May the Force be With You: Investor Power and Company Valuations[*]

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Abstract

This paper re-examines the role of investor power in a model of staged equity financing. It shows how the usual effect where market power reduces valuations can be reversed in later rounds. Once they become insiders, powerful investors may use their market power to increase, not decrease valuations. The critical determinant is whether the insider invests above or below the pro-rata threshold. Even though powerful investors initially lower valuations, companies prefer to bring them inside, to leverage their power in later financing rounds. The paper generates novel predictions about valuations and investor returns.

Keywords: Staged financing, valuation, inside investor, market power.

JEL classification: G32.

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1 Introduction

Much of corporate finance assumes that investors are perfectly competitive, and the supply of capital is perfectly elastic. While this assumption may be suitable for public equity markets, private equity markets typically face an inelastic supply of capital, where a limited number of investors exercise market power. The standard implication of market power is that investors can take larger equity stakes, which drives down company valuations. This result is based on a single-round investment logic, where the investor is an outsider without a prior stake in the company. Many private equity markets, however, are characterized by staged financing where investors fund companies over several rounds. This is common with angel financing, venture capital (VC), growth capital, or private placements. It also happens with publicly-listed companies that raise money in initial public offerings (IPOs), seasoned equity offerings (SEOs), or private investments in public equity (PIPEs). The fundamental difference with staged financing is that while an investor is an outsider the first time he invests, he becomes an insider thereafter. In this paper we pose two main research questions. First, in a staged financing context, what is the effect of investor power on the valuation of a company? Second, when does a company want to bring a powerful investor inside?

To answer these questions, we build a parsimonious theory of staged financing. The company needs two financing rounds. In the base model information is symmetric, and the company faces a supply of capital that is not perfectly elastic. Our focus is on the effect of market power on the valuations of early and late financing rounds. We adopt to our investment context the dominant firm model (DFM), a very common and transparent model of market power in the industrial organization literature. As long as all investors are competitive, the valuation of the late round is independent of the valuation that is obtained in the first round. However, with a powerful investor this is not true, because it matters what stake he obtained in the first round. If he is an outsider, the powerful investor always wants to push down the valuation of the second round. However, if he already has a stake in the company, there is a trade-off. On the one hand, the powerful investor invests new money and therefore wants a lower valuation, just like an outsider. We call this the aggressive outsider logic. On the other hand, he already has a stake in the company, and prefers a higher valuation, just like an insider. We call this the defensive insider logic. The net preference depends on the relative sizes of the existing stake versus the stake in the new round. Our theory derives a simple condition that says that a powerful investor prefers a higher (lower) valuation whenever his second-round investment is below (above) the pro-rata threshold. The important novelty is that with staged financing, market power has an ambiguous effect on valuations. In particular, the model identifies a large range of parameter values where market power leads to higher, not lower valuations in the second-round.

Turning to the first round, we ask about the effect of market power on valuations, and whether the company wants to bring the powerful investor into the first round. We distinguish three scenarios. The benchmark scenario is perfect competition, i.e., no powerful investor. We then look at a scenario where the company obtains first-round funding from the powerful investor, and finally the scenario where the company delays bringing in the powerful investor until the second round. We find that first-round valuations are highest with competition, lower when postponing the investment of the powerful investor, and lowest when the powerful investor participates in the first round. At first glance this result seems to suggest that bringing in the powerful investor up-front is a bad idea, because it generates the lowest first-round valuation. However, as noted above, having an insider can increase second-round valuations. The question becomes whether the higher second-round valuations can justify the lower first-round valuation? We formally show that the company always prefers to bring in a powerful investor right from the start. The key intuition is that, while expensive, bringing in the powerful investor upfront allows the company to leverage his market power. Hence the title of this paper: "May the force be with you."

In our model the source of investor power comes from size, specifically the size of the powerful investor relative to the competitive fringe. This concern for size seems of practical importance in the VC industry. Consider the example of the "Y Combinator Continuity" fund, launched by Y Combinator, a famous Silicon Valley venture accelerator. This is a $1B later stage fund that allows Y Combinator to continue investing in later rounds, to better protect its initial ownership stakes.[1] Along similar lines, Nanda, Sadun, and Hull (2018) report of a local Boston VC firm that aggregates capital from angel investors, in order to form more powerful investment syndicates for the later stage financing of its early stage portfolio.

Our paper builds on several prior literatures. Admati and Pfleiderer (1994) were the first to examine equity valuations in a staged financing context. Their model focuses on identifying robust financial contracts, and shows how an inside investor is neutral with respect to valuations when investing at pro-rata. However, their model artificially fixes the amount of insider funding at the pro-rata level, i.e., their model does not consider inside investors investing below or above pro-rata. Moreover, market power is not directly accounted for in their model. The issue of market power does arise in models of hold-up, such as in the work of Grossman and Hart

[1] According to their website: "YC Continuity is an investment fund dedicated to supporting founders as they scale their companies. Our primary goal is to support YC alumni companies by investing in their subsequent funding rounds [...]." (see `https://www.ycombinator.com/continuity/`).

(1986), Aghion and Tirole (1994), and Hellmann and Thiele (2015). These models are based on incomplete contracting, no such assumption is needed in our model.

Closest to our paper are two recent papers by Khanna and Mathews (2016) and Mella-Barral (2020).[2] Both develop models where the insider has an informational advantage in later financing rounds, but still wants other investors to coinvest. In Khanna and Mathews (2016) the inside investor uses his follow-on investments to signal his private information. This signalling effect reduces his hold-up power and induces the inside investor to offer higher prices. The authors call this "posturing" by insiders. The model of Mella-Barral (2020) has a similar set-up but focuses specifically on the insider's threat to "decertify" a company by withholding investment. The paper asks the interesting question whether the company always wants an insider with the most accurate information. The key finding is that companies prefer insiders with intermediate levels of accuracy: too much accuracy gives insiders too much power, whereas too little accuracy undermines company valuations. Neither of these two papers explicitly consider whether insiders invest above or below pro-rata. We therefore provide a small extension of our model where the powerful investor can signal some private information. We find that signaling behaviour changes below and above pro-rata, along the lines of our main model. Specifically, below pro-rata the powerful insider signals confidence by overinvesting in the good state, much in the spirit of Khanna and Mathews (2016). Above pro-rata, however, the powerful insider signals hesitancy by underinvesting in the bad state, which is in the spirit of Mella-Barral's decertification.[3]

In addition to the literature on staged equity financing, there is a related literature about multi-period debt financing. The seminal work by Sharpe (1990) and Rajan (1992) examines the effects of inside lenders. The advantage is having better information at the time of refinancing, but the disadvantage is that this creates some informational hold-up power. Berglöf and Von Thadden (1994) and Dewatripont and Tirole (1994) further extend this type of analysis to allow for optimal securities and multiple investors. In our base model we assume symmetric information, and therefore abstract away from informational advantages of insiders. Instead we look at size-based market power. This approach is related to Burkhart, Gromb, and Panunzi (1997) who emphasize the importance of size for monitoring incentives. As an extension to our

[2]Further theories about staged financing include the work of Neher (1999), Cornelli and Yosha (2003), Bergemann and Hege (2005), Hellmann and Thiele (2015), and Nanda and Rhodes-Kropf (2017).

[3]It is worth pointing out one difference between our model and those of Khanna and Mathews and Mella-Barral. In their models the inside investor has actual 'hold-up power' in the sense that withholding the investment would prevent the company from raising funds altogether (the 'decertification' effect). In our model, however, the competitive fringe investors can provide all the required funding, just at a higher cost of capital. The powerful investor in the DFM has 'market power', but he does not have any 'hold-up power' in terms of being able to prevent funding altogether.

base model, we also consider the use of debt. We find a rationale for powerful insiders to use equity over debt. With equity they internalize valuation effects at the time of the second round, and therefore use their market power in a way that is more beneficial to the company. With debt, however, there is no need to protect prior stakes, so market power is not used to defend the valuation.

Our model also generates some predictions about the returns to equity investors at different stages. Looking at the final returns of first-round investors, we note that powerful inside investors make the highest returns. However, powerful insiders have lower returns in the second round. Indeed, moderately powerful insiders who invest below pro-rata have even lower returns than the benchmark return of competitive investors. Powerful outsiders, by contrast, have the highest returns of all second-round investors. The analysis also shows that unrealized interim returns need to be interpreted with caution. For example, competitive investors experience low interim returns when the company is financed by a powerful outsider in the second round. However, these are not realized returns. In fact, these competitive investors achieve higher returns between the second round and the date where value is realized, fully recouping their benchmark returns. Our analysis therefore cautions against using unrealized returns as a proxy for expected realized returns.

The structure of the paper is as follows. In the next section we provide a simple numerical example to illustrate how a powerful investor would like to change the company valuation. Section 3 introduces our base model. Section 4 derives the second-stage equilibrium, and Section 5 the first-stage equilibrium. Section 6 discusses model extensions and empirical implications. It is followed by a brief conclusion. All proofs are in the Appendix.

2 A Simple Numerical Example

To illustrate a central insight, we briefly consider a simple numerical example. Suppose a company needs to raise \$10M in a second-round financing. There is a powerful investor who already owns 20% of the company from the first round as well as some new second-round investors. Suppose that there are only two valuations under consideration. Either all the second-round investors jointly receive 50% of the company for their \$10M investment, which implies a post-money valuation of \$20M (= \$10/50%). Or the second-round investors jointly receive 40% of the company for their \$10M investment, which implies a valuation of \$25M (= \$10/40%). What deal would the powerful investor prefer?

Suppose first that the powerful investor plans to invest $2M in the new round. In this case he provides 20% of the new money and receives 20% from the stake of the new investors. However, if he already owns 20% of the existing stake, and now receives 20% of the new stake, his overall stake simply remains at 20%. This is called investing at pro-rata, i.e., investing in the new round an amount that is proportional to the existing stake. It allows the existing investor to exactly maintain his ownership stake.

An insight going back to Admati and Pfleiderer (1994) is that if an investor invests at pro-rata, then he is indifferent between lower and higher valuations, precisely because he always maintains the same ownership stake. While Admati and Pfleiderer fix the investment of the insider at pro-rata, we consider what would happen if the insider invested above or below pro-rata. Suppose the powerful investor invested $3M out of the $10M in the second round. Under the low valuation he would get a stake of 30%*50% = 15%, and his existing stake would be worth 20%*50%=10%, so his overall stake would be 25%. Under the high valuation he would get a stake of 30%*40% = 12%, and his existing stake would be worth 20%*60%=12%, so his overall stake would be 24%. This shows that a powerful investor who invests above pro-rata prefers the low valuation. The outsider logic of investing at a low valuation dominates because the new outsider stake is relatively larger than the existing insider stake.

Consider next the case where the powerful investor invests below pro-rata. Suppose he only provides $1M out of the $10M in the second round. Under the low valuation he would get a stake of 10%*50% = 5%, and his existing stake would be worth 20%*50%=10%, so his overall stake would be 15%. Under the high valuation he would get a stake of 10%*40% = 4%, and his existing stake would be worth 20%*60%=12%, so his overall stake would be 16%. This shows that when the powerful investor invests below pro-rata, he prefers the high valuation. In this case the investment made in the second round is relatively small (below pro-rata) compared the existing stake. Consequently, the insider logic of preferring high valuations dominates.

This example illustrates the way that a powerful insider would like to influence the valuation of the company. Naturally, this simple example is built on several artificial assumptions. It also does not explain how the investor can influence the valuation, and what the equilibrium looks like. For all this we need a proper theory model. We turn to that in the next section.

3 Base Model

Consider a company, called A, which requires two rounds of financing for a project. Specifically, A needs to raise an amount K_1 in the first round, and K_2 in the second round, with

$K_1 < K_2$. The project generates a return $x > 0$ which, for simplicity, is a fixed parameter (although nothing would change if x is the expected value of stochastic returns). For parsimony we assume no discounting. There are three dates: date 1 for the first round, date 2 for the second round, and date 3 for the realization of returns. For simplicity we assume that financing occurs in two stages. In the Appendix we further discuss the theoretical foundations of this assumption.[4]

There are n risk neutral and perfectly competitive investors, which we denote by C. The competitive investors are all price-takers, and choose their investments simultaneously. The prices (α, β, to be introduced shortly) clear the market in a standard Walrasian fashion. In Section 4.2 where we introduce a powerful investor, these competitive investors will become the competitive fringe in a standard 'dominant firm with competitive fringe' model.

We assume that the first round is relatively small and that a single investor provides the entire amount K_1. For simplicity we assume that all competitive investors have the same cost of providing K_1, given by $C_1(K_1) = \mu_1 K_1$, with $\mu_1 > 1$. In return the investor gets an equity share, denoted by α.[5]

Similarly, the second-round investors collectively invest K_2, and jointly receive a total equity share β. The post-money valuations are given by $V_1 = K_1/\alpha$ and $V_2 = K_2/\beta$. We assume that K_2 is large so that the company has to raise funding from multiple investors. This is also known as a syndicated investment round. A competitive investor provides an amount k_2^j, $j \in \{1, ..., n\}$, incurring a cost

$$c_2^j = \mu_2 k_2^j + \frac{\gamma}{2}\left(k_2^j\right)^2.$$

Note that our model uses competitive but not atomistic investors. This means that the market has a finite number of investors, each facing increasing marginal investment costs. This approach will allow us to introduce market power in a simple and intuitive way, as discussed in Section 4.2. The key assumption here is that $\gamma > 0$, so that costs are convex, implying increasing marginal costs of investing. This ensures that the supply of capital has some elasticity.

[4] A prior literature examines the benefits and limitations of staged financing. Neher (1999) derives the need for staged financing in an incomplete contracts framework where the entrepreneur controls the value and can threaten to leave. In contrast, Bergemann and Hege (2005) consider a model where the investors threaten to abandon the project prematurely. Fluck et al. (2005) examine how the staging of financing can undermine entrepreneurial effort. In a related vein, Cumming (2005) includes a critique of Admati and Pfleiderer (1994), noting that their model is not robust to the inclusion of entrepreneurial moral hazard. Finally, Nanda and Rhodes-Kropf (2017) show how the risk of not obtaining competitive refinancing can undermine especially more innovative projects. Note also that in our model K_1 and K_2 are fixed with no possibility of shifting funds from one period to the next. This is a standard assumption in the literature, although Mella-Barral (2020) is a notable exception in this respect.

[5] For simplicity we assume that competitive investors from the first round do not participate in the second round. Allowing for this would make the exposition slightly more cumbersome, but would not change anything, precisely because the competitive investors are always price-takers.

This is needed because assuming an infinitely elastic supply would mechanically eliminate all interesting questions about investor composition and investor power.[6]

4 Second-round Equilibrium

4.1 Competitive Benchmark

We start by deriving the equilibrium investments and company valuation in the second round. To illustrate the effect of market power in our staged financing model, we first consider the benchmark where there are only price-taking competitive investors. In Section 4.2 we introduce a powerful investor who can affect valuations.

Suppose there are n competitive investors in the second round. We assume that the second-round price, as represented by β, clears the market in a Walrasian fashion. We call this the NP case, which stands for "No Powerful" investor.

When investing k_2^j, $j = 1, ..., n$, in the second round, investor j gets the equity share $\frac{k_2^j}{K_2}\beta$. The objective function for each competitive investor is therefore given by

$$\max_{k_2^j} \frac{k_2^j}{K_2}\beta x - \mu_2 k_2^j - \frac{\gamma}{2}\left(k_2^j\right)^2.$$

Each competitive investor is a price-taker and invests an amount $k_2^j(\beta)$. The first-order condition is:

$$\frac{1}{K_2}\beta x = \mu_2 + \gamma k_2^j. \tag{1}$$

To ensure interior solutions, we assume that the relative return x/K_2 is sufficiently large and/or the cost of capital is sufficiently low, so that $k_2^j(\beta) > 0$. The equilibrium equity share for all second-round investors, β, is then defined by the market clearing condition $nk_2^j(\beta) = K_2$.

[6] The specific cost function is chosen to provide the simplest possible model specification. All that matters, however, is that the marginal cost of the fringe (i.e., all competitive investors combined) is increasing in investments. For simplicity we assume symmetric competitive investors with convex costs to give elasticity to the supply of capital. This can also be derived from a model with heterogenous small investors that have different marginal costs of investing. Aligning investors in increasing cost order would immediately generate increasing marginal costs of capital in the competitive fringe. We discuss this further in Section 6.3.

Solving the system of two equations we find that the investment by a competitive investor j is $k_2^{j|NP} = K_2/n$. Moreover, the equilibrium equity share β^{NP} is given by

$$\beta^{NP} = \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n}K_2\right]. \tag{2}$$

This implies the following company valuation:

$$V_2^{NP} = \frac{K_2}{\beta^{NP}} = \frac{x}{\mu_2 + \frac{\gamma}{n}K_2}. \tag{3}$$

For later comparison, we note that the first-round price α does not affect the second-round valuation V_2^{NP}. This is because second-round investors take the first-round price α as given. We will see that this is not true in the model with a powerful investor.

The comparative statics results from the expression of V_2^{NP} are as follows. A higher cost of capital forces A to issue more equity, which implies a lower second-round valuation (i.e., $dV_2^{NP}/d\mu_2$, $dV_2^{NP}/d\gamma < 0$). By contrast, a higher return implies a lower price β, and therefore a higher valuation (i.e., $dV_2^{NP}/dx > 0$). Likewise, if A requires a larger amount in the second round (K_2), then each investor needs to provide more capital (i.e., $dk_2^{j|NP}/dK_2 > 0$). Naturally the round can then only close if A issues more equity, which implies a lower valuation (i.e., $dV_2^{NP}/dK_2 < 0$). We find the opposite with respect to the number of investors n: The presence of more investors induces every single investor to invest less (so that $dk_2^{j|NP}/dn < 0$). This implies a lower total cost of financing across all investors (due to the convexity of c_2^j). The company can then raise the amount K_2 with less equity, which in turn improves the company valuation ($dV_2^{NP}/dn > 0$).

4.2 Powerful Investor Assumptions

We now introduce a powerful investor, called P. We use the classic dominant firm model (DFM) that is commonly used in industrial economics to analyze markets where there is one large firm with substantial market power, and many small price-taking firms, aka the 'competitive fringe'.[7]

In the DFM model, the competitive firms (C) always take the equilibrium price as given, and make their optimal investments accordingly. The dominant firm (P), however, calculates how its own investment quantity affects the equilibrium price. In our model those prices are α

[7] The DFM dates back to the work of Forchheimer (1908), and was further developed by Knight (1921), Stackelberg (1934), Stigler (1940), and later Fudenberg and Tirole (1984). Schenzler, Siegfried, and Thweatt (1992) provide a comprehensive overview.

and β. For each price, P anticipates the investment quantities of the competitive firms C. P then chooses his own investment quantity to achieve the equilibrium price that maximizes his own profits. In equilibrium the quantities of P and all C add up to a total quantity that generates the equilibrium price.

The DFM model always assumes that one player (P in our case) has some exogenous market power. Section 6.4 discusses the possible sources of this market power. Our focus here is to examine its consequences. We focus on relative size as the parameter of market power, i.e., we assume that the larger the powerful investor, the bigger his market power. We further distinguish investor size from any cost advantages. In Section 6.3 we explain how any cost advantages of P affect equilibrium outcomes, but for the base model we deliberately focus on the benchmark case where P has no cost advantage over C. This allows us to isolate a pure size effect, as measured by the parameter m explained below.

In our model the market supply cost curve is upward sloping. To compare market constellations with more or less market power, we want the overall cost structure in the market to remain constant. That is, for a cost-neutral benchmark model we want the market supply curve to remain the same, irrespective of the relative size of the powerful investor. We use the parameter n to describe the size of the overall market, and m to describe the size of the powerful investor.[8] The size of the competitive fringe is given by $(n-m)$ competitive investors.

For the benchmark model we now assume that there are n "funding pots", each with identical costs $c_2(k_2^j) = \mu_2 k_2^j + \frac{\gamma}{2}\left(k_2^j\right)^2$. Each competitive fringe investor j has exactly one funding pot, just like in our base model of Section 4.1. For a powerful investor P of size m, we assume that his investment consists of m funding units each denoted by k_2^P. His total investment is denoted by $K_2^P = mk_2^P$, and his total investment costs are given by $C_2^P = mc_2(k_2^P) = \mu_2 K_2^P + \dfrac{\gamma}{2m}(K_2^P)^2$.[9] The total market cost is thus given by

$$\overline{C}_2 = mc_2(k_2^P) + (n-m)c_2(k_2^j),$$

[8] One limitation of the DFM model worth mentioning here is that it can only accommodate a single powerful investor. Allowing for multiple price-setting investors would greatly increase the complexity of the model, because it requires solving a set of fixed-point equations that no longer have analytical solutions.

[9] In Section 6.3 we discuss in detail what happens when we generalize this cost function. For now, let us just note that one useful simplifying assumption is that P's cost function is based only on the second round investment amount, and does not account for any first round investment amounts. Relaxing this assumption would complicate the analysis by creating a linkage between the investment amount in the first round and the marginal investment cost in the second round. As long as K_1 is small, however, this additional effect is also small.

where $k_2^j = k_2^C$ for all competitive investors j. This yields identical costs to the competitive market outcome whenever $k_2^P = k_2^C$ (as well as for $m = 0$). The question of interest is how the powerful investor uses his market power to strategically alter his investment level K_2^P.

It is also worth noting that we assume that the powerful investor makes investments, but cannot make any transfer payments. Due to our sparse set of assumptions, there is a theoretical possibility in this model that a powerful investor makes a transfer payment to the company, and then owns and runs the entire company. This is clearly unrealistic, and it is easy to augment the model to exclude this possibility.[10]

The model with a powerful investor divides into two cases. First, there is the case of a powerful inside investor, who participated in the first financing round, and therefore already holds a stake in the company. We denote this stake by α^P. We call this the PI case, which stands for "Powerful Insider". Second, there is the case of the powerful investor who only invests in the second round. We call this the PO case, which stands for "Powerful Outsider". In this case a competitive outcome occurs at the first stage investment K_1. At the second stage we can simply set $\alpha^P = 0$. For expositional convenience we initially treat PI and PO as two exogenously separate cases. The easiest way to justify this is to assume that in the case of PI (PO), P has a very low (high) investment cost μ_1^P. In Section 5.3 we will then examine P's endogenous choice, i.e., when he chooses to become a PI or PO.

4.3 The Case of a Powerful Insider

Consider first the case of a powerful insider. When choosing his investment K_2^P, P takes the effect on the investments of the competitive investors (C), and therefore on the equilibrium price β, into account. The price β is defined by the market clearing condition

$$K_2^P + (n - m)\, k_2^j(\beta) = K_2, \quad j = m + 1, ..., n, \tag{4}$$

[10] The assumption of no transfer payments is standard in the literature (see Rajan (1992), or Hellmann (2002) for a discussion). One way to derive this constraint endogenously is to add a simple adverse selection problem. Suppose that in addition to the honest companies described so far, there are dishonest companies that can pretend to have a business that looks identical to the honest ones. The dishonest companies take the transfer payment and disappear, leaving investors with a total loss. If for every honest company there are enough dishonest ones, no investor would ever make a transfer payment, simply because the probability of investing in a dishonest company is too high. Other ways of justifying the absence of transfer payments are based on moral hazard models, where the owner/manager needs to retain as much equity as possible to continue providing effort for the success of the company.

where $k_2^j(\beta) = \frac{1}{\gamma}\left[\frac{1}{K_2}\beta x - \mu_2\right]$ is the amount provided by each competitive investor (which can be derived from (1) in Section 4.1).

P chooses K_2^P to maximize his net return

$$\pi_2^{P|PI}(K_2^P) = (1 - \beta(K_2^P))\alpha^P x + \frac{K_2^P}{K_2}\beta(K_2^P)x - C_2^P(K_2^P), \tag{5}$$

where $\beta(K_2^P)$ is the total equity issued to the second-round investors as a function of P's investment (as defined by the market clearing condition (4)), and K_2^P/K_2 is P's share in the round. Intuitively P's net payoff depends on three components. The second term represent his second-round equity stake ($\frac{K_2^P}{K_2}\beta(K_2^P)$). The third term is his cost of capital. The most interesting component is the first term. P holds an equity stake α^P from his first-round investment K_1. This gets diluted as A needs to issue equity to new investors (including to P). This component therefore captures the fact that P is an inside investor who also cares about preserving his existing stake.

We informally note that the first component is decreasing in β. This means that as an insider P prefers a higher valuation (i.e., a lower β). We call this the "defensive" insider logic. The second component, however, is increasing in β. As an outsider P prefers a lower valuation (i.e., a higher β). We call this the "aggressive" outsider logic. The relative strength of these two logics will play a key role in determining how P exercises his market power. We discuss this in Section 5.

In the Appendix we show that in equilibrium P invests

$$K_2^{P|PI} = \frac{m}{n+m}\left[1 + \alpha^P\right]K_2. \tag{6}$$

We can immediately see that P's investment $K_2^{P|PI}$ depends on his first-round equity stake α^P. This also applies to the second-round valuation V_2^{PI}, given by

$$V_2^{PI} = \frac{K_2}{\beta^{PI}} = \frac{x}{\mu_2 + \gamma\dfrac{n - \alpha^P m}{n^2 - m^2}K_2}. \tag{7}$$

Note that for $m = 0$, this simplifies back to the competitive valuation V_2^{NP} (see (3) in Section 4.1).

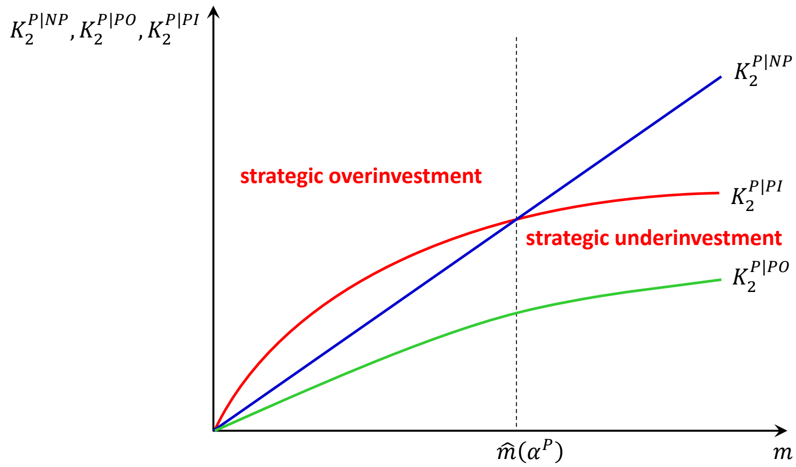

Figure 1: Second-round Investments

An important question is how P's investment $K_2^{P|PI}$ relates to the pro-rata investment. In other words, when does P invest below pro-rata and accept a dilution of his stake, and when does P invest above pro-rata to increase his stake? The next lemma provides a simple condition.

Lemma 1 *P invests above (below) pro-rata whenever $m > (<)\ \widehat{m} \equiv \alpha^P n$.*

Lemma 1 says that P invests above (below) pro-rata whenever his power (m), is above (below) a critical value $\widehat{m}$. This critical value varies with his prior equity stake (α^P). With this we can examine how P's market power (as measured by m), and his first-round equity stake α^P, affect $K_2^{P|PI}$ and V_2^{PI}.

Proposition 1 *Consider the PI case. The investment by P, $K_2^{P|PI}$, is increasing in both m and α^P. Moreover, there exists a threshold m', with $0 < m' < \widehat{m}$, such that the second-round valuation of A, V_2^{PI}, is increasing in m for $m < m'$, and decreasing thereafter. The valuation V_2^{PI} is also increasing in α^P.*

Figures 1 and 2 illustrate the main insights from Proposition 1 (these two figures also compare the PI case with the other cases, which we discuss in more detail in Section 4.5). More market power (m) encourages P to provide a larger share of the required second-round investment K_2. The effect on valuation is non-monotonic. To get an intuition, we first note that P behaves just like a competitive investor in two cases: when m is small, i.e. when there is not much concentration of power, and when $m = \widehat{m}$, i.e., when P maintains exactly his pro-rata. Between these critical values, we find that the valuation first rises and then falls in m. The intuition for the valuation rising initially in m is that P is driven by the defensive insider logic

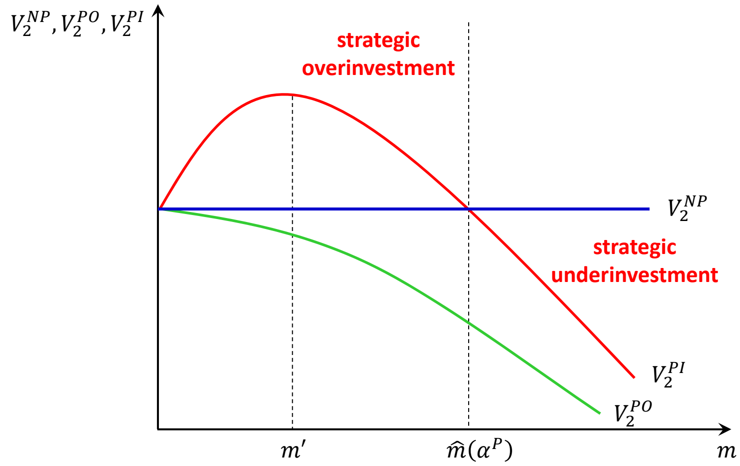

Figure 2: Second-round Valuations

that comes from the first component of equation (5). The intuition why it falls again is that as P invests larger amounts, he is increasingly driven by the aggressive outsider logic from the second component of (5). For $m = \widehat{m}$, the insider and outsider logic exactly cancel out. For $m > \widehat{m}$, the aggressive outsider logic always dominates. Note also that the intuition for why the valuation is always increasing in α^P is that a higher α^P increases the first component, and therefore strengthens the defensive insider logic.

4.4 The Case of a Powerful Outsider

So far we assumed that the powerful investor is an insider, with some ownership stake $\alpha^P > 0$. We now turn to the case of a powerful outside investor. Technically this is a special case of the model from the previous section with a powerful insider where we set $\alpha^P = 0$ in (6) and (7).

Proposition 2 *Consider the PO case. The investment by P, $K_2^{P|PO}$, is increasing m. The second-round valuation, V_2^{PO}, is always decreasing in m.*

In the PO case, P does not hold any equity when participating in the second financing round ($\alpha^P = 0$). This means that P can only invest above pro-rata (since $m > \alpha^P n = 0$). P is not concerned about equity dilution, and only uses his market power to drive the valuation down (see also Figures 1 and 2).

In the Appendix we derive some additional comparative statics results with respect to equilibrium investments and valuations for the PI and PO cases. The cost of capital (μ_2, γ), as well as the return (x), do not affect P's investment ($K_2^{P|PI}$, $K_2^{P|PO}$). However, the cost of capital has a negative effect on valuation (V_2^{PI}, V_2^{PO}), while the return (x) has a positive effect on valuation. Furthermore, a higher capital requirement in the second round (K_2) results in P making a larger investment ($K_2^{P|PI}$, $K_2^{P|PO}$), leading to a lower company valuation (V_2^{PI}, V_2^{PO}). The presence of more investors (n) also implies a lower equilibrium investment by P, and a higher company valuation. Finally, we show in the Appendix that P's participation constraint is always satisfied.[11]

4.5 Comparing Second-round Constellations

We now compare the second-round investments and company valuations when P is an insider versus outsider. We also compare both cases against the competitive benchmark (NP case). For this it is useful to define $K_2^{P|NP} \equiv mk_2^i = \frac{m}{n}K_2$. This is the total amount that P would provide in a (counterfactual) competitive benchmark.[12] All results derived in this section are illustrated in Figures 1 and 2.

We begin with a proposition focussed on the powerful outsider (PO) case.

Proposition 3 *For all $m > 0$, and all $\alpha^P > 0$, a powerful outsider makes the smallest second-round investment, i.e., $K_2^{P|PO} < \{K_2^{P|PI}, K_2^{P|NP}\}$. This also results in the lowest second-round company valuation, i.e., $V_2^{PO} < \{V_2^{PI}, V_2^{NP}\}$.*

Figures 1 and 2 illustrate how the investment and valuation in the PO case compare to the other scenarios. In the PO case, P does not have a stake in A ($\alpha^P = 0$). It is then optimal for P to exploit his market power to drive down the valuation of the company. This can be achieved by investing a smaller amount compared to the competitive benchmark (i.e., $K_2^{P|PO} < K_2^{P|NP}$, so that $V_2^{PO} < V_2^{NP}$). This result is consistent with the standard economics result that market power leads to smaller quantities (here: smaller investment) and higher prices (here: lower valuation, which means higher cost of capital for the company).

To further understand why the powerful insider behaves differently from the powerful outsider, we note that having a stake in the company (PI case with $\alpha^P > 0$) curbs P's temptation to drive the valuation down in the second round. That is, when choosing his second-round investment K_2^P, P trades off the positive effect on his second-round equity stake ($\frac{K_2^P}{K_2}\beta(K_2^P)$), and the negative effect on his first-round stake (α^P). A larger stake from the first financing round

[11]Note that this also implies that a constellation where P participates in the first round, but not the second round, can never arise in equilibrium.

[12]Note that the investment amount of an individual investment "pocket", $k_2^{j|NP}$, does not depend on m. However, the more individual pockets belong to P, the higher the total investment $K_2^{P|NP}$.

(α^P) makes it optimal for P to invest a larger amount in the second round, resulting in a higher valuation (see Proposition 1).

Proposition 3 shows that having a powerful outsider leads to the lowest possible second-round valuation. We also just discussed how the powerful insider increases second-round valuations, relative to the powerful outsider. It remains to be seen how the powerful insider compares to the competitive benchmark (the NP case).

Proposition 4 *Comparing the powerful insider against the competitive benchmark, we distinguish two cases:*

(i) ***Moderately powerful insider:*** *Suppose $m \leq \widehat{m}$. P invests more in the second round compared to the competitive benchmark (i.e., $K_2^{P|PI} \geq K_2^{P|NP}$). The second-round valuation is higher compared to the competitive benchmark (i.e., $V_2^{PI} \geq V_2^{NP}$).*

(ii) ***Highly powerful insider:*** *Suppose $m > \widehat{m}$. P invests less in the second round compared to the competitive benchmark (i.e., $K_2^{P|PI} < K_2^{P|NP}$). The second-round valuation is lower compared to the competitive benchmark (i.e., $V_2^{PI} < V_2^{NP}$).*

The insights from Proposition 4 can be seen in Figures 1 and 2. At the critical level of market power ($m = \widehat{m}$), we know that P invests exactly at pro-rata, and therefore maintains his stake in the company. In this case the outcome is identical to the competitive benchmark, i.e., P provides the same amount of capital in the second round ($K_2^{P|PI} = K_2^{P|NP}$), leading to the same company valuation ($V_2^{PI} = V_2^{NP}$).

If the inside investor is highly powerful ($m > \widehat{m}$), he invests more in absolute terms, but strategically reduces his investment compared to the competitive benchmark ($K_2^{P|PI} < K_2^{P|NP}$), as shown in Figure 1. In doing so P accepts a dilution of his first-round equity stake, but obtains a lower company valuation in the second round ($V_2^{PI} < V_2^{NP}$), as shown in Figure 2. This captures the problem of having an insider that is too powerful. At the same time we note that it is better for A to have the powerful investor as an insider, than as an outsider. This follows from the fact that valuations are even lower with the powerful outsider.

Maybe the most interesting insight pertains to the investment of a moderately powerful insider ($m < \widehat{m}$). Such an inside investor strategically overinvests ($K_2^{P|PI} > K_2^{P|NP}$); see Figure 1. This increases the second-round valuation, and helps to preserve P's stake from the first-round investment, as shown in Figure 2. The moderately powerful insider is therefore beneficial for the company, in the sense that his market power is now used to the benefit of the company: "May the force be with you!"

5 First-round Equilibrium

We now close our staged financing model by deriving the equilibrium investments and company valuation in the first financing round. Ideally we could use the same cost structure in the first as in the second round. Unfortunately, the model becomes too complex to analyze, so we have to resort to some simplifying assumptions. Our main interests concern the interactions between inside and outside investors in the second round. We therefore simplify the model by assuming that it is either the powerful investor or the competitive fringe that provides the first round financing, but not both. Moreover, we assume that investment costs are not quadratic but linear. One way of thinking about this is that the first round is relatively small, and that the cost function can be represented by its first order Taylor approximation. In a dominant firm model with such linear costs, it is easy to show (see Appendix) that the powerful investor either has a sufficient cost advantage to take the entire first round, or else he takes none of it at all. Put differently, the first order Taylor approximation simplifies the first round to a linear cost structure that generates a binary outcome where either the powerful investor or the fringe make the entire investment. With that, the analysis simplifies to three scenarios: (i) only competitive investors participate in the first and second round (the NP case), (ii) the powerful investor only invests in the second round (the PO case), and (iii) the powerful investor participates in both financing rounds (the PI case). With that simplification, we ask how first-round valuations differ across these three scenarios. In addition, we derive whether in equilibrium the powerful investor enters in the first or second round.[13]

5.1 Investment by Competitive Investor

Suppose the required amount K_1 in the first round is provided by a competitive investor. For parsimony we derive the outcome for the NP and PO case jointly (they are very similar, the only difference being β^{NP} versus β^{PO}). The winning investor receives an equity stake α^i ($i \in \{NP, PO\}$). To derive the valuation we consider the zero-profit condition for a competitive investor:

$$\left(1-\beta^i\right)\alpha^i x - \mu_1 K_1 = 0 \quad \Leftrightarrow \quad \alpha^i = \frac{\mu_1 K_1}{\left(1-\beta^i\right)x}. \tag{8}$$

[13] It is worth briefly explaining what we miss out on with our simplifying assumption. The main loss is that in the first round we do not allow for joint funding between the powerful investor P and the competitive fringe. However, it is easy to see that such joint funding would generate second round dynamics that are very similar to the PI case as examined in Section 4.3. We already explained that the fringe investors from the first round do not participate in the second round (although this assumption can easily be relaxed). With joint funding, P can again manipulate the second round valuation along the lines described in Section 4.3. The only difference is that P has a slightly lower prior ownership stake α, because he only provided part but not all of the first round funding.

We note that α^i is increasing in β^i because a competitive investor knows that his stake will get diluted in the second round when A issues β^i to new investors. The equilibrium first-round price α^i is such that the investor's diluted share of the payoff equals his cost of investing K_1.[14]

Using the expressions for β^{NP} and β^{PO}, we get the following first-round competitive equilibrium valuations:

$$
\begin{aligned}
V_1^{NP} &= \frac{K_1}{\alpha^{NP}} = \frac{1}{\mu_1}\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n}K_2\right]\right)x \\
V_1^{PO} &= \frac{K_1}{\alpha^{PO}} = \frac{1}{\mu_1}\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2 - m^2}K_2\right]\right)x.
\end{aligned}
$$

The next proposition summarizes how P's market power (m) as an outsider affects the company valuation when the first-round investment is provided by a competitive investor.

Proposition 5 *Suppose a competitive investor provides the amount K_1 in the first round. If a powerful outsider provides capital in the second round, then the valuation V_1^{PO} is decreasing in m.*

This result is very intuitive, and can be seen from the expression of V_1^{PO}. As noted above, the difference between the company valuations is rooted in the amount of equity issued to the second-round investors (β^{NP} versus β^{PO}). The presence of a more powerful outsider P implies that in equilibrium more equity is being issued to the second-round investors (i.e., $d\beta^{PO}/dm > 0$). This implies that the equity stake of the competitive first-round investor gets more diluted. Thus the company needs to provide a higher α^{PO} to the competitive investors, to compensate for their subsequent dilution. This means that the first-round valuation V_1^{PO} is lower.

From the expressions of V_1^{NP} and V_1^{PO} it is easy to derive that the first and second-round costs of capital (μ_1, μ_2, γ) have a negative effect on the first-round valuation, while the effect of the return (x) is positive. The required amount in the second round (K_2) has a negative effect on the first-round valuation. The presence of more second-round investors (n) implies a higher

[14] Our model is deliberately simple and only has a single state in the second period. Consequently, there is no role for anti-dilution clauses. In a more complex model with multiple states in the second period, investors could ask for anti-dilution clauses (Woronoff and Rosen, 2005). As explained in Da Rin and Hellmann (2020), an anti-dilution clause for a first-round investment means that if the price of the second round lies below the first-round price, the ownership stake of the first-round investors gets readjusted to compensate them for poor company performance. In our model this would mean that α would get readjusted under some realizations of the state variable.

first-round valuation. This is because it requires issuing less equity to second-round investors, and therefore curbs the dilution of the first-round investor's equity stake.[15]

5.2 Investment by Powerful Investor

Now suppose that P wants to finance the first round. For parsimony we assume that company A accepts P's offer as long as A is indifferent between having P investing K_1, and a competitive investor providing K_1.

We know from Section 4.2 that P always wants to participate in the second round. Consequently, P makes the first-round investment K_1 (PI case) as long as it leads to a higher profit compared to waiting for the second round (PO). Formally, P's participation constraint for the first round is given by $\pi^{P|PI}(\alpha^P) \geq \pi^{P|PO}$.

The next proposition characterizes P's offer in the first round.

Proposition 6 *P's offer in the first round is given by α^P, where α^P satisfies*

$$\alpha^P \left(x - K_2 \left[\mu_2 + \frac{\gamma \left(n + \left(1 - \alpha^P \right) m \right)}{n^2 - m^2} K_2 \right] \right) = \mu_1 K_1. \tag{9}$$

The resulting first-round valuation $V_1^{PI} = K_1/\alpha^P$ is decreasing in m.

When submitting a bid to A for the first financing round, P is competing with all the other competitive investors. The equity stake α^P, as defined in Proposition 6, makes A just indifferent between accepting the bid α^P and taking the bid α^{PO} from a competitive investor. And by assumption, A then chooses P to make the first-round investment K_1.

Proposition 6 also shows that a more powerful investor asks for a higher equity stake α^P, which leads to a lower company valuation in the first round (i.e., $dV_1^{PI}/dm < 0$). We know that the powerful outsider fully exploits his market power in the second round to maximize his economic rents. In addition, we know from Propositions 2 and 3 that more market power implies a lower second-round valuation (see also Figure 2). It follows that the powerful outsider is very unattractive for A. This situation improves for A when P becomes an insider. Knowing this, P can ask for a higher equity stake α^P in the first round. This implies a lower valuation

[15]Not surprising we find equivalent effects on company A's expected profit, which we denote by $\pi^{A|i}$, $i \in \{NP, OP\}$. The only exception concerns K_1, which has a negative effect on $\pi^{A|i}$. We provide more details in the Appendix (see Proof of Proposition 8).

V_1^{PI}. Note also that P's participation constraint is always satisfied at α^P, so he always prefers to start investing in the first round (i.e., he prefers PI over PO).[16]

In the Appendix we derive additional comparative statics results. Specifically, we show that the first-round valuation V_1^{PI} is decreasing in the cost of capital (μ_1, μ_2, γ), and increasing in the return (x) and the number of second-round investors (n). Moreover, we find that the required financing amount in the first round (K_1) has a positive effect on V_1^{PI}, while the effect of the required amount in the second round (K_2) is negative.[17]

5.3 Comparing First-round Constellations

So far we considered the case of PO and PI as two exogenously distinct scenarios. We now make this endogenous by looking at P's equilibrium offer at date 1, and whether the company accepts or refuses it.

The next proposition compares the first-round valuations for the different investor constellations.

Proposition 7 *The first-round valuation is lowest with a powerful insider (PI case), higher with the anticipation of a powerful outsider (PO case), and highest when there is no powerful investor (NP case). Formally, $V_1^{PI} < V_1^{PO} < V_1^{NP}$ for all $m > 0$.*

The company valuation in the first round is lowest in the PI case. To see why, we first note that P's offer in the first round makes A marginally better off, compared to having a competitive investor in the first round and P only participating in the second round (PO case). According to Proposition 3, the second-round valuation is the lowest for the PO constellation. P then exploits the lower outside option for A by asking for a higher equity stake, which implies a lower first-round valuation. This also explains why the equilibrium valuation V_1^{PI} in the first financing round is below the competitive benchmark.

A key insight so far is that the presence of a powerful investor can have opposite effects on company valuations across different financing rounds. For example, with a powerful insider (PI case) the first-round valuation is always below the competitive benchmark, while the second-round valuation may be above the benchmark level (see Proposition 4). It therefore remains to compare A's profits under the different investor constellations.

[16] Note that this depends on the assumption that P has the same costs as C. In Section 6.3 we will introduce heterogenous costs and derive conditions under which P prefers PO over PI.

[17] The effects on company A's expected profit, $\pi^{A|PI}$, are equivalent; see Proof of Proposition 8 in the Appendix for more details.

Proposition 8 *Company A's profits for the different investor constellations are as follows: $\pi^{A|NP} > \pi^{A|PI} > \pi^{A|PO}$ for all $m > 0$. Furthermore, A's profit in the PO case ($\pi^{A|PO}$) and the PI case ($\pi^{A|PI}$) is decreasing in m.*

It is intuitive that the presence of a powerful investor always leads to reduced profits for the company. The important insight pertains to the question whether a company should bring in a powerful investor early, thereby making him an insider in later financing rounds. Having an early stake encourages P to use his market power to defend the company valuation in later rounds. However, bringing in P at the beginning comes at the price of a lower valuation. While A gets a higher valuation in the second round, A pays for it up-front. Still, Proposition 8 shows that "having the force with you" leads to a higher profit for the company.

6 Extensions and Discussions

6.1 Investor Returns

So far, our analysis looks at equilibrium valuations at date 1 and date 2. The model therefore provides some insights into the structure of investor returns. In particular, we distinguish three types of returns. Define R_{13} (R_{23}) as the return that a date 1 (date 2) investment generates at date 3 when the company realizes its value. Moreover, define R_{12} as the unrealized interim return (at date 2) for a date 1 investment. In the model these three returns are defined as follows:

$$R_{12} = \frac{V_2 - K_2}{V_1} \qquad R_{23} = \frac{x}{V_2} \qquad R_{13} = \frac{(1-\beta)x}{V_1} = \frac{x}{V_1}\frac{V_2 - K_2}{V_2}.$$

These returns differ across the various model permutations. We distinguish four scenarios. First we look at the benchmark case where there is no powerful investor (NP case). Second, we look at the case with a powerful outsider (PO case). Third, we consider a moderately powerful insider where $m < \widehat{m}$. We call this the MPI case. Finally, there is the highly powerful insider with $m > \widehat{m}$, which we call the HPI case.

We show in the Appendix that the returns compare as follows:

- R_{13}: $R_{13}^{HPI} > R_{13}^{MPI} > R_{13}^{NP} = R_{13}^{PO} = \mu_1$
- R_{23}: $R_{23}^{PO} > R_{23}^{HPI} > R_{23}^{NP} > R_{23}^{MPI}$
- R_{12}: $R_{12}^{MPI} > R_{12}^{NP} > R_{12}^{PO}$ and $R_{12}^{HPI} > R_{12}^{PO}$.

The realized returns of date 1 investors (R_{13}) can be ranked according to the amount of market power exercised. Returns are highest for highly powerful insiders, followed by moderately powerful insiders, followed by competitive investors in the PO and NP case. Note that $R_{13}^{PO} = R_{13}^{NP} = \mu_1$ because these are the returns of the competitive first-round investors, not the powerful investor.

The realized returns of date 2 investors (R_{23}) are also influenced by market power, but rank entirely differently. In particular we find that returns are highest in the PO case. This is because a powerful investor depresses the valuation to generate the highest possible returns to himself. The powerful insider moderates the use of his market power, because of his prior stake in the company, as discussed in Proposition 4. In fact, the realized returns of date 2 investors are lowest in the MPI case, which is when the moderately powerful investor boosts valuation above the competitive benchmark.

The unrealized returns of date 1 investors at date 2 (R_{12}^{PO}) follow yet another pattern. They rank lowest in the PO case. This is not because date 1 investors made bad investments, but because the powerful outside investor depresses the valuation. The final realized returns of these date 1 investors (R_{13}^{PO}) are in fact the same as the returns in the benchmark NP case (i.e., $R_{13}^{PO} = \mu_1$). Empirical researchers have often shied away from treating interim valuations as valid return signals. Our theoretical derivation here justifies this caution, showing how in the presence of market power, unrealized interim returns can be misleading indicators of expected realized returns.[18]

Overall, our model shows the importance of decomposing investor returns by stages of investment, and generating differential predictions for initial versus follow-on investment rounds. To empirical test these predictions the main additional data required are investor returns, including data on exits (Harris et al., 2014).

6.2 The Role of Debt

So far our model considers equity investors. In this section we ask whether our results change in the presence of debt. The main question is whether it would be more efficient for the powerful investor to hold debt instead of equity? To examine this, suppose A wants to raise capital by issuing debt claims to investors. The (endogenous) interest rates are denoted by r_1 and r_2. The

[18]For completeness, note also that date 1 investors in the MPI case achieve both higher unrealized interim returns (R_{12}^{MPI}) and higher realized final returns (R_{13}^{MPI}), compared to the benchmark returns (R_{12}^{NP} and R_{13}^{NP}), and also compared to the PO case (R_{12}^{PO} and R_{13}^{PO}). Finally, comparisons for the HPI case cannot always be signed unambiguously.

expected return from the project is uncertain, and given by $x = \rho x_H + (1-\rho)x_L$, with $x_H > x_L$. For brevity's sake we focus on the case where debt is safe, i.e., $x_L \geq (1+r_1)K_1 + (1+r_2)K_2$.[19]

Consider the second financing round, and assume for a moment that A issued equity to P in the first round – this allows us to focus exclusively on the effect of debt financing in the second round. In the Appendix we formally derive the equilibrium investments in the second round under debt financing, and derive the equilibrium cost of capital r_2^*. We find that the costs of debt and equity financing in the second round are identical, i.e., $(1 + r_2^*)K_2 = \beta^{PI}x$. This is essentially the Modigliani-Miller theorem. In the second round it is irrelevant whether A raises the required amount K_2 through debt or equity, because the financial structure doesn't affect any behaviors.

Now consider the first financing round, and suppose that A issues a debt claim to raise the amount K_1. In the Appendix we establish the following two results. First, when K_1 is raised from a competitive investor (PO case), then debt and equity financing are again equivalent. Second, when the powerful investor provides K_1 (PI case), then equity is optimal. The intuition is that issuing equity in the first round makes the powerful investor defend the valuation in the second round, to limit the dilution of his first-round equity stake. This strategic effect is not present when P holds debt, because in this case the value of his existing stake is not affected by the valuation in the second round.

Our model therefore identifies a new reason for the use of equity, which is similar but not identical to the traditional incentive argument. The traditional argument is that investors need equity to have incentives for providing more value-adding services. This is typically modelled as a two-sided moral hazard problem (see, e.g., Da Rin et al. (2012), and Hong, Serfes, and Thiele (2020)). The reason for using equity in our model is also related to incentives, but not to moral hazard. Instead the argument is that by giving equity to the powerful investor, he takes the value of insiders into consideration when exercising his market power.[20] In terms of empirical prediction, the analysis suggests that adding venture debt weakens the beneficial effects of bringing a powerful investor inside. The stylized fact that venture debt gets mostly used in later rounds is broadly consistent with this prediction.[21]

[19] In this simple model with only two states, any risky debt is equivalent to equity. Hence the combination of safe debt and equity covers all relevant combinations.

[20] This argument is reminiscent of Da Rin and Hellmann (2002) who look at how market power allows a bank to finance "big push" investments.

[21] See Tykvová (2017), Hochberg, Serrano, and Ziedonis (2018), and Fulghieri, García, and Hackbarth (2020) for some analysis of venture debt. Note also that venture investors often use more complex securities than straight equity, especially various convertible securities. Their analyses go beyond the scope of this paper. See Schmidt (2003) and Hellmann (2006) for some foundational theories, and Kaplan and Strömberg (2003) and Gornall and Strebulaev (2020, 2021) for some related empirical work.

6.3 Cost Advantages

Our benchmark model uses a linear-quadratic cost function and imposes cost symmetry between the powerful investor P and the competitive fringe C. This allows us to focus on the size parameter m as the key differentiator between P and C. We now consider the possibility that P has a cost advantage (or disadvantage). Let

$$C_1^P = (1-\zeta_1)\mu_1 k_1^P \quad \text{and} \quad C_2^P = (1-\zeta_2)\mu_2 K_2^P + (1-\vartheta_2)\frac{\gamma}{2m}(K_2^P)^2,$$

where ζ_1 and ζ_2 represent P's marginal cost advantage at date 1 and date 2, respectively. Furthermore, ϑ_2 represents an additional cost advantage related to the slope of the marginal cost curve. For $\zeta_1, \zeta_2, \vartheta_2 < 0$, P has a cost disadvantage, whereas for $\zeta_1, \zeta_2, \vartheta_2 > 0$, P has a cost advantage. Naturally we assume $\zeta_1, \zeta_2, \vartheta_2 < 1$.

In the Appendix we rederive the equilibrium investment levels, equity stakes, and valuations. We find that

$$\frac{dK_2^{P|i}}{d\zeta_2} > 0, \quad \frac{d\beta^i}{d\zeta_2} < 0, \quad \text{and} \quad \frac{dV_2^i}{d\zeta_2} > 0, \quad i = PI, PO.$$

The bigger P's cost advantage, the more he invests ($K_2^{P|i}$). This implies that the competitive fringe investors C invest less. In the DFM, prices (i.e., ownership stakes and valuations) are set by C's marginal costs. Since they invest less, C's marginal costs are lower, resulting in a lower investor stake β^i and a higher valuation V_2^i. This applies both to the case of PO and PI.[22]

If P has a cost advantage over the competitive fringe investors, then this also affects the critical threshold $\widehat{m}$. In the Appendix we show that $\widehat{m}(\zeta_2) < \widehat{m}(0) \equiv \alpha^P n$ for $\zeta_2 > 0$, and conversely $\widehat{m}(\zeta_2) > \widehat{m}(0)$ for $\zeta_2 < 0$. What really matters in the model is whether P wants to invest above or below pro-rata. In our benchmark model with symmetric costs, this switch happens exactly at $\widehat{m}(0) \equiv \alpha^P n$, i.e., exactly when P's relative size (given by $\frac{m}{n}$) equals his pro-rata threshold α^P. However, if P has a cost advantage and thus makes a larger investment, then even if his size m is a little bit below $\alpha^P n$, he may still want to invest above pro-rata. Consequently, the critical size $\widehat{m}(\zeta_2)$ is now below $\widehat{m}(0) \equiv \alpha^P n$. Put differently, once we allow for heterogenous costs, the key issue is not P's relative size by itself, but how much money he actually wants to 'put on the table', and in particular whether he wants to invest above or below pro-rata.

Turning to the date 1 equilibrium, we find that there exists some $\widehat{\zeta_1}$ so that $\pi_1^{P|PI}(\widehat{\zeta_1}) = \pi_1^{P|PO}$. For $\zeta_1 < \widehat{\zeta_1}$ we have $\pi_1^{P|PI}(\zeta_1) < \pi_1^{P|PO}$, so that P does not invest at date 1 (and thus

[22] The comparative statics for ϑ_2 are very similar to those of ζ_2. We provide them in the Appendix.

becomes a powerful outsider at date 2). For $\zeta_1 > \widehat{\zeta}_1$ we have $\pi_1^{P|PI}(\zeta_1) > \pi_1^{P|PO}$, so that P chooses to invest at date 1 and thus becomes a powerful insider. In the latter case, P always makes the lowest possible offer that leaves the company indifferent between taking P's or C's offer. This implies that the equilibrium offer α^P is determined by C's competitive offer. Thus, whenever P's participation constraint is satisfied (i.e., $\zeta_1 > \widehat{\zeta}_1$), his investor stake α^P and the valuation V_1^{PI} do not depend on ζ_1, i.e.,

$$\frac{d\alpha^P}{d\zeta_1} = 0 \quad \text{and} \quad \frac{dV_1^{PI}}{d\zeta_1} = 0.$$

We also consider how date 1 valuations depend on date 2 cost advantages. In the Appendix we show that

$$\frac{d\alpha^P}{d\zeta_2} < 0, \quad \text{and} \quad \frac{dV_1^{PI}}{d\zeta_2} > 0.$$

We already noted that lower marginal costs (higher ζ_2) reduce the investor stake β^i at date 2. At date 1, investors thus expect less future dilution. This explains why their competitive offer α becomes lower (and thus V_1 higher). This argument applies both to the case of PI and PO.[23]

While the focus of this paper is how investor power affects valuations, it should be noted that investor power also affects efficiency. Since returns (x) and capital commitments (K_1, K_2) are all constant in this model, efficiency solely pertains to the overall cost of capital. In the benchmark model with symmetric costs, the competitive outcome is first-best, achieving lowest overall costs. Thus, any deviations from that imply some cost inefficiencies. In the case of $m < \widehat{m}$ ($m > \widehat{m}$) where P overinvests (underinvests), the inefficiency comes from the fact that P's marginal cost is higher (lower) than that of the competitive fringe. Moreover, if there are heterogenous costs at date 1, we may ask whether it is always the lowest cost investor that provides the initial investment. In the Appendix we show that $\widehat{\zeta}_1 < 0$. This means that there exists a parameter range $\zeta_1 \in (\widehat{\zeta}_1, 0)$ where P provides the initial investment, even though P has a cost disadvantage. This is inefficient relative to the first-best.

Finally, let us also briefly address the question of how to generalize our linear-quadratic cost curves. If one uses a generic convex cost function $c_2^j(k_2^j)$ that satisfies $dc_2^j/dk_2^j > 0$ and $d^2c_2^j/d(k_2^j)^2 > 0$, almost all of our main results continue to hold. Formally, all that is needed

[23] One may also ask what happens if there are heterogeneous costs among the fringe investors themselves. While the notation gets cumbersome, conceptually this is very straightforward. All that matters in the DFM is the aggregate supply curve of the competitive fringe. If different investors in the fringe have different cost curves, we can simply add them up, noting that the sum of convex cost curves remains convex too.

is a mild condition on the third derivative of the cost function.[24] Moreover, we already mentioned in footnote 6 that instead of *assuming* increasing marginal costs, this can also be *derived* by assuming heterogeneity among the competitive fringe investors. Another interesting question is whether one can also derive increasing marginal costs for the powerful investor? In the Appendix we develop a simple model extension where instead of assuming convex costs, the powerful investor has a fixed budget that can be invested across multiple companies. Increasing the investment amount in one company requires reducing investments in other portfolio companies. We show that this implies lower returns from these other investments. In fact, this generates convex *opportunity costs* to increasing investment amounts in the focal company. Our assumption of convex costs is thus a tractable simplification that can be interpreted as convex opportunity costs of investing less in other portfolio companies.[25]

6.4 Sources of Investor Power

The key contribution of this paper is to examine how market power affects valuations across multiple investment stages. We examine the opposing incentives of aggressive outsider versus defensive insider stakes. Our key insights pertain to the *consequences* of investor market power. A natural follow-up is to inquire about the *sources* of investor power.

We use the DFM as our benchmark model, because it is a simple and transparent model of market power, with a long tradition in economics. The DFM uses relative size as the key source of market power. This fits well the investment context where it is frequently said that larger investors use "the power of the purse" to price investments to their liking. Our discussion in Section 6.3 further clarifies that market power is related to how much money an investor actually "puts on the table". We now discuss two broad classes of power sources: better access to funds, and informational advantages.

Within our investment context, having a size advantage essentially means having better access to funds. The underlying issue is that the supply of capital is not perfectly elastic (as reflected by our convex cost function). This assumption suits the context of private investment

[24]The formal proof is available from the authors upon request. Note also that it is not necessary to assume that the cost function is convex everywhere. For example, if there are some fixed costs, then average costs fall for an initial range of k_2, and there is a minimum investment level below which no one wants to investment. This does not matter – all that matters is that the average investment costs eventually rise, i.e., that marginal costs increase for larger values of k_2.

[25]Note also that in our model with a single investment, costs are based on the absolute investment amount. However, in a model with multiple investments, there can be portfolio interdependencies along the lines discussed in the Appendix. In that case, the size of the investment relative to other investments in the portfolio may also matter. We will leverage this idea further in the next subsection.

markets, such as venture capital, where the investors are general partners (GPs) who have to raise their funds from limited partners (LPs). A growing literature describes the GPs' challenges of raising funds from LPs. The work of Kaplan and Schoar (2005) and Metrick and Yasuda (2010) shows the importance of having a prior track record. Their evidence also suggests diseconomies to scale that prevent venture funds from becoming too large. Hochberg et al. (2014) explain the challenges of raising funds in terms of imperfect information at the LP level. Lopez-de-Silanes et al. (2015) elaborate on the organizational constraints faced by fund managers. Milosevic (2018) also argues that access to funds is largely determined by social networks. In Section 6.6 we will use this insight to suggest some empirical tests about the sources of investor power.

In this context it is worth briefly mentioning the possibility of collusion among investors. The industrial organization literature recognizes the possibility of collusion among firms (e.g. Jacquemin and Slade (1989), Bernheim and Whinston (1990), and Feuerstein (2005)). In our investment context, collusion could take the form of investors holding a unified front while negotiating a deal with the company. The outcome would look like a syndicated investment deal. However, not every syndicated deal necessarily involves collusion. Potential examples of investor collusion might include angel groups that band together around investment deals, or tight networks of venture capitalists who repeatedly coinvest across multiple companies.[26]

Concerning informational sources of investor power, a prior corporate finance literature (see Sharpe (1990), Rajan (1992), Khanna and Mathews (2016), and Mella-Barral (2020)) notes that insiders are likely to have better information about the true prospects of a company. Their later round investments can reveal private information, and can thus have a signalling effect on outside investors. The insiders' power to withhold investments can also become a source of informational rents. Recognizing that inside information is a *source* of market power, the question becomes how this relates to our analysis of the *consequences* of investor power.

To address this, we derive a model extension with insider information in the Appendix. The main goal is to show that our core insights about aggressive outsiders versus defensive insiders, remains valid. Suppose now that inside investors have better information about the company

[26]It is needless to say that a deeper analysis of investor networks and collusive investor behaviours is beyond the scope of this paper. The interested reader should consider several strands of prior literature that examine various aspects of this. Seminal papers on venture capital networks include Hochberg et al. (2007) and Sorenson and Stuart (2008). The issue of repeated investor interactions is examined by Elfenbein and Zenger (2013), and Du and Hellmann (2021). Casamatta and Haritchabalet (2007) consider the possibility of anti-competitive collusion. Cestone et al. (2007) examine the formation of investment syndicates, arguing that lead investors typically avoid coinvesting with investors who are more experienced than themselves. This prediction is broadly in line with the descriptive evidence in Lerner (1994). A recent review paper by Nanda and Rhodes-Kropf (2019) also discusses frictions within venture capital syndicates.

than outside investors. Specifically, at date 2 there are two possible states for the expected return, denoted by x_L and x_H, with $0 < x_L < x_H$. Assume that only the company and the powerful insider can observe the true state, but outside investors cannot.[27] We are interested in insider signaling, and thus focus on separating equilibria where P invests one amount for the low, and a different amount for the high signal.[28]

Our central finding is that P's signaling strategy depends on whether he wants to invest below or above pro-rata. Specifically, below pro-rata, P wants to signal high expected returns (x_H) in order to increase valuations, and does so by overinvesting in the high state. By contrast, above pro-rata, P wants to signal low expected returns (x_L) in order to decrease valuations, and does so by underinvesting in the low state. This confirms our key message about aggressive outsiders versus defensive insiders. Powerful investors thus make use of their informational advantages in ways that are similar to size advantages. That is, while there may be multiple sources of investor power, their consequences are along the lines of our main results.

6.5 Endogenous Market Power

An interesting question related to the discussion in the previous section concerns the endogeneity of market power, i.e., to what extent investors choose how much power they have? While a general equilibrium model of endogenous market structure is beyond the scope of this paper, the Appendix contains a simple model extension where the market power is determined endogenously. At the core of the argument is a trade-off between concentrating capital into one investment to maximize market power, versus distributing capital across multiple investments to generate higher portfolio returns.

Specifically, suppose P now has an overall investment budget and is considering investments in multiple companies. For simplicity assume that all potential investments have the same returns as described in our base model. For each company, P is facing a fixed number of fringe competitors. Moreover, there is a fixed cost of investing in any new company, which we can think of as a due diligence and transaction cost. P's portfolio optimization problem consists of choosing the number of companies in the portfolio. The more companies P invests in, the less capital is available per company. Using the positive relationship between capital invested ($K_2^{P|PO}$) and market power (m) from Proposition 2, we show that investing in more companies

[27] We assume that the competitive fringe investors from date 1 do not reinvest at date 2, and therefore do not matter here.

[28] In the Appendix we derive sufficient conditions for the separating equilibrium to exist. Moreover, note that any pure pooling equilibrium behaves just like our base model.

naturally results in less market power per company. The portfolio problem thus reduces to a trade-off between the number of investments versus market power per investment.

In the Appendix we show that P's profits from any one company are an increasing but concave function of m.[29] While more market power always increases P's profits, marginal profits are actually decreasing in m. It follows that making one large investment in one company generates less profits than making two half-sized investments in two companies and so on, provided the fixed costs are not too high. Consequently, P does not want to concentrate all his market power into a single company, but prefers spreading it over a diversified portfolio, with the total number of companies limited due to the fixed costs.[30] This model extension thus generates market power endogenously as the outcome of a portfolio optimization problem.

Naturally, there remains a question of where P's overall budget constraint is coming from. In the case of angel investors, the amount of personal (investable) wealth provides a natural budget constraint. In the case of professional investment funds, such as venture capital, the overall budget constraint is determined by the size of the fund. One may thus wonder if LPs always want to create larger funds for market power reasons. Within the confines of our model a powerful investor would always benefit from having more funds available for investments. Thus, our model cannot provide an explanation for the well-documented fact that investment funds eventually face diseconomies of scale. However, the aforementioned literature on LP-GP relations suggests several reasons why increasing fund sizes eventually becomes challenging. Fulghieri and Sevilir (2009) also provide a more comprehensive theoretical framework for determining optimal fund sizes. Their model is based on human capital constraints and focuses on the ability of VCs to managerially support all of their portfolio companies. Their analysis thus provides a justification for why LPs optimally limit the size and scope of VC funds.

6.6 Main Empirical Predictions

Our theory generates several testable predictions about the relationship between investor power and company valuations. The first is related to the initial investments of a powerful investor. Specifically, the model generates an unambiguous prediction that powerful investors drive valuations down when they invest in a company for the first time. The second empirical prediction concerns the follow-up investments of powerful investors. Here the model generates a nuanced pattern of predictions. Specifically, powerful investors drive valuations down if they invest

[29]For tractability reasons we focus on the PO case ($\zeta_1 < \widehat{\zeta}_1$).

[30]This result holds without invoking risk-aversion, and would be even stronger if P was also concerned about risk diversification.

above pro-rata. However, if they invest below pro-rata, the model predicts that they actually drive valuations up. The model thus predicts a non-monotone relationship between investor power and valuations, as depicted in Figure 2.

To empirically test this, it is useful to remember that what matters in the model is how much money the powerful investor actually puts on the table. Put differently, what matters is the powerful investor's share in a new round, and how this compares to his pro-rata share. A direct empirical test of our theory therefore should relate the company's post-money valuation V_2 to an inside investor's stake in a new round (K_2^P/K_2), further distinguishing between stakes above or below the pro-rata threshold (α^P).

Three empirical challenges will need to be overcome. First, there is a data problem. Investments and valuations data is readily available for public markets but much more difficult to obtain for private markets. And while total round investments (K_2) are readily available, reliable breakdowns by investors (required to estimate K_2^P/K_2) are harder to obtain. Second, to estimate over- and under-valuations, one needs a credible benchmark valuation model, including a rich set of control variables. Third, in our model there is only one powerful investor, but in reality there may be more. The empirical analysis thus needs to handle multiple inside investors. Identifying collusive investor groups, as discussed in Section 6.4, is another empirical challenge.

While our main focus is to derive the valuation *consequences* of investor power, Section 6.4 also discusses possible *sources* of investor power. Our DFM model equates investor power with relative size, which we relate to better access to funds. By contrast, the work of Khanna and Mathews (2016) and Mella-Barral (2020) equates investor power with informational advantages. Our discussion in Section 6.4 shows that the core predictions about the *consequences* of investor power are very similar. The interesting question is whether one can empirically distinguish between access and informational sources of investor power. Empirically, this means relating new investment stakes to specific investor characteristics that stem from access versus informational advantages.

To empirically evaluate access to funds, the most direct measure is the amount of funding under control by an investor, such as the size of a VC fund. A useful refinement is to only consider what is known as "dry powder", defined as the amount of money in a fund left for investments. Our portfolio model in Section 6.5 suggests a trade-off between portfolio investments. One can thus use exogenous shocks to the investment demand of other companies in the investor's portfolio as an instrument for changes in available dry powder. This is because these shocks affect the available dry powder of the investor, which then influences the valuation of the focal company. However, the demand shocks from the other companies do not directly influence the focal company's valuation itself, thereby satisfying the exclusion restriction.[31] One might also leverage market conditions at the time of fundraising. The argument is that if an investor raises a fund from limited partners in a hot (cold) market, this generates a positive (negative) shock to his fund size. Finally, what matters in the DFM is the size of the investor relative to the market. Empirically one may thus also want to control for market conditions at the time of the investment round. Using a "money chasing deals" logic (see Gompers and Lerner (2000) and Nanda and Rhodes-Kropf (2013)), we would expect investors to have more (less) power when current market conditions are cold (hot).

Informational-based sources of investor power relate to a very different set of investor characteristics. For this, we would want to measure how closely an investor's expertise matches with the company's current situation. We would expect informational advantages to be larger whenever the investor has deeper experience in a company's sector, geography, or stage. Moreover, we would expect the informational advantage to be larger if the investor also sits on the board of directors.[32]

7 Conclusion

In this paper we develop a model of staged equity financing with investor market power. Standard economic reasoning suggests that a powerful investor obtains lower valuations, and thereby achieves higher returns. We show that while this result holds the first time a powerful investor invests, it may not hold in subsequent financing rounds. As an insider, a powerful investor faces dual motives. One is that higher valuations preserve his existing stake, the other is that lower valuations are more attractive for his new investments. We show that the former motive dominates when the investor is moderately powerful and invests below pro-rata in later financing rounds. In this case the effect of market power is reversed, i.e., the investor uses his market power to increase, not decrease, valuations. We explain how this is an equilibrium behavior, and describe the circumstances under which this result obtains.

[31]This argument is based on the prior work of Bernstein, Lerner, and Mezzanotti (2019) who establish the importance of dry powder, and the work of Kempf, Manconi, and Spalt (2017), Liu, Low, Masulis, and Zhang (2020), and Abuzov (2020), who use a similar instrumental variable strategy that leverages shocks to other companies in an investor's portfolio.

[32]Naturally, it is possible that higher quality investors have more power due to both informational and funding reasons, something any empirical analysis should take into account. In this context one may also consider the empirical approach of Bernstein, Giroud, and Townsend (2016), who use the introduction of new direct airline routes as an instrument for the effective closeness between investors and companies.

Our model also asks whether the company prefers to have the powerful investor up-front, or delay him to a later round. Even though the powerful investor can extract a lower valuation in the first round, the company prefers to bring him in up-front. This is because once he becomes an insider, the company can leverage his power to defend its valuation. Hence the title of the paper: “May the force be with you.”

Appendix

PI Case: Equilibrium Investment and Valuation.

Using $k_2^j(\beta) = \frac{1}{\gamma}\left[\frac{1}{K_2}\beta x - \mu_2\right]$ we can write the market clearing condition (4) as

$$K_2^P + (n-m)\frac{1}{\gamma}\left[\frac{1}{K_2}\beta x - \mu_2\right] = K_2.$$

Solving for β we get

$$\beta(K_2^P) = \frac{K_2}{x}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right]. \tag{10}$$

Using the expression for $\beta(K_2^P)$ and $k_2^i = K_2^P/m$, $i = 1, ..., m$, we can write the objective function of P as follows:

$$\begin{aligned}\max_{K_2^P} \pi_2^{P|PI}(K_2^P) &= \alpha^P x - \alpha^P K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right] + K_2^P\left[\frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2\right] \\ &\quad - \left[\mu_2 K_2^P + \frac{\gamma}{2m}\left(K_2^P\right)^2\right].\end{aligned}$$

The optimal investment, $K_2^{P|PI}$, is then defined by the first-order condition:

$$\alpha^P K_2 \frac{\gamma}{n-m} + \frac{\gamma}{n-m}\left(K_2 - K_2^P\right) + \mu_2 = K_2^P\frac{\gamma}{n-m} + \mu_2 + \frac{\gamma}{m}K_2^P.$$

Solving for K_2^P we get $K_2^{P|PI} = \frac{m}{n+m}\left[1 + \alpha^P\right]K_2$. Substituting $K_2^{P|PI}$ in (10) then yields the equilibrium equity share for all investors:

$$\beta^{PI} = \frac{K_2}{x}\left[\mu_2 + \frac{\gamma\left(n - \alpha^P m\right)}{n^2 - m^2}K_2\right]. \tag{11}$$

Thus, the equilibrium valuation, V_2^{PI}, is given by

$$V_2^{PI} = \frac{K_2}{\beta^{PI}} = \frac{x(n^2 - m^2)}{\gamma\left(n - \alpha^P m\right)K_2 + \mu_2(n^2 - m^2)}.$$

Proof of Lemma 1.

Note that P invests above (below) pro-rata when

$$(1-\beta^{PI})\alpha^P + \frac{K_2^{P|PI}}{K_2}\beta^{PI} > (<)\alpha^P.$$

Using $K_2^{P|PI} = \frac{m}{n+m}\left[1+\alpha^P\right]K_2$, we can write this condition as

$$\frac{m}{n+m}\left[1+\alpha^P\right]\beta^{PI} > (<)\alpha^P\beta^{PI},$$

which can be simplified to $\frac{m}{n} > (<)\alpha^P$. $\square$

Proof of Proposition 1.

We can immediately see that $dK_2^{P|PI}/d\alpha^P > 0$ and $dV_2^{PI}/d\alpha^P > 0$. Moreover,

$$\begin{aligned}
\frac{dK_2^{P|PI}}{dm} &= \frac{n}{[n+m]^2}\left[1+\alpha^P\right]K_2 > 0,\\
\frac{dV_2^{PI}}{dm} &= \frac{-2mx\left[\gamma\left(n-\alpha^P m\right)K_2 + \mu_2(n^2-m^2)\right] + x(n^2-m^2)\left[\gamma\alpha^P K_2 + 2m\mu_2\right]}{\left[\gamma\left(n-\alpha^P m\right)K_2 + \mu_2(n^2-m^2)\right]^2}.
\end{aligned}$$

We have $dV_2^{PI}/dm > 0$ when

$$\begin{aligned}
(n^2-m^2)\left[\gamma\alpha^P K_2 + 2m\mu_2\right] &> 2m\left[\gamma\left(n-\alpha^P m\right)K_2 + \mu_2(n^2-m^2)\right]\\
\Leftrightarrow \quad \gamma\alpha^P n^2 K_2 + 2\mu_2 mn^2 - \gamma\alpha^P m^2 K_2 &> 2\gamma mnK_2 - 2\gamma\alpha^P m^2 K_2 + 2\mu_2 mn^2\\
\Leftrightarrow \quad \underbrace{\alpha^P\left(n^2+m^2\right) - 2mn}_{\equiv Z} &> 0
\end{aligned}$$

Note that this condition is satisfied for $m=0$. Thus, $dV_2^{PI}/dm > 0$ for $m \to 0$. Moreover, for $m = n$ this condition simplifies to $\alpha^P - 1 > 0$, which is clearly violated. Therefore, $dV_2^{PI}/dm < 0$ for $m \to n$. Next, note that $dZ/dm = 2\alpha^P m - 2n < 0$. Consequently, there exists a unique $m' > 0$ so that $dV_2^{PI}/dm > 0$ for $m < m'$, and $dV_2^{PI}/dm \leq 0$ for $m \geq m'$. Finally, evaluating Z at $m = \widehat{m} = \alpha^P n$ we get $\alpha^P n^2\left(\alpha^{P2}-1\right) < 0$. Thus, $\left.\frac{dV_2^{PI}}{dm}\right|_{m=\widehat{m}} < 0$, which implies that $m' < \widehat{m}$. $\square$

Proof of Proposition 2.

Setting $\alpha^P = 0$ in (6) and (7) we get

$$K_2^{P|PO} = \frac{m}{n+m}K_2 \tag{12}$$

$$V_2^{PO} = \frac{x(n^2-m^2)}{\gamma n K_2 + \mu_2(n^2-m^2)}. \tag{13}$$

Thus,

$$\begin{aligned}
\frac{dK_2^{P|PO}}{dm} &= \frac{n}{[n+m]^2}K_2 > 0 \\
\frac{dV_2^{PO}}{dm} &= \frac{-2mx\left[\gamma n K_2 + \mu_2(n^2-m^2)\right] + x(n^2-m^2)2\mu_2 m}{\left[\gamma n K_2 + \mu_2(n^2-m^2)\right]^2} \\
&= -\frac{2\gamma mnxK_2}{\left[\gamma n K_2 + \mu_2(n^2-m^2)\right]^2} < 0.
\end{aligned}$$

□

Second Round: Additional Comparative Statics.

Consider first $K_2^{P|PI}$. We can immediately see that $dK_2^{P|PI}/d\mu_2 = dK_2^{P|PI}/d\gamma = dK_2^{P|PI}/dx = 0$, $dK_2^{P|PI}/dK_2 > 0$, and $dK_2^{P|PI}/dn < 0$. Likewise, for V_2^{PI} we can see that $dV_2^{PI}/d\mu_2$, dV_2^{PI}/dK_2, $dV_2^{PI}/dK_2 < 0$, and $dV_2^{PI}/dx > 0$. Moreover,

$$\begin{aligned}
\frac{dV_2^{PI}}{dn} &= \frac{2xn\left[\gamma\left(n-\alpha^P m\right)K_2 + \mu_2(n^2-m^2)\right] - x(n^2-m^2)\left[\gamma K_2 + 2n\mu_2\right]}{\left[\gamma\left(n-\alpha^P m\right)K_2 + \mu_2(n^2-m^2)\right]^2} \\
&= \frac{\overbrace{\left[n^2 - 2\alpha^P mn + m^2)\right]}^{\equiv Z(\alpha^P)} x\gamma K_2}{\left[\gamma\left(n-\alpha^P m\right)K_2 + \mu_2(n^2-m^2)\right]^2}.
\end{aligned}$$

Note that $Z(\alpha^P)$ is decreasing in α^P. Moreover, evaluating $Z(\alpha^P)$ at $\alpha^P = 1$ we get $Z(1) = (n-m)^2 > 0$. This implies that $Z(\alpha^P) > 0$ for all $\alpha^P \in [0,1]$. Thus, $dV_2^{PI}/dn > 0$.

Next consider $K_2^{P|PO}$ (see (12) in Proof of Proposition 2). Note that $dK_2^{P|PO}/d\mu_2$, $dK_2^{P|PO}/d\gamma$, $dK_2^{P|PO}/dx = 0$, $dK_2^{P|PO}/dK_2 > 0$, and $dK_2^{P|PO}/dn < 0$. In addition, for V_2^{PO} (see (13) in

Proof of Proposition 2) we can immediately see that $dV_2^{PO}/d\mu_2$, $dV_2^{PO}/d\mu_2$, $dV_2^{PO}/dK_2 < 0$, and $dV_2^{PO}/dx > 0$. Furthermore,

$$\begin{aligned}\frac{dV_2^{PO}}{dn} &= \frac{2xn\left[\gamma nK_2 + \mu_2(n^2-m^2)\right] - x(n^2-m^2)\left[\gamma K_2 + 2\mu_2 n\right]}{\left[\gamma nK_2 + \mu_2(n^2-m^2)\right]^2} \\ &= \frac{\left[n^2+m^2)\right]\gamma xK_2}{\left[\gamma nK_2 + \mu_2(n^2-m^2)\right]^2} > 0.\end{aligned}$$

Second Round Participation of Powerful Investor.

Note that $K_2^{P|P-P} > 0$ for all $\alpha^P \in [0,1]$; see (6). Given the structure of P's profit function (see (5)), this implies that P always chooses to invest in the second round for all $\alpha^P \in [0,1]$. Consequently, his participation constraint is always satisfied in the second round.

Proof of Proposition 3.

We can immediately see that $K_2^{P|PO} < K_2^{P|PI}$ for all $\alpha^P > 0$, and $K_2^{P|PO} < K_2^{P|NP}$ for all $m > 0$. Moreover, note that $V_2^{PO} = V_2^{PI}$ at $\alpha^P = 0$, and recall from Proposition 1 that $dV_2^{PI}/d\alpha^P > 0$. Thus, $V_2^{PO} < V_2^{PI}$ for all $\alpha^P > 0$. Finally, $V_2^{PO} < V_2^{NP}$ because

$$\begin{aligned} \frac{x(n^2-m^2)}{\gamma nK_2 + \mu_2(n^2-m^2)} &< \frac{x}{\mu_2 + \frac{\gamma}{n}K_2} \\ \Leftrightarrow \quad (n^2-m^2)\left(\mu_2 + \frac{\gamma}{n}K_2\right) &< \gamma nK_2 + \mu_2(n^2-m^2) \\ \Leftrightarrow \quad n^2 - m^2 &< n^2. \end{aligned}$$

□

Proof of Proposition 4.

We have $K_2^{P|PI} \geq K_2^{P|NP}$ when

$$\frac{m}{n+m}\left[1+\alpha^P\right]K_2 \geq \frac{m}{n}K_2 \quad \Leftrightarrow \quad \alpha^P \geq \widehat{\alpha}^P(m) = \frac{m}{n},$$

which is equivalent to $m \leq \widehat{m} = \alpha^P n$. Moreover, note that $V_2^{PI} \geq V_2^{NP}$ when

$$\begin{aligned}
\frac{x(n^2 - m^2)}{\gamma\,(n - \alpha^P m)\,K_2 + \mu_2(n^2 - m^2)} &\geq \frac{x}{\mu_2 + \frac{\gamma}{n}K_2} \\
\Leftrightarrow \quad (n^2 - m^2)\frac{\gamma}{n}K_2 &\geq \gamma\left(n - \alpha^P m\right) K_2 \\
\Leftrightarrow \quad \alpha^P &\geq \widehat{\alpha}^P(m) = \frac{m}{n},
\end{aligned}$$

which, again, is equivalent to $m \leq \widehat{m} = \alpha^P n$. □

First-round Investments.

When investing k_1^j, $j = 1, ..., n$, in the first round, investor j gets the equity share $\frac{k_1^j}{K_1}\alpha$. Thus, the objective function for each competitive investor is

$$\max_{k_1^j} \frac{k_1^j}{K_1}\left(1 - \beta^i\right)\alpha^i x - \mu_1 k_1^j.$$

The first-order condition is

$$\left(1 - \beta^i\right)\alpha^i x = \mu_1 K_1.$$

Note that the LHS and RHS are both constant in k_1^j. Thus, we get a bang-bang solution: (i) $k_1^j(\alpha^i) = K_1$ if $(1 - \beta^i)\,\alpha^i x > \mu_1 K_1$, and ($ii$) $k_1^j(\alpha^i) = 0$ otherwise.

Likewise, P's objective function in the first round is

$$\max_{k_1^{PI}} = \frac{k_1^{PI}}{K_1}(1 - \beta^{PI})\alpha^P x + \frac{K_2^{P|PI}}{K_2}\beta^{PI} x - \mu_1 k_1^{PI} - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].$$

Note that $K_2^{P|PI}$ and β^{PI} do not depend on k_1^{PI}. Thus, the first-order condition is

$$(1 - \beta^{PI})\alpha^P x = \mu_1 K_1.$$

Again, we get a bang-bang solution: (i) PI chooses $k_1^{PI}(\alpha^P) = K_1$ if $(1 - \beta^{PI})\alpha^P x > \mu_1 K_1$, and (ii) $k_1^{PI}(\alpha^P) = 0$ otherwise.

Participation Constraint of Powerful Investor (First Round).

Using the expressions for $K_2^{P|PI}$ and β^{PI} (see (6) and (11)) we get

$$\begin{aligned} \pi^{P|PI}(\alpha^P) &= (1-\beta^{PI})\alpha^P x + \frac{K_2^{P|PI}}{K_2}\beta^{PI}x - \mu_1 K_1 - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right] \\ &= \left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right]\right)\alpha^P + \frac{m}{n+m}\left[1+\alpha^P\right]\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2^2 \\ &\quad -\mu_1 K_1 - \frac{\gamma m}{2}\frac{1}{[n+m]^2}\left[1+\alpha^P\right]^2 K_2^2. \end{aligned} \tag{14}$$

Likewise, by setting $\alpha^P = 0$ and $\mu_1 K_1 = 0$, we get

$$\pi^{P|PO} = \frac{m}{n+m}\frac{\gamma n}{n^2-m^2}K_2^2 - \frac{\gamma m}{2}\frac{1}{[n+m]^2}K_2^2$$

We then find that $\pi^{P|PI}(\alpha^P) \geq \pi^{P|PO}$ is equivalent to $Z \geq \mu_1 K_1$, where

$$\begin{aligned} Z &= \left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right]\right)\alpha^P - \frac{m}{n+m}\frac{\gamma\alpha^P m}{n^2-m^2}K_2^2 + \frac{m}{n+m}\alpha^P\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2^2 \\ &\quad -\frac{\gamma m}{2}\frac{1}{[n+m]^2}\left[2\alpha^P + \left[\alpha^P\right]^2\right]K_2^2 \\ &= \left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right]\right)\alpha^P - \frac{1}{2}\alpha^P\frac{\gamma m}{(n+m)^2}\frac{\alpha^P}{(n^2-m^2)}K_2^2\left[m^2+2mn+n^2\right] \\ &= \left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\frac{1}{2}\alpha^P m\right)}{n^2-m^2}K_2\right]\right)\alpha^P. \end{aligned}$$

Thus, P's participation constraint can be written as

$$\alpha^P\left(x - K_2\left[\mu_2 + \frac{\gamma\left(n-\frac{1}{2}\alpha^P m\right)}{n^2-m^2}K_2\right]\right) \geq \mu_1 K_1. \tag{15}$$

Proof of Proposition 6.

Let α^P denote the first round equity share which makes A indifferent between accepting α^P from P, and α^{PO} from a competitive investor. Formally, α^P is defined by $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. Using (11) we get

$$\pi^{A|PI}(\alpha^P) = \left(1-\alpha^P\right)\left(1-\beta^{PI}(\alpha^P)\right)x = \left(1-\alpha^P\right)\left(1-\frac{K_2}{x}\left[\mu_2+\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right]\right)x.$$

Moreover, recall from the comparative statics analysis for the PO case (see (21)) that

$$\pi^{A|PO} = \left(1-\frac{K_2}{x}\left[\mu_2+\frac{\gamma n}{n^2-m^2}K_2\right]\right)x-\mu_1 K_1.$$

Using these two expressions we then find that $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$ is equivalent to

$$\alpha^P\left(x-K_2\left[\mu_2+\frac{\gamma\left(n+\left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]\right) = \mu_1 K_1, \tag{16}$$

which defines α^P.

Next we need to show that α^P satisfies P's participation constraint (15). Using (16) we can write (15) as

$$\alpha^P\left(x-K_2\left[\mu_2+\frac{\gamma\left(n-\frac{1}{2}\alpha^P m\right)}{n^2-m^2}K_2\right]\right) \;\geq\; \alpha^P\left(x-K_2\left[\mu_2+\frac{\gamma\left(n+\left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]\right)$$

$$\Leftrightarrow \quad 0 \;\leq\; 1-\frac{1}{2}\alpha^P,$$

which is clearly satisfied for all $\alpha^P \in [0,1]$.

Finally, using (16), we can implicitly differentiate α^P w.r.t. m:

$$\frac{d\alpha^P}{dm} = \frac{\alpha^P\gamma K_2^2\left(\frac{\left(1-\alpha^P\right)\left(n^2-m^2\right)+2m\left(n+\left(1-\alpha^P\right)m\right)}{\left[n^2-m^2\right]^2}\right)}{\underbrace{x-K_2\left[\mu_2+\frac{\gamma\left(n+\left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]}_{>0}+\alpha^P\frac{\gamma m}{n^2-m^2}K_2^2} > 0.$$

Consequently,

$$\frac{dV_1^{PI}}{dm} = \frac{d}{dm}\left[\frac{K_1}{\alpha^P}\right] < 0.$$

Powerful Investor in First Round – Comparative Statics.

Recall that α^P is defined by

$$Z \equiv \alpha^P \left(x - K_2 \left[\mu_2 + \frac{\gamma \left(n + \left(1 - \alpha^P \right) m \right)}{n^2 - m^2} K_2 \right] \right) - \mu_1 K_1 = 0.$$

Moreover, note that

$$\frac{\partial Z}{\partial \alpha^P} = \underbrace{x - K_2 \left[\mu_2 + \frac{\gamma \left(n + \left(1 - \alpha^P \right) m \right)}{n^2 - m^2} K_2 \right]}_{>0} + \alpha^P \frac{\gamma m}{n^2 - m^2} K_2^2 > 0. \tag{17}$$

Using Z we can then implicitly differentiate α^P and get

$$\begin{aligned}
\frac{d\alpha^P}{d\mu_1} &= \frac{K_1}{\frac{\partial Z}{\partial \alpha^P}} > 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{d\mu_1} = \frac{d}{d\mu_1}\left[\frac{K_1}{\alpha^P}\right] < 0 \\
\frac{d\alpha^P}{d\mu_2} &= \frac{\alpha^P K_2}{\frac{\partial Z}{\partial \alpha^P}} > 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{d\mu_2} = \frac{d}{d\mu_2}\left[\frac{K_1}{\alpha^P}\right] < 0 \\
\frac{d\alpha^P}{d\gamma} &= \frac{\alpha^P \frac{n + \left(1-\alpha^P\right) m}{n^2 - m^2} K_2^2}{\frac{\partial Z}{\partial \alpha^P}} > 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{d\gamma} = \frac{d}{d\gamma}\left[\frac{K_1}{\alpha^P}\right] < 0 \\
\frac{d\alpha^P}{dx} &= -\frac{\alpha^P}{\frac{\partial Z}{\partial \alpha^P}} < 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{dx} = \frac{d}{dx}\left[\frac{K_1}{\alpha^P}\right] > 0 \\
\frac{d\alpha^P}{dn} &= -\frac{\alpha^P K_2^2 \frac{\gamma\left(n^2+m^2\right) + 2\gamma\left(1-\alpha^P\right) mn}{\left[n^2-m^2\right]^2}}{\frac{\partial Z}{\partial \alpha^P}} < 0 \quad \Rightarrow \quad \frac{dV_1^{PI}}{dn} = \frac{d}{dn}\left[\frac{K_1}{\alpha^P}\right] > 0.
\end{aligned}$$

Likewise,

$$\frac{d\alpha^P}{dK_1} = \frac{\mu_1}{\frac{\partial Z}{\partial \alpha^P}}, \tag{18}$$

so that

$$\frac{dV_1^{PI}}{dK_1} = \frac{d}{dK_1}\left[\frac{K_1}{\alpha^P}\right] = \frac{\overbrace{\alpha^P - K_1\frac{d\alpha^P}{dK_1}}^{\equiv T}}{\left[\alpha^P\right]^2}.$$

Using (18) with (17), we can write T as

$$\begin{aligned} T &= \frac{\overbrace{\alpha^P\left[x - K_2\left[\mu_2 + \frac{\gamma\left(n + \left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]\right] - \mu_1 K_1}^{=Z=0} + \left(\alpha^P\right)^2\frac{\gamma m}{n^2-m^2}K_2^2}{x - K_2\left[\mu_2 + \frac{\gamma(n+(1-\alpha^P)m)}{n^2-m^2}K_2\right] + \alpha^P\frac{\gamma m}{n^2-m^2}K_2^2} \\ &= \frac{\left(\alpha^P\right)^2\frac{\gamma m}{n^2-m^2}K_2^2}{\underbrace{x - K_2\left[\mu_2 + \frac{\gamma\left(n + \left(1-\alpha^P\right)m\right)}{n^2-m^2}K_2\right]}_{>0} + \alpha^P\frac{\gamma m}{n^2-m^2}K_2^2} > 0. \end{aligned}$$

Thus, $dV_1^{PI}/dK_1 > 0$.

Finally note that α^P is defined by $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. Consequently, we get the same comparative statics results for $\pi^{A|PI}$ as for $\pi^{A|PO}$.

Proof of Proposition 7.

Recall that α^P is defined by (9) (see Proposition 6). For $m = 0$ we can write condition (9) as

$$\alpha^P = \frac{\mu_1 K_1}{\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n}K_2\right]\right)x}.$$

Clearly, $\alpha^P = \alpha^{NP}$ for $m = 0$. This implies that $V_1^{PO} = V_1^{NP}$ for $m = 0$. Moreover, recall from Proposition 6 that $dV_1^{PI}/dm < 0$. Consequently, $V_1^{PI} < V_1^{NP}$ for all $m > 0$.

Next, note that the condition $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$, which defines α^P, can be written as

$$\left(1-\alpha^P\right)\left(1-\beta^{PI}\right) - \left(1-\alpha^{PO}\right)\left(1-\beta^{PO}\right) = 0. \tag{19}$$

Moreover, recall that β^{PI} and β^{PO} are given by

$$\beta^{PI} = \frac{K_2}{x}\left[\mu_2 + \frac{\gamma\left(n - \alpha^P m\right)}{n^2 - m^2}K_2\right] \qquad \beta^{PO} = \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2 - m^2}K_2\right].$$

We can immediately see that $\beta^{PI} = \beta^{PO}$ for $m = 0$. This implies that $\alpha^P = \alpha^{PO}$ for $m = 0$, and therefore $\alpha^P = \alpha^{PO}$ for $m = 0$. Hence, $V_1^{PI} = V_1^{PO}$ for $m = 0$. Moreover, it is easy to see that $\beta^{PI} < \beta^{PO}$ for all $m > 0$. And using (19) we get

$$\frac{\partial \alpha^P}{\partial \beta^{PI}} = -\frac{1 - \alpha^P}{1 - \beta^{PI}} < 0.$$

This implies that $\alpha^P > \alpha^{PO}$ for all $m > 0$. Consequently, $V_1^{PI} < V_1^{PO}$ for all $m > 0$. $\square$

Proof of Proposition 8.

The profit for A is given by

$$\pi^{A|i} = \left(1 - \alpha^i\right)\left(1 - \beta^i\right)x = \left(1 - \beta^i\right)x - \mu_1 K_1,$$

with $i \in \{NP, PO, PI\}$. Using the expressions for β^{NP} (see (2)) and β^{PO} (see (11) with $\alpha^P = 0$) we get

$$\pi^{A|NP} = \left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n}K_2\right]\right)x - \mu_1 K_1 \tag{20}$$

$$\pi^{A|PO} = \left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2 - m^2}K_2\right]\right)x - \mu_1 K_1. \tag{21}$$

It is easy to see that $\pi^{A|NP} = \pi^{A|PO}$ when $m = 0$. Moreover, note that $d\pi^{A|PO}/dm < 0$. Thus, $\pi^{A|NP} > \pi^{A|PO}$ for all $m > 0$.

Likewise, we can immediately see that $d\pi^{A|i}/d\mu_1$, $d\pi^{A|i}/d\mu_2$, $d\pi^{A|i}/d\gamma$, $d\pi^{A|i}/dK_1$, $d\pi^{A|i}/dK_2 < 0$, and $d\pi^{A|i}/dx > 0$, $i \in \{NP, PO\}$. Moreover, $d\pi^{A|NP}/dn > 0$, and

$$\frac{d\pi^{A|PO}}{dn} = -\frac{\gamma\left[n^2 - m^2\right] - 2\gamma n^2}{\left[n^2 - m^2\right]^2}K_2^2 = \frac{\gamma\left[n^2 + m^2\right]}{\left[n^2 - m^2\right]^2}K_2^2 > 0.$$

Next, recall from Proof of Proposition 7 that $\alpha^P = \alpha^{PO}$ when $m = 0$. For $m > 0$ we have $\alpha = \alpha^P$, where α^P is defined by $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. This implies that $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. Consequently, $\pi^{A|NP} > \pi^{A|PI} > \pi^{A|PO}$ for all $m > 0$.

We can immediate see from (20) that $d\pi^{A|NP}/dm = 0$. Moreover, it is easy to see from (21) that $d\pi^{A|PO}/dm < 0$. And because $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$, this also implies that $d\pi^{A|PI}/dm < 0$.

Finally recall that α^P is defined by $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. Consequently, we get the same comparative statics results for $\pi^{A|PI}$ as for $\pi^{A|PO}$. □

Investor Returns - Derivations.

We first derive and compare R_{23} for the different investor constellations. Using the expressions of V_2^{NP}, V_2^{PO}, and V_2^{PI}, we get

$$R_{23}^{NP} = \mu_2 + \frac{\gamma}{n} K_2 \quad R_{23}^{PO} = \mu_2 + \gamma \frac{n}{n^2 - m^2} K_2 \quad R_{23}^{PI} = \mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2.$$

Recall from Propositions 3 and 4 that $V_2^{PO} < V_2^{HPI} < V_2^{NP} < V_2^{MPI}$. Thus, $R_{23}^{PO} > R_{23}^{HPI} > R_{23}^{NP} > R_{23}^{MPI}$.

Next consider R_{12}. Using the expressions of V_1^{PO} and V_2^{PO} we get

$$R_{12}^{PO} = \frac{V_2^{PO} - K_2}{V_1^{PO}} = \frac{\frac{x}{\mu_2 + \gamma \frac{n}{n^2 - m^2} K_2} - K_2}{\frac{1}{\mu_1}\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2\right]\right) x} = \frac{\mu_1}{\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2}.$$

Likewise, using the expressions of V_1^{NP} and V_2^{NP} we find

$$R_{12}^{NP} = \frac{V_2^{NP} - K_2}{V_1^{NP}} = \frac{\frac{\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n} K_2\right]\right) x}{\mu_2 + \frac{\gamma}{n} K_2}}{\frac{1}{\mu_1}\left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma}{n} K_2\right]\right) x} = \frac{\mu_1}{\mu_2 + \frac{\gamma}{n} K_2}.$$

Finally, using V_1^{PI} and V_2^{PI} we get

$$R_{12}^{PI} = \frac{V_2^{PI} - K_2}{V_1^{PI}} = \frac{x - K_2\left[\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2\right]}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2} \frac{\alpha^P}{K_1},$$

where α^P satisfies

$$\alpha^P\left(x - K_2\left[\mu_2 + \frac{\gamma\left(n + \left(1-\alpha^P\right)m\right)}{n^2 - m^2}K_2\right]\right) = \mu_1 K_1. \tag{22}$$

Note that we can rewrite R_{12}^{PI} as

$$R_{12}^{PI} = \frac{\alpha^P\left(x - K_2\left[\mu_2 + \gamma\frac{n + \left(1-\alpha^P\right)m}{n^2 - m^2}K_2\right]\right)\frac{1}{K_1} + \left[\gamma\frac{m}{n^2 - m^2}K_2^2\right]\frac{\alpha^P}{K_1}}{\mu_2 + \gamma\frac{n - \alpha^P m}{n^2 - m^2}K_2}.$$

Using (22) we then get

$$R_{12}^{PI} = \frac{\mu_1 + \gamma\frac{m}{n^2 - m^2}K_2^2\frac{\alpha^P}{K_1}}{\mu_2 + \gamma\frac{n - \alpha^P m}{n^2 - m^2}K_2}.$$

We know from Propositions 3 and 4 that $V_2^{PO} < V_2^{HPI} < V_2^{NP} < V_2^{MPI}$. And according to Proposition 7, $V_1^{PI} < V_1^{PO} < V_1^{NP}$. Moreover, we know from Proposition 6 that $dV_1^{PI}/dm < 0$. Consequently, $V_1^{HPI} < V_1^{MPI} < V_1^{PO} < V_1^{NP}$. This implies that (i) $R_{12}^{HPI} > R_{12}^{PO}$ because $V_2^{PO} < V_2^{HPI}$ and $V_1^{HPI} < V_1^{PO}$, (ii) $R_{12}^{PO} < R_{12}^{MPI}$ because $V_2^{PO} < V_2^{MPI}$ and $V_1^{MPI} < V_1^{PO}$, and (iii) $R_{12}^{NP} < R_{12}^{MPI}$ because $V_2^{NP} < V_2^{MPI}$ and $V_1^{MPI} < V_1^{PO} < V_1^{NP}$. Moreover, it is straightforward to show that $R_{12}^{PO} < R_{12}^{NP}$. Consequently, $R_{12}^{PO} < R_{12}^{NP} < R_{12}^{MPI}$ and $R_{12}^{PO} < R_{12}^{HPI}$.

Finally, using the above expressions for R_{12} and R_{23} we get

$$\begin{aligned} R_{13}^{PO} &= R_{12}^{PO}R_{23}^{PO} = \frac{\mu_1}{\mu_2 + \frac{\gamma n}{n^2 - m^2}K_2}\left[\mu_2 + \gamma\frac{n}{n^2 - m^2}K_2\right] = \mu_1 \\ R_{13}^{NP} &= R_{12}^{NP}R_{23}^{NP} = \frac{\mu_1}{\mu_2 + \frac{\gamma}{n}K_2}\left[\mu_2 + \frac{\gamma}{n}K_2\right] = \mu_1. \end{aligned}$$

Likewise,

$$
\begin{aligned}
R_{13}^{PI} &= R_{12}^{PI} R_{23}^{PI} = \frac{\mu_1 + \gamma \frac{m}{n^2 - m^2} K_2^2 \frac{\alpha^P}{K_1}}{\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2} \left[\mu_2 + \gamma \frac{n - \alpha^P m}{n^2 - m^2} K_2 \right] \\
&= \mu_1 + \underbrace{\gamma \frac{m}{n^2 - m^2} K_2^2 \frac{\alpha^P}{K_1}}_{\equiv Z}.
\end{aligned}
$$

We can immediately see that $R_{13}^{PO} = R_{13}^{NP} = \mu_1$. And because $Z > 0$, we have $R_{13}^{PI} > R_{13}^{PO} = R_{13}^{NP} = \mu_1$. Moreover, recall from Proof of Proposition 6 that $d\alpha^P / dm > 0$ – this implies that $dZ/dm > 0$. Consequently, $R_{13}^{HPI} > R_{13}^{MPI}$. All this implies that $R_{13}^{HPI} > R_{13}^{MPI} > R_{13}^{PO} = R_{13}^{NP} = \mu_1$. $\square$

The Role of Debt – Derivations.

Consider the second financing round. Given r_2 each competitive investor will invest k_2^j, $j = m+1, ..., n$, so that the price per unit of capital equals the marginal cost: $1 + r_2 = \mu_2 + \gamma k_2^j$. Consequently, $k_2^{j|D}(r_2) = [1 + r_2 - \mu_2] / \gamma$. Using $k_2^{j|D}(r_2)$ in the market clearing condition, $K_2^P + (n - m) k_2^{j|D}(r_2) = K_2$, and solving for $1 + r_2$, we get

$$1 + r_2 = \frac{\gamma}{n - m} \left[K_2 - K_2^P \right] + \mu_2. \tag{23}$$

P then chooses K_2^P to maximize his net return

$$\pi_2^{P|D}(K_2^P) = \alpha^P \left(x - (1 + r_2) K_2 \right) + (1 + r_2) K_2^P - \left[\mu_2 K_2^P + \frac{\gamma}{2m} \left(K_2^P \right)^2 \right]. \tag{24}$$

Using (23) we can write (24) as

$$\pi_2^{P|D}(K_2^P) = \alpha^P x + \left(\frac{\gamma}{n - m} \left[K_2 - K_2^P \right] + \mu_2 \right) \left[K_2^P - \alpha^P K_2 \right] - \left[\mu_2 K_2^P + \frac{\gamma}{2m} \left(K_2^P \right)^2 \right].$$

The optimal investment, $K_2^{P|D}$, is then defined by the first-order condition:

$$-\frac{\gamma}{n - m} \left[K_2^P - \alpha^P K_2 \right] + \frac{\gamma}{n - m} \left[K_2 - K_2^P \right] + \mu_2 = \mu_2 + \frac{\gamma}{m} K_2^P.$$

Solving for K_2^P we get

$$K_2^{P|D} = \frac{m}{n+m}\left[1+\alpha^P\right]K_2. \tag{25}$$

Next, using (25) we can rewrite (23) as

$$1+r_2^* = \frac{\gamma}{n-m}\left[K_2 - \frac{m}{n+m}\left[1+\alpha^P\right]K_2\right] + \mu_2 = \mu_2 + \left[\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}\right]K_2. \tag{26}$$

Thus, the cost of debt financing in the second round is given by

$$(1+r_2^*)K_2 = K_2\left[\mu_2 + \left[\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}\right]K_2\right].$$

Moreover, note that the cost of equity financing is given by

$$\beta^{PI}x = K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right].$$

Consequently, $(1+r_2^*)K_2 = \beta^{PI}x$.

Next we derive the outcome for the first financing round. Consider first the PO case (where $\alpha^P = 0$), and let

$$T_2 \equiv (1+r_2^*)K_2 = \beta^{PO}x = K_2\left[\mu_2 + \frac{\gamma n}{n^2-m^2}K_2\right]$$

denote the cost of financing in the second round (which is the same for debt and equity financing). Under debt financing the competitive investor's zero profit condition is given by $(1+r_1)K_1 - \mu_1 K_1 = 0$. Consequently, the equilibrium return is $r_1^C = \mu_1 - 1$. The profit for A, denoted by $\pi^{A|D(PO)}$, is then given by

$$\pi^{A|D(PO)} = x - \left(1+r_1^C\right)K_1 - T_2 = \left(1 - \frac{K_2}{x}\left[\mu_2 + \frac{\gamma n}{n^2-m^2}K_2\right]\right)x - \mu_1 K_1.$$

We can immediately see that $\pi^{A|D(PO)} = \pi^{A|PO}$ (see (21), i.e., for the PO case debt and equity financing are equivalent for A.

Now consider the PI case. The profit for A is then given by

$$\pi^{A|D(PI)} = x - (1 + r_1) K_1 - T_2 = x - (1 + r_1) K_1 - K_2 \left[\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right].$$

Let $\widehat{r}_2$ denote the return which makes A indifferent between accepting $\widehat{r}_2$ from P, and r_1^C from a competitive investor. Formally, $\widehat{r}_2$ is defined by $\pi^{A|D(PI)}(\widehat{r}_2) = \pi^{A|D(PO)}$, which immediately implies that $\widehat{r}_2 = r_1^C = \mu_1 - 1$. Consequently, P offers $r_1^P = \widehat{r}_2 = \mu_1 - 1$. The profit for A is then given by $\pi^{A|D(PI)} = \pi^{A|D(PO)}$. And because $\pi^{A|D(PO)} = \pi^{A|PO}$, we find again that debt and equity financing lead to the same profit for A.

It remains to show that P is strictly better off under equity financing. For this it is sufficient to compare the joint surplus of A and P under equity and debt financing, since $\pi^{A|D(PO)} = \pi^{A|PO}$. Note that P's profit under debt financing can be written as

$$\pi^{P|D(PI)} = \left(1 + r_1^P\right) K_1 + (1 + r_2^*) K_2^P - \mu_1 K_1 - \left[\mu_2 K_2^P + \frac{\gamma}{2m} \left(K_2^P\right)^2 \right].$$

Using $r_1^P = \mu_1 - 1$, (25) with $\alpha^P = 0$, and (26), we can write $\pi^{P|D(PI)}$ as

$$\pi^{P|D(PI)} = K_2^P \left[(1 + r_2^*) - \left[\mu_2 + \frac{\gamma}{2m} K_2^P \right] \right] = \frac{1}{2} \frac{\gamma m}{n^2 - m^2} K_2^2.$$

Thus, the joint surplus for A and P under debt financing, $\Pi^D \equiv \pi^{A|D(PI)} + \pi^{P|D(PI)}$, is given by

$$\begin{aligned} \Pi^D &= x - K_2 \left[\mu_2 + \frac{\gamma n}{n^2 - m^2} K_2 \right] - \mu_1 K_1 + \frac{1}{2} \frac{\gamma m}{n^2 - m^2} K_2^2 \\ &= x - \mu_1 K_1 - \mu_2 K_2 - \frac{\gamma \left(n - \frac{1}{2} m\right)}{n^2 - m^2} K_2^2. \end{aligned}$$

Next we derive the joint surplus for equity financing (with $\alpha^P > 0$). We first note that A's profit is given by

$$\pi^{A|PI} = \left(1 - \alpha^{PI}\right) \left(1 - \beta^{PI}\right) x = \left(1 - \alpha^{PI}\right) \left(1 - \frac{K_2}{x} \left[\mu_2 + \frac{\gamma \left(n - \alpha^P m\right)}{n^2 - m^2} K_2 \right] \right) x.$$

Moreover, the profit for P is given by (14). Thus, the joint surplus under equity financing, $\Pi^E \equiv \pi^{A|PI} + \pi^{P|PI}$, can be written as

$$\begin{aligned}\Pi^E &= x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right] - \mu_1 K_1 + \frac{\gamma\left(1+\alpha^P\right)m}{\left(n+m\right)^2}K_2^2\left[\frac{\left(n-\alpha^P m\right)}{\left(n-m\right)} - \frac{1}{2}\left[1+\alpha^P\right]\right] \\ &= x - K_2\left[\mu_2 + \frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2\right] - \mu_1 K_1 + \frac{1}{2}\frac{\gamma\left(1-\left(\alpha^P\right)^2\right)m}{n^2-m^2}K_2^2.\end{aligned}$$

Finally, we have $\Pi^E > \Pi^D$ if

$$\begin{aligned}-\frac{\gamma\left(n-\alpha^P m\right)}{n^2-m^2}K_2^2 + \frac{1}{2}\frac{\gamma\left(1-\left(\alpha^P\right)^2\right)m}{n^2-m^2}K_2^2 &> -\frac{\gamma\left(n-\frac{1}{2}m\right)}{n^2-m^2}K_2^2 \\ \Leftrightarrow \quad \alpha^P m + \frac{1}{2}\left(1-\left(\alpha^P\right)^2\right)m &> \frac{1}{2}m \\ \Leftrightarrow \quad 2 &> \alpha^P,\end{aligned}$$

which is clearly satisfied. Thus, $\Pi^E > \Pi^D$. And because $\pi^{A|D(PO)} = \pi^{A|PO}$, we can infer that $\pi^{P|D(PO)} < \pi^{P|PO}$.

Cost Advantages versus Cost Disadvantages for Powerful Investor.

Consider first the investment by a powerful insider (PI case) in the second round. Using (10) and C_2^P we get the following expression for P's net return:

$$\begin{aligned}\pi_2^{P|PI}(K_2^P) &= (1-\beta(K_2^P))\alpha^P x + \frac{K_2^P}{K_2}\beta(K_2^P)x - C_2^P \\ &= \alpha^P x - \alpha^P K_2\left[\frac{\gamma}{n-m}\left(K_2-K_2^P\right)+\mu_2\right] + K_2^P\left[\frac{\gamma}{n-m}\left(K_2-K_2^P\right)+\mu_2\right] \\ &\quad - \left[\left(1-\zeta_2\right)\mu_2 K_2^P + \left(1-\vartheta_2\right)\frac{\gamma}{2m}\left(K_2^P\right)^2\right].\end{aligned}$$

The optimal investment $K_2^{P|PI}$ is defined by the first-order condition:

$$\alpha^P K_2 \frac{\gamma}{n-m} + \frac{\gamma}{n-m}\left(K_2 - 2K_2^P\right) = -\zeta_2\mu_2 + \left(1-\vartheta_2\right)\frac{\gamma}{m}K_2^P.$$

Solving for K_2^P we get

$$K_2^{P|PI} = \frac{m}{\gamma\left[n+m-\vartheta_2\left(n-m\right)\right]}\left[\left(1+\alpha^P\right)\gamma K_2+\zeta_2\left(n-m\right)\mu_2\right] \tag{27}$$

Note that $K_2^{P|PI} > 0$ requires that $\left(1+\alpha^P\right)\gamma K_2 > -\zeta_2\left(n-m\right)\mu_2$. We can then immediately see that $\frac{dK_2^{P|PI}}{d\zeta_2} > 0$ and $\frac{dK_2^{P|PI}}{d\vartheta_2} > 0$.

Next, substituting (27) (expression for $K_2^{P|PI}(\zeta_2, \vartheta_2)$) in (10) yields

$$\beta^{PI} = \frac{K_2}{x}\left[\mu_2 + \frac{\gamma\left[n-\alpha^P m-\vartheta_2\left(n-m\right)\right]K_2-\zeta_2 m\left(n-m\right)\mu_2}{n^2-m^2-\vartheta_2\left(n-m\right)^2}\right]. \tag{28}$$

Note that $\frac{d\beta^{PI}}{d\zeta_2} < 0$. And because $V_2^{PI} = \frac{K_2}{\beta^{PI}}$, we have $\frac{dV_2^{PI}}{d\zeta_2} > 0$. Moreover,

$$\begin{aligned}
\frac{d\beta^{PI}(\zeta_2,\vartheta_2)}{d\vartheta_2} &= \frac{K_2}{x}\frac{-\gamma\left(n-m\right)K_2\left[n^2-m^2-\vartheta_2\left(n-m\right)^2\right]}{\left[n^2-m^2-\vartheta_2\left(n-m\right)^2\right]^2} \\
&\quad+\frac{K_2}{x}\frac{\left[\gamma\left[n-\alpha^P m-\vartheta_2\left(n-m\right)\right]K_2-\zeta_2 m\left(n-m\right)\mu_2\right]\left(n-m\right)^2}{\left[n^2-m^2-\vartheta_2\left(n-m\right)^2\right]^2}.
\end{aligned}$$

This is negative if

$$\begin{aligned}
&\left[\gamma\left[n-\alpha^P m-\vartheta_2\left(n-m\right)\right]K_2-\zeta_2 m\left(n-m\right)\mu_2\right]\left(n-m\right)^2 \\
&\qquad< \gamma\left(n-m\right)K_2\left[n^2-m^2-\vartheta_2\left(n-m\right)^2\right],
\end{aligned}$$

which can be simplified to $\gamma\left(1+\alpha^P\right)K_2+\zeta_2\left(n-m\right)\mu_2 > 0$. Note that this condition is satisfied for any $K_2^{P|PI} > 0$. Thus, $\frac{d\beta^{PI}}{d\vartheta_2} < 0$, and therefore, $\frac{dV_2^{PI}}{d\vartheta_2} > 0$.

Now consider the case of a powerful outsider (PO case). Using (27) and (28) with $\alpha^P = 0$, we get

$$K_2^{P|PO} = \frac{m}{\gamma\left[n+m-\vartheta_2\left(n-m\right)\right]}\left[\gamma K_2+\zeta_2\left(n-m\right)\mu_2\right] \tag{29}$$

$$\beta^{PO} = \frac{K_2}{x}\left[\mu_2+\frac{\gamma\left[n-\vartheta_2\left(n-m\right)\right]K_2-\zeta_2 m\left(n-m\right)\mu_2}{n^2-m^2-\vartheta_2\left(n-m\right)^2}\right], \tag{30}$$

where $K_2^{P|PO} > 0$ requires $\gamma K_2 + \zeta_2 (n-m) \mu_2 > 0$. It is then straightforward to show that $\frac{dK_2^{P|PO}}{d\zeta_2} > 0$, $\frac{dK_2^{P|PO}}{d\vartheta_2} > 0$, $\frac{d\beta^{PO}}{d\zeta_2} < 0$, and $\frac{d\beta^{PO}}{d\vartheta_2} < 0$. Consequently, $\frac{dV_2^{PO}}{d\zeta_2} > 0$ and $\frac{dV_2^{PO}}{d\vartheta_2} > 0$.

P invests above pro-rata when $(1-\beta^{PI})\alpha^P + \frac{K_2^{P|PI}(\zeta_2,\vartheta_2)}{K_2}\beta^{PI} > \alpha^P$, which is equivalent to $K_2^{P|PI} > \alpha^P K_2$. Using (27), we can write the pro rata condition as

$$\frac{m}{\gamma\left[n+m-\vartheta_2(n-m)\right]}\left[\left(1+\alpha^P\right)\gamma K_2 + \zeta_2(n-m)\mu_2\right] > \alpha^P K_2.$$

Next, we evaluate this condition at $\alpha^P = \frac{m}{n}$ (where P invests at pro rata in the base model with $\vartheta_2 = \zeta_2 = 0$):

$$\frac{m}{\gamma\left[n+m-\vartheta_2(n-m)\right]}\left[\left(1+\frac{m}{n}\right)\gamma K_2 + \zeta_2(n-m)\mu_2\right] > \frac{m}{n}K_2.$$

This condition simplifies to $\zeta_2 n\mu_2 + \vartheta_2\gamma K_2 > 0$. Note that this condition is satisfied when P has a cost advantage ($\zeta_2, \vartheta_2 > 0$), and violated if P has a cost-disadvantage ($\zeta_2, \vartheta_2 < 0$). Define $\widehat{m}^+$ as the threshold which satisfies $K_2^{P|PI}(\widehat{m}^+) = \alpha^P K_2$ (investment at pro rata) in case of a cost advantage ($\zeta_2, \vartheta_2 > 0$), and define $\widehat{m}^-$ as the threshold which satisfies $K_2^{P|PI}(\widehat{m}^-) = \alpha^P K_2$ in case of a cost-disadvantage ($\zeta_2, \vartheta_2 < 0$). The above derivations then imply that $\widehat{m}^+ < \widehat{m} < \widehat{m}^-$.

Now consider the first round. Suppose for a moment that K_1 is provided by a competitive investor. Recall that $\alpha^{PO} = \frac{\mu_1 K_1}{(1-\beta^{PO})x}$. Because $\frac{d\beta^{PO}}{d\zeta_2} < 0$ and $\frac{d\beta^{PO}}{d\vartheta_2} < 0$, we have $\frac{d\alpha^{PO}}{d\zeta_2} < 0$ and $\frac{d\alpha^{PO}}{d\vartheta_2} < 0$, and therefore, $\frac{dV_1^{PO}}{d\zeta_2} > 0$ and $\frac{dV_1^{PO}}{d\vartheta_2} > 0$.

Now consider the powerful investor P. His participation constraint for the first round is $\pi^{P|PI}(\alpha^P) \geq \pi^{P|PO}$. Recall that P's participation constraint is satisfied for α^P when $\zeta_1 = 0$. Moreover, it is easy to see that $\frac{d\pi^{P|PI}(\alpha^P)}{d\zeta_1} > 0$. Thus, there exists a threshold $\widehat{\zeta}_1$ so that $\pi_1^{P|PI}(\widehat{\zeta}_1) = \pi_1^{P|PO}$. Consequently, $\pi_1^{P|PI}(\zeta_1) < \pi_1^{P|PO}$ for $\zeta_1 < \widehat{\zeta}_1$ (P does not invest at date 1), and $\pi_1^{P|PI}(\zeta_1) > \pi_1^{P|PO}$ for $\zeta_1 > \widehat{\zeta}_1$ (P invests at date 1). If $\zeta_1 > \widehat{\zeta}_1$, P invests K_1 in exchange for the equity stake α^P, which ensures that $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$. Note that $\pi^{A|PI}(\alpha^P) = \left(1-\alpha^P\right)\left(1-\beta^{PI}(\alpha^P)\right)x$ and $\pi^{A|PO} = \left(1-\alpha^{PO}\right)\left(1-\beta^{PO}\right)x$. Using (28), (30), and $\alpha^{PO} = \frac{\mu_1 K_1}{(1-\beta^{PO})x}$, we can write the condition $\pi^{A|PI}(\alpha^P) = \pi^{A|PO}$, which defines α^P, as

$$\begin{aligned} Z \;\equiv\; & \alpha^P\left(x - K_2\left[\mu_2 + \frac{\gamma\left[n+\left(1-\alpha^P\right)m-\vartheta_2(n-m)\right]K_2 - \zeta_2 m(n-m)\mu_2}{n^2-m^2-\vartheta_2(n-m)^2}\right]\right) \\ & -\mu_1 K_1 = 0. \end{aligned}$$

We can immediately see that $\frac{d\alpha^P}{d\zeta_1} = 0$, so that $\dfrac{dV_1^{PI}}{d\zeta_1} = 0$. Moreover,

$$\begin{aligned}
\frac{\partial Z}{\partial \zeta_2} &= \alpha^P K_2 \frac{m(n-m)\mu_2}{n^2 - m^2 - \vartheta_2 (n-m)^2} > 0 \\
\frac{\partial Z}{\partial \alpha^P} &= \underbrace{x - K_2 \left[\mu_2 + \frac{\gamma \left[n + \left(1-\alpha^P\right) m - \vartheta_2 (n-m)\right] K_2 - \zeta_2 m (n-m) \mu_2}{n^2 - m^2 - \vartheta_2 (n-m)^2}\right]}_{= \frac{\mu_1 K_1}{\alpha^P} > 0} \\
&\quad + \alpha^P K_2 \left[\frac{\gamma m K_2}{n^2 - m^2 - \vartheta_2 (n-m)^2}\right] > 0.
\end{aligned}$$

Thus, $\frac{d\alpha^P}{d\zeta_2} = -\frac{\partial Z/\partial \zeta_2}{\partial Z/\partial \alpha^P} < 0$. Consequently, $\frac{dV_1^{PI}}{d\zeta_2} > 0$.

Convexity of Powerful Investor's Opportunity Cost.

Suppose the powerful investor P can invest in two companies: k_A^P in Company A, and k_B^P in Company B, with $k_A^P + k_B^P = \overline{k}^P$. For simplicity we assume that both companies are symmetric, i.e., they generate the same returns $x_A = x_B \equiv x$, and require the same amounts of capital $K_A = K_B \equiv K$. Moreover, suppose there are n_A competitive and symmetric fringe investors, each investing k_A^f in Company A. Likewise, there are n_B competitive and symmetric fringe investors, each investing k_B^f in Company B.

The profit for the powerful investor is given by

$$\pi^P = \frac{k_A^P}{K} \beta_A x + \frac{k_B^P}{K} \beta_B x - C\left(\overline{k}^P\right),$$

where β_i, $i = A, B$, is the total equity issued by Company i, and $C\left(\overline{k}^P\right)$ is the cost of investing the total amount $\overline{k}^P$.

The market clearing conditions for the two companies are

$$k_i^P + n_i k_i^f = K, \quad i = A, B.$$

We know from our base model that $k_i^f = \frac{1}{\gamma}\left[\frac{1}{K}\beta_i x - \mu\right]$. Substituting this expression in the market clearing condition and solving for β_i we get $\beta_i = \frac{K}{x}\left[\frac{\gamma}{n_i}\left(K - k_i^P\right) + \mu\right]$. Using this we can write P's profit function as follows:

$$\pi^P = \underbrace{k_A^P\left[\frac{\gamma}{n_A}\left(K - k_A^P\right) + \mu\right]}_{\equiv R_A} + \underbrace{k_B^P\left[\frac{\gamma}{n_B}\left(K - k_B^P\right) + \mu\right]}_{\equiv R_B} - C\left(\overline{k}^P\right),$$

where R_i is P's return from investing in Company i, $i = A, B$. We know from our base model that R_i is increasing in k_i^P for $k_i^P \leq k_i^{P*}$. We assume that $\overline{k}^P \leq k_i^{P*}$, i.e., P's budget constraint is binding, and he can only invest the optimal amount in one company. Using $k_B^P = \overline{k}^P - k_A^P$ we get

$$\pi^P(k_A^P) = k_A^P\left[\frac{\gamma}{n_A}\left(K - k_A^P\right) + \mu\right] + \left(\overline{k}^P - k_A^P\right)\left[\frac{\gamma}{n_B}\left(K - \overline{k}^P + k_A^P\right) + \mu\right] - C\left(\overline{k}^P\right).$$

Suppose P wants to increase his investment k_A^P in Company A. Note that the return R_B is decreasing in k_A^P. Thus, the marginal opportunity cost for P when increasing the investment k_A^P, is $\frac{dC_O(k_A^P)}{dk_A^P} = -\frac{dR_B(k_A^P)}{dk_A^P}$, with

$$\frac{dC_O(k_A^P)}{dk_A^P} = \frac{\gamma}{n_B}\left(K - \overline{k}^P + k_A^P\right) + \mu - \left(\overline{k}^P - k_A^P\right)\frac{\gamma}{n_B} > 0.$$

Furthermore,

$$\frac{d^2C_O(k_A^P)}{d\left(k_A^P\right)^2} = \frac{\gamma}{n_B} + \frac{\gamma}{n_B} > 0.$$

Thus, P's opportunity cost $C_O(k_A^P)$ is convex increasing in k_A^P.

Model with Insider Information.

Let $\widetilde{x} = \widetilde{x}_i$, $i = L, H$, denote the market belief about the return x_i. Using $k_2^j(\beta) = \frac{1}{\gamma}\left[\frac{1}{K_2}\beta\widetilde{x} - \mu_2\right]$ we can write the market clearing condition (4) as follows:

$$K_2^P + (n - m)\frac{1}{\gamma}\left[\frac{1}{K_2}\beta\widetilde{x} - \mu_2\right] = K_2.$$

Solving for β we get the total equity issued to the second-round investors:

$$\beta(K_2^P, \widetilde{x}) = \frac{K_2}{\widetilde{x}} \left[\frac{\gamma}{n-m} \left(K_2 - K_2^P \right) + \mu_2 \right]. \tag{31}$$

We can infer from (31) that P's signaling strategy depends on whether he wants to invest below or above pro-rata. Below pro-rata ($m < \widehat{m}$), P wants to signal x_H to reduce $\beta(K_2^P, \widetilde{x})$, and therefore to increase valuations. Above pro-rata ($m > \widehat{m}$), P wants to signal x_L to increase $\beta(K_2^P, \widetilde{x})$, and therefore to reduce valuations.

The objective function of P can be written as

$$\begin{aligned}
\max_{K_2^P} \pi_2^{P|PI}(K_2^P) &= (1 - \beta(K_2^P, \widetilde{x}))\alpha^P x + \frac{K_2^P}{K_2} \beta(K_2^P, \widetilde{x}) x - \left[\mu_2 K_2^P + \frac{\gamma}{2m} \left(K_2^P \right)^2 \right] \\
&= \alpha^P x - \alpha^P \frac{x}{\widetilde{x}} K_2 \left[\frac{\gamma}{n-m} \left(K_2 - K_2^P \right) + \mu_2 \right] \\
&\quad + K_2^P \frac{x}{\widetilde{x}} \left[\frac{\gamma}{n-m} \left(K_2 - K_2^P \right) + \mu_2 \right] - \left[\mu_2 K_2^P + \frac{\gamma}{2m} \left(K_2^P \right)^2 \right].
\end{aligned}$$

The optimal investment, denoted $K_2^{P*}(x, \widetilde{x})$, is then defined by the first-order condition:

$$\alpha^P \frac{x}{\widetilde{x}} K_2 \frac{\gamma}{n-m} + \frac{x}{\widetilde{x}} \left[\frac{\gamma}{n-m} \left(K_2 - K_2^P \right) + \mu_2 \right] - K_2^P \frac{x}{\widetilde{x}} \frac{\gamma}{n-m} = \mu_2 + \frac{\gamma}{m} K_2^P.$$

Solving for K_2^P we get

$$K_2^{P*}(X) = \frac{m \left[X K_2 \left[1 + \alpha^P \right] + \frac{n-m}{\gamma} \left[X - 1 \right] \mu_2 \right]}{\left(2X - 1 \right) m + n}, \tag{32}$$

where $X = x / \widetilde{x}$. Moreover,

$$\frac{dK_2^P(X)}{dX} = \frac{m \left[K_2 \left(1 + \alpha^P \right) + \Omega \mu_2 \right] \left(\left(2X - 1 \right) m + n \right) - 2m^2 \left[X K_2 \left(1 + \alpha^P \right) + \Omega \left[X - 1 \right] \mu_2 \right]}{\left[\left(2X - 1 \right) m + n \right]^2},$$

where $\Omega = (n-m)/\gamma$. The derivative is positive if

$$\begin{aligned} K_2\Theta\left((2X-1)m+n\right) + \Omega\mu_2\left((2X-1)m+n\right) &> 2mXK_2\Theta + 2m\Omega\left[X-1\right]\mu_2 \\ \Leftrightarrow \quad K_2\Theta\left(n-m\right) + \Omega\mu_2\left(m+n\right) &> 0. \end{aligned}$$

This condition is satisfied because $m < n$. Thus, $dK_2^P(X)/dX > 0$.

Now consider the case where P wants to invest below pro-rata ($m < \widehat{m}$), and would therefore like to signal a high value x_H to increase the valuation. In a separating equilibrium, P will invest $K_2^P(x_L)$ in the low state $x = x_L$ (instead of $K_2^P(x_H)$), and $K_2^P(x_H)$ in the high state $x = x_H$ (instead of $K_2^P(x_L)$). Thus, in a separating equilibrium the following two truth-telling constraints need to be satisfied:

$$\pi_2^{P|PI}\left(x_L, K_2^P(x_L), \widetilde{x}_L\right) \geq \pi_2^{P|PI}\left(x_L, K_2^P(x_H), \widetilde{x}_H\right) \quad (33)$$

$$\pi_2^{P|PI}\left(x_H, K_2^P(x_H), \widetilde{x}_H\right) \geq \pi_2^{P|PI}\left(x_H, K_2^P(x_L), \widetilde{x}_L\right). \quad (34)$$

Because $\widetilde{x}_L = x_L$ in the separating equilibrium, we can immediately infer from (32) that $K_2^{P*}(x_L)$ equals the equilibrium investment for the full information case, $K_2^{P|PI}$:

$$K_2^{P*}(x_L) = K_2^{P|PI} = \frac{mK_2\left[1+\alpha^P\right]}{m+n}.$$

The optimal investment in the high state $x = x_H$, $K_2^{P*}(x_H)$, is then defined by the binding truth-telling constraint (34).

Next we show that $\pi_2^{P|PI}\left(x_H, K_2^P(x_H) = K_2^{P|PI}, \widetilde{x}_H\right) > \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)$. This would imply that there exists a $K_2^P(x_H) \neq K_2^{P|PI}$ which satisfies the binding truth-telling constraint (34). Note that

$$\begin{aligned}
\pi_2^{P|PI}\left(x_H, K_2^P(x_H) = K_2^{P|PI}, \widetilde{x}_H\right) &= \left(1-\beta(K_2^{P|PI}, x_H)\right)\alpha^P x_H + \frac{K_2^{P|PI}}{K_2}\beta(K_2^{P|PI}, x_H)x_H \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right] \\
\pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right) &= \left(1-\beta(K_2^{P|PI}, x_L)\right)\alpha^P x_H + \frac{K_2^{P|PI}}{K_2}\beta(K_2^{P|PI}, x_L)x_H \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].
\end{aligned}$$

Thus, $\pi_2^{P|PI}\left(x_H, K_2^P(x_H) = K_2^{P|PI}, \widetilde{x}_H\right) > \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)$ is equivalent to

$$\left(\frac{K_2^{P|PI}}{K_2} - \alpha^P\right)\beta(K_2^{P|PI}, x_H) > \left(\frac{K_2^{P|PI}}{K_2} - \alpha^P\right)\beta(K_2^{P|PI}, x_L). \tag{35}$$

Note that

$$\frac{K_2^{P|PI}}{K_2} - \alpha^P = \frac{1}{K_2}\frac{mK_2\left[1+\alpha^P\right]}{m+n} - \alpha^P = \frac{m - n\alpha^P}{m+n}.$$

This is negative if $\alpha^P > m/n$, which is the case when P wants to invest below pro-rata. Thus, condition (35) is equivalent to $\beta(K_2^{P|PI}, x_H) < \beta(K_2^{P|PI}, x_L)$. We can immediately see from (31) that this is always satisfied. Thus, $\pi_2^{P|PI}\left(x_H, K_2^P(x_H) = K_2^{P|PI}, \widetilde{x}_H\right) > \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)$. This implies that there exists a $K_2^{P*}(x_H)$, with $K_2^{P*}(x_H) > K_2^{P|PI}$, which is defined by the binding truth-telling constraint (34).

It remains to derive a sufficient condition so that the truth-telling constraint (33) is satisfied with strict inequality for $K_2^{P*}(x_H)$. We first note that $\pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_L\right)$ has an inverted U-shape, and is maximized at $K_2^P = K_2^{P|PI} = \frac{mK_2\left[1+\alpha^P\right]}{m+n}$. Moreover, for $x_H \to x_L$ the binding truth-telling constraint (34) becomes $\pi_2^{P|PI}\left(x_L, K_2^P(x_H), \widetilde{x}_L\right) = \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_L\right)$, which implies that $K_2^{P*}(x_H) = K_2^{P|PI}$ for $x_H \to x_L$. Consequently, the truth-telling constraint (33) is a strict equality for $x_H \to x_L$. Furthermore, it is easy to see that $d\pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_L\right)/dx_H =$

0. Thus, we need to show that $d\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)/dx_H < 0$. This would imply that the truth-telling constraint (33) is a strict inequality for $x_H > x_L$ and $K_2^{P*}(x_H)$. Note that

$$\begin{aligned}
\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right) &= \left(1-\beta(K_2^{P*}(x_H), x_H)\right)\alpha^P x_L + \frac{K_2^{P*}(x_H)}{K_2}\beta(K_2^{P*}(x_H), x_H)x_L \\
&\quad - \left[\mu_2 K_2^{P*}(x_H) + \frac{\gamma}{2m}\left(K_2^{P*}(x_H)\right)^2\right] \\
&= \left(x_L - \frac{x_L}{x_H}K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P*}(x_H)\frac{x_L}{x_H}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P*}(x_H) + \frac{\gamma}{2m}\left(K_2^{P*}(x_H)\right)^2\right].
\end{aligned}$$

Thus,

$$\frac{d\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)}{dx_H} = \left.\frac{d\pi_2^{P|PI}(\cdot)}{dK_2^P}\right|_{K_2^P = K_2^{P*}(x_H)} \cdot \frac{dK_2^{P*}(x_H)}{dx_H} + \frac{\partial \pi_2^{P|PI}(\cdot)}{\partial x_H}.$$

We immediately get

$$\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)}{\partial x_H} = \left(\alpha^P K_2 - K_2^{P*}(x_H)\right)\frac{x_L}{(x_H)^2}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right].$$

Next, recall that $\pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_H\right)$ is maximized at $K_2^P = K_2^P(X)$, with $X = x/\widetilde{x}$; see (32). Moreover, recall that $dK_2^P(X)/dX > 0$. Clearly, $x_L/x_H < x_L/x_L = x_H/x_H = 1$. Hence, $K_2^P(x_L, \widetilde{x} = x_H) < K_2^{P|PI}$. And because $K_2^{P|PI} < K_2^{P*}(x_H)$, we have $K_2^P(x_L, \widetilde{x} = x_H) < K_2^{P*}(x_H)$. This implies that

$$\left.\frac{d\pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_H\right)}{dK_2^P}\right|_{K_2^P = K_2^{P*}(x_H)} < 0.$$

Next, recall that $K_2^{P*}(x_H)$ is implicitly defined by the binding truth-telling constraint (34). Implicitly differentiating $K_2^{P*}(x_H)$ w.r.t. x_H yields

$$\frac{dK_2^{P*}(x_H)}{dx_H} = -\frac{\frac{\partial \pi_2^{P|PI}\left(x_H, K_2^{P*}(x_H), \widetilde{x}_H\right)}{\partial x_H} - \frac{\partial \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)}{\partial x_H}}{\left.\frac{\partial \pi_2^{P|PI}\left(x_H, K_2^{P}, \widetilde{x}_H\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_H)}}. \tag{36}$$

We have already shown that $K_2^{P*}(x_H) > K_2^{P|PI}$. Therefore,

$$\left.\frac{\partial \pi_2^{P|PI}\left(x_H, K_2^{P}, \widetilde{x}_H\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_H)} < 0.$$

Moreover, using (31) with $X = x_H/\widetilde{x}_H = 1$, we get

$$\begin{aligned}
\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_H), \widetilde{x}_H\right) &= \left(1 - \beta(K_2^{P*}(x_H), x_H)\right)\alpha^P x_H + \frac{K_2^{P*}(x_H)}{K_2}\beta(K_2^{P*}(x_H), x_H)x_H \\
&\quad - \left[\mu_2 K_2^{P*}(x_H) + \frac{\gamma}{2m}\left(K_2^{P*}(x_H)\right)^2\right] \\
&= \left(x_H - K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P*}(x_H)\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P*}(x_H) + \frac{\gamma}{2m}\left(K_2^{P*}(x_H)\right)^2\right].
\end{aligned}$$

Note that $\partial \pi_2^{P|PI}\left(x_H, K_2^{P*}(x_H), \widetilde{x}_H\right)/\partial x_H = \alpha^P$. Moreover, we have

$$\begin{aligned}
\pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right) &= \left(x_H - \frac{x_H}{x_L}K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P|PI}\frac{x_H}{x_L}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].
\end{aligned}$$

Consequently,

$$\frac{\partial \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_L\right)}{\partial x_H} = \alpha^P - \left(\alpha^P K_2 - K_2^{P|PI}\right)\frac{1}{x_L}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right].$$

Using all this we can write (36) as

$$\frac{dK_2^{P*}(x_H)}{dx_H} = \frac{\left(\alpha^P K_2 - K_2^{P|PI}\right)\frac{1}{x_L}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right]}{\underbrace{-\left.\frac{\partial \pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_H\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_H)}}_{>0}}.$$

Note that

$$\alpha^P K_2 - K_2^{P|PI} = \alpha^P K_2 - \frac{mK_2\left[1+\alpha^P\right]}{m+n} = \frac{n\alpha^P - m}{m+n}K_2.$$

This is positive if $\alpha^P > \frac{m}{n}$, which is the case when P invests below pro-rata ($m < \widehat{m}$). Consequently, $dK_2^{P*}(x_H)/dx_H > 0$.

Finally, we can write the total derivative as follows:

$$\frac{d\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)}{dx_H} = \underbrace{\left.\frac{d\pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_H\right)}{dK_2^P}\right|_{K_2^P = K_2^{P*}(x_H)}}_{<0} \cdot \underbrace{\frac{dK_2^{P*}(x_H)}{dx_H}}_{>0} + Z_1,$$

where

$$Z_1 = \left(\alpha^P K_2 - K_2^{P*}(x_H)\right)\frac{x_L}{(x_H)^2}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_H)\right) + \mu_2\right].$$

We have already shown that $\alpha^P K_2 - K_2^{P|PI} > 0$ when P invests below pro-rata. Moreover, recall that $K_2^{P*}(x_H) > K_2^{P|PI}$. Thus, we can either have $Z_1 \leq 0$ or $Z_1 > 0$. We can see that $d\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_H), \widetilde{x}_H\right)/dx_H < 0$ when x_L is sufficiently small. Thus, there exists a threshold $\widehat{x}_L$ so that we have a separating equilibrium for $x_L < \widehat{x}_L$ (which is a sufficient condition) when P invests below pro-rata (i.e., $m < \widehat{m}$).

Next consider the case where P wants to invest above pro-rata ($m > \widehat{m}$), and would therefore like to signal a low value x_L to reduce the valuation. Again, the two truth-telling constraints (33) and (34) must then be satisfied in a separating equilibrium.

In the separating equilibrium we have $\widetilde{x}_L = x_L$, so that $K_2^{P*}(x_H)$ equals the equilibrium investment for the full information case, i.e., $K_2^{P*}(x_H) = K_2^{P|PI}$; see (32) (equation K_2^{P*}). The optimal investment in the low state $x = x_L$ is then defined by the binding truth-telling constraint (33).

Next we show that $\pi_2^{P|PI}\left(x_L, K_2^P(x_L) = K_2^{P|PI}, \widetilde{x}_L\right) > \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)$. We have

$$\begin{aligned}
\pi_2^{P|PI}\left(x_L, K_2^P(x_L) = K_2^{P|PI}, \widetilde{x}_L\right) &= \left(1 - \beta(K_2^{P|PI}, x_L)\right)\alpha^P x_L + \frac{K_2^{P|PI}}{K_2}\beta(K_2^{P|PI}, x_L)x_L \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right] \\
\pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right) &= \left(1 - \beta(K_2^{P|PI}, x_H)\right)\alpha^P x_L + \frac{K_2^{P|PI}}{K_2}\beta(K_2^{P|PI}, x_H)x_L \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].
\end{aligned}$$

Note that $\pi_2^{P|PI}\left(x_L, K_2^P(x_L) = K_2^{P|PI}, \widetilde{x}_L\right) > \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)$ is then equivalent to

$$\left(\frac{K_2^{P|PI}}{K_2} - \alpha^P\right)\beta(K_2^{P|PI}, x_L) > \left(\frac{K_2^{P|PI}}{K_2} - \alpha^P\right)\beta(K_2^{P|PI}, x_H). \tag{37}$$

Recall that $K_2^{P|PI}/K_2 - \alpha^P = \left(m - n\alpha^P\right)/(m+n)$. This is positive if $\alpha^P < m/n$, which is true when P wants to invest above pro-rata. Hence, condition (37) simplifies to $\beta(K_2^{P|PI}, x_L) > \beta(K_2^{P|PI}, x_H)$, which, according to (31), is always satisfied. Consequently, $\pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_L\right) > \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)$. Moreover, it is straightforward to show that $\pi_2^{P|PI}\left(x_L, K_2^P = 0, \widetilde{x}_L\right) < \pi_2^{P|PI}\left(x_L, K_2^P = 0, \widetilde{x}_H\right)$ for all $\alpha^P > 0$. Thus, there exists a $K_2^{P*}(x_L)$, with $K_2^{P*}(x_L) < K_2^{P|PI}$, which is defined by the binding truth-telling constraint (33).

Next we derive a sufficient condition so that the truth-telling constraint (34) is satisfied with strict inequality for $K_2^{P*}(x_L)$. Recall that $\pi_2^{P|PI}\left(x_H, K_2^P(x_H), \widetilde{x}_H\right)$ has an inverted U-shape, and is maximized at $K_2^P = K_2^{P|PI}$. Suppose for a moment that $x_L \to x_H$. The truth-telling constraint (33) then becomes $\pi_2^{P|PI}\left(x_H, K_2^P(x_L), \widetilde{x}_H\right) = \pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_H\right)$, so that $K_2^{P*}(x_L) = K_2^{P|PI}$ for $x_L \to x_H$. Moreover, we can see that $d\pi_2^{P|PI}\left(x_H, K_2^{P|PI}, \widetilde{x}_H\right)/dx_L = 0$. Consequently, we need to show that $d\pi_2^{P|PI}\left(x_H, K_2^P(x_L), \widetilde{x}_L\right)/dx_L > 0$. In this case the truth-telling constraint (34) is a strict inequality for $x_L < x_H$ and $K_2^{P*}(x_L)$.

Note that

$$\begin{aligned}
\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right) &= \left(1-\beta(K_2^{P*}(x_L), x_L)\right)\alpha^P x_H + \frac{K_2^{P*}(x_L)}{K_2}\beta(K_2^{P*}(x_L), x_L)x_H \\
&\quad - \left[\mu_2 K_2^{P*}(x_L) + \frac{\gamma}{2m}\left(K_2^{P*}(x_L)\right)^2\right] \\
&= \left(x_H - \frac{x_H}{x_L}K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P*}(x_L)\frac{x_H}{x_L}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P*}(x_L) + \frac{\gamma}{2m}\left(K_2^{P*}(x_L)\right)^2\right].
\end{aligned}$$

Consequently,

$$\frac{d\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right)}{dx_L} = \frac{d\pi_2^{P|PI}(\cdot)}{dK_2^P}|_{K_2^P = K_2^{P*}(x_L)} \cdot \frac{dK_2^{P*}(x_L)}{dx_L} + \frac{\partial \pi_2^{P|PI}(\cdot)}{\partial x_L}.$$

Note that

$$\frac{\partial \pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right)}{\partial x_L} = \left(\alpha^P K_2 - K_2^{P*}(x_L)\right)\frac{x_H}{(x_L)^2}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right].$$

Recall that $\pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_L\right)$ is maximized at $K_2^P = K_2^P(X)$, with $X = x/\widetilde{x}$; see (32). Furthermore, we have shown that $dK_2^P(X)/dX > 0$. Notice that $x_H/x_L > x_L/x_L = x_H/x_H = 1$. This implies that $K_2^P(x_H, \widetilde{x} = x_L) > K_2^{P|PI}$. Moreover, recall that $K_2^{P*}(x_L) < K_2^{P|PI}$. Thus, $K_2^{P*}(x_L) < K_2^P(x_H, \widetilde{x} = x_L)$, which also implies that

$$\left.\frac{d\pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_L\right)}{dK_2^P}\right|_{K_2^P = K_2^{P*}(x_L)} > 0.$$

Next, using the binding truth-telling constraint (33), which defines $K_2^{P*}(x_L)$, we get

$$\frac{dK_2^{P*}(x_L)}{dx_L} = -\frac{\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^{P*}(x_L), \widetilde{x}_L\right)}{\partial x_L} - \frac{\partial \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)}{\partial x_L}}{\left.\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_L\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_L)}}.$$

Recall that $K_2^{P*}(x_L) < K_2^{P|PI}$. Consequently,

$$\left.\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_L\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_L)} > 0.$$

Moreover, note that

$$\begin{aligned}
\pi_2^{P|PI}\left(x_L, K_2^{P*}(x_L), \widetilde{x}_L\right) &= \left(1 - \beta(K_2^{P*}(x_L), x_L)\right)\alpha^P x_L + \frac{K_2^{P*}(x_L)}{K_2}\beta(K_2^{P*}(x_L), x_L)x_L \\
&\quad - \left[\mu_2 K_2^{P*}(x_L) + \frac{\gamma}{2m}\left(K_2^{P*}(x_L)\right)^2\right] \\
&= \left(x_L - K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P*}(x_L)\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P*}(x_L) + \frac{\gamma}{2m}\left(K_2^{P*}(x_L)\right)^2\right].
\end{aligned}$$

Clearly, $\partial \pi_2^{P|PI}\left(x_L, K_2^{P*}(x_L), \widetilde{x}_L\right)/\partial x_L = \alpha^P$. Furthermore,

$$\begin{aligned}
\partial \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right) &= \left(x_L - \frac{x_L}{x_H}K_2\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right]\right)\alpha^P \\
&\quad + K_2^{P|PI}\frac{x_L}{x_H}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right] \\
&\quad - \left[\mu_2 K_2^{P|PI} + \frac{\gamma}{2m}\left(K_2^{P|PI}\right)^2\right].
\end{aligned}$$

Thus,

$$\frac{\partial \pi_2^{P|PI}\left(x_L, K_2^{P|PI}, \widetilde{x}_H\right)}{\partial x_L} = \alpha^P - \left(\alpha^P K_2 - K_2^{P|PI}\right)\frac{1}{x_H}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right) + \mu_2\right].$$

Therefore we get

$$\frac{dK_2^{P*}(x_L)}{dx_L} = -\frac{\left(\alpha^P K_2 - K_2^{P|PI}\right)\frac{1}{x_H}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P|PI}\right)+\mu_2\right]}{\underbrace{\left.\frac{\partial\pi_2^{P|PI}\left(x_L, K_2^P, \widetilde{x}_L\right)}{\partial K_2^P}\right|_{K_2^P = K_2^{P*}(x_L)}}_{>0}}.$$

It is straightforward to show that $\alpha^P K_2 - K_2^{P|PI} = \frac{n\alpha^P - m}{m+n}K_2$, which is negative if $\alpha^P < m/n$. Note that this is true when P invests above pro-rata ($m > \widehat{m}$). Hence, $dK_2^{P*}(x_L)/dx_L > 0$.

Using all this we can eventually write the total derivative as follows:

$$\frac{d\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right)}{dx_L} = \underbrace{\frac{d\pi_2^{P|PI}\left(x_H, K_2^P, \widetilde{x}_L\right)}{dK_2^P}|_{K_2^P = K_2^{P*}(x_L)}}_{>0}\underbrace{\frac{dK_2^{P*}(x_L)}{dx_L}}_{>0} + Z_2,$$

where

$$Z_2 = \left(\alpha^P K_2 - K_2^{P*}(x_L)\right)\frac{x_H}{(x_L)^2}\left[\frac{\gamma}{n-m}\left(K_2 - K_2^{P*}(x_L)\right)+\mu_2\right].$$

Recall that $\alpha^P K_2 - K_2^{P|PI} < 0$ when P invests above pro-rata. Moreover, we have shown that $K_2^{P*}(x_L) < K_2^{P|PI}$. Hence, it is possible that either $Z_2 \geq 0$ or $Z_2 < 0$. Clearly, we always have $d\pi_2^{P|PI}\left(x_H, K_2^{P*}(x_L), \widetilde{x}_L\right)/dx_L > 0$ when x_H is sufficiently small. This implies that there exists a threshold $\widehat{x}_H$ so that we have a separating equilibrium for $x_H < \widehat{x}_H$ (which, again, is a sufficient condition) when P invests above pro-rata (i.e., $m > \widehat{m}$).

To summarize, P's investments in the separating equilibrium are as follows:

- **Below pro-rata case ($m < \widehat{m}$):** P overinvests in the high state ($K_2^{P*}(x_H) > K_2^{P|PI}$). In the low state, P invests as before ($K_2^{P*}(x_L) = K_2^{P|PI}$).
- **Above pro-rata case ($m > \widehat{m}$):** P underinvests in the low state ($K_2^{P*}(x_L) < K_2^{P|PI}$). In the high state, P invests as before ($K_2^{P*}(x_H) = K_2^{P|PI}$).

Endogenous Market Power.

Consider the PO case and assume that the number of competitive fringe investors, denoted by n^f, is fixed. Moreover, suppose there are $\overline{J} \geq 2$ companies. P can then choose the number J, with $J \leq \overline{J}$, of companies that he wants to invest in. We first assume that there is no fixed cost associated with each investment. We then explain how P's optimal portfolio choice changes with a strictly positive fixed cost. For parsimony we omit the subscripts indicating the second round.

Each competitive investor provides $k_j(\beta_j) = \frac{1}{\gamma}\left[\frac{1}{K}\beta_j x - \mu\right]$, where β_j is the equity issued by company j, and k_j is the amount a competitive investor invests in company j. The market clearing condition for company j is then given by $K_j^P + n^f \frac{1}{\gamma}\left[\frac{1}{K}\beta_j x - \mu\right] = K$, where K_j^P is P's investment in company j, and K is the total capital requirement (which we assume to be the same for all companies). Solving for β_j we get $\beta_j(K_j^P) = \frac{K}{x}\left[\frac{\gamma}{n^f}\left(K - K_j^P\right) + \mu\right]$. We assume full symmetry so that $K^P \equiv K_1^P = K_1^P = ... = K_J^P$ and $\beta \equiv \beta_1 = \beta_2 = ... = \beta_J$.

Suppose that P has the amount $\overline{K}^P$ available for investments. P's budget constraint is therefore $JK^P \leq \overline{K}^P$. For simplicity we assume that P has just enough capital $\overline{K}^P$ to make the optimal investment $K^{P|PO}$ in one company. Formally, $\overline{K}^P = K^{P|PO}$. This implies that the budget constraint is always binding. Thus, $K^P = \frac{\overline{K}^P}{J}$. Using this together with the expression for $\beta_j(K_j^P)$, we can write P's profit function is follows:

$$\pi^{P|PO}(J) = \sum_{j=1}^{J} \frac{K_j^P}{K}\beta_j(K_j^P)x - C^P(\overline{K}^P) = \overline{K}^P\left[\frac{\gamma}{n^f}\left(K - \frac{\overline{K}^P}{J}\right) + \mu\right] - C^P(\overline{K}^P).$$

We can immediately see that $\pi^{P|PO}(J)$ is increasing in J. Consequently, in the absence of any fixed costs, we have $J^* = \overline{J} \geq 2$. With a strictly positive fixed cost associated with each investment, we have $1 \leq J^* \leq \overline{J}$.

Next we show that P's profit from investing in a given company, is increasing and concave in m. Using $\beta(K^P) = \frac{K}{x}\left[\frac{\gamma}{n^f}\left(K - K^P\right) + \mu\right]$ we get the following objective function for P:

$$\pi^{P|PO}(K^P) = \frac{K^P}{K}\beta(K^P)x - C^P(K^P) = K^P\left[\frac{\gamma}{n^f}\left(K - K^P\right) + \mu\right] - \left[\mu K^P + \frac{\gamma}{2m}\left(K^P\right)^2\right].$$

P's optimal investment $K^{P|PO}$ is defined by the first-order condition: $\frac{\gamma}{n^f}\left(K - 2K^P\right) + \mu = \mu + \frac{\gamma}{m}K^P$. Solving for K^P we get $K^{P|PO} = \frac{m}{n^f + 2m}K$. Substituting this expression into P's objective function we get

$$\pi^{P|PO} = \frac{m}{n^f + 2m}K\left[\frac{\gamma}{n^f}\left(K - \frac{m}{n^f + 2m}K\right) - \frac{\gamma}{2m}\frac{m}{n^f + 2m}K\right] = \frac{1}{2}\frac{\gamma m}{n^f\left(n^f + 2m\right)}K^2.$$

Finally, it is straightforward to show that $\pi^{P|PO}$ is increasing and concave increasing in m:

$$\begin{aligned}\frac{d\pi^{P|PO}}{dm} &= \frac{1}{2}\frac{\gamma}{n^f}K^2\frac{n^f+2m-2m}{[n^f+2m]^2}=\frac{1}{2}\gamma K^2\frac{1}{[n^f+2m]^2}>0\\ \frac{d^2\pi^{P|PO}}{dm^2} &= -2\frac{1}{2}\gamma K^2\frac{2}{[n^f+2m]^3}=-2\gamma K^2\frac{1}{[n^f+2m]^3}<0.\end{aligned}$$

Model with Endogenous Staging.

Our base model assumes exogenous staging. Empirically, staging is pervasive in venture financing. From a theoretical lens, staging is a standard assumption in the literature. Still it is useful to briefly discuss its justification. Instead of providing staged financing (K_1 at date 1 and K_2 at date 2), investors could in principle provide all the required capital ($K_1 + K_2$) upfront at date 1. Intuitively, this is imprudent because too much capital could be poured into a loosing project. Formally we only need to add the possibility that, in case of early failure, the company spends the remaining funds inefficiently. This way, no investor would be willing to provide all the capital upfront. For a simple model of this, suppose there is an observable (but not verifiable) signal that the company receives just prior to date 2. If the signal is positive, the project has a positive NPV and deserves an investment K_2, just like in our main model. However, if the signal is negative, the project has a negative NPV. For simplicity, assume that in case of a bad signal, the project has a probability $\varepsilon(K_2)$ of success, where ε is very small. While no investor would want to continue the project after a negative signal, the company would want to do so. This is because in case of termination it receives zero returns, compared to $(1-\alpha)(1-\beta)\varepsilon x$ in case of continuation.[33] Anticipating this, investors prefer staged financing, investing K_1 at date 1, and only investing K_2 at date 2 in case of a positive signal. We conclude that providing all the capital upfront is inefficient since it gives the company incentives to spend it all. Staged financing, by contrast, eliminates this incentive and thus ensures a more efficient capital utilization.

[33]Theoretically it is possible to allow for some renegotiation at this stage. However, this would generate a large wealth transfer to the company in case of a bad signal. Standard moral hazard and adverse selection arguments can then be invoked to explain why investors would want to avoid this.

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