Can goal-free problems be used to support students in solving goal-specific problems in the secondary mathematics classroom?

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Abstract

This research project explores the extent to which goal-free problems can be used to support Year 10 students in solving goal-specific problems in the domain of circle theorems. The study compares the performance of an intervention group working on goal-free problems (n = 51) with a control group (n = 46) on a variety of goal-specific problems at the end of a sequence of ten lessons. Issues within the affective domain, such as student confidence and enjoyment of lessons, are also compared through the use of questionnaires.

Findings from the assessment demonstrate that the intervention had a positive impact on students' performance on multi-application circle theorem problems. However, no statistically significant difference was found between the control group and the intervention group on non-routine, multi-domain goal-specific problems. Analysis of questionnaire responses found that the intervention group found the sequence of lessons to be less challenging than the control group. However, analysis also highlighted several negative perceptions held by students towards goal-free problems which educators should be aware of if they intend to use goal-free problems in the classroom.

Introduction

The importance of problem-solving in mathematics has been globally recognised for hundreds of years and, more recently, problem-solving has become a core component of mathematics education across the world. In the US, The National Council of Teachers in Mathematics (NCTM) have championed problem-solving for several decades (NCTM, 1980; 2010). Problem-solving has played some part in every conference organised by The International Congress on Mathematical Education (ICME) since 1969 (Kaiser, 2016). On an inter-disciplinary level, since 2000 problem-solving capability has been tested by the Programme for International Student Assessment (PISA) (Dossey, 2015).

In the UK, problem-solving has been similarly emphasised over the last half Century. The Cockcroft Report, published in 1982, asserts that 'the ability to solve problems is at the heart of mathematics' (Cockcroft, 1982). Recommendations laid out in the Cockcroft Report led to the development of the National Curriculum (Brown, 2014), which continues to emphasise problem-solving as one of the three main objectives of mathematics teaching at Key Stage 4 (KS4) (DfE, 2014).

In the last decade, partly influenced by global comparisons produced by PISA (Coughlan, 2003), UK governmental policy has further intensified the focus on problem-solving in schools. In its most recent guidance, the examinations regulator OFQUAL (2015) now stipulates that 30% of marks awarded on all higher tier papers and 25% of marks on all foundation tier papers must be designated to problem-solving (previously 15-25% of marks were allocated to problem-solving). Reforms also mean that there is an emphasis on "translating problems into mathematical processes, linking different areas of mathematics and evaluating solutions" where previously students were only required to "interpret and analyse problems" (AQA, 2014).

I first became aware of the difficulties posed by the new problem-solving focus at GCSE whilst working with a high attaining Year 11 class in the year that the reformed examinations were introduced. Analysis of their mock exams highlighted that these students were struggling to access the problem-solving questions on the specimen papers provided by the exam boards. Conversing with these pupils, it also became clear that many students were losing mathematical confidence due to their inability to independently solve the challenging problem-solving questions. Conversations with maths teachers at other schools in the local area have highlighted that these issues are not isolated to students at my school.

Over the last few years, in my role as Head of KS4 (Years 9-11) Mathematics, I have been determined to find ways to better support students with mathematical problem-solving. Initially, I undertook an Action Research Project, in partnership with the Oxford Deanery, with a focus on producing metacognitive prompts to support students with problem-solving (Author,

2018). These prompts, based on the heuristic strategies developed by Polya (1990), were developed over a period of several months in collaboration with my Year 11 students and shared with other members of my department. However, it soon became evident from conversations and observations that many of my colleagues were reluctant to encourage problem-solving in many of their lessons.

In order to better understand this issue, I undertook some practitioner research which looked at various aspects of the espoused mathematical problem-solving beliefs of 17 teachers working in my department (Author, unpublished). This research found that the majority of teachers claimed to use problem-solving as a way of stretching their highest attainers, often at the end of a teaching unit. Very few spoke of problem-solving as a fundamental, integrated part of their mathematics lessons. Follow-up interviews indicated that reluctance to encourage problem-solving was partly a result of time constraints on staff; teachers did not feel there was adequate time to cover the necessary course content and develop students' problem-solving skills. In order to support staff in promoting problem-solving in their lessons, it is important that I find solutions which address these concerns.

Goal-Free Problems

Goal-free problems provide students with a mathematical scenario and encourage them to work out everything they can from the information given. This can be contrasted with goal-specific problems which require students to work towards a specific end-point or unknown. Goal-free problems have recently gained momentum in professional mathematics education literature, appearing in popular books (Barton, 2018) and inspiring blog posts full of ready-to-use examples (Mattock, 2018; Banks, 2018). However, the original idea has its roots in the work of Sweller (1988), who advocated the use of goal-free problems thirty years ago as a consequence of his work on cognitive load theory.

I first recognised the potential of goal-free problems when I attended a BBO MathsHub conference and watched a presentation by White-Zamler (2018), which focused on supporting students with problem-solving at KS4. The speaker spoke positively about her experiences in using goal-free problems with both low and high attaining students. Importantly, she spoke of how this strategy could be used to support understanding of mathematical ideas as well as being an effective strategy to allow all students to engage in problem-solving. This would suggest that goal-free problems could provide an effective way for all teachers in my department to integrate problem-solving into their lessons; simultaneously covering course content and developing students' ability to solve goal-specific problems.

Initial questions:

In order to determine whether goal-free problems are an effective strategy to use in the classroom, it is important to obtain an understanding of the empirical findings related to goal-free problems and the theoretical explanations for these findings. The following three initial questions (IQs) have been formulated to focus my search of the research literature in light of the issues raised above:

IQ1: What impact has goal-free problem-solving been found to have on goal-specific problem-solving in empirical studies?

IQ2: What scientific theories can be used to explain the results of empirical studies into goal-free problem-solving?

IQ3: What are the limitations of existing goal-free problem-solving research and theory?

Literature Review

This literature review begins by exploring the defining principles of human cognitive architecture and other key concepts which are fundamental to the understanding of many of the theories related to goal-free problems. The original justification for the introduction of goal-free problems is then examined before laying out the range of empirical findings related to goal-free problem-solving. It is then considered how different theoretical perspectives can be used to explain these empirical results. Finally, gaps in the literature are identified, as are suggestions for further research.

Human Cognitive Architecture

The mechanics of human thought have been considered for hundreds of years. In the 17th Century, Locke (2001, p.112) distinguished between the concept of contemplation as keeping an idea "actually in view" and the concept of memory as "the power to revive again in our minds those ideas which, after imprinting, have disappeared, or have been as it were laid aside out of sight." This distinction has developed into our modern understanding of working memory and long-term memory which are now considered to be the key components of human cognitive architecture (Cowan, 2014; Sweller et al., 2019). The following sections look at working memory and long-term memory in more detail and explores the connections between the two.

Working Memory

Our working memory (WM) can be thought of as the home of conscious thought (Sweller et. al., 1998; Kirschner, 2002; Cowan, 2014). It is in the WM where information, which could be abstract (e.g. ideas) or concrete (e.g. objects to be enumerated), is consciously processed (de Jong, 2009).

WM is also defined by two key features: it is limited in terms of capacity (how many items can be considered at once) and duration (how long an item can remain in WM without being refreshed) (Cowan, 2014).

Whilst there is wide agreement that WM capacity is limited, there is some uncertainty as to the nature of this limit. Early research into the capacity of WM found that humans are capable of holding around seven chunks of information at any given time (Miller, 1956). However, Cowan (2014) notes that more recent studies, designed to prevent subjects using memory strategies such as chunking information and mentally rehearsing, put the actual capacity of WM closer to four or five chunks of information. Further to this, it must be noted that comparing, contrasting and organising chunks of information also uses WM capacity. This means that, in reality, only two or three elements can be processed at any one time (Sweller et al., 1998; Kirschner, 2002).

The durational limit of WM is more widely disputed in the literature (Cowan, 2014). Some studies have found that the maximum limit for novel information to remain in WM is 30 seconds without being refreshed (Paas & Ayres, 2014). However, it is also suggested that novel information in WM can be lost in a matter of seconds if attention is diverted to other demanding tasks (Ericsson & Kintsch, 1995). Cowan (2014) highlights that the range of empirical findings suggest decay of information in WM is a complicated function of the amount of information to be processed, where attention is focused, and initial consolidation of the information.

Long Term Memory and Schema

In contrast to WM, our Long-Term Memory (LTM) is believed to be effectively limitless in capacity; it acts as a vast store of our past knowledge (Kirschner, 2002). Whilst the exact mechanics of LTM are debated, most psychologists describe how information is stored in our LTM through the concept of schema (Sweller et al., 1998; Cowan, 2014). The American Psychological Association (APA) defines a schema as “a cognitive structure representing a

person's knowledge about some entity or situation, including its qualities and the relationships between these" ('Schema', n.d.). In the mathematics education literature, the analogous notion of a concept image is also used to describe "the total cognitive structure that is associated with [a] concept, which includes all the mental pictures and associated properties and processes" (Tall & Vinner, 1981, p.152).

It is possible to frame learning in terms of the construction of schema. Skemp (1971) argues that new information is learnt only if it is assimilated or accommodated into the existing schema within our LTM. Assimilation describes the process whereby a new idea fits into existing schema and accommodation describes the process whereby schema must be modified in order to incorporate a new idea. In this view of schematic learning, it is beneficial to develop complex schemas with many connections between ideas as this will give us a better chance of assimilating future information (Sweller et al., 1998).

WM and LTM are dependent on each other and so are both vital to human cognition. Humans are not directly conscious of information stored in our LTM; we only become aware of this past information when it is filtered through our WM (Cowan, 2014). When retrieving information in our LTM, WM processes the relevant schema in our LTM and, within this activated portion, focuses attention on a smaller subset of items. This activated portion is analogous to Tall and Vinner's (1981) notion of an evoked concept image. It is believed that schema allow us to process a vast array of complex information as a single element in WM and therefore help us to avoid overloading the limited capacity of our WM (Paas et al., 2003; Schnotz & Kurschner, 2007). This idea is captured in Gray and Tall's (1994) notion of proceptual thinking, which is characterised by "the ability to compress stages in symbol manipulation to the point where symbols are viewed as objects that can be decomposed and recomposed in flexible ways" (p.132). Equally, our LTM is crucial for helping us make sense of information in our WM (Kirschner, 2002); we can only understand new information by relating it to our current schema.

Goal-specific problems and related problem-solving strategies

Sweller and Levine (1982, p.463) describe transformation problems as categories of tasks which are characterised by "an initial state, a goal state, and a set of operators to transform the initial state into the goal state". An example of a legal operator which could be applied to a transformation problem would be an equation which can be applied to the known variables in the initial state to calculate a new piece of information (Paas & Kirschner, 2012). Sweller (1988) argues that the vast majority of questions in mathematics can be considered as transformation problems. In this essay, the term goal-specific problem is used to refer to any mathematical task which could be viewed as a transformation problem.

When working on goal-specific problems, there are two common strategies used by problem-solvers: means-ends analysis and forward working. The APA defines means-ends analysis to be "any problem-solving strategy that assesses the difference between the current state and a desired end state and attempts to discover means to reduce that difference" ('means-ends analysis', n.d.). In contrast, the APA defines forward working to be any problem-solving strategy that "proceeds from the initial conditions to find a path to the desired goal state" ('working forward', n.d.).

Simon and Simon (1978) conducted a case study of two students completing kinematics problems. One student had ample experience in solving kinematics problems previously and was considered an 'expert' whilst the other was new to the topic and hence considered a 'novice'. They concluded that the expert problem-solver tended to use forward working strategies for problem-solving whilst the novice was more likely to use means-ends analysis. A further study conducted by Larkin et al. (1980), comparing eleven 'novice' undergraduates against eleven 'expert' graduates and professors answering a series of dynamics problems, also found means-ends analysis to be the predominant strategy used by the novices.

Owen and Sweller (1985) explain this difference by considering the schema possessed by problem solvers. Expert problem-solvers, within a certain domain, have the ability to categorise a problem and recognise the appropriate actions to move forward from that problem state. This is highlighted in the work of Goldenberg and Mason (2008) who suggest that, through a variety of past experiences, experts in a particular domain have developed a rich example space, which can be drawn upon to recognise the mathematically relevant features of particular problems. This is reflected in numerous video studies of problem-solving behaviour summarised by Schoenfeld (2016), who has found that expert problem-solvers usually dedicate time a significant amount of time to fully understanding a problem before exploring solution paths based on their prior experiences. Novice problem-solvers, who do not have the necessary schema to classify a problem, are only likely to recognise surface features of a problem. Hence novices have to regularly consider whether they are moving closer to the goal state in order to get feedback on their progress with the problem.

When lacking the necessary prior experience of working with problems within a domain, means-ends analysis provides a useful method for novices to approach novel problems (Sweller, 1998). However, numerous authors have criticised means-ends analysis as an ineffective method for solving many problems. Sweller and Levine (1982, p.464) argue that it is a flawed control mechanism because, in most problems, the goal does not provide "unambiguous information concerning the accuracy of individual moves". When working with many goal-specific problems, it is not possible to judge whether a particular operator will bring you closer to the desired goal without previous experience of working with similar problems.

Empirical results related to goal-free problems

In the early 1980s, researchers were considering ways in which they could encourage novice problem solvers to use forward working methods to solve mathematical problems (Sweller & Levine, 1982; Sweller et al., 1983; Owen & Sweller, 1985). Recognising that the goal state of a

problem plays a vital role in the process of means-ends analysis, Sweller and Levine (1982) focused their attention on problems in which the goal state is either not given or not clearly specified. An example of a situation in which the goal is not clearly specified could be a maze problem where the goal is to find the exit, but no useful information is given about the location of this goal. In this essay, as with the majority of the research literature, the term goal-free problem is used to refer to problems in which the goal state is not clearly specified or is omitted entirely.

Goal-free problems were first introduced because their very nature prevents solvers from using means-ends analysis when working on them (Sweller and Levine, 1982; Sweller et al., 1983). For any current problem state, goal-free problems do not have a final goal state for the solver to attempt to minimise the differences between. Sweller and Levine (1982) postulated that, in the absence of a clearly specified goal state, it is more likely that the solver will be influenced by other, more important aspects of the problem structure (Sweller and Levine, 1982). This is because goal-free problems force the solver to consider the current state of the problem and decide which problem operators can be applied to this particular configuration (Paas & Kirschner, 2012).

The first study to explicitly explore effects of goal-free problems was conducted by Sweller and Levine (1982) over a series of similar experiments. The first three experiments involved 20-24 psychology students solving a maze with an underlying pattern to the solution path whilst blindfolded or, in experiment three, on a computer simulating these conditions. An example of the type of maze that students were expected to solve is given in figure 1. In each experiment, half of the students knew the location of the finish position (the goal state) whilst the other half were not told the location of the goal state. A record was kept of errors made by the participants as they attempted to solve the maze.

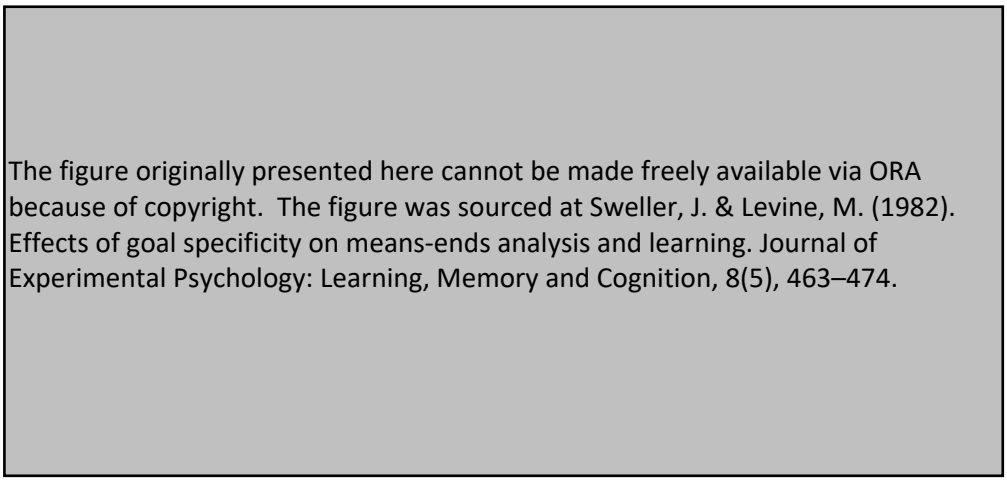


Figure 1 – Example of a maze to be solved by participants
in a study conducted by Sweller and Levine (1982)

Sweller and Levine (1982) found that in each iteration of the experiment, the group who were aware of the goal location made more errors and claimed that this demonstrated that the forward working goal-free group were better able to discern the underlying pattern of the maze. However, there are two reasons to be cautious of these results. Firstly, the solution of the maze was set up in a counterintuitive way that meant it was misleading to know the location of the goal state; participants had to move away from the goal state for much of the journey through the maze in order to follow the correct solution path. Secondly, when participants from both groups were given a second run through the maze, no discernible difference was found between the number of errors made by the two groups. This suggests that successful participants were able to learn the underlying pattern of the maze after a single run through, regardless of the goal-specificity.

Sweller and Levine (1982) ran a final experiment which addresses the criticism that the original mazes were set up to mislead goal-specific participants. In this iteration, a less regular maze (with a solution path which did not consistently move away from the goal location) was given for participants to solve on a computer screen with arrows indicating the viable moves at any position in the maze. The rule that could be induced was that in each case, a solid arrow pointed in the direction of the correct solution path compared with a hollow arrow which pointed to the potential paths leading to dead ends. Of the 20 participants in this experiment, seven goal-free

and two goal-specific participants showed evidence of having induced the rule by completing the maze without making any errors on the second attempt. This final experiment provides more compelling evidence that solving goal-free problems can support rule induction when problem-solving. Further evidence that goal-free problem-solving supports rule induction has also been found in similar maze studies (Paas et al., 2001), studies of problem-solving puzzles (Zanga et al., 2004), and studies of students conducting experiments in Biology (Vollmeyer et al., 1996), Physics (Wirth et al., 2009), and Statistics (Trumpower et al., 2004).

Building on the findings of Sweller and Levine, another set of six experiments were conducted to explore the effects of goal-free problems on novice Australian high school students across two mathematical domains: kinematics and angle geometry (Sweller et al., 1983). Each experiment used between 20 and 34 students with half of the participants allocated to a goal-specific group and half allocated to a goal-free group. The goal-free problems were set up so that students had to find the exact same unknowns regardless of the goal-specificity of the problems; participants could not calculate irrelevant variables or angles. This has the benefit of allowing for a direct comparison of the progress made by the goal-free and goal-specific participants but does not necessarily reflect the majority of problem-solving situations which students may face in the mathematics classroom. This concern is noted by Ayres (1993) who cautions that using a goal-free approach on problem states with a vast number of calculable unknowns may lead to many fruitless calculations.

Sweller, Mawer and Ward (1983) found that, after an initial practice period, the group who had trained on goal-free problems were, on average, faster to solve future goal-specific problems and used fewer equations in order to reach the end goal. They argue that, because these students were better at identifying which equation was needed in different situations, the goal-free group were better able to categorise kinematics and geometry problems according to which equations needed to be applied and in which order. They also suggest that the speed with which these students were identifying the correct equations indicates that the goal-free group were faster to switch from means-ends analysis to a forward working strategy.

Owen and Sweller (1985) conducted a further three experiments with between 20 and 22 high school students in Australia working on trigonometry problems. The students initially underwent an instruction period in which students were given six worked examples which demonstrated how to use the trigonometric ratios to find missing sides of right-angled triangles. Students were then randomly split into two equal groups and underwent a 30-minute acquisition phase with one group practising goal-specific problems and the other practising goal-free problems. This was then followed with a test which consisted of goal-specific questions.

In each iteration of the experiment, Owen and Sweller (1985) found that the goal-free group were more successful in answering follow-up problems requiring multiple applications of trigonometric ratios and an unseen problem which asked participants to draw a triangle based on the values of two trigonometric ratios. They also found that the goal-specific group made significantly more errors in both the acquisition phase and the follow up assessment. This is despite the fact that the goal-free students calculated a greater number of sides on average. Similar results have also been found in studies of two-step Pythagoras problems (Ayres, 1998).

These findings were further explored in a series of experiments conducted by Ayres and Sweller (1990) on Australian High School students with various population sizes. It was found that students working on multi-step goal-specific geometry problems were more likely to make errors when calculating a subgoal than when calculating the final problem goal; a phenomenon which the authors refer to as the 'stage effect'. These findings were replicated by Ayres (1993) in a study of 67 Australian high school students, which found that goal-specific students made more errors than goal-free students on two-step trigonometry problems and that the majority of these errors were made at the subgoal stage.

Whilst some of the sample sizes of experiments in the studies outlined above are relatively small, the fact that these findings have been duplicated across multiple experiments with a variety of populations and mathematical domains makes a more compelling case for the benefits of goal-

free problems (Sweller et al., 2019). Many papers use the term 'goal-free effect' to describe the positive results exhibited in studies of goal-free problems (Sweller et al., 2019). Paas and Kirschner (2012, p.1375) summarise the goal-free effect as the finding that goal-free problem-solving "leads to better learning than practicing by solving problems with a specific goal". However, this summary is unspecific about what is meant by 'better learning' and loses the nuance of the different results that have been found in empirical studies. As such, this essay will not refer to the goal-free effect in general terms but instead refer to specific key findings which are relevant to RQ1, outlined in table 2.

Original study	**Summary of key findings**
Sweller and Levine (1982)	Goal-free problems may be more suitable than goal-specific problems at supporting rule induction during problem-solving.
Sweller, Mawer, and Ward (1983)	Goal-free problems may be more suitable than goal-specific problems at supporting students to categorise problems based on the strategies needed to solve them. Goal-free problems may be more suitable than goal-specific problems at encouraging a shift from means-ends analysis to forward working on goal-specific problems.
Owen and Sweller (1985)	Goal-free problems may be more suitable than goal-specific problems for improving student performance on future goal-specific problems. Goal-free problems may lead to fewer errors than equivalent goal-specific problems.

Table 2: A summary of the key findings
from goal-free problem studies.

Attentional Focus and Goal-Free Problems

Early studies explained some of the positive effects of goal-free problems using the theory of attentional focus (Sweller and Levine, 1982; Sweller et al., 1983). The APA define attentional focus as "the focus of an individual's attention at a particular moment" ("Attentional Focus", n.d.). The mathematics education literature also emphasises the importance of directing students' attention as a means of educating awareness (Gattegno, 1987) through the discipline

of noticing (Mason, 2011). In this section, we explore how these concepts can be used to explain why goal-free problems may support goal-specific problem-solving.

Ingram (2014) notes that learning is dependent on our awareness and our awareness is directed by where our attention is focused. By shifting students' attention, we provide the opportunity for them to notice important things which can then further their understanding (Mason, 2011). In order for students to make connections and perceive relationships between mathematical ideas, it is therefore important to consider where students' attention is focused.

It is argued that goal-free problem-solving shifts students' attention towards the immediate effects of problem-solving operators on different problem states. When solving goal-specific problems using means-ends analysis, one must attend to differences between the current state and the goal state (Paas and Kirschner, 2012). As a result, attention is focused on distal relations between problem states and goal states. In contrast, when solving goal-free problems, attention is focused on how to get from the current problem state to the next attainable state (Trumpower et al., 2004). This helps students to recognise how to move forward when faced with similar problems in the future (Sweller et al., 1983; Paas and Kirschner, 2012).

This focusing of attention on problem states and operators gives a potential explanation for why Sweller et al. (1983) found evidence to suggest that goal-free learners were quicker to categorise problem types than their goal-specific peers. It also helps to explain why studies have shown that students who practise on goal-free problems are quicker to move to a forward working strategy when they are later given goal-specific problems. The ability to recognise problems as belonging to a particular category requiring a particular set of moves renders mean-ends analysis redundant (Sweller, 1988).

It is important, however, to note that the focus of student attention is not the only important factor in the development of student awareness. Mason (2011) also makes a distinction between different structures of attention: holding wholes; discerning important details;

recognising relationships among the discerned details; perceiving properties which could hold in other situations; and reasoning on the basis of these perceived properties. In order to categorise problems appropriately, it is crucial that students are perceiving important properties of problem states (Goldenberg & Mason, 2008). The use of goal-free problems does not guarantee that attention will be structured in this way. Therefore, in order to further support students in categorizing problems, it may also be important to encourage students to consider the range of possible variation within example spaces (Goldenberg and Mason, 2008).

Cognitive Load Theory

Cognitive Load Theory (CLT) has its roots in the 1980s where it developed as theory of instructional design which takes consideration of how the mind processes information. Sweller and colleagues (2019) note that the effects of goal-free problem-solving were the first applications of CLT, however, before looking in detail at this explanation, it is important to first understand some key constructs underlying the theory.

Whilst certain specifics of information storage and processing outlined earlier are debated, the general principle of human cognitive architecture consisting of a finite WM coupled with an effectively infinite LTM is widely accepted and uncontroversial in the literature (Cowan, 2014; Sweller et al., 2019). Building on this understanding of cognitive architecture, cognitive load theory (CLT) was founded on the following core principle: “limited working memory is one of the defining aspects of human cognitive architecture and… [so] the cognitive load imposed by instructional designs should be the pre-eminent consideration when determining design structures” (Sweller et al., 1998, p.262). Hence the initial focus of CLT was on generating and advocating for instructional designs which minimise the cognitive load imposed on learners.

Types of Cognitive Load

Implicit in the desire to minimise cognitive load is the recognition that not all the cognitive load imposed by a learning task is strictly necessary to achieve the learning goal. This is an important realisation that led theorists to make a distinction between three different types of cognitive load: intrinsic, extrinsic and germane.[1]

Intrinsic cognitive load (ICL) is used to refer to the cognitive load imposed as a result of the inherent complexity of the information being processed (Sweller et al., 1998). CLT proposes that this ICL is determined by the level of element interactivity inherent to a concept (Sweller, 1994; Sweller et al., 1998). A concept has low element interactivity if it consists of simple non-interacting elements which can be learnt serially and in isolation from one another. For example, learning a set of nouns in a foreign language would be a low element interactivity task as each noun could be learnt separately and hence this would impose a low ICL. A concept has high element interactivity if it is necessary to understand several interrelated elements simultaneously in order to grasp the overall concept. For example, learning the sentence structure and grammar of a foreign language would require a learner to balance multiple related concepts concurrently which thereby imposes a high ICL (Paas et al., 2003a). Much of mathematics has high element interactivity because there are many links between concepts and often many steps in a solution to a maths problem (Sweller et. al., 1998).

It is important to note that ICL is not just a function of the complexity of the concept but also the prior knowledge of the learner (Paas et al., 2003a). It is not enough to analyse the learning material in order to determine the element interactivity because what constitutes an element is dependent on the learner (Schnotz & Kurschner, 2007). Valcke (2002) explains this using schema theory: learners with relevant prior knowledge will have access to schema in their LTM which are readily available to be processed by their WM as a single element, thus reducing the

[1] The reader should refer Sweller (2010) for a more detailed description of the different types of cognitive load and de Jong (2009) for an exploration of some conceptual issues with these definitions.

ICL. When teaching new concepts, it is important to ensure students possess the necessary schema so that the ICL of the task is does not overload WM capacity (Sweller et al., 2019). This relates closely with Vygotsky's (1978) concept of the zone of proximal development (ZPD), which encourages engagement in tasks which will develop students' current knowledge but are also achievable with the support of a teacher.

It is also important to note that ICL is dependent on the focus of learning. Understanding the division of fractions, to a student with limited prior knowledge, could be considered a difficult concept with high-element interactivity which imposes a high ICL. It is possible to introduce a rule that students can learn which will reduce the cognitive load of answering divisional fraction questions (Sweller et al., 1998). However, if taught in isolation and without further explanation, such a rule would usually be at the expense of deeper understanding; the focus would shift from relational understanding to instrumental understanding (Skemp, 1976). The implication is that ICL can only be adjusted by changing the focus of learning or by changing the prior knowledge of the learner (Sweller et al., 2019).

Extraneous cognitive load (ECL) is described by Sweller et al. (1998) as cognitive load imposed by the format of the instruction rather than the intrinsic characteristics of the concept to be learnt. Sweller (2010) states that if element interactivity can be reduced without altering the learning focus then this additional element interactivity is causing ECL. This implies that ECL is also dependent on the focus of learning. For example, consider a circle theorem problem that is presented with a diagram and further information in separate text. If the learning goal is to understand which circle theorems can be applied to different problem states, then the cognitive load imposed by integrating the information from the text and diagram could be considered extraneous. However, if the learning goal is to be able to successfully answer exam-style questions then the cognitive load imposed by integrating the information from the text and diagram would be intrinsic. When the ICL of a concept is high, it may be necessary to reduce sources of ECL to ensure that WM is not overloaded (Paas et al., 2003a). This suggests that it is

possible for practitioners to reduce cognitive load by making decisions about which task elements are essential to the learning goals.

Sweller (2010) defines germane cognitive load (GCL) as "the working memory resources available to deal with the element interactivity associated with intrinsic cognitive load" (p.126). This formulation does not assume that GCL contributes itself to total cognitive load but instead "has a redistributive function from extraneous to intrinsic aspects of the task" (Sweller et al., 2019, p.264). It has been argued that GCL can be increased by asking questions which require learners to focus more attention on connections between different elements and make explicit the links to prior knowledge (Sweller et al., 1998). This focus on making links between new ideas, prior knowledge and other areas of mathematics is also championed by supporters of connectionist teaching in mathematics education (Askew et al., 1997).

Cognitive Load Theory and Goal-Free Problems

Sweller (1988) was the first to construct an explanation for why goal-free problems might impose a lower cognitive load than goal-specific problems. Students engaging in means-ends analysis are seeking for legal problem-solving operators (for example, equations) to reach subgoals where the difference between the current problem state and the goal state is reduced (Paas & Kirschner, 2012). Sweller (1988) argues that, whilst this method is relatively efficient for novice problem-solvers who have underdeveloped schemas in the problem domain, it also puts a lot of demand on the WM. This is because the problem-solver must constantly consider the current state, the goal state, the relations between the current state and goal state, the potential problem-solving operators and keep track of achieved subgoals in order to move towards the solution (Paas and Kirschner, 2011).

In the absence of a direct measure of cognitive load, Sweller used a computational model to demonstrate the relative information processing capacity required by both forward working and

means-ends analysis based on a similar model by Larkin et al. (1980). Flow diagrams describing the processes involved in forward working and means-ends analysis[2] (appendix 1a and 1b) clearly demonstrate that means-ends analysis requires more decisions to be made and more elements to be held simultaneously in WM in order to make these decisions (Sweller, 1988).

If we assume that increasing the number of elements held concurrently in WM increases instantaneous cognitive load, the cognitive load at a specific point in time, and the necessity to make more decisions increases the average cognitive load over the problem-solving episode, then this provides evidence to suggest that means-ends analysis is more taxing on WM than forward working. The model also suggests that instantaneous cognitive load during means-ends analysis is highest during the calculation of subgoals, which could explain findings that errors are more often made when calculating subgoals than when calculating the final goal (Ayres & Sweller, 1990; Ayres, 1993). It is important, however, to note that there are limitations to this computational model. For example, de Jong (2009) points out that, in reality, people use resources such as pen and paper to offload WM by making notes of knowns, unknowns, goals and subgoals. This theoretical model does not account for such realities and, as such, must be treated with caution.

In order to describe goal-free and goal-specific problem-solving in terms of the three different types of cognitive load, it is important to consider the intention of problem-solving in education. When asking students to solve problems, educators must decide if the intention is merely for students to successfully solve the problems or if the true intention is to gain insights into mathematical concepts and different problem structures. Here, it is assumed that the latter is more important.

If the learning intention is to recognise how to move forward from particular categories of problem state, then identifying similarities and differences between different problem states is

[2] For more details on this computational model, refer to Sweller (1988).

intrinsic to the development of problem-solving schema. In such situations, Sweller (1988, p.283) argues that "goal attainment and schema acquisition may be two largely unrelated and even incompatible processes". Goal-specific problems, particularly when solved via means-ends analysis, impose a high ECL because the requirement to consider relationships between problem states, subgoals and goal states is not usually necessary to gain a deeper insight into particular problem states. By removing the problem goal, this ECL is removed and more WM capacity becomes available for germane processes that make connections between intrinsic elements of the problem and previously encountered problems. This may explain why studies have found that students who practise on goal-free problems are quicker to categorise problems according to the required solution path (Sweller et al., 1983) and hence perform better on follow-up problems (Owen and Sweller, 1985).

Far from acting as competing theories, CLT and attentional focus can be used concurrently to describe the positive effects of goal-free problem-solving (Sweller, 1988). Sweller and colleagues state this explicitly in later research: "The combination of decreasing extraneous cognitive load and at the same time increasing germane cognitive load involves redirecting attention: learners' attention must be withdrawn from processes not relevant to learning and directed toward processes that are relevant to learning and, in particular, toward the construction and mindful abstraction of schemas" (1998, p.264).

Current Limitations of Goal-Free Problem Research

Many authors have noted that terms 'problem' and 'problem-solving' are not used consistently by mathematics teachers or in professional and academic literature (Schoenfeld, 2016; Dossey, 2017). As an example, Polya (1990) makes a distinction between 'routine' problems (tasks which can be solved by following a previous example or known procedure) and 'non-routine' problems (tasks for which the solver has no known strategy to immediately progress towards the solution). This is the same language that is used in the National Curriculum at KS4, which requires students

to be able to solve both routine and non-routine problems (DfE, 2014). However, Schoenfeld (2016, p.4) argues that the term 'problem' is used too broadly in these contexts and that any routine mathematical tasks should be referred to as exercises rather than problems. This opinion is shared by Dossey (2017, p.61) who describes a problem as a task where the individual working on the task "does not immediately know how to proceed or of the existence of an algorithm that will immediately move toward a solution".

These contrasting definitions can create issues when educators interpret advice shared in the literature differently to how the author intended (Foster & Inglis, 2017). In the professional literature, goal-free problems are often recommended as a strategy to support students in solving previously unseen problems that require connections to be made between different areas of mathematics (e.g. Barton, 2018). It is these types of non-routine problems that I am most interested in supporting students to answer. However, most empirical studies focus exclusively on the impact of goal-free problem-solving on routine problem-solving. It is essential that future research determines whether goal-free problem-solving can also support student performance on non-routine mathematical problems.

The majority of goal-free problem research, conducted in the 1980s and 1990s, demonstrated the positive effects of goal-free problem-solving during short acquisition periods on narrow areas of mathematics. Whilst recommendations from trusted mathematics educators might inspire a new generation of teachers to use goal-free problems in the classroom (Barton, 2018), this can only be effective and sustained if teachers have guidance on implementing the strategy in the long-term. There are currently no mid-term or long-term studies into the effects of goal-free problem-solving in realistic classroom settings. Such studies could provide vital information about: whether goal-free problems can have a significant, long-term impact on the learning of students; in which circumstances and with which topics goal-free problems can be beneficial or detrimental; the potential challenges of using goal-free problems in the classroom. Additionally, although current research is limited, there is some evidence to suggest that shifting attention

from problem goals to learning goals[3] could also have positive effects on schema acquisition (Wirth et al., 2009). This is an interesting finding which should be explored further.

Existing research has largely looked at goal-free problem-solving from a cognitive load perspective. However, Schoenfeld (2016) notes that it is important to consider issues of affect when exploring how students engage with mathematical problem-solving. A number of authors have noted that there is a strong link between student confidence and problem-solving performance (e.g. Burton, 2004; Schoenfeld, 2016). McLeod (1988) argues that confidence can influence whether students persevere with potential solution paths or even whether they start to answer a problem they perceive to be difficult. It would therefore be interesting to conduct research that explores whether goal-free problem-solving has an impact on students' reported confidence when solving goal-specific problems. Other areas to explore within the affective domain include the relationship between goal-free problem-solving and student motivation and the relationship between goal-free problem-solving and mathematical anxiety.

There is a lack of knowledge about how goal-free problem-solving interacts with students' beliefs about mathematics and problem-solving. Schoenfeld (2016) argues that student beliefs, defined as "an individual's understandings and feelings" are important because they "shape the ways that the individual conceptualises and engages in mathematical behaviour" (p.26). Over years of working on problem-solving with students in the US, Schoenfeld (2016) found that it is typical for students to believe that mathematical problems have a single solution, which can be found using a single method taught by the teacher. Students value being able to find the correct answer as quickly as possible. By shifting student focus away from goal attainment, it is possible that goal-free problems can positively influence student beliefs by stressing the importance of learning about problem structures and exploring mathematical ideas. Future studies could

[3] An example of a learning goal would be a series of geometry problems in which students were asked to find all the angles they can in order to determine in which circumstances it is impossible to find particular missing angles.

explore the extent to which goal-free problem-solving interventions can impact students' problem-solving beliefs.

Finally, as with any intervention that constitutes a change to students' normal ways of working, it is likely that students will have a range of positive and negative perceptions of goal-free problems. It is important for teachers to be aware of potential negative perceptions of goal-free problems so that these can be addressed to ensure that students are maximally engaged in the intervention. It is therefore important that future research explores student perceptions of goal-free problems.

Summary of Findings

Having explored the relevant areas of the research literature, it is now possible to answer the initial questions outlined in the introduction.

IQ1: What impact has goal-free problem-solving been found to have on goal-specific problem-solving in empirical studies?

Studies have found that goal-free problems can support students in categorizing problems according to the solution path needed to solve these problems. There is also evidence to suggest that students make fewer errors when working on goal-free problems and continue to make fewer errors on subsequent goal-specific problems. These improvements in problem categorization and reduction in errors are reflected in other results which have found that goal-free problem-solving leads to an improved performance on future routine goal-specific problems compared with practising with conventional problems.

IQ2: What scientific theories can be used to explain the results of empirical studies into goal-free problem-solving?

Some empirical findings related to goal-free problem-solving can be explained by considering the focus of students' attention. Goal-free problem-solving shifts students' attention towards the current problem state and the potential problem-solving operators that can be applied to that problem state. This supports students in developing awareness of the similarities and differences in problem states, which is necessary to categorise problems effectively.

It is also possible to explain empirical findings using CLT. Goal-specific problems are often solved using means-ends analysis which imposes a high ECL as this requires students to simultaneously consider the current known variables, the goal and any subgoals needed to reach that goal. Goal-free problems remove the possibility of using means-ends analysis and hence removes this ECL. This reduction in ECL means that a greater proportion of WM resources can be dedicated to germane processes, such as problem categorisation and schema acquisition. It also means that there is more WM capacity available to perform calculations which results in students making fewer errors.

IQ3: What are the limitations of the existing goal-free problem-solving research and theory?

Current empirical research only examines the impact of goal-free problem-solving on the solving of routine problems. More research is needed to explore whether the positive results found in these studies can be extended to non-routine problems that require students to make connections between different areas of mathematics. Most studies are conducted over a short duration in an experimental setting; more research is needed to explore the impact of goal-free problem-solving over a sequence of lessons in a natural classroom setting.

Current theory and research around goal-free problems is clearly rooted in the cognitive domain rather than the affective domain. More research is needed to explore the impact of goal-free problem-solving on issues such as confidence, enjoyment of lessons, and motivation. Furthermore, research is needed to identify student perceptions of goal-free problems and whether goal-free problems can positively impact students' beliefs about mathematical problem-solving.

Research Questions

In light of my findings in the literature review, and in consideration of the current gaps in understanding about goal-free problems, I have identified the following three research questions (RQs) in order to structure my investigation into the utility of goal-free problems in the mathematics classroom:

RQ1: What is the impact of a goal-free problem intervention over a period of three weeks on student ability to solve a variety of goal-specific problems within a mathematical domain?

This is an important question to explore because it closely relates to the issues outlined in my introduction. Whilst current empirical evidence explored in the literature review suggests that goal-free practise may support goal-specific problem-solving in small scale experiments, more evidence is needed in realistic settings and the use of goal-free problems in the teaching of a whole topic. Furthermore, whilst current research focuses only on supporting students in answering routine problems, this research will also aim to explore the effects of goal-free problem-solving on non-routine problem-solving.

RQ2: What is the impact of a goal-free problem intervention over a period of three weeks on student perceptions of a mathematical topic?

As outlined in the introduction, the difficulty of the reformed mathematics GCSE led some students to express that they were losing confidence in their ability to solve mathematical problems. It would be interesting to identify whether goal-free problem-solving has an affect on students' confidence in answering goal-specific problems. Other aspects of student attitudes should also be explored to assess the impact of goal-free problem-solving on students' perceptions of a mathematical topic.

RQ3: What are students' perceptions of goal-free problems in the mathematics classroom?

As identified in the literature review, this is an unexplored area of research into goal-free problems. Understanding students' perceptions of goal-free problems is important to the continued modification and development of the use of goal-free problems in the classroom. If students' perceptions are positive, then this may strengthen the argument for using goal-free problems in mathematics education. If students have negative perceptions of goal-free problems, then it may be important to address these concerns if the strategy is to be successful in the long term or in order to optimise its effectiveness in the classroom.

Methodology

Participants

In order to address my research questions, I decided to undertake some practitioner research exploring the effects of goal-free problems in a natural school setting. This research was conducted in an Academy in Oxfordshire, United Kingdom using 97 students from four parallel classes of Year 10 students. All students were working towards the Higher Tier Mathematics GCSE, and so had prior attainment in the top half of their year group, but the classes were not otherwise set by prior attainment. Year 10 students were chosen as the focus of this study because they will soon be taking mock exams and, in just over a year, will be taking their mathematics GCSE. It is these students which I am looking to support in answering problem-solving questions. Higher-Tier students were chosen both because of the greater problem-solving focus on the higher GCSE (30%) and because at Foundation level, it would not have been possible to study four parallel classes that were not set by prior attainment.

Choice of Mathematical Domain

The topic of circle theorems was chosen as the mathematical domain to be used in this study. The benefits of choosing this topic were fourfold. Firstly, analysis of mock exam data from the previous year suggested that these problems were a weakness for last year's cohort of Year 11s. Secondly, it is a natural extension of the angle geometry problems that have already been explored in the literature (e.g. Sweller et al., 1983; Ayres & Sweller, 1990). Thirdly, the number of circle theorems on the GCSE syllabus (eight) means there is plenty of scope to extend the intervention over a long sequence of lessons. Finally, the complexity of the topic provides plenty of scope to create problem questions which make links to other mathematical domains that

students will have previously encountered (e.g. trigonometry and area), as is required on the new GCSE (AQA, 2014).

Outline of the Practitioner Research

An intervention was set up that mirrored many of the goal-free problem studies which were explored in the literature review (e.g. Owen and Sweller, 1985; Ayres, 1993), except that this study was designed to run over a series of lessons. Two of the classes were randomly chosen to receive the intervention and were designated the 'goal-free group' (n = 51) whilst the other two classes acted as a control group and were designated the 'goal-specific group' (n = 46). Over a sequence of lessons, both groups were introduced to the eight circle theorems (CTs) (table 3). When lesson time was dedicated to problem-solving, the goal-specific group worked on conventional goal-specific problems. The goal-free group were given the same problems as the goal-specific group but with the goal removed. The sequences of lessons concluded with a final assessment which could be used to compare the performance of both groups on goal-specific problems. A questionnaire was then given to students in order to ascertain their perceptions of the lessons and goal-free problems.

Before undertaking this research, ethical approval was obtained from the Central University Research Ethics Committee (CUREC) at Oxford University. Permission was also granted to undertake this research by the Headteacher of the Academy. All students' data and questionnaire responses have been anonymised in accordance with the ethical guidelines outlined by BERA (2011). Whilst conducting the research, changes were made to the teaching practice of the two teachers working with the goal-free classes; students were unable to opt out of these changes. However, since the existing research suggested that this change of practice should at least have the same impact as working on goal-specific problems, this was not a major concern in this study. Further ethical considerations can be found in later sections of the methodology.

	Circle Theorem
CT1	The perpendicular from the centre to the chord bisects the chord.
CT2	The angle between a tangent and a radius is 90^o.
CT3	Tangents from a point outside a circle are equal in length.
CT4	The angle at the centre is twice the angle at the circumference.
CT5	The angle in a semi-circle is 90^o.
CT6	Angles in the same segment are equal.
CT7	Opposite angles in a cyclic quadrilateral add up to 180^o.
CT8	Alternate segment theorem.

Table 3: A list of the eight circle theorems that students are required to know for the Higher Mathematics GCSE.

Prior Knowledge of Students and Pre-Intervention Data

Consideration was taken of the prior knowledge of students before starting the sequence of lessons on circle theorems. Students had all covered the following topics, either in Year 9 or earlier in Year 10: angles in polygons; angles in parallel lines; similarity; congruence; area of circles and sectors; trigonometry (including to find the area of triangles). As such, all students had seen the necessary mathematics to develop the idea of circle theorems and to solve problems that draw on these mathematical domains. However, the extent to which students remembered these concepts and were fluent in applying them will inevitably have varied; all students will possess different schema. One of the difficulties of working in an ordinary school setting is that it is not possible to precisely account for every students' prior knowledge.

All students had, a few months previously, sat a department generated assessment which aimed to cover a range of topics covered in Years 9 and 10 up to that point and included many of the necessary prior knowledge topics highlighted above. Analysis of this pre-intervention assessment indicated a mean of 63.2% for the goal-free group and a mean of 61.5% for the goal-specific group. Although not specifically tailored to assess the necessary prior knowledge for

understanding circle theorems, this indicates that the two groups had a similar starting point in terms of prior attainment.

Lesson Structure and Sequencing

In collaboration with four teachers in my department, a sequence of ten lessons were planned to develop students' understanding of circle theorems. An overview of those lessons, which highlights when each circle theorems was introduced, can be found in table 4. The first lesson began with a short revision exercise requiring students to match key mathematical terms related to circle (e.g. tangents and chords) to their definition. This was to ensure that students had a reference for the necessary language needed to understand each circle theorem. It was decided that CT2 and CT3 would be introduced in the same lesson because CT2 offered a limited variety of new problem-solving questions to be introduced and provided a natural way to introduce CT3. Furthermore, because CT5 is a special case of CT4, it was decided that these theorems would also be introduced in the same lesson.

Lesson	Theorems Introduced	Other Notes
1	CT1	Introduction to key language, Practise CT1
2	CT2 and CT3	Practise CT1 to CT3
3	CT4 and CT5	Practise CT1 to CT5
4		Practise CT1 to CT5
5	CT6	Practise CT1 to CT6
6	CT7	Practise CT1 to CT7
7	CT8	Practise CT1 to CT8
8		Practise CT1 to CT8
9		Practise and Single-Application Assessment
10		Assessment and Questionnaire

Table 4: A breakdown of when each circle theorems was introduced over the ten-lesson sequence.

Lessons in which new circle theorems were introduced were designed to have the format outlined in table 5. Lessons which did not introduce new theorems began with segment 1 before recapping all of the previous circle theorems and then moving to segments 6 and 7. Some aspects of the lesson design were a result of normal working practice. For example, it was part of the Academy policy to start every lesson with a short, silent, and independent starter consisting of four questions that recap previous learning. One of the benefits of this was that it could be used to remind students of circle theorems that had been covered previously and other mathematical concepts that would be required later in the lesson.

	Lesson Segment	**Description**
1	Independent Starter	A starter which recaps previous learning material in the format: last lesson, last week, last topic, last year.
2	Circle Theorem Discovery	Students are given an opportunity to discover a new circle theorem by drawing particular scenarios, measuring angles and comparing results amongst the class.
3	Dynamic Demonstration	The discovered circle theorem is formally introduced and is further exemplified interactively on GeoGebra.
4	Proof of Circle Theorem	The circle theorem is proved using the previous circle theorems covered and/or prior knowledge of angle rules, similarity and congruence.
5	Worked Example	A worked example is used to demonstrate finding multiple unknowns through application of the new theorem, previous theorems and other areas of maths.
6	Problem-Solving Practice	A set of multi-step problems are provided for students to solve.
7	Review of Solutions	Worked solutions are projected on the board for students to check understanding.

Table 5: The general structure of lessons which introduce a new circle theorem.

As suggested by Retnowati and Maulidya (2017) in their recommendations for designing lessons using goal-free problems, it is necessary to have a phase of the lesson which is dedicated to exploring any new concepts in more detail before students engage in problem-solving. Lesson

segments 2 to 5 were designed for this purpose, ensuring that students had some familiarity with new theorems before moving on to problem-solving. Segment 6 was designed to ensure that students had opportunities to apply new theorems to a variety of problems and also required them to use previous theorems covered and prior knowledge from other mathematical domains. Segment 7 was included to ensure that students had a sense of the progress they were making with questions, however, students could also obtain feedback during the lesson from their peers and the class teacher, who circulated and supported students during each problem-solving segment.

The only difference between the lessons provided to the control group and the intervention group were in parts 5, 6, and 7. The goal-specific group went through a worked example which consisted of a question requiring the solver to find three specified unknowns (Figure 6a), whilst the goal-free group went through an example consisting of the same initial conditions but with a statement asking them to find all possible unknowns (Figure 6b). The unknowns were chosen to exemplify a range of possible unknowns that students could be asked to calculate from particular given information. The teachers who demonstrated exploration of the goal-free problem were careful to ensure that they calculated all of the unknowns calculated by the goal-specific groups to ensure that both groups could see the range of unknowns that were calculable.

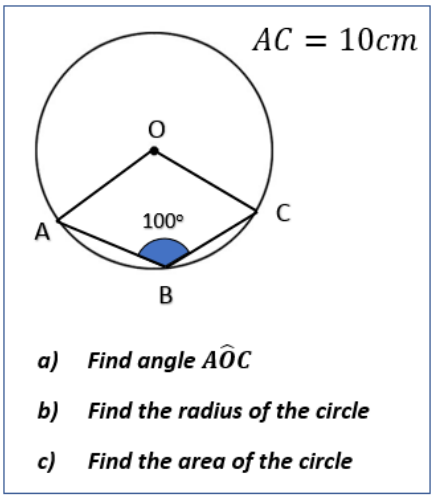


Figure 6a: Worked example given to goal-specific students in lesson 3.

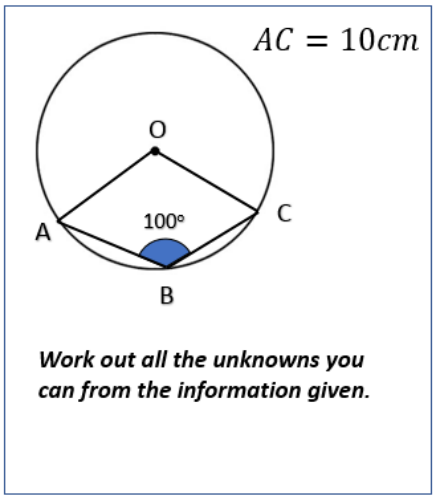


Figure 6b: Worked example given to goal-free students in lesson 3.

For lesson segment 6, the goal-free students were given the same initial problem states as the goal-specific students except that the goal was removed. Instead, all students were asked to find as many unknowns as possible. It is important to note here that, in contrast to many previous studies (e.g. Ayres 1993), no effort was made to ensure that both groups would calculate the same number of unknowns. There was potential for goal-free students to spend a significantly greater amount of time on individual problems. In lesson segment 7, it was not possible to display an exhaustive set of calculable unknowns and worked solutions to goal-free students. Instead, students were shown the solutions to the goal-specific problems and told that these represented a sample of the potential unknowns that students could have calculated.

Revision of the planned lessons

Due to time constraints and the nature of the curriculum, it was not possible to pilot the lessons with any classes in advance of the study. After all four teachers had taught the first two lessons in the sequence, we met as a group to discuss how the lessons were progressing. It became clear that all four teachers felt the lessons were slightly too long and students were not getting enough time to work independently on the problem-solving questions in each lesson. This is one of the drawbacks of planning a whole sequence of lessons in advance; part of being a teacher means adapting plans to the needs of students. It was decided that because the discovery segment of each lesson was quite time consuming and mainly involved students focusing on drawing and measuring, this would be removed from future lessons. This meant that more lesson time could be dedicated to problem-solving and students' attention could be focused on making connections between problem states and circle theorems, which we agreed would be more beneficial to schema acquisition and was more in line with the aims of this study.

Post-Intervention Assessment

As with similar post-tests in the literature (e.g. Ayres, 1993), and the examination that the students are being prepared for, the final assessment consisted only of goal-specific problems. In order to thoroughly address RQ1, it was important that these goal-specific problems could provide an insight into student performance on a variety of different problem types. The four teachers who collaborated in the planning and teaching of the lessons were also interested to see how students would perform on common types of GCSE questions. In order to achieve this, time was taken to analyse the circle theorem questions which have appeared on the five Edexcel exam series since the introduction of the new specification.

Across fifteen papers, it was found that there had been eleven questions which required knowledge of circle theorems, worth a total of 45 marks. The mark schemes allocate these marks according to the three assessment objectives (AOs) of the mathematics GCSE (OFQUAL, 2015). Method marks are allocated to AO1, which test students' ability to "use and apply standard techniques" (p.5). Problem-solving marks are allocated to AO3, which test students' ability to "make and use connections between different parts of mathematics" (p.5). Reasoning marks are allocated to AO2, which test students' ability to "reason, interpret and communicate mathematically" (p.5). Some questions allocate reasoning marks alongside method and problem-solving marks, whilst other questions are exclusively dedicated to reasoning. Table 7, which summarises how circle theorem marks are distributed across the AOs, clearly demonstrates that students need to be prepared to answer circle theorem questions which test all three AOs.

Type of Marks	Number of Marks	Percentage of Marks
AO1 Application Marks	12	24%
AO2 Reasoning Marks	20	44%
AO3 Problem-Solving Marks	13	31%

Table 7: A summary of how of circle theorem related marks are allocated to different assessment objectives across the first five Edexcel exam series since the introduction of the new specification.

The final assessment (appendix 2) was split into four sections which aimed to cover the three AOs outlined above: single-application questions, multi-application questions, reasoning questions, multi-domain questions. In order to ensure that there was adequate time for students to complete all of these sections effectively, the single-application questions were given to students at the end of lesson 9 and then students had the whole of lesson 10 to complete the final three sections.

The eight single-application problems (exemplified in figure 8) were designed to test AO1. These straightforward questions aimed to provide an efficient way to test students' ability to recognise particular configurations of circles, lines and angles as examples of situations where a particular circle theorem could be applied. These questions also function as a test of whether students could accurately apply the circle theorem they had identified to find the required angle. It was also possible to assess whether students could recall the correct wording for each circle theorem.

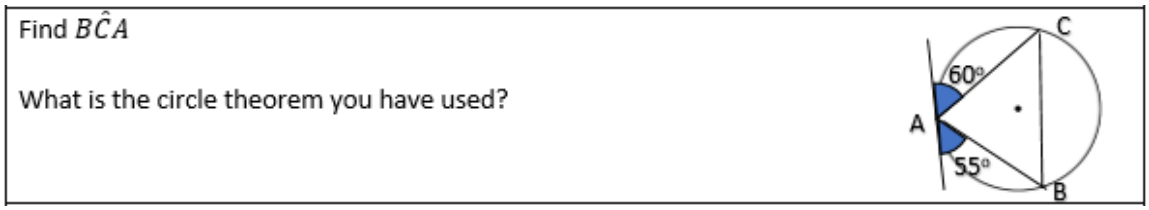


Figure 8: Example of a single-step assessment question testing students understanding of CT8.

The five multi-application problems (exemplified in figure 9) were also designed to test AO1. These problems required students to identify a series of circle theorems, in combination with other angle and circle properties, that could be applied in order to find a missing angle. Due to the nature of the problems, there was sometimes more than one combination of operators that allowed students to find the required unknown. These problems were structured in the same way as many of the practice problems in class and are similar to the routine goal-specific problems which have been explored in previous studies (e.g. Ayres, 1993). As with many of the past exam questions analysed above, students were also asked to communicate their reasoning for each step of their working which meant that these questions could also be used to test AO2.

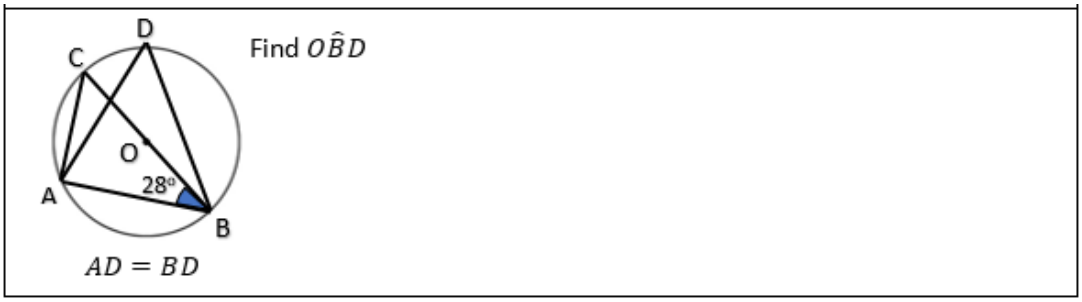


Figure 9: Example of a multi-application assessment question that could be solved using CT5, CT6, and other angles properties.

The reasoning questions were designed to test AO2. Exemplified in figure 10, these questions require students to use their understanding of circle theorems to interpret the information given and construct a chain of mathematical reasoning to justify an answer. These reasoning questions were unlike any of the circle theorem problem-solving questions practised in lessons and have not previously been explored in the literature in the context of goal-free problems.

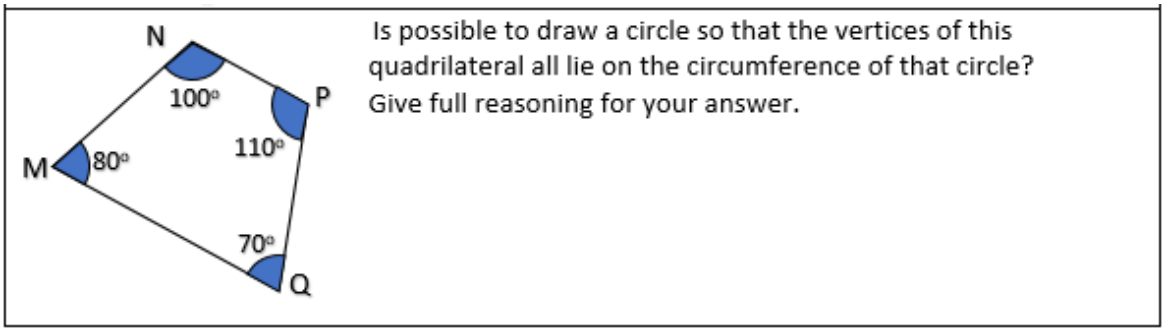


Figure 10: Example of reasoning question testing students understanding of CT7.

The four multi-domain questions were designed to test AO3. All four questions were taken from specimen papers of different GCSE exam boards but reformatted to fit on a single page. The first question requires students to make connections between CT1, Pythagoras and the area of a circle, and was very similar to other problems provided to the goal-specific group from the first lesson. The other three multi-domain questions were all previously 'unseen' in that they were unlike any other problems presented to students in lessons. The first question was an 'algebraic problem' which required students to use circle theorems and algebra to find the missing angle. The second question was a 'two-circle problem' which required students to use circle theorems

and trigonometry to find the length of a line (figure 11). The third question was a '3D problem' which required students to consider circle theorems in three-dimensions.

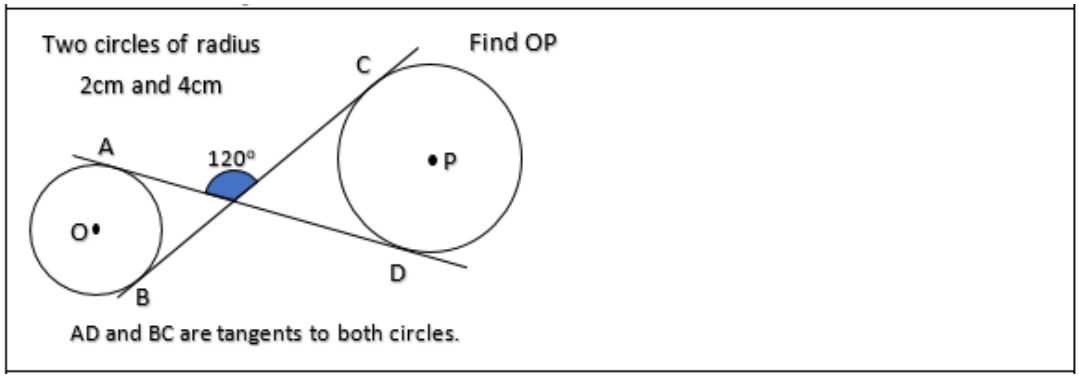


Figure 11: Example of an unseen question that required connections to be made between circle theorems and trigonometry.

One of the key issues that I have previously encountered with students working on new exam questions is that they often struggle to make any progress with novel problems. These unseen questions were specifically designed to be unlike any of the problems previously encountered by students, allowing analysis of the impact of a goal-free problem-solving intervention on performance on these novel problems. It is, however, also recognised that performance on these problems is also be dependent on students' understanding of algebra, trigonometry and 3D space.

Post-Intervention Questionnaire

After the assessment, students were given a questionnaire which included questions aimed to address RQ2. Attitude scales were used to compare students' perceptions of circle theorems. As suggested by Edwards and Talbot (2014), in order to determine the question focus of the attitude scales, preliminary conversations were held in advance of the study with four Year 10 students (two male, two female) asking them to share their attitudes about different mathematical topics recently covered. This made it possible to identify common themes and language used by students to describe different aspects of their attitudes towards these topics,

which are outlined in table 12. It was possible to group student responses into three major categories: enjoyment, challenge and confidence.

Category of Attitude	Common vocabulary used when referring to this attitude
Enjoyment	enjoy, fun, boring, interesting
Challenge	challenging, hard, difficult, easy
Confidence	struggle, can('t), (not) good at

Table 12: Key themes of attitudes identified towards mathematical topics.

The questionnaire (appendix 3a & 3b) asked all students to rate on a five-point Likert scale their perception of the following: enjoyment of the lessons; the challenge of the lessons; their confidence in answering the assessment questions. Each of these categories can be represented by a unipolar scale, which include a zero value at one end and have no conceptual midpoint (Krosnick & Fabrigar, 1997). For example, challenge of lessons is rated on a scale from 'not challenging at all' to 'extremely challenging'. Scale points were verbally (rather than numerically) labelled in order to give respondents a more natural list of options with which to express themselves. Studies have found that verbal labelling of all points on rating scales increases the reliability and validity of the scale and decreases the chance that rating scales will be ambiguous or confusing (Krosnick & Fabrigar, 1997).

Five-point scales were chosen after careful consideration of prior research. Current research into the validity and reliability of rating scales suggests that between 5 and 7 points is optimal (Krosnick & Fabrigar, 1997). The literature also suggests that there must be clear distinctions between scale points in order for respondents to reliably place themselves on the scale. After considering how to verbally label each scale point, it became clear that for any more than five quantifiers (not at all, slightly, moderately, very, extremely), it would be difficult to differentiate between categories.

Goal-free participants had three additional questions to answer that aimed to address RQ3. The first asked them to rate the perceived usefulness of goal-free problems in helping them to answer goal-specific questions on the assessment. Students were then asked if they would like to work on goal-free problems in future lessons. This question gave students a binary choice in order to force students to make a choice. A final open-response question asked students to elaborate on why they did or did not wish to work on goal-free problems in the future.

Students were not required to complete the questionnaire, however, it is important to note that some students may feel obliged to participate due to the power dynamics of teacher-student relationships (Cohen et al., 2007). In order to mitigate this, time was taken to explain the purpose of the questionnaire to students (and its relationship to this study) and students were explicitly told that they did not have to complete the questionnaire, or any individual questions, if they did not want to. It was made clear to students that although students' names were requested as part of the responses, this would be anonymised in any research. As one of the questionnaire items asked students to rate their confidence, there was potential that this could make certain students uncomfortable due to their anxieties. The class teachers were vigilant of this and were prepared to set aside time to have individual conversations to support any students that required it.

Although they have the benefit of being quick to complete, a drawback of using Likert questions is that they may miss some nuance in the attitudes held by individual students (Edwards and Talbot, 2014). In order to address this, students were also prompted to leave any other comments about the lessons that they wished to make on the back of the questionnaire, however, it was recognised that many students would not use this opportunity. Audio-recorded follow-up interviews with a random sample of students were also originally planned to mitigate this issue but, due to time constraints at the end of a school term, it was not possible to complete these interviews.

Another limitation of using a single questionnaire is that attitudes are not fixed in time and are influenced by context (Edwards and Talbot, 2014). The questionnaire was administered after the assessment so that all students could self-report their confidence in answering the assessment questions and so that the goal-free students could report how useful they found goal-free problem-solving in preparing them for the goal-specific questions in the assessment. However, it is important to note that student responses to other questions could have been influenced by their completion of the assessment. For example, students who may have perceived circle theorems to be unchallenging whilst practising in lessons may have changed their minds after struggling with the assessment. Further studies could look more closely at how students' perceptions of a topic develop from lesson to lesson and also elicit student attitudes before and after any final assessment.

Statistical Analysis of Data

As Likert data is ordinal, categories have a natural ranking but differences between ranks may not be equal, it is more appropriate to consider the mode and median responses in order to get a sense of the average for such data. Converting the categories to a numerical value and calculating a mean is generally agreed to be inappropriate and meaningless (Jamieson, 2004).
When analysing differences in assessment and questionnaire data between the goal-free and goal-specific group, a two-tailed Mann-Whitney test is used to compare distributions. This test is useful for comparing a dependent variable measured as ordinal data (such as responses on a Likert scale) or ratio data (such as the number of questions answered successfully on an assessment) between two categorical independent groups (goal-free and goal-specific) (Cohen et al., 2007). Analysis of the Mann-Whitney test has found it to be appropriate for detecting whether there is a statistically significant difference in the mean ranks of responses on attitude scales (de Winter & Dodou, 2010). Each time the test is applied, the following null hypothesis and alternative hypothesis are used:

H_0: the distribution of results for the two groups is equal.

H_1: the distribution of results for the two groups is not equal.

In each case, the null hypothesis is supported unless the Mann-Whitney test suggests that there is a statistically significant difference between the mean ranks of the two groups with $\alpha = 0.05$. If a statistically significant difference is identified, the nature of these differences can be identified by visually inspecting the data through cross-tabulation or graphical representation (Cohen et al., 2007).

Findings and Discussion

In this section, data from the assessment and questionnaire is reported and analysed in order to compare the goal-free group with the goal-specific group. This data analysis is discussed in relation to the research questions outlined previously. The final assessment and questionnaire were completed by 90 students (49 goal-specific and 41 goal-free). This is lower than the original 97 students who participated in the lessons because several students were either off school with illness or involved in a school dance rehearsal over the days when the assessment took place.

RQ1: What is the impact of a goal-free problem intervention over a period of three weeks on student ability to solve a variety of goal-specific problems within a mathematical domain?

Findings from the Single-Application Questions

In order to compare how students performed on the eight single-application questions, students were first categorised according to how many questions they had successfully answered. A student was deemed to be successful on a question if they correctly stated the angle requested or if they labelled it correctly on the diagram.

The median number of successes was 7 for both groups. The mean number of successes was 6.18 for the goal-free group and 6.59 for the goal-specific group. A two-tailed Mann-Whitney test (U = 855.5, p = .230) provided no strong evidence to suggest there was a statistically significant difference between the distributions of the successes of the two groups. However, visual analysis of the graph (figure 13) does show some interesting results that suggest this issue may be worth exploring further. For example, twice as many goal-specific students (34%) managed to answer all eight questions correctly than goal-free students (14%).

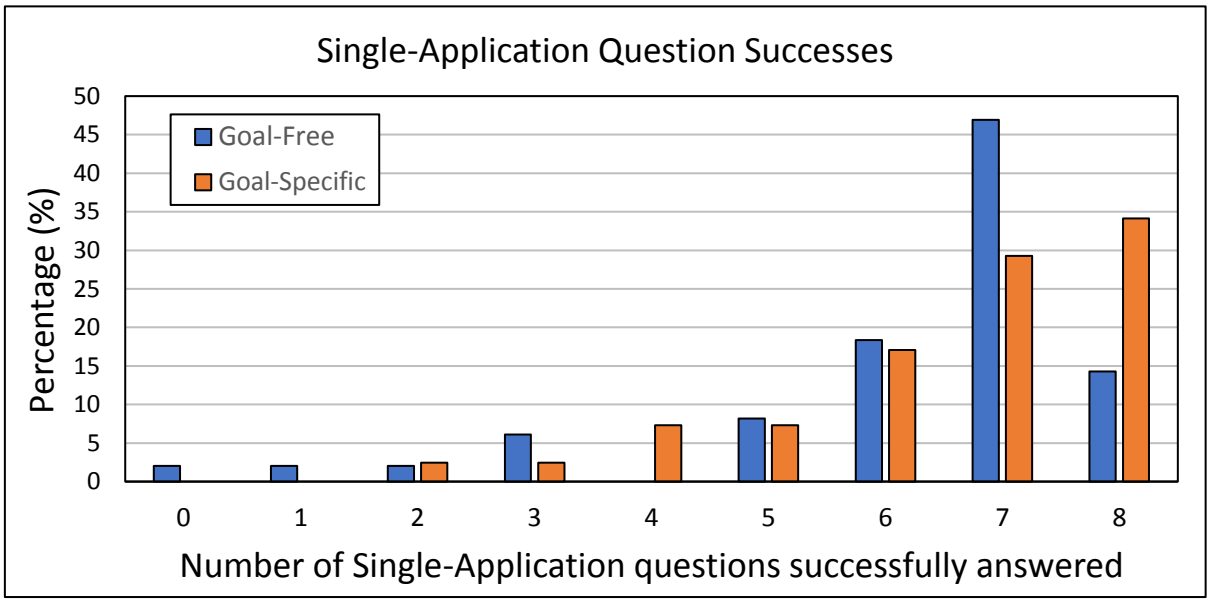


Figure 13: Chart comparing the number of single-application questions answered correctly by students in the two groups.

Responses to the single-application questions were also used to determine the extent to which students could correctly state the circle theorems that could be applied in each question. In order for a circle theorem to be considered as correctly stated, it was not necessary to have identical wording or mathematical language to the formulation shared in class. However, it did need to have all the necessary information to make the statement accurate. For example, when referring to CT1, a common error was for students to state that “a line from the centre to a chord bisects the chord”. This was not considered correct because it is necessary to state that this line must be perpendicular. Conversely, although the response “if a line from the centre goes through the chord at a right-angle then it cuts the chord into two lines of equal length”

does not use the mathematical language shared in class, it is factually correct and therefore marked as such.

The median number of correctly stated circle theorems was 6 for both groups. The mean number of correctly stated circle theorems was 4.98 for the goal-free group and 5.46 for the goal-specific group. A two-tailed Mann-Whitney test (U = 876, p = .395) provided no strong evidence to suggest there was a statistically significant difference between the distributions of the number of circle theorems correctly stated by each group.

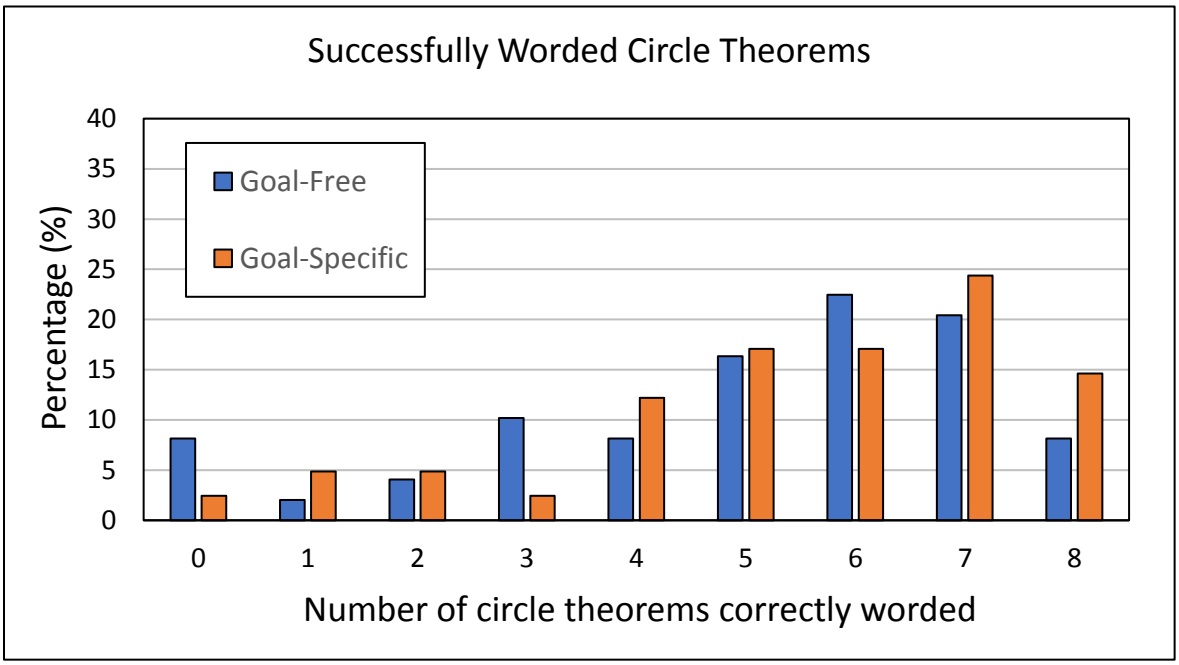


Figure 14: Chart comparing the number of circle theorems correctly worded by students in the single-application section.

Findings from the Multi-Application Questions

In order to compare how students performed on the five multi-application questions, students were first categorised according to how many questions they had successfully answered. For the purposes of this initial analysis, the inclusion of detailed and correct reasoning was ignored. A student was deemed to be fully successful on one of these questions if they correctly stated the angle requested or if they labelled it correctly on the diagram. A student was deemed to have

been partially successful if they applied a circle theorem to correctly find any missing angle on the diagram. As such, the definition of partial success does not differentiate between students who only managed to find a single angle and students who made a single error in an otherwise correct sequence of steps to find the final angle. Rather, it gives an indication of which students were able to make at least some progress with the problem.

The mean number of questions answered fully successfully for the goal-free group was 2.71 compared with a mean of 1.66 for the goal-specific group. The median number of full successes for the goal-free group was 3 compared with a median of 1 for the goal-specific group. A two-tailed Mann-Whitney test ($U = 627$, $p = .002$) suggested that there was a statistically significant difference between the distributions of the full successes of both groups. Visual analysis of the distribution (figure 15) shows that the goal-free students were consistently more successful at answering these multi-application questions with 54% of students answering three or more questions correctly compared with 22% for the goal-specific group.

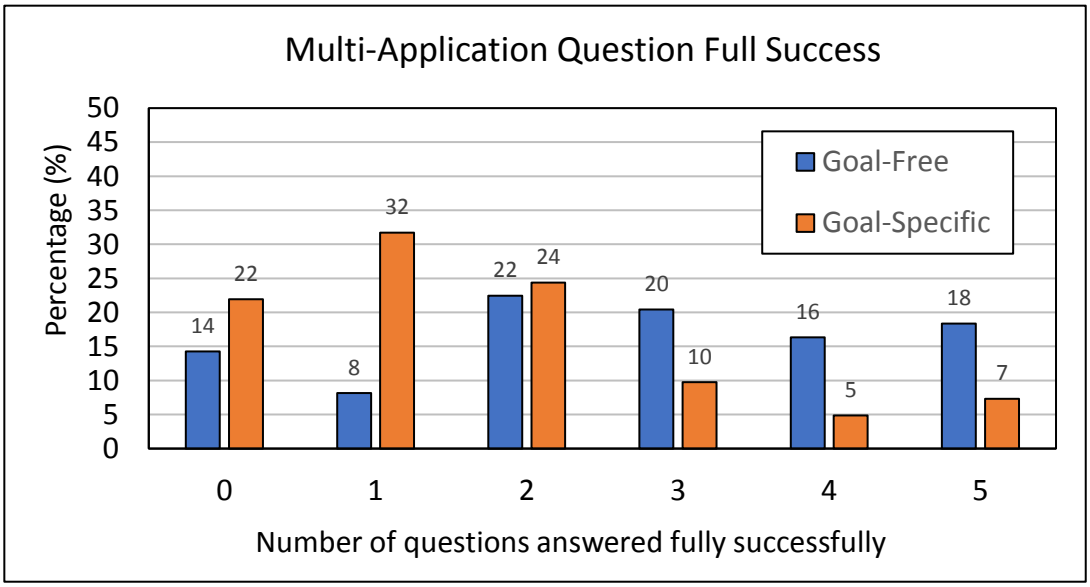


Figure 15: Chart comparing the number of multi-application questions answered fully successfully by students in the two groups.

The mean number of either partial or full successes for the goal-free group was 3.96 compared with a mean of 3.37 for the goal-specific group. The median number of partial or full successes for the goal-free group was 4 compared with a median of 3 for the goal-specific group. A two-

tailed Mann-Whitney test (U = 758, p = .047) suggested that there was a statistically significant difference between the distributions of the partial-full successes of both groups. Looking at a graph of the distribution (figure 16), a key difference was that 45% of the goal-free group had at least partial success on all five questions compared with only 24% of goal-specific students.

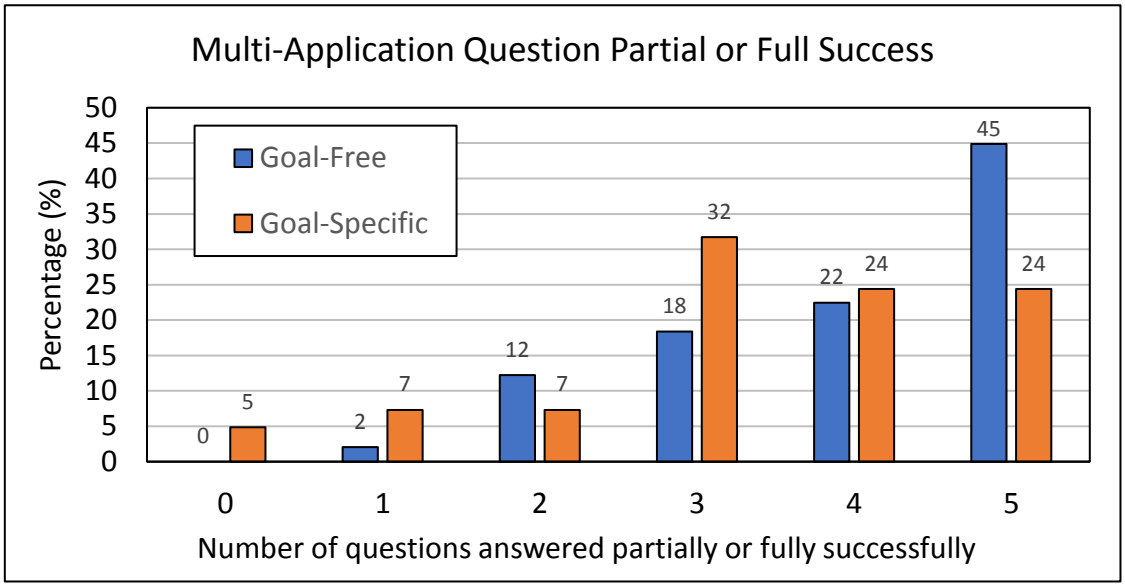


Figure 16: Chart comparing the number of multi-application questions answered partially or fully successfully by students in the two groups.

It is important to note that the increased success, as it is defined here, experienced by the goal-free group may not necessarily translate to increased success on these questions in the maths GCSE. As highlighted earlier (table 7), it is typical for many multi-step questions in the exam to also ask students to state full reasoning at each stage of their working. Therefore, in order to get a sense of how students would perform on these questions in an examination, it was first necessary to allocate marks to each question. Using previous exam mark schemes as a guide, it was decided to allocate four marks to these circle theorem questions. The first mark was available for finding any unknown angle that involved the use of a circle theorem. The second mark was available for finding the unknown specified in the question. The third mark was designated for having at least one piece of correctly stated reasoning. The final mark was available for finding the specified unknown and giving complete correct reasoning.

The total number of marks available for these questions was 20. The median mark for the goal-free group was 10 and the median mark for the goal-specific group was 9. The mean mark for the goal-free group was 10.6 (on average 2.1 marks per question) compared with a mean of 8.6 for the goal-specific group (on average 1.7 marks per questions). Visual analysis of the marks allocated (figure 17) demonstrates that a greater proportion of the goal-free group (26%) scored 15 marks or more compared with the goal-specific group (10%). However, a two-tailed Mann-Whitney test (U = 797.5, p = .095) did not provide evidence to suggest that the difference between the distribution of marks was statistically significant.

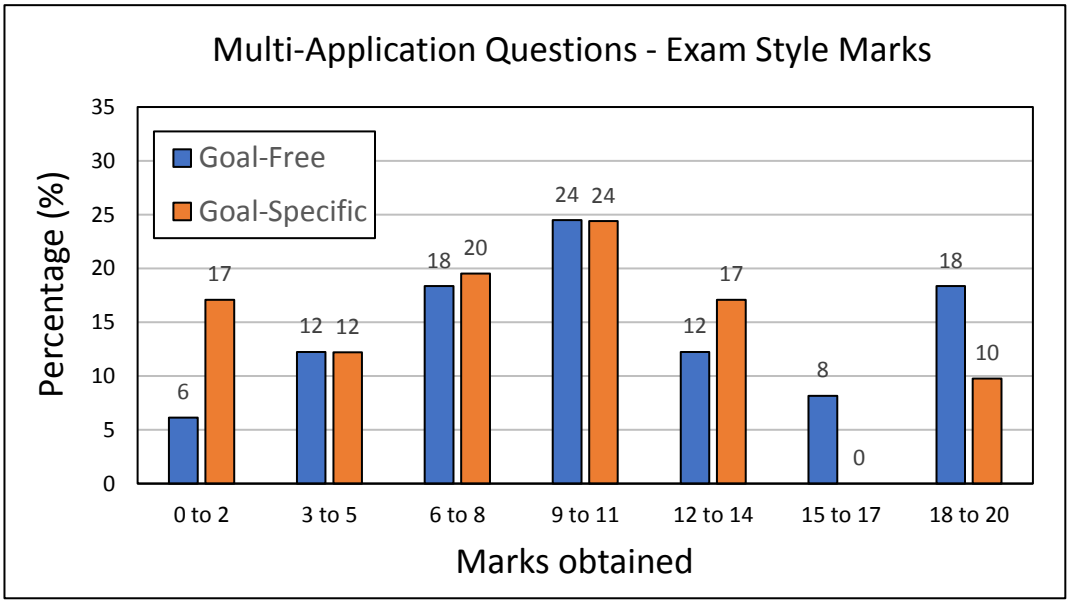


Figure 17: Chart comparing the distribution of marks for the multi-application questions for students in the two groups.

Findings from the Reasoning Questions

The reasoning questions on the assessment are not goal-specific problems and therefore do not directly address RQ1. However, as proof questions, requiring students to effectively communicate reasoning, account for 31% of circle theorem marks available on the new specification maths GCSE papers, my colleagues and I felt it important to consider whether the intervention impacted student performance on these questions. Therefore, findings related to the reasoning questions are outlined briefly below for the interest of the reader.

Each response to a reasoning question was placed into three categories: no success, partial success, full success. A reasoning question was deemed to have been answered fully successful if a complete argument was communicated, using accurately stated circle theorems, to justify a correct answer to the question. Responses were deemed partially successful if students gave a correct answer, which they attempted to justify using the necessary circle theorem. It was not necessary for the circle theorem to be accurately stated but it had to be clear which circle theorem they were referring to. All other responses were deemed to be unsuccessful.

The median number of partial or full successes for the goal-free group was 3 compared with a median of 2 for the goal-specific group. The mean number of full successes for the goal-free group was 2.45 compared with a mean of 1.95 for the goal-specific group. A two-tailed Mann-Whitney test (U = 842.5, p = .190) suggested that there was not a statistically significant difference between the distributions of the partial-full successes of both groups. This finding is consistent with the previous analysis of reasoning communicated in the single- and multi-application questions which found no significant difference in performance between the two groups.

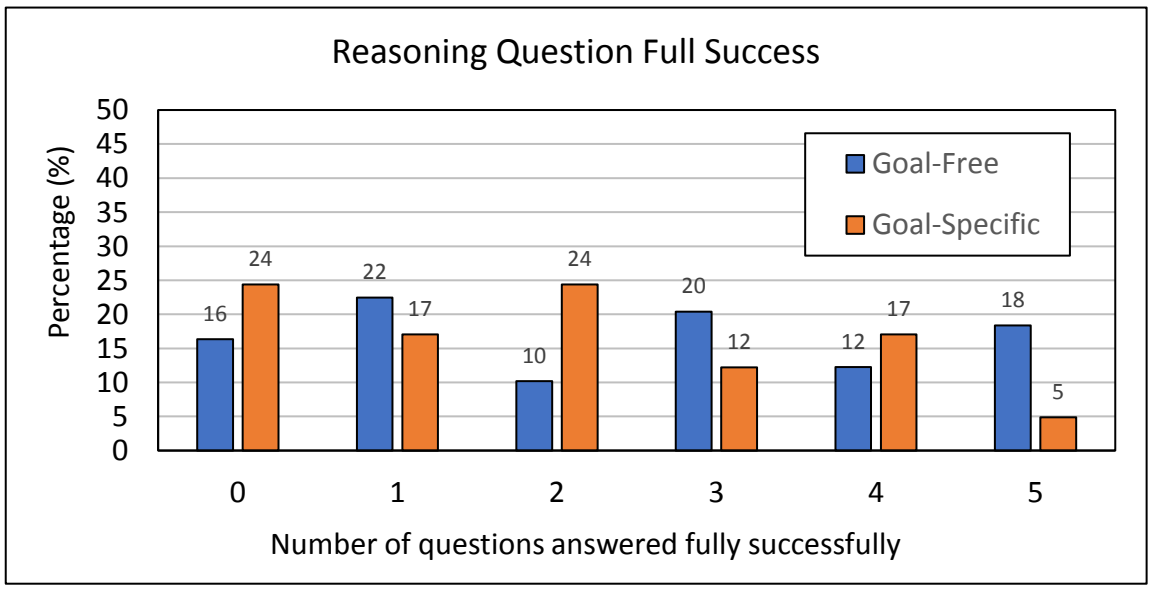


Figure 18: Chart comparing the number reasoning questions answered fully successfully by students in the two groups.

The median number of partial-full successes for the goal-free group was 4 compared with a median of 3 for the goal-specific group. The mean number of either partial or full successes for the goal-free group was 3.29 compared with a mean of 3.37 for the goal-specific group. A two-tailed Mann-Whitney test (U = 996, p = .952) suggested that there was not a statistically significant difference between the distributions of the partial-full successes of both groups.

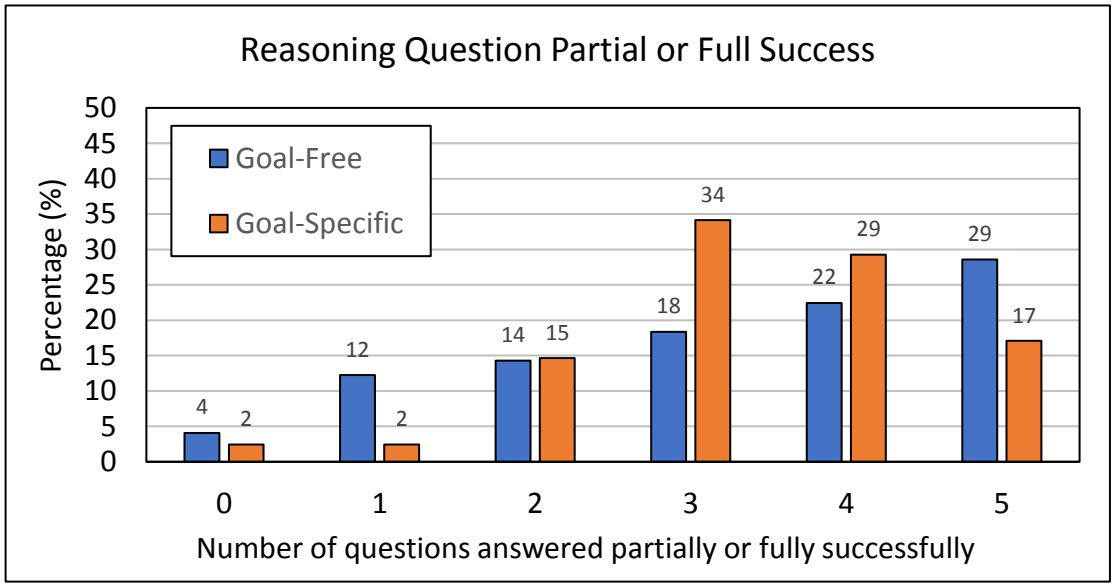


Figure 19: Chart comparing the number reasoning questions in which the necessary circle theorem was identified by the student to justify a response.

Findings from the Multi-Domain Questions

Each multi-domain problem was marked according to the mark scheme provided alongside its source specimen paper and was worth four marks. All marks were problem-solving marks, allocated for making progress towards the final goal, and not for communicating any reasoning. For each question, student responses were placed into three categories: no success (zero marks); partial success (one to three marks); full success (four marks). Performance on each problem is considered separately in the analysis below.

On the circle area problem, the median mark was 2 for the goal-free group and 3 for the goal-specific group. The mean mark for the goal-free group was 2.5 compared with a mean of 2.7 for the goal-specific group. A two-tailed Mann-Whitney test (U = 902.5, p = .412) found that there was no statistically significant difference between the distribution of marks scored by both groups.

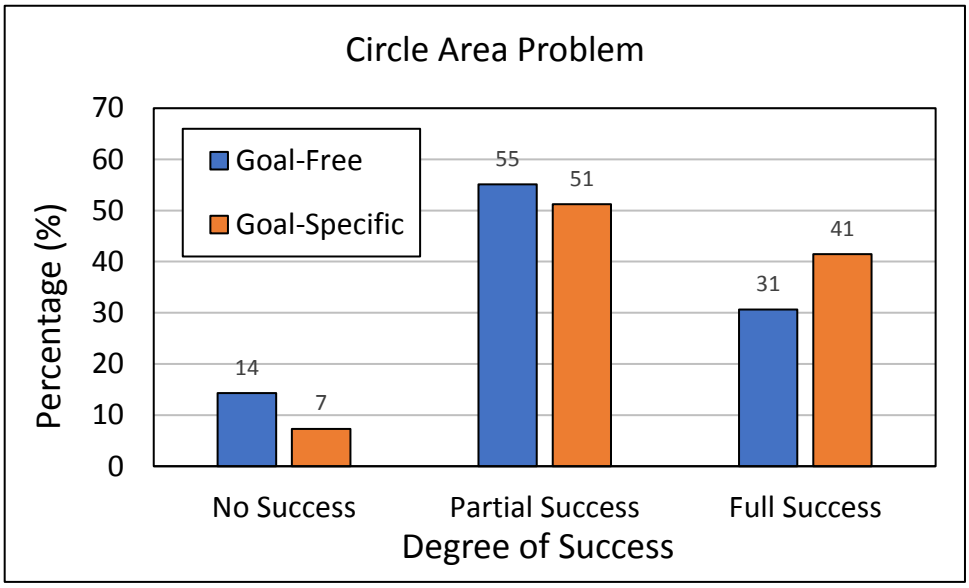


Figure 20: Chart showing the degree of success achieved by students on the circle area problem.

On the algebraic problem, the median mark for both groups was zero. The mean mark for the goal-free group was 1.1 compared with a mean mark of 0.76 for the goal-specific group. Visual analysis of the data (figure 21) shows that 47% of the goal-free group had at least partial success on the problem compared with only 29% of the goal-specific students. However, a two-tailed Mann-Whitney test (U = 836.5, p = .174) found that there was no statistically significant difference between the distribution of marks scored by both groups.

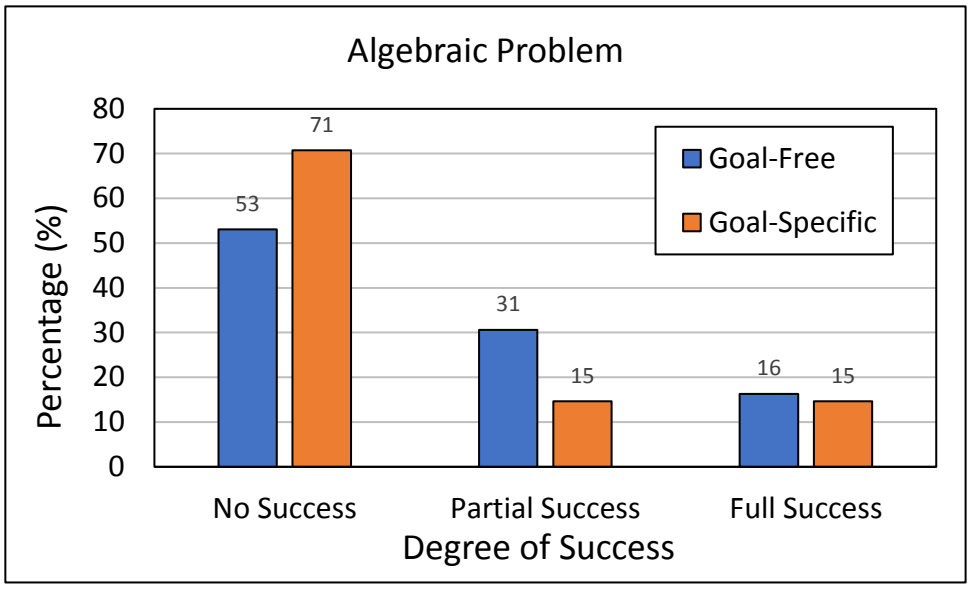


Figure 21: Chart showing the degree of success achieved by students on the algebraic problem.

On the two-circle problem, the median mark for both groups was zero. The mean mark for the goal-free group was 1.1 compared with a mean mark of 0.59 for the goal-specific group. A two-tailed Mann-Whitney test (U = 881.5, p = .322) found that there was no statistically significant difference between the distribution of marks scored by the goal-free and goal-specific group.

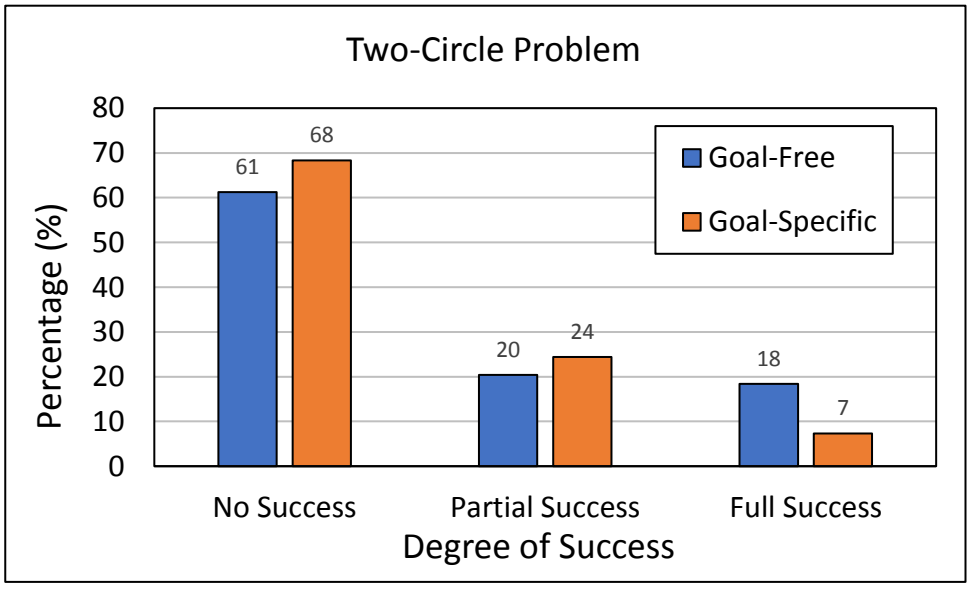


Figure 22: Chart showing the degree of success achieved by students on the two-circle problem.

The 3D-problem was the least successfully answered problem on the assessment. Only one student from either group managed to achieve any marks (by translating the problem to an

equivalent problem in two-dimensions). For this reason, no further analysis of performance on this question was undertaken.

Discussion of Findings related to RQ1

The findings outlined above demonstrate that the goal-free intervention had an impact on student performance on certain types of goal-specific problem but not on others. In this section, the findings are considered in the light of empirical evidence and theory outlined in the literature review.

The finding that there was no statistically significant difference between the performance of the two groups on the single-application questions may initially seem unexpected when considered alongside empirical findings explored in the literature review. Previous studies have found that students who have practised on goal-free problems have performed better on single-step goal-specific problems in post-tests (e.g. Sweller et al., 1983). However, it is more commonly demonstrated in studies that goal-free practise leads to improved performance on multi-step problems (e.g. Owen & Sweller, 1985; Ayres, 1993).

The fact that both groups have performed similarly could be a result of the relative simplicity of these single-application questions; the configuration of lines and angles presented may closely resemble the visual representations of each circle theorem stored in students' schema. It is possible that, whilst the goal-free students may have been quicker to develop visual representations in their schema, there was enough time dedicated to the topic to ensure the goal-specific group were also able to develop this schema. This would be similar to findings in the early maze study by Sweller and Levine (1982) who found that if participants had long enough to explore a maze with an underlying pattern to the solution path, there was no difference in rule induction between goal-free and the goal-specific participants.

The finding that the goal-free group experienced greater success on the multi-application problems on the assessment is consistent with the results of many short-durational studies into the effects of goal-free problem-solving on performance in the domains of kinematics (Sweller et al., 1983), angle geometry (Ayres & Sweller, 1990), Pythagoras (Ayres, 1998), and trigonometry (Owen & Sweller, 1985). Theory would suggest that the goal-free students should have had more opportunities to focus their attention on problem states and the relevant circle theorems that can be applied to these states (Trumpower et al., 2004). No attempt was made to determine whether students were using means-ends analysis or forward working strategies in this study. However, previous studies have found evidence to suggest that, because goal-free students have a better understanding of relevant problem states and operators, they are more likely to shift to a forward working strategy on goal-specific problems (Sweller et al., 1983). This is believed to impose a lower ECL and hence has been used to explain findings in previous studies that practising on goal-free problems can increase success on future goal-specific problems and lead to decreased error rates (Ayres, 1993).

One of the issues that arose from conversations with Year 11 students struggling with problem-solving on the GCSE is that they often did not know how to start answering multi-step questions. Often students would pick up no marks on these questions in mock exams. The fact that goal-free students were more likely than goal-specific students to have partial or full success on these questions suggests that goal-free students were more likely to recognise how to proceed from the initial information provided. Therefore, encouraging students to engage in goal-free problem-solving could be a useful strategy to help students to recognise how to begin answering these types of goal-specific problems.

Whilst goal-free problem-solving is a useful strategy to support the solution of multi-application, goal-specific circle theorems problems, there is evidence to suggest that this may not translate to improved exam performance on these types of questions. When exam style marks were allocated to these problems, which also required students to communicate mathematical reasoning, no statistically significant difference was found between the distribution of marks

obtained by the two groups. This suggests that the implementation of other tasks which aim to improve students' ability to communicate mathematical reasoning (and accurately state circle theorems) are also necessary in order support the answering of exam style multi-application questions.

When considering the theory explored in the literature review, there is little reason to think that goal-free problem-solving would impact students' abilities to communicate mathematical reasoning. This is reflected in the finding that there was no significant difference between the two groups in terms of the number of correctly stated circle theorems. In order to improve students' ability to recall and accurately state circle theorems, students would need to have opportunities to work on tasks which focused their attention on the precise wording of each question. Similarly, no statistically significant difference in performance was found between the two groups on the reasoning questions. In order to make improvements in answering these questions, students would need to be provided with tasks that provide opportunities to form a mathematical argument and communicate mathematical reasoning.

Multi-domain problems, presented in novel contexts or configurations, that account for 30% of marks in the Higher GCSE (OFQUAL, 2015), are of key interest in this research. It is these types of problems which were causing the biggest issues for Year 11 students in mock exams and are the reason why I am looking for strategies to support students with problem-solving. This study found no evidence to suggest that the goal-free intervention had an impact on student performance on these types of circle theorem problems. On the circle area problem, which required students to use their knowledge of Pythagoras and the formula for the area of a circle, 86% of goal-free students and 93% of goal-specific students were able to make at least partial progress on the problem. However, this problem was also similar to other problems practised in lessons. On the remaining three multi-domain problems, which were dissimilar to any questions previously practised in class, over 50% of students had no success at all.

It is possible that students struggled with these other multi-domain problems because they were not fluent enough in the relevant mathematical domains in order to solve these problems. Theory suggests that prior knowledge of students in the mathematical domains relevant to a problem is a key determinant of success on that problem (Schoenfeld, 2016). For example, when answering the algebraic problem, students may not have been familiar enough at labelling angles in terms of an unknowns and using this to form and solve equations. Students have also had very limited experience at working with 3D problems in maths lessons.

This explanation does not explain student performance on the two-circle problem, which required students to draw on their understanding of right-angle trigonometry. A number of problems explored in lessons required students to make similar connections and had been answered successfully by students. Additionally, trigonometry was not required until a later stage in this problem and so this does not explain why so few students were unable to pick up some initial marks (61% goal-free, 68% goal-specific had no success on this problem). It is possible that, because students had not previously seen a configuration which contained multiple circles, they were unable to draw on their previous experiences and were unsure how to proceed. This is consistent with theory which suggests that students should be exposed to a wide variety of problem configurations over a long period of time in order to maximise the chance that students can recognise how to connect their existing knowledge to novel problem configurations (Schoenfeld, 2016).

Overall, these results suggest a complex picture for the utility of goal-free problem-solving as an intervention to support goal-specific problem-solving in the domain of circle theorems. Clear evidence has been found to suggest that goal-free problem-solving can lead to improved performance on multi-step circle theorem problems when compared to conventional goal-specific practice. However, when students are also asked to communicate reasoning for solving these problems, as is often the case in examinations, no evidence was found to suggest a significant difference in the marks obtained by both groups. Furthermore, these types of problems only address AO1 on the mathematics GCSE and only account for one type of problem

that students may face. No evidence has been found to suggest that goal-free problem-solving is more effective than goal-specific problem-solving at preparing students to apply understanding to questions which require them to communicate mathematical reasoning (AO2). More significantly, there is no evidence to suggest that a goal-free problem-solving intervention, as it was implemented in this study, is suitable to support students in answering goal-specific circle theorem problems which are novel in configuration or are linked with other mathematical domains.

RQ2: What is the impact of a goal-free problem intervention over a period of three weeks on student perceptions of a mathematical topic?

Findings from the Questionnaire

The first questionnaire question ("How have much have you enjoyed working on the topic of circle theorems?") aimed to determine whether there was a difference between the goal-free and goal-specific groups in terms of how enjoyable they found the sequence of lessons on circle theorems. As illustrated in figure 23, the distribution of responses was very similar. In addition, both the median and mode response was that the sequence of lessons had been 'moderately enjoyable'. A two-tailed Mann-Whitney test (U = 918.5, p = .490) provided no evidence to suggest there was a statistically significant difference between the distributions of reported enjoyment of the two groups.



Figure 23: Chart comparing self-reported enjoyment on working with circle theorems after working on goal-free and goal-specific problems.

The second questionnaire question ("How challenging have you found working on the topic of circle theorems?") aimed to identify whether goal-specificity impacted the perceived challenge imposed by the sequence of circle theorem lessons. The mode and median response for the goal-specific group indicate that, on average, students found the lessons to be 'very challenging'. In contrast, the mode and median response for the goal-free group suggested that, on average, students found circle theorems to be 'moderately challenging'. A two-tailed Mann-Whitney test (U = 697, p = .013) indicated that there was a statistically significant difference between the distributions of perceived challenge reported by the two groups. The difference between the two distributions is clearly illustrated in figure 24. For example, 73% of goal-specific students expressed that they had found the lessons on circle theorems to be 'very' or 'extremely' challenging compared with only 44% of the goal-free group.

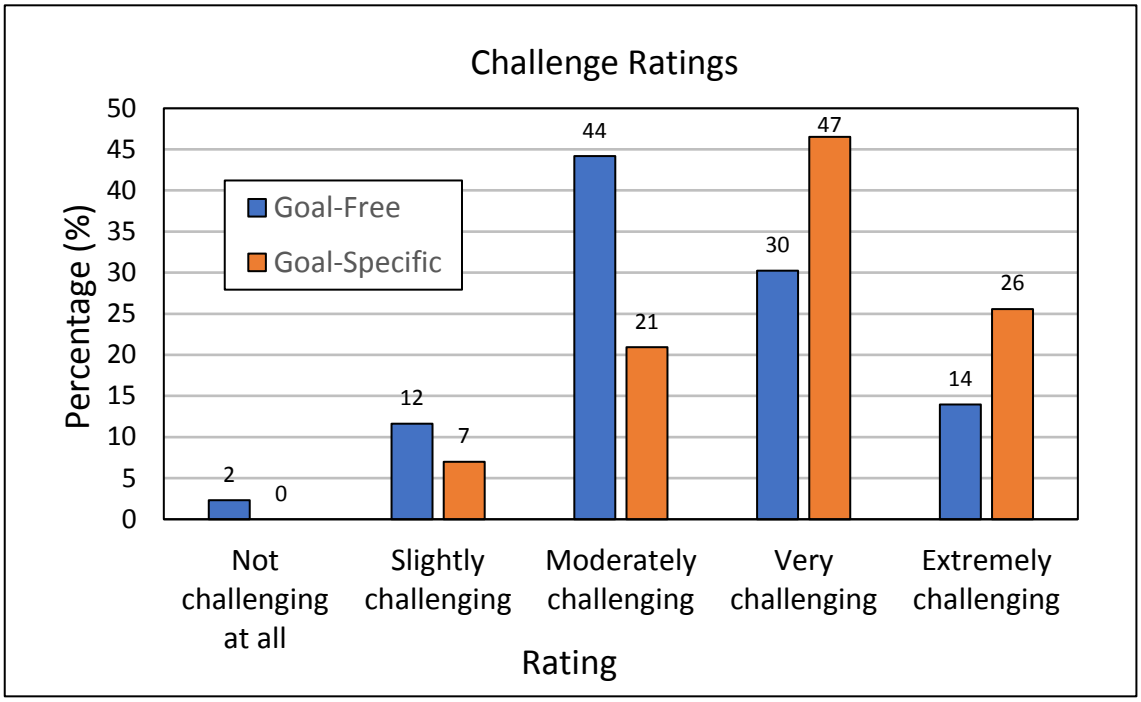


Figure 24: Chart comparing perceived challenge of circle theorems after working on goal-free and goal-specific problems.

The third question on the survey ("How confident were you when answering the circle theorem questions on the assessment?"), aimed to identify whether goal-free problems had any impact on students' reported self-confidence. As illustrated in figure 25, the distribution of responses was very similar. A higher percentage of goal-free students indicated that they were moderately to extremely confident (73%) compared with the goal-specific group (61%). However, both the median and mode response indicated that students felt 'moderately confident'. A two-tailed Mann-Whitney test ($U = 877.5$, $p = .308$) provided no evidence to suggest there was a statistically significant difference between the distributions of reported confidence of the two groups.

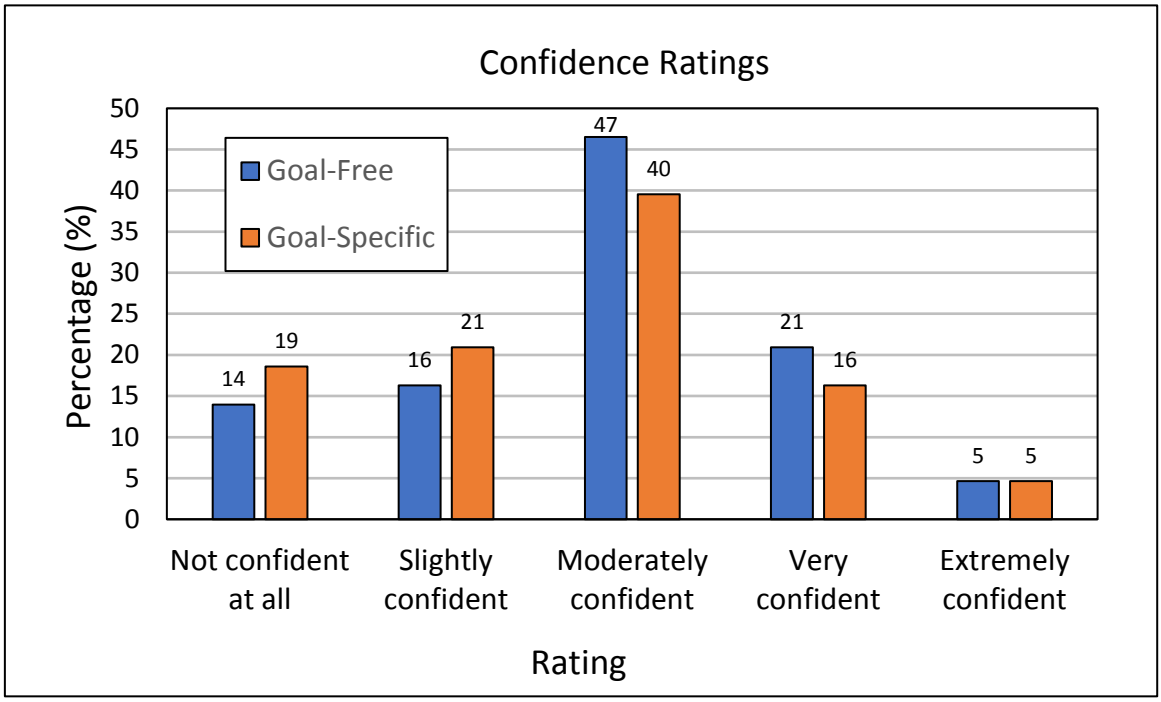


Figure 25: Chart comparing self-reported confidence with circle theorems after completing the assessment.

Discussion of Findings related to RQ2

The analysis above shows that the goal-free intervention had no statistically significant impact on students' reported enjoyment of the circle theorem lessons. Whilst this research was only conducted with students from one school, this finding gives no reason to suggest that teachers need to be concerned that working on goal-free problems will lead to decreased enjoyment of working on circle theorems. This is important because student enjoyment of mathematics lessons is closely linked to their motivation (McLeod, 1988).

The finding that the goal-free intervention had a statistically significant impact on the perceived challenge experienced during the circle theorem lessons is consistent with theory which suggests that goal-free problem-solving reduces cognitive load (Sweller, 1988). Although it is more common to use seven or nine-point scales asking subjects to rate their mental effort, perceived challenge has been used as an indicator of cognitive load in some studies (Paas et al., 2003b). Seven and nine-point scales are usually used to ensure there is enough sensitivity to

detect subtle differences in perceived challenge. The fact that a significant difference has been found on this five-point scale further emphasises the contrast in experienced challenge.

As outlined in the introduction, conversations with Year 11 students had highlighted that some students were losing confidence in their mathematical abilities because they were struggling with difficult problems in mock examinations. Student confidence is particularly important in mathematical problem-solving, with many authors arguing that confidence affects whether students make initial attempt to solve a problem or whether they persevere with a potential solution path (McLeod, 1988). Therefore, when introducing strategies to support students in answering a variety of goal-specific problems, I am also interested in building student confidence when faced with difficult problems.

These results indicate that goal-free problem-solving did not have a significant impact on students' reported confidence. However, as previously noted, attitudes are not fixed and student confidence may have varied between questions on the assessment. Student confidence ratings can instead be thought of as an average of student confidence over the whole assessment. More research could be conducted to explore how student confidence developed between lessons and different sections of the assessment. Deeper analysis may find that goal-free problem-solving did lead to a change in perceived confidence in answering particular types of questions.

Student confidence and assessment performance are likely to have a strong connection and influence each other (Burton, 2004). Students may experience improved confidence after being successful on a problem and decreased confidence after struggling to solve a problem. Importantly, confidence may affect whether students are willing to attempt a difficult question, such as the unseen multi-domain questions on the assessment. Finding ways to improve student confidence could be the key to ensuring that students attempt and make progress with problems such as these.

Overall, there is evidence to suggest that goal-free problem-solving had an impact on one aspect of student perceptions towards circle theorems: perceived challenge of the topic. This fits with theory that goal-free problem-solving can reduce ECL (by inhibiting means-ends analysis) and therefore reduce overall cognitive load (Sweller, 1988). However, no evidence was found to suggest that goal-free problem-solving resulted in students enjoying the topic more or less than students who engaged in goal-specific problem-solving. Importantly, no evidence was found that goal-free problem-solving impacted students' reported levels of confidence after completing the assessment.

RQ3: What are students' perceptions of goal-free problems in the mathematics classroom?

Findings from the Questionnaire

The fourth question on the goal-free questionnaire ("How useful have you found working on goal-free problems in lessons in helping you to understand the topic of circle theorems and to answer questions on the assessment?") aimed to assess students' perception of the utility of goal-free problem-solving. As illustrated in figure 26, the mode and median response was that goal-free problems were 'moderately useful', with 33% of students choosing this option.

The fifth question on the questionnaire was a binary question which asked students: "would you like to work on goal-free problems in future lessons on different topics?". Of the 49 respondents who took this questionnaire, 16 students (33%) indicated that they would like to work on goal-free problems in the future. Students then had the opportunity to share their reasoning for this answer as an open response. These reasons are explored in the discussion below.

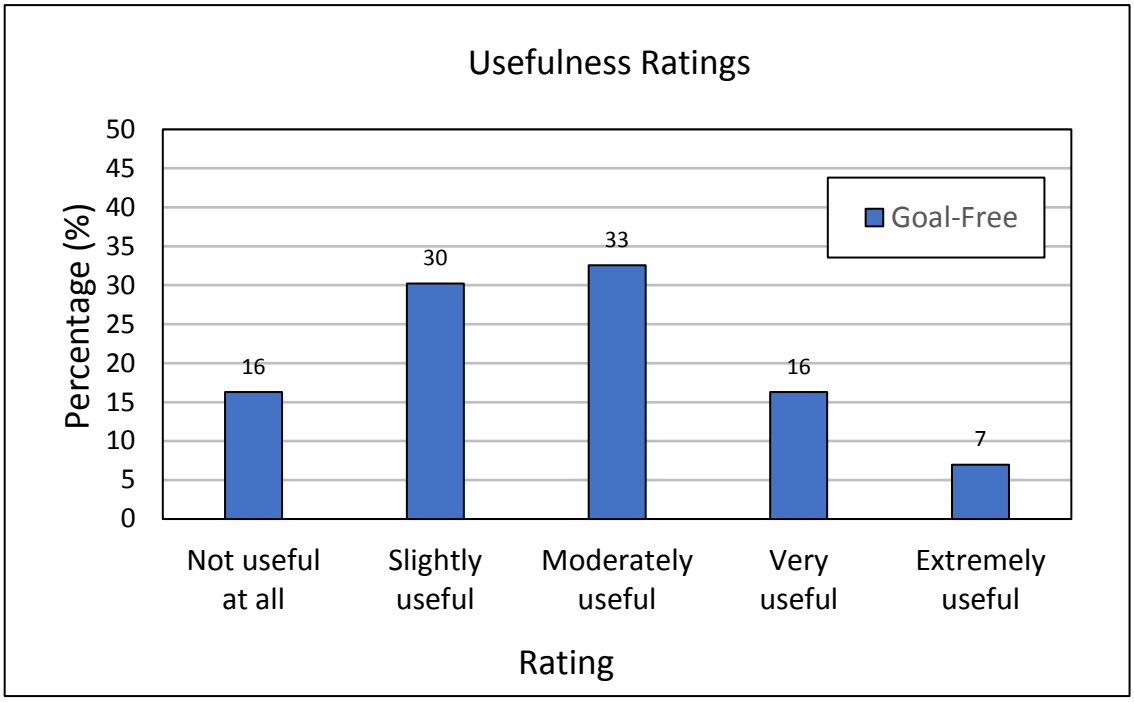


Figure 26: Chart illustrating students' perceived usefulness of goal-free problem-solving in helping them to answer goal-specific questions.

Discussion of Findings related to RQ3

The fact that 46% of students indicated that they believed goal-free problems to be 'slightly useful' or 'not useful at all', clearly indicates that a large proportion of students were not convinced that goal-free problems were worthwhile. These doubts are further exemplified in by the fact that 67% of students indicated that they would not like to work on goal-free problems again in the future. However, whilst these results indicate a mixed response towards goal-free problems, they do not provide any specific detail about students' perceptions of goal-free problems. In contrast, the follow-up question, which asked students to share their reasoning, provoked interesting responses which allow a deeper insight. Some of the clear themes that emerged are explored below.

It was clear that some students enjoyed the change of working with goal-free problems. For example, one student claimed to like the fact that goal-free problems were "a bit different" to what they would usually work on in class, whilst another student noted that these problems

were “more interesting” than traditional goal-specific problems. However, many other students found the lack of a clearly defined goal to be dissatisfying. One student explained that they found the problems “boring as there are no right/wrong answers” whilst another expressed their belief that “you need something to work towards so it is rewarding when you finish”.

These comments could be argued to reflect the fact that the goal-free intervention subverted students’ expectations of a traditional maths lesson. Students are not used to having the freedom to explore problem spaces and choose their own lines of enquiry when working on problems and so this opportunity feels alien to them. Previous research by the author (unpublished) found that the majority of teachers working in this department almost exclusively provide students with multi-step, goal-specific problems with unique solutions. It was also found to be uncommon for teachers to acknowledge multiple solution paths. Schoenfeld (2016) claims that this is typical of how problems are presented in many mathematics classrooms and results in the common student belief that mathematical problems have only one correct answer that can be found by using a single correct method demonstrated by the teacher. Lampert (1990, p.32) suggests that this leads students to associate being successful in mathematics with “being able to get to the right answer, quickly”, which some students begin to value over the process of learning and developing understanding of a mathematical topic.

A number of students were able to recognise some learning benefits of goal-free problem-solving. One student mentioned that goal-free problems helped them to “see the question easier”, whilst another stated that they can help to “understand what you are doing” because these problems “don’t rush you”. Another student argued that answering a goal-free problem could “make another question easier when asked to find something”. One student delved into this idea more specifically and expressed that these problems could help to “use your knowledge on questions from other units/topics”. These responses could suggest that goal-free problem-solving helped these students to focus attention on problem states and perceive the important properties of these problem states: a key requirement for categorising problems and developing example spaces (Goldenberg & Mason, 2008).

It is clear that there were also students who did not recognise any potential benefits of goal-free problems. For example, one student lamented that work on these problems "kinda feels like it is a waste of time", whilst another expressed their belief that "questions with goals are equally as helpful". These responses emphasise the importance of finding ways to engage students in new teaching strategies. In order to avoid influencing student opinions in the questionnaire, teachers did not explicitly share any of the theory behind goal-free problems with students. However, in a general teaching setting, justifying a new teaching strategy by explaining to students the ways in which it may be useful and the sharing the evidence supporting that strategy may help students to engage with the intervention more effectively.

Several students demonstrated appreciation of the open nature goal-free problems. For example, one student relished the opportunity to "test [himself]" by working out difficult unknowns, whilst another enjoyed the fact that these problems were not "limited in difficulty" as there were always opportunities to "learn to find more things out". Students such as these appreciated that there was "a lot to work on" which keeps you "always active in the subject". However, there were a number of comments made by students who disliked the lack of structure imposed by having "no target to work towards". One student mentioned that not having an aim meant she often "lost track of what [she] was trying to do" whilst others mentioned that it was often "difficult to know where to start" and that they did not "know what [they] should be looking for". One student explained that working on problems without a goal made it "hard to concentrate as you were never finished", whilst another student argued that this aimlessness meant he often found himself "not doing much work".

Some students echoed concerns by Ayres (1993) that large problem spaces could by quite time consuming and lead to the calculation of many unnecessary unknowns. A number of students argued that working on goal-free problems "takes too long" because there is "no set finish point". One student lamented that they were often "finding the same things and spent too long on most questions" and suggested that the introduction of time limits could help solve this issue. Some students also recognised that "you could work out loads of unuseful information on one

question but barely anything on another". Another issue arising from this is whether students get a chance to fully explore or expand their understanding of the problem space. One student spoke of how they often ended up "finding out a lot of things [they] could already do… using the same circle theorems every time" but often did not "get onto the harder parts". These findings stress the importance of using goal-free problems that give students experiences of a range of different problem states. The introduction of learning goals could help structure students' attention towards perceiving important properties of these problem states and discourage students from calculating unnecessary unknowns.

Conclusions and Implications

To conclude this essay, the key findings from the goal-free intervention are summarised in relation to the research questions which framed the study. In each case, the implication for students, colleagues and my own practice are considered. Potential avenues for future research, which build on the findings of this study, are also identified.

RQ1: What is the impact of a goal-free problem intervention over a period of three weeks on student ability to solve a variety of goal-specific problems within a mathematical domain?

This practitioner research has allowed insight into the impact of goal-free problem-solving intervention over a period of three weeks on students' ability to solve a variety of goal-specific problems in the domain of circle theorems. On the most straightforward problems which required students to apply a single circle theorem to find the solution (AO1), the goal-free intervention was not found to have had a statistically significant impact on student performance. Similarly, there was no statistically significant evidence to suggest that the intervention

impacted students' ability to accurately communicate (AO2) the correct circle theorem that needed to be applied in these single-application problems.

As with goal-free problem studies conducted over a short duration, it was found that students who had practised solving goal-free problems in lessons were more successful than their peers at solving multi-application goal-specific problems (AO1) on a follow up assessment. However, when exam-style marks were allocated to these problems and students were also required to state full reasoning at each stage of their working (AO2), no statistically significant difference between the marks obtained by the two groups of students was found.

Additionally, it was found that the goal-free intervention had no statistically significant impact on students' performance on multi-domain goal-specific problems (AO3). This result was found on multi-domain problems which were similar to those previously practised in lessons and on multi-domain problems that were dissimilar to any previously practised in lessons. Students from both groups particularly struggled with multi-domain questions which differed in structure to lesson problems or made links to mathematical domains that had not previously been explored in lessons.

Whilst there were several practice problems worked on by students which encouraged connections to be made between circle theorems, trigonometry, Pythagoras and area, the goal-free and goal-specific problems worked on by students focused heavily on multi-application questions that were abstract in nature and remained in the domain of circle theorems. This was deemed necessary in order to focus on developing students' understanding of circle theorems and also mirrored previous goal-free-problem research. Had there been more time to explore the topic, it would have been interesting to present students with problems that gave students experience of: a greater variety of initial problem states; different contexts for the application of circle theorems; linking circle theorems to a greater variety of mathematical domains. More research is needed to explore whether goal-free problem-solving is effective when students

have had access to these wider range of experiences and when students have greater understanding and fluency in other mathematical domains.

These findings suggest that it may be beneficial to continue providing students with goal-free problems in future circle theorem lessons. This is because the evidence suggests that these problems can have positive impacts on multi-application problem performance and there is no evidence to suggest that it decreases performance on other problem types or in communicating mathematical reasoning. However, there is a question of whether a goal-free intervention, as it has been implemented in this study, is feasible for teachers to replicate in normal practice. Whilst a number of professional websites now offer a selection of goal-free problem resources for teachers to use, these are generally a mix of adapted exam questions on various topics. Creating enough goal-free problems to use as the main task for a sequence of ten lessons was a time-consuming process which would not be sustainable for most teachers in the long-term. Further research would need to be conducted to explore whether less regular use of goal-free problems, alongside goal-specific problems, can have a positive impact on students' problem-solving performance.

It is also crucial to note that marks for solving goal-specific problems only account for 56% of the circle theorem marks available on previous exam papers for the current GCSE specification, with the remaining marks allocated to the communication of mathematical reasoning (AO2). Therefore, in order to support students to be successful at answering the full range of circle theorem questions, it is also necessary for teachers to use a range of other tasks to support students in recalling the wording of different circle theorems and communicating their mathematical reasoning effectively. Tasks which require students to annotate a worked-example with the circle-theorem that is used at each step could encourage a greater focus on the theorems themselves over the calculation of angles. Strategies such as self-quizzing could also be useful to support students in recognising which circle theorems they are struggling to recall or inaccurately stating. Tasks which focus students' attention on the precise wording of circle theorems could help them to recognise the importance of key words in the theorems. For

example, lesson time could be dedicated to exploring what happens if certain parts of the theorem are removed and helping students to recognise why each aspect of the theorem is necessary in order for it to hold true.

RQ2: What is the impact of a goal-free problem intervention over a period of three weeks on student perceptions of a mathematical topic?

This practitioner research has allowed insight into the impact of goal-free problem-solving intervention over a period of three weeks on student perceptions of circle theorem lessons. The goal-free intervention was not found to have had any impact on students' enjoyment of the circle theorem lessons, however, it was found that to have decreased the perceived challenge reported by students. This finding is consistent with theory which suggests that goal-free problem-solving decreases cognitive load. It was also found that the goal-free intervention had no impact in the reported confidence of students in answering questions on the assessment.

These findings suggest that it may be beneficial to continue providing students with goal-free problems in future circle theorem lessons in order to reduce the perceived difficulty of the lessons. This could support students to explore mathematical ideas and problem states that may have imposed too high a cognitive load if presented as goal-specific problems.

It is important to note that one of the limitations of this study is that student attitudes were only captured in a single questionnaire at the end of the sequence of lessons. In reality, student attitudes are not fixed in time and are influenced by context (Edwards and Talbot, 2014). Further studies looking at the impact of goal-free problems on student attitudes towards mathematical topics could look more closely at how perceptions of a topic develop from lesson to lesson as well as before and after any assessment. Similarly, it is likely that student confidence would vary between questions on the assessment. Future studies could attempt to identify whether the

goal-free intervention has any impact on student confidence in answering particular types of problem.

RQ3: What are students' perceptions of goal-free problems in the mathematics classroom?

This practitioner research has also allowed insight into student perceptions of goal-free problems in the mathematics classroom. It was found that a significant number of students did not believe that goal-free problems were useful in helping them to solve the goal-specific problems on the assessment and that the majority of students did not want to use goal-free problems in future lessons. This was also demonstrated by a number of responses which highlighted that some students did not see the point of working on goal-free problems.

When analysing students' opinions about the goal-free intervention, it was possible to identify commonalities in their responses. Some students enjoyed the fact that the goal-free problems were different to the conventional problems they would usually work on in class and relished the opportunity to test themselves by finding challenging unknowns. Others noted that the goal-free problems helped them to make connections to previous problems they had looked at and make links to other mathematical domains. There were, however, concerns raised by a number of students which would need to be addressed in order to maximise the success of any future goal-free interventions. Many students felt that working on goal-free problems was unrewarding because there was no right or wrong answer to work towards. A number of students also felt aimless when working on goal-free problems and, as a result, claimed to feel demotivated and unproductive.

As these opinions were obtained from an open questionnaire item, it is difficult to identify whether many of these beliefs were held only by the students who stated them or whether a wider number of students held these beliefs. Now that this study has identified some key themes in students' opinions towards goal-free problems, future studies could include specific

questionnaire items that separately address each of these opinions and this could give a picture of the prevalence of each belief.

The fact that a number of students responded negatively to the introduction of goal-free problems has strong implications for how colleagues and I will look to use goal-free problems with students in future lessons. These findings highlight the importance of being clear with students about the reasons for introducing new teaching strategies. It could be useful to share the results of this study with students, including the positive appraisals given by some participants, so that they recognise the potential benefits of engaging with goal-free problem-solving. It is also important that teachers continue to provide opportunities for students to give feedback on goal-free problem-solving in order to monitor student engagement over time and so that teachers can continue to adapt how these problems are used in lessons with different classes.

It is possible that some concerns expressed by students could be addressed by being more explicit about learning goals when working on goal-free problems. This could help shift student focus from 'getting the right answer' towards progressing in understanding and gaining new insights into mathematical topics. For example, asking students to generalise which angles are calculable from certain initial conditions or to identify what initial information is needed as a minimum in order to calculate particular unknowns. The introduction of learning goals would give students more structure and could remove some of the aimlessness experienced by some pupils. Learning goals could also help direct students' attention to specific aspects of problem structures and ensure that students do not spend too long calculating unnecessary unknowns. There has already been some research which suggests that replacing problem goals with learning goals can have positive effects on schema acquisition and rule induction (e.g. Wirth et al., 2009). In situations where the explicit early introduction of these learning goals imposes too high a cognitive load, it might be possible to introduce the learning goal later after an initial goal-free period: asking students to go back, conjecture and generalise from the problems they have already worked on.

Concluding Remarks

In summary, there is evidence to suggest that a goal-free problem intervention, over a period of three weeks, can be used to support students in solving multi-application, goal-specific circle theorem problems. However, there is no evidence to suggest that goal-free problems will have a significant impact on students' performance on other types of goal-specific problems and their ability to meet other assessment objectives on the maths GCSE. Furthermore, a survey of student beliefs has highlighted a number of concerns held by students towards goal-free problems, which would need to be considered and addressed by teachers if they wish to maximise the impact of the intervention in the classroom.

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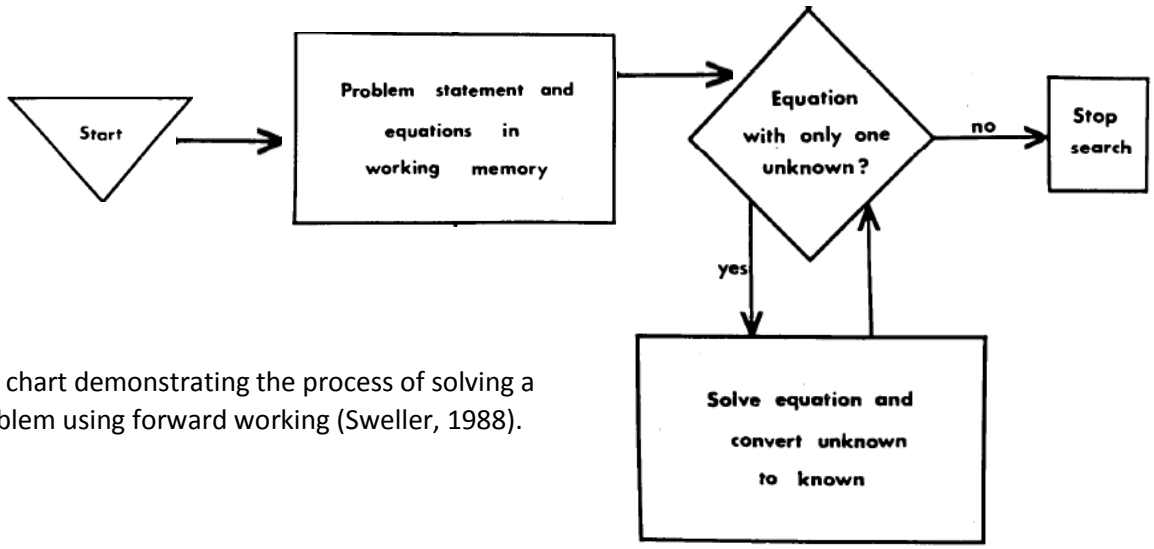
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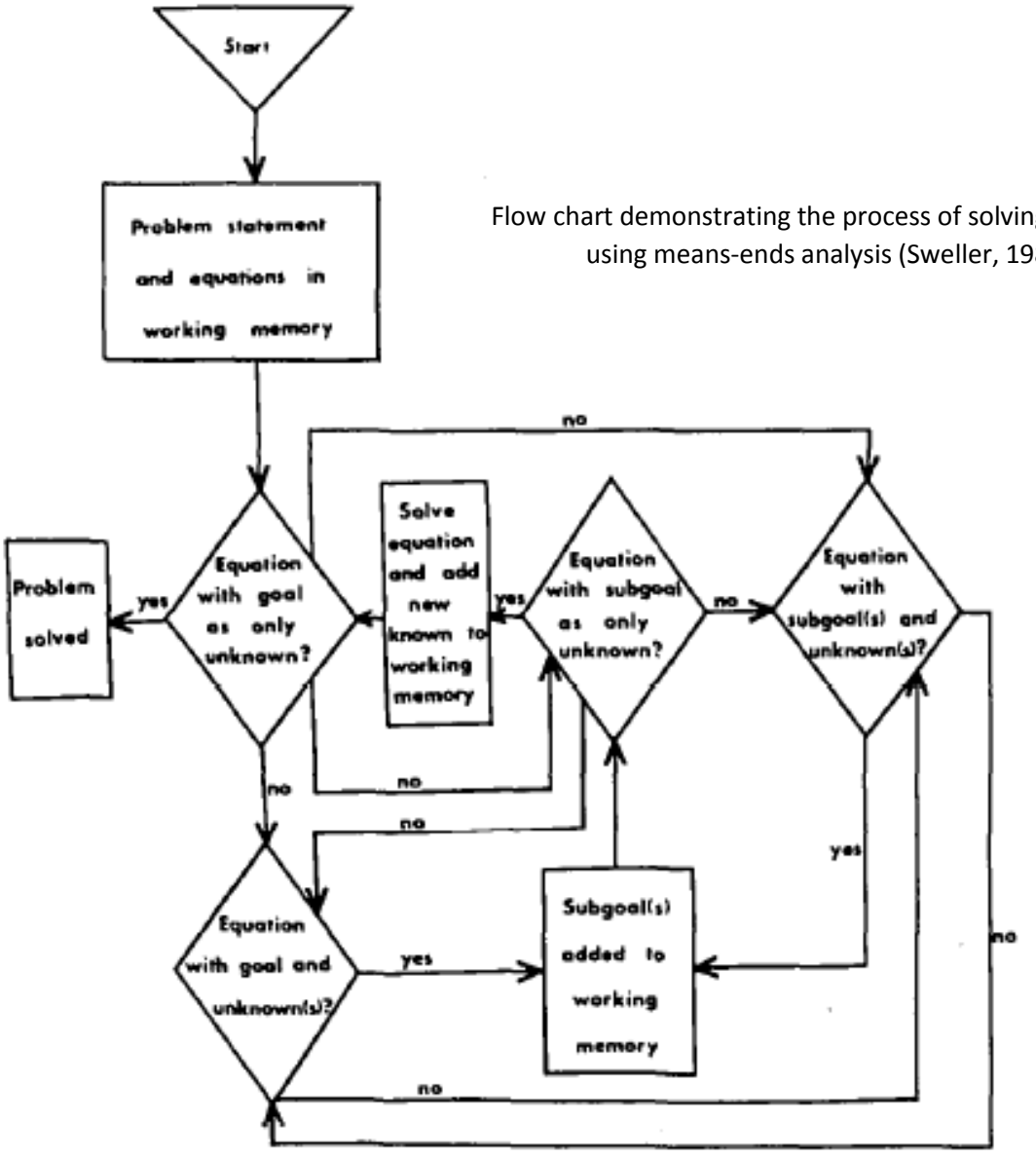
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Appendix 1a: Forward Working Process



Flow chart demonstrating the process of solving a problem using forward working (Sweller, 1988).

Appendix 1b: Means-Ends Analysis Process



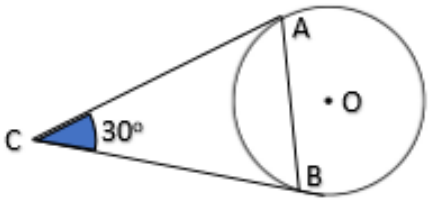
Flow chart demonstrating the process of solving a problem using means-ends analysis (Sweller, 1988).

Appendix 2a: Single-Application Assessment Questions

Circle Theorems: Single-Application

Find $C\hat{A}B$

What is the circle theorem you have used?

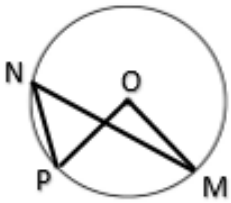


Find $P\hat{N}M$

$P\hat{O}M = 80°$

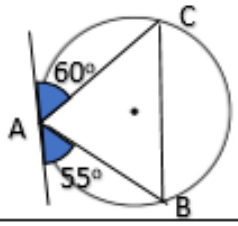
$O\hat{P}N = 60°$

What is the circle theorem you have used?



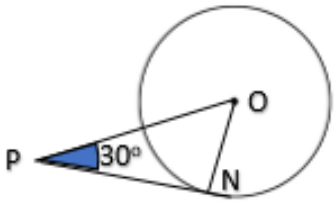
Find $B\hat{C}A$

What is the circle theorem you have used?



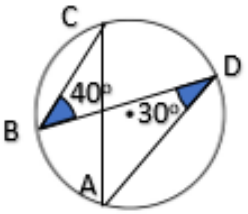
Find $P\hat{O}N$

What is the circle theorem you have used?



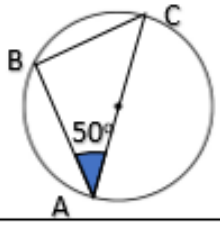
Find $B\hat{C}A$

What is the circle theorem you have used?



Find $B\hat{C}A$

What is the circle theorem you have used?

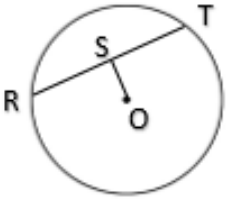


Find ST

RT = 10cm

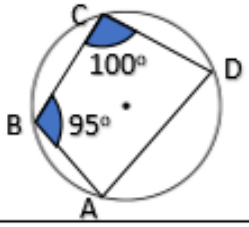
OS is perp. to RT

What is the circle theorem you have used?



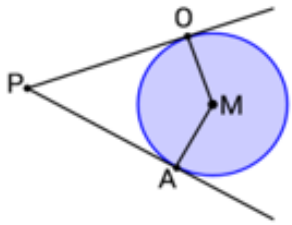
Find $C\hat{D}A$

What is the circle theorem you have used?

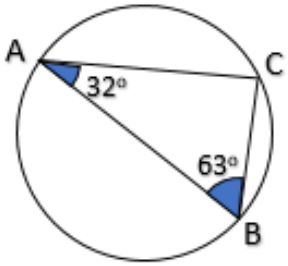


Appendix 2b: Reasoning Assessment Questions

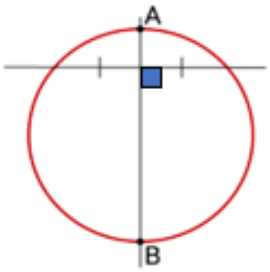
Circle Theorems - Reasoning



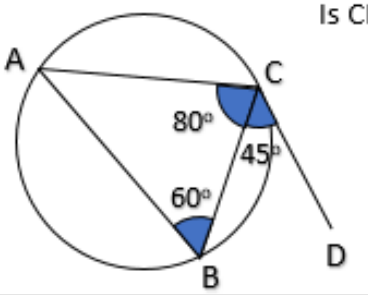
What is $O\hat{P}A + O\hat{M}A$? Give full reasoning for your answer.



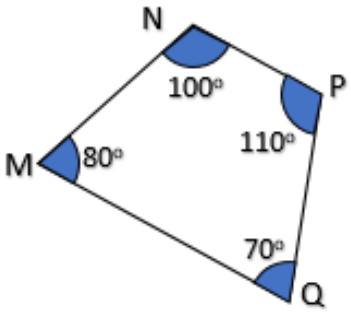
Is AB a diameter of the circle? Give full reasoning for your answer.



Is AB a diameter of the circle? Give full reasoning for your answer.



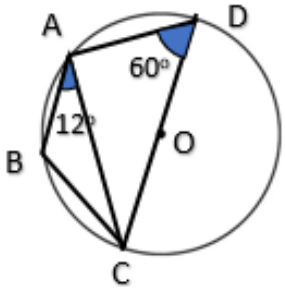
Is CD a tangent to the circle at C? Give full reasoning for your answer.



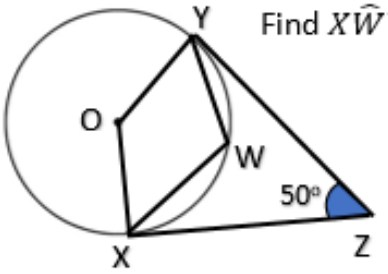
Is possible to draw a circle so that the vertices of this quadrilateral all lie on the circumference of that circle? Give full reasoning for your answer.

Appendix 2c: Multi-Application Assessment Questions

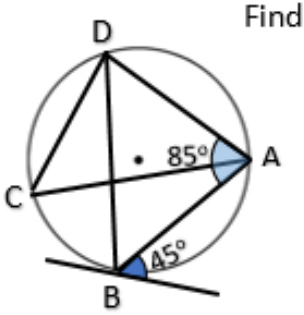
Circle Theorems: Multi-Application



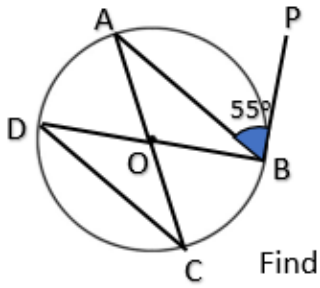
Find $A\hat{C}B$



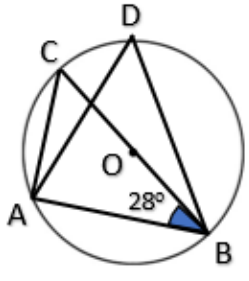
Find $X\widehat{W}Y$



Find $D\hat{C}A$



Find $C\hat{D}B$



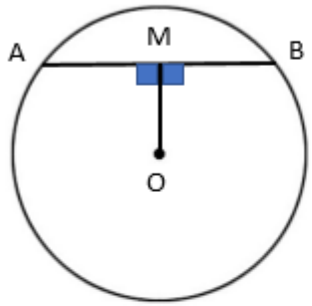
Find $O\hat{B}D$

$AD = BD$

Appendix 2d: Multi-Domain Assessment Questions

Circle Theorems: Multi-Domain Problems

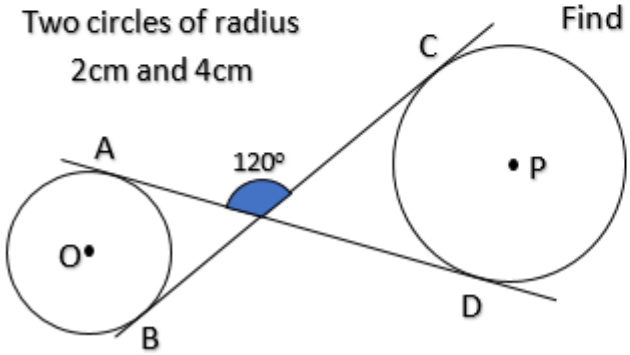
Find the circle area.



AB = 10cm, OM = 2.5cm

Two circles of radius 2cm and 4cm

Find OP



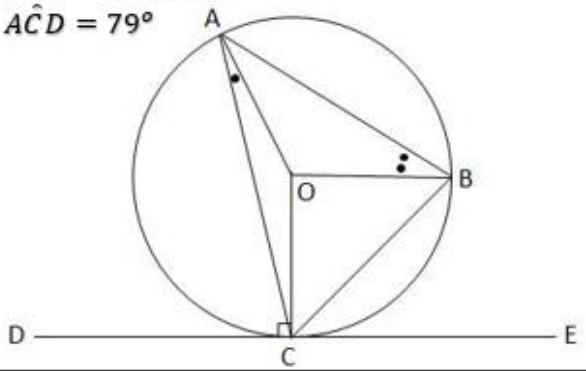
AD and BC are tangents to both circles.

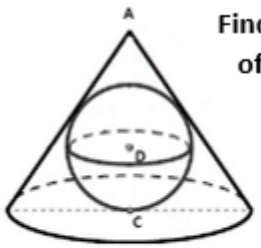
$O\hat{A}C = x$

$O\hat{B}A = 2 \times O\hat{A}C$

$A\hat{C}D = 79°$

Find $O\hat{C}B$





Find the height of the cone.

A sphere is inscribed in a cone.

The radius of the sphere is 6cm.
The radius of the base of the cone is 12cm.

Appendix 3a: Goal-Specific Questionnaire

Circle Theorems Questionnaire Name: _____

How have much have you enjoyed working on the topic of circle theorems? (please circle)

Not Enjoyable at all	Slightly Enjoyable	Moderately Enjoyable	Very Enjoyable	Extremely Enjoyable

How challenging have you found working on the topic of circle theorems? (please circle)

Not Challenging at all	Slightly Challenging	Moderately Challenging	Very Challenging	Extremely Challenging

How confident were you when answering the circle theorem questions on the assessment? (please circle)

Not Confident at all	Slightly Confident	Moderately Confident	Very Confident	Extremely Confident

Any comments? Please write on the back of this sheet.

Appendix 3b: Goal-Free Questionnaire

Circle Theorems Questionnaire Name: _____

How have much have you enjoyed working on the topic of circle theorems? (please circle)

Not Enjoyable at all	Slightly Enjoyable	Moderately Enjoyable	Very Enjoyable	Extremely Enjoyable

How challenging have you found working on the topic of circle theorems? (please circle)

Not Challenging at all	Slightly Challenging	Moderately Challenging	Very Challenging	Extremely Challenging

How confident were you when answering the circle theorem questions on the assessment? (please circle)

Not Confident at all	Slightly Confident	Moderately Confident	Very Confident	Extremely Confident

How useful have you found working on goal-free problems in lessons in helping you to understand the topic of circle theorems and to answer questions on the assessment? (please circle)

Not Useful at all	Slightly Useful	Moderately Useful	Very Useful	Extremely useful

Would you like to work on goal-free problems in future lessons on different topics? Yes / No

Please give a reason for your answer above:

_____

_____

_____

Any other comments? Please write on the back of this sheet.