

ERRATUM TO: "ARITHMETIC CORRELATIONS OVER LARGE FINITE FIELDS"

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It was brought to our attention that Theorem 1.3 and Theorem 1.4 as stated are not completely accurate. It follows that the conditions for Corollary 1.5 also need to be reformulated. There is furthermore a minor correction to equation (65) and (76) which will lead to a minor change in the statement of Theorem 4.2 and the statement of Theorem 4.4. We owe these observations to Ofir Gorodetsky and are most grateful to him for pointing them out.

The corrections are as follows:

There is a problem with restricting the J-sum at the top of page 866 to monics. Instead we will leave it as a sum over all non-zero polynomials of degree j . We omit the line on top of page 866 that begins "We will restrict the J-sum..." and omit equation (31) that follows. Therefore the J-sum (equations 32-35) will be $\sum_{j=0}^h \sum_{\substack{\deg J=j \\ J \neq 0}}$ and we will not have to multiply it by $(q-1)$. As a result of this change, the average on equation (36) will be

$$\frac{1}{q^{h+1}-q^h} \sum_{\substack{\deg J=h \\ J \neq 0}} E(J, n, q) = -\frac{1}{q^{h+1}-q^h} + O\left(\frac{1}{q^{1/2+h}}\right).$$

We also redefine $S_E(k, n, q)$ (equation (21)) to be the sum over all non-zero polynomials of degree k , instead of the sum over monics of degree k . Therefore equation (23) in Theorem 1.3 will be

$$S_E(k, n, q) = -1 + O(1/q^{1/2}).$$

In section 4 we will also apply the same correction which means that in equation (65) we do not restrict the J-sum to monics and do not multiply by $q-1$, so in the statement of Theorem 4.2 the J-sums are over all non-zero polynomials of degree h . Note that this will not change the discussion following theorem 4.2 other than the fact that the number of terms is now $q^h(q-1)$ instead of q^h . The J-sum in equation (76) will not be restricted to monics and will not be multiplied by $(q-1)$ and so in the statement of Theorem 4.4 the J-sums are over all non-zero polynomials of degree h and the sum is of order $O(\frac{1}{\sqrt{q}})$.

Again in section 3, we will not restrict the J-sum to monics, therefore starting equation (41) to the end of the section the sum will be over all non

zero polynomials of degree j and will not be multiplied by $(q - 1)$. As a result we will redefine $\tilde{S}_E(n, q; Q)$ in equation (22) to be the sum over all non-zero of degree j (instead of over monics of degree j), so in Theorem 1.4 and Corollary 1.5 we do not have to divide by $(q - 1)$.

Another issue that was brought to our attention is that while equation (43) is correct, we still need to keep track of the lower order terms. If $Q \neq 1$, we denote by m_Q the degree of the smallest-degree prime factor of Q and by n_Q the number of prime factors of Q of degree m_Q . We therefore have

$$\frac{1}{\Phi(Q)} = \frac{1}{|Q|} \left(1 + \frac{n_Q}{q^{m_Q}} + O(q^{-m_Q-1}) \right)$$

which mean that in the second line of equation (44) and on to the end of the section we have an extra factor of $q^{n-m_Q-\deg Q} + O(\max(\frac{1}{\sqrt{q}}, q^{n-m_Q-\deg Q-1}))$. This leads to a restatement of Theorem 1.4 and Corollary 1.5 as follows:
Theorem 1.4.

Let $Q \in F_q[t]$ a square-free polynomial such that $\deg Q < n$ and let m_Q be the degree of the smallest degree prime factor of Q . Then in the limit $q \rightarrow \infty$,

$$\tilde{S}_E(n, q; Q) = \deg Q - n + q^{n-m_Q-\deg Q} + O(\max(\frac{1}{\sqrt{q}}, q^{n-m_Q-\deg Q-1}))$$

Corollary 1.5.

Let $Q \in F_q[t]$ be a square-free polynomial of degree $n - 1$ with no linear factors. Then in the limit $q \rightarrow \infty$,

$$\sum_{c \in F_q^*} E(cQ, n, q) = -1 + O(1/\sqrt{q})$$

The discussion after corollary 1.5 is still valid and correct. The only changes that are needed are in the number of terms (which will be $(q - 1)$ times the written number), and to replace the line stating $E(1, n, q) = \frac{1}{q-1} + O(1/q^{3/2})$ by $\sum_{c \in F_q^*} E(c, n, q) = -1 + O(1/\sqrt{q})$.

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