemens des mathematiques*, which he attributes to a more skilful man than me, is a collection of all or most authors on this topic; but where the reader is not interrupted by the account of these diverse authors, of which he supposes I make use, save two of my own nation, to use his expression, which are Messrs Descartes and Viète"⁵⁴.

It would be possible to consign this dispute to the list of all the other innumerable skirmishes and battles which characterized a large part of the intellectual exchanges in the Republic of Letters in the second half of the seventeenth century, were it not at the same time an excellent example of just how blind and misguided many early modern scientific disputes were⁵⁵. For all the rhetoric about the over emphasis of French mathematicians here or of the insupportable elevation to pre-eminence of Harriot, Oughtred, and Pell there, the ironic truth is that Wallis and Prestet had much in common: they held exceedingly similar views on the status of arithmetic and on the nature of algebra or analysis. Like Wallis, Prestet considers arithmetic or as he calls it "the science of numbers" to be a universal science on which many others depend. By employing letters for unknowns and coefficients it becomes, as he writes, increasingly more general and more extended, "algebra being nothing but a literal arithmetic, and analysis nothing but an application of algebra in order to solve the questions one proposes on all sorts of magnitudes"⁵⁶. Fundamentally, Prestet identified algebra and analysis just as the Savilian professor did, and he also considered it to be an

54 Jean Prestet: *Nouveaux elemens des mathematiques ou principes generaux de toutes les sciences*, second edition, 2 vols, Paris 1689, vol. II, sig. e1^r: "Au reste qu'on ne soit pas surpris, si je n'entreprends point de faire ici une histoire de l'Algèbre & de l'Analyse. C'est, ce me semble, sans aucun fondement légitime que Monsieur Wallis prétend me chicaner & m'intenter un procès là-dessus, lors qu'il avance dans son grand ouvrage de l'*Algèbre historique & pratique*, que mes premiers *Elemens des Mathématiques*, qu'il attribue à une personne plus habile que moy, sont un recueil de tous ou de la plupart des Ecrivains de ce genre, mais où le Lecteur n'est point interrompu par le récit des noms de ces divers Auteurs, dont il suppose que je me suis servi, excepté de deux de ma nation, pour user de ses termes, qui sont Messieurs Descartes & Viète".

55 For a typology of such disputes, focusing on those involving Wallis, see the discussion in Philip Beeley and Christoph J. Scriba: "Disputed Glory. John Wallis and Some Questions of Precedence in Seventeenth-Century Mathematics", in: *Kosmos und Zahl. Beiträge zur Mathematik- und Astronomiegeschichte, zu Alexander von Humboldt und Leibniz*, ed. by H. Hecht, R. Mikosch et al., Stuttgart 2008, pp. 275-299.

56 Prestet: *Nouveaux elemens des mathematiques*, second edition (see note 54), vol. I, sig. o3^r: "[...] l'Algèbre n'étant autre chose qu'une Arithmétique littérale, & l'Analyse qu'une application de l'Algèbre pour résoudre les questions qu'on propose sur toutes sortes de grandeurs".

art of invention⁵⁷, for, as he almost exuberantly writes in the *Nouveaux elemens*, algebra gives to the mind “the means of discovering the most hidden truths”⁵⁸.

7. Wallis and the New Analysis

Wallis uses his *Treatise of Algebra* to continue to promote the arithmetical method of quadrature and cubature which he had first presented to the academic world some thirty years earlier. Developing the method of Cavalieri in the simplified form he had found it in Torricelli in the 1650s, Wallis had supposed a plane figure to be a collection of indivisible lines, a solid to be a collection of indivisible planes. But whereas Cavalieri and Torricelli had summed geometric indivisibles, Wallis had carried out these summations arithmetically: as sums of infinite sequences of what he conceived as being infinitely small parts or infinitesimal quantities⁵⁹. Looking back on his youthful work, he points out that whereas some in order to look more geometrical had affected lines and figures, he had chosen rather in his *Arithmetica infinitorum* “to demonstrate universally from the nature of Proportions, and regular Progressions; because such Arithmetical Demonstrations are more Abstract, and therefore more universally applicable to particular occasions”⁶⁰. He emphasizes this, too, in his letter to Henry Oldenburg of 20 September 1673, which was effectively a reply to an earlier letter received from the Flemish mathematician René François de Sluse. Answering a question concerning the centrobaric rule of Guldin, Wallis points out that

“the whole of my theory of infinitesimal arithmetic is directed to that end, which is here in view, so that the problems occurring in any topic whatever may be referred back to the principles of arithmetic which, depending upon the bare theory of ratios, abstracted from every material, at once demonstrate those things which may be adapted to countless topics. Which also comprises the whole of analysis and is for that reason most sincerely welcome to me”⁶¹.

In a similar vein, Wallis presents the method of induction which he employed liberally in *Arithmetica infinitorum* as an analytical art of invention. However, in many ways this was an analytical art intimately bound up with Wallis’s own abilities to recognize patterns; what was established to be regular could be

57 See for example Prestet: *Nouveaux elemens des mathematiques*, second edition (see note 54), vol. II, sig. a2^r. Prestet describes the second volume as comprising “l’Analyse, ou l’art d’inventer ce qu’on veut sur toute sorte de grandeurs”.

58 Ibid., vol. I, sig. i2^r: “C’est la facilité qu’elle donne à l’esprit pour découvrir les vérités les plus cachées, & dont il seroit absolument impossible de s’éclaircir par l’Arithmétique & par la Géométrie ordinaire, ni par le secours d’aucune autre science”.

59 See Wallis: *Arithmetica infinitorum*, p. 5; *Opera mathematica*, vol. I, p. 367.

60 Wallis: *Treatise of Algebra*, p. 292; *Opera mathematica*, vol. II, p. 318.

61 Wallis to Oldenburg, 20 September 1673, *Correspondence of John Wallis (1616-1703)* (see note 39), vol. IV, Oxford 2014, p. 260: “Atque eo quidem collimat tota mea de Arithmetica Infinitorum doctrina (quo et haec spectant,) ut quae in quacunque materia occurrunt quaesita, ad Arithmetica principia revocentur; quae nudam *Rationum* doctrinam (ab omni materia abstractam) perpendens, simul et semel ea demonstrat, quae subjectis innumeris accommodentur. Quod et facit Analytica tota; eoque mihi nomine potissimum grata est”.

reasonably assumed to be repeated indefinitely – even if the rule could not be stated⁶². Arguing that such a tool had already been used by Euclid and more recently by Briggs and Viète⁶³, he considered it to be perfectly sufficient to the task of effecting the exact determination of quadratures and cubatures. Precisely for this reason he saw no need to embrace the infinitesimal methods of either Newton or Leibniz, and he remained firmly wedded to the conception of analysis which he had developed already at the beginning of his mathematical career and which he had expounded in most detail in his *Treatise of Algebra*. It is perhaps therefore not surprising that the new terminology of analysis, its identification with the algebra of infinite series in Newton and with infinitesimal calculus in Leibniz had practically no impact whatsoever on Wallis's writings – apart from the late introduction of the term “pars infinitesima”⁶⁴ as an equivalent term to his earlier “pars infinite parva”⁶⁵.

Leibniz repeatedly sought to convince the Savilian professor of the qualitative leap in procedures in quadratures represented by his infinitesimal calculus. In his anonymous review of the first two volumes of Wallis's *Opera mathematica*, which appeared in the *Acta eruditorum*, Leibniz summarizes the approach used by Wallis in the *Arithmetica infinitorum* and compares it to the work of Fermat, Roberval, and Pascal, before arriving at a rather damning conclusion:

“From this it appears that the arithmetic of infinities in the sense of Wallis means something quite different from the analysis of infinities or from differential calculus, which stands in relation to that as specious analysis stands to arithmetic”⁶⁶.

In the correspondence between Leibniz and Wallis in the 1690s the methods of the two men are constantly played out against each other. The Savilian professor argues for the sufficiency of his arithmetic of infinities, while at the same time questioning the validity of some of the conclusions which Leibniz draws from working with his infinitesimal quantities. Wallis also takes the opportunity to describe the approach he originally set out in *Arithmetica infinitorum*, based on induction, as a “new method of investigation” which effectively enabled the

62 Wallis: *Treatise of Algebra*, p. 298: “Those Propositions of my Arithmetick of Infinities, are (some of them) demonstrated by way of Induction: Which is plain, obvious, and easy; and where things proceed in a clear regular Order [...] very satisfactory”; *Opera mathematica*, vol. II, p. 323.

63 See Wallis to Huygens, 12 August 1656, *Correspondence of John Wallis (1616-1703)* (see note 39), vol. I, pp. 193-198, pp. 194-195; Wallis to Digby, 21 November 1657, *Correspondence of John Wallis (1616-1703)* (see note 39), vol. I, pp. 320-342, p. 331;

64 Wallis: *Opera mathematica*, vol. I, p. 367: “pars infinitesima, seu infinite parva”. See Philip Beeley: “Infinity, Infinitesimals, and the Reform of Cavalieri: John Wallis and His Critics”, in: *Infinitesimal Differences: Controversies between Leibniz and his Contemporaries*, ed. by U. Goldenbaum and D. Jesseph, Berlin 2008, pp. 31-52, p. 41.

65 Wallis: *Arithmetica infinitorum*, p. 5: “pars infinite parva”.

66 Leibniz, review of Wallis's *Opera mathematica*, vols. I and II, in: *Acta eruditorum*, June 1696, pp. 249-259, p. 252: “Ex his patet, Arithmetica infinitorum sensu Wallisii longe diversum significare ab Analysis infinitorum, seu calculo differentiali, qui ita se habet ad illam, ut Analysis speciosa ad Arithmetica”.

apagogical proofs of the ancients to be circumvented⁶⁷. Leibniz for his part sets out the reasons why in his opinion the procedures recently developed by him and Newton cannot be compared with those of Wallis, James Gregory, and Mercator⁶⁸.

The motives of Wallis's conservatism are complex and can only be touched on here. In part, he almost stubbornly refused to acknowledge that the modernization he had contributed to bringing about in mid-century had now been itself eclipsed by the work of younger scholars. In part, he was genuinely concerned that rapid developments in mathematical techniques had led to a certain degree of terminological confusion. Here not only the mathematician but also the grammarian and logician are speaking. Questions on the precise use of terminology abound in Wallis's writings on grammar and logic, some of which go back to his student days in Cambridge in the 1630s. In an easily misunderstood and nevertheless partly tongue-in-cheek passage he writes on the supposed equivalence of Newton's and Leibniz's methods in his letter to the German mathematician and philosopher of 6 April 1697:

"Now what I would call continuous approximations, James Gregory calls convergent series, and Newton calls infinite series; but they are the same thing. Similarly, what I would call centre of percussion, Huygens calls (by the new name) centre of oscillation; but they are the same thing. And Fermat's infinite hyperbolae are the same as my reciprocal series. And Galileo's cycloid, Mersenne's trochoid, my cyclois, & Cusanus's curve (by whatever name they are called) are the same thing. Likewise, Neile's rectified curve, & the curve of Heuraet, & finally the curve of Fermat are the same as my semi-cubic parabola. [...] And, if I am not mistaken (it is at least so described to me,) Newton's doctrine of fluxions is the same (or at least very similar) to what by you is called differential calculus"⁶⁹.

67 See Wallis to Leibniz, 1 December 1696, A III, 7, 204-213: "Quod notat de meo per Inductionem processu (quod quadantenus verum est,) de hoc abunde dictum est *Algebrae* Cap. 78 et 79. Sed et recordandum erat, me non tam methodum Demonstrandi tum docere, quam methodum Investigandi, (et quidem novam et minime contemnendam, quod ne quidem adversarii negare poterunt;) cui methodus Inductionem apprime convenit. Quod si, ubi haec ego rite investigaverim, velint alii (demonstrationibus Apagogicis) porro confirmare; per me licet" (A III, 7, 210).

68 See Leibniz to Wallis, [19] March 1697, A III, 7, 348-354, 349-350 (Leibniz quotes his anonymous review of the first two volumes of Wallis's *Opera mathematica* in the *Acta eruditorum*); Leibniz to Wallis, 28 May 1697, A III, 7, 427-434, 428-430. See also A. Rupert Hall: "Leibniz and the British Mathematicians 1673-1676", in: *Leibniz à Paris (1672-1676)*, ed. by A. Heinekamp et al., 2 vols., Wiesbaden 1978, vol. II, pp. 131-152, especially pp. 136-139.

69 Wallis to Leibniz, 6 April 1697, A III, 7, 370-376, 374: "Nam quas ego vocaveram *Continuas Approximationes*, vocat Jacobus Gregorius *series convergentes*, et Newtonus *series Infinitas*: sed res eadem est. Sic, quod ego vocaveram *Centrum Percussionis*, vocat Hugenius (novo nomine) *Centrum Oscillationis*; sed eadem res est. Et Fermatii *Hyperbolae Infinitae* eadem sunt cum meis *Seriebus Reciprocis*. Et Galilaei *Cycloides*, Mersenni *Trochoides*, mea *Cyclois*, et Cusani Curva (quocunque nomine dicatur) sunt res eadem. Sic rectificata Curva *Nelii*, et curva *Heuratii*, et tandem Curva *Fermatii*, sunt eadem curva cum mea *Paraboloide Semicubicali*. Et Gallorum *Socia Cycloidis*, est eadem curva quae (mihi) terminat Figuram *Sinum Rectorum*. Et ni fallor (sic saltem mihi nunciatum est) Newtoni *Doctrina Fluxionum*, res eadem (vel quam simillima) quae vobis dicitur *Calculus Differentialis*".

Leibniz's response was first to distinguish between two kinds of tetragonistic methods, both of which could be traced back to Archimedes, and then to distinguish these from recent infinitesimal techniques. Of the older established methods one, he writes, considers geometrical figures and bodies to be collections of an infinite number of quantities each of which is incomparably smaller than the whole, while in the other quantities remain comparable to the whole and are taken successively in infinite number so as eventually to exhaust that whole⁷⁰. In precisely identifying and describing these two approaches, the former of which corresponded to that employed by the Savilian professor, Leibniz sought to accommodate Wallis's wish for distinctions to be recognized amongst older approaches to quadratures and cubatures, while leaving no doubt about the major difference between them on the one side and Newton's and his analyses of the infinite on the other side⁷¹. Effectively, the concept of analysis with the infinite as its object became the distinguishing factor between the old and new. The method of fluxions of "the most profound Newton", he writes to Wallis on 28 May 1697,

"[...] is related to my differential method, as I have recognized not only after his work and your work have appeared. I have also stated this in the *Acta eruditorum* and elsewhere. I have judged this to correspond not only to my own sincerity, but also to its merit. I am therefore accustomed to designate them commonly as analyses of the infinite; which are broader than tetragonistic methods"⁷².

8. Conclusion

Looking from our perspective now, Wallis appears almost as a victim of his own longevity. By the time the *Treatise of Algebra* had appeared in 1685, it had become a record of his own mathematical developments since the late 1640s. This is perhaps the only sense in which this work can be understood as a history, certainly in the way that we understand this term today – although Newton

70 Leibniz to Wallis, 28 May 1697, A III, 7, 427-434, 428: "Dixi aliquando in Lipsiensibus *eruditorum Actis* mihi omnes methodos Tetragonisticas ad duo summa genera reducendas videri, vel enim colliguntur in unum quantitates infinitae numero, quantitate incomparabiliter minores toto vel semper manetur in quantitatibus toti comparabilibus, quarum tamen numerus infinitus quando totum *exhauriunt*".

71 See Beeley: "Infinity, Infinitesimals, and the Reform of Cavalieri" (see note 64), pp. 44-51.

72 Leibniz to Wallis, 28 May 1697, A III, 7, 427-34, 429: "*Methodum Fluxionum* profundissimi Neutoni cognatam esse methodo meae differentiali non tantum animadverti postquam opus ejus et tuum prodiit, sed etiam professus sum in *Actis eruditorum*, et alios quoque monui; id enim candori meo convenire judicavi, non minus quam ipsius merito. Itaque communi nomine designare soleo, Analyseos Infinitesimalis, quae latius quam methodus tetragonistica patet". See Joseph E. Hofmann: *Leibniz in Paris 1672-1676. His Growth to Mathematical Maturity*, Cambridge 1974, p. 124: "There was the further unfortunate circumstance that by the word Analysis, Newton meant his theory of series, Leibniz his Calculus – and so a virtually inextricable tangle of misunderstanding was created".

and others at the time did refer to it as a “history of algebra”⁷³. At the end of the book, Wallis returns to the promise he had given many years earlier in his *Opus arithmeticum*. It was, he suggests, above all in order to fulfill that obligation that now, after many diversions to other studies, he had finally completed his work on analysis, the *Treatise of Algebra*. As he explains, his aim was to give a brief account of algebra, “from what Principles, and by what steps it hath made its Progress; and to what pass it is arrived at this day”⁷⁴. This was an aim he could justifiably claim to have met. But Wallis also had to make the kind of concession, which many authors make, who wait too long to publish: “Much of what I might then have said, hath been since said by others”⁷⁵. Importantly, this comment did not extend to the concept of analysis itself. As we have seen, Wallis near the end of the seventeenth century felt no need to accommodate the changed understanding of analysis in his own work or to modify the concept of analysis which he had already formulated thirty years earlier. When it came to analysis, it was just not a question of what he might have said earlier. Quite simply, new parameters had been established by others, and in consequence Wallis’s own ideas on the topic had been eclipsed.

Dr. Philip Beeley, Faculty of History, University of Oxford, George Street, Oxford OX1 2RL, United Kingdom, philip.beeley@history.ox.ac.uk

73 See Newton to Leibniz, 16/26 October 1693, A III, 5, 655–658, where Leibniz is informed of the inclusion of letters sent to him via Oldenburg (the so-called *epistola prior* and *epistola posterior*) in the forthcoming second (Latin) edition of Wallis’s “history of algebra”: “idque maxime cum Wallisius noster Historiam Algebrae in lucem denuo missurus nova aliqua e literis inseruit quas olim per manus Dⁿⁱ Oldenburgi ad te conscripsi” (A III, 5, 655). Newton’s description of the book reflects above all the enduring power of the Aristotelian tradition in history. See Beeley, “The Progress of Mathematick Learning” (see note 5), pp. 20–22.

74 Wallis: *Treatise of Algebra*, p. 374.

75 Ibid.