

Enhanced Communication With the Assistance of Indefinite Causal Order

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In quantum Shannon theory, the way information is encoded and decoded takes advantage of the laws of quantum mechanics, while the way communication channels are interlinked is assumed to be classical. In this Letter we relax the assumption that quantum channels are combined classically, showing that a quantum communication network where quantum channels are combined in a superposition of different orders can achieve tasks that are impossible in conventional quantum Shannon theory. In particular, we show that two identical copies of a completely depolarizing channel become able to transmit information when they are combined in a quantum superposition of two alternative orders. This finding runs counter to the intuition that if two communication channels are identical, using them in different orders should not make any difference. The failure of such intuition stems from the fact that a single noisy channel can be a random mixture of elementary, non-commuting processes, whose order (or lack thereof) can affect the ability to transmit information.

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Introduction – Information theory, initiated by the seminal work of Claude Shannon, [1], has given us a framework to understand the fundamental workings of communication, data storage and signal processing. Shannon’s theory was originally formulated with the assumption that the carriers of information and the communication channels are classical. The data were represented by classical random variables and the communication channels were treated as stochastic transition matrices. However, the laws of nature are fundamentally quantum and one can take advantage of these laws to build a new model of information processing. This gave rise to quantum Shannon theory [2], where quantum features such as superposition and entanglement were used to enhance communication, increasing transmission rates [3], providing unconditional security [4], and introducing new means of information transmission [5], just to name a few examples. Nevertheless, quantum Shannon theory is still conservative, in that it assumes that the communication channels are combined in a well-defined configuration. In principle, quantum theory allows for new ways to combine communication channels, by connecting them in a quantum superposition of different configurations. In particular, quantum theory allows the order of application of channels to be entangled with a control system [6], a situation that is sometimes referred to as a *quantum superposition of orders*. Even more generally, quantum theory allows for exotic configurations that are not compatible with any underlying model where the order is definite [7]. Both features could emerge in a theory of quantum gravity [8, 9] and would offer enhancements in a number of tasks, such as testing properties of quantum channels [10, 11], playing non-local games [7], and reducing communication complexity [12]. In this Letter, we show that the ability to combine quantum channels in a superposition of orders can boost the rate of com-

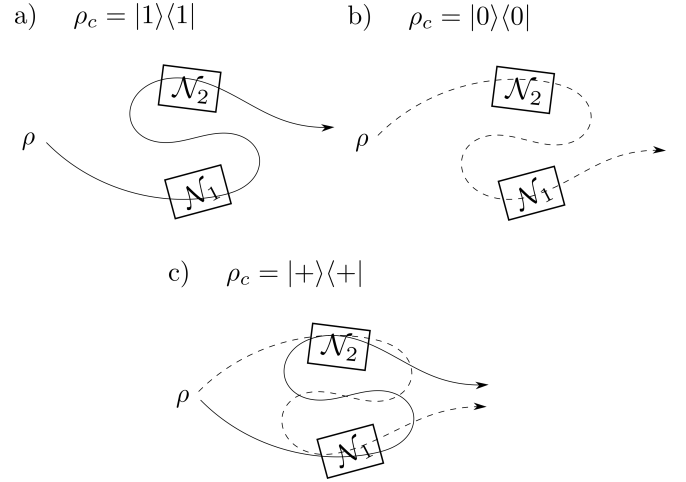


FIG. 1: Fixed order vs superposition of orders. 1(a) A quantum particle, prepared in the state ρ , goes first through channel \mathcal{N}_1 and then through channel \mathcal{N}_2 . This configuration is associated to the state $\rho_c = |1\rangle\langle 1|$ of a control qubit, in which the choice of order is encoded. 1(b) The quantum particle goes first through \mathcal{N}_2 and then through \mathcal{N}_1 . This alternative configuration is associated to the qubit state $\rho_c = |0\rangle\langle 0|$. 1(c) The quantum SWITCH creates a superposition of the two configurations 1(a) and 1(b). It takes a control qubit in a superposition state, such as $\rho_c = |+\rangle\langle +|$, and correlates the order of the two channels with the state of the qubit.

munication beyond the limits of conventional quantum Shannon theory.

Our result is based on a novel quantum primitive, called the quantum SWITCH [6]. The quantum SWITCH is an operation that takes two channels \mathcal{N}_1 and \mathcal{N}_2 as inputs and creates a new channel, which uses the channels \mathcal{N}_1 and \mathcal{N}_2 in an order that is entangled with the state of a control qubit, thus generating a quantum

superposition of two alternative orders. Fig. 1 illustrates different ways of combining the two channels \mathcal{N}_1 and \mathcal{N}_2 , either in a definite order or in a quantum superposition of orders. In [6] it was shown that the quantum SWITCH cannot be realized by any circuit where the order to the two channels \mathcal{N}_1 and \mathcal{N}_2 is fixed. Likewise, the quantum SWITCH cannot be realized as a classical mixture of circuits using channels \mathcal{N}_1 and \mathcal{N}_2 in fixed orders [10]. An even broader sense in which the quantum SWITCH cannot be decomposed into quantum processes with definite order has been discussed in [13].

In this Letter, we introduce a quantum Shannon theoretic task where the quantum SWITCH enables two communicating parties to transmit information. We show that the entanglement of the control system with the order of application of two channels can be used to perform communication tasks that are impossible when the order is fixed, or even correlated with a classical variable. Surprisingly, the advantage can be achieved by switching two copies of the same channel, a phenomenon which we refer to as *self-switching*. Specifically, we show the advantage of switching two copies of the completely depolarising channel, which transforms every quantum state into the maximally mixed state. Clearly, none of the two fixed configurations in Fig. (1) can be used to communicate information. Here we show that the entanglement of these two configurations with a control system can be used to communicate classical information – a phenomenon we call Causal Activation. This phenomenon sheds light on the fact that the properties of channels do not solely depend on the way they are constructed, but also the way they are *combined*: two channels combined in a superposition of different orders behave very differently from the same channels combined in a fixed order.

One of the main pillars of Shannon theory is quantifying the capacity of channels to communicate information. Channel capacity theorems are of fundamental importance both for the theoretical characterisation of channels, as well as the experimental implementation of communication protocols. In this Letter, we derive an analytical expression for the Holevo capacity [14] of the two causally activated depolarizing channels. Quite counter-intuitively, we find that the Holevo capacity is maximum for qubit channels and decreases with the dimension of the input.

Preliminaries – In this section we review the concepts and tools needed to understand our finding.

We use quantum channels to represent transmission lines in a quantum communication network. Mathematically, quantum channels are described by completely positive trace preserving maps (CPTP). We will often use the Kraus decomposition, which allows one to represent the action of a channel \mathcal{N} on a quantum state ρ as $\mathcal{N}(\rho) = \sum_i K_i \rho K_i^\dagger$, where $\{K_i\}$ is a set of operators such that $\sum_i K_i^\dagger K_i = I$.

In a fixed causal structure channels can be composed

either in parallel or series. Two channels \mathcal{N}_1 and \mathcal{N}_2 composed in parallel are represented by the tensor product of the two channels $\mathcal{N}_1 \otimes \mathcal{N}_2$. If used in series, the second channel simply acts on the output of the first channel, *i.e.* $\mathcal{N}_2 \circ \mathcal{N}_1$. More generally, the two channels can be connected in an arbitrary quantum circuit including intermediate operations.

In principle, however, the order does not have to be fixed. Two channels could be combined by the quantum SWITCH operation, ending up in a situation where their relative order is entangled with a control system. Let us denote the Kraus operators of the channel \mathcal{N}_1 as $\{K_i^{(1)}\}$ and \mathcal{N}_2 as $\{K_j^{(2)}\}$. The quantum SWITCH uses an auxiliary quantum system to control the order of the Kraus operators of the two channels in an indefinite causal manner. The Kraus operators of the overall quantum channel resulting from the switching of \mathcal{N}_1 and \mathcal{N}_2 are

$$W_{ij} = K_i^{(2)} K_j^{(1)} \otimes |0\rangle\langle 0|_c + K_j^{(1)} K_i^{(2)} \otimes |1\rangle\langle 1|_c, \quad (1)$$

acting on a target quantum state ρ and a control state ρ_c . The action of the quantum SWITCH is then given by

$$\mathcal{S}(\mathcal{N}_1, \mathcal{N}_2)(\rho \otimes \rho_c) = \sum_{i,j} W_{ij}(\rho \otimes \rho_c) W_{ij}^\dagger. \quad (2)$$

It is easy to check that above definition is independent of the choice of Kraus operators for the channels \mathcal{N}_1 and \mathcal{N}_2 . Mathematically, the quantum SWITCH is a *higher-order operation* [6]: it takes two channels \mathcal{N}_1 and \mathcal{N}_2 as input and creates a quantum channel $\mathcal{S}(\mathcal{N}_1, \mathcal{N}_2)$ as output. Specifically, this higher-order operation combines the two input channels in an order that depends on the state of the control qubit: if the qubit is in the state $\rho_x = |0\rangle\langle 0|$, the channel $\mathcal{S}(\mathcal{N}_1, \mathcal{N}_2)$ will return the state $\mathcal{N}_2 \mathcal{N}_1(\rho) \otimes |0\rangle\langle 0|$, if the qubit is in the state $\rho_c = |1\rangle\langle 1|$, the channel will return the state $\mathcal{N}_1 \mathcal{N}_2(\rho) \otimes |1\rangle\langle 1|$. When the qubit is in a superposition of $|0\rangle$ and $|1\rangle$, the channel returns a correlated state, which can be interpreted as the result of the input channels \mathcal{N}_1 and \mathcal{N}_2 acting on ρ in a quantum superposition of two alternative orders.

Quantum Shannon theory with the assistance of the quantum SWITCH. In quantum Shannon theory, quantum channels represent communication resources. Hence, higher-order operations, like the quantum SWITCH, can be viewed as transformations of resources. Quantum Shannon theory can be cast in the form of a resource theory, by specifying a set of higher-order operations that are regarded as *free* [15]. A basic type of free operation maps an input channel \mathcal{N} into an output channel $\mathcal{D} \circ \mathcal{N} \circ \mathcal{E}$, where \mathcal{E} and \mathcal{D} are two channels, representing encoding and decoding operations at the sender's and receiver's end, respectively. Another type is composition in parallel, whereby two channels \mathcal{N}_1 and \mathcal{N}_2 are combined into the channel $\mathcal{N}_1 \otimes \mathcal{N}_2$. Finally, it is also natural to consider scenarios where the sender sends the information to

the receiver through a repeater, connected to the sender and the receiver through two channels \mathcal{N}_1 and \mathcal{N}_2 , respectively. The corresponding type of free operation is composition in sequence, whereby two input channels \mathcal{N}_1 and \mathcal{N}_2 are combined into the output channel $\mathcal{N}_2 \circ \mathcal{R} \circ \mathcal{N}_1$, where \mathcal{R} represents the operation performed by the repeater. Combining these three types of operations (possibly including free classical correlations), one obtains a resource theory, suitable to describe basic communication tasks involving a single sender and a single receiver.

We now define an extension of standard quantum Shannon theory that includes quantum superpositions of causal orders. We do this in the resource-theoretic framework, by adding the quantum SWITCH to the set of free operations. More precisely, we add the free operation that maps a pair of channels \mathcal{N}_1 and \mathcal{N}_2 to the new channel \mathcal{N}' defined by $\mathcal{N}'(\rho) = \mathcal{S}(\mathcal{N}_1, \mathcal{N}_2)(\rho \otimes \rho_c)$, where \mathcal{S} is defined as in Eq. (2) and ρ_c is a fixed state of the control qubit. Note that the state of the control is part of the way the two channels are combined, and is *not* accessible to the sender: the sender cannot encode classical bit in the state of the control. The control is only accessible to the receiver, who can use it as an aid for decoding. We refer to the extended model as *quantum Shannon theory with the assistance of the quantum SWITCH*. Adding the quantum SWITCH is similar to what is done in other variants of quantum Shannon theory, where one adds free entanglement [3], free symmetric side-channels [16, 17], free no-signalling channels [18, 19] and such like.

In the following we will focus on the communication of classical information. Holevo [14], Schumacher, and Westmoreland [20] (HSW) proved that a single copy of any quantum channel \mathcal{N} can communicate classical information at best at the rate $\chi(\mathcal{N}) := \max_{\{p_x, \rho_x\}} I(X; B)_\sigma$, where $I(X; B)_\sigma$ is the von Neumann Mutual Information, evaluated on a state of the form $\sigma := \sum_x p_x |x\rangle\langle x|_X \otimes \mathcal{N}(\rho_x)_B$ and maximized over all possible ensembles $\{p_x, \rho_x\}$. The quantity $\chi(\mathcal{N})$ is called the Holevo information and has been shown to be in general non-additive [21]. This means that there exist two channels \mathcal{N} and \mathcal{M} such that $\chi(\mathcal{N} \otimes \mathcal{M}) > \chi(\mathcal{N}) + \chi(\mathcal{M})$. Therefore,

if many copies of a channel whose Holevo information is non-additive are available, the Holevo information is a lower bound for the capacity of quantum channels to communicate classical information. Operationally, the lower bound corresponds to the amount of information that can be transmitted if the sender uses only product states in the encoding.

One of the implications of the HSW theorem is that any quantum channel that is not constant can be used to communicate classical information. This is because for a channel \mathcal{N} that is not constant, there exist at least two pure states $|\phi\rangle$ and $|\psi\rangle$, such that $\mathcal{N}(|\phi\rangle\langle\phi|) \neq \mathcal{N}(|\psi\rangle\langle\psi|)$. Using these two states with equal probability it is immediate to see that the Holevo information is positive. On the other hand, the Holevo information of a constant channel is trivially zero. Even if the constant channel is used many times, none of the operations allowed in the standard model of quantum Shannon theory allows to generate a channel that transmits information. In the following we will show that, in contrast, classical communication can become possible with the assistance of the quantum SWITCH.

Main Result – A completely depolarising channel \mathcal{N}^D on a d -dimensional quantum system can be represented by uniform randomisation over d^2 orthogonal unitary operators U_i , such that its action on a state ρ is

$$\mathcal{N}^D(\rho) = \frac{1}{d^2} \sum_{i=1}^{d^2} U_i \rho U_i^\dagger = \text{Tr}[\rho] \frac{I}{d}. \quad (3)$$

Therefore, according to Eq. (1), the overall quantum channel resulting from the quantum SWITCH of two completely depolarising channels has Kraus operators

$$W_{ij} = \frac{1}{d^2} (U_i U_j \otimes |0\rangle\langle 0|_c + U_j U_i \otimes |1\rangle\langle 1|_c). \quad (4)$$

Suppose that the control system is fixed to the state $\rho_c := |\psi_c\rangle\langle\psi_c|$, where $|\psi_c\rangle := \sqrt{p}|0\rangle + \sqrt{1-p}|1\rangle$. If the sender prepares the target system in the state ρ , then the receiver will get the output state

$$\begin{aligned}
\mathcal{S}(\mathcal{N}^D, \mathcal{N}^D)(\rho \otimes \rho_c) &= \frac{1}{d^4} \sum_{i,j} \left(p|0\rangle\langle 0|_c \otimes U_i U_j \rho U_j^\dagger U_i^\dagger + (1-p)|1\rangle\langle 1|_c \otimes U_j U_i \rho U_i^\dagger U_j^\dagger \right. \\
&\quad \left. + \sqrt{p(1-p)}|0\rangle\langle 1|_c \otimes U_i U_j \rho U_i^\dagger U_j^\dagger + \sqrt{p(1-p)}|1\rangle\langle 0|_c \otimes U_j U_i \rho U_j^\dagger U_i^\dagger \right) \\
&= p|0\rangle\langle 0|_c \otimes \frac{I}{d} + (1-p)|1\rangle\langle 1|_c \otimes \frac{I}{d} + \frac{\sqrt{p(1-p)}}{d^2} |0\rangle\langle 1|_c \otimes \sum_j \text{Tr}[U_j \rho] \frac{U_j^\dagger}{d} \\
&\quad + \frac{\sqrt{p(1-p)}}{d^2} |1\rangle\langle 0|_c \otimes \sum_j \text{Tr}[\rho U_j^\dagger] \frac{U_j}{d} \\
&= (p|0\rangle\langle 0|_c + (1-p)|1\rangle\langle 1|_c) \otimes \frac{I}{d} + \sqrt{p(1-p)}(|0\rangle\langle 1|_c + |1\rangle\langle 0|_c) \otimes \frac{\rho}{d^2}. \tag{5}
\end{aligned}$$

The first equality follows from Eq. (4). The second equality is the application of the depolarising channel in Eq. (3). Finally, the last equation follows from the fact that the operators U_j form an orthonormal basis for the set of $d \times d$ matrices.

Eq. (5) shows that the quantum SWITCH of two depolarising channels has a clear dependence on the input state ρ . Therefore, the HSW theorem implies that we can communicate classical information at a non-zero rate.

The quantum SWITCH implements a transfer of information from the input system to the correlations between the output system and the control. Note that the information is not contained in the state of the system alone, nor in the state of the control alone: it is genuinely contained in the correlations. Note also that these correlations must be quantum: if the control decoheres in the basis $\{|0\rangle, |1\rangle\}$, the information is completely lost. In spite of this, the receiver does not need to perform entangled operations in the decoding. Instead, the receiver measure the control system in the Fourier basis $\{|+\rangle, |-\rangle\}$, obtaining the conditional states

$$\langle \pm | \mathcal{S}(\mathcal{N}^D, \mathcal{N}^D)(\rho \otimes \rho_c) | \pm \rangle = \frac{I}{2d} \pm \sqrt{p(1-p)} \frac{\rho}{d^2}. \tag{6}$$

Since these states still depend on ρ , the receiver can use this dependence to extract non-zero information from the target system. In terms of the Kraus operators (4), the postselection on the outcomes $+$ and $-$ generates the noisy channel generalisation of the *quantum superpositions of time evolutions* proposed by Aharonov and collaborators [22].

So far we have shown that the quantum SWITCH allows one to use depolarising channels to communicate at *some* non-zero rate. We now compute the optimal rate in the case of product encodings, by analytically calculating the maximum Holevo information over all input ensembles. We restrict our attention to the case where the control qubit is in the state $\rho_c = |+\rangle\langle +|$, because for such state the communication rate is the highest. The expression for the maximum Holevo information is derived in the Supplemental Material, where, in fact, we derive an even more general expression, valid for arbitrary depolarizing channels, sending an input state ρ to an output state $q\rho + (1-q)I/d$, with $0 \leq q \leq 1$ a generic noise parameter. For the complete depolarizing channel ($q = 0$), we find the Holevo information to be

$$\begin{aligned}
\chi(S(\mathcal{N}^D, \mathcal{N}^D)) &= \log d + H(\tilde{\rho}_c) \\
&\quad + \left\{ \left(\frac{d+1}{2d^2} \right) \log \left(\frac{d+1}{2d^2} \right) + \left(\frac{d-1}{2d^2} \right) \log \left(\frac{d-1}{2d^2} \right) + 2(d-1) \left(\frac{1}{2d} \right) \log \left(\frac{1}{2d} \right) \right\}, \tag{7}
\end{aligned}$$

where $\tilde{\rho}_c = 1/2|0\rangle\langle 0| + 1/2|1\rangle\langle 1| + 1/2d^2(|0\rangle\langle 1| + |1\rangle\langle 0|)$ is the reduced state of the control system. It should not be surprising that the entropy of the control system appears in the expression for the capacity of the switched depolarising channels. This is because the control system is a parameter describing the way in which the depolarising

channels are combined.

Eq. (7) is the best rate one can communicate classical information by switching only two copies of depolarising channel. However, if one has access to more copies of these channels, one may be able communicate more by inputting states that are entangled across the channels.

To show this would require a proof that the overall mapping generated by switching depolarising channels is not additive. Since this question is separate from the main point of this Letter, we leave this task for future investigation.

Conclusions and Discussion – In this work, we explored an extension of quantum Shannon theory where communication channels can be combined in a quantum superposition of orders. In this extended model, we showed that combining two completely depolarizing channels with the quantum SWITCH activates them, allowing the transmission of classical information. In contrast, no such activation is possible in the standard model, where the order is fixed or controlled by a classical random variable.

Strikingly, we showed that the Shannon theoretic advantage can be gained as a result of creating a superposition of a channel with another copy of itself. This result may seem paradoxical, because exchanging two uses of the same channel would not have any observable effect in any ordinary quantum circuit. The resolution of the paradox lies in the fact that noisy quantum channels can be seen as random mixture of different processes, corresponding to different Kraus operators. The advantage of the self-switching arises because some of these processes do not commute with each other, and therefore a quantum control on the order offers a non-trivial resource. We observe that no self-switching effect arises for quantum channels that admit a Kraus decomposition consisting of mutually commuting operators.

Our results are an invitation to investigate a new paradigm of Shannon theory, where the order of the communication channels can be in a quantum superposition. This paradigm may find applications in future quantum communication networks. Consider a situation where a provider connects different communicating parties through a network of quantum channels. In this situation, the provider could opt to connect the channels in a superposition of alternative configurations, thereby boosting the communication rates between parties. Of course, every such application requires a careful analysis of physical resources required for the implementation of the quantum SWITCH. While in this work we treated the quantum SWITCH as an abstract higher-order operation, there exist different ways in which this operation could be realized, including table-top photonic implementations [23, 24], implementations with ion traps [25] and superconducting circuits [26]. The practical extent of the advantage shown in our paper greatly depends on the resources required in each implementation. For example, Oreshkov [27] has recently analyzed the structure of the photonic implementations of [23, 24], showing that an essential ingredient is the ability to delocalize the input channels in time, coherently controlling when the environment interacts with the system. On the other hand, our result provides new motivation to the develop-

ment of experimental techniques for the implementation of the quantum SWITCH.

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