Void and Spatiality

in Epicurus' *Letter to Herodotus*

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Abstract

This thesis analyses the notion of void (*to kenon*) in Epicurus' *Letter to Herodotus*: I argue that void should not be identified with space, but only with the empty intervals between bodies.

The thesis is composed by four chapters. In Chapter 1, I maintain that the Epicurean void is meant to fulfil two different functions (that I call the 'elemental' and the 'spatial' functions): I discuss the historical roots of the attribution of these functions to the void, and suggest how they could be brought together in a unitary, hybrid conception.

In Chapter 2, I examine how void, interpreted in this way, could make motion possible. I focus on the *eixis* of void, which I do not interpret in terms of receptivity, and draw a comparison with a contemporary metaphysical theory of holes to show that the resulting conception is coherent.

In Chapter 3, I discuss the theory of the *minimae partes* in the atom, offering an original interpretation of the notions of 'part' and 'division' on which the theory relies.

In Chapter 4, I analyse the consequences of this theory for the structure of void. I argue that the theory of minima naturally leads to suppose that motion is quantized: because of the Isomorphism Thesis, also time and spatial magnitudes must be composed by minima. However, I conclude that void in itself has not a granular structure, but is affected by the quantization only in a very peculiar and restricted sense.

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My dad likes to talk about his childhood, and there is an anecdote that has always struck me. It is April of 1970, and the world has one eye fixed on the Moon and the other on the TV, following anxiously the fate of Apollo 13 and its three passengers. The interminable wait is filled by the familiar voice of a famous journalist, who covers all lunar expeditions. With his passionate and intriguing voice, he vividly illustrates all the terrible dangers that the three astronauts face: the loss of cabin heat, the shortage of water, the danger of going astray in space… and then, if this were not enough, 'if the wall of the spaceship breaks, *the void comes in!*'

At this point in the story, my dad always pauses for a couple of seconds. Then, looking straight into my eyes, he asks: 'and I mean, how can the void *come in*?!'

My dad's confusion is caused by the clash between what David Sedley calls the 'two conceptions of vacuum':[1] void can be considered either as the space in which things are positioned and in which they move, or as something which occupies the space – something which has a position, a shape, and can be a subject of motion. To visualise this second conception of void in a simple way, think of Torricelli's experiment: the vacuum created in the column of mercury has a well-defined shape, a position, and undoubtedly moves with the column when the latter is carried around. One could also think of a gap in traffic according to the same model.[2] In these cases, void is not identified with space, but rather thought of as an absence, an emptiness which is positioned in space.

Both these conceptions are legitimate ways of understanding the void, and both appear in everyday discourse about it. In spite of this, though, they do not seem to be completely compatible: when they accidentally come into direct contact, a certain tension and confusion arise. This tension is responsible for my dad's puzzlement: if the spaceship

[1] Sedley 1982.
[2] This is one of the examples used by Sedley.

is located, and moves, in the void-as-space, this same void cannot also be the void-as-emptiness which can be the subject of motion and break into the spaceship.

Usually we do not notice this tension, because the context makes clear which function the void should play – and consequently, according to which model it should be conceived. We could sketchily present the two main functions of void in this way:

a) Elemental function: the void constitutes the complement of matter. It is one of the constituents of the universe and enters in compounds;
b) Spatial function: the void makes local motion possible; it provides a frame of reference, defines locations, and hosts bodies.

The two conceptions of void identified by Sedley are associated with those two functions in a very clear and simple way: void-as-emptiness fulfils the elemental function, while void-as-space fulfils the spatial function. Notice that the conception of void as space is not compatible with the conception of void as emptiness because the way in which void-as-space fulfils the spatial function prevents it from also fulfilling the elemental function.

Is it possible to think of void in such a way that it fulfils *both* these functions at the same time? I believe that Epicurus tries to do that: this thesis is an investigation on his peculiar, hybrid conception of void.

In the first part, I will tackle directly a set of fundamental issues linked to the overall coherence of the project, and try to understand what such a conception of void could look like. Notably, in Chapter 1, I will look at the historical roots of his notion, tracing the focus on the elemental function back to the Early Atomists, and the introduction of the spatial function to Aristotle. I will then claim that the two functions are combined by Epicurus in a unitary and original notion. In Chapter 2, I will analyse in further detail how this original conception of void can be presented as a necessary condition for local motion.

The second part of the thesis will be concerned with the properties, characteristics and structure of the Epicurean void. I will focus in particular on the interactions between the hybrid conception of void and some kinematic theses attributed to the Epicureans. Notably, Themistius and Simplicius attribute to Epicurus a quantized theory of motion, time, and spatial magnitudes. Does this imply that the *void* is composed by indivisibles? To assess this claim, I will first discuss, in Chapter 3, the doctrine of the *minimae partes* in the atom: the doctrine of quantized motion is, in fact, a direct consequence and development of this theory. In Chapter 4, I will assess how this kinematic theory enforces a discrete structure of spatial magnitudes, and what the consequences of this are on the structure of void, once we refuse to identify void with space.

Before getting started, a couple of methodological and terminological remarks are in order. First, because of the amplitude of the analysis, and the relatively short space at my disposal, I will limit the inquiry to the analysis of the *Letter to Herodotus*, which is an early work,[3] and give relatively little attention to the other *testimonia* and to Lucretius' *De Rerum Natura*, as well as to the later developments of Epicurus' doctrine.

The second *proviso* is that I am assuming that Epicurus was familiar with Aristotle's scholastic treatises, and that he took Aristotle's objections to the Early Atomists into account while developing his own theory. This is nowadays usually accepted, but is far from being uncontroversial:[4] the reader should bear in mind that I am reading Epicurus' text under this quite heavy assumption.

Finally, on terminology. In what follows, I will use the term 'space' in a restricted sense, to refer uniquely to an infinite, three-dimensional extension, existing independently of bodies and able to host them by allowing for superposition with them. Limiting the use of the term 'space' in such a way is not always correct: there are on the

[3] See Verde 2013: 14-15 for a discussion of the chronology.
[4] See Verde 2013: 4-9 for a useful *résumé* of the state of the debate.

market other theories (such as Leibniz' relational theory of space, for example), which make no reference to such an extension, but are legitimately considered theories *of space*. With this terminological choice, I do not want to claim that a Newtonian conception of space is the only correct or respectable one. On the contrary, this decision is purely guided by the practical needs of the inquiry: the infinite three-dimensional extension plays an important role in the analysis, and it is practical to have a simple term to refer to it. Moreover, such a terminological choice allows us to make sense of, and verify, the claim that Aristotle's universe is a universe 'without space':[5] it is hence coherent with an important tradition in Ancient Philosophy scholarship.

[5] Quarantotto 2017.

Chapter 1. The Nature of Void

In the natural language of everyday conversation, void is presented as able to fulfil two very different functions, that I have called the 'elemental' and the 'spatial' functions. In this chapter, I will first discuss the history of these functions: in §1.1, I will show that the elemental function is central in the Early Atomists' conception of void, which associates void with emptiness; in §1.2, on the contrary, I will claim that Aristotle moves the focus from the elemental to the spatial function, and in so doing he modifies considerably the conception of void, which he associates to place. In §1.3 and §1.4, I will suggest that Epicurus draws from both traditions, and that both functions can be identified in his treatment of void: in §1.5, I will examine whether this lead to a unitary and coherent conception.

§1.1. The Elemental Function: Leucippus, Democritus, and Void-as-Emptiness

As the elemental and the spatial functions of void are not immediately compatible, it should not be surprising to find out that they have very distinct histories, and were not always associated. Notably, there is no trace of a spatial function in the conception of void developed by the Early Atomists, Leucippus and Democritus: as David Sedley has brilliantly demonstrated,[6] they conceive of void as emptiness, as some kind of negative element, the complement of matter.

From an historical perspective, it is easy to understand why Leucippus and Democritus conceive of void not as space, but rather as a sort of negative element – something which is situated in a place, can move and can participate in compounds, but does not have any materiality nor positive nature. Indeed, the Early Atomists' physical

[6] This section relies heavily on the first part of Sedley 1982.

theses should be understood as a reply to the Eleatic denial of the possibility of change. We have two versions of the Eleatic argument against change: on the one hand, Parmenides denies the possibility of change, as it would require the existence of not-being; on the other hand, Melissus denies the possibility of local motion, as it would require the existence of void. Leucippus and Democritus seem to accept the Eleatic arguments, but are ready to bite the bullet: if void and not-being are necessary for change and motion, they exist – and they are the same thing.

Let us first focus on Parmenides. The Parmenidean distinction between being and not-being is evidently a distinction between two contradictory elements: being opposite and specular, they are thought of under the same model. If Democritus is accepting this structure, and simply identifying atoms with being, and void with not-being, the resulting conception of void cannot help but be element-like: void is conceived of as nothingness, as a negative element.

This hypothesis is confirmed by the analysis of the Early Atomists' nomenclature for void and atoms. Sedley notices that they use three pairs of opposites: the existent and the non-existent (*to on kai to me on*); the thing and the nothing (*to den kai to meden*); the full and the empty (*to pleres kai to kenon*). In all three cases, the two terms are placed exactly on the same level, and presented as contradictory and complementary to one and other. This means that the distinction between void and atoms can be understood by analogy with a mathematical partition of a unitary domain into two mutually exclusive and jointly exhaustive sub-sets. The two terms are mirror-like, hence structurally similar: both 'the existent' and 'the thing' are elements that occupy a place, so that their contradictories should be thought as occupying a place too. Only the third pair of terms seems slightly problematic: however, if it is more natural for us to associate 'the empty' with empty *space* rather than with the emptiness located in that space, it seems that this association would cause problems when it comes to the atoms. Indeed, if 'the empty' is empty space,

‘the full’ should be identified with filled space – not with the atoms which fill it! On the contrary, it easy to see that ‘the full’ refers to the atom, which is the principle of being, and is compact, with no internal gaps. Hence, we should take both terms of the pair as referring to the occupants of space: ‘atoms and void’ is the answer to the basic ontological question, ‘what is there in the universe?’, where ‘to be in’ maintains its spatial connotation.

In spite of this, commentators have often interpreted the Early Atomists’ notion of void as space. This is probably due to the fact that Melissus’ argument seems indeed to have a ‘spatial flavour’: if void is supposed to enable local motion, it is quite natural to think of it as empty space. However, the spatial interpretation of the argument, which is mostly due to Aristotle, is surely not the only one possible. The argument itself runs as follows:

> οὐδὲ κενεόν ἐστιν οὐδέν· τὸ γὰρ κενεὸν οὐδέν ἐστιν· οὐκ ἂν οὖν εἴη τό γε μηδέν. οὐδὲ κινεῖται· ὑποχωρῆσαι γὰρ οὐκ ἔχει οὐδαμῆι, ἀλλὰ πλέων ἐστίν. Εἰ μὲν γὰρ κενεὸν ἦν, ὑπεχώρει ἂν εἰς τὸ κενόν· κενοῦ δὲ μὴ ἐόντος οὐκ ἔχει ὅκηι ὑποχωρήσει. πυκνὸν δὲ καὶ ἀραιὸν οὐκ ἂν εἴη. τὸ γὰρ ἀραιὸν οὐκ ἀνυστὸν πλέων εἶναι ὁμοίως τῶι πυκνῶι, ἀλλ’ἤδη τὸ ἀραιόν γε κενεώτερον γίνεται τοῦ πυκνοῦ. κρίσιν δὲ ταύτην χρὴ ποιήσασθαι τοῦ πυκνοῦ. κρίσιν δὲ ταύτην χρὴ ποιήσασθαι τοῦ πλέω καὶ τοῦ μὴ πλέω· εἰ μὲν οὖν χωρεῖ τι ἢ εἰσδέχεται, οὐ πλέων· εἰ δὲ μήτε χωρεῖ μήτε εἰσδέχεται, πλέων. ἀνάγκη τοίνυν πλέων εἶναι, εἰ κενὸν μὴ ἔστιν. εἰ τοίνυν πλέων ἐστίν, οὐ κινεῖται.

> And nothing of it is empty. For what is empty is nothing. Well, what is nothing could not very well exist.
> Nor does it move. For it cannot give way at any point, but is full. For if there were such a thing as empty it would give way into what was empty; but since there is not such a thing as empty, it has no point at which to give way. (Dense and rare could not exist. For what is rare cannot be as full as what is dense, but what is rare already thereby becomes emptier than what is dense. And that is the criterion for distinguishing between what is full and what is not full. Hence if something gives way or accommodates, it is not full,

but if it neither gives way nor accommodates, it is full). So it must be full, if there is no such thing as empty; and so if it is full, it does not move.

Melissus 30 B 7.7-10 (Sedley's transl.)

The core of the argument is that void *gives way* (*hypochorein*) to bodies. This can surely be interpreted, as Aristotle does, in terms of receptivity: void can be thought as an independently existing extension, which is empty but can be filled by bodies – i.e. can host bodies by allowing for superposition and co-location. Other interpretations, however, are possible: for example, as Sedley suggests, we can take the argument to speak of 'an internal *admixture* of void, which would make what exists rare or spongy and thus enable it to 'give way' (*hypochorein*) at some point'.[7] But we can also imagine that void is able to disappear when bodies approach, or to be reconfigured by their motion. We will discuss these possibilities in detail in Chapter 2: for the moment, it is sufficient to stress that the argument only claims that void is necessary for motion because it is able to give way to bodies, but does not necessarily require that void is interpreted as space. Hence, the analysis of the argument does point to a shortcoming of the Early Atomists' arguments, as they do not seem to clarify how precisely void is supposed to make motion possible, but it does not constitute a counter-example to the elemental interpretation of void, nor enforce a spatial interpretation.

Conceiving void as a negative element which constitutes the complement of matter gives to the Early Atomists a powerful argumentative tool to argue for its existence. The symmetry between void and atoms is indeed at the heart of Democritus' famous '*ou mallon* argument': he can claim that 'the existent no more exists than the non-existent' precisely because atoms and void are opposite and specular, but fundamentally similar, and both located in a place.

[7] Sedley 1982: 178. This seems to be the image suggested by the text in brackets: void would be a component, an element of bodies. A higher or lower presence of void in bodies makes them rare or dense: rare bodies, which contain a lot of void, have the ability to give way or accommodate to other bodies.

Moreover, conceiving of the two elements as formal contradictories leads to a very elegant ontological scheme. As Sedley says, 'by relying on the plausible and widespread assumption that to exist is to occupy a place, the atomists could comfortably sit back and allow any existing thing, i.e. any occupant of place, to sort itself out into one or other of their pair of contradictory elements or into a complex of the two. The Law of the Excluded Middle would ensure that nothing escaped classification under the scheme'.[8]

§1.2. The Spatial Function: Aristotle and Void-as-Place

While the elemental function is at the centre of the Early Atomists' conception of void, the spatial function can be traced back to Aristotle. Despite being one of the fiercest adversaries of void, Aristotle plays a fundamental role in the evolution and refinement of the concept: he focuses his criticisms against a very specific conception of void, which he selects from the various conceptions proposed by his opponents. In criticising it, he also refines the notion, which he characterises in fundamentally spatial terms: 'those who say there is void suppose it to be a kind of place and a vessel; it is thought to be a plenum when it contains the extended body it is capable of receiving, and void when deprived [of that body], the supposition being that void and plenum and place are the same thing, though their being is not the same'.[9]

This conception of void is evidently built around the spatial function: the way in which it fulfils it is by being receptive, i.e. able to host bodies by allowing them to penetrate it and superpose with it.[10] Void is conceived as an extension which is empty, but can host bodies: the ability to allow for superposition (receptivity) is so important that Aristotle uses it to determine whether an extension can classify as void. At *Ph.* IV.7,

[8] Sedley 1982: 180.
[9] Arist. *Ph.* IV.6, 213a15-19: οἷον γὰρ τόπον τινὰ καὶ ἀγγεῖον τὸ κενὸν τιθέασιν οἱ λέγοντες, δοκεῖ δὲ πλῆρες μὲν εἶναι, ὅταν ἔχῃ τὸν ὄγκον οὗ δεκτικόν ἐστιν, ὅταν δὲ στερηθῇ, κενόν, ὡς τὸ αὐτὸ μὲν ὂν κενὸν καὶ πλῆρες καὶ τόπον, τὸ δ' εἶναι αὐτοῖς οὐ ταὐτὸ ὄν. All translations from *Physics* IV are Hussey's.
[10] See Hussey 1983: xxxv.

214a10 ff. he specifies that an extension which has colour or sound can count as void only if it could receive tangible bodies.[11]

The fact that Aristotle attributes this view to 'those who say there is void' might seem not completely fair, as we just saw that the Early Atomists' conception of void does *not* involve any reference to receptivity, and is built around the elemental rather than the spatial function. However, it should be noticed that when Aristotle reports the Atomists' views directly, he makes no reference to receptivity: on the contrary, he presents their void as an 'extension, whether separable or in actual operation, distinct from bodies, which separates the sum of body so that it is not continuous'.[12] Here, void is explicitly contrasted to bodies, as if they were two incompatible elements.

Because of this, I prefer not to speak of a misrepresentation or falsification of the Atomists' views in Aristotle's report. Rather, I believe that in *Physics* IV Aristotle is carrying out a sophisticated operation of disambiguation and clarification of the Atomists' conception of void, which focuses on the weakest and more confused aspects of it,[13] and ultimately leads to switch the focus from the elemental to the spatial function of void. Unfortunately, this is not the place where to develop this view, as this would lead us astray from our main focus. In investigating the Aristotelian roots of Epicurus' conception of void, it is besides the point whether Aristotle misrepresents the views of his opponents, or is legitimately exploring the weaknesses of their conceptions by pushing them to their extreme but necessary conclusions. What is really important for us is to make it clear that Aristotle focuses precisely on the aspects which are left unexplored by Leucippus and

[11] Tangibility is also used to distinguish different conceptions of void: as void is fundamentally receptive (i.e. able to host bodies) and potentiality is defined in relation to an actuality, we can distinguish between void as extension which is able to receive a tangible body (i.e. void-as-place, as we will see shortly) and void as unqualified extension which is able to receive determination, i.e. void-as-matter (*Ph.* IV.7, 214a12 ff.) As this latter conception is quickly dismissed, I will not discuss it here.

[12] Arist. *Ph.* IV.6, 213a32-34: διάστημα ἕτερον τῶν σωμάτων, οὔτε χωριστὸν οὔτε ἐνεργείᾳ ὄν, ὃ διαλαμβάνει τὸ πᾶν σῶμα ὥστε εἶναι μὴ συνεχές. The formula appears in a negative context, hence the 'οὔτε'. Notice that the receptivity is also absent from the report of Pythagorean views on void.

[13] As we saw in §1.1, one of the major problems of the Early Atomists' conception is precisely that it is not clear how void should make motion possible: the introduction of receptivity is meant to clarify this aspect.

Democritus, namely how precisely void is supposed to enable local motion: the results are (i) the emphasis on the spatial function on void, (ii) the association of *kenon* and *topos*, and (iii) the distinction of void from nothing.

Aristotle's discussion of void is long and complex: it will be impossible to take all its notable aspects into account here. I only want to point out that the introduction of the feature of receptivity marks the shift from the elemental to the spatial function of void: this can be seen in his interpretation of Melissus' argument. Indeed, Aristotle takes void to enable motion because of its ability to receive bodies, which he spells out in terms of superposition and co-location: void is supposed to be able to do what bodies cannot do, i.e. allowing superposition.

The importance of receptivity is even more evident in the so-called 'argument of the cube':

> καὶ καθ' αὑτὸ δὲ σκοποῦσιν φανείη ἂν τὸ λεγόμενον κενὸν ὡς ἀληθῶς κενόν. ὥσπερ γὰρ ἐὰν ἐν ὕδατι τιθῇ τις κύβον, ἐκστήσεται τοσοῦτον ὕδωρ ὅσος ὁ κύβος, οὕτω καὶ ἐν ἀέρι· ἀλλὰ τῇ αἰσθήσει ἄδηλον. καὶ αἰεὶ δὴ ἐν παντὶ σώματι ἔχοντι μετάστασιν, ἐφ' ὃ πέφυκε μεθίστασθαι, ἀνάγκη, ἂν μὴ συμπιλῆται, μεθίστασθαι ἢ κάτω αἰεί, εἰ κάτω ἡ φορὰ ὥσπερ γῆς, ἢ ἄνω, εἰ πῦρ, ἢ ἐπ' ἄμφω, [ἢ] ὁποῖον ἄν τι ᾖ τὸ ἐντιθέμενον· ἐν δὲ δὴ τῷ κενῷ τοῦτο μὲν ἀδύνατον (οὐδὲν γὰρ σῶμα), διὰ δὲ τοῦ κύβου τὸ ἴσον διάστημα διεληλυθέναι, ὅπερ ἦν καὶ πρότερον ἐν τῷ κενῷ, ὥσπερ ἂν εἰ τὸ ὕδωρ μὴ μεθίστατο τῷ ξυλίνῳ κύβῳ μηδ' ὁ ἀήρ, ἀλλὰ πάντῃ διήεσαν δι' αὐτοῦ. ἀλλὰ μὴν καὶ ὁ κύβος γε ἔχει τοσοῦτον μέγεθος, ὅσον κατέχει κενόν· ὃ εἰ καὶ θερμὸν ἢ ψυχρόν ἐστιν ἢ βαρὺ ἢ κοῦφον, οὐδὲν ἧττον ἕτερον τῷ εἶναι πάντων τῶν παθημάτων ἐστί, καὶ εἰ μὴ χωριστόν· λέγω δὲ τὸν ὄγκον τοῦ ξυλίνου κύβου. ὥστ' εἰ καὶ χωρισθείη τῶν ἄλλων πάντων καὶ μήτε βαρὺ μήτε κοῦφον εἴη, καθέξει τὸ ἴσον κενὸν καὶ ἐν τῷ αὐτῷ ἔσται τῷ τοῦ τόπου καὶ τῷ τοῦ κενοῦ μέρει ἴσῳ ἑαυτῷ. τί οὖν διοίσει τὸ τοῦ κύβου σῶμα τοῦ ἴσου κενοῦ καὶ τόπου; καὶ εἰ δύο τοιαῦτα, διὰ τί οὐ καὶ ὁποσαοῦν ἐν τῷ αὐτῷ ἔσται;
>
> But even if we consider it in itself, that which is called the void will appear void indeed. Just as, if one places a cube in water, water in equal amount to the cube will be displaced, so it is with air too, though this is not obvious to sense-perception, and so too it must always be with every body that admits of change of position: that if it is not compressed

> it changes position, to the extent that its nature allows, (either always downwards, if its motion, like that of earth, is downwards, or upwards, if it is fire, or either way), of whatever kind the inserted object may be. But in the void this is impossible, for it is not any body, and [it must be that] the extension equal [to the cube], which was there previously in the void, has permeated the cube, just as if the water or the air were not to be made to change position by the wooden cube, but were to permeate it at every point. And yet the cube, too, has a magnitude equal in amount to the void it occupies, and even if this – I mean the extended body of the wooden cube – is hot or cold or heavy or light, yet none the less it is different in being from all the qualities even if not separable [from them]. Hence (even if it were separated from all the other things and were neither heavy nor light), it will occupy an equal amount of void and it will be in the same spot as the part of place and the part of void equal to itself. How then will the body of the cube differ from the equal amount of void and place? And if there are two such things, why should there not be any number of things in the same spot?
>
> Arist. *Ph.* IV.8, 216a26-b11

The problem that Aristotle is tackling is: what happens when a body approaches the void? He presents two main possibilities: the void could be displaced, or it could stay still and be penetrated by the body.[14] The first option is immediately refused by Aristotle, as it would require a mechanical interaction between void and the bodies. Aristotle claims that this is impossible because 'the void is not any body': plausibly the reasoning is that the void, not being a body, is intangible, and cannot interact with bodies. The second option, which obviously involves the idea of receptivity, is taken in more serious consideration, and a full-fledged argument is dedicated to its refutation. Unfortunately, the argument itself is quite obscure and difficult to understand completely. It is nonetheless worth looking at it, as it proves insightful regarding the notion of void which Aristotle is attacking.

[14] Sextus Empiricus presents essentially the same argument at *M.* 10. 24-36. Interestingly, he adds a third (unsatisfactory) option to the two selected by Aristotle: void could simply disappear. See also Sedley 1982.

The core idea of the argument is that the interpenetration of the cube and the void would lead to unacceptable consequences.[15] Notably, the superposition of the void and the volume (magnitude) of the cube would make it impossible to distinguish them: it would even become impossible to tell whether in that place there is one thing, or two. As if this were not paradoxical enough, Aristotle adds that this would ultimately lead to jeopardize the general principle of identification of bodies, because if two things can occupy the same spatio-temporal location, how could we tell that there are not *more* things in that very same place? This would lead to the impossibility of counting, and identifying, bodies.

But how does Aristotle arrive at this dramatic conclusion? Christian Pfeiffer claims that he treats the superposition of the void and the bulk of the cube as an infraction of the principle of non-coincidence. This is a principle that does not apply to *everything*, but only to bodies: it can be invoked in this situation only because Aristotle thinks that 'body is a three-dimensionally extended magnitude that is divisible in all three dimensions and is bounded by surfaces'.[16] Even if void is not material, then, 'the way of coincidence between void and body is structurally the same as the coincidence of two bodies'.[17] I will not be able to discuss here the details of Pfeiffer's interpretation. It seems clear to me, however, that he is pointing the finger to an aspect that is fundamental in the argument: void is here treated as a *bounded* extension.

This is a very important point. We said that in *Ph.* IV, Aristotle gradually singles out the particular conception of void that he finds the most interesting and threatening, and concentrates his efforts on refuting exactly this notion. We also said that he takes it to be primarily associated with the spatial function, because void is presented as receptive, hence capable of hosting bodies: this makes it different from the elemental conception of

[15] I rely here on Pfeiffer 2016's interpretation of the argument, which I find the most convincing.
[16] Pfeiffer 2016: 366.
[17] Pfeiffer 2016: 380.

void adopted by the Atomists, which was more or less interchangeable with the notion of nothingness.

The argument of the cube provides the final pieces of information necessary to have a complete picture of this notion: void is supposed to be immobile and receptive, extended and bounded. This is why Aristotle claims that void is a *topos*: it is a limited and well defined place, somewhere in which bodies could be, and which is involved in the explanation of local motion, but it is not space, as it is not coincident with the totality of possible positions.

§1.3. The Spatial Function of Void in Epicurus

It is undoubtable that Epicurus follows Aristotle in attributing a spatial function to the void. The main argument for the existence of void is, after all, a hypothetical argument which invokes two fundamental aspects of the spatial function: the ability to provide location, and the role in the explanation of local motion:

> ἀλλὰ μὴν καὶ τὸ πᾶν ἐστι <σώματα καὶ κενόν>· σώματα μὲν γὰρ ὡς ἔστιν, αὐτὴ ἡ αἴσθησις ἐπί πάντων μαρτυρεῖ, καθ'ἣν ἀναγκαῖον τὸ ἄδηλον τῷ λογισμῷ τεκμαίρεσθαι, ὥσπερ προεῖπον. Τόπος δὲ εἰ μὴ ἦν, ὃν κενὸν καὶ χώραν καὶ ἀναφῆ φύσιν ὀνομάζομεν, οὐκ ἂν εἶκε τὰ σώματα ὅπου ἦν οὐδὲ δι'οὗ ἐκινεῖτο, καθάπερ φαίνεται κινούμενα. Παρά δέ ταῦτα οὐθὲν οὐδ'ἐπινοηθῆναι δύναται οὔτε περιληπτῶς ὔυτε ἀναλόγως τοῖς περιληπτοῖς ὡς καθ'ὅλας φύσεις λαμβανόμενα καὶ μὴ ὡς τὰ τούτων συμτώματα ἢ συμβεβηκότα λεγόμενα.
>
> Moreover, the totality of things is <bodies and void>. That bodies exist is universally witnessed by sensation itself, in accordance with which it is necessary to judge by reason that which is non-evident, as I said above; and if place,[18] which we call 'void', 'room'

[18] The text that I have reported is the one that can be found in Long and Sedley's anthology. However, notice that it is uncertain in two key points: the first one is the dramatic *lacuna* in correspondence with the basic division. Gassendi's integration < σώματα καὶ κενόν > is almost universally accepted over Usener's proposal < σώματα καὶ τόπος >, as it is confirmed by other sources and corresponds to Lucretius 'corpora sunt et inane' at 1.420. The second emendation is more controversial, though: were Long and Sedley follow Usener and read '…ὥσπερ προεῖπον. Τόπος δὲ εἰ μὴ ἦν, ὃν κενὸν…', Gassendi proposes '…ὥσπερ προεῖπον τὸ πρόσθεν. Εἰ <δὲ> μὴ ὃ κενὸν…'. Usener's emendation pushes towards a spatial interpretation

> and 'intangible substance', did not exist, bodies would not have anywhere to be or to move through in the way they are observed to move. Beyond these [i.e. body and void][19] nothing can even be thought of, either by imagination or by analogy with what is imagined, as completely substantial things and not as the things which we call accidents and properties of these.
>
> Epicurus, *Ep. Hdt.* 39-40[20]

This leads Sedley to believe that void has only (or at least primarily) a spatial function, and should hence be interpreted according to the conception that nowadays we tend to associate with the spatial function of void: as *space*. According to Sedley, Epicurus follows Aristotle in associating void with *topos*, and this leads to the 'first clear recognition of geometrical space as a three-dimensional extension which persists whether or not it is occupied by body'.[21] Space is an infinite three-dimensional extension, which is partially occupied by bodies and partially empty, thus constituting that in which bodies are located, and in which they move. The difference between the parts which are occupied by bodies and those which are not is recorded in the language, as we say that only the latter are void, but is not a difference in substance.

Notice that the idea of void-as-space that Sedley attributes to Epicurus is different, and more sophisticated, than Aristotle's notion of void-as-place. Void-as-space would be an *infinite* and unbounded extension, thus coinciding with all possible locations – a much more encompassing concept than void-as-place, which, being a bounded extension, can at best be identified with a proper part of space.

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of void, as it explicitly states that void is to be identified with place. I do not wish to enter the discussion apropos this emendation, since (as it will become clear in §1.5) I do not wish to negate that void fulfils the function of place (and the spatial function in general): I deny that the only way in which it can fulfil it is by being identified with space itself, as suggested by Sedley.

[19] Long and Sedley add 'body' to maintain the analysis with Lucretius' 'corpus', but Konstan 2014: 85-6 notices that it would be more correct to speak of *bodies*: even in Lucretius' poem, 'corpus' is used to refer to a particular body, not to matter in general.

[20] The translations from the *Letter to Herodotus* are from Long&Sedley 1987.

[21] Sedley 1982: 188.

Sedley's interpretation of void-as-space would allow Epicurus to avoid the dramatic consequence hypothesized by Aristotle, i.e. the loss of the principle of individuation of bodies. Aristotle believes that accepting the superposition of a body and of void-as-place woul be like opening Pandora's vase: once we let two things to co-locate, 'why cannot there be any number coinciding?'. In Epicurus' view, the reason would be that tangible substances could co-locate only with intangible ones, and void is the only existing intangible substance.[22]

So, the identification of void with space would provide a way out of the argument of the cube, and in fact the *only* way out that, according to Sedley, an Epicurean could accept. It is not possible to suppose that void is pushed away by bodies, as there is no possible mechanical interaction between the two;[23] it is not even possible to suppose that void disappears, because it is meant to be a stable and eternal principle of reality.

In support of his interpretation, Sedley brings two *testimonia*:

> διὸ προληπτέον ὅτι κατὰ τὸν Ἐπίκουρον τῆς ἀναφοῦς καλουμένης φύσεως τὸ μέν τι ὀνομάζεται κενὸν τὸ δὲ τόπος τὸ δὲ χώρα, μεταλαμβανομένων κατὰ διαφόρους ἐπιβολὰς τῶν ὀνομάτων, ἐπείπερ ἡ αὐτὴ φύσις ἔρημος μὲν καθεστηκυῖα παντὸς σώματος κενὸν προσαγορεύεται, καταλαμβανομένη δὲ ὑπὸ σώματος τόπος καλεῖται, χωρούντων δὲ δι'αὐτῆς σωμάτων χώρα γίνεται. Κοινῶς μέντοι φύσις ἀναφὴς εἴρηται παρὰ τῷ Ἐπικούρῳ διὰ τὸ ἐστερῆσθαι τῆς κατὰ ἀντίβασιν ἁφῆς.
>
> Therefore one must grasp that, according to Epicurus, of 'intelligible substance', as he calls it, one kind is named 'void', another 'place', and another 'room', the names varying according to the different ways of looking at it, since the same substance when empty of all body is called 'void', when occupied by a body is called 'place', and when bodies roam through it becomes 'room'. But generically it is called 'intangible substance' in Epicurus' school, since it lacks resistant touch.
>
> Sextus Empiricus, *Against the Professors* 10.2 (Usener 271, Sedley's transl.)

[22] *Ep. Hdt*. 67.
[23] *Ep. Hdt*. 67.

Ἐπίκουρος ὀνόμασιν [πᾶσιν] παραλλάττειν κενὸν τόπον χώραν.
Epicurus says that the difference between void, place and room is one of name.
Aëtius 1.20.2 (Usener 271, Sedley's transl.)

Epicurus would have coined the general term 'intangible substance' to refer to the whole of space, which is an infinite extension. From a metaphysical perspective, this extension is existentially independent, immutable, immobile, and unaffected by bodies. From a human perspective, however, it can be useful to differentiate the case in which this extension is empty from the case in which it is full: the names 'void', 'place' and 'space' all ultimately refer to the same intangible extension, but specifically designate it in the state of being empty of all body, occupied by a body, or traversed by a body. The difference, however, 'is one of context, not of essence'.[24]

A consequence of this interpretation is that void and atoms are not conceived as perfect contraries anymore: they are very different sort of things, and furthermore many parts of void are occupied by bodies. Sedley believes that this is the reason why Epicurus does not adopt Democritus' terminology, and avoids speaking of void as an element. However, the contraposition between the two is maintained by focusing on their essential attributes: bodies are tangible, while void is intangible. As tangible/intangible are true contradictories, the definitional symmetry and the complementarity of void and atoms is somehow recovered. This pair of contradictory characteristics can then be used to show that no other independent substance can exist:

Praeterea nil est quod possis dicere ab omni
corpore seiunctum secretumque esse ab inani,
quod quasi tertia sit numero natura reperta.
Nam quodcumque erit, esse aliquid debebit id ipsum
augmine vel grandi vel parvo denique, dum sit.

[24] Sedley 1982: 188.

Cui si tactus erit quamvis levis exiguusque,
corporis augebit numerum summamque sequetur.
Sin intactile erit, nulla de parte quod ullam
rem prohibere queat per se transire meantem,
scilicet hoc id erit, vacuum quod inane vocamus.

Beyond these there is nothing which you can call distinct from all body and separate from void, to play the role of a third discovered substance. For whatever will exist will have to be in itself something with extension, whether large or small, so long as it exists. If it has tangibility, however light and faint, it will extend the measure of a body and be added to its sum. Whereas if it is intangible, and unable to prevent anything from moving through it at any point, it will undoubtedly be the emptiness which we call void.

Lucretius 1.430-439 (Sedley's transl.)

However, Sedley's interpretation leads him to posit that 'under the definitional symmetry there now lurks a strong ontological asymmetry',[25] as void and body should be considered as two different 'orders of being', or 'orders of reality'.[26] They are complementary not because they are mutually exclusive and jointly exhaustive, but in the sense that they constitute the only two orders of being that are required to account for the universe: 'all other candidates for the title 'existent', including time, events and properties, can be accounted for as attributes of body, incapable of separate existence. Space alone cannot'.[27]

While I agree that Epicurus attributes a spatial function to the void, I do not accept Sedley's conclusion that it should be identified with the totality of space: in the next section, I will explain why I find this interpretation, and Sedley's argument, unsatisfying.

[25] Sedley 1982: 190.
[26] Sedley 1982: 191; Long&Sedley 1987: 30.
[27] Sedley 1982: 191.

§1.4. The Elemental Function of Void in Epicurus

The passage of the *Letter to Herodotus* quoted above shows clearly that the void is supposed to have spatial functions, as it is *where* atoms are located and *through which* they move. The same passage suggests, however, that Epicurus also attributes the elemental function to the void. This is clear first of all from the way in which the notion is introduced: Epicurus starts by considering what exists, *to pan*, as a unitary domain which he then divides into two classes – atoms and void. That this operation should be understood as division is clear, not only from the passage of the *Letter* quoted above, but also from the reports that we have regarding Epicurus' major physiological work, the *Perì Physeōs*. Both Plutarch and Cicero speak of a 'division' (*diaresis*; *dividit*), and Sextus Empiricus' *testimonium* has the same logical form.[28]

This choice demands attention, as it is not the most intuitive way to introduce a concept: it is plausible to suppose that such an argumentative strategy is conceived as a direct reference to the Early Atomists, and their elemental conception of void as complement of atoms. Moreover, the use that Epicurus makes of the division, and the context in which he introduces it, clearly indicate that void is supposed to be a complement constituent of the universe, to be thought of as contradictory to, and incompatible with, atoms.

Indeed, the division is introduced right after the statement of the principles of conservation: Epicurus claims that 'nothing comes into being out of what is not',[29] and that it is impossible that 'that which disappears were destroyed into what is not'.[30] Together, those principles grant that 'the totality of things was always such as it is now, and always will be'.[31] It is precisely this 'totality of things', *to pan*, which is said to be

[28] Fragments 74, 75 and 92 Usener. See also Inwood 1981: 276-7.
[29] *Ep. Hdt.* 38: οὐδὲν γίνεται ἐκ τοῦ μὴ ὄντος.
[30] *Ep. Hdt.* 38: τὸ αφανιζόμενον εἰς τὸ μὴ ὄν.
[31] *Ep. Hdt.* 38: τὸ πᾶν ἀεὶ τοιοῦτον ἦν οἷον νῦν ἐστι, καὶ ἀεὶ τοιοῦτον ἔσται.

bodies and void. As the principles of conservation are obviously supposed to make sure that the *elements that constitute* the universe are eternal and unchangeable, this statement should be taken as the declaration of what these elements are – not as the introduction of the two different 'orders of being' which are sufficient to *explain* reality.

The 'basic division' (as Long and Sedley call it) is not only a rhetorical artifice: it plays a fundamental part in Epicurus' argument. Notably, it is invoked to prove that no other independent substance can exist, apart from atoms and void: as the division operates a partition of the domain into two disjoint but exhaustive subsets, nothing can exist without falling into the one or the other. This argumentative strategy is natural and intuitive if void is associated with the elemental function – much less if we suppose that it only fulfils the spatial one. As we saw in §1.3, Sedley tries to fit his spatial interpretation into this argumentative strategy: he suggests that it is not necessary that the two contradictory terms are both conceived as constituent elements of reality, as long as they are duly contraposed – and they are, as their essential attributes, i.e. being tangible and being intangible, are true contradictories, and can hence be used as *fundamentum divisionis*.

This solution is surely ingenious, but it does not seem sufficient. On the face of it, it is not at all clear why body and void should be the *only* two 'orders of being' allowed in the system, and appealing to the opposition between tangibility and intangibility leads to this conclusion only if a very loaded premise is added: that whatever exists is extended. Without this premise, in fact, the argument could bring into existence also God, the soul, Platonic Forms, numbers, and many other fishy entities.

This premise seems indeed to be present in Lucretius' argument quoted above,[32] but it is not clear where it comes from: it is quite different from the traditional principles of

[32] At least according to Sedley 1982: 190: most editors transpose lines 434 and 435, but he retains the manuscript's order and takes *augmine* to mean 'extension' precisely in order to have this premise. The

being, such as the Aristotelian 'to be is to be in something',[33] or the principle 'to be is to be causally active' discussed by Plato.[34] Sedley tries to justify the introduction of this premise by comparing it to the premise used by Zeno of Elea: something without magnitude could not exist because when added to something else it could not increase that thing's size.[35] If this is the way in which the premise should be understood, however, it is blatantly incompatible with Sedley's own interpretation of void: void, understood as *space*, does not increase a thing's size when added to it – it should hence not exist.

Rather, I believe that the principle 'being is being extended' is a genuinely Epicurean principle of being, which Lucretius takes as certain, but whose formulation could be traced back to an analysis of the shared characteristics of void and atoms: once void is understood as a constitutive element of reality, it turns out that the only two independently existing substances are both extended. If this is the case, Lucretius' argument is invalid, as it is either incomplete or circular: the premise that being is being extended is either assumed without justification, or derived from (and depending on) the conclusion.

One might think that I am being uncharitable to Lucretius. However, it should be noticed that the argument which immediately follows the one we are taking into consideration has a considerable formal problem: Lucretius claims that void and bodies are the only two existing substances because all existing things are either able to enter into mechanical relations ('act upon something, or itself be acted upon by other things', 1.440-1) or are 'such that things can exist and happen in it' (1.442). This second argument is not valid, as Lucretius does not prove that, nor explain why, the two options should be exhaustive. It is hence plausible that he is making the same mistake also in the argument

LOEB edition, on the contrary, translates simply 'whatever is to be, that must be something in itself'. I will not contest this choice, however, as I think that Sedley's interpretation is problematic anyway.

[33] Arist. *Ph.* IV.1, 208a29-31.

[34] Pl. *Soph.* 246.

[35] Sedley 1982: 190; DK 29 B 2.

that immediately precedes it, and that he is indeed using as *fundamentum divisionis* a pair of characteristics which are not adequate to their role.

Epicurus does stress that void is essentially intangible, and he does contrast void's intangibility and the tangibility proper of the atoms.[36] But I believe that Sedley misrepresents the role that the contraposition tangible/intangible has in Epicurus' overall system. Notably, I reject his claim that Epicurus is forced to rely on this specific contraposition because he cannot make use of Democritus' three pairs of opposites, as they can only refer to space-occupiers and not to space itself. If it is true that he makes no use of the pairs 'being/not-being' and 'thing/nothing', it is not because they refer to space-occupiers: rather, it is because Epicurus tries to distinguish void from nothingness, and dissociate it from the idea of not-being. Indeed, he presents it in positive terms, describing it as a substance existing *per se*.[37] This kind of language is as 'elemental' as Democritus' terminology. Moreover, Epicurus does make use of the third of Democritus' pairs, i.e. the opposition between 'the void' and 'the full', and he uses it in such a way that tends towards the elemental interpretation: fullness is not a characteristic of occupied *space*, but an essential feature of atoms, the property of not having internal gaps.[38]

Fullness also seems to be a more fundamental property than tangibility. As Gábor Betegh notes, tangibility comes in degrees:[39] Lucretius himself points this out, speaking of 'quamvis levis exiguusque' tangible bodies. The different degrees of tangibility of compound bodies result from the different proportions of atoms and void from which they are composed: in itself, tangibility is the perceptible manifestation, through touch, of the more fundamental ability to repel bodies by entering in mechanical interactions with

[36] As a parenthetical remark, I will say that I agree with Konstan 2014: 85-6: Sedley is too quick in switching from atom*s* and bod*ies* to *body* in the sense of matter, as it seems that the plurality of atoms, their being many distinct individual elements, is fundamental. Focusing on the individuality of atoms breaks the symmetry between space and matter as 'orders of being' on which Sedley's interpretation is built. See also ft. 19 above.
[37] *Ep. Hdt.* 40.
[38] See *Ep. Hdt.* 41: πλήρη τὴν φύσιν ὄντα.
[39] Betegh 2014: 399 ff.

them. Compound bodies are tangible (in various degrees) because the atoms which compose them have the peculiar ability to interfere with other atoms' natural motion through collisions. This capacity, which is macroscopically manifested as tangibility, is grounded in the fullness of atoms.[40]

The reason why Epicurus speaks more of tangibility than of fullness is to be found in his peculiar epistemology: according to Epicurus all knowledge derives from experience, and while *all* experiences are always true, touch is the most reliable. This is because tactile experiences are the only ones which are directly concerned with the object itself, and not with its *eidola*: tactile experiences are a direct grasp of the object.[41]

Apart from these considerations, it is difficult to misinterpret, or to explain away, the explicit commitment to the mutual exclusiveness of body and void in Lucretius 1.507-9: 'wherever there is the empty space which we call void, there no body exists, while wherever body is in occupation, there the emptiness of void is totally absent'.

§1.5. Reconciling Spatial and Elemental Functions

The results that this analysis has produced until now are quite puzzling: it seems that the Epicurean notion of void should to fulfil *both* the spatial and the elemental function.

[40] As tangibility comes in degrees, it is more natural to think of intangibility not as a radically different and contradictory way of being, but as the degree zero of the very same perceptible property. To give a perceptual account of tangibility, it is necessary that both tangible and intangible substances are detectable: the ones as resistance, the others as lack of resistance. If we take void to be space, hence to be partially filled by bodies, its intangibility is never perceptually evident: there can be an intangible thing even where the hand meets a resistance. Indeed, according to Sedley's interpretation, there always is an intangible thing where the hand meets resistance. From a logical point of view, there is nothing wrong with this idea: intangibility allows for co-location, precisely because it is unable to offer resistance. However, spelling out the difference between tangible and intangible in those terms destroys its empirical basis: the distinction between tangible and intangible bodies is not perceptually detectable anymore, but is derived in a purely logical way. As such, it is less compatible with the overall Epicurean epistemology.

[41] On the contrary, vision can sometimes be misleading, without being false: visual experiences are experiences of the *eidola* emanated by the object, and *eidola* can be deformed by the clashes with other atoms along their way to the eye. Visual perception is hence always true, but in the same way in which photographs cannot lie: 'it is "true" not because it accurately depicts the shape and colours of the object itself – it may well not do, e.g. because of the distortion undergone by the images in transit, and the insensitivity of sense-organs to some grades of particle in them – but because it accurately reports the state of the images entering the eye, and thus provides *bona fide* evidence about the external object emitting the images' (Long&Sedley 1987: 85).

Before despairing and concluding that there is some deep inconsistency in Epicurus' theory, the reader should recall that the spatial function of void should not necessarily be identified with Sedley's conception of void-as-space, nor the elemental function with the Early Atomists' conception of void-as-emptiness. Those two conceptions are indeed incompatible, but it is maybe possible to find a third conception, which can fulfil both functions.

Let us briefly recapitulate what this hypothetical conception should do:

(i) Constitute a complement of atoms: be only where atoms are not (elemental function);

(ii) Participate in compounds (elemental function);

(iii) Provide a location for atoms, so that atoms are in the void and void is the place of atoms (spatial function);

(iv) Be a necessary condition of motion, providing something through which atoms can move (spatial function).

For the void to fulfil condition (i), it must not be identified with space, but with the empty intervals between atoms. Condition (ii) demands that it is conceived as some sort of space-occupier: we could think of it as an intangible entity, with a definite position and shape. In Chapter 2, I will try to shed light on the features of this entity – for now, I only want to verify that the four conditions do not necessarily lead to contradiction.

Condition (iii) is less problematic than it might seem. As Brad Inwood notices,[42] it is sufficient to go back to Aristotle to see that the fulfilment of this condition does not require any co-location or superposition between extensions: the Aristotelian theory of *topos* does not identify place with the extension occupied by a body, but with the internal boundary of the surrounding body. Epicurus modifies Aristotle's theory of limits, notably

[42] Inwood 1981.

by denying that they lack dimensions:[43] thus, the definition cannot be imported directly. However, in some particular cases, Aristotle relaxes his definition a little, and identifies the place not only with the inner boundary of the surrounding body, but with the surrounding body itself: the harboured boat is *in the river*, not in the inner boundary of the water (as the water moves, the place of the boat would change continuously even if it is harboured, and hence immobile with respect to the land). In the same way, it is surely possible to say that atoms are *in the void* in Epicurus' system, as they are surrounded by the void.[44] This would also explain why Epicurus specifies that the function of void is to separate atoms.[45]

As void is infinite, we can picture it as an infinite extension, which is internally interrupted by the presence of atoms: this means that void has an 'internal' shape (the pattern determined by the position of the atoms),[46] while being infinitely extended in every direction, and not having limits, it does not strictly speaking have an 'external' shape.

Condition (iv) is undoubtedly the most problematic, and I will dedicate the whole of Chapter 2 to the analysis of the connection between void and atoms, and of Epicurean kinematics. Before doing this, though, we have to address a problem that is probably bothering the reader: what should we do with the *testimonia* by Sextus Empiricus and Aëtius that Sedley brings in support of his interpretation?

First of all, it should be noticed that only the *testimonium* by Sextus is really problematic for my interpretation: Aëtius' claim that '*kenon*', '*topos*' and '*chora*' are used as synonyms by the Epicureans is not particularly worrying, as I am arguing that

[43] As we will see in Chapter 3.
[44] A bit like we say that the cereals are in the milk.
[45] *Ep. Hdt.* 44.
[46] I take the idea of the internal shape of void from Konstan 2014: I will clarify it in Chapter 2.

Epicurus' void should also fulfil at least some spatial functions.[47] On the contrary, if the context of the *testimonium* is taken into account, it could even be used against Sedley. In fact, the doxographer is here contrasting the Stoics and the Epicureans, underlying how the former attribute a different meaning to each term, while Epicurus uses them interchangeably.[48]

In the light of this, the reliability of Sextus' claim can be put into doubt. Indeed, he attributes the same doctrine to Stoics and Epicureans, and he does that by using the Stoic jargon: the specific use of the three terms that he attributes to the Epicureans does not map exactly on the *Ep. Hdt.* 40,[49] and in general – as Mansfeld notices – the subtle distinctions that he traces are not really supported by the evidence of Epicurus' authentic sources.[50] They even seem to be at odds with some of Epicurus' physical theses: as we will see in more detail in what follows, Epicurus claims that atoms are in constant motion, and never stand still. This means that the case that, according to Sextus, is associated with the word *topos* (i.e. the case in which an atom stands still and occupies a portion of space) can never occur.[51] Thus, I believe we can safely leave Sextus' *testimonium* on the side, and pass to analyse how Epicurus' hybrid conception of void could fulfil condition (iv), and make motion possible.

[47] This report is only confusing if we associate to the words *topos* and *chora* the heavy assumptions that are associated with their conventional English translations, i.e. 'place' and 'space', but this would be an anachronism that should absolutely be avoided.
[48] See Mansfeld 2014: 191 and 199.
[49] Not even if we accept Usener's emendation and we read '*topos*' in the text. See Verde 2010: 97 and footnote 18 above.
[50] Mansfeld 2014: 191.
[51] This point is made by Inwood 1981: 281.

Chapter 2. Void and Motion

How can the empty intervals between bodies make local motion possible? What are the characteristics that an 'intangible substance' must have, to fulfil this role? In this chapter, I will try to provide an answer to these and related questions. The eventual aim is to spell out the details of the 'hybrid' conception of void that I have attributed to Epicurus.

The chapter is divided in three parts: in §2.1, I will provide the details of my interpretation, focusing on the one hand on the differences between void and atoms, and on the other on the *eixis* of void. In §2.2 I will tackle two main objections to this interpretation: answering them, I will clarify why, and in which sense, void can be considered a necessary condition for atomic motion. Finally, in §2.3, I will discuss whether the void has any other properties, such as directionality.

§2.1. Eixis

The analysis conducted in Chapter 1 led us to identify void with the complement of atoms, their contradictory. The opposition between atoms and void develops in three directions:

(i) void is only where atoms are not;

(ii) void is intangible and incorporeal, while atoms are tangible and solid;

(iii) void is one, while atoms are many.

Point (iii) is especially stressed by Konstan,[52] who notices that the plural is not applicable to void. This means, first of all, that void and atoms are regulated by two different counting principles: 'void' works like a mass noun, while it is essential to the atoms that they constitute a countable plurality. Every atom is a basic unit, complete in itself: atoms cannot be split into parts, nor modified in any way. Nor can they blend

[52] Konstan 2014: 85.

together: even when atoms associate to compose a macroscopic body, they maintain their independence and their individuality – they continue to move and 'vibrate' inside the compound, clashing constantly with the other atoms.[53] On the contrary, one should not think of the single empty intervals between two given bodies as individual and self-sufficient 'voids': we could say that void behaves like 'stuff', while atoms are 'things'.

But there is also another reason why it is inappropriate to apply the plural to void, namely that all the parts of void are connected.[54] As atoms move without interruption, and do not stick together, they 'cannot coalesce to form a solid container around a portion of void'.[55] This means that, even if it is impossible to treat the various parts of void as individuals, there is a sense in which the sum of all empty intervals constitutes a unitary, individual entity. This unitary entity is what Epicurus calls an *anaphē physis*, an intangible nature, or substance. Notice that being a *physis*, it is given a positive characterisation: it can surely be used as a logical subject, and it can be the bearer of properties and attributes. The differentiation between the notions of void and of nothingness is hence complete.

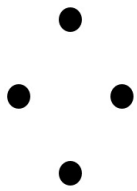
How should this entity be conceived? For sure, to fulfil the basic elemental functions, it should be a space-occupier, albeit intangible and incorporeal. Notably, this means that the void should have a position, a shape and a size. As for the position, we have already established that the void occupies all the positions that are not occupied by atoms. Epicurus is also very explicit about the size of the void, which is infinite. Being infinite, it is difficult to say whether it can have a shape: the notion of shape is linked to the

[53] I will say more on atomic motion in Chapter 4.
[54] Konstan says that this makes void 'continuous'. As 'continuous' is a heavily connoted term, which risks bringing into the picture Aristotelian intuitions, I would rather avoid using it when it is not introduced by Epicurus himself. I believe that speaking of connectedness in this context maintains Konstan's intuition, avoiding the danger of confusion. I will nonetheless discuss the problem of Epicurus' use of the term *sunechès* in Chapter 4.
[55] Konstan 2014: 91.

possession of boundaries and limits – and an infinite extension does not have limits.[56] Notice that also relatively to this feature, atoms and void are in complete opposition: in fact, atoms are individually characterised by their unmodifiable shape.

There is, however, a very specific sense in which it is possible to speak of the shape of the void. To underline that it is a special case, I would like to call this notion the 'internal' shape of the void, to differentiate from the 'external' shape which would imply the possession of limits.

As the void is only where atoms are not, we can think of it as an infinite, three-dimensional extension which is 'pierced' by the presence of atoms.[57] Think of a bunch of atoms in the void: we usually represent them like this:

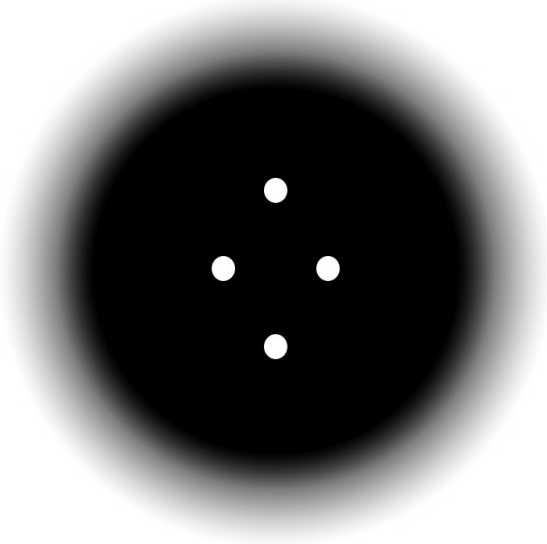


If we shift our attention from the atoms to the void, however, we can see that the spatial arrangement of the atoms imposes an internal shape on the void, which is 'interrupted' by their presence:[58]

[56] *Ep. Hdt.* 41: Ἀλλὰ μὴν καὶ τὸ πᾶν ἄπειρόν ἐστι· τὸ γὰρ πεπερασμένον ἄκρον ἔχει· τὸ δὲ ἄκρον παρ' ἕτερόν τι θεωρεῖται· <ἀλλὰ μὴν τὸ πᾶν οὐ παρ' ἕτερόν τι θεωρεῖται·> ὥστε οὐκ ἔχον ἄκρον πέρας οὐκ ἔχει· πέρας δὲ οὐκ ἔχον ἄπειρον ἂν εἴη καὶ οὐ πεπερασμένον.

[57] My attention was brought to this feature by Konstan 2014. However, he does not distinguish between internal and external shape.

[58] The void is infinite, so it cannot fit in the picture: the reader should imagine it infinitely extended (and three-dimensional) – this is why the coloured patch has fading borders.



This feature will become very useful when explaining why the void constitutes a necessary condition for atomic motion. Before turning to this issue, however, let us continue our review of the characteristics of the void.

We said that void is said to be a *physis*, and as such should be considered as an entity, not as nothingness: Epicurus also claims that it is possible to think of void on its own.[59] Nonetheless, it is an essentially negative entity: thus, it is not surprising to see that Epicurus characterises it *via negativa*. In fact, the only characteristics that are attributed to the void are negative features, privations: besides being intangible and incorporeal,[60] we are told that void is 'unable to furnish any resistance',[61] and that it 'can neither act nor be acted upon, but merely provides bodies with motion through itself'.[62] The scholiast who comments on paragraph 43 gathers all these features under a unitary label, explaining that void παρέχεται εἶξιν, 'gives way'. Even if the term is not present in Epicurus' own

[59] Interestingly, Epicurus claims that the void is the *only* incorporeal thing which can be thought on its own. See *Ep. Hdt.* 67: καθ'ἑαυτὸ δὲ οὐκ ἔστι νοῆσαι τὸ ἀσώματον πλὴν τοῦ κενοῦ.
[60] Respectively, *Ep. Hdt.* 40 and 67.
[61] *Ep. Htd.* 44: τὴν ὑπέρεισιν οὐχ οἵα τε οὖσα ποιεῖσθαι.
[62] *Ep. Hdt.* 67: τὸ δὲ κενὸν οὔτε ποιῆσαι οὔτε παθεῖν δύναται, ἀλλὰ κίνησιν μόνον δι'ἑαυτοῦ τοῖς σώμασι παρέχεται.

text, I will follow this anonymous commentator and call '*eixis*' the peculiar ability of void to 'give way to bodies' which makes its existence necessary for motion.

To understand precisely what *eixis* is, and how it works, it is important to understand to which property it is opposed. According to Keimpe Algra, *eixis* is supposed to be the contrary of *antitypia*, i.e. the impenetrability of bodies.[63] This interpretation is only satisfactory if one accepts Sedley's interpretation of void-as-space: in fact, the contrary of impenetrability is penetrability, which leads to the claim that void is receptive, and allows for superposition with bodies. This matches perfectly Sedley's claim that the difference between bodies and void is that the former, being impenetrable, repel bodies and do not allow for co-location, while the latter does not react to the approaching bodies, and let them penetrate it and superpose with it.

This, however, is not the only way in which the notion of *eixis* can be analysed. It is not even the most natural one, as it does not seem that the notion of *antitypia* has a primary role in the context of the *Letter*: a cognate of the term appears only once in Epicurus,[64] and even there it is assimilated to the idea of tangibility. As we quickly saw in Chapter 1, tangibility comes in degrees, and objects do not need to be completely impenetrable to be tangible: what is conveyed by the idea of tangibility is not primarily the impossibility of co-location, but rather the ability to enter in mechanical relations with other bodies, thus modifying their trajectory.

According to Betegh, tangibility is 'the ability of atomic and composite bodies to enter into causal interactions with one another by touching each other':[65] impenetrability is not part of the definition of this notion, even if it is a consequence of this ability to mechanically interact with bodies. Indeed, it is because bodies can enter in mechanical interactions with other bodies, and modify their trajectories, that they can repel them and

[63] Algra 1995: 53-58.
[64] In *Nat*. 2 118.25–119.2 XXVI 17–20. See Betegh 2014: 401.
[65] Betegh 2014: 401.

avoid being penetrated. But even if impenetrability is always associated with tangibility, the opposite is not always true – as it happens, for example, in the case of sponges, ashes and rare bodies.

This shows that there is a relevant difference between the two notions – a difference that becomes even more evident when we notice that the contrary of tangibility is not penetrability, but intangibility. As Epicurus explicitly defines the void as 'intangible nature', this reading seems closer to the text than Algra's. And if Betegh is right in his characterisation of tangibility, we should symmetrically think of intangibility as the inability to enter in mechanical relations with bodies, and to alter their trajectories. This *can* manifest itself as penetrability, but it is equally legitimate to suggest that void manifests this feature by 'opening' at the passage of a body, letting it pass through.[66]

This interpretation of the Epicurean model of motion, which is first proposed by Inwood and then adopted by Konstan,[67] can be described as a modification of the internal shape of void: when a body approaches an empty interval, the void is not able to resist its passage by repelling or re-directing it, but it is not even penetrated by the body. On the contrary, it changes its internal shape, so that the body passes undisturbed but there is no superposition.

In the next section, I will address the two main objections to this interpretation, and in so doing I will spell out the details of this modification of the internal shape of void.

[66] Notice that Epicurus says that atoms move *through* void, using *dia* + genitive.
[67] Inwood 1981; Konstan 2014. See in particular p. 92: 'The intangibility of space is Epicurus' name for its susceptibility to such alteration, just as the tangibility of bodies is manifested in their resistance to any form of change, save that of position'.

§2.2. Objections

The dynamical model that I am adopting has striking similarities with Aristotle's theory of *antiperistasis*[68] – which radically denies not only the necessity, but even the possibility, of void. How can I be suggesting that void is necessary for motion, if motion happens in this way? The answer lies in the fact that Aristotle and Epicurus have radically different theories of body and matter. This is unfortunately not the place to carry out a complete analysis of Aristotle's theory of body, nor of his kinematics: I will simply draw attention to the features that are most relevant to our problem, i.e. that

(1) the kinematic model adopted by Aristotle is *hydrostatic*: the idealised model of motion is motion in water – the role of the medium is fundamental, and it is conceived as a fluid;[69]

(2) bodies, as well as the distances covered by moving bodies, are continuous – as such, they are infinitely divisible.

Point (2) constitutes a condition of possibility of (1): Aristotle conceives of motion as motion in *fluids*. Both the mover and the medium are infinitely divisible, and this makes it possible for changes to happen smoothly: air, water, and bodies all share the same essential structure. It is true that differences in rareness can explain differences in the resistance that various materials oppose to change, but they are differences in degree, not in nature.

On the contrary, Epicurus' system is built on the radical rejection of Aristotle's theory of continuity, and of the hypothesis (2) that bodies are infinitely divisible. The main characteristic of atoms is that they are full, hence absolutely rigid: not having any degree of fluidity or flexibility, it is impossible to think of their motion according to a hydrostatic model. Not unless something else is added, something which – being conceived as the

[68] Inwood recognises the similarity very explicitly: he claims that void 'behaved very much like the fluids which Aristotle set up as the primary place of objects in motion' (Inwood 1981: 280).
[69] See Ugaglia 2015 for details.

polar opposite of atoms and thus allowing for flexibility and modifications in shape – could behave as a fluid: the void.

It could be objected that this makes Epicurus' argument circular and invalid: the necessity of void would be due to the peculiar features of atoms, but these are only defined on the basis of the partition of what exists into atoms and void. This, however, would be the wrong way to look at the issue. Epicurus is not trying to build his system by refuting step by step Aristotle's theses: he is proposing a rival and radically different physical system. The condition for the success of this attempt is whether all the parts cohere in a unitary picture – *not* whether there is one thesis which can be proved independently and which grounds the entire system.

As a matter of fact, Epicurus is rejecting Aristotle's physics and kinematics *in toto*: being built on the assumption that matter and motion are continuous, they are deeply embedded in the metaphysics of potentiality, which he refuses.[70] The idea of a potentiality which could never be fulfilled is at the heart of Aristotle's characterisation of matter as infinitely divisible, and appears even in his definition of motion as 'actuality of what potentially is, as such'.[71] Epicurus' own *physiologia* should hence be considered a radical alternative to Aristotle's physics of the continuum, one which does not depend on the acceptance of the metaphysics of potentiality, and which is built on the basic division between atoms and void.[72]

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[70] This is pointed out by Furley 1967: 38 with regards to the divisibility of matter: 'the idea of an infinite *potentiality* for division, never realizable, was repugnant to the Epicurean mind'. White 1992 elaborates on this suggestion, and shows that this leads to the rejection of Aristotle's theory of motion.

[71] Arist. *Ph.* III.1, 201a11-12: ἡ τοῦ δυνάμει ὄντος ἐντελέχεια, ᾗ τοιοῦτον, κίνησίς ἐστιν. Unfortunately, I do not have enough space to defend this claim in the adequate way. I will simply point out that – even if one might think that the potentiality is fulfilled by the arrival of the moving body to its destination – the definition of motion focuses on the potentiality *as potentiality*. Thus, this definition uses the same conceptual apparatus that leads to the idea of potential infinity, and which is radically rejected by Epicurus.

[72] In light of this remark, one might wonder whether there is some tension between the importance that Epicurus gives to the ability of void to yield to bodies and his refusal of Aristotelian potentiality. The ability of void to reconfigure its shape accordingly to changes in the atoms' spatial arrangement seems to be a *power*: does this commit Epicurus to the re-introduction of potentiality through the back door? I do not think so. This worry is likely to arise in an Aristotelian or neo-Aristotelian context (such as the one in which the contemporary theory of powers is originally developed), in which every ability is underpinned by a potentiality. However, both in ancient and in contemporary metaphysics, powers that are always active

There is, however, an even more dangerous objection that still awaits an answer. Epicurus says very explicitly that void cannot act nor be acted upon,[73] while the picture that I am drawing seems to suggest that bodies 'push away' the void when they approach it. This is the reason why Aristotle, introducing the argument of the cube, refuses to accept the possibility that void could behave like water or air and be displaced by the incoming body: not being a body itself, void cannot interact mechanically with bodies.

Aristotle rightly refuses the possibility that the spatial reconfiguration of void is due to a mechanical interaction between void and atoms: I have just claimed that this is exactly what Epicurus means when he says that void is intangible. But he is wrong to think that in the absence of mechanical interaction, the void cannot change its internal shape. It is up to the proponent of this interpretation, however, to prove that this can be done – and it is rather disappointing that Inwood and Konstan both fail to provide more details.[74] In particular, without further qualification, Inwood's claim that void can be assimilated to an ideal liquid could be misleading: I want hence to specify that the analogy between void and water should not be understood as a mechanical explanation of how the reconfiguration of void can happen, but only as a way to enlighten the formal characteristics of the dynamic. Now the question is: how to explain and justify this dynamic, without relying on a mechanical interaction between bodies and water? I believe that a comparison with some contemporary metaphysical theories of holes can help us to deal with this issue.

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have been theorised: being always active, they do not rely on Aristotelian potentiality. The *eixis* of void can be understood as a power of this kind: atoms are in constant motion, so that the void is also constantly undergoing reconfigurations of its internal shape. Indeed, Epicurus is not rejecting the idea that bodies and void 'can do things': he is refusing a much more specific and controversial conception, that is specific to Aristotle: namely, the idea that there can be a potentiality which *cannot* be actualised.

[73] *Ep. Hdt.* 67, quoted above in footnote 62.

[74] Inwood 1981: 279 simply claims that 'the problems raised by *antiperastasis* on the phenomenal level do not arise for the ideal fluid, void', but fails to explain why. Konstan 2014: 89 simply states that void 'simply changes its configuration when bodies move within it', without offering further details.

According to the realist theory proposed by Roberto Casati and Achille Varzi, holes are immaterial bodies.[75] They do not coincide with the limit of their material hosts, but are complementary to them:[76]

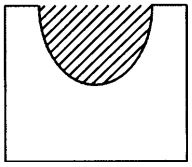

Figure 3.8
Holes can be construed as "immaterial bodies" complementary to their material hosts.

Consequently, they have a proper shape and a position, and – most importantly for our purposes – they can *change* their shape and position, even if 'immaterial bodies cannot interact'.[77] Notably, holes are created, moved, or modified by acting not on them, but on the material body in which they are hosted.[78]

For example, consider a lump of modelling clay. I can surely model it in such a way that it has a hole at the centre. If I then go on playing with the clay, I can surely modify the size, the shape, and even the position of the hole. All I have done, however, is play *with the clay*: it is not necessary to suppose that I have exercised any kind of direct action on the hole, to see that modifying the form of the clay results in a change of shape and position of the hole, too.

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[75] Casati&Varzi 1995: 34 ff. In their opinion, immaterial bodies are made of space. I wish to underline that I am using Casati and Varzi's realist theory of holes just as an analogy, to show that it is possible to think that changes of an immaterial body result as a consequence of changes of, or actions on, material bodies, without having to suppose that there is any mechanical interaction between material and immaterial bodies. I do not wish to import any other substantial thesis from their work: for example, it would be extremely problematic to fit the claim that void is an immaterial body *made of space* in our analysis. The reader should keep in mind that the analogy (as all analogies) has some limits, and fails in a number of occasions. There is in particular one disanalogy that I would like to flag: Casati and Varzi claim that holes can be filled without disappearing, i.e. they believe that when a body is inserted in the hole, there is a superposition between the two. On the contrary, I am here claiming that the Epicurean void does not allow for any superposition: void is *only* where bodies are not.

[76] The picture is taken from Casati&Varzi 1995: 34.

[77] Casati&Varzi 1995: 35. They go on remarking that 'it is not as if you pushed an immaterial body inside a material one, thereby creating a hole'.

[78] See in particular Casati&Varzi 1995: Chapter 10.

The reason why this is possible is that holes are ontologically dependent on their hosts: 'holes are always *in* something', and they cannot exist without a host.[79] Casati and Varzi themselves suggest that this same model can be applied not only to *stricto sensu* holes, but to the shape and disposition of empty intervals between bodies: 'Note also that every time that [the objects in the room] move, the empty space in your room undergoes a rather complex modification'.[80] If this implies that void is ontologically dependent on atoms, it is doubtful that Epicurus would be ready to accept this strategy: as we have seen, atoms and void are supposed to be ontologically independent *physeis*.

Before dismissing this attempt, however, we should consider that there is indeed a feature of void that is strictly associated to the existence and position of the atoms: its internal shape. I believe that Epicurus would agree that the internal shape of void is dependent on the existence of atoms, without making void itself ontologically dependent: if there were no atoms, the void would be internally homogeneous. Konstan points to a very interesting aspect of this dependence: 'the motion of bodies in space is in fact crucial to defining space itself: if bodies did not move, space might well be regarded as a single, huge body'.[81] Konstan's observation is very helpful, I believe, in showing that there is a very specific and restricted sense in which it can be said that the void depends on the atoms. But it still falls short of explaining how this dependence works, and how the reconfiguration of void is brought about.

To solve these puzzles, it is useful to focus on the reverse of Konstan's observation. In fact, not only is it true that the motion of atoms is crucial to defining void itself: the

[79] The 'superficial dependence' (their term) that links a hole to its host is a matter of *de re* necessity (Casati&Varzi 1995: 19). They underline, however, that holes are not *rigidly* dependent on their hosts, so that it is possible to change or substitute the host while leaving the hole intact and unmodified (Casati&Varzi 1995: 137-8).
[80] Casati&Varzi 1995: 168.
[81] Konstan 2014: 91. Notice that Konstan believes that void consists not in the totality of all possible positions, but only in the empty intervals between bodies. However, he claims that this should be taken to be Epicurus' conception of space. This is why in the quote there is 'space' where I would rather say 'void', given the way in which I am using the term 'space'.

converse claim is also true, as changes in the internal shape of void are crucial to define the motion of atoms. Recall that the Epicurean *kosmos* is infinite, without a centre nor any privileged framework: the reconfiguration of the internal shape of void is a fundamental part of what it means for an atom to move. Using a lexicon completely extraneous to the Epicurean tradition, but probably more familiar to the reader, we can say that the modification of the internal shape of void is the manifestation of the atoms' power to move. The manifestation of a power is not its causal product,[82] so that the claim that void cannot be acted upon is not violated. However, the relation between a power and its manifestation is strict enough to grant that the reconfiguration of the internal shape of void can be a function of the spatial arrangement of atoms.

At the same time, this observation provides the final piece of information necessary to complete the picture, and explain how the void – interpreted not as the totality of space, but as the collection of the empty intervals between bodies – can be a necessary condition of motion: generating a reconfiguration of the internal shape of void is part of what it means, for an atom, to be in motion rather than stationary.

§2.3. Directionality

The picture that is emerging from this discussion is that void, in itself, would not have any kind of internal differentiations: these are only due to the presence and motion of atoms. It is well-attested, however, that Epicurus takes the *kosmos* to have a privileged direction, as atoms tend to fall *downwards*. Should we think of this directionality as a structural and intrinsic property of void itself? Those who think that void should be identified with space would answer positively to the question. But what if we reject this association?

[82] For details, see Martin 2008 (esp. p. 51).

This is how Epicurus presents the issue:

> Καὶ μὴν καὶ τοῦ ἀπείρου ὡς μὲν ἀνωτάτω ἢ κατωτάτω οὐ δεῖ κατηγορεῖν τὸ ἄνω ἤ κάτω. […] Ὥστε ἔστι μίαν λαβεῖν φορὰν τὴν ἄνω νοουμένην εἰς ἄπειρον καὶ μίαν τὴν κάτω, ἂν καὶ μυρίακις πρὸς τοὺς πόδας τῶν ἐπάνω τὸ παρ'ἡμῶν φερόμενον <εἰς> τοὺς ὑπὲρ κεφαλῆς ἡμῶν τόπους ἀφικνῆται ἢ ἐπὶ τὴν κεφαλὴν τῶν ὑποκάτω τὸ παρ'ἡμῶν κάτω φερόμενον· ἡ γὰρ ὅλη φορὰ οὐθὲν ἧττον ἑκατέρα ἑκατέρᾳ ἀντικειμένη ἐπ'ἄπειρον νοεῖται.
>
> Moreover in speaking of the infinite we must not use 'up' and 'down' with the implication that they top or bottom. […] Therefore it is possible to take as one motion that which is conceived as upwards to infinity, and as one motion that which is conceived as downwards to infinity, even if that which moves from where we are towards the places above our heads arrives ten thousand times at the feet of those above, or at the heads of those below, in the case of that which moves downwards from where we are. For each of the two mutually opposed motions is none the less, as a whole, conceived as being to infinity.
>
> *Ep. Hdt.* 60

His main concern is to show that the assumption of a privileged direction is compatible with the claim that the universe is infinite: this is why he underlines that the 'up' and 'down' do not correspond to the upper and lower points in the *kosmos*, but are linked to the direction of motion. It is unclear, however, if this directionality should be attributed to the void. To solve the issue, we should look at this property in connection with the corresponding atomic property: weight. Indeed, Epicurus seems to attribute to the atoms the same set of properties selected by Democritus,[83] but with one difference: he claims that atoms also have *weight*.[84]

What is weight? For us, weight is a force: notably, the force that a gravitational field exercises on a body having mass. We hence understand weight as the manifestation of an interaction between a property of the body, and a property of space (or better, of space-

[83] At least according to Plutarch: see fr. 275 Usener.
[84] *Ep. Hdt.* 54: καὶ μὴν καὶ τὰς ἀτόμους νομιστέον μηδεμίαν ποιότητα τῶν φαινομένων προσφέρεσθαι πλὴν σχήματος καὶ βάρους καὶ μεγέθους καὶ ὅσα ἐξ ἀνάγκης σχήματος συμφυῆ ἐστι.

time). Importing this modern intuition into the Epicurean framework, however, risks causing problems. For example, think of a stone thrown into the air. It will move upwards for a while, and then change its direction and start moving downwards. In our view, the stone will change direction without any need for further interactions with bodies; it does so only in virtue of its weight – i.e. because of the force exercised by the gravitational field. If we try to describe this situation in Epicurean terms, a problem arises: as we will see in more detail in Chapter 4, atoms all move constantly at the same speed. This means that we should conceive of the change in direction as an instantaneous, apparently uncaused, switch from motion at maximum speed upwards, to motion at maximum speed downwards. Furley does accept this idea, and claims that there are two ways in which a change in direction can be imposed on the moving atom: either by collision with another atom, or by 'reassertion of weight'. This latter case does not involve any interaction with bodies.[85]

This picture seems quite implausible, though. This is why Konstan suggests a different explanation. Rather than supposing that this change in direction happens spontaneously, he submits that *all* changes in the direction of motion are the result of atomic collision.[86] This is suggested by the last, enigmatic lines of paragraph 61:

> ἐφ' ὁπόσον γὰρ ἂν κατίσχῃ ἑκάτερον, ἐπὶ τοσοῦτον ἅμα νοήματι τὴν φορὰν σχήσει, ἕως ἀντικόψῃ ἢ ἔξωθεν ἢ ἐκ τοῦ ἰδίου βάρους πρὸς τὴν τοῦ πλήξαντος δύναμιν.
>
> For however far along either kind of trajectory it gets, for that distance it will move as fast as thought, until it is in collision, either through some external cause or through its own weight in relation to the force of the impacting body.
>
> *Ep. Hdt.* 61

[85] Furley 1967: 122-123.
[86] See Konstan 1979: 410 ff.

If he is right, weight manifests itself only as a tendency to take a direction rather than another after a collision. Does this presuppose an intrinsic directionality of void? According to Aristotle, it does – but as the void is completely unqualified, and has no internal structure nor privileged direction, he concludes that motion in the void is impossible. Epicurus is evidently reacting to this claim. One possibility is indeed that he simply posits that the void *does have* an internal structure and a privileged direction. But another, more interesting possibility is that he is reversing the order of the explanation proposed by Aristotle. Rather than introducing some differentiation in the void, he might attribute the property of weight to the atoms, conceiving of it as a primitive feature. According to this picture, having weight would simply *mean* tending to move in a certain direction: as a consequence of this tendency, which is a property of the atoms, this direction is selected as the privileged one, and determines where the 'down' of the infinite universe is.

This second possibility, which is the one that Konstan favours, has no better textual evidence than the other. But it is interesting for our purposes to notice that it is viable, and fits much better the interpretation that I am defending, according to which the void is in itself homogeneous and undifferentiated, and that internal differentiations depend on the presence and motion of atoms.

There is one last issue left to discuss. It is common to claim, on the basis of *testimonia* by Simplicius and Themistius, that Epicurean space is discrete – that it is composed by indivisible parts, so that it has a 'quantised' or 'granular' structure.

Under the traditional interpretation, this claim is equally applicable to void, because void is taken to be space. I have been arguing, however, that void does not coincide with the totality of space, but only with the empty intervals between bodies. What should we make of Simplicius and Themistius' reports, in such a context? Should we take void to be quantised as well? What could it mean?

To answer these questions, we have to look at the so-called theory of the *minimae partes*, the minimal parts in the atom. The reason why we have to look at the structure of the atoms to determine whether void is internally structured might not be evident to the modern reader, but I believe it would be obvious to the Ancient thinkers. An enduring legacy from Aristotle, which has been called the 'Isomorphism Thesis', posits that all the fundamental physical magnitudes share the same microstructure. For this reason, I will now turn to the theory of the *minimae partes*, and I will postpone the discussion of the consequences of this theory for the structure of void until Chapter 4.

Chapter 3. (Minimal) Parts in the Atom?

The doctrine of the *minimae partes*, or 'minima',[87] is known as one of the most fascinating, original, and weird parts of Epicurean *physiologia*. Indeed, the *minimae partes* are supposed to be the smallest possible parts in the atom – but as the atom is itself indivisible, it is not completely clear what the parts in the atom might be, nor why, once we allow for the possibility of individualising parts in a physically indivisible atom, there should be a lower limit to this possibility.

In this chapter, I will try to offer an interpretation of the theory which takes seriously the idea that the parts in question are supposed to be *minima*, hence the smallest possible, but does not end up postulating the existence of paradoxical entities, such as spatially extended bodies without shapes.[88] The discussion will be in three parts. §3.1 and §3.2 will be dedicated to the analysis of Epicurus' text, while in §3.3 I will suggest an original interpretation of the notions of part and division which should help us understand what the minima in the atom are.

§3.1. Divisibility paradoxes

Epicurus presents the theory of atomic minima in a very succinct way, in *Ep. Hdt*. 56-59. The location of these paragraphs in the text is important: the theory is introduced when the need for physically indivisible atoms has already been established. As we have seen in §1.4, the 'basic division' results in the identification of the two components of reality: void and atoms. In *Ep. Hdt.* 41, Epicurus explains that the atoms, which are the

[87] Notice that '*minimae partes*' is an expression introduced by Lucretius: Epicurus only says '*elachista*', which translates as 'minima'. As we will see shortly, probably Epicurus consciously avoids speaking of 'parts'. I will sometimes speak of *minimae partes* nonetheless, both because this is term is commonly used in the literature, and because I will discuss at length what kind of parts the minima are.
[88] This is suggested, for example, by Sorabji 1983.

constituents of compound bodies, 'must be uncuttable and unalterable – if all things are not going to be destroyed into the non-existent but be strong enough to survive the dissolution of the compounds – full in nature, and incapable of dissolution at any point or in any way'.[89]

This argument is very sketchy, but it does show that Epicurus follows the Early Atomists in presenting the existence of atoms as a response to the Eleatic criticism of change.[90] Notice that the Eleatic inference from change to not-being is not refuted: the Atomists accept this principle, and they explain the phenomenon of change in such a way that the principle is not violated. The strategy is very simple: it consists in attributing all the qualities of the Eleatic being (notably inalterability) to a *plurality* of elements, the atoms. Those are the principles of being, and cannot perish, come into existence, nor be modified, but they can combine in various ways and thus generate the phenomena of change.

This interpretation is confirmed in *Ep. Hdt.* 54: after discussing other issues (such as perception and the theory of *eidola*), Epicurus analyses the nature and properties of atoms. He begins by explaining that atoms have only three properties: shape (and all which is related to shape), weight and size. This is because all other qualities are mutable, while atoms cannot change, 'since something solid and indissoluble must survive the dissolution of the compounds, to ensure that the changes are not into, or out of, the non-existent, but result from transpositions within many things, and in other cases from additions and subtractions of certain things'.[91] The argument is clearly the same as in paragraph 41, but we are also told that atoms are indivisible in virtue of their solidity: this

[89] ταῦτα δέ ἐστιν ἄτομα καὶ ἀμετάβλητα, εἴπερ μὴ μέλλει πάντα εἰς τὸ μὴ ὂν φθαρήσεσθαι, ἀλλ'ἰσχύοντα ὑπομενεῖν ἐν ταῖς διαλύσεσι τῶν συγκρίσεων πλήρη τὴν φύσιν ὄντα καὶ οὐκ ἔχοντα ὅπῃ ἢ ὅπως διαλυθήσεται.

[90] See Chapter 1 above, especially §1.1.

[91] αἱ δὲ ἄτομοι οὐδὲν μεταβάλλουσιν, ἐπειδή περ δεῖ τι ὑπομένειν ἐν ταῖς διαλύσεσι τῶν συγκρίσεων στερεὸν καὶ ἀδιάλυτον, ὃ τὰς μεταβολὰς οὐκ εἰς τὸ μὴ ὂν ποιήσεται οὐδ' ἐκ τοῦ μὴ ὄντος, ἀλλὰ κατὰ μεταθέσεις ἐν πολλοῖς, τινῶν δὲ καὶ προσόδους καὶ ἀφόδους.

means that their inalterability is ensured by the absence of internal void – not simply by the smallness of the atom, as it is the case according to the Early Atomists.[92]

At paragraph 56, Epicurus points out that only a finite number of atomic sizes can be found in nature. Not only it is unnecessary to suppose that atoms of all sizes exist in nature: it is impossible, because we know that atoms must be too small to be perceptible. This claim should be taken as a partial explanation of (or at least as related to) the claim that there can only be a finite (albeit inconceivably big) number of different atomic shapes.[93]

In the same paragraph, as some sort of follow-up or explanation, Epicurus writes:

> Πρὸς δὲ τούτοις οὐ δεῖ νομίζειν ἐν τῷ ὡρισμένῳ σώματι ἀπείρους ὄγκους εἶναι οὐδ' ὀπηλίκους οὖν. ὥστε οὐ μόνον τὴν εἰς ἄπειρον τομὴν ἐπὶ τοὔλαττον ἀναιρετέον, ἵνα μὴ πάντα ἀσθενῆ ποιῶμεν κἀν ταῖς περιλήψεσι τῶν ἀθρόων εἰς τὸ μὴ ὂν ἀναγκαζώμεθα τὰ ὄντα θλίβοντες καταναλίσκειν, ἀλλὰ καὶ τὴν μετάβασιν μὴ νομιστέον γίνεσθαι ἐν τοῖς ὡρισμένοις εἰς ἄπειρον μηδ' ἐ<πὶ> τοὔλαττον.
> οὔτε γὰρ ὅπως, ἐπειδὰν ἅπαξ τις εἴπῃ ὅτι ἄπειροι ὄγκοι ἔν τινι ὑπάρχουσιν ἢ ὀπηλίκοι οὖν, ἔστι νοῆσαι· πῶς τ' ἂν ἔτι τοῦτο πεπερασμένον εἴη τὸ μέγεθος; πηλίκοι γάρ τινες δῆλον ὡς οἱ ἄπειροί εἰσιν ὄγκοι· καὶ οὗτοι ὀπηλίκοι ἄν ποτε ὦσιν, ἄπειρον ἂν ἦν καὶ τὸ μέγεθος. ἄκρον τε ἔχοντος τοῦ πεπερασμένου διαληπτόν, εἰ μὴ καὶ καθ' ἑαυτὸ θεωρητόν, οὐκ ἔστι μὴ οὐ καὶ τὸ ἐξῆς τούτου τοιοῦτον νοεῖν, καὶ οὕτω κατὰ τὸ ἐξῆς εἰς τοὔμπροσθεν βαδίζοντα εἰς τὸ ἄπειρον ὑπάρχειν κατὰ <τὸ> τοιοῦτον ἀφικνεῖσθαι τῇ ἐννοίᾳ.
>
> Furthermore, we must not consider that the finite body contains an infinite number of parts,[94] not even parts with no [lower] limit to size. Therefore not only must we deny cutting into smaller and smaller parts to infinity, so that we do not make everything weak and be compelled in our conception of complex entities to grind away existing things and

[92] Betegh 2014; Verde 2013: 24.
[93] *Ep. Hdt.* 42.
[94] I depart from Long and Sedley's text and translate '*ogkoi*' as 'parts'. This choice is not uncontroversial, even if it is accepted by Bailey, Furley and Conche. The original translation by Long and Sedley reads 'bits', while Verde translates 'masse' (masses). Other possibilities, discussed by Conche, are 'corpuscles' or 'atoms'. I opt for 'parts' because, as it will become clearer in what follows, I believe that this passage should be read keeping the divisibility paradoxes in mind, which are related to the division in parts. The minima are surely very special kinds of parts, though, and I am dedicating §3.3 to spell out precisely how special they are.

> waste them into non-existence, but also we must not consider that in finite bodies there is traversal to infinity, not even through smaller and smaller parts.
> For, first, it is impossible to conceive how [there could be traversal], once someone says that something contains an infinite number of parts or parts with no [lower] limit to size. Second, how could this magnitude still be finite? For obviously these infinitely many parts are themselves of some size, and however small they may be the magnitude consisting of them would also be infinite. And third, since the finite body has an extremity which is distinguishable, even if not imaginable as existing *per se*, one must inevitably think of what is in sequence to is as being of the same kind, and by thus proceeding forward in sequence it must be possible, to that extent, to reach infinity in thought.
>
> *Ep. Hdt.* 56-57 (Long&Sedley's transl., modified)

This passage, which continues in paragraphs 58 and 59 with the analysis of the minima in perception and in the atom, provides the reasons for the assumption of a lower limit to divisibility. Unfortunately, it is quite obscure. One of the first problems that we face in interpreting these lines is that it is not at all clear whether the 'finite bodies' under examination are supposed to be atoms, macroscopic bodies, or both. Obviously, if Epicurus is here speaking about the atoms, the points that he makes can be easily generalised to all kind of bodies, as macroscopic bodies are ultimately composed by atoms. But the converse does not hold, so that if Epicurus is speaking only of macroscopic bodies, we cannot use this passage to clarify the notion of *minimae partes* in the atom.

As my goal is to explain what the minima in the atom are, it would be extremely helpful to know which problems their introduction is supposed to solve. It is hence very important to determine what is the object of these lines, and whether they can be used in our analysis, because this is the only passage in which this issue is tackled.

Fortunately, it is extremely unlikely that these considerations do not apply to the parts of the atom, for at least three reasons. Firstly, we have already been told twice that there is a lower limit to the physical divisibility of matter, as macroscopic bodies are composed by unperishable, unchangeable and unbreakable atoms. Second, in this paragraph

Epicurus is discussing the number of atomic shapes that can be found in nature: it would not make sense to repeat that *there are* atoms, while it would absolutely make sense to discuss how many parts can be found in the atom itself. Third, the passage continues by discussing the characteristics of the minimum in the atom, so that it is highly plausible to suppose that this passage is an attempt at a proof of the *existence* of a minimum.

Marcel Conche is one of the few interpreters who claim that these observations *cannot* be applied to the minima in the atom, as – he claims – the reasons adduced by Epicurus (in particular the first one) can only be applied to justify the impossibility of physically dividing the atom into separate parts.[95] Let us take a closer look at these reasons, and see why they can be applied also when we speak of the parts that can be individuated (albeit not physically separated) in the atom itself.

The thesis stated by Epicurus is that there cannot be an infinite number of parts in a finite body.[96] This is articulated in two aspects:

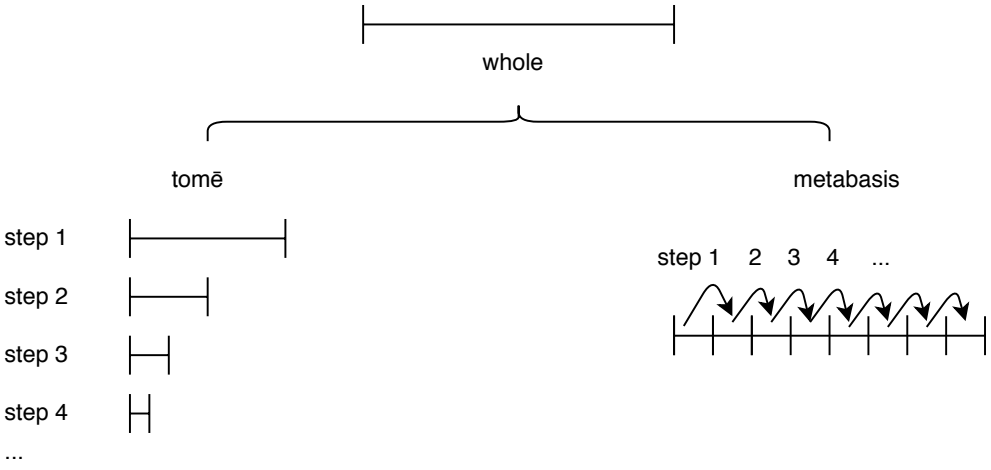
1. There cannot be division (*tomē*) into smaller and smaller part *ad infinitum*;
2. There cannot be traversal (*metabasis*) *ad infinitum*, not even through smaller and smaller parts.

The issue is, obviously, how to understand the two operations, *tomē* and *metabasis*. A possibility is that the first operation corresponds to physical division, while the second one to theoretical division, intended as the identification of parts in a whole, which does not result in the destruction of the whole and the spatial separation of the parts themselves.[97] If this were true, the first one would not be applicable to atoms, and to understand the theory of *minimae partes* we should only focus on the second operation.

[95] Conche 1987: 148.
[96] I agree with Furley 1967 that it makes more sense to take this as a unique thesis articulated in two aspects. But no substantial problem arises if the reader prefers to treat the two statements that (i) there cannot be an infinity of parts, and (ii) there cannot be parts of whatever size, as two distinct theses.
[97] I will discuss at length the notions of physical and theoretical divisions in §3.2: for the moment, an intuitive grasp of the idea will suffice.

This would not be a problem, because the analysis of *metabasis* is sufficient to lead us to interesting results. But I think that we should take both *tomē* and *metabasis* as technical terms which refer to two different ways of identifying parts in a body: the first one is a dichotomy (the division of a whole in two proper parts), while the second one refers to the act of traversing a spatial interval by passing over all its parts in succession (so to the act of identifying parts one in succession to the other, up to the exhaustion of the body).

A diagram will help clarifying the difference:[98]



These two operations appear in a set of problems and paradoxes about divisibility which are usually attributed to Zeno, and which are incredibly similar to the arguments advanced by Epicurus. The peculiarity of the paradoxes is that they appear in very different contexts: to mention the most obvious examples, Aristotle's physical theory is developed by constantly engaging with them, but they are also invoked, in a purely mathematical form, by the supporters of the 'indivisible lines' attacked in the pseudo-Aristotelian *De lineis insecabilibus* (probably Xenocrates).

[98] For simplicity, I have drawn the parts found by *metabasis* as all of the same size. This is not necessary, however: we can think of the parts as decreasing in dimension, for example by keeping fixed the ratio between two consecutive parts, rather than the dimension of the parts. This is why Epicurus specifies that there cannot be infinite traversal 'not even through smaller and smaller parts'.

This means that both *tomē* and *metabasis* can be interpreted as physical *and* as theoretical operations. Indeed, we can find in the tradition examples of paradoxes for each combination:

1) *Tomē ad infinitum:*

   a) Physical: if a body can be physically bisected *ad infinitum*, it will disappear into non-existence. This argument is attributed to Democritus by Aristotle (*GC* I.2, 316a.22 ff.): suppose that the infinite division has been completed. It is impossible that the resulting parts have some magnitude, because this would mean that they could still be divided and that the division had not been completed. But if they have no positive size, the body would be composed out of nothing.

   b) Theoretical: if a finite body is composed by an infinite number of parts,[99] it is both finite and infinite. The root of this paradox is the assumption that any infinite sequence sums up to infinity: if the parts have a positive magnitude, however small they are, their sum will have infinite size. This argument appears in Zeno's argument 'from finite size', reported by Simplicius.[100] A consequence of the argument is the loss of the possibility to discriminate between the predicates 'big' and 'small' lamented in *De lineis insecabilibus* 968a: both big and small bodies will have the same (namely infinite) number of parts, so that the distinction will be lost.

[99] It is commonly assumed, both by Zeno and his opponents, that the operations of division and composition are complementary: everything can be divided exactly into the parts out of which it is composed, and *vice versa*.

[100] *In. Phys.* 139.9-141.2. See also Huggett 2017.

2) *Metabasis ad infinitum:*

a) Physical: Zeno's arguments against motion present a series of variations on the same theme: if a distance is composed by an infinite number of parts, it will be impossible not only to complete (Achilles), but even to start (Dichotomy) the motion across it. In *De lineis insecabilibus* 968a, the problem is cast in these terms: 'Zeno's argument proves that there must be simple magnitudes. For the body, which is moving along a line, must reach a half-way point before it reaches the end. And since there always is a half-way point in any "stretch" which is not simple, motion – unless there be simple magnitudes – involves that the moving body touches successively one-by-one an infinite number of points in a finite time: which is impossible'.

b) Theoretical: if a body is composed by an infinite number of parts, by considering it we would be able to count to infinity. This argument is reported in *De lineis insecabilibus* just after the physical version just presented: 'it follows that thought too would come successively into contact with an infinity of objects in a finite time. And since "thought's coming into contact with objects one-by-one" is counting, we must admit that it is possible to count the units of an infinite sum in a finite time'.

The arguments in *Ep. Hdt.* 56-57 share striking similarities with this tradition. Four main claims are advanced:

(1) If we could operate *tomē ad infinitum,* we would 'make everything weak and be compelled in our conception of complex entities to grind away existing things and waste them into non-existence';

(2) If there were infinitely many parts, *metabasis* would be impossible;

(3) If there were infinitely many parts, 'the magnitude consisting of them would also be infinite';

(4) 'Since the finite body has an extremity which is distinguishable, even if not imaginable as existing *per se*, one must inevitably think of what is in sequence as being of the same kind, and by thus proceeding forward in the sequence it must be possible, to that extent, to reach infinity in thought'.

According to Conche, argument (1) can only be applied to macroscopic bodies, because it is only concerned with physical division (and we know that atoms cannot be physically divided). It is surely possible to interpret the argument in a physical way: this would simply be case (1.a) above. However, there might be more to this argument. Epicurus says that by dividing too many times we would make everything 'weak', and that this would have consequences on our 'conception' (*perilēpsis*) of the bodies: he seems to be concerned with something slightly different than the possibility of physically tearing something apart until it is completely annihilated. Rather, the issue seems linked to the peculiar epistemology developed by Epicurus, according to which concepts are directly derived from experience.[101] According to Furley, the issue is that 'when we tried to get a firm mental grasp (*perilēpsis*) on the atoms, we should find them crumbling away into nothingness. Every time we thought we had arrived at the irreducible minima, we should have to admit that even these minima are divisible. And so our search for the reality of the atoms would be endlessly frustrated'.[102] This epistemic version of the argument seems out of place next to paradoxes concerning ontological issues, but it should be underlined that in the context of Epicurean philosophy, the epistemological and the ontological are not so sharply distinct. Indeed, the ontological analysis is not carried out through *a priori* logical reasoning or through definitions, but it originates from, and

[101] See Long&Sedley 1987: 94 ff.
[102] Furley 1967: 13.

is dependent on, sensible experience: epistemic paradoxes reflect deep ontological problems.

Arguments (2) and (3) are more straightforward. Provided that we consider (2) as an independent argument,[103] it simply points to the physical impossibility of completing an infinite task. Argument (3), on the contrary, relies on the (false, but widely shared in Antiquity) assumption that an infinite sequence sums up to infinity, and so corresponds to case (1.b) above: a body composed by an infinite number of parts would have an infinite size.

Argument (4), finally, evidently falls under case (2.b): the problem raised here is that if a body contains an infinite number of parts, it would be possible to 'reach infinity in thought' (i.e. counting to infinity, presumably) by considering all these parts in succession. Of note is the way in which the argument is presented, which gives us some interesting details on how the *metabasis* is operated, according to Epicurus. Notably, we learn that:

a) The *metabasis* starts from one of the extremities of a body and finishes at the opposite one, passing through all the intermediate parts;
b) The extremity which constitutes the starting point of the *metabasis*, and is the model on the basis of which all the other parts are thought, is distinguishable but not separable from the body: it cannot be thought as existing *per se*;
c) These parts are arranged in succession, one next to the other.

These points will also feature in the description of the visual minima (which are the model for the minima in the atom), so that the suggestion seems to be that vision too should be thought of as a *metabasis*. For the moment, however, the important thing to

[103] I am following Long and Sedley in presenting this argument as associated to the operation of *metabasis*. Some interpreters (for example Furley and Conche), on the contrary, read it as simply stating the impossibility of conceiving a body composed by an infinite number of parts (hence as part of argument 3).

note is that Epicurus maintains that this kind of process cannot involve an infinite number of parts – and hence steps.

§3.2. The Analogy with Visual Minima

Let us continue our analysis of the text. The passage quoted in §3.1 continues with the analogy between the minima in sensation (notably in vision) and the minima in the atom:

> τό τε ἐλάχιστον τὸ ἐν τῇ αἰσθήσει δεῖ κατανοεῖν ὅτι οὔτε τοιοῦτόν ἐστιν οἷον τὸ τὰς μεταβάσεις ἔχον οὔτε πάντῃ πάντως ἀνόμοιον, ἀλλ' ἔχον μέν τινα κοινότητα τῶν μεταβατῶν, διάληψιν δὲ μερῶν οὐκ ἔχον· ἀλλ' ὅταν διὰ τὴν τῆς κοινότητος προσεμφέρειαν οἰηθῶμεν διαλήψεσθαί τι αὐτοῦ, τὸ μὲν ἐπιτάδε, τὸ δὲ ἐπέκεινα, τὸ ἴσον ἡμῖν δεῖ προσπίπτειν. ἐξῆς τε θεωροῦμεν ταῦτα ἀπὸ τοῦ πρώτου καταρχόμενοι καὶ οὐκ ἐν τῷ αὐτῷ, οὐδὲ μέρεσι μερῶν ἁπτόμενα, ἀλλ' ἢ ἐν τῇ ἰδιότητι τῇ ἑαυτῶν τὰ μεγέθη καταμετροῦντα, τὰ πλεῖω πλεῖον καὶ τὰ ἐλάττω ἔλαττον. ταύτῃ τῇ ἀναλογίᾳ νομιστέον καὶ τὸ ἐν τῇ ἀτόμῳ ἐλάχιστον κεχρῆσθαι·
> μικρότητι γὰρ ἐκεῖνο δῆλον ὡς διαφέρει τοῦ κατὰ τὴν αἴσθησιν θεωρουμένου, ἀναλογίᾳ δὲ τῇ αὐτῇ κέχρηται. ἐπεὶ περ καὶ ὅτι μέγεθος ἔχει ἡ ἄτομος, κατὰ τὴν ἐνταῦθα ἀναλογίαν κατηγορήσαμεν, μικρόν τι μόνον μακρὰν ἐκβαλόντες. ἔτι τε τὰ ἐλάχιστα καὶ ἀμερῆ πέρατα δεῖ νομίζειν τῶν μηκῶν τὸ καταμέτρημα ἐξ αὐτῶν πρῶτον τοῖς μείζοσι καὶ ἐλάττοσι παρασκευάζοντα τῇ διὰ λόγου θεωρίᾳ ἐπὶ τῶν ἀοράτων. ἡ γὰρ κοινότης ἡ ὑπάρχουσα αὐτοῖς πρὸς τὰ ἀμετάβατα ἱκανὴ τὸ μέχρι τούτου συντελέσαι, συμφόρησιν δὲ ἐκ τούτων κίνησιν ἐχόντων οὐχ οἷόν τε γίνεσθαι.
>
> As for the minimum in sensation, we must grasp that it is neither of the same kind as that which admits of traversal, nor entirely unlike it; but that while having a certain resemblance to traversable things it has no distinction of parts. Whenever because of the closeness of the resemblance we think we are going to make a distinction in it – one part on this side, the other on that – it must be the same magnitude that confronts us. We view these minima in sequence, starting from the first, neither all in the same place nor touching parts with parts, but merely in their own peculiar way providing the measure of magnitudes – more for a larger magnitude, fewer for a smaller one.
> This analogy, we must consider, is followed also by the minimum in the atom: in its smallness, obviously, it differs from the one viewed through sensation, but it follows the same analogy. For even the claim that the atom has size is one which we made in

> accordance with the analogy of things before our eyes, merely projecting something small onto a large scale. We must also think of the minimum uncompounded limits as providing out of themselves in the first instance the measure of lengths for both greater and smaller magnitudes, using our reason to view that which is invisible. For the resemblance which they bear to changeable things is sufficient to establish this much; but a process of composition out of minima with their own movement is an impossibility.
>
> *Ep. Hdt.* 58-59

The first thing to make clear is that the analogy links four terms, so that it is structured as a proportion:

visible bodies : minima in sensation = atoms : minima in the atom

Being a proportion and not a simple analogy, it can give us much more information: it can indeed be read in many different ways, focusing on the one or the other of the connections that are made between the four terms. In particular, Epicurus exploits the following connections to elucidate the nature of the *minimae partes*:

- Visible body ~ atom
- Visible body ~ minimum in sensation
- Minimum in sensation ~ minimum in the atom

Obviously, this latter analogy is the most important one: the relation between the visible body and the visual minima provides the model to understand the relation between the atom and the *minimae partes*. Before tackling it, however, a brief word on the '=', that is, on what grants that we can use one case to understand the other. Epicurus points out that we can use the visible bodies as models for understanding the atom, because we can 'project the atom onto a large scale', magnify it, so that we can 'see it with the mind'. This is the way in which all knowledge of atoms is gained: even the attribution of size relies on this analogy with visual experience. This way of gaining knowledge of the atom

is reliable: first, because perceptual experience, and analogy with perceptual experience, is a (if not the most) valid source of knowledge in the Epicurean system;[104] second, because the magnification compensates the only difference between the atom and the visible body, which is the smallness of the atom.[105]

It is now time to discuss the elements of the analogy. First, it is clear that 'that which admits of traversal', which could *a priori* refer to any kind of bodies, both the macroscopic, compound ones and the atoms, is supposed to refer to visible, macroscopic bodies. Indeed, this particular body is immediately put into relation with the minimum in sensation, showing that it must be a sensible, macroscopic body. In what follows, moreover, Epicurus develops the analogy with the 'things before our eyes', and focusing on vision as the exemplar case of sensible perception. His analysis of vision respects the features of *metabasis* that I have briefly spelled out at the end of §3.1: we see the minima in succession, starting from the first.

This raises a question that has puzzled the commentators: do we really *see* the visual minima, when we look at a visible object? Purinton strongly suggests that this is the case, and that Epicurus takes vision to be quantized: according to this reading, we see the world in pixels. He pushes this claim to its extreme, asserting that we are even able to count the visual minima in our visual field.[106] This interpretation, however, is highly implausible: Purinton's extreme claim is blatantly falsified by the experience. Rather, I believe that we should take the visual minima as effectively visible only in very special cases: for

[104] See Long&Sedley 1987: 94 ff.

[105] Purinton 1994 takes this analogy as the core of his interpretation, and claims that it would be arbitrary to put a limit to the ability of the mind to magnify the atom: he thinks that perception is always quantised, so that at every level of magnification there will be minimal parts, but that it is always possible to magnify further and find ulterior parts at the next level of magnification. The interpretation that I will lay down in §3.3 avoids this objection, as it turns out that it is impossible to find more than a given number of parts not because we cannot magnify the atom any further, but because there are no more parts.

[106] Purinton suggests a thought experiment, and he asks the reader to imagine to be sitting in a room, looking out on the things within their field of vision: 'In the room in which you sit, there are, let us say, two candles on the table in front of you, each occupying a finite portion of your field of vision. You traverse with your eyes the width of the candles and *determine that each has 100 visibly minimal parts*' (Purinton 1994: 133, my emphasis).

example, when a body appears or disappears at the horizon, we can isolate the minimal visible magnitude. Or again, we can take the minima to be visible at the border of visible bodies: focusing very carefully on the edge of the body, we could see the minimal distinguishable parts. A nice example of this case would be the edge of a needle.[107] As we will see shortly, Epicurus connects strictly the notions of minima and of limits: thus, this way of visualising the minima as borders is probably the better one.

Saying that we cannot see all the individual minima in visible bodies does not mean that visible bodies are not constituted by visible minima. Epicurus makes it very clear that all sensibilia must be integral multiples of the visible minimum: Vlastos points out that this thesis is not advanced on empirical grounds, but deduced from the empirical premise that things must have a minimum size, if they are to be sensed at all.[108]

The second important highlight of this passage is that we are given more details regarding the arrangement of the minima in the whole. Epicurus confirms that they are ordered in succession, but 'in their own special way': the lexicon of this paragraph is exactly the same that Aristotle defines in *Ph.* V.3, where he also claims that indivisible entities cannot be in contact without overlapping completely. As I will show shortly, this is probably one of the reasons that lead Epicurus to posit the existence of parts in the indivisible atom in the first place. Here, however, he is taking a very definite stance on the issue, and claiming that the same problem does not arise with regards to the minima. Why? I believe the thought is that the minima do not exist independently of the body to which they belong. This means that they are not independently existing elements which must be arranged in a given order: rather, they *already are* in the whole, so that the

[107] See also Verde 2013: 58.
[108] Vlastos 1965: 143-144. He also points out that the inference is invalid, but is based on a plausible assumption: that if A + *d*, or A – *d*, is perceived as different from A, then *d* must be perceptible all by itself. Vlastos also underlines that in *De Sensu*, 446A, 10-15, Aristotle manages to refute this assumption only by invoking the special concept of potentiality that Epicurus rejects.

problem of overlapping does not even arise. In §3.3, I will discuss how a coherent interpretation can come from this vague intuition.

The third point, which goes in the same direction as the one just presented, is that the minima are said to be limits (or: the limits are said to be minima),[109] and to provide the measure of magnitudes. This observation is very puzzling, but I think that three lessons can be taken from it:

1. The limit of the *n*-dimensional whole (be it a macroscopic body or an atom), which is the starting point of the *metabasis*, cannot have dimension *n*-1, as Aristotle maintains: it must be a minimum of positive size. This is because it constitutes the model for all the parts involved in the *metabasis*, and because it provides the measure of the magnitude: this means that the magnitude is an integral multiple of it.[110]
2. As the minima provide a unit of measure for the atom, they block the paradox 'of the big and the small' (case 1.b above): Epicurus underlines that there are more minima in bigger bodies, and less in smaller ones.
3. The limit is a very peculiar part, as it is not a constituent of the whole, nor can it be separated from it. Nonetheless, it can be identified and considered in its own right. As Verde underlines,[111] in claiming that minima are 'limits' (rather than 'parts'), Epicurus signals that they are very special kinds of parts: they cannot be imagined as existing independently of the body to which they belong, and so cannot be separated from it. This is important, because it allows Epicurus to break the symmetry between composition and division: the minima are parts that can be identified in the atom, but out of which the atom cannot be composed.[112] This feature is also stressed by

[109] It is debated whether *'perata'* should be taken as predicative or as attributive (see Verde 2013: 69 ff. for a discussion): the one or the other reading might shift slightly the focus of the analysis, but I think that the considerations that I am going to present should be compatible with both versions.
[110] See also Vlastos 1965.
[111] Verde 2013.
[112] Notice that, by using the notion of limit in this way, Epicurus is exploiting a tension internal to Aristotle's discussion of limits. Indeed, Aristotle's claim that indivisibles cannot be in contact relies on the assumption that limits are parts of the body. However, he also claims elsewhere that limits are not properly parts of

Lucretius, who claims that the minimum 'never was or could be separated by itself, since its very existence is as a part of something else'.[113]

The analysis of *Ep. Hdt.* 56-59 shows clearly that to understand Epicurus' theory of minima, it is essential to clarify the notions of division and of part that he is using. I will now try to give an original interpretation of these notions, by taking into account all the elements that I have brought to light.

§3.3. Divisions and Parts

The analysis of the four arguments against the possibility that a limited body contains an infinite number of parts shows that they are directed against both physical and theoretical divisibility. David Furley offers an interpretation which has now become mainstream: Epicurus, being worried by both kinds of divisibility (as the one would endanger the physical constitution of the world, and the other its rationality), postulates a limit to both. Thus, he develops a very peculiar kind of atomism, which is articulated on two levels: atoms constitute the lower limit to physical divisibility, but they are still theoretically divisible. The lower limit to theoretical divisibility, on the contrary, is constituted by the minima in the atom.

At the core of the interpretation is the distinction between the two levels, which Furley presents as a distinction between the two different kinds of division:

> We have to distinguish two kinds of division. I call the first *physical division*: that is, the division of something in such a way that formerly contiguous parts are separated from each other by a spatial interval. This is opposed to *theoretical division*: an object is theoretically divisible if parts can be distinguished within it by the mind, even if the parts

bodies, as they are not separable from them, and they do not augment nor diminish the size of the body – they have one dimension less than the body. Epicurus is here exploiting the tension in Aristotle's treatment, by selecting some specific functions of limits which can fit his own needs.

[113] Lucretius 1.599 ff.

> can never be separated from each other by a spatial interval. […] The *minimae partes* of atoms in Epicurean theory ere theoretically indivisible portions of matter; that is to say, they were such that no parts could be distinguished within them by the mind.
>
> Furley 1967: 4

As I briefly mentioned at the beginning of the chapter, this theory has always been considered suspect. This should not come as a surprise: the idea of a spatially extended interval which is nonetheless not theoretically divisible[114] in parts blatantly contradicts one of the basic assumptions of Euclidean geometry, that it is always possible to individuate a point between any two given points.[115] Moreover, it seems completely arbitrary to impose a lower limit to the capacity of the mind to identify parts on a spatially extended interval, once we allow for the possibility of identifying parts in an atom by 'magnifying' it.[116]

To justify this imposition, it would be necessary to invoke some *a priori*, theoretical principle. The only science able to provide an adequate principle, however, seems to be geometry.[117] Euclidean geometry cannot do the job, obviously: this view can hence be held only if one is ready to accept that Epicurus rejects Euclidean geometry and substitutes it with a finitist, discretist one. It is indeed possible that an alternative geometry is developed in the Garden.[118] This hypothetical alternative geometry, however, is at most attributable to some of his followers, but surely not to Epicurus himself: there is no trace of geometrical considerations in the *Letter,* and Epicurus is well known for his

[114] Throughout this chapter, 'theoretically (in)divisible' will be consistently used to say that something is not divisible in theoretical parts, *not* to say that in theory it could be divisible but there are physical limitations to the actualization of this possibility. I will discuss what theoretical parts are shortly.
[115] Euclidean geometry is built upon the assumption of continuity, even if this is never made explicit in the *Elements*. This is already noticed by Leibniz, and axioms of continuity are added by Hilbert in his new axiomatization of Euclidean geometry.
[116] This criticism is expressed by Purinton 1989. See footnote 105 above.
[117] According to Vlastos 1965, a geometrical, *a priori* reason would be the *only* possible justification for positing a lower limit to theoretical divisibility.
[118] Unfortunately, discussing this issue goes well beyond the scope of this work. For further details, however, see Mau 1973; Sedley 1976; Müller 1982; White 1989; White 1992: 230-51; Bénatouïl 2010; Verde 2013: 249-308.

hatred of geometry.[119] Since he probably studied with Xenocrates, who proposes a theory of indivisible lines, this hatred should not be taken as directed exclusively at *Euclidean* geometry because it is continuous, but at geometry *in general*, as a discipline.

This implies that the physical theory of the *minima* cannot be *dependent on* this hypothetical alternative geometry: on the contrary, the need to emend the principles of geometry might be caused by this revolutionary physical theory. It remains to be explained, however, how the imposition of a lower limit to theoretical divisibility can be justified, if not by appealing to geometry.[120]

The difficulty of this task raises the doubt that Furley's focus on the distinction between physical and theoretical division is misleading – or at least insufficient: it seems that to properly understand the doctrine of the *minimae partes* we should concentrate not only on what kind of division is involved, but also on the notion of a part. It is impossible to discuss this issue by reasoning in an abstract way, though: it is necessary to consider the arguments in their dialectical context, and understand why Epicurus decides to modify Democritus' 'one-tier' atomism by introducing the idea of parts in the atom, despite the physical indivisibility of the atom itself.

Epicurus does not offer any explanation, but a few reasons can be adduced for this modification. The first thing that usually comes to the mind of the modern reader is that Epicurus understands that atoms *cannot* have shape, magnitude and dimensions without having theoretically distinguishable parts: because of this, he would distinguish between physical and theoretical divisibility, denying the former but allowing the latter.

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[119] Cicero, *Academica*, II, 1.

[120] These questions are famously put forward by Vlastos 1965 against the idea that minima are theoretically indivisible. Furley 1967: 4-5 acknowledges that speaking of 'geometrical' or 'mathematical divisibility' is ambiguous: 'since mathematics is one kind of theory, these units can certainly be described as mathematically indivisible. But in another sense a mathematical atomist would be one who denied the principle of infinite divisibility altogether, for every extended magnitude, that is, one who substituted the principle of infinite divisibility in his geometry'. I take this statement as showing an awareness that geometry is just one of the theories which can forbid theoretical divisibility, but Furley fails to specify what other theories could have the same effect, and how theoretical divisibility could be ruled out, if not on the basis of geometrical principles.

This 'geometrical' reason, however, is not adequate. In fact, if this were the reason why Epicurus allows for the theoretical divisibility of the atom, it would be impossible to justify why some of these parts are theoretically indivisible even if they have a shape and a size – and it is quite natural to suppose that minima do have shape and size, as they have positive magnitude.[121] It could even be said that it is the implicit adoption of such a geometrical frame of mind which makes the idea of imposing a lower limit to theoretical divisibility seem so implausible: as soon as the possession of properties such as shape and size is presented as implying theoretical divisibility, the idea of a minimal part which is extended but not theoretically divisible becomes paradoxical.

Let us resist this geometrical intuition, and focus on the *physical* and *dialectical* reasons for the introduction of parts in the atom. Simplicius suggests that the introduction of parts in the atom is a reaction to Aristotle's criticisms of the Early Atomists,[122] and I think that, in light of his suggestion, we can identify three main reasons why parts are introduced:

1) Atoms have *different* shapes and sizes: to account for differences in shapes and sizes, it is not just *possible*, but *inevitable* to identify a part in the atom.[123]
2) Aristotle says that what is indivisible cannot move, because things are in motion only when they are partly in a state and partly in a different state. Consequently, if something has no parts, it cannot be in motion. Indeed, it cannot be moving while it is at its initial position, because it has not started the motion yet, and it cannot be moving while it is at the final position, because at that point it *has moved*, and it is

[121] Since Epicurus never suggests that they do not have shape or size, I believe that we should at least try to give an interpretation of the theory which does not need to suppose that minima lack shapes.
[122] Simplicius, *in Phys*. 925.13.
[123] See also Furley 1967; Long&Sedley 1987. Notice that this point is different from the geometrical reason just rejected. While from a geometrical point of view the possibility of theoretical division is simply dependent on the *possession* of shape and size, here the theoretical division is necessary to account for *differences* in shape and size, so that it becomes necessary only when at least two items are taken into account.

impossible that something is moving and has moved at the same time.[124] However, Epicurus needs to make sure that atoms can move.[125]

3) Aristotle says that what is indivisible cannot be in contact without complete superposition: not having parts, it cannot be in contact part-to-part, but only whole-to-whole. However, for Epicurus it is necessary that atoms can touch without coinciding, as the collision between atoms has a very important role in Epicurean physics.[126]

I believe that once we understand that these are the three main reasons for introducing the theoretical divisibility of the atom, we also gain important information about what kind of division, and what kind of parts, Epicurus is talking about. Indeed, this brief preliminary analysis shows that what is at stake is not the purely geometrical principle according to which it is *possible* to find a point between any two given points. What really matters is that Epicurean physics *does* identify some parts in the atom, because there are functions and roles that these parts must play which cannot be played by the whole atom. This means that we are not just talking about the hypothetical possibility of individuating a part, but of the existence of physical processes and states which do identify a part in the atom.

Let us have a closer look at how this intuition is in play in the three cases I have identified:

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[124] See Furley 1967.

[125] As we will see shortly, this problem is not solved by the introduction of minimal parts in the atoms: the same difficulty appears relative to the minima, and can only be solved by supposing a quantized model of motion. It is hence dubious whether this is really one of the reasons which led Epicurus to introduce the idea of parts in the atom in the first place. Long and Sedley believe that it is, and that he comes to realise that this solution is not strong enough only later in life: at this point, he introduces the theory of quantized motion, which is not present in the *Letter*. As we will see in Chapter 4, it is difficult to determine when the quantized motion is introduced, and some commentators claim that it is already present in the *Letter*. For our purposes, it is not necessary to take a definite stance on the issue: even if the reader rejects reason 2, my argument should work anyway, on the basis of reasons 1 and 3.

[126] Konstan 1979.

1) the difference in shape and size between two atoms forces us to 'see' the parts which correspond to the difference:

A)  B) 

In this case, for example, the 'missing part' in B forces us to see that the corresponding part in A is theoretically divisible from the main body of A.

2) When things move, they are partly in a state and partly in another state: being in two different states, the two parts are differentiated.
3) When an atom touches another atom at just one part, this part has a specific function that cannot be fulfilled by the atom as a whole (if it did, the two atoms would coincide – which is impossible).

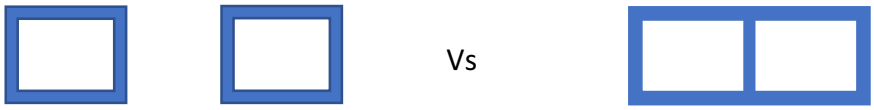
The insight is that what is really at stake is not the abstract capacity to pick out a point between any two given points, but the existence of physical states and processes which involve an internal differentiation of the atom, and hence require the identification of parts in it.

This means that the distinction between physical division and theoretical division traced by Furley is not fine-grained enough. The inadequacy of this distinction is flagged also by Stephen Makin,[127] who points out that every time someone wishes to speak about 'indivisibility', she should specify at least two variables: (i) what counts as a part; (ii) what kind of modality is in play. The problem with Furley's distinction is that the two

[127] Makin 1989. Makin contests the efficacy of this distinction to understand Democritus' conception of the indivisibility of the atom, but I think that his criticism can be easily applied also to the debate on Epicurus' minima.

parameters are confused and collapsed, so that the notion of 'theoretical (in)divisibility' is not completely spelled out. Let us try to clarify it.

Furley presents the distinction between theoretical and physical division as relative to the *kind of division* in question. However, this distinction can be better understood if we separate two factors. The first one is the *outcome* of the division: what kind of parts are produced? A physical division results in physically separated parts (which we can call 'physical parts'), while a theoretical division results in parts which are not physically separated from the whole, and that we can call 'theoretical parts', i.e. parts which are not separated the one from one another by a gap:



Notice that in both cases, what counts as a part is a physical, three-dimensionally extended, solid bulk, which is strictly smaller than the whole. Notably, a 'theoretical part' in the atom is not some kind of mathematical or abstract object – it simply is a part which can be marked out and identified but cannot be separated from the whole.

The second factor is the way in which the division is carried out: a physical division can be operated through a variety of physical means (with scissors, a saw, a knife, bare hands…). But what about a theoretical division? What does it consist in? I believe that there are at least two very different ways in which a theoretical part can be individuated. On the one hand, one can create a part in a previously undifferentiated and homogeneous extension. This is the kind of operation that is at stake when the 'geometrical reasoning' that I have discussed above leads to the claim that it is always possible to freely mark out a point between any two given points, thus individuating a part.

On the other hand, one could think that theoretical division simply consists in recognising and marking out parts which correspond to internal differentiations of the whole. This kind of process seems to fit the three cases that I have discussed above: in all of them, the division into parts maps onto differentiations that are somehow naturally specified by physical factors.

What kind of parts does this division mark out? Very special parts, which, as we have seen in §3.2, share many of the features of limits. I suggest calling them 'functional parts': they are the elements and parts of a whole which *fulfil a function or a role which is specific to them, and cannot be fulfilled by the whole*. They are very special parts, not only because they cannot be separated from the whole, but also because they are in a certain sense dependent on the environment: while an atom on its own might not have any functional part, as soon as it interacts with other atoms and bodies it will become possible to mark out functional parts in it.

In Chapter 4, I will show how this interpretation fits the bigger picture of Epicurean physics and kinematics, and outline the consequences of this theory for the structure of void. Before doing that, however, I want to point out that this interpretation makes the modality involved in the idea of theoretical divisibility not logical, geometrical or metaphysical, but physical. As such, it is absolutely legitimate to suppose that there is a lower limit to the theoretical divisibility: it is impossible to mark out more than a finite number of parts, because the division consists in mapping the internal differentiations required and enforced by physical processes. As the number of these differentiations happen to be finite, the division must terminate.

The strategy that I am following (i.e. taking the theory of minima to be a physical law, rather than a geometrical one) is similar to Vlastos',[128] but I wish to point to two

[128] Vlastos 1965.

main differences. The first one, which is the most important, is related to the idea of part of an atom. Vlastos believes that Epicurus is using the technical, geometrical sense of 'part' which appears in Euclid: something is a part of a body *Z iff Z* is an integral multiple of it. As it happens, I accept Vlastos' point that the minima provide a common unit of measure for all atoms, but I think that we should not take this geometrical sense of part as the primary one. This leads to the second point: Vlastos thinks that the physical law he is spelling out (i.e. that there is a minimal atomic length in nature, and all atoms are multiples of it) has no influence on the possibility of individuating theoretical parts with the mind, and in general that theoretical divisibility has no physical meaning nor consequence. On the contrary, I believe that it does: the originality of Epicurus' theory consists precisely in claiming that the process of individuating parts with the mind does in fact have a physical basis, and that the *minimae partes* are indeed very specific, individual, physical bits of matter.

Chapter 4. Discontinuous Motion and the Structure of Void

The analysis of the nature and structure of the atoms that I have been conducting can be summarised in the following four points:

(i) Atoms are constitutionally primitive;

(ii) They are the constitutive elements of all compound bodies;

(iii) They do not undergo any kind of change or alteration, apart from changes in their location;

(iv) They contain parts, but they are not constituted by these parts. I have called them 'functional parts', as they are only identifiable because they fulfil a specific role or function which cannot be fulfilled by the atom as a whole, but requires some internal functional differentiation. They cannot be separated from the atom, and there is only a finite number of them in every atom.

Hence, the fact that an atom cannot be divided *ad infinitum* does not have to do with a logical or geometrical modality, but with the laws of physics: it is impossible to find more than a finite number of functional parts in the atom, because only a finite number of functional differentiations are required by the physical laws. When considering the atom, the mind cannot create functional parts; it can only individuate them.

In this chapter, I will explore the consequences of the theory of minima, thus interpreted, for the structure of void. The questions that I will be concerned with are: is void structured in a quantized way? Does the theory of finite divisibility also apply to void, and if so, how? Are there minima in void?

In the context of my interpretation of the theory, these questions become non-trivial. Had we concluded that the minima constitute the lower limit to *geometrical* divisibility, they would have obviously been present in the void too: from a purely geometrical point of view, dividing a material or an empty extension makes no difference. As the existence

of the minima is dependent on physical, rather than geometrical or otherwise *a priori* laws, however, the question must be tackled by checking first whether there are physical laws which require functional differentiations in the void, and second whether the number of such differentiations is finite.

On the face of it, there is nothing in the *Letter to Herodotus* which suggests that void should be internally differentiated. On the contrary, in Chapter 2, I have suggested that the void is a unitary, connected, intangible entity which lacks every positive characterization. Simplicius and Themistius, however, attribute to Epicurus the introduction of quantized motion, as well as space- and time-atoms. What should we do of their *testimonia*? In this chapter, I will suggest that, though probably introduced at a later point, the theory of quantized motion is a direct consequence, and a natural development of the physical theses expressed in the *Letter*. This is why it is important to take this theory into account: even accepting the hypothesis of an evolution in the physical theses held by Epicurus (and by his successors), I believe that, when possible, an interpretation that provides a coherent story of the development of the theory as a progressive sophistication and radicalisation of some theses should be preferred over a discontinuist interpretation, which posits the rejection and radical modification of some points.

For this reason, the first aim of this chapter is to prove that the theory of quantized motion is not in conflict with the *Letter*, but the natural development of it, as it represents the extreme consequence of the premises laid down in that early work. To do this, I first analyse the physical meaning of the minima in the atom (§4.1), and show how a generalization of arguments that we find in the *Letter* might lead to the introduction of a quantized theory of motion (§4.2). I then show that this theory, even if absent from the *Letter*, is compatible with the theses on atomic motion there stated (§4.3).

Having shown this, I discuss how quantized motion impacts on the structure of void (§4.4). The results that I obtain can be projected back on the *Letter*, on the assumption that the introduction of quantized motion does not amount to a radical modification of the other notions and theses stated in this early work.

§4.1. The Physical Meaning of Minima

To understand why the theory of minima in the atom might naturally lead to the assumption of a quantized theory of motion, it is first of all necessary to discuss what the minima are from a physical point of view, focusing notably on their physical properties.

I have suggested that minima should be understood as the minimal functional parts of the atom, but this is not very informative without further qualification. All we know at this point is that:

a) The minima are internally functionally undifferentiated: they act as a whole, and there is no process which involves only a part of a minimum, rather than the minimum itself;

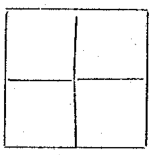
b) The minima are not existent (they are not even conceivable) independently of the atom. The atom is the basic building block of material bodies, and the minima can only be identified as its functional parts, but they are essentially parts.

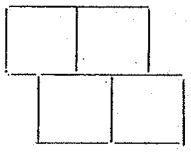
Up to now, however, I have not stressed enough another very important aspect: the minima are physical, solid, full entities – not abstract nor fictional ones. This is because they are parts of atoms, which are themselves physical, full and solid entities. The fact that we can only individuate them on the basis of functional differentiations does not make them less physical.

Accordingly, the minima have some physical properties: notably, they have shape and size, and they are arranged in a specific way in the atom.[129] As one of the things that lead us to identify functional parts is the comparison between the size and the shape of different elements, it must be the case that all the minima have *the same* shape and size. This might seem counter-intuitive, but it surely more acceptable than the idea of a spatial extension that cannot be geometrically divided, or of a three-dimensionally extended physical entity without a shape.[130]

As for the spatial arrangement, we know that there cannot be any empty space between the minima, as the atom is the basic physical element, and we have already said that it is absolutely full, solid and tangible.[131] I have also already claimed, in §3.2, that the problem of contact does not arise in relation to the minima: they are 'neither all in the same place nor touching parts with parts', but are arranged 'in their own peculiar way'.[132]

There is however something deeply counter-intuitive about the arrangement of the minima in the atom. This is linked to the fact that, for the minima to exist, they must not be further differentiable. Consequently, the minima cannot be positioned in any way that involves half a minimum, or in general that leads to the identification of further parts in the atom. As Furley shows,[133] this means that not all possible spatial arrangements are allowed: minima can be disposed as in figure A, but not as in figure B:

A) 

B) 

[129] It does not seem that minima have weight. Indeed, minima cannot exist independently of the atom, so that they cannot even move independently of the atom. If we accept (as I think we should) Konstan's interpretation of weight as a tendency to exit from a collision in a privileged direction, then it is easy to see why they cannot have weight: minima have no independent tendency to move at all. See Konstan 1979 and §2.3 above.
[130] As suggested by Sorabji 1983 and White 1992.
[131] Minima are in a certain sense 'fully-packed' in the atom: see Bicknell 1990 and Lucretius 1.599 ff.
[132] *Ep. Hdt.* 58.
[133] Furley 1967. These two diagrams are also by Furley.

This counter-intuitive consequence, however, is precisely the feature that makes this interpretation plausible and interesting. Indeed, it helps make sense of some quite bizarre and apparently unjustified statements that can be found in the *Letter*. Notably, with regards to the size of atoms, Epicurus claims plainly that there is only a limited (albeit very large) number of atomic sizes, as all atoms must be smaller than the threshold of visible dimensions.[134] Now, passing from the thesis that all atoms must be smaller than *n*, to the statement that there are only a limited number of allowed sizes, is not justified without further assumptions. If there is no restriction on the possible combinations of the minima, an infinite number of different shapes and sizes can indeed be possible, even remaining under a certain size threshold.[135]

Moreover, this picture seems to fit Lucretius' description:

> Quod quoniam docui, pergam conectere rem quae
> Ex hoc apta fidem ducat, primordia rerum
> Finita variare figurarum ratione.
> Quod si non ita sit, rursum iam semina quaedam
> Esse infinito debebunt corporis auctu.
> Namque in eadem una cuiusvis iam brevitate
> Corporis inter se multum variare figurae
> Non possunt: face nim minimis e partibus esse
> Corpora prima tribus, vel paulo pluribus auge;
> nempe ubi eas partis unius corporis omnis,
> summa atque ima locans, transmutans dextera laevis,

[134] *Ep. Hdt.* 62. See Chapter 3 above.

[135] Vlastos tries to explain how Epicurus might pass from the one statement to the other without attributing any physical meaning to individual minima (if fact, he takes the theory of minima to be nothing more than the claim that all atomic sizes are commensurable. The theory is a physical law, but the minima are not taken to be individually specified physical items, but simply a common unit of measurement). However, his attempt is not completely successful, as he attributes to Epicurus two very bad arguments. The first one is simply besides the point: if, as Vlastos maintains, Epicurus is not at all concerned with divisibility paradoxes, but (i) he is just trying to demonstrate that allowing for an infinity of atomic sizes would contradict the phenomena, and (ii) he is allowing for the possibility of continuing the division of the *minimae partes ad libidum,* it is useless to appeal to the assumption that an infinite sequence sums up to infinity: this could only demonstrate that there cannot be an infinity of parts of different sizes *in the same atom*. But it would still be possible to have an infinity of atoms, all of different sizes. The second argument is, by admission of Vlastos himself, fallacious. Moreover, both arguments are geometrical in nature, which seems suspicious given Epicurus' dislike for geometry.

omnimodis expertus eris, quam quisque det ordo
formai speciem totius corporis eius,
quod superset, si forte voles variare figuras,
addendum partis alias erit; inde sequetur,
assimili ratione alias ut postulet ordo,
si tu forte voles etiam variare figuras:
ergo formarum novitatem corporis augmen
subsequitus.

Now that I have explained this, I will proceed to link it with another truth which depending on it draws its proof from it: that the first-beginnings have a finite number of differing shapes. If that were not so, it would once more follow that some of the seeds will necessarily be of infinite size. For within the small measure of one given body the shapes cannot differ much from one another: suppose, for instance, the first bodies to consist of three smallest parts, or increase that number by a few more; naturally, when you take all those parts of one body, and by placing them top or bottom, and transposing right and left, you have tried in all possible ways what shape of that whole body each order gives, if after all you wish perhaps to vary the shapes, other parts must be added; and it will follow that in like manner the arrangement will demand other parts, if you perhaps wish to vary the shapes even further. Therefore novelty of shapes implies increase of size.

Lucretius 2.478-496

It should be noticed, however, that though speaking of 'forbidden arrangements', or saying that a certain combination is 'not allowed', is practical because easily conceivable and understandable, it is not the correct way of presenting the issue and might be misleading. Once again, the confusion is due to the tendency to think of the minima as building blocks that can be used to constitute the atom. We have already seen that this tendency should be resisted, as the atom is not composed by the minima, and the minima have no independent existence. So, it is not the case that some combinations of minima are forbidden in virtue of some *a priori* reason. Rather, the point is that minima do exist precisely because the sizes and shapes of atoms are limited, and atoms are made in such a way that it is possible to identify only a finite number of functional parts, that are

arranged as in figure A. If configuration B were found in nature, it would simply mean that a further level of functional distinctions should be added, and the minima would be smaller than we thought. Epicurus tells us that this cannot go on forever: after a certain degree of smallness, only atoms with regular shapes, which can be divided as in A, can be found.

§4.2. Quantized Motion

If one takes seriously the idea that the minima are physical bits of the atom in which no further functional distinction can be traced, it becomes easier to see how the theory of minima could lead someone to posit a quantized theory of motion, time and spatial magnitude like the one that Simplicius attributes to Epicurus:[136]

> ὅτι δὲ οὐ πάντῃ ἀπίθανον ταύτην τέθεικε τὴν ἔνστασιν, δηλοῖ τὸ καὶ θέντος αὐτὴν καὶ διαλύσαντος τοὺς περὶ Ἐπίκουρον ὅμως ὕστερον γενομένους οὕτω λέγειν τὴν κίνησιν γίνεσθαι· ἐξ ἀμερῶν γὰρ καὶ τὸ μέγεθος καὶ τὴν κίνησιν καὶ τὸν χρόνον εἶναι λέγοντες ἐπὶ μὲν τοῦ ὅλου μεγέθους τοῦ ἐξ ἀμερῶν συνεστῶτος κινεῖσθαι λέγουσι τὸ κινούμενον, καθ' ἕκαστον δὲ τῶν ἐν αὐτῷ ἀμερῶν οὐ κινεῖσθαι, ἀλλὰ κεκινῆσθαι, διὰ τὸ εἰ τεθείη καὶ ἐπὶ τούτων κινεῖσθαι τὸ ἐπὶ τοῦ ὅλου κινούμενον διαιρετὰ αὐτὰ ἔσεσθαι.
>
> That this obstacle which he [Aristotle] has formulated is itself not entirely beyond belief is shown by the fact that despite his having formulated it and produced his solution, the Epicureans, who came along later, said that this is precisely how motion does occur. For they say that motion, magnitude and time have partless constituents and that over the whole magnitude composed of partless constituents the moving object moves, but at each of the partless magnitudes contained in it it does not move but *has* moved; for if it were laid down that the object moving over the whole magnitude moves over these too, they would turn out to be divisible.
>
> Simplicius, *in Phys*. 934, 23-30 (Long&Sedley's transl.)

[136] Themistius' testimony, which is contained in a comment to the same passage of Aristotle's *Physics*, is very similar. I omit the quotation for reasons of space, but it can be found at Usener 278.

εἰ γὰρ μὴ ἦν πᾶν μέγεθος διαιρετόν, οὐχ οἷόν τε ἦν τὸ βραδύτερον ἀεὶ ἐν τῷ ἴσῳ χρόνῳ ἔλαττον κινεῖσθαι τοῦ θάττονος. τὸ γὰρ ἄτομον καὶ τὸ ἀμερὲς ἐν τῷ αὐτῷ χρόνῳ καὶ τὸ θᾶττον διέρχεται καὶ τὸ βραδύτερον· εἰ γὰρ ἐν πλείονι, ἐν τῷ ἴσῳ ἔλαττόν τι τοῦ ἀμεροῦς διελεύσεται. διὸ καὶ τοῖς περὶ Ἐπίκουρον ἀρέσκει ἰσοταχῶς πάντα διὰ τῶν ἀμερῶν κινεῖσθαι, ἵνα μὴ τὰ ἄτομα αὐτῶν διαιρούμενα μηκέτι ἄτομα ᾖ.

For unless every magnitude were divisible, it would not be possible for the slower always to move less distance than the faster in equal time; for that which is atomic, and that which is partless, are traversed by the faster in the same time as they are by the slower. For if the slower takes a longer time, in the equal time it is going to traverse a distance smaller than the partless. That is why Epicurus' school holds that all things move at the same speed through the partless places, lest their atoms be divided and thus no longer atoms.

Simplicius, *in Phys*. 938, 17-22 (Long&Sedley's transl.)

There is an ongoing debate as to whether this theory is already present in the *Letter*,[137] or if it is introduced later, either by Epicurus himself or by some of his followers.[138] I will not enter in this debate, as my argument does not depend on it. In fact, if quantized motion *is* theorized in the *Letter*, then the analysis of its consequence for the structure of space is surely relevant and should be considered. But even if it is *not* present, I am now going to show that the introduction of it is a natural expansion of the theses stated there – and looking at the direct consequences of the theses explicitly stated in the *Letter* is also an important part of the analysis.

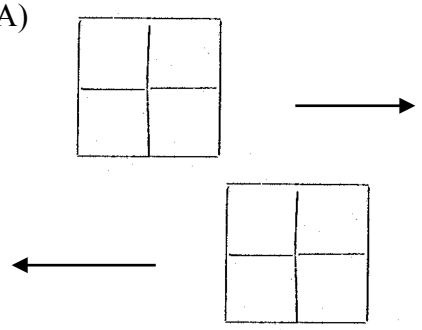
So, how is the 'staccato theory' of motion related to the minima in the atom? Taking seriously the physical nature of the minima is paramount to understanding it. Indeed, if one does not pay due attention to the physical reality of minima, one could think that *for each* possible function or physical process, only a limited number of functional parts can

[137] As Konstan maintains. Notably, one could think that paragraph 62 mentions time-atoms, even if this theory is drastically opposed by Cajoulle-Zaslawski 1980. I will not tackle this issue, because the theory of time goes well beyond the scope of this work.
[138] As claimed by Sedley, Furley, Bicknell.

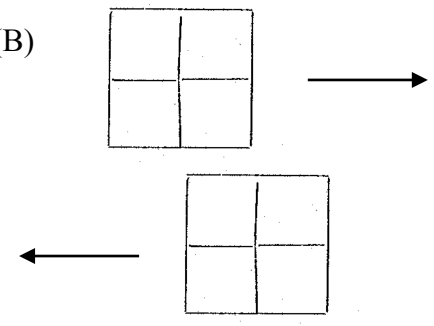
be identified in the atom. This could leave open the possibility that different functions identify different functional parts which do not coincide, so that, even if each function only identifies a limited number of functional parts, taken all together they might sum up to infinity. For the minima to be real physical entities, however, it must be the case that, given *all* the functions and physical processes, and considering them all together, a limited number of parts is identified.

As we have to consider *all* physical processes, we should apply the same kind of reasoning which led us to see that configuration A above was acceptable while B was not, not only to the arrangement of minima in the atom, but also to the relative positions of the atoms themselves. Indeed, it must not be the case that the atoms are positioned in such a way that the one 'cuts' the minima of the other. It immediately follows that some configurations are not acceptable, and that atoms cannot be found in this arrangement. Applied to atoms in motion, this analysis leads to claim that atoms must pass from the legitimate configuration (A) to the next legitimate one (B), without passing by the forbidden configuration (C):

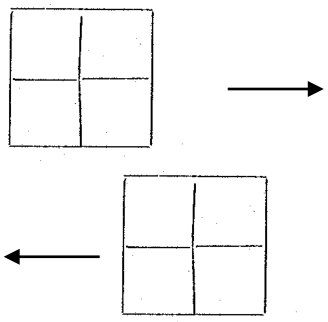
(A)



(B)



(C)



The idea that atoms pass from one position to the other by 'jumps' or 'jerks', without occupying the intermediate states, is incredibly counter-intuitive. However, it does figure between the options that are considered in the ancient debate on the structure of motion, and is often discussed – albeit sometimes only invoked for dialectical and polemical reasons. Locating Epicurus' theory in the broader context of the debate on the structure of physical magnitudes is helpful to see the connections between his various physical theses, and to understand the implications of the quantized theory of motion. In particular, I believe that this theory should be read bearing in mind Aristotle's influence. Therefore, I will now discuss briefly the two most influential Aristotelian theses: the continuity of motion, and the Isomorphism Thesis.[139]

The Isomorphism Thesis, whose main formulation can be found at *Physics* VI.1, 231b18-20,[140] states that the three main physical magnitudes – spatial magnitude (*megethos*),[141] time, and motion – all share the same structure: either the three are discrete, or they are all continuous. This network of dependencies is widely accepted throughout Antiquity and Hellenistic period,[142] and constitutes the theoretical frame of the debate: it can be used to argue for one outcome or for the other, but it is widely held that the structure of one physical magnitude will end up influencing the structure of the others. Let us briefly see why.

Aristotle's proof (which covers the first two chapters of *Physics* VI) can be divided into two statements: first, he claims that if the trajectory is composed by indivisibles, then also the motion over it will be composed by indivisibles.[143] The idea here is that if motion

[139] So called by Miller 1982.
[140] τοῦ δ' αὐτοῦ λόγου μέγεθος καὶ χρόνον καὶ κίνησιν ἐξ ἀδιαιρέτων συγκεῖσθαι, καὶ διαιρεῖσθαι εἰς ἀδιαίρετα, ἢ μηθέν.
[141] The translation of '*megethos*' is rather problematic, as it is not completely clear which of the spatial magnitudes it refers to. I will translate it as 'distance' or 'trajectory': the idea is that the path followed by the moving object must share the same structure than the motion itself.
[142] And later: one could argue that it has been definitely rejected only with Quantum Mechanics.
[143] Arist. *Ph.* VI.1, 231b21 ff: εἰ γὰρ τὸ μέγεθος ἐξ ἀδιαιρέτων σύγκειται, καὶ ἡ κίνησις ἡ τούτου ἐξ ἴσων κινήσεων ἔσται ἀδιαιρέτων.

were divisible, it would be possible to stop the mobile half-way through the distance – which is impossible, if the latter is indivisible.

The second leg of the argument takes as premise the continuity of motion, and proves that, on this assumption, the continuity of time and of *megethos* are interdependent. The demonstration makes use of some wide-spread intuitions about velocity: namely, the idea that if a mobile A is faster than a mobile B, then (i) it moves over the same distance as B in a shorter time, and (ii) in the time that B needs to move over a given distance, A moves over a longer one. Aristotle also assumes that it is always possible to move faster (or slower) than a given mobile. Given these premises, it is clear that the divisibility of motion leads to the divisibility of time: if A is faster than B, and B moves over the distance FG in time XY, then A will cover the same distance in a shorter time, for example XZ. Dividing time in this way causes a division also in the distance: in time XZ, in fact, B has moved across a shorter distance than FG – let us say FE.

Notice that the kind of division invoked across this demonstration is one that Epicurus would accept (if I am right in my reconstruction): we can say that the comparison between different motions, distances and times implies the functional distinguishability of parts in them. It is not a physical division in the sense that the magnitude is not *broken* into two, but it is a physical process which determines and prompts the theoretical division operated by the mind: the resulting parts qualify as functional parts.

Notice also that, in Aristotle's system, the central aspect – the one result on which all the others depend – is the continuity of motion. This is not simply assumed, but the argument that Aristotle gives is quite peculiar, and shows that he takes infinite divisibility to be a fundamental feature of motion.[144] The argument rests on the assumption that it is

[144] I am using 'continuity' and 'infinite divisibility' as if they were synonyms. This is not completely correct. However, a full analysis of Aristotle's theory of continuity would be out of place in this work, and for our purposes the most important feature is that continuity implies infinite divisibility: I hence hope that the reader will excuse my lack of precision.

impossible for something to *have moved* (κεκινῆσθαι) without ever *moving* (κινεῖσθαι) – which seems almost a grammatical observation. This is exactly what would happen in the case of staccato motion: if a mobile X passes instantaneously from position A to position B, disappearing from the first and appearing in the latter without passing through the intermediate positions, one can say that X has moved without ever moving. Indeed, it cannot be moving in A, because the motion is not started yet, and it cannot be moving in B, because when the mobile is in B the motion is already finished.

Interestingly, Diodorus Cronus challenges Aristotle by attacking this main assumption, and producing examples in which is it possible to say that something has happened without ever happening. His examples are usually considered bizarre and besides the point: how are we supposed to progress in the analysis of the structure of motion by observing that 'These men have married' may be true even if, as they married at different times, it is never true to say of them that 'These men are marrying'?[145] Before dismissing this strategy as sophistic, one should notice that it succeeds in pointing out that the reason why Aristotle assumes that things cannot have moved without ever moving is not grammatical, but internal to Aristotle's theory and definition of motion – and that it could hence be rejected with it.

In particular, Diodorus uses this argument to refute the actuality of motion: he takes motion to have always already happened, without ever be actual. This succeeds in getting rid of Aristotle's theory of potentiality, in which his theory of motion is deeply embedded.

The relation between Diodorus and Epicurus is difficult to pin down, as the chronology is unclear: it is uncertain whether Epicurus knows Diodorus' work. The striking similarities in their terminological choices seem to suggest some sort of contact, but this is far from conclusive.[146] If we suppose that Epicurus comes indeed into contact

[145] See Sextus Empiricus, *M.* X 97–100, and Sedley 2013.
[146] See Verde 2013: 210-237 for a discussion.

with Diodorus' theses, it is probable that this encounter prompts him to inquire further on the issue of motion, thus leading him to see that his own premises point to a very similar picture. Notably, Epicurus might be led by Diodorus to see that a complete rejection of Aristotle's theory of potentiality cannot but lead to the complete rejection of his theory of motion. But the assumption of direct contact between the two is not strictly necessary: it is equally possible that the two develop a similar position independently of each other.

§4.3. Atomic Motion in the Letter

In the *Letter*, Epicurus makes two main claims about atomic motion: (i) that atoms move continuously (*sunechōs*),[147] and (ii) that they all move at the same speed (the doctrine of *isotacheia*).[148]

Claim (i) might seem to indicate that my attempt to show that a quantized theory of motion is the natural development of the theses expressed in the *Letter* is bound to fail. Luckily, it is commonly held that the term '*sunechōs*' is not here used in the technical, Aristotelian sense, as opposed to 'discretely'. Rather, Epicurus is simply claiming that the atoms move without interruption, all the time.

Besides being very convenient for my purposes, this interpretation seems indeed the most plausible one. Notice that I am not suggesting that Epicurus does not know that the term '*sunechès*' is defined in a very technical way by Aristotle: the term is at the centre of the network of definitions of *Physics* V.3, and we saw in §3.2 that Epicurus is conscious of Aristotle's analysis. Rather, I believe that Epicurus is rejecting, or consciously ignoring, Aristotle's characterisation of the term. This is not unusual: it is commonly noticed that Epicurus shows a certain dislike for others' technical jargon, and prefers to stick to the common usage of the terms.

[147] *Ep. Hdt.* 43.
[148] *Ep. Hdt.* 61.

In this particular case, moreover, the term in question is one that is used also by Parmenides,[149] so that Epicurus' choice to avoid Aristotle's usage could also be taken as referring back to the Eleatic tradition. If, as I have suggested, he is proposing an atomistic physical theory at least in part as a response to Eleatic problems, this lexical choice would be understandable.

To cast away the last doubts of the sceptical reader, I will also add that the adverbial form '*sunechōs*' is sometimes used by Aristotle himself in a non-technical way (as opposed to the adjective '*sunechès*', whose use is much more controlled), even in the passage of *Physics* V.3 where '*sunechès*' is defined. The definition of '*sunechès*', being the last one of the series, is dependent on all the terms defined previously – notably, it is dependent on the definition of 'in between':

> ἐν ἐλαχίστοις δ' ἐστὶ τὸ μεταξὺ τρισίν· ἔσχατον μὲν γάρ | ἐστι τῆς μεταβολῆς τὸ ἐναντίον,> | μεταξὺ δὲ εἰς ὃ πέφυκε πρότερον ἀφικνεῖσθαι τὸ μεταβάλλον ἢ εἰς ὃ ἔσχατον μεταβάλλει κατὰ φύσιν συνεχῶς μεταβάλλον. [ἐν ... εναντιον.] | συνεχῶς δὲ κινεῖται τὸ μηθὲν ἢ ὅτι ὀλίγιστον διαλεῖπον τοῦ πράγματος—μὴ τοῦ χρόνου (οὐδὲν γὰρ κωλύει διαλείποντα, καὶ εὐθὺς δὲ μετὰ τὴν ὑπάτην φθέγξασθαι τὴν νεάτην) ἀλλὰ τοῦ πράγματος ἐν ᾧ κινεῖται.
>
> What is between involves three things at least; for the contrary is a last point in change, and that which a changing thing, changing continuously and naturally, naturally reaches before it reaches that to which it changes last, is between. A thing is moved continuously (*sunechōs*) if it leaves no gap or only the smallest possible gap in the material—not in the time (for a gap in the time does not prevent things moving continuously, while, on the other hand, there is nothing to prevent the highest note sounding immediately after the lowest) but in the material in which the motion takes place.
>
> Arist. *Ph.* V.3, 227a8-226b29 (*ROT* transl.)

If *sunechōs* had the same meaning as *sunechès*, the definition of the latter would be circular, because *sunechōs* appears in it, and plays a substantial role. However, this does

[149] DK 28 B 8.25: 'the whole is continuous, for what is clings close to what is'. See also Makin 1989: 135.

not seem to be the case: Aristotle explicitly claims that something is 'moving continuously' *iff* it moves without leaving gaps, so in such a way that it cannot be further refined.[150] As long as the motion passes through all the distinguishable states, it could be said to happen *sunechōs*, even if it is discontinuous.[151]

This is a considerably weaker condition than the ones dictated by the theory of 'continuous motion' in *Physics* VIII: we can hence conclude that Aristotle himself was ready to use '*sunechōs*' in the same non-technical, weak sense that we find in Epicurus' text. Obviously, this kind of textual consideration should be taken with care, as it is improbable that Epicurus had direct access to the *text* of the *Physics* as we know it. However, if a weak, non-technical use of the term '*sunechōs*' is registered in the text, it is also plausible to suppose that it used in the same way by Peripatetics in Epicurus' time.

With these terminological provisos, we can safely assume that claim (i) simply reduces to the claim that atoms are *always* in motion, they never rest. Notice that a consequence of this thesis is that atoms do not properly speaking constitute the compound bodies by aggregation. Indeed, they keep moving, or 'vibrating' all the time: the higher concentration of atoms makes clashes happen more often, and 'traps' the atoms in a limited area, thus constituting the macroscopic body.[152]

Moreover, read in this way, claim (i) becomes a limit case of claim (ii), which says that all atoms move at the same speed (indeed, if atoms could rest, there would be variation in speed). The doctrine of *isotacheia* is quite particular, and its details are not completely clear.[153] The general idea is that all atoms move at the same speed, in many

[150] See Panza 2015: 719-720.
[151] The value of the example in brackets is debated. Discussing it, however, would lead us astray from the focus: bringing up this passage, I am not suggesting that Epicurus is conforming to this specific use of the term. Notably, I do not want to suggest that temporal interruptions are allowed in Epicurus' system. I am only using this example to point out that 'moving continuously' (συνεχῶς κινεῖσθαι) is a weak and non-technical formulation even in Aristotle's own exposition.
[152] See *Ep. Hdt.* 43-44.
[153] Notably, it is unclear how the overall motion of macroscopic bodies is supposed to emerge from the individual motions of the atoms. For some related problems, see Drabkin 1938.

different directions, and that the variations in speed that we observe at the level of macroscopic bodies are just the result of the clashes and changes in direction which happen at the atomic level.

Even when not coupled with a quantized theory of motion, this theory is highly counter-intuitive. What leads Epicurus to formulate it? The explanation internal to the *Letter*, which only draws on the theoretical resources developed in that work, is related to the fact that atoms move through void, and void does not resist their passage. This is one of the reasons why Aristotle denies the existence of void: as the speed of a body in motion is a function of its weight and of the resistance offered by the medium, the hypothesis of motion in the void would lead to the implausible conclusion that all bodies move at the same, infinite speed. Epicurus rejects the idea that the speed of the atoms is infinite, but takes them to be inconceivably fast – so fast that there is no well-defined ratio between atomic speed and the velocities of macroscopic bodies that we are able to perceive.[154]

There is hence no need to posit a quantized theory of motion to make sense of the theory of *isotacheia*. It is, however, remarkable that this theory can be applied without any difficulty also to quantized motion. And it is even more remarkable that once one switches from a continuist to a quantized theory of motion, *isotacheia* allows one to solve many problems which would otherwise be insoluble. First, it blocks at least part of Aristotle's argument for the divisibility of physical magnitudes: as we saw, the argument invokes differences in speed to mark distinctions in time and spatial magnitudes – if there is *no* difference in speed, the argument cannot be applied.[155] This strongly supports my

[154] Sedley 1976: 84. *Ep. Hdt* 61 says that atoms are 'as fast as thought'.

[155] Notice also that one of Aristotle's arguments against the possibility of motion at the instant (the one presented at *Ph*. VI.3, 234a24-31) makes substantial use of the assumption that it is always possible to have variations in speed. Also this argument is blocked by the doctrine of *isotacheia*. It is not the only argument that Aristotle presents on this issue, so that blocking it is not conclusive, but it is still interesting to see how rejecting this particular assumption allows Epicurus to dismiss a huge part of Aristotle's arguments.

claim that quantized motion is a natural evolution of the theses expressed in the *Letter*, and should be regarded as a direct consequence of these.

§4.4. Quantized Void?

Now that I have argued for the relevance of the theory of quantized motion for our analysis, let us explore the consequences of this kinematics for the issue which interests us, the structure of void.

Both Simplicius and Themistius' *testimonia* make reference to the Isomorphism Thesis:[156] the quantized structure of motion is shared also by the other two fundamental physical magnitudes involved, i.e. time and distance. As a result, the theory posits a physically indivisible atom which moves at a constant rate of one space-atom per time-atom, disappearing from one position and re-appearing immediately in the following one.

We have evidence that this is the kind of model accepted by Epicurus thanks to Sextus Empiricus, who presents the Epicureans with a paradox that can only apply to a theory of quantized motion so conceived, and notably which involves a quantization of the trajectory of motion.[157] The setting of the paradox is very simple. Take two atoms, placed at a distance of nine spatial minima, and moving at the constant speed of one spatial minimum per time-atom towards each other. After one time-atom, they will be separated by seven spatial minima, then five, then three, then one… and then? What happens? According to Sextus, none of the possible options is viable: none of the atoms can stop (and anyway it would be implausible to suppose that the one 'knows' that it must stop *before* colliding with the other), it is impossible that they meet halfway because the spatial minimum is indivisible, but it is equally impossible that they continue without colliding, because they are both solid and cannot pass through each other.

[156] They come from comments to Aristotle's *Physics* VI, so this should not be surprising.
[157] *M.* X 142-147. See also Bicknell 1990.

Some possible solutions to the paradox have been proposed,[158] but I am here not as interested in solving it, as in understanding the details of the theory it is addressed to, and so clarifying what the spatial minima are supposed to be, and how they are related to void. Does the presence of spatial minima imply that void is composed by indivisible atoms as well?

First of all, it should be noticed that the physical magnitude involved in the discussion is not necessarily space itself. Recall that the Isomorphism Thesis is first given in the Aristotelian setting, where no similar notion is present: the Thesis states that the *distance between two given positions*, not space as a whole, must be structured in a discrete way. From here, to the assumption of a quantized theory *of space*, there is a gap that can be closed only by positing that (i) space is an entity with independent properties, and that (ii) all possible trajectories are somehow connected, so that they must all fit in the same grid.

Commentators tend to overlook the importance of these two additional assumptions, and take the theory to require the imposition of a unitary spatial grid.[159] This is justified only if one accepts Sedley's identification of void with space: this identification grants both assumptions, because void-as-space is a three-dimensional, infinite and unitary extension existing independently. In so doing though, one is also led to assume that the direction of dependence goes from the general structure of space, to the structure of the trajectory of a given motion, to the structure of motion. However, this is not the only possible account. The only thing that the Thesis requires is that the structure of the three magnitudes is isomorphic: it does not impose an order of dependence. Though it is

[158] For example by Bicknell 1990; Konstan 2014.
[159] The only exception I am aware of is Purinton, who however links the quantization to perception, not to the *thing*: he claims that every measurement and every situation can give rise to a different quantization, because this is a structural feature of the human way of perceiving the world, not of the world itself. This is, however, not the kind of discourse that I am doing: I am claiming that the (im)possibility of dividing a magnitude in parts is dependent on the objective features of physical processes.

plausible to suggest that the structure of the trajectory is imposed by some intrinsic property of space, it is equally possible to claim that it is the quantized motion which has this effect: it might be because motion is quantized that the trajectory ends up being composed by minimal distances which are traversed 'in a jump'. Applied to Sextus' case, this means that the problem is not that space is structured in such a way that an atom cannot be found half-way through a spatial minimum, but rather that atoms move in such a way that they cannot end up there.

If one sticks to the picture that I have suggested, where the void is not identified with the totality of space but only with the empty spaces between bodies, the quantization of the trajectories of moving atoms falls short of imposing a unitary quantized structure to the whole of space. In fact, it might be the case that between a moving atom and the other there is an empty space which has a dimension incommensurable with the minima of the two trajectories.

To sum up, the interpretation that I have developed so far takes the void to be a completely independent and autonomous entity, which occupies all and only the positions which are not occupied by atoms. It has no positive features, but is characterised by a lack of all the defining characteristics of atoms, thus as being completely opposite to them: it has no borders nor external shape, it is unable to oppose resistance to the atoms, it is intangible. The only feature of void that is linked to and influenced by the presence of atoms is the internal shape. Consequently, this is the only feature that is influenced by the structure of atoms and of their motion. Indeed, void in itself lacks every structure, but its internal shape changes according to the local motion of atoms: if those move in a quantized way, the changes in the internal shape of void also happen in a quantized way.

This means that the internal shape of void is modified by jerks, at the rate of one spatial minimum per time-atom.[160]

This leaves open the possibility that the starting points of two atoms are at a distance which is incommensurable with the size of the minima, and that there exist stretches or patches of void which have incommensurable sizes. This conceptual possibility is very interesting, because it might be used to solve some deep problems in Epicurean physics. For example, it might explain why incommensurable quantities can be found in the macroscopic world, even if they are rigorously banned at the atomic level: as atoms do not coalesce to form compound bodies, it would be possible to suppose that in a compound there is a patch of void of incommensurable size, which is responsible for the emergence of incommensurability. Exploring this possibility, however, is matter for another study.

[160] Notice that atoms are allowed to move only along the three principal axes: up/down, left/right, backwards/forwards: diagonal motion is not permissible (see Lucretius 2.243-250; Long&Sedley 1987: 50 ff.; White 1992: 250). Again, this can be justified without supposing that space is independently structured: diagonal motion can be forbidden because it would lead to spatial arrangements that violate the indivisibility of atomic minima, or even on the basis of *isotacheia*, because an atom moving diagonally would be faster than the others. Unfortunately, I cannot tackle this issue with the due attention here, nor can I discuss the doctrine of the swerve, as it is not the focus of my analysis. For our purposes, it is sufficient to notice that the internal shape of the void will change accordingly, without needing to suppose that the void itself is structured.

Conclusion

In this work, I have analysed the notion of void as it figures in Epicurus' *Letter to Herodotus*. This hybrid notion tries to conciliate two very different functions, that I have called the 'elemental' and the 'spatial' functions: as such, it should not be identified with the totality of space, but only with the empty intervals between bodies.

In Chapter 1, I have discussed the historical roots of the notion: I have shown that the elemental function is central in the Early Atomists' conception of void, while the spatial function is introduced by Aristotle and presented in terms of receptivity. I have then suggested that a conciliation of the two functions is possible, provided that the feature of receptivity is dropped.

Without appealing to receptivity, however, it is difficult to see how void can be presented as a necessary condition of motion. In Chapter 2, I have addressed this problem, focusing in particular on the *eixis* of void, its ability to 'give way' to bodies. I have argued that it can be interpreted as a reconfiguration of the 'internal shape' of void, and I have drawn a comparison with a contemporary metaphysical theory of holes to show that this can lead to a coherent picture. Then, I have proceeded to show that the void lacks any intrinsic structure, arguing that the directionality of the universe is not due to a property of void.

To conclude the analysis of the structure of void, I had to take into account the theory of the *minimae partes* in the atom. In Chapter 3, I have offered an original interpretation of the notions of part and division used by Epicurus. I have claimed that the minima should be understood as functional parts, i.e. parts which cannot be physically separated from the atom but fulfil a function or a role that cannot be fulfilled by the atom as a whole.

In Chapter 4, I have analysed the consequences of the theory of the *minimae partes* for the structure of void. I have claimed that the theory of quantized motion is a natural

development of the theory of minima, and should be regarded as a direct consequence of it. I have also maintained that the quantization of motion implies the quantization of time and spatial magnitudes: however, this does not mean that void has a quantized structure. On the contrary, void is completely unqualified and unstructured in itself: the only feature of void which is affected by the quantization of motion is the rate of change of its internal shape.

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