Targeted Carbon Tax Reforms*

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Abstract

In the presence of intersectoral linkages, sector-specific carbon tax changes can have complex general equilibrium effects. In particular, a carbon tax on the emissions of a sector can lead to an increase in aggregate emissions. We analytically characterise how incremental taxes on the emissions of any set of sectors affect aggregate emissions. We show that carbon tax reforms that target sectors based on their position in the production network can achieve a greater reduction in aggregate emissions than reforms that target sectors based on their direct emissions alone. We illustrate the effects of carbon tax reforms by calibrating our intersectoral network model to the economies of two countries.

Keywords: emissions tax, carbon tax, pollution tax, climate change, environmental tax reform, input-output linkages, intersectoral network.

JEL Classification: D51, D62, H23, Q54.

*We would like to thank Daron Acemoglu, David Baqaee, Ozan Candogan, Justin Caron, Emanuele Ciola, Luís Fonseca, Cameron Hepburn, Ernest Liu, Linus Mattauch, Adrian Odenweller, Rick van der Ploeg, two anonymous referees, and participants at the 2018 Symposium on Social and Economic Networks (Monash Business School) and at the 2018 EEA-ESEM Congress for their comments. Teytelboym is grateful for the support of the Oxford Martin Programme on the Post-Carbon Transition.

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Final accepted version: September 2019.

1 Introduction

To have a reasonable chance of avoiding the dangerous effects of climate change, cumulative anthropogenic carbon emissions must be limited to one trillion tonnes (Allen et al., 2009, Meinshausen et al., 2009). Achieving this target requires coordinated global action and a substantial increase in the price of carbon (High-Level Commission on Carbon Prices, 2017). However, current carbon pricing policies around the world are piecemeal and are imposed only on certain sectors (World Bank and Ecofys, 2018). In this paper, we analyse the effects of incremental (or marginal) sector-specific carbon tax changes, which we refer to as *carbon tax reforms*. In the presence of intersectoral linkages, such reforms can have complex general equilibrium effects on aggregate emissions.

We analyse the effects of carbon tax reforms in an *intersectoral input-output network* in which firms trade intermediate inputs and sell their goods to a representative consumer. A sector's use of different intermediate inputs produces emissions directly and indirectly. For example, a factory might use coal, electricity, and labour. For every unit of coal that it uses, the factory produces emissions directly. But when the factory uses electricity, it does not produce emissions directly; rather the utility company that supplies the factory is likely to produce emissions in the process of electricity generation. Therefore, by using electricity the factory also produces emissions indirectly. The impact of taxing the emissions of a sector on *aggregate* emissions will therefore be determined by all the intersectoral linkages.

We assume that the government can only implement *incremental* sector-specific carbon taxes. The main contribution of our paper is to track the propagation of the effects of such taxes throughout the economy. Our analysis focuses on marginal tax changes and ignores damages resulting from emissions.[1]

Our main result characterises the effects of carbon tax reforms in the presence of intersectoral linkages. We show that the effect of any multi-sector carbon tax reform can be linearly decomposed into the *aggregate emissions impacts* of the taxed sectors. The aggregate emissions impact of a sector is the change—which can be positive or negative—in aggregate emissions that results from incrementally taxing the emissions of that sector alone.

[1] In contrast, the analysis of optimal, non-marginal carbon taxes would require the policy-maker either to know the damages from emissions or to specify an emissions reduction target. The calculation of optimal carbon taxes is usually done with sophisticated Integrated Assessment Models (IAMs) that combine a {multi-sectoral, multi-regional, dynamic, stochastic, computable} general equilibrium model of the economy and an earth science model (Metz et al., 2001). These models are broadly of two types. Policy evaluation IAMs, such as GCAM, IMAGE, and MESSAGE, used by the Intergovernmental Panel on Climate Change, calculate the most cost-effective paths to a fixed emissions target (e.g. net zero carbon emissions by 2050). Policy optimisation IAMs, such as DICE/RICE, PAGE, and FUND additionally include a damage function (that maps emissions to the effects on output and consumption) which allows them to calculate the socially optimal level of cumulative emissions.

We refer to sectors with negative aggregate emissions impacts as *key sectors*. Therefore, a policymaker, who is interested in emissions reduction but is (politically or technologically) constrained to incrementally taxing the emissions of only certain sectors, targets key sectors with the greatest aggregate emissions impacts (in absolute value). The carbon tax reform that delivers the greatest reduction in aggregate emissions—the *most effective* carbon tax reform—imposes a carbon tax on a sector if and only if it is a key sector.

A sectoral carbon tax affects aggregate emissions in three ways. First, a taxed sector's production mix switches from more polluting to less polluting direct inputs. The fall in the sector's input demands reduces the output and emissions of all direct and indirect suppliers to the sector. Second, the taxed sector's output price rises which causes its buyers to switch away from using the sector's good as an input and reduces the output and emissions of all its direct and indirect buyers. The magnitudes of these upstream and downstream influences of a sector are a function of the sector's position within the intersectoral production network. Third, the tax revenue is rebated to the consumer who is therefore able to consume more from *every* sector. This *tax rebate effect* (or aggregate demand channel) increases sectoral output and aggregate emissions.

Whether taxing the emissions of a sector has a positive or negative impact on aggregate emissions depends on (i) the tax rebate effect, (ii) the sector's intersectoral influence on emissions, and (iii) the sector's level of emissions relative to aggregate emissions. If the sector's emissions are sufficiently high, the sector will always be a key sector. But even sectors with low emissions might have high intersectoral influence on emissions in the intersectoral network, and could therefore be key sectors. However, if the sector's emissions are low, then the tax rebate effect can exceed the other effects, netting a positive effect on aggregate emissions following a sectoral carbon tax.

We calibrate our model to the economies of the United States and Pakistan and estimate the aggregate emissions impacts of each industrial sector. For Pakistan's economy, we find that while the non-metallic minerals sector (e.g. cement) emits approximately three times less carbon than the electricity production sector, taxing the emissions of the non-metallic minerals sector would result in a greater fall in aggregate emissions than taxing the emissions of the electricity sector. The fact that the non-metallic minerals sector has a greater aggregate emissions impact (in absolute value) than the electricity sector is a result of the centrality of the non-metallic minerals sector in the intersectoral network. Moreover, taxing the emissions of the non-metallic minerals and electricity sectors alone would deliver most of the emissions reduction from the most effective carbon tax reform.

Our paper connects two strands of the literature: intersectoral input-output networks and environmental tax reforms. The first strand is a rapidly growing literature that looks at the propagation of shocks in an interconnected economy. In the zero-tax benchmark, our general equilibrium model is a static version of Long Jr. and Plosser's (1983) real business cycles model which is analysed by Acemoglu et al. (2012). In this paper, we analyse the effects of adding emissions and emissions taxes to that model. Sectors produce emissions according to an emissions function which is assumed to have constant returns to scale in the inputs to production and we place no restrictions on the structure of the input-output network. We therefore generalise the insights of Baylis et al. (2013) and Baylis et al. (2014) who pointed out that carbon taxes in some sectors can lead to emissions reductions in other sectors thereby causing "negative leakage".[2,3] The upstream and downstream effects of carbon tax reforms in our model are similar to the structure of technology shock propagation in other models (Shea, 2002, Baqaee, 2018, Huremovic and Vega-Redondo, 2016).[4] In an important contribution, Baqaee (2016) showed that in the presence of intersectoral linkages fiscal policy could be targeted at certain sectors in order to maximise impact on employment and output; our results have a similar flavour. Our set-up is also related to models used in the input-output analysis of carbon content of consumption and production (e.g. Turner et al., 2007, Wiedmann et al., 2007, Wiedmann, 2009, Davis and Caldeira, 2010, Caron et al., 2017).

The second strand is a rich literature on environmental tax reforms. This work stems from the classic public economics literature on tax reforms (Buchanan, 1976, Feldstein, 1976, Guesnerie, 1977, Weymark, 1981).[5] One of the key questions in the environmental tax reforms literature is whether the shift of the tax burden away from employment and income towards consumer-harming pollution makes the consumer better off (Copeland, 1994, Bovenberg and De Mooij, 1994, Bovenberg and van der Ploeg, 1994, Bovenberg and Goulder, 1996, Bovenberg and van der Ploeg, 1996).[6] In contrast, in our model, emissions do not affect utility directly i.e. emissions are not an externality. In the absence of externalities or other distortions, optimal carbon taxes are zero in our benchmark economy. To motivate the reason for government intervention, we imagine that the government simply has an exogenous reason to reduce carbon emissions (e.g. adhering to the Nationally Determined Contribution

[2] The possibility of an overall positive effect on aggregate emissions resulting from sectoral taxes in our model is different from the channels described in carbon leakage (i.e. offshoring of production due to domestic emission pricing; see, for example, Babiker, 2005). The tax rebate to the consumer is central to our results, but it plays little role in carbon leakage.

[3] Jarke and Perino (2017) discuss negative leakage in the context of incomplete cap-and-trade schemes.

[4] Since taxes introduce non-linearities into our model, we solve our model using the first-order approach suggested by Acemoglu et al. (2015).

[5] See Myles (1995, Chapter 6) for a summary. In this spirit, Allouch (2017) and Allouch and King (2019) examine the social welfare impacts of small and incremental budget-balanced transfers in networks with private provision of local public goods. Galeotti et al. (2018) analyse targeting policies in networks in which a planner has a budget to affect the marginal benefits of agents' actions.

[6] See Bovenberg and Goulder (2002, Section 3) for a summary.

as part of the Paris Agreement) and is constrained to implementing incremental sector-specific carbon taxes. Moreover, we assume that all the tax revenue is rebated in full to the consumer and that the economy has no unemployment or distortions (such as income or commodity taxes) prior to the introduction of a carbon tax reform. However, we expect that the crux of our analysis would go through even in an economy with preexisting distortions (Drèze and Stern, 1987, 1990).

This paper is organised as follows. Section 2 introduces the model. Section 3 provides the benchmark solution in the absence of emissions taxation. Section 4 considers the effect of carbon tax reforms on sectoral consumption, labour demand, intermediate input use, output, and emissions. Section 5 introduces aggregate emissions impacts, identifies the key sectors, and characterises the most effective carbon tax reform. Section 6 presents our calibration results. Section 7 concludes. The Appendix provides the proofs and further details of the calibration.

2 Model

2.1 Sectors

Our model and notation build directly on Acemoglu et al. (2012). Each of n competitive sectors produces a distinct good. Sector i produces output Y_i according to the Cobb-Douglas production function

$$Y_i = l_i^{1-\alpha} \prod_{j=1}^{n} x_{ij}^{\alpha w_{ij}}, \tag{2.1}$$

where $w_{ij} \geq 0$ is the share of expenditure on input j in sector i's expenditure on intermediate inputs, $\alpha \in (0,1)$ is the share of intermediate goods in production, x_{ij} is the input demand of sector i for the good produced by sector j, and l_i is sector i's demand for labour.[7] The matrix $W = [w_{ij}]$ is the economy's *input-output* matrix. We assume that $\sum_{j=1}^{n} w_{ij} = 1$ for all i which ensures constant returns to scale in production.

The emissions of sector i are determined by a constant returns to scale emissions function $E_i(x_{i1}, ..., x_{in})$ that is increasing (and differentiable) in intermediate inputs.[8] Sectoral emissions are a function of inputs directly rather than a function of sectoral output. Therefore, depending on its input mix, a sector can produce the same level of output at different levels of emissions.

[7]Since our focus is not on total factor productivity shocks, we normalise the usual TFP parameter to 1.

[8]We will take the second-order conditions of the profit maximisation problem to be satisfied e.g. when E_i is linear.

The profit of (a representative firm in) sector i is given by

$$\pi_i = p_i Y_i - \sum_{j=1}^{n} p_j x_{ij} - \omega l_i - \lambda_i t \omega E_i, \tag{2.2}$$

where ω is the competitive wage rate and t is the tax rate on emissions. We anchor the value of the tax to the wage so $t\omega$ is a per-unit tax on emissions. Here, $\lambda_i \in \{0, 1\}$ is a parameter such that sector i taxed if and only if $\lambda_i = 1$. By selecting an appropriate vector $\lambda \in \{0, 1\}^n$ we can analyse the effects of emissions taxes on any set of sectors.

2.2 Consumer

A representative consumer inelastically supplies one unit of labour ($l = 1$) that can be hired by the sectors, and has Cobb-Douglas preferences over consumption with utility function

$$U(C_1, ..., C_n) = \prod_{i=1}^{n} C_i^{1/n}. \tag{2.3}$$

The utility function does not depend on the level of emissions. Emissions are not an externality in our model but rather act as a friction on the production side in the presence of emissions taxes. We take the government's reasons to reduce emissions as exogenously given and we are not attempting to estimate the *socially optimal level* of emissions.

We assume perfect mobility of labour across sectors so there is a unique competitive wage ω. The government redistributes the emissions tax revenue in full to the consumer. Therefore, the consumer's budget constraint is given by

$$\sum_{i=1}^{n} p_i C_i \leq \omega l + T = \omega + T, \tag{2.4}$$

where $T = t\omega \sum_{i=1}^{n} \lambda_i E_i$ is the total emissions tax revenue. There are no profits from sector ownership due to the constant returns to scale exhibited by production and emissions.

3 Zero-tax benchmark

When the emissions tax is zero, our economy essentially coincides with the economy presented in Acemoglu et al. (2012).

Definition 1. A *competitive equilibrium* consists of prices $p_1, \ldots, p_n$, a wage ω, consumption levels $C_1, \ldots, C_n$, labour demands $l_1, \ldots, l_n$, and intermediate input quantities x_{ij} for all i, j, such that (i) the consumer maximises her utility subject to her budget constraint, (ii) the sectors maximise their profits, (iii) the markets for each good and labour clear, that is,

$$\sum_{i=1}^{n} l_i = l = 1, \tag{3.1}$$

and, for each i,

$$Y_i = C_i + \sum_{j=1}^{n} x_{ji}. \tag{3.2}$$

In order to characterise the competitive equilibrium, we will employ standard definitions from the input-output literature (Leontief, 1966).

Definition 2. The matrix $V = [v_{ij}] = (I - \alpha W')^{-1}$ is the economy's *Leontief inverse.*[9]

We can alternatively write the Leontief inverse as

$$V = I + \alpha(W') + \alpha^2(W')^2 + \dots \tag{3.3}$$

The ik element of the matrix W', w_{ki}, measures how much of sector k's production depends directly on the use of the good produced by sector i. In Figure 3.1 we represent this dependence as a thin arrow going from i to k (only links with positive weight are shown). The ik element of the matrix $(W')^2$ aggregates all weighted walks of length two going from i to k: it measures how reliant k's production is on the use of good i through its use of the intermediate input j. In the network shown in Figure 3.1, the ik element of $(W')^2$ is equal to the product $w_{kj}w_{ji}$. Similar reasoning applies to the elements of $(W')^h$ for $h > 2$. The element v_{ik} then measures how reliant sector k's production is on the output of sector i taking *all direct and indirect* effects into account. We therefore refer to the term as the *downstream influence* of i on k. We will also interchangeably refer to this term as the *upstream influence* of k on i since v_{ik} also measures how reliant sector i's production is on input demand by sector k taking all direct and indirect effects into account. In Figure 3.1, $v_{ik} = \alpha w_{ki} + \alpha^2 w_{kj} w_{ji}$ is represented by a thick arrow going from i to k.

The Bonacich centrality of sector i is its average downstream influence on others or, equivalently, the average upstream influence of other sectors on i. In other words, it measures direct and indirect reliance of other sectors on sector i's output (Bonacich, 1987, Ballester et al., 2006, Jackson, 2008).

Definition 3. The *Bonacich centrality* of sector i is given by $b_i = \frac{1}{n}\sum_{j=1}^{n} v_{ij}$.

[9] V is well-defined because $\alpha \in (0, 1)$ and W is row-stochastic. Acemoglu et al. (2015) refer to the transposed matrix V' as the *Leontief matrix.*

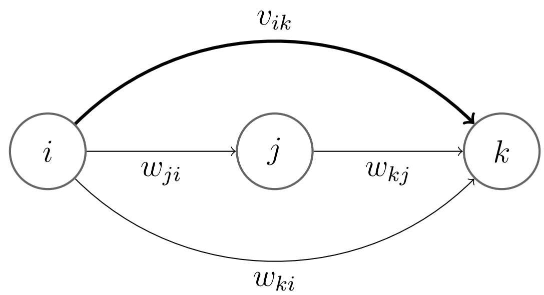

Figure 3.1: Thin arrow $i \xrightarrow{w_{ki}} k$ represents how much of sector k's production depends *directly* on good i. Thick arrow $i \xrightarrow{v_{ik}} k$ represents how much sector k's production depends *directly and indirectly* (via j) on good i.

Proposition 1 shows that b_i is the (relative) value of the sales of sector i in the zero-tax equilibrium (Hulten, 1978).

Proposition 1 (Acemoglu et al., 2012)**.** When $t = 0$ there is a unique competitive equilibrium and it is characterised by

$$p_i^* = \exp\left(-\frac{\theta}{1-\alpha} - \alpha \sum_{j=1}^{n} v_{ji} \sum_{h=1}^{n} w_{jh} \ln(w_{jh}) + \ln(\omega)\right), \quad (3.4)$$

$$C_i^* = \omega/(np_i^*), \quad (3.5)$$

$$s_i^* = \omega b_i, \quad (3.6)$$

$$Y_i^* = s_i^*/p_i^*, \quad (3.7)$$

$$x_{ij}^* = s_i^* \alpha w_{ij}/p_j^*, \quad (3.8)$$

$$l_i^* = s_i^*(1-\alpha)/\omega, \quad (3.9)$$

$$U^* = \omega/p^*, \quad (3.10)$$

where $\theta = (1-\alpha)\ln(1-\alpha) + \alpha\ln(\alpha)$, $p^* = n\prod_{i=1}^{n}(p_i^*)^{1/n}$ is the ideal price index, and the sales of sector i are defined by $s_i = p_i Y_i$. The equilibrium emissions of sector i when $t = 0$ are given by $E_i^* = E_i(x_{i1}^*, ..., x_{in}^*)$.

We anchor all values to the nominal wage ω. Note that in the zero-tax equilibrium utility is simply equal to the real wage (and to real GDP).

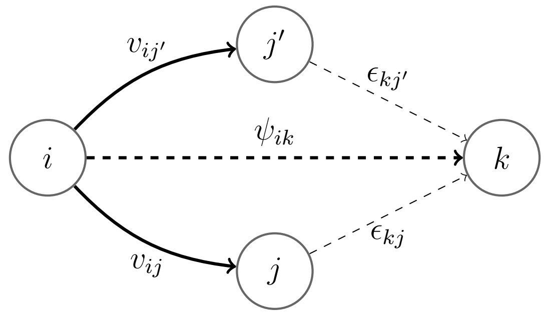

Figure 4.1: Thin dashed arrow $j \xrightarrow{\epsilon_{kj}} k$ represents input j's elasticity of emissions for sector k. Thick dashed arrow $i \xrightarrow{\psi_{ik}} k$ represents how much a change in k's demand for inputs affects sector i following a tax on sector k's emissions.

4 Carbon tax reforms

We now consider the impact of introducing a carbon tax reform around our benchmark zero-tax equilibrium.[10]

Definition 4. For any sector i, *input j's elasticity of emissions* evaluated at $t = 0$ is

$$\epsilon_{ij} = \frac{x_{ij}^*}{E_i^*} \frac{\partial E_i}{\partial x_{ij}}\bigg|_{t=0}. \tag{4.1}$$

For each i and j we assume that $\epsilon_{ij} > 0$ only if $w_{ij} > 0$. We refer to $\mathcal{E} = [\epsilon_{ij}]$ as the *emissions elasticity* matrix or as the *emissions network*.

Emissions elasticities will be crucial in determining how the effects of emissions taxes propagate through intersectoral linkages.

Definition 5. The *downstream emissions influence* of sector i on sector k or, equivalently, the *upstream emissions influence* of sector k on sector i is

$$\psi_{ik} = \sum_{j=1}^{n} v_{ij} \epsilon_{kj}. \tag{4.2}$$

Note that ψ_{ik} is the ik element of the matrix $\Psi = [\psi_{ij}] = V\mathcal{E}'$.

Let us now illustrate ψ_{ik} as upstream emissions influence of k on i (Figure 4.1).[11] Recall that ϵ_{kj} measures how sensitive the emissions of sector k are to the use of input j. Following

[10] When emissions taxes are strictly positive, the system of equations that determines competitive equilibria in our economy becomes highly non-linear. Therefore, we cannot give a closed-form analytical solution to the effects of a tax reform around an arbitrary positive emissions tax.

[11] In general, sector i could be *both* upstream and downstream from sector k. In that case, sector i will be subject to both upstream emissions influence ψ_{ik} and downstream emissions influence ψ_{ki} of k on i. In our analysis of tax reforms these effects turn out to be completely separable.

a tax on its emissions, sector k changes its demand for input j. Since demand for good j has changed, sector j adjusts its demand for its direct (and indirect) inputs. Recall that v_{ij} measures the effect of j's input demand change on sector i (i.e. upstream influence of j on i). Aggregating over all of k's direct inputs (i.e. j and j' in Figure 4.1), ψ_{ik} is a measure of how an emissions tax on sector k would affect the demand for sector i's good via the changes in demand for all of k's direct and indirect inputs.

Armed with precise interpretations of the ψ_{ik} and v_{ik} terms, we are now in a position to trace the effects of carbon tax reforms on relative prices and on real variables (intermediate input use, labour demand, sectoral output, sectoral emissions, and consumption).

Proposition 2. Consider any sectoral tax vector $\lambda \in \{0, 1\}^n$, then[12]

$$\left.\frac{\partial \ln(p_i)}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \frac{v_{hi}}{b_h}, \tag{4.3}$$

$$\left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i} - \frac{v_{hj}}{b_h}\right) - \lambda_i \frac{E_i^*}{b_i} \frac{\epsilon_{ij}}{\alpha w_{ij}}, \tag{4.4}$$

$$\left.\frac{\partial \ln(l_i)}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i}\right), \tag{4.5}$$

$$\left.\frac{\partial \ln(Y_i)}{\partial t}\right|_{t=0} = (1-\alpha) \left.\frac{\partial \ln(l_i)}{\partial t}\right|_{t=0} + \alpha \sum_{j=1}^{n} w_{ij} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}, \tag{4.6}$$

$$\left.\frac{\partial \ln(E_i)}{\partial t}\right|_{t=0} = \sum_{j=1}^{n} \epsilon_{ij} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}, \tag{4.7}$$

$$\left.\frac{\partial \ln(C_i)}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{v_{hi}}{b_h}\right), \tag{4.8}$$

where b_i is the Bonacich centrality of sector i, v_{hj} is the downstream influence of sector h on sector j (equivalently, the upstream influence of j on h), and ψ_{ik} is the downstream emissions influence of sector i on sector k (equivalently, the upstream emissions influence of k on i).

We can unpack the contents of Proposition 2 by considering a special case in which only sector k's emissions are taxed (so $\lambda_k = 1$ and $\lambda_i = 0$ for all $i \neq k$). Let us consider the effect on prices first. Equation (4.3) shows that, when sector k's emissions are taxed, only the prices of goods produced by sectors downstream from k increase relative to the wage. The sectors downstream from k face a new set of relative input prices, which distorts their input mix, thereby reducing their output and increasing their prices. However, the first-order

[12]The derivative of input j with respect to tax for sector i is zero when $w_{ij} = 0$, but in order to economise on notation, we omit this fact in the proposition.

conditions that determine the input choices of sectors upstream (but not downstream) from k do not change: upstream sectors simply face a new level of demand by sector k (directly or indirectly) for their output. In response to the new demands for their goods, sectors upstream from k simply scale the use of *all* their inputs thereby maintaining the same prices and zero profits for all upstream sectors.

Now let us examine the change in sector i's demand for good j, again when only sector k's emissions are taxed (and $i \neq k$). This change is given by the first term in equation (4.4) which, in this case, we can write as

$$E_k^* \Bigg(\underbrace{1}_{\substack{\text{tax}\\ \text{rebate}\\ \text{effect}}} - \underbrace{\frac{\psi_{ik}}{b_i}}_{\substack{\text{relative}\\ \text{upstream}\\ \text{emissions}\\ \text{influence}}} - \underbrace{\frac{v_{kj}}{b_k}}_{\substack{\text{relative}\\ \text{downstream}\\ \text{influence}}} \Bigg). \tag{4.9}$$

The term E_k^* outside the parentheses states the obvious: the higher the emissions of sector k, the greater its impact on the whole network. The first term inside the parentheses is the tax rebate effect (aggregate demand channel): since the consumer has a higher income, she can consume more of good i thereby increasing sector i's demand for all its inputs. The second term is the *relative* upstream emissions influence of sector k on sector i. The reduction in sector k's demand for inputs dampens sector i's demand for inputs. However, the magnitude of this effect is attenuated by the sales (Bonacich centrality) of i. The third term is the *relative* downstream influence of sector k on sector j. When sector k is taxed, its output price rises. This raises the price of downstream sector j's good which, in turn, reduces i's demand for good j. The magnitude of this effect is attenuated by the sales of sector k since taxing the emissions of a sector with large sales (relative to a fixed level of emissions E_k^*) will have a small impact on its price and therefore on the prices of its downstream sectors.

To understand the relevance of the final term in equation (4.4) we need to consider a setting in which sector i's emissions are among those that are taxed (so $\lambda_i = 1$). This final term is always negative and measures the additional *self-distortion* to the first-order conditions of sector i from a tax on its own emissions.

Let us now look at the effects of a tax reform on the remaining variables in the case in which only sector k's emissions are taxed. The labour demand of sector i is affected only by the tax rebate effect and by the relative upstream emissions influence of k on i (see equation 4.5). Since the wage is our anchor price, it remains unchanged, and therefore there is no downstream (or price) effect of an emissions tax on labour demand. The impact of the tax reform on sectoral output and sectoral emissions can be decomposed into weighted sums of changes in the use of the relevant inputs (see equations 4.6 and 4.7). In the case of output, the weights are simply expenditure shares on factors of production, and, in the case of emissions, the weights are emissions elasticities. Finally, from equation (4.8), we can observe that the change in the consumption of good i depends only on the tax rebate effect and on the relative downstream influence of sector k on sector i. This is because the consumer cares only about relative prices of goods and only the prices of goods produced by sectors downstream from k change following an emissions tax on k.

The effect of incremental taxation on the consumer's utility is zero because the tax rebate exactly offsets the relative price changes. Indeed, by equation (4.8) and the definition of Bonacich centrality,

$$\left.\frac{\partial \ln(U)}{\partial t}\right|_{t=0} = \sum_{i=1}^{n} \frac{1}{n} \left.\frac{\partial \ln(C_i)}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \sum_{i=1}^{n} \frac{1}{n} \left(1 - \frac{v_{hi}}{b_h}\right) = 0. \tag{4.10}$$

Since real GDP is equal to utility by Proposition 1, an incremental emissions tax has no effect on real GDP. Of course, if taxes were away from the margin or if utility were a function of emissions, then emissions taxation might have consequences for real GDP and welfare.

5 Targeted carbon tax reforms

In this section we analyse the impact of carbon tax reforms on *aggregate* emissions. We first characterise the effect of taxing the emissions of individual sectors. We introduce two new concepts: *key sectors* and *aggregate emissions impact*. We then consider the effects of taxing the emissions of any set of sectors. Finally, we draw implications for policy.

5.1 Key sectors: taxing the emissions of individual sectors

Let us first characterise the effect of taxing the emissions of one sector on aggregate emissions. Define Υ_i to be the *aggregate emissions impact* of sector i, that is,

$$\Upsilon_i = \left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}},$$

where e_i denotes the ith standard basis vector. Of course, the aggregate emissions impact of a sector i is different from the change in the emissions of sector i following a carbon tax on i alone (that is, $\left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}} \neq \left.\frac{\partial \ln(E_i)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}}$). The following proposition characterises the aggregate emissions impact of sector i.

Proposition 3. The aggregate emissions impact of sector i is given by

$$\Upsilon_i = E_i^* - \sum_{j=1}^{n} \frac{E_i^* E_j^*}{E^*} \left(\frac{\psi_{ji}}{b_j} + \frac{\psi_{ij}}{b_i} \right) - \frac{(E_i^*)^2}{E^*} \left(\sum_{j=1}^{n} \frac{\epsilon_{ij}^2}{\alpha w_{ij} b_i} \right). \tag{5.1}$$

Let us now unpack the terms of equation (5.1). The first term is simply the tax rebate effect. The higher the emissions of sector i, the greater the amount of tax revenue that the government can raise following a tax. As before, this effect makes the consumer wealthier which induces greater spending across the economy, higher output, and higher emissions. This is the only positive term in the expression.[13] We call the second term sector i's *intersectoral emissions influence*: it is a weighted sum of relative upstream and relative downstream emissions influences. Note that the relative downstream emissions influence part is new to Proposition 3 (and did not appear in equation (4.4) of Proposition 2): when i's emissions are taxed, i's price rises which distorts the input mix of its downstream sectors, thereby reducing their output and their emissions. The final term comes from the familiar self-distortion to sector i when its own emissions are taxed (cf. equation 4.4).

In general, Υ_i, the aggregate emissions impact of sector i, can be positive or negative (in the sense that one can construct theoretical examples in which some sectors have a strictly positive aggregate emissions impact and others have a strictly negative impact). It should be clear that if the size of the tax rebate term outweighs the intersectoral emissions influence and the self-distortion terms then the aggregate emissions impact of a sector will be positive.

Definition 6. Sector i is a *key sector* if its aggregate emissions impact is strictly negative.

Using equation (5.1), we can see that a sector is more likely to be a key sector if it has a large intersectoral emissions influence or high emissions. Indeed, the aggregate emissions impact of sector i is generally inverse-U-shaped in sector i's emissions. So, for low levels of sector i's emissions, the tax rebate term might outweigh the self-distortion term (potentially rendering the aggregate emissions impact positive), whereas when sector i's emissions are sufficiently high, the (squared) self-distortion term will dominate the other two effects.

To appreciate how intersectoral linkages affect the relationship between a sector's emissions and its aggregate emissions impact, let us consider what happens in an economy without any intersectoral linkages.

Proposition 4. Assume that $W = I$. Then, for any key sectors i and j,

$$|\Upsilon_i| \geq |\Upsilon_j| \quad \text{if and only if} \quad E_i^* \geq E_j^*.$$

[13] In a partial equilibrium setting in which only a fraction of the tax is rebated to the consumer, this term would be scaled down correspondingly.

When $W = I$, there are no intersectional linkages in the economy. Each sector uses only labour and its own output as inputs.[14] In this case, $\epsilon_{ii} = 1$ for all sectors. When a tax is imposed on the emissions of sector k, the price of good k rises relative to all other prices, while k's input use, output, emissions, and the consumption of good k all decrease. Conversely, the input use, output, and consumption of all other goods ($i \neq k$) rise. Following a sectoral carbon tax on k, the consumer spends the tax rebate equally on the goods of all sectors and the emissions of all sectors but k rise. Therefore, when the emissions of a high-emitting sector k are taxed, the reduction in k's emissions is large and, at the same time, the consumer spends the tax rebate on the goods of sectors less-emitting on average than k. In fact, Proposition 4 shows that, in this economy, a key sector has a higher aggregate emissions impact (in absolute terms) than another key sector if and only if it has higher emissions in the zero-tax equilibrium. Therefore, if a policymaker were interested in reducing emissions by taxing one sector, they could simply look at the key sector with the greatest sectoral emissions. In the presence of intersectoral linkages, however, Proposition 4 no longer holds.[15] Therefore, the sector with the highest aggregate emissions impact might not have the highest sectoral emissions in general.[16]

5.2 Taxing the emissions of multiple sectors

Having characterised the effects of taxing the emissions of any single sector, we now consider the effect of taxing the emissions of any set of sectors. For any set of sectoral emissions taxes $\mu \in \{0, 1\}^n$ we define the aggregate emissions impact of a carbon tax reform as

$$\Upsilon(\mu) = \left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=\mu \\ t=0}}.$$

[14] Acemoglu et al. (2017) also consider a counterfactual "simple economy" without intersectoral linkages by setting $\alpha = 0$. This implies that every sector uses only labour as an input. This counterfactual economy is less appropriate in our setting since we have assumed that emissions do not depend on labour directly so this economy would produce no emissions.

[15] In general, Proposition 4 would also break down in the presence of heterogeneity in consumer preferences, sectoral production functions, or sectoral emissions elasticities. See Remark 1 after the proof of Proposition 4 in Appendix C.

[16] In Appendix E, we study an extension of the economy without intersectoral linkages considered in Proposition 4 that allows for heterogeneity in the consumer's preferences. As consumer expenditure shares are no longer assumed to be equal, the monotonic relationship between sectoral emissions and aggregate emissions impacts predicted in Proposition 4 no longer holds. Yet, we show that accounting for intersectoral linkages remains important for calculating aggregate emissions impacts. To see this, we match sectoral sales of the interconnected economy with those of the economy without linkages by adjusting the consumer's preference weights. Despite having identical sales, the aggregate emissions impacts are quantitatively and qualitatively different in the two economies.

Notice that $\Upsilon_i \equiv \Upsilon(e_i)$. The effect on aggregate emissions of taxing the emissions of any set of sectors can be linearly decomposed into the aggregate emissions impacts of individual sectors.

Proposition 5. For any sectoral tax vector $\lambda \in \{0, 1\}^n$,

$$\Upsilon(\lambda) = \sum_{i=1}^{n} \lambda_i \Upsilon_i. \tag{5.2}$$

As Proposition 5 shows, the impact of a multi-sector carbon tax reform on aggregate emissions is the sum of the aggregate emissions impacts of each taxed sector. In other words, the aggregate emissions impact of taxing the emissions of sectors i and k is equal to the aggregate emissions impact of sector i plus the aggregate emissions impact of sector k. Therefore, taxing the emissions of any key sector will reduce emissions while taxing the emissions of any additional non-key sector will dampen this reduction.

5.3 Policy implications

Let us consider the implications of our analysis for policy. We say that the *most effective* carbon tax reforms are the sets of incremental sectoral taxes that produce the greatest reduction in aggregate emissions (around the zero-tax equilibrium). Let us define the most effective carbon tax reforms formally.

Definition 7. The *most effective* carbon tax reforms are vectors λ^* which satisfy

$$\lambda^* \in \arg\min_{\mu \in \{0,1\}^n} \Big\{ \Upsilon(\mu) \text{ such that } \Upsilon(\mu) < 0 \Big\}. \tag{5.3}$$

Recall that any carbon tax reform leads to no loss in real GDP or utility, but the most effective carbon tax reforms will lead to the greatest reduction in emissions at the margin. The most effective carbon tax reform is, of course, not the socially optimal carbon tax; instead we want to find the steepest gradient of emissions in the direction of an incremental tax.

Taxing the emissions of *all* sectors will unambiguously reduce aggregate emissions as the following proposition shows.

Proposition 6. Taxing the emissions of all sectors reduces aggregate emissions, i.e.,

$$\Upsilon(\mathbf{1}) \leq 0. \tag{5.4}$$

However, because some sectors may have a positive aggregate emissions impact, taxing the emissions of all sectors might not constitute the most effective carbon tax reform, as the following corollary of Proposition 5 shows.

Corollary 1. In the most effective carbon tax reform, sector i's emissions are taxed if and only if i is a key sector.[17]

Corollary 1 gives a stark characterisation of the most effective carbon tax reform by focusing only on whether a sector is key or non-key. However, in practice, due to political or technological constraints, the most effective carbon tax reform may not be achievable. Nevertheless, the *magnitudes* of aggregate emissions impacts (derived in Proposition 3) can serve as a guide to the policymaker who is interested in effective emissions reduction. The policymaker can use the aggregate emissions impacts of sectors to rank sectors according to their importance in reducing emissions in the economy. And, recalling our discussion of Proposition 4, in the presence of intersectoral linkages, the policymaker cannot simply rely on a ranking by sectoral emissions as a basis for the most effective carbon tax reform because sectors with highest emissions might not have the greatest aggregate emissions impact (in absolute value). As Proposition 5 shows, the greatest emissions reduction following a carbon tax reform, is achieved by targeting as many of the key sectors that have the highest aggregate emissions impact (in absolute value) as possible.[18]

6 Calibration

We now illustrate our main results by tracing the effects of carbon tax reforms in a version of our model that is calibrated to real-world data. In the calibration, we will maintain our competitive equilibrium assumption but introduce more heterogeneity in the sectoral production functions and in the representative consumer's preferences. We therefore assume

[17] If the government were also able to *subsidise* certain sectors then the most effective carbon policy reform would be

$$\lambda^* \in \underset{\mu \in \{-1,0,1\}^n}{\arg\min} \Big\{ \Upsilon(\mu) \text{ such that } \Upsilon(\mu) < 0 \text{ and } T > -\omega \Big\}.$$

That is, the government can choose which sectors to tax and subsidise subject to the constraint that the consumer's income remains strictly positive. The solution to this problem is an immediate consequence of Proposition 5: tax all the key sectors and subsidise as many sectors with the largest positive aggregate emissions impacts as possible. The reason subsidies for non-key sectors reduce emissions in our model is that the reduction in the consumer's income (the negative tax rebate effect) exceeds the (positive) emissions influence and self-distortion terms. This policy reform relies heavily on the assumption of fixed technology: if firms can adjust their production functions, they could easily exploit the subsidy while increasing emissions.

[18] Rather than having to optimise over 2^n possible subsets of taxed sectors!

that

$$Y_i = l_i^{1-\alpha_i} \prod_{j=1}^{n} x_{ij}^{\alpha_i w_{ij}}, \tag{6.1}$$

where $\alpha_i \in (0, 1)$ for all sectors i, and that

$$U(C_1, ..., C_n) = \prod_{i=1}^{n} C_i^{\gamma_i}, \tag{6.2}$$

where without loss of generality $\sum_{i=1}^{n} \gamma_i = 1$. Moreover, we assume that the emissions function for each sector i is linear, i.e.

$$E_i(x_{1i}, ..., x_{ni}) = \sum_{j=1}^{n} \eta_{ij} x_{ij}, \tag{6.3}$$

where $\eta_{ij} \geq 0$ measures the emissions intensity of sector i's use of input j.

To calibrate α_i, w_{ij}, γ_i, and η_{ij}, we use data from the Global Trade Analysis Project (GTAP 8) database which contains consistent information on input-output and emissions networks for 57 sectors across many developing and developed economies in 2007 (Narayanan et al., 2012).[19]

In particular, we use data on sectoral sales and intersectoral spending patterns to calibrate the production and consumption sides of our economy, and we use the data on intersectoral emissions to back out sectoral carbon emissions (in tons) as well as the emissions elasticities. The details of our calibration method are given in Appendix D.

We should stress that our calibration is for illustrative purposes only and we ignore important factors such as other taxes and government spending, international trade, and capital investment. We focus on the United States and Pakistan. We chose these countries mainly because they have: (i) a relatively low fraction of trade/GDP (28% and 33% of GDP, respectively); (ii) relatively low general government final consumption expenditure (15% and 10% of GDP, respectively); and (iii) no substantial, explicit, and nationwide carbon taxes.[20]

Since our comparative static results apply only to incremental taxes, we calculate the effects of a small carbon tax of $10 per ton of carbon emissions. We do not suggest that this is an appropriate carbon tax. The World Bank High-Level Commission on Carbon Prices led by Nicholas Stern and Joseph Stiglitz recommended a tax of between $40 and $80 per ton of CO_2 emissions by 2020 (High-Level Commission on Carbon Prices, 2017), which,

[19]The data are available at https://www.gtap.agecon.purdue.edu/databases/archives.asp.
[20]World Bank Open Data: https://data.worldbank.org/.

adjusting for inflation, would correspond to roughly \$32–\$64 per ton of carbon in 2007. Our derivative calculations may not extrapolate well to analysing the effects of carbon taxes of this magnitude. To estimate the percentage change in aggregate emissions as a result of the \$10 tax, we extrapolate linearly around the estimated zero-tax equilibrium using an expression for the aggregate emissions impact that is derived for the generalised model in Appendix D.

Panel 1 in Figure 6.1 plots sectoral emissions against the absolute value of aggregate emissions impacts for all sectors in the United States (on a log scale).[21] We can see that the emissions and the aggregate emissions impact of the electricity production sector are higher than those of any other sector. In general, however, the relationship between emissions and aggregate emissions impacts is not one-to-one: for example, the air transport sector is more polluting than the non-metallic minerals sector, yet the aggregate emissions impact of the latter exceeds that of the former.[22] Panel 2 in Figure 6.1 shows the estimated percentage reduction in aggregate emissions from *individually* taxing the emissions of any of the top twenty key sectors with the highest aggregate emissions impacts. For example, taxing the emissions of the electricity production sector alone gives a 4.893 percent reduction in aggregate emissions. Panel 3 in Figure 6.1 shows the estimated percentage reduction in aggregate emissions of *cumulatively* taxing the emissions of the key sectors with the highest aggregate emissions impacts. For example, taxing the emissions of the electricity sector alone gives a 4.893 percent reduction in emissions while taxing the emissions of the electricity *and* transport sectors together gives a 5.106 percent reduction in emissions.[23] Note that taxing the emissions of the top twenty sectors (out of 57) with the highest aggregate emissions impacts delivers most of the aggregate emissions reduction (of around 6.05%) from an economy-wide \$10 carbon tax reform.

Panel 1 in Figure 6.2 plots sectoral emissions against the absolute value of aggregate emissions impacts for all the key sectors in Pakistan (on a log scale).[24] The emissions of the electricity production sector are around three times higher than those of the non-metallic

[21]Sector codes and descriptions are available on the GTAP website: `https://www.gtap.agecon.purdue.edu/databases/v8/v8_sectors.asp`.

[22]`ely` corresponds to "Electricity: production, collection and distribution". `nmm` corresponds to "Non-Metallic Minerals: cement, plaster, lime, gravel, concrete". `atp` corresponds to "Air transport". `otp` corresponds to "Other Transport: road, rail; pipelines, auxiliary transport activities; travel agencies", which we simply refer to as "transport".

[23]In Appendix D, we explain why the estimated percentage change in aggregate emissions from a tax on the emissions of the electricity (4.893%) and transport sectors (0.224%) individually does not add up exactly to the estimated percentage change from a tax on their emissions jointly (i.e. $4.893+0.224\neq5.106\%$).

[24]The only non-key sector – dwellings – has zero recorded emissions and therefore an aggregate emissions impact of zero. While theoretically possible, we do not find empirical evidence of non-key sectors with a strictly positive aggregate emissions impact in our data.

minerals sector. However, the aggregate emissions impact of the non-metallic minerals sector is approximately four times higher than that of the electricity production sector. From Panels 2 and 3, one can see that, similarly to the US, taxing the emissions of the top twenty sectors would deliver most of the carbon reduction (of 4.35%) from an economy-wide $10 carbon tax reform.

In summary, multi-sector carbon tax reforms in these countries may be poorly judged if they were made on the basis of sectoral emissions alone rather than by looking at the sectoral aggregate emissions impacts. In Appendix E we show that the effects of carbon taxes in our calibration are driven by the structure of the production network and not solely by the sector size distribution.

7 Conclusion

This paper formally analyses carbon taxes in the presence of intersectoral linkages. Our results highlight the importance of considering general equilibrium effects when implementing carbon tax reforms. We provide closed-form expressions for the network effects of carbon tax reforms on output, labour demand, consumption, intermediate input use, as well as aggregate and sectoral emissions. We show that a carbon tax reform imposed on all sectors may not reduce aggregate emissions by the greatest amount. The most effective carbon tax reform involves taxing key sectors: those with a negative aggregate emissions impact. Taxing additional non-key sectors dampens the reduction in aggregate emissions due to the tax rebate effect. We also showed that the magnitudes of the aggregate emissions impacts—rather than sectoral emissions alone—matter a great deal for any effective carbon tax reform.

Our formal analysis is valid only for small changes in carbon taxes: since the system defining our economy is non-linear in the presence of distortionary taxes, the derivatives that we present may not extrapolate well to large carbon taxes. However, the basic logic of our results—that targeted sectoral taxation might be more effective than economy-wide taxation—might well extend to non-marginal tax changes. This suggests that the quantitative gains of adopting targeted sectoral taxation should be considered as a policy scenario in more sophisticated integrated assessment models.

A marginal analysis similar to the one in this paper could be used to examine two policy levers beyond the sector-specific emissions taxes that we have focussed on. Firstly, commonly advocated and used *cap-and-trade* schemes might have complex aggregate and distributional effects in the presence of intersectoral linkages (Montgomery, 1972, Stavins, 2008, Hepburn et al., 2013). Modifying the model to analyse a cap-and-trade scheme would allow us to investigate the economy-wide impacts of two policy changes: either tightening the aggregate emissions cap, or making changes to the allocation of sector-specific emissions permits.[25] Secondly, one could also consider direct taxation of *polluting inputs* in which each sector i could be subjected to input taxes (or subsidies) τ_{ij} that apply to the dirtiness of each input $(\frac{\partial E_i}{\partial x_{ij}} x_{ij})$.[26]

Moreover, in an international trade version of our model, input-specific taxes could allow us to trace the effects of border carbon adjustments (Fischer and Fox, 2012, Helm et al., 2012) and to investigate carbon leakage (Babiker, 2005).

There are several ways to enrich the production side of our economy. One could include firm profits (Baqaee, 2018, Huremovic and Vega-Redondo, 2016), financial frictions (Bigio and La'O, 2016) or other fundamental market distortions (Liu, 2017), firm entry and exit (Baqaee, 2018), dynamics and unemployment (Baqaee, 2016), distributional concerns (Klenert et al., 2016), international trade (Antweiler et al., 2001, Davis and Caldeira, 2010, Bosker and Westbrock, 2014), and production network formation (Acemoglu and Azar, 2017, Oberfield, 2018). Further work could also examine the extent to which our results are affected by existing taxes (Goulder, 1995, Bovenberg and Goulder, 1996, Parry et al., 1999), partial rebates of tax revenue (Metcalf, 2009), and technological progress induced by changes in relative prices (Di Maria and Van der Werf, 2008).

[25] Suppose that the government allocated each sector i an emissions permit allowance $\sigma_i Q$, where Q is the overall emissions cap and σ_i is sector i's share of the cap ($\sum_{i=1}^n \sigma_i = 1$). Permits are competitively traded across firms at price p_Q and the market for emissions permits clears when $\sum_{i=1}^n E_i = Q$. Hence (a representative firm in) each sector i chooses l_i and x_{ij} to maximise $p_i Y_i - \sum_{j=1}^n p_j x_{ij} - \omega l_i - p_Q E_i + p_Q \sigma_i Q$.

[26] In the presence of polluting input taxation, (a representative firm in) each sector i chooses l_i and x_{ij} to maximise $p_i Y_i - \sum_{j=1}^n p_j x_{ij} - \omega l_i - \sum_{j=1}^n \tau_{ij} \frac{\partial E_i}{\partial x_{ij}} x_{ij}$.

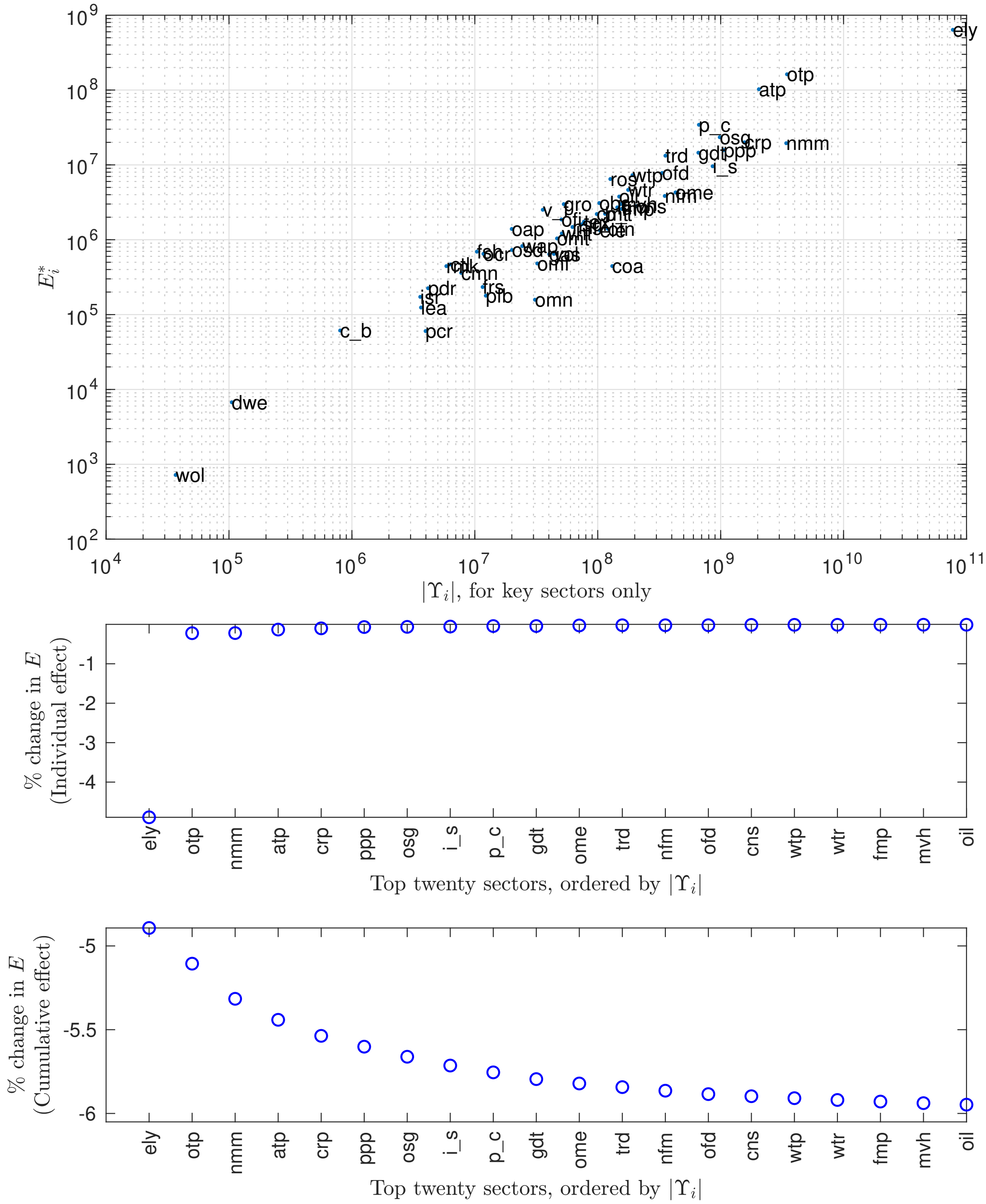

Figure 6.1: Calibration for the United States

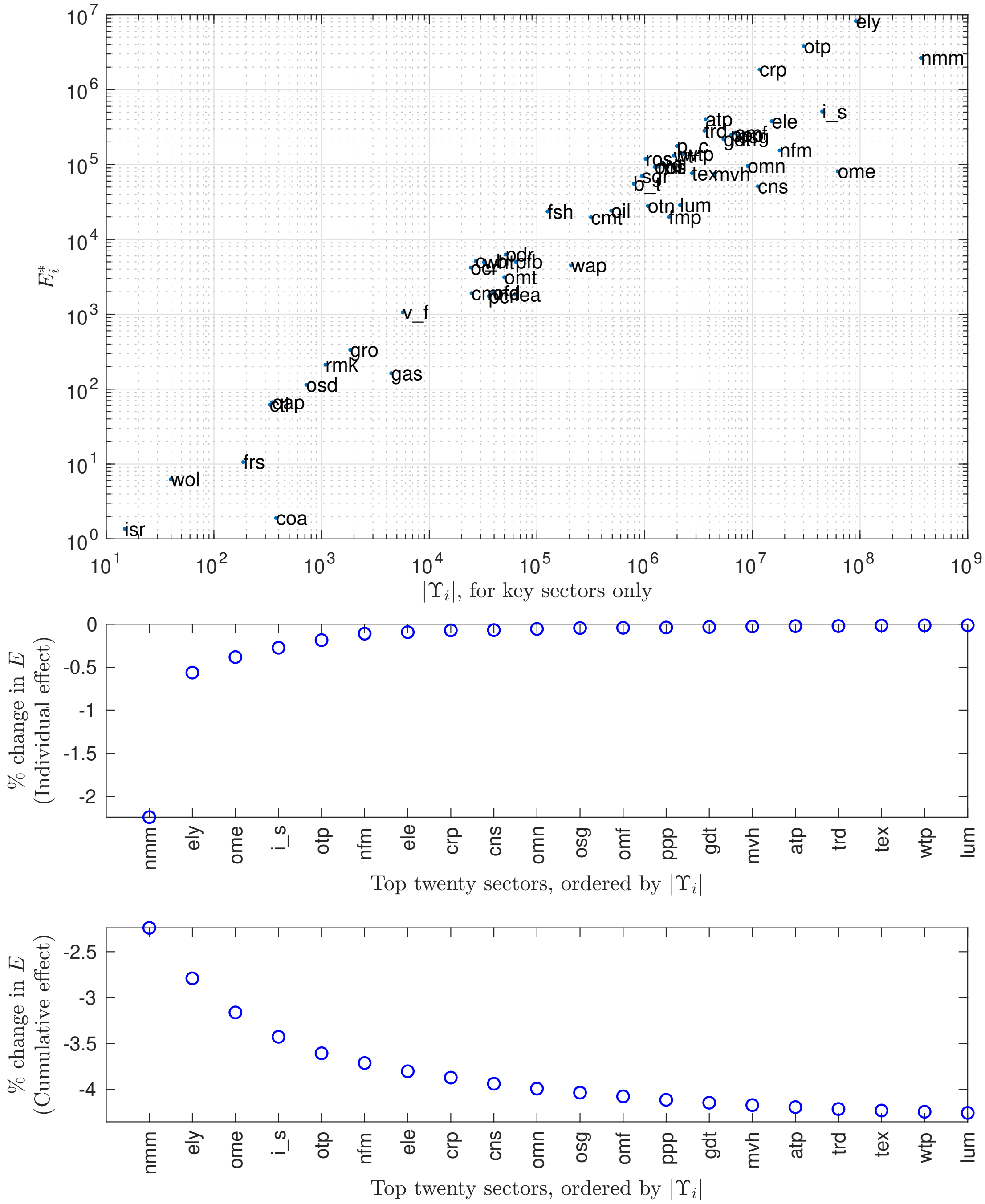

Figure 6.2: Calibration for Pakistan

A Proof of Proposition 1

The steps of this proof closely follow Acemoglu et al. (2015).

The consumer's problem. Due to Cobb-Douglas preferences, the consumer spends a fixed proportion of her income on each good:

$$p_i C_i = \frac{1}{n}(\omega + T). \tag{A.1}$$

Sectoral profit maximisation. The first-order conditions of (a representative firm in) sector i are given by[27]

$$p_i \alpha w_{ij} \frac{Y_i}{x_{ij}} = p_j + \lambda_i t \omega \frac{\partial E_i}{\partial x_{ij}}, \tag{A.2}$$

$$p_i (1-\alpha) \frac{Y_i}{l_i} = \omega. \tag{A.3}$$

Equilibrium prices. For each sector i, define i's sales as $s_i = p_i Y_i$, and let $z_{ij} = \lambda_i t x_{ij} \frac{\partial E_i}{\partial x_{ij}}$. Then we can re-write the sectoral first-order conditions as

$$s_i \alpha w_{ij} = p_j x_{ij} + \omega z_{ij}, \tag{A.4}$$

$$s_i (1-\alpha) = \omega l_i. \tag{A.5}$$

Now take a log of the production function in equation (2.1) to obtain

$$\ln(Y_i) = (1-\alpha)\ln(l_i) + \alpha \sum_{j=1}^{n} w_{ij} \ln(x_{ij}). \tag{A.6}$$

Plug in the first-order conditions (A.4) and (A.5) to get

$$\begin{aligned} \ln(Y_i) = {} & (1-\alpha)\ln(s_i(1-\alpha)) - (1-\alpha)\ln(\omega) \\ & + \alpha \sum_{j=1}^{n} w_{ij} \ln(s_i \alpha_i w_{ij} - \omega z_{ij}) - \alpha \sum_{j=1}^{n} w_{ij} \ln(p_j). \end{aligned} \tag{A.7}$$

[27] We take the second-order conditions to be satisfied, e.g. when E_i is linear.

Subtract $\ln(s_i)$ from both sides and then multiply both sides by -1 to obtain

$$\begin{aligned}\ln(p_i) = \ln(s_i) - (1-\alpha)\ln(s_i(1-\alpha)) + (1-\alpha)\ln(\omega) \\ - \alpha\sum_{j=1}^{n} w_{ij}\ln(s_i\alpha w_{ij} - \omega z_{ij}) + \alpha\sum_{j=1}^{n} w_{ij}\ln(p_j).\end{aligned} \tag{A.8}$$

At $t = 0$ equation (A.8) becomes

$$\begin{aligned}\ln(p_i^*) = -(1-\alpha)\ln(1-\alpha) - \alpha\ln(\alpha) + (1-\alpha)\ln(\omega) \\ - \alpha\sum_{j=1}^{n} w_{ij}\ln(w_{ij}) + \alpha\sum_{j=1}^{n} w_{ij}\ln(p_j^*).\end{aligned} \tag{A.9}$$

Let $\theta = (1-\alpha)\ln(1-\alpha) + \alpha\ln(\alpha)$. We can now re-write equation (A.9) in matrix form

$$\ln(p^*) = -\theta\mathbf{1} - \alpha(W \circ \ln(W))\mathbf{1} + (1-\alpha)\ln(\omega)\mathbf{1} + \alpha W\ln(p^*), \tag{A.10}$$

where $\circ$ denotes the Hadamard product. Solving equation (A.10) one obtains

$$\ln(p^*) = -V'[\theta\mathbf{1} + \alpha(W \circ \ln(W))\mathbf{1} - (1-\alpha)\ln(\omega)\mathbf{1}]. \tag{A.11}$$

The ith entry of the equilibrium log price vector is therefore given by

$$\ln(p_i^*) = -\theta\sum_{j=1}^{n} v_{ji} - \alpha\sum_{j=1}^{n} v_{ji}\sum_{h=1}^{n} w_{jh}\ln(w_{jh}) + (1-\alpha)\sum_{j=1}^{n} v_{ji}\ln(\omega). \tag{A.12}$$

Now notice that $V'\mathbf{1} = \left(I + \sum_{k=1}^{\infty}\alpha^k W^k\right)\mathbf{1} = (1-\alpha)^{-1}\mathbf{1}$, where the second step follows from the fact that W is row stochastic. It follows that $\sum_{j=1}^{n} v_{ji} = (1-\alpha)^{-1}$ for each i. Therefore,

$$\ln(p_i^*) = -\frac{\theta}{1-\alpha} - \alpha\sum_{j=1}^{n} v_{ji}\sum_{h=1}^{n} w_{jh}\ln(w_{jh}) + \ln(\omega). \tag{A.13}$$

Equilibrium sales. Multiply both sides of the market clearing condition in equation (3.2) by p_i to obtain that

$$p_iY_i = p_iC_i + \sum_{j=1}^{n} p_ix_{ji}. \tag{A.14}$$

Plugging in the consumer's demands from equation (A.1) we have

$$s_i = \frac{1}{n}(\omega + T) + \sum_{j=1}^{n} p_i x_{ji}. \tag{A.15}$$

Using the sectoral first-order conditions in (A.4) and (A.5) we get

$$s_i = \frac{1}{n}(\omega + T) + \alpha \sum_{j=1}^{n} s_j w_{ji} - \sum_{j=1}^{n} z_{ji}\omega. \tag{A.16}$$

In matrix form,

$$s = \frac{1}{n}(\omega + T)\mathbf{1} + \alpha W' s - \omega Z' \mathbf{1}. \tag{A.17}$$

And so,

$$s = V\left[\frac{1}{n}(\omega + T)\mathbf{1} - \omega Z' \mathbf{1}\right]. \tag{A.18}$$

The ith entry of the equilibrium sales vector is therefore given by

$$s_i = (\omega + T)b_i - \omega \sum_{j=1}^{n} v_{ij} \sum_{h=1}^{n} z_{hj}. \tag{A.19}$$

Clearly, when $t = 0$, we have that $s_i = \omega b_i$. We can therefore use this result in conjunction with the equilibrium prices in equation (A.13) and the sectoral first-order conditions (A.4) and (A.5) to obtain the results stated in Proposition 1. The fact that $C_i^* = \omega/(np_i^*)$ follows from equation (A.1). We obtain U^* by plugging the equilibrium consumption values into the utility function.

B Derivatives of essential terms

For a given vector $\lambda \in \{0, 1\}^n$ we need to evaluate the derivatives of consumption, labour, inputs, sectoral outputs, and sectoral emissions with respect to t around the zero-tax benchmark. To do this, we first evaluate the derivatives of essential terms such as z_{ij} and T (among others) in this section. Throughout, ω is treated as a constant: we use it as the anchoring price and therefore hold its level fixed. The proofs of the propositions that are stated in the main text are derived in section C of the appendix.

Derivative of z_{ij}. Recall that $z_{ij} = \lambda_i t x_{ij} \frac{\partial E_i}{\partial x_{ij}}$. Therefore

$$\frac{\partial z_{ij}}{\partial t} = \lambda_i x_{ij} \frac{\partial E_i}{\partial x_{ij}} + \lambda_i t \frac{\partial}{\partial t} \left(x_{ij} \frac{\partial E_i}{\partial x_{ij}} \right). \tag{B.1}$$

Evaluating at $t = 0$ we obtain

$$\left. \frac{\partial z_{ij}}{\partial t} \right|_{t=0} = \lambda_i E_i^* \epsilon_{ij}. \tag{B.2}$$

Derivative of T. We have that $T = t\omega \sum_{i=1}^{n} \lambda_i E_i$. Therefore

$$\left. \frac{\partial T}{\partial t} \right|_{t=0} = \omega \sum_{i=1}^{n} \lambda_i E_i^*. \tag{B.3}$$

Derivative of $\ln(s_i)$. The equilibrium sales for sector i satisfy equation (A.19). Therefore

$$\left. \frac{\partial s_i}{\partial t} \right|_{t=0} = b_i \left. \frac{\partial T}{\partial t} \right|_{t=0} - \omega \sum_{j=1}^{n} v_{ij} \sum_{h=1}^{n} \left. \frac{\partial z_{hj}}{\partial t} \right|_{t=0}. \tag{B.4}$$

Using equations (B.3) and (B.2), we obtain

$$\left. \frac{\partial s_i}{\partial t} \right|_{t=0} = \omega b_i \sum_{h=1}^{n} \lambda_h E_h^* - \omega \sum_{j=1}^{n} v_{ij} \sum_{h=1}^{n} \lambda_h E_h^* \epsilon_{hj} = \omega \sum_{h=1}^{n} \lambda_h E_h^* (b_i - \psi_{ih}). \tag{B.5}$$

Notice that since $s_i = \omega b_i$ when $t = 0$,

$$\left. \frac{\partial \ln(s_i)}{\partial t} \right|_{t=0} = \frac{1}{\omega b_i} \left. \frac{\partial s_i}{\partial t} \right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i} \right). \tag{B.6}$$

Derivative of $\ln(s_i \alpha w_{ij} - \omega z_{ij})$. This term appears in equation (A.8) which the equilibrium prices must satisfy. Taking a derivative of this term with respect to t yields

$$\frac{\partial \ln(s_i \alpha w_{ij} - \omega z_{ij})}{\partial t} = \frac{\frac{\partial s_i}{\partial t} \alpha w_{ij} - \omega \frac{\partial z_{ij}}{\partial t}}{s_i \alpha w_{ij} - \omega z_{ij}}. \tag{B.7}$$

Evaluating at $t = 0$, using equations (B.6) and (B.2) and the fact that $s_i = \omega b_i$ at $t = 0$, we obtain

$$\left. \frac{\partial \ln(s_i \alpha w_{ij} - \omega z_{ij})}{\partial t} \right|_{t=0} = \frac{1}{\omega b_i} \left. \frac{\partial s_i}{\partial t} \right|_{t=0} - \frac{\lambda_i E_i^* \epsilon_{ij}}{\alpha w_{ij} b_i} = \left. \frac{\partial \ln(s_i)}{\partial t} \right|_{t=0} - \frac{\lambda_i E_i^* \epsilon_{ij}}{\alpha w_{ij} b_i}. \tag{B.8}$$

C Proofs of Propositions 2 to 6

Proof of Proposition 2.

Prices. Taking the derivative of equation (A.8) with respect to t yields

$$\frac{\partial \ln(p_i)}{\partial t} = -\left((1-\alpha)\frac{\partial \ln(s_i(1-\alpha))}{\partial t} + \alpha\sum_{j=1}^{n} w_{ij}\frac{\partial \ln(s_i\alpha w_{ij} - \omega z_{ij})}{\partial t}\right) + \alpha\sum_{j=1}^{n} w_{ij}\frac{\partial \ln(p_j)}{\partial t} + \frac{\partial \ln(s_i)}{\partial t}. \tag{C.1}$$

Using equation (B.8), the term inside the parentheses evaluated at $t=0$ is equal to

$$\left.\frac{\partial \ln(s_i)}{\partial t}\right|_{t=0} - \frac{\lambda_i E_i^*}{b_i}\left(\sum_{j=1}^{n}\epsilon_{ij}\right) = \left.\frac{\partial \ln(s_i)}{\partial t}\right|_{t=0} - \frac{\lambda_i E_i^*}{b_i}, \tag{C.2}$$

where the second part follows from Euler's theorem for homogeneous equations (emissions are homogeneous of degree one). Substituting this back into equation (C.1) we obtain

$$\left.\frac{\partial \ln(p_i)}{\partial t}\right|_{t=0} = \frac{\lambda_i E_i^*}{b_i} + \alpha\sum_{j=1}^{n} w_{ij}\left.\frac{\partial \ln(p_j)}{\partial t}\right|_{t=0}. \tag{C.3}$$

Solving this linear system yields

$$\left.\frac{\partial \ln(p_i)}{\partial t}\right|_{t=0} = \sum_{j=1}^{n} v_{ji}\frac{\lambda_j E_j^*}{b_j}. \tag{C.4}$$

Consumption. Taking a derivative of the consumer's demand in equation (A.1) with respect to t yields

$$\frac{\partial \ln(C_i)}{\partial t} = \frac{1}{C_i}\frac{1}{np_i}\left(\frac{\partial T}{\partial t} - (\omega + T)\frac{\partial \ln(p_i)}{\partial t}\right). \tag{C.5}$$

Evaluating at $t=0$, using the fact that $C_i^* = \omega/(np_i^*)$, and using equations (B.3) and (C.4), we obtain

$$\left.\frac{\partial \ln(C_i)}{\partial t}\right|_{t=0} = \frac{1}{\omega}\left(\left.\frac{\partial T}{\partial t}\right|_{t=0} - \omega\left.\frac{\partial \ln(p_i)}{\partial t}\right|_{t=0}\right) = \sum_{j=1}^{n}\lambda_j E_j^*\left(1 - \frac{v_{ji}}{b_j}\right). \tag{C.6}$$

Labour demand. To obtain the result on labour demand, take a derivative of the log of both sides of equation (A.5) with respect to t to obtain

$$\left.\frac{\partial \ln(l_i)}{\partial t}\right|_{t=0} = \left.\frac{\partial \ln(s_i(1-\alpha))}{\partial t}\right|_{t=0} = \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i}\right), \tag{C.7}$$

where the second step follows from substitution of equation (B.6).

Inputs. For the intermediate inputs take a derivative of the log of both sides of equation (A.4) with respect to t to obtain

$$\begin{aligned} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0} &= \left.\frac{\partial \ln(s_i \alpha w_{ij} - \omega z_{ij})}{\partial t}\right|_{t=0} - \left.\frac{\partial \ln(p_j)}{\partial t}\right|_{t=0} \\ &= \sum_{h=1}^{n} \lambda_h E_h^* \left(1 - \frac{\psi_{ih}}{b_i} - \frac{\upsilon_{hj}}{b_h}\right) - \frac{\lambda_i E_i^* \epsilon_{ij}}{\alpha w_{ij} b_i}. \end{aligned} \tag{C.8}$$

where the second step follows from substitution of equations (B.6), (B.8), and (C.4).

Sectoral outputs and emissions. For sectoral outputs, observe that

$$\left.\frac{\partial \ln(Y_i)}{\partial t}\right|_{t=0} = \frac{l_i^*}{Y_i^*} \cdot \left.\frac{\partial Y_i}{\partial l_i}\right|_{t=0} \left.\frac{\partial \ln(l_i)}{\partial t}\right|_{t=0} + \sum_{j=1}^{n} \frac{x_{ij}^*}{Y_i^*} \left.\frac{\partial Y_i}{\partial x_{ij}}\right|_{t=0} \cdot \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}. \tag{C.9}$$

From the production function in equation (2.1) the elasticity of output i with respect to labour is $(1-\alpha)$ while the elasticity of output i with respect to intermediate input j is αw_{ij}. This gives us the desired result. A similar derivation allows us to obtain the derivative of log emissions. Namely,

$$\left.\frac{\partial \ln(E_i)}{\partial t}\right|_{t=0} = \sum_{j=1}^{n} \epsilon_{ij} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}. \tag{C.10}$$

□

Proof of Propositions 3 and 5. Since $E = \sum_{i=1}^{n} E_i$, the derivative of log aggregate emissions evaluated at $t = 0$ is given by

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} = \sum_{i=1}^{n} \frac{E_i^*}{E^*} \left.\frac{\partial \ln(E_i)}{\partial t}\right|_{t=0} = \sum_{i=1}^{n} \frac{E_i^*}{E^*} \left(\sum_{j=1}^{n} \epsilon_{ij} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}\right), \tag{C.11}$$

where the second step follows from substitution of equation (C.10). Plugging equations (C.7) and (C.8) into the above and evaluating yields

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} = \sum_{i=1}^{n} \lambda_i \left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}}, \tag{C.12}$$

where

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}} = E_i^* - \sum_{j=1}^{n} \frac{E_i^* E_j^*}{E^*}\left(\frac{\psi_{ji}}{b_j} + \frac{\psi_{ij}}{b_i}\right) - \frac{(E_i^*)^2}{E^*}\left(\sum_{j=1}^{n} \frac{\epsilon_{ij}^2}{\alpha w_{ij} b_i}\right). \tag{C.13}$$

□

Proof of Proposition 4. When $W = I$ it is easy to show that $\psi_{ij} = (1-\alpha)^{-1}$ if $i = j$ and $\psi_{ij} = 0$ if $i \neq j$, and that $b_i = n^{-1}(1-\alpha)^{-1}$ for all i. The aggregate emissions impact of sector i is then

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}} = E_i^* - \frac{n}{E^*}\frac{1+\alpha}{\alpha}(E_i^*)^2 = E_i^* - \kappa(E_i^*)^2, \tag{C.14}$$

where $\kappa = \frac{n}{E^*}\frac{1+\alpha}{\alpha}$. The aggregate emissions impact of sector i is concave in E_i^* with roots at 0 and $1/\kappa$ and reaches a maximum somewhere in between. When $E_i^* > 1/\kappa$, the absolute value of the aggregate emissions impact is strictly increasing in E_i^*. A sector i is a key sector if $E_i^* > 1/\kappa$, so the result follows. □

Remark 1. Notice that, in the proof of Proposition 4, if the aggregate emissions impact of sector i were dependent on i other than through E_i^* then ranking key sectors by their direct emissions and ranking key sectors by the absolute value of their aggregate emissions impact will in general produce different results. For example, see the expression for a sector's aggregate emissions impact in the economy analysed in Appendix E (in which the consumer's preferences are heterogeneous). We should therefore expect different rankings whenever there is heterogeneity in intersectoral linkages, emissions elasticities, consumer preferences, or production functions.

Proof of Proposition 6. Set $\lambda_i = 1$ for all i and let $M = \max_{i,j} \frac{E_i^*}{E^*}\epsilon_{ij}$ (and notice that $M \geq 0$). Then by equation (C.11),

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} \leq M \sum_{i=1}^{n}\sum_{j=1}^{n} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0}. \tag{C.15}$$

Since we set $\lambda_i = 1$ for each i, by equation (C.8) we also know that

$$\left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0} \leq \sum_{h=1}^{n} E_h^* \left(1 - \frac{\psi_{ih}}{b_i} - \frac{\upsilon_{hj}}{b_h}\right) = E^* - \sum_{h=1}^{n} E_h^* \frac{\psi_{ih}}{b_i} - \sum_{h=1}^{n} E_h^* \frac{\upsilon_{hj}}{b_h}. \tag{C.16}$$

Let us now sum the above expression over all j to obtain

$$\sum_{j=1}^{n} \left.\frac{\partial \ln(x_{ij})}{\partial t}\right|_{t=0} \leq nE^* - n\sum_{h=1}^{n} E_h^* \frac{\psi_{ih}}{b_i} - \sum_{j=1}^{n}\sum_{h=1}^{n} E_h^* \frac{\upsilon_{hj}}{b_h} \tag{C.17}$$

$$= nE^* - n\sum_{h=1}^{n} E_h^* \frac{\psi_{ih}}{b_i} - n\sum_{h=1}^{n} E_h^* \tag{C.18}$$

$$= -n\sum_{h=1}^{n} E_h^* \frac{\psi_{ih}}{b_i}. \tag{C.19}$$

The second line follows from the fact that $\sum_{j=1}^{n} \upsilon_{hj} = nb_h$. Since the term in the final line is weakly negative, we obtain the desired result. □

D Details of the calibration

One can follow the steps of Appendix A-Appendix C to show that, in the generalised model with heterogeneous consumer preferences and heterogeneous labour shares in production, when $t = 0$ there is a unique competitive equilibrium and it is characterised by

$$p_i^* = \exp\left(-\sum_{j=1}^{n} \nu_{ji}\vartheta_j - \sum_{j=1}^{n} \nu_{ji}\alpha_j \sum_{h=1}^{n} w_{jh}\ln(w_{jh}) + \sum_{j=1}^{n} \nu_{ji}(1-\alpha_j)\ln(\omega)\right), \tag{D.1}$$

$$C_i^* = \gamma_i \omega / p_i^*, \tag{D.2}$$

$$s_i^* = \omega\beta_i, \tag{D.3}$$

$$x_{ij}^* = s_i^* \alpha_i w_{ij} / p_j^*, \tag{D.4}$$

$$l_i^* = s_i^*(1-\alpha_i)/\omega. \tag{D.5}$$

The equilibrium emissions of sector i when $t = 0$ are given by $E_i^* = E_i(x_{i1}^*, ..., x_{in}^*)$. Furthermore, for any $\lambda \in \{0,1\}^n$,

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{t=0} = \sum_{i=1}^{n} \lambda_i \left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}}, \tag{D.6}$$

where the aggregate emissions impact of sector i is given by

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i\\ t=0}} = E_i^* - \sum_{j=1}^{n} \frac{E_i^* E_j^*}{E^*}\left(\frac{\phi_{ji}}{\beta_j} + \frac{\phi_{ij}}{\beta_i}\right) - \frac{(E_i^*)^2}{E^*}\left(\sum_{j=1}^{n} \frac{\epsilon_{ij}^2}{\alpha_i w_{ij} \beta_i}\right). \tag{D.7}$$

The new terms are defined as follows: firstly, $\vartheta_i = (1-\alpha_i)\ln(1-\alpha_i) + \alpha_i \ln(\alpha_i)$. Secondly, define A to be an $n \times n$ matrix with the ith diagonal entry equal to α_i and every other entry equal to zero. Let $\mathcal{V} = (I - (AW)')^{-1}$, and define $\beta = \mathcal{V}\gamma$ and $\Phi = \mathcal{V}\mathcal{E}'$. One can verify that the derivative of utility with respect to tax at the zero-tax benchmark is still zero here. Notice also that when for all i, $\gamma_i = 1/n$ and $\alpha_i = \alpha$ for some scalar α, then $\vartheta_i = \theta$ for all i, $[\nu_{ij}] = \mathcal{V} = V = [v_{ij}]$, $\beta = b$, and $[\phi_{ij}] = \Phi = \Psi = [\psi_{ij}]$, which therefore recovers equation (5.1), the expression for aggregate emissions impact when consumer preferences and labour shares in production are homogeneous.

In order to calibrate the production and consumption sides of our model we use the GTAP variables `TVOM`, `VDPM`, and `VDFM`. For any sector i and country c, $\texttt{TVOM}(i,c)$ are the sales of domestic product (of i) in country c at market prices. The sales are made to households or to other sectors. For any sectors i and j in country c, $\texttt{VDFM}(i,j,c)$ are the domestic purchases by sector j of i's good in country c at market prices. For any sector i in country c, $\texttt{VDPM}(i,c)$ are the domestic purchases by households of i's good in country c at market prices. All the variables are measured in millions of dollars.

We describe how we calibrate the production and consumption sides of our economy below. The description applies in the same way for each country. We therefore omit the country-indexing of the variables.

Since the consumer's preference weight γ_i is precisely the share of the consumer's spending on the good produced by sector i (see equation (D.2)), we calibrate the consumer's preference weights as $\gamma_i = \texttt{VDPM}(i)/\sum_{i=1}^{n}\texttt{VDPM}(i)$. Similarly, for any sector i, $(1-\alpha_i)$ is precisely the spending of that sector on labour as a fraction of i's sales (see equation (D.5)). Since $\sum_{j=1}^{n}\texttt{VDFM}(j,i)$ is a measure of i's spending on all intermediate inputs and $\texttt{TVOM}(i)$ measures i's sales, we obtain i's spending on labour as $\texttt{TVOM}(i) - \sum_{j=1}^{n}\texttt{VDFM}(j,i)$. It follows that we can calibrate α_i via $1-\alpha_i = (\texttt{TVOM}(i) - \sum_{j=1}^{n}\texttt{VDFM}(j,i))/\texttt{TVOM}(i)$. Finally, having calibrated α_i we can work out the input-output matrix W as follows: since $\alpha_i w_{ij}$ is the spending of sector i on good j as a fraction of i's sales (see equation (D.4)), we obtain the calibration $w_{ij} = (\texttt{VDFM}(j,i)/\texttt{TVOM}(i))/\alpha_i$.

The GTAP database contains information on imports and exports. We ignore international trade in our calibration: we employ variables that report only spending by domestic households and sectors on goods produced by domestic sectors. A full analysis that accounts for international trade is beyond the scope of this paper and may yield different numerical results.

With the calibration for the production and consumption sides of the economy that is described above, we can work out the equilibrium input values under a zero-tax regime. These can straightforwardly be calculated using equations (D.1) and (D.4). Working out the equilibrium inputs allows us to calibrate the emissions side of our economy.

GTAP 8 contains data on sectoral carbon emissions as part of the core database. (GTAP 7 and its associated CO_2 emissions data for 2004 have been used in previous studies of sectoral emissions; see, e.g., Davis and Caldeira (2010)).[28] For any sectors i and j in country c, the variable $\texttt{MDF}(i,j,c)$ measures the CO_2 emissions (in million tons) from purchases of domestic energy commodity i by sector j in country c.[29] From this variable we create a new variable $\texttt{C}(i,j,c)$ which measures carbon emissions (in tons) from energy commodity i produced in country c and used by sector j in country c. We obtain this variable via the transformation $\texttt{C}(i,j,c) = (10^6/3.67) \times \texttt{MDF}(i,j,c)$, which uses the standard IPCC conversion factor.[30]

Below, we describe how we calibrate the emissions side of our economy. Since the description applies in the same way to each country, we once again drop the indexing by country. Since we have worked out the equilibrium input levels x^*_{ij}, we can use equation (6.3) to calibrate the emissions coefficients via $\eta_{ij} = \texttt{C}(j,i)/x^*_{ij}$. In other words, the emissions that sector i produces from using sector j's good as an input increases linearly with i's use of sector j's good. Finally, we obtain the elasticities via $\epsilon_{ij} = \eta_{ij} x^*_{ij}/E^*_i = \texttt{C}(j,i)/\sum_{j=1}^{n} \texttt{C}(j,i)$.

Equation (D.6) only provides us with a derivative for the log of aggregate emissions with respect to a tax reform. To estimate the effect of targeted carbon tax reforms on aggregate emissions, we use the derivative at the zero-tax equilibrium to linearly extrapolate the change in emissions from a given carbon tax change. To do this, let $E(t,\lambda)$ denote aggregate emissions when the tax rate is t and the tax reform is λ. Now integrate equation (D.6) with respect to t and use the boundary condition that aggregate emissions are given by E^* when $t = 0$ to obtain

$$E(t,\lambda) = E^* \exp\left\{ t \sum_{i=1}^{n} \lambda_i \frac{\partial \ln(E)}{\partial t}\bigg|_{\substack{\lambda=e_i \\ t=0}} \right\}. \tag{D.8}$$

[28] Lee (2008) provides details of the emissions estimation for GTAP 7. See also https://www.gtap.agecon.purdue.edu/resources/res_display.asp?RecordID=1143 .

[29] The energy commodities in the dataset are: coal, crude oil, natural gas, petroleum products, and gas distribution. The dataset also reports emissions by the government and by households. These emissions are ignored in the calibration reported here.

[30] See http://www.ipcc.ch/ipccreports/tar/wg3/index.php?idp=477.

It follows that for any tax reform $\lambda \in \{0,1\}^n$ the percentage change in aggregate emissions is given by

$$\left(\frac{E(t,\lambda)}{E^*} - 1\right) \times 100. \tag{D.9}$$

We use the equation above as our estimate of the percentage change in aggregate emissions when the tax rate is t. Strictly speaking, our *analytical* results apply only for infinitessimal tax rates and our estimates may worsen rapidly with large tax rates if the economy is highly non-linear. Note that in general

$$\left(\frac{E(t,e_i)}{E^*} - 1\right) + \left(\frac{E(t,e_j)}{E^*} - 1\right) \neq \left(\frac{E(t,e_i+e_j)}{E^*} - 1\right), \tag{D.10}$$

i.e. the estimated percentage change in aggregate emissions of taxing individual sectors is not additive.

To work out the tax *rate* we need to normalise by nominal GDP. We estimate nominal GDP, or ω, using equation (D.3) since $\omega = \sum_{i=1}^n s_i^* / \sum_{i=1}^n \beta_i = \sum_{i=1}^n \texttt{TVOM}(i) / \sum_{i=1}^n \beta_i$, and we obtain a nominal GDP of roughly 15,445,088 million dollars for the USA and of 162,421 million dollars for Pakistan.[31] For our \$10 tax we therefore set the tax rate t to $10/(15445088 \times 10^6)$ for the USA and to $10/(162421 \times 10^6)$ for Pakistan.

E Role of intersectoral linkages

In this section we show that the aggregate emissions impacts estimated in Section 6 are not driven solely by the distribution of sector sizes; indeed, the structure of the whole production network matters for our results. To do this we analyse an economy without intersectoral linkages (in which sectors use only labour and their own output as inputs) and we calibrate consumer preference weights to match the sector size distribution of the interconnected economy of Pakistan examined in Section 6. We show that, while the economy without an intersectoral network has the same sector size distribution as the economy with an intersectoral network, the effects of sectoral carbon taxes differ across the two economies.

The economy without intersectoral linkages. The production function of sector i is given by

$$Y_i = l_i^{1-\alpha} x_{ii}^{\alpha}. \tag{E.1}$$

[31] According to World Bank estimates, in 2007, the nominal GDP in the USA was 14,451,858 million dollars and the nominal GDP in Pakistan was 152,385 million dollars. See https://data.worldbank.org/indicator/NY.GDP.MKTP.CD?end=2007&start=2006&year_high_desc=true.

This implies that there are no intersectoral linkages and $W = I$. The profit function of sector i is given by

$$\pi_i = p_i Y_i - \omega l_i - p_i x_{ii} - \lambda_i t \omega E_i, \tag{E.2}$$

where $E_i = \eta_{ii} x_{ii}$. The consumer's preferences are given by

$$U(C_1, \ldots, C_n) = \prod_{i=1}^{n} C_i^{\gamma_i}, \tag{E.3}$$

where $\sum_{i=1}^{n} \gamma_i = 1$. This economy is essentially identical to the one studied in Proposition 4 with the exception that there is now heterogeneity in the consumer's preference parameters. Using the notation of Section D we have that $\beta_i = \gamma_i/(1-\alpha)$; $\phi_{ij} = 1/(1-\alpha)$ if $i = j$ and is zero otherwise; and $\epsilon_{ij} = 1$ if $i = j$ and is zero otherwise. Using equation (D.7) we therefore obtain the aggregate emissions impact of sector i:

$$\left.\frac{\partial \ln(E)}{\partial t}\right|_{\substack{\lambda=e_i \\ t=0}} = E_i^* - \frac{1+\alpha}{\alpha \gamma_i E^*}(E_i^*)^2. \tag{E.4}$$

When $t = 0$, using equation (D.3), the sales of sector i are given by $\omega \frac{\gamma_i}{1-\alpha}$.

Calibration of the economy without intersectoral linkages. We set nominal GDP, namely ω, to match the nominal GDP for Pakistan's economy that we obtained in Section D. We calibrate the emissions directly to match sectoral emissions in Pakistan's economy. Finally, we calibrate the consumer's preference weights γ_i by equating the sales of each sector i, $\omega \frac{\gamma_i}{1-\alpha}$, to the sales in Pakistan's economy. In other words, γ_i is proportional to the sales of sector i in Pakistan's interconnected economy and the labour share, $1-\alpha$, is set to satisfy the normalisation that consumer preference weights sum to one.[32] This calibration allows us to compare the aggregate emissions impacts in the economies with and without intersectoral linkages while keeping the sales distribution identical across the two economies.

Our results for the key sectors in the economy without intersectoral linkages are shown in Figure E.1. The top panel shows that the aggregate emissions impacts of the key sectors in the economy without intersectoral linkages are correlated with those of their counterparts in Pakistan's interconnected economy. There are, however, important differences: when intersectoral linkages are accounted for (as was done for our calibration of Pakistan's economy), the non-metallic minerals sector is ranked first in terms of its aggregate emissions impact. When linkages are ignored, this sector becomes far less important. The bottom panel of Figure E.1 shows that the aggregate emissions impacts of the key sectors in the economy

[32] In this calibration, the labour share is 0.6. The aggregate labour share in Pakistan's data is also 0.6.

without linkages are correlated with sectoral emissions, however, because of heterogeneity in consumer preferences the monotonic relationship predicted in Proposition 4 is not present here (see Remark 1 in Appendix C).

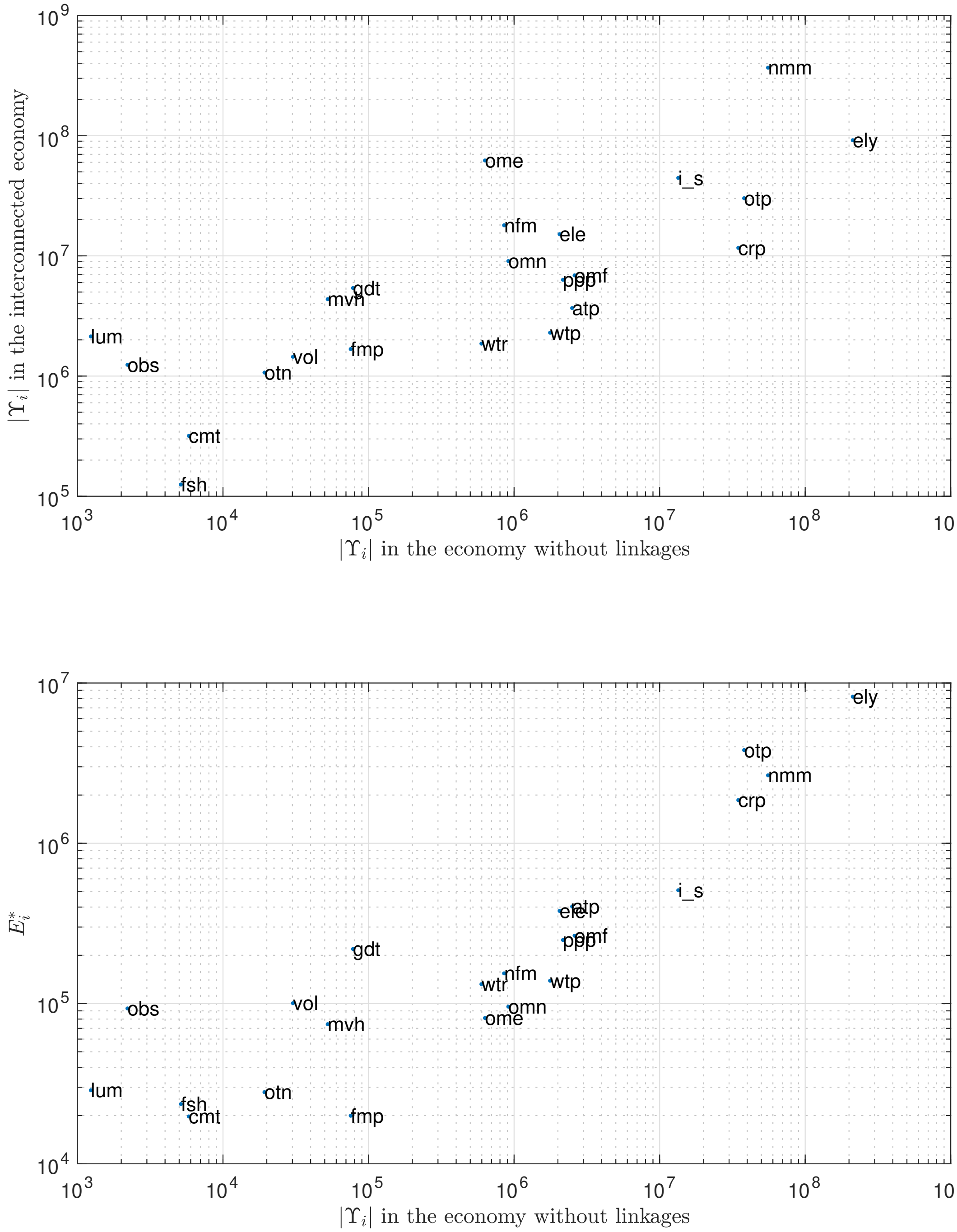

Figure E.1: Aggregate emissions impacts of the key sectors in the economy without intersectoral linkages. The sales distribution of sectors is calibrated to match that of Pakistan's interconnected economy of Section 6.

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