

Numerical Modelling of Laterally Loaded Piles for Offshore Wind Turbines



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A thesis submitted for the degree of
Doctor of Philosophy
Trinity 2017

Acknowledgements

First and foremost, I would like to thank my supervisors Prof Harvey Burd and Prof Guy Houlsby for their support and guidance throughout my period in Oxford. They have provided me generously with advice in many meetings and supported my personal and professional development.

My DPhil was supported by the PISA project, funded by the UK Department for Energy and Climate Change and the PISA Industry Partners through the Carbon Trust led Offshore Wind Accelerator, involving DONG Energy, Alstom Wind, E.ON, EDF Energies Nouvelles, Iberdrola, innogy, SSE, Statkraft, Statoil, Van Oord and Vattenfall. The project was successfully led by the principal investigator Prof Byron Byrne. I would like to thank him for giving me the chance to contribute to this project and for the input he had in various stages of my doctorate. Dr Ross McAdam played a key role in the project as lead research assistant, his insights from the field tests have helped greatly in analysing the large amount of data. More generally I would like to acknowledge the many technical discussions through the PISA project with the academic and industry participants.

I would also like to thank the British Geotechnical Association for awarding me the Cooling prize in my first year which, amongst other things, supported me to present at the European Conference on Soil Mechanics and Geotechnical Engineering in Edinburgh.

Throughout my period in Oxford, I have been actively involved in the student life at St. Catherine's College. I will cherish the memories and the friends I made during this period. In particular I would like to thank Julian Malisano, who I had the pleasure of being co-president with for the Middle Common Room. The travels across Europe with Christoph Dorn and Serkan Birgel brought refreshing breaks from work. I have been active in the college rowing club, rowing in college bumps races, British rowing competitions and with a highlight of competing in Japan for the Oxford Shield Regatta. I would like to thank the many people in the boat club who made all this possible. St. Catherine's College has generously offered me the Light Senior Scholarship which gave me the chance to dine at the high table at many occasions.

I have also been active in the Oxford University Belgo-Luxembourgish Society. As president I had the chance of organising the Annual Black Tie Banquet with honorary guests including the Belgian Ambassador and Flemish Representative to the UK. I would like to thank the friends I worked with to organise many events for the society.

And finally but immensely, I would like to thank my family for their support throughout this period.

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The offshore wind market has grown rapidly over recent years. The most widely used foundation type for offshore wind turbines is the monopile. Current design guidance for monopile foundations dates back to the 1950s and 1960s and was originally developed for the oil and gas industry, where both the pile dimensions as well as the design load conditions are different than for offshore wind.

The PISA project was aimed at investigating the behaviour of monopile foundations and reducing the conservatism in design. The project used state of the art numerical modelling, which was validated through a field testing campaign. The numerical modelling was focused on capturing the monotonic response of the foundation to failure. The field testing provided valuable additional data for the rate effects and the cyclic behaviour of monopile foundations.

Data analysis methods are presented to accurately interpret the foundation response. The data analysis is focused on capturing the ground level foundation response as well as the pile behaviour below ground. Additionally, the cyclic behaviour is analysed in detail. These results are used as a basis for the development of numerical models for capturing the observed behaviour in the field tests.

This thesis outlines plasticity models which are integrated in a generalised Winkler model. The model development makes use of the Hyperplasticity framework. These numerical models include: (1) the kinematic hardening model to capture plastic unloading; (2) the rate effect model to capture increased foundation capacity with increasing load rate; (3) the ratcheting model to capture accumulated rotation under cyclic loading; (4) the combined rate and ratcheting model and (5) the gapping model to capture gapping on the active side of the pile. Each of the models is calibrated to the PISA field tests illustrating the capabilities and limitations of the models for capturing the field test response.

Finally, the kinematic hardening model is integrated in software to calculate the dynamic behaviour of an operating wind turbine. Integrating more accurate soil-structure interaction models could lead to improved predictions of the turbine behaviour and reduce the cost of monopiles.

Contents

1	Introduction	1
1.1	Background	1
1.2	Problem definition	4
1.3	Current foundation design method	5
1.4	Thesis outline	9
2	The PISA project	11
2.1	Site descriptions and characterisation	13
2.1.1	Cowden	14
2.1.2	Dunkirk	16
2.2	Test outline	20
2.2.1	Cowden	21
2.2.2	Dunkirk	22
2.3	PISA soil reaction curves	25
2.3.1	Normalised variables	26
2.3.2	Mathematical expression for the soil reaction curves	27
2.3.3	Parameters for the soil reaction curves	29
2.4	PISA beyond monotonic	31
2.5	Conclusions	35
3	Data analysis	36
3.1	Test setup	37
3.2	Analysis tools	39
3.2.1	Calculation of the ground level response	39
3.2.2	Calculation of the embedded pile response	49
3.3	Cyclic loading analysis	58
3.3.1	Work and dissipated energy	58
3.3.2	Foundation stiffness and energy dissipation	60

3.3.3	Accumulated pile rotation	61
3.4	Foundation behaviour	62
3.4.1	Monotonic field response	62
3.4.2	Rate effects	64
3.4.3	Behaviour under 1-way cyclic loading	69
3.4.4	Behaviour under 2-way cyclic loading	80
3.5	Conclusion	88
4	Foundation models	89
4.1	Finite element model of Winkler foundation	90
4.1.1	Virtual work equations	92
4.1.2	Displacement interpolation for the beam and soil elements . .	94
4.1.3	Finite element analysis	95
4.1.4	Non-linear solution scheme	97
4.2	Hyperplasticity framework	100
4.2.1	Outline	100
4.2.2	Single yield surface model	103
4.3	Kinematic hardening model	107
4.3.1	Masing rules	107
4.3.2	Constitutive model	109
4.3.3	Numerical implementation	110
4.3.4	Model output	112
4.4	Rate effect model	114
4.4.1	Rate process theory	115
4.4.2	Constitutive model	118
4.4.3	Numerical implementation	120
4.4.4	Model output	121
4.5	Ratcheting model	123
4.5.1	Development of ratcheting models	123
4.5.2	Constitutive model	124
4.5.3	Numerical implementation	126
4.5.4	Calibration of the strength and stiffness parameters	128
4.5.5	Model output	128
4.6	Combined rate and ratcheting model	131
4.6.1	Constitutive model	131
4.6.2	Numerical implementation	132

4.6.3	Model output	134
4.7	Gapping model	135
4.7.1	Generalised Winkler model	135
4.7.2	Constitutive model	137
4.7.3	Numerical implementation	139
4.7.4	Model output	141
4.7.5	Model extensions	143
4.8	Conclusions	145
5	Calibration and results	146
5.1	Introduction	146
5.2	Recalibration of the soil reaction curves	147
5.3	Kinematic hardening model	150
5.4	Rate effect model	153
5.4.1	Rate effects in Cowden	153
5.4.2	Rate effects in Dunkirk	157
5.5	Ratcheting model	160
5.5.1	Formulation of the ratcheting strain	160
5.5.2	Ratcheting in Cowden	169
5.5.3	Ratcheting in Dunkirk	173
5.6	Combined rate and ratcheting model	176
5.7	Gapping model	179
5.7.1	Calibration of the gapping element	179
5.7.2	Gapping in Cowden	184
5.7.3	Gapping in Dunkirk	185
5.8	Conclusion	191
6	Integration into commercial software	193
6.1	2D soil reaction models	195
6.1.1	Series and parallel model comparison in 2D	195
6.1.2	Analytical integration of a 2D yield surface	198
6.1.3	Extension to 2D for other models	200
6.2	Integration in HAWC2	203
6.2.1	Finite element interpolation and integration	203
6.2.2	Interface between HAWC2 and Matlab	205
6.3	Offshore wind turbine simulation	207
6.3.1	Backbone curve of the soil reaction	207

6.3.2	Soil reaction under cyclic loading	208
6.3.3	Rotor stop simulation	209
6.4	Discussion	212
6.5	Conclusion	215
7	Conclusions	216
7.1	Data analysis	217
7.2	Model development	218
7.3	Calibration	219
7.4	Integration into commercial software	220
7.5	Recommendations for future work	220
7.6	Summary	223
	Bibliography	224

Nomenclature

Common variables

Greek alphabet

α	[-]	Strain-type hardening variable (hyperplasticity)
	[-]	Roughness pile-soil interface
$\boldsymbol{\alpha}, \mathbf{A}$		Vector/matrix of internal hardening variables
β	[N/m ²]	Work-type hardening variable (hyperplasticity)
γ	[N/m ³]	Soil unit weight
	[-]	Shear strain
δ	[-]	Logarithmic decrement
ϵ	[-]	Strain
ζ_b	[-]	Cyclic load magnitude parameter
ζ_c	[-]	Cyclic load shape parameter
η	[-]	Local coordinate (finite element formulation)
	[-]	Damping parameter (viscosity in case of linear damper)
η_{10}	[-]	Logarithmic (10 log) damping parameter
θ	[rad]	Overall pile rotation
κ	[-]	Shear coefficient
λ	[m ²]	Shear factor ($EI/GA\kappa$)
Λ	[-]	Lagrange multiplier (hyperplasticity)
μ	[-]	Friction coefficient
ν	[-]	Poisson's coefficient
ξ	[-]	Damping ratio
ρ	[kg/m ³]	Density
σ	[N/m ²]	Stress
ϕ	[rad]	Angle in the x-y plane (gapping model)
	[rad]	Friction angle
χ	[N/m ²]	Dissipative generalised stress
$\bar{\chi}$	[N/m ²]	Generalised stress
ψ	[rad]	Cross-section pile rotation
ω	[rad/s]	Angular frequency

Latin alphabet

$a-b$		Optimisation parameters for the ground level response
$a-c$		Quadratic equation parameters
$a-f$		Optimisation parameters for the below ground response
A	$[m^2]$	Cross section area
	$[-]$	Correction factor (calibration / current design method)
A		Matrix of boundary conditions
b		Vector of boundary conditions
B		Backbone curve
c	$[Ns/m]$	Dashpot value
		Constraint equation (hyperplasticity)
C		Matrix of constraint equations
C_1 to C_3	$[-]$	Coefficients for the soil reaction (current design method)
d	$[N/m^2s]$	Dissipation potential (hyperplasticity)
d		Vector of constraint equations
D	$[m]$	Pile Diameter
E	$[N/m^2]$	Young's modulus / tangent stiffness
	$[Nm]$	Energy (Cyclic analysis)
f	$[1/s]$	Cycle frequency
	$[N/m^2]$	Helmholz free energy (hyperplasticity)
f		Force vector (finite element formulation)
G	$[N/m^2]$	Shear modulus
h	$[m]$	Stick-up height / height of load application
H	$[N]$	Horizontal force
	$[N/m^2]$	Hardening modulus (hyperplasticity)
i or j	$[-]$	Index
I	$[m^4]$	Second moment of area
I_R	$[-]$	Rigidity index
I_S	$[-]$	Sand stiffness coefficient
J	$[-]$	Jacobian (finite element formulation)
k		Stiffness (units depend on which stiffness is considered)
	$[N/m^3]$	Initial modulus of soil reaction (current design method)
	$[N/m^2]$	Slider strength (hyperplasticity)
K_0	$[-]$	Earth pressure coefficient
K		Stiffness matrix
L	$[m]$	Embedded pile length
L_e	$[m]$	Element length (finite element formulation)
m	$[N]$	Distributed moment
	$[-]$	Power coefficient
M	$[Nm]$	Moment
n	$[-]$	Curvature
	$[-]$	Number/amount
N	$[-]$	Cycle number
	$[-]$	Shape function (finite element formulation)

N_n, N_t, N_{σ_0}	[-]	Normalised normal strength, tangential strength and initial stress coefficients
p	[N/m]	Distributed lateral load
p'	[N/m ²]	Mean effective stress
p_w	[N/m ²]	Pore water pressure
R	[-]	Ratcheting coupling parameter (hyperplasticity)
	[m]	Pile radius
S	[N]	Shear force
s_u	[N/m ²]	Undrained shear strength
t	[m]	Wall thickness
	[s]	Time
T	[s]	Cycle period (cyclic analysis)
T_b	[-]	Cyclic accumulation function of ζ_b
T_c	[-]	Cyclic accumulation function of ζ_c
$\mathbf{u}$		Displacement vector (finite element formulation)
V_w	[m/s]	Wind speed
w	[-]	Weight factor
W	[Nm]	Work
u, v, w	[m]	Displacement in the respective x, y and z direction
$\mathbf{x}$		Vector of unknown parameters
y	[-]	Yield parameter
z	[N/m ² s]	Force potential (hyperplasticity)
x, y, z	[m]	Coordinate system
z_i	[m]	Interface depth

Common subscripts, superscripts and diacritics

X_0	Initial Pile neutral axis
X_1	Updated / refers to the end of the step
$X_{0.1D}$	Value at pile failure displacement of $0.1D$
$X^{(1)}, X^{(2)}$	With respect to the first or second node (finite element formulation)
X_b	Boundary elements
X_B	Pile base
X_d	Damping
X_g	Gapping Gauss point
X_G	Ground level
X_h	Hardening
X_h	Horizontal
X_{HB}	Base shear
X_m	Distributed moment
X_{MB}	Base moment

X_n	Normal
X_p	Lateral distributed load
X_r	Ratcheting
	Reversal (Masing rules)
X_s	Soil
	Spring-slider element (hyperplasticity)
X_t	Tangential
X_T	Pile top
X_u	Ultimate
X_v	With respect to the load-displacement curve
X_v	Vertical
X_x, X_y, X_z	In the respective x, y and z direction
X^y	Yielding
X_ψ	With respect to the moment-rotation curve
X_{acc}	Accumulated
X_{aim}	Aim / target
X^{bound}	Boundary
X_{diss}	Dissipated
X_{el}	Elastic
X_{ext}	External
X_{eq}	Equivalent
X_{int}	Internal
X_{loc}	Local
X_{max}	Maximum
X_{min}	Minimum
X_{qs}	Quasi-static
X_{tot}	Total
X^{trial}	Trial stress
$\dot{X}$	Time derivative
X'	Effective / submerged
	Spatial derivative
X^*	Corrected
	Temporary variable (algorithm notation)
	Augmented potential (hyperplasticity)
$\bar{X}$	Normalised
$\mathbf{X}$	Vector or matrix
$\mathbf{X}^T$	Transpose of vector or matrix
$\mathbf{X}_s$	Soil vector or matrix
$\mathbf{X}_p$	Pile vector or matrix
$\mathbf{X}^e$	Element vector or matrix
$\mathbf{X}^w$	Weighted vector or matrix
$\mathbf{X}^*$	Reduced vector or matrix

Abbreviations

Field test piles

CL1-2	Cowden large diameter piles
CM1-9	Cowden mid-diameter piles
CR1	Cowden reaction pile
CS1-4	Cowden small diameter piles
DL1-2	Dunkirk large diameter piles
DM1-9	Dunkirk mid-diameter piles
DR1	Dunkirk reaction pile
DS1-4	Dunkirk small diameter piles

Pile instrumentation

AH	Active high LVDT location
AL	Active low LVDT location
FOS	Fibre optic strain gauge
IPE	In place extensometer
IPI	In place inclinometer
LCL	Load cell
LVDT	Displacement transducer
PH	Passive high LVDT location
PL	Passive low LVDT location
PZT	Piezometer
SUI	Stick-up inclinometer

Other

1D, 2D, 3D	One, two and three dimensional
AWG	Academic Work Group
BRE	Building Research Establishment
CDM	Current Design Method
CPT	Cone Penetration Test
FE	Finite Element
ICL	Imperial College London
KHM	Kinematic Hardening Model
PISA	Pile Soil Analysis
SCPT	Seismic Cone Penetration Test
TXC	Triaxial Compression

Functions

$$S(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ \in [-1, 1] & \text{if } x = 0 \end{cases} \quad \text{Generalised signum function}$$

$$< x > = \begin{cases} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases} \quad \text{Macaulay brackets}$$

$$E(\phi, x) = \int_0^\phi \sqrt{1 - x^2 \sin^2 \theta} d\theta \quad \text{Elliptic integral of the second kind}$$

$$E(x) = E\left(\frac{\pi}{2}, x\right) \quad \text{Complete elliptic integral}$$

$$I_C(x) = \begin{cases} 0 & \text{if } x \in C \\ +\infty & \text{if } x \notin C \end{cases} \quad \text{Indicator function of a set } C$$

$$N_C(x) = \begin{cases} \{0\} & \text{if } x \in \text{int } C \\ \{y | (x - x')y \geq 0, \forall x' \in C\} & \text{if } x \in \text{bdy } C \\ \emptyset & \text{if } x \notin C \end{cases} \quad \text{Normal cone of a set } C$$

Chapter 1

Introduction

1.1 Background

A world-wide commitment to sustainable development, dating back to the Brundtland Commission of 1987 (WCED, 1987), and more recently the need to address the effects of climate change, has focused the attention of many developed countries on seeking out and exploiting renewable energy resources. As a consequence, offshore wind energy is becoming one of the major alternative energy sources in Europe. The UK, Germany and Denmark have taken the lead in this field, with 1,472 operational offshore wind turbines installed off the UK's coast, 947 off Germany's coast and 517 off Denmark's coast at the end of 2016 (WindEurope, 2017). Figure 1.1 gives an overview of the annual and cumulative capacity of offshore wind power connected to the grid, illustrating the growth of the sector in recent years.

EWEA (2015) estimates that by 2030 there will be more than 11,000 offshore wind turbines installed across Europe. This growth in the sector would lead to the creation of more than 180,000 jobs and a total investment of over £100bn. Activity is also increasing in the USA, Japan and China. Technological advances in the last few years have meant that increasingly large and more efficient wind turbines are being

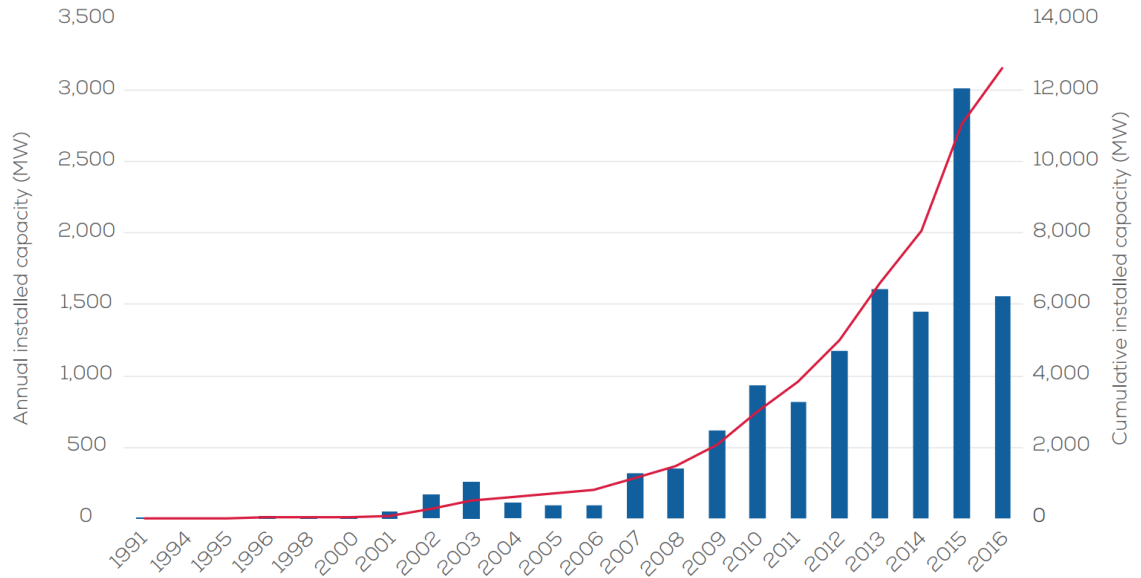


Figure 1.1: Annual (blue) and cumulative (red) MW of offshore wind turbines connected to the grid in Europe (WindEurope, 2017).

installed (Figure 1.2).

Offshore wind energy has several advantages over onshore wind: at sea, the wind blows more steadily and the average wind speed is higher, making turbines more efficient. Moreover, since these wind turbines do not disturb local communities and their environment (the so-called NIMBY effect), larger wind turbines – and more of them – can be installed. The additional installation and maintenance costs of construction offshore need to be minimised to make offshore wind financially viable. This means improving design methods and is one of the drivers of this research.

The foundation can account for up to 35% of the total wind turbine installation cost (Byrne and Houlsby, 2003). The current design guidelines for foundations are considered to be overly conservative and so lead to sub-optimal design. By improving current design methods, potential savings for monopile foundations are estimated to be as much as 25% (Kallehave et al., 2015), leading to significant savings in the total project cost.

Figure 1.3 shows the range of foundation and support structure concepts for off-



Figure 1.2: Evolution of the turbine size since 1991 (DONG Energy, 2016).

shore wind turbines. In waters up to 35m deep, the monopile foundation – a hollow steel tube driven into the soil – is the preferred option, accounting for over 80% of currently installed turbine structures (WindEurope, 2017). The tower and turbine are installed on top of the monopile. Other foundation methods are also being installed (*e.g.* Figure 1.3: gravity based foundations, tripods, jackets, *etc.*) and each has different advantages and disadvantages: installability (*e.g.* pile drivability), costs, construction limitations, geotechnical stability, and so on. However, for most wind farm designs, these foundation methods are less economical, resulting in a comparatively low market share of 7.5% for gravity based foundations, 6.6% for jacket structures and 3.3% for tripods (WindEurope, 2017).

The larger offshore wind turbines need larger foundations. Early designs were based on monopiles with a diameter in the region of 4m. In more recent installations the typical diameter is in the region of 6m; future designs are likely to be based on monopiles with diameters of up to 10m or even more (Kallehave et al., 2015).

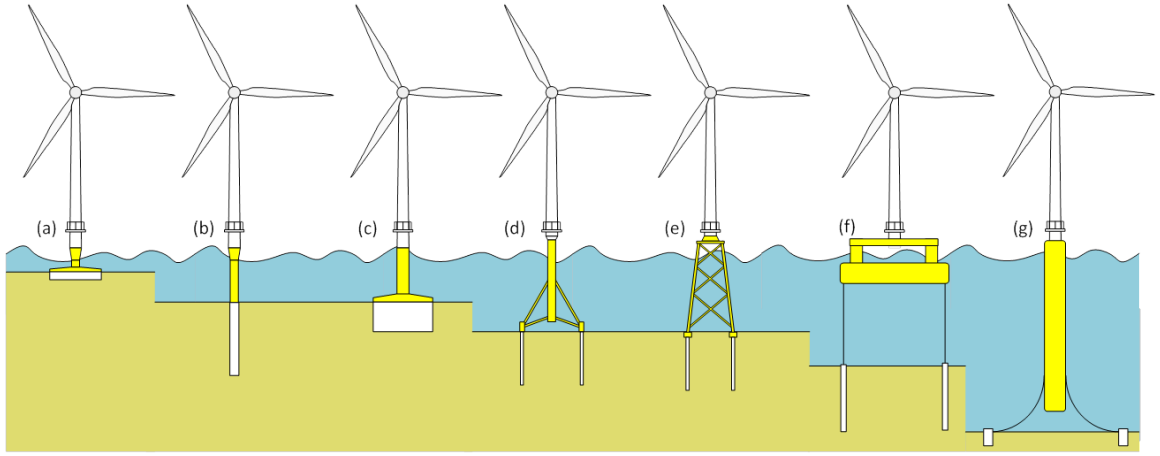


Figure 1.3: Support structures for offshore wind turbines. (a) Gravity based foundation, (b) monopile, (c) suction caisson, (d) tripod, (e) jacket, (f) tension leg platform and (g) ballast stabilised spar buoy (Houlsby and Byrne, private communication).

1.2 Problem definition

Foundations for offshore wind turbines are subjected to substantial overturning moments from the action of wind on the turbine and tower, as well as wave and current loading on the substructure. A schematic illustration of the design problem is shown in Figure 1.4. This thesis focuses on the behaviour of a monopile under the lateral loads acting on the structure.

The design cases for the foundation include: ultimate capacity of the foundation under storm loading conditions, accumulated rotation of the foundation after a lifetime of load cycles and steel fatigue in the monopile, where the foundation stiffness and damping are important design drivers. In these design cases, the reaction of the foundation can be analysed as quasi-static and independent of the dynamics of the superstructure. This thesis describes numerical models to capture the foundation behaviour for these design scenarios.

There are significant potential savings for the structural design of offshore wind turbines by using an integrated design approach, where the foundation design is optimised at the same time as the superstructure design (Hagi, 2012). This thesis also

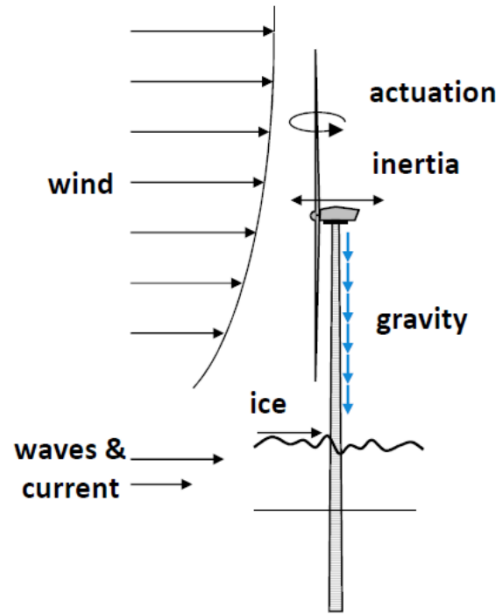


Figure 1.4: Loads acting on an offshore wind turbine (Segeren and De Vries, 2013).

sets out an integrated design approach, where the soil-structure interaction models can be integrated into software for simulating the dynamic behaviour of offshore wind turbines.

1.3 Current foundation design method

The design of a monopile foundation is commonly performed using the so-called ‘ $p-y$ method’. This method is based on a generalised Winkler foundation model, where beam elements representing the foundation pile are supported by non-linear lateral springs that represent the soil reaction. In the Winkler approach, the lateral reaction p of each of the lateral springs depends only on the local pile displacement v . Although the lateral springs can be calibrated to capture the soil reaction on a beam, this restriction remains one of the main limitations of the model. The method is illustrated in Figure 1.5, where the lateral displacement in the y -direction is denoted v rather than y as in the original $p-y$ method. H_G and M_G are the ground level lateral load and overturning moment.

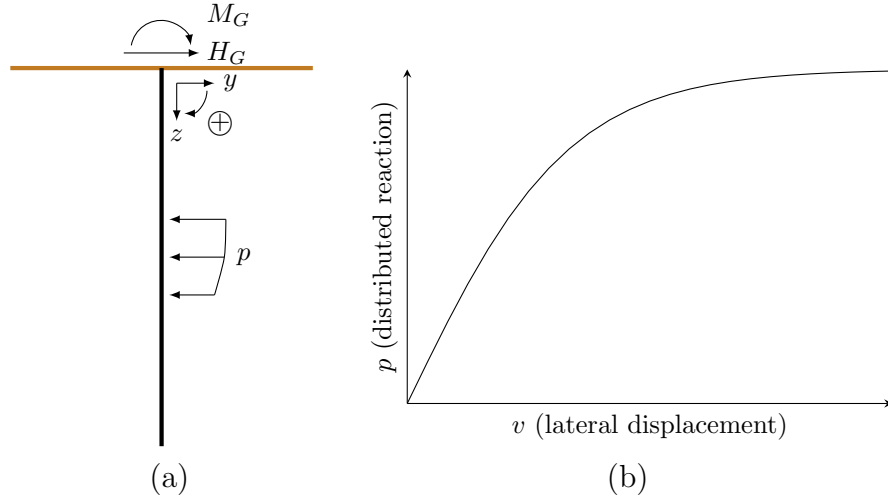


Figure 1.5: (a) The $p - y$ modelling approach, and (b) a typical $p - y$ curve for an element of the pile.

Historically, the $p - y$ curves were developed for the design of foundation piles in the oil and gas industry dating back to the 1950s and 1960s. The method for laterally loaded piles in offshore design guidelines (API, 2010; DNV, 2014) refers to publications of Matlock (1970), Reese et al. (1975) and O'Neill and Murchison (1983), who developed the $p - y$ curves for soft clay, stiff clay and sand respectively. The field tests for calibrating these soil reaction curves were typically carried out with foundation pile diameters around 0.6m and a length to diameter ratio (L/D) of over 30. This high L/D ratio typically results in flexible behaviour of the foundation pile. The main design consideration for oil and gas structures is to avoid ultimate failure of the foundation, though design considerations have been put in place for cyclic events with a limited cycle number, which would typically occur during storm loading conditions. The evaluation of the cyclic response in API (2010); DNV (2014) uses a decreased ultimate resistance with respect to the static $p - y$ curves to evaluate the maximum pile displacement or rotation under cyclic loading.

There are considerable differences between foundation piles for oil and gas structures and monopiles for offshore wind turbines, both in terms of pile dimensions and design loading conditions. Monopile diameters have grown increasingly large and the

L/D ratio has been decreasing for these large foundation piles, resulting in less flexible behaviour. These differences raise questions as to whether the $p - y$ method can be used for monopile foundations. There are three specific areas of concern for the application of the $p - y$ method for the design of monopile foundations (Byrne et al., 2017):

1. Definition and calibration of the $p - y$ curves

The current definition of the $p - y$ curves is based on generic formulations, which in turn depend on a limited set of soil parameters. There are questions relating to how these parameters should be identified, whether these parameters are sufficient to define the soil response, and the applicability of the generic formulations to the increasingly large diameter foundation piles employed for wind turbine structures. Advanced soil testing techniques, both *in situ* (*e.g.* seismic cone tests) and in the laboratory (*e.g.* bender element tests), are currently not integrated into the design method. If applied, they would provide a more informed definition of the soil response, which could contribute to cost savings in the design.

2. Components of the soil reaction curves

Various authors, including Davidson (1982), Lam and Martin (1986), Gerolymos and Gazetas (2006) and Lam (2013) have identified a set of additional soil reaction components, which contribute increasingly to the overall foundation behaviour as the L/D ratio of a foundation pile decreases (Byrne et al., 2017). These additional components include a distributed moment term, a base shear and a base moment term. Identifying these additional soil reaction components leads to a more rigorous approach towards capturing the mechanics of the problem.

3. Extensions to cyclic loading

Current extensions of the $p - y$ curves to cyclic loading (API, 2010; DNV, 2014) effectively decrease the ultimate resistance of the static soil reaction curve by an empirically defined factor. These cyclic soil reaction curves can be used to calculate the maximum displacement of the foundation pile under cyclic loading. This method assumes that equilibrium is reached after a limited number of load cycles and does not account for additional ratcheting of the foundation under high number of load cycles. While this is a pragmatic method and can be implemented easily as an extension to the static $p - y$ curve reaction, the method does not reflect the mechanical processes of the foundation reaction as the pile is loaded cyclically. The method does not provide a rigorous way to calculate effects like soil damping or accumulated pile rotation with cycle number. Nor does this method capture the effect of loading rate on the soil behaviour.

The limitations of the current design guidance have been recognised in an update to DNV (2014) in note F.2.4.1, discussing the applicability of the $p - y$ curves for monopile foundations.

F.2.4.1 The nonlinear p - y curves recommended in [F.2.2] and [F.2.3] are meant primarily for analysis of piles for evaluation of lateral pile capacity in the ULS. These p - y curves have been calibrated for long slender jacket piles with diameters of up to 1.0 m. They have not been calibrated for monopiles with larger diameters and are in general not valid for such monopiles. P - y curves to be used for monopile design should be validated for such use, e.g. by FE analysis.

1.4 Thesis outline

The structure of this thesis is outlined below.

Chapter 1 (*Introduction*) describes the shortcomings of the current design guidance when applied to monopile foundations for offshore wind turbines. The chapter sets out the problem definition for the PISA project and the additional research aspects in this thesis.

Chapter 2 (*The PISA project*) gives an overview of the field tests performed in the PISA project, with a description of the soil conditions at the test sites and the pile tests performed. The proposed design methodology for monopile foundations is given as a starting point for further numerical modelling.

Chapter 3 (*Data analysis*) outlines methods to analyse the instrumentation data in the PISA project, which are then used to interpret accurately the behaviour of the field tests. The data analysis in this thesis focuses on additional aspects of the pile behaviour beyond the monotonic backbone curve.

Chapter 4 (*Foundation models*) presents a 1D methodology to calculate the foundation response using the reaction components in the PISA method. The soil reaction curves are extended to plasticity models, to incorporate effects including: rate, ratcheting and gapping.

Chapter 5 (*Calibration and results*) describes how the various soil reaction models can be calibrated to fit the behaviour of the PISA field tests. The effect of each of the foundation models in the previous chapter is analysed in detail.

Chapter 6 (*Integration into commercial software*) describes the integration of one of the previously described soil reaction models into commercial software to simulate the wind turbine response in the time domain. Since these simulations allow displacement of the foundation in both lateral directions, the extensions of the foundation models to 2D are also described.

Chapter 7 (*Conclusions*) summarises the key findings of the previous chapters.

Chapter 2

The PISA project

The Pile Soil Analysis (PISA) project was developed and led by DONG Energy, in partnership with ten other companies active in the offshore wind sector with the aim of addressing the shortcomings of the current design methodology described in Chapter 1 (Byrne et al., 2015). The project was run as a discretionary project through the Carbon Trust’s Offshore Wind Accelerator programme. The principal activities were managed by the Academic Work Group (AWG) led by Oxford University, partnering with Imperial College London (ICL) and University College Dublin. The activities of the AWG were critically reviewed by an independent technical review panel to substantiate the developed design methodology in the PISA project. The project employed a number of consultants and contractors to execute elements of the work (*e.g.* site investigation, field test execution). Figure 2.1 illustrates the structure of the PISA project.

The development of the new design methodology used state of the art Finite Element (FE) modelling for monopile foundations which was validated through a field testing campaign at two test sites. The numerical modelling used historical knowledge of these sites, assisted by a more recent site investigation as part of the PISA project to determine the parameters for the constitutive soil models (Zdravković

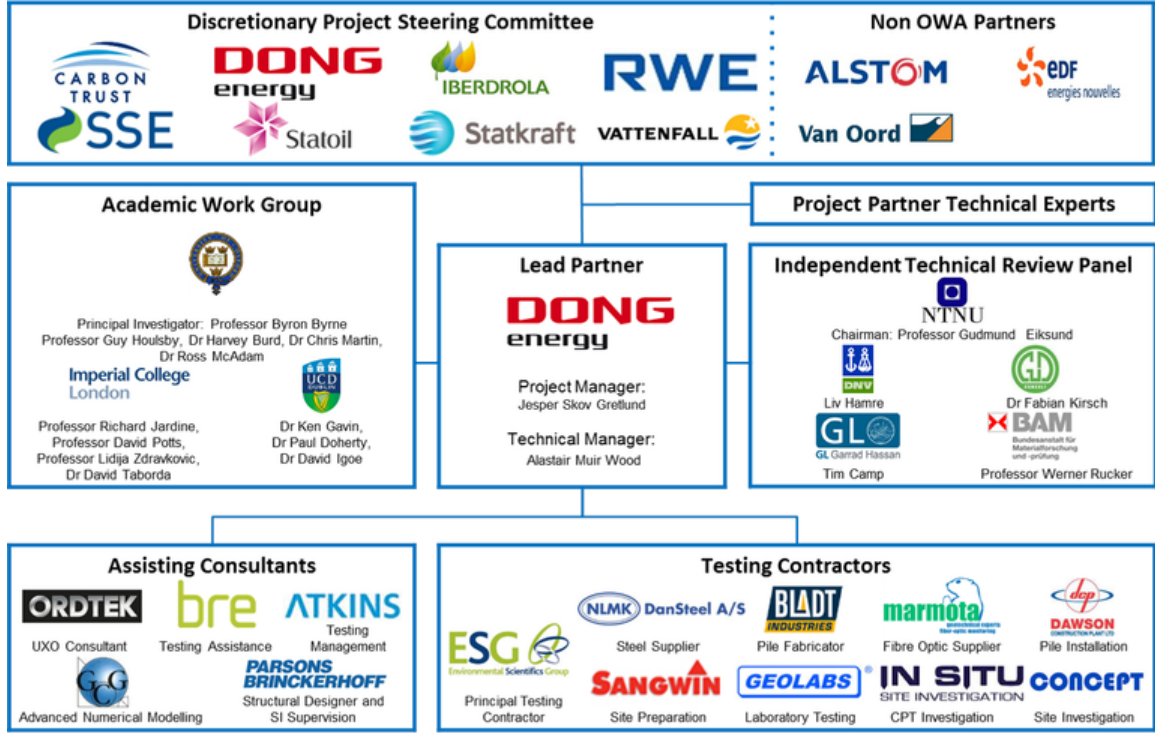


Figure 2.1: Structure of the PISA project (PISA AWG, 2016b).

et al., 2015). A detailed schematic view of the project execution is given in Figure 2.2.

From 3D FE simulations, soil reaction components can be extracted along the foundation pile. These reactions can be approximated by parameterised curves, which are then applied in a generalised Winkler model. This approach is called the parameterised design method. The PISA design methodology covers two streams (Byrne et al., 2017). (1) A ‘numerical-based’ method which requires extensive site investigation to calibrate an appropriate constitutive soil model for the 3D FE simulations. The parameterised design method can then be applied to these FE simulations. (2) A ‘rule based’ method, where the parameters for the parameterised curves are determined from a lookup table. This method requires that the parameterised curves are determined for similar soil conditions and within a range of pile geometries. The numerical-based method has been applied in the PISA project to extract the parameterised soil reaction curves for both test sites. Careful engineering judgement will be

required when using the derived parameters in the design for a specific offshore wind farm, as is the case with using any design method.

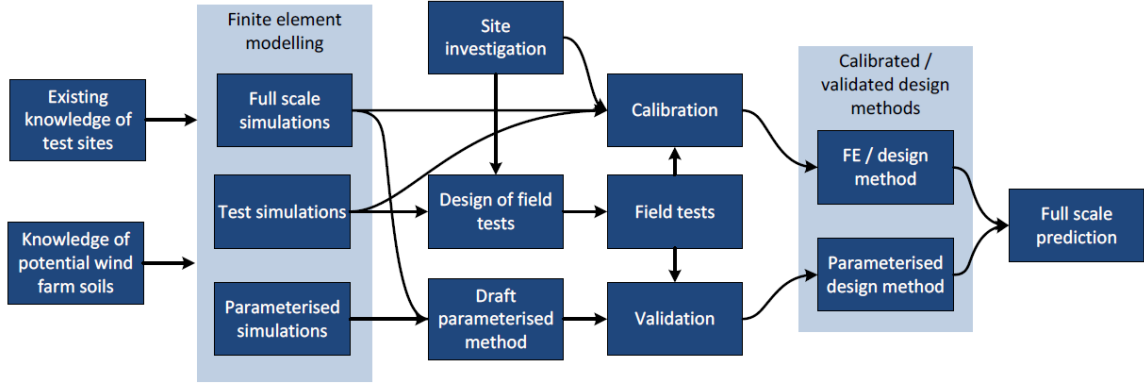


Figure 2.2: Detailed schematic showing project execution (Byrne et al., 2015).

In agreement with the project partners, the scope of the project deliverables for the PISA project was limited to the foundation behaviour under monotonic loading at the two reference test sites. The majority of the field tests were therefore subjected to a monotonic loading routine. A number of field tests have been loaded cyclically and at varying load rates, providing data for further research on these effects.

The PISA design framework provides rigorous methods for determining the soil reactions under monotonic loading. The framework addresses concerns regarding the current design methods without dramatically changing the calculation processes, so that the methods could be adopted within industry practice. A special consideration was put into possible future extensions of the monotonic response, to capture additional effects including rate-effects and the behaviour under cyclic loading.

2.1 Site descriptions and characterisation

The research within the PISA project was focused on two reference soil profiles representing typical sand and clay profiles for offshore wind farms in the North Sea. Two onshore test sites were selected to perform the field tests: (1) the Cowden test site in

North-East England with a stiff over-consolidated low plasticity clay till profile and (2) the Dunkirk test site in the North of France with a dense marine sand profile. Both test sites have been used previously for geotechnical research and have been characterised using both field and laboratory testing. This makes them suitable as reference sites for the PISA project.

2.1.1 Cowden

The location of the Cowden test site and a satellite image are shown in Figure 2.3 and 2.4. The PISA test site lies near the Building Research Establishment (BRE) test site, which has been investigated extensively since 1976. Most of the historic field and laboratory investigations date back to the 1980s and are published in Powell and Butcher (2003). The data from these historic tests were used for preliminary 3D FE analyses of the PISA field tests (Zdravković et al., 2015). As part of the PISA project, additional site investigation and testing were commissioned to supplement the historic data and derive improved parameters for the final numerical analyses.

The stratigraphy at the Cowden test site is presented in Table 2.1 with the initial pore water pressure p_w and earth pressure coefficient at rest K_0 in Figure 2.5. The ground water table lies 1m below ground surface and the measurements indicate an under-drained pore water pressure distribution in the top 12m. Below this depth, a hydrostatic pore water pressure is assumed in the adopted profile for the FE simulations conducted by ICL. The historic test data indicate a very high earth pressure coefficient in the top 5m of the soil profile. These historic data were predominantly obtained using oedometer tests, where the one-dimensional nature of the test may not be representative of processes induced by glacier advances. As these high values are considered unrealistic, the initial earth pressure coefficient has been set to 1.5 in the top 4m of the profile for the numerical simulations.

The normalisation of the parameterised curves for the Cowden till is based on the



Figure 2.3: Location of the Cowden test site (PISA AWG, 2016b).



Figure 2.4: Satellite image of Cowden test site (PISA AWG, 2016b).

Table 2.1: Stratigraphy at Cowden (PISA AWG, 2016b).

Depth	Material	Description
0-5m	Weathered till	Weathered brown stiff stony clay till. Some fissures.
5-40m	Unweathered till	Unweathered dark grey brown stiff stony clay till. Becoming less stony with depth. Stone mainly chalk. With silty sand/gravel bands at approximately 12m and 18m depth, as detected by CPT profiling.
40m +	Chalk	-

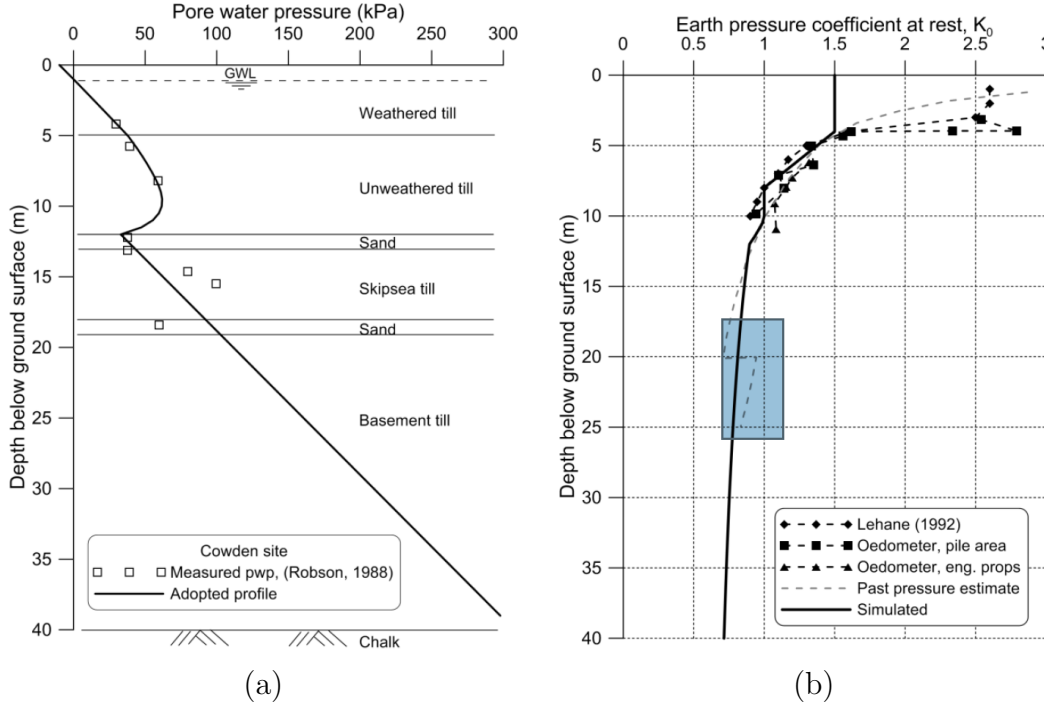


Figure 2.5: Profiles of (a) pore water pressure and (b) earth pressure coefficient of Cowden Till from Powell and Butcher (2003) (PISA AWG, 2016b).

undrained shear strength s_u and the initial shear modulus G_0 . At low loads, the pile response is governed by the initial shear modulus and the shear strength governs the response at ultimate failure loading. The shear strength profile for undrained triaxial compression (TXC) and the initial shear modulus are given in Figure 2.6 (a) and (b) respectively. The simulated initial stiffness profile is calculated as a proportion of the mean effective stress p' .

2.1.2 Dunkirk

The location of the Dunkirk test site and a satellite image are shown in Figure 2.7 and 2.8. The PISA test site is located on the extended beach area, which is owned by the Port Authority of Dunkirk. The test site has been used for a number of geotechnical research projects with field testing dating back to the 1980s. The projects include:

- the French CLAROM (Club for Research Activities on Offshore Structures)/IFP

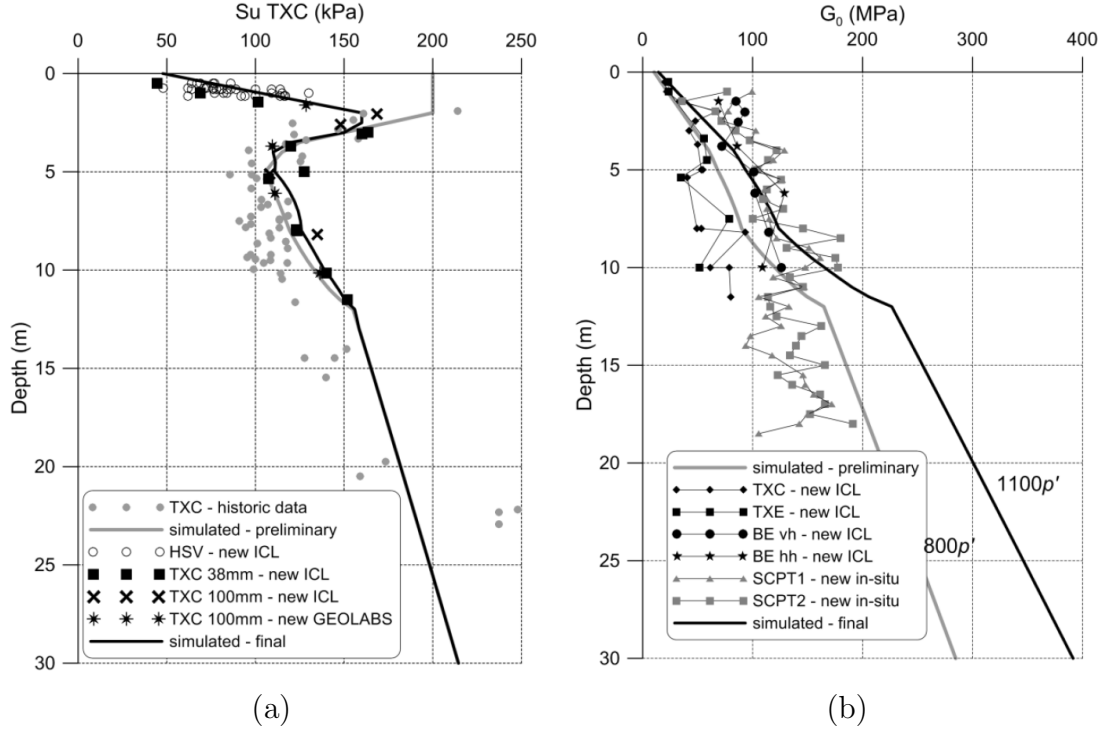


Figure 2.6: Profiles of (a) shear strength and (b) initial shear modulus of Cowden Till with historical data from Powell and Butcher (2003) (PISA AWG, 2016b).

(French Institute of Petroleum) project;

- the ICL and BRE tests in the mid-1990s;
- the GOPAL (Grouted Offshore Piles for Alternating Loading) project in the late 1990s; and
- the French SOLCYP project (joint industry project for piles under axial and lateral cyclic loads) in the late 2000s and still on-going, which has used a test site very close to the PISA Dunkirk site at Loon-Plage.

Soil testing from these projects can be used to calibrate a constitutive soil model. The historic data were used in preliminary 3D FE analyses by Chow (1997) based on the ICL and BRE field investigation data from the mid-1990s; and by Kuwano (1999) who performed laboratory testing focussing on the small strain behaviour. Recently, additional soil testing has been carried out at ICL by Aghakouchak (2015) to derive



Figure 2.7: Location of the Dunkirk test site (PISA AWG, 2016b).



Figure 2.8: Satellite image of the Dunkirk test site (PISA AWG, 2016b).

the critical state parameters for Dunkirk sand using three undrained triaxial tests to ultimate failure. The stratigraphy of the Dunkirk test site is summarised in Table 2.2.

Table 2.2: Stratigraphy at Dunkirk (PISA AWG, 2016b).

Depth	Material	Description
0-3m	Hydraulic Fill	Sand fill that was dredged from the offshore Flandrian deposits, and placed to raise the ground level. No compaction or surcharging has taken place.
3-30m	Flandrian Sand	Marine sand deposited during three local marine transgressions. These sands are often separated by organic layers which accumulated between transgressions. A 600mm thick organic layer is found at around 8m depth, separating the Flandrian sand into upper and middle units.
30m +	Yprésienne Clay	An Eocene marine clay (also known as London Clay and Argile de Flandres) which extends beneath the southern North Sea.

Figure 2.9 shows the pore water pressure profile and the initial shear stiffness profile at the Dunkirk test site. Based on the pore water pressure measurements from CPT tests at all test pile locations, the ground water level is estimated at a depth of 5.4m below which the profile is hydrostatic. The pore pressure above the water table remains uncertain. However, during the PISA field test stable gaps were observed at the active side of the test piles. This is potentially caused by the effective stresses due to suction in the unsaturated sand. Based on these observations, the pore pressure distribution for the FE simulations in the PISA project is adjusted as shown in the figure. The adopted shear stiffness profile for the FE simulations, which depends on the mean effective stress p' , matches the new SCPT tests.

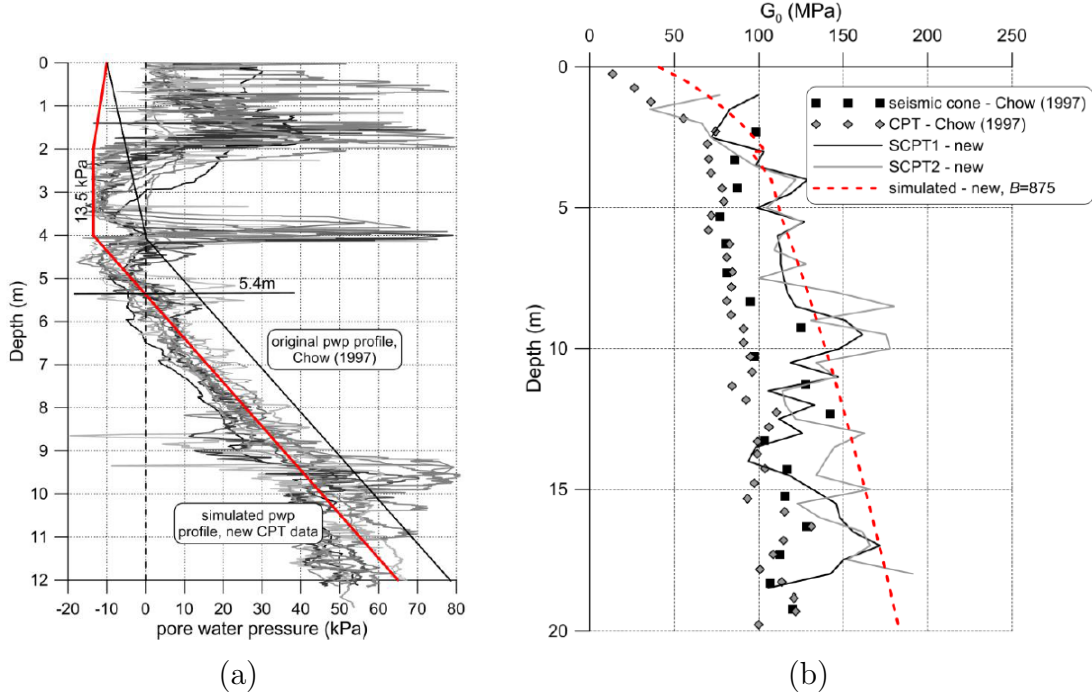


Figure 2.9: Profiles of interpreted (a) initial pore water pressure and (b) initial shear modulus of Dunkirk Sand based on experimental data in the PISA project (PISA AWG, 2016b).

2.2 Test outline

Validation of the 3D FE results and the design method in the PISA project was achieved through a number of field tests. The test program is centred around a base pile geometry with a diameter of 0.762m and a L/D ratio of 5.25 (Figure 2.10). From this base geometry, the diameter was varied for the large (2.0m) and small (0.273m) diameter field tests. In the short, long and extra-long tests, the L/D ratio has been varied. The effect of increasing the wall thickness t on the pile response was evaluated using the thick pile test. The height of load application h was 5m for the small diameter piles and 10m for the mid and large diameter piles.

Within the test program, not only the geometries, but also the loading has been changed. Most tests were focused on the monotonic response, where the load is increased using a number of load steps followed by hold phases. After reaching the

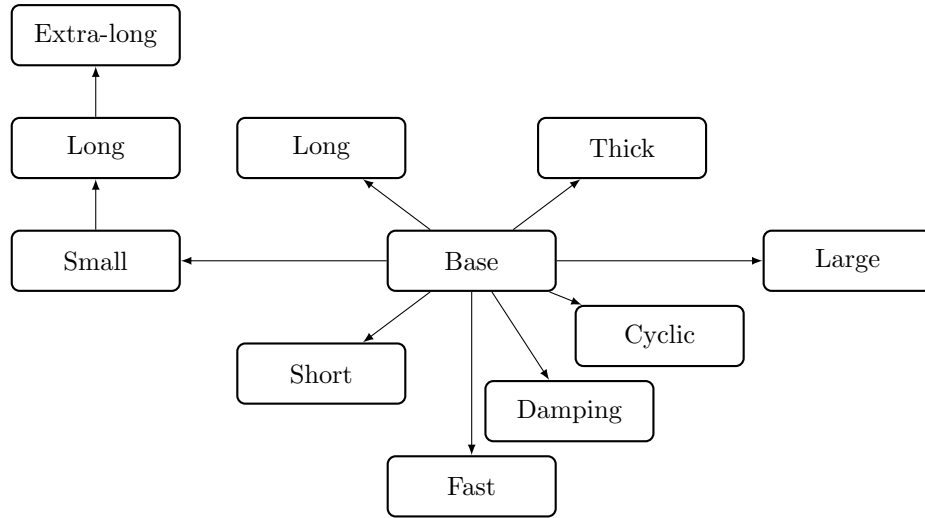


Figure 2.10: Schematic overview of the different tests within the PISA project.

ultimate capacity, a limited number of unload and reload steps were applied. At both test sites, 3 test piles have specifically been dedicated to different loading regimes.

(1) The loading rate has been varied in the ‘fast’ test to capture the capacity increase of the foundation with increasing load rate. (2) The 1-way cyclic (‘cyclic’) test was focused on capturing the accumulation of pile rotation with cycle number and load amplitude. (3) The 2-way cyclic (‘damping’) test was focused on the stiffness and damping of the foundation.

2.2.1 Cowden

The layout of the field test piles in Cowden is given in Figure 2.11. The small and mid-diameter piles were loaded against a reaction pile and therefore, the layout of these piles is centered around the reaction pile. For the small diameter piles (CS1 - CS4), the mid-diameter pile CM9 was used and for the mid-diameter piles (CM1 - CM9), the large diameter reaction pile CR1 was used. The large diameter test piles (CL1 and CL2), were loaded against each other.

A detailed overview of the dimensions, additional instrumentation and load description of the test piles is given in Table 2.3. Each of the test piles was instrumented

Table 2.3: Pile geometry and instrumentation types for the tests at Cowden (PISA AWG, 2015a).

Pile location	D [m]	L [m]	L/D	t [mm]	D/t	Below ground data	Piezos	Description
CS1	0.273	1.43	5.25	7	39			Fast monotonic test
CS2	0.273	1.43	5.25	7	39			Repeat monotonic test
CS3	0.273	2.18	8	7	39			Monotonic test
CS4	0.273	2.73	10	7	39			Monotonic test
CM2	0.762	2.29	3	10	76	*		Monotonic test
CM8	0.762	2.29	3	10	76			Repeat monotonic test
CM5	0.762	4.00	5.25	11	69	*	*	1-way cyclic test
CM6	0.762	4.00	5.25	11	69	*	*	Control geometry
CM7	0.762	4.00	5.25	11	69	*		2-way cyclic test
CM9	0.762	4.00	5.25	11	69			Repeat control geometry
CM1	0.762	4.00	5.25	15	51	*		Monotonic test
CM3	0.762	7.62	10	25	30	*		Monotonic test
CR1	2	8.00	4	25	80			Reaction pile
CL1	2	10.50	5.25	25	80	*	*	Monotonic and cyclic test
CL2	2	10.50	5.25	25	80	*	*	Monotonic and cyclic test

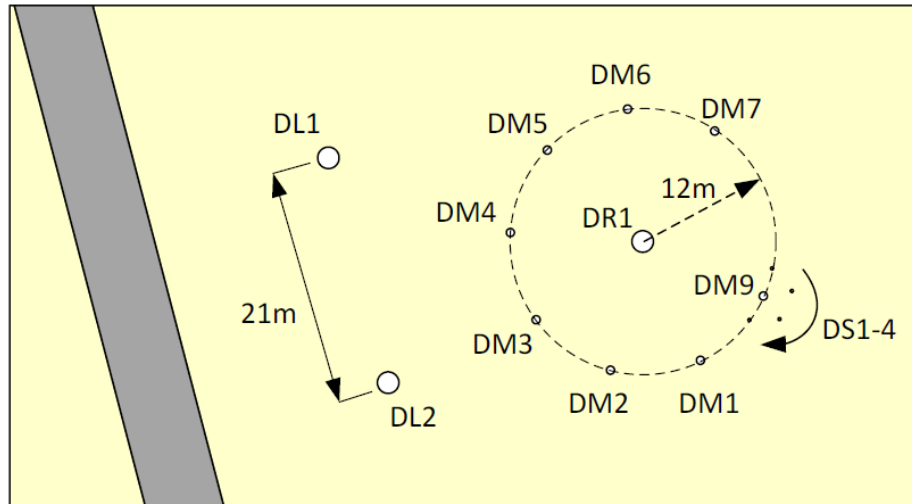


Figure 2.12: Dunkirk site layout (PISA AWG, 2015a).

A detailed overview of the pile geometries, additional instrumentation and loading routine of the test piles are given in Table 2.4. Similarly to the test piles in Cowden, a selection of mid and large diameter piles have been instrumented with below ground inclinometers, extensometers and strain gauges to calculate the pile behaviour below ground. Test piles DM1 and DM2 have been loaded with a backstay system to allow for accurate control in the cyclic loading.

Table 2.4: Pile geometry and instrumentation types for the tests at Dunkirk (PISA AWG, 2015a).

Pile location	D [m]	L [m]	L/D	t [mm]	D/t	Below ground data	Description
DS1	0.273	1.43	5.25	7	39		Fast monotonic test
DS2	0.273	1.43	5.25	7	39		Repeat monotonic test
DS3	0.273	2.18	8	7	39		Monotonic test
DS4	0.273	2.73	10	7	39		Monotonic test
DM5	0.762	2.29	3	10	76		Repeat monotonic test
DM7	0.762	2.29	3	10	76	*	Monotonic test
DM2	0.762	4.00	5.25	14	54	*	1-way cyclic test
DM4	0.762	4.00	5.25	14	54	*	Control geometry
DM9	0.762	4.00	5.25	14	54		Repeat control geometry
DM1	0.762	4.00	5.25	14	54	*	2-way cyclic test
DM6	0.762	4.00	5.25	19	40	*	Monotonic test
DM3	0.762	6.10	8	25	30	*	Monotonic test
DR1	2	8.00	4	25	80		Reaction pile
DL1	2	10.50	5.25	38	53	*	Monotonic and cyclic test
DL2	2	10.50	5.25	38	53	*	Monotonic and cyclic test

2.3 PISA soil reaction curves

The parameterised design method in the PISA project is based on the existing $p - y$ approach in current design guidance, where distributed lateral soil reaction curves are given as a function of the lateral displacement of a 1D beam. This methodology uses the Winkler assumption, where the soil reaction at a specific depth is a function only of the local pile displacement and rotation. The design method is extended by including distributed moment curves, a base shear curve and a base moment curve as illustrated in Figure 2.13. Each of these soil reaction curves is fitted to the reaction component of a 3D FE simulation of the foundation pile. The importance of the additional components increases for decreasing L/D ratios (Byrne et al., 2017). In the proposed methodology, the soil reaction curves are expressed as mathematical functions which relate the reaction component to the local pile displacement or rotation.

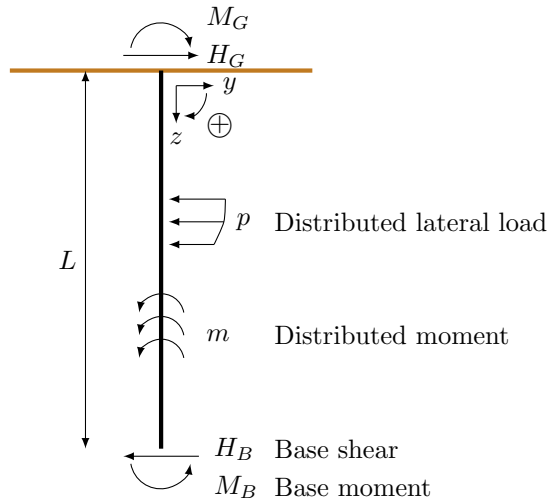


Figure 2.13: Overview of the assumed soil reactions on a monopile in the PISA project.

The lateral soil reaction as a pile section moves horizontally, is captured using the distributed lateral load curves (units of p : [N/m]). The distributed moment is mobilised as the pile section rotates, where the shear stresses along the pile boundary result in a moment (units of m : [Nm/m]). Shear friction along the pile is also

mobilised as a wedge mechanism forms on the passive side of the pile (Figure 2.14a), resulting in an additional moment component. The base shear force is mobilised as the pile base displaces laterally (units of H_B : [N]) and similarly, the base moment is mobilised as the pile base rotates (units of M_B : [Nm]).

The soil reaction curves have been calibrated against 3D FE simulations of the foundation pile conducted by ICL, where the effect of gapping is incorporated using interface elements along the pile boundary. The effect of the gap on the lateral soil reaction is illustrated in Figure 2.14 and the formation of the gap for pile test CM3 is illustrated in Figure 2.15.

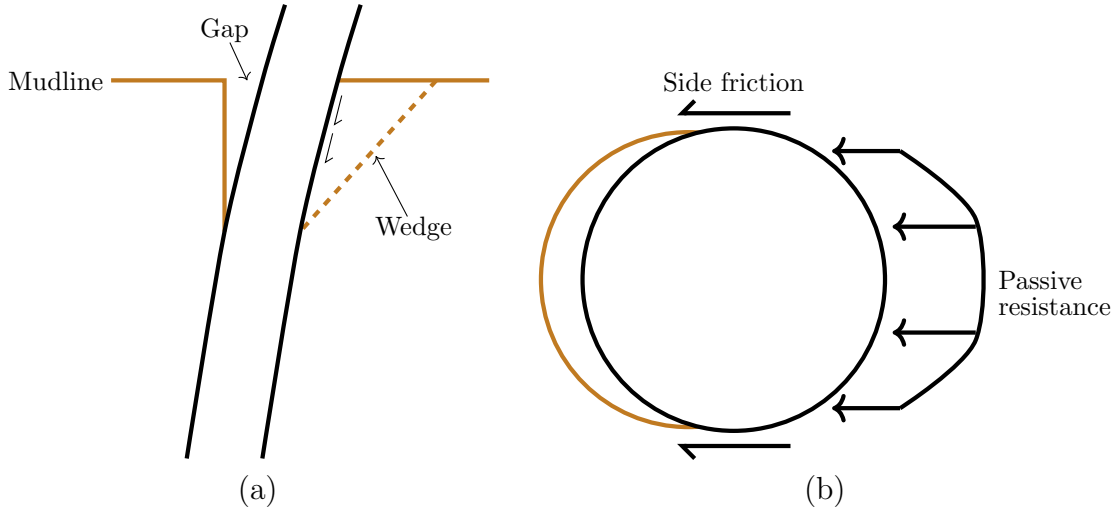


Figure 2.14: Illustration of (a) the side view of the foundation pile and (b) components of the lateral soil reaction.

2.3.1 Normalised variables

The soil reaction curves for the 1D model are normalised in such a way that the reactions scale appropriately with the dimensions of the foundation pile. The normalisations for the clay and sand site are given in Table 2.5. In this table, $I_R (= G_0/s_u)$ is the rigidity index, $I_S (= G_0/(\sqrt{\sigma'_{v0}p_a}))$ is the sand stiffness coefficient, σ'_{v0} is the initial vertical effective stress and p_a is the atmospheric pressure. The parameters of



Figure 2.15: Image of the gap on the active side of CM3 (PISA AWG, 2016b).

the reaction curves can then be expressed as functions of these normalised variables.

Table 2.5: Normalisation of pile reaction components (PISA AWG, 2016b).

Component	Clay normalisation	Sand normalisation
Distributed load, $\bar{p}$	$\frac{p}{s_u D}$	$\frac{p}{\sigma'_{v0} D}$
Lateral displacement, $\bar{v}$	$\frac{v}{D} I_R$	$\frac{v}{D} I_s \sqrt{\frac{p_a}{\sigma'_{v0}}} \left[= \frac{v}{D} \frac{G_0}{\sigma'_{v0}} \right]$
Distributed moment, $\bar{m}$	$\frac{m}{s_u D^2}$	$\frac{m}{ p D}$
Pile cross section rotation, $\bar{\psi}$	ψI_R	$\psi I_s \sqrt{\frac{p_a}{\sigma'_{v0}}} \left[= \psi \frac{G_0}{\sigma'_{v0}} \right]$
Base shear load, $\bar{H}_B$	$\frac{H_B}{s_u D^2}$	$\frac{H_B}{\sigma'_{v0} D^2}$
Base moment, $\bar{M}_B$	$\frac{M_B}{s_u D^3}$	$\frac{M_B}{\sigma'_{v0} D^3}$

2.3.2 Mathematical expression for the soil reaction curves

The soil reactions on the foundation pile are approximated using a simplified expression. To capture the soil reaction stiffness, curvature and ultimate resistance, the

PISA project makes use of a conic function:

$$-n \left(\frac{\bar{\sigma}}{\bar{\sigma}_u} - \frac{\bar{\epsilon}}{\bar{\epsilon}_u} \right)^2 + (1-n) \left(\frac{\bar{\sigma}}{\bar{\sigma}_u} - \frac{\bar{\epsilon}k_0}{\bar{\epsilon}_u} \right) \left(\frac{\bar{\sigma}}{\bar{\sigma}_u} - 1 \right) = 0 \quad (2.1)$$

where $\bar{\sigma}$ refers to a normalised load variable in Table 2.5 and $\bar{\epsilon}$ is the corresponding normalised displacement variable. The real positive roots of $\bar{\sigma}$ are calculated as:

$$\begin{aligned} \bar{\sigma} &= \bar{\sigma}_u \frac{2c}{-b + \sqrt{b^2 - 4ac}} & \text{for } 0 < \bar{\epsilon} \leq \bar{\epsilon}_u \\ \bar{\sigma} &= \bar{\sigma}_u & \text{for } \bar{\epsilon}_u < \bar{\epsilon} \end{aligned} \quad (2.2)$$

where:

$$\begin{aligned} a &= 1 - 2n \\ b &= 2n \frac{\bar{\epsilon}}{\bar{\epsilon}_u} - (1-n) \left(1 + \frac{\bar{\epsilon}k_0}{\bar{\sigma}_u} \right) \\ c &= \frac{\bar{\epsilon}k_0}{\bar{\sigma}_u} (1-n) - n \frac{\bar{\epsilon}^2}{\bar{\epsilon}_u^2} \end{aligned}$$

The conic expression is defined by 4 parameters: $\bar{\epsilon}_u$, $\bar{\sigma}_u$, k_0 and n , where $0 \leq n \leq 1$ and $k_0\bar{\epsilon}_u > \bar{\sigma}_u$. The shape of the conic curve is illustrated in Figure 2.16.

The expression of Equation 2.2 gives a positive normalised stress for a positive normalised strain. In case a negative strain is applied, the stress can be calculated by applying the positive magnitude of strain in Equation 2.2 and correcting the sign of the resulting stress. Another approach is to inherently correct the solution for the sign of the strain:

$$\begin{aligned} \bar{\sigma} &= S(\bar{\epsilon}) \bar{\sigma}_u \frac{2c}{-b + \sqrt{b^2 - 4ac}} & \text{for } |\bar{\epsilon}| \leq \bar{\epsilon}_u \\ \bar{\sigma} &= S(\bar{\epsilon}) \bar{\sigma}_u & \text{for } |\bar{\epsilon}| > \bar{\epsilon}_u \end{aligned} \quad (2.3)$$

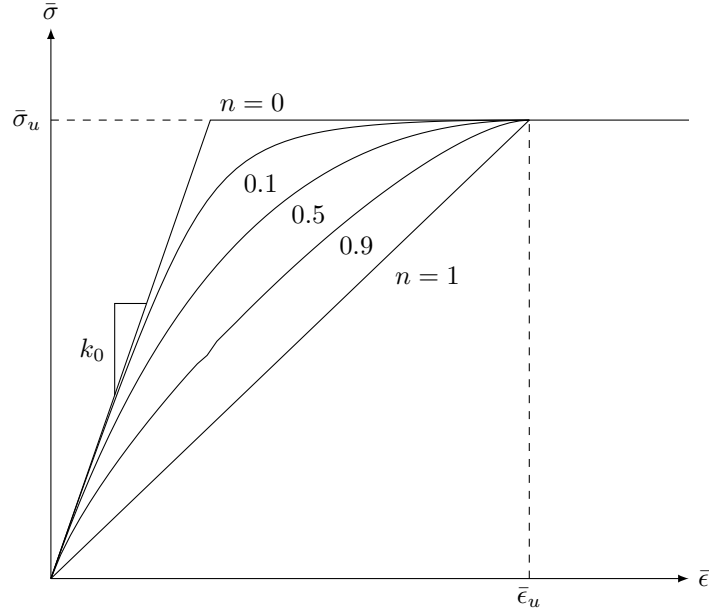


Figure 2.16: Illustration of the conic curve.

where:

$$\begin{aligned}
 a &= 1 - 2n \\
 b &= 2n \frac{|\bar{\epsilon}|}{\bar{\epsilon}_u} - (1 - n) \left(1 + \frac{|\bar{\epsilon}| k_0}{\bar{\sigma}_u} \right) \\
 c &= \frac{|\bar{\epsilon}| k_0}{\bar{\sigma}_u} (1 - n) - n \frac{\bar{\epsilon}^2}{\bar{\epsilon}_u^2}
 \end{aligned}$$

2.3.3 Parameters for the soil reaction curves

Based on 3D FE simulations for full scale monopiles in idealised profiles of the Cowden till and Dunkirk sand, soil reaction curves have been extracted for a parameterised 1D model. The parameters for these soil reaction curves are given in Table 2.6. These parameters are calibrated within the parameter space: $2 < L/D < 6$, $5\text{m} < D < 10\text{m}$, $5 < h/D < 15$, $60 < D/t < 110$ and the equations are valid for $0 < z/D < 6$. Since an idealised profile is used for these parameterised curves and since the full scale calibration piles in the PISA programme lie outside the parameter space of the field test piles, the parameters do not give an accurate prediction for the field tests.

However, these expressions can be used as a starting point for calibration of soil reaction models in Chapter 5.

The normalised values of the ultimate distributed lateral load in clay can be compared with results derived by Murff and Hamilton (1993) (Figure 2.17) using the upper-bound method, where the ultimate reaction is calculated for a fully rough and smooth pile soil interface.

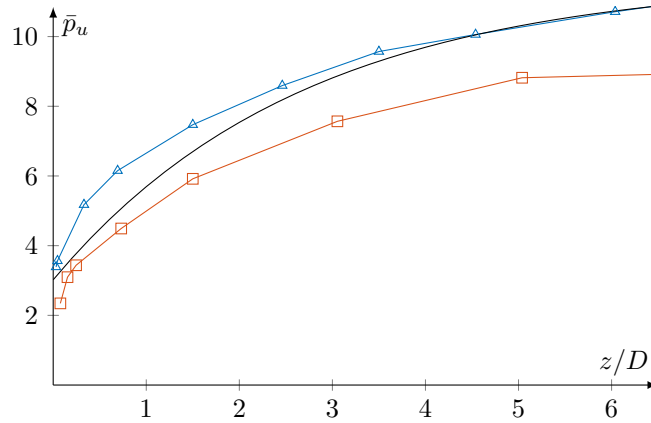


Figure 2.17: Comparison of the normalised ultimate distributed lateral load using (—) the PISA calibrated lateral soil reaction and the prediction by Murff and Hamilton (1993) for (—□—) smooth and (—△—) rough pile soil interface.

At shallow depths, a wedge type failure mechanism governs the ultimate soil reaction. At large depths, a flow-around mechanism occurs and the ultimate normalised reaction tends towards a constant value. Randolph and Houlsby (1984) derived the ultimate reaction for the flow-around mechanism using the method of characteristics. This results in the following expression:

$$\frac{p_u}{s_u D} = \pi + 2\Delta + 2 \cos \Delta + 4 [\cos(\Delta/2) + \sin(\Delta/2)] \quad (2.4)$$

where $\Delta = \sin^{-1} \alpha$ and α defines the fraction of the shear strength mobilised at the pile-soil interface (αs_u). For a fully rough interface, $\alpha = 1$, the normalised resistance becomes: $\bar{p} = 11.94$ and for a sliding interface, $\alpha = 0$, the normalised resistance is lower: $\bar{p} = 9.14$. This upper-bound solution was improved slightly by Martin and

Randolph (2006) for the case where $\alpha < 1$. The correction leads to a resistance of $\bar{p} = 9.20$ for $\alpha = 0$. The maximum normalised lateral reaction using the parametrised reaction curves is 11.66, which is closely comparable to the upper bound results of the ultimate reaction using the flow-around mechanism with a fully rough interface. For the base shear in clay, it can be expected that the maximum resistance is comparable to $H_B = s_u D^2 \pi / 4$, so that $\bar{H}_B = 0.79$, which is comparable to the value in Table 2.6.

For the ultimate resistance of the distributed lateral pressure in sand, the expression can be compared to the solution of Broms (1964):

$$\frac{p_u}{\gamma' z D} = 3 \frac{1 + \sin \phi}{1 - \sin \phi} \quad (2.5)$$

where ϕ is the friction angle of sand and γ' is the submerged unit weight. The initial vertical effective stress in a uniform submerged soil can simply be calculated as: $\sigma'_{v0} = \gamma' z$. Using an approximate friction angle of 40° , the right hand side of the equation is equal to 13.8, which is in the same order of magnitude as the ultimate lateral distributed resistance in Table 2.6. The ultimate base reaction in a frictional material could be calculated analytically as: $H_B = \tan(\phi) \sigma'_{v0} D^2 \pi / 4$, so that $\bar{H}_B = 0.66$ which is comparable to the value in Table 2.6.

2.4 PISA beyond monotonic

The design methodology presented in the PISA project focuses on the monotonic behaviour of monopile foundations. The current expressions for the soil reactions are therefore presented as curves, which directly give the value for the reaction variable in function of the displacement variable without integrating plasticity in the soil reaction. This results in a non-linear elastic response of the foundation.

This thesis focuses on integrating plasticity models in the soil reaction curves to capture various aspects of the soil reaction which have been observed in the PISA field

Table 2.6: Parameters for the soil reaction curves (PISA AWG, 2016b).

Soil reaction component	Parameter	Clay expression	Sand expression
Distributed lateral load, $\bar{p}$	Ultimate strain, $\bar{v}_u$	200	53.1
	Initial stiffness, k_{0p}	$-1.11 \frac{z}{D} + 8.17$	$-0.85 \frac{z}{D} + 7.46$
	Curvature, n_p	$-0.07 \frac{z}{D} + 0.92$	0.944
	Ultimate reaction, $\bar{p}_u$	$11.66 - 8.64e^{-0.37 \frac{z}{D}}$	$-0.05 \frac{z}{L} + 21.61$
Distributed moment, $\bar{m}$	Ultimate rotation, $\bar{\psi}_u$	10	20
	Initial stiffness, k_{0m}	$-0.12 \frac{z}{D} + 0.98$	20
	Curvature, n_m	0	0
	Ultimate reaction, $\bar{m}_u$	$-0.05 \frac{z}{D} + 0.38$	$-0.05 \frac{z}{L} + 0.21$
Base shear, $\bar{H}_B$	Ultimate strain, $\bar{v}_{Bu}$	300	$-0.29 \frac{L}{D} + 2.31$
	Initial stiffness, k_{0HB}	$-0.32 \frac{L}{D} + 2.58$	$-0.38 \frac{L}{D} + 3.02$
	Curvature, n_{HB}	$-0.04 \frac{L}{D} + 0.76$	$-0.05 \frac{L}{D} + 0.94$
	Ultimate reaction, $\bar{H}_{Bu}$	$0.07 \frac{L}{D} + 0.59$	$-0.07 \frac{L}{D} + 0.62$
Base moment, $\bar{M}_B$	Ultimate rotation, $\bar{\psi}_{Bu}$	200	50
	Initial stiffness, k_{0MB}	$-0.002 \frac{L}{D} + 0.19$	0.29
	Curvature, n_{MB}	$-0.15 \frac{L}{D} + 0.99$	0.89
	Ultimate reaction, $\bar{M}_{Bu}$	$-0.07 \frac{L}{D} + 0.65$	$-0.05 \frac{L}{D} + 0.38$

tests. The starting point of these models is the kinematic hardening model, which results in the extended Masing rules upon unloading and reloading as illustrated in Figure 2.18 (a). In the PISA field tests, a rate effect has been observed which results in an increased foundation capacity at fast load rates. Simulated behaviour using the rate effect model developed in this thesis is illustrated in Figure 2.18 (b). As the foundation pile is loaded cyclically, accumulated displacement has been observed. The behaviour is captured using the ratcheting model as illustrated in Figure 2.18 (c). The rate and ratcheting models can be combined as illustrated in Figure 2.18 (d), where the rate effect results in creep displacement when the load is held constant. For each of these models, the plasticity models are integrated in the soil reaction curves, such that the modelling is consistent with the PISA 1D modelling approach. Additionally, a soil reaction model is developed to capture the gapping observed in the PISA field tests. Distributed reaction curves along the foundation pile using the gapping model are illustrated in Figure 2.19. The gapping effect results in bow-tie shaped reaction curves.

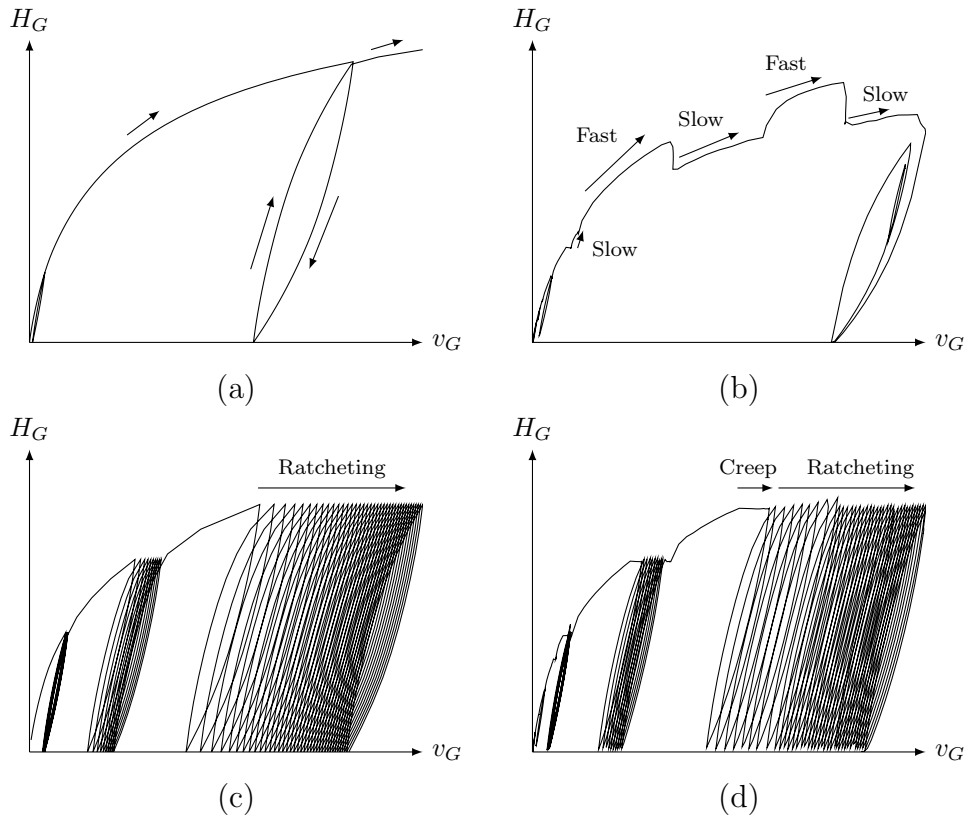


Figure 2.18: Illustration of (a) the kinematic hardening model, (b) the rate effect model, (c) the ratcheting model and (d) the combined rate and ratcheting model.

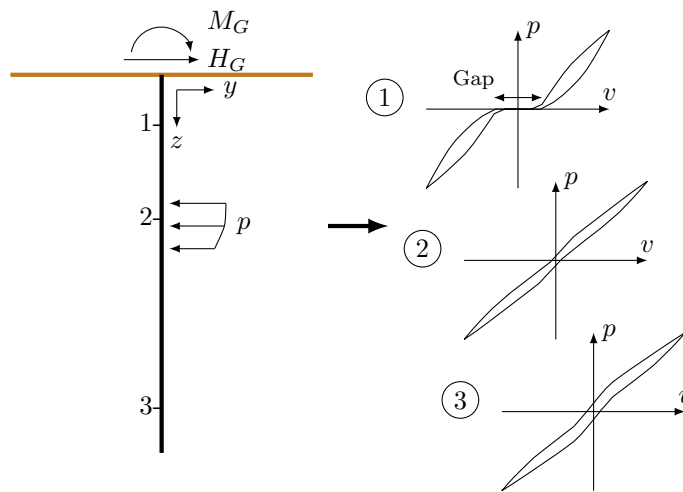


Figure 2.19: Illustration of the distributed lateral reaction using the gapping model.

2.5 Conclusions

The PISA project presents a framework for the design method for monopile foundations. The current methodology focuses on the monotonic response of the foundation. The formulation does, however, present a solid baseline for further modifications to capture the additional effects seen in the field tests.

Within the PISA project, a large number of monopiles have been tested with extensive instrumentation. The majority of the field tests focus on capturing the backbone curve using a monotonic test routine. At each test site, a fast load test was performed to capture the foundation response under varying load rate. Additionally a 1-way cyclic test was performed with a focus on the ratcheting effect under cyclic loading and a 2-way cyclic test with a focus on the stiffness and damping of the foundation. The results from these tests are used in this thesis to develop and calibrate soil reaction models which capture plastic unloading and reloading, rate effects, ratcheting and gapping.

Chapter 3

Data analysis

Within the PISA project over 28 pile tests have been performed with up to 130 simultaneous instrument measurements per test pile. To interpret accurately the large amount of data, innovative methods have been developed. These methods have allowed improved analysis and understanding of the foundation behaviour. The analysis methods combine different sets of instrumentation (load, displacement, rotation and strain measurements), such that a rigorous interpretation can be found of the pile response. The methods can also confirm consistency between the different data sets.

The following sections give an overview of these methods and give a detailed description of the foundation behaviour, which is then used as a basis for developing new foundation models as described in Chapter 4.

3.1 Test setup

The test setup for the base geometry test pile is given in Figure 3.1. As shown in the figure, a large number of sensors have been placed on the test pile which can be used to interpret the test pile behaviour. These include: a load cell (LCL), stick-up inclinometers (SUI), displacement transducers (LVDT), in place extensometers (IPE), in place inclinometers (IPI), fibre optic strain gauges (FOS) and piezometers (PZT). The LVDT locations are denoted Passive High (PH), Passive Low (PL), Active High (AH) and Active Low (AL).

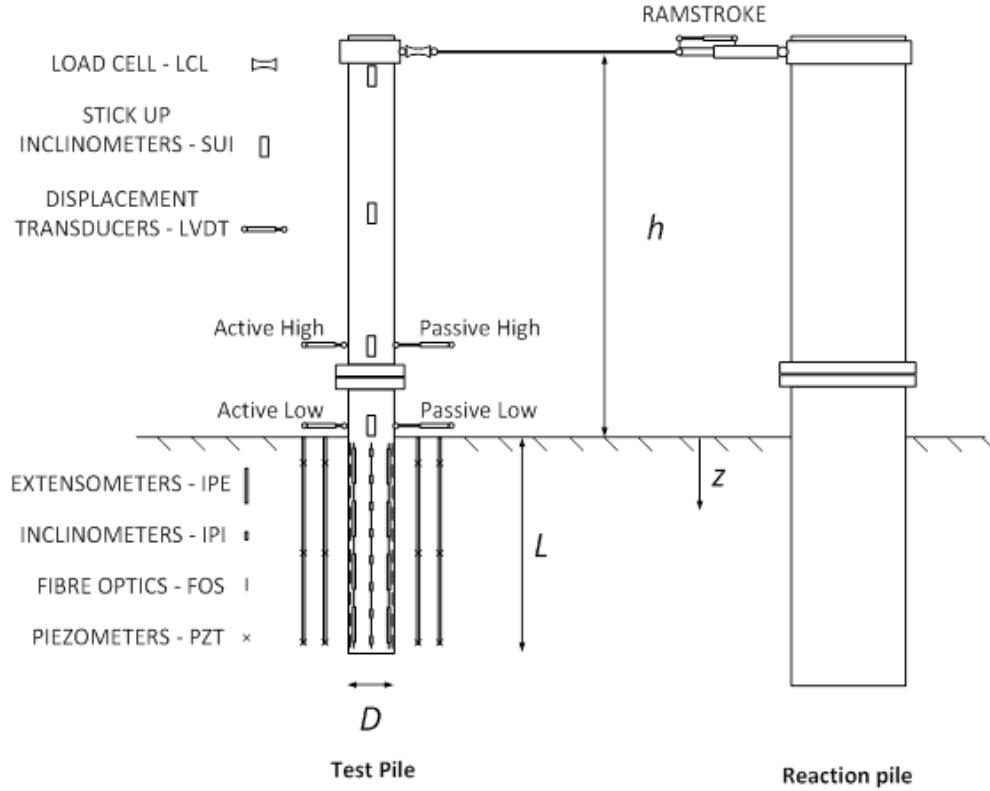


Figure 3.1: Setup for the mid-diameter test piles with below ground instrumentation (PISA AWG, 2015a).

It should be noted that not all mid-diameter test piles have been equipped with below ground instrumentation. A comprehensive list of the test piles with and without instrumentation is given in Table 2.3 and 2.4. The test setup for small and large

diameter piles is given in Figure 3.2 (a) and (b). For the cyclic test, a setup with a backstay loading system has been used (Figure 3.2 (c)). The cyclic test setup allows accurate load control upon unloading in the 1-way cyclic tests, even at the high loading frequencies. The weight of 8 tonnes allows for 2-way loading with a maximum amplitude of approximately 40kN. The net load on the test pile is calculated as $H_{\text{net}} = H - T \cos \theta$.

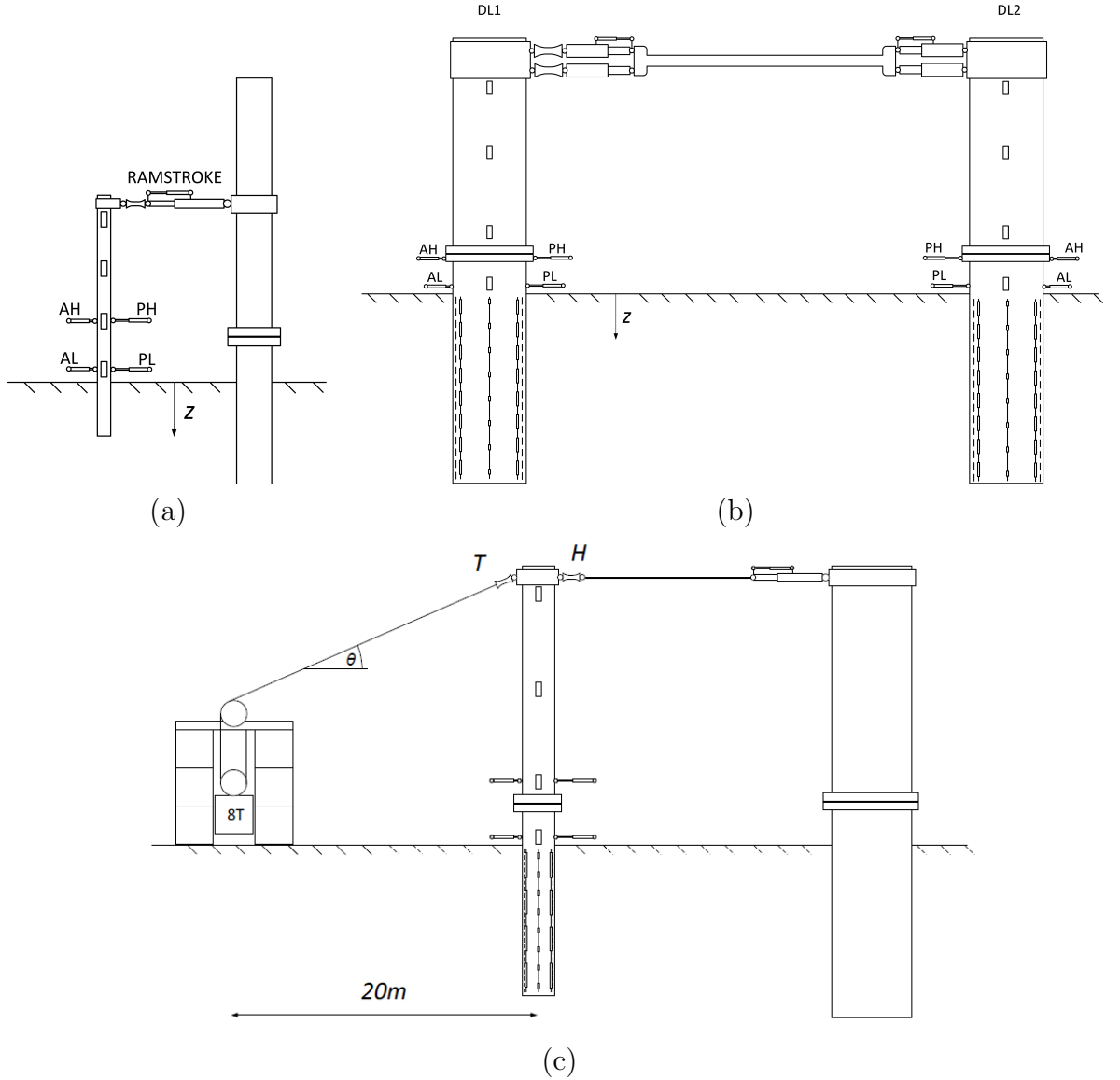


Figure 3.2: Test setup for the (a) small, (b) large and (c) mid-diameter test piles subjected to cyclic loading (PISA AWG, 2015a).

3.2 Analysis tools

In this section, two analysis tools are outlined which are based on a combination of Timoshenko beam theory and constrained least squares optimisation.

1. The first method uses the above ground level instrumentation to determine the ground level response. This method can be applied to all test piles and therefore gives a consistent approach to analyse all pile tests.
2. The second method uses the below-ground instrumentation to determine the pile behaviour below ground level. This method can be applied to a subset of mid and large diameter test piles to obtain more in-depth understanding of the foundation response below ground level.

3.2.1 Calculation of the ground level response

The foundation response is commonly visualised in terms of the ground level response. The two characteristic curves for the interpretation of the foundation response are the horizontal load-displacement curve and the moment-rotation curve at ground level. Since the instrumentation does not directly measure the ground level displacement and rotation, these values are interpreted from the above ground instrumentation: four LVDT measuring points and rotation at the various inclinometer locations.

The method presented here to determine the ground level response is based on Timoshenko beam theory and a least squares optimisation to fit both sets of instrumentation data. For the small diameter test piles, the cross section of the stick-up is uniform and therefore it can be modelled using a single section. In the case of the mid and large diameter piles, the stick-up is modelled using 3 sections: the foundation pile, the flanged connection and the transition piece. Continuity between the sections is modelled using constraint equations. Since different instrumentation types are used, appropriate weighting of the data sets is used. The inclinometer data have significant

noise, and therefore the data from these instruments are smoothed to obtain more reliable results. The calculation method for the ground level response also includes the updated geometry of the LVDTs under large pile deflections.

Governing equations for the Timoshenko model of the stick-up

The governing equations for the stick-up, based on Timoshenko beam theory, are:

$$M = -EI \frac{d\psi}{dz} \quad (3.1)$$

$$\psi = - \left(\frac{dv}{dz} + \frac{S(z)}{\kappa AG} \right) \quad (3.2)$$

where M is the bending moment (a positive moment causes tension on the left-hand side of the stick-up), S is the shear force, ψ is the (clockwise positive) rotation of the beam cross-section, E is the Young's modulus, I is the cross-section second moment of area, κ is the shear coefficient ($\kappa = \frac{2(1+\nu)}{4+3\nu}$ for thin walled circular tubes, Cowper (1966)), A is the cross-section area and G is the shear modulus ($G = \frac{E}{2(1+\nu)}$), where ν is the Poisson's ratio. In the analyses, it is assumed that $E = 200\text{GPa}$ and $\nu = 0.3$, similarly to the 3D analysis in the PISA Final Report (PISA AWG, 2016b).

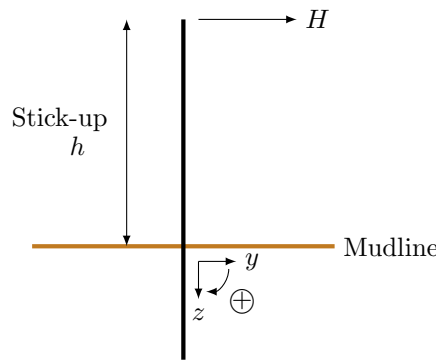


Figure 3.3: Timoshenko beam model for the pile stick-up.

The bending moment and shear force in the stick-up, illustrated in Figure 3.3,

are:

$$M = H(h + z) \quad (3.3)$$

$$S = \frac{dM}{dz} = H \quad (3.4)$$

The rotation of the neutral axis, θ (clockwise positive), and the rotation of the beam cross-section ψ , are related by:

$$\psi = \theta - \frac{H}{\kappa AG} \quad (3.5)$$

Note that for the moment-rotation curves, the cross section rotation is used. This representation is more appropriate for further use in FE modelling and can be used for energetic interpretation of the foundation response (Section 3.3). Combining Equations 3.1 to 3.4 gives:

$$\frac{d^2v}{dz^2} = \frac{H}{EI}(h + z) \quad (3.6)$$

Integrating Equation 3.6 (for the case of uniform section properties) gives the rotation of the neutral axis of the beam θ and the lateral displacement v :

$$\theta = -\frac{dv}{dz} = -\left(\frac{H}{EI}\left(hz + \frac{z^2}{2}\right) + a\right) \quad (3.7)$$

$$v = \frac{H}{EI}\left(\frac{hz^2}{2} + \frac{z^3}{6}\right) + az + b \quad (3.8)$$

where a and b are the unknowns to be determined on the basis of the measured displacements and rotation. Equations 3.7 and 3.8 can be used directly for the analysis of small diameter piles. The development of the model for cases where the stick-up properties are not uniform is given below.

Stick-up model for the mid and large diameter piles

The stick-up of the mid and large diameter piles consists of three connected sections

illustrated in Figure 3.4: the above ground section of the foundation pile (section 1), the flanged connection (section 2) and the transition piece (section 3). Each section is modelled separately using Timoshenko beam theory, with continuity of displacement and cross-section rotation (modelled with constraint equations) at the locations where the sections connect. The resulting number of undetermined coefficients in the Timoshenko model remains unaffected since the two unknowns for each additional section are matched by the two constraint equations.

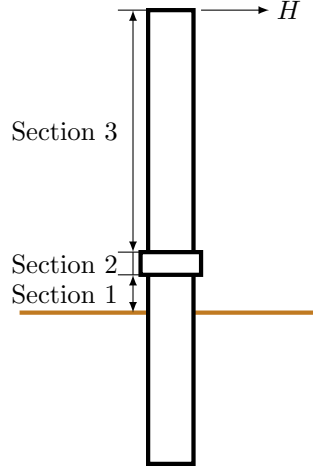


Figure 3.4: Sections for the stick-up model for the mid and large diameter piles.

The continuity of cross-section rotation, ψ , and lateral displacement, v , at the connection of two sections (denoted i and $i + 1$ where $i = 1, 2$) gives:

$$\begin{aligned} \psi_i = \psi_{i+1} &\rightarrow -\frac{H}{(EI)_i} \left(h z_i + \frac{z_i^2}{2} \right) - \frac{H}{(\kappa AG)_i} - a_i \\ &= -\frac{H}{(EI)_{i+1}} \left(h z_i + \frac{z_i^2}{2} \right) - \frac{H}{(\kappa AG)_{i+1}} - a_{i+1} \end{aligned} \quad (3.9)$$

$$\begin{aligned} v_i = v_{i+1} &\rightarrow \frac{H}{(EI)_i} \left(\frac{h z_i^2}{2} + \frac{z_i^3}{6} \right) + a_i z_i + b_i \\ &= \frac{H}{(EI)_{i+1}} \left(\frac{h z_i^2}{2} + \frac{z_i^3}{6} \right) + a_{i+1} z_i + b_{i+2} \end{aligned} \quad (3.10)$$

where z_i is the coordinate of the interface between section i and section $i + 1$ and $(EI)_i$, $(\kappa AG)_i$ are respectively the flexural and shear stiffnesses of the i^{th} section.

Boundary conditions

The measured data from the LVDTs and SUIs are used as boundary conditions to solve for the unknown parameters, together with the continuity conditions. Note that the SUIs measure the rotation of the neutral axis of the stick-up. The boundary conditions can be written as:

$$\theta_{\text{SUI}} = -\frac{H}{(EI)_i} \left(h z_{\text{SUI}} + \frac{z_{\text{SUI}}^2}{2} \right) - a_i \quad (3.11)$$

$$v_{\text{LVDT}} = \frac{H}{(EI)_i} \left(\frac{h z_{\text{LVDT}}^2}{2} + \frac{z_{\text{LVDT}}^3}{6} \right) + a_i z_{\text{LVDT}} + b_i \quad (3.12)$$

where θ_{SUI} is the rotation measured by a SUI at height z_{SUI} and v_{LVDT} is a LVDT measurement made at height z_{LVDT} . The optimisation process for the ground level response uses all 4 LVDTs and the lowest inclinometer (except in cases where the readings from one or more of these instruments was judged to be unreliable, in which case a reduced data set was used). This set of data provides accurate and reliable results for the ground level response throughout the majority of the pile tests.

Instrument weighting

In the least squares optimisation, both types of instrumentation should have an appropriate weighting such that the solution is not biased towards a specific set of instrumentation. To achieve this, the weighting factors reflect the anticipated magnitudes of measurements in each set of data. The expected displacement or rotation at failure are estimated using a rigid pile rotation of $\psi_G = 2^\circ$ and a ground displacement of $v_G = 0.1D$, illustrated in Figure 3.5. The lowest inclinometer is valued as much as one pair of LVDTs resulting in an additional factor of 2 in the inclinometer weighting.

$$w_{\text{INCL}} = \frac{2}{2^\circ} \quad (3.13)$$

$$w_{\text{LVDT}i} = \frac{1}{0.1D - z_{\text{LVDT}i} \tan 2^\circ} \quad (3.14)$$

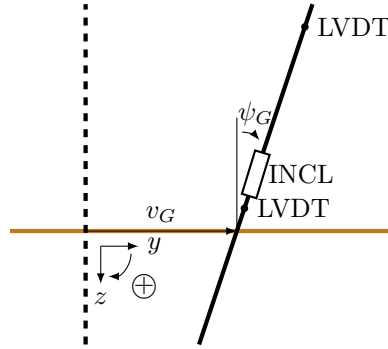


Figure 3.5: Illustration of pile rotation for weighting factor calculation.

Least-squares solution to the problem

(a) Small diameter piles

In this case the stick-up consists of a single section with flexural stiffness EI and shear stiffness κAG . The boundary condition equation is $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{x}^T = \begin{bmatrix} a & b \end{bmatrix}$ and:

$$\mathbf{A} = \begin{bmatrix} z_{\text{LVDT1}} & 1 \\ z_{\text{LVDT2}} & 1 \\ z_{\text{LVDT3}} & 1 \\ z_{\text{LVDT4}} & 1 \\ -1 & 0 \end{bmatrix} ; \mathbf{b} = \begin{bmatrix} v_{\text{LVDT1}} - \frac{H}{EI} \left(\frac{hz_{\text{LVDT1}}^2}{2} + \frac{z_{\text{LVDT1}}^3}{6} \right) \\ v_{\text{LVDT2}} - \frac{H}{EI} \left(\frac{hz_{\text{LVDT2}}^2}{2} + \frac{z_{\text{LVDT2}}^3}{6} \right) \\ v_{\text{LVDT3}} - \frac{H}{EI} \left(\frac{hz_{\text{LVDT3}}^2}{2} + \frac{z_{\text{LVDT3}}^3}{6} \right) \\ v_{\text{LVDT4}} - \frac{H}{EI} \left(\frac{hz_{\text{LVDT4}}^2}{2} + \frac{z_{\text{LVDT4}}^3}{6} \right) \\ \theta_{\text{SUI1}} + \frac{H}{EI} \left(hz_{\text{SUI1}} + \frac{z_{\text{SUI1}}^2}{2} \right) \end{bmatrix} \quad (3.15)$$

In this case (since the stick-up is formed from a uniform section) a separate continuity condition is not required.

(b) *Mid and large diameter piles*

For the mid and large diameter piles (for which the stick-up is modelled as three connected sections) the boundary conditions (Equation 3.11 and 3.12) are written as $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{x}^T = [a_1 \ b_1 \ a_2 \ b_2 \ a_3 \ b_3]$ is a vector of undetermined parameters and $\mathbf{A}$, $\mathbf{b}$ are:

$$\mathbf{A} = \begin{bmatrix} z_{\text{LVDT1}} & 1 & 0 & 0 & 0 & 0 \\ z_{\text{LVDT2}} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & z_{\text{LVDT1}} & 1 & 0 & 0 \\ 0 & 0 & z_{\text{LVDT1}} & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} v_{\text{LVDT1}} - \frac{H}{(EI)_1} \left(\frac{hz_{\text{LVDT1}}^2}{2} + \frac{z_{\text{LVDT1}}^3}{6} \right) \\ v_{\text{LVDT2}} - \frac{H}{(EI)_1} \left(\frac{hz_{\text{LVDT2}}^2}{2} + \frac{z_{\text{LVDT2}}^3}{6} \right) \\ v_{\text{LVDT3}} - \frac{H}{(EI)_2} \left(\frac{hz_{\text{LVDT3}}^2}{2} + \frac{z_{\text{LVDT3}}^3}{6} \right) \\ v_{\text{LVDT4}} - \frac{H}{(EI)_2} \left(\frac{hz_{\text{LVDT4}}^2}{2} + \frac{z_{\text{LVDT4}}^3}{6} \right) \\ \theta_{\text{SUI1}} + \frac{H}{(EI)_1} \left(hz_{\text{SUI1}} + \frac{z_{\text{SUI1}}^2}{2} \right) \end{bmatrix} \quad (3.16)$$

Note that this equation is for the case where the upper LVDTs are placed on the flanged connection. A slightly modified form of Equation 3.16 is adopted for the case where the upper LVDTs are placed on the transition piece. The continuity equations (equation 3.9 and 3.10) are written as $\mathbf{Cx} = \mathbf{d}$, where:

$$\mathbf{C} = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 & 0 \\ z_1 & 1 & -z_1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & z_2 & 1 & -z_2 & -1 \end{bmatrix} \quad \mathbf{d} = H \begin{bmatrix} \left(\frac{1}{(EI)_1} \left(hz_1 + \frac{z_1^2}{2} \right) + \frac{1}{(\kappa AG)_1} \right) - \left(\frac{1}{(EI)_2} \left(hz_1 + \frac{z_1^2}{2} \right) + \frac{1}{(\kappa AG)_2} \right) \\ \frac{1}{(EI)_1} \left(\frac{hz_1^2}{2} + \frac{z_1^3}{6} \right) - \frac{1}{(EI)_2} \left(\frac{hz_1^2}{2} + \frac{z_1^3}{6} \right) \\ \left(\frac{1}{(EI)_2} \left(hz_2 + \frac{z_2^2}{2} \right) + \frac{1}{(\kappa AG)_2} \right) - \left(\frac{1}{(EI)_3} \left(hz_2 + \frac{z_2^2}{2} \right) + \frac{1}{(\kappa AG)_3} \right) \\ \frac{1}{(EI)_2} \left(\frac{hz_2^2}{2} + \frac{z_2^3}{6} \right) - \frac{1}{(EI)_3} \left(\frac{hz_2^2}{2} + \frac{z_2^3}{6} \right) \end{bmatrix} \quad (3.17)$$

(c) Weighted least-squares solution

The weighting on the instrumentation is applied to the boundary conditions as: $\mathbf{A}^{\mathbf{w}} = \text{diag}(\mathbf{w})\mathbf{A}$ and $\mathbf{b}^{\mathbf{w}} = \text{diag}(\mathbf{w})\mathbf{b}$, where $\text{diag}(\cdot)$ denotes the diagonal matrix of a vector and $\mathbf{w}^T = \begin{bmatrix} w_{\text{LVDT1}} & w_{\text{LVDT2}} & w_{\text{LVDT3}} & w_{\text{LVDT4}} & w_{\text{INCL}} \end{bmatrix}$ is the vector of weighting factors. The undetermined parameters are now determined by finding the optimal solution to:

$$\text{minimise : } \|\mathbf{A}^{\mathbf{w}}\mathbf{x} - \mathbf{b}^{\mathbf{w}}\|^2 \quad (3.18)$$

where $\|\cdot\|$ is the norm of a vector. For the small diameter piles an unconstrained solution to Equation 3.18 is found, where $\mathbf{A}$ and $\mathbf{b}$ are given by Equation 3.15. In Matlab, the function *mldivide* can be used to find the solution. For the mid and large diameter piles, $\mathbf{A}$ and $\mathbf{b}$ are given by Equation 3.16. In this case the least squares solution (Equation 3.18) is constrained by $\mathbf{C}\mathbf{x} = \mathbf{d}$ where $\mathbf{C}$ and $\mathbf{d}$ are given in Equation 3.17. The solution can be found using the *lsqlin* function available in Matlab.

Inclinometer data smoothing

Since the raw inclinometer data are subject to noise, these data are smoothed before being adopted in the calculation of the ground displacement and rotation. This smoothing is performed using a zero phase digital filter, which applies a Gaussian window of 41 samples width (4.1 seconds width) in both the forward and reverse filtering directions. For tests in which the pile loading and displacement were varying rapidly (including CS1, CM6, CM7, DS2, DM1 and DM6), a smaller Gaussian window was used, as described in Appendix A of the PISA factual report (PISA AWG, 2015a). Whilst smaller window widths result in greater fluctuations of the calculated displacements and rotations, these values retain an accurate reflection of the measured signals.

Displacement transducer rotation

The analysis described above is based on the assumption that the LVDTs remain horizontal as the pile is rotated. However, since the LVDTs are pinned to the pile, some rotation will occur (see Figure 3.6). As a consequence, the movement recorded by each LVDT will not correspond precisely to the lateral movement v_{LVDT} of the pile. To account for this in the optimisation procedure, an iterative approach is adopted in which the length of the displacement transducers, accounting for the rotation of the pile, is computed from the current results of the optimisation. The solution is then iterated until the computed displacement transducer lengths converge to their measured values. The ground level displacement v_G and rotation θ_G are derived from this converged solution.

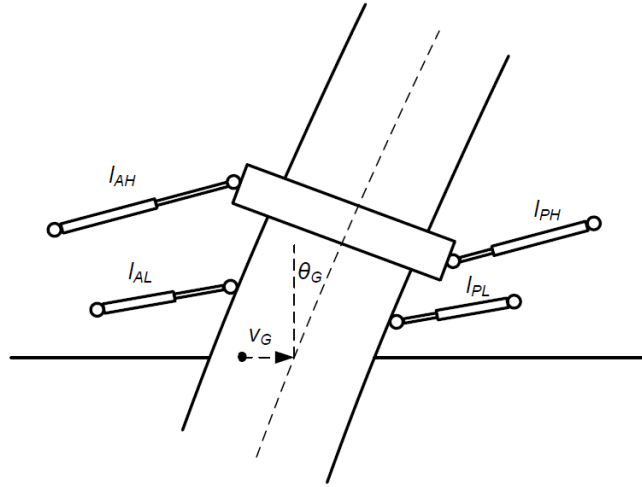


Figure 3.6: Illustration of exaggerated pile and transducer rotation (PISA AWG, 2015a).

Example calculation

The approximation of the stick-up deflection and rotation can be compared to the measurements of the remaining instruments. This is demonstrated in Figure 3.7.

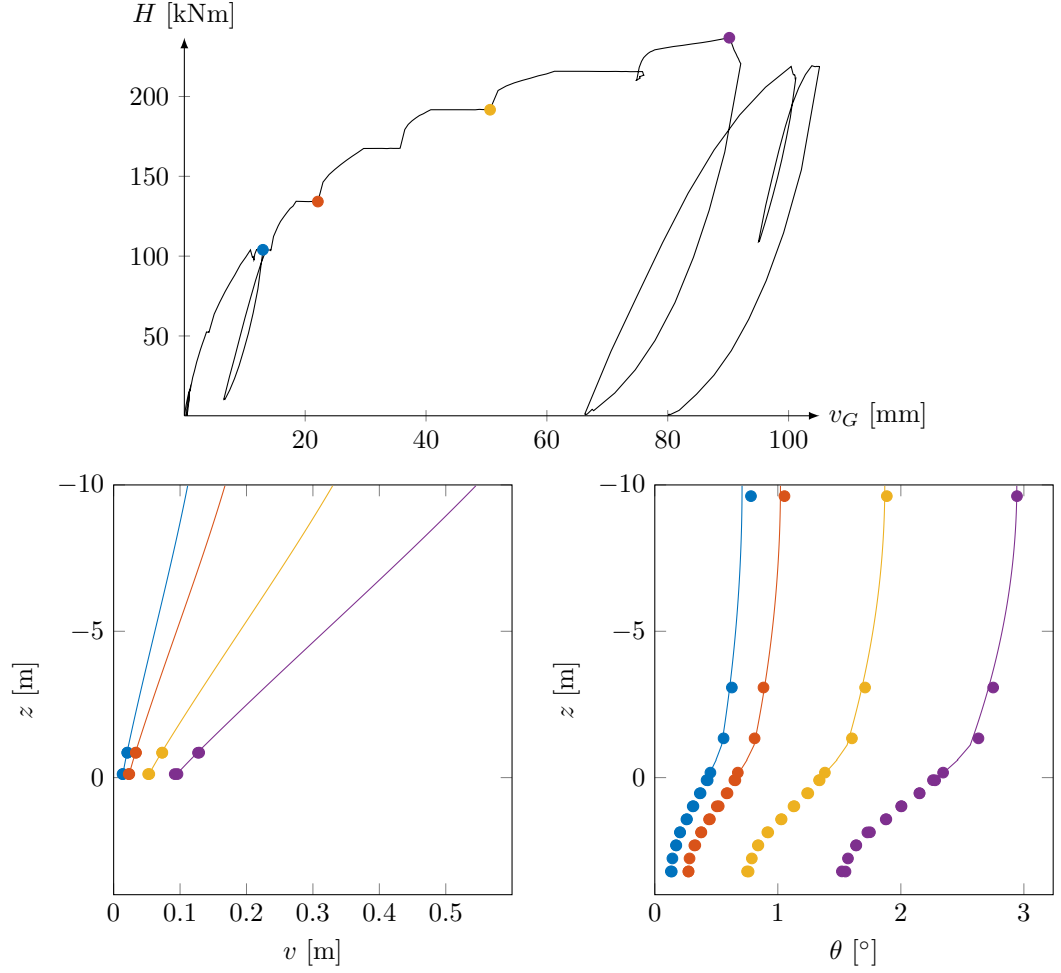


Figure 3.7: Calculation of the ground level response for DM4 and illustration of the fit to data above ground level. Dots represent the measured values and solid lines represent the Timoshenko beam approximation of the pile deflection.

3.2.2 Calculation of the embedded pile response

For the PISA tests, a subset of the mid-diameter piles and all large diameter piles were instrumented with inclinometers, extensometers and fibre optic strain gauges, as illustrated in Figure 3.8. The data from these different sets of instrumentation, in combination with the calculated ground level displacement and rotation, allow for the calculation of the pile behaviour below ground level. A procedure to interpret the pile behaviour, based on Timoshenko beam theory and least squares optimisation, is described below.

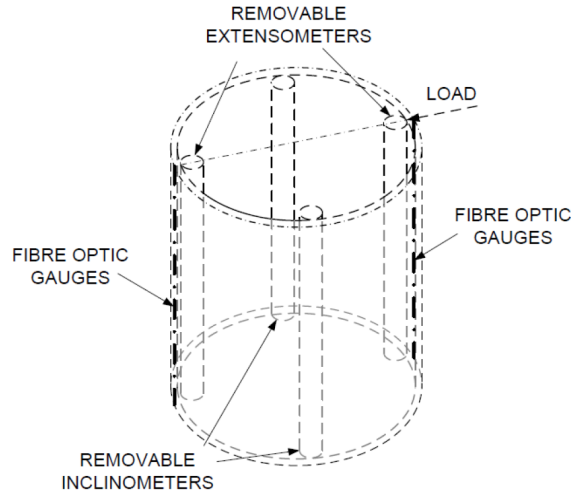


Figure 3.8: Illustration of the instrumentation below ground level (PISA AWG, 2016b).

Governing equations of the foundation model

The pile is divided into n equal sections along its depth (Figure 3.9), where the equivalent soil reaction is defined by:

$$p_{eq} = -(az + b) \quad (3.19)$$

It is noted that there are insufficient data to determine the distributed moment reaction along the pile and therefore this term is not carried through in the analysis.

The distributed moment term is therefore ignored and p_{eq} represents the equivalent distributed lateral soil reaction to fit the measured data. The shear force and moment along the foundation pile are found by integrating the equivalent soil reaction equation:

$$S = - \int p_{eq} dz = a \frac{z^2}{2} + bz + c \quad (3.20)$$

$$M = \int S dz = a \frac{z^3}{6} + b \frac{z^2}{2} + cz + d \quad (3.21)$$

Combining this with the governing equations of the Timoshenko beam gives:

$$\frac{M}{EI} = \left(\frac{d^2v}{dz^2} + \frac{1}{\kappa AG} \frac{dS}{dz} \right) \quad (3.22)$$

$$EI \frac{d^2v}{dz^2} = M - \lambda \frac{dS}{dz} = a \frac{z^3}{6} + b \frac{z^2}{2} + cz + d - \lambda(az + b) \quad (3.23)$$

where $\lambda = EI/\kappa AG$ is a shear factor. The adopted flexural rigidity (EI) of the foundation pile includes the effect of the tubes for extensometers and inclinometers. For the shear stiffness (κAG), only the circular section of the foundation pile is considered as the tubes are assumed to have a negligible influence on the shear stiffness. The cross section rotation, rotation of the neutral axis and lateral displacement are calculated by:

$$-EI\psi = a \frac{z^4}{24} + b \frac{z^3}{6} + c \frac{z^2}{2} + dz + e \quad (3.24)$$

$$-EI\theta = a \frac{z^4}{24} + b \frac{z^3}{6} + c \frac{z^2}{2} + dz + e - \lambda \left(a \frac{z^2}{2} + bz + c \right) \quad (3.25)$$

$$EIv = a \frac{z^5}{120} + b \frac{z^4}{24} + c \frac{z^3}{6} + d \frac{z^2}{2} + ez + f - \lambda \left(a \frac{z^3}{6} + b \frac{z^2}{2} + cz \right) \quad (3.26)$$

In the above equations, variables a to f are unknowns which have to be determined for each of the n sections of the pile.

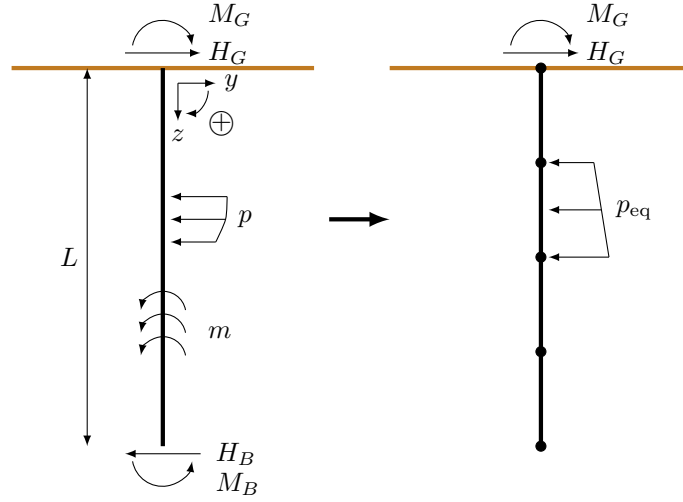


Figure 3.9: Illustration of the discretisation of the pile.

Continuity between the sections

At the interface of two sections, continuity is enforced for the displacement, cross-section rotation, moment, shear force and equivalent lateral soil reaction. This results in the following equations:

$$v_i = v_{i+1} \rightarrow a_i \frac{z_i^5}{120} + b_i \frac{z_i^4}{24} + c_i \frac{z_i^3}{6} + d_i \frac{z_i^2}{2} + e_i z_i + f_i - \lambda \left(a_i \frac{z_i^3}{6} + b_i \frac{z_i^2}{2} + c_i z_i \right) \quad (3.27)$$

$$\begin{aligned} &= a_{i+1} \frac{z_i^5}{120} + b_{i+1} \frac{z_i^4}{24} + c_{i+1} \frac{z_i^3}{6} + d_{i+1} \frac{z_i^2}{2} + e_{i+1} z_i + f_{i+1} \\ &\quad - \lambda \left(a_{i+1} \frac{z_i^3}{6} + b_{i+1} \frac{z_i^2}{2} + c_{i+1} z_i \right) \\ \psi_i &= \psi_{i+1} \rightarrow a_i \frac{z_i^4}{24} + b_i \frac{z_i^3}{6} + c_i \frac{z_i^2}{2} + d_i z_i + e_i \end{aligned} \quad (3.28)$$

$$\begin{aligned} M_i &= M_{i+1} \rightarrow a_i \frac{z_i^3}{6} + b_i \frac{z_i^2}{2} + c_i z_i + d_i \\ &= a_{i+1} \frac{z_i^3}{6} + b_{i+1} \frac{z_i^2}{2} + c_{i+1} z_i + d_{i+1} \end{aligned} \quad (3.29)$$

$$S_i = S_{i+1} \rightarrow a_i \frac{z_i^2}{2} + b_i z_i + c_i = a_{i+1} \frac{z_i^2}{2} + b_{i+1} z_i + c_{i+1} \quad (3.30)$$

$$p_i = p_{i+1} \rightarrow a_i z_i + b_i = a_{i+1} z_i + b_{i+1} \quad (3.31)$$

where z_i is the depth of the interface between sections i and $i + 1$. Note that the continuity of the equivalent soil reaction is a modelling choice to reduce the number of variables. For each additional section, the five additional equations provide six unknowns. The degree of non-determination of the foundation model is therefore equal to $\text{DoND} = 5 + n$.

Physical constraints

In the approach of finding a solution where the soil reaction is governed by the lateral soil reaction along the pile, the soil reactions at the base of the foundation are ignored and set to zero through constraint equations.

$$H_B = 0 = a_n \frac{L^2}{2} + b_n L + c_n \quad (3.32)$$

$$M_B = 0 = a_n \frac{L^3}{6} + b_n \frac{L^2}{2} + c_n L + d_n \quad (3.33)$$

Due to errors in the measurement data, the optimised solution for the soil reaction may give a reaction in the same direction as the displacement at ground level which is physically unrealistic. The lateral soil reaction at ground level is therefore also constrained to zero:

$$p_G = 0 = b_1 \quad (3.34)$$

Using these three additional constraints, the degree of non-determination reduces to: $\text{DoND} = 2 + n$.

Boundary conditions

The displacement, rotation of the neutral axis, moment and shear force at ground level (obtained from the ground level optimisation process in Section 3.2.1) provide boundary conditions which are used in combination with the measured data along

the foundation pile to solve for the unknowns:

$$v_G = \frac{f_1}{EI} \quad \theta_G = -\frac{e_1 - \lambda c_1}{EI} \quad M_G = d_1 \quad S_G = c_1 \quad (3.35)$$

Below ground level, the pile response is measured using inclinometers, extensometers and strain gauges. The extensometer data is found to be generally consistent with the fibre optic data. However, as the fibre optic system provides more accurate and reliable data, the extensometer data are not employed in the optimisation process. The boundary conditions from the strain gauges and inclinometers are written as:

$$\theta_{\text{IPI}} = -\frac{1}{EI} \left(a_i \frac{z_{\text{IPI}}^5}{120} + b_i \frac{z_{\text{IPI}}^4}{24} + c_i \frac{z_{\text{IPI}}^3}{6} + d_i \frac{z_{\text{IPI}}^2}{2} + e_i z_{\text{IPI}} + f_i \right) \quad (3.36)$$

$$-\lambda \left(a_i \frac{z_{\text{IPI}}^3}{6} + b_i \frac{z_{\text{IPI}}^2}{2} + c_i z_{\text{IPI}} \right) \quad (3.37)$$

$$M_{\text{FOS}} = a_i \frac{z_{\text{FOS}}^3}{6} + b_i \frac{z_{\text{FOS}}^2}{2} + c_i z_{\text{FOS}} + d_i \quad (3.38)$$

where θ_{IPI} is the rotation measured by the inclinometers and M_{FOS} is the bending moment determined from strains measured by the fibre optic system. Subscript i indicates which pile section the instrument is located on.

In finding an optimal fit to the measured data, an appropriate ratio needs to be used of the number of specified boundary conditions over the degree of non-determination. It is found that if this ratio is larger than 2 (discarding the number of measurements at the same depth on opposite sides of the pile) then the optimised fit gives good results. This limits the number of elements that can be employed.

Instrument weighting

In the least squares optimisation, it is desirable that the weighting given to the ground level data, inclinometer data and fibre optic data should be broadly comparable. Moreover, the weightings employed for the instruments are required to normalise the

data, to reflect the anticipated magnitudes of the measurements in each set of data.

The weighting factors using this approach are specified below:

$$w_v = \frac{1}{0.1D} \quad w_\theta = \frac{1}{2^\circ} \quad w_M = \frac{1}{M_{0.1D}} \quad w_S = \frac{1}{H_{0.1D}} \quad (3.39)$$

where the moment $M_{0.1D}$ and horizontal force $H_{0.1D}$ at ground displacement $v_G = 0.1D$ are extracted from the ground level response (PISA AWG, 2015a). This ground level displacement is considered to represent failure of the foundation pile.

Least-squares solution to the problem

Values of the undetermined parameters a_1 to f_n are calculated by finding a least-squares solution subject to the boundary conditions and continuity equations. The equations are expressed as $\mathbf{Ax} = \mathbf{b}$, for the boundary conditions and $\mathbf{Cx} = \mathbf{d}$ for the continuity equations and physical constraints. In these equations $\mathbf{x}^T = [\mathbf{x}_1 \dots \mathbf{x}_n]$, with $\mathbf{x}_i = [a_i \ b_i \ c_i \ d_i \ e_i \ f_i]$ is the vector of unknowns to be determined.

The matrix for the continuity equations and physical constraints $\mathbf{C}$ is a $5(n-1)+3$ by $6n$ matrix and $\mathbf{b}$ is a vector with $5(n-1)+3$ elements. These are constructed as:

$$\mathbf{C}_i = \begin{bmatrix} \frac{z_i^5}{120} - \lambda \frac{z_i^3}{6} & \frac{z_i^4}{24} - \lambda \frac{z_i^2}{2} & \frac{z_i^3}{6} - \lambda z_i & \frac{z_i^2}{2} & z_i & 1 \\ \frac{z_i^4}{24} & \frac{z_i^3}{6} & \frac{z_i^2}{2} & z_i & 1 & 0 \\ \frac{z_i^3}{6} & \frac{z_i^2}{2} & z_i & 1 & 0 & 0 \\ \frac{z_i^2}{2} & z_i & 1 & 0 & 0 & 0 \\ z_i & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{d}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.40)$$

$$\mathbf{C}_G = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{d}_G = \begin{bmatrix} 0 \end{bmatrix} \quad (3.41)$$

$$\mathbf{C}_B = \begin{bmatrix} \frac{L^2}{2} & L & 1 & 0 & 0 & 0 \\ \frac{L^3}{6} & \frac{L^2}{2} & L & 1 & 0 & 0 \end{bmatrix} \quad \mathbf{d}_B = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (3.42)$$

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_G & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \hline \mathbf{C}_1 & -\mathbf{C}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 & -\mathbf{C}_2 & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{C}_{n-1} & \mathbf{C}_{n-1} \\ \hline \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{C}_B \end{bmatrix} \quad \mathbf{d} = \begin{bmatrix} \mathbf{d}_G \\ \hline \mathbf{d}_1 \\ \mathbf{d}_2 \\ \vdots \\ \mathbf{d}_{n-1} \\ \hline \mathbf{d}_B \end{bmatrix} \quad (3.43)$$

For the ground level boundary conditions, the rows in the $\mathbf{A}_G$ and $\mathbf{b}_G$ matrices are written as:

$$\mathbf{A}_G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1/EI \\ 0 & 0 & \lambda/EI & 0 & -1/EI & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{b}_G = \begin{bmatrix} v_G \\ \theta_G \\ M_G \\ S_G \end{bmatrix} \quad (3.44)$$

For the weighting of the different ground level values, the previous matrices are multiplied with a weighting matrix: $\mathbf{A}_G^w = \text{diag}(\mathbf{w}_G)\mathbf{A}_G$ and $\mathbf{b}_G^w = \text{diag}(\mathbf{w}_G)\mathbf{b}_G$, where $\mathbf{w}_G = \begin{bmatrix} w_v & w_\theta & w_M & w_S \end{bmatrix}$. For each data point of the inclinometers and the fibre optics, a row in the boundary condition matrices can be written as:

$$\mathbf{A}_i^{\text{IPI}} = -\frac{1}{EI} \begin{bmatrix} \frac{z_{\text{IPI}}^4}{24} - \lambda \frac{z_{\text{IPI}}^2}{2} & \frac{z_{\text{IPI}}^3}{6} - \lambda z_{\text{IPI}} & \frac{z_{\text{IPI}}^2}{2} - \lambda & z_{\text{IPI}} & 1 & 0 \end{bmatrix} \mathbf{b}_i^{\text{IPI}} = \begin{bmatrix} \theta_{\text{IPI}} \end{bmatrix} \quad (3.45)$$

$$\mathbf{A}_i^{\text{FOS}} = \begin{bmatrix} \frac{z_{\text{FOS}}^3}{6} & \frac{z_{\text{FOS}}^2}{2} & z_{\text{FOS}} & 1 & 0 & 0 \end{bmatrix} \mathbf{b}_i^{\text{FOS}} = \begin{bmatrix} M_{\text{FOS}} \end{bmatrix} \quad (3.46)$$

The total weighted matrices are assembled as:

$$\mathbf{A}_i^w = \begin{bmatrix} w_\theta \mathbf{A}_i^{\text{IPI}} \\ w_M \mathbf{A}_i^{\text{FOS}} \end{bmatrix} \quad \mathbf{b}_i^w = \begin{bmatrix} w_\theta \mathbf{b}_i^{\text{IPI}} \\ w_M \mathbf{b}_i^{\text{FOS}} \end{bmatrix} \quad (3.47)$$

$$\mathbf{A}^w = \begin{bmatrix} \mathbf{A}_G^w & \mathbf{0} & \cdots & \mathbf{0} \\ \hline \mathbf{A}_1^w & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2^w & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{A}_n^w \end{bmatrix} \quad \mathbf{b}^w = \begin{bmatrix} \mathbf{b}_G^w \\ \hline \mathbf{b}_1^w \\ \mathbf{b}_2^w \\ \vdots \\ \mathbf{b}_n^w \end{bmatrix} \quad (3.48)$$

The unknowns are determined by finding the solution to the following problem which can be solved using the *lsqlin* function in Matlab.

$$\text{minimise : } \|\mathbf{A}^w \mathbf{x} - \mathbf{b}^w\|^2 \quad (3.49)$$

$$\text{subject to : } \mathbf{C} \mathbf{x} = \mathbf{d} \quad (3.50)$$

Example calculation

The method described above has been applied to DM4 at the 4 load steps indicated in Figure 3.7. The results using the approach are visualised in Figure 3.10.

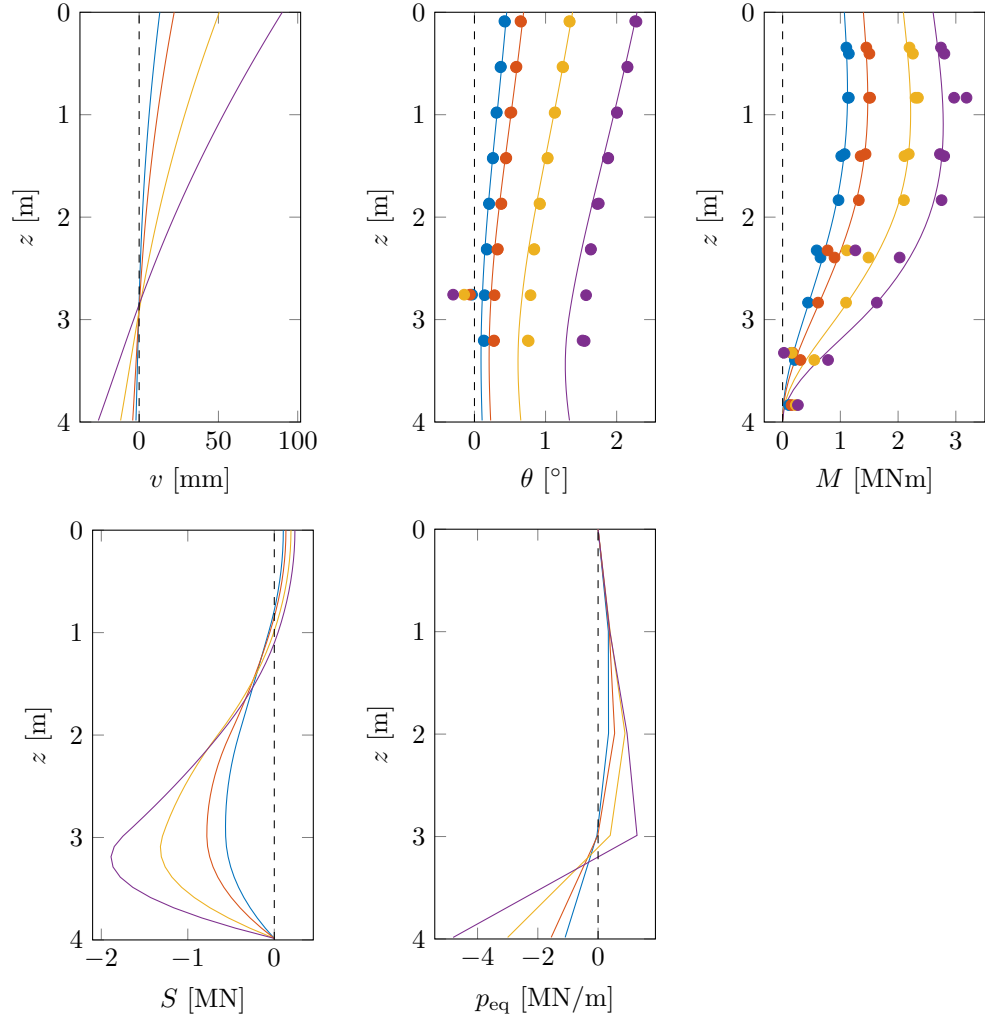


Figure 3.10: Calculation of the below ground level response for DM4 and illustration of the fit to the inclinometer and strain gauge data. Colours refer to the points in Figure 3.7. Dots are measured values and solid lines represent the fitted behaviour of the foundation pile.

3.3 Cyclic loading analysis

The analysis of pile behaviour under cyclic loads covers accumulated displacement and rotation, change of stiffness, and energy dissipated in each load cycle. Firstly the load cycles are identified between two minima in the load history as shown in Figure 3.11.

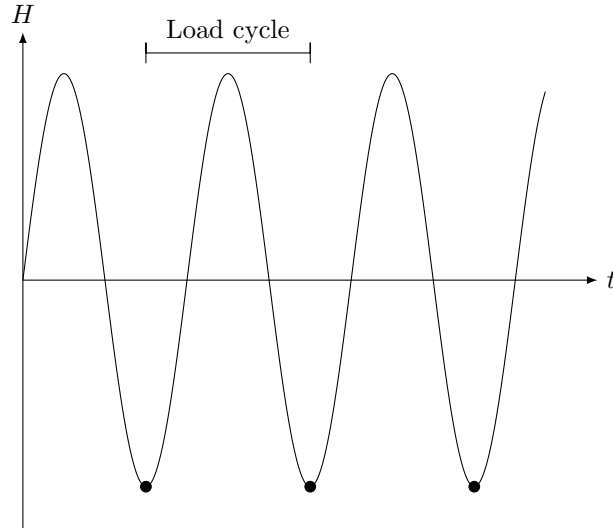


Figure 3.11: Identification of a load cycle.

3.3.1 Work and dissipated energy

The dissipated energy can be derived from the external work on the stick-up pile. An increment in external work is given by:

$$dW_{\text{ext}} = -(H(v)dv + M(\psi)d\psi) \quad (3.51)$$

At the pile head, no external moment is applied, however if a cross section at the ground level is considered, the moment is equal to: $M_G = hH$. The dissipated energy

in a load cycle can be calculated by integrating the work in each cycle:

$$E_{\text{diss}} = - \int_{\text{Cycle}} dW_{\text{ext}} = \int_{\text{Cycle}} H(v)dv + \int_{\text{Cycle}} M(\psi)d\psi \quad (3.52)$$

The dissipated energy considered at the pile head is equal to the dissipated energy considered at ground level, assuming there is no energy loss in the stick-up pile. The dissipated energy is expected to mainly result from the non-linearity in the soil-structure interaction. The internal work of the foundation pile is given by:

$$\begin{aligned} dW_{\text{int}} = & \int_0^L \left[d \left(\frac{d\psi}{dz} \right) EI \frac{d\psi}{dz} + d \left(\frac{dv}{dz} + \psi \right) GA \kappa \left(\frac{dv}{dz} + \psi \right) \right] dz \\ & + \int_0^L [p(z, v)dv + m(z, \psi)d\psi] dz + H_B(v_B)dv_B + M_B(\psi_B)d\psi_B \end{aligned} \quad (3.53)$$

The dissipated energy could in theory also be calculated by integrating these terms over a load cycle. However, the measurements are generally not sufficiently resolved to recreate accurate force-displacement and moment-rotation curves at the pile base. Moreover, not enough measured components of section forces are available to extract a distributed moment-rotation curve. Using the approach in section 3.2.2, an equivalent distributed load can be calculated which ignores the effect of the distributed moment and base terms. Assuming no change in the internal energy of the pile after a load cycle (the pile is modelled elastically and does not dissipate energy), the dissipated energy from the internal work can be approximated by integrating the hysteresis loops of the equivalent distributed soil-reaction:

$$E_{\text{diss}} = \int_{\text{Cycle}} dW_{\text{int}} = \int_0^L \int_{\text{Cycle}} p_{\text{eq}}(z, v) dv dz \quad (3.54)$$

3.3.2 Foundation stiffness and energy dissipation

The stiffness k for each load cycle is identified by fitting a line through the data points in the load-displacement and moment-rotation plot using a least squares fit as shown in Figure 3.12 (a). The stiffness is then equal to the slope of the line. In further analysis, the stiffness of a load-displacement loop is denoted k_v and k_ψ for the moment-rotation loop. The elastic stored energy under symmetric loading is calculated by:

$$E_{\text{el}} = \frac{\max(\epsilon\sigma)}{2} \approx \frac{\sigma_{\text{max}}^2}{2k} \quad (3.55)$$

where the parameter ϵ indicates displacement (or rotation) and the parameter σ denotes load (or moment). In this calculation, the results are most consistent if the latter half of the equation is used to calculate the elastic stored energy, using the target applied load/moment as σ_{max} . In the case of non symmetric cyclic loading, the elastic energy is calculated as: $E_{\text{el}} \approx \frac{(\sigma_{\text{max}} - \sigma_{\text{min}})^2}{8k}$ (under symmetric loading, this reduces to: $\frac{(\sigma_{\text{max}} + \sigma_{\text{min}})^2}{8k} = \frac{(\sigma_{\text{max}})^2}{2k}$).

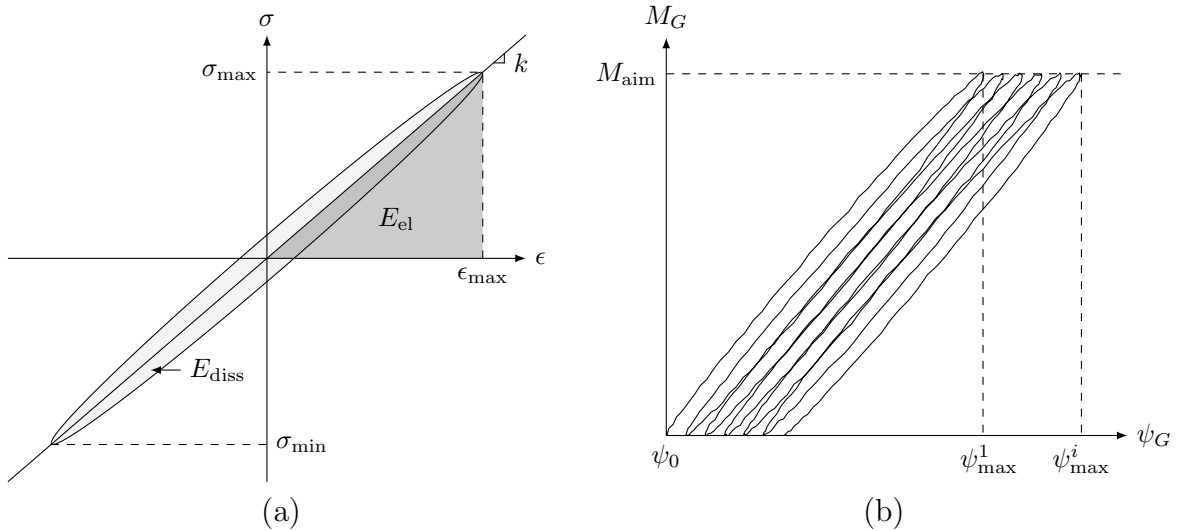


Figure 3.12: (a) Illustration of the stiffness, elastic and dissipated energy. (b) Illustration of the accumulation of rotation.

The dissipated energy in the hysteresis loop is calculated by integrating over the

loop and reconnecting the last point to the first: $E_{\text{diss}} = \int \sigma(\epsilon) d\epsilon$. The damping ratio is then defined as:

$$\xi = \frac{1}{4\pi} \frac{E_{\text{diss}}}{E_{\text{el}}} \quad (3.56)$$

The damping ratio of a load-displacement loop is denoted ξ_v and ξ_ψ for the moment-rotation loop at ground level. Even though the damping ratio in the moment-rotation response is often lower than the damping ratio in the load-displacement response, the contribution to the total dissipated energy can be higher. The total elastic and dissipated energy at ground level are the combination of these values from the load-displacement and moment-rotation response:

$$E_{\text{el}}^{\text{tot}} = E_{\text{el}}^v + E_{\text{el}}^\psi \quad (3.57)$$

$$E_{\text{diss}}^{\text{tot}} = E_{\text{diss}}^v + E_{\text{diss}}^\psi \quad (3.58)$$

The damping ratio can also be defined for the total energy at ground level: $\xi_{\text{tot}} = \frac{1}{4\pi} \frac{E_{\text{diss}}^{\text{tot}}}{E_{\text{el}}^{\text{tot}}}$. Though this value has a clear physical meaning, instrumentation errors may affect the results of load-displacement more than the moment-rotation loops or vice versa. The damping ratios are therefore presented separately for the load-displacement and moment-rotation. These separate values could also be used in the development of marco-element foundation models at ground level.

3.3.3 Accumulated pile rotation

A schematic illustration of the moment-rotation response is given in Figure 3.12 (b), where M_{aim} is the target ground level moment for the cyclic load set. The normalised accumulated rotation is defined as:

$$\bar{\psi}_{\text{acc}} = \frac{\psi_{\text{max}}^{\star i} - \psi_{\text{max}}^{\star 1}}{2^\circ} \quad (3.59)$$

where 2° is defined as the failure rotation of the pile in the PISA project and the corrected maximum rotation is:

$$\psi_{\max}^{\star i} = \psi_{\max}^i + (M_{\text{aim}} - M_{\max})/k_\psi \quad (3.60)$$

The corrected maximum rotation accounts for errors in the load control, where the maximum ground level moment in a cycle $M_{\max}$ can vary from the target moment M_{aim} . This can have a significant effect on the cycles at low loading amplitudes.

3.4 Foundation behaviour

Whilst the PISA project was aimed at capturing the monotonic backbone curve, a series of additional effects have been explored to improve the understanding of the foundation behaviour. In this section, the test procedure for the monotonic tests is outlined first. This provides a baseline for comparison of tests where the loading routine has been varied. This is followed by an analysis of rate effects and of the effects during one-way and two-way cyclic loading.

3.4.1 Monotonic field response

The monotonic tests are conducted as a series of load steps followed by load hold phases. The load steps are performed at constant displacement rates and the creep displacement during the load holds provide information for the viscosity effects of the foundation response. The general loading routine for the monotonic tests in the PISA project is given below (PISA AWG, 2015*a*):

- A small initial load-unload loop at a displacement rate of $\dot{v}_G = \frac{D}{500\text{min}}$ to $v_G = D/1000$ was performed before the start of the main load phase. The load was held for 5 minutes, which allowed time to check if the instrumentation was

functioning as expected.

- The main load phase consists of load steps and load holds. The load steps were performed to have a constant displacement rate of $\dot{v}_G = \frac{D}{300\text{min}}$ until the target displacement of the load step was reached (Table 3.1).
- After the target displacement was reached, the load was held constant to allow for creep in the soil. The load hold was sustained until the creep displacement rate slowed down to $\dot{v}_G < \frac{D}{100000\text{min}}$ or for a maximum period of 30 minutes, before starting the next load step.
- After the third load step, an unload-reload loop was performed at a loading rate of $\dot{v}_G = \frac{D}{500\text{min}}$. The pile was unloaded to approximately 10% of the load in the third load step. This load was sustained for 10 minutes before reloading to the same load as in load step 3, which was then maintained for another 10 minutes before the start of the next load step.
- If the creep displacement during a load hold resulted in exceeding the target displacement of the next load step, that load step was skipped and the subsequent target displacement was used.
- If the failure rotation of 2° was not reached in the final load step, an additional load increment was performed until the failure rotation was reached.
- After the final load increment, the pile was unloaded at a rate of $\dot{v}_G = \frac{D}{500\text{min}}$. The pile was then monitored for another period of approximately 30 minutes to capture the creep displacement after the load test.

Following the monotonic loading routine, most foundation piles were subjected to a number of additional load-unload loops. For illustration, the load history of CM9 is given together with a load-displacement plot of the behaviour at ground level in

Figure 3.13. Note that CM9 was the first mid-diameter pile which was tested in the PISA programme and that the load control was still being optimised for more stable load control in later tests.

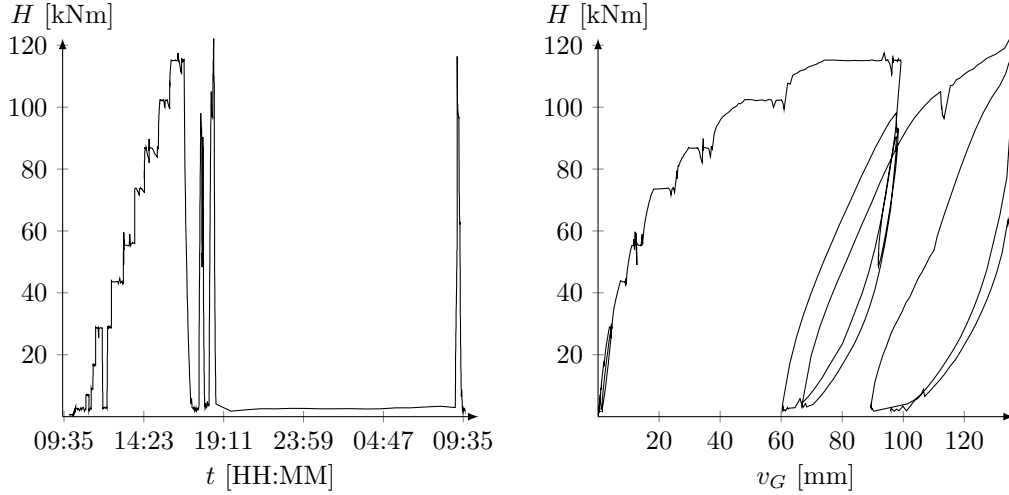


Figure 3.13: Applied load history and ground level load-displacement response of CM9.

3.4.2 Rate effects

The rate effect on piles can be observed in the hold phases of the monotonic tests and at the high rate tests. First, an overview of test results in Cowden is given, together with an initial analysis of the response with respect to rate effects. The final paragraph gives an analysis of the rate effects in Dunkirk. Tatsuoka et al. (2008) presents an overview of different viscosity effects in the behaviour of soils. Four different effects are identified (Figure 3.14): (1) *Isotache viscosity*, where the current stress σ is a function of the current strain ϵ and strain rate $\dot{\epsilon}$. In isotache viscosity, an increase in the strain rate results in an increased soil resistance. (2) *TESRA viscosity* (temporary or transient effects of strain rate and strain acceleration), where the changes in strain rate result in a temporary increase of the soil resistance with respect to the monotonic loading (ML) curve. (3) *Isotache-and-TESRA-Combined*,

Table 3.1: Target displacement for the load steps in the monotonic load test routine (PISA AWG, 2015a).

	Cowden	Dunkirk
Step No.	v_G/D (%)	v_G/D (%)
0	0.1	0.1
1	0.125	0.125
2	0.25	0.5
3	0.5	1.5
Unload-reload	-	-
4	1	2.5
5	1.5	4
6	2.5	5.5
7	4	6.75
8	6.5	8.25
9	10	10

where both isotache and TESRA behaviour are observed. (4) *Positive and Negative (P & N) viscosity*, where a step increase of the strain rate results in a temporary increase of the soil resistance followed by decrease with respect to the ML curve *i.e.* a positive TESRA type component and a negative isotache type component.

Cowden

At each test site, a pile was subjected to load steps at much higher displacement rates than in the monotonic tests. The load-displacement at ground level for the fast test (CM1) is given in Figure 3.15, compared to the monotonic test of CM9. The load-displacement response of CM1 indicates that isotache viscosity is the dominant viscosity type for this test. Isotache curves are added to CM1 based on the field test response, illustrating the difference in capacity at the different loading rates. The higher load rates in these tests could represent the impact of a storm wave or boat loading impact. At the faster load rates, a capacity increase of approximately 21% is seen for CM1. This corresponds to $8.7\%/\log_{10}(\dot{v}_G)$. Note that at pile failure, defined as $v_G = 0.1D$, not all the surrounding soil is failing, so that the relative increase

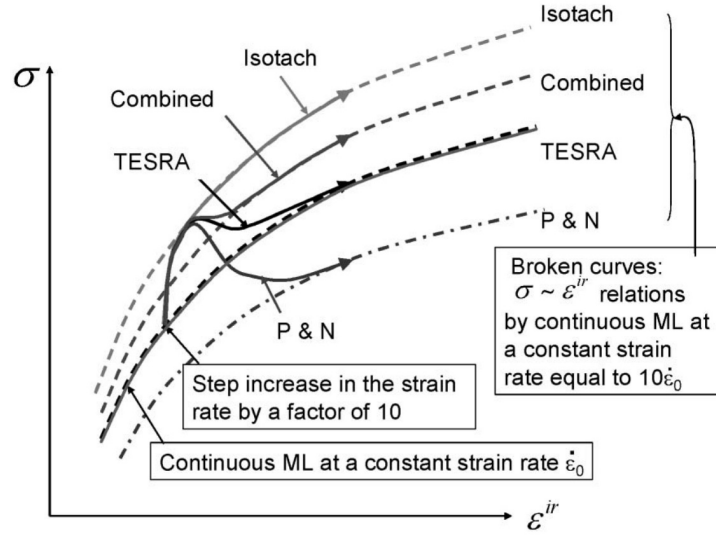


Figure 3.14: Schematic stress-strain curves for four viscosity types of soils in shear (Tatsuoka, 2007).

of strength does not directly reflect the increase of shear strength with strain rate in the soil. Nevertheless, this result compares quite well to rate tests for a CPT in clay by Dayal and Allen (1975), who observed an increase of the cone resistance of $10 - 25\% / \log_{10}(\dot{w})$ ($\dot{w}$ is the velocity of the CPT cone).

Accidentally, the first small diameter test pile CS1 was loaded at a very high rate, which resulted in an increased resistance compared to CS2, as illustrated in Figure 3.16. The comparison indicates an increase in strength of $24\% / \log_{10}(\dot{v}_G)$, which is much higher than seen in the mid-diameter test piles. It should be noted that rate effects need to be analysed based on the normalised displacement rate: $\dot{v}_G/D$. The corresponding strain rates in the soil are expected to be approximately 3 times higher for CS1 than for the fast rates in CM1.

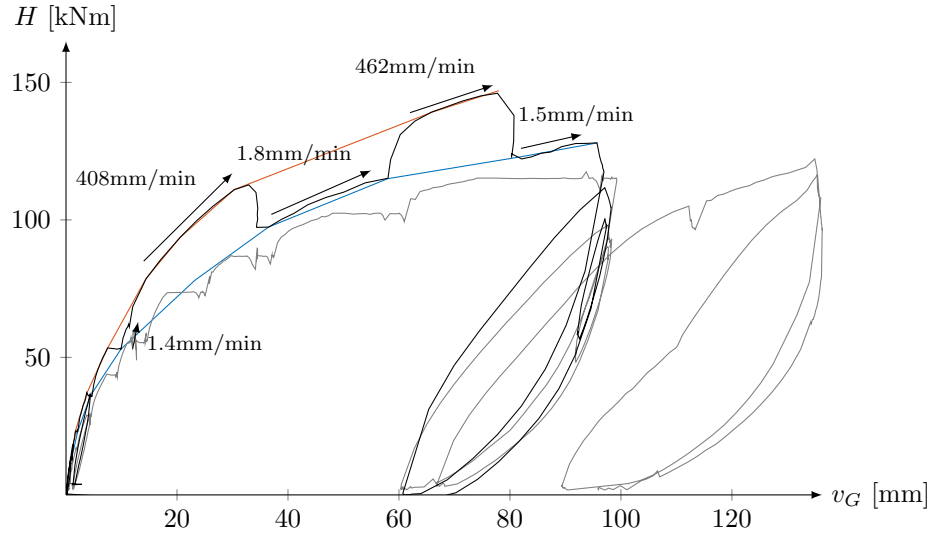


Figure 3.15: Comparison of the ground level load-displacement response of CM1 (—) and CM9 (—), along with a qualitative illustration of isotache curves fitted to CM1 at a slow rate (—: $\sim 1.5\text{mm/min}$) and a fast rate (—: $\sim 430\text{mm/min}$).

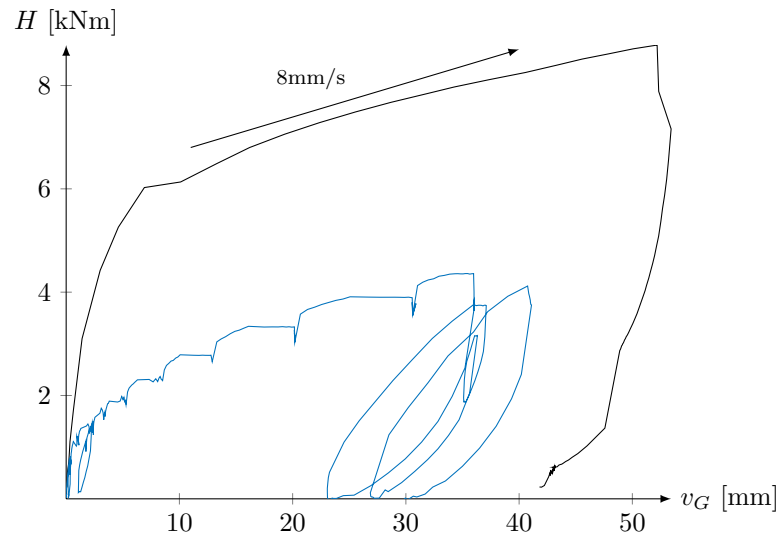


Figure 3.16: Comparison of the ground level load-displacement response of CS1 (—) and CS2 (—).

Dunkirk

Figure 3.17 compares test DM6, where fast loading is applied, to the monotonic tests of DM4 and DM9. The load-displacement response of DM6 indicates that it can be reasonably approximated using isotache viscosity, though combined isotache and TESRA viscosity may result in slightly improved results. Analysis of the response of DM6 indicates a 12% increase in resistance at the higher load rates, corresponding to an increase of $4.2\%/\log_{10}(\dot{v}_G)$. This rate effect is less significant than in the fast load test in Cowden. As noted by Mitchell (1976), strain rate effects are present for element testing of sand, however, they are considerably smaller than for clay.

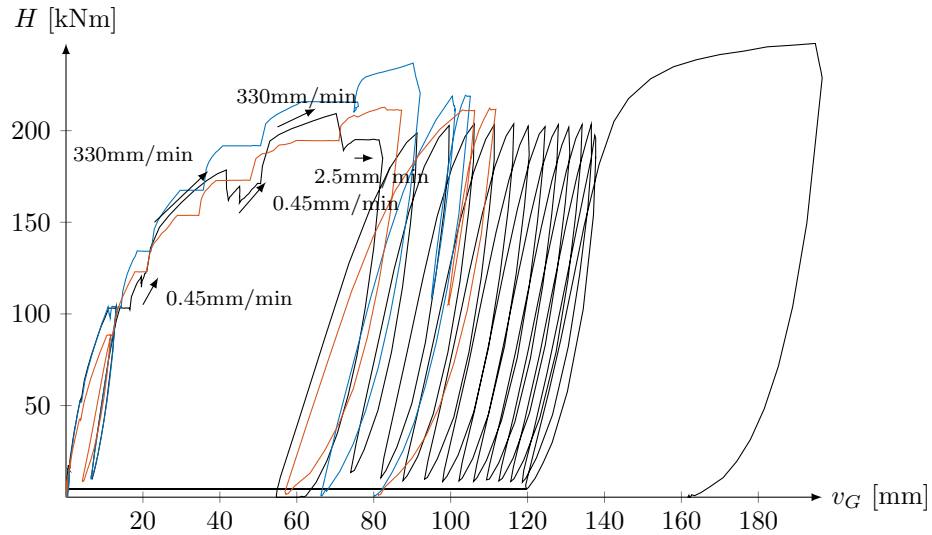


Figure 3.17: Ground level load-displacement response of DM6 (—) compared to DM4 (—) and DM9 (—).

Figure 3.18 shows a comparison of DS1 with DS2. Test DS1 was loaded over a period of 4.5h, resulting in an average load rate of 0.11mm/min. The comparison with DS2 therefore indicates a capacity increase of $6.2\%/\log_{10}(\dot{v}_G)$, which is comparable to the increase seen in DM6.

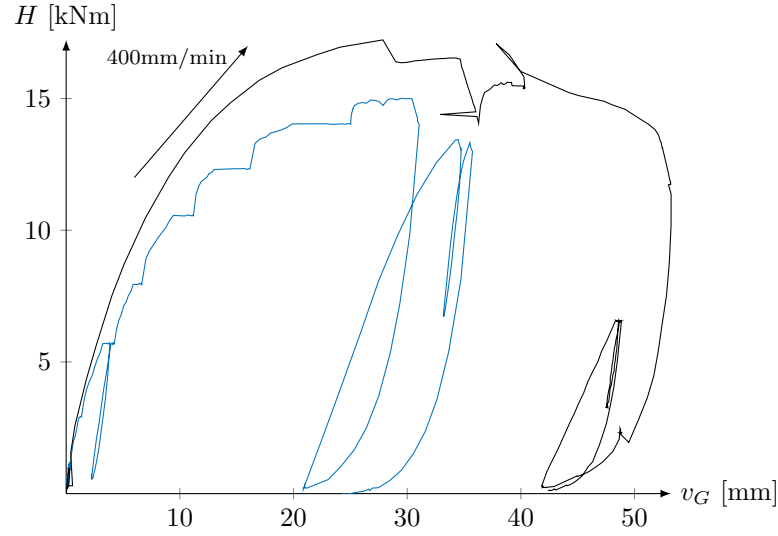


Figure 3.18: Ground level load-displacement response of DS1 (—) and DS2 (—).

3.4.3 Behaviour under 1-way cyclic loading

Within the PISA test programme, a subset of the piles have been subjected to 1-way cyclic loading (Table 3.2). The main focus of these tests is to capture the accumulation of the pile rotation with increasing cycle number. This effect (often referred to as ratcheting) could be a potential design driver for offshore wind turbines. For the structural fatigue analysis of offshore wind turbines, it is also necessary to have an accurate representation of the foundation stiffness and damping. The setup in the PISA project also allows for analysis of these aspects throughout the loading cycles.

Table 3.2: Test piles subjected to 1-way cyclic loading.

Test	Pile	D [m]	t [mm]	L [m]	L/D	M/H [m]	Cycles
15	CM6	0.762	11	4.00	5.25	10	56
18	CM5	0.762	11	4.00	5.25	10	27650
30	DM2	0.762	14	4.00	5.25	10	38110
40	DL1	2.00	38	10.50	5.25	10	588
	DL2	2.00	38	10.50	5.25	10	588
43	CL1	2.00	25	10.50	5.25	10	1060
	CL2	2.00	25	10.50	5.25	10	1060

The ratcheting behaviour for cyclic laterally loaded piles has been described by various authors using logarithmic expressions (Hettler, 1981; Verdure et al., 2003),

power expressions (Little and Briaud, 1988; Long and Vanneste, 1994; LeBlanc et al., 2010) or log-linear expressions (Grabe and Dührkop, 2008; Dührkop, 2010). Cuéllar (2011) presents an overview of these methods describing their applicability for the long term cyclic behaviour of monopile foundations. Currently, the most widely used expression for the cyclic accumulation of monopile foundations is given by LeBlanc et al. (2010), who propose the following expression for the accumulation of rotation $\Delta\theta$:

$$\Delta\theta = \theta_s T_b(\zeta_b) T_c(\zeta_c) N^{m_N} \quad (3.61)$$

where θ_s is the static rotation at the maximum applied load, T_b is a function of the maximum applied load ($\zeta_b = \frac{M_{\max}}{M_{\text{ult}}}$), T_c is a function of the load shape ($\zeta_c = \frac{M_{\min}}{M_{\max}}$), N is the cycle number and m_N is the exponent of the power law. The experimental tests by LeBlanc et al. (2010) were performed for small scale rigid piles in sand, where the cross section rotation ψ is the same as the overall rotation of the pile θ . A power of $m_N = 0.31$ was found to fit consistently with the tests performed at varying T_b and T_c .

Cowden

In Cowden, the CM6 test pile was subjected to a combination of the monotonic loading routine and cycles at three of the load steps (Table 3.3). The ground level moment-rotation response and the analysed results are given in Figure 3.19. The load is applied at a height of $h = 10\text{m}$ and therefore the maximum moment in the load sets is: $M = hH$. Figure 3.19 shows that the rotation accumulation increases, the rotational stiffness decreases and the damping ratio increases with amplitude of loading. The increase in damping ratio with load amplitude is consistent with the increase of damping ratio in soils for increasing strain rates (Vucetic and Dobry, 1991). After the third cyclic load set, the foundation pile was loaded to 112kN. The rate of loading for this final load was slower than in the load cycles, resulting in a

softer behaviour upon loading.

Table 3.3: Cyclic loading routine for CM6.

Load set	Max. Load [kN]	ζ_c	Cycles	Comment
1	43.5	0	10	Sinusoidal
2	70	0	10	Sinusoidal
3	90	0	38	Sinusoidal

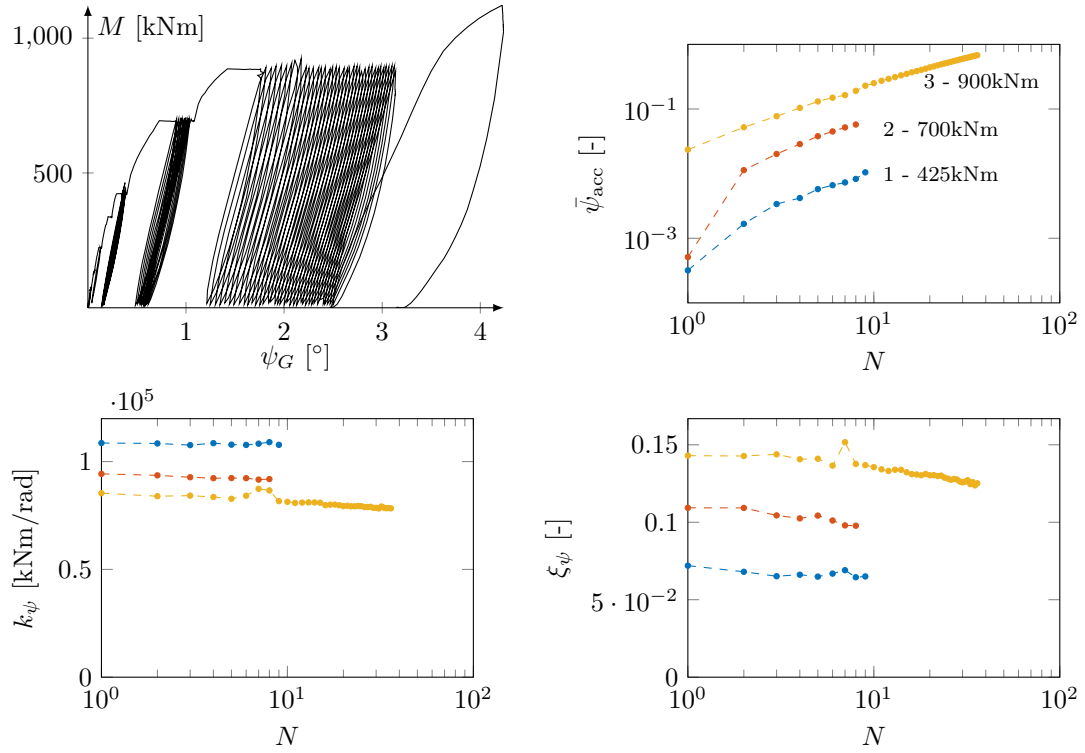


Figure 3.19: Moment rotation response of CM6 and accumulated rotation, stiffness and damping ratio in load set 1 (-●-), 2 (-●-) and 3 (-●-).

Test pile CM5 was subjected to a larger number of load cycles to investigate the effect of sustained cycling on the foundation behaviour. The load cycles are summarised in Table 3.4, with an illustration of the foundation response throughout the cycles in Figure 3.20. Halfway through load set 6, the load was accidentally dropped to -28kN. The analysed accumulated rotation, stiffness and damping for load sets 1 to 5 and 11 are given in Figure 3.21.

At the low load levels, the results are subject to measurement errors, which mainly

affect the interpreted rotation accumulation and damping. Qualitatively, the accumulation of rotation increases with load amplitude. However, at the highest load level (90kN), the rotation accumulation is lower for CM5 than for CM6. This effect might be attributed to a hardening process, where the previously applied load cycles affect the behaviour afterwards. Load sets 4 and 5 show little accumulation of rotation, which again indicates that the hardening process from the previous load sets affects the accumulation in the later load sets. The moment-rotation plot for load set 3 shows that the load cycles were applied without waiting for the strain relaxation from the rate effects to slow down. This leads to high accumulation of rotation in the first cycles, which is caused by a combination of rate and ratcheting effects. The accumulation in the initial cycles of load set 3 can therefore not be compared directly to the accumulation in the other load sets where the rate effect is less important.

The accumulation of rotation is compared to a power curve with a power coefficient of 0.31, which was observed in the small scale tests on monopiles in sand of LeBlanc et al. (2010). The trend of the power curve agrees reasonably well with the rotation accumulation of load sets 1, 2 and 3 at high cycle number. Note that the curvature of the accumulated rotation for load set 11 indicates that the curve at higher cycle numbers will tend to a smaller slope angle.

Both tests show a decreasing foundation stiffness with load amplitude. After load set 3 in CM5, the stiffness is not recovered for the lower load amplitudes. This effect is probably caused by gapping, which has been observed on site. The damping ratio shows some variation, but generally has an increasing trend with load amplitude.

Test piles CL1 and CL2 were subjected to cyclic loading following the monotonic test procedure. The cyclic loading routine is summarised in Table 3.5. The ground level moment-rotation response and analysed results are given in Figure 3.22. This figure shows again that the hardening process from the previous monotonic loading results in little accumulation of the pile rotation for the low amplitude load sets. The

Table 3.4: Cyclic loading routine for CM5.

Load set	Max. Load [kN]	ζ_c	Cycles	Comment
1	10	0	7000	Sinusoidal (+triangles)
2	20	0	7000	Sinusoidal
3	60	0	2500	Sinusoidal (+triangles)
4	20	0	1100	Sinusoidal (+triangles)
5	30	0	3300	Sinusoidal (+triangles)
6	40	0	1400	Sinusoidal (+triangles)
7	20	0	500	Varying loading frequency
8	40	0.5	1000	Sinusoidal
9	20-30-40	0	1500	Triangular
10	20-30-40-60	0	1000	Triangular
11	90	0	145	Sinusoidal (+triangles)

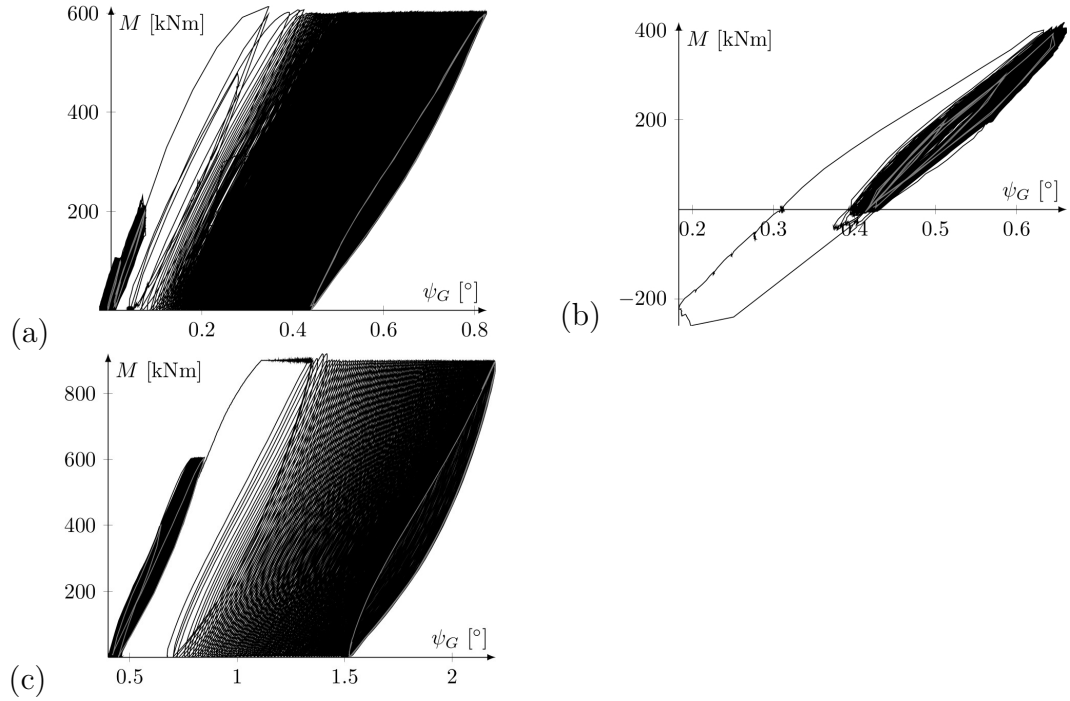


Figure 3.20: Moment-rotation response of CM5 for load sets (a) 1 to 3, (b) 4 to 8 and (c) 9 to 11.

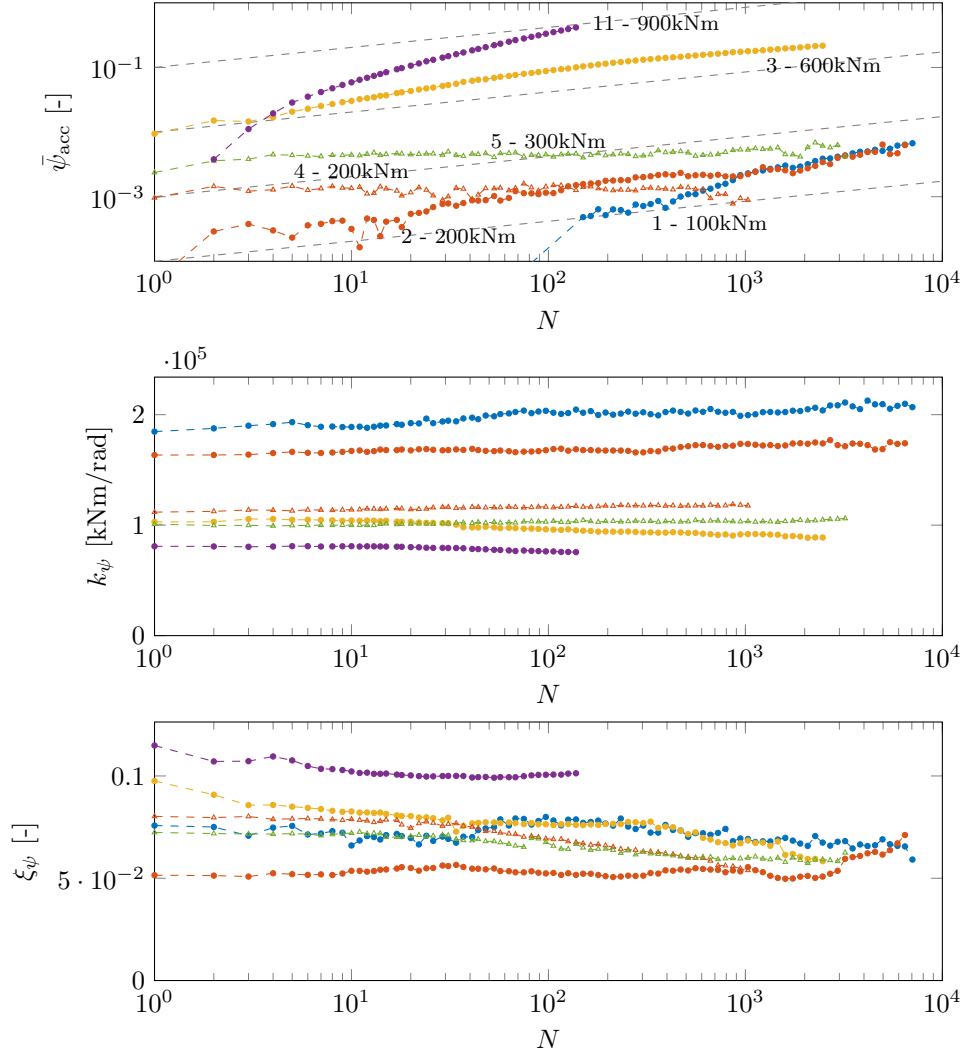


Figure 3.21: Accumulated rotation, stiffness and damping ratio of CM5 for load set 1 (—•—), 2 (—•—), 3 (—•—), 4 (—•—), 5 (—•—), 11 (—•—) and power curves with a power coefficient of 0.31 (---) as reported in the tests by LeBlanc et al. (2010).

stiffness decreases and the damping ratio generally increases with cycle amplitude, which was previously observed in CM5 and CM6.

Table 3.5: Cyclic loading routine for CL2.

Load set	Max. Load [kN]	ζ_c	Cycles	Comment
1	215	0	430	Sinusoidal
2	430	0	115	Sinusoidal
3	860	0	61	Sinusoidal
4	1720	0	16	Sinusoidal

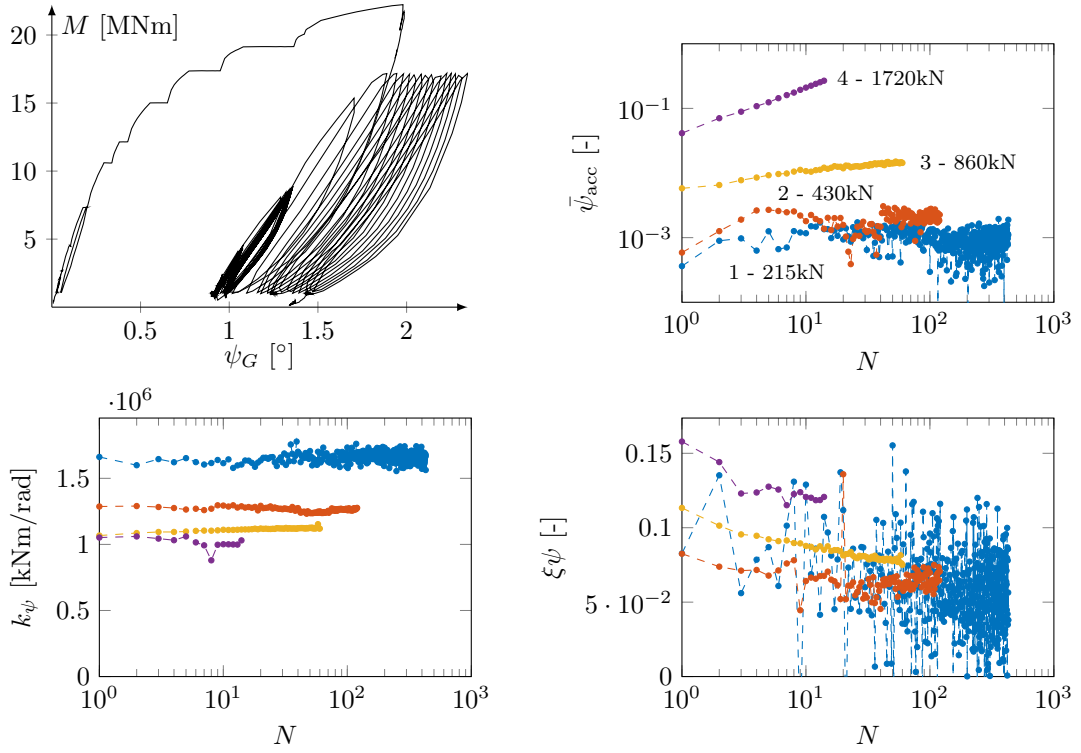


Figure 3.22: Moment rotation response of CL2 and accumulated rotation, stiffness and damping ratio in load set 1 (—•—), 2 (—•—), 3 (—•—) and 4 (—•—).

Dunkirk

Similarly to CM5, test pile DM2 was subjected to a large number of load cycles to investigate the effect of sustained cycling on the foundation behaviour at the Dunkirk test site. The load cycles are summarised in Table 3.6 with moment-rotation plots of the different load sets in Figure 3.23. The analysed results for a subset of the load sets are given in Figure 3.24. The results in Figure 3.23 show that the majority of the rotation accumulation comes from the load cycles where the amplitude is bigger than previously applied cycles. This can be attributed to the hardening effect, which was also observed for CM5. The rotation accumulation in these load sets can consistently be approximated using a power law with a power coefficient of 0.31, as indicated on the figure. This value is consistent with the results of LeBlanc et al. (2010) on small scale tests of monopiles in sand.

The effect of gapping results in a decreased rotational stiffness after high amplitude load cycles are applied, similar to the tests in Cowden. The damping ratio shows a decreasing trend with load amplitude, which is inconsistent with the results in Cowden. The moment-rotation response in Figure 3.23 shows stiffening of the foundation behaviour at increasing load for the large amplitude cyclic load sets, which is indicative of gapping. The gapping effect may contribute to the higher damping ratios at lower load levels since there can be a proportionally higher energy loss from the side friction when the reaction at the front and backside of the pile is not mobilised under cyclic loading.

Table 3.6: Cyclic loading routine for DM2.

Load set	Max. Load [kN]	ζ_c	Cycles	Comment
1	10	0	5100	Sinusoidal (+triangles)
2	20	0	3300	Sinusoidal (+triangles)
3	40	0	8100	Sinusoidal (+triangles)
4	80	0	11110	Sinusoidal (+triangles)
5	10	0	1000	Sinusoidal
6	20	0	1000	Sinusoidal
7	40	0	1000	Sinusoidal
8	10-20-40	0	1000	Triangular
9	10-20-40-80	0	1000	Triangular
10	80	0	3000	Sinusoidal
11	160	0	31	Sinusoidal
12	10	0	100	Triangular
13	20	0	200	Triangular
14	40	0	200	Triangular
15	80	0	200	Triangular
16	10-20-40-80	0	1000	Triangular
17	40	0.5	250	Sinusoidal
18	80	0.5	250	Sinusoidal
19	80	0.25	250	Sinusoidal

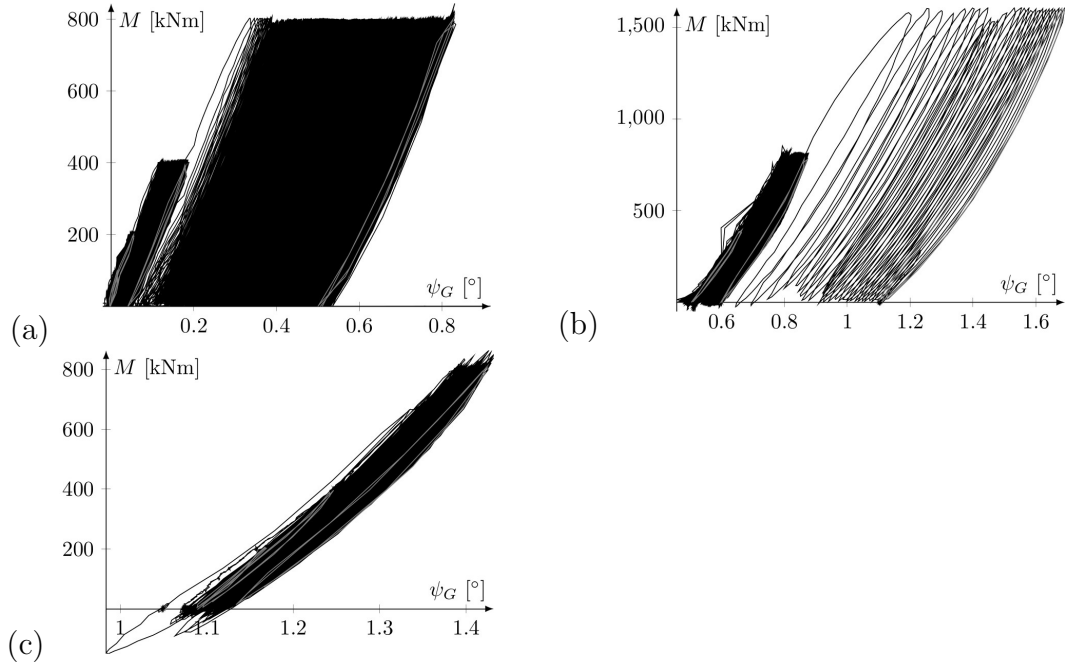


Figure 3.23: Moment-rotation response of DM2 for load sets (a) 1 to 4, (b) 5 to 11 and (c) 12 to 19.

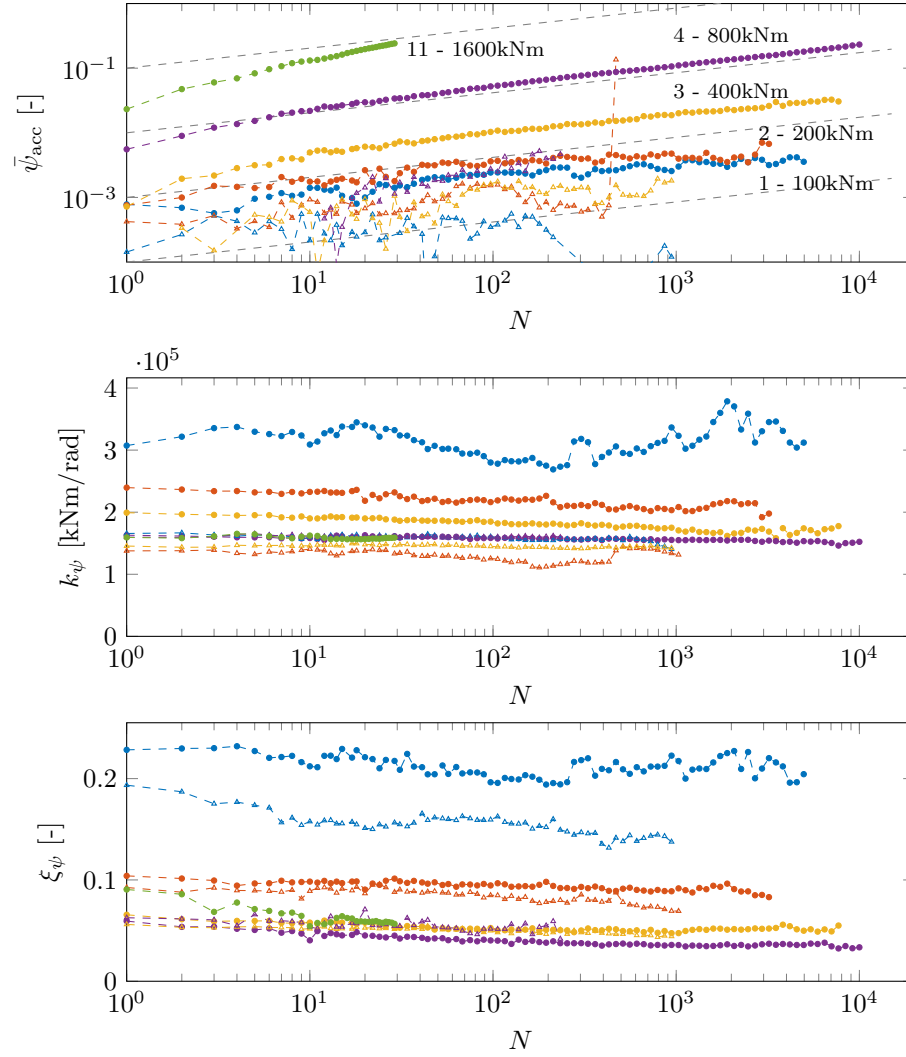


Figure 3.24: Accumulated rotation, stiffness and damping ratio of DM2 for load set 1 (-●-), 2 (-●-), 3 (-●-), 4 (-●-), 5 (-●-), 6 (-●-), 7 (-●-), 10 (-●-), and 11 (-●-), together with power curves with a power coefficient of 0.31 (---) as reported in the tests by LeBlanc et al. (2010).

Test piles DL1 and DL2 were subjected to cyclic loading following the monotonic test procedure. The cyclic loading routine is summarised in Table 3.7. The ground level moment-rotation curve and analysed results are given in Figure 3.25. The rotation accumulation is compared to a power law with the power coefficient of 0.31. For the higher load cycles, the results are relatively consistent with this power law. At the lowest load level, the hardening effect from the monotonic load leads to lower accumulation. The foundation stiffness and damping show similar values for each of the load sets.

Table 3.7: Cyclic loading routine for DL2.

Load set	Max. Load [kN]	ζ_c	Cycles	Comment
1	420	0	200	Sinusoidal
2	840	0	100	Sinusoidal
3	1680	0	50	Sinusoidal
4	3360	0	31	Sinusoidal

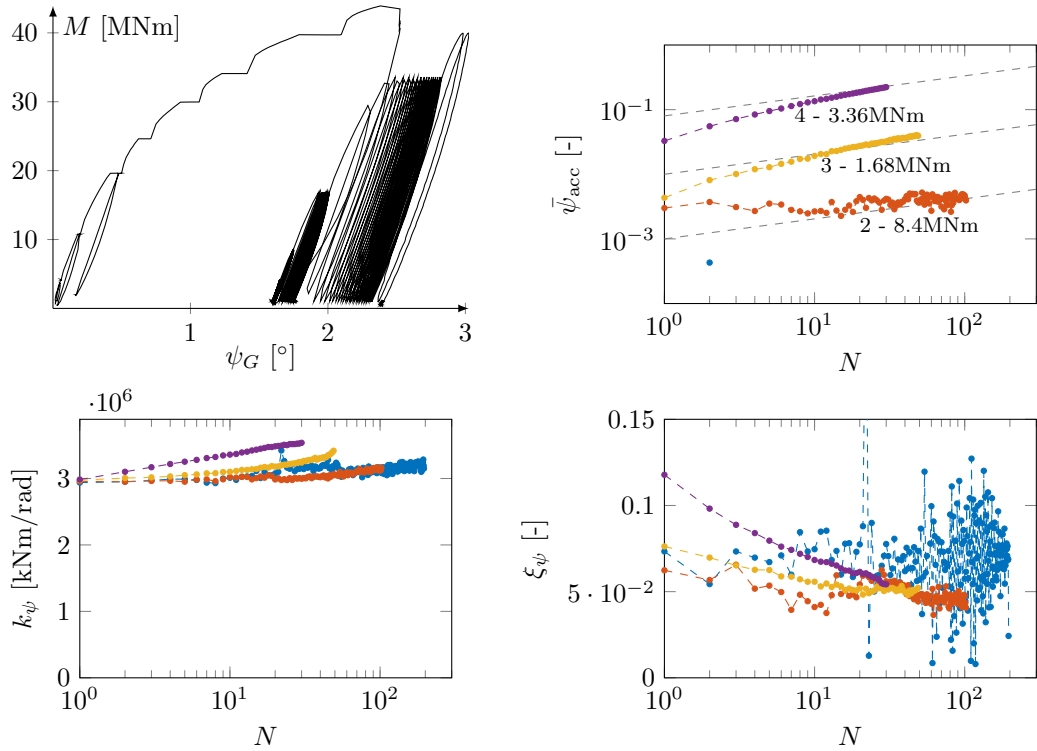


Figure 3.25: Moment rotation response of DL2 and accumulated rotation, stiffness and damping ratio in load set 1 (—•—), 2 (—•—), 3 (—•—) and 4 (—•—).

3.4.4 Behaviour under 2-way cyclic loading

In addition to the 1-way cyclic load tests, one mid-diameter pile at each test site was tested in 2-way symmetric cyclic loading as part of the ‘PISA Damping Project’ (PISA AWG, 2016a). The focus for these tests was to identify the changes in stiffness and damping throughout the applied cycles. Additionally, the effect of the loading frequency on the behaviour was studied.

Cowden

In Cowden, the CM7 test pile was subjected to 2-way cyclic loading. An overview of the loading regime is summarised in Figure 3.26. This figure shows the history of load cycles with sequences of increasing and decreasing load cycles. At several load amplitudes, the loading frequency has been varied. A detailed overview of these cycles is given in the ‘PISA Field Test Factual Report Damping’ (PISA AWG, 2015b).

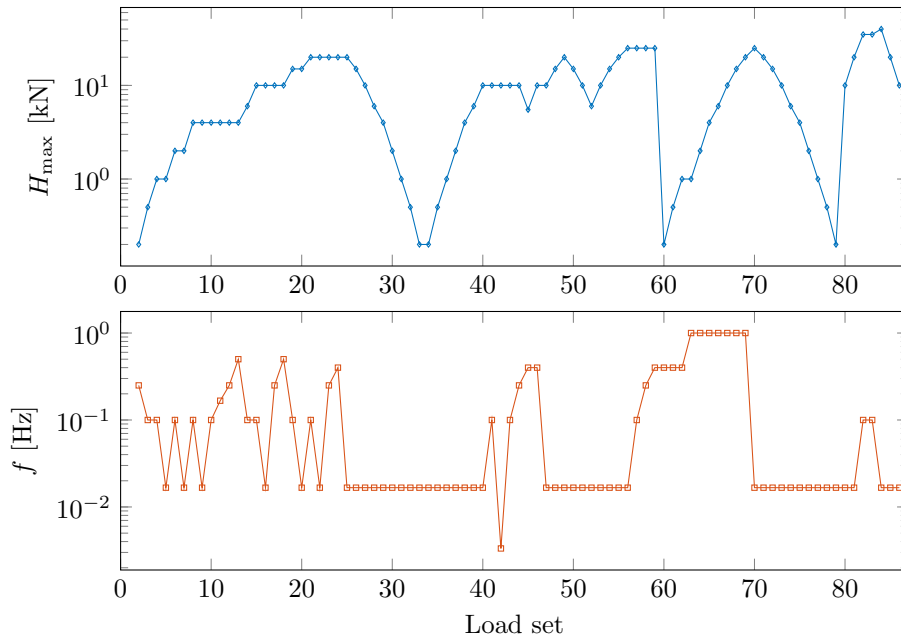


Figure 3.26: Load amplitude and load frequency for the cycles in CM7.

Ground level load-displacement loops are given in Figure 3.27, focusing on the initial load increase and the subsequent load decrease sequence with a load period

of $T = 1/f = 60$ s. The ground level values have been determined based on a combination of Section 3.2.1 and 3.2.2, so that the below ground instrumentation is incorporated in the analysis of the rotation at ground level. Some of the LVDT's have a 'sticky' behaviour, where the measured displacement remains constant around the peak load. In the analysis, only the PL LVDT has been used in the optimisation process since this LVDT was least affected.

Figure 3.27 clearly shows that after the higher loads are applied, the foundation stiffness does not return to its initial value. The evolution of the foundation stiffness and damping are illustrated in Figure 3.28 for a larger number of load sets with the same loading frequency. For the analysis of the damping, the results of ξ_v show increasing damping ratios at smaller load levels for some load sets. This effect is probably attributable to instrumentation error in the LVDT's. In particular, the 'sticky' behaviour of the LVDT's may result in additional measured damping which becomes more important for smaller displacement cycles. The results of the ξ_ψ are not affected by this effect and generally show an increasing damping ratio with load amplitude.

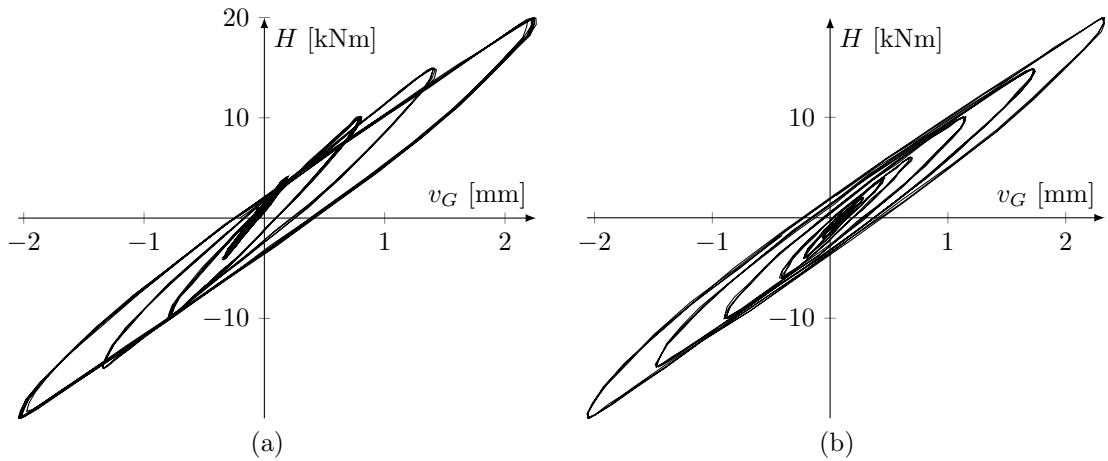


Figure 3.27: Load-displacement loops in (a) the initial load increase (load set 5, 7, 9, 16, 20 and 22) and (b) the following decrease (load set 25 to 33) for CM7.

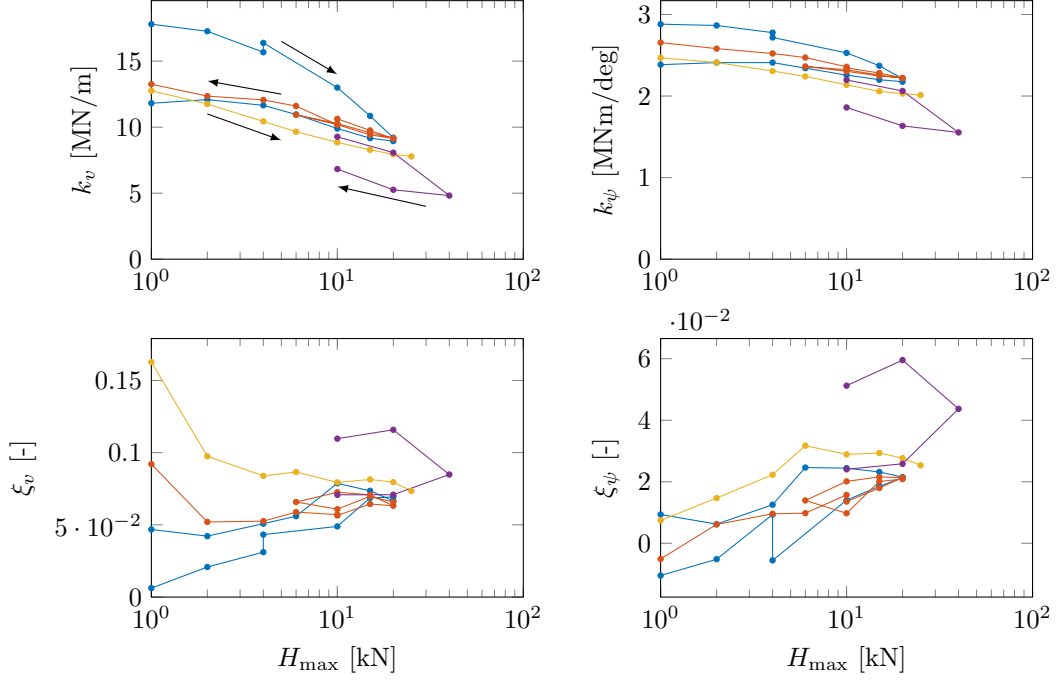


Figure 3.28: Stiffness and damping values of CM7 for (—●—) load sets 5, 7, 9, 16, 20, 22 and 25 to 33, (—●—) load sets 34 to 40 and 47 to 55, (—●—) load sets 70 to 79 and (—●—) load sets 80, 81 and 84 to 86 .

The variation of stiffness and damping with loading frequency is illustrated in Figure 3.29. This figure shows a very low effect of loading frequency on the behaviour. The (slight) variation is expected to result mainly from the rate effects of the soil behaviour. The ground displacement rate in these sets ranges from $\dot{v}_G \approx 4 \frac{4\text{kN} \cdot 0.02\text{Hz}}{16\text{MN/m}} = 0.005\text{mm/s}$ to $\dot{v}_G \approx 4 \frac{25\text{kN} \cdot 0.4\text{Hz}}{7.5\text{MN/m}} = 5.3\text{mm/s}$. Based on CM1, the rate effect could result in a 26% increase of the capacity for 3 orders of magnitude difference of ground displacement rates. However, since cyclic loading is applied at load amplitudes which are much lower than the ultimate capacity, a lower rate effect can be expected on the stiffness. The dimensionless loading frequency is defined as:

$$\bar{f} = fD\sqrt{\frac{\rho}{G_0}} \quad (3.62)$$

where ρ is the soil density. The dimensionless frequency remains very low ($\bar{f} < 0.01$) throughout all load sets so that radiation damping is not expected to influence the

results.

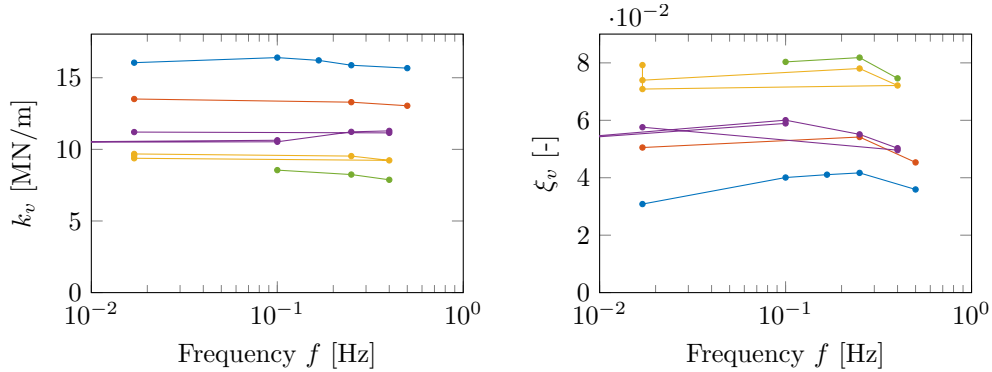


Figure 3.29: Stiffness and damping variation with frequency of CM7 for (—●—) load sets 8 to 13 at 4kN, (—●—) load sets 15 to 18 at 10kN, (—●—) load sets 21 to 25 at 20kN, (—●—) load sets 40 to 44 and 46 to 47 at 10kN and (—●—) load sets 56 to 59 at 25kN.

Using the analysis method for the pile behaviour below ground in Section 3.2.2, equivalent load-displacement loops can be plotted along the pile length. Figure 3.30 shows loops plotted for load set 25 at three different depths along the pile. At $z = 0.5\text{m}$, the soil reaction shows steepening of the tangent stiffness towards the peak load, which is indicative of gapping. The gapping effect was confirmed through visual observation on site. The other two locations show non-linear hysteresis behaviour.

Dunkirk

In Dunkirk, test pile DM1 was subjected to 2-way symmetric loading. Figure 3.31 gives an overview of the load sets, where different sequences of increasing and decreasing load cycles are applied. Again, at a number of load amplitudes, the loading frequency was varied to study its effect on the foundation behaviour.

Figure 3.32 gives ground level load-displacement loops, focusing on the initial increasing load sequence and the subsequent decreasing load sequence. In a way similar to CM7, the initial stiffness is not recovered after a higher load has been applied. Additionally, the load-displacement loops show a slight steepening of the tangent stiffness towards the peak load, which is a characteristic of the gapping effect.

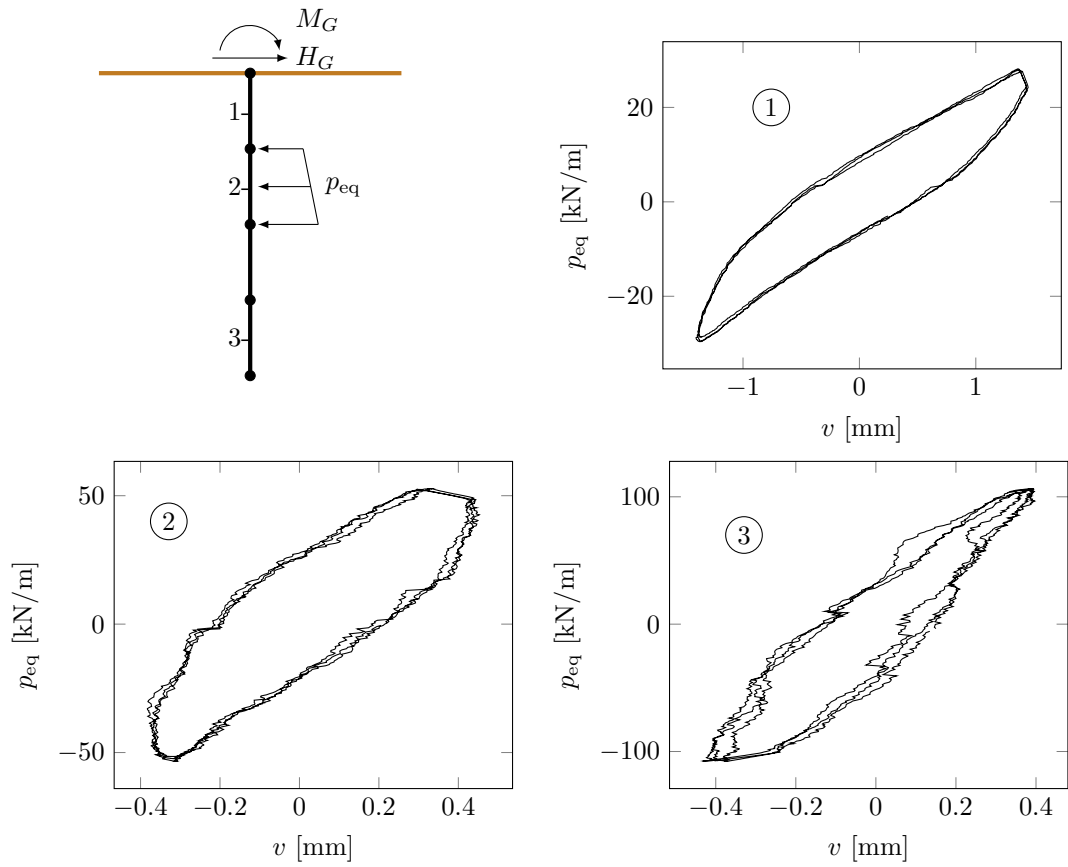


Figure 3.30: Soil reactions at 0.5m, 1.5m and 3.5m below ground level during load set 25 of CM7.

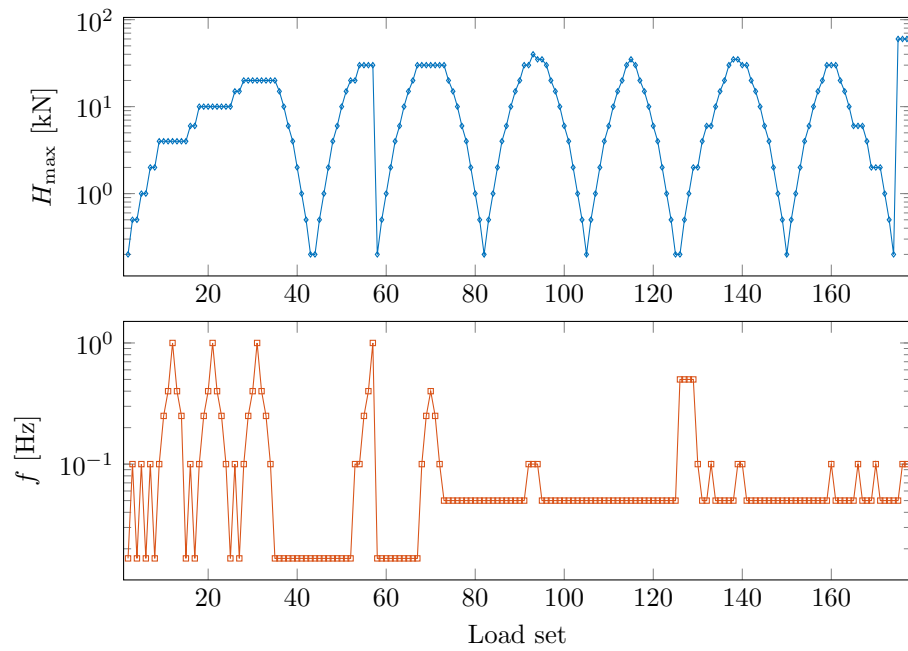


Figure 3.31: Load amplitude and frequency for the load sets of DM1.

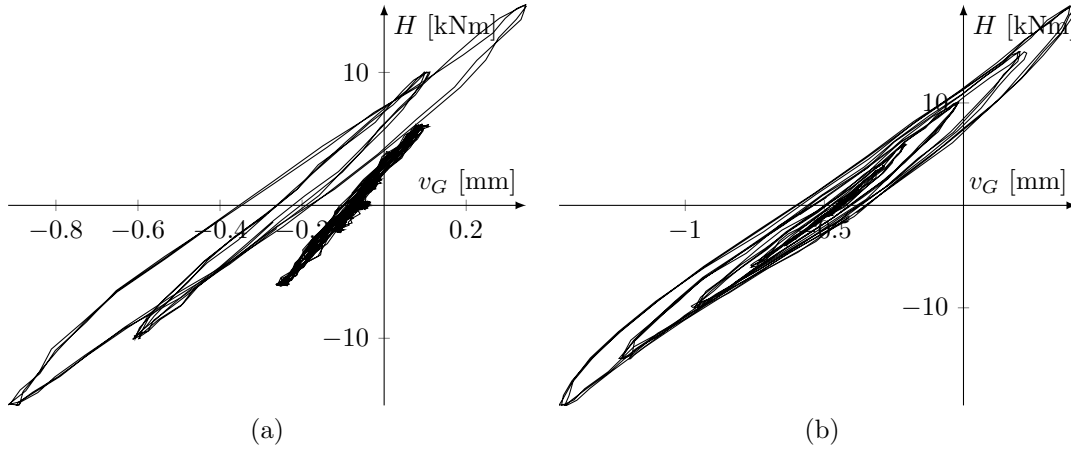


Figure 3.32: Load-displacement loops for DM1 in (a) the initial load increase, load sets 2, 4, 6, 8, 12 and 15 and (b) the following load decrease, load sets 35 to 40.

Figure 3.33 gives the resulting foundation stiffness and damping for a larger number of load sets. The results of stiffness show that due to gapping, the stiffness is not recovered. After load set 57, the load cell cable was broken and the control system pulled the pile to approximately 105kN. This results in the shift in stiffness for the later load sets. The results for the damping ratio show some scatter, but generally give an increasing trend with load amplitude. The damping values are generally lower for DM1 compared to CM7. From empirical damping curves (Vucetic and Dobry, 1991), a higher damping ratio is expected in sand compared to clay at similar strain levels. However, it should also be considered that the strain levels for DM1 are lower than for CM7 since the foundation behaviour is roughly twice as stiff. This may result in the lower observed damping values in DM1 compared to CM7.

The loading frequency was varied at various load amplitudes. The load-displacement stiffness and damping values are illustrated in Figure 3.34. This figure shows little effect of the loading frequency on the foundation response. Rate effects at this site are lower than in Cowden, and therefore the frequency dependency is also expected to be lower. Also, the normalised frequency (Equation 3.62) is very low, so that radiation damping does not affect the foundation behaviour.

Figure 3.35 shows the analysed soil reaction along the foundation pile for load set

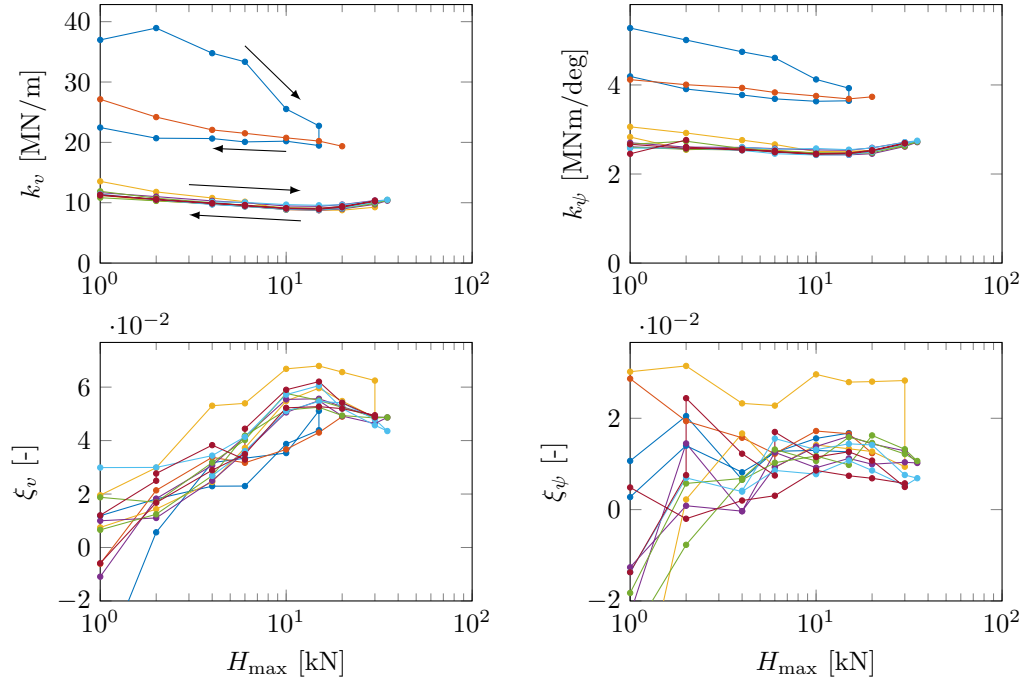


Figure 3.33: Stiffness and damping values of DM1 for load sets (—•—) 2, 4, 6, 8, 12, 15 and 35 to 40, (—•—) 44 to 52, (—•—) 58 to 67 and 73 to 82, (—•—) 83 to 104, (—•—) 105 to 125, (—•—) 131 to 149 and (—•—) 150 to 174.

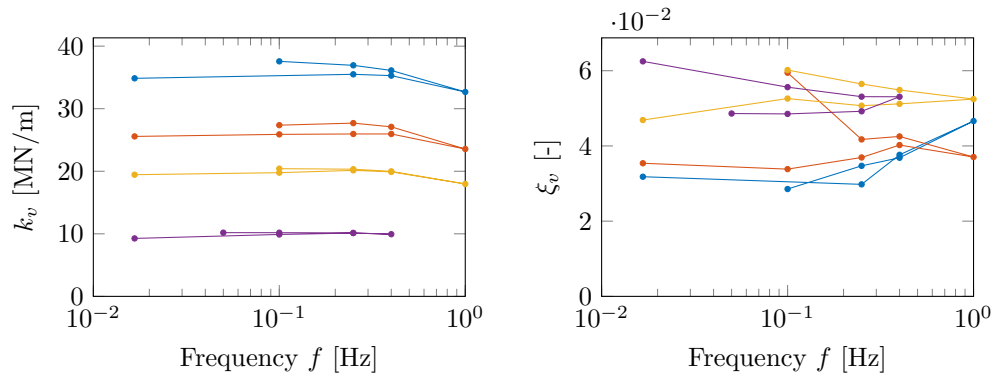


Figure 3.34: Stiffness and damping variation with frequency of DM1 for load sets (—•—) 9 to 15 at 4kN, (—•—) 18 to 25 at 10kN, (—•—) 28 to 35 at 20kN and (—•—) 67 to 73 at 30kN.

67 using the method described in Section 3.2.2. At $z = 0.5\text{m}$ and $z = 1.5\text{m}$, the effect of gapping is very clear, showing an increased tangent stiffness as the foundation pile reaches the end of the gap at both sides. The results at $z = 3.5\text{m}$ show a quasi linear response. The gapping was also confirmed through visual observation at the test site.

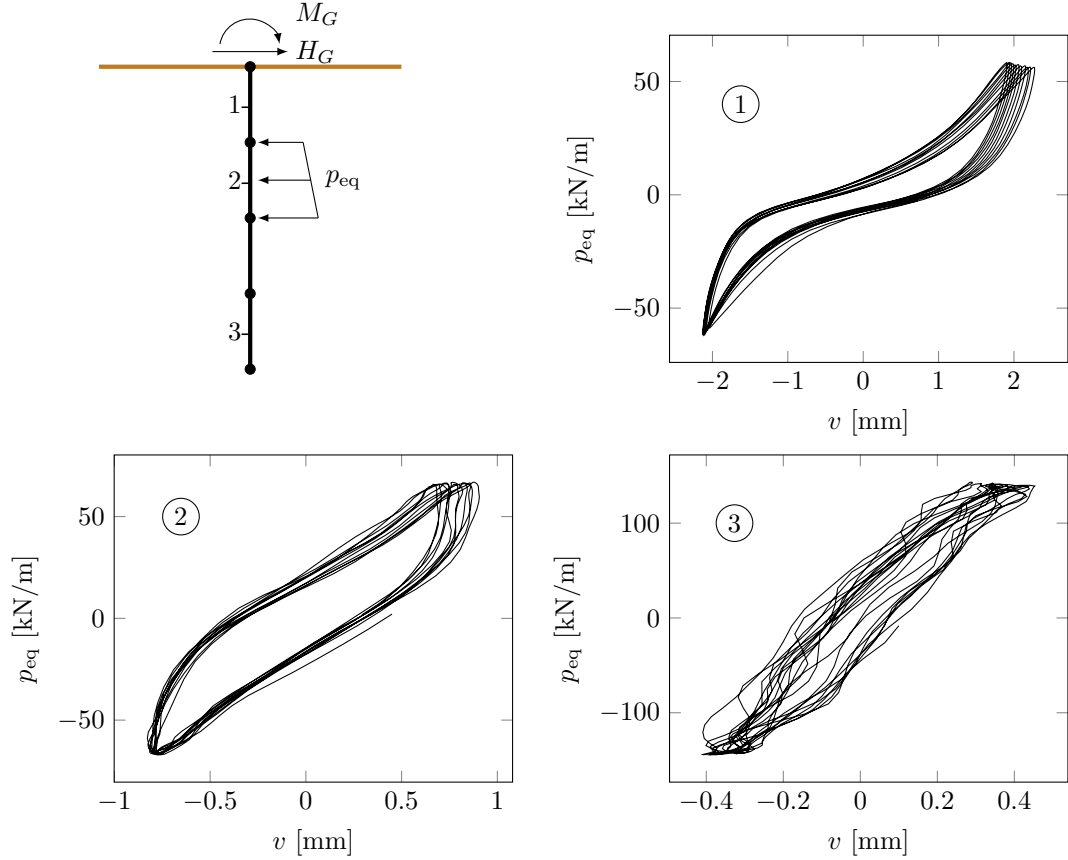


Figure 3.35: Soil reactions at 0.5m, 1.5m and 3.5m below ground level of DM1 for load set 67.

3.5 Conclusion

This chapter presents an overview of the test setup and the methods for analysis of the foundation piles in the PISA test programme. These methods present an approach to interpret the pile behaviour using the various sets of instrumentation data. They are applied to the various tests to provide an understanding of the foundation behaviour. The main foundation aspects observed in the PISA test programme are given below.

1. Rate effects: both in Cowden and Dunkirk, a capacity increase of the foundation was seen at increasing load rates. In Cowden, a relative capacity increase of approximately $8.7\%/\log_{10}(\dot{v}_G)$ at $v_G = 0.1D$ was observed, and in Dunkirk the relative increase was approximately $4.2\%/\log_{10}(\dot{v}_G)$.
2. Ratcheting: in the 1-way cyclic tests at both test sites, accumulation of rotation was seen with increasing cycle number. The results from the tests in Cowden and Dunkirk show a trend of the rotation accumulation with cycle number, which can be approximated using a power law with an exponent of 0.31, as observed in the small scale test piles in sand by LeBlanc et al. (2010).
3. Gapping: this effect was seen at both test sites through a decrease in foundation stiffness at the lower load cycle amplitudes, after a higher load cycle amplitude had been applied. The analysis of the pile behaviour below ground shows the gapping effect near the surface through stiffening of the tangent stiffness of the soil reaction curves towards peak load.
4. No significant rate effects on stiffness or damping were observed in the cyclic tests.

The results from the PISA pile tests form a suitable basis for the model development in Chapter 4 to capture these effects.

Chapter 4

Foundation models

A wide range of numerical models can be used to capture the foundation response of piled foundations. A brief comparison of the different approaches is given in Table 4.1 with their respective advantages and disadvantages.

Table 4.1: Numerical models for piled foundations.

Macro-element	Winkler Foundation	3D FE
→ Increasing computational complexity →		
Entire soil model and structure interaction is captured in a single model at the soil surface.	Interaction between the foundation pile is captured using soil-structure interaction models along the depth of the foundation, which act independently of each other.	3D soil and structural models which capture the soil behaviour and the interaction with the structure.
Calibration based on empirical dataset or derived from 3D FE simulations, combined with element and/or in-situ testing. The calibration often includes theoretically derived stiffness values and limits for soil resistance.		Calibration based on soil element testing and/or in-situ testing.
The limited modelling flexibility makes it most suitable for (but not strictly limited to) rigid foundations.	More flexible modelling of the soil-structure interaction, the approach is most suitable for piled foundations.	Flexible for any foundation geometry, soil-structure interaction is captured in detail.

In the PISA project, it was decided early on that the Winkler approach is most suitable in engineering design for monopile foundations. The Winkler approach assumes that the soil reaction depends only on the local pile displacement and rotation, which simplifies the evaluation of the soil reaction on the foundation. The approach allows for evaluating the response of a large number of monopile geometries within limited computational time and can therefore be used for optimising the design. The Winkler approach is regarded as a reasonable compromise between model accuracy and computational cost. It should be noted that the accuracy of the prediction made with a Winkler model depends largely on the calibration of the soil-structure interaction models.

Within the PISA project, the focus has been on capturing the monotonic response of the foundation pile upon initial loading. This has resulted in soil-reaction curves for a Winkler foundation. The nature of these curves leads to a non-linear response of the foundation. In this chapter, first an outline is given of the FE representation of a Winkler model. This is followed by a set of plasticity models which can be applied to the soil-structure interaction in a Winkler approach, in order to capture the different aspects of the soil response seen in the PISA tests.

4.1 Finite element model of Winkler foundation

The modelling approach for the analysis of monopile foundations in the PISA project is based on the Winkler model. In a Winkler model, the foundation pile is modelled as a beam and the lateral soil reaction at each depth is directly related to the pile displacement at that location. This method is adopted in the current design guidance for monopile foundations (API, 2010; DNV, 2014) and is often referred to as the $p-y$ method. The nomenclature of the $p-y$ method dates back to the 1970s and can be confusing in modern symbol conventions as y commonly refers to an axis in the

coordinate system rather than a displacement. The term $p - y$ will therefore refer to the methodology in the design guidance rather than the soil reaction curves.

Within the PISA project, an extended version of the Winkler model has been used to allow for distributed moments along the pile as a function of the local rotation and to allow for a base shear force and moment reaction as functions of the base displacement and rotation. These extensions have previously been introduced by Davidson (1982), Lam and Martin (1986), Lam (2013) and McVay and Niraula (2004) for other piling applications with similarly low L/D ratios.

The pile and stick-up is modelled as a 1D beam as illustrated in Figure 4.1. At the top of the stick-up, a horizontal force H is applied. The coordinate system and sign conventions used in the formulation are also indicated in the figure. The height of load application h results in a moment at ground level of $M_G = h H$.

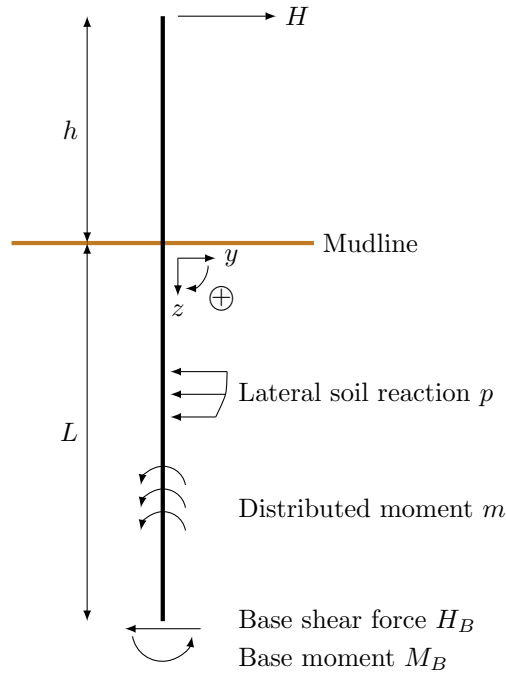


Figure 4.1: Key features of the 1D FE model, after PISA AWG (2016b).

The soil reaction components consist of a lateral distributed load p , a distributed moment m , a base horizontal force H_B and a base moment M_B . The FE representation of this model is derived from a virtual work framework. This approach results in

a consistent way of incorporating the soil reaction components, where the distributed soil reactions are integrated along the elements.

The foundation pile and stick-up are modelled as a line of Timoshenko beam elements. The soil reaction components are directly integrated in this method and can therefore be regarded as a separate set of soil elements, which follow the exact same displacement as the beam elements. In this formulation, the Winkler assumption is maintained: the local soil reaction values are assumed to be functions only of the local pile displacements.

4.1.1 Virtual work equations

The displacements within the pile are represented using Timoshenko beam theory as indicated in Figure 4.2. The assumed displacement field (for the case with no vertical displacement of the neutral axis) is:

$$w(y, z) = y_{\text{loc}} \psi(z) \quad v(y, z) = v_0(z) \quad (4.1)$$

where y_{loc} is a local pile coordinate, w is the vertical displacement, $v_0(z)$ is the lateral displacement of the pile neutral axis and $\psi(z)$ is the rotation of the pile cross-section.

The corresponding strains are:

$$\epsilon_{zz} = \frac{\partial w}{\partial z} = y_{\text{loc}} \frac{d\psi}{dz} \quad \epsilon_{yy} = \frac{\partial v}{\partial y} = 0 \quad \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = \psi + \frac{dv}{dz} \quad (4.2)$$

The bending moment in the pile (a positive moment causes tension on the left hand side of the pile/tower as indicated in Figure 4.1) is:

$$M = - \int E \epsilon_{zz} y_{\text{loc}} dA = -EI \frac{d\psi}{dz} \quad (4.3)$$

where E is the elastic modulus, A is the area and I is the second moment of area of

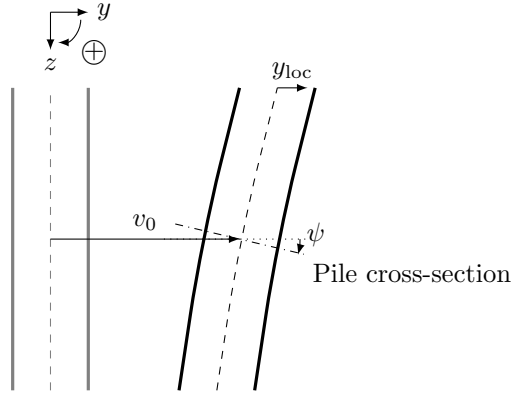


Figure 4.2: Timoshenko beam deformations.

the pile section. The shear force in the pile (sign convention: shear force is positive in the tower for a horizontal load applied as shown in the Figure 4.1) is:

$$S = -GA\kappa \left(\psi + \frac{dv}{dz} \right) \quad (4.4)$$

where G is the shear modulus and κ is the shear factor. In all of the 1D analyses presented in this thesis an elastic modulus $E = 200\text{GPa}$ and a Poisson's ratio $\nu = 0.3$ are adopted to calculate the shear stiffness $\left(G = \frac{E}{2(1+\nu)} \right)$ and the shear factor $\left(\kappa = \frac{2(1+\nu)}{4+3\nu} \right)$ (Cowper, 1966). Using the terminology of Reddy (2002), at equilibrium the sum of the internal and external virtual work is zero. This gives:

$$0 = \delta W_{\text{ext}} + \delta W_{\text{int}} \quad (4.5)$$

where δW_{ext} is the external virtual work, given by:

$$\delta W_{\text{ext}} = -\delta v_T H \quad (4.6)$$

with δv_T , the virtual displacement at the top of the tower. The internal virtual work,

δW_{int} , is:

$$\delta W_{\text{int}} = \int \left[-\delta \frac{d\psi}{dz} M - \delta \gamma S + \delta v p + \delta \psi m \right] dz + \delta v_B H_B + \delta \psi_B M_B \quad (4.7)$$

Expanding this equation for the stiffness terms gives:

$$\begin{aligned} \delta W_{\text{int}} = \int & \left[\delta \frac{d\psi}{dz} EI \frac{d\psi}{dz} + \delta \left(\psi + \frac{dv}{dz} \right) GA \kappa \left(\psi + \frac{dv}{dz} \right) \right. \\ & + \delta v \left(\frac{\partial p}{\partial v} dv + \frac{\partial p}{\partial \psi} d\psi \right) + \delta \psi \left(\frac{\partial m}{\partial v} dv + \frac{\partial m}{\partial \psi} d\psi \right) \Big] dz \\ & + \delta v_B \left(\frac{\partial H_B}{\partial v_B} dv_B + \frac{\partial H_B}{\partial \psi_B} d\psi_B \right) + \delta \psi_B \left(\frac{\partial M_B}{\partial v_B} dv_B + \frac{\partial M_B}{\partial \psi_B} d\psi_B \right) \end{aligned} \quad (4.8)$$

where the integration is taken over the entire length of the pile and tower.

4.1.2 Displacement interpolation for the beam and soil elements

The formulation employed for the FE model used to represent the pile is based on the 2-noded 5 degree-of-freedom element given in Astley (1992). The approach used to interpolate the displacement and rotation within these beam elements is specified below. The same interpolation approach is employed for the 2-noded elements that are used to model the soil. The lateral displacement within each beam (and soil) element is given by the interpolation equation:

$$v(\eta) = N_1 v^{(1)} + N_2 \theta^{(1)} + N_3 v^{(2)} + N_4 \theta^{(2)} \quad (4.9)$$

where η is a local coordinate in the element ranging from 0 to 1. $v^{(1)}$, $v^{(2)}$, $\theta^{(1)}$ and $\theta^{(2)}$ indicate the values of the lateral displacement and overall rotation of the first and

second node. N_i are the conventional set of Hermite cubic interpolation functions:

$$\begin{aligned} N_1 &= (1 + 2\eta)(1 - \eta)^2 & N_2 &= \eta(1 - \eta)^2 L_e \\ N_3 &= \eta^2(3 - 2\eta) & N_4 &= \eta^2(\eta - 1)L_e \end{aligned} \quad (4.10)$$

In these functions, L_e is the length of the element in the global coordinate system. Following Astley (1992), it is assumed that within each element the shear strain, γ_0 , is constant. On this basis, the overall rotation within each element is $\theta = -(dv/dz) = \psi - \gamma_0$. The interpolated displacements may therefore be written as:

$$v(\eta) = N_1 v^{(1)} + N_2 \psi^{(1)} - (N_2 + N_4) \gamma_0 + N_3 v^{(2)} + N_4 \psi^{(2)} \quad (4.11)$$

where $\psi^{(i)}$ are the nodal values of the cross-section rotation (assumed to be continuous at the element nodes) and γ_0 is treated as an additional element degree-of-freedom.

4.1.3 Finite element analysis

The integrals that arise from the discretised form of equation 4.7 and 4.8 are evaluated using Gauss integration with four Gauss points per element. The location and weights related to fourth order Gauss-Legendre quadrature are given in Table 4.2.

Table 4.2: Locations and weights of the Gauss points using fourth order Gauss-Legendre quadrature.

i	η_i	w_i
1	$\frac{1}{2} - \frac{1}{2}\sqrt{\frac{3}{7}} + \frac{2}{7}\sqrt{\frac{6}{5}}$	$\frac{18-\sqrt{30}}{72}$
2	$\frac{1}{2} - \frac{1}{2}\sqrt{\frac{3}{7}} - \frac{2}{7}\sqrt{\frac{6}{5}}$	$\frac{18+\sqrt{30}}{72}$
3	$\frac{1}{2} + \frac{1}{2}\sqrt{\frac{3}{7}} - \frac{2}{7}\sqrt{\frac{6}{5}}$	$\frac{18+\sqrt{30}}{72}$
4	$\frac{1}{2} + \frac{1}{2}\sqrt{\frac{3}{7}} + \frac{2}{7}\sqrt{\frac{6}{5}}$	$\frac{18-\sqrt{30}}{72}$

The displacements from equation 4.11 can be written in the following way:

$$v = \mathbf{N}_v \cdot \mathbf{u}^e \quad \psi = \mathbf{N}_\psi \cdot \mathbf{u}^e \quad \gamma_0 = \left(\psi + \frac{dv}{dz} \right) = \mathbf{N}_\gamma \cdot \mathbf{u}^e \quad \frac{d\psi}{dz} = \mathbf{N}_{d\psi} \cdot \mathbf{u}^e \quad (4.12)$$

where the element displacement vector is given by $\mathbf{u}^e = [v^{(1)} \ \psi^{(1)} \ \gamma_0 \ v^{(2)} \ \psi^{(2)}]^T$ and the vectors of shape functions $\mathbf{N}$ are defined as:

$$\begin{aligned} \mathbf{N}_{vi} &= \begin{bmatrix} N_1(\eta_i) & N_2(\eta_i) & -(N_2(\eta_i) + N_4(\eta_i)) & N_3(\eta_i) & N_4(\eta_i) \end{bmatrix} \\ \mathbf{N}_{\psi i} &= -\frac{1}{L_e} \begin{bmatrix} N'_1(\eta_i) & N'_2(\eta_i) & -(N'_2(\eta_i) + N'_4(\eta_i) + 1) & N'_3(\eta_i) & N'_4(\eta_i) \end{bmatrix} \\ \mathbf{N}_{\gamma i} &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \end{bmatrix} \\ \mathbf{N}_{d\psi i} &= -\frac{1}{L_e^2} \begin{bmatrix} N''_1(\eta_i) & N''_2(\eta_i) & -(N''_2(\eta_i) + N''_4(\eta_i)) & N''_3(\eta_i) & N''_4(\eta_i) \end{bmatrix} \end{aligned} \quad (4.13)$$

Noting that the differentiation refers to differentiation with respect to η in the local coordinate system. The element stiffness matrix $\mathbf{K}^e$ (based on equation 4.8) is defined as:

$$\begin{aligned} \mathbf{K}^e &= \sum_{i=1}^{n_g} w_i \left[\mathbf{N}_{d\psi i}^T E I \mathbf{N}_{d\psi i} + \mathbf{N}_{\gamma i}^T G A \kappa \mathbf{N}_{\gamma i} \right. \\ &\quad \left. + \begin{bmatrix} \mathbf{N}_{vi}^T & \mathbf{N}_{\psi i}^T \end{bmatrix} \begin{bmatrix} \frac{\partial p}{\partial v} & \frac{\partial p}{\partial \psi} \\ \frac{\partial m}{\partial v} & \frac{\partial m}{\partial \psi} \end{bmatrix} \begin{bmatrix} \mathbf{N}_{vi} \\ \mathbf{N}_{\psi i} \end{bmatrix} \right] J \end{aligned} \quad (4.14)$$

The Jacobian $J = \frac{dz}{d\eta}$ is introduced to convert the integration over the element from local to global coordinate system. In this case, the Jacobian is equal to the length of the element ($J = L_e$). The stiffness matrix with only the terms of the pile stiffness will be referred to as the pile stiffness matrix $\mathbf{K}_p$ and similarly for the soil stiffness

matrix $\mathbf{K}_s$. The vector of internal forces is calculated by:

$$\begin{aligned}\mathbf{f}_{\text{int}}^e &= \sum_{i=1}^{n_g} w_i \left[-M\mathbf{N}_{d\phi i}^T - S\mathbf{N}_{\gamma i}^T + p\mathbf{N}_{vi}^T + m\mathbf{N}_{\psi i}^T \right] J \\ &= \mathbf{K}_p^e \mathbf{u}^e + \sum_{i=1}^{n_g} w_i \left[p\mathbf{N}_{vi}^T + m\mathbf{N}_{\psi i}^T \right] J\end{aligned}\quad (4.15)$$

Since the pile is modelled as linear elastic, the internal forces from the pile are simply calculated as $\mathbf{K}_p^e \mathbf{u}^e$. The global displacement vector, force vectors and the stiffness matrix are assembled from the element vectors and matrices as is commonly done in the FE method. In shorthand notation, this is expressed as:

$$\mathbf{u} = \mathbf{A}\mathbf{u}^e \qquad \mathbf{f} = \mathbf{A}\mathbf{f}^e \qquad \mathbf{K} = \mathbf{A}\mathbf{K}^e \quad (4.16)$$

where $\mathbf{A}$ is the assembly operator for the elements. The soil base reactions can be added to the internal force vector at the relevant locations and, similarly, the stiffness of the soil base reactions can simply be added to the relevant locations in the stiffness matrix. The simulations in this thesis only use an external horizontal force applied at the top node of the tower element, which is therefore the only non-zero element in the external force vector. The virtual work expression of equation 4.8 can now be written as:

$$\delta W_{\text{ext}} = -\delta \mathbf{u} \cdot \mathbf{f}_{\text{ext}} \qquad \delta W_{\text{int}} = \delta \mathbf{u} \cdot \mathbf{f}_{\text{int}} = \delta \mathbf{u} \cdot (\mathbf{K}\mathbf{u}) \quad (4.17)$$

so that $\mathbf{f}_{\text{ext}} = \mathbf{K}\mathbf{u}$ for a linear system.

4.1.4 Non-linear solution scheme

In the application of the PISA project and further developments in this thesis, the soil reaction is non-linear. The soil reaction vector and stiffness matrix therefore depend

on the pile displacement and the displacement history, which is captured using a matrix of hardening variables ($\mathbf{A}$). This matrix contains the vectors of hardening variables ($\boldsymbol{\alpha}$) for the soil reactions at the Gauss points and for the base reactions. Evaluating the soil reaction for a displacement update also results in an update of the hardening variables:

$$[\mathbf{f}_{\text{int}}, \mathbf{A}_1] = f(\mathbf{u}_1, \mathbf{u}_0, \mathbf{A}_0) \quad (4.18)$$

where f is shorthand notation for a generic function. The stiffness matrix also takes into account the displacement update and history:

$$\mathbf{K} = f(\mathbf{u}_1, \mathbf{u}_0, \mathbf{A}_0) \quad (4.19)$$

To easily evaluate the soil stiffness terms in the stiffness matrix, a numerical approximation is made where:

$$\begin{aligned} \begin{bmatrix} \frac{\partial p}{\partial v} & \frac{\partial p}{\partial \psi} \\ \frac{\partial m}{\partial v} & \frac{\partial m}{\partial \psi} \end{bmatrix} &= \begin{bmatrix} \frac{p_1^{\Delta v} - p_1}{\Delta v} & \frac{p_1^{\Delta \psi} - p_1}{\Delta \psi} \\ \frac{m_1^{\Delta v} - m_1}{\Delta v} & \frac{m_1^{\Delta \psi} - m_1}{\Delta \psi} \end{bmatrix} \\ \begin{bmatrix} \frac{\partial H_B}{\partial v_B} & \frac{\partial H_B}{\partial \psi_B} \\ \frac{\partial M_B}{\partial v_B} & \frac{\partial M_B}{\partial \psi_B} \end{bmatrix} &= \begin{bmatrix} \frac{H_{B1}^{\Delta v B} - H_{B1}}{\Delta v_B} & \frac{H_{B1}^{\Delta \psi B} - H_{B1}}{\Delta \psi_B} \\ \frac{M_{B1}^{\Delta v B} - M_{B1}}{\Delta v_B} & \frac{M_{B1}^{\Delta \psi B} - M_{B1}}{\Delta \psi_B} \end{bmatrix} \end{aligned} \quad (4.20)$$

with $[p_1, \boldsymbol{\alpha}_{v_1}] = f(v_1, v_0, \boldsymbol{\alpha}_{v_0})$, $[m_1, \boldsymbol{\alpha}_{\psi_1}] = f(\psi_1, \psi_0, \boldsymbol{\alpha}_{\psi_0}, p_1)$, $[H_{B1}, \boldsymbol{\alpha}_{v_{B1}}] = f(v_{B1}, v_{B0}, \boldsymbol{\alpha}_{v_{B0}})$ and $[M_{B1}, \boldsymbol{\alpha}_{\psi_{B1}}] = f(\psi_{B1}, \psi_{B0}, \boldsymbol{\alpha}_{\psi_{B0}})$. The superscripts Δv , $\Delta \psi$ indicate that the reactions are evaluated at $v_1^* = v_1 + \Delta v$ and $\psi_1^* = \psi_1 + \Delta \psi$ rather than v_1 and ψ_1 . The incremental displacement vector for the non-linear solution scheme is now written as:

$$\Delta \mathbf{u} = \mathbf{K}^{-1}(\mathbf{f}_{\text{ext}} - \mathbf{f}_{\text{int}}) \quad (4.21)$$

The solution for the pile displacement is found using an iterative method based on the Newton-Raphson scheme. The method is modified by only updating the harden-

ing variables at the end of each load step when the solution has converged. This is appropriate in plasticity models so that the hardening variables are not affected by the strain paths within iterations for convergence.

```

Initialise variables
 $\mathbf{u}_1 \leftarrow \mathbf{0}, \mathbf{A}_1 \leftarrow \mathbf{0}$ 
for each load step  $\Delta \mathbf{f}_{\text{ext}}$  do
     $\mathbf{u}^* \leftarrow \mathbf{u}_1, \mathbf{u}_0 \leftarrow \mathbf{u}_1, \mathbf{A}_0 \leftarrow \mathbf{A}_1$ 
    while  $\text{RES} > \text{TOL}$  do
         $\mathbf{K} \leftarrow f(\mathbf{u}^*, \mathbf{u}_0, \mathbf{A}_0)$ 
         $\Delta \mathbf{u} \leftarrow \mathbf{K}^{-1}(\mathbf{f}_{\text{ext}} - \mathbf{f}_{\text{int}})$ 
         $\mathbf{u}^* \leftarrow \mathbf{u}^* + \Delta \mathbf{u}$ 
         $[\mathbf{f}_{\text{int}}, \mathbf{A}^*] \leftarrow f(\mathbf{u}^*, \mathbf{u}_0, \mathbf{A}_0)$ 
         $\text{RES} \leftarrow \frac{\max |\mathbf{f}_{\text{ext}} - \mathbf{f}_{\text{int}}|}{H_{\text{max}}}$ 
    end while
     $\mathbf{u}_1 \leftarrow \mathbf{u}^*, \mathbf{A}_1 \leftarrow \mathbf{A}^*$ 
end for

```

The convergence criterion uses the residual of the unbalanced forces normalised by the maximum applied load in the simulation H_{max} . To achieve convergence, an appropriate tolerance limit is applied. In the pseudo-code, the notation $*$ indicates that the value for the variable is not final and $\leftarrow$ indicates that a variable is updated, which is a subtle difference from the use of the equal symbol ($=$).

4.2 Hyperplasticity framework

4.2.1 Outline

Hyperplasticity is a rigorous and consistent framework for developing plasticity models which satisfy thermodynamic principles. The roots of the method were developed by Ziegler (1977,1983) and the method was further developed by Houlsby (1981) and Collins and Houlsby (1997). The framework has been shown to be successful for the description of many engineering materials, notably in the area of geotechnical engineering, where the material often exhibits so-called non-associated flow. Examples of these soil models have been derived by Houlsby and Puzrin (2000) and Houlsby and Puzrin (2002). The main advantages of using the hyperplasticity framework to derive plasticity models are:

1. The framework guarantees thermodynamic acceptability;
2. The constitutive material behaviour is entirely captured in two potential functions. The response can be derived from these potentials using standardised procedures;
3. The consistent framework allows comparison of competing material models and facilitates the development of new material models.

In this subsection, the essential elements of the hyperplasticity framework are outlined. This will form a basis for the further model developments in this chapter. A complete representation of the framework for rate-independent models can be found in Houlsby and Puzrin (2000) and Puzrin and Houlsby (2001). Although the model developments in this chapter only briefly touch on the rate-dependent hyperplasticity framework, the interested reader can also refer to Houlsby and Puzrin (2002) and Puzrin and Houlsby (2003). Finally, Houlsby and Puzrin (2006) presents a comprehensive description of the entire framework with examples of applications.

The constitutive behaviour will be described in stress-strain (σ, ϵ) space, however, in other models different variables can take the role of the stress and strain variables. For the application of the soil reaction curves in the PISA project, these could be the lateral distributed load-displacement (p, v) , the distributed moment-rotation $(m - \psi)$, the base horizontal force-displacement (H_B, v_B) or the base moment-rotation (M_B, ψ_B) .

The state of the model can be described as a function of the stress or strain plus internal variables, denoted α for a strain-type variable or β for a work-type variable. In the FE method, all hardening variables are placed in the vector of hardening variables $\boldsymbol{\alpha}$. The description of the material behaviour is achieved through the definition and derivation of two scalar functions: one defining the energy stored in the system and one defining the rate of dissipation.

The energy function can either be the internal energy u , the Helmholtz free energy f , the enthalpy h or the Gibbs free energy g . All of these functions are related by Legendre-Fenchel transforms, which means that the models developed with any of the energy functions are equivalent (see Houlsby and Puzrin (2006) for details). For rate independent behaviour, the dissipative potential can either be the dissipation function d or the canonical yield function $\bar{y}$. These two potentials are also related and the expression of only one of them is sufficient for the model development. In the case of rate dependent behaviour, the dissipation potential is written in terms of the force potential z or the flow potential w which are related by a Legendre-Fenchel transform.

In this chapter, the material behaviour will be described using the Helmholtz free energy f together with the dissipation function d for rate independent and the force potential z for rate dependent material behaviour. Both dissipation potentials are related by:

$$d = \frac{\partial z}{\partial \dot{\boldsymbol{\alpha}}} \cdot \dot{\boldsymbol{\alpha}} \quad (4.22)$$

In the case where d is homogeneous and first order in $\dot{\boldsymbol{\alpha}}$, then the material behaviour is rate independent and $z \equiv d$. The constitutive behaviour is derived from the following equations:

$$\sigma = \frac{\partial f}{\partial \epsilon} \quad \bar{\boldsymbol{\chi}} = - \frac{\partial f}{\partial \boldsymbol{\alpha}} \quad \boldsymbol{\chi} = \frac{\partial z}{\partial \dot{\boldsymbol{\alpha}}} \quad (4.23)$$

In the above equations, $\bar{\boldsymbol{\chi}}$ is the generalised stress and $\boldsymbol{\chi}$ is the dissipative generalised stress. The second law of thermodynamics implies that $(\bar{\boldsymbol{\chi}} - \boldsymbol{\chi}) \cdot \dot{\boldsymbol{\alpha}} = 0$. The hyperplasticity framework presented by Houlsby and Puzrin (2006) makes use of a stronger assumption:

$$\bar{\boldsymbol{\chi}} = \boldsymbol{\chi} \quad (4.24)$$

This statement, also known as the Ziegler's orthogonality principle (Ziegler, 1977,1983), enforces the dissipation to be maximal, and is more restrictive than the Second Law of Thermodynamics. It is therefore usually considered as a classifying hypothesis. However, this assumption applies to the behaviour of a remarkably wide range of materials (Houlsby and Puzrin, 2006), and using this principle, the entire constitutive response can be derived without any further assumptions being necessary. Consequently, the following work is developed with the use of Equation 4.24.

In the case where additional constraints apply to the rates of the kinematic internal variables ($c = f(\epsilon, \boldsymbol{\alpha}, \dot{\boldsymbol{\alpha}}) = 0$ or $c = f(\sigma, \boldsymbol{\alpha}, \dot{\boldsymbol{\alpha}}) = 0$), the dissipation potential can be augmented using a Lagrange multiplier Λ to give:

$$z^* = z + \Lambda c \quad (4.25)$$

The dissipative generalised stress is now derived from the augmented dissipation function ($\boldsymbol{\chi} = \frac{\partial z^*}{\partial \dot{\boldsymbol{\alpha}}}$), and the Lagrange multiplier is eliminated by use of the constraint $c = 0$.

4.2.2 Single yield surface model

Rate independent model

In this subsection, the principles of the hyperplasticity theory are applied to one of the simplest plasticity models: linear elastic - perfectly plastic in 1D of loading. A more elaborate description of the model can be found in Puzrin and Houlsby (2001). The model consists of an elastic element with hardening modulus H and a slider with strength k . The model is illustrated in Figure 4.3.

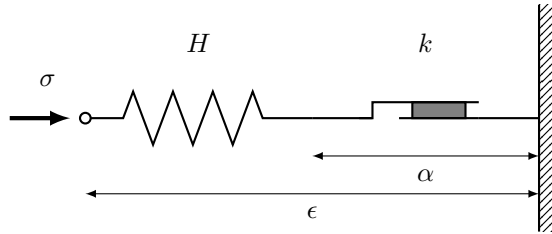


Figure 4.3: Rate independent single yield surface model.

As long as the applied stress is smaller than the capacity of the slider, the displacement is purely elastic ($\sigma = H(\epsilon - \alpha)$). Once the stress reaches the capacity of the slider k , additional strain is manifested as plastic strain. The energy function and dissipation potential are written as:

$$f = \frac{H}{2}(\epsilon - \alpha)^2 \quad (4.26)$$

$$d = k|\dot{\alpha}|$$

From these expressions it can be understood that f is the energy stored in the system and d is the rate of energy dissipation by the system. The stress can be found as:

$$\sigma = \frac{\partial f}{\partial \epsilon} = H(\epsilon - \alpha) \quad (4.27)$$

which gives the same response as previously identified. The generalised stress and

the dissipative generalised stress are given by:

$$\begin{aligned}\bar{\chi} &= -\frac{\partial f}{\partial \alpha} = H(\epsilon - \alpha) \\ \chi &= \frac{\partial z}{\partial \dot{\alpha}} \equiv \frac{\partial d}{\partial \dot{\alpha}} = S(\dot{\alpha})k\end{aligned}\tag{4.28}$$

where $S(\cdot)$ is the generalised Signum function. Since d is homogeneous and first order in $\dot{\alpha}$, the dissipation is equal to the force potential ($z \equiv d$). Applying Ziegler's orthogonality condition, we find:

$$\bar{\chi} = \chi \rightarrow H(\epsilon - \alpha) = S(\dot{\alpha})k\tag{4.29}$$

This model can easily be implemented using a return mapping algorithm, which in this case gives the exact response. The method uses a trial stress based on the strain update. In the case where this stress state would be inadmissible, the hardening variable is modified so that the stress returns to the yield surface. In this case, the direction of hardening strain is always the same as the generalised stress, therefore: $S(\dot{\alpha}) = S(\chi^{\text{trial}})$. The algorithm is specified below.

```
Input:  $\epsilon_1, \alpha_0, k, H$ 
 $\chi^{\text{trial}} \leftarrow H(\epsilon_1 - \alpha_0)$ 
if  $|\chi^{\text{trial}}| > k$  then
     $\alpha_1 \leftarrow \epsilon_1 - S(\chi^{\text{trial}})\frac{k}{H}$ 
else
     $\alpha_1 \leftarrow \alpha_0$ 
end if
 $\sigma_1 \leftarrow H(\epsilon_1 - \alpha_1)$ 
Output:  $\sigma_1, \alpha_1$ 
```

Rate dependent model

A rate dependent version of this model is illustrated in Figure 4.4. A more elaborate description of the model can be found in Houlsby and Puzrin (2002).

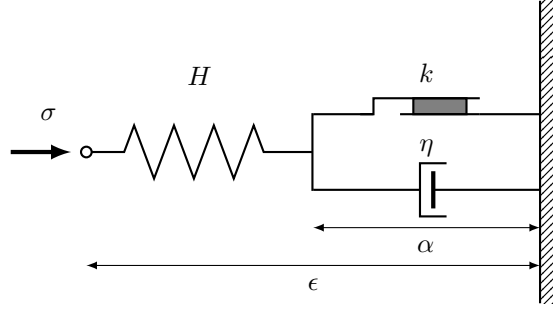


Figure 4.4: Rate dependent single yield surface model.

The energy potential remains the same as in the previous model, however, the dissipation function now includes the effect of the viscous damper.

$$d = k|\dot{\alpha}| + \eta\dot{\alpha}^2 \quad (4.30)$$

where η is the viscosity. Using equation 4.22, the force potential becomes:

$$z = k|\dot{\alpha}| + \frac{\eta}{2}\dot{\alpha}^2 \quad (4.31)$$

The generalised stress and the dissipative generalised stress are given by:

$$\begin{aligned} \bar{\chi} &= -\frac{\partial f}{\partial \alpha} = H(\epsilon - \alpha) \\ \chi &= \frac{\partial z}{\partial \dot{\alpha}} = S(\dot{\alpha})k + \eta\dot{\alpha} \end{aligned} \quad (4.32)$$

Using the equation of the dissipative stress, we can show that if $|\chi| > k$, the signs of χ and $\dot{\alpha}$ must be the same. With Ziegler's orthogonality principle, we find:

$$\bar{\chi} = \chi \rightarrow H(\epsilon - \alpha) = S(\dot{\alpha})k + \eta\dot{\alpha} \quad (4.33)$$

Note that $\dot{\alpha} = 0$ if the generalised stress is within the yield surface and $\dot{\sigma} = H\dot{\epsilon}$. Rearranging equation 4.33, considering the case of $\dot{\alpha} = 0$, results in the incremental form of the hardening variable:

$$\dot{\alpha} = \frac{\langle |\chi| - k \rangle}{\eta} S(\chi) \quad (4.34)$$

The complete incremental response is summarised in the algorithm below.

Input: $\epsilon_1, \epsilon_0, \alpha_0, k, H, \eta, \Delta t$

$$\sigma_0 \leftarrow H(\epsilon_0 - \alpha_0)$$

$$\chi \leftarrow H(\epsilon_1 - \alpha_0)$$

$$\dot{\epsilon} \leftarrow (\epsilon_1 - \epsilon_0)/\Delta t$$

$$\dot{\alpha} \leftarrow \frac{\langle |\chi| - k \rangle}{\eta} S(\chi)$$

$$\dot{\sigma} \leftarrow H(\dot{\epsilon} - \dot{\alpha})$$

$$\sigma_1 \leftarrow \sigma_0 + \dot{\sigma}\Delta t$$

$$\alpha_1 \leftarrow \alpha_0 + \dot{\alpha}\Delta t$$

Output: σ_1, α_1

The stress update using the rate dependent formulation in the hyperplasticity formulation can be implemented in a standardised algorithm, which is very useful for model development. To mimic a rate independent response, the viscosity η can be set to an artificially low value. The convergence of the rate dependent model is constrained by a maximum time step. In this simple example, the maximum time step is calculated by (Houlsby, 2016):

$$\frac{2\eta}{H\Delta t} > 1 \quad (4.35)$$

The limitation increases the computational cost of evaluating the stress for large strain increments. This chapter therefore presents algorithms based on the rate independent

formulation, which can be applied to large strain steps and can be used to calculate the response of a Winkler foundation efficiently.

4.3 Kinematic hardening model

4.3.1 Masing rules

The Masing type hysteretic behaviour is often referred to as a kinematic hardening material model, since it is modelled using yield surfaces which translate but do not expand or contract upon yielding. Kinematic hardening can be described by the extended Masing rules (Masing, 1926; Pyke, 1979), which are well established descriptors of realistic hysteretic behaviour. In initial loading the stress-strain curve can generally be described by a function B , also known as the backbone curve:

$$B(\sigma, \epsilon) = 0 \quad (4.36)$$

The unloading and reloading branches of the steady-state hysteretic response are geometrically similar to the initial loading curve except for a two-fold magnification and a reversal around the reversal points (σ_r, ϵ_r) :

$$B\left(\frac{\sigma - \sigma_r}{2}, \frac{\epsilon - \epsilon_r}{2}\right) \quad (4.37)$$

Note that the backbone curve B should satisfy the symmetry condition:

$$B(-\sigma, -\epsilon) = B(\sigma, \epsilon) = 0 \quad (4.38)$$

so that the initial stress-strain curve is rotationally symmetric around the origin. To apply the kinematic hardening model, the backbone curve must also be monotonically increasing and the slope must be monotonically decreasing for $\epsilon > 0$. The behaviour

of the steady-state response is illustrated in Figure 4.5.

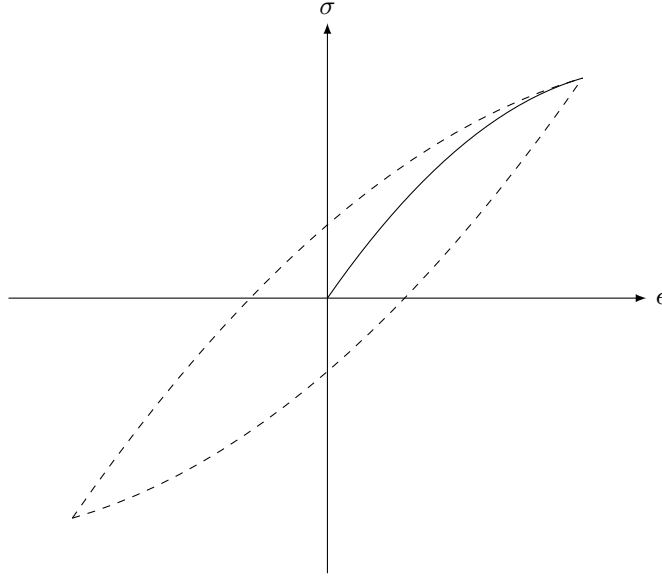


Figure 4.5: Initial backbone curve (—) and steady state hysteretic response (---) using the Masing rules.

To describe the full range of loading and unloading, the following extended Masing rules also apply (Pyke, 1979):

1. The unloading and reloading curves should follow the initial loading curve (backbone curve) if the previous maximum stress is exceeded;
2. If the current loading or unloading curve intersects the curve described by a previous loading or unloading curve, the stress-strain relationship follows the previous curve.

The extended Masing rules have been found to be applicable for the hysteretic soil response observed in experimental work including cyclic element tests by Pyke (1979). They have also been found to apply to the cyclic response for suction caisson foundations of up to 1 diameter embedment (Byrne and Houlsby, 2004) and appear to be a reasonable model for the cyclic response of small-scale pile foundations in sand under a limited amount of cycles (Hajjalilue-Bonab et al., 2013; Abadie, 2015). The field

test setup in the PISA project allows for further investigation of whether this model is applicable for monopile foundations.

4.3.2 Constitutive model

The extended Masing rules for the non-linear soil reaction curves are reproduced using the kinematic hardening model, also known as the Masing-Iwan model (Iwan, 1967). The constitutive behaviour of this model has been described previously using the hyperplasticity framework in Einav (2004) and is given here as guidance to the reader. The constitutive model for the parallel Masing-Iwan model can be visualised as shown in Figure 4.6.

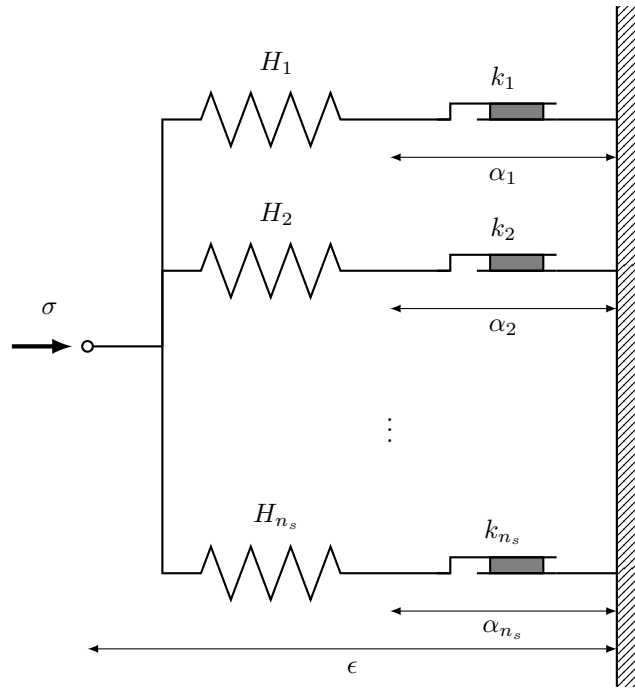


Figure 4.6: Parallel form of the kinematic hardening model.

The energy and dissipation functions are written as:

$$f = \sum_{i=1}^{n_s} \frac{H_i}{2} (\epsilon - \alpha_i)^2 \quad (4.39)$$

$$d = \sum_{i=1}^{n_s} k_i |\dot{\alpha}_i| \quad (4.40)$$

where n_s is the number of spring-slider elements in the model. The equations show how the kinematic hardening model can easily be framed within the hyperplasticity framework. The total stored and dissipated energy is simply the sum of that of the individual elements. Differentiating these expressions gives the stress and plastic behaviour:

$$\sigma = \sum_{i=1}^{n_s} H_i(\epsilon - \alpha_i) \quad (4.41)$$

$$\chi_{\alpha_i} = \bar{\chi}_{\alpha_i} \rightarrow H_i(\epsilon - \alpha_i) = S(\dot{\alpha}_i)k_i \quad (4.42)$$

The second equation shows that each of the plastic elements acts independently of the others, which is useful for the numerical implementation in the strain-driven case.

4.3.3 Numerical implementation

For the numerical implementation, two elements are discussed: (i) the discretisation of the backbone curve and (ii) a numerical algorithm to solve for the stress update.

Discretisation of the backbone curve

The backbone curve is discretised by linear interpolation of points on the backbone curve as shown in Figure 4.7. The parameters of the kinematic hardening model can be derived by solving the following equations:

$$E_i = \frac{\sigma_{i+1} - \sigma_i}{\epsilon_{i+1} - \epsilon_i} \quad i = 0 \dots n_s - 1 \quad (4.43)$$

$$E_i = \sum_{j=i+1}^{n_s} H_j \quad i = 0 \dots n_s - 1 \quad (4.44)$$

$$\sigma_i = \sum_{j=1}^i k_j + \epsilon_i \sum_{j=i+1}^{n_s} H_j \quad i = 1 \dots n_s \quad (4.45)$$

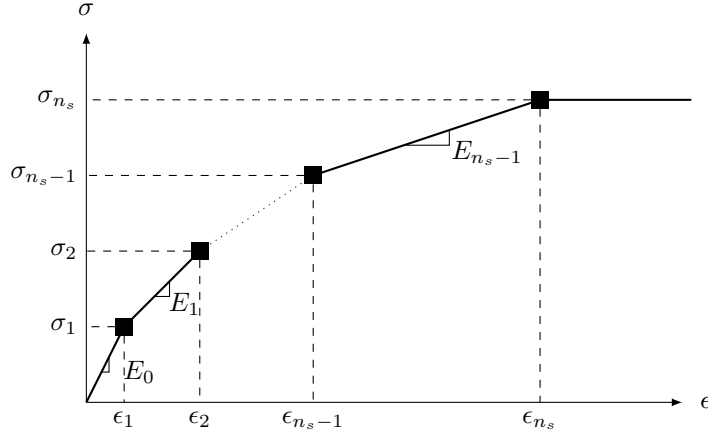


Figure 4.7: Discretisation of the backbone curve.

Numerical algorithm

Within each iteration step in the Newton-Raphson scheme for the FE method, a displacement of the pile is given and the constitutive relationship needs to return the resulting reaction. This ‘strain driven’ problem is suitable for a parallel version of the kinematic hardening model since the applied displacement is the same in each of the spring-slider elements. The algorithm for determining the reaction is simply based on that for the rate independent single yield surface model, and is given below:

Input: $\epsilon_1, \alpha_{0i}, k_i, H_i, i = 1 \dots n_s$

for $i = 1 \dots n_s$ **do**

$\chi_i^{\text{trial}} \leftarrow H_i(\epsilon_1 - \alpha_{0i})$

if $|\chi_i^{\text{trial}}| > k_i$ **then**

$\alpha_{1i} \leftarrow \epsilon_1 - S(\chi_i^{\text{trial}}) \frac{k_i}{H_i}$

else

$\alpha_{1i} \leftarrow \alpha_{0i}$

end if

end for

$\sigma_1 \leftarrow \sum_{i=1}^{n_s} H_i(\epsilon_1 - \alpha_{1i})$

Output: $\sigma_1, \alpha_{1i}, i = 1 \dots n_s$

4.3.4 Model output

This subsection presents an overview of the output from the kinematic hardening model. The backbone curve is fitted to a conic curve described in Equation 2.1 with the following parameters: $k_0 = 1$, $\sigma_u = 1$, $\epsilon_u = 3$ and $n = 0.5$. The discretisation with the kinematic hardening model is done using 10 elements, where σ_i is evenly spaced along the stress axis.

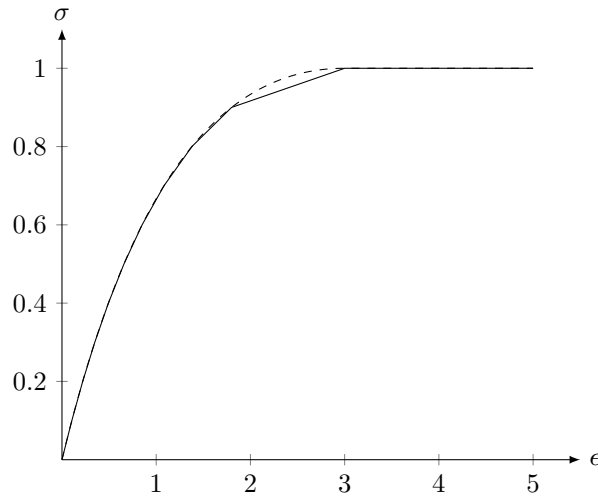


Figure 4.8: Illustration of the (—) kinematic hardening model upon initial loading and (---) comparison with the analytical form of the conic curve.

For a monotonically increasing strain, the kinematic hardening model follows the backbone curve as illustrated in Figure 4.8, showing the effect of the discretisation into a finite number of plastic elements. Figure 4.9 shows the response when a sinusoidal displacement is applied with varying amplitudes. Upon initial loading the model follows the backbone curve, in the unloading and reloading stages the Mas-ing behaviour can clearly be seen with an increasing loop size for increasing strain amplitudes.

Finally, the behaviour of the model is shown for an applied displacement of $\epsilon = 0 \rightarrow 1 \rightarrow -1 \rightarrow 2$ in Figure 4.10. After the cycle, the response follows the backbone curve

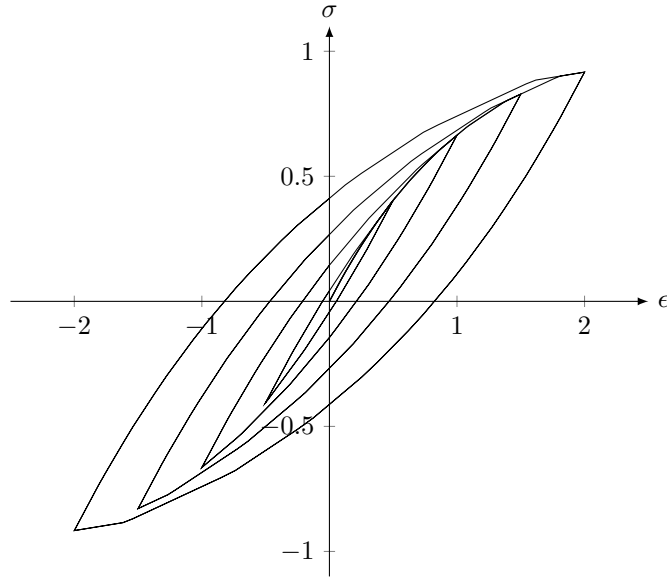


Figure 4.9: Illustration of the kinematic hardening model upon cyclic loading at varying strain amplitudes.

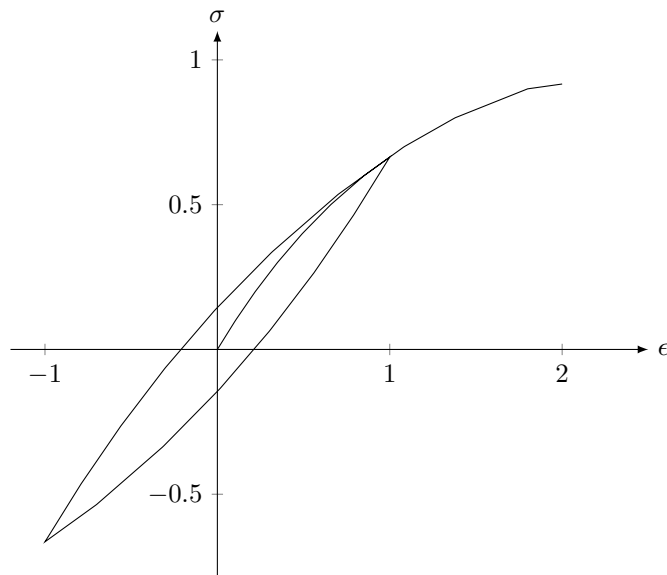


Figure 4.10: Illustration of the kinematic hardening model for an applied displacement of $\epsilon = 0 \rightarrow 1 \rightarrow -1 \rightarrow 2$.

again as specified in the extended Masing rules. It can be shown generally that any one-dimensional plasticity model that employs pure kinematic hardening will conform to the extended Masing rules, and these do not need to be introduced as a separate feature of the model.

4.4 Rate effect model

Conventional implementation of constitutive models follows the underlying physical model *i.e.* a rate independent physical model is modelled using a rate independent numerical method and similarly, for a rate dependent physical model (see Table 4.3). As shown in section 4.2, a rate independent model can also be modelled using a rate dependent numerical implementation by setting the viscosity to an artificially low value. The numerical implementation of such rate dependent models follows standardised methods, which makes them suitable for model development. Setting the viscosity to an artificially low value also limits the time step for each stress update. The implementation of these models can therefore become computationally expensive when integrated in FE methods.

Table 4.3: Modelling techniques for rate dependent and rate independent models.

		Underlying physical model	
		Rate independent	Rate dependent
Numerical implementation	Rate independent	Conventional	Modified method
	Rate dependent	Model development	Conventional

In this section, the inverse approach will be adopted: a rate independent numerical implementation will be introduced for a rate dependent physical model. This section shows that for specific physical models, a rate independent formulation can be found

which captures the rate effect to a reasonable degree for engineering applications, with the advantage that it is computationally more efficient.

The rate effect on the soil strength is often described using a power law (Gibson and Coyle, 1968; Heerema, 1979), a logarithmic law (Dayal and Allen, 1975; Bea, 1982) or using an inverse hyperbolic sine expression (Murayama and Shibata, 1966; Mitchell, 1976). Brown (2004) provides an overview of these expressions in the context of fast axial load tests on piles in fine grained soils.

4.4.1 Rate process theory

The behaviour of many systems can be captured using rate process theory. Mitchell (1976) presents an overview on time dependency of the mechanical soil behaviour and shows that the stress and strain and relaxation behaviour of both sand and clay soils can be captured using the theory. The theory has been introduced in the hyperplasticity framework by Houlsby and Puzrin (2002). In rate process theory, the rate of activation is a function of the activation force. These variables are identified in the hyperplasticity framework as the hardening strain α and the generalised stress χ so that:

$$\dot{\alpha} = A \sinh(\chi/B) \quad (4.46)$$

The viscous dissipated energy can now be written as: $\chi\dot{\alpha} = \dot{\alpha}B \sinh^{-1}(\dot{\alpha}/A)$. The dissipation potential is obtained by adding the dissipation of a frictional slider. After changing the constants for further application, the dissipation function and the generalised stress are written as:

$$d = k \left[|\dot{\alpha}| + \dot{\alpha} \eta \sinh^{-1} \left(\frac{\dot{\alpha}}{\dot{\alpha}_{qs}} \right) \right] \quad (4.47)$$

$$\chi = k \left[S(\dot{\alpha}) + \eta \sinh^{-1} \left(\frac{\dot{\alpha}}{\dot{\alpha}_{qs}} \right) \right] \quad (4.48)$$

In these equations, η is the viscous constant and $\dot{\alpha}_{\text{qs}}$ is the quasi-static strain rate. For $\frac{\dot{\alpha}}{\dot{\alpha}_{\text{qs}}} \gg 1$, the strength increase due to the dashpot is related to $\frac{\chi}{k} - 1 \sim \eta \log \left(2 \frac{\dot{\alpha}}{\dot{\alpha}_{\text{qs}}} \right) = \eta_{10} \log_{10} \left(2 \frac{\dot{\alpha}}{\dot{\alpha}_{\text{qs}}} \right)$, with $\eta_{10} = \eta \log(10)$. Note that the energy potential remains unchanged with respect to the single spring-slider element. This formulation of the dissipation function cannot be decomposed as pseudo-homogeneous in the conventional way (at least without recourse to an infinite series). However, in the case of a 1D model, we can apply equation 4.22. This results in the following force potential:

$$z = k \left[|\dot{\alpha}| + \eta \left(\dot{\alpha} \sinh^{-1} \left(\frac{\dot{\alpha}}{\dot{\alpha}_{\text{qs}}} \right) - \sqrt{\dot{\alpha}^2 + \dot{\alpha}_{\text{qs}}^2} + \dot{\alpha}_{\text{qs}} \right) \right] \quad (4.49)$$

The incremental form for the hardening strain is obtained by inversion of equation 4.48:

$$\dot{\alpha} = \dot{\alpha}_{\text{qs}} \sinh \left(\frac{\langle |\chi| - k \rangle}{k\eta} \right) S(\chi) \quad (4.50)$$

The energy function used here is the same as in the single yield surface model ($f = \frac{H}{2}(\epsilon - \alpha)^2$), which results in $\sigma = \bar{\chi} = \chi$. For a test at constant strain rate, the incremental stress is now written as:

$$d\sigma = H \left(1 - \frac{\dot{\alpha}_{\text{qs}}}{\dot{\epsilon}} \sinh \left(\frac{\langle |\sigma| - k \rangle}{k\eta} \right) S(\sigma) \right) d\epsilon \quad (4.51)$$

This equation can be integrated to give the plastic part of the curve:

$$\frac{\sigma}{k} = S(\epsilon) \left[1 + 2\eta \tanh^{-1} \left\{ x \tanh \left(\left(\frac{H|\epsilon| - k}{2k\eta} \right) x + \tanh^{-1} \left(\frac{\dot{\alpha}_{\text{qs}}}{x|\dot{\epsilon}|} \right) \right) - \frac{\dot{\alpha}_{\text{qs}}}{|\dot{\epsilon}|} \right\} \right] \quad (4.52)$$

with $x = \sqrt{1 + \frac{\dot{\alpha}_{\text{qs}}^2}{\dot{\epsilon}^2}}$. The resulting normalised stress-strain curves are given in Figure 4.11 for various constant normalised strain rates and with $\eta_{10} = 0.2$.

Note that the above derivation follows the developments in Houlsby and Puzrin (2002). Figure 4.12 shows that for constant strain rates, there are 3 zones which can be identified: (1) an elastic zone where no dissipation occurs, (2) an elastic-visco-

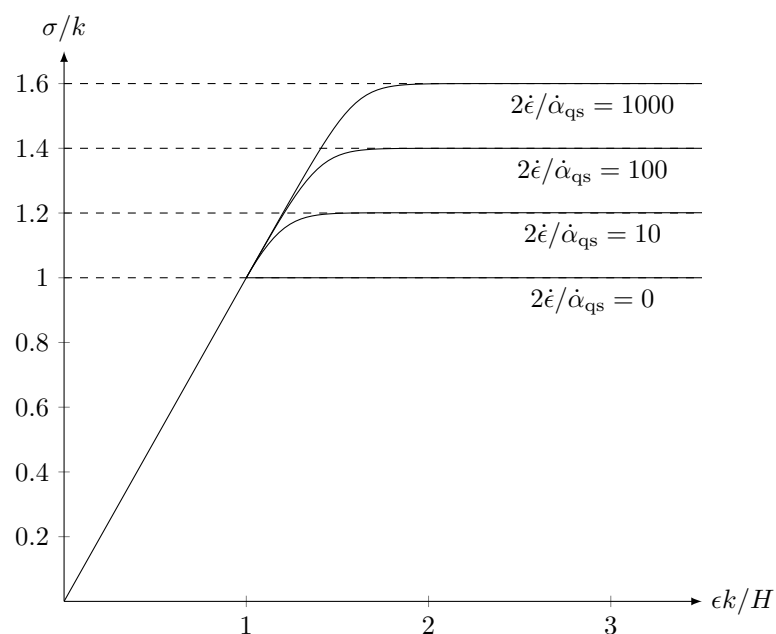


Figure 4.11: Stress-strain curves for the single spring-slider-dashpot element.

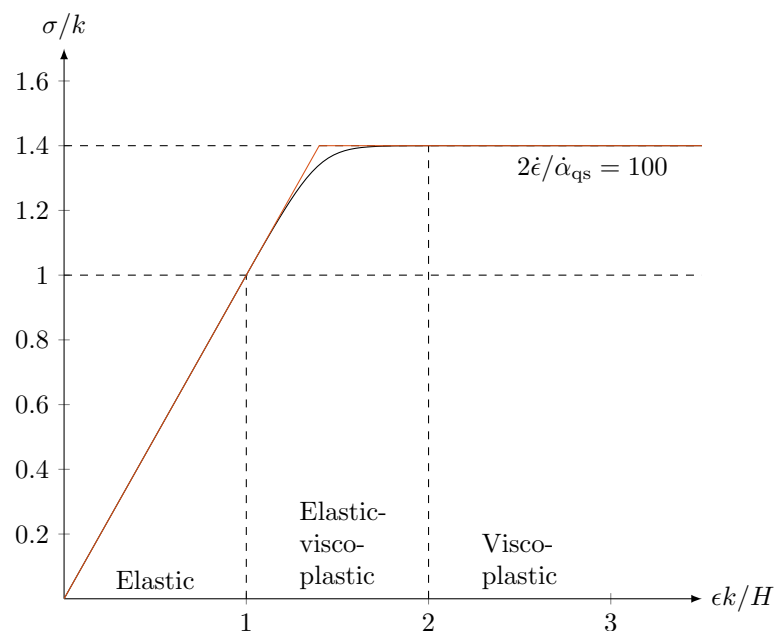


Figure 4.12: Comparison of (—) the bi-linear approximation with (—) the rate process theory model.

plastic zone where both the elastic energy increases and energy is dissipated through the dashpot and slider and (3) a zone where the elastic energy does not increase and the energy is dissipated through the dashpot and slider. The reaction can closely be approximated using a bilinear curve with the same stiffness and a modified strength of:

$$k_d(\dot{\epsilon}) = k \left(1 + \eta \sinh^{-1} \left(\frac{|\dot{\epsilon}|}{\dot{\alpha}_{qs}} \right) \right) \quad (4.53)$$

The bilinear approximation is used to efficiently compute the rate dependent reaction with reasonable accuracy. The derived response, however, is not within the hyperplasticity framework. The dissipation function of the single yield surface model can be modified using this strain rate dependent strength:

$$d = k_d(\dot{\epsilon})|\dot{\alpha}| \quad (4.54)$$

4.4.2 Constitutive model

The two versions of the rate effect model can be integrated in a multi-surface model. The resulting constitutive model can be visualised as shown in Figure 4.13.

Rate process theory model

The energy and dissipation functions for the model based on rate process theory are written as:

$$f = \sum_{i=1}^{n_s} \frac{H_i}{2} (\epsilon - \alpha_i)^2 \quad (4.55)$$

$$z = \sum_{i=1}^{n_s} k_i \left[|\dot{\alpha}_i| + \eta \left(\dot{\alpha}_i \sinh^{-1} \left(\frac{\dot{\alpha}_i}{\dot{\alpha}_{qs}} \right) - \sqrt{\dot{\alpha}_i^2 + \dot{\alpha}_{qs}^2} + \dot{\alpha}_{qs} \right) \right] \quad (4.56)$$

Note that in this expression, the variables η and $\dot{\alpha}_{qs}$ are taken as constants for convenience instead of a varying parameter for each spring-dashpot-slider element. Differ-

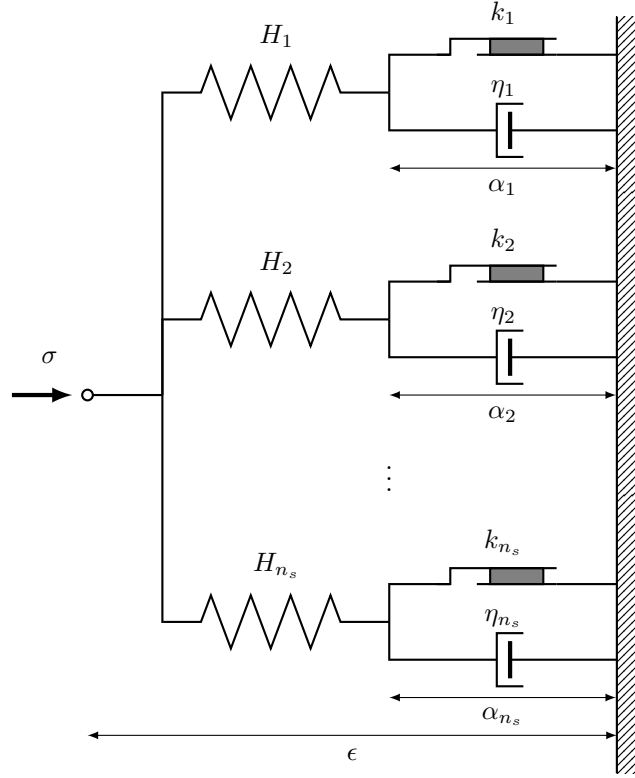


Figure 4.13: Rate effect model.

entiation of these expressions gives the stress and plastic behaviour from this model:

$$\sigma = \sum_{i=1}^{n_s} H_i(\epsilon - \alpha_i) \quad (4.57)$$

$$\chi_{\alpha_i} = \bar{\chi}_{\alpha_i} \rightarrow H_i(\epsilon - \alpha_i) = k_i \left[S(\dot{\alpha}_i) + \eta \sinh^{-1} \left(\frac{\dot{\alpha}_i}{\dot{\alpha}_{qs}} \right) \right] \quad (4.58)$$

The above equations show that each of the parallel elements acts independently of the others. The derivation from rate process theory can therefore be used in the numerical implementation of this model.

Modified rate effect model

In the modified rate effect model the energy potential remains the same function,

however, the dissipation function becomes:

$$d = \sum_{i=1}^{n_s} k_{di}(\dot{\epsilon}) |\dot{\alpha}_i| \quad (4.59)$$

with $k_{di}(\dot{\epsilon}) = k_i \left(1 + \eta \sinh^{-1} \left(\frac{|\dot{\epsilon}|}{\dot{\alpha}_{qs}} \right) \right)$. Although the following is not derived within the hyperplasticity framework, the expression below gives the plastic response of the model, effectively reproducing the bi-linear approximation of Figure 4.12 for each of the parallel elements:

$$H_i(\epsilon - \alpha_i) = S(\dot{\alpha}_i) k_{di}(\dot{\epsilon}) \quad (4.60)$$

The approximation provides a good fit to the reaction and is computationally more efficient, which is essential for implementation in a Winkler model. The numerical implementation can follow the implementation of the kinematic hardening model.

4.4.3 Numerical implementation

The numerical implementation for the rate process theory model is based on equation 4.51. The resulting algorithm is given below:

```

Input:  $\epsilon_1, \epsilon_0, \Delta t, \alpha_{0i}, k_i, H_i, \eta, \dot{\alpha}_{qs}, i = 1 \dots n_s$ 

 $\Delta\epsilon \leftarrow \epsilon_1 - \epsilon_0$ 

 $\dot{\epsilon} \leftarrow \frac{\Delta\epsilon}{\Delta t}$ 

for  $i = 1 \dots n_s$  do
     $\chi_{0i} \leftarrow H_i(\epsilon_0 - \alpha_{0i})$ 
     $\Delta\chi_i \leftarrow H_i \left( 1 - \frac{\dot{\alpha}_{qs}}{\dot{\epsilon}} \sinh \left( \frac{\langle |\chi_{0i}| - k_i \rangle}{k_i \eta} \right) S(\chi_{0i}) \right) \Delta\epsilon$ 
     $\chi_{1i} \leftarrow \chi_{0i} + \Delta\chi_i$ 
     $\alpha_{1i} \leftarrow \epsilon_1 - S(\chi_{1i}) \frac{k_i}{H_i}$ 
end for
    
```

$$\sigma_1 \leftarrow \sum_{i=1}^{n_s} H_i(\epsilon_1 - \alpha_{1i})$$

Output: $\sigma_1, \alpha_{1i}, i = 1 \dots n_s$

This algorithm requires small strain steps to achieve convergence. Using the modified rate effect model, large strain steps can be taken and the solution is found using a single evaluation of the stress-strain response. The algorithm follows the implementation of the kinematic hardening model.

Input: $\epsilon_1, \epsilon_0, \Delta t, \alpha_{0i}, k_i, H_i, \eta, \dot{\alpha}_{qs}, i = 1 \dots n_s$

$$\dot{\epsilon} \leftarrow \frac{\epsilon_1 - \epsilon_0}{\Delta t}$$

$$k_{di} \leftarrow k_i \left(1 + \eta \sinh^{-1} (|\dot{\epsilon}| / \dot{\alpha}_{qs}) \right), i = 1 \dots n_s$$

for $i = 1 \dots n_s$ **do**

$$\chi_i^{\text{trial}} \leftarrow H_i(\epsilon_1 - \alpha_{0i})$$

if $|\chi_i^{\text{trial}}| > k_{di}$ **then**

$$\alpha_{1i} \leftarrow \epsilon_1 - S(\chi_i^{\text{trial}}) \frac{k_{di}}{H_i}$$

else

$$\alpha_{1i} \leftarrow \alpha_{0i}$$

end if

end for

$$\sigma_1 \leftarrow \sum_{i=1}^{n_s} H_i(\epsilon_1 - \alpha_{1i})$$

Output: $\sigma_1, \alpha_{1i}, i = 1 \dots n_s$

4.4.4 Model output

The rate process theory model and the modified rate effect model are compared in Figure 4.14 upon initial loading. The backbone curve is discretised similarly to the example in the kinematic hardening model and η_{10} is set to 0.2. The modified model gives a close match to the rate process theory model.

Figure 4.15 gives a comparison of the models upon cyclic loading. Again, the

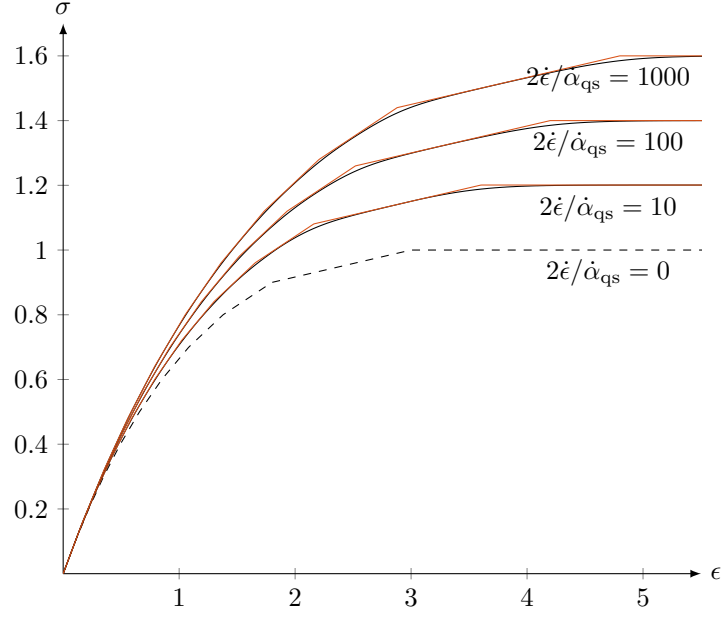


Figure 4.14: Comparison of (---) the static backbone curve, (—) the rate process theory model and (—) the modified rate effect model upon initial loading for three different loading rates.

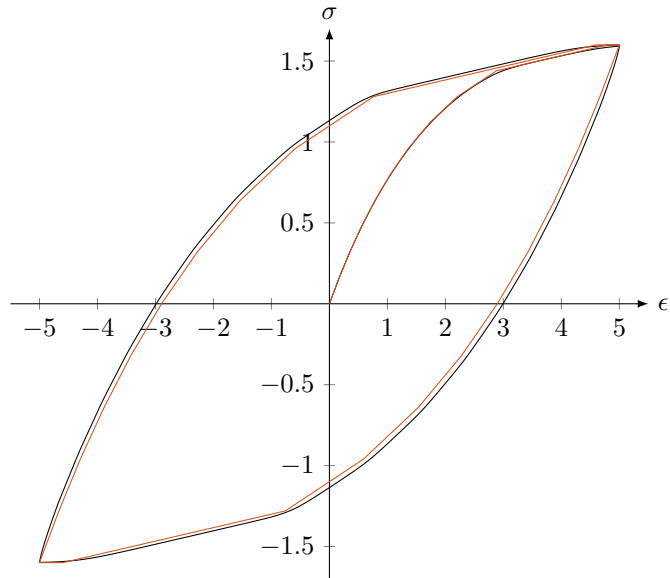


Figure 4.15: Comparison of (—) the rate process theory model and (—) the modified rate effect model upon cyclic loading with $2|\dot{\epsilon}|/\dot{\alpha}_{qs} = 1000$.

modified model gives a close match to the rate process theory model. Upon initial unloading, the rate process theory model is slightly softer because of the unloading in the elastic-visco-plastic phase, where the damper still has an effect on the unloading. Note that the algorithm for the rate process theory model requires small steps in order to give an accurate solution, whereas the modified rate effect model allows for calculating the stress-strain response of any strain step size in one evaluation.

4.5 Ratcheting model

4.5.1 Development of ratcheting models

An extensive overview of the methods used for modelling ratcheting behaviour can be found in Bari (2001). Most ratcheting models found in the literature seem to find their roots in the Armstrong-Frederick model (Armstrong and Frederick, 1966). This model has been reformulated in the hyperplasticity framework by Houlsby et al. (2017). A schematic representation of the original Armstrong-Frederick model is given in Figure 4.16.

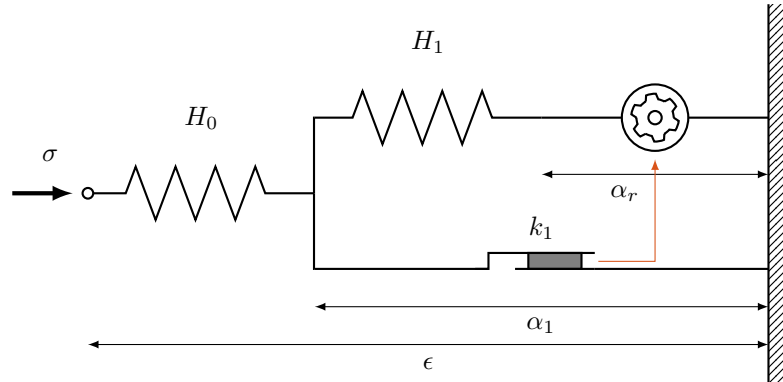


Figure 4.16: Armstrong Frederick model.

The functions describing this model in the hyperplasticity framework can be written

as:

$$f = \frac{H_0}{2}(\epsilon - \alpha_1)^2 + \frac{H_1}{2}(\alpha_1 - \alpha_r)^2 \quad (4.61)$$

$$d = k_1|\dot{\alpha}_1| + \bar{\chi}_r\dot{\alpha}_r \quad (4.62)$$

$$c = \dot{\alpha}_r - \frac{\bar{\chi}_r}{\chi^*}|\dot{\alpha}_1| \quad (4.63)$$

where H_0 , H_1 are constants and k_1 , χ^* are either constants or functions of state. The ratcheting element in this model is placed in parallel with a slider element. Houlsby et al. (2017) have developed an alternative approach, where the ratcheting element is put in series with a kinematic hardening model. This approach leads to a clear decoupling of the ratcheting effect from the kinematic hardening effect. The constitutive model is outlined below.

4.5.2 Constitutive model

The constitutive model can be visualised as shown in Figure 4.17.

The energy and dissipation functions are written as:

$$f = \sum_{i=1}^{n_s} \frac{H_i}{2}(\epsilon - \alpha_i - \alpha_r)^2 \quad (4.64)$$

$$d = \sum_{i=1}^{n_s} k_i|\dot{\alpha}_i| + \sigma\dot{\alpha}_r \quad (4.65)$$

where α_r is the ratcheting strain in the model. The ratcheting strain is activated by the plastic strain in the slider elements. Therefore an additional constraint needs to be written:

$$c = \sigma\dot{\alpha}_r - |\sigma| \sum_{i=1}^{n_s} R_i|\dot{\alpha}_i| = 0 \quad (4.66)$$

with R_i coupling parameters between the slider elements and the ratcheting element.

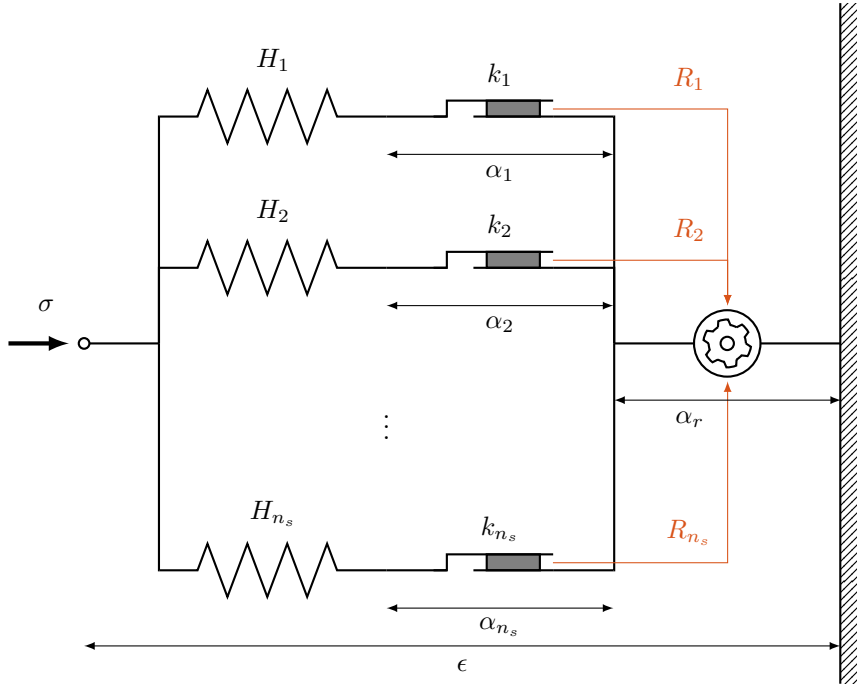


Figure 4.17: Ratcheting model.

The augmented dissipation function is now written as:

$$d^* = \sum_{i=1}^{n_s} k_i |\dot{\alpha}_i| + \sigma \dot{\alpha}_r + \Lambda \left(\sigma \dot{\alpha}_r - |\sigma| \sum_{i=1}^{n_s} R_i |\dot{\alpha}_i| \right) \quad (4.67)$$

Differentiating these expressions gives the stress and plastic behaviour:

$$\sigma = \sum_{i=1}^{n_s} H_i (\epsilon - \alpha_i - \alpha_r) \quad (4.68)$$

$$\chi_{\alpha_i} = \bar{\chi}_{\alpha_i} \rightarrow H_i (\epsilon - \alpha_i - \alpha_r) = S(\dot{\alpha}_i) k_i + \Lambda (|\sigma| R_i S(\dot{\alpha}_i)) \quad (4.69)$$

$$\chi_{\alpha_r} = \bar{\chi}_{\alpha_r} \rightarrow \sum_{i=1}^{n_s} H_i (\epsilon - \alpha_i - \alpha_r) = \sigma + \Lambda \sigma \quad (4.70)$$

From the last equation, it can be concluded that $\Lambda = 0$. This result shows that the ratcheting element is decoupled from the kinematic hardening model. A history effect is also introduced using a hardening variable. Several options are available for the definition of the hardening variable including strain-type quantities ($\dot{\alpha}_h = |\dot{\alpha}_r|$)

or $\dot{\alpha}_h = \sum_{i=1}^{n_s} |\dot{\alpha}_i|$) and work-type quantities ($\dot{\beta}_h = \sigma|\dot{\alpha}_r|$, $\dot{\beta}_h = \sum_{i=1}^{n_s} k_i|\dot{\alpha}_i|$ or $\dot{\beta}_h = \sum_{i=1}^{n_s} k_i|\dot{\alpha}_i| + \sigma|\dot{\alpha}_r|$). For the simulations in this thesis, the hardening strain rate is defined as the magnitude of the ratcheting strain rate:

$$\dot{\alpha}_h = |\dot{\alpha}_r| \quad (4.71)$$

This choice of hardening variable accurately captures the response when using an accelerated modelling approach (Abadie, 2015). The accelerated modelling approach (see Section 5.5.1.3) can be used to capture the effect of many cycles using a reduced number of simulated cycles. The hardening variable can also be used to modify the slider strengths k_i and the coupling parameters R_i , reflecting the effect of the loading history on the cyclic response.

4.5.3 Numerical implementation

For the numerical implementation, the ratcheting constraint and the history effect are rewritten as:

$$\dot{\alpha}_r = S(\sigma)|\dot{\alpha}| \sum_{i=1}^{n_s} y_i R_i \quad (4.72)$$

$$\dot{\alpha}_h = |\dot{\alpha}_r| = |\dot{\alpha}| \sum_{i=1}^{n_s} y_i R_i \quad (4.73)$$

where y_i indicates if a spring-slider element is yielding ($y_i = 1$) or not ($y_i = 0$). For all the elements which are yielding, the individual plastic strain rates are the same: $\dot{\alpha}_i = \dot{\alpha} y_i$. Rewriting the equation for the plastic behaviour of a slider element gives:

$$\dot{\epsilon} = \dot{\chi}_i / H_i + \dot{\alpha}_i + \dot{\alpha}_r \quad (4.74)$$

$$\dot{\epsilon} = \dot{\alpha} \left(1 + S(\sigma) S(\dot{\alpha}) \sum_{i=1}^{n_s} y_i R_i \right) \quad \text{for } y_i = 1 \quad (4.75)$$

In all practical cases, the term $1 + S(\sigma)S(\dot{\alpha}) \sum_{i=1}^{n_s} y_i R_i$ is bigger than 0 and therefore the direction of the hardening strain is in the same direction as the total strain ($S(\dot{\alpha}) = S(\dot{\epsilon})$). The increase in hardening strain can then be rewritten as:

$$\dot{\alpha} = \frac{\dot{\epsilon}}{1 + S(\sigma)S(\dot{\epsilon}) \sum_{i=1}^{n_s} y_i R_i} \quad (4.76)$$

For the numerical implementation, an iterative process is used where the maximum incremental strain step is determined as the minimum between a set maximum strain ($\sum_{i=1}^{n_s} k_i / (200 \sum_{i=1}^{n_s} H_i)$) and the strain where the next yield surface is reached:

$$\Delta\epsilon_{\max} = \min \begin{cases} \sum_{i=1}^{n_s} k_i / (200 \sum_{i=1}^{n_s} H_i) \\ (1 + S(\sigma)S(\Delta\epsilon) \sum_{i=1}^{n_s} y_i R_i) \Delta\alpha^{\text{bound}} \end{cases} \quad (4.77)$$

The internal strain step to reach the next yield surface is determined by $\Delta\alpha^{\text{bound}} = S(\Delta\epsilon) \min(S(\Delta\epsilon)(-\epsilon + \alpha_i + \alpha_r + S(\Delta\epsilon)k_i/H_i) > 0)$. In the case where all surfaces yield or the remaining strain step is smaller than the strain step until the next yield surface, the strain step is determined using the remaining strain in the step.

Input: $\epsilon_1, \epsilon_0, \alpha_{0i}, \alpha_{0r}, \alpha_{0h}, k_i, H_i, i = 1 \dots n_s$

$\epsilon^* \leftarrow \epsilon_0, \alpha_i^* \leftarrow \alpha_{0i}, \alpha_r^* \leftarrow \alpha_{0r}, \alpha_h^* \leftarrow \alpha_{0h}$

while $\epsilon^* \neq \epsilon_1$ **do**

$\sigma^* \leftarrow \sum_{i=1}^{n_s} H_i(\epsilon^* - \alpha_i^* - \alpha_r^*)$

Specify $\Delta\epsilon$

$\epsilon^* \leftarrow \epsilon^* + \Delta\epsilon$

for $i = 1 \dots n_s$ **do**

$\chi_i^{\text{trial}} \leftarrow H_i(\epsilon^* - \alpha_i^* - \alpha_r^*)$

if $|\chi_i^{\text{trial}}| - k_i > 0$ **then**

```

         $y_i \leftarrow 1$ 
    else
         $y_i \leftarrow 0$ 
    end if
end for

 $\Delta\alpha \leftarrow \frac{\Delta\epsilon}{1+S(\sigma^*)S(\Delta\epsilon)\sum_{i=1}^{n_s} y_i R_i(\sigma^*, \alpha_h^*)}$ 
 $\alpha_i^* \leftarrow \alpha_i^* + y_i \Delta\alpha, i = 1 \dots n_s$ 
 $\alpha_r^* \leftarrow \alpha_r^* + \Delta\alpha \sum_{i=1}^{n_s} y_i R_i(\sigma^*, \alpha_h^*)$ 
 $\alpha_h^* \leftarrow \alpha_h^* + |\Delta\alpha| \sum_{i=1}^{n_s} y_i R_i(\sigma^*, \alpha_h^*)$ 
end while

 $\epsilon_1 \leftarrow \epsilon^*, \alpha_{1i} \leftarrow \alpha_i^*, \alpha_{1r} \leftarrow \alpha_r^*, \alpha_{1h} \leftarrow \alpha_h^*$ 
 $\sigma_1 \leftarrow \sum_{i=1}^{n_s} H_i(\epsilon_1 - \alpha_{1i} - \alpha_{1r})$ 
Output:  $\sigma_1, \alpha_{1i}, \alpha_{1r}, \alpha_{1h}, i = 1 \dots n_s$ 

```

4.5.4 Calibration of the strength and stiffness parameters

As the ratcheting element becomes activated more and more on initial loading, it decreases the tangent stiffness with respect to the kinematic hardening model. The strength and stiffness parameters of the ratcheting model should therefore be calibrated such that the behaviour follows a specified backbone curve on initial loading. In the current implementation, this calibration uses a numerical fitting where the backbone curve of the kinematic hardening model is specified incrementally such that it falls down on the target backbone curve when the ratcheting element is activated. This process is illustrated in Figure 4.18.

4.5.5 Model output

The model is illustrated using two simplified examples. The backbone curve is fitted to the conic curve described as given in Section 4.3.4 on the kinematic hardening

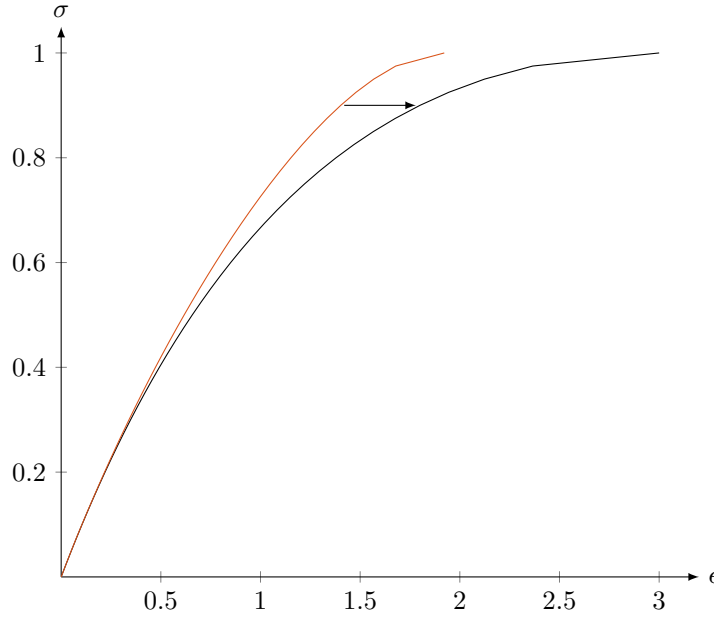


Figure 4.18: Illustration of the calibration process for the strength and stiffness parameters of the ratcheting model, where the backbone curve of the kinematic hardening model (—) falls down onto the target backbone curve (—) when the ratcheting element is activated.

model. The ratcheting parameter $R_i(\sigma, \alpha_h)$ for the first example is modelled as a constant: $R_i = R_0 \frac{k_i}{\sigma_u}$; $i = 1 \dots n_s$ with $R_0 = 2$. The number of ratcheting elements is taken as $n_s = 40$. The first 10 cycles are shown for one-way cycling to $0.7\sigma_u$ in Figure 4.19. The resulting ratcheting strain in this simulation is constant for each load cycle.

A decrease in ratcheting rate can be introduced using the hardening parameter α_h . The ratcheting parameter can be modified to: $R_i = R_0 \frac{k_i}{\sigma_u} \left(\frac{\alpha_h}{\alpha_{h0}} \right)^{m_h}$; $i = 1 \dots n_s$, where m_h is a negative power. This formulation requires an initial value for the hardening parameter α_{h0} , which is set to a fraction of the ultimate strain ($\alpha_{h0} = \epsilon_u/100$). For Figure 4.20, the model parameters are chosen as: $R_0 = 2$ and $m_h = -0.7$. The 100 simulated loops in this figure show a clear decrease of ratcheting rate with increasing cycle number.

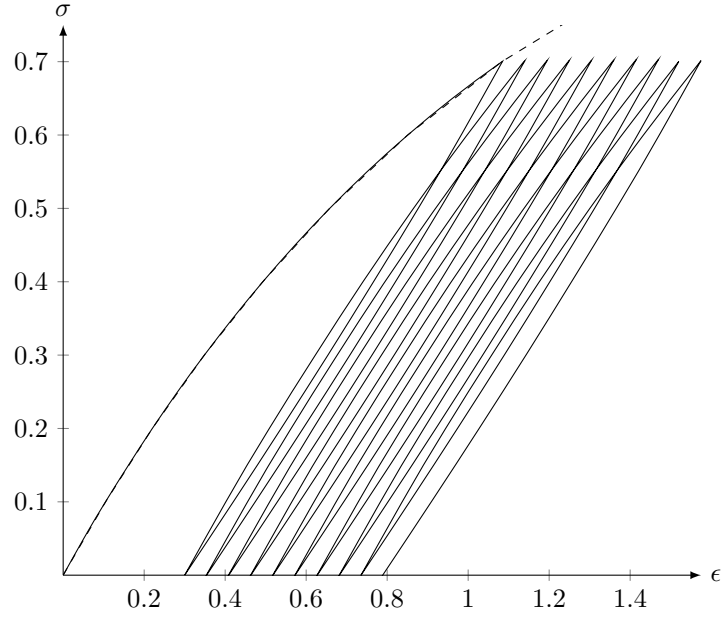


Figure 4.19: Model output with a constant ratcheting parameter of $R_i = R_0/n_s$ $i = 1 \dots n_s$ with $R_0 = 0.5$.

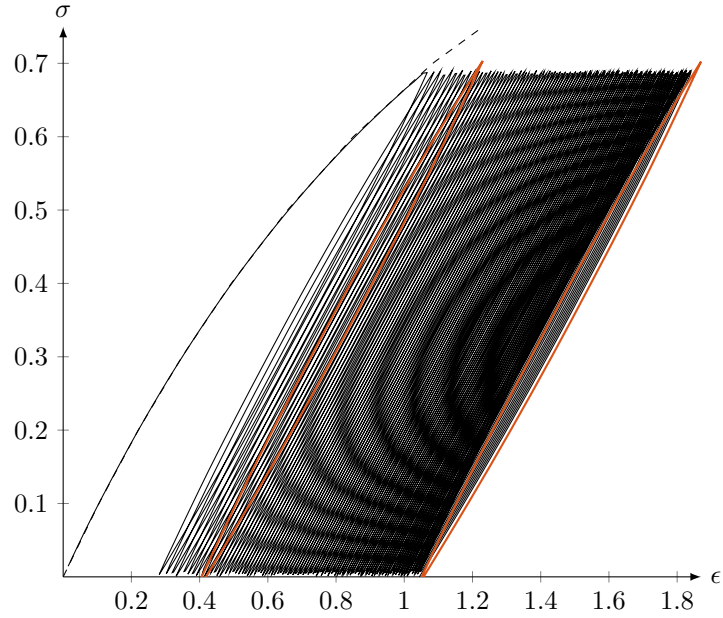


Figure 4.20: Model output with a ratcheting parameter of $R_i = R_0 \frac{k_i}{\sigma_u} \left(\frac{\alpha_h}{\alpha_{h0}} \right)^{m_h}$ $i = 1 \dots n_s$ with $R_0 = 2$ and $m_h = -0.7$. Cycle number 10 and 100 are highlighted in red.

4.6 Combined rate and ratcheting model

This section outlines a combined rate and ratcheting model, based on the developments in the previous sections on the rate model and the ratcheting model. The combined model is developed to capture the creep periods in the PISA tests as well as the ratcheting effect.

4.6.1 Constitutive model

The constitutive model can be visualised as shown in Figure 4.21. The energy function in this model is the same as for the ratcheting model ($f = \sum_{i=1}^{n_s} \frac{H_i}{2} (\epsilon - \alpha_i - \alpha_r)^2$). Using rate process theory, the force potential is given by:

$$z = \sum_{i=1}^{n_s} k_i \left[|\dot{\alpha}_i| + \eta \left(\dot{\alpha}_i + \sinh^{-1} \left(\frac{\dot{\alpha}_i}{\dot{\alpha}_{qs}} \right) - \sqrt{\dot{\alpha}_i^2 + \dot{\alpha}_{qs}^2} + \dot{\alpha}_{qs} \right) \right] + \sigma \dot{\alpha}_r \quad (4.78)$$

which is the combined effect of the rate and ratcheting model. With the modified rate effect model, the dissipation function is written as:

$$d = \sum_{i=1}^{n_s} k_{di} |\dot{\alpha}_i| + \sigma \dot{\alpha}_r \quad (4.79)$$

For both dissipation potentials an additional constraint is given to relate the hardening strains in the slider elements with the ratcheting element

$$c = \sigma \dot{\alpha}_r - |\sigma| \sum_{i=1}^{n_s} R_i |\dot{\alpha}_i| \quad (4.80)$$

In this thesis, the modified model is pursued for reasons of computational cost. The stress and plastic behaviour of this model are given by:

$$\sigma = \sum_{i=1}^{n_s} H_i (\epsilon - \alpha_i - \alpha_r) \quad (4.81)$$

Input: $\epsilon_1, \epsilon_0, \alpha_{0i}, \alpha_{0r}, \alpha_{0h}, k_i, H_i, \eta, \dot{\alpha}_{qs}, \Delta t, i = 1 \dots n_s$

$\epsilon^* \leftarrow \epsilon_0, \alpha_i^* \leftarrow \alpha_{0i}, \alpha_r^* \leftarrow \alpha_{0r}, \alpha_h^* \leftarrow \alpha_{0h}$

$\dot{\epsilon} = (\epsilon_1 - \epsilon_0) / \Delta t$

$k_{di} \leftarrow k_i (1 + \eta \sinh^{-1} (|\dot{\epsilon}| / \dot{\alpha}_{qs})) , i = 1 \dots n_s$

while $\epsilon^* \neq \epsilon_1$ **do**

$\sigma^* \leftarrow \sum_{i=1}^{n_s} H_i(\epsilon^* - \alpha_i^* - \alpha_r^*)$

Specify $\Delta\epsilon$ using k_{di} instead of k_i

$\epsilon^* \leftarrow \epsilon^* + \Delta\epsilon$

for $i = 1 \dots n_s$ **do**

$\chi_i^{\text{trial}} \leftarrow H_i(\epsilon^* - \alpha_i^* - \alpha_r^*)$

if $|\chi_i^{\text{trial}}| - k_{di} > 0$ **then**

$y_i \leftarrow 1$

else

$y_i \leftarrow 0$

end if

end for

$\Delta\alpha = \frac{\Delta\epsilon}{1 + S(\sigma^*)S(\Delta\epsilon) \sum_{i=1}^{n_s} y_i R_i(\sigma^*, \alpha_h^*)}$

$\alpha_i^* \leftarrow \alpha_i^* + y_i \Delta\alpha, i = 1 \dots n_s$

$\alpha_r^* \leftarrow \alpha_r^* + \Delta\alpha \sum_{i=1}^{n_s} y_i R_i(\sigma^*, \alpha_h^*)$

$\alpha_h^* \leftarrow \alpha_h^* + |\Delta\alpha| \sum_{i=1}^{n_s} y_i R_i(\sigma^*, \alpha_h^*)$

end while

$\epsilon_1 \leftarrow \epsilon^*, \alpha_{1i} \leftarrow \alpha_i^*, \alpha_{1r} \leftarrow \alpha_r^*, \alpha_{1h} \leftarrow \alpha_h^*$

$\sigma_1 \leftarrow \sum_{i=1}^{n_s} H_i(\epsilon_1 - \alpha_{1i} - \alpha_{1r})$

Output: $\sigma_1, \alpha_{1i}, \alpha_{1r}, \alpha_{1h}, i = 1 \dots n_s$

4.6.3 Model output

The model response of the combined rate and ratcheting model is compared to the rate independent ratcheting model in Figure 4.22. The ratcheting parameters are defined as $R_i = R_0 \frac{k_i}{\sigma_u}$; $i = 1 \dots n_s$ with $R_0 = 2$ and $n_s = 40$. The rate parameters are $\eta_{10} = 0.2$ and $\dot{\alpha}_{qs} = 1$. The cycles are applied at a displacement rate of $|\dot{\epsilon}|/\dot{\alpha}_{qs} = 50$. The figure shows that the loops of the cycles are smaller because of the increased strength parameters from the rate effect, which results in reduced ratcheting strain for each cycle.

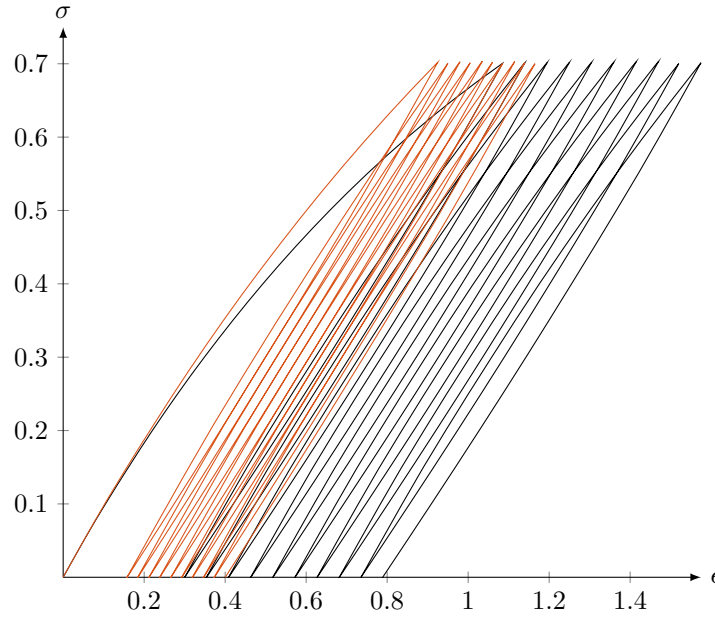


Figure 4.22: Comparison of (—) the rate dependent ratcheting model with (—) the rate independent version under cyclic loading.

4.7 Gapping model

Various authors have developed models to introduce the gapping effect for Winkler foundations (*e.g.* Boulanger et al. (1999), El Naggar et al. (2005), Nogami et al. (1992), Taciroglu et al. (2006), Hededal and Klinkvort (2010), Gerolymos and Gazetas (2006)). The development of these models is often motivated by the field of earthquake engineering. All the models only consider lateral pile displacement in one direction and cannot be generalised to two directions.

4.7.1 Generalised Winkler model

In Houlsby et al. (2005), a method is presented where the soil reaction on a flat footing is specified across the interface between the footing and soil. The soil reaction can then be integrated across the footing to give the global reaction. The soil reaction model in the paper is linear-perfectly plastic with a gapping element. The model presented here builds on this model for the footing by applying the soil reaction along the boundary of the piled foundation. Using this approach, gapping in both lateral directions can be considered. The extensions of the constitutive model include the addition of an initial stress and the extension to a multi-surface approach.

Since the PISA pile tests only considered one loading direction, the problem becomes symmetric and only half of the pile boundary needs to be discretised (Figure 4.23). Evaluating the soil reaction along the entire pile boundary would allow evaluation of the reaction in both lateral directions.

At the pile boundary, the constitutive model for the soil reaction is given in the local coordinate system (x_n, x_t) , where subscript n and t denote the normal and tangential directions with respect to the pile boundary. The displacements from the global coordinate system can be transformed to displacements in the local coordinate

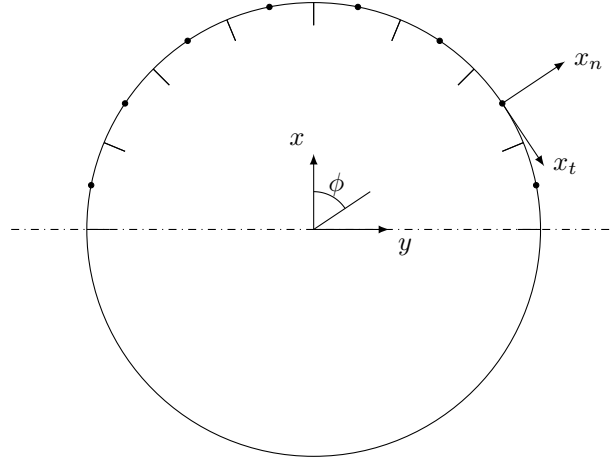


Figure 4.23: Discretisation of the pile boundary for the gapping model.

system at the pile boundary by:

$$\begin{Bmatrix} u_n \\ u_t \end{Bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix} \quad (4.83)$$

where ϕ is the angle between the local coordinate and the x axis. Based on the displacements in the local coordinate system, the soil reaction can be evaluated using the constitutive model. The stress at the boundary can be transformed back into the global coordinate system.

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \end{Bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{bmatrix}^{-1} \begin{Bmatrix} \sigma_n \\ \sigma_t \end{Bmatrix} \quad (4.84)$$

The soil reaction for the pile section can then be integrated along the pile boundary:

$$p_y = 2 \int_{-\pi/2}^{\pi/2} \sigma_y(\phi) R d\phi \approx 2 \sum_{j=1}^{n_b} \sigma_{jy} \frac{R\pi}{n_b} \quad (4.85)$$

with n_b the amount of sections along the boundary and σ_{jy} the stress in the y direction evaluated at the centre of the j^{th} boundary section.

4.7.2 Constitutive model

The constitutive model for the reaction at the boundary considers both the normal and the shear force. The model cannot easily be visualised in these two dimensions of loading. The case of 1D is visualised in Figure 4.24. The constitutive behaviour along the pile boundary is developed in stress-strain space (σ, ϵ) . In the application for the gapping model at the pile boundary, the displacement at the boundary is treated as the strain-like variable and the stress-like variable is the resulting pressure on the pile boundary (σ, u) .

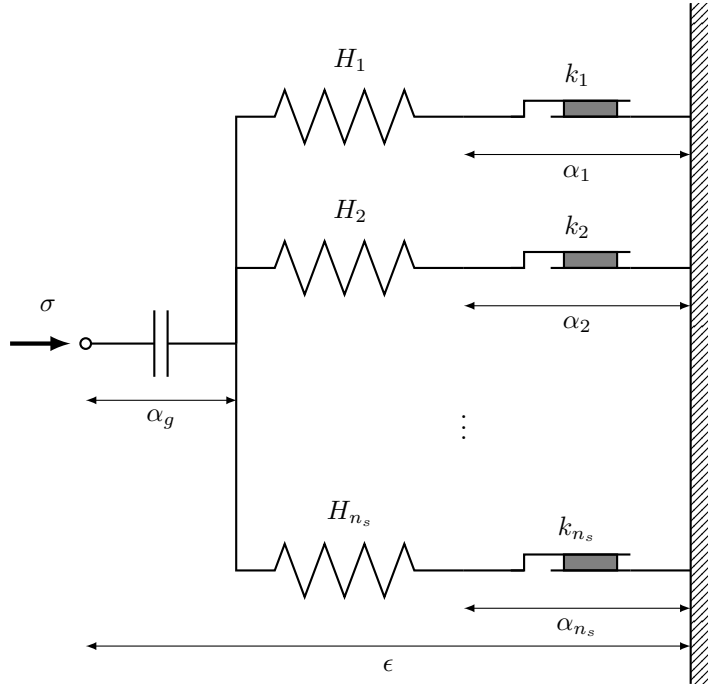


Figure 4.24: Gapping model in 1D.

The energy and dissipation function are written as:

$$f = \sum_{i=1}^{n_s} \left[\frac{H_{in}}{2} (\epsilon_n - \alpha_{in} - \alpha_{gn})^2 + \frac{H_{it}}{2} (\epsilon_t - \alpha_{it} - \alpha_{gt})^2 \right] \quad (4.86)$$

$$d = \sum_{i=1}^{n_s} k_i \sqrt{N_n^2 \dot{\alpha}_{in}^2 + N_t^2 \dot{\alpha}_{it}^2} + \mu \sigma_n |\dot{\alpha}_{gt}| \quad (4.87)$$

with I the indicator function, μ the friction coefficient, N_n and N_t scale the yield strength in the normal and tangential direction, α_{gn} is the gap in the normal direction, α_{gt} is the tangential displacement of the gap element, either by sliding or as displacement at zero stress and σ_{0n} is the initial stress in the normal direction. The main differences with the constitutive model of Houlsby et al. (2005) are (1) the multi-surface approach, (2) the addition of the initial stress and (3) the explicit notation for the gapping in the normal direction, which is required for extending the model to multiple surfaces. Differentiation of these expressions gives the stress and plastic behaviour:

$$\begin{aligned}\sigma_n &= \sum_{i=1}^{n_s} H_{in}(\epsilon_n - \alpha_{in} - \alpha_{gn}) + \sigma_{0n} \\ \sigma_t &= \sum_{i=1}^{n_s} H_{it}(\epsilon_t - \alpha_{it} - \alpha_{gt})\end{aligned}\tag{4.88}$$

$$\begin{aligned}\chi_{\alpha_{in}} = \bar{\chi}_{\alpha_{in}} \rightarrow H_{in}(\epsilon_n - \alpha_{in} - \alpha_{gn}) &= k_i \frac{N_n^2 \dot{\alpha}_{in}}{\sqrt{N_n^2 \dot{\alpha}_{in}^2 + N_t^2 \dot{\alpha}_{it}^2}} \\ \chi_{\alpha_{it}} = \bar{\chi}_{\alpha_{it}} \rightarrow H_{it}(\epsilon_t - \alpha_{it} - \alpha_{gt}) &= k_i \frac{N_t^2 \dot{\alpha}_{it}}{\sqrt{N_n^2 \dot{\alpha}_{in}^2 + N_t^2 \dot{\alpha}_{it}^2}}\end{aligned}\tag{4.89}$$

$$\begin{aligned}\chi_{\alpha_{gn}} = \bar{\chi}_{\alpha_{gn}} \rightarrow \sum_{i=1}^{n_s} H_{in}(\epsilon_n - \alpha_{in} - \alpha_{gn}) + \sigma_{0n} - N_{[-\infty;0]}(\alpha_{gn}) &= 0 \\ \chi_{\alpha_{gt}} = \bar{\chi}_{\alpha_{gt}} \rightarrow \sum_{i=1}^{n_s} H_{it}(\epsilon_t - \alpha_{it} - \alpha_{gt}) &= \mu \sigma_n S(\dot{\alpha}_{gt})\end{aligned}\tag{4.90}$$

where N is the normal cone function. The yield surfaces in 2D become ellipses with a radius of $k_i N_n$ in the normal direction and $k_i N_t$ in the tangential direction, centred around the initial normal stress σ_{0n} (Figure 4.25). The frictional element does not allow negative normal stresses ($\sigma_n = N_{[-\infty;0]}(\alpha_{gn})$) and limits the shear stress to $|\sigma_t^{\max}| = \mu \sigma_n$.

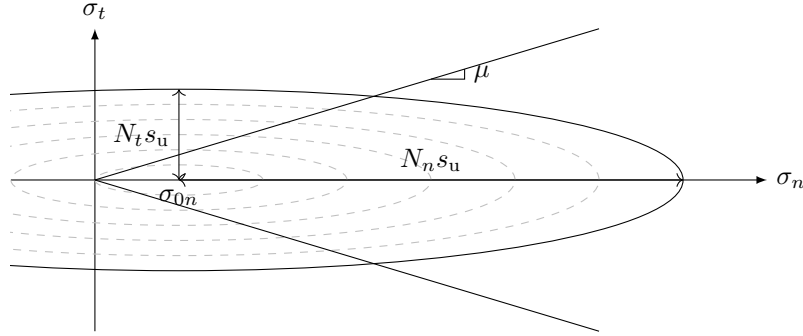


Figure 4.25: Illustration of the yield surface of the gapping model.

4.7.3 Numerical implementation

The constitutive model can be implemented in various ways. To have a fast solution process with accurate results, a method is proposed with 3 main steps: (1) if there is no contact, move the frictional element until there is contact (2) use return mapping on the elliptical yield surfaces (3) use return mapping on the frictional element. In each of these steps only one set of hardening variables is updated: in step 2, the frictional element is assumed to be fixed, and similarly the elliptical yield surfaces are assumed fixed for steps 1 and 3.

The return mapping for the elliptical yield surfaces uses the analytical solution which maps a point outside of the ellipse to the closest point on the ellipse:

$$(\chi_{in}, \chi_{it}) = \left(\frac{(N_n \chi_{in}^{\text{trial}})^2}{k_i(N_n^2 - x)}, \frac{(N_t \chi_{it}^{\text{trial}})^2}{k_i(N_t^2 - x)} \right) \quad (4.91)$$

where x is the real negative solution to

$$\frac{(N_n \chi_{in}^{\text{trial}})^2}{(N_n^2 - x)^2} + \frac{(N_t \chi_{it}^{\text{trial}})^2}{(N_t^2 - x)^2} = k_i^2 \quad (4.92)$$

The algorithm is given below:

Input: $\epsilon_{1n}, \epsilon_{1t}, \epsilon_{0n}, \epsilon_{0t}, \alpha_{0in}, \alpha_{0it}, \alpha_{0gn}, \alpha_{0gt},$

$k_i, H_{in}, H_{it}, N_n, N_t, \sigma_{0n}, i = 1 \dots n_s$

Step 1

$\Delta\epsilon_n \leftarrow \epsilon_{1n} - \epsilon_{0n}; \Delta\epsilon_t \leftarrow \epsilon_{1t} - \epsilon_{0t}$

if $\alpha_{0gn} < 0$ **then**

if $\alpha_{0gn} + \Delta\epsilon_n > 0$ **then**

$\alpha_{gn}^* \leftarrow 0; \alpha_{gt}^* \leftarrow \alpha_{0gt} - (\alpha_{0gn}/\Delta\epsilon_n)\Delta\epsilon_t$

else

$\alpha_{1gn} \leftarrow \alpha_{0gn} + \Delta\epsilon_n; \alpha_{1gt} \leftarrow \alpha_{0gt} + \Delta\epsilon_t; \sigma_{1n} \leftarrow 0; \sigma_{1t} \leftarrow 0$

$\alpha_{1in} \leftarrow \alpha_{0in}; \alpha_{1it} \leftarrow \alpha_{0it}, i = 1 \dots n_s$

return: $\sigma_{1n}, \sigma_{1t}, \alpha_{1in}, \alpha_{1it}, \alpha_{1gn}, \alpha_{1gt}, i = 1 \dots n_s$

end if

else

$\alpha_{gn}^* \leftarrow \alpha_{0gn}; \alpha_{gt}^* \leftarrow \alpha_{0gt}$

end if

Step 2

for $i = 1 \dots n_s$ **do**

$\chi_{in}^{\text{trial}} \leftarrow H_{in}(\epsilon_{1n} - \alpha_{0in} - \alpha_{gn}^*); \chi_{it}^{\text{trial}} \leftarrow H_{it}(\epsilon_{1t} - \alpha_{0it} - \alpha_{gt}^*)$

if $\left(\frac{\chi_{in}^{\text{trial}}}{N_n}\right)^2 + \left(\frac{\chi_{it}^{\text{trial}}}{N_t}\right)^2 > k_i^2$ **then**

$(\chi_{in}, \chi_{it}) \leftarrow \left(\frac{(N_n \chi_{in}^{\text{trial}})^2}{k_i(N_n^2 - x)}, \frac{(N_t \chi_{it}^{\text{trial}})^2}{k_i(N_t^2 - x)}\right)$ where $\frac{(N_n \chi_{in}^{\text{trial}})^2}{(N_n^2 - x)^2} + \frac{(N_t \chi_{it}^{\text{trial}})^2}{(N_t^2 - x)^2} = k_i^2$

$\alpha_{1in} \leftarrow \epsilon_{1n} - \chi_{in}/H_{in} - \alpha_{gn}^*; \alpha_{1it} \leftarrow \epsilon_{1t} - \chi_{it}/H_{it} - \alpha_{gt}^*$

else

$\alpha_{1in} \leftarrow \alpha_{0in}; \alpha_{1it} \leftarrow \alpha_{0it}$

end if

end for

Step 3

```

 $\chi_{gn}^{\text{trial}} \leftarrow \sum_{i=1}^{n_s} H_{in}(\epsilon_{1n} - \alpha_{1in} - \alpha_{gn}^*) + \sigma_{0n}; \chi_{gt}^{\text{trial}} \leftarrow \sum_{i=1}^{n_s} H_{it}(\epsilon_{1t} - \alpha_{1it} - \alpha_{gt}^*)$ 
if  $\chi_{gn}^{\text{trial}} < 0$  then
     $\alpha_{1gn} \leftarrow \epsilon_{1n} + \frac{\sigma_{0n} - \sum_{i=1}^{n_s} H_{in}\alpha_{1in}}{\sum_{i=1}^{n_s} H_{in}}$ 
else
     $\alpha_{1gn} \leftarrow \alpha_{gn}^*$ 
end if
 $\sigma_{1n} \leftarrow \sum_{i=1}^{n_s} H_{in}(\epsilon_{1n} - \alpha_{1in} - \alpha_{1gn}) + \sigma_{0n}$ 
if  $|\chi_{gt}^{\text{trial}}| > \mu\sigma_{1n}$  then
     $\alpha_{1gt} \leftarrow \epsilon_{1t} - \frac{\sum_{i=1}^{n_s} E_{it}\alpha_{1it} + \mu\sigma_{1n}S(\chi_{gt}^{\text{trial}})}{\sum_{i=1}^{n_s} E_{it}}$ 
else
     $\alpha_{1gt} \leftarrow \alpha_{gt}^*$ 
end if
 $\sigma_{1t} \leftarrow \sum_{i=1}^{n_s} H_{it}(\epsilon_{1t} - \alpha_{1it} - \alpha_{1gt})$ 
Output:  $\sigma_{1n}, \sigma_{1t}, \alpha_{1in}, \alpha_{1it}, \alpha_{1gn}, \alpha_{1gt}, i = 1 \dots n_s$ 

```

4.7.4 Model output

Two examples are given of the model output: one for a single element under normal applied strain and one example of the reaction of an entire pile section as it is displaced laterally. Note that the strain-like variable is treated as a displacement for the pile section. The stiffness H_{in} and strength parameters $k_i N_n$ in the normal direction are determined by discretising the same backbone curve as for the examples in the kinematic hardening model, using $n_s = 5$. For simplicity N_n is taken equal to 1. The stiffness parameters in the tangential direction are assumed the same as in the normal direction ($H_{in} = H_{it}$) and the strength in the tangential direction is assumed to be 20% of the strength in the normal direction ($N_t = 0.2$). The initial normal stress is taken equal to $\sigma_{0n} = 0.25$. The reaction of a single element under an applied strain of $\epsilon = 0 \rightarrow 2 \rightarrow -2 \rightarrow 4$ is given in Figure 4.26. This figure shows that the resulting

stress is always greater than or equal to zero. The multi-surface approach captures energy dissipation in the unload-reload loop.

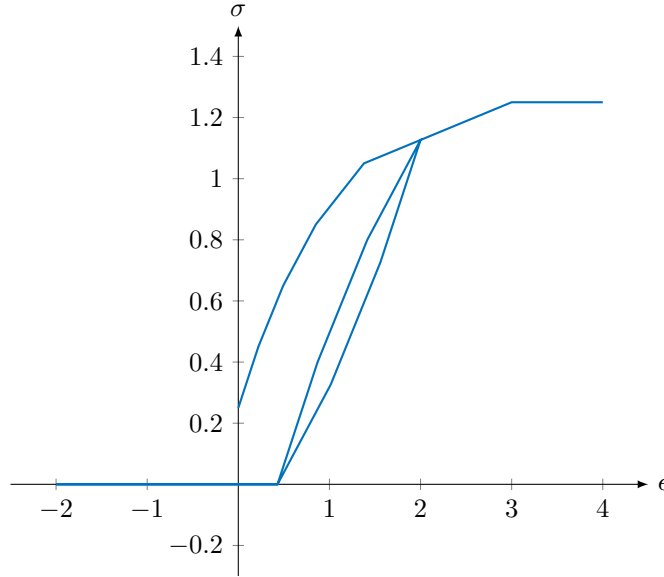


Figure 4.26: Reaction of the gapping model under an applied strain of $\epsilon = 0 \rightarrow 2 \rightarrow -2 \rightarrow 4$.

The integrated reaction along the boundary is calculated with a friction coefficient of $\mu = 0.3$ and a diameter of $D = 1\text{m}$ for cycles with a normalised amplitude of $v/D = 2$, with an initial stress of $\sigma_{0n} = 0.25$ and $\sigma_{0n} = 0.75$ (Figure 4.27). Comparison of both curves clearly shows the effect of the initial stress on the soil reaction, with Masing response when $\sigma_{0n} \geq N_n \sum_{i=1}^{n_s} k_i$. The pile boundary is discretised using $n_b = 8$ sections. Note that the strain variable of the constitutive model is treated as the displacement at the pile boundary.

The initial soil reaction gives a stiff response. As the pile is displaced, a gap is formed at the active side of the pile. This results in a reduced stiffness. At the middle of the cycles, the friction along the sides of the pile results in a reaction on the pile section.

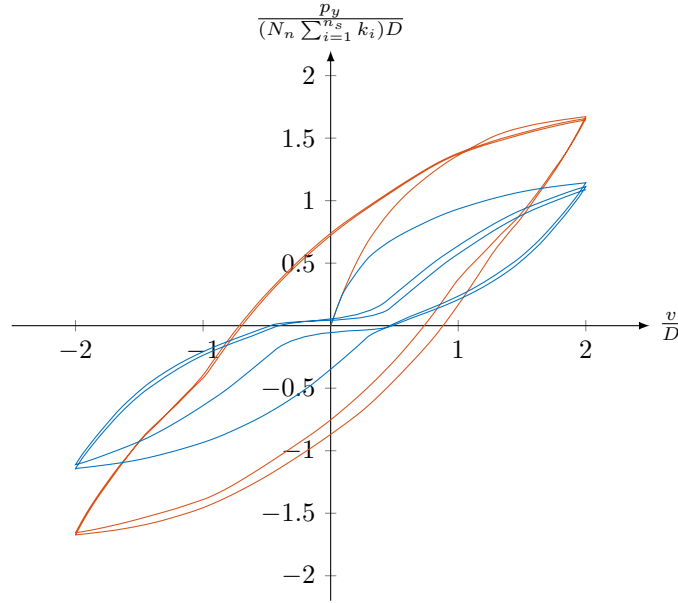


Figure 4.27: Soil reaction on the pile using the gapping model under displacement cycles at (—) high initial stress and at (—) low initial stress conditions.

4.7.5 Model extensions

A first extension of the gapping model would be to extend it to two tangential directions. The extension of the constitutive model is straightforward. When integrated in a Winkler foundation, this allows for the calculation of the axial and moment-rotation soil reaction along the pile. A model for the combined axial and lateral response of foundation piles has also been described by Levy (2007) using a kinematic hardening soil reaction model. The addition of the gapping effect could provide improved modelling of the reaction at the pile-soil interface. In the case where sliding and gapping at the pile base would be considered, the constitutive model could be applied across the pile-base section, in a similar way to the model in Houlsby et al. (2005).

The gapping model can also be extended to include rate and ratcheting effects. The model is not developed further in this thesis, but a graphical representation of a combined gapping, rate and ratcheting model in 1D of loading is given in Figure 4.28. The expressions for the model in 1D of loading are given below and can be extended

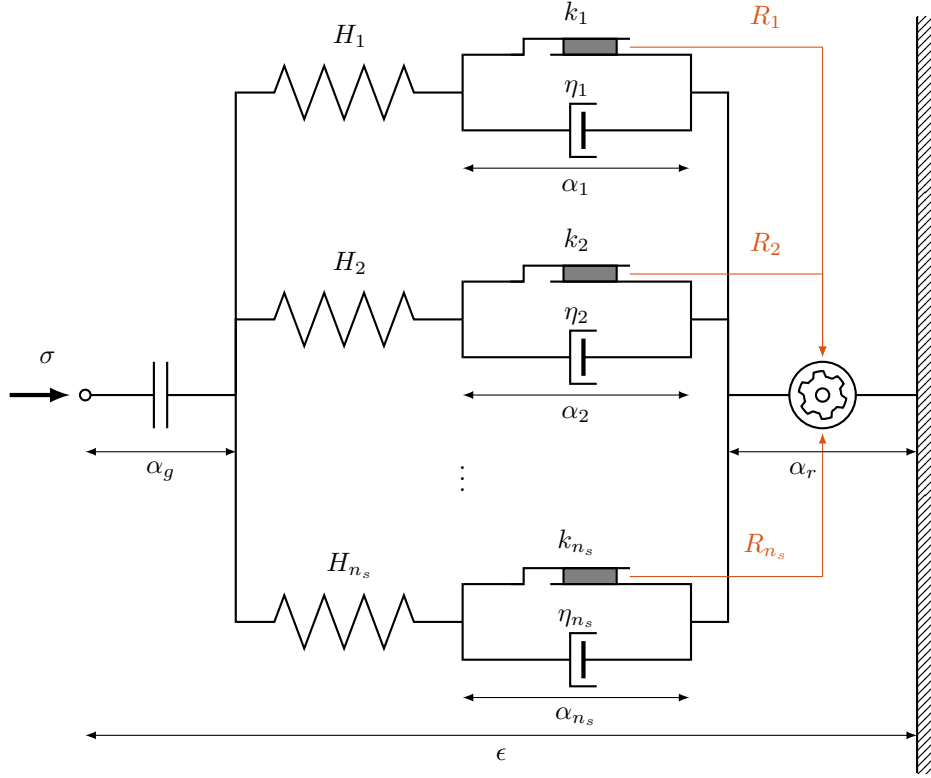


Figure 4.28: Gapping model in 1D with rate and ratcheting effects.

to 2 or 3 dimensions.

$$f = \sum_{i=1}^{n_s} \frac{H_i}{2} (\epsilon - \alpha_i - \alpha_g - \alpha_r)^2 + \sigma_0 (\epsilon - \alpha_g - \alpha_r) + \mathbf{I}_{[-\infty; 0]}(\alpha_g) \quad (4.93)$$

$$d = \sum_{i=1}^{n_s} k_i \left[|\dot{\alpha}_i| + \dot{\alpha}_i \eta \sinh^{-1} \left(\frac{\dot{\alpha}_i}{\dot{\alpha}_{qs}} \right) \right] + \sigma \dot{\alpha}_r \quad (4.94)$$

4.8 Conclusions

This chapter presented a FE framework for calculating the behaviour of laterally loaded piles using the Winker assumption. A series of constitutive models have been presented to capture the plastic soil response starting with the kinematic hardening model which captures the Masing response. Following this, rate effects and ratcheting effects have been introduced. A combined rate and ratcheting model is also presented to capture both effects. Finally, a gapping model is presented where the soil response is given at the pile boundary instead of having an integrated reaction curve for each depth.

Chapter 5

Calibration and results

5.1 Introduction

In this chapter, the soil reaction models of Chapter 4 are applied to the foundation piles of the PISA field tests. The chapter focuses on the effects of rate dependency of the soil reaction, accumulation of pile rotation under cyclic loading and gapping at the active side of the pile.

The aim of this chapter is to calibrate the soil reaction models in such a way that it reproduces the field response of the PISA test piles as closely as possible. The starting point for each of these models is the static backbone curve. The PISA soil reaction curves in Table 2.6 were determined from a calibration study for full size monopile foundations, which was separate from the range of parameters employed in the field tests. Although the soil reaction curves were determined for the two test sites specifically, there are a number of reasons why these curves do not accurately represent the static backbone curve of the test piles:

1. Most test pile geometries lie outside the parameter space of the calibrated soil reaction curves.
2. The soil reaction curves were calibrated based on a load rate which was deemed

appropriate for the field tests. This rate may not represent the static soil reaction used in the soil reaction models.

3. The variability of the soil across the test site results in a degree of uncertainty for prediction of a specific test pile.

To have appropriate soil reaction curves for the specific test piles, first a recalibration of the soil reactions is done. Subsequently, the various soil models are applied. Note that in a design case, where no field tests are performed, the soil reaction models would need to be applied to the initial estimate for the soil reaction curves.

5.2 Recalibration of the soil reaction curves

Using the parametrised soil reaction curves of Table 2.6 and the soil profiles in Figure 2.6, the load-displacement backbone curve for CM9 is given in Figure 5.1 along with a comparison with the field test. The finite element model of the foundation pile uses 20 elements below ground level. The soil reactions are integrated using four Gauss points per element, so that the 80 evaluations of the distributed soil reactions provide an accurate representation of the variation along the pile length.

To accommodate for the fact that the stiffness and ultimate resistance of the backbone curve are not predicted well using the PISA soil reaction curves, the following modifications are made, scaling the curves in both the load and displacement directions.

$$\bar{\sigma}_u^* = A_u \bar{\sigma}_u \quad (5.1)$$

$$k_0^* = A_0 k_0 \quad (5.2)$$

$$\bar{\epsilon}_u^* = \frac{A_u}{A_0} \bar{\epsilon}_u \quad (5.3)$$

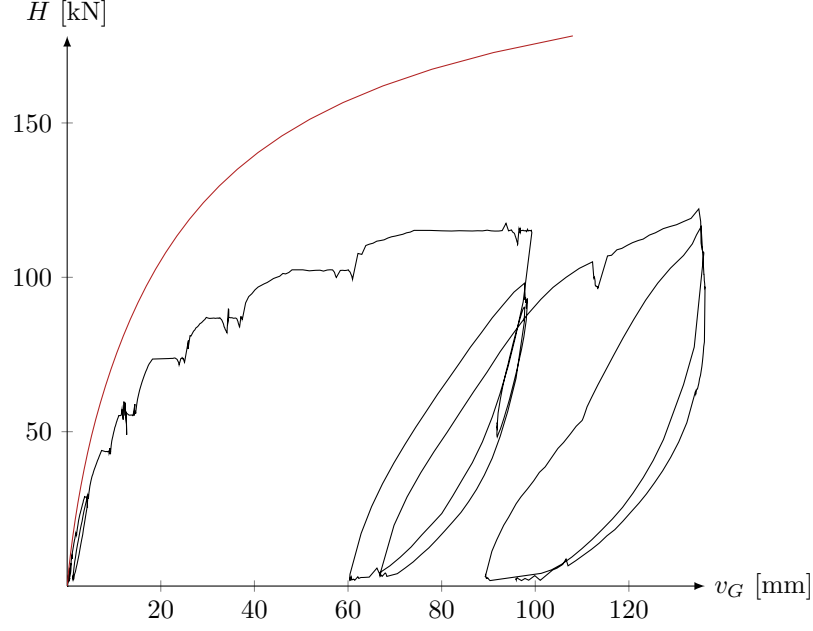


Figure 5.1: Comparison of (—) the calculated backbone curve using the initial parametrised soil reaction curves with (—) the field test response of CM9.

where $\bar{\sigma}$ represents the stress-like variable in the parametrised soil reaction curves: $\bar{p}$, $\bar{m}$, $\bar{H}_B$ or $\bar{M}_B$; and $\bar{\epsilon}$ represents the strain-like variable, which are respectively $\bar{v}$, $\bar{\psi}$, $\bar{v}_B$ or $\bar{\psi}_B$. The same correction factors on the ultimate resistance A_u and on the initial stiffness A_0 are applied to all four components. The correction factors are calculated using an optimisation process where the difference of ground level displacement at the same applied load is minimised. The objective function is given below:

$$\text{minimise : } \sum_{i=1}^{n_c} (v_{Gi}^{\text{test}} - v_{Gi}^{\text{calculated}})^2 \quad (5.4)$$

where n_c is the number of ground displacement points which are compared. The optimisation procedure is implemented using the *fmincon* function in Matlab. Figure 5.2 illustrates the points at the end of the hold phases, which are used for the minimisation problem along with the modified backbone curve, where the optimised correction factors of $A_u = 0.7152$ and $A_0 = 0.4161$ are applied. This figure shows good agreement with the test results at the end of the hold phases.

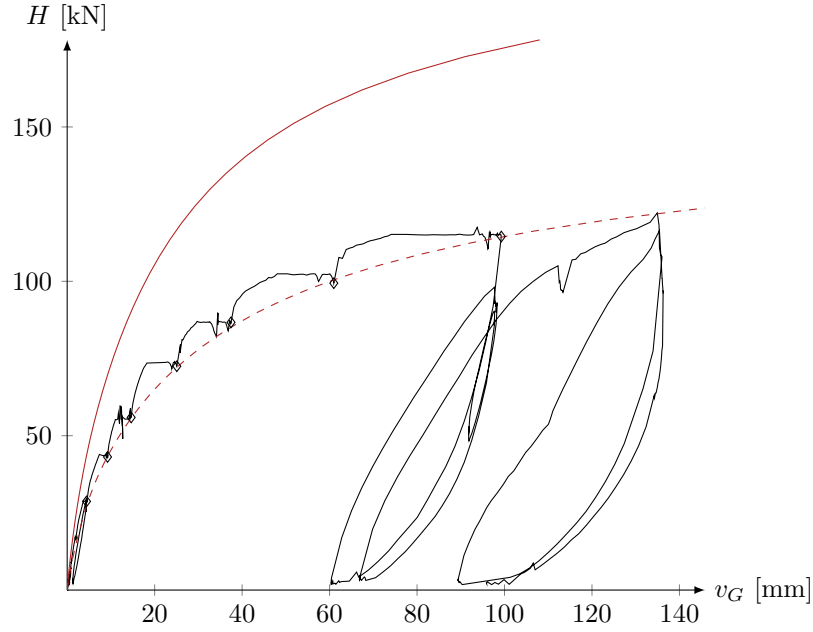


Figure 5.2: Comparison of (—) the field test response for CM9 with calculated backbone curve using (—) the initial and (---) the modified parametrised soil reaction curves, calibrated to the points at the end of the hold phases ($\diamond$).

The calibration has been repeated for selected tests, which are analysed in this chapter. The correction factors for these tests are given in Table 5.1.

Table 5.1: Correction factors for the backbone curve of the PISA pile tests.

Pile	CS2	CM9	CL2	CM1	CM6
A_0	0.1518	0.4161	0.9424	0.5713	0.2777
A_u	0.4108	0.7152	0.6927	0.6720	0.6268

Pile	DM9	DS1	DL2	DM6
A_0	0.7059	0.5004	0.7264	0.5652
A_u	1.2231	1.5992	1.0500	1.2178

5.3 Kinematic hardening model

The calibration of the kinematic hardening model follows directly from the soil reaction curves. The parameters of the kinematic hardening model can be calculated using the equations in Section 4.3. This calibration is performed using 50 spring-slider elements to accurately capture the non-linearity of each of the soil reaction curves. Figure 5.3 shows a comparison of test CM9 with the kinematic hardening model applied to the modified backbone curves, where an unload-reload loop is applied after load step 3 and 9.

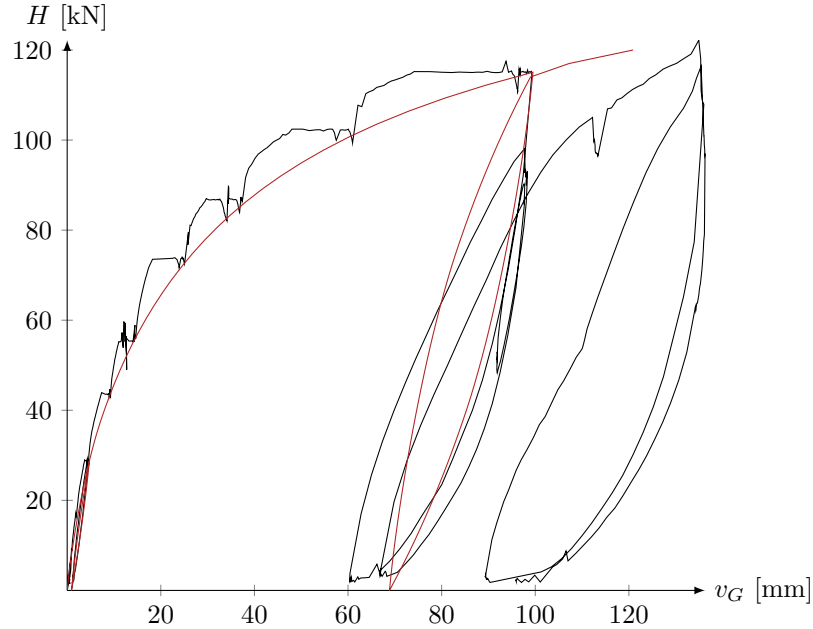


Figure 5.3: Comparison of (—) the field test response of CM9 with (—) the Masing response using the kinematic hardening model on the modified backbone curve.

Figure 5.4 illustrates the distributed lateral reaction throughout the simulation at three depths along the foundation.

In the FE calculations, it should be noted that the plastic strain-like variables need to be passed on throughout the iterations. This can be done using the normalised values. It should also be noted that the different normalisation for the soil-reaction curves in sand does not affect the storage of the internal strain-like variables. Figure

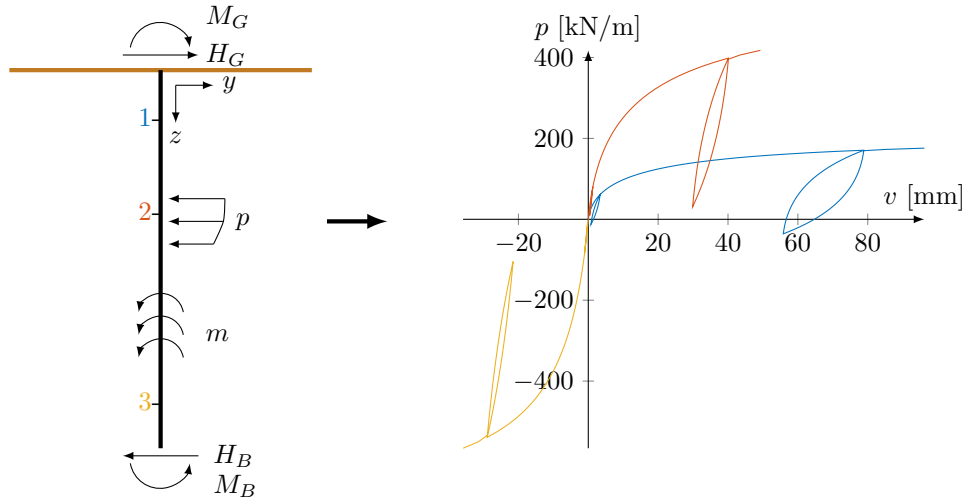


Figure 5.4: Simulated lateral soil reaction curves for CM9 using the kinematic hardening model at (1) $z=0.5\text{m}$, (2) $z=1.5\text{m}$ and (3) $z=3.5\text{m}$.

5.5 gives a comparison of DM9 with the kinematic hardening model applied to the modified backbone curves with unload-reload loops after load step 2 and 7. The distributed lateral reaction throughout the simulation is shown in Figure 5.6 at three depths along the foundation.

Comparing the modelled response with the field test for CM9 and DM9 illustrates a few key additional effects:

1. At the loading rate applied in the load steps, the pile behaves more stiffly than the model and additional displacement occurs during the creep phases.
2. Upon reloading after unloading, ratcheting of the pile displacement occurs which is not captured in the Masing response generated by the kinematic hardening model.
3. When unloading after a high applied load, the pile behaves slightly less stiffly than using the Masing response. This can probably be attributed to gapping at the active side of the pile.

Each of these three effects will be treated in the following sections.

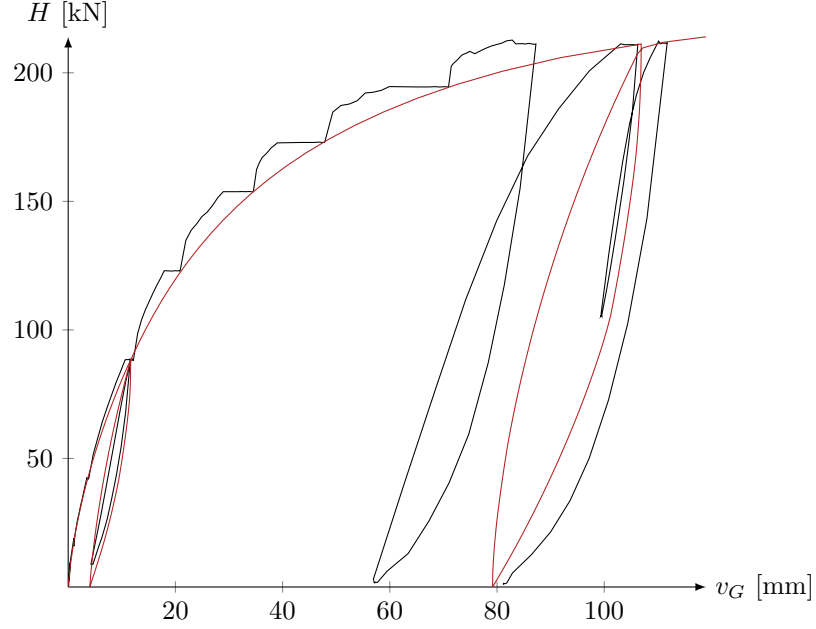


Figure 5.5: Comparison of (—) the field test response of DM9 with (—) the Masing response using the kinematic hardening model on the modified backbone curve.

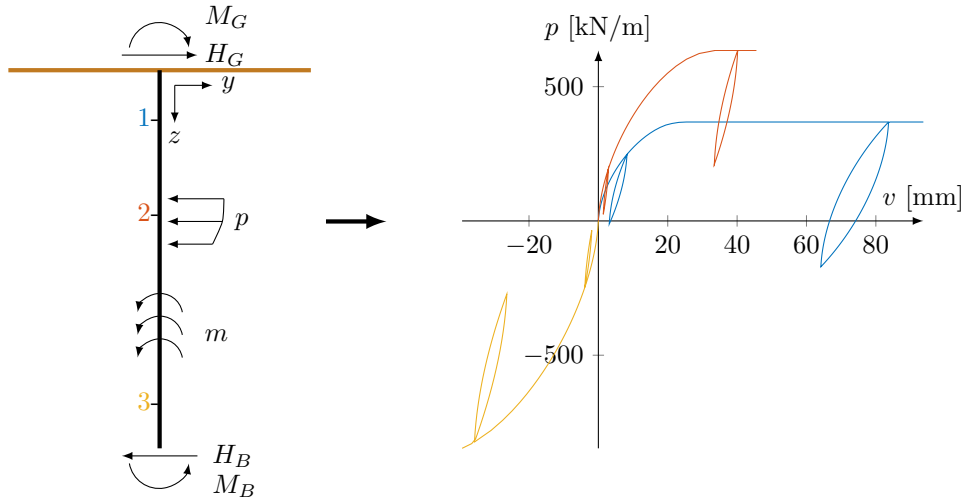


Figure 5.6: Simulated lateral soil reaction curves for DM9 using the kinematic hardening model at (1) $z=0.5$ m, (2) $z=1.5$ m and (3) $z=3.5$ m.

5.4 Rate effect model

The rate effect is added using the modified rate effect model introduced in Section 4.4. The calibration of the stiffness and strength parameters for this model follows the same approach as for the kinematic hardening model. The addition of the rate effect introduces two new variables: the quasi-static strain velocity $\dot{\alpha}_{\text{qs}}$ and the relative capacity increase per log increase of plastic strain velocity η_{10} . The quasi-static strain velocity needs to be determined for lateral displacement (superscript v) and for cross-section rotation (superscript ψ). In the following analyses, this parameter is defined by:

$$\dot{\alpha}_{\text{qs}}^v = \frac{0.1D}{6h} \quad (5.5)$$

$$\dot{\alpha}_{\text{qs}}^\psi = \frac{2^\circ}{6h} \quad (5.6)$$

which are the average rates of ground level displacement and rotation during the monotonic tests, performed over a time of $\sim 6h$. The calibration of the backbone curve at the end of the hold phases can be interpreted as the slow isotache curve using this value for the quasi-static strain rate.

The field tests can be modelled using a limited amount of steps, where each load step and load hold is discretised into 10 sub-steps. This reduced load history can then be applied to the model, to calculate efficiently the response with the rate effect model. Note that using this approach, not all sub-steps are applied over the same time interval and that for each step, the associated duration must be transferred throughout the computation.

5.4.1 Rate effects in Cowden

Firstly, the monotonic tests in Cowden are modelled and the parameter η_{10} is calibrated for tests CS2, CM9 and CL2. With a factor of $\eta_{10} = 0.12$, the simulated

response provides a close match to the test results (Figure 5.7). This value is consistent with the increase of shear strength in clay with strain rate, where an increase of $10 - 25\% / \log(\dot{\epsilon})$ is typically observed (Dayal and Allen, 1975). Further work could provide insight on whether the rate parameter determined from element shear tests can be applied directly to the soil reaction models in a Winkler model, or whether a correction factor is needed.

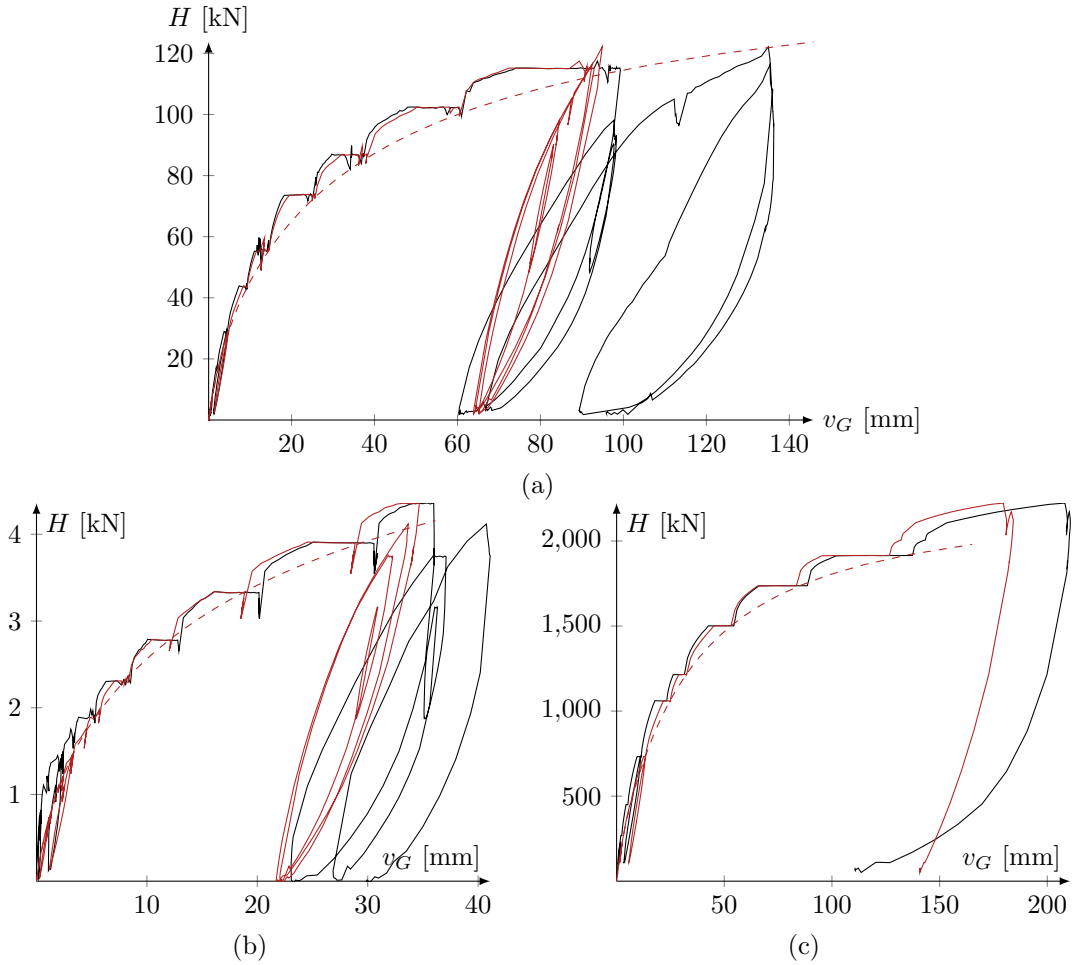


Figure 5.7: Comparison of (—) the field test response for (a) CM9, (b) CS2 and (c) CL2 with (---) the recalibrated backbone curve and (—) the simulated response using the modified rate effect model.

Figure 5.7 shows a good agreement with the rate effect during the load steps and the hold phases in the monotonic routine. However, in the subsequent unload-reload loops, it is clear that the ratcheting behaviour is not captured using this model.

In Figure 5.8, the rate effect model is applied to CM1. The static backbone curve is fitted to the points illustrated in Figure 5.8 at the end of the load holds of step 1 to 4 and to points which lie 10% below the start and endpoints of the slow load steps (6 and 8). Since the load rate is much faster in CM1 compared to CM9, the load-displacement response lies above the static backbone curve. The results of two simulations are plotted using $\eta_{10} = 0.12$ and $\eta_{10} = 0.20$, illustrating the sensitivity of the results to this parameter. The simulation with $\eta_{10} = 0.12$ provides the best match with the field test results. The distributed lateral reaction using this model is given in Figure 5.9 at three depths along the foundation pile.

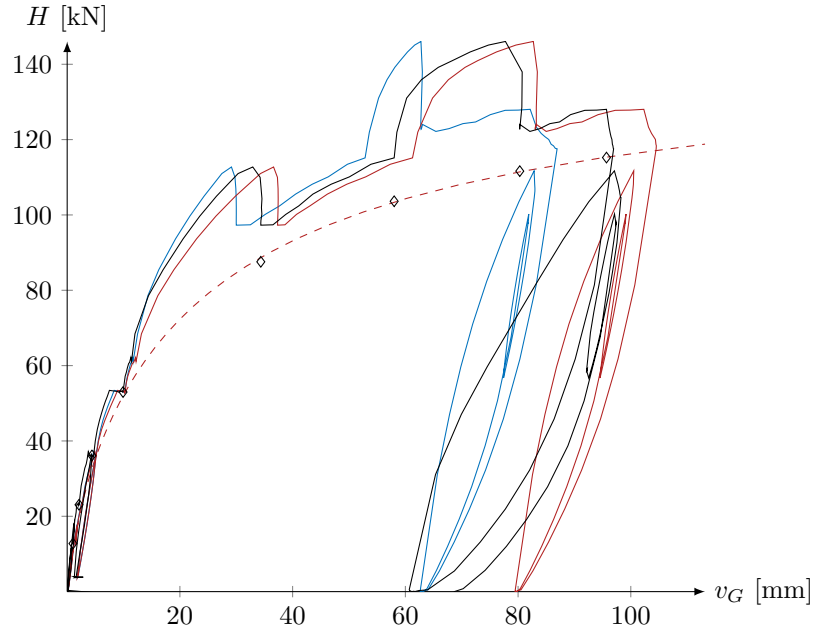


Figure 5.8: Comparison of (—) the field test response for CM1 with (---) the recalibrated backbone curve to ($\diamond$) the calibration points and the simulated response using the modified rate effect with $\eta_{10} = 0.12$ (—) and $\eta_{10} = 0.20$ (—).

Figure 5.10 compares test CS1 to the modelled response using the static backbone curve of CS2 and $\eta_{10} = 0.12$. This figure shows that the strength increase cannot be modelled using the value of η_{10} . Given the good agreement of the previous tests, the difference in strength is probably attributed to the spatial variability of the soil characteristics rather than the rate effect.

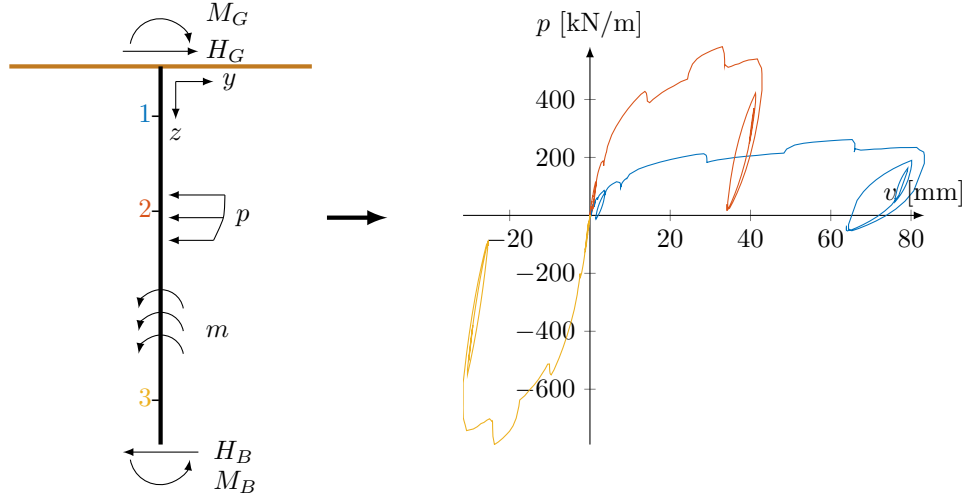


Figure 5.9: Simulated lateral soil reaction curves for CM1 using the rate effect model at (1) $z=0.5\text{m}$, (2) $z=1.5\text{m}$ and (3) $z=3.5\text{m}$.

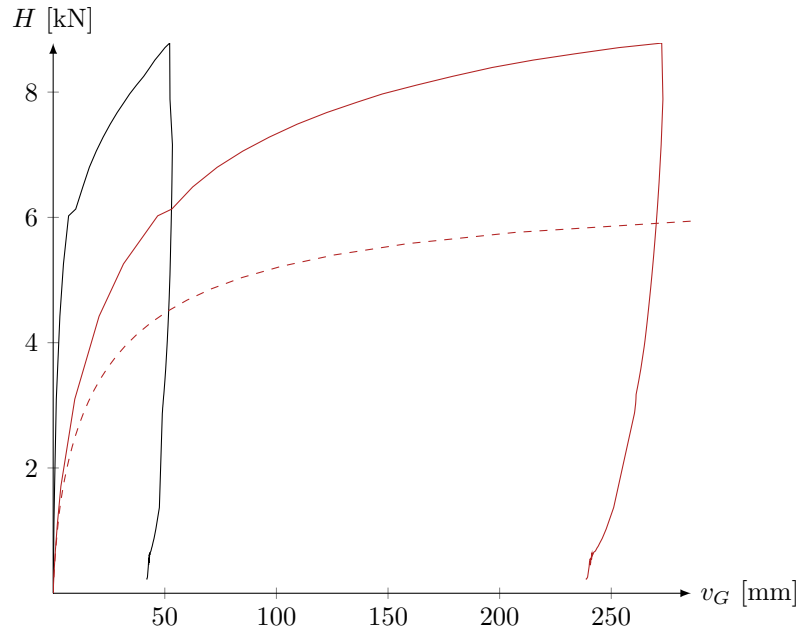


Figure 5.10: Comparison of (—) the field test response for CS1 with (---) the recalibrated backbone curve and (—) the simulated response using the modified rate effect model.

5.4.2 Rate effects in Dunkirk

In Dunkirk, η_{10} has been calibrated to DM9, DS1 and DL2. Using a value of $\eta_{10} = 0.06$ gives results which closely align with the field test response, as shown in Figure 5.11. The lower value of η_{10} compared to the tests in Cowden is in line with the lower strength increase at the high rate tests, discussed in Section 3.4.2.

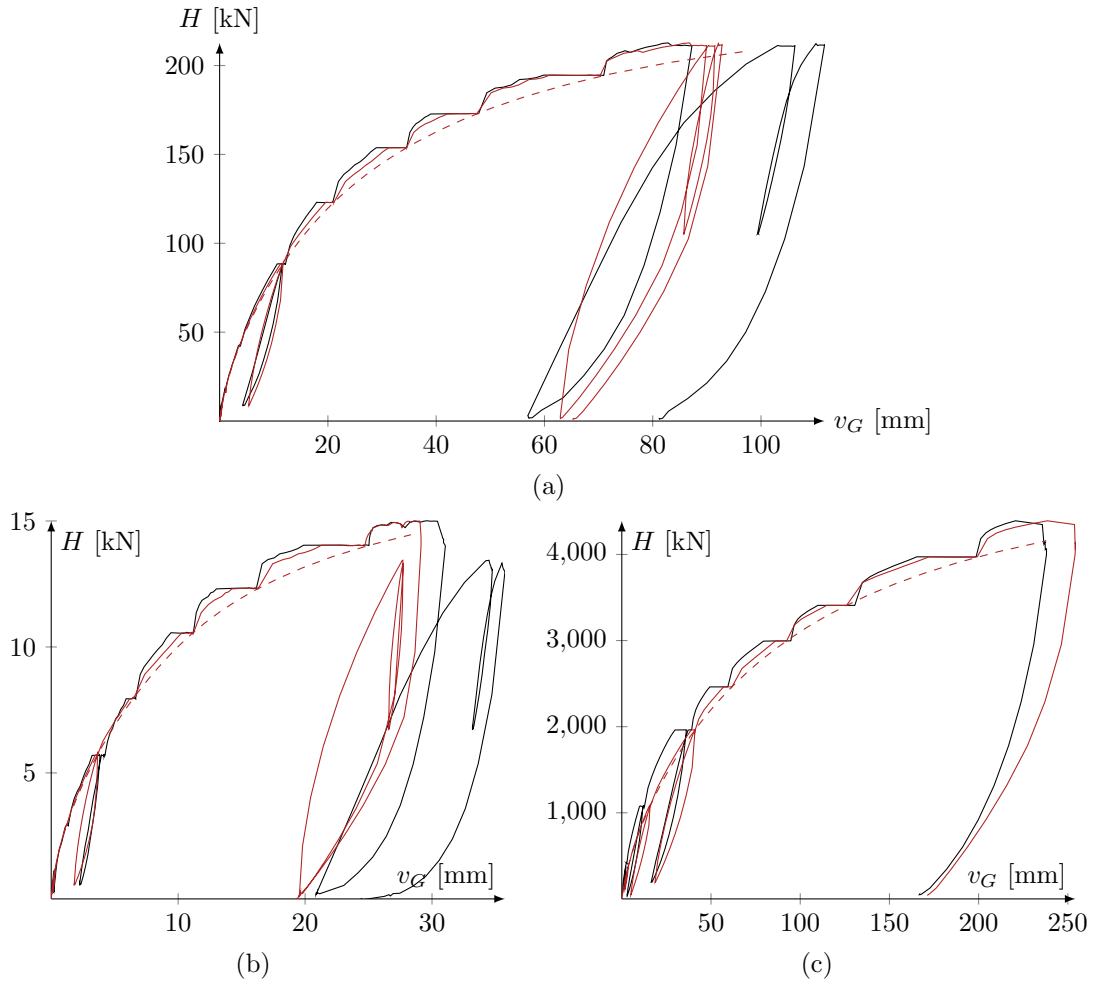


Figure 5.11: Comparison of (—) the field test response for (a) DM9, (b) DS1 and (c) DL2 with (---) the recalibrated backbone curve and (—) the simulated response using the modified rate effect model.

The rate effect model has also been applied to the fast rate test in Dunkirk (DM6) with a value of $\eta_{10} = 0.06$ in Figure 5.12. For comparison, a simulation is added with $\eta_{10} = 0.10$, illustrating the sensitivity of the results to this parameter. The simulation

with $\eta_{10} = 0.06$ provides the best match with the field test results. The backbone curve for this test has been calibrated to fit the points illustrated in Figure 5.12 at the end of the load holds of load steps 2 and 3 and to points which lie 3% below the beginning of load steps 4 and 6. The resulting modelled response in Figure 5.12 agrees well with the field test response before the start of the cyclic loading. The ratcheting effect during the cyclic loading will be discussed in Section 5.5. The distributed lateral reaction for this simulation is illustrated in Figure 5.13 at three depths along the foundation pile.

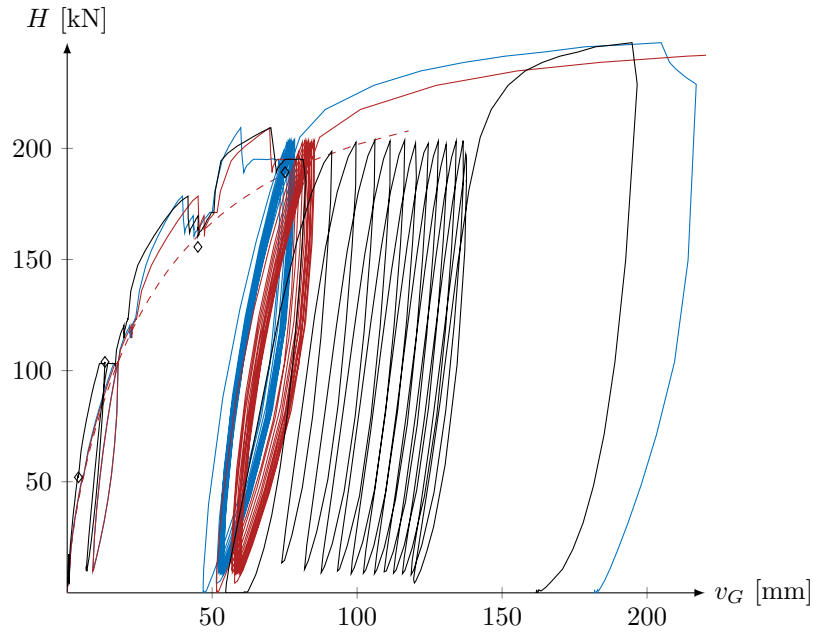


Figure 5.12: Comparison of (—) the field test response for DM6 with (---) the recalibrated backbone curve to ($\diamond$) the calibration points and the simulated response with the modified rate effect model using $\eta_{10} = 0.06$ (—) and $\eta_{10} = 0.10$ (—).

Figure 5.14 gives the modelled response using the backbone curve from DS1 and applying the rate effect model with the load history of DS2. The resulting capacity increase due to the rate increase agrees well between the field test and the modelled results.

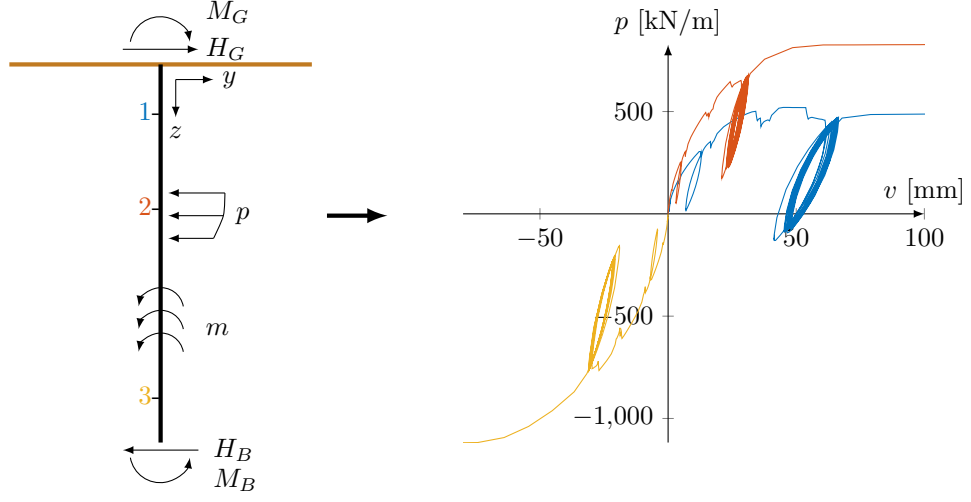


Figure 5.13: Simulated lateral soil reaction curves for DM6 using the rate effect model at (1) $z=0.5$ m, (2) $z=1.5$ m and (3) $z=3.5$ m.

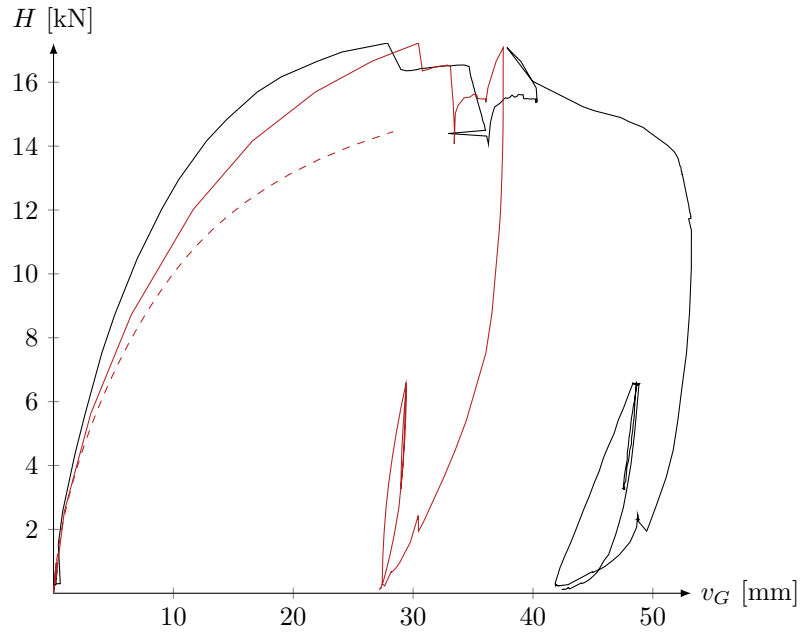


Figure 5.14: Comparison of (—) the field test response for DS2 with (---) the recalibrated backbone curve and (—) the simulated response using the modified rate effect model.

5.5 Ratcheting model

Figure 5.15 shows the results of the LeBlanc et al. (2010) cyclic accumulation tests for varying T_b and T_c (Equation 3.61). Figure 5.16 illustrates the variation of $T_b(\zeta_b)$ and $T_c(\zeta_c)$ along with fitted curves. Similar small scale experiments are described by Nicolai and Ibsen (2014), Zhu et al. (2012) and Foglia et al. (2014) though the latter two of these studies were performed for caisson foundations with a smaller L/D ratio compared to monopile foundations. The variation of $T_c(\zeta_c)$ from these experimental tests is compared in Table 5.2, which show a smaller peak of $T_c(\zeta_c)$ compared to the result of LeBlanc et al. (2010). In the PISA field tests, the cyclic loading was applied mostly with $\zeta_c = 0$ for the ‘cyclic’ tests and $\zeta_c = -1$ for the ‘damping’ tests. Since there are no data at intermediate values of ζ_c , $T_c(\zeta_c)$ cannot be reconstructed for the field tests.

Table 5.2: Nondimensional parameters achieved by various authors for $N = 10^7$, $\zeta_b = 0.3$ and $\zeta_c = -0.5$ (Nicolai and Ibsen, 2014).

Authors	T_b	T_c	m_N	$\Delta\theta/\theta_s$	Soil	Foundation
LeBlanc et al. (2010)	0.047	4	0.31	27.8	Loose sand	Monopile
Nicolai and Ibsen (2014)	0.65	1.35	0.13	7.13	Dense sand	Monopile
Zhu et al. (2012)	0.04	1.5	0.39	32.2	Loose sand	Caisson
Foglia et al. (2014)	0.3	1.5	0.19	9.6	Dense sand	Caisson

5.5.1 Formulation of the ratcheting strain

To reflect the variation of the accumulated rotation with N , ζ_c and ζ_b , the coupling parameters R_i for the ratcheting element introduced in Section 4.5 are written as:

$$R_i = R_0 \left(\frac{|\sigma|}{\sigma_u} \right)^{m_\sigma} \left(\frac{\sum y_i k_i}{\sigma_u} \right)^{m_k} \left(\frac{\alpha_h}{\alpha_{h0}} \right)^{m_h} \frac{k_i}{\sigma_u} \quad (5.7)$$

where the combination of $\left(\frac{|\sigma|}{\sigma_u} \right)^{m_\sigma}$ and $\left(\frac{\sum y_i k_i}{\sigma_u} \right)^{m_k}$ is used to reflect the effect of load magnitude ($T_b(\zeta_b)$) and load shape ($T_c(\zeta_c)$) as will be described in Section 5.5.1.2

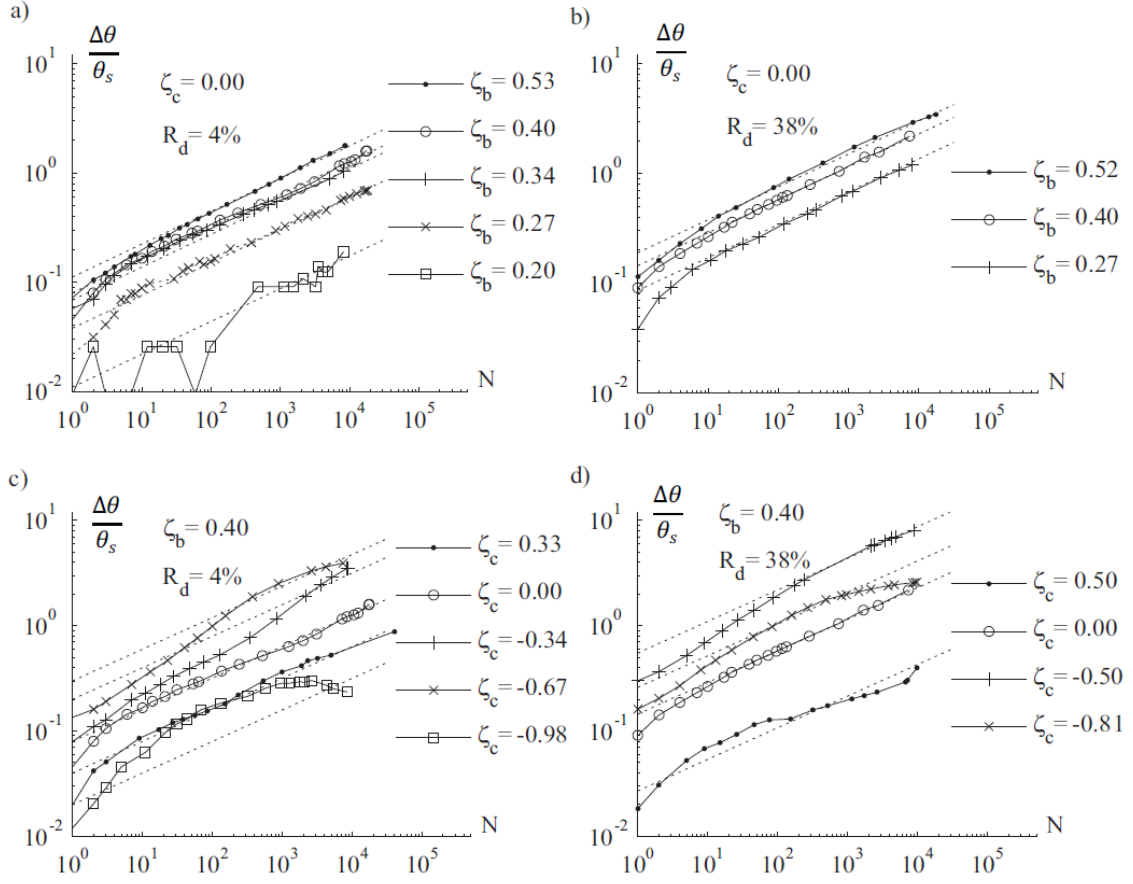


Figure 5.15: Measured accumulation of rotation as a function of N , R_d , ζ_b and ζ_c (LeBlanc et al., 2010).

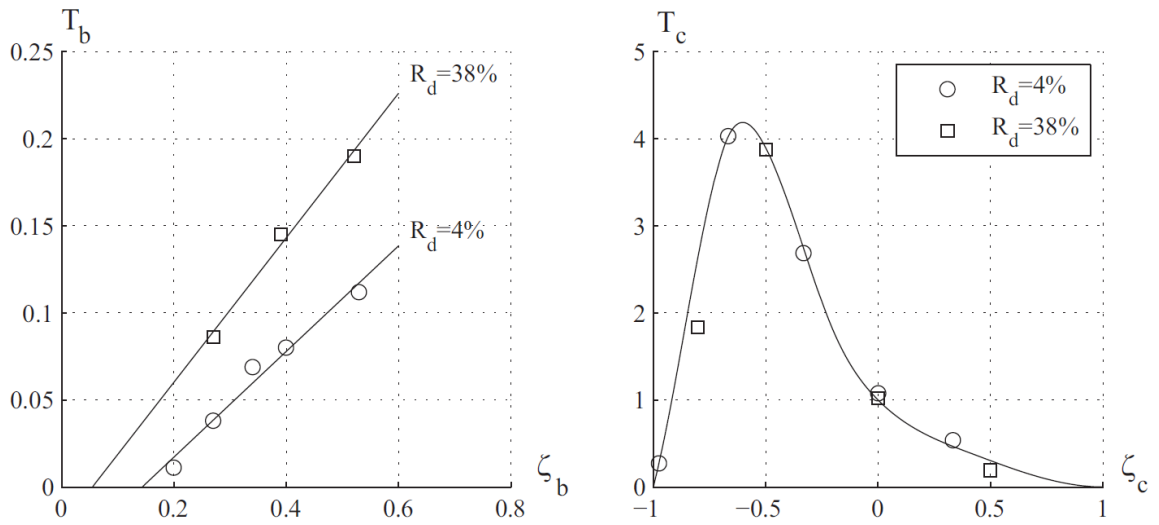


Figure 5.16: Functions relating T_b and T_c to the relative density, R_d , and the characteristics of the cyclic load in terms of ζ_b and ζ_c (LeBlanc et al., 2010).

and 5.5.1.3. The expression $\left(\frac{\alpha_h}{\alpha_{h0}}\right)^{m_h}$ introduces the history effect as described in Section 5.5.1.1. Combining this with Equation 4.66 and 4.71, the accumulation of the ratcheting strain and the hardening strain are written as:

$$\dot{\alpha}_r = S(\sigma) R_0 \left(\frac{|\sigma|}{\sigma_u}\right)^{m_\sigma} \left(\frac{\sum y_i k_i}{\sigma_u}\right)^{m_k} \left(\frac{\alpha_h}{\alpha_{h0}}\right)^{m_h} \frac{\sum k_i |\dot{\alpha}_i|}{\sigma_u} \quad (5.8)$$

$$\dot{\alpha}_h = |\dot{\alpha}_r| \quad (5.9)$$

where $\left(\frac{|\sigma|}{\sigma_u}\right)^{m_\sigma}$ is referred to as the stress component, $\left(\frac{\sum y_i k_i}{\sigma_u}\right)^{m_k}$ the strength component, $\left(\frac{\alpha_h}{\alpha_{h0}}\right)^{m_h}$ the hardening component and $\frac{\sum k_i |\dot{\alpha}_i|}{\sigma_u}$ the dissipation component.

The calibration of the ratcheting model follows the assumption that the accumulated rotation at ground level is reflected in the ratcheting element ($\Delta\alpha_r \sim \Delta\psi_G$), so that we can apply the same formulation for the coupling parameters to all four components of the soil reactions. The calibration study focuses on Test CM6, illustrated in Figure 5.17 along with the recalibrated backbone curve and a power law fit, which is used in the calibration of the ratcheting element.

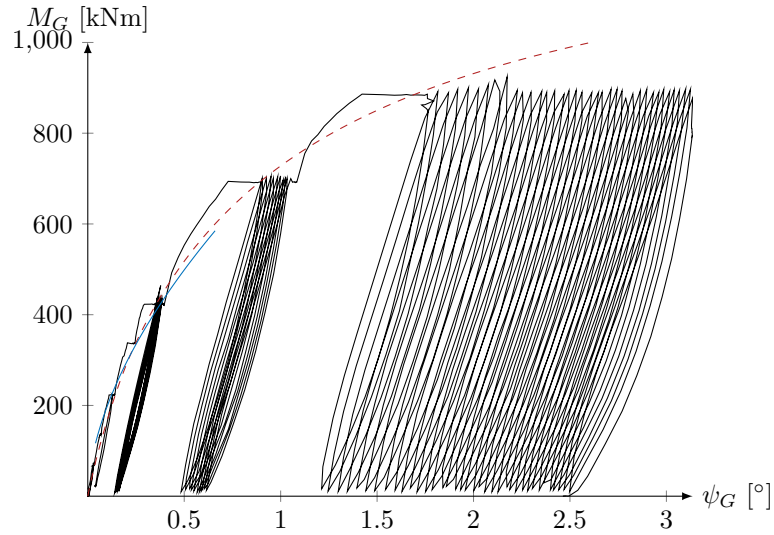


Figure 5.17: Comparison of CM6 field tests (—) with the recalibrated backbone curve (---) and a fitted power law expression $M_G \sim 742\psi_G^{0.575}$ (—).

5.5.1.1 Calibration of the load history effect

At large cycle number, where the hysteresis loops are almost closed, the stress component and the dissipation component do not vary significantly with cycle number. The hardening parameter can be written as: $\alpha_h = \int_0^N |\dot{\alpha}_r| dN + \alpha_{h0}$, where the constant can be neglected at high cycle numbers if α_{h0} is small compared to the later accumulation of strain. The ratcheting strain can therefore be expressed as a function of the cycle number by:

$$\Delta\psi_G \sim \Delta\alpha_r = \int_0^N |\dot{\alpha}_r| dN \rightarrow \dot{\alpha}_r \sim N^{m_N-1} \quad (5.10)$$

$$\dot{\alpha}_r \sim \alpha_h^{m_h} \rightarrow N^{m_N-1} \sim N^{m_N m_h} \rightarrow m_h = \frac{m_N - 1}{m_N} \quad (5.11)$$

with a value of $m_N = 0.31$, which is consistent with the tests of LeBlanc et al. (2010), the exponent for the hardening expression is equal to $m_h = -2.2$.

The initial hardening parameter α_{h0} defines the location of the transition zone where the slope of the accumulated rotation decreases to the value of m_N . The data in the PISA field tests do not provide enough information to determine accurately this parameter. An estimated value for this parameter of $\alpha_{h0} = \epsilon_u/100$, where ϵ_u represents the ultimate displacement or rotation in the soil reaction curve, results in reasonable modelling of the ratcheting behaviour and this value has therefore been used in further analysis. The inability to calibrate this parameter despite the amount of information in the PISA project is a shortcoming of this formulation since in most practical cases there will not be the benefit of field data.

5.5.1.2 Calibration of the load magnitude effect

The effect of load magnitude on the ratcheting response can be calibrated using a similar methodology. The increase of accumulated rotation with load magnitude can

generally be described by:

$$\Delta\psi_G \sim \tilde{T}_b(\zeta_b) = \theta_s T_b(\zeta_b) \quad (5.12)$$

In the experimental tests by Abadie (2015), $\tilde{T}_b(\zeta_b)$ has been found to fit well with a power exponent of 4 ($\Delta\psi_G \sim \zeta_b^{m_t}$ with $m_t = 4$). The extracted values for CM6 and the results by Abadie (2015) are compared in Figure 5.18 along with the power law fit showing that this value is consistent between both sets of data.

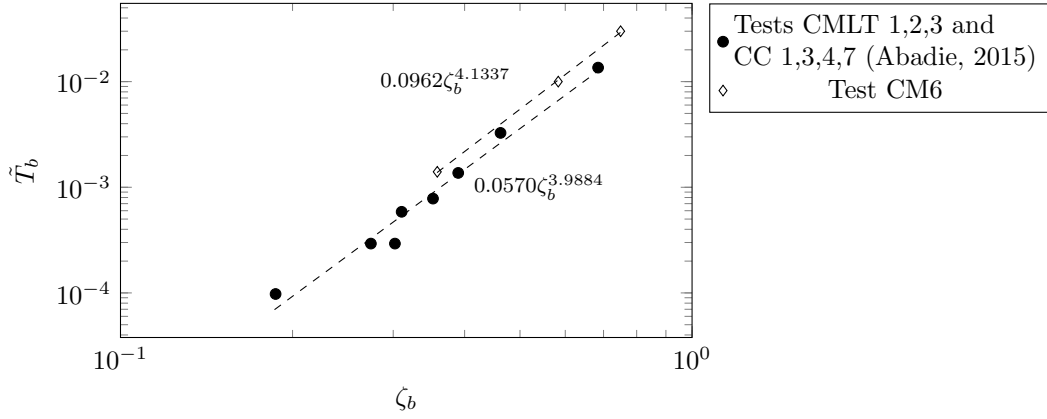


Figure 5.18: $\tilde{T}_b(\zeta_b)$ for test CM6 and comparison with the results by Abadie (2015). Data points are fitted with power law expressions.

The behaviour of the ratcheting element with cycle amplitude is analysed for a single cycle, where $\dot{\alpha}_i = y_i \dot{\alpha}$ (see Section 4.5.3):

$$\begin{aligned} \Delta\psi_G \sim \int_{\text{Cycle}} \dot{\alpha}_r dt \sim & \int_0^{\sigma_{\max}} \left(\frac{|\sigma|}{\sigma_u} \right)^{m_\sigma} \left(\frac{\sum y_i k_i}{\sigma_u} \right)^{m_k+1} \left(\frac{\alpha_h}{\alpha_{h0}} \right)^{m_h} \frac{d|\alpha|}{d\sigma} d\sigma \\ & + \int_{\sigma_{\max}}^0 \left(\frac{|\sigma|}{\sigma_u} \right)^{m_\sigma} \left(\frac{\sum y_i k_i}{\sigma_u} \right)^{m_k+1} \left(\frac{\alpha_h}{\alpha_{h0}} \right)^{m_h} \frac{d|\alpha|}{d\sigma} d\sigma \end{aligned} \quad (5.13)$$

In the unload part of the cycle, the accumulated ratcheting is very low as the effects of the stress component and the dissipation component reach their maxima at opposite points during unloading. Therefore, the analysis can be carried through with reason-

able accuracy by only considering the upward part of the load cycle. To analyse the effect of the stress magnitude on the ratcheting element, the evolution of each of the individual components needs to be identified.

Since $\alpha_h \sim \int_0^N |\dot{\alpha}_r| dN$ at high cycle numbers, both terms increase with the same exponent with respect to cycle amplitude. Therefore, we can write: $\alpha_h \sim \sigma^{m_t}$. The stress term simply increases with its exponent m_σ . The evolution of the dissipation term with cycle amplitude can be evaluated by analysing the evolution of dissipated energy by the foundation within a load cycle using the kinematic hardening model. The dissipated energy for varying cycle amplitudes $\left(\zeta_I = \frac{M_{\max} - M_{\min}}{2M_{\text{ult}}}\right)$ is illustrated in Figure 5.19, which can be approximated using a power law fit as: $E_{\text{diss}}^{\text{tot,G}} \sim \zeta_I^{m_d}$ where $m_d = 3.4$. Since it is expected that this dissipation is reflected in each of the soil reaction curves, we can write that:

$$|\sigma - \sigma_r|^{m_d} \sim \int_{\sigma_r}^{\sigma} \sum_{i=1}^{n_s} \frac{k_i y_i}{\sigma_u} \frac{d|\alpha|}{d\sigma} d\sigma \rightarrow \sum_{i=1}^{n_s} \frac{k_i y_i}{\sigma_u} \frac{d|\alpha|}{d\sigma} \sim |\sigma - \sigma_r|^{m_d-1} \quad (5.14)$$

where σ_r is the point of stress reversal in the cyclic loading. As the hardening process accumulates and the incremental ratcheting strain becomes negligible compared to the strain step, we can write $\dot{\alpha} \approx \dot{\epsilon}$. Using the power law fit for the backbone curve in Figure 5.17, we can write $\epsilon \sim \sigma^{m_b}$ with $m_b = 1/0.575 = 1.74$ (the power of 0.575 is taken from the fit of the backbone in Figure 5.17). For an unload or reload loop using the kinematic hardening model, we can now write: $\frac{d\epsilon}{d\sigma} = |\sigma - \sigma_r|^{m_b-1}$. The evolution of the strength term with cycle amplitude can therefore be approximated as:

$$\sum_{i=1}^{n_s} \frac{k_i y_i}{\sigma_u} \sim |\sigma - \sigma_r|^{m_d-m_b} \quad (5.15)$$

Combining the effects of all terms of the ratcheting element, we arrive at:

$$\sigma_{\max}^{m_t} \sim \sigma_{\max}^{m_\sigma + m_k(m_d - m_b) + m_h m_t + m_d} \quad (5.16)$$

$$\rightarrow m_\sigma + m_k(m_d - m_b) = m_t(1 - m_h) - m_d \quad (5.17)$$

With values of $m_t = 4$, $m_h = -2.2$ and $m_d = 3.4$, the right hand side is equal to: $m_t(1 - m_h) - m_d = 9.4$. Using an appropriate combination of m_σ and m_k , the increase of the ratcheting with load magnitude can be achieved.

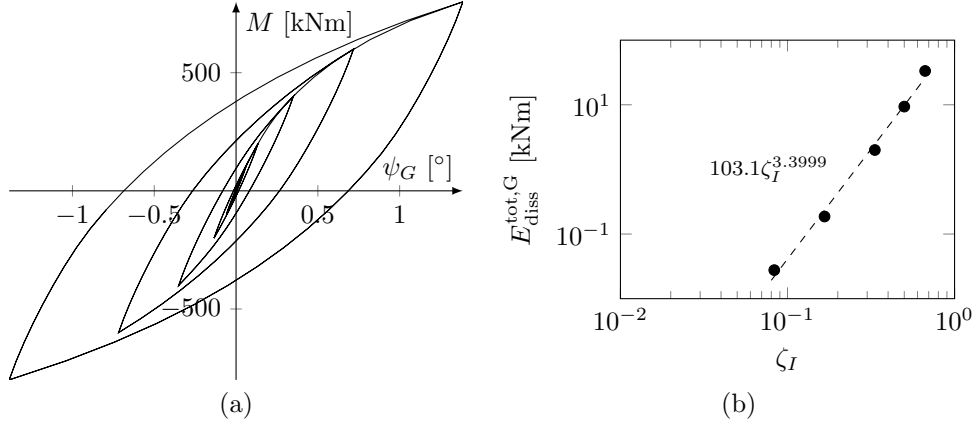


Figure 5.19: (a) Hysteresis loops for CM6 at different load amplitudes using the kinematic hardening model and (b) evolution of the dissipated energy and fitted curve using a power expression.

5.5.1.3 Calibration of the load shape effect

The calibration of the load shape effect is based on a numerical study where the ratcheting model is applied to the moment-rotation response of the recalibrated backbone curve for CM6. The ultimate strength for the ratcheting model is defined where the ground level rotation reaches 2° . Varying the power on the stress m_σ and the strength component m_k results in a change of the $T_c(\zeta_c)$ curve. To capture the same increase of ratcheting with load magnitude, $m_\sigma + m_k(m_d - m_b)$ is kept constant at 9.4, with $m_d = 3.4$ and $m_b = 1.74$. The other ratcheting parameters for the simulations are: $R_0 = 3000$, $m_h = -2.2$ and $\alpha_{h0} = 0.02^\circ$.

In the simulation, 10^5 cycles are simulated using the accelerated modelling approach introduced by Houlsby et al. (2017), where the coupling terms R_i can be

accelerated by a factor N_{accel} . This factor is gradually increased throughout the cycles (10 cycles at $N_{\text{accel}} = 1$, 9 cycles at $N_{\text{accel}} = 10$, 9 cycles at $N_{\text{accel}} = 100$, *etc.*). This approach allows the ratcheting response after 10^5 load cycles to be calculated with only 46 simulated cycles, resulting in a significantly lower computational time. An example of these simulations is illustrated in Figure 5.20, where $m_\sigma = 9.4$, $m_k = 0$ and $\zeta_c = 0$. The simulations are repeated for varying ζ_c values and the resulting $T_c(\zeta_c)$ curves are illustrated in Figure 5.21. This figure shows that with values of $m_\sigma = 9.4$ and $m_k = 0$, a peak in the $T_c(\zeta_c)$ curve is achieved of 1.5, which is consistent with the tests by Zhu et al. (2012), Foglia et al. (2014) and Nicolai and Ibsen (2014). By varying m_σ , a higher peak can be achieved which would be consistent with the results by LeBlanc et al. (2010). When $m_\sigma = 0$, the ratcheting strain rate changes drastically at the change of stress direction. This results in a drop in $T_c(\zeta_c)$ at low negative values of ζ_c . The increase of T_c at $\zeta_c = -1$ with $m_\sigma = 0$ is a numerical problem caused by the acceleration process in the first peak after the acceleration is increased. Since this first peak is always in the same direction, the ratcheting is dominated by this direction.

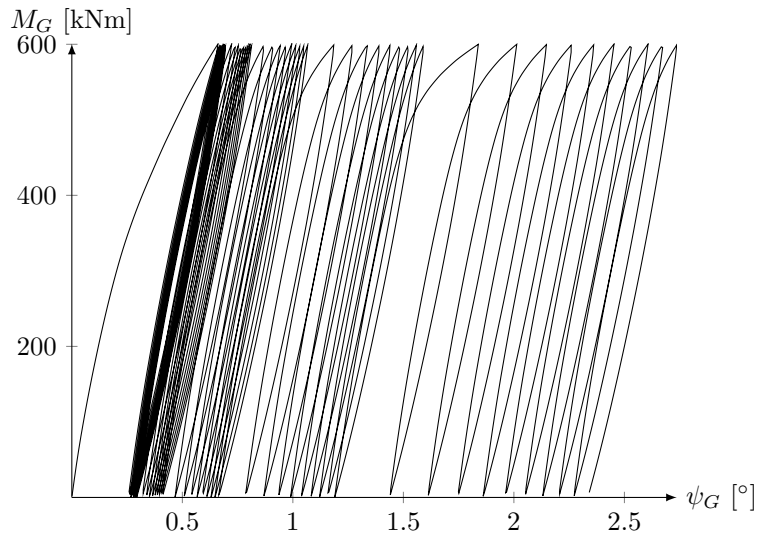


Figure 5.20: Example simulation of the cyclic response for the calibration of the load shape effect with $m_\sigma = 9.4$, $m_k = 0$ and $\zeta_c = 0$, where 10^5 cycles are simulated.

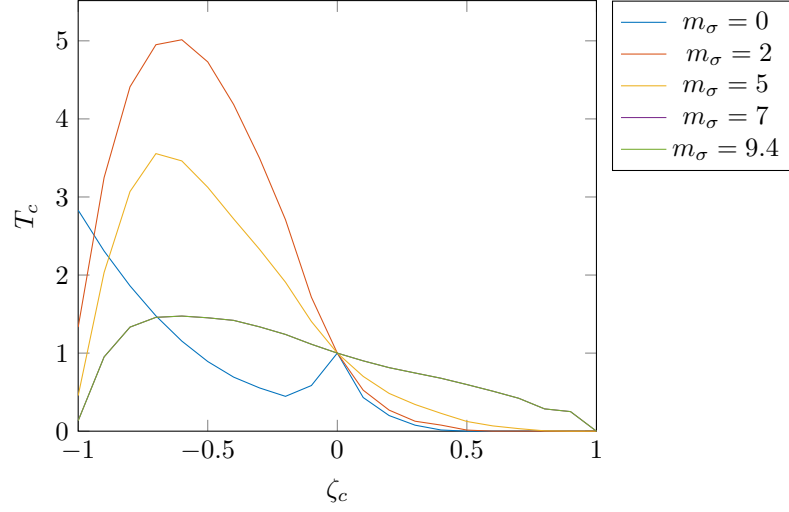


Figure 5.21: T_c as a function of ζ_c evaluated using the numerical study with $m_\sigma + m_k(m_d - m_b) = 9.4$.

5.5.1.4 Simplified expression for the ratcheting strain

With the choice of $m_k = 0$, the formulations of the ratcheting strain and hardening strain reduce to:

$$\dot{\alpha}_r = S(\sigma) R_0 \left(\frac{|\sigma|}{\sigma_u} \right)^{m_\sigma} \left(\frac{\alpha_h}{\alpha_{h0}} \right)^{m_h} \frac{\sum k_i |\dot{\alpha}_i|}{\sigma_u} \quad (5.18)$$

$$\dot{\alpha}_h = |\dot{\alpha}_r| \quad (5.19)$$

where only four variables need to be calibrated: R_0 , m_σ , m_h and α_{h0} . As illustrated in Figure 5.21, this expression results in a reasonable variation of $T_c(\zeta_c)$. This choice of the ratcheting strain and hardening strain is particularly interesting since it can be applied to a series version of the model (Houlsby et al., 2017), resulting in the same ratcheting behaviour if both models are calibrated for the same backbone curve. The reason for this is that the dissipated energy in the slider elements $\sum k_i |\dot{\alpha}_i|$ is consistent between the two versions of the model.

5.5.2 Ratcheting in Cowden

The simulated ground level moment-rotation response and accumulated rotation for CM6 are given in Figure 5.22, using the simplified ratcheting expression with a choice of parameters given in Table 5.3. Similarly to the kinematic hardening model, the stiffnesses and strengths of the spring-slider elements have been calibrated using 50 elements. The applied load in the simulation consists of 10 cycles at 43.5kN, followed by 10 cycles at 60kN and 38 cycles at 90kN, in a similar way to the applied load in the field test. To capture the ratcheting increase with load amplitude, m_σ is reduced slightly from 9.4 to 9.0. The calibrated model shows close agreement with the experimental results. The distributed lateral reaction in the simulation is illustrated in Figure 5.23.

Table 5.3: Ratcheting parameters for the calibration of CM6.

R_0	m_σ	m_h	α_{h0}
3000	9.0	-2.2	$\epsilon_u/100$

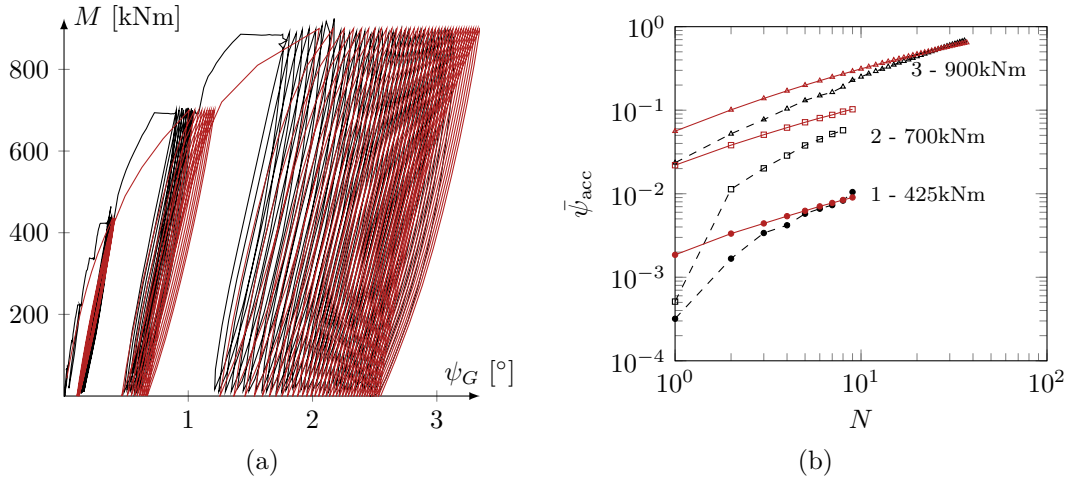


Figure 5.22: (a) Simulated ground level moment-rotation response of CM6 using the ratcheting model (—) compared to field test response (—) and (b) comparison of the accumulated rotation.

The same calibration for the soil reaction curves and ratcheting model is applied to CM5. The simulation uses the accelerated modelling introduced in Section 5.5.1.3.

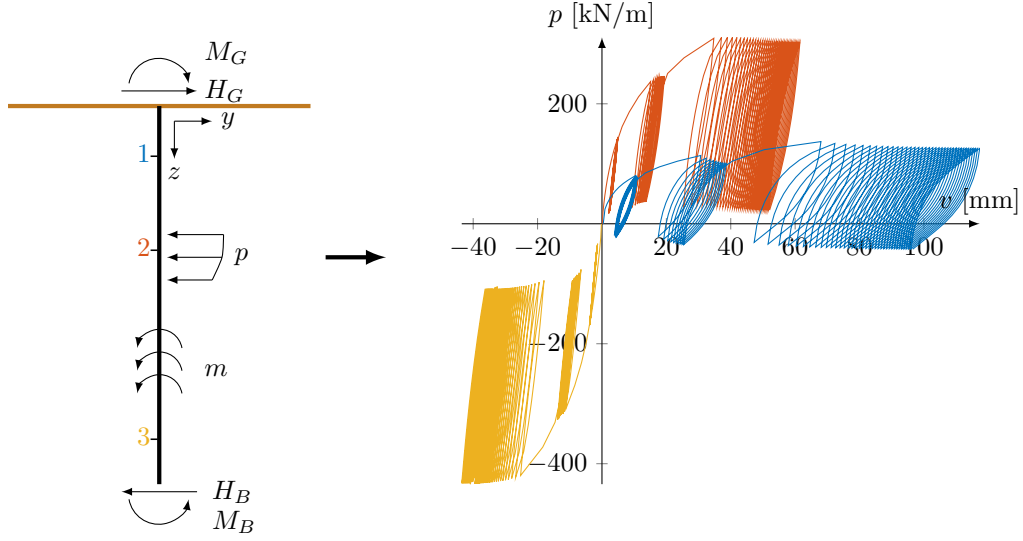


Figure 5.23: Simulated lateral soil reaction curves for CM6 using the ratcheting model at (1) $z=0.5\text{m}$, (2) $z=1.5\text{m}$ and (3) $z=3.5\text{m}$.

The simulated load sets are summarised in Table 5.4, which is a reduced version of the applied load sets for CM5. The measured rotation accumulation in load sets 4 to 10 was very limited compared to the accumulation in the previous load sets. Since the hardening variable increases with the magnitude of the ratcheting strain, the effect of load sets 4 to 10 on the later load sets is limited, which supports the use of the reduced load sets.

Table 5.4: Simulated load sets for CM5.

Load set	Max. Moment [kNm]	Simulated cycles
1	100	7000
2	200	7000
3	600	2500
11	900	150

The simulated results of CM5 are compared with the field test in Figure 5.24. The comparison of the accumulated rotation in Figure 5.25 shows that the increase with cycle number and with load amplitude is captured reasonably well. However, the simulated response shows an overprediction of the ratcheting effect in load sets 3 and 11 by a factor of approximately 3. In Section 3.4.3, it was noted that instrumentation

error probably affected the results at the low load levels (load sets 1 and 2). For the cycles at 60kN in load set 3, a significant rate effect was observed in the initial load cycles resulting in an additional rotation accumulation component from the rate effect in the first few load cycles. The initial loading to 600kNm in Figure 5.24 illustrates that the backbone curve used in this simulation does not correspond accurately to the backbone curve for test CM5. The calibration of the backbone curve affects the ratcheting response and a stiffer backbone curve would lead to lower accumulation of rotation in the simulation.

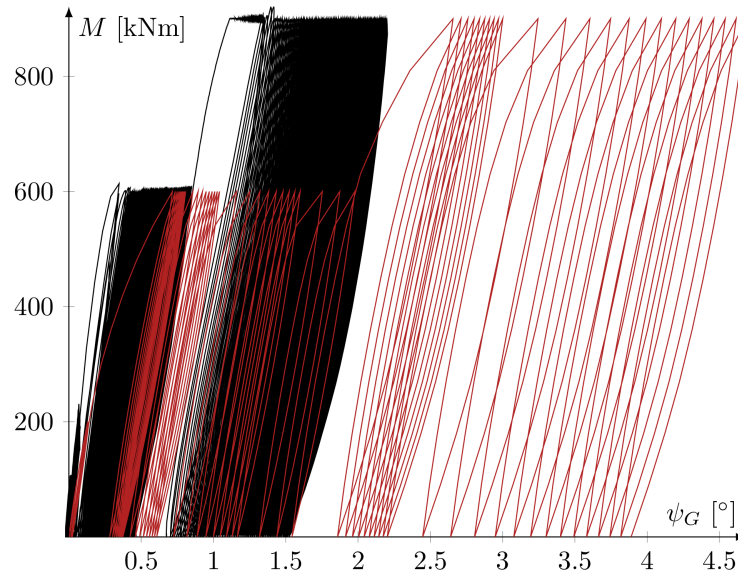


Figure 5.24: Comparison of the simulated and field test response of CM5.

Table 5.5: Simulated load sets for CL2.

Load set	Max. Moment [MNm]	Simulated cycles
0	20	1
1	2.15	400
2	4.3	100
3	8.6	60
4	17.2	16

For the large pile test in Cowden, the cyclic loads were applied after the monotonic test procedure. The load history for the simulated cycles is summarised in Table

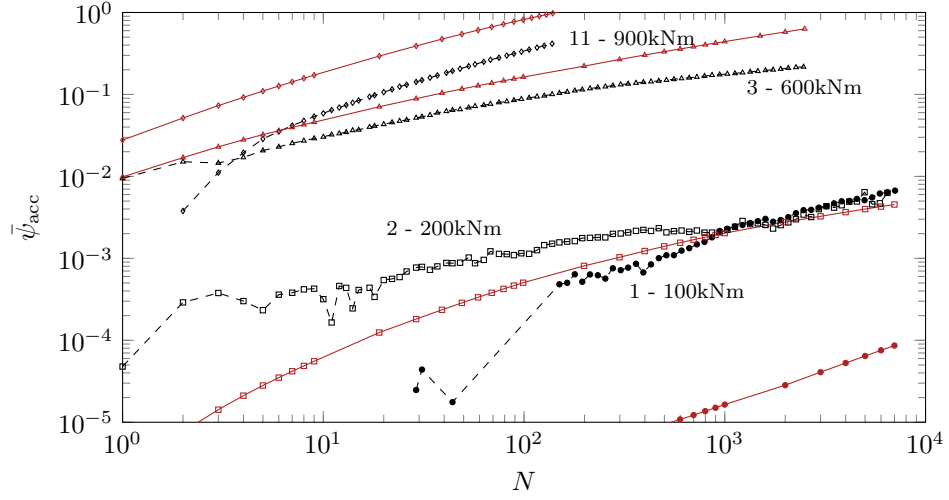


Figure 5.25: Simulated accumulated rotation of CM5 (—) compared to the field test response (—).

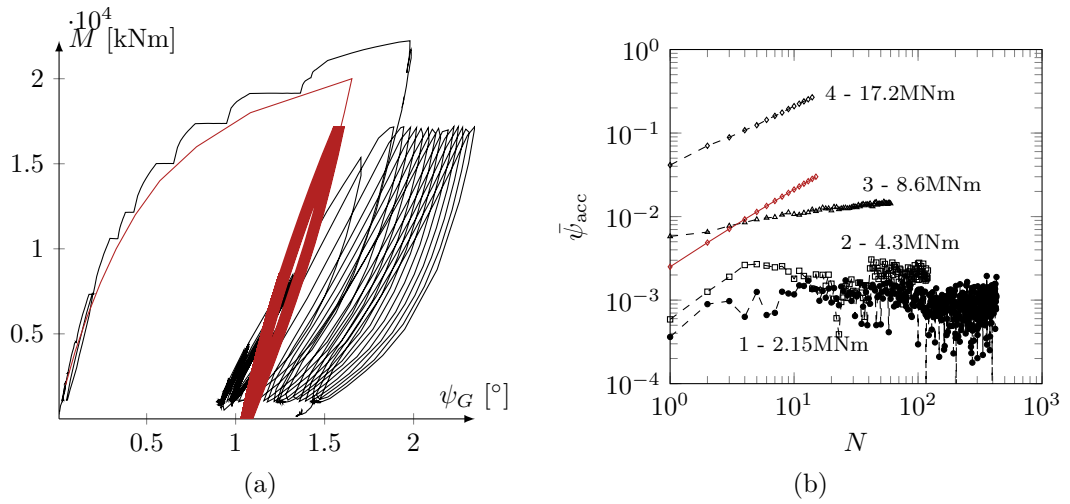


Figure 5.26: (a) Simulated ground level load displacement response of CL2 (—) compared to field test response (—) and (b) comparison of the accumulated rotation.

5.5. The ground level moment-rotation response and the accumulated rotation of the foundation are given in Figure 5.26, compared with the field test results. The simulation uses the same calibration for the ratcheting element as CM6. The simulation shows small negative accumulation of rotation in the first three load sets and the accumulated rotation in load set 4 is underpredicted by roughly one order of magnitude. This indicates that the hardening process is not captured accurately using this model and might also indicate that the scaling of the ratcheting is not properly captured. The reduction in stiffness after the monotonic loading to failure is also not captured using this model. This stiffness reduction is probably the result of the gapping effect.

5.5.3 Ratcheting in Dunkirk

The R_0 value for the ratcheting element has been calibrated for DM2 using the recalibrated backbone curve of DM9. The other ratcheting parameters were chosen as the same as for CM6, resulting in a good agreement between the simulated results and the field tests for both the load history and the load magnitude effect. The values for the ratcheting element are given in Table 5.6. The higher R_0 value for DM2 compared to CM6 and CM5 is considered to be the effect of the stiffer response of DM2 upon cyclic loading.

Table 5.6: Ratcheting parameters for the calibration of DM2.

R_0	m_σ	m_h	α_{h0}
500000	9.0	-2.2	$\epsilon_u/100$

For the simulation, a reduced number of load sets have been applied which are summarised in Table 5.7. Similarly to CM5, the observed accumulation of rotation in load sets 4 to 10 was low compared to the earlier load sets. The resulting hardening effect on load set 11 is considered to be limited, which supports the use of the reduced load sets. The moment-rotation response is given in Figure 5.27 and the resulting accumulation of rotation is illustrated in Figure 5.28. In the first two load sets,

the simulated accumulated rotation is lower than the measured values, though the measured rotation accumulation at these low load levels is likely to be affected by instrumentation error. The simulated ratcheting response in load set 3, 4 and 11 matches closely with the field test results.

Table 5.7: Simulated load sets for DM2.

Load set	Max. Moment [kNm]	Simulated cycles
1	100	5000
2	200	3000
3	400	8000
4	800	11000
11	1600	30

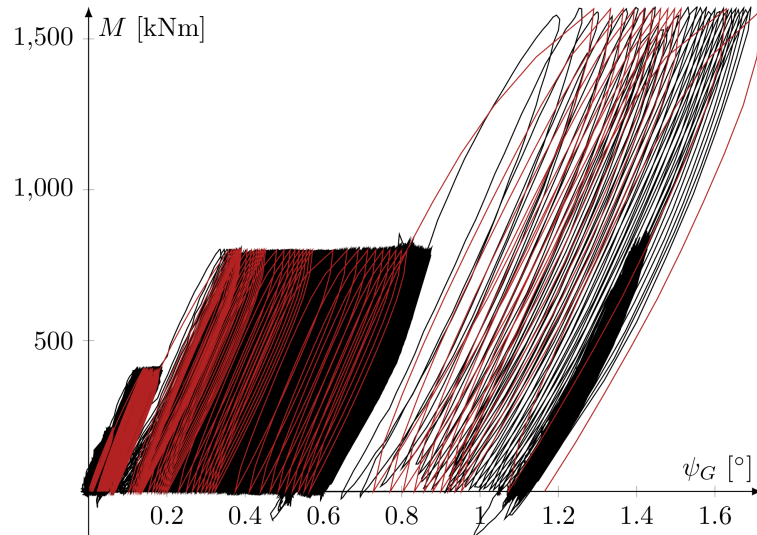


Figure 5.27: Comparison of the simulated and field test response of DM2.

For the large pile test in Dunkirk, the cyclic loads have been applied after the monotonic test procedure. The load history for the simulated cycles is summarised in Table 5.8. The simulation is compared to the field test in Figure 5.29 using the calibration of the ratcheting model for DM2. The simulated response shows no accumulation in the first three load sets. The accumulation of rotation in load set 4 is too low by a factor of roughly 3. These results indicate that the hardening response may not be captured well using this model or that scaling effects are not captured

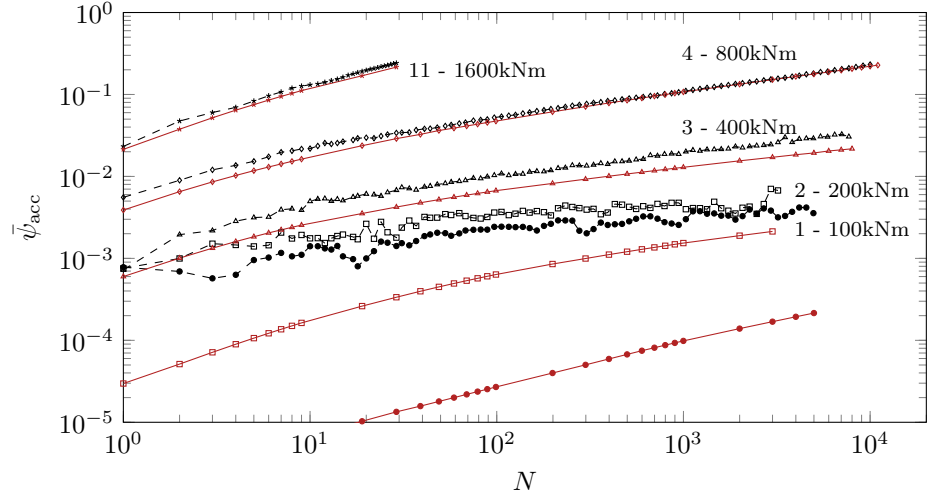


Figure 5.28: Simulated accumulated rotation of DM2 (—) compared to the field test results (—).

Table 5.8: Simulated load sets for DL2.

Load set	Max. Moment [MNm]	Simulated cycles
0	40	1
1	4.2	200
2	8.4	100
3	16.8	50
4	33.6	30
5	40	2

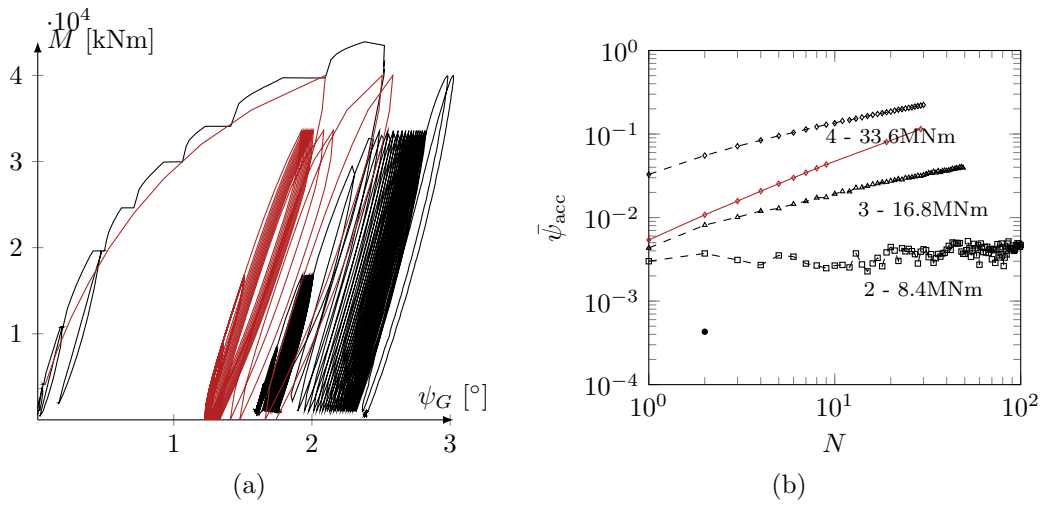


Figure 5.29: (a) Simulated ground level load displacement response of DL2 (—) compared to field test response (—) and (b) comparison of the accumulated rotation.

appropriately for the ratcheting model. The simulations of CL2 and DL2 show the importance of capturing events where the hardening variable increases ($\dot{\alpha}_h = |\dot{\alpha}_r|$) on the response of the later load sets.

5.6 Combined rate and ratcheting model

The combined rate and ratcheting model has been applied to CM6 and DM6. In these tests, both the rate and ratcheting are clearly observed with a limited amount of cycles. Since the time history of a specific test is applied to the simulation, modelling large numbers of cycles would exceed reasonable computational time. The parameters for the rate and ratcheting model are a combination of the previous calibrations. As discussed in Section 3.6.3, the ratcheting effect using this model is lower because of the smaller hysteresis loop caused by the rate effect. The R_0 factor has therefore been increased in the simulations for CM6. The model parameters are given in Table 5.9

Table 5.9: Rate and ratcheting parameters for the calibration of CM6.

R_0	m_σ	m_h	α_{h0}	η_{10}	$\dot{\alpha}_{qs}^v$	$\dot{\alpha}_{qs}^\psi$
10000	9.0	-2.2	$\epsilon_u/100$	0.12	$D/(10 \cdot 6h)$	$2^\circ/6h$

The simulated behaviour is compared to the field test response in Figure 5.30. The simulated behaviour compares very well to the field test response, matching both the ratcheting and the rate effect. The distributed lateral reaction is illustrated in Figure 5.31.

The rate and ratcheting parameters for the simulation of DM6 are given in Table 5.10. The simulated behaviour is compared to the field test result in Figure 5.32. The simulated response compares well to the field test response, both in terms of the rate as well as the ratcheting effect. The ultimate capacity is not captured as precisely, though this is due to the calibration of the backbone curve rather than the rate and ratcheting effect. The distributed lateral reaction is illustrated in Figure 5.33.

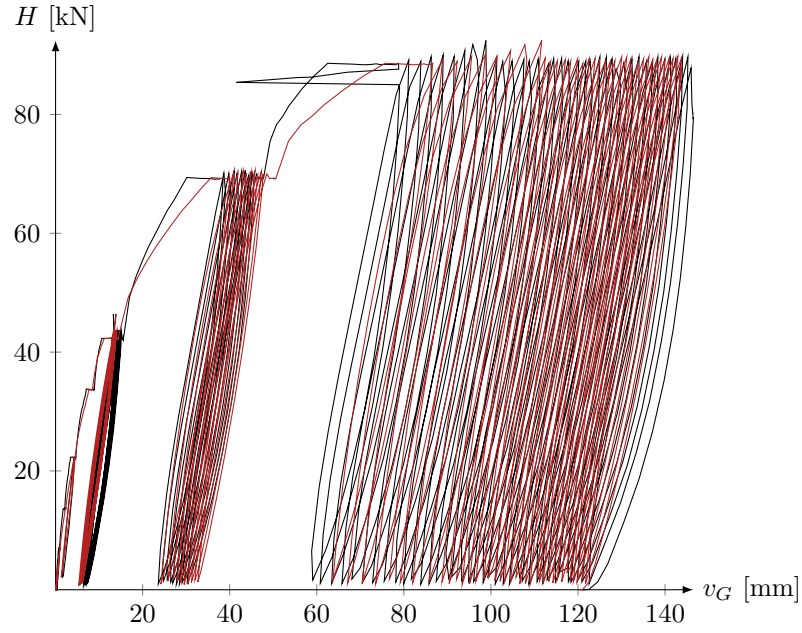


Figure 5.30: Simulated response of CM6 (—) compared to the field test response (—).

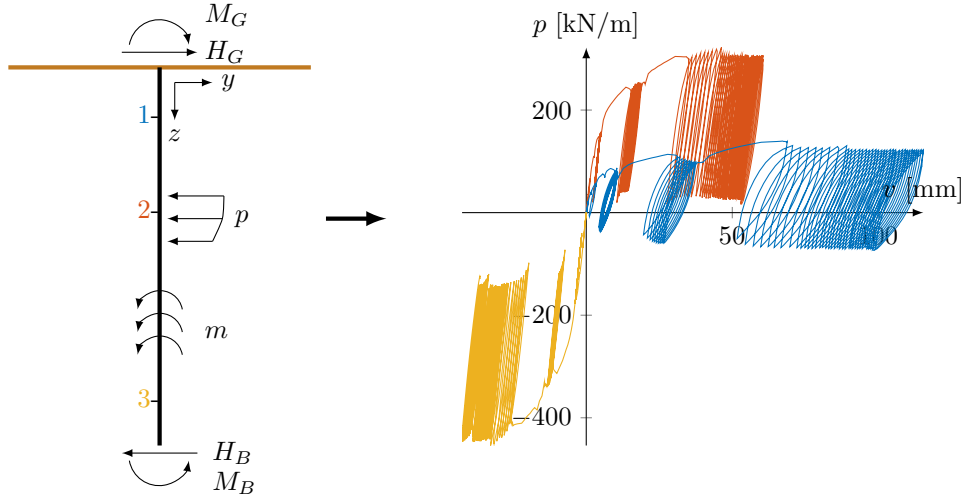


Figure 5.31: Simulated lateral soil reaction curves for CM6 using the combined rate and ratcheting model at (1) $z=0.5\text{m}$, (2) $z=1.5\text{m}$ and (3) $z=3.5\text{m}$.

Table 5.10: Rate and ratcheting parameters for the calibration of DM6.

R_0	m_σ	m_h	α_{h0}	η_{10}	$\dot{\alpha}_{\text{qs}}^v$	$\dot{\alpha}_{\text{qs}}^\psi$
$2 \cdot 10^6$	9.0	-2.2	$\epsilon_u/100$	0.06	$D/(10 \cdot 6\text{h})$	$2^\circ/6\text{h}$

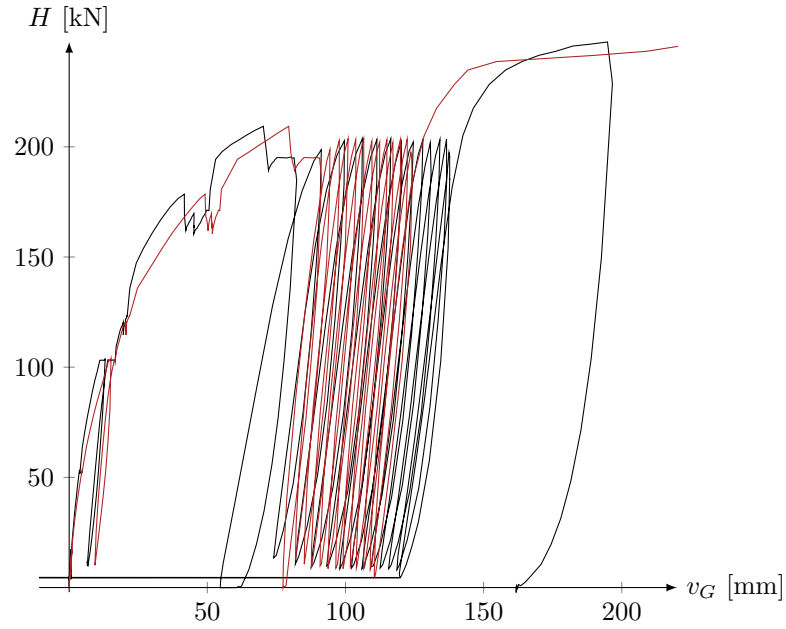


Figure 5.32: Simulated response of DM6 (—) compared to the field test response (—).

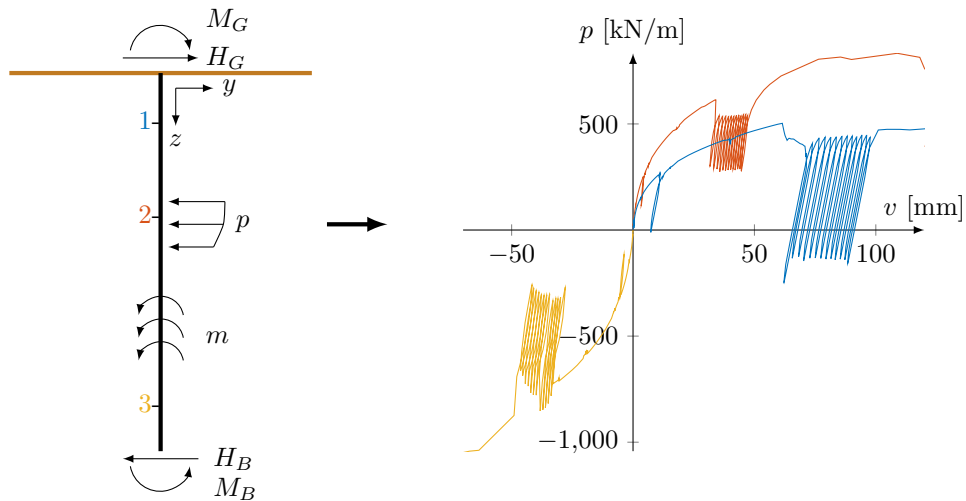


Figure 5.33: Simulated lateral soil reaction curves for DM6 using the combined rate and ratcheting model at (1) $z=0.5\text{m}$, (2) $z=1.5\text{m}$ and (3) $z=3.5\text{m}$.

5.7 Gapping model

5.7.1 Calibration of the gapping element

The gapping model is calibrated to the distributed load-displacement curves from the PISA tests. To capture similar behaviour with the gapping element as with the load-displacement soil reaction curve, the model needs to be calibrated for (1) initial stiffness, (2) ultimate resistance and (3) curvature in the initial loading. An additional component in the gapping element is the initial stress. As indicated in Section 4.7.4, the initial stress affects the resulting behaviour under cyclic loading.

Since the aim is to reproduce the monotonic behaviour of the field tests in the PISA project, the calibration is done using the correction factors on the backbone curves in Table 5.1. For CM7, the correction factors of CM9 are used, since both piles have the same geometry. Similarly for DM1, the correction factors of DM9 are used.

Initial stiffness

Using the model in Section 4.7, the lateral soil reaction is evaluated by:

$$p_y = D \int_{-\pi/2}^{\pi/2} \sigma_y d\phi \quad (5.20)$$

At small displacements, we can write that $\sigma_{0n} = E_{0n}u_n$ and $\sigma_{0t} = E_{0t}u_t$. Using equation 4.84 and 4.85, this results in:

$$\begin{aligned} p_y &= D \int_{-\pi/2}^{\pi/2} -\sin(\phi)\sigma_{0n} + \cos(\phi)\sigma_{0t} d\phi \\ &= D \int_{-\pi/2}^{\pi/2} -\sin(\phi)E_{0n}u_n + \cos(\phi)E_{0t}u_t d\phi \\ &= D \int_{-\pi/2}^{\pi/2} \sin^2(\phi)E_{0n}v + \cos^2(\phi)E_{0t}vd\phi \end{aligned} \quad (5.21)$$

so that

$$k_{0p} = \frac{p_y}{v} = \pi R(E_{0n} + E_{0t}) \quad (5.22)$$

At small displacements, the entire pile boundary remains in contact with the soil if σ_{0n} is greater than 0. To relate the shear stiffness parameters to the normal stiffness parameters, an analogy with a circular surface footing with radius R can be used. The vertical stiffness k_v and the horizontal stiffness k_h are function of the shear modulus G_0 and the Poisson's ratio ν (Siefert and Cevaer, 1992):

$$k_v = \frac{4G_0R}{1-\nu} \quad (5.23)$$

$$k_h = \frac{8G_0R}{2-\nu} \quad (5.24)$$

It can be expected that the relationship between these two parameters is similar to that for the pile-soil boundary:

$$\frac{E_{0t}}{E_{0n}} = \frac{k_h}{k_v} = 2 \frac{1-\nu}{2-\nu} \quad (5.25)$$

The Poisson's ratio ν will be assumed equal to 0.2 for sand and 0.5 for clay. The initial normal stiffness for the soil reaction in the gapping model is therefore calculated as:

$$E_{0n} = \frac{k_y}{\pi R \left(1 + 2 \frac{1-\nu}{2-\nu}\right)} \quad (5.26)$$

Ultimate resistance

The yield surface of the gapping element at the pile boundary is illustrated in Figure 4.25. For the ultimate resistance of a pile section, the maximum stress which can be developed along the boundary in the x -direction is given by (assuming the pile stays

in contact with the soil):

$$\sigma_x^{\max} = s_u \left(\sqrt{N_n^2 \cos^2 \phi + N_t^2 \sin^2 \phi} + N_{\sigma_0} \cos \phi \right) \quad (5.27)$$

where $N_{\sigma_0} = \sigma_{0n}/s_u$. Assuming $\mu = \infty$, the normal stress must always be greater than 0, otherwise gapping occurs and there is no contribution to the lateral reaction. This occurs at the critical angle along the pile section ϕ_c . Integrating the maximum soil reaction along the pile boundary gives:

$$\frac{p^{\max}}{s_u D} = \int_0^{\pi/2 + \arctan \phi_c} \left(\sqrt{N_n^2 \cos^2 \phi + N_t^2 \sin^2 \phi} + N_{\sigma_0} \cos \phi \right) d\phi \quad (5.28)$$

$$\begin{aligned} &= N_n E \left(1 - \frac{N_t^2}{N_n^2} \right) + \frac{N_{\sigma_0}}{\sqrt{1 + \phi_c^2}} \\ &+ N_n \left(-E \left(1 - \frac{N_t^2}{N_n^2} \right) + E \left(\frac{1}{2} (\pi + i \log(1 - i\phi_c) - i \log(i + i\phi_c)), 1 - \frac{N_t^2}{N_n^2} \right) \right) \\ &\approx (N_n + N_{\sigma_0}) E \left(1 - \frac{N_t^2}{N_n^2} \right) \end{aligned} \quad (5.29)$$

$$\phi_c = \frac{N_t}{N_n} \frac{N_{\sigma_0}}{\sqrt{N_n^2 - N_t^2}} \quad (5.30)$$

where $E(\cdot)$ is the complete elliptic integral and $E(\cdot, \cdot)$ is the elliptic integral of the second kind. In the case where N_{σ_0} is greater than N_n , the ultimate resistance becomes:

$$\frac{p^{\max}}{s_u D} = \int_0^{\pi} \left(\sqrt{N_n^2 \cos^2 \phi + N_t^2 \sin^2 \phi} \right) d\phi = 2N_n E \left(1 - \frac{N_t^2}{N_n^2} \right) \quad (5.31)$$

The calibration of the ultimate resistance of a pile section in clay makes use of the solution for a cylindrical cavity expansion problem, originally derived by Gibson and Anderson (1961) for the application of a pressuremeter test. The limiting pressure is calculated by:

$$N_n = \frac{\sigma_n^{\text{ult}} - \sigma_{0n}}{s_u} = 1 + \log \left(\frac{G_0}{s_u} \right) \quad (5.32)$$

Using a ratio of G_0/s_u of 500, which is representative for the soil profile in the Cowden test site for $z < 4\text{m}$ (Figure 2.6), results in a value of $N_n = 7.21$. The maximum shear stress which can be developed along the pile boundary can be assumed equal to the soil shear strength $N_t s_u = s_u$ when the gapping element is not activated, so that $N_t = 1$ and the ratio between the tangential and the normal strength is equal to $\frac{N_n}{N_t} = 7.21$. The friction angle between the soil and pile is assumed equal to 20° so that $\mu = \tan(20^\circ) = 0.36$. Note that in the present formulation of the gapping model, the gapping element uses a frictional sliding mechanism which is different from the maximum mobilised shear used commonly for a clay-steel interface ($\tau_{\max} = \alpha s_u$).

For the calibration of the ultimate resistance of a pile in sand, a comparison is used with experiments on a spudcan in dense sand by Cassidy (1999). The experimental results are fitted with a macro-model of the foundation behaviour called ‘Model C’. The yield surface in V/H (vertical/horizontal force) space is ‘cigar’ shaped between the origin and $V_{\max}$. Around $V = V_{\max}/2$, the maximum horizontal force is equal to $H_{\max} = 0.116V_{\max}$. This ‘cigar’ shaped yield surface is approximated to a reasonable degree with the yield surface of the gapping model. Using the same $\frac{N_t}{N_n}$ ratio as for clay (0.139), the ratio between maximum normal and shear stress is similar to the $\frac{H_{\max}}{V_{\max}}$ in Model C, taking into consideration that the friction coefficient lowers the maximum shear resistance slightly.

Using the previous equations, the following relationship can be found :

$$N_n \approx \max \left\{ \frac{\bar{p}_u}{E \left(1 - \left(\frac{N_t}{N_n} \right)^2 \right)} - N_{\sigma_0}, \frac{\bar{p}_u/2}{E \left(1 - \left(\frac{N_t}{N_n} \right)^2 \right)} \right\} \quad (5.33)$$

The initial effective horizontal stress σ'_{h0} , which is used as the initial stress in the gapping model σ_{0n} , is calculated by:

$$\sigma'_{h0} = K_0(z) (\sigma_{v0}(z) - p_w(z)) \quad (5.34)$$

where the profiles of $K_0(z)$ and $p_w(z)$ in Cowden are given in Figure 2.5. The vertical stress is calculated by $\sigma_{v0} = \int_0^z \gamma(z)dz$, with a constant saturated bulk unit weight $\gamma(z) = 21.19\text{kN/m}^3$. For Dunkirk, the pore water pressure profile is given in Figure 2.9 (a) and the K_0 is taken as constant at 0.4. For the calculation of the initial vertical stress, a bulk unit density of $\gamma = 17.1\text{kN/m}^3$ is used above the water table and $\gamma = 19.9\text{kN/m}^3$ below.

Curvature

Once the initial stiffness and the ultimate capacity have been defined, the stiffness and strength distribution of the multi-surface gapping model (H_{in}, k_i) are fitted to match the monotonic soil reaction curve upon monotonic loading. Here, the values are determined by fitting to the conic curve in Equation 2.1. This calibration has been performed using 10 spring-slider elements, providing a reasonable approximation for the fitting of the conic curves. The integration along the pile boundary is performed using 8 sections (Figure 4.23). Varying the ultimate displacement and curvature allows fitting of the reaction of the cross section to the distributed load-displacement curve defined in the PISA project. The comparison of the reaction using the multi-surface gapping model at two depths is shown in Figure 5.34 where the Cowden clay reaction normal to the pile boundary is calibrated with $n = 0.7$ and $\bar{v}_u = 20$ and the Dunkirk sand reaction is calibrated with $n = 0.8$ and $\bar{v}_u = 5$. The reaction of the gapping model with a single spring-slider element is given for comparison. In the analysis, the friction coefficient μ is taken equal to 0.36.

This figure shows a good match between the multi-surface gapping model with curvature and the soil reaction curves. The multi-surface approach allows capturing of the curvature in the intermediate response of the distributed load-displacement curves much better than the single surface approach.

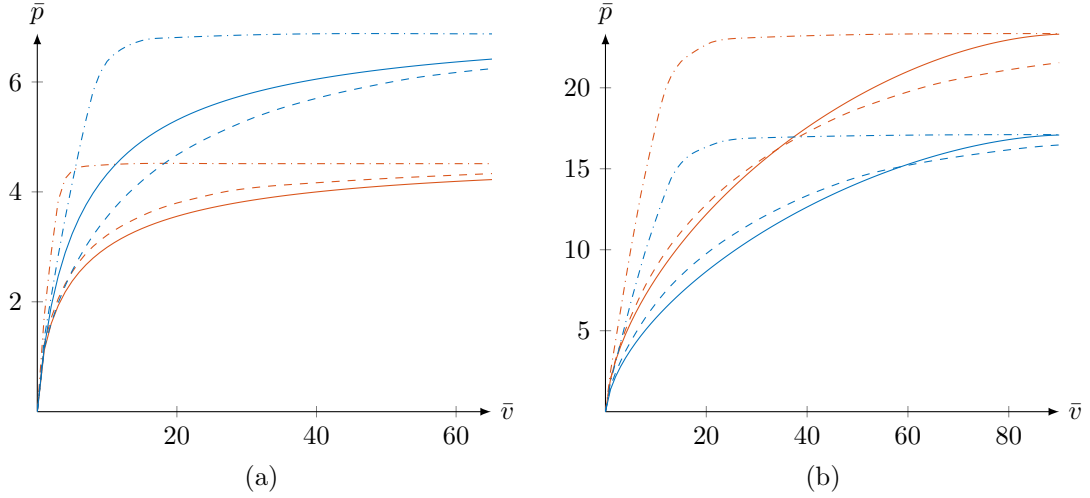


Figure 5.34: Comparison of the normalised soil reaction for (a) CM5 and (b) DM1 at (red) $z/L = 1/4$ and (blue) $z/L = 3/4$ for (full) the initial soil reaction curves, (dashdotted) the single surface gapping model and (dashed) the multi-surface gapping model with curvature.

5.7.2 Gapping in Cowden

The gapping model is applied to CM7, where symmetric cyclic loading is applied in a sequence described in Figure 5.35, which is similar to the sequence in the field test. For each load set, 2 cycles were simulated. The resulting ground level load displacement is illustrated in Figure 5.36. This figure shows the hysteresis behaviour of the model throughout the various load sequences. In ramping down the load amplitude after a maximum load of 20kN is applied, the foundation reaction recovers to a similar behaviour as the initial ramping up in load. However, after the load magnitude of 40kN is applied, the gapping effect in the model results in a lower secant stiffness in the cycles. The variation of stiffness and damping throughout the load sets is illustrated in Figure 5.37, where the stiffness is compared to the measured values in the field tests.

The decrease in stiffness after the load set of 40kN agrees qualitatively with the results seen in CM7 (Figure 3.28). However, the calibration of the model does not result in the same stiffness reduction after the maximum load set of 20kN was applied.

It can be expected that with more detailed calibration of the model, this effect could be captured better. The variation in damping ratio after load set 27 also explains the higher damping ratios seen in the later load sets in field test CM7. The additional damping effect is a result of the hysteresis in the side friction, which becomes more dominant once a gap has been formed at the front and back of the foundation pile. The magnitude of the damping ratio, however, is under predicted by a factor of approximately 5 compared to measured values in Figure 3.28.

The soil reactions at three locations along the pile length throughout load set 27 are illustrated in Figure 5.38. This figure clearly shows a ‘bow tie’ shaped hysteresis loop in the upper soil reaction curve, which indicates that, as expected, the gapping effect is more important near the surface.

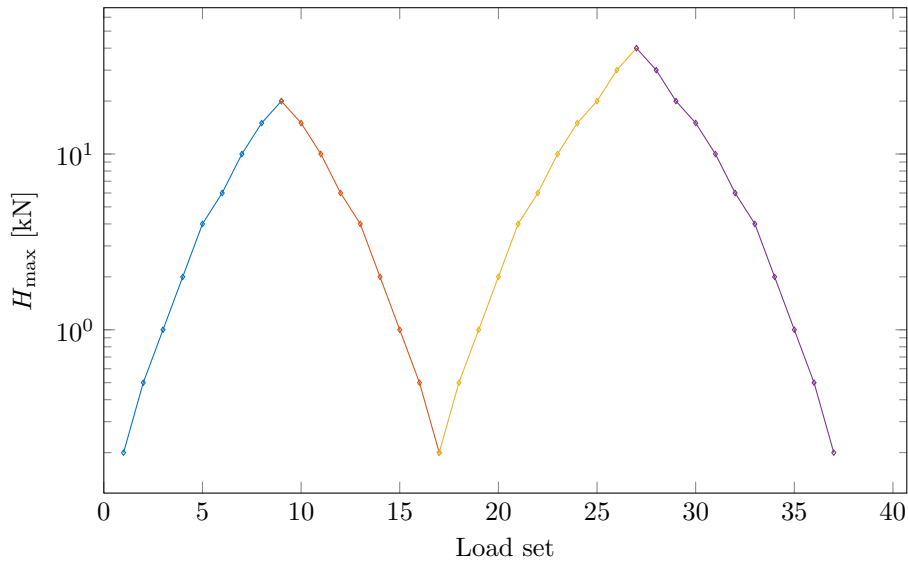


Figure 5.35: Simulated load amplitude for the cycles in CM7.

5.7.3 Gapping in Dunkirk

For the simulation of DM1, the same load sequence is applied as for CM7, illustrated in Figure 5.35. The resulting ground level load displacement is illustrated in Figure 5.39. This figure shows clearly the ‘bow tie’ shaped curves which are illustrative of

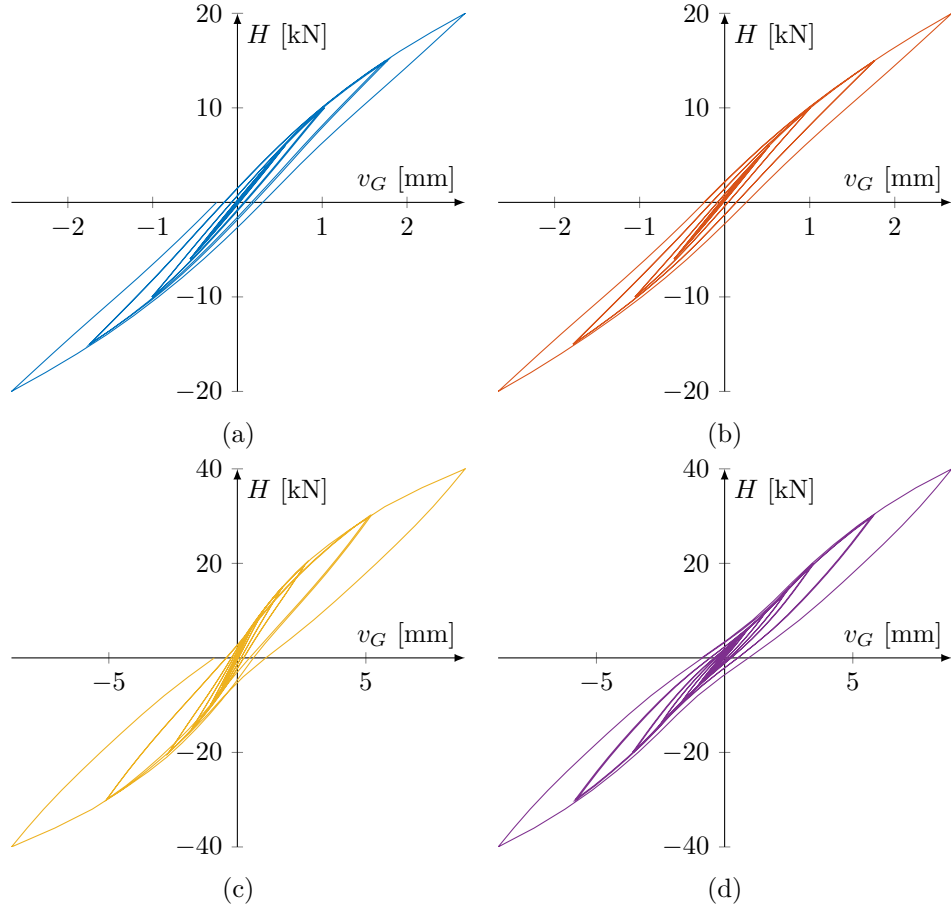


Figure 5.36: Simulated response of CM7 for load sets (a) 1 to 9, (b) 9 to 17, (c) 17 to 27 and (d) 27 to 37.

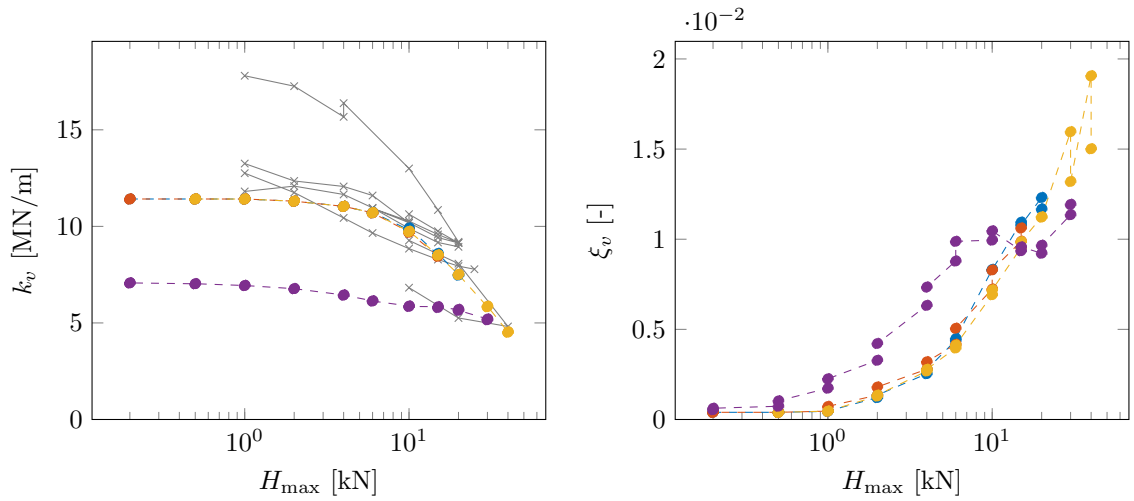


Figure 5.37: Simulated (a) stiffness and (b) damping ratio for CM7 for load sets (—•—) 1 to 9, (—•—) 9 to 17, (—•—) 17 to 27, (—•—) 27 to 37 and a comparison with the measured stiffness (—×—).

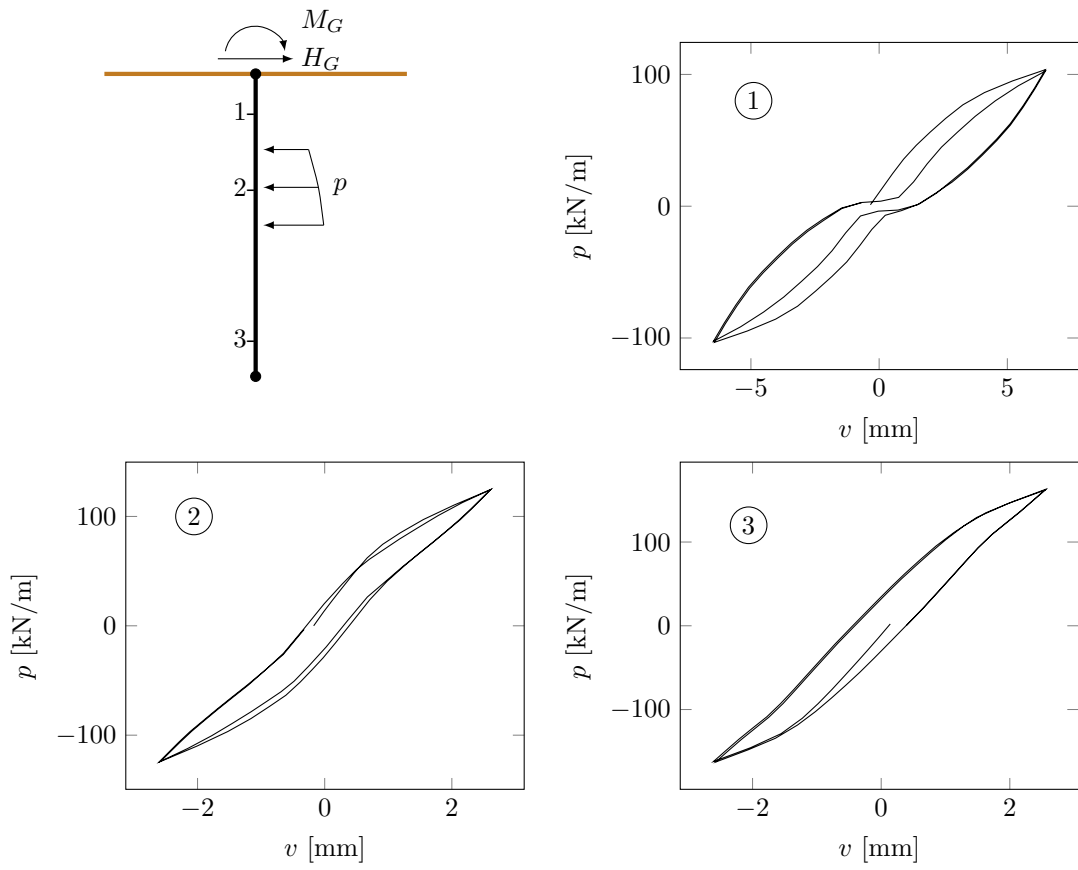


Figure 5.38: Simulated soil reactions at 0.5m, 1.5m and 3.5 below ground level of CM7 for load set 27 with the gapping model.

the gapping effect, especially after the cycles with load magnitude of 40kN have been applied. The variation of stiffness and damping throughout the load sets is illustrated in Figure 5.40. This figure shows a decrease in stiffness after higher amplitude load cycles have been applied, which compares qualitatively to the field test results of DM1 (Figure 3.33). In a similar way to CM7, the variation in damping ratio at low amplitude load sets after a higher amplitude has been applied, can be attributed to the hysteresis in the side friction. This effect plays a more dominant role once a gap has been formed at the front and back of the foundation pile. The magnitude of the damping ratio, however, is under predicted by a factor of approximately 5 compared to the test results in Figure 3.33.

The soil reactions at three locations along the pile length throughout load set 27 are illustrated in Figure 5.41. The ‘bow tie’ shaped hysteresis loop in the upper soil reaction curve is even more pronounced than in the simulation of CM7. This shows that the gapping effect is significant near the soil surface.

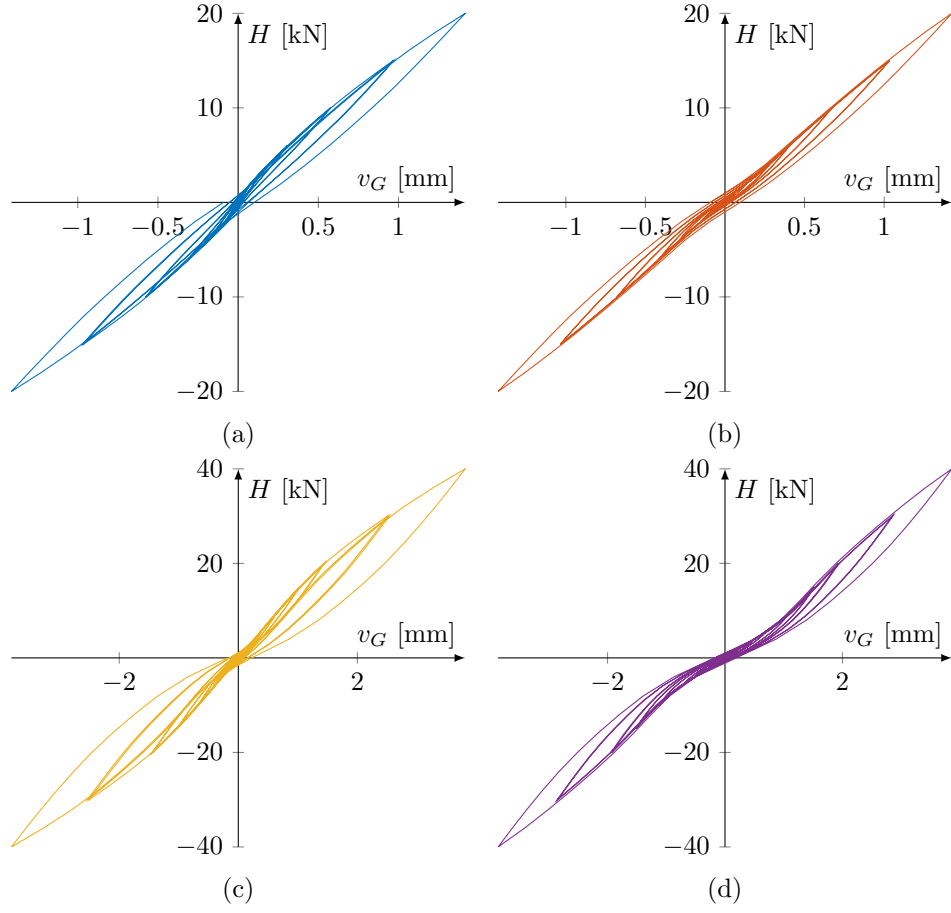


Figure 5.39: Simulated response of DM1 for load sets (a) 1 to 9, (b) 9 to 17, (c) 17 to 27 and (d) 27 to 37.

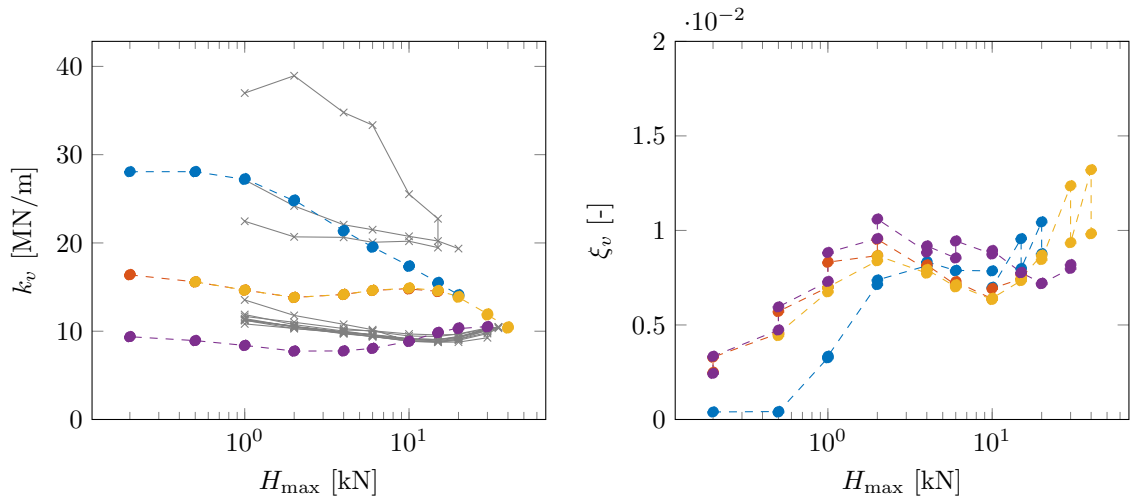


Figure 5.40: Simulated (a) stiffness and (b) damping ratio for DM1 for load sets ($\text{---}\bullet\text{---}$) 1 to 9, ($\text{---}\circ\text{---}$) 9 to 17, ($\text{---}\diamond\text{---}$) 17 to 27, ($\text{---}\times\text{---}$) 27 to 37 and a comparison with the measured stiffness ($\text{---}\times\text{---}$).

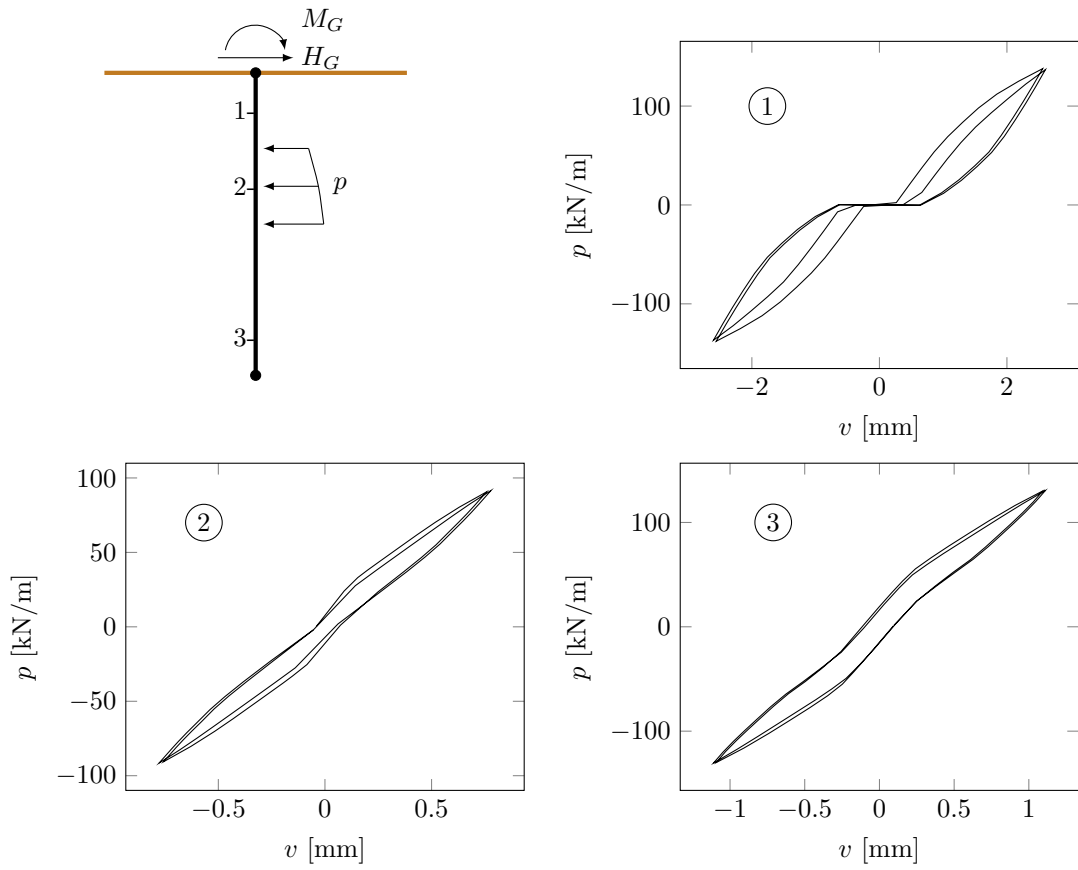


Figure 5.41: Simulated soil reactions at 0.5m, 1.5m and 3.5 below ground level of DM1 for load set 27 with the gapping model.

5.8 Conclusion

This chapter presents the calibration of the various models described in Chapter 4 to the field tests in the PISA project. The backbone curves are recalibrated to model accurately the quasi-static monotonic response. The kinematic hardening model is applied to the soil reaction curves, introducing Masing behaviour to the soil reactions. This model is used as a basis for the other models in this chapter.

The rate effect model is calibrated to both the PISA tests in Cowden and Dunkirk. In the Cowden tests, a rate parameter of $\eta_{10} = 0.12$ captures the increase in capacity at higher loading rates. For the Dunkirk tests, a rate parameter of $\eta_{10} = 0.06$ results in good agreement between the field test and the simulated response. The calibrated fit of the rate effect model depends largely on the backbone curves for the soil reactions, but with appropriate calibration, a single set of rate parameters ($\eta, \dot{\alpha}_{qs}$) appears to capture the rate effects for test piles with varying diameters.

The ratcheting model is applied to the 1-way cyclic loading tests in the PISA programme. The ratcheting model parameters ($R_0, m_\sigma, m_h, \alpha_{h0}$) have been calibrated to CM6 and DM2, resulting in a good fit for the accumulation of rotation for both tests. Using the same backbone curve and ratcheting parameters for CM5 as for CM6, results in an over prediction of the rotation accumulation by a factor of approximately 3. This over prediction could potentially be attributed to the stiffer backbone curve observed in test CM5, which would decrease the ratcheting effect. Applying the ratcheting parameters from CM6 to CL2 and similarly from DM2 to DL2 results in an under prediction of the rotation accumulation during the cyclic load sets after the monotonic test. It can be argued that this is caused by the choice of the hardening variable in the ratcheting model. Further work would be required to investigate whether a different choice of the hardening variable could result in a better match for the ratcheting response after a monotonic load test. The combined rate and ratcheting effect model is calibrated to tests CM6 and DM6, resulting in good agreement between

the field test response and the simulated results both in terms of the rate effect and the ratcheting effect.

Finally, the gapping model has been calibrated to capture the recalibrated lateral distributed soil-reaction curves for CM5 and DM1. Both test piles were subjected to 2-way cyclic loading, to evaluate the foundation stiffness and damping throughout the loading history. The simulated response with the gapping model results in similar qualitative results as observed in the field tests in terms of the stiffness decrease and damping increase at low amplitude cyclic loading after higher amplitude cyclic loading has been applied.

Chapter 6

Integration into commercial software

In simulations for offshore wind turbines, where the dynamics of the rotor and the interaction with wind and wave loading are simulated, the soil reaction along the foundation is usually simplified to linear spring and dashpot elements. This chapter presents a method to incorporate plasticity in the soil reaction models, such that the models in Chapter 4 can be integrated in the foundation response. Integrating more accurate foundation models in software for these simulations would help align the structural with the geotechnical design calculations. This could contribute towards further optimisation of the design.

The external loading from wind and wave and the dynamic interaction with the structure lead to lateral displacements of the monopile in both lateral directions (x and y). The soil reaction models therefore also need to be extended to 2D to provide an accurate representation of the plasticity in the soil reaction model. Figure 6.1 illustrates the simplified structural model for an offshore wind turbine with the coordinate system.

The application in this chapter focuses on the kinematic hardening model applied

to the $p - y$ curves in the current design guidance. Applying the kinematic hardening model to the additional soil reaction components (from the parameterised design method of the PISA project) would follow a straightforward extension. Integrating the other soil reaction models described in Chapter 4 could use the integration method for the plastic soil reaction described in this chapter. However, further work is required to develop fast and accurate stress update algorithms for the 2D extensions of these soil reaction models and to validate the resulting behaviour.

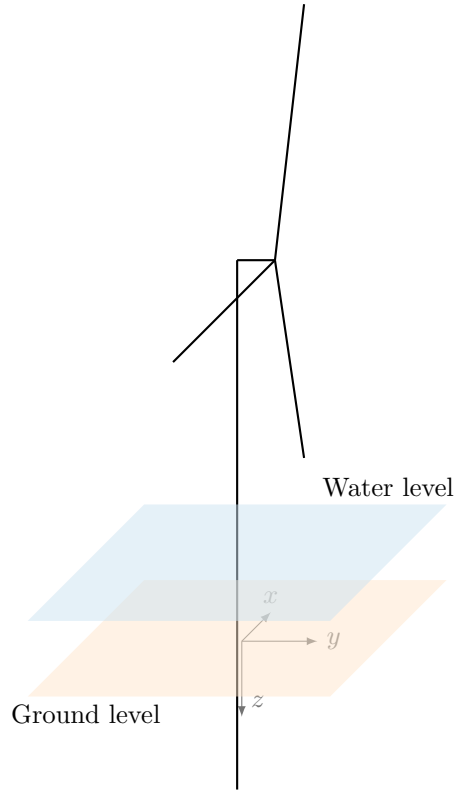


Figure 6.1: Illustration of the offshore wind turbine structural model.

There are several commercial programs available to simulate the dynamics of offshore wind turbines including ADAMS (MSC Software, 2016), HAWC2 (DTU Wind Energy, 2017), Bladed (DNV-GL, 2016), FLEX5 (Øye, 1996) and FAST (NREL, 2016). Jonkman et al. (2007) give a detailed comparison of these different programs for the application of an offshore wind turbine on a monopile foundation. For the application in this chapter, HAWC2 is used. This program allows for data transfer

between Matlab and HAWC2, which is used to apply the nonlinear soil reaction on the foundation pile.

6.1 2D soil reaction models

6.1.1 Series and parallel model comparison in 2D

In Chapter 4, the kinematic hardening model is introduced, which results in the Masing behaviour for 1D loading. The formulation uses a set of parallel spring-slider elements as illustrated in Figure 4.6. The parallel model, determined by Equations 4.39 and 4.40, can be extended for reactions in 2D by using a straightforward extension in hyperplasticity theory. The energy function and dissipation function in 2D are given by:

$$f = \sum_{i=1}^{n_s} \frac{H_i}{2} [(\epsilon_x - \alpha_{ix})^2 + (\epsilon_y - \alpha_{iy})^2] \quad (6.1)$$

$$d = \sum_{i=1}^{n_s} k_i \sqrt{\dot{\alpha}_{ix}^2 + \dot{\alpha}_{iy}^2} \quad (6.2)$$

Masing behaviour can also be modelled in 1D using a set of spring-slider elements in series (Puzrin and Houlsby, 2001). A schematic representation of the model in 1D is given in Figure 6.2. The extension of the energy and dissipation function for the series model in 2D is given by:

$$f = \frac{H_0}{2} \left[\left(\epsilon_x - \sum_{i=1}^{n_s} \alpha_{ix} \right)^2 + \left(\epsilon_y - \sum_{i=1}^{n_s} \alpha_{iy} \right)^2 \right] + \sum_{i=1}^{n_s-1} \frac{H_i}{2} (\alpha_{ix}^2 + \alpha_{iy}^2) \quad (6.3)$$

$$d = \sum_{i=1}^{n_s} k_i \sqrt{\dot{\alpha}_{ix}^2 + \dot{\alpha}_{iy}^2} \quad (6.4)$$

The series and parallel versions of the kinematic hardening model in 2D are compared for a simple example. Figure 6.3 gives the response for both models where

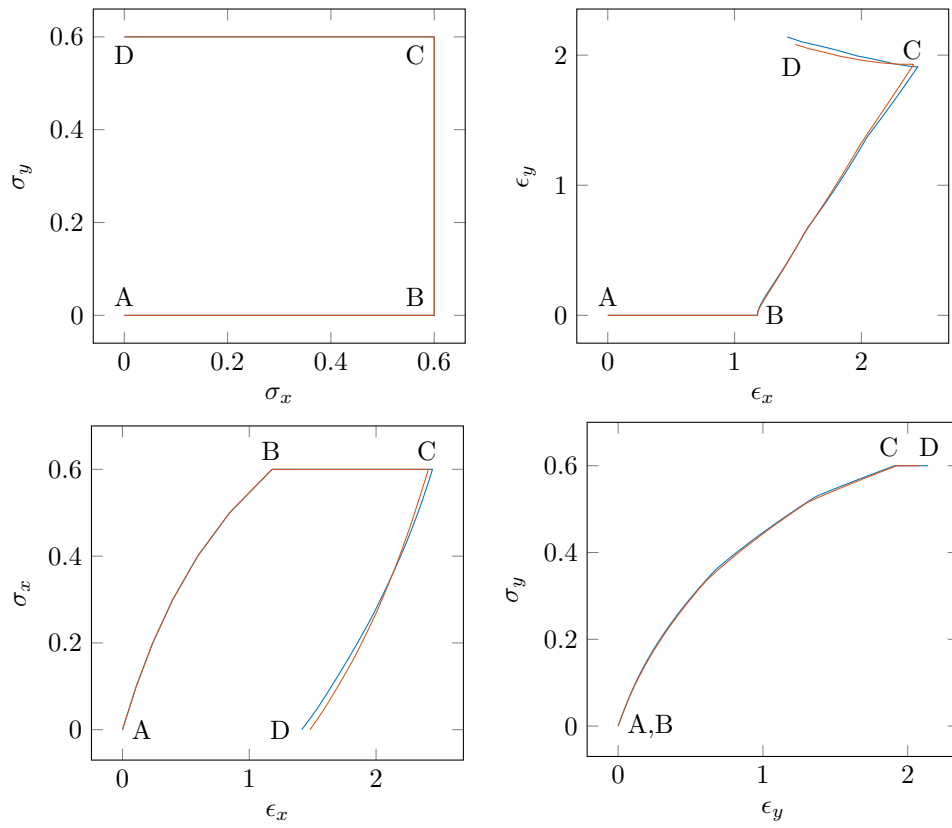


Figure 6.3: Response of the series (—) and parallel (—) model for an applied stress path of $\{\sigma_x, \sigma_y\} = (A)\{0, 0\} \rightarrow (B)\{0.6, 0\} \rightarrow (C)\{0.6, 0.6\} \rightarrow (D)\{0, 0.6\}$.

6.1.2 Analytical integration of a 2D yield surface

The numerical implementation for the plastic behaviour of the kinematic hardening model uses the analytical solution by Krieg and Krieg (1977). Their solution defines the plastic behaviour for a single yield surface in 2D assuming a straight strain path. Since each of the yield surfaces in the kinematic hardening model is in parallel, the response of the model is the sum of each of the individual yield surfaces. In vectorial notation, this is written as: $\boldsymbol{\sigma} = \sum_{i=1}^{n_s} H_i(\boldsymbol{\epsilon} - \boldsymbol{\alpha}_i)$, where $\boldsymbol{\sigma} = \{\sigma_x, \sigma_y\}$, $\boldsymbol{\epsilon} = \{\epsilon_x, \epsilon_y\}$ and $\boldsymbol{\alpha}_i = \{\alpha_{ix}, \alpha_{iy}\}$.

The generalised stress of an individual element is written as $\boldsymbol{\chi}_i = H_i(\boldsymbol{\epsilon} - \boldsymbol{\alpha}_i)$, with $\boldsymbol{\chi}_i = \{\chi_{ix}, \chi_{iy}\}$, where the individual yield surface is defined as: $\|\boldsymbol{\chi}_i\| = k_i$. Within the yield surface, the response is purely elastic. As the generalised stress reaches the yield surface ($\boldsymbol{\chi}_i^y$), the rate of plastic strain is orthogonal to the yield surface: $\dot{\boldsymbol{\alpha}}_i = 2\Lambda_i\boldsymbol{\chi}_i$, where Λ_i is a scalar multiplier. Combining the equations for the generalised stress and the plastic strain becomes: $\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\chi}}_i/H_i + 2\Lambda_i\boldsymbol{\chi}_i$. The analytical solution to this equation is given by:

$$\eta_i\boldsymbol{\chi}_i = H_i\xi_i\dot{\boldsymbol{\epsilon}} + \boldsymbol{\chi}_i^y \quad (6.5)$$

where $\dot{\boldsymbol{\epsilon}}$ is the proportion of the strain step after yield surface intersection. The variables ξ_i and η_i are defined as:

$$\xi_i = \frac{k_i}{\beta_i} \left(\sinh \left(\frac{\beta_i t}{k_i} \right) + \zeta_i \cosh \left(\frac{\beta_i t}{k_i} \right) - \zeta_i \right) \quad (6.6)$$

$$\eta_i = \dot{\xi}_i = \cosh \left(\frac{\beta_i t}{k_i} \right) + \zeta_i \sinh \left(\frac{\beta_i t}{k_i} \right) \quad (6.7)$$

and the following constants are defined:

$$\beta_i = H_i \|\dot{\boldsymbol{\epsilon}}\| \quad (6.8)$$

$$\zeta_i = \frac{H_i}{k_i \beta_i} \chi_i^y \cdot \dot{\epsilon} \quad (6.9)$$

The solution in Equation 6.5 can be proven by differentiating the equation with respect to time:

$$\dot{\eta}_i \chi_i + \eta_i \dot{\chi}_i = H_i \dot{\xi}_i \dot{\epsilon} = H_i \eta_i \dot{\epsilon} = H_i \eta_i (\dot{\epsilon} - \dot{\alpha}_i) + H_i \eta_i \dot{\alpha}_i = \eta_i \dot{\chi}_i + H_i \eta_i \dot{\alpha}_i \quad (6.10)$$

so that $\dot{\eta}_i \chi_i = H_i \eta_i \dot{\alpha}_i$. Therefore, with $2\Lambda_i = \dot{\eta}_i / (H_i \eta_i)$, the flow rule is satisfied. Squaring Equation 6.5 gives:

$$\begin{aligned} \eta_i^2 \|\chi_i\|^2 &= \|H_i \xi_i \dot{\epsilon} + \chi_i^y\|^2 = H_i^2 \xi_i^2 \|\dot{\epsilon}\|^2 + 2H_i \xi_i \dot{\epsilon} \cdot \chi_i^y + \|\chi_i^y\|^2 \\ &= \beta_u^2 \xi_i^2 + 2k_i \beta_i \zeta_i \xi_i + k_i^2 = k_i^2 \left(\left(\frac{\beta_i \xi_i}{k_i} \right)^2 + 2\zeta_i \frac{\beta_i \xi_i}{k_i} + 1 \right) \end{aligned} \quad (6.11)$$

and expanding η_i^2 gives:

$$\eta_i^2 = \cosh^2 \left(\frac{\beta_i t}{k_i} \right) + 2\zeta_i \cosh \left(\frac{\beta_i t}{k_i} \right) \sinh \left(\frac{\beta_i t}{k_i} \right) + \zeta_i^2 \sinh^2 \left(\frac{\beta_i t}{k_i} \right) \quad (6.12)$$

Expanding the term in brackets on the right hand side of Equation 6.11 gives:

$$\begin{aligned} \left(\left(\frac{\beta_i \xi_i}{k_i} \right)^2 + 2\zeta_i \frac{\beta_i \xi_i}{k_i} + 1 \right) &= (S_i + \zeta_i C_i - \zeta_i)^2 + 2\zeta_i (S_i + \zeta_i C_i - \zeta_i) + 1 \\ &= S_i^2 + \zeta_i^2 C_i^2 + \zeta_i^2 + 2\zeta_i C_i S_i - 2\zeta_i^2 C_i - 2\zeta_i S_i + 2\zeta_i S_i + 2\zeta_i^2 C_i - 2\zeta_i^2 + 1 \\ &= (1 + S_i^2) + 2\zeta_i C_i S_i + \zeta_i^2 S_i^2 = \eta_i^2 \end{aligned} \quad (6.13)$$

where $\sinh \left(\frac{\beta_i t}{k_i} \right)$ and $\cosh \left(\frac{\beta_i t}{k_i} \right)$ are abbreviated to S_i and C_i respectively. Combining Equation 6.11 and 6.13 results in $\|\chi_i\|^2 = k_i^2$, which proves that the yield condition is satisfied throughout the plastic strain path. The implementation of the 2D kinematic hardening model follows the following steps:

Input: $\epsilon_1, \epsilon_0, \alpha_{0i}, k_i, H_i, i = 1 \dots n_s$

for $i = 1 \dots n_s$ **do**

$\chi_i^{\text{trial}} \leftarrow H_i(\epsilon_1 - \alpha_{0i})$

if $\|\chi_i^{\text{trial}}\| > k_{di}$ **then**

Find intersection with yield surface χ^y

Solve: $\eta_i \chi_i = H_i \xi_i \epsilon + \chi_i^y$ for χ_i

$\alpha_{1i} \leftarrow \epsilon - \chi_i / H_i$

else

$\alpha_{1i} \leftarrow \alpha_{0i}$

end if

end for

$\sigma_1 \leftarrow \sum_{i=1}^{n_s} H_i(\epsilon_1 - \alpha_{1i})$

Output: $\sigma_1, \alpha_{1i}, i = 1 \dots n_s$

6.1.3 Extension to 2D for other models

Section 6.1.1 explains how the kinematic hardening model can be expanded for 2D loading, so that the soil reaction components in both directions along the pile can be calculated rigorously. This section describes the extensions of the potential functions to 2D for the other models described in Chapter 4. Further work is needed to develop fast and accurate numerical algorithms for the stress update in the calculation steps in order to integrate these models into HAWC2.

Rate effect model

Extending the rate effect model to 2D results in:

$$f = \sum_{i=1}^{n_s} \frac{H_i}{2} \|\epsilon - \alpha_i\|^2 \quad (6.14)$$

$$z = \sum_{i=1}^{n_s} k_i \left[\|\dot{\boldsymbol{\alpha}}_i\| + \eta \left(\|\dot{\boldsymbol{\alpha}}_i\| \sinh^{-1} \left(\frac{\|\dot{\boldsymbol{\alpha}}_i\|}{\dot{\alpha}_{qs}} \right) - \sqrt{\|\dot{\boldsymbol{\alpha}}_i\|^2 + \dot{\alpha}_{qs}^2} + \dot{\alpha}_{qs} \right) \right] \quad (6.15)$$

so that the generalised stress becomes:

$$\boldsymbol{\chi}_{\alpha_i} = \bar{\boldsymbol{\chi}}_{\alpha_i} \rightarrow H_i(\boldsymbol{\epsilon} - \boldsymbol{\alpha}_i) = k_i \frac{\dot{\boldsymbol{\alpha}}_i}{\|\dot{\boldsymbol{\alpha}}_i\|} \left(1 + \eta \sinh^{-1} \left(\frac{\|\dot{\boldsymbol{\alpha}}_i\|}{\dot{\alpha}_{qs}} \right) \right) \quad (6.16)$$

which is consistent with the 1D approach and this result shows that the nonlinearity in the viscous term depends on the magnitude of the plastic strain rate.

Ratcheting model

For the ratcheting model, the extension to 2 directions becomes:

$$f = \sum_{i=1}^{n_s} \frac{H_i}{2} \|\boldsymbol{\epsilon} - \boldsymbol{\alpha}_i - \boldsymbol{\alpha}_r\|^2 \quad (6.17)$$

$$d = \sum_{i=1}^{n_s} k_i \|\dot{\boldsymbol{\alpha}}_i\| + \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\alpha}}_r \quad (6.18)$$

resulting in the following generalised stresses:

$$\boldsymbol{\chi}_{\alpha_i} = \bar{\boldsymbol{\chi}}_{\alpha_i} \rightarrow H_i(\boldsymbol{\epsilon} - \boldsymbol{\alpha}_i - \boldsymbol{\alpha}_r) = k_i \frac{\dot{\boldsymbol{\alpha}}_i}{\|\dot{\boldsymbol{\alpha}}_i\|} \quad (6.19)$$

$$\boldsymbol{\chi}_{\alpha_r} = \bar{\boldsymbol{\chi}}_{\alpha_r} \rightarrow \sum_{i=1}^{n_s} H_i(\boldsymbol{\epsilon} - \boldsymbol{\alpha}_i - \boldsymbol{\alpha}_r) = \boldsymbol{\sigma} \quad (6.20)$$

which is again consistent with the 1D approach. The constraint for the ratcheting is written as:

$$c = \dot{\boldsymbol{\alpha}}_r - \frac{\boldsymbol{\sigma}}{\|\boldsymbol{\sigma}\|} \sum_{i=1}^{n_s} R_i \|\dot{\boldsymbol{\alpha}}_i\| = 0 \quad (6.21)$$

Using this constraint function, the direction of the ratcheting strain rate is the same as the stress, such that the ratcheting work term in the dissipation potential could also be written as $\|\boldsymbol{\sigma}\| \|\dot{\boldsymbol{\alpha}}_r\|$ instead of $\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\alpha}}_r$. The extension of the hardening variable

to 2D becomes:

$$\dot{\alpha}_h = \|\dot{\alpha}_r\| \quad (6.22)$$

Other options for the hardening variable could be extended to 2D as: $\dot{\alpha}_h = \sum_{i=1}^{n_s} \|\dot{\alpha}_i\|$, $\dot{\beta}_h = \sum_{i=1}^{n_s} k_i \|\dot{\alpha}_i\|$, $\dot{\beta}_h = \boldsymbol{\sigma} \cdot \dot{\alpha}_r$ or $\dot{\beta}_h = \sum_{i=1}^{n_s} k_i \|\dot{\alpha}_i\| + \boldsymbol{\sigma} \cdot \dot{\alpha}_r$.

Combined rate and ratcheting model

Combining the energy and force potential of the rate and ratcheting model gives:

$$f = \sum_{i=1}^{n_s} \frac{H_i}{2} \|\boldsymbol{\epsilon} - \boldsymbol{\alpha}_i - \boldsymbol{\alpha}_r\|^2 \quad (6.23)$$

$$z = \sum_{i=1}^{n_s} k_i \left[\|\dot{\alpha}_i\| + \eta \left(\|\dot{\alpha}_i\| \sinh^{-1} \left(\frac{\|\dot{\alpha}_i\|}{\dot{\alpha}_{qs}} \right) - \sqrt{\|\dot{\alpha}_i\|^2 + \dot{\alpha}_{qs}^2} + \dot{\alpha}_{qs}^2 \right) \right] + \boldsymbol{\sigma} \cdot \dot{\alpha}_r \quad (6.24)$$

The same constraint functions from the ratcheting model for the ratcheting and hardening effects can be applied to this set of equations.

Gapping model

In the gapping model, the soil reaction is described along the boundary of a section of the foundation pile. In the case with only one lateral displacement, the soil reaction is symmetric along the axis of loading and therefore only half of the section needs to be discretised (Figure 4.23). When loading in both lateral directions is applied, this simplification cannot be made any more and the entire boundary of a pile section needs to be discretised. The soil reaction is evaluated by integrating the stress along the boundary:

$$\mathbf{p} = \int_0^{2\pi} \boldsymbol{\sigma}(\phi) R d\phi \approx \sum_{j=1}^{n_b} \boldsymbol{\sigma}_j \frac{R\pi}{n_b} \quad (6.25)$$

where $\boldsymbol{\sigma}_j$ is the stress at the j^{th} section of the pile boundary and n_b is the total number of sections.

6.2 Integration in HAWC2

This section outlines how the plastic soil reaction can be integrated into an aero-elastic code for the simulation of offshore wind turbines. The model could be integrated directly into the source code of the software or through the interface with an external program which can be used to compute the soil reactions. In this chapter, the latter approach is used since the source code of most software packages cannot be modified directly. The soil reaction model is integrated in HAWC2. However, integration into other commercial software could follow a similar approach.

6.2.1 Finite element interpolation and integration

During each iteration step in the HAWC2 simulation, the FE model computes the nodal displacements and rotations. For each element, this can be written as:

$$\mathbf{u}^e = \begin{bmatrix} u^{(1)} & v^{(1)} & \psi_x^{(1)} & \psi_y^{(1)} & u^{(2)} & v^{(2)} & \psi_x^{(2)} & \psi_y^{(2)} \end{bmatrix}^T \quad (6.26)$$

where superscript (1) and (2) indicate the nodal displacement or rotation. Based on the nodal displacements and rotations, the lateral displacements of the elements along the foundation can be calculated using shape functions:

$$\mathbf{u}(\eta) = \mathbf{N}(\eta)\mathbf{u}^e \quad (6.27)$$

where η is a local coordinate of the element ranging from 0 at the first node to 1 at the second. The shape functions are based on the superconvergent shape functions for Timoshenko beam theory by Reddy (2002). The matrix of shape functions is defined

as:

$$\mathbf{N} = \begin{bmatrix} N_1 & 0 & 0 & N_2 & N_3 & 0 & 0 & N_4 \\ 0 & N_1 & N_2 & 0 & 0 & N_3 & N_4 & 0 \end{bmatrix} \quad (6.28)$$

$$N_1 = \frac{1}{1 + \phi} (1 + \phi - \phi\eta - 3\eta^2 + 2\eta^3) \quad (6.29)$$

$$N_2 = \frac{L_e}{1 + \phi} \left(\frac{2 + \phi}{2} \eta - \frac{\phi + 4}{2} \eta^2 + \eta^3 \right) \quad (6.30)$$

$$N_3 = \frac{1}{1 + \phi} (\phi\eta + 3\eta^2 - 2\eta^3) \quad (6.31)$$

$$N_4 = \frac{L_e}{1 + \phi} \left(-\frac{\phi}{2} \eta + \frac{\phi - 2}{2} \eta^2 + \eta^3 \right) \quad (6.32)$$

where $\phi = \frac{12EI}{\kappa AGL_e^2}$ is a dimensionless factor based on the bending stiffness EI , shear stiffness κAG and the length of the element L_e . When this factor equals zero, the shape functions are the conventional set of Hermite cubic interpolation functions for Bernoulli beams, where no shear deformation is taken into account. The nodal soil reaction for each foundation element:

$$\mathbf{f}_s^e = \begin{bmatrix} H_x^{(1)} & H_y^{(1)} & M_x^{(1)} & M_y^{(1)} & H_x^{(2)} & H_y^{(2)} & M_x^{(2)} & M_y^{(2)} \end{bmatrix}^T \quad (6.33)$$

is calculated by integrating the distributed soil reaction over each element:

$$\mathbf{f}_s^e = \int_0^1 \mathbf{N}^T \mathbf{p} \frac{dz}{d\eta} d\eta \quad (6.34)$$

where $\mathbf{p} = \begin{bmatrix} p_x & p_y \end{bmatrix}^T$ is the soil reaction along the element. Using numerical integration, this integral can be written as a summation over the Gauss points:

$$\mathbf{f}_s^e = \sum_{i=1}^{n_g} w_i \mathbf{N}_i^T \mathbf{p}_i J \quad (6.35)$$

where subscript i refers to the Gauss point. The location and weights of the Gauss points are evaluated using fourth order Gauss-Legendre quadrature (Table 4.2). Similarly, the element soil stiffness matrix is evaluated by:

$$\mathbf{K}_s^e = \int_0^1 \mathbf{N}^T \left(\frac{d\mathbf{p}}{d\mathbf{u}} \right) \mathbf{N} \frac{dz}{d\eta} d\eta \quad (6.36)$$

which can be evaluated using numerical integration by:

$$\mathbf{K}_s^e = \sum_{i=1}^{n_g} w_i \mathbf{N}_i^T \left(\frac{d\mathbf{p}_i}{d\mathbf{u}} \right) \mathbf{N}_i J \quad (6.37)$$

The global soil reaction vector $\mathbf{f}_s$ and the soil stiffness matrix $\mathbf{K}_s$ are assembled from the element vectors as given in Equation 4.16.

6.2.2 Interface between HAWC2 and Matlab

In HAWC2, the dynamic response of the wind turbine is calculated using the Newmark solution scheme. In order to converge efficiently to a solution within each time-step, the soil stiffness should be added to the stiffness matrix of the structure. The approach described in Section 6.2.1 for calculating the soil stiffness matrix $\mathbf{K}_s$ results in off-diagonal terms, which cannot be added externally to HAWC2. Therefore, a reduced soil stiffness matrix is added using lateral support elements at the nodes of the foundation (Figure 6.4). The reduced stiffness matrix can be written as $\mathbf{K}_s^*$.

For the simulations in Section 6.3, the stiffness of the lateral support elements is evaluated by multiplying the length of an estimated influence zone by the initial stiffness of the soil reaction at the average depth of the influence zone. This influence zone is defined as the combination of the zones between the node and the average depth of the elements surrounding the node which are below ground.

The corresponding non-linear soil reaction $\Delta \mathbf{f}_s^* = \mathbf{f}_s - \mathbf{K}_s^* \mathbf{u}$, can be applied as an external force to the structure within each iteration step of the calculation. The use

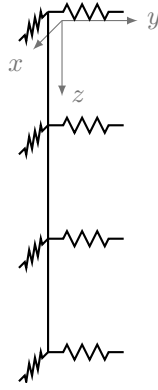


Figure 6.4: Illustration of the stiffness elements for the reduced stiffness matrix.

of the reduced stiffness matrix does not, however, affect the final converged solution in the Newmark scheme for the dynamics of the structure. However, it does influence the number of iterations needed to achieve a converged solution.

The dynamic calculations of the wind turbine using HAWC2 are controlled by a bespoke Matlab script. This script initiates the HAWC2 calculations, transfers data to and from HAWC2 using a TCP/IP protocol and calculates the soil reaction at each iteration step for the dynamic calculation of the wind turbine as described in Figure 6.5. This separation of the programs allows for the storage of the plastic strains and all parameters for the multiple yield surface model within Matlab. Therefore, the data transfer between HAWC2 and Matlab for each iteration step is minimal. In each iteration step of the Newmark solution scheme, HAWC2 outputs the displacement and rotation at the nodes of the foundation. The displacements at the Gauss points are calculated by evaluating $\mathbf{u}_i(\eta) = \mathbf{N}(\eta_i)\mathbf{u}^e$. The soil reactions at the Gauss points are then updated using the analytical solution for the kinematic hardening model. Integrating these soil reactions over the elements gives the non-linear nodal forces which are then used for the next iteration step in HAWC2.

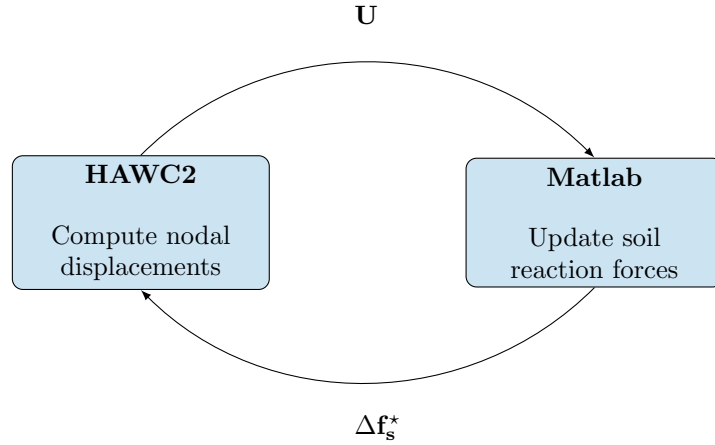


Figure 6.5: Flow chart describing the interaction between HAWC2 and the bespoke Matlab script.

6.3 Offshore wind turbine simulation

6.3.1 Backbone curve of the soil reaction

The soil reaction for this example simulation is based on the current offshore design method for laterally loaded piles in sand. In the current design method, only the lateral soil reaction component is considered (API, 2010; DNV, 2014). The soil reaction is defined as:

$$p_y = Ap_u \tanh\left(\frac{kz}{Ap_u}v\right) \quad (6.38)$$

where $A = 0.9$ for cyclic loading and $A = 3.0 - 0.8z/D$ for static loading. In the simulations in this section, cyclic loading is applied and therefore $A = 0.9$ has been used. The initial modulus of soil reaction, k , depends on the friction angle, and a distinction is made between saturated and unsaturated sand. The ultimate soil resistance p_u is calculated by:

$$p_u = \min \begin{cases} (C_1z + C_2D) \gamma' z \\ C_3D \gamma' z \end{cases} \quad (6.39)$$

where the coefficients C_1 to C_3 depend on the friction angle and γ' is the effective soil unit weight. Equation 6.38 can be generalised to 2D loading as:

$$\mathbf{p} = Ap_u \frac{\mathbf{u}}{\|\mathbf{u}\|} \tanh\left(\frac{kz}{Ap_u} \|\mathbf{u}\|\right) \quad (6.40)$$

where $\mathbf{p} = \{p_x, p_y\}$ is the reaction in both lateral directions. The equation specifies that the reaction is always in the same direction as the applied displacement.

6.3.2 Soil reaction under cyclic loading

Though the implementation of the kinematic hardening model follows the method as described in Section 6.1.2, the cyclic behaviour of the soil reaction is presented analytically in this section. The analysis illustrates the effect of cycle amplitude on the stiffness and damping of the soil reaction. Applying the Masing rules to the backbone curve (Equation 6.38), the steady state cyclic response in the y -direction is specified by:

$$\frac{p_y - p_r}{2} = -Ap_u \tanh\left(\frac{kz}{Ap_u} \frac{v - v_r}{2}\right) \quad (6.41)$$

This curve is geometrically similar to the initial loading curve, but reversed, factored by 2 and translated to the reversal pressure p_r and displacement u_r . These steady state hysteresis loops are illustrated in Figure 6.6 for varying displacement amplitudes. Using this analytical solution, the normalised secant stiffness $\bar{k}_p$ and damping ratio ξ_p can be described as a function of the normalised displacement amplitude $\bar{v} = v \frac{kz}{Ap_u}$:

$$\bar{k}_p = \frac{k_p}{kz} = \frac{\tanh(\bar{v})}{\bar{v}} \quad (6.42)$$

$$\xi_p = \frac{1}{4\pi} \frac{E_{\text{diss}}}{E_{\text{el}}} = \frac{4}{\pi} \frac{(\log(\cosh(\bar{v})) - \tanh(\bar{v})\bar{v}/2)}{\tanh(\bar{v})\bar{v}} \quad (6.43)$$

where E_{el} is the maximum stored energy and E_{diss} is the dissipated energy as illustrated in Figure 3.12 (a). The equations for the normalised stiffness and damping

ratio are illustrated in Figure 6.7. As the displacement amplitude increases, the secant stiffness decreases and the area within the hysteresis loop increases, resulting in larger damping values.

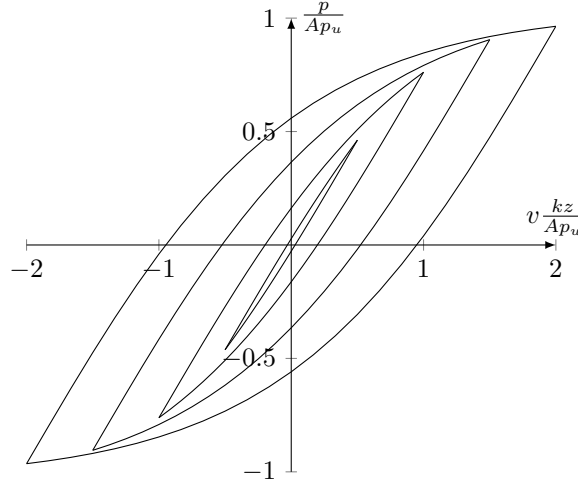


Figure 6.6: Steady state hysteresis loops based on the backbone curve in Equation 6.38 at displacement amplitudes of $\bar{v} = 0.5, 1, 1.5$ and 2.

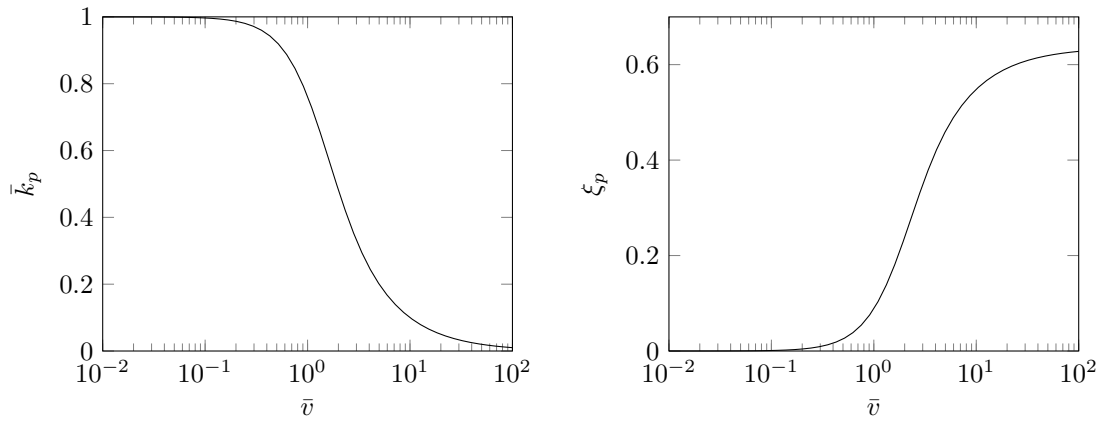


Figure 6.7: Normalised stiffness and damping ratio in function of the normalised displacement amplitude based on the backbone curve in Equation APIReaction.

6.3.3 Rotor stop simulation

To illustrate the effect of the kinematic hardening model on the damping of the structure, three rotor stops are simulated at constant wind speeds V_w of 8m/s, 10m/s

and 12m/s along the structure. To simplify the analysis of the structural response, no wave loading is simulated. The response using the kinematic hardening model is compared with the response using Equation 6.40, where the soil reaction is non-linear elastic. The structural definition is based on the fictitious 5MW NREL reference wind turbine with the extended monopile support structure (Jonkman et al., 2007). The monopile has an embedded length of 36m and a diameter of 4.2m. In this existing model, Rayleigh damping is used to simulate the structural damping. The interaction of the structure with the wind leads to an additional aerodynamic damping which is inherent to the aeroelastic calculations. The soil is modelled as a sand with a friction angle of 36° and an effective unit weight of 9kN/m^3 . When the kinematic hardening model is applied, this results in an additional damping on the structure due to soil hysteresis.

The wind loading is applied at the start of the simulation. After a period of 25s, sufficient for transient effects to have ended, the rotor stop is simulated by turning the pitch angle of the blades from 0° to 90° over a period of 10s. By feathering the blades, the wind load on the structure drops significantly. This results in a vibration of the structure, which is dominated by the first resonance mode in the fore-aft direction, illustrated in Figure 6.8.

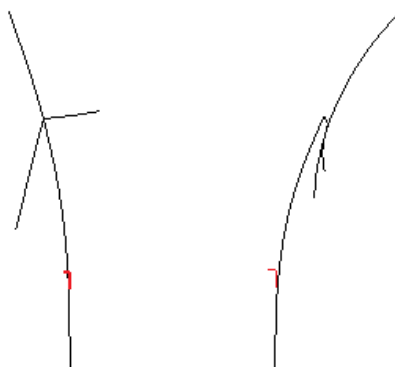


Figure 6.8: First mode of the structure in the side-side (left) and the fore-aft (right) direction.

Figure 6.9 illustrates the pitch angle of the blades, the rotor thrust, and the displacement of the nacelle for the three load cases using the kinematic hardening model and the non-linear elastic response from the backbone curve in the current design method (Equation 6.40). The damping on the first resonance mode is estimated using the logarithmic decrement, which is the natural log of the ratio of amplitudes of any two successive peaks:

$$\delta = \frac{1}{n} \log \left(\frac{v(t)}{v(t+nT)} \right) \quad (6.44)$$

where $v(t)$ is the amplitude of the nacelle displacement in the fore-aft direction at time t and $v(t+nT)$ is the amplitude of the peak n periods away. It should be noted that the damping with the kinematic hardening model is amplitude-dependent and decreases as the vibration amplitude decreases. The estimated logarithmic decrement on the first resonance mode is given in Table 6.1 using the first peak of the nacelle displacement after the rotor stop and the peak after 3 periods.

Using the current design method (based on a non-linear elastic soil response), the damping on the structure is similar for the three load cases. The kinematic hardening model adds damping to the structure which is amplitude-dependent. As the cycle amplitude increases, the area in the hysteresis loops of the soil reaction increases. This results in an additional soil damping of 0.71% for $V_w=8\text{m/s}$, 0.88% for $V_w=10\text{m/s}$ and 2.29% for $V_w=12\text{m/s}$. The kinematic hardening model can therefore be used to capture the amplitude-dependent damping of the foundation reaction.

Table 6.1: Logarithmic decrement for the three load cases using the kinematic hardening model (KHM) and the non-linear elastic response from the backbone curve in the current design method (CDM).

V_w [m/s]	8	10	12
CDM δ [$\cdot 10^{-2}$]	2.45	2.31	2.66
KHM δ [$\cdot 10^{-2}$]	3.16	3.19	4.95

The overall foundation response can be captured by plotting the moment against rotation at the mudline (Figure 6.10). This figure shows an increased hysteresis loop using the kinematic hardening model at higher load levels. The foundation reaction using the current design method shows a slight S-shape from the $p - y$ curves. The Rayleigh damping in the simulations results in additional damping in the foundation and therefore the results with the current design method do not exactly trace back the original curve. In these examples, the applied overturning moment is far from the ultimate capacity of the foundation ($\sim 2,250\text{MNm}$), resulting in relatively small hysteresis loops. As the load level increases, the area in the hysteresis loop with the kinematic hardening model grows, resulting in more energy dissipation in the foundation. This effect can dominate the overall damping under high loading conditions. However, under low loading conditions, the large strain material damping only makes a minor contribution to the overall damping on the structure.

6.4 Discussion

This chapter presents a method to integrate plasticity in the soil reaction for simulations for offshore wind turbines. In structural analysis, the reaction is often approximated using linear spring and dashpot elements. The equivalent dashpot value c_p can be calculated based on the damping ratio and secant stiffness of the soil reaction by:

$$\xi_p = \frac{\omega}{4\pi} \frac{c_p}{k_p} \quad (6.45)$$

where ω is the angular frequency. The secant stiffness of the soil reaction is used as the stiffness of the linear spring. With a specific value of the dashpot, the damping value for higher and lower frequencies will vary from the damping which the dashpot was evaluated for. Therefore a careful selection of the representative loading frequency also needs to be assessed. This technique is used by Skau et al. (2015) for

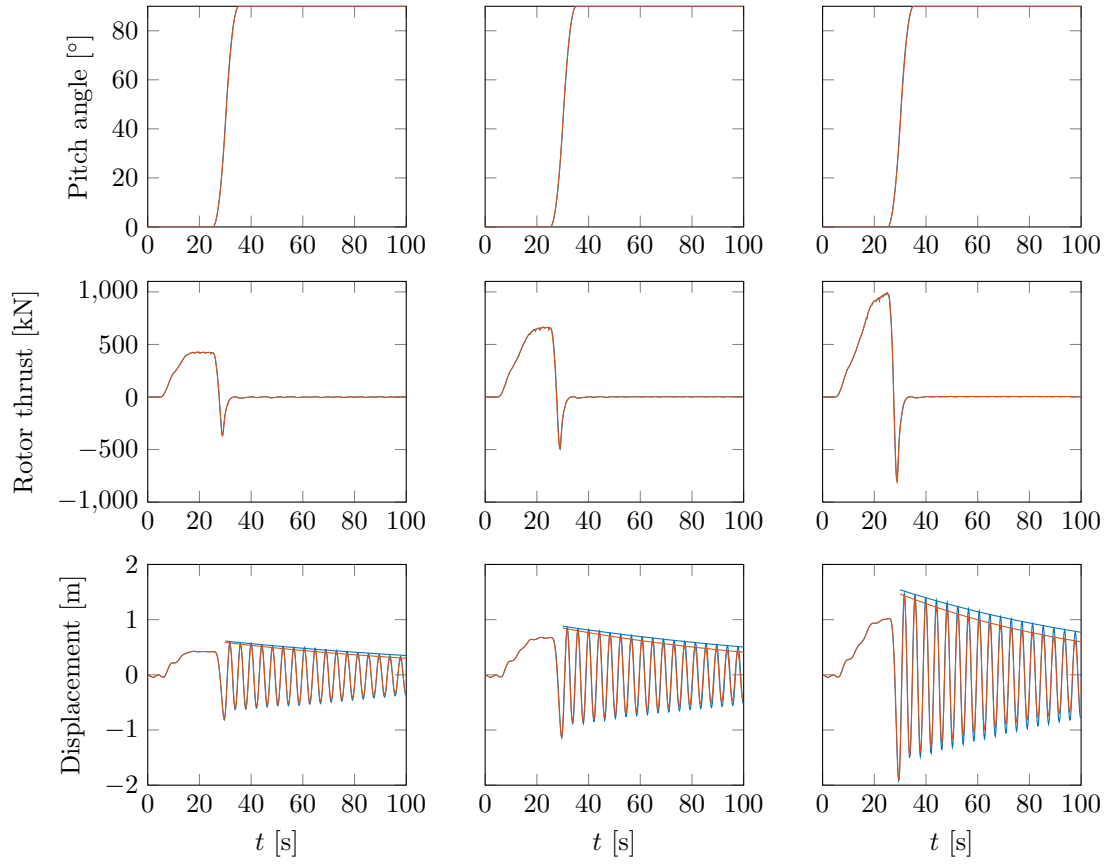


Figure 6.9: Pitch angle of the blades, the rotor thrust and the displacement of the nacelle for $V_w=8\text{m/s}$ (column 1), $V_w=10\text{m/s}$ (column 2) and $V_w=12\text{m/s}$ (column 3) using the kinematic hardening model (—) and the current design method (—).

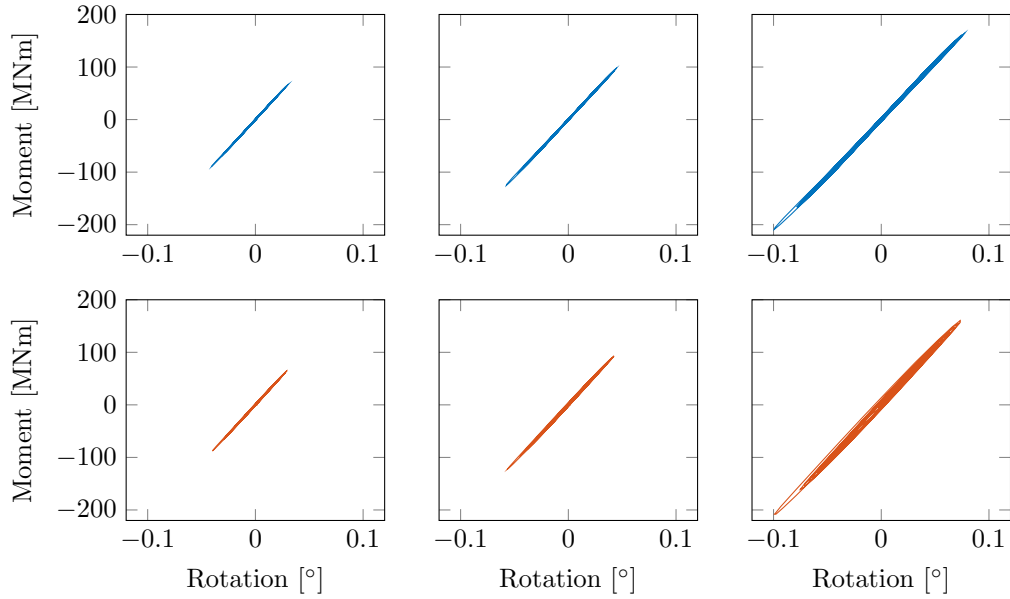


Figure 6.10: Ground level moment rotation curves for the simulation with $V_w=8\text{m/s}$ (column 1), $V_w=10\text{m/s}$ (column 2) and $V_w=12\text{m/s}$ (column 3) using the kinematic hardening model (—) and the current design method (—).

the foundation of a jack-up structure, where the entire soil reaction is described by a single rotational spring and dashpot. However, evaluating equivalent soil springs and dashpots along the pile length, where the lateral displacement varies along the pile length, is inefficient. Applying the kinematic hardening method within the dynamic calculations would therefore lead to simpler implementation of the large strain material damping in the design. As the underlying damping mechanism is plastic rather than viscous, it makes more sense to implement it as such.

6.5 Conclusion

This chapter presents the extensions of the potential functions to 2D loading for the soil reaction models in Chapter 4. A numerical algorithm is presented for the kinematic hardening model in 2D, which allows it to be integrated into simulations for offshore wind turbines where loading in both lateral directions is applied. The kinematic hardening model is applied to the soil reaction during a rotor stop simulation of an offshore wind turbine. This simulation illustrates the effect of amplitude-dependent damping in the soil with the kinematic hardening model, where the underlying damping is plastic rather than viscous.

Chapter 7

Conclusions

This thesis presents work completed as part of the PISA project and further developments which complement the project scope. The overarching aim is to increase understanding of the behaviour of monopile foundations and develop numerical methods which can be used in design to capture the foundation response. The contributions in this research cover:

- Developing **data analysis** tools and applying them to the field test data obtained in the PISA project;
- **Developing models** to capture various aspects of monopile foundations including rate effects, ratcheting and gapping;
- **Calibrating** the soil reaction models to the PISA test results;
- Extending a kinematic hardening model to 2D and **integrating it into commercial software** for time domain simulations of offshore wind turbines.

Each of these aspects is elaborated in the following sections. The contributions are followed by a section on future work and the chapter concludes with a summary of the achievements of this research.

7.1 Data analysis

The analysis tools in Chapter 3 present rigorous methods for analysing the data from the PISA field tests using the combination of a structural model and an optimisation process. These methods are used for accurate interpretation of the field tests. The interpretation presented in this thesis focuses on the rate effects and the behaviour under cyclic loading with the monotonic response as a reference.

Increasing the loading rate results in an additional capacity of the foundation. This effect was observed at both test sites. However, the rate effect was found to be more important in the Cowden till profile than in the Dunkirk sand profile. The rate effect was demonstrated through creep periods in the load holds of the monotonic loading routine and was confirmed in the fast load rate tests at both sites (Cowden: Figure 3.15 - Dunkirk: Figure 3.17).

Under cyclic loading conditions, the foundation stiffness, damping and accumulated rotation are the main geotechnical design drivers. These effects are analysed in detail for the 1-way cyclic tests at both test sites. The accumulation of rotation strongly depends on the magnitude of loading and the applied load history. The rotation accumulation on the cyclic load sets (with higher load amplitude than the previously applied load sets) shows an increase with cycle number, which can be approximated using a power law at high cycle numbers. The exponent for this power law appears consistent with the small scale experimental tests by LeBlanc et al. (2010). Foundation stiffness and damping were found to be almost constant throughout each load set (Cowden: Figure 3.21 - Dunkirk: Figure 3.24).

Additionally, the foundation behaviour of the 2-way cyclic tests was analysed with a focus on foundation stiffness and damping. The results at both test sites show stiffness decrease and damping increase as load amplitude increases. However, after a high amplitude load set was applied, stiffness did not recover to its original value when lower magnitude cyclic loads were applied (Cowden: Figure 3.28 - Dunkirk: Figure

3.33). The analyses of the soil reaction along the pile length show an increased tangent stiffness towards the peaks in the cyclic loading, which is illustrative of the gapping effect (Cowden: Figure 3.30 - Dunkirk: Figure 3.35). This is considered to be the governing effect behind stiffness reduction at small amplitude cyclic loading after high amplitude cyclic loading is applied. At both test sites, the loading frequency did not have a significant impact on foundation stiffness and damping.

7.2 Model development

In the PISA project, the following soil reaction components were identified for a laterally loaded pile: distributed lateral load, distributed moment, base shear force and base moment. In Chapter 4, a number of soil reaction models are proposed to extend these reaction components beyond static monotonic behaviour. The modelling of the foundation pile is set out in the finite element framework, where each of the soil reaction components is consistently added using the virtual work expressions.

The soil reaction models are derived within the hyperplasticity framework, which guarantees thermodynamic acceptability. The models include: (1) the kinematic hardening model to capture plastic unloading; (2) the rate effect model to capture increased capacity with increasing load rate; (3) the ratcheting model to capture accumulated rotation under cyclic loading; (4) the combined rate and ratcheting model and (5) the gapping model to capture gapping on the active side of the pile. For each of the models, a numerical implementation is proposed to efficiently compute the foundation response.

The gapping model sets out a new method to capture the gapping effect in the foundation response. Rather than having a single model for the soil reaction at a certain depth, the soil reaction is calculated at multiple locations along the boundary of a pile section, where a gapping element captures the pile-soil interface.

7.3 Calibration

The calibration of the soil reaction models provides insight into the possibilities and limitations of the various models. Comparison of the calibration parameters at the test sites illustrates the importance of the various effects at both locations. The kinematic hardening model is applied to the soil reaction curves, providing a good baseline model for further developments. Comparison with the monotonic test illustrates that although the plastic unloading is captured reasonably well, rate, ratcheting and gapping effects contribute to the foundation behaviour at both test sites.

The rate effect model is calibrated to the field tests capturing both the capacity increase at the high loading rates in the ‘fast’ tests and the creep periods in the monotonic test routine (Cowden: Figure 5.7 and 5.8 - Dunkirk: Figure 5.11 and 5.12). As expected from the data analysis, the calibrated rate parameter η_{10} is higher for the field tests in the Cowden till than for those in the Dunkirk sand.

The ratcheting model is calibrated to capture the increase of rotation accumulation with cycle number and load amplitude observed in the field tests. The model also captures a dependence of the ratcheting on the load history applied to the pile. Since no significant changes of foundation stiffness and damping were observed within the load sets, these effects were not calibrated for in the ratcheting model. With appropriate calibration, the modelled response agrees reasonably well with the field test results (Cowden: Figure 5.22 and 5.25 - Dunkirk: Figure 5.28).

The combined rate and ratcheting model is calibrated to a pile at both test sites where a combination of the monotonic loading routine and cyclic loading was applied. The model is capable of capturing both the rate and ratcheting effects observed in these tests (Cowden: Figure 5.30 - Dunkirk: Figure 5.32).

The simulations using the gapping model show qualitatively similar effects to the 2-way cyclic tests in terms of stiffness decrease, confirming that gapping is the driving mechanism for this effect (Cowden: Figure 5.37 - Dunkirk: Figure 5.40). The

gapping model introduces the effect of initial horizontal stress on cyclic behaviour and is calibrated to the distributed lateral reaction curves upon monotonic loading.

7.4 Integration into commercial software

Chapter 6 presents a method to integrate the soil reaction models into software to simulate the behaviour of offshore wind turbines in the time domain. Since the simulation allows for displacement of the foundation pile in both lateral directions, the soil reaction models are extended to 2D. The behaviour of a wind turbine is simulated for a simplified rotor stop at different wind speeds using the kinematic hardening soil reaction model applied to the $p-y$ curves in the current design method. The simulations illustrate the effect of the kinematic hardening model on the overall response of the turbine.

The other soil reaction models described in this study could also be integrated into design software using the 2D extension described in Chapter 6. Integrating more sophisticated models for the foundation could result in more accurate modelling of the foundation response and could lead to savings in design.

7.5 Recommendations for future work

The offshore wind industry is a relatively new industry with the first offshore wind turbine dating back to 1991. There are still many areas of research which could be investigated to further optimise the design. In terms of the geotechnical design, these areas range from identifying seabed changes throughout the lifetime to understanding the mechanical behaviour of the foundation pile. A few key aspects relating to the work in this thesis are described below.

Calibration process

The calibration process of the various models starts with identifying which tests could be undertaken to calibrate the parameters of the model. Through element testing, the constitutive behaviour could be calibrated for a rate and/or ratcheting model. The parameters of the rate and ratcheting model for 1D simulations could then be inferred from 3D FE simulations. Another approach is to identify the scaling of the rate and ratcheting effects. Using appropriate scaling, the model parameters could be derived from small scale tests. In this thesis, the gapping model is calibrated to the 1D reaction curves described in the PISA project. The model could also be calibrated more directly to 3D FE simulations of monopiles.

Further model development

The gapping model could be developed further to include the shear term along the vertical axis of the foundation plus extensions for rate and ratcheting effects. Adding the shear term in the vertical axis would also require modelling the axial behaviour of the foundation pile in the 1D model. This extension has the potential of capturing the foundation pile behaviour under axial loads if it is developed further. The extensions could be calibrated to the results of the PISA field tests.

Identifying 2D reaction behaviour

In Chapter 6, the subtle difference between the series and parallel approach for the kinematic hardening model is elaborated when the model is extended to 2D. Laboratory tests could potentially indicate which would be more appropriate for the reaction components on monopile foundations. The slight difference between both approaches would also have an effect on the rate, ratcheting and gapping models.

Aligning structural and geotechnical simulations

Simulations for the dynamics of an offshore wind turbine typically use a simplified soil reaction model, where the soil reaction is captured using linear spring and dashpot elements. Integrating more accurate foundation models in software for these simulations would help align the structural with the geotechnical design. Further work is required to integrate the rate, ratcheting and gapping models in these simulations.

7.6 Summary

This study brings insights into the behaviour of monopile foundations and provides modelling options to capture plastic unloading, rate, ratcheting and gapping effects for laterally loaded piles. The various models are calibrated to match the soil reaction curves of the PISA parameterised method upon monotonic loading. The models are subsequently calibrated to the PISA field tests. The results provide valuable insights into the capabilities and limitations of the models.

Integrating these effects into the design of monopile foundations can serve to optimise their design. A method is also proposed to integrate plastic soil reaction models into dynamic simulations for offshore wind turbines, which is demonstrated using the kinematic hardening soil reaction model. Aligning structural with geotechnical design calculations could result in additional cost savings for the design of offshore wind turbines.

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