

INFLUENCE OF EDDIES ON VERTICAL TRANSPORT AND NUTRIENTS IN SUBTROPICAL GYRES



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Abstract

The presence of large-scale Ekman pumping associated with the climatological wind-stress curl is the textbook explanation for low biological activity in the subtropical gyres. Using an idealised model it is shown that Eulerian-mean Ekman pumping may be opposed by an eddy-driven circulation, analogous to the way in which the atmospheric Ferrel cell and the Southern Ocean Deacon cell are opposed by eddy-driven circulations. Potential vorticity fluxes, Lagrangian particle tracking, and depth-density streamfunctions are used to show that, in the model, the rectified effect of eddies acts to largely cancel the Eulerian-mean Ekman downwelling. To distinguish this effect from eddy compensation, it is proposed that the suppression of Eulerian-mean downwelling by eddies be called “eddy cancellation.”

Eddy cancellation highlights that the thermocline forms a barrier between the surface waters and the abyssal ocean. The presence of this barrier suggests that the nutrient budget of the subtropical mode water and euphotic zone should be considered together. An idealised two-layer axisymmetric model of nutrient concentration in subtropical gyres is developed. This model is used to explore the concept of a unified nutrient budget for the euphotic zone and mode water of subtropical gyres. The steady-state nutrient distribution and fluxes from the idealised model are compared with observations and previous studies. These comparisons show that the solutions of the idealised model are reasonable. Following this validation the sensitivity of the model to changes in parameter values is explored. The model predicts a non-monotonic response to changes in the residual Ekman pumping velocity, with a productivity minimum at 30 m year^{-1} . The model also predicts a positive relationship between primary productivity and mode water thickness. The predicted relationship between primary productivity and mode water thickness is tested using observational datasets, which provide some evidence to support the conclusion that thicker mode waters lead to higher productivity in the oligotrophic gyres.

Contents

1	Introduction	1
1.1	Background and motivation	1
1.2	Key questions	4
1.3	Thesis outline	5
2	Equations of motion	7
2.1	Material derivative	7
2.2	Conservation of mass	8
2.3	Conservation of tracers	9
2.4	Conservation of momentum	9
2.5	Equation of state	13
2.6	Navier-Stokes equations	14
2.7	Approximations	14
2.7.1	Boussinesq approximation	14
2.7.2	Hydrostatic approximation	15
2.7.3	Hydrostatic primitive equations	17
2.7.4	Linear equation of state	19
2.7.5	Beta plane	19
2.8	Summary of key points	20
3	Computational tools	21
3.1	The MITgcm	21
3.1.1	Hydrostatic approximation	21
3.1.2	Equation of state	22
3.1.3	Advection scheme	22
3.1.4	Free surface	23
3.1.5	Mesoscale eddy parametrisation	24

3.1.6	Viscosity	25
3.1.7	Biogeochemical package	25
3.2	Python module for the MITgcm	26
3.2.1	The model object	27
3.2.2	Core classes	27
3.2.3	Iso-surfaces	29
3.2.4	Streamlines and particle tracking	29
3.2.5	Visualisation	30
3.3	Summary of key points	30
4	Potential vorticity dynamics of a wind-driven gyre	33
4.1	Potential vorticity fluxes	34
4.1.1	Instantaneous potential vorticity flux	35
4.1.2	Time-mean potential vorticity flux	39
4.2	Vertical PV flux	40
4.2.1	Computing the PV flux terms	44
4.3	Mode water	48
4.4	Hypotheses	48
4.5	Potential vorticity fluxes in simulations	49
4.5.1	Vertical fluxes in an eddy-resolving gyre	49
4.5.2	Vertical fluxes in an eddy-permitting simulation	56
4.5.3	Vertical fluxes in a model with unresolved mesoscale eddies	56
4.6	Concluding remarks	61
5	Potential vorticity dynamics of an idealised North Atlantic	63
5.1	Hypotheses	64
5.2	Overview of the simulations	64
5.3	Multiple gyres	67
5.3.1	Eddy-resolving with multiple gyres	67
5.3.2	Eddy-permitting with multiple gyres	72
5.3.3	Non-eddying resolution with multiple gyres	76
5.4	Including a meridional overturning circulation	79
5.4.1	Eddy-resolving resolution	81
5.4.2	Eddy-permitting resolution	85
5.4.3	Non-eddying resolution	88
5.5	Concluding remarks	90

6	Eddy mean-flow cancellation in wind-driven gyres	93
6.1	Averaging flows and vanishing circulations	95
6.1.1	Averaging flows	95
6.1.2	Vanishing circulations	97
6.2	Hypotheses	99
6.3	Lagrangian particle tracking	100
6.3.1	Parametrised eddies	101
6.3.2	Resolved eddies	102
6.4	Overturning streamfunctions	106
6.4.1	Defined around streamlines	106
6.4.2	Depth-density overturning streamfunction	109
6.5	Concluding remarks	115
7	Nutrient transport in the oligotrophic ocean	121
7.1	Biological productivity in a subtropical gyre	123
7.1.1	Hypotheses	124
7.1.2	Nutrient in the mode water	124
7.2	Nutrients in a cylindrical gyre	126
7.2.1	Process driven fluxes in the euphotic zone	138
7.2.2	Sensitivity testing	143
7.3	The influence of mode water thickness on biological activity	162
7.3.1	North Atlantic subtropical gyre	163
7.3.2	Other subtropical gyres	168
7.4	Concluding remarks	177
8	Conclusions and future work	179
8.1	Summary of key results	179
8.2	Future work	181
8.2.1	Potential vorticity fluxes	181
8.2.2	Numerical estimates of the Generalised Lagrangian Mean	182
8.2.3	Eddy cancellation and adjustment timescales	182
8.2.4	Biogeochemistry in the oligotrophic gyres	183
8.3	Concluding remarks	184
	References	186

Introduction

Mesoscale eddies are a ubiquitous feature of geophysical flows. These eddies alter the large-scale structure of the fluid and the flow. While the importance of eddies in the atmosphere and the Southern Ocean is now well recognised, the dynamics of wind-driven gyres are still predominantly described by time-mean theories that neglect the rectified effect of mesoscale eddies. As well as their dynamical effects, mesoscale eddies are believed to play a role in the nutrient budget of subtropical gyres. However, the nutrient budget of subtropical gyres is poorly constrained, as is our understanding of the importance of mesoscale eddies within this budget. The central theme of this thesis is that including the rectified effects of eddies in our conceptual models is crucial to improving our understanding of the circulation and biogeochemistry of ocean gyres.

While the overarching aim of this thesis is to examine the rectified effect of mesoscale eddies in ocean gyres and their impact on the large-scale circulation and tracer distributions, each of the chapters in this thesis takes a somewhat different approach to the topic. Because of this diversity each chapter includes a largely self-contained introduction. Therefore, this chapter provides a more general introduction before finishing with an outline of the thesis.

1.1 Background and motivation

The circulation of subtropical gyres has often been described with linear theories based on low Rossby number dynamics. For example Sverdrup balance relates the time-mean gyre circulation to the climatological wind-stress curl; in the gyre the change in planetary vorticity due to meridional motion of fluid parcels is balanced by vorticity

input from the wind (Vallis, 2006). Stommel (1948) used a homogeneous model to show that variation of the Coriolis parameter with latitude is a necessary condition for the formation of western boundary currents. Munk (1950) expanded upon this work to show that the surface circulation of the ocean gyres could be explained by the climatological wind-stress curl.

The large-scale atmospheric and oceanic flow is adequately represented by quasi-geostrophic dynamics, which describe the conservation of large-scale potential vorticity (Pedlosky, 1987). Rhines and Young (1982b) presented one of the earliest studies to suggest that eddies play an important role in the large-scale circulation of gyres. Rhines and Young used a quasigeostrophic model to argue that eddies are important for correctly representing the potential vorticity distribution on subsurface density surfaces. They suggested that eddy fluxes were crucial for setting the potential vorticity distribution on density surfaces that are isolated from wind and surface buoyancy forcing. In the quasigeostrophic model used by Rhines and Young (1982b), eddies redistribute potential vorticity in the horizontal and momentum in the vertical, and once the homogenisation of potential vorticity on a density surface has been achieved the eddies need only be vanishingly weak to maintain this state. Rhines and Young (1982a) use this result to explain the vertical structure of the wind-driven gyres. They show that eddies are able to prevent baroclinic planetary waves from continuously concentrating the flow near the surface through the formation of regions that are surrounded by closed contours of potential vorticity. The eddies isolate these regions from high vertical wave number planetary waves since contours of potential vorticity are characteristics of the wave equation. More recently Williams (1991) showed that variations in mixed layer depth and outcropping latitude could explain the homogenisation of potential vorticity on density surfaces within the ventilated thermocline. However, eddies were still required to homogenise the potential vorticity within the western unventilated regions.

As well as analysing the distribution of potential vorticity, it is possible to use potential vorticity fluxes to probe the structure and circulation of ocean gyres. Haynes and McIntyre (1987, 1990) showed that the potential vorticity evolution equation may be expressed in flux form, which relates the local change in potential vorticity to the divergence of a potential vorticity flux vector. They also showed that surfaces of constant density are impermeable to potential vorticity. Schär (1993) extended Bernoulli's theorem to show that in statistically steady diabatic flows, iso-surfaces of Bernoulli Potential are impermeable to potential vorticity fluxes, provided Eulerian-mean quantities are used to construct the flux vectors. Marshall and Nurser (1992)

used potential vorticity fluxes to examine the structure of the subtropical thermocline and the subduction of fluid through the thermocline. More recently Deremble et al. (2014) examined potential vorticity budgets in the North Atlantic. They found that the dominant feature of their potential vorticity budget was recirculation with relatively small fluxes at the surface or the boundaries. Marshall (2000) constructed an area integrated budget for vertical fluxes of potential vorticity. This framework was used and extended by Polton and Marshall (2003, 2007) to examine similarities between the residual overturning circulation in the Southern Ocean and intergyre overturning circulations in the midlatitude gyres. The budget dictates no net flux of potential vorticity through a depth surface within a closed contour of Eulerian-mean Bernoulli potential. A substantial advantage of this framework over the potential vorticity budget described by Deremble et al. (2014) is that it ignores the rotational fluxes of potential vorticity associated with the gyre circulation. In chapters 4 and 5 I use the framework developed by Marshall (2000) to examine the role of eddies in balancing Eulerian-mean advective fluxes of potential vorticity in subtropical gyres.

Taking an Eulerian-mean of the circulation results in a description of the flow that ignores the contribution of correlations between time-varying components. The Eulerian-mean circulation in the midlatitudes includes an intergyre overturning cell linking Ekman pumping in subtropical gyres with Ekman suction in subpolar gyres. This Ekman cell shares many similar features with the atmospheric Ferrel cell, and the Deacon cell in the Southern Ocean. Like the Ferrel and Deacon cells, the Ekman cell is a thermally indirect circulation in which warmer fluid sinks and colder fluid rises. This means that the circulation cannot be sustained by density differences and requires the input of energy to maintain (Vallis, 2006). The Ekman cell also requires diabatic fluid transport across the subtropical thermocline, which is at odds with the adiabatic nature of the ocean interior. The discrepancy between the diabatic flow implied by the Ekman cell and the adiabatic ocean interior motivates our study of the balance between the Eulerian-mean flow and eddy-induced transport in subtropical gyres in chapter 6.

The nutrient budget of the oligotrophic ocean, the surface regions of subtropical gyres with low nutrient concentrations, is poorly constrained (Williams and Follows, 2011). In these regions the presence of large-scale Ekman pumping is often invoked as an explanation of the low nutrient concentrations; the downwards velocity induced by Ekman pumping removes fluid and nutrients from the euphotic zone and inhibits the vertical transfer of nutrients up towards the surface. Jenkins (1982) used observations of tritium and helium to infer the age of subsurface water masses, and hence,

the oxygen utilisation rate. These observations implied a productivity, and associated nutrient fluxes, that were substantially larger than previous estimates. McGillicuddy and Robinson (1997) proposed nutrient fluxes from eddy pumping, mesoscale eddies lifting density surfaces and nutrients into the sunlit upper ocean, could provide the missing nutrients. However, since the sinking of particulate organic matter represents a locally diapycnal transport, eddy pumping relies on a mechanism to replenish nutrients at depth to make a steady-state contribution to the nutrient budget (Oschlies, 2002a).

There are many processes that are believed to play a role in the nutrient budget of the euphotic zone in subtropical gyres, including eddy pumping, horizontal Ekman transport, Ekman pumping, and the biological pump. Although the euphotic zone is a distinct region biologically, it is not dynamically separate from the underlying fluid. However, the thermocline forms a boundary separating the mode water and euphotic zone above from the abyssal water below, which means that it may be more appropriate to consider a nutrient budget for the fluid above the thermocline. This motivates the development of an idealised toy model to represent nutrient concentrations in the subtropical mode water and euphotic zone, which is presented in chapter 7. This idealised model parametrises the key processes in the nutrient budget and is a useful tool for examining the dependency of the nutrient concentrations on these parameters and creating predictions to test with observational datasets.

1.2 Key questions

There are five key questions that this thesis seeks to address. We begin by examining the dynamics of wind-driven gyres and budgets of vertical potential vorticity fluxes to address the first key question of this thesis.

Do the vertical fluxes of potential vorticity within wind-driven gyres exhibit a similar eddy mean-flow balance to the Southern Ocean?

Having observed a balance in the potential vorticity flux budget, we begin exploring the balance between Eulerian-mean advection and eddy-induced circulations in order to examine the second of the key questions.

Does the Eulerian-mean flow field provide a misleading description of the vertical motion within wind-driven gyres?

The possibility that the Ekman pumping in subtropical gyres is opposed by an eddy-induced circulation, like the Deacon cell in the Southern Ocean and the Ferrel cell in the atmosphere, leads to the third key question for this thesis.

If the Eulerian-mean downwelling due to Ekman pumping is opposed by an eddy-induced circulation, what is the large-scale residual vertical motion within subtropical gyres?

The modelling results show a small residual vertical velocity within the subtropical gyre. Since there exist strong vertical gradients in biogeochemical species, any alteration to our assumptions regarding the large-scale vertical motion could have profound implications on our understanding of the biogeochemistry. This leads to the fourth and fifth key questions for the thesis.

What is the impact of this small residual vertical motion on the distribution of nutrients within subtropical gyres?

How sensitive is the large-scale nutrient distribution to changes in the circulation?

1.3 Thesis outline

This thesis is structured as follows. The equations of motion and a number of approximations used to simplify these equations are derived and discussed in chapter 2. Chapter 3 introduces the numerical tools that will be used to approximately solve discretised versions of the equations of motion, and analyse the output of these models. The potential vorticity flux budget developed by Marshall (2000) and Polton and Marshall (2003, 2007) is used to analyse an isolated subtropical gyre in chapter 4, while chapter 5 extends this analysis to a larger domain with multiple gyres and a meridional overturning circulation. Both chapter 4 and chapter 5 show a balance between vertical potential vorticity fluxes due to Eulerian-mean advection and eddy induced fluxes. The balance between the Eulerian-mean circulation and eddies is explored in chapter 6 using Lagrangian particle tracking and depth-density overturning stream functions. These analyses show that Eulerian-mean Ekman pumping is opposed by an eddy-induced circulation. To explore the implications of reduced Ekman pumping chapter 7 presents the development of an axisymmetric advection-diffusion model for phosphate in subtropical gyres. The sensitivity of this model to changes in its parameters, including Ekman pumping and mode water thickness, is tested. These predictions are evaluated against previous theoretical predictions and observational

datasets. Chapter 8 presents conclusions that may be drawn from the work contained in this thesis and some avenues for future work to build upon these results.

Equations of motion

At their most basic level the equations of motion describe the conservation of mass, momentum, and buoyancy. When deriving and describing the equations of fluid dynamics it is helpful to consider a fluid parcel. This hypothetical fluid parcel is much smaller than the length scale of the flow but large enough that the fluid may be considered as a continuum, that is the molecular nature of the fluid may be ignored.

2.1 Material derivative

The following derivations, and much of the rest of this thesis, will make extensive use of the material derivative of fluid properties. This section provides a brief introduction to the material derivative and derives several useful identities that will be relied on in other derivations.

Fluid dynamics can be thought of from a Lagrangian perspective, in which properties are defined following fluid parcels, or from an Eulerian perspective, in which properties within the region of interest are defined as fields. The material derivative provides a useful bridge between these perspectives. The material derivative of a fluid property describes the rate of change of that property following a fluid parcel, but expresses this quantity as a field. If we consider the fluid property ϕ of a fluid parcel initially at location (x, y, z) , the material derivative of ϕ is given by,

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial x} \frac{dx}{dt} + \frac{\partial\phi}{\partial y} \frac{dy}{dt} + \frac{\partial\phi}{\partial z} \frac{dz}{dt}, \quad (2.1)$$

which, if we equate the time derivatives of the coordinates with velocities, can be generalised to become

$$\frac{d\phi}{dt} = \frac{\partial\phi}{\partial t} + \frac{\partial\phi}{\partial x}u + \frac{\partial\phi}{\partial y}v + \frac{\partial\phi}{\partial z}w = \frac{\partial\phi}{\partial t} + \mathbf{u} \cdot \nabla\phi, \quad (2.2)$$

where $\mathbf{u}$ is the three dimensional velocity vector $u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$. To distinguish the material derivative from other derivatives, the following standard notation will be used for the rest of this thesis. The material derivative of ϕ is written as

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + \mathbf{u} \cdot \nabla\phi. \quad (2.3)$$

The following derivations will make use of the material derivative of an integral over a fluid volume. Rather than derive the expression each time, it will be presented here. As shown in Vallis (2006), the material derivative of a volume integral is given by

$$\frac{D}{Dt} \iiint_{V(t)} \phi dV = \iiint_{V(t)} \left(\phi \nabla \cdot \mathbf{u} + \frac{D\phi}{Dt} \right) dV, \quad (2.4)$$

where $V(t)$ is the volume over which the integral is being taken, ϕ is a property of the fluid and $\mathbf{u}$ is again the three-dimensional velocity vector.

2.2 Conservation of mass

Conservation of mass requires that the mass of a fluid parcel remains unchanged regardless of how the fluid parcel is deformed or otherwise affected by the flow. If we consider a volume of fluid V , with density ρ , the mass of this volume, M , is given by,

$$M = \iiint_{V(t)} \rho dV, \quad (2.5)$$

and must be conserved.

Using equation (2.4), the material derivative of mass is

$$\frac{DM}{Dt} = \iiint_{V(t)} \left(\rho (\nabla \cdot \mathbf{u}) + \frac{D\rho}{Dt} \right) dV = 0. \quad (2.6)$$

Since the volume $V(t)$ is arbitrary, the integrand must be identically zero. This result,

$$\frac{D\rho}{Dt} + \rho (\nabla \cdot \mathbf{u}) = 0, \quad (2.7)$$

is the continuity equation, and defines the conservation of mass within the fluid. By expanding the material derivative, this can also be written as

$$\frac{\partial \rho}{\partial t} = \nabla \cdot (\rho \mathbf{u}). \quad (2.8)$$

2.3 Conservation of tracers

Any quantity that is transported by a fluid may be considered a tracer. Tracers can be divided into active tracers, those that affect the flow field, and passive tracers, those whose presence does not alter the flow. Examples of active tracers are density and potential vorticity, while dissolved gasses are passive tracers. If we define ϕ to be some tracer, then away from sources and sinks of ϕ the total quantity

$$\Phi = \iiint_{V(t)} \phi \, dV, \quad (2.9)$$

must be constant. This implies, using equation (2.4) to calculate the material derivative of a volume element, that

$$\frac{D\phi}{Dt} + \phi \nabla \cdot \mathbf{u} = 0. \quad (2.10)$$

However, this has ignored the effect of diffusion because it doesn't alter the integrated quantity of tracer. If we include a term on the right hand side of equation (2.10) that describes diffusion as well as the sources and sinks of ϕ , then we can generalise equation (2.10) to become

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\mathbf{u}\phi) = Q_\phi, \quad (2.11)$$

where Q_ϕ represents the effect of sources, sinks, and diffusion. When ϕ represents density this tracer equation is known as the 'buoyancy equation.'

2.4 Conservation of momentum

The equation for the conservation of momentum per unit mass, $\mathbf{u}$, can be constructed by considering the material derivative of momentum and the body forces acting on an arbitrary volume, $V(t)$, of the fluid.

The momentum of the fluid in the volume $V(t)$ is given by

$$\iiint_{V(t)} \rho \mathbf{u} dV. \quad (2.12)$$

Taking the material derivative of momentum and using the identity in equation (2.4), gives

$$\frac{D}{Dt} \iiint_{V(t)} \rho \mathbf{u} dV = \iiint_{V(t)} \left(\rho \mathbf{u} (\nabla \cdot \mathbf{u}) + \frac{D\rho}{Dt} \mathbf{u} \right) dV, \quad (2.13)$$

which can be further simplified using the product rule on the final term to become

$$\frac{D}{Dt} \iiint_{V(t)} \rho \mathbf{u} dV = \iiint_{V(t)} \left(\rho \mathbf{u} \nabla \cdot \mathbf{u} + \mathbf{u} \frac{D\rho}{Dt} + \rho \frac{D\mathbf{u}}{Dt} \right) dV \quad (2.14)$$

$$= \iiint_{V(t)} \left(\mathbf{u} \left(\rho \nabla \cdot \mathbf{u} + \frac{D\rho}{Dt} \right) + \rho \frac{D\mathbf{u}}{Dt} \right) dV, \quad (2.15)$$

in which first term on the right hand side can be identified as the continuity equation which is identically zero. Using Newton's second law this must equal the integral of forces acting on the fluid, giving

$$\iiint_{V(t)} \rho \frac{D\mathbf{u}}{Dt} dV = \iiint_{V(t)} \mathbf{F} dV, \quad (2.16)$$

where $\mathbf{F}$ represents all of the forces acting on the fluid parcel. These forces include pressure, gravity, the centrifugal and Coriolis forces, wind stress, viscosity, and frictional drag on the boundaries.

The first force we consider is pressure. Pressure is the normal force per unit area, and is defined within the fluid as well as at the boundary. The pressure force on a volume of fluid is the integral of pressure on the surface, S , of the volume,

$$\iiint_{V(t)} \mathbf{F}_p dV = - \oint_{S(t)} p d\mathbf{S}, \quad (2.17)$$

where p is the pressure, and the negative sign comes from the directional definition of the normal to the surface. If we consider an arbitrarily small cube, shown in figure 2.1, the pressure force on the cube is given by

$$\begin{aligned} \mathbf{F}_p \delta V = & - (P_1 \delta y \delta z \mathbf{i} - P_2 \delta y \delta z \mathbf{i} + P_3 \delta x \delta z \mathbf{j} - P_4 \delta x \delta z \mathbf{j} \\ & + P_5 \delta x \delta y \mathbf{k} - P_6 \delta x \delta y \mathbf{k}), \end{aligned} \quad (2.18)$$

in which δx , δy , and δz are the side lengths of the cube, δV is the volume of the cube, and P_1 to P_6 are the pressures on the faces. Equation (2.18) can be rearranged to give

$$\mathbf{F}_p \delta V = - \left(\frac{P_2 - P_1}{\delta x} \mathbf{i} + \frac{P_4 - P_3}{\delta y} \mathbf{j} + \frac{P_6 - P_5}{\delta z} \mathbf{k} \right) \delta x \delta y \delta z, \quad (2.19)$$

which in the limit $\delta x, \delta y, \delta z \rightarrow 0$ becomes

$$\mathbf{F}_p dV = -\nabla p dV, \quad (2.20)$$

and can be integrated over the volume $V(t)$ to give

$$\iiint_{V(t)} \mathbf{F}_p dV = \iiint_{V(t)} -\nabla p dV. \quad (2.21)$$

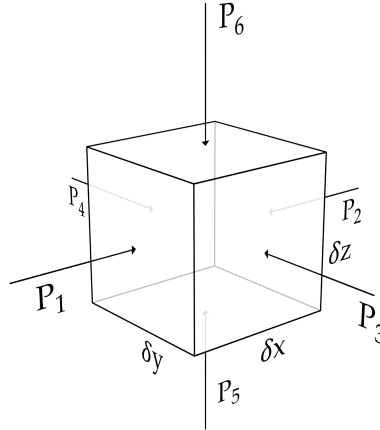


Figure 2.1: *Schematic of the pressure forces on an arbitrary cube in the fluid*

If the reference frame is non-inertial, such as the surface of a rotating sphere, we require the presence of two additional forces in the momentum equation to ensure the conservation of momentum. These are the centrifugal force and the Coriolis force.

The centrifugal force is much smaller than gravity and can be included into the gravitational term by constructing a geoid, the surface of which is everywhere perpendicular to the sum of the gravitational and centrifugal forces, and using the tangent plane of the geoid as the locally horizontal plane. For the geophysical flows considered in this thesis the strength of the gravitational force and centrifugal force can be considered a constant, thus we add

$$- \iiint_{V(t)} \rho g \mathbf{k} \, dx dy dz \quad (2.22)$$

to the right hand side of the momentum equation, where g represents the sum of gravity and the centrifugal force. The negative sign comes from our choice regarding the sign of the vertical axis; z increases upwards.

The second additional force in the momentum equation is known as the Coriolis force, and is the cross product of the Coriolis parameter and the momentum, taking the form

$$\iiint_{V(t)} -\mathbf{f} \times \rho \mathbf{u} dV. \quad (2.23)$$

Combining these terms results in the momentum equation. Since the volume, $V(t)$, is arbitrary we can drop the integral and the momentum equation becomes

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{f} \times \rho \mathbf{u} + \nabla p = \mathbf{F}' - \rho g \mathbf{k}, \quad (2.24)$$

where ρ is the density, and $\mathbf{F}'$ represents the effect of forcings, e.g. viscosity, bottom and side drag, and wind.

Using the identities,

$$\mathbf{u} \cdot \nabla \mathbf{u} = \frac{1}{2} \nabla (\mathbf{u} \cdot \mathbf{u}) - \mathbf{u} \times (\nabla \times \mathbf{u}), \quad (2.25)$$

and

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a} \quad (2.26)$$

the $\rho \mathbf{u} \cdot \nabla \mathbf{u}$ and $\mathbf{f} \times \rho \mathbf{u}$ terms in equation (2.24) can be rewritten as

$$\frac{\rho}{2} \nabla (\mathbf{u} \cdot \mathbf{u}) + (\mathbf{f} + \boldsymbol{\zeta}) \times \rho \mathbf{u}, \quad (2.27)$$

in which $\boldsymbol{\zeta}$ is the curl of the velocity field, referred to as the relative vorticity. If we substitute equation (2.27) into equation (2.24) it becomes

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \frac{\rho}{2} \nabla (\mathbf{u} \cdot \mathbf{u}) + (\mathbf{f} + \boldsymbol{\zeta}) \times \rho \mathbf{u} + \nabla p = \mathbf{F}' - \rho g \mathbf{k}, \quad (2.28)$$

which can be simplified further if the two gradient terms are combined into a single term and we convert from momentum per unit volume to momentum per unit mass by dividing each term by the density. Combining the two gradient terms requires

the Boussinesq approximation, which is described in the upcoming section 2.7.1, and gives

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{f} + \boldsymbol{\zeta}) \times \mathbf{u} + \nabla \Pi = \frac{\mathbf{F}'}{\rho_0} + b\mathbf{k}, \quad (2.29)$$

where Π is the Bernoulli potential, $(\mathbf{u} \cdot \mathbf{u})/2 + p/\rho_0$, and b is the buoyancy given by $-g\delta\rho/\rho_0$. In this form the non-linearity from the material derivative has been moved into the Bernoulli Potential and the vorticity term. An alternative form that retains the material derivative and does not require the Boussinesq approximation is

$$\frac{D\mathbf{u}}{Dt} + \mathbf{f} \times \mathbf{u} + \frac{1}{\rho} \nabla p = \frac{\mathbf{F}'}{\rho_0} - g\mathbf{k}. \quad (2.30)$$

As written, $\mathbf{F}'$ encompasses all of the other forcings in the momentum equation including viscosity, side-drag, bottom-drag, and wind forcing. The viscous effects in the forcing term, $\mathbf{F}'$, are usually expressed as a Laplacian, but the exact form depends on the choice of viscosity used in the simulation.

2.5 Equation of state

The equation of state defines density as a function of temperature, salinity, and pressure, written in general terms as

$$\rho = \rho(\theta, S, p), \quad (2.31)$$

where θ is temperature, S is salinity, and p is pressure.

The relationship between temperature, salinity, pressure, and density in the ocean is complex and non-linear. The accepted best relationship between these variables in the ocean is given by the recent Thermodynamic Equation Of Seawater (TEOS-10) (IOC et al., 2010). However, a range of equations of state are used in modelling studies depending on the intention of the study. Idealised modelling studies often use a linear equation of state, and may even neglect the effect of salinity on density (e.g. Abernathey et al., 2011). The simulations presented in this thesis will use an idealised linear equation of state, which means that we ignore non-linear effects such as cabbeling, and density is a materially conserved tracer in these simulations.

2.6 Navier-Stokes equations

The full set of equations describing fluid motion, equations (2.7), (2.28), and (2.31), are collectively referred to as the Navier-Stokes equations, or the Euler equations if the effects of viscosity are neglected.

$$\frac{D\rho}{Dt} + \rho(\nabla \cdot \mathbf{u}) = 0, \quad (2.32)$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \frac{\rho}{2} \nabla (\mathbf{u} \cdot \mathbf{u}) + (\mathbf{f} + \boldsymbol{\zeta}) \times \rho \mathbf{u} + \nabla p = \mathbf{F}' - \rho g \mathbf{k}, \quad (2.33)$$

$$\rho = \rho(\theta, S, p), \quad (2.34)$$

2.7 Approximations

The Navier-Stokes equations apply at all length- and timescales, which means that everything from sound waves to ocean gyres can be described by the same set of equations. This vast range of time and length scales makes numerical simulation of the full equations extremely difficult. To alleviate this challenge many approximations are used. The approximations used in the numerical simulations discussed in this thesis are described below.

2.7.1 Boussinesq approximation

To derive the Boussinesq approximation we begin by partitioning density into a reference value and perturbations from this value (Vallis, 2006). That is,

$$\rho(x, y, z, t) = \rho_0 + \delta\rho(x, y, z, t), \quad (2.35)$$

where ρ_0 is the reference value, and $\delta\rho$ is the perturbation from this value that depends on all spatial and temporal coordinates. The Boussinesq approximation requires that $\delta\rho \ll \rho_0$.

In the Boussinesq approximation density is replaced by a reference value everywhere density appears in the equations of motion except for the terms involving gravity (Olbers et al., 2012). Therefore the momentum per unit volume of fluid depends only on the velocity, rather than the velocity and the density.

The Boussinesq approximation is widely used in numerical ocean models and has an error of 2-3% (Olbers et al., 2012). The anelastic approximation, which is more

accurate than the Boussinesq approximation, involves replacing the full density with the reference profile except in the gravitational term (Ogura and Phillips, 1962). That is $\rho_0(z)$ instead of ρ_0 as is done in the Boussinesq approximation.

2.7.2 Hydrostatic approximation

The hydrostatic approximation reduces the vertical momentum equation to a balance between the gravitational term and the vertical pressure gradient. This means that vertical velocity is calculated from the divergence of the horizontal velocity field and is a diagnostic variable, rather than a prognostic one.

The hydrostatic approximation is primarily a consequence of the aspect ratio of geophysical flows. It is because the horizontal length scale is much larger than the vertical length scale that we are able to ignore the rest of the terms in the vertical component of the momentum equation.

To show this we begin by non-dimensionalising the horizontal components of the momentum equation,

$$\frac{D\mathbf{u}}{Dt} + (\mathbf{f} \times \mathbf{u}) + \frac{1}{\rho} \nabla p = \mathbf{F} - g\mathbf{k}, \quad (2.36)$$

to obtain a scaling for the pressure term. If we consider the horizontal component of equation (2.36), neglect the effect of viscosity and other forcings since they are not dominant in oceanic flows, as shown by the large Reynolds number which is given by the ratio of inertial to viscous forces, and non-dimensionalise the variables with the following scalings

$$(u, v) = U(\hat{u}, \hat{v}) \quad (2.37)$$

$$w = W\hat{w} \quad (2.38)$$

$$(x, y) = L(\hat{x}, \hat{y}) \quad (2.39)$$

$$z = H\hat{z} \quad (2.40)$$

$$t = T\hat{t} = \frac{L}{U}\hat{t} = \frac{H}{W}\hat{t} \quad (2.41)$$

$$f = \Omega\hat{f} \quad (2.42)$$

$$p = P\hat{p} \quad (2.43)$$

$$g = G\hat{g} \quad (2.44)$$

$$\rho = R\hat{\rho} \quad (2.45)$$

we get

$$\frac{U}{T} \frac{\partial \hat{\mathbf{v}}}{\partial \hat{t}} + \frac{U^2}{L} (\hat{\mathbf{v}} \cdot \hat{\nabla}) \hat{\mathbf{v}} + \frac{WU}{H} \hat{w} \frac{\partial}{\partial \hat{z}} \hat{\mathbf{v}} + \Omega U (\hat{\mathbf{f}} \times \hat{\mathbf{v}}) + \frac{1}{\rho_0 \hat{\rho}} \frac{P}{L} \nabla_{\mathbf{h}} \hat{p} = 0, \quad (2.46)$$

in which $\mathbf{v} = (u, v, 0)$ represents the horizontal velocity components.

Using the advective timescale each of the first three terms have a magnitude of U/T . Comparing the magnitude of the rest of these terms shows that the pressure term must have a magnitude of

$$P = \rho_0 L U \Omega \max(1, \frac{1}{\Omega T}). \quad (2.47)$$

This means that for flows with a long time-scale in which $\Omega T \gg 1$ the pressure term balances the Coriolis term, as happens for the large-scale ocean flow. In flows where $\Omega T \ll 1$ the pressure term is balanced by the acceleration term, as happens in tidal currents (Olbers et al., 2012).

If we now non-dimensionalise the vertical momentum equation, and calculate the ratio of the coefficients to the pressure term, we get

$$\begin{aligned} \text{Time derivative: } \left(\frac{W}{T} \right) / \left(\frac{P}{\rho_0 H} \right) &= \frac{H W}{L U \Omega T \max(1, 1/\Omega T)} \\ &= \frac{H^2}{L^2} \min\left(\frac{1}{\Omega T}, 1\right) \end{aligned} \quad (2.48)$$

$$\begin{aligned} \text{Advection term: } \left(\frac{U W}{L} \right) / \left(\frac{P}{\rho_0 H} \right) &= \left(\frac{U}{L \Omega} \right) \left(\frac{H W}{U L \max(1, 1/\Omega T)} \right) \\ &= R_o \frac{H^2}{L^2} \min(1, \Omega T) \end{aligned} \quad (2.49)$$

$$\begin{aligned} \text{Coriolis term: } (U \Omega) / \left(\frac{P}{\rho_0 H} \right) &= \frac{\Omega U \rho_0 H}{\rho_0 L U \Omega \max(1, 1/\Omega T)} \\ &= \frac{H}{L} \min(1, \Omega T) \end{aligned} \quad (2.50)$$

$$\text{Pressure term: } \left(\frac{P}{\rho_0 H} \right) / \left(\frac{P}{\rho_0 H} \right) = 1 \quad (2.51)$$

$$\text{Gravitational term: } \left(\frac{G R}{\rho_0} \right) / \left(\frac{P}{\rho_0 H} \right) = \frac{G R H}{P}, \quad (2.52)$$

in which the Rossby number, R_o , is defined as $U/L\Omega$.

Each of these relative magnitudes, apart from the gravitational term, contain the ratio H/L . For large-scale oceanic flows the horizontal length scale is several

orders of magnitude larger than the vertical length scale. This indicates that only the gravitational term is able to balance the vertical pressure gradient.

Therefore, if the aspect ratio of the flow, H/L , is small, the dimensional vertical momentum equation reduces to

$$\frac{\partial p}{\partial z} = -\rho g. \quad (2.53)$$

Equation (2.53) can also be derived by considering a fluid at rest, but the derivation presented shows more clearly that the hydrostatic approximation is a consequence of the aspect ratio of oceanic flows, rather than their velocities.

2.7.3 Hydrostatic primitive equations

The hydrostatic primitive equations are commonly used as the basis for numerical ocean models (e.g. Adcroft et al., 2008; Madec, 2011; Marshall et al., 1997). The derivation of these equations from the full equations of motions involves conversion to a spherical geometry and the application of three approximations:

- the hydrostatic approximation;
- the thin-shell approximation; and
- the traditional approximation.

The first of these, the hydrostatic approximation has been covered in section 2.7.2. As shown in Olbers et al. (2012), expressing the equations of motion on the surface of a sphere results in the following set of equations:

$$\left(\frac{Du}{Dt} + \frac{uw}{r} - \frac{uv}{r} \tan \phi - f_v + f_h w \right) = -\frac{1}{\rho} \frac{1}{r \cos \phi} \frac{\partial p}{\partial \lambda} + F_u \quad (2.54)$$

$$\left(\frac{Dv}{Dt} + \frac{wv}{r} + \frac{u^2}{r} \tan \phi + f_u \right) = -\frac{1}{\rho} \frac{1}{r} \frac{\partial p}{\partial \phi} + F_v \quad (2.55)$$

$$\left(\frac{Dw}{Dt} - \frac{u^2 + v^2}{r} - f_h u \right) = -\frac{1}{\rho} \frac{\partial p}{\partial r} - g + F_w \quad (2.56)$$

$$\frac{1}{r \cos \phi} \left(\frac{\partial u}{\partial \lambda} + \frac{\partial v \cos \phi}{\partial \phi} \right) + \frac{1}{r^2} \frac{\partial r^2 w}{\partial r} = 0, \quad (2.57)$$

in which r the vertical coordinate is the distance from the centre of the sphere, ϕ is latitude, λ is longitude, f is the Coriolis parameter, f_h is the locally horizontal component of the Coriolis parameter, and the other terms have their standard meanings.

Equations (2.54) - (2.56) are respectively the zonal, meridional, and vertical momentum equations, and equation (2.57), is the continuity equation.

The first step in the thin-shell approximation is to replace the vertical coordinate, r , in these equations with $z + a$, so that the vertical derivatives become $\partial/\partial z$. The second step uses the fact that the depth of the fluid is small compared to the radius of the earth to approximate $r = z + a$ with the constant value a . The third approximation, the so called “traditional approximation” neglecting the terms involving the horizontal component of the Coriolis parameter and the metric terms uw/r and vw/r . The thin-shell and traditional approximations should be used together or not at all, since the use of one without the other results in a system of equations that is not internally consistent (Vallis, 2006).

If the remaining metric terms are incorporated into the material derivative, the hydrostatic primitive equations can be written as

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} + \frac{1}{\rho}\nabla_z p = \mathbf{F} \quad (2.58)$$

$$\frac{\partial p}{\partial z} = -g\rho \quad (2.59)$$

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \nabla_z \cdot \mathbf{v} + \frac{\partial w}{\partial z} = 0 \quad (2.60)$$

$$\frac{D\theta}{Dt} = Q_\theta \quad (2.61)$$

$$\frac{DS}{Dt} = Q_S \quad (2.62)$$

$$\rho = \rho(\theta, S, p) \quad (2.63)$$

in which

$$\frac{D\mathbf{v}}{Dt} = \mathbf{i}\frac{Du}{Dt} + \mathbf{j}\frac{Dv}{Dt} + u\frac{D\mathbf{i}}{Dt} + v\frac{D\mathbf{j}}{Dt} \quad (2.64)$$

where the final two terms represent the metric terms and $\mathbf{v}$ is the horizontal velocity. The hydrostatic primitive equations can also be written in a vector invariant form in which there are no advective derivatives of the coordinate system. The horizontal components of the momentum equation become

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{k}f + \mathbf{k}\zeta) \times \mathbf{v} + w\frac{\partial \mathbf{v}}{\partial z} + \frac{1}{\rho}\nabla_z p + \frac{1}{2}\nabla(\mathbf{v} \cdot \mathbf{v}) = \mathbf{F}, \quad (2.65)$$

in which ∇_z is the gradient operator at constant depth, and $\mathbf{v}$ again represents the horizontal velocity components.

2.7.4 Linear equation of state

The general equation of state, discussed briefly in section 2.5, can be expanded as a Taylor series in temperature, salinity, and pressure to give

$$\rho = \rho_0 + \left. \frac{\partial \rho}{\partial \theta} \right|_{\substack{\theta_0 \\ S_0 \\ p_0}} (\theta - \theta_0) + \left. \frac{\partial \rho}{\partial S} \right|_{\substack{\theta_0 \\ S_0 \\ p_0}} (S - S_0) + \left. \frac{\partial \rho}{\partial p} \right|_{\substack{\theta_0 \\ S_0 \\ p_0}} (p - p_0) + \dots, \quad (2.66)$$

which can be simplified by ignoring higher order terms and the pressure dependence to produce a linear equation of state that depends only on temperature and salinity. The coefficients, $\partial \rho / \partial \theta$ and $\partial \rho / \partial S$ are known as the thermal expansion coefficient and the haline contraction coefficient, and will be designated, throughout the rest of this thesis as α_T and β_S respectively. Using this notation the linear equation of state can be written as

$$\rho \approx \rho_0 + \alpha_T (\theta - \theta_0) + \beta_S (S - S_0). \quad (2.67)$$

A linear equation of state represents a substantial simplification of the true relationship between temperature, salinity, pressure, and density. In reality the coefficients in equation (2.67) are themselves functions of temperature, salinity, and pressure. Provided the range of temperatures and salinities encountered is small enough the non-linearities in the equation of state may be small enough that they can be ignored. Because of their limitations linear equations of state are generally used only in idealised modelling studies.

2.7.5 Beta plane

Variation of the Coriolis parameter with latitude was identified by Stommel (1948) as a key cause of the westward intensification of wind-driven gyres. The simplest geometry on which this effect is retained, and the one used by Stommel, is the beta-plane. The vertical component of the Coriolis parameter, given by $2\Omega \sin(\phi)$, where Ω is the angular velocity of the Earth and ϕ is latitude, can be expanded as a Taylor series around a chosen latitude to give

$$f = f_0 + \left. \frac{\partial f}{\partial \phi} \right|_{\phi_0} (\phi - \phi_0) + \dots, \quad (2.68)$$

where ϕ_0 represents the latitude around which the expansion is centred. Rather than using spherical geometry, a beta-plane is a Cartesian coordinate system, which means

latitude is converted into meridional distance from the expansion point. If latitude is expressed in radians, then meridional distance from the expansion point, y , is given by $(\phi - \phi_0) R_e$, where R_e is the radius of the Earth. Converting equation (2.68) from latitude into meridional distance gives

$$f(y) \approx f_0 + \left. \frac{\partial f}{\partial y} \right|_{y=0} \left. \frac{\partial y}{\partial \phi} \right|_{\phi=\phi_0} \frac{y}{R_e}, \quad (2.69)$$

in which the derivatives can be expressed as

$$\left. \frac{\partial f}{\partial y} \right|_{y=0} = \left. \frac{\partial 2\Omega \sin \left(\frac{y}{R_e} + \phi_0 \right)}{\partial y} \right|_{y=0} = 2\Omega \cos(\phi_0) \frac{1}{R_e} = \beta \quad (2.70)$$

$$\left. \frac{\partial y}{\partial \phi} \right|_{\phi=\phi_0} = R_e, \quad (2.71)$$

which allows equation (2.69) to be written as

$$f(y) \approx f_0 + \beta y. \quad (2.72)$$

Equation (2.72) is used to represent the variation of the Coriolis parameter with latitude on a beta-plane.

2.8 Summary of key points

In this chapter we have derived the material derivative for fluid properties and used an identity for the material derivative of a volume integral to derive the equations of motion for fluids. The conservation of mass, equation (2.7), and momentum, equation (2.28), and the equation of state, equation (2.31) are together known as the Navier-Stokes equations. These equations are valid at all length- and timescales, which means that numerically evaluating solutions to the full equations is extremely difficult. To this end numerous approximations to the Navier-Stokes equations have been developed, some of which were discussed in this chapter. These approximate equations form the basis of the numerical models that will be introduced in the next chapter.

Computational tools

Throughout this thesis I use a range of numerical tools to model the ocean and to analyse the output of those models. This chapter introduces and describes these tools so that the explanations do not distract from the content of the results chapters presented later.

3.1 The MITgcm

The MITgcm (Marshall et al., 1997) is a highly configurable general circulation model that can be used for a range of applications including state estimation (e.g. Mazloff et al., 2010; Wunsch and Heimbach, 2013; Wunsch et al., 2009), free-running numerical simulations (e.g. Thomas and Zhai, 2013), and idealised modelling of oceans or atmospheres (e.g. Munday et al., 2013). The MITgcm can be used to simulate a range of scales from non-hydrostatic convection, to large-scale ocean dynamics.

In this section I describe the configurations used for the simulations presented in the rest of this thesis.

3.1.1 Hydrostatic approximation

The simulations presented in this thesis all have a horizontal length scale that is substantially larger than the vertical length scale. As described in section 2.7.2 a large aspect ratio means that the hydrostatic approximation is appropriate. All of the MITgcm simulations presented in this thesis use the hydrostatic approximation.

3.1.2 Equation of state

Since this thesis focuses on analysing the dynamics of large-scale ocean flows, rather than their thermodynamics, a linear equation of state that depends only on temperature has been used. This simplifies both the simulation and the analysis. However, the omission of haline and thermobaric terms in the equation of state will prevent results from being generalised to regimes in which those effects are important. The use of a linear equation of state precludes the simulation of non-linear effects such as cabelling.

3.1.3 Advection scheme

The tracer equation, discussed earlier in section 2.3, governs the transport and diffusion of tracers. Two advection schemes were used in the simulations presented in this thesis. The simplest of the advection scheme uses the centred second-order discretisation in space and a second-order Adams-Bashforth time stepping algorithm. This is the default advection scheme for the MITgcm (Adcroft et al., 2008). The algorithm computes the advective tendency by calculating the advective flux through each face of the tracer grid cell, an example of which is shown in figure 3.1. The flux of the tracer T through the western face of the grid cell is given by the mass flux, $u_i \times \Delta y \times \Delta z \times h_w$, multiplied by the concentration of the tracer on the face, $(T_{(i-1,j,k)} + T_{(i,j,k)})/2$. The advective contribution to the rate of change of the tracer in this cell is calculated by computing the divergence of these fluxes. This gives the advective contribution to the time derivative as

$$\begin{aligned}
 G_{adv}^T = & u_i \Delta y \Delta z h_w \frac{(T_{(i-1,j,k)} + T_{(i,j,k)})}{2} - u_{i+1} \Delta y \Delta z h_w \frac{(T_{(i,j,k)} + T_{(i+1,j,k)})}{2} + \\
 & v_j \Delta x \Delta z h_s \frac{(T_{(i,j-1,k)} + T_{(i,j,k)})}{2} - v_{j+1} \Delta x \Delta z h_s \frac{(T_{(i,j,k)} + T_{(i,j+1,k)})}{2} + \\
 & w_k \Delta x \Delta y \frac{(T_{(i,j,k-1)} + T_{(i,j,k)})}{2} - w_{k+1} \Delta x \Delta y \frac{(T_{(i,j,k)} + T_{(i,j,k+1)})}{2},
 \end{aligned} \tag{3.1}$$

where the grid spacings Δx , Δy , and Δz may be functions of location, and the h_w and h_s terms denote the proportion of the western and southern faces that are not blocked by the bathymetry.

The centred second-order advection scheme is used for the single gyre simulations. The other simulations use a second order moments conserving scheme by Prather (1986). This scheme represents tracer concentration as a quadratic polynomial within each grid cell. This means that the scheme produces very little numerical diffusion

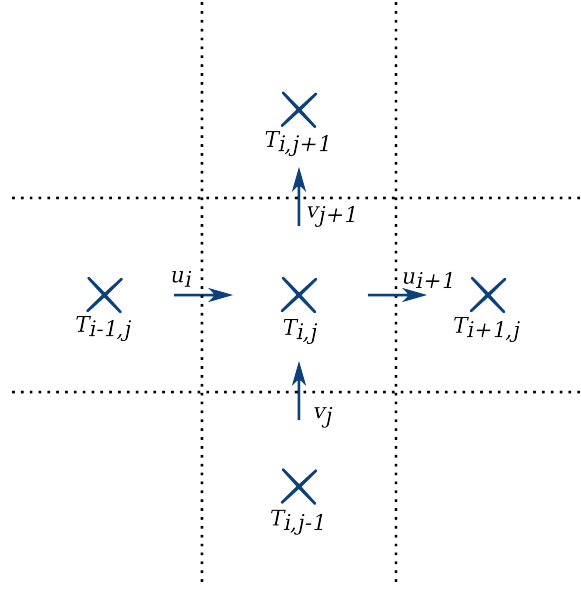


Figure 3.1: A small section of a two-dimensional Arakawa C-grid, with the tracer variable, T , at the centre of the grid cell marked by a blue cross, and the zonal and meridional velocity variables, u and v , located at the left and bottom edges respectively of the grid cell.

or dispersion (Hill et al., 2012). Prather’s advection algorithm is computationally efficient, but does require more arrays to be stored in memory (Prather, 1986).

3.1.4 Free surface

The simulations presented in this thesis use an implicit linear free surface. The free surface elevation is calculated by solving the depth integrated continuity equation implicitly. Integrating the continuity equation over the full depth of the fluid gives

$$\frac{\partial \eta}{\partial t} + \frac{\partial H \hat{u}}{\partial x} + \frac{\partial H \hat{v}}{\partial y} = P - E, \quad (3.2)$$

in which η is the free surface anomaly, $H \hat{u} = \int_{-H}^{\eta} u \, dz$ is the depth integral of the zonal velocity, $H \hat{v} = \int_{-H}^{\eta} v \, dz$ is the depth integral of the meridional velocity, and $P - E$ is the mass source due to precipitation and evaporation. In the configurations used in this thesis $P - E$ is zero.

This equation is discretised as

$$\eta^{n+1} + \Delta t \frac{\partial H \widehat{u}^{n+1}}{\partial x} + \Delta t \frac{\partial H \widehat{v}^{n+1}}{\partial y} = \eta^n, \quad (3.3)$$

in which the superscripts denote the time level, and the spatial derivatives are discretised as centred first-order differences so that they are evaluated on the tracer grid point. Because the discretisation uses velocity data at the $n + 1$ time level to compute the free surface elevation at the $n + 1$ time level it is an implicit algorithm. The greatest advantage of implicit algorithms is that they are extremely stable, often unconditionally so, although the accuracy decreases with increases in the length of the time step (Press et al., 1990). Solving this equation implicitly also has the advantage that fast surface gravity waves, which would impose a severe restriction on the maximum allowable time step, are strongly damped without having their energy aliased into slower modes (Adcroft et al., 2008).

3.1.5 Mesoscale eddy parametrisation

The simulations presented in this thesis are run across a range of resolutions. Some of the resolutions are sufficiently high that the first baroclinic deformation radius is adequately resolved and the effect of mesoscale variability is directly simulated. To improve the fidelity of the simulations which are not able to resolve mesoscale variability the model is augmented by the inclusion of a mesoscale eddy parametrisation in the tracer equation.

The two rectified effects of eddies in the tracer equation are a down-gradient diffusion along isopycnal surfaces (Redi, 1982), and the addition of an eddy-induced transport velocity to the advection term (Gent and McWilliams, 1990; Gent et al., 1995). Since the down-gradient diffusion is constrained to act along isopycnals, and density is constant on isopycnal surfaces by definition, the mixing cannot alter the density field except through non-linearities in the equation of state, which may lead to cabelling. However, this effect is important for other tracers. The presence of an eddy-induced transport velocity means that tracers are not advected by the model velocity field, but rather by a residual velocity field that is the sum of model velocity and the eddy-induced transport velocity.

Gent et al. (1995) show that the Gent-McWilliams eddy parametrisation introduced by Gent and McWilliams (1990) acts adiabatically to slump density surfaces and redistribute water masses. Advection of density by the bolus velocity represents a sink of available potential energy, and mimics the flattening of isopycnals by baroclinic instability.

The MITgcm contains code to implement the Gent-McWilliams eddy parametrisation either as a bolus velocity in the advection scheme, the method in which it was

originally presented, or as a skew flux in the diffusion term, as described by Griffies (1998).

The simulations presented in this thesis use the original advective form of the Gent-McWilliams eddy parametrisation because much of the analysis relies on separating the mean and eddy contributions to advection.

3.1.6 Viscosity

The simulations presented in this thesis use Laplacian and biharmonic implementations of the Smagorinsky eddy viscosity scheme (Smagorinsky, 1963). The biharmonic version is described by Griffies and Hallberg (2000). The Smagorinsky viscosity scheme is not strictly valid for the coarse resolution simulations presented in this thesis. However, since these simulations require unjustifiably large viscosities to remain numerically stable, the use of a numerically convenient viscosity scheme seems reasonable.

Laplacian and biharmonic viscosities are used in combination since this allows for lower values of viscosity to be used, leading to less over-dissipated simulations. Because the Laplacian and biharmonic operators target different scales, using a combination of the two allows for a more scale selective viscosity operator. The benefits of using both Laplacian and biharmonic viscosity operators is discussed by Chassignet and Marshall (2008) in the context of western boundary current separation.

3.1.7 Biogeochemical package

Modelling biological productivity requires the addition of a biogeochemical package to the physical model. The MITgcm biogeochemical package models the evolution of multiple biogeochemical tracers (Dutkiewicz et al., 2005). The biogeochemical package used in this thesis represents two forms of a single nutrient, in this case phosphate and dissolved organic phosphate (DOP).

The package assumes a steady state in order to avoid simulating additional biogeochemical species. Rather than explicitly solving for the concentration of phytoplankton, zooplankton, and detritus the model parametrises their effects on the steady state nutrient concentration. For example, the effect of sinking and remineralisation of detritus is encapsulated by a vertical transfer of nutrients following an empirical power law following Najjar and Orr (1998) and tuned using sediment trap data from Martin et al. (1987).

The package solves the tracer equation for the two forms of phosphate, with additional terms to represent the biological sources and sinks of the nutrient. The source and sink terms for the biogeochemical package are

$$S_{PO_4} = -f_{DOP}J_{prod} - \frac{\partial F_P}{\partial z} + \kappa_{remin}[DOP] \quad (3.4)$$

$$S_{DOP} = f_{DOP}J_{prod} - \kappa_{remin}[DOP], \quad (3.5)$$

where S_{PO_4} and S_{DOP} represent sources and sinks of PO_4 and DOP respectively, f_{DOP} is fraction of productivity that remains suspended in the water column as DOP, J_{prod} is the net community productivity, F_P represents the vertical particulate flux of biological material, κ_{remin} is rate with which DOP remineralises back to PO_4 , and the square parentheses, $[\]$, represent concentrations. The fraction of productivity that does not remain suspended as DOP is instantaneously exported to deeper levels of the model by the particulate flux F_P . As such, $\partial F_P / \partial z$ represents the divergence of the particulate flux due to biological productivity and redistributes PO_4 in the water column using an empirical power law following Najjar and Orr (1998) tuned somewhat using the sediment trap observations of Martin et al. (1987) (Dutkiewicz et al., 2005).

Net community production is calculated as

$$J_{prod} = \alpha \frac{[PO_4]}{[PO_4] + \kappa_{PO_4}} \frac{I}{I + \kappa_I}, \quad (3.6)$$

in which α is the maximum net community production in mol phosphate $\text{m}^{-3} \text{s}^{-1}$, $[PO_4]$ is the concentration of phosphate in mol m^{-3} , κ_{PO_4} is the half-saturation value for phosphate in mol m^{-3} , I is the incident photosynthetically available radiation as a function of depth in W m^{-2} , and κ_I is the half-saturation value for light in W m^{-2} . The biogeochemical package for the MITgcm is described in more detail in Dutkiewicz et al. (2005) and Adcroft et al. (2008). The simulations presented in this thesis use the parameter values shown in table 3.1.

3.2 Python module for the MITgcm

During the course of my PhD I have developed a python package for working with the output of MITgcm simulations. The source code and documentation for this package can be found at <https://doddridge.me/code>.

Table 3.1: *Parameter values used for the biogeochemical package of the MITgcm. These are the default values described in Adcroft et al. (2008) and Dutkiewicz et al. (2005).*

Parameter	Symbol	Value	Units
Maximum net community production	α	2×10^{-3}	$\text{mol PO}_4 \text{ m}^{-3} \text{ year}^{-1}$
Fraction of production converted into dissolved organic phosphate (DOP)	f_{DOP}	0.67	
Remineralisation rate for DOP	γ_{remin}	2	years^{-1}
Phosphate half saturation	λ_{PO_4}	5×10^{-4}	mol m^{-3}
Light half saturation	λ_I	30	W m^{-2}

The model output must be in NetCDF format, as described in Rew and Davis (1990), but may be distributed across multiple files, as occurs automatically when running the MITgcm on multiple processors. The module requires access to a single grid file that spans the model domain, which can be easily constructed using the python script ‘gluemncbig.py’ that is included with the MITgcm source code distribution.

The rest of this section provides an overview of the functionality of the module and includes example code snippets along with their associated output. These examples assume that the modules have been imported with the following standard aliases:

```
import numpy as np
import matplotlib.pyplot as plt
import mitgcm
```

3.2.1 The model object

This module is built around a model object, which is an instantiation of the *MITgcm_Simulation* class and is used to store both model data and parameters. When the model object is created a number of parameters need to be specified. The parameters that must be specified include: the type of grid, currently spherical polar and Cartesian are supported; the number of tiles in each direction, this allows the package to automatically stitch multiple NetCDF files together into arrays; and the equation of state used in the simulation, the package currently only supports linear equations of state. The example in code snippet 3.1 shows a model object being instantiated.

3.2.2 Core classes

The module contains classes for model fields on each point of the Arakawa C-grid. These classes contain bound methods to take derivatives in all three spatial directions on both Cartesian and spherical polar grids. The derivatives are discretised as centred

3.2.3 Iso-surfaces

The module contains a function for computing iso-surfaces from model output. The function extracts the depth at which the field reaches the given value at each grid point and is written assuming the fields are monotonic in the vertical direction. This situation arises naturally for density when the fluid is gravitationally stable. The function is unsuitable for computing iso-speed surfaces, since these fields are not monotonic. It may also be unsuitable for computing temperature surfaces in the presence of stable inversions, such as the temperature inversion found in the Arctic halocline (Talley et al., 2011). To calculate the depth of the 17 °C isotherm and save it to an array called T_17 we execute the following command:

```
T_17 = mitgcm.functions.calc_surface(m.temterature[ 'THETA' ][:], 17, m.grid
    [ 'Z' ][:], method='linear')
```

The module also contains a function for extracting the value of a three-dimensional array on a given depth surface. This could, for example, be used to calculate the potential vorticity on a temperature surface, as shown in the following example:

```
PV_on_T_17 = mitgcm.functions.extract_on_surface(m.PV[ 'PV' ][:], T_17, m.
    grid[ 'Z' ][:])
```

3.2.4 Streamlines and particle tracking

The module includes functions for computing streamlines and performing Lagrangian particle tracking. The streamline and particle tracking algorithms implement a fourth order Runge-Kutta time-stepping scheme, and allow particles to be traced forwards or backwards in time.

Separate functions are provided to track particles in two or three spatial dimensions with constant velocity fields, or three spatial dimensions with time-varying velocity fields. Velocity fields are linearly interpolated in space and time to each particle's location, and a fourth-order Runge-Kutta scheme is used to numerically integrate the three coupled ordinary differential equations for each particle's location as a function of time.

The module also contains functions for extracting model data along trajectories. This means that it is possible to examine buoyancy and other budgets along streamlines or contours.

3.2.5 Visualisation

To aid in the exploration and presentation of model data the module includes several functions for visualising flow fields. These functions are based on the line integral convolution algorithm presented by Cabral and Leedom (1993). This algorithm computes streamlines and uses a convolution kernel to produce an image in which pixels are highly correlated along streamlines and uncorrelated across streamlines. This means that the resulting image provides information about the streamlines of the vector field, but not the direction of flow along those streamlines. However, if the convolution kernel is anisotropic, then the resulting image can provide information about the flow direction as well as the flow topology (Wegenkittl et al., 1997), and is known as ‘oriented line integral convolution’.

The functions in this module use a sparse noise field and an anisotropic convolution kernel to produce an image with a specified number of traces that vary in intensity along their length. The variation in intensity provides information about the flow direction and speed. This algorithm is computationally less expensive than the algorithm presented by Cabral and Leedom (1993) because it requires fewer streamlines to be computed. The computational cost of oriented line integral convolution is determined by the number of traces computed; computing more traces raises the computational cost. Oriented line integral convolution is a particularly powerful technique for visualising geophysical flows in which the direction is as important as the magnitude.

The module includes two functions for performing oriented line integral convolution calculations. The first is intended to produce a single image, while the second is able to produce a sequence of images suitable for turning into an animation. By continuously seeding new traces, and removing old ones, compelling animations of the flow field can be produced. This python module contains an automated method for producing oriented line integral convolution animations of time-mean or time-varying flow fields.

The commands shown in code snippet 3.3 produce a single figure with many traces showing the direction and magnitude of the velocity field on a near-surface depth level. The output of these commands is shown in figure 3.2.

3.3 Summary of key points

This chapter has introduced the numerical tools that were used to produce the results that will be presented in this thesis. The MITgcm is a highly configurable general

Code snippet 3.3: *This code produces a figure using oriented line integral convolution to display the direction and magnitude of the flow field.*

```

output = mitgcm.visualisation.LIC2_sparse(
    m.zonal_velocity[ 'UVEL' ][2,:,:],
    m.meridional_velocity[ 'VVEL' ][2,:,:],
    points=5000,grid_object=m.grid,
    trace_length=30*86400,
    kernel='anisotropic_linear',
    delta_t=3600)

CM_var_alpha = mitgcm.visualisation.create_cmap_vary_alpha(colour='w')

fig = plt.figure(figsize=(12,8))
ax = fig.add_subplot(111)

plt.ylim(15,40)
plt.xlim(0,30)
ax.set_aspect('equal')

im = plt.pcolormesh(m.grid[ 'X' ][:, :], m.grid[ 'Y' ][:, :],
    m.temperature[ 'THETA' ][2,:,:], cmap='OrRd')

skip = 3
for i in xrange(output.shape[1]):
    plt.scatter(output[0,i,::skip], output[1,i,::skip],
        c=output[2,i,::skip]/4., cmap=CM_var_alpha, lw=0, vmin=0,
        vmax=1, s=5)

CB = plt.colorbar(im, shrink=1)

CB.ax.tick_params(labelsize=25)
CB.set_label( '\\textbf{Temperature} (\\circ C)', fontsize=25)

ax.tick_params(labelsize=25)

plt.xlabel( '\\textbf{Longitude} (degrees)', fontsize=25)
plt.ylabel( '\\textbf{Latitude} (degrees)', fontsize=25)

plt.savefig( 'OLIC_example.png', dpi=200)

```

circulation model and this chapter has introduced the specific configurations and packages that will be used in the simulations presented in this thesis. The simulations are all hydrostatic with an implicit linear free-surface. Simulations in which the model does not resolve mesoscale variability employ the GM eddy parametrisation (Gent and McWilliams, 1990; Gent et al., 1995) in its original advective formalism. Viscosity is implemented as a combination of Laplacian and biharmonic Smagorin-

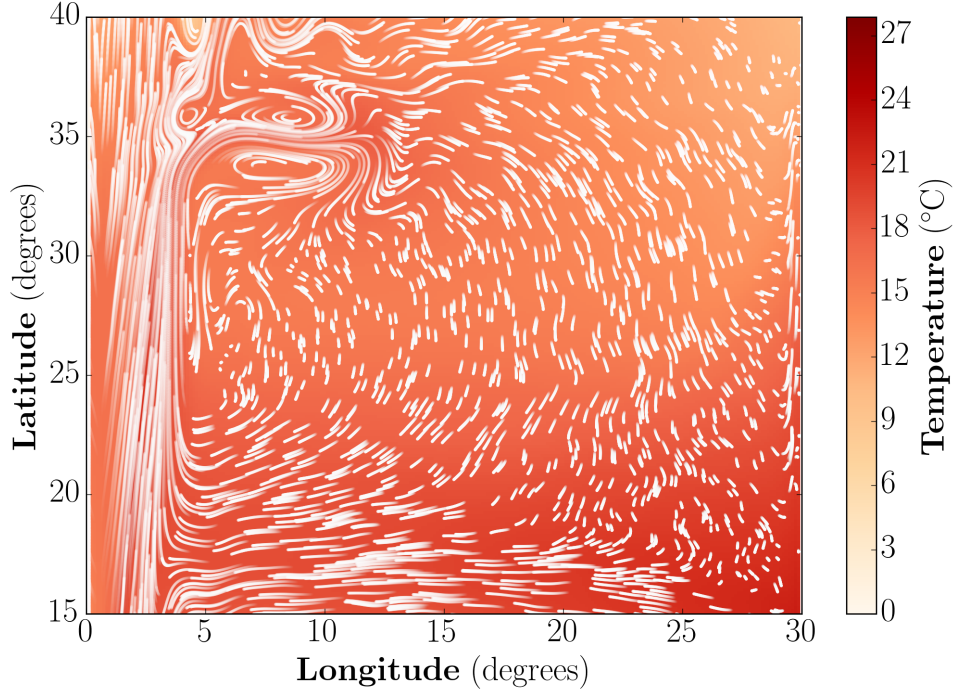


Figure 3.2: *The figure produced by the Oriented Line Integral Convolution example shown in code snippet 3.3. The colours show temperature near the surface in the subtropical gyre, and the white traces represent the motion of particles in the near-surface Eulerian-mean velocity field.*

sky viscosities. A simple biogeochemical package is implemented by considering two biologically active tracers, phosphate and dissolved organic phosphate.

This chapter also introduced a python package that was developed to aid in the analysis of output from the MITgcm simulations. This package provides functions to take derivatives at each location on an Arakawa C-grid, compute iso-surfaces, extract data on non-trivial surfaces, perform Lagrangian particle tracking experiments, and visualise model output.

Potential vorticity dynamics of a wind-driven gyre

It is, according to Pedlosky (1987, p. 42) “hard to exaggerate the importance of the theorem of potential vorticity conservation.” Potential vorticity, usually written as Q , is the fundamental variable for much of geophysical fluid dynamics. In the simplest case of a homogeneous fluid the potential vorticity is given by the ratio of absolute vorticity and thickness, $(f + \zeta)/h$, where f is the planetary vorticity, ζ is the relative vorticity, and h is the thickness of the fluid. In stratified fluids the potential vorticity is given by

$$Q = -\frac{1}{\rho_0}(f + \zeta)\frac{\partial \rho}{\partial z}, \quad (4.1)$$

where ρ is the fluid density, ρ_0 is a reference density, and z is the vertical coordinate.

Conservation of potential vorticity forms the basis for large-scale non-dissipative motions of the ocean and atmosphere (Pedlosky, 1987). Our understanding of wind-driven gyres is built on Sverdrup balance (e.g. Munk, 1950; Stommel, 1948), a linear, time-mean solution to the vorticity equation. For an ocean gyre in Sverdrup balance the change in planetary vorticity in the ocean interior due to meridional motion is balanced by vorticity input from the wind stress. Western boundary currents are invoked to close the mass and vorticity budgets of these gyre solutions (e.g. Pedlosky, 1987; Rhines and Young, 1982a; Talley et al., 2011; Young and Rhines, 1982). This interpretation of subtropical gyres describes a non-local potential vorticity balance. The potential vorticity of fluid parcels decreases in the interior as they move equatorwards in Sverdrup balance and increases in the western boundary current so that fluid parcels may smoothly re-enter the interior flow at the poleward edge of the western

boundary current. Historically the mechanism for the change in potential vorticity within the western boundary current was considered to be viscosity and friction (e.g. Munk, 1950; Stommel, 1948). However, recent work analysing the barotropic vorticity budget has suggested that interactions between the flow and the bathymetry are the dominant mechanism responsible for closing the barotropic vorticity budget (Hughes, 2000; Hughes and de Cuevas, 2001; Jackson et al., 2006). The implications of this work on the potential vorticity dynamics of baroclinic western boundary currents are not yet clear.

Potential vorticity fluxes can also be used to explore the structure and dynamics of geophysical flows.

4.1 Potential vorticity fluxes

Potential vorticity flux vectors have been used in a number of studies to examine the dynamics of oceans and atmospheres. Marshall and Nurser (1992) studied thermocline ventilation by using potential vorticity fluxes to estimate the rate at which fluid crossed the base of the mixed layer. Marshall (2000) constructed an integral constraint on vertical fluxes of potential vorticity within closed contours of Bernoulli Potential to examine the structure of the thermocline and the Antarctic Circumpolar Current. The budget was based on Haynes and McIntyre (1987, 1990) who showed that potential vorticity substance is constrained not to cross surfaces of constant density, and Schär (1993) who generalised Bernoulli's theorem to show that surfaces of constant Bernoulli Potential were also impermeable to potential vorticity. The framework developed by Marshall (2000) was used by Polton and Marshall (2003, 2007) to explore the vertical structure of subtropical gyres and to examine dynamical similarities between these gyres and the Southern Ocean.

Polton and Marshall (2007) identified a three-way balance between potential vorticity fluxes from mean advection, eddies, and buoyancy forcing. They analysed output from a run of the Ocean Circulation and Climate Advanced Modelling Project (OCCAM) model, as described in Webb et al. (1998). The simulation they analysed used an eddy-permitting horizontal resolution of one-quarter of a degree. The diffusivity required for a stable simulation at this resolution may have artificially increased the importance of diffusion in their results.

Deremble et al. (2014) evaluated a potential vorticity budget for the North Atlantic, with a particular focus on the subtropical gyre. They concluded that the

dominant feature of the potential vorticity dynamics is recirculation, with comparatively small fluxes through the sea surface and the boundaries. Despite the small magnitude of these fluxes Deremble et al. were able to show the importance of topographic interactions in the region of the Gulf Stream, and of buoyancy loss in the Gulf Stream extension to the potential vorticity budget. Deremble and Dewar (2013) used the potential vorticity budget developed by Deremble et al. (2014) to examine the formation of mode water in the subtropical gyre.

4.1.1 Instantaneous potential vorticity flux

Ertel's theorem (Ertel, 1942) takes the vorticity conservation equation,

$$\frac{D}{Dt} \left(\frac{\mathbf{q}}{\rho} \right) = \left(\frac{\mathbf{q}}{\rho} \cdot \nabla \right) \mathbf{u} + \frac{1}{\rho^3} \nabla \rho \times \nabla p + \frac{1}{\rho} \nabla \times \mathbf{F} \quad (4.2)$$

in which $\mathbf{q}$ is the absolute vorticity, ρ is the density, $\mathbf{u}$ is the three-dimensional velocity, p is the pressure, and $\mathbf{F}$ represents the forcings in the momentum equation, and projects it on to the vector field given by $\nabla \chi$, where χ is a tracer that satisfies $\rho D\chi/Dt = G_\chi$. As described in Olbers et al. (2012), this can be simplified with the use of a non-intuitive vector identity to give

$$\rho \frac{D}{Dt} \left(\frac{\mathbf{q}}{\rho} \cdot \nabla \chi \right) = \mathbf{q} \cdot \left(\frac{G_\chi}{\rho} \right) + \frac{1}{\rho^2} \nabla \chi \cdot \nabla \rho \times \nabla p + \nabla \chi \cdot (\nabla \times \mathbf{F}), \quad (4.3)$$

where G_χ represents sources and sinks of χ . If χ is a function of ρ and p only, then the gradient triple product term in equation (4.3) is identically zero. Potential density has traditionally been used as the tracer in Ertel's equation. For this reason the variable is called potential vorticity.

Using the Boussinesq approximation in an incompressible fluid where ρ is a materially conserved tracer, the potential vorticity evolution equation from Ertel (1942) becomes

$$\rho_0 \frac{D}{Dt} \left(\frac{\mathbf{q}}{\rho_0} \cdot \nabla \rho \right) = \mathbf{q} \cdot \nabla B + \nabla \rho \cdot \left(\nabla \times \left(\mathbf{F} - \frac{\rho - \rho_0}{\rho_0} g \mathbf{k} \right) \right), \quad (4.4)$$

where ρ is the fluid density, ρ_0 is a reference density, $\mathbf{q}$ is the absolute vorticity, B represents all non-advective terms in the buoyancy equation, $\mathbf{F}$ represents the mechanical and body forcings acting on the fluid, such as viscosity, surface wind forcing, and bottom and side drag, and g is the gravitational acceleration.

Rewriting equation (4.4) using incompressibility to simplify the material derivative gives,

$$-\frac{\partial \rho_0 Q}{\partial t} - \rho_0 \mathbf{u} \cdot \nabla Q = \mathbf{q} \cdot \nabla B + \nabla \rho \cdot \left(\nabla \times \left(\mathbf{F} - \frac{\rho - \rho_0}{\rho_0} g \mathbf{k} \right) \right), \quad (4.5)$$

where $Q = -\frac{\mathbf{q}}{\rho_0} \cdot \nabla \rho$ is the potential vorticity, and the other terms are as defined above. The negative sign in the definition of potential vorticity ensures that the large scale potential vorticity is positive in the Northern Hemisphere and hence that the advective flux in the simulations we consider is in the same direction as the fluid velocity, rather than anti-parallel. As described earlier, the potential vorticity, Q , is usually approximated as

$$-\frac{1}{\rho_0} (f + \zeta) \frac{\partial \rho}{\partial z}, \quad (4.6)$$

where f and ζ represent the vertical component of planetary and relative vorticity respectively. For particularly large-scale or weak-flow applications, this may be simplified even further by ignoring the contribution from relative vorticity.

As noted by Haynes and McIntyre (1987, 1990), equation (4.5) may be recast into a flux form equation which relates the local time derivative of potential vorticity to the divergence of a potential vorticity flux vector, $\mathbf{J}$. Using the identity

$$\nabla \cdot (\psi \mathbf{A}) = \psi \nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla \psi \quad (4.7)$$

and assuming a non-divergent flow field, the $\rho_0 \mathbf{u} \cdot \nabla Q$ term can be rewritten as $\nabla \cdot (\rho_0 Q \mathbf{u})$.

The $\mathbf{q} \cdot \nabla B$ term can be rewritten as $\nabla \cdot (\mathbf{q} B)$ by using the identity in equation (4.7) and confirming that the $B \nabla \cdot \mathbf{q}$ term is identically zero. $\mathbf{q}$ is the sum of the Coriolis parameter and the relative vorticity. The divergence of the Coriolis parameter is zero since under the traditional approximation we consider only the vertical component, which varies only in the meridional direction, hence all of the terms in the divergence evaluate to zero. The divergence of the curl of any vector field is identically zero. Since the relative vorticity is the curl of the velocity field, its divergence is identically zero.

Using the scalar triple product identity,

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \mathbf{b} \cdot (\mathbf{c} \times \mathbf{a}) = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}), \quad (4.8)$$

the $\nabla \rho \cdot \left(\nabla \times \left(\mathbf{F} - \frac{\rho - \rho_0}{\rho_0} g \mathbf{k} \right) \right)$ term becomes $\nabla \cdot \left(\left(\mathbf{F} - \frac{\rho - \rho_0}{\rho_0} g \mathbf{k} \right) \times \nabla \rho \right)$.

Now equation (4.5) can be written in flux form,

$$\frac{\partial \rho_0 Q}{\partial t} + \nabla \cdot (\rho_0 Q \mathbf{u}) = -\nabla \cdot (\mathbf{q} B) - \nabla \cdot \left(\left(\mathbf{F} - \frac{\rho - \rho_0}{\rho_0} g \mathbf{k} \right) \times \nabla \rho \right), \quad (4.9)$$

which can be further simplified to

$$\frac{\partial}{\partial t} (\rho_0 Q) + \nabla \cdot (\rho_0 Q \mathbf{u}) = -\nabla \cdot \mathbf{J}_N, \quad (4.10)$$

by defining $\mathbf{J}_N = \mathbf{q} B + \left(\mathbf{F} - \frac{\rho - \rho_0}{\rho_0} g \mathbf{k} \right) \times \nabla \rho$ as the non-advective potential vorticity flux vector.

Combining the advective and non-advective contributions to the potential vorticity flux into a single flux vector, $\mathbf{J}$, allows equation (4.10) to be written more succinctly as

$$\frac{\partial}{\partial t} (\rho_0 Q) + \nabla \cdot \mathbf{J} = 0, \quad (4.11)$$

where $\mathbf{J}$ is the full potential vorticity flux vector that includes advective and non-advective flux terms. As defined in equation (4.11) only the divergence of the potential vorticity flux vector is constrained. Therefore, the potential vorticity flux may also contain any arbitrary divergence free vector field; this represents a gauge invariance for the potential vorticity flux vector field. Since the divergence of the curl of an arbitrary vector field is identically zero, we include a gauge term in this form. The resulting potential vorticity flux vector field is given by

$$\mathbf{J} = \rho_0 Q \mathbf{u} + \mathbf{q} B + \left(\mathbf{F} - \frac{\rho - \rho_0}{\rho_0} g \mathbf{k} \right) \times \nabla \rho + \nabla \times \mathbf{A}, \quad (4.12)$$

where $\nabla \times \mathbf{A}$ represents the gauge.

The form presented in equation (4.12) explicitly shows the advective and non-advective contributions to the potential vorticity flux. Each of the terms in equation (4.12), apart from the gauge, describe the flux of potential vorticity due to a physical processes:

$$\rho_0 Q \mathbf{u} = \text{advection} \quad (4.13)$$

$$\mathbf{q} B = \text{buoyancy forcing} \quad (4.14)$$

$$\mathbf{F} \times \nabla \rho = \text{momentum forcing} \quad (4.15)$$

$$\frac{\rho - \rho_0}{\rho_0} g \mathbf{k} \times \nabla \rho = \text{gravity under the Boussinesq approximation} \quad (4.16)$$

To derive an alternate formalism, we begin with the hydrostatic, Boussinesq momentum equation written in terms of the Bernoulli Potential and take the cross product of each term with the gradient of density. This gives

$$\frac{\partial \mathbf{v}}{\partial t} \times \nabla \rho + \mathbf{q} \times \mathbf{u} \times \nabla \rho + \nabla \Pi \times \nabla \rho = \left(\mathbf{F} - \frac{\Delta \rho}{\rho_0} g \mathbf{k} \right) \times \nabla \rho \quad (4.17)$$

in which $\mathbf{v} = (u, v, 0)$, and $\Pi = (u^2 + v^2)/2 + p/\rho_0 + gz$ consistent with the hydrostatic approximation.

The triple cross product term, $\mathbf{q} \times \mathbf{u} \times \nabla \rho$ can be simplified using the identity

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C}) \mathbf{B} - (\mathbf{B} \cdot \mathbf{C}) \mathbf{A}, \quad (4.18)$$

which gives

$$\frac{\partial \mathbf{v}}{\partial t} \times \nabla \rho + (\mathbf{q} \cdot \nabla \rho) \mathbf{u} - (\mathbf{u} \cdot \nabla \rho) \mathbf{q} + \nabla \Pi \times \nabla \rho = \left(\mathbf{F} - \frac{\Delta \rho}{\rho_0} g \mathbf{k} \right) \times \nabla \rho. \quad (4.19)$$

The buoyancy equation can be used to replace the $\mathbf{u} \cdot \nabla \rho$ term with $B - \partial \rho / \partial t$, and $\mathbf{q} \cdot \nabla \rho$ can be simplified to $-\rho_0 Q$, which results in

$$\frac{\partial \mathbf{v}}{\partial t} \times \nabla \rho - \rho_0 Q \mathbf{u} - \left(B - \frac{\partial \rho}{\partial t} \right) \mathbf{q} + \nabla \Pi \times \nabla \rho = \left(\mathbf{F} - \frac{\Delta \rho}{\rho_0} g \mathbf{k} \right) \times \nabla \rho. \quad (4.20)$$

This can be rearranged to give

$$\mathbf{q} \frac{\partial \rho}{\partial t} + \left(\frac{\partial \mathbf{v}}{\partial t} + \nabla \Pi \right) \times \nabla \rho = \rho_0 Q \mathbf{u} + \mathbf{q} B + \left(\mathbf{F} - \frac{\Delta \rho}{\rho_0} g \mathbf{k} \right) \times \nabla \rho = \mathbf{J} - \nabla \times \mathbf{A}, \quad (4.21)$$

which shows that the potential vorticity flux can also be written as

$$\mathbf{J} = \mathbf{q} \frac{\partial \rho}{\partial t} + \left(\frac{\partial \mathbf{v}}{\partial t} + \nabla \Pi \right) \times \nabla \rho + \nabla \times \mathbf{A}. \quad (4.22)$$

This form makes explicit the impermeability of density and Bernoulli surfaces in the steady-state solution, provided that the gauge is chosen such that $\nabla \times \mathbf{A}$ is equal to zero. Haynes and McIntyre (1987, 1990) were the first to generalise Ertel's theorem to show that isentropic surfaces are impermeable to potential vorticity. This is known as "the impermeability theorem". Schär (1993) was the first to generalise Bernoulli's theorem to show that at a statistically steady-state potential vorticity fluxes are constrained to lie along the intersection of surfaces of potential temperature and Bernoulli potential, even in a viscous, diabatic, time-varying flow.

4.1.2 Time-mean potential vorticity flux

Equations (4.12) and (4.22) describe the instantaneous flux of instantaneous potential vorticity. It is also possible to examine time-mean potential vorticity fluxes by applying a suitable time filter to the instantaneous equations. To derive the time-mean potential vorticity fluxes we start by filtering the momentum and buoyancy equations into time-mean, $\bar{X}$, and time varying, X' , components. The filter should average over a period of time long enough to ensure that the influence of individual eddies and the initial state are sufficiently reduced. This requires that the time period considered is substantially longer than the lifetime of the average eddy. The time filters used in this thesis average the model fields over at least five years of model time. Using the above notation the time filtered momentum equation takes the form

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \bar{\mathbf{q}} \times \bar{\mathbf{u}} + \overline{\mathbf{q}' \times \mathbf{u}'} + \nabla \tilde{\Pi} + \nabla \frac{\overline{\mathbf{v}' \cdot \mathbf{v}'}}{2} = \bar{\mathbf{F}} - \frac{\bar{\Delta \rho}}{\rho_0} g \mathbf{k}, \quad (4.23)$$

where the tildes, $\tilde{X}$, denote variables constructed from time-mean quantities. The time derivative now represents changes in the velocity field over timescales comparable with, or longer than, the averaging period. Equation (4.23) can be simplified if the eddy contributions are grouped into a single term $\mathbf{F}_R = -\overline{\mathbf{q}' \times \mathbf{u}'} - \nabla \frac{\overline{\mathbf{v}' \cdot \mathbf{v}'}}{2}$ to give

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \bar{\mathbf{q}} \times \bar{\mathbf{u}} + \nabla \tilde{\Pi} = \bar{\mathbf{F}} + \mathbf{F}_R - \frac{\bar{\Delta \rho}}{\rho_0} g \mathbf{k}. \quad (4.24)$$

Applying the same filter to the buoyancy equation results in

$$\frac{\partial \bar{\rho}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \bar{\rho} = \bar{B} - \nabla \cdot \overline{\mathbf{u}' \rho'}, \quad (4.25)$$

where the non-divergence of the velocity field has been used to simplify the second term on the right-hand side.

If these time-filtered momentum and buoyancy equations are used to construct the potential vorticity flux vectors, then we get

$$\tilde{\mathbf{J}} = \rho_0 \tilde{Q} \bar{\mathbf{u}} + \bar{q} \left(\bar{B} - \nabla \cdot \overline{\mathbf{u}' \rho'} \right) + \left(\bar{\mathbf{F}} - \mathbf{F}_R - \frac{\overline{\Delta \rho}}{\rho_0} g \mathbf{k} \right) \times \nabla \bar{\rho} + \nabla \times \mathbf{A} \quad (4.26)$$

$$\tilde{\mathbf{J}} = \bar{q} \frac{\partial \bar{\rho}}{\partial t} + \left(\frac{\partial \bar{v}}{\partial t} + \nabla \tilde{\Pi} \right) \times \nabla \bar{\rho} + \nabla \times \mathbf{A}. \quad (4.27)$$

These equations have the same form as the instantaneous equations, shown in equations (4.12) and (4.22), except that $\mathbf{F}$ and B have been replaced by $\mathbf{F} + \mathbf{F}_R$ and $B - \nabla \cdot \overline{\mathbf{u}' \rho'}$ respectively to take into account the rectified effect of eddies.

To preserve the impermeability of surfaces of constant density and Bernoulli Potential, the gauge used throughout the rest of this thesis will be chosen such that $\nabla \times \mathbf{A} = \mathbf{0}$.

4.2 Vertical PV flux

Following Marshall (2000) we use the impermeability of surfaces of constant density and Bernoulli Potential to create volumes into which there is no net flux of potential vorticity. We construct a volume like this by taking an isosurface of Bernoulli Potential and enclosing a volume with a surface of constant depth. This is demonstrated in figure 4.1, in which a closed volume is formed by the intersection of a Bernoulli Potential isosurface and a depth surface. Since there can be no net flux of potential vorticity across the Bernoulli Potential isosurface, there must also be no net flux across the depth surface that forms the closed volume, provided that the amount of potential vorticity substance inside the volume is constant. It is trivial to repeat this exercise with arbitrary isosurfaces of Bernoulli Potential and depth surfaces. This leads to the conclusion that the area integral of potential vorticity flux through any depth surface between closed contours of Bernoulli Potential must be zero. While the net flux is constrained to zero by construction, it is not clear *a priori* which processes should dominate the dynamical balance.

The integral may be performed with either of the expressions for potential vorticity flux, equations (4.26) and (4.27), or any linear combination of the two.

When performing this integral the $\nabla \tilde{\Pi} \times \nabla \bar{\rho}$ term vanishes. This was shown by Marshall (2000) who used the vector identity

$$\nabla F \times \nabla G = \nabla \times (F \nabla G) = -\nabla \times (G \nabla F), \quad (4.28)$$

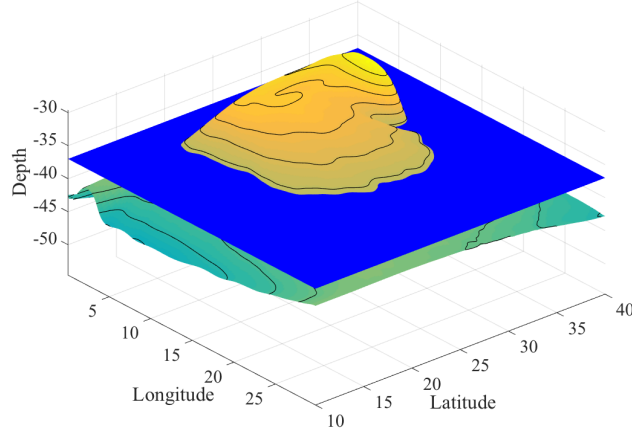


Figure 4.1: Volume with zero net flux of potential vorticity through its faces. The blue surface is a constant depth surface, and the other surface is a Bernoulli Potential isosurface with contours and shading representing depth.

to convert

$$\iint_{\lambda} (\nabla \tilde{\Pi} \times \nabla \bar{\rho}) dA \quad (4.29)$$

into

$$\iint_{\lambda} -\nabla \times (\bar{\rho} \nabla \tilde{\Pi}) dA \quad (4.30)$$

which can be converted to a line integral using Stokes' theorem, and becomes

$$-\oint_{\partial\lambda} \bar{\rho} \nabla \tilde{\Pi} \cdot d\mathbf{r}, \quad (4.31)$$

which must equal zero, since the edge of the area is defined as a line of constant Bernoulli Potential, and so $\nabla \tilde{\Pi} \cdot d\mathbf{r}$ is identically zero everywhere on $\partial\lambda$. The only other terms in equation (4.27) include time derivatives, which can be used to check the impact of model drift on the results. These drift terms relate to changes in the storage of potential vorticity substance within the constructed volume.

We obtain our potential vorticity flux diagnostics by integrating the vertical component of equation (4.26) minus equation (4.27) between closed contours of Bernoulli Potential on a depth surface. The integral has the form

$$\iint_{\lambda} \mathbf{k} \cdot \left[\rho_0 \tilde{Q} \bar{\mathbf{u}} + \bar{\mathbf{q}} \left(\bar{B} - \nabla \cdot \overline{\mathbf{u}' \rho'} \right) + \left(\bar{\mathbf{F}} + \mathbf{F}_R - \frac{\bar{\Delta \rho}}{\rho_0} g \mathbf{k} \right) \times \nabla \bar{\rho} - \bar{\mathbf{q}} \frac{\partial \bar{\rho}}{\partial t} - \frac{\partial \bar{\mathbf{v}}}{\partial t} \times \nabla \bar{\rho} \right] dA = 0, \quad (4.32)$$

where λ is defined as the area between two closed contours of Bernoulli Potential.

The terms in equation (4.32) describe the vertical potential vorticity flux due to specific physical processes. This can be made explicit by rewriting equation (4.32) as

$$\iint_{\lambda} (\mathbf{J}_{adv} + \mathbf{J}_{buoy} + \mathbf{J}_{fric} + \mathbf{J}_{eddy} + \mathbf{J}_{drift}) dA = 0, \quad (4.33)$$

where

$$\mathbf{J}_{adv} = \rho_0 \tilde{Q} \bar{\mathbf{v}} \quad (4.34)$$

is the potential vorticity flux from advection by the time-mean vertical velocity,

$$\mathbf{J}_{buoy} = \bar{q}^z \bar{B} \quad (4.35)$$

is the flux due to buoyancy forcing,

$$\mathbf{J}_{fric} = \mathbf{k} \times \bar{\mathbf{F}} \cdot \nabla \bar{\rho} \quad (4.36)$$

is the flux due to viscosity and the boundary conditions of the momentum equation,

$$\mathbf{J}_{eddy} = \mathbf{k} \times \bar{\mathbf{F}}_R \cdot \nabla \bar{\rho} - \bar{q}^z \nabla \cdot \overline{\mathbf{u}' \rho'}, \quad (4.37)$$

is the eddy induced flux, and

$$\mathbf{J}_{drift} = -\mathbf{k} \times \frac{\partial \bar{\mathbf{v}}}{\partial t} \cdot \nabla \bar{\rho} - \bar{q}^z \frac{\partial \bar{\rho}}{\partial t}, \quad (4.38)$$

is the flux due to model drift. In each of these the superscript z refers to the vertical component.

As described in Polton and Marshall (2007), the terms described in equations (4.34) - (4.38) can be evaluated either as area integrals, or converted to contour integrals. Polton and Marshall (2007) provide a brief discussion on evaluating these terms as contour integrals, and highlight the fact that these contour integrals can be used to construct a time-integrated buoyancy budget around closed Bernoulli Potential

contours. A brief discussion of this is presented below for the sake of completeness, but further implications will not be explored.

We begin by defining a recirculation velocity along contours of Bernoulli Potential. The leading order solution to the momentum equation is,

$$\bar{v}_{recirc} \approx \frac{1}{\bar{q}^z} \frac{d\tilde{\Pi}}{dn}, \quad (4.39)$$

where dn is in the direction normal to the contour. The recirculation velocity calculated in equation (4.39) is not quite the geostrophic velocity since we have evaluated the derivative of the Bernoulli Potential on a depth surface not a density surface - $\bar{v}_{recirc}$ can be interpreted as the geostrophic recirculation velocity around the Bernoulli contour at constant depth.

By definition

$$\bar{v}_{recirc} = \frac{ds}{dt}, \quad (4.40)$$

where ds is always tangential to the contour. By combining equations (4.39) and (4.40) the area element can be written as

$$dA = ds dn \approx \frac{\delta\tilde{\Pi}}{\bar{q}^z} dt, \quad (4.41)$$

which can be substituted into equation (4.32) to convert it from an area integral into a closed contour integral. If it is also assumed that the eddy buoyancy flux can be represented as advection of the mean buoyancy gradient by a bolus velocity, then equation (4.32) can be rewritten as

$$\oint_{\partial\lambda} \left(\frac{\partial\bar{\rho}}{\partial t} + (\bar{\mathbf{v}} - \bar{\mathbf{v}}_{recirc} + \mathbf{v}^*) \cdot \nabla_{\mathbf{z}} \bar{\rho} + (\bar{w} + w^*) \frac{\partial\bar{\rho}}{\partial z} - \bar{B} \right) dt \approx 0, \quad (4.42)$$

where $\mathbf{v}^*$ and w^* represent the eddy induced bolus velocity, and $\nabla_{\mathbf{z}}$ represents the gradient operator on a constant depth surface.

Equation (4.42) is a time integrated buoyancy budget that includes time-mean and eddy-induced advection as well as buoyancy forcing.

4.2.1 Computing the PV flux terms

In the ocean or in models with complex thermodynamics it is important to use an appropriate estimate of density, for example, neutral density (McDougall, 1987). However, the simplified thermodynamics of my model mean that such concerns are unnecessary, and density as computed from the linear equation of state should be used to evaluate the potential vorticity.

Each of the terms in equation (4.33) was calculated on the desired depth surface, and then an integral was performed using masks to limit the area of integration to the area between adjacent contours of Eulerian-mean Bernoulli Potential. The results were normalised by the area of integration to give a potential vorticity flux intensity. Normalising the results by the area allows for more transparent comparisons between regions with different areas.

4.2.1.1 Advection

The advective flux,

$$\mathbf{J}_{adv} = \bar{\rho} \tilde{Q} \bar{w} \quad (4.43)$$

was straightforward to calculate. Each term was estimated at the tracer points of the MITgcm model grid and then the terms were multiplied together.

4.2.1.2 Buoyancy

The potential vorticity flux due to buoyancy forcing includes contributions from two processes. These are the diffusion of density within the fluid, and surface buoyancy fluxes. Splitting these processes allows this term to be more transparently written as

$$\mathbf{J}_{buoy} = \bar{q}^z \bar{B} = \bar{q}^z (\nu \nabla^2 \bar{\rho} + \bar{B}_{surf}), \quad (4.44)$$

where $\bar{B}_{surf}$ represents the mean buoyancy flux through the sea surface, and ν is the diffusion.

The component of the buoyancy forcing due to diffusion can be calculated two ways; either by considering the divergence of the diagnosed diffusive fluxes of temperature from the model, or by calculating the second-order derivatives of the density field. Because the idealised simulations presented in this thesis use a linear equation of state that depends only on temperature, it is trivial to convert between fluxes of

temperature and density. Calculating this term using each method for a test simulation gave indistinguishable results. For the results presented here the diffusion term was calculated from the diffusive flux fields output by the model.

Near the surface of the model, the potential vorticity flux due to buoyancy forcing must also include the effect of surface buoyancy forcing. If a non-local convective parametrisation, e.g. the K-Profile Parametrisation (Large et al., 1994), were used in the simulation, then the surface buoyancy forcing could be communicated to substantial depths within the model domain. Most of the simulations presented in this thesis avoided using a convective parametrisation so that the difficulties associated with non-local density fluxes could be avoided. The MITgcm is able to crudely parametrise convection by artificially increasing vertical diffusion in the presence of a gravitationally unstable density profile. Using the diffusive flux fields to calculate buoyancy forcing means that this effect is correctly represented in the results.

4.2.1.3 Mechanical forcing

Calculating the potential vorticity flux due to mechanical forcing,

$$\mathbf{J}_{fric} = \mathbf{k} \times \overline{\mathbf{F}} \cdot \nabla \bar{\rho} \quad (4.45)$$

requires computing the time-mean mechanical forcing, $\overline{\mathbf{F}}$ from equation (4.23). The time-mean mechanical forcing includes viscous drag, side drag, bottom drag, and surface wind forcing, which can all be diagnosed directly from the simulation.

4.2.1.4 Eddies

The eddy contribution to the potential vorticity flux

$$\mathbf{J}_{eddy} = \mathbf{k} \times \overline{\mathbf{F}}_R \cdot \nabla \bar{\rho} - \bar{q}^z \nabla \cdot \overline{\mathbf{u}' \rho'} \quad (4.46)$$

consists of the rectified effect of eddies in the momentum equation and the buoyancy equation. Calculating the eddy contribution to the time-mean momentum equation is straightforward since the terms may be estimated by differencing fields that can be directly saved from the simulation. The two vorticity components $-\overline{q^{z'} u'}$ and $-\overline{q^{z'} v'}$ from the meridional and zonal momentum equations respectively, may be estimated by calculating

$$\overline{q^{z'} u'} = \overline{q^z u} - \bar{q}^z \bar{u} \quad (4.47)$$

$$\overline{q^{z'} v'} = \overline{q^z v} - \bar{q}^z \bar{v}, \quad (4.48)$$

which requires only fields that can be directly saved from the model simulation. The $\nabla \overline{u' \cdot u'} / 2$ term can be calculated in a similar manner.

The eddy contribution to the buoyancy equation needs to be computed differently depending on whether or not the simulation used a mesoscale eddy parametrisation, e.g. the Gent-McWilliams eddy parametrisation (Gent and McWilliams, 1990; Gent et al., 1995).

In simulations with a sufficiently high resolution such that a mesoscale eddy parametrisation was not required, the eddy buoyancy flux can be estimated by decomposing the time-mean advection of buoyancy as

$$\nabla \cdot \overline{\mathbf{u} \rho} = \nabla \cdot \overline{\mathbf{u}} \bar{\rho} + \nabla \cdot \overline{\mathbf{u}' \rho'}, \quad (4.49)$$

where the primes denote deviations from time-mean values. Using diagnostic output from the model simulations it is possible to calculate two of these terms directly, and estimate the third as the difference between them. The left-hand side term, $\nabla \cdot \overline{\mathbf{u} \rho}$, can be calculated as the divergence of the time mean advective buoyancy fluxes, which can be saved directly from the simulation. The first term on the right-hand side, $\nabla \cdot \overline{\mathbf{u}} \bar{\rho}$ can be calculated from the time-mean velocity and density fields. Once these two divergences have been calculated, the divergence of the eddy density flux, $\nabla \cdot \overline{\mathbf{u}' \rho'}$, can be estimated as the difference between them. This means that any numerical artefacts from the advection scheme, which are included in the model diagnostics used to calculate the left-hand side, are aliased into the eddy term. The eddy term may also be calculated directly from the density and velocity fields if they are saved with a high enough temporal resolution. This method requires much more storage and computational power. In a verification test the two methods gave indistinguishable results, suggesting that either method may be used, provided the fields are saved sufficiently regularly.

The simulations presented in this thesis that use a mesoscale eddy parametrisation implemented GM in its original advective form, rather than as a skew-flux, as described by Griffies (1998). In the advective form of GM the eddy contribution is calculated as a bolus velocity that is added to the model velocity fields prior to invoking the advection scheme. The skew-flux form proposed by Griffies (1998) adds the eddy contribution as a skew flux in the dissipation term. The advective form allows for a cleaner separation of the model velocity and the eddy bolus velocity than the skew-flux formalism.

If the simulation used the GM eddy parametrisation then the diagnosed time-mean advection of density, $\nabla \cdot \overline{\mathbf{u} \rho}$, includes both the model velocity and the bolus

velocity contributions. Separating the two velocities when decomposing the time-mean advection of buoyancy leads to four terms, compared with the two shown in equation (4.49). The time-mean advection of density can be written as

$$\nabla \cdot (\overline{\mathbf{u}_m + \mathbf{u}_{GM}} \rho) = \nabla \cdot (\overline{\mathbf{u}_m} \bar{\rho}) + \nabla \cdot \overline{\mathbf{u}'_m \rho'} + \nabla \cdot (\overline{\mathbf{u}_{GM}} \bar{\rho}) + \nabla \cdot \overline{\mathbf{u}'_{GM} \rho'}, \quad (4.50)$$

where primes once again denote deviations from time-mean values, the subscript m denotes model velocity, and the subscript GM denotes Gent-McWilliams bolus velocity. The terms on the right-hand side of this decomposition are respectively the advection of mean density by the mean model velocity, the rectified effect of resolved variability, the mean advection by the parametrised bolus velocity, and the rectified effect of variability in the GM bolus velocity.

The left hand side of equation (4.50), $\nabla \cdot (\overline{\mathbf{u}_m + \mathbf{u}_{GM}} \rho)$, is calculated from the time-mean advective density fluxes that are output by the model's advection scheme.

The advection of time-mean density by the time-mean model velocity, $\nabla \cdot \overline{\mathbf{u}_m} \bar{\rho}$, is computed directly from the time-mean density and model velocity fields.

To calculate the rectified effect of resolved variability requires an estimate of $\nabla \cdot \overline{\mathbf{u}_m \rho}$ so that the decomposition shown in equation (4.49) can be performed. However, the use of a non-linear advection scheme means that the component of the advective flux due to the model velocity cannot be cleanly separated from the total advective flux. Therefore the value of $\nabla \cdot \overline{\mathbf{u}_m \rho}$ cannot be directly diagnosed from the advection scheme. It was instead estimated from the divergence of the Eulerian-mean of $(\mathbf{u}_m \rho)$, which could be directly saved from the simulations. The rectified eddy divergence was then calculated as

$$\nabla \cdot \overline{\mathbf{u}'_m \rho'} = \nabla \cdot \overline{\mathbf{u}_m \rho} - \nabla \cdot \overline{\mathbf{u}_m} \bar{\rho}. \quad (4.51)$$

The time-mean contribution from the parametrised eddies, $\nabla \cdot (\overline{\mathbf{u}_{GM}} \bar{\rho})$, was calculated from the time-mean bolus velocity and time-mean density fields.

The rectified effect of Gent-McWilliams parametrised eddies, $\nabla \cdot \overline{\mathbf{u}'_{GM} \rho'}$, is calculated as the residual of the other terms. This means that any numerical artefacts from the non-linear advection scheme and other sources are aliased into this term. It would be preferable to have these effects aliased into equivalent terms in each scenario, since that would make direct comparisons easier. However, this final term is generally negligible, making it unlikely that the aliasing of advection scheme artefacts into the eddy term has had a significant impact on the results.

4.2.1.5 Model drift

The contribution of model drift to the vertical potential vorticity flux,

$$\mathbf{J}_{drift} = -\mathbf{k} \times \frac{\partial \bar{\mathbf{v}}}{\partial t} \cdot \nabla \bar{\rho} - \bar{q}^z \frac{\partial \bar{\rho}}{\partial t} \quad (4.52)$$

is calculated from the change in velocity and density fields over the period of the time averaging filter. The magnitude of this term indicates whether the simulation was in a statistically steady state. The model drift term was small in all of the simulations discussed in this thesis, which indicates that they were at a statistically steady state.

4.3 Mode water

This analysis presented in this chapter examines the potential vorticity flux within the mode water of the subtropical gyre. The mode water is the weakly stratified mass of nearly homogeneous fluid above the subtropical thermocline. These water masses are called mode waters because they form a mode of the volume distribution in temperature-salinity space. The North Atlantic subtropical mode water, dubbed “Eighteen Degree Water” (EDW) by Worthington (1958), was identified in observations taken during the *Challenger* expedition in 1873 (Deremble and Dewar, 2013). The mode water associated with each of the subtropical gyres has a unique salinity and temperature that allows it to be identified in observational datasets (Talley et al., 2011). These mode waters form on the lower density side of separating boundary currents (Hanawa and Talley, 2001), and their formation is generally associated with wintertime convective mixing due to surface buoyancy losses (Hanawa and Talley, 2001).

4.4 Hypotheses

This chapter sets out to test multiple hypotheses about which terms will dominate an integrated budget of vertical potential vorticity fluxes.

This chapter seeks to test the following hypotheses:

- as the resolution is decreased, the contribution of eddies will decrease;
- as the resolution is increased, the diffusive contribution will decrease;

- at an eddy-resolving resolution the dynamical balance will be between eddy fluxes and mean advection, similar to the regime identified by Polton and Marshall (2007) in the Southern Ocean; and
- at non eddy-resolving resolutions the eddy bolus velocity computed from the Gent-McWilliams eddy parametrisation will provide the potential vorticity flux required to balance mean advection.

4.5 Potential vorticity fluxes in simulations

To test the hypotheses proposed in section 4.4, we require simulations at a range of resolutions. The results discussed in the following sections are from simulations on Cartesian grids with horizontal resolutions of 4 km, 25 km, and 100 km. The model domain is a square basin with side length of 2,000 km and depth of 4,000 m. The vertical resolution is consistent between all of the simulations with 40 unevenly spaced levels that have a thickness of 10 m at the surface and increase monotonically to 360 m for the deepest level. The highest resolution simulations are therefore run on a grid that with 500 x 500 x 40 grid points. The simulations in this chapter were spun up using progressively finer grids. At each resolution the simulation was integrated until reaching either a steady state or a statistically steady state. The Coriolis parameter is chosen such that these simulations are in the midlatitudes where the first baroclinic Rossby deformation radius is approximately 40 km. The western portion of the domain contains an idealised continental shelf above which the fluid is approximately 200 m deep. The shelf extends 500 km into the domain, before descending steeply to 4,000 m depth.

The single gyre simulations discussed in this chapter were forced with an idealised sinusoidal zonal wind forcing, and a specified surface heat flux. The forcings are shown in figure 4.2.

4.5.1 Vertical fluxes in an eddy-resolving gyre

With a horizontal resolution of 4 km, the first baroclinic radius of deformation is well resolved in this simulation. Because mesoscale variability is well resolved, we expect to find eddies as the dominant flux balancing Eulerian-mean advection.

To reduce the numerical cost of this high-resolution simulation, a computationally cheap advection scheme was used. The use of a centred second-order discretisation for the advection-diffusion equation necessitates some explicit diffusion to keep the

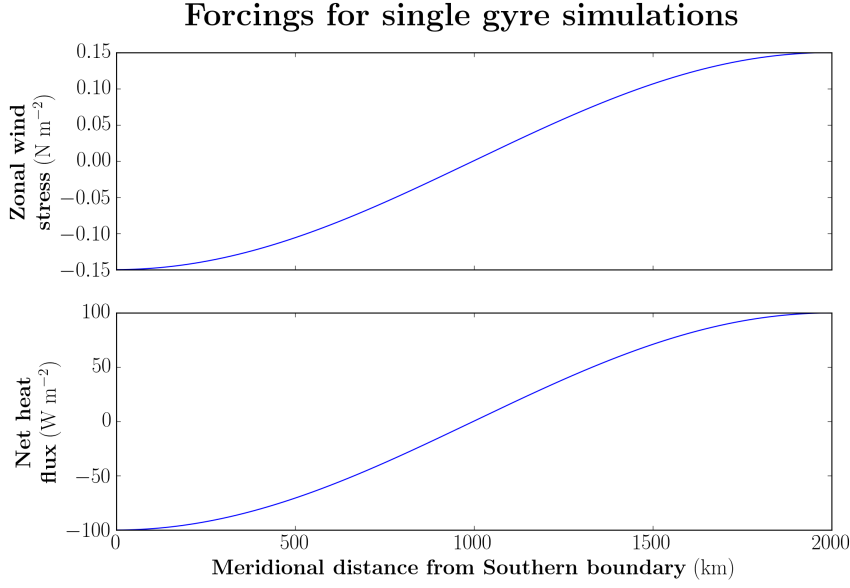


Figure 4.2: *Forcings for the single gyre simulations.*

model fields smooth (Adcroft et al., 2008). The net effect of this discretisation and the explicit numerical diffusion is to render the interior flow more diffusive than desired. However, this provides a slightly more stringent test of our hypothesis; if the vertical potential vorticity flux dynamics still show a balance between mean downwelling and eddy-induced upwelling in a model with excessive diffusion, then it is likely that diffusion will play an even smaller role in other models and in the ocean. Six model years of output were averaged for the analysis presented in this section.

4.5.1.1 At 100 m depth

At 100 m depth the potential vorticity flux due to time-mean advection is strongly downwards, as shown in figure 4.3. Figure 4.3 and all similar figures plot the area averaged flux intensity between neighbouring contours of Eulerian-mean Bernoulli Potential. Each region is represented by a single value

This aligns with our expectations from Ekman pumping and previous results (e.g. Polton and Marshall, 2007; Spall, 1992).

This downwards flux is balanced almost entirely by an upwards eddy induced flux, shown in figure 4.4. The sum of the flux terms, which is zero across almost all regions in the gyre, is shown in figure 4.5. A synopsis of the potential vorticity fluxes is shown in figure 4.6, where it can also be seen that the contribution from buoyancy forcing is small for almost all regions. Because the Eulerian-mean Bernoulli Potential on this

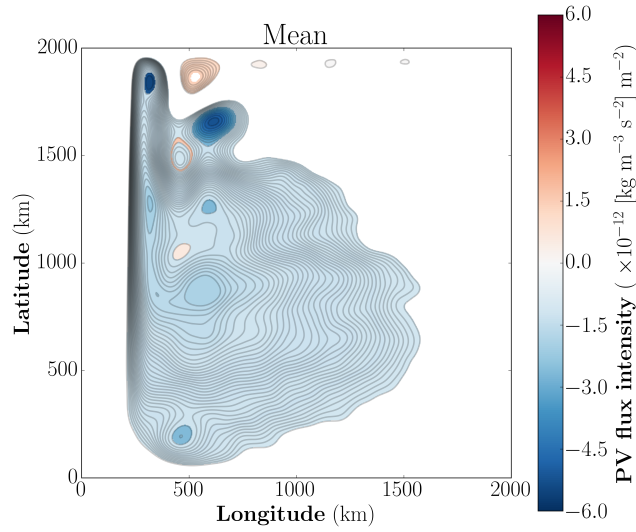


Figure 4.3: *Vertical flux intensity of potential vorticity due to Eulerian-mean advection at 100 m depth in a simulation with a horizontal resolution of 4 km. The area averaged flux intensity is calculated between neighbouring contours of Eulerian-mean Bernoulli Potential.*

depth surface contains multiple local maxima and minima, there are multiple regions bounded by contours with the same value. This is why figure 4.6, and other similar figures, have multiple flux intensities for a single value of Bernoulli Potential.

Figures 4.4 and 4.3 both show that the potential vorticity flux balance also holds over small regions. While the mathematics which underpin the framework are not dependent on scale, the physical mechanism by which it holds at these small scales is not obvious. The two regions in the north-west of the domain are located within the extension of the western boundary current, and thus one expects the dynamics there to be dominated by non-linear effects. The elevated kinetic energy in this region may have contributed to the substantially larger values computed for both Eulerian-mean and eddy-induced advection. However, no such argument can be made for the small region in the south-west of the domain. It is possible that these localised extrema are due to the impact of the initial conditions or a single large fluctuation, in which case averaging for a longer period of time may reduce the magnitude of the fluxes within these regions.

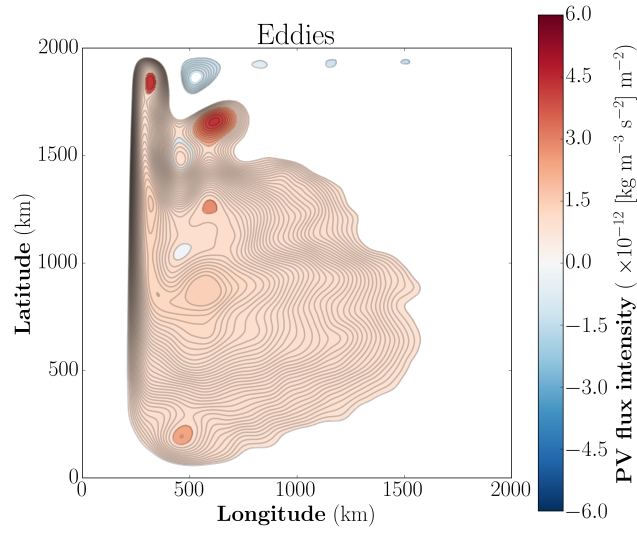


Figure 4.4: Vertical flux intensity of potential vorticity due to eddies at 100 m depth in a simulation with a horizontal resolution of 4 km. The area averaged flux intensity is calculated between neighbouring contours of Eulerian-mean Bernoulli Potential.

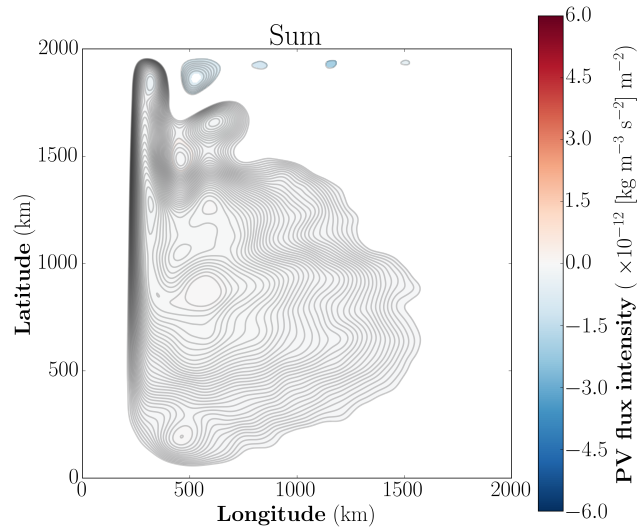


Figure 4.5: Sum of the vertical flux intensities of potential vorticity at 100 m depth in a simulation with a horizontal resolution of 4 km.

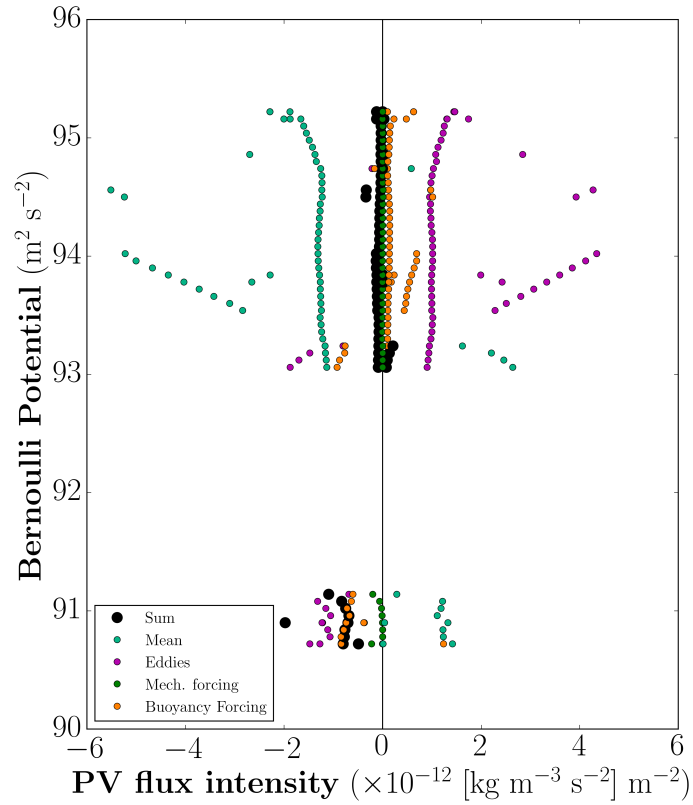


Figure 4.6: *Synopsis of the vertical flux intensities of potential vorticity at 100 m depth in a simulation with a horizontal resolution of 4 km. There are multiple intensities for some values of Bernoulli Potential because there are multiple regions bounded by Bernoulli Potential contours with the same value.*

4.5.1.2 At 450 m depth

At 450 m depth the balance is not materially altered; time-mean advection fluxes potential vorticity downwards, and this flux is balanced by an upwards eddy-induced flux. A synopsis of the fluxes is shown in figure 4.7, and the eddy-induced potential vorticity flux is shown in figure 4.8.

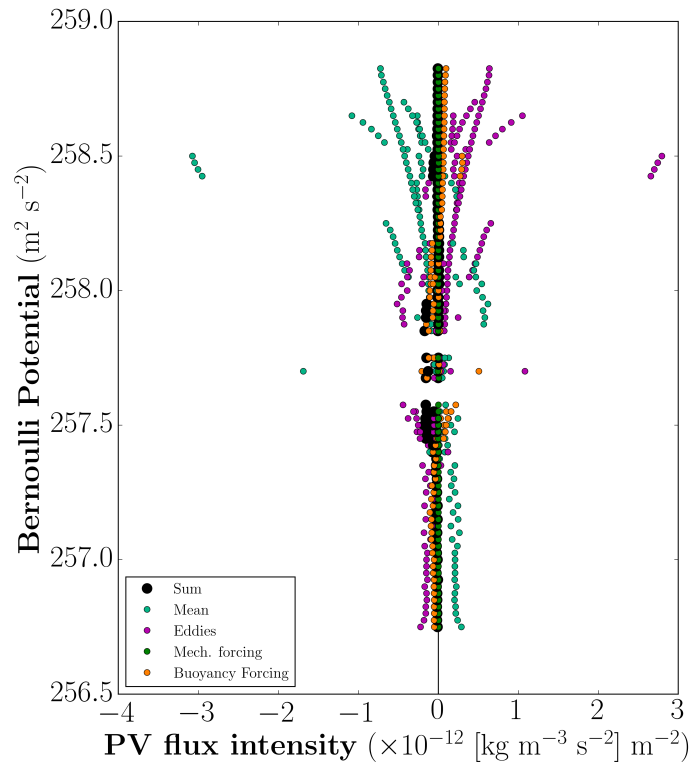


Figure 4.7: Synopsis of the vertical flux intensities of potential vorticity at 450 m depth in a simulation with a horizontal resolution of 4 km. There are multiple intensities for some values of Bernoulli Potential because there are multiple regions bounded by Bernoulli Potential contours with the same value.

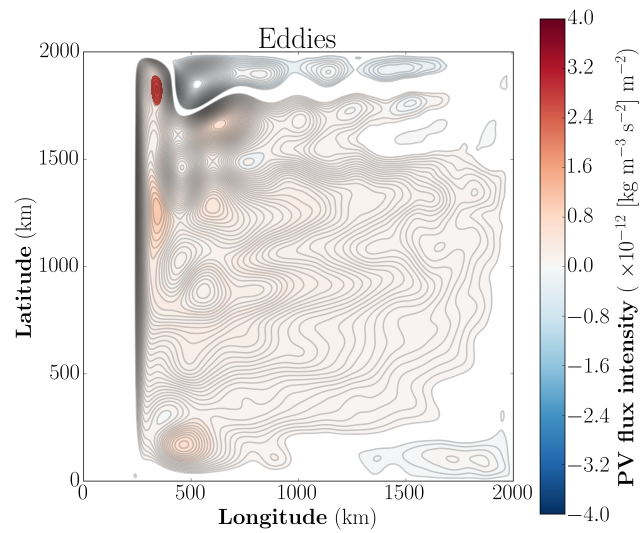


Figure 4.8: Vertical flux intensity of potential vorticity due to eddies at 450 m depth in a simulation with a horizontal resolution of 4 km. The area averaged flux intensity is calculated between neighbouring contours of Eulerian-mean Bernoulli Potential.

4.5.2 Vertical fluxes in an eddy-permitting simulation

A horizontal resolution of 25 km is comparable to a resolution of one quarter of a degree in the midlatitudes. With a horizontal resolution of 25 km our model contains mesoscale variability, albeit with a reduced amplitude. Models with this resolution are known as “eddy-permitting” because their grid is fine enough to resolve some mesoscale variability. However, the mesoscale eddy field is not fully resolved, and models with this resolution are generally unable to match the levels of eddy kinetic energy observed in the ocean (see e.g. Oschlies and Garçon, 1998). This resolution is comparable with the model used by Polton and Marshall (2007). Because of the similarity in resolutions, we expect that potential vorticity fluxes due to buoyancy changes will make a significant contribution to the balance. The simulation analysed in this section did not include a mesoscale eddy parametrisation since this would damp the marginally resolved eddy field Hallberg (2013). This allows for the impact of a barely resolved mesoscale eddy field on the potential vorticity flux dynamics to be assessed.

The dominant terms in the potential vorticity flux balance were found to be time-mean advection and eddy-induced fluxes, but the contribution from buoyancy forcing is now significant in some regions. Figure 4.9 shows the potential vorticity flux intensity due to buoyancy forcing. For some of the contours there is a three-way balance between time-mean advection, eddy-induced fluxes, and buoyancy forcing. The three-way balance is shown in figure 4.10. The emergence of a three-way balance supports our hypothesis that the impact of buoyancy changes would increase with decreasing horizontal resolution.

4.5.3 Vertical fluxes in a model with unresolved mesoscale eddies

At horizontal resolutions comparable with current climate models, $\Delta x \approx 100$ km, mesoscale eddies are not resolved. This simulation allows us to test our hypothesis that the Gent-McWilliams eddy parametrisation will be able to provide a substitute for the eddy contribution to the dynamical balance identified in section 4.5.1.

As described in section 4.2.1.4 the potential vorticity flux due to unresolved mesoscale eddies is split into a time-mean component and a rectified component. The time-mean term represents the contribution of the Eulerian-mean bolus velocity calculated by Gent-McWilliams mesoscale eddy parametrisation. The rectified effect due to correlations between the variations in this bolus velocity and the density field are diagnosed as a residual, and is generally small. The potential vorticity flux due to

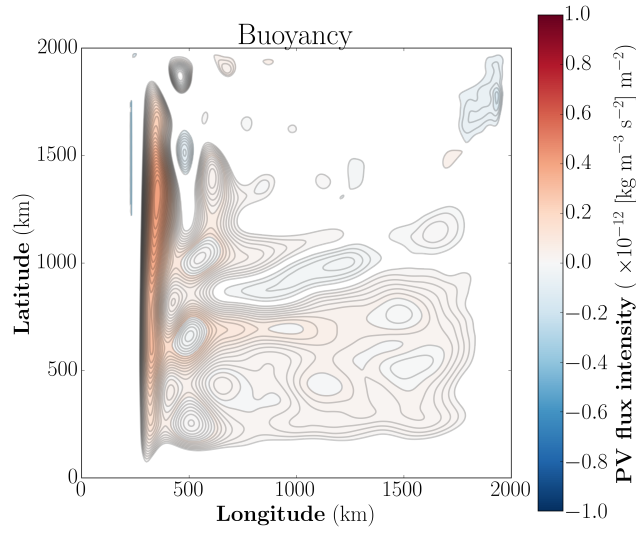


Figure 4.9: *Vertical flux intensity of potential vorticity due to buoyancy forcing at 450 m depth in a simulation with a horizontal resolution of 25 km. The area averaged flux intensity is calculated between neighbouring contours of Eulerian-mean Bernoulli Potential.*

the Eulerian-mean bolus velocity calculated by the Gent-McWilliams eddy parametrisation is upwards throughout the gyre at a depth of 450 m, as shown in figure 4.11. At this resolution the gyre lacks inertial recirculations and other non-linear features, as shown by the shape of the Bernoulli Potential contours.

Figure 4.12 shows a synopsis of the fluxes due to each of the physical processes by contour. The balance between Eulerian-mean advection and the GM eddy parametrisation is clear. The potential vorticity flux budget does not quite close at this resolution, but the residual is substantially smaller than either of the dominant terms.

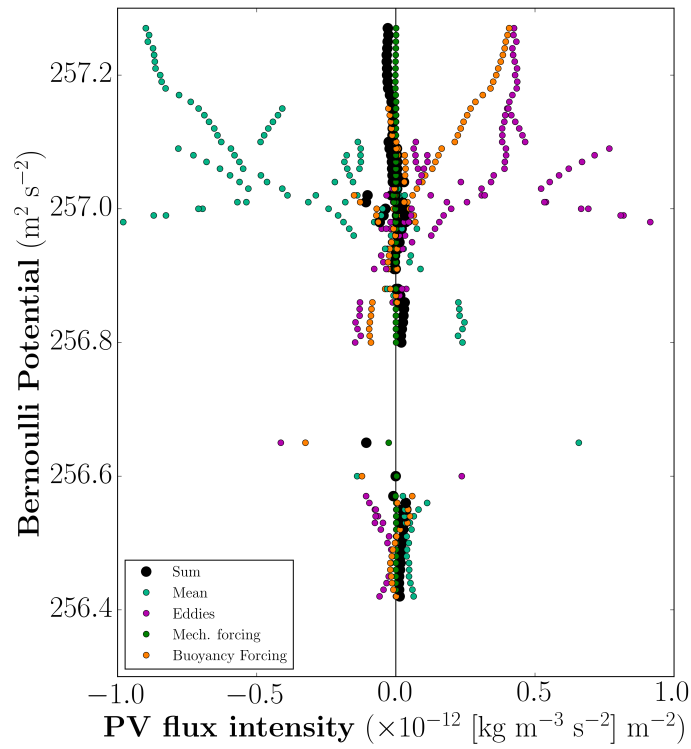


Figure 4.10: Synopsis of the vertical flux intensities of potential vorticity at 450 m depth in a simulation with a horizontal resolution of 25 km. Of particular interest is the contribution from buoyancy forcing on some of the contours that pass through the western boundary current. There are multiple intensities for some values of Bernoulli Potential because there are multiple regions bounded by Bernoulli Potential contours with the same value.

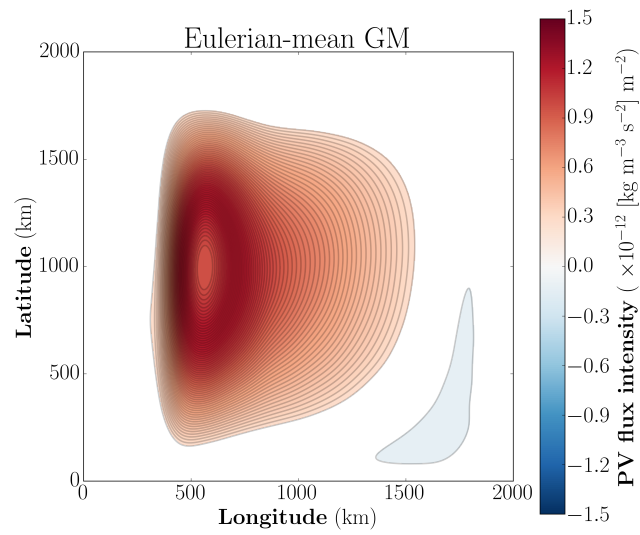


Figure 4.11: *Vertical flux intensity of potential vorticity due to advection by the Eulerian-mean bolus velocity from the Gent-McWilliams mesoscale eddy parametrisation at 450 m depth in a simulation with a horizontal resolution of 100 km. The area averaged flux intensity is calculated between neighbouring contours of Eulerian-mean Bernoulli Potential.*

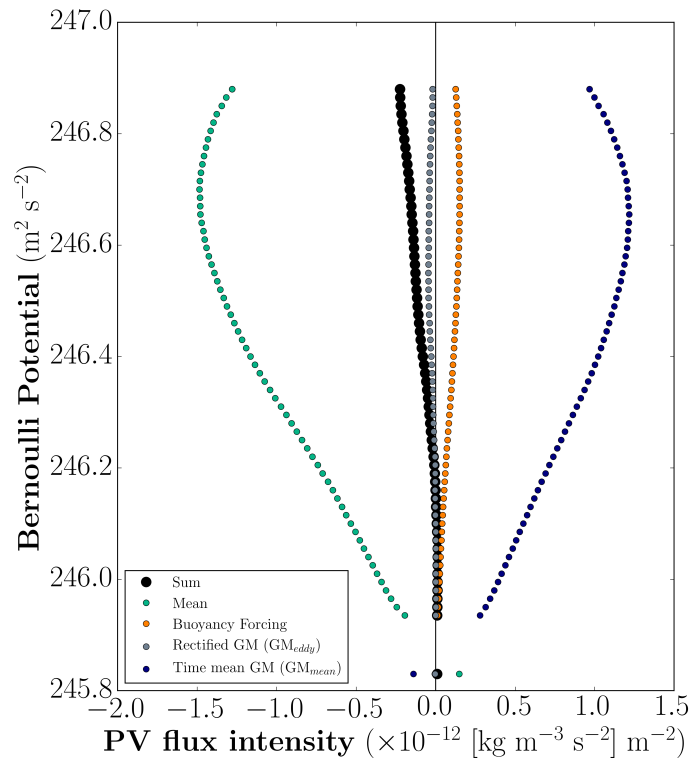


Figure 4.12: A synopsis of the intensity of vertical potential vorticity fluxes at 450 m depth in the 100 km resolution simulation. The drift, eddy, and mechanical forcing terms are all approximately zero, and are not included in this figure for the sake of clarity.

4.6 Concluding remarks

Our hypotheses for this chapter were supported. We identified a balance between vertical potential vorticity fluxes due to Ekman driven Eulerian-mean advection and eddy induced fluxes. As the resolution of the model was reduced from eddy-resolving to eddy-permitting there was a modest increase in the contribution from buoyancy forcing. However, as the resolution was further reduced the dominant balance was between the mesoscale eddy parametrisation and Eulerian-mean advection.

The balance identified in the potential vorticity flux framework can be physically understood in the context of Eulerian-mean Ekman pumping depressing isopycnals in the centre of the gyre and, through the tilting of isopycnals, storing potential energy. Baroclinic eddies release this potential energy by flattening the isopycnals, which produces an eddy-induced circulation to oppose the Eulerian-mean Ekman pumping. The baroclinic eddies need not act to flatten isopycnals across the entire gyre at the same moment, the framework is after all constructed from time-averaged fields. Mathematically the framework must hold regardless of the length of time covered by the time-averaging filter. However, as the length of time considered is shortened, the magnitude of the drift term will likely increase, since this term relates to the change in storage of potential vorticity substance. For timescales less than a year I speculate that this term would dominate the budget.

The main dynamical balance, fluxes due to Ekman driven advection balancing eddy-induced fluxes, is equivalent to the results found by Polton and Marshall (2007) in the Southern Ocean below the mixed layer. However, in the ocean, subtropical gyres do not exist in isolation. By constraining our simulation to a domain with only a single gyre, it is possible that we have excluded some dynamics that should have been important to the potential vorticity flux balance. Simulations with a less restrictive domain will be explored in the next chapter.

Potential vorticity dynamics of an idealised North Atlantic

In the previous chapter we used an integral constraint for vertical fluxes of potential vorticity to examine the balance between Eulerian-mean Ekman pumping and eddy-induced fluxes in an isolated subtropical gyre. However, the domain used in chapter 4 may have affected the results by artificially restricting the dynamics of the simulation. In this chapter a similar analysis will be performed on simulations in less restrictive domains that include a subpolar gyre, and in some cases a meridional overturning circulation. This allows us to test whether the domain used in chapter 4 constrained the flow in such a way as to ensure that the only mechanism available to balance the potential vorticity flux due to Eulerian-mean advection was eddy driven fluxes.

Including a subpolar gyre in the model domain allows for possibility of an Ekman driven intergyre overturning circulation commonly known as an Ekman cell (see e.g. Marshall, 2000). The Ekman cell is a diabatic overturning circulation linking downwelling in subtropical gyres driven by Ekman convergence with upwelling in the subpolar gyres driven by Ekman divergence. As described in Polton and Marshall (2007) the descending arm of this overturning circulation must have a diabatic component, since there are no adiabatic pathways for the descending fluid to follow. In contrast, the upwelling arm in the subpolar gyre could be largely adiabatic since the density surfaces move closer to the sea surface the farther north the fluid parcel travels. A schematic of the Ekman cells, adapted from Marshall (2000), is shown in figure 5.1.

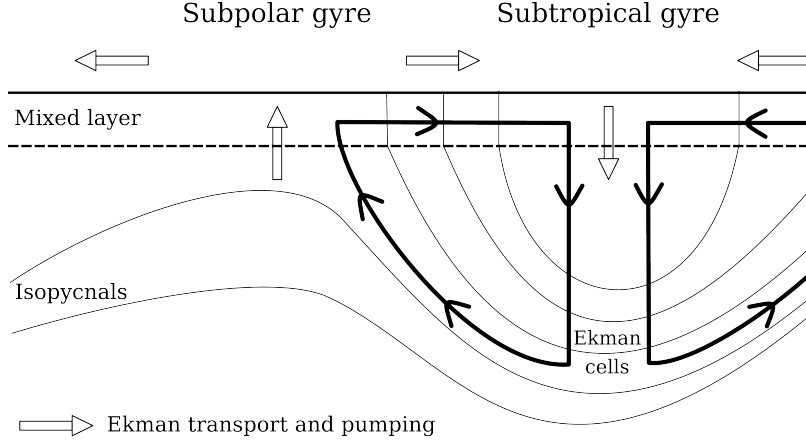


Figure 5.1: Schematic showing the Ekman cells. Fluid is downwelled in the subtropical gyre by Ekman pumping, passes diabatically through the thermocline and moves adiabatically towards the surface either in the subpolar or tropical regions. The white arrows represent Ekman transport and pumping, the thick black lines show the Ekman cells, and the thin black lines are isopycnals. Figure adapted from Marshall (2000).

5.1 Hypotheses

In this chapter we vary the resolution and background flow of a model with multiple wind-driven gyres to test the robustness of the eddy-mean potential vorticity flux balance identified in the previous chapter. In particular, we seek to test the following hypotheses.

- In the subtropical gyre, we hypothesise the existence of a diabatic three-way balance between a downwards Eulerian-mean advective flux, an upwards eddy-driven flux, and an upwards flux due to diffusion.
- In the subpolar gyre, we hypothesise an adiabatic balance between an upwards mean advective flux and a downwards eddy-driven flux.
- We hypothesise that in the simulations employing a mesoscale eddy parametrisation, the parametrised eddies will provide a vertical flux of potential vorticity to balance Eulerian-mean advection.

5.2 Overview of the simulations

Our model domain covers sixty-five degrees of latitude and thirty degrees of longitude. The basin has a continental shelf in the west that is five degrees wide and slopes gently

downwards until reaching 200 m depth, at which point it drops steeply to 4000 m depth. This shelf is equivalent to the one used in chapter 4. The rest of the domain is a uniform depth of 4000 m. Our model employs a single variable linear equation of state that depends only on temperature. A sinusoidal wind forcing is used to drive the horizontal circulation. Sea surface temperature is relaxed to a linear function based on mean sea surface temperatures in the North Atlantic from the NOAA World Ocean Atlas 2013 (Locarnini et al., 2013).

Several of the simulations discussed in this chapter include a meridional overturning circulation. In these simulations the temperature field is also relaxed to an exponential profile within a sponge region occupying the southernmost five degrees of the domain. The relaxation profile has the form

$$T(z) = (T_{max} - T_{min}) \frac{\exp(z/h) - \exp(-H/h)}{1 - \exp(-H/h)} + T_{min}, \quad (5.1)$$

where H is the maximum depth of the model domain, T_{max} and T_{min} are the maximum and minimum temperatures for the profile, and h is a vertical scale parameter that was chosen so that this profile closely fitted equatorial profiles from the NOAA World Ocean Atlas 2013 (Locarnini et al., 2013). The scale height used for these simulations is $h = 220$ m, the maximum temperature, T_{max} , is 30°C , and the minimum temperature, T_{min} is 2°C . The sponge region does not alter any other model variables.

The model domain and forcings are shown in figures 5.2 and 5.3. The Coriolis parameter is determined by the latitude on the model grid, so that the model latitudes are directly comparable with the equivalent latitude on earth. We use the MITgcm (Marshall et al., 1997) to numerically solve the equations of motion on a spherical grid with forty unevenly spaced vertical levels. The vertical resolution is 10 m at the surface and increases by ten percent at each depth level until it reaches 360 m at the bottom. The variable vertical resolution can be seen in figure 5.3 c). The simulations presented in this chapter were performed at multiple horizontal resolutions so that the effect of mesoscale eddies could be more easily isolated. The horizontal resolutions used were one eighth of a degree, one quarter of a degree, and one half of a degree. These resolutions span a range of numerical regimes from eddy-resolving through eddy-permitting to non-eddy resolving. At a horizontal resolution of one eighth of a degree midlatitude mesoscale variability is well resolved by our model, as evidenced by the strong inertial recirculation regions surrounding the zonal extension of the western boundary currents shown in figure 5.2. The model was integrated to a steady, or statistically steady, state using a series of progressively finer grids. The coarse resolution simulations were run for centuries of model time to ensure that the model

had equilibrated. Each analysis was performed after the simulation had stabilised. The shortest equilibration period was approximately 60 model years for the highest resolution simulation.

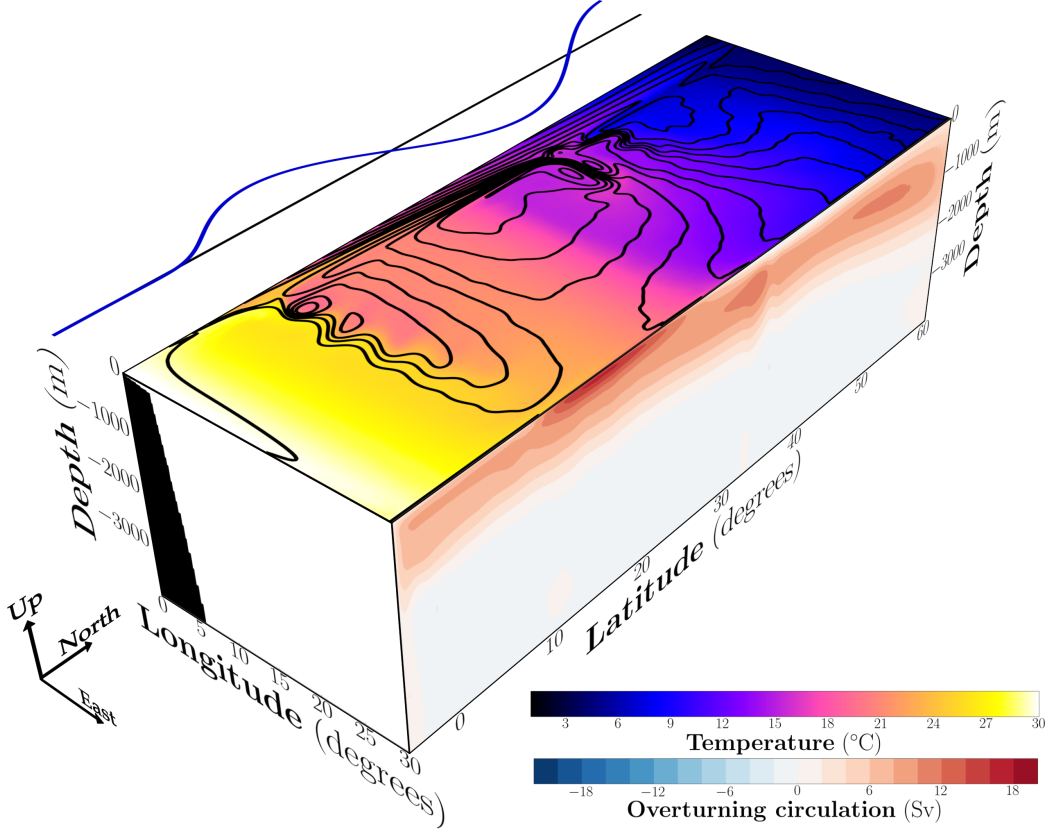


Figure 5.2: *Schematic of the model. Model fields from the one eighth degree horizontal resolution simulation are shown here: the surface shows the Eulerian-mean temperature and Bernoulli Potential contours, while the eastern face shows depth-latitude overturning circulation calculated from the Eulerian-mean fields. The southern face shows the bathymetry and the momentum forcing is shown to the west of the model domain.*

The final difference between the simulations presented in this chapter, and those of the previous chapter is the choice of advection scheme. The simulations discussed in this chapter used the Prather advection scheme (Prather, 1986). The Prather advection scheme, which was briefly discussed in section 3.1.3 results in very little numerical diffusion of tracers. The low numerical diffusion, coupled with the low values for explicit horizontal and vertical diffusion, means that we expect the ocean interior to be largely adiabatic in these simulations.

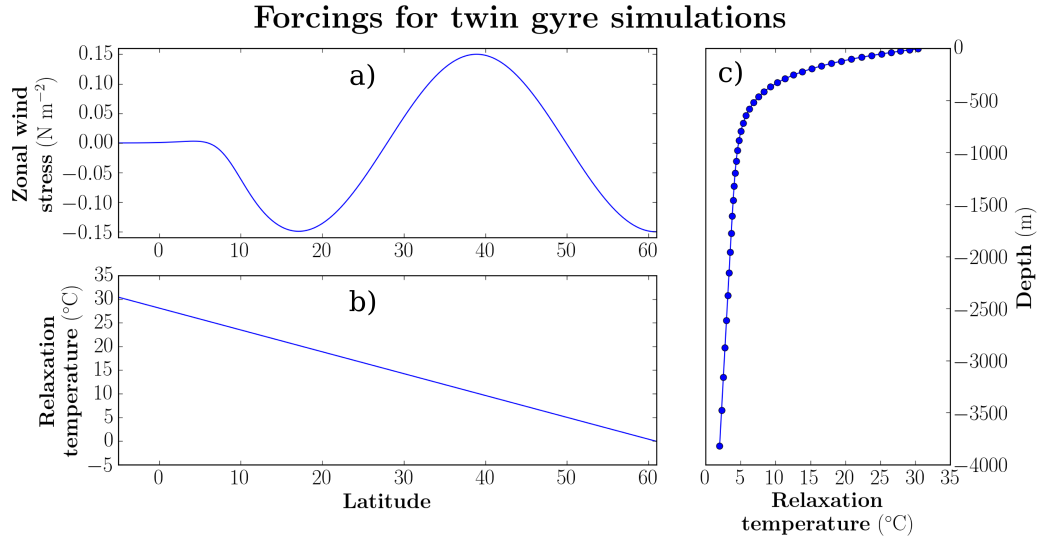


Figure 5.3: *Forcing fields for the simulations presented in this chapter. a)* the zonal wind stress as a function of latitude, *b)* the relaxation field for sea surface temperature as a function of latitude, and *c)* the temperature profile used in the southernmost five degrees of the domain to force a meridional overturning circulation in some of the simulations. The circles show the location of the centre of the tracer cells and hence the variable vertical resolution of the model.

5.3 Multiple gyres

Ultimately our interest lies in understanding the dynamics of vertical transport in the ocean. By expanding the domain to include a subpolar gyre our model becomes a better analogue of the ocean. Subpolar gyres are regions of large-scale Ekman divergence, which leads to a broad region of upwelling in the Eulerian-mean velocity fields. This upwelling is believed to form part of a basin-scale Ekman Cell that forms a closed circulation with the Ekman driven downwelling in the subtropical gyre (see e.g. Marshall, 2000). The sinking limb of the Ekman cell requires that fluid parcels experience a change in density as they progress through the thermocline. As described in the hypotheses section, we expect buoyancy forcing to make a substantial contribution to the potential vorticity flux budget in the subtropical gyre.

5.3.1 Eddy-resolving with multiple gyres

Solving the equations of motion on a numerical grid with a horizontal resolution of one eighth of a degree means that mesoscale variability is well resolved over most of the model domain.

As expected, Eulerian-mean advection, shown in figure 5.4, provides a downwards flux of potential vorticity across most of the subtropical gyre. In the subtropical gyre, the potential vorticity flux due to Eulerian-mean advection, given by $\rho_o \tilde{Q} \bar{w}$, is predominantly balanced by eddy-induced fluxes, given by $\mathbf{k} \times \bar{\mathbf{F}}_R \cdot \nabla \bar{\rho} - \bar{q}^z \nabla \overline{\mathbf{u}' \rho'}$, shown in figure 5.5. There is also a contribution from buoyancy forcing in the subtropical gyre, which can be seen in the synopsis shown in figure 5.6. In the subpolar gyre the potential vorticity flux due to mean advection is upwards, but the flux intensity is extremely small.

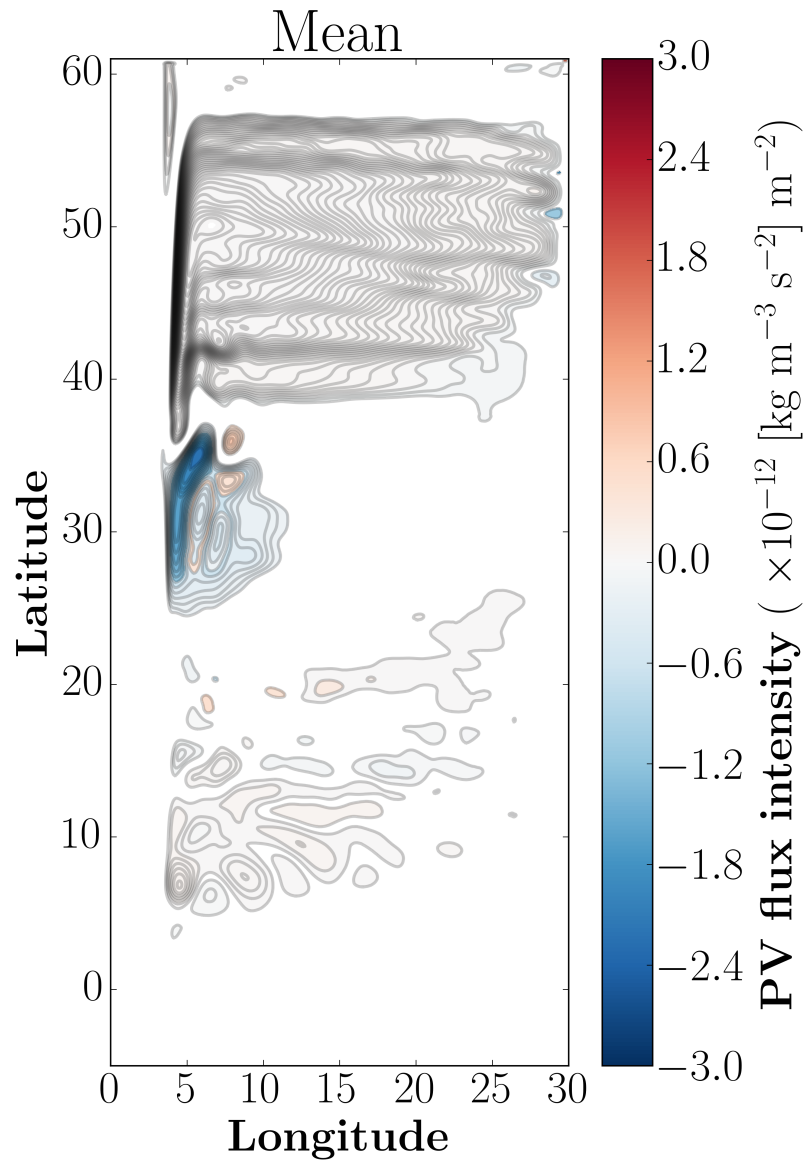


Figure 5.4: Vertical flux of potential vorticity due to Eulerian-mean advection, given by $\rho_o \bar{Q} \bar{w}$, at a depth of 450m in a 1/8th degree idealised simulation with multiple wind-driven gyres.

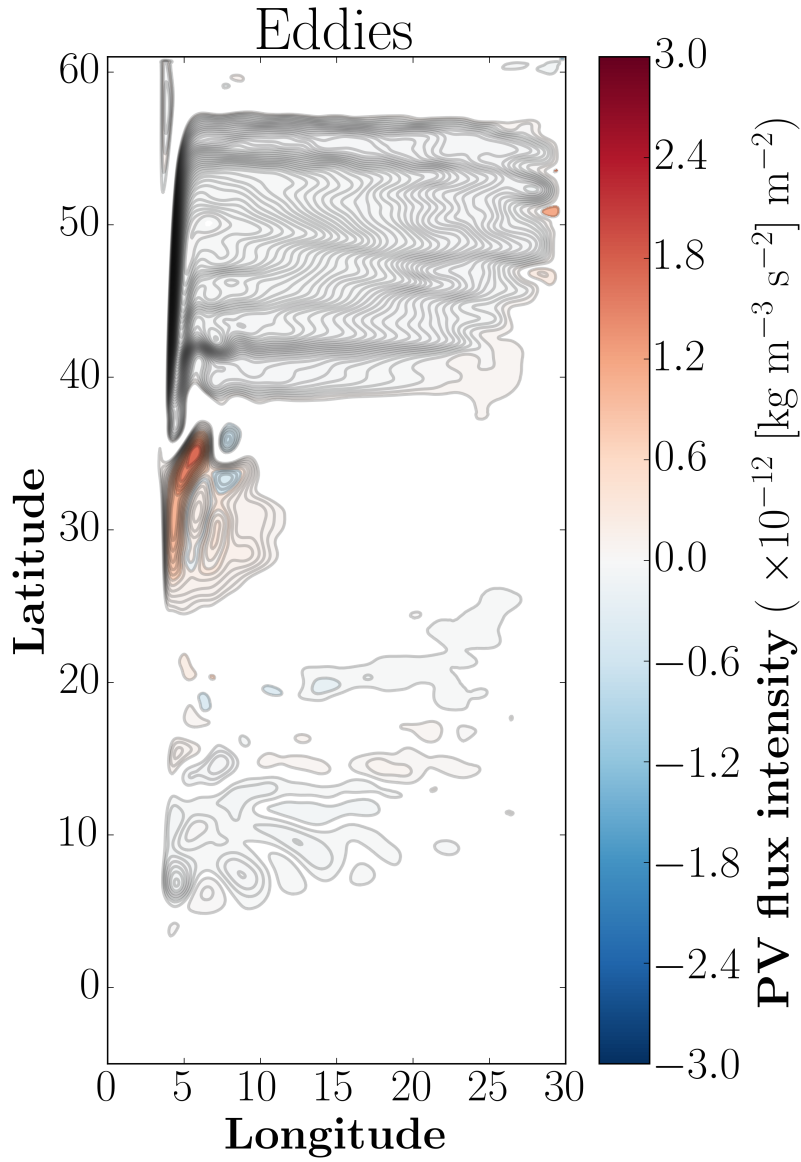


Figure 5.5: Vertical flux of potential vorticity due to eddy activity, given by $\mathbf{k} \times \overline{\mathbf{F}_R} \cdot \nabla \bar{\rho} - \bar{q}^z \nabla \overline{\mathbf{u}' \rho'}$, at a depth of 450m in a 1/8th degree idealised simulation with multiple wind-driven gyres.

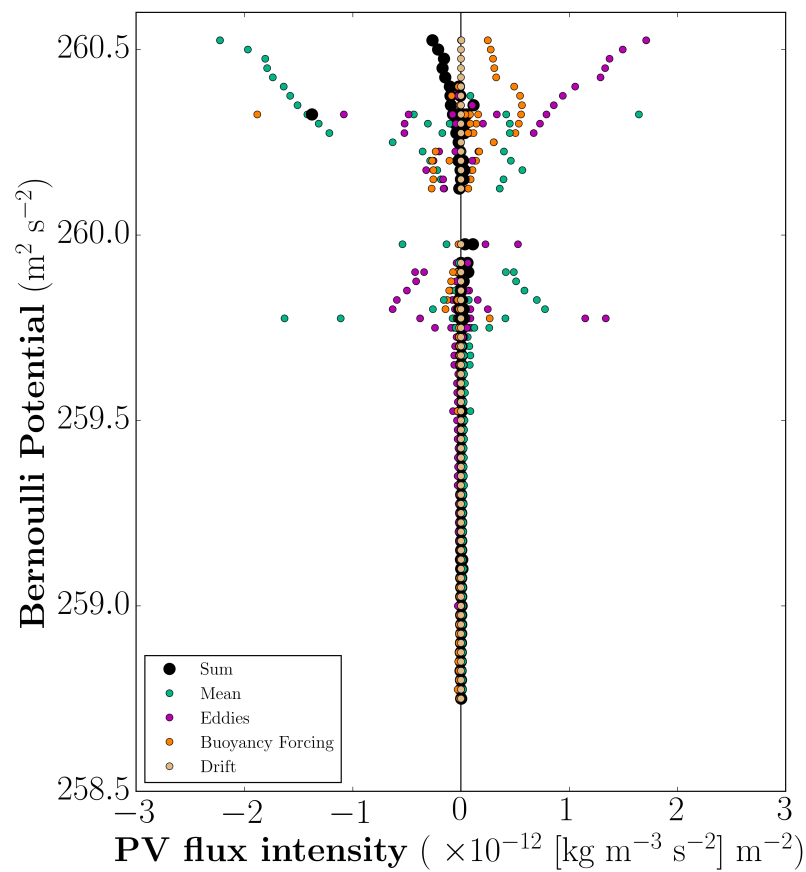


Figure 5.6: Synopsis of the vertical fluxes of potential vorticity at 450 m depth in a 1/8th degree simulation with multiple wind-driven gyres. The contours with higher values of Bernoulli Potential are in the subtropical gyre, while the contours with lower values are predominantly in the subpolar gyre.

5.3.2 Eddy-permitting with multiple gyres

At one quarter of a degree horizontal resolution the model is able to partially resolve eddies in the lower latitudes. However, mesoscale eddies remain unresolved across most of the domain. The Gent-McWilliams mesoscale eddy parametrization (Gent and McWilliams, 1990; Gent et al., 1995), discussed in section 3.1.5, is used to parametrise the effect of these unresolved mesoscale eddies. The vertical potential vorticity flux due to parametrised mesoscale eddies, derived in section 4.2.1.4, is calculated from the eddy buoyancy flux. As described earlier, we are able to split the advective component into contributions from the model velocity and the bolus velocity. The use of a mesoscale eddy parametrisation in an eddy-permitting simulation leads to the suppression of eddies, even in regions where the model grid is able to resolve eddies (Hallberg, 2013). Thus we expect the contribution from eddies to be small in this simulation, despite the eddy-permitting horizontal resolution. The choice to include a mesoscale eddy parametrisation in this simulation is somewhat at odds with the 25 km resolution simulation presented in chapter 4, which used a similar horizontal resolution but no parametrisation. The GM scheme is included here to allow the results to be more easily compared with those of Polton and Marshall (2007) who analysed an eddy-permitting ocean model that included a mesoscale eddy parametrisation.

Figure 5.7, a synopsis of the fluxes at 450 m depth, shows that the inclusion of a mesoscale eddy parametrisation has substantially weakened the effect of eddies. In the subtropical gyre the fluxes due to Eulerian-mean advection, figure 5.8, are predominantly negative. The Eulerian-mean bolus velocity generated by the GM mesoscale eddy parametrisation produces a vertical flux of potential vorticity, given by $-\bar{q}^z \nabla \cdot \bar{\mathbf{u}}_{GM} \bar{\rho}$, and shown in figure 5.9. This parametrised eddy flux largely balances the flux from Eulerian-mean advection. Figure 5.7 also shows very weak fluxes in the subpolar gyre similar to the results of the previous section.

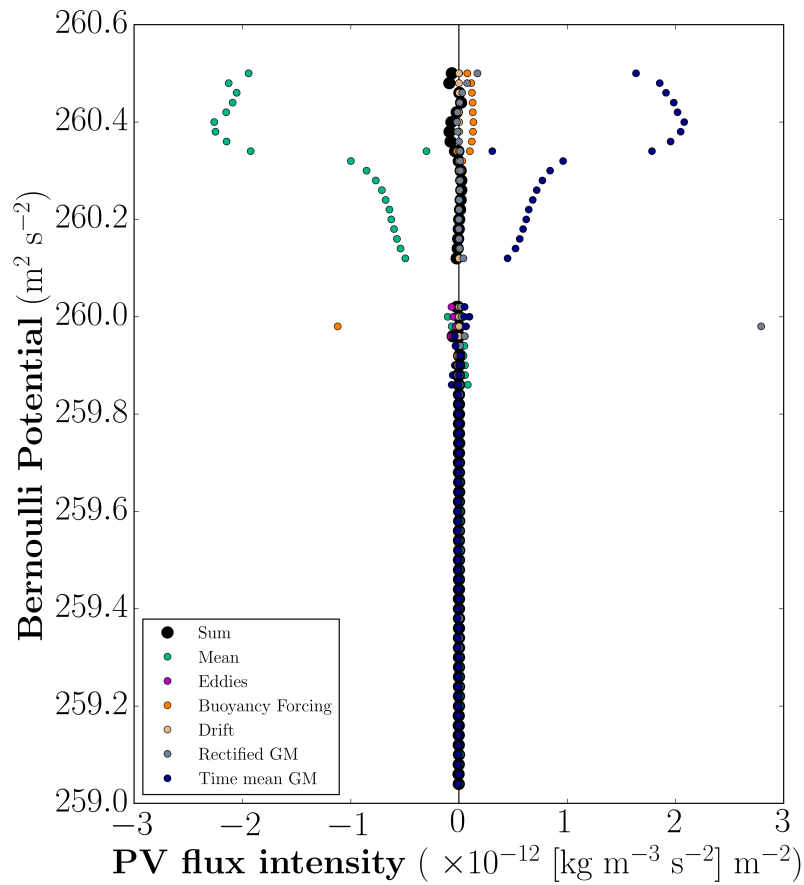


Figure 5.7: Vertical flux intensities of potential vorticity at a depth of 450 m in a one quarter degree idealised simulation with multiple wind-driven gyres. The contours with higher values of Bernoulli Potential are in the subtropical gyre, while the contours with lower values are predominantly in the subpolar gyre.

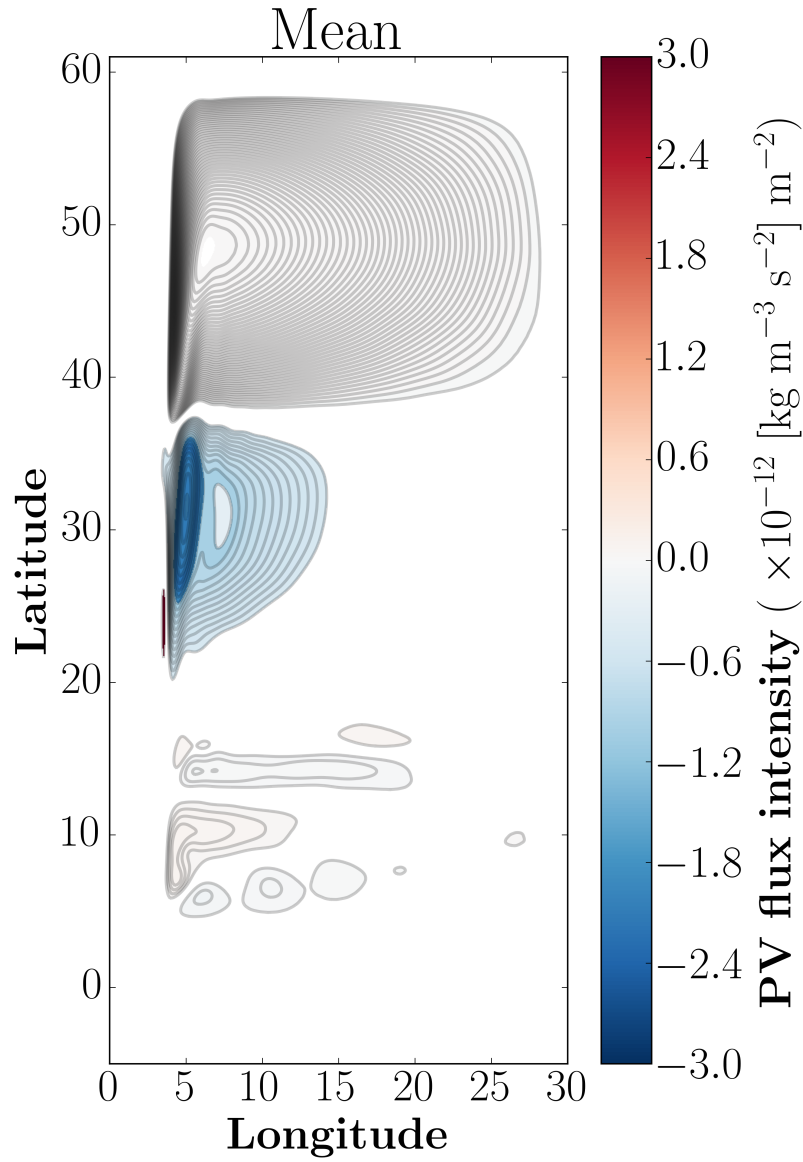


Figure 5.8: Vertical flux intensity of potential vorticity due to Eulerian-mean advection, given by $\rho_o \tilde{Q} \bar{w}$, at a depth of 450m in a one quarter degree idealised simulation with multiple wind-driven gyres.

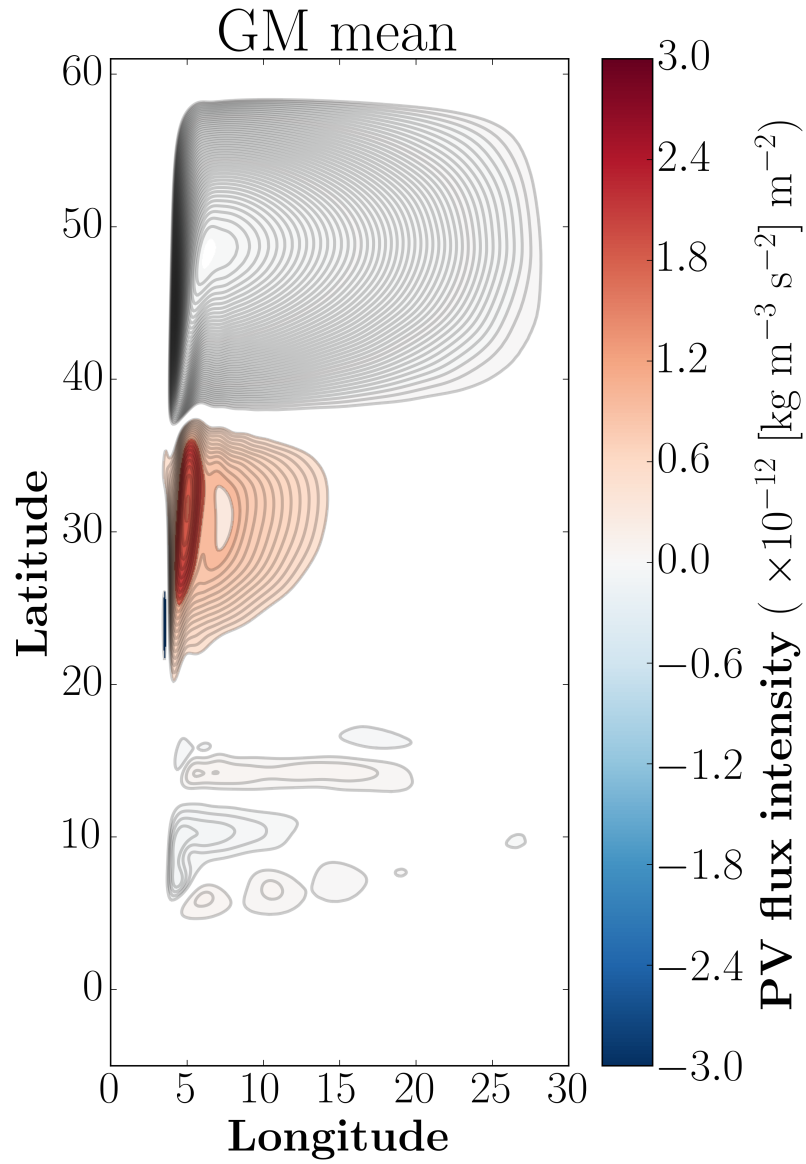


Figure 5.9: Vertical flux intensity of potential vorticity due to the Gent-McWilliams mesoscale eddy parametrisation at a depth of 450m in a one quarter degree idealised simulation with multiple wind-driven gyres.

5.3.3 Non-eddy resolution with multiple gyres

At this resolution the budget does not reliably close, as can be seen in figure 5.10. However, the potential vorticity flux budget of a half degree simulation follows the same pattern seen previously, namely, the Eulerian-mean advective flux is opposed by the parametrised eddies with a small contribution from buoyancy forcing. Once again, the flux intensity in the subpolar gyre is very small. Figure 5.11 shows the flux due to Eulerian-mean advection, and highlights that the non-zero fluxes seen in figure 5.10 are from the subtropical gyre and the additional closed contours in the southern portion of the domain.

Despite averaging 100 years of model data to produce the contours in figure 5.11 they are less rounded than those from the quarter degree resolution simulation, shown in figure 5.8. The shape of these contours is not due to non-linear effects, since the resolution and viscosity of this simulation prevent such effects. The southern portion of the domain is covered by small regions of closed Bernoulli Potential contours. The spacing of these regions is larger than the horizontal grid spacing, which suggests they are not simply the remnants of grid-scale noise that has been smoothed by the interpolation process. While noisy model fields may have contributed to the unexpected shape of these contours, it is unlikely to be the sole explanation. It is possible that the contours are due to standing Rossby waves at the boundary between the subtropical gyre and the anomalous gyre to the south, but it is unclear why this effect should appear only in this simulation.

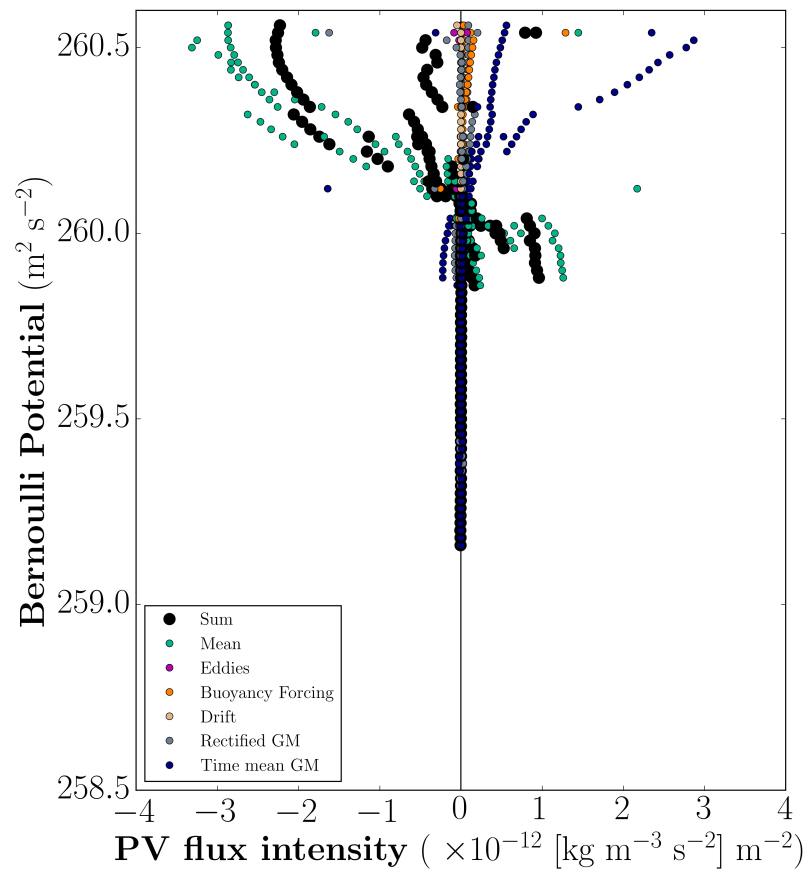


Figure 5.10: Synopsis of potential vorticity flux intensities at 450 m depth in a half degree resolution simulation.

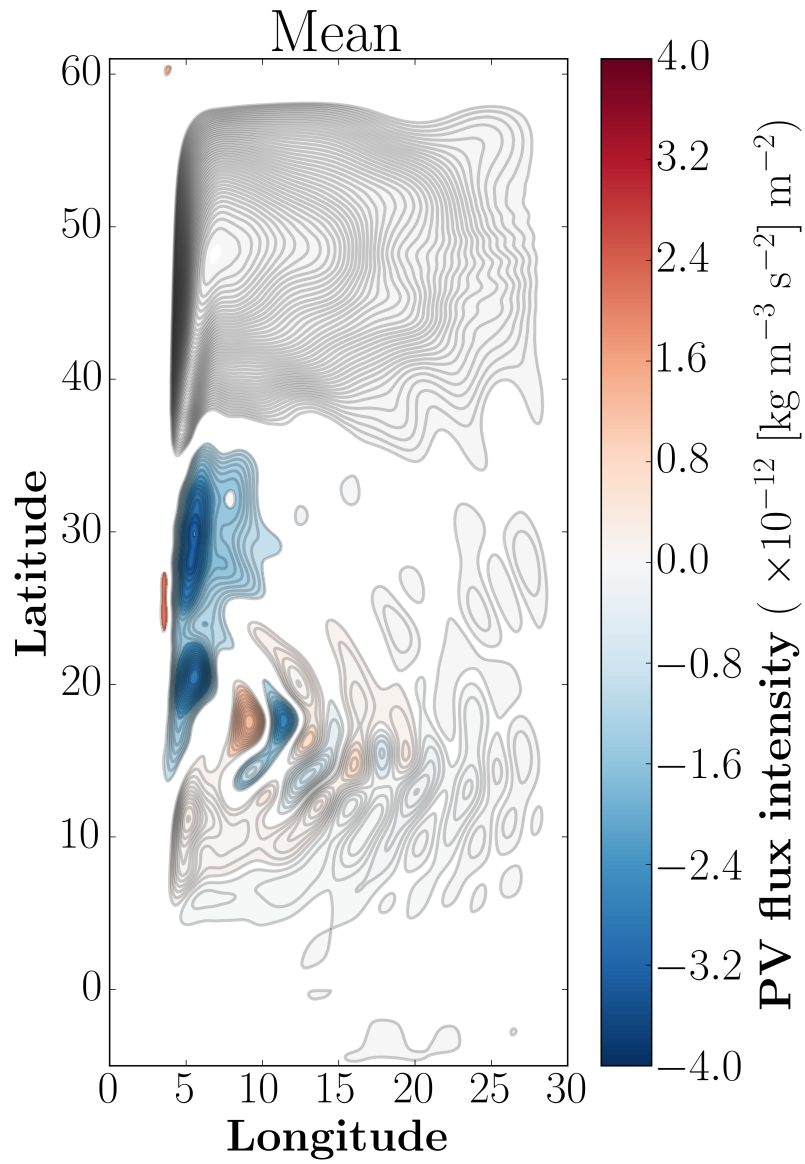


Figure 5.11: *Potential vorticity flux intensity due to Eulerian-mean advection, given by $\rho_o \overline{Qw}$, at 450 m depth in a half degree simulation.*

5.4 Including a meridional overturning circulation

As shown in the previous section, the presence of multiple wind-driven gyres does not materially alter the balance between potential vorticity fluxes due to Eulerian-mean advection and eddies. In this section we test whether the presence of a meridional overturning circulation alters this balance. The inclusion of a meridional overturning circulation makes our model a better analogue of the North Atlantic.

Relaxing the density field at the equator to a prescribed stratification profile can induce a meridional overturning circulation. This is analogous to Southern Ocean models that relax density to a given profile at the northern boundary (e.g. Abernathey et al., 2011). The strength of the overturning circulation and the depth of maximum overturning can both be tuned by varying the equatorial profile, the surface density relaxation field, and their associated relaxation timescales. Since this model is intended as an idealisation, neither the strength nor the depth of the overturning circulation have been exhaustively tuned. The maximum strength varies somewhat with resolution but remains between 10 Sv and 20 Sv, which is roughly consistent with the observed strength of the Atlantic meridional overturning circulation (Smeed et al., 2014). The overturning circulation from a one eighth degree simulation plotted in depth-latitude coördinates is shown in figure 5.12.

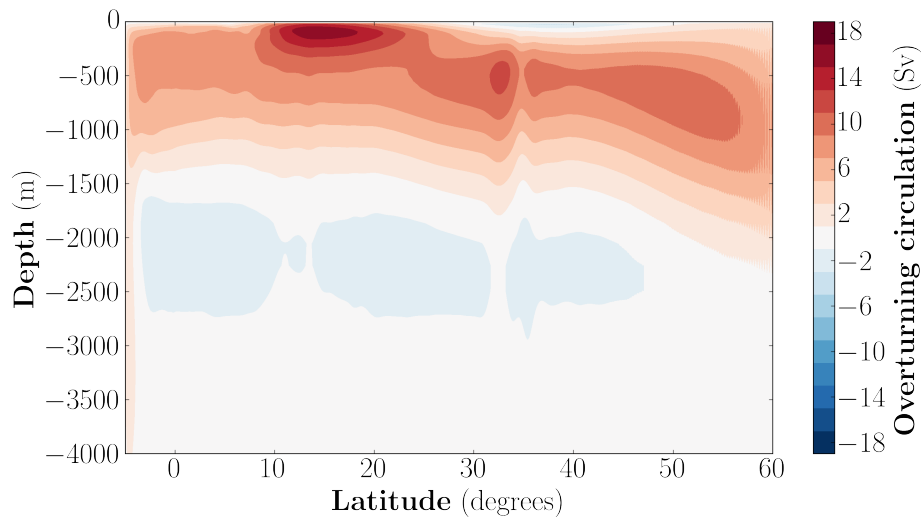


Figure 5.12: *The meridional overturning circulation calculated in depth-latitude space from the Eulerian-mean fields of a one eighth degree horizontal resolution simulation. The red lobes rotate clockwise and represent northwards transport of warm surface water.*

The southwards return flow of the overturning circulation in figure 5.12 occupies nearly 2000 m of the water column but only a very small range in density space, as

shown in figure 5.13. As reported by Zhang (2010), calculating the meridional overturning circulation (MOC) in density-latitude space produces a stronger circulation in the high-latitudes than calculating the MOC in depth-latitude space. The overturning circulation in this model is slightly shallower and weaker than the circulation in the model used by Zhang (2010), but is well within the range exhibited by other global models (see e.g. Hirschi et al., 2013; Weaver et al., 2012).

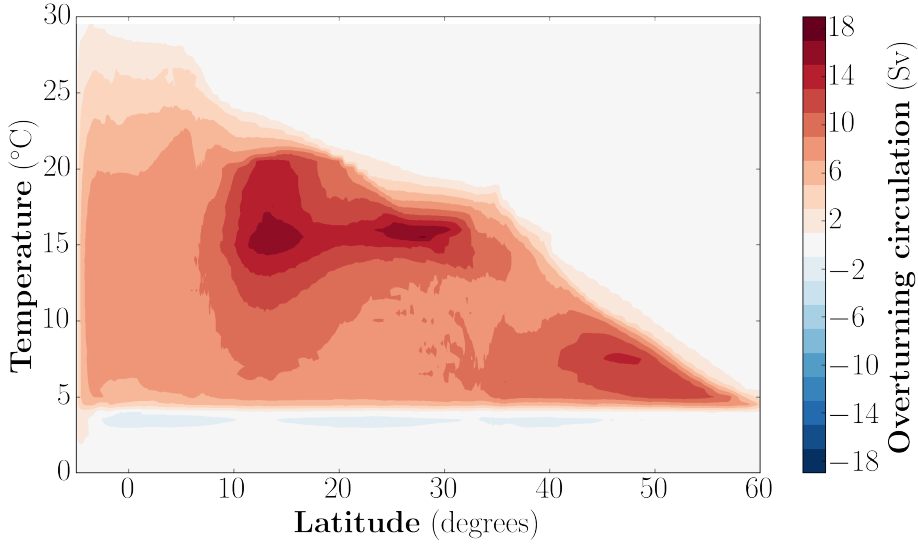


Figure 5.13: Meridional overturning circulation in temperature-latitude space for the 0.125 degree horizontal resolution simulation calculated from the Eulerian-mean temperature and velocity fields. The red lobes rotate clockwise.

5.4.1 Eddy-resolving resolution

The eddy-resolving version of this model exhibits a balance between potential vorticity fluxes due to Eulerian-mean advection, eddies, and buoyancy diffusion. The presence of a meridional overturning circulation has not materially altered the result found in section 5.3.1. However, the presence of a meridional overturning circulation in this simulation has substantially increased the flux intensity in the subpolar gyre.

As expected, the mean advection of potential vorticity provides a downwards flux in the subtropical gyre and an upward flux in the subpolar gyre, as shown in figure 5.15. These fluxes are particularly strong in the inertial recirculation regions.

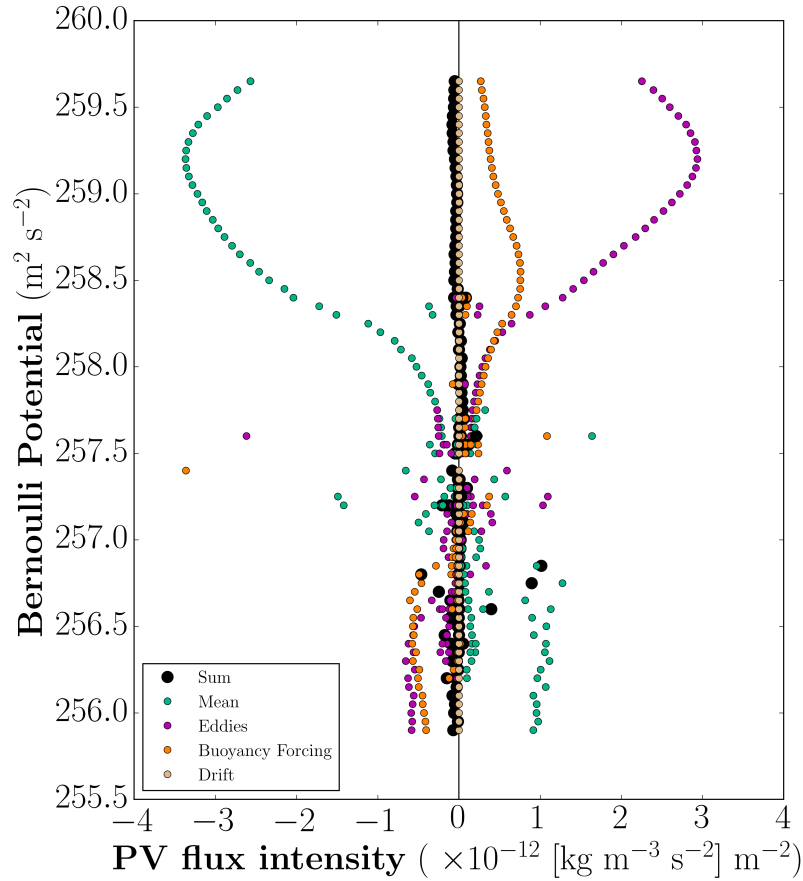


Figure 5.14: Vertical flux of potential vorticity at a depth of 450 m in an idealised North Atlantic with multiple wind-driven gyres and a meridional overturning circulation. The contours with higher values of Bernoulli Potential are in the subtropical gyre, while the contours with lower values are predominantly in the subpolar gyre.

The eddy induced vertical flux of potential vorticity opposes the Eulerian-mean advective flux. The rectified eddy contribution is upwards in the subtropical gyre

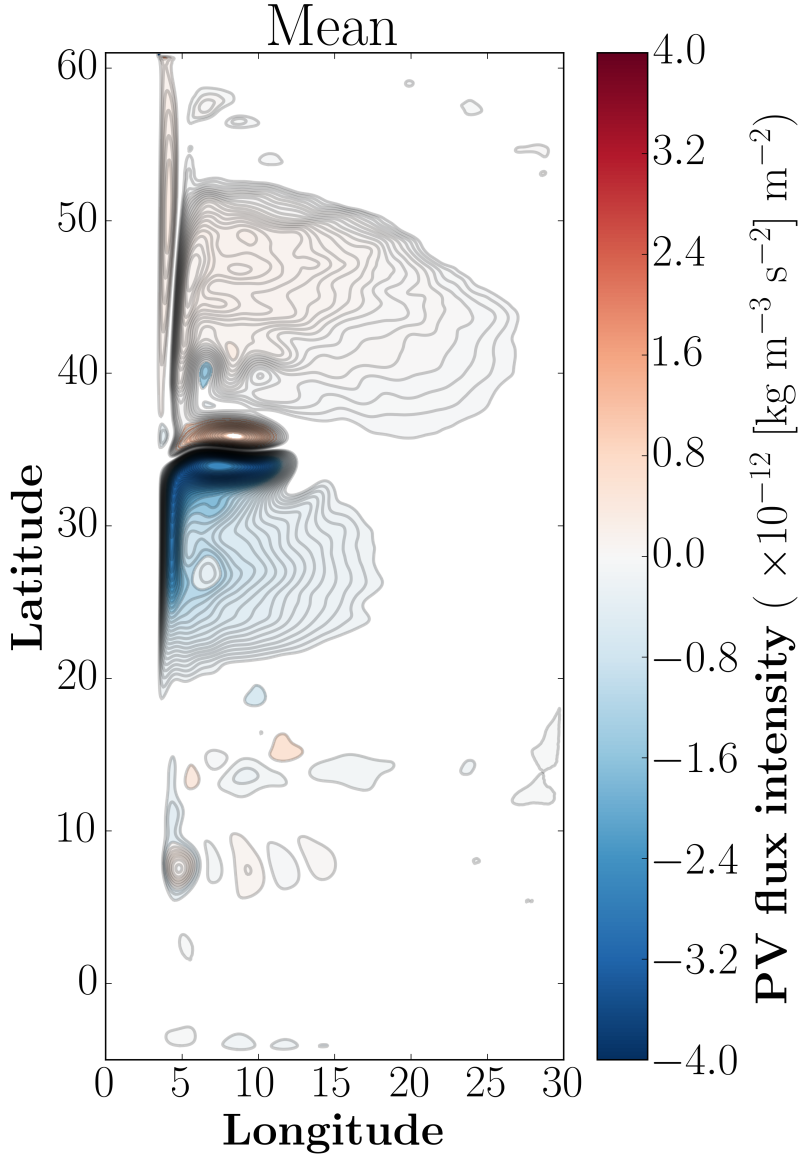


Figure 5.15: *Vertical flux of potential vorticity due to Eulerian-mean advection, given by $\rho_o Q \bar{w}$, at a depth of 450 m in an idealised North Atlantic with multiple wind-driven gyres and a basin scale overturning circulation.*

and downwards in the subpolar gyre, as shown in figure 5.16. The vertical potential vorticity flux induced by buoyancy forcing has the same magnitude and direction as the eddy induced flux in the subpolar gyre, as shown in figure 5.17. In the subtropical gyre the eddy induced flux is substantially larger than the flux due to buoyancy forcing, but they are both positive.

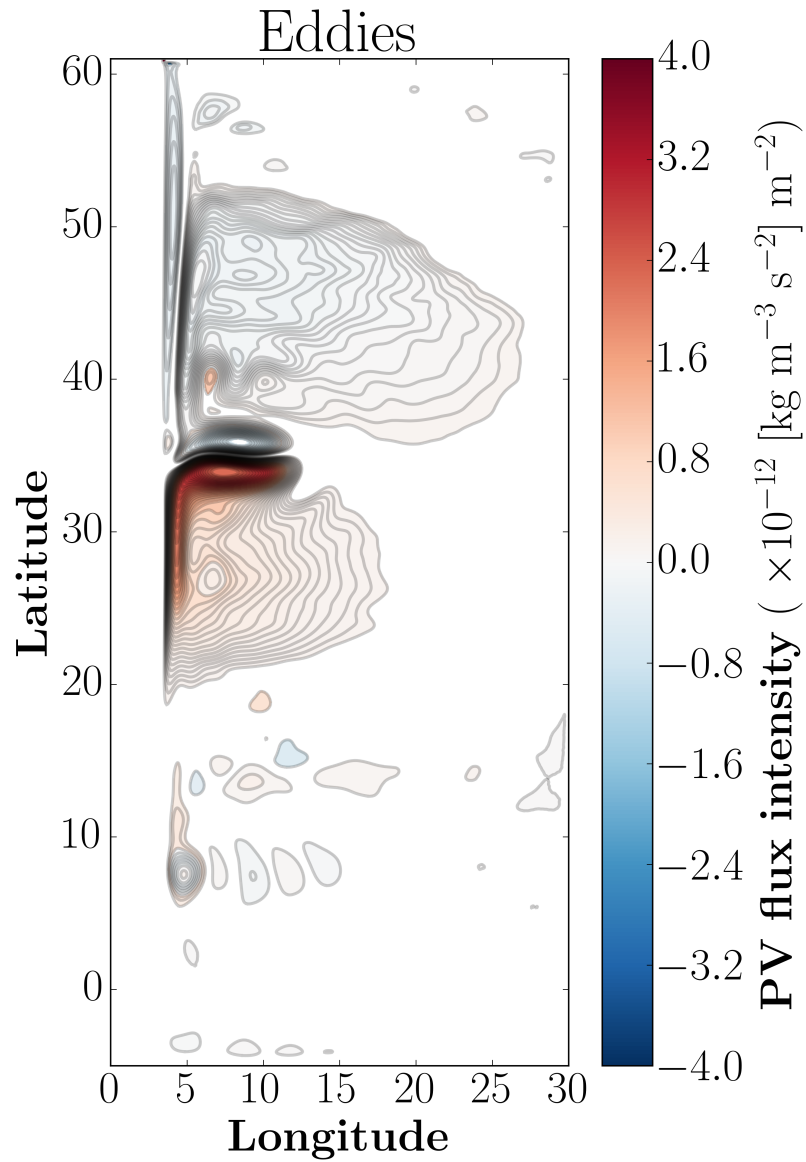


Figure 5.16: Vertical flux of potential vorticity due to eddy activity, given by $\mathbf{k} \times \overline{\mathbf{F}_R} \cdot \nabla \bar{\rho} - \overline{q^z \nabla \cdot \mathbf{u}' \rho'}$, at a depth of 450 m in an idealised North Atlantic with multiple wind-driven gyres and a meridional overturning circulation.

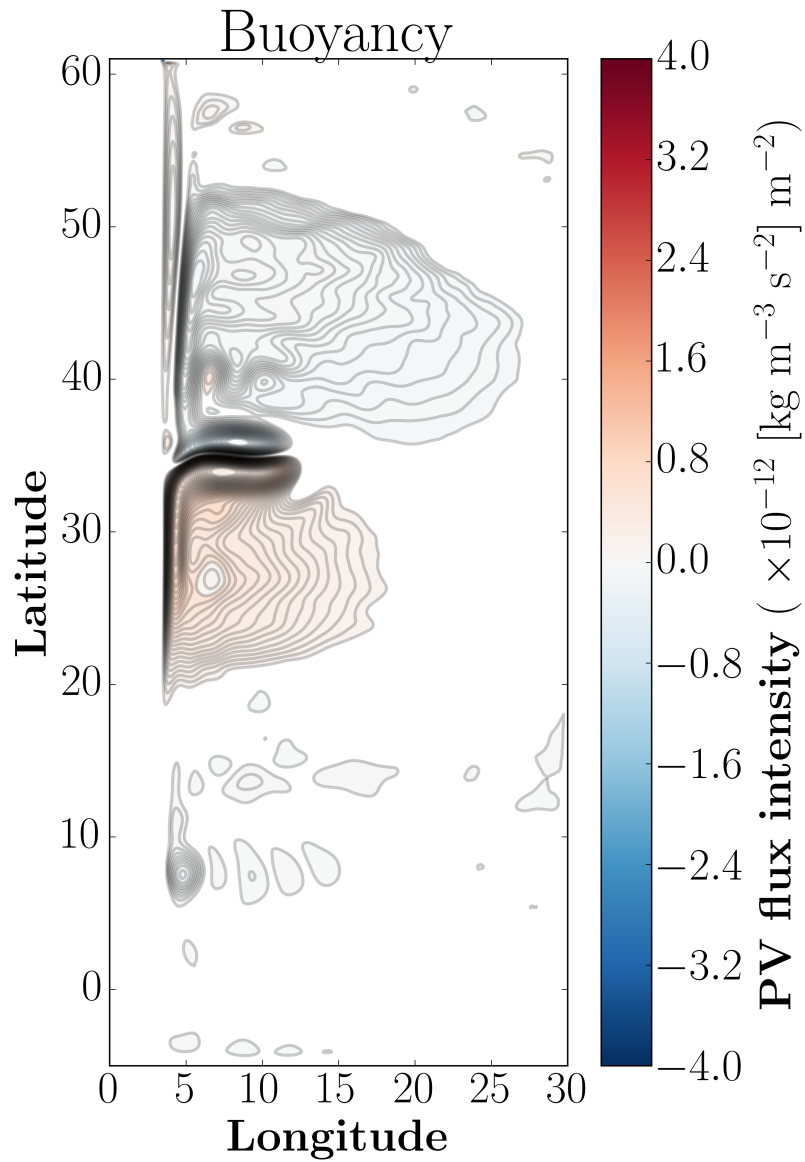


Figure 5.17: *Vertical flux of potential vorticity due to buoyancy forcing at a depth of 450 m in an idealised North Atlantic with multiple wind-driven gyres and a meridional overturning circulation.*

5.4.2 Eddy-permitting resolution

At one quarter of a degree horizontal resolution the model is able to partially resolve mesoscale eddies in the lower latitudes. However, mesoscale eddies remain unresolved across much of the domain, and as described previously, the use of a mesoscale eddy parametrisation suppresses mesoscale eddies even in regions where they are resolved.

As found in the previous simulations, the mean advection of potential vorticity provides a downwards flux in the subtropical gyre. The parametrised eddy induced vertical flux of potential vorticity opposes the Eulerian-mean advective flux, with a small contribution from buoyancy forcing. Once again, the flux intensities are smaller in the subpolar gyre.

The flux intensities shown in figures 5.18 and 5.19 are much larger than the intensities found in other simulations. I speculate that this may be due to the model grid being fine enough to resolve much of the large-scale flow, but not fine enough to resolve the transfer of energy from the large-scale flow to eddies. Thus, the flux intensity due to Eulerian-mean advection is much larger than the other simulations, and correspondingly the flux due to the mesoscale eddy parametrisation is also larger. However, if this is the reason for the larger intensities, it is unclear why a similar effect was not found in section 5.3.2.

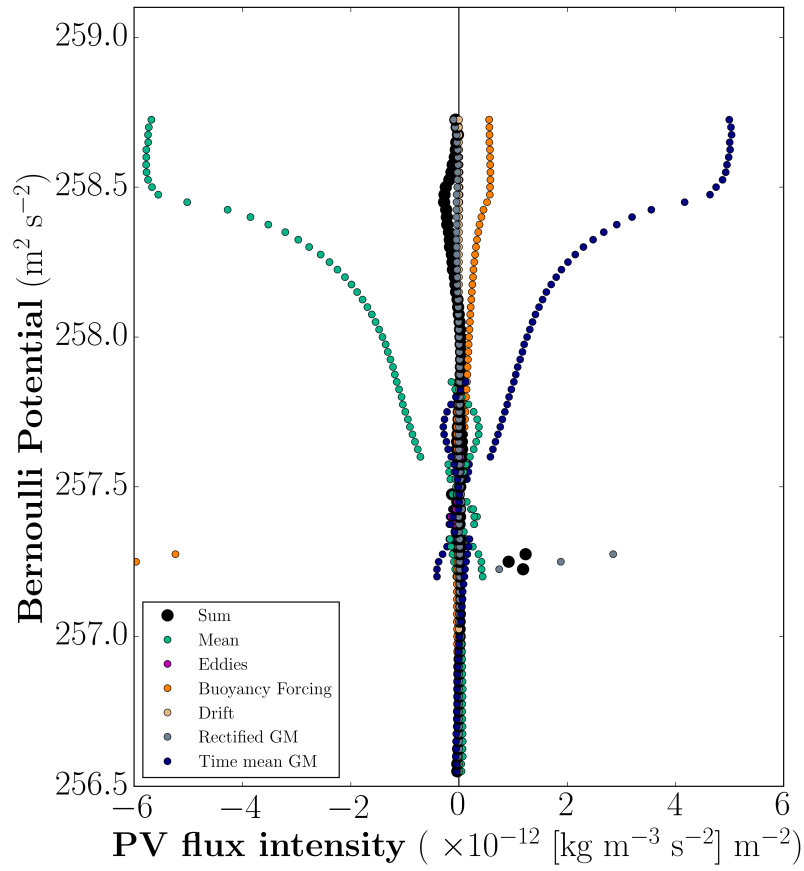


Figure 5.18: Vertical flux of potential vorticity at a depth of 450m in an idealised North Atlantic simulation run at a horizontal resolution of 0.25 degrees, with multiple wind-driven gyres and a meridional overturning circulation.

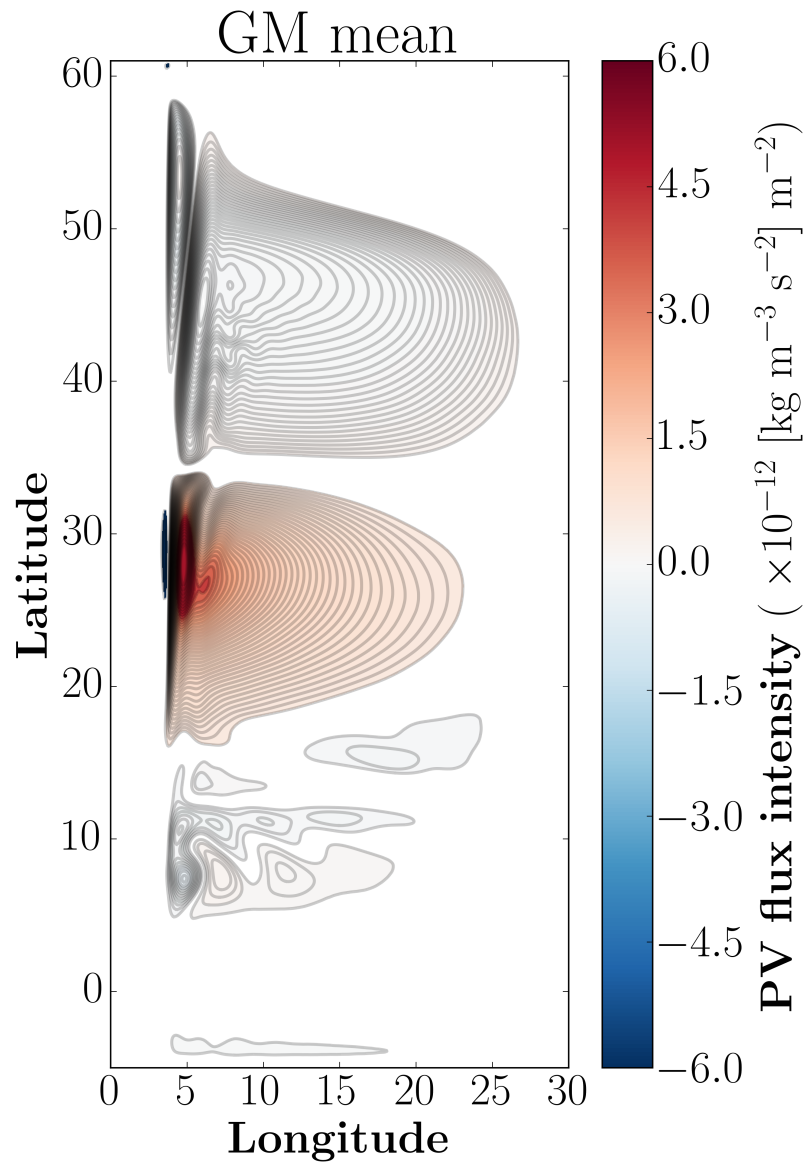


Figure 5.19: *Vertical flux intensity of potential vorticity due to the Gent-McWilliams mesoscale eddy parametrisation at a depth of 450m in an idealised North Atlantic simulation run at a horizontal resolution of 0.25 degrees, with multiple wind-driven gyres and a meridional overturning circulation.*

5.4.3 Non-eddying resolution

In this simulation the two main terms in the balance are still Eulerian-mean advection and parametrised eddies, as shown in the synopsis in figure 5.20. Despite the coarse resolution used for this simulation, equivalent to the grid used in section 5.3.3, the potential vorticity flux budget almost closes. The similarity between the residual in the budget and the rectified GM term, which was diagnosed as a residual from the advective terms, suggests that the residual in this budget likely arises from the derivatives and interpolations required to estimate the advective contribution. Figure 5.21 shows the potential vorticity flux intensity from Eulerian-mean advection. Consistent with the other results presented, this flux is negative in the subtropical gyre and weakly positive in the subpolar gyre.

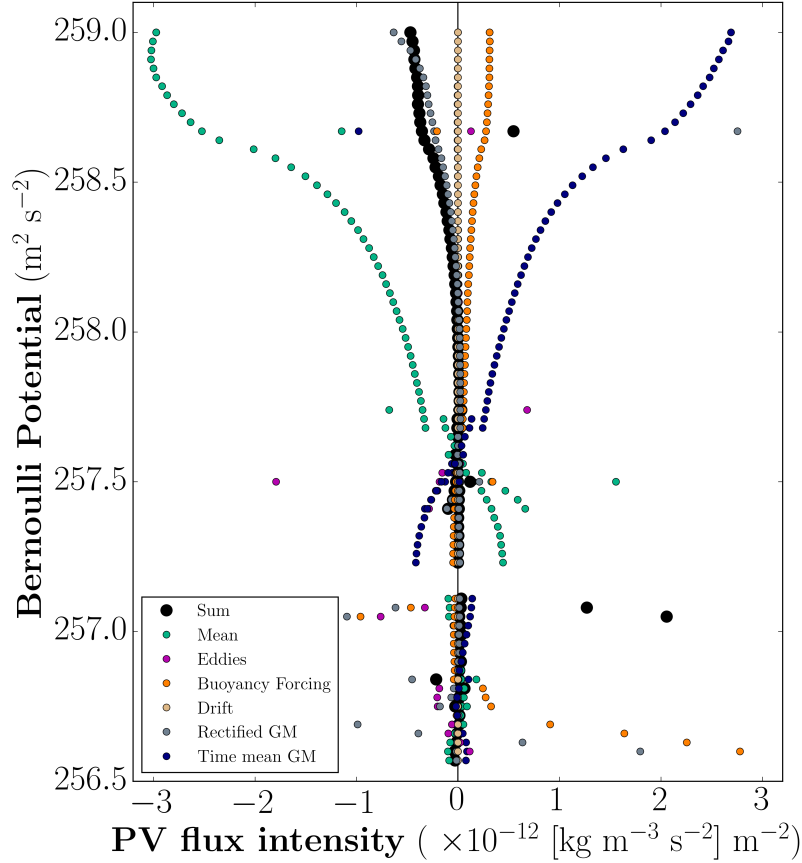


Figure 5.20: *Synopsis of vertical potential vorticity flux intensities at a depth of 450 m in an idealised North Atlantic simulation run at a horizontal resolution of 0.5 degrees, with multiple wind-driven gyres and a meridional overturning circulation.*

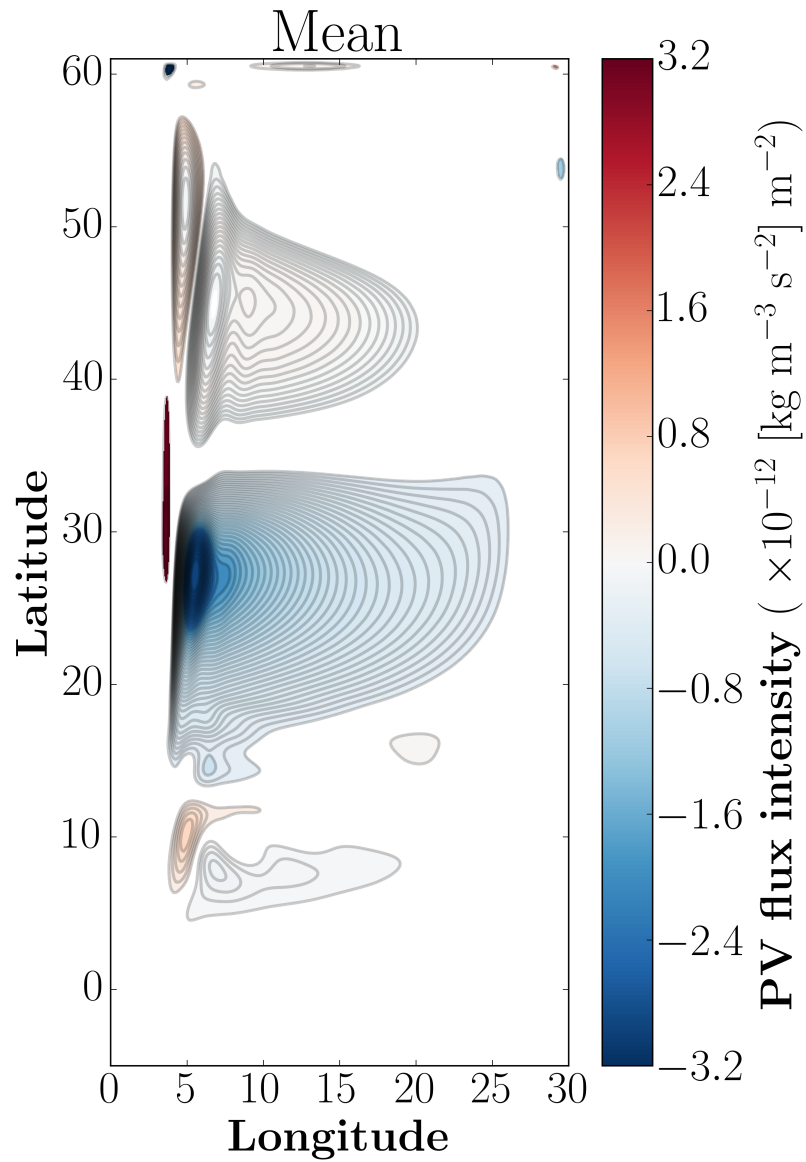


Figure 5.21: Vertical potential vorticity flux intensity due to Eulerian-mean advection, given by $\rho_o \tilde{Q} \bar{w}$, at a depth of 450 m in a non-eddying idealised North Atlantic with multiple wind-driven gyres and a basin scale overturning circulation.

5.5 Concluding remarks

The simulations presented in this chapter suggest that there is a three-way balance in the vertical potential vorticity flux budget in the subtropical gyre. The flux due to Eulerian-mean advection is predominantly balanced by an opposing flux due to eddies, either resolved or parametrised, while the residual between these two terms is balanced by the flux due to diffusion of density. This result is consistent with the idealized numerical experiments of Radko and Marshall (2004a) who found a balance between mean advection and eddies in global buoyancy and potential vorticity budgets. Our results are also consistent with Henning and Vallis (2004) who found a small residual between diapycnal mass fluxes due to eddies and mean advection across the internal thermocline. The residual identified by Henning and Vallis (2004) was balanced by diffusion. The three-way balance identified in the subtropical gyre is robust to changes in domain, resolution, background flow, and to the inclusion of a mesoscale eddy parametrisation. Given that this balance is robust to these changes, it is likely to be robust across a range of models. These results provide evidence to support our hypothesis that the integrated budget for vertical fluxes of potential vorticity in the subtropical gyre is a three-way balance between mean advection, eddy-driven upwelling, and diffusion.

Our hypothesis that the dynamical balance in the subpolar gyre would be largely adiabatic was not supported. In most of the simulations the flux intensities were extremely small in the subpolar gyre. The eddy-resolving simulation with a meridional overturning circulation was the only simulation in which these fluxes were of a comparable intensity to those in the subtropical gyre. In this simulation the potential vorticity flux budget in the subpolar gyre shows a substantial contribution from diffusion. The flux due to buoyancy diffusion is largely restricted to the inertial recirculation region. This suggests that the diabatic processes associated with this flux are linked with the non-linear dynamics of the inertial recirculation region, but this does not explain why the equivalent simulation without a meridional overturning circulation did not show similar dynamics. Further work is required to understand why the flux intensities in the subpolar gyre are so much lower than those in the subtropical gyre.

Our third hypothesis, that the Gent-McWilliams mesoscale eddy parametrisation would act to oppose the potential vorticity flux due to Eulerian-mean advection was supported. All of the simulations in which we used the Gent-McWilliams parametrisation showed the parametrised eddy-induced fluxes opposing the flux due to Eulerian-

mean advection. Given the robustness of the eddy-mean balance to changes in resolution and background flow, it is likely that this result would hold in other ocean models and perhaps even in the ocean.

At coarser horizontal resolutions the residual in our potential vorticity flux budget becomes larger; at a horizontal resolution of one half of a degree the residual is substantial. The residual increases with the horizontal grid spacing, which suggests that it is likely due to errors introduced by the discretisation. Constructing this budget requires evaluating a number of derivatives and the staggered nature of the C-grid means that variables must be averaged onto new locations within the grid cell before they can be multiplied, or compared. Computing the eddy contribution to this budget uses variables that are stored on a number of different locations within the grid cell and requires the largest amount of horizontal averaging. This means that the eddy term is likely the largest source of numerical errors when evaluating this budget. This remains true even when using the GM mesoscale eddy parametrisation. The other terms require fewer averaging operations. For example the Eulerian-mean advection requires two terms that are defined on the tracer point and only the vertical velocity must be averaged in the vertical to move it to the correct location.

The inertial recirculation regions exhibit much larger flux intensities in these budgets than other regions within the gyres. This suggests that the non-linear dynamics and larger velocities present in these regions contribute to increasing the flux intensity. The energetic mesoscale eddy field in these regions is the likely reason for the increased intensity of the eddy-induced fluxes, while the increased velocities likely contribute to the heightened Eulerian-mean advection term. These larger velocities also advect fluid in more rapidly, which likely increases the tracer gradients, which may explain the increased magnitude of the buoyancy diffusion term in these regions. Given that the dynamics in these regions are much more energetic than elsewhere in the domain, it is perhaps unsurprising that they dominate the vertical potential vorticity budget.

These two chapters have addressed the first of the key questions for this thesis. We have found a similar balance in the potential vorticity flux budget of wind-driven gyres to the balance identified in the Southern Ocean. That there exists a balance between Eulerian-mean advection and rectified eddy effects in the potential vorticity flux budget raises the prospect of a balance in other budgets. The next chapter seeks to address the second and third key questions of this thesis, and to test whether the rectified effect of eddies is able to oppose Eulerian-mean vertical mass transport in subtropical gyres.

Eddy mean-flow cancellation in wind-driven gyres

The potential vorticity flux diagnostics presented in Chapters 4 and 5 indicate a balance between vertical potential vorticity fluxes from Eulerian-mean advection and the fluxes due to eddies. This chapter aims to test the impact of rectified eddy transports on the vertical movement of fluid and other tracers. The text of this chapter is largely drawn from the paper “Eddy cancellation of the Ekman cell in subtropical gyres” published in the *Journal of Physical Oceanography* (Doddridge et al., 2016).

There exist a number of circulations in the ocean and atmosphere that are prominent in Eulerian-mean velocity fields, but are inconsistent with observed tracer distributions, for example, the Deacon cell in the Southern Ocean (Döös and Webb, 1994). When the effect of eddies is included in the averaging operator, these circulations are dramatically altered or disappear entirely.

Large-scale sinking due to Eulerian-mean Ekman pumping is believed to dominate the vertical motion in subtropical gyres (see e.g. Oschlies, 2002b; Williams and Follows, 2011) and to form part of an intergyre overturning circulation known as the Ekman cell. A schematic of the Ekman cell is shown in figure 6.1. The diabatic circulation of the Ekman cell is difficult to reconcile with the adiabatic nature of the ocean interior. Cancellation of large-scale downwelling in subtropical gyres by an eddy induced circulation would remove the substantial diapycnal transport across the subtropical thermocline associated with the Ekman cell. As described by Griffies et al. (2015), if the ocean interior is adiabatic and temperature surfaces are approximately aligned with density surfaces, then we expect the net vertical transfer of heat

to be small and the downwards transport of heat by the mean flow to be opposed by vertical eddy transports.

Downwelling in the subtropical gyres is also believed to remove nutrients from the oligotrophic surface waters (Williams and Follows, 2011). Cancellation of the Ekman cell by eddies may help settle the long-standing discrepancy between new production and estimated nutrient supply in the gyre-scale nutrient budget (see e.g. McGillicuddy, 2016; Williams and Follows, 1998, 2011). Because such a large proportion of the ocean surface is covered by subtropical gyres, these regions contribute approximately half of all marine export production (Jenkins and Doney, 2003; Williams and Follows, 2011). This large contribution means that improving our understanding of vertical transports in subtropical gyres has implications for the global nutrient and carbon budgets. The presence of an eddy induced circulation balancing time-mean Ekman downwelling also has implications for mode water formation and the maintenance and structure of the thermocline (see e.g. Henning and Vallis, 2004; Marshall, 2000; Marshall et al., 2002; Polton and Marshall, 2003; Radko and Marshall, 2004b; Xu et al., 2016).

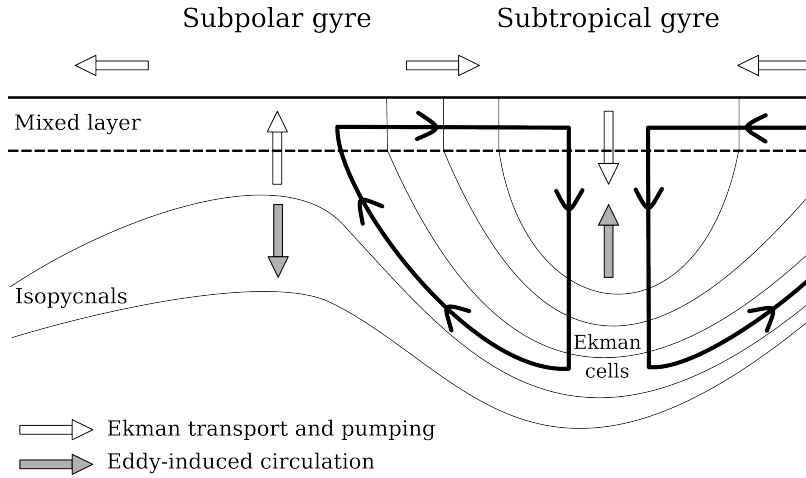


Figure 6.1: Schematic showing Ekman cells and eddy-induced advection. Adapted from Marshall (2000).

This chapter is organized as follows. Section 6.1 describes the atmospheric Ferrel cell and the Deacon cell in the Southern Ocean, their sensitivity to the choice of averaging technique, and some theoretical background regarding different methods for averaging flows. Section 6.2 contains the hypotheses that we seek to test in this chapter. In section 6.3 we present the results of our Lagrangian particle tracking

experiments. Section 6.4 quantifies the extent to which eddies oppose the Eulerian-mean circulation through the use of depth-density overturning streamfunctions. In section 6.5 we present our concluding remarks and some possibilities for future work that could expand upon our current results.

6.1 Averaging flows and vanishing circulations

To develop an understanding of the chaotic and turbulent patterns of circulation commonly observed in the ocean and atmosphere we need to take an average of the flow. However, the way in which we take this average can have a dramatic impact on the structure, strength, and even direction of the diagnosed circulation (see, e.g., Plumb and Mahlman, 1987). In turbulent flows, such as those analysed in geophysical fluid dynamics, the transport of mass and tracers due to fast, small-scale, time-varying features of the flow field can be substantial. If our averaging operator does not account for these transports, then there may be substantial differences between the transport of the averaged flow and the averaged transport of the flow. While taking an average cannot create a flow where none exists, it can provide a misleading description of the flow if considered in isolation.

The rectified effect of eddies is a combination of down-gradient diffusion along isopycnals and advective transport (Gent et al., 1995; Plumb, 1979). In certain geometries this advective transport can lead to up-gradient transport (Lee et al., 1997), and can oppose circulations in the Eulerian-mean fields. There are multiple studies showing that correctly representing eddies is a necessary condition for accurately modelling the distribution of tracers. For example, Follows and Marshall (1996) modelled the distribution of a transient radioisotope of carbon, and found that models without mesoscale eddies were unable to reproduce the observed distribution.

6.1.1 Averaging flows

We restrict our attention to two averaging techniques because they are conceptually simple and encompass the range of behaviours that we wish to discuss. For a more rigorous and mathematically demanding approach to averaging the equations of motion the reader is referred to Young (2012) and Maddison and Marshall (2013)

6.1.1.1 Eulerian-mean

Taking the Eulerian-mean of a field is straightforward: a time averaging operation is performed independently at each spatial location. A variable X is separated into

a time-mean component, usually denoted $\bar{X}$, and time-varying component, denoted X' . The effect of eddies and other variability is taken into account by calculating the cross-correlations between terms. For example, if we take the inviscid Boussinesq zonal momentum equation and apply a time averaging operator, the equation becomes

$$\frac{\partial \bar{u}}{\partial t} + \bar{\mathbf{u}} \cdot \nabla \bar{u} - f \bar{v} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} - \nabla \cdot \overline{\mathbf{u}' u'}, \quad (6.1)$$

in which u is the zonal velocity, $\mathbf{u}$ is the three-dimensional velocity field, f is the Coriolis parameter, v is the meridional velocity, ρ_0 is the reference density, p is the pressure, x and t are the zonal and temporal coordinates, and the final term represents the effect of the variability on the time-mean field, known as the Reynolds stress. We retain the time derivative of the time-mean zonal velocity to describe changes over timescales comparable with, or longer than, the timescale of the time-mean operator. The Eulerian-mean formalism splits the tendency terms into contributions from time-mean and time-varying components. It is this artificial separation that lead Dunkerton (1980) to state that the Eulerian-mean “can be and has been misleading.”

Within the Eulerian-mean framework it is possible to identify circulations that are incompatible with observed tracer distributions, such as the Deacon cell in the Southern Ocean (Danabasoglu et al., 1994; Döös and Webb, 1994; Marshall, 1997). The location of inconsistencies between Eulerian-mean circulations and tracer distributions highlights areas where the time-varying contribution is non-negligible.

Another example of a situation in which the Eulerian-mean velocity field is misleading may be found by considering the propagation of planar waves across the surface of a fluid otherwise at rest. The fluid velocity at the wave crests is in the direction of wave propagation, and the particles are lifted and moved forward as a crest passes their position. As a trough passes their location, the particles are lowered and advected in the opposite direction. The magnitude of the wave velocity decays with depth, which leads to the particles experiencing a net displacement after each wave period, despite the fact that below the level of the wave troughs the Eulerian-mean velocity is identically zero. The difference between the Eulerian-mean velocity and the velocity following a particle is known as the Stokes’ drift velocity. For further discussion of Stokes’ drift the reader is referred to chapter 3 of Williams and Follows (2011).

6.1.1.2 Generalised Lagrangian mean

The mathematics of the Generalised Lagrangian-Mean (GLM) framework is formidable, but in this case we require only an intuitive understanding of its implications. GLM

was developed as a framework to analyse wave-mean flow interactions in the stratosphere (Andrews and McIntyre, 1978a, b). The GLM velocity is not defined following a single fluid parcel, but rather following the centre-of-mass of a collection of fluid parcels that span the region being averaged (Andrews and McIntyre, 1978a). For example, to calculate a zonal average, a tube of particles would be initialized along a line of latitude. However, after some finite period of time the particle distribution can be sufficiently altered by the fluid motion that interpretation of the velocity and location of the centroid of the distribution becomes difficult. Figure 6.2 shows this problem schematically. In the initial circular configuration, the location and momentum of the centroid may be easily associated with the average motion of the surrounding fluid. However, once advection has sufficiently distorted that distribution, shown schematically in the right-hand portion of figure 6.2, it is no longer clear that the GLM over the parcel is a meaningful quantity. The issue of reinitialization within GLM theory is briefly discussed by Dunkerton (1980), who recommends a case-by-case approach.

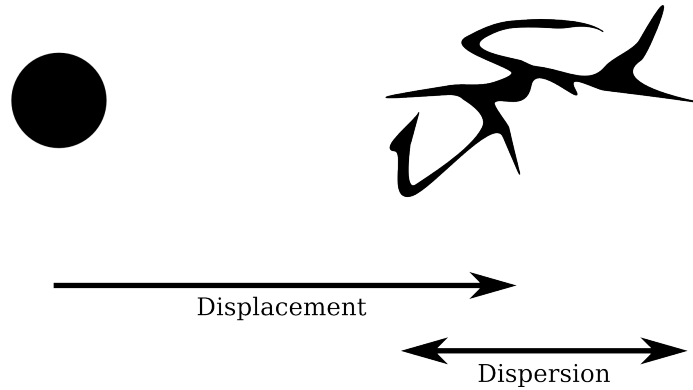


Figure 6.2: A schematic showing the issue of reinitialisation in Generalised Lagrangian-Mean (GLM) theory. The initially circular distribution of particles (left) is deformed by the flow field, and after some finite period of time is substantially altered (right). Once the distribution is sufficiently altered the location and velocity of the centroid are no longer meaningful quantities.

6.1.2 Vanishing circulations

6.1.2.1 Ferrel cell

The Ferrel cell is an atmospheric circulation with air rising in the subpolar latitudes and sinking in the subtropical latitudes. Because this circulation involves colder air rising and warmer air sinking it cannot be sustained by density differences due to temperature. Circulations such as this are known as thermally indirect and require the

input of energy to maintain (Vallis, 2006). The presence of a Ferrel cell in atmospheric circulations is dependent on the averaging process used to analyse the atmosphere (Dunkerton, 1978). Plumb and Mahlman (1987) provide an excellent illustration of how sensitive the existence of the Ferrel cell is to averaging. Plumb and Mahlman present time- and zonal-means of the velocity fields from a general circulation model in which there are three circulation cells in each hemisphere. However, when Plumb and Mahlman compute the effective transport circulation, which is similar to the GLM but allows for spatially inhomogeneous diffusion, the circulation pattern shows only two cells in each hemisphere, and the Ferrel cell is no longer present. Furthermore, when comparing the two transport fields, they give opposite results for the direction of meridional transport at the surface at a number of latitudes.

6.1.2.2 Deacon cell

The Deacon cell is a meridional circulation in the Southern Ocean characterized by upwelling of deep water south of the Antarctic Circumpolar Current (ACC), northwards movement of this fluid in the surface Ekman layer, and downwelling north of the ACC (see e.g. Speer et al., 2000). A Deacon cell can be clearly seen in the time- and zonal-mean circulations of the coarse resolution ocean model used by Danabasoglu et al. (1994). When Danabasoglu et al. include an eddy-induced bolus velocity in their analysis, there is a near-complete cancellation of the Deacon cell. While most models do not reproduce such near-complete cancellation, they do show a substantially smaller residual overturning circulation that is directed along isopycnals and consistent with the observed water masses (Marshall, 1997; Marshall and Radko, 2003). The existence and strength of the Deacon cell, which are dependent not only on the flow field but also the choice of averaging operator, are relevant to our understanding of subduction, the formation of deep water, and to tracer advection (see, e.g., Abernathey et al., 2011; Dufour et al., 2012; Marshall, 1997; Marshall and Radko, 2003).

The similarities between the Ferrel cell in the atmosphere and the Deacon cell in the Southern Ocean have been explored previously. For example, Karoly et al. (1997) showed that both the Ferrel and Deacon cells are the result of correlations between zonal variations in density and meridional flow, not zonal mean buoyancy forcing. This means that if the averaging is performed using density as the vertical coordinate, neither circulation cell is apparent. Several other studies have also found that the use of density as the vertical coordinate for their time- and zonal-mean averaging

operations resulted in the disappearance of the Deacon cell (Döös and Webb, 1994; Hirst et al., 1996).

6.1.2.3 Ekman cell

The Ekman cell is a diabatic circulation hypothesized to link Ekman driven downwelling in subtropical gyres with Ekman upwelling in the tropics and subpolar regions. Figure 6.1, which has been adapted from Marshall (2000), shows a schematic of the Ekman cell. The concept of Ekman cells is used extensively in the biogeochemistry literature. For example, Najjar and Keeling (2000) describe a fluid parcel traversing the Ekman cell and use the changes in nutrient concentration and light availability to explain observed productivity and oxygen fluxes. Even when the Ekman cell is not invoked, the surface Ekman flows and the resulting upwelling and downwelling are often used when analysing the nutrient budget of the subtropical regions (e.g. Williams and Follows, 1998).

The geometry of subtropical gyres is more complex than the near zonal flows in the atmosphere and Southern Ocean. The presence of non-linear inertial recirculations and variation in gyre shape with depth makes it more difficult to directly diagnose and analyse the Ekman cell.

6.2 Hypotheses

As described above, there are multiple thermally indirect circulations that appear in the Eulerian mean fields and are largely opposed by eddy-driven circulations. Given that the Ekman cell shares many characteristics with these circulations, we hypothesise the existence of an analogous eddy-driven circulation in the subtropical gyres that opposes Ekman downwelling.

In this chapter we seek to test the following two hypotheses:

- we hypothesise the existence of an eddy-driven circulation that opposes the Eulerian-mean circulation of the Ekman cell; and
- we hypothesise that the residual circulation - the net effect of the Ekman cell and eddy-driven circulation - will be small

6.3 Lagrangian particle tracking

To compute the combined effect of eddies and the mean flow we use the GLM framework. However, because the shape of wind-driven gyres varies substantially with depth we use Lagrangian particle tracking techniques to compute an approximate GLM velocity numerically. As described in section 6.1.1.2, the GLM velocity is the mean velocity following a collection of parcels that span the region being averaged. We estimate the GLM velocity in the subtropical mode water of our model by seeding a cloud of particles there. The subtropical mode water in our model is the mass of weakly stratified fluid above the thermocline in the subtropical gyre (Worthington, 1958). We deal with the issue of reinitialization, mentioned in section 6.1.1.2 and discussed by Dunkerton (1980), pragmatically. Our Lagrangian particles are observed to begin leaving the region of interest after three months of model integration. Thus we track particles for three months before calculating the centre-of-mass of the particles' distribution and reseeding new particles within an ellipsoid around this point. Only the location of the ellipsoid changes; its size remains constant throughout the experiments.

We use a fourth-order Runge-Kutta scheme to numerically solve the three coupled ordinary differential equations for each particle. Velocity fields are linearly interpolated in space and time to each particle's location. The code we use is described in more detail in section 3.2.4 and is part of the Python module that can be found at <http://doddridge.me/code/>. To test for sensitivity to the time-step chosen in our algorithm we computed a test set of pathways using a time-step of one minute. Using any time-step up to one hour gives results that are indistinguishable for ten years of model time; hence we use a time step of one hour.

Similarly to Pennel and Kamenkovich (2014), we perform two different particle tracking experiments and artificially alter the velocity field used to advect some of the particles. In the first particle tracking experiment we advect the particles using the full time-varying velocity fields, which have been saved every five model days. Using the full velocity fields to advect our Lagrangian particles provides us with a numerical estimate of the GLM velocity of the weakly stratified subtropical mode water. In the other, we use the Eulerian-mean velocity fields. Following Lagrangian particles in the Eulerian-mean velocity fields provides us with a velocity that is conceptually similar to the GLM velocity, but with the effect of eddies removed. The difference between these two results is an estimate of the rectified effect of eddies on the bulk transport of fluid in the subtropical gyre.

One disadvantage of using an offline Lagrangian particle tracking algorithm is that the temporal resolution of the velocity data is reduced. In an eddy-resolving simulation the velocity fields evolve with time, and if the data are not saved regularly enough, then the Lagrangian particles will not follow the correct pathways. As stated above, the velocity fields were saved every five model days. This temporal resolution was chosen after inspecting the velocity fields of the highest resolution simulation and noting that the change between subsequent snapshots was small. This suggests that using velocity snapshots saved every five model days is appropriate for the simulations presented in this thesis.

Particles were initially seeded uniformly throughout the mode water. The centre-of-mass and spread of these particles were used to construct an ellipsoid and approximately five thousand particles were then seeded uniformly throughout this ellipsoid. As described above, our Lagrangian particles are observed to begin leaving the region of interest after three months of model integration. Thus we track the particles for three months before calculating the centre-of-mass of the particles' new distribution and reseeding new particles within an ellipsoid around this point. The Lagrangian particle tracking experiments were performed in simulations that included a meridional overturning circulation.

6.3.1 Parametrised eddies

At a horizontal resolution of one half of a degree these simulations do not resolve the first baroclinic radius of deformation and show very little temporal variability. The path mapped out by the centre-of-mass of the particles changes dramatically depending on whether the particles are advected by the model velocity or the residual velocity. The residual velocity is calculated as the sum of the model velocity and the parametrised eddy-induced bolus velocity from the Gent-McWilliams parametrisation scheme (Gent and McWilliams, 1990; Gent et al., 1995). Figures 6.3 and 6.4 show the pathways followed by the centre-of-mass of the particle distributions from the two particle tracking experiments. The horizontal motion in the two experiments is similar; each centre-of-mass flows clockwise around the gyre. However, the vertical motion differs substantially between the experiments. Including the eddy bolus velocity prevents the centre-of-mass from downwelling through the thermocline. The results shown in figures 6.3 and 6.4 use the time-varying velocity fields, but almost identical results are obtained if the Eulerian-mean values are used. This occurs because the coarse resolution of the model grid leads to very little temporal variability, and so the instantaneous velocities are almost identical to the Eulerian-mean values.

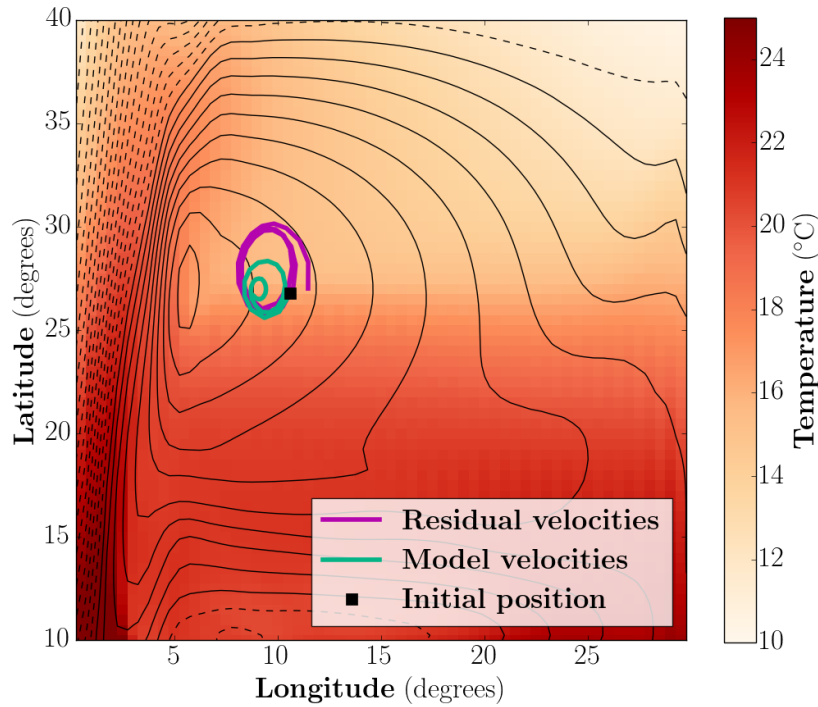


Figure 6.3: Results of the Lagrangian particle tracking experiments in the half-degree resolution simulation plotted on top of the Eulerian-mean sea surface temperature (colours) and free surface height (contours, with a contour interval of 5 cm). Green represents the centre-of-mass of the particles advected using the model velocities, purple shows the centre-of-mass of the particles advected by the residual velocity fields. These results were calculated using the time-varying velocity fields; the trajectories are almost identical if the Eulerian-mean fields are used.

6.3.2 Resolved eddies

In a simulation with a horizontal resolution of one eighth of a degree the model is able to resolve mesoscale variability in the subtropical gyre. This allows us to examine the rectified effect of resolved mesoscale eddies on our Lagrangian particles. Once again, approximately five thousand particles were seeded inside an ellipsoid in the mode water. The path mapped out by the centre-of-mass of the particles advected by the Eulerian-mean velocities is vastly different to that from the particles advected by the time-varying velocities. Figures 6.5 a) and b) show the pathways followed by the centre-of-mass of the particles in a latitude-longitude plane and latitude-depth plane respectively, while figure 6.6 shows the pathways in time-depth coordinates. A video of the two particle tracking experiments can be found at <https://doddridge.me/publications/dmh2016/>.

As shown in figure 6.6, the subduction rate of the centre-of-mass in the Eulerian-

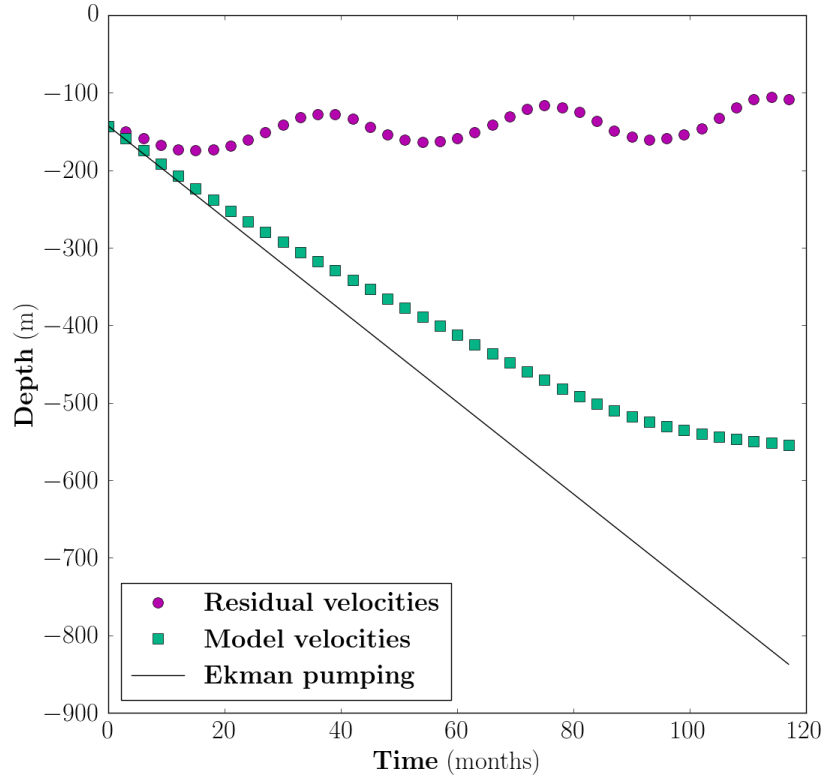
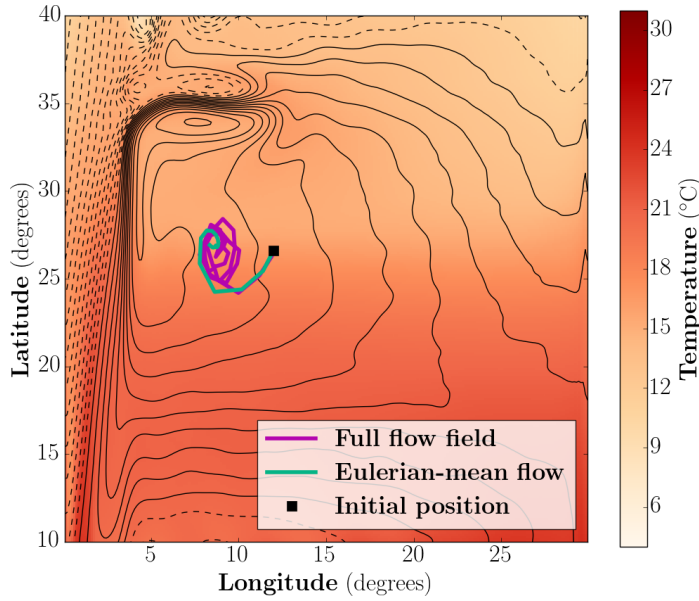
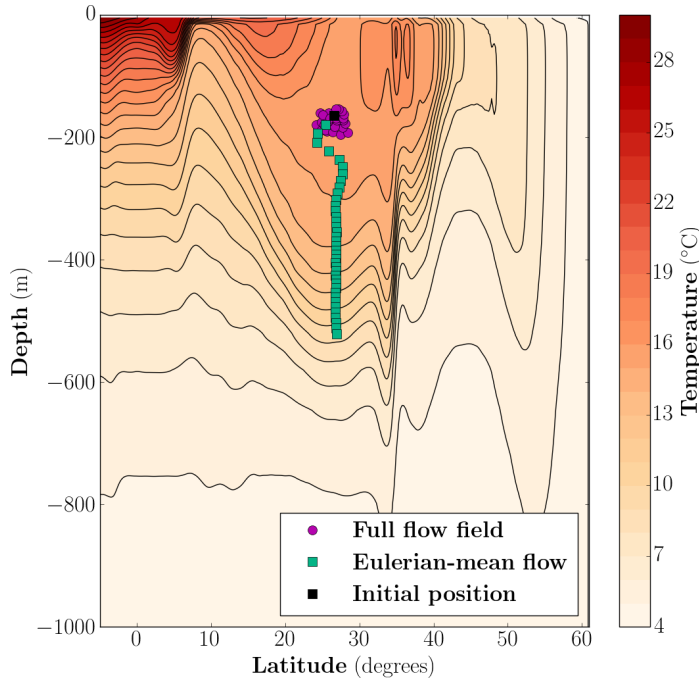


Figure 6.4: *Pathway for centre-of-mass of the Lagrangian particles tracked for ten years of model time in the half-degree resolution simulation. Each dot represents the location after three months of model time. Green represents the centre-of-mass of the particles advected using the model velocities, purple shows the centre-of-mass of the particles advected by the residual velocity fields. These results were calculated using the time-varying velocity fields; the trajectories are almost identical if the Eulerian-mean fields are used.*

mean experiment is initially comparable with the Ekman pumping velocity averaged over the gyre. After the first six months the rate of subduction slows to approximately half of the gyre-averaged Ekman pumping velocity.



(a) Pathway of the centre-of-mass of the particles shown in the latitude-longitude plane plotted on top of the Eulerian-mean sea surface temperature (colours) and free surface height (contours, with a contour interval of 5 cm).



(b) Pathway of the centre-of-mass in the latitude-depth plane. Each dot represents the location after three months of model time. The background colours and contours represent the Eulerian-mean temperature field at 10°E with a contour interval of one $^\circ\text{C}$.

Figure 6.5: Results of the Lagrangian particle tracking experiments in the one-eighth-degree resolution simulation. Green represents the centre-of-mass of the particles advected by the Eulerian-mean velocity fields, purple shows the centre-of-mass of the particles advected with the time-varying fields.

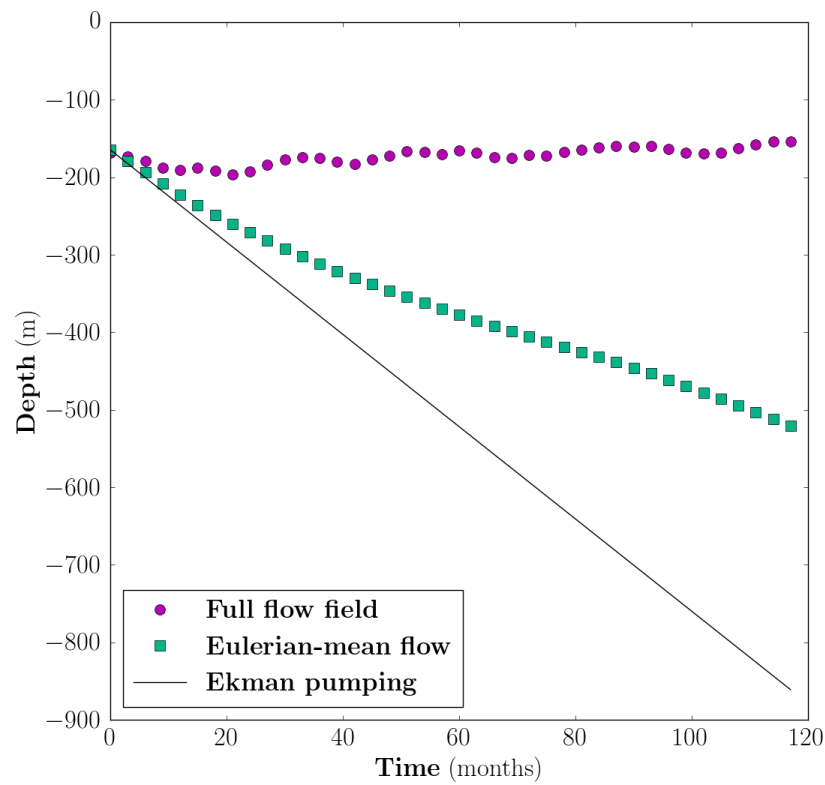


Figure 6.6: Pathways for centre-of-mass of the Lagrangian particles over 10 years of model time in the one-eight-degree simulation. Each dot represents the location after three months of model time. Green represents the centre-of-mass of the particles advected by the Eulerian-mean velocity fields, purple shows the centre-of-mass of the particles advected with the full flow field.

6.4 Overturning streamfunctions

Our Lagrangian particle tracking experiments and analysis of potential vorticity fluxes have provided strong evidence for an eddy-driven circulation that opposes the Eulerian-mean Ekman downwelling. We now seek to quantify this effect through the use of an overturning streamfunction.

As discussed previously, these ideas have also been applied to the Ferrel cell in the atmosphere and the Deacon cell in the Southern Ocean. The residual meridional overturning circulation in the Southern Ocean can be described as a balance between an Ekman driven mean overturning circulation and an eddy-driven overturning circulation. These two circulations largely cancel, leaving a small residual overturning circulation. This residual overturning circulation is what the fluid experiences, as shown by tracer release experiments in numerical simulations (see e.g. Abernathey et al., 2011; Hill et al., 2012). We now seek to define an overturning streamfunction around gyre streamlines.

To verify that our overturning streamfunctions are capturing the Ekman pumping, it will be useful to have an estimate of the volume of fluid we expect to be subducted through Eulerian-mean Ekman pumping in the subtropical gyre. The vertical velocity from Ekman pumping is given by

$$\frac{1}{\rho_0} \mathbf{k} \cdot \left(\nabla \times \frac{\boldsymbol{\tau}}{f} \right), \quad (6.2)$$

where $\boldsymbol{\tau}$ is the wind stress vector field (Vallis, 2006; Williams and Follows, 2011). In the simulations analysed for this chapter, the Ekman pumping velocity reaches a maximum of approximately $3 \times 10^{-6} \text{ m s}^{-1}$, or about 90 m year^{-1} , over the subtropical gyre. Taking the horizontal integral of the Ekman pumping velocity within the subtropical gyre gives a vertical transport of approximately 10 Sv.

6.4.1 Defined around streamlines

We begin by constructing an overturning streamfunction around geostrophic streamlines of the gyre circulation. This will provide an estimate of overturning within the gyre that is analogous to the overturning circulation Marshall and Radko (2003) calculated around streamlines of the Antarctic Circumpolar Current (ACC).

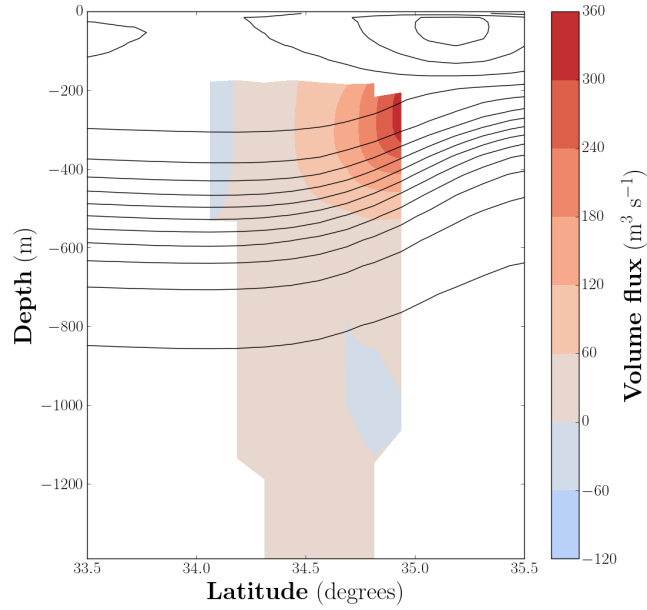
Integrating the normal component of the velocity field along a closed contour is equivalent to an area integral of the divergence of the horizontal velocity field within the closed contour. Therefore, to simplify the calculation we compute

$$\begin{aligned}
\oint_C \mathbf{u} \cdot d\mathbf{n} &= \iint_S \nabla \cdot \mathbf{u} dA \\
&= \iint_{\text{everywhere}} \lambda \nabla \cdot \mathbf{u} dA,
\end{aligned} \tag{6.3}$$

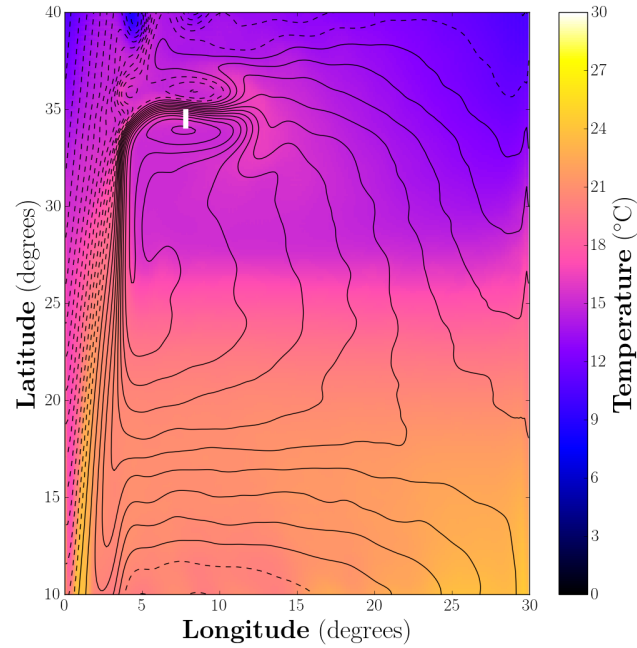
where λ is one inside the contour, and zero elsewhere.

Choosing the streamlines to use to define the gyre circulation is less straightforward than in the case of the ACC. Unlike the ACC the shape of the gyre varies substantially with depth. The other complicating factor is the presence of inertial recirculation regions, which lead to changes in the topology of the gyre flow with depth due to the presence of multiple regions containing closed streamlines. These two factors complicate the definition of an unambiguous coördinate system to serve as the gyre equivalent of latitude-depth for the overturning streamfunction calculated around ACC streamlines.

Despite these difficulties we begin by calculating closed contours of Eulerian-mean Bernoulli Potential on Eulerian-mean density surfaces to obtain the time-mean geostrophic streamlines. We then use equation (6.3) to calculate the flow crossing these streamlines. To visualise the resulting circulation we project the streamfunction on to a depth-latitude plane that cuts through the separating western boundary current. Because these streamlines are defined on density surfaces and the near-surface density varies with latitude, there are no closed streamlines in the upper 180 m of the water column. We intend to use this streamfunction to examine the balance between Ekman pumping and an eddy induced circulation. Therefore, the fact that the Ekman layer is contained within the region that our streamfunction is unable to cover is problematic. The streamfunction, shown in figure 6.7, shows some flow moving downwards through the thermocline. However, the volume flux of $400 \text{ m}^3 \text{ s}^{-1}$ is substantially smaller than the expected 10 Sv of Ekman pumping. This is most likely due to the fact that the streamfunction is only able to capture the horizontally divergent component of the flow field within closed streamlines, and does not capture the convergence of Ekman transports in the fluid near the surface. Thus the diagnosed streamfunction is only able to detect a small proportion of the sinking within the gyre.



(a) Overturning streamfunction around Eulerian-mean geostrophic streamlines in subtropical gyre (colours) plotted over Eulerian-mean temperature (contours with a contour interval of 1°C). The streamfunction is plotted on the meridional slice shown by the thick white line in panel b).



(b) Sea surface temperature (colours) and free surface height (contours, with a contour interval of 5 cm). The thick white line at 35°N and 10°E shows the location where the overturning streamfunction in panel a) has been projected.

Figure 6.7: Streamwise overturning circulation in the subtropical gyre.

6.4.2 Depth-density overturning streamfunction

Our attempt to construct an overturning circulation around time-mean geostrophic streamlines, similar to the method described by Marshall and Radko (2003) for the Antarctic Circumpolar Current, was unsatisfactory. We now turn to the depth-density overturning streamfunction developed by Nurser and Lee (2004). Using the depth-density formalism obviates the need to specify an explicit geometry for our streamfunction. Since our model employs a linear equation of state that depends only on temperature, there is a one-to-one correspondence between density and temperature. Henceforth, we will refer to these as depth-temperature streamfunctions. The streamfunctions presented in this section were all calculated from the simulations with a horizontal resolution of one eighth of a degree and a meridional overturning circulation.

As described in Nurser and Lee (2004), the depth-temperature streamfunction, $\Psi(z, T)$, is calculated by considering a depth surface at depth z , masking all regions with a temperature greater than T , and integrating the vertical transport through the unmasked portion of the depth surface. This is shown schematically in figure 6.8, which is a portion of figure 6 from Nurser and Lee (2004). As well as providing transport streamfunctions this method can also be used to calculate the advective heat flux of the flow.

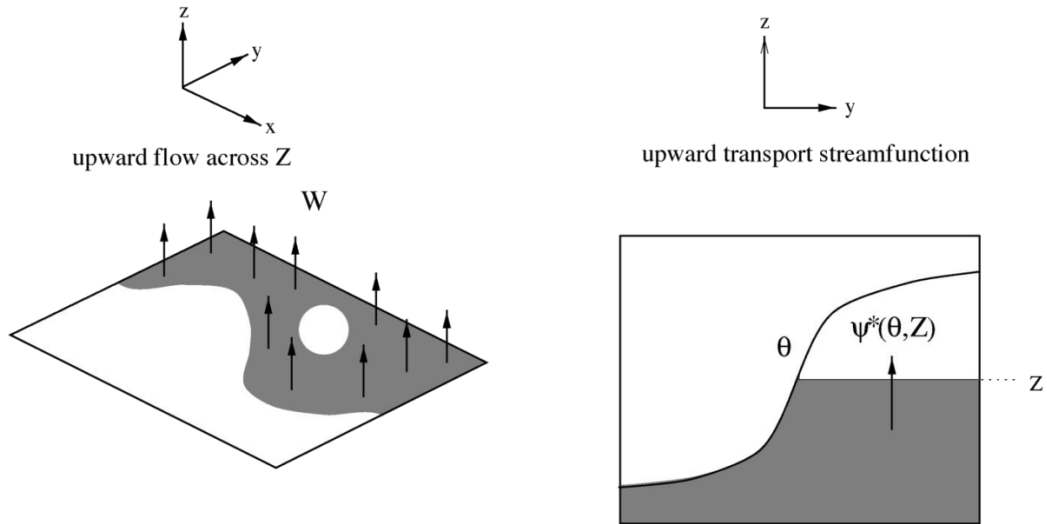


Figure 6.8: A portion of figure 6 from Nurser and Lee (2004) showing how to calculate the depth-temperature streamfunction. The white section of the left hand panel represents the portion of the depth surface with temperature greater than T .

6.4.2.1 Eulerian mean fields

The depth-temperature overturning streamfunction calculated using the Eulerian-mean velocity and temperature fields is shown in figure 6.9. There is evidence of two circulations superimposed on top of each other. The blue lobes, rotating counter-clockwise, represent a thermally direct meridional overturning circulation driven by dense water formation in the north of the domain and upwelling in the sponge at the southern boundary along the imposed temperature profile shown by the blue line. This meridional overturning circulation spans the entire domain and will henceforth be referred to as the MOC. The red lobes, which rotate clockwise, show fluid sinking in the 15-20 °C range corresponding to the surface temperatures in the subtropical gyre, and upwelling at colder temperatures. The location of the downwelling, the depth, and the strength all suggest that the red lobe at 15 °C is part of an Ekman cell.

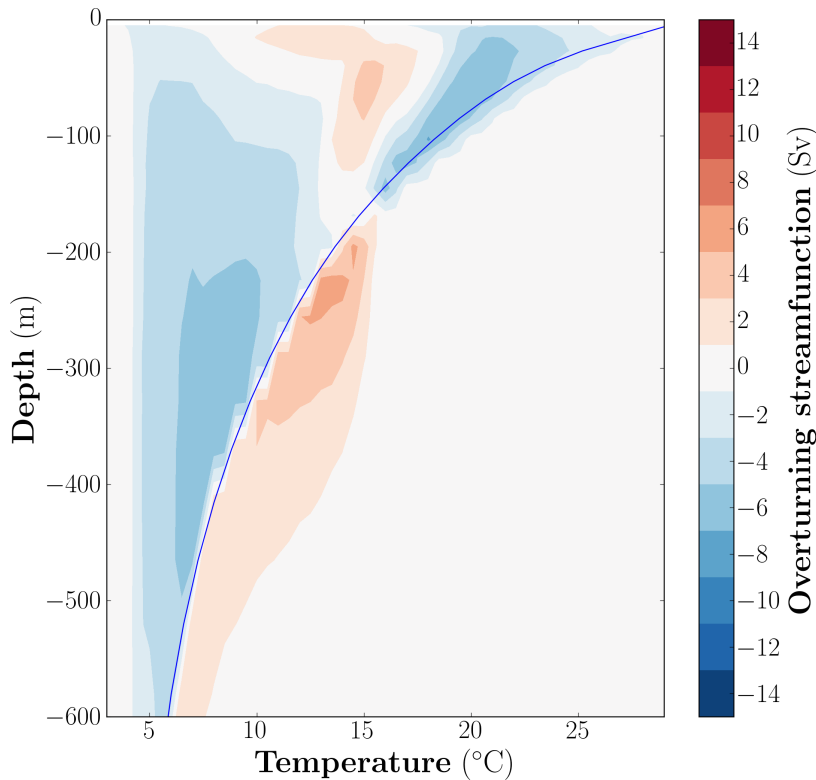


Figure 6.9: Depth-temperature overturning streamfunction calculated from Eulerian-mean fields. The red lobes rotate clockwise, the blue ones counter-clockwise. The blue line represents the temperature profile in the sponge region, and is thus the profile along which we expect upwelling in the sponge region.

Figure 6.9 excludes the deep ocean below 600 m depth. The circulation below this

depth is less interesting, with the streamfunction closing at around 1500 m depth.

6.4.2.2 Time mean of instantaneous overturning

The Lagrangian particle tracking results presented in section 6.3 suggest that the overturning streamfunction would look very different if it were calculated using the instantaneous fields, and then averaged. The time-mean of the depth-temperature overturning streamfunction calculated from ten years of model snapshots saved every five days is shown in figure 6.10. The 12 Sverdrup MOC in figure 6.10 is stronger than the circulation of 6 Sverdrups from figure 6.9. The overturning streamfunction calculated from the instantaneous fields is also more adiabatic; the water mass transformations occur primarily at the surface or in the southern sponge region.

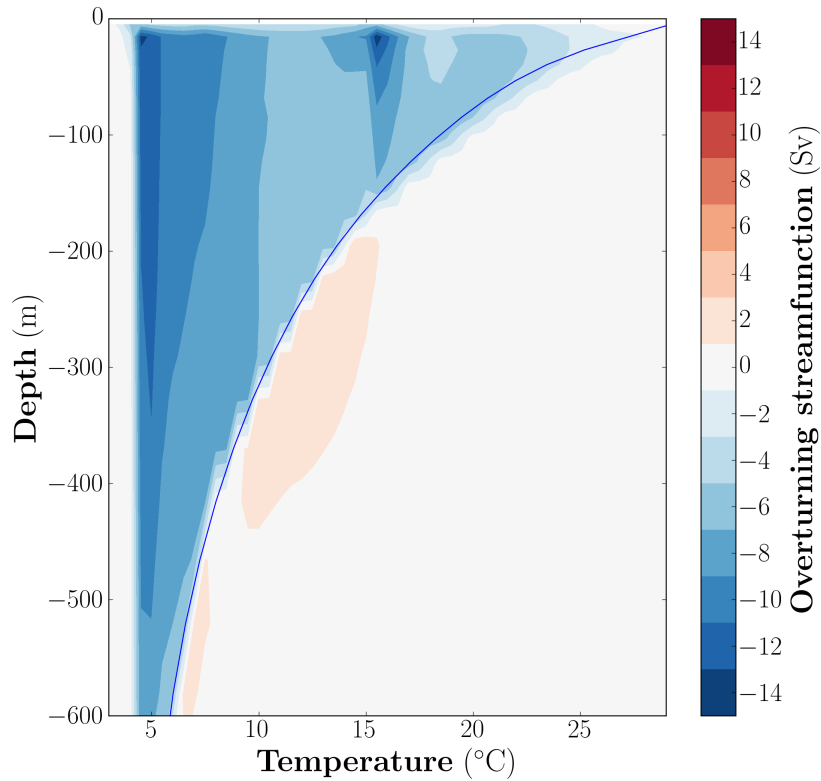


Figure 6.10: Depth-temperature overturning streamfunction calculated from instantaneous velocity and temperature fields. The red lobes rotate clockwise, the blue ones counter-clockwise. The blue line represents the temperature profile in the sponge region, and is thus the profile along which we expect upwelling in the sponge region.

Since the only difference between figures 6.9 and 6.10 is the time varying component, this provides a way to quantify the rectified transport due to eddies. We define our eddy-driven depth-temperature overturning streamfunction as

$$\Psi_{\text{eddy}} = \overline{\Psi_{\text{inst}}} - \Psi_{\text{mean fields}}, \quad (6.4)$$

where the overbar again represents a time-average operator, and $\overline{\Psi_{\text{inst}}}$ is calculated at each time level in the saved data, and then averaged. The eddy-driven overturning streamfunction is plotted in figure 6.11, which shows that the eddy-driven overturning circulation is particularly strong in the temperature and depth ranges found in the subtropical gyre.

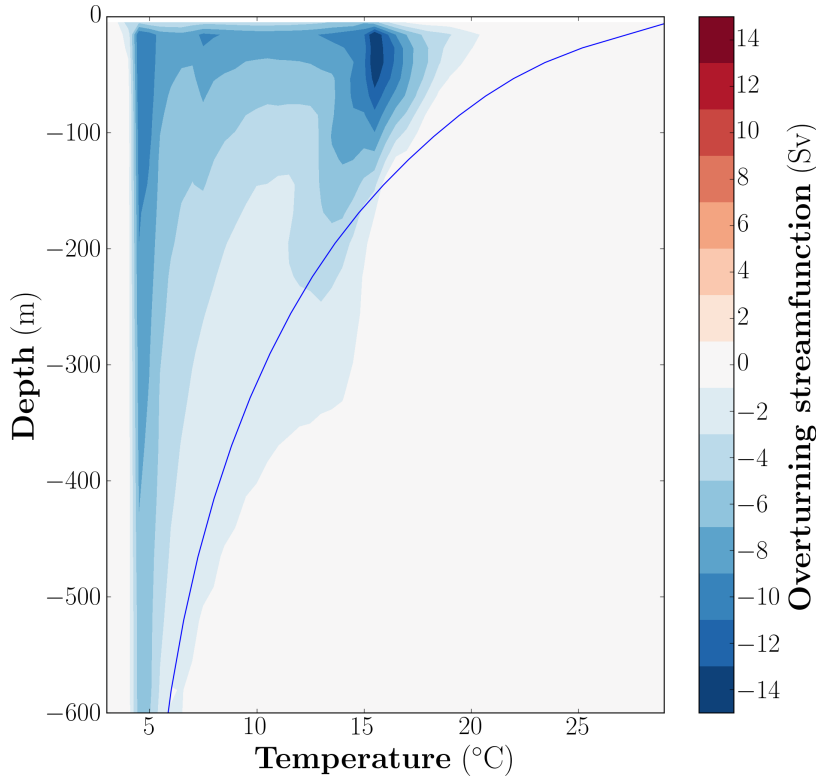


Figure 6.11: Eddy contribution to the depth-temperature overturning streamfunction as defined by equation (6.4). Blue lobes rotate counter-clockwise. The blue line represents the temperature profile in the sponge region, and is thus the profile along which we expect upwelling in the sponge region.

The vertical structure of eddy-induced circulation in the subtropical gyre, shown by the minimum at 15°C in figure 6.11, is similar to the vertical structure of the eddy-induced meridional overturning circulation found by Morrison and Hogg (2013) in their idealised Southern Ocean model. Both circulations show a near surface maximum and a decrease in magnitude with depth. However, the vertical scale of the eddy-induced circulation identified by Morrison and Hogg (2013) is substantially

larger than the circulation shown in figure 6.11. This is likely due to differences in stratification between the models.

We expect the horizontal Ekman transport to be confined to the Ekman layer near the surface of the model. The eddies are also surface intensified, but penetrate deeper into the fluid. Thus we expect the eddy-induced circulation to penetrate deeper than the Ekman layer and to cancel the Eulerian-mean circulation at a depth between the bottom of the Ekman layer and the top of the subtropical thermocline. The vertical structure of the eddy-induced circulation in figure 6.11 supports this speculation.

The plots shown in figures 6.10 and 6.11 exclude the deep ocean below 600 m depth. The circulation below this depth is less interesting, with the streamfunctions closing between 1000 and 2000 m depth.

6.4.2.3 Masked overturning streamfunctions

The Ekman cell in figure 6.9 is somewhat obscured by the MOC. We can remove the MOC by appropriately masking the domain when calculating the depth-temperature streamfunction. Masking the sponge removes the upwelling component of the MOC. It is less straightforward to mask the downwelling limb of the MOC. We have tested both latitude based criteria, masking northwards of a specified latitude, and temperature based criteria, masking regions with a temperature that is less than a specified cut-off temperature. Neither of these methods was able to completely isolate the Ekman cell from the MOC, but both gave similar results. Since using a threshold latitude is conceptually simpler, we mask the northern- and southernmost ten degrees of latitude. Figure 6.12 shows the Eulerian-mean free-surface elevation (contours) and sea surface temperature (colours) of the masked domain.

Figure 6.13 shows the depth-temperature overturning streamfunction calculated excluding the southern- and northern-most ten degrees of latitude. In the masked streamfunction the two red lobes from figure 6.9 are now connected and form a single, thermally indirect, circulation. The masked streamfunction does not close because we are considering an open domain. We have truncated the streamfunction at the dashed line in figure 6.13, which represents the maximum temperature in our model at each depth. The 10 Sv of sinking in the 15-20°C range is consistent with the expected value of Ekman pumping over the subtropical gyre.

Figure 6.14 shows the masked instantaneous depth-temperature overturning streamfunction. The sinking in the subtropical gyre now occurs in a much smaller range of temperatures at approximately 15 °C and is still consistent with the expected 10

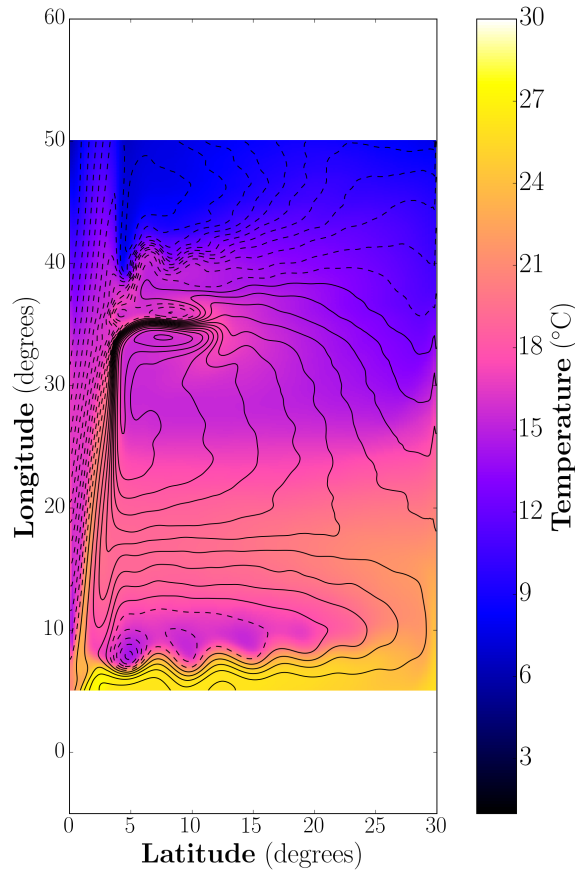


Figure 6.12: *Sea surface temperature (colours) and free-surface height (contours, with a contour interval of 5 cm) of the masked domain. Excluding the northern- and southernmost ten degrees of latitude removes most of the MOC from the overturning streamfunction diagnostics.*

S_v of Ekman pumping from our wind-stress curl. However, as shown by this streamfunction, the Ekman pumping is unable to penetrate deeper than 200 m below the surface. The other feature to note is that this circulation is thermally direct; cold water is downwelled, and warmer water is upwelled. In this framework, the eddy-induced overturning is stronger than the Eulerian-mean overturning, leading to the subduction of colder water and the upwelling of warmer water.

The eddy contribution to the masked depth-temperature streamfunction is shown in figure 6.15. Comparing figures 6.11 and 6.15 shows that the eddy contribution centred at approximately 15°C does not occur in the northern- or southernmost ten degrees of latitude. The eddy contribution is thus consistent with an eddy-induced circulation that opposes the Eulerian-mean Ekman cell.

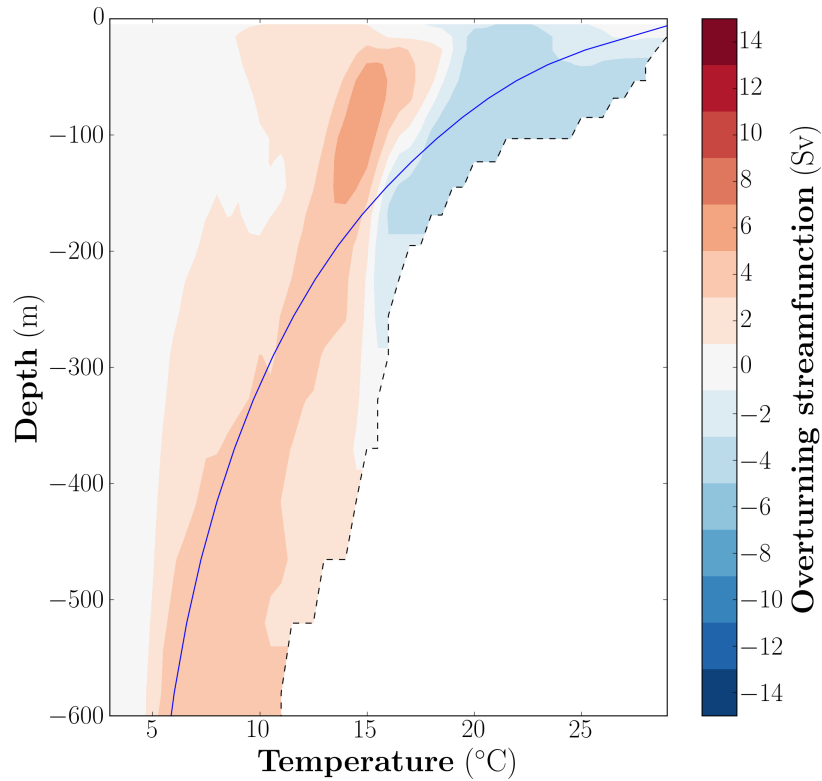


Figure 6.13: *Depth-temperature overturning streamfunction calculated from the masked Eulerian-mean fields. Blue lobes rotate counter-clockwise. The blue line represents the temperature profile in the sponge region, and is thus the profile along which we expect upwelling in the sponge region. Since the streamfunction is calculated in an open domain, it is not constrained to close. Hence, the streamfunction has been truncated at the dashed line, which represents the maximum temperature in our model at each depth.*

6.5 Concluding remarks

The results of our Lagrangian particle tracking experiments show that the rectified effect of eddies acts to suppress Ekman-driven downwelling. Advecting our particles with the velocity fields that include the effects of eddies, either the residual velocity in the case of parametrised eddies or the time-varying velocities in the eddy-resolving simulation, leads to those particles remaining in the mode water of our subtropical gyre. In comparison, the particles advected without the effect of eddies, using the model velocities in the coarse simulation or Eulerian-mean velocities in the eddy-resolving case, are downwelled through the thermocline. The average vertical velocity of these particles is initially comparable with the Ekman pumping velocity computed from the wind-stress curl, but as they are downwelled the vertical motion slows. In the one-eighth of a degree simulation, the vertical velocity of the centroid is approximately

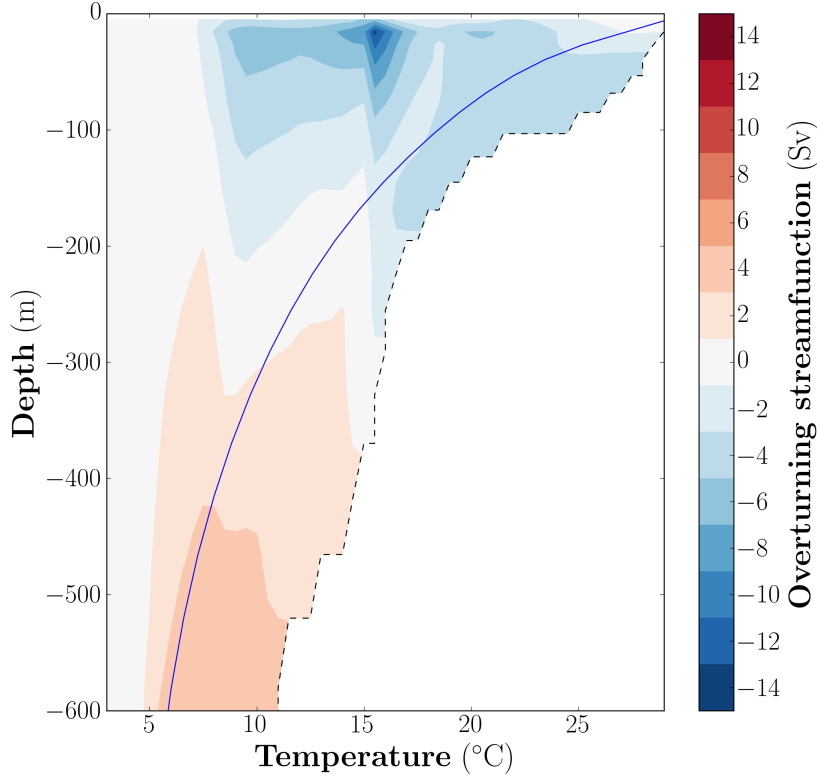


Figure 6.14: Time-mean of depth-temperature overturning streamfunction calculated from the masked instantaneous fields. Blue lobes rotate counter-clockwise. The blue line represents the temperature profile in the sponge region, and is thus the profile along which we expect upwelling in the sponge region. Since the streamfunction is calculated in an open domain, it is not constrained to close. Hence, the streamfunction has been truncated at the dashed line, which represents the maximum temperature in our model at each depth.

half of the gyre averaged Ekman pumping velocity. The trajectory of the particles in the full velocity fields supports our hypothesis that there exists an eddy-induced circulation which opposes the Eulerian-mean downwelling. The experiment using a coarser resolution version of our model that included a Gent-McWilliams mesoscale eddy parametrization (Gent and McWilliams, 1990; Gent et al., 1995) indicates that the GM mesoscale eddy parametrization is able to mimic the eddy dynamics that are responsible for this result.

We have found that the eddy-driven upwelling is able to suppress large-scale Ekman driven downwelling in subtropical gyres. Our depth-temperature overturning streamfunctions indicate that an eddy-driven circulation opposes the Eulerian-mean Ekman cell and prevents the downwelled fluid from penetrating deeper than 200 m below the surface.

The main conclusion of this chapter is that eddies play a substantial role in setting

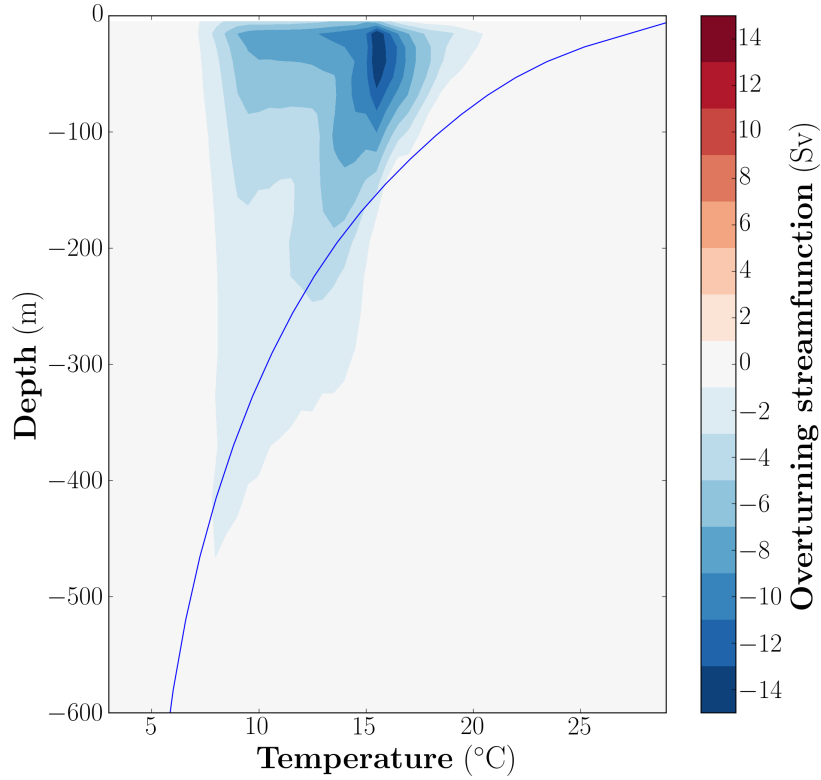


Figure 6.15: *Eddy contribution to the masked depth-temperature overturning streamfunction as defined by equation (6.4). Blue lobes rotate counter-clockwise. The blue line represents the temperature profile in the sponge region, and is thus the profile along which we expect upwelling in the sponge region.*

the residual overturning circulation within wind-driven gyres, which have long been described with time-mean linear theories. While the idealised nature of our model may have enhanced this effect and contributed to the near-complete cancellation of the Ekman-driven downwelling, we expect some level of cancellation to be a robust result. Near-complete cancellation between eddies and the Eulerian-mean flow is consistent with the results of Griffies et al. (2015), who found a balance between vertical heat fluxes from Eulerian-mean advection and eddy advection throughout their eddy-resolving global model.

Although we show a compensation between an eddy-driven overturning circulation and an Eulerian-mean circulation, we wish to highlight that this effect is subtly different to the concept of “eddy compensation.” Within the Southern Ocean literature, the term “eddy compensation” is used to refer to the dynamic change of the eddy-driven overturning circulation in response to a change in the wind stress (Viebahn and Eden, 2010). The eddy-driven overturning circulation partially compensates for

changes in the surface Ekman transport (Morrison and Hogg, 2013). Eddy compensation is part of the dynamic adjustment of the residual overturning circulation towards a new equilibrium. Unlike eddy compensation, the phenomenon discussed in this paper is an equilibrium balance between an Eulerian-mean circulation and an eddy-induced circulation. To emphasize the difference between the dynamic concept of eddy compensation and the equilibrium concept discussed here, we suggest “*eddy cancellation*” as the name for the phenomenon discussed in this chapter.

Sverdrup balance predicts that the horizontal gyre transport will increase with the wind-stress curl. Within the Southern Ocean literature, there are numerous studies showing that the presence of eddies in models of the Southern Ocean reduces the sensitivity of the residual meridional overturning circulation to changes in wind stress (see e.g. Abernathey et al., 2011; Hallberg and Gnanadesikan, 2006; Henning and Vallis, 2005; Munday et al., 2013; Viebahn and Eden, 2010). It may therefore be of interest to investigate how the Eulerian-mean downwelling and eddy-driven upwelling in eddy-resolving models of subtropical gyres respond to changes in the wind stress. For example, if the wind stress were suddenly doubled, the Ekman transport would respond quickly, likely leading to a steepening of the thermocline while the eddy response spins up over a longer period of time, possibly up to a decade (see e.g. Davis et al., 2014; Manucharyan and Spall, 2016). However, it is unclear how the equilibrium vertical transport would respond or the exact time-scale over which this equilibrium would occur. Understanding the time-scale of the eddy-driven response should provide insight into the dynamical and biogeochemical response of subtropical gyres to changes in the wind stress.

The balance between Ekman pumping and an eddy-induced circulation presented in this chapter is likely to be relevant to other wind driven gyres. Davis et al. (2014) used an idealised reduced gravity model to show that the accumulation of fresh water in the Beaufort Gyre could be represented as a balance between Ekman pumping increasing the storage and eddy fluxes removing freshwater. Due to the idealised nature of the model used by Davis et al. the eddy fluxes were parametrised as a thickness diffusion. Manucharyan and Spall (2016) built on this work and the ideas presented by Radko and Marshall (2004a) to examine the freshwater content of the Beaufort Gyre in the Arctic using a less idealised model that was able to simulate baroclinic eddies. Manucharyan and Spall also found that the quantity of freshwater stored in the gyre was controlled by a balance between Ekman pumping and eddy fluxes.

The existence of an eddy-driven circulation which opposes the Eulerian-mean Ekman cell would explain the unexpected eddy-driven downwelling in the subpolar gyre found by McGillicuddy et al. (2003). Since our results show that an eddy-driven circulation opposes the Ekman velocity, we expect to see eddy-driven downwelling in the subpolar gyre. Our result may also help to explain the low correlation between maps of biological activity and time-mean Ekman pumping (see e.g. Palter et al., 2005).

This chapter has addressed the second and third key questions set out in the introduction: the Eulerian-mean flow field does provide a misleading description of vertical motion within wind-driven gyres, and the residual vertical motion is small. If large-scale Ekman driven downwelling is effectively cancelled at depth in subtropical gyres, this has substantial implications for our understanding of the nutrient budget of these regions. Ekman-driven downwelling is invoked as a mechanism to remove nutrients from the subtropical gyres (e.g. Jenkins and Doney, 2003; Kähler et al., 2010; McGillicuddy et al., 1998; Williams and Follows, 1998). The presence of a gyre-scale rectified transport opposing this downwelling would allow nutrients to be recycled more easily within subtropical gyres thereby reducing the quantity of new nutrients required to supply the observed levels of productivity. Although Lee and Williams (2000) examined the balance between eddy diffusion, eddy advection, and Ekman transport in the horizontal movement of nutrients, the idea of eddy cancellation in the vertical is a qualitatively different role for eddies than has been previously proposed (e.g. McGillicuddy, 2016; McGillicuddy et al., 2003, 1998). By lowering the required supply of nutrients this could settle the long standing discrepancy between estimates of nutrient supply and export production in subtropical gyres. The biogeochemical implications of these results will be explored in the next chapter.

Nutrient transport in the oligotrophic ocean

The level of biological productivity varies dramatically across the ocean (McClain et al., 2004). The surface of the subtropical ocean is dominated by large regions of low nutrient concentration and low productivity. These areas of low nutrient concentration are referred to as oligotrophic. Despite the low productivity in these oligotrophic regions they contribute substantially to globally integrated marine production because they cover a large fraction of the ocean (Jenkins and Doney, 2003).

The nutrient budget of the oligotrophic ocean has been an area of active research for several decades. Jenkins (1982) used observations of tritium and helium to infer oxygen utilisation rates. The results suggested substantially larger downward fluxes of carbon from the upper sunlit region of the ocean, the euphotic zone, than had previously been assumed, and thus also a larger nutrient demand. There have since been many studies analysing nutrient supply and productivity in the oligotrophic oceans in general, and the Sargasso Sea in particular (see e.g. Lee and Williams, 2000; Lévy, 2008; Lévy et al., 2012; Martin, 2003; McClain et al., 2004; McGillicuddy and Robinson, 1997; McGillicuddy et al., 2003, 1998; Oschlies, 2002b; Pelegrí et al., 2006; Williams and Follows, 1998; Williams et al., 2006). Despite these efforts there are still large uncertainties in the nutrient budget of the oligotrophic ocean. In particular, the nutrient flux due to mesoscale eddies moving nutrients upwards into the euphotic zone is poorly constrained.

McGillicuddy and Robinson (1997) suggested that mesoscale eddies may be the dominant mechanism supplying nutrients to the euphotic zone of the oligotrophic ocean, delivering $0.35 \pm 0.10 \text{ mol N m}^{-2} \text{ year}^{-1}$. However, McGillicuddy and Robinson

(1997) used a simplified biogeochemical model that relaxed nutrient concentrations to climatological values below the euphotic zone, so their result is likely an overestimation of the effect of mesoscale eddies. As eddies lift a fluid parcel into the euphotic zone its nutrients are consumed and there is a flux of particulate matter downwards. This represents a diapycnal flux of nutrients downwards through the water column. Oschlies (2002a) noted that for eddies to produce a steady state flux of nutrients into the euphotic zone, there needs to be a process to replenish the nutrient content of the fluid below the euphotic zone. Jenkins and Doney (2003) proposed the “subtropical nutrient spiral” as a solution to this problem. Nutrients within the subtropical gyre are consumed and subsequently remineralised deeper within the fluid column. This fluid is advected into the western boundary current where enhanced diapycnal diffusion moves the nutrients upwards towards the surface and they are returned to the euphotic zone in the gyre. The subtropical nutrient spiral from Jenkins and Doney (2003) is shown in figure 7.1. However, this spiral is contradicted by the particle tracking experiments of Burkholder and Lozier (2014) who found that Lagrangian particles move deeper in the water column as they circulate around the North Atlantic subtropical gyre.

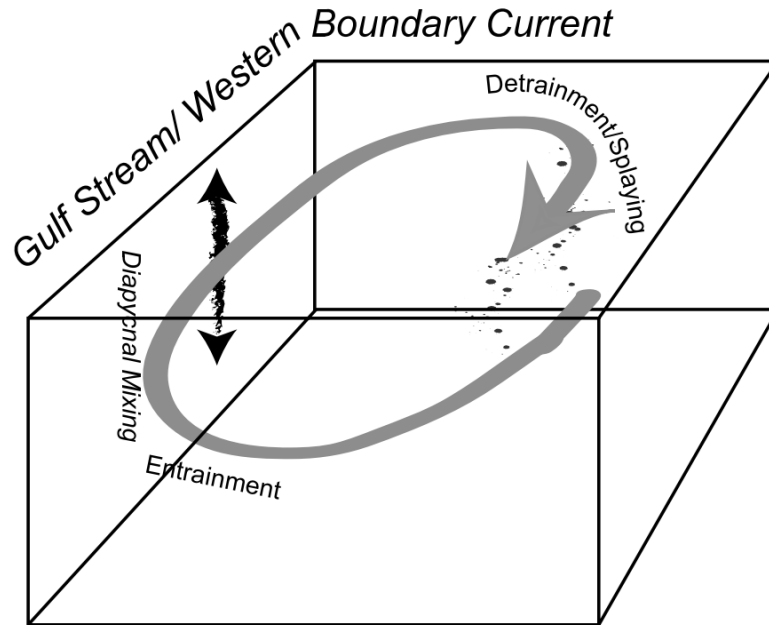


Figure 7.1: *Subtropical nutrient spiral from Jenkins and Doney (2003).*

If the vertical transfer of nutrients by eddies is larger than the replenishment of nutrients at depth, then the effectiveness of eddies will diminish with time. This

effect is shown by Lévy et al. (2012) who run two versions of their model at different horizontal resolutions: one is a mesoscale resolving simulation at $1/9$ th of a degree; the other is a submesoscale permitting simulation at $1/54$ th of a degree. In the submesoscale permitting simulation the eddy activity is more vigorous, but after a transient peak the gyre averaged productivity is lower. This is due to the depletion of nutrients at depth, which reduces the efficiency of vertical eddy transfers. Martin (2003) computed the efficiency of eddy pumping in the subtropical North Atlantic and concluded that it is approximately 40-50 %, implying that previous altimetry based estimates of the eddy contribution to the nutrient budget had substantially overestimated the nutrient supply from eddy pumping.

This chapter builds on the results of the previous chapter, in which it was shown that the rectified effect of eddies opposes Eulerian-mean Ekman pumping, to investigate the implications of this result on nutrient supply in subtropical gyres.

7.1 Biological productivity in a subtropical gyre

Eddy cancellation of the Ekman cell suggests that there should be little or no residual transport downwards through the thermocline due to Ekman pumping. This means that the only mechanism to move nutrients through the thermocline is the biological pump. Figure 7.2 shows two limits for advective transport in a subtropical gyre: figure 7.2 a) shows the Eulerian-mean vertical transport in which Ekman pumping moves fluid downwards from the euphotic zone into the mode water and through the thermocline; figure 7.2 b) shows vertical transport in the limit with a small residual flow. A small residual flow would mean that nutrients were not transported downwards out of the gyre. Retaining nutrients within the gyre for longer should allow for more efficient recycling of nutrients and support a higher level of productivity for a given flux of nutrients into the gyre.

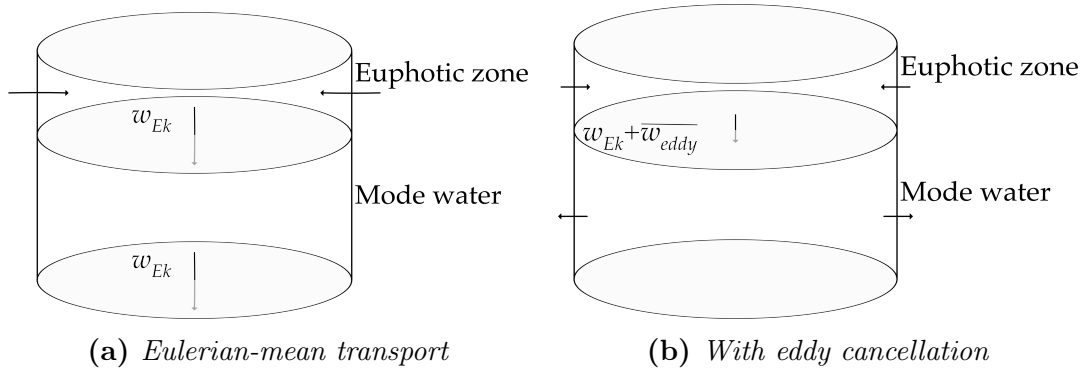


Figure 7.2: Schematic of the two limiting cases for advective transport in a subtropical gyre. If the eddies are ignored, then Eulerian-mean Ekman pumping, w_{Ek} , dominates. If eddy cancellation by the rectified effect of eddies, $\overline{w_{eddy}}$, is included then there is reduced transport from the euphotic zone into the mode water and no advective transport through the base of the mode water.

7.1.1 Hypotheses

Based on the eddy cancellation results presented in the previous chapter we seek to test the following hypotheses in this section.

- Ekman downwelling in the Eulerian mean velocity fields will suppress biological productivity relative to the equivalent simulation with the full velocity fields.
- The simulation with time-varying velocities will retain nutrients within the subtropical gyre more effectively, and therefore sustain higher rates of productivity for longer.

7.1.2 Nutrient in the mode water

Previous results discussed in chapter 6 indicated that the rectified effect of mesoscale eddies opposes Eulerian-mean downwelling due to Ekman pumping. To test the implications of this on gyre-scale biological activity, I perform two idealised simulations using a simplified biogeochemical model. In one simulation the velocities are allowed to vary and evolve with time as the simulation progresses. In the other simulation the velocities are held fixed at the Eulerian-mean values from the first simulation. The simulations are initialised with nutrients in the mode water. This provides a method to test our hypotheses that eddies will aid nutrient retention and recycling, and lead to higher productivity within the gyre. Our simulations are similar to work presented by Kr  m  ur et al. (2009), who artificially increased the nutrient concentration in the

mode water of their model to explore the role of mode water in regulating productivity. As well as modifying the nutrient content of the mode water, we also alter the velocity fields for one of the simulations to explore the impact of the Ekman cells on the biogeochemical response. The integral of net community production within the gyre will be used to compare the different simulations.

The nutrient is initially located in the subtropical mode water, which is identified using a temperature criterion. The nutrient is restricted to waters between 15°C and 16°C. We also use depth and latitude criteria to ensure that the nutrient is placed only in the subtropical gyre. The initial conditions for the simulation using the Eulerian-mean velocity and temperature fields are shown in figure 7.3.

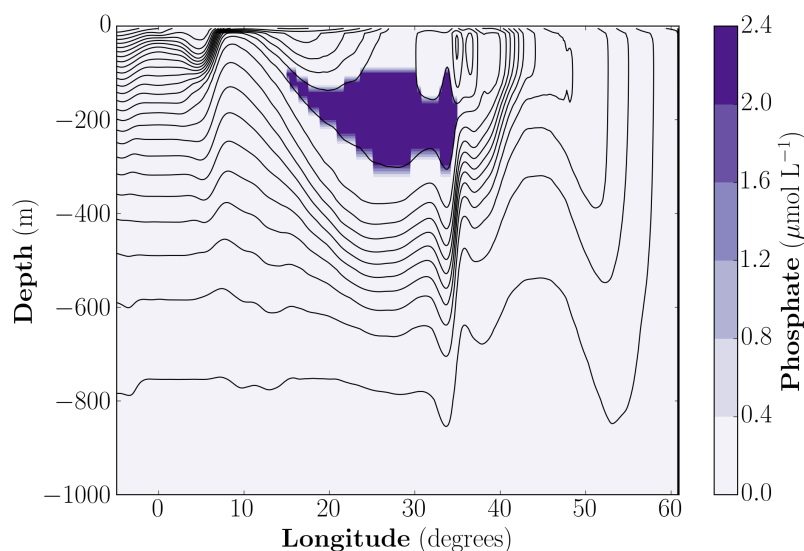


Figure 7.3: *Initial conditions for the biogeochemical simulation using the Eulerian-mean fields. The contours represent temperature with a contour interval of 1 °C and the colours represent phosphate concentration.*

The initial conditions for the time-varying simulation are shown in figure 7.4. The nutrient location was computed using the same criteria as for the Eulerian-mean simulation. Because of differences between the instantaneous snapshot and the Eulerian-mean fields the initial conditions are equivalent but not identical.

A comparison of the two runs is shown in figure 7.5. As expected the simulation with time-varying velocities has much higher productivity, and, even when normalised to the same peak productivity as the Eulerian-mean simulation, the productivity of the time-varying simulation decays more slowly.

These biogeochemical simulations provide further evidence for eddy cancellation of the Ekman Cell. In the simulation using the Eulerian-mean velocity fields, the

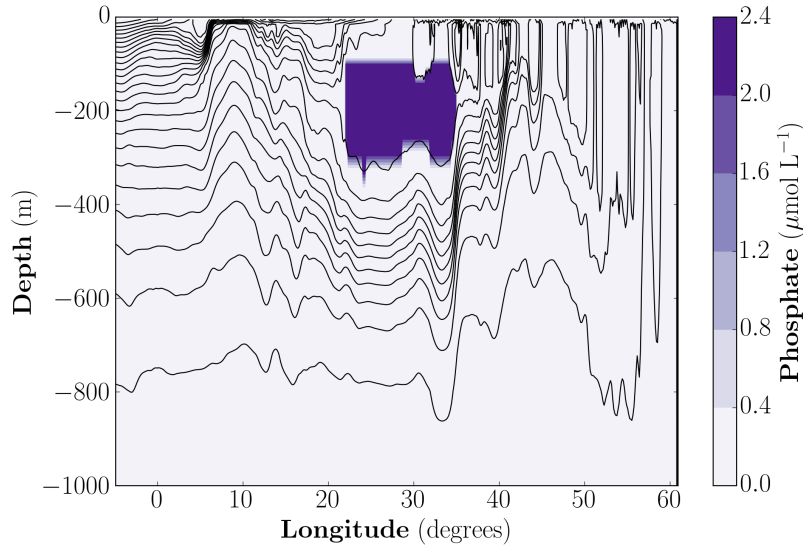


Figure 7.4: *Initial conditions for the biogeochemical simulation using the full velocity and density fields. The contours represent temperature with a contour interval of 1 °C and the colours represent phosphate concentration.*

nutrient is downwelled in the subtropical gyre. The equivalent simulation using the time-varying velocities exhibits a qualitatively different evolution; the nutrient remains in the mode water region for longer and exits the subtropical gyre horizontally near the surface as a result of eddy induced mixing, rather than vertically through the thermocline.

7.2 Nutrients in a cylindrical gyre

When constructing nutrient budgets most previous studies have focussed on budgets for the euphotic zone. However, mode water has been implicated in the nutrient budget, with Palter et al. (2005) even suggesting that mode water was the dominant control on the system. Subtropical mode water is considered to be an interannual store of nutrients and heat (Fratantoni et al., 2013). Autonomous drifters designed to follow the 18.5°C isotherm released by Fratantoni et al. (2013) in the North Atlantic subtropical gyre remained within the mode water for at least 2.5-3.5 years, confirming the possibility that mode water could act as an interannual store of nutrients and heat. The results of the previous chapter, in which the Ekman cell was largely cancelled by an eddy-induced circulation, suggest that Ekman pumping should not be regarded as a sink of nutrients if the entire water mass above the thermocline is considered. The results also suggest that the mode water could act as a reservoir of nutrients

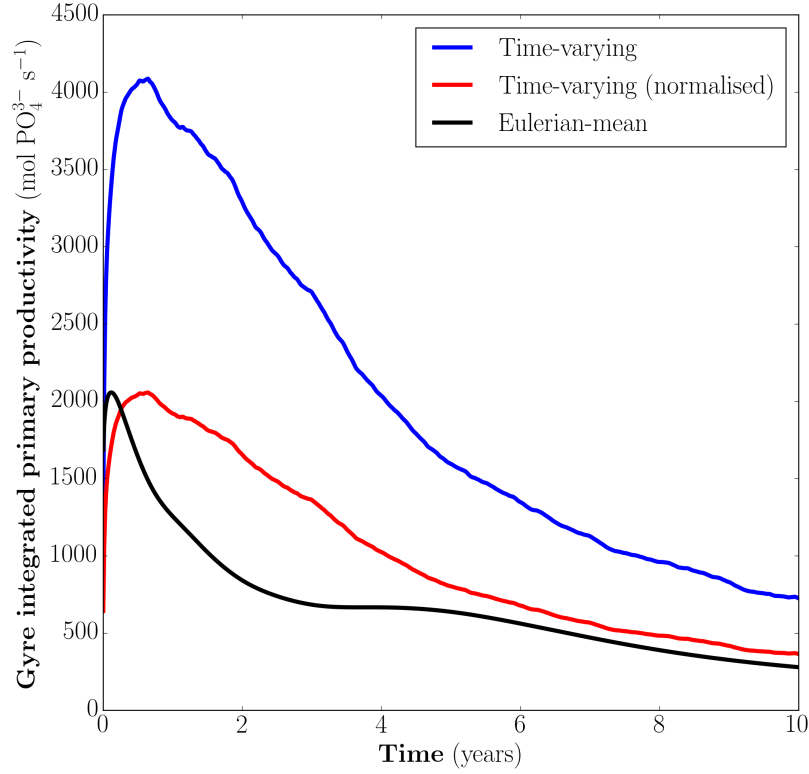


Figure 7.5: Comparison of the gyre integrated biological activity in the two simulations. The time-varying simulation has much higher productivity, and even when normalised to the maximum of the Eulerian-mean simulation, the higher productivity persists through time.

with eddies bringing these nutrients up into the euphotic zone where phytoplankton can consume them. Johnson et al. (2010) examined the nutrient content of the upper 250 m of the water column in the North Pacific subtropical gyre and found that the nutrient supply was approximately equivalent to the nutrient demand over seasonal timescales.

Therefore, we seek to model the entire gyre system with an idealised two layer model. The two layers consist of a euphotic zone, and below that a mode water layer. Any nutrients that are transported downwards through the base of the mode water layer are considered to be lost from the model.

The idealised model is based on the advection-diffusion equation. As discussed in section 2.3, the generic advection-diffusion equation for a tracer C in a non-divergent flow field is

$$\frac{\partial C}{\partial t} + \nabla \cdot (\mathbf{u}C) = \nabla \cdot (\kappa \nabla C) + S_C, \quad (7.1)$$

where S_C represents sources and sinks of the tracer C . In cylindrical coordinates this becomes

$$\begin{aligned} \frac{\partial C}{\partial t} + \frac{1}{r} \frac{\partial r u_r C}{\partial r} + \frac{1}{r} \frac{\partial u_\phi C}{\partial \phi} + \frac{\partial u_z C}{\partial z} = & \frac{1}{r} \frac{\partial}{\partial r} \left(\kappa r \frac{\partial C}{\partial r} \right) \\ & + \frac{1}{r} \frac{\partial}{\partial \phi} \left(\kappa \frac{\partial C}{\partial \phi} \right) \\ & + \frac{\partial}{\partial z} \left(\kappa \frac{\partial C}{\partial z} \right) + S_C, \end{aligned} \quad (7.2)$$

where (u_r, u_ϕ, u_z) are the three components of the velocity vector in the radial, r , azimuthal, ϕ , and vertical, z , directions respectively.

To construct a model of nutrient transport within subtropical gyres we begin by assuming that within these gyres the ocean consists of two layers. The upper layer represents the euphotic zone and contains all of the biological activity. The lower layer represents the weakly stratified mode water of the gyre. Due to the deep penetration of light through the clear waters of the subtropical gyres, the nutricline is generally deeper than the mixed layer (Williams and Follows, 2011, p.268), as can be seen in the Seaglider data collected by Hemsley et al. (2015). This means that nutrients do not have to move vertically through a large density gradient to move from the mode water layer to the euphotic zone.

As a first approximation we will use an axisymmetric model in radial coordinates; that is none of the variables depend on ϕ . This is a substantial idealisation of gyres, since it ignores both western boundary currents and inertial recirculation regions. However, if we restrict our attention to the open ocean regions, where the mean flow is weak and the kinetic energy budget is dominated by eddies (Ferrari and Wunsch, 2009), then the inconsistency between our axisymmetric assumption and the mean circulation is less problematic. Another reason that we do not expect contours of mean dynamic topography and biological activity to align throughout these regions is the presence of wintertime convection in the poleward portions of subtropical gyres, which provides a substantial source of nutrients.

The biological activity in the oligotrophic ocean decreases away from the tropical and subpolar regions and land. Although not circular, the contours of chlorophyll concentration do form concentric rings, as shown in figures 7.6 a) and b), which suggests that our axisymmetric assumption is an acceptable approximation.

The model parametrises many processes that affect the distribution and concentration of nutrients. These are shown schematically in figure 7.7.

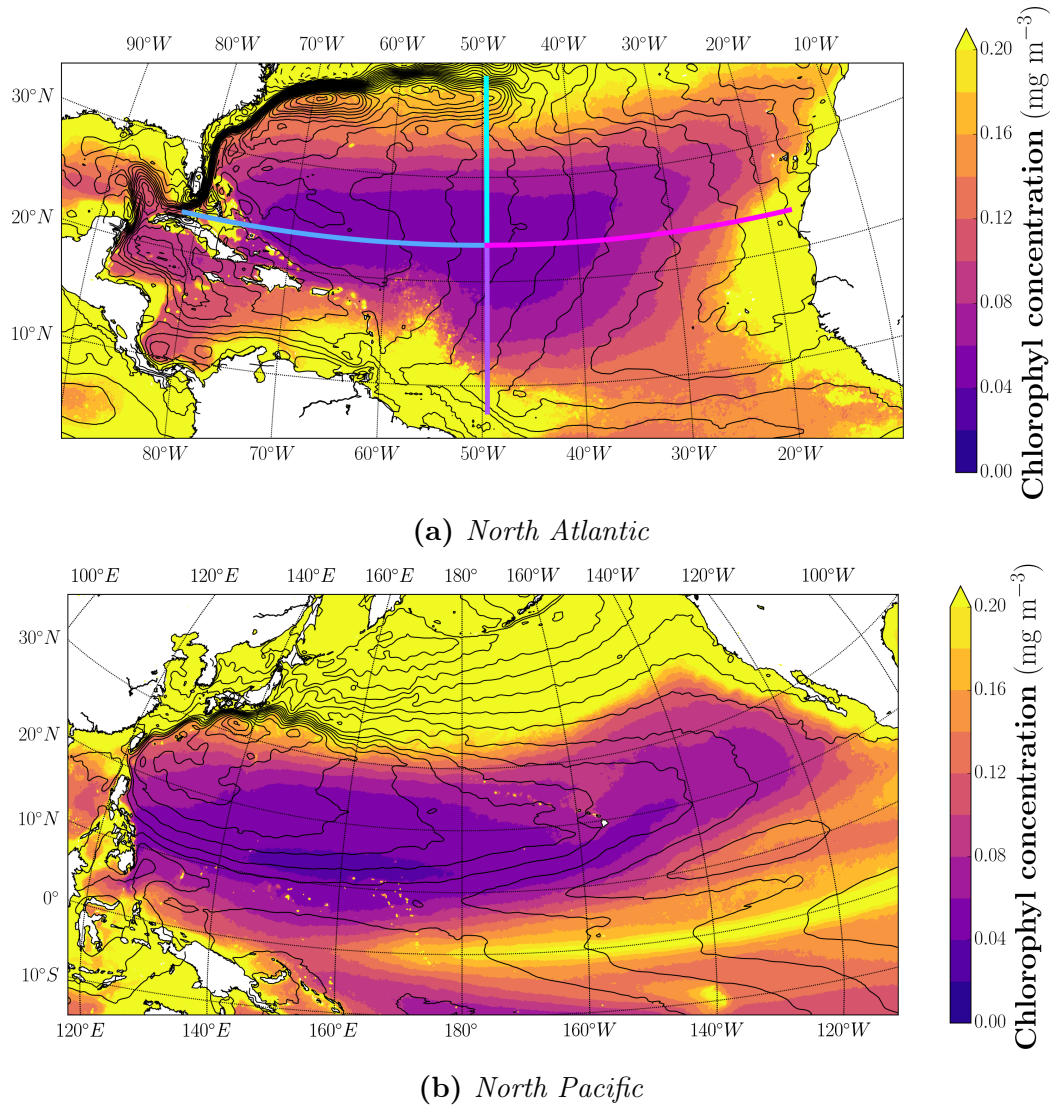


Figure 7.6: Mean chlorophyll concentration in the subtropical North Pacific from MODIS Aqua data (NASA Ocean Biology Processing Group, 2015) spanning January 2004 to January 2016, with contours of mean dynamic topography from CNES CLS09 (Rio et al., 2011) overlaid using a contour interval of 10 cm. The zonal and meridional lines in a) show sections along which data will be extracted for comparison with the idealised modelling results.

If we ignore the rotational velocity associated with the gyre circulation, which is consistent with our assumption that the fields are axisymmetric, then the advective component of the material derivative reduces to advection by the residual vertical velocity between Ekman pumping and the eddy-induced circulation, with a corresponding radial mass flux to enforce conservation of mass. The effect of light limitation on biological productivity will be ignored, since these regions of the ocean are nutrient limited not light limited. Thus, if we begin with the equation for biological

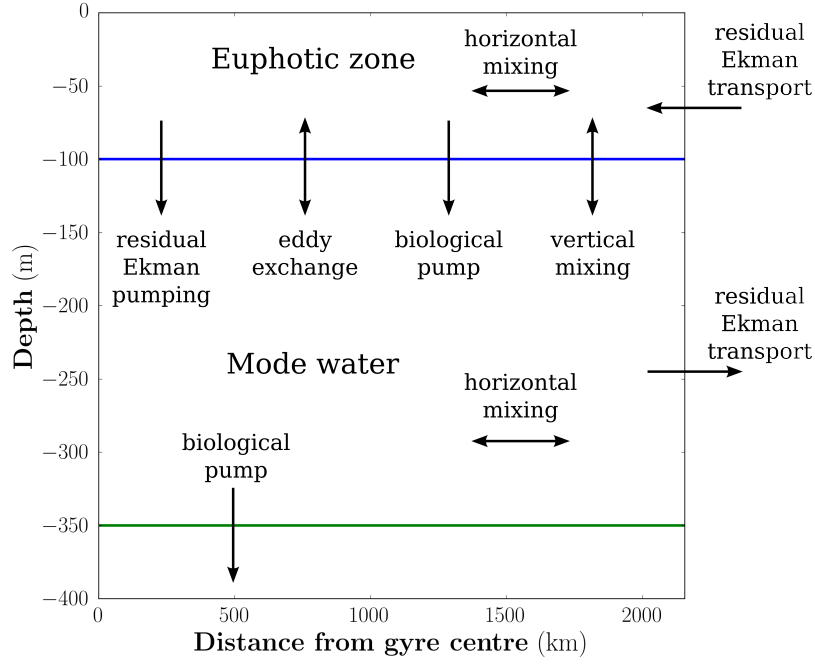


Figure 7.7: Schematic of the cylindrical coordinate model showing the two-layer domain and the processes included in the model.

productivity from the MITgcm biogeochemical package, equation (3.6), we replace the $I/(I + \kappa_I)$ term with the nondimensional constant I_c .

Like the biogeochemical package for the MITgcm, our idealised model parametrises the particulate flux due to biological activity as an instantaneous redistribution of phosphate to depth. At a steady state, this approximation is valid, but for time-varying simulations this amounts to an instantaneous transport of nutrients vertically in the water column. Hence, we run this idealised model to a steady state for the analysis presented here.

Combining these terms and the other processes shown in figure 7.7, the equation for the concentration of phosphate in the euphotic zone is

$$\begin{aligned}
h_{euph} \frac{\partial [PO_4]}{\partial t} = & \frac{1}{r} \frac{\partial}{\partial r} \left(\kappa_h r h_{euph} \frac{\partial [PO_4]}{\partial r} \right) - \kappa_z \left(\frac{[PO_4] - [PO_{4mode}]}{h_{euph}} \right) \\
& - \frac{1}{2r} \frac{\partial ((w_{Ek} + \overline{w_{eddy}}) r^2 [PO_4])}{\partial r} + (w_{Ek} + \overline{w_{eddy}}) [PO_4] \\
& - \alpha h_{euph} I_c \frac{[PO_4]}{[PO_4] + \lambda_{PO_4}} + h_{euph} \gamma_{remin} [DOP] \\
& + w_{rms} A_{eddy} ([PO_{4mode}] - [PO_4]),
\end{aligned} \tag{7.3}$$

in which h_{euph} is the thickness of the euphotic zone, $[PO_4]$ is the vertically averaged concentration of phosphate in the euphotic zone, κ_h is the horizontal diffusivity and parametrises the horizontal mixing effects of eddies, κ_z is the vertical diffusivity, $[PO_{4mode}]$ is the vertically averaged concentration of phosphate in the mode water layer, w_{Ek} is the Eulerian-mean Ekman pumping velocity, $\overline{w_{eddy}}$ is the representative vertical velocity from eddy cancellation which opposes Ekman pumping, α is the maximum net community production, I_c represents the impact of light limitation and is set to one for the simulations in this thesis, λ_{PO_4} is the half saturation concentration for phosphate, γ_{remin} is the remineralisation timescale for dissolved organic phosphate, $[DOP]$ is the vertically averaged concentration of dissolved organic phosphate in the euphotic zone, w_{rms} is the representative vertical velocity for eddy exchange between the two layers, and A_{eddy} is the fraction of the ocean covered by eddies. The vertical mixing term represents the diffusive flux of phosphate between the two layers, hence it is a first derivative rather than a second derivative. As can be inferred from equation (7.3) the impact of eddies is parametrised in three ways in this idealised model: eddy cancellation is included through modification of the residual Ekman pumping velocity by $\overline{w_{eddy}}$, horizontal stirring by eddies is parametrised through the horizontal mixing term, and the eddy pumping mechanism of McGillicuddy and Robinson (1997) and McGillicuddy et al. (2003, 1998) is parametrised as a vertical exchange of fluid between the two layers. Thus, our toy model includes three eddy effects; lateral mixing, vertical exchange, and suppression of Eulerian-mean Ekman pumping. Since these three terms are each parametrising a different effect of the same eddy field it is worth considering whether all three are required. A large part of the motivation for this model is that our conceptual models of the nutrient budget of oligotrophic gyres do not generally provide closed budgets, and are therefore unsatisfactory. This model seeks to improve these conceptual nutrient budgets through the inclusion of a heretofore excluded process, eddy cancellation. Because both eddy cancellation and eddy pumping involve vertical transport, it may appear that a model with both effects is double counting

the impact of eddies. However, eddy pumping is an intrinsically diabatic process that relies on the vertical motion of isopycnals, while eddy cancellation acts to reduce the adiabatic vertical motion observed in the Eulerian-mean velocity fields. These two processes are distinct and should both be represented. A key argument of this chapter is that previous calculations which did not include eddy cancellation misrepresented the effect of eddies on the nutrient budget of the oligotrophic ocean.

The equation for the concentration of phosphate in the mode water layer is structurally similar, and has the form

$$\begin{aligned}
 h_{mode} \frac{\partial [PO_{4mode}]}{\partial t} = & \frac{1}{r} \frac{\partial}{\partial r} \left(\kappa_h r h_{mode} \frac{\partial [PO_{4mode}]}{\partial r} \right) + \kappa_z \frac{[PO_4] - [PO_{4mode}]}{h_{euph}} \\
 & + \frac{1}{2r} \frac{\partial ((w_{Ek} + \overline{w_{eddy}}) r^2 [PO_{4mode}])}{\partial r} - (w_{Ek} + \overline{w_{eddy}}) [PO_4] \\
 & + (1 - f_{DOP}) \alpha h_{euph} I_c \frac{[PO_4]}{[PO_4] + \lambda_{PO_4}} \left(1 - \left(\frac{h_{mode} + h_{euph}}{h_{euph}} \right)^{-0.988} \right) \\
 & + h_{mode} \gamma_{remin} [DOP_{mode}] - w_{rms} A_{eddy} ([PO_{4mode}] - [PO_4]), \tag{7.4}
 \end{aligned}$$

in which $(1 - f_{DOP}) \alpha h_{euph} I_c \frac{[PO_4]}{[PO_4] + \lambda_{PO_4}} \left(1 - \left(\frac{h_{mode} + h_{euph}}{h_{euph}} \right)^{-0.988} \right)$ represents the particulate flux of organic matter through the base of the mode water layer using the empirical power law derived by Martin et al. (1987), and all other symbols have the same meanings as before.

These two equations are coupled with each other, and with two similar equations for dissolved organic phosphate (*DOP*). The equation for the concentration of DOP in the euphotic zone, $[DOP]$, is

$$\begin{aligned}
 h_{euph} \frac{\partial [DOP]}{\partial t} = & \frac{1}{r} \frac{\partial}{\partial r} \left(\kappa_h r h_{euph} \frac{\partial [DOP]}{\partial r} \right) - \kappa_z \left(\frac{[DOP] - [DOP_{mode}]}{h_{euph}} \right) \\
 & - \frac{1}{2r} \frac{\partial ((w_{Ek} + \overline{w_{eddy}}) r^2 [DOP])}{\partial r} + (w_{Ek} + \overline{w_{eddy}}) [DOP] \\
 & + f_{DOP} \alpha h_{euph} I_c \frac{[PO_4]}{[PO_4] - \lambda_{PO_4}} \\
 & - h_{euph} \gamma_{remin} [DOP] \\
 & + w_{rms} A_{eddy} ([DOP_{mode}] - [DOP]), \tag{7.5}
 \end{aligned}$$

while the equation for the concentration of DOP in the mode water layer, $[DOP_{mode}]$, is

$$\begin{aligned}
h_{mode} \frac{\partial [DOP_{mode}]}{\partial t} = & \frac{1}{r} \frac{\partial}{\partial r} \left(\kappa_h r h_{mode} \frac{\partial [DOP_{mode}]}{\partial r} \right) + \kappa_z \left(\frac{[DOP] - [DOP_{mode}]}{h_{euph}} \right) \\
& + \frac{1}{2r} \frac{\partial ((w_{Ek} + \overline{w_{eddy}}) r^2 [DOP_{mode}])}{\partial r} - (w_{Ek} + \overline{w_{eddy}}) [DOP] \\
& - h_{mode} \gamma_{remin} [DOP_{mode}] \\
& - w_{rms} A_{eddy} ([DOP_{mode}] - [DOP]).
\end{aligned} \tag{7.6}$$

Together, these four equations form a set of coupled non-linear ordinary differential equations that can be solved numerically. The solutions are radial profiles of phosphate and DOP in the euphotic zone and mode water layer. At the centre of the gyre a zero-gradient boundary condition is enforced on all concentrations. At the gyre edge the concentration of phosphate in the euphotic zone is held at a fixed value chosen from climatological maps of phosphate concentration (Garcia et al., 2013). For phosphate in the mode water layer and dissolved organic phosphate in both layers neither the concentration nor the radial gradient are prescribed at the gyre edge; the boundary value is obtained each time step by extrapolating one grid point beyond the edge of the gyre using the gradient between the final two grid points. This allows the model to set the value and flux of these tracers at the boundary in a dynamically self-consistent manner.

The euphotic zone is assumed to have constant thickness in both time and space. A thickness of 100 m is used as the default value, which is based on previous modelling studies and observations (Hemsley et al., 2015; Lévy et al., 2012; Martin, 2003; Oschlies, 2002a; Oschlies and Garçon, 1998; Williams and Follows, 1998). The mode water layer is assumed to have a constant thickness of 250 m.

The idealised geometry of the model means that there is a residual Ekman transport in to the domain in the euphotic zone, and that all of this fluid is advected out through the edge of the domain in the mode water layer. This means that everywhere in the mode water layer there is a horizontal velocity directed outwards. In reality the convergence of fluid due to residual Ekman transport could be balanced by a range of spatially and temporally varying processes. The inability of our toy model to simulate these processes necessitates the presence of an outwards velocity in the mode water layer. This may lead to the mode water layer being flushed in an unrealistic way and reduce its capacity to act as a store of nutrients.

Numerically solving this model with the parameter values shown in table 7.1 produces the profiles shown in figure 7.8. The agreement between the model and the

climatological phosphate concentration in the euphotic zone, taken as the average concentration in the upper 100 m, is satisfactory. However, in the mode water layer, taken as depths between 100 - 300 m, there is a noticeable discrepancy between the model and the data. The largest disagreement between the model result and the data in the mode water region occurs for the meridional section going south from the gyre centre (shown in purple). This disagreement may be due to the fact that our two layer assumption is no longer valid in this region, since the mode water layer becomes thinner and less distinct moving south of the gyre centre, so that much of the water between 100 m and 300 m is qualitatively different from subtropical mode water. The best agreement is with the data going polewards from the gyre centre, which represents the direction in which our two layer assumption is the most valid.

It would be possible to modify the model such that a boundary value for dissolved organic phosphate in the euphotic zone must be specified. If the specified value is similar to the value which the model would have solved for, then this boundary condition will have only a small impact on the solution. If the specified value is substantially larger, then it will increase the flux of DOP in to the domain and result in a slower decrease of nutrient concentration towards the centre of the domain. If the boundary value is substantially smaller, then it will result in smaller DOP fluxes and a much sharper decrease in nutrient concentration near the edge of the domain. Torres-Valdés et al. (2009) found concentrations of dissolved organic phosphate of approximately $0.05 - 0.15 \mu\text{mol L}^{-1}$ in the upper 100 m of the North Atlantic. The solutions shown in figure 7.8 solved for a value of $0.07 \mu\text{mol L}^{-1}$ in the euphotic zone at the edge of the domain. The consistency between the observed values and the model solution suggests that assigning a boundary condition to dissolved organic phosphate in the euphotic zone will not substantially alter the results.

At steady-state the model exhibits a balance between competing processes, as can be seen in the tendency terms shown in figure 7.9. In the euphotic zone, figure 7.9 a), the biological activity consumes nutrients, and is therefore always acting to reduce the concentration of phosphate in the model. The negative tendencies from biological activity and residual Ekman pumping are predominantly opposed by the vertical flux due to eddy exchange, the remineralisation of dissolved organic phosphate, and the horizontal flux from residual Ekman transport. The importance of the residual Ekman transport term near the gyre edge is consistent with the results of Williams and Follows (1998), as is the fact that this term becomes small in the gyre interior. The impact of the horizontal and vertical diffusion terms is substantially smaller.

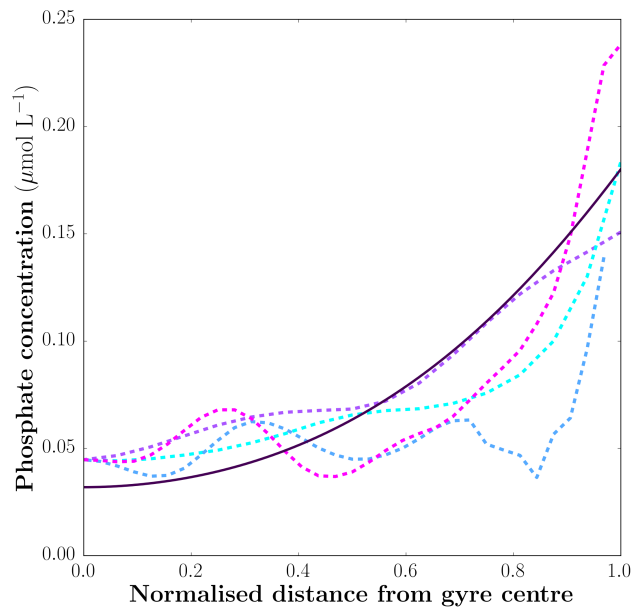
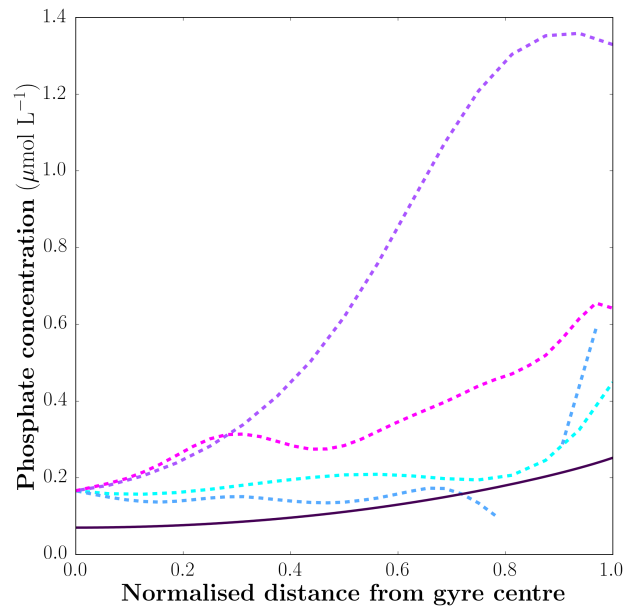
(a) *Euphotic zone*(b) *Mode water layer*

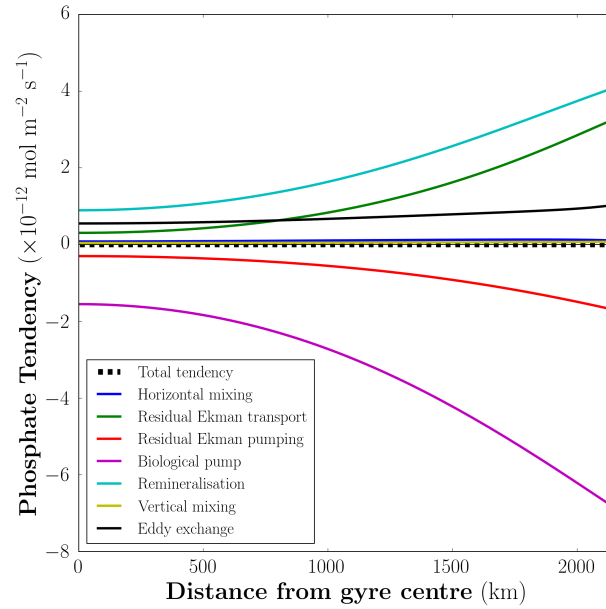
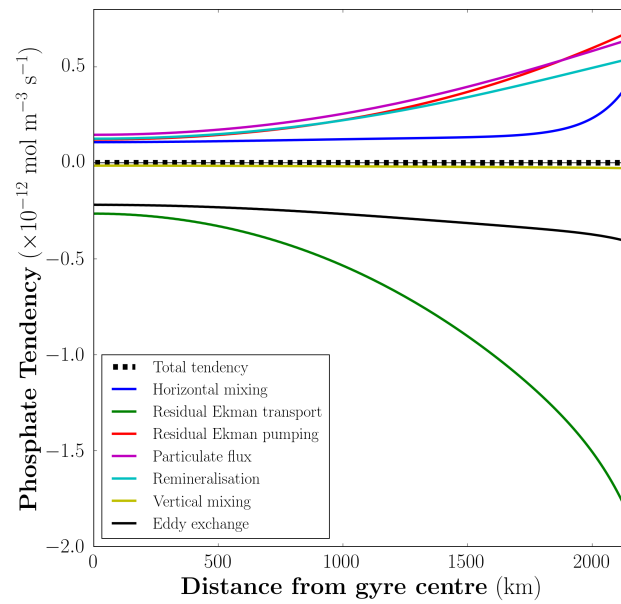
Figure 7.8: Phosphate concentration in the the model (black), compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

Table 7.1: *Parameter values for the two layer advection-diffusion model. The parameters for the biological productivity have been taken from the MITgcm biogeochemical model described in Dutkiewicz et al. (2005). The horizontal eddy diffusion is taken from Marshall et al. (2006). The Ekman pumping velocity is estimated from the scatterometer derived wind stress described in Risien and Chelton (2008) and Risien (2011). The representative eddy velocity and fractional area are taken from Brannigan (2016). The phosphate concentration at the edge of gyre is estimated from the World Ocean Atlas climatology described in Garcia et al. (2013).*

Parameter	Symbol	Value	Units
Depth of euphotic zone	h_{euph}	100	m
Depth of mode water	h_{mode}	250	m
Maximum net community production	α	2×10^{-3}	mol P m ⁻³ year ⁻¹
Fraction of production converted to dissolved organic phosphate (DOP)	f_{DOP}	0.67	
Remineralisation rate for DOP	γ_{remin}	2	years ⁻¹
Phosphate half saturation	λ_{PO_4}	5×10^{-4}	mol m ⁻³
Horizontal diffusion	κ_h	800	m ² s ⁻¹
Vertical diffusion	κ_z	10^{-5}	m ² s ⁻¹
Ekman pumping velocity	w_{Ek}	-80	m year ⁻¹
Eddy cancellation velocity	$\overline{w_{eddy}}$	50	m year ⁻¹
Light limitation coefficient	I_c	1	
Eddy pumping velocity	w_{rms}	2.5	m day ⁻¹
Fractional area covered by eddies	A_{eddy}	0.05	
Phosphate concentration in euphotic zone at gyre edge		0.18×10^{-3}	mol m ⁻³
Gyre radius	R	2.15×10^6	m

The model tendencies in the mode water layer are shown in figure 7.9 b). In the mode water the largest tendency is due to the horizontal advective flux by the residual Ekman transport, which is negative since it is transporting fluid from low nutrient regions near the centre of the gyre to high nutrient regions near the edge of the gyre. The other negative term is due to eddies which are transporting nutrients from the higher concentration mode water to the lower concentration euphotic zone. These two terms are opposed by a combination of processes. The tendencies due to the particulate flux, vertical Ekman transfer, and remineralisation are all positive, and of approximately the same magnitude. The tendency due to horizontal diffusion is also positive, but slightly smaller than the previous three terms. Once again, vertical diffusion makes a small contribution.

When comparing figures 7.9 a) and 7.9 b) it appears as though the horizontal diffusion term makes a much larger contribution in the mode water layer. While the impact of horizontal diffusion is somewhat larger in this layer, the difference is not as

(a) *Euphotic zone*(b) *Mode water layer***Figure 7.9:** *Tendency terms from the idealised nutrient model.*

substantial as these two figures might suggest. Figure 7.10 plots the two horizontal diffusion terms on the same axis, from which it can be clearly seen that although the tendency in the mode water layer is larger, the difference is not as extreme as figures 7.9 a) and 7.9 b) suggest.

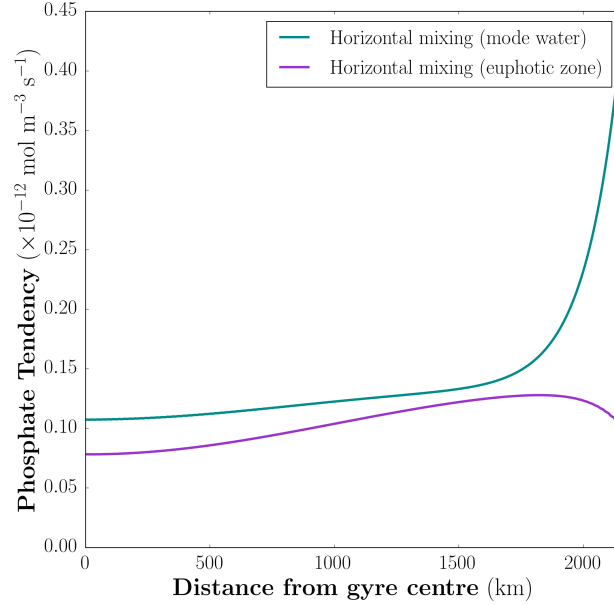


Figure 7.10: *Phosphate tendency due to horizontal diffusion in the idealised model.*

This model does not include the vertical transport of nutrients from the abyss in to the mode water layer through diffusion. This means that the mechanism described by Palter et al. (2005) in which a thicker mode water layer reduces the vertical diffusive flux of nutrients from the abyss to the euphotic zone.

7.2.1 Process driven fluxes in the euphotic zone

The contribution from each process in the model can be analysed to compute a nutrient flux due to that process. These fluxes will be compared with observational and modelling estimates from the literature to assess the validity of this highly idealised model.

7.2.1.1 Residual Ekman pumping

Ekman pumping provides a sink of nutrients from the euphotic zone. The magnitude of the sink depends on the residual Ekman pumping velocity, given by $w_{Ek} + \overline{w_{eddy}}$,

and the nutrient concentration. Averaged over the gyre the loss of nutrients per unit area is

$$\frac{\int_0^{2\pi} \int_0^R (w_{Ek} + \overline{w_{eddy}}) [PO_4] r dr d\phi}{\int_0^{2\pi} \int_0^R r dr d\phi} = 2(w_{Ek} + \overline{w_{eddy}}) \frac{\int_0^R [PO_4] r dr}{R^2}, \quad (7.7)$$

where R is the radius of the gyre, $(w_{Ek} + \overline{w_{eddy}})$ is the residual Ekman pumping velocity, $[PO_4]$ is the concentration of phosphate, r is the radial coordinate, and ϕ is the azimuthal coordinate. Evaluating equation (7.7) for the default parameters gives an estimate of the nutrient sink due to vertical Ekman transfer of -9.40×10^{-11} mol P $m^{-2} s^{-1}$ averaged over the gyre.

7.2.1.2 Residual Ekman transport

The residual Ekman pumping requires a horizontal mass flux to conserve volume in the model. The horizontal velocity in the euphotic zone at radius r is given by the vertical volume flux through a circle of radius r , divided by the area of a surface given by the depth of the euphotic zone multiplied by the circumference of a circle with radius r , which simplifies to

$$\frac{-(w_{Ek} + \overline{w_{eddy}}) \pi r^2}{2\pi r h_{euph}} = \frac{-(w_{Ek} + \overline{w_{eddy}}) r}{2h_{euph}}. \quad (7.8)$$

Therefore the horizontal flux of phosphate through a circle of radius r is given by

$$[PO_4](r) \times \frac{-(w_{Ek} + \overline{w_{eddy}}) r}{2h_{euph}} \times h_{euph} 2\pi r = -[PO_4](r) (w_{Ek} + \overline{w_{eddy}}) \pi r^2. \quad (7.9)$$

Evaluating equation (7.9) at $r = R$ and dividing by the area gives the gyre averaged contribution of the nutrient flux from the residual horizontal Ekman transport. Using the default parameters for the model this is 1.71×10^{-10} mol P $m^{-2} s^{-1}$.

7.2.1.3 Total residual Ekman contribution

Residual Ekman pumping is a sink of nutrients from the euphotic zone, while residual Ekman transport is a source. This shows that the nett contribution of Ekman dynamics to the nutrient budget of the euphotic zone is the residual between two different processes, one of which acts as a sink of nutrients and the other as a source. Summing the two Ekman driven processes yields the total contribution of Ekman

dynamics to the nutrient budget of the euphotic zone of the model. The total contribution is $7.72 \times 10^{-11} \text{ mol P m}^{-2} \text{ s}^{-1}$ or converting from phosphate into nitrate using the Redfield ratio (Redfield et al., 1963; Williams and Follows, 2011), this is equivalent to $1.24 \times 10^{-9} \text{ mol N m}^{-2} \text{ s}^{-1}$. Williams and Follows (1998) evaluated the contribution of horizontal and vertical Ekman transports to the nutrient budget of the subtropical North Atlantic gyre. They estimated a source of $1.2 \times 10^{-9} \text{ mol N m}^{-2} \text{ s}^{-1}$ using the extrapolation method of Glover and Brewer (1988), or $1.5 \times 10^{-9} \text{ mol N m}^{-2} \text{ s}^{-1}$ using the integral method of Glover and Brewer (1988) to calculate mixed layer nitrate concentration. There is a close agreement between the estimates by Williams and Follows (1998) and our toy model.

7.2.1.4 Horizontal mixing

Horizontal mixing provides a flux of nutrients into the euphotic zone from the outer edge of the model domain. This term parametrises the horizontal flux from lateral eddy-driven mixing. The nutrient concentration at the boundary is held fixed, a Dirichlet boundary condition, which means that the diffusive flux into the domain adjusts as the model is integrated.

The domain averaged contribution of the diffusive flux at the edge of the gyre is given by

$$\frac{\kappa_h \left. \frac{\partial [PO_4]}{\partial r} \right|_R \times 2\pi R h_{euph}}{\pi R^2} \quad (7.10)$$

which simplifies to $\kappa_h \left. \frac{\partial [PO_4]}{\partial r} \right|_R 2h_{euph}/R$. Evaluating this expression with the default parameters leads to a gyre averaged contribution of $1.23 \times 10^{-11} \text{ mol P m}^{-2} \text{ s}^{-1}$ from horizontal mixing, or less than one tenth of the contribution of the residual horizontal Ekman transport. This term parametrises the lateral mixing due to eddy stirring as a down gradient flux, and represents part of the contribution of eddies to the nutrient budget.

7.2.1.5 Vertical eddy exchange

The vertical transfer of nutrients by eddies, as proposed by McGillicuddy and Robinson (1997), relies on raising higher nutrient water from below the euphotic zone in to the euphotic zone. Once these higher nutrient waters are uplifted in to the euphotic zone there is an increase in productivity and the nutrients are consumed. The water parcel then moves downwards again and out of the euphotic zone. This mechanism is not a permanent exchange between the two regions. However, some

submesoscale instabilities do result in the vertical exchange of fluid. For example, symmetric instability, as described by Brannigan (2016), produces small overturning cells that transport fluid vertically. Given that this term parametrises both the effects of mesoscale eddies and submesoscale dynamics, the parametrisation of this term as an exchange of fluid between the two layers of the model seems reasonable.

The gyre averaged value for vertical eddy transfer is given by

$$\frac{2\pi w_{rms} A_{eddy} \int_0^R ([PO_{4mode}] - [PO_4]) r dr}{\pi R^2}, \quad (7.11)$$

which evaluates to 6.98×10^{-11} mol P m⁻² s⁻¹, equivalent to 0.035 mol N m⁻² y⁻¹. This is substantially smaller than the estimate of 0.35 ± 0.10 mol N m⁻² y⁻¹ from McGillicuddy and Robinson (1997). However, the model used by McGillicuddy and Robinson (1997) relaxed nitrate to climatological values below the euphotic zone, and thus represents an upper bound and is likely an overestimate. Lévy et al. (2012) ran two idealised simulations with a full biogeochemical model. One simulation used a submesoscale permitting resolution of 1/54°, the other used a mesoscale resolving resolution of 1/9°. Their estimate of the eddy nitrate flux was approximately 0.03 mmol N m⁻² day⁻¹, or 0.011 mol N m⁻² y⁻¹, in both simulations. Our toy model provides an eddy flux to the euphotic zone that is approximately one tenth of the value computed by McGillicuddy and Robinson (1997), and three times larger than the value calculated by Lévy et al. (2012). Thus, although the value computed from the idealised model is within the range of values found in the literature, this does not provide a particularly stringent test of our model.

7.2.1.6 Vertical mixing

The flux of phosphate into the euphotic zone due to vertical diffusion is given by

$$\frac{\int_0^{2\pi} \int_0^R \kappa_z ([PO_4] - [PO_{4mode}]) r dr d\phi}{h_{euph} \pi R^2}, \quad (7.12)$$

which evaluates to 4.83×10^{-12} mol P m⁻² s⁻¹, equivalent to 0.0024 mol N m⁻² y⁻¹.

Estimates for the flux of nitrate into the euphotic zone of subtropical gyres by vertical mixing are generally low due to the small vertical gradients at the base of the euphotic zone and weak diapycnal diffusion. Lewis et al. (1986) used observational measurements of turbulent kinetic energy dissipation and vertical profiles of nitrate and density to estimate vertical nitrate fluxes due to diffusion in the oligotrophic

ocean. They found a vertical flux of $0.14 \text{ mmol nitrate m}^{-2} \text{ day}^{-1}$, which is equivalent to $0.0511 \text{ mol N m}^{-2} \text{ y}^{-1}$. Dietze et al. (2004) also estimated the flux due to diffusive processes as $0.05 - 0.15 \text{ mol N m}^{-2} \text{ y}^{-1}$. The estimate from Dietze et al. (2004) was based on internal wave breaking and double diffusion. Both of these estimates are substantially larger than the value calculated by this model. The sensitivity of the model to changes in the vertical diffusivity will be explored in section 7.2.2.1.

7.2.1.7 Nutrient budget of the model

It is possible to use these process driven fluxes to construct a nutrient budget for the idealised model by considering the fluxes into and out of the entire domain. After integrating the model for six centuries of model time the nutrient budget closes to approximately one part in a thousand, as shown in table 7.2. The largest source is the horizontal supply of nutrients by residual Ekman transport in the euphotic zone, which provides $2486.6 \text{ mol PO}_4^{3-} \text{ s}^{-1}$ through the horizontal advection of phosphate into the euphotic zone and $892.8 \text{ mol P s}^{-1}$ through the horizontal advection of dissolved organic phosphate (DOP). Horizontal mixing also supplies some nutrients through the gyre edge in both layers, but is much smaller than the horizontal Ekman flux. At a steady state most of these nutrients are removed by the horizontal residual Ekman transport in the mode water layer. The particulate flux also provides a sink of nutrients, although it is a smaller contribution than the residual Ekman transport in the mode water layer.

Table 7.2: *Nutrient budget for the two layer advection-diffusion model.*

Process	Nutrient flux (mol P s^{-1})
Residual Ekman transport (PO_4^{3-} in euphotic zone)	2486.6
Residual Ekman transport (DOP in euphotic zone)	892.8
Horizontal mixing (PO_4^{3-} in euphotic zone)	165.8
Horizontal mixing (DOP in euphotic zone)	38.0
Residual Ekman transport (PO_4^{3-} in mode water layer)	-3477.9
Residual Ekman transport (DOP in mode water layer)	-118.4
Horizontal mixing (PO_4^{3-} in mode water layer)	595.6
Horizontal mixing (DOP in mode water layer)	13.8
Particulate flux through thermocline	-602.2
Sum	-5.8

7.2.2 Sensitivity testing

Our highly idealised model includes a number of physical parameters for which reasonable values have been chosen from the literature. However, the sensitivity of this model to the chosen parameter values should be evaluated. The response of the model to changes in these parameters may also be used to make predictions for the biogeochemical response of the ocean to similar changes on interannual to decadal timescales.

7.2.2.1 Vertical mixing

Vertical diffusion is believed to play a relatively minor role in the nutrient budget of the oligotrophic ocean (Williams and Follows, 2011) because of the weak vertical gradients in nutrient concentrations at the base of the euphotic zone and the consistently low values of vertical diffusivity measured in the ocean.

Our idealised model is not especially sensitive to changes in the vertical diffusion coefficient, as shown in figure 7.11. Reducing the value to zero, or increasing it by several orders of magnitude, has a relatively small effect on the steady-state distribution of phosphate in the euphotic zone, until the physically implausible value of $10^{-3} \text{ m}^2 \text{ s}^{-1}$ is reached. Increasing the vertical diffusion does decrease the phosphate concentration in the mode water layer, but once again this effect is only appreciable when the vertical diffusion coefficient is increased beyond the physically implausible value of $10^{-3} \text{ m}^2 \text{ s}^{-1}$.

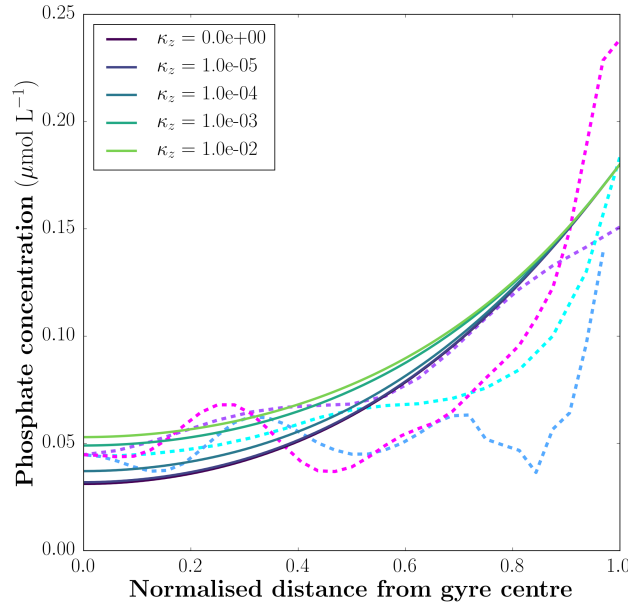
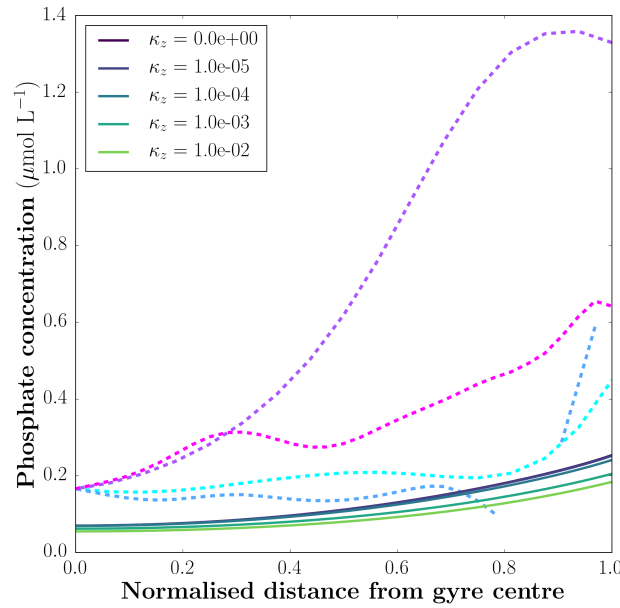
(a) *Euphotic zone*(b) *Mode water layer*

Figure 7.11: Solutions for the concentration of phosphate for a series of simulations with different values of vertical diffusivity, κ_z , compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

7.2.2.2 Horizontal mixing

The nutrient concentration at the outer edge of the upper layer is fixed at a constant value, which means horizontal diffusion can provide a source of nutrients to the rest of the domain by mixing those nutrients inwards. The horizontal diffusivity used in this model is substantially larger than the molecular diffusivity of water because the horizontal diffusion term parametrises lateral mixing by eddies. Estimates of horizontal eddy diffusivity are on the order of $800 \text{ m}^2 \text{ s}^{-1}$ for geostrophic eddies (Marshall et al., 2006), but are believed to vary substantially with location (Rypina et al., 2012). Rypina et al. (2012) estimated values between 200 and $12,000 \text{ m}^2 \text{ s}^{-1}$ in the North Atlantic, with the higher values found in the strongly eddying western portion of the basin, near the western boundary current. We might, therefore, expect the best agreement between our idealised model and the data when we use a value for κ_h at the lower end of the range found by Rypina et al. (2012) that is representative of the open ocean.

Varying the eddy diffusivity from $10 \text{ m}^2 \text{ s}^{-1}$ to $4,000 \text{ m}^2 \text{ s}^{-1}$ makes a substantial difference to the steady-state nutrient distributions, as shown in figure 7.12. The sensitivity of this model to κ_h suggests that interannual variability in the horizontal diffusion of nutrients by eddies may have a substantial effect on the nutrient budget of oligotrophic subtropical gyres.

Lee and Williams (2000) examined the diffusive and advective contributions of eddies to nutrient transport in an idealised model. They concluded that in the euphotic zone the combination of Ekman transport and eddy diffusion was substantially stronger than the advective transport due to eddies. Our toy model is only able to represent the advective transport of eddies through the modification of the Ekman pumping velocity, which will be explored next.

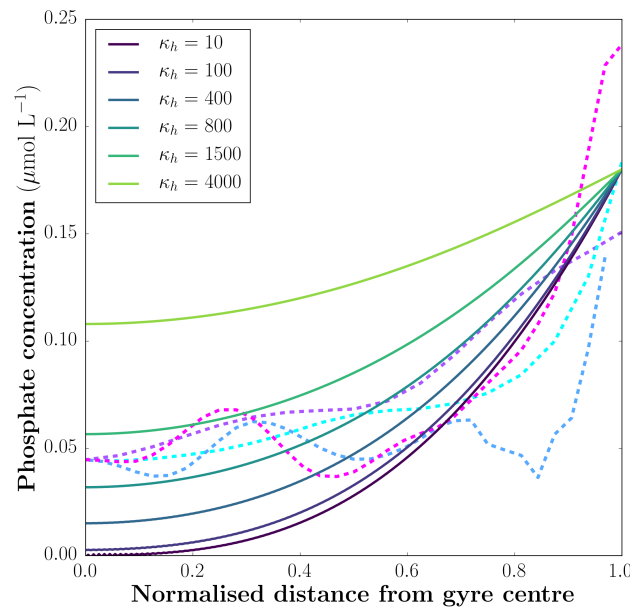
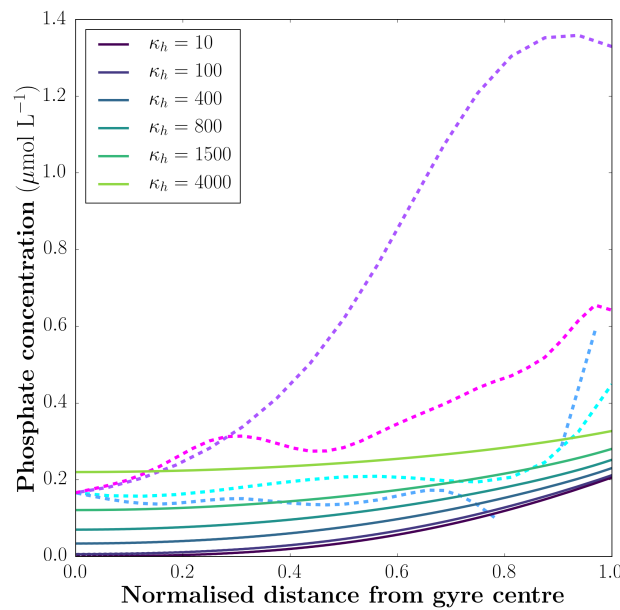
(a) *Euphotic zone*(b) *Mode water layer*

Figure 7.12: Solutions for the concentration of phosphate for a series of simulations with different values of horizontal diffusivity, κ_h , compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

7.2.2.3 Residual Ekman pumping velocity

Ekman pumping is the textbook explanation for the low levels of biological activity in the subtropical gyres (Williams and Follows, 2011). It is believed that the large scale Ekman-driven downwelling inhibits biological activity by suppressing the upwards flux of nutrients from deeper waters. McClain et al. (2004) used satellite observations of ocean colour to compare biological activity with various physical parameters averaged within subtropical gyres. They found that at times of peak Ekman pumping the oligotrophic regions of the subtropical gyres shrank. This observation is consistent with the horizontal Ekman flux of nutrients proposed by Williams and Follows (1998). Varying the residual Ekman pumping velocity is the mechanism by which our toy model is able to represent the eddy cancellation result from previous chapters, or equivalently the advective eddy transport discussed by Lee and Williams (2000).

In this highly idealised model Ekman pumping is parametrised as a mass flux from the euphotic zone in to the mode water layer. Therefore, it represents a continual movement of fluid and nutrients out of the euphotic zone. However, to conserve mass the model also includes a horizontal flux of fluid. In the euphotic zone this is a flow towards the centre of the gyre, and thus represents a horizontal flux of nutrients in from the edge.

By varying the magnitude of residual Ekman pumping from 0 to 80 m per year we can span the range from complete eddy cancellation to no eddy cancellation and can investigate the impact of this process on the phosphate distribution. The results shown in figure 7.13 plot the radial phosphate profiles for a range of residual Ekman pumping velocities.

Our toy model is sensitive to changes in the residual Ekman pumping velocity. It also suggests that the initial increase from 0 to 30 m y⁻¹ results in a decrease of biological activity because the nutrients are being removed from the euphotic zone. However, increasing the residual Ekman pumping beyond 30 m y⁻¹ actually increases the availability of nutrients in the euphotic zone by increasing the horizontal flux of nutrients in to the gyre. This is equivalent to the horizontal Ekman flux of nutrients discussed by Williams and Follows (1998), and consistent with the observational findings of McClain et al. (2004). The best fit between the idealised model and the climatological nutrient distribution is obtained with a residual Ekman pumping of 10-20 m year⁻¹. This is substantially less than the -78 m year⁻¹ value computed from the scatterometer derived wind stress climatology (Risien, 2011; Risien and Chelton, 2008) shown in figure 7.14. While the need to use a lower value for the Ekman pumping velocity lends further support to the eddy cancellation of Ekman

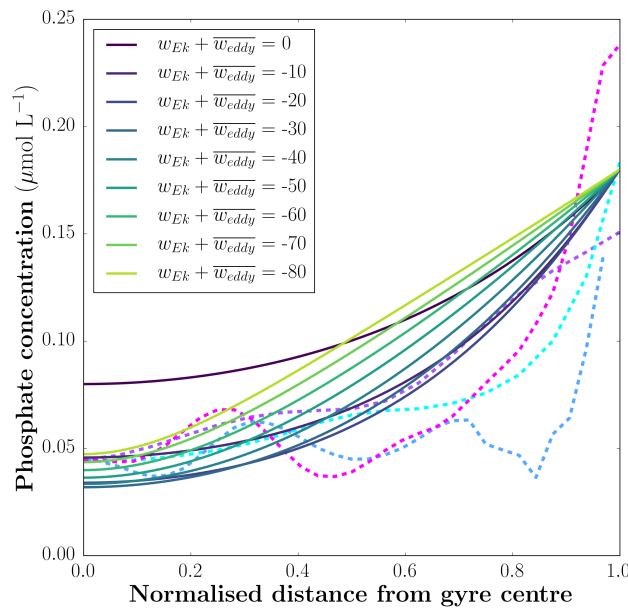
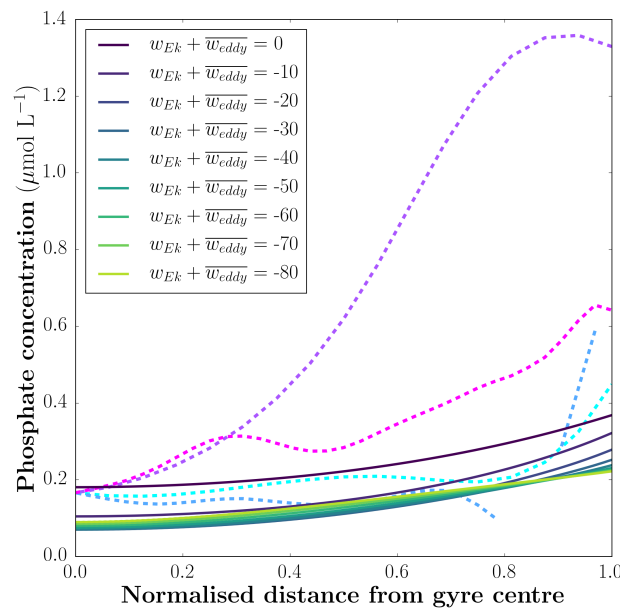
(a) *Euphotic zone*(b) *Mode water layer*

Figure 7.13: Solutions for the concentration of phosphate for a series of simulations with different values of residual Ekman pumping velocity, $w_{Ek} + \overline{w_{eddy}}$, compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

pumping discussed in chapter 6, it should be noted that this model contains many parametrised processes and numerous parameters. As such, the exact value of residual Ekman pumping which produces the best fit for the nutrient concentration is unlikely to be robust. This is also true of the value at which the nutrient minimum occurs. It is of more interest that such a minimum is present than the exact parameter value at which it occurs.

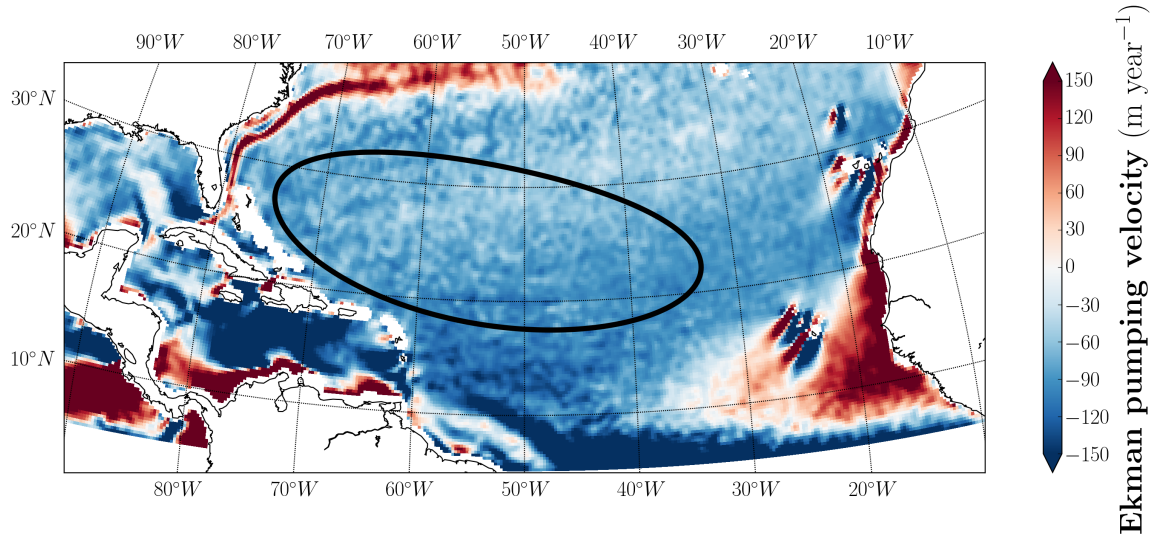


Figure 7.14: *Ekman pumping velocity over the North Atlantic computed from climatological wind stress (Risien, 2011; Risien and Chelton, 2008). The area averaged value within the black ellipse is -78 m year^{-1} .*

7.2.2.4 Thickness of the mode water layer

In this idealised model, the thickness of the mode water layer determines the efficiency of the biological pump at removing nutrients from the model domain. A thin mode water layer means that more of the parametrised particulate flux passes through the mode water layer and exits the domain. This leads to a loss of nutrients from the model. We therefore expect a thicker mode water layer to lead to higher nutrient concentrations in the mode water, which would then be transferred to the euphotic zone through eddy activity.

As can be seen in figure 7.15, increasing the thickness of the mode water layer leads to a monotonic increase in the concentration of phosphate in both the euphotic zone and the mode water layer. This result is contrary to the prediction made by Palter et al. (2005) and will be explored further using observational datasets in section 7.3.

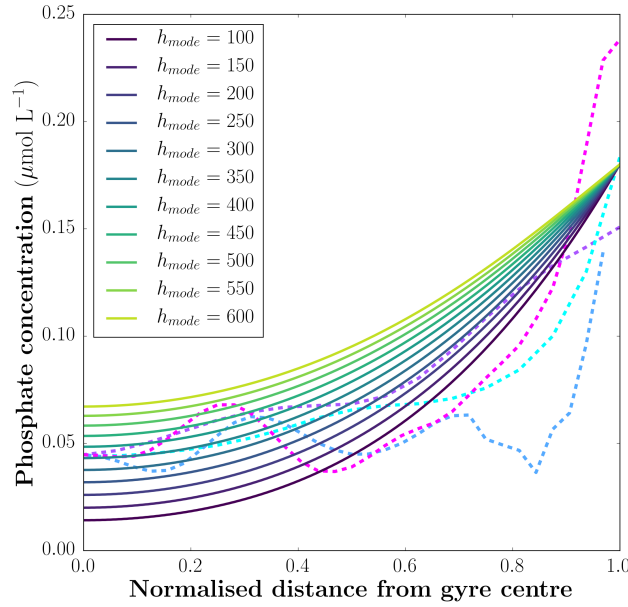
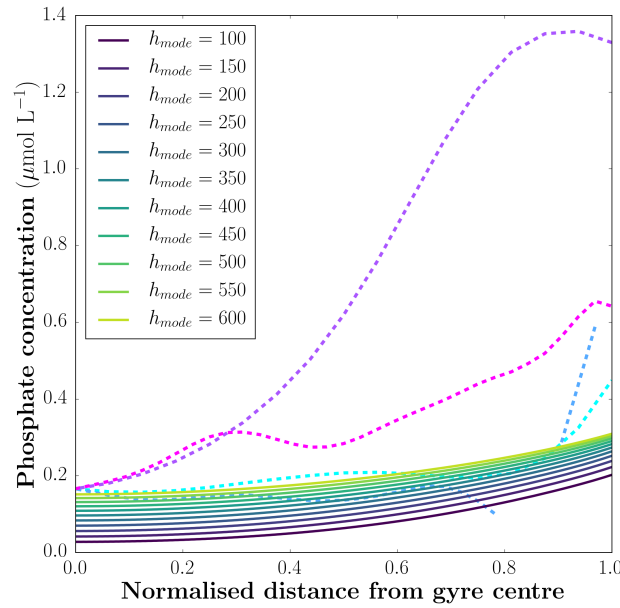
(a) *Euphotic zone*(b) *Mode water layer*

Figure 7.15: Solutions for the concentration of phosphate for a series of simulations with different values for the thickness of the mode water layer, h_{mode} , compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

7.2.2.5 Thickness of the euphotic zone

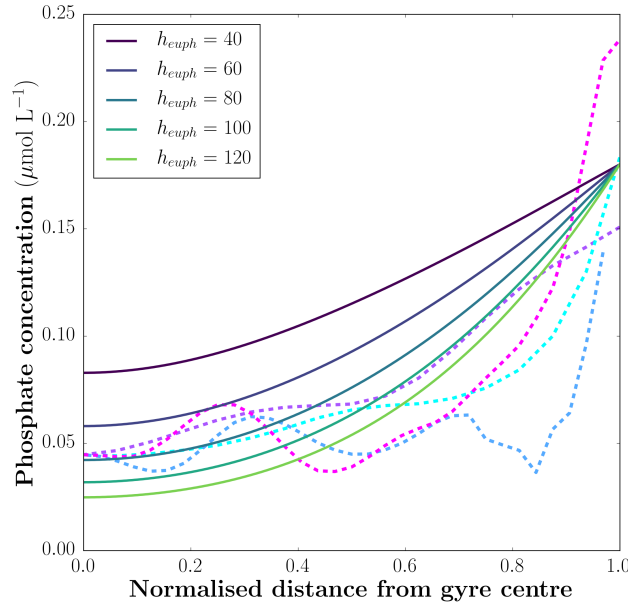
The thickness of the euphotic zone influences the total amount of biological productivity. A thinner euphotic zone means lower productivity since the parameter α limits productivity to a given value per cubic metre, and a thinner euphotic zone contains fewer cubic metres of water. A thinner euphotic zone also implies a larger horizontal Ekman velocity since in this model the thickness of the Ekman layer and the euphotic zone are equivalent, which implies a faster horizontal advection of phosphate and DOP. Since the timescale over which DOP remineralises into phosphate remains unchanged, this means that the DOP is advected farther in towards the centre of the domain before remineralising, thereby increasing the nutrient concentration in the centre of the gyre. Estimates of the euphotic zone thickness, defined as the depth at which the incident radiation is 1% of the surface value, vary from 40 m to 100 m (Hemsley et al., 2015; Martin, 2003), while many idealised modelling studies use a value between 70 m and 120 m (see e.g. Lévy, 2008; Lévy et al., 2012; McGillicuddy and Robinson, 1997). The sensitivity testing shows that a thinner euphotic zone leads to higher nutrient concentrations in the toy model.

7.2.2.6 Eddy exchange

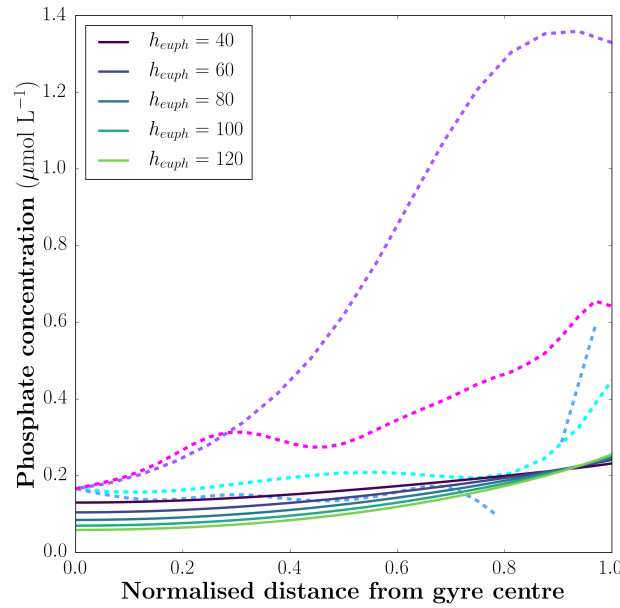
Upwelling due to mesoscale eddies is believed to provide a source of nutrients to the euphotic zone (McGillicuddy and Robinson, 1997; McGillicuddy, 2016; McGillicuddy et al., 2003, 1998), as is upwelling due to submesoscale features (Brannigan, 2016). We expect an increase in eddy driven exchange to lead to higher phosphate concentrations and higher biological activity in the euphotic zone of our model, and lower phosphate concentrations in the mode water layer. Nutrients in the mode water layer will be depleted through increased exchange with the low nutrient water of the euphotic zone.

There are two parameters in this idealised model that control the vertical eddy driven exchange of fluid. The representative vertical velocity, w_{rms} , and the fraction of the ocean covered by eddies, A_{eddy} . The idealised model multiplies the representative vertical velocity and the fractional area together. This means that there is a degeneracy between these two terms and only changes in their product affect the model.

Brannigan (2016) estimates the representative vertical velocity from upwelling caused by symmetric instability as 2.5 m s^{-1} . The mechanism identified by Brannigan (2016) only occurs in anticyclones and is weakened by the presence of a restratifying



(a) Euphotic zone



(b) Mode water layer

Figure 7.16: Solutions for the concentration of phosphate for a series of simulations with different values of the thickness of the euphotic zone, h_{euph} , compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

heat flux as occurs in summer. Brannigan (2016) calculated that 10 % of the Sargasso Sea was covered by anticyclones in the summer of 2013. However, since the submesoscale instability discussed by Brannigan (2016) is suppressed by restratifying heat fluxes this implies that the annual fraction of the gyre over which it is active is more like 5 %.

In a recent review McGillicuddy (2016) provides an overview of the physical mechanisms by which mesoscale eddies modulate biological activity in the ocean. These include vertical displacement during the intensification and decay of eddies as well as eddy wind interactions, the strongest of which is eddy Ekman pumping with a characteristic vertical velocity of 1 m s^{-1} .

Since the global abundance of cyclones and anticyclones is relatively similar (Chelton et al., 2011, 2007), we can use Brannigan’s estimate of eddy abundance to infer that approximately 20 % of the subtropical North Atlantic is covered by eddies. However, as noted by Martin (2003) it is implausible that every eddy contributes to the vertical flux of nutrients, that is, the efficiency of eddy pumping is likely to be substantially less than 100 %. Martin (2003) estimate an efficiency of just under 60 % for the eddy events observed near the Bermuda Atlantic Time Series site. Using either the estimates from Brannigan (2016) of 2.5 m s^{-1} and 5 % coverage, or a representative vertical velocity of 1 m s^{-1} , a fractional area of 20 %, and an efficiency of 60 %, the product of A_{eddy} , w_{rms} , and the efficiency is approximately 0.12 m s^{-1} . The processes described by McGillicuddy (2016) and Brannigan (2016) are physically distinct and there is no reason to consider them mutually exclusive, so our toy model may be underestimating the effect of eddies.

Because of the degeneracy between w_{rms} and A_{eddy} only the sensitivity of the model to changes in w_{rms} will be shown here. Equivalent changes to A_{eddy} give identical results. Figure 7.17 shows the phosphate concentration in the euphotic zone and mode water layer for a series of simulations with different representative vertical velocities and a value of 5 % for A_{eddy} . These show that the effect of eddy exchange saturates, and that above a threshold value the model is not particularly sensitive to the parametrised eddy exchange.

A similar effect was noticed by Lévy et al. (2012): in the simulations presented by Lévy et al. (2012) the larger vertical velocities in the higher resolution simulation were compensated by an increase in the depth of the nutricline, or equivalently a decrease in the nutrient concentration at a fixed depth. Despite showing a low sensitivity to w_{rms} , our toy model is unable to reproduce this effect; as w_{rms} increases there is only

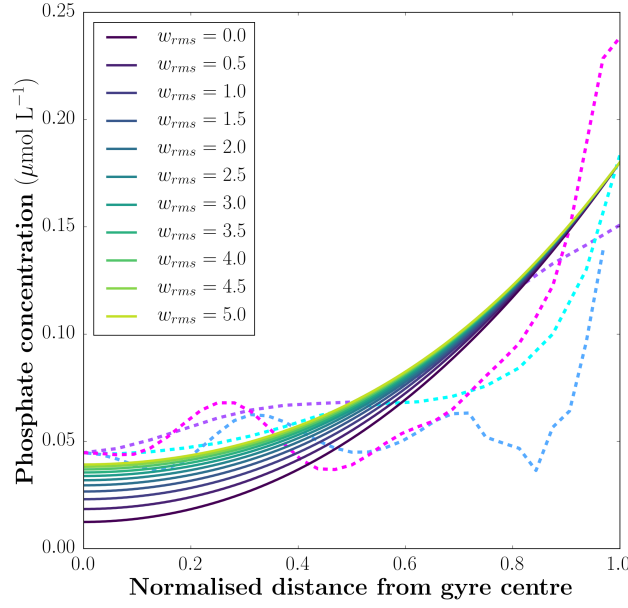
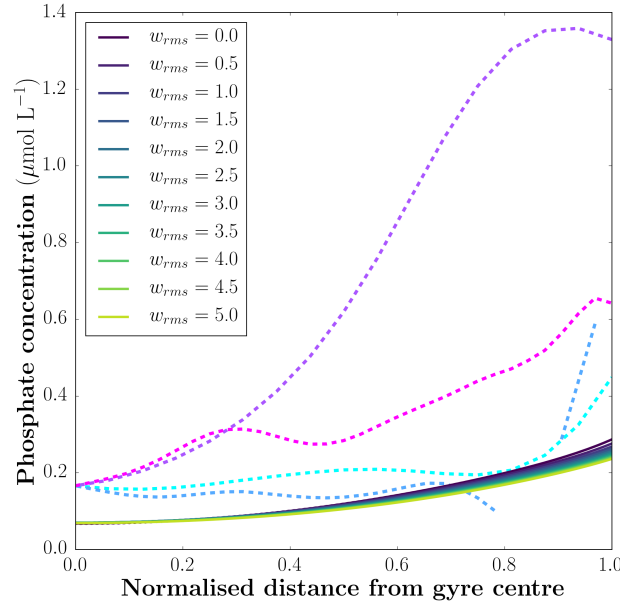
(a) *Euphotic zone*(b) *Mode water layer*

Figure 7.17: Solutions for the concentration of phosphate for a series of simulations with different values of the representative eddy velocity, w_{rms} , compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

a slight decrease in phosphate concentration near the gyre edge in the mode water layer.

The form of the eddy exchange term in the model is simplistic, but the lack of vertical resolution in the toy model precludes a more complex parametrisation. Recent work exploring the impact of submesoscale dynamics (see e.g. Brannigan, 2016; Lévy et al., 2012) adds support to the use of the chosen form, since symmetric instability does produce a vertical exchange of fluid, unlike the classical mesoscale eddy pumping mechanism described in McGillicuddy and Robinson (1997).

7.2.2.7 Nutrient concentration at the edge of the gyre

The nutrient concentration at the edge of the gyre is specified as a boundary condition for phosphate in the euphotic zone. By setting the concentration of the phosphate at the edge of the model domain, this boundary condition has a large impact on the availability of phosphate throughout the model domain. Increasing the boundary value results in increased concentrations everywhere in the domain. The magnitude of the effect is smaller at the gyre centre.

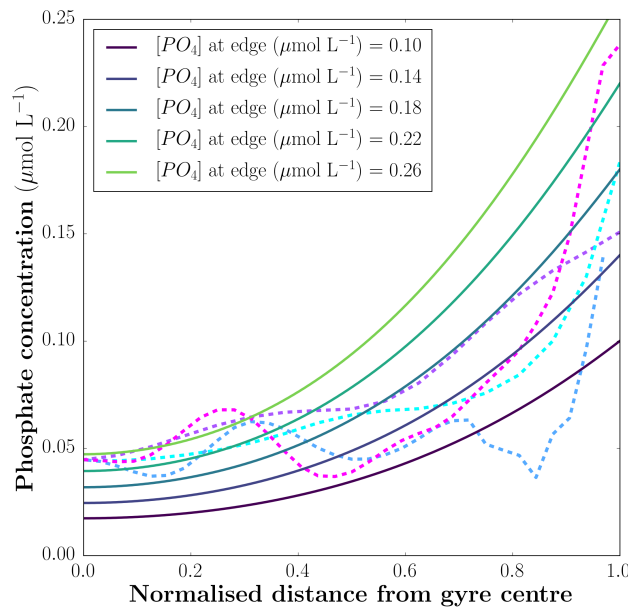
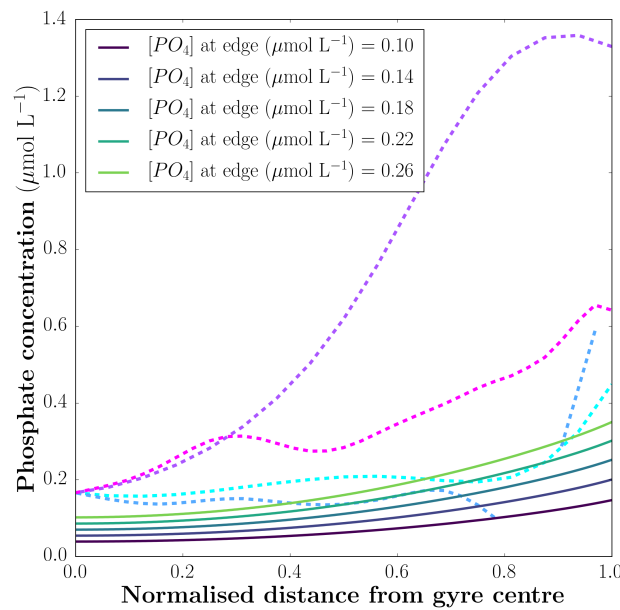
(a) *Euphotic zone*(b) *Mode water layer*

Figure 7.18: Solutions for the concentration of phosphate for a series of simulations with different boundary values for phosphate in the euphotic zone, compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

7.2.2.8 Maximum community production

The maximum value for net community production is given by α . Increasing this value will allow higher levels of biological activity, which will result in faster consumption of the nutrients in the euphotic zone and a stronger biological pump transporting nutrients into the mode water layer and out of the domain through the base of the mode water layer.

Figure 7.19 shows that if there is no biological activity the model predicts uniform phosphate concentrations. In the euphotic zone we see a monotonic decrease in phosphate concentration as α is increased. However, the response in the mode water layer is non-monotonic. The concentration at the gyre centre decreases initially due to exchanges with the low nutrient euphotic zone, but increases slightly for very large values of α because of nutrient fluxes from the biological pump. At the gyre edge the response in the mode water layer is again monotonic; increasing community production leads to higher nutrient concentrations in the mode water because of the biological pump and the fixed concentration of phosphate at the gyre edge in the euphotic zone.

7.2.2.9 Phosphate half saturation

The half saturation coefficient describes the concentration at which the net community production reaches half of its maximum value. As the nutrient concentration increases beyond this value the net community production asymptotically approaches its maximum value.

Figure 7.20 shows the phosphate concentration for a series of simulations using different values for λ_{PO_4} . For low values of λ_{PO_4} the biology is able to utilise nutrients in extremely low concentrations and remove almost all nutrients from the euphotic zone. As λ_{PO_4} is increased the concentration of phosphate in the euphotic zone increases because the biology is unable to utilise the nutrients as efficiently. The response in the mode water depends on location. Near the centre of the gyre the concentration is determined by exchanges with low nutrient fluid from the euphotic zone. Near the gyre edge the response is dominated by the biological pump. Since the boundary value in the euphotic zone is held at a fixed value the productivity becomes very large for small values of λ_{PO_4} and large quantities of nutrients are transported to the mode water layer by the biological pump.

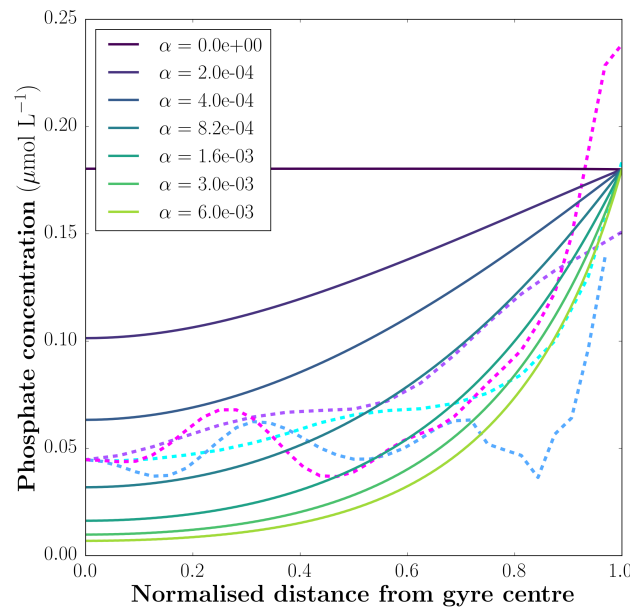
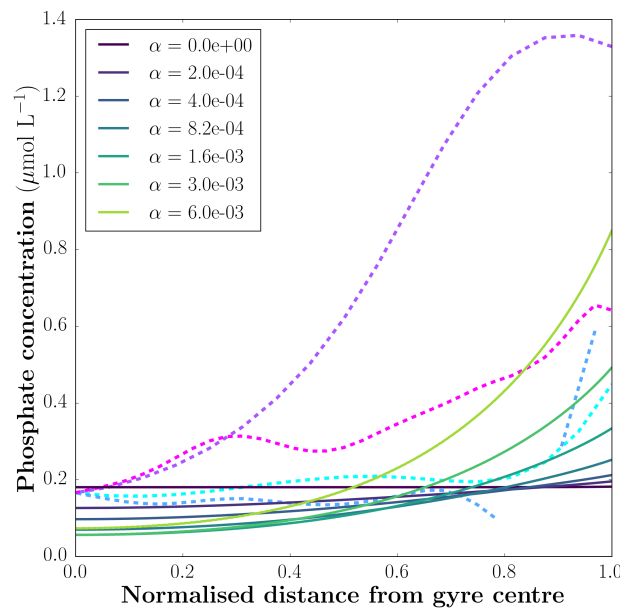
(a) *Euphotic zone*(b) *Mode water layer*

Figure 7.19: Solutions for the concentration of phosphate for a series of simulations with different values of maximum community production, α , compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

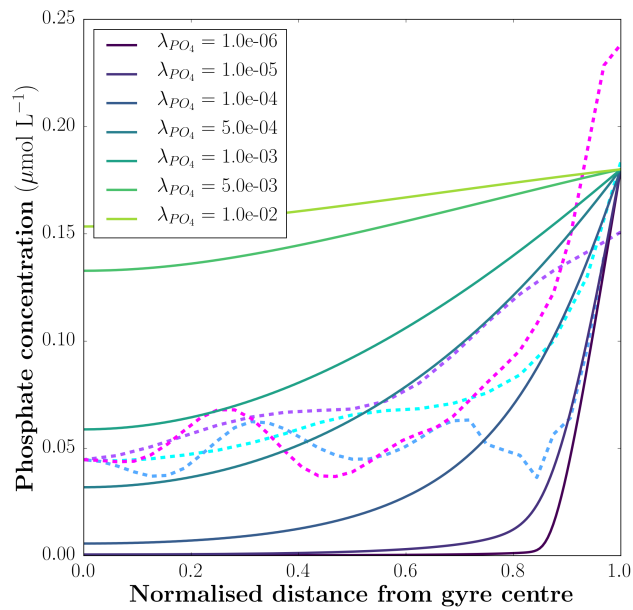
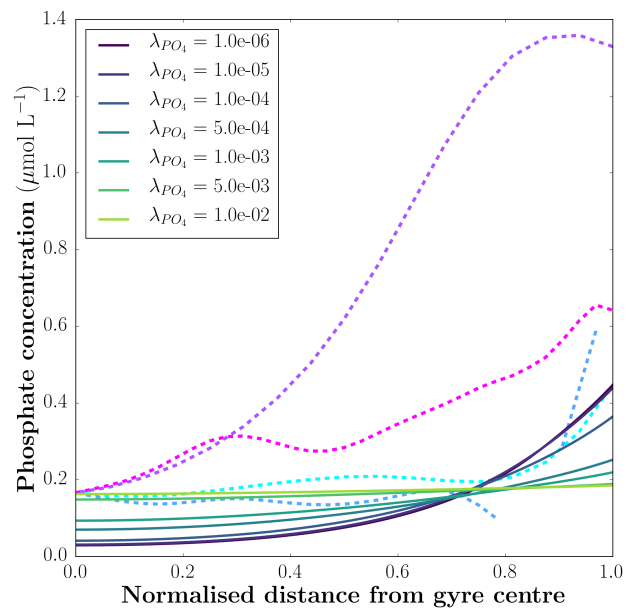
(a) *Euphotic zone*(b) *Mode water layer*

Figure 7.20: Solutions for the concentration of phosphate for a series of simulations with different values of phosphate half saturation concentration, λ_{PO_4} , compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

7.2.2.10 Remineralisation timescale

The timescale for the remineralisation of dissolved organic phosphate (DOP) into phosphate is given by $1/\gamma_{remin}$. If γ_{remin} is increased then the rate at which DOP converts into biologically accessible phosphate is increased.

Figure 7.21 shows the results of multiple simulations with different remineralisation timescales. In the simulations with a long timescale, small γ_{remin} , the concentration of phosphate in the euphotic zone is increased. This is partly due to the increased lateral transport of DOP within the euphotic zone and partly to the increased concentration of phosphate in the mode water layer. The concentration in the mode water layer increases with smaller γ_{remin} because the eddy and Ekman transports have a longer period of time in which to transport the DOP from the euphotic zone into the mode water layer before it is remineralised and because of the enhanced flux from the biological pump due to elevated nutrient concentrations in the euphotic zone.

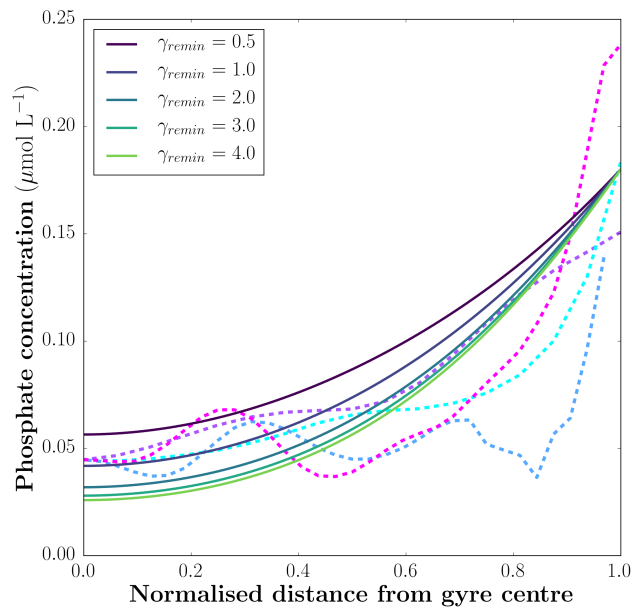
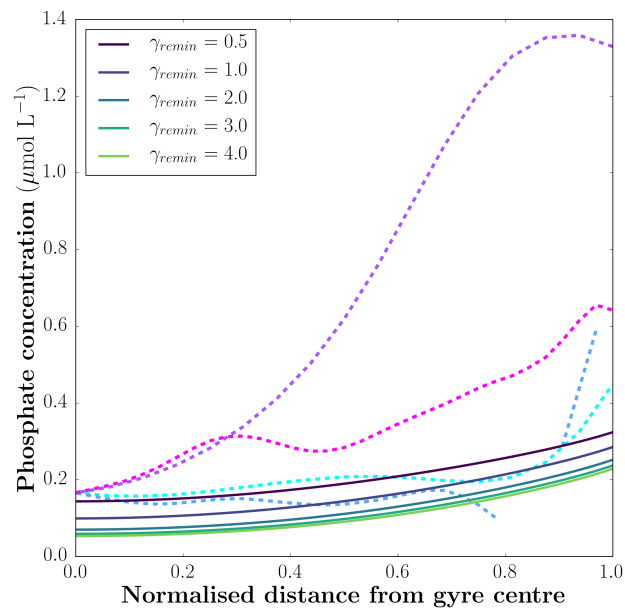
(a) *Euphotic zone*(b) *Mode water layer*

Figure 7.21: Solutions for the concentration of phosphate for a series of simulations with different values of the remineralisation timescale, γ_{remin} , compared with phosphate concentrations from the World Ocean Atlas climatology (Garcia et al., 2013) along the sections shown in figure 7.6 a) (dashed lines, colours matched to each section).

7.3 The influence of mode water thickness on biological activity

In section 7.2.2.4 we found that the idealised model predicts that an increase in the mode water thickness will lead to an increase in nutrient concentration and biological activity. This result from our highly idealised model is at odds with the prediction of Palter et al. (2005) who suggested that a thick mode water layer would inhibit biological activity by inserting a wedge of low nutrient water between the euphotic zone and the nutricline, thus inhibiting the vertical transfer of nutrients. Krémeur et al. (2009) tested the impact of the nutrient content of subtropical mode water on primary productivity by artificially altering the nutrient concentration in the subtropical mode water of their idealised model. Krémeur et al. (2009) concluded that the nutrient content of subtropical mode water impacted the primary productivity of the subpolar region and the subtropical western boundary current, but had relatively little influence over the productivity within the gyre. However, the time-mean primary productivity in their simulated subtropical gyre, shown in figure 7.22, does not display the characteristic concentric rings of decreasing productivity that are apparent in the observations, as shown in figure 7.6.

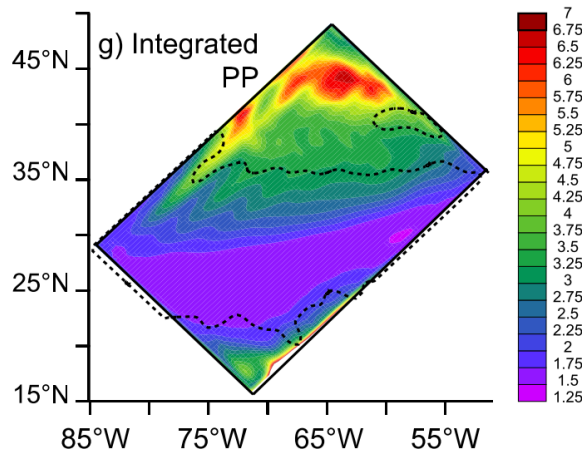


Figure 7.22: Time-mean primary productivity from Krémeur et al. (2009) in $\text{mmol nitrate m}^{-2} \text{ day}^{-1}$. The dotted black line represents the zero contour of the barotropic streamfunction and is taken as the boundary of the subtropical gyre.

Based on the predictions from Palter et al. (2005), the results from Krémeur et al. (2009), and the sensitivity of the idealised model we have three different and incompatible relationships between mode water and biological activity within subtropical gyres. Palter et al. (2005) predict a decrease in biological activity with an increase in

the thickness of the mode water. Krémeur et al. (2009) predict very little relationship between mode water and primary productivity within the subtropical gyre, and our idealised model predicts an increase in primary productivity with mode water thickness.

In the following sections we seek to test these predictions against observational datasets. Mode water thickness is estimated using a one-degree gridded Argo product that is a continuation of the dataset described by Roemmich and Gilson (2009). The dataset can be downloaded from http://sio-argo.ucsd.edu/RG_Climatology.html and is updated regularly with the latest observations. The relationship between temperature and salinity in mode waters is very robust (Fratantoni et al., 2013), which means that the temperature field alone is adequate for diagnosing the thickness of mode water.

Primary productivity within the subtropical gyres is taken from a satellite derived dataset hosted at

<http://www.science.oregonstate.edu/ocean.productivity/index.php> by Oregon State University (O'Malley, 2016). This dataset is produced by using the Vertically Generalized Production Model of Behrenfeld and Falkowski (1997) and satellite derived estimates of chlorophyll concentration (NASA Ocean Biology Processing Group, 2015) to estimate ocean primary productivity. The algorithm presented by Behrenfeld and Falkowski (1997) uses a temperature dependent function to convert satellite chlorophyll retrievals in to an estimate of primary productivity. Their algorithm was able to explain 86% of the variability of in situ measurements of productivity obtained using carbon-14 fixation. The analysis presented in this chapter focusses on assessing changes in productivity, which means that the presence of bias within the productivity estimate is only a problem if that bias changes with time, as could occur if the species of phytoplankton in an area change. To assess the impact of biases in the productivity algorithm, equivalent calculations are also performed with MODIS-Aqua chlorophyll concentrations (NASA Ocean Biology Processing Group, 2015) that were downloaded from http://oceandata.sci.gsfc.nasa.gov/MODIS-Aqua/Mapped/Monthly/9km/chlor_a.

7.3.1 North Atlantic subtropical gyre

Figure 7.23 shows mean ocean productivity in the subtropical North Atlantic between January 2004 and December 2015, as well as the ellipse within which the observational quantities are averaged. The area over which the averages are taken encompasses

most of the gyre and the productivity minimum. This region is chosen to cover the area of low productivity and avoid regions of strong wintertime convection since this convection represents a substantial source of nutrients that are not accounted for in our toy model.

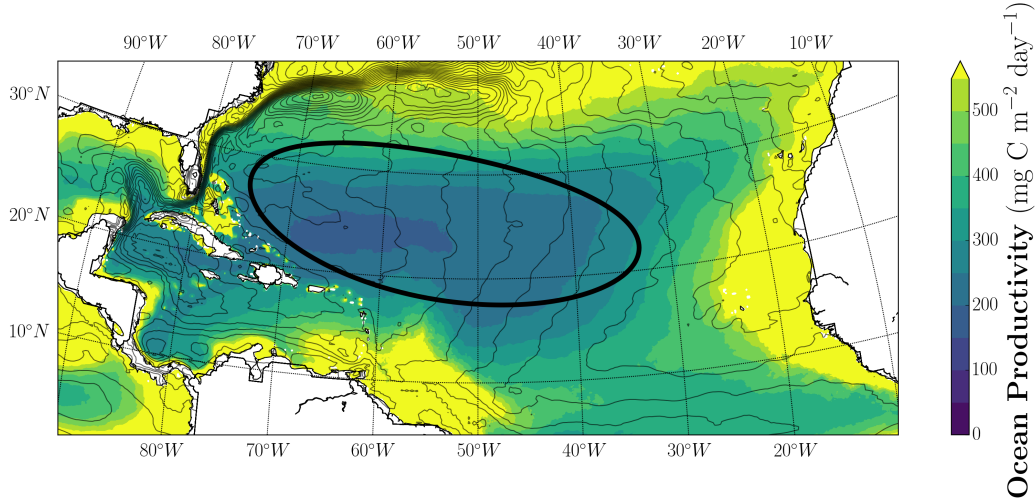
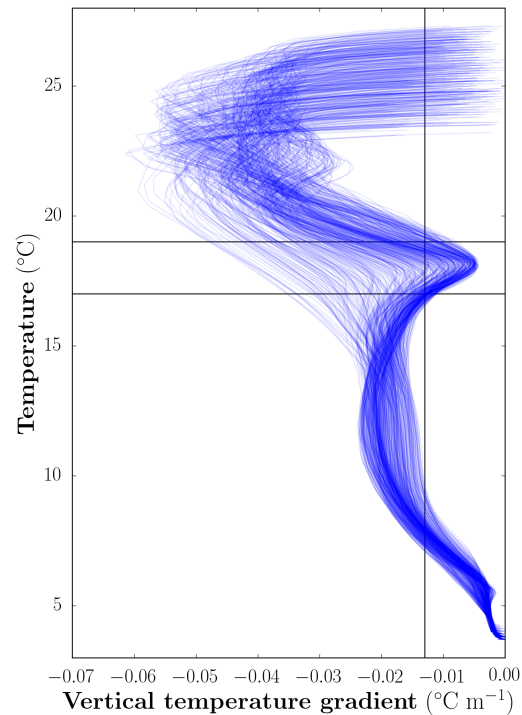
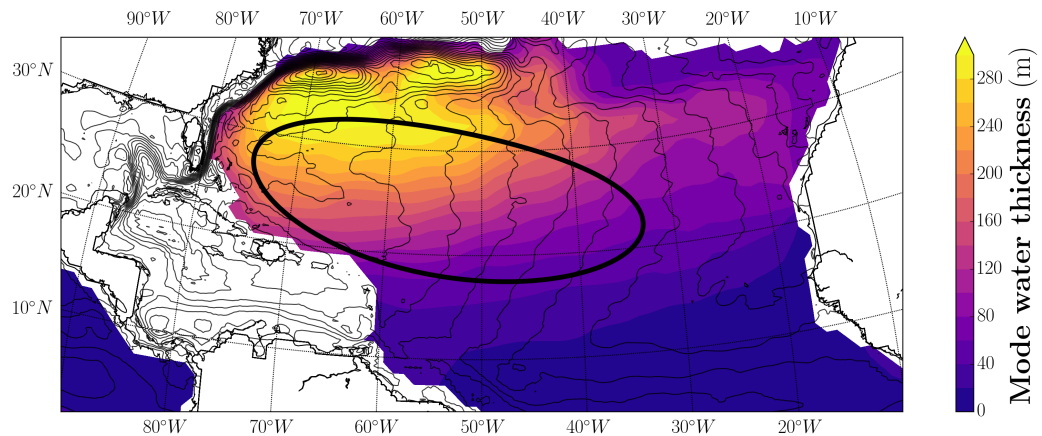


Figure 7.23: Mean ocean primary productivity ($\text{mg C m}^{-2} \text{ day}^{-1}$) in the North Atlantic subtropical region. The thin black lines are contours of mean dynamic topography in 5 cm intervals and the thick black ellipse shows the area over which the observational quantities are averaged.

Worthington (1958) described a volume of weakly stratified fluid that could be found across much of the North Atlantic. This water mass is known as eighteen degree water and is the mode water of the North Atlantic subtropical gyre. Recent definitions of eighteen degree water include a vertical stratification criterion (Alfultis and Cornillon, 2001; Klein and Hogg, 1996; Kwon and Riser, 2004) as well as a temperature range. For the calculations presented here, the thickness of the North Atlantic mode water is determined as the difference in depth between the 17°C and 19°C isotherms. The choice of 17°C and 19°C was informed by the stratification minimum shown in figure 7.24 a). Our calculation of mode water thickness ignores the vertical gradient condition used by Klein and Hogg (1996) and Alfultis and Cornillon (2001). The vertical gradient criterion is ignored since the stratification minimum is slowly eroded as the mode water is advected southwards in the gyre (Palter et al., 2005) and we wish to capture the thickness of these older mode water masses in the diagnostic. Figure 7.24 b) shows the diagnosed mode water thickness over the subtropical North Atlantic and the ellipse within which the observational quantities are averaged.



(a) Profiles of vertical temperature gradient plotted against temperature for each one degree box within the ellipse shown in figure 7.23 using the Eulerian-mean temperature field for the period 2004 to 2015 (Roemmich and Gilson, 2009). The stratification minimum within the mode water can be clearly seen at approximately 18 °C. The horizontal lines are at 17 °C and 19 °C, and are used to define the limits of the mode water.



(b) Mode water thickness (m) in the North Atlantic subtropical region diagnosed as the depth difference between the 17 °C and 19 °C isotherms in the 2004-2015 Eulerian-mean temperature field. Regions in which one, or both, of the isotherms does not exist have been masked. The thin black lines are contours of mean dynamic topography in 5 cm intervals and the thick black ellipse shows the area over which the observational quantities are averaged.

Figure 7.24: North Atlantic

Figure 7.25 shows the averaged quantities plotted against each other with the ordinary least squares regression fit to the data. The proportion of variance explained is 0.18, or 18 %. The slope of the regression line is positive, but not significantly different from zero. The coefficients and their confidence intervals are given in table 7.3. In figure 7.25 there are two data points that appear as outliers: the year with an unusually thin mode water layer (2015); and the year with unusually high productivity (2010). Repeating the regression analysis excluding either of these points results in a larger estimate for the slope and a smaller estimate for the intercept. However, the changes are not substantial. Repeating the analysis averaging over different regions within the gyre produces very similar results; although not statistically significant this result is suggestive.

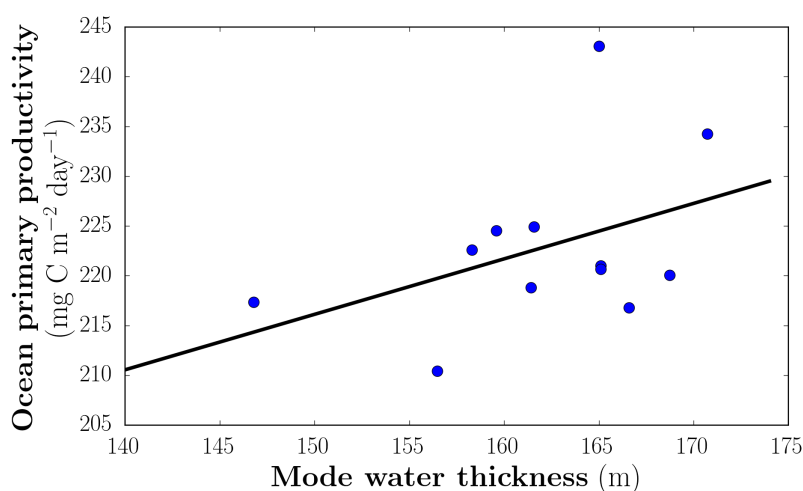


Figure 7.25: Annual mean ocean productivity ($\text{mg C m}^{-2} \text{ day}^{-1}$) plotted against annual mean mode water thickness (m). Each quantity has been averaged over the ellipse shown in figures 7.23 and 7.24 b). The black line shows the ordinary least squares regression fit to the data. The slope is not significantly different from zero.

Table 7.3: Coefficient and error estimates from the ordinary least squares regression of ocean productivity with mode water thickness in the North Atlantic.

	Estimate	Standard error	<i>p</i> -value	95 % confidence interval	
Intercept	132.52	61.89	0.058	-5.378	270.422
Slope	0.557	0.381	0.175	-0.293	1.407

Using chlorophyll concentration instead of ocean productivity yields similar results; there is a positive trend that is not statistically significant and varying the region over which the averages are taken does not materially alter the conclusion.

The annual mean of chlorophyll concentration is plotted against mode water thickness in figure 7.26 with the regression line shown in black.

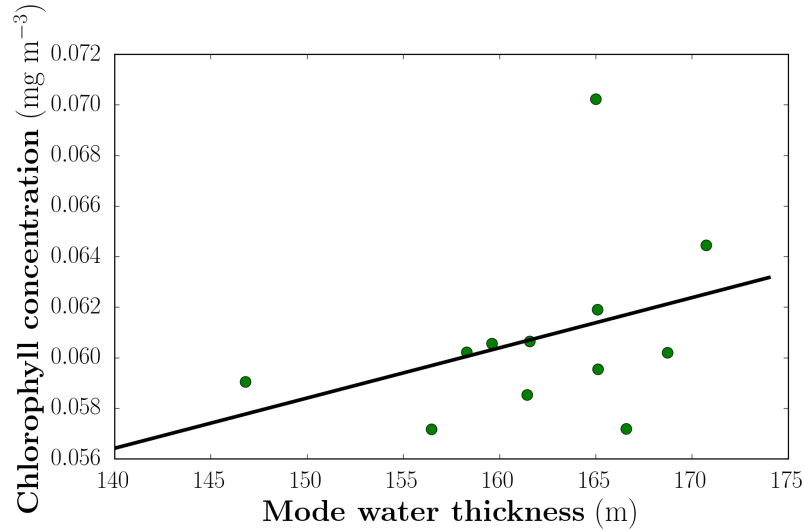


Figure 7.26: Annual mean chlorophyll concentration (mg m^{-3}) plotted against mode water thickness (m). Each quantity has been averaged over the ellipse shown in figures 7.23 and 7.24 b)d. The black line shows the ordinary least squares regression fit to the data. The slope is not significantly different from zero.

7.3.2 Other subtropical gyres

The North Atlantic subtropical gyre is unusual because the dynamical centre of the gyre is close to the biological centre of the gyre. The dynamical centre of the North Pacific subtropical gyre is several thousand kilometres from the biological centre of the gyre (McClain et al., 2004). If the averages were computed over the region with the lowest productivity values then very little of the mode water would be sampled. For this reason the averages are performed in the ellipse shown in figures 7.27 and 7.28b.

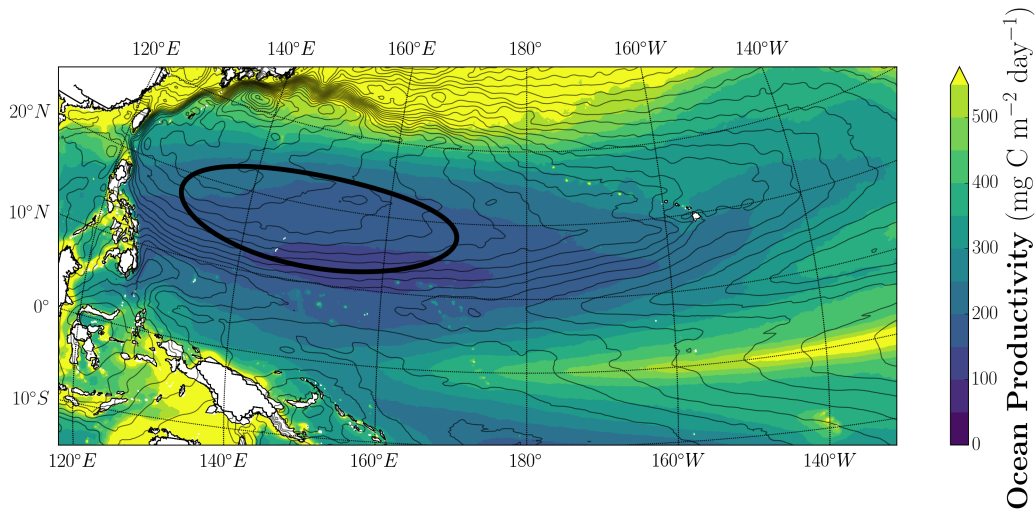
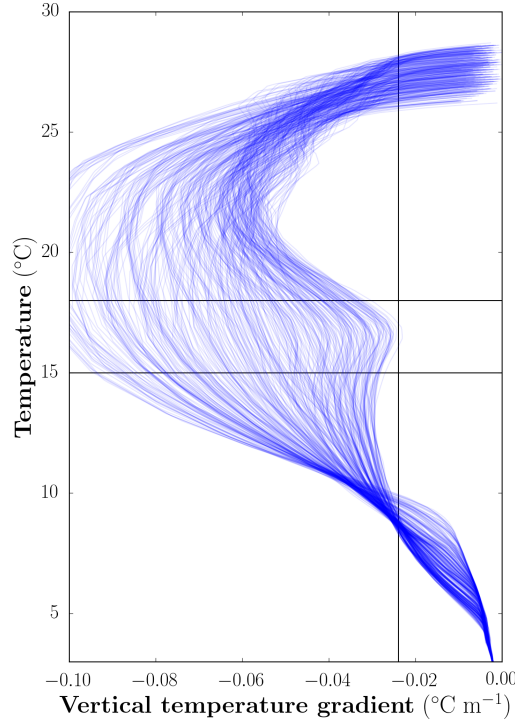
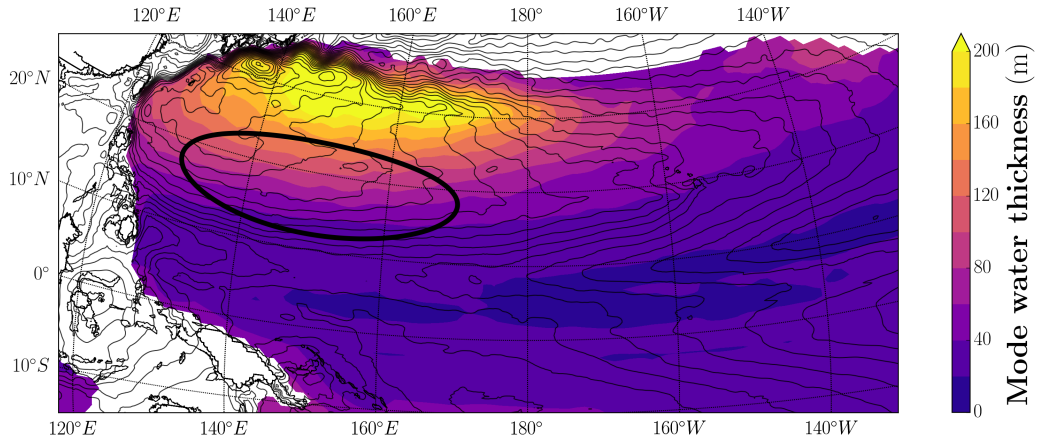


Figure 7.27: Ocean primary productivity ($\text{mg C m}^{-2} \text{ day}^{-1}$) in the North Pacific subtropical region. The thin black lines are contours of mean dynamic topography in 5 cm intervals and the thick black ellipse shows the area over which the observational quantities are averaged.

The stratification minimum in the North Pacific is not as well defined as the minimum in the North Atlantic. Figure 7.28 a) shows that some of the profiles have a clear mode water signal at approximately 16°C , but many do not. If the averaging region is moved further south, towards the productivity minimum, the mode water signal weakens.



(a) Profiles of vertical temperature gradient plotted against temperature for each one degree box within the ellipse in the North Pacific subtropical gyre shown in figure 7.27 using Eulerian-mean temperature fields for the period 2004 to 2015 (Roemmich and Gilson, 2009). A stratification minimum can be seen at approximately 16 °C. The horizontal lines are at 15 °C and 18 °C, and are used to define the limits of the mode water.



(b) Mode water thickness (m) in the North Pacific subtropical region diagnosed as the depth difference between the 15 °C and 18 °C isotherms. Regions in which one or both of the isotherms does not exist have been masked. The thin black lines are contours of mean dynamic topography in 5 cm intervals and the thick black ellipse shows the area over which the observational quantities are averaged.

Figure 7.28: North Pacific

In the subtropical South Pacific there are two regions of low productivity, as shown in figure 7.29. The South Pacific Subtropical Mode Water is the weakest of the subtropical mode waters (Talley et al., 2011). The lack of a clear stratification minimum can be seen in figure 7.30 a). Some of the profiles in figure 7.30 a) show a local minimum between 15 °C and 19 °C. Figure 7.30 b) shows the thickness of the 15-19 °C layer in the South Pacific on the same colour scale as figure 7.28 b).

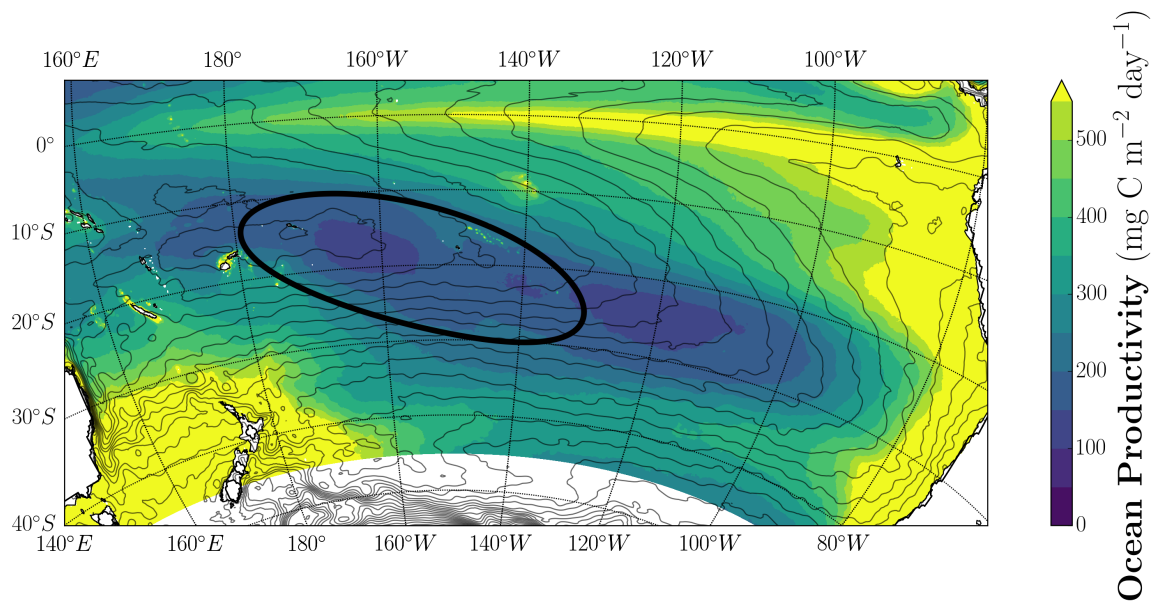
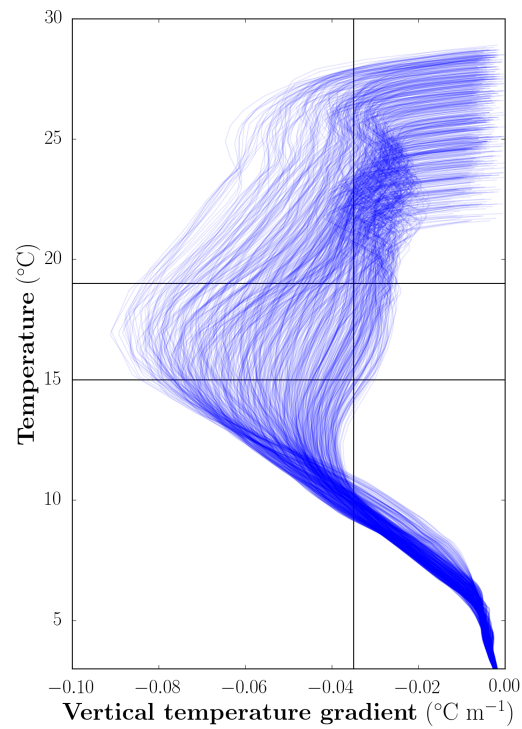
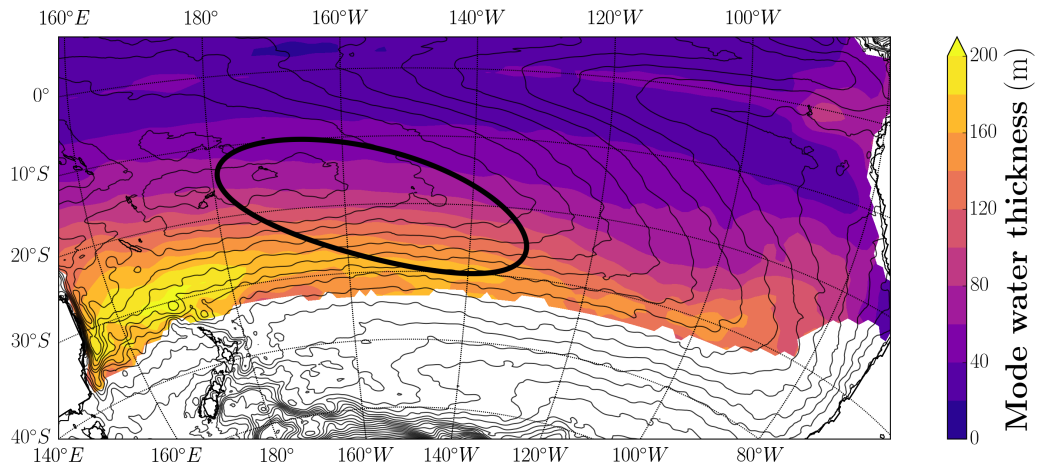


Figure 7.29: Ocean primary productivity ($\text{mg C m}^{-2} \text{ day}^{-1}$) in the South Pacific subtropical region. The thin black lines are contours of mean dynamic topography in 5 cm intervals and the thick black ellipse shows the area over which the observational quantities are averaged.



(a) Profiles of vertical temperature gradient plotted against temperature for each one degree box within the ellipse in the South Pacific subtropical gyre shown in figure 7.29 using Eulerian-mean temperature fields for the period 2004 to 2015 (Roemmich and Gilson, 2009). A weak stratification minimum can be seen in some of the profiles at approximately 16 °C. The horizontal lines are at 15 °C and 19 °C, and are used to define the limits of the mode water.



(b) Mode water thickness (m) in the South Pacific subtropical region diagnosed as the depth difference between the 15 °C and 19 °C isotherms. Regions in which one or both of the isotherms does not exist have been masked. The thin black lines are contours of mean dynamic topography in 5 cm intervals and the thick black ellipse shows the area over which the observational quantities are averaged.

Figure 7.30: South Pacific

Primary productivity in the South Atlantic is shown in figure 7.31, which also shows a substantial latitudinal offset between the biological gyre and the dynamical gyre. The South Atlantic subtropical gyre is associated with multiple mode waters due to the presence of multiple currents and fronts in that region (Talley et al., 2011). Figure 7.32 shows a meridional slice of temperature in the South Atlantic. Two of the mode waters described by Talley et al. (2011) are in the 12-15 °C range. These water masses are well below the bottom of the euphotic zone, and the water above is stratified. This makes the unambiguous identification of the relevant mode water layer difficult, as can be seen in the profiles of stratification shown in figure 7.33 a). Thus, the mechanism identified in the idealised model may be less relevant to the South Atlantic subtropical gyre.

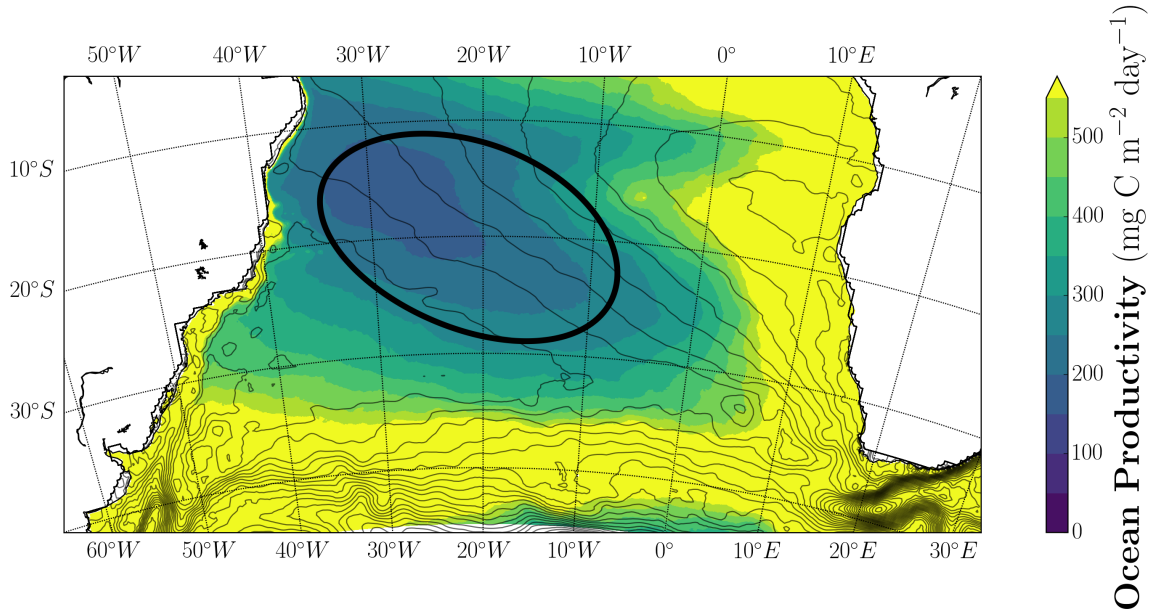


Figure 7.31: Ocean primary productivity ($\text{mg C m}^{-2} \text{ day}^{-1}$) in the South Atlantic subtropical region. The thin black lines are contours of mean dynamic topography in 5 cm intervals and the thick black ellipse shows the area over which the observational quantities are averaged.

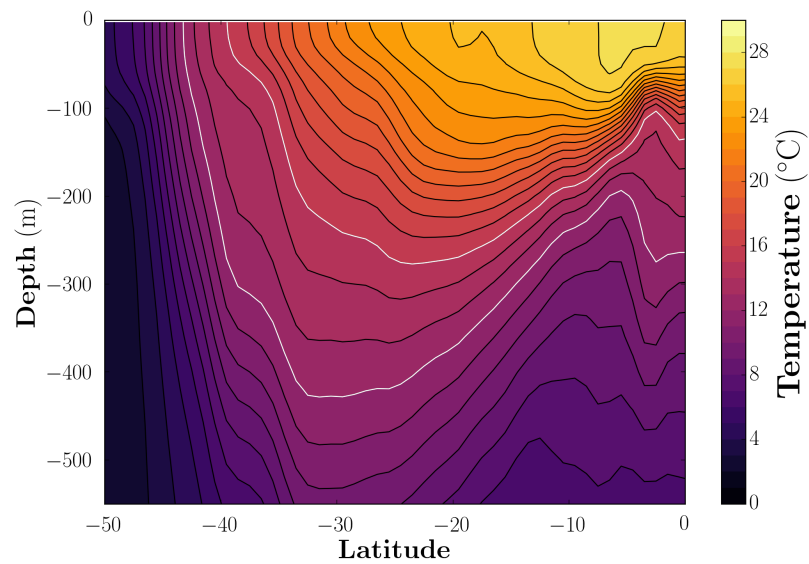
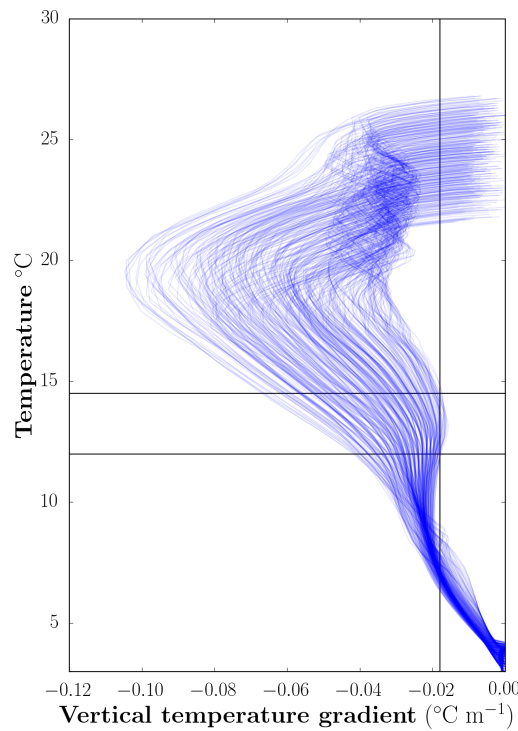
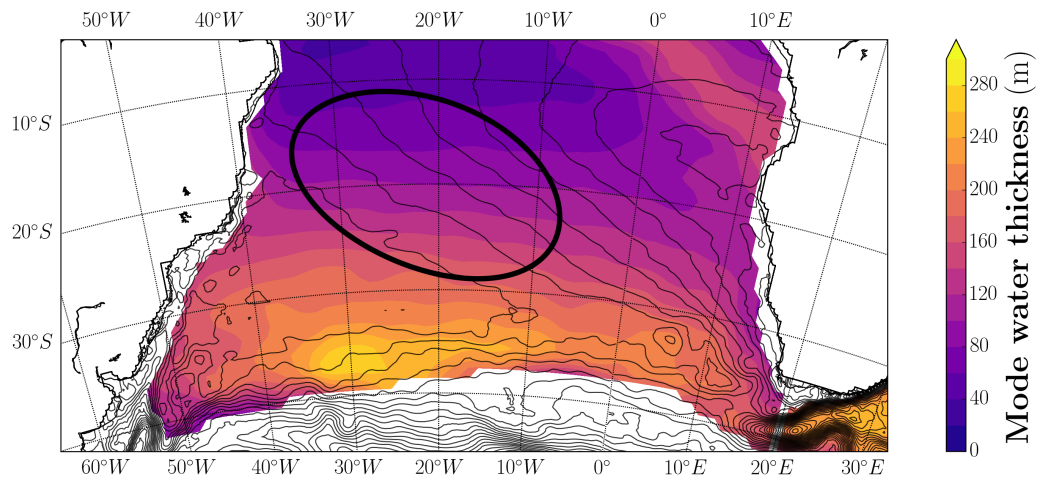


Figure 7.32: Meridional slice of temperature at 26.5W in the South Atlantic. The two white contours are for 12 °C and 15 °C. Note the absence of a well defined mode water layer near the base of the euphotic zone at 100 m depth.



(a) Profiles of vertical temperature gradient plotted against temperature for each one degree box within the ellipse in the South Atlantic subtropical gyre shown in figure 7.31 using Eulerian-mean temperature fields for the period 2004 to 2015 (Roemmich and Gilson, 2009). A weak stratification minimum can be seen at approximately 14°C . The horizontal lines are at 12°C and 14.5°C , and are used to define the limits of the mode water.



(b) Mode water thickness (m) in the South Atlantic subtropical region diagnosed as the depth difference between the 12°C and 14.5°C isotherms. Regions in which one, or both of the isotherms, does not exist have been masked. The thin black lines are contours of mean dynamic topography in 5 cm intervals and the thick black ellipse shows the area over which the observational quantities are averaged.

Figure 7.33: South Atlantic

Figure 7.34 shows annual mean mode water thickness plotted against annual mean productivity in each of the subtropical gyres. Performing a regression analysis on each of the gyres individually produces a positive, but not statistically significant, relationship between mode water thickness and productivity, suggesting that thicker mode water leads to higher productivity within the oligotrophic gyres. Plotting all of the gyres on the same axes, as has been done in figure 7.34, is not meant to imply that a regression analysis should be performed on this combined dataset. Because of the offset between the region of maximum mode water thickness and minimum productivity, if the ellipse in each gyre were moved polewards, then the value for productivity in that gyre would increase and the mode water thickness would increase. This means that the signal we are looking for is built in to the background state, and so a comparison between the different gyres is not an appropriate method for testing our prediction. Since the choice of ellipse within each gyre largely determines the mean values of the productivity and the mode water thickness, it is inappropriate to draw a line of best fit through the data plotted in figure 7.34. However, placing all of the data on the same axes allows other comparisons to be made. It is of interest to note that the variation in productivity is of a similar magnitude in all of the gyres, but the variation in mode water thickness is not. The two gyres with the least well defined mode water layers are also the two that show the least variation in mode water thickness. This suggests that the analysis is sampling the background stratification, not interannually varying mode water thickness.

Comparisons between the gyres also provide weak support for the hypothesis that thicker mode water leads to higher productivity. There are however many parameters that vary between the gyres, and which are not accounted for in figure 7.34. The values of mode water thickness and primary productivity shown in figure 7.34 are somewhat sensitive to the location and size of the area over which the averages are taken. Changing the analysis region alters the specific values, but not the trend of higher productivity with thicker mode water.

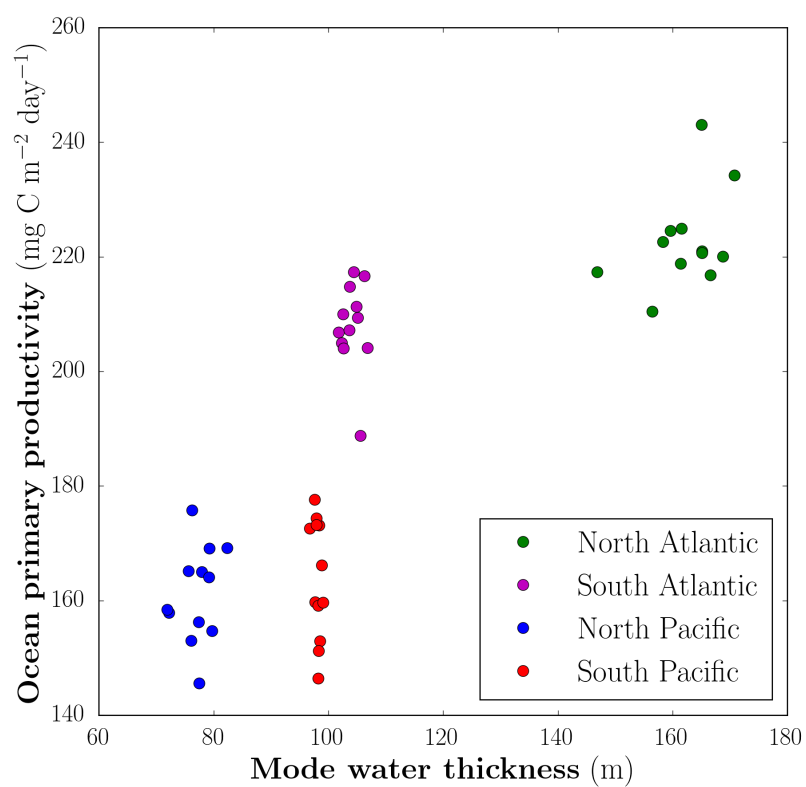


Figure 7.34: *Mode water thickness plotted against ocean productivity in the subtropical gyres.*

7.4 Concluding remarks

This chapter has examined the transport and distribution of nutrients in oligotrophic subtropical gyres. The nutrient budget in these regions has been the subject of considerable research. Variability on a range of scales from submesoscale (e.g. Brannigan, 2016) to mesoscale (e.g. McGillicuddy, 2016) has been proposed as a mechanism to close the budget. However, large uncertainties remain. The role of mode water as an interannual store of heat and nutrients has received some attention (e.g. Kwon and Riser, 2004; Palter et al., 2005). Palter et al. (2005) predict that a thicker mode water layer would inhibit productivity by incorporating a wedge of low nutrient water between the euphotic zone and the high nutrient concentrations at depth.

We have developed an idealised model of nutrients within subtropical gyres and validated it against observations and published estimates of nutrient fluxes. This has allowed us to explore the fourth and fifth key questions regarding the impact of a small residual vertical velocity, and the sensitivity of nutrient distributions to changes in this velocity. Our toy model incorporates the concept of eddy cancellation from chapter 6 through modification of the residual Ekman pumping velocity, which allows us to test the sensitivity of our toy model to changes in the residual Ekman pumping velocity. The model displays a non-monotonic response; for small values of $w_{Ek} + \overline{w_{eddy}}$ the phosphate concentration in the euphotic zone decreases, reaching a minimum at approximately 30 m y^{-1} , as the residual Ekman pumping velocity increases beyond this value the phosphate concentration begins to increase. While the magnitude of residual Ekman pumping at which this change in behaviour occurs is likely to be sensitive to the values chosen for other parameters in this model, this non-monotonic response warrants further investigation.

The simple model also predicts an increase in productivity with increases in mode water thickness. This occurs because a thicker mode water layer reduces the effectiveness of the particulate flux through the thermocline. Since more of the particulate flux is remineralised within the mode water, the particulate flux of nutrients out of the domain is reduced leading to a higher phosphate concentration. However, it should be noted that the mechanism proposed by Palter et al. (2005) is excluded from this model since there is no diffusive transport of nutrients from the abyss in to the mode water layer. Future work should modify the model to include this mechanism.

We tested the prediction that primary productivity would increase with thicker mode water using observational temperature data from the Argo array (Roemmich and Gilson, 2009) and satellite derived estimates of ocean productivity (Behrenfeld

and Falkowski, 1997; O'Malley, 2016) and chlorophyll concentration (NASA Ocean Biology Processing Group, 2015). These data products show a positive, but weak, relationship between annual mean mode water thickness and productivity. Testing the predicted response to changes in the residual Ekman pumping velocity requires an understanding of the timescale over which eddy cancellation operates, and represents an exciting avenue for future research.

Conclusions and future work

The research presented in this thesis has explored the large-scale effects of mesoscale eddies in ocean gyres. In particular I have examined the balance between Eulerian-mean and eddy-induced circulations, and the implications of this balance on the biogeochemistry of subtropical gyres.

8.1 Summary of key results

The equations of motion were derived in chapter 2, along with a number of commonly used approximations to the full equations of motion. The simulations presented in this thesis focus on large scale dynamics, which means that the Navier-Stokes equations could be approximated to reduce the computational cost of the simulations. For the parameter space occupied by the oceanic circulation the Boussinesq and hydrostatic approximations are appropriate simplifications. Although these simplified equations of motion are able to reproduce only a subset of the motions allowed by the full equations, they are simpler to solve numerically while still describing motions typical of the large scale oceanic circulation. The computational tools used to calculate approximate solutions to the equations of motion and to analyse output from these numerical models were presented in chapter 3.

The work presented in chapter 4 used the potential vorticity framework described in Marshall (2000) and Polton and Marshall (2003, 2007) to examine vertical fluxes of potential vorticity in an isolated subtropical gyre. The results showed a balance between vertical potential vorticity fluxes due to Eulerian-mean advection and eddy-induced fluxes. Polton and Marshall (2007) reported a similar balance in the Southern Ocean. The simulations for this chapter were performed in a limited domain that

consisted of an isolated subtropical gyre. The restricted nature of this domain may have contributed to the very clean balance between mean flow and eddies identified in the potential vorticity flux budget. To test the sensitivity of this result to changes in the domain more simulations were performed using a larger domain with multiple gyres.

The simulations presented in chapter 5 expand the model domain to include a subpolar gyre and in some cases a meridional overturning circulation. By making these changes to the domain our model becomes a better analogue of the ocean. Despite substantial changes to the model domain, and some changes to the numerics, the results of these simulations corroborate the previously identified balance between potential vorticity fluxes due to Eulerian-mean advection and eddy induced fluxes. In the simulations that were too coarse to resolve mesoscale variability, the potential vorticity flux produced by the bolus velocity of the GM mesoscale eddy parametrisation (Gent and McWilliams, 1990; Gent et al., 1995) was able to balance the vertical potential vorticity flux due to Eulerian-mean advection. Chapter 5 showed that the balance between eddy-induced fluxes and Eulerian-mean advection is robust to changes in the domain, the resolution, and the numerical details of the model.

Chapter 6 builds upon the results of chapters 4 and 5 by exploring the impact of averaging techniques on diagnosed flows, considering similarities between gyre circulations and flows in the atmosphere and the Southern Ocean, and continuing to probe the balance between Eulerian-mean flow and eddy-induced circulations in subtropical gyres. By using Lagrangian particle tracking techniques to numerically estimate the Generalised Lagrangian Mean velocity I was able to infer the rectified effect of eddies on the subtropical mode water. The Lagrangian particle tracking experiments showed downwelling in the Eulerian-mean velocities consistent with the presence of Ekman cells. Using the time-varying velocity fields to estimate the Generalised Lagrangian Mean velocity of the mode water showed that the rectified effect of eddies was able to cancel this downwelling. To quantify the impact of eddies, I used depth-density streamfunctions to examine the gyre scale overturning circulation. The depth-density streamfunction calculated from the Eulerian-mean velocity and temperature fields shows a thermally indirect circulation consistent with an Ekman cell. The downwelling occurs in the temperature range found in the subtropical gyre and the strength is equal to the expected magnitude of Ekman pumping. Calculating the overturning circulation with the instantaneous fields led to a very different result, with the thermally indirect circulation replaced by a thermally direct circulation that

is unable to penetrate below 200 m depth. The cancellation of the Eulerian-mean circulation by an eddy-induced circulation removes the large diabatic transport across the subtropical thermocline associated with the Ekman cell. In the paper describing this work my co-authors and I suggest “eddy cancellation” as the name for this phenomenon.

The results presented in chapter 6 suggest that rather than constructing a nutrient budget for the euphotic zone of subtropical gyres, it may be prudent to consider the nutrient budget of the entire water mass above the thermocline. This idea was explored in chapter 7 by deriving a simple two-layer axisymmetric model of nutrient distribution in a subtropical gyre. This idealised model provides an efficient means to test the sensitivity of the nutrient distribution to various parameters. Of particular interest is the response to the residual Ekman pumping velocity. As the residual Ekman pumping velocity is increased the nutrient concentration initially decreases, until reaching a minimum at approximately 30 m y^{-1} , after which the nutrient concentration increases with increasing Ekman pumping, consistent with McClain et al. (2004). The thickness of the mode water layer also has a substantial impact on nutrient concentrations in the idealised model, with a thicker mode water layer leading to higher nutrient concentrations. Testing the predicted relationship between mode water thickness and nutrient concentration using observational data provided some evidence to support the prediction that thicker mode water leads to higher productivity.

8.2 Future work

The experiments discussed in this thesis were designed to test the hypotheses described in each chapter. However, having tested these hypotheses, the results presented raise further questions. In this section I discuss some of the possibilities for future research that could build upon the methods and results presented.

8.2.1 Potential vorticity fluxes

The potential vorticity framework used in chapters 4 and 5 considers vertical fluxes of potential vorticity, but could be adapted to horizontal fluxes. By constructing volumes with density and Bernoulli isosurfaces it may be possible to use a variant of this formalism to study time-mean potential vorticity fluxes into and out of western boundary currents. In homogeneous models it is well established that friction in the western boundary is an important component of the potential vorticity budget (see

e.g. Fox-Kemper, 2005; Fox-Kemper and Pedlosky, 2004). However, other studies using three dimensional models have suggested that bottom pressure torque is important in the zonally integrated barotropic vorticity budget (Hughes, 2000; Hughes and de Cuevas, 2001; Jackson et al., 2006). Deremble et al. (2014) examined a potential vorticity budget of the North Atlantic, and inferred that some potential vorticity enters the gyre through interactions between the flow and the bathymetry in the western boundary current. By carefully constructing appropriate volumes over which to apply a horizontal version of the flux formalism it may be possible to determine regions in which the gyre and the western boundary current exchange potential vorticity, and the degree to which the two budgets are linked.

8.2.2 Numerical estimates of the Generalised Lagrangian Mean

The results presented in chapter 6 used Lagrangian particle tracking techniques to explore the rectified effect of eddies on the Generalised Lagrangian Mean velocity of the mode water. However, because the particles began to leave the gyre after some finite period of time the particle tracking algorithm needed to be regularly restarted. It would be preferable to dynamically reseed individual particles when they move beyond a threshold distance from the centre-of-mass. By altering this threshold distance one could probe the rectified effect of eddies over a range of length scales. It may also be possible to use density and Bernoulli potential as coördinates to define this distance, thereby allowing this algorithm to probe the rectified effect of eddies along geostrophic streamlines.

Since Lagrangian particle tracking is model agnostic, it is possible to apply this technique to the outputs of other models. In particular, it would be of interest to explore the residual vertical velocity in the subtropical gyres of an eddy-resolving global model to test for eddy cancellation. This would provide further evidence to support or negate the eddy cancellation mechanism identified in this thesis. Using Lagrangian particle tracking to estimate Generalised Lagrangian Mean velocities would also provide a new method for investigating the residual flow throughout a global model, potentially leading to additional insight into the residual transport of an eddying ocean.

8.2.3 Eddy cancellation and adjustment timescales

Eddy cancellation, the suppression of Ekman driven downwelling, was shown to apply at a statistically steady state, but it may also be of interest to explore the adjust-

ment timescale associated with this effect. Examining changes in the eddy-mean balance under changing winds is analogous to the concept of eddy compensation in the Southern Ocean literature. Knowledge of the timescale over which the eddy response adjusts to oppose the Eulerian-mean Ekman downwelling may provide insight into the transient response of subtropical gyres to changing winds. Numerical models could be used to explore the timescale by varying the imposed wind-stress curl and monitoring how long the eddy response takes to equilibrate. There has been some work examining this problem in the context of freshwater accumulation in the Beaufort Gyre in the Arctic (see e.g. Davis et al., 2014; Manucharyan and Spall, 2016), but not subtropical gyres.

8.2.4 Biogeochemistry in the oligotrophic gyres

Understanding the timescale of the eddy response to Ekman pumping may provide insight into the biogeochemical response of the oligotrophic ocean, and observing the biological response may be helpful in probing the timescale of the eddy response. As the eddies begin to cancel the Ekman pumping, the horizontal convergence of nutrients should slow causing the size of the oligotrophic region to stabilise.

The non-monotonic response of nutrient concentration to residual Ekman pumping in the idealised model presented in chapter 7 suggests that the relationship between Ekman pumping and nutrient concentration deserves further study. There has been some research on this topic; McClain et al. (2004) found that the area of ocean with low chlorophyll concentration decreased during periods of anomalously high wind-stress curl. However, there is scope for substantially more research to be done. For example, I plan to construct an Ekman pumping index consisting of the area averaged wind-stress curl over the gyre and examine spatial and temporal correlations between primary productivity, chlorophyll concentration, and this index. By exploring time-lagged spatial correlations between this index and chlorophyll concentration and primary productivity I hope to probe the timescale of the eddy response and its influence on the biogeochemistry of the oligotrophic ocean. The Ekman pumping index would also facilitate the creation of composite maps of chlorophyll concentration or primary productivity during large wind-stress curl events. These composites will hopefully provide insight into the average biogeochemical and dynamical response within the gyre. However, the criterion used to define the gyre may have an impact on the outcome of this analysis. If the gyre is defined using a static region, perhaps a contour of mean dynamic topography or productivity, then shifts in the location of the gyre will be aliased in to the results. As discussed previously, this may pose

a problem since the gradients in the background mode water thickness and productivity contain a signal that is equivalent to the prediction of the idealised model. A similar problem would still be present if the gyre were defined using a contour of productivity or chlorophyll concentration, as was done by McClain et al. (2004). The use of temporally variable regions may alleviate this problem, but introduces further complexity in to the analysis. It may also be possible to perform local comparisons between productivity and mode water thickness using datasets such as those from the Bermuda Atlantic Time-Series Study (BATS), which can be downloaded from <http://bats.bios.edu/>.

The impact of mode water thickness on oceanic productivity was briefly explored in chapter 7, and while the data suggest a positive relationship, the results were somewhat inconclusive. In the future I would like to further explore this relationship by developing a scaling that would allow more robust comparisons between different subtropical gyres.

8.3 Concluding remarks

This thesis set out to explore five key questions concerning mesoscale eddies and wind-driven gyres. By identifying a balance between mean advection and eddies in the vertical potential vorticity flux budget, we answered our first key question: the potential vorticity flux budget of eddy-resolving wind-driven gyres is dynamically similar to the Southern Ocean. This allowed us to draw on a substantial body of literature to further our understanding of the residual vertical transport within wind-driven gyres.

We showed that the Ekman cell, just like the Deacon cell in the Southern Ocean and the Ferrel cell in the atmosphere, provides a misleading description of the flow. This answered our second key question, and prompted further investigation of the residual vertical motion within subtropical gyres. This investigation addressed the third key question and led to the most significant result contained in this thesis: the residual vertical velocity within our subtropical gyre is substantially smaller than the Ekman pumping velocity. We hope that this result will spur further research into the dynamics of eddying gyres and that this will advance our understanding of the residual flow in wind-driven gyres.

The absence of large-scale downwelling at depth in the subtropical gyre led to the development of a novel two-layer toy model to describe nutrient concentrations in subtropical gyres. Using this toy model we set out to answer the final two key

questions, and found that the nutrient distribution is sensitive to changes in the residual vertical velocity. We also found that mode water thickness has a substantial impact on the nutrient concentration: a prediction that was somewhat supported by the observational data.

The research presented in this thesis has answered the five key questions that we set out to address. It has contributed to our understanding of the dynamics of wind-driven gyres and used these new insights to improve our understanding of the biogeochemistry of these oligotrophic regions.

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