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COMPETITION IN POSTED PRICES WITH BARGAINING

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Competition in Posted Prices With Bargaining ^{*}

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Abstract

In this paper we study price competition between firms when some consumers attempt to bargain while others buy at the public list or posted prices. Even though bargainers succeed in negotiating discounts off the list prices, their presence dampens competitive pressure in the market by reducing the incentive to undercut a rival's list price, thus raising all prices and increasing profits. Welfare falls because of the uncertainty in the bargaining process, which generates some misallocation of products to consumers. We also find that the bargainers facilitate collusion by reducing the market share that can be gained from a deviation.

Keywords: Posted prices; list prices; collusion; bargaining; negotiation; haggling; discounts; outside option; price takers; Hotelling line.

JEL Classification: C78; D43; L13.

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1 Introduction

Firms often set public list or posted prices that some consumers will seek to bargain down. In this paper, we study the impact that such bargainers have on price competition between firms. We find that even though bargainers secure reductions off list prices, their presence raises all prices and causes a misallocation of goods to consumers that lowers total welfare. Furthermore, when the firms interact repeatedly, the bargainers facilitate collusion by the firms.

Bargained discounts off public list prices are commonplace, and can be significant. For example, private automobile buyer discounts tend to be large on average,¹ as are discounts off realtor fees in the United Kingdom,² while public commentators have referred to evidence of bargaining off the list price in retail stores such as jewellers, shoe shops, travel agents, furniture stores and electrical retailers, further emphasizing the relevance of our research question.³

We develop a tractable model of differentiated product price competition in which two firms simultaneously set list prices that become common knowledge. ‘Price takers’ dislike bargaining, or don’t appreciate that discounts might be available, and so buy at the list prices. ‘Bargainers’, on the other hand, ask both firms for a discount off their list price. Price takers and bargainers differ only in their propensity to bargain: both categories of consumer are uniformly distributed along a Hotelling line, and so share a common view of product differentiation.

In reality, bargaining is uncertain with consumers ignorant in advance of how large a reduction, if any at all, they will receive. The psychological costs and tension of bargaining and the danger of frayed emotions lead to the possibility that negotiations between the sales representative and the consumer break down. To capture this uncertainty, we assume that a bargainer receives a particular firm’s reduced price offer (which acts as a final take-it-or-leave-it offer) with probability less than one.⁴

The presence of the bargainers in the market affects the optimal pricing strategy in important ways. Bargainers succeed in negotiating discounts off the list prices since each firm is worried that a bargainer will be tempted by a more attractive offer from the rival firm. However, bargaining

¹The Competition Commission (2000) offer evidence that private automobile buyer discounts in the United Kingdom in 1997/98 averaged 11% off Ford’s list prices, averaged 12% off Vauxhall’s list prices, and averaged 10% off Fiat’s list prices, with some discounts in the 30-40% range in each case (see in particular Appendices 7.1 and 7.2). Yet many consumers received no reduction at all (e.g., 13% in the case of Ford). A similar pattern holds for the automobile market in the United States: according to Goldberg (1996, p. 641), “the data reveal substantial variation in dealer discounts, a large fraction of which cannot be explained just by financing, or model- and time-specific variables.”

²The Office of Fair Trading (2004) report on the United Kingdom estate agency (realtor) market found that almost 50% of house sellers using an estate agent had tried to negotiate fees, with 80% of those receiving a reduction (Section 4.48).

³See, for instance, Sunday Times (2008) and Daily Telegraph (2009).

⁴We are grateful to the Editor for suggesting this helpful way of modeling the bargaining process. Bargaining models with an exogenous probability of breakdown have been studied by, for instance, Binmore et al. (1986), Stole and Zwiebel (1996) and de Fontenay and Gans (2005).

also affects the firms' incentives when they set their list prices. The firms anticipate how their chosen list prices will affect the competition for bargainers. If a firm undercuts its rival's list price, it will win some price takers. However, it will also improve the outside option of all the bargainers and so lower the equilibrium bargained price offers. The presence of the bargainers therefore dampens the competitive pressure that exists in the market for price takers. As a result, list prices rise above the standard equilibrium level without bargainers. By worsening the bargainers' outside option, the higher list prices in turn allow bargained prices to rise.

The moderating influence on competition brought about by bargainers not only increases prices, but also raises profits and lowers consumer surplus and welfare. The higher prices are simply a transfer from consumers to firms, and so are welfare neutral. However, the presence of the bargainers also lowers welfare because of the uncertainty inherent in the bargaining process. When bargaining breaks down with one of the firms, a bargainer might be left with the choice between paying a high list price for the product she prefers and a lower bargained price for the less attractive product. The consumer can, in effect, be bribed to accept a less ideal product. This generates some misallocation of products to consumers, which lowers the efficiency of the market. Furthermore, as the proportion of bargainers in the market increases, prices rise and welfare falls at an increasing rate. This means that as we increase the number of bargainers, the marginal effect of adding yet more bargainers is exacerbated.

We also study the impact of bargainers on the ability of the industry to sustain collusive outcomes, thus extending our work to a dynamic setting. To simplify the analysis, we consider the case where the products are perfect substitutes. However, we also extend our analysis by allowing any number of firms $N \geq 2$ to compete in the market. We find that the presence of bargainers facilitates collusion. The mechanism is that bargainers lower the profits available from deviating on a collusive agreement. If a firm deviates only in the deal it offers to bargainers, it foregoes any increase in the market share of price takers. If, instead, a firm deviates by lowering its publicly posted list price, then the rival firms can respond by pricing more aggressively to the bargainers. In either case, the increase in market share available to firms from a deviation is reduced, thus allowing collusion to be sustained at lower discount factors than would be possible without bargainers.

The paper proceeds as follows: Section 2 relates our results to the existing literature; Section 3 sets out the model; Section 4 characterizes the equilibria; Section 5 conducts comparative statics on prices, profits and welfare; Section 6 considers collusion; and Section 7 concludes. All proofs are relegated to the Appendix.

2 Relation to the Literature

Our work adds to the small and recent literature that considers the impact of bargaining when some consumers take list or posted prices as given. Korn (2007) and Zeng et al. (2007) consider monopolists, and so are silent about the implications of bargainers on competitive outcomes, which is the focus of this study.⁵ Desai and Purohit (2004) and Zeng et al. (2007) focus on the marketing decision of whether to permit bargaining or not. In Desai and Purohit (2004), when both firms permit bargaining, the list prices are irrelevant to the bargainers since they are never effective outside options; thus the strategic interaction between list prices and bargained prices is severed. Raskovich (2007) finds that a big enough proportion of bargainers causes list prices to jump from marginal cost to the monopoly price. The mechanism is different to ours: firms setting higher list prices are assumed to be weaker bargainers and so are more attractive to bargaining consumers. Finally, Gill and Thanassoulis (2009) present a homogeneous product quantity-setting model in which a Cournot auctioneer sets a binding public list price. There, bargaining was modeled as an application of Burdett and Judd (1983) search, resulting in mixed-strategy equilibria instead of the pure-strategy equilibria that we find here, and welfare results were not available.

Our new tractable model allows us to extend this literature by characterizing the impact of bargainers in markets with list-price-setting competition on bargained and list prices, profits and welfare. To the best of our knowledge, the mechanism by which bargaining lowers welfare by causing a misallocation of goods to consumers is new. We are also the first to study dynamic repeated competition in markets with bargaining and list prices, and so the mechanism by which bargaining facilitates collusion is also, as far as we are aware, completely new.

Our result that bargainers facilitate collusion in markets with public list prices complements the existing literature that shows how collusion can be facilitated when firms operate in more than one market. Bernheim and Whinston (1990) find that competing in multiple markets can make collusion easier since firms are able to transfer excess punishment capacity from one market to another. Spector (2007) shows that if a firm is a monopolist in one market but competes in another, then bundling can help collusion by shrinking the demand available in the competitive market.⁶ In both cases, linkages across markets make collusion more sustainable, while our complementary results show that linkages across segments within a single market can also make collusion easier to sustain.

⁵Kuo et al. (2011) and Kuo et al. (2012) also consider monopolists, with a focus on, respectively, inventory management and supply chain relationships. As well as setting a posted price, in these papers the monopolist is allowed to commit to a lower bound below which it will never sell.

⁶In contrast, Montero and Johnson (2012) identify a setting with multiple markets in which collusion is inhibited by bundling.

Our analysis also complements a broader literature in which firms sell to two different types of consumer. In Stahl (1989)’s model of search, consumers have high or low search costs. In Rosenthal (1980), Varian (1980) and Narasimhan (1988), some consumers only consider buying from one firm while others buy at the lowest price. In these papers, the two categories of consumer give rise to mixed-strategy pricing equilibria because of the inability of the firms to price discriminate.⁷ In this paper, the competing firms are able to discriminate between the consumer segments (bargainers and price takers) by setting different list and bargained prices. Compared to these earlier papers, this allows strategic interaction between the price choices for the different segments since the list prices form the outside option for the bargainers. The product differentiation in our model yields pure-strategy pricing equilibria. When we study collusion with perfect substitutes in Section 6, we do find that immediately following a deviation in list prices, and so off the equilibrium path, the firms set prices for the bargaining segment according to a mixed strategy. The mechanism giving rise to mixing for this segment is related to that which gives rise to mixing for the whole market in these earlier papers.

Much of the rest of the literature exploring bargaining in consumer markets examines the choice between committing to a fixed price and allowing consumers to bargain in the absence of a posted price (e.g., Bester, 1993, Wang, 1995, Arnold and Lippman, 1998, Camera and Delacroix, 2004, and Myatt and Rasmusen, 2009.) There is also a small literature on bargaining below a posted price when all consumers bargain (e.g., Chen and Rosenthal, 1996a, 1996b and Camera and Selcuk, 2009).

3 The Model

Two competing firms sell a differentiated product and compete in prices. The firms have the same constant marginal cost of production $c \geq 0$, have no fixed costs, and seek to maximize their expected profits. To capture product differentiation, we adopt the standard Hotelling framework: the two firms are located at the ends of a Hotelling line of length 1 with a uniform density of consumers along it, and the consumers have a linear Hotelling ‘transport cost’ $t > 0$. As in Hotelling (1929), every consumer purchases exactly one unit and the market is always covered. There are two types of consumers. A proportion $\mu \in (0, 1)$ are labeled ‘price takers’, and the remaining proportion $1 - \mu$ are labeled ‘bargainers’. A consumer’s propensity to bargain is independent of her location on the Hotelling line, and firms cannot observe a consumer’s location. We capture bargaining through the following two-stage game:

⁷In Burdett and Judd (1983)’s model of non-sequential search, consumers vary in how many firms they search, again giving rise to mixed-strategy equilibria.

1. List-price-setting stage: The firms simultaneously choose publicly posted list prices $l_i \geq 0$ and $l_j \geq 0$. Each price taker purchases at the list price that gives her the highest surplus net of transport costs.
2. Bargaining stage: The bargainers ask both firms for a better price offer. The firms simultaneously choose the bargained price $p_i \leq l_i$ to offer to bargainers, which acts as a final take-it-or-leave-it offer. Each bargainer receives a particular firm's price offer with probability $\beta \in (0, 1)$; if the bargainer does not receive the price offer, she is still able to purchase at the public list price.⁸ Each bargainer buys at the available price that gives her the highest surplus net of transport costs.⁹

Our simple bargaining model is tractable and reflects real-world bargaining in that bargainers actively approach a seller to ask for a better price than the one posted. The assumption that bargained price offers are received with probability β captures in a simple way the fact that bargaining is uncertain: the psychological costs and tension of bargaining and the danger of frayed emotions lead to the possibility that negotiations between the sales representative and the consumer break down. However, the list prices are binding and thus are always available.

The difference between price takers and bargainers can be motivated by consumers having either high or low personal costs of bargaining.¹⁰ We can think of price takers as consumers who suffer significant bargaining costs: the costs could be real, e.g., time costs, or psychological, e.g., the embarrassment of starting a negotiation; alternatively, the price takers are not aware that discounts might be available. Bargainers, on the other hand, have low costs of bargaining and are aware that firms are willing to negotiate. Given their low costs of bargaining, it is natural to assume that the bargainers approach both firms for a better price offer.

4 Equilibrium Analysis

For any given list prices, we first characterize pure-strategy Nash equilibria of the bargaining stage that are interior, i.e., in which each firm sells to a strictly positive share of the bargainers whenever its price offer is received. We then characterize symmetric pure-strategy Nash equilibria of the list-price-setting stage, under the maintained assumption that interior pure-strategy Nash equilibria are played at the bargaining stage. The solution concept is subgame-perfect

⁸The random draws that determine whether price offers are received are independent across firms and bargainers.

⁹All consumers randomize in the event of a tie.

¹⁰This heterogeneity in bargaining costs parallels the heterogeneity in personal search costs adopted in the search literature. For example, in Stahl (1989) search costs are low or high. Note, however, that our bargainers are doing more than searching: they are actively inviting sellers to beat their publicly posted list prices.

Nash equilibrium.¹¹ We restrict attention to the case where $t > 0$, i.e., there is some product differentiation. In Section 6, we will consider the case where, instead, the products are undifferentiated.

Proposition 1 describes the equilibrium bargained price offers for any list prices.

Proposition 1 *Given any list prices $l_i \geq l_j$, any interior pure-strategy Nash equilibrium of the bargaining stage must be given by:*

1. $\left\{ p_i^* = \frac{1}{2-\beta} (t+c) + \left(1 - \frac{1}{2-\beta}\right) \left(\frac{2l_j+\beta l_i}{2+\beta}\right), p_j^* = \frac{1}{2-\beta} (t+c) + \left(1 - \frac{1}{2-\beta}\right) \left(\frac{2l_i+\beta l_j}{2+\beta}\right) \right\}$
when $\frac{1}{2-\beta} (t+c) + \left(1 - \frac{1}{2-\beta}\right) \left(\frac{2l_i+\beta l_j}{2+\beta}\right) \leq l_j$; and
2. $\left\{ p_i^* = \min \left\{ \frac{t+c+l_j}{2}, l_i \right\}, p_j^* = l_j \right\}$ when $\frac{1}{2-\beta} (t+c) + \left(1 - \frac{1}{2-\beta}\right) \left(\frac{2l_i+\beta l_j}{2+\beta}\right) > l_j$.

We can see that the bargained price offers are increasing in the list prices. The higher the list prices, the less of a competitive constraint they impose on price offers at the bargaining stage. In particular, as list prices rise the outside option for a bargainer who fails to receive a bargained price offer from the rival firm becomes worse.

The Corollary to Proposition 1 describes the equilibrium bargained price offers for symmetric list prices.

Corollary 1 (to Proposition 1) *Given symmetric list prices $l_i = l_j = l$, any interior pure-strategy Nash equilibrium of the bargaining stage must be given by:*

1. $p_i^* = p_j^* = \frac{1}{2-\beta} (t+c) + \left(1 - \frac{1}{2-\beta}\right) l < l$ when $l > t+c$; and
2. $p_i^* = p_j^* = l$ when $l \leq t+c$.

We can see that when the list prices are above the standard Hotelling level of $t+c$, the bargained price offers are a weighted average of the standard Hotelling price and the list prices. Thus, when responding to the bargainers, the firms push their bargained price offers away from the list prices and towards the standard Hotelling level. How far the bargained price offers are from the list prices depends upon the probability β that the consumers receive the firms' bargained price offers. The higher is this probability, the less relevant the list prices become as an outside option, and so the closer are the bargained price offers to the standard Hotelling level.

We now turn to the firms' decision about what list prices to set. If the firms considered only the price takers, prices would be set just as in the standard Hotelling model, and so the equilibrium list prices would be $l^* = t+c$. However, when thinking about how to set list prices,

¹¹The two list prices are common knowledge; furthermore, in the equilibria we study they are equal. This holds throughout the manuscript. Nonetheless, we need to solve the bargaining stage given unequal list prices to allow us to consider the impact of deviations at the list-price-setting stage.

the firms also anticipate how the chosen list prices will affect the competition for the bargainers. We saw above that the higher the list prices, the worse the outside option for the bargainers, and so the higher the bargained price offers that can be supported in equilibrium. The presence of the bargainers moderates competitive forces in the market by reducing the incentive to undercut any given rival list price because of the impact of the list price reduction on the bargained prices. Proposition 2 describes equilibria in which this moderating force allows list prices to rise above the standard Hotelling level.

Proposition 2 *Any symmetric pure-strategy Nash equilibrium of the list-price-setting stage in which $l^* > t + c$ must be given by*

$$l^* = t + c + t \left[\frac{2(2 - \beta)(1 - \beta)\beta}{\left(\frac{\mu}{1 - \mu}\right)(4 - \beta^2)(2 - \beta) + 2(1 - \beta)(4 - 2\beta + \beta^2)} \right]. \quad (1)$$

Furthermore, Proposition 3 shows that no symmetric equilibrium can exist in which list prices are below the standard Hotelling level.

Proposition 3 *There can be no symmetric pure-strategy Nash equilibrium of the list-price-setting stage in which $l^* < t + c$.*

We have now characterized the unique symmetric equilibrium with list prices in excess of the standard Hotelling level, that is with $l > t + c$, and we have also shown that no equilibrium can exist with list prices below the standard Hotelling level, that is with $l < t + c$. Appendix B considers existence of the equilibrium with $l > t + c$. We cannot rule out an equilibrium in which the firms set list prices at the standard Hotelling level, that is with $l = t + c$.¹² However, from the firms' perspective such an equilibrium is Pareto dominated by the equilibrium with $l > t + c$, and when $l = t + c$ list prices are unaffected by the presence of bargainers. Thus, in the next section, we focus on the more interesting and Pareto dominant equilibrium with $l > t + c$ in which the bargainers do in fact affect competition between the firms, and we analyze the properties of this equilibrium.

5 Prices, Profits and Welfare

We described in Section 4 how the presence of bargainers moderates competition in the market. We saw there that the list prices act as an outside option when consumers bargain, and so the

¹²Briefly, the reason is that when $l = t + c$, both firms' equilibrium bargained price offers are constrained by their list prices, and hence an upward deviation in a firm's list price cannot induce a corresponding upward shift in the rival's bargained price offer.

presence of bargainers reduces the incentive to undercut the rival's list price. The higher list prices in turn allow bargained prices to rise. In this section, we analyze the properties of the equilibrium with list prices above the standard Hotelling level, that is with $l > t + c$, outlined in Proposition 2.¹³ Using Part 1 of the Corollary to Proposition 1, the equilibrium bargained price offers are given by

$$p = \frac{1}{2-\beta} (t + c) + \left(1 - \frac{1}{2-\beta}\right) l, \quad (2)$$

and so are a weighted average of the list prices and the standard Hotelling price.

Proposition 4 *Compared to the benchmark with only price takers, the presence of bargainers: (i) raises the list prices and bargained price offers; (ii) raises the firms' profits; (iii) lowers consumer surplus; and (iv) lowers total welfare.*

As formalized in Proposition 4, the moderating influence on competition brought about by the bargainers not only increases prices, but also raises profits and lowers welfare. Since both the bargained price offers and the list prices are higher in the presence of bargainers, profits must rise given the market is covered. The higher prices are simply a transfer from consumers to firms, and so are welfare neutral. Welfare nonetheless falls because of the uncertainty inherent in the bargaining process. When bargaining breaks down with one of the firms, a bargainer might be left with the choice between paying a high list price for the product she prefers and a lower bargained price for the less attractive product. This generates some misallocation of products to consumers, which lowers the efficiency of the market.

So far we have compared a market with a positive fraction of bargainers to the standard case where nobody bargains. Next, we turn to a comparative statics analysis of how prices, profits and welfare vary in the proportion of consumers who bargain.

Proposition 5 *As the proportion of bargainers increases: (i) the list prices rise; (ii) the bargained price offers rise; and (iii) the difference between the list prices and the bargained price offers also rises. Furthermore, all three rise at an increasing rate.*

Proposition 5 considers the impact on prices. The moderating influence of bargainers on competition means that both list prices and bargained price offers go up as we increase the proportion of bargainers in the market. The list prices become increasingly set to allow profits to be made from the bargainers. The proposition further tells us that the list prices go up faster than the bargained prices, so the difference between the list prices and the bargained prices also increases in the proportion of bargainers. Finally, we can see that the prices go up at an increasing rate: as bargainers become more prevalent in the population of consumers, the reduction in the competitive pressure on list prices becomes increasingly powerful, resulting in

¹³Throughout this section and the related proofs, for notational clarity we omit the stars when referring to equilibrium prices.

a convex increase in the list prices. Since the bargained prices are a weighted average of the list prices and the standard Hotelling price, the bargained prices also inherit this convexity. The convexity means that as we increase the number of bargainers, the marginal effect on prices of adding yet more bargainers to the market is compounded.

Proposition 6 *As the proportion of bargainers increases, total welfare falls. Furthermore, total welfare falls at an increasing rate.*

Proposition 6 tells us that welfare is always decreasing in the proportion of bargainers. Since the market is covered, and so prices are just a transfer from consumers to firms, the effect of bargainers on welfare depends only on how the bargainers affect the misallocation of goods to consumers. We just saw that the greater the proportion of bargainers, the greater the difference between the list prices and the bargained prices. This increasing disparity makes bargainers who receive a bargained price offer from only one of the firms more likely to settle for a lower priced but less attractive product, thus worsening the misallocation of goods and hence lowering welfare. The proposition also tells us that welfare falls at an increasing rate: this concavity of welfare in the proportion of bargainers follows from the convexity of the difference between list prices and bargained prices reported in Proposition 5. Summarizing, increasing the number of bargainers raises prices, lowers welfare, and exacerbates the marginal effect on prices and welfare of adding yet more bargainers. Figure 1 shows graphically how prices and welfare change in the proportion of consumers who bargain.

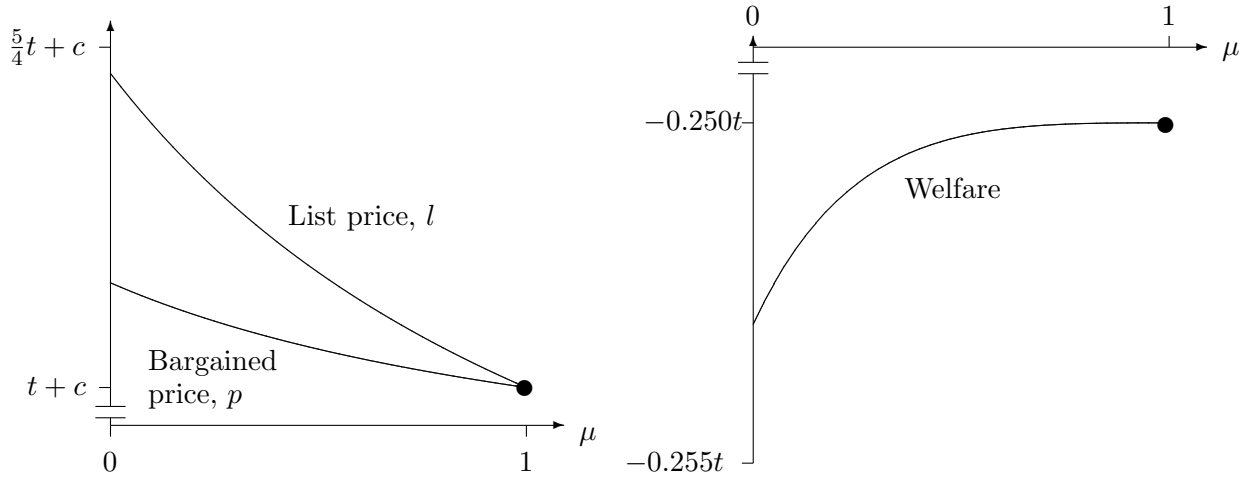


Figure 1: Prices and Welfare as Proportion of Bargainers Changes

Notes: The proportion of bargainers increases from 0 to 1 as the proportion of price takers μ falls from 1 to 0. The dot in each graph gives the benchmark case of Hotelling competition with only price takers. Both graphs are drawn to scale, with $\beta = 1/2$. The welfare graph normalizes surplus from a product at a consumer's exact location on the Hotelling line to zero.

Finally, we consider how profits and consumer surplus change in the proportion of bargainers. We have just seen that total welfare always decreases as the number of bargainers goes up. The impact on profits and consumer surplus is not so clear cut. Proposition 5 shows that increasing the number of bargainers raises both bargained prices and list prices. Clearly, this hurts consumers who remain price takers or were already bargaining. On the other hand, the consumers who become bargainers switch from a high list price to a lower bargained price and so pay less. It is possible to construct examples in which profits and consumer surplus are not monotonic in the proportion of bargainers. However, Proposition 7 shows that when the probability β that the consumers receive the firms' bargained price offers is not too large, profits always go up and consumer surplus always declines in the proportion of bargainers.¹⁴ In that case, the price reduction secured by new bargainers is always outweighed by the increase in prices for the other consumers.

Proposition 7 *When the bargaining technology is such that $\beta \in [0, 0.62]$, the firms' profits rise and consumer surplus falls as the proportion of bargainers increases.*

6 Collusion

Thus far we have analyzed one-shot competition and demonstrated that the presence of bargainers reduces the competitive pressure on firms when setting their list prices, thus allowing list prices, and subsequently bargained prices, to rise. In this section we study dynamic competition in markets with both bargainers and price takers, and we will conclude that the presence of bargainers can facilitate collusion by lowering the critical discount factor above which collusion can be sustained.

6.1 Dynamic List Pricing and Bargaining Model

We approach the question of collusion in the standard way of seeking a symmetric subgame-perfect Nash equilibrium in which the firms collude on a common price $z > c$. As our analysis allows for both list price setting and bargaining, we investigate collusion in which the firms collude on the same price z at both the list-price-setting stage and the bargaining stage.¹⁵ To simplify the analysis, we consider the case where $t = 0$, i.e., the products are perfect substitutes.¹⁶ However, we extend our analysis by allowing any number of firms $N \geq 2$ to compete in the market. As is common in collusion analyses, we focus on collusive equilibria supported by the

¹⁴The proof shows only that the range of β is sufficient for profits to increase and consumer surplus to fall in the proportion of bargainers. Numerical analysis suggests that the result extends to a wider range of β .

¹⁵If we introduce a common maximum willingness to pay $v > c$, then our analysis applies for any collusive price $z \in (c, v]$. When $z = v$, the firms collude on the monopoly price.

¹⁶At the end of this section, we discuss how our results extend when we allow some product differentiation.

threat of reversion to the lowest-payoff non-collusive symmetric equilibrium. Formally, the two-stage one-shot game of Section 3 is assumed to be infinitely repeated, with all firms sharing a discount factor δ .

6.2 One-shot Game Analysis

Our first task is to establish equilibrium behavior when the two-stage game is played only once. Just as in Section 4, we first consider equilibrium behavior in the bargaining stage for any list prices, and then use this to work out the equilibrium at the list-price-setting stage.

Lemma 1 demonstrates that the equilibrium of the bargaining stage involves the firms setting bargained prices according to a mixed strategy.¹⁷

Lemma 1 *Suppose that the list prices are given by $\{l_j : j \in \{1, 2, \dots, N\}\}$ and suppose that $n \geq 1$ firms, including firm i , set the lowest list price $\underline{l} \equiv \min \{l_j\} > c$. Then there is a unique symmetric Nash equilibrium of the bargaining stage, in which all firms offer prices*

$$p \in ((1 - \beta)^{N-1}(\underline{l} - c) + c, \underline{l})$$

drawn from the distribution

$$F(p) = \frac{1}{\beta} - \left(\frac{1 - \beta}{\beta} \right) \left(\frac{\underline{l} - c}{p - c} \right)^{\frac{1}{N-1}}. \quad (3)$$

Firm i makes expected profits from the bargainers of:

$$\pi_i = (l_i - c) (1 - \beta)^{N-1} \left(\frac{1 - \beta}{n} + \beta \right).$$

This recourse to mixed strategies is a consequence of the assumption of perfect substitutes: if a firm sets a bargained price of $p_i > c$ for sure, then a rival could gain the business of all the bargainers who receive both bargained price offers (and no lower offers from other firms) at essentially zero cost by just undercutting p_i . In the mixed-strategy equilibrium, a firm trades off the incentive to price high to profit from bargainers who receive few bargained price offers from the rival firms against the incentive to price low to increase the probability of selling to bargainers who receive many price offers.

Lemma 2 studies the symmetric equilibrium of the two-stage one-shot game.

Lemma 2 *The unique symmetric pure-strategy Nash Equilibrium of the one-shot game has all prices at marginal cost and profits are zero.*

¹⁷Our preferred interpretation is that for every bargainer a firm draws a price from the pricing distribution. It makes no difference to the analysis, however, if a firm draws a single price from the pricing distribution, which it then offers to all bargainers.

Since the products are perfect substitutes, in the standard way a firm which undercuts its rivals' list price by an arbitrarily small amount secures the business of all the price takers. In addition, Lemma 1 shows that undercutting the rivals' list price also increases profits from the bargainers: the reason is that some bargainers will fail to receive bargained price offers from any firm, and these consumers will all buy from the firm with the lower list price. This remorseless Bertrand logic pushes list prices down to marginal cost, and so the bargained prices are forced down to cost also.

6.3 Dynamic Game Analysis

We are now in a position to consider collusion in the dynamic game. Proposition 8 shows that bargainers facilitate collusion.

Proposition 8 *Consider a market with $N \geq 2$ firms. When goods are perfect substitutes, the presence of bargainers facilitates collusion compared to the benchmark with only price takers: the critical discount factor that allows collusion to be subgame perfect is strictly lower with bargainers.*

The presence of the bargainers means that there are two ways for firms to deviate on a collusive equilibrium. Despite this, the presence of the bargainers reduces the profits available from deviation, thus making collusion easier to sustain. First, a colluding firm could deviate at the list-price-setting stage. In the period in which the deviation occurred, rival firms, observing this deviation, would be in a position to respond by bargaining more aggressively (Lemma 1 gives the rivals' bargained price distribution immediately after the deviation). Alternatively, a colluding firm could maintain the collusive list prices, but deviate when bargaining. In that case the deviant firm would be able to sell to only $1/N$ of the price takers. With or without bargainers, the continuation payoff in the periods after the deviation is zero (Lemma 2 proves this for the case with bargainers; the case without is standard). Which of the two possible deviations is optimal depends upon the parameters; in both cases, however, the deviant firm will be unable to secure the entirety of the market.

The limiting case of perfect substitutes yields clear results in an otherwise complicated setting. When products are differentiated, it remains the case that the presence of the bargainers reduces the market share gained from a deviation, since the rivals can respond at the bargaining stage to list price deviations, and deviations at the bargaining stage fail to increase the market share of price takers. With product differentiation, a countervailing force now comes into play: as we saw earlier, the presence of the bargainers increases profits in the one-shot game above zero, and so reduces the punishment cost. When the degree of product differentiation is small,

however, this countervailing effect is also small because profits in the one-shot game are close to zero.

7 Conclusion

Bargained discounts off public list prices are commonplace. In this paper we have developed and analyzed a model of price competition between firms when some consumers bargain while others buy at list prices. We extend the literature by characterizing the impact of bargainers in markets with list-price-setting competition on bargained and list prices, profits and welfare. We find that the presence of bargainers dampens competition for price takers and so allows prices and profits to rise. Furthermore, we document that bargaining lowers welfare through a new misallocation channel: uncertainty in the bargaining process leads to a loss of allocative efficiency with some consumers buying their less preferred product. We also provide an entirely new characterization of how bargainers affect the ability of an industry to sustain collusion. The presence of bargainers facilitates collusion because the profits available from deviating are reduced: if a cartel member deviates on the list price, then it can expect an immediate aggressive retaliation when competing for bargainers; if, instead, the deviation is to bargainers, then deviant profits from list price takers are forgone.

We have used a simple but realistic bargaining formulation that captures in a tractable way the fact that bargaining can break down. However, we believe that the specific way in which bargaining is modeled is not the source of our results. Rather the main driver is the link between the list prices and the bargainers' outside option. Future work could profitably extend our theoretical analysis to more complex bargaining environments with, for example, alternating offers. Empirical and experimental work could also test whether, in practice, markets with the possibility for bargaining below a list price increase firms' ability to agree and stick to collusive agreements as we predict.

Appendix

Appendix A: Proofs

Proof of Proposition 1. At the bargaining stage, p_i only affects profits in the case when both bargained price offers are received and the case where only the firm's offer is received. Thus, relevant profits in the interior are given by

$$(\text{constant}) + \beta^2 (p_i - c) \left(\frac{t + p_j - p_i}{2t} \right) + \beta (1 - \beta) (p_i - c) \left(\frac{t + l_j - p_i}{2t} \right),$$

and are strictly concave in p_i . The first-order condition,

$$\beta^2 [(p_i - c)(-1) + (t + p_j - p_i)] + \beta (1 - \beta) [(p_i - c)(-1) + (t + l_j - p_i)] = 0,$$

gives the linear reaction function $p_i = [t + c + (1 - \beta) l_j + \beta p_j] / 2$. Solving simultaneously for p_i and p_j gives

$$\hat{p}_i = \frac{1}{2 - \beta} (t + c) + \left(1 - \frac{1}{2 - \beta} \right) \left(\frac{2l_j + \beta l_i}{2 + \beta} \right) \text{ and } \hat{p}_j = \frac{1}{2 - \beta} (t + c) + \left(1 - \frac{1}{2 - \beta} \right) \left(\frac{2l_i + \beta l_j}{2 + \beta} \right).$$

Note that $\hat{p}_i \leq \hat{p}_j$, given $l_i \geq l_j$ and $\beta < 1$. Thus, when $\hat{p}_j \leq l_j$, $p_i^* = \hat{p}_i$ and $p_j^* = \hat{p}_j$. When instead $\hat{p}_j > l_j$, the fact the reaction functions intersect only once implies that at least one firm must be constrained by its list price. We cannot have an interior equilibrium with $p_i^* = l_i$ and $p_j^* < l_j$ since then $p_i^* > p_j^* = [t + c + (1 - \beta) l_i + \beta p_i^*] / 2 > [t + c + (1 - \beta) l_j + \beta p_j^*] / 2$, so firm i would want to deviate downward given concavity of profits in the interior. Instead, any interior equilibrium must be given by $p_j^* = l_j$ and

$$p_i^* = \min \left\{ \frac{t + c + (1 - \beta) l_j + \beta l_j}{2}, l_i \right\} = \min \left\{ \frac{t + c + l_j}{2}, l_i \right\}.$$

Note that firm i will hit its constraint when $(t + c + l_j) / 2 > l_i$ given concavity of profits in the interior. ■

Proof of Proposition 2. We use the maintained assumption that interior pure-strategy Nash equilibria are played at the bargaining stage. If the interior pure-strategy Nash Equilibrium of the bargaining stage given in Proposition 1 exists at list prices $\{l_i, l_j\}$, then firm i 's profits are given by

$$\begin{aligned}
2t \cdot \pi_i(l_i, p_i^*(l_i), p_j^*(l_i)) &= \left[\mu + (1 - \mu)(1 - \beta)^2 \right] (l_i - c)(t + l_j - l_i) \\
&+ (1 - \mu)\beta(1 - \beta)(p_i^* - c)(t + l_j - p_i^*) \\
&+ (1 - \mu)(1 - \beta)\beta(l_i - c)(t + p_j^* - l_i) \\
&+ (1 - \mu)\beta^2(p_i^* - c)(t + p_j^* - p_i^*). \tag{4}
\end{aligned}$$

The first line gives firm i 's profits from the proportion μ of price takers, and from the bargainers who receive neither bargained price offer; firm i 's market share is given by $(t + l_j - l_i) / 2t$. The second (third) line gives profit from the bargainers who receive only firm i 's (firm j 's) offer. The final line gives profits from the bargainers who receive both offers. The total derivative of profits with respect to the list price l_i is thus given by

$$\begin{aligned}
2t \cdot \frac{d\pi_i}{dl_i} &= \left[\mu + (1 - \mu)(1 - \beta)^2 \right] (t + l_j - 2l_i + c) \\
&+ (1 - \mu)\beta(1 - \beta)(t + l_j - 2p_i^* + c) \frac{dp_i^*}{dl_i} \\
&+ (1 - \mu)(1 - \beta)\beta \left[(t + p_j^* - 2l_i + c) + (l_i - c) \frac{dp_j^*}{dl_i} \right] \\
&+ (1 - \mu)\beta^2 \left[(t + p_j^* - 2p_i^* + c) \frac{dp_i^*}{dl_i} + (p_i^* - c) \frac{dp_j^*}{dl_i} \right]. \tag{5}
\end{aligned}$$

Using Proposition 1 and its Corollary, and the assumption that $l > t + c$, at a symmetric list price equilibrium: (i) $l_i = l_j = l$; (ii) $p_i^* = p_j^* = \frac{1}{2-\beta}(t + c) + \left(1 - \frac{1}{2-\beta}\right)l$; and (iii)

$$\frac{dp_i^*}{dl_i} = \frac{\beta(1 - \beta)}{4 - \beta^2} \text{ and } \frac{dp_j^*}{dl_i} = \frac{2(1 - \beta)}{4 - \beta^2}. \tag{6}$$

Note also that at a symmetric list price equilibrium with $l > t + c$, the profit function is strictly concave in l_i .¹⁸ The result follows by substituting (6) into the first-order condition given by setting (5) = 0, and then solving for l . ■

Proof of Proposition 3. We use the maintained assumption that interior pure-strategy equilibria are played at the bargaining stage. Suppose a symmetric equilibrium exists with $l < t + c$. If the interior pure-strategy Nash Equilibrium of the bargaining stage given in Proposition 1 exists, we know from Proposition 1 and its Corollary that $p_i^* = p_j^* = l$ and that, if firm i raises its list price to $l_i = l + \varepsilon$ for some small $\varepsilon > 0$, then p_j^* remains unchanged at l

¹⁸Some tedious algebra reduces the second derivative to $2t(4 - \beta^2)^2 \frac{d^2\pi_i}{d(l_i)^2} = -2\mu(4 - \beta^2)^2 - 2(1 - \mu)(1 - \beta)[(4 - \beta^2)(4 - 2\beta + \beta^2) - \beta^3(1 - \beta)]$. Thus $\frac{d^2\pi_i}{d(l_i)^2} < 0$ since $4 - \beta^2 > \beta^3$ and $4 - 2\beta + \beta^2 > 1 - \beta$ given $\beta \in (0, 1)$.

while p_i^* rises to $l_i = l + \varepsilon$. Since $l < t + c$, that is below the standard Hotelling equilibrium level, a deviation by firm i to $l + \varepsilon$ therefore strictly increases expected profits, yielding the desired contradiction. ■

Proof of Proposition 4. From (1) and (2) and given $\beta \in (0, 1)$, when $\mu < 1$ the list prices and bargained price offers rise above the standard Hotelling price in the absence of bargainers of $t + c$. Since the market is covered, every consumer buys at a price above $t + c$, and the equilibrium is symmetric, each firm's profits must exceed the standard Hotelling level of $t/2$. Since the market is covered, total welfare falls linearly in transport costs. Transport costs are higher than the standard Hotelling level, since the indifferent bargainer who receives only one bargained price offer lies away from $1/2$ on the Hotelling line. Finally, since total welfare is lower and profits are higher, consumer surplus must be lower. ■

Proof of Proposition 5. The proportion of bargainers rises as the proportion μ of price takers falls. Let

$$g \equiv \mu (4 - \beta^2) (2 - \beta) + 2 (1 - \mu) (1 - \beta) (4 - 2\beta + \beta^2).$$

Given $\mu \in (0, 1)$ and $\beta \in (0, 1)$, $g > 0$. Using (1),

$$\frac{dl}{d\mu} = -2\beta (1 - \beta) (2 - \beta)^2 (4 - \beta^2) (tg^{-2}) < 0; \quad (7)$$

$$\frac{d^2l}{d\mu^2} = 4\beta^2 (1 - \beta) (2 - \beta)^2 (4 - \beta^2) [3\beta^2 + 8(1 - \beta)] (tg^{-3}) > 0. \quad (8)$$

(7) gives (i), while the convexity of list prices shown in (8) implies that the list prices rise at an increasing rate. Using (2), (7) and (8),

$$\frac{dp}{d\mu} = \left(\frac{1 - \beta}{2 - \beta} \right) \frac{dl}{d\mu} < 0 \quad \text{and} \quad \frac{d^2p}{d\mu^2} = \left(\frac{1 - \beta}{2 - \beta} \right) \frac{d^2l}{d\mu^2} > 0; \quad (9)$$

hence (ii) holds and the bargained price offers rise at an increasing rate. Finally, using (1), (2), (7) and (8),

$$\frac{d(l - p)}{d\mu} = \frac{d}{d\mu} \left(\frac{1}{2 - \beta} [l - (t + c)] \right) = \left(\frac{1}{2 - \beta} \right) \frac{dl}{d\mu} < 0; \quad (10)$$

$$\frac{d^2(l - p)}{d\mu^2} = \left(\frac{1}{2 - \beta} \right) \frac{d^2l}{d\mu^2} > 0. \quad (11)$$

Therefore (iii) holds and the difference rises at an increasing rate. ■

Proof of Proposition 6. Since the market is covered, welfare falls linearly in transport costs.

Denoting transport costs by T ,

$$T = \left\{ \mu + (1 - \mu) \left[(1 - \beta)^2 + \beta^2 \right] \right\} \left(\frac{t}{4} \right) + (1 - \mu) \beta (1 - \beta) 2 \left(\int_0^{\frac{1}{2} + \frac{l-p}{2t}} t x dx + \int_{\frac{1}{2} + \frac{l-p}{2t}}^1 t (1 - x) dx \right).$$

The first line captures the average transport cost of those who choose between two equal prices.

The second line captures the transport cost of the bargainers who receive only one bargained price offer. Differentiating and simplifying yields:

$$\begin{aligned} \frac{dT}{d\mu} &= \frac{\beta(1-\beta)}{t} \left[-\frac{(l-p)^2}{2} + (1-\mu)(l-p) \frac{d(l-p)}{d\mu} \right]; \\ \frac{d^2T}{d\mu^2} &= \frac{\beta(1-\beta)}{t} \left\{ \left[-2(l-p) + (1-\mu) \frac{d(l-p)}{d\mu} \right] \frac{d(l-p)}{d\mu} + (1-\mu)(l-p) \frac{d^2(l-p)}{d\mu^2} \right\}. \end{aligned}$$

Using (10), $dT/d\mu < 0$, and so total welfare falls in the proportion of bargainers. Using (10) and (11), $d^2T/d\mu^2 > 0$, and so total welfare falls at an increasing rate. ■

Proof of Proposition 7. Using (4) to give a firm's profit π and differentiating:

$$\begin{aligned} \pi &= \frac{1}{2} [\beta\mu + (1-\beta)] (l-c) + \frac{1}{2} (1-\mu) \beta (p-c) - \frac{1}{2} (1-\mu) (1-\beta) \beta t^{-1} (l-p)^2; \text{ and} \\ \frac{d\pi}{d\mu} &= \frac{1}{2} \beta (l-c) + \frac{1}{2} [\beta\mu + (1-\beta)] \frac{dl}{d\mu} - \frac{1}{2} \beta (p-c) + \frac{1}{2} (1-\mu) \beta \frac{dp}{d\mu} \\ &\quad + \frac{1}{2} (1-\beta) \beta t^{-1} (l-p)^2 - (1-\mu) (1-\beta) \beta t^{-1} (l-p) \frac{d(l-p)}{d\mu}. \end{aligned}$$

Using (1), (2), (7), (9) and (10), and after some manipulation,

$$\frac{\frac{d\pi}{d\mu}}{\frac{dl}{d\mu}} = \frac{1}{2} + \left(\frac{1-\mu}{2-\beta} \right) \frac{\beta}{2} \left\{ 1 - \underbrace{\left[\mu + (1-\mu) \frac{(2-2\beta)(4-2\beta+\beta^2)}{(2-\beta)(4-\beta^2)} + 2 \right]}_{(i)} \left[1 + (1-\beta) \left(\frac{l-p}{t} \right) \right] \right\}.$$

Since $2 - \beta > 2 - 2\beta$ and $4 - \beta^2 > 4 - 2\beta + \beta^2$, $(i) < 3$. Furthermore, from the expression for $l - p$ on the left-hand side of (15), $\sup \{l - p : \mu \in (0, 1)\} = \beta (4 - 2\beta + \beta^2)^{-1} t$, and so $(l - p)/t < \beta (4 - 2\beta + \beta^2)^{-1}$. Hence we can determine a bound:

$$\begin{aligned} \frac{\frac{d\pi}{d\mu}}{\frac{dl}{d\mu}} &> \frac{1}{2} + \left(\frac{1-\mu}{2-\beta} \right) \frac{\beta}{2} \left[-2 - 3 \frac{(1-\beta)\beta}{(4-2\beta+\beta^2)} \right] \\ &= \frac{8 - 16\beta + 5\beta^2 + \mu\beta(8 - \beta - \beta^2)}{2(2-\beta)(4-2\beta+\beta^2)}. \end{aligned} \tag{12}$$

Now (12) > 0 for any $\mu > 0$ if $8 - 16\beta + 5\beta^2 > 0$, which in turn requires $\beta < (8 - 2\sqrt{6})/5 \approx 0.6202$. From (7) $dl/d\mu < 0$, and so for $\beta \in [0, (8 - 2\sqrt{6})/5]$ we have $d\pi/d\mu < 0$, and hence profits increase in the proportion of bargainers. From Proposition 6, total welfare always falls in the proportion of bargainers, and hence consumer surplus falls when profits increase. ■

Proof of Lemma 1. We start by showing that any symmetric equilibrium must be mixed with: (i) $F(\underline{l}) = 1$; (ii) no mass points in the density function; and (iii) $F(p) < 1$ for $p < \underline{l}$.

(i) Recall from Section 3 that $p_i \leq l_i$, and by assumption firm i is one of the $n \geq 1$ firms setting the lowest list price $\underline{l}$, so $p_i \leq \underline{l}$. Thus, in a symmetric equilibrium $F(\underline{l}) = 1$.

(ii) If there were a mass point at price $p > c$, a firm could deviate profitably by lowering its bargained price to $p - \varepsilon$ just below the mass point whenever it would have offered p . This increases sales by a discrete amount (when the bargainer also receives an offer of p from a rival firm and receives no lower offers) in return for a vanishingly small loss and so is a profitable deviation. If there were a mass point at $p = c$, a firm would deviate upward to sell at a strictly positive profit to bargainers who receive its offer and not any of the rivals’.

(iii) Suppose that the support of F is bounded above at $p < \underline{l}$. From (ii), the probability that any of the rival firms offers this highest price to the bargainers is zero. Thus any firm offering the highest bargained price could deviate profitably by raising price towards $\underline{l}$ since the firm will continue to sell at the offered price if and only if the bargainer receives its offer and not any of the rivals’.¹⁹

Now suppose that firm i offers a bargained price $p < \underline{l}$, and the rival firms draw their bargained prices from the same distribution F . If a bargainer does not receive firm i ’s offer, then the firm sells at l_i with probability $1/n$ when the bargainer also fails to receive any of the other firms’ offers. If, instead, a bargainer does receive the offer, then the firm sells at p when p is below any other offers received by the bargainer. The probability that a bargainer receives k of the $N - 1$ rival firms’ offers is

$$\beta^k (1 - \beta)^{N-1-k} \binom{N-1}{k}$$

where the binomial coefficient counts the number of (unordered) combinations of k rivals that can be constructed out of a set of $N - 1$. Combining, we can write firm i ’s expected profit from

¹⁹Even if the density is zero at the highest price in the support of the distribution, by continuity profit at this price must be the same as for prices in the interior of the mixing distribution.

the bargainers at any offered price $p < l_i$ as

$$\pi_i(p) = (l_i - c)(1 - \beta)^N \frac{1}{n} + (p - c)\beta \sum_{k=0}^{N-1} \beta^k (1 - \beta)^{N-1-k} \binom{N-1}{k} (1 - F(p))^k.$$

Using the Binomial Theorem (e.g., Kreyszig, 1993, p. 1165), we have

$$\begin{aligned} \pi_i(p) &= (l_i - c)(1 - \beta)^N \frac{1}{n} + (p - c)\beta [1 - \beta + \beta(1 - F(p))]^{N-1} \\ &= (l_i - c)(1 - \beta)^N \frac{1}{n} + (p - c)\beta (1 - \beta F(p))^{N-1}. \end{aligned} \quad (13)$$

For firm i to be willing to randomize, its profit must be constant at all points in the support of F . To find π_i , we consider firm i 's profit from setting a price which tends to the upper bound of the support of F , that is $l_i = \underline{l}$:

$$\begin{aligned} \pi_i &= \lim_{p \rightarrow \underline{l}_i} \pi_i(p) = (l_i - c)(1 - \beta)^N \frac{1}{n} + (l_i - c)\beta (1 - \beta)^{N-1} \\ &= (l_i - c)(1 - \beta)^{N-1} \left(\frac{1 - \beta}{n} + \beta \right). \end{aligned} \quad (14)$$

Equating (13) and (14) yields

$$\left(\frac{l_i - c}{p - c} \right) (1 - \beta)^{N-1} = (1 - \beta F(p))^{N-1}.$$

This can be solved to yield (3). The lower bound of the support $\underline{p}$ can then be determined by setting $F(\underline{p}) = 0$.

Clearly, firm i has no incentive to deviate downward from $\underline{p}$: from (ii) there can't be a mass point at $\underline{p}$, so the firm would continue to sell to the same proportion of bargainers. It is readily confirmed that any firm j whose list price is above $\underline{l}$ has the same pricing distribution since the profit for such a firm j is the same as for firm i , except that firm j makes no profit when its offer is not received. We also have to check that such a firm j has no incentive to deviate to $p_j \in [\underline{l}, l_j]$: at $p_j = \underline{l}$ the firm would sell to only $1/(n+1)$ of the bargainers who receive only its offer; at $p_j > \underline{l}$ the firm would fail to sell to any of the bargainers when its offer is received. ■

Proof of Lemma 2. Suppose first that there is a symmetric equilibrium with list prices $l^* > c$. A given firm i could deviate profitably by lowering its list price to $l^* - \varepsilon$. The firm would then sell to all the price takers. From Lemma 1, profits from bargainers would also rise, since there would then be $n = 1$ firms with the lowest list price as opposed to $n = N$. Symmetric list prices $l^* = c$ form an equilibrium, since profits from price takers and bargainers are zero at $l_i \geq \left\{ l_j^* : j \neq i \right\} = c$. ■

Proof of Proposition 8. Recall that we are looking for subgame-perfect Nash equilibria in which the firms collude on a price z at the list-price-setting stage and the bargaining stage, supported by the threat of reversion to the lowest-payoff non-collusive symmetric equilibrium. In the benchmark case with only price takers, it is well-known that such collusion can be sustained by the threat of reversion to the zero-profit one-shot equilibrium when the discount factor $\delta \geq 1 - 1/N$.

Lemma 1 with $n = 1$ gives profits in the unique non-collusive symmetric equilibrium in the bargaining stage during a period in which a deviation from $z > c$ occurred at the list-price-setting stage. By Lemma 2, the lowest-payoff non-collusive symmetric equilibrium of the one-shot game must give profits of zero in the periods after a deviation occurred. Deviating at the list-price-setting stage to $l_i = z - \varepsilon$ wins all the price takers. Using Lemmas 1 and 2, total profit from this deviation is given by

$$\pi^{\text{dev1}} \simeq \mu(z - c) + (1 - \mu)(z - c)(1 - \beta)^{N-1} = (z - c) \left[\mu + (1 - \mu)(1 - \beta)^{N-1} \right].$$

An alternative deviation would be to deviate at the bargaining stage to $p_i = z - \varepsilon$, instead of deviating at the list-price-setting stage. The deviant firm would capture all the bargainers whenever its price offer was received and $1/N$ of the bargainers otherwise. Thus, total profit from this deviation (including profit at the list price-setting-stage preceding the deviation) is given by

$$\pi^{\text{dev2}} \simeq \mu \left(\frac{z - c}{N} \right) + (1 - \mu) \left[\beta(z - c) + (1 - \beta) \left(\frac{z - c}{N} \right) \right] = \left(\frac{z - c}{N} \right) [1 + (1 - \mu)\beta(N - 1)].$$

Hence the collusion can be sustained at all discount factors $\delta \geq \delta^\dagger$, where

$$\begin{aligned} \frac{z - c}{N(1 - \delta^\dagger)} &= \left(\frac{z - c}{N} \right) \max \left\{ N \left[\mu + (1 - \mu)(1 - \beta)^{N-1} \right], 1 + (1 - \mu)\beta(N - 1) \right\}, \text{ i.e.,} \\ \delta^\dagger &= 1 - \frac{1}{\max \left\{ N \left[\mu + (1 - \mu)(1 - \beta)^{N-1} \right], 1 + (1 - \mu)\beta(N - 1) \right\}}. \end{aligned}$$

Clearly, $\delta^\dagger < 1 - 1/N$, since

$$\max \left\{ N \left[\mu + (1 - \mu)(1 - \beta)^{N-1} \right], 1 + (1 - \mu)\beta(N - 1) \right\} < N$$

given $\mu \in (0, 1)$ and $\beta \in (0, 1)$. ■

Appendix B: Existence

Proposition 9 *The unique candidate interior pure-strategy Nash equilibrium of the bargaining stage exists for the symmetric list prices given by (1).*

Proof. Since $l > t + c$, any interior pure-strategy Nash equilibrium of the bargaining stage must be given by Part 1 of the Corollary to Proposition 1. To show existence, we must first show that p_i^* does in fact lie in the interior. As the firms set the same bargained prices in the candidate equilibrium, they must share the bargainers who receive both offers. Thus, for p_i^* to lie in the interior, we just need that $p_i^* \in (l - t, l + t)$. Clearly, $p_i^* < l + t$ since $p_i^* < l$. Next, using the expression for p_i^* from Part 1 of the Corollary to Proposition 1,

$$l - t < p_i^* \Leftrightarrow l - p_i^* < t \Leftrightarrow \frac{1}{2 - \beta} [l - (t + c)] < t.$$

Using (1), this inequality holds if and only if

$$\frac{1}{2 - \beta} t \left[\frac{2(2 - \beta)(1 - \beta)\beta}{\left(\frac{\mu}{1 - \mu}\right)(4 - \beta^2)(2 - \beta) + 2(1 - \beta)(4 - 2\beta + \beta^2)} \right] < t \Leftrightarrow \quad (15)$$

$$\frac{2(1 - \beta)\beta}{\left(\frac{\mu}{1 - \mu}\right)(4 - \beta^2)(2 - \beta) + 2(1 - \beta)(4 - 2\beta + \beta^2)} < 1. \quad (16)$$

Since $\left(\frac{\mu}{1 - \mu}\right)(4 - \beta^2)(2 - \beta) > 0$ given $\mu \in (0, 1)$ and $\beta \in (0, 1)$, (16) holds so long as $\frac{2(1 - \beta)\beta}{2(1 - \beta)(4 - 2\beta + \beta^2)} < 1$. This simplifies to $0 < 4 - 3\beta + \beta^2$, which holds given $\beta \in (0, 1)$.

From the proof of Proposition 1, the firm does not wish to deviate to any p_i in the interior, since profits are strictly concave in the interior. The firm will not deviate to $p_i \leq l - t$, since the profit function in the proof of Proposition 1 overstates profits in that case because it assumes that market share continues to increase beyond 1, and even with that profit overstatement the firm has no incentive to deviate given concavity. If firm i deviated to $p_i \geq p_j + t$, all bargainers who receive the rival's bargained offer would purchase from the rival. The profits to firm i from the bargainers who only receive its offer would be

$$\frac{\beta(1 - \beta)(p_i - c)(t + l - p_i)}{2t}, \quad (17)$$

dropping to zero if $p_i \geq l + t$. This profit function is concave and maximized at $p_i = (t + c + l)/2$. A sufficient condition for the firm to not wish to deviate is that (17) be decreasing in the range $p_i \geq p_j + t$, which in turn requires that the maximand lies below this range. This holds if $l < 2p_j + t - c$. Using the expression for p_j^* from Part 1 of the Corollary to Proposition 1 and

re-arranging, the sufficient condition becomes

$$l < \frac{(4 - \beta)t + \beta c}{\beta}. \quad (18)$$

Using (1) and noting that $\left(\frac{\mu}{1-\mu}\right)(4 - \beta^2)(2 - \beta) > 0$, at the symmetric list prices in Proposition 2, (18) must hold whenever

$$t + c + t \left[\frac{2(2 - \beta)(1 - \beta)\beta}{2(1 - \beta)(4 - 2\beta + \beta^2)} \right] < \frac{(4 - \beta)t + \beta c}{\beta}.$$

This simplifies to

$$0 < \left(\frac{2 - \beta}{\beta}\right) \left(\frac{8 - 4\beta + \beta^2}{4 - 2\beta + \beta^2}\right) t,$$

which holds given $\beta \in (0, 1)$ and $t > 0$. ■

Proposition 10 *The unique candidate symmetric pure-strategy Nash equilibrium of the list-price-setting stage in which $l^* > t + c$, characterized in Proposition 2, exists as a local Nash equilibrium.*

Proof. From Proposition 9, the unique candidate interior pure-strategy Nash equilibrium of the bargaining stage exists at the symmetric list prices. By continuity, the interior equilibrium of the bargaining stage continues to exist for small deviations in the list price. Since the proof of Proposition 2 shows that profit is strictly concave in l_i , firm i has no local incentive to deviate from $l^* > t + c$. ■

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