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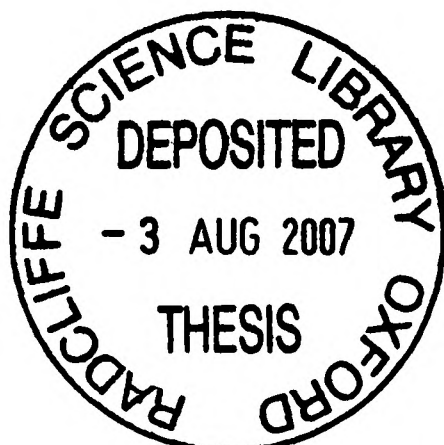
# Improving Instruments for Infrared Remote Sensing

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# Improving Instruments for Infrared Remote Sensing

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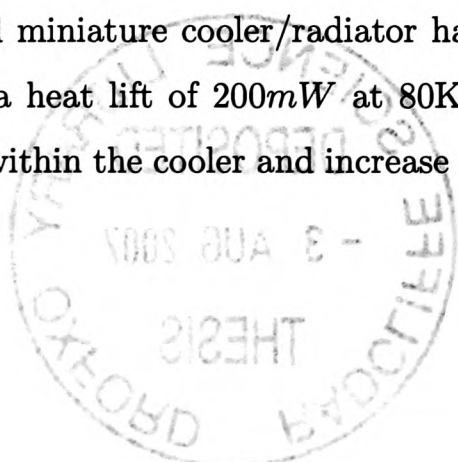
## Abstract

Remote sensing of the Earth's atmosphere, typically performed in the infrared region of the spectrum, plays an important role in scientific research. In the past the instruments used to perform these observations have been large, massive devices and correspondingly have only been able to be placed on large satellites. There is currently a trend toward smaller Earth observing platforms, so-called micro-satellites, and there is therefore a need for smaller, less massive instruments. Typically these instruments utilise a semiconductor device that responds to incoming infrared radiation in a known way. Such devices are subject to a number of noise sources that reduce their performance. By cooling them to temperatures around 80K it is possible to significantly reduce the amplitude of this noise compared to the incoming radiation of interest, thus increasing the signal-to-noise ratio (SNR). Typically this cooling is performed by a mechanical cooler, but currently many of them are too massive and require too much power to be suitable for use on a small remote sensing satellite.

By considering a typical application, a performance target for a miniature cooler was determined to be a heat lift of  $200mW$  at a cold tip temperature of 80K. Hardware has been created to investigate the feasibility of achieving this aim with a hybrid cooler/radiator. The cooler is a miniature integral Stirling machine and uses flat spiral flexures with a newly designed linear motor to drive the compressor piston; the displacer is driven pneumatically.

The prototype initially underwent characterisation without first being pre-cooled by the radiator. Although significant cooling was observed (to below 170K), initial characterisation highlighted a low thermal resistance between the warm end of the cooler and the cold tip. With pre-cooling the cold tip was able to reach a minimum no-load cold tip temperature of 92K, and with  $200mW$  applied to the cold tip a temperature of 122.4 K was sustainable. Attempts were made to increase the thermal resistance between the warm end and cold tip by introducing thermal breaks into the regenerator, and whilst these did increase the thermal resistance, the overall performance of the cooler decreased.

The concept of a hybrid miniature cooler/radiator has been shown to be feasible. To achieve the target performance of a heat lift of  $200mW$  at 80K further work needs to be performed to characterise loss processes within the cooler and increase the thermal resistance between the warm end and cold tip.



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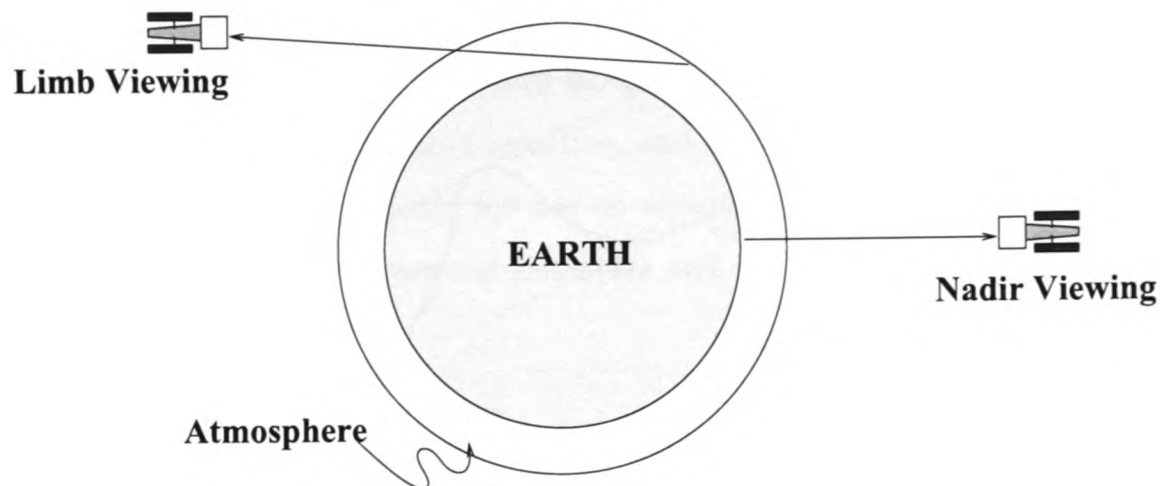
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# Chapter 1

## Infrared remote sensing

The infrared region of the spectrum holds much potential for the exploration and observation of our environment, the Earth's surface, oceans and atmosphere as well as observations of our Solar System, the Milky Way and even remote galaxies [Sandau, R. et al. (1996)]. In atmospheric science the goals are to achieve better understanding of the chemical and physical processes in the atmosphere. In astronomy, the chemical processing and energy budget of the interstellar medium as well as star formation are amongst the phenomena investigated.

In atmospheric science, detailed measurements of density, temperature, composition etc, and how they change with time, are needed to constrain complex models of the atmosphere and oceans. Such measurements can be made from a variety of platforms; weather stations, balloons, aircraft etc have all been utilised in efforts to make such measurements in an efficient and repeatable way. They all, however, have the disadvantage that they can only measure quantities in their locality, restricting the area over which useful data can be gathered. Of course this drawback can be mitigated by using a large number of such devices to gather data over a large area, but this introduces the need for consistent calibration between the instruments. Remote sensing offers an alternative whereby a single well-calibrated instrument carried on an orbiting platform can provide measurements over large areas for much longer time periods. This method, of course, has its drawbacks too; increased complexity of the device, higher cost and the need to interpret radiative signals being the most obvious. Spectrographic measurements (wherein the intensity of the radiation detected by an instrument is observed as a function of wavelength) comprise the majority of remote sensing methods. The radiation emitted or scattered by the atmosphere at a particular wavelength is determined by the emission and absorption by its component molecules and this makes it possible to retrieve valuable information about the atmosphere from the spectrum of the detected radiation. Consider an instrument on an Earth orbiting satellite receiving infrared

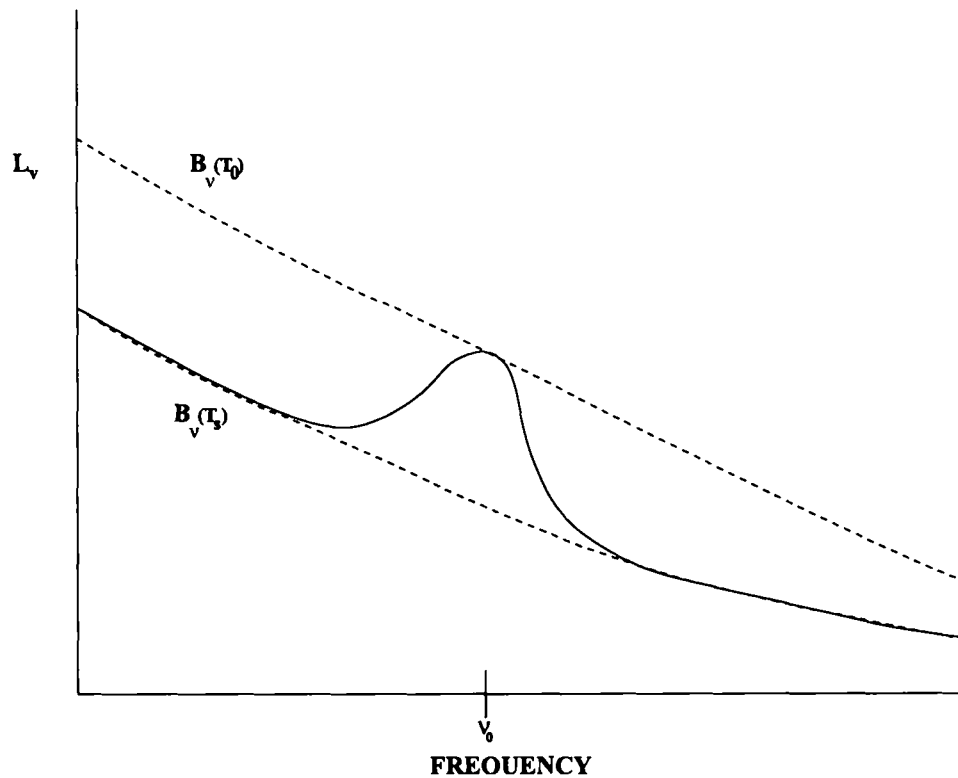


**Fig. 1.1:** Diagram illustrating nadir and limb viewing geometries. After Andrews [Andrews, D.G. (2000)].

radiation that has been emitted by the Earth along a nadir path (see fig 1.1 for explanation of viewing geometries). We can derive equations for the radiation received by the instrument in terms of the properties of the atmosphere. Assuming local thermodynamic equilibrium the spectral radiance  $L_\nu$  at frequency  $\nu$  received by the satellite instrument is given by [Andrews, D.G. (2000)]:

$$L_\mu = \int_0^\infty B_\nu(T(z)) \frac{\partial \tau_\nu(z, \infty)}{\partial z} dz + B_\nu(T_s) \tau_\nu(0, \infty) \quad (1.1)$$

Where  $T(z)$  is the atmospheric temperature at height  $z$ ,  $\tau_\nu(z, \infty)$  is the spectral transmittance between height  $z$  and the satellite,  $T(s)$  is the surface temperature. Here it is assumed that the surface acts as if it is a black body. The spectral absorptance between height  $z$  and the satellite is  $A_\nu(z, \infty) = 1 - \tau_\nu(z, \infty)$ . The form of  $A_\nu$  near a spectral line at  $\nu = \nu_0$  is shown in figure 1.2. The precise details of the line depend on whether it is a Doppler or Lorentz line shape, on the quantity and distribution of the absorbers and scatterers in the path between the height  $z$  that we are interested in, and the satellite instrument, as well as the temperature and pressure variations along the path. The diagram in figure 1.2 shows a simplified Lorentz-broadened spectral line for an isothermal atmosphere (at constant temperature  $T_0$ ) where the surface (at temperature  $T_s$ ) is cooler than the atmosphere, the increase in radiance at  $\nu_0$  is known as an emission line. If the surface is warmer than the atmosphere, however, then a reduction in the radiance is seen by the satellite instrument when observing radiance near  $\nu_0$ , this is called an absorption line. These features are easily explained by substituting the (isothermal and constant) temperatures of the



**Fig. 1.2:** Diagram showing satellite observed radiance near a spectral line. After Andrews [Andrews, D.G. (2000)].

surface and atmosphere into equation 1.1 so that it can be evaluated explicitly:

$$L_\nu = B_\nu(T_0) [1 - \tau_\nu(0, \infty)] + B_\nu(T_s) \tau_\nu(0, \infty) \quad (1.2)$$

Which simplifies to:

$$L_\nu = B_\nu(T_s) + [B_\nu(T_0) - B_\nu(T_s)] A_\nu(0) \quad (1.3)$$

In the case where  $T_0 > T_s$  then  $B_\nu(T_0) > B_\nu(T_s)$  and so the quantity in the square parentheses in equation 1.1 is positive (since  $\partial B_\nu / \partial T > 0$  for all values of  $\nu$  and  $T$ ) so as the absorbance  $A_\nu$  increases around  $\nu_0$  an emission line is formed ( $L_\nu$  increases). In the case where  $T_0 < T_s$  the quantity in the square parentheses becomes negative around  $\nu_0$  and so an absorption line is formed ( $L_\nu$  decreases). Of course in reality the situation is never so simple as having the atmosphere all at one temperature and the surface at another, and so the corresponding radiance measurements are also more complicated.

Satellites used for remote sensing are usually large and very expensive. Not only that, once they are in orbit they must operate reliably without maintenance throughout their lifetimes. If smaller, cheaper satellites with high reliability could be developed it would be of tremendous benefit to future remote sensing endeavours. This thesis is concerned with the development of a miniature Stirling cooler and radiator that would cool down infrared detectors on board a small remote sensing satellite. The next section of this thesis will give a brief overview of infrared detectors and why they need to be cooled.

## 1.1 Infrared detectors

Detectors that need to be cooled to cryogenic temperatures are the driving force behind the development of cryogenics for space. They determine not only the required operating temperature but also the temperature stability and architecture of the payload system [Linder, M. et al. (2001)]. Colder detectors are generally faster, more sensitive, and usable to longer wavelengths. There is a wide range of infrared detectors available for use on remote sensing missions, although these can be broadly divided into two types: *Thermal Detectors* and *Photon Detectors*. A brief summary of each of these types follows.

### 1.1.1 Thermal detectors

Thermal detectors operate on the principle that when heated by incoming infrared radiation their temperature rises and this can be measured by any phenomenon dependent on temperature in a known way. Some common types of thermal detectors are:

- Bolometer: The electrical resistance of the bolometer changes due to a change in its temperature caused by absorption of radiant energy [Jamieson, J.A. et al. (1963)].
- Thermovoltaic device: Heating of a thermocouple junction by incident radiation generates a potential difference across it. Often a number of junctions are wired in series to form a thermopile.
- Pyroelectric devices: Spontaneous polarisation in certain materials is used to detect temperature changes due to IR heating.
- Thermopneumatic cells: The expansion of a gas due to heating by incident radiation can be measured. Golay cells (in which the expansion of the gas causes distortion of a flexible mirror) are of this type [Smith, R.A. et al. (1968)].

Thermal detectors typically have time constants in the millisecond range, much longer than photon detector times [Rogalski, A. (2003)]. They have been used extensively in the past and since the early 1990s there has been a resurgence of interest in un-cooled thermal detectors for use with non-scanned imagers with 2-D detectors.

### 1.1.2 Photon detectors

#### Photo-emissive detectors

The emission of electric charge from surfaces exposed to radiation was observed by Hertz in 1887 [Smith, R.A. et al. (1968)], many years before the discovery of the electron itself. The nature of



photo-electric emission was explained in terms of quantum theory in 1905 [Einstein, A. (1905)]. Photo-electric emission is used as the basis of the photo-multiplier wherein electrons emitted from a photo-cathode due to the radiation striking its surface go on to strike another electrode (called a “dynode”) from which secondary electrons are emitted, these then go on to cause yet more electron emission from a third electrode and so on. The material from which the electrode surfaces are made determines the wavelength to which the photomultiplier is sensitive. When there is no radiation incident on the photocathode an anode current still flows due to thermionic emission of electrons from the photocathode. This “dark current” of thermally excited electrons can be reduced by cooling the photomultiplier [Sherring, C.W. (1970)], [Naray, Z. and Varga, P. (1957)] and [Naray, Z. (1956)]. Despite the extremely high sensitivity in their working range (near infrared to ultraviolet) the use of photomultipliers in the infrared region is limited, since at wavelengths longer than around  $1.7\mu m$  the quantum efficiency falls dramatically. This is because photons incident on the photocathode must have energy greater than the work function necessary to eject an electron from the photocathode [Biswas, A. and Farr, W.H. (2003)]. As wavelength increases beyond the near-infrared region the photons incident on the photocathode do not have enough energy to produce any photo-electrons and so the photomultiplier cannot be used. It is possible that developments in the production of photocathode materials could extend the usefulness of photomultipliers to longer wavelengths.

### **Photoconductive and photovoltaic detectors**

Photoconductive (PC) and Photovoltaic (PV) detectors are semiconductor devices in which electron-hole pairs are generated by incoming photons. These detectors have a cut-off wavelength above which no electron-hole pairs are created, this is due to the fact that the charge carriers in the semiconductor must be excited across an energy gap. If the wavelength of the incoming photon is too long then no carriers will have sufficient energy to bridge this gap. In PV detectors a p-n junction is utilised to detect incoming photons, these photons generate electron-hole pairs which migrate to opposite sides of the junction thus producing a voltage (and also a current if the device is in a circuit). In PC mode a bias voltage is applied across a bar of material, incoming photons generate electron-hole pairs which increase conductance of the material and thus the current that flows is proportional to the light intensity over a wide range. Recent developments in novel IR detectors include Quantum Well Infrared Photodetectors (QWIP) detectors, in which charge carriers are excited from a bound state in the quantum well. In focal plane arrays (FPAs), described below, pixel yield and uniformity are key issues [Rogalski, (2002)] and QWIP detectors promise to excel in these areas, although they have a lower quantum efficiency than HgCdTe [Rogalski, A. (2003)] which is used in many conventional photon detectors. This is currently the preferred ma-



terial for photon detectors because of its high absorption coefficient and relatively low thermal generation rate [Rogalski, A. (2003)]. There are however other materials that are used, see table 1.1.

| Detector         | $E_{gap}(meV)$ | $\lambda_c(\mu m)$ | Typical operating temp (K) |
|------------------|----------------|--------------------|----------------------------|
| Ge(PV)           | 670            | 1.85               | 77-100                     |
| Si(PV) CCD       | 1100           | 1.125              | 215-300                    |
| SiIn             | 160            | 7.75               | 30-77                      |
| GaAs             | 1500           | 0.84               | 300                        |
| InSb             | 200            | 6.25               | 12-77                      |
| $Pb_8Sn_2Te(PC)$ | 100            | 12.4               | 4.2-77                     |
| PbS              | 440            | 2.8                | 77-300                     |
| $Hg_8Cd_2Te(PV)$ | 88             | 14                 | 65-90                      |

**Table 1.1:** Semiconductor materials used in photon detectors. After [McMurray, R.E. (1994)].

### Charge Coupled Devices

The use of Charge Coupled Devices (CCDs) as detectors at infrared wavelengths is limited as the quantum efficiency declines steeply at wavelengths longer than approximately  $1\mu m$  [Groom, D.E. et al. (2000) ]. Since CCDs have limited uses in infrared remote sensing what follows is only a brief description of their operation and how cooling can benefit them. A CCD works when photons incident on the CCD excite bound electrons to a conduction band. In a CCD the charge produced by the excited electrons, and the holes they leave in the valence band, can be measured. A CCD is split into an array of pixels and the charge gathered at each pixel is proportional to the number of photons incident on the pixel in a specified integration time. The charge gathered at each pixel is then “read-out” and can be used to build up an image. Since the operation of CCDs depends upon electron-hole pairs being created by incident photons it is clear that thermally excited electrons will constitute a source of noise. Even when no photons are incident on the CCD some electrons will be thermally excited from the valence band. This is known as “dark current” and by cooling the CCD this can be reduced.

#### 1.1.3 Focal plane arrays

A 2-dimensional scene can be built up from 2-D scanning of a single pixel detector, 1-D scanning of a linear array of detectors or use of a 2-D “staring array” with no scanning. There has been an

| Temperature (K) | Limiting $D^*$       |
|-----------------|----------------------|
| 300             | $1.8 \times 10^{10}$ |
| 90              | $3.6 \times 10^{11}$ |
| 40              | $2.7 \times 10^{12}$ |

**Table 1.2:** Theoretical  $D^*$  limited by photon noise after [Brasunas, J.C. Lakew, B. (2003)].

increase in the size of multi-element arrays as manufacturing techniques have improved and sizes in excess of  $10^6$  elements for the 2-D arrays [Rogalski, A. (2003)] are now achievable. Detector pixels can be mounted on CCDs either by mating each detector pixel to a CCD pixel through an indium bump bond or by fabricating the detectors directly into the silicon or other substrate [Burt W.W. (1990)].

## 1.2 Detector noise

In an ideal photon detector each incoming photon with an energy higher than the ionisation energy produces a charge carrier. In reality, the performance is limited by a number of noise sources (as well as the quantum efficiency of the detector), and this appears in the detector output as a random component equal to the RMS sum of the individual noise sources. Brasunas and Lakew [Brasunas, J.C. Lakew, B. (2003)] provide table 1.2 which shows theoretically attainable values of  $D^*$  (area and bandwidth normalised sensitivity) at various detector temperatures.

## 1.3 Noise sources

There are five main noise sources that need to be considered when discussing infrared detectors:

1. Photon noise
2. Johnson noise
3. Flicker or  $1/f$  noise
4. Generation-combination noise
5. Shot noise

These noise sources add in quadrature and determine the overall noise level of the detector. The ideal situation is for a detector's performance only to be limited by the fluctuations in the arrival rate of photons from the target of interest. For example, if the background (sources of

photons other than the target) could be at absolute zero and the detector had a unit quantum efficiency then the detectability of a signal against the background would be dependent upon the statistical nature of the arrival of the signal photons [Kruse, P.W. et al. (1962)]. Success in reducing the background photon noise below the level of the target (or “signal”) photon noise leads to the Signal Limited Infrared Photodetector (SLIP) condition. In practice, reducing the background to this level is rarely achieved and designers commonly aim to achieve the Background Limited Infrared Photodetector (BLIP) condition. This condition occurs when the background photon noise (fluctuations in the arrival rate of the photons from background sources) dominates over all the other noise sources so that any further improvements that reduce noise sources internal to the detector will have no effect on its performance. In other words the “ultimate limit for detector noise and hence the ultimate detectivity of photodetectors is determined by these fluctuations” [Jamieson, J.A. et al. (1963)]. A more thorough discussion of photon noise and other noise sources follows.

### Photon noise

There is noise inherent in a stream of photons incident on a detector which is completely independent from the detector structure and characteristics. The photons may originate from the target being viewed, background sources in the field of view, or even from elements in the instrument’s optical scheme (filters, lenses etc), although this can be minimised by cooling these elements. We now consider the nature of this noise source.

The mean density of photons in the frequency range  $\nu, \nu + \delta\nu$  in a cavity with walls that act as a perfect blackbody is given by the well-known equation [Jamieson, J.A. et al. (1963)]:

$$\overline{n(\nu, \delta\nu)} = \frac{8\pi\nu^2}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \quad (1.4)$$

Where  $c$  is the speed of electromagnetic radiation in vacuum,  $h$  is Planck’s constant,  $k$  is Boltzmann’s constant and  $T$  is temperature. One quarter of these are crossing a surface in unit volume in either direction and thus the density of the stream of photons is given by:

$$\overline{n(\nu, \delta\nu)_p} = \frac{2\pi\nu^2}{c^3} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \quad (1.5)$$

Where  $\overline{n(\nu, \delta\nu)_p}$  is the density of the stream of photons. The number of photons that arrive in a short time interval  $\Delta t$  on an element of area  $\Delta A$  were, at the beginning of the time interval, in a sphere of radius  $c\Delta t$ . Those coming from a solid angle  $\Delta\Omega$  around a direction  $\theta$  to the normal with  $\Delta A$  were in a slant cone of volume  $c \cos\theta \Delta A \Delta\Omega \Delta t$ . Thus the average number of photons passing through  $\Delta A$  in time interval  $\Delta t$  is given by:

$$\overline{N(\nu, \delta\nu)_p} = \frac{2\pi\nu^2}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \cos\theta \Delta A \Delta\Omega \Delta t \quad (1.6)$$

The fluctuation about this mean value is given by:

$$\overline{\Delta N(\nu, \delta\nu)_p^2} = \overline{N(\nu, \delta\nu)_p} \left(1 + \frac{1}{e^{\frac{h\nu}{kT}} - 1}\right) \delta\nu \quad (1.7)$$

Therefore:

$$\overline{\Delta N(\nu, \delta\nu)_p^2} = \frac{2\pi\nu^2}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \cos\theta \Delta A \Delta\Omega \Delta t \left(1 + \frac{1}{e^{\frac{h\nu}{kT}} - 1}\right) \delta\nu \quad (1.8)$$

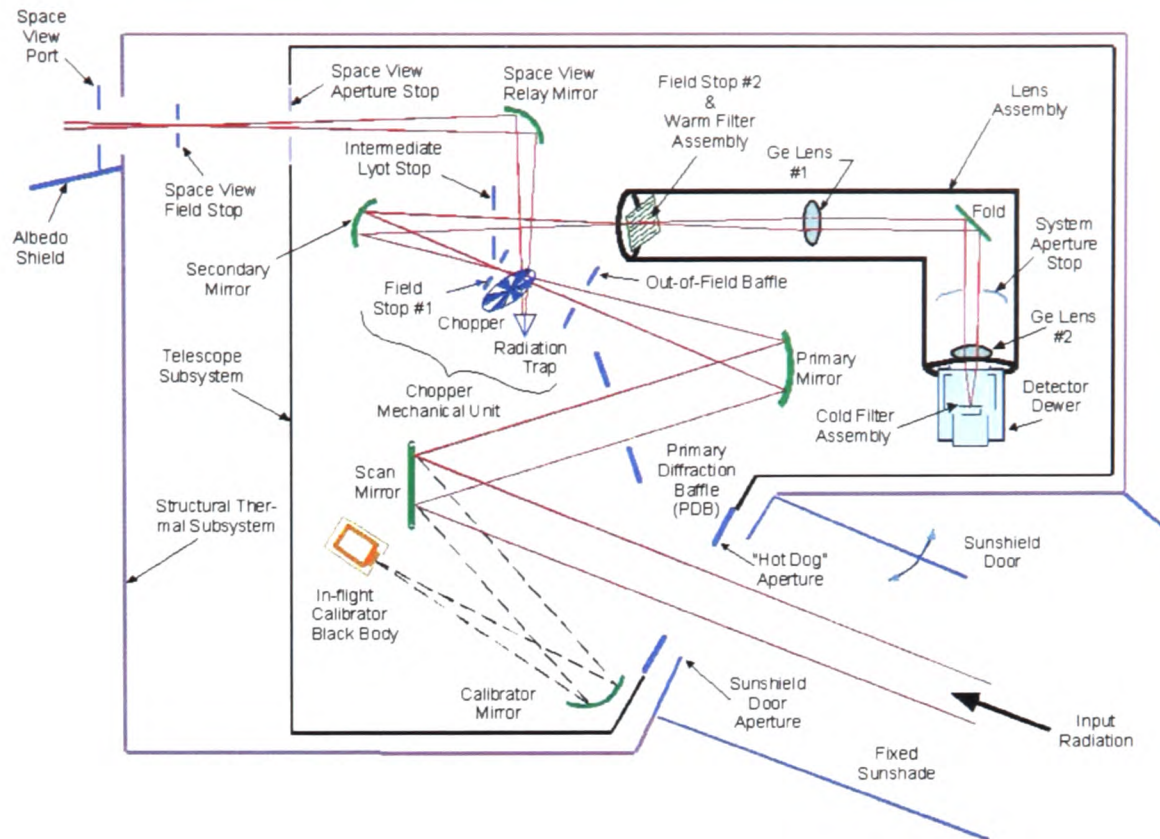
The factor  $1 + \frac{1}{e^{\frac{h\nu}{kT}} - 1}$  arises from the quantum theory of radiation that states that photons at the same frequency cannot be identified individually. The fluctuations of such quanta are described by Bose-Einstein rather than Poisson statistics, hence the inclusion of this factor. If they did obey Poisson statistics we would expect the fluctuation to be given by the square root of the expectation value [Smith, R.A. et al. (1968)]:

$$\overline{\Delta N(\nu, \delta\nu)_p^2} = \overline{N(\nu, \delta\nu)_p} \quad (1.9)$$

Looking at equation 1.8 we see that this particular noise source is dependent on the temperature of the source of the photons not the detector itself. If the other noise sources discussed later in this section can be reduced below the level of the photon noise then this will limit the performance of the detector even if further improvements are made to it. Since the background in most cases acts as a grey-body emitter, the photon flux from it is dependent upon its temperature and emissivity [Jamieson, J.A. et al. (1963)]. To reduce background noise a detector element can be surrounded with a cooled enclosure having an aperture through which the detector can view its target. In this way the detector is made to view as little as the background as possible [Kruse, P.W. et al. (1962)].

The photon noise produced by elements of the instrument in the optical path can be minimised by cooling them. For example, cooling of filters; If the spectral response of the detector extends to wavelengths much greater than those at which the source of interest radiates, then the detector may see a significant amount of photon noise from the background at wavelengths of no interest. This can be reduced by placing a filter over the detector which only passes the relevant region of the spectrum to the detector. By cooling such a filter its contribution to the background photon noise can be minimised [Moorhouse, J. (2003)].

The High Resolution Dynamics Limb Sounder (HIRDLS) instrument provides a useful example in which a filter as well as the detector itself was cooled. Diagram 1.3 shows an optical schematic for the HIRDLS instrument that was flown on the AURA spacecraft. HIRDLS is a 21 channel filter radiometer for the measurement of infrared emission from the limb of the Earth [Moorhouse, J. (2003)]. Note the cooled filter and detector dewar on the right hand side of the figure. The cooling was performed by two Stirling coolers manufactured by Ball Aerospace. Diagram 1.4 shows an exploded view of the HIRDLS instrument; the coolers can be seen at the bottom left of the diagram.



**Fig. 1.3:** Optical schematic of the HIRDLS instrument.

By reducing the photon noise from optical elements such as filters, the instrument may reach a condition whereby its performance is only limited by the photon noise from sources external to it. These will be both “signal” photons from the target of interest and also photons radiating from other background sources in the field of view. It is the need to detect these signal photons in the presence of this background photon noise which establishes the performance limit of photon detectors [Kruse, P.W. (1995)].

### Johnson noise

Johnson (Nyquist) noise originates from thermal fluctuations in any resistive elements of a detector and its associated electronics. Even when no direct current is flowing through a resistor there is still a small fluctuating voltage across its end due to the random motion of electric charge carriers within the resistor itself [Smith, R.A. et al. (1968)].

The well-known expression for the Johnson noise is [Jamieson, J.A. et al. (1963)]:

$$v_{rms}^2 = [4kTR\Delta f]^{\frac{1}{2}} \quad (1.10)$$

Which we can write as:

$$N_j = (4kTR)^{\frac{1}{2}} \quad \text{volts}/(Hz)^{\frac{1}{2}} \quad (1.11)$$

Where R is the resistance of the resistive device in question at a temperature T. It is clear from



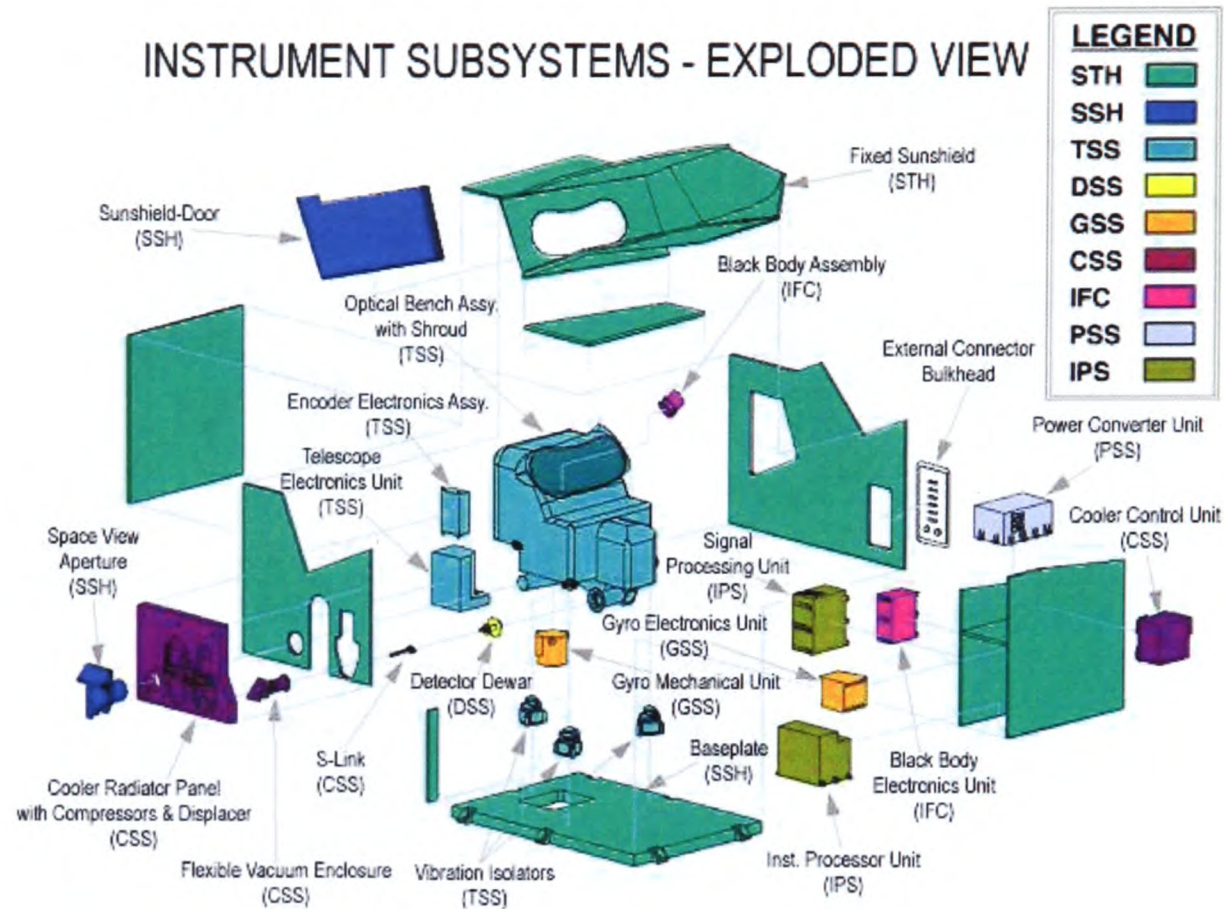


Fig. 1.4: Exploded view of the HIRDLS instrument.

the explicit dependence on temperature in equation 1.11 that cooling the detector will reduce this noise source.

**Flicker noise**

Flicker noise is created by frequency dependent noise mechanisms. The magnitude of the noise voltage is inversely proportional to some power of the frequency. It can be caused by trapping states in the detector bulk or surface and by carrier trapping in the channel of a Field Effect Transistor (FET) amplifier. This noise can dominate at low frequencies, and is given by:

$$N_f = \frac{V_n(T)}{f^n} \quad \text{volts}/(\text{Hz})^{\frac{1}{2}} \quad (1.12)$$

Where  $V_n(T)$  is a temperature dependent constant of proportionality. Flicker noise is found in all conductors but can be minimised through good production methods and in photovoltaic detectors by using a slightly negative bias voltage.

### Generation-Recombination (g-r) noise

The number of charge carriers in a semiconducting material fluctuates about some mean value. There are two types of charge carriers in a semiconductor; electrons and holes. Because of the need to maintain space-charge neutrality, except under severe trapping conditions, a change  $\Delta N$  in the total number of electrons implies an equal change  $\Delta p$  in the number of holes [Smith, R.A. et al. (1968)]. These fluctuations in turn cause fluctuations in the resistance  $R$  of the semiconductor which show up as voltage fluctuations when a current is flowing through it. So, unlike Johnson noise described earlier g-r noise is only present when a current is flowing in the semiconducting element. The following expression can be derived for the magnitude of the g-r noise in a semiconductor:

$$\overline{\Delta N_{g-r}^2} = \frac{4\tau^2 G \Delta f}{1 + 4\pi^2 f^2 \tau^2} \quad (1.13)$$

Where  $G$  is the rate at which carriers are being created in the semiconductor,  $\overline{\Delta N_{g-r}}$  is the fluctuation in the number of charge carriers and  $\tau$  is the carrier lifetime. Even when no radiation is incident on the detector some carriers will be thermally excited to the conduction band so that  $G \neq 0$  in equation 1.13. By cooling the detector the rate at which these thermally excited carriers are produced, and therefore  $\overline{\Delta N_{g-r}}$ , can be minimised.

#### 1.3.1 Shot noise

Shot noise arises due to the discrete nature of charge carriers which form a current. An electrical current consists of a flow of charge in a conductor and is defined as the rate at which charge passes a point in that conductor. Such a definition is strictly only applicable to a continuous distribution of charge. If we count the number of charges passing a point in a time  $\tau$  we obtain a number  $n_\tau$ . If we repeat this count over a large number of intervals we will not get the same value of  $n_\tau$  as the charges are not flowing at regularly spaced intervals but in a random fashion. We can determine the average value of charge carriers passing our chosen point,  $\bar{n}_\tau$ . If we keep repeating this experiment we will find values that differ from  $\bar{n}_\tau$  by small amounts and so we say the current is “steady”. The value of this steady current  $i_0$  we define as the average number of charges times the elementary charge,  $e$ , divided by the counting time  $\tau$  resulting in:

$$i_0 = \frac{e\bar{n}_\tau}{\tau} \quad (1.14)$$

For steady conditions it is apparent that  $\bar{n}_\tau$  is proportional to the length of  $\tau$ . We may write:

$$\frac{\bar{n}_\tau}{\tau} = n \quad (1.15)$$

Where  $n$  is the average number of charge carriers passing in unit time, so we can write:

$$i_0 = ne \quad (1.16)$$

Which we see is independent of  $\tau$ . The actual counts  $n_\tau$  in a series of experiments will deviate from  $\bar{n}_\tau$  by a certain amount and it is this deviation that causes the fluctuation in current. So we can write:

$$\overline{(\Delta n_\tau)^2} = \overline{(n_\tau - \bar{n}_\tau)^2} = \bar{n}_\tau \quad (1.17)$$

If we define the actual current as:

$$i_\tau = \frac{en_\tau}{\tau} \quad (1.18)$$

We then have:

$$\overline{\Delta(i_\tau)^2} = \overline{(i_\tau - i_0)^2} = \frac{e^2}{\tau^2} \overline{(n_\tau - \bar{n}_\tau)^2} \quad (1.19)$$

Substituting in our result from equation 1.17 we find:

$$\overline{\Delta(i_\tau)^2} = \frac{e^2 \bar{n}_\tau}{\tau^2} \quad (1.20)$$

Substituting in our result from equation 1.15 we then find:

$$\overline{\Delta(i_\tau)^2} = \frac{e^2 n}{\tau} \quad (1.21)$$

Since  $i_0 = ne$  (equation 1.16) we may write:

$$\overline{\Delta(i_\tau)^2} = \frac{ei_0}{\tau} \quad (1.22)$$

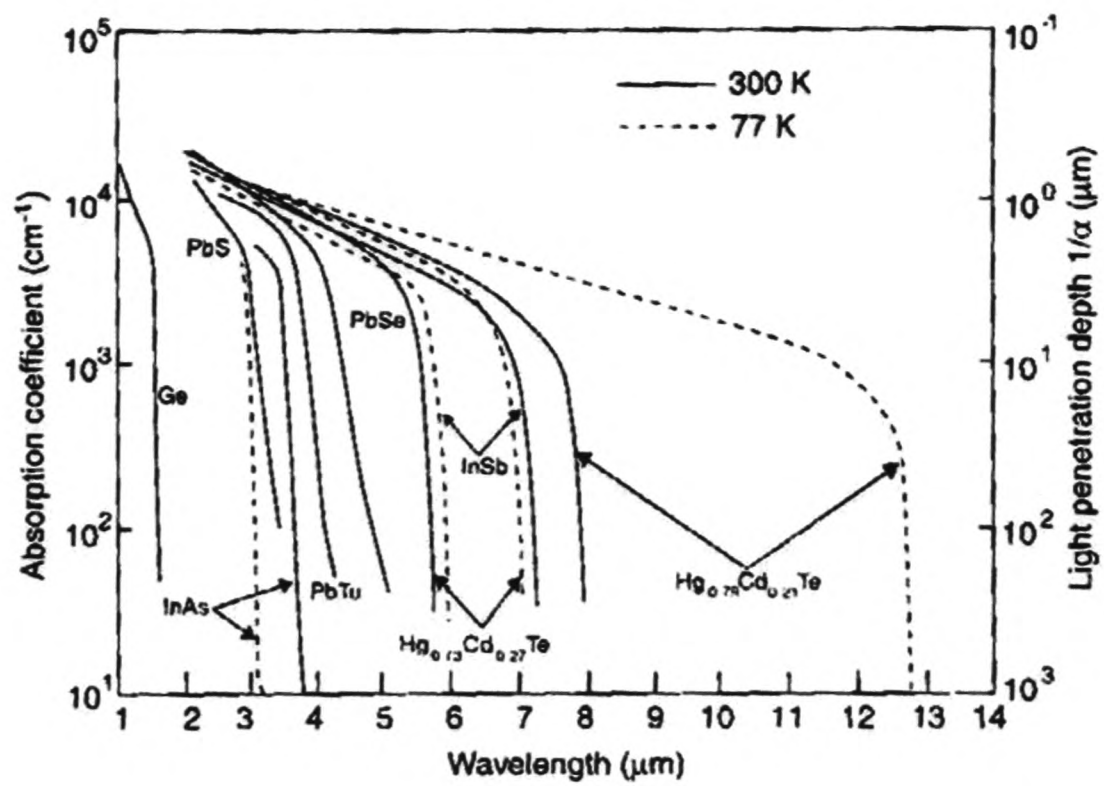
So for a given value of  $i_0$  the mean square fluctuation in the quantity  $i_\tau$  is proportional to the elementary charge  $e$ . This equation is not representative of shot noise in semiconductor detectors, it does, however, illustrate the mechanism in which the discrete nature of charge characters impacts on the noise characteristics of detectors. The equation for shot noise in such devices is given by [McMurray, R.E. (1994)]:

$$N_s = (2eI)^{(\frac{1}{2})} \quad \text{ampere}/(Hz)^{\frac{1}{2}} \quad (1.23)$$

## 1.4 Summary

Remote sensing of the Earth and other planets in the infrared region of the electromagnetic spectrum utilises a range of detectors. Invariably the performance of these detectors is improved by cooling them as this reduces the noise and increases the SNR. With the development of 2-D “staring” FPAs where no scanning mechanism is necessary, smaller more reliable detector systems are being developed. A suitable cooling system for such arrays would enable the production of small, reliable infrared remote sensing instruments capable of high quality infrared observations.





**Fig. 1.5:** Variation in absorption coefficient and cut-off wavelength for common photon detector materials. After [Rogalski, A. (2003)].

## Chapter 2

# Cooling options for space-borne detectors

### 2.1 Introduction

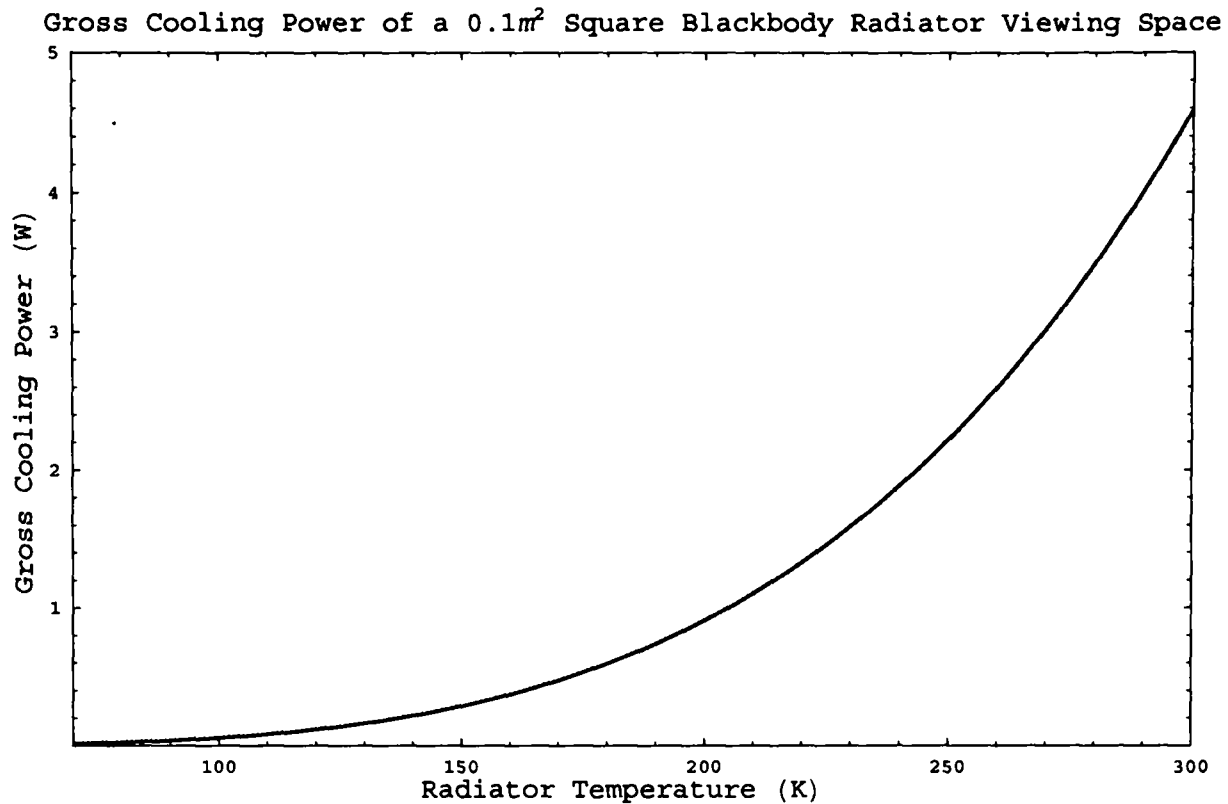
The need for cooling has been discussed in the previous chapter, but there are a myriad of technologies that can be applied to produce low temperatures. The application of these technologies is not limited to space science. Medical imagers require cooling as do the infrared sensors used by rescue teams at earthquake sites. Cooling of computer hardware and Superconducting Quantum Interference Devices (SQUIDs) are further examples. This chapter will only cover those cooling options suitable for space-borne infrared detectors.

### 2.2 Cooling technology

#### 2.2.1 Principle of cryogenic systems for space missions

A cryogenic system removes heat from a cold region and rejects it to a warm region. In the case of a satellite or spacecraft this heat will eventually be radiated to space. Radiators are the simplest and most reliable cooler. They are governed by Stefan's law that all objects emit radiant energy at a rate proportional to their surface area, emissivity, and to the fourth power of their temperature [Halliday, D. et al. (1997)], and on the fact that space is very cold ( $\approx 2.73K$ ). The gross cooling power of a radiator viewing only space is thus:

$$Q_{rad} = \sigma A \epsilon (T^4 - T_0^4) \approx \sigma A \epsilon T^4 \quad (2.1)$$



**Fig. 2.1:** Example of the gross cooling power of a space-viewing radiator as a function of radiator temperature.

Where  $A$  is the area of the radiator,  $\sigma$  is Stefan's constant and  $T$  and  $T_0$  ( $\approx 2.7K$ ) are the temperature of the radiator and of the region of space it is viewing respectively. The temperature at which a radiator settles occurs when the gross cooling power equals the total heat load arising from:

- Conduction through radiator supports.
- Radiation from objects in field of view, and from the spacecraft itself.
- Conduction from thermal links to the instrument to be cooled.

The gross cooling power (as calculated by equation 2.1) of a  $0.1m^2$  square blackbody radiator as a function of radiator temperature is shown in figure 2.1. As the radiator temperature decreases the gross cooling power declines very rapidly. The parasitic heat loads, through the radiator's connections to the spacecraft, will be proportional to the temperature difference between it and the spacecraft. They will, therefore, increase linearly as the radiator temperature decreases and will become an increasingly significant part of the heat load on the radiator. This limits the final temperature of the radiator and the useful cooling it can perform. Further limitations arise when considering the required orientation of the radiator, as it must be shielded from solar radiation and from Earth infrared and albedo radiation and this is not always possible for some orbits.

If heat is not radiated directly to space via radiators at the desired low temperature some work has to be performed to pump the energy between the two temperature levels, from a cold to a warm level. Such heat pumping can be done according to an open cycle configuration or in a closed cycle [Collaudin, B. and Rando, N. (2000)]. The open cycle means the use of stored cryogen, in this case the work is done before the launch of the mission (in liquefaction of the cryogen). The cold sink is provided by the evaporation of the cryogen. In this case there is no energy to be radiated away from the spacecraft but there is gas to be released. The lifetime is limited by the mass of cryogen that can be carried on-board the spacecraft. For a small remote sensing platform where the total mass should be kept as small as possible the extra mass of the stored cryogen is not an attractive option. The closed cycle, corresponding to a mechanical cooler, differs in that work is done continuously throughout the mission rather than beforehand. The same working fluid is driven repeatedly through a closed thermodynamic cycle and so large amounts of cryogen are not needed, thus reducing mass and providing the possibility of longer lifetime missions. Heat removed from the cold tip of the mechanical cooler is rejected at the warm end and can be radiated away to space.

Mechanical coolers enable infrared detectors to be cooled without the need for large amounts of stored cryogen to be placed in orbit. However, there are very stringent requirements put upon these coolers by the space-borne environment [Chan, C.K. (1991)]. Spacecraft cooler requirements differ quite dramatically from tactical (military) cooler applications due to the imposition of a requirement for long life and continuous operation [Davis, T.M. et al. (1999)]. The requirements for a spacecraft-borne cooler include:

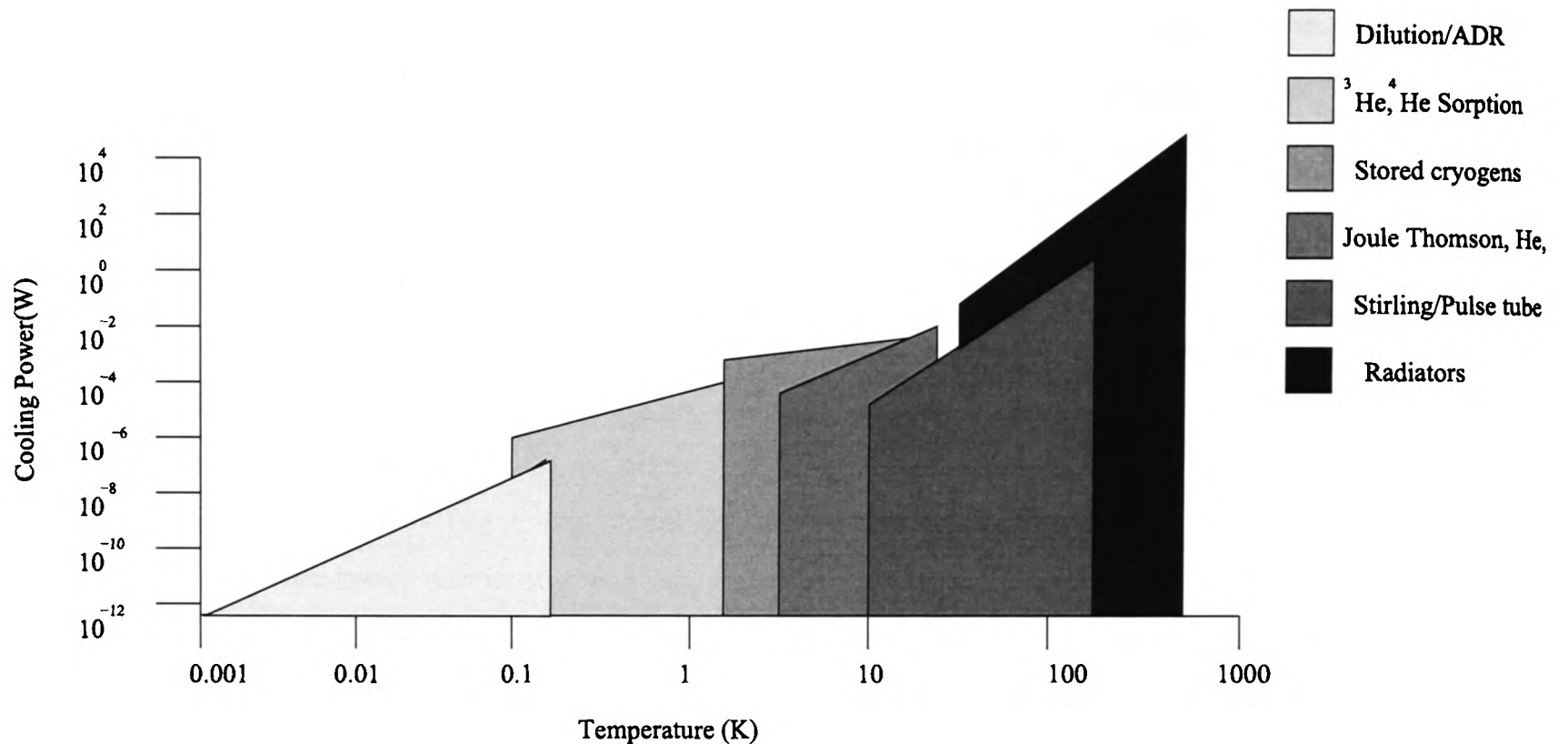
- Required cooling power at the appropriate low temperature with low input power.
- Long lifetime.
- Reliable and maintenance free operation.
- Minimum vibration.
- Compactness and low mass.

### 2.2.2 Brief overview of types of mechanical coolers

- **Regenerative coolers** are based on a pressure wave, generated by a compressor and a cold finger, using a mobile (Stirling, Gifford) or fixed (pulse tube) regenerator. Heat is extracted at the cold tip as the gas expands and is rejected at the warm end as it is compressed.
- **Recuperative coolers** use counterflow heat-exchangers (unlike the regenerators found in Stirling coolers) in which separate channels are used for the hot and cold working fluid

streams. Brayton coolers are of this type and use a cold turbine to expand the working gas adiabatically [Collaudin, B. and Rando, N. (2000)] and [Walker G. (1983a)] before passing it through a cold tip heat exchanger where heat is abstracted from its surroundings. Joule-Thomson coolers utilise expansion of the working gas through an orifice and the Joule-Thomson effect to achieve cooling [Zemansky, M.W. (1957)].

- **Sorption coolers** are based on the principle that large amounts of gas can be adsorbed on certain solids such as highly porous carbon [Burger, J.F. et al. (2002)]. If a pressure vessel is filled with such a material and the working gas is adsorbed at a low temperature and pressure, then a high pressure can be created inside the closed vessel by increasing the temperature of the material. Next, a controlled gas flow out of the vessel can be maintained at a high pressure by further increase of the temperature until most of the gas is desorbed. In the cold stage, compressed gas coming out of the compressor moves through a counterflow heat exchanger to the cold side where Joule-Thomson expansion to low pressure occurs through an orifice. After evaporation by the thermal load, the low pressure vapour returns through the counterflow heat exchanger to the compressor unit. Helium-3 sorption coolers are used to achieve temperatures down to 0.3K [Kittel et al. (2002)].
- **Adiabatic demagnetisation** is a cooling system based on the orientation of electron spins of paramagnetic salts. The principle is as follows: The salt is first magnetised in an isothermal environment (i.e. increasing a magnetic field while the salt is thermally coupled to a thermal bath, at 2 or 4K). The salt is then de-magnetised adiabatically (i.e. lowering the magnetic field applied to the insulated salt). This action produces a cooling effect because the energy required to disorient the electron spins can only be taken from the salt thermal bath [Barron, R.F. (1985)]. Adiabatic Demagnetisation Refrigerators (ADRs) are not continuous cooling systems, but the hold time can be very long (depending on applied heat loads). Interfaces and operations are very simple, and are associated with the control of the cooling magnet.
- **Dilution coolers** use both helium-3 and helium-4 to achieve cooling. Helium-3 gas is condensed in a bath of liquid helium-4, it then expands through a capillary tube and is cooled further still. The helium-3 is again cooled in a heat exchanger before entering a mixing chamber where it is mixed with helium-4 at temperatures between 0.005K and 0.05K. In the mixing chamber the liquid separates into two phases; a less dense concentrated helium-3 mixture and a more dense dilute helium-3 mixture. Because the helium-3 has a higher entropy in the dilute phase than in the pure phase, heat will be absorbed when helium-3 passes from the pure phase to the dilute one in a reversible manner and this is how useful cooling is produced.



**Fig. 2.2:** Comparison of various types of coolers.

Figure 2.2 is based on information from Collaudin [Collaudin, B. and Rando, N. (2000)] and gives a comparison of the operational temperature ranges and cooling power of cooling technologies available to space machines. A more in-depth comparison can be found in Glaister [Glaister, D.S. et al. (1999)].

## 2.3 Choice of cryogenic system

The choice of cooler depends on many factors:

1. **Temperature and heat lift.** As discussed earlier in this thesis, the operating temperature is set by the BLIP temperature of the detector in use, temperatures of 80K or below are typical operating temperature for an IR detector used in remote sensing [Kim, H. et al. (2000)]. The heat lift required to achieve this temperature is set by the heat dissipation of the detector/pre-amplifier/cooled optics as well as parasitic heat loads from the spacecraft structure.
2. **Efficiency.** Power on board any spacecraft is always limited by the output of the batteries or solar cells available. Devices destined for such craft must always be constructed with high efficiency in mind.
3. **Mass.** Increasing the mass of a satellite increases the cost to launch it and instruments rarely have a mass exceeding 200kg [Reed, J. (2004)]. The less of this mass that is taken up

| Cooler           | Temp. (K) | Cooling | Advantages                                      | Disadvantages                                           |
|------------------|-----------|---------|-------------------------------------------------|---------------------------------------------------------|
| Radiator         | 80K       | 0.5W    | Reliable, low vibration. Long lifetime          | Complicates orbit<br>Large size                         |
| Stored Cryogens  | 4K        | 0.05W   | Stable, low vibration                           | Short lifetime, Out-gassing<br>Large mass complex.      |
| 1 Stage Stirling | 80K       | 0.8W    | Efficient mature tech.                          | Vibrations                                              |
| 2 Stage Stirling | 20K       | 0.06W   | Intermediate temp available                     | Not mature<br>Still developing                          |
| Pulse tube       | 80K       | 0.8W    | Lower cold head induced vibration c.f. Stirling | Reduced efficiency c.f. Stirling                        |
| Peltier          | 170K      | 1W      | Low mass                                        | High temp. low efficiency                               |
| Joule-Thompson   | 4K        | 0.01W   | Low vibrations                                  | Hybrid design<br>Pre-cooling via alternative mechanism. |
| Sorption         | 10K       | 0.1W    | Low vibrations                                  | Under development                                       |
| Reverse Brayton  | 65K       | 8W      | High capacity                                   | Complex                                                 |
| ADR              | 0.05K     | 0.01mW  | Access to extremely low temperature             | Requires large magnetic field                           |

Table 2.1: Advantages and disadvantages of different types of cooler.



with cooling apparatus the more can be used to increase the performance of the instrument's other elements.

4. **Reliability.** The typical duration for an Earth observing satellite could be estimated as 3-5 years and any cooling system must be able to operate consistently (although not necessarily continuously) throughout this time in zero gravity. The expected lifetime of a system is quantified by its mean time between failures (MTBF) and this should be compatible or ideally exceed the mission duration.
5. **Thermal Insulation.** The low temperature equipment must be properly supported, insulated from the higher temperature (usually room temperature) spacecraft bus and protected from solar/earth radiation. The lower the cryogenic operating temperature the more stringent the demands on the thermal insulation.
6. **Accessibility.** The cold parts need to be accessible (e.g. connected to the detectors, signal wires, sensor wires temperature sensors etc) and wiring needs to be routed between the cold payload and the satellite bus for processing (A/D conversion, data handling) before transmission.
7. **Ancillary cryogenic equipment.** Cryogenic ancillary equipment needs to be used to operate the cryogenic payload (e.g. heat links, heat switches etc).
8. **Testability.** Pre-flight testability is likely to have an impact on architecture as it is essential to ensure that the instrument and the payload cooling system can operate as planned.
9. **Vibrations.** The instrument and cooling system must be able to survive the vibrations experienced during launch. This requirement has a large impact on the instrument design, and results in a compromise between the large support structure that would ensure safety from vibration damage and little support that would result in better thermal insulation.

There is a large number of peer reviewed papers that give an overview of available cryogenic systems for space ([Glaister, D.S. et al. (1999)] and [Collaudin, B. and Rando, N. (2000)] for example). Rather than repeat the conclusions of these papers a simple summary is presented in table 2.1 which gives a general overview of popular coolers, typical cold tip temperatures, their disadvantages and advantages.

## 2.4 The need for a new type of cooler

Earth observing missions are currently undergoing a revolution due to the development of so-called micro-satellites (Microsats) up to 50kg whose aim is to provide faster access to space at



much lower cost than is possible with conventional programs [Turner, R.F. (1998)]. This move toward smaller satellites has been in progress for some time. Saundau et al. note that, “Due to the progress in different fields of technologies used for detector development and for instrument design, the application of small satellites for earth observation becomes increasingly attractive” [Sandau, R. et al. (1996)] . Reed cites Tsinghua-1 launched in June 2000 as an example of such a micro-satellite [Reed, J. (2004)]. The use of radiators to cool down devices on such satellites is limited, as the maximum possible radiator surface area imposed by the small satellite size may lead to an unacceptably high operating temperature at the radiator cold patch.

Deep space missions have not used active coolers because of mass and power considerations, and so in the past radiators have been used (with the Composite Infrared Spectrometer (CIRS) on the Cassini mission, for example). It is clear that the flexibility and also the quality of observations could be improved by the use of a miniature cooler with low mass and high efficiency. With these two types of missions requiring a new type of cooler it was decided to examine the requirements such a cooler would be needed to fulfill:

- **Cold tip temperature.** To achieve BLIP performance from the detectors they must be cooled to temperatures equal to or below 80K. In some cases the final stages of the optical system may need to be cooled as well.
- **Heat lift.** The heat lift requirements will differ from one instrument to another, and they are determined by the electrical power dissipation in the detector electronics and by parasitic heat loads from warmer parts of the instrument and the spacecraft (see figure 2.3). As mentioned above the heat load is mission specific; a typical heat load might be  $200mW$  [Reed, J. (2004)] and therefore this was selected as the target cooling performance for this development effort.

Currently there are coolers in existence that could reach both targets for cold tip temperature and heat load set out above. However, none has the required small mass and high efficiency required for use on-board a micro satellite. For example, if we look at the Stirling coolers (Ball Aerospace SB160) used on the EOS-AURA satellite to provide cooling for the HIRDLS instrument we find that each cooler weighed, not including drive electronics, 10.6kg [Ball Aerospace Website (2005)] (rising to 14.7kg if they are included). Compare this with the instrument mass budgets on a typical micro-satellite or deep space mission:

- **MicroSat-70.** This micro-satellite produced by Surrey Satellite Technology Ltd. has a total mass ranging from 50 – 70kg and can carry a payload of 25kg. Assuming the SB160 mass is typical, more than half of this payload mass would have been used up simply with the

cooler and its electronics. An enhanced version of this satellite (the Microsat-100) is capable of payload masses up to  $40\text{kg}$  but the cooler and associated electronics would still approach 40% of this mass. What is more, the Aura satellite had a power budget of around  $220\text{W}$ , whereas these micro-satellites have a budget around  $20\text{W}$ . The SB160 Ball cooler typically uses  $65\text{W}$  ( $75\text{K}$  cold tip temperature for a  $2\text{W}$  heat load) far beyond what is available on a micro-satellite.

- **Cassini.** The CIRS instrument on the Cassini spacecraft has a total mass of around  $39\text{kg}$  of which only  $2.5\text{kg}$  was consumed by the cooling system (a passive radiator) and so a cooler with associated electronics massing  $14.6\text{kg}$  would be totally unsuitable alternative. The CIRS instrument had a power budget of around  $15\text{W}$ , much lower than what is needed by the HIRDLS coolers.
- **Clementine.** Tactical coolers (so called because they are typically developed for military applications) were used on the Clementine mission to cool infrared detectors. However, these coolers, although miniature, still consumed  $14\text{W}$  of power and had relatively short operational lifetimes (less than 900 hours) [Priest, R.E. et al. (1994)]. What is more these coolers were of a rotary design which requires more complex mechanical linkages between the motor and piston and can lead to increased rubbing between the piston bush and compression cylinder. Typically Oxford-developed coolers have used a compressor piston driven by a linear motor which avoids such problems. Linearly driven tactical coolers are available but have larger input powers than the rotary versions. What is more they are larger and/or are available only in a split, rather than integral geometry [Hymatic (2006)] and [Ricor (2006)]. Since tactical coolers are developed for military applications they may not be designed with the long-lifetime and reliability requirements of a space mission in mind and are therefore unsuitable for many remote sensing applications. A new miniature cooler developed specifically for remote sensing missions, with long-lifetime and reliability in mind from the outset could overcome this problem. It may also be possible with good design and a period of optimisation that a new cooler could consume less power than tactical coolers with similar cooling performance.

## 2.5 Summary

Although the above review is brief, it does illustrate the point that current mechanical coolers are, by-and-large, not suitably designed for use on the latest micro-satellites proposed for remote sensing missions or for deep space missions where power budgets are very stringent. Reviews of available cooling technologies strongly suggest that there is at present no suitable cooling system for use on

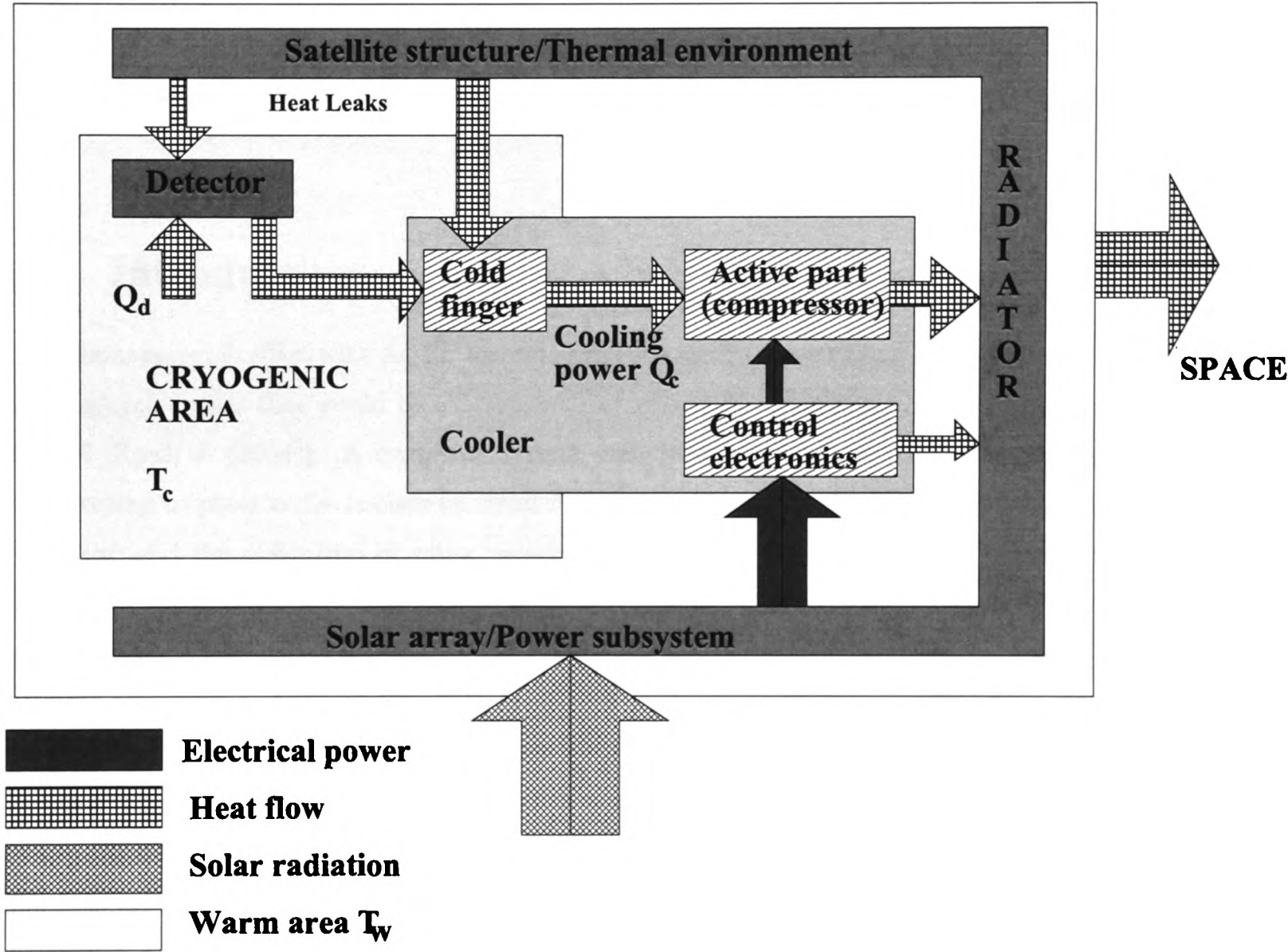


Fig. 2.3: Block diagram of satellite-borne cooler.

such missions. Collaudin and Rando's "Technology road-map for cryogenics in space" [Collaudin, B. and Rando, N. (2000)] identifies both high Coefficient Of Performance (c.o.p.), miniaturised active coolers and low vibration Stirling coolers as two of the key areas that need to be explored in order to make significant advances in the field of cryogenics for space applications. Since the sub-department of Atmospheric, Oceanic and Planetary Physics (AOPP) has a long heritage of designing and building Stirling coolers and considering the advantages and disadvantages of the various cooling technologies available, it was decided that the most suitable choice for a small remote sensing or deep space spacecraft would be a miniature Stirling cooler/radiator hybrid.

# Chapter 3

## Design preliminaries

### 3.1 Introduction

A previous research effort with AOPP identified the possibility of creating a hybrid radiator/miniature Stirling cycle cooler that would be able to produce a heat lift of 200mW at a cold tip temperature of 80K [Reed, J. (2004)]. A cryogenic system such as this would overcome the problems faced when trying to place active coolers on small remote sensing platforms (high power demands, large mass etc) and the difficulties in using radiators (significant parasitic losses at low temperatures, large size). The radiator would reduce the cooler's power demands by increasing its coefficient of performance (see subsection 3.1.1). Furthermore, the radiator would operate not at the final cold tip temperature but at the cooler warm end temperature so the parasitic heat loads to the radiator from a room temperature spacecraft would be reduced. A prototype Stirling cooler made by Reed [Reed, J. (2004)] showed potential in achieving this goal but suffered from design flaws that ultimately prevented the target performance from being achieved. The prototype designed by Reed re-used many components remaining from the ISAMS Stirling cooler development effort that had taken place in the department some years previously [Werrett S.T.R. (1991)] and whilst this had enabled the cooler to be built in a shorter time than would have otherwise been possible, it did mean the prototype design had to be created in such a way as to accommodate these components. Moreover, the linear motor used in the ISAMS cooler was also re-used in this prototype despite advances in magnet technology since their creation. Rather than repeat this attempt at re-using a previous cooler's components, a decision was made to re-design the cooler completely, which allowed much more freedom in design and also the opportunity to create a much more efficient linear motor. Before proceeding with a detailed description of the new prototype it is worth examining the thermodynamic cycle on which the cooler relies: The Stirling cycle.

### 3.1.1 The Stirling cycle

A Stirling machine operates on a closed thermodynamic regenerative cycle in which the same working fluid is repeatedly compressed and expanded at different temperatures. This results in a net conversion of heat to work or vice-versa. The Stirling cycle can be used in machines operating as cooling engines, prime movers, or heat pumps [Walker G. (1980)]. The first use of the Stirling cycle as a cooling engine was described by Alexander Kirk in 1874, although as early as 1834 John Herschel had conceived the closed cycle regenerative cooling engine [Walker G. (1983b)]. The original Stirling engine, however, pre-dates even this, being invented by the Reverend Robert Stirling in 1817.

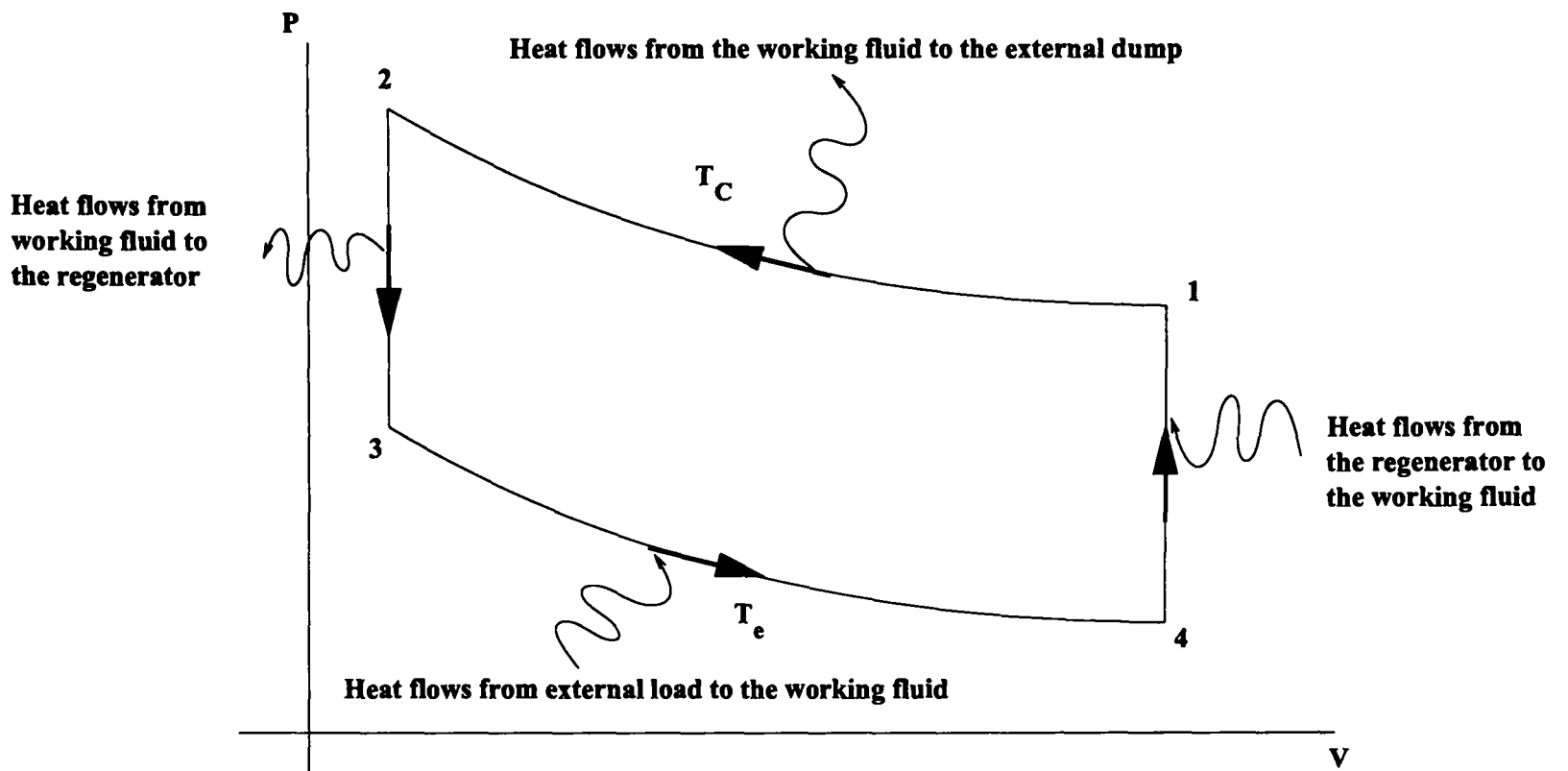
Although Stirling cycle machines can come in many different configurations depending upon their intended application, the underlying thermodynamic cycle remains the same. The Stirling cycle comprises two isothermal processes and two constant volume (isochoric) processes. The cycle involves a *regenerator* which absorbs heat during one part of the cycle and releases it during another part. The processes are described in turn below and shown in figure 3.1:

- 1-2 Isothermal compression at the warm temperature  $T_c$ . In this part of the cycle heat flows from the working fluid to an external heat dump.
- 2-3 Constant volume cooling. Heat flows from the working fluid to the regenerator as the fluid is transferred from the warm compression space to the cold expansion space.
- 3-4 Isothermal expansion at cold temperature  $T_e$ . Heat flows to the working fluid from an external load.
- 4-1 Constant volume heating. Heat flows to the working fluid from the regenerator. The same amount of heat flows to the working fluid from the regenerator as flowed from the working fluid to the regenerator in path 1-2.

The Stirling cycle is similar to the Carnot cycle, in that heat is exchanged with the surroundings at *constant temperatures* and so it has the same c.o.p. as the Carnot cycle. Examining equation 3.1 [Finn, C.B.P. (1986)] for the c.o.p. of a refrigerator operating on the Carnot or Stirling cycle between cold reservoir temperature  $T_e$  and warm reservoir temperature  $T_c$  it is immediately clear that the c.o.p. can be increased by decreasing the warm reservoir temperature for a fixed value of  $T_e$ .

$$\eta_C = \frac{T_e}{T_c - T_e} \quad (3.1)$$

Where  $\eta_C$  is the Carnot c.o.p.



**Fig. 3.1:** The PV diagram of the Ideal Stirling Cycle.

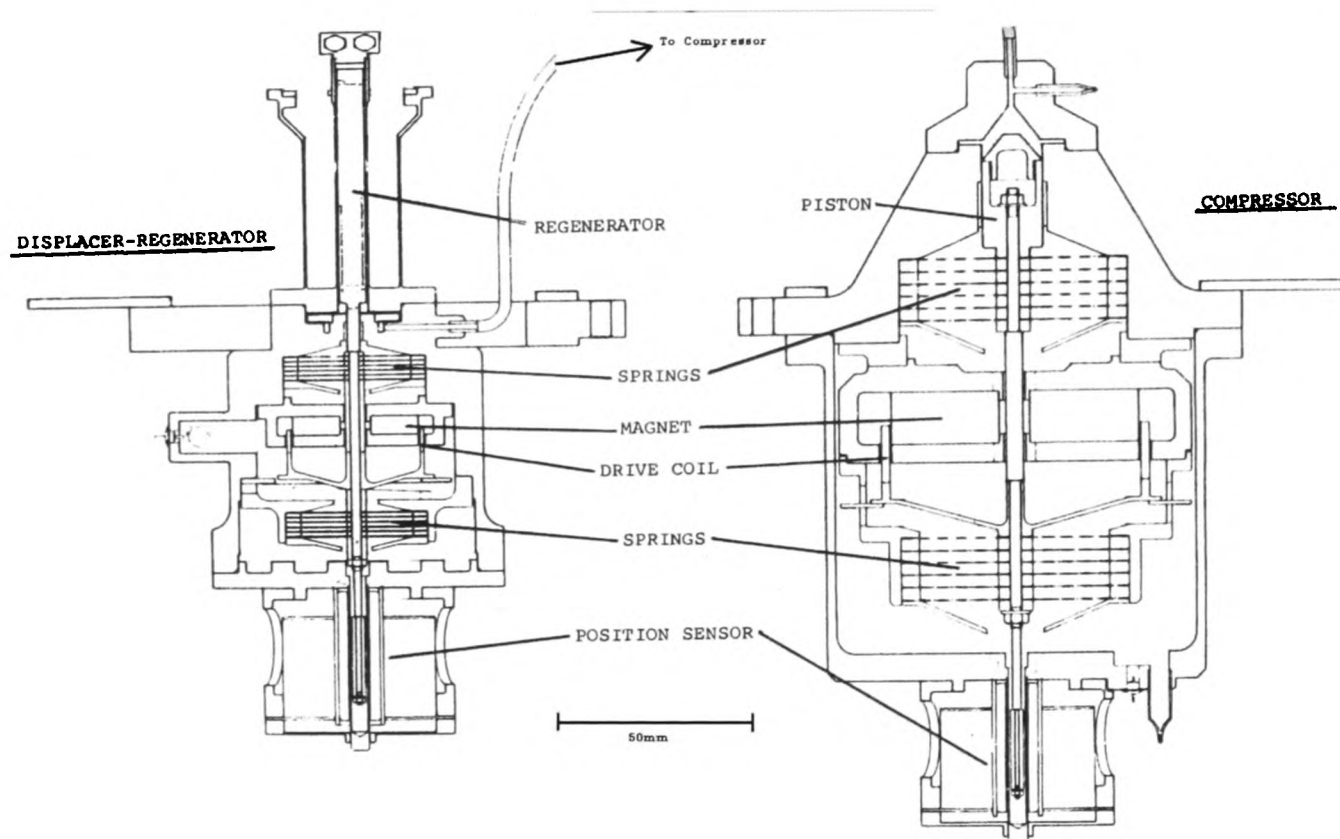
The hybrid design investigated by Reed [Reed, J. (2004)] suggested that using a radiator to reduce the cooler warm-end temperature could be a practical method of achieving this increase.

It cannot be stressed enough that the Stirling cycle described above is the ideal cycle and is never realised due to numerous practical problems that are encountered in constructing a working Stirling machine. For example, the ideal cycle above requires each process to be quasi-static whereas in reality Stirling machines run at frequencies in the range of tens of Hertz. What is more, there are mechanical losses such as friction to be considered. Some attempts have been made to take into account the finite time that the thermodynamical processes occur within ([Blank, D.A. (1999)] and [Erbay L.B, and Yavuz, H. (1998)]) but there is, as yet, not a consensus as to whether these attempts have been successful. Another importance difference is that in real machines the moving parts move with sinusoidal motion rather than the discontinuous motion that would be necessary for the sharp cornered ideal PV loop shown in figure 3.1. These loss processes mean that real refrigerators rarely approach the Carnot c.o.p. value. Section 4.2 gives a brief summary of the major losses in a miniature Stirling cooler.

### 3.1.2 Stirling cooler geometry

Before discussing the various possible forms a Stirling machine can take, it is useful to define the terms used to describe the various components used within them. To aid in this description figure 3.2 shows a schematic of the Stirling coolers used in the ISAMS instrument. Various components



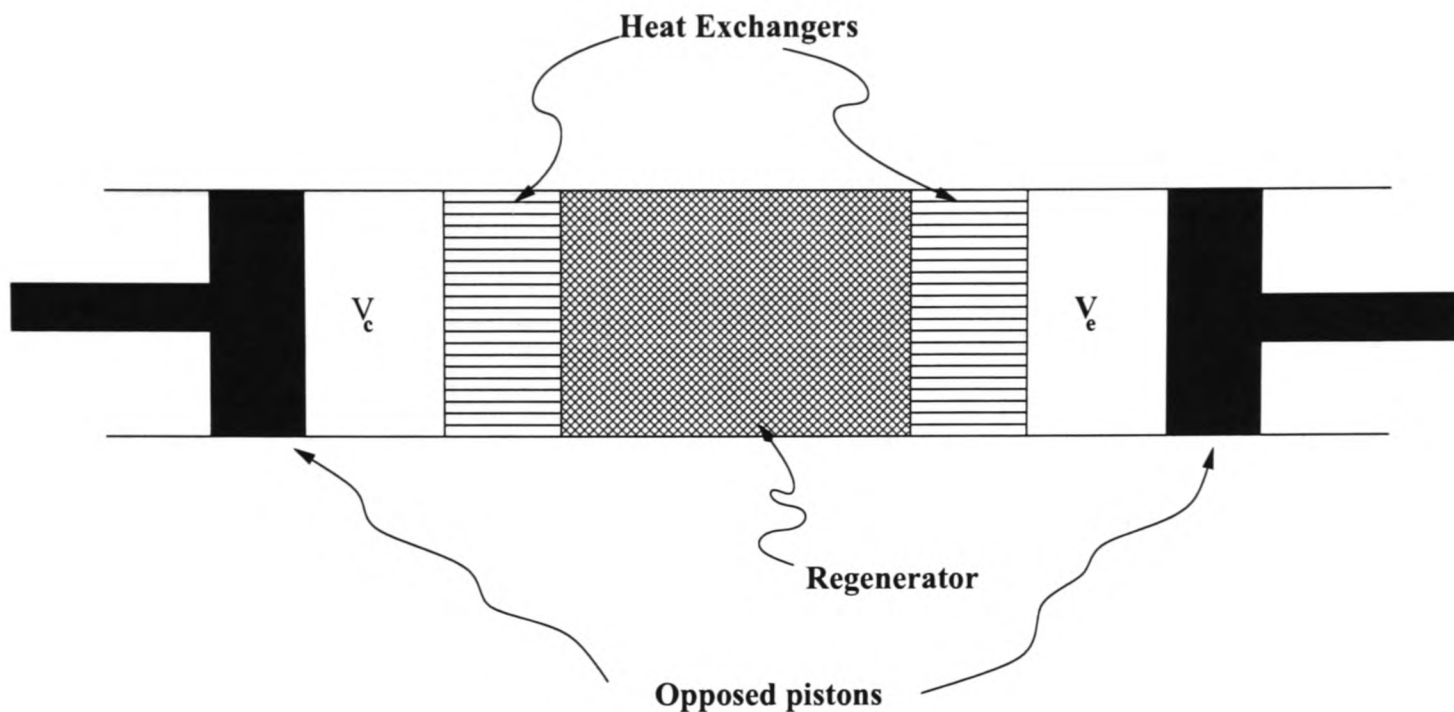


**Fig. 3.2:** Schematic diagram of the ISAMS coolers.

are indicated on the diagram and the following bulleted list will describe the key components.

- **Compressor and piston.** As detailed in subsection 3.1.1, the working fluid within a Stirling cooler undergoes compression, this is produced by a piston driven by a motor. As shown in figure 3.2 the piston can be driven by a linear motor but it may also be driven by a rotary motor. The compressor piston utilises flat spiral springs in a suspension system, such springs were also used as suspension for the displacer/regenerator in the ISAMS coolers.
- **Displacer.** Once the working fluid has been compressed it is displaced from the warm compression space, through a porous regenerator (described below) to the lower temperature expansion space, at a later point in the cycle the working fluid is then displaced back to the compressions space so the cycle can repeat. This displacement is produced by a second moving element within the cooler. In some designs of Stirling cooler the displacer is just a second piston but in many spaceborne coolers a displacive regenerator is used which combines the functionality of the displacer and regenerator into one element, the ISAMS cooler shown in figure 3.2 was of this type.
- **Regenerator.** As the working fluid is displaced from the warm compression space to the lower temperature expansion space it passes through the regenerator, this acts as a “heat





**Fig. 3.3:** Schematic of a simple “alpha” configuration Stirling machine.

sponge” absorbing heat in this part of the cycle and transferring back to the working fluid when it is displaced back from the compression to expansion space.

Stirling machines can take a number of forms. So called “alpha” machines have opposed pistons to vary the expansion and compression space volumes. Both pistons can be connected to the same crankshaft to recover the work done on the expansion piston, thus increasing the efficiency of the machine. So-called “Beta” type Stirling machines, in which a regenerative displacer and piston exist in a coaxial arrangement (this is in fact the configuration that Robert Stirling described in his patent application of 1816 [Organ, A.J. (1997)] ) are favoured over “alpha” type machines for space applications as they have a reduced number of parts and seals. Figure 3.3 is a schematic of a simple alpha configuration Stirling machine with a fixed regenerator. Although Stirling’s original patent was for a beta machine it had a rotary drive rather than the linear motors found in the common geometry of space coolers. Rotary drives are seldom used in space (a notable exception are the coolers on the Rosetta mission which have rotary drives) because the associated side forces lead to wear on the seals. In this configuration the displacer can either be driven by a linear motor or by pneumatic forces, in which case only one motor is needed for the machine. By only having one motor the mass and complexity of the cooler can be reduced. This suggests that a pneumatically driven displacer combined with a linear motor driven piston would be the basis for a small reliable cooler.

Depending on whether the piston and displacer/regenerator are housed within a single cylinder or separate cylinders the machine is termed “integral” or “split” respectively. By choosing an integral design the size of the cooler can be reduced although there are penalties: The cold tip

| Fluid          | $\sqrt{M^2 c^3}$ |
|----------------|------------------|
| Hydrogen       | 104              |
| Helium         | 44               |
| Air            | 29               |
| Carbon dioxide | 31               |
| Water vapour   | 44               |
| Nitrogen       | 30               |

**Table 3.1:** Comparative heat transfer for various working fluids. After Walker [Walker G. (1983b)].

will be close to the oscillating mass of the compressor piston and coil and so may suffer from larger vibrations than in a split design. These vibrations could be minimised with a well-designed balancing mechanism.

### 3.1.3 Choice of working fluid

Various working fluids may be used in Stirling machines [Walker G. (1980)], however space-borne coolers have traditionally used helium. The reason for one fluid being preferable to another is due to considerations of how the fluid properties effect heat transfer and fluid friction losses within the machine. At extremely low temperatures the heat capacity of the regenerator material compared to that of the working gas must be considered, but for temperatures down to 80K this is not necessary since the heat capacity of the phosphor bronze or stainless steel (typical regenerator materials) in the regenerator will be much greater than that of the helium at typical working pressures and will only become comparable at temperatures below 80K ([Qiu, L. et al. (2005)] and [Lemmon E.W. et al. (2005)] ). The importance of having an effective regenerator is illustrated in subsection 4.2.4. Walker devotes a chapter of one of his books to discussing the merits of various working fluids (and combinations of them) and derives a simple relationship between the heat transfer in the cooler and the thermophysical properties of the gas for a given set of conditions within a machine [Walker G. (1980)]:

$$Q_h \propto \sqrt{M^2 c^3} \quad (3.2)$$

Where  $Q_h$  is the heat transferred during any heat transfer processes within the machine,  $M$  is the molecular mass of the working fluid and  $c$  is its specific heat capacity. Table 3.1 shows the quantity  $\sqrt{M^2 c^3}$  (and thus their comparative heat transfer ability) for a range of working fluids [Walker G. (1980)]. Clearly hydrogen is the preferred working fluid but its extreme flammability in air means that it is usually excluded on safety grounds. Water vapour and helium are the next most favourable but water vapour cannot be used at cryogenic temperatures and so helium remains as

the working fluid of choice.

## 3.2 Summary

This chapter has described the Stirling cycle on which the proposed cooler will operate as well as possible forms Stirling machines can take. It has been shown that, when considering the ideal cycle, reducing the warm end reservoir temperature leads to an increase in the c.o.p. of the cooler for a given cold tip temperature. To reduce mass and complexity an integral design with a pneumatically driven displacer was chosen. The compressor piston will be driven by a linear motor rather than a rotary drive as this reduces seal wear and simplifies the drive mechanism. Considerations of thermophysical properties and safety aspects led to a choice of helium as the working fluid.

## Chapter 4

# A simple linear model

### 4.1 Introduction

As mentioned in the previous chapter, there are various loss mechanisms within Stirling machines that drive them away from the theoretical maximum c.o.p. The goal in designing a new miniature Stirling cooler was to reduce these loss processes as much as possible. This is a difficult task as many of the loss processes are not well understood and the complex gas flow within the cooler makes it very challenging to model ([ Sun, F. and Carrington, C.G. (1996) ] and [Organ, A.J. (1997)]). Since the prototype was being built from entirely new components it meant that the length of the cold finger, size of regenerator, diameter of piston etc could be determined completely by the designer so as to minimise losses. A simple linear model was developed to aid in this process. The initial sizing of the piston was based on results from a previous research effort conducted at AOPP [Reed, J. (2004)] which had involved an integral Stirling cooler. PV loops from this cooler showed that a compressor piston of 17.5mm diameter with a peak-to-peak stroke of 5mm could deliver in excess of 3W to the working gas at 7bar filling pressure with an operating frequency close to 32Hz. It is possible to achieve a c.o.p. above 10% of the Carnot value for a Stirling cooler ([Davey, G. and Orłowska, A.H. (1987)] and [Smedley, G.T. et al. (1995)]) thus an input power to the working gas of 3W results in a refrigeration power of 200mW (assuming a cold tip temperature of 80K and a compression space temperature of 200K in a cooler with 10% of the Carnot c.o.p.) and so a target of 3W input power to the working gas was chosen.

Since the new prototype was likely to be of a different design to that described by Reed [Reed, J. (2004)] the data available was of limited use. However, it did suggest that for a similar compression ratio, the piston diameter could be reduced and still achieve an input power to the gas of at least 3W. Furthermore the data indicated that if the frequency of operation could be increased, further

reductions in the piston size (and hence overall cooler size) might be possible. The simple linear model described here helped explore these possibilities.

The entire design of the mechanical cooler was performed by the author during the course of this research effort. The cooler was constructed entirely from titanium because of its high strength, low density and low thermal conductivity which reduces the flow of heat from the warm to cold end of the cold finger. Calculations were performed to ensure that thermal contraction would not cause moving components within the cooler to seize up when the cooler body was precooled by the radiator or when the cold tip temperature dropped. Further thermal analysis, of the various parasitic heat loads on the cold tip (shuttle heat loss, conduction through the regenerator), is described in the following chapter.

#### 4.1.1 A simple linear model

A simple linear model of the compressor used in the miniature Stirling cooler is possible and is described below. The compressor is modeled as a damped harmonic oscillator. The damping term takes into account the work done on the working fluid throughout a cycle. The compressor is driven by a linear motor. An alternating current passes through the coil which is sitting in the field of a permanent magnet, this results in a force perpendicular to the magnetic field and current directions. It is this force which causes the oscillatory motion of the compressor piston. The moving mass of the compressor is suspended on flat spiral flexures which provide the mechanical restoring force in the harmonic oscillator equation. Figure 4.1 shows simplified views of the linear motor from the side and from above. The model has been implemented in the Mathematica<sup>tm</sup> computer algebra package. The following two tables (Tables 4.1 to 4.4) explain the symbols and subscripts used in the governing equations. A number of groups of thermophysical quantities are defined within the text and so are omitted from the tables but all those symbols that are used repeatedly throughout the governing equations should be found in these tables. The areas of various components described as “face” areas are perpendicular to the direction of the fluid flow within the cooler (and hence they present a “face” to it).

| Symbol     | Meaning                                                                                 |
|------------|-----------------------------------------------------------------------------------------|
| $m_p$      | Mass of piston.                                                                         |
| $m_d$      | Mass of displacer.                                                                      |
| $b$        | Gas damping due to working gas on piston.                                               |
| $k_p$      | Mechanical spring constant of piston springs.                                           |
| $k_d$      | Mechanical spring constant of displacer springs.                                        |
| $B$        | Air gap flux density in magnet assembly.                                                |
| $l_b$      | Length of motor coil wire in motor air gap.                                             |
| $R_c$      | Motor coil resistance.                                                                  |
| $p_c$      | Gas pressure in compression space.                                                      |
| $p_b$      | Gas pressure in body of cooler.                                                         |
| $p_{mean}$ | Mean cycle gas pressure.                                                                |
| $A_p$      | Piston face area.                                                                       |
| $A_R$      | Face area of displacer rod.                                                             |
| $A_D$      | Face area of displacer.                                                                 |
| $A_t$      | Face area of regenerator sleeve.                                                        |
| $A_{reg}$  | Regenerator free-flow area.                                                             |
| $x_p$      | Piston displacement from mid-point of motion, +ve toward cold tip of cooler.            |
| $X_{p0}$   | Max. piston displacement.                                                               |
| $X_{d0}$   | Max. displacer/regenerator displacement.                                                |
| $\phi$     | Phase angle between piston and displacer motion.                                        |
| $x_d$      | Displacer/regenerator displacement from mid-point of motion, toward cold tip of cooler. |
| $\omega$   | Angular frequency of piston oscillation.                                                |
| $S$        | Linear position coefficient(s) defined in text.                                         |
| $D$        | Damping coefficient defined in text.                                                    |
| $\Delta p$ | Pressure drop along regenerator.                                                        |
| $C$        | Pressure position coefficient(s) defined in text.                                       |
| $V_c$      | Compression space volume.                                                               |
| $V_b$      | Buffer space volume.                                                                    |
| $V_r$      | Regenerator void volume.                                                                |
| $n$        | Number of mols of gas.                                                                  |
| $R$        | Universal gas constant.                                                                 |
| $L$        | Length of regenerator.                                                                  |

Table 4.1: Symbols used in the following analysis.

| Symbol         | Meaning                                                                             |
|----------------|-------------------------------------------------------------------------------------|
| $d_h$          | Hydraulic diameter of regenerator mesh.                                             |
| $r_h$          | Hydraulic radius of regenerator mesh.                                               |
| $\beta$        | Heat transfer coefficient (Klein's analysis).                                       |
| $\lambda$      | Heat conductivity in regenerator.                                                   |
| $G_z$          | Mass flux through regenerator.                                                      |
| $\mu$          | Fluid viscosity.                                                                    |
| $a_v$          | Regenerator heat exchange area.                                                     |
| $T_e$          | Expansion space temperature.                                                        |
| $T_c$          | Compression space temperature.                                                      |
| $E_{ed}$       | Max. allowable forward displacer displacement (toward cold tip).                    |
| $E_{cp}$       | Distance between max. forward piston position and displacer when at mid point.      |
| $E_{cd}$       | Max. allowable displacer/regenerator backwards displacement (toward warm end).      |
| $\rho$         | Density of helium at mean cycle temperature.                                        |
| $f_1$          | Friction factor for flow through the regenerator.                                   |
| $f_{st}$       | Friction factor for steady flow through regenerator.                                |
| $f_r$          | Regenerator friction factor in the linear approximation.                            |
| $Re$           | Reynolds number.                                                                    |
| $u$            | Fluid velocity.                                                                     |
| $\bar{u}_{rr}$ | Mean fluid flow velocity through regenerator relative to regenerator velocity.      |
| $U_{rr}$       | Peak fluid flow velocity through regenerator relative to regenerator velocity.      |
| $Pr$           | Prandtl number.                                                                     |
| $\alpha$       | Thermal diffusivity.                                                                |
| $D$            | Displacer/regenerator diameter.                                                     |
| $\zeta$        | Dimensionless length.                                                               |
| $T_s$          | Temperature of solid in regenerator matrix.                                         |
| $T_g$          | Temperature of gas in regenerator matrix.                                           |
| $\delta$       | Gap between displacer/regenerator and cold tube.                                    |
| $c$            | Specific heat capacity.                                                             |
| $k$            | Thermal conductivity.                                                               |
| $l$            | Length.                                                                             |
| $z$            | Direction along the axis of the regenerator, +ve toward the cold tip of the cooler. |
| $v$            | Velocity.                                                                           |

Table 4.2: Symbols used in the following analysis-continued.



| Symbol         | Meaning                                          |
|----------------|--------------------------------------------------|
| $h$            | Heat transfer coefficient (De Monte's analysis). |
| $W$            | Regenerator wetted perimeter.                    |
| $\lambda_{sl}$ | Sliding friction damping factor.                 |
| $\lambda_g$    | Gas damping friction factor.                     |
| $\lambda_d$    | Total damping factor.                            |
| $\chi$         | Piston/drive voltage phase angle.                |

Table 4.3: Symbols used in the following analysis-continued.

| Symbol | Meaning                                                                                       |
|--------|-----------------------------------------------------------------------------------------------|
| $pg$   | Meaning that the value of the subscripted symbol relates to the gas at constant pressure.     |
| $g$    | Gas.                                                                                          |
| $s$    | Solid.                                                                                        |
| $ch$   | Cold tip.                                                                                     |
| $dis.$ | Displacer.                                                                                    |
| $g1$   | Gas during half period in which there is inlet of warm gas into the regenerator.              |
| $g2$   | Gas during half period in which there is inlet of cold gas into the regenerator.              |
| $SH$   | Shuttle heat loss.                                                                            |
| $rt$   | Regenerator tube, (Vespel tube containing wire mesh).                                         |
| $0$    | Entire cycle                                                                                  |
| $b$    | Blow period (gas forced through regenerator toward cold tip).                                 |
| $r$    | Reverse part of cycle (gas moves from cold tip to warm end).                                  |
| $f$    | Fluid.                                                                                        |
| $w$    | Regenerator matrix.                                                                           |
| $pp$   | Meaning that the subscripted symbol is a coefficient related only to the piston.              |
| $dd$   | Meaning that the subscripted symbol is a coefficient related only to the displacer.           |
| $pd$   | Meaning that the subscripted symbol is a coefficient related to the piston and the displacer. |

Table 4.4: Subscripts used in the following analysis.



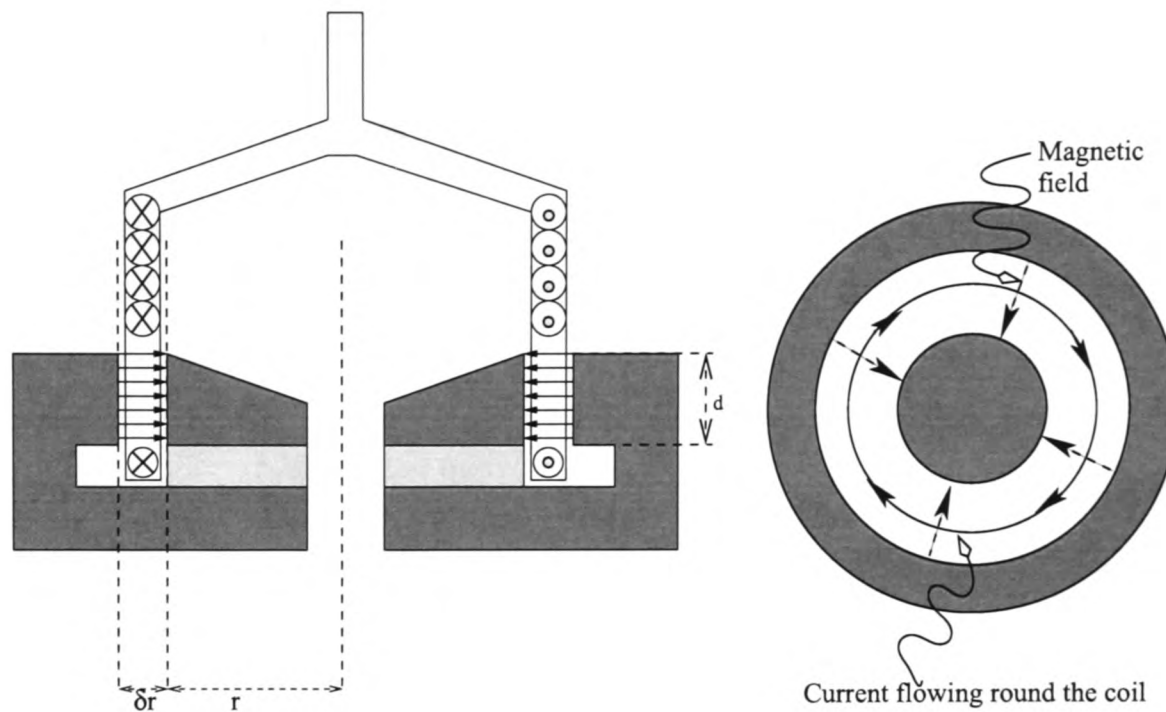


Fig. 4.1: Simplified diagram of the linear motor.

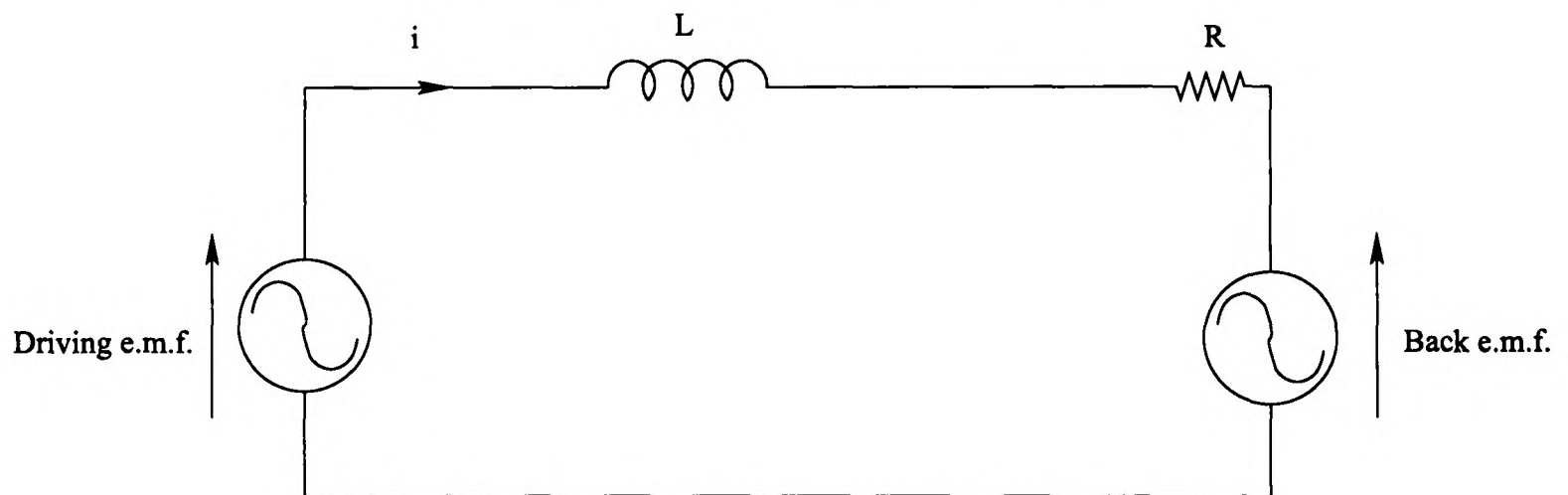


Fig. 4.2: Circuit diagram for the linear motor.

### Initial Equations

The first step in constructing the model is to consider the back e.m.f. generated as the coil moves in the magnetic field. The driving e.m.f. and back e.m.f. are represented by complex exponentials. Figure 4.2 shows the current and e.m.f. directions. For the circuit containing the linear motor:

$$\epsilon_v - \epsilon_{back} = i(R + j\omega L) \quad (4.1)$$

If  $L$  can be shown to be negligible for the size of coil being considered then:

$$\epsilon_v - \epsilon_{back} = iR \quad (4.2)$$

It is a simple matter to calculate  $\epsilon_{back}$  using Faraday-Henry-Lenz's law:

$$\epsilon_{back} = \dot{x}Bl_b \quad (4.3)$$

So equation 4 can be written as:

$$\epsilon_v - \dot{x}Bl_b = iR \quad (4.4)$$

So we can write an expression for the current in the circuit:

$$i = \frac{1}{R}(\epsilon_{v0} \exp j\omega t - \dot{x}Bl_b) \quad (4.5)$$

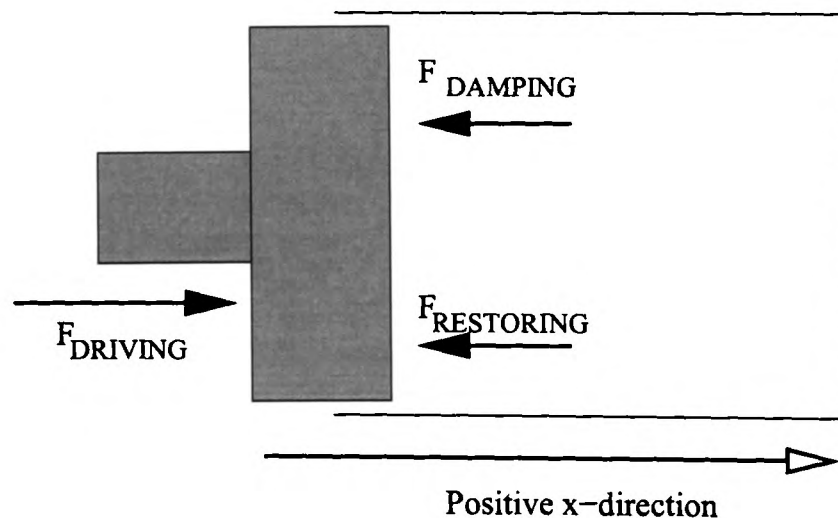
It is now only necessary to find an expression for the force on the coil due to the magnetic field before constructing an equation for the forced damped harmonic oscillator. The force on the coil,  $F$ , is given by:

$$F = Bil_b \quad (4.6)$$

We can eliminate  $i$  by writing:

$$F = Bl_b \frac{(\epsilon_v - \dot{x}Bl_b)}{R} \quad (4.7)$$

This equation represents the driving force in the harmonic oscillator equation. We now consider the other forces in the model. The forces on the moving mass (piston) in the case of the linear



**Fig. 4.3:** Simple diagram showing the forces acting on the piston as it travels in the positive  $x$  direction.

motor are defined as shown in Figure 4.3. At first we assume there is no gas being compressed and the only damping arises from friction between the piston and the cylinder walls. The damping force is assumed to be proportional to the velocity of the piston ( $F_{Damping} = -\lambda_{sl}\dot{x}_p$  where  $\lambda_{sl}$  is the sliding friction damping coefficient) and the restoring force is assumed to be proportional to the displacement of the piston ( $F_{Restoring} = -kx$ ). The equation of motion is:

$$m_p\ddot{x}_p = F_{Restoring} + F_{Damping} + F_{Driving} \quad (4.8)$$

Which, if we use our expression for the force on the coil derived previously, can be written as:

$$m_p \ddot{x}_p + \lambda_{sl} \dot{x}_p + k_p x_p = Bl_b \frac{(\epsilon_v - \dot{x}_p Bl_b)}{R_c} \quad (4.9)$$

If we now consider the situation when there is gas in the cylinder there is a term missing from this equation; the force due to the pressure difference across the piston. For now this is written in terms of the pressures in front of the piston ( $P_c$ ), the pressure behind the piston ( $P_b$ ) and the area of the piston face ( $A_p$ ) but this will eventually be written in terms of the piston and displacer positions and velocities.

$$m_p \ddot{x}_p + k_p x_p + (p_c - p_b) A_p + \lambda_{sl} \dot{x}_p = Bl_b \frac{(\epsilon_v - \dot{x}_p Bl_b)}{R_c} \quad (4.10)$$

$$m_p \ddot{x}_p + k_p x_p + (p_c - p_b) A_p + \left(\lambda_{sl} + \frac{B^2 l_b^2}{R_c}\right) \dot{x}_p = Bl_b \frac{\epsilon_{v0} e^{j\omega t}}{R_c} \quad (4.11)$$

$$\ddot{x}_p = -\frac{k_p}{m_p} x_p - \frac{(p_c - p_b) A_p}{m_p} - \frac{\left(\lambda_{sl} + \frac{B^2 l_b^2}{R_c}\right)}{m_p} \dot{x}_p + Bl_b \frac{\epsilon_{v0} e^{j\omega t}}{R_c m_p} \quad (4.12)$$

Assuming that the compression space pressure is a linear function of the piston and displacer positions only, and that the buffer space pressure is constant (which is approximately true since the volume of the body of the cooler behind the piston is much larger than the compression space) we now state:

$$\frac{-k_p}{m_p} x_p - \frac{(p_c - p_b) A_p}{m_p} = s_{pp} x_p + s_{pd} x_d \quad (4.13)$$

$$s_{pp} x_p + s_{pd} x_d = s_{pp} x_p + \Psi s_{pd} x_p e^{j\phi} \quad (4.14)$$

Wherein the following substitution has been made:

$$x_d = \psi x_p e^{j\phi} \quad (4.15)$$

In which  $\phi$  is the piston/displacer phase angle and  $\Psi$  is the ratio of the displacer amplitude to the piston amplitude:

$$\Psi = \frac{X_{d0}}{X_{p0}} \quad (4.16)$$

We then substitute this into equation 4.12 to eliminate any terms with  $x_d$ :

$$\ddot{x}_p = s_{pp} x_p + \Psi s_{pd} x_p e^{j\phi} - \frac{1}{m_p} \left(\lambda_{sl} + \frac{B^2 l_b^2}{R_c}\right) \dot{x}_p + \frac{Bl_b \epsilon_{v0} e^{j\omega t}}{m_p R_c} \quad (4.17)$$

Which has the steady state solution:

$$x_p = \frac{Bl_b \epsilon_{v0} e^{j\omega t}}{m_p R_c} \frac{1}{\left[-\omega^2 + \frac{j\omega}{m_p} \left(\lambda_{sl} + \frac{B^2 l_b^2}{R_c}\right) - s_{pp} - \Psi s_{pd} e^{j\phi}\right]} \quad (4.18)$$

When  $-\omega^2 - s_{pp} + \mathbf{Re}[\Psi s_{pd}e^{j\phi}] = 0$  we find;

$$x_{pRes} = \frac{Bl_b\epsilon_{v0}e^{j\omega t}}{R_c} \frac{1}{j\omega(\mathbf{Im}[-\Psi s_{pd}e^{j\phi}] + \lambda_{sl} + \frac{B^2l_b^2}{R_c})} \quad (4.19)$$

i.e. that the driving force and displacement are  $\pi/2$  out of phase, the well known result for a harmonic oscillator at resonance. Now  $\mathbf{Im}(-\Psi s_{pd}e^{j\phi}) = -\Psi s_{pd}\sin[\phi]$  and thus:

$$x_{pRes} = \frac{Bl_b\epsilon_{v0}e^{j\omega t}}{R_c} \frac{1}{j\omega(\lambda_{sl} + \frac{B^2l_b^2}{R_c} - \frac{m_p\Psi s_{pd}\sin(\phi)}{\omega})} \quad (4.20)$$

Where  $x_{pRes}$  is the displacement of the piston at resonance. The piston velocity at resonance is given by:

$$\dot{x}_{pRes} = \frac{Bl_b\epsilon_{v0}e^{j\omega t}}{R_c} \frac{1}{(\lambda_{sl} + \frac{B^2l_b^2}{R_c} - \frac{m_p\Psi s_{pd}\sin\phi}{\omega})} \quad (4.21)$$

The term  $\frac{m_p\Psi s_{pd}\sin\phi}{\omega}$  represents the gas damping on the piston and we denote this  $\lambda_g$ .

$$x_{pRes} = \frac{Bl_b\epsilon_{v0}e^{j\omega t}}{R_c} \frac{1}{j\omega(\lambda_{sl} + \frac{B^2l_b^2}{R_c} + \lambda_g)} \quad (4.22)$$

$$\dot{x}_{pRes} = \frac{Bl_b\epsilon_{v0}e^{j\omega t}}{R_c} \frac{1}{(\lambda_{sl} + \frac{B^2l_b^2}{R_c} + \lambda_g)} \quad (4.23)$$

The terms  $[-s_{pp} + \mathbf{Re}(\Psi s_{pd}e^{j\phi})] x_p$  in equation 4.17 represent the restoring force due to the mechanical springs and the component of the gas spring force that is in phase with the mechanical spring restoring force. Thus the resonant frequency,  $\omega_0$  is given by  $\sqrt{-s_{pp} + \mathbf{Re}(\Psi s_{pd}e^{j\phi})}$ . An expression for the gas spring is derived later in this section (see equation 4.60 onward). It is clear to see from equation 4.13 that if there is no gas in the system then  $\omega_0^2 = -s_{pp}$  and it can be seen immediately that this leads to  $\omega_0 = \sqrt{\frac{k}{m}}$  which is the well known result for an undamped harmonic oscillator [Kleppner, D. Kolenkow, R.J. (1978)]. If we collect the sliding friction term and the gas damping term together into a “total external damping term” (external in the sense that it is damping that occurs due to processes external to the linear motor unlike the damping term  $\frac{B^2l_b^2}{R_c}$ ) we obtain:

$$x_p = \frac{Bl_b\epsilon_{v0}e^{j\omega t}}{m_p R_c} \frac{1}{\left[(\omega_0^2 - \omega^2) + j\omega(\lambda_d + \frac{B^2l_b^2}{R_c})\right]} \quad (4.24)$$

Where:

$$\lambda_d = \lambda_g + \lambda_{sl} \quad (4.25)$$

With these expressions we can now write a simple expression for the piston displacement and velocity:

$$x_p = X_{p0}e^{j(\omega t + \chi)} \quad (4.26)$$

$$\dot{x}_p = j\omega X_{p0} e^{j(\omega t + \chi)} \quad (4.27)$$

Where:

$$X_{p0} = \frac{Bl_b \epsilon_{v0}}{m_p R_c} \frac{1}{\left[ (\omega_0^2 - \omega^2)^2 + \omega^2 \left( \frac{B^2 l_b^2}{R_c} + \lambda_d \right)^2 \right]^{\frac{1}{2}}} \quad (4.28)$$

and:

$$\chi = \text{Arctan} \left( \frac{-\omega \left( \lambda_d + \frac{B^2 l_b^2}{R_c} \right)}{\omega_0^2 - \omega^2} \right) \quad (4.29)$$

Where  $\chi$  is the phase angle between the piston's motion and the driving voltage. Recalling equation 4.5:

$$i = \frac{1}{R} (\epsilon_{v0} \exp j\omega t - \dot{x} Bl_b) \quad (4.30)$$

Substituting equation 4.27 into equation 4.30 we find:

$$i = \frac{\epsilon_{v0}}{R_c} \left[ 1 - \frac{j\omega X_{p0} e^{j(\omega t + \chi)}}{\epsilon_{v0}} \right] e^{j\omega t} \quad (4.31)$$

From this we can derive:

$$\frac{i}{\epsilon_{v0}} = \frac{1}{R_c} \sqrt{1 + 2 \frac{\omega^2 X_{p0}^2 B^2 l_b^2}{\epsilon_{v0}} + \frac{2\omega X_{p0} Bl_b}{\epsilon_{v0}} \sin \chi} \quad (4.32)$$

From this equation it is clear that when  $\chi = -\pi/2$  the quantity  $i/v$  will be minimised and from equation 4.29 we see that this occurs when  $\omega^2 = \omega_0^2$ . At this point the resistive losses in the compressor motor will be minimised and so one may consider this point as the “power resonance”. From equation 4.28 it is clear that this does not correspond to maximising the piston stroke (which occurs at the amplitude resonance). The term “resonance” in this thesis refers to the frequency at which the resistive losses are minimised unless otherwise stated.

One aspect of the cooler performance that has not been examined yet is the motor power output. The force produced by the motor is given by:

$$F = Bl_b \frac{\epsilon_{v0} - \dot{x} Bl_b}{R} \quad (4.33)$$

Which can be expanded to:

$$F = \frac{Bl_b \epsilon_{v0}}{R} - \frac{\dot{x} B^2 l_b^2}{R} \quad (4.34)$$

We can now write an expression for the motor output power as:

$$Power_{motor} = \frac{1}{t_p} \int_0^{t_p} \left( \frac{Bl_b \epsilon_{v0}}{R} - \frac{\dot{x} B^2 l_b^2}{R} \right) \dot{x} \quad (4.35)$$

Since:

$$Power_{motor} = \frac{1}{t_p} \int_0^{t_p} F \dot{x} \quad (4.36)$$

We now progress to the displacer harmonic oscillator equation:

$$m_d \ddot{x}_d = -k_d x_d - (p_c - p_b) A_R - \Delta p A_{reg} \quad (4.37)$$

We assume that the pressure drop across the regenerator is proportional to the difference between the fluid velocity in the displacer/regenerator and the displacer/regenerator's velocity itself (in other words the fluid velocity relative to the displacer/regenerator's velocity). We then take this "relative" fluid velocity to be a linear function of the velocities of the displacer/regenerator and piston. We can then make the substitution as follows:

$$-\Delta p A_{reg} = C_p \dot{x}_p + C_d \dot{x}_d \quad (4.38)$$

We can then re-write equation 4.37 as:

$$\ddot{x}_d = -\frac{k_d x_d}{m_d} - \frac{(p_c - p_b) A_R}{m_d} + \frac{C_p \dot{x}_p}{m_d} + \frac{C_d \dot{x}_d}{m_d} \quad (4.39)$$

In a similar way to the substitution we made for the piston equation (equation 4.13) we assume the following to be true:

$$-\frac{p_c - p_b}{m_d} - \frac{k_d}{m_d} x_d = S_{dp} x_p + S_{dd} x_d \quad (4.40)$$

So we can re-write 4.37:

$$\ddot{x}_d = S_{dp} x_p + S_{dd} x_d + D_{dp} \dot{x}_p + D_{dd} \dot{x}_d \quad (4.41)$$

Where the S's and D's are linear position coefficients and damping coefficients respectively, with the damping coefficients being given by:

$$D_{dp} = \frac{C_p}{m_d} \quad D_{dd} = \frac{C_d}{m_d} \quad (4.42)$$

Where  $C_p$  and  $C_d$  will be derived later. We now re-write equation 4.41 in terms of the displacer motion only:

$$\ddot{x}_d = s_{dp} \frac{x_d}{\Psi} e^{-j\phi} + s_{dd} x_d + \frac{D_{dp} \dot{x}_d}{\Psi} e^{-j\phi} + D_{dd} \dot{x}_d \quad (4.43)$$

Since (from equation 4.15):

$$x_p = \frac{x_d}{\Psi} e^{-j\phi} \quad (4.44)$$

Where  $\phi$  is the piston displacer phase angle. Re-arranging equation 4.43 and canceling the common term  $x_d$  we find:

$$(-\omega^2 - s_{dd} - D_{dd} j\omega) = \left[ \frac{s_{dp}}{\Psi} - \frac{2D_{dp} \omega j}{\Psi} \right] e^{-j\phi} \quad (4.45)$$

From this we can find expressions for the displacer/piston stroke ratio  $\Psi$ :

$$\Psi = \frac{X_{d0}}{X_{p0}} = \frac{(s_{dp} - jD_{dp}\omega)}{-\omega^2 - s_{dd} - D_{dd}j\omega} \quad (4.46)$$

For the piston/displacer phase angle:

$$\tan(\phi) = \frac{\omega [D_{dp}(\omega^2 + s_{dd}) + s_{dp}D_{dd}]}{(D_{dd}D_{dp})\omega^2 - s_{dd}s_{dp}} \quad (4.47)$$

### 4.1.2 Linear and damping coefficients

We can now begin to derive expressions for the linear coefficients (the S's) and the damping coefficients (D's). The first step in doing this is to derive an expression for the compression space pressure  $p_c$ . We start by stating that:

$$\frac{pV}{T} = \text{constant} \quad (4.48)$$

The volume occupied by the working gas consists of compression space, expansion space and regenerator void volume, each of which has a characteristic temperature and are considered to be isothermal, so we write:

$$p = \frac{nR}{\left[ \frac{V_{comp.}}{T_c} + \frac{V_r \text{Log}(T_e/T_c)}{T_e - T_c} + \frac{V_e}{T_e} \right]} \quad (4.49)$$

We can write the volume of the compression space in terms of the piston position, displacer position and their areas:

$$V_{comp.} = (E_{cd} + x_{cd})(A_d - A_R) + (E_{cp} - x_p)(A_p - A_R) \quad (4.50)$$

The regenerator void volume is given by:

$$V_r = (A_d - A_t)L\epsilon = A_{reg}L\epsilon \quad (4.51)$$

Where  $A_t$  is the area of the regenerator sleeve, and the expansion space volume is given by:

$$V_e = (E_{ed} - x_d)A_D \quad (4.52)$$

Substituting into equation 4.49 we find:

$$p = nR \left[ \frac{(E_{cd} + x_{cd})(A_d - A_R) + (E_{cp} - x_p)(A_p - A_R)}{T_c} + \frac{(A_{reg})L\epsilon \text{Log}(T_e/T_c)}{T_e - T_c} + \frac{(E_{ed} - x_d)A_D}{T_e} \right]^{-1} \quad (4.53)$$

We now expand this using the binomial expansion and neglect the second order terms in the displacer and piston displacements to arrive at:

$$p = \frac{nR}{q} \left( 1 - \left[ \frac{x_d(A_d - A_R) - x_p(A_p - A_R)}{qT_c} - \frac{x_d A_D}{qT_e} \right] \right) \quad (4.54)$$

Where:

$$q = \frac{E_{cd}(A_d - A_R) + E_{cp}(A_p - A_R)}{T_c} + \frac{V_r \log(T_e/T_c)}{T_e - T_c} + \frac{E_{ed}A_D}{T_e} \quad (4.55)$$



Which simplifies to:

$$p = p_{mean} \left( 1 - \frac{x_d(A_d - A_R) - x_p(A_p - A_R)}{qT_c} + \frac{x_d A_D}{qT_e} \right) \quad (4.56)$$

Neglecting the second order and higher terms in the binomial expansion to produce equation 4.54 does introduce errors but it can be easily verified that the second order term is only about 5% the size of the first order term and higher order terms are even smaller. Since this analysis neglects pressure gradients, we follow the example of Urieli and Berchowitz and take equation 4.56 to represent the spatial average of pressure at any instant [Urieli, I. Berchowitz, D.M. (1998)]. In fact, equation 4.56 is probably a more accurate representation of some intermediate pressure between the compression space and expansion space pressures. It is not a simple matter to derive an expression for the exact compression space pressure since the gas is being compressed and also flowing through the porous media of the displacer. Urieli and Berchowitz make the assumption that:

$$p = p_c. \quad (4.57)$$

In this analysis the same assumption will be made and the pressure gradient along the regenerator treated separately (equation 4.68 onward). Now we substitute equation 4.56 into equation 4.13 and compare coefficients of  $x_p$  and  $x_d$  to find expressions for  $S_{pp}$  and  $S_{pd}$ :

$$S_{pp} = \frac{-p_{mean}(A_p - A_R)^2}{qT_c M_p} - \frac{k_p}{m_p} \quad (4.58)$$

$$S_{pd} = \frac{p_{mean}(A_p - A_R)}{qM_p} \left[ \frac{(A_D - A_R)}{T_c} - \frac{A_D}{T_e} \right] \quad (4.59)$$

Recalling that at resonance:

$$-\omega^2 - s_{pp} - \text{Re}[-\Psi s_{pd} e^{j\phi}] = 0 \quad (4.60)$$

We can now substitute equations 4.59 and 4.58 into equation 4.60:

$$\omega^2 = \frac{k_p}{m_p} + \frac{(A_p - A_R)^2 p_{mean}}{qT_c m_p} - \frac{\Psi P_{mean}(A_p - A_R)}{q m_p} \left[ \frac{A_D - A_R}{T_c} - \frac{A_D}{T_e} \right] \cos\phi \quad (4.61)$$

It can be shown that for typical conditions the third term in the right hand side of equation 4.61 is less than 5 % of the other two terms and so we neglect it to leave:

$$\omega^2 = \frac{k_p}{m_p} + \frac{(A_p - A_R)^2 P_{mean}}{qT_c m_p} \quad (4.62)$$

Thus we have an expression for the resonant frequency in terms of the compressor mechanical and gas springs:

$$\omega = \sqrt{\left[ \frac{k_p}{m_p} + \frac{(A_p - A_R)^2 P_{mean}}{qT_c m_p} \right]} \quad (4.63)$$



Similar expressions for the gas spring in Stirling machines have been derived elsewhere [Reed, J. (2004)] and include a factor  $\alpha$ . This factor arises because the compression is unlikely purely isothermal (which has been assumed in this derivation). If we do not make this assumption and allow the pressure volume relationship to be  $p = \frac{c}{V^\alpha}$  we find the following expression:

$$\omega = \sqrt{\left[ \frac{k_p}{m_p} + \frac{\alpha(A_p - A_R)^2 P_{mean}}{qT_c m_p} \right]} \quad (4.64)$$

We now consider the remaining coefficients ( $s_{dp}$ ,  $s_{dd}$ ). We substitute equation 4.56 into equation 4.40 and compare coefficients to arrive at:

$$S_{dp} = \frac{-P_{mean}(A_p - A_R)A_R}{qT_c M_d} \quad (4.65)$$

$$S_{dd} = \frac{p_{mean}A_R}{qM_d} \left[ \frac{(A_D - A_R)}{T_c} - \frac{A_D}{T_e} \right] - \frac{k_d}{M_d} \quad (4.66)$$

We now derive expressions for  $C_p$  and  $C_d$ , our starting point being equation 4.38;  $-\Delta p A_{reg} = C_p \dot{x}_p + C_d \dot{x}_d$ . The pressure drop due to a fluid flowing through a regenerator made of wire mesh screens is given by Ju et al. [Ju, Y. et al. (1998)]:

$$\Delta p_{max} = \frac{f_{max} \rho u_{max}^2 L}{d_h} \quad (4.67)$$

Where  $L$  is the total length of the stack of meshes and  $f_{max}$  is the maximum pressure drop factor,  $u_{max}$  is the maximum cross-sectional mean flow velocity in the regenerator and  $d_h$  is the hydraulic diameter of the meshes. So for the pressure drop across the regenerator in the prototype cooler we write:

$$\Delta p = \frac{\rho f_1 u_{rr} |u_{rr}| L}{2d_h} \quad (4.68)$$

Where  $\Delta p$  is the instantaneous pressure drop along the regenerator for a fluid velocity relative to the regenerator  $u_{rr}$ .

In equation 4.68  $f_1$  is the pressure drop friction factor. Data on friction factors for steady uni-directional flows through wire meshes are readily available [Kays W.M. and London A.L. (1964)] but information on such factors for the conditions found in Stirling coolers is harder to come by, and although there are a number of papers detailing experimental work on oscillating flows through regenerator meshes ([Ju, Y. et al. (1998)], [Ju, Y. (2003)], [Cheng, P. and Zhao, T.S. (1998)], [Nam, K. and Jeong, S. (2005)] for example) there is not a definitive treatment of them. Ju et al. [Ju, Y. et al. (1998)] conducted an experimental study of the oscillating flow for a regenerator within a pulse tube cooler. It was found that the pressure drops for an oscillating flow was two to three times the size of that for steady flow at the same Reynolds number (based on the inter-wire distance of the meshes). Zhao and Cheng [Zhao, T.S. and Cheng P. (1996)] found that the pressure drops were between 4.5 – 5.8 times the steady flow pressure drops at the same Reynolds number

based on the inter-wire distance of the wire meshes. Urieli and Berchowitz [Urieli, I. Berchowitz, D.M. (1998)] describe a method of deriving expressions that can be used to produce estimates of the pressure drop across the regenerator in a Stirling engine and it is this method that is described below. The first step is to define the friction factor in terms of the Reynolds number for steady flows through a stack of woven screens ([Miyabe, H. et al. (1982)] in [Ju, Y. et al. (1998)]):

$$f_{st} = \frac{33.6}{Re} + 0.337 \quad (4.69)$$

Where  $Re$  is the Reynolds number based on the distance between wires in the screen.

Equation 4.68 is non-linear and so we must find a suitable linear expression for the pressure drop before we can proceed. The pressure drop is a damping loss on the motion of the displacer and therefore energy is being dissipated. As long as we ensure that the linear approximation dissipates the same amount of energy in one quarter cycle as does the non-linear expression then we can be confident that the approximation is valid. Assuming a linear damper type relationship we can state:

$$\Delta p = f_r \bar{u}_{rr} \quad (4.70)$$

We assume that the fluid velocity varies sinusoidally with maximum velocity  $U_{rr}$ , we then integrate equations 4.68 and 4.70 between 0 and  $\pi/2$  to find the energy dissipated per unit flow area. For equation 4.68 we find:

$$E_1 = \frac{L\rho f_1 U_{rr}^3}{3\omega d_h} \quad (4.71)$$

For the linear damper:

$$E_2 = \frac{\pi f_r U_{rr}^2}{4\omega} \quad (4.72)$$

We then equate the  $E_1$  to  $E_2$  and re-arrange to find an expression for  $f_r$  the regenerator friction factor:

$$f_r = \frac{4L\rho f_1 U_{rr}}{3\pi d_h} \quad (4.73)$$

We then substitute this expression into equation 4.70 and find:

$$\Delta p = \frac{4L\rho f_1 U_{rr} \bar{u}_{rr}}{3\pi d_h} \quad (4.74)$$

We now need to find expressions for  $U_{rr}$  and  $u_{rr}$ , we start by writing expressions for the rate at which the volume of compression and expansion spaces change:

$$\dot{v}_c = -(A_p - A_R)\dot{x}_p + (A_D - A_R)\dot{x}_d \quad (4.75)$$

$$\dot{v}_e = -A_d\dot{x}_d \quad (4.76)$$

So the overall rate of volume change can be written as:

$$\dot{v} = -(A_p - A_R)\dot{x}_p + (A_D - A_R)\dot{x}_d + A_d\dot{x}_d \quad (4.77)$$

From this we can derive the fluid velocity in the regenerator by dividing by the regenerator free flow area  $A_{reg}$ :

$$u = \frac{-(A_p - A_R)\dot{x}_p + (A_D - A_R)\dot{x}_d + A_d\dot{x}_d}{A_{reg}} \quad (4.78)$$

We actually need the fluid velocity relative to the regenerator motion (rather than to the fixed geometry of the cooler)  $u_{rr}$  and this is obtained by subtracting the displacer velocity ( $\dot{x}_d$ ) from the previous equation which yields:

$$u_{rr} = \frac{-(A_p - A_R)\dot{x}_p + (2A_D - A_R - A_{reg})\dot{x}_d}{A_{reg}} \quad (4.79)$$

From this it is a simple matter to derive the maximum velocity through the regenerator (relative to the regenerator):

$$U_{rr} = \frac{\omega}{A_{reg}} [(A_p - A_R)^2 X_{p0}^2 + (2A_D - A_R - A_{reg})^2 X_{d0}^2 + 2(A_p - A_R)(2A_D - A_R - A_{reg})X_{d0}X_{p0}\cos[\phi]]^{\frac{1}{2}} \quad (4.80)$$

We then substitute equation 4.79 into equation 4.74 and find:

$$\Delta p = \frac{4L\rho f_1 U_{rr}}{3\pi d_h} \frac{-(A_p - A_R)\dot{x}_p + (2A_D - A_R - A_{reg})\dot{x}_d}{A_{reg}} \quad (4.81)$$

If we now substitute equation 4.81 into equation 4.38 we find:

$$\frac{-4L\rho f_1 U_{rr}}{3\pi d_h} [-(A_p - A_R)\dot{x}_p + (2A_D - A_R - A_{reg})\dot{x}_d] = C_p\dot{x}_p + C_d\dot{x}_d \quad (4.82)$$

We now compare coefficients of  $x_p$  and  $x_d$  to find expressions for  $C_p$  and  $C_d$ :

$$C_p = \frac{4L\rho f_1 U_{rr}(A_p - A_R)}{3\pi d_h} \quad C_d = -\frac{4L\rho f_1 U_{rr}(2A_D - A_R - A_{reg})}{3\pi d_h} \quad (4.83)$$

Substituting these expressions into equation 4.42 we find:

$$D_{dp} = \frac{4L\rho f_1 U_{rr}(A_p - A_R)}{3\pi d_h m_d} \quad D_{dd} = \frac{-4L\rho f_1 U_{rr}(2A_D - A_R - A_{reg})}{3\pi d_h m_d} \quad (4.84)$$

We now have a complete set of coefficients for the harmonic oscillator equation that describe the motions of the piston and displacer. We must now consider the various loss processes within the cooler that were mentioned in the previous chapter.

## 4.2 Losses in the cooler

The previous section takes into account dissipative loss as the working fluid flows through the regenerator. The departure from the discontinuous motion of the piston and displacer assumed by the isothermal cycle is inherent in the previous analysis and so an explicit loss term for this does not need to be applied. The effect of dead volume is also included. However there are many

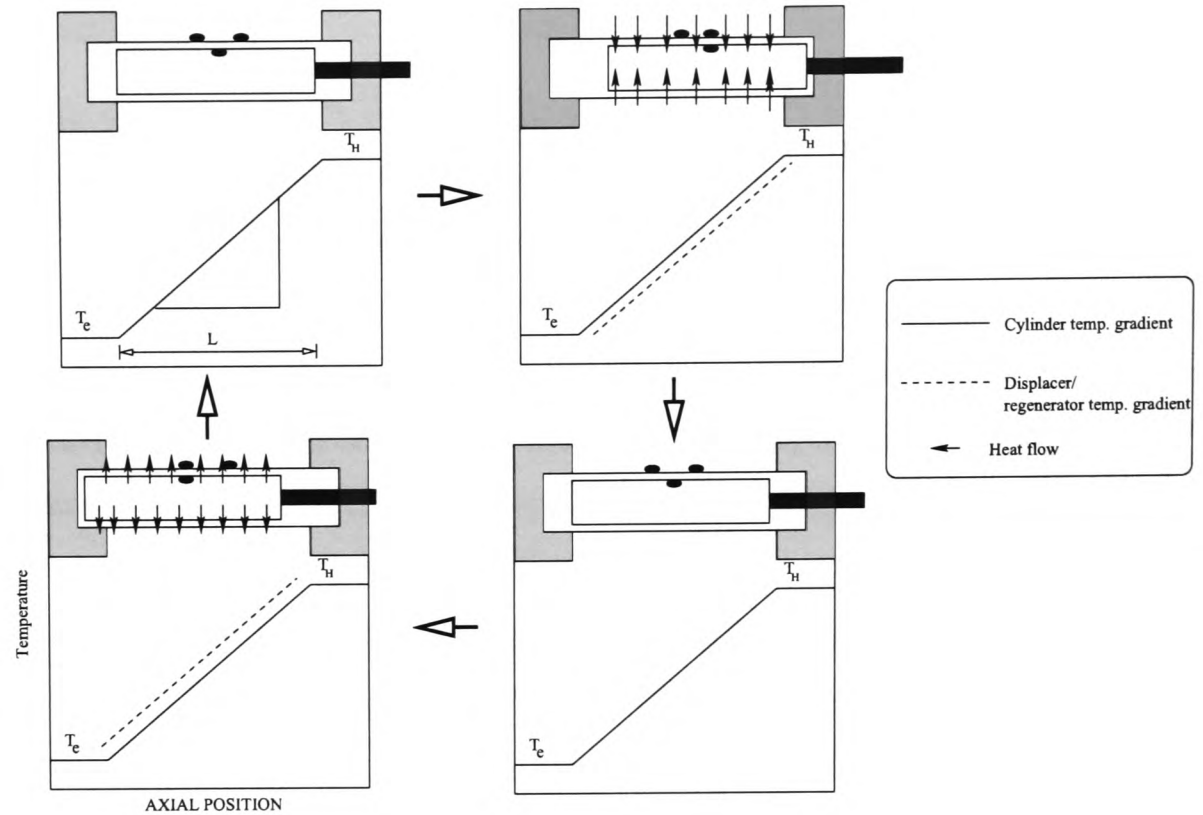
more loss mechanisms that will effect the cooler's performance, and these can be divided into two types: dynamic losses and static losses. Static losses exist whether the cooler is operating or not (conduction through the regenerator mesh, regenerator sleeve, working gas) whereas dynamic losses only occur when the cooler is operating. The following list is of the major losses within a cooler.

- Shuttle heat loss
- Conduction loss
- Radiation loss
- Regenerator ineffectiveness
- Windage
- Heat exchanger loss
- Pumping loss
- Compressor seal leakage
- Turbulence

We shall examine each of these in turn, and where possible derive equations to predict their effect on the cooler's performance:

#### 4.2.1 Shuttle heat loss

The displacer/regenerator in the prototype will oscillate in a cylinder (the cold finger) of length  $L$ , that has a temperature gradient along its length. The motion of the displacer/regenerator causes an extra heat transfer from the high temperature region of the cooler (the compression space) to the cold region (the expansion space). Figure 4.4 illustrates the mechanism for this heat transfer. The dot midway along the length of the displacer/regenerator indicates its axial position relative to the cylinder (when the displacer dot is midway between the two dots on the cylinder it is at its mid stroke position). When the displacer is moving toward the compression space from its mid-stroke position the displacer/regenerator wall has a lower temperature than the cylinder wall so that heat is transferred from the cylinder to the displacer. When the displacer/regenerator is moving toward the expansion space it has a higher wall temperature than the cylinder and so heat is transferred from it to the expansion space. As the cooler operates the displacer/regenerator repeatedly "shuttles" heat from the compression space to the expansion space in this manner creating a extra heat



**Fig. 4.4:** Simplified diagram of shuttle heat transfer.

load to the cold tip. There have been many papers published containing expressions for quantifying the amount of heat shuttled in this fashion ([Zimmermann F.J, Longworth R.C. (1970)] and [Baik, J.H. (1995)] for example), many assume that the heat transferred between the cylinder and displacer is proportional to the temperature difference between the cylinder and displacer/regenerator wall, but as Chang points out this may not be the case in machines with rapidly oscillating machines [Chang, H.M. et al. (2000)]. The reason for this is that the heat is transferred through the working fluid in the gap between the displacer wall and cylinder and this will be oscillating in the axial direction. So whilst conventional convective heat transfer is based on steady fluid flow the assumption that the heat transfer is proportional to the temperature difference may be valid in low speed (low frequency) machines it becomes dubious in machines operating at higher frequencies. Chang recently published a paper containing an expression for the shuttle heat transfer that takes into account the properties of the fluid in the displacer-cylinder gap and this is the expression 4.85 that will be used to predict this loss mechanism in the prototype [Chang, H.M. et al. (2000)]:

$$Q_{SH} = \frac{T_c - T_e}{L} \frac{RE[1 - \psi] - IM[1 - \psi]}{1 - Pr} \frac{\pi DS^2 k_{dis.}}{8} \sqrt{\frac{\omega}{2\alpha_{dis.}}} \quad (4.85)$$

Where:

$$\psi = \frac{\left( \left\{ \sigma_2 \sinh \left[ (1+i) \sqrt{\frac{\omega\delta}{2\alpha}} \right] + \cosh \left[ (1+i) \sqrt{\frac{\omega\delta}{2\alpha}} \right] \right\} \left[ \sigma_1 \sinh \left[ (1+i) \sqrt{\frac{\omega\delta}{2\mu}} \right] + \sqrt{Pr} \cosh \left[ (1+i) \sqrt{\frac{\omega\delta}{2\mu}} \right] - \sqrt{Pr} \right] \right)}{\left( \sinh \left[ (1+i) \sqrt{\frac{\omega\delta}{2\alpha}} \right] \left\{ (1 + \sigma_1 \sigma_2) \left( \sinh \left[ (1+i) \sqrt{\frac{\omega\delta}{2\alpha}} \right] + (\sigma_1 + \sigma_2) \cosh \left[ (1+i) \sqrt{\frac{\omega\delta}{2\alpha}} \right] \right\} \right) \right)} \quad (4.86)$$

Where:

$$Pr = \frac{\mu}{\alpha} \quad (4.87)$$

$$\sigma_1 = \sqrt{\frac{k_1 \rho_1 C_1}{k \rho C}} \quad (4.88)$$

$$\sigma_2 = \sqrt{\frac{k_2 \rho_2 C_2}{k \rho C}} \quad (4.89)$$

These last three equations represent the thermo-physical properties of the fluid in the gap, the displacer/regenerator wall, and the cylinder wall respectively. Estimates can be made of the various values of viscosity, thermal diffusivity etc by looking up appropriate values in tables of thermo-physical data [Lemmon E.W. et al. (2005)]. The gap width (clearance between the displacer and cylinder) can be estimated but will be dependent on good machining of components and careful alignment during assembly.

#### 4.2.2 Conduction loss

The following equation accounts for conduction of heat from the warm end (compression space) along the cold finger to the cold tip (expansion space). This will put an added heat load on the cold tip:

$$Q_{ch} = k_2 \left( \frac{T_e - T_c}{l_{ch}} \right) A_{ch} \quad (4.90)$$

However, there will also be conduction through the regenerator tube and the regenerator mesh itself. We can use equation 4.90 to estimate the conduction through the regenerator tube by simply replacing the  $A_{ch}$  with  $A_{rt}$ , the conduction area of regenerator tube, and  $k_2$  with  $k_{rt}$  the thermal conductivity of Vespel Sp3 from which the regenerator tube is made.

Conduction through the regenerator mesh is dependent on the packing of the regenerator mesh and is hard to predict. Brito made measurements of the thermal conductivity of a similar design of regenerator with two different packing densities, and it was found that an 8% increase in packing resulted in a doubling of the thermal conductivity of the regenerator mesh [Bruto, M. (1999)]. It was also found that with a packing density of 270 stainless steel discs (400 mesh) per cm the conductivity was less than 4% of the basic mesh material (stainless steel). So we can reasonably expect that for a tightly packed regenerator the conduction through the regenerator mesh will be less than 5% of the thermal conductivity of the basic material. So as an approximation we can use equation 4.90 with an appropriate estimate of the thermal conductivity for the regenerator mesh and the conduction area.

### 4.2.3 Radiation loss

The cold tip of the cooler will be at  $80K$  but the warm end will be at  $200K$  and the vacuum tank will be at room temperature. This means that there will be a heat load applied to the cold tip by radiation from the warm end and vacuum tank. The load applied to the cold tip can be significantly reduced by the use of multiple layer insulation (MLI) composed of aluminised polyester films (Mylar sheets). Conduction through the blanket is minimised using low conductivity spacers to prevent contact between adjacent layers. The difference between the energy absorbed and emitted by the cold tip is:

$$Q_{rad} = \sigma A_{ce} \epsilon_{ce} (T_W^4 - T_C^4) \quad (4.91)$$

Where  $\sigma$  is Stefan's constant,  $A_{ce}$  is the effective area of the cold tip,  $\epsilon_{ce}$  is the emissivity,  $T_W$  and  $T_C$  are the temperatures of the surroundings and the cold tip respectively. It is extremely difficult to estimate the emissivity of the cold tip when the MLI insulation is used and so producing estimates of the radiation heat load on the cold end is very difficult. Brito made measurements of the radiation loss in a small pulse tube cooler and found the radiation loss was only 4% of the static heat load in that cooler [Brito, M. (1999)]. There will also be a heat load applied to the cold tip by gas in the vacuum tank that has not be extracted, although below pressures of  $1 \times 10^{-4}$  mbar the convective heat load is negligible (see section 6.3 and [Reed, J. (2004)]).

### 4.2.4 Regenerator ineffectiveness

The study of regenerators and the development of the theory surrounding them has a long history. The earliest papers (published in German) on the matter by Rummel, Nusselt and Hausen still form the basis for many analyses [Willmott, A.J. (2005)]. Robert Stirling, after whom the Stirling engine is named, actually patented the regenerator (not the Stirling engine) in 1816 and built an engine to demonstrate the principle [Organ, A.J. (1997)]. Since this time there have been huge advances in mathematics, engineering, computing (which, in the modern sense, did not exist at all in Stirling's day) and thermodynamics. However, regenerator theory has not become a matter of routine by any means [Organ, A.J. (1997)].

The regenerator is the crucial component for a Stirling machine and is the heart of small cryogenic refrigerators [Kush, P.K. and Thirumaleshwar M. (1989)]. It must operate with an effectiveness close to 100% for the thermodynamic cycle to have a high c.o.p. and so the lack of a reliable analytical model is an obstacle in the design of Stirling machines. The heat load applied to the cold tip due to regenerator ineffectiveness is given by [Gifford, W.E. et al. (1969)]:

$$Q_{ri} = (1 - \epsilon) \dot{m}_g c_v (T_c - T_e) \quad (4.92)$$



Where  $\epsilon$  is the regenerator effectiveness  $\dot{m}_g$  is the mass flow rate of the working gas,  $c_v$  is the specific heat capacity of the working gas,  $T_c$  and  $T_e$  are the compression space and expansion space temperatures respectively. Barron gives an illuminating example of quite how important regenerator effectiveness is [Barron, R.F. (1985)]. This example is repeated here (equations 4.93 to 4.95):

If we assume that the working fluid behaves as an ideal gas the energy removed from the cold tip is given by:

$$Q_{ideal} = (\gamma - 1)\dot{m}_g c_v T_e \ln\left(\frac{ve_4}{ve_3}\right) \quad (4.93)$$

Where  $\gamma$  is the ratio of the specific heats for the working fluid,  $ve_n$  is the volume of the expansion space as shown at each apex of the ideal PV diagram as shown in figure 3.1. If we divide equation 4.92 by 4.93 we find the fraction of the ideal refrigeration that is wasted due to regenerator ineffectiveness.

$$\frac{Q_{ir}}{Q_{ideal}} = \frac{1 - \epsilon}{\gamma - 1} \frac{T_c/T_e - 1}{\ln(ve_4/ve_3)} \quad (4.94)$$

As a numerical example suppose we have helium as the working fluid for which  $\gamma = 1.67$ ,  $ve_4/ve_3 = 1.5$   $T_c = 300K$  and  $T_e = 78K$  we find that:

$$\frac{Q_{ir}}{Q_{ideal}} = \frac{1 - \epsilon}{0.67} \frac{300/78 - 1}{\ln(1.5)} = 10.48(1 - \epsilon) \quad (4.95)$$

So even if the regenerator was 99 % effective we find that 10.48% of the ideal refrigeration is lost. To reach high levels of effectiveness a large surface area for heat transfer and a large heat capacity are required. The void volume should also be small to maintain the pressure ratio in the system [Radebaugh, R. (1984)]. Typically wire mesh screens stacked in hollow plastic tube are used to form a regenerator with these characteristics. A recent paper by Klein and Eigenberger [Klein, H. and Eigenberger G. (2001)] includes a new treatment of regenerators and includes the effect of heat conductivity in the regenerator material parallel to the flow of the working fluid. Within a regenerator structure temperature changes both spatially and with time. If the regenerator material has a small thickness (i.e. its thickness is small compared to the thermal penetration depth) then the temperature variation perpendicular to the fluid flow can be neglected. Klein and Eigenberger go on to construct a one dimensional, dynamic, two-phase model of the regenerator described by the following differential equations:

$$\epsilon \rho_g c_{pg} \frac{\delta T_g}{\delta t} = \mp G_z c_{pg} \frac{\delta T_g}{\delta z} + \beta a_V (T_s - T_g) \quad (4.96)$$

$$(1 - \epsilon) \rho_s c_s \frac{\delta T_s}{\delta t} = \lambda_z \frac{\delta^2 T_s}{\delta z^2} - \beta a_V (T_s - T_g) \quad (4.97)$$



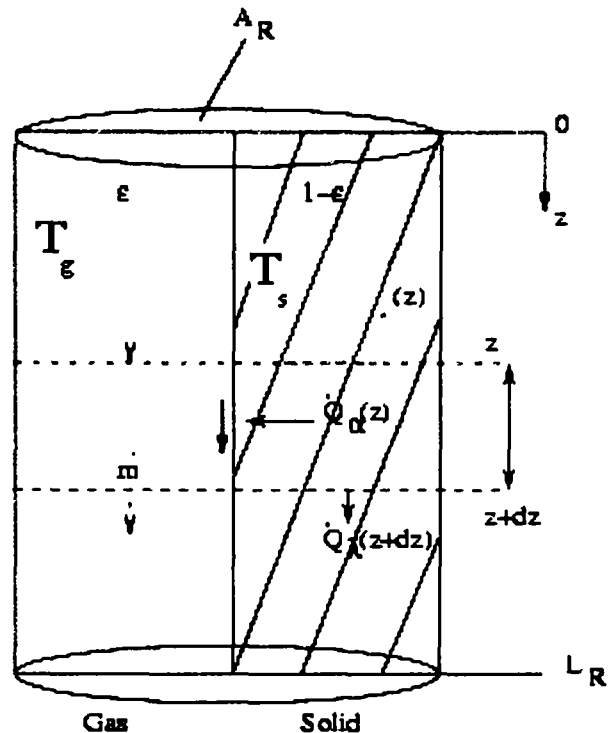


Fig. 4.5: Volume element from Klein's regenerator analysis.

Boundary conditions are formulated by considering the solid matrix material to be thermally insulated at both boundaries. The inlet temperatures are also assumed to be constant for both half periods (in other words during the inlet of gas into the regenerator from the compression space the inlet gas is assumed to be at the compression space temperature). Figure 4.5 shows a volume element within the regenerator model. This diagram reflects the underlying assumption of this analysis that the regenerator can be treated as a recuperator in which the flow direction switches rapidly. It shows gas flowing ( $\dot{m}$ ) in a volume (given by the porosity  $\epsilon$ ) past a solid wall (of volume  $1 - \epsilon$ ) to and from which there is heat transfer (indicated by  $\dot{Q}$ , the subscripts  $\lambda$  and  $\alpha$  denote heat transfer in the longitudinal and transverse directions respectively).

The mass flow rate is assumed to be constant within each half period (for example the half period when the piston is moving from its mid-point position to its forward-most position) as is the heat capacity of the gas in each half period. Based on these assumptions Klein and Eigenberger derive the energy balance equation (equation 4.100) for a regenerator with a high value of the product  $\sigma\Gamma$ . Where:

$$\sigma = \frac{c_s}{c_p} \quad (4.98)$$

and:

$$\Gamma = \frac{(1 - \epsilon)\rho_s L}{2\Delta t G_z} \quad (4.99)$$

Where  $\Delta t$  is half the cycle time and  $\rho_s$  is the density of the matrix material. The product  $\sigma\Gamma$  represents the ratio of heat storage capacity of the solid matrix of the regenerator to that of the gas flowing through it during one complete period. If this product is large then the following equation

can be derived:

$$0 = \frac{1}{Pe} \frac{\delta^2 T_s}{\delta \zeta^2} - NTU(T_s - T_{g2}) - NTU(T_s - T_{g1}) \quad (4.100)$$

Where  $Pe$  is the Peclet number, which is the ratio of heat exchanged through convection to that exchanged through conduction. If the condition of a large value of the product  $\sigma\Gamma$  is satisfied then the following system of ordinary differential equations can be stated for a regenerator:

$$\frac{1}{Pe} \frac{d^2 T_s}{d\zeta^2} = NTU(T_s - T_{g2}) + NTU(T_s - T_{g1}) \quad (4.101)$$

$$\frac{dT_{g1}}{d\zeta} = 2NTU(T_s - T_{g1}) \quad (4.102)$$

$$-\frac{dT_{g2}}{d\zeta} = 2NTU(T_s - T_{g2}) \quad (4.103)$$

With boundary conditions:

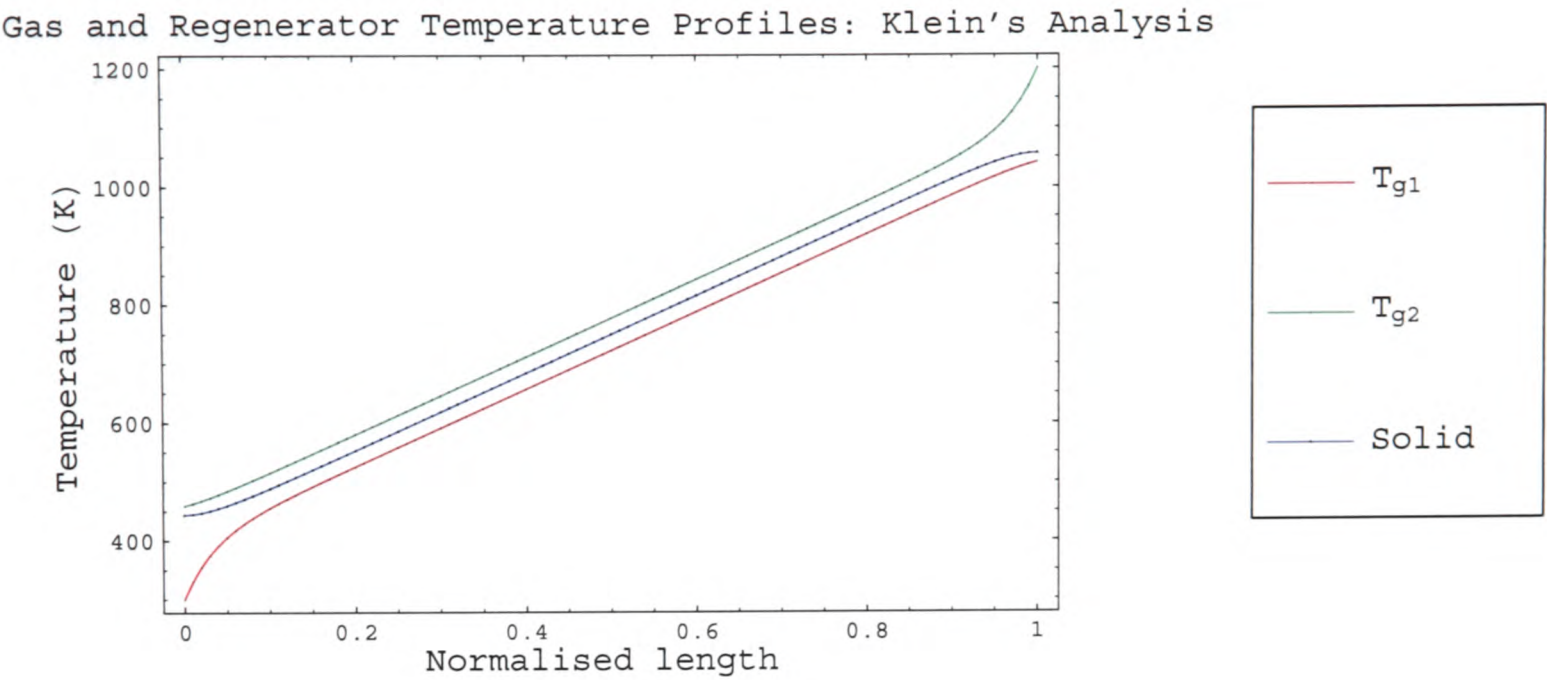
$$T_{g1}(\zeta = 0) = T_{1in} \quad (4.104)$$

$$T_{g2}(\zeta = 1) = T_{2in} \quad (4.105)$$

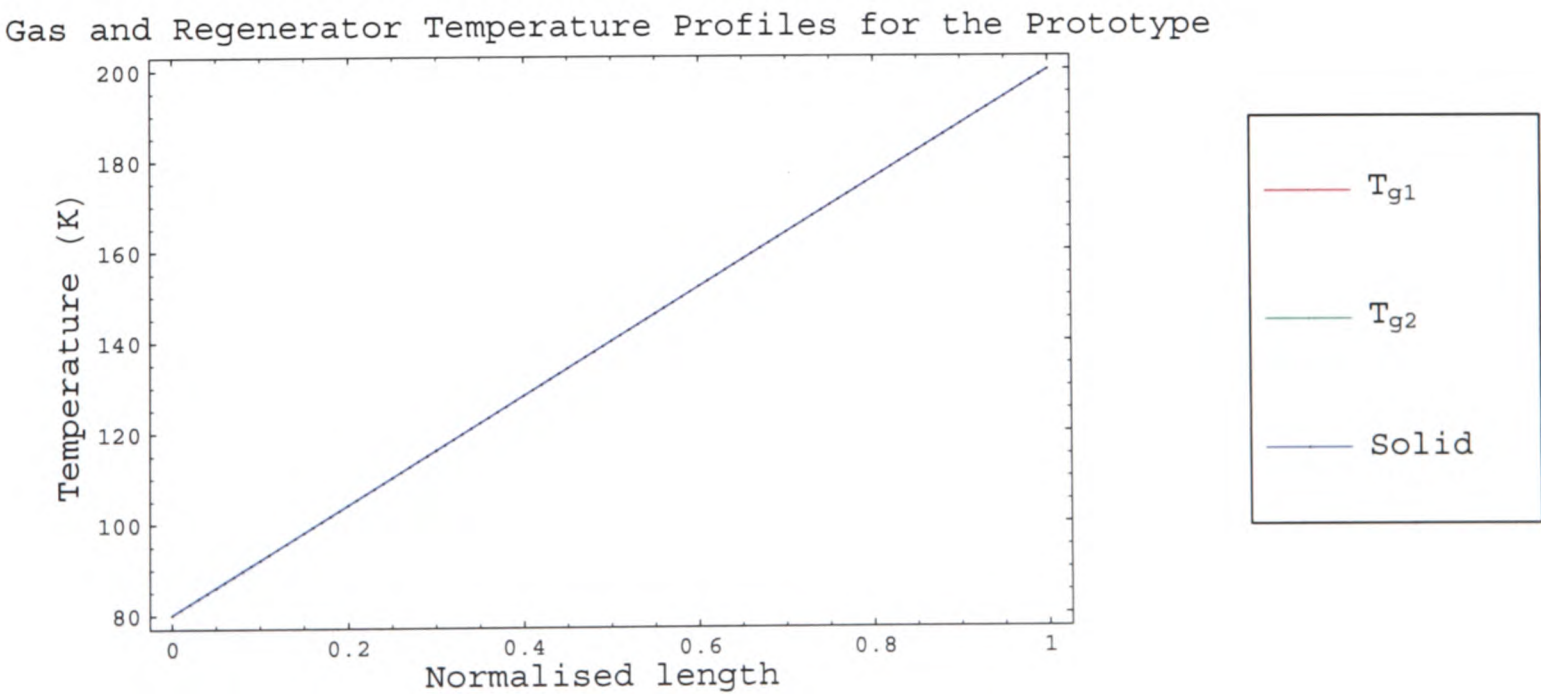
$$\frac{dT_s(\zeta = 0)}{d\zeta} = 0 \quad (4.106)$$

$$\frac{dT_s(\zeta = 1)}{d\zeta} = 0 \quad (4.107)$$

The first two boundary conditions state that during the inlet of warm gas the temperature of the gas at the warm end ( $\zeta = 0$ ) must be equal to the warm end temperature ( $T_{1in}$ ) and during the inlet of cold gas the gas temperature at the regenerator cold end ( $\zeta = 1$ ) is equal to the cold end inlet temperature ( $T_{2in}$ ). Applying these boundary conditions (equations 4.104 - 4.107) it is possible to solve this system of equations. Klein and Eigenberger [Klein, H. and Eigenberger G. (2001)] solve the same set of equations for a regenerator used in a radically different environment (inlet temperatures of 1200K and 300K,  $NTU = 11.95$  and  $Pe = 12.59$ ) the temperature profiles found are shown in figure 4.6. If the solutions to this set of differential equations are plotted for the expected conditions within the prototype cooler, assuming a regenerator of length 50mm filled with 250 mesh phosphor-bronze screens, the temperature profiles of the fluid are so close to each other and to the solid matrix temperature profile as to be indistinguishable from each other. This can be seen in figure 4.7. What this indicates is that the proposed design of regenerator will ensure that the gas is in good thermal conduct with the matrix material and also that the temperature gradient across the regenerator is linear. The solutions to the system of differential equations (equations 4.101 to 4.103) provide expressions for the temperature of the fluid as functions of time and space and these can then be use to derive an expression for the effectiveness of the regenerator which is given by equation 4.108 [Walker G. (1983b)].



**Fig. 4.6:** Temperature profiles for gas and solid (regenerator matrix) produced from Klein's analysis, with  $NTU = 11.95$ ,  $Pe = 12.59$  and inlet temperatures  $1200K$  and  $300K$ .



**Fig. 4.7:** Temperature profiles produced using the method of Klein and Eigenberger for the expected conditions within the cooler.

$$\eta = \frac{Q_R}{Q_R^{ideal}} = \frac{T_{1in} - \bar{T}_{1out}}{T_{1in} - T_{2in}} = \frac{\bar{T}_{2out} - T_{2in}}{T_{1in} - T_{2in}} \quad (4.108)$$

Where  $T_{1in}$  is the (fixed) warm end temperature and  $T_{2in}$  is the (fixed) cold end temperature (in the case of the proposed cooler these are 200K and 80K respectively).  $\bar{T}_{1out}$  and  $\bar{T}_{2out}$  represent the gas exit temperature during the inlet of hot gas and during the inlet of cold gas respectively. The fluid temperatures (assuming the product  $\sigma\Gamma$  approaches infinity) are given by:

$$T_{g1} = \frac{(T_{1in} + T_{2in}) \left( \frac{e^{k_1-1}}{e^{k_1+1}} k_2 + 1 \right) + Pe \left( T_{1in} + \frac{T_{1in}}{NTU} \right)}{2 + Pe + \frac{Pe}{NTU} + 2k_2 \frac{e^{k_1-1}}{e^{k_1+1}}} + \frac{T_{2in} - T_{1in} \left( Pe\zeta - \frac{Pe}{2NTU} k_2 \frac{e^{k_1(1-\zeta)} - e^{k_1\zeta}}{e^{k_1+1}} \right)}{2 + Pe + \frac{Pe}{NTU} + 2k_2 \frac{e^{k_1-1}}{e^{k_1+1}}} \quad (4.109)$$

$$T_{g2} = \frac{(T_{1in} + T_{2in}) \left( \frac{e^{k_1-1}}{e^{k_1+1}} k_2 + 1 \right) + Pe \left( T_{1in} + \frac{T_{2in}}{NTU} \right)}{2 + Pe + \frac{Pe}{NTU} + 2k_2 \frac{e^{k_1-1}}{e^{k_1+1}}} + \frac{T_{2in} - T_{1in} \left( Pe\zeta - \frac{Pe}{2NTU} k_2 \frac{e^{k_1(1-\zeta)} - e^{k_1\zeta}}{e^{k_1+1}} \right)}{2 + Pe + \frac{Pe}{NTU} + 2k_2 \frac{e^{k_1-1}}{e^{k_1+1}}} \quad (4.110)$$

Where  $k_1$  is given by:

$$k_1 = \sqrt{2NTU} \sqrt{2NTU + Pe} \quad (4.111)$$

and  $k_2$  is given by:

$$k_2 = \frac{\sqrt{2NTU}}{\sqrt{2NTU + Pe}} \quad (4.112)$$

We substitute equations 4.109 and 4.110 into equation 4.108 to derive an expression for the effectiveness of the regenerator in terms of  $k_2$ ,  $k_1$ , NTU and Pe which are themselves functions of the regenerator length, diameter porosity etc. The regenerator effectiveness is given by:

$$\eta = \frac{Pe + 2k_2 \left( \frac{e^{k_1-1}}{e^{k_1+1}} \right)}{2 + Pe + \left( \frac{Pe}{NTU} \right) + 2k_2 \left( \frac{e^{k_1-1}}{e^{k_1+1}} \right)} \quad (4.113)$$

In traditional analyses of regenerators modeled as counter-current heat exchangers that do not take into account axial conduction the effectiveness is given by [Kays W.M. and London A.L. (1964)] :

$$\eta = \frac{NTU}{1 + NTU} \quad (4.114)$$

If we assume that Pe approaches infinity in equation 4.113 (i.e. we assume that axial conduction is negligible compared to the heat exchanged through convection) then we recover equation 4.114 as we expect.

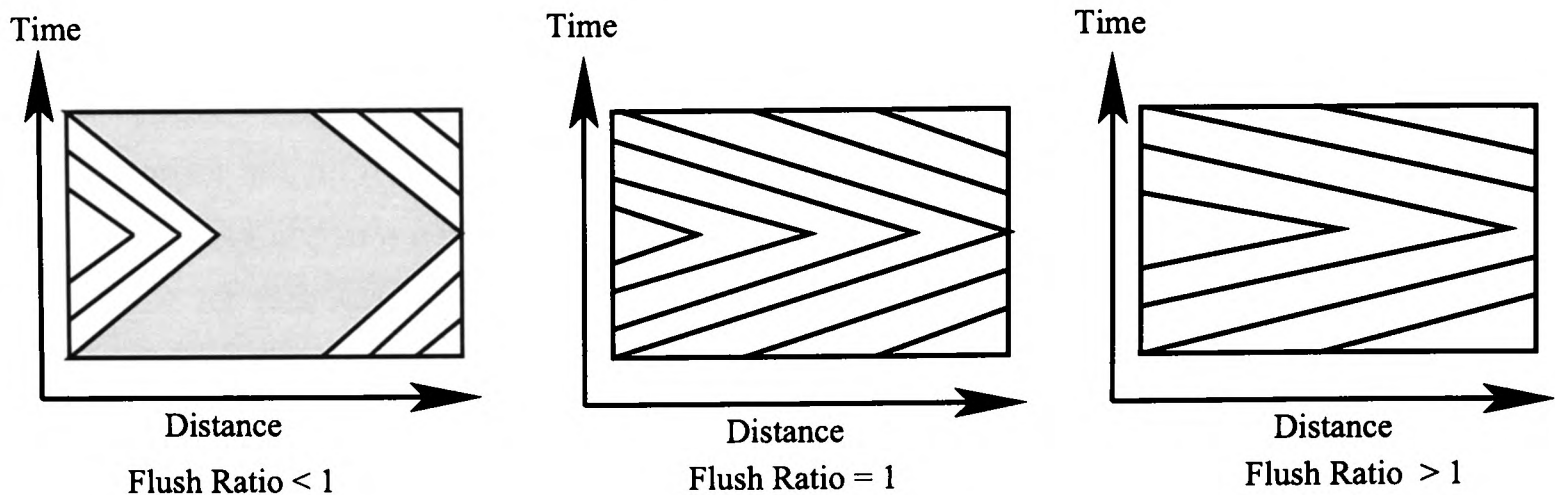
At first it seems that this treatment of regenerators is extremely useful in sizing the regenerator needed for the prototype, however there is a major assumption inherent in the analysis that makes its applicability to Stirling regenerators highly suspect. The analysis assumes that the energy of the gas within the void volume of the regenerator can be regarded as negligible compared to the

energy stored within the solid matrix. By making this assumption, time dependent changes in the gas temperature are neglected within each half period (each direction of fluid motion), the temperature profile “jumps” as the flow direction changes but within the following half period (in which the direction is the same) the gas temperature is then purely a function of its position along the length of the regenerator. This assumption is valid if the time for which the fluid is flowing in one direction (the blow time) is larger than the time it takes for fluid to traverse the length of the regenerator (the residence time). However in miniature Stirling machines such as the proposed cooler the residence time is comparable to the blow time and so this assumption is not valid. Models have been developed that do take into account the dynamic behavior of the gas within the regenerator. Such models introduce what is known as the *flushing phase*, that is they consider the gas that remains in the regenerator from one period to the next. Organ [Organ, A.J. (1997)] and De Monte [De Monte, F. (1999)] have both published analyses that take into account this effect. A brief summary of the method of De Monte is presented here and the limitations of this analysis and its applicability to the proposed cooler discussed.

Figure 4.8 illustrates the difference between the conditions in large industrial regenerators and those found in Stirling machine regenerators. The flushing ratio indicated in the figure is defined as ratio of mass throughput,  $M_i - m_r$  to resident mass,  $m_r$ , where  $M_i$  is the mass of gas that directly interacts with the regenerator matrix and  $m_r$  is the mass of gas contained within the regenerator at a point within the cycle (this will vary throughout the cycle in step with the pressure variation but the peak mass content is used)[Organ, A.J. (1997)]. In figure 4.8 particle trajectories for gas within three idealised regenerators are displayed. In the left hand diagram the flush ratio is less than one and following the particle trajectory for an element of gas that just enters the regenerator at the beginning of the forward blow (when the gas is being forced through the regenerator by the piston’s motion toward the regenerator) we see that that the gas does not pass through the regenerator but moves part way through before reversing its direction on the reverse blow as the piston moves away from the regenerator. Any gas that is within the grey shaded area in the left hand diagram oscillates back and forth within the regenerator without ever exiting. This contrasts with the right hand diagram where the flush ratio is much larger and so gas that enters the regenerator at the beginning of the forward blow traverses the length of the regenerator before switching its direction of motion on the reverse blow.

#### 4.2.5 The method of De Monte

De Monte starts the analysis by stating the governing equations and then making assumptions that allow them to be simplified into a form that have analytical solutions. The full analysis is not



**Fig. 4.8:** Particle trajectories for working fluid in regenerators with differing flush ratios.

repeated here but the governing equations, assumptions and resulting solutions are.

#### 4.2.6 The governing equations

The first set of assumptions are:

- The regenerator is one-dimensional (no variation in properties perpendicular to the direction of fluid flow).
- Thermal conductivity of the matrix and fluid are zero parallel to the direction of fluid flow and infinitely large normal to the flow.
- The matrix specific heat and density are independent of temperature.
- The cross sectional free flow area of the regenerator is constant.
- The specific heats  $c_{p,f}$  and  $c_{v,f}$  are independent of temperature.

We see that, unlike Klein and Eigenberger [Klein, H. and Eigenberger G. (2001)], this analysis neglects axial conduction (conduction parallel to the fluid flow direction).

For the matrix material within the regenerator the basic energy equation is:

$$hW [T_f(x, t) - T_w(x, t)] = c_w \rho_w A_{reg} \left( \frac{1 - \epsilon}{\epsilon} \right) \frac{\delta T_w(x, t)}{\delta t} \quad (4.115)$$

Fluid mass continuity equation:

$$\frac{\delta}{\delta x}(\rho_f u) + \frac{\rho_f}{\delta t} = 0 \quad (4.116)$$

Fluid momentum equation:

$$\rho_f \frac{\delta u}{\delta t} + \rho_f u \frac{\delta u}{\delta x} + \frac{\delta p}{\delta x} + f \rho_f \frac{u^2}{2r_h} = 0 \quad (4.117)$$

Fluid energy equation:

$$hW[T_W - T_f(x, t)] = c_{vf}A_f \frac{\delta}{\delta t}[\rho_f T_f(x, t)] + c_{pf}A_f \frac{\delta}{\delta x}[\rho_f u T_f(x, t)] \quad (4.118)$$

Substituting equation 4.116 into equation 4.118, applying the ideal gas law and re-arranging we find an expression for the gas energy balance:

$$c_{pf}\rho_f \frac{\delta T_f}{\delta x} + c_{pf}\rho_f \frac{\delta T_f}{\delta t} = \frac{\delta \rho}{\delta t} - \frac{h}{\tau_h}(T_f - T_w) \quad (4.119)$$

This equation, (equation 4.119) along with equations, 4.115, 4.116, and 4.118 form a set of partial differential equations in the unknowns  $u$ ,  $p$ ,  $T_f$ ,  $T_w$  depending on  $x$  and  $t$ . Any solution must also satisfy certain cyclical conditions if they are to represent a regenerator working in the cyclic steady state:

$$\int_{n\tau_0}^{n\tau_0+\tau_b} h_b(T_w - T_f)d\tau = \int_{n\tau_0+\tau_b}^{(n+1)\tau_0} h_r(T_f - T_w)d\tau \quad (4.120)$$

Equation 4.120 says that the heat exchanged due to convection between the gas and the regenerator matrix at any given location during the blow period must be equal in magnitude to the heat exchanged during the reverse period.

$$c_w\rho_w \oint A_f \rho_f u dt = 0 \quad (4.121)$$

Equation 4.121 says that over a complete cycle there can be no net storage by the regenerator.

$$\oint A_f \rho_f u dt = 0 \quad (4.122)$$

Equation 4.122 ensures that there is no change in mass of the gas over a cycle.

$$\oint \frac{\delta p}{\delta t} dt = 0 \quad (4.123)$$

Equation 4.123 simply ensures that the pressure variation is cyclical.

To ensure that there is an analytical solution to this set of partial differential equations, De Monte makes another set of assumptions:

- The regenerator matrix has infinite thermal capacity per unit volume ( $c_w\rho_w \rightarrow \infty$ ).
- The gas velocity,  $u$  and pressure,  $p$  are constant throughout the length of the regenerator.
- The flow velocity and pressure are quasi stationary ( $\frac{\delta u}{\delta t} = 0$  and  $\frac{\delta p}{\delta t} = 0$ ).
- The fluid density,  $\rho_f$  is constant and evaluated at the average temperature,  $\frac{T_c+T_e}{2}$ .
- The coefficient of convective heat transfer,  $h$ , is constant.
- The gas flow switches in equal intervals:  $\tau_b = \tau_r = \frac{\tau_0}{2}$ .



It is worth considering these assumptions in turn to see how applicable this analysis is to the regenerator in the proposed cooler. The first assumption is applicable to the proposed cooler since the thermal capacity of the gas in the regenerator during the blow period is much smaller than the thermal capacity of the regenerator itself. The second and third assumptions are highly suspect and there is a great deal of literature on pressure drops for steady and oscillating flows through wire meshes (see [Ju, Y. et al. (1998)] and [Kays W.M. and London A.L. (1964)] for example). What is more, the pressure drop across the regenerator is a loss mechanism within the cooler and cannot be completely ignored when designing a regenerator for such an application. However these assumptions are also implied in the analysis of Klein [Klein, H. and Eigenberger G. (2001)] and most other regenerator analyses, few of which take into account the flushing phase. Bearing this in mind it seems that the assumptions are acceptable up to a point but that any results must be carefully considered in view of these simplifications.

Solutions for the gas and matrix temperature of the regenerator during the blow and reverse periods can be found once these assumptions have been made and temperature profiles for the fluid and matrix can be made either by setting the length along the regenerator as fixed and plotting the temperature at this point with time over a cycle or by looking at the temperature distribution along the regenerator length at a particular moment in time. Once expressions for the fluid and wall temperatures have been found it is possible to calculate the effectiveness of the regenerator via equation 4.108. Section 4.3 details how this analysis of the regenerator along with analysis of the other loss mechanisms in the cooler have enabled a computer model of the cooler to be created.

#### 4.2.7 Windage

The motion of the oscillating elements through the working fluid within the cooler, both in the working spaces and in the cooler body itself can cause the dissipation of energy due to friction. Peters found that the windage loss in Stirling coolers developed for the Improved Stratospheric And Mesospheric Sounder (ISAMS) instrument was less than  $40mW$  at  $40Hz$  for a piston stroke of  $5mm$  [Peters, M. (1994)]. The design of the oscillating components within the new cooler will be different (generally smaller) to those in the ISAMS coolers and so we can reasonably expect the windage loss to be less. The coil former that oscillates within the cooler body has been designed with reducing the windage loss in mind; this can be seen in section 5.4.2.

#### 4.2.8 Heat exchanger loss

In the compression space heat must be transferred from the working gas to the cylinder walls and in the expansion space heat must flow from the metal cold tip to the working fluid if any cooling



is to occur. For these heat transfer processes to occur there must be temperature differences between the gas and surrounding metal (i.e. in the compression space cylinder the gas must be at a higher temperature than the cylinder walls). These temperature differences reduce the amount of total refrigeration produced by the cooler for a given amount of work done on the gas by the compressor piston. Equation 4.124 is the Carnot c.o.p. for a refrigerator operating with cold tip and compression space temperatures  $T_e$  and  $T_c$  respectively but with temperature differences  $\Delta T_c$  between the gas in the compression space and the compression space cylinder and temperature difference  $\Delta T_e$  between gas in the expansion space and the metal cold tip.

$$\eta = \frac{T_e - \Delta T_e}{(T_c + \Delta T_c) - (T_e - \Delta T_e)} \quad (4.124)$$

It is clear that the temperature differences  $\Delta T_c$  and  $\Delta T_e$  reduce the c.o.p. and thus have a detrimental effect on cooler performance.

#### 4.2.9 Pumping loss

There must be a clearance between the displacer/regenerator tube (containing the wire mesh) and the cylinder within which it oscillates. This means there is a volume around the displacer/regenerator into which gas will flow as it is being compressed in the compression space. During the compression part of the cycle if the displacer is at the cold end of the cooler warm gas will flow into this volume and will constitute an additional load on the cold tip. However the gap between the displacer/regenerator and the cylinder is so small (it can be smaller than the inter-wire distance in the regenerator mesh if the displacer/regenerator sleeve and cold finger are well machined) that the annulus has a regenerative effect [Peters, M. (1994)], and so the pumping loss is likely to be a very small effect.

#### 4.2.10 Compressor piston seal leakage

The cooler design does not include a conventional seal around the piston but uses a clearance seal. A very small gap (of the order of tens of microns) will exist between the piston bush and the wall of the cylinder in which the piston is oscillating. This gap will allow gas to flow from the compression space past the piston and into the cooler body during the cooler's operation. This will reduce the amount of power being delivered into the working fluid and so reduce the available cooling. The only way to reduce this loss is to ensure that the clearance seal (made from Fluorosint) is the best fit it can be within the compression cylinder without causing rubbing (which would constitute another loss mechanism). This can be done by running the cooler with the piston bearing in place until sufficient material has been worn away to allow free movement (no rubbing). A model of the compressor seal leakage has been derived and incorporated into a computer model of the cooler.

The basic equation for the gas flow is:

$$\nabla p = \mu \nabla^2 u \quad (4.125)$$

The clearance between the Fluorosint bush around the piston and the cylinder it oscillates in will be much smaller than the circumference of the cylinder and so we can simplify the situation by assuming that the clearance gap can be considered as a rectangular gap with its short side equal to the gap width and the long side equal to the circumference of the cylinder inner wall (this assumes that the piston is concentric within the bore of the cylinder). In the steady case where by symmetry the gas flow must be along the  $z$  axis (no pressure gradients in the  $x$  or  $y$  directions) equation 4.125 may be written as:

$$\frac{d^2 u}{dx^2} + \frac{d^2 u}{dy^2} = \frac{1}{\mu} \frac{dP}{dz} \quad (4.126)$$

By symmetry there can be no  $y$  dependence of  $u$  (the gas velocity past through the gap) and so simplifying further we find that:

$$\frac{d^2 u}{dx^2} = \frac{1}{\mu} \frac{dP}{dz} \quad (4.127)$$

Assuming that the gas at the walls of the cylinder and the piston bush have the velocity of the walls they are adjacent to we can apply the following boundary conditions:

$$u = 0 \text{ at } x = d \text{ and } u = v \text{ at } x = 0 \quad (4.128)$$

We can then solve equation 4.127 to find an expression for the velocity of the gas past the piston seal in the  $z$ -direction at a point  $x$  is given by:

$$u_z = \frac{1}{2\mu} \frac{dp}{dz} x^2 - \left( \frac{d}{2\mu} \frac{dp}{dz} + \frac{v}{d} \right) x + v \quad (4.129)$$

The mass flow past the piston seal is given by:

$$\dot{m} = \int_0^d u_z \rho(x, z) c dx \quad (4.130)$$

Where  $c$  is the piston bush circumference. Since the steady state has already been assumed there can be no pressure gradients in the  $x$  or  $y$  directions so equation 4.129 can be substituted into equation 4.130 and the integration performed to give:

$$\dot{m} = c \rho d \left[ \frac{d^2}{12\mu} \frac{dP}{dz} - \frac{v}{2} \right] \quad (4.131)$$

Equation 4.131 gives the total mass flow at any point  $z$  in terms of the local pressure gradient  $\frac{dP}{dz}$ . It is possible to replace this term with an approximation  $\frac{\Delta p}{L}$ . However it is also possible to integrate this equation over the length of the seal to obtain an exact expression for the pressure gradient. A time-stepping method can be applied to calculate the mass flow past the piston seal throughout

the piston's motion. It should be noted that as the piston moves forward into the compression space the pressure in the compression space increases which, if we assume the pressure in the space behind the piston remains constant (which it will given the much larger volume of gas behind the piston), will cause the mass flow past the piston to increase which will act to reduce the increase in pressure caused by the piston's forward motion. Taking this effect into account requires finding an equation for the pressure gradient along the length of the piston seal. Since mass flow rate through the gap is constant and applying the ideal gas law it can be stated that:

$$\dot{m} = \frac{\text{constant}}{p} \quad (4.132)$$

If this is equated to equation 4.131 it can be seen that the following is true:

$$\frac{dP}{dz} = \frac{\kappa}{p} \quad (4.133)$$

Where  $\kappa$  is a constant. There are two solutions to this equation:

$$p = -\sqrt{2}\sqrt{(\kappa) + K_1} \quad (4.134)$$

and

$$p = \sqrt{2}\sqrt{(\kappa) + K_1} \quad (4.135)$$

Where  $\kappa$  is the same constant as in equation 4.133 and  $K_1$  is a constant yet to be determined. The solution shown in equation 4.134 can be disregarded as it would produce negative pressures which is un-physical.  $K_1$  and the constant in the acceptable solution (4.135) can be found by assuming that the pressure behind the piston (i.e. at  $z = 0$ ) is equal to and constant at the filling pressure of the cooler,  $P_{fill}$ , and that at the far end of the piston seal ( $x = L_p$ ) the pressure is equal to the compression space pressure,  $p_c$  (given by equation 4.56). Stating these as boundary condition equations:

$$p = p_{fill} \text{ at } x = 0 \text{ and } p = p_c \quad (4.136)$$

This leads to the following expressions for  $\kappa$  and  $k_1$ :

$$k_1 = \frac{p_{fill}^2}{2} \quad (4.137)$$

and

$$\kappa = \frac{1}{2L} \left( \frac{p_c^2}{2} - \frac{p_{fill}^2}{2} \right) \quad (4.138)$$

This leads to the following expression for the pressure along the piston seal:

$$p = \sqrt{\frac{1}{L} \left( \frac{p_c^2}{2} - \frac{p_{fill}^2}{2} \right) x + \frac{p_{fill}^2}{2}} \quad (4.139)$$

This is then differentiated to provide an expression for  $\frac{dp}{dz}$ :

$$\frac{dp}{dz} = \frac{\frac{1}{2} (p_c^2 - p_{fill}^2)}{L_p \left( p_{fill}^2 + \left( \frac{(p_c^2 - p_{fill}^2)x}{L_p} \right) \right)^{0.5}} \quad (4.140)$$

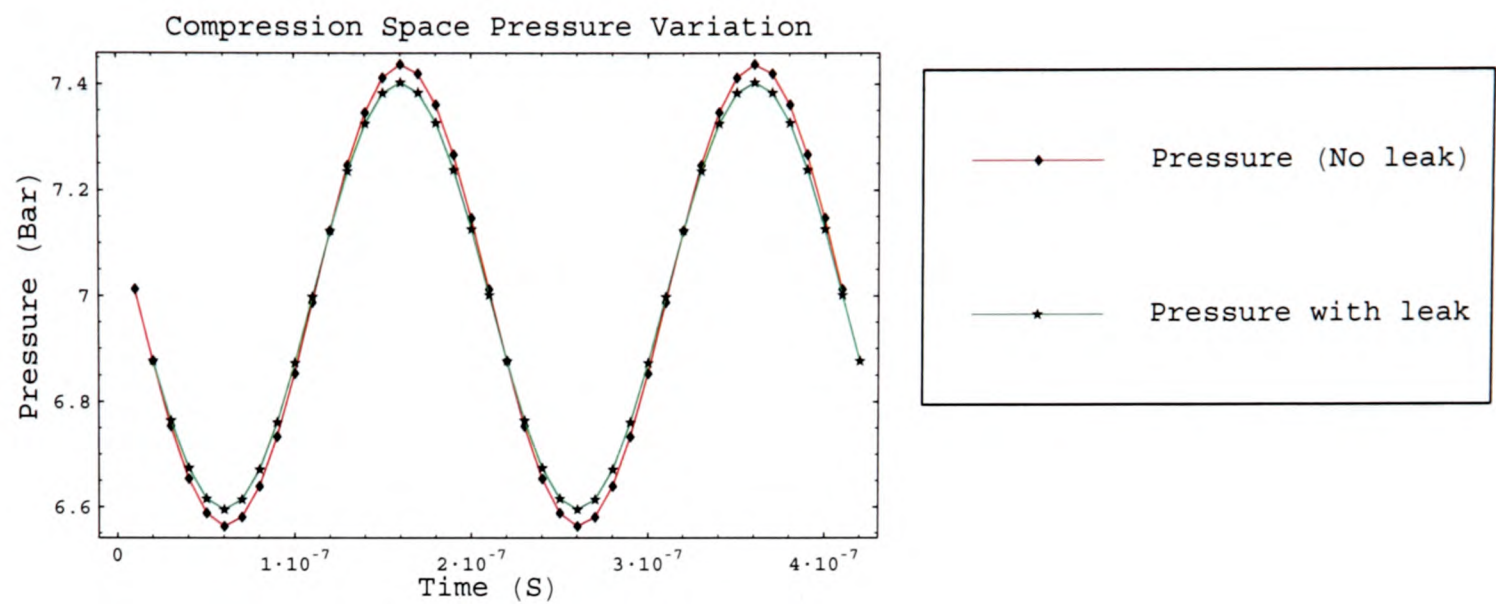
This is then substituted into equation 4.131:

$$\dot{m} = c\rho d \left[ \frac{d^2}{12\mu} \frac{\frac{1}{2} (p_c^2 - p_{fill}^2)}{L_p \left( p_{fill}^2 + \left( \frac{(p_c^2 - p_{fill}^2)x}{L_p} \right) \right)^{0.5}} - \frac{v}{2} \right] \quad (4.141)$$

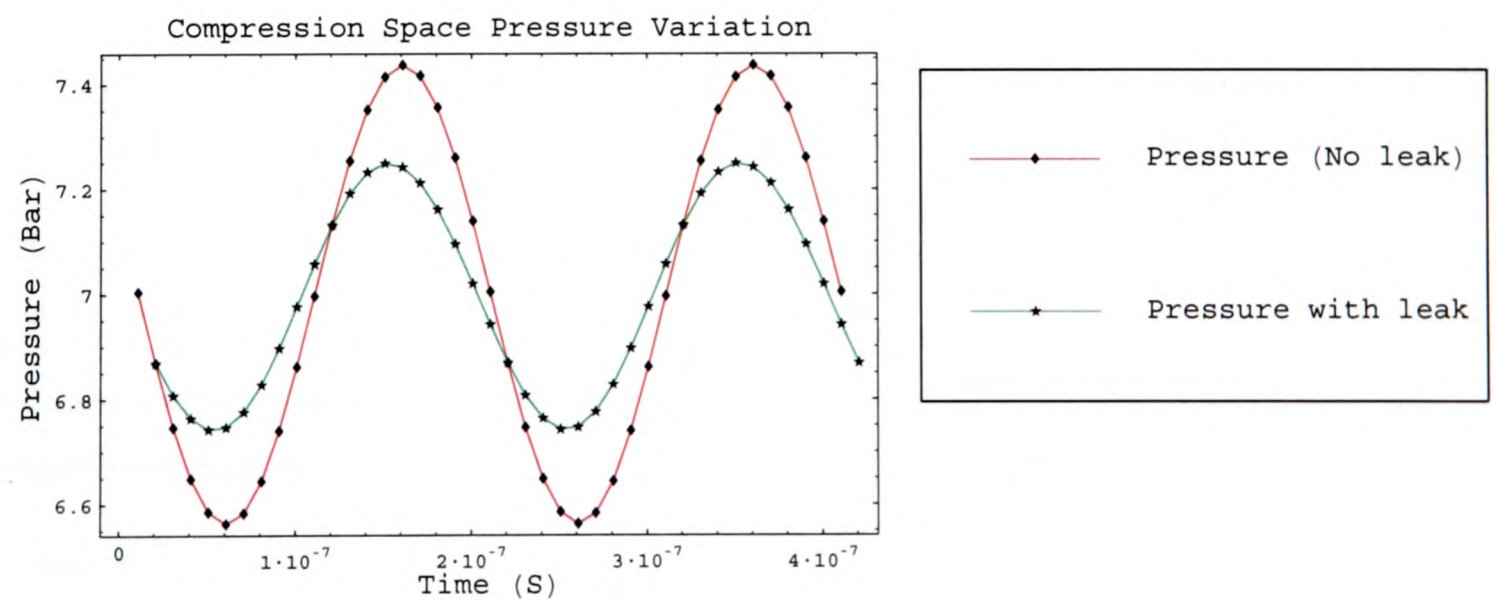
The process for calculating the mass flow is as follows:

1. Set time to be  $t_{initial}$  when piston is at midpoint of its motion.
2. Calculate  $p_c$  at  $t = t_{initial}$ .
3. Calculate mass flow at this instant (equation 4.141) and multiply by length of time step  $\Delta t$  to get mass flow over this time step.
4. Set  $t = t_{initial} + \Delta t$ , calculate new value of  $p_c$  at this instant using equation 4.56.
5. Calculate the drop in  $p_c$  due to the mass flow in time step  $\Delta t$  (calculated in step 3), subtract this from  $p_c$  (calculated in step 4) and set the answer to be the new value of  $p_c$ .
6. Loop back to step 3 until  $t = t_p$ , where  $t_p$  is the period of the piston motion.

This method allows for the effect of piston seal leakage to be incorporated into the computer model of the cooler. Figure 4.9 shows the variation of compression space pressure with time when gas leakage past the piston seal is ignored and also when it is included. The two plots are phase shifted from each other and the piston seal leakage reduces the amplitude of the pressure swing. Figure 4.9 was plotted assuming a gap of 28 microns, the resulting decrease in the pressure swing amplitude is visible but the phase change is not so apparent. However, figure 4.10 shows a plot which compares the compression space pressure swing with no leakage and the pressure swing if we assume a gap of 50 microns. In this case the drop in the pressure swing amplitude and the phase shift are clearly visible. The phase shift arises because as the piston compresses the gas to increasingly high pressures during its forward (toward the cold tip) movement there will be increased gas flow past the piston through the clearance seal. At some point during this forward motion of the piston this leakage flow will become so great that the pressure in the compression space no longer increase despite the compressing action of the piston. Figure 4.11 shows an example of a PV plot for the compression space in the cooler produced by the computer model, this particular plot was produced assuming a clearance seal gap of 60 microns to make the effect of the gap more clearly



**Fig. 4.9:** Pressure swing in the compression space assuming 28 micron gap around the piston bush compared to the “no leakage” case.



**Fig. 4.10:** Pressure swing in the compression space assuming 50 micron gap around the piston bush compared to the “no leakage” case.

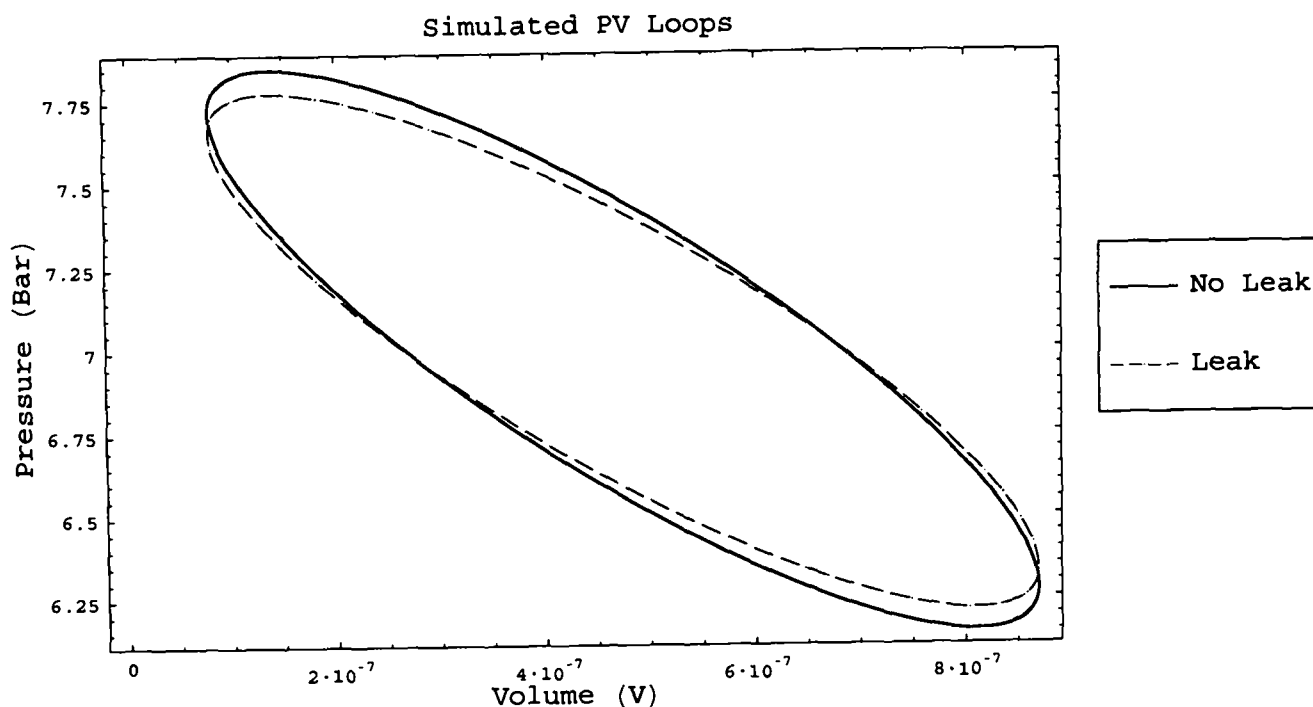


Fig. 4.11: PV plots for the compression space produced by the computer model.

visible in the plot, this is larger than the gap in the current prototype cooler is expected to be (for comparison, Werrett found the clearance seal gap in the ISAMS developmental cooler to be 28 microns [Werrett S.T.R. (1991)]). Results suggest that for a clearance seal gap of 28 microns would reduce the PV power by less than 5%.

### 4.3 Implementation of the computer model

The equations derived in the previous section are the basis for a computer model that has been used to obtain an estimate of the regenerator size needed for the cooler to obtain its performance targets. Due to the complex gas flows within the cooler and the simplifying assumptions used to make the systems of differential equations that describe the situation within the regenerator tractable the results of the model can only be regarded as an estimate. That is not to say that the above analysis and computer model are not useful as they highlight which aspects of the design have a large impact on the cooler's performance and aid in understanding its operation.

The basic procedure that the computer model follows is:

- Thermophysical quantities (specific heats, densities etc) and fixed cooler geometry input as well as quantities such as the linear motor flux density, coil resistance and length.
- Expressions for various "secondary" aspects of the cooler are defined (regenerator free flow area, heat exchange area etc) in terms of the fixed geometry of the cooler and those yet to be decided (piston stroke, regenerator length).

- Solve equation for the piston stroke required to produce 3W of output power from the compressor (Equation 4.35).
- Piston stroke substituted into expression for the maximum gas flow velocity through the regenerator if the regenerator/displacer is oscillating with maximum amplitude.
- Substitute the gas flow velocity into expression for dimensionless groups such as the Reynolds number.
- Define expressions for the various loss processes: shuttle heat loss, regenerator ineffectiveness (based on Klein's method) etc. The pressure drop along the length of the regenerator is given by equation 4.38 with  $C_p$  and  $C_d$  given by equation 4.83.
- Solve equation 4.63 for the mechanical spring constant required to produce resonance at 50Hz.
- Combine expressions for loss processes into one expression, a function of regenerator length, and find the value of the regenerator length that minimises this function. Set this as the optimum regenerator length. Plot graphs of how the various loss processes and the combined loss vary with regenerator length (an example of such a plot is shown in figure 4.12).
- Substitute the value of the optimum regenerator length into the set of differential equations derived by Klein and solve for the gas and matrix temperature profiles.
- Define the "reduced length" and "flushing ratio" and substitute the optimum regenerator length into these expressions, define the various coefficients used in the expressions for matrix and fluid temperature derived by De Monte.
- Substitute coefficients derived by De Monte into gas and matrix temperature profile solutions and plot them along the length of the regenerator.
- Calculate the regenerator effectiveness at the regenerator optimum length using both the method of Klein and of De Monte.
- Output the magnitudes of the various loss processes and the total loss predicted by the model.

The computer model can be used to calculate the optimum regenerator length, required piston stroke for a given input power to the gas, required spring constant for operation at a given frequency. The model also produces estimates of the regenerator effectiveness via two different methods. The model can be easily adapted to take into account changes in the cooler (for example if the porosity of the regenerator mesh or the filling pressure is changed).



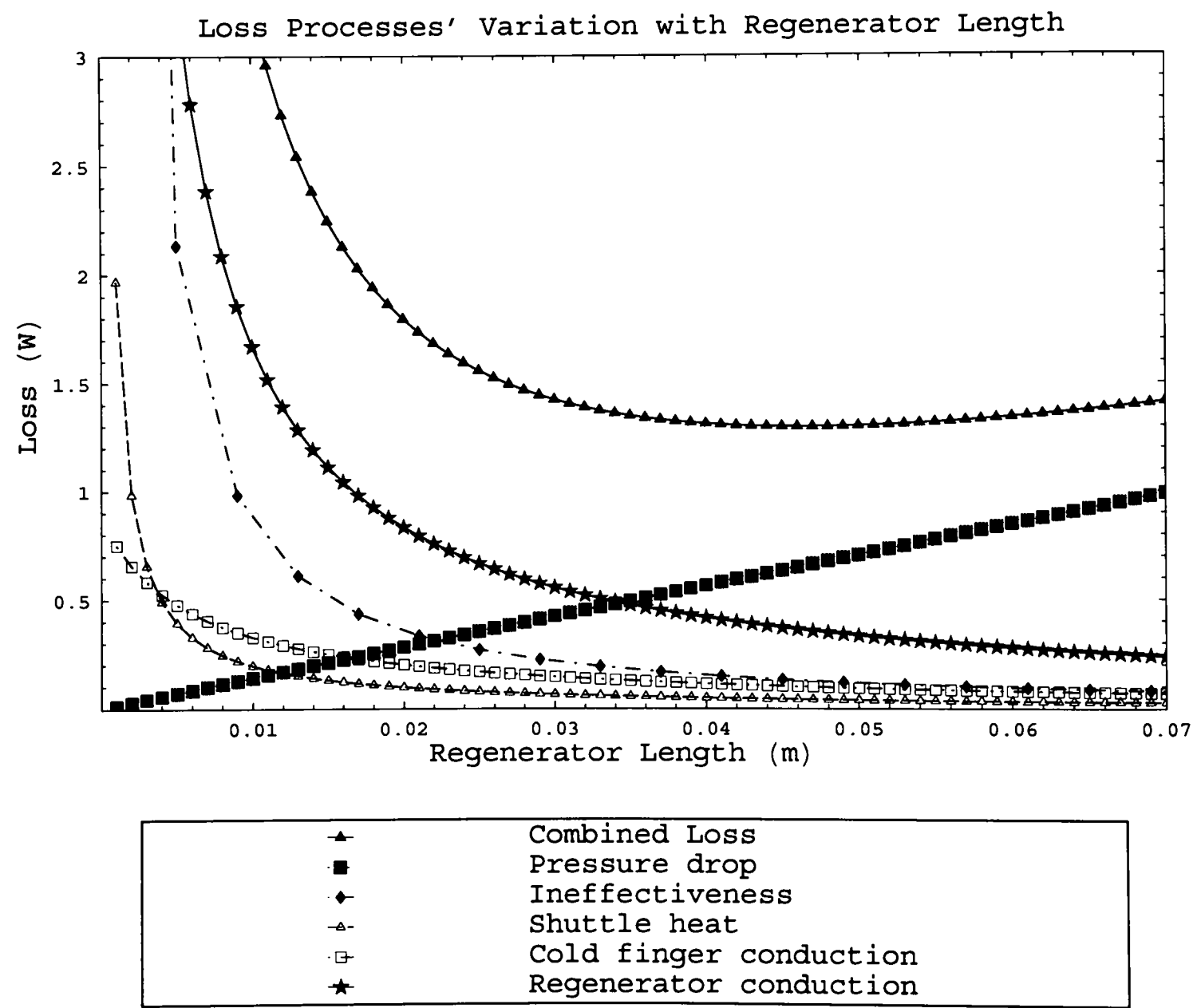


Fig. 4.12: Output graphs from the Mathematica program showing how the various modeled loss processes vary with regenerator length.



### 4.3.1 Limitations of the model

The model described in this section can be used to estimate the length of regenerator needed in the prototype cooler. However, it must be stressed that the number of simplifying assumptions means that the output from the model must only be used as a guide to aid the design of the prototype. These simplifying assumptions are described below.

- **Averaging of Thermophysical Properties** The specific heats and densities of the regenerator materials and of the working gas will change as their temperature changes, the model only takes account of this in a very approximate way by taking the values of such properties at the cycle average temperature  $(T_c - T_e)/2$ .
- **Pressure Drop Along the Regenerator** The pressure drop across the regenerator is calculated using standard methods for calculating pressure drops for fluid flows through porous media, however these depend on correlations derived for steady flows rather than the oscillating flows we are concerned with, and they are also being used in a situation where there is a large temperature gradient along the regenerator. There is no clear way of accurately predicting the pressure drop inside the cooler and so the pressure drop loss can only be regarded as an estimate.
- **Fluid velocity** The fluid velocity is calculated by considering the rate at which the compression and expansion space volumes change as the piston and displacer/regenerator move. The fluid is assumed to be incompressible and the velocity is assumed to be uniform throughout the regenerator, entrance and exit effects are assumed to be negligible.
- **Heat exchanger loss** The temperature differences between working gas and its surroundings (the compression cylinder walls and the metal cold tip) reduce the cooler's performance (see equation 4.124) however the size of these temperature differences is unknown. Peters calculated them for an ISAMS development model cooler and found that the gas in the expansion space was 5.6K lower than the copper cold tip and that the temperature of the gas in the compression space was 63K higher than the compression cylinder itself although he considered this to be an overestimation [Peters, M. (1994)]. Including these temperature difference in the model is questionable as it is not clear how they will change with a different design of cooler but omitting them completely would mean ignoring a significant loss process. As a compromise the model was run with the heat exchanger loss term omitted (i.e. the temperature of the working gas in the expansion space and compression space were taken to be the temperatures of the compression space cylinder and the copper cold tip respectively) and then the model was run again with the temperature differences between the working

gas and its surroundings taken to be those found by Peters. The predictions of the optimum regenerator length from these two runs were then averaged and this value was used in the prototype design.

- **Conduction through regenerator mesh** Conduction through the regenerator mesh is dependent on how tightly packed the meshes are within the regenerator tube. Brito found that the regenerator meshes when tightly packed had a coefficient of thermal conduction around 4% of the solid material's coefficient but exactly how the conduction varies with packing is not clear [Brito, M. (1999)].
- **Compression loss** Previous studies of the losses within Stirling coolers [Peters, M. (1994)] and [Werrett S.T.R. (1991)] have identified irreversible compressor loss as the single largest loss mechanism. Despite this the actual mechanism itself is not well understood. Equations for calculating these losses have been suggested ([Urieli, I. Berchowitz, D.M. (1998)] and [Cooke-Yarborough E.H. and Ryden, D.J. (1985)] for example) but they are not well supported by experimental results nor do they all agree as to what the loss process is. Due to this large uncertainty, and lack of analytical expressions or a large number of empirical results on which to base an estimate of the magnitude of this loss within the prototype cooler design it was difficult to include it in the model. It was found that 48% of the input power to the working gas was wasted by the compression loss in an ISAMS development model cooler [Peters, M. (1994)]. This figure was used to calculate how much of the input power of the gas would be wasted and thus how much gross refrigeration would be available in the prototype. It is not included in figure 4.12 as it is not known to vary with regenerator length and so does not effect the choice of optimum regenerator length. If the sum of the power consumed by the compression loss and the loss processes plotted in figure 4.12 equal or exceed the input power to the gas the cooler will not be able to cool an external load. The uncertainty in the size of the compression loss thus gives rise to an uncertainty in the predictive ability of the model.

## 4.4 Summary

The feasibility of a miniature integral Stirling cooler able to produce  $200mW$  of cooling at  $80K$  with a compression space pre-cooled to  $200K$  by a radiator has been investigated previously [Reed, J. (2004)]. This analysis, backed up by preliminary results gained from a developmental cooler built by Reed, suggest that the performance target is realistic and this is supported by the new analysis contained within this section. This new analysis resulted in a computer model that allowed

| Parameter            | Value                    |
|----------------------|--------------------------|
| Piston diameter      | 14.5mm                   |
| Piston Stroke (p-p)  | 5mm                      |
| Operating Frequency  | 50Hz                     |
| Regenerator Length   | 50mm                     |
| Regenerator Diameter | 9mm                      |
| Regenerator Material | Phosphor-Bronze 250 mesh |

**Table 4.5:** Optimum parameters as suggested by the computer model.

variables such as the dimensions of the compression and expansion spaces, regenerator and piston to be varied and their effect on the cooler’s performance to be predicted. This model was used to select what was believed to be the optimum set of parameters (see table 4.5) for the cooler.

One of the crucial elements of the cooler is the linear motor and particular attention was paid to developing a new linear motor. This is detailed in the next chapter along with the general design of the cooler.

## Chapter 5

# Design and construction of the cooler

### 5.1 Introduction

Previous research into miniature Stirling coolers within AOPP conducted by Reed [Reed, J. (2004)] had used a prototype constructed predominantly from components originally created for the ISAMS coolers, however when this prototype was run with a pre-cooled compression space a number of design flaws caused the cooler to cease operating. From the experiments performed with this cooler and a computer model developed alongside the cooler, Reed made a number of suggestions that a future miniature cooler should adopt:

- **Reduction in compressor piston size.** Reducing the piston diameter enables the cooler's size to be reduced.
- **Increased operating frequency.** If the piston diameter is reduced then at a given filling pressure and stroke the input power to the working fluid is reduced, thus the operating frequency must be increased to maintain a constant input power to the working fluid. What is more, analysis performed by during this research effort indicates that a higher operating frequency improves the linear motor's efficiency (see section 5.3). Reed suggested an operating frequency of approximately 45Hz but the optimum operating frequency will depend on exactly how the various dynamic loss processes vary with frequency.
- **Improved design of linear motor.** Reed found that the efficiency of the linear motor in his prototype had a maximum of approximately 19% and was thus in need of extensive modification if the cooler was to be made more efficient.

- **Reduce the number of materials used in construction of the cooler.** Differences in the coefficient of thermal expansion (CTE) between materials within Reed's cooler led to moving components within it coming into contact and eventually the piston seized around the displacer shaft. Reed suggested titanium as a suitable construction material. Titanium has the advantage of low density, high strength and lower values of CTE and thermal conductivity than other suitable construction materials such as aluminium or steel.

Stirling coolers require the working fluid in the compression space to undergo repeated cycles of compression and expansion. Traditionally in Stirling machines this compression is caused by a piston connected to a rotary drive. However, such mechanisms cause side forces to be applied by the piston on the cylinder in which it is oscillating and so increases friction within the machine. This can be avoided if a linear motor is used to drive the piston. Such motors have been in use in coolers for around twenty years and operate on the same principle as a loudspeaker: A coil of wire subject to a time varying driving voltage oscillates in a magnet field created by a permanent magnet. Recent development efforts within AOPP had neglected the linear motor and had simply re-used components from the ISAMS coolers. Since the ISAMS development program there have been advances not only in magnetic materials but also in the modeling of magnet assemblies with finite element software and so a new linear motor was designed and manufactured for the current research effort.

When designing a linear drive cooler there is a choice to be made between having a moving coil or moving magnet motor. Moving coil machines need sliding or flexing contacts to the coil [Redlich, R. (1996)] and therefore moving magnet motors have often been favoured because the electrical connections to the stationary coil are not required to flex (which can cause them to break) and so improve the reliability of the linear motor. However, the large moving mass of the magnet can lead to large exported vibrations. If the magnet is held stationary and the coil (which is generally significantly lighter than the magnet) moves, then these vibrations can be reduced but at the cost of having flexible electrical connections. The ISAMS coolers used flexible copper photo-etched arms to make the electrical connections and these performed well, however they do mean that extra volume must be included in the cooler body. The new cooler described in this chapter chose a different method of making the electrical connections: the spiral flexure bearings themselves. Since the spiral flexures have been shown to be reliable in numerous coolers and since they are necessary for the cooler's operation regardless of what method of electrical connection is used, it is clear that by using them we keep the number of components in the cooler body to a minimum (thus increasing reliability) and simplify assembly. The method by which the flexures are used as electrical contacts is described in section 5.2. Figure 5.1 shows a schematic overview

| Flexure Type       | Spring Constant (N/m) |
|--------------------|-----------------------|
| Compressor         | $361.991 \pm 1.59$    |
| Original Displacer | $547.833 \pm 4.94$    |

Table 5.1: Spring Constants.

of the prototype cooler.

## 5.2 Use of spiral flexures as electrical feedthroughs

Figure 5.2 shows a diagram of the two compressor flexure stacks that are used as the electrical feedthroughs to the linear motor coil. Notice that the flexure stacks are only connected to current carrying wires from the drive supply at their outer circumference which does not move, so these wires are not required to flex repeatedly throughout the cooler's operation. Figure 5.2 also shows how the arrangement of mica components effectively insulated the spring stacks from each other. A compressor flexure is shown in figure 5.3 and a displacer/regenerator flexure is shown in figure 5.4. The flexures were designed by writing a simple program in Mathematica that allowed equations to be entered that defined each flexure arm, the central hole, the outer diameter, bolt holes etc (the equations for the spiral arms can be found in [Reed, J. (2004)] amongst others). Holes were added to the end of the spirals to better distribute stresses and were blended into the spirals with splines (this has been done on previous cooler flexure designs [Reed, J. (2004)]). The output from this program was a postscript file which could then be used to produce high-resolution transparencies for use in photo etching the springs out of 0.30mm thick spring steel.

The flexures' spring constants were determined using the apparatus shown in figure 5.6. The springs were clamped around the outer rim as would be the case when they were fixed within the cooler. Various masses were then attached to the spring through the centre hole and the deflection of the spring then measured using a *Mitutoyo<sup>tm</sup>* height gauge accurate to 0.01mm. The masses were measure using a digital scale accurate to 0.01g. The flexures were found to behave linearly to a very good degree and an example of the deflection plotted against applied force can be seen in figure 5.5. Table 5.1 shows the spring constants derived from these results. At present there is no computer program available within AOPP for the prediction of the spring constants for spiral flexure designs and so only once they have been produced and the moving mass of the compressor measured can the resonant frequency of the compressor be determined.

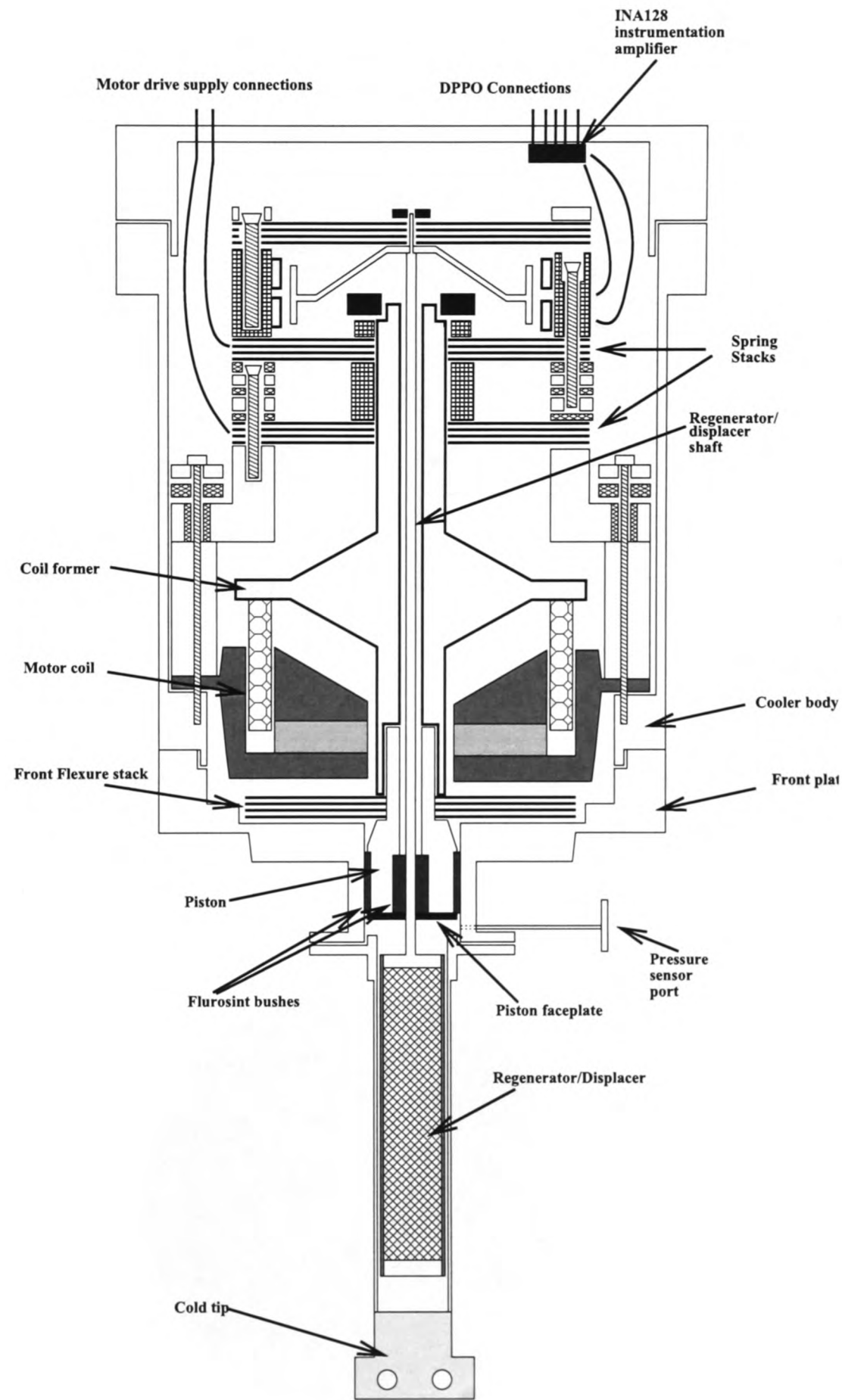
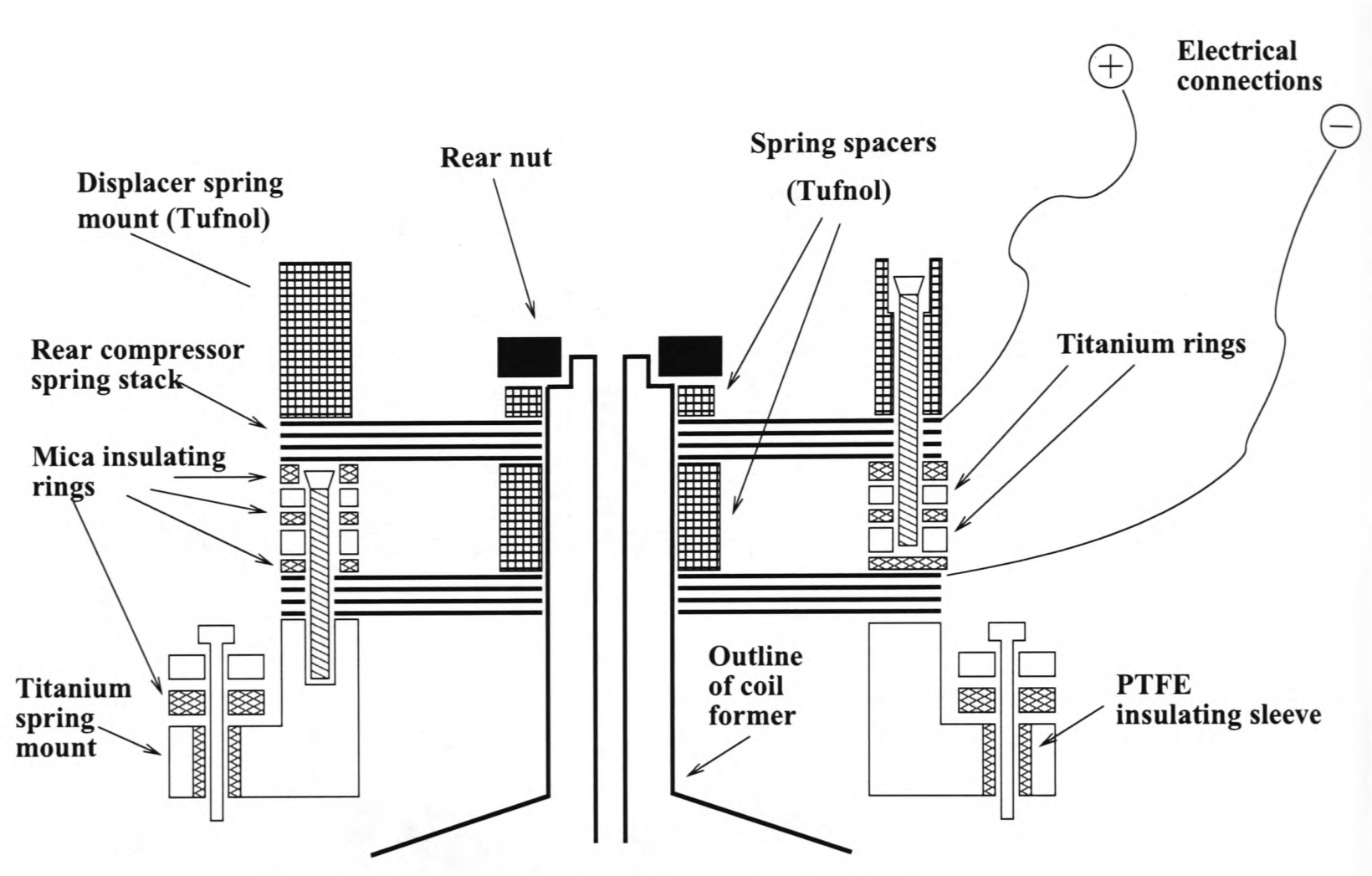
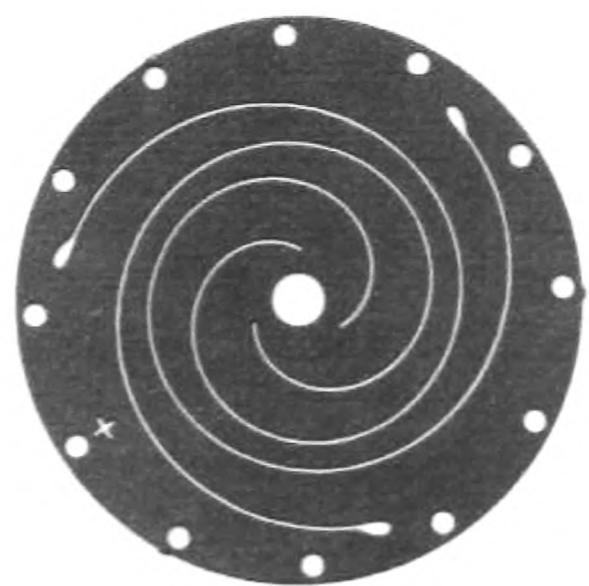


Fig. 5.1: Schematic overview of the cooler.





**Fig. 5.2:** Diagram showing how the compressor flexure stacks were used as electrical feedthroughs to the linear motor.



**Fig. 5.3:** Compressor flexure.



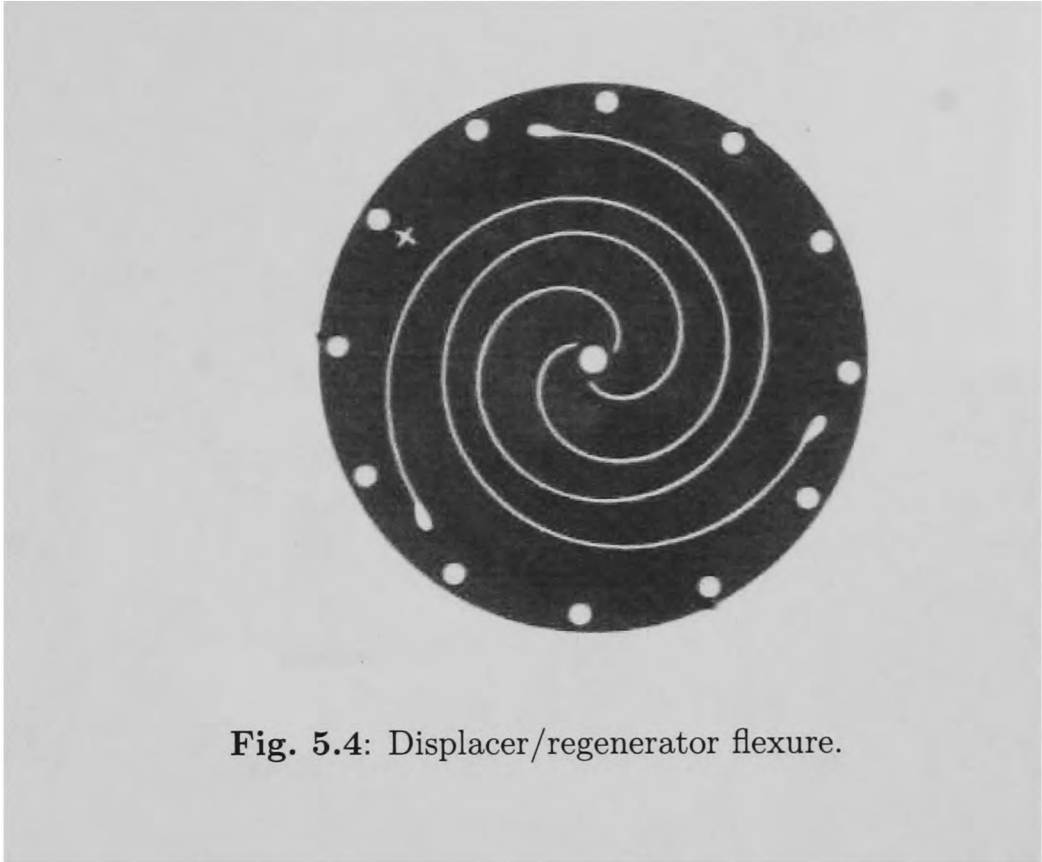


Fig. 5.4: Displacer/regenerator flexure.

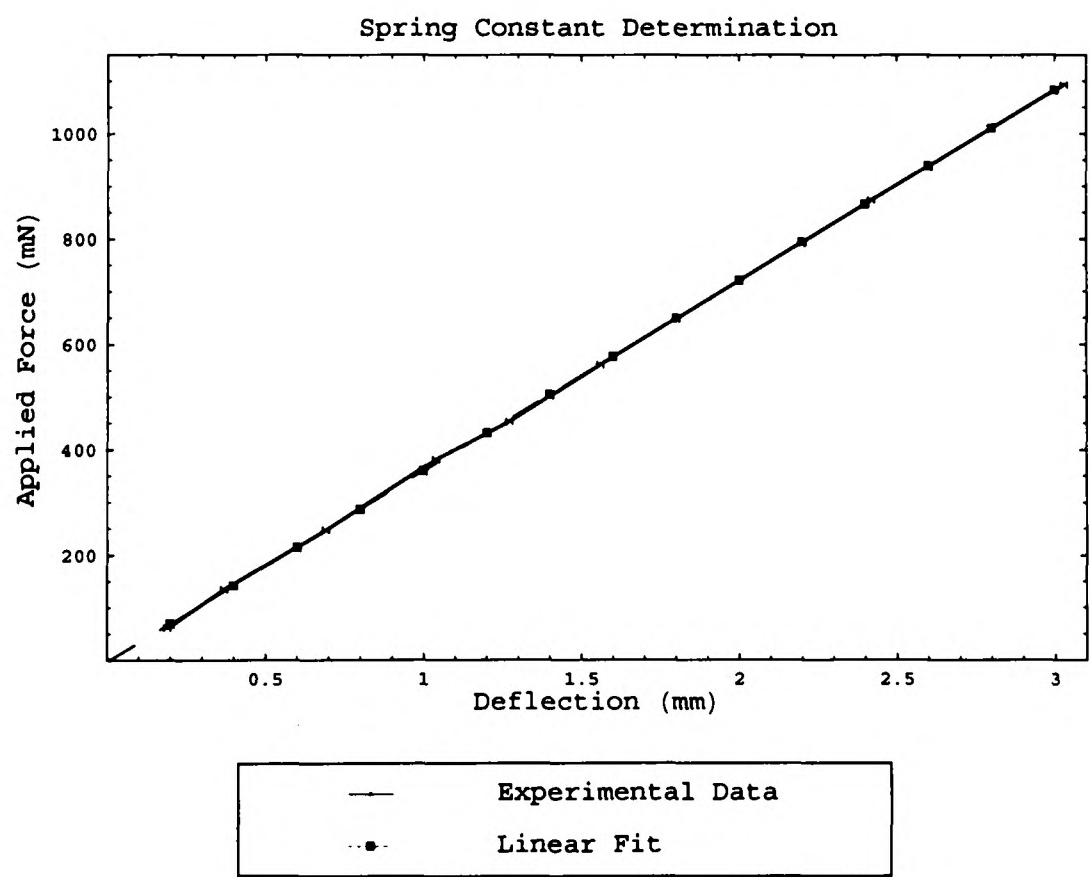


Fig. 5.5: Graph showing data from a spring constant determination experiment.

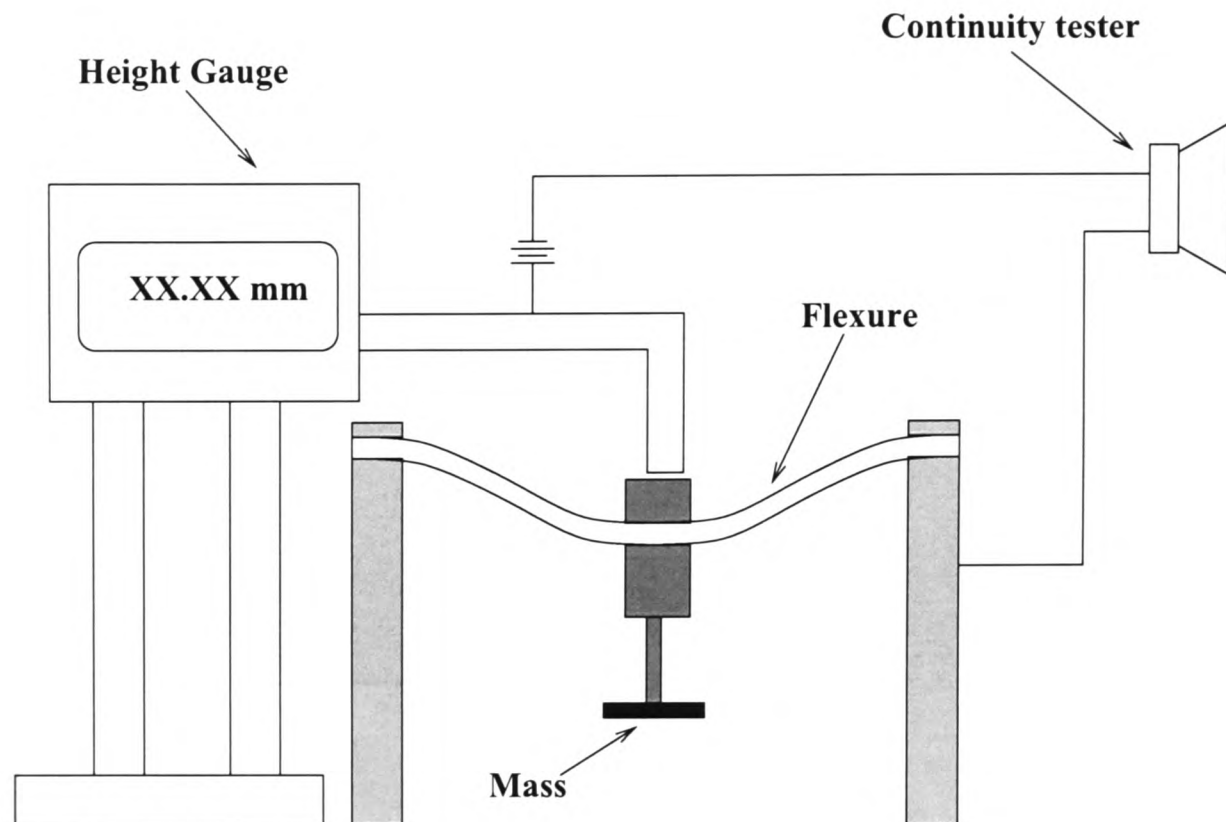


Fig. 5.6: Diagram of the experimental arrangement to measure the flexures' spring constants.

## 5.3 Construction of the linear motor

### 5.3.1 Motor efficiency equations

The linear motor design was based on equations derived from the harmonic oscillator model of the cooler derived earlier. The following section derives an efficiency equation for a linear motor in terms of its fundamental properties (magnetic field strength, resistivity of the coil material etc) this could then be used to predict the efficiency of any given design.

We can write equation 4.19 in terms of the fundamental properties of the motor (filling factor, coil length etc):

$$x_{pRes} = \frac{BfA}{Nj\omega} \epsilon_0 e^{j\omega t} \frac{1}{(\lambda_d \rho + B^2 f A 2\pi a n / N)} \quad (5.1)$$

Now the work done by the motor against any damping coefficient  $\lambda$  is given by:

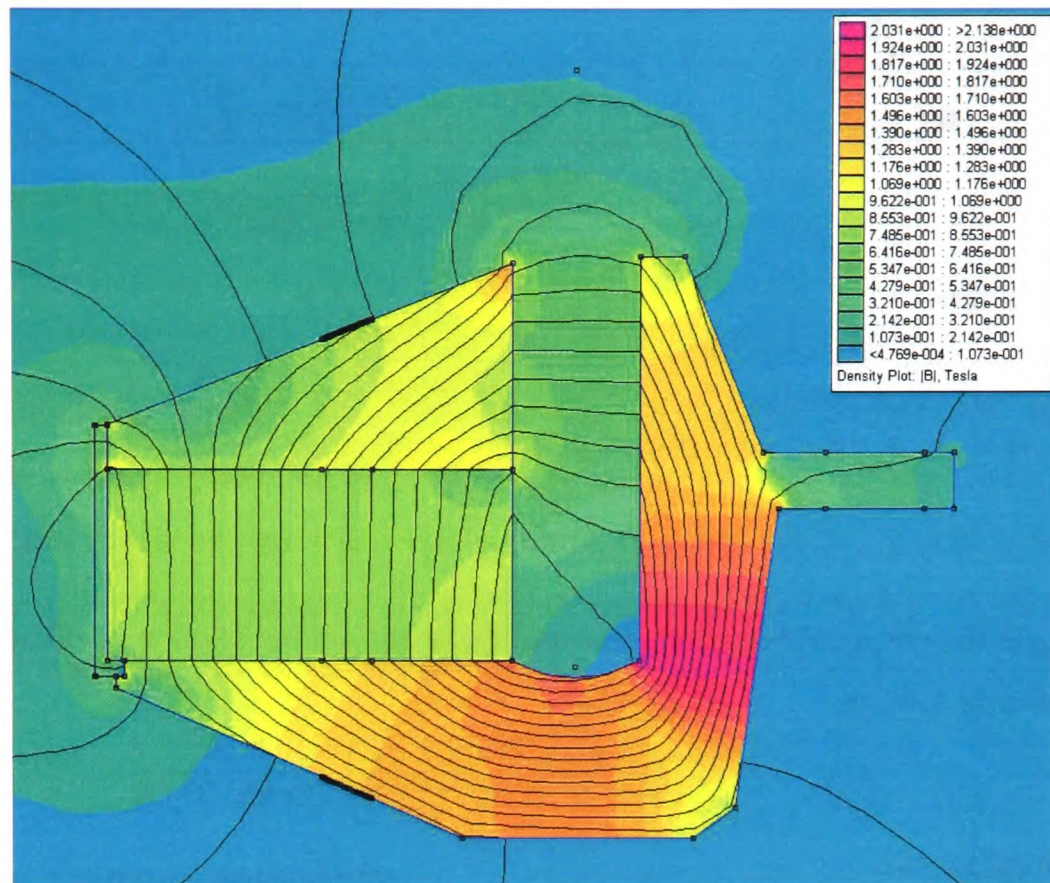
$$W_\lambda = \frac{1}{2} \text{Re}(\lambda \omega^2 X_{p0}^2) \quad (5.2)$$

So if we are concerned only with the work done on the gas we can write:

$$W_g = \frac{\lambda_g B^2 f^2 A^2 \epsilon_0^2}{2N^2 \lambda_d^2 \rho^2} \left( \frac{1}{1 + \frac{B^2 f A 2\pi a n}{\lambda_d \rho N}} \right)^2 \quad (5.3)$$

Now we derive an expression for the power dissipated in the coil,  $W_r$ . We begin by stating the expression for the current through the coil:

$$i = \frac{1}{R_c} (\epsilon_{v0} e^{j\omega t} - \dot{x}_p B l_b) \quad (5.4)$$



**Fig. 5.7:** Finite element simulation of magnet assembly.

Now we want to express equation 5.4 in terms of the fundamental properties of the motor. We do this by stating the following two equations and then substituting into 5.4:

$$l_b = 2\pi an \quad R_c = \frac{\rho 2\pi a N n}{f A}$$

With some re-arrangement we find that:

$$i = \frac{\lambda_d \epsilon_{v0} e^{j\omega t}}{(2\pi an)^2 B^2} \frac{1}{\left( \frac{N \lambda_d \rho}{B^2 f A 2\pi an} + 1 \right)} \quad (5.5)$$

The power dissipated in the coil is given by:

$$W_r = \frac{1}{2} \text{Re}(ii^* R_c) \quad (5.6)$$

Substituting 5.5 into 5.6 we find:

$$W_r = \frac{\epsilon_0^2 f A}{2(2\pi an) N \rho} \frac{1}{\left( \frac{B^2 f A 2\pi an}{N \lambda_d \rho} + 1 \right)^2} \quad (5.7)$$

The motor efficiency  $\eta$  is given by:

$$\eta = \frac{W_g}{W_g + W_r} \quad (5.8)$$

Or

$$\eta^{-1} = \frac{W_g + W_r}{W_g} = 1 + \frac{W_r}{W_g} \quad (5.9)$$

So we first find the ratio  $W_r/W_g$ :

$$\frac{W_r}{W_g} = \frac{\frac{\epsilon_0^2 f A}{2(2\pi a n) N \rho} \frac{1}{\left(\frac{B^2 f A 2\pi a n}{N \lambda_d \rho} + 1\right)^2}}{\frac{\lambda_g B^2 f^2 A^2 \epsilon_0^2}{2N^2 \lambda_d^2 \rho^2} \left(\frac{1}{1 + \frac{B^2 f A 2\pi a n}{\lambda_d \rho N}}\right)^2} \quad (5.10)$$

Which simplifies to:

$$\frac{W_r}{W_g} = \frac{N \rho \lambda_d^2}{(2\pi a n) B^2 f A \lambda_g} \quad (5.11)$$

Recalling that  $\lambda_d$  is the combined damping factor given by:

$$\lambda_d = \lambda_g + \lambda_{sl} \quad (5.12)$$

Where  $\lambda_g$  is the gas damping coefficient and  $\lambda_{sl}$  is the sliding friction coefficient. We can therefore write equation 5.11 as:

$$\frac{W_r}{W_g} = \left( \frac{\lambda_{sl}^2 + 2\lambda_g \lambda_{sl} + \lambda_g^2}{\lambda_g} \right) \frac{N \rho b^2}{(2\pi a n) B^2 f A} \quad (5.13)$$

Which simplifies to:

$$\frac{W_r}{W_g} = \left( \frac{\lambda_{sl}^2}{\lambda_g} + 2\lambda_{sl} + \lambda_g \right) \frac{N \rho b^2}{(2\pi a n) B^2 f A} \quad (5.14)$$

Recalling equation 5.9:

$$\eta^{-1} = 1 + \frac{W_r}{W_g} \quad (5.15)$$

Therefore:

$$\eta = \frac{1}{1 + \frac{W_r}{W_g}} \quad (5.16)$$

Substituting equation 5.14 into 5.15 we find:

$$\eta = \frac{1}{1 + \left( \frac{\lambda_{sl}^2}{\lambda_g} + 2\lambda_{sl} + \lambda_g \right) \frac{N \rho b^2}{(2\pi a n) B^2 f A}} \quad (5.17)$$

With good alignment during assembly of the cooler and running in of the piston bush, the sliding friction factor  $\lambda_{sl}$  will be negligibly small compared to the gas damping factor  $\lambda_g$  and so we can write:

$$\eta = \frac{1}{1 + \frac{\lambda_g N \rho}{2\pi a n B^2 f A}} \quad (5.18)$$

Recalling equation 5.2 it is clear that we may state:

$$\lambda_g = \frac{2W_g}{\omega^2 X_{p0}^2} \quad (5.19)$$

So we can write equation 5.18 as:

$$\eta = \frac{1}{1 + \frac{W_g N \rho}{A \pi a n B^2 f X_{p0}^2 \omega^2}} \quad (5.20)$$

Now “a” in the above equation is the average radius of the coil which we may write as  $a = r + \delta r/2$  where  $r$  is the radius of the inner pole pieces and  $\delta r$  is the width of the air gap. “A” represents the cross sectional area of the air gap so can be expressed as  $A = d\delta r$  where  $d$  is the pole piece height. Substituting into equation 5.20:

$$\eta = \frac{1}{1 + \left( \frac{W_g N \rho}{d\pi n B^2 f X_{p0}^2 \omega^2 \delta r (r + \delta r/2)} \right)} \quad (5.21)$$

An estimate of the required power into the gas ( $W_g$ ) needed for 200 mW of cooling at 80K is 3W and the required piston stroke,  $X_{p0}$  to produce this has been calculated as 2.5mm (see Chapter 4). Thus it can be seen from equation 5.21 that maximising the efficiency is a matter of minimising the following group when designing the motor (even if the sliding friction factor  $\lambda_{sl}$  is not considered negligible this is still true):

$$\frac{N \rho}{d\pi n B^2 f \omega^2 \delta r (r + \delta r/2)} \quad (5.22)$$

### 5.3.2 The magnet assembly

Initial estimates of the required magnet size were obtained from examining the de-magnetisation curves for possible magnet materials. The de-magnetisation curve for 44MGOe neodymium-iron-boron is shown below in figure 5.9. By tracing a line from the origin through the “knee” of the curve and seeing where it intercepts the top axis (the  $B/\mu_0 H$  ratio) it is possible to find the value of the B/H ratio that is required for the magnet to operate at its  $BH_{max}$  point. This is demonstrated in figure 5.8. The  $B/\mu_0 H$  ratio that a magnet operates at is completely defined by dimensional factors, not the material that it and the pole pieces are composed of. If the  $B/\mu_0 H$  ratio which a magnet will be operating at is to be calculated the total magnetic circuit must be considered.

As a starting point to deriving the  $B/\mu_0 H$  ratio at which a magnet will be operating at we consider the simple geometry of a torus of magnetic material of length  $l$  with a section cut out of it to produce an air gap of length  $g$ . The magnetomotive force in this magnetic circuit is zero [Duffin, W.J.(1998)] so we write:

$$H_g g = H_m l \quad (5.23)$$

Where  $H$  is the magnetic field strength, and the subscripts  $g$  and  $m$  represent gap and the magnetic material and  $g$  and  $l$  are the lengths of the air gap and magnetic material respectively. We state [Duffin, W.J.(1998)]:

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \quad (5.24)$$

If we initially assume no flux leakage:

$$B_m A_m = B_g A_g \quad (5.25)$$



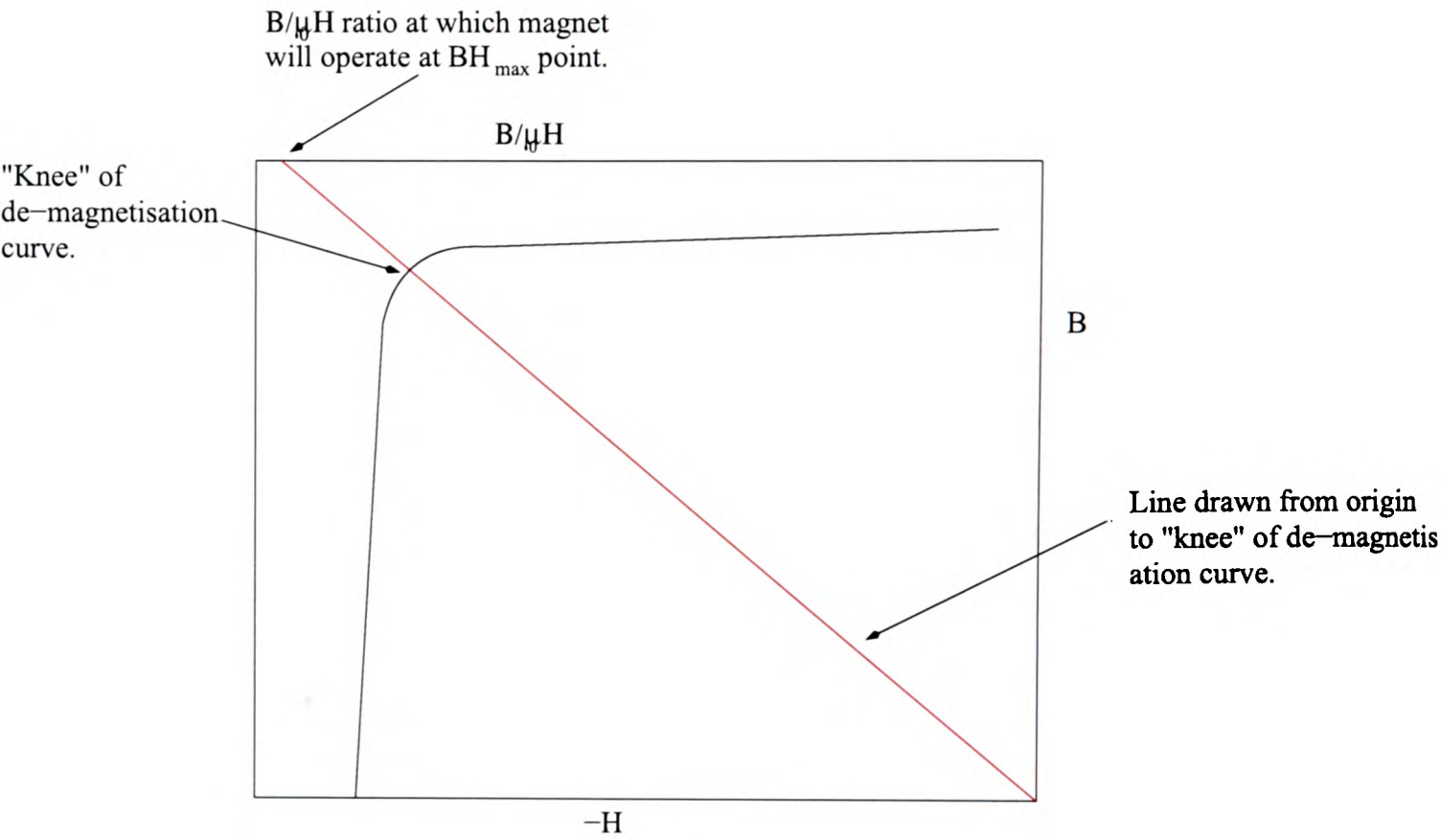


Fig. 5.8: Finding the  $B/\mu_0 H$  ratio.

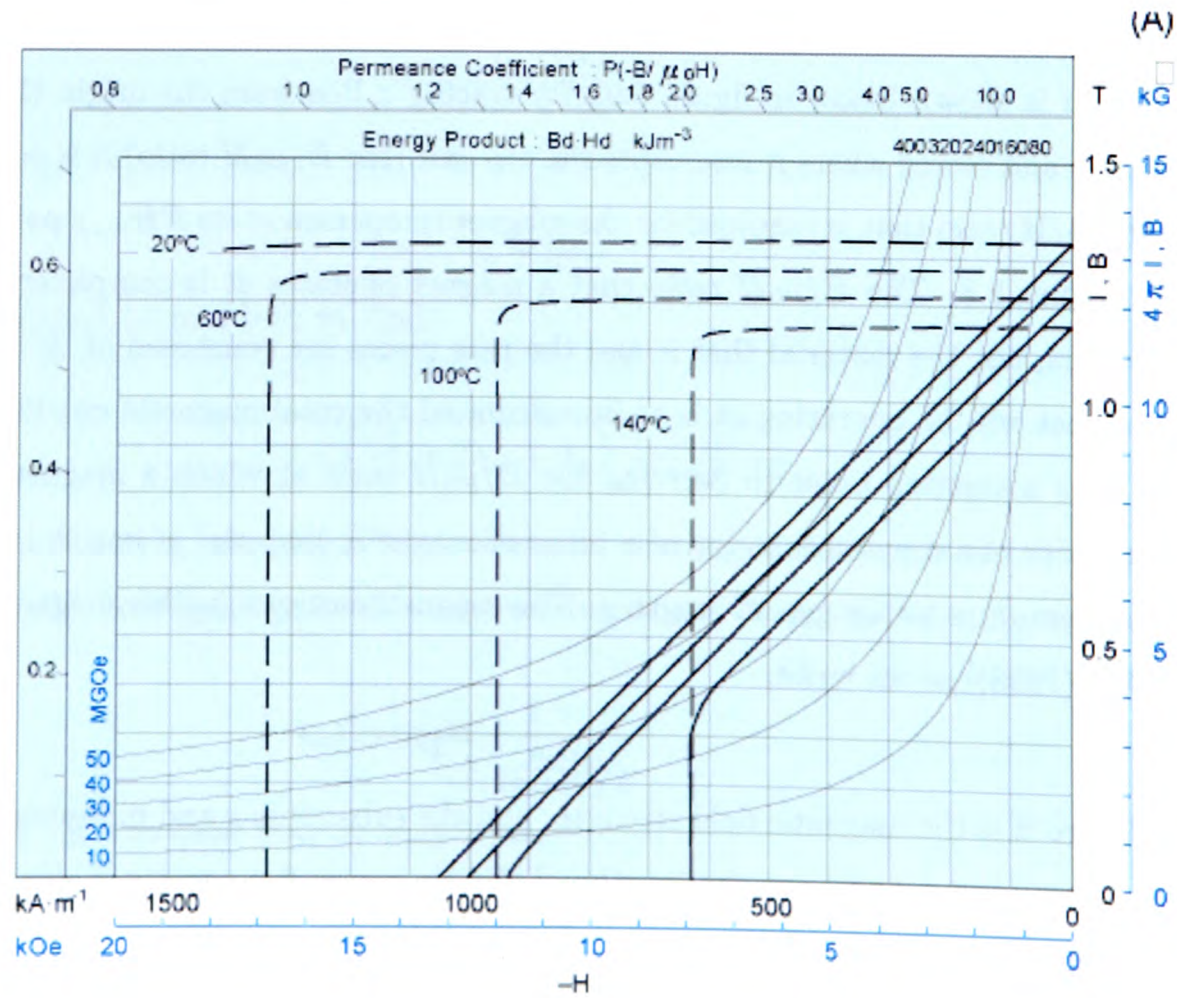


Fig. 5.9: Demagnetisation curve for 44MGOe neodymium-iron-boron (From Sumitomo Special metals [Sumitomo Inc. (2003)]).

Where  $B$  and  $A$  are flux densities and areas respectively. If we substitute these previous two equations into equation 5.23 we find:

$$H_m L_m = \frac{B_m A_m L_g}{\mu_0 A_g} \quad (5.26)$$

We then re-arrange this to give:

$$\frac{B_m}{\mu_0 H_m} = \frac{L_m A_g}{A_m L_g} \quad (5.27)$$

By substituting the various magnet assembly dimensions into equation we can find the  $B/H$  ratio of a particular magnet assembly design. We can then alter the design to make the ratio equal to that which will enable the magnet to operate at its  $BH_{max}$  point. In other words the  $B/\mu_0 H$  ratio can only be calculated once the dimensions of the magnetic circuit have been defined. Finite element software allows the dimensions of the proposed magnet design to be altered easily and the predicted effect on the air gap flux density to be observed. However the change in pole piece dimensions changes the  $B/\mu_0 H$  ratio of the magnet in the circuit and so the  $B/\mu_0 H$  ratio should be re-calculated each time the design is changed.

Finite element simulations were used to provide estimates of the average magnetic field in the motor air-gap for each design. The software used was F.E.M.M. (Finite Element Method Magnetics) a two dimensional finite element solver that is freely available<sup>1</sup>. The equation for the efficiency of the linear motor in terms of the air gap width, coil height etc. (equation 5.21) has been incorporated into a Mathematica program that enables the output of the finite element simulations to be used directly in the calculation of the motor efficiency. This is done by instructing the finite element program to output values of the flux density along the path of the motor coil's motion and then storing them in a text file. Mathematica can then read out this data and average the flux density over the coil's motion. This can then be used to calculate the motor efficiency by substituting it into the equation for the motor efficiency along with the dimensions of the magnet assembly. The program then plots the motor efficiency over a suitable range of coil lengths and finds the coil length that will provide the maximum efficiency. This has enabled a magnet assembly and coil to be designed with a predicted efficiency of approximately 80%. Figure 5.7 shows a typical finite element simulation of the magnet assembly. Once an optimum design had been finalised the magnet was purchased. It was a neodymium-iron-boron sintered magnet with a quoted  $BH_{max}$  value of 44MGOe. Previous coolers developed at AOPP have used samarium cobalt (SmCo) magnets in their linear motors. A neodymium-iron-boron (NdFeB) magnet was chosen for the new cooler because of the advantages these magnets have over SmCo magnets which are summarised in table 5.2 [Hanitsch, R. (1989)]. The pole pieces for the linear motor were machined by the author to the dimensions of the optimum finite element simulation and are made of pure iron. It

<sup>1</sup>F.E.M.M. can be downloaded from <http://femm.foster-miller.net/>

| Property                          | NdFeB                | SmCo:              |
|-----------------------------------|----------------------|--------------------|
| Maximum energy product $BH_{max}$ | 25 – 45 $MGOe$       | 16 – 32 $MGOe$     |
| Residual flux density ( $B_r$ )   | 10.3 – 13.0 $KG$     | 8.2 – 11.6 $KG$    |
| Intrinsic coercive force          | 11.0 – 25.0 $KOe$    | 6.2 – 20.0 $KOe$   |
| Density                           | 7.3 – 7.5 $g/cm^3$   | 8.2 – 8.4 $g/cm^3$ |
| Resistance to cracking            | Solid, hard to break | Brittle            |
| Deflection strength               | 25 $kg/mm^2$         | 15 $kg/mm^2$       |
| Compressive strength              | 110 $kg/mm^2$        | 82 $kg/mm^2$       |
| Tensile strength                  | 7.5 $kg/mm^2$        | 3.6 $kg/mm^2$      |

Table 5.2: Comparison of NdFeB and SmCo magnets.

would be possible to increase the efficiency and/or reduce the mass of the pole pieces by using Vanadium Permendur, however this is an expensive material and is extremely difficult to machine (it is nick-named “crack-alloy” because of its extreme proclivity to crack during machining) and so it was decided that the time and funds that would have to be expended to produce pole pieces out of this material was not feasible for a developmental cooler. Figure 5.11 is a photo of assembled magnet and pole pieces.

## 5.4 Effect of cryogenic temperatures on magnet performance

The performance of permanent magnets can be greatly effected by temperature ([Trout S.R. (2004)] and [Trout S.R. (2001)]) and generally deteriorates as temperature increases. As temperature decreases there are beneficial effects on magnetic performance. The flux output from neodymium-iron-boron magnets increases as temperature decreases, however at 135K NdFeB undergoes a spin re-orientation (the preferred direction of magnetization changes), which would result, for example, in a reduction in the flux density in the air gap of a motor assembly. The change in flux density with temperature for *NdFeB* and *SmCo* magnets is illustrated in figure 5.10[Trout S.R. (2004)].

### 5.4.1 Measurements of the magnet assembly’s flux density

Once the magnet had been assembled with its pole pieces, a gaussmeter was used to measure the flux density in the air gap. The results are shown, along with the predicted output from the finite element simulation, in figure 5.12. It was immediately apparent that the measured flux density was significantly below that predicted by the finite element simulation. To rule out the possibility



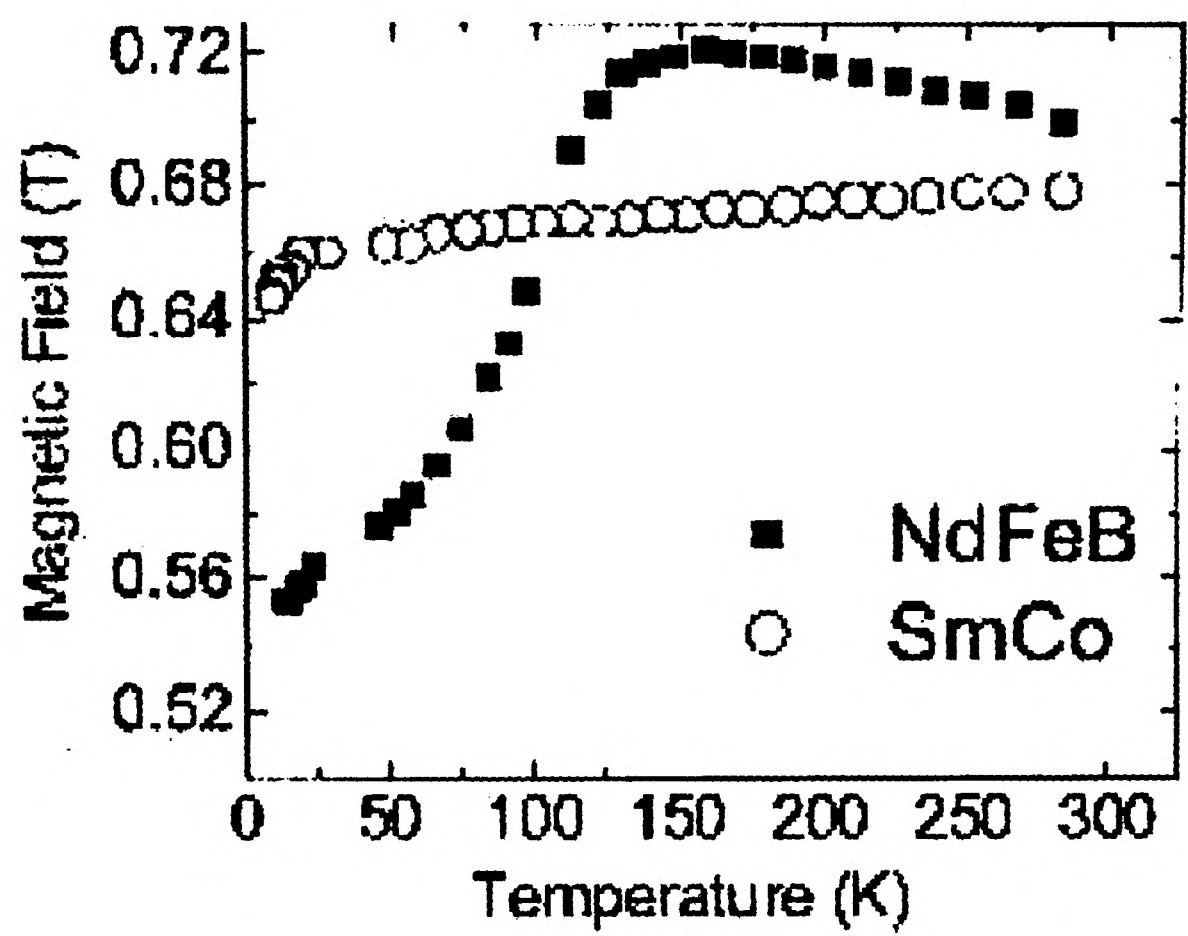


Fig. 5.10: Effect of cryogenic temperatures on permanent magnets.

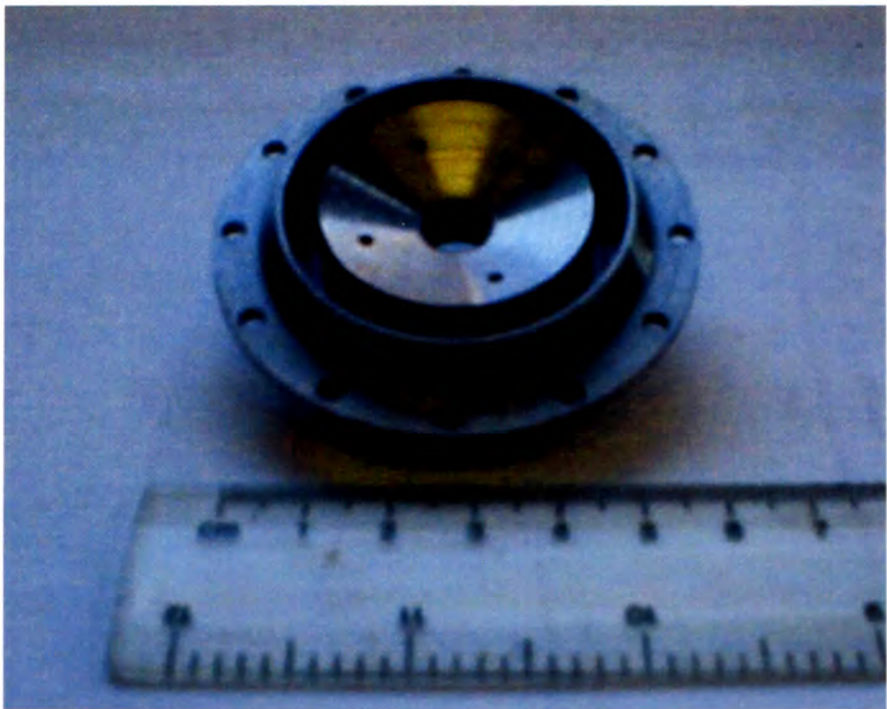


Fig. 5.11: Photograph of the magnet assembly.

that it was the gaussmeter “under-reading” two further gaussmeters were used to corroborate the measurements. Further measurements showed that the flux density around the magnet when it was not assembled with the iron pole pieces was below that predicted by finite element simulations of the same situation, furthermore the leakage flux around the magnet assembly was lower than that predicted by the simulations which suggested that the low air gap flux density was not due to a higher than expected amount of flux leakage through parts of the pole pieces away from the air gap. After repeated measurements of the flux density in the air gap it became apparent that the mismatch between the predicted and measured flux densities was not caused by the measuring equipment or a larger amount of flux leakage than expected. This seemingly left two possible causes; the magnet itself was, for some reason, not behaving as expected or the software was producing inaccurate predictions. To ensure that the magnet was in fact a sintered neodymium-iron-boron magnet with a  $44\text{MgOe } BH_{max}$  the magnet was taken back to the suppliers (MMG MAGDEV, Swindon U.K.) who confirmed its composition but identified only partial magnetisation of the neodymium-iron-boron during the manufacturing process as the most likely cause of the lower than expected flux density. To improve the air gap flux density the assembly was placed within a capacitive magnetiser which at its centre has a coil capable of producing a maximum flux density of 3.2 Tesla. The re-magnetisation had a distinct effect on the performance of the magnet and this is distinctly shown in figure 5.13 which shows the flux measurements before and after re-magnetisation. After re-magnetisation the flux density in the assembly’s air gap was found to lie within approximately ten percent of that predicted by the finite element software. This is illustrated in figure 5.14. Although the flux density in the air gap was still lower than the finite element simulation had predicted, it had originally been assumed that the simulations would only be accurate to within 10% and therefore it was decided to proceed with the manufacture of the remaining cooler components.

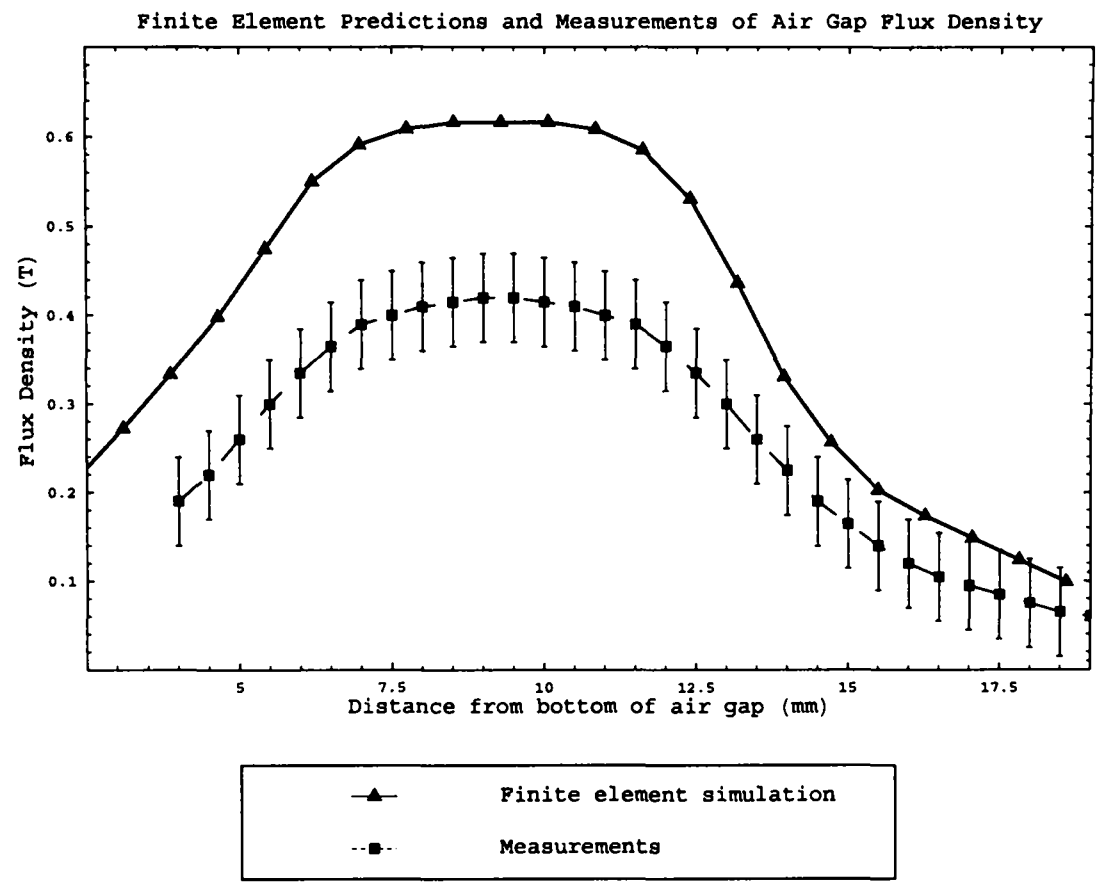


Fig. 5.12: Comparison of measured flux density to values predicted by the finite element software.

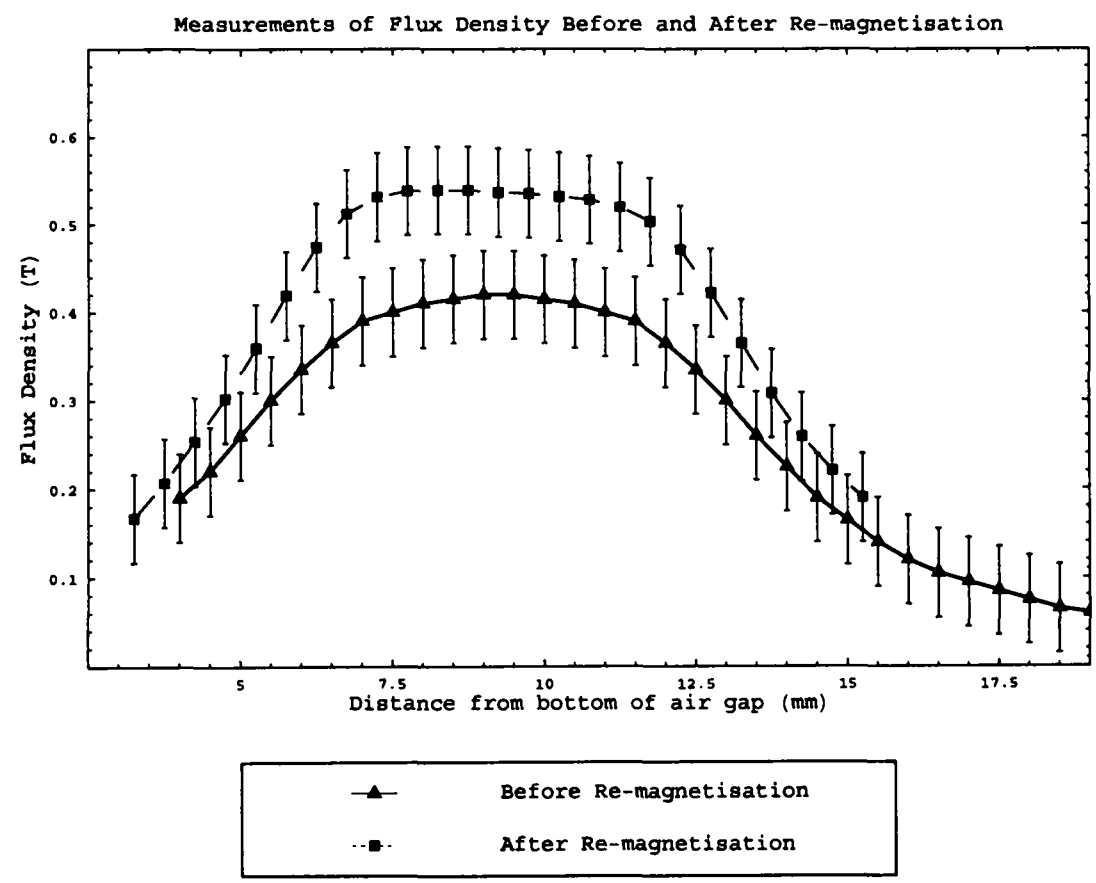


Fig. 5.13: Comparison of flux density before and after re-magnetisation.

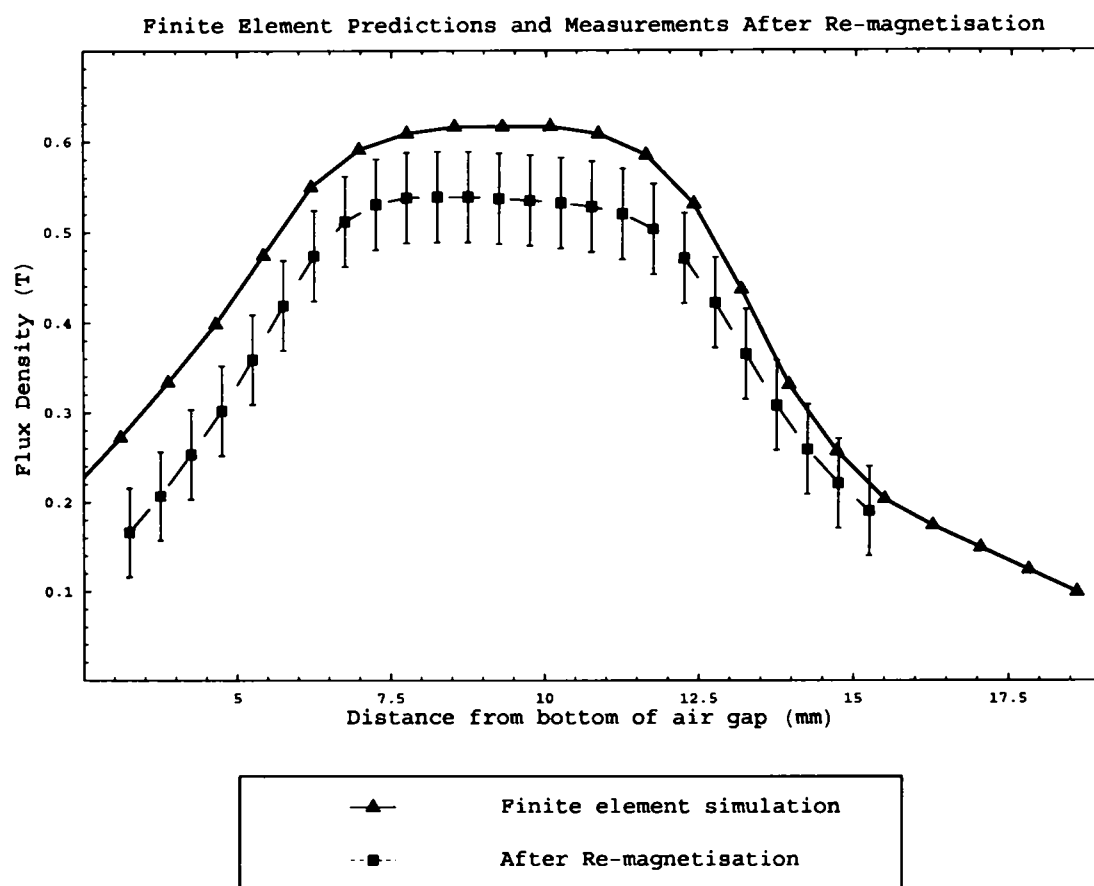
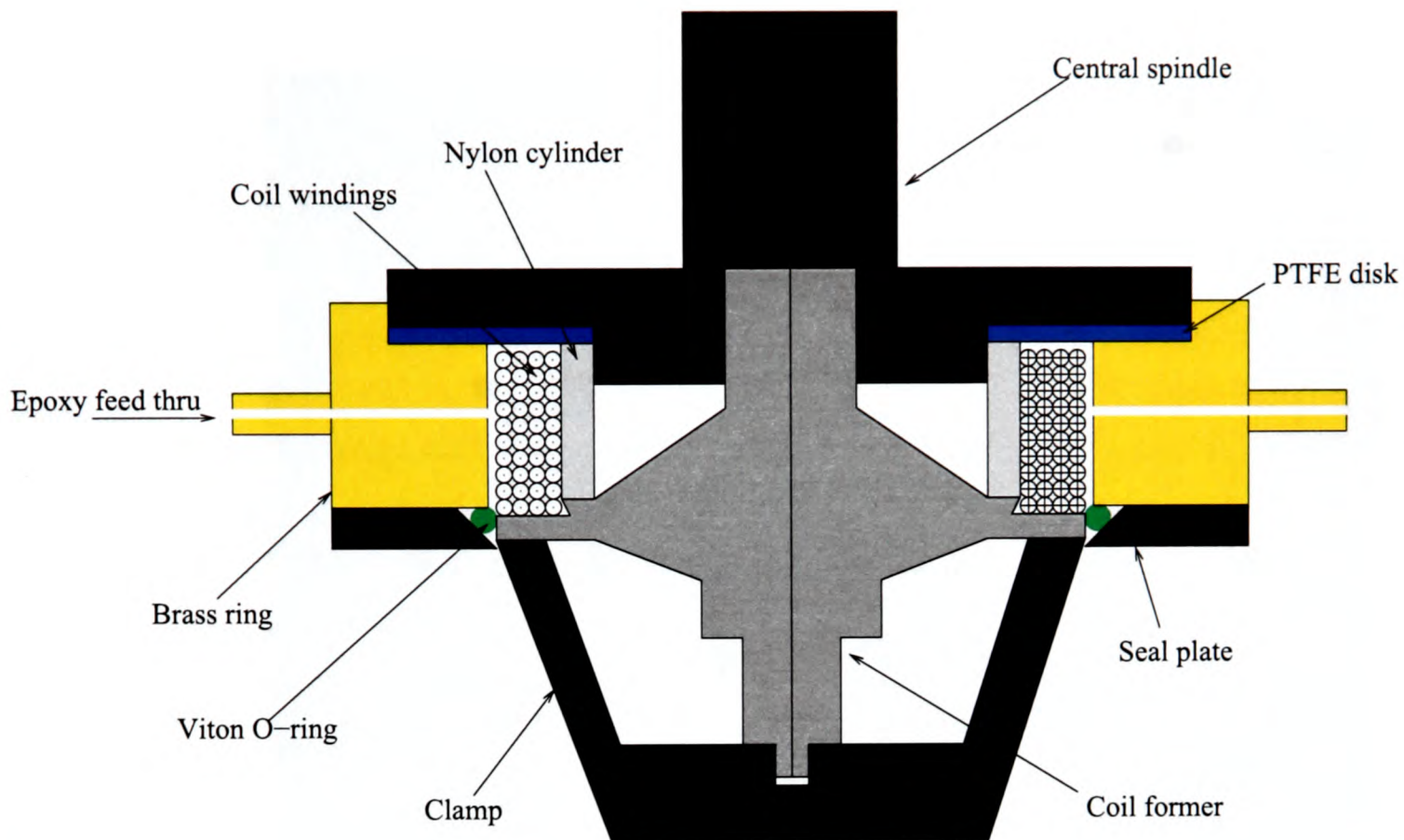


Fig. 5.14: Comparison of measured flux density after re-magnetisation to predicted values.

#### 5.4.2 The motor coil and coil former

As mentioned in section 5.3.2 a Mathematica program was combined with the output of the finite element software to enable the optimum coil length to be calculated for a given magnet assembly design. Taking this length and the width of the air gap defined by the design of the magnet assembly, the dimensions of the coil are defined. The coil is free standing in that the wire is encased in epoxy which holds it in shape once cured. The new coil is significantly different in size from those used on the ISAMS coolers and this necessitated constructing a jig to enable the winding and potting of the coil.

Figure 5.15 shows the coil former within the potting assembly, also indicated are the coil windings. This drawing is not to scale and the number of coil turns does not bear any relation to the number that were wound although it does illustrate how the potting will take place. The coil is wound onto the nylon ring which is located on the central spindle and successive layers built up until the correct number of turns have been wound. The wire is then led up specially designed grooves in the coil former. The assembly is then completed by sealing off the volume where the coil has been wound using a seal plate, Viton O-ring and clamp. This volume can then be flooded with epoxy which is then heated until it has cured. Heating is performed by placing the potting assembly in a metal framework (not shown in figure 5.15) to which large resistors are secured.

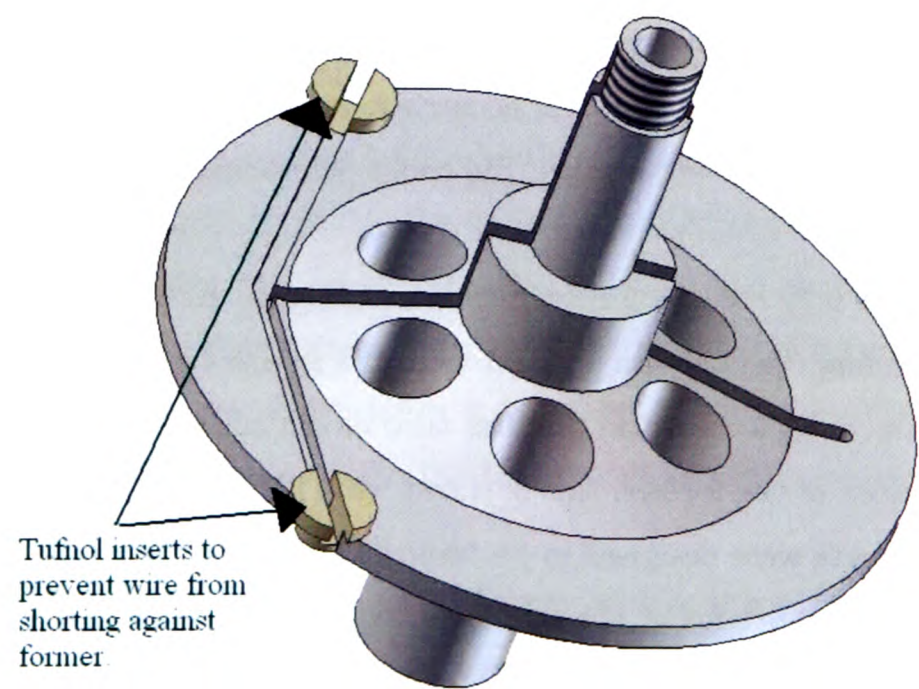
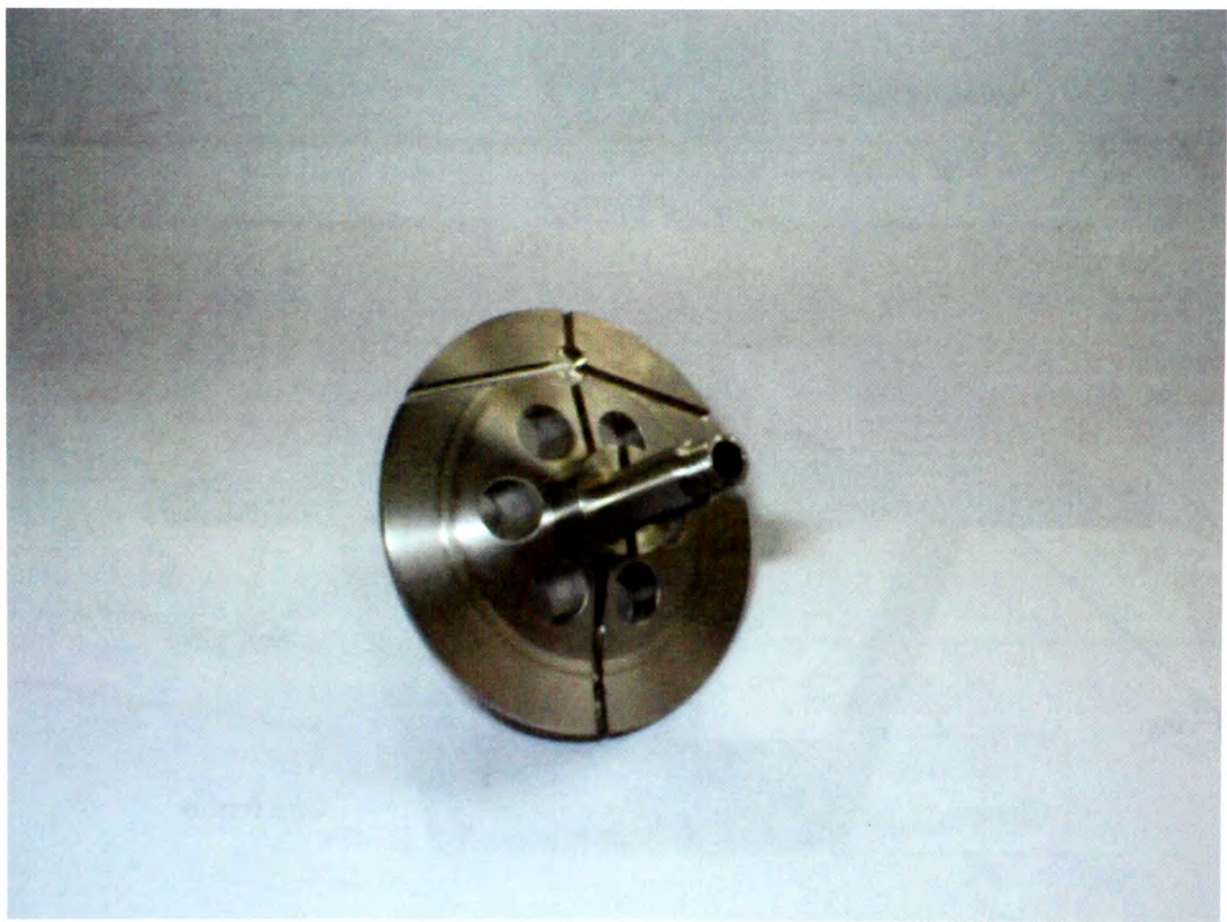


**Fig. 5.15:** Coil potting assembly.

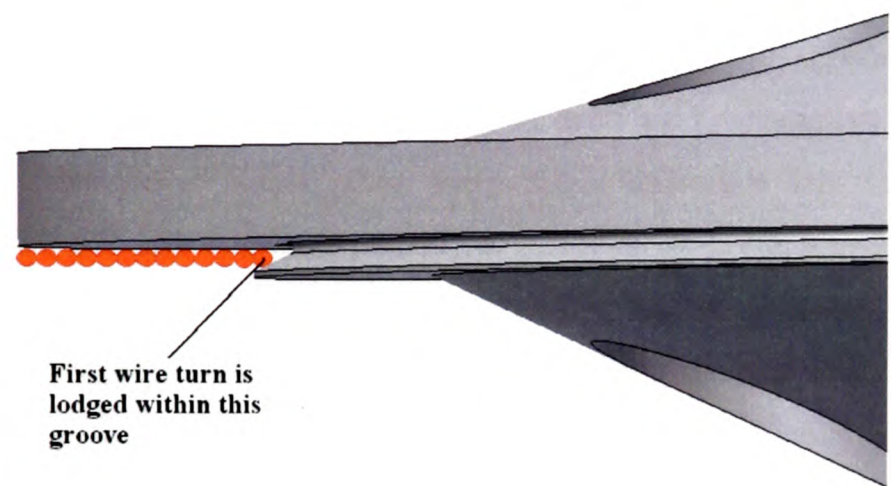
Current is then passed through these resistors to provide a heat source. The epoxy is de-gassed before potting the coil to reduce the risk of air bubbles occurring. If air bubbles are present in the epoxy around the coil serious damage can occur when the cooler is evacuated before being filled with helium.

As mentioned earlier (Section 5.2) the coil former has been designed to allow the coil to be connected to the springs by running the coil wire up grooves in the former (see figure 5.16, not to scale). The former has also been designed so that the first turn of the coil is held firmly in place by a groove cut into the underside of the former, this is shown on a CAD drawing in figure 5.17. Epoxy cotton (Tufnol 6f/45) inserts were designed to be inserted into drilled holes at the end of the grooves where they meet the outer edge of the coil former (see figures 5.16 and 5.18). This was done because the coil wire must be bent up round the edge of the coil former and into the groove, which presents a very real possibility that the insulating varnish on the coil wire will be broken and so the coil would short onto the coil former. The addition of the Tufnol inserts means that the coil is only bent over the Tufnol inserts which will prevent any shorting at this point. Figure 5.18 shows the coil former during a test winding, annotations show the Tufnol inserts and various other features of the coil and former. Figure 5.17 shows a side view of the coil former with the underside magnified so that the mechanism by which the coil is “keyed” onto the coil former is shown.

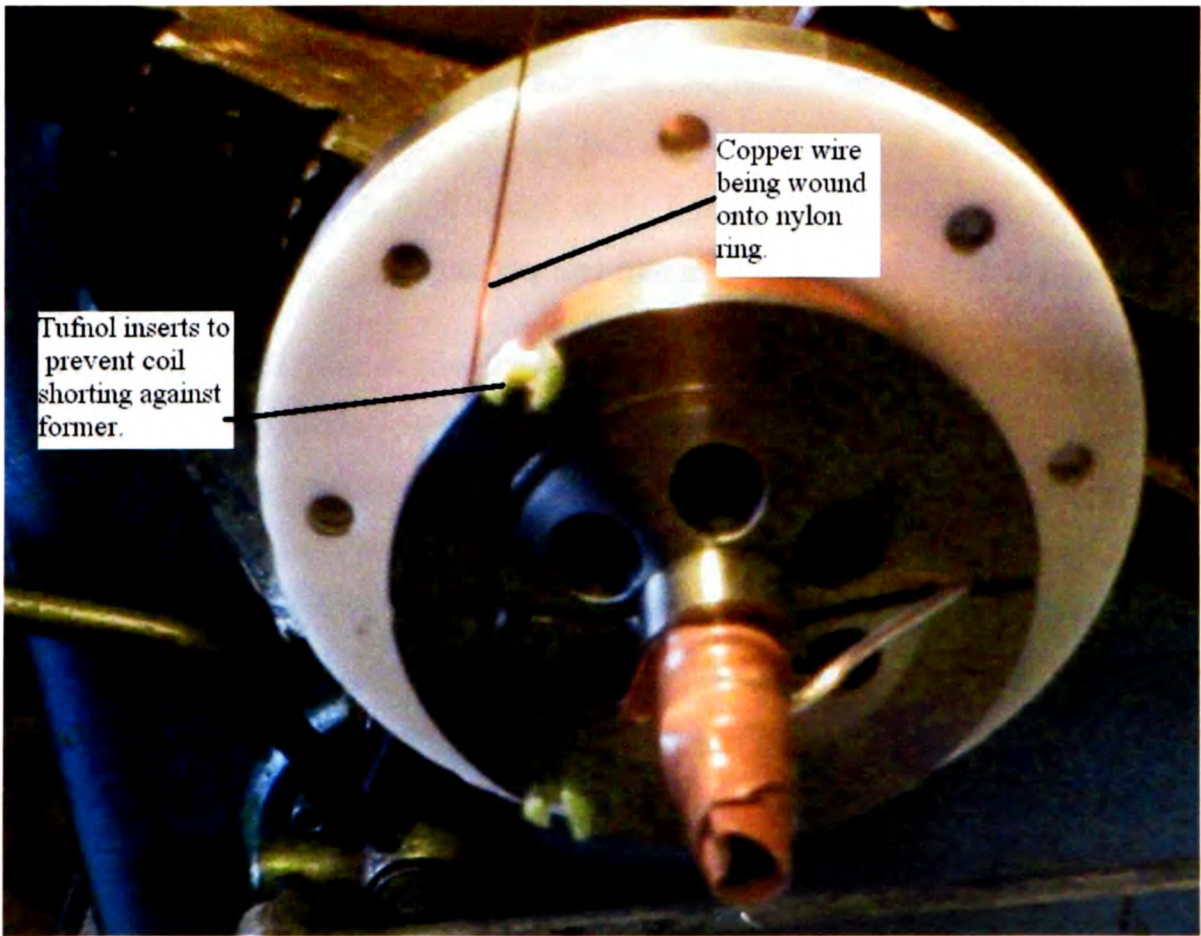




**Fig. 5.16:** Photo of the former without the inserts (top) 3-D graphic of the coil former with Tufnol inserts (bottom).



**Fig. 5.17:** View of the coil former showing how the coil wire is “keyed” onto the coil former (not to scale).



**Fig. 5.18:** Photo of the coil former during a test winding of the coil.

## 5.5 Improvements to the coil potting jig

During assembly and initial assembly of the cooler it became clear that there was a significant degree of rubbing occurring when the piston was driven by the linear motor. It was found that the Flurosint piston bush was not concentric with the coil former and so was not centrally located within the compression cylinder. An attempt to rectify this situation was made and this involved machining an oversize piston bush (one that could fit properly over the piston but that was too large to fit in the cylinder). This bush could then be machined so that its outer surface (that would be in contact with the compression space walls) would be concentric with the coil former. It was thought that this could be achieved by placing the bush over the piston and then screwing the piston into the coil former. The coil former would be held in a lathe and the piston bush then lightly machined to give an outer surface concentric with the coil former. Unfortunately during this process the coil was irreparably damaged. At first it was believed that the coil could be removed from the coil former with acetone and a new coil wound and potted onto the former, however closer inspection revealed that the coil former had been bent at the same time that the coil was damaged and so could not be reused.

### 5.5.1 The second coil potting jig

Whilst causing a significant delay to the progress of developing the prototype the damage to the coil and former did allow the lessons learnt in the construction of the first coil to be put to immediate use in designing an improved coil winding jig. The design of the new jig is shown in a simplified form below. By comparing figure 5.15 to figure 5.19 it can be seen that there are some major differences between the first and second coil potting jigs:

1. *Epoxy feedthroughs*

In the original jig the feedthroughs were brazed into the brass ring, which meant that after curing hardened rods of epoxy were left in these feedthroughs connected to the mass of epoxy around the coil. Removing the brass ring by simply pushing it away from the central spindle would involve snapping these rods and this risked damaging the epoxy around the coil. To avoid this the feedthrough pipes were sawn off where they emerged from the brass ring and then the brass ring itself was machined away to allow access to the coil. Obviously this was an extremely wasteful method of producing the coil as it meant destroying part of the jig. When the second coil potting jig was designed it was decided to find a design in which all the components could be preserved. This was achieved by moving the epoxy feedthroughs from the circumference of the brass ring to the rear of the stainless steel centre piece. After the coil had been potted it could be removed and any hardened epoxy in the feedthroughs would



be able to be pulled out of the feedthroughs and removed from the coil by hand. The epoxy was prevented from adhering to the inside of the feedthroughs by lining them with PTFE.

## 2. *Double O-ring seal*

An extra O-ring was included in the second jig design to ensure epoxy could not flow into the central void of the jig between the centre piece and the titanium former.

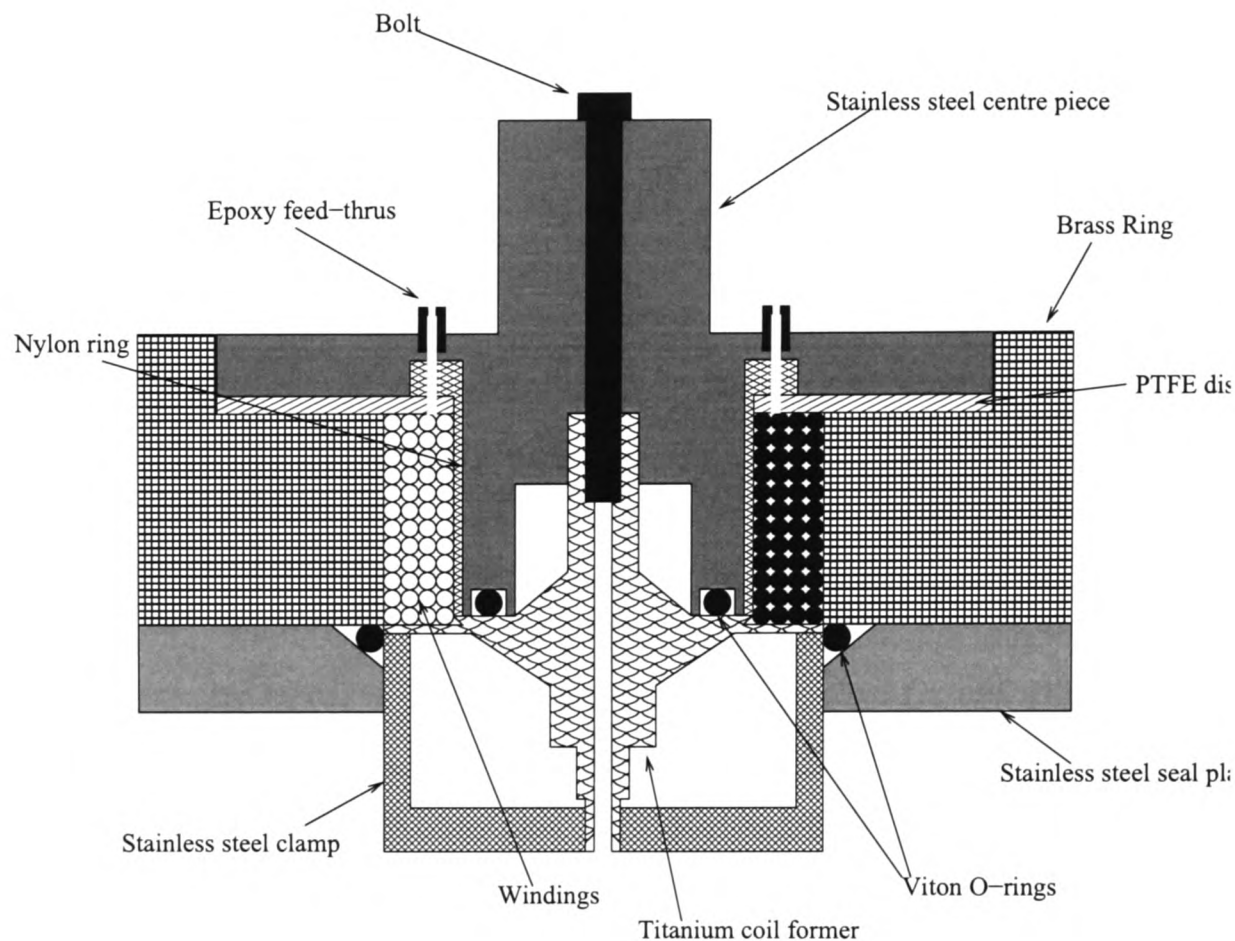
## 3. *Concentricity and squareness*

Whereas in the original jig the coil former was pushed up against the surface of the nylon-ring in the second jig the coil former is pushed up against the centre piece. The nylon ring was a lot more flexible than the steel centre piece and so could not allow the coil former to be held square to the centre piece to the degree that the arrangement in the second jig allowed. In the second jig the nylon ring was first screwed in place onto the centre piece after the centre piece had been machined but before it had been removed from the lathe. The outer diameter of the nylon was then machined to size. This method ensured that the outer diameter of the nylon ring was concentric with all the cylindrical surfaces on the centre piece. In the original jig the nylon ring was machined separately from the centre piece and was pushed on over it which meant that the degree of concentricity between the outer diameter of the nylon ring and the centre piece was poor.

# 5.6 Miniature radiator

## 5.6.1 Introduction

At the beginning of the third year of the development of the prototype cooler at AOPP, Professor Deping Dong of the Shanghai Institute of Technical Physics (SITP) joined the department and agreed to develop a radiator suitable for coupling with the cooler. After three months he returned to SITP but a second Chinese visitor, Professor Guotong Hong of the Technical Institute of Physics and Chemistry (TIPC) continued his work. The design and construction of the radiator was carried out by Professor Dong and Professor Hong. Some assistance was given by the author during the design process as it was necessary to ensure the radiator could be easily fixed to the cooler and mounted within the vacuum tank. The author was also involved in the manufacture of the radiator cone, cooler interface patch and space view. What follows is a brief description of the radiator and how it connects to the radiator, but detailed description of the design of the radiator is not included as it was not performed by the author as part of the D.Phil.



**Fig. 5.19:** Simplified diagram of the second coil potting jig.

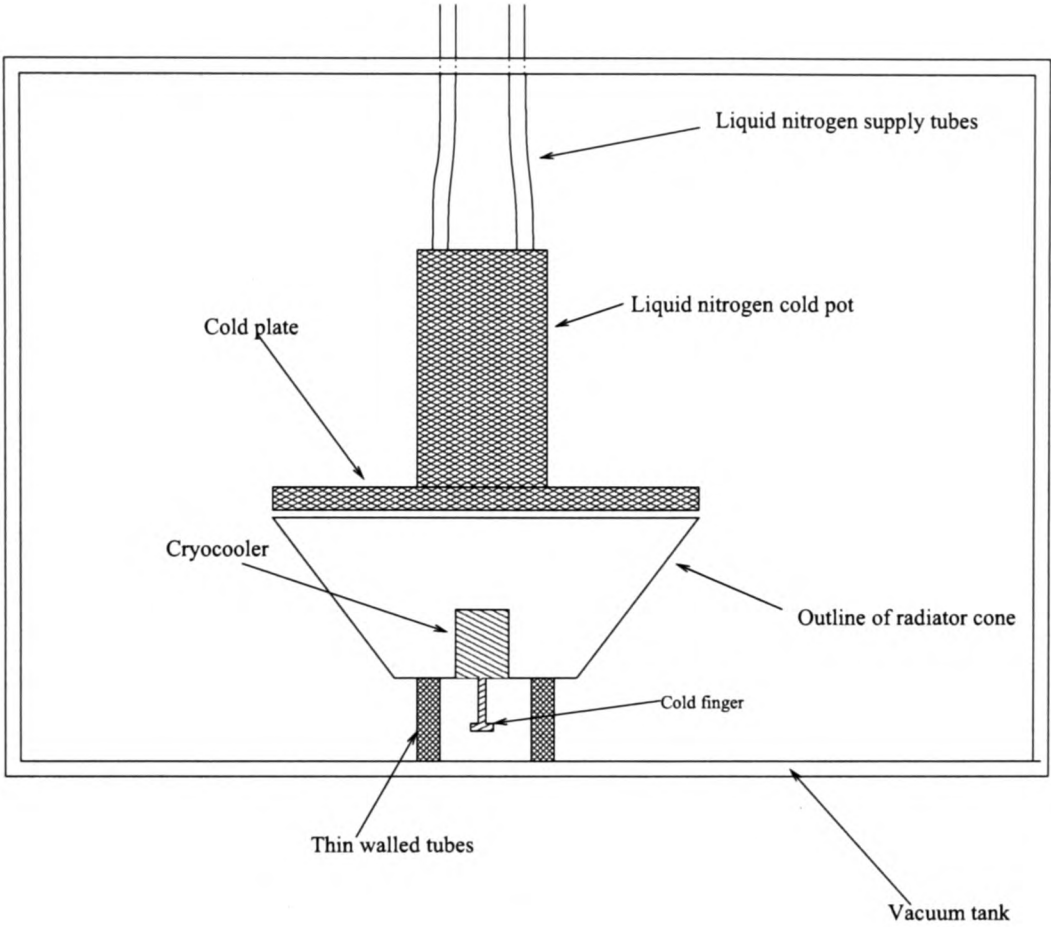
### 5.6.2 Design and installation of the radiator

The basic layout of the radiator, cooler and space view within the vacuum tank is shown in figure 5.20 and a photo of the same can be seen in figure 5.21. The space view was simply a copper plate brazed to the bottom of a brass reservoir suspended from the vacuum tank ceiling. The reservoir was filled with liquid nitrogen from outside the vacuum tank to cool the copper plate down.

## 5.7 Summary

A design for a miniature integral Stirling cooler was developed that utilised spiral flexures not only as a suspension system for the moving coil linear motor but also as electrical feedthroughs. The linear motor itself (along with the rest of the cooler) was of an entirely new design which utilised a NdFeB magnet rather than a SmCo of the type used in all previous coolers developed at AOPP. Difficulties were encountered in producing the required motor coil but these were overcome by improving the coil potting jig. The spiral flexures were designed using a Mathematica program developed for that purpose, and were photo-etched from spring steel. The spring constants of these flexures were determined by experiment.

A miniature radiator for pre-cooling the cooler was designed and fabricated and a “space view”



**Fig. 5.20:** Simplified diagram of the radiator/cold plate arrangement (not to scale).

| Property                         | Value for new cooler                        | Value for existing cooler   |
|----------------------------------|---------------------------------------------|-----------------------------|
| Cooler Type                      | Miniature Integral Stirling/radiator hybrid | Miniature Integral Stirling |
| Mass (radiator and cooler)       | 2.69Kg                                      | N/A                         |
| Cooler height                    | 174mm                                       | 229mm                       |
| Cooler diameter (Max.)           | 78mm                                        | 141mm                       |
| Radiator width (across diagonal) | 480mm                                       | N/A                         |
| Total height                     | (cooler and radiator) 200mm                 | 229mm                       |

**Table 5.3:** Summary of cooler and radiator properties.

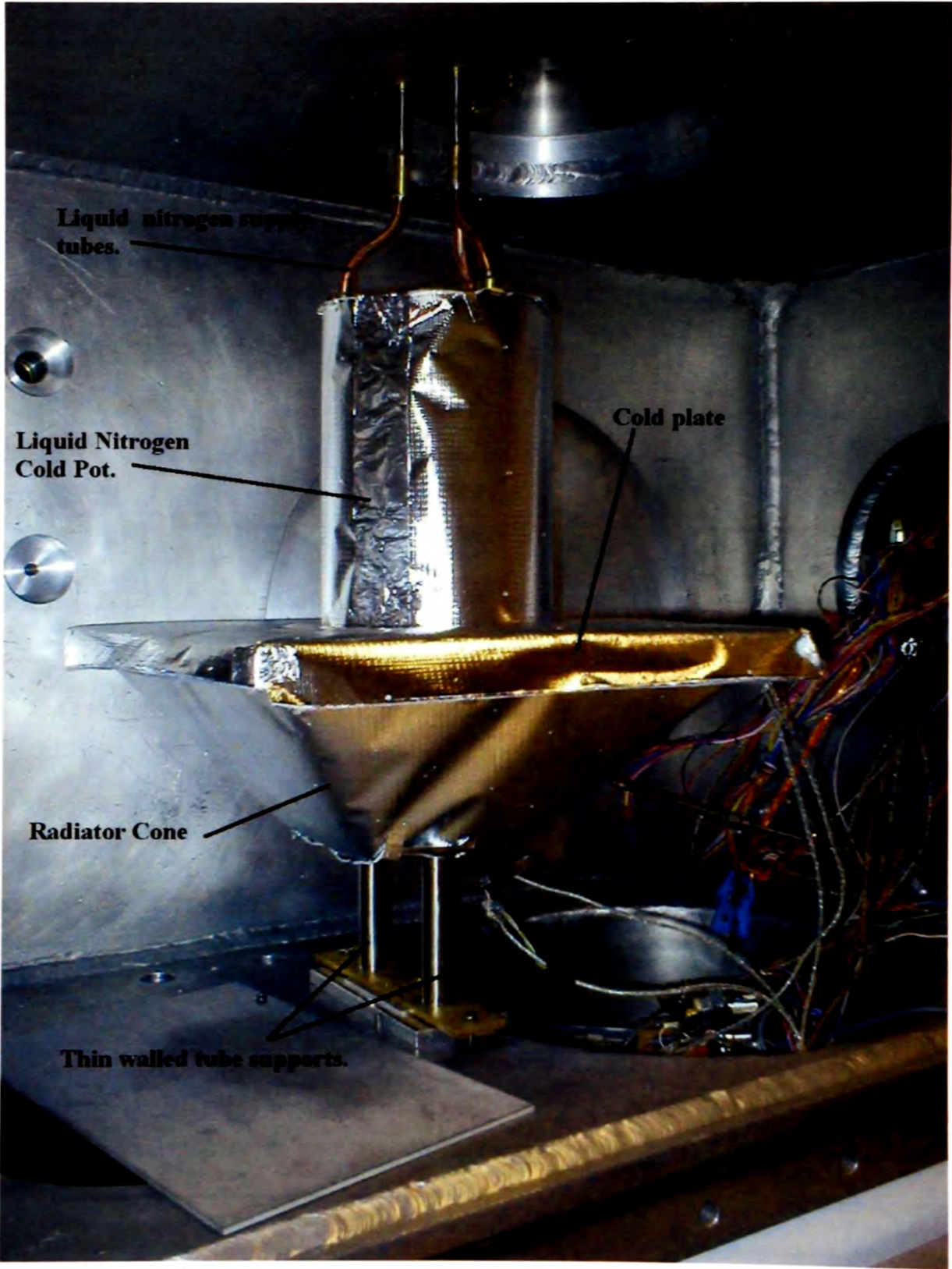


Fig. 5.21: Photo of the radiator and cold plate arrangement within the vacuum tank.



to which the radiator could reject heat was created and installed within the vacuum chamber. Table 5.3 gives an overview of the cooler and radiator properties. Heat was rejected from the compression space to the radiator through the titanium walls of the compression cylinder. The low thermal conductivity of titanium and the need for there to be heat transfer across the wall (from the warm working fluid to the radiator at 200K) means that there must be a temperature gradient across the wall. Unfortunately the design did not allow for a temperature sensor to be mounted within the compression space and so this gradient could not be measured. Peters [Peters, M. (1994)] calculated the temperature gradient through the compression cylinder walls in the ISAMS coolers as 63K but this calculation required the temperature of the working fluid in the expansion space to be known (rather than the temperature of the copper cold tip itself) as well as the magnitude of each and every loss process within the cooler. This information is not known for the new cooler and so the same analysis cannot be performed. If, however, we assume that all the work done on the working fluid by the compressor is lost as heat by conduction through the compression cylinder walls to the radiator patch (during the pre-cooled runs) or to the aluminium cylinder that supports the cooler during room temperature runs we can make an estimate of the temperature gradient. Equation 5.28 [Holman, J.P. (1986)] describes one-dimensional heat flow ( $q$ ) through a hollow cylinder of length  $L$ , inner radius  $r_i$  and outer radius  $r_o$  with thermal conductivity  $k$ , this is shown in figure 5.22.

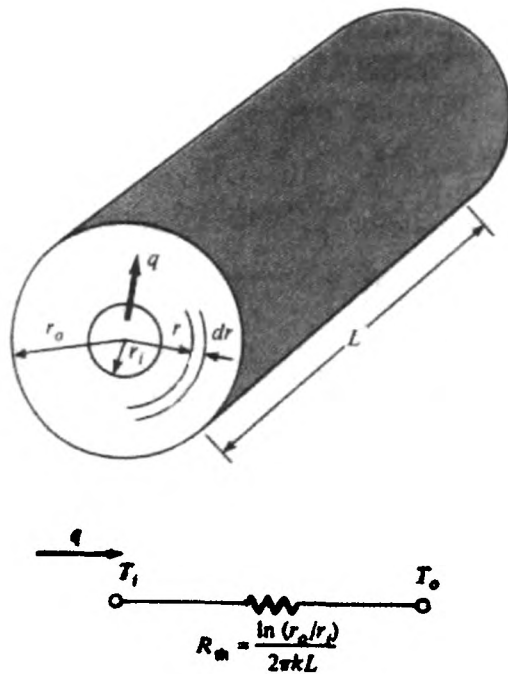
$$q = \frac{2\pi k L \Delta T}{\ln\left(\frac{r_o}{r_i}\right)} \quad (5.28)$$

This can be re-arranged to give:

$$\Delta T = \frac{q \ln\left(\frac{r_o}{r_i}\right)}{2\pi k L} \quad (5.29)$$

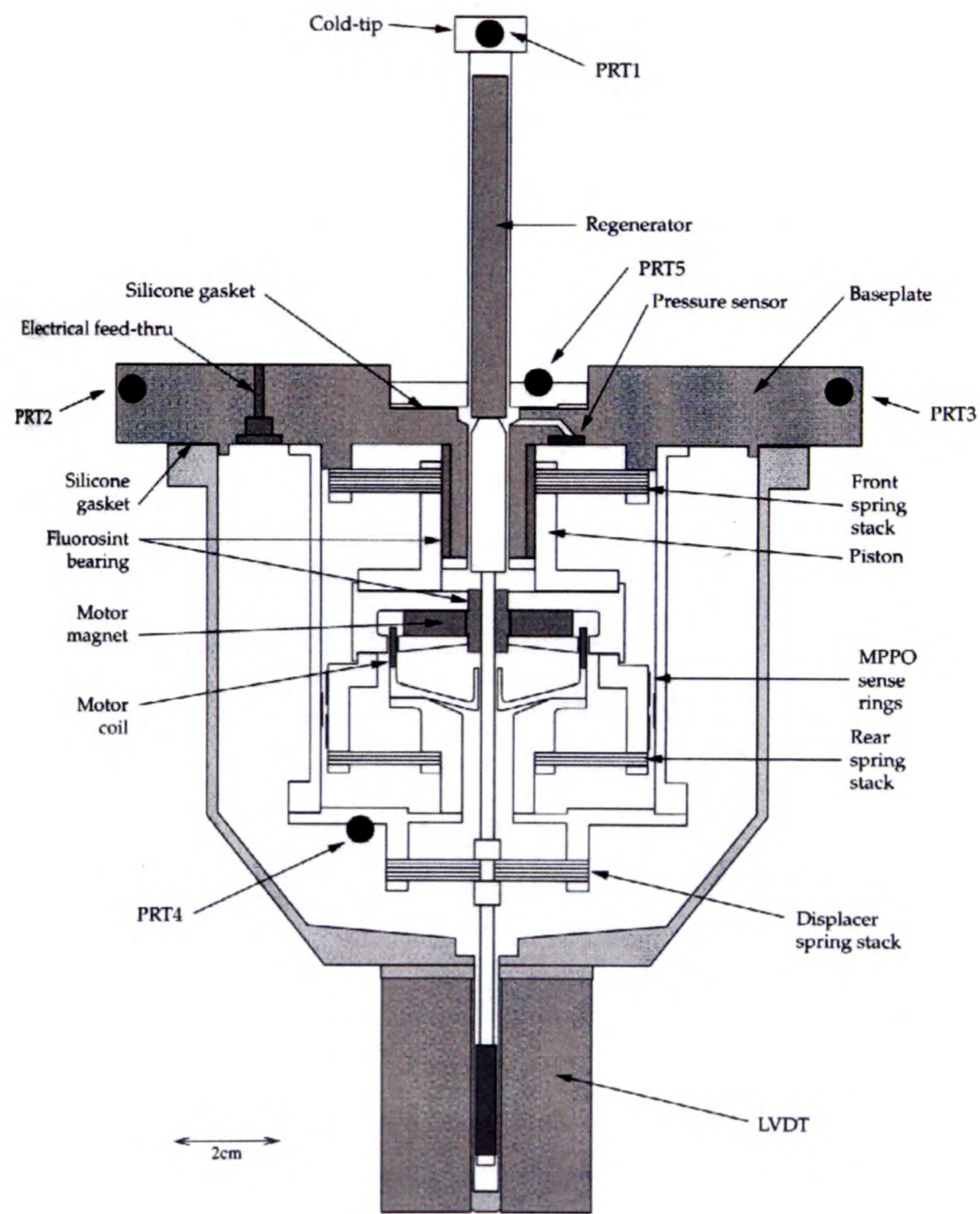
Applying equation 5.29 to the titanium compression space cylinder we find that the temperature difference between the inner and outer surfaces of the compression cylinder is approximately 3.8K. However if we also include heat conduction through the piston itself to the working fluid behind the piston (in the cooler body containing the linear motor etc) this rises to 8.8K although this does assume that the temperature difference along the piston length and through the cylinder walls are the same. This brief analysis neglects any conduction along the cold finger or through the displacer shaft so can only be regarded as a first step in estimating the temperature difference between the inside of the compression space and the external wall. However it can be seen from equation 5.29 that increasing the thermal conductivity of the cylinder walls would aid in the conduction of heat away from the working fluid in the compression space. This could be achieved by using aluminium instead of titanium (typically increasing the thermal conductivity by a factor of ten) for the compression space cylinder although this might then introduce thermal contraction issues.

Compared to the prototype cooler created by Reed [Reed, J. (2004)] the current cooler has a



**Fig. 5.22:** Basic geometry used to model the temperature gradient through the compression cylinder walls. After [Holman, J.P. (1986)].

smaller maximum diameter and has a reduced length. Reed's cooler utilised a Samarium-Cobalt magnet whereas the new cooler uses a Neodymium-Boron-Iron magnet as these have a higher energy density. What is more Reed's cooler simply re-used a linear motor originally created for the ISAMS coolers whereas the new cooler has a newly designed motor. The same is true for the regenerator, Reed's prototype used one created for the ISAMS cooler, the new cooler's regenerator was the product of detailed modelling (see subsection 4.2.4 and section 4.3) and was constructed by the author during this research effort. Figure 5.23 shows Reed's cooler and figure 5.24 shows the design of the new cooler for comparison. It's clear that the design of the two cooler's are very different, for example it can be seen that Reed's cooler is of a moving magnet design whereas the new cooler uses a moving coil compressor (see section 5.1 for the advantages of moving coil compressors). Table 5.3 compares various aspects of the two coolers.



**Fig. 5.23:** Schematic of the existing prototype cooler created by Reed [Reed, J. (2004)].

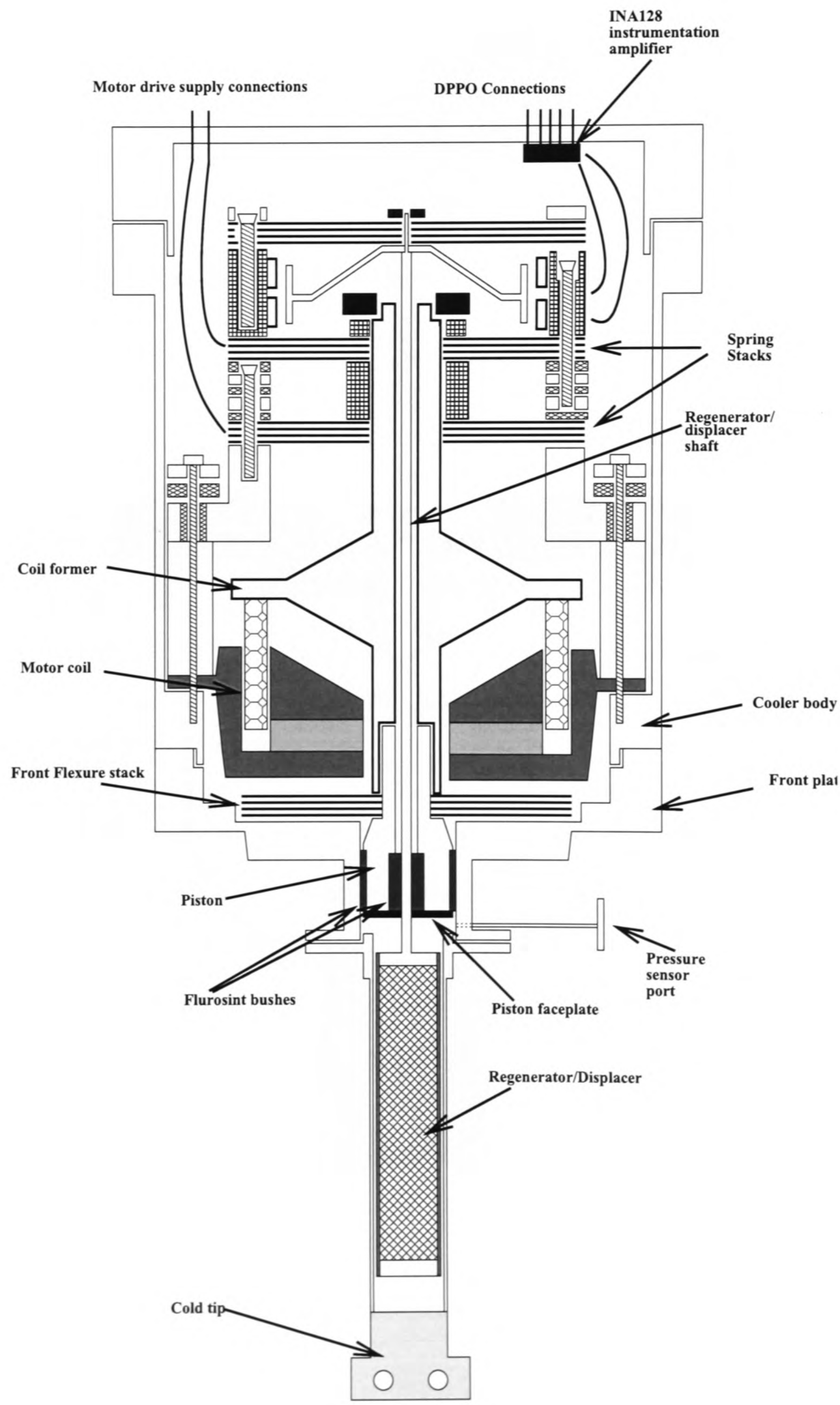


Fig. 5.24: Schematic of the new cooler.



## Chapter 6

# Instrumentation and support systems

### 6.1 Introduction

To assess the cooler's performance adequately requires a range of sensors. These measure values such as the pressure within the cooler compression space, temperature at various points in the cooler and the positions of the oscillating components within the cooler (the displacer and the piston). The data received from these sensors needs to be readily available for analysis and so a data logging system is also required. This chapter describes the design and construction of the sensors and the data logging system.

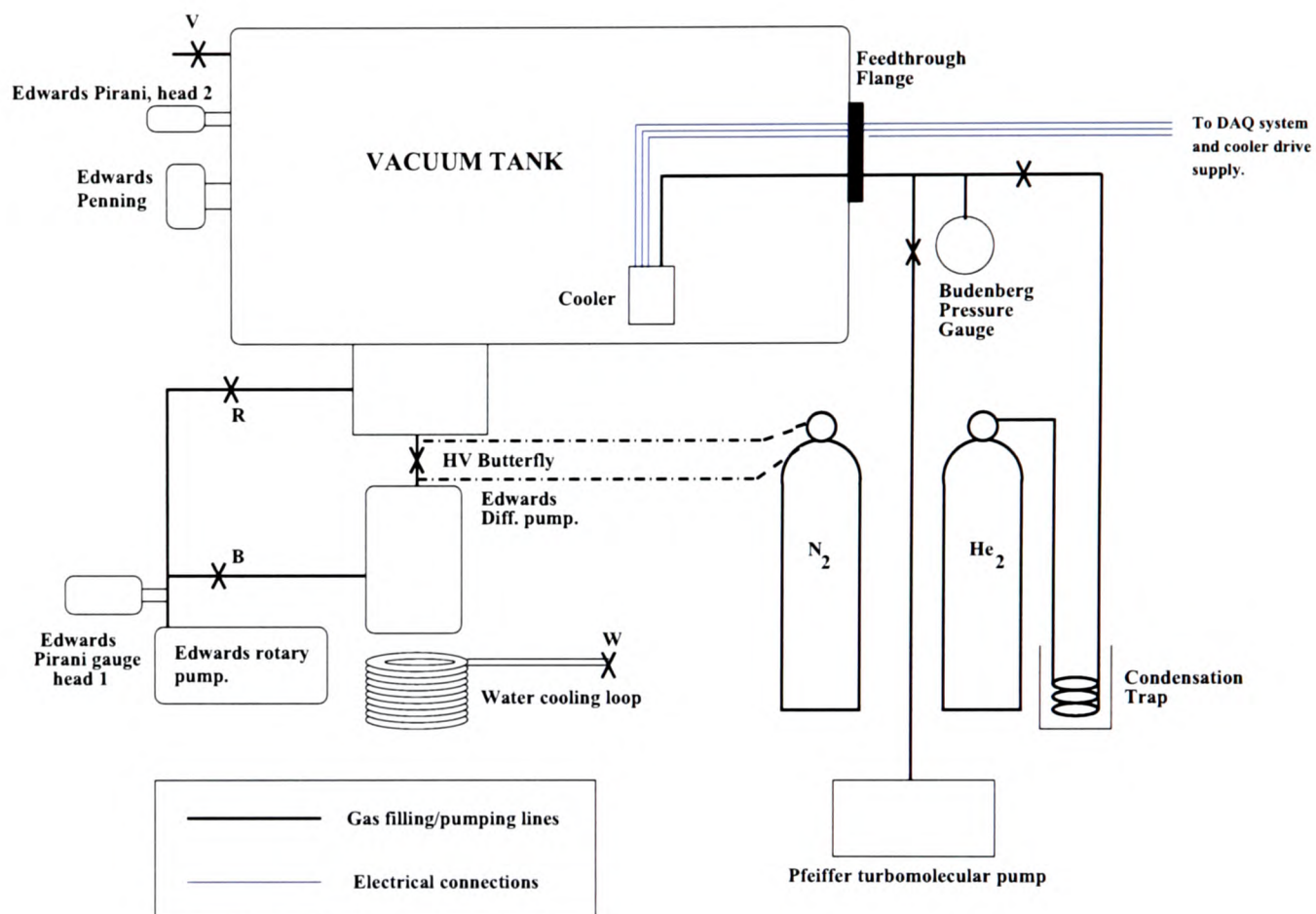
### 6.2 Data acquisition

Data is gathered from the sensors described above using a PCI-DAS1000 data acquisition card manufactured by Measurement Computing Inc. This card has 12 bit resolution, 16 single-ended or 8 differential inputs, and a maximum sample rate of 250KHz <sup>1</sup>. The DAQ card was calibrated using the supplied calibration software (Instacal<sup>tm</sup>).

The data acquisition software was written using the Labview<sup>tm</sup> graphical programming environment. The software was implemented on a desktop PC and allowed the operator to select which sensor outputs to sample, the sampling rate, sampling period, and time interval between sampling periods. The software produced two files for each sampling period; one file contained the

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<sup>1</sup>This is the maximum sample rate if only one channel is being sampled, if all 16 are sampled then the maximum sampling rate is 15.625KHz per channel



**Fig. 6.1:** Diagram of the vacuum tank and filling rig.

un-calibrated (“raw”) sensor outputs and the other contained the calibrated outputs. Each file was automatically given a unique name which included a time stamp. A typical file name for a file containing calibrated sensor outputs is :“calibrated\_channels.05 – 11 – 30.1655.lvm”. The software was also written so that it continually displayed numeric and graphical data on the PC monitor in real time which allowed the operator to continuously monitor the cooler performance.

### 6.3 Vacuum tank and filling rig

Figure 6.1 is a diagram of the vacuum tank and filling rig used during this research effort. It had been used in previous cooler research within AOPP but required some refurbishment before being put back into use (replacing o-ring seals, creating fixtures to hold the radiator and space view etc). The tank was evacuated using a rotary pump (Edwards E1M40) to rough out the tank and then to back a diffusion pump (Edwards 160/700F Diffstak). It was found that the pressure could reach approximately  $5 \times 10^{-5}$  mbar when the cooler was also evacuated (the cooler was evacuated using a Pfeiffer turbomolecular pump and could reach a pressure of approximately  $3 \times 10^{-5}$  mbar). When the cooler was filled with dried helium to a pressure of 7 bar the tank pressure was seen to increase

and the pressure that could be maintained in the tank increased to approximately  $8 \times 10^{-5}$  mbar indicating that the tank ultimate pressure was dominated by a leak from the cooler. This was not unexpected as elastomeric rather than metal seals were being used for ease of disassembly and re-usability. Brito [Brito, M. (1999)] derived the following expression for the heat transfer to the cold tip of a cooler from its surroundings:

$$Q_{vac} = 0.598A\sqrt{\frac{R}{M}}\frac{\Delta T}{\sqrt{T_w}}P \quad (6.1)$$

Where A is the surface area of the cold tip, M is the molar mass of the residual gas in the vacuum tank, R is the gas constant,  $T_w$  is the temperature of the surroundings,  $\Delta T$  is the temperature difference between the cold tip and the surroundings. Assuming that the dominant residual gas in the vacuum tank is helium we find that for a value of  $\Delta T$  equal to 214K (from 294K-80K) the heat load is less than a milliwatt at pressures of  $1 \times 10^{-4}$  mbar and so can be considered negligible.

## 6.4 Instrumentation

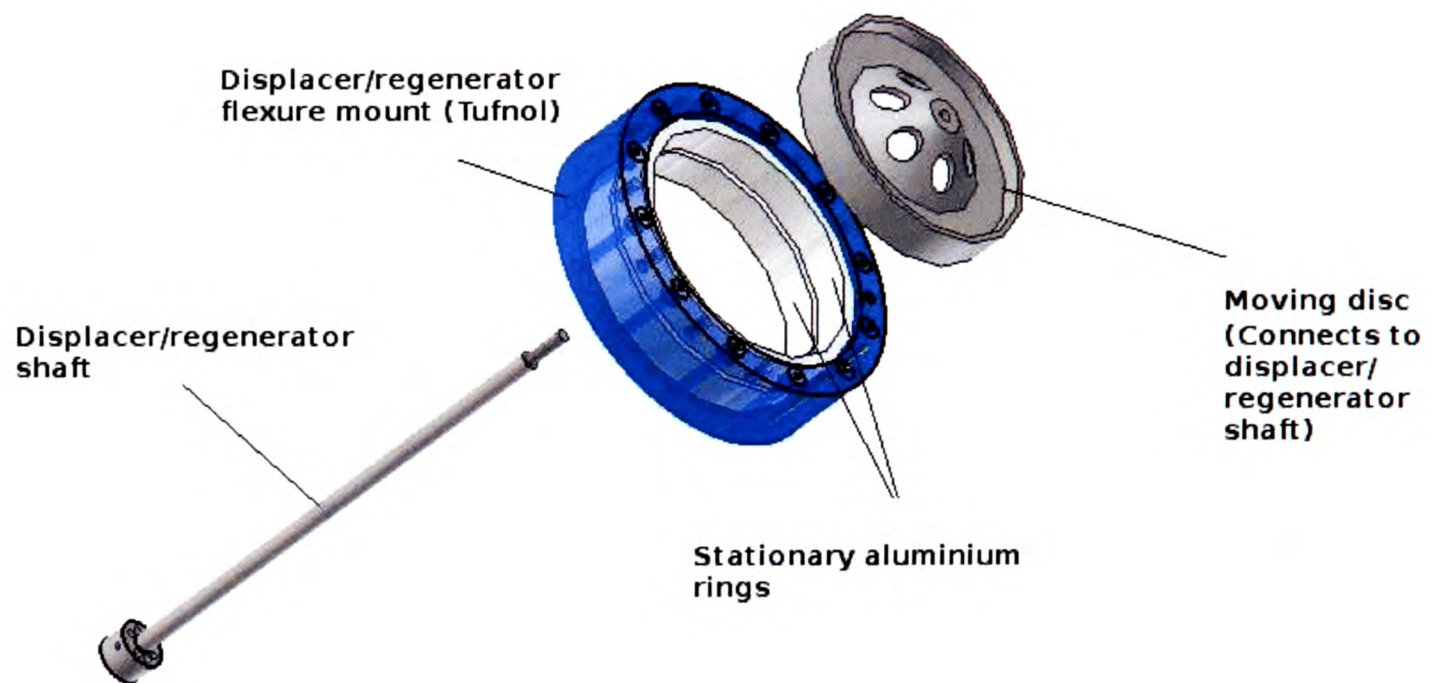
### 6.4.1 Displacer position pick-off (DPPO)

The DPPO uses the changing capacitance between two components to produce a voltage proportional to the displacement of the displacer. An exploded view of the displacer/regenerator position sensor produced from a 3D CAD model of the cooler is shown in figure 6.2. A simple diagram of the sensor is shown below in figure 6.4. As the DPPO disc oscillates up and down (because it is attached to the displacer/regenerator shaft) the capacitance between it and the aluminum rings changes (in fact not only does it change in magnitude but in the sign of the phase difference between the output voltage and the voltage supplied to the sensor -see analysis later in this subsection). Figure 6.3 shows a diagram of the capacitive DPPO. The left hand part of the figure shows the sensor ring at the midpoint position. In this situation the capacitance between the moving disc and either of the aluminum rings is given by:

$$c_0 = \frac{2\pi r \frac{l_d}{2} \epsilon_0}{d} \quad (6.2)$$

Where r is the radius of the capacitor plates,  $l_d$  is the length of each of the aluminum rings, d is the clearance between the moving DPPO disc and aluminum rings,  $\epsilon_0$  is the permittivity of free space.

The right hand diagram in figure 6.3 shows the situation when the displacer moves by  $x_d$ . This corresponds to the moving disc shifting so that the fraction of its area overlapping one of the stationary aluminum rings increases and decreases over the other. In the right hand diagram



**Fig. 6.2:** Exploded view of the DPPO assembly produced from the 3D CAD model of the cooler.

of figure 6.3 the moving disc shifts to the right and so the capacitance between the right-hand aluminum ring and the moving disc is given by:

$$c_{xd} = \frac{2\pi r \epsilon_0 \left( \frac{l_d}{2} + x_d \right)}{d} \quad (6.3)$$

Which we write as:

$$c_{xd} = c_0 \left( 1 + \frac{2x_d}{l_d} \right). \quad (6.4)$$

Similarly the capacitance between the DPPO disc and the left-hand ring is given by:

$$c_{xd} = \frac{2\pi r \epsilon_0 \left( \frac{l_d}{2} - x_d \right)}{d} \quad (6.5)$$

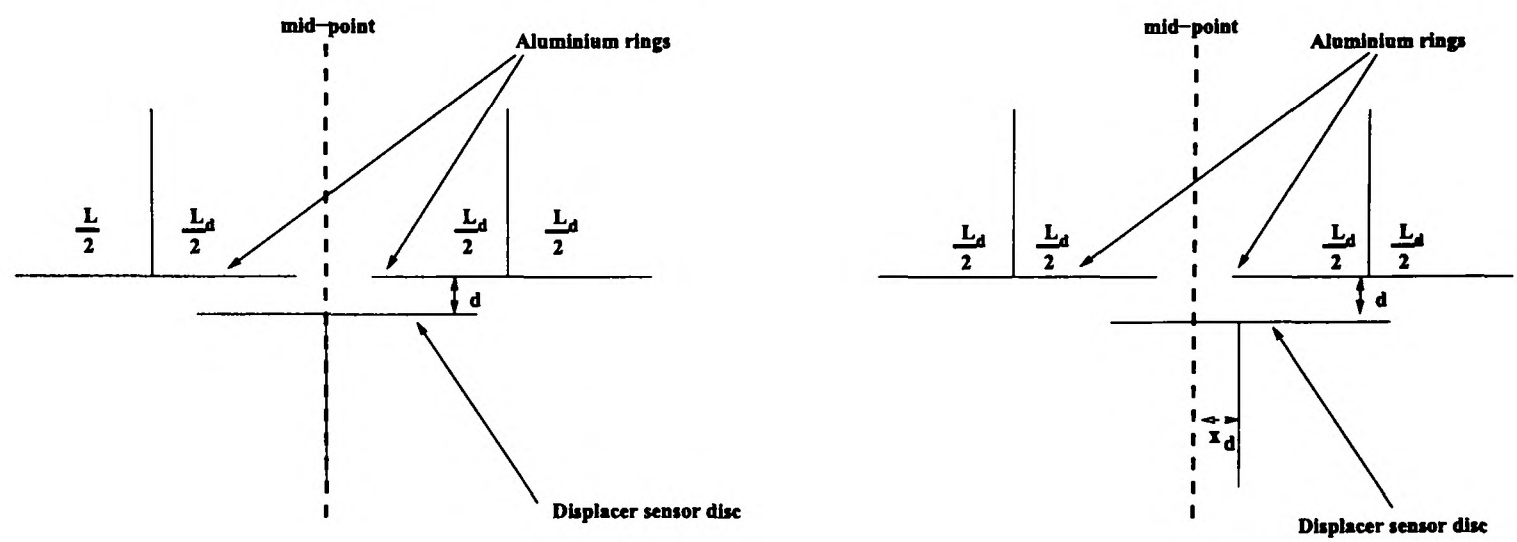
Which we write as:

$$c_{xd} = c_0 \left( 1 - \frac{2x_d}{l_d} \right) \quad (6.6)$$

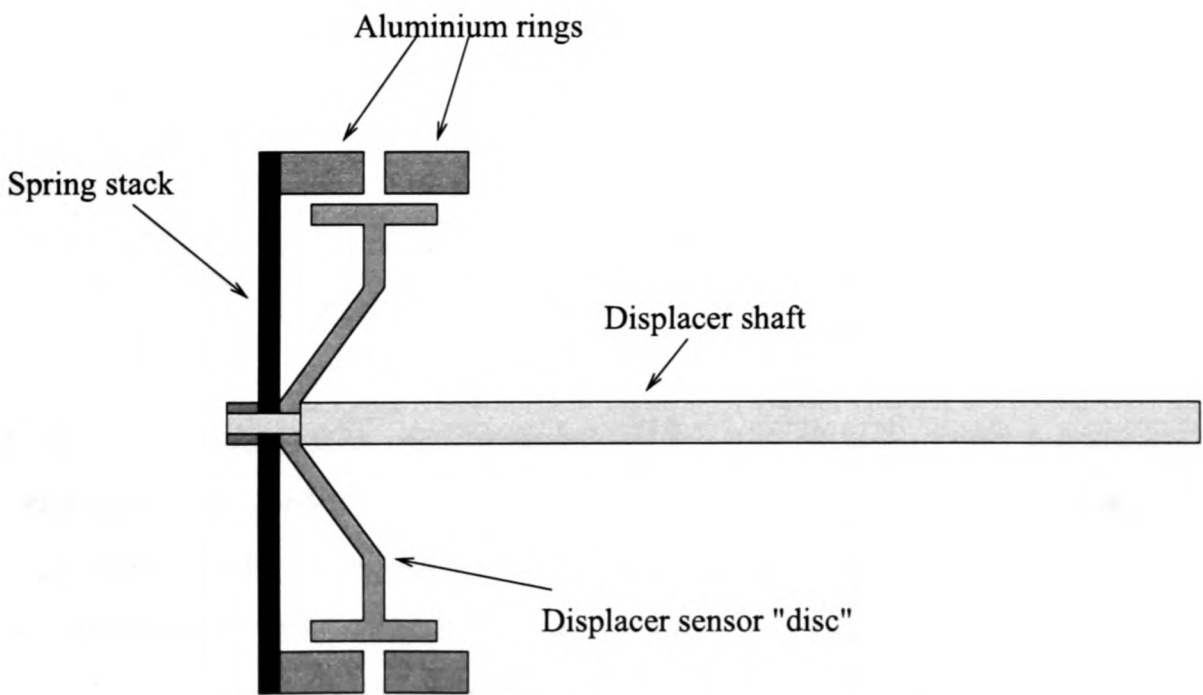
The change in capacitance as the displacer/regenerator moves can be detected and from this its motion can be sensed. A complete analysis of the DPPO's operation is given in Appendix A.

#### 6.4.2 Motor position pick off (MPPO)

The MPPO will be based on the circuit shown in figure 6.5. As the motor coil moves up and down within the air gap of the magnet assembly an e.m.f. is induced that acts to oppose the e.m.f. provided by the power supply. This “back e.m.f.” is given by  $\dot{x}Bl_b$  and so this leads to the



**Fig. 6.3:** Diagrams of the capacitive displacer position sensor at midpoint position (left diagram) and with displacer displaced by  $x_d$  to the right (right diagram).



**Fig. 6.4:** Schematic of the DPPO.



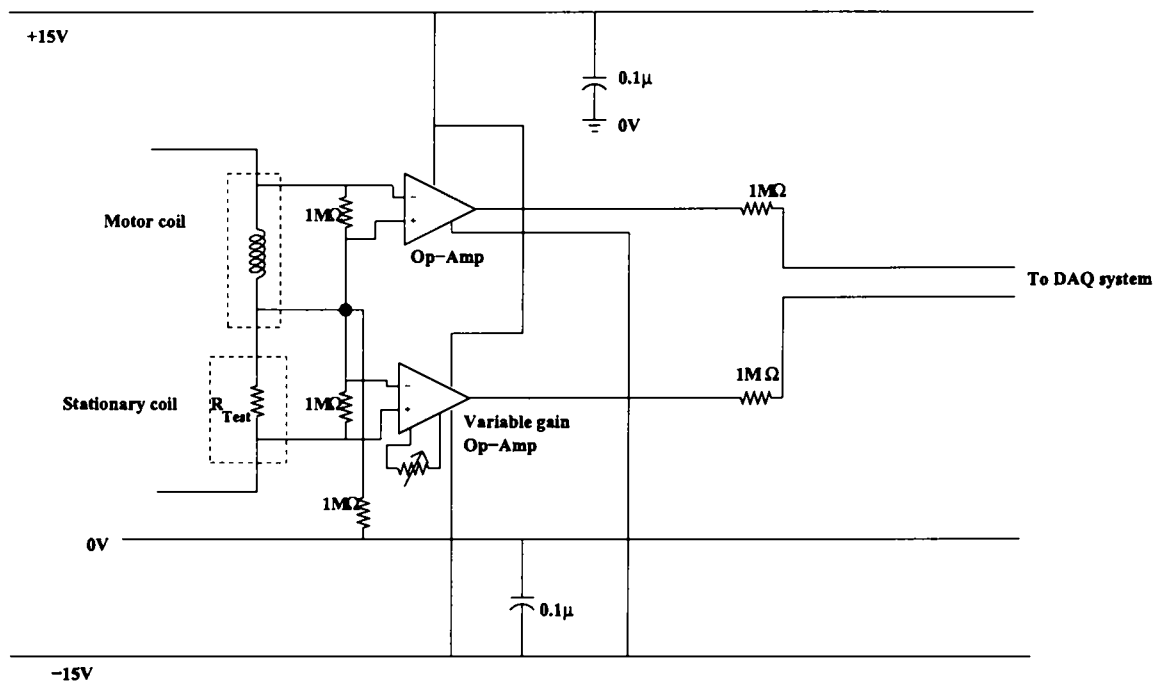


Fig. 6.5: Circuit diagram for the MPPO.

following expression:

$$\varepsilon_v - \dot{x}Bl_b = iR \quad (6.7)$$

Where  $i$  is the current through the motor coil,  $\varepsilon_v$  the voltage applied across the motor coil by the power supply,  $R$  is its resistance,  $B$  is the magnetic field in the air gap,  $\dot{x}$  is the velocity of the coil's motion and  $l_b$  is the length of the coil in the magnetic field. Re-arranging equation 6.7:

$$\varepsilon_v = iR + \dot{x}Bl_b \quad (6.8)$$

In other words the voltage drop across the motor coil is equal to the sum of the resistive drop and the inductive voltage drop (back e.m.f.) due to the coil's motion. If the voltage drop due the resistive component can be measured and then subtracted from the total voltage drop across the motor coil the back e.m.f. can be found. Since the back e.m.f. is given by  $\dot{x}Bl_b$ , it is clear that integrating this gives a signal proportional to the coil position. The circuit shown in figure 6.5 enables this process to be carried out. A small, stationary coil made from the same wire <sup>2</sup> as the motor coil is wound around one of the Tufnol components within the cooler. This coil is connected in series with the motor coil. When a DC voltage is applied to the series combination of the two coils there will be voltage drops across each of them in proportion to their resistances. A fixed gain op-amp samples the voltage drop across the motor coil whereas a variable gain op-amp samples

<sup>2</sup>Making the test coil from the same material as the motor coil ensures that the effect of temperature on the resistivity of the wire will not effect the accuracy of the position sensor as the effect will be the same for both coils. By placing the test coil close to the motor coil it is ensured that the temperatures of the two coils should be very similar.

that across the stationary coil. The gain of the stationary coil op-amp can then be altered so that its output is equal to that of the op-amp across the motor coil, thus when the difference in the output of the two op-amps is measured by the data acquisition system the result is zero. If an AC signal is then applied to the series combination the resistive voltage drops across the motor coil and stationary coil still cancel out, however there will also be a voltage across the motor coil (and not the stationary coil) due to the back e.m.f. This will be the result of taking the difference between the two op-amp outputs when the series combination is subjected to an AC signal. The difference between the two op-amp outputs can then be integrated to give a signal proportional to the coil's position. This integration was originally intended to be performed by an integrator circuit but it was realised that it could be performed in the Labview data acquisition software. Multiplying the voltage drop across the stationary coil, so that it equals the resistive voltage drop across the motor coil, can be done either by altering the gain of the op-amp that samples the voltage across it (as in figure 6.5) or by creating a simple control in the data acquisition software.

### 6.4.3 Pressure sensor

The pressure sensor monitors the pressure in the compression space. The sensor itself was mounted at the end of a titanium pipe of 1mm diameter as shown in figure 6.6. The pressure sensors are strain gauge sensors (Sensym SCC300AH), which are Wheatstone bridge circuits packaged in a metal T05 transistor package. This package was machined away as it represents extra dead volume. When the sensors are subjected to an increase in pressure the bridge deforms, which changes the resistance in the branches of the bridge thus producing a measurable voltage. The circuit for the pressure sensors is shown in figure 6.7.

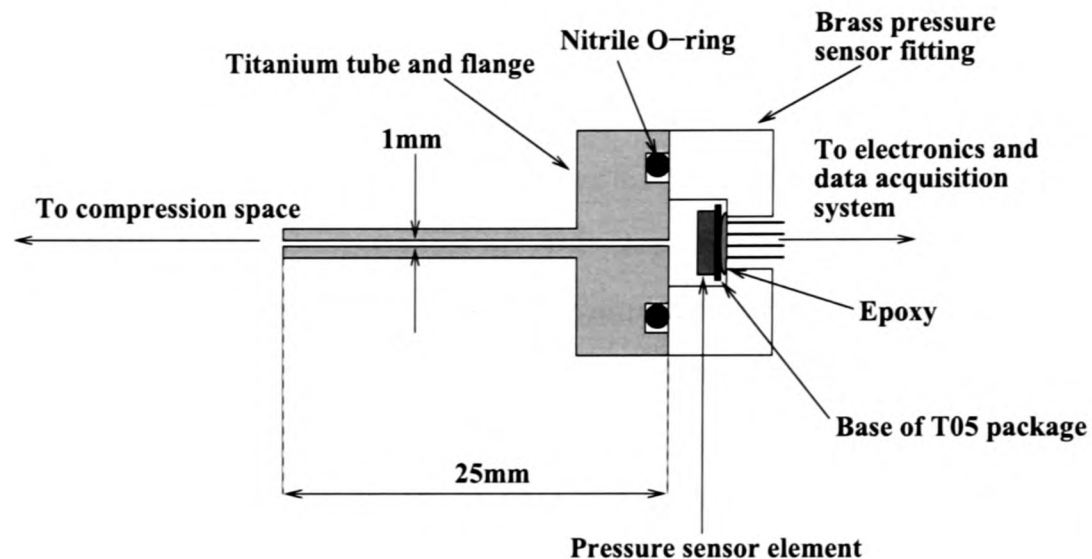
It is possible to model this arrangement as a simple Helmholtz resonator [Morse, P.M. (1948)] with a resonant frequency given by:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{c^2 A}{l_e v}} \quad (6.9)$$

Where  $c$  is the speed of sound in helium,  $A$  is the cross-sectional area of the titanium tube,  $l_e$  is the effective length of the tube and  $v$  is the volume of the cavity in front of the pressure sensor. Calculations reveal that the resonant frequency in the expected operational temperature range of the cooler (200-310K) is greater than 1.5KHz, far beyond the expected operational frequency of the cooler (50Hz). The possibility that the pipe could introduce a phase change into the pressure signal was also examined. The phase change introduced by the pipe can be calculate via [Morse, P.M. (1948)]:

$$\Omega = \cot^{-1} \left[ \frac{\cot(\frac{2\pi l}{\lambda}) - M(w)}{1 - 2 \frac{J_1(w)}{w}} \right] \quad (6.10)$$





**Fig. 6.6:** Diagram of how the pressure sensor was mounted on the cooler.

Where  $l$  is the position along the tube as measured from the compression space,  $\lambda$  is the wavelength of the pressure oscillation,  $w$  is  $4\pi a/\lambda$ ,  $a$  is the tube inner radius,  $J_1(w)$  is a Bessel function and  $M(w)$  is the acoustic reactance ratio as defined in [Morse, P.M. (1948)]. For the dimensions of the tube in this case we find that  $w$  is very small (around  $4 \times 10^{-3}$ ) and so  $\Omega$  is a vanishingly small fraction of a degree. Similarly we find that for the tube dimensions the pressure drop is negligible, this is calculated via [Morse, P.M. (1948)]:

$$\left(\frac{P_l}{P_0}\right) = \left\{ \left[1 - 2\frac{J_1(w)}{w}\right]^2 \sin^2\left(\frac{2\pi l}{\lambda}\right) + \left[M(w)\sin\left(\frac{2\pi l}{\lambda}\right) - \cos\left(\frac{2\pi l}{\lambda}\right)\right]^2 \right\}^{-\frac{1}{2}} \quad (6.11)$$

As  $w \rightarrow 0$ ,  $\frac{P_l}{P_0} \rightarrow 1$  i.e. the pressure drop along the tube is negligible.

#### 6.4.4 Temperature sensors

The temperature of the cooler cold tip and compression space external surfaces were measured with platinum resistance thermometers (PRTs), which were connected to a circuit identical to that used with the pressure sensors and so the circuit diagram is not repeated here. These PRTs are bonded to the outside of the cooler with thermally conducting elastomer to provide good thermal contact. Aluminum tape was used to further secure them in place and they were then covered with MLI shielding. PRTs have been used in past investigations of cooler performance [Brito, M. (1999)] and [Peters, M. (1994)]. Figure 6.8 shows the positions of the PRTs on the cooler and

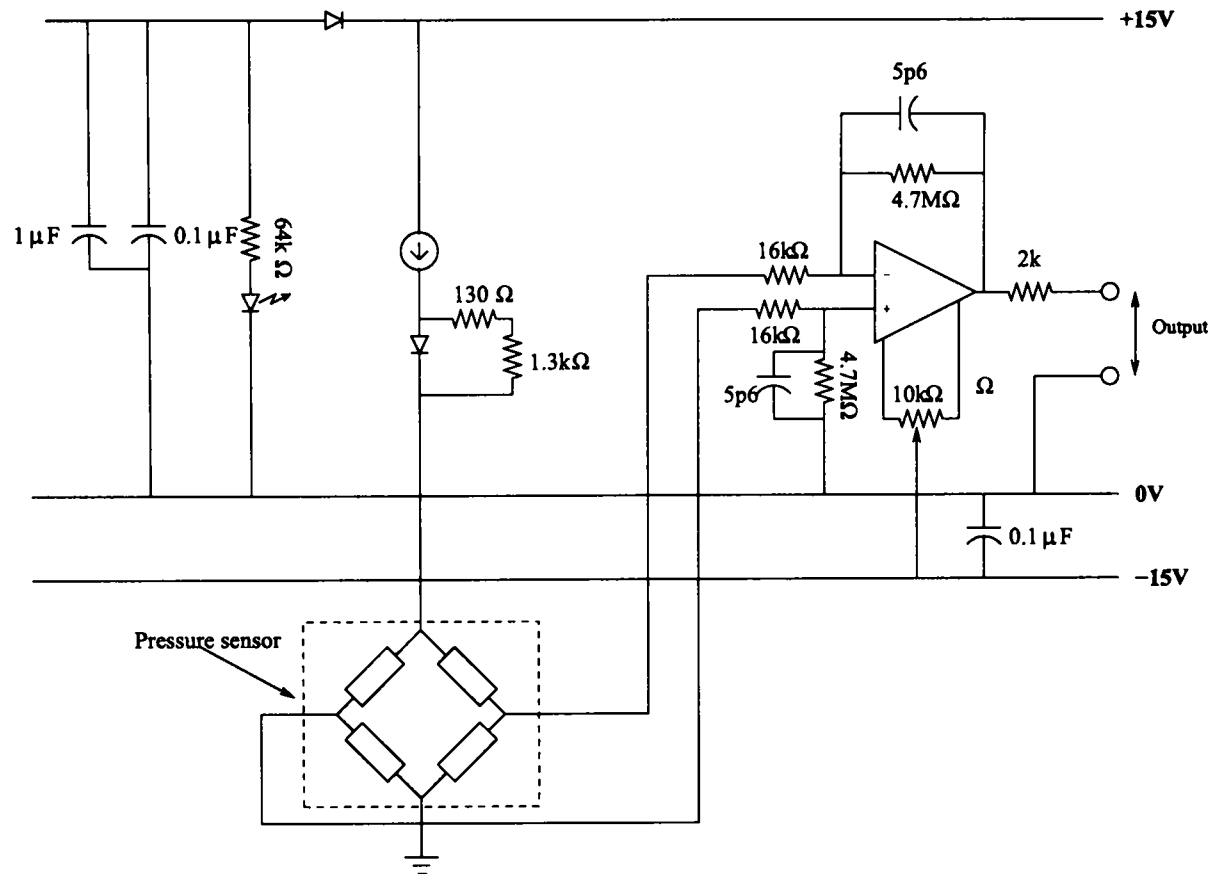


Fig. 6.7: Pressure sensor circuit diagram.

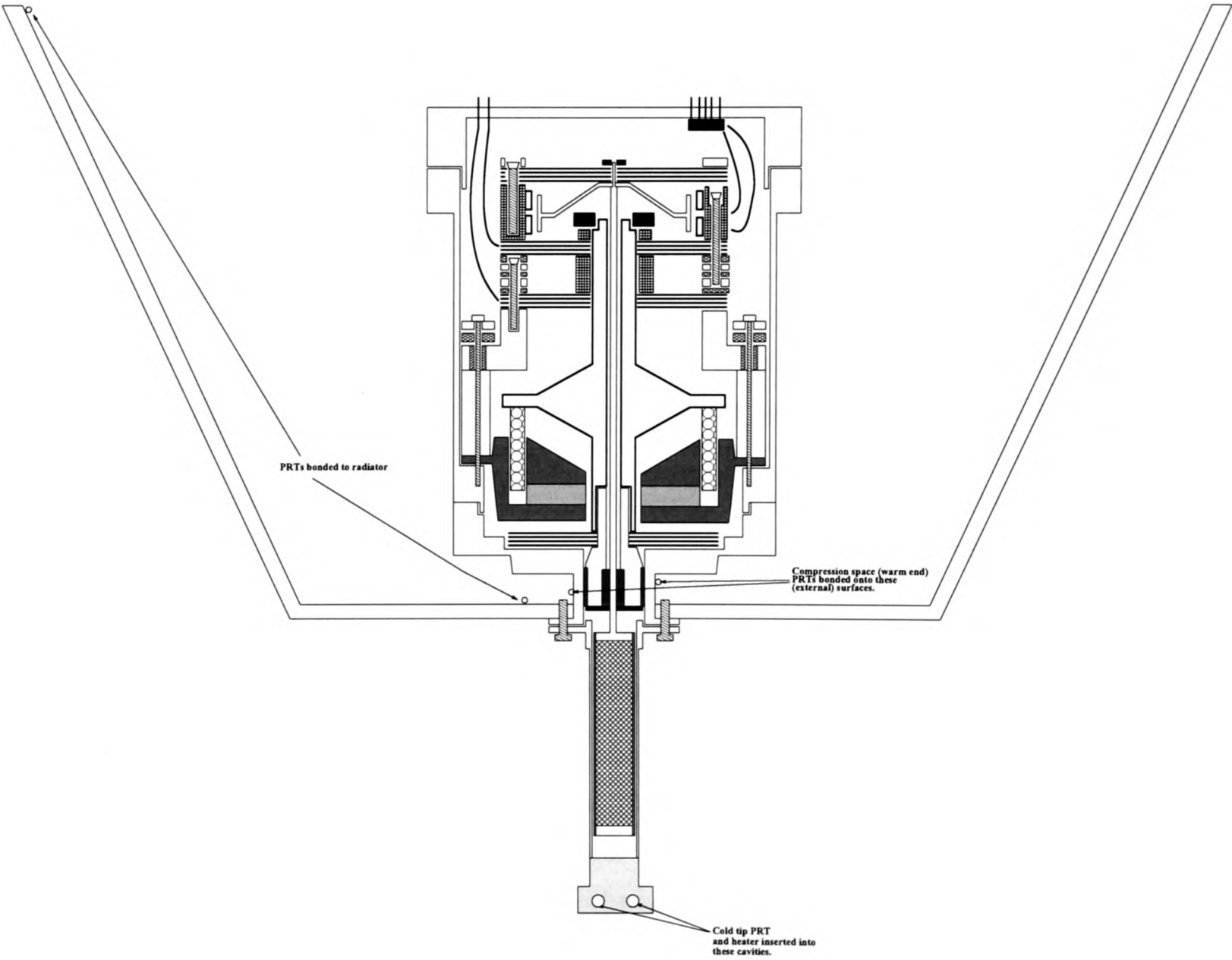
the radiator. The average of the outputs of the two PRTs mounted on the walls of compression cylinder was used as the value for the compression space temperature during analysis and plotting of results.

#### 6.4.5 Voltage and current sensors

The power supplied to drive the compressor motor was measured by recording the voltage and current in the coil. This was achieved by measuring the potential differences over the motor coil and over a  $0.503\Omega$  resistor connected in series with the motor coil. The resistor was of the aluminum housed wire wound type rated to  $15W$  with a maximum quoted temperature coefficient of resistance equal to  $100\text{ ppm}/^\circ\text{C}$ . The potential differences over the motor coil and the current sampling resistor were sampled by AD620 instrumentation amplifiers.

### 6.5 Sensor calibration

Each sensor was calibrated before being used to characterise the cooler and radiator's performances. The data acquisition system was used extensively in this process and initially there were a number of problems with it. The data acquisition card would seem to automatically un-install itself between one boot up of the PC and the next. After a number of new installs and trying the card in a different



**Fig. 6.8:** Positions of the PRTs on the cooler and radiator (Radiator not shown to scale)

PC it was concluded that the data acquisition card was faulty. This was confirmed by the supplier once the card had been returned to them for analysis. A replacement card was purchased and after a new calibration had been performed using the manufacturer-supplied software (Instacal) the data acquisition system worked without any further problems.

### 6.5.1 Voltage and current sensors

The voltage and current sensors were calibrated using a  $4.7\Omega$  test resistance as shown in figure 6.9. A test resistance was used in place of the cooler so that the full range of voltages could be investigated without risking damage to the cooler or involving any frequency responses the cooler could introduce. The test resistance was mounted on an aluminum block so that thermally induced resistance changes were kept to a minimum. The resistor was quoted as having a maximum temperature coefficient of  $50ppm/C^\circ$  so bolting to the aluminum block should be unnecessary but as it was a simple matter it was done anyway. The quoted temperature coefficient leads to an increase in the resistance of 0.125 % for a  $25C^\circ$  temperature. The Fluke 8010A digital multimeters used in the calibration were set on 2V/20V ranges for the voltmeter and 200ma/2000mA with an expected accuracy of 0.5% and 1% respectively. A range of AC currents with frequencies from 10 to 80 Hz were applied (in 10 Hz steps). At each frequency 5 files were created by the data acquisition system each recording the output of the current and voltage sensors at a sample rate of 1KHz. Correspondingly at each frequency five manual readings of the drive voltage and current were taken from the Fluke 8010A multimeters. These data files could then be used to calculate (using a simple Mathematica routine) the RMS output from the drive voltage and current sensors ( $V_{COIL}$  and  $I_{COIL}$ ), the Labview software also enables RMS values to be calculated from the sensor outputs and this typically agreed with the Mathematica calculation to within 10mV and so was used as a “double check” for the calculated RMS values. Figure 6.10 shows the variation of the voltage sensor with increasing drive voltage at frequencies from 10 to 80Hz. As expected there was no variation of the sensor output with frequency up to 80Hz. Figure 6.11 shows the drive voltage plotted against the drive sensor output at the frequencies tested. As expected the sensor output is linearly proportional to the magnitude of the drive voltage. Since there was no variation in the sensors’ performance with frequency for a given voltage, data points at each drive voltage were combined and averaged and a fit made to the resulting data to obtain the sensors calibration and gain. The voltage sensor calibration residuals are shown in figure 6.12. The voltage and current sensors simply used instrumentation amplifiers to measure voltage drops across various resistances and circuits of a very similar design had been used on developmental coolers in AOPP previously [Reed, J. (2004)]. This combined with their linear behaviour during calibration led to a high degree

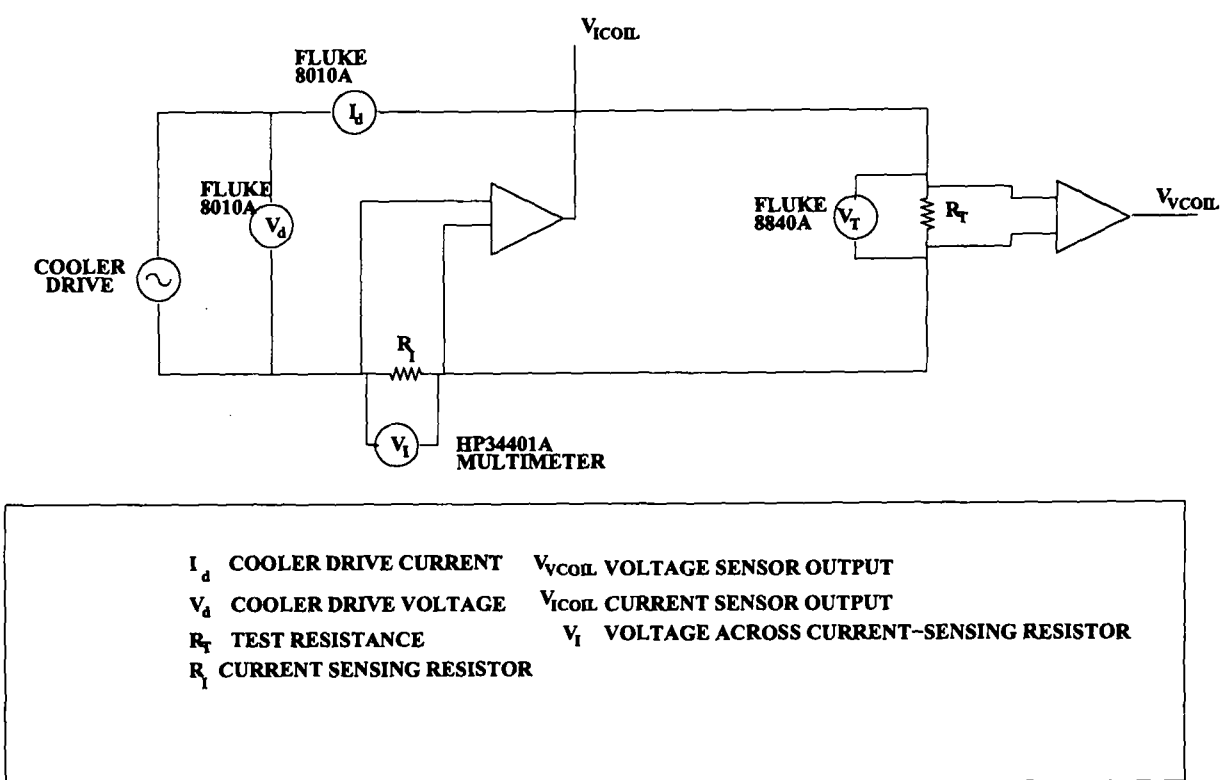


Fig. 6.9: Voltage and current sensor calibration circuit.

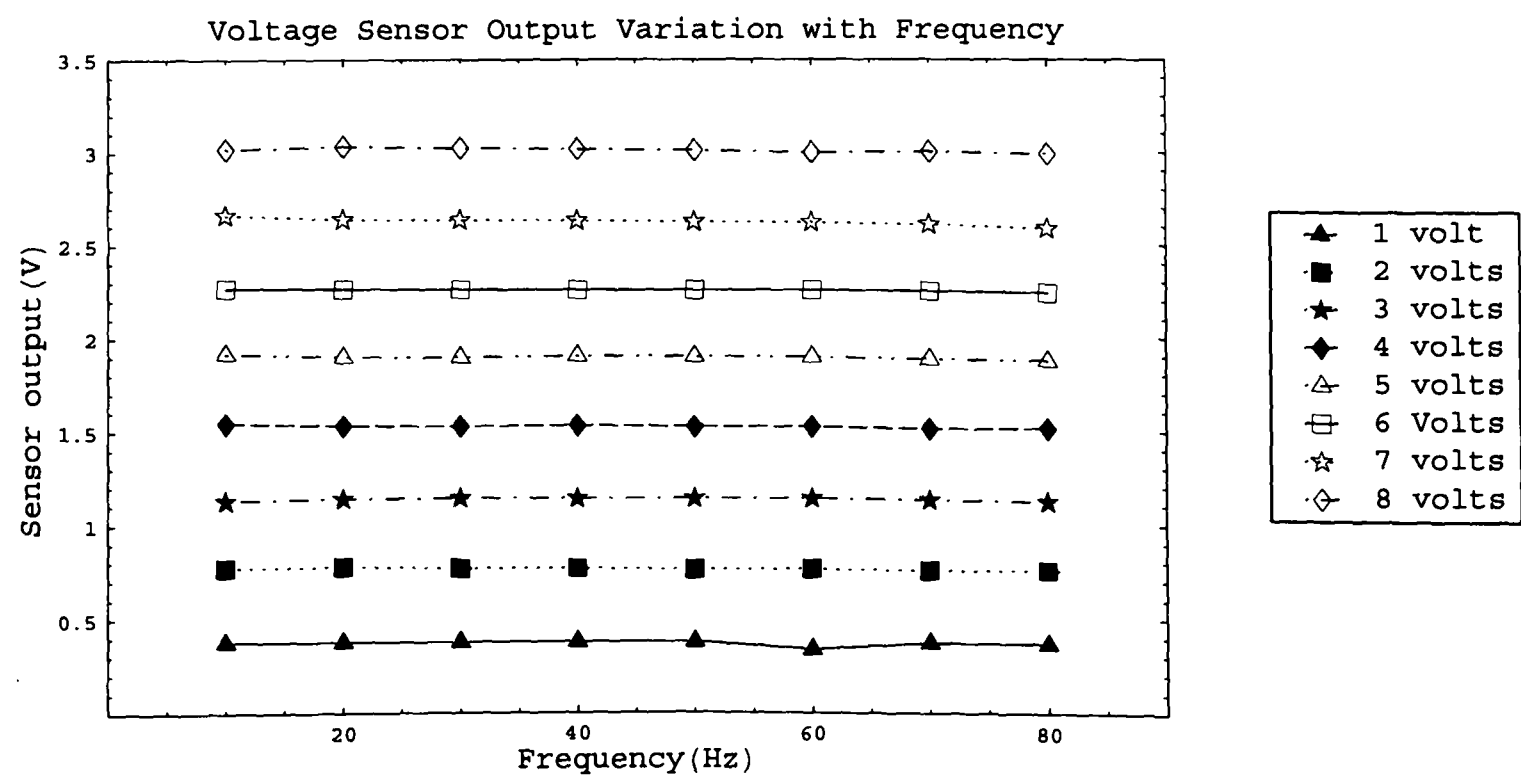


Fig. 6.10: Voltage sensor output variation with AC drive voltage frequency. Error bars too small to be visible at this scale.

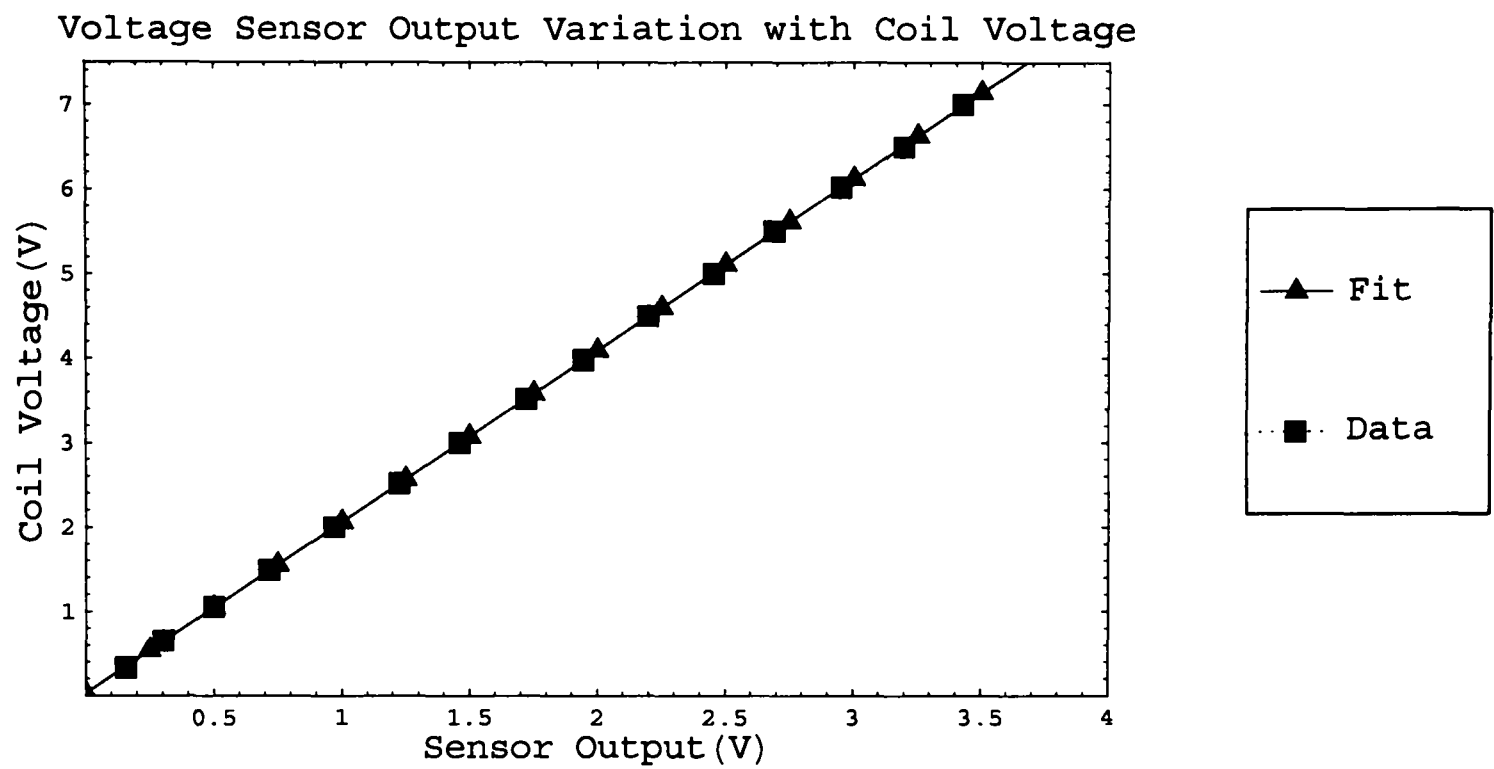


Fig. 6.11: Voltage sensor calibration. Error bars too small to be visible at this scale.

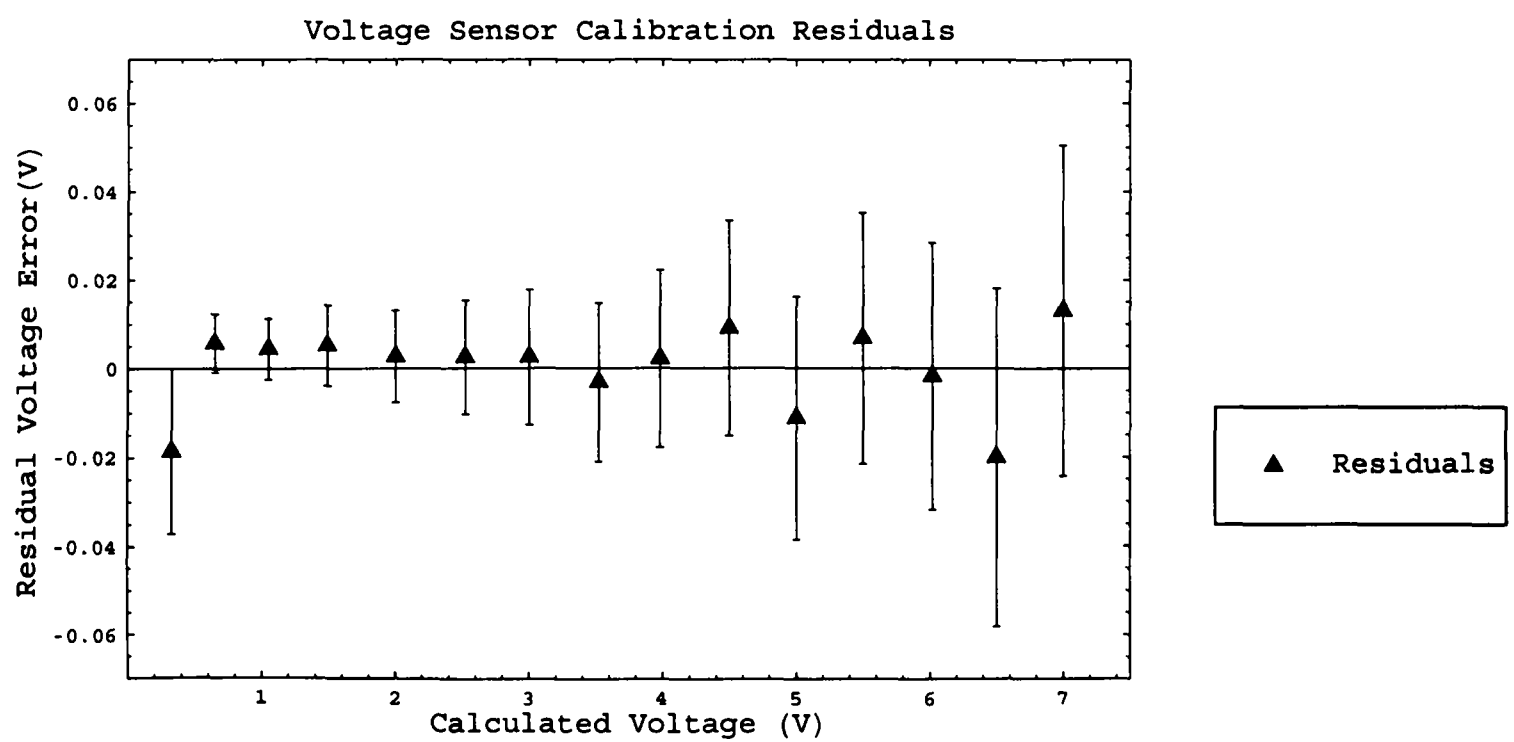


Fig. 6.12: Voltage sensor calibration residuals.

of confidence in their ability to produce reliable information about the cooler's performance.

The power supplied to the cooler could be calculated from the outputs of the current and voltage sensors via equation 6.12.

$$W_m = \int_0^P (V_m \times I_m) dt \quad (6.12)$$

Where  $W_m$  is the electrical power supplied to the cooler's linear motor,  $P$  is the period of the drive waveform,  $V_m$  is the instantaneous value of the voltage across the motor terminals as measured by the voltage sensor described above,  $I_m$  is the instantaneous value of the current through the motor coil as measured by the current sensor described above. A Mathematica routine was written by the author which took data files containing the outputs of the current and voltage sensors and used them to perform the integral shown in equation 6.12.

### 6.5.2 Pressure sensor

The pressure sensor was similar to that used in a number of previous research efforts within the department and so it was expected to be suitable for use in the current research effort. A calibration process was performed whereby the cooler was filled with helium in increments of 0.5bar from 1 bar to 9.5 bar. The pressure sensor was calibrated against a Budenberg standard test gauge that had been recently calibrated and was traceable to national standards, it was accurate to 0.025 bar. The output of the pressure sensor was sampled 5 times at each pressure for 1 second at a sampling rate of 1Khz. The output at each pressure was then averaged and the results are shown in figure 6.13 along with a linear fit to the data. Figure 6.14 shows the residuals from this calibration process. The fit was accurate to better than 0.04bar over the entire pressure range.

### 6.5.3 Temperature sensor calibration

Each PRT was immersed in liquid nitrogen and the output from the temperature sensor electronics recorded by the data acquisition system for a sampling period of one second every minute for five minutes. The sampling rate within each sampling period was 1KHz. The output from each PRT was seen to vary by less than 0.5% over the five minutes that data was recorded and so an average output was calculated. This procedure was then repeated with each PRT being immersed in a water-ice mixture and also at room temperature. Linear fits were then made to the data and were found to be accurate to better than 1K in all cases. Figure 6.15 shows a typical fit for one of the temperature sensors. Room temperature was measured by a mercury thermometer (accurate to  $\pm 0.5K$ ) as was the temperature of the water-ice mixture, the liquid nitrogen was assumed to be at 77.4K.



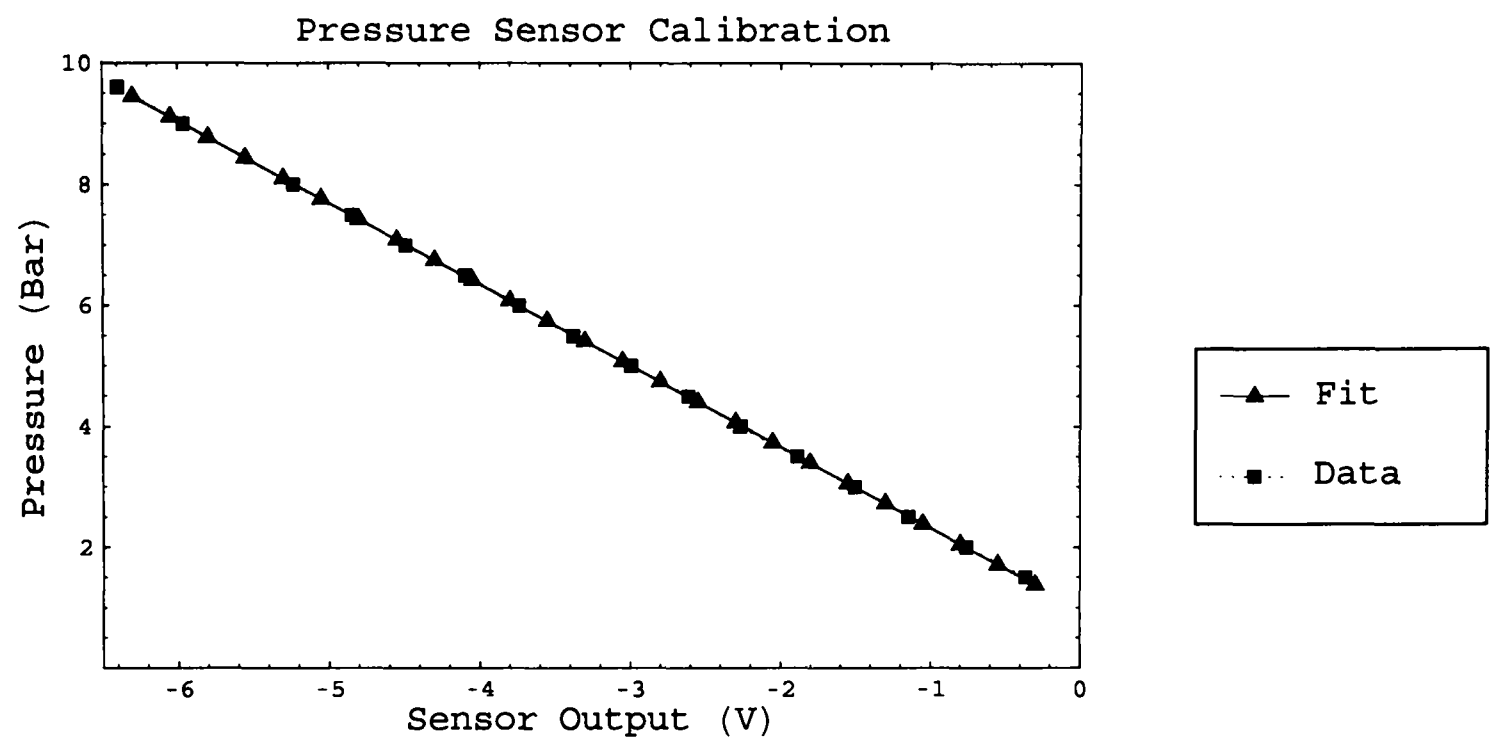


Fig. 6.13: Pressure sensor calibration.

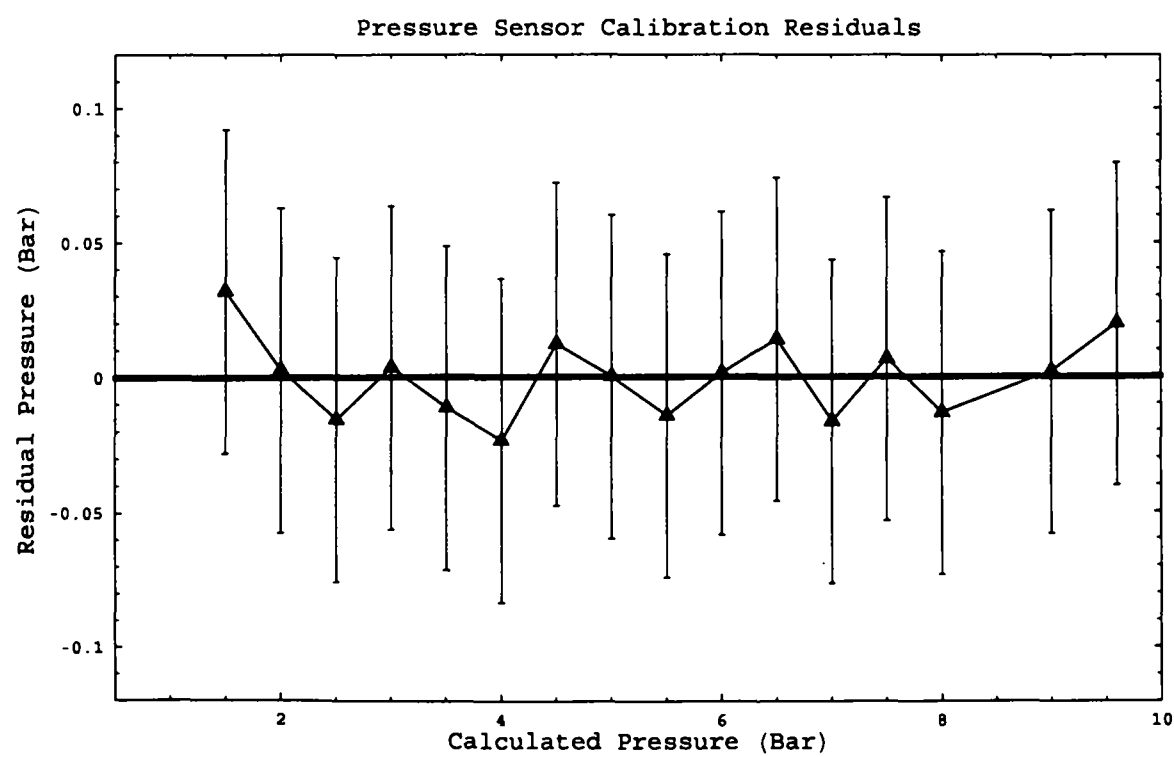


Fig. 6.14: Pressure sensor calibration residuals.

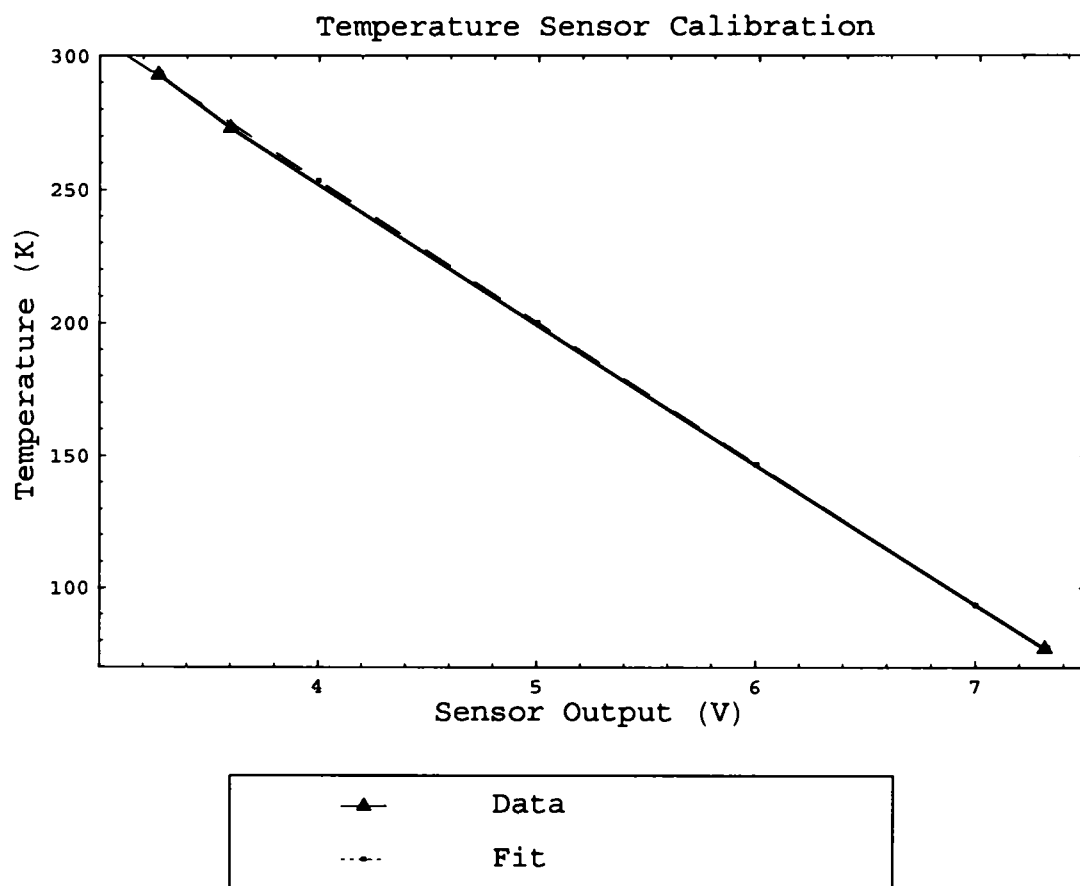


Fig. 6.15: Temperature sensor calibration.

#### 6.5.4 DPPO and MPPO calibration

The displacer position pick-off (DPPO) was calibrated by shifting the regenerator/displacer away from its mean position by an amount measured with a height gauge accurate to 0.01mm. The experimental set-up is shown in figure 6.16. Figures 6.17 and 6.18 show the results of the calibration procedure. It was found that a linear fit was accurate to better than 0.08mm.

The experimental set-up for the MPPO calibration is shown in figure 6.19. The MPPO only produces a useful output when the motor coil is moving and so it cannot be calibrated by moving the coil to various positions and recording the change in the MPPO output. Calibration was performed by measuring the peak displacement of the coil in either direction for a variety of input voltages and frequencies. The peak displacement in the direction of the cold tip was measured by a vernier screw gauge and in the opposite direction by a height gauge. During calibration the MPPO produced unexpected results. Simply multiplying the voltage drop across the stationary coil by the correct factor and subtracting it from the motor coil voltage drop should yield a sensor output that should vary linearly with frequency for a fixed stroke. The MPPO output for a fixed stroke is shown in figure 6.20 along with the expected dependence on frequency. Since the DPPO had been calibrated and seen to behave as expected an adaptor was created that allowed the moving part of the DPPO to be attached to the rear of the coil former. This enabled the DPPO to be used to

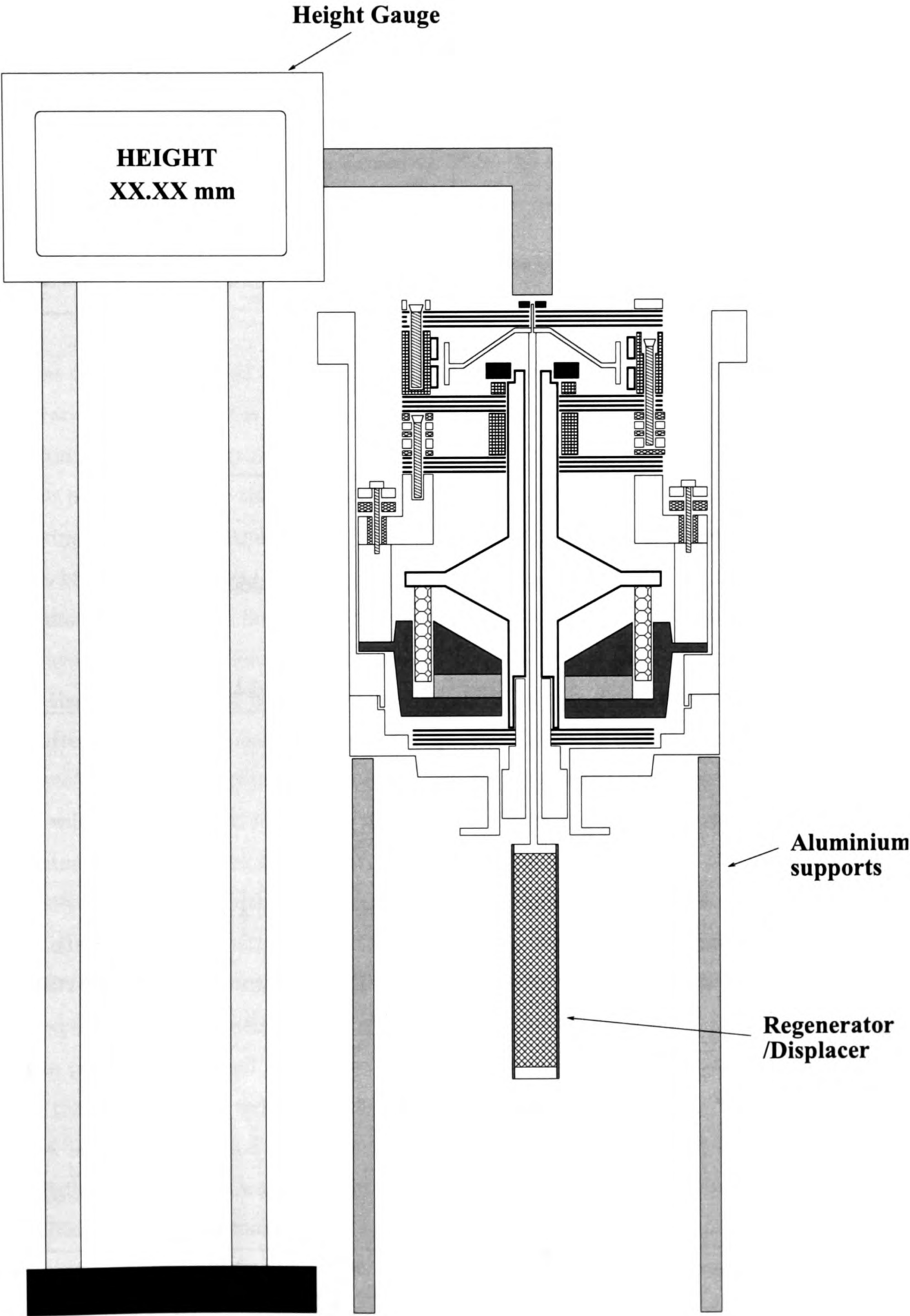


Fig. 6.16: Experimental set-up for the DPPO calibration.

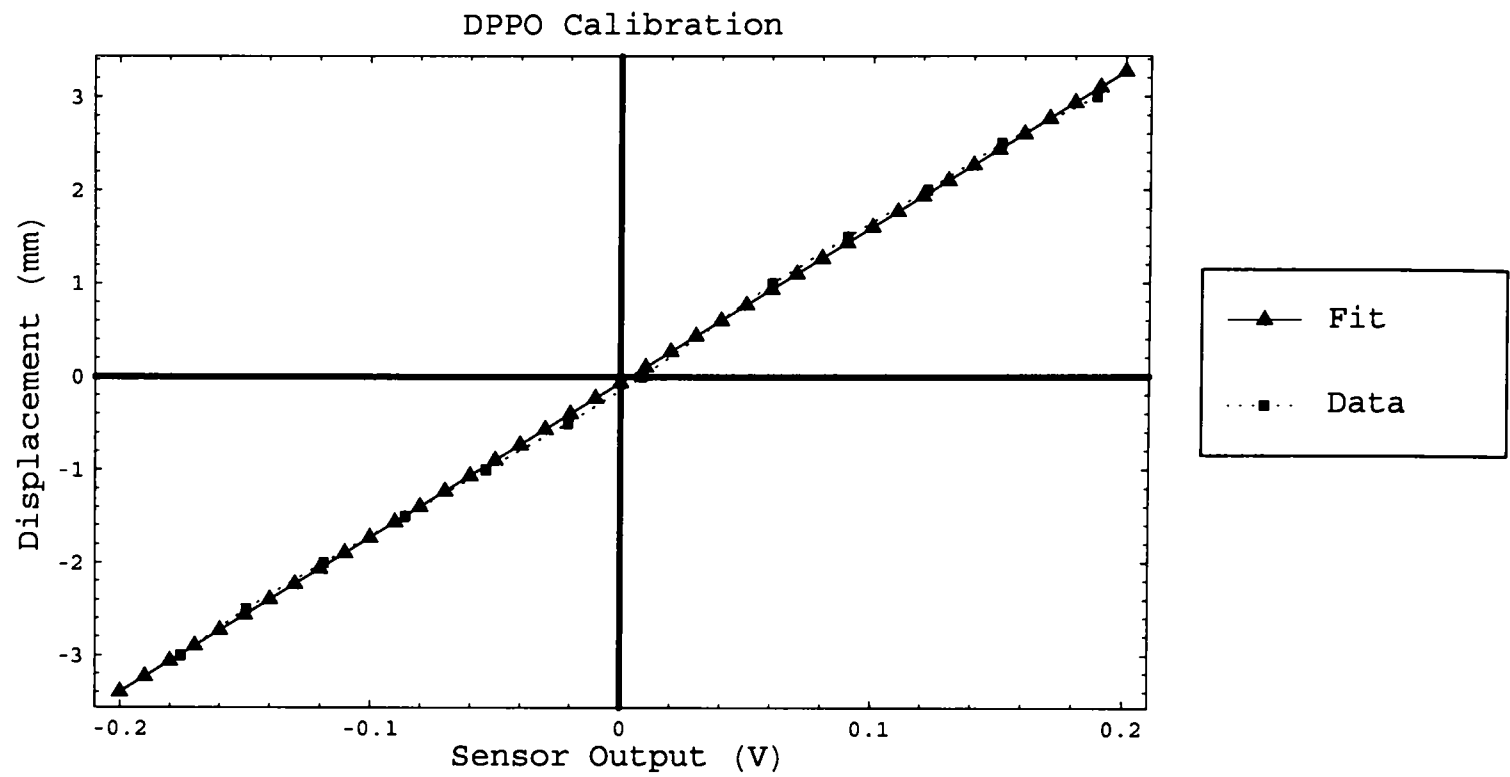


Fig. 6.17: DPPO calibration results.

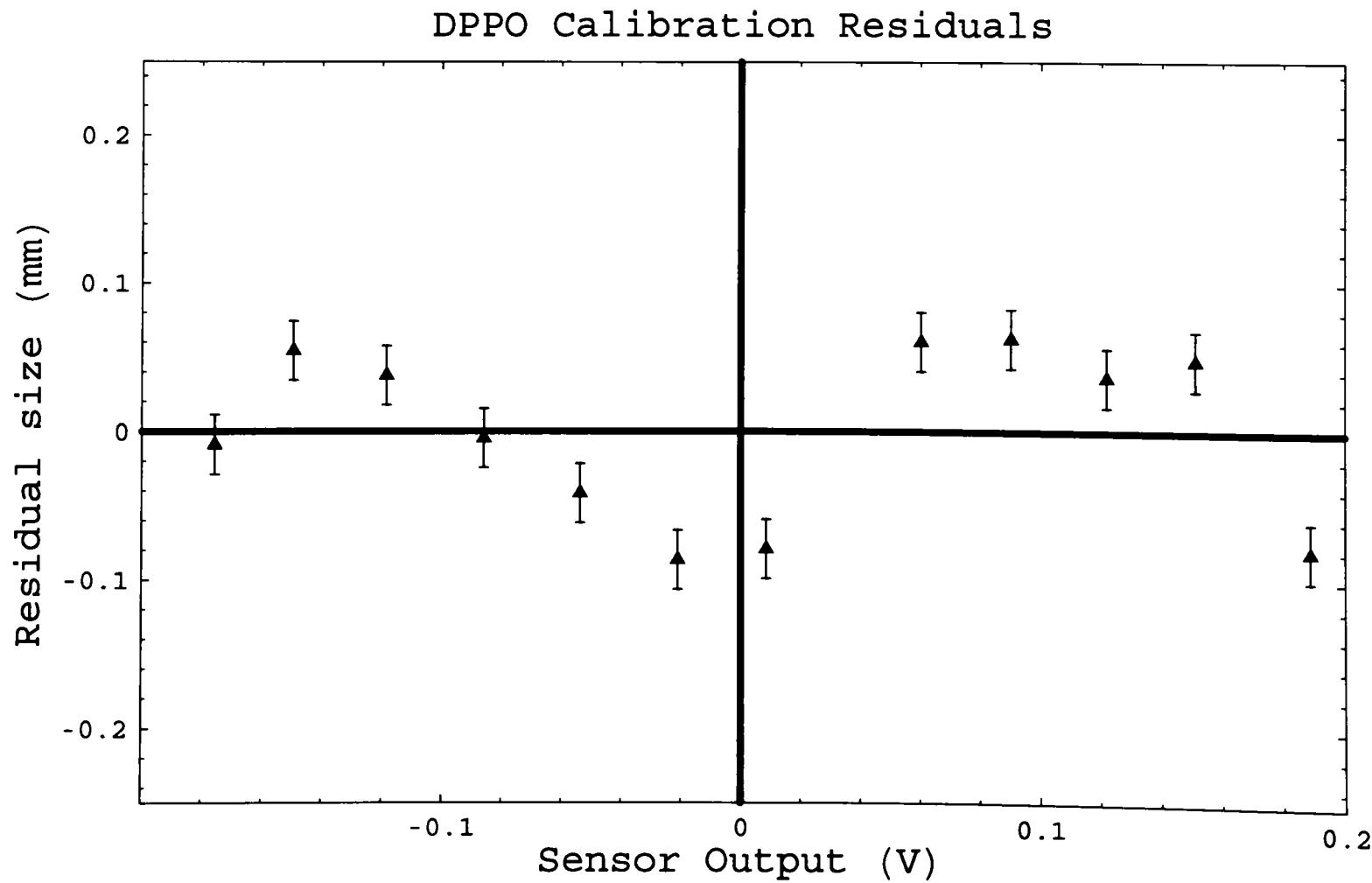


Fig. 6.18: DPPO calibration residuals.

sense the coil former's (and therefore the piston's) motion. This arrangement is shown in figure 6.21 and could be used to measure the piston's motion continuously during the MPPO calibration (rather than just measuring the peak displacement). The MPPO calibration was then repeated using this arrangement but the unexpected MPPO behaviour still remained.

In an attempt to explain the departure from the expected behaviour of the MPPO, the assumptions that were made during its inception were re-examined. Recalling equation 6.7 for the motor coil with an applied driving voltage  $\varepsilon_v$ :

$$\varepsilon_v - \dot{x}Bl_b = iR \quad (6.13)$$

It was originally assumed that  $B$ , the magnetic flux density could be assumed constant for all motor operating conditions, but this is not, in fact, the case. If the motor coil oscillates about a mean position, with a fixed stroke then, for all frequencies, it will see the same average flux density. If the mean position changes then the average flux density experienced by the coil will change too thus altering the MPPO output for a given stroke and frequency. Compressor pistons in coolers have been known to drift away from their nominal mean position [Reed, J. (2004)]. If this happened in the prototype we would find that for two otherwise identical sets of operating conditions (stroke and frequency) but with different mean positions the peak-to-peak MPPO output amplitude would vary but the DC level would not change, so it would be impossible to determine whether the decrease occurred because the mean position of the coil had shifted or because the peak to peak stroke had reduced. Even if the mean position of the coil did not vary the average B-field experienced by the coil will change with stroke. If the stroke is small then the average B-field it experiences is fairly constant, however if the stroke is increased the coil's motion extends into regions where the flux density is lower and so the average flux density it experiences will be reduced. Neither of these two effects (B-field experienced by the coil changing with mean position or changing with stroke) could account for the form of the MPPO output shown in figure 6.20, as this would require the coil to experienced a decreasing B-field with increasing frequency which would only be possible if the mean position of the coil had moved away from the magnet assembly as frequency was increased and this was not observed during the calibration process.

A second assumption that was made about the motor was that the coil's inductance was negligible. Belatedly it was realised that this was untrue; the coil was found to have an inductance of 4.02mH when it was measured in isolation with a 50 Hz excitation current and this rose to 8.60mH when the coil was fully inserted into the magnet assembly. These measurements were taken with a HP 4284A Precision LCR meter. Measurements of the inductance were taken with the coil gradually being removed from the magnet assembly to investigate how the inductance would vary throughout the coil's motion. The coil was held in a collet secured in a milling machine, and the

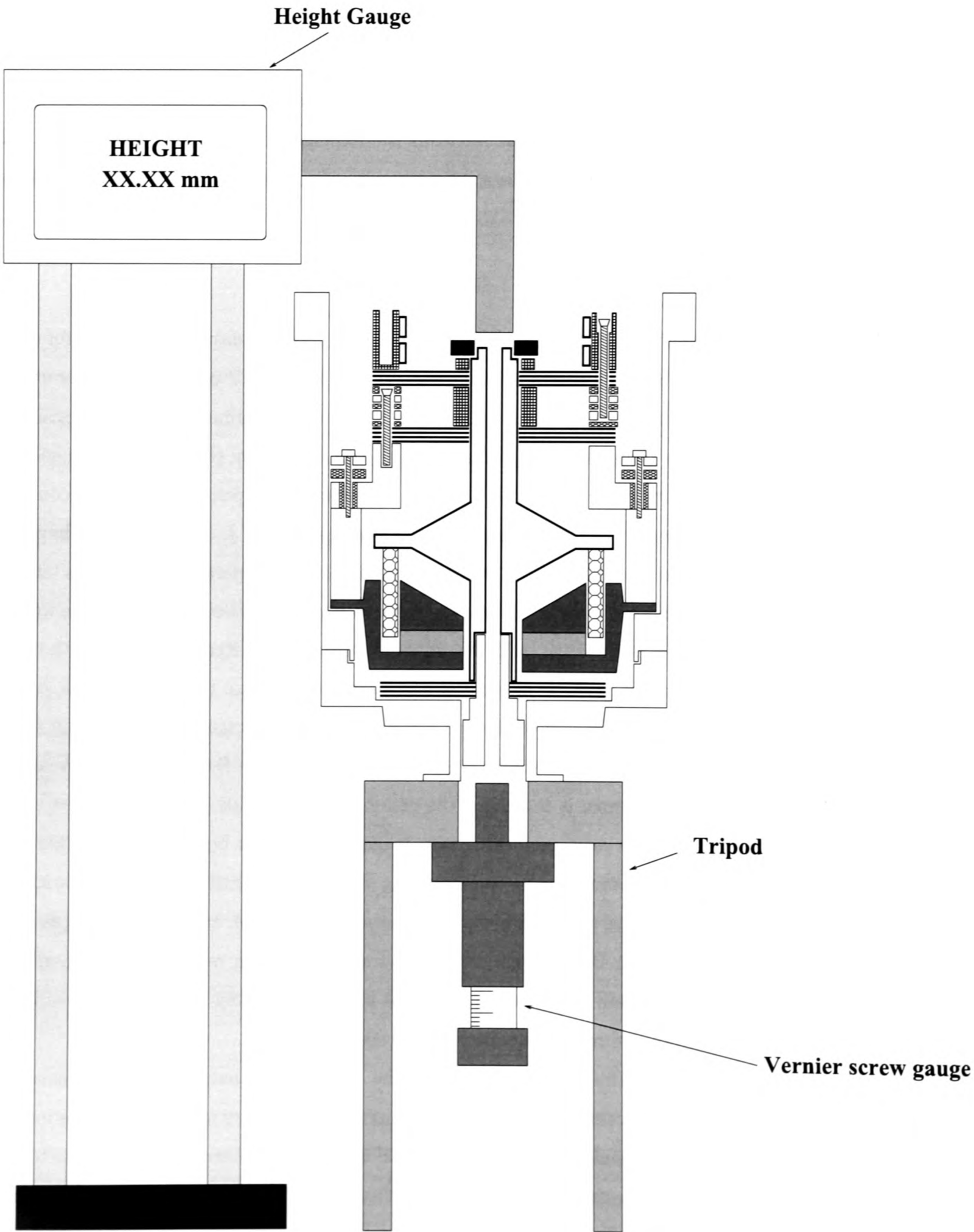
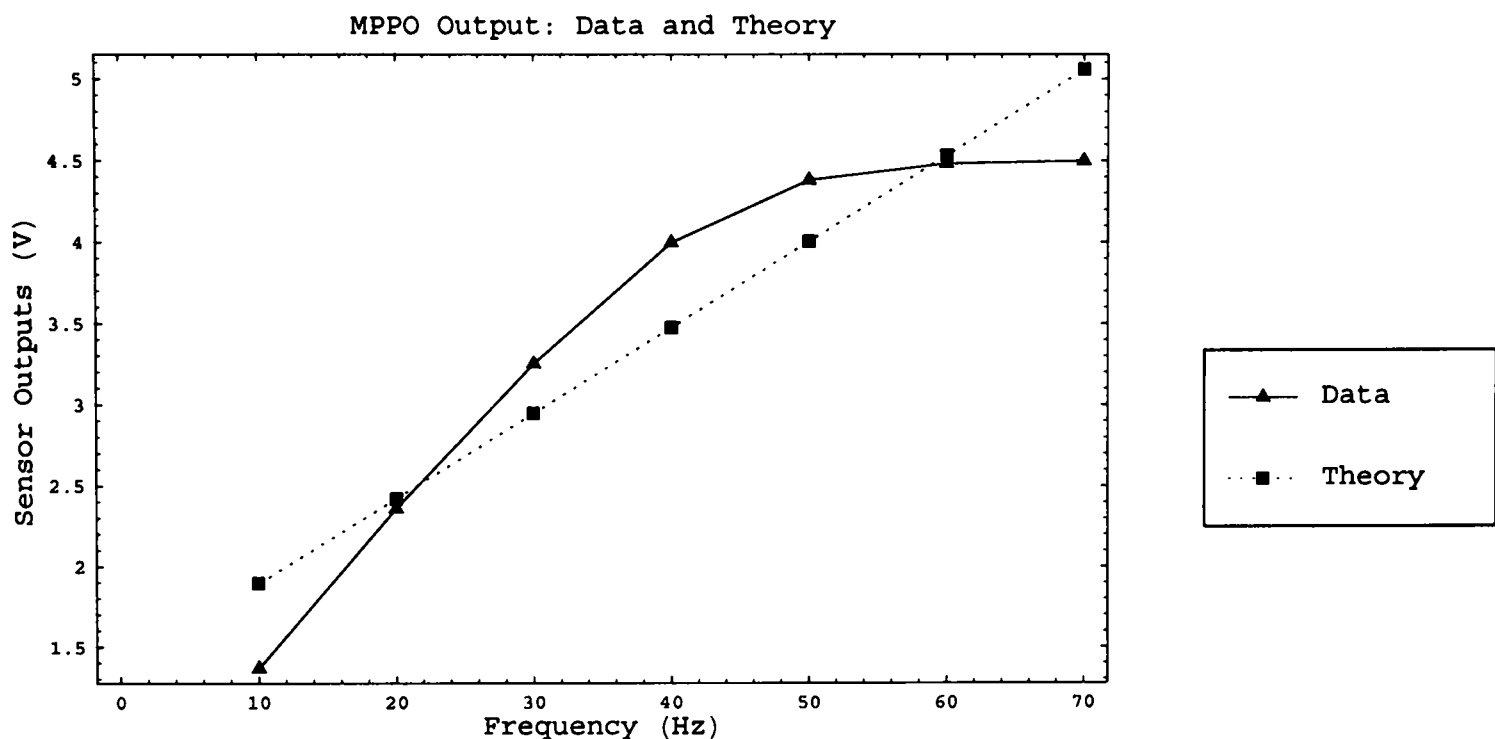


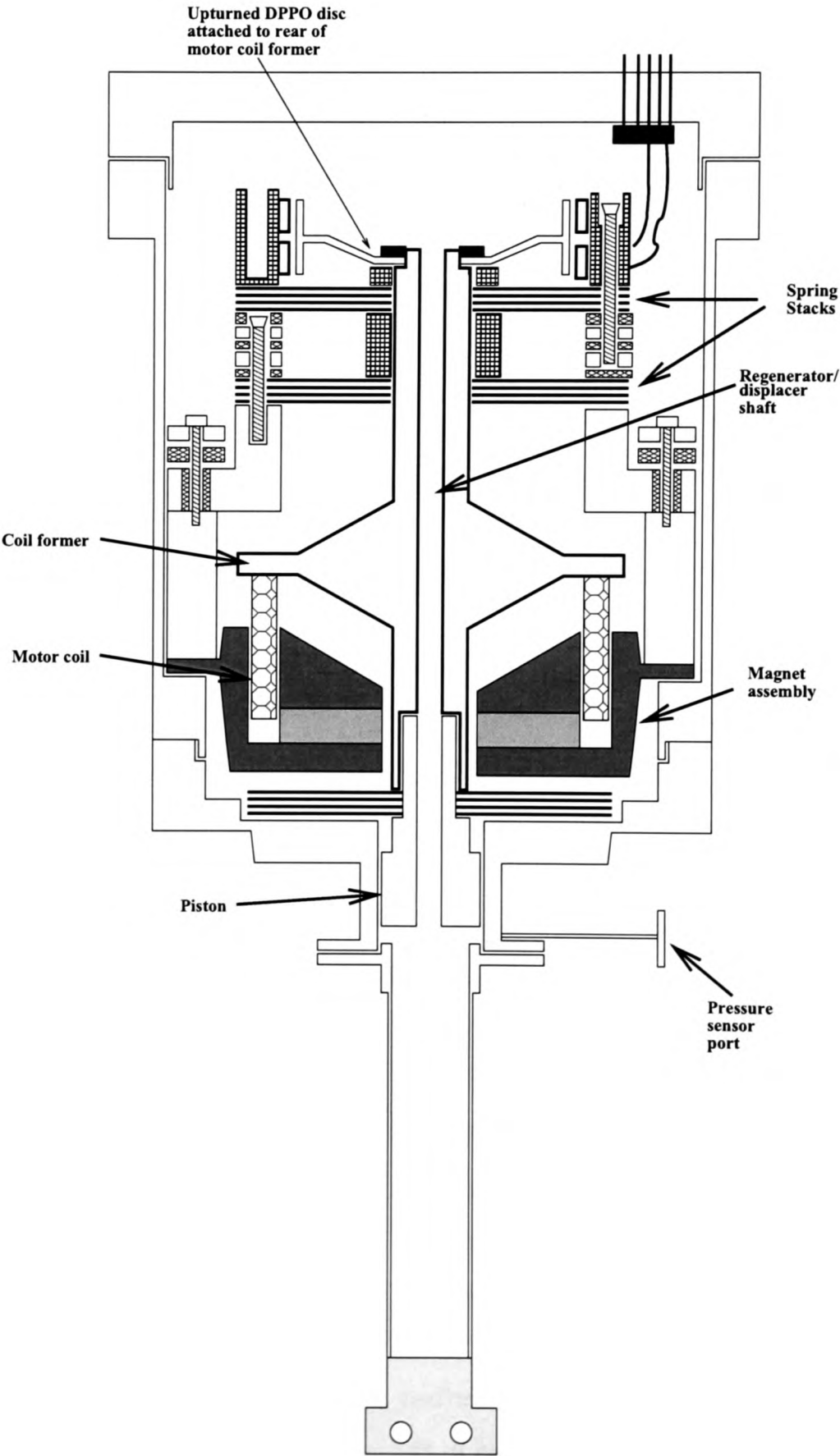
Fig. 6.19: Experimental set-up for the MPPO calibration.



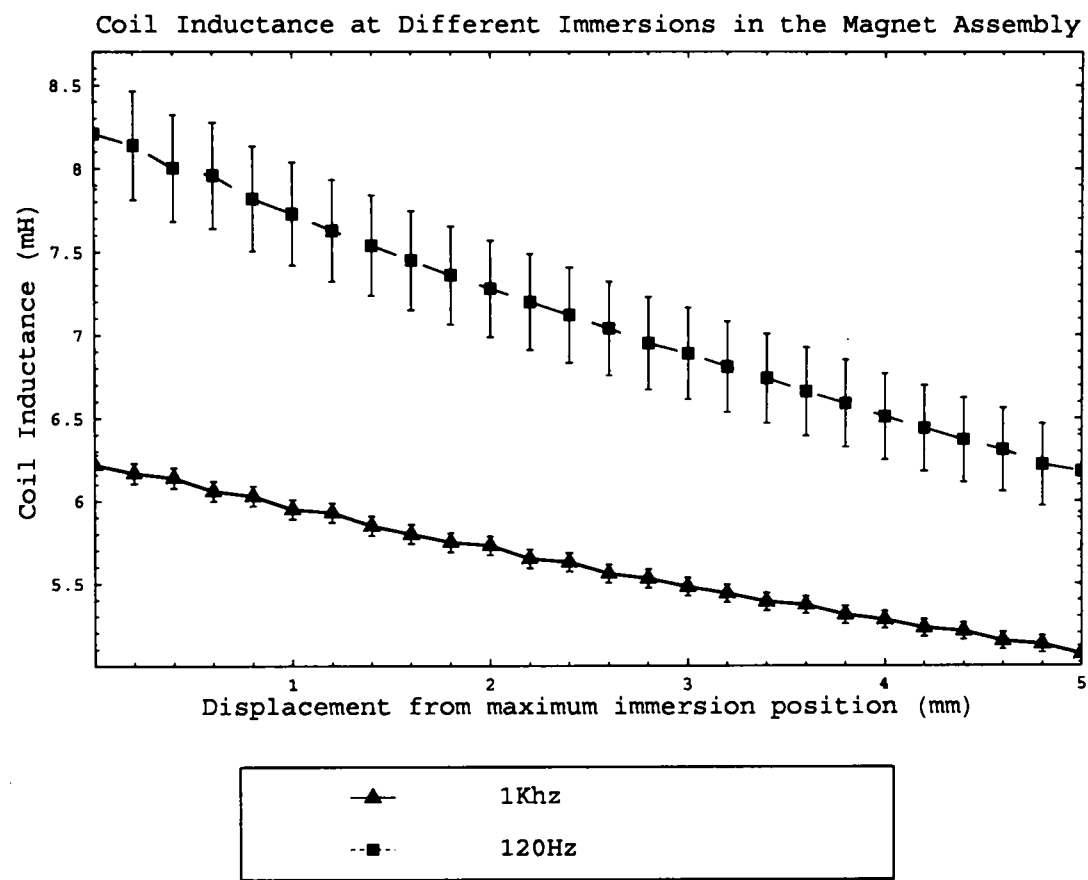
**Fig. 6.20:** Variation in MPPO output with frequency, theory and experiment.

magnet assembly was secured to the bed of the milling machine which could be raised or lowered to alter the degree of coil immersion in the magnet assembly. It was observed that the inductance decreased the more the coil was removed from the magnet assembly. This was to be expected as it is a similar situation to removing a ferrite rod from the centre of a wire-wound inductor. Figure 6.22 shows how the inductance varied as the coil was removed from the magnet assembly, the zero point on the x-axis represents the coil being fully inserted into the magnet assembly so that the bottom of the coil is in contact with the iron pole piece at the bottom of the air gap. Plots are shown at excitation frequencies of 120Hz and 1KHz as these measurements were taken with a small hand-held LCR meter which only had these frequency settings; the high precision meter (which had frequency settings from 20Hz to many KHz) used for the earlier measurements was kept in a clean room and could not be removed for use in the workshop where the milling machine was kept. At 120Hz and 1KHz the LCR meters agreed to better than 4% and 1% respectively. From figures 6.22 and 6.23 it is clear that the coil inductance is not constant but varies with coil position relative to the magnet assembly and also frequency. If the inductance of the coil is not negligible then the MPPO set-up process described in section 6.4.2 will not result in the MPPO output simply being the back e.m.f. induced in the coil but the sum of the induced back e.m.f. and an out of phase voltage drop across the coil due to its inductance. Assuming an operating frequency of 50Hz and a coil inductance of 8.6mH we can calculate the value of  $j\omega L$  to be  $2.7\Omega$ , which is more than a third of the room temperature resistance of the coil and therefore non-negligible. Equation 6.14 represents the MPPO output if the inductance is non-negligible,  $L$  is the inductance of the

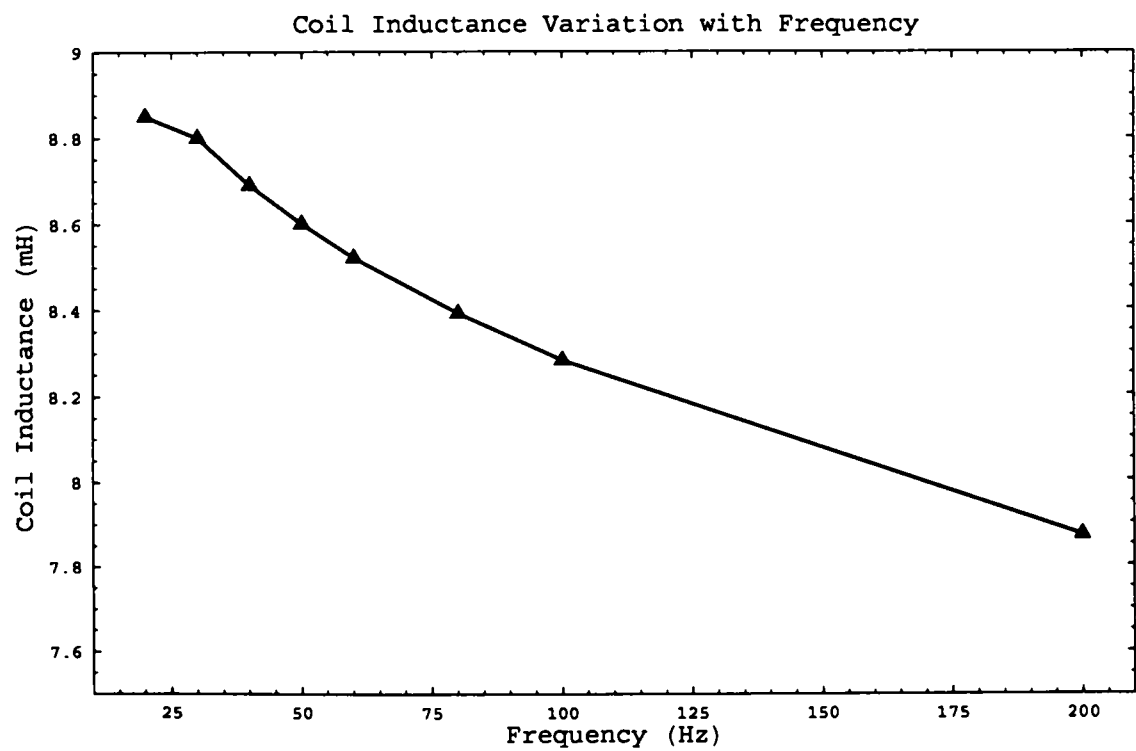




**Fig. 6.21:** Using the DPPO to measure the piston’s motion whilst investigating the MPPO.



**Fig. 6.22:** Variation in coil inductance with immersion in the magnet assembly. Zero on the x-axis represents the maximum immersion of the coil in the magnet assembly.



**Fig. 6.23:** Variation in coil inductance with frequency of excitation.

motor coil, all other symbols have been previously defined.

$$-\dot{x}Bl_b - (j\omega L)i \quad (6.14)$$

Where  $i$  is the current through the motor coil given by:

$$i = \frac{\epsilon_{v0} + \epsilon_{back}}{R + j\omega L} \quad (6.15)$$

Substituting equation 6.15 into equation 6.14 we find:

$$-\dot{x}Bl_b - (j\omega L)\frac{\epsilon_{v0} + \epsilon_{back}}{R + j\omega L} \quad (6.16)$$

Writing the oscillatory terms explicitly:

$$-j\omega X_{po}Bl_b e^{j\omega t} - (j\omega L)\left(\frac{\epsilon e^{j\omega t + \chi} - j\omega X_{po}Bl_b e^{j\omega t}}{R + j\omega L}\right) \quad (6.17)$$

Which simplifies to:

$$\left[-j\omega X_{po}Bl_b - (j\omega L)\left(\frac{\epsilon_{v0}e^{\chi} - j\omega X_{po}Bl_b}{R + j\omega L}\right)\right] e^{j\omega t} \quad (6.18)$$

Multiplying out and collecting real and imaginary parts together. Real part:

$$-\frac{B L l_b R x_{po} \omega^2}{R^2 + L^2 \omega^2} - \frac{L^2 R \epsilon_{v0} \omega \cos(\chi)}{R^2 + L^2 \omega^2} + \frac{L R \epsilon_{v0} \omega^2 \sin(\chi)}{R^2 + L^2 \omega^2} + \frac{L^2 \epsilon_{v0} \omega^2 \sin(\chi)}{R^2 + L^2 \omega^2} \quad (6.19)$$

Imaginary part:

$$j \left[ -\frac{B l_b R^2 x_{po} \omega}{R^2 + L^2 \omega^2} - \frac{L R \epsilon_{v0} \omega \cos(\chi)}{R^2 + L^2 \omega^2} - \frac{L^2 \epsilon_{v0} \omega^2 \sin(\chi)}{R^2 + L^2 \omega^2} \right] \quad (6.20)$$

This has magnitude given by:

$$\left( \frac{\omega^2 (B^2 l_b^2 R^2 x_{po}^2 + L^2 \epsilon_{v0}^2 + 2 B L l_b R x_{po} \epsilon_{v0} \cos(\chi))}{R^2 + L^2 \omega^2} \right)^{\frac{1}{2}} \quad (6.21)$$

The phase angle  $\theta$  is given by:

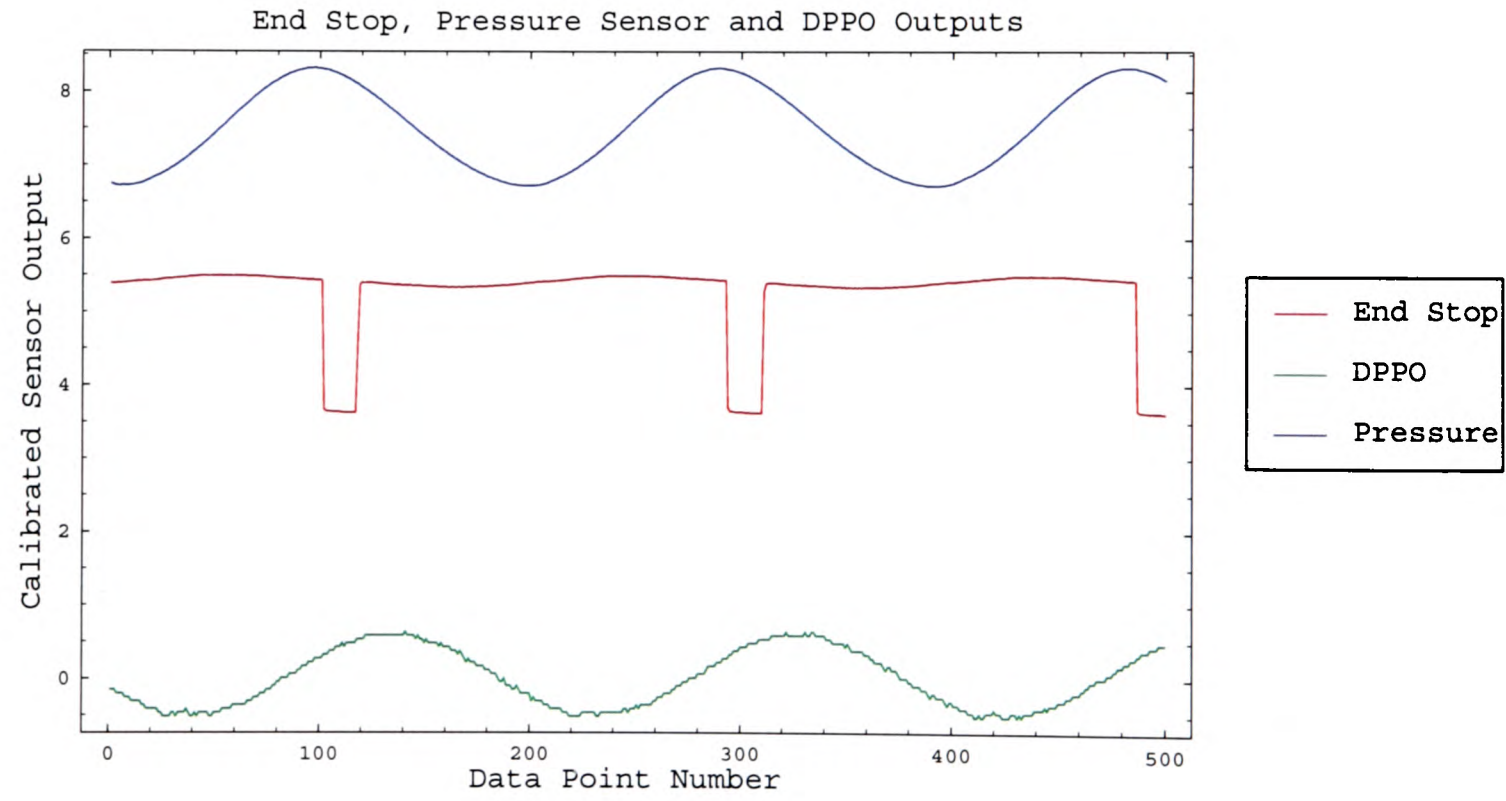
$$\theta = \text{Arctan} \left[ \frac{B l_b R^2 x_{po} + L R \epsilon_{v0} \cos(\chi) + L^2 \epsilon_{v0} \omega \sin(\chi)}{B L l_b R x_{po} \omega + L^2 \epsilon_{v0} \omega \cos(\chi) - L R \epsilon_{v0} \sin(\chi)} \right] \quad (6.22)$$

It had become clear that subtracting the resistive part of the voltage drop across the motor coil would not simply leave the back e.m.f. but a combination of the back e.m.f. and a voltage drop proportional to the inductance of the motor coil. Since this “inductive voltage drop”,  $V_L$  is given by  $j\omega Li$  and  $i$  (the current through the motor coil) is itself a function of the back e.m.f, driving voltage and the inductance  $L$ , it will have components in phase and out of phase with the back e.m.f. as shown in the above analysis. The complicated form of  $V_L$  and its dependence on the coil's inductance,  $L$ , which varies according to the coil's position, make it difficult to calculate. Even if the inductance was known, the dependence of  $V_L$  on the the phase angle between the coil's motion

and the driving voltage,  $\chi$  (which can only be measured if accurate position data is available) presents an obstacle. Initially it was thought that by introducing a second inductor in series with the stationary sample coil and between the input terminals of the variable gain op-amp the motor coil inductance could be accounted for. By introducing a stationary inductor in the MPPO circuit an inductive voltage drop in phase but with opposite sign to the inductive voltage drop across the motor coil could be created. This could then be subtracted from the voltage drop across the motor coil so that only the voltage drop due to the back e.m.f. remained. However figures 6.22 and 6.23 show that the inductance will not be constant with stroke or frequency so this simple method for canceling out the inductive voltage drop would not work. It was decided that until a solution to the problems with the MPPO could be found or an alternative sensor could be created, detecting the extremes of the coil's displacement would be sufficient. This was achieved by inserting two beryllium copper rings with tags that would be touched by the coil former at each extreme of its motion. Each ring was electrically insulated from the rest of the cooler and a voltage was applied to them and this was monitored by the data acquisition system. When the coil former came into contact with one of the rings it would ground it. When this happened a distinct reduction could be seen in the voltage data for the ring as seen by the data acquisition system. This simple system only allows the coil motion to be detected when it is large enough to hit these rings but it was thought that since the cooler would always be run at full stroke this would not present a problem. Of course having continuous coil/piston position data throughout the cycle would be an advantage but the design of the cooler could not be adapted to allow for a conventional position sensor such as a LVDT. Figure 6.24 shows typical data from one of the MPPO end-stops, the DPPO and the pressure sensor. The sudden reductions in the end stop voltage indicate that the piston is achieving one extreme of motion and is grounding one of the beryllium copper rings.

## 6.6 Summary

This chapter has described the instrumentation and support systems created for use with the prototype cooler. The array of sensors developed for use with the cooler have been calibrated and during this process the MPPO displayed serious flaws which necessitated its replacement by a much simpler method of detecting the piston's motion. The other sensors worked as expected. A newly created data acquisition system allowed the output of these sensors to be displayed in real-time on a personal computer as well as storing them for later analysis.



**Fig. 6.24:** Typical outputs from the DPPO, MPPO end-stop and pressure sensor.

## Chapter 7

# Initial compressor characterisation

### 7.1 Introduction

The overall efficiency of the cooler is considered as the product of the efficiency of the compressor and the efficiency of the thermal cycle. This chapter considers optimising the compressor at room temperature whereas thermal cycle optimisation is considered in chapter 8. However, the performance of the thermal cycle determines what performance the compressor must produce if the cooler is to achieve its target performance. It has been estimated (Section 4.1) that an input power of 3W into the working gas from the compressor would allow its performance target to be reached. Determination of the operating conditions that would allow 3W to be input to the gas from the compressor piston are described within this chapter, as are measurements of the compressor sliding friction factor and resonant frequency.

### 7.2 Compressor sliding friction

It was possible to use the DPPO to measure the piston motion if the displacer/regenerator was removed and the moving DPPO disk fitted to the back of the coil former via an adaptor fabricated for this purpose (see figure 6.21). This enabled continuous piston position data to be obtained during the initial characterisation tests. The first of these tests was performed to ascertain the sliding friction factor of the compressor. A large amount of sliding friction may occur if there is significant contact between the compressor piston seal and the compression cylinder. The seal is made from Flurosint (PTFE filled with mica) and initially it is a tight fit to the compression cylinder. Gradually it wears away as the compressor runs so that a non-contact seal remains around the piston. To determine the sliding friction factor a step voltage is applied to the compressor motor

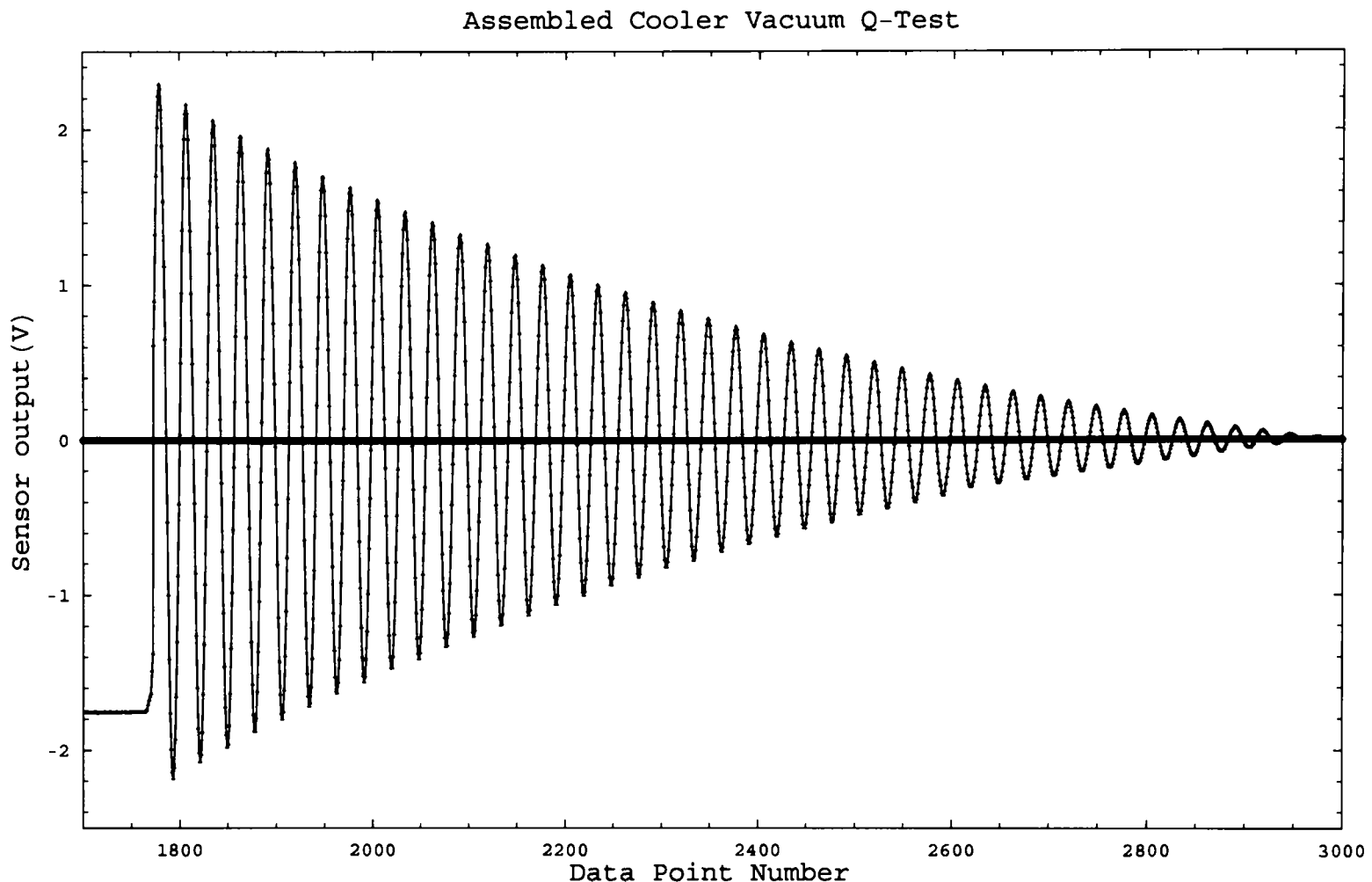


Fig. 7.1: Q-test on the assembled cooler when evacuated.

several times when the cooler is evacuated. The decaying oscillations of the coil motion with the coil open circuited were recorded using the data acquisition system described previously. Figure 7.1 shows a typical data acquisition scan for this procedure. The Q-value for the compressor is then calculated via the following equation:

$$Q = \frac{\pi(m - n)}{\ln(A_n/A_m)} \quad (7.1)$$

Where  $A_n$  and  $A_m$  are the amplitudes of the oscillations  $n$  and  $m$ . Applying this equation to successive peaks in the data shown in figure 7.1 and averaging the results the Q-value was calculated as 74.6 (to 3.s.f).

The sliding friction factor could then be calculated using this Q-value and the following equation:

$$\lambda_{sl} = \frac{2m_c\omega_{cr}}{Q} \quad (7.2)$$

$\lambda_{sl}$  was found to be 0.48 Ns/m for the q-test shown in figure 7.1. Recalling equation 5.2 for the work done by the motor against any damping coefficient  $\lambda$ :

$$W_\lambda = \frac{1}{2}\text{Re}(\lambda\omega^2 X_{p0}^2) \quad (7.3)$$

From this the power lost to sliding friction at full stroke can be calculated: 180mW.



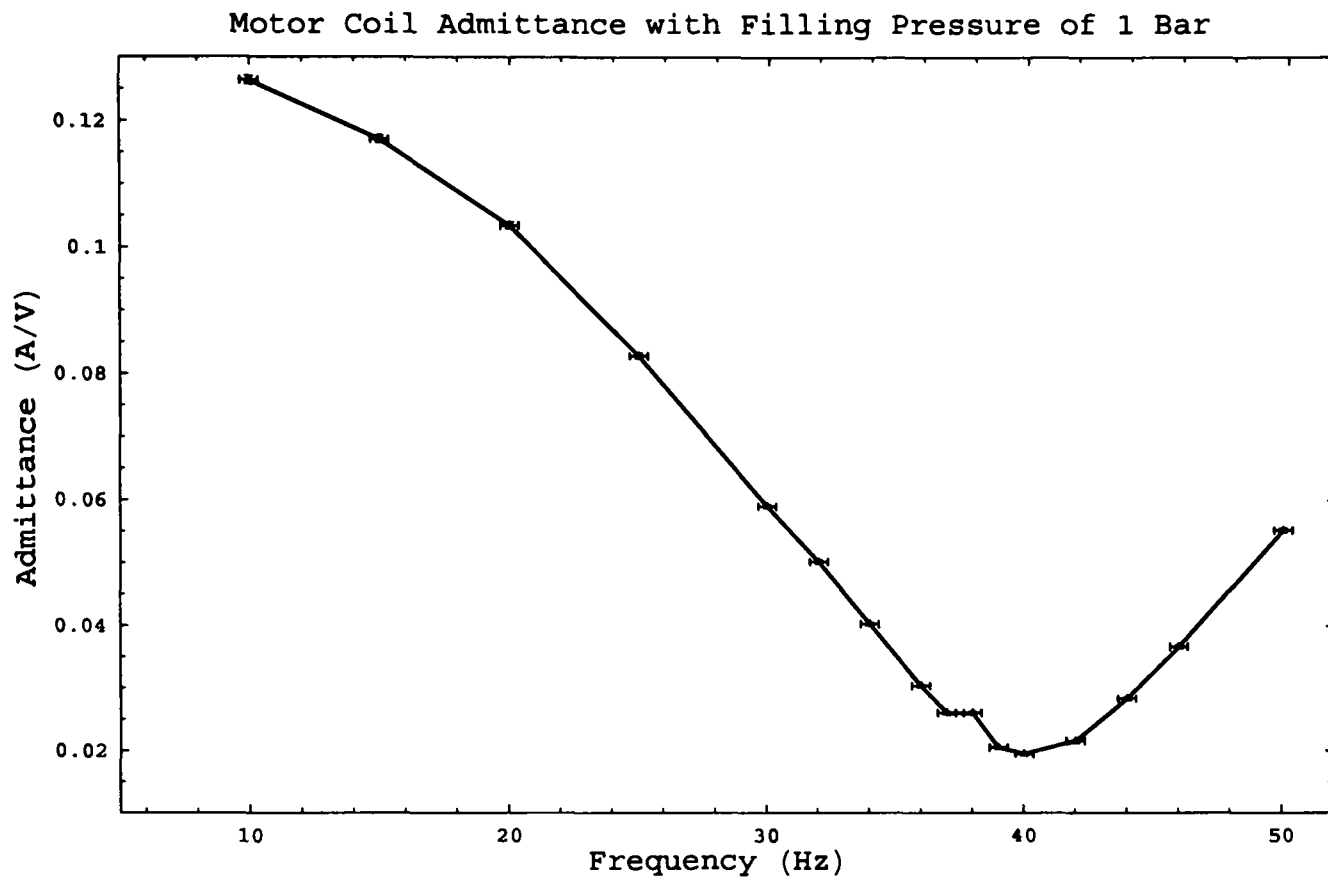


Fig. 7.2: Variation in coil admittance with frequency at 1 bar filling pressure.

### 7.3 Characterisation procedure

After performing the q-test the prototype cooler was re-assembled with the displacer inserted. The cooler was then mounted on the warm heat sink in the vacuum chamber and filled with helium at pressures between 0–8 bar. The motor drive voltage was then swept through a range of frequencies from 10 to 70 Hz and high frequency data (1 KHz) from all the cooler sensors (other than the PRTs) was recorded. Figures 7.2 to 7.5 show how the coil admittance varied with frequency for filling pressures of between 1 and 7 bar. It can be seen that, as expected, increasing the filling pressure leads to an increase in the frequency at which resonance occurs. The variation of the compressor resonance with filling pressure is shown in figure 7.6. Equation 4.64 was fitted to the data and resulted in values of  $\alpha = 1.12$  (suggesting the compression is somewhere between adiabatic and isothermal) and  $k_p = 4777.6 \text{ N/m}$  which differs by 1.5% from the experimentally determined value of the mechanical spring constant<sup>1</sup>. Measuring the power input to the gas was problematic, originally it was intended that a PV loop would be created using the MPPO and pressure sensor output but since the MPPO had been replaced by the simple end stops these were the only way to gather data on the piston position. As shown in figure 6.24, when the piston reaches one extreme of its motion a sudden reduction is produced in the voltage output of one of the end stops. Similarly

<sup>1</sup>Equation 4.64 assumes that the coil inductance is negligible, the effect of non-negligible inductance is examined in Appendix B



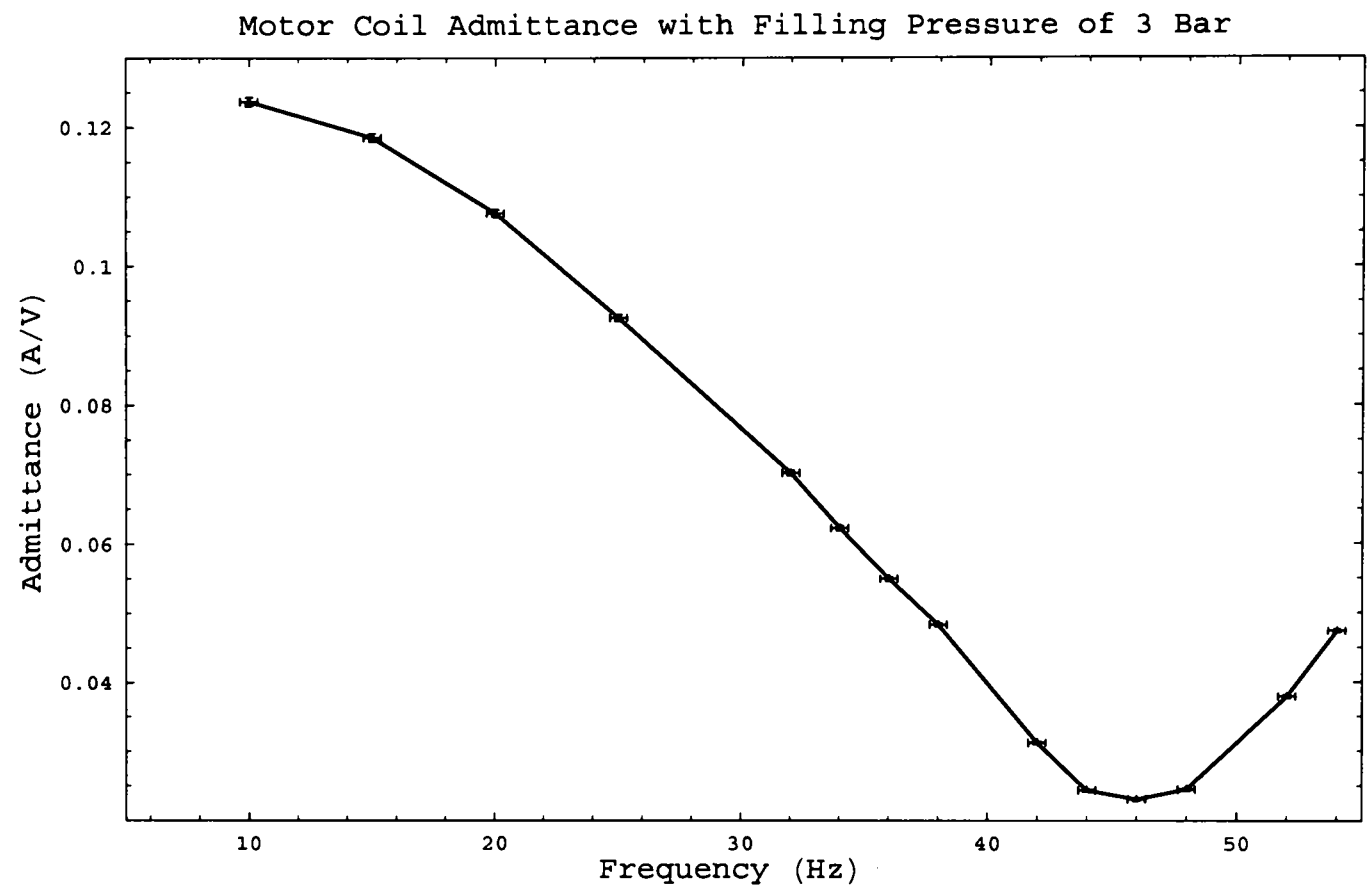


Fig. 7.3: Variation in coil admittance with frequency at 3 bar filling pressure.

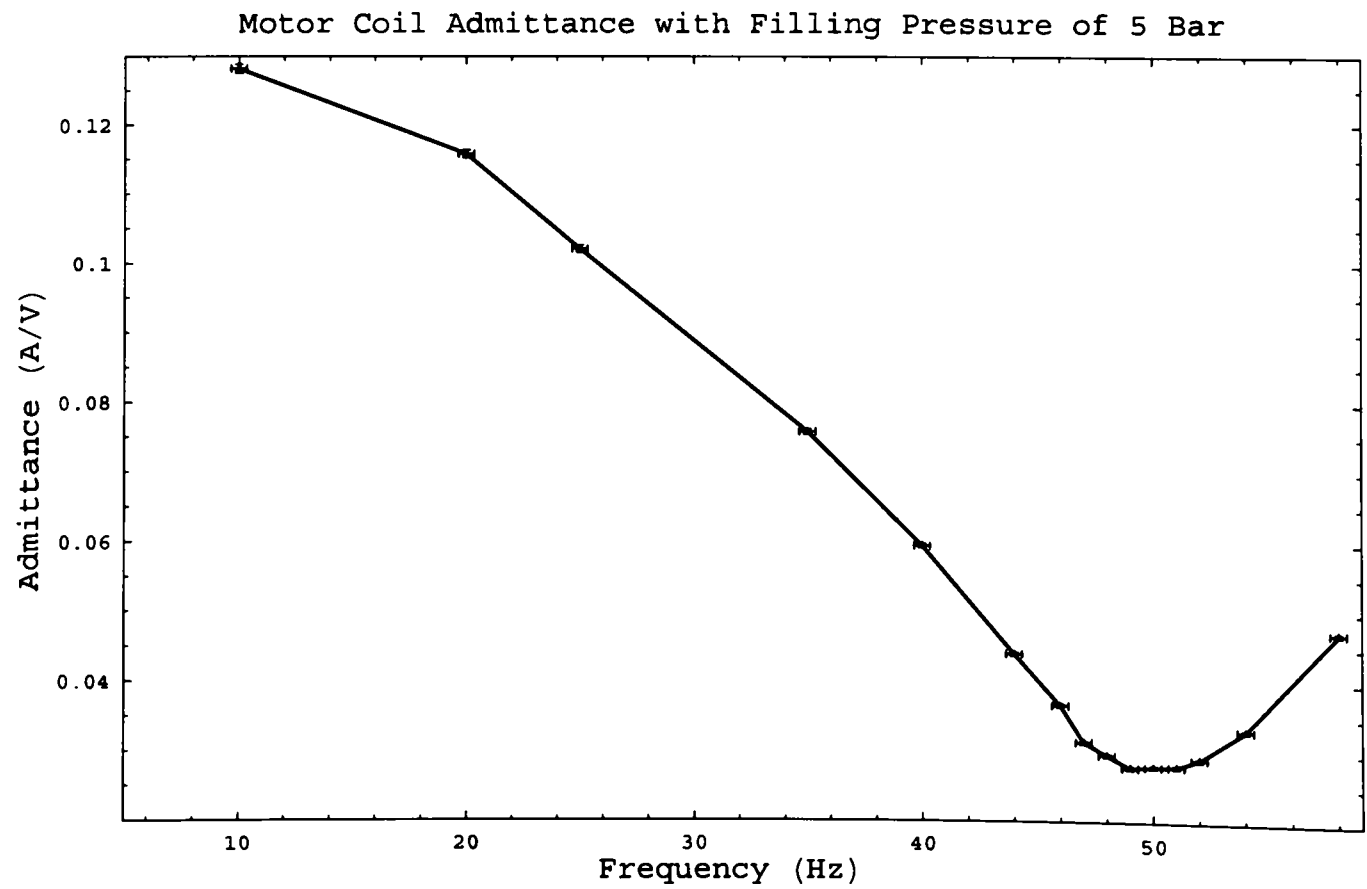


Fig. 7.4: Variation in coil admittance with frequency at 5 bar filling pressure.

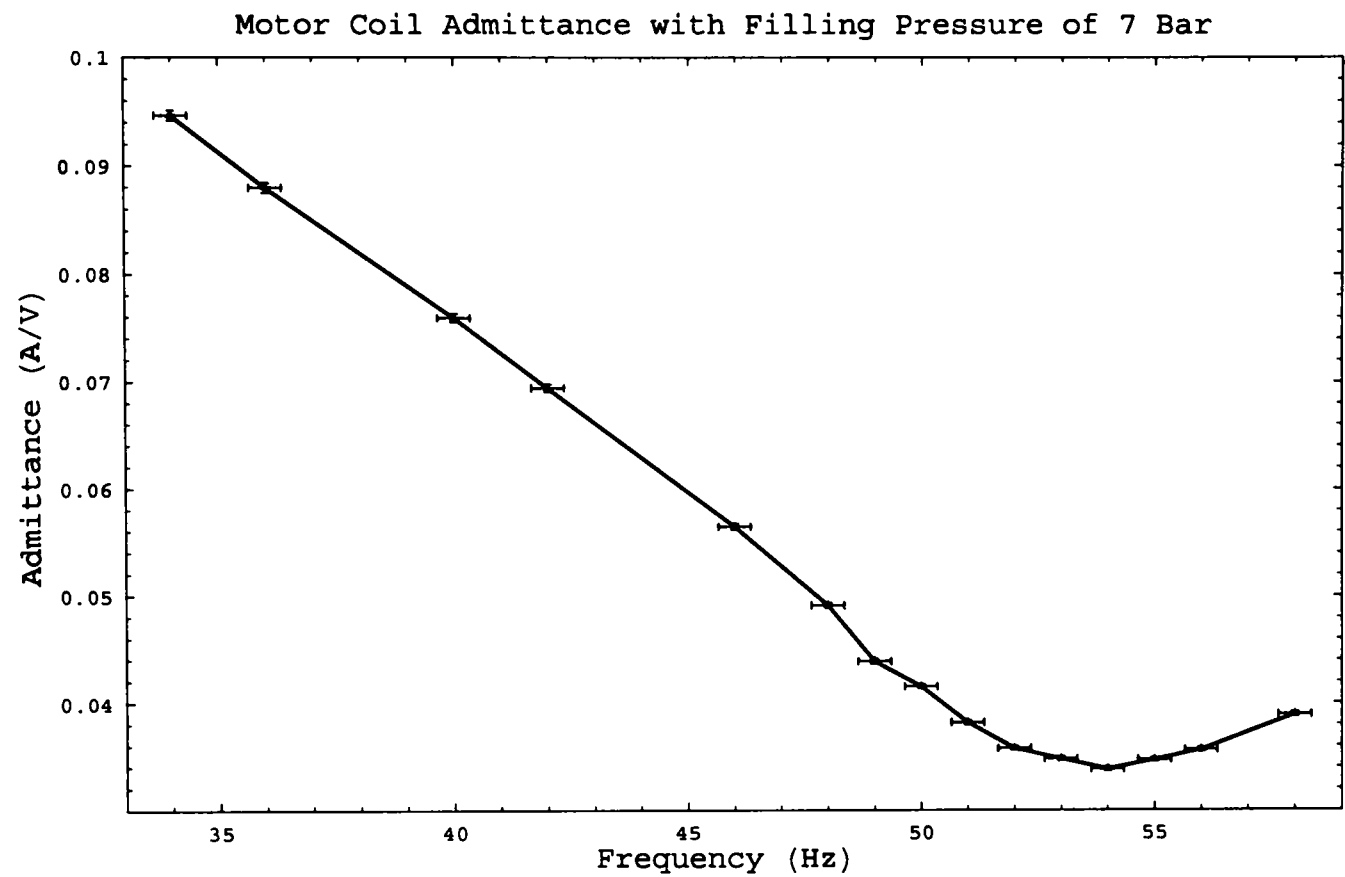


Fig. 7.5: Variation in coil admittance with frequency at 7 bar filling pressure.

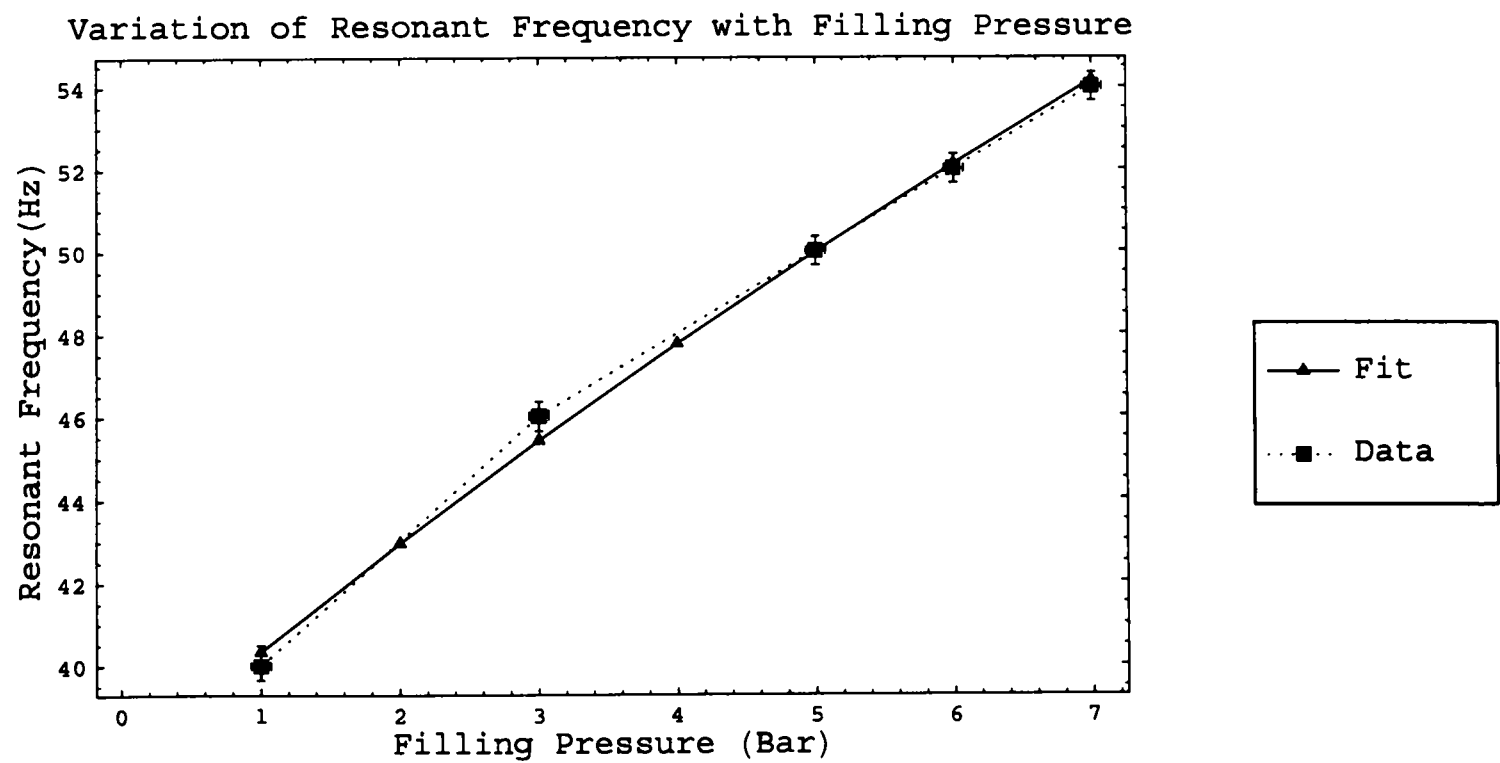


Fig. 7.6: Variation in the compressor resonant frequency with filling pressure.

when it reaches the other extreme of motion a reduction is seen in the output of the other end stop. Originally it was intended that by fitting a sine wave so that its peaks coincided with these reductions in the end stop voltages the piston motion could be inferred. Whilst this may give a rough estimate of the piston position the width of the reductions in the end stop voltages meant there was a large degree of uncertainty in the exact point at which the coil achieves its maximum displacement. This leads a large uncertainty in the phase angle between the piston's motion and the compression space pressure variation. The area of the PV loop is very sensitive to the phase angle between the volume variation (i.e. the piston's motion) and the pressure variation and so this presented a problem in getting reliable measurements of the power into the gas. By very carefully adjusting the drive amplitude so that both end limits were only just hit, the width of the sudden voltage reductions could be reduced. This measurement was repeated many times at each filling pressure and the results averaged. It was found that to achieve an input power of at least 3W to the gas the compressor needed to run at full stroke with a filling pressure of 7 bar.

The method outlined above would not be suitable for calculating the power into the gas during cooling runs as it requires the drive amplitude to be constantly tweaked so that the end stops are just being touched and it is difficult to judge how wide the reductions in the end limit output voltages are when they are being viewed in real-time via the data acquisition software on the laboratory computer. An alternative method for calculating the power delivered to the gas involved subtracting the power lost to Joule heating of the motor coil and any other residual losses, from the electrical input power to the compressor motor. This was thought to be more suitable for calculating the power into the gas throughout cooling runs.

## 7.4 Residual losses

To produce an accurate calculation of the power delivered to the gas it was important to know if any power was lost to processes other than Joule heating, gas and sliding friction. These residual losses may arise from windage, vibration of the cooler or eddy current losses. To investigate whether these were significant the cold finger was removed from the cooler so that the compressor was open to air at atmospheric pressure. Under these conditions the viscosity and density of air are similar to that of helium at 7bar ( $\rho_{He}(7bar)/\rho_{air}(atm) = 0.93$ ,  $\mu_{He}(7bar)/\mu_{air}(atm) = 1.1$ ) thus windage losses should be close to those experienced under operational conditions. Since no compression is taking place the motor input power is consumed only by Joule heating of the motor coil, frictional and residual losses. Figure 7.7 shows how the residual loss varied with piston stroke. The residual loss was calculated via equation 7.4.

$$W_{res} = W_{in} - W_{ir} - W_{sl} \quad (7.4)$$

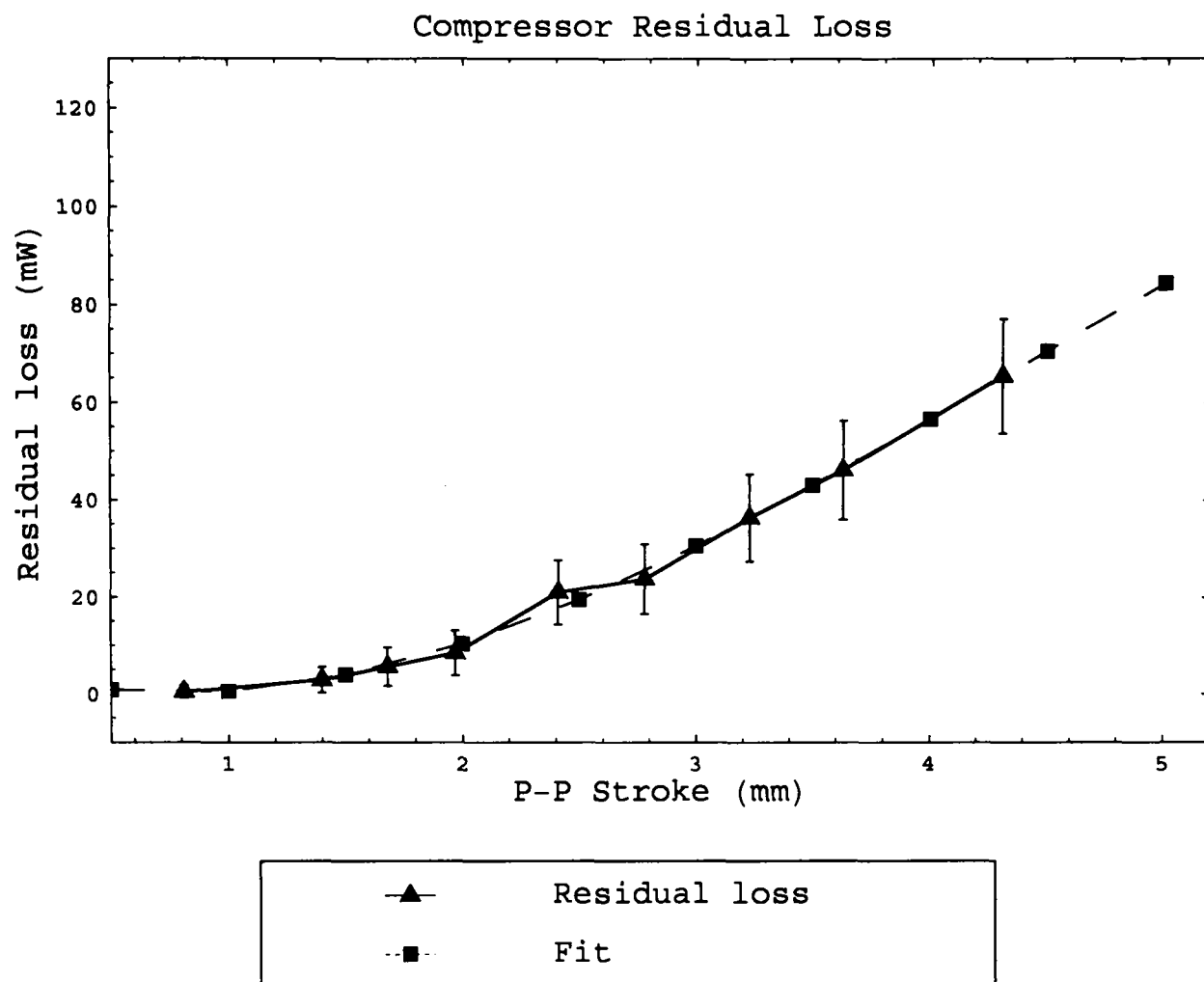


Fig. 7.7: Variation in residual loss with piston stroke in air at 54Hz.

Where  $W_{res}$  is the residual loss,  $W_{in}$  is the electrical input power to the motor,  $W_{ir}$  is the power lost to Joule heating and  $W_{sl}$  is the power lost to sliding friction.

The compressor was run at 54Hz over the full range of strokes available. Before making a measurement the coil resistance was measured and then again immediately after each measurement. This reduced uncertainty in the coil resistance when calculating the loss due to Joule heating. Measurements of the current and voltage supplied to the motor were measured by the data acquisition system at a rate of 4000 samples per second per channel. In air at 1 bar the compressor's resonance is below 40Hz and so driving the compressor at full stroke at 54Hz when open to the air would have required a large drive voltage. Since most of the input power would have been dissipated as heat in the coil it was felt unwise to attempt this. Instead the residual loss at full stroke (5mm) was found from a polynomial fit to the data shown in figure 7.7. It can be seen that at full stroke (5mm p-p) the residual loss is around 90mW. Using this value for the residual loss  $W_{res}$  it was possible to calculate the power being delivered to the gas at full stroke at resonance at 7bar via equation 7.5:

$$W_{gas} = W_{in} - W_{ir} - W_{sl} - W_{res} \quad (7.5)$$

## 7.5 Summary

The compressor has been characterised and as expected its resonant frequency (as measured by the coil admittance) was seen to increase with filling pressure. The data acquired during characterisation enabled a fit to the model predictions of resonant frequency to be made. The result of this fit indicated that the gas was undergoing compression that was neither completely adiabatic nor isothermal but somewhere in between. The value of the mechanical spring constant produced by this fit was in good agreement with the experimentally obtained value. Caution should be exercised when interpreting the meaning of  $\alpha$  since it assumes all the gas is behaving in the same manner. In reality it is likely that gas close to the cylinder walls will be isothermal whilst gas in the centre of the compression space will be more nearly adiabatic with a polytropic transition region in between. Measurements indicated that an input power of 3W to the working gas from the compressor could be achieved at resonance with a filling pressure of 7bar. It was possible to adapt the DPPO so that continuous piston position data could be obtained during investigation of the compressor sliding friction factor and it was found that a Q value in excess of 70 was possible with the cooler evacuated.

## Chapter 8

# Thermodynamic optimisation

### 8.1 Introduction

The goal of the optimisation is to achieve the lowest possible cold tip temperature with no external load applied. Consider the energy balance equation for the cooler operating between a cold tip temperature  $T_e$  and compression space temperature  $T_c$ :

$$C_{ct} \frac{dT_e}{dt} = Q_{leak} + Q_{ext} - W_{gas} \epsilon_i \beta \quad (8.1)$$

Where  $Q_{leak}$  represents the parasitic heat load on the cold tip, which can be split into *dynamic losses* that are only present when the cooler is operating and *static losses* that are present whether the cooler is running or not.  $Q_{ext}$  represents any external heat load applied to the cold tip.  $W_{gas}$  is the input power to the working fluid supplied by the compressor and  $\epsilon_i$  is the Carnot coefficient of performance (c.o.p.) at the values of  $T_e$  and  $T_c$  that the cooler is operating between.  $\epsilon_i$  is given by:

$$\epsilon_i = \frac{T_e}{T_c - T_e} \quad (8.2)$$

$W_{gas} \epsilon_i$  represents the maximum possible power available for refrigeration. In real refrigerators irreversible processes mean that the power available for refrigeration is reduced. The fraction of  $W_{gas} \epsilon_i$  available for refrigeration is  $\beta W_{gas} \epsilon_i$  where  $\beta$  is the “ideality factor” the value of which represents the degree to which the refrigerator approaches the ideal case (i.e.  $\beta = 1$  would represent an ideal refrigerator).

If we consider the cooler with no external heat load applied to the cold tip ( $Q_{ext}=0$ ) equation 8.1 becomes:

$$C_{ct} \frac{dT_e}{dt} = Q_{leak} - W_{gas} \epsilon_i \beta \quad (8.3)$$

$Q_{leak}$  will be frequency dependent due to the dynamic heat load, as will  $W_{gas}$  and  $\beta$  is expected to be as well. There will be some optimum frequency at which  $Q_{leak} - W_{gas}\epsilon_i\beta$  will be at its most negative, and examining equation 8.3 it is clear that this can be identified by determining the frequency at which the greatest rate of decrease of the cold tip temperature  $T_e$  occurs. This optimum frequency corresponds to the volume variation in the cold expansion space leading the volume variation in the warm compression space by a particular phase angle, typically around  $90^\circ$  in Stirling coolers [Radebaugh R. (1999)].

### 8.1.1 Optimisation procedure

The cooler was filled with dried helium to a pressure of 7 bar, then run at full stroke, just hitting the end stops, at the resonant frequency with one displacer spring installed. These operating conditions were maintained until the cold tip temperature became steady (a rate of change less than 0.2K/per min for a period of ten minutes). The frequency was then altered by 1Hz, and the cold tip observed until a new steady state was reached. If this was at a lower temperature than the original steady state then the frequency was increased again, and so on until no further decrease in the cold tip steady state temperature was observed. These initial measurements (when one displacer flexure was in place) indicated that the optimum cooling frequency was approximately 42Hz.

The frequency at which the highest cooling rate was observed could then be “shifted” close to the resonant frequency. To understand how this was done we consider a simple harmonic oscillator model of the displacer: It is assumed that the resonant frequency of the displacer (when referring to the displacer, resonance refers to the frequency at which its displacement is maximised) would be dominated by the mechanical flexures, and to a good approximation would have a resonant frequency given by:

$$f_{dr} = \frac{1}{2\pi} \sqrt{\frac{n_d k_d}{m_d}} \quad (8.4)$$

Where  $n_d$  is the number of displacer flexures,  $k_d$  is the axial spring constant of a single displacer flexure (as determined by experiment earlier in this study), and  $m_d$  is the displacer mass. By changing the number of displacer flexures its resonant frequency (and also the optimum cooling frequency) can be altered. Figure 8.1 shows diagrammatically how this procedure works. The displacer springs were very stiff and so only provided a very coarse method of shifting the optimum cooling frequency. There seemed to be two solutions to this problem:

- Create new springs with a lower axial spring constant so that the frequency could be shifted in smaller increments.



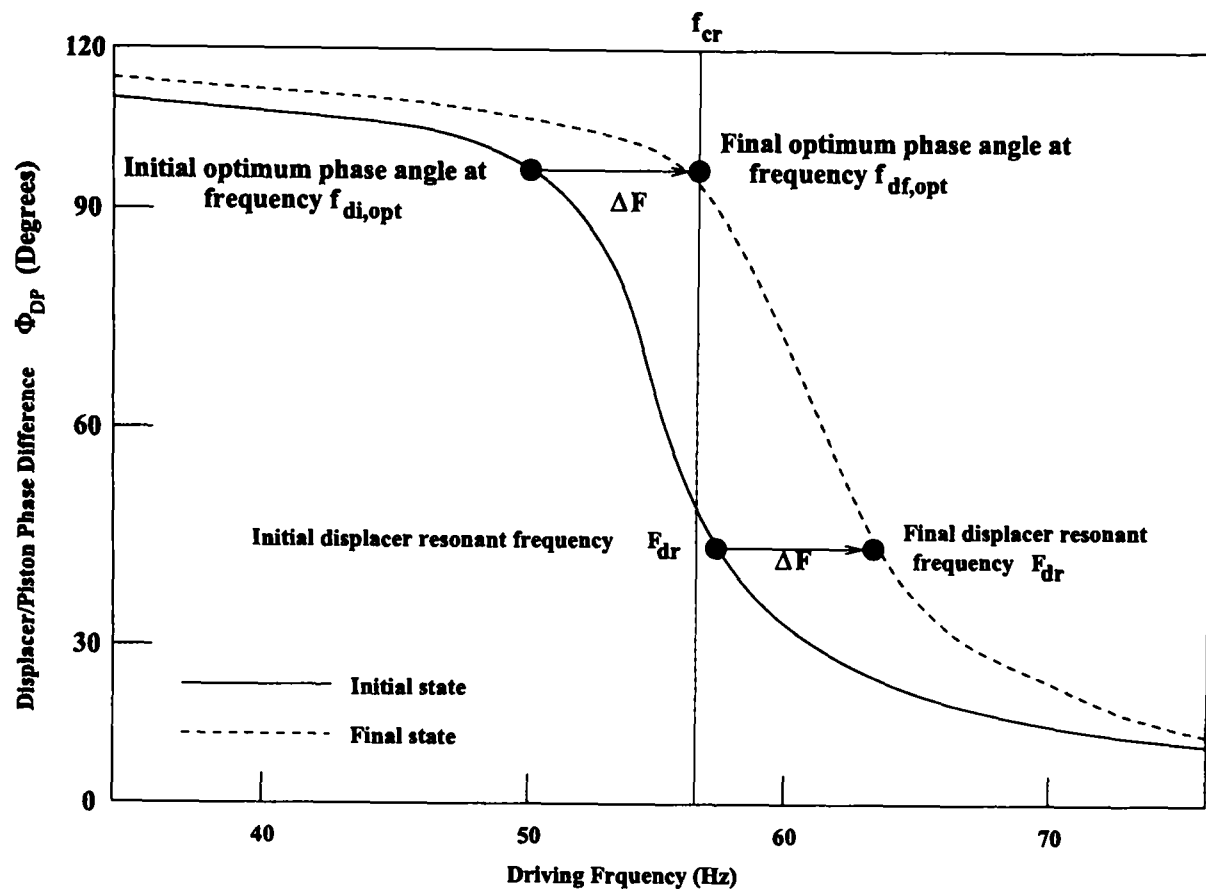


Fig. 8.1: Diagram showing how the optimum frequency for cooling can be shifted close to the compressor resonant frequency.

- Use two displacer springs and “mass load” the displacer to shift its frequency back toward the compressor resonance.

The availability of a photo-etching facility within the department meant it was a relatively simple matter to alter the design of the springs and have a new batch produced, so the former option was chosen. Only one thickness of spring steel was available and so the flexure’s spring constant was reduced by decreasing the width of the arms, this can be seen by comparing the original displacer flexure shown in figure 8.2 with the lower spring constant flexure shown in figure 8.3. The new springs’ spring constant was ascertained by the method described for the compressor springs and found to be  $435.141 \pm 3.32 \text{ N/M}$  compared to the original displacer flexures value of  $547.833 \pm 4.94 \text{ N/M}$ . Once re-assembled with the new displacer flexures, further cooling runs were performed and the ideal cooling rate was found to lie in the range  $54 - 61 \text{ Hz}$ , close to the compressor resonance. The precise frequency at which the greatest cooling rate occurred was observed to change between one assembly of the cooler and the next and this was due to poor alignment of the displacer causing a large amount of rubbing and impeding its movement. As experience was gained in assembling the cooler the scatter in the frequency at which the greatest cooling rate was found was reduced. Later cooling runs, when the cooler was being assembled with greater skill, indicated that the optimum frequency for cooling was in the range  $59 - 61 \text{ Hz}$ . Table





**Fig. 8.2:** Original displacer spring.



**Fig. 8.3:** Example of a displacer from the second batch (reduced stiffness).

8.1 contains the optimum operating conditions under which cooling runs were performed.

## 8.2 Summary

This chapter contains a simple thermal analysis of the cooler which has allowed for a simple procedure to determine the frequency at which the cooler's performance is maximised.

A procedure for optimising the cooler has been described which involved shifting the resonant frequency of the displacer/regenerator by altering the number of spiral flexures on which it was suspended. A new batch of displacer/regenerator flexures with lower spring constants were produced to provide finer control in shifting the displacer/regenerator resonant frequency. It became clear during the optimisation procedure that there were a number of flaws with the cooler design

| Parameter           | Value     |
|---------------------|-----------|
| Filling Pressure    | 7 bar     |
| Piston Stroke (P-P) | 5mm       |
| Frequency (Hz)      | 59 – 61Hz |

**Table 8.1:** Optimum Cooler Conditions.

that had not been foreseen during the design phase, most notably the displacer/regenerator is only aligned with the rest of the cooler geometry at the very back of the cooler far away from the cold tip. This caused problems in aligning the cooler in a repeatable way between successive assemblies (hence the variation in optimum frequency for cooling as described above) and so led to large variations in displacer/regenerator stroke and cooler performance under otherwise identical operating conditions. What is more, the displacer/regenerator spring stack was secured onto a Tufnol (Epoxy-cotton) mount which was not as rigid as the other titanium components in the cooler and the threaded holes suffered severely from wear after only a few assemblies. These design flaws, amongst others are discussed in a chapter 12.

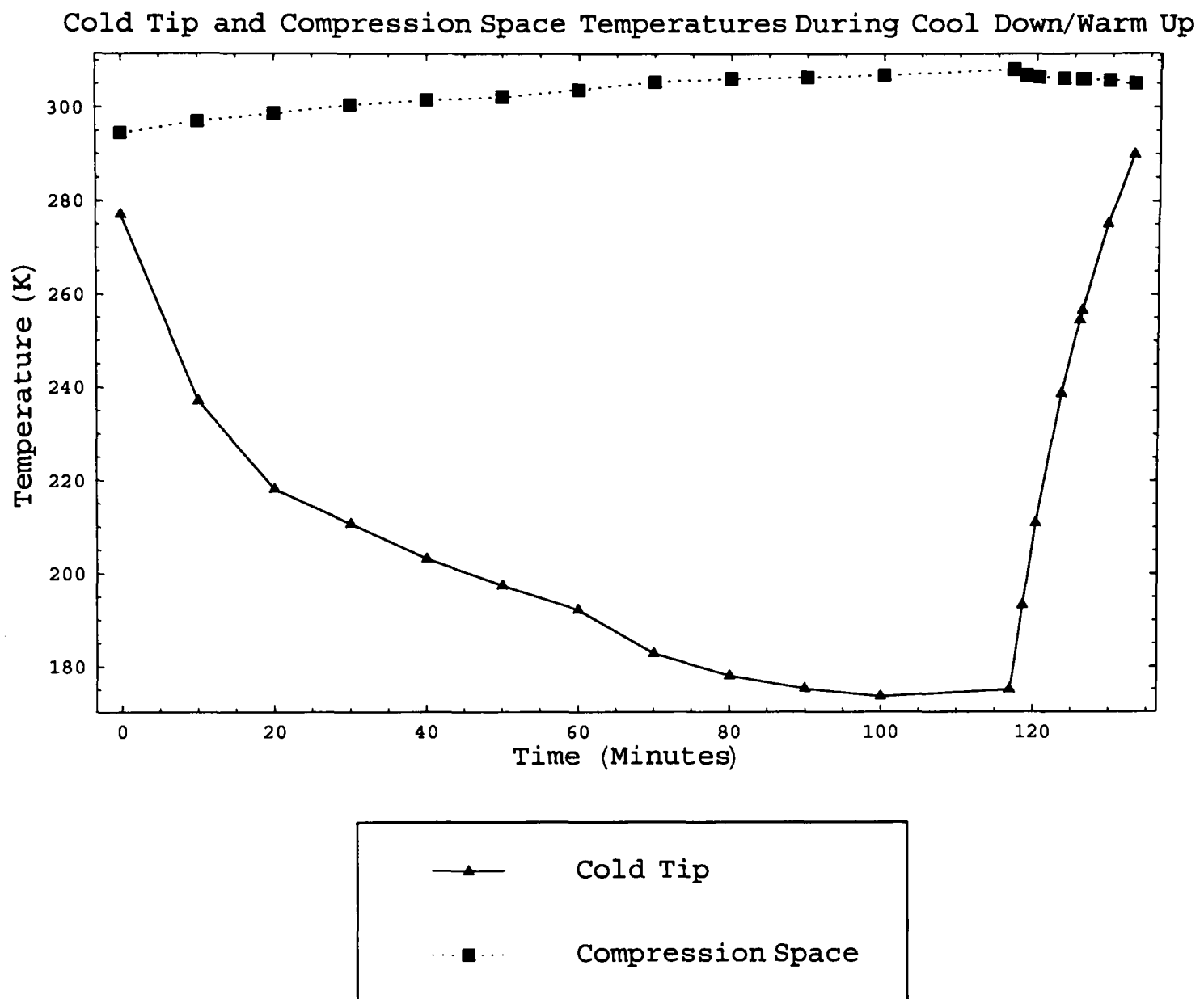
## Chapter 9

# Room temperature runs

### 9.1 Introduction

The initial cooling runs were performed with the cooler body mounted on a rigid aluminium cylinder that in turn was bolted to the bottom inside surface of the vacuum chamber. This provided a rigid mount for the cooler when the body was not being cooled by the radiator. The cold tip was surrounded by 10 layers of multi-layer insulation (MLI) to reduce the radiation loading on the cold tip from its room temperature surroundings. The cold tip temperature was measured by a single PRT inserted into a cavity in the cold tip copper block, and good thermal contact was ensured by the use of thermally conducting elastomer. The body temperature rose as the cooler operated, this could be detected by three platinum resistance thermometers located on the cooler body, two on the outside of the compression space cylinder and one on the main body (containing the motor, springs, displacer/regenerator shaft etc). Again these were all bonded to the cooler with thermally conducting elastomer and were further secured with adhesive aluminium tape.

During the initial tests the cooler was filled with dried helium to a pressure of 7 bar. The cooler was left connected to the helium bottle throughout the warm and cooled body runs. The data acquisition system was used to record the sensor outputs throughout each cooling run for later analysis. Typically the sensor outputs would be recorded every minute for a period of 1 second at a sampling rate of ten thousand samples per second. Once the cold tip temperature had stabilised the power supply to the cooler was switched off and an external heat load applied to the cold tip. The data acquisition system was programmed to take scans of the PRT outputs every 20 seconds as the cold tip warmed up. Section 9.3 describes how these measurements can be used to calculate the cold tip heat capacity,  $C_{ct}$  and the thermal resistance between the compression space and cold tip  $r_{ct}$ .



**Fig. 9.1:** Cold tip and compression space temperatures during a cooling run.

## 9.2 Results

Figure 9.1 shows temperature data from a typical cooling run/warm-up. The increase in cooling rate at around 60 minutes is due to an increase in the operating frequency, indicating that the optimum frequency for cooling at that cold-end temperature was higher than the frequency the cooler had initially been operating at. During cooling runs the input current and voltage to the cooler were recorded by the data acquisition system. This data could then be analysed to produce plots of the cooler's power consumption, power lost to resistive heating and motor efficiency. Figure 9.2 shows the cold tip and compression space temperatures from another cooling run and 9.3 shows the corresponding motor input power and resistive loss for that cooling run. A number of cooling runs were performed with the body allowed to warm up from room temperature as the motor operated (i.e. it was not being pre-cooled by the radiator). This allowed data to be gathered so

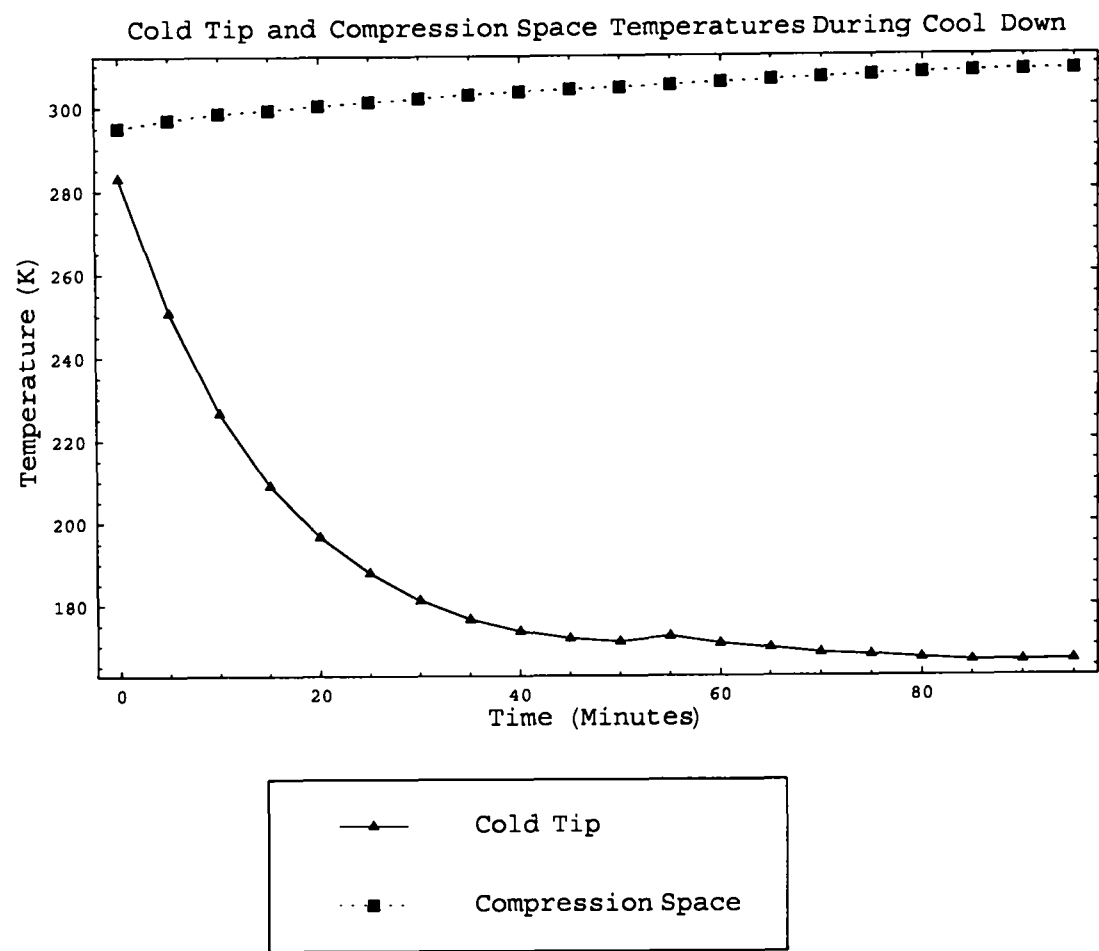


Fig. 9.2: Cold tip and compression space temperatures during a cooling run.

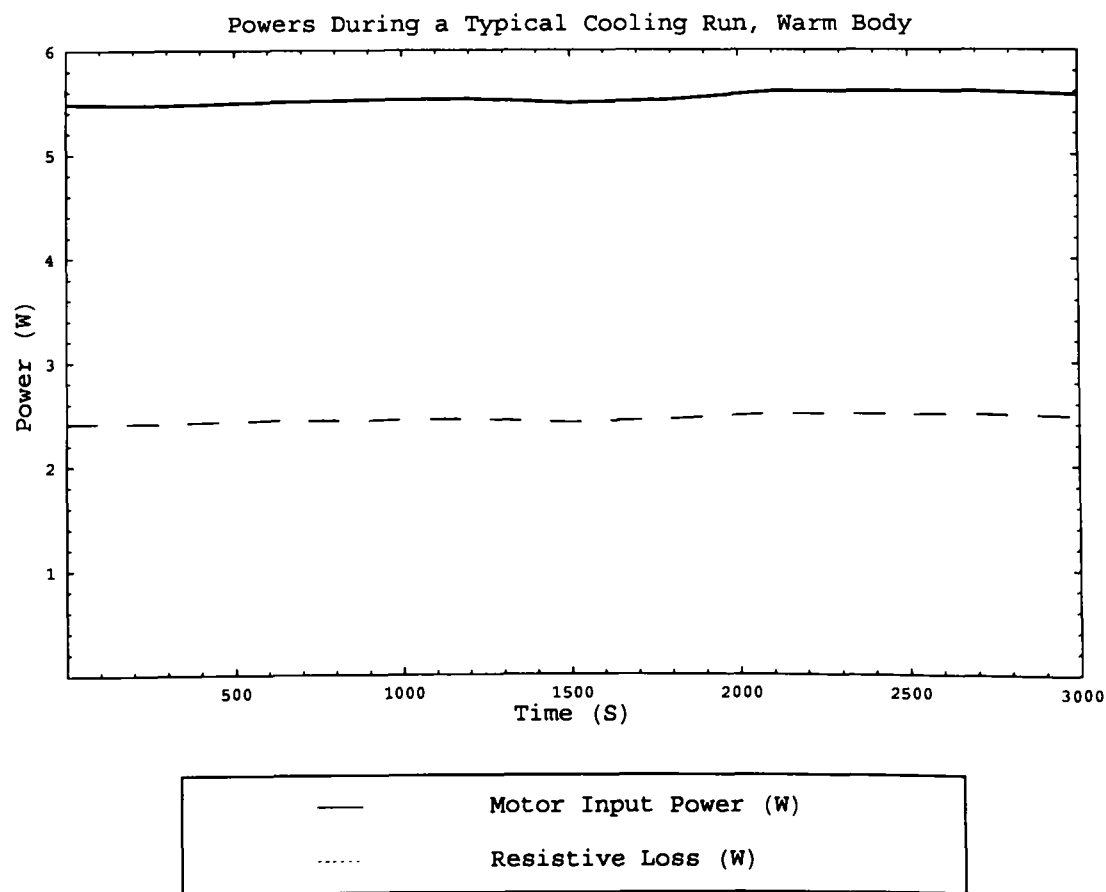


Fig. 9.3: Motor input power and power lost to Joule heating in the motor coil during a cooling run.

that both the static heat load parameters and the lowest cold tip temperature could be found. The results from these runs are shown in section 9.4.

### 9.3 Measuring the static heat load

A complete investigation of the loss processes within the cooler was not possible in the time available and although the loss processes can be separated into static and dynamic losses, further separating these two categories is a lengthy process. It was, however, felt that measurements of the static heat load could be performed within the time constraints of the research effort and this would be a useful measurement as a large static heat load could significantly reduce the cooler's performance.

The static heat load is constant for given values of  $T_c$  and  $T_e$  and is present even if the cooler is switched off. It is possible to measure the static loss relatively simply by employing a commonly used method ([Kotsubo, V. et al. (1999)] and [Orlowska A. and Davey. G. (1987)] for example). If the cooler is not running then both  $W_{gas}$  and the dynamic loss are zero and equation 8.1 becomes:

$$C_{ct} \frac{dT_e}{dt} = \dot{Q}_{ext} + \frac{T_c - T_e}{r_{ct}} \quad (9.1)$$

The term  $\frac{T_c - T_e}{r_{ct}}$  in equation 9.1 represents the static parasitic heat load on the cold tip. Contributions to this heat load arise from conduction through the regenerator and cold finger as well as convective loading from residual gas in the vacuum tank and radiation (thought to be negligible due to MLI shielding of the cold tip and the low pressure in the vacuum tank). To simplify the analysis we make the assumption that the static heat load is proportional to the temperature difference between the cold tip and the cooler body (plausible since radiation loading is assumed negligible). If, in equation 9.1 we assume that  $T_c$  represents some average of temperature of the parts of the cooler from which heat is being conducted to the cold tip, then we can find values of the thermal resistance  $r_{ct}$  and the heat capacity of the cold tip  $C_{ct}$ .

An example of the procedure for doing this is as follows:

1. Run the cooler with no heat load applied to the cold tip until the cold tip reaches its lowest possible temperature, for example 170K.
2. Stop the cooler, apply an external heat load to the cold tip and record the time taken for the cold tip to warm up 10 degrees K (in this example 180K).
3. Repeat the previous two steps for a variety of different external heat loads.

The static heat load on the cold tip at (in this example) 175K is then obtained from the intercept of a graph of  $\frac{1}{t_w}$  against  $\dot{Q}_{ext}$  where  $t_w$  is the time taken to warm up 10 degrees from

170K to 180K. The heat capacity can be obtained from the gradient of the graph since it is equal to  $C_{ch}\Delta T$  where  $C_{ch}$  is the heat capacity of the cold tip and  $\Delta T$  is the temperature change in the time  $t_w$  (10 degrees Kelvin in the example above).

The external heat loads were applied to the cold tip using a resistor embedded in the cold tip. It was assumed any power dissipated in the resistor was applied to the cold tip. This was thought to be a safe assumption as the cold tip was designed with two cylindrical cavities in it. This meant that the cold tip heater resistor  $R_h$  and the cold tip PRT could be inserted fully into the cold tip. Both the PRT and resistor were covered with thermally conducting elastomer before being inserted to further ensure good thermal contact. This meant that the resistor (and indeed the PRT) would be surrounded by the cold tip material and thus any heat emerging from the resistor should flow into the cold tip. The applied heat load from the heater is given by:

$$\dot{Q}_L = i_h^2 R_h \quad (9.2)$$

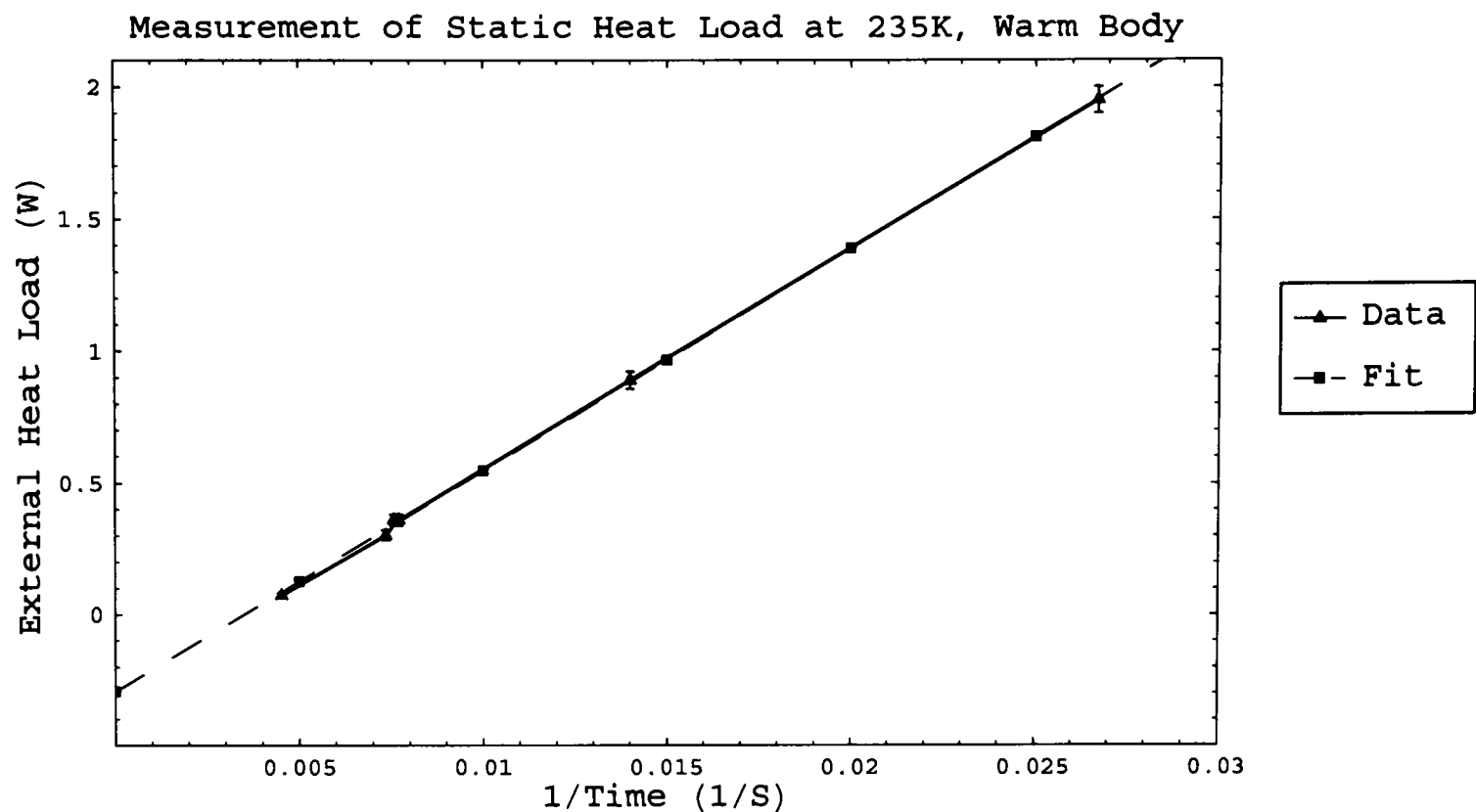
Where  $i_h$  is the (DC) amplitude of the constant current supplied to the cold tip resistor. This assumes that the value of  $R_h$  will not change with temperature which is not strictly speaking true, however the value of the resistance could be measured immediately after the warm up process using a high precision HP multimeter and could be observed to have changed by less than  $0.1\Omega$  during the warm up and so the error caused by using the room temperature throughout the calculations would be negligibly small compared to other measurement errors.

## 9.4 Results: static heat load parameters

Figures 9.4 and 9.5 are a sample of the graphs of external heat load applied to the cold tip plotted against  $1/\Delta t$  where  $\Delta t$  is the time taken for cold tip to warm up by ten degrees. This procedure was performed at a nine different cold tip temperatures in the range 225-265K so that any temperature dependence of the thermal resistance and/or cold tip heat capacity could be ascertained.

Figure 9.6 shows how the cold tip heat capacity varied with cold tip temperature. The cold tip heat capacity was higher than the value determined by simply calculating the mass of copper in the cold tip and multiplying it by the specific heat capacity at the relevant temperature. This could be explained by the contribution made to the cold tip heat capacity by part of the regenerator close to the cold tip. If the heat capacity of 20% of the regenerator material plus the heat capacity of the aluminium “plug” in the cold tip of the regenerator (that retains the meshes within the regenerator sleeve) is added to the heat capacity calculated for the cold tip alone then there is reasonable agreement (within 6%) between the experimentally determined value and the calculated value at 265K. As expected the experimentally determined value of the cold tip heat capacity decreased at





**Fig. 9.4:** Measurement of the static heat load at 235K with warm body.

lower cold tip temperatures. This is because the heat capacity of copper, aluminium and phosphor bronze decrease with decreasing temperature.

Figure 9.7 shows how the thermal resistance varied with cold tip temperature and as can be seen, it was constant within the experimental errors with an average value of 222 K/W. The resulting static heat loads were larger than the model predictions (see figure 9.8) which were obtained by assuming the thermal conductivity of the stack of regenerator meshes was 4% of the bulk material (phosphor bronze) [Brito, M. (1999)]. Increasing this to 4.7 % improves the model predictions (see figure 9.9). The thermal resistance was lower than values found for similar coolers (Reed found a value of 352K/W for a cooler with a similar cold finger and regenerator [Reed, J. (2004)] ).

## 9.5 Summary

This chapter describes the first cooling runs performed with the new cooler. The body of the cooler was not cooled by the radiator during these runs. The lowest cold tip temperature achieved during these runs was 166K when no heat load was applied. Measurements of the cold tip heat capacity and the thermal resistance between the warm end and cold tip were made. The thermal resistance remained constant over the temperature range investigated but was lower than that found in similar coolers. It was realised that this low value of the thermal resistance would present a problem if the cooler was to achieve its performance target. By comparison, Peters [Peters, M.



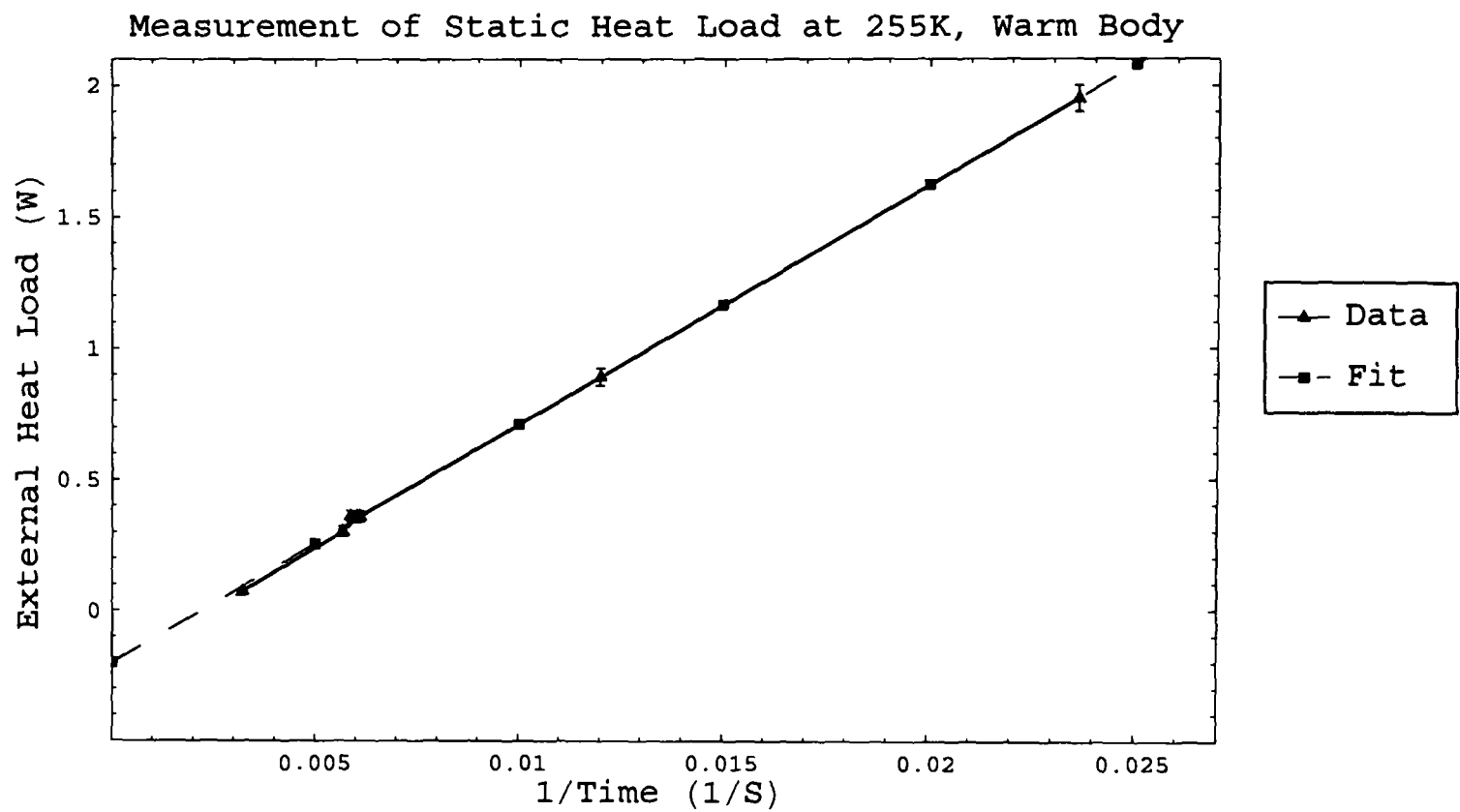


Fig. 9.5: Measurement of the static heat load at 255K with warm body.

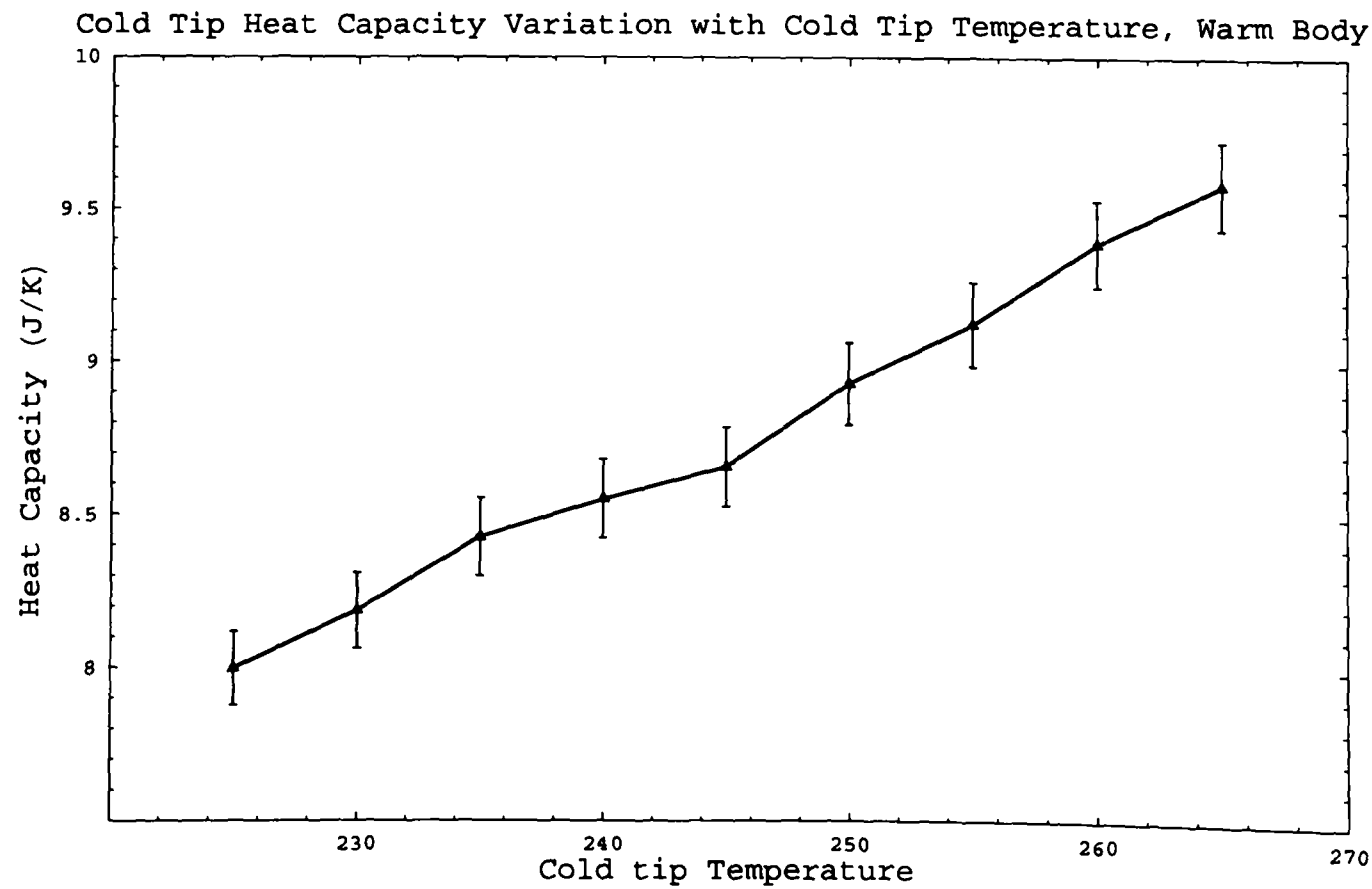


Fig. 9.6: Variation of heat capacity with cold tip temperature.

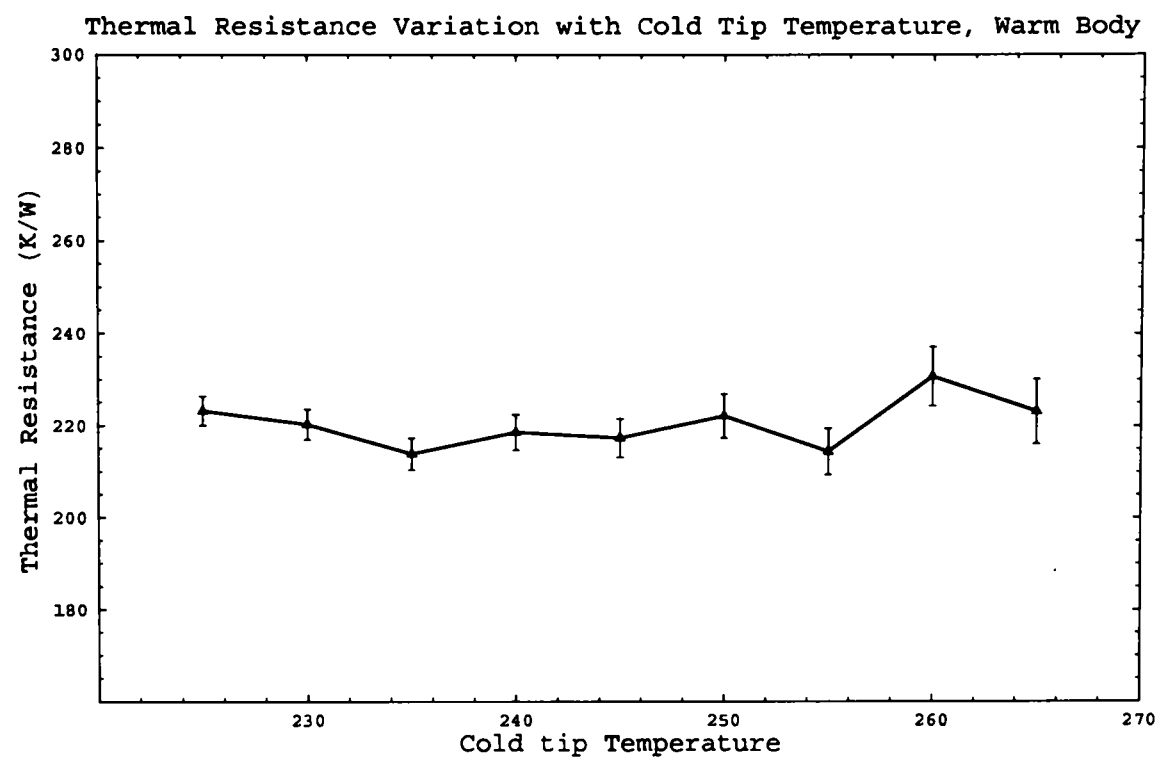


Fig. 9.7: Variation of thermal resistance with cold tip temperature.

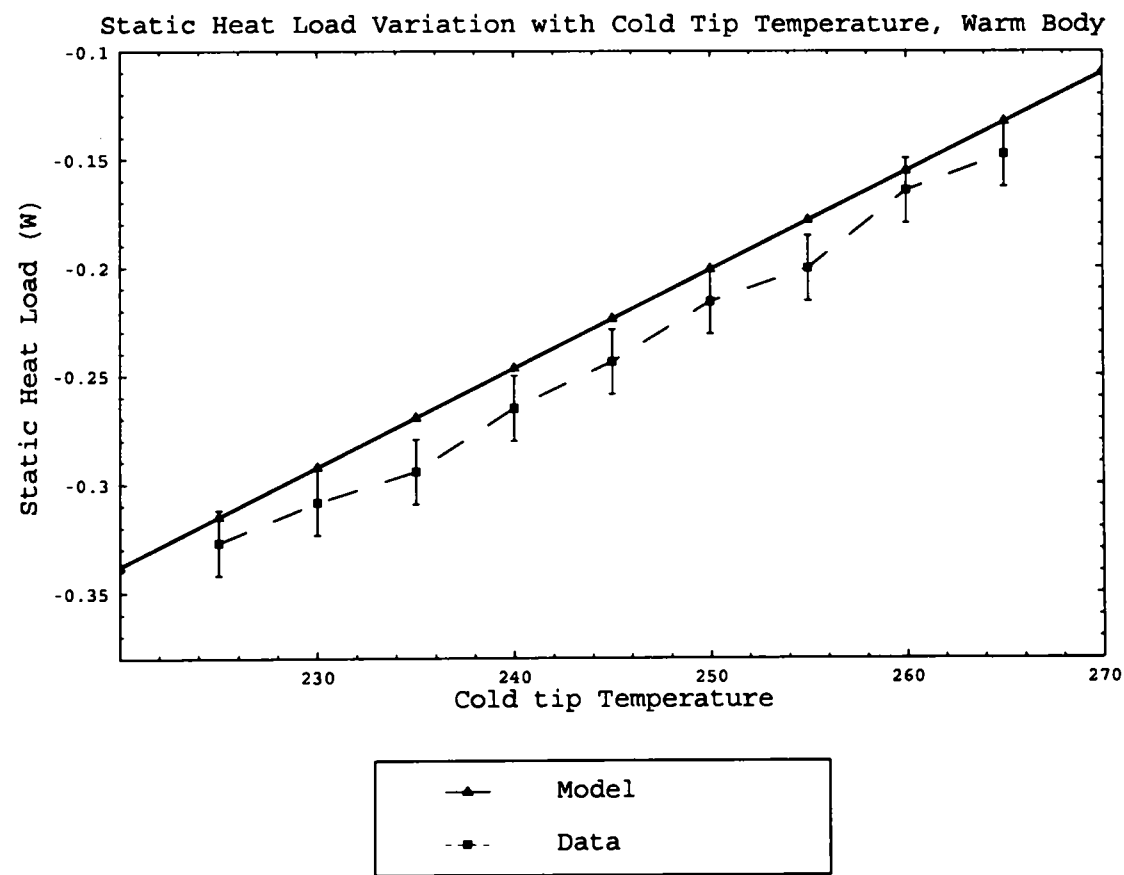
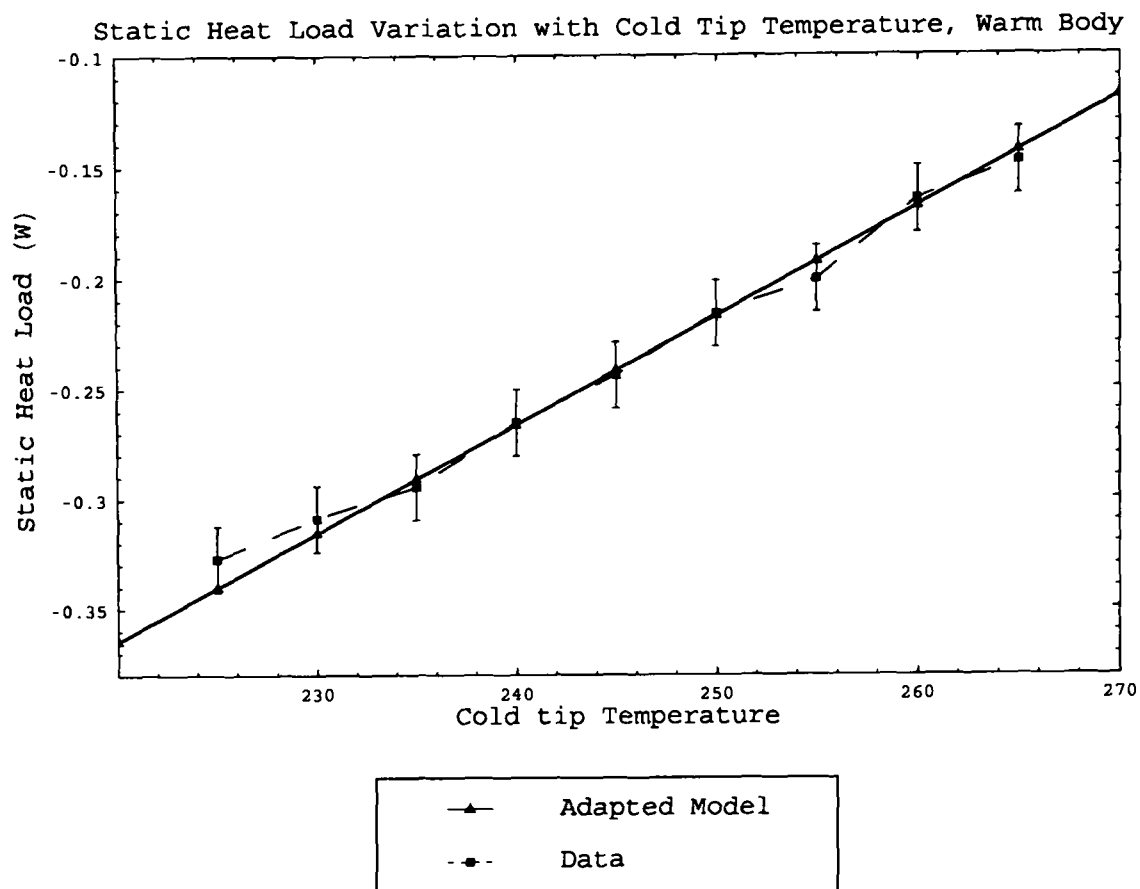


Fig. 9.8: Variation of the static heat load with cold tip temperature. Also shown are predictions from the model.



**Fig. 9.9:** Variation of the static heat load with cold tip temperature. Also shown are predictions from the adapted model.

(1994)] found that for an Oxford split-Stirling cooler operating between 80K and room temperature the static heat load was 0.4W which constituted only 4 % of the power supplied to the gas by the compressor. The measurements described above indicate that between the same temperature limits the cooler under development would have a static heat load of 0.99W or 33% of the input power to the gas. Of course, the present cooler is expected to run with the body pre-cooled to 200K and, assuming the thermal resistance did not change, this would reduce the static heat load to 0.54W (or 18% of the input power to the gas) but this is still an extremely significant loss process. Nevertheless, a decision was made to proceed with cooled body experiments before addressing the problem of the low thermal resistance value so that any problems associated with cooled body operation could be determined as early as possible in the experimental stage of the research. What is more, the thermal conductivity of the material used to construct the regenerator and cold finger was expected to decrease with decreasing temperature thus increasing the value of  $r_{ch}$  as the body was pre-cooled. Decreasing cold tip temperatures have been associated with increases in thermal resistance  $r_{ct}$  ([Kotsubo, V. et al. (1992)] in [Reed, J. (2004)]) in the cold tip temperature range 170K to 40K and so it was expected that the thermal resistance  $r_{ct}$  would also increase if pre-cooling of the cooler body enabled similar cold tip temperatures to be reached.

# Chapter 10

## Pre-cooled body runs

### 10.1 Introduction

It was shown in previous chapters that the prototype cooler behaved in an understandable way and produced significant cooling when the body was not pre-cooled (initially at room temperature and typically rising to temperatures in the range 306-308K). By pre-cooling the body with a radiator facing a space view it was expected that there would be improvements in the coefficient of performance of the cooler. Pre-cooling the body was also expected to reveal any problems associated with operating the compressor at lower temperatures (e.g. thermal contraction causing moving parts to seize up, which had been observed in a previous prototype with a cooled body [Reed, J. (2004)]).

### 10.2 Pre-cooled sensors

Before the performance of the cooler could be investigated with a cooled body the sensors had to be calibrated for use at this lower temperature range. Since the drive voltage and current sensors were remote from the cooler these were not required to be calibrated at lower temperatures and so it was only the following sensors that needed calibrating for use in the pre-cooled body experiments:

- Displacer/regenerator position pick-off sensor (DPPO)
- Compression space pressure sensor

### 10.2.1 DPPO calibration

The displacer/regenerator position pick-off (DPPO) was sensitive to the capacitance between a moving plate connected to the rear of the displacer/regenerator shaft and two fixed rings on a stationary part of the cooler as described in subsection 6.4.1. The sensitivity is dependent on temperature via two mechanisms:

1. Dimensional changes (i.e. thermal contraction) of the moving and stationary plates leading to a change in their separation.
2. Changes in the gain and offset of the op-amp mounted withing the cooler body.

The change in permittivity of helium over the range of temperatures considered is negligible [Lide, D.R. (1995)] and the effect of the change in permittivity on a similar capacitive position sensor has previously been shown to be negligible [Reed, J. (2004)]. Ideally a full calibration similar to that performed on the DPPO at room temperature would be performed but the need for the cooler to be cooled to 200K and filled with helium at 7bar prevented the displacer/regenerator's position from being measured in this way since the cooler had to be sealed up and cooled in a vacuum tank. Instead the displacer/regenerator was fixed at an arbitrary position relative to the fixed DPPO plates and the cooler then sealed up and cooled using the radiator as in a normal cooled body cooling run without the cooler drive supply being turned on. The output from the DPPO was recorded via the data acquisition system and this process repeated for different positions of the displacer/regenerator. This was a time consuming process and did not resolve the magnitude of the contributions of each of the mechanisms listed above to the change in the DPPO output (dimensional changes were expected to be negligible which would suggest that it was changes in the op-amp's gain at low temperatures that made the largest contribution). Figure 10.1 shows how the DPPO output varied when the displacer/regenerator was fixed at a particular displacement for two different cool downs (the positions were different for each cool down). It can be seen that the output changed by the same amount for both positions. It was found that if the body was allowed to warm up after being cooled the DPPO output did not change in the range 200 – 240K (see figure 10.2) and since the body remained close to 200K throughout the pre-cooled body tests it was sufficient to simply subtract the observed increase in DPPO output from the room temperature calibration. It is not clear why the DPPO output did not start to approach its room temperature value as it warmed up from 200K as shown in figure 10.2, but as this did not affect the cooler's operation further investigation was not performed.

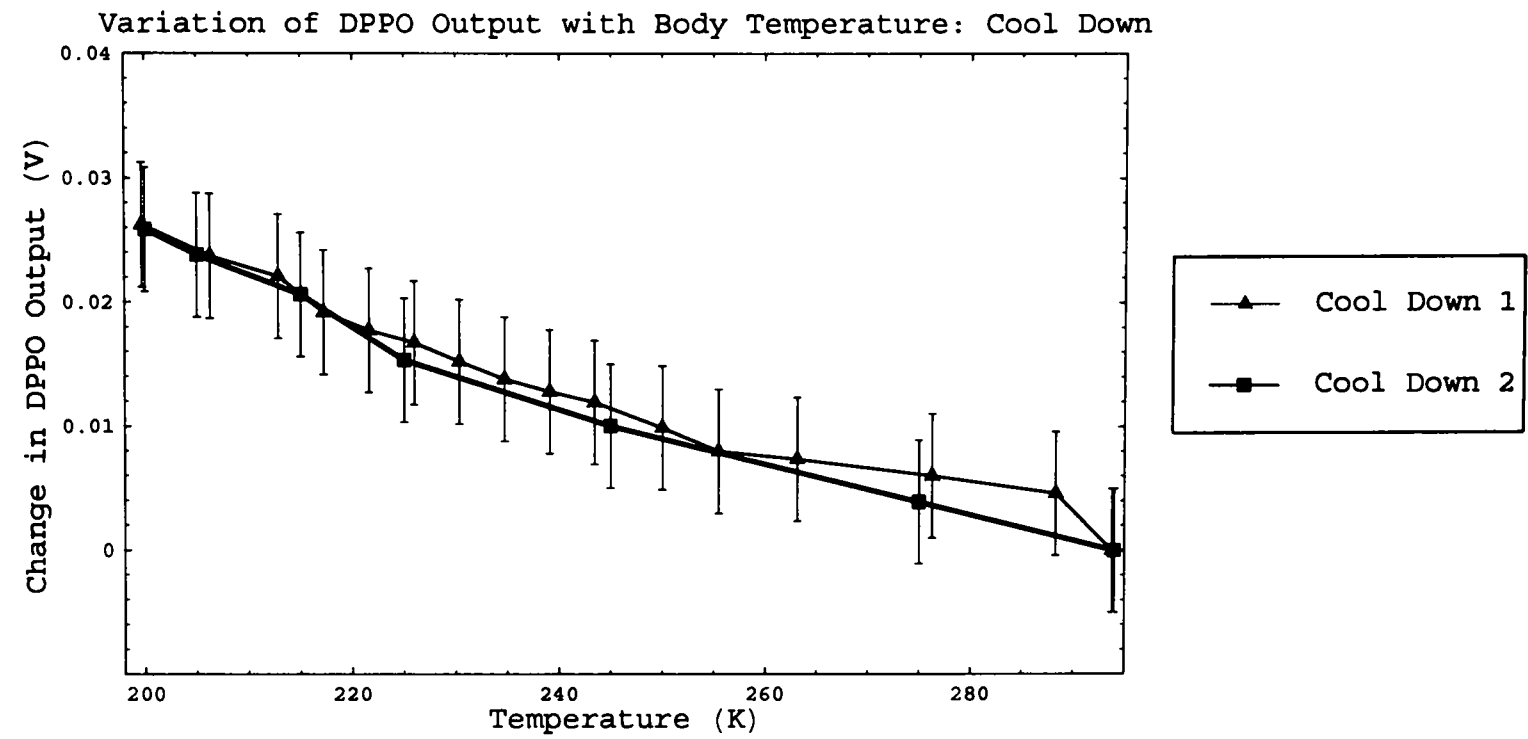


Fig. 10.1: Variation of DPPO output for two different fixed displacements during cool down.

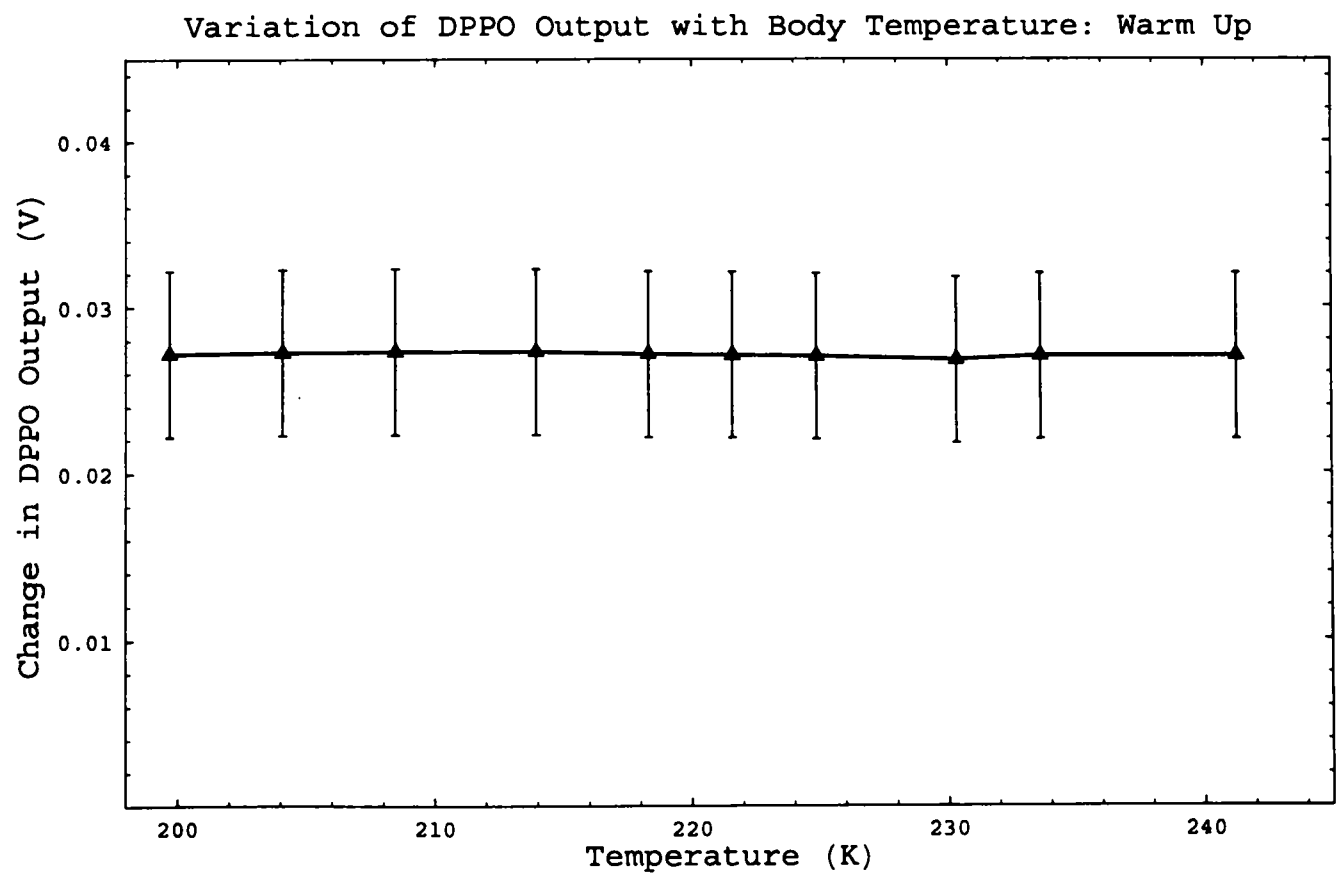


Fig. 10.2: Variation of DPPO output during warm up from 200 – 240K.

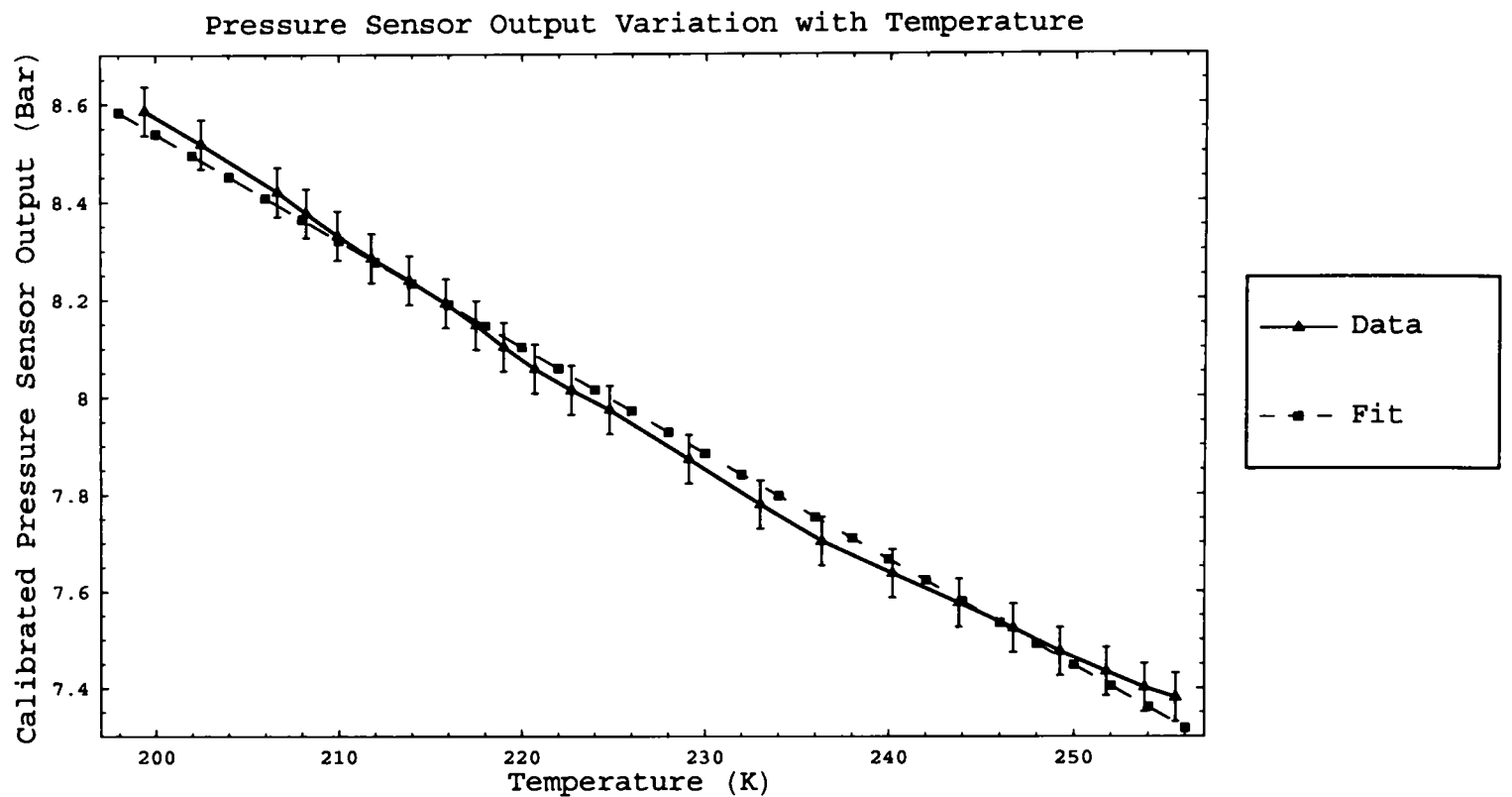


Fig. 10.3: Variation in the pressure sensor output with temperature.

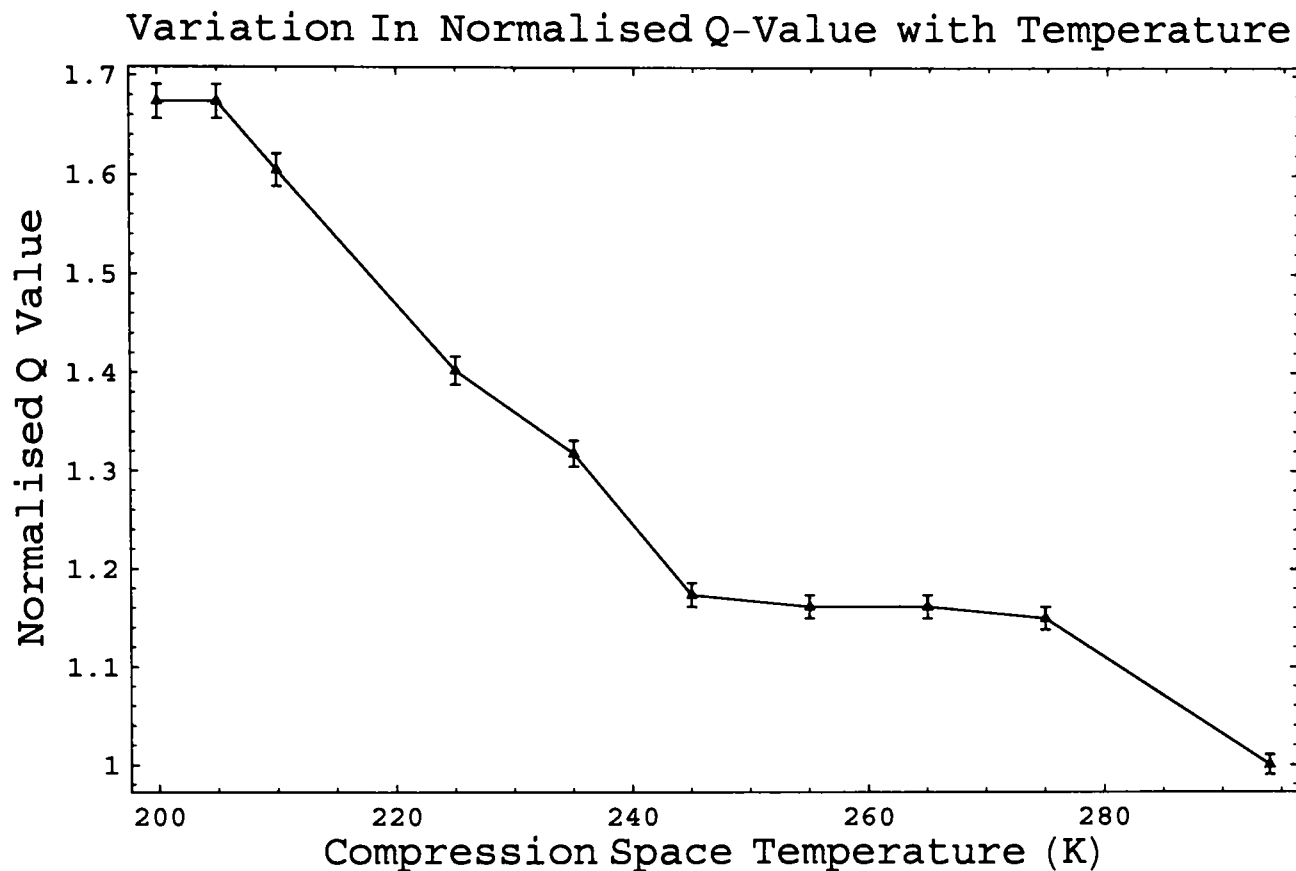
### 10.2.2 Compression space pressure sensor

The effect of temperature on the compression space pressure sensor was determined by filling the cooler with dry helium at 7 bar and then cooling the body in the evacuated vacuum tank using the radiator as would be done in a typical pre-cooled cooling run. The cooler body remained connected to the gas bottle at all times and the pressure could be monitored on a mechanical Budenberg gauge external to the tank that was at constant temperature. This enabled the pressure of the helium in the cooler to be maintained at 7 bar (accurate to 0.05 bar) throughout the experiment. The output of the pressure sensor was recorded as the body cooled and can be seen in figure 10.3. This data could then be used to calibrate the sensor for use with the cooler during pre-cooled body runs.

## 10.3 Pre-cooled body measurements of piston freedom

Once the sensors had been calibrated for use at low temperatures the prototype was brought back into use. The first measurements made were to determine how the piston freedom changed with decreasing body temperature. The prototype cooler was mounted on the radiator and the whole assembly placed in position under the space view in the vacuum tank. The tank and cooler were then evacuated using the pumping system described earlier. The cooler body was then cooled by first cooling down the space view. This was achieved by filling the liquid nitrogen reservoir





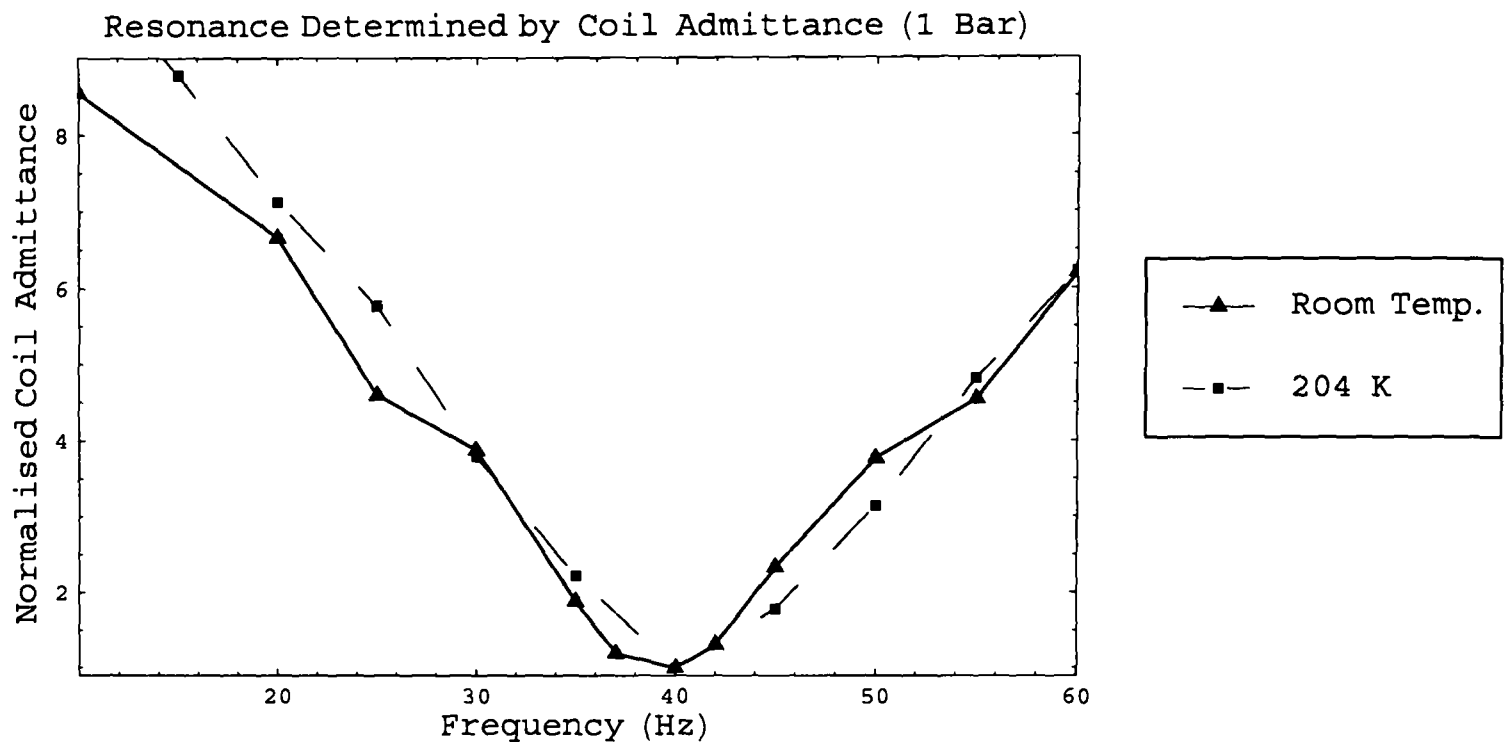
**Fig. 10.4:** Variation of compressor Q-value with compression space temperature. Normalised q-value is the q-value at the temperature of interest divided by the q-value at room temperature.

with liquid nitrogen and waiting for the radiator to reduce the cooler's temperature to 200K. As the cooler body's temperature decreased a "q-test" (as described in chapter 7) was performed periodically and the temperature of the compression space at that time recorded. It is clear that there is a trend for the piston freedom to increase as the compression space temperature decreases. The trend is not linear as it can be seen that the q-value seems to plateau at temperatures between 245 – 275K before increasing at a faster rate until 205K where it appeared to plateau again. An increase in piston freedom for an integral cooler with a pre-cooled body has been observed elsewhere [Reed, J. (2004)].

## 10.4 Pre-cooled body measurements of compressor resonance

Figures 10.5 and 10.6 show how the resonant frequency varied between a compression space temperature of approximately 200K and room temperature when the cooler was filled with dried helium to 1bar and to 7 bar. It can be seen that the resonant point does not change over this temperature at either of the filling pressures. A previous research effort in the department [Reed, J. (2004)], had observed a decrease in the resonant frequency of a pre-cooled compressor as its temperature was reduced. This observation had not been explained by theory as the required change in the type



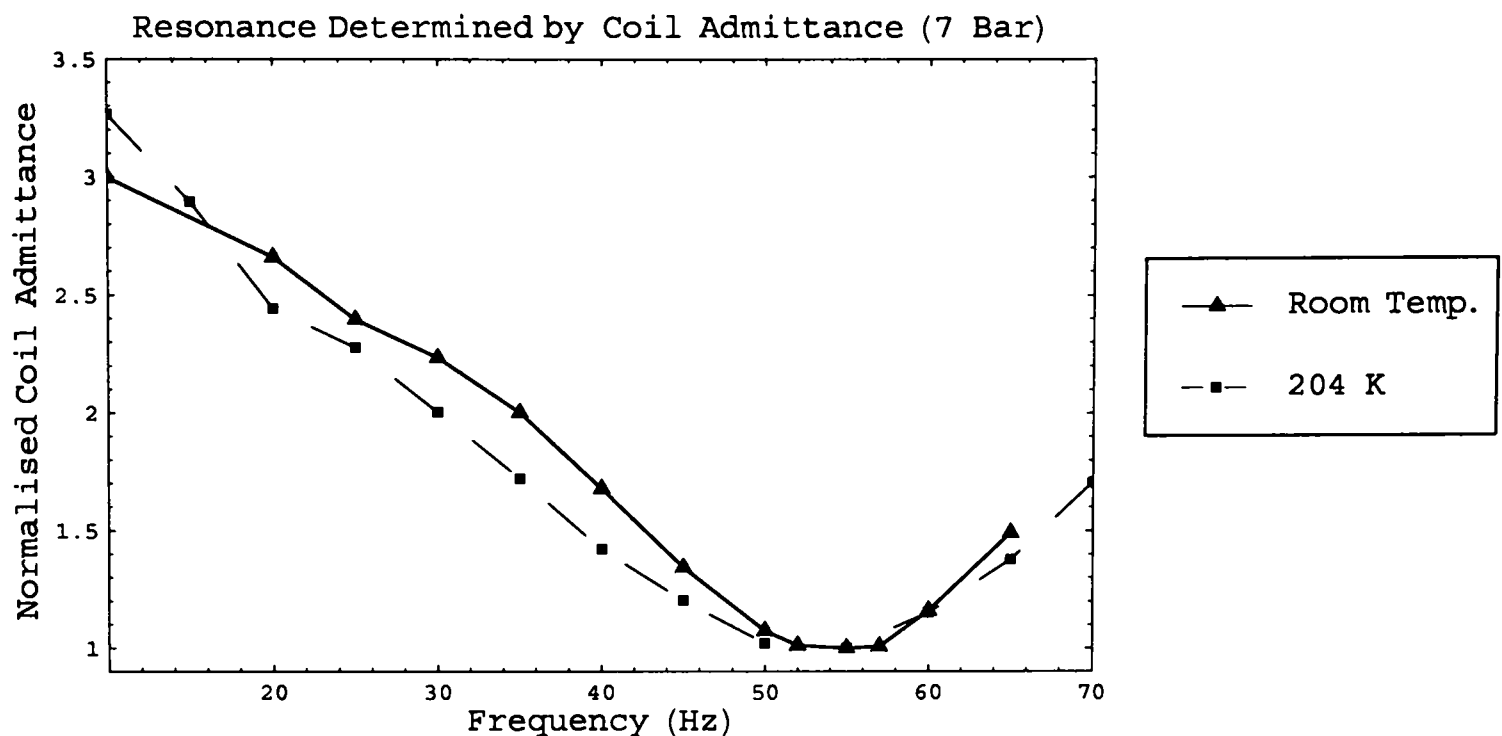


**Fig. 10.5:** Variation of compressor resonance with compression space temperature with the cooler filled with dried helium to 1 bar.

of compression (it would have had to become more isothermal as the temperature decreased to explain the decrease in resonant frequency) contradicted earlier observations that indicated that the compression had become more adiabatic as the compressor temperature dropped. It is not clear what differences between the design of the current prototype and that described by Reed lead to this difference in the variation of resonance with temperature, and further investigations into this were beyond the scope of the current experiments.

## 10.5 Seals during cooled body runs

During the initial pre-cooled body tests described above a significant problem was discovered with the cooler that had not been observed during room temperature operation: The ability of the elastomeric seals used on the cooler to prevent leaks reduced significantly as the cooler body temperature decreased. As the cooler body decreased in temperature the vacuum tank pressure would rise as the leak rate from the cooler increased. Generally the tank pressure could be kept below  $10^{-4}$  mbar (the cut-off pressure below which convective loading of the cold tip was considered negligible -see section 6.3) until the cooler body temperature reached 230 – 240K . Similar problems have been experienced with this type of seal before [Reed, J. (2004)] but this had not been appreciated before the cooled body tests had begun. To solve this problem the cooler was dis-assembled and the elastomeric seals replaced by indium seals. Indium seals have

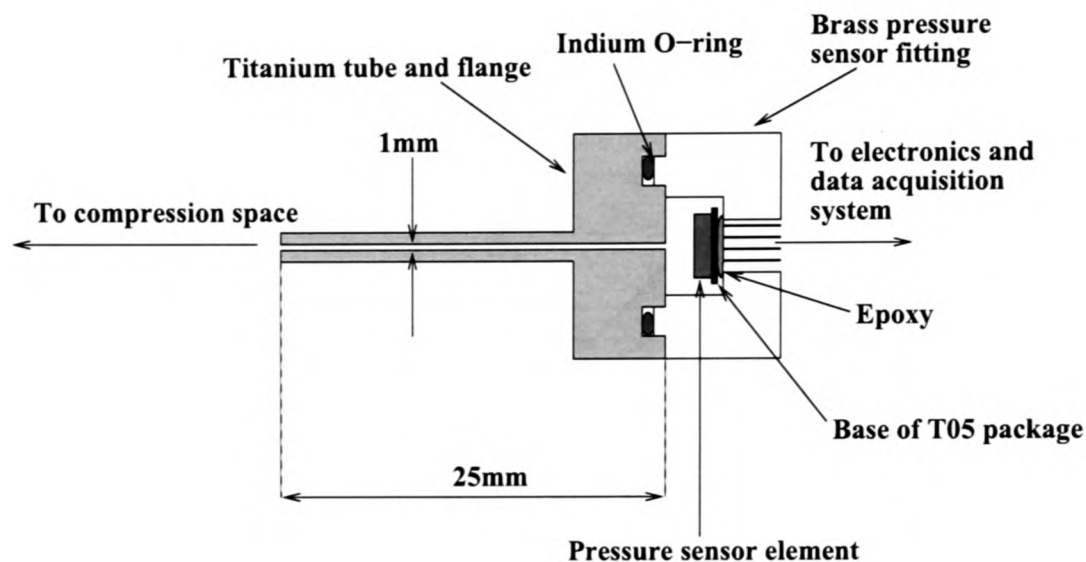


**Fig. 10.6:** Variation of compressor resonance with compression space temperature with the cooler filled with dried helium to 7 bar.

been demonstrated to be effective at temperatures below 200K [Sharma, R. et al. (2006)]. Initial tests showed that even with indium seals the leakage from the cooler was still too large and after numerous assemblies of the cooler this problem was traced to the pressure sensor flange. Tracing the leak to the pressure sensor was done by filling the cooler to 7 bar with helium when the body was at 200K with the pressure sensor in place. The lowest vacuum tank pressure <sup>1</sup>that could be maintained was then recorded. This procedure was then repeated but with the pressure sensor removed and the pressure sensor tap covered by a brass plate and sealed with indium. This allowed the tank pressure to drop below  $10^{-4}$ mbar when the cooler was filled with helium to 7 bar.

The pressure sensor fitting had been designed to accommodate a standard nitrile O-ring and could not accommodate an indium seal. To solve this problem a new pressure sensor fitting was designed and built that could be used with an indium seal. This is shown in figure 10.7. Initial cooled body tests with the new pressure sensor fitting enabled the cooler body to approach 200K without the vacuum tank pressure rising above  $10^{-4}$ mbar however a gradual rise in the vacuum tank pressure as the cooler body dropped below 220K indicated that the cooler leak was gradually becoming worse. This increase in the leakage from the cooler arose from a gradual degradation of the epoxy used to secure the pressure sensor in its fitting. Repeated applications of epoxy resin (CREST) met with little success and invariably the leak became worse after the cooler body had been cycled to 200K and then back to room temperature. The lowest tank pressure (approximately

<sup>1</sup>The vacuum tank pressure was measured by Penning and Pirani gauges fixed to the tank (see figure 6.1)

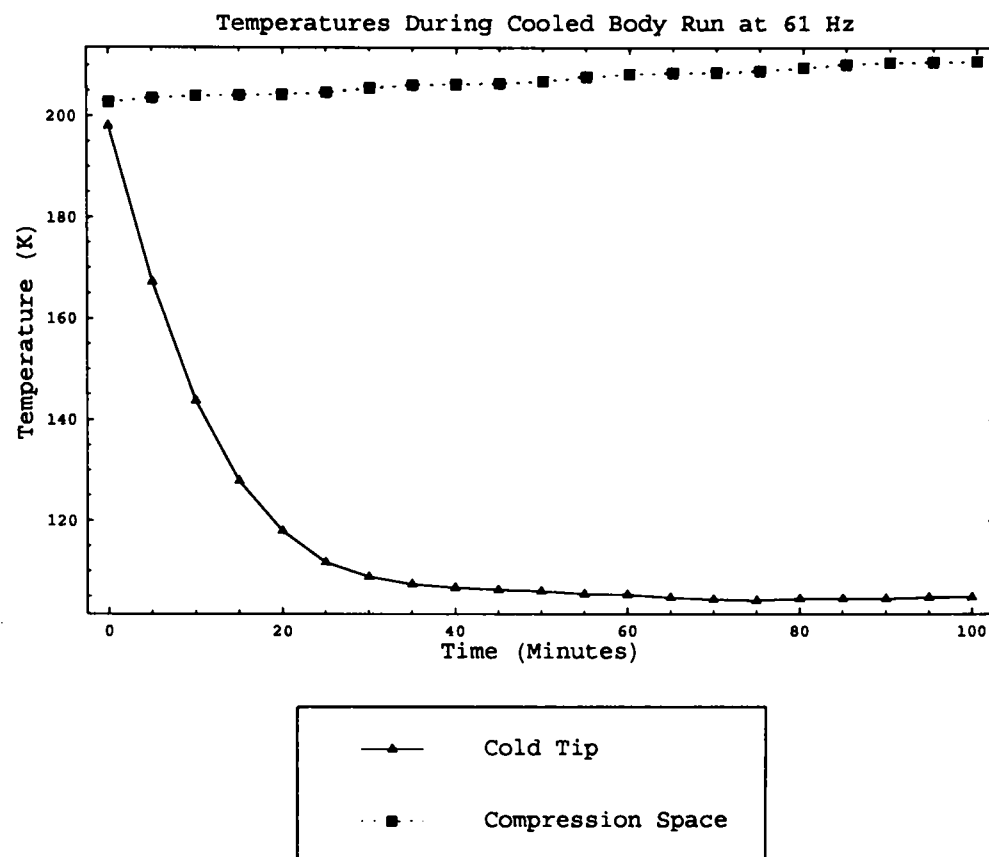


**Fig. 10.7:** Diagram showing the new pressure sensor fitting with indium o-ring.

$10^{-6}$  mbar as measured on the vacuum tank penning gauge) during pre-cooled body cooling runs occurred when the pressure sensor tap was blanked off. Cooling runs could be performed with the pressure sensor in place but this meant that the vacuum tank pressure would be above  $10^{-4}$  mbar and so convective loading of the cold tip would prevent the lowest cold tip temperature from being reached. Blanking off the pressure sensor tap and sealing it with indium allowed the leak from the cooler to be minimised and the vacuum tank could be maintained at a pressure at which convective loading of the cold tip became negligible. This meant that lower cold tip temperatures could be achieved but no pressure data could be obtained during these runs.

## 10.6 Pre-cooled body cooling runs

When the temperature of the compression space was approaching 200K the cooler was filled to 7bar with dried helium and then the compression space was allowed to reach the target of 200K. The cooler power supply was then switched on at the apparent resonant frequency. The cooler cold tip temperature was observed until its temperature stopped decreasing (defined as a decrease in temperature of less than 0.2 K/min for a period of not less than ten minutes). The drive frequency was then altered and the cold tip temperature was observed until it became stable at a new temperature. This procedure was repeated until the frequency that produced the lowest



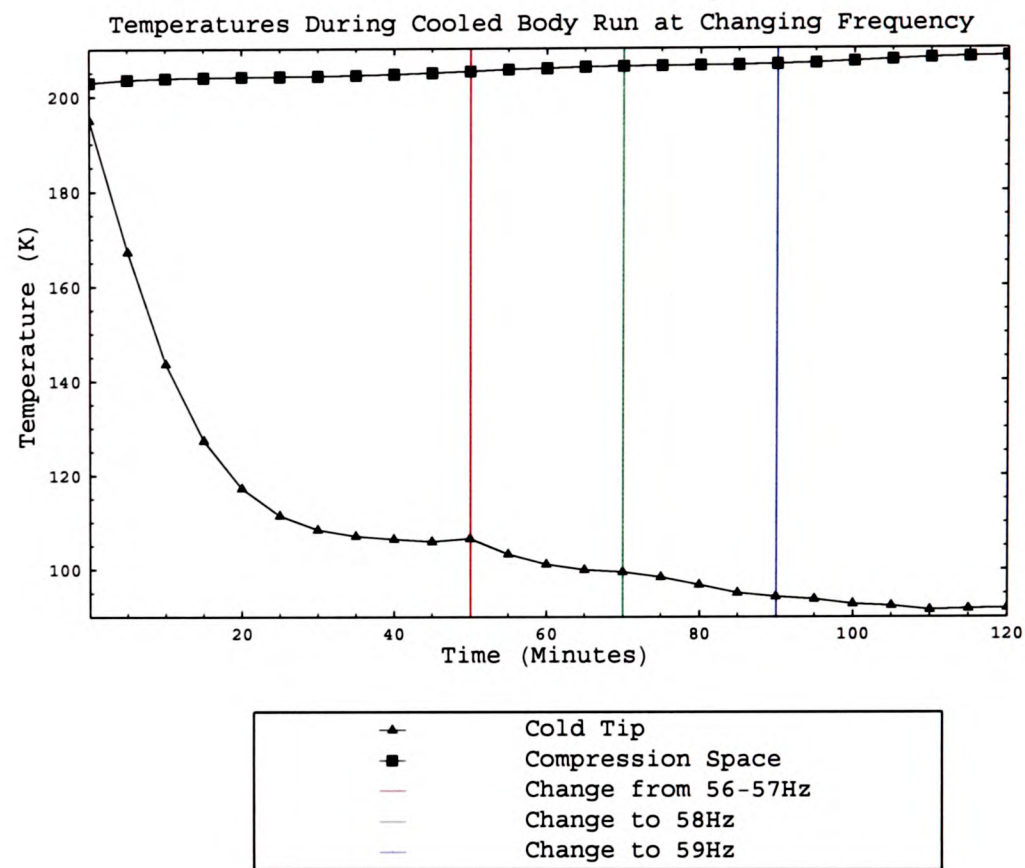
**Fig. 10.8:** Cold tip and compression space temperatures during pre-cooled body cooling run when the drive frequency was kept constant at 61Hz.

ultimate cold tip temperature was found.

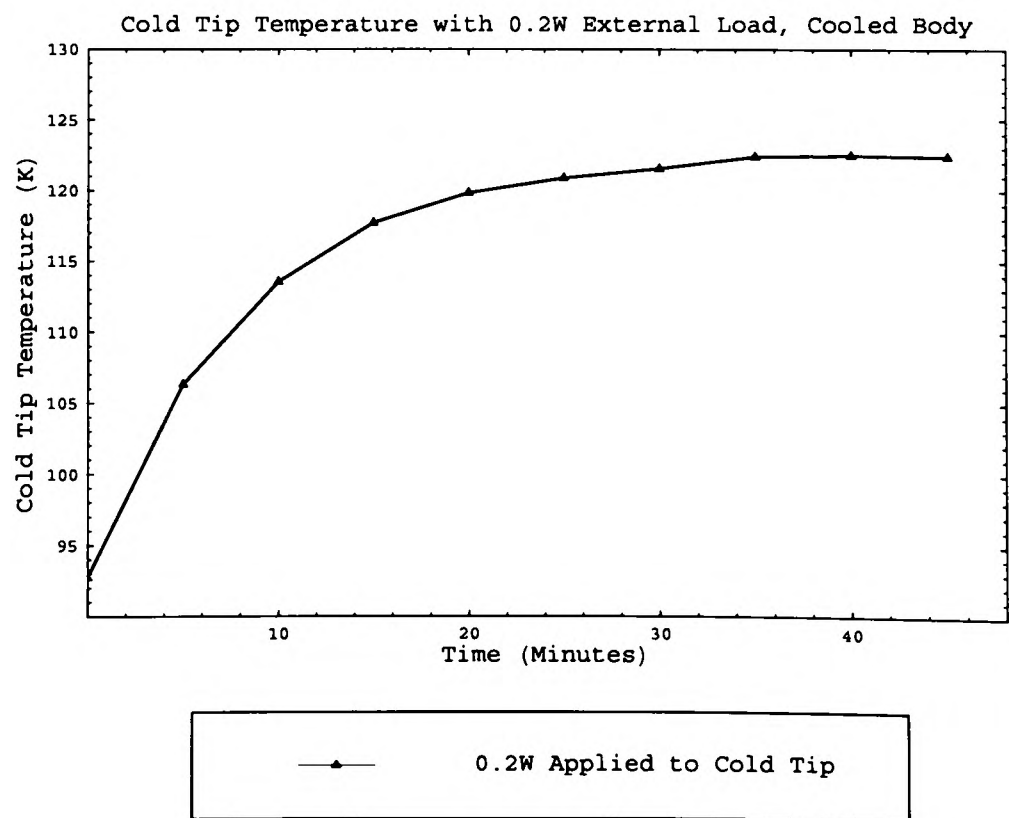
Figure 10.8 shows cold tip and compression space temperatures for a typical pre-cooled body cooling run performed with the cooler drive supply at 61Hz. This cooling run achieved an ultimate cold tip temperature of 104K with no load applied to the cold tip. Figure 10.9 shows cold tip and compression space temperatures for a pre-cooled body cooling run in which the cooler drive frequency was increased in 1Hz steps. The lowest cold tip temperature was 92K and was found to occur with a drive frequency of 59Hz. Once the cold tip temperature had stabilised a heat load was applied to the cold tip using the cold tip heater as described earlier. Figure 10.10 shows how the temperature increased when a heat load of 0.2W was applied to the cold tip. The equilibrium cold tip temperature with 0.2W applied to it was 122.4K. A number of cooling runs were performed but none of these were able to maintain a lower temperature than 122.4K with a 0.2W external heat load applied to the cold tip.

## 10.7 Motor efficiency results

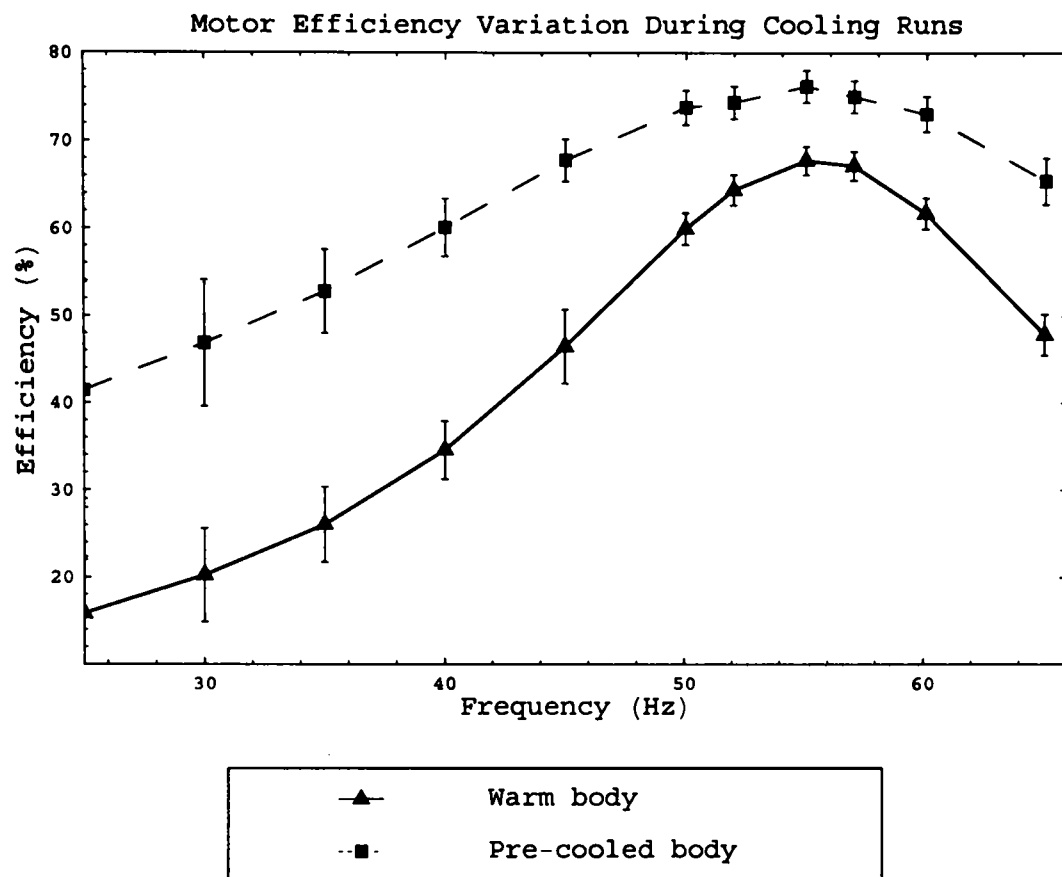
A number of experiments were performed with both the cooler body pre-cooled and at room temperature to determine the linear motor efficiency. The motor efficiency was calculated using



**Fig. 10.9:** Cold tip and compression space temperatures during a pre-cooled body run in which the frequency was altered.



**Fig. 10.10:** Graph showing the temperature of the cold tip when an external load of 0.2W was applied to it during a cooling run (ultimate no-load temperature was 92K).

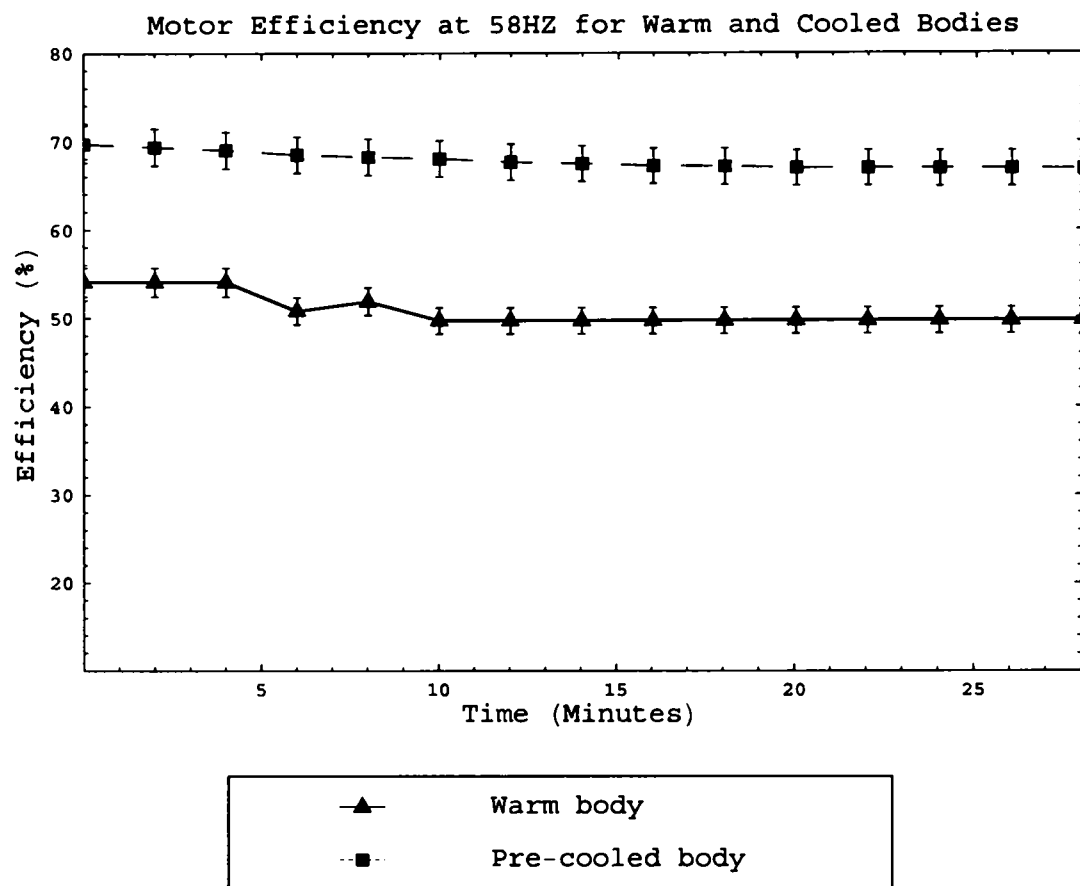


**Fig. 10.11:** Linear motor efficiency variation with frequency for warm and pre-cooled body experiments.

equation 10.1:

$$\epsilon_m = \frac{P_m - P_{ir}}{P_m} \quad (10.1)$$

Where  $P_m$  is the net electrical power supplied to the linear motor and  $P_{ir}$  is the resistive power loss (Joule heating) in the linear motor coil. Figure 10.11 shows the variation of the motor efficiency with frequency for a room temperature body and pre-cooled body ( $\approx 200\text{K}$ ) experiment; both show a peak at approximately 54Hz as expected. The room temperature efficiency peaks at just over 66 % whereas the pre-cooled body efficiency peaks at approximately 76%. The increase in efficiency results from a decrease in the coil resistance due to the reduction in temperature caused by pre-cooling the body of the cooler. Measurements of the coil resistance are described in section 10.8. There may also be a slight contribution from the improved performance of NdFeB magnets at the cooled body temperature compared to room temperature (see section 5.4). Figure 10.12 shows the linear motor efficiency during typical cooling runs with the cooler body at room temperature and with the body pre-cooled to 200K. Both runs were performed with the cooler being run at 58Hz. Note that the efficiency is lower than the peak efficiencies in both the pre-cooled and room temperature experiments because the optimum cooling rate did not occur at the same frequency (58Hz) as the efficiency maximum (54Hz).



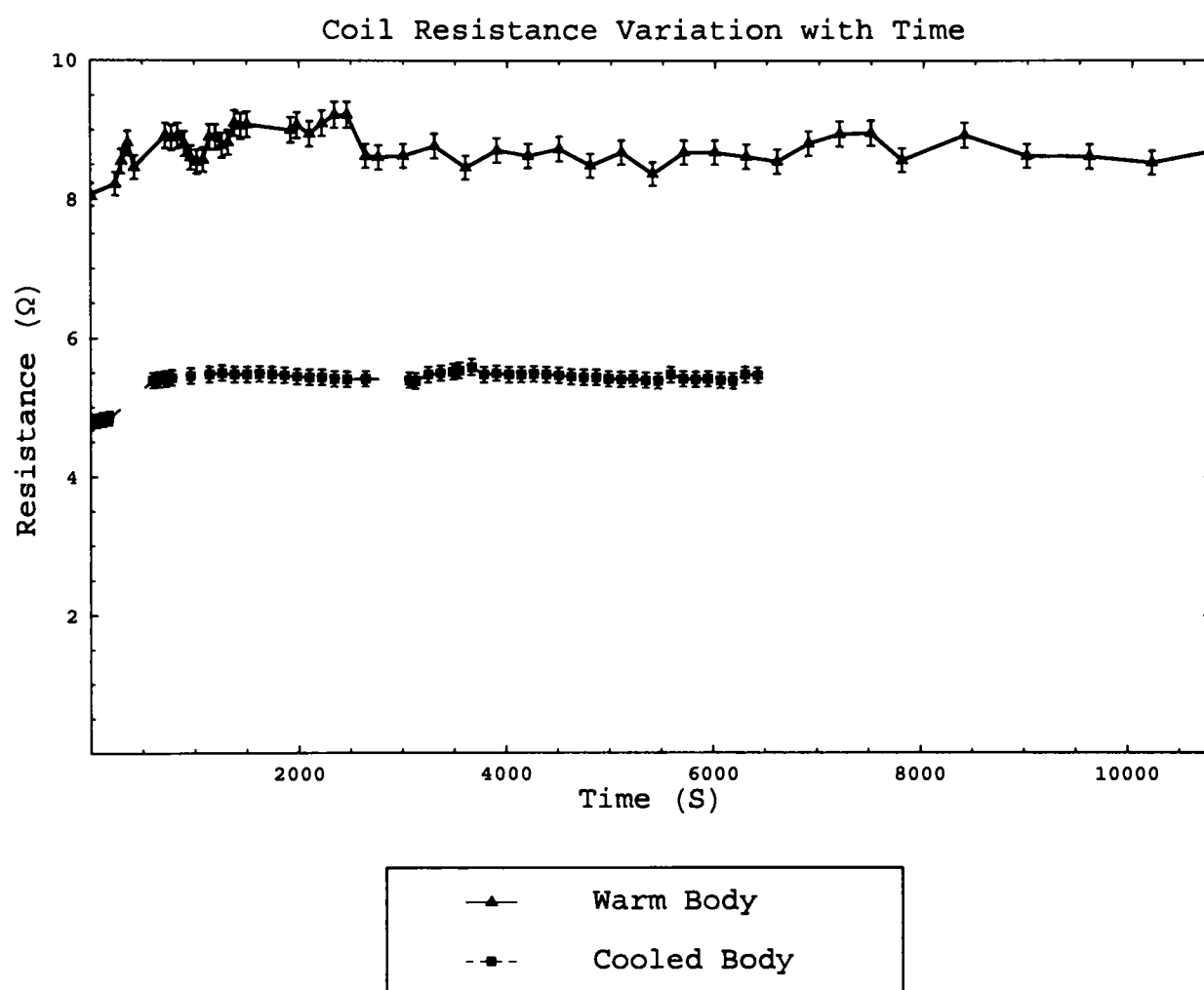
**Fig. 10.12:** Linear motor efficiency throughout room temperature body and pre-cooled body cooling runs.

## 10.8 Coil resistance

The motor coil will increase in temperature as the cooler is operating and this will lead to an increase in its resistance. Due to the complex geometry of the coil and its environment within the cooler it is not a simple matter to predict what the temperature rise, and corresponding increase in resistance, will be. A simpler approach was adopted in which a DC voltage was applied to the motor coil so that the current flowing in the coil matched a value typical of the RMS current during a cooling run. Measurements of the coil resistance could then be made with the data acquisition system by recording the potential difference across the motor coil and the current flowing through it. Figures 10.13 to 10.15 show the results of these experiments with the warm and pre-cooled cooler bodies. The warm body measurements were consistently noisier than the cooled body measurements but the same general trend can be seen in both sets of measurements. Looking at figure 10.13 we see that in both cases the resistance increase rapidly at the beginning and then drops slightly to a fairly constant value for the rest of the measurements (especially in the cooled body case). In the warm body case the resistance settled at a value  $0.75 \pm 0.13\Omega$  above its non-operating room temperature value. In the cooled body case the resistance settled at a value  $0.60 \pm 0.13\Omega$  above its 200K non-operating value. Since the cooler is operated with a AC drive



signal it was not possible to collect resistance data throughout cooling runs but periodically the resistance was checked by turning off the AC drive and applying a DC signal. It was found that the resistance was always close to the value to which the coil settled in figure 10.14 for the warm body and 10.15 for the cooled body, rarely differing by more than 2% of the total coil resistance. The elevated values of the resistance in both the warm and cooled body case were used in all subsequent analyses rather than the non-operating values.

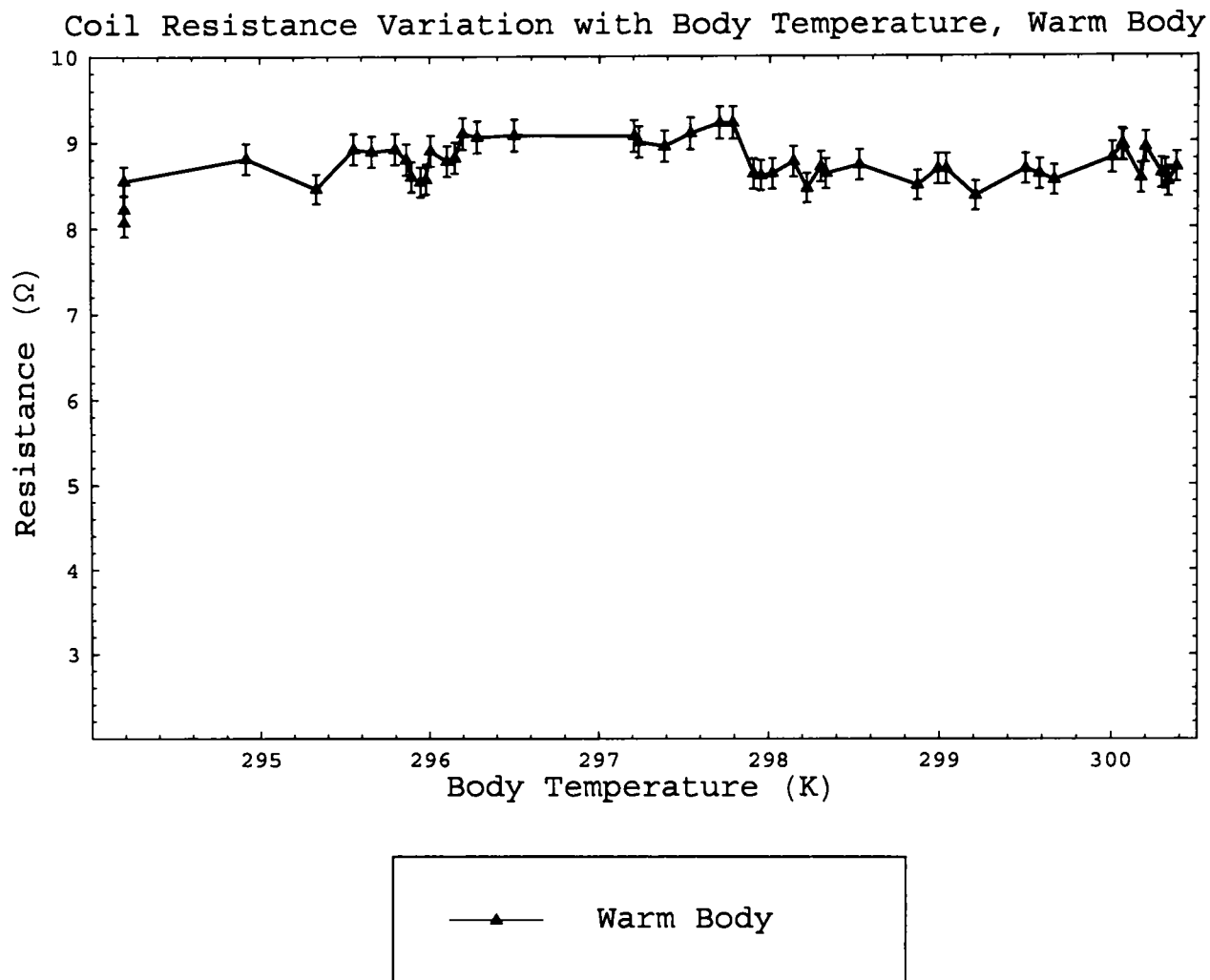


**Fig. 10.13:** Variation in coil resistance with time during warm and pre-cooled body cooling runs.

## 10.9 Results summary

Pre-cooling the body produced a significant increase in the performance of the cooler, in terms of its ultimate cold tip temperature. The cooler was able to reach a cold tip temperature of 92K with no load applied and 122.4K with a 0.2W load applied. The original aim was for the cooler to be able to maintain a cold tip temperature of 80K with a heat load of 0.2W applied. It was thought that the low thermal resistance between the cold tip and warm end was a significant factor in the cooler's failure to reach its performance target.



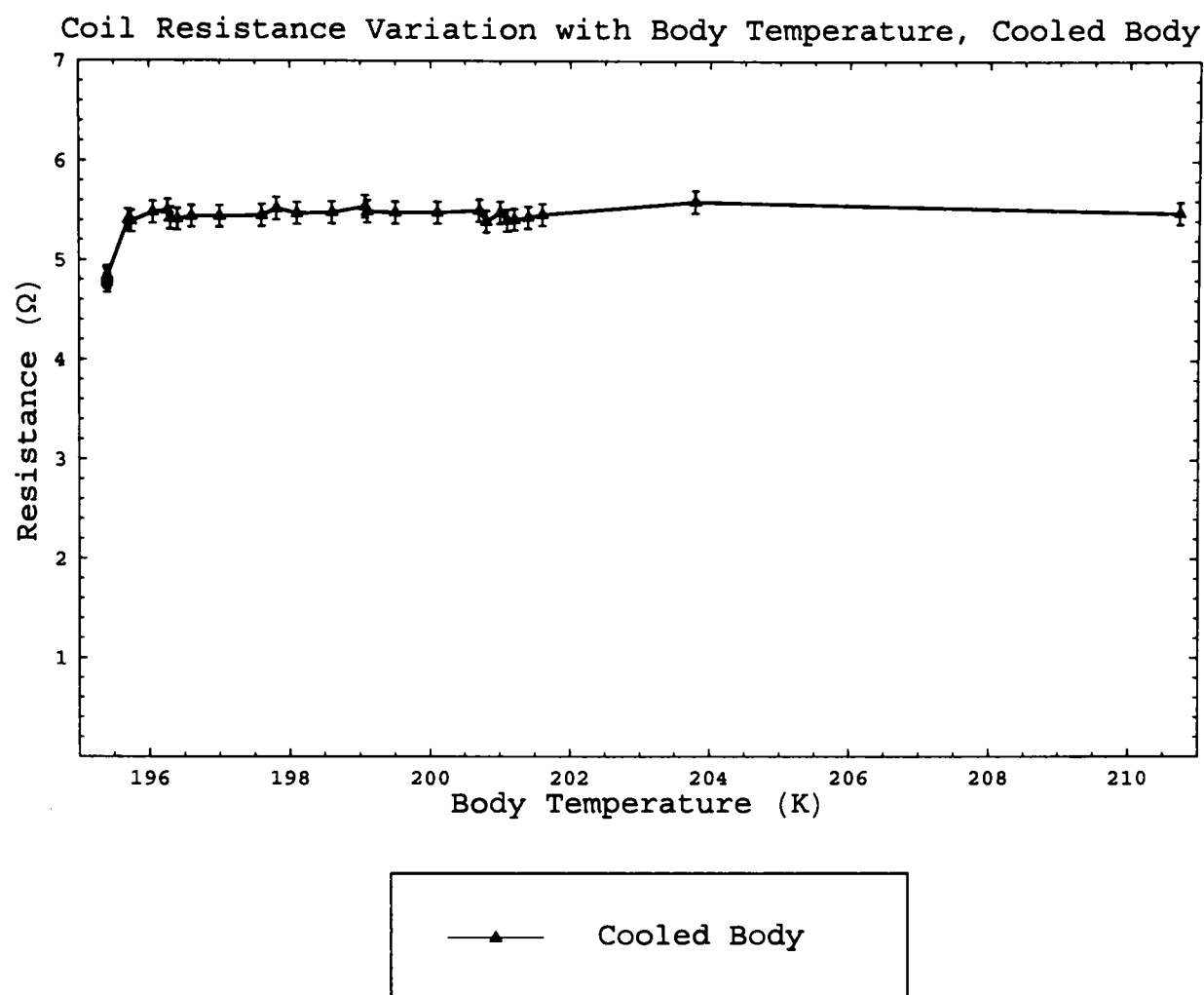


**Fig. 10.14:** Variation in coil resistance with body temperature during a warm body cooling runs.

With the limited time remaining to the project a decision had to be made whether to characterise the cooler with its current performance or to concentrate on achieving the performance target. Due to the failure of the original MPPO and, during pre-cooled runs, the pressure sensor, PV loops could not be produced and so the power into the gas was calculated via:

$$W_{gas} = W_{in} - W_{ir} - W_{sl} - W_{res} \quad (10.2)$$

For the warm body runs  $W_{res}$  was calculated from experiments performed with the compressor open to the air at room temperature (see section 7.4), under these conditions the viscosity and density of air is similar to that of helium at 7 bar and so it was believed the residual loss calculated by this method would be applicable to conditions in the cooler. However if we look at the viscosity and density of helium at 200K at 7 bar we find that they are significantly different from air at room temperature and pressure ( $\rho_{He}(7bar, 200K)/\rho_{air}(atm, 294K) = 1.4$ ,  $\mu_{He}(7bar, 200K)/\mu_{air}(atm, 294K) = 0.88$ ) and so the value  $W_{res}$  calculated for the warm body runs cannot be applied to the cooled body runs. Without knowledge of the size of  $W_{res}$  the power input to the gas cannot be calculated and so detailed characterisation of the cooler's performance was not achievable. Considering this obstacle it was felt that attempting to improve the cooler's



**Fig. 10.15:** Variation in coil resistance with body temperature during a cooled body cooling run.

performance by increasing the thermal resistance between the cold tip and warm end of the cooler would be the more fruitful avenue to pursue as the thermal resistance can be measured without the need for a compression space pressure sensor or MPPO.

The first step in improving the thermal resistance was to measure it when the body was pre-cooled to see if there had been any change from the value measured during the warm body experiments. This was done in the same way as had been done when measuring the warm body value (see subsection 9.3). Figures 10.16 to 10.17 show a sample of the results from this procedure. Figure 10.19 shows how the thermal resistance was calculated at four cold tip temperatures when the cooler body was pre-cooled. For comparison the average value of the thermal resistance measured during the warm-body experiments is also plotted. Figure 10.20 shows the static heat load on the cold tip calculated from the values of thermal resistance shown in figure 10.19. From figure 10.19 it is clear that the thermal resistance had decreased significantly when the cooler body was pre-cooled. This was a surprising result as, in the past, lower cold tip temperatures have been associated with increases in the thermal resistance [Kotsubo, V. et al. (1992)]. One possible explanation was that radiation loading of the cold tip had increased between the warm and cold

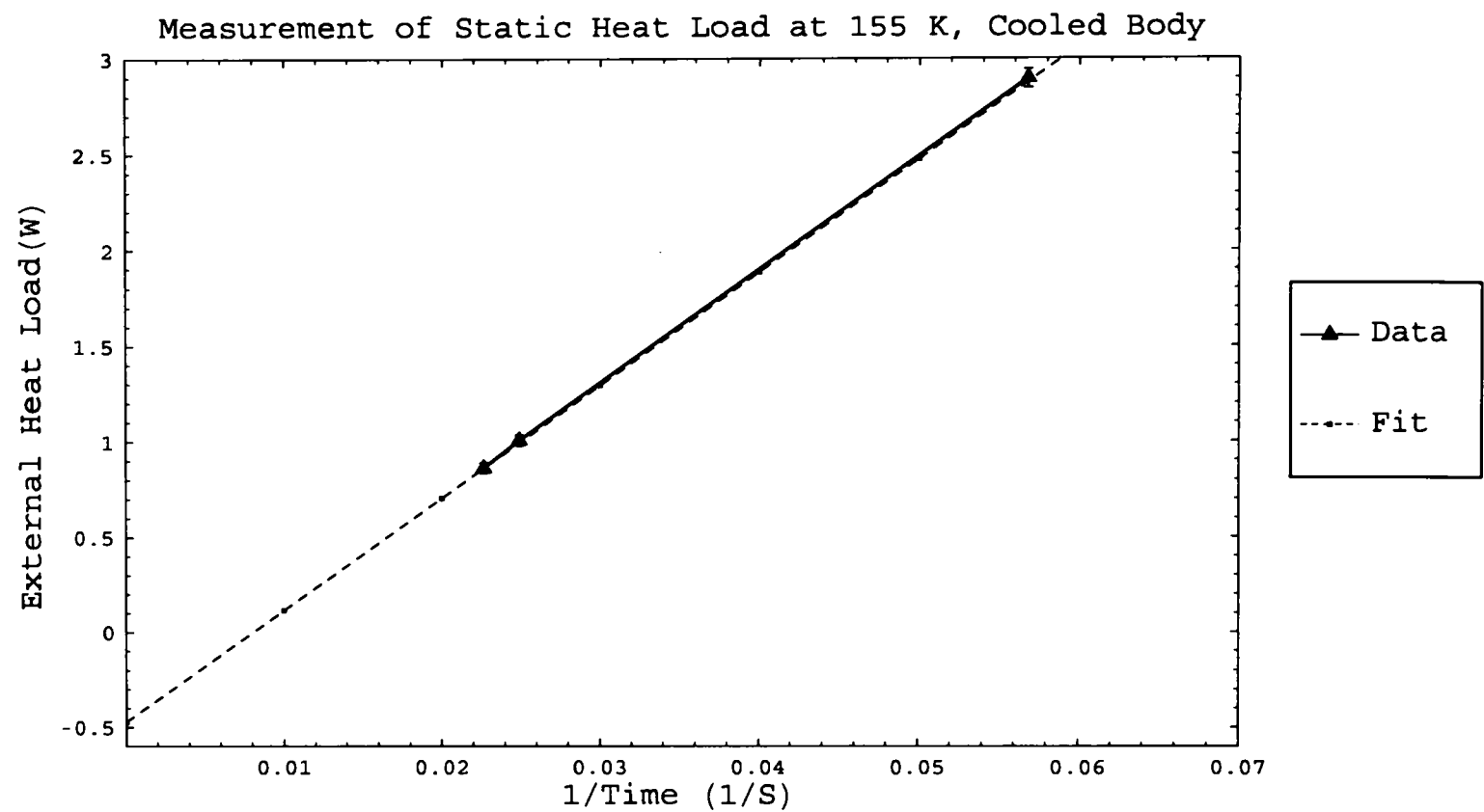


Fig. 10.16: Measurement of the static heat load at 155K with cooled body.

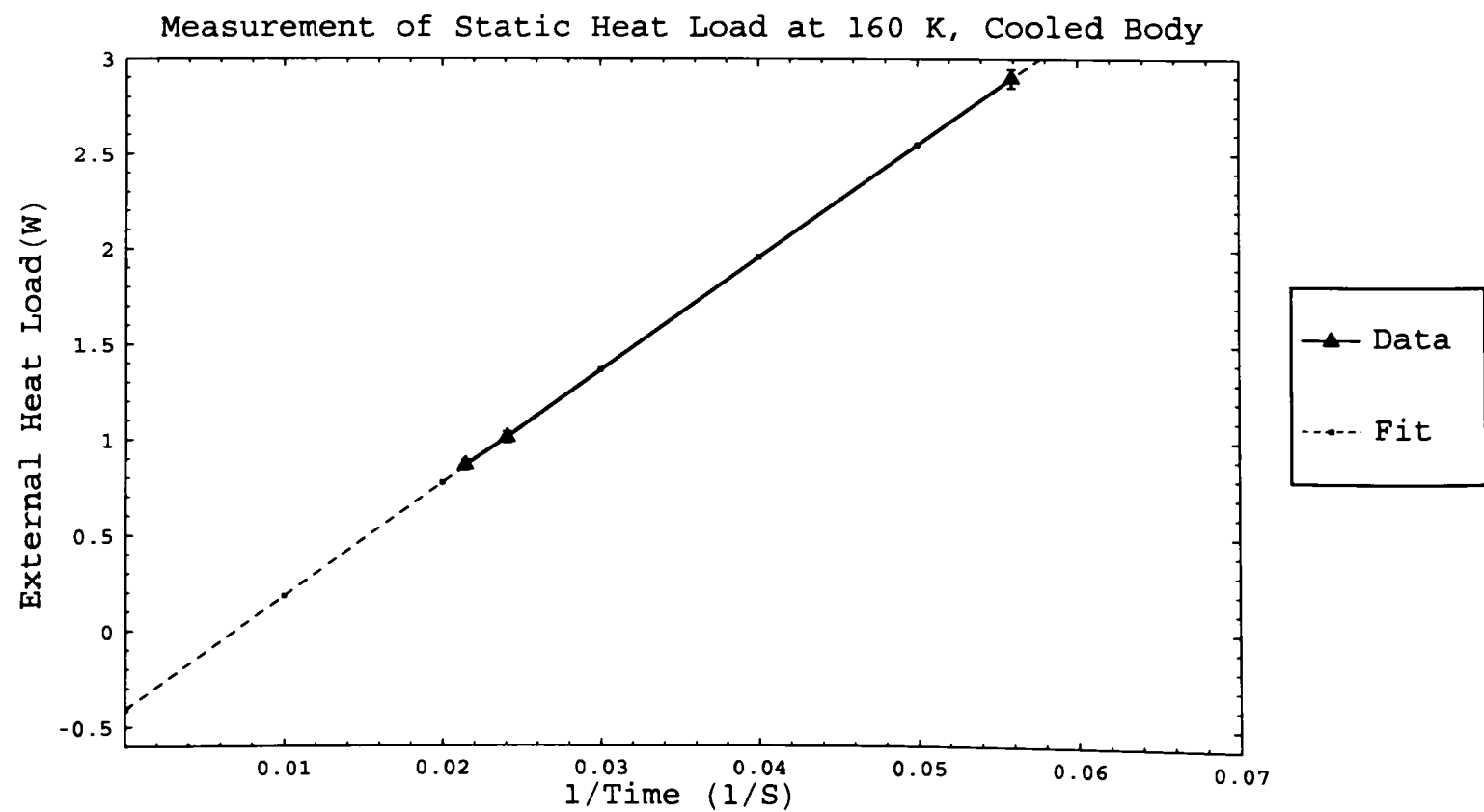
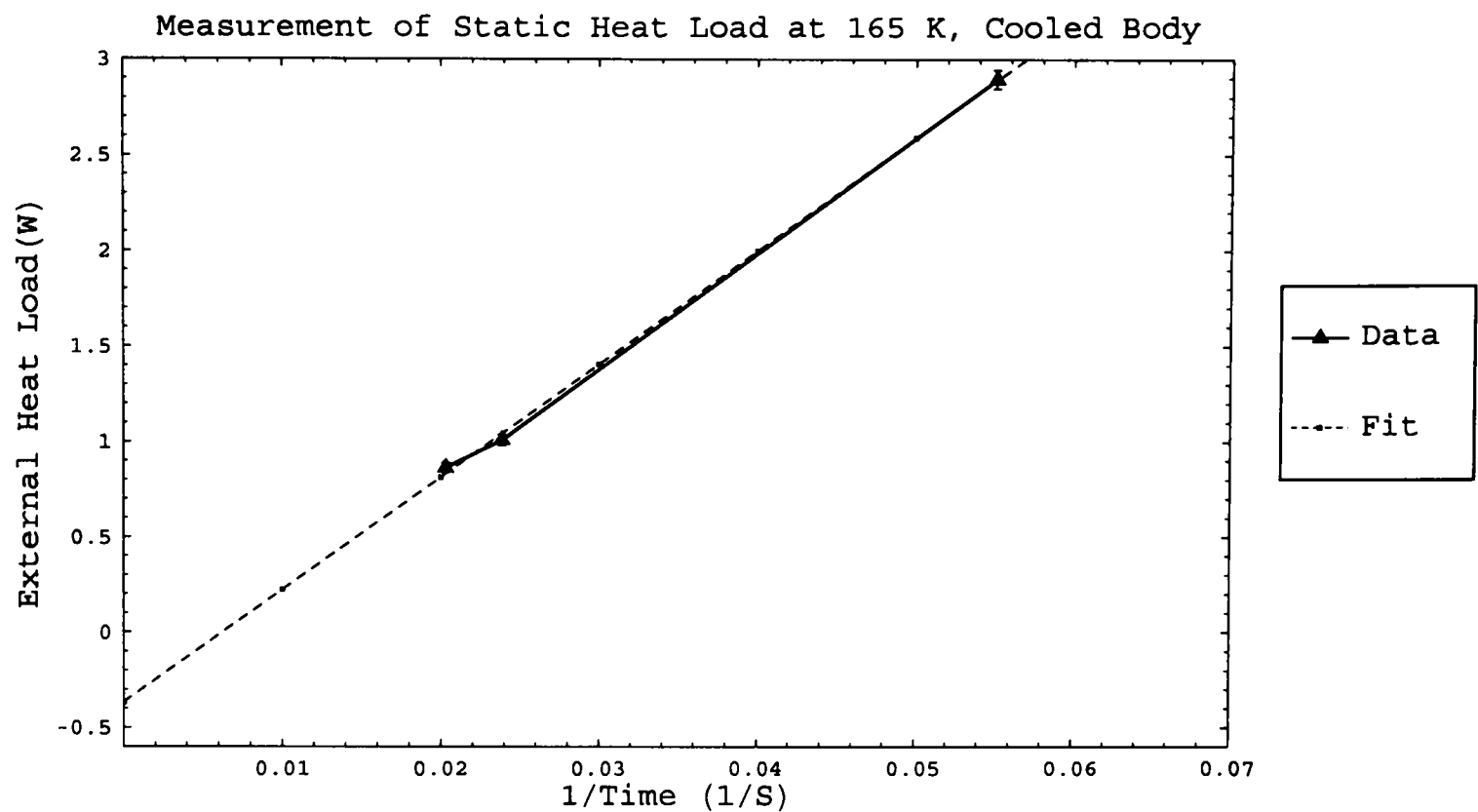


Fig. 10.17: Measurement of the static heat load at 160K with cooled body.



**Fig. 10.18:** Measurement of the static heat load at 165K with cooled body.

body measurements. This had been assumed to be negligible because the cold tip and cold finger were covered by ten layer MLI but it was decided an investigation should be made to see if this assumption was safe.

## 10.10 Investigation of radiation loading of the cold tip

As was discussed in subsection 9.3 the static heat load model assumes that the heat load on the cold tip when the cooler is not running is given by:

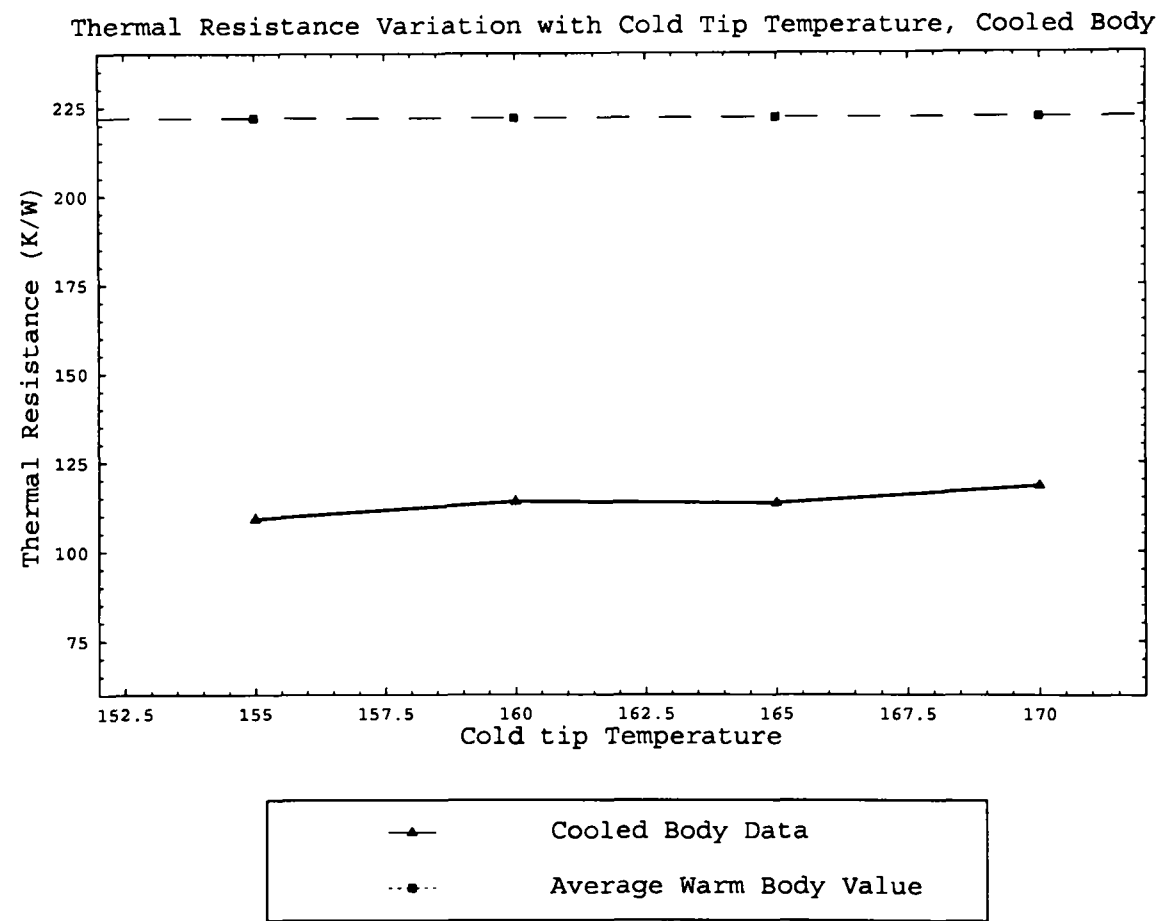
$$Q_{static} = \frac{\Delta T}{r_{ch}} \quad (10.3)$$

Where  $\Delta T$  is the temperature difference between the cold tip and the warm end of the cooler. This assumption is only valid if the non-linear contribution to the radiation loss is negligible. The radiation loss (i.e. the heat load applied to the cold tip) is given by:

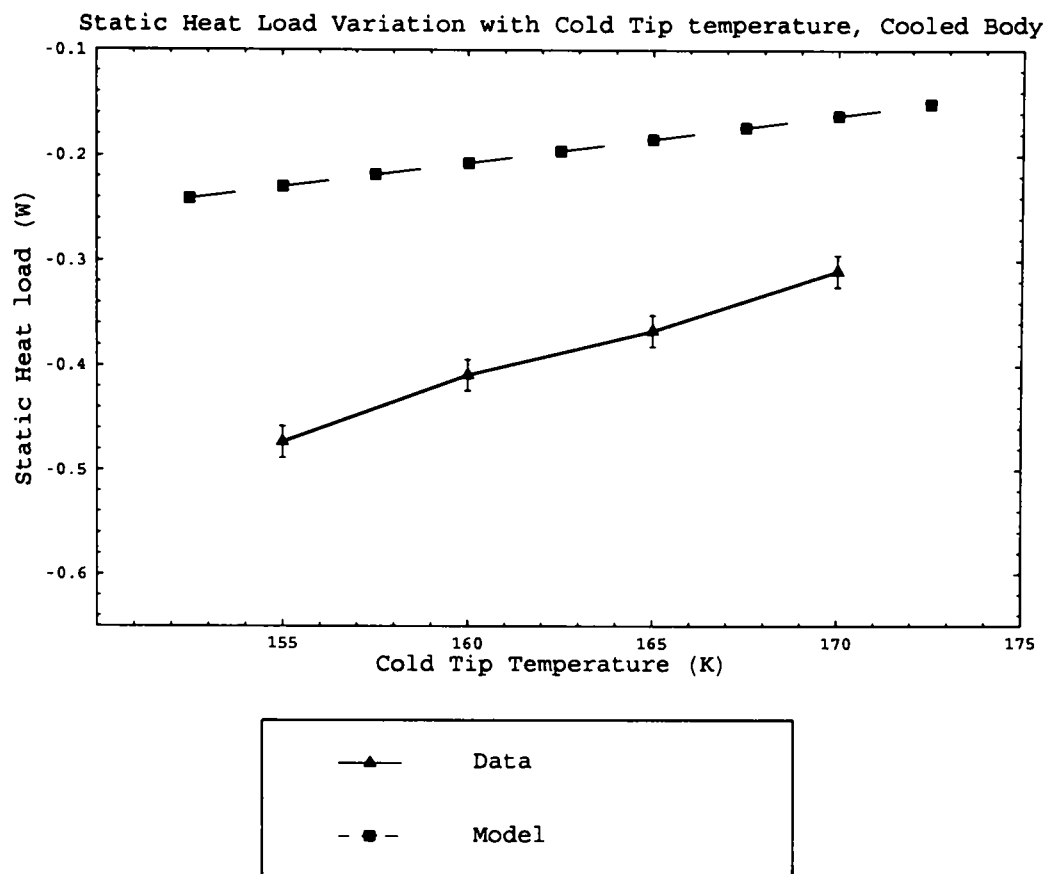
$$Q_{rad} = \sigma \epsilon A_{ch} [(T_{\infty}^4 - T_e^4)] \quad (10.4)$$

Where  $T_{\infty}$  is the temperature of the source of radiation impinging on the cold tip,  $T_e$  is the cold tip temperature,  $A_{ch}$  is the cold tip's surface area,  $\epsilon$  is its emissivity and  $\sigma$  is Stefan's constant. If we substitute  $T_{\infty} = T_e + \Delta T$ , this can be expanded to:

$$Q_{rad} = \sigma \epsilon A_{ch} [4T_e^3 \Delta T + 6T_e^2 \Delta T^2 + 4T_e \Delta T^3 + \Delta T^4] \quad (10.5)$$



**Fig. 10.19:** Thermal resistance for cold tip temperatures between 155 – 170K with the cooler body pre-cooled.

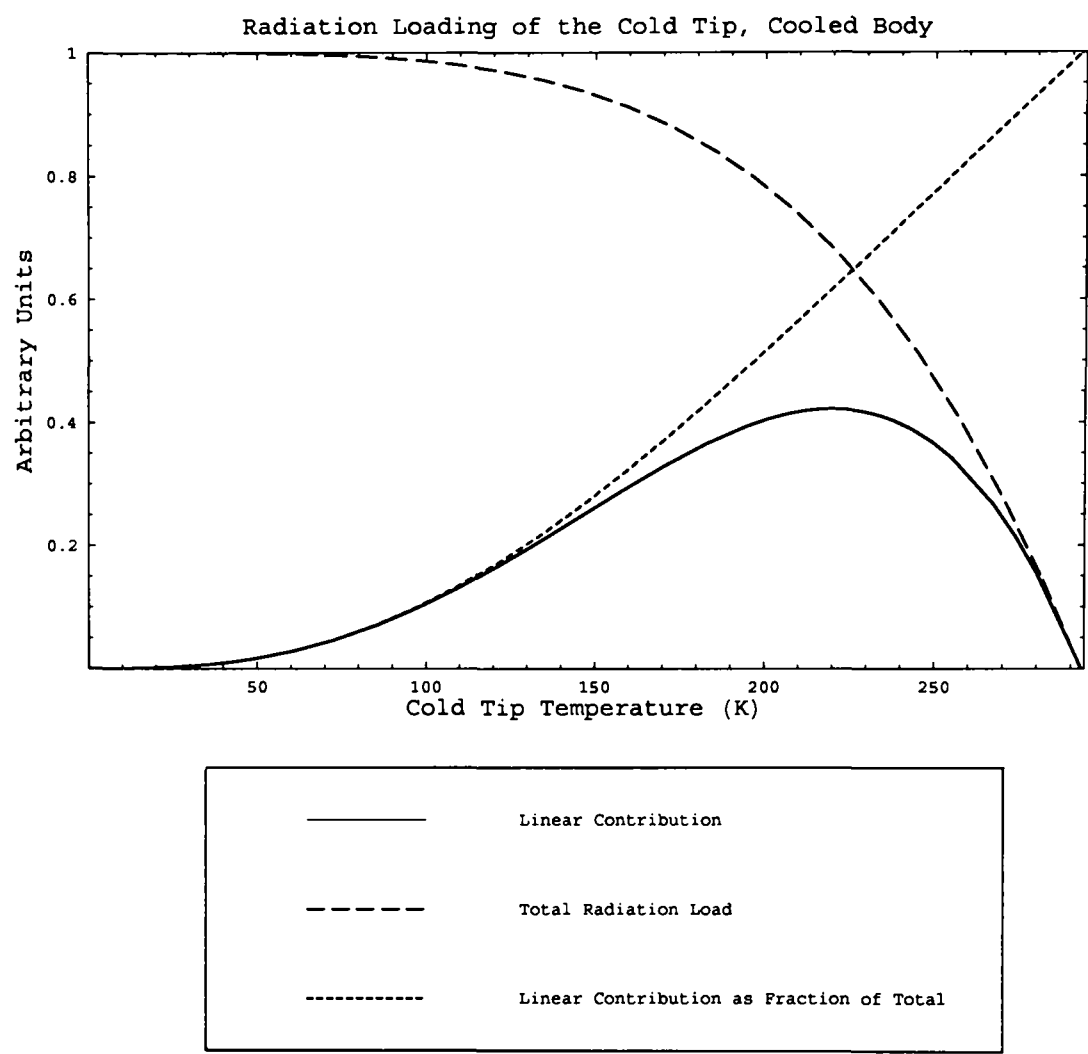


**Fig. 10.20:** Static heat loads with the cooler body pre-cooled. Also shown are model predictions of the static heat load produced using the warm-body value of the thermal resistance.

So we see from equation 10.5 that there is a term ( $4Te^3\Delta T$ ) that is linear in  $\Delta T$  and the remaining terms are non-linear in  $\Delta T$ . The contribution to the static heat load by this linear term will be included in the experimental measurement of the static heat load. If the contribution was significant it could give rise to a substantially larger static heat load than conduction through the cold finger and regenerator alone. By looking at the residuals for the static heat load measurements for the cooled body we can determine an upper limit on the non-linear component of the radiation loss. Re-arranging equation 10.5 we find:

$$Q_{rad,total} = \frac{Q_{rad,non-linear}}{1 - \frac{Q_{rad,linear}}{Q_{rad,total}}} \quad (10.6)$$

It is a simple matter to calculate what fraction of the total radiation loss is due to the linear contribution ( $\frac{Q_{rad,linear}}{Q_{rad,total}}$ ) and the variation of this fraction with cold tip temperature is shown in Figure 10.21 in which it is assumed  $T_{\infty} = 294K$ . Since we also have an upper limit for the non-linear contribution from the residuals of the static heat load measurements it is possible to calculate the total radiation loss and linear contribution via equation 10.6. With a pre-cooled body there are two sources of radiation; the cooled body at around 200K and the vacuum tank walls at room temperature. Depending on the assumed temperature of the radiation source, we find that the total radiation loss at a cold tip temperature of 155K is between 4 and 7 mW and a linear



**Fig. 10.21:** Variation in the radiation loading of the cold tip.

contribution in the range 1.2 – 4.4 mW. These calculations strongly suggest that the radiation loss was negligible and was not making a significant contribution to the measured static heat load and corresponding thermal resistance. Another possible heat load to the cold tip could arise from the cold tip heater and PRT. Both had wires connecting them to their current supplies and the data acquisition system and these could apply an additional heat load to the cold tip, but since the wires were of a small diameter (0.012mm) and were heat sunk to the space view (so that it could act as “heat interceptor”) this was rejected as an explanation. It seemed probable that the low thermal resistance between the cold tip and compression space was due to thermal conduction through the regenerator. It was therefore decided that if the cooler performance was to be improved then the thermal resistance of the regenerator needed to be increased.



# Chapter 11

## Improved regenerator design

### 11.1 Introduction

Measurements of the thermal resistance had shown that it was lower than had been expected and that this led to a large static heat load. In similar coolers the axial conduction through the regenerator has been the largest contribution to the static heat load and the conduction through the regenerator calculated by the computer model described in Chapter 4 supports this claim. With this in mind it seems logical to concentrate on minimising conduction through the regenerator if an attempt is to be made to decrease the static heat load.

There are numerous papers on regenerator theory and design, yet it is still not a routine matter to optimise a regenerator. If a regenerator is to work well then it must have an effectiveness approaching 100%. This can be achieved by ensuring that the regenerator material is in good thermal contact with the working gas. Typically this is done by packing the regenerator densely with fine wire meshes made of a material such as stainless steel or phosphor bronze. However, densely packing the regenerator increases the pressure drop along the length of the regenerator and reduces its thermal resistance (i.e. increases the contribution of the regenerator to the static heat load) thus reducing the cooler's performance. Optimising the regenerator thus becomes a matter of finding a design which allows it to be extremely effective but without introducing significant conduction or a large pressure drop along its length.

### 11.2 Increasing the regenerator's thermal resistance

By packing the regenerator less densely it is possible to reduce axial conduction along its length. However, this is not without penalties as it reduces the effectiveness of the regenerator. Brito [Br



| Regenerator | Mesh size | Packing % | $R_s$ | $R_r$ | Description               |
|-------------|-----------|-----------|-------|-------|---------------------------|
| 1           | 250       | 86.7      | 227.3 | 149.3 | Standard                  |
| 2           | 250       | 93.3      | 151.5 | 250   | Standard                  |
| 3           | 250       | 93        | 714.3 | 72.5  | Thermal Breaks            |
| 4           | 250/500   | 92.7      | 188.7 | 96.2  | Thermal Breaks and Graded |
| 5           | 250/500   | 71.0      | 294.1 | N/A   | Graded                    |
| 6           | 250/500   | 79        | 156.3 | N/A   | Graded                    |
| 7           | 250/500   | 86.3      | 117.6 | 126.6 | Graded                    |
| 8           | 250/500   | 91.0      | 90    | N/A   | Graded                    |

**Table 11.1:** Thermal resistances for different regenerator configurations, after [Brito, M. (1999)].

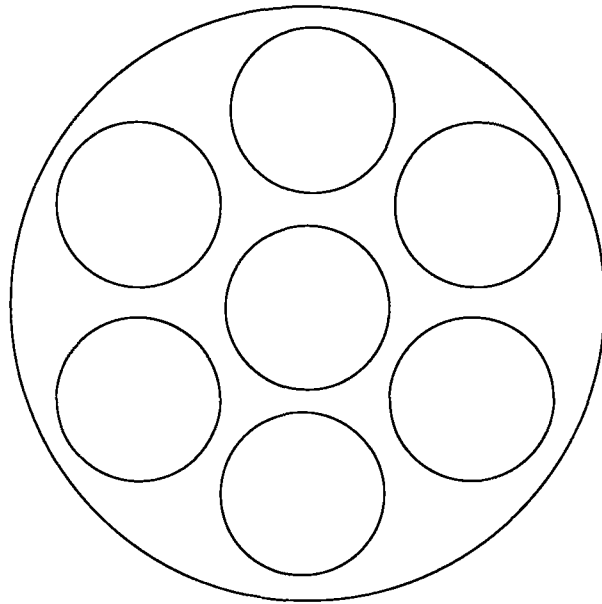
M. (1999)] performed an experimental study on a regenerator for use in an prototype miniature pulse tube and his results are summarised in table 11.1.  $R_s$  is the static thermal resistance,  $R_r$  is the thermal resistance derived from the experimentally determined regeneration loss (the higher the resistance the more effective the regenerator). Mesh size is given in wires per inch. Packing is defined as:

$$Packing = \frac{2dN}{L} \quad (11.1)$$

Where  $d$  is the diameter of the mesh wire,  $N$  is the number of discs packed in the regenerator and  $L$  is the regenerator length. The graded regenerators were comprised of alternating sections of 250-mesh and 500-mesh discs. Regenerators 3 and 4 had three thermal breaks (made from glass fibre re-enforced plastic) inserted at regular intervals along the regenerator length. Looking at regenerators 1 and 2 in table 11.1 we can see that an increase in packing of 6.6% led to a decrease of 33% in the thermal resistance. Comparing regenerators 2 and 3 (which have similar if not exactly the same packing) we see that adding thermal breaks had an extremely significant effect on the thermal resistance increasing it by 371 %. However introducing thermal breaks decreases the effectiveness of the regenerator thus increasing the regeneration loss. We can see that the thermal breaks in regenerator 3 lead to  $R_r$  decreasing by 71 % compared to regenerator 2. It is clear from these results that thermal breaks can be extremely effective at increasing the thermal resistance of a regenerator and so an attempt was made to utilise them in the prototype's regenerator.

### 11.2.1 Design of the regenerator's thermal breaks

Figure 11.1 shows a diagram of a thermal break used in the regenerator. Each break was 0.6mm thick and was made of glass fibre re-enforced plastic. The regenerator was dis-assembled and the phosphor-bronze discs removed from the Vespel sleeve of the regenerator. The whole assembly was



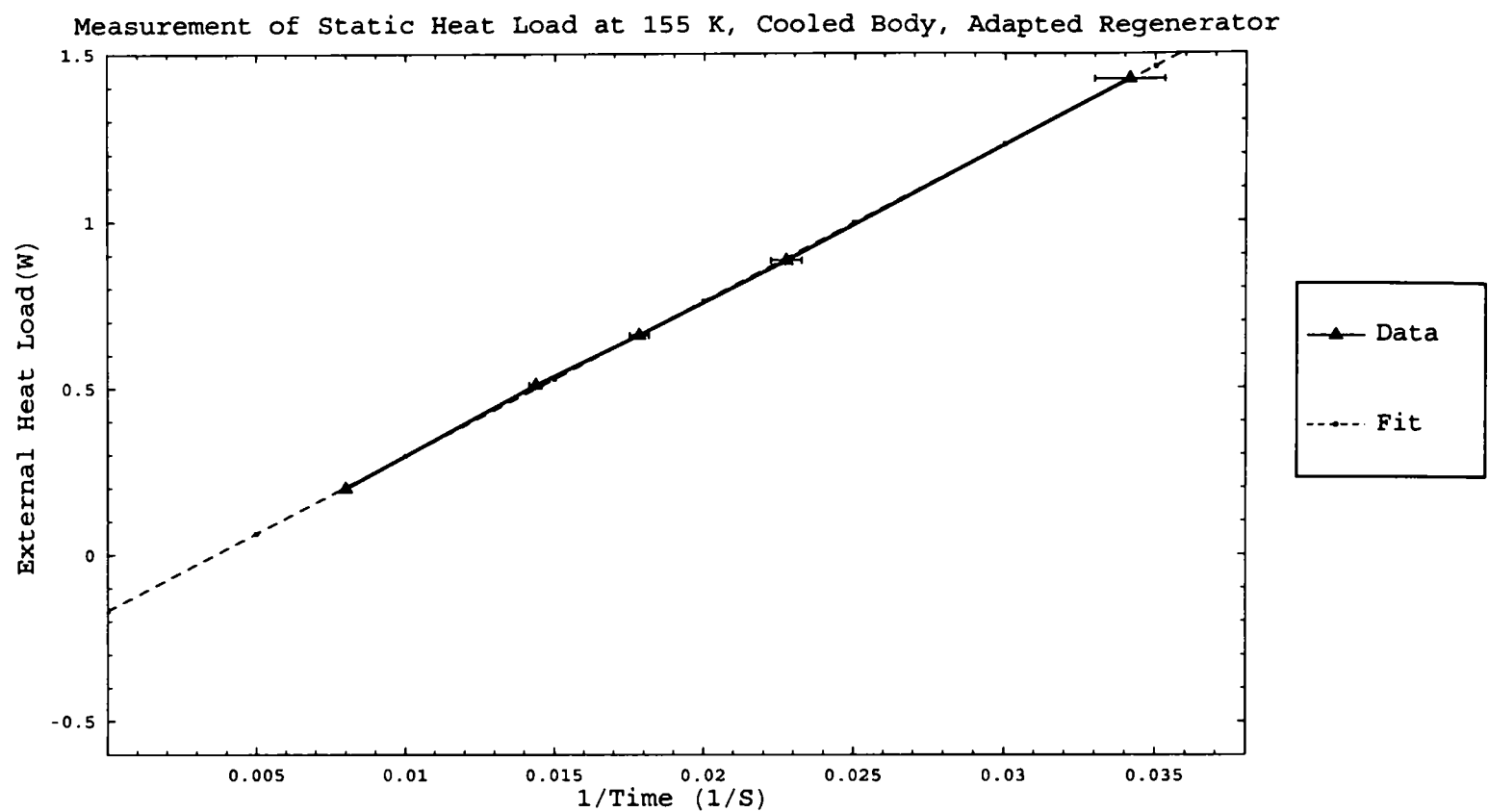
**Fig. 11.1:** Diagram of a glass-fibre re-enforced plastic thermal break (not to scale).

ultrasonically cleaned whilst immersed in acetone as were the thermal breaks. The regenerator was then re-assembled with 4 thermal breaks spaced equidistantly along the stack of phosphor bronze disks. The regenerator was then baked at approximately 343K for twelve hours to rid the meshes of any trapped moisture. So as to reduce any possible shift in the ideal optimum cooling frequency an effort was made to maintain the regenerator at the same mass as its original configuration. Since glass fibre re-enforced plastic is less dense than phosphor bronze, less than 5% of the discs needed to be removed to keep the regenerator mass constant. The thermal breaks are designed to provide as little resistance to gas flow through the regenerator as possible, Brito used a similar design and found that they had a negligible effect on the pressure drop across the regenerator [Brito, M. (1999)].

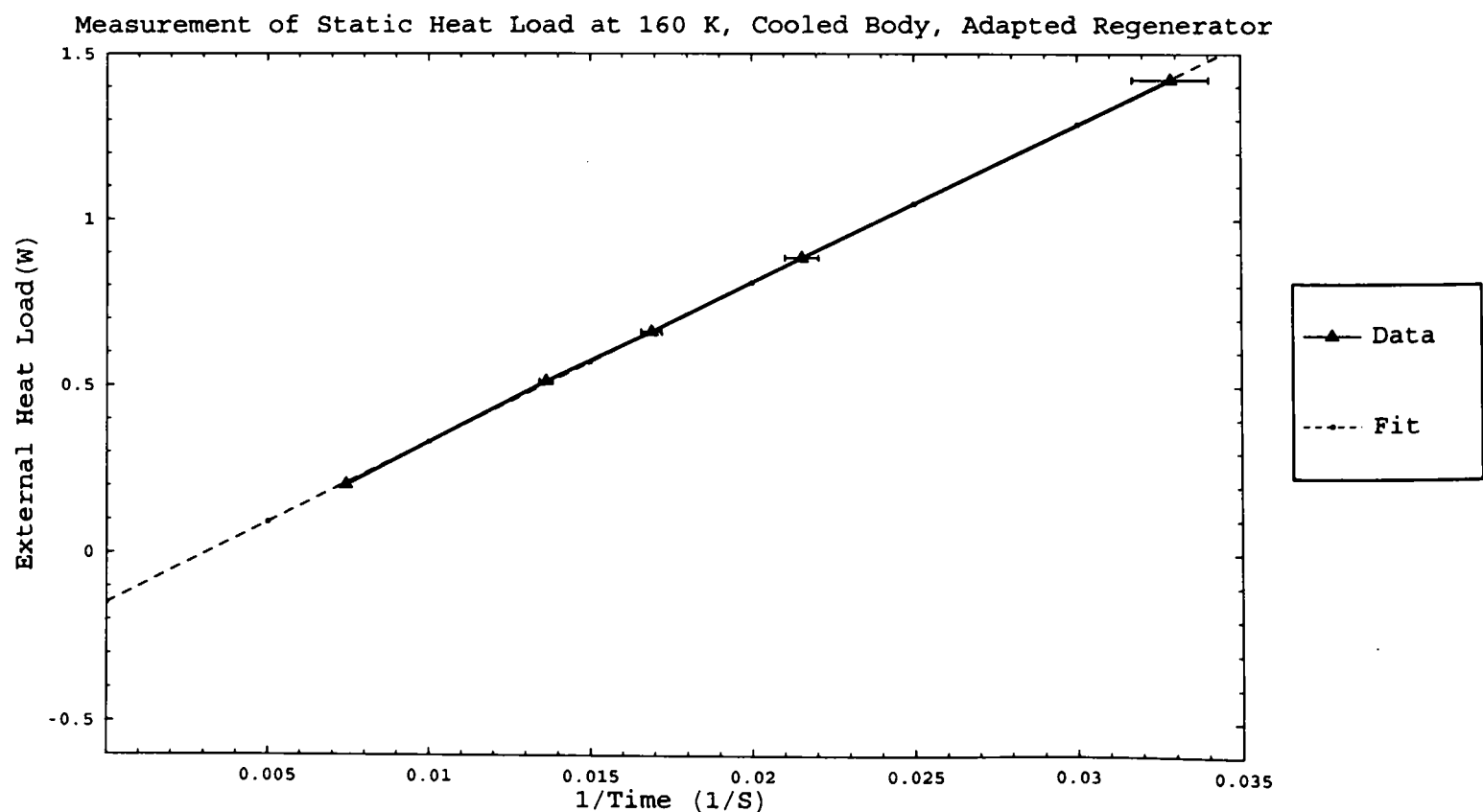
### 11.3 Static heat load with the adapted regenerator

The cooler was re-assembled with the adapted regenerator and the thermal resistance of the new configuration determined via the method described in subsection 9.3. Figures 11.2 to 11.5 show graphs derived from this procedure at cold tip temperatures from 155K to 170K with pre-cooling of the cooler body performed by the radiator.

The static heat load derived from these measurements are shown in figure 11.6 and thermal resistance at each temperature is shown in figure 11.7. As before the thermal resistance is reasonably stable over the range of cold tip temperatures considered, having an average value 306.15K/W. It is possible that there is a trend for the thermal resistance to decrease with increasing temperatures but within the measurement errors the thermal resistance is constant in this range of cold tip temperatures. This is an increase of almost 157% (a factor of 2.57) over the thermal resistance



**Fig. 11.2:** Measurement of the static heat load at 155K with cooled body and the adapted regenerator.



**Fig. 11.3:** Measurement of the static heat load at 160K with cooled body and the adapted regenerator.

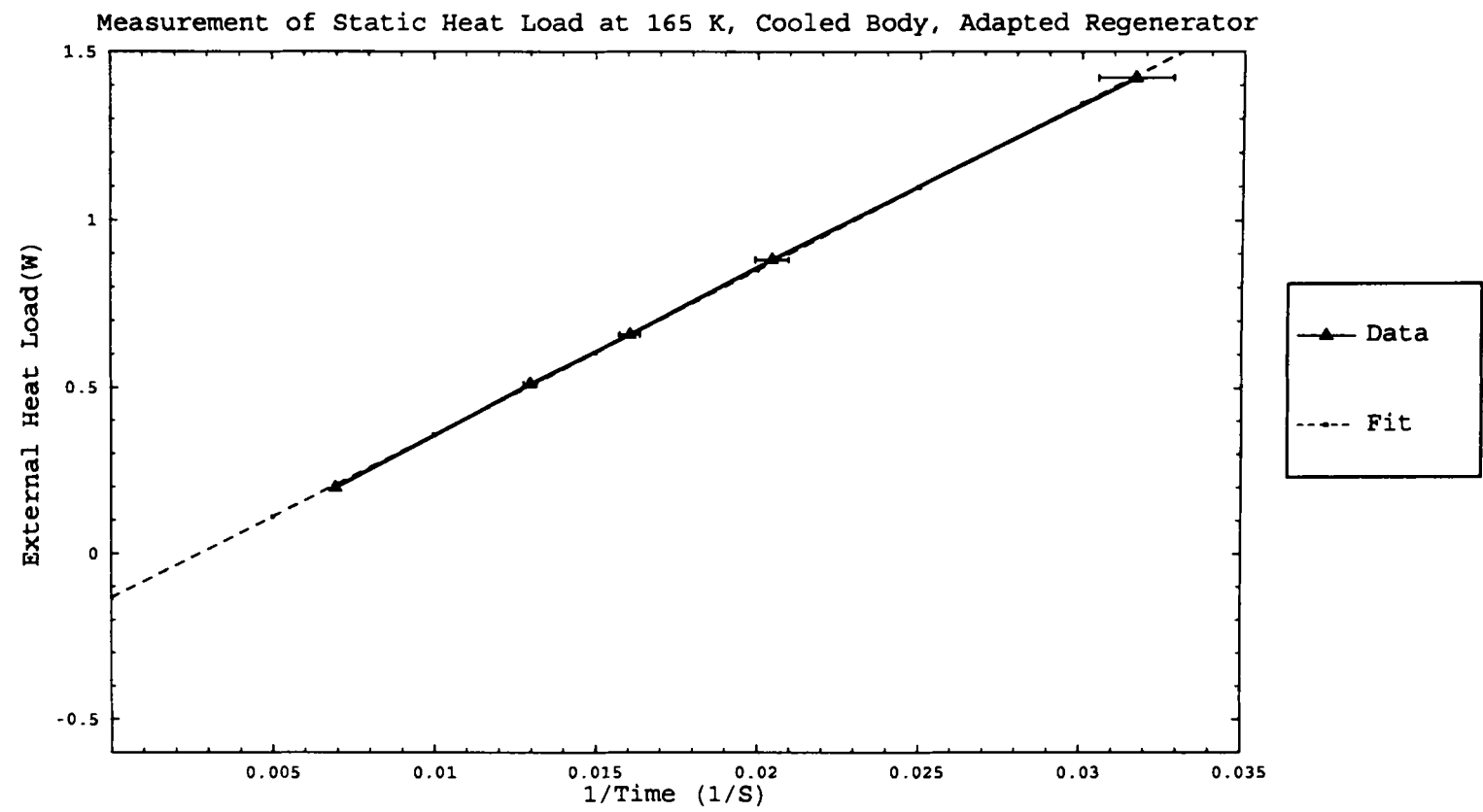


Fig. 11.4: Measurement of the static heat load at 155K with cooled body and the adapted regenerator.

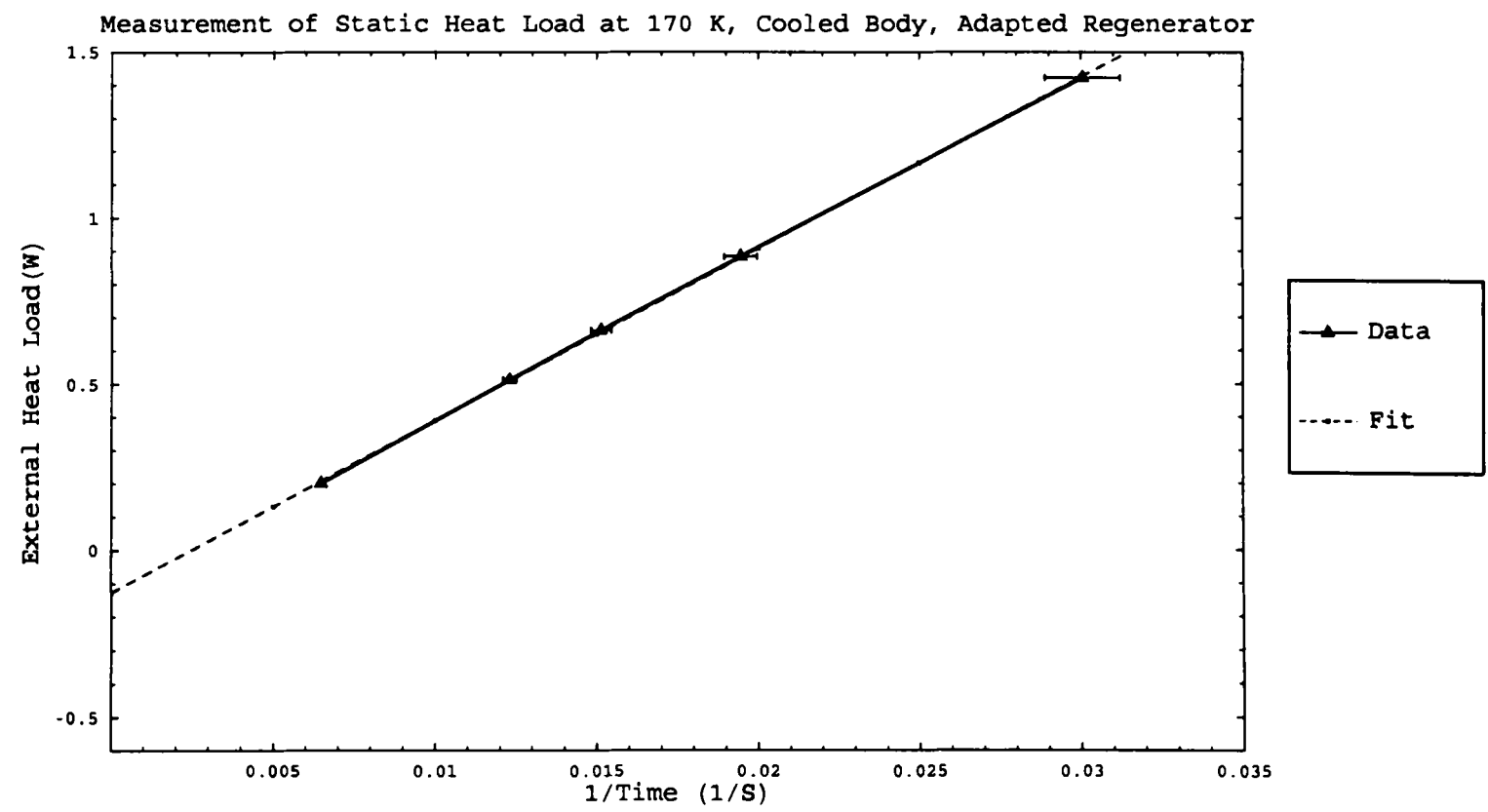
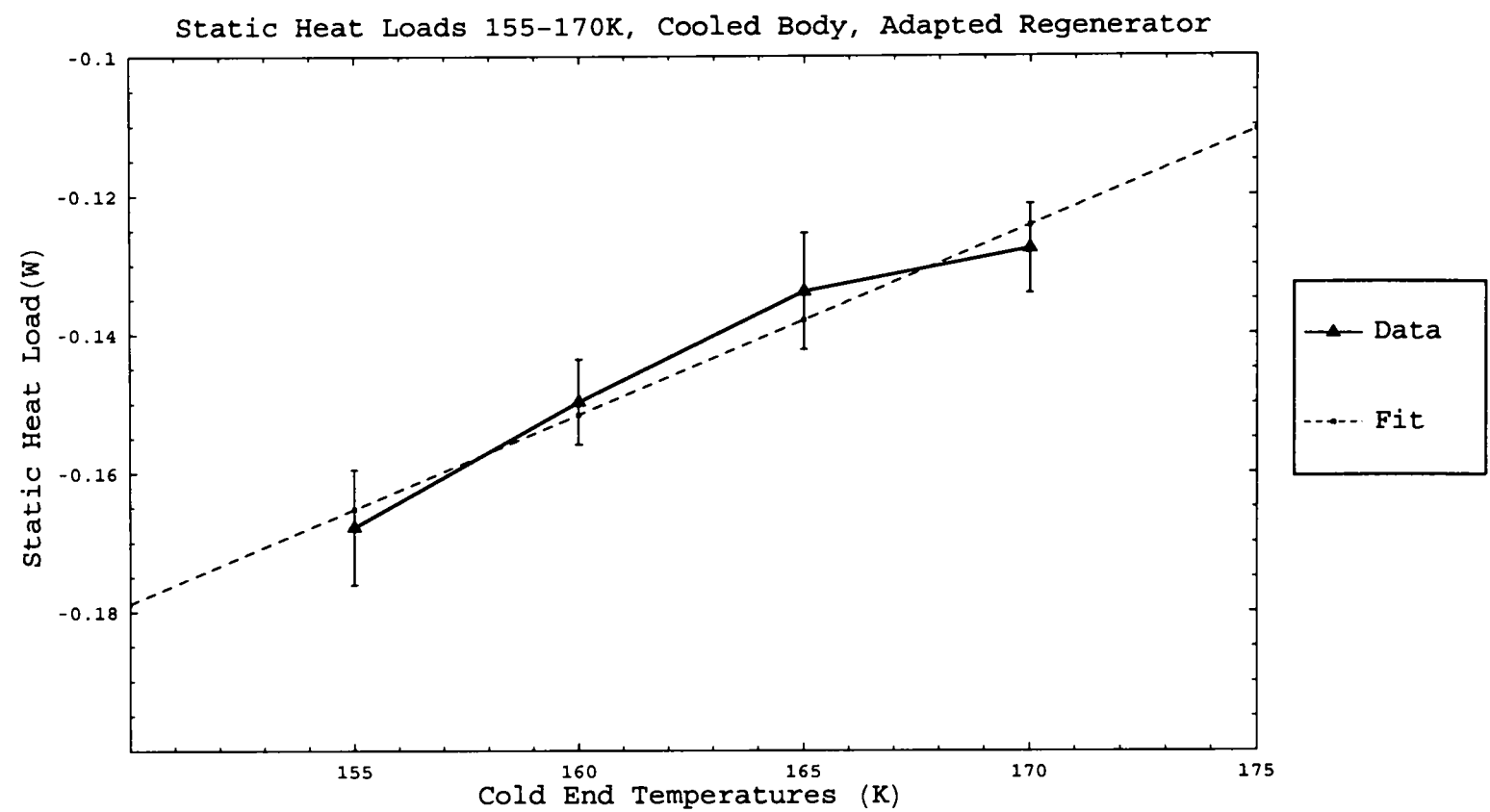
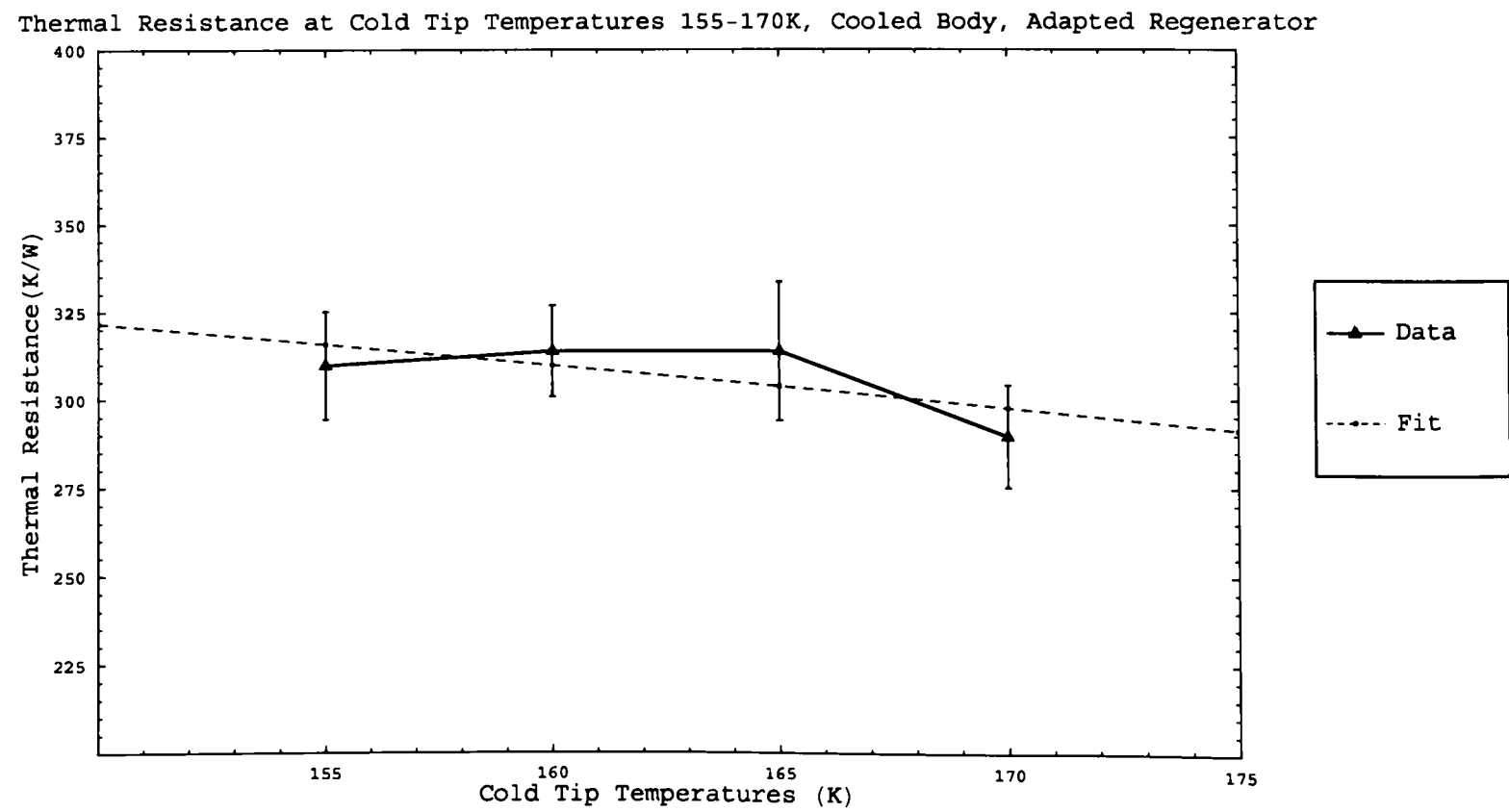


Fig. 11.5: Measurement of the static heat load at 170K with cooled body and the adapted regenerator.



**Fig. 11.6:** Static heat load with the adapted regenerator over the range of cold tip temperatures 155K-170K.



**Fig. 11.7:** Thermal resistance with the adapted regenerator in place over the range of cold tip temperatures 155K-170K.

when measured with the original regenerator indicating that the thermal breaks had a significant effect on the thermal resistance of the regenerator. Typically the glass fibre re-inforced plastic used for the thermal breaks has a thermal conductivity in the range of  $0.25 - 0.6 \text{ W m}^{-1} \text{ K}^{-1}$  [Kasen, M.B. (1975)]. This leads to a total thermal resistance for the thermal breaks of  $140 - 338 \text{ K/W}$  calculated via equation 11.2:

$$R_{th} = \frac{\Delta x}{kA} \quad (11.2)$$

Where  $R_{th}$  is the thermal resistance,  $\Delta x$  is the thickness of the material in the direction of heat conduction,  $k$  is the thermal conductivity in the direction of conduction and  $A$  is the cross sectional area. If we add the calculated range of thermal resistance of the thermal breaks to the thermal resistance originally measured for the regenerator during cooled body experiments, we find a predicted thermal resistance for the adapted regenerator in the range  $259 - 457 \text{ K/W}$  which agrees with the experimentally derived value of  $306.15 \text{ K/W}$ .

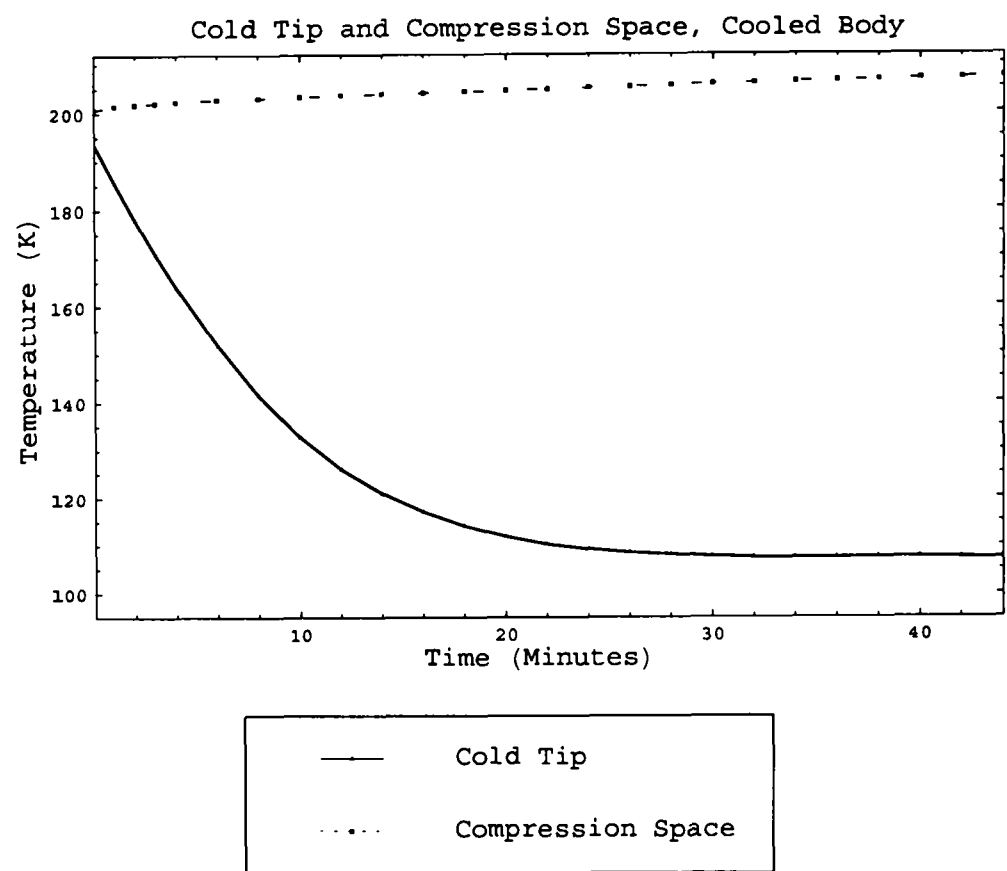
## 11.4 Pre-cooled body cooling runs with the adapted regenerator

With the adapted regenerator in place and measurements indicating that the thermal resistance had been significantly increased over its original value, further cooling runs were performed to see if a lower ultimate cold tip temperature could be attained. The cold tip and compression space temperatures for the cooling run in which the lowest ultimate cold tip temperature was reached is shown in figure 11.8.

The lowest cold tip temperature attained with the adapted regenerator was  $107.4 \text{ K}$  (compared to  $92 \text{ K}$  with the original regenerator). Five pre-cooled body cooling runs were performed to see if this performance could be improved but at no point did the cold tip temperature drop below this temperature. A cooling run without the body being pre-cooled led to the cold tip temperature reaching a minimum of  $185.6 \text{ K}$  indicating that in both the warm and cooled body cases the adapted regenerator had been detrimental to the cooler's performance.

## 11.5 Summary

Measurements had indicated that the low thermal resistance between the cold tip and compression space was causing a large static heat load and this was preventing the cooler from reaching its target performance. Based on a previous experimental study [Brito, M. (1999)] it was thought that introducing thermal breaks into the regenerator would increase its thermal resistance and reduce



**Fig. 11.8:** Cold tip and compression space temperatures during the lowest ultimate cold tip cooling run with the adapted regenerator.

the static heat load. Experimental results confirmed this and an increase of approximately 157% was observed in the thermal resistance. This improvement in the thermal resistance did not translate into an overall improvement in the cooler’s performance. It is thought that the introduction of thermal breaks detrimentally effected the regenerator’s effectiveness as this behaviour has been observed before [Brito, M. (1999)]. It is possible that the thermal breaks increased the pressure drop along the regenerator and so reduced the cooling power of the prototype, however due to time constraints it was not possible to determine exactly how they had effected the regenerator within the current research effort.



## Chapter 12

# Conclusions and suggested further work

### 12.1 Conclusion

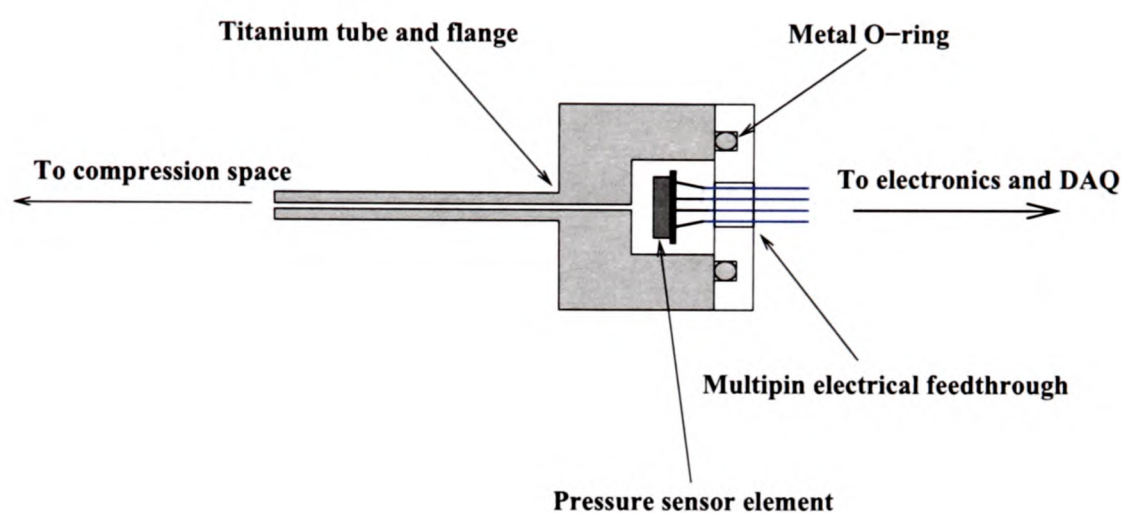
A prototype free-piston integral Stirling cooler combined with a radiator has been developed to investigate the feasibility of such a hybrid design for use on small remote sensing platforms. All the components used in the cooler were manufactured within the department and all but the detailed machining of the coil former was performed by the author. This consumed more than 75% of the time available to the project. Test facilities, sensors, and data acquisition software was developed to allow preliminary investigation of the cooler's performance with and without pre-cooling by the radiator. The cooler has achieved a minimum cold tip temperature of 92K when the warm end is pre-cooled to approximately 206K and can maintain a cold tip temperature of 122K with a 0.2W external heat load applied to the cold tip when pre-cooled. The original target of maintaining a cold tip temperature of 80K with a 0.2W external heat load applied was not achieved and some evidence has been found to suggest that this may be due to a low value of thermal resistance between the cold tip and warm end. Further work is required to investigate the low thermal resistance, other loss mechanisms within the cooler and to better characterise its performance. A number of flaws in the design of the cooler and also the sensors used to gather data about its performance need to be addressed before these can commence. These flaws, and possible solutions are discussed in section 12.2 below.



## 12.2 Future work

### 12.2.1 New MPPO

There have been a number of problems encountered during this research effort that have limited the performance of the cooler and hampered attempts to characterise its performance. The first to be encountered was the failure of the original MPPO. The original design had assumed that the motor coil would have a negligible inductance as this had been true for previous miniature cooler motors [Reed, J. (2004)]. However the current prototype had a newly designed coil and the inductance was not negligible. What is more the inductance varied depending on the degree of the coil's insertion into the linear motor magnet assembly. This non-negligible variable inductance had a significant effect on the MPPO's ability to provide accurate position data. This presented a significant barrier to the development of the cooler as data on the compressor piston's motion was needed to analyse the cooler's behaviour. Other sensing techniques were considered but the cooler design had focused on miniaturising the compressor and so there was little available space or flexibility to implement a new sensor. The co-axial design of the cooler prevented the use of a Linear Variable Differential Transducer (LVDT) which have been implemented as position sensors in previous Oxford coolers [Peters, M. (1994)]. The displacer/regenerator shaft runs through the centre of the piston and coil former (see figure 12.3) and so an LVDT attached to the rear of the coil former would have prevented the displacer/regenerator from being in place. The possibility of using another capacitive sensor similar to the DPPO was considered but there was too little space within the cooler body unless the design was altered significantly. As time constraints did not allow for an extensive re-design of the cooler a solution was found whereby the extremes of the piston motion were detected by insulated metal end stops with a voltage applied to them as described in section 6.5. This solution was far from perfect as it did not allow the piston motion's waveform to be observed. Further, the piston's motion could only be detected when it was running at full stroke (i.e. hitting the end stops) which was not always possible when the compressor was being run at frequencies away from its amplitude resonance. If the cooler's design is not to be substantially altered to allow a conventional LVDT piston position sensor to be fitted it seems that a capacitive sensor based on the DPPO would be a convenient method. Figure 12.2 shows how this could be achieved without increasing the size of the cooler although it may well complicate the assembly.



**Fig. 12.1:** Diagram showing a possible improvement to the pressure sensor fitting.

### 12.2.2 Improved pressure sensor fitting

In the prototype the pressure sensor was secured into a brass fitting with epoxy. This fitting could then be screwed onto the pressure sensor port that led to the compression space with sealing provided by a nitrile o-ring. During pre-cooled body experiments the pressure sensor fitting caused a significant leak which caused the vacuum tank pressure to rise above  $1 \times 10^{-4}$ . Repeated efforts to fix this problem were unsuccessful and in future work it seems likely that an alternative method of securing the pressure sensor in its fitting will be needed. One possibility is to use a vacuum feedthrough as shown in figure 12.1. This method does not rely on epoxy to produce any sealing and so should overcome the problems experience with the prototype.

### 12.2.3 Improvements to aid assembly

#### Altering the cooler body

In the current design the regenerator is first inserted into the cold finger, then the piston into the compression space cylinder followed by the flexure bearings, magnet assembly and coil etc. Other than the flexures nearest the compression space all the components stack on top of each other with the magnet assembly at the bottom where it is screwed into the cooler body itself. Assembly in this fashion presents a problem as the space between the components and the inner wall of the cooler body is very small (less than 1cm) and so it is difficult to secure each component as it is

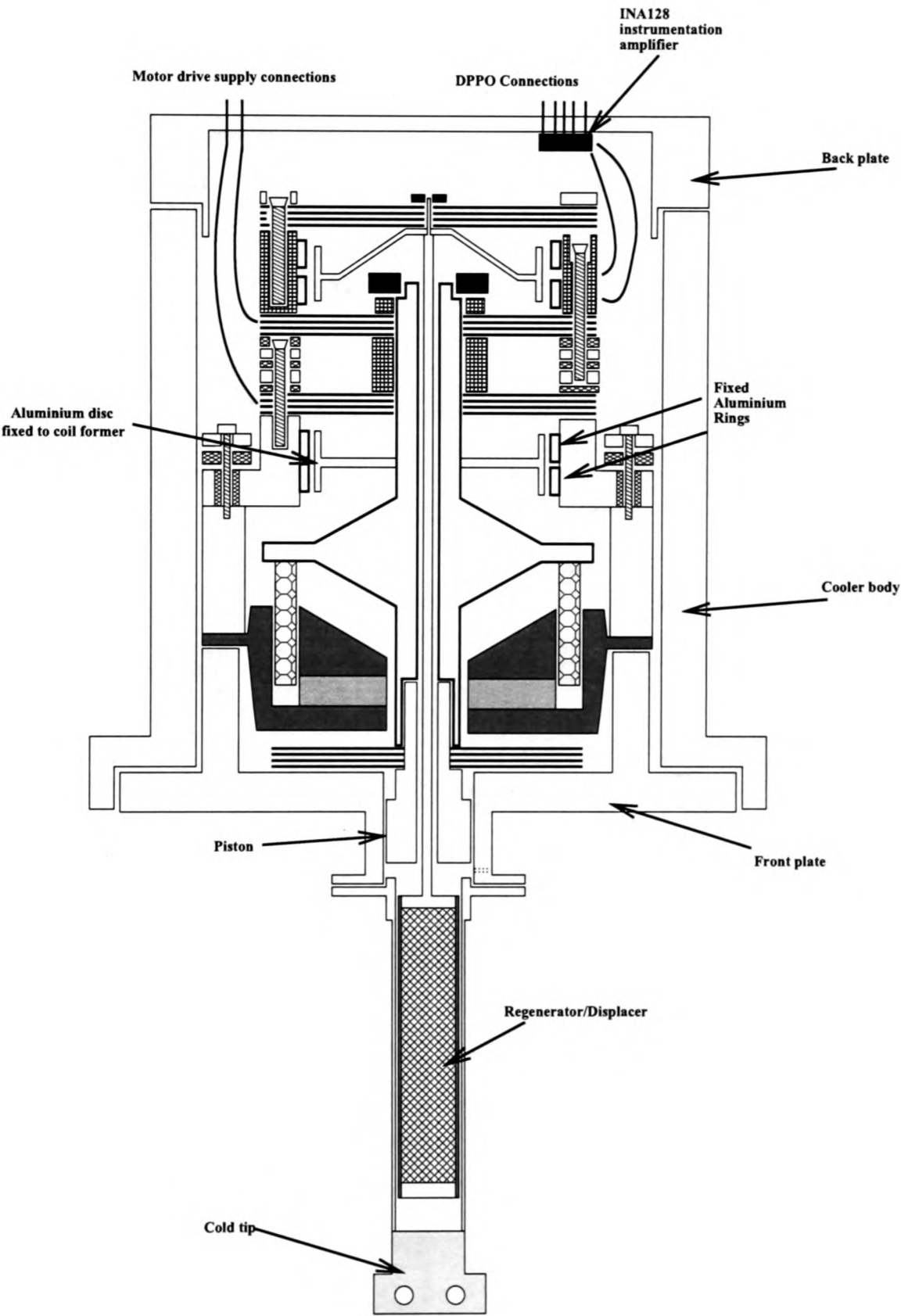
inserted into the body during assembly. It also means that electrical connections to the flexures and to the DPPO are difficult to access. Figure 12.2 illustrates how this problem could be avoided by securing the magnet assembly to the front plate rather than the body, the current prototype design is shown in figure 12.3 for comparison. The body would only be placed over the stack of components and secured to the front plate at the end of the assembly thus allowing access to the components and electrical connections until the very last stages of assembly.

### **Improving displacer/regenerator alignment**

Another significant problem with the cooler was that the displacer/regenerator spring stack was at the very other extent of the cooler from the displacer/regenerator itself, which led to poor alignment within the cold finger. This meant that between one assembly and the next the displacer/regenerator could experience very different degrees of rubbing between itself and the cold finger. This caused variation in the displacer/regenerator stroke, the frequency at which optimum performance was achieved, and overall performance of the cooler. This could be overcome by very careful assembly but the problem was exacerbated by the use of Tufnol for the displacer/regenerator flexure stack mount. It was found that the tapped holes in the Tufnol wore out after two or three assemblies and the component had to be re-made with helicoil inserts for the tapped holes. Tufnol was used because an insulator was needed to mount the two stationary DPPO rings, but future coolers that attempt to use a DPPO of this design should consider other materials for this component. One possible solution is a composite spring mount (see figure 12.4), this would provide a rigid mount for the spring stack (made from titanium) that could have a Tufnol component inserted within its inner diameter to provide the insulation for the stationary DPPO rings. The outer diameter of proposed titanium spring mount could be increased at the rear so that it was a close fit to the inner diameter of the cooler body. This would make it easy to locate the spring mount concentrically with the fixed cooler geometry as shown in figure 12.5. Of course, if the body was adapted as shown in figure 12.2 this may no longer be necessary.

### **Improving the electrical insulation of the flexure stacks**

The use of the flexures as electrical connections was successful although the method of insulating the flexure stacks could be improved. In the prototype rings of mica around the circumference of the flexures insulated them from the rest of the cooler (see figure 5.2). The rings were 0.5mm thick and were very fragile, often cracking during assembly which caused the motor coil to short circuit. The prototype was assembled and dis-assembled many times during this research effort and so this was a constant problem. An attempt was made to overcome this obstacle by making the flexure mounts from an electrically insulating material (Tufnol) but the flexibility of the material and its



**Fig. 12.2:** Suggested design modifications for a future cooler based on the prototype, including an MPPO based on the current DPPO.

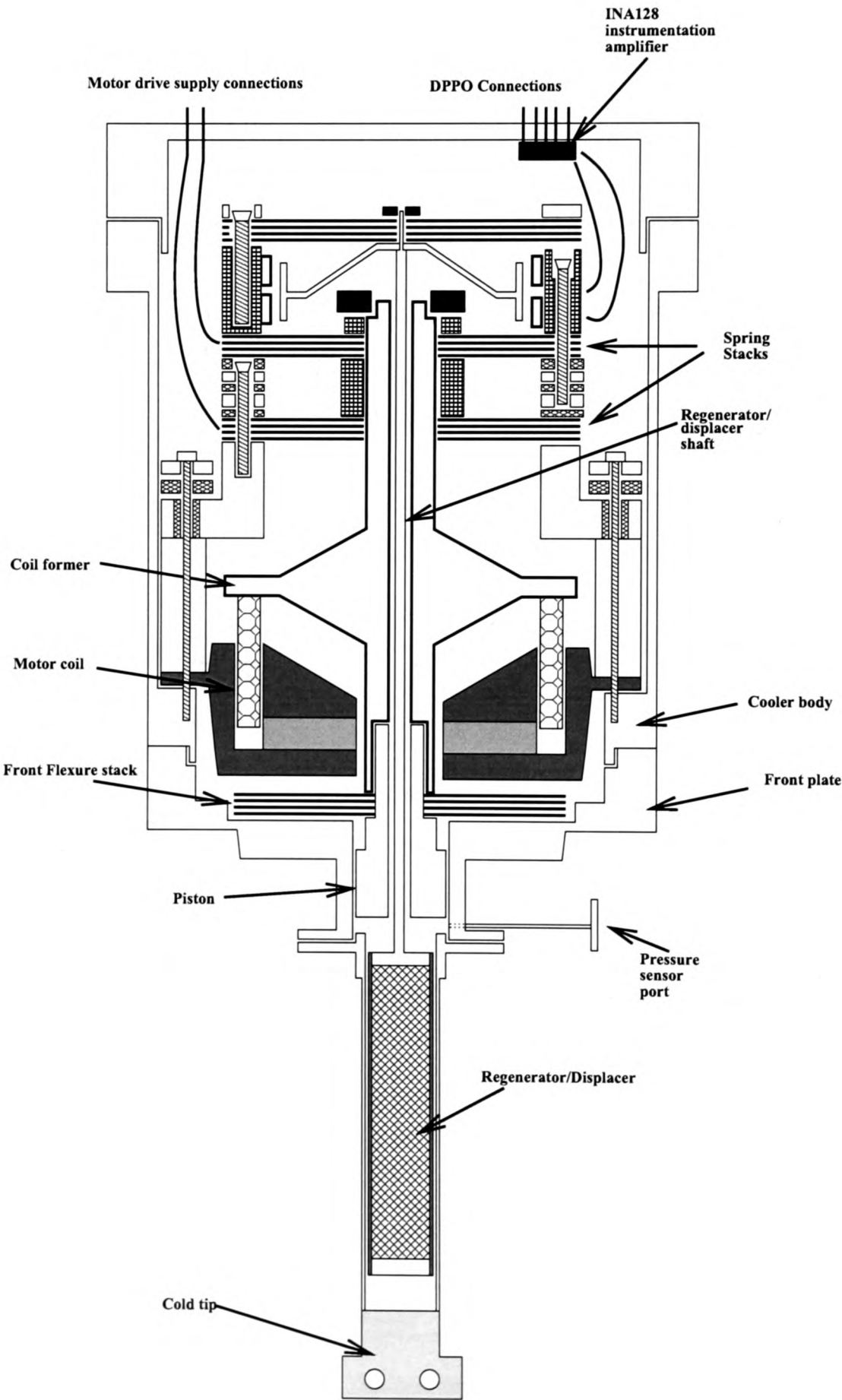
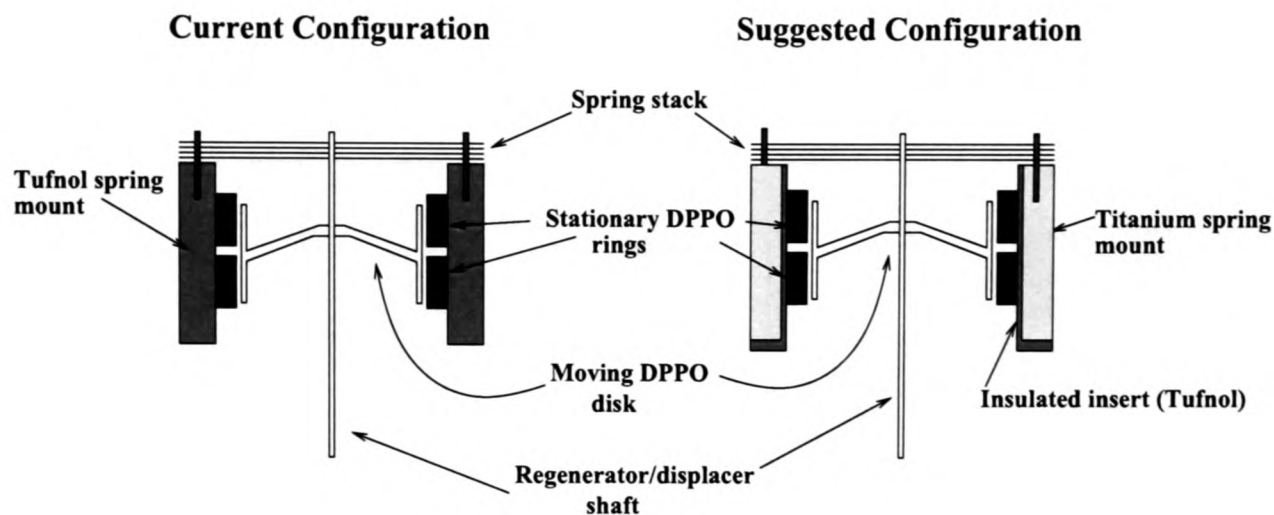


Fig. 12.3: Current design of the prototype.



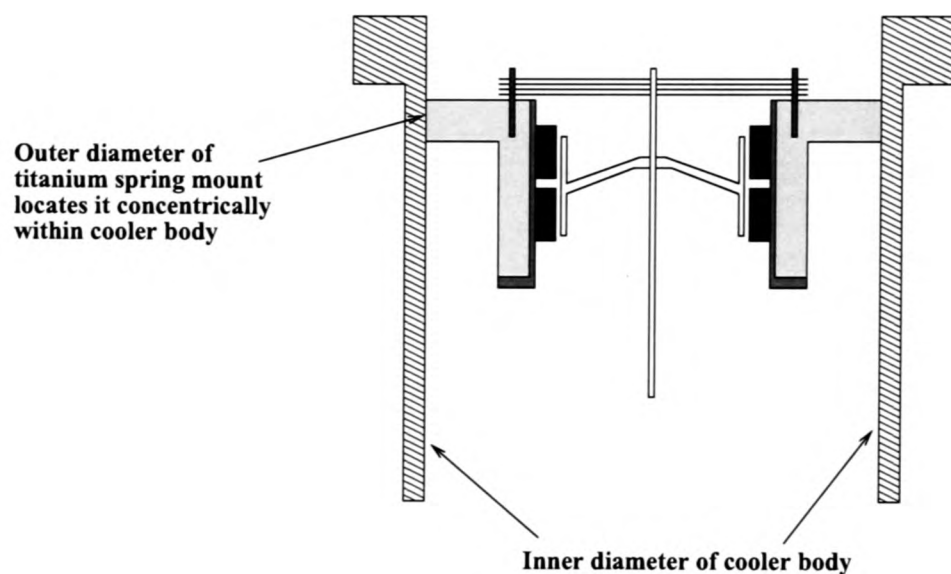
**Fig. 12.4:** Diagrams of the current DPPO configuration (left) and a suggested configuration for an improved version (right).

poor ability to retain threaded holes meant it was unsuccessful. Other suitable insulators should be explored in future designs if the flexures are still to be used as electrical feedthroughs.

#### 12.2.4 Further investigation of the thermal resistance

A significant problem with the cooler was the low value of thermal resistance between the compression space and cold tip. Calculations show that this is largely determined by the displacer/regenerator. When the warm end of the cooler was pre-cooled by the radiator the thermal resistance was seen to decrease. This was an unexpected result as in the past lower cold tip temperatures have been associated with an increase in thermal resistance (Kotsubo [Kotsubo, V. et al. (1992)] in Reed [Reed, J. (2004)]). Since cooling the warm end allows lower cold tip temperatures to be reached, one might expect the thermal resistance to increase with pre-cooling. The method used to determine the thermal resistance produced consistent results and is one that has been used before in cooler research ([Kotsubo, V. et al. (1992)], [Orlowska A. and Davey. G. (1987)]) and was therefore considered to be reliable. Further research into the cause of this decrease in thermal resistance could be very useful in determining ways to reduce the static heat load. During this research effort attempts to increase the thermal resistance by introducing thermal breaks into the regenerator were successful in reducing the static heat load but had a detrimental effect on the overall cooler's





**Fig. 12.5:** Diagram showing how an improved DPPO design could involve a feature that would aid concentricity during assembly (apart from the body the rest of the cooler is not shown).

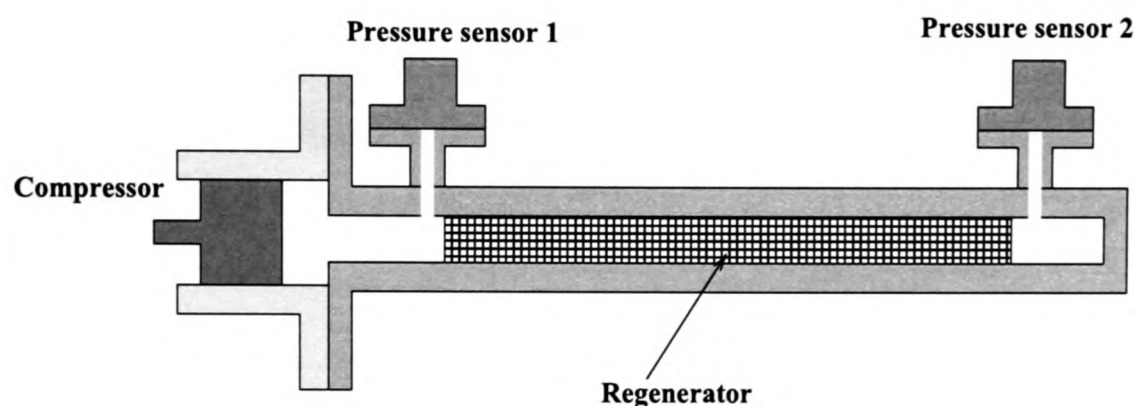
performance. There are two likely reasons for this:

1. The thermal breaks reduced the effectiveness of the regenerator.
2. The thermal breaks increased the pressure drop along the regenerator.

The latter of these two possibilities could be investigated fairly simply by designing and making a test apparatus similar to that shown in figure 12.6. By measuring the pressure swing with pressure sensors 1 and 2 for different regenerator configurations it would be possible to measure the effect on the pressure drop across the regenerator, not just of thermal breaks, but of different meshes, packing force etc.

Brito [Brito, M. (1999)] used a similar apparatus and found the thermal breaks had no measurable effect on the pressure drop along the regenerator, but this may not be true for the current regenerator as it is shorter, has a smaller diameter and uses different meshes to the regenerator used by Brito. Similar experimental arrangements have been used in the past to investigate pressure drops along cooler regenerators (e.g.[Choi, S. et al. (2004)], [Ju, Y. et al. (1998)], [Nam, K. Sangkwon, J. (2003)], [Zhao, T.S. and Cheng P. (1996)]).

The second possible explanation for the decrease in the cooler's performance when thermal breaks were used in the regenerator is that they had a detrimental effect on the regenerator's effectiveness. It is extremely difficult to measure such a loss in a cooler such as the prototype



**Fig. 12.6:** Regenerator pressure drop experimental set-up.

since it is one of a number of loss processes that only occur when the cooler is running; so called “dynamic losses” (see section 4.2). In the past methods have been devised to separate various contributions to the total dynamic heat loss, for example Peters [Peters, M. (1994)] describes methods for determining the various contributions to the total dynamic heat loss in a split Stirling cooler but these methods depend on being able to drive the displacer and compressor separately (in the cooler Peters was investigating the displacer was moved by a separate linear motor to the compressor piston unlike the current prototype wherein the regenerator is driven purely by the pneumatic force created by the compression and expansion of the working gas).

One possible dynamic loss that could be investigated with the current prototype is the compression loss, this is typically the largest dynamic loss process in Stirling coolers, Peters [Peters, M. (1994)] found that under some operating conditions it accounted for almost 60% of the net input power. By holding the displacer/regenerator stationary this could be done by replacing the displacer/regenerator flexures with rigid metal plates during these experiments and running the compressor refrigeration is prevented, so any power dissipated by the compressor is due to compression loss. To investigate the compression losses it would be necessary to have accurate piston position data over a wide range of operating conditions. It is clear that the current piston position sensing technique is not adequate as it can only detect the extremes of the piston’s motion over



a narrow band of frequencies close to the amplitude resonance and is poor at resolving the phase of the piston's motion. Furthermore a pressure sensor fitting that remains leak-tight during the cooled body runs would also be required.

Although the cooler did not attain its target performance of maintaining a cold tip temperature of 80K with an external heat load of 200mW, it has been shown that a hybrid radiator/cooler can produce significant cooling. With further work the performance of the cooler may be improved upon and the original target reached. The small size and low power consumption of the cooler would make it an attractive option for use on small remote sensing satellites or other small spacecraft.

# Appendix A

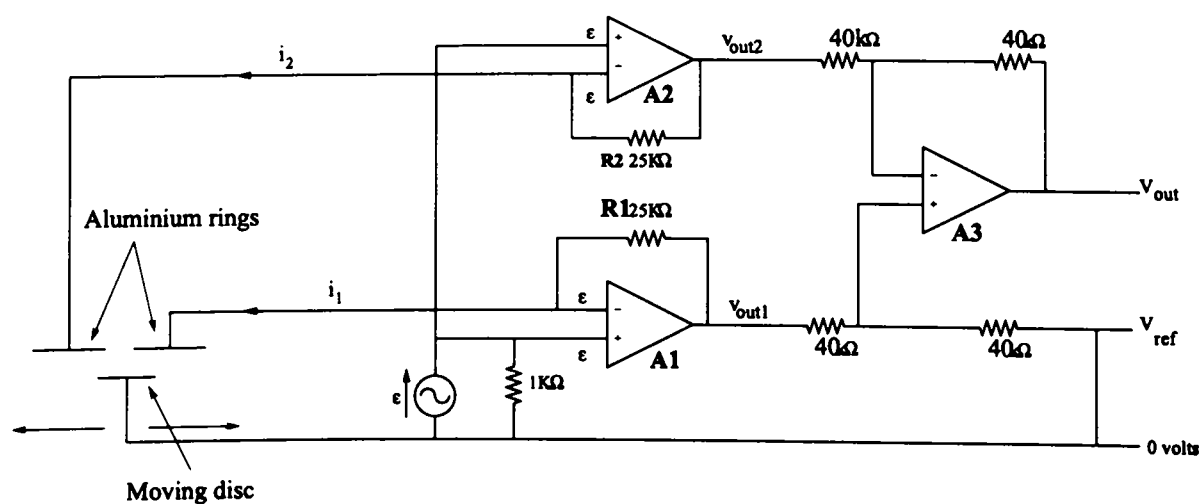
## DPPO analysis

Figure A.1 shows the circuit diagram for the DPPO (in fact this is only part of the circuit the rest is shown later in figure A.3) The sensor is excited with an emf,  $\epsilon$ , of frequency  $\omega_\epsilon$  (2 V P-P at 50 KHz). The current through the right-hand capacitor (which is comprised of the moving disc and the right-hand aluminium ring shown in figure 6.4) in figure A.1, is given by [Duffin, W.J.(1998)]:

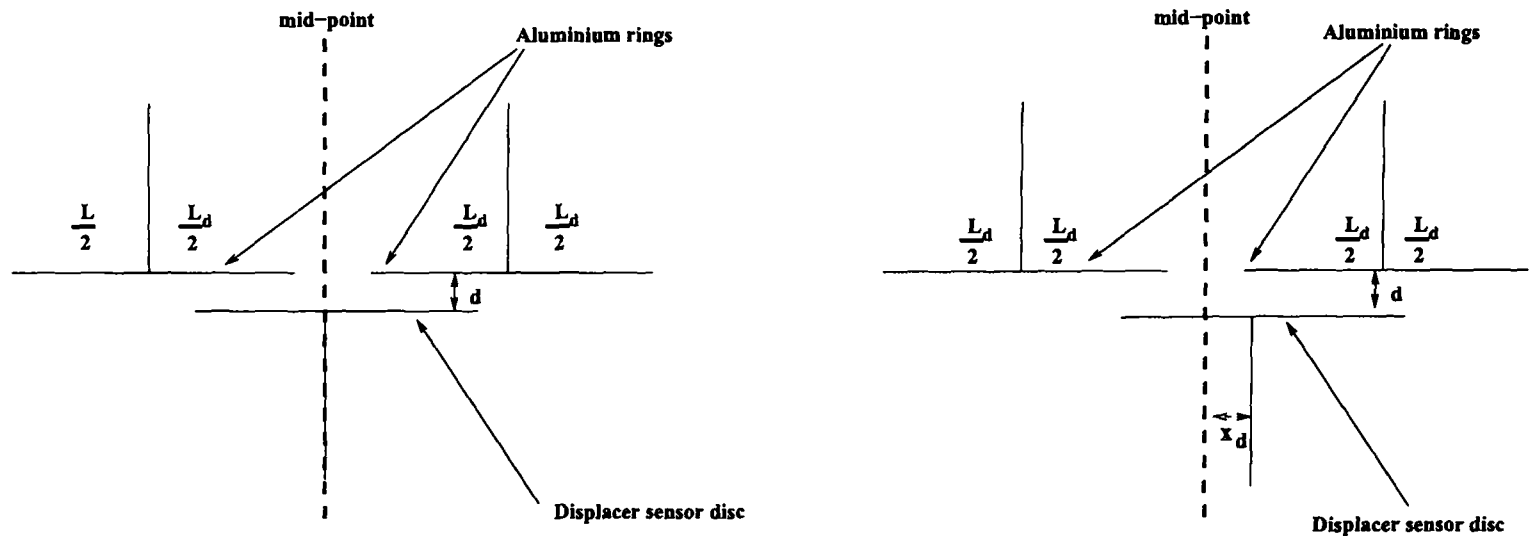
$$\epsilon = \frac{i}{j\omega c} \quad (\text{A.1})$$

If we consider the situation when the moving disc has moved to the right by a displacement  $x_d$  from its mid-point position (see figure A.2) then the capacitance between the moving disc and the right-hand stationary ring is given by:

$$c_{xd} = c_0 \left(1 + \frac{2x_d}{l_d}\right) \quad (\text{A.2})$$



**Fig. A.1:** Circuit diagram of the DPPO. The blue arrows represent the direction of movement of the DPPO disc.



**Fig. A.2:** Diagrams of the capacitive displacer position sensor at midpoint position (left diagram) and with displacer displaced by  $x_d$  to the right (right diagram).

Thus we can see that the voltage across the right-hand aluminum ring and the sensor disc will be:

$$\epsilon = \frac{i_1}{j\omega c_0(1 + \frac{2x_d}{l_d})} \quad (\text{A.3})$$

Similarly if the displacer/regenerator has moved by a displacement  $x_d$  to its left we would find the voltage across the left hand aluminum ring and the sensor disc will be:

$$\epsilon = \frac{i_2}{j\omega c_0(1 - \frac{2x_d}{l_d})} \quad (\text{A.4})$$

Where  $x_d$  is the magnitude of the moving disc's (and hence the displacer/regenerator itself) displacement. Re-arranging A.3 we find that the current through the right hand capacitor is given by:

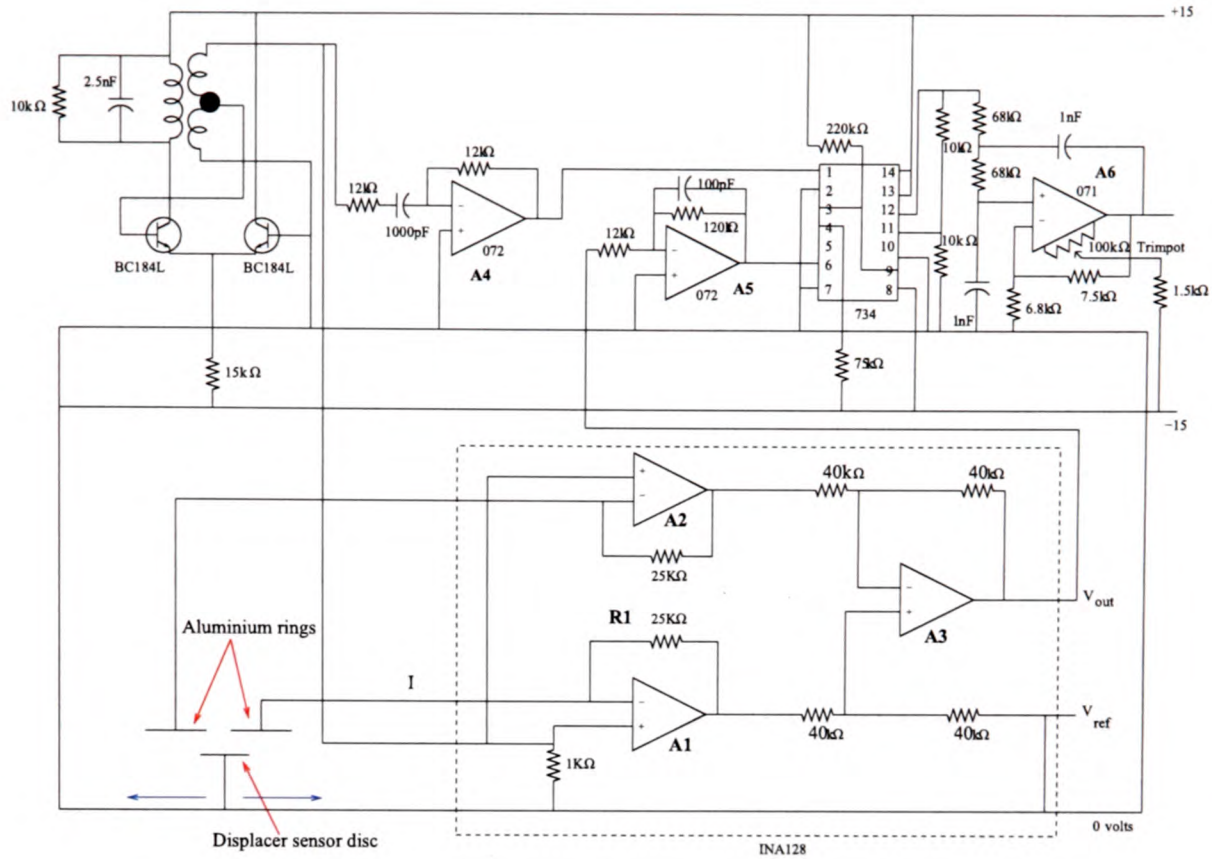
$$i_1 = \epsilon j\omega c_0(1 + \frac{2x_d}{l_d}) \quad (\text{A.5})$$

Similarly for the left hand capacitor we find:

$$i_2 = \epsilon j\omega c_0(1 - \frac{2x_d}{l_d}) \quad (\text{A.6})$$

If we now turn our attention to the op-amp labeled **A1** in A.1 we can find an expression for the output voltage when the DPPO disc is in this position. The amplifier “sees” a voltage at its non-inverting input and alters its output voltage (part of which is fed back to the inverting input) so as to make the voltage at the inverting input equal to that at the non-inverting one. Assuming that the inputs of the op-amp draw no current [Horowitz, P. and Hill, W. (1980)] we find that any current fed back from the output must flow through the resistor (**R1**) so we can write:

$$v_{out1} - \epsilon = i_1 R1 \quad (\text{A.7})$$



**Fig. A.3:** Circuit diagram of the complete DPPO incorporating the circuit shown in figure A.1.

Applying the same reasoning to amplifier **A2** we find:

$$v_{out2} - \epsilon = i_2 R_2 \quad (\text{A.8})$$

Since this current flows in the aluminum ring we can now substitute our expression for current, equation A.5, into equation A.7 to find:

$$v_{out1} - \epsilon = j\omega c_0 R_1 \left(1 + \frac{2x_d}{l_d}\right) v \quad (\text{A.9})$$

Re-arranging to give:

$$v_{out1} = \epsilon + j\omega c_0 R_1 \left(1 + \frac{2x_d}{l_d}\right) v \quad (\text{A.10})$$

Similarly  $v_{out2}$ , the output of op-amp **A2** in figure A.1 when the DPPO disc is displaced by  $x_d$  to the right is given by:

$$v_{out2} = \epsilon + j\omega c_0 R_1 \left(1 - \frac{2x_d}{l_d}\right) v \quad (\text{A.11})$$

Subtracting these two output voltages from each other we find that:

$$v_{out1} - v_{out2} = j\omega c_0 \epsilon R_1 \frac{4x_d}{l_d} \quad (\text{A.12})$$

The output from the circuit in figure A.1 is phase shifted by 90 degrees from the sensor excitation emf  $\epsilon$ . It remains to pass this signal to a phase sensitive rectifier to obtain a voltage proportional to the displacement of the displacer/regenerator. To use the excitation e.m.f. as the phase reference

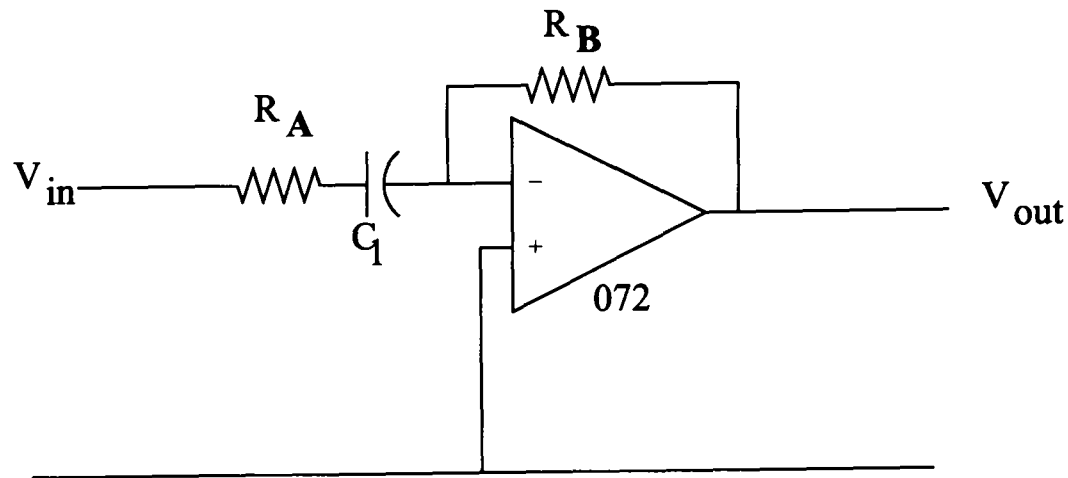


Fig. A.4: Op-amp A4 in figure A.3.

it must be shifted by  $\frac{\pi}{2}$ . The sensor excitation e.m.f output is also fed to an op-amp in series with a capacitor and resistor (the left-most op-amp labeled A4) in figure A.3 this produces a phase shifted output. We now consider how this phase shift can be removed.

Consider the diagram in figure A.4: The non-inverting input of the op-amp is at ground and so we can state:

$$0 - V_{out} = iR_B \quad (\text{A.13})$$

Re-arrange to get:

$$i = \frac{-V_{out}}{R_B} \quad (\text{A.14})$$

This current must flow through the resistor  $R_A$  and the capacitor  $c_1$ , this combination has an impedance given by:

$$z = \left( R_A + \frac{1}{j\omega c_1} \right) \quad (\text{A.15})$$

Now,  $V_{in}$  is given by the voltage drop across the series combination of  $C_1$  and  $R_A$  (since the voltage at the inverting input of the op-amp is at ground) so we can write:

$$V_{in} = i \left( R_A + \frac{1}{j\omega c_1} \right) \quad (\text{A.16})$$

Substituting equation A.16 into equation A.14 and re-arranging:

$$\frac{V_{out}}{V_{in}} = - \frac{R_B}{\left( R_A + \frac{1}{j\omega C_1} \right)} \quad (\text{A.17})$$

With a bit of algebra the following expression can be derived:

$$\frac{V_{out}}{V_{in}} = - \frac{j\omega c_1 R_B}{j\omega c_1 R_A + 1} \quad (\text{A.18})$$

The phase angle between the input and output voltages can easily be found, remembering that  $z = \mathbf{R} + j\mathbf{X}$  and thus  $\phi = \text{Arctan}\left(\frac{\mathbf{X}}{\mathbf{R}}\right)$  [Duffin, W.J.(1998)] we find that in the case where  $R_A = R_B$ :

$$\phi_1 = \text{Arctan}\left(\frac{1}{R_A \omega C_1}\right) \quad (\text{A.19})$$

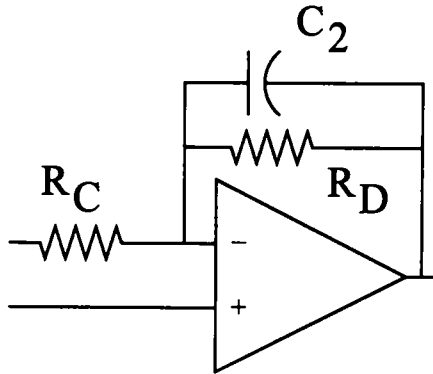


Fig. A.5: Op-amp A5 from figure A.3.

Where  $\phi_1$  is the phase shift between the input and output voltages of the first op-amp in figure A.3. Now consider the op-amp labelled **A5** in the circuit diagram of figure A.3, also shown figure A.5. We can state:

$$V_{in} = iR_C \quad (\text{A.20})$$

And:

$$\frac{-V_{out}}{Z} = \frac{V_{in}}{R_C} \quad (\text{A.21})$$

Where  $Z$  is the impedance of the parallel combination of the resistor  $R_D$  and the capacitor  $C_2$  given by  $\frac{-jR_D}{C_2\omega(R_D - \frac{j}{C_2\omega})}$ . From this an expression for the phase shift between the input and output voltages is derived:

$$\phi_2 = \text{Arctan}(-R_DC_2\omega) \quad (\text{A.22})$$

Where  $\phi_2$  is the phase shift between the input and output voltages of the second op-amp in figure A.3. The outputs of the two op-amps are multiplied together and if the resistors and capacitors,  $R_A, R_B, R_C, R_D, C_1$  and  $C_2$  are chosen so that  $R_DC_2\omega = R_AC_1\omega$  then we find that  $\phi_1 = -(90 - \phi_2)$  degrees. Thus on multiplying the outputs of the op-amps together the 90 degree phase shift in the sensor signal (from the oscillator signal) is removed. The output from the multiplier chip is then passed through a 2 pole Chebyshev filter to produce the output of the DPPO.

## Appendix B

# Effect of non-negligible inductance on resonance

Equation 4.18 repeated here as equation B.1 assumes that the inductance of the motor coil was negligible, however it was later discovered that this was not true.

$$x_p = \frac{Bl_b \epsilon_{v0} e^{j\omega t}}{m_p R_c} \frac{1}{\left[ -\omega^2 + \frac{j\omega}{m_p} \left( \lambda_{sl} + \frac{B^2 l_b^2}{R_c} \right) - s_{pp} - \Psi s_{pd} e^{j\phi} \right]} \quad (\text{B.1})$$

Thus at resonance:  $-\omega^2 - s_{pp} + \text{Re}[\Psi s_{pd} e^{j\phi}] = 0$ . From which was derived (see equations 4.61 to 4.64):

$$\omega = \sqrt{\left[ \frac{k_p}{m_p} + \frac{\alpha(A_p - A_R)^2 P_{mean}}{q T_c m_p} \right]} \quad (\text{B.2})$$

We now consider how the non negligible inductance of the coil alters these equations:

$$x_p = \frac{Bl_b \epsilon_{v0} e^{j\omega t}}{m_p (R_c + j\omega L)} \frac{1}{\left[ -\omega^2 + \frac{j\omega}{m_p} \left( \lambda_{sl} + \frac{B^2 l_b^2}{(R_c + j\omega L)} \right) - s_{pp} - \Psi s_{pd} e^{j\phi} \right]} \quad (\text{B.3})$$

And at resonance:

$$-\omega^2 - s_{pp} + \text{Re}[\Psi s_{pd} e^{j\phi}] + \text{Re}\left[\frac{j\omega}{m_p} \left( \lambda_{sl} + \frac{B^2 l_b^2}{R_c + j\omega L} \right)\right] = 0 \quad (\text{B.4})$$

Where:

$$\text{Re}\left[\frac{j\omega}{m_p} \left( \lambda_{sl} + \frac{B^2 l_b^2}{R_c + j\omega L} \right)\right] = \frac{1}{m_p} \frac{\omega^2 B^2 l_b^2 L}{R_c^2 + \omega^2 L^2} \quad (\text{B.5})$$

And so equation B.2 becomes:

$$\omega = \sqrt{\left[ \frac{k_p}{m_p} + \frac{\alpha(A_p - A_R)^2 P_{mean}}{q T_c m_p} + \frac{1}{m_p} \frac{\omega^2 B^2 l_b^2 L}{R_c^2 + \omega^2 L^2} \right]} \quad (\text{B.6})$$



This was then solved for  $\omega$  (using the Mathematica computer algebra package). The inductance of the coil varies depending on the degree of immersion in the magnet assembly and with frequency, therefore  $L$  will not be a constant in equation B.6 so there is significant uncertainty about the validity of this model. However as an exercise, a typical value of the coil's inductance was chosen ( $8mH$ ) and a fit made to the experimental data using equation B.6. This resulted in values of  $\alpha = 1.02$  and  $k = 4266.1$ , suggesting that the compression of the working gas was almost isothermal which contrasts with the original fit (which neglected inductance) in Chapter 7 which suggested the compression was somewhere between isothermal and adiabatic. The value of the mechanical spring constant  $k$  produced from the fit is approximately 9% lower than the value calculated from the measured value of the compressor flexures and this is difficult to interpret. It may well arise from value of inductance that was chosen in this fit, as selecting lower values lead to fitted values of  $k$  that approach the calculated value of the mechanical spring constant. It has been noted [Reed, J. (2004)] that models of the compressor similar to the one used here are an oversimplification and the value of  $\alpha$  should not be relied upon as an indicator that the compression space is isothermal. Its value in this research effort was in the computer model where it could be used to predict how the resonant frequency of the compressor would change with filling pressure and the number of compressor springs installed.



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