THE ROBUSTNESS OF MULTI-ARRAY CROSS-DIRECTIONAL CONTROL SYSTEMS

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ABSTRACT

This paper considers the design of systems that simultaneously determine the inputs to multiple arrays of actuators for regulating cross-directional (CD) variations of properties such as basis weight, moisture and calliper on paper machines. Control designs based on large scale optimization have been proposed, but this paper extends previous work based on an approach that exploits knowledge of the spatial response of the actuators. The design a CD controller for a system with two actuator arrays and a single sensor array is described, which decomposes the design problem into the controllable subspaces of each actuator array into a series of two-input, two output; two-input, one output and single-input, single-output problems. The robustness of the controller to uncertainties in the responses of the arrays is determined using small gain arguments. This robustness analysis can be extended to the case where there are constraints on the control inputs, as is commonly the case for CD controllers that are installed on paper-machines, using the concept of Integral Quadratic Constraints (IQC).

INTRODUCTION

On many paper machines, cross-directional (CD) variations in a number of properties (basis weight, moisture and caliper) are regulated simultaneously by multiple arrays of actuators. These arrays of actuators often have a significant degree of interaction between their responses and control designs based on large scale optimization [1, 2, 3] have been proposed to address the interactions. A previous paper [4] used a different approach that exploits knowledge of the spatial response of the actuators to design a CD controller for a system with two actuator arrays and a single sensor array, which decomposes the design problem into the controllable subspaces of each actuator array. This decouples the control design into a series of two-input, two output; two-input, one output and single-input, single-output problems. This paper shows how the approach can be extended to determine the robustness of the controller to uncertainties in the responses of the arrays, particularly when there are constraints on the control inputs, as is commonly the case for the CD controllers that are installed on paper-machines. For control synthesis we propose a simple heuristic that determines whether or not to include modes according to simple geometric criteria. Standard small gain (and structured singular value) analysis confirms that this leads to a controller that is robust in closed loop. The heuristic can be extended to the case where the constraints associated with actuator saturation are significant. In this case the robustness measures can be obtained via the concept of Integral Quadratic Constraints (IQC).

The robustness analysis is based upon the concept of Integral Quadratic Constraints (IQC) that was applied to CD controllers in [5, 6, 7]. The results provide insight into how the control effort should be divided between actuator arrays that have different spatial responses and different dynamic responses in the case where each array is subject to its own input constraints.

We adopt an internal model control approach to our analysis and design. The structure is highly amenable to robustness analysis [8], and has long been applied in the unconstrained robustness analysis of single-array cross directional controllers [9]. It is also closely related to the Dahlin controller, recommended for plants such as paper machines whose responses are subject to significant delay [10].

SINGLE-ARRAY SYSTEMS

Internal Model Control Approach

We will frame our analysis and design in terms of internal model control. In addition we will decompose the profile into modes. We will begin by reviewing the case where the controller is designed to determine the inputs to a single array that is regulating a single variable (i.e. one of basis weight, moisture or calliper). The approach will then be extended to consider the interaction between two actuator arrays that have an effect on the variable. Suppose our nominal plant model is given by

$$y(t) = g(z)\, Bu(t) + d(t)$$

where $y(t) \in \Re^m$ represents the profile of the variable to be controlled, which is sampled across the sheet at m measuring positions, $d(t) \in \Re^m$ represents the process disturbance, and $u(t) \in \Re^n$ represents the inputs (set points) applied to the n actuators in the CD actuator array. The object $B \in \Re^{m\times n}$ is the (static) interaction matrix, while $g(z)$ is a single-input single-output discrete transfer function representing the actuator and process dynamics. By absorbing the dc gain of the dynamics into the interaction matrix B, without loss of generality we can assume that $g(1) = 1$.

Typically, the controller is designed to make the CD profile flat, so that the aim is to achieve $y(t) = 0$ (it is straightforward to extend the analysis to accommodate non-flat target profile). We will use the approach given in [5,7], which is based on internal model control (IMC), where the control law takes the form

$$\hat{d}(t) = y(t) - g(z)Bu(t)$$
$$u(t) = -\tilde{Q}(z)\hat{d}(t)$$

with $\tilde{Q}(z) \in \mathbb{C}^{n\times m}$ being some (as yet) unspecified transfer function matrix.

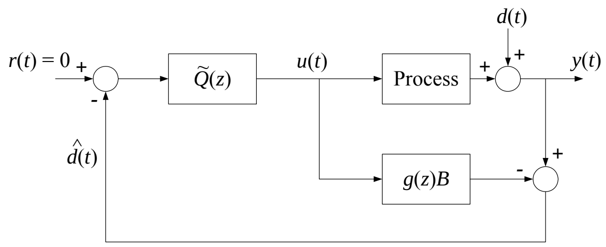

Figure 1. Schematic for internal model control.

The nominal closed-loop response is then

$$y(t) = \left[I - g(z)B\tilde{Q}(z)\right]d(t)$$

Suppose the reduced singular value decomposition (SVD) of the interaction matrix B is given by

$$B = U\,S\,V^{\mathrm{T}}$$

with $U \in \Re^{m\times n}$, $S \in \Re^{n\times n}$ and $V \in \Re^{n\times n}$ satisfying $U^{\mathrm{T}}U = I$, $V^{\mathrm{T}}V = I$, and S diagonal. We choose to control the first r modes (heuristics for choosing r will be considered later). If the diagonal matrix containing the singular values is

$$S = \begin{bmatrix} s_1 & & & & & \\ & \ddots & & & & \\ & & s_r & & & \\ & & & s_{r+1} & & \\ & & & & \ddots & \\ & & & & & s_n \end{bmatrix}$$

then we define

$$S_r = \begin{bmatrix} s_1 & & & & & \\ & \ddots & & & & \\ & & s_r & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{bmatrix}$$

so that

$$S_r^{-1}S = \begin{bmatrix} I_r & \\ & 0 \end{bmatrix}$$

Suppose we choose

$$\tilde{Q}(z) = VS_r^{-1}Q(z)U^T$$

with $Q(z)$ diagonal so that it takes the form

$$Q(z) = \begin{bmatrix} q_1(z) & & & & & \\ & \ddots & & & & \\ & & q_r(z) & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{bmatrix}$$

then the closed-loop response can be written

$$y(t) = [I - g(z)U\,Q(z)\,U^{\mathrm{T}}]\,d(t)$$

or equivalently

$$\tilde{y}(t) = \left[I - g(z)Q(z)\right]^{\mathrm{T}}\tilde{d}(t)$$

where $\tilde{y}(t) = U^{\mathrm{T}}y(t)$ and $\tilde{d}(t) = U^{\mathrm{T}}d(t)$

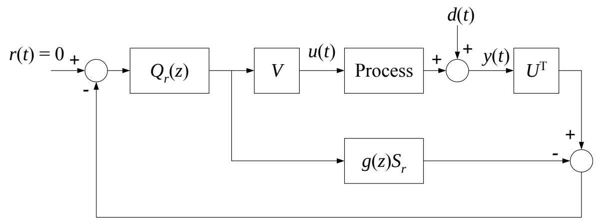

Figure 2. Schematic for internal model control with decomposition into modes. Both S_r and $Q_r(z)$ are diagonal so the control may be designed for each mode as an independent single-input single-output loop.

Design Heuristic

We now use a simple heuristic for designing our controller:

For each mode, $i = 1, \ldots, n$:

- if s_i is large and/or well-known, choose $q_i(z)$ such that $[g(z)\ q_i(z)]$ is fast (*i.e.* has high bandwidth);
- if s_i is medium and/or partially known, choose $q_i(z)$ such that $[g(z)\ q_i(z)]$ is slow (*i.e.* has low bandwidth);
- if s_i is small and/or poorly known, reduce r so that $r < i$ and choose $q_i(z)$ to be zero.

It is convenient to write

$$B = B_r + B_\Delta$$

with

$$B_r = U_r\,S_r\,V_r^{\mathrm{T}}$$

and where $U_r \in \Re^{m\times r}$ consists of the first r columns of U, $V_r \in \Re^{n\times r}$ consists of the first r columns of V and $S_r \in \Re^{r\times r}$ consists of the first r columns of S. We call B_r

the reduced mode representation of the full interaction matrix B that includes r modes. The steady state responses of the remaining modes are described by $B_\Delta \in \Re^{m\times(n-r)}$. We can then write

$$\widetilde{Q}(z) = V_r S_r^{-1} Q_r(z) U_r^T$$

where $Q_r(z)$ is a (diagonal) transfer function matrix of dimension $r\times r$ that contains the first r columns and rows of $Q(z)$. The closed-loop response can now be written

$$y(t) = \left[I - g(z)\, U_r\, Q_r(z)\, U_r^{\mathrm{T}}\right] d(t)$$

Robustness Analysis

The robustness of such a heuristic can easily be verified by rigorous analysis. Suppose the true plant behaviour is

$$y(t) = P(z)u(t) + d(t)$$

$$P(z) = g(z)\, B_r + \Delta_p(z)$$

where $\Delta_p(z)$ represents the plant uncertainty. The response due to the higher order modes, $g(z)\ B_\Delta$, is included within $\Delta_p(z)$.

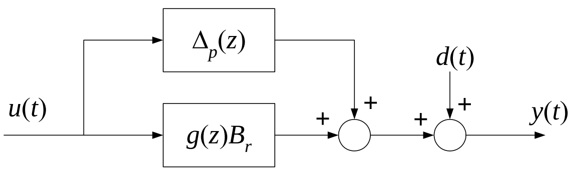

Figure 3. Plant model with uncertainty.

Then the loop is robustly stable provided

$$\left\| \widetilde{Q}\, \Delta_P \right\|_\infty < 1$$

Suppose $\Delta^p(z)$ can be represented as

$$U_r^{\mathrm{T}}\, \Delta^p(z)\, V_r = W^L(z)\, \Delta(z)\, W^R(z)$$

where both $W^L(z) \in \mathbb{C}^{r\times r}$ and $W^R(z) \in \mathbb{C}^{r\times r}$ are known weighting transfer function matrices, while $\Delta(z) \in \mathbb{C}^{r\times r}$ represents uncertainty in the reduced mode space, which satisfies

$$\left\| \Delta \right\|_\infty \le 1$$

The loop is robustly stable provided that [11,12]

$$\left\| W^R S_r^{-1} Q_r W^L \right\|_\infty < 1$$

The design heuristic ensures that this condition is satisfied provided the weighting transfer function matrices $W^L(z)$ and $W^R(z)$ are not pathological [12].

If the mode space is correctly known (*i.e*. the only uncertainty only lies in the values of S_r and the scalar dynamics $g(z)$) then $W^L(z)$, $W^R(z)$ and $\Delta(z)$ are all diagonal. In this case, it suffices to check that the diagonal elements of $W^R(z)S_r^{-1}Q_r(z)W^L(z)$ all have infinity norm less than one.

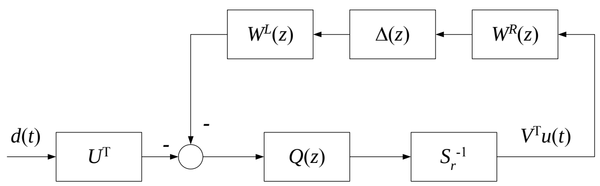

Figure 4. Equivalent feedback loop for robust stability analysis.

MULTI-ARRAY SYSTEMS

Controller Structure

The analysis so far has considered the case where a single array of actuators is being used to control a single variable. Suppose that we now have two actuator arrays so the nominal plant model becomes

$$y(t) = g_1(z)\, B_1\, u_1(t) + g_2(z)\, B_2\, u_2(t) + d(t)$$

with $u_1(t) \in \Re^{n_1}$ and $u_2(t) \in \Re^{n_2}$ representing the inputs applied to two distinct arrays of actuators. We also assume that both B_1 and B_2 are the reduced mode representations of their respective interaction matrices with r_1 and r_2 modes respectively. We can assume without loss of generality that $r_1 \ge r_2$ and that B_1 and B_2 are scaled so that

$$g_1(1) = 1 \quad \text{and} \quad g_2(1) = 1$$

which means that the dc gains of the dynamics are unity. As in the analysis for the single array case, the responses due to the modes higher than r will be incorporated into the plant uncertainty.

An internal model controller for such a multi-array system takes the form

$$\hat{d}(t) = y(t) - g_1(z)B_1u_1(t) - g_2(z)B_2u_2(t)$$
$$u_1(t) = -\widetilde{Q}_1(z)\hat{d}(t)$$
$$u_2(t) = -\widetilde{Q}_2(z)\hat{d}(t)$$

so that the nominal closed-loop response is

$$y(t) = \left[I - g_1(z)B_1\widetilde{Q}_1(z) - g_2(z)B_2\widetilde{Q}_2(z)\right]d(t)$$

The challenge is to find an appropriate modular decomposition and then to design $\widetilde{Q}_1(z)$ and $\widetilde{Q}_2(z)$.

Suppose B_1 and B_2 have reduced singular value decompositions

$$B_1 = U_1\, S_1\, V_1^{\mathrm{T}} \quad \text{and} \quad B_2 = U_2\, S_2\, V_2^{\mathrm{T}}$$

where $U_1 \in \Re^{m\times r1}$, $S_1 \in \Re^{r1\times r1}$, $V_1 \in \Re^{n1\times r1}$, $U_2 \in \Re^{m\times r2}$, $S_2 \in \Re^{r2\times r2}$ and $V_2 \in \Re^{n2\times r2}$. We then use Algorithm 12.4.3 in [13] to factorise the controllable subspaces for the two arrays, so that

$$U_1 = P_1 N_1 \quad \text{and} \quad U_2 = P_2 N_2$$

where $N_1 \in \Re^{r_1 \times r_1}$ and $N_2 \in \Re^{r_2 \times r_2}$, while $P_1 \in \Re^{m \times r_1}$ and $P_2 \in \Re^{m \times r_2}$ are orthogonal matrices satisfying

$$P_1^T P_1 = I_{r_1} \text{ and } P_2^T P_2 = I_{r_2}$$

so that

$$N_1 = P_1^{\mathrm{T}} U_1 \quad \text{and} \quad N_2 = P_2^{\mathrm{T}} U_2$$

Also

$$P_1^T P_2 = \begin{bmatrix} C \\ 0 \end{bmatrix}$$

with $C \in \Re^{r_2 \times r_2}$ being diagonal

$$C = \begin{bmatrix} \cos(\theta_1) & & \\ & \ddots & \\ & & \cos(\theta_{r_2}) \end{bmatrix}$$

where the θ_i are the angles between the principal vectors spanning U_1 and U_2, the controllable sub-spaces of B_1 and B_2.

We can express the nominal-closed loop response as

$$y(t) = \left[I - \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} g_1(z) N_1 S_1 V_1^{\mathrm{T}} \widetilde{Q}_1(z) \\ g_2(z) N_2 S_2 V_2^{\mathrm{T}} \widetilde{Q}_2(z) \end{bmatrix} \right] d(t)$$

so that if we write

$$\widetilde{Q}_1(z) = V_1 S_1^{-1} Q_1(z) N_1^{-1} T_1$$

and

$$\widetilde{Q}_2(z) = V_2 S_2^{-1} Q_2(z) N_2^{-1} T_2$$

for some diagonal $Q_1(z)$ and $Q_2(z)$ and for some matrices $T_1 \in \Re^{r1 \times m}$ and $T_2 \in \Re^{r2 \times m}$, then we obtain

$$y(t) = \left[I - \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} g_1(z) N_1 Q_1(z) N_1^{-1} T_1 \\ g_2(z) N_2 Q_2(z) N_2^{-1} T_2 \end{bmatrix} \right] d(t)$$

The diagonal transfer function matrices $Q_1(z)$ and $Q_2(z)$ can be chosen according to a similar design heuristic as for single array cross-directional design. In particular, if we choose $Q_1(z)$ and $Q_2(z)$ so that they are diagonal with unity dc gain

$$Q_1(1) = I \text{ and } Q_2(1) = I$$

then in steady state

$$y_{ss} = \left[I - \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} \right] d_{ss}$$

Similar to [4], pre-multiplying both sides

$$\begin{bmatrix} P_1^{\mathrm{T}} \\ P_2^{\mathrm{T}} \end{bmatrix} y_{ss} = \begin{bmatrix} P_1^{\mathrm{T}} \\ P_2^{\mathrm{T}} \end{bmatrix} d_{ss} - \begin{bmatrix} P_1^{\mathrm{T}} \\ P_2^{\mathrm{T}} \end{bmatrix} \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} d_{ss}$$

and setting

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = H \begin{bmatrix} P_1^T \\ P_2^T \end{bmatrix}$$

with

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

for some $H_{11} \in \Re^{r_1 \times r_1}$, $H_{12} \in \Re^{r_1 \times r_2}$, $H_{21} \in \Re^{r_2 \times r_1}$ and $H_{22} \in \Re^{r_2 \times r_2}$, then because P_1 and P_2 are orthogonal matrices, this leads to

$$\widetilde{y}_{ss} = \left[I - \begin{bmatrix} I & P_1^T P_2 \\ P_2^T P_1 & I \end{bmatrix} \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \right] \widetilde{d}_{ss}$$

where

$$\widetilde{y}_{ss} = \begin{bmatrix} P_1^T \\ P_2^T \end{bmatrix} y_{ss} \text{ and } \widetilde{d}_{ss} = \begin{bmatrix} P_1^T \\ P_2^T \end{bmatrix} d_{ss}$$

Design Heuristic for Decoupling Matrix

Using

$$P_1^T P_2 = \begin{bmatrix} C \\ 0 \end{bmatrix}$$

then

$$\begin{bmatrix} I & P_1^T P_2 \\ P_2^T P_1 & I \end{bmatrix} = \begin{bmatrix} I & 0 & C \\ 0 & I & 0 \\ C & 0 & I \end{bmatrix}$$

If this matrix is denoted by M, then the ideal design would be to make H equal to the inverse of M and because M has a block diagonal structure, this gives

$$H = M^{-1} = \begin{bmatrix} \overline{D} & 0 & \overline{D} \\ 0 & I & 0 \\ \overline{D} & 0 & \overline{D} \end{bmatrix} \begin{bmatrix} I & 0 & -C \\ 0 & I & 0 \\ -C & 0 & I \end{bmatrix}$$

where $\overline{D} \in \Re^{r_2 \times r_2}$ is a diagonal matrix with elements

$$\left[\overline{D}\right]_{ii} = \frac{1}{1 - \cos^2(\theta_i)}$$

However, difficulties arise when $\cos(\theta_i) \approx 1$, which corresponds to the case when θ_i is small, reflecting the fact that the angle between the subspaces is small. Instead we use a heuristic for choosing the elements of H_{11}, H_{12}, H_{21} and H_{22}. Specifically, set non-diagonal elements in all four matrices to zero. Then, for each mode $i = 1, \ldots, r_2$:

- If $\cos(\theta_i) = 0$, set the corresponding diagonal elements of H_{11} and H_{22} to 1 and set the corresponding diagonal elements of H_{12} and H_{21} to 0.
- If $\cos(\theta_i)$ is small, set the corresponding diagonal elements of H_{11} and H_{22} to 1/(1-

$\cos^2(\theta_i)$) and set the corresponding diagonal elements of H_{12} and H_{21} to $-\cos(\theta_i)/(1-\cos^2(\theta_i))$

- If $\cos(\theta_i)$ is close to one, set the corresponding diagonal element of either H_{11} or H_{22} to 1 and set the other to 0; set the corresponding diagonal elements of H_{12} and H_{21} to 0.
- If $\cos(\theta_i) = 1$, set the corresponding diagonal element of either H_{11} or H_{22} to 1, while setting the diagonal elements of the other matrix to 0; set the corresponding diagonal elements of H_{12} and H_{21} to 0.

Note that there are other possibilities. When $\cos(\theta_i) = 1$, the control effort can be split between the first actuator array and the second actuator array by setting the corresponding diagonal elements of H_{11} and H_{22} to (say) 0.5. It is also possible to make the diagonal elements transfer functions; in this way functionality such as mid-ranging control can be introduced [14].

Robustness Analysis

Suppose the true plant behaviour is given by

$$y(t) = G_1(z)u_1(t) + G_2(z)u_2(t) + d(t)$$
$$G_1(z) = g_1(z)B_1 + \Delta_1^p(z)$$
$$G_2(z) = g_2(z)B_2 + \Delta_2^p(z)$$

where $\Delta_1^p(z) \in \Re^{m\times n1}$ and $\Delta_2^p(z) \in \Re^{m\times n2}$ includes both the additive uncertainty in the response of the two arrays and the components neglected by truncating the responses to the first r_1 and r_2 modes, respectively.

Then the controller is robustly stable provided

$$\left\| -\begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \end{bmatrix} \begin{bmatrix} \Delta_1^p & \Delta_2^p \end{bmatrix} \right\|_\infty < 1$$

If we write

$$\begin{bmatrix} \tilde{Q}_1(z) \\ \tilde{Q}_2(z) \end{bmatrix} = \begin{bmatrix} V_1 S_1^{-1} Q_1(z) N_1^{-1} & 0 \\ 0 & V_2 S_2^{-1} Q_2(z) N_2^{-1} \end{bmatrix} H \begin{bmatrix} P_1^T \\ P_2^T \end{bmatrix}$$

and also express the uncertainty in the two actuator arrays as

$$\begin{bmatrix} \Delta_1^p(z) & \Delta_2^p(z) \end{bmatrix} = \begin{bmatrix} P_1 & P_2 \end{bmatrix} W^L(z) \begin{bmatrix} \Delta_{11}(z) & \Delta_{12}(z) \\ \Delta_{21}(z) & \Delta_{22}(z) \end{bmatrix}.$$
$$W^R(z) \begin{bmatrix} V_1^{\mathrm{T}} & 0 \\ 0 & V_2^{\mathrm{T}} \end{bmatrix} + \begin{bmatrix} \Delta_1^\perp(z) & \Delta_2^\perp(z) \end{bmatrix}$$

where

$$W^L(z) = \begin{bmatrix} W_1^L(z) & 0 \\ 0 & W_2^L(z) \end{bmatrix}$$

and

$$W^R(z) = \begin{bmatrix} W_1^R(z) & 0 \\ 0 & W_2^R(z) \end{bmatrix}$$

with $\Delta_{11}(z) \in \Re^{r1\times r1}$, $\Delta_{12}(z) \in \Re^{r1\times r2}$, $\Delta_{21}(z) \in \Re^{r2\times r1}$, $\Delta_{22}(z) \in \Re^{r2\times r2}$, representing the plant uncertainty lying within the subspace that is controllable by the combination of the two actuator arrays, while $W_1^R(z) \in C^{r_1\times r_1}$, $W_1^L(z) \in C^{r_1\times r_1}$, $W_2^R(z) \in C^{r_2\times r_2}$ and $W_2^L(z) \in C^{r_2\times r_2}$ are the transfer functions of known weighting matrices. $\Delta_1^\perp(z) \in C^{m\times n_1}$ and $\Delta_2^\perp(z) \in C^{m\times n_2}$ are the components of the uncertainties orthogonal to controllable subspaces of B_1 and B_2 (*i.e.* lying in the null spaces of P_1 and P_2) such that

$$P_1^T \Delta_1^\perp(z) = 0 \quad \text{and} \quad P_2^T \Delta_2^\perp(z) = 0$$

then using these decompositions and the fact that V_1 and V_2 are orthogonal, the robustness condition becomes

$$\left\| -\begin{bmatrix} S_1^{-1} Q_1 N_1^{-1} & 0 \\ 0 & S_2^{-1} Q_2 N_2^{-1} \end{bmatrix} HMW^L \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix} W^R \right\|_\infty < 1$$

For any uncertainty satisfying

$$\left\| \begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{bmatrix} \right\|_\infty < 1$$

the closed loop system will be robustly stable provided that

$$\left\| W^R \begin{bmatrix} S_1^{-1} Q_1 N_1^{-1} & 0 \\ 0 & S_2^{-1} Q_2 N_2^{-1} \end{bmatrix} HM\, W^L \right\|_\infty < 1$$

It is straightforward to check that our design heuristic ensures such a condition is met, provided the uncertainty is not pathological. In particular, the design ensures that we do not attempt to invert the matrix

$$\begin{bmatrix} 1 & \cos(\theta_i) \\ \cos(\theta_i) & 1 \end{bmatrix}$$

when θ_i is small, as it would be highly ill-conditioned in this case.

MULTI-ARRAY SYSTEMS WITH CONSTRIANTS

Up to this point, the analysis has not considered the effect of constraints on the allowable inputs that can be applied to the actuator arrays. In practice, the actuator arrays in all cross-directional control systems have limits on the allowable range of inputs that can be applied to the actuators and many actuator arrays also have constraints on the relative values of the inputs applied to adjacent actuators (for example, the bending constraints on the inputs to slice lip arrays used to control CD basis weight variations.) The standard approach to determining the inputs in the presence of constraints is based upon large scale optimisation [1,2,3], but an alternative approach is based upon an Internal Model Control (IMC) structure. Using this approach, the constraints are incorporated by solving a quadratic programme at each step. It can be shown [7] that this approach provides the same steady-state performance as large scale optimisation and for processes such as paper machines with relatively straightforward dynamic responses (first order response plus time delay), the dynamic performances of the two approaches are also similar [5,6]. The IMC approach has a number of advantages including the fact that it is possible to express the problem in terms of the controllable subspaces of the actuator arrays, which preserves the underlying structure of the design of the controller [5]. The controller dynamics $\tilde{Q}(z)$ are implemented in an "anti-windup" structure and the nonlinearity associated with finding the optimal inputs that satisfy the constraints can be expressed in terms of a sector bounded non-linearity [5,6]. Since the IQC approach can address system containing sector bounded nonlinearities, this means that the robustness analysis developed for the unconstrained case can be extended to determine the robustness of a multi-array CD controller when there are constraints on the inputs to each of the arrays.

CONCLUSION

This paper has considered the design of CD control systems where multiple arrays of actuators are used to regulate a given variation in the sheet. Rather than using a controller based upon large-scale optimisation, the approach described here is based upon Internal Model Control and exploits the underlying structure of the controllable subspaces of the actuator arrays. This has the benefit of allowing the control effort to be split between the two arrays in a way that exploits the different dynamics of the arrays. The paper presents an analysis of the robustness to model uncertainty based upon the IQC approach. The analysis can be extended to determine the robustness of a controller that combines the IMC design with a constrained optimisation to handle the case where the actuator inputs are constrained.

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