

# A Discontinuity Model of Technological Change: Catastrophe Theory and Network Structure

Torsten Heinrich<sup>1</sup>

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## Abstract

Discontinuities as a crucial aspect of economic systems have been discussed both verbally - particularly in institutionality theory - and formally, chiefly using catastrophe theory. Catastrophe theory has, however, been criticized heavily for lacking micro-foundations and has mainly fallen out of use in economics and social sciences. The present paper proposes a simple catastrophe theory model of technological change with network externalities and reevaluates the value of such a model by adding an agent-based micro layer. To this end an agent-based variant of the model is proposed and investigated specifically with regard to the network structure among the agents. While the macro level of the model produces a classical cusp catastrophe - a result that is preserved in the agent-based form - it is found that the behavior of the model changes locally depending on the network structure, especially if networks with features that resemble social networks (low diameter, high clustering, power law distributed node degree) are considered.

While the present work investigates merely an aspect out of a large possibility space, it encourages further research using agent-based catastrophe theory models especially of economic aspects to which catastrophe theory has previously successfully been applied; aspects such as technological and institutional change, economic crises, or industry structure.

*Keywords:*

network structures; agent-based modeling; catastrophe theory; information and communication technology; preferential attachment networks; technological change

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## 1. Introduction

Path dependence and discontinuity have been central issues for institutional and evolutionary economics since at least Gunnar Myrdal's introduction of the idea of Circular Cumulative Causation (see, e.g., [16]), possibly much longer (specifically since Thorstein Veblen's [28, 29] writings on Cumulative Causation and path dependence in history, institutions, and fashion goods). A formal framework to capture these concepts presented itself with René Thom's [27] analysis of catastrophe theory. It has since been applied to different aspects of economic systems [32, 14, 15, 19, 10] both with and without the framework of institutionalist theory and evolutionary economics. Catastrophe theory models in economics do, however, generally operate at

12 an aggregate level only; in fact, they became controversial in the 1990s (for  
13 an overview, see, e.g., [20]). Consideration of, for instance, group or network  
14 structures would transform the model into a set of distinct but overlapping  
15 models, thereby increasing the system's complexity by orders of magnitude.

16 The present paper reevaluates a simple catastrophe theory model of tech-  
17 nological change with network externalities by adding an agent-based micro  
18 layer. To this end an agent-based variant of the model is proposed and in-  
19 vestigated with specific regard to the network structure among agents. The  
20 technological-institutional layer of the model is very simple; a base technol-  
21 ogy is contrasted with a new innovative technology that they may adopt.  
22 The latter generates network externalities for the adopters but they also in-  
23 cur (fixed) costs that arise periodically. The model uses a simple evolutionary  
24 mechanism (a replicator dynamic with a capacity boundary) for defining the  
25 probabilities for agents to adopt the new technology. The network external-  
26 ity enters this function as a standard intensity term while the periodic costs  
27 are applied additively. This yields a polynomial of degree 3 which can be  
28 reduced to the canonical equation of a cusp catastrophe (for overviews on  
29 cusp catastrophes, see, e.g., [23, 21, 20]) by applying the linear Tschirnhaus  
30 transformation. While the macro level of the model thus produces a clas-

31 sical cusp catastrophe - a result that is preserved in the agent-based form  
32 - it is found that the behavior of the model changes locally with the net-  
33 work structure. The network structure affects the distribution of the agent's  
34 neighborhood sizes (number of directly connected agents). The agent will  
35 then base her decisions to adopt or abandon the new technology on her  
36 neighborhood rather than the entire population. Depending on the network  
37 structure, different local neighborhoods may persist in different adoption  
38 states (specifically for small world networks) and the theoretical equilibria of  
39 the underlying macro-level equation may exhibit different degrees of stabil-  
40 ity and sensitivity depending on the network structure. In turn, the network  
41 structure affects the general catastrophic or non-catastrophic outcomes as  
42 well when the slow variables (the catastrophe parameters, namely the cost  
43 and the capacity boundary) are included in the simulation, i.e. when the  
44 catastrophe is allowed to happen.

45 While the present work investigates merely an aspect out of a large pos-  
46 sibility space, it encourages further research using agent-based catastrophe  
47 theory models especially of economic aspects to which catastrophe theory  
48 has previously successfully been applied; aspects such as technological and  
49 institutional change, economic crises, or industry structure.

50 In section 2, a more detailed overview of the literature on both network  
51 externalities and catastrophe theory in economic systems will be given. Sec-  
52 tion 3 proceeds with a simple theoretical model of an industry subject to  
53 network externality that results in a classical cusp catastrophe. This model  
54 will then be extended to an agent-based version and the role of the network  
55 structure and other aspects on the behavior of the model will be studied in  
56 section 4. section 5 concludes.

## 57 **2. Literature Review**

58 Catastrophe theory was established and developend as a field of math-  
59 ematics in the 1970s, mainly by René Thom [27]. It immediately enjoyed  
60 some popularity and applications to many fields including economics and so-  
61 cial sciences (see, e.g. Woodcock and Davis [32]) were developed but it was  
62 heavily criticized and has largely fallen out of use. One of the main critiques  
63 focusses on it being rooted in simple macro-level dynamic systems without  
64 relating to a micro level (microfoundation). For a brief overview, see Rosser  
65 [19] However, this should neither disqualify an entire class of models if it  
66 is able to describe or approximate observed phenomena nor is it generally  
67 impossible to add micro-foundations to such a model.

68       The current paper attempts to describe technological change with net-  
69       work externalities using a catastrophe model (and to add proper microfoun-  
70       dations with an agent-based version in section 4). This field has long been  
71       recognized as falling out of the domain of most earlier models in economics  
72       since there are increasing returns to scale driven by and increased willing-  
73       ness to pay higher prices for access to more well-established networks. This  
74       leads to path dependent technology choice processes as extensively analyzed  
75       by, e.g., Arthur and Ermoliev, and Kaniovski [2, 1], as well as (in a less  
76       formal-mathematical way) David [7]. The basic argument is that switching  
77       of technologies is costly and can, if it depends on others switching as well  
78       only be done with coordination - even if the intent is to switch to a new  
79       technology that would clearly be better than the old one. It was, however,  
80       long before this that it was recognized that technological change occurs in  
81       waves, as paradigm change rather than as a continuous process. The original  
82       idea is mainly due to Schumpeter, but the field was advanced greatly in the  
83       1980s by, among others, Dosi [9] and Freeman and Perez [12] whose papers  
84       also include discussions of the earlier literature. More recent research has  
85       brought these two approaches (path dependent technology choice and tech-  
86       nological change) together [31, 25, 8] and has applied this to the research on

87 and explanation of growth cycles [24].

88 While the discontinuous nature of this phenomenon is obvious, catastrophe  
89 models have rarely been applied to the field of technological change. Herbig  
90 [14] proposed that this could be done without, however, providing a model.  
91 Two different later approaches, those by Lange et al. [15] and by Dou and  
92 Ghose [10], followed Herbig's idea and added catastrophe models for specific  
93 cases, software adoption (Dou and Ghose) and online retail trade (Lange et  
94 al.) respectively. The two models are, however, not particularly close to  
95 the present one and use on different effects to derive the catastrophe equa-  
96 tion. Both also remain focussed on their particular case studies and neither  
97 comment on potential further uses of catastrophe models in the field of tech-  
98 nological change or in social and economic systems as a whole nor do they  
99 address the earlier criticism of catastrophe models lacking microfoundation  
100 (which is why they do not attempt to proceed to replicate the macro-level  
101 catastrophe model in an agent-based approach).

### 102 **3. A Simple Model**

103 Similar to Dou and Ghose's approach [10], the basis of the present model  
104 is the canonical replicator dynamic equation for population dynamics with



105 capacity boundary (for a general introduction and detailed explanation of  
106 this approach, see e.g. Nowak [18])

$$\frac{dS}{dt} = \alpha S \left( 1 - \frac{S}{z} \right)$$

107 Here,  $S$  is the size of the current user base,  $z$  is the capacity of the in-  
108 dustry (i.e. the size of the potential user population, the maximum number  
109 of users), and  $\alpha$  is the growth rate (or rather the fitness term that affects the  
110 growth rate as long as the capacity boundary is not approached) of the user  
111 base.

112 Different from Dou and Ghose, the present model introduces network exter-  
113 nalities by making the growth rate dependent on the population size, here  
114 simply  $\alpha = S$ . The result is the dynamic system described by the first order  
115 differential equation (consisting of a third order polynomial)

$$\frac{dS}{dt} = S^2 \left( 1 - \frac{S}{z} \right) = -\frac{1}{z}S^3 + S^2$$

116 Here, two simple fixed points (only the second one stable) can be found  
117 at

$$S = 0$$

$$S = z$$

118 But so far, the system does not involve chatastrophic transitions. It is  
 119 well known that the bifurcations become supercritical (see figure 1) if a neg-  
 120 ative  $S$ -intercept is added [23, 21, 20]. Other than in Dou and Ghose, this  
 121 is not introduced as a market interaction polynomial with a denominator of  
 122 order  $S^2$ ,<sup>2</sup> but in a more straightforward and simple way as the periodic costs  
 123 of using a technology. This modification is plausible since costs of infrastruc-  
 124 ture, maintenance, and expert knowledge increases linearly in the number of  
 125 different technologies an agent uses while it the additional costs from more  
 126 intensive use of an existing technology (that is, the respective infrastructure)  
 127 are decreasing, possibly even zero.

128

129 The resulting equation is

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<sup>2</sup>Dou and Ghose [10] use such a term to model network effects (and network externalities). The equilibrium set of this model is identical to one given by a polynomial of order  $S^4$  (and without negative powers of  $S$ ) which they then reduce to cubic order by dividing by  $S$  to develop it into the classical cusp model using the Tschirnhaus transformation in much the same way as outlined below.

$$\frac{dS}{dt} = \alpha S \left(1 - \frac{S}{z}\right) - \beta = -\frac{1}{z}S^3 + S^2 - \beta \quad (1)$$

130 Since it is the fixed points, the stability and potential catastrophic change  
131 that is under investigation here, the equilibrium set (i.e.  $\frac{dS}{dt} = 0$ ) is what is  
132 of primary interest in this system. The equilibrium set is

$$\left(\frac{dS}{dt} =\right) 0 = -\frac{1}{z}S^3 + S^2 - \beta$$

133

$$0 = S^3 - zS^2 + z\beta \quad (2)$$

134 Using a Tschirnhaus transformation

$$S = x + \frac{z}{3}$$

135 the equation becomes

$$0 = + \left(x + \frac{z}{3}\right)^3 - z \left(x + \frac{z}{3}\right)^2 + z\beta$$

$$0 = x^3 - \frac{z^2}{3}x - \frac{2z^3}{27} + z\beta$$

136 which can be developed into<sup>3</sup>

$$0 = x^3 + ax + b,$$

137 the classical cusp catastrophe equation [23, 21, 20].

138

139 For the original system (equation 1), the equilibrium surface as stated  
 140 above (equation 2) yields two supercritical bifurcations (together called the  
 141 bifurcation set). They are obtained as the solution of<sup>4</sup>

$$0 = -\frac{1}{z}S^3 + S^2 - \beta$$

$$0 = -\frac{3}{z}S^2 + 2S$$

142 as

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<sup>3</sup>By substituting

$$a = -\frac{1}{3}z^2$$

and

$$b = -\frac{2z^3}{27} + z\beta$$

<sup>4</sup>That is, the intersection of the equilibrium surface  $\frac{dS}{dt} = 0$  with the set of marginal stability  $\lambda = \frac{\partial(dS/dt)}{\partial S} = 0$ . The bifurcation set therefore contains all points on the equilibrium surface that are marginally stable, indicating that the system's stability properties change in the vicinity of the set (or rather at this very point) in the control space (i.e. the  $z$ - $\beta$ -plane.)

$$\begin{array}{ll} \beta = 0 & z = S \\ \beta = \frac{1}{3}S^2 & z = \frac{3}{2}S \end{array},$$

143 more conveniently written (with  $S$  eliminated) as  $\beta = 0$  and  $4z^2 = 27\beta$ .

144 The system is shown in figures 1 (transition for changing  $\beta$ ), 2 (bifurcation  
145 set in control space) and 3 (equilibrium surface). Note that for the cusp  
146 area (between the two bifurcation sets), there are three equilibria, the lower  
147 and the upper of which are stable while the middle one is not. Outside  
148 the cusp area, there is only one equilibrium. Consequently, if an additional  
149 dynamic which modifies the parameters  $b$  and  $Z$  is introduced, catastrophic  
150 changes occur, when either of the bifurcations is reached from the stable  
151 equilibrium plane which does not continue on the other side of the bifurcation.  
152 Specifically, the interpretation of the two bifurcations is

- 153 1.  $b = 0$ : The size of the user base is zero (lower stable equilibrium) and  
154 the costs become zero - since, e.g. the vendor is supplies the technology  
155 for free to the first couple of users. The system will then jump to the  
156 upper stable equilibrium.
- 157 2.  $4z^2 = 27\beta$ : The system is in the upper stable equilibrium and either  
158 the costs increase beyond the threshold (for given capacity  $z$ ) or the

159 population shrinks (for given  $\beta$ ) below the bifurcation threshold (or a  
160 combination of the two). In this case, using the technology becomes  
161 uneconomic for all its users and the network will collapse (to the lower  
162 stable equilibrium, i.e. zero size).

163 It should be noted, that many modifications to the fitness (growth rate)  
164 or network externality term  $\alpha = S$  leave the properties of the model intact,  
165 i.e. that this model is generalizable at least to some degree. For instance,  
166 if the term is a positive linear function of  $S$  (with factor  $f$ ),  $\alpha = fS$ , the  
167 system results in the same pattern of equilibrium surfaces and bifurcations  
168 with the specific bifurcation set (1)  $\beta = 0$  and (2)  $4z^2 = 27\frac{\beta}{f}$ .

169 It is nevertheless obvious, that this is too simple a model to capture in-  
170 teractive usage decisions and other socio-economic system processes relevant  
171 to network technologies. It certainly can serve for identifying a potential key  
172 mechanism that may arise in and be relevant to the economics of innovation  
173 and technology. However, it must now be shown that the effect is preserved  
174 in systems which take the micro-layer into account, i.e. agent-based models.  
175 In that case, not only the network externality and the technology diffusion  
176 (following for instance the above population dynamics) are important factors,  
177 but the network structure and its properties as well as potential heterogeneity

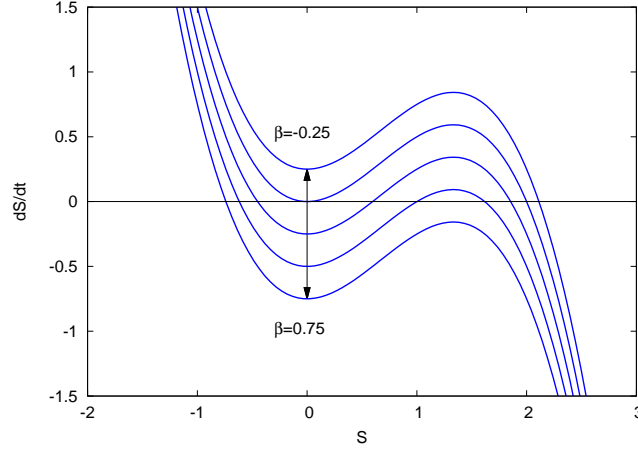


Figure 1: The cusp catastrophe: Bifurcations of the system with  $z = 2$ , for increasing  $\beta$ . For  $0 < \beta < \frac{16}{27} (= \frac{4}{27}z^2)$  there are three equilibria ( $dS = 0$ ), above and below this interval, two of the equilibria vanish.

178 of agents become relevant to the dynamic properties and the outcome of the  
 179 resulting system. This will be the focus of section 4.

#### 180 4. An Agent-Based Simulation

181 This section reconsiders the model developed in section 3 above and trans-  
 182 forms it into an agent based model. Here, the behavior of the population is  
 183 not governed any more by macro-level equations, but all agents make their  
 184 own decisions. These decisions follow the same model (and equations) as  
 185 above but the reference usage share that the agents base their decision on is  
 186 (generally) not the entire population but only the immediate neighborhood

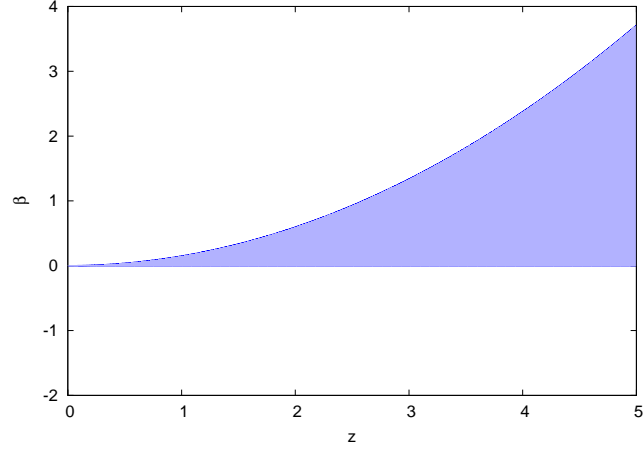


Figure 2: Bifurcation set and cusp area of the model.

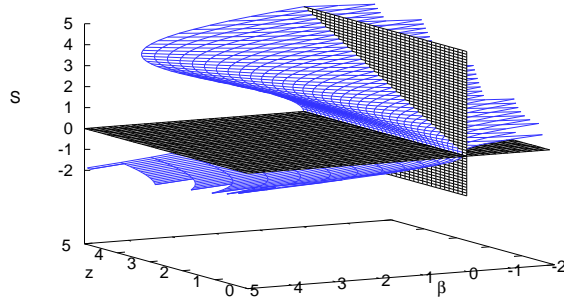


Figure 3: Equilibrium surface of the model including cusp area with three equilibrium surface points for every point in control space. Note that though the lower equilibrium surface is below zero, negative network sizes  $S$  are not allowed and the equilibrium is instead at  $S = 0$ . Also note that other than in figures 6 through 10, the vertical axis is absolute network sizes  $S$ .



187 of the respective agent. As a reference point, the section starts with the  
 188 complete network (i.e. every agent is direct neighbor to every other agent), a  
 189 setting that unsurprisingly yields the same result as the analytical macro-level  
 190 model above, before more complex network topologies are considered. The  
 191 agent-based version of the technology choice model on non-trivial networks  
 192 is related to but not the same as the variously studied models of contagion  
 193 between agents on networks as discussed in the literature review section 2 (of  
 194 particular interest in this respect, Barash et al. [4] investigate discontinuities,  
 195 bifurcation points, etc. of contagion models in Barabási-Albert networks).

196 The agents base their decision on the same polynomial as in equation 1  
 197 above,

$$-\frac{1}{z}S^3 + S^2 - \beta,$$

198 this time, however, the resulting quantity is not directly the dynamic  
 199 change of the usage share (since the agent does not control this quantity)  
 200 but the agent's inclination to join the network (if she is not currently a

201 subscriber)<sup>5</sup>

$$\varrho_+ = \min \left( z, \max \left( 0, -\frac{1}{z}S^3 + S^2 - \beta \right) \right) \quad (3)$$

202 or to leave the network (if she currently is a subscriber).

$$\varrho_- = \min \left( z, \max \left( 0, -\frac{1}{z}S^3 + S^2 - \beta \right) \right) \quad (4)$$

203 Three network structures are considered besides the complete network  
204 (examples of the three network structures are shown in figure 4):

- 205 1. The grid network (figure 4a): Agents are arranged in a 1-dimensional  
206 order; they are directly connected with their  $m$  predecessors and  $m$   
207 successors in this order. Grid networks have a high clustering coefficient,  
208 but a high diameter (average path length between random nodes). The  
209 first property resembles social networks, the second does not.
- 210 2. The preferential attachment network (figure 4b): The network is con-  
211 structed consecutively; every new agent is connected to  $m$  agents who  
212 are already part of the network; she chooses these agents with a prob-

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<sup>5</sup>The min and max operations guarantee that the resulting probability is between 0 and 1, if  $\varrho$  already falls into these limits, the probabilities are more conveniently written as  $\varrho$  and  $1 - \varrho$ .

213 ability proportional to their current degree, i.e. well connected nodes  
 214 have a much higher probability to gain even more connections (hence,  
 215 a Barabási-Albert network, [3]). The mechanism is known to lead to  
 216 a power law degree distribution. This is not not generally a property  
 217 of social networks as there seems to be a limit to the number of one's  
 218 stable social relations<sup>6</sup>, but it may be a realistic way to model net-  
 219 works among social groups, organizations, or firm networks, especially  
 220 in network industries.<sup>7</sup> Further, preferential attachment networks are  
 221 characterized by small diameter<sup>8</sup> but also (if no other modifications  
 222 have been made to the generating process) by low clustering coefficient.  
 223 The former (small diameter) is also realistic for networks observed in  
 224 reality (e.g. in networks among firms) while the latter (low clustering

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<sup>6</sup>Dunbar [11] for instance proposed a number of around 300. There are some models that nevertheless propose using Barabási-Albert networks as a model of social networks, e.g. [4] - the argument of networks among social groups (as opposed to within social groups) may be relevant to this discussion.

<sup>7</sup>This has been suggested directly in some models [6, 26], but since both the size of firms (profits, capital, number of employees) and the degrees of the internet (but also the sizes of urban centers and many other quantities related to network technologies in one way or another) are known to be power law distributed [17, 13], this is generally a plausible assumption related to other stylized facts.

<sup>8</sup>The diameter of a network is the longest distance (shortest path) between two nodes in the network. A small diameter compared to the size of the network (number of nodes) this means that the network are relatively well-connected, more specifically having Watts and Strogatz' small world property [30] (or having a huge number of links compared to the number of nodes, thus being a complete or almost complete network as this would also lead to a small diameter).

coefficient) is not as networks between firms, organizations, but also between individuals are generally highly clustered.<sup>9</sup>

3. A preferential attachment network with triadic closure (figure 4c). In order to obtain a network that does not only have a small diameter and a degree distribution following a power law, but also a generally higher clustering, one can apply start with a Barabási-Albert preferential attachment network and apply triadic closure. After the Barabási-Albert process completes, a predefined number of open triads - groups of three nodes (agents) where one is connected to the other two but the other two are not linked directly - are selected and closed (thereby linking the unconnected nodes of the triad directly). This network type has other interesting properties. It is more likely to contain components that are internally well-connected but poorly connected to other components of the network (though not isolated), something that is also observed in real social networks. Formally, the betweenness centrality<sup>10</sup> distri-

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<sup>9</sup>These two properties (and the conjunction of these two properties) in social networks has received particular attention in the small-world network literature [30]; for an extensive overview, see [22].

<sup>10</sup>The betweenness centrality of a node  $i$  is the share of shortest path  $sp_{j,k}$  in the network between any two nodes ( $j$  and  $k$ ) of which it is part (denoted  $sp_{j,k}(i)$ )

$$bc(i) = \sum_{j \neq k \neq i} \frac{sp_{j,k}(i)}{sp_{j,k}}.$$

240 bution of the network changes such that some nodes (the ones with  
241 a connection to other closely connected components of the network)  
242 will have extraordinarily high betweenness centrality while most of the  
243 nodes score very low in this measure. In the other types of networks  
244 considered here (complete, grid, and preferential attachment without  
245 triadic closure) the distributions of betweenness centrality are substan-  
246 tially more even<sup>11</sup> (See figure 5).<sup>12</sup>

247 All simulations in this section have been conducted in networks of 1250  
248 agents. Figures 6a through 10a for illustration of the simulations show the  
249 entire equilibrium surface as derived in the simulation. For the stable equilib-  
250 rium surfaces (darker grey), the simulation is straightforward and accurate;  
251 for the unstable equilibria (repeller surface, light grey), the location was ap-  
252 proximated from the approximate borders of the basins of attraction of the  
253 stable equilibria. Note that other than in figure 3 the vertical axis gives the

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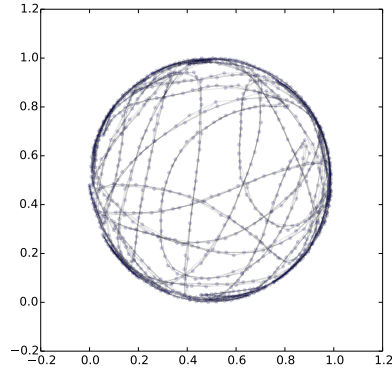
<sup>11</sup>In fact, the distributions of the node's betweenness centrality seem to decay according to a power law for the preferential attachment network and the preferential attachment network with triadic closure, but with different tail slopes.

<sup>12</sup>Note that this module uses a breadth-first search algorithm to obtain a single shortest path between any two nodes. As in regular grids there are always many equally short paths, one is selected randomly, therefore the computed betweenness centralities differ slightly between the nodes; the correct result would be a constant and equal value for all nodes.

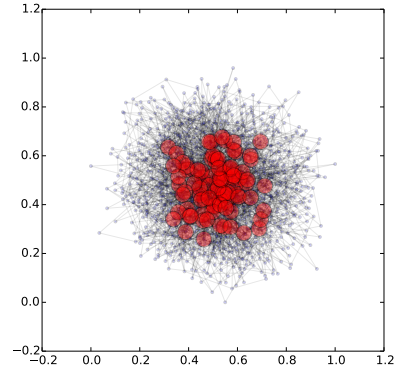
usage network sizes relative to the capacity (population size),  $\frac{S}{z}$ . To show the effect of the network structure in more detail, the results of simulations with constant  $z$  and initial value ( $z = 4$  and  $S_0/z = 0.5$ ) are also shown (figures 6b through 10b). This differs from the simulations shown in figures 6a through 10a in that this does not show the entire equilibrium surface but just the convergence point for a specific starting value ( $S_0/z = 0.5$ ).

Unsurprisingly, the simulation of the complete network (figure 6) results in the same picture as obtained from the macro-level development equation (figure 6). Note that the  $S/z$ -value of the upper sheet of the equilibrium surface again first declines smoothly in  $\beta$  (like in the theoretical model above); this is because the constant cost ( $\beta$ ) makes the participation undesirable for a part of the population. When the bifurcation set from the theoretical model is reached, the triple equilibrium vanishes and just one of the equilibrium surfaces continues to exist (here visible is the part for which the lower equilibrium continues to exist; towards the observer).

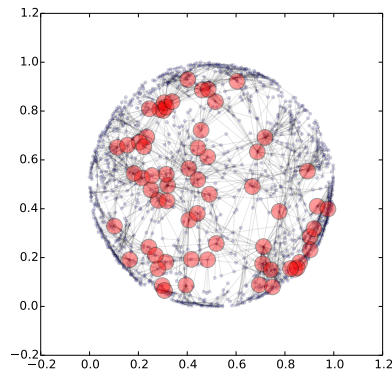
This result is still preserved in the case of the grid network (figure 7) with the slight difference that the upper equilibrium surface is much lower. This is a numerical issue resulting from the low degree of the nodes in this network (shown in figure 7 is a network with even node degree 4): If all



(a) Grid (double ring)



(b) Preferential attachment



(c) Preferential attachment with triadic closure

Figure 4: Networks of the same size (1250 agents, ca. 3750 links) but different types (grid, preferential attachment, and preferential attachment with triadic closure). Nodes (agents) with particularly high betweenness centrality highlighted; illustration generated by the python-networkx module.

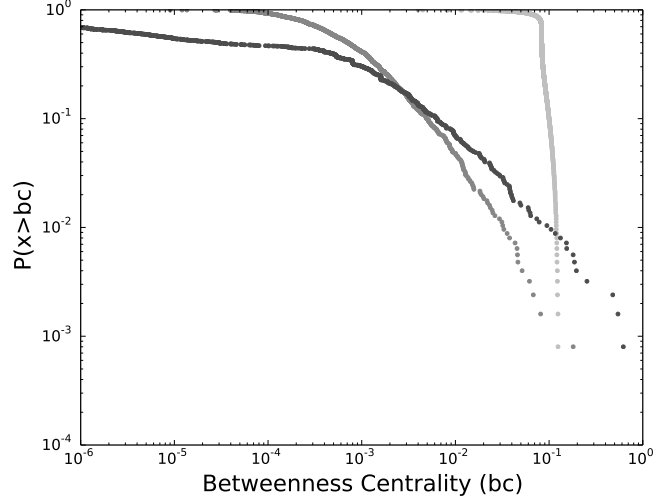


Figure 5: Complementary cumulative distribution of betweenness centrality of the vertices (agents) in the three networks in figure 4 (log-log plot): Grid (double ring, 4a) light grey, preferential attachment (4b), grey, and preferential attachment with triadic closure (4c), dark grey. Betweenness centrality computed with the python-networkx module.

273 the neighbors of a node are subscribers but the node is not, the dynamic  
 274 (equations  $eq : abm : 1$ ) yields 0 and the agent will not subscribe; if  $\beta > 0$ ,  
 275 there is even a positive probability for subscribers to unsubscribe - which is as  
 276 it should be, but at the macro-level this should result in a very small number  
 277 of unsubscriptions while here it matches every 3rd agent (who is between 2  
 278 subscribers to the left and 2 subscribers to the right). Indeed, this effect  
 279 is much less prominent with a higher neighborhood degree (see figure 8 for  
 280 comparison); the such denser networks will, however, increase computation



time substantially.<sup>13</sup> In both grid networks, the catastrophe continues to exist and is identical to the theoretical model.

In a preferential attachment network (figure 9), this seems to change slightly. The catastrophe (i.e. the bifurcation, the switch between a regime with 1 equilibrium and one with 3 equilibria) is still present, but it seems to happen slightly less sudden. This is an illusion. It results from the fact that for the area around the bifurcation set in parameter space, different parts of the network may converge to different equilibria (yielding a network average between the upper and lower equilibria). This is even more pronounced in the case of preferential attachment networks with triadic closure (see below). Further, the effect described for grid networks, a shift of the upper equilibrium surface to a lower level) is also present here (and also more pronounced because the median neighborhood size is even smaller than for the grid network).

Finally, in a preferential attachment network with triadic closure (figure 10) the transition seems even more smooth as different parts of the network are not strongly connected. This also holds for preferential attachment

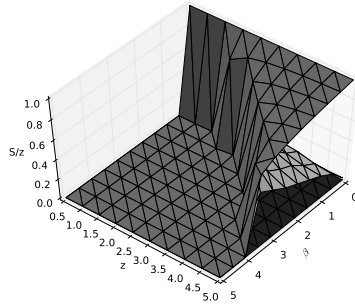
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<sup>13</sup>Further, for the network 7, the number of links is such that the network is directly comparable to a Barabási-Albert preferential attachment network of the same size (figure 9).

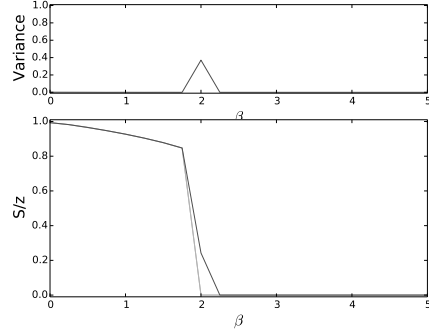
298 networks with  $m = 1$  which are entirely loop free. For this case it is obvious  
299 from figure 10b that even for low values of  $\beta$  there are increasingly large parts  
300 of the network that converge to the lower equilibrium. Figure 11 shows how  
301 this changes with the amount of triadic closure. While it is clear that if triadic  
302 closure were applied permanently for a long time the graph would eventually  
303 converge to a complete network (with the associated characteristics shown  
304 above), the figure shows that for even a substantial amount (roughly 3 times  
305 the number of links present before is added through triadic closure) the  
306 characteristics of the process are preserved. While the catastrophe is still  
307 present - and the macro-level model shown above is therefore a suitable first  
308 approximation of the situation - the network structure may in this case lead  
309 to different outcomes in different parts of the network.

## 310 5. Conclusion

311 The present work attempted to apply a catastrophe theory model to the  
312 problem of network industries. A development equation is defined for the  
313 development of the user base of a technology with network externalities.  
314 Those not part of the user base are assumed to use a very simple and free  
315 standard technology or no equivalent technology at all. With the size of the

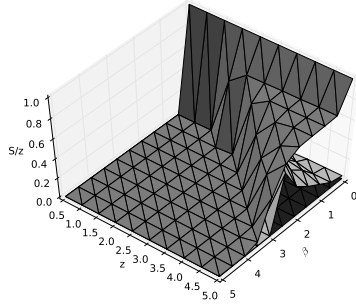


(a) Equilibrium surface (dark grey), repeller surface (light grey)

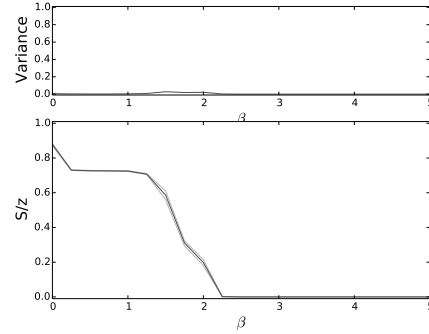


(b) Average outcome across 10 runs for  $z = 4$ ,  $s_0 = 2$

Figure 6: Agent based model on a complete network (1250 agents,  $1250 \times 1249 = 1561250$  links).

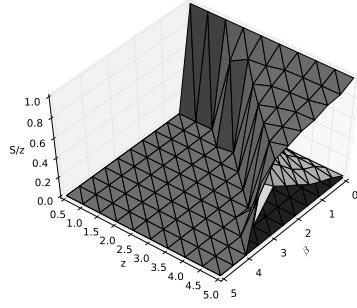


(a) Equilibrium surface (dark grey), repeller surface (light grey)

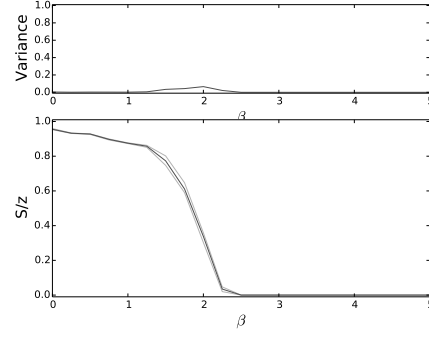


(b) Average outcome across 10 runs for  $z = 4$ ,  $s_0 = 2$

Figure 7: Agent based model on a grid (double ring) network (1250 agents, 2500 links).

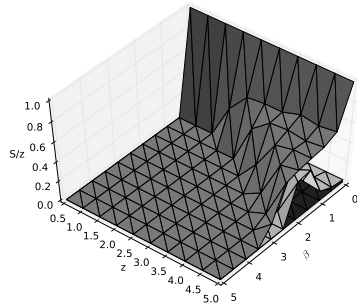


(a) Equilibrium surface (dark grey), repeller surface (light grey)

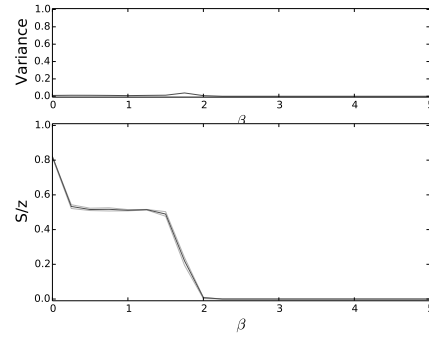


(b) Average outcome across 10 runs for  $z = 4$ ,  $s_0 = 2$

Figure 8: Agent based model on a grid (double ring) network (1250 agents, 12500 links).

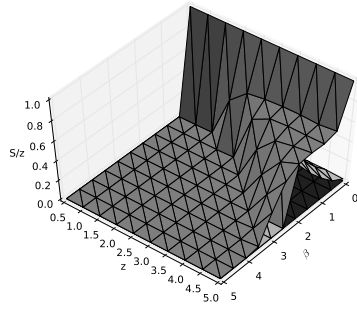


(a) Equilibrium surface (dark grey), repeller surface (light grey)

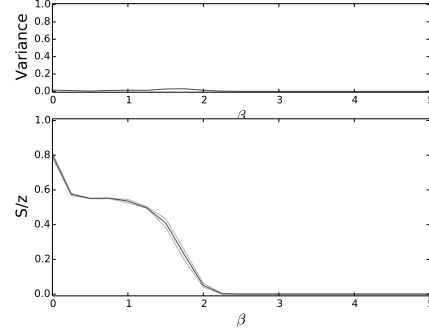


(b) Average outcome across 10 runs for  $z = 4$ ,  $s_0 = 2$

Figure 9: Agent based model on a preferential attachment network (1250 agents, 2497 links; Barabási-Albert with  $m = 2$ ).



(a) Equilibrium surface (dark grey), repeller surface (light grey)



(b) Average outcome across 10 runs for  $z = 4$ ,  $s_0 = 2$

Figure 10: Agent based model on a preferential attachment network with triadic closure (1250 agents, 2499 links, Barabási-Albert with  $m = 1$ , thus 1249 links, and 1250 random open triads closed, thus  $1249 + 1250 = 2499$  links).

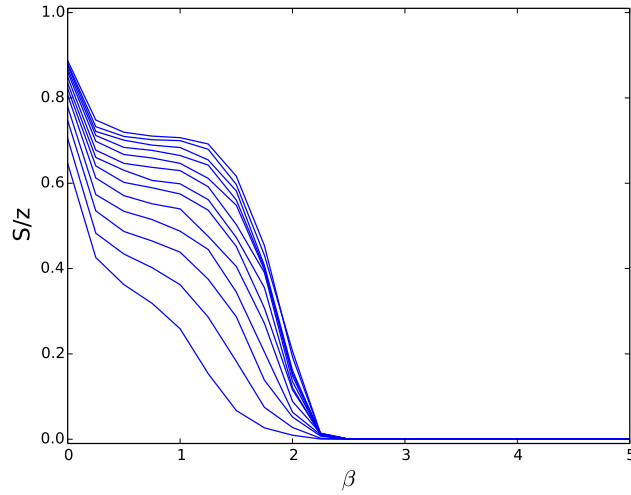


Figure 11: Average outcome across 10 runs for  $z = 4$ ,  $s_0 = 2$  for preferential attachment networks (Barabási-Albert with  $m = 1$ , thus 1250 agents, 1249 links) with different degrees of triadic closure (between 0 additional links, lowest graph, and 3750 additional links, uppermost graph).

316 user base as the only variable, the characteristics of the model depend on  
317 two control variables, the total population size (capacity boundary) and the  
318 periodic fixed costs of using the technology.

319 The model follows a simple but generalizable layout that includes a pop-  
320 ulation dynamic with capacity boundary, the network externality in the form  
321 of a growth rate of the user base<sup>14</sup> proportional to the size of the user base,  
322 and the cost term as an intercept. As was shown in section 3, the model pro-  
323 duces a cusp catastrophe and it is, in fact, possible to transform the model  
324 into the classical cusp catastrophe equation. The bifurcations of the model  
325 occur (1) at zero costs and (2) at a specific relation of costs and the capacity  
326 boundary. It can be shown, that any factors applied to the network exter-  
327 nality term would in effect shift this second bifurcation (dividing the cost  
328 term).

329 Further, it was established using an agent-based simulation, that the  
330 findings of this model are robust with respect to the introduction of a true  
331 micro-level (interdependent agents) for several network structures (grid net-  
332 works, Barabási-Albert preferential attachment networks, and preferential

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<sup>14</sup>Or rather, the fitness term that dominates the growth rate as long as the capacity boundary is not reached.

333 attachment networks with triadic closure). While the general behavior - the  
334 bifurcation, the catastrophe - is preserved, there were some critical changes to  
335 the macro-level for some network structures. More realistic approximations  
336 of social networks (and specifically firm networks, with small diameter, high  
337 clustering, and scale free degree distribution) such as preferential attachment  
338 networks with triadic closure found that different parts of the network may  
339 converge to different equilibria. The structure of such networks with several  
340 well-connected subgraphs that are poorly interconnected through a couple of  
341 bottleneck nodes, appears to facilitate this outcome.

342     As sudden and intense changes in the use of network technologies have  
343 repeatedly been observed (so in the case of Microsoft's takeover of the PC  
344 operating system market in the 1980s, but also in the case of the rise and fall  
345 of MySpace, in the rise of tablet computers, etc.) catastrophe models may  
346 indeed be a promising approach to explaining the specific dynamics of these  
347 sectors. This is particularly relevant as it is clear that they are subject to  
348 increasing returns to the size of the user base (which allows the introduction  
349 of many non-trivial strategies on the part of the market participants), that  
350 many standard models focusing on equilibrium concepts are therefore not  
351 applicable, and that the reasons for the asymmetric industry structure in

almost all of these sectors are not entirely understood.<sup>15</sup>

- [1] Arthur, W. B., 1989. Competing technologies, increasing returns, and lock-in by historical events. *Economic Journal* 99 (394), 116–31.
- [2] Arthur, W. B., Ermoliev, Y. M., Kaniovski, Y. M., 1987. Path dependent processes and the emergence of macro-structure. *European Journal of Operational Research* 30, 294–303.
- [3] Barabási, A.-L., Albert, R., 1999. Emergence of scaling in random networks. *Science* 286 (5439), 509–512.
- [4] Barash, V., Cameron, C., Macy, M., 2012. Critical phenomena in complex contagions. *Social Networks* 34 (4), 451 – 461.
- [5] Buendía, F., 2013. Self-organizing market structures, system dynamics, and urn theory. *Complexity* 18 (4), 28–40.
- [6] Dahui, W., Li, Z., Zengru, D., 2006. Bipartite producer–consumer networks and the size distribution of firms. *Physica A: Statistical Mechanics and its Applications* 363 (2), 359 – 366.

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<sup>15</sup>However, some promising approaches exist; these include urn schemes [2, 5] and network externality based models.



- 367 [7] David, P. A., 1985. Clio and the economics of QWERTY. American  
368 Economic Review 75 (2), 332–337.
- 369 [8] Dolfma, W., Leydesdorff, L., 2009. Lock-in and break-out from tech-  
370 nological trajectories: Modeling and policy implications. Technological  
371 Forecasting and Social Change 76 (7), 932 – 941.  
372 URL <http://www.sciencedirect.com/science/article/B6V71-4W1SGCH-1/2/40b6d67cf6>
- 373 [9] Dosi, G., 1982. Technological paradigms and technological trajectories:  
374 A suggested interpretation of the determinants and directions of techni-  
375 cal change. Research Policy 11 (3), 147 – 162.
- 376 [10] Dou, W., Ghose, S., 2006. A dynamic nonlinear model of online retail  
377 competition using cusp catastrophe theory. Journal of Business Research  
378 59 (7), 838 – 848.
- 379 [11] Dunbar, R. I. M., 1993. Coevolution of neocortical size, group size and  
380 language in humans. Behavioral and Brain Sciences 16 (04), 681–694.  
381 URL <http://dx.doi.org/10.1017/S0140525X00032325>
- 382 [12] Freeman, C., Perez, C., 1988. Structural crisis of adjustment, business  
383 cycles and investment behaviour. In: Dosi, G., Freeman, C., Nelson, R.,

- 384 Silverberg, G., Soete, L. (Eds.), Technological Change and Economic  
385 Theory. Pinter Publishers, London, N.Y., pp. 38–66.
- 386 [13] Gatti, D. D., Guilmi, C. D., Gaffeo, E., Giulioni, G., Gallegati, M.,  
387 Palestrini, A., 2005. A new approach to business fluctuations: hetero-  
388 geneous interacting agents, scaling laws and financial fragility. Journal  
389 of Economic Behavior & Organization 56 (4), 489 – 512, festschrift in  
390 honor of Richard H. Day.
- 391 [14] Herbig, P. A., 1991. A cusp catastrophe model of the adoption of an  
392 industrial innovation. Journal of Product Innovation Management 8 (2),  
393 127 – 137.
- 394 [15] Lange, R., Mcdade, S. R., Oliva, T. A., 2004. The estimation of a cusp  
395 model to describe the adoption of Word for Windows. Journal of Product  
396 Innovation Management 21 (1), 15–32.
- 397 [16] Myrdal, G., 1974. What is development? Journal of Economic Issues  
398 8 (4), 729–736.
- 399 [17] Newman, M. E. J., 2005. Power laws, Pareto distributions and Zipf’s  
400 law. Contemporary Physics 46 (5), 323–351.

- 401 [18] Nowak, M. A., 2006. Evolutionary dynamics: exploring the equations of  
402 life. Belknap Press of Harvard Univ. Press, Cambridge, Mass.
- 403 [19] Rosser, J. B., 2000. From Catastrophe to Chaos: A General Theory of  
404 Economic Discontinuities: Mathematics, Microeconomics and Finance.  
405 Springer, New York.
- 406 [20] Rosser, J. B., 2007. The rise and fall of catastrophe theory applications  
407 in economics: Was the baby thrown out with the bathwater? Journal  
408 of Economic Dynamics and Control 31 (10), 3255 – 3280.
- 409 [21] Saunders, P. T., 1980. An introduction to catastrophe theory. Cambridge  
410 University Press, Cambridge.
- 411 [22] Schnettler, S., 2009. A structured overview of 50 years of small-world  
412 research. Social Networks 31 (3), 165 – 178.
- 413 [23] Sewell, M. J., 1977. Elementary catastrophe theory. In: Branin Jr., F.,  
414 Huseyin, K. (Eds.), Problem Analyses in Science and Engineering. Aca-  
415 demic Press, New York, pp. 391–426.
- 416 [24] Silverberg, G., Lehnert, D., 1993. Long waves and ‘evolutionary  
417 chaos’ in a simple Schumpeterian model of embodied technical change.

- 418        Structural Change and Economic Dynamics 4 (1), 9 – 37.
- 419        URL <http://www.sciencedirect.com/science/article/B6VFN-45D0S8G-3/2/8d133ab9f6>
- 420 [25] Silverberg, G., Verspagen, B., 2005. A percolation model of innovation
- 421        in complex technology spaces. Journal of Economic Dynamics and
- 422        Control 29 (1-2), 225 – 244.
- 423        URL <http://www.sciencedirect.com/science/article/B6V85-4BT19PR-2/2/9caf868d9a>
- 424 [26] Stephen, A. T., Toubia, O., 2009. Explaining the power-law degree dis-
- 425        tribution in a social commerce network. Social Networks 31 (4), 262 –
- 426        270.
- 427 [27] Thom, R., 1975. Structural stability and morphogenesis: an outline of
- 428        a general theory of models. The Advanced Book Program. W. A. Ben-
- 429        jamin, Reading, MA, USA, translated from French by D.H. Fowler.
- 430 [28] Veblen, T. B., 1898. Why is economics not an evolutionary science? The
- 431        Quarterly Journal of Economics 12 (4), 373–397.
- 432 [29] Veblen, T. B., 2008 [1899]. Theory of the Leisure Class. Project Guten-
- 433        berg.
- 434        URL <http://www.gutenberg.org/ebooks/833>

- 435 [30] Watts, D. J., Strogatz, S. H., Jun. 1998. Collective dynamics of 'small-  
436 world' networks. *Nature* 393 (6684), 440–442.  
437 URL <http://dx.doi.org/10.1038/30918>
- 438 [31] Windrum, P., 2003. Unlocking a lock-in: Towards a model of technologi-  
439 cal succession. In: Saviotti, P. P. (Ed.), *Applied Evolutionary Economics*  
440 *New Empirical Methods and Simulation Techniques*. Edward Elgar Pub-  
441 lishing, Inc., Cheltenham, UK.
- 442 [32] Woodcock, A., Davis, M., 1978. *Catastrophe Theory: A Revolutionary*  
443 *New Way of Understanding How Things Change*. Dutton, New York.