Essays on the Marriage Market

Chloe Qianzi Zeng
Nuffield College
University of Oxford

A thesis submitted for the degree of
Doctor of Philosophy

Hilary Term 2015

Acknowledgments

To my supervisor Professor Martin Browning for his immense and invaluable guidance, who has inspired me and taught me how to do research. To my co-supervisor Doctor Margaret Meyer for her extremely helpful insights and encouragements, who guided me into microeconomic theory.

To Professor Ian Crawford and Dr Victoria Prowse for their valuable comments, suggestions and help.

To my parents and my husband, for their love and support.

To my friends and many others who have helped me along the way writing this thesis, for their kindness.

The financial support from the Clarendon Scholarship, the Nuffield Studentship, and the J.K. Swire Memorial Scholarship is gratefully acknowledged.

Abstract

This thesis consists of a short introduction and three self-contained chapters.

Chapter 1 develops a model of intra-household specialization and human capital formation for couples, taking into account of assortative sorting on income potentials in the marriage market. I assume people are matching on potential wage growth rates which differ across individuals and are realized through actual work experiences. The model is estimated by a simulated minimum distance estimator with PSID data from 1968 to 2011. I find there is strong positive assortative matching on wage growth rates, which helps explain the correlated wage growth residuals of married couples. If matching is switched to random, there will be more variation in household specialization arrangements and higher observed wage growth rates. The estimated elasticity of substitution between market goods and home production is approximately 0.37. Husband's time and wife's time turn out to be complements in the home production function.

Chapter 2 studies a marriage market with two-sided information asymmetry in which the gains from marriage are stochastic. Contracts specify divisions of ex-post realized marital surplus. I first study a game in which one side of the matching market offers contracts, and then study a social planner's problem, finding necessary and sufficient conditions for a truthful direct revelation mechanism to achieve matching efficiency. These conditions become more stringent as the number of agents in the matching market increases.

Chapter 3 examines the relationship between women's preference towards marriage and her marital outcomes. I propose using the mother's marital status as a proxy for her daughter's ex-ante preference towards marriage. Using 1980 and 2008 U.S. Census data, I estimate the impact of women's preference towards marriage and their educational attainments on their probability of getting married, and with Heckman correction, the impact on their husbands' earnings conditional on being married.

Contents

List of Tables

List of Figures

Introduction

This thesis consists of a short introduction and three self-contained analytical chapters on the marriage market.

Chapter 1 studies intra-household specialization and human capital accumulation for married couples, taking into account of the assortative matching in the marriage market. I argue there is marital sorting on income potentials: in the marriage market, people are matching on potential wage growth rates which differ across individuals. Assuming perfect foresight, a married couple jointly decide their time divisions between market work and home production till retirement. Potential wage growth rates are realized through actual work experience, and couples take the human capital accumulation into account when they make their specialization decisions. The model is estimated by a simulated minimum distance estimator with PSID data from 1968 to 2011. Couples are followed through from marriage to the last time they were in the survey.

I find there is strong positive assortative matching on potential wage growth rates, which helps explain the correlated wage growth residuals for husbands and wives. In the absence of assortative matching where couples are matched randomly, the observed average wage growth rates of households would be higher, and are mainly due to an increase among low educated groups. Households' market intensity remains largely the same, but there will be more variation in the household specialization arrangements. Second, I find that the observed work hours of husbands and wives can be rationalized by a model in which their potential wage growth rates follow the same distribution. The fact that on average married women work less than their husbands is partially due to a lower human capital depreciation rate for women. Third, the estimated elasticity of substitution between market goods and home goods is approximately 0.37. This is different from other estimates in the literature where the elasticity of substitution between market goods and home goods are usually found to be greater than one. However, restricting the elasticity greater than one in my structural estimation gives a worse data fit and fails to capture the correlated wage growth residuals and observed wage growth rates of husbands and wives. Moreover, the estimated elasticity of substitution between wife's and husband's time-use in home production is around 0.3, indicating that the husband's time and wife's time are actually complements in home production.

Chapter 2 studies a marriage market with two-sided information asymmetry in which the gains from marriage are stochastic. Contracts specify divisions of ex-post realized marital surplus. I first study a game in which one side of the matching market offers contracts. I show that when expected marital surplus is strictly monotonic in agents' types, no separating equilibrium that achieves matching efficiency exists. I then study a social planner's problem, finding necessary and sufficient conditions for a truthful direct revelation mechanism to achieve matching efficiency. These conditions become more stringent as the number of agents in the matching market increases.

Chapter 3 examines the relationship between women's preference towards marriage and her marital outcomes. I propose using the mother's marital status as a proxy for her daughter's preference towards marriage. Using 1980 and 2008 U.S. Census data, I estimate the impact of women's preference towards marriage and their educational attainments on their probability of getting married, and with Heckman correction, the impact on their husbands' earnings conditional on being married. I find that conditional on education, women's preference towards marriage is positively related to the probability of getting married, and conditional on being married, negatively correlated with their husbands' annual earnings. Women's educational investments are negatively correlated with the likelihood of marriage, and are positively correlated with their husbands' income in 1980 and are negatively correlated with their husbands' income in 2008.

Chapter 1

Assortative Matching on Income Potentials, Intra-Household Specialization and Human Capital Formation

1.1 Introduction

In Becker's (1973, 1974) classical marriage models, people sort into marriages because together they generate mutual gains from marriage, due to interactions of their attributes. A natural question then arises: exactly what are the important attributes people match on in the marriage market? Empirical evidence suggests many: there is a positive correlation in a couple's intelligence, education, height, attractiveness, race and age, and a negative correlation in earning power and in the propensity to dominate. In a simple one-dimensional setting, economists so far have focused mainly on either wage rates or education. Matching on wage rates is typically found in the matching with search framework (Shimer and Smith, 2000; Jacquemet and Robin, 2013): a person faces a sequential arrival of potential mates with different wage rates. He or she decides whether to accept or reject the marriage offer just like employees decide whether to take or decline a firm's wage offer.

One drawback of assuming sorting is based on people's wage rates at marriage is that if men choose and accept marriage offers based solely on wage rates of their potential wives, it would be difficult to justify why some married women choose not to participate in the labour market afterwards. Second, and more importantly, the wage rate just prior to marriage is likely to be an inaccurate measure of a person's lifetime earning power. Wages of young workers grow rapidly, and the fact that people with high education usually have a much steeper wage growth trajectory than less-educated people has been well documented in the human capital literature[1]. However wage levels at marriage do not capture such delayed effects of education on people's wage growth rates. This led to a number of papers studying who marries whom based on their schooling (Mare, 1991; Pencavel, 1998; Wong[2], 2003; Chiappori et al, 2009; Furtado, 2012; Greenwood et al. 2014), as education might be a better long run determinant of a person's expected lifetime wealth[3]. A shortcoming of using years of education to study assortative mating is that years of education only captures a small fraction of the variation in people's wage growth rates.

This paper develops a novel model assuming couples match on potential wage growth rates rather than wage levels at marriage. Individuals differ in their potential wage growth rates, and couples sort into marriages based on their income potentials. As couples value both market consumption and home goods, gains from marriage arise from intra-household specialization. Their efficient time allocation between home production and labour market work depends on three factors: spouses' potential wage growth rates relative to one another, who has a comparative advantage in home production, and how important they value home goods over market con-

[1] In both cross-sectional data (e.g., Mincer 1974; Carroll and Summers 1991; Card 1994) and panel data (e.g., Lillard and Weiss 1979; Hause 1980). See Card (2012) for a full survey.

[2] Wong (2003) constructed a marriage index that contained both wage rates at marriage and education.

[3] An alternative explanation which is not discussed here is that as the age at first marriage is raising, it becomes more likely that people meet their future spouses while still at school.

sumption. In turn, their time allocation decisions influence the couple's actual work experience in the labour market and hence the realization of their potential income.

I formally model the heterogeneity of people's potential wage growth rates and allow the wage growth to depend on an individual's acquired schooling. Researchers examining wage dynamics, often with longitudinal data (Lillard and Willis, 1978; MaCurdy, 1982; Abowd and Card 1989; Baker, 1997; Alvarez, Browning and Ejrnaes, 2010; Gustavsson and Osterholm, 2014), have concluded that there is substantial heterogeneity in wage growth rates. What has not been explored is that wage growth rates of married couples could be correlated.

To the best of my knowledge, this paper is the first study examining the degree of assortative matching on *income potentials*. Such matching is twofold: first, following the traditional Becker argument, if home production is important, anticipating specialization may lead to negative assortative matching on potential income. A person with a high income potential will tend to marry a person with a low income potential: the latter will specialize more in home production as the cost of him or her not working in the labour market is small, so the spouse with high income potential can devote more energy, time and effort to his or her market work. As marriage duration increases, the more human capital one spouse accumulates and the less attachment to the labour market the other spouse has: there will be further specialization. On the other hand, positive assortative matching on wage growth rates could be a countervailing force to intra-household specialization when 1) market goods are relatively much more important, so couples desire to maximize their total wealth, or when 2) home production can be easily outsourced or home goods become more available to be purchased in the market, or when 3) home production requires inputs from both the husband and the wife, and their inputs are not highly substitutable so that specialization is not feasible. In short, if husbands and wives experience a rising substitutability between market goods and home goods, and a rising complementarity in their home production, the positive matching effect may dominate the negative specialization effect.

I use structural modelling to empirically examine the model. The structural parameters of the model are estimated by a simulated minimum distance estimator using a sample of white married couples from the PSID 1968-2011 data. The parameter estimates are then used to simulate counter-factuals to disentangle the effects from matching and intra-household specialization.

1.1.1 The Marriage Premium Puzzle

My work helps address the marriage premium puzzle. The puzzle asks why everything else equal[4], married men earn approximately 10-50% more than their single counterparts. Researchers have been arguing over two main hypotheses: a marriage leads men to be more productive (the specialization effect) or men with higher earnings are more likely to get married (the selection effect).

The selection effect states that there exists some time-invariant fundamental differences between married men and single men, and those differences contribute positively to married men's wage growth. Ideally we would like to compare people stayed in their first marriages throughout and people never getting married. However, it is difficult to pin down the pool of individuals never getting married: for example, the youngest batch observed by Gray (1997) were from age 24 to 28, and the youngest batch observed by Stratton (2000) were from age 18 to 25. Even if they were single in the sample period, there is a fair chance they will get married later. Virtually all men eventually marry, those who never marry are a very small and likely unusual fraction of the population (Gupta, Smith and Stratton, 2007). Furthermore, researchers have mainly applied fixed effect estimations to examine the selection effect[5]. Hence, their samples often contain a large proportion of divorces

[4]The Mincer equation $lnW = \beta X + \gamma M + u$ estimated on cross-sectional data reported a positive and significant γ ranging from 10-50%. InW is men's log hourly earnings. M is the dummy variable equals to one if he is married. X includes schooling, experience, race, gender, family background, geographical area and industries worked for. See appendix B for a more detailed view of the marriage premium literature.

[5]See Korenman and Neumark, 1991; Cornwell and Rupert, 1997; Gray, 1997; Hersch and Strat-

and remarriages as a fixed effect model requires observing changes in the marital status during the observation period. No distinction has been made between an individual's first and later marriages. These issues make the comparison difficult.

More importantly, a fixed effect model does not address the selection on income potentials. If the actual process is the selection of men with higher income potential which most likely leads to higher expected earnings into marriage, such a mechanism cannot be estimated by a fixed effect model.

The specialization argument suggests that a wife's time spent on housework or the number of domestic chores she does contributes positively to her husband's earnings. However, assuming they are exogenous controls in the earning regression of their husbands inevitably suffers from the reverse causality bias. What is more likely in reality is that a couple make joint decisions on their labour supply and the degree of specialization when they get married, hence the husband's current wage rate and potential wage growth will also affect the wife's decision on the specialization. There is not much room to find a good instrument for the wife's hours. Jacobsen and Rayack (1996) and Chun and Lee (2001) used wife's predicted working hours which depend on the number of children in the household, the age of the youngest child, the wife's age and dummies for the wife's educational attainment. Similarly Gray (1997) used an infant dummy to instrument[6] for wives' work hours. Are these instruments affect the husband's wage only through the wife's work hours? Gupta, Smith and Stratton (2007) argued otherwise, as greater household responsibilities may induce the husband to work harder. A better instrument proposed by Gray (1997) is to use a four-point scale of attitudes toward gender roles (whether men should share housework responsibilities with women) as an instrument to capture the degree of specialization, however, the attitude question is only asked once in the NLS survey and cannot be used together with a fixed effect model. Moreover, Lundberg

ton, 2000; Hewitt, Western and Baxter, 2002; Loughran and Zissimopoulos, 2003; Richardson, 2000; Stratton, 2002; Gupta, Smith and Stratton, 2007.

[6] A rejection of the over-identifying restrictions by the author is not surprising as the infant dummy is likely to be an endogenous factor in the household decision.

and Rose (1999) have found significant heterogeneity in the effects of children on the household behavior. This finding further casts doubt on the IV method, as the IV estimation is not appropriate if the marriage or the children effect varies across individuals (Ribar, 2004).

To the best of my knowledge, this paper is the first analysis using a structural modeling method to examine the specialization effect and the selection effect related to the marriage premium. A structural model avoids the endogeneity bias which often arises in a reduced form regression examining couples' behaviors. The mechanisms in my model are explicit: there is sorting on income potentials in the marriage market. Couples utilize their comparative advantages in either home or market production, the spouse specializing in market production is going to accumulate more human capital and have a higher wage growth. To structurally estimate the model, I focus on a relatively homogenous sample which consists couples who have only married once.

1.1.2 Married Couple's Labour Supply Decisions

My work is also related to the literature on married couples' labour supply. When the focus is on married women (Mincer and Polachek, 1974; Heckman and Macurdy, 1980; Francesconi 2002; Jones et al., 2003; Olivetti, 2006), husbands' wages are often treated as exogenous, or solely determined by wife's pre-marriage demographics (Van Der Klaauw, 1996; Eckstein and Wolpin, 1989; Francesconi, 2002; Bredemeier and Juessen, 2013). This treatment stands in contrast to the marriage premium literature where wives' activities are explanatory variables for husbands' wages.

Only a few empirical papers study the interrelationship between husbands and wives. Lundberg (1988) estimates a general linear simultaneous equations model and showed that data rejects the hypothesis that in families with young children, either husband's or wife's hours are exogenous with respect to their spouse's labour supply decision. Similarly, Lundberg and Rose (2000) found that husbands' and wife's hours *both* change upon the arrival of their first child in a random effect(s) model. Bloemen and Stancanelli (2014) examined French data and showed that there is significant gender differences in cross-hour effects. More recent studies examining labour supply in a collective model also looked at both married men and women[7]. For example, Jacquemet and Robin (2013) found that leisure is an inferior good for men and a normal good for women. However, in terms of studying work hours of husbands and wives, empirical work with collective models so far usually exclude children[8] and do not take human capital accumulation into consideration. Moreover, when matching is taken into data, often repeated cross-sectional data (Shimer and Smith, 2000; Jacquemet and Robin, 2013; Bredemeier and Juessen, 2013), couples married for 20 years long are treated the same as newly married couples, the marriage duration which should affect the degree of the specialization process and couple's human capital accumulation is being ignored[9].

I model human capital accumulation and work hour decisions for both husbands and wives. A man can fully realize his income potential only if he works full time. If he works part time, his actual wage growth will be less than his full potential wage growth. If he does not participate in the labour market, his potential wage growth will not be realized, and on top of that, his wage rate will be lower due to human capital depreciation. The same applies to a woman.

I find that the observed work hours of husbands and wives can be rationalized by a model in which their potential wage growth rates follow the same distribution. The fact that on average married women work less than their husbands is due to a lower human capital depreciation rate for female workers. Not participating in the labour market for one year will lower a husband's wage rate by approximately 4%

[7]See Bourguignon and Chiappori, 1992; Chiappori, 1992; Fortin and Lacroix,1997; Browning and Chiappori, 1998; Del Boca and Flinn, 2005; Iyigun, 2005; Blundell et al., 2007; Dominique et al., 2014.

[8]See Choo et al. 2008 for an exception.

[9]See Gemici and Laufer (2014) for an exception.

and lower a wife's wage rate by 1.9%. On average without a college degree a person's wage will grow by 3.9% a year if he or she works full time. With a college degree, his or her expected annual wage growth will be approximately 5.3%. While there is no difference in the degree of heterogeneity in the wage growth across gender groups, I find there is more variation among high educated groups. The standard deviation of potential wage growth rates is 0.022 for people without a college degree, and 0.035 for people with a college degree.

1.1.3 The Degree of Assortative Marital Sorting

I find there is strong positive assortative matching on potential wage growth rates. In the absence of assortative matching, where couples just match randomly in the marriage market, the observed average wage growth rate of households would be higher. The intuition is the following: consider a man ranked first in the male wage growth distribution, and a woman also ranked first in the female wage growth distribution. In a perfect assortative matching setting, they will be matched together. For simplicity assume specialization only depends on relative wage growth rates. If the woman has a higher wage growth rate than the man, the husband will specialize more in home production. However if matching is random, since the underlying wage growth distribution is the same for both husbands and wives, there is a large probability that he is going to be matched with a woman who has a lower wage growth rate than him since he is ranked high in his gender group; as a result he will specialize in the labour market. Following this argument, random matching allows people with high income potential to have a higher chance of meeting someone with low income potential and allows them to specialize more in market work, as a result on average the observed wage growth rates would be higher. In this sense, random matching helps offset the income inequality between households.

This paper contributes to the literature on evaluating the importance of assortative matching. Hryshoko et al. (2015) find that marital sorting and the coordination of labour supply decisions of couples played a minor role in the earnings inequality among households. On the other hand, Greenwood et al. (2014) find positive assortative matching is a major contributing factor to rising household income inequality. I find that positive matching is important in terms of explaining the correlated wage growth residuals of married couples, and if matching is random, on average the observed husbands' wage growth rate will increase from 2.8% to 7.7% and for wives it will increase from 4.9% to 7.9%. The rise in the actual wage growth is significantly mainly for people without a college degree, so in this sense, random matching helps offset inequalities across high educated and less educated groups.

Apart from household inequalities and wage growth rates, Bredemeier and Juessen (2013) find that assortative mating is important in terms of explaining married women's work hours. Due to the raising trend in positive assortative matching on wage levels, wives of men in the middle of the wage distribution work the most. I find otherwise, when the positive matching effect is removed, the distribution of households' total market hours and the distribution of wives' work hours do not change significantly. The intuition is simple: consider the simplest case that there is extreme specialization, the spouse with a higher potential wage growth rate will work full time in the labour market, and the spouse with lower income potential will specialize fully in home production. Whether there is random matching or perfect assortative sorting, the total market hours of each household will be the same, there is just one spouse per household working full time.

1.1.4 Outline

This paper proceeds as follows: Section 1.2 presents the data used and lists how the sample is constructed. Section 1.3 presents the theoretical framework, followed by an analytical example. Section 1.4 presents the estimation strategy. Section 1.5 presents estimation results and section 1.6 concludes.

1.2 Data

1.2.1 Sample Selection

The data used in this analysis is the Panel Study of Income Dynamics (PSID) data from 1968 to 2011. Table 1.1 and 1.2 show how I construct the sample. Table 1.1 uses data from the PSID individual file, where detailed information of an individual is given and each individual is uniquely identified in the survey. Table 1.2 uses data from the PSID family file, where earnings and wage information of only the household head and his wife were reported. The final data set merges the individual file with a history of his and his wife's wage records. I include only white couples who were married or cohabited between year 1969 and 2011. Marriage duration ranges from 2 to 40 years and the age at marriage ranges from 18 to 35 for both husbands and wives. As one main aim of this paper is to understand specialization within the household, I make no distinction between legal marriages and cohabitation, that is, I assume specialization starts when couples start living together. Although this may seem to be a strong assumption, here the number of unmarried couples is small. The final data set constitutes 650 households in total, 180 couples have cohabited for some time before their marriage, and there are only 10 cohabited couples who have not married yet.

Wage rates are computed as annual total earnings divided by annual total labour market hours and for each individual I use only the first and the last wage rates observed. Table 1.2 shows how I clean the wage data in the 2011 survey, where wage rates in 2010 were reported. If a person's wage rate is below the minimum wage rate in that year, I ignore the wage rate and record a missing for the wage. In 2010 the minimum wage is approximately 7 dollars per hour, which is close to the 10th percentile value of the wage distribution. Wages are then deflated by the consumer price index of the year 1982. Wage rates of individuals whose annual hours are positive but below 100 are ignored (the number of observations is 73, I

Table 1.1: Dataset Construction

	Data construction procedure	No. of obs dropped	Sample size
			73251
1	Drop the obs if a person was married more than once or was married before 1969	38229	35022
2	Drop if a person was widowed, divorced or separated	4810	30212
3	Drop if a person was not surveyed after age 20	4921	25291
4	Drop if annual age records are inconsistent	2197	23094
5	Keep the obs if the household head is male[1]	11927	11167
6	White couples only	6740	4427
7	Drop disabled, in prison, or employment status not available throughout	579	3848
8	Merge with 1968-2011 income data, keep if couples' income records were both available at least once	1730	2118
9	Drop couples who were interviewed only once and then dropped out of the survey	193	1925
10	Drop couple who joined the PSID only after their marriage[2]	384	1541
11	Drop if the wage at marriage was missing due to one spouse not working	209	1332
12	Drop if one spouse stops working in the labour market after marriage	86	1246
13	Merge with education data, drop if the education data are not available	184	1062
14	Keep if marriage duration is between 2 and 40, the age at marriage is between 18 and 35, not a student during marriage	125	937
15	Merge with the actual hours of work data	88	849
16	Drop if the person has less than 12 years of schooling	59	790
17	Merge with the children data, drop if children records are missing or inconsistent[3]	140	650

1 The wage data are from the PSID main family data, wages are recorded only for household heads and their wives.
2 This could also be the case where the couple did not form a new household immediately after marriage.
3 In the sample all children were born after cohabitation.

record a missing for those wages). Individuals whose annual hours exceeded 3500 were censored at 3500, as I take 70 hours a week 50 weeks a year as the maximum of full time work hours[10]. The same procedures are applied to the wage data in other years, individuals in the main file are then merged with their corresponding wage histories.

Table 1.2: Wage Data Construction

Clean 2010 wage rates record	No. of obs dropped	sample size
Households interviewed in 2011		11805
Keep the observation if the husband is the household head	5503	6302
Drop if the husband's wage rate is smaller than the minimum wage in 2010	523	5779
Drop if the wife's wage rate is smaller than the minimum wage in 2010	275	5504
Drop if the husband is neither in the labour market nor keeping house[1]	997	4507
Drop if the wife is neither in the labour market nor keeping house	282	4225
Drop if the husband's total work hours is positive but less than 100	31	4194
Drop if the wife's total work hours in 2010 is positive but less than 100	42	4152
Number of husbands worked more than 3500 hours a year		90
Number of wives worked more than 3500 hours a year		20

1 I drop the observation if the husband's employment status is retired, disabled, a student, in prison or not applicable in 2010. I keep the observation if he is currently working, temporally out of work, unemployed, or keeping house. Same for the wives.

Wage rates are missing in some years for those who did not participate in the labour market at that time. In this analysis, as long as I can observe an individual's wage rates twice, one at the year she (he) was married, and the other one at some point during her (his) marriage, I will be able to examine her (his) wage growth. This

[10] In 2010, the 95th percentile value of work hours is 3120 for the husband, and the 99th percentile value of work hours is 3107 for the wife. As seen in the last two rows of table 1.2, the number of people whose work hours are censored is quite small.

two-wage requirement mainly excludes 1) households in which only one spouse did participate in the labour market both before and after marriage; and 2) households in which one spouse specializes fully in home production after marriage and never returns to work. As row 11 and 12 in table 1.1 suggest, in total there are 295 single-earner households out of 1541 families. This group has extreme specialization hence are useful in terms of understanding specialization within the household, but due to the missing wage rates, it is not examined in this paper, addressing them should be an important future work.

Apart from the two wage rates, I also need an individual's actual work experience since marriage in order to examine his (her) wage growth. For each individual, I sum his (her) annual labour market hours (zero or some positive value) since marriage till the last time I observe him (her) in the data. Couples who were not in the survey for a continuous period since their first interview are dropped out. As table 1.1 row 15 shows, there were 88 couples out of 937 being dropped out. Another issue with the accumulated work hours is that starting in 1997 the PSID surveys are recorded in every two years, so annual hours of work are not recorded in between. I impute the missing work hours as the average of work hours in adjacent years: for example, I impute work hours in year 2005 as the average of work hours in 2004 (in the 2005 survey) and work hours in 2006 (in the 2007 survey)[11].

1.2.2 Data description

The average marriage duration in the sample is 12.7 years and the median duration is 10 years. The average age at marriage is 25.4 for husbands and 24.1 for wives. Key variables examined in this analysis include annual hours of work, the (log of the real) hourly wage rate, educational attainment (whether a person has obtained a college degree), and the number and timing of children. Table 1.3 provides a basic

[11] I estimate the model by a simulated minimum distance estimator. For the simulated data set, I do the exact same imputation as I did to the actual data to mitigate the missing data problem due the switching from annual to biennial sampling.

description of market hours and wage ratios. On average husbands work one and half times (1.52) as many hours as their wives. The average annual work hours are computed as the total work hours since marriage divided by marriage duration. On average husbands work 2234 hours a year (approximately 45 hours a week) and wives work 1708 hours a year (approximately 34 hours a week). Husbands' annual hourly wage rates are also approximately one and half times higher (1.36 times higher at marriage and 1.66 times higher after being married for some time). At the beginning of marriage, the mean of hourly real wage rates for husband is 9.76 and for wives is 8.22. The average hourly wage rates after marriage is 14.9 for husbands and 11.5 for wives. Wives have larger variation (480 v.s 352) in the work hours and husbands have larger variation (5.4 v.s. 4.8 and 10.2 v.s. 7.3) in the wage rates.

Table 1.3: Labour Market Hours and Wage Rates

		mean	s.d.	min	max
hours ratio (husb/wife)		1.52	1.06	0.47	14.7
average annual work hours	husb	2234	352	928	3378
	wife	1708	480	138	2993
wage ratio at marriage (husb/wife)		1.36	0.79	0.11	6.49
wage rate at marriage	husb	9.76	5.4	3.3	48.8
	wife	8.22	4.8	2.6	54.7
wage ratio last observed (husb/wife)		1.66	1.65	0.20	22.2
wage rate last observed	husb	14.9	10.2	3.47	90.3
	wife	11.5	7.3	2.42	48.2

Matching in terms of Educational Attainments Table 1.4 shows assortative matching in the marriage market in terms of educational attainments. There are 255 couples (approximately 39%) in the data that both are without a college degree, and 230 couples (about 35%) that both the husband and the wife have obtained at least a college degree. There are 77 households (12%) in which the wife is "marrying up", only the husband has at least a college degree. And there are 88 couples (14%) where the wife with at least a college degree is "marrying down".

Table 1.4: Matching in terms of educational attainment

		Husband		
		Non-College	College	total
Wife	Non-College	255	77	343
	College	88	230	307
total		332	318	650

Table 1.5: Differences in the hours worked in labour market and children effects

		Husband Non-College	Husband College
	No. of children	Annual Hours Difference	
Wife Non-College	**all**	**553**	**603**
	0	285	190
	1	486	415
	>1	695	781
Wife College	**all**	**469**	**491**
	0	271	160
	1	349	315
	>1	591	646

Market hours difference Table 1.5 shows the differences between husbands and wives in terms of their work experiences after marriage, and the effects of children on those differences. First consider the "all" row, for couples both without a college degree, on average husbands works 553 hours more per year than the wives. When both have obtained at least a college degree, on average husbands work 491 hours more. The largest difference is when the husband has at least a college degree and the wife did not attend college, on average husbands work 603 hours more. The least difference is when the wife has a higher degree than her husband, still, husbands in that category work 469 hours more per year than their wives.

Children Effects Approximately 80% of the couples in my data set have at least one child during the observation period. Children have a strong impact on the work hour differences: they further enlarge the gender difference in the work hours after marriage. In all four educational matching categories in table 1.5, the difference in hours increases hugely when women enter motherhood, and increases further when the number of children increases. The impact of children on specialization is particularly large for couples where the husband has a higher degree, without children husbands only work 190 hours more than their wives, with one child they work 415 hours more than their wives and work 781 hours more per year when they have more children.

Variation across households I define market intensity as the sum of husband's and wife's hours worked since marriage divided by their marriage duration. The more (less) value attached to market (home) goods, the higher (lower) the market intensity we would observe. Specialization is defined as the wife's hours worked in the labour market since marriage divided by the sum of her husband's and her work hours. The smaller her share, the more she specializes in home production.

Figure 1.1: Specialization and Market Intensity

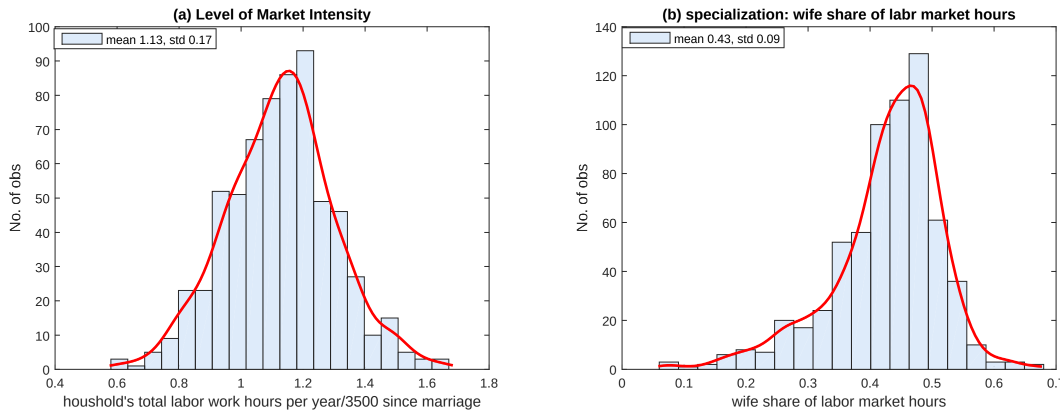
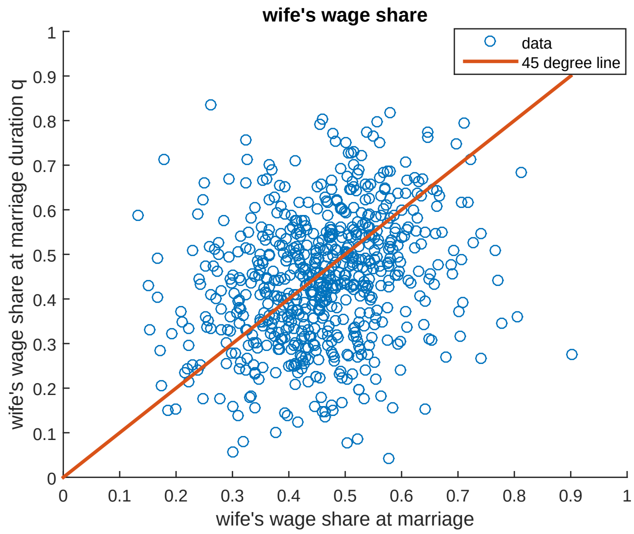


Figure 1.1 a) and b) restates what we have already seen from previous tables: on average a household will work $3500 \times 1.13 = 3955$ hours a year, and the wife and husband's work hour ratio is 0.43:0.57 which suggests that wives specialize more in home production than their husbands. Apart from these obvious mean effects, there is significant variations across households. Figure 1.1 shows consideration dispersion

in specialization and market intensity across households. The standard deviation is 0.17 and 0.09 respectively.

Figure 1.2: wife's wage share



marriage duration	std of wife's wage share
0-5	0.142
5-10	0.158
10-15	0.171
15-39	0.182

Figure 1.2 depicts the wide variation in wife's *wage shares* since marriage across households. The wife's wage share is calculated as her wage rate divided by the sum of her husband's and her own wage rate. The x-axis is wife's wage share in the year she just gets married, and the y-axis is her wage share the last time we observe her wage rate record in the data. As figure 1.2 shows, there are almost equal number of data points above and below the 45-degree line, those points above (below) the 45-degree line indicate that wife's wage share has increased (decreased) after marriage. What is the driving force behind these changes? This paper offers three possible candidates: specialization within the household together with effects of children, human capital accumulation, and measurement errors. The impact of the former two forces will likely depend on marriage duration. The longer a couple has been married, the more changes of the wife's share they may experience. These could be represented by the data points which are further away from the 45 degree line. Figure 1.2 also reports the standard deviation of wife's wage share change when I divide households into four groups according to the length of their marriage duration. The standard deviation increases from 0.14 to 0.18 when the marriage

duration increases. I shall present a model that aims to identify the driving forces behind.

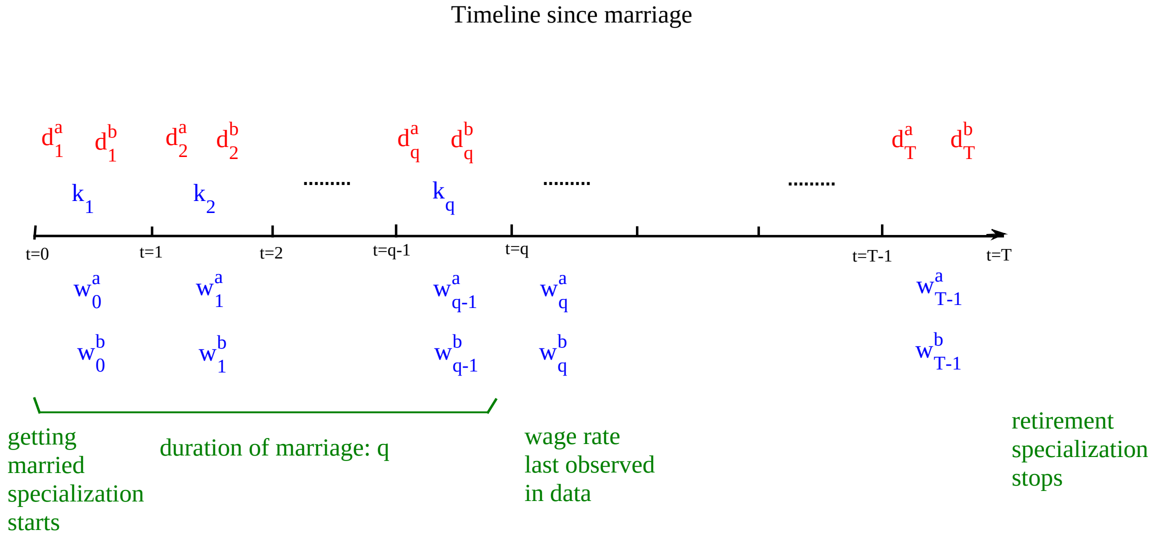
1.3 The Model

I present the decision process for married couples. Consider a household consists of two spouses, a husband a and a wife b. Figure 1.3 shows the time-line since marriage. At $t = 0$, the husband with wage rate w_0^a and the wife with wage rate w_0^b are married. Specialization starts as they make joint decisions on how to divide their time between working in the labour market and home production. I denote d_t as the proportion of time spent on home production at period t. $1 - d_t$ is the proportion of time spent in the labour market. Following, for example, Greenwood and Hercowitz (1991), Ramey and Francis (2009) and Bredemeier and Juessen (2013), I assume leisure is fixed for individuals in the main working age[12]. The specialization process stops until one of the couple retires. I take 65 as the retirement age, so the total number of specialization periods which denoted by T is $65 - max(age_0^a, age_0^b)$, where age_0 denotes a person's age at marriage. For most couples I do not observe them until retirement, rather I observe their marriage duration, denoted by q, and their wage rates last observed in the survey, denoted by w_q.

Couples know that their wage rates at beginning of each period will be fixed for that period, and their current wage rates together with their labour supply decisions at that period will affect their future wage rates. When couples make time allocation decisions for each period, they take human capital accumulation into consideration. Intuitively, a spouse with a comparative advantage in home production will devote more time to produce home goods, while the spouse with a better income prospect will specialize more in market production. Hence we would observe a spouse who works more in the labour market accumulates relatively more human capital over

[12] A number of other studies chose to divide discretionary time into three components: market work, homework and leisure, see Gronau (1997) for an overview.

Figure 1.3: Timeline Since Marriage



time and has a higher wage growth rate than his or her partner. The detailed wage dynamics are presented in section 3.2.

Moreover, at each period, children may arrive the household and influence the couple's decision process. In this model, the arrival of children denoted by k is assumed to be exogenous. Children affect a couple's decision in two ways. First, the couple's preference with respect to home production may change when there are young children present in the household. Second, the relative efficiency in home production may change in terms of rising and taking care of children. Couples will have to adjust their division of time accordingly. To examine these effects, I now present the parametric model.

1.3.1 Preferences and Technology

At each period, married couples consume two types of commodities: a market consumption good denoted by c_t and a home consumption good denoted by z_t.

Assume no saving or borrowing, market goods can be purchased with the couple's labour income:

$$c_t = w^a_{t-1} H_{max}(1 - d^a_t) + w^b_{t-1} H_{max}(1 - d^b_t)$$

w is the hourly wage rate and H_{max} is the maximum number of total hours that couples may divide between work and home production. I take $H_{max} = 3500$ in this analysis. Home production, z_t, is produced within the household and requires both husband's and wife's time:

$$z_t = \left[(1 - \theta_t)(d^a_t)^{(s_1 - 1)/s_1} + \theta_t (d^b_t)^{(s_1 - 1)/s_1} \right]^{s_1/(s_1 - 1)} \tag{1.1}$$

where $s_1 \neq 1$ and $s_1 \in (0, +\infty)$ denotes the constant elasticity of substitution between husband's time and wife's time in home production. $\theta_t \subset [0, 1]$ denotes the wife's relative efficiency of time-use in home production at period t. It depends on whether there are young children at that period:

$$\theta_t = \theta_0 + \theta_1 \widetilde{k}_t$$

$\widetilde{k}_t$ is a dummy variable equal to one if there is at least one child under age six in the household at period t and zero otherwise. $\theta_0 \subset [0, 1]$ is the relative efficiency of time-use in home production for wives without young children. $\theta_1 \subset [0, 1]$ denotes the wife's additional efficiency of time-use in home production when there are young children present[13].

Couples maximize their joint utility in a cooperative framework:

$$\max_{d^a_t, d^b_t} \sum_{t=1}^{T} p^t \left[(1 - \lambda_t)^{1/s_2} c_t^{(s_2 - 1)/s_2} + \lambda_t^{1/s_2} z_t^{(s_2 - 1)/s_2} \right]^{s_2/(s_2 - 1)} \tag{1.2}$$

where $0 < p < 1$ denotes the discount factor. d^a_t and d^b_t are the choice variables at

[13]In the estimation, I restrict the value of $\theta_0 \subset [0, 1]$ and $\theta_1 \subset [0, 1]$, but did not restrict θ_t to be between 0 and 1. The estimation results suggest that θ_0 is approximately 0.5 and θ_1 is approximately 0.15. The condition $\theta_t \subset [0, 1]$ is satisfied at the optimum. Same for λ_0, λ_1 and λ_t.

period t for each couple's specialization within marriage. $\lambda_t \subset [0,1]$ is the weight attached to home goods at period t and it depends on whether there are young children present in the family at that period:

$$\lambda_t = \lambda_0 + \lambda_1 \widetilde{k_t}$$

where $\lambda_0 \subset [0,1]$ is the weight attached to home goods for households without young children, or the unconditional taste for home goods. $\lambda_1 \subset [0,1]$ determines the additional importance of home goods when there are young children present in the household. $s_2 \neq 1$ and $s_2 \in (0, +\infty)$ denotes the constant elasticity of substitution between market consumption good and home produced good, which indicates how much people are willing to substitute between market goods and home goods.

1.3.2 Human Capital Accumulation

As mentioned earlier, human capital accumulation, which takes the form of *actual* labour market experience in this paper, is important. Now I introduce a key ingredient to the model: potential wage growth rates. The observed wage rate records w_t^a and w_t^b at period t are a combination of the true wage rates and some measurement error which I will specify later. The true wage dynamics, w_t^{a*} and w_t^{b*}, follow the process below:

$$w_t^{a*} = w_{t-1}^{a*}(1+g^a)^{(1-d_t^a)}(1-\delta^a d_t^a) \quad (1.3)$$

$$w_t^{b*} = w_{t-1}^{b*}(1+g^b)^{(1-d_t^b)}(1-\delta^b d_t^b) \quad (1.4)$$

where $g^a \geq 0$ and $g^b \geq 0$ denote the annual potential wage growth rates of husbands and wives respectively. δ^a and δ^b denote the human capital depreciation rates for husbands and wives. Take the first period for example, if the husband worked full time during the first period, $d_1^a = 0$, his wage will grow at rate g^a and his wage next period will be $w_0^{a*}(1+g^a)$. If he spent all his time in home production, $d_1^a = 1$, his wage next period will not increase and depreciate to $w_1^{a*}(1-\delta^a)$, which is smaller than w_1^{a*}. If he worked part time, say $d_1^a = 0.5$, his wage will grow at rate $(1+g^a)^{0.5} \simeq 0.5g^a$, and will depreciates at rate $0.5\delta^a$. Likewise for the wife.

In short, given the potential wage rate and the human capital depreciation rate, whether a person's *actual* wage growth rate is negative, zero or positive depends on his or her labour supply decision. The more accumulated stock of human capital, the higher his or her actual wage rate. This endogenous formation of human capital via the specialization decision per period is a key element of the model.

I assume there is no uncertainty in the wage dynamics, couples have perfect foresight and are able to make full-commitment to their future plans since marriage. The rational of this assumption comes from the earnings dynamic literature: when heterogeneity in trends is allowed, the unit root hypothesis is strongly rejected (Lillard and Weiss, 1979; Browning et al., 2010). In the context of long-run models, the effects of random shocks are washed out over time, and the wage trend is stationary and exhibits mean reversion. Following this strand, I now introduce heterogeneity in the wage growth to the model.

1.3.3 Heterogeneity of Wage Growth Rates and Assortative Matching

Assume each person has an individual-specific potential wage growth rate g_i that is positive and drawn from an inverse Gaussian distribution[14]:

$$\begin{aligned} g^a &\sim IG(\mu_{ga},\, \phi_{ga}) \\ g^b &\sim IG(\mu_{gb},\, \phi_{gb}) \end{aligned}$$

[14]The choice of inverse Gaussian distribution is made on the basis of how well the data appear to be fitted. The natural candidates to model skewed positive data are: the log normal distribution, the inverse Gaussian distribution, the gamma distribution and Weibull distribution. Some of the alternative distribution specifications are presented in section 1.5.3.

$\mu_{ga} > 0$ and $\mu_{gb} > 0$ are the mean parameters and $\phi_{ga} > 0$ and $\phi_{gb} > 0$ are the shape parameters. As the name suggests, the shape of an inverse Gaussian distribution depends on ϕ only[15] and is invariant under scale transformations of g. The variance of g is $\frac{\mu_g^3}{\phi_g}$. The average wage growth rates depend on the level of education:

$$\begin{aligned} \mu_{ga} &= g_0^a + g_1^a * college^a \\ \mu_{gb} &= g_0^b + g_1^b * college^b \end{aligned}$$

where $college^a$ and $college^b$ are dummy variables equal to one if the person has obtained at least a college degree, and equal to zero otherwise. g_0^a denotes the average wage growth rate for low educated husbands who work full time. g_1^a denotes an additional effect of having obtained at least a college degree. Likewise for wives. Note that since the variance of g is $\frac{\mu_g^3}{\phi_g}$, by construction the variance of potential wage growth rates differ across education groups, for high educated group the variance is $(g_0 + g_1)^3/\phi_g$, and for low educated group the variance is g_0^3/ϕ_g.

I now introduce a parameter governing the rank correlation of the potential wage growth rates between husbands and wives:

$$\rho_g^k = rank\ corr(g^a,\ g^b)$$

A rank correlation measures the degree of similarity between two rankings, so ρ_g^k here measures the relative positions of husbands' and wives' rankings in terms of their potential wage growth rates in their gender groups[16].

I assume each individual knows his or her own potential wage growth rate, and is also aware of wage growth rates of his or her potential partners. The sign and magnitude of ρ_g^k reveal the direction and the degree of assortative matching in the

[15]The inverse Gaussian can present a highly skewed (when $\phi \to 0$) to an almost normal distribution (when $\phi \to \infty$). Chhikara and Forlks (1977) made the point that compared to the log normal distribution, in some circumstances, the inverse Gaussian describes data better in terms of both peak and the extreme values.

[16]Kendall's τ (the rank correlation) is also used in Greenwood et al. (2014).

marriage market. $\rho_g^k = 0$ can be considered as there is just random matching on potential wage growth rates. If $\rho_g^k > 0$, it indicates that there is positive assortative matching that both men and women prefer a partner with a high potential wage growth rate, and men and women of similar ranks ar matched together. If $\rho_g^k < 0$, it is likely due to people's anticipation of intra-household specialization: a woman with a low potential wage growth rate prefers to marry a man with a high potential wage growth rate, as she desires the income he could bring to the family. The man at the same time is also looking for a wife to liberate him from housework so he can fully concentrate on his job. A woman with a low potential growth rate is more wiling to give up her wage growth than a woman with a very high potential wage growth rate. We can regard $\rho_g^k < 0$ as the case that the specialization effect dominates the positive assortative matching effect. Analogously, we can think $\rho_g^k = 0$ as the case that the specialization effect and the positive matching effect happen to just offset each other, hence the matching process can be viewed as if couples are matching randomly in the marriage market.

To simulate correlated inverse Gaussian distributed wage growth rates, I first simulate correlated standard uniform random variables $u^a \sim U(0,1)$ and $u^b \sim U(0,1)$ with a *linear* correlation coefficient denoted by ρ_u. I then apply the Copula method to construct dependent variables g^a and g^b: $g^a = IG^{-1}(u^a | \, \mu_g, \phi_g)$ and $g^b = IG^{-1}(u^b | \, \mu_g, \phi_g)$ where IG denotes the cumulative distribution of an inverse Gaussian random variable with specified μ_g and ϕ_g. The proof of g^a and g^b follow the inverse Gaussian distribution is straightforward:

$$Pr(g \leq x) = Pr(IG(g) \leq IG(x)) = Pr(IG[IG^{-1}(u)] \leq IG(x)) = Pr(u \leq IG(x)) = IG(x)$$

The dependence between g^a and g^b by construction is determined by the linear correlation ρ_u, but as I applied nonlinear transformations of u^a and u^b, the linear correlation of g^a and g^b is not exactly ρ_u anymore. Fortunately the rank correlation,

ρ_g^k, is invariant under monotonic transformations hence preserves the rank correlation of u^a and u^b. In the later section of estimation results, both ρ_u and the implied rank correlation ρ_g^k will be reported.

1.3.4 Measurement Error

To introduce measurement error, I take:

$$\begin{aligned} w_t^a &= w_t^{a*} exp(e_t^a) \qquad & e_t^a &\overset{i.i.d}{\sim} N(\mu_{me},\ \sigma_{me}^2)\,\forall t \\ w_t^b &= w_t^{b*} exp(e_t^b) \qquad & e_t^b &\overset{i.i.d}{\sim} N(\mu_{me},\ \sigma_{me}^2)\,\forall t \end{aligned}$$

where w_t^{a*} and w_t^{b*} denote husband's and wife's true wage rates at period t, and w_t^a and w_t^b denote their observed wage rates in the data. e_t^a and e_t^b are transitory measurement errors which are drawn independently from an identical normal distribution with mean μ_{me} and standard deviation σ_{me}. The key wage variables I will be examining later is (the log of) wage ratios, as the measurement error in different periods has the same mean, the observed log wage ratio is unbiased:

$$E\left(log\frac{w_q^a}{w_0^a}\right) = log\frac{w_q^{a*}}{w_0^{a*}} + E(e_q^a - e_0^a) = log\frac{w_q^{a*}}{w_0^{a*}}$$

Note that with measurement error and human capital depreciation, the *observed* wage growth rates are allowed to be negative.

1.3.5 An Analytical Special Case

To better understand the economics of couples' behavior, I first present a simple model before estimating the full model. There are three special cases of couple's utility function specified in (1.2) depending on the value of s_2, the constant elasticity of substitution between market goods and home goods. When $s_2 \to \infty$, the two commodities are considered to be perfect substitutes. When $s_2 \to 0$, the two commodities are considered to be perfect complements and we have the Leontief utility function. When $s_2 \to 1$, the utility function is reduced to a Cobb-Douglas function. I consider the case where $s_2 \to 1$, and per-period utility is a log linear form of the Cobb-Douglas utility function[17]:

$$\max_{d_t^a, d_t^b} \sum_{t=1}^{T} p^t [(1-\lambda) log(c_t) + \lambda log(z_t)]$$

Assume there is no children effect, hence no time subscript for λ.

Similarly, there are three special cases of the home production function specified in (1.1): if the constant elasticity of technical substitution, s_1, approaches to 0, husband's time and wife's time spent in home production are perfect complements and we have a Leontief production function where $z_t = Min[(1-\theta)d_t^a,\ \theta d_t^b]$. If $s_1 \to 1$, we have a Cobb-Douglas production function where $z_t = (d_t^a)^{(1-\theta)}(d_t^b)^{\theta}$. The third special case which I use in this example is that when $s_1 \to \infty$, that is, husband's time and wife's time are perfect substitutes:

$$z_t = (1-\theta)d_t^a + \theta d_t^b$$

Again assume there is no children effect, hence no time subscript for θ.

Assume no measurement error and no human capital depreciation, wage dynamics follow:

$$\begin{aligned} w_t^a &= w_{t-1}^a (1+g^a)^{(1-d_t^a)} \\ w_t^b &= w_{t-1}^b (1+g^b)^{(1-d_t^b)} \end{aligned} \tag{1.5}$$

Potential wage growth rates are heterogeneous and follow the inverse Gaussian

[17]Strictly speaking I should consider $\max_{d_t^a, d_t^b} \sum_{t=1}^{T} \beta^t [c_t{}^{(1-\lambda)} z_t^{\lambda}]$ which is not exactly equal to $\max_{d_t^a, d_t^b} \sum_{t=1}^{T} \beta^t [(1-\lambda) log(c_t) + \lambda log(z_t)]$, I chose the former for computation convenience. The relationship between the optimal behavior and the key structural parameters will hold in both cases.

distribution:

$$\begin{aligned} g^a &\sim IG(\mu_{ga},\, \phi_{ga}) \\ g^b &\sim IG(\mu_{gb},\, \phi_{gb}) \end{aligned}$$

Assume no education effect, so μ_{ga} and μ_{gb} do not depend on education. ϕ_{ga} and ϕ_{gb} are the shape parameters. The rank correlation between g^a and g^b, ρ_g^k, captures assortative matching on potential wage growth rates in the marriage market.

1.3.5.1 Propositions and an Example

Proposition 1.1 *In a single period where there is no human capital accumulation, when* $\frac{w_{t-1}^b}{w_{t-1}^a+w_{t-1}^b} < \theta$ *, optimal household specialization is the following:*

$$[d_t^{a*},\, d_t^{b*}] = \begin{cases} [\frac{\lambda-\theta}{1-\theta},\ 1] & when\ \theta < \lambda < 1 \\ [0,\ 1] & when\ \frac{w_{t-1}^b}{w_{t-1}^a+w_{t-1}^b} \le \lambda \le \theta \\ [0,\ \lambda\frac{w_{t-1}^a+w_{t-1}^b}{w_{t-1}^b}] & when\ 0 < \lambda < \frac{w_{t-1}^b}{w_{t-1}^a+w_{t-1}^b} \end{cases}$$

Proof. See Appendix. ■

For simplicity let us consider the case $\theta > \frac{1}{2}$ (a wife is relatively more efficient in home production than her husband), and $w_{t-1}^a > w_{t-1}^b$ (husband is the high income earner), then the condition $\frac{w_{t-1}^b}{w_{t-1}^a+w_{t-1}^b} < \theta$ is satisfied. Proposition 1.1 is hence intuitive: the spouse with a higher wage rate should devote more time to market work, and the spouse who is relatively more efficient in home production should spend more time at home, the optimal time allocation is always $d_t^{a*} < d_t^{b*}$[18].

Holding everything else constant, a household's total time devoted to home production, $d_t^{a*} + d_t^{b*}$, depends on λ, the parameter characterizing couples' preference towards home goods. The higher λ, the larger $d_t^{a*} + d_t^{b*}$. First consider the middle

[18] Note that we do not need the two simplifying assumptions to hold at the same time, either $\theta = \frac{1}{2}$, $w_t^a > w_t^b$ or $\theta > \frac{1}{2}$, $w_t^a = w_t^b$ will lead to the specialization result in Proposition 1.1.

case where $\frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \leq \lambda \leq \theta$, there is a full specialization where the husband spends all his time on labour market work, $d^{a*}_t = 0$; and the wife does not participate in the labour market and fully specializes in home production: $d^{b*}_t = 1$, so $d^{a*}_t + d^{b*}_t = 1$. When $\lambda > \theta$, this is when home goods are highly valued, in addition to wife's full specialization, the husband will also join home production: $d^{a*}_T = \frac{\lambda-\theta}{1-\theta} > 0$, hence $d^{a*}_t + d^{b*}_t > 1$. When $\lambda < \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}}$, that is when wife's wage share is higher than the relative importance of home goods, in addition to husband's full specialization in market work, the wife will also participate in the labour market, $d^{b*}_t = \lambda\frac{w^a_{T-1}+w^b_{T-1}}{w^b_{T-1}}$, which is proportional to λ and inversely proportional to her wage share, we have $d^{a*}_t + d^{b*}_t < 1$.

Proposition 1.2 *In a single period where there is no human capital accumulation, wife's share of the household total home production time is the following:*

$$\frac{d^{b*}_t}{d^{a*}_t + d^{b*}_t} = \begin{cases} 1 & \text{when } \theta \geq \lambda \text{ and } \theta \geq \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \\ \frac{1}{1+\frac{\lambda-\theta}{1-\theta}} & \text{when } \theta < \lambda \text{ and } \theta \geq \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \\ \frac{1}{1+\frac{\theta}{\lambda-(1-\theta)}} & \text{when } \theta \geq 1-\lambda \text{ and } \theta < \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \\ 0 & \text{when } \theta < 1-\lambda \text{ and } \theta < \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \end{cases}$$

where $\frac{1}{1+\frac{\lambda-\theta}{1-\theta}} > \frac{1}{1+\frac{\theta}{\lambda-(1-\theta)}}$.

Proof. See Appendix. ■

Proposition 1.2 states that the wife's share of total home production time increases with θ, the parameter characterizing her efficiency in home production. The larger θ, the more efficient she is at home production, therefore the more time she should devote to it than her husband.

Household specialization becomes more complicated when there is human capital accumulation, I shall illustrate with the example below.

Example 1.1 *Consider a simple two-period model, the husband has a lower initial* *wage rate at marriage:* $w_0^a = 10 < w_0^b = 11$*. The husband and his wife are equally efficient in home production:* $\theta = \frac{1}{2}$*. He will experience a positive wage growth if he works full time:* $g^a = 0.23 > 0$*, while there is no potential wage growth for his wife:* $g^b = 0$*. When* $\lambda = 0.53$*, the optimal household specialization is* $d_1^{a*} = 0$*,* $d_1^{b*} = 1$*,* $d_2^{a*} = \frac{\lambda-\theta}{1-\theta} > 0$ *and* $d_2^{b*} = 1$*.*

Example 1.1 shows that when home goods are highly valued and when the husband's potential wage growth is significantly high, the optimal household specialization should be that the wife devotes more time to home production than her husband in both periods: $d_t^{b*} > d_t^{a*}$ for $t = 1, 2$, despite the fact that she has a higher initial wage rate and is **no more** efficient in home production than her husband.

1.3.5.2 A Simulation

In section 1.4, I use a simulation based method to estimate the parameters of the full model. For the time being, I would like to take the simple model to highlight the relationships between some structural parameters and their corresponding moments in a simulated data-set. I will present several tables (table 1.7-1.13) in which I hold all but one of the model parameters constant at some benchmark values and show the effects of changing just one model parameter on the observable statistics.

Table 1.6 lists the structural parameters and the chosen observable statistics. q_i denotes marriage duration for household i. H^a and H^b denote husband's and wife's total labour market hours from marriage till period q. As marriage durations are different for different couples, for each household i, I divide their total hours spent on market work $H_i^a + H_i^b$ by q_i. I also divide it by 3500 as I take 3500 hours a year as the full time benchmark, hence, $\frac{H^a+H^b}{3500*q}$ is the actual work experience per household per year.

According to Proposition 1.1 and 1.2, in a single period model, if I change the parameter values of λ and θ, household's annual labour market work should decrease when λ increases, and the wife's share of market work should decrease when θ

increases. Will this result hold when I extend the model to a multi-period setting? Table 1.7 shows that holding everything else constant, the household annual market work experience decreases with an increasing λ. There is significant variation in the simulated moment mean of $\frac{H^a+H^b}{3500*q}$. Likewise, table 1.8 confirms the negative relationship between the wife's efficiency in home production and her market work hours. As θ increases, wife's share of market work decreases significantly, ceteris paribus.

Table 1.6: Structural Parameters and Key Variables in the Simple Model

Parameter	Interpretation
p	discount factor
λ	weight attached to home consumption good
θ	relative efficiency of wife's time-use in home production
μ_{ga}	average potential wage growth rate for husbands
μ_{gb}	average potential wage growth rate for wives
ϕ_{ga}	shape parameter of the husband's potential wage growth rate
ϕ_{gb}	shape parameter of the wife's potential wage growth rate
ρ_u	linear correlation between standard uniform random variables
(ρ_g^k)	implied rank correlation between husbands' and wives' potential wage growth rates
Key Variable	Interpretation
$\frac{H^a+H^b}{3500*q}$	annual actual years of work experience per household
$\frac{H^b}{H^a+H^b}$	share of wife's work hours since marriage
$\frac{log(w_q^a/w_0^a)}{(H^a/3500+1)}$	husband's (approximate) actual wage growth rate
$\frac{log(w_q^b/w_0^b)}{(H^b/3500+1)}$	wife's (approximate) actual wage growth rate

In theory, $H^a = \sum_{t=1}^{q} 3500 \times (1 - d_t^{*a})$ and $H^b = \sum_{t=1}^{q} 3500 \times (1 - d_t^{*b})$, where d_t^{*a} and d_t^{*b} are couple's optimal choices at period t. Following (1.5) and using the fact that $log(1+x) \simeq x$ when x is small, we have:

$$\begin{aligned} w_q^a &= w_0^a(1+g^a)^{\sum_{t=1}^{q}(1-d_t^{a*})} \\ log\left(\frac{w_q^a}{w_0^a}\right) &= \sum_{t=1}^{q}(1-d_t^{a*})log(1+g^a) \simeq \frac{H^a}{3500}g^a \Rightarrow \frac{log(w_q^a/w_0^a)}{H^a/3500} \simeq g^a \end{aligned}$$

Similarly, we have $\frac{log(w_q^b/w_0^b)}{H^b/3500} \simeq g^b$. Note that in simulation, for some values of the structural parameters, it is possible that simulated hours of work is 0: an individual with a low wage growth rate may fully specialize in home production. In that case $\frac{log(w_q/w_0)}{H/3500}$ is undefined. I let the denominator to be $\frac{H}{3500}+1$ instead in the simulation to avoid undefined wage growth values.

To understand the effect of μ_{ga}, ϕ_{ga}, μ_{gb} and ϕ_{gb}, we shall look at the mean and the standard deviation of observed wage growth rates. Recall the standard deviation is $\frac{\mu_g^3}{\phi_g}$. When I increase μ_{ga}, the mean and the standard deviation of $\frac{log(w_q^a/w_0^a)}{H^a/3500+1}$ both increase. Likewise when I increase μ_{gb} (see table 1.9 and 1.10). When I increase ϕ_{ga}, table 1.11 shows that the standard deviation of $\frac{log(w_q^a/w_0^a)}{H^a/3500+1}$ falls. Likewise for ϕ_{gb} (see table 1.12).

Last but not least, table 1.13 shows how the correlation of simulated husband wage growth and simulated wife's wage growth when ρ_u changes. As discussed in section 1.3.3, ρ_u is the linear correlation between standard uniform random variables that is used for simulation, ρ_g^k is the corresponding rank correlation between g^a and g^b. There is significant variation in the correlation of couples' observed wage growth, and it increases with ρ_u (hence ρ_g^k). The observed correlation of $\left(\frac{log(w_q^a/w_0^a)}{1+H^a/3500}, \frac{log(w_q^b/w_0^b)}{1+H^b/3500}\right)$ is always negative when $\rho_u < 0$. Moreover, even when $\rho_u = \rho_g^k = 0$, we observe negative correlation because of the specialization effect. Last, note that the observed wage growth correlation can be either positive or negative when $\rho_u > 0$, the sign depends on the structural parameters chosen.

Table 1.7: Changing λ, the value attached to home goods

	$\lambda = 0.1$	$\lambda = 0.2$	$\lambda = 0.3$	$\lambda = 0.4$	$\lambda = 0.5$	$\lambda = 0.6$	$\lambda = 0.7$	$\lambda = 0.8$	$\lambda = 0.9$
Moments									
mean of $\frac{H^a+H^b}{3500*q}$	1.81	1.58	1.34	1.15	1.07	1.02	1.00	0.94	0.62
mean of $\frac{H^b}{H^a+H^b}$	0.53	0.55	0.55	0.54	0.50	0.46	0.38	0.28	0.13
mean of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.07	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.05
mean of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	0.11	0.11	0.10	0.09	0.08	0.08	0.07	0.05	0.03
std of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.07	0.07	0.07	0.07	0.07	0.07	0.06	0.06	0.06
std of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	0.08	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.09
corr$\left(\frac{log(w_q^a/w_0^a)}{1+H^a/3500}, \frac{log(w_q^b/w_0^b)}{1+H^b/3500}\right)$	0.51	0.47	0.40	0.25	0.09	-0.13	-0.31	-0.36	-0.28

Note: The imposed parameter values are $\theta = 0.7$, $\mu_{ga} = 0.1$,$\mu_{gb} = 0.2$, $\sigma_{ga} = 0.05$ $\sigma_{gb} = 0.15$ and $\rho = 0.6$.

Table 1.8: Changing θ, the wife's efficiency of time-use in home production

	$\theta = 0.1$	$\theta = 0.2$	$\theta = 0.3$	$\theta = 0.4$	$\theta = 0.5$	$\theta = 0.6$	$\theta = 0.7$	$\theta = 0.8$	$\theta = 0.9$
Moments									
mean of $\frac{H^a+H^b}{3500*q}$	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2	1.2
mean of $\frac{H^b}{H^a+H^b}$	0.80	0.77	0.74	0.71	0.65	0.60	0.54	0.47	0.42
mean of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.06
mean of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	0.11	0.11	0.10	0.10	0.10	0.10	0.09	0.09	0.09
std of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
std of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10
corr$\left(\frac{log(w_q^a/w_0^a)}{1+H^a/3500}, \frac{log(w_q^b/w_0^b)}{1+H^b/3500}\right)$	0.32	0.30	0.25	0.25	0.25	0.25	0.25	0.27	0.27

Note: The imposed parameter values are $\lambda = 0.4$, $\mu_{ga} = 0.1$,$\mu_{gb} = 0.2$, $\sigma_{ga} = 0.05$ $\sigma_{gb} = 0.15$ and $\rho = 0.60$.

Table 1.9: Changing μ_{ga}, the mean of husband's wage growth rates

	$\mu_{ga} = 0.02$	$\mu_{ga} = 0.04$	$\mu_{ga} = 0.06$	$\mu_{ga} = 0.08$	$\mu_{ga} = 0.1$	$\mu_{ga} = 0.12$	$\mu_{ga} = 0.14$	$\mu_{ga} = 0.18$
Moments								
mean of $\frac{H^a+H^b}{3500*q}$	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.2
mean of $\frac{H^b}{H^a+H^b}$	0.72	0.67	0.62	0.56	0.54	0.52	0.49	0.47
mean of $\frac{log(w^a_q/w^a_0)}{1+H^a/3500}$	**0.01**	**0.02**	**0.03**	**0.04**	**0.05**	**0.06**	**0.07**	**0.08**
mean of $\frac{log(w^b_q/w^b_0)}{1+H^b/3500}$	0.10	0.10	0.10	0.09	0.09	0.09	0.09	0.09
std of $\frac{log(w^a_q/w^a_0)}{1+H^a/3500}$	**0.008**	**0.024**	**0.04**	**0.06**	**0.07**	**0.08**	**0.09**	**0.10**
std of $\frac{log(w^b_q/w^b_0)}{1+H^b/3500}$	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10
corr$\left(\frac{log(w^a_q/w^a_0)}{1+H^a/3500}, \frac{log(w^b_q/w^b_0)}{1+H^b/3500}\right)$	0.04	0.04	0.13	0.17	0.25	0.27	0.29	0.29

Note: The imposed parameter values are $\lambda = 0.4$, $\theta = 0.7$,$\mu_{gb} = 0.2$, $\sigma_{ga} = 0.05$ $\sigma_{gb} = 0.15$ and $\rho = 0.6$.

Table 1.10: Changing μ_{gb}, the mean of wife's wage growth rates

	$\mu_{gb} = 0.05$	$\mu_{gb} = 0.10$	$\mu_{gb} = 0.15$	$\mu_{gb} = 0.2$	$\mu_{gb} = 0.22$	$\mu_{gb} = 0.25$	$\mu_{gb} = 0.3$	$\mu_{gb} = 0.35$
Moments								
mean of $\frac{H^a+H^b}{3500*q}$	1.1	1.1	1.1	1.2	1.2	1.2	1.2	1.2
mean of $\frac{H^b}{H^a+H^b}$	0.14	0.34	0.46	0.54	0.55	0.58	0.61	0.63
mean of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
mean of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	**0.016**	**0.047**	**0.074**	**0.093**	**0.098**	**0.106**	**0.116**	**0.123**
std of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
std of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	**0.02**	**0.06**	**0.08**	**0.10**	**0.10**	**0.11**	**0.12**	**0.12**
corr$\left(\frac{log(w_q^a/w_0^a)}{1+H^a/3500}, \frac{log(w_q^b/w_0^b)}{1+H^b/3500}\right)$	0.02	0.12	0.21	0.25	0.25	0.23	0.23	0.25

Note: The imposed parameter values are $\lambda = 0.4$, $\theta = 0.7$, $\mu_{ga} = 0.1$, $\sigma_{ga} = 0.05$ $\sigma_{gb} = 0.15$ and $\rho = 0.6$.

Table 1.11: Changing ϕ_{ga}, the shape parameter of husband's wage growth rates

	$\phi_{ga} = 0.01$	$\phi_{ga} = 0.03$	$\phi_{ga} = 0.05$	$\phi_{ga} = 0.07$	$\phi_{ga} = 0.09$	$\phi_{ga} = 0.12$	$\phi_{ga} = 0.15$	$\phi_{ga} = 0.20$
Moments								
mean of $\frac{H^a+H^b}{3500*q}$	1.1	1.2	1.2	1.2	1.1	1.1	1.1	1.1
mean of $\frac{H^b}{H^a+H^b}$	0.63	0.57	0.54	0.52	0.50	0.49	0.49	0.47
mean of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.04	0.05	0.05	0.05	0.06	0.06	0.06	0.06
mean of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	0.09	0.10	0.09	0.09	0.09	0.09	0.09	0.09
std of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	**0.081**	**0.074**	**0.069**	**0.064**	**0.060**	**0.057**	**0.052**	**0.049**
std of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	0.09	0.09	0.10	0.10	0.10	0.10	0.097	0.10
corr$\left(\frac{log(w_q^a/w_0^a)}{1+H^a/3500}, \frac{log(w_q^b/w_0^b)}{1+H^b/3500}\right)$	0.28	0.28	0.25	0.20	0.17	0.13	0.056	0.034

Note: The imposed parameter values are $\lambda = 0.4$, $\theta = 0.7$, $\mu_{ga} = 0.1$,$\mu_{gb} = 0.2$, $\sigma_{gb} = 0.15$ and $\rho = 0.6$.

Table 1.12: Changing ϕ_{gb}, the shape parameter of wife's wage growth rates

	$\phi_{gb} = 0.05$	$\phi_{gb} = 0.10$	$\phi_{gb} = 0.15$	$\phi_{gb} = 0.2$	$\phi_{gb} = 0.25$	$\phi_{gb} = 0.30$	$\phi_{gb} = 0.35$
Moments							
mean of $\frac{H^a+H^b}{3500*q}$	1.14	1.14	1.2	1.2	1.2	1.2	1.2
mean of $\frac{H^b}{H^a+H^b}$	0.38	0.47	0.54	0.58	0.61	0.64	0.65
mean of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.06	0.05	0.05	0.05	0.05	0.05	0.05
mean of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	0.07	0.09	0.09	0.10	0.10	0.10	0.11
std of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.07	0.07	0.07	0.07	0.07	0.07	0.07
std of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	**0.106**	**0.101**	**0.095**	**0.091**	**0.0874**	**0.0843**	**0.0815**
corr$\left(\frac{log(w_q^a/w_0^a)}{1+H^a/3500}, \frac{log(w_q^b/w_0^b)}{1+H^b/3500}\right)$	0.21	0.20	0.24	0.24	0.210	0.215	0.2333

Note: The imposed parameter values are $\lambda = 0.4$, $\theta = 0.7$, $\mu_{ga} = 0.1$,$\mu_{gb} = 0.2$, $\sigma_{ga} = 0.05$ and $\rho = 0.6$.

Table 1.13: Changing ρ_u, the linear correlation between two standard uniform variables

	$\rho_u = -1$ $\Downarrow$ $\rho_g^k = -1$	$\rho_u = -0.8$ $\Downarrow$ $\rho_g^k = -0.58$	$\rho_u = -0.6$ $\Downarrow$ $\rho_g^k = -0.40$	$\rho_u = -0.2$ $\Downarrow$ $\rho_g^k = -0.13$	$\rho_u = 0$ $\Downarrow$ $\rho_g^k = 0$	$\rho_u = 0.5$ $\Downarrow$ $\rho_g^k = 0.3$	$\rho_u = 0.6$ $\Downarrow$ $\rho_g^k = 0.38$	$\rho_u = 0.7$ $\Downarrow$ $\rho_g^k = 0.47$	$\rho_u = 1$ $\Downarrow$ $\rho_g^k = 1$
Moments									
mean of $\frac{H^a+H^b}{3500*q}$	1.1	1.1	1.1	1.1	1.1	1.1	1.2	1.2	1.2
mean of $\frac{H^b}{H^a+H^b}$	0.54	0.54	0.54	0.53	0.53	0.53	0.54	0.54	0.63
mean of $\frac{log(w_q^a/w_0^a)}{H^a/3500}$	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05
mean of $\frac{log(w_q^b/w_0^b)}{H^b/3500}$	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10
std of $\frac{log(w_q^a/w_0^a)}{1+H^a/3500}$	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07	0.07
std of $\frac{log(w_q^b/w_0^b)}{1+H^b/3500}$	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.10
corr$\left(\frac{log(w_q^a/w_0^a)}{1+H^a/3500}, \frac{log(w_q^b/w_0^b)}{1+H^b/3500}\right)$	-0.59	-0.54	-0.48	-0.29	-0.18	0.16	0.25	0.35	0.78

Note: The imposed parameter values are $\lambda = 0.4$, $\theta = 0.7$, $\mu_{ga} = 0.1$,$\mu_{gb} = 0.2$, $\sigma_{ga} = 0.05$ and$\sigma_{gb} = 0.15$.

1.4 Structural Estimation

So far the model has presented a number of important factors simultaneously affecting couples' work hours and wages after marriage: couples' preference towards consuming home produced goods, their relative efficiency of time-use in home production, and their potential income relative to each another. Simple reduced form regressions are clearly insufficient to test the model and disentangle these effects, a better estimation strategy is to use structural estimation.

Structural estimation exploits the fact that different values of underlying structural parameters will generate different patterns in the simulated data, and aims to find parameter values which provide the best match between features of a simulated data set and the corresponding features of an empirical data set. To describe features of a data-set, auxiliary parameters are introduced. They are some chosen statistics providing useful information about the underlying structural parameters. Most often they are moments of a data-set, for example in this paper the mean of a wife's share of hours worked since marriage would tell us something about θ, the wife's relative efficiency in home production, as suggested by proposition 1.2. They can also be coefficients from a reduced form regression. For example, the coefficient which is often being interpreted as the actual return to experience in the Mincer wage equation, could be an important auxiliary parameter as it helps identify an individual's potential wage growth. Essentially, the auxiliary parameters are a function of the underlying structural parameters, and they should aim to provide a good description of the data at the same time allowing us to recover the structural parameters. The chosen auxiliary parameters are discussed in details in section 1.4.2.

1.4.1 The Indirect Inference Estimator

Let Θ denote the set of unknown structural parameters in the model. The simulated indirect inference method minimizes the discrepancy between a vector containing auxiliary parameters calculated from an empirical data-set, denoted by $\widehat{\Phi}^D$, and a vector containing simulated auxiliary parameters, denoted by $\widehat{\Phi}_r^S$. $\widehat{\Phi}_r^S(\Theta)$ contains the same auxiliary parameters as for $\widehat{\Phi}^D$ but calculated from a simulated data-set of the structural model for some values of Θ. The simulated auxiliary parameters are a function of Θ as they depend crucially on the structural parameters imposed in each round of simulation. Following Gourieroux, Monfort and Renault (1993), the simulated indirect inference estimator or the minimum distance (MD) estimator solves a minimum distance problem in a weighted quadratic sense:

$$\widehat{\Theta}_{MD} = arg\,\underset{\Theta}{min}\left(\frac{1}{R}\sum_{r=1}^{R}\widehat{\Phi}_r^S(\Theta) - \widehat{\Phi}^D\right)^{'}\hat{V}^{-1}\left(\frac{1}{R}\sum_{r=1}^{R}\widehat{\Phi}_r^S(\Theta) - \widehat{\Phi}^D\right)$$

where R denotes the number of replications of the model that we are going to draw for each simulation, A large R helps minimize simulation errors. $\widehat{V}$ is an optimal weighting matrix:

$$\hat{V} = (1 + \frac{1}{R})\widehat{V}_J$$

where $\widehat{V}_J$ is a positive definite covariance matrix calculated with a nonparametric bootstrap with J replications.

$$\widehat{V}_J = \frac{1}{J}\sum_{j=1}^{J}\left(\widehat{\Phi}^D - \overline{\Phi}^D\right)\left(\widehat{\Phi}^D - \overline{\Phi}^D\right)^{'}$$

where $\overline{\Phi}^D = \frac{1}{J}\sum_{j=1}^{J}\widehat{\Phi}^D$, and J denotes the number of bootstraps, I let $J = 500$.

If the model is correctly specified, given a large N, J and R, it can be shown that the value of the criterion at the optimum, denoted by $\hat{M}$, follows a chi-square distribution with a degree of freedom equal to the difference between the number of auxiliary parameters and the number of structural parameters:

$$\hat{M} = \left(\frac{1}{R}\sum_{r=1}^{R}\widehat{\Phi}_r^S(\widehat{\Theta}_{MD})-\widehat{\Phi}^D\right)' \hat{V}^{-1}\left(\frac{1}{R}\sum_{r=1}^{R}\widehat{\Phi}_r^S(\widehat{\Theta}_{MD})-\widehat{\Phi}^D\right) \sim \chi^2[dim(\widehat{\Phi})-dim(\Theta)]$$

This allows us to test the model and recover the structural parameters. By looking at the gap between auxiliary parameters of the data-set and their simulated counterparts, I can tell which features of the data the model fails to fit so that I can correct the model accordingly to see whether the fit can be improved.

After the estimation, hypothesis tests on parameters of interest can be conducted. The structural parameters can be partitioned into two sets: Θ_1 and Θ_2, where Θ_1 contains only parameters of interest and Θ_2 contains the remaining structural parameters. Take $\Theta_1 = 0$ as the null hypothesis for example, Two simulated minimum distance criteria need to be obtained: $\widehat{M_{unc}}$, the unconstrained criterion, and $\widehat{M_{con}}$, the constrained criterion where $\Theta_1 = 0$ is imposed. The difference between these two criteria follows a chi-square distribution with degrees of freedom being the dimension of Θ_1 under the null hypothesis:

$$\widehat{M_{cons}} - \widehat{M_{unc}} \sim \chi^2[dim(\Theta_1)]$$

I choose the indirect inference method to estimate the model because of the following reasons: first, there is no reduced form regression that allows us to examine the correlation of potential wage growth where couples take into consideration matching in the marriage market. Structural estimation allows us to clearly specify the mechanism and conduct experiment to quantitatively pin down the magnitude of different effects. Second, although the maximum likelihood estimation makes full use of the structural model and the specified relationships between unobservable distributions, it is computationally too demanding for this model. There is no closed form binding functions, and the switch to biennial sampling makes the likelihood function even more complex. Third, the indirect inference method helps mitigate the missing data problem due to the switch to biennial sampling. For example, the labour market hours in 2001 is not recorded, I have to impute the hours as an average of the hours worked in 2000 and 2002. In simulation I have couple's optimal hours in each year, I can ignore the simulated hours in 2001 and impute it as the average of the simulated hours in 2000 and 2002 just like what I did to the actual data.

1.4.2 Structural Parameters

Table 1.14 lists the set of parameters Θ that I aim to estimate, which can be divided into four categories. The first category consists of parameters characterizing the preference and technology: couples' preference towards home goods, the relative efficiency of time use in home production, the elasticity of substitution between market consumption goods and home goods, and the elasticity of substitution between husband's and wife's time spent in home production. I denote $PT = (\lambda_0, \lambda_1, \theta_0, \theta_1, s_1, s_2)$. PT can be thought as mainly determining intra-household specialization. The second category consists of parameters characterizing the heterogeneity of wage growth rates and human capital depreciation rates. Denote $PG = (g_0^a, g_0^b, g_1^a, g_1^b, \delta^a, \delta^b, \phi_{ga}, \phi_{gb})$. The third category consists of parameter(s) governing assortative matching on wage growth rates in the marriage market. Denote $PM = (\rho_u, \rho_g^k)$, where ρ_g^k is the implied rank correlation of growth rates according to ρ_u. The last category is miscellaneous, containing a parameter characterizing the standard deviation of the measurement error, σ_{me}. Besides these 16 structural parameters, there is an additional parameter that is assumed exogenous which also affects couple's specialization decision, the time discount factor p. For all the simulations I set $p = 0.97$[19].

[19] Asiedu and Villamil (2000) presents estimates of the discount factor for 40 countries, 0.95 is a common value used. With PSID data, Gourinchas and Parker (2002) structurally estimates the discount factor and reports values around 0.96.

Table 1.14: Structural Parameters

Parameter	Definition
	Specialization Effect
λ_0	weight attached to home goods for households without young children
λ_1	additional weight attached to home goods when young children were present in the households
θ_0	wife's relative efficiency of time-use in home production
θ_1	wife's additional efficiency of time-use when young children were present
s_1	elasticity of substitution between husband's and wife's time in home production
s_2	elasticity of substitution between market goods and home goods
	Human Capital Effect
g_0^a	husband's annual wage growth rate without a college degree
g_0^b	wife's annual wage growth rate without a college degree
g_1^a	husband's additional wage growth rate with at least a college degree
g_1^b	wife's additional wage growth rate with at least a college degree
δ^a	husband's annual human capital depreciation rate
δ^b	wife's annual human capital depreciation rate
ϕ_{ga}	shape parameter of husband's annual wage growth rate
ϕ_{gb}	shape parameter of wife's annual wage growth rate
	Matching on Wage Growth
ρ_u	linear correlation between standard uniform random variables
(ρ_g^k)	(implied rank correlation between husband's and wife's potential wage growth rates from ρ_u)
	Miscellaneous
σ_{me}	standard deviation of measurement error
p	time discount factor

1.4.3 Auxiliary Regressions:

I run 6 auxiliary regressions (regression 1.6, 1.7, 1.8. 1.9, 1.10 and 1.11) to identify the 16 theory parameters (ρ_g^k is not counted) in table 1.14. The regressor $Dchild \subseteq [0, 1]$ denotes the proportion of time in marriage that there is at least one young (age under 6) child present in the household:

$$\frac{H_i^a + H_i^b}{3500 * q_i} = \gamma_1 + \gamma_2 Dchild_i + v_{1i} \tag{1.6}$$

γ_3 measures the standard deviation of residuals from regression 1.6:

$$\widehat{\gamma_3} = std(\widehat{v_1})$$

$$\frac{H_i^b}{H_i^a + H_i^b} = \gamma_4 + \gamma_5 Dchild_i + v_{2i} \tag{1.7}$$

γ_6 measures the standard deviation of residuals from regression 1.7:

$$\widehat{\gamma_6} = std(\widehat{v_2})$$

$$log\left(\frac{w_q^a}{w_0^a}\right) = \alpha_0 + \alpha_1 \frac{H^a}{3500} + \alpha_2 college^a \frac{H^a}{3500} + \alpha_3 q + v_3 \tag{1.8}$$

$$log\left(\frac{w_q^b}{w_0^b}\right) = \beta_0 + \beta_1 \frac{H^b}{3500} + \beta_2 college^a \frac{H^b}{3500} + \beta_3 q + v_4 \tag{1.9}$$

Denote $\widehat{v_3} = log(\frac{w_q^a}{w_0^a}) - \widehat{\alpha_0} - \widehat{\alpha_1}\frac{H^a}{3500} - \widehat{\alpha_2}\frac{H^a}{3500}college^a - \widehat{\alpha_3}q$, then run the regression:

$$\left(\widehat{v_3}\right)^2 = \alpha_4 + \alpha_5 \left(\frac{H^a}{3500}\right)^2 + \alpha_6 * college^a * \left(\frac{H^a}{3500}\right)^2 + v_5 \tag{1.10}$$

Similarly denote $\widehat{v^b} = log(\frac{w_q^b}{w_0^b}) - \widehat{\beta_0} - \widehat{\beta_1}\frac{H^b}{3500} - \widehat{\beta_2}\frac{H^b}{3500}college^b - \widehat{\beta_3}q$, then run the regression:

$$\left(\widehat{v_4}\right)^2 = \beta_4 + \beta_5 \left(\frac{H^b}{3500}\right)^2 + \beta_6 * college^b * \left(\frac{H^b}{3500}\right)^2 + v_6 \tag{1.11}$$

κ_{ab} measures the correlation of the residuals in regression 1.8 and 1.9:

$$\widehat{\kappa_{ab}} = corr(\widehat{v_3},\ \widehat{v_4})$$

1.4.4 Identification

The non-linearity of the model setup makes it difficult to determine a well-specified binding function that maps exactly each theory parameter to a corresponding auxiliary parameter. Nevertheless, as I have illustrated with a simplified case in section 1.3.5, with some parametric assumptions, the theory parameters listed in table 1.14 can be identified through various variation in the data.

1.4.4.1 Identification of Parameters Governing Intra-household Specialization

The intercept in regression 1.6, γ_1, measures the average total labour market time in a year for households without children. γ_2 measures the additional children effect. Looking at couple's utility function, we see that λ indicates how people value home goods relative to market goods. Intuitively, the bigger λ, the more time a household will devote to home production, hence we expect less labour market participation on average. A bigger λ_0 and λ_1 implies a smaller γ_1 and γ_2, hence γ_1 and γ_2 help identify λ_0 and λ_1.

γ_3 measures the standard deviation of the average household labour market time and help identify s_2, the elasticity of substitution between home goods and market goods. If s_2 is large, consider s_2 is greater than 1, home goods and market goods are highly substitutable, we would expect to see more variation in the household actual labour market time. Couples who worked much more in the labour market and couples who spend much more time at home may have the same preference towards home goods (the same λ), and their differed choices may simply due to a large s_2.

Analogously, γ_4 and γ_5 in regression 1.7 measures the average wife's share of labour market hours[20] without and with young children and help identify θ_0 and θ_1. If the wife is relatively more efficient in home production (a higher θ), due to specialization we would expect her to have a smaller share of labour market hours (smaller γ_4 and γ_5). γ_6 measures the standard deviation of the wife's share of labour market hours and help identify the elasticity of substitution between husband's time and wife's time as inputs of home production. If s_1 is small, consider s_1 is smaller than 1, husband's time and wife's time are not highly substitutable, we would expect to see a relatively small variation in the wife's share of labour hours. This could help explain why specialization is less common for husbands devoting more time to home production, even though their wives have a higher wage or higher wage growth rate. Due to a small s_1, they stuck at home production and were unable to swap with their husbands to work in the labour market.

1.4.4.2 Identification of Parameters Related to Wage Growth

Following (1.3) and re-arrange the equation, we have:

$$\begin{aligned} w_q^{a*} &= w_0^{a*}(1+g^a)^{\sum_{t=1}^{q}(1-d_t^{*a})}\prod_{t=1}^{q}(1-\delta^a d_t^{*a}) \\ log\left(\frac{w_q^{a*}}{w_0^{a*}}\right) &= log(1+g^a)\sum_{t=1}^{q}(1-d_t^{*a}) + \sum_{t=1}^{q} log(1-\delta^a d_t^{*a}) \\ &= \frac{H^a}{3500} log(1+g^a) + \sum_{t=1}^{q} log(1-\delta^a d_t^{*a}) \end{aligned}$$

where $H^a = \sum_{t=1}^{q} 3500(1-d_t^{*a})$. Using the fact that $log(z) \simeq z-1$ for any real number $z \subseteq (0, 2)$:

$$log\left(\frac{w_q^{a*}}{w_0^{a*}}\right) \simeq \frac{H^a}{3500}g^a - \delta^a \sum_{t=1}^{q} d_t^{*a} = \frac{H^a}{3500}(g^a+\delta^a) - \delta^a q$$

[20] Looking at the utility function, we see that at any period, spending all time spent on home production (that is $d_t^a = 1$ and $d_t^b = 1$) is always suboptimal, therefore $H^a + H^b$ should always be positive, hence I can divide H^b by $H^a + H^b$ without worrying the denominator being 0.

Adding the measurement error:

$$\begin{aligned} log\left(\frac{w_q^a}{w_0^a}\right) &= log\left(\frac{w_q^{a*}\nu_q^a}{w_0^{a*}\nu_0^a}\right) = log\left(\frac{w_q^{a*}}{w_0^{a*}}\right) + log\left(\frac{e_q^a}{e_0^a}\right) \\ &= \frac{H^a}{3500}(g^a + \delta^a) - \delta^a q + v_{me}^a \end{aligned}$$

where v_{me}^a denotes the log ratio of measurement errors in the two periods (for husbands' wages). Substitute in $g^a = g_0^a + g_1^a * college^a$:

$$\begin{aligned} log\left(\frac{w_q^a}{w_0^a}\right) &= \frac{H^a}{3500}[g_0^a + g_1^a * college^a + \delta^a] - \delta^a q + v_{me}^a \\ &= [g_0^a + \delta^a]\frac{H^a}{3500} + g_1^a * \frac{H^a}{3500} * college^a - \delta^a q + v_{me}^a \end{aligned}$$

Comparing equations above and the auxiliary regression 1.8, it is clear that the coefficients α_1, α_2 and α_3 help identify parameters for the growth rate, g_0^a and g_1^a , and the human capital depreciation rate, δ^a. Likewise for wives.

To examine the heterogeneity of wage growth rates, we can regard the wage growth equation as a random slope model without an intercept:

$$\begin{aligned} y_i &= (\zeta_0 + \zeta_i)x_i + u_i \\ E(y_i|x_i) &= \zeta_0 x_i \\ E\Big((y_i - \widehat{\zeta_0}x_i)^2|x_i\Big) &= Var(\zeta_i)x_i^2 + E(u_i^2) \end{aligned}$$

assuming x and u are independent. ζ_0 is the mean effect and ζ_i implies heterogeneity of ζ across individuals. In order to identify the variance of ζ_i, we need to take the square of the residuals from the OLS regression y on x and regress them on a constant and the square of x. Regression 1.10 and 1.11 take this procedure, so α_5 and α_6 help to identify the heterogeneity of husband's growth rate σ_{ga}, and α_4 helps identify the standard deviation of the measurement error σ_{me}.

Lastly but not least, let's consider κ_{ab} which measures the correlation of residual v_3 and v_4 from regression 1.8 and 1.9 respectively. According to the theory, we can regard v_3 as a combination of the heterogeneous wage effect $f(\sigma_{ga})\frac{H^a}{3500}$ and some measurement error v^a_{me}. Since the theory assumes measurement errors are independent across time and person, the correlation of v_3 and v_4 therefore should help identify the correlation of husband's and wife's wage growth rates[21].

1.4.5 Estimates of Auxiliary Parameters

Table 1.15 reports results of regression 1.6 and 1.7. Estimates of regression 1.6 show that without young children a household spends approximately $1.18 \times 3500 = 4130$ hours a year working in the labour market. If throughout the marriage duration ($t = 0$ to $t = q$) there are always young children present in the household, on average they spend $0.15 \times 3500 = 525$ hours more at home (less in the labour market) per year.

Table 1.15: Household Specialization

	market intensity	wife's share of work hours
	$\frac{H^a_i+H^b_i}{3500q_i}$ (regression 1.6)	$\frac{H^b_i}{H^a_i+H^b_i}$ (regression 1.7)
Dchild	-0.15***	-0.073***
	(0.02)	(0.01)
Intercept	1.18***	0.45***
	(0.01)	(0.004)
root MSE	0.17	0.085
R^2	0.06	0.06

standard errors in parentheses. N=650. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Estimates of egression 1.7 show that without children, wife's share of labour market hours is approximately 45%, while the husband works more (55%). For an average household, the wife's annual labour market hours are $4130 \times 0.45 \simeq 1856$,

[21] In the literature, the wage residuals in a typical Mincer earning regressions would represent the unobserved characteristics related to the earning potential. With PSID data (1983-89), Nakosteen and Zimmer (2001) found the Mincer residuals immediately after marriage are positively correlated between husbands and wives. The correlation is small, they argue that the effect of assortative mating on unobservables is significant at 0.09 level. The channel from matching to the wage residuals is not clearly specified though.

and the husband work approximately $4130 \times 0.55 = 2272$ hours a year in the labour market. If there are young children present throughout the observation period ($t = 0$ to $t = q$), she will work $(4130 - 525) \times (0.45 - 0.073) \simeq 1359$ hours a year, which means having children reduced her work hours by 497 per year. The husband will work approximately 2246 hours a year, and having children has a very small effect on his labour market work as he will work only 26 hours less per year.

Table 1.16: Wage Growth and Depreciation

	Husband	Wife
	$log\frac{w_q^a}{w_0^a}$ (1.8)	$log\frac{w_q^b}{w_0^b}$ (1.9)
total hours/3500	0.025	0.052***
	(0.01)	(0.01)
total hours/3500 *College	0.021***	0.013*
	(0.004)	(0.006)
duration of marriage	-0.013	-0.019***
	(0.01)	(0.005)
Intercept	0.24***	0.20***
	(0.03)	(0.03)
R^2	0.08	0.06

standard errors in parentheses. N=650. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 1.16 reports results of regression 1.8 and 1.9 which measure husband's and wife's returns to actual work experience respectively. For a husband without a college degree, the return to his work experience is not significantly different from 0. For a husband with at least a college degree, full-time work will increase his wage by approximately 2.1% a year. For a wife without a college degree, her wage will increase by 5.2% a year if she works full time, and if with at least a college degree, her return to experience will further increase by 1.3%. Coefficients of marriage duration measure the annual depreciation rate of human capital if a person does not participate in the labour market. We see that the husband's wage rates do not depreciate significantly, this is most likely due to the fact that most husbands work full time, their actual work experience after marriage are similar to their marriage duration, hence the regressor marriage duration does not have any additional explanatory power. This is not the case for wives. If a wife does not participate in the labour market after marriage, her wage will depreciate by approximately 1.9% each year.

1.5 Estimation Results

1.5.1 Estimates of the Structural Parameters

Table 1.17: Parameter Estimates

Parameter	Notation	Estimates	Std.Error.	$\rho = 0$
Specialization				
weight attached to home goods	λ_0	0.61	0.002	0.60
children effect	λ_1	0.09	0.05	0.09
wife's efficiency in home production	θ_0	0.54	0.07	0.56
children effect	θ_1	0.26	0.185	0.23
elasticity between husband and wife's time	s_1	0.30	0.04	0.33
elasticity between market and home goods	s_2	0.37	0.044	0.33
Human Capital				
mean growth rate	g_0	0.039	0.004	0.024
education effect	g_1	0.014	0.004	0.013
husband's human capital depreciation rate	δ^a	0.044	0.0136	0.023
wife's human capital depreciation rate	δ^b	0.019	0.006	0.01
shape parameter of wage growth rates	ϕ_g	0.12	0.0312	0.09
Matching				
linear correlation btw standard uniform r.v.	ρ_u	0.99	0.0016	0
implied rank correlation	(ρ_g^k)	(0.88)		(0)
Measurement Error	σ_{me}	0.46	0.0175	0.47
Number of model parameters		13 (χ_6^2)		12 (χ_7^2)
critical value of χ^2 at 5% level		12.59		14.07
Simulated minimum distance:		4.52		11.90

Imposed discount factor: $p = 0.97$.

Table 1.17 presents estimates of the structural parameters. Overall the theory provides a good fit to the data. There are 13 structural parameters and 19 auxiliary parameters, the simulated distance hence follows a chi-square distribution with 6 degrees of freedom. The simulated minimum distance is approximately 4.52 which is much smaller than 12.59, the critical chi-square value at 5% significant level.

Estimates of λ_0 cannot be interpreted as any scaling of the units of market goods would alter the estimated value of λ_0. λ_1 is not significantly different from zero, this suggests that the arrival of children do not change couples' preference towards home goods. This finding is different from the estimation result in van Klaveren et al. (2008), where the authors find that women value home production more when there are young children present in the household.

Estimates of θ provide information about the wife's comparative advantage in home production relative to her husband. The fact that $\theta_0 = 0.54$ (θ_0 is significantly different from 0.5) leads to the conclusion that wives are more efficient in home production. In addition, having young children in the households further enlarges the efficiency difference as θ_1 is significantly greater than zero. The estimates presented here, $\theta_0 = 0.54$ and $\theta_1 = 0.26$ are in line with Bredemeir and Juessen (2013), where the authors calibrate $\theta = 0.7$ and interpret it as the female elasticity in home production. With data on Japanese households, Lise and Yamada (2014) find that wives are not necessary more productive in home production (the mean of the structurally estimated share parameter for the wife is 0.56, but it is *not* significantly different from 0.5).

The estimated elasticity of substitution between wife's and husband's time-use in home production is around 0.3, indicating that the husband's time and wife's time are actually complements in home production. This is different from Lise and Yamada (2014) where the authors find the elasticity of substitution is 2.3 in Japan. Here I interpret all non-market time as home production, but it could also contain leisure, and this complementarity result could be due to spousal complementarity in leisure, where a couple value each other's companionship (Aguiar and Hurst, 2007; Goux et al. 2014).

The estimated elasticity of substitution between market goods and home goods, s_2, is 0.37, indicating the two types of goods are not highly substitutable. The estimate of s_2 is different from values estimated by others in the literature. With micro data, Rupert et al. (1995) report that the elasticity of substitution is between 1 and 2. Aguiar and Hurst (2007) report an estimate of 1.8. Fang and Zhu (2012) find it to be 2.1. Using Macro data, MaGrattan et al. (1997) find an elasticity in the range $1.7 - 1.8$, and Chang and Schorfheide (2003) find a higher value of approximately 2.3. In the later robustness check section (section 1.5.3), table 1.22 column 4 presents the parameter estimates when I restrict $s_2 > 1$, the simulated minimum distance is 20.39. Although with $s_2 > 1$ there exist some parameter values that can fit the household specialization patterns in the data well, it fails to recover a matching parameter ρ which explains the correlated wage growth residuals and the observed wage growth for couples. This provides evidence that $s_2 < 1$ is preferred. The elasticity of substitutions between market good and home good is essential in determining whether home production should be considered in economic models, and this could have important implications on macroeconomic models with the household sector (see Becker, 1988; McGratthan et al. 1997 for example). My finding shows that it is important to take matching into account when estimating this elasticity of substitution.

The wage growth rates of husbands and wives turn out to have the same underlying distribution (the tests for $g_0^a = g_0^b$, $g_1^a = g_1^b$ and $\phi_{ga} = \phi_{gb}$ are presented in section 1.5.3 table 1.22 column 1): without a college degree their average potential wage growth rate is close to 3.9%, and having a college degree significantly increases their potential wage growth by approximately 1.4 percentage points. The mean parameters g_0, g_1 and the shape parameter ϕ_g together imply the standard deviation is 0.022 for low educated individuals and 0.035 for high educated individuals[22].

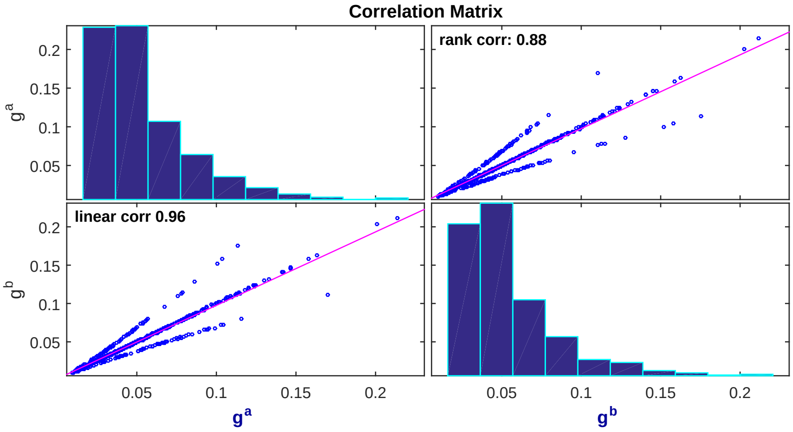
The human capital depreciation rates turn out to be significant and different across gender groups (again the test for $\delta^a = \delta^b$ are presented in section 1.5.3 table 1.22 column 2). Furthermore, there is no association between depreciation rates and education (estimation results are in section 1.5.3 table 1.22 column 3). For husbands the depreciation rate is approximately 4.4% and for wives it is approximately 2%. These values are in line with the estimates in the literature survey by Browning et al. (1999). In more recent work, Attanasio et al. (2008) using PSID data calibrate the annual wage loss as 3% for women. Huggett et al. (2011) set the average depreciation rate to 2% a year. With structural estimation, Blundell et al. (2013) find a depreciation rate around 11% for women using UK BHPS data. Adda et al. (2015) using German data find an estimate of 0.06-6.9% for one year interruption in women's career.

Last and foremost, together with estimated values of ρ_u, g_0, g_1, and ϕ_g, the implied linear and rank correlation between growth rates g^a and g^b is approximately 0.96 and 0.88, respectively. The standard error is 0.0016, indicating the estimate is precise and highly significant. As discussed in Section 1.3.3, a positive large ρ_g^k reflects a strong positive effect of assortative matching on couple's potential wage growth rates in the marriage market.

Figure 1.4 shows the relationship between the potential wage growth rates of husbands and wives, and separated histograms of g^a and g^b respectively. The pink line has a slope of 0.96 which indicates the points along the line are households in which the potential wage growth rates of husbands and wives are almost equal. For those households, the specialization decision would depend only on their initial wage ratio at marriage. Points above (below) the pink line indicate matches in which $g^a > g^b$ ($g^a < g^b$): the husband has a higher (lower) potential wage growth rate than the wife. For those households, presumably the spouse with a higher growth rate

[22] Recall the variance of an inverse Gaussian random variable is $\frac{\mu^3}{\phi}$ where μ is the mean parameter and λ is the shape parameter.

Figure 1.4: Correlated Potential Wage Growth Rates

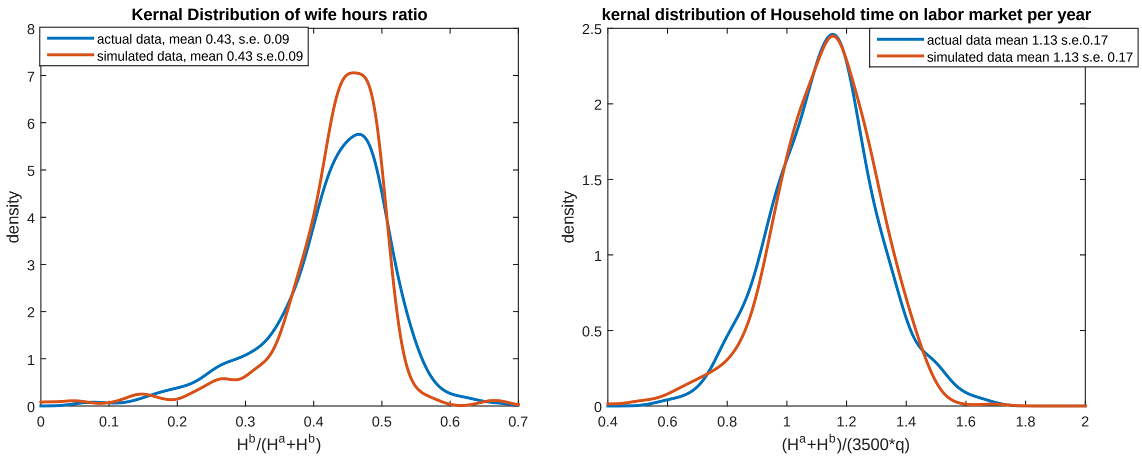


will specialize more in market work.

1.5.1.1 Model Fit

The prediction of intra-household specialization and their market intensity levels is close to the data, as shown in figure 1.5, both mean and variance are closely matched, except that in terms of wife's hours ratio, the theory does not predict the fatter tails of the actual data.

Figure 1.5: specialization and market intensity



The first five columns in Table 1.18 show that the simulated auxiliary parameters are all matched to the data auxiliary parameters.

Figure 1.6 compares the wife's wage share since marriage which shows that the

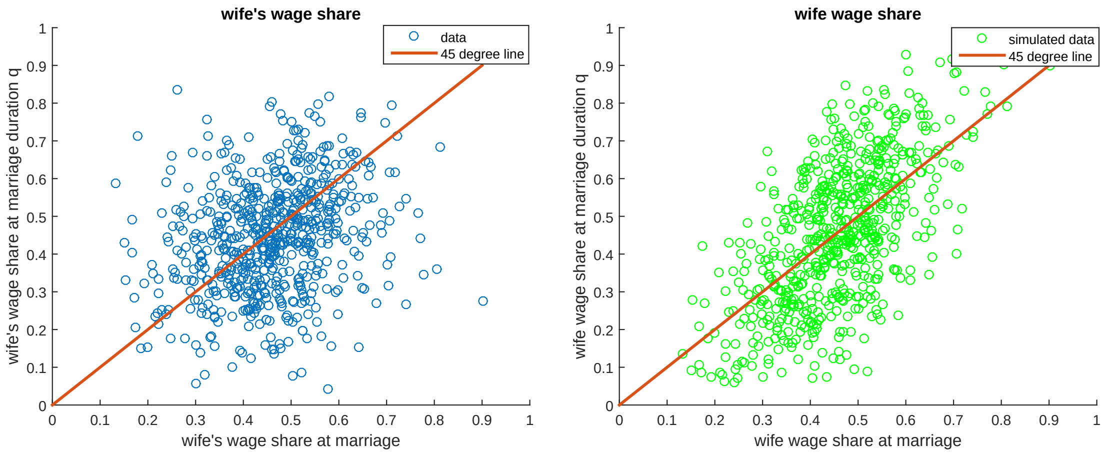
Table 1.18: Auxiliary Parameter Estimates

	Benchmark model					$\rho = 0$		
	empirical	simulated	s.e.	t-value	M	simulated	t-value	M
Specialization								
γ_1	1.18	1.18	0.01	0.10	1	1.18	0.03	1
γ_2	-0.15	-0.14	0.02	0.12	1	-0.15	0.04	1
γ_3	0.17	0.17	0.005	0.18	1	0.17	0.14	1
γ_4	0.45	0.45	0.005	0.10	1	0.46	0.45	1
γ_5	-0.073	-0.07	0.012	0.33	1	-0.07	0.54	1
γ_6	0.085	0.084	0.003	0.26	1	0.085	0.07	1
Human Capital Accumulation								
α_1	0.025	0.028	0.018	0.16	1	0.030	0.28	1
α_2	0.021	0.017	0.005	0.98	1	0.016	1.08	1
α_3	-0.013	-0.012	0.012	0.03	1	-0.014	0.08	1
α_4	0.22	0.25	0.023	1.30	1	0.25	1.09	1
α_5	0.0003	0.0001	0.0002	1.55	1	-0.000	2.27	0
α_6	0.0002	0.0004	0.0002	0.82	1	0.0003	0.46	1
β_1	0.052	0.049	0.012	0.29	1	0.049	0.27	1
β_2	0.013	0.017	0.006	0.70	1	0.019	0.98	1
β_3	-0.019	-0.019	0.005	0.06	1	-0.020	0.05	1
β_4	0.25	0.21	0.027	1.48	1	0.20	1.90	0
β_5	-0.0001	0.0002	0.0002	1.21	1	0.0003	1.84	0
β_6	0.0006	0.0003	0.0004	0.79	1	0.0006	0.09	1
Assortative Matching								
κ_{ab}	0.094	0.11	0.043	0.34	1	0.001	2.17	0

A t-value is computed as the difference between a simulated auxiliary parameter and the corresponding data auxiliary parameter divided by its standard error. M equals to one if the parameter is matched and equals to zero otherwise. A parameter is matched to the data if the t value is smaller than 1.64.

simulated model captures the variation to a large extent: Both in the PSID data and simulated data, there is large variation in the wife's share. There are almost equal number of points above and below the 45 degree line. What the theory did not capture are points in the northwest and southeast quadrant, those are the households where the wife's wage share changed most significantly since marriage, and they are also likely to be the fatter tails in figure 1.5.

Figure 1.6: wife's wage share comparison



1.5.2 Positive Assortative Matching

1.5.2.1 Evidence for Positive Assortative Matching

The structural estimation finds strong evidence for positive assortative matching. If I let $\rho = 0$, but allow other structural parameters to be re-estimated, I obtain a set of new estimates of the simulated minimum distance. The new estimates are shown in the last column in table 1.17(the case $\rho = 0$). Parameters related to intra-household specialization and human capital accumulation are still identified and close to the previous estimates. Nevertheless, the simulated minimum distance increases to 11.90, the quasi-likelihood ratio test statistic is therefore $11.90 - 4.52 = 7.38$ which is greater than 3.84, the critical chi-square value with one degree of freedom, hence the model with $\rho > 0$ is preferred and I conclude that ρ is significantly positive.

Take the new estimates in the last column of table 1.17, the new corresponding auxiliary parameters are shown in the last three columns of table 1.18(for the case $\rho = 0$). Note that in the last row, the auxiliary parameter κ_{ab} which measures the correlation between the wage growth residuals is not matched anymore, $\rho = 0$ implies that the simulated correlation of the wage growth residuals is *not* significantly different from zero, while for the actual data the positive correlation is significant.

1.5.2.2 Assortative versus Random Matching

To examine the effect of positive assortative matching, I take the structural estimates in column 3 of table 1.17 which I use as the benchmark model, and turn off the matching effect by imposing $\rho = 0$.

Effects on observed wage growth rates Table 1.19 records the simulated auxiliary parameters of the model when I impose $\rho = 0$. For both husbands and wives, regardless of their educational attainments, the simulated wage growth rates are always higher when $\rho = 0$ than in the benchmark model in which $\rho > 0$. Note that both α_1 and β_1 increase significantly, suggesting that the impact of changing from positive assortative matching to random matching is significant mainly for low educated people.

Why are the observed wage growth rates higher in the random matching setting? The intuition is the following: with random matching, it is more likely that high potential wage growth rates are being realized and low potential wage growth rates are being dominated. Therefore we would expect higher overall wage growth rates when matching is random.

I present a simple example to illustrate this point. Consider the case where there are three men and women, each being ranked according to his or her potential wage growth rate in their gender groups, we have $g_1^a > g_2^a > g_3^a$ and $g_1^b > g_2^b > g_3^b$.

Table 1.19: Counter-Factual simulation $\rho = 0$

Simulated Auxiliary Parameter Estimates				
	Benchmark Model	$\rho = 0$	s.e.	t-value
Specialization				
γ_1	1.18	1.18	0.01	0.28
γ_2	-0.14	-0.15	0.02	0.25
γ_3	0.17	0.17	0.005	0.93
γ_4	0.45	0.46	0.004	0.71
γ_5	-0.07	-0.07	0.012	0.29
γ_6	0.084	0.096	0.003	3.80
Human Capital Accumulation				
α_1	0.028	0.077	0.018	2.71
α_2	0.017	0.019	0.005	0.41
α_3	-0.012	-0.043	0.012	2.61
β_1	0.049	0.079	0.012	2.54
β_2	0.017	0.022	0.006	0.85
β_3	-0.019	-0.032	0.0054	2.33
Assortative Matching				
κ_{ab}	0.11	0.02	0.04	2.0

The t-values are computed as (ap$_{simulated}$ -ap$_{simulated\,\rho=0}$)/s.e.

The relationship between g^a and g^b is shown in the "Ranking" column in table 1.20. Perfect positive assortative matching (PAM) implies that people with the same rank of potential wage growth rates are matched together. For simplicity let us ignore the relative efficiency parameter θ and initial wage levels for the time being. Assuming full specialization, the actual wage growth rates we would observe afterwards are

Table 1.20: A simple example of PAM v.s. Random Matching

Ranking	Positive Assortative Matching
$g_1^a < g_1^b$	$g_1^a \leftrightarrow g_1^b \; (g_1^b)$
$g_2^a > g_2^b$	$g_2^a \leftrightarrow g_2^b \; (g_2^a)$
$g_3^a < g_3^b$	$g_3^a \leftrightarrow g_3^b \; (g_3^b)$

Random Matching

(1)	(2)	(3)
$g_1^a \leftrightarrow g_1^b \; (g_1^b)$	$g_1^a \leftrightarrow g_1^b \; (g_1^b)$	$g_1^a \leftrightarrow g_2^b \; (g_1^a)$
$g_2^a \leftrightarrow g_2^b \; (g_2^a)$	$g_2^a \leftrightarrow g_3^b \; (g_2^a)$	$g_2^a \leftrightarrow g_1^b \; (g_1^b)$
$g_3^a \leftrightarrow g_3^b \; (g_3^b)$	$g_3^a \leftrightarrow g_2^b \; (g_2^b)$	$g_3^a \leftrightarrow g_3^b \; (g_3^b)$
(4)	**(5)**	**(6)**
$g_1^a \leftrightarrow g_2^b \; (g_1^a)$	$g_1^a \leftrightarrow g_3^b \; (g_1^a)$	$g_1^a \leftrightarrow g_3^b \; (g_1^a)$
$g_2^a \leftrightarrow g_3^b \; (g_2^a)$	$g_2^a \leftrightarrow g_2^b \; (g_2^a)$	$g_2^a \leftrightarrow g_1^b \; (g_1^b)$
$g_3^a \leftrightarrow g_1^b \; (g_1^b)$	$g_3^a \leftrightarrow g_1^b \; (g_1^b)$	$g_3^a \leftrightarrow g_2^b \; (g_2^b)$

Table 1.21: Observed average growth rates: Random Matching v.s. PAM

	Random Matching						PAM
	(1)	(2)	(3)	(4)	(5)	(6)	
husb avg growth	g_2^a	g_2^a	g_1^a	g_2^a	$\frac{g_1^a+g_2^a}{2}$	g_1^a	g_2^a
wife avg growth	$\frac{g_1^b+g_3^b}{2}$	$\frac{g_1^b+g_2^b}{2}$	$\frac{g_1^b+g_3^b}{2}$	g_1^b	g_1^b	$\frac{g_1^b+g_2^b}{2}$	$\frac{g_1^b+g_3^b}{2}$

shown in the brackets. Take the first couple for example. Both have the highest potential wage growth in their gender groups, but since $g_1^a < g_1^b$, only the wife will work in the labour market and realize her potential growth rate g_1^b. The husband will give up his wage growth and specialize in home production. In this simple three pairs example, positive assortative matching implies that the average wage growth rate observed for women is $\frac{g_1^b+g_3^b}{2}$ and for men it is g_2^a, as the spouse specializing in home production would have no observed wage growth.

Now consider the random matching case. There are six possible matching scenarios shown in table 1.20. The observed wage growth rate for each pair is shown in the brackets. Accordingly, table 1.21 (1)-(6) calculate the *observed average* growth in each case of the random matching setting. Clearly for both husbands and wives, the observed growth rates are either higher or at least equal in the random matching setting than in the perfect sorting case.

The reasoning is the following: in both the perfect sorting and random matching setting, the person with the highest wage growth rate, in this case g_1^b, will always engage in labour market work and realize her full potential growth no matter who she is matched with. The person with the lowest wage growth rate, in this case g_3^a, will stay at home and never realize his wage growth rate. Now consider their partners whose potential wage growth is g_1^a and g_3^b respectively. Even when g_1^a is high, say only ϵ smaller than g_1^b where ϵ is very small, it will not be realized when matched with g_1^b. Even when g_3^b is small, say only ϵ bigger than g_3^a, it will always be realized when matched with g_3^a. For the random matching case, there is some probability that g_1^a is going to be realized, when it is not matched with g_1^b (table 1.20 (3), (5) and (6)), and there is some probability that g_3^b is going to be dominated when it is not matched with g_3^a (table 1.20 (2), (4), (5) and (6)). Since g_1^a is large and g_3^b is small, the overall growth rates would be higher in the random matching setting.

This simple example helps explain the empirical conterfactuals: consider a woman with a potential wage growth rate that is highly ranked among women in the marriage market. In the perfect positive sorting case, she is going to be matched with a man of the same rank, how much she works in the labour market depends on whether this man's wage growth rate is higher or lower than hers. Under random matching, there is a higher probability that she is going to be matched with someone below her rank since she is ranked high, so she will be more likely to specialize in market work and realize her high potential growth rate. The same story applies to men with highly ranked wage growth rates. Now consider a woman with a potential wage growth rate which is ranked nearly in the bottom. Again under perfect sorting, she is going to be matched with a man of the same rank, whether she will work in the labour market depends on their relative wage growth rate. Under random matching, there is a higher probability that she is going to be matched with someone ranked higher than her since she is ranked in the bottom, she will be more likely to specialize in home production, hence her *observed* wage growth would be low. The same story applies to men. In summary, with positive assortative (random) matching, it is more (less) likely that some high potential wage growth rates are being dominated and some low potential wage growth rates are being realized. Therefore we would expect lower (higher) overall wage growth rates when matching is positive assortative (random).

Now I explain why the increase in the observed wage growth rates are mainly significant for low educated groups. Recall education contributes positively to the potential wage growth rate and high education group has a larger variance. Consider a person's potential growth rate as the following: $g_i = \mu_g + \sigma_{gi}$, where μ_g denotes the mean wage growth rate, and σ_{gi} denotes the departure from the mean growth rate for person i. Consider only two possible positive values: σ_{g_1} amd σ_{g2} where $\sigma_{g_1} > \sigma_{g2}$. For low educated people, their wage growth could be either $g_i = \mu_g + \sigma_{g2}$ or $g_i = \mu_g - \sigma_{g2}$. For high educated people, their wage growth could be either $g_i = g_0 + g_1 + \sigma_{g1}$ or $g_i = g_0 + g_1 - \sigma_{g1}$. We already know that people with very high potential growth rates or very low potential growth rates are not affected much when matching switches from positive sorting to random matching. These groups are likely to be people with high education and high values of σ_{gi} (consider for example $g_0 + g_1 + \sigma_{g1}$), and people with low education and also low values of σ_{gi} (consider fore example $g_0 - \sigma_{g2}$). For people in the middle of the spectrum, those with low education but relatively high values of σ_{gi} (consider for example $g_0 + \sigma_{g2}$) and those with high education but relatively low potential wage growth rates (consider for example $g_0 + g_1 - \sigma_{g1}$), they will be affected most when matching changes. Both groups are more likely to be dominated by their partners and hence specialize in home production when matching is positive assortative. When matching is random, there is a higher probability that they will work in the labour market. When there are more people with low education but relatively high values of σ_{gi} working in the labour market, the overall wage growth rates observed for low educated group will increase. When there are more people with high education but relatively low values of σ_{gi} working in the labour market, the overall wage growth rates observed for high educated group will fall. This explains why the wage growth rates are mainly significant for the low educated groups.

Following this argument, it is then not surprising to see α_3 and β_3 become more negative when $\rho = 0$. α_3 and β_3 measure the wage depreciation rates if a person's work experience is zero. When matching switches from positive assortative to random matching, there will be more people with relatively high potential wage growth rates working in the labour market (that's why we observe higher wage growth rates). If they do not participate in the labour market, there will be more wage growth forgone, hence α_3 and β_3 become more negative.

Effects on Specialization within the household There is no significant change in $\gamma_1 - \gamma_5$, the parameters governing the household market intensity and intra-household specialization, except that the standard deviation of the wife's share in total household work hours, γ_6, has increased from 0.084 to 0.096. The change suggests there is more variation in the household specialization arrangements under random matching.

The matching mechanism does not affect households' market intensity. This reasoning is the following: consider the simplest case that there is extreme specialization, the spouse with a higher potential wage growth rate will work full time in the labour market, and the spouse with lower income potential will specialize fully in home production. Whether there is random matching or perfect assortative sorting, the total market hours of each household will be the same, that there is just one spouse working full time. This helps explain why parameters $\gamma_1 - \gamma_3$ do not change when matching switches from assortative to random matching.

Under perfect positive assortative matching, for the three households in table 1.20, we observe two households in which the wife specializes fully in labour market work (g_1^b and g_3^b), and in one household the wife specializes in home production (g_2^b). Under random matching, in case (1) and (3) the specialization arrangements are the same as in the PAM setting. In case (2), (4), (5) and (6) we observe different specialization arrangements. Clearly the random matching setting allows all possible matches while the PAM setting restricts certain matches, it is not surprising there is more variation in the wife's share of labour market hours under random matching and we observe a higher γ_6.

1.5.3 Robustness checks

Table 1.22 presents estimates with other specifications. Column 1 estimates the model in which I allow both the potential wage growth rates and human capital depreciation rates differ across genders. The estimates of g_0^a and g_0^b are 0.37 and 0.38, g_1^a and g_1^b are 0.14 and 0.11, and the shape parameters ϕ_{ga} and ϕ_{gb} are 0.11 and 0.09, respectively. When I impose $g_0^a = g_0^b$, $g_1^a = g_1^b$ and $\phi_{ga} = \phi_{gb}$ (the estimates are presented earlier in section 5.1), the simulated minimum distance does not change much (it increases from 4.43 to 4.52). This leads to the conclusion that the potential wage growth rates of husbands and wives follow the same distribution. Column 3 presents the estimates when I further impose $\delta^a = \delta^b$, the simulated minimum distance increases to 10.22 from 4.52, which provides evidence that the human capital depreciation rates are different between husbands and wives. Column 4 presents estimates when I allow the depreciation rate depends on education, that is I specify $\delta = \delta_0 + \delta_1 * college$. Both δ_1^a and δ_1^b are not significantly different from zero. Besides, the simulated minimum distance is 4.18 which is close to 4.5. It suggests that it

Table 1.22: Parameter Estimates

	Notations	(1) $g^a \neq g^b$ $\delta^a \neq \delta^b$	(2) $g^a = g^b$ $\delta^a = \delta^b$	(3) $\delta(edu)$	(4) $s_2 > 1$	(5) Log N	(6) Gamma
No. of para		16	12	15	13	15	16
Specialization	λ_0	0.61	0.59	0.61	0.44	0.45	0.58
children effect	λ_1	0.09	0.09	0.09	0.04	0.05	0.10
wife's efficiency	θ_0	0.55	0.55	0.55	0.52	0.57	0.57
children effect	θ_1	0.25	0.24	0.26	0.15	0.15	0.23
elasticities	s_1	0.31	0.33	0.30	0.47	0.43	0.39
	s_2	0.37	0.38	0.36	1.32	1.24	0.29
Human Capital	g_0^a	0.037	0.028	0.037	0.016	-4.30	0.05
edu effect	g_1^b	0.014	0.011	0.015	0.017	0.83	0.004
	g_0^b	0.038				-4.00	0.05
edu effect	g_1^b	0.011				0.56	0.001
depreciation	δ_0^a	0.04	0.016	0.04	0.02	0.01	0.001
	δ_1^a			0.0001			
	δ_0^b	0.02		0.017	0.01		0.000
	δ_1^b			0.005			
shape para	ϕ_{ga}	0.11	0.057	0.12	0.15	0.13	0.13
shape para	ϕ_{gb}	0.09				0.50	0.19
Matching	ρ_u	0.99	0.88	0.99	0.33	0.99	0.83
(rank corr)	(ρ_g^k)	0.88	(0.66)	(0.87)	(0.24)	(0.54)	(0.62)
measure. error	σ_{me}	0.46	0.46	0.46	0.49	0.49	0.45
sim. mim.dist		4.43	10.22	4.18	20.39	12.27	36.98
distribution		χ_3^2	χ_7^2	χ_4^2	χ_6^2	χ_4^2	χ_3^2
critical value 5%		7.82	14.07	9.49	12.59	9.49	7.82

is not the case where educated people have higher depreciation rates. Column 5 presents estimates when I impose the elasticity between home goods and market consumption to be greater than one. As discussed in section 1.5, several studies suggest elasticity of substitution is greater than one. The simulated distance is 20.39 which is greater than the critical chi-square value with 6 degrees of freedom. Compared with estimates where I restrict $s_2 < 1$, having $s_2 > 1$ gives a much worse data fit and it fails to capture the correlated wage growth residuals and observed wage growth of husbands and wives[23]. Column (6) and (7) presents estimates when the wage growth rates follow the log normal distribution and the Gamma distribution respectively. The simulated minimum distance is 12.27 and 36.98, indicating an inverse Gaussian distribution provides a much better fit to the data than the log normal distribution and the Gamma distribution.

1.6 Conclusion

In summary, this paper finds that: first, there is strong assortative matching on potential wage growth rates in the marriage market. The rank correlation between potential wage growth rates of husbands and wives is around 0.88. Positive assortative matching is important in terms of explaining the correlated wage growth residuals of married couples, and if matching is random, the observed average wage growth rate of households would be higher, and are mainly due to an increase in the average wage growth rates among low educated groups. In addition, the switch from assortative matching to random matching does not affect households' total market hours, but there will be more variation in intra-household specialization under random matching.

Second, work hours of both husbands and wives can be rationalized by a model in which their underlying potential wage growth rates follow the same distribution. One of the reasons we observe on average wives' market hours are lower than their husbands' is due to a lower human capital depreciation rate for female workers. The other two reasons are that wives are slightly more efficient in home production than

[23]The fit of the auxiliary parameters are not reported here, but the results are qualitatively similar to the case where I restrict $\rho = 0$.

their husbands, and the difference in the efficiency enlarges when there are young children present in the households.

Contrary to other studies where estimates of the elasticity of substitution between market work and home production is greater than one, I find an estimate of around 0.37. Restricting the elasticity greater than one in my structural estimation gives a worse data fit and fails to capture the correlated wage growth residuals and observed wage growth of husbands and wives. Moreover, the estimate of the elasticity between husband's time and wife's time is 0.30, suggesting that husbands' time and wife's time are actually complements in home production, while in the literature, they seem to be substitutes. These new estimates provide evidence that it is important to take into matching into account.

In my analysis the switch from positive assortative matching to random matching does not change households' arrangements much. Even after I experiment with introducing a gender gap in potential wage growth rates, and impose husbands always work full time, still there is no significant difference between households arrangements under the two matching settings. It would be interesting to explore whether this result still holds if the model is extended to a collective model setting, where specialization in home production may change a person's bargaining power and his or her private consumption.

Chapter 2

Matching with Contracts: An Efficient Marriage Market?

2.1 Introduction

This paper studies who marries whom and how couples share their resources within marriage. In a transferable utility model, I examine the possibility of achieving an efficient matching outcome, that is, an outcome in which the aggregate gains from marriage are maximized. I focus on settings where i) there is two-sided information asymmetry, and ii) the gains from marriage are stochastic.

2.1.1 Motivation

Marriages are not formed randomly. Marriage patterns observed in society suggest that individuals often sort themselves into marriages based on the attributes of partners. Starting with Becker, economists have long pointed out that marriages are formed because interactions between individuals' attributes generate mutual gains from marriage. To study who marries whom, Becker (1973) and Shapley and Shubik (1971) analyzed a marriage market with transferable utility and no frictions (no information asymmetry, no search cost). With transferable utility, the problem of obtaining an efficient and stable matching simplifies to a problem of maximizing the aggregate match output. Therefore complementarity of traits, that is, supermodularity of the match production function, must lead to positive assortative matching (PAM), where people of similar traits are matched together.

However, in real life the matching process is characterized by scarcity of information about potential matches. One strand of the literature focuses on the meeting technology and assumes search costs (Diamond, 1982; Moretensen, 1988; Burdett and Coles, 1997 and 1999, Shimer and Smith, 2000 and Smith, 2006). The alternative approach to adding realism to the matching model is to introduce information asymmetry (Damiano and Li, 2007; Hoppe, Moldovanu and Sela, 2009). Individuals' attributes may not be fully revealed to one another before a match. Even after a match has formed, not all attributes are observable to researchers.

This paper takes the second approach: The type of each agent is only privately known. Assuming supermodularity of the match production function, I examine whether PAM can be achieved in the presence of information asymmetry.

Unlike previous literature, I analyze an incomplete-information model where the sharing rules and the matching outcomes are jointly determined within the model. In models of frictionless marriage markets (Gale and Shapley, 1962; Becker, 1973; Browning et al., 2003), the division of marital output and matching outcomes are determined simultaneously. In a negotiation over the sharing of the match surplus, bargaining power comes from one's ability to replace his or her current spouse with another. An efficient and stable matching must be supported by sharing rules specifying how to divide the match surplus. However, in the literature on matching markets with information asymmetry, how to split the shares is often pre-imposed, so there is no room for negotiation. For instance, both Damiano and Li (2007) and Hoppe et al. (2009) examined the setting in which, when a man x matches with a woman y, together they produce $2xy$ with certainty and each partner gets half of the produced output. The half-half sharing rule is fixed and known ex-ante to all types of agents. It seems inconsistent that, whereas in models of frictionless marriage markets, the sharing rule and the matching outcome are simultaneously determined, in models of markets with information asymmetry, the sharing rule is exogenously imposed.

The second novelty of this paper's analysis is that the gains from marriage are stochastic. The stochastic component of marital surplus is often ignored in the literature on matching markets with information asymmetry. There, an individual's uncertainty about match surplus comes only from uncertainty about the type of his or her partner. Such an assumption ignores the possibility of shocks to the marital surplus after matching, yet the shocks can have important consequences. In fact, in most of the literature on divorce and re-marriage, shocks to marital output play an important role. Browning et al. (2014) show that high aggregate divorce rates can be beneficial because they facilitate the recovery from negative shocks to match quality, allowing couples to replace bad marriages by better ones. This paper assumes divorce is hugely costly; therefore before any match, people use the available information as much as possible in order to achieve a good matching outcome and avoid future separation. I show that the stochastic component of the marital surplus is important in determining whether or not a mechanism achieving efficient matching exists.

A related paper which also relates noise to efficiency is by Bhaskar and Hopkins (2014) who examine pre-marital investment with stochastic returns. The key differences are that their main focus is on efficiency of pre-marital investment rather than efficient matching, and their setting is a frictionless marriage market with non-transferable utility.

Another related work by Dizdar and Moldovanu (2014) asks questions in the same spirit as this paper: What sharing rules are compatible with a truthful direct revelation mechanism that achieves matching efficiency? This paper differs in three key aspects from theirs. First, unlike their paper, where agents of multi-dimensional types are allowed to compete with ex-ante monetary transfers before the match process, I exclude any ex-ante monetary transfer and investigate whether matching efficiency can be achieved *by endogenous sharing rules alone*, Second, while their analysis rules out stochastic shocks to match output and examines an ex-post equilibrium, I allow marital gains to be stochastic and characterize the conditions under which a Bayesian Nash equilibrium may be found. Third, they focused on studying agents of multi-dimensional types, where types are one-dimensional and binary in my setting.

To examine how the sharing rules and the matching outcomes are endogenously determined in a model with two-sided information asymmetry, I adopt the standard assumption in the marriage literature[1] regarding how couples bargain over the surplus: Prospective couples can make *binding, costlessly enforceable, prenuptial agreements* to transfer resources within marriage. This assumption has been labeled Binding Agreements on the Marriage Market (BAMM).[2]

Since contracts specifying how to divide the match surplus are credible, binding and costlessly enforceable, this paper asks: Do there exist contracts that achieve an efficient matching outcome? Note that individuals in this setting face two types of uncertainty. The first type arises from asymmetry of information; ex-ante a woman (or a man) cannot observe her (his) potential partners' types, so she (he) does not know whom to match with and therefore is not sure about the amount of marital output that will be produced by a match. When agents on one side of the matching market offer contracts, the sharing rules proposed need to serve two purposes: revealing one's type (signaling) and attracting a suitable partner (screening). There may exist trade-offs between these two functions. The second type of uncertainty arises because the marital output contains random elements; even without infor-

[1]See Gale and Shapley, 1962; Becker, 1973; Choo and Siow, 2006; Chiappori, Iyigun and Weiss, 2006; Nick and Walsh, 2007; Browning, Chiappori and Weiss, 2014.

[2]This assumption is not as strong as it sounds. Any transfer that (i) is decided before marriage, and (ii) can be used to alter the spouse's respective bargaining positions after marriage is considered as a BAMM framework (Browning, Chiappori and Weiss, 2014).

mation asymmetry, after a match has formed, the total marital output produced is subject to a random shock. Therefore contracts can only specify how to divide each ex-post *realized* surplus. This worsens the trade-offs between the signaling and the screening functions of a contract. I show that when the marital surplus is stochastic and there is two-sided information asymmetry, prenuptial contracts specifying divisions of ex-post realized marital surplus may be *ineffective* in achieving matching efficiency.

2.1.2 Outline

This paper is organized as follows: Section 2.2 studies a game in which one side of the matching market offers contracts specifying sharing rules, and the agents on the other side then decide which contract to accept. I examine conditions under which matching efficiency is achievable, and show that matching efficiency cannot be achieved when expected marital surplus is strictly monotonic in agents' types.

Section 2.3 analyzes the problem of a social planner who aims to maximize the aggregate matching surplus. He commits both to a matching rule and contracts specifying the division of the surplus, conditional on the reported types and the realization of the surplus of each matched pair. A social planner can do strictly better in terms of achieving matching efficiency than the equilibrium of the game in which one side offers contracts. I derive necessary and sufficient conditions for the existence of a truthful direct revelation mechanism that achieves matching efficiency. I show that these conditions become more stringent as the number of agents in the matching market increases. In the limits as the number of agents goes to infinity, matching efficiency is not achievable even in the social planner's setting.

Comparisons of alternative models and some extensions are discussed in Section 2.4. Section 2.5 concludes.

2.2 Game with One Side Offering Contracts

There are N women and N men, all are risk neutral. Each one is independently drawn from a distribution with probability $\frac{1}{2}$ to be a high type agent and probability $\frac{1}{2}$ to be a low type agent. Each agent observes only his or her own type. The payoff of an unmatched agent is assumed to be 0 regardless of types.

2.2.1 Benchmark Model: Matching Surplus is Deterministic

For each possible combination of types, we have the following:

$$\begin{aligned} W^H + M^H &= z_2 \\ W^H + M^L &= z_1' \\ W^L + M^H &= z_1 \\ W^L + M^L &= z_0 \end{aligned}$$

The first equation denotes that when a high type woman, denoted by W^H, matches with a high type man, denoted by M^H, together they will produce a matching surplus of magnitude z_2. Similarly for the other three combinations of types. The match surplus z_2, z_1', z_1 and z_0 are different values[3], and satisfy the supermodularity condition:

$$z_2 + z_0 \geq z_1 + z_1' \tag{2.1}$$

which implies that positive assortative matching (PAM) is efficient. Without loss of generality I assume $z_2 > z_0$.

The game with one side offering contracts proceeds in the following:

First consider $N = 2$. Each woman, having observed only her own type, offers a contract specifying a sharing of each possible realization of the surplus. An example of a contract she offers is illustrated below, which suggests that a man only gets ε when the total surplus turns out to be z_2, in all other cases he gets 0.

[3] The special case where $z_1 = z_1'$ is discussed in Section 4 and Appendx 6.2.

Surplus	Woman	Man
z_2	$z_2 - \varepsilon$	ε
z_1'	z_1'	0
z_1	z_1	0
z_0	z_0	0

Each man, having observed his own type and the two offered contracts, updates his beliefs about the types of the two women, and uses these updated beliefs to calculate his expected payoff for each contract offered. A man will first go to a contract which gives him a higher expected payoff. If not successful, he will then go to the second contract if the second contract offers him a non-negative payoff. He will not sign any contract which gives him a negative expected payoff but to remain single. If a man is indifferent between being single and signing a contract, assume he will sign the contract. If both men go for the same contract, the woman offering the contract will randomly pick one to match with.

2.2.1.1 No Competition between W^H and W^L

Proposition 2.1 *When $z_1' \leq z_2$ and $z_1 \leq z_0$, there exists a separating equilibrium that achieves matching efficiency where W^H offers contract A and W^L offers contract B:*

Contract A		
Surplus	*Woman*	*Man*
z_2	z_2	0
z_1'	z_1'	0
z_1	0	z_1
z_0	0	z_0

Contract B		
Surplus	*Woman*	*Man*
z_2	0	z_2
z_1'	0	z_1'
z_1	z_1	0
z_0	z_0	0

let $\mu(W^k|contract\ i)$ denotes men' belief on the probability assigned to a woman is of type k when she offers contract i.

Equilibrium beliefs: $\mu(W^H|contract\ A) = 1$ and $\mu(W^L|contract\ B) = 1$

Equilibrium response rule: When being indifferent between payoffs of contract A and contract B, M^H goes for contract A and M^L goes for contract B.

Off-equilibrium beliefs: $\mu(W^H|$*any other off-equilibrium contracts*$)\in [0,1]$

Off-Equilibrium response rule: When being indifferent between an equilibrium contract and an off-equilibrium contract, both M^H and M^L always go to an ***off-equilibrium*** *contract first.*

Proof. When the four agents turn out to be exactly one W^H, one W^L, one M^H and one M^L, and when W^H offers contract A and W^L offers contract B, M^H will go for contract A and M^L will go for contract B. Matching efficiency is achieved.

Table 2.1: Matching Outcomes of Equilibrium Contracts

Prob	Other three agents	Contract A	Contract B
$\frac{1}{4}$	W^L; M^H, M^L	M^H	$\frac{1}{2}M^H + \frac{1}{2}M^L$
$\frac{1}{4}$	W^H; M^H, M^L	$\frac{1}{2}M^H + \frac{1}{2}M^L$	M^L
$\frac{1}{8}$	W^L; M^H, M^H	M^H	M^H
$\frac{1}{8}$	W^H; M^H, M^H	M^H	M^H
$\frac{1}{8}$	W^L; M^L, M^L	M^L	M^L
$\frac{1}{8}$	W^H; M^L, M^L	M^L	M^L
Expected matching outcome		$\frac{5}{8}M^H + \frac{3}{8}M^L$	$\frac{3}{8}M^H + \frac{5}{8}M^L$

Table 2.1 calculates the expected matching outcomes of offering the two equilibrium contracts respectively. Given a woman knows her own type, she needs to first work out the possible scenarios she will face before offering a contract. The first line in Table 2.1 summarizes the following: The probability that of the other three agents, one turns out to be a low type woman, one a high type man and one a low type man is $\frac{1}{4}$, given that each agent is independently drawn from a population with probability $\frac{1}{2}$ to be a high type agent and probability $\frac{1}{2}$ to be a low type agent. Since the other woman is of low type, and W^L offers contract B in the equilibrium, if the woman offers contract A, she will be regarded as a high type woman and hence attract only M^H. If she offers contract B, then men believe there are two low type women and are indifferent between them. She has half probability to be

matched with M^H and half probability to be matched with M^L. The reasoning is the same for other scenarios. Comparing the two expected matching outcomes, we see that offering contract A has a higher probability of matching with M^H and offering contract B has a higher probability of matching with M^L.

Remark 2.1 *Only when there is exactly one M^H and one M^L, offering different contracts may make a difference to the matching outcome. If the two men turn out to be of the same type, all contracts offering men non-negative payoffs achieve the same matching outcome and there is* ***no*** *efficiency loss in matching due to information asymmetry.*

Therefore in subsequent analysis, I mainly focus on scenarios with exactly one M^H and one M^L. To prove such a separating equilibrium exists, we need to show that given the other woman is offering her equilibrium contract, the woman will not deviate from offering her equilibrium contract.

1. There is *no competition between W^H and W^L* as they desire different partners in terms of total surplus produced. $z_1 \leq z_0$ implies a low type woman prefers a low type man to a high type man in terms of total expected matching surplus produced. $z_1' \leq z_2$ means a high type woman prefers a high type man to a low type man. When the two women turn out to be one W^H and one W^L, and there are one M^H and one M^L, each woman gets her desired partner by offering her equilibrium contract. It is clear that W^L does not have any incentive to compete against W^H for a high type man, and vice versa.

2. Now consider *competition between women of the same type*. To prevent deviations, I specify men's indifference behavior as the following: When being indifferent between an equilibrium contract and an off-equilibrium contract, both M^H and M^L always go to an ***off-equilibrium*** contract first.

2.1. When there are two W^H, one M^H, and one M^L, in the equilibrium both women offer contract A, so each has half probability matching with M^H and half probability matching with M^L. A high type woman may deviate to offer an off-equilibrium contract in order to "attract only M^H" against the other W^H who offers contract A. If the off-equilibrium contract successfully achieves "attract only M^H", it suggests a better matching outcome — matching with M^H with probability 1. However, such contract does not exist as any deviating contract will also attract a low type man. As M^L gets 0 from contract A, given the specified men's indifference behavior, anything equal to or greater than 0 will attract him over from contract A. There does not exist a deviating contract achieves "attract only M^H", hence no profitable deviation for W^H.

2.2. When there are two W^L, one M^H and one M^L, similar analysis as in 2.1, no deviating contract achieves "attract only M^L", therefore no profitable deviation for W^L.

From 1 and 2, we conclude that when $z_1' \leq z_2$ and $z_1 \leq z_0$, there exists a separating equilibrium that achieves matching efficiency. ■

Remark 2.2 *When the four agents turn out to be exactly one W^H, one W^L, one M^H and one M^L, given the supermodularity condition $z_2 + z_0 \geq z_1 + z_1'$, a separating equilibrium that achieves matching efficiency requires that a high type woman matches with a high type man, and a low type woman matches with a low type man. In the presence of information asymmetry, the contracts offered serve two purposes: First they allow women to signal their types, offering contract A signals a woman is of high type, and offering contract B signals a woman is of low type. Second they serve as a screening device for women to attract a partner of the same type, contract A attracts M^H and contract B attracts M^L. Matching efficiency is then achieved.*

As the surplus value reveals information about the type of each agent with certainty, signaling (punishing a mimic) is costless. When we observe z_2, we know it must be produced by a couple of high type agents; when we observe z_1', we know it must be produced by a high type woman and a low type man. As z_1 and z_0 will never be produced by a high type woman, in order to signal her type, she could offer men the full amount conditional on the output turns out to be z_1 or z_0. By doing so a low type woman will not deviate to mimic her, as offering contract A means W^L always gets 0 surplus. Such signaling is costless for W^H as her actual payoff which is conditioned on z_2 or z_1', is not affected. Similarly for a low type woman, to signal her type (punishing a high type woman mimicking to be a low type woman), she offers full amount to men conditional on the output which turns out to be z_2 or z_1', which does not affect her actual payoff neither.

Second, when $z_1' \leq z_2$ and $z_1 \leq z_0$, screening (attracting a partner of the same type) is costless for women. There is no competition between W^H and W^L as they desire different types of men: $z_1' \leq z_2$ and $z_1 \leq z_0$ implies that W^H prefers M^H and W^L prefers M^L in terms of total surplus produced. In a separating equilibrium that achieves matching efficiency, both W^H and W^L obtain the highest possible payoff by i)matching with a partner of the desired type with the highest possible probability (best possible matching outcome), and ii)extracting all the matching surplus since they are the side offering contracts.

2.2.1.2 Competition between W^H and W^L

Proposition 2.2 *When $z_1 > z_0$, there exists a separating equilibrium that achieves matching efficiency where W^H offers contract A and W^L offers contract B:*

Contract A		
Surplus	*Woman*	*Man*
z_2	$z_2 - \beta_2$	β_2
z_1'	z_1'	0
z_1	0	z_1
z_0	0	z_0

Contract B		
Surplus	*Woman*	*Man*
z_2	0	z_2
z_1'	0	z_1'
z_1	z_1	0
z_0	z_0	0

where $\beta_2 \in [\frac{z_1-z_0}{4}, \frac{2(z_2-z_1')}{5}]$.

The best separating equilibrium for W^H is to offer $\beta_2 = \frac{z_1-z_0}{4}$.

Equilibrium beliefs: $\mu(W^H|contract\ A) = 1$ *and* $\mu(W^L|contract\ B) = 1$

Equilibrium response rule: M^H *strictly prefers contract A to contract B. When being indifferent between payoffs in contract A and contract B,* M^L *goes for contract B.*

Off-equilibrium beliefs: $\mu(W^H|contract\ C\ or\ F) = 0$ *and* $\mu(W^H|contract\ E) = 1$

Off-Equilibrium response rule: When being indifferent between an equilibrium contract and an off-equilibrium contract, M^H *will always go to an **equilibrium** contract first and* M^L *will always go to an **off-equilibrium** contract first.*

We divide all the possible off-equilibrium contracts[4] *into 3 categories according to the amount offered to men conditional on the total surplus turns out to be* z_1*. Contract C offers 0 to men when total surplus is* z_1*, contract F offers men a strictly positive amount but no greater than* β_2*, and contract E offers men an amount strictly greater than* β_2 *but no greater than* z_1*.*

Contract C		
Surplus	*Woman*	*Man*
z_2		
z_1'		
z_1	z_1	0
z_0		

Contract F		
Surplus	*Woman*	*Man*
z_2		
z_1'		
z_1	$z_1 - \gamma_1^F$	$0 < \gamma_1^F \leq \beta_2$
z_0		

Contract E		
Surplus	*Woman*	*Man*
z_2		
z_1'		
z_1	$z_1 - \gamma_1^E$	$\beta_2 < \gamma_1^E \leq z_1$
z_0		

Proof. When the four agents turn out to be exactly one W^H, one W^L, one M^H

[4]Note because they are off-equilibrium contracts, Contract C will not be the same as Contract B, and Contract E will not be the same as Contract A.

and one M^L, and when W^H offers contract A and W^L offers contract B, M^H will go for contract A and M^L will go for contract B. Matching efficiency is achieved.

To examine whether a deviation—either by offering an off-equilibrium contract or a "wrong" equilibrium contract (W^H offers contract B, or W^L offers contract A)—is profitable, I categorize all the contracts in terms of matching outcomes achieved when competing against contract A and against contract B (see Table 2.2). "attract both" means both types of men go for that contract first, only when not successfully matched, a man will go for the remaining equilibrium contract. "attract only M^H" means only M^H prefers the contract to the other contract (one of the equilibrium contracts), while M^L prefers an equilibrium contract. Similar definitions for "attract only M^L" and "attract both".

Table 2.2: Categorization of Contracts
in terms of Matching Outcome Achieved when Competing Against Equilibrium Contracts

W^H/W^L		Competing against contract A: Attract neither	Attract only M^L	Attract only M^H	Attract both
Competing against contract B	Attract neither	Not Exist	Not Exist	Not Exist	Not Exist
	Attract only M^L	Not Exist	Contract C Contract E_1	Not Exist	Not Exist
	Attract only M^H	Not Exist	Not Exist	Not Exist	Contract A
	Attract both	Not Exist	Contract F Contract E_2 Contract B	Not Exist	Contract E_3

There does not exist a deviating off-equilibrium contract that achieves "attract neither" or "attract only M^H" when competing against contract B or contract A. Since both contracts offer 0 to M^L, for any deviating contract offering M^L a non-negative payoff, he prefers the deviating contract to an equilibrium contract. This explains the "Not Exist" in row 1, row 3, column 1 and column 3 in table 2.2.

Only the equilibrium contract A achieves "attract only M^H" when competing against contract B. When the other woman also offers contract A, there is half probability the woman gets M^H and half probability she gets M^L. I put contract A into the category "attract both" when competing against contract A. Similarly only contract B achieves "attract only M^L" when competing against contract A and I put it int he category of"attract both" when competing against contract B.

As contract B offers 0 to M^H, to achieve "attract only M^L" against contract B, a deviating contract must also offer M^H 0 expected payoff. Contract C and contract E_1 below achieve "attract only M^L" when competing against contract B, and contract A[5]:

Contract C		
Surplus	Woman	Man
z_2		
z_1'		
z_1	z_1	0
z_0		

Contract E_1		
Surplus	Woman	Man
z_2	z_2	0
z_1'		
z_1	$z_1 - \gamma_1^{E_1}$	$\gamma_1^{E_1} > \beta_2$
z_0		

Any deviating contract offering M^H a positive expected payoff will attract him over from contract B. Therefore contract E_2, contract F and contract E_3 achieve "attract both" when competing against contract B. As the expected payoff in contract E_2 and contract F is smaller than or equal to a high type man's payoff of matching with contract A,[6] they achieve "attract only M^L" when competing against contract A:

[5] $\mu(W^H|\text{contract C}) = 0$, and $\mu(W^H|\text{contract } E_1) = 1$. $P_C^{M^H} = 0$; $P_{E_1}^{M^H} = 0$, both are equal to $P_B^{M^H}$ and smaller than $P_A^{M^H}$, where P_K^i denotes the expected payoff of matching with contract K for agent i.

[6] $\mu(W^H|\text{contract } E_2) = 1$ and $\mu(W^H|\text{contract F}) = 0$. $P_{E_2}^{M^H} = \gamma_2^{E_2} \leq \beta_2 = P_A^{M^H}$; $P_F^{M^H} = \gamma_1^F \leq \beta_2$.

Contract E_2		
Surplus	Woman	Man
z_2	$z_2 - \gamma_2^{E_2}$	$0 < \gamma_2^{E_2} \leq \beta_2$
z_1'		
z_1	$z_1 - \gamma_1^{E_2}$	$\gamma_1^{E_2} > \beta_2$
z_0		

Contract F		
Surplus	Woman	Man
z_2		
z_1'		
z_1	$z_1 - \gamma_1^F$	$\gamma_1^F \leq \beta_2$
z_0		

Contract E_3 achieves "attract both" when competing against contract A, and against contract B[7].

Contract E_3		
Surplus	Woman	Man
z_2	$z_2 - \gamma_2^{E_3}$	$\gamma_2^{E_3} > \beta_2$
z_1'		
z_1	$z_1 - \gamma_1^{E_3}$	$\gamma_1^{E_3} > \beta_2$
z_0		

Table 2.3: Matching Outcomes of Offering Different Contracts

Prob	Three other agents	A	B	C & E_1	F & E_2	E_3
$\frac{1}{4}$	W^L; M^H, M^L	M^H	$\frac{1}{2}M^H + \frac{1}{2}M^L$	M^L	$\frac{1}{2}M^H + \frac{1}{2}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$
$\frac{1}{4}$	W^H; M^H, M^L	$\frac{1}{2}M^H + \frac{1}{2}M^L$	M^L	M^L	M^L	$\frac{1}{2}M^H + \frac{1}{2}M^L$
$\frac{1}{8}$	W^L; M^H, M^H	M^H	M^H	M^H	M^H	M^H
$\frac{1}{8}$	W^H; M^H, M^H	M^H	M^H	M^H	M^H	M^H
$\frac{1}{8}$	W^L; M^L, M^L	M^L	M^L	M^L	M^L	M^L
$\frac{1}{8}$	W^H; M^L, M^L	M^L	M^L	M^L	M^L	M^L
Expected matching		$\frac{5}{8}M^H + \frac{3}{8}M^L$	$\frac{3}{8}M^H + \frac{5}{8}M^L$	$\frac{1}{4}M^H + \frac{3}{4}M^L$	$\frac{3}{8}M^H + \frac{5}{8}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$

Following table 2.2, given the other woman is offering her "right" equilibrium contract, Table 2.3 summarizes the different expected matching outcomes when a woman offers different contracts. To prove such a separating equilibrium exists, I proceed to show that the most profitable contract for W^H is contract A for $\beta_2 \in$

[7] $\mu(W^H|\text{contract } E_3) = 1$, $P_{E_3}^{M^H} = \gamma_2^{E_3} > \beta_2 = P_A^{M^H}$.

$[\frac{z_1 - z_0}{4}, \frac{2(z_2 - z_1')}{5}]$, and contract B dominates any other contract for W^L. See appendix.

■

When $z_1 > z_0$, W^L may prefer to match with M^H since they together produce a larger surplus than if she is matched with M^L. Furthermore, $z_1 > z_0$ and $z_2 + z_0 \geq z_1 + z_1'$ imply $z_2 > z_1'$, which means W^H also prefers M^H to M^L in terms of total expected matching surplus produced. **There exists competition for M^H between W^H and W^L, attracting a high type man therefore becomes costly**: In an equilibrium that achieves matching efficiency, a high type woman needs to offer a certain amount in order to attract M^H, the amount offered prevents a low type woman from attempting to win the high type man over. Here W^H has to offer at least $\frac{z_1 - z_0}{4}$ to M^H, anything less than this amount will lead a low type woman to deviate to offer up to $\frac{z_1 - z_0}{4} - \varepsilon$ in order to attract M^H over and achieve a better matching outcome, and subsequently a higher expected payoff.

Second, for a high type woman, deterring a low type man is costless as she offers 0 to M^L conditional on z_1. M^H's payoff of matching with contract A is not affected.

Remark 2.3 *In the benchmark model, even we do not observe agents' types before a match, after observing the surplus, we can deduce the type of each agent with certainty. This enables different types of women to signal their types costlessly. When there is no competition between women of different types, women (the side offering contracts) extract all the surplus. When there is competition, men of the type desired by both women will get a positive amount while the not-preferred type gets 0 surplus*[8]*. A separating equilibrium with an efficient matching outcome can* ***always be found*** *in the contracts-offering game.*

[8] I have shown in proposition 2 when $z_1 > z_0$, high type men are desired by both women hence M^H gets a positive payoff in contract A. I omit the analysis of the case $z_1 < z_0$ and $z_2 < z_1'$ where a low type man is desired by both types of women, the analysis is essentially the same, where M^L prefers contract B and gets a positive payoff and M^H matches with contract A and gets 0.

2.2.2 The Model: Matching Surplus is Stochastic

For each possible combination of types, we have the following:

$$\begin{array}{rcl} W^H + M^H & : & p_2 Y_G + (1-p_2)Y_B = z_2 \\ W^H + M^L & : & p_1' Y_G + (1-p_1')Y_B = z_1' \\ W^L + M^H & : & p_1 Y_G + (1-p_1)Y_B = z_1 \\ W^L + M^L & : & p_0 Y_G + (1-p_0)Y_B = z_0 \end{array}$$

Where $Y_G > Y_B \geq 0$, denote the good and the bad realization of surplus respectively. $p_i \in [0,1]$, where $i = 0, 1, 1', 2$, denotes probability of realizing the good surplus state. The first equation represents that when a high type woman matches with a high type man, they will realize the good surplus state with probability p_2 and the bad surplus state with probability $1-p_2$. The expected match surplus is z_2. Similarly for the rest combinations of types. As before, the expected match surplus z satisfies the supermodularity condition:

$$z_2 + z_0 \geq z_1 + z_1'$$

which implies PAM is efficient. Without loss of generality I assume $z_2 > z_0$.

The set-up captures the stochastic nature of the marital surplus. Even for a given partner's type, there is still uncertainty about the match surplus. Unlike the benchmark model, the ex-post surplus value no longer contains any information about agents' types. Although the different values of z represent different combinations of types as before, they are now magnitudes only *in expectation.* Remark 2.4 explains the implications.

Remark 2.4 *In the benchmark model, each combination of types generates a distinct value of surplus with certainty. Sharing rules conditional on different values indicate clearly which type of agent gets what amount. For example, the sharing of surplus conditional on z_1 specifies how much a woman, who must be of low type, gets out of z_1; and a man, who must be of high type, gets the rest of z_1. However, when* *random shocks are introduced into the marital surplus, although each combination of types still generates a distinct value of surplus, the values are only in expectation. The actual amount of surplus that could be conditioned on in a contract has to be the* ***realized*** *surplus state, Y_G and Y_B, which can be observed and verified ex-post. As Y_G and Y_B could be realized by any combination of types,* ***any sharing rule conditional on any surplus state affects all types of agents****. In contrast with the benchmark model, the sharing of surplus no longer indicates which type of agents gets what amount.*

The game with one side offering contracts proceeds exactly as in the benchmark model, except that now each woman offers a contract specifying *a sharing of each possible realization of the surplus.* An example of a contract is illustrated below, a man gets ε when the total surplus turns out to be Y_G and gets 0 when it turns out to be Y_B.

Surplus	Woman	Man
Y_G	$Y_G - \varepsilon$	ε
Y_B	Y_B	0

2.2.2.1 No Competition between W^H and W^L

Proposition 2.3 *When $z_1 < z_0$ and $z_1' < z_2$, there exists a separating equilibrium that achieves matching efficiency where W^H offers contract A and W^L offers contract B:*

Contract A		
Surplus	*Woman*	*Man*
Y_G	$Y_G - \frac{p_1}{p_2}\varepsilon$	$\frac{p_1}{p_2}\varepsilon$
Y_B	Y_B	0

Contract B		
Surplus	*Woman*	*Man*
Y_G	$Y_G - \varepsilon$	ε
Y_B	Y_B	0

Equilibrium beliefs: $\mu(W^H|contract\ A) = 1$ and $\mu(W^L|contract\ B) = 1$

Equilibrium response rule: M^L prefers contract B to contract A. M^H is indifferent between payoffs of contract A and contract B, he goes for contract A.

Off-equilibrium beliefs: $\mu(W^H|$*any off-equilibrium contracts*$)= \frac{p_0-p_1}{p_2+p_0-p_1'-p_1} \in [0,1]$

*Off-equilibrium response rule: When being indifferent between an equilibrium contract and an off-equilibrium contract, both M^H and M^L will always go to an **off-equilibrium** contract first.*

Proof. When the four agents turn out to be exactly one W^H, one W^L, one M^H and one M^L, and when W^H offers contract A and W^L offers contract B, a high type man's expected payoff of matching with contract A is $p_2\frac{p_1}{p_2}\varepsilon = p_1\varepsilon$, and is also $p_1\varepsilon$ when being matched with contract B. He is indifferent between the payoffs and will go for contract A. A low type man's expected payoff of matching with contract A is $\frac{p_1p_1'}{p_2}\varepsilon$, which is smaller than $p_0\varepsilon$, the expected payoff of being matched with contract B, since $z_1 < z_0$ and $z_1' < z_2$ implies $p_1 < p_0$ and $p_1' < p_2 \Rightarrow p_1p_1' < p_0p_2$. He will go for contract B. Matching efficiency is achieved.

To show there is no profitable deviation for women to offer other contracts, the reasoning is essentially the same as in Proposition 2.1 in the benchmark model. Competition exists only between women of the same type. The off-equilibrium beliefs and response rule help to ensure that there does not exist an off-equilibrium contract that achieves a better matching outcome for women. See appendix. ■

I draw the similarities and differences of the model with the benchmark model in the case $z_0 > z_1$ and $z_2 > z_1'$.

Similar to the benchmark model:

1. Screening (attracting a partner of the same type) is costless for women. $z_0 > z_1$ and $z_2 > z_1'$ suggests that there is *no competition between W^H and W^L* as they desire different types of men in terms of the expected total surplus produced. A woman desires a partner of the same type, and she is able to match with her desired type with the maximum probability in a separating equilibrium that achieves matching efficiency. This leads to women, the side offering contracts, to extract all the surplus.

2. When there is no competition between women of different types, a separating equilibrium that achieves matching efficiency always exists, both in the benchmark

model and in the model with stochastic marital surplus. No restrictions on the parameters of the model are required to find a separating equilibrium.

In contrast with the benchmark model: Signaling (punishing a mimic) is costless in the benchmark model. Offering men the full amount conditional on z_1 and z_0 prevents a low type woman from mimicking, and W^H's payoff is not affected. However, in the model with stochastic marital surplus, since any sharing rule will affect all types of agents, there is no costless sharing rule to signal a woman's type. Despite that, since there is no competition between women of different types, the cost of signaling is very small: It only cost ε for low type women and $\frac{p_1}{p_2}\varepsilon$ for high type women. Offering a very small but different amount to men is enough to signal women's types.

2.2.2.2 Competition between W^H and W^L

Theorem 2.4 *When $z_1 > z_0$, a necessary condition for a separating equilibrium that achieves matching efficiency to exist is $p_1' = 0$.*

Proof. For a separating equilibrium to achieve matching efficiency, we need women of different types offer different contracts, a high type man prefers a contract offered by a high type woman, and a low type man prefers a contract offered by a low type woman. Suppose there exists such a separating equilibrium with W^H offering contract A and W^L offering contract B:

Contract A		
Surplus	Woman	Man
Y_G	$Y_G - \beta_1$	β_1
Y_B	$Y_B - \beta_0$	β_0

Contract B		
Surplus	Woman	Man
Y_G	$Y_G - \alpha_1$	α_1
Y_B	$Y_B - \alpha_0$	α_0

where $\beta_1, \alpha_1 \in [0, Y_G], \quad \beta_0, \alpha_0 \in [0, Y_B]$

Equilibrium beliefs: $\mu(W^H|\text{contract A}) = 1$ and $\mu(W^L|\text{contract B}) = 1$

M^H prefers contract A to contract B if and only if

$$P_A^{M^H} = p_2\beta_1 + (1-p_2)\beta_0 \geq p_1\alpha_1 + (1-p_1)\alpha_0 = P_B^{M^H} \tag{2.2}$$

M^L prefers contract B to contract A if and only if

$$P_B^{M^L} = p_0\alpha_1 + (1-p_0)\alpha_0 \geq p_1'\beta_1 + (1-p_1')\beta_0 = P_A^{M^L} \tag{2.3}$$

For contract A to be different from contract B, we need $\beta_1 \neq \alpha_1$ or $\beta_0 \neq \alpha_0$, or both.

As $z_1 > z_0$ and $z_2 > z_1'$,[9] both W^H and W^L prefer M^H to M^L in terms of the total expected matching surplus produced. To prevent a profitable deviation to the largest extent, I specify men's indifference behavior as the following: When being indifferent between expected payoffs of contracts, M^H will always first go for an *equilibrium* contract (contract A or contract B), and M^L will always first go for an *off-equilibrium* contract.

Step 1: *First I show that* $p_0\alpha_1 + (1-p_0)\alpha_0 = 0$ *and* $p_1'\beta_1 + (1-p_1')\beta_0 = 0$ *are necessary conditions for such a separating equilibrium to exist.*

Proof of Step 1: Prove by contradiction. Suppose $p_0\alpha_1 + (1-p_0)\alpha_0 > 0$, then there exists a profitable deviation for W^L to offer contract C which offers 0 to men:

Contract C		
Surplus	Woman	Man
Y_G	Y_G	0
Y_B	Y_B	0

Given specified men's indifference behavior, when a high type man sees an equilibrium contract and contract C which offers him nothing, he will always go for an equilibrium contract first. Contract C never achieves the matching outcome "attracts W^H".

[9] $z_1 > z_0$ and $z_2 + z_0 \geq z_1 + z_1' \Leftrightarrow z_2 - z_1' \geq z_1 - z_0 > 0 \Leftrightarrow z_2 - z_1' > 0$.

When a low type man sees contract B and contract C, he compares $p_0\alpha_1 + (1 - p_0)\alpha_0$ and 0. Since $p_0\alpha_1 + (1 - p_0)\alpha_0 > 0$, he will first go for contract B. Contract C achieves "attracts neither" when competing against contract B.

When M^L sees contract A and contract C, he compares $p_1'\beta_1 + (1 - p_1')\beta_0$ and 0. If $p_1'\beta_1 + (1 - p_1')\beta_0 > 0$, he will first go for contract A; then contract C achieves "attracts neither" when competing against contract A. If $p_1'\beta_1 + (1 - p_1')\beta_0 = 0$, he will first go for contract C, then contract C achieves "attracts only M^L" when competing against contract A. Table 2.4 summarizes the matching outcomes of offering contract C against equilibrium contracts. Table 2.5 then summarizes the different matching outcomes when W^L offers contract B and contract C, given the other woman is offering her equilibrium contract.

Table 2.4: Contract C's Matching Outcome when Competing Against Equilibrium Contracts

Contract C's matching outcome	$p_0\alpha_1 + (1 - p_0)\alpha > 0$
① $p_1'\beta_1 + (1 - p_1')\beta_0 > 0$	against contract B: attracts neither against contract A: attracts neither
② $p_0'\beta_1 + (1 - p_1')\beta_0 = 0$	against contract B: attracts neither against contract A: attracts only M^L

Table 2.5: Matching outcome of W^L Offering Contract B and Contract C

Prob	Other three agents	Contract B	Contract C ①	Contract C ②
$\frac{1}{4}$	W^L; M^H, M^L	$\frac{1}{2}M^H + \frac{1}{2}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$
$\frac{1}{4}$	W^H; M^H, M^L	M^L	$\frac{1}{2}M^H + \frac{1}{2}M^L$	M^L
$\frac{1}{8}$	W^L; M^H, M^H	M^H	M^H	M^H
$\frac{1}{8}$	W^H; M^H, M^H	M^H	M^H	M^H
$\frac{1}{8}$	W^L; M^L, M^L	M^L	M^L	M^L
$\frac{1}{8}$	W^H; M^L, M^L	M^L	M^L	M^L
Expected matching		$\frac{3}{8}M^H + \frac{5}{8}M^L$	$\frac{1}{2}M^H + \frac{1}{2}M^L$	$\frac{3}{8}M^H + \frac{5}{2}M^L$
Profitable deviation			Yes	Yes

Table 2.5 suggests that contract C is a profitable deviation when $p_0\alpha_1 + (1 - p_0)\alpha_0 > 0$. By offering contract C, the low type woman's chances of matching with

M^H is either improved in case ① or remain unchanged in case ②, so her expected payoff becomes higher as contract C allows women to extract all the surplus[10]. Therefore for a separating equilibrium to exist, we need $p_0\alpha_1 + (1-p_0)\alpha_0 = 0$. This means a low type man gets 0 expected payoff from contract B.

From inequality (2.3), we must have $p_1'\beta_1 + (1-p_1')\beta_0 = 0$, that M^L also gets 0 payoff from contract A.

Step 2: *Discuss two cases: either* $\alpha_1 = \alpha_0 = 0$ *or* $\alpha_0 = 0$, $\alpha_1 > 0$.

Since $z_0 < z_1 \Rightarrow 0 \leq p_0 < p_1 \leq 1 \Rightarrow 1 - p_0 > 0$, for $p_0\alpha_1 + (1-p_0)\alpha_0 = 0$ we need $\alpha_0 = 0$. $z_2 - z_1' > 0 \Rightarrow (p_2 - p_1')(Y_G - Y_B) > 0 \Rightarrow p_2 - p_1' > 0 \Rightarrow p_1' < p_2 \leq 1 \Rightarrow 1 - p_1' > 0$. For $p_1'\beta_1 + (1-p_1')\beta_0 = 0$ we need $\beta_0 = 0$. In order to find values of β_1 and α_1, I discuss two cases: $\alpha_1 = \alpha_0 = 0$ or $\alpha_0 = 0$, $\alpha_1 > 0$.

Case 1: $\alpha_1 = \alpha_0 = 0$

Since $\beta_0 = 0$, we need $\beta_1 > 0$ for contract A to be different from contract B.

Case 2: $\alpha_0 = 0$, $\alpha_1 > 0$.

For $p_0\alpha_1 + (1-p_0)\alpha_0 = 0$, we need $p_0 = 0$. Condition (2.2) implies $p_2\beta_1 \geq p_1\alpha_1 > 0$ since $p_1 > p_0 \geq 0$ and $\alpha_1 > 0$. Therefore we need $p_2 > 0$ and $\beta_1 > 0$.

Both cases need $\beta_1 > 0$. For $p_1'\beta_1 + (1-p_1')\beta_0 = 0$, it must be the case that $p_1' = 0$. ■

The requirement for efficiency when the marital surplus is stochastic is that we need $p_1' = 0$ or equivalently $W^H + M^L = Y_B$. Only if the combination of an agent ranked high in the side offering contracts and an agent ranked low in the side choosing contracts, leads to *a production of the lowest possible marital surplus with certainty*, an efficient matching outcome may exist. The reasoning is explained below:

In a separating equilibrium that achieves matching efficiency, when there exists competition for a high type man, a low type woman always loses the competition to

[10] In expectation she gets $\frac{3}{8}z_1 + \frac{5}{8}z_0$ in case ② and $\frac{1}{2}z_1 + \frac{1}{2}z_0$ in case ①, while her expected payoff from offering the equilibrium contract B is $\frac{3}{8}z_1 + \frac{5}{8}z_0 - \frac{3}{8}\left[p_1\alpha_1 + (1-p_1)\alpha_0\right] - \frac{5}{8}\left[p_0\alpha_1 + (1-p_0)\alpha_0\right]$ which is strictly less than $\frac{3}{8}z_1 + \frac{5}{8}z_0$ and $\frac{1}{2}z_1 + \frac{1}{2}z_0$ when $p_0\alpha_1 + (1-p_0)\alpha > 0$.

a high type woman. She matches with M^H with a positive probability only when there are two M^H or the other woman is also of low type. Therefore the best for her is to offer 0 to M^H. As M^L is *not* desired by W^L, she offers 0 to M^L. A low type woman therefore offers 0 to men in the equilibrium. In an efficient matching outcome, offering nothing to men achieves the highest possible expected surplus for W^L.

For a high type woman, in order to attract only high type men and prevent low type men from coming over from contract B, she needs to offer 0 expected surplus to M^L, as any positive expected payoff will attract M^L since he only gets 0 from W^L. However, in order to signal her type, W^H needs to offer a contract different from W^L. This is possible only if $p_1' = 0$ or $W^H + M^L = Y_B$. When a low type man and a high type woman produce Y_B with certainty, it allows W^H to offer M^L 0 conditional on Y_B, and offer M^H some positive amount conditional on Y_G to attract M^H, such contract will attract only M^H. Only when $p_1' = 0$, there is no trade-off between screening (attracting a man only of the same type) and signaling (offering a contract different from a low type woman) for W^H.

Theorem 2.5 *When $z_1 > z_0$ and $p_1' = 0$, there exists a separating equilibrium that achieves matching efficiency where W^H offers contract A and W^L offers contract B:*

Contract A		
Surplus	*Woman*	*Man*
Y_G	$Y_G - \beta_1$	β_1
Y_B	Y_B	0

Contract B		
Surplus	*Woman*	*Man*
Y_G	Y_G	0
Y_B	Y_B	0

where $\beta_1 \in [\frac{2(z_1-z_0)}{5p_1+3p_0}, \frac{2}{5}(Y_G - Y_B)]$.

The best separating equilibrium for W^H is to offer $\beta_1 = \frac{2(z_1-z_0)}{5p_1+3p_0}$.

Equilibrium beliefs: $\mu(W^H|\text{contract } A) = 1$ *and* $\mu(W^L|\text{contract } B) = 1$

Equilibrium response rule: M^H will go for contract A as the expected payoff is strictly higher than that in contract B. M^L is indifferent between payoffs of contract A and contract B, he goes for contract B.

Off-equilibrium beliefs: $\mu(W^H|\textit{any off-equilibrium contracts}) = \frac{1}{2}$

Off-equilibrium response rule: When being indifferent between an equilibrium contract and an off-equilibrium contract, M^H will always go to an ***equilibrium*** *contract first and M^L will always go to an* ***off-equilibrium*** *contract first.*

Proof. Consider the case that the four agents turn out to be exactly one W^H, one W^L, one M^H and one M^L. When W^H offers contract A and W^L offers contract B, a high type man's expected payoff of matching with contract A is $p_2\beta_1$, and is 0 when being matched with contract B. He prefers contract A since $p_2\beta_1 > 0$. A low type man's expected payoff of matching with contract A is $p_1'\beta_1 = 0$ since $p_1' = 0$, and is also 0 when being matched with contract B. He is indifferent between the expected payoffs and will go for contract B. Matching efficiency is achieved.

To show there is no profitable deviation for women to offer other contracts when there exists competition for M^H, the proof is technically more complicated than that in Proposition 2.2 as now there is uncertainty about the payoff, but the reasoning is essentially the same. First I categorize all the contracts in terms of matching outcomes achieved when competing against contract A and against contract B. Then I show that given the specified off-equilibrium beliefs and off-equilibrium response rule, the most profitable contract for W^H is contract A with $\beta_2 = \frac{2(z_1-z_0)}{5p_1+3p_0}$, and contract B dominates any other contract for W^L. See appendix. ■

I draw the similarities and differences of the model with the benchmark model in the case $z_1 > z_0$.

Similar to the benchmark model:

1. When there exists competition for M^H between women of different types, a low type woman always offers nothing to men. Because she always loses the competition to a high type woman in the equilibrium, and M^L is not her desired type, she offers 0 to both M^H and M^L to maximize her expected payoff. A high type woman needs to offer a positive amount to M^H to prevent a low type woman from attempting to win him over, and to offer 0 to M^L to deter him. In both the

benchmark model and the model with stochastic marital surplus, high type men's expected payoff is strictly positive and low type men's payoff is 0.

2. In the benchmark model, the amount offered to M^H cannot be greater than $\frac{2}{5}(z_2-z_1')$. Similarly in the model with stochastic marital surplus, the amount offered to M^H conditional on Y_G cannot be greater than $\frac{2}{5}(Y_G - Y_B)$, this is equivalent to M^H's expected payoff being no greater than $p_2\frac{2}{5}(Y_G - Y_B) = \frac{2}{5}(z_2 - z_1')$. The upper bound is to prevent a high type woman from giving up attracting high type men: As the cost of attracting M^H becomes too high, high type women may find offering nothing to men becomes a profitable deviation despite a worse matching outcome.

In contrast to the benchmark model:

1. In the benchmark model, a separating equilibrium that achieves matching efficiency always exists. However, in the model with stochastic marital surplus, a separating equilibrium exists only if $p_1' = 0$. When random shocks are introduced into the marital surplus with *non-trivial* probabilities (neither 0 nor 1), there is no separating equilibrium in the contracts-offering game[11]. As pointed out in Remark 2.4, since Y_G and Y_B can be realized by any combination of types, *any sharing rule conditional on any surplus state affects all types of agents.* A high type woman needs to deter a low type man by offering him 0, which means she has to offer men 0 conditional on both Y_G and Y_B. However, such a contract does not signal her high type since a low type woman also offers 0 to men in the equilibrium. The conflict between deterring a low type partner (offering him zero amount) and signalling her high type (offering a contract different from a low type woman) renders contracts ineffective in achieving matching efficiency.

2. In the benchmark model, the amount offered to a high type man by a high type woman needs to be at least $\frac{z_1-z_0}{4}$. This is to prevent a low type woman from

[11]In this case I have specified women to be be the side offering contracts, there could exist a separating equilibrium if we let men offer contracts instead in this case, see Figure 2.2. However in the men offering contracts game, we then require $p_1 = 0$ when $z_1' > z_0$. The main message is that even if we can let either side offer contracts, in the case $z_2 > z_1 > z_0$ and $z_2 > z_1' > z_0$, there is still no separating equilibrium that achieves matching efficiency, see Claim 2.1.

attempting to offer slightly more to win the high type man over. *It is not related to preventing a mimic as women have costlessly signaled their types.*

In the model with stochastic marital surplus, if $p_2 > p_1$, preventing a mimic becomes costly for high type women. Consider a low type woman deviates to mimic M^H by offering contract A. She will be regarded as a high type woman: When there is only one high type man, not only she wins the competition against another W^L, but also she has half chance of matching with M^H when competing against W^H.

In fact, offering contract A always enables women to match with M^H with a higher probability than offering contract B. In the benchmark model, this better matching outcome does not lead to a higher expected payoff for a deviating W^L, since W^H can costlessly punish a mimic by offering full amount to men conditional on z_1 and z_0. However, in the model with stochastic marital surplus, the only way she can punish a mimic is to offer a large amount to men conditional on Y_G, as she has to offer 0 to men conditional on Y_B.

Suppose W^H propose to offer β to M^H conditional on Y_G in contract A. M^H calculates his expected payoff as $p_2\beta$ when he sees contract A. A deviating M^L offering contract A will actually offer M^H expected payoff of only $p_1\beta$. If $p_2 > p_1$, then low type women gain from mimicking. As a result high type women have to bear the cost of signalling, offering a larger[12] expected amount to high type men than what she offers M^H in the benchmark model, in order to prevent W^L from mimicking and competing for M^H.

Corollary 2.6 *When $z_1 > z_0$, a necessary and sufficient condition for finding a separating equilibrium that achieves matching efficiency is ${p'}_1 = 0$.*

Figure 2.1 summarizes the range of z that a separating equilibrium with matching efficiency can be found in the game with *women* offering contracts. The vertical red line represents Corollary 2.6: Only when $p'_1 = 0(z'_1 = Y_B)$, there exists a separating

[12] $p_2\frac{2(z_1-z_0)}{5p_1+3p_0} > \frac{z_1-z_0}{4}$ if $p_2 > p_1$. This is analogous to the classical Spence's signalling model that high ability workers have to **over-invest** in education in order to signal their types.

equilibrium with matching efficiency. Similarly the horizontal red line represents a symmetric case where women of different types desire a low type man[13]: Only when $p_1 = 0$ ($z_1 = Y_B$), there exists a separating equilibrium. The red region represents Proposition 2.3: When there is no competition between women of different types, a separating equilibrium that achieves matching efficiency can always be found.

Figure 2.2 is the mirror image of Figure 2.1 when *men* become the side offering contracts.

Suppose we have the freedom to choose which side to offer contracts, then combining Figure 2.1 and Figure 2.2, Figure 2.3 represents the largest range of z where a separating equilibrium with matching efficiency can be found. As Figure 2.3 suggests, when the probability of realizing the good surplus state is *non-trivial (neither 0 nor 1)*, and when $z_2 > z_1 > z_0$ and $z_2 > z_1' > z_0$, that is, the expected marital surplus is strictly monotonic in agents' types, there is *no* separating equilibrium that achieves matching efficiency in the contracts-offering game.

To summarize main findings of the model:

Claim 2.1 *When there is i) two-sided information asymmetry, ii) the marital surplus is stochastic, and iii) PAM achieves matching efficiency; let one side of the market, say women, offer contracts specifying the sharing rule of the realized surplus to men:*

1. *When there exists competition between women of different types, a necessary and sufficient condition for finding a separating equilibrium that achieves matching efficiency is that the combination of a man of the not-desired type and a woman of a different type, leads to a production of the **lowest possible surplus with certainty**.*

2. *When expected surplus is strictly monotonic in agents' types, or agents of different types have the same rank of the types of potential partners in terms of*

[13]The proofs and reasoning are essentially the same as Theorem 2.4 and Theorem 2.5 where there exists competition for M^H.

Figure 2.1: Game with Women Offering Contracts

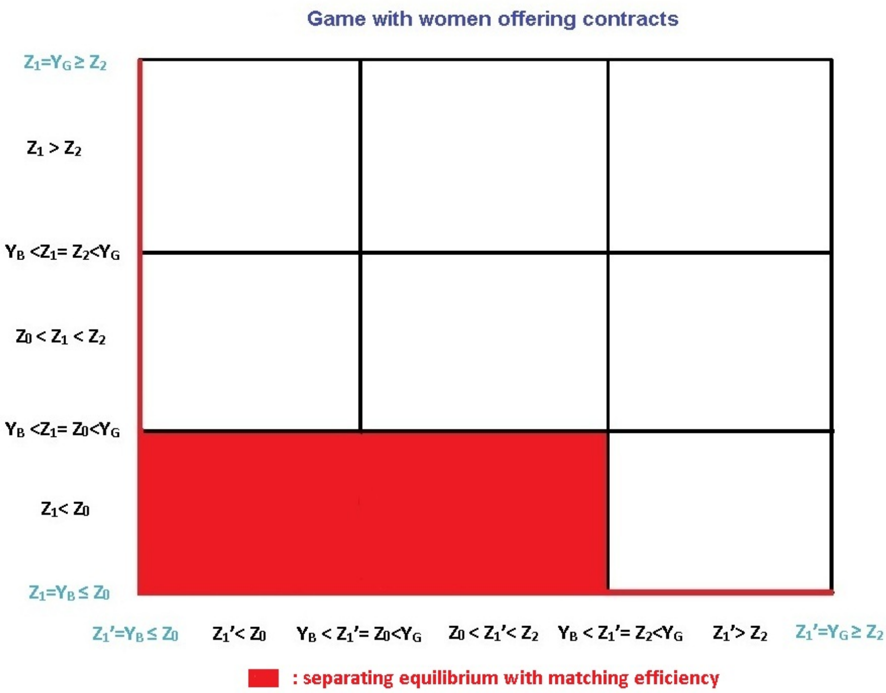


Figure 2.2: Game with Men Offering Contracts

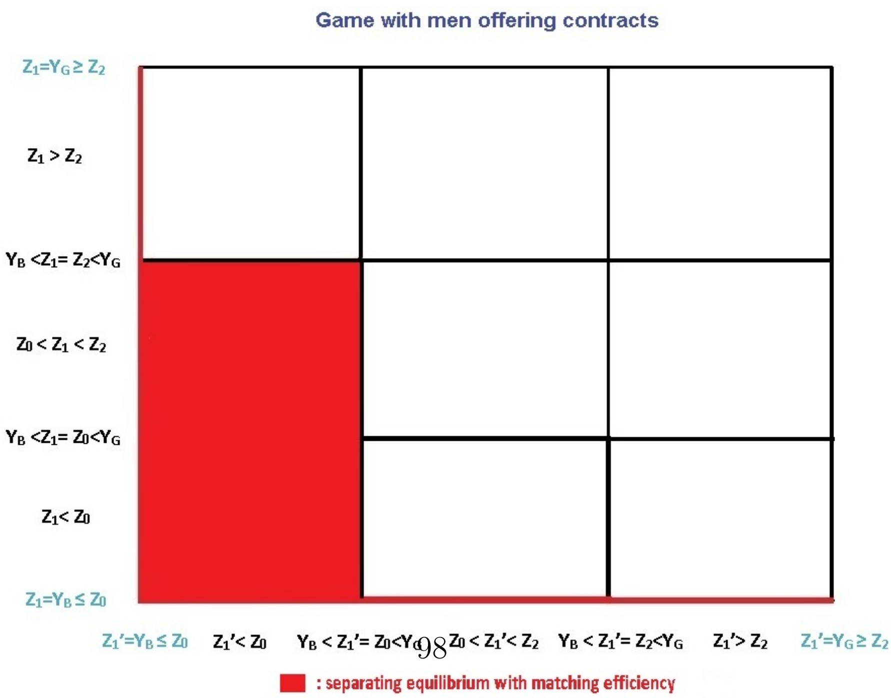
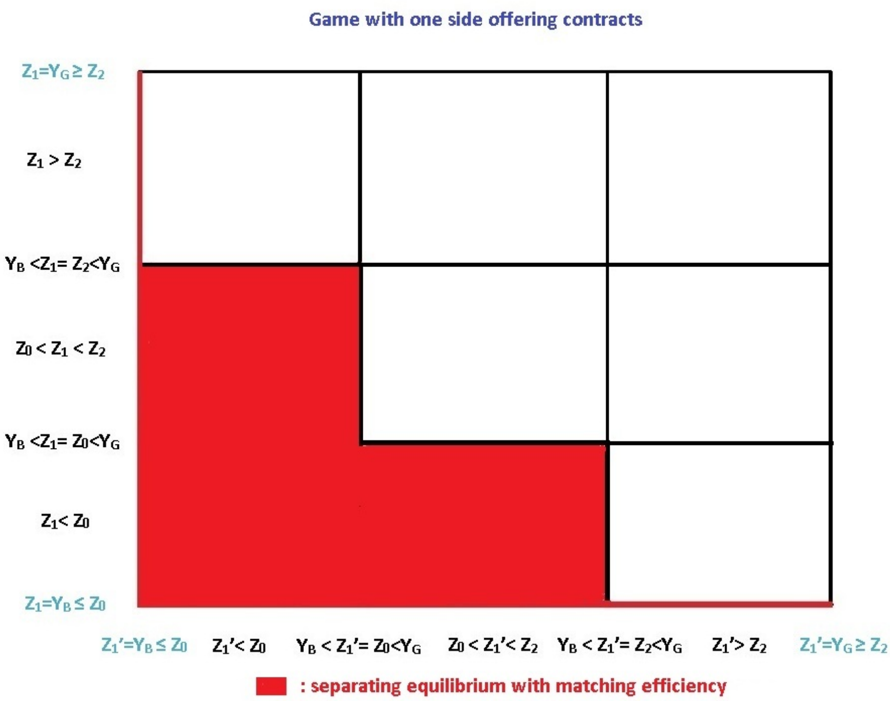


Figure 2.3: Game with Either Side Offering Contracts



generating the total expected match surplus, there is ***no*** *separating equilibrium that achieves an efficient matching outcome.*

2.3 Social Planner's Mechanism

Claim 2.1.2 states that when the probabilities of realizing the good surplus state are non-trivial (neither 0 nor 1), and when there exists competition between agents of different types on the side offering contracts, the contracts-offering game in section 2.2 cannot achieve matching efficiency. Section 2.3 then asks is there any other mechanism able to achieve matching efficiency?

The set-up of a social planner's problem is the following:

Each agent, having observed only his or her own type, could either simultaneously and costlessly report his or her type to a social planner or choose not to report. A social planner commits to a mechanism specifying who matches with whom and how each surplus is shared as a function of the reported types. The social planner designs the following contracts, choosing $\beta_1 \in [0, Y_G]$, $\beta_0 \in [0, Y_B]$ and similarly for μ, τ, and α to help achieve matching efficiency:

Contract C_1: $W^H + M^H$		
Surplus	Woman	Man
Y_G	$Y_G - \beta_1$	β_1
Y_B	$Y_B - \beta_0$	β_0

Contract C_2: $W^H + M^L$		
Surplus	Woman	Man
Y_G	$Y_G - \mu_1$	μ_1
Y_B	$Y_B - \mu_0$	μ_0

Contract C_4: $W^L + M^L$		
Surplus	Woman	Man
Y_G	$Y_G - \alpha_1$	α_1
Y_B	$Y_B - \alpha_0$	α_0

Contract C_3: $W^L + M^H$		
Surplus	Woman	Man
Y_G	$Y_G - \tau_1$	τ_1
Y_B	$Y_B - \tau_0$	τ_0

For N=2, if there is exactly one reported "high" type agent on each side of the matching market, the social planner matches "W^H" with "M^H" with the sharing rule specified in contract C_1 and matches "W^L" with "M^L" with the sharing rule specified in contract C_4. Matching efficiency is achieved. In other cases the social planner randomly matches the agents, as any re-arranging of the matching does not affect matching efficiency.

For general N, the social planner matches all the reported "high" type agents on the side which has fewer "high" type agents with "high" type agents on the other side.

Each agent, having observed his or her own type and knowing the contracts and matching rules, chooses whether to report his or her type truthfully.

By the *Revelation Principle*, Section 3 restricts attention to a direct revelation mechanism where each agent chooses to report truthfully to the social planner.

2.3.1 The Two men-Two Women Case

Proposition 2.7 *In a symmetric setting ($z_1 = z_1'$), if matching efficiency can be achieved, it can be achieved by **a symmetric mechanism** in which $\beta_1 = \alpha_1 = \frac{Y_G}{2}$, $\beta_0 = \alpha_0 = \frac{Y_B}{2}$ and $\mu_1 = Y_G - \tau_1$, $\mu_0 = Y_B - \tau_0$.*

Contract C_1: $W^H + M^H$		
Surplus	*Woman*	*Man*
Y_G	$\frac{Y_G}{2}$	$\frac{Y_G}{2}$
Y_B	$\frac{Y_B}{2}$	$\frac{Y_B}{2}$

Contract C_2: $W^H + M^L$		
Surplus	*Woman*	*Man*
Y_G	$Y_G - \mu_1$	μ_1
Y_B	$Y_B - \mu_0$	μ_0

Contract C_4: $W^L + M^L$		
Surplus	*Woman*	*Man*
Y_G	$\frac{Y_G}{2}$	$\frac{Y_G}{2}$
Y_B	$\frac{Y_B}{2}$	$\frac{Y_B}{2}$

Contract C_3 : $W^L + M^H$		
Surplus	*Woman*	*Man*
Y_G	μ_1	$Y_G - \mu_1$
Y_B	μ_0	$Y_B - \mu_0$

Proof. See appendix. ■

Intuitively, when the genders are symmetric in terms of generating the surplus[14], a symmetric mechanism which treats genders equally supports a separating equilibrium with matching efficiency *to the largest extent.* If a symmetric mechanism

[14]A high type woman matches with a low type man, or a low type woman matches with a high type man, both pairs generate the same amount of surplus: $z_1 = z_1'$.

cannot achieve matching efficiency in this setting, no asymmetric mechanism can. Therefore I focus the analysis on a symmetric mechanism: When the agents of the same types are matched, they always share the surplus equally. When agents of different types are matched, the high type agent gets the same amount regardless of one's gender. Hence, when $z_1 = z_1'$, the problem simplifies to can we find some $\mu_1 \in [0, Y_G]$ and some $\mu_0 \in [0, Y_B]$ for a truthful direct revelation mechanism (TDRM) to achieve matching efficiency?

Proposition 2.8 *In a symmetric setting ($z_1 = z_1'$) with N=2, there exists a truthful direct revelation mechanism that achieves matching efficiency if and only if*

$$\frac{2(p_1^2 - p_2 p_0)}{3(p_2 - p_0)} \leq \frac{Y_B}{Y_G - Y_B}.$$

Proof. See appendix. ■

The condition $\frac{2(p_1^2 - p_2 p_0)}{3(p_2 - p_0)} \leq \frac{Y_B}{Y_G - Y_B}$ is to ensure that the smallest μ_0^*, which satisfies both high type and low type agents' incentive compatibility constraints, is no greater than Y_B.

Remark 2.5 *With $N = 2$, a social planner can do* ***strictly better*** *in terms of achieving matching efficiency than the equilibrium of the game in which one side offers contracts.*

Whenever the contracts-offering game achieves matching efficiency with W^H offering contract A and W^L offering contract B, by replicating the contracts, matching efficiency can always be achieved in the social planner's setting. In addition, the social planner restricts the possible deviations of sharing the surplus to a much smaller set. In the contracts-offering game, for a separating equilibrium to exist, not only we need to show a woman will not deviate to mimic the other type, but also need to show that she will not deviate to offer *any other* contracts. In the social planner's setting, signaling becomes a *binary* choice as an agent can only report either high or low. This is equivalent to restricting a woman to offer either contract A or contract B in the contracts-offering game.

When expected surplus is strictly monotonic in agents' types, the social planner achieves an efficient matching under the condition $\frac{2(p_1^2-p_2p_0)}{3(p_2-p_0)} \leq \frac{Y_B}{Y_G-Y_B}$, even the one side offering contracts game does not.

Combining the two cases, I conclude with Remark 2.5.

2.3.2 The N Men-N Women Case

Proposition 2.9 *Let P_N denote the probability of matching with a high type man, M^H, when a woman reports "high" type to the social planner.*

$$P_N = 1 - \frac{1}{2^{2N-1}} \begin{pmatrix} 2N-1 \\ N \end{pmatrix}$$

It has the following properties:

1. $P_N > \frac{1}{2}$;
2. P_N *increases with* N *and* $\frac{2P_N-1}{1-P_N}$ *increases with* N;
3. $1 - P_N \approx \frac{1}{\sqrt{\pi N}}$ *when* N *is large.*

Proof. Table 2.6 explains calculation of P_N, the probability of matching with

Table 2.6: Probability of Matching with M^H after Reporting as W^H

		No. of high type men, M^H							
		0	1	2	3	...	$N-2$	$N-1$	N
Total No. of reported W^H	1	0	1	1	1	...	1	1	1
	2	0	$\frac{1}{2}$	1	1	...	1	1	1
	3	0	$\frac{1}{3}$	0	1	...	1	1	1
	...	...	...	...	...	...	...	...	...
	N−2	0	$\frac{1}{N-2}$	$\frac{2}{N-2}$	$\frac{3}{N-2}$	...	1	1	1
	N−1	0	$\frac{1}{N-1}$	$\frac{2}{N-1}$	$\frac{3}{N-1}$	...	$\frac{N-2}{N-1}$	1	1
	N	0	$\frac{1}{N}$	$\frac{2}{N}$	$\frac{3}{N}$	...	$\frac{N-2}{N}$	$\frac{N-1}{N}$	1

M^H when a woman reports "high" type to the social planner. Given other agents are reporting truthfully, whether she can be matched with M^H depends on the number of total high type men and the number of other high type women. For instance, when all the other women are of low type, the woman becomes the only one reporting "high" type, she will be matched with M^H with probability 1 when the number of M^H equal to or greater than one (see line 1). If all the other $N-1$ women are of high type, and there is only one M^H, then her probability of matching with M^H becomes $\frac{1}{N}$ (see the last line, column 2). To calculate P_N, I sum up all the conditional probabilities. See appendix. ■

The properties of P_N are quite intuitive. In a random matching, a woman will be matched with M^H with probability half. In a mechanism that aims to achieve matching efficiency, a woman reporting "high" type will be matched with M^H with a probability greater than half as the social planner attempts to match W^H with M^H whenever it is possible. As the number of agents increases on both sides of the market, a woman is more likely to face the scenario that there are $\frac{N-1}{2}$ high type women out there and $\frac{N}{2}$ high type men. The probability of the woman reporting "high" type and then matching with M^H becomes $\frac{N/2}{1+(N-1)/2}=\frac{1}{1+\frac{1}{N}}$ which increases with N. As N becomes very large, the probability approaches 1.

Corollary 2.10 *The probability of matching with a low type man, M^L, when a woman reports "low" type to the social planner, is also P_N.*

Theorem 2.11 *In the symmetric setting, there exists a truthful direct revelation mechanism that achieves matching efficiency if and only if*

$$\frac{(2P_N-1)\left(p_1^2-p_2p_0\right)}{(1-p_N)\left(p_2-p_0\right)} \leq \frac{Y_B}{Y_G-Y_B} \tag{2.4}$$

and

$$(1-P_N)\,z_1 \geq P_N\left(\frac{z_1-z_0}{2}\right) \tag{2.5}$$

where $P_N = 1 - \frac{1}{2^{2N-1}} \left(\begin{array}{c} 2N-1 \\ N \end{array} \right)$.

Similar to the $N = 2$ case, condition (2.4) is to ensure that the smallest μ_0^*, which satisfies both high type and low type agents' incentive compatibility constraints, is no greater than Y_B. As N increases, $\frac{2P_N-1}{1-P_N}$ increases, the inequality is harder to sustain. Similarly for condition (2.5): $\frac{P_N}{1-P_N} \leq \frac{2z_1}{z1-z0}$. The two conditions become more stringent as the number of agents increases.

Condition (2.5) represents low type agents' incentive compatibility constraint:

$$\underbrace{(1-P_N)\, z_1}_{\text{Largest expected loss by mis-reporting as “H”}} \geq \underbrace{P_N \left(\frac{z_1 - z_0}{2} \right)}_{\text{Expected gain by mis-reporting as “H”}}$$

The right-hand side of inequality (2.5) represents the expected gain from mis-reporting as a “high” type agent. Because in a symmetric mechanism, agents of the same reported types share the surplus equally, and each agent has probability P_N matching with an agent reporting the same type. Therefore, when a woman reports as “low” type (same analysis for a low type men), her expected payoff is $P_N \frac{z_0}{2}$; when she mis-reports as “high” type, she will match with a high type man with probability P_N and share the surplus z_1 equally. Therefore her expected gain from mis-reporting is $P_N(\frac{z_1-z_0}{2})$.

The left-hand side represents the largest expected loss from mis-reporting as a “high” type agent. In a symmetric mechanism, the most severe punishment to a deviating low type agent is to set $\mu_1 = Y_G$ and $\mu_0 = Y_B$: It specifies that when a high type agent matches with a low type agent, the high type agent gets 0 surplus and the low type agent gets the full surplus. Hence it provides the low type agents the largest incentive to report truthfully. After reporting truthfully as “low” type, a woman matches with M^H with probability $1 - P_N$ and gets z_1; after mis-reporting as “high” type, she matches with M^L with probability $1 - P_N$ and gets 0. The largest expected loss from mis-reporting is therefore $(1 - P_N)z_1$. Condition (2.5) is necessary to prevent low type agents from mis-reporting.

When N is large, P_N approaches 1, condition (2.5) becomes $0 \geq \frac{z_1 - z_0}{2}$ which does not hold when $z_1 > z_0$. Matching efficiency therefore cannot be achieved in the social planner's setting. This is because when a low type woman mis-reports as "high" type, she will match with M^H with probability almost equal to 1 and gets an expected surplus of $\frac{z_1}{2}$. It is therefore always profitable for her to mis-report when N is large.

To summarize main findings of the social planner's problem:

Claim 2.2 *When there is two-sided information asymmetry and the marital surplus is stochastic, consider a direct revelation mechanism where a social planner designs contracts specifying the sharing of the realized surplus:*

1. *With finite number of agents, the social planner* ***can do strictly better*** *than the contracts-offering game.*

2. *His ability to achieve matching efficiency* ***decreases*** *with the number of agents in the matching market.*

3. *When N is very large, an efficient matching cannot be achieved.*

2.4 Discussion

1. First Mover Advantage in the contracts-offering game. When there is no competition between women of different types, and if *women* are the side offering contracts, they are able to extract almost all the surplus. The signalling costs for women, or men's expected payoffs, are very small. When there is competition for M^H, in the equilibrium W^L extracts all the surplus, and W^H offers a positive amount to M^H conditional on Y_G. Still, M^H's expected payoff is *always strictly less* than W^H's payoff.[15] Similar results hold when there is competition for M^L.

[15] $P_A^{M^H} = p_2\beta 1 \leq \frac{2p_2(Y_G - Y_B)}{5} = \frac{2(z_2 - z_1')}{5} < \frac{5}{8}z_2 + \frac{3}{8}z_1' = P_A^{W^H}$.

Hence, a separating equilibrium that achieves matching efficiency in the *women offering contracts game* is *strictly preferred* to a *men offering contracts game* ***by all women***. There is *first mover advantage* in the contracts-offering game. This is analogous to the first mover advantage in the Gale-Shapley Algorithm, that in a non-transferrable utility model, *for all women*, a stable matching outcome obtained when women propose to men (weakly) *Pareto dominates* the stable matching when men propose to women.

2. In the contracts-offering game, the harder to verify agents' types from the surplus value, the more difficult to achieve matching efficiency. Section 2 shows that in the benchmark model where each realized surplus state fully reveals the combination of agents' types, a separating equilibrium that achieves matching efficiency *can always be found*. However, in the model with stochastic marital surplus, the realized surplus states do not reveal any information about agents' types. When the expected marital surplus is strictly monotonic in agents' types, *no* separating equilibrium that achieves matching efficiency exists. The set-up below provides an intermediate case with partial verifiability of agents' types. Comparing the different results suggests that the harder to verify agent's types, the more difficult to achieve matching efficiency.

$$\begin{aligned} W^H + M^H &= z_2 \\ W^H + M^L = W^L + M^H &= z_1 \\ W^L + M^L &= z_0 \end{aligned}$$

The partial non-verifiability comes from z_1: z_1 reveals that it is must be produced by a combination of a high type agent and a low type agent, but it does not tell *which agent is of low type*. As a result any sharing rule conditional on z_1 affects agents of all types. It can be shown that in the intermediate case, the range of z supporting matching efficiency is larger than that in the model with stochastic marital surplus, and smaller than the range of z in the benchmark model.[16]

[16]The analysis is essentially the same as in Section 2. In the equilibrium W^L offers 0 to men.

3. Alternative "global" sharing rule. This paper focuses analysis on sharing rules only *within*, but not across couples. There is no transfer of resources between couples. Section 3 shows that an agent's incentive compatibility constraint is not always satisfied in a truthful DRM, hence matching efficiency is not always achievable. An alternative approach to deal with agents' incentives is to propose a "global" sharing rule. For example, the social planner can specify that any agent will get $\frac{1}{4}$ of the *aggregate* matching surplus in the two men-two women case. Such a global sharing rule aligns agents' incentives with the social planner's. However, such analysis has to assume that the social planner can observe and verify all couples' marital output. If it is not the case, then a couple may hide their assets and always claim the bad surplus state even if they have reached the good surplus state, since by doing so they avoid subsidizing the other couple and may even get transfers from them. The analysis in this paper does not require such an assumption. For instance, even if a high type woman wants to hide the assets and claim the bad surplus state, her partner will stop her from doing so as he has an incentive to reveal the true surplus state to get a larger amount. Since there is no transfer across couples, no couple has an incentive to hide their assets.

4. Type-independent v.s. type-dependent outside option. This paper assumes that the payoff of an unmatched agent is 0 regardless of types. It automatically puts lower and upper bounds on the amount can be shared between a couple. A woman can not offer a negative amount to men, neither can she offer an amount greater than the total surplus produced. Second, it implies that any amount of surplus is verifiable, hence contracts specifying sharing rules can condition on the realized surplus states.

Such an assumption rules out type-dependent outside options. A possible exten-

In order to deter W^L, W^H offers 0 conditional on z_1. However, this will induce W^L to mimic as now she is able to get the full surplus when she matches with M^H. To punish a mimic, W^H offers the full amount to men conditional on z_0. Therefore, to sustain a separating equilibrium with matching efficiency, the benefit of mimicking, z_1, cannot be too large relative to z_0, the cost of mimicking. With $N = 2$, we need $z_1 \leq \frac{5}{2}z_0$. When N is very large, the probability of matching with W^H approaches 1, W^L will always mimic. See Proposition B.1 in appendix.

sion is to assume a high type agent can have a better outside option than a low type agent. Then if agents' types are only privately known, an agent's outside option also becomes private information. Although the absolute amount of a couple's marital output can be verified, the marital surplus—total output subtracting the sum of the two agents' outside options—is no longer verifiable. As a result, sharing rules have to condition on the total marital *output* instead. *The bounds on what can be shared enlarges.* As now in some states an agent may get less than what he or she produces when being single, the new analysis needs to take into account each agent's participation constraint: Ex-ante a participating agent expects non-negative gains from marriage.

The qualitative conclusions remain. In the contracts-offering game, whether matching efficiency can be achieved depends on to what extent the marital output is informative about agents' types. In the social planner's mechanism, there exists a larger range of marital output supporting matching efficiency in the truthful DRM.[17] However when N becomes large, matching efficiency is still not achievable.

2.5 Conclusion

This paper examines a marriage market with two-sided information asymmetry and in which the match surplus is stochastic. My main focus is on how endogenous determination of the sharing rule can help to achieve an efficient matching. I find that when marital surplus is deterministic, matching efficiency can be achieved by a simple contracts-offering game, where contracts specifying the sharing of the surplus serve as a credible signal to reveal one's type, and as a screening device to attract a suitable partner. However, when random shocks are introduced into marital surplus, matching efficiency is not always achievable, especially when the number of agents

[17] Previously, the smallest μ_0^*, the amount a low type agent gets when being matched with a high type agent conditional on Y_B, which satisfies all agents' incentive compatibility constraints, needs to be no greater than Y_B. Now this condition can be relaxed to some extent, that μ_0^* can be greater than Y_B to compensate a low type agent more.

is large. A natural next step would be to examine some costly pre-marital signals that affects agents of different types differently and helps to achieve an efficient matching. Matching efficiency with costly pre-marital signals when marital surplus is stochastic, can then be related to the literate on efficiency of pre-marital investment.

Chapter 3

Women's Preference towards Marriage

3.1 Introduction

In the household economics, there is a large literature on potential material gains from marriage (Becker, 1973, 1991; Browning et al. 2014). However, the non-material benefit of marriage has often been neglected or modelled as an unimportant parameter since it is difficult to measure and quantify[1]. Little is known about the role of the non-material benefit of marriage in the family economics. In this paper, I interpret the non-material gains from marriage for a woman as her perceived emotional satisfaction of having her own family, namely her ex-ante preference towards marriage. I examine the relationship between a woman's preference towards marriage, her investment in schooling and her marital outcomes.

The relationship between women's educational investments and their preference towards marriage is twofold. On the one hand, women with a strong desire for marriage tend to prioritize marriage and family life over their educational investments and later careers. In the NLSY79 youths survey, approximately 25% of women an-

[1]Except for Weiss and Willis (1997) in which the authors model the match quality of marriage and examined its effect on the timing of children and divorce divisions.

swered "getting married, pregnancy, or home responsibilities" as their main reason for dropping out of school (Ge, 2011). This is in sharp contrast with men being only 5%. On the other hand, women may find investing more in education leads to a better marriage outcome. Goldin (1992) argues that many women graduating from 1945 to the early 1960 in the U.S. attended college mainly for finding a college-educated husband. A man with a college degree or more often has a good income potential, and his wife would benefit from his momentary transfer within marriage. Ge (2011) finds that without such marriage considerations, the college attendance for women would be 8% lower, and the graduation rates would be 2% higher. Furthermore, Chiappori et al. (2009) have argued that investing in education brings an additional marriage return for women in the sense that a woman with high education can get a larger share of the resources within households compared to a woman with low education. In addition, Huang et al. (2009) found that there is significant cross-productivity effect from a husband to his wife. Following these arguments, marriage expectations presumably are more important in determining women's educational investment than men, and women who value marriage highly would have more incentive to invest more in education in order to find a good husband.

Women's preference towards marriage surely affects their marital outcomes. Consider a spectrum of the marriage preference, at the one end a woman always prefers to get married regardless of the quality of her marriage offers. At the other end of the spectrum a woman always prefers to be single. Women in between have different thresholds of whether to enter into marriage and the threshold would depend on their relative value of getting married and being single. Women with a high threshold would only find marriage attractive if she receives a very good marriage offer. Hence we would expect that although women with high threshold find it more difficult to get married, but conditional on being married, their marriage outcomes for example their husbands' earnings may be better than women with a high preference towards marriage who are less choosy in the marriage market.

Although economists have long noticed the positive relationship between spousal education and a person's own earnings (Neuman and Ziderman, 1992; Keane and Wolpin 1997, 2001; Eckstein and Wolpin, 1999; Lefgren and McIntyre, 2006), the fact that the selection into marriage is not random always complicates the analysis (Boulier and Rosenzweig 1984). As discussed earlier, when women's preferences towards marriage are the unobservables determining both the probability of getting married and their husbands' earnings, the OLS estimates of the relationship between earnings and spousal education would be biased and inconsistent. Rose (2006) has argued that marriage decisions and the choice of the spouse should be modeled simultaneously. She has applied a selection model to study the effect of a woman's education on her husband's education, using parent's religion and the sex composition of her children as the exclusion variables.

In line with these studies, I propose a proxy for women's preference towards marriage to mitigate the omitted variable bias. I construct a dummy variable capturing whether a woman's mother has had a stable marriage. This proxy captures whether the daughter grew up in a stable family. I defined women's preference towards marriage as the emotional and psychological support and commitment from another individual to start a family. I expect that women who grew up in a broken family to have a higher preference towards marriage, as the lack of a proper family in her childhood would potentially make them want to have their own family more badly than others. To the best of my knowledge, this is the first paper examining women's preference towards marriage with such a proxy on women's martial outcomes.

I apply the Heckman selection model to take into account of the selection into marriage. The first stage probit regression examines how a woman's preference towards marriage affects her probability of ever getting married. The second regression examines how her marriage preference and her educational investments are associated with the husband's earnings, conditional on being married.

With 1980 and 2008 U.S. Census data, the empirical findings of this paper are the following: First, conditional on educational attainments, the probability of ever getting married *increases* with a woman's preference towards marriage. Conditional on the preference towards marriage, a woman's educational attainment is significantly *negatively* associated with the probability of getting married. Second, conditional on being married, the preference towards marriage is *negatively* associated with the husband's annual earnings, and the association between higher education and husbands' income are *positive* in 1980 and *negative* in 2008.

The remainder of this paper proceeds as follows: in section 3.2 I present the data and then introduce the proxy for women's preference towards marriage. Section 3.3 discusses the estimation results of a two stage regression with the Heckman correction. Section 3.4 concludes.

3.2 Data

The data used is from the U.S. Census 1980 5% sample and 2008 American Community Survey (ACS) data USA IPUMS (Integrated Public Use Microdata Series). The sample consists white women who were born and have worked in the 51 states in the United States. I restrict the analysis to women between age 35 and 45, since in this range most of the women have finished their educational investment, and there is a fair chance that a woman would have already been married by this time if she desires to get married. Moreover, in this range the majority of their husbands have completed their education and have not retired yet, which allow us to explore the relationship between husbands' earnings and women's educational investments and preference towards marriage. The key variables examined in the paper are whether a woman has ever been married, her education attainment, her preference towards marriage (proxyed by her mother's marriage outcome), and her husband's income if she is married.

In both 1980 and 2008 there is information on the number of times of a person's marriage, for both a woman and her mother. This information is essential to code the proxy for women's preference towards marriage and it is available for the first time in the 2008 ACS since the 1980 Census. The 2008 data is attractive also in the sense that it provides accurate details on women's educational attainments. It differentiates high school graduates from who have 12 years of education but did not get a diploma. This allows a more accurate classification of a person's educational attainment. In the 2008 data I classify all people without a high school diploma as "less than high school". While in 1980, all respondents were classified according to the highest year of school completed, due to a lack of the detailed information on whether the respondent has received a degree. Despite the less detailed information on educational attainments, the 1980 data is special in the sense that it is the latest year recording the most detailed information on one's marriage outcome. It reports whether the first marriage of a woman's mother was ended by the death of her spouse. This information is useful for coding the proxy for women's preference towards marriage.

Table 3.1 shows the summary statistics of key variables examined in this paper. The average age for women is 39.6 in 1980 and 39.9 in 2008. In 1980, the average annual income of women is $10,168, which is equivalent to 56% of the average husbands' annual income which is $18,086. In 2008, the average annual income of women is $31,354, which is equivalent to about 65% of the average husbands' income which is $ 48,009. This suggests that the income gap between husbands and wives have been narrowed since 1980. Over 90% of the husbands are currently in labour force (approximately 96% in 1980 and 93% 2008).

In terms of educational attainments, the majority of the women (86.4% in 1980 and 94.7% in 2008) have completed at least high school. The most significant change from 1980 to 2008 is the proportion of women completing some college or receiving a bachelor degree. Percentage of women attended some college has increased from approximately 18% in 1980 to 37% in 2008, while the percentage of women receiving

Table 3.1: summary statistics of education, age and income

	1980		2008	
average female age	39.6	(3.18)	39.9	(3.16)
women's average annual wages	10168	(6744)	31354	(30501)
maximum annual wage	75000		472000	
husband's age	43.3	(5.62)	42.3	(5.73)
age range	23-65		22-65	
husbands in labour force				
no, not in labour force	60	4.36%	70	6.58%
yes in labour force	1316	95.64%	994	93.42%
husband's average annual wages	18086	(11419)	48,009	(46139)
maximum annual wage	75000		575000	
mother's age	68.1	(6.82)	66.6	(6.87)
age range	51-90		50-95	
wife's educational attainment	No. of obs	percentage		
Less than High School	803	13.6%	263	5.29%
High School Graduates	3033	51.4%	1603	32.2%
Some College	1058	17.9%	1853	37.3%
Bachelor Degree	481	8.14%	849	17.1%
Masters & PhDs	532	9.01%	406	8.16%
husband's educational attainment				
Less than high school	288	20.9%	97	9.12%
High school graduates	575	41.8%	371	34.9%
Some college	251	18.2%	353	33.2%
Bachelor degree	140	10.2%	172	16.2%
Masters & Phds	122	8.87%	71	6.67%
total number of women	5907		4974	
total number of husbands	1376		1064	

Note: The sample includes white women between age 35 and 45 who were born and worked in the 51 states in the United States. Standard deviation are in parentheses a bachelor degree has increased from approximately 8% in 1980 to 17% in 2008. The proportion of women obtaining a Master or a PhD degree stays roughly the same. Husband's education attainments follow the same trend. There are approximately 18% of men who have attended some college in 1980, and the proportion has increased to 33% in 2008. Likewise for receiving a bachelor degree, the percentage has increased from 10% in 1980 to about 16% in 2008.

3.2.1 The Preference towards Marriage

Table 3.2 shows the summary statistics of women's marital status and their mothers' marital status in 1980 and 2008. The sample consists of 5907 women in 1980 and 4974 women in 2008. Nearly 53.6% have been married at least once in 1980 and 57.3% in 2008, although only 23.3% are "currently married with a spouse present" in 1980 and 21.4% in 2008. In 1980 the largest proportion (57.3%) of mothers' current marital status is "widowed", while in 2008, the largest proportion (41.2%) of mothers' current marital status is "married, spouse present". Furthermore, there is a significant increase in the proportion of mothers getting divorced, it has increased from approximately 7% in 1980 to approximately 22% in 2008.

Table 3.3 shows more details of the mother's marital outcomes: the number of times she has been married. I construct a dummy variable capturing whether a woman's mother has had a stable marriage. The dummy variable equals to zero if her mother was only married once and her current marital status is "married, spouse present", or her mother's first marriage was ended by the death of her spouse, she married twice with current marital status being "married, spouse present". (This information is only available in 1980 data).

This proxy captures whether a woman grew up in a family with a stable marriage. I define women's preference towards marriage as the non-material benefit associated with marriage. The preference towards marriage can also be regarded as the emotional and psychological support and commitment from another individual

Table 3.2: summary statistics of marital status

	1980		2008	
Marital Status	No. of obs	percentage	No. of obs	percentage
Married, spouse present	1376	23.3%	1064	21.4%
Married, spouse absent	66	1.12%	167	3.36%
Separated	219	3.7%	249	5.01%
Divorced	1366	23.1%	1311	26.4%
Widowed	142	2.40%	60	1.21%
Never married/Single	2,738	46.4%	2,123	42.7%
Mothers' Current Marital Status				
Married, Spouse Present	1947	33.0%	2048	41.2%
Married, Spouse absent	35	0.59%	92	1.85%
Separated	103	1.75%	89	1.79%
Divorced	414	7.01%	1071	21.5%
Widowed	3384	57.3%	1634	32.9%
Never Married	24	0.41%	40	0.80%
No. of observations	5907		4974	

The sample includes white women between age 35 and 45 who were born and worked in the 51 states in the United States.

to start a family. At this stage I expect that women who grew up in a broken family to have a higher preference towards marriage, as the lack of a proper family in her childhood would potentially make a woman want to have her own family more badly than otherwise. However, on the other hand, it has also often been suggested that children growing up in a broken family tend to have difficulties in maintaining relationships; they are more likely to marry early, but are also more likely to get divorced. This makes the relation between a woman's true preference towards marriage and whether she grew up in a broken family complicated.

Another concern would be the group of mothers whose current marital status are "widowed" and they only married once. Due to a lack of data on the year of a mother being widowed, I cannot differentiate mothers who became widowed only recently, which would indicate her daughter grew up in a two-parent, intact family, or the daughter has lost her father from an early age. In the baseline regressions I code the proxy to be one for widowed mothers. In the robustness check I try the alternative of coding it to zero. The results suggest that the empirical findings are

Table 3.3: The mother's current marital status in 1980

		No. of marriages			
		zero	once	twice or more	total
Married, spouse present			1812	135	1,947
First marriage ended by the	yes			52	
death of spouse	no			83	
Married, spouse absent			24	11	35
Separated			75	28	103
Divorced			287	127	414
Widowed			2,963	421	3,384
Never married/single		24	0	0	24
Total		24	5,161	722	5,907

This table contains information on the mother's marriages for white women aged 35-45 and were born and worked in the 51 states in the United States in 1980.

robust to the alternative way of coding women's preference towards marriage.

Table 3.4 shows how the dummy variable relates to a woman's other characteristics. The dummy variable is negatively correlated with some of the higher education degrees. This could be due to the fact that a woman grew up in a broken family is more likely to perform poorly in school because of the lack of family support. Having said that, the negative correlation with high educational investments (a bachelor degree or more) could also due to women with a low preference towards marriage tends to invest fully in her education as she needs to be financially independent. A woman with a high preference towards marriage who desires to get married may under-invest in her education.

The dummy variable is positively correlated with age in both 1980 and 2008. This positive correlation could be due to the fact that the older a woman is, the earlier her parents have been married and hence more likely being divorced or separated. The distribution of women who grown up in a broken family is also related to the region she was born. In later regressions I control for both age and the state of a woman's birth place.

Table 3.4: Proxy for Preference towards Marriage and other controls

correlation	preference towards marriage θ_f	
	1980	2008
Educational attainment		
Less than high school	0.042*	0.003
	(0.001)	(0.823)
High school graduate	0.010	-0.011
	(0.430)	(0.447)
Some college	0.008	0.043*
	(0.518)	(0.002)
Bachelor Degree	-0.051*	-0.030*
	(0.0001)	(0.035)
Masters & PhDs	-0.031*	-0.019
	(0.018)	(0.178)
Age	0.169*	0.095*
	(0.000)	(0.000)
Region of Birth state		
Northeast Region	-0.029*	-0.015
	(0.028)	(0.283)
Midwest (formerly	0.008	-0.020
North Central) Region	(0.531)	(0.153)
South Region	0.023*	0.021
	(0.081)	(0.127)
West Region	-0.0001	0.014
	(0.996)	(0.315)
No. of Observation	5907	4974

This table reports correlations between the the dummy variable whether a woman grew up in a broken family and a number of controls in our later regressions. * p<0.1

3.3 Econometric Specification

I run two regressions. The first stage regression examines how a woman's preference towards marriage and her educational attainment are associated with her probability of getting married:

$$P(M=1\,|X_f) = \Phi(\mu_0 + \mu_1\theta_f + \mu_2 S_f + \mu_3 Q_f + \epsilon_f \geq 0\,|X_f) = \Phi(X_f\mu + \epsilon_f \geq 0\,|X_f) \tag{3.1}$$

where the dummy variable M equals to zero if a woman has never been married and equals to one otherwise. X_f=[1 θ_f S_f Q_f] contains women's characteristics that may influence the probability of getting married. θ_f is a dummy variable equals to one if a woman's mother has married more than once[2]. S_f denotes the woman's education category with high school being the reference category. Q_f contains the woman's age and dummy variables representing the state of her birth place. $\mu = [\mu_0 \ \mu_1 \ \mu_2 \ \mu_3]$. ϵ_f represents the unobservables that may influence her probability of getting married. Φ is the cumulative distribution function of a standard normal distribution.

With Heckman correction, the second stage regression tests that conditional on being married, how her husband's wage rate is associated with the woman's preference towards marriage and her schooling:

$$log(w_m) = \gamma_0 + \gamma_1\theta_f + \gamma_2 S_f + \gamma_3 Z_m + u \tag{3.2}$$

where Z_m denotes the husband's individual characteristics: his educational attainment, the state of his work place, years of work experience (proxyed by his age).

The hypothesis is that a woman with relatively high θ_f is less choosy, hence are more likely to get married. They are also more willing to accept a man with low income. Therefore conditional on getting married, the average husband's wage rate of women with high θ_f is likely to be lower:

$$H_0 : \mu_1 = 0 \quad and \quad \gamma_1 = 0 \qquad v.s. \qquad H_1 \quad \mu_1 > 0 \quad and \quad \gamma_1 < 0$$

3.3.1 The Heckman Selection Model

The Selection Rule: Women's participation in the marriage market

I assume every woman in the marriage market has a reservation wage rate for her potential husband. It is the threshold of wage rates that a woman is just indifferent between getting married and staying single. Different women may have different

[2] or married twice because her first marriage was ended by the death of a spouse (this is a small proportion in the sample).

thresholds. I specify this underlying latent variable, denoted by w_m^*, as the following:

$$ln(w_m^*|\ X_f) = \beta_0 + \beta_1\theta_f + \beta_2 S_f + \beta_3 Q_f + \epsilon_f = X_1\beta + \beta_3 Q_f + \epsilon_f$$

where $X_1 = [1\ \ \theta_f\ \ S_f]$ and $\beta = (\beta_0\ \ \beta_1\ \ \beta_2)'$. Note here all the regressors are women's characteristics: a woman's age, the state in which she was born, and more importantly, her education level and her preference towards marriage. Assume $\epsilon_f \sim N(0, {\sigma_f}^2)$ where v_f represents women's unobserved characteristics that may influence their reservation husband wage.

The actual wage rate of a woman's potential husband is denoted by w_m. Similarly, I model w_m as the following:

$$ln(w_m) = \gamma_0 + \gamma_1\theta_f + \gamma_2 S_f + \epsilon_m = \gamma_0 + \gamma_1\theta_f + \gamma_2 S_f + \epsilon_m \tag{3.3}$$

where $\epsilon_m \sim N(0,\ \sigma_m^2)$ denotes the husband's characteristics affecting his wage rate. We cannot observe those characteristics for *all* potential husbands in the first stage. I assume ϵ_m is uncorrelated with women's educational attainment and their preference towards marriage:

$$corr(X_1,\ \epsilon_m) = 0$$

In other words, I assume apart from a woman's own characteristics, a man becomes her potential husband only directly through his wage rate (indirectly through ϵ_m). If a potential husband's wage rate exceeds a woman's reservation husband wage rate, she gets married and only then we observe her husband's income:

$$M = \begin{cases} 1 & if\ ln(w_m) \geq ln(w_m^*) \\ 0 & if\ ln(w_m) < ln(w_m^*) \end{cases}$$

Therefore, the probability of getting married is the following:

$$
\begin{aligned}
Prob(M = 1 \mid X_f) &= Prob(w_m \geq w_m^*) \\
&= Prob(\gamma_0 + \gamma_1\theta_f + \gamma_2 S_f + \epsilon_m \geq \beta_0 + \beta_1\theta_f + \beta_2 S_f + \beta_3 Q + \epsilon_f) \\
&= Prob(\gamma_0 - \beta_0 + (\gamma_1 - \beta_1)\theta_f + (\gamma_2 - \beta_2)S_f - \beta_3 Q + \epsilon_m - \epsilon_f \geq 0) \\
&= Prob(X_f\mu + v \geq 0)
\end{aligned}
$$

where $\mu_0 = \gamma_0 - \beta_0$, $\mu_1 = \gamma_1 - \beta_1$, $\mu_2 = \gamma_2 - \beta_2$, $\mu_3 = -\beta_3$, and $\mu = (\mu_0\ \mu_1\ \mu_2\ \mu_3)'$. $v = \epsilon_m - \epsilon_f$, hence $v \sim N(0,\ \sigma_m^2 + \sigma_f^2)$.

Biased and Inconsistent OLS estimates

Since $ln(w_m)$ is only observed among married women, there is a selection problem in the OLS regression:

$$E(ln(w_m)|X_1,\ M = 1) = E(X_1\gamma + \epsilon_m|X_f\mu + v \geq 0) = X_1\gamma + E(\epsilon_m \mid v \geq -X_f\mu)$$

In order to have consistent OLS estimates, we need $E(\epsilon_m \mid v \geq -X_f\mu) = 0$ which is equivalent to assuming no correlation between v and ε_m. However $v = \epsilon_m - \epsilon_f$ is by construction, thus $Corr(\epsilon_m,\ v) \neq 0$ and $E(\epsilon_m \mid v \geq -X_f\mu) \neq 0$. As women are not randomly selected into marriage, the observed husband's log wage rates have unobservables which differ systematically from the unobservables of the population containing all potential husbands. I apply the Heckman selection model (Heckman,1979) to find consistent estimates[3].

Exclusion restriction: I assume the state of a woman's birth place affects her preference towards marriage and her later marriage decision, but does not directly affect her potential husbands' wage rates[4].

[3]Note that compare to equation 3.3, there is an additional explanatory variable Z_m in regression 3.6. Including Z_m in the second stage regression would help improve the accuracy to the estimates, the Heckman correction takes into account that we do not observe Z_m for all potential husbands.

[4]Later I report the exclusion restriction tests which suggest the relevance condition is satisfied: state of birth place are jointly significant in determining the first stage probability of getting married in both 1980 and 2008 data.

3.4 Estimation Results

3.4.1 The First Stage Probit Regression

Table 3.5 presents the estimates of regression 3.1, the first stage probit regression. Column 2 and 4 show the estimates of regression coefficients. Column 3 and Column 5 calculate the associated marginal effects. The estimates suggest that: First, conditional on the educational attainment, a woman's preference towards marriage is positively associated with her probability of ever getting married. Second, conditional on her marriage preference, her schooling is negatively associated with her probability of being married.

The dummy variable of women's preference towards marriage is highly significant and positive in both 1980 and 2008. In 1980, a woman with a high preference towards marriage is approximately 21 percent points more likely to get married than a woman with a low preference. In 2008, she is approximately 14 percentage points more likely to get married. Although both are highly significant, its importance seems to have declined from 1980 to 2008.

I propose two main reasons accounting for this change: a changing social norm that on average women nowadays have a lower preference towards marriage that the zero-one dummy proxy did not capture, and a changing social norm that men nowadays are also less likely to get married, therefore women with a high preference towards marriage find it harder to actually get married.

Due to the changing social norms, on average women have a lower preference towards marriage than in the past. Nowadays divorce rates are higher and women marry later (In the 1980 data the average age at first marriage is 21, and in 2008 data age at first marriage is 26). Women nowadays are more likely to have a career and participate in the labour market, getting married and having children at an older age than before. As a result of this social change, whether women were growing up in a broken family or not, they now both converge to have a lower preference towards

Table 3.5: Probit estimation for the probability of ever being married

	Ever married			
	1980		2008	
	Estimates	Marginal Effects	Estimates	Marginal Effects
preference towards marriage θ	0.54***	0.21***	0.36***	0.14***
	(0.037)	(0.141)	(0.039)	(0.015)
education category				
less than high school	0.144***	0.06***	0.15*	0.058*
	(0.052)	(0.020)	(0.086)	(0.033)
some college	0.04	0.017	0.043	0.017
	(0.046)	(0.018)	(0.044)	(0.017)
college graduates	-0.26***	-0.10***	-0.21***	-0.08***
	(0.063)	(0.025)	(0.054)	(0.022)
Masters or more	-0.66***	-0.25***	-0.15***	-0.058**
	(0.063)	(0.023)	(0.071)	(0.028)
Age	0.016***	0.006	0.024***	0.010***
	(0.005)	(0.002)	(0.006)	(0.002)
intercept	-0.60***		-0.68***	
	(0.253)		(0.281)	
No. of obs	5907	5907	4974	4974

This table contains the estimated coefficients and associated marginal effects of women's preference towards marriage and their educational attainments on the probability of ever getting married. The sample consists white women between age 35 and 45 years who were born and worked in the 51 states in the United States, taken from 1980 and 2008 Census data. Standard errors are in parentheses. *** $p<0.01$, ** $p<0.05$, *$p<0.10$. Completing high school is the reference category. All regressions include a full set of state of birth dummies.

marriage. Therefore this proxy in our 2008 data has a weaker link to women's true preference towards marriage than our 1980 data due to the change in distribution of women's preference towards marriage.

Not only women but also men are having a lower preference towards marriage compared to the past. While the fact that men are less likely to get married would have a small impact on women who do not want to get married anyway, the impact is mainly on women who have high preference towards marriage. They would find it harder to get married because there are less men available in the marriage market than three decades ago. This helps explain why women's preference towards marriage plays a smaller role in actually getting married in 2008.

Educational Attainment Table 3.5 shows that higher educational qualifications are negatively associated with the probability of ever getting married in both 1980 and 2008 data. Compared to high school graduates, having a bachelor degree is associated with approximately 10 percentage points less likely to get married in 1980 and approximately 8 percentage points in 2008. Having a Master or a PhD degree is associated with approximately 25 percentage points less likely to get married in 1980 and approximately 6 percentage points less likely in 2008. Conditional on the preference towards marriage, highly educated women are less likely to get married.

This is probably due to the income pooling of married couples. Holding the non-material benefit of marriage fixed, a highly educated woman would have high income potential. Marriage would be attractive if she meets a man also with high income potential. She would be less willing to marry a man with low income potential as she has to share her income with him. A woman's reservation husband wage increases with her education, as a result her probability of getting married decreases.

Table 3.5 also shows significant credential effects (Hungerford and Solon, 1987) prevailing in the marriage market. In both 1980 and 2008, the probability of being married does not differ between women who are high school graduates and women attended college but did not obtain a college degree in both 1980 and 2008. Not finishing high school is associated with approximately 6 percentage points more likely to get married. We observe a substantial decrease in the marriage probability only with a degree completion. This credential effect in marriage market is presumably due to the credential effect in the return to education in the labour market, that people only get a large pay rise if they obtained a higher degree.

In addition, the less significant relationship between "less than high school" and "ever married" could be due to a relatively small number of women in that group in 2008 than in 1980. In 1980, there is approximately 14% of women belong to " less than high school " group, while in 2008, there is only approximately 5% of women belong to "less than high school".

Having a Master or a PhD degree has a much stronger negative association with the probability of ever getting married in 1980 than in 2008 (25 versus 6 percentage points). The falling costs of education for women and the less income disparity between men and women may have caused this change.

The falling costs of education enable women with a high preference towards marriage to invest more in her education. Previously a woman may not be able to enter into tertiary education because she values family and gets married in an early age, but now with the legalization of abortion and the introduction of contraceptive pills (Goldin et al 2006), women with a high preference towards marriage are able to plan their educational investment better. There is now less conflict between educational investment and marriage. In 1980 only 30% of women with a Master or a PhD degree were married, while in 2008, the percentage increased to 50%. As more and more women entering tertiary education also getting married, we would find a smaller negative association of high education with the likelihood of marriage.

There is less income disparity between men and women with similar educational attainment in 2008 compared to 1980. The average annual income of married women with a Master or a PhD degree is equivalently to approximately 65% of their husbands' income in 1980, and has increased to approximately 112% in 2008. Clearly married woman with more than a college degree earn more than their husbands on average in 2008. On one hand, this would favour women with a high preference towards marriage. They are more likely to find a husband and get married since their reservation husband wage could be lower when they are more financially independent. On the other hand, men would find marrying high educated women attractive as their schooling has a stronger association with high income.

3.4.2 The Second Stage Regression with Heckman Correction

Table 3.6 reports both the OLS estimates and estimates with the Heckman correction in 1980 and 2008. In the OLS regression, all the wife's characteristics seem insignificant. When the husband's education levels and work experiences (proxyed by his age) are controlled for, there is no significant relationship between a woman's preference towards marriage and her educational attainment with her husband's earnings. In contrast, the dummy variable proxying women's preference towards marriage is highly significant with the Heckman correction, and some of women's educational attainments are significant too. This provides evidence that the selection into marriage is not random. Furthermore, the likelihood ratio tests suggest that ρ is highly significant, indicating a strong selection bias, hence a Heckman selection model is more appropriate.

Conditional on educational attainments of both the husband and the wife, a woman's preference towards marriage is negatively associated with her husband's income. In 1980 a high preference towards marriage is associated with 70% less of husbands' income and with 50% less in 2008. The reasoning of this strong negative relationship is the following: the higher preference towards marriage, the lower a woman's reservation husband wage since she values the non-material benefit of marriage more. As a result, we observe the large difference in the husband's annual income between the group of women with a high preference towards marriage and the group with a low preference.

Conditional on the preference towards marriage, a woman's educational attainment is positively correlated with her husband's income in 1980 and is negatively correlated with her husband's earnings in 2008. In 1980, having a college degree is associated with approximately 15% *increase* of the husband's earnings compared to high school graduates, and having a Master or a PhD degree is associated with approximately 28% *increase* of the husband's annual income. In contrast, having a college degree is associated with approximately 11% *decrease* of the husband's

Table 3.6: OLS and Heckman Estimation of the log of husbands' annual earnings

	1980		2008	
	OLS	Heckman correction	OLS	Heckman correction
marriage preference θ_f	0.073	-0.69***	0.026	-0.50***
	(0.068)	(0.066)	(0.057)	(0.075)
wife's education				
less than high school	-0.09	-0.03	-0.13	-0.20
	(0.064)	(0.069)	(0.128)	(0.135)
some college	-0.037	0.06	0.03	-0.16**
	(0.054)	(0.063)	(0.062)	(0.069)
college graduates	0.055	0.15*	-0.094	-0.11***
	(0.085)	(0.090)	(0.078)	(0.087)
Masters or more	-0.172	0.28***	0.07	-0.24**
	(0.120)	(0.105)	(0.097)	(0.107)
husband's education				
less than high school	-0.306***	-0.163***	-0.29***	-0.27***
	(0.056)	(0.042)	(0.110)	(0.087)
some college	0.06	0.073*	0.11***	0.10**
	(0.047)	(0.041)	(0.057)	(0.049)
college graduates	0.26***	0.23***	0.31***	0.34***
	(0.061)	(0.051)	(0.076)	(0.0563)
Masters or more	0.130	0.28***	0.73***	0.76***
	(0.095)	(0.059)	(0.096)	(0.093)
Joint Significance of		62.67		78.47
Exclusions χ^2 test		[0.09]		[0.006]
ρ		-0.976		-0.942
		(0.003)		(0.011)
LR($\rho = 0$)		419.14		107.46
		[0.00]		[0.00]
total number of obs		5907		4974
censored	1252	4655	933	4041
uncensored		1252		933

Note: Robust standard errors are in parentheses; p-values of the F-statistics are in square brackets. *** p<0.01, ** p<0.05, * p<0.1 Completing high school is the reference category. The regression also control for woman's age, the husband's age and age square and a full set of state of husband's work place dummies. The exclusion variables are women's state of birth dummies.

annual earnings in 2008. Having a Master or a PhD degree is associated with approximately 24% *decrease* of the husband's annual earnings.

The positive relationship between women's high educational attainments and their husbands' income in 1980 could be due to the fact that there is not much participation in the labour market even for women with relatively high education. Presumably this positive association is due to a meeting technology: Women with high education are more likely to meet potential husbands with good income. There are more men investing in high education than women in 1980, the imbalanced sex ratio in college could lead to a higher probability of getting married with a man who has a decent income.

As discussed earlier, the falling cost of education in 2008 enables more women to invest more in education. In 1980 the proportion of women obtaining at least some college experience is 35% of the female population in the sample. While the proportion has increased to 62% in 2008, the proportion of men investing in college education or beyond has remained roughly the same. Hence there are more women investing in higher education than men, and they also participate more often in the labour market and are more likely to have higher income than in the past. The reason we observe a significant negative relationship between women's educational attainments and their husbands' income in 2008 could be the following: women with high educational investments which usually are associated with higher income find it more affordable to marry a husband with relatively lower income.

3.4.3 Robustness Checks

Table 3.7 presents results for three robustness tests. Row 1 and 5 are the benchmark estimates of the probit and the Heckman selection model. Row 2 and 6 show the results with a different coding of the proxy for women's preference towards marriage. Row 3 and 7 limit the sample to women who finished their schooling before marriage. Row 4 and 8 show results for a smaller age group (women age between

40 and 45).

Table 3.7: Robustness Tests

ever married	marriage preference 1980	2008	college graduate 1980	2008	masters or PhDs 1980	2008
1 Baseline Probit	0.542***	0.359***	-0.262***	-0.205***	-0.658***	-0.147**
	(0.0369)	(0.0388)	(0.0634)	(0.0542)	(0.0635)	(0.0714)
2 Alternative coding for proxy of θ_w	0.505***	0.265***	-0.273***	-0.193***	-0.658***	-0.139*
	(0.0457)	(0.0375)	(0.0630)	(0.0542)	(0.0632)	(0.0711)
3 Excluding married before education	0.537***	0.353***	-0.489***	-0.261***	-1.099***	-0.317***
	(0.0391)	(0.0397)	(0.0709)	(0.0557)	(0.0789)	(0.0764)
4 Age 40-45	0.504***	0.381***	-0.165*	-0.314***	-0.597***	-0.186*
	(0.0567)	(0.0559)	(0.0937)	(0.0754)	(0.0909)	(0.102)
log of husband's annual income	1980	2008	1980	2008	1980	2008
5 Baseline Heckman	-0.690***	-0.504***	0.146	-0.110	0.283***	-0.236**
	(0.0665)	(0.0746)	(0.0895)	(0.0866)	(0.105)	(0.107)
6 Alternative coding for proxy θ_w	-0.231***	-0.213***	0.171*	-0.105	0.320***	-0.235**
	(0.0571)	(0.0561)	(0.0899)	(0.0869)	(0.106)	(0.107)
7 Excluding married before education	-0.695***	-0.504***	0.158	-0.113***	0.833***	-0.207*
	(0.0695)	(0.0786)	(0.100)	(0.091)	(0.153)	(0.120)
8 Age40-45	0.0886	-0.614***	0.0278	-0.0206	0.0415	-0.315**
	(0.104)	(0.117)	(0.0998)	(0.123)	(0.125)	(0.152)

Note: This table contains the estimated probit and Heckman coefficients of women's preference towards marriage and educational attainment (a college degree or more) on the probability of ever getting married, and on the log of husbands' annual income. The specification is identical to those in table 3.5 and 3.6 except for the difference noted. Standard errors are in parentheses. ***p<0.01, **p<0.05, *p<0.1.

As mentioned earlier in section 3.2, due to a lack of data on the year of the mother being widowed, it is not possible to differentiate between the group of women whose mothers became widowed only recently (which would imply the woman grew up in a two-parent, intact family), and the group of women who lost their fathers since an early age. In the baseline specification I have coded women whose mothers married only once and current status being "widowed" as if they grew up in a broken family and hence they have a high preference towards marriage. Now I code these women differently, assuming their mother only became widowed recently. This together with the baseline estimates give us a range of the possible effects. The different coding has no substantial effect on the estimated impact of women's marriage preference on the probability of ever getting married. The estimates of its association with husbands' income become smaller but are still highly significant in both 1980 and 2008. Estimated effects of educational attainments change little from the baseline specification, suggesting the results are robust to a different way of coding the preference towards marriage.

Lefgren and McIntyre (2006) point out that the relationship between a woman's education level at the time of marriage and her marriage outcome could be stronger than the relationship between her completed education and her marriage outcome, as the later could lead to a significant timing bias. In the robustness check I exclude those women who get married before finishing their schooling[5]. Row 3 and 7 suggest that the estimated impact of women's preference towards marriage do not change much. Obtaining a bachelor degree or more before marriage becomes much more negatively correlated with the probability of getting married, while the relationship with husbands' income remained largely unchanged from the baseline specification, except that having a Master or a PhD degree is much more positively correlated with husbands' income.

Row 4 and 8 show results for a smaller group of women between age 40 and 45. The first stage probit estimates change little. Examining its relationship with the husband's income, we see the preference towards marriage is not significant in 1980. Having a college degree or above is not significantly associated with husbands' income except for a Master or a PhD degree in 2008.

[5] In 1980 there is data available on the age at the first marriage, I therefore exclude women whose age at first marriage is smaller than their years of education plus 6. In 2008 there is data on the year of last married, the same procedure is applied to women who were only married once.

Overall, the robustness checks suggest that a woman's preference towards marriage is always positively associated with the probability of getting married and is often negatively related to her husband's income conditional on getting married. Having a college degree or more is always negatively correlated with the probability of getting married in both 1980 and 2008. Women's educational attainments is usually associated with a significant increase in husbands' earnings in 1980, while in 2008, having a college degree or more is negatively associated with husbands' earnings, conditional on being married.

3.5 Conclusion

An important caveat on this paper is that it often been suggested that children growing up in broken families tend to have difficulties in maintaining relationships; they are more likely to marry as teenagers, but are also likely to get divorced. This makes the relationship between a woman's true preference towards marriage and whether she grew up in a broken family more complicated, especially at low educational levels. In addition, I restrict the analysis to only one side of the marriage market, ignoring the potential consequences of men taking into account of marriage considerations when invest in schooling. A more careful theoretical approach should take these issues into account. Furthermore, the Census data available dropped significantly when I use the additional information on a woman's mother's marital status. If given more time, I would like to investigate further why the data has dropped 2/3 with mothers' information missing and seems not missing randomly. Another important caveat is that there likely exists significant measurement error in this binary proxy, and this could lead to biases in the probit estimation. As I apply the Heckman selection model to the analysis on husbands' income, I impose a strong assumption that a woman's educational attainment does not directly affect her potential husband's income but only through his income. While it has always been know that highly educated women tend to marry highly educated men, this assumption ignores this potential assortative mating effect.

Appendix A

Matching on Income Potentials and Specialization within the Household

A.1 Proofs of Propositions and Lemmas

Proof of Proposition 1.1

Proof. Couple's utility maximization is equivalently to the following:

$$\max_{d_t^a, d_t^b} \sum_{t=1}^{T} p^t \left[(1-\lambda)log\Big(\underbrace{w_{t-1}^a(1-d_t^a) + w_{t-1}^b(1-d_t^b)}_{c_t}\Big) + \lambda log\Big(\underbrace{(1-\theta)d_t^a + \theta d_t^b}_{z_t}\Big)\right]$$

Consider the last period $t = T$: given wage rates w_{T-1}^a and w_{T-1}^b, the first order conditions are

$$\frac{\partial U_T}{\partial d_T^a} = \frac{\lambda(1-\theta)c_T - (1-\lambda)w_{T-1}^a z_T}{c_T z_T} \tag{A.1}$$

$$\frac{\partial U_T}{\partial d_T^b} = \frac{\lambda(1-\theta)c_T - (1-\lambda)\frac{1-\theta}{\theta}w_{T-1}^b z_T}{c_T z_T \frac{1-\theta}{\theta}} \tag{A.2}$$

If $\frac{\partial U_T}{\partial d_T^a} = 0$, which means $\lambda(1-\theta)c_T = (1-\lambda)w_{T-1}^a z_T$, the first order condition regarding wife's time division (A.2) becomes

$$\frac{\partial U_T}{\partial d_T^b} = \frac{(1-\lambda)z_T}{c_T z_T \frac{1-\theta}{\theta}}\left(w_{T-1}^a - \frac{1-\theta}{\theta}w_{T-1}^b\right) > 0 \quad if\ \frac{w_{T-1}^b}{w_{T-1}^a} < \frac{\theta}{1-\theta}$$

When $\frac{w_{T-1}^b}{w_{T-1}^a} < \frac{\theta}{1-\theta}$, or when $\frac{w_{T-1}^b}{w_{T-1}^a + w_{T-1}^b} < \theta$: the two first order conditions cannot both equal to 0, we have either $\frac{\partial U_T}{d_T^a}|_{d_T^{a*}} = 0$ and $\frac{\partial U_T}{d_T^b} > 0$ which implies we have a corner solution $d_T^{b*} = 1$, or $\frac{\partial U_T}{d_T^b}|_{d_T^{b*}} = 0$ and $\frac{\partial U_T}{d_T^a} < 0$ which implies we have a corner solution $d_T^{a*}=0$. Similarly, when $\frac{w_{T-1}^b}{w_{T-1}^a} > \frac{\theta}{1-\theta}$, or when $\frac{w_{T-1}^b}{w_{T-1}^a+w_{T-1}^b} > \theta$, we have either $\frac{\partial U_T}{d_T^a}|_{d_T^{a*}} = 0$ and $\frac{\partial U_T}{d_T^b} < 0$ which implies we have a corner solution $d_T^{b*}=0$, or $\frac{\partial U_T}{d_T^b}|_{d_T^{b*}} = 0$ and $\frac{\partial U_T}{d_T^a} > 0$ which implies we have a corner solution $d_T^{a*}=1$. Now consider the four cases where there exists a corner solution:

- $d_T^{b*}=1$, that is the wife spends all her time on home production and does not participate in the labor market, utility function becomes:

$$\begin{aligned} U_T &= (1-\lambda)log[w_{T-1}^a(1-d_T^a)] + \lambda log((1-\theta)d_T^a+\theta) \\ &= (1-\lambda)log(w_{T-1}^a) + (1-\lambda)log(1-d_T^a) + \lambda log[(1-\theta)d_T^a+\theta] \end{aligned}$$

$$\begin{aligned} \frac{\partial U_T}{\partial d_T^a} &= \frac{\lambda(1-\theta)(1-d_T^a) - (1-\lambda)[(1-\theta)d_T^a+\theta]}{(1-d_T^a)[(1-\theta)d_T^a+\theta]} \\ &= \frac{\lambda-\theta-(1-\theta)d_T^a}{(1-d_T^a)[(1-\theta)d_T^a+\theta]} \begin{cases} = 0 \quad and \quad d_T^{a*} = \frac{\lambda-\theta}{1-\theta} \quad when \quad \lambda \geq \theta \\ < 0 \quad and \quad d_T^{a*} = 0 \quad when \quad \lambda < \theta \end{cases} \end{aligned}$$

Therefore

$$U_T(d_T^{a*},1) \begin{cases} = \quad (1-\lambda)log(w_{T-1}^a\frac{1-\lambda}{1-\theta}) + \lambda log(\lambda) & when \quad \lambda \geq \theta \\ = \quad (1-\lambda)log(w_{T-1}^a) + \lambda log(\theta) & when \quad \lambda < \theta \end{cases}$$

- $d_T^{a*}=0$, that is the husband spends all his time on labor market work. Utility function becomes:

$$U_T = (1-\lambda)log[w_{T-1}^a + w_{T-1}^b(1-d_T^b)] + \lambda log(\theta) + \lambda log(d_T^b)$$

$$\begin{aligned} \frac{\partial U_T}{\partial d_T^b} &= \frac{\lambda w_{T-1}^a + \lambda w_{T-1}^b(1-d_T^b) - (1-\lambda)w_{T-1}^b d_T^b}{d_T^b[w_{T-1}^a + w_{T-1}^b(1-d_T^b)]} \\ &= \frac{\lambda(w_{T-1}^a + w_{T-1}^b) - w_{T-1}^b d_T^b}{d_T^b[w_{T-1}^a + w_{T-1}^b(1-d_T^b)]} \begin{cases} = 0 \quad and\ d_T^{b*} = \lambda\frac{w_{T-1}^a+w_{T-1}^b}{w_{T-1}^b} \\ \quad when\ \frac{w_{T-1}^b}{w_{T-1}^a+w_{T-1}^b} \geq \lambda \\ > 0 \quad and\ d_T^{b*} = 1 \\ \quad when\ \frac{w_{T-1}^b}{w_{T-1}^a+w_{T-1}^b} < \lambda \end{cases} \end{aligned}$$

Therefore

$$U_T(0\,,\,d_T^{b*}) \begin{cases} = \quad log(w_{T-1}^a + w_{T-1}^b) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda\theta}{w_{T-1}^b}\ when \quad \frac{w_{T-1}^b}{w_{T-1}^a+w_{T-1}^b} \geq \lambda \\ = \quad (1-\lambda)log(w_{T-1}^a) + \lambda log(\theta)\ when\ \frac{w_{T-1}^b}{w_{T-1}^a+w_{T-1}^b} < \lambda \end{cases}$$

- $d_T^{a*}=1$, the husband spends all his time in home production, utility function becomes:

$$\begin{aligned} U_T &= (1-\lambda)log[w_{T-1}^b(1-d_T^b)]+\lambda log[(1-\theta)+\theta d_T^b] \\ &= (1-\lambda)log(w_{T-1}^b)+(1-\lambda)log(1-d_T^b))+\lambda log[(1-\theta)+\theta d_T^b] \end{aligned}$$

$$\begin{aligned} \frac{\partial U_T}{\partial d_T^b} &= \frac{\lambda\theta(1-d_T^b)-(1-\lambda)[(1-\theta)+\theta d_T^b]}{(1-d_T^b)[(1-\theta)+\theta d_T^b]} \\ &= \frac{\lambda+\theta-1-\theta d_T^b}{(1-d_T^b)[(1-\theta)+\theta d_T^b]}\begin{cases} =0 \quad and \quad d_T^{b*}=\frac{\lambda+\theta-1}{\theta} \quad when \quad \lambda\geq 1-\theta \\ <0 \quad and \quad d_T^{b*}=0 \quad when \quad \lambda<1-\theta \end{cases} \end{aligned}$$

Therefore

$$U_T(1,\, d_T^{b*})\begin{cases} = & (1-\lambda)log(w_{T-1}^b\frac{1-\lambda}{\theta})+\lambda log(\lambda) \quad when \quad \lambda\geq 1-\theta \\ = & (1-\lambda)log(w_{T-1}^b)+\lambda log(1-\theta) \quad when \quad \lambda<1-\theta \end{cases}$$

- $d_T^{b*}=0$, the wife spends all her time on labor market work, utility becomes:

$$\begin{aligned} U_T &= (1-\lambda)log[w_{T-1}^a(1-d_T^a)+w_{T-1}^b]+\lambda log[(1-\theta)d_T^a] \\ &= (1-\lambda)log[w_{T-1}^a(1-d_T^a)+w_{T-1}^b]+\lambda log(d_T^a)+\lambda log(1-\theta) \end{aligned}$$

$$\begin{aligned} \frac{\partial U_T}{\partial d_T^a} &= \frac{\lambda[w_{T-1}^a(1-d_T^a)+w_{T-1}^b]-(1-\lambda)w_{T-1}^a d_T^a}{d_T^a[w_{T-1}^a(1-d_T^a)+w_{T-1}^b]} \\ &= \frac{\lambda(w_{T-1}^a+w_{T-1}^b)-w_{T-1}^a d_T^a}{d_T^a[w_{T-1}^a(1-d_T^a)+w_{T-1}^b]}\begin{cases} =0 \quad and\ d_T^{a*}=\lambda\frac{w_{T-1}^a+w_{T-1}^b}{w_{T-1}^a} \ when\ \frac{w_{T-1}^a}{w_{T-1}^a+w_{T-1}^b}\geq\lambda \\ >0 \quad and\ d_T^{a*}=1\ when\ \frac{w_{T-1}^a}{w_{T-1}^a+w_{T-1}^b}<\lambda \end{cases} \end{aligned}$$

Therefore

$$U_T(d_{T-1}^{a*},\, 0)\begin{cases} = & log(w_{T-1}^a+w_{T-1}^b)+(1-\lambda)log(1-\lambda)+\lambda log\frac{\lambda(1-\theta)}{w_{T-1}^a}\ when\ \frac{w_{T-1}^a}{w_{T-1}^a+w_{T-1}^b}\geq\lambda \\ = & (1-\lambda)log(w_{T-1}^b)+\lambda log(1-\theta)\ when\ \frac{w_{T-1}^a}{w_{T-1}^a+w_{T-1}^b}<\lambda \end{cases}$$

Step 1: Compare $U_T(d_T^{a*},1)$ and $U_T(0\,,\,d_T^{b*})$:

when $\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}} < \lambda$ and $\lambda \geq \theta$, using the fact that $x = \lambda$ is the global maximum of the function[1] $f(x) = \lambda log(x) + (1-\lambda) log(1-x)$, $U_T(d^{a*}_T, 1)$ is preferred to $U_T(0\,,\, d^{b*}_T)$:

$$\begin{aligned}
U_T(d^{a*}_T, 1) &\geq U_T(0\,,\, d^{b*}_T) \\
(1-\lambda)log(w^a_{T-1}\frac{1-\lambda}{1-\theta}) + \lambda log(\lambda) &\geq (1-\lambda)log(w^a_{T-1}) + \lambda log(\theta) \\
(1-\lambda)log(\frac{1-\lambda}{1-\theta}) &\geq \lambda log(\frac{\theta}{\lambda}) \\
\lambda log(\lambda) + (1-\lambda)log(1-\lambda) &\geq \lambda log(\theta) + (1-\lambda)log(1-\theta)
\end{aligned}$$

If $\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}} < \lambda$ and $\lambda < \theta$, i.e $\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}} \leq \lambda < \theta$, we have full specialization where $d^{a*}_T = 0$ and $d^{b*}_T = 1$ and:

$$\begin{aligned}
U_T(d^{a*}_T, 1) &= U_T(0\,,\, d^{b*}_T) \\
(1-\lambda)log(w^a_{T-1}) + \lambda log(\theta) &= (1-\lambda)log(w^a_{T-1}) + \lambda log(\theta)
\end{aligned}$$

when $\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}} \geq \lambda$, using $\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}} < \theta$, $U_T(0\,,\, d^{b*}_T)$ is preferred to $U_T(d^{a*}_T, 1)$:

$$\begin{aligned}
U_T(0\,,\, d^{b*}_T) &> U_T(d^{a*}_T, 1) \\
log(w^a_{T-1} + w^b_{T-1}) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda\theta}{w^b_{T-1}} &> (1-\lambda)log(w^a_{T-1}) + \lambda log(\theta) \\
\lambda log(\lambda) + (1-\lambda)log(1-\lambda) &> \begin{array}{c} (1-\lambda)log(\frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}}) \\ +\lambda log(\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}}) \end{array}
\end{aligned}$$

In a single period model:

$$U_t(d^{a*}_t,\ d^{b*}_t)\left\{\begin{array}{lcr} = & (1-\lambda)log(w^a_{t-1}) + (1-\lambda)log(\frac{1-\lambda}{1-\theta}) + \lambda log(\lambda) & when\ \lambda \geq \theta \\ = & (1-\lambda)log(w^a_{t-1}) + \lambda log(\theta) & when\ \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \leq \lambda \leq \theta \\ = & log(w^a_{t-1} + w^b_{t-1}) + (1-\lambda)log(1-\lambda) + \lambda log(\frac{\lambda\theta}{w^b_{t-1}}) & when\ \lambda < \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \end{array}\right. \tag{A.3}$$

Step 2: Compare $U_T(d^{a*}_T, 0)$ and $U_T(1\,,\, d^{b*}_T)$, when $\frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}} < \lambda$, we have:

$$\begin{aligned}
U_T(d^{a*}_T, 0) &< U_T(1\,,\, d^{b*}_T) \\
(1-\lambda)log(w^b_{T-1}) + \lambda log(1-\theta) &< (1-\lambda)log(w^b_{T-1}\frac{1-\lambda}{\theta}) + \lambda log(\lambda) \\
(1-\lambda)log(\theta) + \lambda log(1-\theta) &< \lambda log(\lambda) + (1-\lambda)log(1-\lambda)
\end{aligned}$$

[1] $f'(x) = \frac{\lambda}{x} - \frac{1-\lambda}{1-x} = \frac{\lambda - x}{x(1-x)} = 0$ if $x = \lambda$ and $f''(x) = -\frac{\lambda(1-x)^2 + (1-\lambda)x^2}{x^2(1-x)^2} = -\frac{1}{\lambda(1-\lambda)} < 0$ when $x = \lambda$, therefore $x = \lambda$ is the global maximum.

when home goods are highly valued, $U_T(1\,,\,d_T^{b*}) = (1-\lambda)log(w_{T-1}^b\frac{1-\lambda}{\theta}) + \lambda log(\lambda)$

When $\lambda < 1-\theta$, we have: $U_T(d_T^{a*},0) = log(w_{T-1}^a + w_{T-1}^b) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda(1-\theta)}{w_{T-1}^a}$

$$\begin{aligned}
U_T(d_T^{a*},0) &> U_T(1\,,\,d_T^{b*}) \\
log(w_{T-1}^a + w_{T-1}^b) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda(1-\theta)}{w_{T-1}^a} &> (1-\lambda)log(w_{T-1}^b) + \lambda log(1-\theta) \\
\lambda log(\lambda) + (1-\lambda)log(1-\lambda) &> \begin{array}{c}\lambda log\left(\frac{w_{T-1}^a}{w_{T-1}^a+w_{T-1}^b}\right) \\ +(1-\lambda)log\left(\frac{w_{T-1}^b}{w_{T-1}^a+w_{T-1}^b}\right)\end{array}
\end{aligned}$$

Step 3: assume $w_{T-1}^a > w_{T-1}^b$, compare $U_T(d_T^{a*},1)$ and $U_T(1\,,\,d_T^{b*})$ when $\lambda > \frac{w_{T-1}^a}{w_{T-1}^a+w_{T-1}^b}$ and $\lambda > \theta$:

$$\begin{aligned}
U_T(d_T^{a*},1) &> U_T(1\,,\,d_T^{b*}) \\
(1-\lambda)log(w_{T-1}^a\frac{1-\lambda}{1-\theta}) + \lambda log(\lambda) &> (1-\lambda)log(w_{T-1}^b\frac{1-\lambda}{\theta}) + \lambda log(\lambda) \\
(1-\lambda)log\left(\frac{w_{T-1}^a}{w_{T-1}^b}\right) &> (1-\lambda)log\left(\frac{1-\theta}{\theta}\right)
\end{aligned}$$

compare $U_T(d_T^{a*},1)$ and $U_T(1\,,\,d_T^{b*})$ when $\lambda > \frac{w_{T-1}^a}{w_{T-1}^a+w_{T-1}^b}$ and $\frac{w_{T-1}^b}{w_{T-1}^a+w_{T-1}^b} < \lambda < \theta$, using the fact that $w_{T-1}^a > w_{T-1}^b$

$$\begin{aligned}
U_T(d_T^{a*},1) &> U_T(1\,,\,d_T^{b*}) \\
(1-\lambda)log(w_{T-1}^a) + \lambda log(\theta) &> (1-\lambda)log(w_{T-1}^b\frac{1-\lambda}{\theta}) + \lambda log(\lambda) \\
(1-\lambda)log\left(\frac{w_{T-1}^a}{w_{T-1}^b}\right) + \lambda log(\theta) &> (1-\lambda)log(1-\lambda) + \lambda log(\lambda)
\end{aligned}$$

Now I compare when $\lambda < 1-\theta$, and $\lambda < \frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}}$, we have

$$\begin{aligned}
U_T(0,\ d^{b*}_T) &> U_T(d^{a*}_T,\ 0)\\
log(w^a_{T-1}+w^b_{T-1}) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda\theta}{w^b_{T-1}} &> \begin{array}{r} log(w^a_{T-1}+w^b_{T-1}) + (1-\lambda)log(1-\lambda)\\ \lambda log\lambda(1-\theta) - \lambda log(w^a_{T-1})\end{array}\\
\lambda log\frac{\theta}{w^b_{T-1}} &> \lambda log\frac{(1-\theta)}{w^a_{T-1}}\\
\frac{\theta}{1-\theta} &> \frac{w^b_{T-1}}{w^a_{T-1}}
\end{aligned}$$

Now I compare when $\lambda < 1-\theta$, and $\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}} < \lambda < \theta$, we have

$$\begin{aligned}
U_T(0,\ d^{b*}_T) &> U_T(d^{a*}_T,\ 0)\\
(1-\lambda)log(w^a_{T-1}) + \lambda log\theta &> \begin{array}{r} log(w^a_{T-1}+w^b_{T-1}) + (1-\lambda)log(1-\lambda)\\ +\lambda log\lambda(1-\theta) - \lambda log(w^a_{T-1})\end{array}\\
log\frac{w^a_{T-1}}{(w^a_{T-1}+w^b_{T-1})} + \lambda log\frac{\theta}{1-\theta} &> (1-\lambda)log(1-\lambda) + \lambda log\lambda\\
log\frac{w^a_{T-1}}{(w^a_{T-1}+w^b_{T-1})} + \lambda log\frac{\theta}{\lambda} &> (1-\lambda)log(1-\lambda) + \lambda log(1-\theta)
\end{aligned}$$

since $\theta > \lambda$ and $\frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}} > 1-\lambda > 1-\theta$.

when $1-\theta < \lambda < \frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}}$, and $\lambda \geq \theta$:

$$\begin{aligned}
U_T(d^{a*}_T, 1) &> U_T(d^{a*}_{T-1},\ 0)\\
(1-\lambda)log(w^a_{T-1}\frac{1-\lambda}{1-\theta}) + \lambda log(\lambda) &> log(w^a_{T-1}+w^b_{T-1}) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda(1-\theta)}{w^a_{T-1}}\\
log\frac{w^a_{T-1}}{(w^a_{T-1}+w^b_{T-1})} &> log(1-\theta)
\end{aligned}$$

when $1-\theta < \lambda < \frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}}$, and $\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}} < \lambda < \theta$, using the fact $\theta > 1-\theta$ and $\frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}} > max(\lambda\ 1-\lambda)$

$$\begin{aligned}
U_T(d^{a*}_T, 1) &> U_T(d^{a*}_{T-1},\ 0)\\
(1-\lambda)log(w^a_{T-1}) + \lambda log(\theta) &> log(w^a_{T-1}+w^b_{T-1}) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda(1-\theta)}{w^a_{T-1}}\\
log\left(\frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}}\right) + \lambda log(\theta) &> (1-\lambda)log(1-\lambda) + \lambda log\lambda + \lambda log(1-\theta)
\end{aligned}$$

when $1-\theta < \lambda < \frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}}$, and $\lambda \geq \theta$:

$$\begin{aligned}
U_T(d^{a*}_T, 1) &> U_T(1,\ d^{b*}_{T-1}) \\
(1-\lambda)log(w^a_{T-1}\frac{1-\lambda}{1-\theta}) + \lambda log(\lambda) &> (1-\lambda)log(w^b_{T-1}\frac{1-\lambda}{\theta}) + \lambda log(\lambda) \\
log\left(\frac{\theta}{1-\theta}\right) &> log\left(\frac{w^b_{T-1}}{w^a_{T-1}}\right)
\end{aligned}$$

when $1-\theta < \lambda < \frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}}$, and $\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}} < \lambda < \theta$, using the fact $\theta > 1-\theta$ and $\frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}} > max(\lambda\ 1-\lambda)$

$$\begin{aligned}
U_T(d^{a*}_T, 1) &> U_T(1,\ d^{b*}_{T-1}) \\
(1-\lambda)log(w^a_{T-1}) + \lambda log(\theta) &> (1-\lambda)log(w^b_{T-1}\frac{1-\lambda}{\theta}) + \lambda log(\lambda) \\
(1-\lambda)log(\frac{w^a_{T-1}}{w^b_{T-1}}) + log(\theta) &> (1-\lambda)log(1-\lambda) + \lambda log(\lambda)
\end{aligned}$$

using the fact that $\theta > \lambda$ and $\theta > 1-\lambda$ and $w^a_{T-1} > w^b_{T-1}$.

Step 4: assume $w^a_{T-1} < w^b_{T-1}$, and still we have $\frac{w^b_{T-1}}{w^a_{T-1}} < \frac{\theta}{1-\theta}$, this two conditions lead to $\theta > \frac{1}{2}$

when $\frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}} < \lambda < \frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}}$,

$$\begin{aligned}
U_T(0,\ d^{b*}_T) &> U_T(1,\ d^{b*}_T) \\
log(w^a_{T-1}+w^b_{T-1}) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda\theta}{w^b_{T-1}} &> \begin{array}{r}(1-\lambda)log(w^b_{t-1}) + (1-\lambda)log(1-\lambda) \\ -(1-\lambda)log\theta + \lambda log(\lambda)\end{array} \\
log(w^a_{T-1}+w^b_{T-1}) + \lambda log\frac{\theta}{w^b_{T-1}} &> (1-\lambda)log(w^b_{t-1}) + (1-\lambda)log(\frac{1}{\theta}) \\
log(\theta) &> log\left(\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}}\right)
\end{aligned}$$

when $1-\theta < \lambda < \frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}}$, using the fact that $\lambda > 1-\theta$, $\theta > \frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}}$ and $(1-\lambda) > \frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}}$, we have:

$$\begin{aligned}
U_T(0,\ d^{b*}_T) &> U_T(1,\ d^{b*}_T) \\
log(w^a_{T-1}+w^b_{T-1}) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda\theta}{w^b_{T-1}} &> (1-\lambda)log(w^b_{T-1}) + \lambda log(1-\theta) \\
(1-\lambda)log(1-\lambda) + \lambda log\lambda + \lambda log(\theta) &> log\left(\frac{w^b_{T-1}}{w^a_{T-1}+w^b_{T-1}}\right) + \lambda log(1-\theta)
\end{aligned}$$

when $\lambda < \frac{w^a_{T-1}}{w^a_{T-1}+w^b_{T-1}}$, we have

$$\begin{aligned}
U_T(0,\ d^{b*}_T) &> U_T(d^{a*}_T,\ 0) \\
log(w^a_{T-1}+w^b_{T-1}) + (1-\lambda)log(1-\lambda) + \lambda log\frac{\lambda\theta}{w^b_{T-1}} &> \begin{array}{r} log(w^a_{T-1}+w^b_{T-1}) + (1-\lambda)log(1-\lambda) \\ +\lambda log(\lambda(1-\theta)) - \lambda log(w^a_{T-1}) \end{array} \\
log\frac{\theta}{w^b_{T-1}} &> log\frac{(1-\theta)}{w^a_{T-1}} \\
log\frac{\theta}{(1-\theta)} &> log\frac{w^b_{T-1}}{w^a_{T-1}}
\end{aligned}$$

■

Proof of Proposition 1.2

Proof. From proposition 1.1, we have:

$$\frac{d^{b*}_t}{d^{a*}_t + d^{b*}_t} = \begin{cases} 1 & when\ \theta \geq \lambda\ and\ \theta > \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \\ \frac{1}{1+\frac{\lambda-\theta}{1-\theta}} & when\ \theta < \lambda\ and\ \theta > \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \end{cases} \tag{A.4}$$

Similar to proposition 1.1, when $\theta < \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}}$

$$[d^{a*}_t,\ d^{b*}_t] = \begin{cases} [1,\ \frac{\lambda-(1-\theta)}{\theta}] & when\ \lambda \geq 1-\theta \\ [1,\ 0] & when\ \frac{w^a_{t-1}}{w^a_{t-1}+w^b_{t-1}} \leq \lambda \leq 1-\theta \\ [\lambda\frac{w^a_{t-1}+w^b_{t-1}}{w^a_{t-1}},\ 0] & when\ \lambda < \frac{w^a_{t-1}}{w^a_{t-1}+w^b_{t-1}} \end{cases}$$

Hence

$$\frac{d^{b*}_t}{d^{a*}_t + d^{b*}_t} = \begin{cases} \frac{1}{1+\frac{\theta}{\lambda-(1-\theta)}} & when\ \theta \geq 1-\lambda\ and\ \theta < \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \\ 0 & when\ \theta < 1-\lambda\ and\ \theta < \frac{w^b_{t-1}}{w^a_{t-1}+w^b_{t-1}} \end{cases} \tag{A.5}$$

Proposition 1.2 combines the two cases. In addition, we have $\frac{1}{1+\frac{\theta}{\lambda-(1-\theta)}} < \frac{1}{1+\frac{\lambda-\theta}{1-\theta}}$ since $\lambda < 1$. ■

A.2 The Marriage Premium Literature

The marriage premium puzzle states that everything else equal, married men earn approximately 10-50% more than their single counterparts. Two main hypotheses have been proposed to explain the marriage premium: marriage leads men to be more productive (the specialization effect) and/or men with higher earnings are more likely to get married (the selection effect).

The specialization effect: According to Becker's (1973, 1974): while the wife specializes in domestic housework: cooking, laundry and bearing children, the husband is able to devote more energy, time and effort to his market work.

Direct empirical evidence on the specialization effect is mixed. On the one hand, wife's housework time or the number of her domestic chores is found to be positively related to her husband's labor-market hours (Hersch and Stratton, 1994; Chun and Lee, 2001; Bardasi and Taylor, 2008; Jacobsen and Rayack, 1996). On the other hand, South and Spitze (1994), Loh (1996) and Hersch and Stratton (2000) found that the wage premium is unaffected after controlling for time spent on housework by husband or wife's labor supply decisions. While Gray and Vanderhart (2000) suggested that a decline in the wage premium after 1970 is due to a decline in household specialization when divorce gets unilateral, and Gupta, Smith and Stratton (2007) suggested that marriage premium is less than 4% in Denmark because families there have lower intra-household specialization, Loh (1996) pointed out that in contrast to what specialization effect suggests, married self-employed men earn less than single self-employed men, and cohabited men earn the same marriage premium as married men.

Kenny (1983) suggested an alternative mechanism which follows the Mincer's (1974) human capital theory to explain the marriage premium (also in Becker,1985): the cost of investment in human capital is lower during one's married years than during his single years. A man will be able to accumulate a certain level of investment in a shorter period compared to being single when time and effort are constrained by housework or wife-searching. Again, direct empirical evidence is not conclusive: Lynch (1992) indicated that formal on-the-job training is concentrated among white married unionized males. Hill (1979) reported that on average married men spent more time in training than single men, Arulampalam and Booth (1998) and Shileds (1998) suggested that marriage significantly raises the likelihood of men receiving training. On the other hand, Rodgers and Stratton (2010) reported that controlling for training does not alter the marriage premium much.

The human capital accumulation mechanism sounds similar to the specialization effect mentioned earlier, and indeed several researchers provided evidence for (or against) the human capital mechanism is also consistent with (or against) the specialization hypothesis. Kenny (1983), Koreman and Neumark (1991), Gray (1997) and Stratton (2002) found the positive return is to martial tenure instead of marital status, and concluded that the wage premium is due to a faster growth rate during marriage. In contrast, Cornwell and Rupert (1997) examining the US data, Richardson (2000) examining Swedish households, and Bardasi and Taylor (2004) examining UK data all argued that return to marriage is just an intercept shift, whereas time spent in marriage has no effect on wages.

The selection effect: There are some characteristics of a man, such as disciplined personal habits, amiability, a sense of responsibility and strong interpersonal skills, are deemed attractive by his potential wives and/or[2] employers but are not observable to econometricians. Marital status therefore act as a proxy picking up positive effects of those characteristics in the (log) wage earning regression.

Direct empirical evidence is rare due to a lack of accurate measurements of those skills. Cappelli, Constantine and Chadwick (2000) examined high school students' attitudes toward marriage and family[3] and found that giving priority to a good family life pays off, indicating such preference leads to success in both labor and marriage market.

Assuming those characteristics are time-invariant and affect log earnings in a linear manner, researchers using panel data with fixed effect estimations provided a wide spectrum of the magnitude of the selection effect: Nakosteen and Zimmer (1987), Jacobsen and Rayack (1996), Gray (1997) and Rodgers and Stratton[4] (2010) found that marriage premium disappeared after removing individual fixed effects. Petersen, Penner and Hogsnes (2011) reported that 80% of the marriage premium is due to selection in Norway. Cornwell and Rupert (1997), Gupta, Smith and Stratton (2007), and Bardai and Taylor (2008) found that the use of panel data methods reduces the marriage wage premium by more than 50%. At the other end of the spectrum, Hersch and Stratton (2000) conclude that fixed effects explain less than half of the marriage premium. Stratton (2002) pointed out that the marriage premium enjoyed by cohabited men is purely due to selection, while for married men it is not. Koreman and Neumark (1991) argued that at most 20% of the marriage premium is due to selection. Kenny (1983) argued there is no individual difference in the initial human capital level or the propensity to invest in human capital between married and single men. Loh (1996) found that wife's education contributes positively to her husband's wage rate, but a multi-logit test concludes that men with higher wage rate in the past were *not* more likely to marry better-educated women.

[2]It can also be that marital status acts as a signal to potential employers as they assume married men are more likely to have those skills.

[3]In NLS 72, high school students were asked to rate "how important is to find the right person to marry and having a happy family life" on a three-point scale. This additional control is positively and significantly related to hourly earnings 14 years later.

[4]Rodgers and Stratton (2010) found that controlling for individual fixed effects, marriage premium for white men disappeared and for black men it is still significant.

Focusing on data which are more likely free of selectivity bias is another alternative to examine the selection effect, still, researchers disagree on the degree of its importance: Ginther and Zavodny (2001) found that the premium for shotgun marriages (16%) is similar to that of non-shotgun marriages (16.4%), and concluded that at most 10% of the marriage premium is due to selection. Antonovics and Town (2004) using data on monozygotic twins found that marriage premium increased from 19% (cross-sectional estimates) to 26% (within twin estimates), which suggests a strong rejection of the selection effect.

Appendix B

Matching with Contracts: an Efficient Marriage Market?

B.1 Proof of Propositions, Lemmas and Theorems

Proof of Proposition 2.2: **To check whether there is any profitable deviation for W^H and W^L,** Proposition 2.1 has established that signaling (punishing a mimic) is costless in the benchmark model, W^H will not mimic W^L by offering contract B, and W^L will not mimic W^H by offering contract A.

For W^L, conditional on the expected matching outcome, contract B achieves the highest possible payoff for W^L as it offers 0 to men. Hence contract B dominates contract C, contract E_1, contract F_1 and contract E_2, as they achieve no better expected matching outcome. W^L will prefer contract B to contract E_3 iff $P_{E_3}^{W^L} \leq P_B^{W^L} \Leftrightarrow \frac{1}{2}(z_1 - \beta_2) + \frac{1}{2}z_0 \leq \frac{3}{8}z_1 + \frac{5}{8}z_0 \Leftrightarrow \beta_2 \geq \frac{z_1 - z_0}{4}$.

For W^H, the best contract E_2 (contract in the form of E_2 that achieves the highest possible payoff for W^H) is strictly dominated by the best contract F, as both contracts achieve the same expected matching outcome, but W^H could offer 0 to men conditional on both z_2 and z_1' in contract F. The best contract C and contract E_1 are strictly dominated by the best contract F as they achieve no better matching outcome. W^H will not deviate to offer contract F iff $\frac{3}{8}(z_2) + \frac{5}{8}(z_1') \leq \frac{5}{8}(z_2 - \beta_2) + \frac{3}{8}z_1' \Leftrightarrow \beta_2 \leq \frac{2}{5}(z_2 - z_1')$; and will not deviate to offer contract E_3 iff $\frac{1}{2}(z_2 - \beta_2) + \frac{1}{2}z_1' \leq \frac{5}{8}(z_2 - \beta_2) + \frac{3}{8}z_1' \Leftrightarrow \beta_2 \leq z_2 - z_1'$.

We conclude that $\beta_2 \in [\frac{z_1 - z_0}{4}; \frac{2}{5}(z_2 - z_1')]$ supports a separating equilibrium that achieves matching efficiency.

Proof of Proposition 2.3: Similar reasoning as in Proposition 2.1, we only need to consider competition between women of the same type. a). **When there are two W^H, one M^H and one M^L**: There exists competition between the two high type women to try to win the only high type man over. i.e. W^H may deviate to offer contract H in an attempt to achieve "attract only M^H":

Contract H		
Surplus	Woman	Man
Y_G	$Y_G - \gamma_1^H$	γ_1^H
Y_B	$Y_B - \gamma_0^H$	γ_0^H

where $\gamma_1^H \in [0, Y_G]$ and $\gamma_0^H \in [0, Y_B]$ achieve "attract both" when competing against contract A. Calculate men's expected payoffs of being matched with contract H: $P_H^{M^H} = \mu_H[p_2\gamma_1^H + (1-p_2)\gamma_0^H] + (1-\mu_H)[p_1\gamma_1^H + (1-p_1)\gamma_0^H]$ and $P_H^{M^L} = \mu_H[p_1'\gamma_1^H + (1-p_1')\gamma_0^H] + (1-\mu_H)[p_0\gamma_1^H + (1-p_0)\gamma_0^H]$. I specify the off-equilibrium belief μ_H such that both high type and low type men's expected payoff when being matched with contract H is the same: $\mu_H = \frac{p_0 - p_1}{p_2 + p_0 - p_1' - p_1} \in [0, 1] \Leftrightarrow P_H^{M^H} = P_H^{M^L}$. Hence there does exist a deviating contract that achieves "attract only W^H" when competing against contract A.

b). **When there are two W^L, one M^H and one M^L**: Similarly there exists competition between the two low type women to attempt to win the only low type man over. However for whatever W^L offers in the off-equilibrium contract H such that $P_H^{M^L} \geq p_0\varepsilon$, we also have $P_H^{M^H} \geq p_0\varepsilon > p_1\varepsilon$ since $z_0 > z_1$ implies $p_0 > p_1$. Therefore both M^H and M^L will be attracted over. It is not possible for W^L to achieve "attract only W^L" by deviating when competing against contract B.

Proof of Theorem 2.5: $\beta_1 \in [\frac{2(z_1 - z_0)}{5p_1 + 3p_0}, \frac{2}{5}(Y_G - Y_B)]$ satisfy the six conditions below, therefore a separating equilibrium that achieves matching efficiency always exists when $z_1 > z_0$ and $p'_1 = 0$.

1. $(p_2 + p_1)Y_G \geq 2(p_2\beta_1 + \varepsilon)$, i.e $p_2\beta_1 \leq \frac{p_2+p_1}{2} Y_G$, then there exists a contract F_2

Contract F_2		
Surplus	Woman	Man
Y_G	$Y_G - \gamma_1^{F_2}$	$\gamma_1^{F_2}$
Y_B	Y_B	0

   where $\gamma_1^{F_2} = \frac{2(p_2\beta_1+\varepsilon)}{p_2+p_1} \in [0, Y_G]$ maximizes women's expected payoff subject to the constraint that it "attracts both" when competing against contract A and contract B.

2. $p_2\beta_1 \geq \frac{p_2+p_1}{8(p_1+p_0)}(z_1 - z_0)$, for W^L not to deviate to offer contract F_2.

3. $\left(5 - \frac{8p_2}{p_2+p_1)}\right) p_2\beta_1 \leq z_2 - z_1'$, for W^H not to deviate to offer contract F_2.

4. $\beta_1 \geq \frac{2(z_1-z_0)}{5p_1+3p_0}$), for W^L not to deviate to offer contract A mimicking W^H.

5. $\beta_1 \leq \frac{2}{5}(Y_G - Y_B)$, for W^H not to deviate to offer contract B mimicking W^L.

6. $\beta_1 \in [0, Y_G]$

Proof of Proposition 2.7 and 2.8: When $N = 2$, same as the contract-offering game, if a woman reports "high" (which is equivalent to offering contract A), her probability of being matched with M^H is $\frac{5}{8}$ and with M^L is $\frac{3}{8}$.

In a symmetric setting where $z_1 = z_1'(p_1 = p_1')$, when a W^H reports truthfully as a "high" type woman, her expected payoff is $\frac{5}{8}z_2 + \frac{3}{8}z_1' - \frac{5}{8}[p_2\beta_1 + (1-p_2)\beta_0] - \frac{3}{8}[p_1'\mu_1 + (1-p_1')\mu_0]$. When she mis-reports as a "low" type woman, her expected payoff is $\frac{3}{8}z_2 + \frac{5}{8}z_1' - \frac{3}{8}[p_2\tau_1 + (1-p_2)\tau_0] - \frac{5}{8}[p_1'\alpha_1 + (1-p_1')\alpha_0]$. W^H will report truthfully if and only if

$$\begin{array}{l} 5[(p_2\beta_1 + (1-p_2)\beta_0) - (p_1\alpha_1 + (1-p_1)\alpha_0)] \\ +3[(p_1\mu_1 + (1-p_1)\mu_0) - (p_2\tau_1 + (1-p_2)\tau_0)] \end{array} \leq 2(z_2 - z_1) \tag{B.1}$$

Similarly W^L will report truthfully if and only if

$$\begin{array}{l} 5[(p_1\beta_1 + (1-p_1)\beta_0) - (p_0\alpha_1 + (1-p_0)\alpha_0)] \\ +3[(p_0\mu_1 + (1-p_0)\mu_0) - (p_1\tau_1 + (1-p_1)\tau_0)] \end{array} \geq 2(z_1 - z_0) \tag{B.2}$$

M^H will report truthfully if and only if

$$5[(p_2\beta_1 + (1-p_2)\beta_0) - (p_1\alpha_1 + (1-p_1)\alpha_0)] \geq 3[(p_2\mu_1 + (1-p_2)\mu_0) - (p_1\tau_1 + (1-p_1)\tau_0)] \tag{B.3}$$

M^L will report truthfully if and only if

$$5[(p_0\alpha_1 + (1-p_0)\alpha_0) - (p_1\beta_1 + (1-p_1)\beta_0)] \geq 3[(p_0\tau_1 + (1-p_0)\tau_0) - (p_1\mu_1 + (1-p_1)\mu_0)] \tag{B.4}$$

Consider a ***symmetric*** mechanism: $\beta_1 = \alpha_1 = \frac{Y_G}{2}, \beta_0 = \alpha_0 = \frac{Y_B}{2}; \mu_1 = Y_G - \tau_1, \mu_0 = Y_B - \tau_0$

Condition (B.1) and (B.3) becomes: $(p_2 + p_1)\mu_1 + (2 - p_2 - p_1)\mu_0 \leq \frac{5}{6}z_2 + \frac{1}{6}z_1$

Condition (B.2) and (B.4) becomes: $(p_2 + p_1)\mu_1 + (2 - p_2 - p_1)\mu_0 \geq \frac{5}{6}z_1 + \frac{1}{6}z_0$

Re-arranging we obtain $12(p_2 - p_0)\mu_0 \geq (6p_2 - 4p_1 - 2p_0)Y_B + 4(p_1z_1 - p_0z_2)$. With the requirement $\mu_0 \leq Y_B$, we need $\frac{Y_B}{Y_G - Y_B} \geq \frac{2(p_1^2 - p_0p_2)}{3p_2 - 3p_0}$. Furthermore, when this condition holds, there always exists some $\mu_1^* \in [0, Y_G]$ that satisfy the two inequalities above. Therefore $\frac{Y_B}{Y_G - Y_B} \geq \frac{2(p_1^2 - p_0p_2)}{3p_2 - 3p_0}$ is the necessary and sufficient condition for the DRM to achieve matching efficiency.

Now consider an ***asymmetric*** mechanism. By Condition (B.1) and (B.3), we obtain:

$$(p_1+p_2)\mu_1+(2-p_1-p_2)\mu_0 \leq \frac{2}{3}(z_2-z_1)+(p_1+p_2)\tau_1+(2-p_1-p_2)\tau_0 \quad \text{(B.5)}$$

By condition (B.2) and (B.4), we obtain:

$$(p_1+p_0)\mu_1+(2-p_1-p_0)\mu_0 \geq \frac{2}{3}(z_1-z_0)+(p_1+p_0)\tau_1+(2-p_1-p_0)\tau_0 \quad \text{(B.6)}$$

Multiply (B.6) by (p_1+p_2) then subtract (B.5) multiplied by (p_1+p_0), we get: $2(p_2-p_0)(\mu_0-\tau_0)\geq\frac{2}{3}(2p_1^2-2p_2p_0)(Y_G-Y_B)\Rightarrow\mu_0-\tau_0\geq\frac{2}{3}\left(\frac{p_1^2-p_2p_0}{p_2-p_0}\right)(Y_G-Y_B)$. Since $max(\mu_0-\tau_0)=Y_B$, a necessary condition for a general **asymmetric** mechanism to achieve matching efficiency is $Y_B\geq\frac{2}{3}\left(\frac{p_1^2-p_2p_0}{p_2-p_0}\right)(Y_G-Y_B)\Rightarrow\frac{Y_B}{Y_G-Y_B}\geq\frac{2}{3}\left(\frac{p_1^2-p_2p_0}{p_2-p_0}\right)$. Recall the necessary and sufficient condition for a **symmetric** mechanism that achieves matching efficiency is $\frac{Y_B}{Y_G-Y_B}\geq\frac{2}{3}\left(\frac{p_1^2-p_2p_0}{p_2-p_0}\right)$. We conclude that in a symmetric set-up $(z_1=z_1')$, a symmetric mechanism is the best mechanism. If the symmetric mechanism cannot achieve matching efficiency, then no other mechanism can.

Proof of Proposition 2.9:

$$P_N=1-\frac{1}{2^{2N-1}}\begin{pmatrix}2N-1\\N\end{pmatrix}$$

The entries in Table 2.6, a_{ij}, is the probability of marrying a high type man, M^H, where i is the total number of women offering contract A and j is the number of total high type men.

$$a_{ij}=\begin{cases}\frac{j}{i}\text{ if } j\leq i\\ 1\text{ if } j>i\end{cases}$$

.

$$\begin{aligned}P_N &= \sum_{1\leq i\leq N}^{N}\sum_{0\leq j\leq N}^{N}\frac{\begin{pmatrix}N-1\\i-1\end{pmatrix}\begin{pmatrix}N\\j\end{pmatrix}}{2^{2N-1}}a_{ij}\\ &= \frac{1}{2^{2N-1}}\left[\sum_{1\leq j\leq i\leq N}^{N}\begin{pmatrix}N-1\\i-1\end{pmatrix}\begin{pmatrix}N\\j\end{pmatrix}a_{ij}+\sum_{1\leq i<j\leq N}^{N}\begin{pmatrix}N-1\\i-1\end{pmatrix}\begin{pmatrix}N\\j\end{pmatrix}\right]\\ &= \frac{1}{2^{2N-1}}\left[\underbrace{\sum_{1\leq j\leq i\leq N}^{N}\begin{pmatrix}N\\i\end{pmatrix}\begin{pmatrix}N-1\\j-1\end{pmatrix}}_{X}+\underbrace{\sum_{1\leq i<j\leq N}^{N}\begin{pmatrix}N-1\\i-1\end{pmatrix}\begin{pmatrix}N\\j\end{pmatrix}}_{Y}\right]\end{aligned}$$

$$X = \sum_{1\le j\le i\le N}^{N} \begin{pmatrix} N \\ i \end{pmatrix} \begin{pmatrix} N-1 \\ j-1 \end{pmatrix} = \sum_{0\le \widetilde{j} < i \le N}^{N} \begin{pmatrix} N \\ i \end{pmatrix} \begin{pmatrix} N-1 \\ \widetilde{j} \end{pmatrix} \quad \text{denote } \widetilde{j} = j-1$$

so we could rewrite X as $X = \sum_{0\le j<i\le N}^{N} \begin{pmatrix} N \\ i \end{pmatrix} \begin{pmatrix} N-1 \\ j \end{pmatrix}$. Imagine there are $N-1$ white balls, $W_1, W_2 ... W_{N-1}$ and N black balls, $B_1, B_2 ... B_N$, each ball is distinct. Now we consider the number of ways to select these balls, for each ball we could either select it or not. X represents the number of ways to select the balls such that there are more black balls than white balls. To compute X, we need to count the number of ways such that $|B| > |W|$. There are two situations: 1) B_N is selected, then we should have $|B| \ge |W|$ in $(N-1)$ black balls $v.s$ $(N-1)$ white balls case, and 2) B_N is not selected, then we should have $|B| > |W|$ in $(N-1)$ black balls $v.s$ $(N-1)$ white balls case.

$$\begin{aligned} X &= \sharp\{|B| \ge |W|\, in (N-1)\ v.s\, (N-1)\} + \sharp\{|B| > |W|\, in (N-1)\ v.s\, (N-1)\} \\ &= \sharp\{|B| \ge |W|\, in (N-1)\ v.s\, (N-1)\} + \sharp\{|B| < |W|\, in (N-1)\ v.s\, (N-1)\}, \text{ by symmetry} \\ &= \sharp\{\text{any way to choose } 2N-2 \text{ balls}\} = 2^{2N-2} \end{aligned}$$

$$\begin{aligned} Y &= \sum_{1\le i<j\le N}^{N} \begin{pmatrix} N-1 \\ i-1 \end{pmatrix} \begin{pmatrix} N \\ j \end{pmatrix} = \sum_{1\le i\le j\le N}^{N} \begin{pmatrix} N-1 \\ i-1 \end{pmatrix} \begin{pmatrix} N \\ j \end{pmatrix} - \sum_{1\le i=j\le N}^{N} \begin{pmatrix} N-1 \\ i-1 \end{pmatrix} \begin{pmatrix} N \\ j \end{pmatrix} \\ &= X - \underbrace{\sum_{k=1}^{N} \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} \begin{pmatrix} N \\ k \end{pmatrix}}_{Z} \end{aligned}$$

$$Z = \sum_{k=1}^{N} \begin{pmatrix} N-1 \\ k-1 \end{pmatrix} \begin{pmatrix} N \\ k \end{pmatrix} = \sum_{k=1}^{N} \begin{pmatrix} N-1 \\ N-k \end{pmatrix} \begin{pmatrix} N \\ k \end{pmatrix} = \begin{pmatrix} 2N-1 \\ N \end{pmatrix}$$

since Z represents the number of ways to choose N balls out of 2N-1 balls. Therefore $P_N = \frac{1}{2^{2N-1}}(X+Y) = \frac{1}{2^{2N-1}}(X+X-Z) = 1 - \frac{1}{2^{2N-1}} \begin{pmatrix} 2N-1 \\ N \end{pmatrix}$.

1. P_N increases with N: $P_{N+1} - P_N = \frac{1}{2^{2N}} \cdot \frac{(2N-1)!}{(N-1)!(N+1)!} > 0,\ N = 2, 3, 4....$

2. $P_N > \frac{1}{2}$, $\frac{P_N}{2P_N - 1} > 1$: $P_2 = 1 - \frac{1}{2^3} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{5}{8} > \frac{1}{2}$ with N=2 and P_N increases with N.

3. $1 - P_N \approx \frac{1}{\sqrt{\pi N}}$ when N is large: apply Stirling's formula. $\frac{1}{2^{2N-1}} \cdot \binom{2N-1}{N} = \frac{1}{2^{2N-1}} \cdot \frac{(2N-1)!}{(N-1)!N!} = \sqrt{\frac{1}{\pi N}} \cdot \left(1 + \frac{(-\frac{1}{2})}{N}\right)^N \left(1 + \frac{\frac{1}{2}}{(N-1)}\right)^{N-1} \approx \sqrt{\frac{1}{\pi N}} \cdot e^{-\frac{1}{2}} \cdot e^{\frac{1}{2}}$ using the fact $\lim_{n\to\infty}(1 + \frac{x}{n})^n = e^x$. Therefore when N is large, P_N is approximately $1 - \sqrt{\frac{1}{\pi N}}$.

Intermediate Case with partial non-verifiability about agents' types ($z_1 = z_1'$)

Proposition B.1 *In the N men-N women case, when $z_1 \leq \frac{P_N}{2P_N-1}z_0$, there exists a separating equilibrium where W^H offers contract A and W^L offers contract B*

Contract A		
Surplus	*woman*	*man*
z_2	$z_2 - \gamma_2^A$	γ_2^A
z_1	z_1	0
z_0	0	z_0

Contract B		
Surplus	*woman*	*man*
z_2	0	z_2
z_1	z_1	0
z_0	z_0	0

where $\gamma_2^A \in \left[(2P_N - 1)(z_1 - z_0), \frac{(2P_N-1)}{P_N}(z_2 - z_1)\right]$, $P_N = 1 - \frac{1}{2^{2N-1}}\begin{pmatrix} 2N-1 \\ N \end{pmatrix}$

Equilibrium beliefs: $\mu(W^H|\text{contract } A) = 1$ *and* $\mu(W^L|\text{contract } B) = 1$

Equilibrium response rule: When being indifferent between payoffs of equilibrium contracts (contract A and contract B), M^H goes for contract A, M^L goes for contract B.

Off-equilibrium beliefs: $\mu(W^H|\text{any other off-equilibrium contracts}) = 0$

*Off-equilibrium response rule: A man will first go for a contract which gives him the maximum payoff. If he did not get it, he will then go for the next contract which gives him the second largest payoff, and so on. When being indifferent between an equilibrium contract and an off-equilibrium contract: M^L will go for an **off-equilibrium** contract first; M^H will go for an **equilibrium** contract first.*

Theorem B.2 $z_1 \leq \frac{P_N}{2P_N-1}z_0$ *is a necessary condition for the existence of a separating equilibrium with matching efficiency.*

The condition $z_1 \leq \frac{P_N}{2P_N-1}z_0$ is to ensure that W^L will not mimic M^H by offering contract A. When N=2, we require $z_1 \leq 2.5z_0$. When N is very large, the condition becomes $z_1 \leq z_0$.

Bibliography

[1] Abowd, J. M., & Card, D. (1989). On the Covariance Structure of Earnings and Hours Changes. Econometrica, 57(2), 411-445.

[2] Adda, J., Dustmann, C., & Stevens, K. (2015). The career costs of children. Working Paper.

[3] Aguiar, M., & Hurst, E. (2007). Life-Cycle Prices and Production. American Economic Review, 97(5), 1533-1559.

[4] Aguiar, M., & Hurst, E. (2007). Measuring Trends In Leisure: The Allocation Of Time over Five Decades. Technology, 969.

[5] Ahituv, A., & Lerman, R. I. (2007). How do marital status, work effort, and wage rates interact?. Demography, 44 (3), 623-647.

[6] Antonovics, K., & Town, R. (2004). Are all the good men married? Uncovering the sources of the marital wage premium. American Economic Review, 317-321.

[7] Arrow, K. J., Chenery, H. B., Minhas, B. S., & Solow, R. M. (1961). Capital-labor substitution and economic efficiency. The Review of Economics and Statistics, 225-250.

[8] Arulampalam, W., & Booth, A. L. (1998). Training and Labour Market Flexibility: Is There a Trade-off?. British Journal of Industrial Relations, 36 (4), 521-536.

[9] Asiedu, E., & Villamil, A. P. (2000). Discount factors and thresholds: Foreign investment when enforcement is imperfect. Macroeconomic Dynamics, 4(01), 1-21.

[10] Baker, M. (1997). Growth-rate heterogeneity and the covariance structure of life-cycle earnings. Journal of Labor Economics, 338-375.

[11] Bardasi, E., & Taylor, M. (2008). Marriage and wages: A test of the specialization hypothesis. Economica, 75(299), 569-591.

[12] Bartlett, R. L., & Callahan, C. (1984). Wage determination and marital status: Another look. Industrial Relations: A Journal of Economy and Society, 23 (1), 90-96.

[13] Becker, G. S. (1973). A theory of marriage: Part I. The Journal of Political Economy, 813-846.

[14] Becker, G. S. (1974). A theory of marriage. In Economics of the family: Marriage, children, and human capital (pp. 299-351). UMI.

[15] Becker, G. S. (1985). Human capital, effort, and the sexual division of labor. Journal of labor economics, S33-S58.

[16] Becker, G. S. (1991). A treatise on the family (Enl. ed.). Cambridge, Mass: Harvard.

[17] Benham, L. (1974). Benefits of women's education within marriage. In Economics of the Family: Marriage, Children, and Human Capital (pp. 375-394). UMI.

[18] Ben-Porath, Y. (1967). The production of human capital and the life cycle of earnings. The Journal of Political Economy, 352-365.

[19] Bhaskar, V., & Hopkins, E. (2014). Marriage as a Rat Race: Noisy Pre-Marital Investments with Assortative Matching.

[20] Bielby, W. T., & Bielby, D. D. (1989). Family ties: Balancing commitments to work and family in dual earner households. American Sociological Review, 776-789.

[21] Blackburn, M., & Korenman, S. (1994). The declining marital-status earnings differential. Journal of Population Economics, 7 (3), 247-270.

[22] Bloemen, H. G., & Stancanelli, E. G. (2014). Market hours, household work, child care, and wage rates of partners: An empirical analysis. Review of Economics of the Household, 12(1), 51-81.

[23] Blundell, R., Chiappori, P. A., Magnac, T., & Meghir, C. (2007). Collective labour supply: Heterogeneity and non-participation. The Review of Economic Studies, 74(2), 417-445.

[24] Boulier, B. L., & Rosenzweig, M. R. (1984). Schooling, search, and spouse selection: Testing economic theories of marriage and household behavior. The Journal of Political Economy, 712-732.

[25] Bourguignon, F., & Chiappori, P. A. (1992). Collective models of household behavior: An introduction. European Economic Review, 36(2), 355-364.

[26] Bredemeier, C., & Juessen, F. (2013). Assortative mating and female labor supply. Journal of Labor Economics, 31(3), 603-631.

[27] Browning, M., Chiappori, P. A., & Weiss, Y. (2003). A simple matching model of the marriage market. University of Chicago, unpublished manuscript.

[28] Browning, M., & Chiappori, P. A. (1998). Efficient intra-household allocations: A general characterization and empirical tests. Econometrica, 1241-1278.

[29] Browning, M., Hansen, L.P., Heckman, J.J. (1999). Micro data and general equilibrium models J.B. Taylor, M. Woodford (Eds.), Handbook of Macroeconomics, North-Holland, Amsterdam

[30] Browning, M., Chiappori, P. A., & Lechene, V. (2006). Collective and unitary models: A clarification. Review of Economics of the Household, 4(1), 5-14.

[31] Browning, M., & Carro, J. (2007). Heterogeneity and microeconometrics modeling. Econometric Society Monographs, 43, 47.

[32] Browning, M., Ejrnaes, M., & Alvarez, J. (2010). Modelling income processes with lots of heterogeneity. The Review of Economic Studies, 77(4), 1353-1381.

[33] Browning, M., Chiappori, P. A., & Weiss, Y. (2014). Economics of the Family. Cambridge University Press.

[34] Burdett, K., & Coles, M. G. (1997). Marriage and class. The Quarterly Journal of Economics, 141-168.

[35] Burdett, K., & Coles, M. G. (1999). Long-term partnership formation: Marriage and employment. The Economic Journal, 109(456), 307-334.

[36] Card, D. (1994). Earnings, schooling, and ability revisited (No. w4832). National Bureau of Economic Research.

[37] Card, D. (2012). Earnings, Schooling, and Ability Revisited. 35th Anniversary Retrospective (Research in Labor Economics, Volume 35) Emerald Group Publishing Limited, 35, 111-136

[38] Cappelli, P., Constantine, J., & Chadwick, C. (2000). It pays to value family: Work and family tradeoffs reconsidered. Industrial Relations: A Journal of Economy and Society, 39 (2), 175-198.

[39] Carroll, C. D., & Summers, L. H. (1991). Consumption growth parallels income growth: Some new evidence. In National saving and economic performance (pp. 305-348). University of Chicago Press.

[40] Chang, Y., & Schorfheide, F. (2003). Labor-supply shifts and economic fluctuations. Journal of Monetary Economics, 50(8), 1751-1768.

[41] Chiappori, P. A. (1992). Collective labor supply and welfare. Journal of political Economy, 437-467.

[42] Chiappori, P. A. (1997). Introducing household production in collective models of labor supply. Journal of Political Economy, 191-209.

[43] Chiappori, P. A., Fortin, B., & Lacroix, G. (2002). Marriage market, divorce legislation, and household labor supply. Journal of political Economy, 110(1), 37-72.

[44] Chiappori, P. A., Iyigun, M., & Weiss, Y. (2006). Spousal matching, marriage contracts and property division in divorce. Tel Aviv University, unpublished manuscript.

[45] Chiappori, P. A., Iyigun, M., & Weiss, Y. (2009). Investment in Schooling and the Marriage Market. American Economic Review, 99(5), 1689-1713.

[46] Choo, E., Seitz, S., & Siow, A. (2008). Marriage matching, risk sharing and spousal labor supplies. University of Toronto WP, 332.

[47] Choo, E., & Siow, A. (2006). Who marries whom and why. Journal of Political Economy, 114(1), 175-201.

[48] Chun, H., & Lee, I. (2001). Why do married men earn more: Productivity or marriage selection?. Economic Inquiry, 39 (2), 307-319.

[49] Cornwell, C., & Rupert, P. (1997). Unobservable individual effects, marriage and the earnings of young men. Economic Inquiry, 35 (2), 285-294.

[50] Damiano, E., & Li, H. (2007). Price discrimination and efficient matching. Economic Theory, 30(2), 243-263.

[51] Del Boca, D., & Flinn, C. J. (2005). Household time allocation and modes of behavior: A theory of sorts.

[52] Diamond, P. A. (1982). Wage determination and efficiency in search equilibrium. The Review of Economic Studies, 49(2), 217-227.

[53] Dizdar, D., & Moldovanu, B. (2014). On the Importance of Uniform Sharing Rules for Efficient Matching. working paper.

[54] Duncan, G. J., & Holmlund, B. (1983). Was Adam Smith right after all? Another test of the theory of compensating wage differentials. Journal of Labor Economics, 366-379.

[55] Eckstein, Z., & Wolpin, K. I. (1989). Dynamic labour force participation of married women and endogenous work experience. The Review of Economic Studies, 56(3), 375-390.

[56] Eckstein, Z., & Wolpin, K. I. (1999). Why youths drop out of high school: The impact of preferences, opportunities, and abilities. Econometrica, 67(6), 1295-1339.

[57] Erosa, A., Fuster, L., & Restuccia, D (2012). Human Capital and the Gender Gap in Wages.

[58] Fang, L., & Zhu, G. (2012). Home production technology and time allocation: empirics, theory, and implications (No. 2012-19). Working Paper, Federal Reserve Bank of Atlanta.

[59] Folks, J. L., & Chhikara, R. S. (1978). The inverse Gaussian distribution and its statistical application–a review. Journal of the Royal Statistical Society. Series B (Methodological), 263-289.

[60] Fortin, B., & Lacroix, G. (1997). A test of the unitary and collective models of household labour supply. The economic journal, 107(443), 933-955.

[61] Francesconi, M. (2002). A joint dynamic model of fertility and work of married women. Journal of Labor Economics, 20(2), 336-380.

[62] Furtado, D. (2012). Human capital and inter-ethnic marriage decisions. Economic inquiry, 50(1), 82-93.

[63] Gale, D. (2001). Signaling in markets with two-sided adverse selection. Economic Theory, 18(2), 391-414.

[64] Gale, D., & Shapley, L. S. (1962). College admissions and the stability of marriage. American mathematical monthly, 9-15.

[65] Gale, D., & Shapley, L. S. (2013). College admissions and the stability of marriage. The American Mathematical Monthly, 120(5), 386-391.

[66] Ge, S. (2011). Women's college decisions: how much does marriage matter?. Journal of Labor Economics, 29(4), 773-818.

[67] Gemici, A., & Laufer, S. (2014). Marriage and cohabitation. Working Paper.

[68] Ginther, D. K., & Zavodny, M. (2001). Is the male marriage premium due to selection? The effect of shotgun weddings on the return to marriage. Journal of Population Economics, 14 (2), 313-328.

[69] Goldin, C. (1992). The meaning of college in the lives of american women: The past one-hundred years (No. w4099). National bureau of economic research.

[70] Goldin, C., Katz, L. F., & Kuziemko, I. (2006). The homecoming of American college women: The reversal of the college gender gap (No. w12139). National Bureau of Economic Research.

[71] Gray, J. S. (1997). The fall in men's return to marriage: Declining productivity effects or changing selection?. Journal of Human Resources, 481-504.

[72] Gray, J. S., & Vanderhart, M. J. (2000). The Determination of Wages: Does Marriage Matter?. The ties that bind: Perspectives on marriage and cohabitation, 356-367.

[73] Greenwood, J., & Hercowitz, Z. (1991). The allocation of capital and time over the business cycle. journal of Political Economy, 1188-1214.

[74] Greenwood, J., Guner, N., Kocharkov, G., & Santos, C. (2014). Marry Your Like: Assortative Mating and Income Inequality. The American Economic Review, 104(5), 348-353.

[75] Gronau, R. (1997). The theory of home production: The past ten years. Journal of Labor Economics, 197-205.

[76] Gourieroux, C., Monfort, A., & Renault, E. (1993). Indirect inference. Journal of applied econometrics, 8(S1), S85-S118.

[77] Gourinchas, P. O., & Parker, J. A. (2002). Consumption over the life cycle. Econometrica, 70(1), 47-89.

[78] Goux, D., Maurin, E., & Petrongolo, B. (2014). Worktime Regulations and Spousal Labor Supply. American Economic Review, 104(1), 252-76.

[79] Gupta, N. D., Smith, N., & Stratton, L. S. (2007). Is marriage poisonous? Are relationships taxing? An analysis of the male marital wage differential in Denmark. Southern Economic Journal, 412-433.

[80] Gustavsson, M., & Österholm, P. (2014). Does the labor-income process contain a unit root? Evidence from individual-specific time series. Journal of Economic Dynamics and Control, 47, 152-167.

[81] Hause, J. C. (1980). The fine structure of earnings and the on-the-job training hypothesis. Econometrica: Journal of the Econometric Society, 1013-1029.

[82] Heckman, J. J. (1979). Sample selection bias as a specification error. Econometrica: Journal of the econometric society, 153-161.

[83] Heckman, J. J., & MaCurdy, T. (1982). Corrigendum on a life cycle model of female labour supply. The Review of Economic Studies, 49(4), 659-660.

[84] Hersch, J., & Stratton, L. S. (1994). Housework, wages, and the division of housework time for employed spouses. The American Economic Review, 120-125.

[85] Hersch, J., & Stratton, L. S. (2000). Household specialization and the male marriage wage premium. Indus. & Lab. Rel. Rev., 54, 78.

[86] Hersch, J., & Stratton, L. S. (2002). Housework and wages. Journal of Human Resources, 217-229.

[87] Hill, M. S. (1979). The wage effects of marital status and children. Journal of Human Resources, 579-594.

[88] Hoppe, H. C., Moldovanu, B., & Sela, A. (2009). The theory of assortative matching based on costly signals. The Review of Economic Studies, 76(1), 253-281.

[89] Hryshko, D., Juhn, C., & McCue, K. (2014). Trends in Earnings Inequality and Earnings Instability among US Couples: How Important Is Assortative Matching?.

[90] Huang, C., Li, H., Liu, P. W., & Zhang, J. (2009). Why Does Spousal Education Matter for Earnings? Assortative Mating and Cross-Productivity. Journal of Labor Economics, 27(4), 633-652.

[91] Huggett, M., Ventura, G. and A. Yaron (2011), Sources of Lifetime Inequality, American Economic Review, 101, 2923-54.

[92] Hungerford, T., & Solon, G. (1987). Sheepskin effects in the returns to education. The review of economics and statistics, 175-177.

[93] Iyigun, M. (2005). Bargaining and specialization in marriage.

[94] Jacobsen, J. P., & Rayack, W. L. (1996). Do men whose wives work really earn less?. The American Economic Review, 268-273.

[95] Jacquemet, N., & Robin, J. M. (2013). Assortative matching and search with labour supply and home production (No. CWP07/13). cemmap working paper, Centre for Microdata Methods and Practice.

[96] Jones, L. E., Manuelli, R. E., & McGrattan, E. R. (2003). Why are married women working so much? (No. 317). Federal Reserve Bank of Minneapolis.

[97] Keane, M. P., & Wolpin, K. I. (1997). The career decisions of young men. Journal of political Economy, 105(3), 473-522.

[98] Keane, M. P., & Wolpin, K. I. (2001). The effect of parental transfers and borrowing constraints on educational attainment. International Economic Review, 1051-1103.

[99] Kenny, L. W. (1983). The accumulation of human capital during marriage by males. Economic Inquiry, 21 (2), 223-232.

[100] Korenman, S., & Neumark, D. (1991). Does marriage really make men more productive?. Journal of Human Resources, 282-307.

[101] Lefgren, L., & McIntyre, F. (2006). The relationship between women's education and marriage outcomes. Journal of labor Economics, 24(4), 787-830.

[102] Lillard, L. A., & Willis, R. J. (1978). Dynamic aspects of earning mobility. Econometrica: Journal of the Econometric Society, 985-1012.

[103] Lillard, L. A., & Weiss, Y. (1979). Components of variation in panel earnings data: American scientists 1960-70. Econometrica: Journal of the Econometric Society, 437-454.

[104] Lise, J., & Yamada, K. (2014). Household sharing and commitment: Evidence from panel data on individual expenditures and time use (No. W14/05). IFS Working Papers.

[105] Loh, E. S. (1996). Productivity differences and the marriage wage premium for white males. Journal of Human Resources, 566-589

[106] Loscocco, K. A., & Leicht, K. T. (1993). Gender, work-family linkages, and economic success among small business owners. Journal of Marriage and the Family, 875-887.

[107] Lundberg, S. (1985). The added worker effect. Journal of Labor Economics, 11-37.

[108] Lundberg, S. (1988). Labor supply of husbands and wives: A simultaneous equations approach. The Review of Economics and Statistics, 224-235.

[109] Lundberg, S., & Rose, E. (1999). The determinants of specialization within marriage. School of Economics and Finance, University of Hong Kong.

[110] Lundberg, S., & Rose, E. (2000). Parenthood and the earnings of married men and women. Labour Economics, 7(6), 689-710.

[111] Lundberg, S. (2005). Men and islands: dealing with the family in empirical labor economics. Labour Economics, 12(4), 591-612.

[112] Lynch, L. M. (1992). Differential effects of post-school training on early career mobility (No. w4034). National Bureau of Economic Research.

[113] MaCurdy, T. E. (1982). The use of time series processes to model the error structure of earnings in a longitudinal data analysis. Journal of econometrics, 18(1), 83-114.

[114] Mare, R. D. (1991). Five decades of educational assortative mating. American sociological review, 15-32.

[115] Mazzocco, M. (2007). Household inter-temporal behaviour: A collective characterization and a test of commitment. The Review of Economic Studies, 74(3), 857-895.

[116] Mazzocco, M., Ruiz, C., & Yamaguchi, S. (2014). Labor Supply and Household Dynamics. The American Economic Review, 104(5), 354-359.

[117] McGrattan, E. R., Rogerson, R., & Wright, R. (1997). An equilibrium model of the business cycle with household production and fiscal policy. International Economic Review, 267-290.

[118] Mincer, J. (1974). Schooling, Experience, and Earnings. Human Behavior & Social Institutions No. 2.

[119] Mincer, J., & Polachek, S. (1974). Family investments in human capital: Earnings of women. In Marriage, family, human capital, and fertility (pp. 76-110). NBER.

[120] Mortensen, D. T. (1988). Matching: finding a partner for life or otherwise. American Journal of Sociology, S215-S240.

[121] Nakosteen, R. A., & Zimmer, M. A. (1987). Marital status and earnings of young men: A model with endogenous selection. Journal of Human Resources, 248-268.

[122] Nakosteen, R. A., & Zimmer, M. A. (1997). Men money and marriage: are high earners more prone than low earners to marry?. Social Science Quarterly, 78 (1), 66-82.

[123] Nakosteen, R. A., Westerlund, O., & Zimmer, M. A. (2004). Marital Matching and Earnings Evidence from the Unmarried Population in Sweden. Journal of Human Resources, 39(4), 1033-1044.

[124] Neuman, S., & Ziderman, A. (1992). Benefits of women's education within marriage: results for Israel in a dual labor market context. Economic Development and Cultural Change, 413-424.

[125] Nick, M., & Walsh, P. R. (2007). Building the family nest: Premarital investments, marriage markets, and spousal allocations. The Review of Economic Studies, 74(2), 507-535.

[126] Olivetti, C. (2006). Changes in women's hours of market work: The role of returns to experience. Review of Economic Dynamics, 9(4), 557-587.

[127] Olovsson, C. (2015). Optimal taxation with home production. Journal of Monetary Economics, 70, 39-50.

[128] Parsons, D. O. (1976). Health, family structure, and labor supply.

[129] Pencavel, J. (1998). Assortative mating by schooling and the work behavior of wives and husbands. American Economic Review, 326-329.

[130] Petersen, T., Penner, A. M., & Høgsnes, G. (2011). The male marital wage premium: Sorting vs. differential pay. Industrial and Labor Relations Review, 283-304.

[131] Pollak, R. A. (2003). Gary Becker's contributions to family and household economics. Review of Economics of the Household, 1(1-2), 111-141.

[132] Ragan, K. (2005). Fiscal policy and the family: explaining labor supply in a model with household production. Manuscript, Univ. Chicago.

[133] Ramey, V. A. (2008). Time spent in home production in the 20th century: New estimates from old data (No. w13985). National Bureau of Economic Research.

[134] Ramey, V. A., & Francis, N. (2009). A Century of Work and Leisure. American Economic Journal: Macroeconomics, 1(2), 189-224.

[135] Reed, W. R., & Harford, K. (1989). The marriage premium and compensating wage differentials. Journal of Population Economics, 2 (4), 237-265.

[136] Ribar, D. C. (2004). What Do Social Scientists Know About the Benefits of Marriage? A Review of Quantitative Methodologies (No. 998). Institute for the Study of Labor (IZA).

[137] Richardson, K. (2000). The evolution of the marriage premium in the Swedish labor market 1968-1991 (No. 2000: 5). Working Paper, IFAU-Institute for Labour Market Policy Evaluation.

[138] Rodgers, W. M., & Stratton, L. S. (2010). Male marital wage differentials: Training, personal characteristics, and fixed effects. Economic Inquiry, 48(3), 722-742.

[139] Rose, E. (2006). A Joint Econometric Model of Marriage and Partner Choice (No. UWEC-2004-23).

[140] Rubinstein, Y., & Weiss, Y. (2006). Post schooling wage growth: Investment, search and learning. Handbook of the Economics of Education, 1, 1-67.

[141] Rupert, P., Rogerson, R., & Wright, R. (1995). Estimating substitution elasticities in household production models. Economic theory, 6(1), 179-193.

[142] Shapley, L. S., & Shubik, M. (1971). The assignment game I: The core. International Journal of Game Theory, 1(1), 111-130.

[143] Schwartz, Christine R. and Robert D. Mare (2005). Trends in Educational Assortative Marriage from 1940 to 2003. Demography, 42, 621-646

[144] Shields, M. (1998). Changes in the Determinants of Employer-funded Training for Full-time Employees in Britain, 1984–94. Oxford Bulletin of Economics and Statistics, 60(2), 189-214.

[145] Shimer, R., & Smith, L. (2000). Assortative matching and search. Econometrica, 68(2), 343-369.

[146] Smith, L. (2006). The marriage model with search frictions. Journal of Political Economy, 114(6), 1124-1144.

[147] Smoluchowsky, M. V. (1915a). Notiz uiber die Berechnung der Brownschen Molekularbewegung bei der Ehrenhaft-Millikanschen Versuchsanordnung. Phys. Z., 16, 318-321. (1915b). Nachtrag. Phys. Z., 16, 375.

[148] South, S. J., & Spitze, G. (1994). Housework in marital and nonmarital households. American Sociological Review, 327-347.

[149] Spence, M. (1973). Job market signaling. The quarterly journal of Economics, 355-374.

[150] Stephens, M. J. (2002). Worker Displacement and the added worker effect. Journal of Labor Economics, 20(3).

[151] Stratton, L. S. (2002). Examining the wage differential for married and cohabiting men. Economic Inquiry, 40 (2), 199-212.

[152] Tiefenthaler, J. (1997). Fertility and family time allocation in the Philippines. Population and Development Review, 377-397.

[153] Van der Klaauw, W. (1996). Female labour supply and marital status decisions: A life-cycle model. The Review of Economic Studies, 63(2), 199-235.

[154] Van Klaveren, C., Van Praag, B., & van den Brink, H. M. (2008). A public good version of the collective household model: an empirical approach with an application to British household data. Review of Economics of the Household, 6(2), 169-191.

[155] Waite, L. J., & Gallagher, M. (2001). Case for marriage: Why married people are happier, healthier, and better off financially . New York: Doubleday

[156] Weiss, Y., & Willis, R. J. (1997). Match quality, new information, and marital dissolution. Journal of Labor Economics, S293-S329.

[157] Wong, L. Y. (2003). Structural estimation of marriage models. Journal of Labor Economics, 21(3), 699-727.

[158] Perli, R. (1998). Indeterminacy, home production, and the business cycle: a calibrated analysis. Journal of Monetary Economics, 41(1), 105-125.

[159] Panel Study of Income Dynamics, public use dataset. Produced and distributed by the Survey Research Center, Institute for Social Research, University of Michigan, Ann Arbor, MI (year 1968-2011).