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Modelling Investment Optimization on Smallholder Farms through Multi-criteria Decision Approaches: An Example from Ethiopia

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Abstract. We use data from the Ethiopia Rural Household Survey and the Ethiopian Central Statistics Agency to demonstrate a set of techniques for estimating optimal investment allocation in smallholder farming. The approaches treat farming tasks, constraints, and investments as a portfolio problem, characterized by multiple competing objectives. We formulate several versions of the multi-objective problem and solve them in three alternative ways; 1) using standard Markowitz portfolio optimization, 2) using a weighted goal programming model, and 3) a multi-horizon mean variance goal programming model, estimating all model parameters using real data. The main benefit of the goal programming formulation is the possibility to simplify in a single criterion problem complex situations in which the Decision Maker (DM) faces a trade-off between two or more objectives. We discuss the importance of portfolio allocations for smallholder farmers in minimizing risk and increasing return, and discuss how these approaches provide a framework that can be extended to practical applications in smallholder farming.

1 Introduction

Multi-objective portfolio diversification is the process by which a Decision Maker (DM) allocates recourses efficiency. In the literature on financial management, various approaches have been formalized using the tools of portfolio optimization, and using investment variety to mitigate risk while increasing return is widely accepted as best practice among financial professionals.

The motivation behind applying the concept to the smallholder farming context is simple; like investors in stocks or bonds, farmers evaluate activities with the goal of making a return, and are constrained by limited resources. They are moreover exposed to a variety of risks that threaten to reduce their returns on investments. Particularly in globalized markets, the goods that farmers produce can vary widely in price and demand for reasons that are, from the point of view of some producers, remote. Understanding these fluctuations in detail is a substantial and at times insurmountable challenge for analysts and businesses with a wealth of information available to them, let alone farmers with access to much less information.

Policy-oriented research has repeatedly demonstrated the benefits of efficient allocation of income generating activities in high-risk contexts (Ellis, 2000). Experimental and survey analyses also show that the psychological impacts of unmanaged downside risks from farming can be severe. In addition to the negative effects from lower income, individuals that experience high volatility in earnings have lower life satisfaction (Caria & Falco, 2013), and as Vijayakumar *et al*. (2005) report for developing economies, income volatility from farming can be related to depression and stress-related mental illness.

Smallholder farmers in developing countries -- as is the case for people with low incomes more generally -- often respond to the threat of severe downside risk by being very conservative in their investment decisions, and are characterized by strong loss aversion (Caria & Falco, 2013). In Ethiopia in particular, experimental evidence shows agricultural households have an exceptionally high aversion to risk (Yesuf & Bluffstone, 2009). These risk preferences lead to allocation decisions with systematically lower returns and investment, and is also evident in a general reluctance to take out loans (Karlan *et al*, 2011).

There is a growing literature on the ways in which farmers can manage the risks associated with small-scale farming, in addition to how they might maximize returns given conservative risk preferences. Insurance can put a floor on losses for farmers that have access to fair products, and social protection schemes help farmers in many countries deal with adverse conditions. Intercropping and crop diversification strategies are also popular. Many farmers apply some form of risk mitigating practices, and most diversify their investments to some extent. Still, in the absence of costly interventions, most smallholder farmers make allocation decisions with incomplete information about the risk mitigation potential of their portfolios.

At the same time, information about agricultural markets has never been so available or so rich. In this paper we exploit some of this publically available data from Ethiopia and we use stylized applications of portfolio optimization and goal programming techniques to the investment allocation problems associated with smallholder farming. Using widely available optimization software such as Lindo and CPLEX simplifies the process of optimizing the allocation of investments according to available information.

The intent of this paper is twofold. Firstly, we review portfolio optimization as it applies to smallholder farmers. Secondly, we demonstrate easily replicated systems, using widely available software, which allow for a variety of assumptions to be tested and considered in a flexible framework of optimal farming investment allocations. Such tools could potentially be used in development contexts to advise farmers on ideal risk mitigation strategies.

The paper is organized as follows: Section 2 provides an introduction to portfolio optimization and goal programming. Section 3 provides applied examples of portfolio optimization and describes the data from which the model parameters are estimated. Section 4 concludes.

2 Portfolio Optimization

The standard portfolio optimization model from Markowitz (1952) has been widely applied in both theoretical and applied contexts. The model is based on the assumption that investors balance risk and return. Formally, the model for portfolios of n risky assets is commonly expressed as:

$$Min \ \frac{1}{2}\sum_{i,h=1}^{n} x_i x_h \sigma_{ih} \tag{1}$$

Subject to:

$$\sum_{i=1}^{n} x_i r_i = \bar{r}$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x \in D$$

Where $\bar{r}$ the fixed and feasible rate of return is desired by the Decision Maker (DM), and σ_{ih} is the minimal covariance possible for a given rate of return. A solution is a portfolio $(x_1, x_2, \ldots x_n)$ that minimizes variance while offering the expected rate of return. D is a fixed subset of R^n which takes into account all possible extra constraints.

One drawback to this approach is the fixed nature of the desired rate of return. For applied decisions concerning the allocation of finite resources, the objectives of a DM are usually complex and restricted by multiple considerations, rather than a straightforward and fixed desired rate of return. Allowing small deviations to either the desired rate of return or the acceptable level of variance may be more appropriate than a level that is artificially set.

The above model (1) can be then reformulated into a multi-criteria setting in the following manner:

$$Min \ \frac{1}{2}\sum_{i,h=1}^{n} x_i x_h \sigma_{ih} \tag{2}$$

$$Max \ \sum_{i=1}^{n} r_i x_i$$

Subject to:

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \geq 0 \quad i = 1 \ldots n$$

$$x \in D$$

Goal Programming (GP) is a common computational method used to solve such multi-criteria questions, either as an alternative to the standard

Markowitz formulation, or a compliment to it. In the empirical section of this paper, we will demonstrate the estimation procedure for the standard portfolio maximization approach to underline the differences between it and the goal programming alternative. The latter allows for greater flexibility in the trade-off between multiple objectives.

Assume for the sake of simplicity that the DM wants to optimize a portfolio of n risky assets minimizing the deviations from expected risk (objective 1) and expected return (objective 2):

$$Min\ w_1^+\delta_1^+ + w_1^-\delta_1^- + w_2^+\delta_2^+ + w_2^-\delta_2^- \tag{3}$$

Subject to:

$$\frac{1}{2}\sum_{i,h=1}^{n} x_i x_h \sigma_{ih} - \delta_1^+ + \delta_1^- = g_1$$

$$\sum_{i=1}^{n} x_i r_i - \delta_2^+ + \delta_2^- = g_2$$

$$\sum_{i=1}^{n} x_i = 1$$

$$x_i \geq 0 \quad i = 1 \ldots n$$

$$\delta_1^+, \delta_1^-, \delta_2^+, \delta_1^- \geq 0$$

Taking a sample we estimate the covariance σ_{ih}, the return r_i, and the two goals g_1 and g_2. For completeness, equation 3 includes weights $w_{1,2}^{+,-}$, which the DM can vary to account for any individual preferences with respect to deviations of different types.[1]

An extension of the goal programming approach can explicitly consider varying goals the DM aims to achieve with respect to distinct time periods. An example of such a case may be where an abnormal year has significantly increased the return for some investment(s) over which the DM is seeking to efficiently allocate resources. The DM may believe that such information is less reliable than the long-term evidence with respect to returns, to take an illustrative example. Such discretion can be exercised within the context of a goal programming model by calculating the returns and correlations in returns separately for the different time periods in question, and subsequently assigning goals to each period taken as a distinct component of the optimization problem. These goals enter as additional possible deviations to be minimized in the objective function, in the same fashion that the problem is formalized in equation 3.

[1] Such preferences are not considered in the empirical section below.

$$Min \quad \delta_1^+ + \delta_1^- + \delta_2^+ + \delta_2^- + \delta_3^+ + \delta_3^- + \delta_4^+ + \delta_4^- \tag{4}$$

Subject to:

$$\sum_{i,h=1}^{n} x_i x_h \sigma_{ih} - \delta_1^+ + \delta_1^- = g_1$$
$$\sum_{i=1}^{n} r_i x_i - \delta_2^+ + \delta_2^- = g_2$$
$$\sum_{i,h=1}^{n} x_i x_h \hat{\sigma}_{ih} - \delta_3^+ + \delta_3^- = g_3$$
$$\sum_{i=1}^{n} \hat{r}_i x_i - \delta_4^+ + \delta_4^- = g_4$$
$$\sum_{i=1}^{n} x_i = 1$$
$$x_i \geq 0 \quad i = 1 \ldots n$$
$$\delta_1^+, \delta_1^-, \delta_2^+, \delta_2^-, \delta_3^+, \delta_3^-, \delta_4^+, \delta_4^- \geq 0$$

We estimate this `multi-horizon goal programming' approach explicitly in Section 3.3, also using real data.

3 Examples from Smallholder Farmers in Ethiopia

To provide examples of how one might structure the applied use of traditional portfolio optimization to smallholder farming; in the following sections we use data from Ethiopian farming households and national statistics from the Ethiopian Central Statistics Agency. The household data used here come from the Ethiopia Rural Household Survey, a dataset made available by Economics Department of Addis Ababa University, the Centre for the Study of African Economies at the University of Oxford, and the International Food Policy Research Institute. The data are extracted from the 2009 wave of the survey, which covered 15 villages.

3.1 Data Description

Major cereals – including teff, wheat, maize, sorghum and barley – dominate agricultural production in Ethiopia.

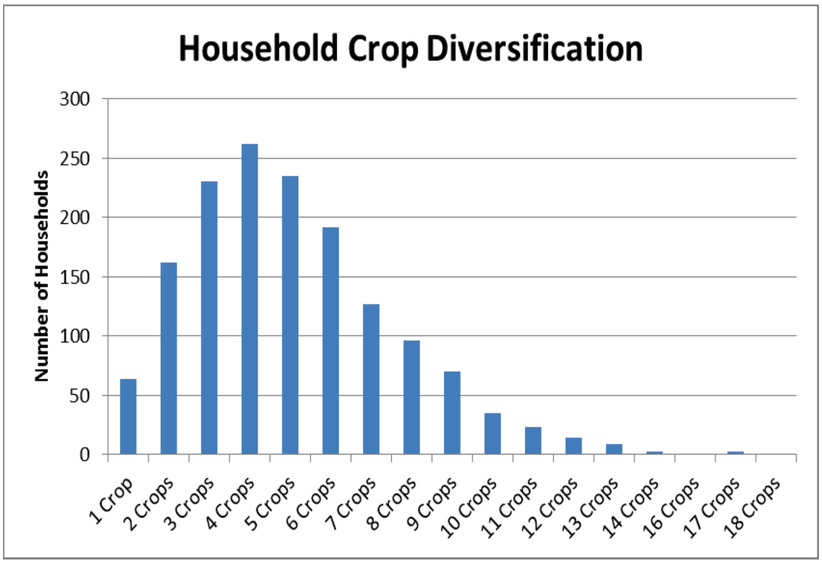

Figure 1: Agricultural Household Crop Diversification

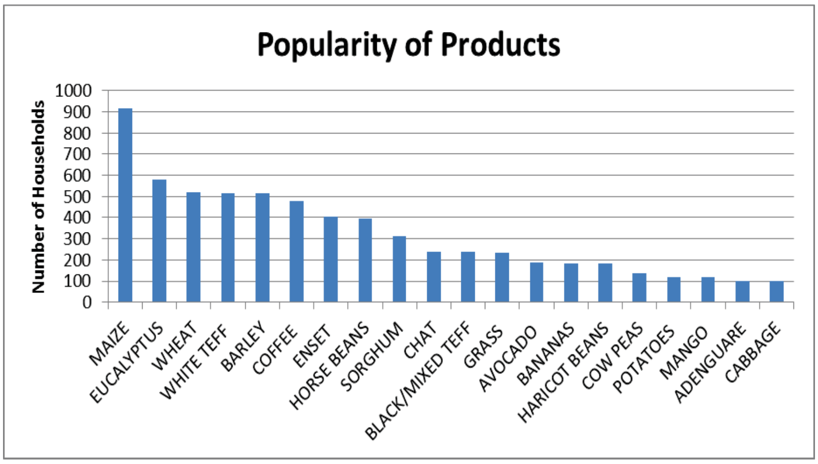

Figure 2: Number of Households by Product Types

As can be seen in figure 2 cereals such as maize, teff, and wheat are popular products for most households, alongside some cash crops such as eucalyptus and coffee. It is also clear from figure 1 that many farmers diversify between 3-5 different crop types on their land. This suggests that at least in the context of the surveyed farmers, diversification is a common strategy.

3.2 Estimating Variations on the Investment Portfolio

We must first estimate the costs of producing each individual good from the set of feasible crops from which the farmer chooses. Examples of costs include investments in seeds and fertilizer, the value of the labour expended for producing each crop, and other inputs that are required to realize a return on the investment in any particular crop.[2]

To estimate the amount of seed required for each product per hectare, we extract the figures reported in table 1 detailing nationwide production statistics for seven crops in Ethiopia for 2007-2008 from Ethiopian Central Statistics agency.

Average Indigenous Seed Use

Type	Holder	Hectare	Quintal (100Kg)	Qntl/Hctr
Barley	3806114	977955	1757089	1.797
Wheat	4057525	1381942	2625410	1.900
Maize	7060071	1421279	643412	0.453
Sorghum	4255998	1529047	399821	0.261
Finger millet	1352715	398340	175051	0.439
Oats / 'Aja'	265031	30526	50313	1.648
Faba Beans	3586452	518785	760440	1.466

Table 1: Estimates of Seed Use

For simplicity, we assume that the farmer in this optimization example cultivates a single hectare, experiences constant returns to scale, and that the cost of seeds for the next year are equivalent the cost of seeds in the previous year. Thus, estimates of the cost of busying seeds for the entire hectare of each product can be derived by multiplying the amount used by an average farmer per hectare by the price in Birr[3]. The result is reported in the following table:

Estimated Seed Cost

	Birr/Kg (2011)	Qntl/Hct	total cost
Barley	4.567238	1.797	820.59
Wheat	5.515317	1.900	1047.80
Maize	2.756598	0.453	124.79
Sorghum	3.43218	0.261	89.75
Finger millet	3.662143	0.439	160.93
Oats / 'Aja'	5.1475	1.648	848.41
Faba Beans	7.044762	1.466	1032.76

Table 2: Estimated Seed Cost

To estimate future prices of products, we extract historical producer commodity prices from the 2013 CountrySTAT Ethiopia database, in which yearly data are available from 1999-2011.

The average number of labour hours per hectare according Pender & Fafchamps (2001) is 177.25 per growing season. For hired labour, average wages were 73 Birr a day in 2009, thus we estimate labour costs per hectare of

[2] In Ethiopia, most seeds are not enhanced varieties that must be purchased each season, so for this example we use prices for indigenous seed.

[3] The Ethiopian currency; about 19.20 = 1 USD in January, 2014

agricultural production at about 1617.5 Birr for this example.[4][5] Fertilizer is used widely in Ethiopia and, according to the 2009 Ethiopia Rural Household Survey; the mean cost of fertilizer per hectare is approximately 686.71 Birr.

Taking these costs together with the cost of seeds per hectare for each crop type, and in turn subtracting these from expected earnings gives the expected return for each crop. These are reported in figure 3.

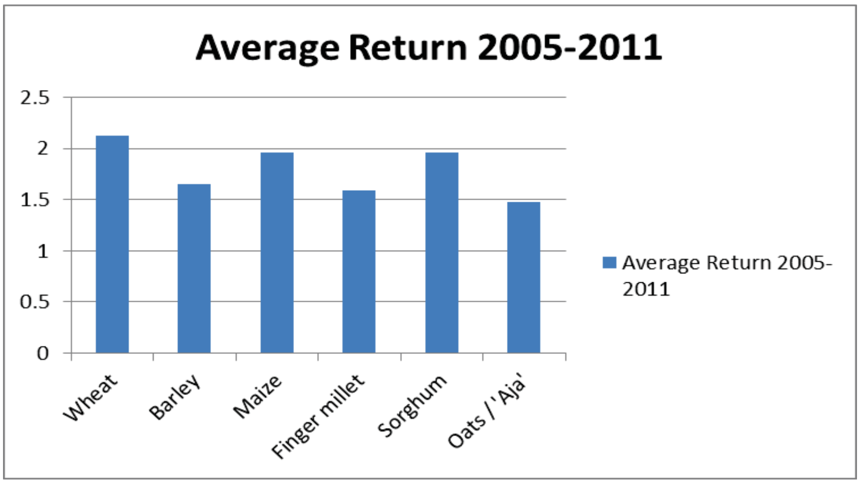

Figure 3: Average Return for 6 Grain Types

The following table provides the covariance matrix for the goods included in a hypothetical portfolio:

Full Year Coverage (2005-2011) Covariance Matrix

	Wheat	Barley	Maize	Finger millet	Sorghum	Oats / 'Aja'
Wheat	0.305789					
Barley	0.271181	0.250341				
Maize	0.35916	0.333082	0.479123			
Finger millet	0.285783	0.26828	0.366302	0.294656		
Sorghum	0.39092	0.361478	0.498598	0.396908	0.542804	
Oats / 'Aja'	0.202352	0.188782	0.221407	0.190769	0.25697	0.199377

Table 3: Covariance Matrix of Returns for 2005-2011

More easily interpretable is the correlation matrix of returns reported in table 4, which highlights the extent to which returns for different crops are highly correlated with one another.

[4] Alternative measures include Guido Gryseels and Michael R. Goe, who calculate an average of 440 hr/ha/year, bounded at 600 hr/ha/year for the labor-intensive Teff, and 300 hr/ha/year for pulses. These latter measures cover between two and four growing seasons in Ethiopia.

[5] For the returns estimated in this analysis, labour and fertilizer costs are assumed to be stable, and to vary only with inflation.

Full Year Coverage (2005-2011) Correlation Matrix						
	Wheat	Barley	Maize	Finger millet	Sorghum	Oats / 'Aja'
Wheat	1					
Barley	0.9801	1				
Maize	0.9383	0.9617	1			
Finger millet	0.9521	0.9878	0.9749	1		
Sorghum	0.9595	0.9806	0.9777	0.9925	1	
Oats / 'Aja'	0.8195	0.845	0.7164	0.7871	0.7811	1

Table 4: Correlation Matrix of Returns 2005-2011

For the sake of this example, suppose that the estimated budget is set to 1. In the standard Markowitz portfolio optimization model we minimize the covariance of returns based subject to a budget constraint. Such a model can be written:

$$Min\ \frac{1}{2}\sum_{i,h=1}^{n} x_i x_h \sigma_{ih} \tag{5}$$

Subject to:

$$\sum_{i=1}^{6} \hat{r}_i x_i = 1.6$$

$$\sum_{i=1}^{6} x_i = 1$$

x_i is positive

Where r_i is return for good x_i. The optimal allocation for this example is 19% in wheat, and 81% in oats.

3.3 A Goal Programming Example

Suppose now that instead of a return aspiration, the DM wishes to minimize departures from multiple objectives simultaneously. Suppose that levels for the covariance and return of the portfolio are 1.2 and 1.6 respectively.

The goal programming model allows deviations from multiple objectives:

$$Min\ Z = (\delta_1^+ + \delta_1^- + \delta_2^+ + \delta_2^-) \tag{6}$$

Subject to:

$$\sum_{i,h=1}^{6} x_i x_h \sigma_{ih} - \delta_1^+ + \delta_1^- = 1.2$$

$$\sum_{i=1}^{6} r_i x_i - \delta_2^+ + \delta_2^- = 1.6$$

$$\sum_{i=1}^{6} x_i = 1$$

x_i is positive

The optimal allocation using this approach is 98% in millet and 2% in maize.

3.3 A multi time horizon example

In some instances, the DM may be balancing considerations of yet another type with respect to their investment allocations. In some instances, the DM may value information from one time period more highly than another. Rather than an arbitrary choice of time horizon, it can in some cases be useful to split periods into units and include average returns and correlations for each as individual components of the optimization problem.

A common concern for small hold farmers, for example, is weighing more recent returns more heavily than returns from returns in the more distant past. Such problems can be analysed in the context of multiple horizon goal programming, an approach that allows for varying preferences of the DM with respect to weighting over of returns estimated over different time periods.[6]

In the following exercise, we assume that the information regarding more recent returns is valued equally with those in the longer term, although this is not a requirement. Considering several time horizons requires return estimates and covariance matrices calculated individually over the time windows in question. For the first time horizon in this example, we use the returns and covariance over the years from 2005 to 2008. We then calculate similar estimates over the period from 2009 to 2011. The returns and covariance matrix for these windows are presented in figure 4 and table 5 respectively.

[6] The methodology employed here was formalized in Ballestero & Garcia-Bernabeu (2012)

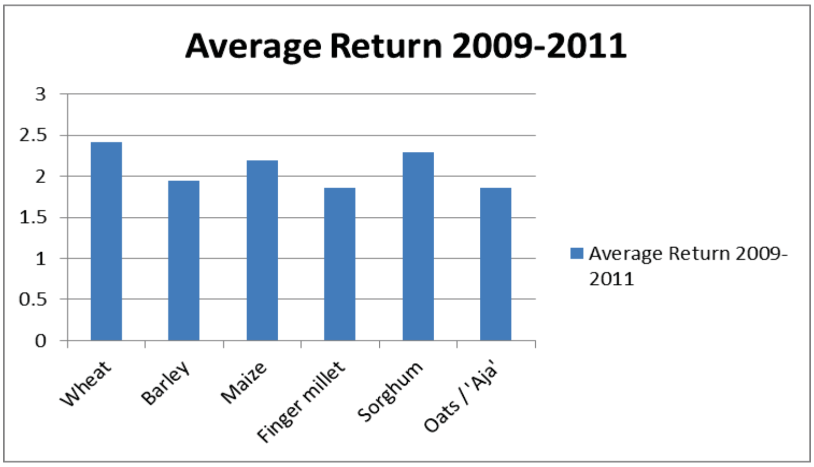

Figure 4: Average Return over Recent Window

Period A (2009-2011) Covariance Matrix

	Wheat	Barley	Maize	Finger millet	Sorghum	Oats / 'Aja'
Wheat	0.201922					
Barley	0.19124	0.193016				
Maize	0.299728	0.301639	0.471453			
Finger millet	0.196247	0.210237	0.327715	0.240679		
Sorghum	0.241556	0.251306	0.392217	0.280937	0.331647	
Oats / 'Aja'	0.142084	0.124865	0.19641	0.118205	0.151591	0.107896

Table 5: Covariance Matric of Returns 2009-2011

Period B (2005-2008) Covariance Matrix

	Wheat	Barley	Maize	Finger millet	Sorghum	Oats / 'Aja'
Wheat	0.324558					
Barley	0.264798	0.224234				
Maize	0.399494	0.347887	0.551015			
Finger millet	0.300506	0.258322	0.404628	0.299834		
Sorghum	0.445798	0.383095	0.599062	0.445503	0.663554	
Oats / 'Aja'	0.113637	0.100987	0.158556	0.122078	0.18742	0.073874

Table 6: Covariance Matrix of Returns for 2005-2008

If r_i and σ_{ih} are the returns and the correlations relating to the first time horizon while $\hat{r}_i$ and $\hat{\sigma}_{ih}$ are the returns and the correlations relating to the second time horizon, the model with two time horizons is stated as follows:

$$Min \quad \delta_1^+ + \delta_1^- + \delta_2^+ + \delta_2^- + \delta_3^+ + \delta_3^- + \delta_4^+ + \delta_4^- \tag{7}$$

Subject to:

$$\sum_{i,h=1}^{6} x_i x_h \sigma_{ih} - \delta_1^+ + \delta_1^- = 1.2$$

$$\sum_{i=1}^{6} r_i x_i - \delta_2^+ + \delta_2^- = 1.6$$

$$\sum_{i,h=1}^{6} x_i x_h \hat{\sigma}_{ih} - \delta_3^+ + \delta_3^- = 1.3$$

$$\sum_{i=1}^{6} \hat{r}_i x_i - \delta_4^+ + \delta_4^- = 1.7$$

$$\sum_{i=1}^{6} x_i = 1$$

Using this multiple time horizon approach to estimate an optimal portfolio provides a recommendation for 78% investment in maize, and 22% in millet.

4 Conclusion

Smallholder farmers face complicated and risky decisions when deciding what to grow, and how to allocate limited resources. Like portfolio managers in other contexts, they desire large returns but also strive to minimize risk. Indeed, there is evidence that smallholder farmers are exceptionally risk averse. Portfolio optimization is one way to more efficiently allocate resources and productive capacity to investments with respect to a DM's preferences that provide an effective risk return profile given available information.

In the preceding sections we introduce optimization problems as applied to smallholder farming. These problems are then explored using real data to highlight several approaches to solutions using classical Markowitz Portfolio optimization, goal programming, and multi-horizon goal programming.

These examples highlight methods that could potentially be applied in developing country contexts to more efficiently allocate investments given available information. Depending on the preferences of farmers with respect to risk and return, portfolio optimization techniques could more systematically take into account the competing interests that farmers consider. Although it is unlikely that individual farmers could leverage such approaches individually, advice through agricultural extension or professional support for smallholder farmers could incorporate the lessons of such analyses. Using widely available optimization computer programs, the techniques presented in this paper are flexible, and could help in developing more ideal investment portfolios for smallholder farmers than are available without such aids.

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Appendix A

Lingo Solver Results

Objective value:	0.1218541
Infeasibilities:	0
Total solver iterations:	28
Model Class:	NLP
Total variables:	6
Nonlinear variables:	6
Integer variables:	0
Total constraints:	3
Nonlinear constraints:	1
Total nonzeros:	18
Nonlinear nonzeros:	6

Variable	Value	Reduced Cost
TARGET	1.6	0
AMT(WHEAT)	0.1905212	0
AMT(BARLEY)	0	2.45E-02
AMT(MAIZE)	0	0.1446722
AMT(MILLET)	0	2.34E-02
AMT(SORGHUM)	0	5.07E-02
AMT(OATS)	0.8094788	0

	Row	Slack or Surplus Dual
VAR	0.1218541	-1
BUDGET	0	-5.05E-02
RETURN	0	-0.1207283

Appendix B

Lingo Solver Results Model 2

Objective value:	0.9556729
Infeasibilities:	0
Total solver iterations:	24
Model Class:	NLP
Total variables:	10
Nonlinear variables:	6
Integer variables:	0
Total constraints:	4
Nonlinear constraints:	1
Total nonzeros:	26
Nonlinear nonzeros:	6

Variable	Value	Reduced Cost
TARGET	1.6	0
AMT(WHEAT)	0	0.3413241
AMT(BARLEY)	0	9.13E-02
AMT(MAIZE)	2.08E-02	0
AMT(MILLET)	0.9791839	0
AMT(SORGHUM)	0	9.19E-02
AMT(OATS)	0	0.1902718
N(1)	0	2
N(2)	0	1.4761
P(1)	1.355673	0
P(2)	0	0.5239001

	Row	Slack or Surplus Dual Price
OBJECTIVE	1.355673	-1
VAR	0	-1
RETURN	0	0.4760999
BUDGET	0	-0.2731057

Appendix C

Lingo Solver Results Model 3

Objective value:		2.122139
Infeasibilities:		0
Total solver iterations:		26
Model Class:	Class:	NLP
Total variables:	variables:	14
Nonlinear variables:	variables:	6
Integer variables:	variables:	0
Total constraints:	constraints:	6
Nonlinear constraints:	constraints:	2
Total nonzeros:	nonzeros:	46
Nonlinear nonzeros:	nonzeros:	12

Variable	Value	Reduced Cost
TARGETA	1.6	0
TARGETB	1.7	0
AMT(WHEAT)	0	0.890948
AMT(BARLEY)	0	0.2696568
AMT(MAIZE)	0.7756934	0
AMT(MILLET)	0.2243066	0
AMT(SORGHUM)	0	0.1081058
AMT(OATS)	0	0.6117057
N(1)	0	2
N(2)	0	2
N(3)	0.5193982	0
N(4)	0	0.4851392
P(1)	0.7901769	0
P(2)	0.8125635	0
P(3)	0	2
P(4)	0	1.514861

	Row	Slack or Surplus Dual Price
OBJECTIVE	2.122139	-1
VARA	0	-1
VARB	0	-1
RETURNA	0	1
RETURNB	0	0.5148608
BUDGET	0	-1.200142