PATENT RACES AND MARKET STRUCTURE

Thesis Submitted for the Degree of D.Phil.

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February 1985

Abstract

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D.Phil Thesis
Hilary Term 1985

This thesis is a theoretical study of relationships between patent races and market structure. The outcome of a patent race can be an important determinant of market structure. For example, whether or not a new firm enters a market may depend upon its winning a patent race against an incumbent firm already in that market. Moreover, market structure can be a major influence upon competition in a patent race. In the example, the asymmetry between incumbent and potential entrant has an effect upon their respective incentives in the patent race.

Chapter I discusses models of R and D with uncertainty. We show that, as the degree of correlation between the uncertainties facing rival firms increases, R and D efforts increase under some, but not all, conditions, and the number of active competitors falls. Chapter II discusses the approach of representing patent races as bidding games. We examine a model in which several incumbent firms compete with a number of potential entrants in a patent race, and ask whether the incumbents have an incentive to form a joint venture to deter entry. They do so if and only if the patent does not offer a major cost improvement. In Chapter III we examine the strategic interactions between competitors during the course of a race, in an attempt to clarify (for different types of race) the idea that a race degenerates when one player becomes 'far enough ahead' of his rivals, in a sense made precise. In Chapter IV we examine the evolution of market structure in a duopoly model when there is a sequence of patent races. The nature of competition in the product market is shown to determine whether one firm becomes increasingly dominant as industry leader, or whether there is 'action - reaction' between firms.

ACKNOWLEDGEMENTS

James Mirrlees, my supervisor, gave invaluable help and criticism throughout the preparation of this thesis. I have learned a great deal from Partha Dasgupta and Christopher Harris. I have received helpful comments on parts of the thesis from Terence Gorman, Donald Hay, Colin Mayer, Kevin Roberts, Amartya Sen, Paul Stoneman, John Sutton, and participants in the Warwick workshop on technological change in July 1984. Humaira Ahmed typed the manuscript with great patience and skill. The Warden and Fellows of All Souls, and the Warden and Fellows of Nuffield, gave me the opportunity to carry out this research. I am most grateful to them all.

PATENT RACES AND MARKET STRUCTURE

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INTRODUCTION

PATENT RACES AND MARKET STRUCTURE

This thesis is concerned with the relationships between patent races and market structure. The influences of patent races on market structure, and vice versa, can be complex. As an illustration of this, consider the following example, which will be discussed at some length in Chapters II and III.

Suppose that firm A is contemplating entry into a market currently monopolised by firm B. Firm A will gain entry if and only if it wins the patent for a technology that will enable it to compete with firm B. One could imagine that the patent is for a product that would be a substitute for that currently supplied by firm B. Alternatively, one could regard the patent as being for a new process for making firm B's product. There is to be a race for the patent. The outcome of the race determines the resulting market structure: if A wins he gains access to the market and either duopoly results, or the innovation is so 'drastic' that B is displaced altogether; if B wins he retains his monopoly position. In this case, B may or may not wish actually to employ the innovation.

Market structure is not only influenced by the outcome of patent races. It is also an influence upon the outcome of patent races, and R and D competitions generally. By the 'outcome' of an R and D competition is meant not only the winner, but also the speed at which the competitors research, the number of active participants in the contest, the expected duration of the patent race, and so on.

In the entrant-versus-incumbent example above, there are reasons why the asymmetrical position of firms A and B, due to the initial market structure, might affect the patent race itself. In particular, a subject of recent debate has been whether the desire of the incumbent monopolist (firm B) to retain his monopoly position has the effect of thwarting the rivalry from firm A. If this proposition is true, then it would appear to be an important ingredient of a theory of the persistence of monopoly: the original market structure is preserved because its effect upon the strategic interaction between the firms in the patent race is such that the incumbent wins the race.

The above example was intended to illustrate the nature of some of the influences of market structure on R and D competitions, and vice-versa. It was also meant as a sample of the sort of question to be addressed in the four Chapters that follow. Indeed, the example, and variations on its theme, will be discussed later in full.

Patent races are an important type of R and D competition, but not the only type. Dasgupta (1982) distinguishes between 'tournament' and 'nontournament' forms of R and D competition. Patent races are tournaments - they have winners and losers. Therefore they are modelled as discontinuous games. In models of non-tournament forms of R and D competition it is typically assumed that a firm's cost level depends in a continuous way upon its own R and D (and upon that of other firms if there are spillovers): there is no such thing as a winner or loser. This thesis is restricted to tournament forms of R and D competition. For non-tournament forms of R and D competition, see in particular Dasgupta and Stiglitz (1980a), Brander and Spencer (1983), Spence (1984), and Tandon (1984).

Patent races are not the only 'tournaments' of economic interest. Another example is to be found in the literature on the principal-agent problem: see e.g. Lazear and Rosen (1981), Holmstrom (1982), and Nalebuff and Stiglitz (1983a, 1983b). This literature examines the properties of incentive schemes in which an agent is rewarded according to his performance relative to that of other agents. An example of such a scheme is one in which an agent's rank determines his reward. Simplest of all is a scheme in which a prize (e.g. promotion to Vice-President of the corporation) is won by the agent who performs best. There is a clear parallel with patent races. Indeed some of the models to follow could be reinterpreted as contests between agents competing for some reward. Perhaps the main difference between the literature on patent races and that on the principal-agent problem is that the latter is concerned with the design of incentive schemes, which has no counterpart in the former problem. The aspect of the patent race problem considered here is the interplay between patent races and market structure. This has no parallel in the principal-agent problem.

Chapter I: Models of R and D with Uncertainty

The earliest and most influential game theoretic models of R and D are those of Loury (1979), Dasgupta and Stiglitz (1980b), and Lee and Wilde (1980). These models, which are static in the sense that each agent chooses his level of R and D once-and-for-all, have been extended by Reinganum's (1981,1982) papers on dynamic games of R and D.

This literature has been surveyed, and to some extent generalised, by Kamien and Schwartz (1982, Chapter 5); see also Reinganum (1984b). It is not my purpose to duplicate these surveys. However, it is necessary briefly to discuss these models, in part because in later Chapters (for example Chapter II on the persistence of oligopoly) I shall sometimes employ Loury's, or Lee and Wilde's models, of R and D as convenient constituent parts of larger models. More important, Chapter I seeks also to criticise and develop these models. First, it is shown that some of the results of Reinganum's sophisticated dynamic game of R and D depend critically on her (unusual) assumption that players do not discount the value fo the prize for which they are competing, whereas they do discount their R and D costs. Equilibrium in Reinganum's model is calculated when this assumption is relaxed, and it is shown to be equivalent to a special case of Lee and Wilde's much simpler model when the time horizon becomes large. It is argued that this justifies the restriction of attention to the simpler formulation, at any rate as far as this type of model is concerned.

Secondly, the models in the literature above are ill-equipped to capture the 'race-like' features of many R and D competitions. The reasons for this are explained in the Introduction to Chapter III. It is precisely these features that the models in that Chapter attempt to capture. One reason is that it is typically assumed in these models that the technological uncertainty facing rival players is completely uncorrelated. The general case of partial correlation of technological uncertainty is worthy of formal investigation, and the final section of Chapter I attempts to provide this.

Chapter II: Patent Races as Bidding Games

This Chapter discusses a separate strand of the literature on patent races, namely that which has used the simplificatory device of representing patent races as bidding games. This has provided a straightforward approach to issues such as the entrant-versus-incumbent example described at the beginning of this Introduction. An important paper is that by Gilbert and Newbery (1982) on 'Preemptive Patenting and the Persistence of Monopoly', which has provoked a heated debate recently in the American Economic Review. Gilbert and Newbery proposed a simple model of a patent race as a bidding game in which the incumbent monopolist has a greater incentive to win the patent than the potential entrant (see also Dasgupta (1982)). The incumbent is therefore prepared to outbid his rival, and so his monopoly persists.

The stark simplicity of the Gilbert-Newbery model is at once its strength and its weakness. On the one hand, it is easily developed to analyse patent races in which there are several players, or several prizes at stake, and one can see how conclusions regarding the persistence of monopoly are modified in the light of these variations. Moreover, a central part of Chapter II concerns preemptive patenting and the persistence or otherwise of oligopoly. It is shown how free rider problems for incumbent oligopolists may overturn the conclusions previously adduced regarding the persistence of monopoly. Next joint ventures are introduced into the analysis, and it is shown how these may overcome the incumbents' free rider problem and so serve to deter entry.

On the other hand, the Gilbert-Newbery framework has drawbacks. Reinganum (1983) has argued that its exclusion of uncertainty is a serious flaw, and that the conclusions of the model may even be reversed once uncertainty is introduced. Gilbert and Newbery have countered that the exclusion of uncertainty is not responsible for the different results, but that assumptions regarding move order are important. After considering this debate, and its bearing on other issues in this thesis, I conclude that there is no entirely satisfactory assumption regarding move order in models where firms have to make once-and-for-all decisions about their level of R and D at the oustet of the race.

Another criticism of Gilbert and Newbery's framework is that it cannot capture important strategic interactions between competitors during the course of a patent race. This is because each player has just one bid, which is equivalent to a once-and-for-all commitment to R and D being made at the outset of the race: thus there is no scope for action-and-reaction between players as the race unfolds. Models designed to capture these strategic interactions are the subject of Chapter III.

Chapter III: Perfect Equilibrium in Patent Races

The first two parts of this Chapter contain my account of joint work with Christopher Harris of Nuffield College. They follow the pattern of, and develop, two papers that we have written jointly:

'Perfect Equilibrium in a Model of a Race', (Harris and Vickers (1983)), and

'Patent Races and the Persistence of Monopoly', (Harris and Vickers (1984)).

Throughout, these papers will be denoted HV1 and HV2 respectively.

Our work on HV1 stemmed from a dissatisfaction with models of the sort discussed in Chapter II that allowed players just one move or 'bid' in the patent race (i.e. the strategy sets were R_+). We attempted to develop a framework in which each player could make a sequence of moves, each one being made in response to those of his rivals, and with a view to influencing his rivals in turn (a related model is that of Fudenberg et al. (1983, section 4), which I discuss). In particular, our aim was to verify (or falsify) the conjecture that if one player was 'far enough ahead' of his rival (in some sense) then the rival would give up and the race would degenerate into an effective monopoly. Above all, we wished to establish, necessarily in a stylised setting, exactly what it was to be far enough ahead to cause the rival to give up. In order to examine the structure of threats and counter-threats in the race, and to rule out incredible threats, we naturally used Selten's (1965) perfect equilibrium solution concept.

The result of the analysis of the model in HV1 is that the conjecture is confirmed that usually one player has strategic supremacy in the race: it is as though his rival did not exist. If the presence of the rival affects behaviour in the race, it does so only at the outset of the race and not thereafter. Being far enough ahead to deter one's rival from active competition is not just a matter of literal distance from the finishing line. Rather, it depends also on the player's respective valuations of the prize, their discount rates and their efficiency at R and D. Taken in conjunction with the claim (mentioned above) that an incumbent monopolist is likely to be keener than any potential entrant to win a patent race, the model in HV1 therefore suggests that the incumbent's original monopoly in the product market may yield him considerable strategic advantage in the race for the new patent, by virtue of its effect on incentives (and therefore upon credible threats) in the race.

The model in HV2 develops this theme by analysing a model of a race in which one players concern is not directly to win the patent, but to ensure that his rival does not win. We call this an asymmetrical race in contrast to the standard race in which both players positively wish to win. In some economic contexts (perhaps including examples of entry deterrence already discussed), there is an asymmetrical race rather than a standard race. The analysis of the model in HV2 shows that the player whose concern is to stop his rival derives some strategic advantage from that fact. This provides an extra reason why an incumbent monopolist might enjoy supremacy in the race for a new patent.

The least satisfactory aspect of the models in HV1 and HV2 is that they are models of complete and perfect information: there is no uncertainty. Thus in some sense they are at the opposite extreme to the models of Lee and Wilde, Reinganum, and others discussed in Chapter I. The latter have uncertainty but are unable to capture strategic interactions during the course of the race, whereas HV1 and HV2 (and Fudenberg et al.) attempt to do the latter but require certainty for the sake of tractability. This is not to denigrate either type of model: they are designed to capture quite different aspects of R and D competition.

To incorporate incomplete and/or imperfect information into the HV models appears to be beyond reach, for reasons that are discussed. Therefore perhaps the most sensible approach is to develop models independently that attempt, as simply as is possible, to incorporate both uncertainty and the possibilities for strategic interaction as the race unfolds. To this end, the last two sections of Chapter III study 'multistage patent races'.

In the first of these (which is perhaps more a model of a tug-of-war than of a patent race) a prize is awarded to the first player to get a certain number of stages ahead of his rival. The form of uncertainty is that the probability that a player completes a stage in the next interval of time depends upon his current effort level. The model has the convenient mathematical property that there is only one state variable: the 'gap' between the players at the current time. In fact the model is completely tractable. It shows how the players compete vigorously if and only if they are 'close' to each other. Once a player obtains a lead over his rival, effort rates decline markedly, although both players continue to make some positive effort until the race is won. Once a player has a lead, it becomes very likely that he will be the eventual winner.

The final model of a multistage patent race is of more economic interest, but is harder to analyse. The model is a generalisation and extension of that in Fudenberg et al. (1983, Section 3). The winner of the patent is the first firm to complete a given number of stages of research. As in the previous model, the probability that a firm completes the next stage of research in the next interval of time depends upon its current effort rate. Again it turns out that firms compete most vigorously when they are close to each other, and when they have few stages to go. As well as some analytical results, a numerical computation is given which illustrates these effects clearly.

In conclusion, multistage patent races appear to combine uncertainty and strategic interaction in a promising way.

Chapter IV: R and D Competition with a Sequence of Innovations

This Chapter moves away from looking at patent races in isolation in order to consider the relationship between market structure (and its evolution) and R and D competitions when a series innovative opportunities arises over time. The additional complexity here is that the outcome of one patent race has an influence upon the likely course of the next, with market structure evolving all the while. Sophisticated rivals are aware of these effects and make their R and D efforts accordingly. With a sequence of patent races we can ask whether market structure is likely to evolve in such a way that one firm emerges preeminent, or whether there is likely to be action-and-reaction with firms leap-frogging one another so that there are frequent changes of market leadership.

A model is developed which shows how the evolution of market structure depends upon the nature of the firms' profit functions in the product market, and hence upon the nature of competition between them. It emerges that intense competition in the product market can lead to increasing concentration over time, whereas less intense competition can lead to action-reaction. This contrast between static and dynamic competition has intriguing Schumpeterian overtones.

CHAPTER I

MODELS OF R AND D WITH UNCERTAINTY

INTRODUCTION

Kamien and Schwartz (1982) identify two approaches used in modern theories of market structure and innovation:

(a) the 'decision theoretic approach', and

(b) the 'game theoretic approach'.

Following Barzel (1968), Kamien and Schwartz are themselves perhaps the main exponents of (a): see the list of their papers on p. 108 of their (1982) book. Chapter 4 of that book generalises previous decision theoretic models. In these models there is only one decision-maker, who chooses a dynamic strategy in an environment that is usually assumed to be uncertain. Sometimes certain aspects of the environment are labelled 'rivals' behaviour', for example, but the 'rivals' are not optimising agents - their behaviour is specified in advance, often as a function of the state of the environment.

In approach (b), however, there are several optimising agents. At the (Nash) equilibrium of the game, each is behaving optimally given the behaviour chosen by the other agents. This thesis deals only with the game-theoretic approach, because there are at least two decision-making agents in all the R and D competitions studied here. Indeed, it might be argued that a <u>competition</u> necessarily involves two or more players, and therefore that approach (b), and not (a), must be employed to study competitions. The presence of several decision-making agents makes (b) more complex to analyse than (a). In view of this complexity, in approach (b) each player is usually assumed simply to choose his level of R and D (i.e. strategy sets are R_+) whereas dynamic strategies (i.e.

functions of time or the state of the environment) are available to the player in approach (a)).

Reinganum claims that her (1981,1982) model 'combines' approaches (a) and (b). However, this is slightly misleading. Approach (a), with one optimising agent, cannot be combined with approach (b) in which there are several. Reinganum is squarely within approach (b), although agents in her model do choose dynamic strategies (like the agent in approach (a)).

In all the models discussed in this chapter there is technological uncertainty regarding the relationship between R and D effort and the timing of innovation. By contrast, some other models of R and D competition exclude technological uncertainty, in order to focus sharply upon other issues: see the next Chapter for example. These latter models involve game theoretic uncertainty - in that one player does not know what the others will do - but this is to be distinguished from intrinsic technological uncertainty, which exists whether there is one player or many.

The plan of this Chapter is as follows. First, we describe the three most important models of R and D with uncertainty, namely those developed by

(i) Loury (1979), and Dasgupta and Stiglitz (1980b),

(ii) Lee and Wilde (1980), and

(iii) Reinganum (1981,1982)

Some criticisms are made of Reinganum's model, and the equilibrium of a modified version of it is shown to be equivalent to Lee and Wilde's much simpler formulation. Secondly, there is a discussion of the degree of correlation between the technological uncertainties facing rival firms. We attempt to relax the common assumption that there is zero correlation of uncertainties, and to examine the general case in which the degree of correlation is a parameter that can be varied in comparative statics exercises. Part of the interest of this analysis is that the limiting case of perfect correlation of uncertainty is in many ways like the case of no uncertainty. Indeed it is often trivial to reinterpret models with no uncertainty as models with perfectly correlated uncertainty.[1] The common assumption of zero correlation of uncertainty is one (but not the main) reason why models of the type reviewed in this Chapter - even Reinganum's dynamic formulation - are unable to capture important aspects of patent _races_: see the Introduction to Chapter III below for an explanation of why this is so.

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1. See Dasgupta and Stiglitz (1980b), p.11.

1. A BRIEF REVIEW OF THE LITERATURE

The purpose of this Section is not to duplicate the existing surveys by Kamien and Schwartz (1982, Chapter 5) and Reinganum (1984b). Rather, it is to set out the basic models that are discussed later in this Chapter, and which are the constituent parts of more elaborate models studied in other Chapters.

Before describing these models, it is necessary to clear up the following point. Some authors (e.g. Reinganum (1982)) assume that firms choose rates of acquisition of knowledge. Other authors (e.g. Loury (1979) and Dasgupta and Stiglitz (1980)) assume that firms choose rates of expenditure on R and D. In the models described below it makes no difference which specification is adopted, since a firm's rate of acquisition of knowledge[1] is a strictly increasing function of its R and D expenditure. We employ the Reinganum specification throughout, because it simplifies the analysis in various places. (It also brings out the parallel between some R and D models and some quantity-setting oligopoly models.)

As to the relationship between the rate of knowledge acquisition and the cost of R and D, let x be the rate of aquisition of knowledge of some firm. The cost of acquiring knowledge at rate x is then given by some function c(x), where it is assumed that

(1) $$c'(x) > 0 \qquad \text{for } x > 0$$

(2) $$c(0) = 0$$

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1. Sometimes we shall speak of a firm's rate of 'R and D effort' as meaning its rate of acquisition of knowledge. This phrase is looser but shorter.

(3) $\lim_{x \to \infty} c'(x) = \infty$

(4) There exists $\bar{x} \geq 0$ such that $c''(x) \gtrless 0$ as $x \gtrless \bar{x}$

The assumptions state that cost is a strictly increasing function of the rate of acquisition of knowledge; that no cost is incurred if no effort is made; that the marginal cost of effort goes to infinity; and that there may be increasing returns to scale initially, but that there are decreasing returns after some point. A convenient example is $c(x) = \frac{1}{2}x^2$ (note that $\bar{x} = 0$ here.) These assumptions ensure that firms' optimisation problems have well-defined solutions.

Using the alternative approach, let u be the R and D expenditure of some firm. If technological uncertainty is exponential (see (9) below), this determines the 'hazard rate' for the firm according to a given function h(u). The hazard rate is the instantaneous probability of discovering the invention conditional upon not having discovered it before. It is assumed (see Kamien and Schwartz (1982, pp. 180-1)) that

(5) $h'(u) > 0$ for $u > 0$

(6) $h(0) = 0$

(7) $\lim_{u \to \infty} h'(u) = 0$

(8) There exists $\bar{u} \geq 0$ such that $h''(u) \gtrless 0$ as $u \lessgtr \bar{u}$

Note the parallel between (1) - (4) and (5) - (8).

Henceforth we adopt the former approach for the reasons described above. We now proceed to describe the most important models in the literature.

Model (i): That of Loury (1979) and Dasgupta and Stiglitz (1980b)

There are $n \geq 2$ firms competing to be the first to make a particular innovation. The winner receives a discounted flow of profits worth V, valued at the date of innovation. The losers receive nothing. At time $t = 0$ each firm decides the level of its R and D effort. Let x_i denote the R and D effort of firm i, and let $c(x_i)$ be the (lump-sum) cost of R and D effort of x_i, where c has properties (1) - (4) above. The discount rate common to all players is r.

There is technological uncertainty. In particular, if x_i is the R and D effort of firm i, then the probability that i makes the innovation at or before time t is assumed to have the exponential form:

$$(9) \qquad F(t;\, x_i) \quad = \quad 1 - e^{-x_i t}$$

Then the probability that i makes the innovation in the small interval $(t,\, t + \varepsilon)$, conditional upon not having succeeded before t, is

$$(10) \qquad \varepsilon F'/(1 - F) \quad = \quad \varepsilon x_i$$

Thus x_i is firm i's hazard rate.

A neat rationalization of this approach is to define $z_i(t)$ as firm i's accumulated knowledge at t, and to define $F_i(z)$ as the probability that firm i innovates with accumulated knowledge no greater than z. Assuming that $z_i(0) = 0$,[1] we have $z_i(t) = x_i t$, because x_i is the rate at which firm i acquires knowledge (sometimes called his rate of R and D). If we specify that $F_i(z) = 1 - e^{-z}$, then the probability that i innovates at or before time t is $1 - e^{-x_i t}$, which is equation (9).

On the assumption that the uncertainties facing the firms are uncorrelated, it follows that the expected payoff of firm i is

$$\Omega_i = \int_0^\infty x_i V e^{-t(r + \sum x_j)} dt - c(x_i)$$

$$(11) \qquad = \frac{x_i V}{r + \sum x_j} - c(x_i)$$

The expected time until some firm makes the innovation is $1/\sum x_j$: see Kamien and Schwartz (1982, p.200) for a proof. The probability that firm i is the first to innovate is $x_i/\sum x_j$.

Firm i chooses x_i to maximise Ω_i. The first-order condition is

$$(12) \qquad \frac{\partial \Omega_i}{\partial x_i} = \frac{(r + \sum_{j \neq i} x_j)V}{(r + \sum x_j)^2} - c'(x_i) = 0$$

The second derivative is

$$(13) \qquad \frac{\partial^2 \Omega_i}{\partial x_i^2} = -\frac{2(r + \sum_{j \neq i} x_j)V}{(r + \sum x_j)^3} - c''(x_i)$$

1. This assumption is made purely for the sake of convenience. Even if firms' initial knowledge levels are different (let alone non-zero), given that no firm has yet innovated it makes no difference to assume $z_i(0) = 0$. The reasons for this are explained on p. 123 below.

Therefore Ω_i is strictly concave in x_i on $[\bar{x}, \infty)$, where $\bar{x}$ is defined in (4). We shall often employ functional forms such that $c''(x) > 0$ for all $x > 0$, such as $\frac{1}{2}x^2$. Then Ω_i is strictly concave on R_+. The first derivative of Ω_i at the origin is positive if

$$(14) \qquad V \quad > \quad (r + \sum_{j \neq i} x_j)\, c'(0)$$

~~This certainly holds if V/r > c'(0).~~ We often assume functional forms such that $c'(0) = 0$, in which case the condition is met automatically. At the symmetric Nash equilibrium ($x_i = x$ for all i) (12) yields

$$(15) \qquad (r + (n - 1)x)V = (r + nx)^2\, c'(x)$$

The comparative static results are that $dx/dn < 0$, $d(nx)/dn > 0$ and $dx/dV > 0$. As the number of firms increases, each firm undertakes less R and D, intuitively because his chance of winning is lower - but aggregate R and D rises.

With free entry we also have the zero profit condition $\Omega_i = 0$, or $xV = (r + nx)c(x)$. With (15) this implies $c'(x) < c(x)/x$. This is the result that there is 'excess capacity' in R and D at equilibrium. There is a close similarity between this R and D model and a quantity-setting oligopoly model.[1] If x_i is reinterpreted as the quantity supplied by firm i, then (11) is the profit function for a firm supplying a homogeneous good in an industry with inverse demand function $p(X) = V/(r + X)$, where X is industry output. This parallel with

1. The similarity is not so evident if R and D expenditure is taken as primitive in the specification of the model. This is another reason why the change of variables is helpful.

quantity-setting models will be explored further in Section 2 of this Chapter.

Dasgupta and Stiglitz (1980b) do not calculate the Nash equilibrium as above. They follow the simpler route of assuming that each firm ignores its own influence upon $\sum x_j$ (Alternatively one could make an equivalent assumption about 'conjectural variations'). This is akin to price-taking behaviour in the reinterpretation as quantity-setting oligopoly. This gives the first-order condition $V = (r + \sum x_j)c'(x_i)$, which together with the zero profit condition implies $c'(x) = c(x)/x$. Dasgupta and Stiglitz compare the equilibrium outcome in the market with free entry with the optimum outcome in a socially managed economy. In the latter, the planner's problem is to choose the number of research units, $\hat{n}$, and R and D effort for each research unit, $\hat{x}$. One of the conditions is $c'(\hat{x}) = c(\hat{x})/\hat{x}$. Therefore if firms in the market ignore their effect upon $\sum x_j$, then they research at the socially efficient rate. If, however, they take account of that effect, then they do too little R and D. Whether or not $\hat{n}$ exceeds the number of firms at the market equilibrium is ambiguous: see Dasgupta and Stiglitz (1980 b, pp. 16ff for details).

Model (ii): That of Lee and Wilde (1980)

In the model just described, all R and D expenditure is incurred at time $t = 0$. Kamien and Schwartz (1982) call this the case of contractual cost. By contrast, Lee and Wilde (1980) study the non-contractual cost case, in which R and D expenditure is incurred as a flow cost until some firm succeeds in making the innovation.[1]

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1. To avoid a proliferation of notation, their model will be described using the same symbols as for model (i), although some of them have slight differences of meaning. For example, $c(x)$ was a lump-sum expenditure in model (i), whereas here it will be a flow cost.

There are $n \geq 2$ firms competing to be the first to make a particular innovation. The winner receives a discounted flow of profits worth V, valued at the date of innovation. At time $t = 0$, each firm decides the rate of its R and D effort. Let x_i be the rate of R and D effort of firm i, and let the associated cost be a fixed lump-sum cost $f \geq 0$ plus a flow cost $c(x_i)$ which continues to be paid until some firm makes the innovation. The discount rate common to all players is r.

Technological uncertainty is assumed to be of the exponential form assumed before. Thus the probability that firm i with R and D rate x_i makes the innovation at or before time t is $1 - e^{-x_i t}$. Thus x_i is firm i's hazard rate.

Assuming zero correlation between the uncertainties facing rival firms, the expected payoff of firm i is

$$\Omega_i = \int_0^\infty [x_i V - c(x_i)] e^{-t(r + \sum x_j)} dt - f$$

$$(16) \qquad = \frac{x_i V - c(x_i)}{r + \sum x_j} - f$$

We find the Nash equilibrium. The first-order condition for firm i is

$$(17) \qquad \frac{\partial \Omega_i}{\partial x_i} = \frac{(r + \sum x_j)(V - c'(x_i)) - (x_i V - c(x_i))}{(r + \sum x_j)^2}$$

$$= 0$$

This implies the simple expression

$$(18) \qquad c'(x_i) = V - \Omega_i - f$$

The right-hand derivative at the origin has the same sign as V - c'(0), which is assumed to be positive. The maximum therefore satisfies the first-order condition (17), provided that $\Omega_i \geq 0$ - otherwise it would be better to do no R and D, and save the fixed cost f. For many instance of c(·), such as $c(x) = \frac{1}{2}x^2$, Ω_i is strictly concave in x for x > 0. Then (17) uniquely defines the maximum, provided that $\Omega_i > 0$.

Comparative statics exercises, using the further condition that equilibrium is stable, show that at the symmetric Nash equilibrium dx/dV > 0 and dx/dn > 0. The latter result at first appears to be the reverse of that obtained for model (i) above. Indeed Kamien and Schwartz (1982, p.193) say that it is 'opposite to that obtained for the contractual cost case'. This is somewhat misleading, because it involves a comparison between unlike objects. In model (i) R and D expenditure is lump-sum, whereas in model (ii) it is a flow cost. As Reinganum (1984b, p. 62), observes, it is quite possible in model (ii) that the expected total R and D expenditure of a firm falls as the number of firms in the market increases, because the decrease in expected discovery time outweights the increase in the flow cost of R and D.

With free entry, we have the zero expected profit condition $\Omega = 0$. From (18), this implies $c'(x) > V$. From (16) we have $xV > c(x)$. Therefore $c'(x) > c(x)/x$. This result is opposite to the 'excess capacity' result obtained in model (i). At the social optimum $c'(x) = c(x)/x$.

Discussion of models (i) and (ii)

We are left in the rather unsatisfactory position of having two models which have dissimilar results but which are hard to choose between. In each model, a player's strategy set is R_+. Much of this thesis (especially Chapters III and IV) is concerned with models that have richer strategy sets. For example, in Reinganum's model, described below, players choose time paths for R and D effort. One simple extension in a framework that embraces both models (i) and (ii) would be to have strategy sets as R^2_+ as follows. Let the R and D effort of firm i depend upon a fixed factor K_i and a variable factor L_i. Suppose that the fixed factor is (say) research equipment or laboratories that are a sunk cost made at the outset. Let the variable factor be (say) researchers who are paid until some firm makes the innovation. Let $x(L_i, K_i)$ be firm i's rate of R and D effort, let $c(L_i)$ be the flow cost of employing L_i units of the variable factor, and let $f(K_i)$ be the (lump-sum) cost of K_i units of the fixed factor.

By the same reasoning as before, the expected payoff of firm i is

$$\Omega_i = \int_0^\infty [x(L_i,K_i)V - c(L_i)]e^{-t(r + \Sigma x_j)} dt - f(K_i)$$

$$(19) \qquad = \frac{x(L_i,K_i)V - c(L_i)}{r + \sum x(L_j,K_j)} - f(K_i)$$

Models (i) and (ii) are special cases of (19). Model (i) is the case in which

$$x(L_i,K_i) = K_i$$

$$f(K_i) = c(K_i)$$

and

$$c(L_i) = 0$$

Model (ii) is the case in which

$$x(L_i,K_i) = L_i$$

$$f(K_i) = f$$

In the general case firm i chooses L_i and K_i to maximise Ω_i, given the choices of other firms. The first order conditions are

$$(20) \qquad \frac{c'(L_i)}{x_{L_i}} = V - \Omega_i - f(K_i)$$

and

$$(21) \qquad \frac{V(r + \sum_{j \neq i} x_j) + c(L_i)}{(r + \sum x_j)^2} = \frac{f'(K_i)}{x_{K_i}}$$

With free entry we have the zero profit condition $\Omega_i = 0$, which implies

$$(22) \qquad \frac{c'}{x_L} > \frac{c}{x} \quad , \quad \text{and}$$

$$(23) \qquad \operatorname{sgn}\left[\frac{f}{x} - \frac{f'}{x_K}\right] = \operatorname{sgn}\left[f - \frac{c}{x}\right]$$

Condition (22) reflects the earlier result of Lee and Wilde for model (ii). However (23) does not necessarily imply the excess capacity result obtained for model (i). Rather than attempt a full comparative statics exercise, let us adopt the simplification used by Dasgupta and Stiglitz (1980b) described on p. 18 above, and assume that firm i ignores its own effect upon Σx_j. Then the first-order conditions and symmetry imply

$$(24) \qquad x_L V = c' \quad , \quad \text{and}$$

$$(25) \qquad x_K V = (r + nx)f'$$

With the further assumption that c and f are linear functions, total differentiation of (24) and (25) implies

$$(26) \qquad x_{LL}dL + x_{LK}dK = 0 \quad , \quad \text{and}$$

$$(27) \qquad x_{KL}dL + x_{KK}dK = \psi\{n(x_K dK + x_L dL) + x dn\},$$

where ψ is a positive constant. Since x (·) is rather like a production function, it is natural to assume x_L, x_K, $x_{LK} > 0$ and x_{LL}, $x_{KK} < 0$. Then dL/dn and dK/dn have the same sign, but the direction of the sign is ambiguous. This more general formulation has not yielded concrete results, but it has perhaps served to clarify the relationship between models (i) and (ii).

Model (iii): that of Reinganum[1]

There are $n \geq 2$ firms competing to be the first to make a particular innovation. The value of the innovation to the winner is V, valued at time zero and independent of the timing of the innovation. $T < \infty$ is the date of Doomsday. The discount rate common to all players is r. Firm i's accumulated knowledge at time t is $z_i(t)$. By assumption $z_i(0) = 0$. The rate at which firm i acquires knowledge at t is denoted by $x_i(t)$. $x_i(t)$ is bounded above by B. Thus $\dot{z}_i(t) = x_i(t)$. The flow cost - valued at time t - of acquiring knowledge at rate x_i is $c(x_i)$ which is specified as $\frac{1}{2}x_i^2$. The probability that z is sufficient knowlede for firm i to innovate is $F_i(z) = 1 - e^{-z}$. Therefore the hazard rate for firm i at t is $x_i(t)$. A strategy for firm i is a function of time and current knowledge levels (For technical reasons, the strategy space is restricted to continuous functions satisfying a Lipschitz condition). The solution to the differential game is a Nash equilibrium in these strategies.

The Nash equilibrium strategies are

$$x^*(t, \underline{z}) = \frac{2(n-1)Ve^{rt}}{2n - 1 - \exp\{V(n-1)(e^{rt} - e^{rT})/r\}} \tag{28}$$

Note that equilibrium strategies are independent of knowledge levels - they are functions only of time. R and D effort x^* increases over time. For given $t < T$, x^* is increasing in n. This mirrors the result obtained for model (ii) above.

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1. In Reinganum's model it is assumed that imitation is costless and immediate, and that losers in the R and D competition receive some payoff. We ignore this possibility for the sake of comparability with other models. We also ignore Reinganum's parameter λ in the probability distribution function, because we are not concerned with comparative statics using that term.

Although the date of Doomsday is finite in this model, the infinite horizon case can be examined as $T \to \infty$. From (28) we have

$$\lim_{T\to\infty} x^*(t, \underline{z}) = \frac{2(n-1)Ve^{rt}}{2n-1} \quad . \tag{29}$$

Hence x^* not only increases with time but goes to infinity.

Discussion and modification of model (iii)

Perhaps the most curious feature of the specification of Reinganum's model is that the value of making the innovation is not discounted, but R and D expenses are discounted. Therefore, valued at time t, the innovation is worth Ve^{rt} - whereas in models (i) and (ii) it was worth V, which is more natural. This curious asymmetry is an obvious reason why x^* is increasing in t, because the value of the innovation increases exponentially relative to the cost of pursuing it. It explains why, in the limit, x^* increases without bound.

What happens if costs and benefits are treated on a par? Henceforth suppose that the value of the innovation to the winner is V, valued at time t. Otherwise let the model be the same as before. The objective functional of firm i is

$$\Omega_i = \int_0^T (x_i(t)V - \tfrac{1}{2}(x_i(t))^2) \exp\{-rt - \Sigma z_j(t)\}\, dt \tag{30}$$

The Euler equation is

$$\dot{x}_i + (V - x_i)(r + \Sigma x_j) = x_i V - \tfrac{1}{2} x_i^2 \quad . \tag{31}$$

With symmetry this implies

(32) $$\dot{x} = (n - \tfrac{1}{2})x^2 + (r - (n-1)V)x - rV \quad .$$

Factorising the quadratic on the RHS of (32) we get

(33) $$2(2n-1)\dot{x} = [(2n-1)x - \mu - \beta][(2n-1)x - \mu + \beta]$$

where

(34) $$\mu = (n-1)V - r \quad , \text{ and}$$

(35) $$\beta = \{(n-1)^2V^2 + 2nrV + r^2\}^{1/2}$$

From (33) we have

(36) $$\frac{(2n-1)\,dx}{(2n-1)x - \mu - \beta} - \frac{(2n-1)\,dx}{(2n-1)x - \mu + \beta} = \beta dt$$

Integrating both sides with respect to t we obtain

(37) $$\ln\left[\frac{(2n-1)x - \mu - \beta}{(2n-1)x - \mu + \beta}\right] = \beta t + \text{const.}$$

The terminal condition is $c'(x(T)) = x(T) = V$. This enables the constant in (37) to be found. We obtain

$$\text{(38)} \qquad \ln\left[\frac{(2n-1)V - \mu - \beta}{(2n-1)V - \mu + \beta} \right] - \ln\left[\frac{(2n-1)x(t) - \mu - \beta}{(2n-1)x(t) - \mu + \beta} \right] = \beta(T - t)$$

Using the definition of μ, this implies

$$\text{(39)} \qquad \left[\frac{(2n-1)x(t) - \mu - \beta}{(2n-1)x(t) - \mu + \beta} \right] = \left[\frac{nV + r - \beta}{nV + r + \beta} \right] \exp\{\beta(t - T)\}$$

Let the RHS of (39) be denoted by θ. Then we have

$$\text{(40)} \qquad x(t) = \frac{\mu + \beta + (\beta - \mu)\theta}{(2n - 1)(1 - \theta)}$$

As $T \to \infty$, $\theta \to 0$. Therefore

$$\text{(41)} \quad \lim_{T\to\infty} x(t) = \frac{\mu + \beta}{2n-1} = \frac{(n-1)V - r + \{(n-1)^2V^2 + 2nrV + r^2\}^{1/2}}{(2n - 1)} .$$

Let us now compare this result with that obtained from Lee and Wilde's model - i.e. model (ii) - when $c(x) = \frac{1}{2}x^2$ and $f = 0$. The expected payoff of firm is then

$$\Omega_i = \int_0^\infty (x_iV - \tfrac{1}{2}x_i^2)e^{-t(r + \Sigma x_j)}dt$$

(42) $$= \frac{x_iV - \frac{1}{2}x_i^2}{r + \Sigma x_j}$$

Equation (18) and symmetry imply

(43) $$x = V - \frac{xV - \frac{1}{2}x^2}{r + nx}$$

Therefore

(44) $$(n - \tfrac{1}{2})x^2 - ((n-1)V - r)x - rV = 0$$

The solution of this quadratic is

(45) $$x = \frac{\mu \pm \beta}{2n - 1}$$

which is identical to (41).

We have shown that when Reinganum's model is reformulated so that the value of the innovation is discounted in the same way as the cost of R and D, the solution in the limit as $T \to \infty$ is identical to the corresponding special case of Lee and Wilde's model. This limiting case is perhaps the most interesting one to study. Reinganum has finite T in her model for technical, not economic, reasons.[1]

It is not surprising that Reinganum's model, when reformulated, is in some sense equivalent to Lee and Wilde's model in the infinite horizon case. If the prize is discounted and if there is no finite Doomsday, then the game is in some sense stationary. If no firm is fortunate enough to innovate in the interval $[0,\varepsilon)$, then the subgame beginning at time ε is a replica of the game that began at time zero. Therefore one would expect equilibrium strategies for the interval $[\varepsilon, 2\varepsilon)$ to be the same as for the interval $[0, \varepsilon)$. This being so for all $\varepsilon > 0$, strategies in equilibrium are constant.

<u>Conclusion</u>

Although model (iii) is more subtle and sophisticated than models (i) and (ii), it has some disadvantages. Its most curious economic feature is that the value of the innovation is not discounted - although costs are. This is one reson why in equilibrium R and D increases with time. The other reason for this is that there is a finite Doomsday date in model (iii) - for technical reasons. A further drawback of model (iii) is the complexity of deriving the solution, even when the special form $c(x) = \frac{1}{2}x^2$ is assumed (Reinganum (1982, p.673) notes that generalisation to any isoelastic cost function is straightforward, but this is still a narrow class). By contrast, the simplicity of models (i) and (ii) permits the economics to be illuminated more clearly, and a

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1. It is a simple matter to find equilibrium in the version of Lee and Wilde's model with a finite horizon. The range of integration is 0 to T, rather than 0 to ∞ as in (16). The fraction on the RHS of (16) is then multiplied by $\zeta = (1 - e^{-rT})$. The first-order condition (17) holds as before. When $c(x) = \frac{1}{2}x^2$ the equilibrium solution is $\tilde{x} = (\tilde{\alpha} + \tilde{\beta})/(2n - \zeta)$, where $\tilde{\alpha} = (n - \zeta)V - r$ and $\tilde{\beta} = \{(n - \zeta)^2V^2 + 2nrV + r^2\}^{1/2}$. Compare with (45) which is the special case when $\zeta = 1$, i.e. $T = \infty$.

more general cost function can be employed. We have shown further that the solution to model (iii) with economically desirable amendments is equivalent to a special case of model (ii). Together these reasons imply that although model (iii) has great theoretical interest, it is not appropriate for application to particular topics such as those investigated in later chapters of this thesis. For those purposes we shall employ versions of models (i) and (ii).

Finally it must be emphasised that in these models the hazard rate depends upon current effort levels, owing to the 'memorylessness' of the exponential distribution. History - in particular the experience that firms have acquired - plays no part. Fethke and Birch (1982) introduce a less restrictive formulation and consider the implications of a nondecreasing (as opposed to constant) hazard rate. Fudenberg et al. (1983) study models in which a firm's hazard rate depends upon its current level of experience. However, the models described earlier in this section possess considerable mathematical convenience and are therefore attractive to use in many applications. It must be remarked that, properly speaking, they are not models of races, for in races past efforts do matter. This point is discussed more fully in the Introduction to Chapter III, where models designed to capture particular race-like features are presented.

2. R AND D COMPETITION WITH CORRELATED UNCERTAINTIES

In all the models so far it has been assumed that the uncertainties facing rival firms are completely uncorrelated. This assumption is very common in the literature. It is most convenient, since if $F(z_i)$ is the probability that z_i is sufficient knowledge for firm i to innovate, then the probability that no firm has innovated when knowledge levels are $z_1,\ldots,z_n$ is $\prod_i (1 - F(z_i))$. When $F(z_i) = 1 - \exp\{z_i\}$, this latter expression is simply $\exp\{\Sigma z_i\}$.

Another branch of the literature on R and D competition[1] assumes that there is no technological uncertainty or - which is much the same thing - that the uncertainties facing rival firms are perfectly correlated. In the latter case the probability that no firm has innovated when knowledge levels are $z_1,\ldots,z_n$ is $1 - \text{Max}\{F(z_1),\ldots,F(z_n)\}$.

The degree to which uncertainties are correlated is an important influence upon R and D competition. Dasgupta and Stiglitz (1980b, p. 26, point (4)) identify it as the critical determinant of the number of firms engaged in R and D, although they examine only the polar cases of zero and perfect correlation. Our aim in this section is to explore the intermediate - or rather the general - case of incomplete correlation. Specifically, we see how the equilibrium of the R and D competition varies with a 'correlation parameter' σ, which varies between 0 and 1.

It is a property of the models used that if units of knowledge, or the acquisition of knowledge, are appropriately redefined, then partial correlation of uncertainty can be represented as zero correlation of uncertainty but with a different R and D cost function. In some sense the case of (apparently) zero correlation therefore covers cases of partial correlation. There is a striking parallel with the issue of

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1. See Chapter II for, inter alia, a discussion of this literature.

product differentiation in oligopoly theory. For example, in simple models of product differentiation, the inverse demand for product i is sometimes a function of the quantity of that product x_i and of $\sum_j \emptyset(x_j)$ where $\emptyset$ is a monotonic function. Defining $y_k = \emptyset(x_k)$ the profit function can be written in terms of y's to yield a homogeneous goods model. The cost function c(x) becomes $c(\emptyset^{-1}(y))$. Thus the differentiated goods model is made formally equivalent to a homogeneous goods model with a different cost function (see Spence (1984, p.104, fn. 8). This equivalence is not only interesting in its own right, for it also enables one to see how the degree of product differentiation corresponds to (say) the degree of convexity of the cost function. The same is true with the degree of correlation of uncertainty.

The Analysis

After some introductory remarks, we analyse versions of models (i) and (ii) in turn to see how the degree of correlation influences the nature of R and D competition.

As before, let x_i denote the R and D effort of firm i. When there is no correlation of uncertainty, the probability that firm i is first to innovate is $x_i / \sum x_j$. When there is a perfect correlation, the probability is zero if $x_j > x_i$ for some j, and otherwise is $\frac{1}{k}$ where k firms have $x = x_i$. These are the polar extremes of a more general case where the probability is

$$(1) \qquad P_i = x_i^{\rho} / \sum x_j^{\rho} \qquad , \rho \geq 1.$$

The correlation parameter is $\sigma = (\rho - 1)/\rho$, which varies between 0 (zero correlation) and 1 (perfect correlation).

Version of Model (i)

The basic description is as in the first paragraph of the description of model (i) on p. 15 above (n players, prize worth V, etc.) Let $z_i(t) = x_i t$ be the knowledge level of firm i at time t, and let $\underline{z} = (z_1, \ldots z_n)$. Let $G(\underline{z})$ be the probability that for all i, z_i is insufficient knowledge for i to innovate. G is given the specific form

$$G(\underline{z}) = \exp\{-(\Sigma z_j^{\rho})^{1/\rho}\}$$

$$(2) \qquad = \exp\{-t(\Sigma x_j^{\rho})^{1/\rho}\}$$

$$= e^{-t\alpha} \qquad , \text{say} \quad .$$

In model (i) above we had $G(\underline{z}) = \prod_i (1 - F(z_j)) = \exp\{-\Sigma z_j\}$, which is a special case of (2).

The expected payoff of firm i is

$$\Omega_i = V \int_0^{\infty} - \frac{\partial G}{\partial z_i} x_i \, e^{-rt} dt - c(x_i)$$

$$= V \int_0^{\infty} x_i^{\rho} (\Sigma x_j^{\rho})^{-\sigma} \, e^{-t(r+\alpha)} \, dt - c(x_i)$$

$$(3) \qquad = \frac{V x_i^{\rho}}{r\alpha^{\rho-1} + \alpha^{\rho}} - c(x_i)$$

The probability that i wins is $x_i^{\rho} / \Sigma x_j^{\rho}$, justifying (1).

The expected overall discovery time is $\int t \, F' dt = \int G \, dt = 1/\alpha$ (see Kamien and Schwartz pp. 199-200). When n firms each research at rate x, this is $1/n^{1/\rho} x$. When n = 1, this is 1/x, as one would expect in a consistent framework. For given x, expected discovery time is 1/nx for zero correlation, and goes to 1/x as correlation becomes more complete. In the limit, expected discovery time is independent of n because the firms are completely 'duplicating' one another's efforts.

It will prove convenient to define

$$(4) \qquad y_i = x_i^{\rho} \qquad \text{and} \qquad Y = \sum y_i$$

Then (3) can be expressed as

$$(5) \qquad \Omega_i = \frac{Vy_i}{rY^{\sigma} + Y} - c(y^{1/\rho})$$

Note that the RHS of (5) can be interpreted alternatively as the profit function of an oligopolist in a homogeneous industry with inverse demand given by $p = V/(r\, Y^{\sigma} + Y)$, where Y is industry output. Equally, the RHS of (3) can be regarded as the profit function of an oligopolist in a differentiated goods industry where inverse demand for product i is $p_i = Vx_i^{\rho-1}/(r\alpha^{\rho-1} + \alpha^{\rho})$. This is an illustration of the transformation trick described above.

Rather than attempt a full analysis of Nash equilibrium in the model, we shall employ two simplifying assumptions in turn.

Subcase (a): Each firm assumes that it does not affect the overall expected discovery time[1]

That is, firm i assumes that Y is independent of y_i, or, equivalently, that α is independent of x_i. The first-order condition, ignoring subscripts, is

$$(6) \qquad \frac{V}{rY^{\sigma} + Y} = y^{-\sigma} c' / \rho$$

with symmetry, (4) and (6) imply

$$(7) \qquad \rho V = (rn^{\sigma} + nx)c'$$

The second order condition is

$$(8) \qquad \left[\frac{y^{-\sigma}}{\rho} \right]^2 c'' > \frac{\rho - 1}{\rho^2} \, y^{\frac{1}{\rho} - 2} c' ,$$

which is equivalent to

$$(9) \qquad \frac{xc''}{c'} > \rho - 1$$

1. This is akin to the Dasgupta-Stiglitz assumption: see p. 18 above. It is like an assumption of price-taking behaviour: see Dixit and Stiglitz (1977) in another context.

The LHS of (9) is the elasticity of marginal cost with respect to output. We shall sometimes focus on specific form

$$(10) \qquad c(x) = a + x^{\gamma} \qquad , \qquad \gamma > 1$$

See figure 1. Then (9) becomes $\gamma > \rho$: for the second-order condition to be met, returns to scale must increase fast enough relative to the degeree of correlation of uncertainty. Condition (9) reveals why Nash equilibrium is likely to fail to exist when uncertainties are highly correlated, and does not exist when correlation is perfect.

From (7) it is clear that with ρ fixed, x is decreasing in n. With n fixed, differentiation of (7) yields

$$(11) \qquad [V - rn^{\sigma}(\ln n)\, c'/\rho^2]d\rho = \{nc' + \rho Vc''/c'\}dx$$

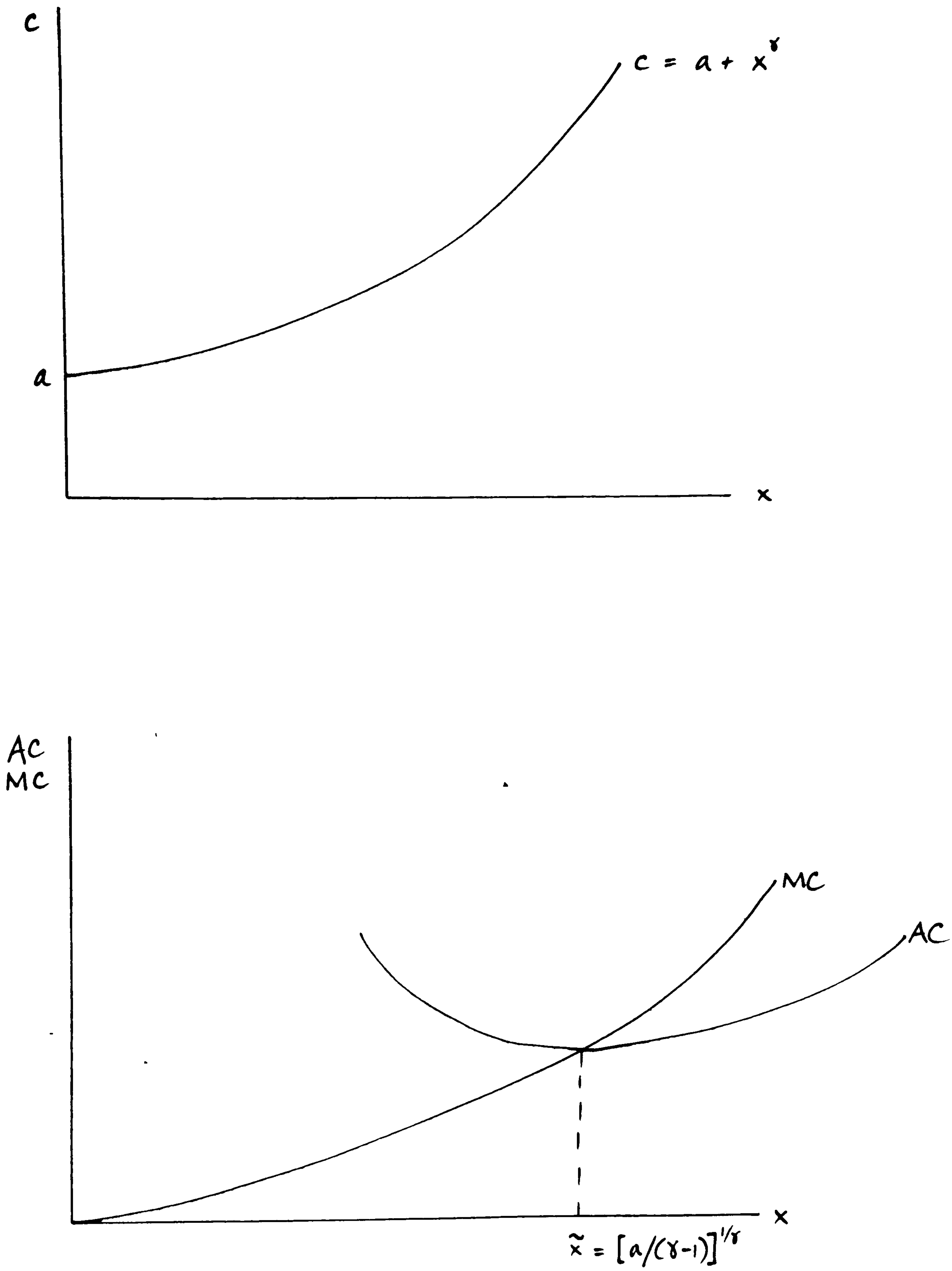
The term $\{\cdot\}$ on the RHS is positive, but the sign of $[\cdot]$ on the LHS is ambiguous. For example, if $c(x) = x^2$ and $\rho = 1$, then, from (7)

$$x = \frac{-r + \sqrt{r^2 + 2nV}}{2n}$$

whereupon $[\cdot]$ has the same sign as $V - 2rx(\ln n)$, which has the same sign as

$$\left[\frac{nV}{r(\ln n)} + r\right] - \sqrt{r^2 + 2nV}$$

Figure 1



The Cost Function

Squaring both terms we have that $dx/d\rho$ has the sign of

$$(12) \qquad nV - 2r^2 (\ln n) [(\ln n) - 1] \quad ,$$

which is ambiguous. The intuitive explanation for this ambiguous result would appear to be as follows. Suppose that initially we have a symmetric equilibrium. Let ρ increase by a small amount, and suppose that all players other than i keep their R and D effort as before. Since uncertainties are more correlated than before, a small increase in x_i increases the probability that i wins by more than it would have done before. This force encourages i to increase x_i. But there is an opposing force. Since uncertainties are more correlated, the expected time until discovery is higher. With positive discounting, the expected value of winning is therefore lower. This tends to discourage R and D spending by i. The latter force dominates the former if r is large enough.

Since firms are making non-negative profits we have from (5) that

$$(13) \qquad Vx \geq (rn^{\sigma} + nx)c$$

With (7) this is equivalent to

$$(14) \qquad c' \geq \rho c/x$$

Since $\rho \geq 1$ it follows that MC $\geq$ AC and therefore that firms operate where there are decreasing returns in R and D.

If n is determined endogenously by the free entry condition that expected payoffs are zero, then (13) and (14) hold as equalities (ignoring integer problems). Then x is an increasing function of ρ, independent of n. To see that x is increasing in ρ, totally differentiate (14) as an equality to find that dx/dρ has the same sign as

$$1 - \frac{xc'}{c} + \frac{xc''}{c'}$$

The first two terms sum to 1 - ρ. By (9) the third term exeeds ρ - 1. Therefore dx/dρ > 0. It follows that equilibrium n is decreasing in ρ, for (13) as an equality can be written as

$$(15) \qquad V = (rn^{\sigma} + nx)c/x$$

As ρ increases so too does σ, x and c/x (because MC $\geq$ AC it follows that AC rises with x, which increases with ρ). Therefore n must fall to maintain the equality of (15). We have established that if uncertainties are more correlated then there are fewer firms at equilibrium, but each does more R and D.

Subcase (b): Cournot Behaviour and r = 0

We now assume Cournot behaviour but for the sake of simplicity assume that there is no discounting. From (3) we see that the expected payoff of firm i is

$$(16) \qquad \Omega_i = \frac{Vx_i^\rho}{\Sigma x_j^\rho} - c(x_i) \quad .$$

The first term on the RHS is simply V times the probability that i is the winner. The first-order condition implies

$$(17) \qquad \rho(n - 1)V/n^2 = xc'$$

at the symmetric equilibrum. We shall assume that the second-order condition holds. (Condition (9) is suffficient for it to do so).

From (17) it is clear that, for given ρ, x is decreasing in n. For given n, x is increasing in ρ. Thus R and D effort increases as the correlation of uncertainties is greater. This is because the extent to which a firm improves it probability of winning by a small increase in R and D is greater when uncertainties are more correlated. The fact that, for given levels of R and D effort, expected discovery time is greater, is of no consequence, since $r = 0$ in this subcase (cf. the discussion following (12)).

If n is determined endogenously by the free entry zero profit condition, then

$$(18) \qquad V = nc \quad .$$

From (17) and (18) we have

(19) $\rho(n - 1)/n = xc'/c$

Let $\tilde{x}$ denote the rate of knowledge acquisition at which cost per unit of knowledge acquisition is minimised (In one sense - but perhaps not the proper sense (see below) - this is the most efficient rate of R and D effort). Then (19) implies that $x \gtrless \tilde{x}$ according as $\rho \lessgtr n/(n-1)$.

As to comparative statics, it follows from (18) that

(20) $c\,dn + nc'\,dx = 0$

$dx/d\rho$ and $dn/d\rho$ have opposite signs. Total differentiation of (19) yields

$$(1 - 1/n)d\rho = \left[\frac{cc' + cxc'' - xc'c'}{c^2}\right] dx - \rho\,\frac{dn}{n^2}$$

(21) $$= \left[1 + \frac{xc''}{c} - \frac{xc'}{c} + \frac{\rho}{n}\right]\frac{c'}{c}\,dx \quad ,$$

using (20). From (16), the expression [·] on the RHS of (21) is equal to

(22) $$\frac{xc''}{c'} + 1 - \rho + \frac{2\rho}{n} \quad .$$

By the second-order condition (see (9)), this is positive. Therefore $dx/d\rho > 0$ and $dn/d\rho < 0$. With greater correlation of uncertainty, equilibrium sustains fewer firms, but each does more R and D. Total R and D effort increases (decreases) with ρ if $\rho < (>)\ n/(n-1)$. This is because (19) and (20) imply

$$d(nx) = x\,dn + n\,dx$$

$$= [1 - xc'/c]n\,dx$$

(23) $$= [n - (n-1)\rho]\,dx$$

and $dx/d\rho > 0$.

With the the specific functional form (10), the explicit solution is

(24) $$n = \frac{V - K + \sqrt{V^2 + K^2 - 2KV + 4aK}}{2a}$$

(25) $$x = \left[\frac{V}{n} - a\right]^{\frac{1}{\gamma}}$$

where $K = V\rho/\gamma$

To summarise, we have shown that with greater correlation of uncertainty, equilibrium sustains fewer firms, but each does more R and D. The intuition is that when uncertainties are more correlated, the expected benefit of doing slightly more R and D than ones rivals is higher. At equilibrium, each therefore does more R and D. Because efforts are greater, fewer firms can enjoy expected non-negative profits.

Social Optimality

Let the social value of the discovery be W. It is reasonable to suppose that W > V, because the innovation would be more fully used at the social optimum than at the monopolist's optimum, and the monopoly welfare loss would be eliminated: see Dasgupta and Stiglitz (1980 b, p. 5). By previous reasoning it follows that the expected net social benefit of m firms (or laboratories) each acquiring knowledge at rate x is

$$(26) \qquad U = \frac{W x m^{1/\rho}}{r + x m^{1/\rho}} - mc(x)$$

The first-order conditions imply that

$$(27) \qquad \frac{rWm^{-1-1/\rho}}{(rm^{-1/\rho} + x)^2} = \rho c/x = c'$$

Assuming an interior maximum, social optimality therefore requires $xc'/c = \rho$, i.e. operation at a point where there are diminishing returns to R and D. This condition is exactly met when firms assume that they do not affect the overall expected discovery time and when there is free entry: see the discussion of (14) above. Cournot firms each do too little R and D, relative to the social optimum: see e.g. equation (19). This is despite the fact that each firm might be operating at a point of diminishing returns. Social optimality requires MC = AC <u>only when</u> there is no correlation of the uncertainties facing different labs. When some correlation exists there is a tradeoff between the efficiency of each lab and the number of labs - which arises because there is a kind of 'duplication' of efforts. There is an analogy with product differentiation and optimum product diversity. With differentiated products there is a tradeoff between the number of products and the efficiency with which each product is made. Social optimality does not require MC = AC, but involves 'excess capacity' (see Dixit and Stiglitz (1977)). The extent of departure from MC = AC depends on the degree of differentiation between products. The parallels are very close.

<u>Version of Model (ii)</u>

So far we have explored the question of the correlation of uncertainties using the Loury-Dasgupta-Stiglitz 'contractual cost' model of R and D. We now employ a version of the Lee-Wilde 'flow cost' model. The notation is the same as that used in the version of model (i) above, but there is no risk of confusion.

The model is the same as before, except that R and D costs are incurred as a flow until some firm succeeds in innovating. The expected payoff of firm i is

$$\Omega_i = \int_0^\infty - Vx_i \frac{\partial G}{\partial z_i} e^{-rt} - c(x_i)e^{-t(r+\alpha)}\, dt$$

$$= \frac{Vx_i^\rho \alpha^{-(\rho-1)} - c(x_i)}{r + \alpha} \qquad (28)$$

Consider two subcases.

Subcase (a): each firm assumes that it does not affect the overall expected discovery time

Firm i assumes that α is independent of x_i. The first-order condition is

$$\rho V(x/\alpha)^{\rho-1} = c' ,$$

or

$$\text{(29)} \qquad \rho V = n^{\sigma} c'$$

with symmetry. The second-order condition is

$$\text{(30)} \qquad \frac{xc''}{c'} > \rho - 1 ,$$

which is identical to (9).

From (29) it is clear that x is decreasing in n for given ρ. When n is fixed, differentiation of (29) reveals that $dx/d\rho$ has the same sign as $(\rho - \ln n)$. Therefore the sign of $dx/d\rho$ is ambiguous. The intuitive explanation is as follows. When uncertainties are more correlated, an extra amount of R and D effort by firm i improves its chance of winning by more than it would have done previously. On the other hand, α falls, i.e. the expected time until discovery is greater. From (28) it can be seen that this increases expected costs by more than expected benefits. This tends to offset the former effect. Hence the overall effect is ambiguous.

Non-negative profit requires

$$(31) \qquad Vx \geq n^{\sigma}c$$

which with (29) implies

$$(32) \qquad c' \geq \rho c/x \quad .$$

This is identical to (14). Firms operate where there are decreasing returns in R and D.

When n is determined endogenously by free entry, and hence the zero profit condition, then (31) and (32) hold as equalities. Then x is an increasing function of ρ, independent of n. Since AC is increasing in x (because MC ≥ AC), it follows from (31) as an equality that n is decreasing in ρ. Thus we have established the same comparative statics results as with the version of model (i) above. As the correlation of uncertainties increases, there are fewer firms at equilibrium, but each does more R and D.

In the special case (10), (31) and (32) as equalities imply that

$$(33) \qquad x = [a/(\gamma - \rho)]^{1/\gamma} \quad \text{, and}$$

$$(34) \qquad n = \left[\frac{\rho V}{\gamma} \left(\frac{\gamma - \rho}{a}\right)^{1-1/\gamma} \right]^{1/\sigma}$$

Subcase (b): Cournot behaviour[1]

Consider firm i's objective function (28) as a fraction of the form N/D (N for numerator, D for denominator), where N and D are fractions of x_i . The first-order condition is that N'D = D'N. The zero profit conditon implied by free entry is N = 0, so the first-order condition is simply N'=0. This implies

$$V[\rho(x_i/\alpha)^{\rho-1} - (\rho-1)(x_i/\alpha)^{2\rho-1}] = c' \tag{35}$$

With symmetry (35) implies

$$V[\rho n^{-\sigma} - (\rho-1)n^{-1-\sigma}] = c' \tag{36}$$

The zero profit condition is

$$Vxn^{-\sigma} = c \tag{37}$$

From (36) and (37) we have

$$\frac{xc'}{c} = \frac{(n-1)\rho+1}{n} \tag{38}$$

1. This subcase is easy enough to analyse without the simplifying assumption that r = 0, which was made for the analysis of the corresponding subcase of version (i) above.

which exceeds 1 (but is less than ρ) if there is some correlation of uncertainties. Firms therefore operate at a point of decreasing returns on the c function.

As to comparative statics, total differentiation of (37) and (38) yields

$$(39)\qquad 0 = n^{\sigma}\, d(c/x) + \sigma n^{\sigma-1}\,(c/x)dn + n^{\sigma}\rho^{-2}(c/x)(\ln n)d\rho$$

$$(40)\qquad d(xc'/c) = (1 - n^{-1})d\rho + n^{-2}\,(\rho - 1)dn$$

Simplification and elimination of dn yields

$$(41)\qquad \left[\frac{d(xc'/c)}{dx} + \frac{\rho}{n}\,\frac{d(c/x)}{dx}\right] dx = \left[1 - \frac{1}{n} - \frac{\ln n}{n\rho}\right] d\rho$$

The expression [·] on the LHS is positive because (xc'/c) and (c/x) are both increasing in x. The expression [·] on the RHS is positive because $n - 1 > \ln n$ for $n > 1$. Therefore $dx/d\rho > 0$. From (39) it follows immediately that $dn/d\rho < 0$. Again we have the result that as correlation of uncertainty increases there are fewer firms in equilibrium, but each does more R and D.

We now consider n to be given exogenously, and find the Nash-Cournot equilibrium. For simplicity assume that r=0 (as in subcase (b) of version (i) above). Using (4), (28) can be written as

$$(42)\qquad \Omega_i = \frac{Vy_i}{Y} - \frac{c(y_i^{1/\rho})}{Y^{1/\rho}}$$

The first-order condition is

$$\text{(43)} \qquad \frac{V(Y - y_i)}{Y^2} = \frac{Y^{1/\rho} y_i^{-\sigma} c' - Y^{-\sigma} c}{\rho Y^{2/\rho}} .$$

With symmetry, (4) and (43) imply

$$\text{(44)} \qquad (n - 1)\rho V x = n^{\sigma}(nxc' - c)$$

For example, if $c(x) = x^{\gamma}$, then from (44) we have

$$\text{(45)} \qquad x^{\gamma-1} = \frac{(n - 1)\rho V}{n^{\sigma}(n\gamma - 1)} .$$

Non-negative profits imply that

$$\text{(46)} \qquad Vx \geq n^{\sigma} c$$

From (44) and (46) we have

$$\text{(47)} \qquad \frac{xc'}{c} \geq \frac{(n - 1)\rho + 1}{n}$$

This exceeds 1 when there is some correlation of uncertainty. Firms therefore operate at a point of diminishing returns.

With n fixed, total differentiation (44) yields

$$(48) \qquad (n-1)Vn^{-\sigma}\,(1 - (\ln n)/\rho)d\rho = \left[nc'' - \frac{d(c/x)}{dx}\right] dx$$

The RHS of (48) is positive. Therefore the sign of $dx/d\rho$ is the same as that of $(\rho - \ln n)$. The sign is ambiguous.

The reason for the ambiguity is the same as before. When uncertainties are more correlated, the effect of extra R and D on the probability of winning is greater, but the expected time until discovery lengthens. These effects work in opposite directions.

Social Optimality

As before, let the social value of the discovery be W. The expected net social benefit of m labs each making R and D effort x is

$$(49) \qquad U = \frac{Wxm^{1/\rho} - mc(x)}{r + xm^{1/\rho}}$$

The first-order conditions with respect to x and m are

$$(50) \qquad \frac{(Wm^{1/\rho} - mc')}{m^{1/\rho}} = U \qquad , \text{ and}$$

$$(51) \qquad \frac{(Wx\rho^{-1}m^{-\sigma} - c)}{x\rho^{-1}m^{-\sigma}} = U$$

From (50) and (51) we have

$$(52) \qquad \frac{xc'}{c} = \rho$$

Assuming an interior maximum, when there is some correlation of uncertainties facing different labs, social optimality therefore requires operation at a point of diminishing returns on the c function. Comparison with (32) shows that if each firm takes overall expected discovery time as given, then if expected payoffs are positive each firm does too much R and D relative to the social optimum ; if expected payoffs are zero, each firm does the right amount. Comparison with (38) shows that if Cournot firms have zero profits, then each does too little R and D. If Cournot firms have positive profits it is ambiguous whether they do too much or too little, relative to the social optimum.

Summary and Conclusion

This section has addressed the issue of the degree of correlation between the uncertainties facing rival firms. A CES functional form was adopted, partly for reasons of analytical tractibility and also so as to see how equilibrium varied with a 'correlation parameter'. We examined in turn versions of the Loury-Dasgupta-Stiglitz model and the Lee-Wilde model (i.e. models (i) and (ii) in Section 1 of this chapter). In respect of each version we made a variety of assumptions. Each version had two subcases:

(a) each firm assumes that it does not affect overall expected discovery time; and

(b) Cournot behaviour (and, sometimes, r=0).

We explored the cases of endogenous n (determined by the free entry zero profit condition) and of exogenous n.

The table below is a self-explanatory summary of some of the principal results.

Table : Summary of Results

	Version	(i)		(ii)	
	Subcase	(a)	(b)	(a)	(b)
Zero profit	$\frac{dx}{d\rho}$	+	+	+	+
	$\frac{dn}{d\rho}$	−	−	−	−
	$\frac{xc'}{c}$	$= \rho$	$< \rho$	$= \rho$	$> 1, < \rho$
Fixed n	$\frac{dx}{d\rho}$	?	+	?	?

Under all the assumptions tried, at equilibrium with free entry R and D per firm increases with the increased correlation of uncertainty, and the number of firms falls. The robust nature of these two results is comforting. With fixed n, the effect of correlation upon R and D is ambiguous: it depends on the values of other parameters. The ambiguity of the final result is perhaps rather a surprise. Its explanation would appear to be that that greater correlation of uncertainty encourages and R and D inasmuch as a firm's probability of winning becomes more sensitive to its R and D, but discourages it inasmuch as the expected time until discovery becomes greater.

CHAPTER II

PATENT RACES AS BIDDING GAMES

INTRODUCTION

For the most part, the models of R and D with uncertainty discussed in the previous Chapter have been games with n identical players. In principle those models can easily be modified to address cases in which players are not symmetrically placed: see for example Reinagum (1983). However, a rather simpler approach has been adopted to study R and D competitions in which there is some asymmetry between players. This is to model patent races as bidding games, in which the prize (i.e. the patent) is awarded to the highest bidder (i.e he who does most R and D). This type of approach was used in Dasgupta and Stiglitz (1980b), Dasgupta (1982), Gilbert (1981), Gilbert and Newbery (1982). Section 1 below briefly reviews this literature. Several criticisms have been made against representing patent races as bidding games. notably by Reinganum (1983) and Salant (1984). The lively debate resulting from these criticisms, and some lessons to be drawn from it, are the subject of Section 2 of this Chapter.

There are many asymmetries of economic interest. The one to have received most attention is that between an incumbent firm and a potential entrant into his market. The story is that the patent for which the firms are competing is for a new product or process such that the challenger will gain entry into the incumbent's market if and only if he wins the patent.

However this is not the only interesting asymmetry. A natural extension of the above example is to the case of m incumbents and n potential entrants. This is the subject of Section 3 of this Chapter, where it is shown that results that hold when m = 1 may be reversed when m > 1. With several incumbents one can also consider the effects, e.g. upon entry, of the formation of a joint venture by the incumbents.

Another important asymmetry is that between firms with different intial cost levels competing for a patent for a new superior technology. This is the subject of Section 4 of this Chapter, which explores a duopoly model. It is shown that the degree of competitiveness between the firms (measured by a 'conjectural variation' term), as well as the initial disparity of costs and the extent of the superiority of the new technology, is an important influence upon incentives in - and hence the likely outcome of - the R and D competition.

1. PATENT RACES AS BIDDING GAMES

Gilbert and Newbery (1982, p.517) write:

> 'A formal model of the patent competition is that of an auction market. Each firm enters a bid which is the maximum present-value amount that the firm will spend on research and development.'

In Gilbert and Newbery's basic model, only the winner of the prize (i.e. the patent) loses his bid. If players are fully informed, and if participation in the auction is costless,[1] then, loosely speaking, the outcome is that the player prepared to pay most for the prize wins it with a bid equal to the maximum amount that the player next most keen to win would have been prepared to pay.

Dasgupta and Stiglitz (1980b, pp. 9 et seq.) adopt a similar approach. In their model, the time at which a firm would make the innovation depends inversely upon its R and D expenditure. There is symmetry between firms. Therefore the firm to spend most on R and D innovates soonest and therefore wins the patent. All players forgo their R and D expenditures. This game possesses no Nash equilibrium in pure strategies. If there were an equilibrium, at most one player would make a positive bid.[2] For if player 1 made a positive bid less than 2's bid, then 1 would do better to bid zero. If 1 and 2 were tied, then 1 would do better to bid a tiny bit more than 2 to win the prize outright (unless he were already bidding more than the prize was worth to him, in which case he would do better to bid zero). Moreover, if all players except, say, 1 are bidding zero, then 1's best reply is not well-defined: it is the smallest possible strictly positive bid.

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1. See Milgrom and Weber (1982) for a study of various auction schemes under more general conditions.
2. 'Bid' here stands for 'R and D expenditure'.

One response to this non-existence problem is to allow mixed strategies (for a general analysis, see Dasgupta and Maskin (1982)). If n players bid for an object worth V, there is a symmetric equilibrium in which each player's mixed strategy is defined by the cumulative distribution function

$$F(a) = (a/V)^{1/(n-1)} \qquad 0 \leq a \leq V.$$

Another response to the non-existence problem is that adopted by Dasgupta and Stiglitz (1980b, p.9). Following the Schumpeterian idea of competition as to do with the threat of entry, they model 'active firms' as Stackelberg leaders with respect to potential entrants. With this move order, equilibrium exists: one of the active firms makes a bid equal to the full value of the prize and all others bid zero. It is not clear exactly what 'active firm' means. Presumably it means the same as 'incumbent' - as opposed to potential entrant - but then there appears to be no reason to suppose that symmetry exists as between all firms. Indeed, the asymmetries that exist between an incumbent and potential entrants has been a major theme of much of the literature.[1]

Dasgupta (1982, p.32) makes this move order explicit. He considers two firms - let us call them C and I. Imagine I to be the sitting monopolist, or incumbent, currently protected from entry, perhaps by a patent. There is to be a race for a new patent. This patent is for a technology for a product (or process) competitive with I's existing product (or process). The technology may or may not be superior to that already possessed by I. Firm C is a challenger. If he wins the patent he will enter I's market and duopoly will result. If, on the other hand, I

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1. Section 3 of this Chapter examines these asymmetries when there are several incumbents, rather than just one.

wins the patent, his monopoly persists. Let V denote the discounted present value of I's profits if he wins the patent. Let V_I and V_c denote the discounted present value of the profits of the incumbent (I) and the challenger (C) if C wins the patent.[1] Monopoly profit exceeds the sum of the duopolists' profits. Therefore in the event of duopoly we have

$$V > V_I + V_c \quad .$$

C is prepared to bid up to V_c for the patent, whereas I will bid up to $V - V_I$, which exceeds V_c.

As to move order, Dasgupta (1982, p.32) writes:

> 'By definition the sitting monopolist is already in the industry. Thus I let the monopolist make the first move. This is followed by a bid from [the challenger]. The monopolist is therefore a Stackelberg leader'.

This echoes Dasgupta and Stiglitz (1980b, p.10, fn.13) who say that there is 'a natural assumption' that the monopolist is in a position to make first move.

With the incumbent as Stackelberg leader, the outcome of the game is that the incumbent makes a bid of V_c, i.e. the maximum that the challenger would be prepared to bid. This is the basic preemption result. V_c is the smallest bid that successfully preempts entry. The interpretation is that the incumbent wins the patent the instant before the challenger could profitably innovate.

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1. Von Ungern-Sternberg (1980) has shown how the respective incentives of incumbent and challenger depend on the magnitude of the innovation. The incumbent is especially keen to win minor innovations.

The preemption result is similar to that obtained in the literature on brand proliferation: see Hay (1976), Eaton and Lipsey (1979, 1980), and Schmalensee (1978). If demand is growing in a market with differentiated products, there comes a time when there is 'room' for another brand in product space. It is more costly to introduce the new brand sooner than it is to do so later. However, the asymmetry of incentives between incumbent monopolist and potential entrant is such that the incumbent can profitably introduce the new brand before a would-be entrant could profitably do so. The asymmetry exists because the incumbent remains a monopolist if he introduces the new brand, whereas competition results if the entrant does so. The analogy with preemptive patenting is exact: substitute 'innovate' for 'introduce the new brand' in the previous three sentences.

Note that _if_ the player keenest to win the prize is Stackelberg leader, then the outcome is equivalent to Gilbert and Newbery's auction in which only the winner forgoes his bid. This is despite the fact that in Dasgupta's formulation all bids are forfeit.

However, if move order is reversed, so that the challenger is Stackelberg leader, a very different outcome obtains. For the challenger would be deterred from making any positive bid in the knowledge that the incumbent would outdo any reasonable bid, by virtue of his greater incentive to win the prize. This would leave the incumbent able to win the prize with an infinitesimally small bid. The outcome is therefore very sensitive to move order. The incumbent would far prefer to be Stackelberg follower than to be leader. It is unfortunate that second move should be advantageous in a model of a race. A fuller discussion of the merits of various orders of moves is on pp. 79ff below. It is argued there that if players' strategy sets are simply R_+, then there is no completely unobjectionable move order. This suggests that games of this sort should have a more detailed sequential structure of moves, and the models in Chapter III attempt to provide this.

These difficulties about move order do not arise when the patent race is modelled as a bidding game in which only the winner forgoes his bid. However, the straightforward auction model proposed by Gilbert and Newbery is open to the objection that an essential feature of R and D competition is that R and D expenditures are sunk and irrecoverable.

A further criticism of this general approach is that it excludes uncertainty: players are fully informed about one another and about the world. It is true that with mixed strategy equilibria the outcome is not certain, but this form of uncertainty is not intrinsic to the problem. By contrast, technological uncertainty - concerning the relationship between R and D expenditures and discovery of time - is an essential part of the models reviewed in Chapter I above.

A full discussion of the incorporation of uncertainty into models of R and D is to be found on p.149 below. Briefly, the justification of the exclusion of uncertainty is to focus as clearly as possible upon other aspects of the problem at hand. To regard patent races as bidding games has the great merit of being neat and simple. In particular, it focusses attention upon firms' respective <u>incentives</u> to win the patent, and is therefore well-suited to the study of asymmetries between firms. Crudely, it might be said that the literature discussed in Chapter I is about technological uncertainty and symmetrically placed firms, whereas the literature reviewed here is directed mainly at asymmetries and excludes technological uncertainty. It is quite possible to combine asymmetry and uncertainty in one model (as in (Reinganum (1983)) but as a first, simplifying step it is convenient not to.

Being so parsimonious, the framework of patent races as bidding games can readily be extended to address more complicated examples than that above. In Section 3 of this Chapter, the incumbent/challenger asymmetry is examined when there are m incumbents and n challengers. This general (m,n) case has results which differ in interesting ways from the (1,1) case commonly considered. Section 4 of this Chapter explores another type of asymmetry - that betwen existing duopolists with different initial cost levels.

The remainder of this Section is devoted to other extensions of the basic model. Dasgupta (1982, pp.33-4) asks what happens when there are several challengers to a single incumbent. In previous terminology this is the (1,n) case. The incumbent has first move,[1] after which the n challengers simultaneously choose their bids. In equilibrium the incumbent once again makes a bid of V_c, i.e. the value of the patent to a challenger. One way to interpret this is that the incumbent innovates just before any challenger could profitably do so. Thus the same preemption result holds as before.

Multiple Patents[2]

We now relax the implicit assumption that there is only one patent at stake. The analysis below shows that the persistence of monopoly result is not robust when several patent opportunities exist. Suppose that there are N patents. There are two players - A, who will be thought of as a challenger, and the sitting incumbent, B. Our main objective is to find conditions under which the incumbent would preempt all N patents.

To simplify, suppose that the patents are identical in their economic effects (although of course they represent different product or process technologies). Let the payoff to the incumbent (ignoring all R

1. Again unfortunately for him!
2. The outlines of this model when N=2 were described to me by Partha Dasgupta early in 1982. He and I then generalised and analysed the model simultaneously but independently from one another. The model is also described in Dasgupta (1982, Section 5.3). Our analysis was carried out before the appearance of Gilbert and Newbery's (1982) paper, which contains some analysis of multiple patent races.

and D expenses) if he has k patents be denoted I(k). Similarly, let C(k) denote the challenger's payoff if he has k patents.

It is reasonable to assume:

(1) $I(k) \geq I(k - 1) \quad : \quad 1 \leq k \leq N$

(2) $C(k) \geq C(k - 1) \quad : \quad 1 \leq k \leq N$

(3) $C(0) = 0$

(4) $I(k) > C(k)$

Conditions (1) and (2) are akin to free disposal assumptions. Condition (3) says that having no patents is worth nothing to the rival. Condition (4) reflects the incumbent's greater incentive to win.

As to move order, let us first consider the case in which the monopolist announces his bid for each prize before the challenger bids. This corresponds to the move order in the one prize game discussed above. Instead of an exhaustive analysis, let us examine two cases:

(i) the challenger's payoff function is superadditive: C(k)/k is increasing in k. The average value of patents to the challenger does not decrease with their number.

(ii) the challenger's payoff function is subadditive: C(k)/k is decreasing in k.

(i) <u>The challenger's payoff function is superadditive.</u>

If the monopolist decides to win exactly k patents he should bid nothing for N-k of them and for each of the rest he should pay

$$(5) \qquad r(k) = \frac{C(n) - C(N - k)}{k}$$

If the incumbent were to bid anything less than kr(k) for the rest, the challenger would successfully bid for them all, since kr(k) is precisely the value to the rival of having all N patents rather than only N-k. There is no need for the incumbent to bid any more than kr(k) for the rest since the challenger would not pay more than that for all the rest. And nor would the challenger be prepared to pay more than r(k) per patent for any subset of the remaining k patents, because of the superadditivity of his payoff function. Therefore, if the incumbent bids kr(k) in total, equally spread across k patents, he will win all of them.

Thus the net benefit to the incumbent of gaining k patents is

$$(6) \qquad I(k) - kr(k) = I(k) + C(N - k) - C(N)$$

The incumbent chooses k ($0 \leq k \leq N$) to maximise this expression. From (6) it is apparent that it is better for the incumbent to gain k patents rather than k - 1 if and only if

$$(7) \qquad I(k) + C(N - k) > I(k - 1) + C(N - k + 1)$$

This implies that the incumbent prefers to gain k patents rather than k - 1 if and only if the value to the incumbent of the k^{th} patent exceeds the value to the challenger of the $(N - k + 1)^{th}$ patent. Perhaps more simply, the condition is that the joint payoff (ignoring R and D expenses) is higher when the incumbent has k patents than when he has k - 1, with the challenger having the remainder.

The incumbent will gain all patents, i.e. pre-empt completely, if (7) holds for all values of k ($0 \leq k \leq N$). This condition is sufficient but not necessary. If the incumbent's payoff function I is also superadditive, (2) will hold for all k if

$$(8) \qquad I(1) - I(0) \geq C(N) - C(N - 1)$$

Condition (8) is sufficient for complete preemption but is a very strong condition.

(ii) <u>The challenger's payoff function is subadditive</u>

When $C(k)/k$ is decreasing in k, if the incumbent decides to win exactly k patents, he should bid nothing for (N - k) patents and for each of the rest he should pay

$$(9) \qquad s(k) = C(N - k + 1) - C(N - k)$$

The value to the challenger of having (N - k + 1) rather than (N - k) patents is precisely s(k). The incumbent would fail to win at least one of the k patents for which he bids if he bid less than s(k) for any of them. By bidding s(k) for each of the k patents he is sure to win all of them, because of the subadditivity of the challenger's payoff function:

the challenger would not be prepared to pay $js(j)$ for a subset of j of the k patents. The net benefit to the incumbent of gaining k patents is therefore

$$(10) \qquad I(k) - ks(k) = I(k) + kC(N - k) - kC(N - k + 1)$$

Gaining k patents is therefore at least as good for the incumbent as gaining $k-1$ if and only if

$$(11) \qquad I(k) - I(k-1) \geq (2k - 1)C(N - k + 1) - kC(N - k) - (k - 1)C(N - k + 2)$$

A sufficient - though not necessary - condition for (11) is

$$(12) \qquad I(k) - I(k - 1) \geq (2k - 1)[C(N - k + 1) - C(N - k)],$$

because C is increasing. Therefore complete preemption (i.e. the incumbent winning all patents) will occur if, but not only if, (12) holds for all k.

Consider now the case where each patent is for a technology equivalent to that already possessed by the incumbent. Then the patent is simply an 'entry ticket': the only relevant question is whether or not the challenger enters the market: duopoly results if the challenger wins some patent, and monopoly persists only if the incumbent wins all patents. Let payoffs (ignoring R and D expenses) be

$$(13) \qquad I(k) = \begin{cases} V & \text{for } k = N \\ V_I & \text{otherwise} \end{cases}$$

for the incumbent, and

$$(14) \qquad C(k) = \begin{cases} 0 & \text{for } k = 0 \\ V_c & \text{otherwise} \end{cases}$$

for the challenger. By the previous argument, assume that $V > V_I + V_c$. However, when $k > 1$ this latter condition is by no means sufficient for complete preemption. Equation (14) is an instance of a subadditive payoff function. From (10) it follows that if the incumbent decides to prevent the entry of the challenger - i.e. win all the patents - then he must bid a total of

$$(15) \qquad Ns(N) = N[C(1) - C(0)] = NV_c$$

This is profitable for the incumbent if and only if

$$(16) \qquad V - V_I \geq NV_c$$

Unless $N = 1$ this is a rather strong condition. Whether or not it is met depends in part upon the 'degreee of competitiveness' in the product market. For example, in linear duopoly with demand given by $p = A - Q$ and unit costs of c for the incumbent's existing technology and for the new technology, we have

$$(17) \qquad V = (A - c)^2/4$$

and

$$(18) \qquad V_I = V_c = \frac{(1 + \lambda)(A - c)^2}{(3 + \lambda)^2}$$

where λ is $(-1 < \lambda \leq 0)$ is the 'conjectural variation' term, which can perhaps be regarded as an index of competitiveness (see p.115). From (16), (17) and (18) we have that:

<u>there is a complete preemption by the incumbent if and only if</u>

$$(19) \qquad (3 + \lambda)^2 \geq 4(1 + N)(1 + \lambda)$$

With Cournot behaviour, $\lambda = 0$, and (19) becomes $N \leq 5/4$. As we approach price-taking behaviour $\lambda \to -1$, and condition (19) holds. Condition (19) is equivalent to

$$(20) \qquad \lambda \leq 2N - 1 - 2\sqrt{N^2 - 1}$$

When (20) holds as a strict equality, λ is decreasing in N and goes to -1 as N becomes large.

In so far as λ is an index of competitiveness, we have the result that for all N, preemption occurs if and only if there is - or, rather, would be - sufficient competitiveness.

Gilbert and Newbery (1982, Section III, C) also consider multiple competitive threats. In their model (p.523), the expected number of new patent opportunities each period is $\bar{N}$. The probability that attempted preemption fails to prevent entry because the incumbent misses some patent opportunity is denoted μ. The discount factor is ρ. Profits per period are π to the incumbent if no challenger enters, and π_I to the incumbent and π_c to the challenger if a challenger enters. Preemption is worthwhile for the incumbent if and only if

$$(21) \qquad \pi - \pi_I \geq \frac{\bar{N}\,\pi_c}{(1-\rho)(1-\mu)}$$

Compare this with (16). Assuming, as is reasonable, that π's and V's stand in constant proportion, (16) can be written:

$$(16') \qquad \pi - \pi_I \geq N\pi_c$$

Compared with (21) there are three differences:

(i) In (16') preemption always works, so $\mu = 0$

(ii) In (21) there are $\bar{N}$ patent opportunities <u>each period</u>: the model is dynamic. In (16') there are N patent opportunities <u>in total</u>: the model is static, properly speaking.

(iii) N in (16') is given whereas $\bar{N}$ in (21) is an expected number.

Of these, perhaps, (ii) is the most interesting. In the dynamic setting, the idea is that 'a preemptor's work is never done'. If the incumbent prevents entry this period he still has other threats to contend with next period.

These analyses of multiple patent races shows that the persistence of monpoly result is not robust when several patent opportunities exist. In sum there is a conflict between two incumbent/challenger asymmetries. On the one hand, the incumbent has a stronger desire for no entry than a challenger does for entry. But, on the other hand, the incumbent must win <u>all</u> patents to stop entry, whereas a challenger needs only <u>one</u> patent to gain entry.

2. THE DEBATE IN THE AMERICAN ECONOMIC REVIEW

The persistence of monopoly result has been subject to at least two lines of attack. The essential argument for the persistence of monopoly result was

(i) that the incumbent has a greater incentive to win the patent than a potential entrant, and

(ii) that the player with the greater incentive wins - or is likely to win - the patent.

The two criticisms to be considered here are directed at (i) and (ii) respectively.

As to (i), Salant (1984) has argued that it is not true that the incumbent has the greater incentive, when account is taken of the possibility for a patent holder to license or sell the exclusive use of his patent. As to (ii), Reinganum has argued that when uncertainty is introduced into the formulation of the R and D competition, the Gilbert-Newbery result might even be reversed, due to another asymmetry between incumbent and entrant - the incumbent, but not the entrant, is currently receiving a flow of profits from the market.

These are not the only criticisms of the basic model used to derive the persistence of monopoly result. Indeed much of the next Chapter is devoted to the development of a model of a patent race with a more sophisticated structure of moves than in the basic model. Moreover, the conclusion that monopoly is likely to persist is not robust to certain variations of the basic model - even if the basic framework is retained. This was shown in the discussion in the previous section of the case in which there are several patents at stake, any one of which would permit access to the market for a new entrant. The following section of this Chapter considers another extension of the basic model, and asks whether an existing <u>oligopoly</u> is likely to persist when there is one patent at stake. In a large class of cases the answer is No, and the persistence of monopoly result therefore appears somewhat special, even in the context of the basic framework from which it was derived.

<u>Incentives to Preempt when patent rights are transferrable</u>

In the exposition of the simple preemption result in the previous section, it was implicitly assumed that a firm could employ the new technology if and only if it won the patent. No account was taken of the possibility that the winner of the patent race could sell the exclusive right to use the patent to his erstwhile rival. How does the result alter when this possibility is allowed?

Salant (1984) takes issue with Gilbert and Newbery's claim that asymmetries of incentive are such that 'it may still pay the monopolist to preempt more efficient rivals' (1982, p.518). Let us assume explicitly that the incumbent is less efficient than his rival. Let Δ be the extent to which the incumbent is less efficient than his (most efficient) rival. Then the incumbent wins the patent if and only if he is prepared to pay Δ more for the patent than his challenger. If V is the discounted value of the profits of the incumbent if he has the patent, and if V_c and V_I are, respectively, the discounted values of the profits of the challenger and the incumbent if the challenger has the patent, then the incumbent will win if and only if

$$V - V_I - \Delta \geq V_c$$

if patent rights cannot be transferred. In the previous section, we assumed that the firms were equally efficient at R and D (i.e. $\Delta = 0$), and by virtue of the argument that $V > V_c + V_I$, the above inequality was always met.

Now suppose that patent rights are transferrable. If transactions costs are zero, it follows that for all $\Delta > 0$, the incumbent will not win the patent. Salant's argument (p.248) is that there will be monopoly after the patent race whoever wins the patent race: one firm or the other will somehow acquire both patents. If the rival wins, he will either license the new patent to the incumbent or he will license the incumbent's old patent himself. Suppose, for example, that the incumbent would licence the patent at cost β if the rival won it. Then if the incumbent wins the patent his payoff will be V less R and D costs. If he loses, his payoff will be $V-\beta$. The incumbent will therefore be prepared to pay up to β for the patent. So too will the rival, since β is precisely his reward for winning the patent. But to win the patent, the incumbent must pay Δ more than the rival. The incumbent will win if and only if $\beta - \Delta \geq \beta$, which is impossible if $\Delta > 0$. This holds for all β. The argument is essentially the same if the rival licenses the incumbent's old technology if he wins the patent. Thus Salant's conclusion is that the most efficient firm at R and D will win the patent: preemptive patenting by an inefficient incumbent will not take place.

Salant introduces positive transactions costs and argues that preemptive patenting by an inefficient incumbent can occur - but <u>only</u> in cases where the inefficiency of the incumbent is <u>smaller</u> than the transactions costs. Under the heading of 'transactions costs' must come the possibility of antitrust interference, especially refusal by the authorities to allow the transfer of patent rights that results in monopoly. In this case the persistence of monopoly result carries through, and, Salant argues, antitrust policy can have the perverse effect of failing to prevent monopoly but inducing the relatively inefficient incumbent to carry out the R and D.

In their (1984b) rejoinder to Salant, Gilbert and Newbery argue that Salant's argument is flawed by its incompleteness (they also criticise his 'creative journalism'! (p.251)). In particular, Gilbert and Newbery contend that 'If firms can negotiate in the patent market, they can also negotiate in markets for inputs to the R and D process' (p.251). They find that the incumbent would sell his original patent to the rival, rather than bargain for the rival's superior R and D technology at the outset, only if transactions costs in the market for R and D inputs, α_1, exceed the transactions costs associated with the transfer of the patent, α_2. If this condition does not hold, then the incumbent does best to acquire the rival's R and D inputs and then to patent preemptively (unless, of course, transactions costs are so high that he does better to preempt with his existing technology). In sum, the original Gilbert and Newbery argument is good if

either transactions costs are high in relation to any disadvantage in R and D efficiency of the incumbent, i.e. $\alpha_2 > \Delta$

or transactions costs in the R and D input market are below those involved with the transfer of patent rights, i.e. $\alpha_1 < \alpha_2$.

It has been necessary to consider this debate at some length because of its bearing on many issues central to this thesis. What conclusions are to be drawn? First, it must be emphasised (as it is by Gilbert and Newbery and by Salant) that the debate is conducted in the context of a very simple but restrictive model. The following should be understood as being subject to that proviso.

(a) Disparities in efficiency at R and D: It is common ground that Salant's criticism applies only when the rival is more efficient at R and D than the incumbent and when transactions costs are exceeded by the disparity in efficiency. In many instances it is plausible that, by virtue of his greater experience, the incumbent is more efficient at R and D than the rival, and certainly no less so. In these cases, Salant's point does not apply: there is preemption, but the more efficient firm (the incumbent) does the R and D. However, it is not always the case that the incumbent is likely to be more efficient at R and D. For example, the oil producer may well not have the advantage in the development of nuclear energy technologies. In short, in many, but not all, cases we have $\Delta \leq 0$.

(b) Transactions costs involved with the transfer of patent rights: Especially in view of antitrust implications, α_2 is likely to be very high. Exclusive licensing of patenting is not per se illegal under US, UK or EEC competition law, but when monopolisation is at stake I believe that it is very unlikely that such licensing would be permitted. In the context of a special case of a special model, this makes antitrust policy appear to have a somewhat perverse effect, i.e. it encourages the less efficient firm to do the R and D without making any difference to the identity of the eventual patent holder. However, as Salant (p.250) recognises, this does not imply that antitrust policy has perverse effects in other cases.

(c) Transaction costs in R and D input markets: If the rival is more efficient at R and D than the incumbent then maybe the incumbent can acquire the superior technology either by obtaining the superior inputs or by acquisition of the rival itself. The latter course is likely to face antitrust difficulties similar to those discussed above. In any event, the policy of acquisition is applicable only in the case where there is just one rival. If there are many then such a policy is futile. The former course -

that of buying the R and D inputs - may also counter antitrust problems. Moreover, it need not be the case that technological efficiency is due to R and D inputs. It might be that the quality of inputs is the same for all firms, but that one firm's 'production function', relating R and D input to R and D output is superior to that of another firm. This might be due to the presence of complementary fixed factors of production that are not tradeable, or to accumulated experience, for example. In these cases, there is every reason to expect transaction costs α_1 to be prohibitively high. Gilbert and Newbery (1984b, p.251) explicitly assume that disparities in efficiency are not due to managerial economies or diseconomies. Interpreted very broadly, that phrase can be taken to cover the various instances mentioned here. However, if differences in efficiency at R and D have to do with factors that are not variable in the short run, let alone tradeable, then perhaps it is inappropriate to assume that they are due to differences in the quality of inputs.

In conclusion, notwithstanding the interesting considerations advanced by Salant, the Gilbert and Newbery argument holds good in the vast majority of cases. Using earlier notation, this is more because $\alpha_2 > \Delta$ than because $\alpha_2 > \alpha_1$. For antitrust, and other reasons, both α_1 and α_2 are likely to be very high. And in many instances there is no reason to suppose $\Delta > 0$. The reverse inequality is perhaps more to be expected, because of the incumbent's experience.

<u>Uncertain Innovation and the Persistence of Monopoly</u>

The title of this subsection is the title of Reinganum's (1983) paper commenting on Gilbert and Newbery (1982). She uses a model of R and D with uncertainty to show that for a sufficiently revolutionary innovation, the incumbent is <u>less</u> likely than the rival to make the innovation. This result is in stark contrast to that of Gilbert and Newbery, and prompts Reinganum (p.746) to say that 'the assumption of certainty in the inventive process is not an innocuous one'. It is important to assess Rienganum's claim not only for its own sake, but also because of the suggestion that results based on assumptions of certainty are not robust (the models in the first two Sections of the next Chapter have no uncertainty). Reinganum's (1983) paper is also the prelude to her (1984c) paper on innovation and industry evolution, discussed in Section 2 of Chapter IV below (we also present an independent proof of Reinganum's main result). What follows is therefore also background to that discussion.

Reinganum's formulation of the R and D competition is on the lines of Lee and Wilde (see pp. 18ff), and so is quite different to Gilbert and Newbery's bidding game. Suppose that the new innovation is drastic, i.e. so superior that whoever wins the patent becomes a monopolist, earning profits with discounted value V. Suppose that the incumbent is currently receiving profit flow $\pi > 0$. The challenger, being outside the market, receives no current profit flow. Technological uncertainty takes the exponential form explained on p. 19 above. The (flow) cost of R and D at rate x is c(x), where $c' > 0$, $c'' > 0$, $c(0) = 0$, and $c(x) \to \infty$ as $x \to \infty$. The incumbent chooses his rate of R and D, x_I, to maximise

(1) $$\Omega_I(x_I, x_c) = \frac{\pi + x_I V - c(x_I)}{x_I + x_c + r}$$

and the challenger chooses his rate of R and D, x_c to maximise

(2) $$\Omega_c(x_I, x_c) = \frac{x_c V - c(x_c)}{x_I + x_c + r}$$

Let asterisks denote equilibrium values. The first-order conditions are

(3) $$c'(x_I^*) = V - \Omega_I^*$$

and

(4) $$c'(x_c^*) = V - \Omega_c^*$$

See Reinganum (1983, pp.743-4) for details of the second-order conditions and proof that equilibrium exists.

Proposition: $x_c^* > x_I^*$ and $\Omega_I^* > \Omega_c^*$.

Proof:[1] $c'' > 0$, (3) and (4) imply:

(5) $$x_I^* \geq x_c^* \iff \Omega_I^* \leq \Omega_c^*$$

Assume for a contradiction that $x_I^* \geq x_c^*$. Then $\Omega_I^* \leq \Omega_c^*$, from (5). However, we have

(6) $$\Omega_I^* \geq \Omega_I(x_c^*, x_c^*) > \Omega_c(x_c^*, x_c^*) \geq \Omega_c^*$$

which is a contradiction. The three inequalities hold, respectively, because

(i) x_I^* is by hypothesis a best response to x_c^*,

1. This proof of Reinganum's result is different from hers, but perhaps simpler. It also serves to establish that $\Omega_I > \Omega_c$, which is subject to separate proof in Reinganum (1984c).

(ii) $\pi > 0$: see (1) and (2),

(iii) Ω_c is decreasing in x_I: see (2).

This contradiction completes the proof.

Q.E.D.

Because the challenger invests more in R and D than the incumbent, the probability that he innovates is greater than a half. It is a simple matter to extend the proof to the case of n - 1 symmetrically placed challengers to show that the probability that the incumbent innovates is less than 1/n.

The intuitive explanation for the result is that the incumbent has a lower marginal incentive to invest in R and D than the challenger because he receives flow profits of π until someone makes the innovation. By reducing slightly his R and D he reduces his chance of winning but also prolongs the expected duration of this profit flow. No such factor operates for the challenger. Models without uncertainty lack this feature altogether.

The claim proved above applies to the case of drastic innovations. If the innovation is not drastic, let V_I and V_c denote, respectively, the present value of the profits of the incumbent and challenger if the challenger wins the patent. Let V continue to denote the discounted value of the profits of a monopolist with the new technology. By the usual argument (see p. 57), we have $V > V_I + V_c$ for a non-drastic innovation. Instead of (1) the objective function of the incumbent is now

$$\text{(7)} \qquad \Omega_I = \frac{\pi + x_I V_I + x_c V - c(x_I)}{x_I + x_c + r}$$

The challenger's objective function remains as in (2). Now there are two forces at work. The fact that $\pi > 0$ encourages the challenger to invest more than the incumbent, but an opposite infuence derives from the fact that $V - V_I > V_c$. Speaking informally, the latter influence weakens as the innovation becomes nearer to being drastic (the measure of an innovation is its unit cost level; for a drastic innovation $V_I = 0$ and $V_c = V$, so that the strict inequality ceases to hold). By an obvious continuity argument, Reinganum (1983, Corollary 1, p.745) establishes that for any $\pi > 0$ here is a nonempty set of non-drastic innovations such that $x_I < x_c$ at equilibrium. Reinganum concludes that

> 'for sufficiently radical innovations... it is precisely the assumption of certainty vs. uncertainty is responsible for the discrepancy between [Reinganum's] results and those of Gilbert and Newbery' (p.745)

This conclusion is overstated.[1] Uncertainty is not the only major respect in which Reinganum's model differs from that of Gilbert and Newbery. In particular there is difference in <u>move order</u>. Gilbert and Newbery model a patent race as a simple auction or bidding game, in which only the winner forfeits his bid (see the quotation on p. 55 above). This is equivalent to assuming (as did Dasgupta and Stiglitz (1980b): see p. 56 above) that the incumbent has first move and that the rival acts as Stackelberg follower. Reinganum, however, has a structure of simultaneous moves. Whatever the relative merits of the various assumptions about move order, it is quite plain that the incumbent might preempt in a model with uncertainty if he had first move and could credibly commit his level of R and D. Gilbert and Newbery (1984a, pp.238-9) construct an example with simultaneous moves and deterministic R and D in which the challenger is more likely than the incumbent to win

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1. The same criticism of Reinganum was made by Gilbert and Newbery in their Comment on Reinganum in the March 1984 AER.

the patent at the (mixed strategy) equilibrium. Therefore deterministic R and D is neither necessary nor sufficient for the preemption result.

Gilbert and Newbery claim that other differences also account for the disparity between their result and Reinganum's, namely the absence of free entry into the R and D competition, and assumed managerial diseconomies. It is not clear that the criticisms levelled at Reinganum on these scores are wholly justified: see Reinganum's (1984a) reply. In any event, the various assumptions regarding move order appear to account in large part for the differences in result.

Whether or not move order, rather than uncertainty, is primarily responsible for the difference in the results, it is important to ask which assumptions regarding move order are most appropriate to the problem at hand. As Gilbert and Newbery (1984a, p.238) note, there is great similarity between the outcome of a bidding game formulated as a standard auction, and that of a game in which the incumbent is Stackelberg leader. Gilbert and Newbery (1984a, p.238) write that in their (1982) paper they 'assumed that the incumbent had a first-mover advantage, and that he could credibly commit himself to any strategy that would make entry unprofitable'. They go on (p.242) to argue 'that the incumbent has a natural temporal advantage since after all he is the incumbent, in which case the central issue is one of credibility'.

Several remarks are in order. First, as was shown on p. 58 above, it is no advantage to the incumbent to have first move in this framework. Indeed, if the challenger has first move he is automatically deterred from actively competing because the incumbent is perpared to outdo any bid that the challenger could afford to make. The incumbent then wins the patent at an infinitesimally small cost. Therefore first move, rather than last move, is a positive - and probably very large - disadvantage as far as the incumbent is concerned. Secondly, it is by no means clear that temporal priority should be accorded to the incumbent. In her Reply (1984a, p.246), Reinganum states that

> 'One could argue equally persuasively the obverse claim that the potential entrant should have the first move, since the incumbent may not be aware of its existence or intent to invest until it actually does so.'

The purist's response to Reinganum's observation would be to say that her story is about a game of incomplete information (i.e. the intentions and even the existence of the challenger are not known for certain by the incumbent) and therefore not strictly applicable to the Gilbert and Newbery model. However Reinganum's point is a powerful one. In the literature on predatory pricing, for example, first move is invariably accorded to the entrant, who decides whether or not to enter the market before the incumbent chooses whether or not to fight entry (see Dixit (1982, p.13, fig.1)). This is quite natural, and in the context of predatory pricing it would be wholly inappropriate to accord first move to the incumbent on the grounds that 'he has a natural temporal advantage since after all he is the incumbent'. Instead of a leader-follower structure of moves, another possibility is simultaneous moves. As was seen above (p. 55), the difficulty here is that equilibrium fails to exist in pure strategies. Indeed, it was this consideration that led Dasgupta and Stiglitz (1980b) to adopt the leader-follower structure.

This debate about the appropriate structure of moves is perhaps artificial. It rests on the assumption that each player in the R and D competition can make just one move: he chooses his level of R and D once-and-for-all. In formal terms, his strategy set is R_+. In a two-player game there are therefore only three logically possible move structures -

(i) A then B ,

(ii) B then A , or

(iii) simultaneous moves.

We have seen that when A is a potential entrant and B is an incumbent each of these has drawbacks. Rather than argue about the relative merits of (i), (ii) and (iii), perhaps a more profitable route is to examine other, broader possibilities.

To accord to each player just one move is tantamount to saying that each player makes a once-and-for-all decision about his level of R and D. For some purposes this assumption may be a reasonable one to make, but for other purposes it is too stylised. Thus the unpleasantness of the choice between (i), (ii) and (iii) may be due in part to the restrictiveness of the strategy sets. The approach taken in Chapter III is to accord to each player a sequence of moves. At a series of points during the course of the R and D competition, a player can decide how much R and D to undertake during the next part of the competition. Each such decision willl be contingent upon the moves that the players have made previously in the competition. We adopt a structure of alternating moves.[1] Thus the move order is: A, then B, then A again, then B again, and so on. The game has no fixed horizon: it lasts until some player succeeds in making the innovation. This structure enables us to analyse in precise terms certain feature of strategic interaction that take place during the course of a race. For example, we are able to see how and why one player might give up in the race once his rival was in some sense 'far enough ahead'. Such features cannot be examined when strategy sets are R_+.

The pattern of alternating moves does not avoid the question: Who has first move? However, it does make the outcome of the race in equilibrium less sensitive to the assumption about who goes first. Indeed, it often makes no difference who has first move given the

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1. Fudenberg et al (1983) explore a model in which there is a sequence of simultaneous moves. See pp. 149ff below for a detailed comparison of their model with that in Sections 1 and 2 of Chapter III.

structure of alternating moves. In the limit, as the interval between moves becomes short, it almost never makes any difference who goes first. Finally, in the models in Chapter III with alternating moves, first move is never a disadvantage and sometimes an advantage. This is what one would hope for in a model of a race. However, we have just seen that when strategy sets are R_+, first move is never an advantage and often a (large) disadvantage.

Conclusion:

The debate between Reinganum and Gilbert and Newbery in the American Economic Review has shown that assumptions about uncertainty and about move order are both critical for the persistence of monopoly result. As to the former, Reinganum has succeeded in showing an interesting reason why an incumbent might invest less in R and D than a potential entrant - the prolongation of his current profit flow. It is therefore not true that the move order assumption is all that matters. However, it is clearly very important. If strategy sets are R_+, there is a choice between three unattractive move structures. The choice can be avoided, for example by employing a richer move structure, as in Sections 1 and 2 of Chapter III.

3. PRE-EMPTIVE PATENTING, JOINT VENTURES, AND THE PERSISTENCE OF OLIGOPOLY

Most of the previous discussion in this chapter has been about patenting and the persistence of monopoly. The basic model has had one incumbent and one potential entrant who are competing for a patent for a product (or process) that is in some sense substitutable with that currently supplied (or used) by the current incumbent. The simplest version of the Gilbert and Newbery argument is (i) that the incumbent has a greater incentive than his rival to win the patent; and (ii) that he will therefore win it, or be likely to do so. Notwithstanding the criticisms by Salant (1984) and Reinganum (1983a) of (i) and (ii), respectively (see the previous section), we shall accept this simple framework, and use it to address the question of patents and the persistence, or otherwise, of oligopoly. One could label the situation with one incumbent and one entrant as the (1,1) case; we shall examine the (m,n) case, where $m > 1$.[1]

The motivation for this is not just the desire to generalise. Rather, it is that when there are initially several incumbent firms (i.e. $m > 1$), the simple argument for the persistence of monopoly (steps (i) and (ii) above) does not necessarily go through. This is for two reasons:

(a) The fall in the profits of an incumbent if new entry occurs need not be greater than the rise in the profits of an entrant who gains entry into the market. Therefore an incumbent may not have a greater incentive than an entrant to win the patent.

(b) For the incumbents, entry deterrence has some of the characteristics of a public good (see Gilbert and Vives

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1. The (1,n) case has been studied by Dasgupta (1982, pp.33-4).

(1983) for a discussion of this point). Quite independently of point (a), there may therefore be underinvestment by the incumbents in entry deterrence, i.e. in R and D, thereby increasing the probability of entry.

Whatever may be their initial plausibility, both these influences require detailed examination, because there are other factors at work. For example, if the patent is for a technology that is markedly superior to that already employed by the incumbents, then each incumbent will be more eager to win it. This force can overcome factor (a). Moreover, the superiority of the new technology implies a negative externality between incumbent firms: if incumbent i wins the patent, then incumbent j is hurt. This might outweigh the positive entry deterrence externality (i.e. if i wins then j does not suffer competition from the new firm).

The following analysis employs the familiar model of linear Cournot oligopoly. The main objective is to show, for given demand and initial cost conditions, how two parameters (m, the number of incumbents, and s, the size of the new innovation) determine whether there is under- or over-investment in R and D by the incumbent firms, and whether or not entry is likely to occur. The analysis uses different specifications of the form of patent races.

So far it has been assumed that the R and D competition is played as a non-cooperative game. However, factors (a) and (b) above suggest that a <u>joint venture</u> in R and D might be a useful entry deterring device. It circumvents factor (a) by pooling the incumbents' incentives, and factor (b) is avoided because one decision is taken for all incumbents. it emerges that if the innovation is minor, then a joint venture is a good way for the incumbents to deter entry. If it is major, then a joint venture can in fact increase the probability of entry.

A Simple Illustration

We begin by looking at factor (a), the way in which the profits of an incumbent and an entrant vary according to who wins the patent. Throughout we assume an industry supplying a homogeneous good facing linear demand

$$(1) \qquad p = A - Q$$

where Q is aggregate industry output. Initially there are m incumbent firms, all with cost function

$$(2) \qquad c(q) = cq \qquad ; \quad c < A$$

There is a patent for a new technology which also has cost function (2). The incumbent firms and some potential entrants compete for the patent. If an incumbent wins, there will remain m firms in the market, each with profit flow

$$(3) \qquad \pi(m) = \left[\frac{A - c}{m + 1} \right]^2$$

If a new entrant wins, there will be m + 1 firms, each with profit flow

$$(4) \qquad \pi(m+1) = \left[\frac{A - c}{m + 2} \right]^2$$

An entrant's incentive to win the patent is $E = \pi(m+1)$, whereas $I = \pi(m) - \pi(m+1)$ is an incumbent's incentive.[1] Equations (3) and (4) imply that

---

1. 'Incentive' really means 'change in profits'. The model here is single period, but it can readily be made multi-period by regarding E and I as proportional to the d.p.v. of profits. This affects none of the argument.

$$
\begin{aligned}
I \lessgtr E \quad &\Leftrightarrow \quad \pi(m) \lessgtr 2\pi(m+1) \\
(5) \qquad &\Leftrightarrow \quad (m+2)^2 \lessgtr 2(m+2)^2 \\
&\Leftrightarrow \quad \sqrt{2} \lessgtr m
\end{aligned}
$$

Therefore, in this illustration, an incumbent's incentive is greater if and <u>only if</u> he is initially a monopolist. If the patent race is won by the player with the greater incentive (as in the basic Gilbert and Newbery model), then entry occurs unless there is initially monopoly. In this simple setting, the persistence of monopoly result is therefore a special case.

However, if the incumbent firms formed a joint venture to do R and D, the picture is very different. Assuming non-cooperative behaviour in the product market (despite cooperative R and D), the joint venture's incentive to win the patent (i.e. the amount that the joint venture would be prepared to bid for the patent) is $J = mI$. Thus we have

$$
\begin{aligned}
(6) \qquad J > E \Longleftrightarrow m\pi(m) &> (m+1)\,\pi(m+1) \\
\Longleftrightarrow m(m+2)^2 &> (m+1)^3 \\
\Longleftrightarrow m^2 + m - 1 &> 0 \qquad ,
\end{aligned}
$$

which holds for all $m \geq 1$. The condition is simply that industry profits are higher without entry than if entry occurs.[1]

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1. Seade (1980, p.479) considers under more general demand conditions and using a conjectural variation term, whether industry profits rise or fall with entry. He finds that they fall if there are non-increasing returns to scale, but that it is conceivable that entry causes industry profits to rise. The latter possibility arises, because if returns to scale are decreasing, the new entrant produces some low cost output.

A More General Illustration:

Assume that demand is as (1) above, and that the initial cost levels of the incumbents are given by (2). Assume now that the patent is for a technology with associated cost function

$$(7) \qquad \bar{c}(q) = (c-s)q \qquad ; \quad s < c$$

We can now see how the incentives of the incumbents and entrants vary with s as well as m. There are several cases to consider.

Case 1: The new innovation is said to be drastic if $s \geq (A - c)$. In this case the winner of the patent becomes effectively a monopolist with the new cost level, and charges a price of $\frac{1}{2}(A + c - s)$, which is less than c, and so no other firm produces. In this case

$$(8) \qquad E = I = \tfrac{1}{4}(A - c + s)^2$$

which is the profit for a monopolist with unit' cost level c - s. In the case of drastic innovation all firms are on a par as far as incentives are concerned: incumbents and entrants are equally competitors for the prospective monopoly. There is no incentive for the incumbents to form a joint venture to attempt to deter entry. If the joint venture were to win the new patent and make the new technology available to all incumbents, then each incumbent would subsequently earn profits equal to $(A - c + s)^2/(m + 1)^2$. Multiplied by $m \geq 2$ this expression is less than expression (8). This result for drastic innovations is in some respects the opposite of that obtained in the previous illustration when $s = 0$.

Case 2: Assume now that $(A - c) > s \geq 0$. If entry occurs there will be m + 1 firms with cost level c and one with cost level c - s. All firms will be active at equilibrium. The entrant's profits are

$$(9) \qquad E = \left[\frac{A - c + (m + 1) s}{m + 2}\right]^2$$

(for this and subsequent results, see the Appendix). An incumbent's incentive to win the patent rather than let a new entrant win is

$$(10) \qquad I = \left[\frac{A - c + ms}{m + 1}\right]^2 - \left[\frac{A - c - s}{m + 2}\right]^2$$

The first term on the RHS is the profit of an incumbent who wins the patent and so has cost level c - s against his m - 1 rivals with cost level c. The second term is the profit of an incumbent when a new firm enters the market with cost level c - s having won the patent.

Expression (10) is the amount that an incumbent would bid in Gilbert and Newbery's basic model to prevent a new firm from winning the patent. This exceeds the amount he would be prepared to bid to get the patent rather than let one of his fellow incumbents win it. This latter amount equals (10) but with m + 1 in place of m + 2 in the denominator. The reason why an incumbent prefers a fellow incumbent to win, rather than a new entrant, is that in the latter event an extra firm is in the market.

The relevant comparison for our purposes is between E and I as defined by (9) and (10). Define B = A - c. Then

$$(11) \qquad I \gtrless E \iff (m + 2)^2(B + ms)^2 \gtrless (m + 1)^2\{(B - s)^2 + (B + (m + 1)s)^2\}$$

$$\iff (m + 2)^2(B^2 + 2msB + m^2s^2) \gtrless (m + 1)^2(2B^2 + 2msB + (m^2 + 2m + 2)s^2)$$

$$\Longleftrightarrow B^2(m^2 + 4m + 4 - 2m^2 - 4m - 2) + 2msB(2m + 3) + s^2(m^2(2m + 3) - 2(m + 1)^3) \lessgtr 0$$

$$\Longleftrightarrow B^2(2 - m^2) + 2msB(2m + 3) - s(3m^2 + 6m + 2) \lessgtr 0$$

$$\Longleftrightarrow (-B^2 + 4B - 3s^2)m^2 + (6sB - 6s^2)m + 2B^2 - 2s^2 \lessgtr 0$$

$$\Longleftrightarrow (B - s)[-(B - 3s)m^2 + 6sm + 2(B + s)] \lessgtr 0$$

Under case 2, $B > s$. Therefore we have

$I \lessgtr E$ according as

(12) $$(B - 3s)m^2 - 6sm - 2(B + s) \lessgtr 0$$

If $s \geq \frac{1}{3}(A - c)$, then $I > E$ for all m. Otherwise, if m is large enough, then $E > I$. Solving the quadratic, we can find this critical level of m, denoted $\bar{m}$, at which $I = E$:

(13) $$\bar{m} = \frac{3s + \sqrt{9s^2 + 2(B + s)(B - 3s)}}{B - 3s}$$

$$= \frac{3s + \sqrt{3s^2 - 4(A - c)s + 2(A - c)^2}}{A - c - 3s}$$

If $s < \frac{1}{3}(A - c)$ then $I > E$ according as $\bar{m} > m$. Total differentiation of (12) as an equality yields

(14) $$[2m(B - 3s) - 6s]\frac{d\bar{m}}{ds} = 3m^2 + 6m + 2$$

The expression in square brackets equals twice the square root in (13), which is positive. Therefore (14) implies that $d\bar{m}/ds > 0$.

Now consider how much a joint venture of the m incumbent firms would be prepared to pay for the patent, rather than permit a new entrant to obtain it. Assuming that, if the joint venture is successful, then all its members enjoy the low cost technology, the profit of each incumbent will subsequently be

$$\left[\frac{A - c + s}{m + 1} \right]^2$$

The amount that the joint venture would pay is therefore

$$(15) \qquad J = m \left[\frac{A - c + s}{m + 1}\right]^2 - \left[\frac{A - c - s}{m + 2}\right]^2$$

In the simple illustration at the beginning of this section, when $s = 0$, the incentive of the joint venture was exactly m times that of an incumbent (I in equation (10)). As to the relationship between I and J, routine calculations using (10) and (15) imply that

$$I \gtrless J \text{ according as}$$

$$(16) \qquad (m + 1)^2 (A - c - s)^2 - (m + 2)^2 [(A - c)^2 - ms^2] \gtrless 0$$

If the innovation is sufficiently large (small) in relation to the size of the market, then a single incumbent has a greater (smaller) incentive to win the patent than a joint venture of incumbents.

In summary, the joint venture has two effects.

(i) it increases the effectiveness of the incumbents' desire to win the patent inasmuch as it combines their incentives; but

(ii) it decreases the effectiveness of their desire to win insofar as it removes the incentive of one incumbent to gain relative to his fellow incumbents.

Effect (i) reflects a positive externality between incumbents, whereas effect (ii) is a negative externality. If the new patent offers a relatively minor improvement over the incumbents' original technology then effect (i) dominates (ii). If the new innovation is major, then the reverse holds. If s is close to A − c then we approach Case 1, analysed above. On the borderline between Cases 1 and 2 we have from (8) that:

$$(17) \qquad I \simeq \tfrac{1}{4}(A - c + s)^2 \simeq (A - c)^2$$

and

$$J \simeq \frac{m(A - c + s)^2}{(m + 1)^2} \simeq 4m \left(\frac{A - c}{m + 1}\right)^2$$

in which case it is clear that $I > J$ if $m > 1$.

Case 3: Assume finally that $s < 0$. That is, the new technology is inferior to that already possessed by the incumbent firms. Therefore if an incumbent − or joint venture of incumbents − won the patent, it would remain unused. A potential entrant may nevertheless have an incentive to win the patent, provided that −s is not too large and that m is not too large. From (9) we see that a potential entrant would have no interest in the patent if $-(m + 1)s \geq A - c$. In this subcase of Case 3 no-one would be motivated to make any effort for the patent. Assume henceforth that this subcase does not hold. Then a potential entrant's incentive to win the patent is given by (9):

$$(9) \qquad E = \left[\frac{A - c + (m + 1)s}{m + 2}\right]^2$$

An incumbent's incentive is

$$(20) \qquad I = \left[\frac{A - c}{m + 1}\right]^2 - \left[\frac{A - c - s}{m + 2}\right]^2$$

(cf. equation (10)). Manipulations using (9) and (20) reveal that

$$(21) \qquad \frac{E - I}{(m+1)^2(m+2)^2} = (A - c)^2(m^2 - 2) + 2ms(A - c)(m + 1)^2 + s^2(m^2 + 2m + 2)(m + 1)^2$$

When s=0 this yields the condition in the original illustration that $E \gtrless I$ as $m^2 \gtrless 2$. Then for $m > 1$ we always have $E > I$. Given the assumption that $0 < -(m + 1)s < A - c$, the expression on the RHS of (21) exceeds

$$(22) \qquad \begin{aligned} &(m + 1)^2[s^2(m^2 - 2 + m^2 + 2m + 2) + 2ms(A - c)] \\ = \; &2(m + 1)^2ms[s(m + 1) + A - c] > 0 \end{aligned}$$

Therefore E > I. But in the limit (E - I) $\rightarrow$ 0 as $-s \rightarrow (A-c)/(m+1)$. In conclusion, if there are two or more incumbents, an entrant has a stronger incentive than an incumbent to win the patent, even though it is for a technology inferior to that originally possessed by the incumbents. When s is close to $-(A - c)/(m + 1)$, an entrant's incentive is weak. However, an incumbent's incentive to thwart entry is weaker still.

Finally, in Case 3 it is simple to verify that a joint venture would always be prepared to outbid a potential entrant.

Summary: So far we have examined the incentives of incumbents and potential entrants in a competition for a new technology, and also how the incumbent's incentives can be modified by their forming a joint venture. In the framework of 'patent races as bidding games' we found that in the absence of joint ventures:

(i) if the new technology is equivalent to that currently employed by the incumbents, then an entrant would bid more than an incumbent.

(ii) if the technology is superior, but not 'drastic', then an incumbent would bid more than an entrant for a major innovation, but less for a minor innovation.

(iii) if the technology is inferior, then a potential entrant would pay more than an incumbent.

(iv) for a drastically superior technology, the incentives are the same for all firms.

A joint venture has two opposing effects on incumbents' incentives. If the new technology is not a major improvement upon the existing technology, then the formation of a joint venture enhances the incumbents' incentives to win the patent by pooling them. For major innovations however, it weakens their incentive by removing the competitive stimulus between incumbents.

Result (i), and to some extent also results (ii) to (iv), are reminiscent of the 'Cournot merger paradox': see Salant et al. (1983). Equation (5) stated that an incumbent's incentive is stronger than that of a potential entrant if and only if $\pi(m) > 2\pi(m + 1)$. This is precisely the condition for a merger between two firms in an m + 1 firm industry to be profitable, since $2\pi(m + 1)$ is the sum of the firms' pre-merger profits and $\pi(m)$ is their joint profit after merger. Our result (i) is therefore equivalent to the Cournot merger result (the other results are broader because they allow for $s \neq 0$). Craswell (1981) in his comments on Gilbert (1981) observed that the persistence of monopoly result shows an incentive for duopolists to merge. What we have shown is that in the case of preemptive patenting, just as in the case of merger, the results for duopoly can be reversed when there are initially more than two incumbents.

However, all this is subject to an important proviso, namely that we have assumed Nash-Cournot behaviour on the part of firms. Deneckere and Davidson (1983) have shown that the Cournot merger paradox ceases to hold for price-setting firms in a market for differentiated products. Essentially the reason is that, in general, reaction curves are upward sloping for price-setting firms and downward-sloping for quantity-setting firms. If two firms merge in a market with price-setting behaviour, they benefit from two effects: first, they internalise the previous externality between them by raising prices somewhat, and secondly, this causes other firms at equilibrium also to have higher prices. We must therefore conclude this discussion with the observation that the above results do not necessarily hold without the Nash-Cournot assumption. The assumption is none the less a common one (if not the usual one). In any event it has been established that results concerning asymmetries of incentive between incumbent and entrant do not necessarily extend from the case of one incumbent to that of many.

The Possible Free Rider Problem

We now turn to an independent reason why incumbents, when several in number, might underinvest in R and D in such a way that entry into their market is likely to occur. So far we have just looked at the incentives of different types of firm; or, to put it another way, how much they would bid for the patent in a deterministic patent race modelled as a bidding game in the fashion of Gilbert and Newbery (1982) or Dasgupta (1982). When uncertainty is introduced into the R and D competition an additional free rider problem may exist.[1] The problem can be seen in its clearest form in the case of a patent for a technology equivalent to that already possessed by the m incumbents. If no entry occurs then each incumbent receives a payoff $\pi(m)$. If entry takes place then each of the resulting firms receives a payoff $\pi(m + 1)$. The previous analysis concerned the form of $\pi(m)$, but what follows is independent of that. Here we are concerned with the following asymmetry:

(i) an incumbent receives $\pi(m)$, rather than $\pi(m + 1)$, if and only if <u>some</u> incumbent wins the patent; it matters not which incumbent;

(ii) a potential entrant receives $\pi(m + 1)$, rather than zero, if and only if <u>he</u> wins the patent.

We shall analyse this asymmetry in terms of a model of R and D under uncertainty in the style of Loury (1979) and Dasgupta and Stiglitz (1980 b): see pp. 15ff above.

Assume that there are m incumbent firms, indexed by i, and n potential entrants, indexed by j. Let the rate of R and D of incumbent i

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1. Gilbert and Vives (1983) explore a similar problem in a quantity-setting oligopoly model.

be denoted x_i, and that of potential entrant j be denoted y_j. Define X = x_i and Y = y_j. Uncertainty is assumed to be exponential and completely uncorrelated. Therefore the probability that i wins the patent is $x_i/(X + Y)$, and the probability that j wins is $y_j/(X + Y)$. Let the cost of carrying out R and D of x be denoted c(x), and to make the analysis as simple as possible, assume constant returns to scale in R and D and normalise so that c(x) = x.

First we shall analyse the case in which the patent is for a technology equivalent to that originally possessed by the incumbents. Define $U = \pi(m) - \pi(m + 1)$ as the difference between an incumbent's payoff if no entry occurs and his payoff if entry does occur. Define $W = \pi(m + 1)$ as a potential entrant's payoff in the event of entry. (Perhaps the most natural way to think of $\pi(m)$ here is as the pdv of a firm's profit flow when there are m firms in the industry.) The overall payoff of incumbent i is therefore

$$I_i = \frac{X\pi(m) + Y\pi(m + 1)}{X + Y} - x_i$$

$$(23) \qquad = W + \frac{XU}{X + Y} - x_i$$

And the overall payoff of entrant j is

$$(24) \qquad E_j = \frac{y_j W}{X + Y} - y_j$$

Firm i chooses x_i to maximise I_i and j chooses y_j to maximise E_j. From (23) and (24) we have

$$(25) \qquad \frac{dI_i}{dx_i} = \frac{YU}{(X + Y)^2} - 1$$

and

$$(26) \qquad \frac{dE_j}{dy_j} = \frac{(X + Y - y_j)W}{(X + Y)^2} - 1$$

The second derivatives are

$$(27) \qquad \frac{d^2 I_i}{dx_i^2} = \frac{-2YU}{(X + Y)^3}$$

and

$$(28) \qquad \frac{d^2 E_j}{dy_j^2} = -\frac{2(X + Y - y_j)W}{(X + Y)^3}$$

Since negative R and D is impossible, (27) and (28) show that I_i and E_j are concave over the relevant domain. If setting the RHS of (25) to zero yields a positive x_i it is therefore a global maximum of I_i, given the levels of R and D chosen by others. If, however, it yields a negative x_i, then the constraint $x_i \geq 0$ bites and $x_i = 0$ is optimal. Similarly for E_j and y_j.

First assume that $nU \geq (n - 1)W$. We proceed to find the equilibrium, which is symmetric as between incumbents and symmetric as between potential entrants. With symmetry, let $x_i = x$ for all i, and $y_j = y$ for all j. Then from (25) and (26) the first-order conditions can be written

$$(29) \qquad nyU = (mx + (n-1)y)W = (mx + ny)^2$$

Define $z = U/W$. Then the equality of the first and second terms in (29) implies

$$(30) \qquad mx = y(nz - n + 1) \quad .$$

Therefore

$$(31) \qquad mx + ny = y(nz + 1) \quad ,$$

and hence (29) and (31) imply

$$(32) \qquad y = \frac{nU}{(nz + 1)^2}$$

and

$$(33) \qquad x = \frac{n(nz - n + 1)U}{m(nz + 1)^2} \quad .$$

From (31) it follows that the probability of entry is

$$(34) \qquad P = \frac{ny}{mx + ny} = \frac{n}{nz + 1}$$

This is independent of m, due to our assumption about functional form, and goes to $1/z$ as the number of potential entrants rises.

If we assume the other possibility, i.e. that $nz < n-1$, then at equilibrium all incumbents do no R and D, and all potential entrants have

$$(35) \qquad y = \frac{(n-1)W}{n^2}$$

(set $x = 0$ in (29)). The second-order conditions hold. First, it is obvious that $E_j > 0$, since $W/n > y$. Secondly, if some incumbent made a best interior response to n potential entrants each bidding y, his payoff would be

$$(36) \qquad I = W + x\left(\frac{U}{x + ny}\right) - 1$$

where $x + ny = nU/(nz + 1)$. This payoff is worse than W, which is achieved by $x = 0$, if $nz < n - 1$. If all incumbents do no R and D it is obvious that the probability of entry P is unity.

The following table summarises the analysis so far

	$nz \geq n - 1$	$nz < n - 1$
x	$\frac{n(nz - n + 1)U}{m(nz + 1)^2}$	0
y	$\frac{nU}{(nz + 1)^2}$	$\frac{(n-1)W}{n^2}$
P	$\frac{n}{nz + 1}$	1

We can now examine what would happen if the m incumbents formed a joint venture for R and D. In a more general analysis it would be necessary to specify whether the joint venture consists just of one project, or of one per firm, or whether the number of projects conducted by the joint venture is a choice variable. Fortunately, our functional form $c(x) = x$ allows us to sidestep this question because all three turn out to be equivalent. This enables us therefore to focus on the main issues at hand.

Suppose that the joint venture operates one R and D project. Then each incumbent would wish the joint venture to choose X to maximise

(37) $$J = mW + \frac{mUX}{X + Y} - X$$

Cf. (23). The first-order condition is

(38) $$mUY = (X + Y)^2$$

Each entrant has the same problem as in the previous analysis, and so parallel to (29) we have

(39) $$mnyU = (X + (n-1)y)W = (X + ny)^2$$

Define $z = U/W$ as before. Then (39) yields

(40) $$y = \frac{mnU}{(mnz + 1)^2}$$

(41) $$X = \frac{mn(mnz - n + 1)U}{(mnz + 1)^2}$$

The probability of entry is now

$$(42) \qquad P = \frac{n}{mnz + 1}$$

which is less than the probability of entry in the absence of the joint venture. This probability goes to zero as m gets large, and goes to 1/mz as the number of potential entrants becomes large.

The second-order condition is now that $z \geq (n - 1)/mn$. The range of values of z for which the joint venture undertakes positive R and D is wider than that for which an incumbent would do positive R and D in the noncooperative game.

Finally let us state equilibrium payoffs for the cases of no joint venture and joint venture, assuming that in each case the second order conditions hold.

<u>(i) No joint venture</u>

$$(43) \qquad I^N = W + \frac{(nz - n + 1)(nz + 1 - \frac{n}{m})U}{(nz + 1)^2}$$

$$(44) \qquad E^N = \frac{W}{(nz + 1)^2}$$

<u>(ii) Joint Venture</u>

$$(45) \qquad I^J = W + \left[\frac{nmz - n + 1}{nmz + 1}\right]^2 U$$

$$(46) \qquad E^J = \frac{W}{(mnz + 1)^2}$$

It is straightforward to verify that $I^N < I^J$ and $E^N > E^J$, as one would expect.

A More General Cost Function

The nature of the free rider problem is seen most clearly if z = 1, i.e. U = W. Then the only asymmetry is purely that an incumbent gains W (relative to what he would otherwise get) if and only if some incumbent wins, whereas an entrant gains W if and only if he himself wins. The simplification that U = W permits us to generalise in another direction. Therefore, instead of assuming that the cost of R and D of x is c(x) = x, let us assume that $c(x) = x^{\gamma}$, where $\gamma \geq 1$. If $\gamma > 1$ there are decreasing returns to scale to R and D effort.

The problem facing incumbent i now is to choose x_i to maximise

$$(47) \qquad I_i = \frac{XW}{X + Y} - x_i^{\gamma}$$

and the problem facing entrant j is to choose y_j to maximise

$$(48) \qquad E_j = \frac{y_j W}{X + Y} - y_j^{\gamma}$$

The first-order conditions are

$$(49) \qquad YW/\gamma x_i^{\gamma-1} = (X + Y - y_j)W/\gamma y_j^{\gamma-1} = (X + Y)^2$$

With symmetry (49) implies

$$(50) \qquad (y/x)^{\gamma-1} = \frac{mx + (n-1)y}{ny}$$

Let $\omega = y/x$. Then (50) is equivalent to

$$(51) \qquad n\omega^{\gamma} = m + (n-1)\omega$$

From (51) it follows that $m > 1$ implies $\omega > 1$ (i.e. $y > x$), otherwise we would have

$$(52) \qquad n\omega \geq n\omega^{\gamma} = m + (n-1)\omega$$

and hence $\omega \geq m$ contrary to $\omega \leq 1$. The probability of entry is

$$P = \left[\frac{mx}{ny} + 1\right]^{-1}$$

$$(53) \qquad = \frac{n}{n\omega^{\gamma-1} + 1}$$

from (50). As one would expect, P is increasing in n, the number of potential entrants. Since $\omega > 1$, it is clear that P is decreasing in γ. Thus the more sharply that returns to scale decrease in R and D, the less likely entry becomes, although of course the probability of entry always exceeds $n/(n + m)$, since $\omega > 1$.

In the special case in which there is just one potential entrant, (51) and (53) combine to yield

$$(54) \qquad P = \frac{1}{m^{\frac{\gamma-1}{\gamma}} + 1}$$

As γ approaches unity this approaches $\frac{1}{2}$. As γ approaches infinity, P approaches $1/(m + 1)$.

A Superior New Technology

So far it has been assumed that the patent is for a technology equivalent to that already possessed by the incumbents.[1] This has allowed us to address the free rider question in its purest form. However, it is important to relax this assumption to consider the general case in which an incumbent is not indifferent between all states of affairs in which some incumbent wins the patent. Assume henceforth that the payoffs are as follows:

K to an incumbent if he wins the patent;

L to an entrant if he wins the patent;

M to an incumbent if another incumbent wins the patent; and

N to an incumbent if a new entrant wins the patent.

Since we have in mind the case of a superior new technology it is natural to expect

$$(55) \qquad \begin{aligned} &K > L, \\ &K > M, \\ &M > N, \text{ and} \\ &L > N \geq 0 \end{aligned}$$

1. Strictly speaking, the previous analysis applies also to the case in which the patent is for a technology inferior to that already possessed, for in that case too an incumbent's payoff is the same so long as some incumbent wins.

These inequalities can be justified in terms of the linear Cournot oligopoly studied earlier. Intuitively the argument is that K > L and M > N because in each case the player in question has fewer rivals, and K > M and L > N because it is better to win the patent than for a rival to win it. So far we have confined attention to K = M = U + V and L = N = V: see (23). The case of a drastic innovation would of course be K = L and M = N = 0.

Incumbent i chooses x_i to maximise.

$$(56) \qquad I_i = \frac{x_i K + (X - x_i)M + YN}{X + Y} - x_i$$

Entrant j chooses y_j to maximise

$$(57) \qquad E_j = \frac{y_j L}{X + Y} - y_j$$

The first-order conditions are

$$(58) \qquad [K(X + Y) - x_i K - (X - x_i)M - YN]$$

$$= [X + Y - y_j]L = (X + Y)^2$$

With symmetry, rearrangement of (58) yields

$$(59) \qquad x\{mL - (m - 1)(K - M)\} = y\{n(K - N) - (n - 1)L\}$$

The second-order conditions for interior maxima are that both expressions $\{\cdot\} \geq 0$. If however $(m - 1)/m > L/(K - M)$, then potential entrants bid zero and there is never entry. If $(n - 1)/n > (K - N)/L$ then incumbents bid zero and entry is certain. Assuming that the conditions for interior maxima hold, then

$$x \lesseqgtr y \text{ according as}$$

$$(m + n - 1)(K - L) \lesseqgtr nN + (m - 1)M \quad ,$$

or, alternatively, according as

$$(60) \qquad n(K - L - N) \gtreqless (m - 1)(L + M - K)$$

Note in particular that it is quite possible to have $x > y$, if for example M is low enough.

The final step is to consider how much R and D a joint venture would undertake. This depends upon the payoffs received by the incumbent if their joint venture is successful. Let us suppose that the payoff is:

G to an incumbent if the joint venture wins the patent.

The considerations set out earlier in this Section would lead us to expect $K > G > M$ in the case of a new superior technology. That is, it would be better to be the sole possessor of the technology than to share it with the other incumbents, but to share it is better than for a rival to have it. We shall continue to suppose that an incumbent receives N if a new entrant wins the patent. The joint venture's problem is to choose X to maximise

$$(61) \qquad J = \frac{mGX + mNY}{X + Y} - X$$

Entrant j's problem is still as stated in (57). The first-order conditions are

(62) $$mY(G - N) = (X + Y - y_j)L = (X + Y)^2$$

With symmetry, i.e. $y_j = y$ for all j, we have

(63) $$XL = y\{nm(G - N) - (n - 1)L\}$$

The second-order condition is that $\{\cdot\} \geq 0$; otherwise the joint venture bids zero.

Assuming that the second-order conditions hold, from (59) and (63) it follows that entry is less likely with the joint venture than without if and only if

(64) $$\frac{nm(G - N) - (n - 1)L}{mL} > \frac{n(K - N) - (n - 1)L}{mL - (m - 1)(K - M)}$$

The LHS of (64) is the ratio of X/m to y when there is a joint venture. The RHS is the ratio of x to y when there is not. Rearrangement of (64) yields

(65) $$\frac{(m - 1)(K - M)}{mL} < \frac{nm(G - N) - n(K - N)}{n(K - N) - (n - 1)L}$$

as the condition for the joint venture to reduce the probability of entry. The condition need not hold. For example, if K is large relative to G, or if M is small, then entry is more likely with the joint venture. The reason is that the joint venture removes the incentive of an incumbent to gain a position of superiority over the other incumbents; and it removes the fear that some other incumbent will gain such a position over oneself. However, it is apparent from (65) that if m is large enough - i.e. if there are enough incumbents originally - then formation of the joint venture does increase the probability that entry will not take place.

Furthermore, it is not necessarily the case that it is in the incumbent's interests to form a joint venture. The explanation is akin to the merger paradox. If the formation of the joint venture causes a reduction in incumbent's R and D, the potential entrants will invest more in R and D, and so the probability of entry rises. The badness of this latter effect may outweigh the benefit of the lower expenditure on R and D for an incumbent. As before, which of these effects predominates is likely to depend at least in part upon the magnitude of the innovation in question.

If the new patent offers no technological advance then we have the case already analysed on pp. 95ff above. In symbols that is the case where $K = M = G > L = N$. Here (65) holds because its RHS is positive (assuming an interior maximum) and its LHS zero. If on the other hand the new innovation is drastic then we have $K = L > mG > M = N = 0$. The reason why $mG < K$ is that if <u>one</u> firm holds the new patent then it receives monopoly profits, whereas if m firms operate according to the new patent their joint profits are lower than the monopoly profits. In this case two things are possible. One is that the second-order condition associated with (63) fails, in which case a joint venture would do no R and D at equilibrium, so that entry would be certain (unlike the case of no joint venture, when the probability of entry is $n/(n+m)$ because the firms are symmetrically placed for a drastic innovation). The other possibility is that the second-order condition does hold. Then the LHS of (65) is positive, but the RHS is $n(mG - K)/K$, which is negative. Therefore (65) fails and the joint venture makes entry more likely.

<u>Summary</u>

Whereas much attention has been given to the asymmetries between an incumbent <u>monopolist</u> and a potential entrant, and their consequences for R and D competitions, this section has explored asymmetries between incumbent <u>oligopolists</u> and potential entrants. The principal conclusions to emerge are that for a minor innovation an incumbent has less incentive than a potential entrant to win the patent, but for a major innovation the opposite is true. Similarly, if the incumbents form a joint venture, this may reduce the probability of entry when small innovations are at stake, but increase it when major innovations are in question. The intuitive explanation is that, for other incumbents, R and D by one incumbent is a public good inasmuch as it reduces the probability of entry, and a public bad inasumch as it increases the chance that the other incumbents will become technologically inferior. The larger the innovation, the more that the latter effect comes to outweigh the former.

4. ASYMMETRY WITHIN DUOPOLY

Imagine two duopolists H and L competing for a patent for a new technology. Initially, H (the High cost firm) is less efficient than L (the Low-cost firm). The patent is for a technology better than either firm's initial technology. Will H win the patent and leapfrog L, or will L win it and extend his advantage over H? In general the answer to this question must depend upon the precise formulation of the patent race (e.g. whether it is deterministic or has uncertainty). Here we abstract from these considerations and ask simply whether H or L has the greater incentive to win the patent. If patent races are bidding games, the player with the greater incentive will also be the winner; if they are not, it is still interesting to see who has the greater incentive. We show that with Cournot behaviour the higher cost firm has the greater incentive if the initial cost disparity and the 'size' of the innovation are small; otherwise the lower cost firm has the greater incentive. The lower cost firm is more likely to have the greater incentive as product market behaviour becomes more 'competitive'.

To tackle the question we adopt an approach very similar to that employed in the previous section. All technologies are assumed to have constant returns to scale, so that the level of unit costs is a well-defined measure of efficiency. Demand is assumed to take the linear form

$$(1) \qquad p = A - Q ,$$

where Q is industry output. Firm L's initial unit cost level c will be taken as reference point. Firm H's initial cost disadvantage is denoted $b \geq 0$, and so H's initial unit cost level is $c + b$. The new patent offers cost improvement $s > 0$, relative to L's initial cost level, i.e. the winner will have cost level $c - s$. The table below summarises:

	H wins	L wins
H's cost level	$c - s$	$c + b$
L's cost level	c	$c - s$

A player's incentive to win the patent is measured by the difference between his profits[1] if he wins and his profits if he loses the patent.

Cournot Behaviour: We shall begin by assuming that firms behave as Cournot duopolists in the product market. Having analysed this case exhaustively, we shall then investigate alternative behavioural assumptions.

There are three cases to consider

Case 1: $s \geq A - c$

Case 2: $A - c > s \geq A - c - 2b$

Case 3: $s < A - c - 2b$

We are assuming that both firms were active initially; therefore the RHS of Case 3 is strictly positive.

1. In the following analysis the measure uses single-period profits. This is reasonable also in multi-period models inasmuch as single-period profits are proportional to payoffs in the longer game.

Case 1: Whoever wins the new patent becomes a monopolist. Even if H wins, L would not wish to supply any output, because the new monopoly price of $\frac{1}{2}(A + c - s)$ is less than his cost level c. In this case each firm's incentive to win the patent is the same.

Case 2: If L wins the patent he becomes a monopolist, since H cannot afford to supply at the new monopoly price. But if H wins, L remains active. H's incentive to win the patent is

$$h = \frac{(A - c + 2s)^2}{9} \tag{2}$$

See the Appendix for the justification of these results. L's incentive is

$$\ell = \frac{(A - c + s)^2}{4} - \frac{(A - c - s)^2}{9} \tag{3}$$

The first term on the RHS of (3) is L's (monopoly) profit if he wins the patent. The second term is L's profit if H wins. It is convenient to define B = A − c. Then

$$36(h - \ell) = 4(B + 2s)^2 + 4(B - s)^2 - 9(B + s)^2$$

$$= -B^2 - 10Bs + 11s^2$$

$$= -(B - s)(B + 11s) \tag{4}$$

This is negative because $B = A - c > s > 0$ by assumption. Thus in Case 2, the low-cost firm has a greater incentive than the high cost firm to win the patent.

Case 3: Here both firms remain active whoever wins the patent. Let h and ℓ denote H's and L's <u>incentives</u> to win the patent. Then we have

$$(5) \qquad 9h = (B + 2s)^2 - (B - s - 2b)^2$$

$$(6) \qquad 9\ell = (B + b + 2s)^2 - (B - s)^2$$

Therefore,

$$(7) \qquad 9(h - \ell) = -b(2B + b + 4s) + 2b(2B - 2s - 2b)$$

$$= b(2B - 5b - 8s)$$

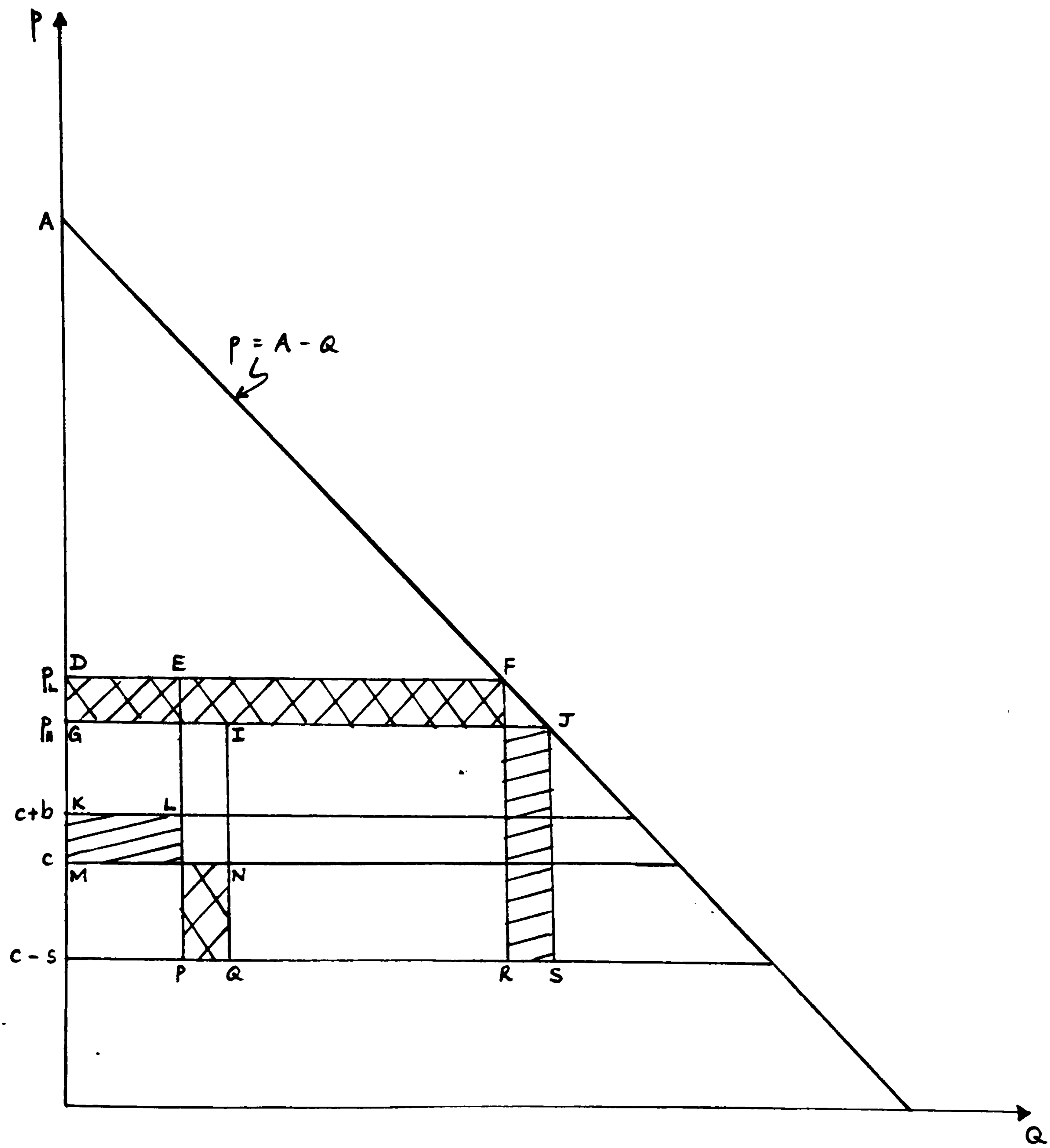
Therefore

$$(8) \qquad h \gtrless \ell \text{ according as } 2A \gtrless 5b + 2c + 8s$$

This condition says that the originally higher-cost firm has a greater incentive than the lower cost firm if the inital cost disparity (b) and the superiority of the new technology (s) are not too large. Otherwise the lower cost firm has the greater incentive.

Figure 1 illustrates the respective incentives.

Figure 1



Profits with Cournot Behaviour

If L wins the equilibrium price is $p_L = (A + 2c + b - s)$. L's profits are the area EFPR, and H's profits are the area DEKL.

If H wins the equilibrium price is $p_H = (A + 2c - s)$. H's profits are the area IJQS and L's profits are the area GIMN.

The difference between joint profits if L wins and joint profits if H wins is the cross-hatched area minus the diagonally shaded area. This difference is equal to $\ell - h$. The diagram illustrates the various influences at work. If L wins price is higher and so is output produced by the new technology. If H wins total ouptut is higher and there is no very high cost output. The overall effect given by (8) is ambiguous.

Non-Cournot Behaviour: We now generalise part of the above analysis by dropping the Cournot assumption. It is clear from the above how an exhaustive analysis would proceed, but we shall concentrate on the case in which both firms remain active irrespective of who wins the patent[1] (i.e. Case 3 above).

Let $\lambda = dq_i/dq_j$ be the common 'conjectural variation' term. One interpretation is that λ is the rate at which i (= H,L) expects j ($\neq$ i) to expand his output in response to a unit increase in output by i. Thus Cournot is $\lambda = 0$ and 'Bertrand' is $\lambda = -1$. The use of conjectural variation terms is of course controversial, but we shall engage in none of the controversy here. Rather, λ will simply be treated as an index of competitiveness in the market. As λ increases, the market becomes less competitive and more cooperative. We wish to see how the players' respective incentives vary with λ.

Using the methods employed in the Appendix, it is simple to verify that the profits of duopolist i are

$$(9) \qquad \pi_i = \frac{\{(1 + \lambda)A + c_j - (2 + \lambda)c_i\}^2}{(1 + \lambda)(3 + \lambda)^2}$$

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1. In the notation to follow, this assumption is $(2 + \lambda)b < B - s$.

where c_i and c_j are the cost levels of i and j, respectively. Using this formula, if h is H's incentive to win the patent, then

$$(10) \qquad (1 + \lambda)(3 + \lambda)^2 h = (B + (2 + \lambda)s)^2 - (B - s - (2 + \lambda)b)^2,$$

where $B = (A - c)(1 + \lambda)$; cf. (5). And if ℓ is L's incentive to win the patent, then

$$(11) \qquad (1 + \lambda)(3 + \lambda)^2 \ell = (B + b + (2 + \lambda)s)^2 - (B - s)^2$$

Subtracting (11) from (10) it follows that $h - \ell$ is directly proportional to

$$\xi = -b(2B + b + 2(2 + \lambda)s)$$

$$+ (2 + \lambda)b(2B - 2s - (2 + \lambda)b)$$

$$(12) \qquad = b\{2(1 + \lambda)B - (5 + 4\lambda + \lambda^2)\, b - 4(2 + \lambda)s]$$

Equation (7) is the special case of (12) when $\lambda = 0$. Once again, $h > \ell$ if b and s are small enough relative to $(A - c)$.

The derivative with respect to λ of the expression $\{\cdot\}$ on the RHS of (12) is

(13) $4B - (4 + 2\lambda)b - 4s,$

which is positive by virtue of the assumption that both firms are active at equilibrium whoever wins the patent. It follows further that $d(h - \ell)/d\lambda$ is positive because $(h - \ell)$ is proportional to ξ, with constant of proportionality $(1 + \lambda)(3 + \lambda)^2$, and therefore $d\xi/d\lambda > 0$.

We have established that H's incentive minus L's incentive increases with λ, i.e. as behaviour becomes less 'competitive' and more 'cooperative'.

Finally we shall consider cases other than those in which both players remain active at equilibrium irrespective of who wins the patent. In general these are not so straightforward to analyse when $\lambda \neq 0$. With the Cournot assumption, the best response when one's rivals output is zero is the monopoly output. This is not true of the best response function when $\lambda \neq 0$. Let us therefore focus on the case of Bertrand, price-setting equilibrium. Figure 2 below illustrates this case. The monopoly price with the new technology is $p_M = \frac{1}{2}(A + c - s)$. If the new technology is so drastic that $p_M \leq c$ then whoever wins the patent will subsequently earn monopoly profits equal to $\frac{1}{4}(A - c + s)^2$. The loser earns no profits. The players are therefore on an equal footing in this case.

If $c < p_M < c + b$ then if L wins the patent he behaves subsequently as a monopolist and earns profits of $\frac{1}{4}(A - c + s)^2$. If H wins he subsequently charges a price equal to c,[1] his rival's cost level, and earns profits of $s(A - c)$, i.e. GHIK in figure 2.
Figure 2 sketches the remaining case: $p_M > c + b$. Whoever wins the patent subsequently sets his price equal to his rivals cost level. Therefore, if H wins the patent his profits equal area GHIK, which

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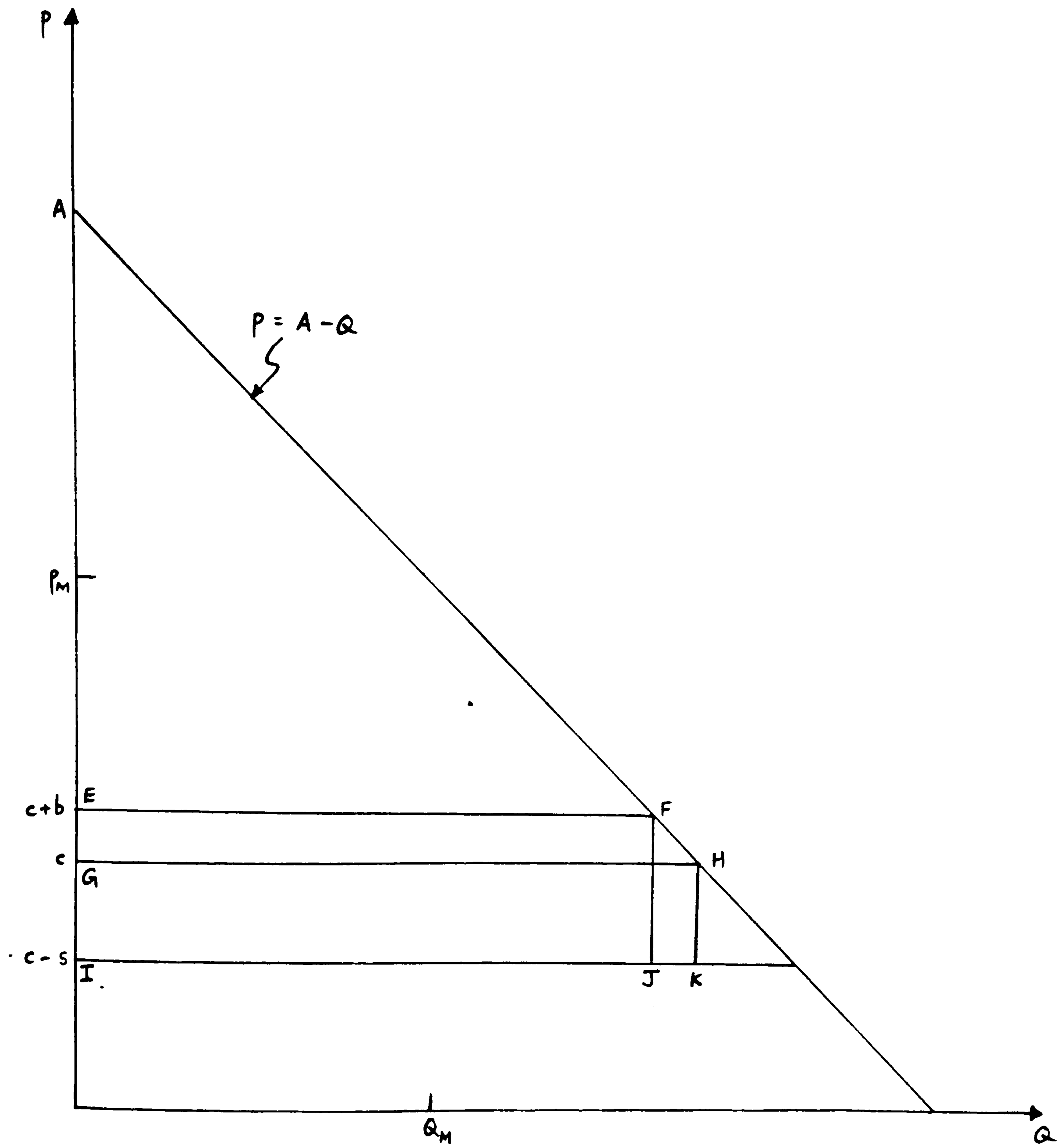
1. Profit is strictly concave in p; therefore there is no incentive to charge less than c.

Figure 2



Profits with Bertrand Behaviour

equals s(A - c); and if L wins the patent, his profits are represented by area EFIJ, which equals (s + b)(A - c - b). L's incentive exceeds H's incentive. This is true in general, because L's choice set exceeds H's. The simple linear case was chosen mainly for illustrative purposes.

Conclusion: This section has employed the linear Cournot model to investigate which of the high- and low-cost duopolists has the greater incentive to win a patent for a new, superior technology. With Nash-Cournot behaviour, the higher cost firm has the greater incentive if the inital cost disparity and the 'size' of the innovation are small; otherwise the lower cost firm has the greater incentive. When a 'conjectural variation' term is introduced as an index of the degree of competitiveness between the firms, it emerges that the lower cost firm has the greater incentive if 'competitiveness' is great enough. In price-setting equilibrium, the lower cost firm always has the greater incentive unless the innovation is so drastic that the monopoly price with the new technology is less than the initial cost level of the more efficient firm. In this latter case, the firms have equal incentives.

Much of Chapter IV is concerned with the question of whether, when there is a <u>sequence</u> of patent races, one firm becomes increasingly dominant by winning all patents, or whether there is action-and-reaction between firms. If the currently low cost firm always wins the next patent then there is increasing dominance. If the currently high cost firm is the winner then there is action-and-reaction. The present section has looked only at competitions for single patents. Nevertheless the analysis above serves as a useful ingredient for the models in Chapter IV.

CHAPTER III

PERFECT EQUILIBRIUM IN PATENT RACES

INTRODUCTION

The main purpose of this Chapter is to examine the strategic nature of competition in patent races, and to consider the consequences of this analysis for some issues in the theory of market structure, notably the question of the persistence of monopoly. This introduction gives an overview of the Chapter and argues that much of the existing literature on R and D competition does not capture the features to be studied here.

It may be useful to begin by considering a simple running race in order to see some of the motivation for the models in this Chapter. Suppose that running is painful, and that pain increases with speed. Then a player would compete in the race only if he thought he had a reasonable chance of winning the valuable prize at stake. If he began the race some distance behind his rival, he might reason that effort was futile, and so would not bother to compete actively. This would leave the leader in the race in the happy position of not needing to run particularly fast in order to win. Equally, if a player knew that his rival was a much better runner than himself, he might be deterred from actively competing in the race because the rival would be expected to outstrip any efforts that he made. Similarly, if the rival were especially keen to win the race, this might also deter the first player from making much effort in the race. In all these cases, it appears that the rival might be left to proceed more or less at his own pace to the finishing line in the race.

The ideas in the last paragraph are only conjectures: the problem is to make them good - or alternatively to falsify them. In particular, we would like to know <u>why</u> the conjectures are true, if indeed they are so. The answer seems to have to do with credible threats and counter-threats. That is, the reason why one player is deterred from competing appears to be that he reckons that his rival <u>would</u> outdo any effort that he might reasonably make. What deters is therefore the threat, or series of threats, that the rival can credibly make. The natural way to explore this structure of threats formally is Selten's (1965) perfect equilibrium solution concept.

The Chapter has four sections. The first two of these contain a detailed account of joint work that I have done with Christopher Harris of Nuffield College, Oxford. Their purpose is to pin down in precise terms the remarks made above about the running race. The first model is of what we call a <u>standard</u> race - that is, a race in which each player positively wishes to win the prize at stake. The second Section of this Chapter contains a model of what we call an <u>asymmetrical</u> race - that is, a race in which one player's sole concern is that his rival does <u>not</u> win the prize at stake. There are important differences between these kinds of race, which are relevant to the question of the persistence of monopoly. The structure of our models of a race enables these differences to be brought out, which it is not possible to do in simpler models. A <u>hybrid</u> race - intermediate between standard and asymmetrical - is also considered.

The third and fourth sections of this chapter look at patent races in which there is technological uncertainty. This desirable advance upon the models in the first two sections is achieved obviously at some cost: the models with uncertainty are much less subtle and general (e.g. in respect of functional forms). Nevertheless it is argued that their analysis offers some illumination. In particular, conclusions emerge that are reasonably similar to those resulting from the models with certainty: this is comforting.

The remaining task in this Introduction is to explain the relationship of Chapters I and II to Chapter III. Perhaps this can best be done in terms of the example of the running race described at the beginning of this Chapter. Recall that a number of conjectures were made - for example that a player would not compete actively in the race if his rival began sufficiently far ahead of him in the race, or if the rival were a sufficiently better runner, or if the rival were especially keen to win. The claim is that these conjectures - and others like them - are worth investigating (especially bearing in mind their implications for the economics of patent races), but that the models in Chapters I and II are ill-equipped to analyse them. None of the following comments are intended as severe criticisms of those models: they were designed for different purposes. The point is simply that there are features of races which they do not capture.

In the models of Loury (1979), Dasgupta and Stiglitz (1980b) and Lee and Wilde (1980), each player's strategy set is R_+: each player chooses the level (or rate) of its R and D expenditure once-and-for-all in the game. It is argued below that the strategic interactions during the course of a race, which are the subject of this Chapter, simply do not exist in models in which each player's strategy set is R_+. In Reinganum's (1981,1982) dynamic game of R and D, each firm's strategy is a function relating its rate of R and D to the vector of players' knowledge levels and time. However, the stationary structure of Reinganum's model is such that firm's equilibrium strategies are functions only of time. Moreover, it was argued (p. 28, above) that if the prize in the competition is discounted - as are costs - then as the time horizon becomes large, equilibrium in Reinganum's model comes to coincide with that in Lee and Wilde's much simpler formulation. In some contexts this perhaps justifies the restriction of attention to Lee and Wilde's formulation, rather than getting involved with Reinganum's very sophisticated version. Be that as it may, the following remarks apply to all the literature mentioned above.

There are at least two reasons why these models do not capture the strategic interactions that we wish to analyse. First, the uncertainties facing rival firms are assumed to be completely uncorrelated: see further Chapter I Section1for an attempt to relax this assumption and a discussion of the success or otherwise of that attempt. The effect of this assumption is that firm i's hazard rate is independent of what its rivals are doing. Secondly, and more significantly, the exponential form of technological uncertainty in these models implies that firm j's present position (i.e. state of accumulated knowledge) does not affect firm i's current calculations.[1] If the race is still in progress, the firms' levels of accumulated knowledge are irrelevant.

To see this, suppose, as is typical, that technological uncertainty takes the following form. Let $1 - e^{-z}$ be the probability that z is sufficient knowledge to innovate. Then e^{-z} is the probability that z is insufficient knowledge. Suppose that a player already has u 'units' of knowledge, but that he has not yet innovated. Then the probability that z units more knowledge is insufficient for him to innovate, given that u is insufficient, is e^{-z-u}/e^{-u}, which equals e^{-z}. This is so for all u. In short, whatever level of knowledge a player has, if he has not yet innovated, then the probability that z more units of knowledge is sufficient for him to innovate is $1 - e^{-z}$. Another way to make the point is that player i's hazard rate is independent of his - and all others' - levels of accumulated knowledge.

Since players' positions are therefore irrelevant to their calculations in these models, one cannot sensibly speak of, say, one player being 'ahead' of another in the competition. To the extent that

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1. For other discussions of this point, see Fethke and Birch (1982) and Fudenberg et al (1983).

they ignore such essential features of races, these models are therefore not really models of <u>races</u> at all.

Suppose by contrast that the technological uncertainties facing rival firms are to some extent correlated and that the players' respective positions (i.e. knowledge levels) do matter. Then if A is ahead of B, it follows that B has a smaller chance than of A of innovating in the near future. In the special case of completely correlated uncertainty B would have no chance of innovating in the immediate future in those circumstances. To increase his probability of success, B therefore has to gain ground on A. But realising this, A is concerned to keep his lead over B; and so on. It is strategic interactions of this sort, which occur as the race unfolds, that we are out to capture.

The models of patent races as simple bidding games (e.g. Dasgupta (1982) and Gilbert and Newbery (1982)) that were discussed in Chapter II are also unable to reflect these strategic interactions. The reason is that each player can make just one bid: this is equivalent to irreversible, once-and-for-all commitments being made at the outset of the race. Thus, there is no scope for action-and-reaction during the course of the race since players make just one move (as with Loury, Dasgupta and Stiglitz, and Lee and Wilde the strategy sets are R_+). Such models do not capture the fact that each competitor can make a series of decisions during the course of the race, each decision being made in the light of the moves made by his rivals and with a view to influencing the moves that they make in their turn.

What is required to remedy this shortcoming, and so to explore the conjectures about the running race described at the beginning, is a more elaborate structure of moves. This Chapter considers two ways in which this might be done. The models in the following two sections have a structure of alternating moves: first A moves, then B, then A, then B

.... Initially Christopher Harris and I developed a model in which this sequence of alternating moves was finite, with an exogenously given number of moves accorded to each player. It proved possible to solve the model, but it was in some ways rather unsatisfactory: the duration of a race should be endogenous, not exogenous, and rather too much hinged on who had first move. The model described in Section I below is certainly superior, and so no attention is paid to the former model in this thesis (see Vickers (1983, pp.83-6) for a brief account).

Instead of alternating moves, a sequential structure of simultaneous moves would provide another way to study strategic interaction in races. This is the approach of Fudenberg et al. (1983, Section 4), whose paper is discussed fully below on pp. 149ff. In essence, it is also the approach used in Sections 3 and 4 of the present Chapter. In these models of multistage patent races, the R and D competition can be in any one of a number of 'states'. Players' strategies specify their effort rates for each state. These effort rates determine the probabilities of transition from state to state. These models are an attempt to combine technological uncertainty with a move structure in which action-and-reaction is possible.

This Introduction has attempted to explain part of the motivation behind the work in the following sections.[1]

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1. In this footnote I attempt to describe, to the extent that I can, the respective contributions of Christopher Harris (CH) and myself (JV) to the joint work reported in the following two sections. (i) 'Perfect Equilibrium in a Model of a Race' (hereafter called 'HV1'). The first version of this paper was completed in February 1983. Initially JV had doubts about modelling patent races as bidding games in which each player has just one bid: see p 81 above for the reasons behind this concern. JV conjectured, and wanted to make precise the idea that some player would often have strategic supremacy in the race and that the presence of his rival would make no difference. After discussing this with CH, CH and JV together formulated a model in which each player had a given number of bids: this approach was unsatisfactory. Then CH and JV together formulated the more satisfactory model reported in Section III.1 below. The elegant method of solution of the model is due to CH. The first draft of HV1 was written approximately half-and-half by CH and JV: CH writing the more 'technical' parts

1 (contd):

(including proofs in Appendices, not included in this thesis) and JV wrote the more 'economic' parts. CH and JV revised the paper jointly.
(ii) 'Patent Races and the Persistence of Monopoly' (hereafter called 'HV2'). CH and JV both realised that HV1 was not an appropriate model of an 'asymmetrical race': see Section III.2 below. Both attempted to find perfect equilibrium in a model of an asymmetrical race based on HV1, with discounting. This was not conveniently tractable but it was evident to both how the methods used to analyse HV1 could readily be used to find equilibrium in the asymmetrical race if there were no discounting. JV sketched out much of the preliminary draft of HV2, but it was in every sense a joint product. CH studied rigorously the limiting case of the model and proved some existence results. He wrote up this work in an Appendix to HV2, but it has not been included in this thesis.

1. PERFECT EQUILIBRIUM IN A MODEL OF A RACE

The model in this section can be summarised as follows. Two players are in competition for an indivisible prize, which they may value differently. At the outset of the race they are at certain distances (not necessarily the same) from the finishing line. The prize goes to the first player to reach the finishing line. Progress towards the finishing line depends upon the (costly and irrecoverable) efforts, called 'bids', made by each player. The players bid alternately. At each of his turns a player must decide how much to bid, i.e. how much ground to cover, at that turn. His decision will depend upon the respective positions of the players at that stage of the race. Moreover, the decision is made in the knowledge that the other player will make his next decision on the basis of the state of the race at his turn; and so on.

It is shown that if one player is far enough ahead of the other, in a sense defined in terms of factors (i) to (iv) below, then the latter gives up completely, leaving the former to move to the finishing line at his own pace. The four factors are:

(i) the players' respective valuations of the prize;

(ii) their discount rates;

(iii) their efficiency at making progress; and

(iv) their initial distances from the finishing line.

Note that the phrase 'far enough ahead' is used loosely here: it is quite possible for a player to be literally further away from the finishing line than his rival and yet cause the rival to give up. This could happen if the former player put a sufficiently higher value on the prize than his rival; then the rival would know that, despite his relative proximity to the finishing line, any reasonable effort he made to approach the finishing line would be outdone by the player more keen to win the prize.

It may be the case that one player is far enough ahead of his rival at the outset of the race. Then the race is no real contest: it is just as if the rival had never existed. If this is not the case, then, assuming that the prize is worth pursuing at all, the player who has the first turn to bid immediately moves to a position far enough ahead of his rival. Thereafter it is as if the rival did not exist. Thus the winner's behaviour is just as if he could proceed at his own pace, unless the players are 'close' at the outset of the race, where closeness is measured in terms of factors (i) to (iv) above. Next the formal analysis is presented to justify these conclusions. Then there is a discussion of the limit of the equilibrium of the model as the intervals between his bids become short. Sometimes this allows one to derive easily a helpful formula for the equilibrium in the limit: this is done for a particular example. Then the relationship between our model and that in Section 4 of Fudenberg et al is discussed in full. Finally, and also as a prelude to Section 2 of this Chapter, some economic consequences of the results of the foregoing analysis are examined.

There are several respects in which it would be desirable to extend the model. Section 2 of this Chapter concerns one extension (of a slightly simpler version of the model), namely to an asymmetrical race - i.e. a race in which one player's concern is that his rival does not win the prize, irrespective of whether he himself does win it. More desirable still would be to incorporate uncertainty into the model (in the language of game theory, incomplete and/or imperfect information). This possibility is discussed at the end of Section 2, rather than in the present section, because it is the natural lead into Sections 3 and 4 of this Chapter. However, the discussion of the incorporation of uncertainty is entirely relevant to the present section.

The Formal Analysis of the Model

Two players, A and B, are in competition for a single indivisible prize. They value the prize at V_A and V_B respectively, where V_A, $V_B > 0$. At the outset of the game A and B are respectively at distances X_0 and Y_0 from the finishing line, where X_0, $Y_0 > 0$. Progress towards the finishing line depends upon the bids made by each player. The players bid alternately, first A, then B, then A again and so on. All bids are forfeit. The prize is awarded to the first player (if any) to reach the finishing line.

Precisely, if player A makes a bid of a he moves a distance of $w_A(a)$ in the direction of the finishing line, where w_A is strictly increasing, continuous, and $w_A(0) = 0$. Thus after A has made his k^{th} bid a_k the respective positions are

$$X_{2k-1} = X_{2k-2} - w_A(a_k)$$

and

$$Y_{2k-1} = Y_{2k-2}$$

Similarly, after B has made his k^{th} bid b_k the respective positions are

$$X_{2k} = X_{2k-1}$$

and

$$Y_{2k} = Y_{2k-1} - w_B(b_k)$$

If n is the smallest integer such that either $X_n \leq 0$ or $Y_n \leq 0$ then the prize is awarded to A (B) if $X_n \leq 0$ ($Y_n \leq 0$). Neither player wins the prize if there is no n such that $X_n \leq 0$ or $Y_n \leq 0$.

Player A (B) applies a discount factor ρ_A (ρ_B) to all costs and benefits, where $0 < \rho_A, \rho_B < 1$. The discount factor relates to the interval between consecutive bids of any one player. Thus if A wins the prize with his kth bid he receives a payoff of

$$\rho_A^{k-1} V_A - \sum_{i=1}^{k} \rho_A^{i-1} a_i$$

If he does not win the prize his payoff is

$$- \sum_{i=1}^{k} \rho_A^{i-1} a_i$$

B's payoffs are defined analogously.

Each player's strategy is an infinite sequence of functions specifying a bid at each turn contingent upon any possible sequence of previous bids. Formally, let S denote the closed half-line $[0, \infty)$, and write $f = (f_1, f_2, \ldots)$ and $g = (g_1, g_2, \ldots)$ for the strategies of A and B respectively. Then

$$f_1 \in S$$

$$f_k: S^{2k-2} \longrightarrow S \qquad (k > 1),$$

$$g_k: S^{2k-1} \longrightarrow S \qquad (k \geq 1).$$

The solution concept is subgame perfection (see Selten (1965, 1975)). A strategy pair (f, g) is in perfect equilibrium if its restriction to any subgame is in Nash equilibrium. In the present context a subgame is the game which ensues after some sequence of previous bids.

We obtain a simple characterization of perfect equilibrium in the game by defining for each player a sequence of critical distances from the finishing line. These sequences are used to partition the space of pairs of distances from the finishing line into four areas. In the first (second) area, no matter who has first move, A (B) wins the prize with his bids the same as he would make if he faced no rivalry. In the third area the prize is won by the player who has first move; after his bid that player's bids are as if he faced no competition. In the fourth (possibly empty) area it is not worthwhile for either player to make a positive bid.

We begin by defining the sequence and state three of their important properties. Then we state and prove the theorem, illustrating it diagrammatically. Finally we make some observations about the theorem.

As a prelim inary we adopt the following simplifying convention: if a player is indifferent between winning the prize with an overall payoff of zero and not winning the prize, then he will choose to win the prize. This convention does not affect the argument, as will be clear.

We denote the sequences of critical distances from the finishing line for A and B by $\{C_n\}_{n=0}^{\infty}$ and $\{D_n\}_{n=0}^{\infty}$, respectively. The sequence $\{C_n\}$ is defined purely in relation to A. Roughly speaking, C_1 is the maximum distance that A can cover with one bid and obtain a non-negative payoff overall. Then C_2 is the maximum distance from the finishing line that A can cover, in any way he pleases, subject to covering at least $C_2 - C_1$ with his first bid and obtaining a non-negative payoff overall. And so on. In formal terms:

DEFINITION: $C_0 = 0$. For $n \geq 1$, C_n is the maximum X such that the inequalities

(1a) $$a_i \geq 0 \; (1 \leq i \leq s)$$

(1b) $$\sum_{i=1}^{s} w_A(a_i) \geq X$$

(1c) $$w_A(a_1) \geq X - C_{n-1}$$

(1d) $$\rho_A^{s-1} V_A - \sum_{i=1}^{s} \rho_A^{i-1} a_i \geq 0$$

can be satisfied jointly in $a_1, a_2, \ldots a_s$ for some s.

Thus C_n is the maximum distance from the finishing line that A can cover with a sequence of non-negative bids, subject to moving to within C_{n-1} from the finishing line with his first bid, and without spending more than the prize is worth to him (in discounted terms).

In order to obtain a somewhat different perspective on $\{C_n\}$ let us define $\Omega_A(X)$ as the value to A of being a distance X from the finishing line when he is not subject to any constraint. Sometimes we call $\Omega_A(X)$ the 'value of a free run for A from X'. Then, if $C_{n+1} > C_n$, we have

$$C_n - C_{n-1} = w_A(\Omega_A(C_{n-1})\rho_A)$$

The RHS of this expression is the distance that A would travel if he bid the full value to him of a (discounted) free run from C_{n-1}.

That the definition is consistent is verified in the Appendix to HV1, which also contains proofs of the following properties.

PROPERTY 1: The sequence $\{C_n\}$ is increasing.

PROPERTY 2: If $\{C_n\}$ ever fails to be strictly increasing it remains constant thereafter,

PROPERTY 3: It is possible for A to cover distance h and obtain a non-negative payoff overall if and only h $\leq C_n$ for some n.

The above definitions and properties apply <u>mutatis mutandis</u> to player B and the sequence $\{D_n\}$. The properties will be used implicitly in the proof of the Theorem, which we now state.

THEOREM: Suppose that the strategies of A and B are in perfect equilibrium. Suppose that A and B are respectively at distances X and Y from the finishing line. Then there are four exhaustive and exclusive possibilities:

(I) For some $n \geq 1$, $X \leq C_n$ and $Y > D_n$. Then A wins. His bids are those he would make in the absence of rivalry from B. B always bids zero.

(II) For some $n \geq 1$, $X > C_n$ and $Y \leq D_n$. Then B wins. His bids are those he would make in the absence of rivalry from A. A always bids zero.

(III) For some $n \geq 0$, $C_n < X \leq C_{n+1}$ and $D_n < Y \leq D_{n+1}$. Then, if it is A's (B's) turn to bid, A (B) wins. His bids are those he would make if, in isolation, he were subject to moving to within C_n (D_n) of the finishing line with his first bid. B(A) always bids zero.

(IV) For all $n \geq 0$, $X > C_n$ and $Y > D_n$. Then neither player wins. Both players always bid zero.

In what follows, if (I) is the case we say that (X,Y) belongs to A's 'safety zone'; if (II) is the case (X,Y) belongs to B's safety zone: and if (III) is the case (X,Y) belongs to a 'trigger zone'.

PROOF: We proceed by partitioning the space of pairs of distances from the finishing line (the positive orthant of R^2_+).

Suppose that the strategies of A and B are in perfect equilibrium.

Consider first the case where $X > C_n$ and $Y > D_n$ for all n (this is case (IV)). From Property 3 we know that neither player can reach the finishing line and obtain a non-negative payoff overall. Hence it cannot be optimal for either player to win the prize. Both players therefore always bid zero.

Consider next the case of $0 < X \leq C_1$, and $0 < Y \leq D_1$ (this is a sub-case of (III)). Suppose that it is A's turn to bid. A prefers to win the prize in one move than not win it. Therefore A wins the prize. For if B were to win it, A would have done better to win it in one move. Moreover, A wins in one move. Otherwise B would find himself in the same situation as that which A faces, in which case B would win – a contradiction. Similar remarks apply when it is B's turn to bid. In short, the player whose turn it is to bid wins the prize immediately. We call this case a trigger zone.

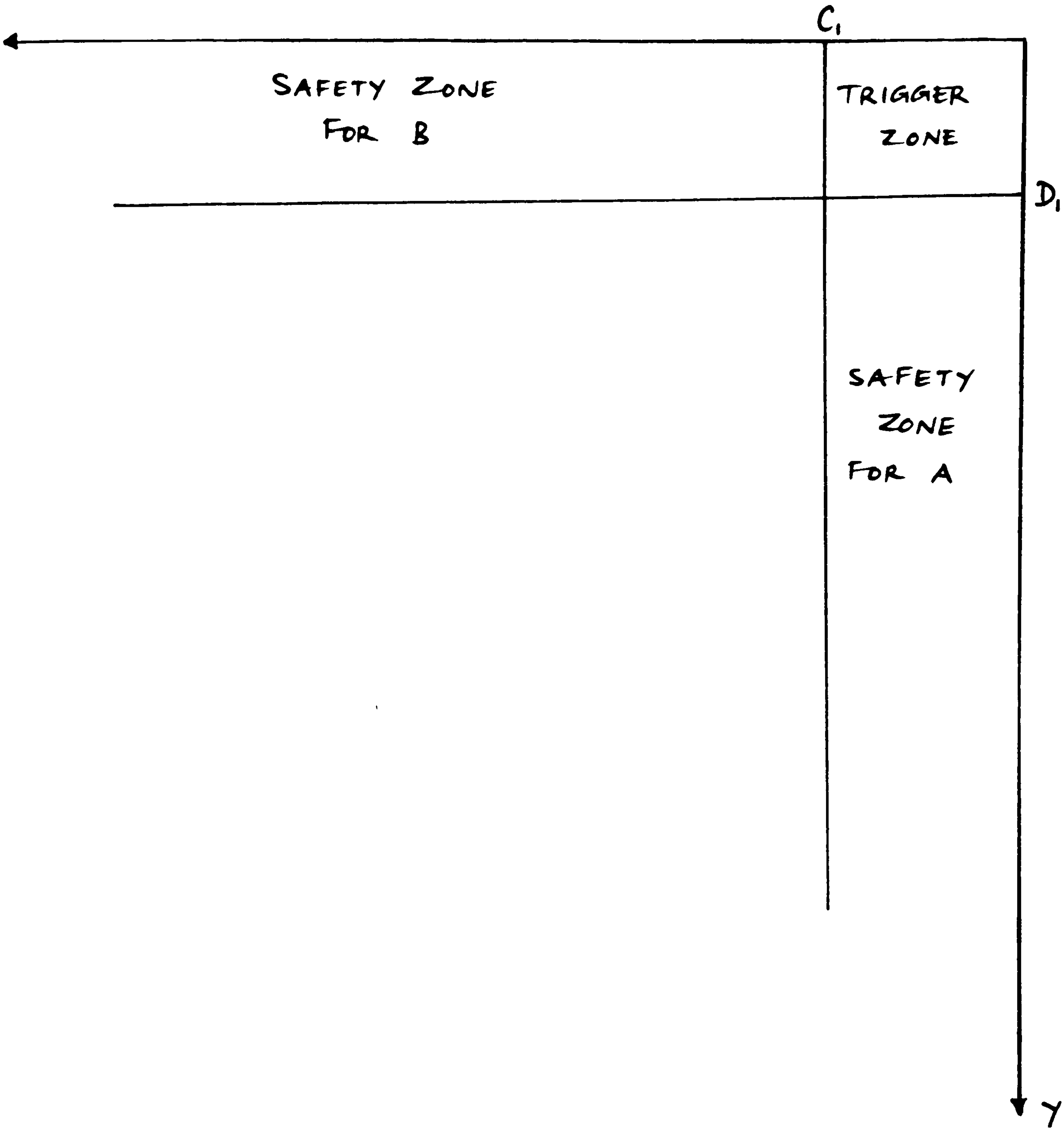
Consider next the case $0 < X \leq C_1$, and $Y > D_1$ (this is a sub-case of (I)). B cannot win the prize in one move without obtaining a strictly negative payoff. If B were to win the prize in more than one move it would at some point be A's turn to bid at a point in the trigger zone discussed above. At this point A would win the prize - a contradiction. Hence B cannot win the prize in any event, so he always bids zero. A therefore proceeds as if he faced no rivalry from B. We say that this case belongs to A's safety zone.

Similar remarks apply to the case $X > C_1$ and $0 \leq Y \leq D_1$, which belongs to B's safety zone. Figure 1 summarizes the argument so far.

Consider now the case $C_1 < X \leq C_2$ and $D_1 < Y \leq D_2$ (also a sub-case of (III)). Suppose that it is A's turn to bid. It is possible for A to move to a distance no greater than C_1 from the finishing line with one bid and to obtain a non-negative payoff overall. Therefore, A wins the prize. For if B were to win it, A would have done better to move immediately to his safety zone (discussed above) and then to proceed to win the prize as if he faced no further rivalry from B. Moreover, A moves to his safety zone immediately. Otherwise B would find himself in a situation the same as that which A now faces, in which case B would win - a contradiction. Similar remarks apply when it is B's turn to bid. In short, the player whose turn it is to bid moves to his safety zone immediately and thereafter proceeds as if in isolation. In particular, that player behaves as a player who faces no rivalry but who is subject to moving at least to within C_1 (in the case of A) or D_1 (in the case B) of the finishing line with his first bid.

Just as above, it now follows that the case $C_1 < X \leq C_2$ and $Y > D_2$ belongs to A's safety zone, and that the case $X > C_2$ and $D_1 < Y \leq D_2$ belongs to B's safety zone.

Figure 1



The Beginning of the Partition of the Space of Distances from the Finishing Line

These arguments can be iterated. The Theorem follows by induction.

Q.E.D.

Figure 2 illustrates the partition of the space of distances from the finishing line.

The converse of the Theorem is obvious - any pair of strategies satisfying the description contained in the statement of the Theorem is in perfect equilibrium. Thus we have completely characterized perfect equilibrium in the game.

If w is strictly concave then the game has an unique perfect equilibrium point, because each of the implicit maximization problems has an unique solution.

For a given initial position (X_0, Y_0) the outcome of the game depends upon how much A and B value the prize and upon their discount factors. Thus we state two further properties of the sequence $\{C_n\}$.

PROPERTY 4: For all $n \geq 1$, C_n is strictly increasing in V_A.

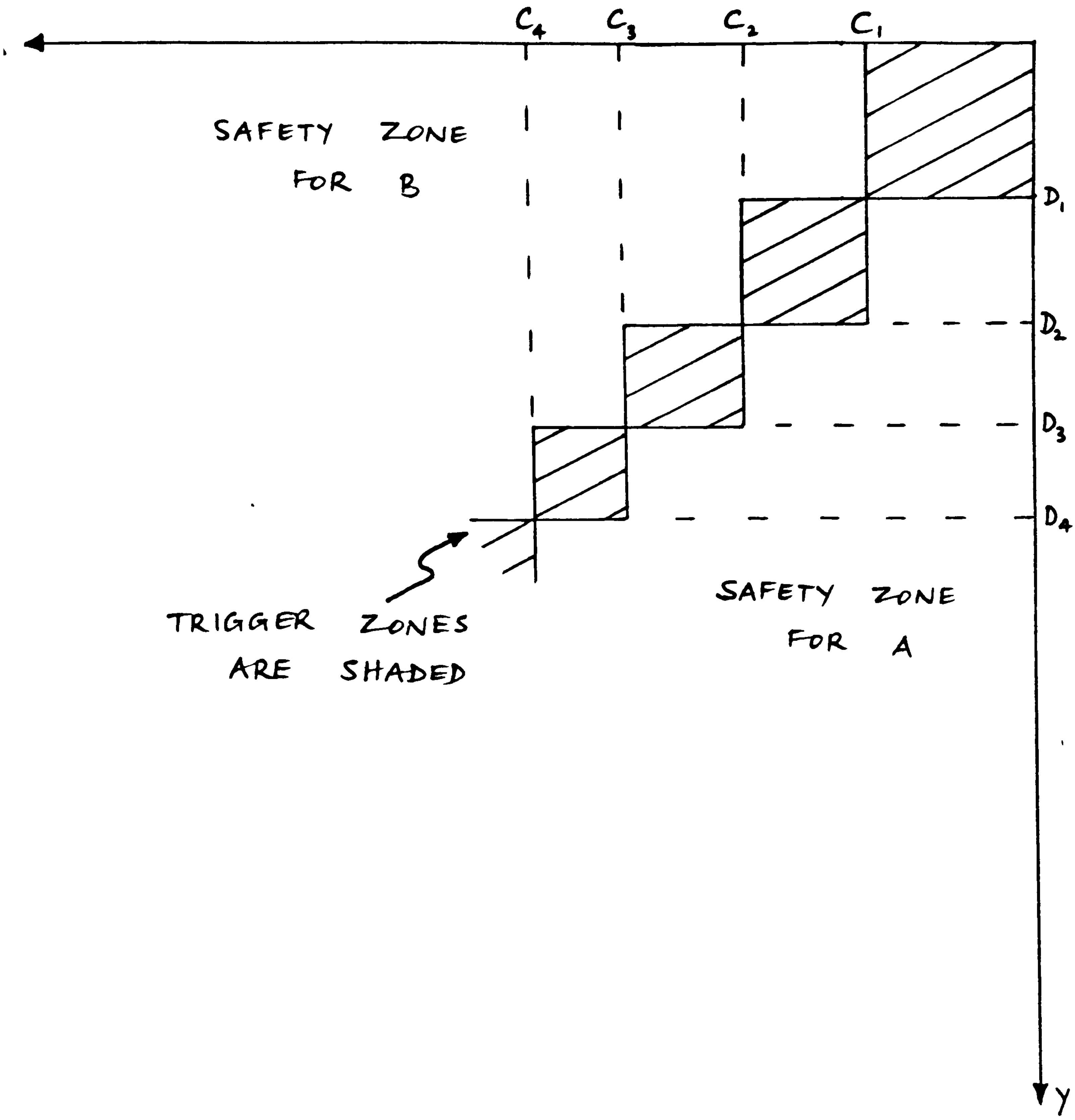
PROPERTY 5: For all $n \geq 2$, C_n is increasing in ρ_A.

That these properties hold is demonstrated in Appendix to HV1. They apply <u>mutatis mutandis</u> to B and the sequence $\{D_n\}$.

As a footnote to Property 5. it should be added that if V_A is regarded as a discounted profit flow, then V_A will be increasing in ρ_A. This provides an extra reason why C_n is increasing in ρ_A.

Note that the model and the Theorem have a straightforward generalization to the case of n players. The results are therefore not special to the case of two players.

Figure 2



The Partition of the Space of Distances from the Finishing Line

<u>The Limit of the Equilibrium: an Example</u>

It would be helpful to work out an explict example of the model, partly to see how the factors (I) to (IV) combine in a concrete case to determine who has strategic supremacy in the race. Unfortunately this cannot be done conveniently in the discrete time model. Consider just the simple problem facing a player who has no rival. It is not hard to find his optimal path to the finishing line when the number of bids that he is allowed is fixed. (If w is differentiable, the first-order condition is that $\rho w'(a_k) = w'(a_{k+1})$, subscripts having been dropped: this is effectively the principle that the ratio of marginal cost to marginal benefit is the same for each bid in the sequence). But it is not so easy to find the optimal number of bids to take to reach the finishing line. This integer problem should suffice to show that any expression for the $\{C_n\}$ sequence in the discrete game would be ugly and probably unilluminating.

An indirect approach to this problem is to take the limit of the equilibria of a sequence of models of the race in which the interval between bids becomes short. In the limit, the integer problem vanishes. Moreover, under certain (plausible) conditions, the trigger zones collapse. This is of interest inasmuch as it implies that with probability zero it makes no difference who has first move. Sometimes one is able to characterize the curve to which the trigger zones collapse in a helpful way. However, this is often not possible, because the limit of the string of trigger zones either is ill-defined or collapses into one of the axes. Later an explicit example is worked out. First, however, there is a fairly intuitive account of how to characterize the limit of the equilibrium. A slightly more detailed account is to be found in Section 5 of HV1; see also Appendix B of HV2.

We wish to consider a sequences of races like that already analysed, in which the interval between bids goes to zero. To do this we need a common framework in which to place the sequence of models. We do this by reference to a continuous time forumlation of players' progress technologies.

If, in the continuous time case, player A's bid rate at time t is a(t) then his total progress over the interval [0,T] is

(1) $$\int_0^T w_A(a(t))\,dt$$

and the cumulative cost to him over the same interval

(2) $$\int_0^T e^{-r_A t} a(t)\,dt$$

where r_A is his discount rate (as opposed to discount factor, which we used in the previous analysis).

In the δ^{th} game in the sequence, we think of time as being divided up into intervals of length δ. Player A chooses a bid rate for each interval of time. So if A chooses bid rate a for some interval, say (t, t + δ), his progress during that interval is $\delta w_A(a)$, and the cost incurred is $\int_t^{t+\delta} ae^{-r_A \tau}\,d\tau$. As δ gets small, this last expression approaches $\delta ae^{r_A t}$. Therefore the progress made by the sequence of bid rates $a_1, \ldots, a_n$ is

(3) $$\sum_{i=1}^{n} \delta w_A(a_i)$$

and the total discounted cost is

$$(4) \qquad \sum_{i=1}^{n} \delta a_i e^{-r_A i\delta}$$

It is now straightforward to define $\{C_n^\delta\}$, the sequence of critical distances for A in the δ^{th} game. Likewise we define the sequence $\{D_n^\delta\}$ for B. Define $\Omega_A^\delta(X)$ as the value, discounted by one period, of a free run for A from distance X in the δ^{th} game, and similarly $\Omega_B^\delta(Y)$ for B.

For the next step in the analysis, we impose two conditions on w_A and w_B in addition to those already made. These assumptions are sufficient to justify the method used below to derive the equation for the curve to which the trigger zones collapse.

(i) $\delta w_J\,(1/\delta) \longrightarrow 0$ as $\delta \to 0$, for J =A,B.

(ii) There exists h: $(0, \infty) \to (0, \infty)$ and functions $\bar{w}_A$ and $\bar{w}_B$ continuous on $(0, \infty)$ such that $a > 0$ implies $\bar{w}_J(a) > 0$, and such that

$$h(\delta)\ \delta w_J(a/\delta) \longrightarrow \bar{w}_J(a) \text{ as } \delta \longrightarrow 0$$

for J = A,B and all $a > 0$

Condition (ii) ensures that the heights and widths of A's and B's trigger zones converge to zero at comparable rates, and that, rescaled by h, they converge to a stable limit. Condition (ii) is a technical assumption and is stated here for the sake of completeness: it will not be discussed further.

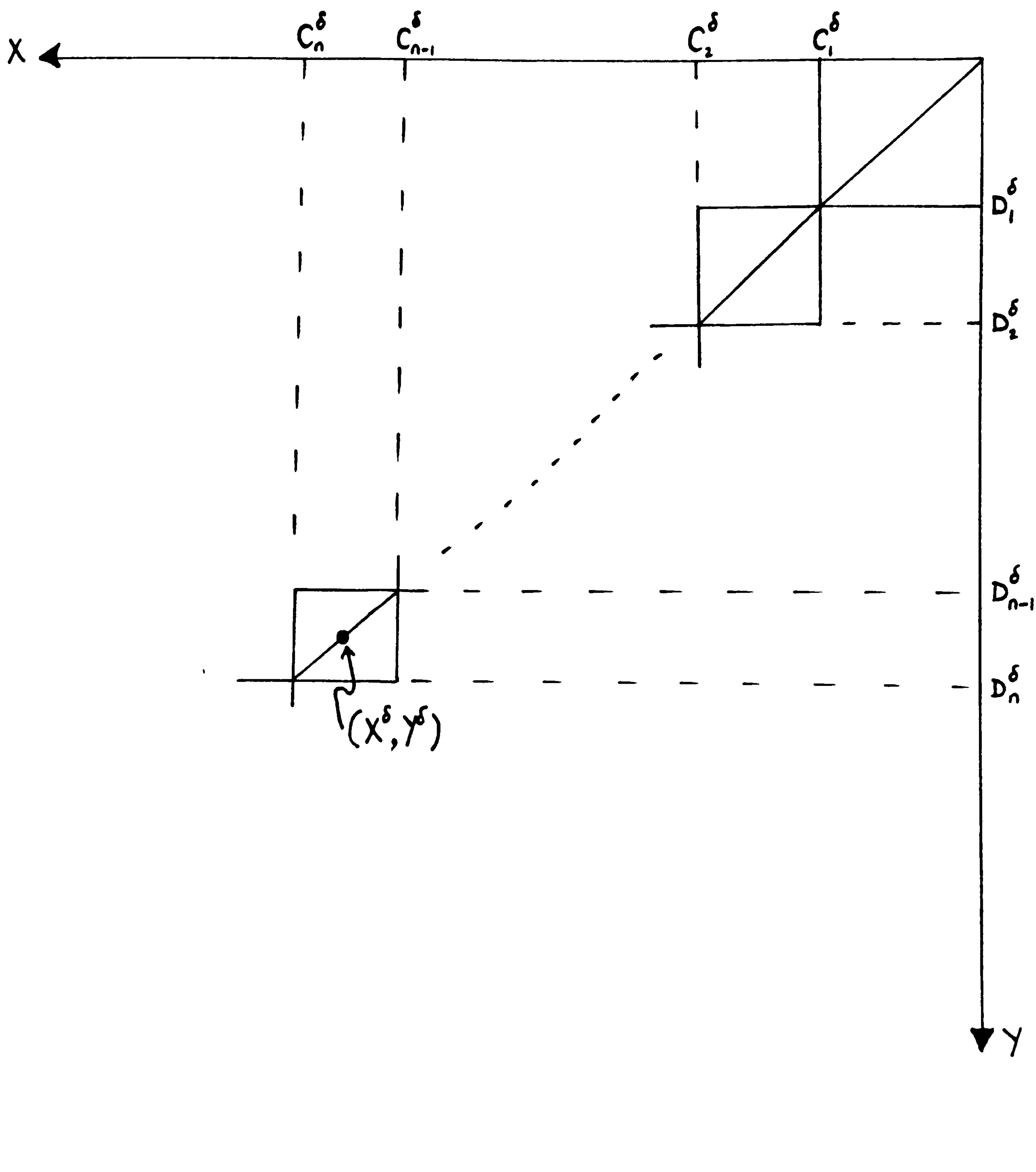
Condition (i) guarantees that the trigger zones collapse as $\delta \rightarrow 0$. To see this, note that A would never spend more than V_A at any of his turns. Thus, in the δ^{th} game in the sequence, he would never bid at a rate exceeding V_A/δ. Otherwise he would spend more at that turn than the prize was worth to him. Therefore the distance that A can cover with one bid is bounded by $\delta w_A(V_A/\delta)$. Condition (i) implies that this expression goes to zero as $\delta \longrightarrow 0$. The same being true for B, we have established that the heights and widths of the trigger zones converge to zero in the limit. We say that the trigger zones <u>collapse</u>. We saw earlier that there is effective rivalry between players in the race only if the initial distances lie in a trigger zone. That the trigger zones collapse therefore implies that in the limit there is almost never effective rivalry in the race.

We now want to characterise the curve to which the trigger zones collapse. We shall refer to this as 'the limiting curve'. For any game in the sequence, consider the curve formed by the diagonals of the trigger zones. For each δ let (X^δ, Y^δ) be a point on the δ^{th} curve, and suppose that (X^δ, Y^δ) converges to (X, Y). If (X^δ, Y^δ) lies on the diagonal of the n(δ)th trigger zone, the slope of the $n(\delta)^{th}$ curve at (X^δ, Y^δ) is

(5)
$$\frac{D_n^\delta - D_{n-1}^\delta}{C_n^\delta - C_{n-1}^\delta}$$

See Figure 3. In (5) and in (6) below, n is a function of δ, since the trigger zones are collapsing. We stated previously (p. 132above) that if the C_n sequence is strictly increasing at n, then $C_n - C_{n-1} = w_A(\Omega_A(C_{n-1})\rho_A)$. Therefore the expression (5) for the slope equals

Figure 3



The Diagonals of Trigger Zones in the δth Game

$$(6) \qquad \frac{\delta w_B(\Omega_B^\delta\ (D_{n-1}^\delta)/\delta)}{\delta w_A(\Omega_A^\delta\ (C_{n-1}^\delta)/\delta)}$$

Because the trigger zones collapse, $C_{n-1}^\delta \rightarrow X^\delta$ and $D_{n-1}^\delta \rightarrow Y^\delta$ as $\delta \rightarrow 0$. Multiplying top and bottom of (6) by $h(\delta)$ and using condition (ii), if (X,Y) lies on the limiting curve then the limiting curve satisfies the differential equation

$$(7) \qquad \frac{dY}{dX} = \frac{\bar{w}_B\ (\Omega_B^o(Y))}{\bar{w}_A\ (\Omega_A^o(X))}$$

where $\Omega_A^o(X)$ is the value of a free run for A from distance X in the limit as $\delta \rightarrow 0$, i.e. in the continuous time setting. As we would expect, $\Omega_A^\delta \rightarrow \Omega_A^o$ as $\delta \rightarrow 0$.

We can solve (7) explicitly for a concrete example. Let the progress functions for A and B be identical and isoelastic:

$$(8) \qquad w_A(a) = a^\eta \quad \text{and} \quad w_B(b) = b^\eta \quad ; \quad 0 < \eta < 1$$

Condition (i) is met, since $\delta w_I\ (1/\delta) = \delta^{1-\eta}$, which clearly goes to zero as $\delta \rightarrow 0$. Condition (ii) is met if we set $h(\delta) = \delta^{\eta-1}$. Then $\bar{w}_A(a) = a^\eta$ and $\bar{w}_B(b) = b^\eta$. Therefore (7) becomes:

$$(9) \qquad \frac{dY}{dX} = \left[\frac{\Omega_B^o(Y)}{\Omega_A^o(X)} \right]^\eta$$

and the only remaining task is to find Ω_J^o. Since the problem is symmetrical, it will cause no confusion to drop subscripts. For further convenience of notation, let $\Theta(X) = \Omega_J^o(X)$. Consider the optimisation problem facing A if he has a free run from X. His problem is to maximise

$$(10) \qquad Ve^{-rT} - \int_0^T a(t)\, e^{-rt}\, dt \qquad \text{, where } T$$

is defined by $\int_0^T w(a\,(t))dt = X$

Because of the stationarity of the problem, we can think of A's optimal bid path as a function of X. The boundary condition is that $\theta(0) = V$. Standard techniques of dynamic programming imply that the Bellman equation is

$$(11) \qquad \theta = \max_a \left[- a - r\theta - \theta' w\right]$$

The first order condition for a maximum of the RHS of (11) is

$$(12) \qquad 1 + \theta' w' = 0$$

which with (11) gives

$$(13) \qquad a + r\theta = \frac{w}{w'},$$

with $w(a) = a^{\eta}$, (12) and (13) are

$$(12') \qquad \theta' = -\left(a^{1-\eta}\right) / \eta \qquad \text{, and}$$

(13') $\theta = a\ (1 - \eta)/r\eta$

Differentiation of (13') and (12') implies

(14) $a' = \frac{-r\ a^{1-\eta}}{(1-\eta)}$

the solution of which is

(15) $a^{\eta} + \frac{r\ \eta\ X}{1 - \eta} = \text{const.}$

Since $\theta(0) = V$, (13') implies that

(16) $a(0) = \frac{r\ \eta\ V}{1 - \eta}$

Therefore, using (15)

(17) $a(X) = \left[\left(\frac{r\ \eta\ V}{1 - \eta}\right)^{\eta} - \frac{r\ \eta\ X}{1 - \eta}\right]^{1/\eta}$

And with (13') this implies

(18) $\theta(X) = \left[V^{\eta} - \left(\frac{r\eta}{1 - \eta}\right)^{1-\eta} X\right]^{1/\eta}$

If the expression on the RHS is negative, then A would of course choose not to make any positive effort.

It remains only to substitute (18) - with subscripts reinserted - into (9). Let $\lambda = (\eta/1 - \eta)^{1-\eta}$. The the limiting curve is characterised by the differential equation:

$$\text{(19)} \qquad \frac{dY}{dX} = \frac{V_B^{\eta} - \lambda\, r_B^{1-\eta}\, Y}{V_A^{\eta} - \lambda\, r_A^{1-\eta}\, X}$$

with the initial condition X = Y = 0. The solution of the equation is the straight line

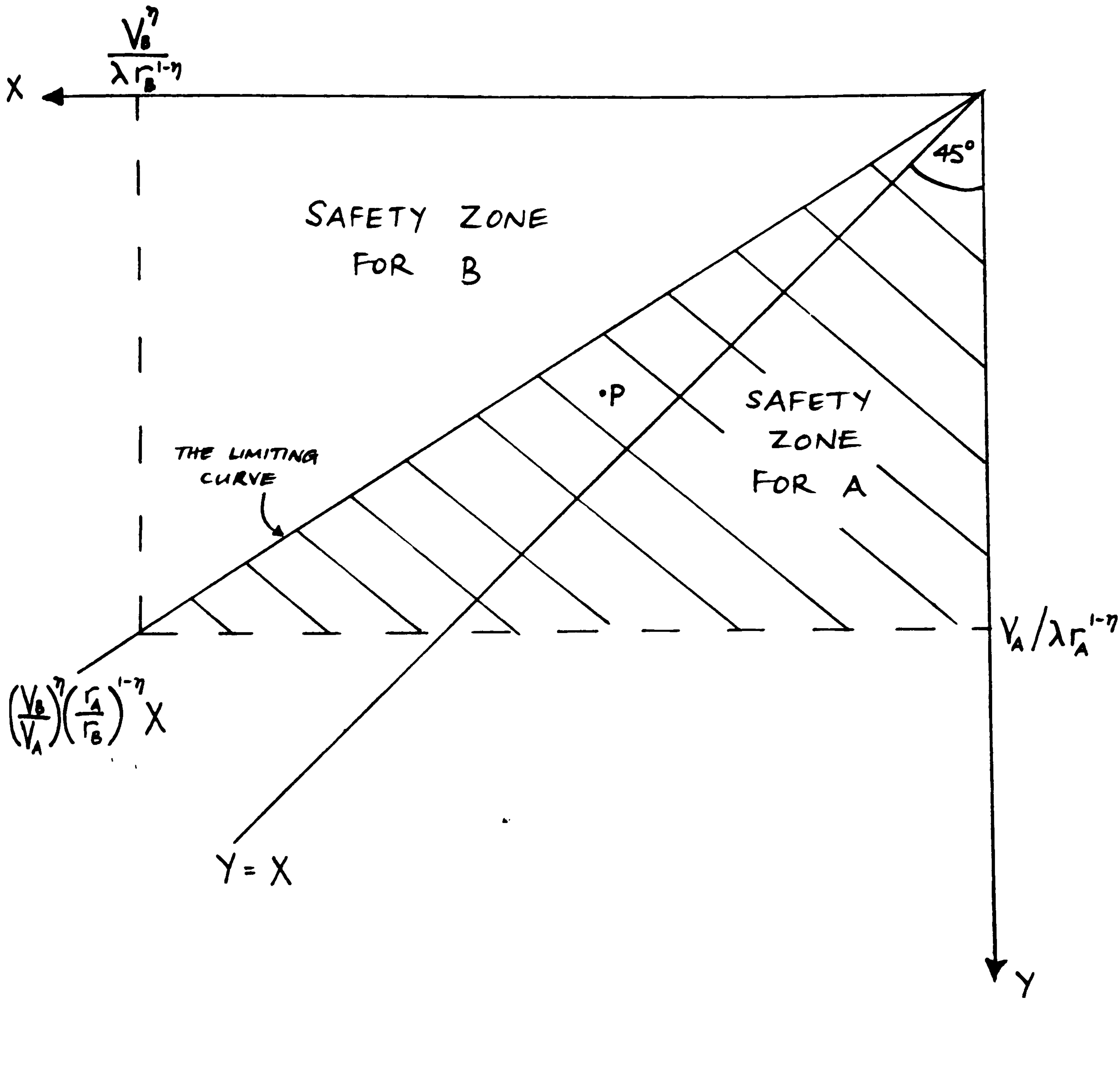
$$\text{(20)} \qquad Y = \left(\frac{V_B}{V_A}\right)^{\eta} \left(\frac{r_A}{r_B}\right)^{1-\eta} X$$

depicted in Figure 4. The formula shows neatly how a player's safety zone expands the more he wants the prize - relative to his rival - and the less heavily he discounts the future. An even simpler version of (20) occurs if V_I represents the discounted value of a constant profit flow π_I. Then $V_I = \pi_I / r_I$ and (20) becomes

$$\text{(20')} \qquad Y = \left(\frac{\pi_B}{\pi_A}\right)^{\eta} \left(\frac{r_A}{r_B}\right) X$$

Figure 4 has been drawn on the assumption that $r_A^{1-\eta} / V_A^{\eta} < r_B^{1-\eta} / V_B^{\eta}$. That is, relative to B, A values the prize highly and/or has a low discount rate. If the players begin equidistant from the finishing line, A has strategic supremacy and it is as though B did not exist. The same may be true even if B is literally closer to the finishing line than A, for example at point P in Figure 4. This illustrates how the conventional notion of distance must be modified to take account of the player's discount rates and their respective valuations of the prize.

Figure 4



The Limiting Curve in the Illustrative Example

Relationship with other Literature: Some Modelling Issues

The work most closely related to the previous model is by Fudenberg et al. (1983), hereafter referred to as FGST. The first version of HV1 was written in February 1983 and incorporated in our M.Phil Theses submitted in April of that year. It was not until the summer of 1983 that we became aware of FGST when Professor Gilbert kindly sent us copies of FGST, having seen our own paper. The purpose of this section is to consider in some detail the links between our paper and FGST. The Introduction to Chapter III has already discussed the relationship with some other literature, in connexion with the motivation behind our own work - e.g. the dissatisfaction with models not able to capture certain features of strategic interaction in races.

FGST describe their paper as follows:

> 'This paper analyzes patent races as dynamic games. We focus on determining when such races will be characterized by vigorous competition and when they will degenerte into monopoly with 'blockaded entry'. In particular, we examine the conditions that allow a firm with an arbitrarily small headstart in the race to preempt its rivals, which we call 'ε - preemption'.' (p. 3)

One of their aims is to present patent races whose equilibria lie between competition and ε - preemption.

FGST address this in two ways. The first of these is a model of a multi-stage patent race in Section 3 of their paper; this is discussed below in Section 4 of the present Chapter. It is their second approach that is the present concern. This is the model of 'A Patent Race with Information Lags' in Section 4 of FGST.

The model can be described as follows. There eare two firms in competition for a patent worth V. R and D competition takes place in discrete time. Discovery of the patent occurs when a firm has N 'units of experience'. There is no discounting. Each period each firm decides whether to buy 0, 1 or 2 units of experience. It is more costly to travel two steps at a time than one step at a time. At each period there are simultaneous moves. Mixed strategies are allowed. The main results are summarised in FGST's Proposition 3:

> In any perfect equilibrium, if the follower is two or more steps behind, it drops out. If it is one step behind, and if the number of stages left is not too large, it randomizes between dropping out and incurring the high cost; the leader randomizes between the high and the low effort levels. If both firms are tied, they compete vigorously (i.e. incur the high cost) if the number of steps is [small enough], and otherwise they randomize. The equilibrium is unique if the number of steps remaining for the leader is [small enough].

There are several differences between the models in FGST and HV1. In FGST, the firms are identical, except perhaps for their respective distances from the finishing line. In particular, they value the prize equally and are equally efficient at R and D. Distance is measured in discrete 'steps'. Progress is possible only at rates of 0, 1 or 2 steps. There is no discounting: it is assumed that there is a lexicographic preference for winning sooner rather than later. By contrast, in HV1 firms may differ in respect of

(i) their valuations of the prize;

(ii) their discount rates;

(iii) their efficiency at R and D; and

(iv) their initial distances from the finishing line.

Distance is a continuous variable, and there is a general progress function with domain R_+. As well as being richer in its specification, HV1 has a broader purpose. An important part of its motivation is to show how factors (i) to (iv) above combine to determine who has strategic supremacy in the race. FGST do not allow for asymmetries (i) to (iii): their focus is just on distance measured in the literal way. In HV1 it is shown how literal measures of distance must be modified in the light of factors (i) to (iii) in order to consider who has strategic supremacy in the race. And asymmetries (i) to (iii) are worthy of study not for the sake of generality, but because there are important economic reasons to expect such asymmetries to exist, for example between an incumbent firm and a potential entrant.

Furthermore, the methods of proof are different in FGST and HV1. We hope that in HV1 the proof using diagrams brings out clearly the structure of threats that deter the loser from active participation in the race. We hope that the picture shows how and <u>why</u> A would do so-and-so if B were to do such-and-such, as it were. None of this is so apparent (to me, at any rate), in FGST. There is another important difference between the models, namely move structure: FGST have simultaneous moves whereas in HV1 move order is sequential. We chose a structure of alternating moves in order to have a game of complete and perfect information: a parallel is with Rubinstein's (1982) bargaining model. The assumption of alternating moves permitted the neat analysis of equilibrium. With simultaneous moves, equilibrium in pure strategies fails to exist. (This consideration led Dasgupta and Stiglitz (1980b) to adopt a sequential move structure even in their very simple model: see p. 56 above). Mixed strategies are therefore needed, but we (and others) have been unable to find a mixed strategy equilibrium. The much simpler structure of the model in FGST enables mixed strategy equilibrium to be found in that model.

It would be worrying if it made a great difference in our model who had first move (as it certainly does in Dasgupta-Stiglitz (1980a): see p. 58 above). In the safety zones it makes no difference. Only if the race begins in a trigger zone does it matter who goes first, and in the limit the trigger zones vanish. (This is rather like FGST's finding that the follower always drops out in the limit of shorter information lags, i.e. period lengths).

For all these differences of specification, a number of results in FGST and HV1 are similar. We both find that there is vigorous competition only if the players are close, and tht the race degenerates otherwise. In FGST it is possible that both firms make positive efforts simultaneously, but this depends upon the employment of mixed strategies, and not due to any basic economic reason.

In conclusion, we regard the main advantage of HV1 over FGST as being that that it brings out clearly, and in a reasonably general framework, how factors (i) to (iv) above interact to determine who has strategic supremacy in the race, and how conventional notions of distance must be modified in the light of these influences.

Conclusions and Economic Implications

This section has analysed perfect equilibrium in a model of a race. It was shown that if one player is far enough ahead of his rival then the rival does not actively participate: the race ceases to be a real contest. The rival realises that any reasonable effort that he made would be outdone by the first player. This leaves the first player free to proceed to the finisihing line at his own pace. Being far enough ahead in this sense is not simply a question of literal distance from the finishing line. It depends also upon the players' respective valuations of the prize, their ability at R and D and their discount rates. It was shown how these factors combine to determine who has strategic advantage, and how they require literal notions of distance to be modified when one speaks of the 'leader' or 'follower' in a race. For example, if a player was keener than his rival to win the prize, then he might have strategic supremacy despite starting literally further from the finishing line than his rival.

The stark nature of the results is in part a consequence of the game having complete and perfect information. The assumption that players are fully informed about one another and about the environment yields a model that is, we hope, illuminating as well as tractible. At least it enables us to make precise the conjectures about strategic interaction in races that were described in the Introduction to this Chapter. A model with incomplete and/or imperfect information would represent patent races in a more 'realistic' way, but the analysis of such a model would appear to be formidably difficult at the prsent state of knowledge. A discussion of this is postponed until the end of the following section.

What are the implications of the results of the present model for the theory of technological competition? Recall that in Chapter II we considered asymmetries between firms engaged in R and D competitions, including asymmetries between duopolists with different initial cost levels, and asymmetries between an incumbent monopolist and a potential entrant into his market.

Strictly speaking, the model so far analysed is not a proper representation of the case in which one firm (an incumbent monopolist, say) is principally concerned that his rival (a potential entrant into his market, say) does not win the patent. This case, which we call an asymmetrical race, is the subject of the next Section of this Chapter. Although the results so far obtained do not apply directly to the asymmetrical race, they are at least suggestive.

To illustrate the implications of the results so far obtained, consider the following example. Firm B is an existing firm, currently supplying a product through an existing sales network, which is a sunk cost. Firm A is a new firm which has no existing sales network. The patent for which B and A are competing is for a new product that B will be able to sell through his existing sales network if he is the winner.

Firm A, on the other hand, will have to invest in a sales network in order to sell the new product if he is the winner. The result is that B has a greater incentive than A to win the patent, since for him the sunk cost of the sales network is a bygone.

Moreover, firm B, by virtue of its experience may have more R and D experience or may have acquired a greater ability at R and D than the new firm A. In the language of the model B may be closer to the finishing line, or may have a more favourable w function than A. It is possible also that discount rates differ: B, the established firm, may discount the future less heavily than A by virtue of better access to finance.

According to our analysis, these factors all conspire to give strategic advantage to the existing firm in the race for the new technology. The new firm, in the example, might therefore be deterred even from competing for the new technology.

We shall return to these themes at the end of the next section after analysing a model of an asymmetrical race.

2. PATENT RACES AND THE PERSISTENCE OF MONOPOLY

The relationship between patenting and the persistence of monopoly has been the subject of much recent work (see, for example, Gilbert (1981), Gilbert and Newbery (1982) and Dasgupta (1982)). The basic argument might be labelled: 'patents as entry tickets'. It can be summarised as follows: see pp. 55ff. above for a fuller discussion. For a new firm, a patent can be an entry ticket into the market. For an incumbent firm already in the market, patents for related technologies can ensure that no new firm enters. It is argued that this gives an incentive for preemptive patenting - the strategic acquisition of patents by an incumbent firm solely to prevent potential rivals from entering the market.

Consideration of the asymmetry between an incumbent firm and a potential entrant (or 'challenger') indicates that the former will have a greater incentive than the latter to win the patent. If the incumbent wins he will remain a monopolist. If the challenger wins, there will be duopoly (unless the innovation in question is so drastic as to displace the incumbent if the challenger wins). Unless there is perfect and costless collaboration between the duopolists, the monopolist's profits exceed the sum of the duopolists' profits. Then the incumbent's incentive to win the patent exceeds the challenger's incentive.

Unfortunately, the model in the previous section does not properly apply to this case. In that model, the prize was of positive value to each player. However, the rationale for the incumbent/entrant asymmetry just described is that the incumbent is concerned that his rival should <u>not</u> win the patent, rather than that he himself should win it: the incumbent is out to deny his rival the entry ticket.

The purpose of the present section is to address this case directly, by using a version of the model analysed in the previous section. The natural way to do this would be to make only one modification to that model. Let B be the player whose concern is to stop his rival; A positively wants to win the patent. Instead of saying that B's payoff is

$$(1) \qquad \rho_B^{k-1} \; V_B - \sum_{i=1}^{\infty} \rho_B^{i-1} \, b_i$$

if he wins the patent with his k^{th} bid, and

$$(2) \qquad -\sum_{i=1}^{\infty} \rho_B^{i-1} \, b_i$$

if he does not win the patent, the modification would be to say that B's payoff is:

$$(3) \qquad - \; \rho_B^{k-1} \; V_B - \sum_{i\,=\,1}^{\infty} \rho_B^{i-1} b_i$$

if A wins the patent with his k^{th} bid, and expression (2) if A never wins the patent. Unfortunately the resulting model is very difficult to analyse. The reasons for this are explained towards the end of this section. Essentially the difficulties arise because of discounting: B's calculations depend upon when A would win. In order to shortcut these difficulties, discounting is ignored in the present section. If some extra assumptions are made about the progress function w, this enables a clean analysis to be carried out.

The plan of this section is as follows. We are interested in comparing

(i) a <u>standard race</u>, in which both A and B want to win the prize; and

(ii) an <u>asymmetrical race</u>, in which A ('the challenger') wants to win the prize, but B's ('the incumbent's') sole concern is that A should not win the prize.

First, the model is described. It is similar - but not identical - to the model in the previous section. Within this framework, results are stated for the standard race because it provides the basis for direct comparison with the model of the asymmetrical race which follows. The latter model is then analysed fully. An explicit example is calculated - both for the standard race and the asymmetrical race - in order to see the difference between the two types of race. Next there is a discussion of

(iii) a <u>hybrid race</u>, in which A ('the challenger') wants to win the prize, but B ('the incumbent') is concerned that A should not win the prize and also has some positive incentive to win the prize himself.

In some sense (i) and (ii) are special cases of (iii), but it is more illuminating to compare (i) and (ii) directly (which in any event makes clear how a detailed analysis of (iii) would proceed).

Next there is a brief account of the difficulties of incorporating discounting into the model of an asymmetrical race. Then there is a discussion of the possibility of extending the models in this and the previous section to include incomplete and/or imperfect information.

This serves as a prelude to Sections 3 and 4 of this Chapter. Finally there are conclusions of the analysis so far examined.

A Model of a Standard Race

Apart from the specification of incentives, the only differences between the model in this section and that in the previous section are

(i) that there is no discounting

(ii) that extra assumptions are made about the progress function.

Briefly, there are two players A and B competing for a prize valued at V_A and V_B respectively. They begin at distances X_o and Y_o from the finishing line. Players bid alternately: first A, then B, then A again, and so on. The sequence of bids is denoted by a_1, b_1, a_2,... The prize is awarded to the first player to reach the finishing line. If at one of his turns A bids a he moves $w_A(a)$ closer to the finishing line. We assume that w_A is continuous, increasing, and that it exhibits increasing returns to scale for low effort levels. We assume also that $w_A(0) = 0$. Figure 1 illustrates one such function.

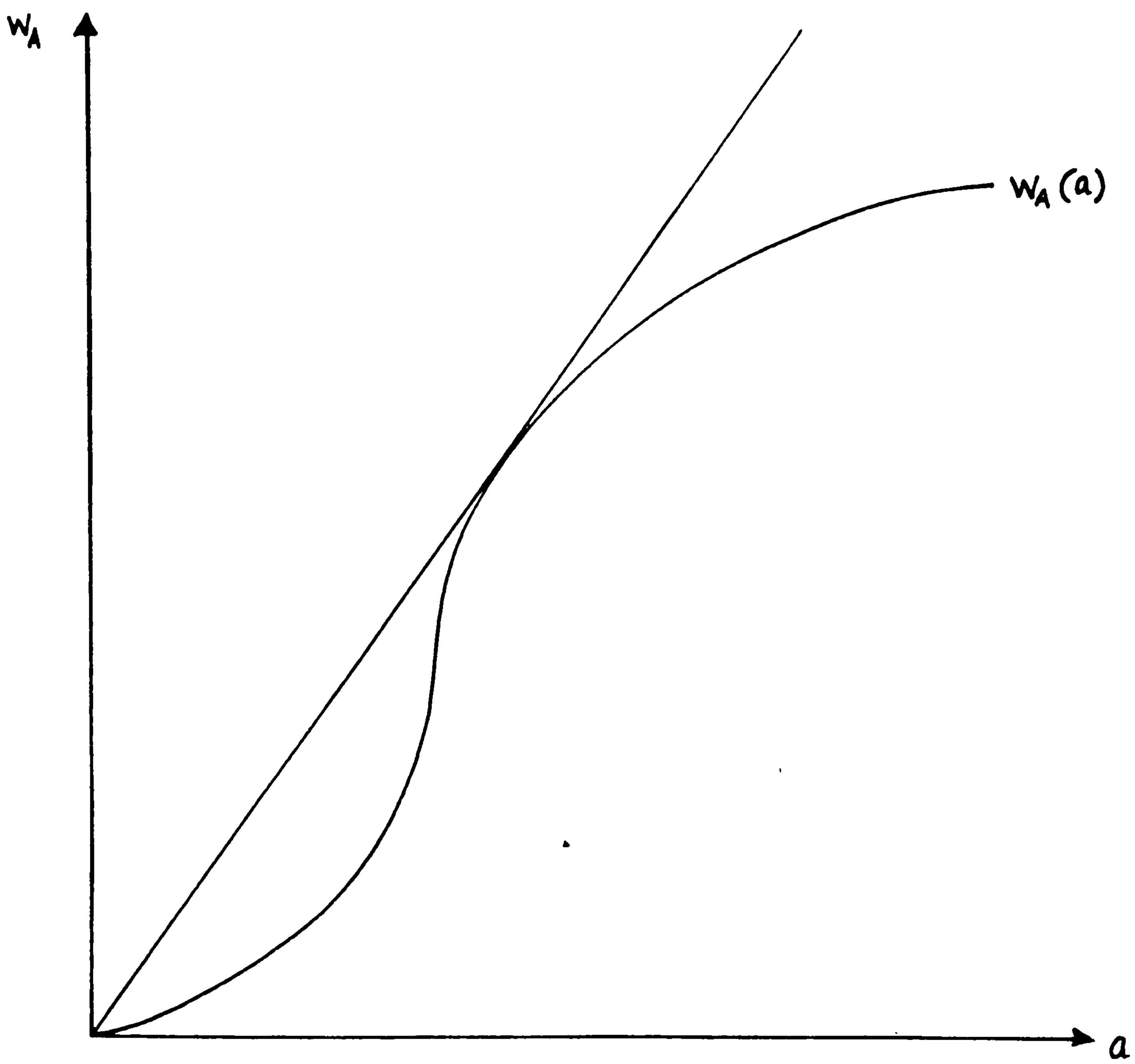
The assumption on w_A embodied in fig. 1 are very similar to those typically made about the hazard function in the literature on R and D under uncertainty: see e.g. Kamien and Schwartz (1982, p.181). Similar assumptions are made in respect of player B.

The above assumptions are shared by the model of an asymmetrical race which follows later.

In the standard race, A's payoff is $V_A - \sum a_i$ if he wins the prize and $-\sum a_i$ if not. Likewise, B's payoff is $V_B - \sum b_i$ if he wins the prize and $-\sum b_i$ if not.

The game is analysed in the same way as the model in the previous section. Perfect equilibrium is found by defining for each player a sequence of critical distances from the finishing line, $\{C_n\}$ for A and

Figure 1



The Progress Function: An Example

$\{D_n\}$ for B. The sequence $\{C_n\}$ is defined formally below in the context of the present model. The solution to the model is the same as that stated in the Theorem on p. 133 of Section 1 of this Chapter and illustrated on p. 138.

In the <u>asymmetrical race</u>, A's payoff is $V_A - \sum a_i$ if he wins the prize and $-\sum a_i$ if not. This is as in the standard race. However, B's payoff is $-V_B - \sum b_i$ if <u>A</u> wins the prize and $-\sum b_i$ if not. Thus B avoids the loss of V_B if and only if A does not win; the loss is avoided if no one wins.

Once again the perfect equilibrium of the game is characterised by partitioning the space of pairs of distances from the finishing line. The sequence $\{C_n\}$ relating to A ('the challenger') is defined as before (but with $\rho_A = 1$).

DEFINITION I: $C_o = 0$. For $n \geq 1$ C_n is the maximum X such that there exists a sequence $a_1, a_2, \dots a_s$, of non-negative bids such that

$$\sum_{i=1}^{s} w_A(a_i) \geq X$$

$$w_A(a_1) \geq X - C_{n-1}$$

$$V_A \geq \sum_{i=1}^{s} a_i$$

More intuitively, if $\Omega_A(X)$ is defined as the value to A of a free run from X, then, if $C_{n+1} > C_n$, we have

$$(4) \qquad C_n - C_{n-1} = w_A(\Omega_A(C_{n-1}))$$

The RHS of this expression is the distance that A would travel if he bid the full value to him of a free run from C_{n-1}: see further p. 132 above.

The critical sequence $\{E_n\}$ for player B ('the incumbent') is defined very simply:

DEFINITION II: $E_n = nw_B(V_B)$

Note that $E_n - E_{n-1} = w_B(V_B)$, whereas $C_n - C_{n-1} < w_A(V_A)$ for $n > 1$. This is because $C_n - C_{n-1}$ also reflects the cost of getting from C_{n-1} to the finishing line. If the firms are identical except for the incumbent/challenger asymmetry, i.e. $V_A = V_B$ and $w_A = w_B$, then $E_n > C_n$ for $n > 1$. (Moreover, the ratio E_n/C_n increases with n.) This shows the advantage to B that arises purely from the incumbent/challenger asymmetry.

Using the sequences $\{C_n\}$ and $\{E_n\}$ we can characterise the outcome of the asymmetrical race in a fashion similar to that used to analyse the standard race in the previous section. See Figure 2.

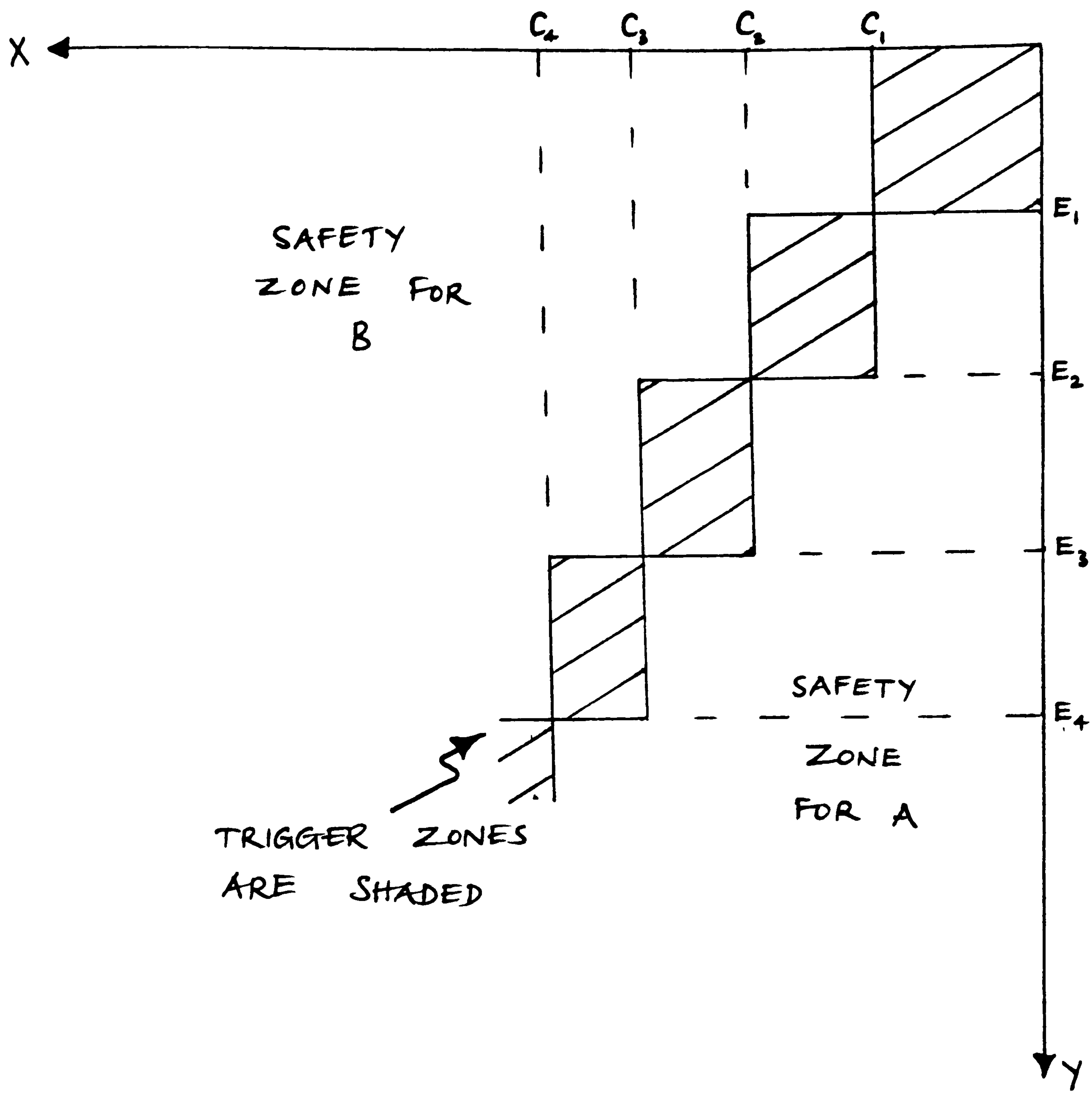
We again use the terminology of safety zones and trigger zones.

PROPOSITION 2: Suppose that the players' strategies are in equilibrium.

(i) If (X, Y) lies in player A's safety zone then A wins the prize. His bids are those he would make in the absence of rivalry from B. B always bids zero.

(ii) If (X, Y) lies in B's safety zone both players bid zero and no one wins the prize.

Figure 2



The Partition of the Space of Distances from the Finishing Line in the Asymmetrical Race

(iii) If (X, Y) lies in a trigger zone and it is A's move then A wins the prize. His bids are those he would make if, in isolation, he were subject only to reaching his safety zone with his first bid. B always bids zero

(iv) If (X, Y) lies in a trigger zone and it is B's move then no one wins the prize. B moves immediately to the boundary of his safety zone and both players bid zero thereafter.

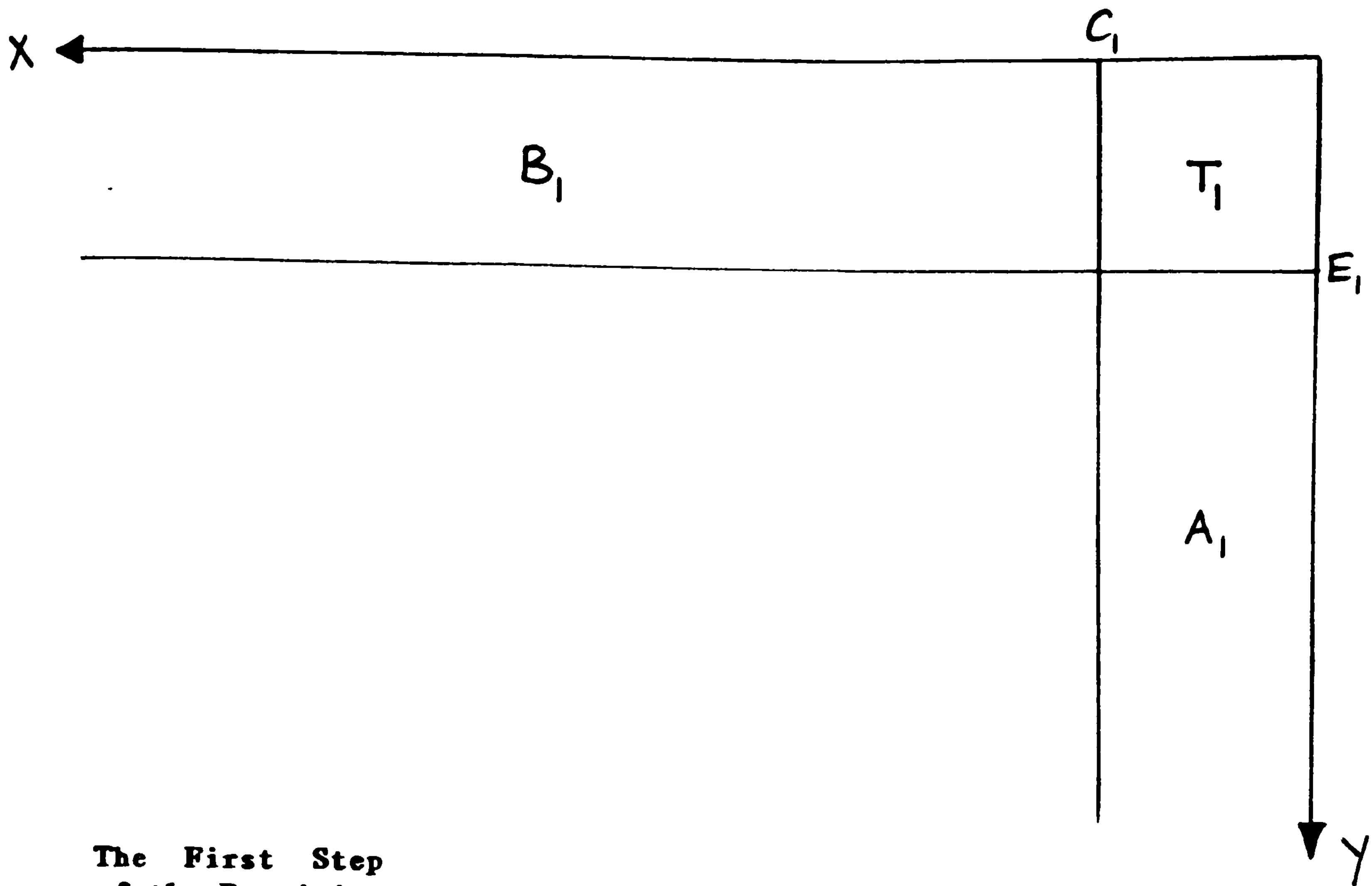
PROOF: The proof is by induction. It is similar to the proof of Proposition 1 in Section 1 of this Chapter above, but is rather more delicate, due to the asymmetry between players.

Figure 3 shows the first trigger zone, T_1, and the first parts of A's and B's safety zones, denoted by A_1 and B_1.

If it is A's turn to bid in $T_1 = \{X,Y | 0 < X \leq C_1 \text{ and } 0 < Y \leq E_1\}$ then A wins the prize. Otherwise his payoff is at best zero, whereas he obtains a positive payoff by moving directly to the finishing line. Moreover, A moves immediately to the line. For otherwise it would become B's turn to move in T_1, in which case A would not win since B would do better to move to the line himself and prevent A's victory than to allow it. Finally, if it is B's turn to move in T_1 he moves immediately to the line. If he did not, it would become A's turn to move in T_1 in which case A would win: then B would get at most $-V_B$ which is worse than moving immediately to the line himself. Thus we have shown that whoever's turn it is to move in T_1 moves immediately to the line.

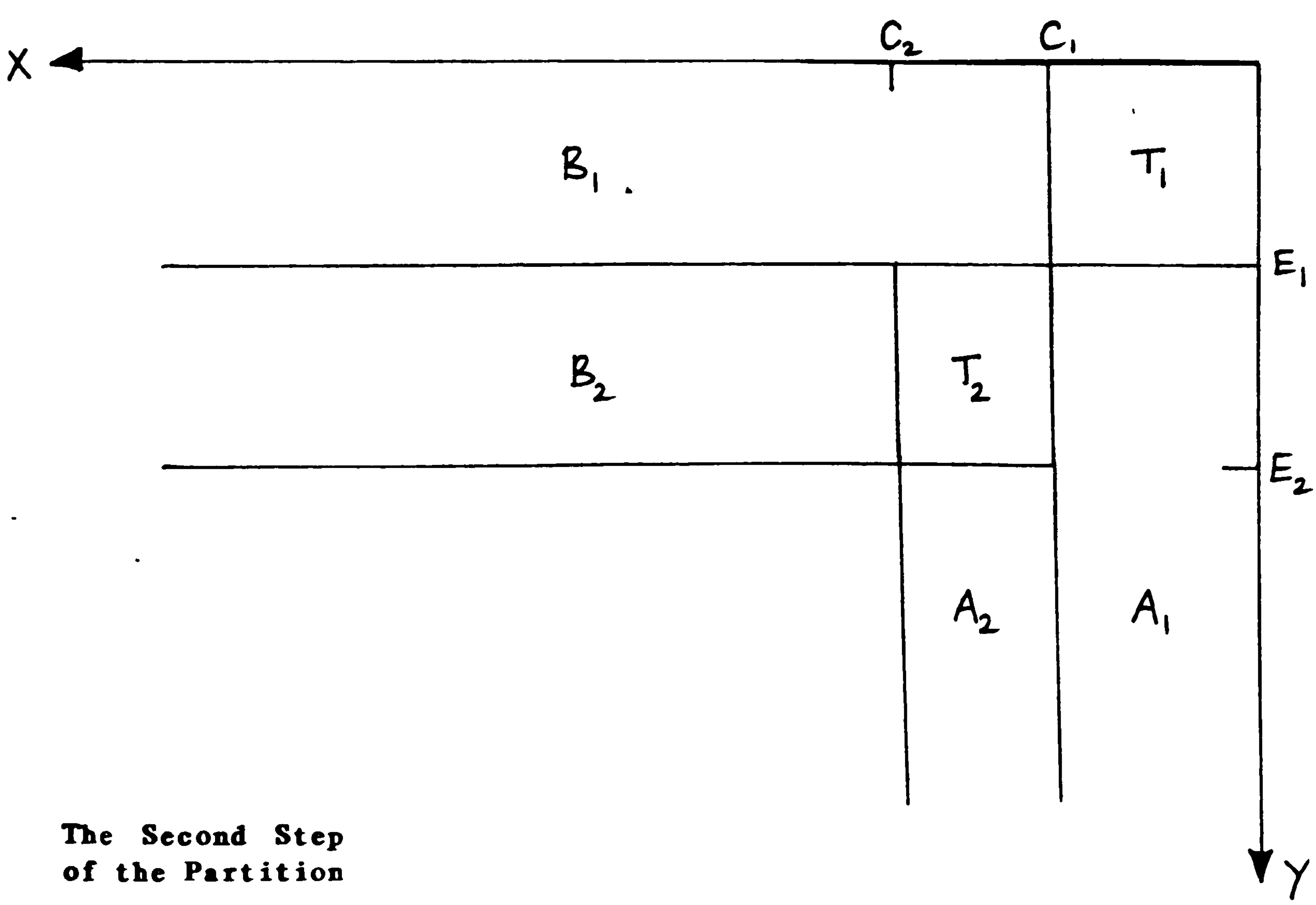
Next we show that A is safe in $A_1 = \{X,Y | 0 < X \leq C_1 \text{ and } Y > E_1\}$. It can never be optimal for B to finish in one move from any point in A_1. For to do so would cost him more than V_B, whereas $-V_B$ is the worst payoff that B can get if he does nothing. Nor can it be optimal for B to

Figure 3



The First Step
of the Partition

Figure 4



The Second Step
of the Partition

move into T_1 from A_1. For if he did so, A would finish immediately, and B's payoff would be $-V_B$ less the cost of his move. This is again worse than bidding zero always. Since B never moves out of A_1, A's behaviour is unaffected by B, and his bids are those he would make if he acted in isolation. Since B cannot affect A's behaviour he always bids zero.

Next we show that B is safe in $B_1 = \{X,Y|X < C_1 \text{ and } 0 < Y \leq E_1\}$. By the same argument as in the previous paragraph, it is not optimal for A to finish in one move from any point in B_1, or for him to move from B_1 into T_1. In either case his payoff would be negative, whereas complete inaction yields a payoff of zero. Since A never moves out of B_1, he never moves, and B faces no effective rivalry. B therefore bids zero, and achieves a payoff of zero, the best payoff he can possibly obtain.

Figure 4 shows the second trigger zone, T_2, and the second sections of A's and B's safety zones, A_2 and B_2.

If it is A's turn to bid in $T_2 = \{X,Y|C_1 < X \leq C_2 \text{ and } E_1 < y \leq E_2\}$ then A wins. If he did not win, his payoff would be at most zero, whereas he can obtain a positive payoff by moving directly to A_1. Moreover, A moves out of T_2 immediately. For if he did not, B would be motivated to preempt A by moving directly to B_1 where B is safe. By preempting A in this way, B's payoff would exceed $-V_B$, since he can reach B_1 at a cost less than V_B. This would be better for B than allowing A to win. Moreover, if it is B's turn to bid in T_2, he moves immediately to B_1. For otherwise A would win, by the previous argument. Thus we have shown that whoever is to move from T_2 moves immediately to his safety zone, A_1 or B_1 as the case may be.

Next we show that A is safe in $A_2 = \{X,Y|C_1 < x \leq C_2 \text{ and } Y > E_2\}$. It cannot be optimal for B to move from A_2 to B_1, since this would cost more than V_B. Nor can it be optimal for B to move from A_2 to T_2, since A would then win, which is worse for B than doing nothing. Therefore A faces no effective rivalry and proceeds as if in isolation. B always bids zero.

Finally we show that B is safe in $B_2 = \{X,Y|X > C_2 \text{ and } E_1 < Y \leq E_2\}$. It is not optimal for A to move from B_2 to A_1, since this would cost more than V_A. Nor is it optimal for A to move from B_2 to A_1, since B would then move to the safety to B_1. Therefore A bids zero, since he cannot win the prize. And B secures his maximum possible payoff by bidding zero himself.

These arguments can be iterated. Thus the proof is complete.

Q.E.D.

Several remarks are in order. In the incumbent's (i.e. B's) safety zone the 'race' is at a standstill. Even if the challenger (A) could profitably reach the finishing line if he were alone, he makes no attempt to do so since B credibly threatens to outdo any such attempt and beat A to the line. Of course B need never carry out this threat, and it is pointless for him to make any effort at all, since A is completely stymied. If the race begins with B to move from within a trigger zone, he makes an immediate dash to safety after which nò player makes any further effort. In A's safety zone, A proceeds at his own pace to the finishing line, because if B attempted to thwart A's victory then A would foil B's attempt by increasing his efforts.

Note that the location of the trigger and safety zones depends upon the players' valuations of the prize (i.e. the V's) and upon their efficiency at R and D (ie. the w functions). Not surprisingly, C_n increases the more that A wants to win the prize and the more efficient he is at R and D. Likewise E_n increases the more that B wants to avoid A winning, and the more efficient he is.

What is striking is the size of the incumbent's safety zone. Increments in the $\{C_n\}$ sequence (i.e. $C_n - C_{n-1}$) decrease and ultimately vanish, and the $\{C_n\}$ sequence is bounded above. However none of these things is true of the $\{E_n\}$ sequence, the increments of which are constant.

It follows from this that A might be deterred from challenging the incumbent even though the incumbent was further from the line, less efficient at R and D, and less concerned to stop A's victory than A was to achieve it. This result stands in contrast to the theme of much of the literature on preemptive patenting and the persistence of monopoly, namely that the incumbent's incentive to win the patent is greater than his rival's.

The intuition behind our result is that the incumbent does not need to go all the way to the finishing line to prevent the rival from winning the prize. That objective can be secured with both players some distance from the line. Thus the incumbent does not have far to go to reach safety. The effect of this is that the incumbent's safety zone is relatively large. This point is illustrated by an example in the following section.

An Example

The purpose of the example in this section is to illustrate the difference between the equilibria of the standard and asymmetrical races. It has already been stated, on p. 161 above, that if the firms are identical except for the incumbent/challenger asymmetry then $E_n > C_n$ for $n > 1$, and the ratio E_n/C_n increases with n. Therefore we expect B's safety zone to be in some sense larger in the asymmetrical race than in the standard race. However, we would like to see more generally, and diagrammatically, the nature of his asymmetry.

To do so, we calculate the limit of the equilibria of the models for a particular example. The motivation for this, and the justification of the techniques involved, have already been explained in Section I on pp. 139ff above. The main motivation is that simple explicit solutions can be obtained for the limiting case, whereas this is not so in the discrete case. Nevertheless the equilibrium in the limit can act as a convenient shorthand representation of the equilibrium in the discrete case.

We shall assume that w_J has the form

$$(5) \qquad w_J(a) = \text{Max}\ \{0,\ a^{\eta} - \varepsilon\}$$

for J =A,B, where $\varepsilon > 0$ and $0 < \eta < 1$. Figure 5 illustrates.

This function satisfies condition (i) on p. 141 above, since $\delta w_J(1/\delta) = \text{Max}\ \{0,\ \delta^{1-\eta} - \varepsilon\}$, which becomes zero as $\delta \to 0$. Condition (ii) on p.141 above is also satisfied if we set $h(\delta) = \delta^{\eta-1}$. Then

$$(6) \qquad \bar{w}_J(a) = \lim_{\delta \to 0} \text{Max}\ \{0,\ a^{\eta} - \varepsilon\delta^{\eta}\} = a^{\eta}$$

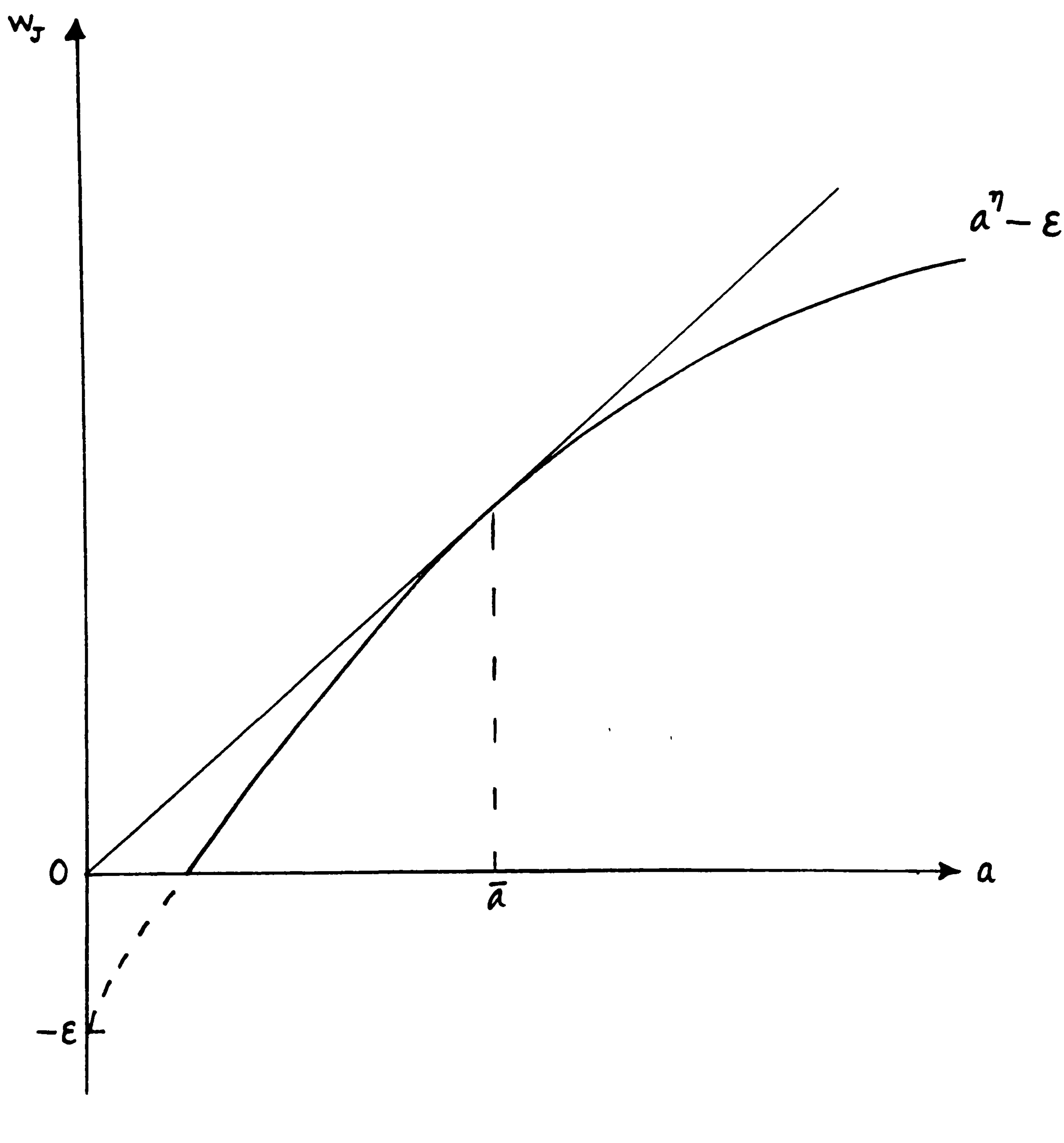
In the <u>standard race</u>, if (X,Y) lies on the limiting curve, then the limiting curve satisfies the differential equation

$$(7) \qquad \frac{dY}{dX} = \frac{\bar{w}_B\ (\Omega_B^o\ (Y))}{\bar{w}_A\ (\Omega_A^o\ (X))} = \left[\frac{\Omega_B^o\ (Y)}{\Omega_A^o\ (X)}\right]^{\eta}$$

See equation (7) of Section I of this Chapter.

Next let us find $\Omega_A^o(X)$. Since there is no discounting, a player would always proceed to the finishing line at the cost minimising speed,

Figure 5



The Progress Function in the Example

i.e. at rate $\bar{a}$ in Fig. 5. For w(a)/a to be maximised, we require that w(a)/a = w'(a). This condition is

$$(8) \qquad a^{\eta-1} - \frac{\varepsilon}{a} = \eta \, a^{\eta-1} \quad \text{, or}$$

$$(9) \qquad \bar{a} = [\varepsilon / (1-\eta)]^{1/\eta}$$

The second-order condition is guaranteed by the concavity of w. The value to A of a free run from X is therefore

$$\Omega_A^o(X) = V_A - \frac{X\bar{a}}{w_A(\bar{a})}$$

$$= V_A - \frac{X}{\eta}\left(\frac{\varepsilon}{1-\eta}\right)^{1/\eta - 1}$$

$$(10) \qquad = V_A - \gamma X \quad \text{, say.}$$[1]

The same applies for player B. Therefore, from (1) it follows that the limiting curve is characterised by

$$(11) \qquad \frac{dY}{dX} = \left(\frac{V_B - \gamma Y}{V_A - \gamma X}\right)^{\eta} = \left(\frac{\beta - Y}{\alpha - X}\right)^{\eta}$$

where $\alpha = V_A/\gamma$ and $\beta = V_B/\gamma$. The boundary condition is X = Y = 0. To solve (11), note that

---

1. Strictly speaking $\Omega_A(X) = \text{Max}\ \{0, V_A - \gamma X\}$, since if $V_A < \gamma x$, A would choose not to make any effort, thereby obtaining a zero payoff.

$$(12) \qquad \int \frac{dY}{(\beta - Y)^{\eta}} = -\eta \int u^{-1/\eta}\, du = \frac{u^{(\eta-1)/\eta}}{1-\eta} = \frac{(\beta - Y)^{1-\eta}}{(1-\eta)},$$

where $u = (\beta - Y)^{\eta}$, hence $du/dY = -\eta u^{(\eta-1)/\eta}$. From (12) it follows that

$$(13) \qquad (\beta - Y)^{1-\eta} = (\alpha - X)^{1-\eta} + \text{const}.$$

The boundary condition implies that const $= \beta^{\eta-1} - \alpha^{\eta-1}$. Therefore the equation of the limiting curve is

$$(14) \qquad Y = \beta - [(\alpha - X)^{1-\eta} - \alpha^{1-\eta} + \beta^{1-\eta}]^{\frac{1}{1-\eta}}$$

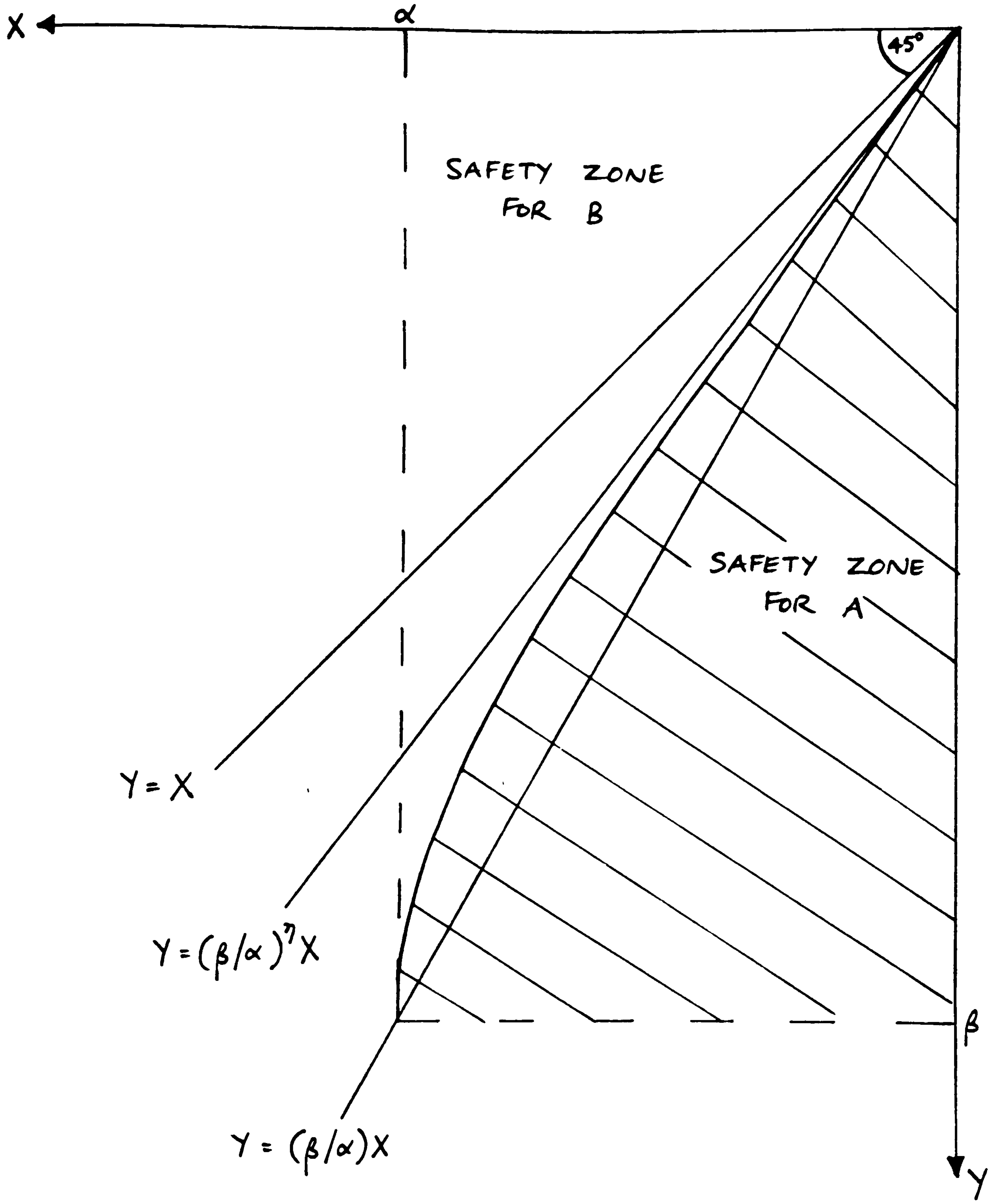
Assuming for instance that $\beta > \alpha$, the slope dY/dX increases with X and approaches infinity as $X \to \alpha$. At the origin slope is $(\beta/\alpha)^{\eta}$. The curve thus lies entirely to one side of the 45° line reflecting the assumption that $\beta > \alpha$. Figure 6 sketches the curve.

We now turn to the <u>asymmetrical race</u>. Here the equation of the limiting curve is given by

$$(15) \qquad \frac{dY}{dX} = \left(\frac{V_B}{V_A - \gamma X}\right)^{\eta} = \left(\frac{\beta}{\alpha - X}\right)^{\eta}$$

In the asymmetrical race, V_B is in effect the value to B of a free run from Y for all Y. That is, if the rival does not exist, then B, the

Figure 6



The Limiting Curve in the Standard Race

incumbent avoids the loss of V_B. Having regard to equations (12) and (13) we see that the solution to (15) is

$$(16) \qquad \frac{(\alpha - X)^{1-\eta}}{1 - \eta} = \frac{Y}{\beta^{\eta}} + \text{const}$$

with the boundary condition X = Y = 0. Therefore the equation is

$$(17) \qquad Y = \frac{\beta}{1 - \eta} \left[\alpha^{1-\eta} - (\alpha - X)^{1-\eta}\right]$$

The slope of this curve is $(\beta/\alpha)^{\eta}$ at the origin, is increasing in X, and goes to infinity as $X \to \alpha$. The curve is sketched in Figure 7.

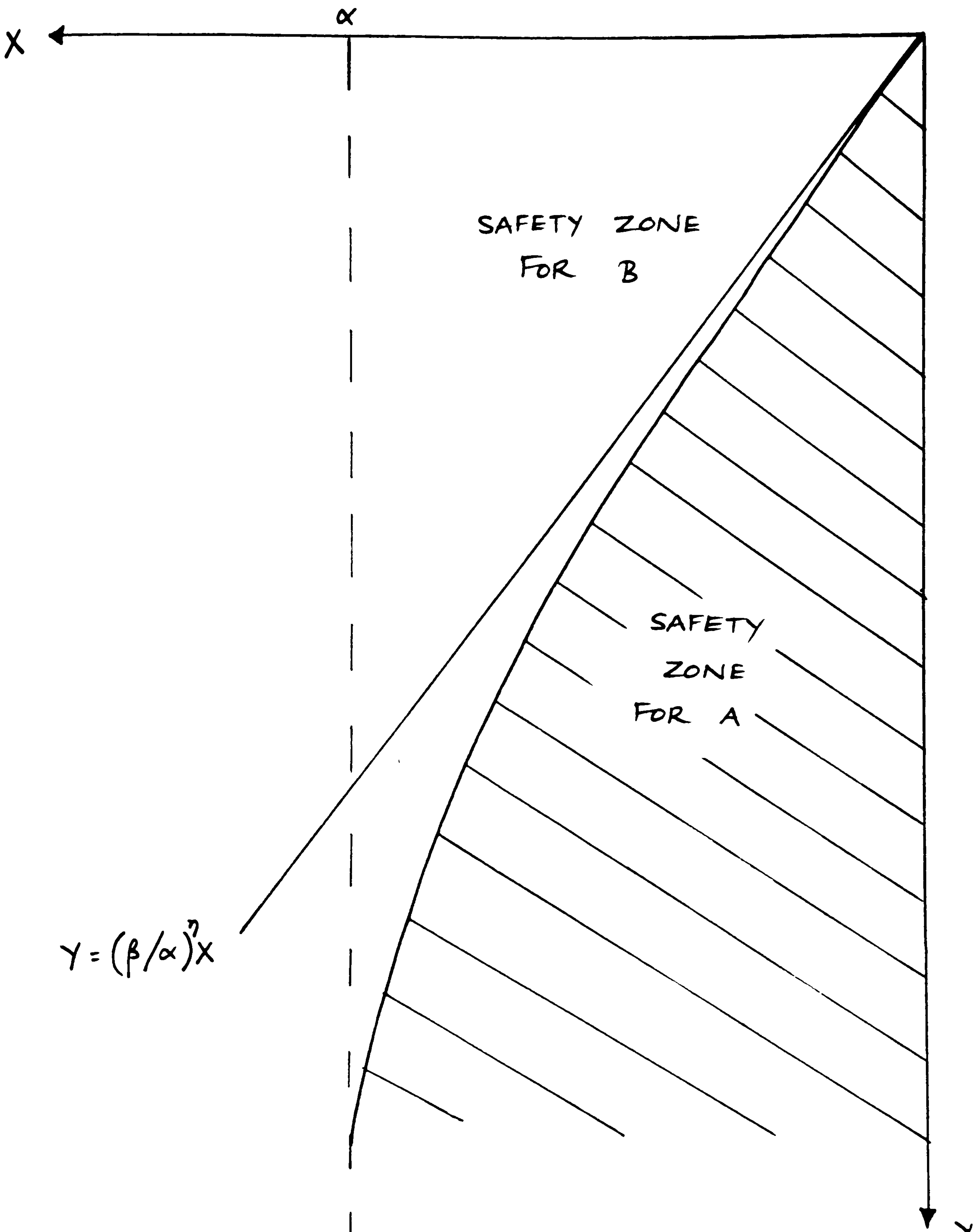
The slope dY/dX for the asymmetrical race is everywhere steeper than it is for the standard race. The value of Y at which $X = \alpha$ is therefore greater, and B's safety zone is correspondingly larger. In Figure 8 both curves are plotted for the case in which $V_A = V_B$ (i.e. $\alpha = \beta$).

This diagram displays the extra advantage enjoyed by B that derives from his desire being to stop A winning, rather than being to win himself.

The Hybrid Case

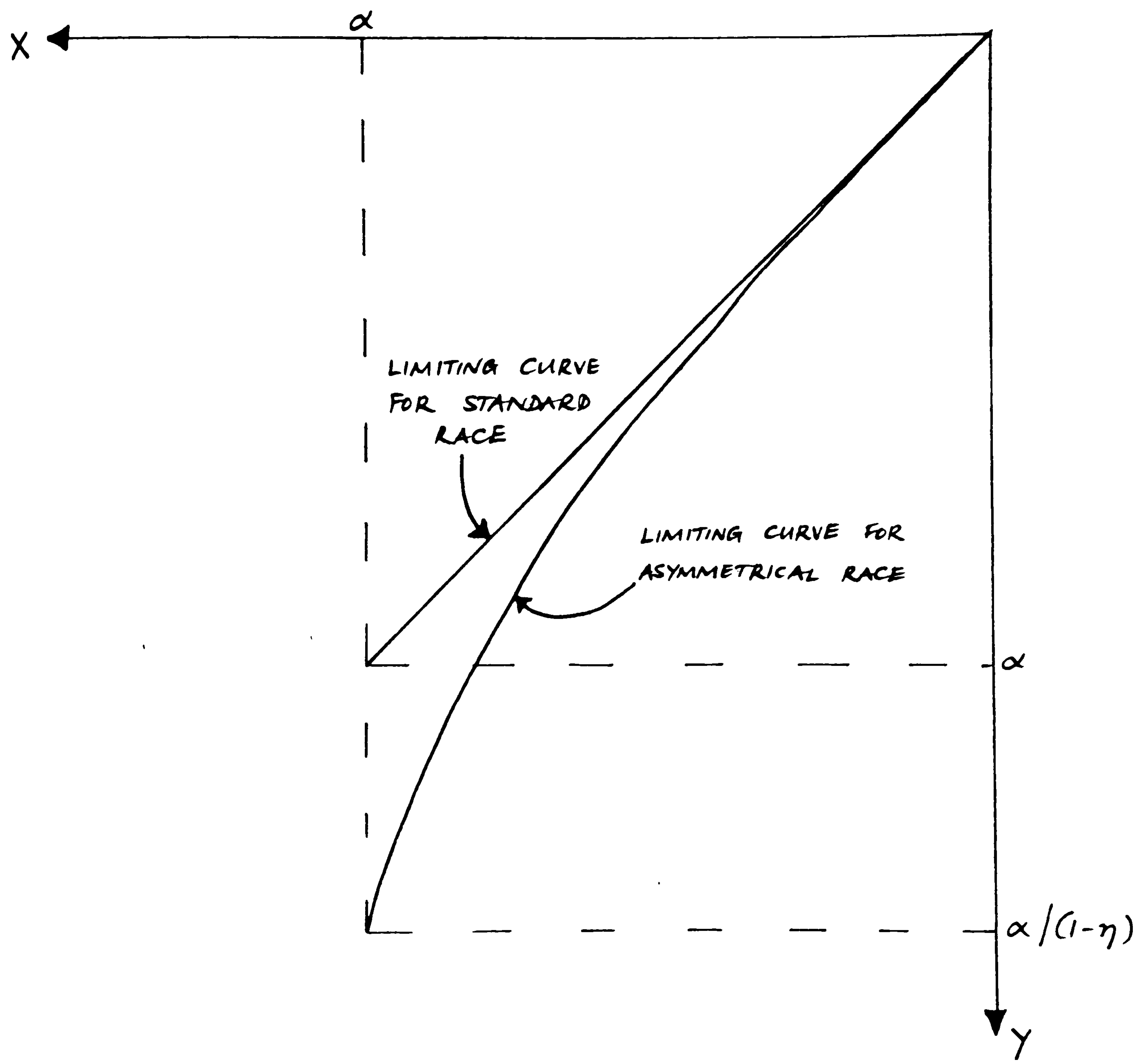
So far in this section we have concentrated on the 'pure' case of an asymmetrical race, in which the incumbent's only concern is to prevent his rival from winning the patent. This is appropriate if, for example, the patent is for a technology no better than that already possessed by the incumbent. If, however, the new technology is for a superior patent, then the incumbent has an incentive to win the patent in addition to his desire that his rival should be denied the patent.

Figure 7



The Limiting Curve in the Asymmetrical Race

Figure 8



Comparison between the Two Limiting Curves

The presence of this extra incentive in this 'hybrid' case strengthens our conclusions regarding the case of a 'pure' asymmetrical race.

It should be clear from the foregoing analysis how a more detailed treatment of the hybrid case would proceed; what follows is merely an informal sketch. The basic structure model is just as before: see pp. 158ff above. The payoffs are defined as follows. A receives $V_A - \sum a_i$ if he wins the patent, and $-\sum a_i$ if he does not. B receives:

$V_z - \sum b_i$ if he wins the patent;

$-\sum b_i$ if no one wins the patent; and

$-V_B - \sum b_i$ if A wins the patent

The case in which $V_B = 0$ is a standard race, and if $V_z = 0$ we have an asymmetrical race. The only real difference between the hybrid and asymmetrical races is that in the former B gains something if he actually wins the prize as well as avoiding a loss if his rival wins.

Again, perfect equilibrium in the race is found by defining sequences of critical distances for A and B. For A the sequence $\{C_n\}$ is defined exactly as before. For B let us characterise a sequence $\{F_n\}$ informally as follows:

(18) $F_o = 0$. For $n \geq 1$, $F_n = F_{n-1} + w_B(V_B + \Omega_B(F_{n-1}))$

where $\Omega_B(Y)$ is the value to B of a free run from Y in a standard race where the prize is worth V_z. This informal characterisation is in the style of equation (4) rather than the rigorous Definition I on p. 160.

Using the sequences $\{C_n\}$ and $\{F_n\}$, the space of pairs of distances from the finishing line (i.e. R^2_+) is partitioned as before into safety zones for A and B, and trigger zones. However there is one novel feature: see Figure 9.

B's safety zone has two portions, labelled BI and BII in Fig.9. These portions are divided by the line $Y = \bar{Y}$, where $Y = \text{Min}\ \{Y \mid \Omega_B(Y) = 0\}$. In area BI player B will move to the finishing line to collect the prize. In area BII he will not do so, but will remain still (of course A is nevertheless deterred from actively competing for the prize, because he knows that B would be motivated to react so as to thwart any such attempt). In an asymmetrical race BII is empty.

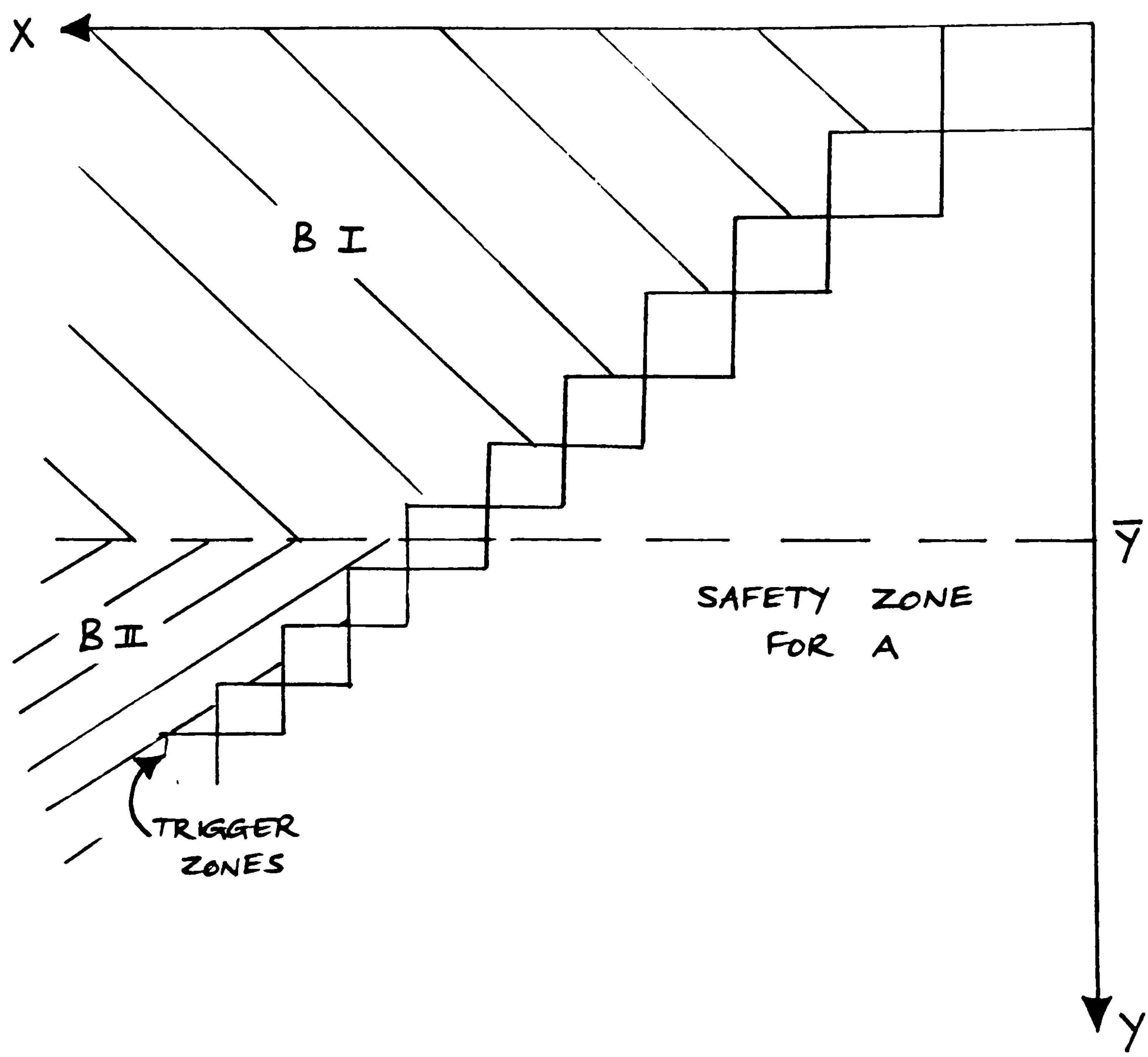
There are further possibilities that could be explored, for example a 'double hybrid' model in which A, as well as B, has payoffs of the form described for B above. It turns out that this is rather more difficult to analyse, and no further discussion is attempted here. As far as the incumbent/potential entrant asymmetry is concerned, the main cases of interest have already been explored anyhow. The payoff structure for a potential entrant will take the simple form because his only interest is in winning and thereby gaining entry into the market.

<u>Positive Discounting in the Asymmetrical Race</u>

As stated at the beginning of this Section, it would have been desirable to take the model of a race from Section 1 as it stands to analyse the asymmetrical race. That model included positive rates of discount, whereas in this section we have assumed no discounting. Below it is briefly explained why this was done. It is wrong to regard the assumption of zero discounting as a fatal drawback of the model asymmetrical race above: it facilitated straightforward analysis (at the expense of some loss of generality) and it was possible to study fairly clearly the features of interest.

With discounting, the payoffs would have been as follows:

Figure 9



The Partition of the Spcae of Distances in the Hybrid Race if A wins the patent with his k^{th} bid, then

A receives $\rho_A^{k-1} V_A - \sum \rho_A^{i-1} a_i$, and

B receives $-\rho_B^{k-1} V_B - \sum \rho_B^{i-1} b_i$;

if A never wins, A gets $-\sum \rho_A^{i-1} a_i$ and B gets $-\sum \rho_B^{i-1} b_i$

With these payoffs, it clearly matters to B <u>when</u> A wins (or would win). If A's victory is far away in time, and/or if B discounts the future heavily, then B will not be so strongly motivated to thwart A's victory. Similarly for A, if the prospect of victory is far away, then he will be less willing to compete for the prize, so making it more likely that B does not suffer the loss associated with A's victory.

In this setting, the critical sequence of distances appropriate to A remains $\{C_n\}$ as defined on p. 132 in Section 1. But the sequence appropriate to B is more complicated (Note how simple is the definition of the $\{E_n\}$ sequence on p. 161 above when there is no discounting). For the analysis to make any progress, it appears necessary to make an extra assumption. Let m(X) be the optimal number of moves for A to take to reach the finishing line when he is not subject to any constraint, i.e. when he has a free run. Assume this function to be well-defined and increasing in X. (These assumptions can be justified in terms of certain prior assumptions.) Let $m_n = m(C_n)$. Let the sequence $\{G_n\}$ be characterised informally by:

$$G_o = 0. \text{ For } n \geq 1, \; G_n = G_{n-1} + w_B(V_B \, \rho_B^{m_{n-1}}),$$

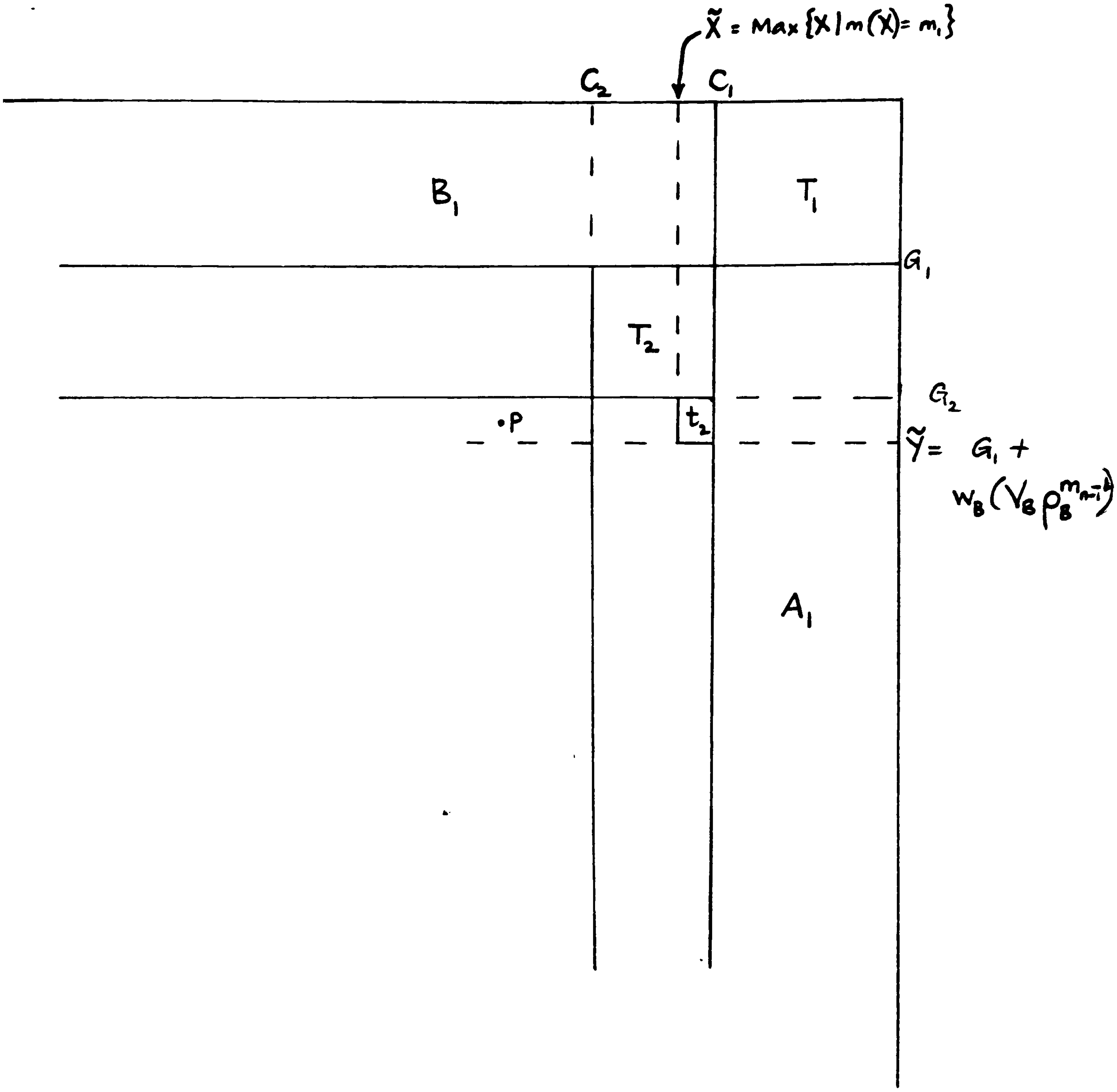
where $m_o = 0$ by convention.

Now consider Fig. 10, below. It is easy to show that T_1 is a trigger zone by repetition of earlier arguments (briefly: if A is to move he must get the patent; if B is to move it cannot be that A gets the patent; therefore whoever moves first goes immediately to the finishing line). Correspondingly, A_1 and B_1 are safety zones for A and B. Moreover T_2 is a trigger zone (briefly: suppose that B has first move but that A eventually wins the prize. Suppose that A leaves T_2 k moves before he reaches the finishing line. Then $k \leq m_1 + 1$, or else A would not be acting optimaly once in A_1. Therefore B's payoff, evaluated at the time of A's departure from T_2 is at most $-V_B \rho_B^{m_1}$. But B can improve on this by moving straight to B_1. Therefore, in equilibrium, if B is to move from T_2, it cannot be that A wins. On the other hand, if A is to move from T_2 it must be that he wins, since he can afford to move immediately to the safety of A_1. Therefore, T_2 is a trigger zone).

Now the complexities begin. Consider area t_2 in Fig 10. From any point in this area, if A proceeded optimally to the finishing line he would take precisely m_1 moves (from T_2 we stated that he would take <u>at most</u> $m_1 + 1$ moves). B would pay up to $V_B \rho_B^{m_1 -1}$ to stop this (i.e. to get to the safety of B_1), and therefore t_2 is also a trigger zone. The analysis now becomes messy. For example, if A is to proceed to the finishing line from a point such as P in Fig. 10, he must avoid area t_2, or else B will jump to B_1.

Complications of this sort appear to place a full analysis beyond reach. It is nevertheless possible to show <u>some</u> properties of perfect equilibrium (not included here); and there are several more or less plausible conjectures about the solution, which we have not yet been able to prove. In view of these difficulties, the most sensible approach is, I believe, to rest content with the (tractible) model without discounting, since it does show the main features that we were out to capture. The complexities that arise from the analysis of the model with

Figure 10



The Complexities when Discounting is Introduced discounting are interesting, even if they are not capable of resolution. For example, they begin to show the subtle ways in which the timing of the rival's possible victory affects the calculations of the incumbent concerned to stop that event.

Imperfect and Incomplete Information

The following remarks apply just as much to the model in Section 1 of this Chapter as to the present Section. The races so far analysed have been modelled as games of complete and perfect information. This has enabled us to use Selten's (1965) concept of subgame perfection to analyse our model. The argument for studying these models in their stark form is that we were concerned to capture features of races that other models were not designed to study (see the remarks at the beginning of this Chapter).

What are the prospects for incorporating uncertainty into the model? Different sorts of uncertainty must be distinguished.

First, the game would cease to be one of complete and perfect information if moves were simultaneous rather than alternating. This has already been discussed in the context of the paper by Fudenberg et al. (1983): see pp. 149ff above.

Secondly, there are several other types of imperfect information. For example, the progress function w_J might be stochastic (but this possibility has not been studied even in the context of models with a considerably simpler structure). A more interesting case of imperfect information is that of uncertain discovery: the players do not know their distances from the finishing line. More formally, think of (X_o, Y_o) as being distributed on R^2_+ according to distribution function F. This covers a wide range of possibilities, of which perhaps the most interesting is the case of perfectly correlated uncertainty (in which the players know the distance between themselves, but not the distance to the finishing line). In this case, and for all continuous F, perfect equilibrium exists: this follows from Theorem 1 of Harris (1983) and the fact that the game with uncertain discovery has a certainty equivalent. However, we have so far been unable to characterise equilibrium.

Thirdly, there is incomplete information, i.e. the players are not fully informed about each other's characteristics. This is a common feature of races. It would for example be interesting to study the case in which B does not know A's valuation of the prize V_A. However, there is as yet no definitive solution concept for the games with incomplete information. The most popular solution concept is sequential equilibrium (Kreps and Wilson (1982)). One application of sequential equilibrium is by Rubinstein (1983) to his (1982) bargaining model. Although his model has a structure that is extremely simple compared with our model of a race, Rubinstein's analysis is very complex and shows that there is a chronic multiplicity of sequential equilibria (despite there being a unique perfect equilibrium in the case of complete information). We therefore doubt whether incomplete information can usefully be analysed in our model of a race, given the present state of knowledge about games with incomplete information. Rather, we believe that the appropriate strategy is first to study incomplete information in games that have much simpler structures.

To summarise, the state of the art in modelling games with incomplete information is not sufficiently advanced to make it possible to incorporate incomplete information into our model. It would be desirable to study the model with simultaneous moves. It would be especially interesting to introduce uncertain discovery. However, it is not at present apparent how to proceed in these directions given the rather complex structure of the model and considering that the solution concept of subgame perfection is applicable only in games of complete and perfect information. However, in the case of uncertain discovery, we do know that under certain conditions perfect equilibrium exists; but this is far short of a characterisation.

This being so, it is perhaps wiser to study models with uncertainty that have a simpler structure than the model of a race so far analysed. This is attempted in the following two sections.

Summary and Conclusion

In a patent race between a firm already in a market and a firm attempting to enter it, there are several reasons why the incumbent and potential entrant might be in an asymmetrical position. First, the two firms might value the patent differently. In particular, it is probable that the incumbent values the patent more highly than his rival, since monopoly persists if he wins the patent whereas there is competition if the rival is successful. Secondly, the firms might begin the race at different distances from the finishing line. For example, due to his past experience. the incumbent might have more knowledge than his rival at the outset of the race. Thirdly, the firms might differ in terms of R and D efficiency - that is, in terms of their ability to transform R and D effort into R and D output.

As well as these factors, there is another important asymmetry, which this Section has sought to explore. It is that the incumbent's aim is to prevent his rival from winning the patent, and not necessarily to win the patent himself. The rival, on the other hand, is out to win the patent. The analysis of our model reveals that this last asymmetry can be rather important. Indeed, it can happen that the rival is deterred from actively competing in the patent race even though he has the advantage over the incumbent in terms of all the three asymmetries described in the previous paragraph.

This section has concentrated mainly on the 'pure' case, in which the incumbent's only concern is to prevent his rival from winning the patent. This is appropriate if, for example, the patent is for a technology no better than that already possessed by the incumbent. If, however, the new technology is for a superior patent, then the incumbent has an incentive to win the patent in addition to his desire to stop the rival winning. The presence of this extra incentive in the 'hybrid' case strengthens our conclusions regarding the pure case.

In summary, there are several sources of strategic advantage that an incumbent firm might enjoy in a patent race against a potential entrant into his market. An important one of these is that the incumbent is concerned to prevent the rival from winning, and not necessarily to obtain the patent for himself. This asymmetry appears to favour the incumbent considerably - especially if the players begin far from the finishing line. It is suggested as a possible ingredient for a theory of the persistence monopoly.

3. A FIRST MODEL OF A MULTISTAGE PATENT RACE

Perhaps the main criticism of the models described in the previous two sections is their exclusion of uncertainty. However, it has already been argued (see pp. 122ff) that R and D models with uncertainty - such as those described in Chapter I - are ill-equipped to capture important 'race-like' features such as the strategic interaction between competitors as the race unfolds. In particular, we would like to employ models with uncertainty to examine the claims that the race tends to degenerate once one competitor is far enough ahead of his rival, and that competition is keen if and only if the players are 'close'. The models in this section and the next are directed to that end.

The are both models of 'multistage patent races'. In the competition modelled in this section, the winner is the first player to get a certain number of stages ahead of his rival. As shorthand we call this 'a model of a tug-of-war'. At any time there is therefore just one relevant state variable in the model - the gap between the players. This makes the model fully tractable in a neat fashion. In the competition modelled in the next section, the winner is the first player to complete a given number of stages of research.[1] This is in the spirit of Roberts and Weitzman (1981). This model is in terms of the economics more satisfactory than the model of a tug-of-war, but, unfortunately, less convenient to analyse.

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1. Fudenberg et al. (1983, Section 3) present a simple model of a multistage patent race with two stages. Their particular aim is to show that a player might compete at the first stage even though he is behind his rival, since there is a chance that he would leapfrog the rival. At the second stage, R and D activity is fixed intensity. The model described in the present section is more general, has a broader purpose (see below) and has variable intensity of R and D, since a major objective is to see how effort rates vary as the race unfolds.

In both models there is technological uncertainty - similar to that in the models reviewed in Chapter I. The probability that a player completes a stage in the next small interval of time depends upon his current rate of R and D effort.

A Model of A Tug of War

The Basic Model: There are two players, A and B, in competition for a prize worth V. The winner is the first player to get N stages ahead of his rival. At any time the race can be in any one of 2N-1 'states': A can be n stages ahead of B for $n = -N+1, \ldots, 0, \ldots, N-1$ (when $n < 0$ it would perhaps be more natural to say that A is $-n$ stages behind B, but we shall say equivalently that A is n stages ahead). There is no discounting.

If A researches at rate x, the probability that he completes a stage within t units of time is assumed to be $1 - e^{-xt}$. Therefore the hazard rate is x. Similarly, the hazard rate for B is his rate of research y. For convenience we assume also that the cost of researching at rate x is $\frac{1}{2}x^2$; similarly for B. As will shortly be seen, this assumption about the nature of the cost function allows a particularly simple solution of the model. Moreover, its convexity and zero derivative at the origin take care of second-order conditions.

A's strategy $\underline{x} = (x_{-N+1}, \ldots, x_{N-1})$ specifies his rate of research for each of the possible 2N-1 states of the race. Thus x_n is his rate of research when he is n stages ahead of B, at which point B's rate of research is y_{-n} (for he is n stages behind A).

We use simple dynamic programming techniques to solve the model. Let Ω_n be the value of being n stages ahead of one's rival in the race, i.e. the (expected) payoff in the subgame beginning with one n stages ahead of one's rival Ω_n is increasing. If A is currently n stages ahead of B. the competition between them can be regarded as a competition in which A gets a prize worth Ω_{n+1} if he is next to complete a stage, and Ω_{n-1} if B is next to complete a stage. Similarly, B gets Ω_{-n+1} if he is next to complete a stage, and Ω_{-n-1} if A does so. We can therefore write

$$\Omega_n = \underset{x_n}{\text{Max}} \left[\int_0^{\infty} (x_n \Omega_{n+1} + y_{-n} \Omega_{n-1} - \tfrac{1}{2} x_n^2) e^{-(x_n + y_{-n})t} dt \right]$$

$$(1) \qquad = \underset{x_n}{\text{Max}} \left[\frac{x_n \Omega_{n+1} + y_{-n} \Omega_{n-1} - \frac{1}{2} x_n^2}{x_n + y_{-n}} \right]$$

In short, we are considering the race when A is n stages ahead as a simple Lee-Wilde type of race: see pp. 18ff above for details.

The first derivative of the expression in square brackets on the RHS of (1) is:

$$(2) \qquad \frac{(x_n + y_{-n})(\Omega_{n+1} - x_n) - (x_n \Omega_{n+1} + y_{-n} \Omega_{n-1} - \frac{1}{2} x_n^2)}{(x_n + y_{-n})^2}$$

When zero, this implies the first-order condition:

$$(3) \qquad x_n = \Omega_{n+1} - \Omega_n$$

The second-order conditions are met because Ω_n has a positive derivative at the origin (see (2)) and is strictly concave in x_n . To see this, note from (2) that the second derivative of Ω_n with respect to x_n is the same as that of

$$-(x_n + y_{-n})^2 - 2\left[y_{-n}(\Omega_{n+1} - \Omega_{n-1}) - x_n y_{-n} - \tfrac{1}{2}x_n^2\right]$$

$$= \quad -y_{-n}^2 - 2y_{-n}\,(\Omega_{n+1} - \Omega_{n-1}) \quad ,$$

which is strictly negative because Ω_n is increasing.

Rewrite (1) as

$$(4) \qquad y_{-n}(\Omega_n - \Omega_{n-1}) = x_n(\Omega_{n+1} - \Omega_n) - \tfrac{1}{2}x_n^2$$

and use (3) to get

$$(5) \qquad y_{-n}\, x_{n-1} = \tfrac{1}{2}x_n^2$$

The reasoning for B is the same. Symmetry therefore implies $x_n = y_n$ for all n, whereupon (5) implies the basic equation

$$(6) \qquad x_{-n}\, x_{n-1} = \tfrac{1}{2}x_n^2$$

Rewrite (6) with 1-n in place of n to obtain

$$(7) \qquad x_{n-1}\, x_{-n} = \tfrac{1}{2}x_{1-n}^2$$

Together (6) and (7) imply

$$(8) \qquad x_n = x_{1-n}$$

Equation (8) with $-n$ in place of n, together with (6) implies

$$(9) \qquad \frac{x_{n+1}}{x_n} = \frac{1}{2} \frac{x_n}{x_{n-1}}$$

Equation (8) with n = 0 implies

$$(10) \qquad x_0 = x_1$$

Therefore, using (9):

$$\frac{x_2}{x_1} = \frac{1}{2} \frac{x_1}{x_0} = \frac{1}{2}$$

$$\frac{x_3}{x_2} = \frac{1}{2} \frac{x_2}{x_1} = \frac{1}{4}$$, and generally

$$(11) \qquad \frac{x_n}{x_{n-1}} = 2^{-n+1}$$

Using (9) we have, for n > 0:

$$\frac{x_n}{x_0} = \frac{x_n}{x_{n-1}} \times \frac{x_{n-1}}{x_{n-2}} \times \ldots \times \frac{x_1}{x_0}$$

$$= \quad 2^{-[(n-1)\ +\ (n-2)\ +\ \ldots\ +\ 1]}$$

(12) $$= \quad 2^{-\frac{1}{2}n(n-1)}$$

If n < 0, (8) and (12) imply

(13) $$\frac{x_n}{x_0} = \frac{x_{-n+1}}{x_0} = 2^{-\frac{1}{2}n(n-1)}$$

Therefore, for all n

(14) $$x_n = x_0 2^{-\frac{1}{2}n(n-1)}$$

Next note that (8) and (11) imply that

(15) $$\frac{x_n}{x_{-n}} = \frac{x_n}{x_{n+1}} \times \frac{x_{n+1}}{x_{-n}} = 2^n$$

Therefore if A is n stages ahead of B, the probability that A, rather than B, is next to complete a stage is $2^n/(1 + 2^n)$. For positive n this rapidly approaches unity.

It remains to find x_0 explicitly. By definition, $\Omega_N = V$ and $\Omega_{-N} = 0$. Therefore

$$V = \Omega_N - \Omega_{-N}$$

$$= (\Omega_N - \Omega_{N-1}) + \ldots + (\Omega_{-N+1} - \Omega_{-N})$$

(16) $$= x_{N-1} + \ldots + x_{-N+1} + \Omega_{-N+1}$$

Equation (1) with n = -N+1 implies

$$\Omega_{-N+1} = \tfrac{1}{2}x^2_{-N+1} \;/\; x_{N-1}$$

$$= \tfrac{1}{2}x_{-N+1}2^{-N+1}$$

(17) $$= 2^{-N}\, x_{-N+1} \qquad > 0$$

Equations (14), (16) and (17) therefore imply that

$$\frac{V}{x_0} = 2^{-\frac{1}{2}(N-1)(N-2)} + 2^{-\frac{1}{2}(N-2)(N-3)} + \ldots.$$

(18) $$\ldots. + 2^{-\frac{1}{2}(2-N)(1-N)} + 2^{-\frac{1}{2}(1-N)(-N)} + 2^{-\frac{1}{2}(-N)(-N-1)}$$

More explicitly

When N = 1

$$x_0 = V/(1 + \tfrac{1}{2}) = \tfrac{2}{3}V$$

When N = 2

$$x_0 = V/(1 + 1 + \tfrac{1}{2} + \tfrac{1}{8}) = \frac{8V}{21}$$

When N = 3

$$x_0 = V/(\tfrac{1}{2} + 1 + 1 + \tfrac{1}{2} + \tfrac{1}{8}) = \frac{8V}{25}$$

When $N \geq 5$

$$(19) \qquad x_0 \simeq K = V\Big/\sum_{n=-\infty}^{\infty} 2^{-\frac{1}{2}n(n-1)} \simeq 0.3046V \quad ,$$

Correct to at least four places of decimals: the series converges very quickly. Equation (14) now implies

$$(20) \qquad x_n \simeq 2^{-\frac{1}{2}n(n-1)} K$$

and we have completely characterised equilibrium in the game.

Let us recapitulate the main results to emerge from the analysis. Players' efforts are highest when they are level with one another. As the gap between them widens, both players exert less effort, although the leader's effort rate is many times greater than that of the follower. As the gap between the players becomes large, each player's effort rate goes to zero. Note that 'zero' is the rate at which a monopolist would research, since there is no discounting. (In fact there is a familiar closure problem: the monopolist would have to research at some $\varepsilon > 0$ to complete stages, but (say) $\frac{1}{2}\varepsilon$ would give him a higher payoff than ε.)

Finally, consider a sequence of competitions like the above, in which N varies. As N becomes larger, there is a sense in which the race degenerates to near monopoly more quickly. Suppose that the prize is awarded to the first player to get a certain distance ahead of his rival. N becoming large corresponds to finer division of that distance. Thus for a given lead measured in distance, corresponding effort rates go to zero as distance is more finely divided.

4. A SECOND MODEL OF A MULTISTAGE PATENT RACE

In the previous section we described a model of a 'tug-of-war' in which a prize is won by the first player to get N stages ahead of his rival. Because there was a single state variable - the gap between the players - the model was easily soluble. However, from the economic point of view the model is not wholly satisfactory, because there are few competitions that can be represented as a tug-of-war.

In this section we present a natural development of the model. The winner is the first player to complete a given number of stages of research. Thus there are two state variables, one for each player, representing the number of stages that each player has still to complete. Otherwise the model is much the same as before. It is a second attempt to combine technological uncertainty with strategic interaction in an R and D competition.

The Model: There are two players A and B in competition for a prize worth V. The winner is the first player to complete N stages of research. At any time the competition can be in any of N^2 states. A state of the competition is a pair (m, n), where m (resp. n) is the number of stages that A (resp. B) still has to complete. A's strategy specifies his rate of R and D x(m,n) for each possible state. Likewise, B's rate of R and D when he has n stages to go and A has m stages to go is denoted y(n, m). The flow cost of researching at rate x is $\frac{1}{2}x^2$. As in the previous model, technological uncertainty is assumed to be of exponential form, so that A's hazard rate when he researches at rate x is x. Similarly for B. Firms maximise expected undiscounted profits minus R and D expenses.

Define the value function $\Omega(m,n)$ as A's expected payoff in the subgame beginning at state (m,n). According to previous argument (see p. 189) we have

$$\text{(1)} \qquad \Omega(m,n) = \underset{x(m,n)}{\text{Max}} \left[\frac{x(m,n)\Omega(m-1,n) + y(n,m)\Omega(m,n-1) - \frac{1}{2}x(m,n)^2}{x(m,n) + y(n,m)} \right]$$

The numerator is the instantaneous probability of transition to state (m-1,n) multiplied by the value of the subgame starting at that state, plus the same thing for state (m, n-1), minus R and D expenses. The boundary conditions are:

$$\text{(2)} \qquad \Omega(0,n) = V \qquad \text{and} \qquad \Omega(m,0) = 0$$

for all m,n. These are, respectively, the values of winning and losing. The first-order condition for a maximum of (1) is

$$\text{(3)} \qquad x(m,n) = \Omega(m-1,n) - \Omega(m,n)$$

The second-order conditions are met, as in the previous section, by virtue of the cost function's concavity and zero derivative at the origin. A and B are symmetrically placed. Therefore $x(m,n) = y(m,n)$. Hence (1) and (3) imply the relationship:

$$\text{(4)} \qquad \tfrac{1}{2}x(m,n)^2 = x(n,m)[\Omega(m,n) - \Omega(m, n-1)]$$

It is easy to characterise $\Omega(m,n)$. Equations (2) and (3) imply that

$$(5) \qquad \Omega(m,n) = V - \sum_{i=1}^{m} x(i,n)$$

Combining (4) and (5) we obtain the basic relationship

$$(6) \qquad \tfrac{1}{2}x(m,n)^2 = \begin{cases} x(1,m)\left[V - \sum_{i=1}^{m} x(i,1)\right] & \text{for } n = 1 \\ x(n,m)\left[\sum_{i=1}^{m} x(i,n-1) - x(i,n)\right] & \text{for } n \geq 2 \end{cases}$$

In principle, (6) defines x(m,n) for all m and n. Further analysis becomes difficult, but it is illuminating to compute x(m,n) for small values of m and n.

The computation involves solving a particular form of cubic equation. To see the general form of this equation, let a denote x (m,n) and let b denote x(n,m). Define f as follows:

$$(7) \qquad f = \begin{cases} V - \sum_{i=1}^{m-1} x(i,1) & \text{for } n = 1 \\ \sum_{i=1}^{m} x(i,n-1) - \sum_{i=1}^{m-1} x(i,n) & \text{for } n \geq 2 \end{cases}$$

Then (6) can be written as

$$(8) \qquad \tfrac{1}{2}a^2 = fb - ab$$

Now define g in the same manner as f, but with m in place of n and with n in place of m. Then, from (6) with m and n reversed, we have

$$(9) \qquad \tfrac{1}{2}b^2 = ga - ab$$

Combining (8) and (9) we have

$$(10) \qquad b = \frac{a^2}{2(f - a)} = -a + \sqrt{a^2 + 2ag}$$

Now define $Q = f/a - 1$ and $R = g/f$. Then dividing (10) through by a we have

$$(11) \qquad \frac{1}{2Q} = -1 + \sqrt{1 + 2R(1 + Q)}$$

Adding 1 to both sides of (11) and squaring, we obtain

$$(12) \qquad \frac{1}{4Q^2} + \frac{1}{Q} = 2R(1 + Q) \quad ,$$

or

$$(13) \qquad 4 = 8RQ^2 + \frac{3}{Q+1} \quad .$$

Our approach is to use (13) as the basis for computing approximate solutions to x(m,n) for small values of m and n.

<u>x(1,1)</u>

Here f = g = V, and therefore R = 1. From (13), Q = f/a - 1 = $\frac{1}{2}$. Therefore a, which here stands for x(1,1), equals 2V/3.

<u>x(2,1)</u>

Here f = V - x(1,1) = V/3 and g = 2V/3. Therefore R = 2. From (13), Q = 0.331. Therefore x(2,1) = 0.250V.

<u>x(1,2)</u>

From (10), this equals x(2,1)/2Q, where Q is as for x(2,1). Therefore x(1,2) = 0.378V.

<u>x(2,2)</u>

Here f = g = x(1,1) + x(2,1) - x(1,2) = 0.539V. R = 1, so Q = $\frac{1}{2}$. Hence x(2,2) = 0.359V.

<u>x(3,1)</u>

Here f = V - x(1,1) - x(2,1) = 0.083V. And g = x(1,2) = 0.378V. Therefore R = 4.554. From (13), Q = 0.203. Therefore x(3,1) = 0.069V.

<u>x(1,3)</u>

From (10), this equals x(3,1)/2Q = 0.170V.

<u>x(3,2)</u>

Here $f = x(1,1) + x(2,1) + x(3,1) - x(1,2) - x(2,2) = 0.249V$. And $g = x(1,2) + x(2,2) - x(1,3) = 0.558V$. Therefore $R = 2.241$. From (13), $Q = 0.308$. Therefore $x(3,2) = 0.190V$.

<u>x(2,3)</u>

From (10), this equals $x(3,2)/2Q = 0.308V$.

<u>x(3,3)</u>

Here $f = g = x(1,2) + x(2,2) + x(3,2) - x(1,3) - x(2,3) = 0.449V$. Since $R = 1$, $Q = \frac{1}{2}$. Hence $x(3,3) = 0.299V$.

In principle these calculations could carry on ad infinitum. However, since our purpose is only to illustrate the nature of the solution to the game, a halt will be called here. The Table below summarises the results obtained.

Table: x(m,n) = V times the nunmber in the grid

m \ n	1	2	3
1	0.667	0.378	0.170
2	0.250	0.359	0.308
3	0.069	0.190	0.299

The Table states the solution to the 3-stage patent race. This of course contains the solution to the 2-stage patent race.

Again it emerges that competition is keenest when the players are close to each other, and especially when they are close to discovery of the patent. The leader undertakes more R and D than the follower, and is strong favourite to win. For example when A has one step to complete and B has two setps to complete, the probability that A is next to complete a stage is 378/(250 + 378) = 0.602. If, however, B is next to complete a stage, then each has an even chance. Therefore the overall probability that A wins is about 80 per cent. Note that it is not necessarily true that x(m,n) is decreasing in m, i.e. that a player's equilibrium effort rate is lower the further he is from discovery. For example, x(2,3) > x(1,3). In this case the player's proximity to each other spurs greater effort by the leader than proximity to the finishing line.

The model in this section employs a special functional form (in fact it is the one used in Reinganum (1982)). However, this does enable the solution to the model to be computed explicitly, which is perhaps illuminating. With more general functional forms it is not clear how to proceed after writing down the first-order conditions. Although it is therefore not possible to claim generality for the results, there is no reason to suppose that they are unique to a narrow class of functional forms. The way to regard the model in this section is as a tentative step towards an integrated analysis of dynamic strategic interaction in patent races with uncertainty. It is simply an illustration of what might be done in that direction.

CHAPTER IV

R AND D COMPETITION WITH A SEQUENCE OF INNOVATIONS

INTRODUCTION

Until this point we have been concerned with single innovations. The same is true of most of the theoretical literature on R and D. This Chapter, however, is devoted to considering the relationship between market structure (and its evolution) and R and D competitions when a number of innovative opportunities arise over time. Previous Chapters have explained how market structure influences the incentives at work in patent races,[1] and how the outcome of a patent race affects market structure in turn. When a series of innovative opportunities is in prospect, these reciprocal effects become more complicated. For the outcome of one patent race affects the likely course of the next, and market structure evolves correspondingly. If firms are sophisticated, and perceive these factors, then their calculations with respect to any one patent race will take account not only of the immediate effect of the race, but also of the second, third and fourth round effects upon subsequent patent races.

In the 'Summary and Prospectus' to their book <u>Market Structure and Innovation</u>, Kamien and Schwartz briefly sketch a framework in which to discuss this issue:

> 'The main question to be addressed in this framework is how market structure evolves through time. Does an industry initially composed of firms of comparable size and strength remain that way through time or does it evolve into an industry with only a few, or only one, dominant firm and many small rivals? And, most importantly, what happens to the pace of innovation in this industry through time?' (1982, p.219)

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1. 'Patent race' is used as a shorthand for R and D competition generally.

As to the prospects for resolving these issues, they warn that

> 'Formulation and analysis of a comprehensive model of interaction between market structure and the process of innovation poses formidable technical difficulties, involving methods and concepts (for example, the theory of repeated games), which are themselves in the process of development and refinement' (p.220).

Nevertheless, some progress has been made. The first section of this Chapter reviews the contributions of Futia (1980), Flaherty (1980), Dasgupta and Stiglitz (1981), Reinganum (1984c), and Nelson and Winter (e.g. 1977, 1978, 1982a, 1982b). Of these, Reinganum's work is closest to the approach attempted later in this Chapter, and is therefore evaluated in some detail.

The remainder of the Chapter investigates one aspect of the question of industry evolution when several innovations are in prospect - the evolution of firms' relative sizes. In particular, are firms that currently have low costs more or less likely than high cost firms to succeed in making the next innovation? If this is so - perhaps due to the low cost firms having a greater incentive to win - then there will be a tendency towards more concentrated market structure. Indeed, the prospect of this market power might be precisely why the low cost firms have the greater incentive. On the other hand, the reverse might be true. For example, higher cost firms might be more likely to make the next innovation than their currently more efficient rivals because of a greater need to do so. In this case there will tend to be a process of 'action-and-reaction' in which victory in patent races switches from one firm to another over time, in a process of leapfrogging in which technological superiority is continually changing hands.

A duopoly model is employed to study this problem. The use of a two-firm example permits a helpful simplification, since the question becomes: who is likely to make the next innovation, the low- or the high-cost firm? Moreover, it is natural to use a two-firm case to look at the evolution of the relative sizes of firms. Our question is not about the evolution of the number of firms over time, but about the disparity between them: does one firm become increasingly dominant, or is there action-and-reaction? Thus we are singling one out of the several dimensions of 'market structure'.

In the model, time consists of a given number of periods. At the beginning of each period there is a competition for a patent for a technology better than any previously available. The winner of this competition discards his old inferior technology in favour of the new one; the loser keeps his existing technology. The competition for the patent is modelled as a simple bidding game, following e.g. Gilbert and Newbery (1982): see Section 1 of Chapter II above. During the period itself, the duopolists operate in the product market, their respective profits in the period depending upon the pair of levels of technology that they have. At the beginning of the next period there is a competition for a yet superior technology, and so on.

The main feature to emerge from the analysis is how the nature of evolution of market structure depends upon the nature of competition between the duopolists in the product market. For example, if duopoly equilibrium is Bertrand (roughly speaking), then the low cost firm wins all R and D competitions and becomes increasingly dominant. On the other hand, there are conditions - likely to be met if duopoly equilibrium is Cournot and if each innovation is not too revolutionary - which imply that the high cost firm will always be the one to win the next patent, which implies action-and-reaction, and therefore a tendency towards the equality of firm size. The model is special in nature, and it would be interesting to pursue more generally the ideas suggested by it.[1] Of particular interest is the idea that keen competition in the product market gives rise to a tendency to increasing disparity of firm size, whereas more accommodating behaviour is associated with action-and-reaction.

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1. In further work I have investigated other models of innovation and industry evolution, which have been specified differently from the model below. Some progress has been made with those models, but not enough to warrant their inclusion in this thesis.

1. EXISTING WORK ON INNOVATION AND INDUSTRY EVOLUTION

This section describes four strands of work on innovation and industry evolution, namely Futia (1980), Flaherty (1980), Reinganum (1984c), and Nelson and Winter (e.g. 1977, 1978, 1982a, 1982b). Most attention is given to Reinganum.

Futia's Model: Futia's paper,[1] 'Schumpeterian Competition' concerns the long-term stochastic equilibrium of an industry. A short-run position of the industry is described by a pair (n, h), where n is the number of firms at that date, and h is the state-of-the-art technique of production available to all n firms. The profits of each firm at that date depend upon n and h. At the beginning of each period there is a patent race for the next technique of production, h + 1, which is superior to h. The patent for h + 1 may or may not be won - if not, then there is another competition for it the following period. If firm i wins the patent, he is the only 'sure survivor' into the following period. If some firm other than i wins the patent, then i is not a sure survivor. If no firm wins the patent then all existing firms surely survive. The number of firms that actually exists in the next period depends (in an unexplained way) stochastically upon the number of sure survivors, and upon a parameter v that represents 'ease of entry'. The probability that firm i wins the patent race depends upon his R and D effort in the race. Sufficient assumptions are made to imply that firm i has a dominant strategy in the patent race. His R and D effort and the probability that he wins therefore depend only on n (and v). The result is that there exists a function p(n,m; v) which gives the probability that there will be m firms next period given that there are n this period. This defines a Markov process with transition probabilities given by p. There

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1. See Kamien and Schwartz (1982, p.220) and Reinganum (1984c, p.23) for some brief comments on Futia's model.

exists a unique, stable, long-run stochastic equilibrium. Futia shows how the average number of firms and average speed of research vary with v, the ease-of-entry parameter. He obtains the 'Schumpeterian' result that the speed of research varies inversely with the number of firms (that is to say, n increases, whereas the speed of research declines, as entry becomes easier).

Futia's paper is a most elegant study of the equilibrium of stochastic processes. But it is not principally about R and D competition. Rather, the story about patent races should be seen as a step towards the derivation of the function which defines the Markov process. R and D competition is not a central topic of analysis, as is indicated by the fact that assumptions are made which ensure that there are dominant strategies in the patent race - thus strategic effects are denied (one of these assumptions is that firms are concerned only with next period's profits, whereas in the analysis to follow it is central that firms make far-sighted calculations). Moreover, in Futia's model, existing firms are always symmetrically placed. Thus there is no way to study asymmetries between them, which are again a major focus in that which is to follow.

To summarise, Futia's is the first and one of the most important formal analyses of innovation and industry evolution. However, it is chiefly a study of long-run stochastic equilibrium, and not of R and D competition. The dimension of market structure that it examines is the number of firms, not the disparity between firms.

Flaherty's Model: Flaherty's (1980) paper presents a discrete time model of <u>cost-reducing</u> R and D. It is not about a sequence of patent races. The interesting result is obtained that, although firms are inherently identical, there are stable steady-state equilibria that are asymmetric. A criticism of the model is that only open loop equilibrium is found, which is certainly different from perfect equilibrium which is in this context a much more attractive solution concept.

The Model of of Dasgupta and Stiglitz: Dasgupta and Stiglitz (1981) analyse the steady state of an industry which sustains an indefinite sequence of innovations. Demand in the industry is growing at a given rate. The rate of technical progress and the rate of change of innovation costs are also assumed to be exogenous. A firm is characterised by its vintage, i.e. the date of its entry, which determines its cost level. Each firm can innovate at most once. Firms exit when they cease to make operating profits. It is shown how the exogenous parameters determine the number of active firms and the time between successive innovations in the steady-state equilibrium.

The main criticism of the model, which Dasgupta (1982, p. 4, fn. 6) himself makes, is the ad hoc nature of certain assumptions, notably the exogeneity of technical progress and the inability of a firm to innovate more than once. This latter assumption is especially far removed from the models to follow in this chapter, because in those models we see what happens when firms face a series of innovative opportunities.

Reinganum's Model: Reinganum (1984c) considers a market in which at any time there is an incumbent monopolist; the other firms are challengers. The current incumbent remains the incumbent until one of the challengers overthrows him by making an innovation. There is a sequence of innovations, so if a firm successfully innovates, it enjoys temporary monopoly power until it is displaced by a rival's innovation. This is in the spirit of Schumpeter's (1942) 'process of creative destruction'.

More formally the basic model is as follows.[1] There is a sequence of innovations with associated profit flows $\{\pi_t\}_{t=0}^{T}$, where π_t is the profit flow from the current innovation when there are t innovations remaining. Consider the race at stage t. (The subscript t will be dropped because there is no risk of ambiguity.) At stage t, n firms are competing to make the next innovation. The winner will become the incumbent until the next innovation occurs. The current incumbent is one of the n firms competing to make the next innovation. The only respect in which he differs from the challengers is that he is currently receiving profit flow π. Let Ω_W and Ω_L be the terminal values for the winner and a loser of the current race. That is, Ω_W is the expected present value of the winner in the continuation game beginning immediately after his victory. The common discount rate is r. Let x_i be the R and D effort of firm i in the race. The flow cost of this effort rate is $c(x_i)$ and the lump-sum fixed cost is $f \geq 0$, where c has the properties described on p. 13 above. (Equivalently one could define a hazard function in terms of R and D investment, as Reinganum (1984b) does. Cf Reinganum (1981, 1982) to whom we are closer) Define $z_i = \sum_{j \neq i} x_j$. Without loss of generality let the current incumbent be firm 1. Technological uncertainty is assumed to have the usual exponential form, with no correlation of the uncertainties facing rival firms.

Thus the expected payoff to firm 1 is

$$(1) \qquad \Omega_1 = \frac{x_1\Omega_W + z_1\Omega_L + \pi - c(x_1)}{x_1 + z_1 + r} - f$$

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1. The notation is different from Reinganum's. I have firms choosing their rates of R and D output rather than R and D input. This makes no essential difference, but is more convenient mathematically.

and the expected payoff to firm $i \neq 1$ is

$$(2) \qquad \Omega_i = \frac{x_i\Omega_W + z_i\Omega_L - c(x_i)}{x_i + z_i + r} - f$$

It is shown in Reinganum's Proposition 1 that there exists a Nash equilibrium in the current R and D competition, in which each firm has a unique interior best response. The existence proof depends not only upon the reasonable assumptions stated in Reinganum's text, but also upon an assumption that is relegated to footnote 4, namely that $\pi < r\Omega_W$. This guarantees that equilibrium is interior. The assumption is about the nature of the solution of the model - it is in no sense a basic assumption. Moreover the assumption is important for the economics because it is responsible for Reinganum's only truly dynamic result: see below. The other main results for the basic model are:

PROPOSITION 2: The incumbent invests less than each challenger in the current stage. Therefore the probability that the incumbent wins the next patent is less than 1/n.

PROPOSITION 3: The value of being the incumbent is greater the smaller the number of remaining innovations.

PROPOSITION 4: Each firm would prefer to be the incument in the current stage than a challenger.

PROPOSITION 5: The effort rate of challengers is increasing in r, Ω_W and n, and decreasing in π and Ω_L.

PROPOSITION 6: Ditto for the effort rate of the incumbent.

Propositions 5 and 6 rely upon a stability condition.[1] All except Proposition 3 are essentially static results. They would hold true in a single race in which the winner receives Ω_w and the loser recieves Ω_L (note the similarity with the payoffs in Reinganum (1981 and 1982)). Hence Proposition 3 is the only really dynamic result. Its proof is simply that _if_ the incumbent's maximisation problem has an interior solution, then it is given by

$$(3) \qquad \Omega_1 = \Omega_w - c'(x_1) - f$$

Since $f \geq 0$ and $c'(x_1) > 0$ if $x_1 > 0$ (because c is differentiable and strictly increasing) it follows immediately that $\Omega_w > \Omega_1$, which is Proposition 3. This relies heavily on the technical assumption in Reinganum's footnote 4. It is therefore perhaps not surprising that Reinganum finds the intuition behind the result 'somewhat murky'.

The following variant of the model may clarify this point. Consider Reinganum's model in 'stationary form'. Instead of a finite sequence of innovations, let there be an infinite sequence, such that the current incumbent at any time receives profit flow π. To simplify the illustration let $f = 0$, and let $c(x) = \frac{1}{2}x^2$. Let Ω_I and Ω_c be the values of being an incumbent and a challenger respectively. Then (1) and (2) become

$$(1') \qquad \Omega_I = \frac{x_I\Omega_I + z_I\Omega_c + \pi - \frac{1}{2}x_I^2}{x_I + z_I + r}$$

1. Reinganum (p.19) warns that care is needed in interpreting these comparative statics results because they apply only to the current stage. Indeed, the example immediately below shows how some of the results are reversed in a fully dynamic setting.

(2') $$\Omega_c = \frac{x_c\Omega_I + z_c\Omega_c - \frac{1}{2}x_c^2}{x_c + z_c + r}$$

The first-order conditions are

(4) $$\Omega_I = \Omega_I - x_I \Rightarrow x_I = 0, \text{ and}$$

(5) $$\Omega_c = \Omega_I - x_c$$

When there are (n - 1) challengers, symmetry implies

(6) $$r\Omega_c = \frac{1}{2}x_c^2$$

(7) $$\pi - r\Omega_I = (n-1)x_c^2$$

Add (6) and (7), and use (5) to obtain

(8) $$(n-\frac{1}{2})x_c^2 + rx_c - \pi = 0$$

Therefore

(9) $$x_c = \frac{-r + \sqrt{r^2 + 2\pi(2n-1)}}{(2n-1)}$$

The comparative static results here are not the same as those obtained by Reinganum when the incumbent's optimisation problem had an interior solution. We have

$$(10) \qquad \frac{\partial x_c}{\partial \pi} > 0$$

$$(11) \qquad (2n - 1)\frac{\partial x_c}{\partial r} = -1 + \frac{r}{\sqrt{r^2 + 2\pi(2n-1)}} < 0$$

$$(12) \qquad (n - \tfrac{1}{2})\frac{\partial x_c}{\partial n} = \frac{\pi}{\sqrt{\cdot}} - x_c$$

$$= \frac{r\sqrt{\cdot} - \{r^2 + (2n-1)\pi\}}{(2n-1)\sqrt{\cdot}}$$

Squaring both terms in the numerator of this expression it is simple to show that $\partial x_c/\partial n < 0$.

All these results are the opposite of Reinganum's. The reasons are twofold. First, the incumbent was active at the equilibrium of Reinganum's model, due to the assumption guaranteeing interiority. Therefore x_I varied with π, r, etc., and x_c varied with x_I via the best-reply function. Secondly, for Reinganum, π is the profit flow associated with the current patent, whereas for us it is the profit flow associated with each innovation. The results that we have obtained in a dynamic model with an infinite horizon perhaps have more intuitive appeal than Reinganum's comparative statics for π and r. The explanations for (10) and (11) are simple: the higher the profit flow of an incumbent the more effort a challenger will make to become an incumbent. With a higher discount rate, the present expected value of making the innovation is lower and so less effort will be made to win it. The explanation for (12) is that when there are more firms, the value of being incumbent is reduced due to the greater number of challengers. Therefore each challenger is less keen to become incumbent.

The infinite horizon version that we have just examined is appealing. It enables time essentially to be ignored in the analysis -

unlike in finite horizon models. A firm can be in one of two states: incumbent or challenger, i.e. 'insider' or 'outsider'. The model is properly dynamic, however. In particular, the incentive for a challenger to become an incumbent reflects the probable speed with which he would later lose his incumbency as a result of a rival making a subsequent innovation.

We have evaluated critically Reinganum's (1984c) model of innovation and industry evolution, which is in the same spirit as the model to follow inasmuch as it has fully optimising, forward looking strategic behaviour, and it focusses on R and D competition more than, say, Futia (1980). Her model was criticised because a technical assumption (of unclear economic merit) was responsible for several economic results, including the major dynamic result. Reinganum's model is unlike that which follows in that the product market is always monopolised - a firm is either 'in' or 'out'. In the model to follow, one firm can have varying degrees of advantage or disadvantage relative to another.

The Work of Nelson and Winter

A remarkable attempt to study the relationship between innovation and industry evolution is the work of Nelson and Winter (see, inter alia, Nelson and Winter (1977, 1978, 1982a, 1982b). For evaluations of this work see Kamien and Schwartz (1982, pp.221-2), Stoneman (1983, pp. 183ff and 225) and Elster (1983, ch. 6)). They employ techniques of numerical simulation to see how patterns of concentration change over time depending upon technological opportunity, ease of imitation, firms' aggressiveness and so on. Their work is a contribution to the behavioural theory of the firm - firms are not optimising agents, nor are they motivated by strategic considerations. Although directed at similar issues, the work of Nelson and Winter therefore represents an approach very different from all the other work discussed here.

2. A MODEL OF A SEQUENCE OF PATENT RACES

It was seen in Chapter II above that asymmetries between firms, for example their valuations of the patent at stake, are important influences upon patent races. In Section 4 of Chapter II we asked whether, in a race between two duopolists for a patent for a new, superior technology, the currently high- or low-cost firm would have the greater incentive to win the patent. It was seen that the answer to this question depended partly upon the nature of interaction between the firms in the product market. In the example with linear demand and costs, it was shown that the higher cost firm would have the stronger incentive if there were Cournot equilibrium in the product market and if cost disparities were not too large. But with price-setting firms, the stronger incentive was always with the lower cost firm.

We now wish to pose similar questions in an explicitly dynamic framework in which there is a sequence of patent races. If the higher cost firm wins the current patent race he will then become the lower cost firm. The outcome of the current patent race will affect the next one, the one after that, and so on. In particular we wish to characterise the conditions under which

(a) the currently high cost firm is favourite to win the next race ('action-and-reaction'); or

(b) the currently low cost firm is favourite to win the next race ('increasing dominance').

The implications for the evolution of market structure over time are clear.

In designing a model to study this topic, at least the following must be specified:

(i) the timing of patent races (i.e. when do they occur?);

(ii) the form of patent races (e.g. do they incoporate uncertainty or not?)

(iii) the type of patent: does the winner become the technological leader in the industry, or, if he were originally inferior to his rival in terms of technology, does the patent merely enable him to catch up somewhat?

(iv) the way in which firms' profit flows depend upon the patents they currently have.

In the model, it is assumed that there is a patent race at the beginning of each period. Following e.g. Gilbert and Newbery (1982), patent races are regarded as simple bidding games: see Chapter II. And a firm's current profit flow is assumed to depend upon the levels of technology represented by the two firms' most recent patents.

Before proceeding with the model, it must be emphasised that whichever firm wins the next patent is assumed to become technologically superior to his rival, i.e. it 'leapfrogs'.

The Basic Model: There are two players A and B. Initially they have cost levels c_A and c_B. The game has T periods $t = T, T-1, \ldots, 2, 1$, which it is convenient to number in reverse order. Thus in period t there are t periods remaining. There is a sequence of cost levels $\{c_t\}_{t=T}^{1}$, one for each period. Define $c_{T+1} = c_A$ and $c_{T+2} = c_B$ for convenience. It is assumed that

$$c_1 < c_2 < \ldots < c_T < c_{T+1} < c_{T+2}$$

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1. More generally, c_t can be regarded as a parameter of the cost function.

At the beginning of the period t there is a race for the patent that gives the right to employ cost level c_t. Before the race for patent t one of the players has cost level t + 1, and the other player has cost level t + k, where k > 1. Let H denote the player who is currently high-cost (i.e. the one with cost level t + k) and let L denote the currently low cost player.

$\pi(s,t)$ is the profit (ignoring R and D expenses) of a firm in a period when it has cost level c_s and its rival has cost level c_t. It is assumed, naturally, that $\pi(s, t)$ is decreasing in s and increasing in t. That is, profits increase as one's own technology improves, and decrease as that of one's rival improves. It is assumed also that $\pi(s,t) \geq 0$ for all s and t. Define $\sigma(s, t) = \pi(s,t) + \pi(t,s)$ as the firms' joint profits in a period when the pair of cost levels is s and t.

It is assumed that a patent race is like a simple auction in which the patent is won by the highest bidder. The winner pays the maximum that the other player would have been prepared to pay in order to have the patent rather than not to have it. The player who fails to win the auction does not forgo his bid. Firms maximise the sum of their profits minus bid costs (i.e. R and D expenses) over time. Later we shall consider what happens when firms discount the future.

Assume that the game has a unique solution. (It will later emerge that this is indeed the case.) Let $\Omega(s,t)$ be the net payoff to a player in the subgame immediately after a race in which he finished cost level c_s and his rival with c_t. Thus Ω is like a value function.

Now consider the race for patent t. H's incentive to win patent t is

$$(1) \qquad h = \Omega(t, t+1) - \Omega(t+k, t)$$

The first term on the RHS is H's payoff in the remainder of the game after he has won race t. The second term is H's payoff in the remainder of the game if L wins race t. Similarly, L's incentive to win patent t is

$$(2) \qquad \ell = \Omega(t,\ t+k) - \Omega(t+1,\ t)$$

By our assumption that patent races are like bidding games, H wins the patent if and only if $h \geq \ell$ (assuming for convenience that he wins if $h = \ell$). It is convenient to define $\Delta(s,t) = \Omega(s,t) + \Omega(t,s)$ as the sum of the value functions. Thus we have

$$(3) \qquad h \geq \ell \quad \text{iff} \quad \Delta(t,t+1) \geq \Delta(t,\ t+k)$$

If $h \geq \ell$, then H's winning bid equals ℓ. Therefore his payoff in the subgame beginning immediately after the previous patent race (i.e. the race for patent t+1) is

$$(4) \qquad \pi(t+k,\ t+1) + \Omega(t,\ t+1) - \ell$$
$$= \pi(t+k,\ t+1) + \Delta(t,\ t+1) - \Omega(t,\ t+k)$$

If however H does not win patent t (i.e. if $h < \ell$), then his payoff in the subgame beginning immediately after the race for patent t + 1 is simply

$$(5) \qquad \pi(t+k,\ t+1) + \Omega(t+k,\ t)$$

Having developed this apparatus, the analysis could now proceed in various ways. Next we find a sufficient condition on π for there to be action-and-reaction (i.e. alternating winners). The condition guarantees that, for all possible pairs of cost levels c_{t+1} and c_{t+k} $(k > 1)$, the race for patent t will be won by the high cost player, i.e. the one with cost level c_{t+k}. We solve for Ω in terms of π.

Then we ask about the converse: what must be the case for it to be true that, for all pairs of cost levels c_{t+1} and c_{t+k} $(k > 1)$, the race for patent t will be won by the low cost player? Thus we find a necessary condition for increasing dominance (more precisely, a necessary condition for there to be increasing dominance from whichever possible pair of cost levels the game begins).

A Sufficient Condition for Action-Reaction: We make the ('egalitarian') assumption that the single period joint profits of the firms are higher when the costs of the higher cost firm are lower:

Assumption A1: For all $t = 1,\dots, T+2$ and $k = 2,\dots, T+2-t$ $\sigma(t,t+1) > \sigma(t, t+k)$

To show that (A1) is sufficient for action-reaction we use an inductive argument:

(i) we show that the higher cost firm in race 1 wins that race

(ii) we assume that the higher cost firm in each of the races $1,\dots, s-1$ wins those races

(iii) given (ii) we show that the higher cost firm in race s wins that race.

Step (i): Since race 1 is the final race, it follows from the definition of Ω that $\Omega(1,k) = \pi(1,k)$ and $\Omega(k,1) = \pi(k,1)$ for all $k > 1$. From (3) we have that the high-cost firm will win race 1 if

$$\Delta(1,2) > \Delta(1,1+k)$$

i.e. if

(6) $\sigma(1,2) > \sigma(1,1+k)$

Assumption (A1) guarantees condition (6), and therefore the high cost firm wins the final race.

Step (ii): Assume that for all $t = 1,\dots,s-1$ and $k = 2,\dots, T+2-t$:

(7) $\Delta(t,t+1) > \Delta(t,t+k)$

This says that races s - 1 onwards are won by the lower cost firms in each race, because that firm is prepared to outbid his rival: see equation (3)

Step (iii): Now consider the race for patent s. At the beginning of the race, let firm L (=A,B) have cost level s + 1, and let H ($\neq$L) have cost level s + k, where k > 1. The aim is to show that H wins the race, i.e. that

(8) $\Delta(s,s+1) > \Delta(s, s+k)$

First, note that

(9) $\Omega(s, s+k) = \pi(s, s+k) + \Omega(s, s-1)$

and

(10) $\Omega(s, s+1) = \pi(s, s+1) + \Omega(s, s-1)$

Equations (9) and (10) say that whoever wins race s does not win - and therefore spends no R and D in - the race for patent s-1. This follows from the assumption at step (ii). The value of winning race s is therefore the profit in the immediate ensuing period plus the value of the continuation game when one has cost level s and one's rival has cost level s-1.

Secondly, we have

$$\Omega(s+k,\ s) = \pi(s+k,\ s) + \Omega(s-1,\ s) - \{\Omega(s-1,\ s+k) - \Omega(s,\ s-1)\} \tag{11}$$

This is H's payoff in the subgame beginning immediately after rae s if L wins race s. The three terms on the RHS of (11) are

- H's profit in the period after race s;
- H's payoff in the subgame immediately after race s-1 (which he wins, by the assumption at step (ii)); and
- the expression {·}, which is the amount that H bids in race s-1. The expression {·} is how much L would have been prepared to pay for patent s-1, i.e. the value of winning it (=Ω(s-1, s+k)) minus the value of not winning it (=Ω(s,s-1)).

Exactly the same reasoning implies that

$$\Omega(s+1,\ s) = \pi(s+1,s) + \Omega(s-1,s) - \{\Omega(s-1,\ s+1) - \Omega(s,s-1)\} \tag{12}$$

Adding (9) and (11) we get

$$\Delta(s,\ s+k) = \sigma(s,\ s+k) + \Omega(s-1,\ s) - \Omega(s-1,\ s+k) + 2\Omega(s,\ s-1) \tag{13}$$

Similarly adding (10) and (12) we get

$$\Delta(s,\ s+1) = \sigma(s,\ s+1) + \Omega(s-1,\ s) - \Omega(s-1,\ s+1) + 2\Omega(s,\ s-1) \tag{14}$$

Subtract (14) from (13):

$$\Delta(s,\ s+k) - \Delta(s,\ s+1) = \sigma(s,\ s+k) - \sigma(s,\ s+1) + \Omega(s-1,\ s+1) - \Omega(s-1,\ s+k) \tag{15}$$

The reasoning used to obtain (9) and (10) implies that

$$(16)\qquad \Omega(s-1,\ s+1) = \pi(s-1,\ s+1) + \Omega(s-1,\ s-2)$$

and

$$(17)\qquad \Omega(s-1,\ s+k) = \pi(s-1,\ s+k) + \Omega(s-1,\ s-2)$$

Equations (15), (16) and (17) now imply the central equation

$$(18)\qquad \Delta(s,\ s+k) - \Delta\ (s,\ s+1) = [\sigma(s,\ s+k) - \sigma(s,\ s+1)] + [\pi(s-1,\ s+1) - \pi(s-1,\ s+k)]$$

This is negative because both terms in square brackets are negative - the first term because of assumption A1 and the second term because π is increasing in its second argument. The negativity of (18) is equivalent to condition (8). This completes the proof.

We have shown that if π meets condition (A1) then there is always action-reaction. Intuitively, the crux of the proof is as follows. Whether H or L wins patent s depends upon whether joint payoffs in the subgame after race s are higher if H wins race s or if L wins it. By the inductive hypothesis, whoever wins race s loses race s-1. Therefore after race s-1, the pair of cost levels will be c_s and c_{s-1}, irrespective of who wins race s. By A1 we know that joint profits in the period after race s are higher if H wins that race than if L wins it. The rest of the proof shows that the winning bid in race s-1 is lower if H wins race s than if L does so. Both factors imply that joint payoffs are higher if H wins race s than if L does.

We can now express Ω in terms of π. First we find an expression for $\Omega(s,\ s-1)$. From (12), (16) and the fact that

$$(19)\qquad \Omega(s-1,\ s) = \pi(s-1,s) + \Omega(s-1,\ s-2)$$

it follows that

(20) $$\Omega(s+1, s) = \Omega(s,s-1) + \pi(s+1, s) + \pi(s-1, s) - \pi(s-1, s+1)$$

Define

(21) $$\mu(s) = \Omega(s+1, s) - \Omega(s, s-1)$$

Then (20) is

(22) $$\mu(s) = \pi(s+1, s) + \pi(s-1, s) - \pi(s-1, s+1)$$

Hence

(23) $$\Omega(s+1,s) = \mu(s) + \mu(s-1) + \dots + \mu(2) + \Omega(2,1)$$

Therefore

(24) $$\Omega(s+1, s) = \pi(2,1) + \sum_{t=2}^{s} [\pi(t+1, t) + \pi(t-1,t) - \pi(t-1, t+1)]$$

(25) $$\Omega(s+1,s) = \pi(s+1,s) + \sum_{t=2}^{s} [\pi(t, t-1) + \pi(t-1, t) - \pi(t-1, t+1)]$$

Therefore

(26) $$\Omega(s+1, s) = \pi(s+1, s) + \sum_{t=2}^{s} [\sigma(t, t-1) - \pi(t-1, t+1)]$$

The expression in square brackets is positive because it is greater than $\pi(t+1, t-1)$ by (A1). Of course we would expect $\Omega(s+1, s)$ to be positive. From (9) and (26):

$$\Omega(s,\ s+k) = \pi(s,\ s+k) + \pi(s,\ s-1) + \sum_{t=2}^{s-1} [\sigma(t,\ t-1) - \pi(t-1,\ t+1)] \tag{27}$$

This holds for all s and k $\geq$ 1. Employing the steps used to derive (20) from (11) we have

$$\Omega(s+k,s) = \Omega(s,\ s-1) + \pi(s+k,\ s) + \pi(s-1,\ s) - \pi(s-1,\ s+k) \tag{28}$$

Using (26) it follows that

$$\Omega(s+k,\ s) = \pi(s+k,\ s) + \sigma(s-1,s) - \pi(s-1,\ s+k) + \sum_{t=2}^{s-1} [\sigma(t,\ t-1) - \pi(t-1,\ t+1)] \tag{29}$$

Equations (27) and (29) completely characterise Ω in terms of π.

Next we can find the size of the winning bid in race s. If H initially has cost level s+k and L has cost level s+1, then H's winning bid equals

$$\begin{aligned} \Omega(s,\ s+k) - \Omega(s+1,\ s) &= \pi(s,\ s+k) + \Omega(s,\ s-1) - \Omega(s+1,\ s) \\ &= \pi(s,\ s+k) - \mu(s) \end{aligned} \tag{30}$$

from (21). Since we have alternating winners, the relevant value for k in (30) is 2. Hence the winning bid equals

$$\pi(s,\ s+2) - \pi(s+1,\ s) - \pi(s-1,\ s) + \pi(s-1,\ s+1) \tag{31}$$

This is positive: the sum of the first two terms > 0 because π is decreasing in its first argument and increasing in its second argument; the sum of the third and fourth terms is positive because π is increasing in its second argument.

To summarise: Condition (A1) is sufficient for there to be action-reaction. Given that assumption, it is possible completely to solve how the game unfolds. Later we shall discuss the economic implications of the result, and also the possibilities for developing the basic model, for example to incorporate positive discounting.

A Necessary Condition for Increasing Dominance

Having found a condition for action-reaction, we now explore whether conditions can be found for the converse to hold. An initial step would be to reverse the inequality in (A1) and ask whether the resulting condition was sufficient for increasing dominance. In fact it is not, and it is interesting to see why. However, it is possible to find a necessary condition for the low-cost firm always to win. Having done this, we shall look at the special case in which the high-cost firm earns zero prófits. (This corresponds in some sense to Bertrand behaviour in the product market - whereas the 'egalitarian' assumption (A1) above was seen earlier to result from Cournot behaviour in the product market under certain conditions.)

Let us ask what must be true for it always to be true that the low cost firm wins the current race. Consider the race for patent s. At the outset of the race L(=A,B) has cost level s+1 and H ($\neq$ L) has cost level s+k, where k > 1. What must be true for L to win race s? That is, what is implied by

(32) $\Delta(s, s+k) > \Delta(s, s+1)$ for all s and k > 1?

Whoever wins race s will be the low cost firm in race s-1 and so wins race s-1, by assumption. Therefore

$$(33) \qquad \Omega(s, s+k) = \pi(s, s+k) - \{\Omega(s-1, s) - \Omega(s+k, s-1)\}$$

The term $\{\cdot\}$ is of course the amount that L will have to bid to win race s-1, if he wins race s.

In fact it is true for all $t \leq s$ that

$$(34) \qquad \Omega(t, s+k) = \pi(t, s+k) + \Omega(t-1, s+k) - \{\Omega(t-1, t) - \Omega(s+k, t-1)\}$$

If H loses race s, then, by assumption, he loses all subsequent races, so that for all $t \leq s$

$$(35) \qquad \Omega(s+k, t) = \pi(s+k, t) + \Omega(s+k, t-1)$$

Combining (34) and (35):

$$(36) \qquad \Delta(t, s+k) = \sigma(t,s+k) + \Delta(t-1,s+k) + \Omega(s+k,t-1) - \Omega(t-1,t)$$

The same is true when $k = 1$. Therefore for $t \leq s$

$$(37) \qquad \Delta(t, s+1) = \sigma(t,s+1) + \Delta(t-1,s+1) + \Omega(s+1,t-1) - \Omega(t-1,t)$$

Define the function

$$(38) \qquad \phi(t) = \Delta(t, s+k) - \Delta(t, s+1).$$

Subtract (37) from (36):

$$(39)\qquad \phi(t) - \phi(t-1) = \sigma(t,s+k) - \sigma(t,s+1) + \Omega(s+k,t-1) - \Omega(s+1,t-1)$$

Whoever loses race s loses all subsequent races. Therefore

$$(40)\qquad \Omega(s+k,\ t-1) = \pi(s+k,\ t-1) + \ldots + \pi(s+k,\ 1)$$

Therefore (39) can be written as

$$(41)\qquad \phi(t) - \phi(t-1) = \sigma(t,\ s+k) - \sigma(t,\ s+1) + \sum_{i=1}^{t-1} [\pi(s+k,\ i) - \pi(s+1,\ i)]$$

It follows that

$$\phi(s) = \sum_{t=2}^{s} [\phi(t) - \phi(t-1)] + \phi(1)$$

$$= \sum_{t=1}^{s} [\sigma(t,\ s+k) - \sigma(t,\ s+1)]$$

$$(42)\qquad + \sum_{t=1}^{s-1} (s-t)[\pi(s+k,\ t) - \pi(s+1,\ t)]$$

Condition (32) is equivalent to $\phi(s) > 0$. This is seen to be quite a strong condition. For example, the reverse of assumption (A1) above is insufficient for this condition to hold. This is because the second term in (42) is not positive - although the first term in (42) is positive if the reverse of assumption (A1) is assumed.

A great simplifcation results from assuming that at any time the high cost firm makes zero profits, i.e.

$$(43)\qquad \text{For all } s \text{ and } k \geq 1 : \pi(s+k,\ s) = 0$$

This assumption is made henceforth; it could be justified in terms of an assumption of Bertrand behaviour in the product market. Under (43) it follows that the low cost firm always wins. It wins race 1 because for all $k > 2$ we have

$$(44) \qquad \Delta(1,2) = \pi(1,2) < \pi(1,k) = \Delta(1,k)$$

because π is increasing in its second argument. Assume now that the low cost firm wins races 1, ..., s-1. Then (42) holds, and together with (43) it implies

$$(45) \qquad \emptyset(s) = \sum_{t=1}^{s} \pi(t, s+k) - \pi(t, s+1)$$

where $k \geq 2$. Since π is increasing in its second argument $\emptyset(s)$ is positive, i.e. condition (32) holds. This suffices to show that the low cost firm wins all races if condition (43) holds.

We can find Ω in terms π. Condition (43) implies that for all s and $k \geq 1$:

$$(46) \qquad \Omega(s+k, s) = 0$$

This is because the high cost firm never wins, so always remains the high cost firm and never makes any profit: see equation (40). Equation (34) now implies that for all $t \leq s$ and $k \geq 1$:

$$(47) \qquad \Omega(t, s+k) = \pi(t, s+k) + \Omega(t-1, s+k) - \Omega(t-1, t)$$

Sum both sides of (47) from 2 to s:

(48) $$\sum_{t=2}^{s} \Omega(t,\ s+k) = \sum_{t=2}^{s} \pi(t,\ s+k) + \sum_{t=1}^{s-1} \Omega(t,\ s+k) - \sum_{t=1}^{s-1} \Omega(t,\ t+1)$$

Subtract $\sum_{t=2}^{s-1} \Omega(t,\ s+k)$ from both sides of (48) and note that $\Omega(1,\ s+k) = \pi(1,\ s+k)$ to get

(49) $$\Omega(s,\ s+k) = \sum_{t=1}^{s} \pi(t,\ s+k) - \sum_{t=1}^{s-1} \Omega(t,\ t+1)$$

Set $k = 1$ in (49) to obtain

(50) $$\sum_{t=1}^{s} \Omega(t,\ t+1) = \sum_{t=1}^{s} \pi(t,\ s+1)$$

Rewrite (50) with s−1 in place of s:

(51) $$\sum_{t=1}^{s-1} \Omega(t,\ t+1) = \sum_{t=1}^{s-1} \pi(t,\ s)$$

Subtract (51) from (50):

(52) $$\Omega(s,\ s+1) = \sum_{t=1}^{s} \pi(t,\ s+1) - \sum_{t=1}^{s-1} \pi(t,\ s)$$

Together with (49) this implies

(53) $$\Omega(s,\ s+k) = \sum_{t=1}^{s} \pi(t,\ s+k) - \sum_{t=1}^{s-1} \left[\sum_{i=1}^{t} \pi(i,\ t+1) - \sum_{i=1}^{t-1} \pi(i,\ t) \right]$$

The first term on the RHS is the sum of profits received; the other term comprises the s - 1 bids that the low cost player makes in the remaining stages. In fact (53) can be simplified, because

$$(54) \qquad \sum_{t=1}^{s-1} \Big[\sum_{i=1}^{t} \pi(i,\ t+1) - \sum_{i=1}^{t-1} \pi(i,\ t) \Big]$$

$$= \sum_{t=1}^{s-1} \Big[\sum_{i=1}^{t} \pi(i,\ t+1) \Big] - \sum_{t=0}^{s-2} \Big[\sum_{i=1}^{t} \pi(i,\ t+1) \Big]$$

$$= \sum_{i=1}^{s-1} \pi(i,\ s)$$

From (53) and (54) it follows that

$$(55) \qquad \Omega(s,\ s+k) = \sum_{t=1}^{s} \pi(t,\ s+k) - \sum_{t=1}^{s-1} \pi(t,\ s)$$

The first term on the RHS is the sum of the profits of the low cost firm in all subsequent periods. The second term equals the sum of the bids (i.e. R and D expenses) in all subsequent periods; this is equal to the product market profits that the other firm would get if it were to win race s - 1.

The winning bid in race s is given by (52). If this is subtracted from (55) we obtain the low cost firm's payoff in the continuation game beginning immediately before race 2 as:

$$(56) \qquad \sum_{t=1}^{s} \pi(t,\ s+k) - \pi(t,\ s+1)$$

(whereas Ω is the payoff in the subgame after a race, i.e. the value of the patent at stake). Equation (56) is easily interpreted: the low cost firm's payoff in the subgame beginning just before a race is equal to the difference between the sum of his product market profits if he wins all future races, and the sum of his rival's profits if he were to win all future races.

An instance of (56) is that for the game as a whole, A's (i.e. the originally low cost player's) payoff is

$$\sum_{t=1}^{T} \pi(t, B) - \pi(t, A)$$

using obvious notation. B's payoff is zero of course.

Discounting: It has so far been assumed that firms maximise the sum of their per period profits less R and D expenses. It would be desirable to incorporate discounting into their calculations. This is especially so since we are considering a finite horizon game. With discounting these remote effects carry less importance. Fortunately, much of the analysis requires no substantial modification when discounting is introduced. Of course certain quantitative features change - such as the formulae for Ω. But the qualitative results remain the same.

For example, condition (A1) remains sufficient for action-reaction. The following sketches why this is so. Let ρ be the common discount factor. Regard Ω as the payoff in the subgame immediately after a race evaluated at that instant. Parallel to the previous analysis we have

(9') $$\Omega(s, s + k) = \pi(s, s + k) + \rho\Omega(s, s - 1)$$

and

(11') $$\Omega(s + k, s) = \pi(s + k, s) + \rho[\Omega(s-1), s) - \Omega(s-1, s+k) + \Omega(s, s-1)]$$

The same holds true for k = 1, and the argument in steps (13) to (17) eventually implies that

$$(18') \qquad \Delta(s, s+k) - \Delta(s, s+1) = [\sigma(s, s+k) - \sigma(s, s+1)] + \rho[\pi(s-1, s+1) - \pi(s-1, s+k)]$$

which is negative just as before. Therefore (A1) implies action-reaction also when there is discounting. Similarly, condition (43) implies increasing dominance just as before. Explicit formulae, e.g. for Ω, are not stated for $\rho \neq 1$ since they would add little extra insight and are in any event very similar to their counterparts already stated.

Discussion: The above model is a simple attempt to examine the question of innovation and industry evolution in the context of duopoly. No attempt was made in the above to model the product market explicitly; this was 'blackboxed' and a general function π was employed. Nevertheless the nature of this function was seen to be a vital determinant of industry evolution. Perhaps the clearest illustration of this is the contrast between the analysis under condition (A1), and that under the (extreme) condition (43) that the high cost firm makes zero profits. We have already suggested (p. 216) possible links between (A1) and 'Cournot' behaviour in the product market, and between (43) and 'Bertrand' behaviour in the product market. It would be wrong to place too much reliance on those possible connexions, particularly given the stylised nature of the above model, and in the absence of a proper formulation of what happens in the product market. However, the analysis so far does suggest the following. These remarks should be regarded as conjectures to be explored further, rather than as conclusions.

Price competition in the product leads to increasing dominance. In any period the low cost firm charges either the monopoly price given his current cost level, or sets price equal to his rival's cost level, whichever is the lower. Quantity competition (à la Cournot) in the product market satisfies condition (A1), at least under certain conditions. Therefore quantity competition in the product market is associated with action-reaction over time. Price falls over time.

In static terms, the Cournot equilibrium is usually considered to be less 'competitive' than the Bertrand equilibrium: it is associated with higher prices and therefore lower consumer welfare, at least according to partial equilibrium analysis. However, the dynamic effects may be rather different, Cournot involving action-reaction and falling prices over time, whereas Bertrand leads to increasing dominance. Although costs fall in the Bertrand case, price may fail to do so.

This interesting contrast between the static and dynamic properties of the two types of behaviour in the product market is not without intuition. In the Cournot case it is not disastrous for the high cost firm to be overtaken (at any rate temporarily): he remains able to make positive profits. Moreover, he may easily regain the advantage - in part because his rival is not strongly motivated to thwart him, for the above reason. The argument is symmetric for the other player and so action-reaction occurs. On the other hand, in the Bertrand case it is more serious to lose technological advantage since profits become zero. The low cost firm is therefore strongly motivated to retain his advantage, and becomes increasingly dominant.

The idea that some static inefficiency promotes dynamic benefits has an obvious Schumpeterian flavour. I hope to explore it further in future research.

APPENDIX

This Appendix derives some results concerning Nash-Cournot equilibrium for an n-firm oligopoly when costs and demand are linear. The simplicity of this example makes it a useful illustration of several points in the thesis.

THE NASH-COURNOT EQUILIBRIUM IN AN N-FIRM OLIGOPOLY WHEN UNIT COSTS ARE CONSTANT

Assume that there are n firms in an industry supplying a homogeneous product. Let π_i, q_i and c_i denote, respectively, the profits, quantity supplied, and unit costs of firm i. Let the inverse demand function be $p = p(Q)$, where $Q = \sum q_i$. We have:

(A1) $$\pi_i = (p - c_i)q_i$$

The first-order condition for a maximum is

(A2) $$p - c_i + q_i \frac{dp}{dq_i} = 0$$

At the Nash-Cournot equilibrium, $dq_i/dQ = 1$. We can therefore write (A2) as

(A3) $$q_i = -(p - c_i)/p'$$

where p' is dp/dQ. Adding (A3) across firms, we have:

$$\text{(A4)} \qquad Q = -(np - \sum c_j)/p'$$

It follows from (A1) and (A3) that, in equilibrium,

$$\text{(A5)} \qquad \pi_i = -[(p(Q^*) - c_i)^2/p'(Q^*)] = -q_i^{*2}p'(Q^*),$$

where asterisks denote equilibrium values.

Now assume that p(Q) has the specific form p(Q) = A-bQ. In this case p' = -b.Therefore, from (A4), we have

$$\text{(A6)} \qquad Q = (np - \sum c_j)/b$$

Therefore

$$\text{(A7)} \qquad A - p = np - \sum c_j, \text{ in which case}$$

$$\text{(A8)} \qquad p^* = \frac{A + \sum c_i}{n + 1}$$

In conjunction with (A3) this yields

(A9) $$q_i^* = \frac{1}{b}\left[\frac{A + \sum c_i}{n + 1} - c_i\right]$$

which, together with (A5), yields

(A10) $$\pi_i = \frac{1}{b}\left[\frac{A + \sum c_i}{n + 1} - c_i\right]^2$$

In the symmetric case, $c_i = c$ for all $i = 1,\ldots,n$. Then (A8) to (A10) become

(A8′) $$p^* = \frac{A + nc}{n + 1}$$

(A9′) $$q_i^* = \frac{A - c}{b(n + 1)}$$

(A10′) $$\pi = \frac{1}{b}\left(\frac{A - c}{n + 1}\right)^2$$

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NOTATION

The following lists contain the main symbols used in each chapter. The thesis as a whole contains discussion of many models. It is therefore inevitable that some symbols are used to denote different objects in different chapters. Every effort has been made to minimise this occurrence, and to eliminate any possible confusion. In view of this, separate lists of notation are provided for each chapter. Symbols that appear only briefly are omitted from the lists.

Notation

CHAPTER I: MODELS OF R AND D WITH UNCERTAINTY

a	: fixed cost in the example: see equation (10) in I.2
c(x)	: cost of R and D effort of x.
F	: cumulative distribution function
f	: fixed cost of R and D
h(u)	: hazard rate given R and D expenditure u
i	: index for firms
K_i	: fixed factor input to R and D effort
L_i	: variable factor input to R and D effort
m	: number of laboratories chosen by social planes
n	: number of firms
P_i	: the probability that firm i wins
r	: discount rate
T	: date of Doomsday
t	: time
U	: net social benefit
u	: R and D expenditure
V	: value of innovation to firm
W	: value of innovation to society
x_i	: R and D effort of firm i
X	$=\sum x_i$
y_i	$= x_i^{\rho}$
Y	$=\sum y_i$
z_i	: knowledge level of firm i
α	$= \{\sum x_i^{\rho}\}^{1/\rho}$
β	: function of (n,V,r) defined at (35) in I.1

Notation

γ	:	returns to scale parameter in example of R and D cost function.
λ	:	R and D efficiency parameter
μ	:	function of (n,V,r) defined at (34) in I.1
ρ	:	parameter representing degree of correlation of uncertainties
σ	:	conclation parameter $=(\rho-1)/\rho$
Ω_i	:	payoff of firm i

Notation

CHAPTER II: PATENT RACES AS BIDDING GAMES

A	: intercept of demand curve in linear oligopoly model.
B	= A-c in linear oligopoly model
b	: high-cost duopolist's initial unit cost disadvantage
C	: challenger (subscript has same meaning).
C(k)	: challenger's payoff when he has k patents
c	: unit cost level in linear oligopoly model
c(x)	: flow cost of researching at rate x
E	: potential entrant's incentive to win
E_j	: (expected) payoff of potential entrant;
F	: cumulative distribution function
G	: incumbent's payoff if joint venture wins
H	: initially high-cost duopolist
h	: H's incentive to win
I	: incumbent (subscript has same meaning)
	: incumbent's incentive to win
I_i	: payoff of incumbent i
I(k)	: incumbent's payoff when he has k patents
i	: index for incumbents
J	: joint venture's incentive to win
j	: index for potential entrant j
K	: incumbent's payoff if he wins
L (in II.3)	potential entrant's payoff if he wins
L (in II.4)	initially low-cost duopolist
ℓ	: L's incentive to win
M	: incumbent's payoff if another incumbent wins
m	: number of incumbents
N (in II.1)	number of patents

Notation

N (in II.4) incumbent's payoff if a potential entrant wins

P : probability of entry

p : price

Q : industry output

q_i : output of firm i

r : interest rate

s : unit cost improvement offered by new technology

U : incubment's payoff if no entry minus incumbent's payoff if entry

V : value of prize

W : potential entrant's payoff if he enters

X $=\sum x_i$

x_i : R and D rate of incumbent i

Y $=\sum y_j$

y_j : R and D rate of potential entrant j

z $= U/W$

α_1 : transactions costs in R and D input market

α_2 : transactions costs for transfer of patent

β : licence fee for patent

γ : parameter for decreasing returns to R and D effort

Δ : degree of incumbent's ineffeciency at R and D

λ : conjectural variation term

π : profit (sometimes subscripted)

Ω : payoff (sometimes subscripted)

ω : y/x

Notation

CHAPTER III: PERFECT EQUILIBRIUM IN PATENT RACES

A,B	:	players
a_k	:	A's k^{th} bid
b_k	:	B's k^{th} bid
$\{C_n\}$, $\{D_n\}$, $\{E_n\}$, $\{F_n\}$, $\{G_n\}$		sequences of critical distances, defined in the text
$h(\delta)$	:	scaling function
J	:	index = A,B
m(X)	:	optimal number of steps for A t otake from distance X.
N	:	number of stages lead for victory in tug-of-war
r_J	:	discount rate
t	:	time
V_J	:	value of prize to J
w_J	:	J's progress function
X	:	A's distance from the finishing line (subscript 0 indicates initial distance)
Y	:	B's distance from the finishing line (subscript 0 indicates initial distance)
x_n	:	A's rate of R and D when n stages ahead in the tug-of-war
y_n	:	B's rate of R and D when n stages ahead in the tug-of-war
x(m,n)	:	A's rate of R and D when he has m steps to complete and his opponent has n steps to complete.
y (m,n)	:	B's rate of R and D when he has m steps to complete and his opponent has n steps to complete.

Notation

α : V_A/γ

β = V_B/γ

γ = function of ε and η: see (10) in III.2.

δ : interval between bids (sometimes a superscript)

ε,η : parameters of progress function in example of asymmetrical race

λ : function of η: see (19) in III.1.

ρ_J : J's discount factor.

θ : value function in continuous time formulation.

$\Omega_J(X)$: value of a free run for J from distance X.

Ω_n : Expected payoff in tug-of-war when n steps ahead

$\Omega(m,n)$: Expected payoff in multistage race when one has m steps remaining and one's rival has n steps to go.

Notation

CHAPTER IV: R AND D COMPETITION WITH A SEQUENCE OF INNOVATIONS

A,B	: players
$c(x)$	: cost of R and D effort of x.
C	: (subscript): challenger
f	: fixed cost of R and D
H	: currently high-cost firm
h	: H's incentive to win the next patent
I	: (subscript) incumbent
i	: index for firms
L	: currently low-cost firm
ℓ	: L's incentive to win the next patent
n	: (in IV.1) : number of firms
r	: discount rate
s,t	: index for cost levels (in IV.2)
T	: number of patents in the sequence
x_i	: R and D effort of firm i
z_i	$= \sum_{j \neq i} x_j$
$\Delta(s,t)$	$= \Omega(s,t) + \Omega(t,s)$
$\mu(s)$	: defined at equation (21) of IV.2
π	: profit flow
$\pi(s,t)$	: profit flow to player with cost level s when his rival has cost level t.
$\phi(t)$	: defined at equation (38) of IV.2
Ω_I	: payoff to incumbent (in IV.1)
Ω_c	: payoff to challenger (in IV.1)
Ω_w	: payoff to winner of the current race (in IV.1)
Ω_L	: payoff to loser of the current race (in IV.1)

Notation

$\Omega(s,t)$: payoff to player with cost level s when his rival has cost level t in the subgame following the current patent race.