BANK CREDIT AND LEGAL STATUS IN MOROCCAN MANUFACTURING

A thesis submitted for the degree of
D.PHIL IN ECONOMICS

by

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Abstract

Moroccan manufacturing firms generally choose to incorporate under one of two legal forms: ‘Société Anonyme’ (SA) and ‘Société À Responsabilité Limitée’ (SARL). This thesis is about that choice and its consequence for firms’ access to bank overdraft facilities.

In 2001, Morocco made a radical change to its company law regime: it replaced a company law dating from 19th-century France with modern standards of corporate governance and accountability. In Chapter One, I use the two-period FACS/ICA panel to analyse that reform and to evaluate its impact upon manufacturing firms’ access to bank credit. I find that the reform induced a substantial share of SA firms to switch to SARL, and that — relative to firms remaining in the SA status — this caused a significant and substantial withdrawal of bank overdraft facilities.

In Chapter Two, I develop a theoretical model in which an agent signals its continuous type by using a variable that may take one of only two values (a ‘binary signal’); this is intended to represent a firm’s choice of legal status. I show that this binary signal provides only ‘coarse information’, and I consider the consequences of this coarseness; I solve for equilibrium conditions and I consider both the role of a principal’s risk aversion and the role of other observable agent characteristics (‘indices’).

Chapter Three uses the results of Chapter Two to develop a new structural methodology for the separate identification of information and incentive effects. I apply the method to the data used in Chapter One, on the subset of firms having an overdraft facility in both survey periods (approximately two-thirds of the total sample). I find that, among that limited sample, there is no relevant information asymmetry. I estimate the potential welfare loss and conclude that, in the 95% confidence region of potential information effects and incentive effects, the maximum median welfare loss from information asymmetry is equivalent to approximately only 3% of the median bank overdraft limit. For the sample of firms having an overdraft facility in both survey periods, this challenges the common narrative that information asymmetry is an important reason for bank credit market failure.
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Finally, I must thank my friends and my family for their support and their love, both during this research process and for the many years before it. I am — and always will be — a guy who grew up playing backyard cricket in Brisbane. That I now submit a doctoral thesis at the University of Oxford is a testament to the extraordinary generosity of many people, and I hope that I will always be very grateful for their help.
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**INTRODUCTION**

Moroccan manufacturing firms generally choose to incorporate under one of two legal forms: ‘Société Anonyme’ (SA) and ‘Société À Responsabilité Limitée’ (SARL). This thesis is about that choice and its consequence for firms’ access to bank overdraft facilities.

**Motivation.** In 2001, Morocco introduced a radical reform to the SA legal form; this reform was a central part of Morocco’s attempts to modernise its legal framework in order to meet western standards of corporate governance. An evaluation of the nature and consequences of that reform is important both for understanding the challenges facing Moroccan manufacturing firms and, more generally, for illustrating the potential costs and benefits of legal modernisation in emerging economies.

However, the distinction between the SA and SARL form is not merely important for understanding the consequences of legal reform; it may also shed light on the problem of information asymmetry. Information asymmetry plays an important role in narratives about credit market inefficiencies in Morocco. In 2006, I visited Morocco as part of this research; I heard first-hand from several bankers that manufacturing firms’ inability to provide credible information impedes credit provision to the sector. The distinction between the SA and SARL forms is valuable for assessing this claim. Because the SA status is more costly than SARL, firms’ choice of SA can be an informative signal of unobservable firm characteristics, such as management quality.
Introduction

It is valuable to understand the extent to which banks use legal status as an informative signal of firms’ management; this provides a general insight into the importance of information asymmetry.

Outline and contribution. This thesis comprises three chapters, each of which is presented as a distinct research paper. Chapter One evaluates the 2001 company law reform. In that chapter, I collate a variety of Moroccan texts in order to summarise the timing and the substance of the reform, and I use a representative two-period panel of manufacturing firms to evaluate the impact of the reform on access to bank credit. I find that the reform induced a substantial share of SA firms to switch to SARL, and that this caused a significant and substantial negative effect upon provision of bank overdraft facilities, relative to firms remaining in the SA status.

The contribution of Chapter One, therefore, is primarily empirical. A growing empirical literature argues that better legal protection for creditors improves access to credit. But Chapter One emphasises the importance of a country’s general legal infrastructure to support such legal reform. There are at least two explanations as to why firms switching from SA to SARL would lose their bank overdraft provision. First, it may be that legal status is an important proxy (that is, ‘signal’) for firm characteristics that a bank cannot observe (such as management quality). Under this interpretation, banks withdrew overdraft facilities from firms switching status because banks continued to use outdated ‘rules of thumb’ to assess firms’ creditworthiness, rather than adopting new assessments in light of the new institutional regime. In that case, the reform process caused a withdrawal of overdraft facilities because of the lack of an effective accompanying campaign to educate banks about the more onerous obligations brought by the reforms. This explanation may be described in terms of legal status having a substantial ‘information effect’.

Simon Quinn
Second, it may be that banks withdrew overdraft facilities from firms switching status because banks viewed the SA status as inherently beneficial to firm performance. Under this interpretation, banks may have understood the new legal regime well, and withdrew credit after assessing that firms migrating to the SARL form would be genuinely more risky as recipients of credit. In this case, the reform process caused a withdrawal of overdraft facilities because policymakers did not anticipate adequately that, by requiring SA firms to adhere to more onerous legal obligations, many firms would merely abandon those requirements in favour of the less costly stipulations of the SARL. This explanation may be described in terms of legal status having a substantial ‘incentive effect’.

Chapter One therefore argues that the 2001 reform may have been counter-productive, in that it induced a substantial share of SA firms to migrate to the SARL status, and that those firms were then significantly more likely to lose overdraft facilities (relative to firms remaining in the SA status). In this way, the chapter suggests an important caveat to assertions that more rigorous legal regimes necessarily imply generally higher standards of corporate governance and lender confidence. However, as explained, Chapter One does not identify the mechanism by which legal status reassures lenders — and, therefore, the reason for which migrating firms would be significantly more likely to lose their overdraft than firms remaining as SA.

In Chapter Two, I develop the insight that legal status may act as a costly signal of firm management quality. I explore a general theoretical model in which an informed agent makes a binary choice in order to signal its ‘type’ to an uninformed principal; for comparability to the traditional signalling literature, I frame this in terms of a contracting
relationship between a student and an employer, but all of the insights developed apply to circumstances of binary signalling generally.

Chapter Two seeks to make several contributions to theoretical literature on signalling. The literature on signalling is vast, but very few previous papers have directly considered the consequences of ‘signal coarseness’ — in the sense of a signal being able to reveal only limited information to a principal. The chapter first considers a model in which the uninformed principal is allowed only a binary response (to contract or not to contract); I use this model to study the conditions for pooling and separating equilibria, the consequences of the principal’s risk-aversion and the role of other observable firm characteristics (‘indices’). I then extend the model to allow the principal a continuous response; this models a circumstance in which an employer can choose a student’s wage, or a bank can choose the level of overdraft limit that it provides a firm. I derive a sufficient condition for a unique equilibrium under this extension and I characterise the solution.

This solution leads naturally to Chapter Three. In Chapter Three, I build and estimate a structural model in which manufacturing firms use legal status to signal their management quality. I estimate using the same dataset as used for Chapter One, on the subset of firms having an overdraft facility in both periods (approximately two-thirds of the total sample). In this chapter, I seek to make both methodological and empirical contributions.

Methodologically, I show how, in a context of binary signalling, a structural model may incorporate directly a principal’s conditional expectation of an agent’s ‘unobserved type’; to my knowledge, previous literature on the estimation of information
asymmetry has never used such an approach. Further, I then show how to test (i) the robustness of the model to firm fixed-effects, (ii) the identifying assumption of mutual knowledge between principal and agent, and (iii) the sufficient condition for existence of a unique equilibrium. All of these insights are of general relevance to the structural estimation of models of information asymmetry; to my knowledge, none of these points has been noted in earlier literature. Finally, I show how the model may be used to estimate not only the significance of information asymmetry but also the consequent welfare loss.

Empirically, I use the structural model to estimate that, for the subset of firms having a positive overdraft in both periods, there is no relevant information asymmetry between firms and banks — and that, in the 95% confidence region of potential information effects and incentive effects, the maximum median welfare loss from information asymmetry is approximately only 3% of the median bank overdraft limit. For the sample of firms having an overdraft facility in both periods, this challenges the common narrative that information asymmetry is an important problem for bank credit market failure.

This result is important for two reasons. First, it identifies the likely mechanism for the behaviour noted in Chapter One; it suggests that banks withdrew overdraft facilities from firms switching status because banks viewed the SA status as inherently beneficial (an ‘incentive effect’) rather than as an important proxy/signal of unobservable firm characteristics (an ‘information effect’).

More generally, the result suggests caution to common claims that information asymmetry is an important reason for credit market failure in emerging economies. To be sure, lenders in emerging economies may find it substantially more costly than lenders
in developed economies to assess the creditworthiness of potential clients. However, this chapter suggests that lenders may nonetheless act as if having sufficient information about their clients.

It is often accepted that legal reforms to introduce modern western standards are valuable for firm outcomes in an emerging economy; Chapter One shows that this is not necessarily the case, particularly if the reform encourages a substantial share of firms to choose a less onerous legal status. It is often assumed that insights from standard signalling models — in which an agent chooses a signal from a continuous distribution — apply as a metaphor for information asymmetries generally; Chapter Two, however, shows that, where the signal is binary, the conditions under which contracts are formed may be substantially more restrictive. Finally, it is often asserted that information asymmetry is an important reason for credit market inefficiency in emerging economies. But Chapter Three shows that, even in emerging economies, lenders can act as if having full information about prospective borrowers. In short, this thesis challenges some commonly-held views on credit markets in emerging economies.
Chapter I: Moroccan company law reform and manufacturing firms’ access to bank credit

MOROCCAN COMPANY LAW REFORM AND MANUFACTURING FIRMS’ ACCESS TO BANK CREDIT: A BEFORE/AFTER PANEL EVALUATION

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April 25, 2010

Abstract

In 2001, Morocco made a radical change to its company law regime: it replaced a company law dating from 19th-century France with modern standards of corporate governance and accountability. This paper analyses that reform and evaluates its impact upon manufacturing firms’ access to bank credit. I use panel data from 2000 and 2004 to test the effect upon bank overdraft provision of a firm’s legal obligations (that is, the firm’s choice of SA status rather than the less onerous SARL). I find that the reform induced a substantial share of SA firms to switch to SARL, and that — relative to firms remaining in the SA status — this caused a significant and substantial withdrawal of bank overdraft facilities.

JEL codes: O12, K22, G21.

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Chapter I: Moroccan company law reform and manufacturing firms’ access to bank credit

1 Introduction

Researchers and policymakers increasingly recognise the importance of formal institutions for corporate governance. Strong institutions for governance, it is argued, are essential for reassuring and protecting lenders and investors, especially in developing economies (for example, see The World Bank (2001, Ch.3) and The World Bank and the IFC (2008, 29–33)). However, the role of such institutions is often difficult to evaluate empirically. In particular, persuasive strategies to control for endogeneity of legal obligations can be difficult to develop with cross-sectional data, while panel datasets bridging corporate law reforms are scarce, particularly in the context of developing economies.

In this paper, I report an empirical evaluation of one such reform: Morocco’s introduction of a new company law regime in 2001. First, I examine the details and timing of the reform process; to my knowledge, the summary in this paper is the only one available in English.¹ I then use a representative two-period panel dataset to test the importance of the reform process for banks’ assessment of Moroccan manufacturing firms. Specifically, I use OLS and fixed-effect estimation to test the importance of a firm’s legal status for bank overdraft provision, including under heterogeneous-effect specifications. I find that the reform induced a substantial share of SA firms to switch to SARL, and that this caused a significant and substantial negative effect upon provision of bank overdraft facilities.

In short, I find that the legal reform caused a significant reduction of credit to the manufacturing sector while failing in its key objective of improving bank confidence in well-governed firms.

The research joins a growing literature on the importance of legal institutions in

¹ There are some very brief references to the reforms in The World Bank (2002) and The World Bank (2006), but neither source contains a summary of the key provisions of the statutes, nor of the reform process.
developing economies. Several papers have exploited cross-sectional variation to evaluate economic consequences of legal institutions: Laeven and Woodruff (2007), for example, use variation across Mexican regions to show a positive effect of legal quality on firm size, and Horioka and Sekita (2010) use region variation in judicial efficacy to estimate consequences of legal enforcement for household credit rationing in Japan. Similarly, La Porta, Lopez-de-Silanes, Shleifer, and Vishny (1997) and Qian and Strahan (2007) both use cross-country evidence to argue that better legal protection for creditors leads to improved lending outcomes (see also Djankov, Hart, McLiesh, and Shleifer (2008), Johnson, McMillan, and Woodruff (2002), and Levine (1998)). The paper also relates to a recent literature on the role of formalisation, particularly among micro-enterprises and small enterprises. For example, Bruhn (2008) shows a small positive employment and competition effect from the introduction of business registration in Mexico (see also Kaplan, Piedra, and Seira (2007)). Similarly, McKenzie and Seynabou Sakho (2009) show that tax registration among micro-enterprises and small enterprises in Bolivia increased profits for firms with between two and five workers, but decreased profits both for smaller and larger firms.

This paper seeks to contribute in several ways to these literatures. First, the paper deals with medium and large enterprises (median firm size is 60 permanent employees), and considers a sweeping reform designed to change significantly the internal governance of those firms. This complements the business registration literature by providing a point of comparison about the way that larger firms and formal bank lenders may respond to legal reform. Second, the paper uses a representative firm-level two-period panel dataset; the analysis, then, can control directly for firm fixed effects. Third, as the next section explains, the critical date of legal reform lay between the two rounds of the panel; the panel therefore allows a ‘before/after’ analysis of the change. This allows for a firm-level com-
The paper is organised as follows. In section 2, I summarise Moroccan company law and the process of its reform. In section 3, I discuss the testing strategy; I specify a Linear Probability Model and outline the Mundlak (1978) fixed-effect method. In section 4, I describe and summarise the available data. I report primary results in section 5; in section 6, I show that those results are robust to alternative choice of explanatory variables, to endogenous attrition and to a regression discontinuity specification. Finally, in section 7, I consider two extensions: heterogeneity of legal status effects and consequences for the level of the overdraft limit.

2 Background: Moroccan company law

In Morocco, standards of corporate governance are implemented through two main legal forms: the ‘Société Anonyme’ (SA) and the ‘Société À Responsabilité Limitée’ (SARL). The SA form is designed for larger, more formal enterprises. Almost all incorporated Moroccan companies choose to exist as one of these two legal forms (other forms include co-operatives and partnerships); for example, the census of manufacturing (discussed shortly) shows that, in 2000, over 95% of incorporated manufacturing companies chose either SA or SARL status. A firm’s choice of legal form is fundamental in determining its structure, its internal governance and its relations with outside parties. The reform of Moroccan company law was a reform of these two legal statuses; specifically, it involved imposing substantially more onerous obligations on firms choosing to exist as SA. In this section, I analyse Moroccan legislation and commentary in order to

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2 It is rare to have panel datasets bridging legal reforms in developing economies. I have been able only to find Klonner and Rai (2006) (where data is at the level of the ROSCA), Banerjee, Gertler, and Ghatak (2002) (where data is at the level of the district), Tybout, Gauthier, Navaretti, and De Melo (1997) and Gauthier, Soloaga, and Tybout (2002) (where data is at the level of the firm).
Chapter I: Moroccan company law reform and manufacturing firms’ access to bank credit

explain the history, justification and substance of that reform process.

2.1 The process of legal reform

This section summarises the process of legal reform. The section explains that the reform brought very substantial changes to standards of corporate governance in Morocco and shows, crucially, that the critical date for the universal application of the new regime was 1 January 2001.3

From 11 August 1922 until 20 January 1997, Moroccan SA companies were governed by French law; a dahir of 1922 applied to Morocco the relevant provisions of the French commercial statute of 24 July 1867 (Lazrak 2004, Slaoui and Lecerf 2000, Chbani Idrissi 1996).4 That law reflected very early notions of corporate governance and accountability; a prominent French lawyer commented in 2005 that the earlier regime allowed “anarchy” (L’Economiste 2005), while one Moroccan businessman interviewed for the present research described the former legal regime as “a jungle”.5 SARL companies were governed by a regime of the same era: a dahir implemented in September 1926 (Lofti 1996).

However, as the end of the last century neared, Moroccan legislators came to recognise the need for a new corporate law regime to reflect and support the modernisation both of Moroccan law and of the Moroccan economy. Thus, on

3 It is difficult to identify precisely the changes made by the reforms. Unlike the new SA and SARL statutes (which can be purchased even from street vendors in Casablanca), the text of the old law is not readily available (and therefore has not been directly consulted for this summary). Similarly, and perhaps somewhat incongruously, academic literature on the reform (such as that cited in this summary) provides little detail on the previous legal regime. Therefore, this summary draws primarily upon general discussions of the reform provided in L’Economiste, the leading Moroccan business newspaper.

4 A dahir is a Royal Decree; among other functions, it is the legal mechanism by which the Government of Morocco formally brings laws into effect. It is broadly analogous to the Royal Assent in a Westminster system or a presidential signature in the United States.

5 Interview with the author, Casablanca, September 2006. (Name withheld from publication here.)
2 July 1996, the Chambre des Représentants (the lower chamber of parliament) adopted Law 17-95: a new regime to regulate SA companies. The authors of the new statute (two French lawyers) took as their starting point the French company law of 24 July 1966 (L’Economiste 1999b, Lazrak 2004). Law 17-95 was promulgated by dahir on 30 August 1996. However, rather than being immediately applicable from that date, the law created a dual regime for its implementation. New SA companies would be required to have corporate statutes in compliance with the law if created after the entry into force of relevant provisions relating to the register of commerce (Article 443, Law 17-95); SA companies created prior to that date would receive an additional grace period to harmonise their corporate statutes with the law (Article 444, Law 17-95). Progress of the new SARL law followed a similar trajectory. A statute to replace the 1926 law was adopted on 7 January 1997 and promulgated by dahir on 13 February of that year. This law (Law 05-96) was subject to a similar dual regime (under Articles 121, 128 and 129 of that law).

The relevant provisions relating to the register of commerce were duly brought into force on 20 January 1997. Thus, law 17-95 was first applicable from this date, for new companies created in the SA status (Maatouk 2002, L’Economiste 1999c, L’Economiste 1998a). Law 05-96 would have been operative from the same time (Article 120, Law 05-96), but was not promulgated until 13 February; it therefore applied immediately from this later date. Deciding on the required date of harmonisation for existing companies was much less straightforward. The relevant provisions of each law originally provided that the harmonisation of existing company statutes would occur within two years. However, there was confusion from the outset (apparently arising because of an imprecise translation from

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6 Dahir no 1-96-124, preamble.
7 Dahir no. 1-96-124. Dates and details cited are provided in the preamble to the dahir.
8 Existing SA companies were entitled to bring their corporate statutes into compliance with the law before this date; however, there were no sanctions for not doing so.
the (official) Arabic text to the French version) as to when the two-year period
would commence (L’Economiste 2000a, L’Economiste 1999c); as a consequence,
interpretations differed between experts and even between government authori-
ties in different cities (L’Economiste 1999a). Eventually, consensus settled upon
the more generous interpretation; namely, that the dual-regime articles allowed
for a grace period of two calendar years following the year that the statutes were
first applicable (that is, 1997).

It was anticipated, then, that law 17-95 and law 05-96 would have universal ap-
lication from 1 January 2000. However, with just three days remaining until the
deadline, the parliament intervened by amending the relevant articles of both laws
to extend the grace period for a further year (L’Economiste 2000b); the amend-
ment was promulgated by dahir the following day.9 Despite calls for another
similar amendment a year later, the grace period finally expired on 31 December
2000. Thus, from 20 January 1997 until 31 December 2000, new SA companies
created were required to comply with law 17-95, while existing SA companies
could ‘opt in’ to law 17-95 or remain under the old law (that is, the dahir of
1922). The same principle applied to SARL companies.10 It was not until 1
January 2001, more than four years after its promulgation, that law 17-95 finally
applied universally to all SA firms (and, similarly, law 05-96 to all SARL firms).
Crucially — thanks to the additional delay — this implementation occurred af-
ter the completion of the bulk of the the FACS survey of manufacturing firms in
2000 (a survey which was completed during the very earliest adjustment to the
new regime). For this reason, the FACS-ICA panel (outlined shortly) provides
a before/after comparison of the effects of the implementation of the new legal

10 There is no information available in the survey data (summarised shortly) about whether particular
firms chose to ‘opt in’ or not.
Figure 1 summarises the process of reform.

2.2 A comparison between SA and SARL under the new legal regime

A comparison between law 17-95 (regarding the SA) and law 05-96 (regarding the SARL) is a comparison between two very different sets of legal obligations; indeed, the very character of the two forms is fundamentally different. “[T]he SA was designed as a joint stock company drawing upon public saving. It it thus distinguished very clearly from the SARL, a structure intended for companies of average size and/or of a family character, where the personality of the contributor is generally more important than the capital contributed.”

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11 Article 449 of law 17-95 and Article 126 of law 05-96 provided the consequences of failing to harmonise the corporate statute: a fine of 2000 to 10 000 dirhams could be imposed upon the relevant individuals and a court-imposed grace period of less than six months allowed; if the subsequent grace period was not followed, an additional fine of 10 000 to 20 000 dirhams was allowed (in addition, presumably, to all of the other consequences specified for operating in breach of the relevant laws).

12 Original quote: “...la société anonyme a vocation à être une société de capitaux et un instrument de drainage de l’épargne publique. Elle se distingue ainsi très nettement de la SARL, structure destinée aux entreprises de taille moyenne et/ou à caractère familial, où c’est la personnalité de l’apporteur qui compte en principe davantage que le capital apporté.”
In that sense, the SARL can be understood as “a hybrid form which in parts draws its rules from joint stock companies and in parts from partnerships. It is an ideal form for the operation of small- and medium-sized enterprises.”\(^{13}\) (Msalha 2005, p.109) This distinction — between a large formal enterprise comprising many contributors (the société anonyme, literally the ‘anonymous company’) and smaller, more informal firms — underpins the many distinctions in substantive law. Table 1 summarises these differences, which are discussed in more detail in the appendix.

### Table 1: SA and SARL: Key legal differences

<table>
<thead>
<tr>
<th></th>
<th>Société Anonyme</th>
<th>Société À Responsibilité Limitée</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capital</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum capital</td>
<td>300 000 or 3 000 000 MAD</td>
<td>100 000 MAD</td>
</tr>
<tr>
<td>Share transferability</td>
<td>Freely transferrable</td>
<td>Limited (subject to approval)</td>
</tr>
<tr>
<td>Granting security</td>
<td>May grant all forms</td>
<td>Limits on moveable interests; limits on negotiable securities</td>
</tr>
<tr>
<td><strong>Governance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Governance structure</td>
<td>Dualist</td>
<td>Unitary</td>
</tr>
<tr>
<td><strong>Transparency</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appointment of auditors</td>
<td>Always at least one</td>
<td>Generally unnecessary</td>
</tr>
</tbody>
</table>

#### 2.3 The old and the new: reforms to the SA form

The reform process introduced new statutes both for the SA and the SARL forms. However, the law on SARL firms, while reformulated, has remained largely unchanged; the SARL statute “has not raised much interest”\(^{14}\) (El Hammoumi

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\(^{13}\) Original quote: “...la société à responsabilité limitée s’apparente à la fois aux sociétés de capitaux et aux sociétés de personnes. C’est un type hybride qui puise ses règles tantôt des sociétés de capitaux, tantôt des sociétés de personnes. C’est un outil idéal pour l’exploitation des petites et moyennes entreprises.”

\(^{14}\) Original quote: “La loi n° 05-96, relative aux autres formes de sociétés commerciales, n’a pas soulevé autant d’intérêt.”
2005), as a survey of *L'Economiste* articles reveals.

In contrast, the reform to the *SA* form was a radical change to make the form substantially more onerous than it was previously; in this way, the reform process significantly increased the *relative cost* of retaining the *SA* status. Table 2 summarises the key reforms, which are also discussed in more detail in the appendix.

<table>
<thead>
<tr>
<th>Governance</th>
<th>Before</th>
<th>After</th>
</tr>
</thead>
<tbody>
<tr>
<td>Management structure</td>
<td>Unitary</td>
<td>Dualist</td>
</tr>
<tr>
<td></td>
<td>Only the corporate object</td>
<td>System of review</td>
</tr>
<tr>
<td></td>
<td>Allowed ‘sleeping directors’</td>
<td>Encourages active involvement</td>
</tr>
<tr>
<td>Transparency</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Auditors</td>
<td>Played a relatively lesser role</td>
<td>Active and ongoing role</td>
</tr>
<tr>
<td></td>
<td>Almost none</td>
<td>Extensive and ongoing</td>
</tr>
<tr>
<td></td>
<td>Clear conflicts allowed</td>
<td>Authorisation required</td>
</tr>
<tr>
<td>Rights of interest</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedures</td>
<td>Almost none</td>
<td>May convene a meeting</td>
</tr>
<tr>
<td>Third parties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk of corporate statute breach</td>
<td>Borne by the third party</td>
<td>Borne by the company</td>
</tr>
<tr>
<td>Corporate legal personality</td>
<td>Upon constitutive meeting</td>
<td>Upon registration</td>
</tr>
</tbody>
</table>

### 2.4 The institutional context: A summary

In summary, the institutional context was as follows.

(i) Manufacturing firms in Morocco are overwhelmingly governed under one of two legal statuses: *SA* and *SARL*. *SA* is more onerous and is designed for larger and more formal firms.

(ii) In 1996 and 1997, Morocco introduced new legislation governing the *SA* and *SARL* forms respectively. The new legislation made the *SA* status
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...substantially more onerous, but made minimal changes to the SARL status. The reform is therefore an important example of improving standards of corporate governance in an emerging economy.

(iii) The new laws applied to new companies from January and February 1997. However, for existing companies, the laws applied from 1 January 2001. That date fell between the two rounds of a panel survey of manufacturing firms. That panel therefore presents an opportunity to understand the consequences of the reform process.

There are many good reasons, then, to expect the distinction between SA and SARL status to be important for a bank’s assessment of a manufacturing firm. The SA/SARL distinction is a prominent one in dealings with any Moroccan manufacturing firm (for example, the status must be indicated prominently on all important documents emanating from the firm\(^\text{15}\)), and the distinction clearly represents fundamental differences in a company’s standards of corporate governance. Moreover, there are many good reasons to believe that the change in obligations — particularly the more onerous obligations placed upon the SA form — would be reflected in increased bank confidence for firms remaining in that status. These intuitions, then, form the basis of the empirical investigation.

3 Testing strategy

3.1 Specifying the model

I am interested in the effect of legal status (denoted \(S_i\), a dummy for whether a firm chooses SA) upon whether or not a firm has a bank overdraft facility (denoted \(y_i\), a dummy for having such a facility). Given that legal status is not assigned randomly, I will control for other firm characteristics, including a time-

\(^{15}\) Article 4, law 17-95; article 45, law 05-96.
invariant fixed effect. I therefore specify a Linear Probability Model (‘LPM’),

\[ y_{it} = \gamma \cdot S_{it} + \delta_1 \cdot x_{1it} + \delta_2 \cdot x_{2i} + \eta_i + \epsilon_{it}, \]  

where \( x_{1it} \) is a vector of time-varying parameters (including a time dummy), \( x_{2i} \) is a vector of time-invariant parameters (including the constant 1) and \( \eta_i \) is the firm fixed effect.

Equation 1 can be estimated either by OLS or by a fixed-effect estimator. For both OLS and fixed-effect estimations, I assume that the time-variant unobservable is linearly independent of the firm’s choice of legal status and of other explanatory variables: \( \mathbb{E}(\epsilon_{it} | S_{i0}, S_{i1}, x_{1i0}, x_{1i1}, x_{2i}) = 0 \). An OLS estimation of equation 1 will produce a consistent estimate of \( \gamma \) if the fixed effect is also linearly independent:

\[ \mathbb{E}(\eta_i | S_{i0}, S_{i1}, x_{1i0}, x_{1i1}, x_{2i}) = 0. \]  

If the condition in equation 2 holds, the OLS estimate of \( \gamma \) will be a consistent estimate of the average effect of legal status upon overdraft provision for all firms. However, whether or not the condition holds, I can estimate \( \gamma \) consistently by a fixed-effect estimator. In that case, the estimate of \( \gamma \) will be a consistent estimate of the effect of the legal reform for those firms changing their legal status between survey rounds. In effect, the legal reform can be considered as a treatment affecting all firms, with \( \hat{\gamma}_{FE} \) estimating the local average treatment effect upon overdraft provision.

I use a Linear Probability Model in order to control for fixed effects and, in doing so, to be able to test whether the condition in equation 2 holds. It is well known that the linear probability structure induces heteroskedasticity that biases the stan-
standard errors; I deal with this by using robust standard errors (clustered at the level of region × sector) (see Wooldridge (2002a, p.454)). An alternative to the LPM would be to use logit and fixed-effect logit models. However, the Mundlak test (outlined shortly) does not extend to that approach; further, the fixed-effect logit provides only conditional estimates. For these reasons, I do not report logit estimations; however, logit and fixed-effect logit estimations on this data provide very similar conclusions to those from the LPM.

### 3.2 Fixed effects

I want to test whether the condition in equation 2 holds. The Hausman (1978) test is standard, but is problematic in this context. In this context, the Hausman test requires a comparison between fixed-effect and random-effect specifications, and requires the random-effects estimator to be efficient. The efficiency of the random-effects estimator, however, is not tested by the Hausman statistic; moreover, it implies a specific form on the correlation between errors, whereas I would prefer to allow a fully-robust correlation structure (see Imbens and Wooldridge (2009)).

I therefore use Mundlak’s (1978) alternative fixed-effect estimator; I simply include \( \eta_i \) in the regression using:

\[
\eta_i \equiv \alpha \cdot \bar{S}_i + \beta \cdot \bar{x}_i,
\]

where \( \bar{S}_i \) and \( \bar{x}_i \) are the firm means of \( S_{i0} \) and \( S_{i1} \) and of \( x_{i0} \) and \( x_{i1} \) respectively. This produces identical estimates to the traditional fixed-effect estimator: Wooldridge (2002a, pp.290–291), Wooldridge (2009) (see also Ahn and Low (1996) and Baltagi and Liu (2007)). However, under this specification, I can perform fully robust estimation and test the condition in equation 2 by testing
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\[ H_0 : \alpha = 0; \beta = 0 \] (see Wooldridge (2002a, p.291), Wooldridge (2009)).

3.3 Robustness checks

I will use both OLS and fixed-effect approaches to estimate \( \gamma \). I will then perform several robustness checks; specifically, (i) I will change the explanatory variables used (discussed shortly), (ii) I will use a regression discontinuity design and (iii) I will use an Inverse Probability Weighting to allow for endogeneity of panel attrition.

4 Data

4.1 The census of manufacturers

A census of manufacturers is conducted annually in Morocco by the Ministry of Commerce, Industry and Telecommunications (‘MCIT’). It has run since 1985 and its coverage is almost universal. This paper does not use the census for substantive empirical analysis, since that census does not record information on firm–bank relations.\(^{16}\) Instead, the census is used here to show two specific trends, in aggregate terms. For this, I use the pooled set of firms recorded in the census from 1996–2003. First, Figure 2 shows the trend in aggregate sales from 1996 to 2003; it shows a relatively steady decline in sales, so that median sales fell by 24.4% from 1996 to 2003 (though declined only 4.9% from 2000 to 2003). This is not directly relevant to the estimation strategy — or to the interpretation of the empirical results — but nonetheless forms an important stylised fact about the general manufacturing climate at the time.

\(^{16}\) However, note that Fafchamps and Schündeln (2010) combine the census with a 1997 World Bank survey with information on the presence of financial institutions; the authors use these combined datasets to show the importance of credit availability for firm expansion, particularly among smaller firms.
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Figure 2: Aggregate trend in sales: The census data

The graph shows the trend in median sales, as calculated separately from each round of the manufacturing census. Bands are for the 33rd–67th quantiles and the 45th–55th quantiles respectively.

More importantly, the census also shows, in aggregate terms, the effect of the legal reform process on manufacturing firms’ choice of legal status. This is important for confirming that the legal reforms under consideration were indeed as important as the legal texts suggest. As outlined earlier, the reforms were designed to increase substantially the onerousness of the obligations of the SA status; in doing so, the reforms were designed to make the SA status a more restrictive and more exclusive form than it had previously been. The extent to which firms chose to migrate from SA to SARL status is therefore an indication of firms’ practical perception of the effect of the new legal regime. Table 3 shows that about 40% of all SA manufacturing firms recorded in the census during that time are also recorded as having changed to SARL status.
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Table 3: Status changes in the census (1996–2003)

<table>
<thead>
<tr>
<th>Status Changes</th>
<th>Number of Firms</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SA \rightarrow SA$</td>
<td>2,214</td>
<td>20.6%</td>
</tr>
<tr>
<td>$SA \rightarrow SARL$</td>
<td>1,513</td>
<td>14.1%</td>
</tr>
<tr>
<td>$SARL \rightarrow SA$</td>
<td>27</td>
<td>0.3%</td>
</tr>
<tr>
<td>$SARL \rightarrow SARL$</td>
<td>3,985</td>
<td>37.1%</td>
</tr>
<tr>
<td>Multiple switches recorded, $SA \leftrightarrow SARL$</td>
<td>88</td>
<td>0.8%</td>
</tr>
<tr>
<td>Other status (predominantly unincorporated firms)</td>
<td>2,907</td>
<td>27.1%</td>
</tr>
<tr>
<td></td>
<td>10,734</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3: Migration from $SA$ to $SARL$ status: The census data
Figure 3 shows that, of the 1513 firms changing from SA to SARL status, the vast majority (over 1100) changed in the year 2000 or 2001. This was a direct response to the legal reform, and was understood as such by the Moroccan business community. For example, Azzedine Benmoussa (Founding Director of KPMG Morocco) noted in 2002 that “a large proportion of SA companies, essentially of the family type, transformed themselves into SARL with the entry into application of law 17-95 on the société anonyme (SA)” (Shamamba 2002). This conclusion accorded closely with the responses of Moroccan businesspeople interviewed for this research; indeed, no other explanation for the migration was ever suggested. However, the migration did not begin with the universal application of the law. Rather, as one would have anticipated, many companies abandoned the SA form in anticipation of the application of the new regime. Indeed, this was noted in L’Economiste as early as 1998 (L’Economiste 1998b).

The reforms had sought to make the SA status a more rigorous and more exclusive form; for better or for worse, they clearly had this effect.

4.2 The FACS-ICA panel

To analyse the bank-firm relationship, I use the two-period FACS-ICA panel. The FACS (‘Firm Analysis and Competitiveness’) Survey was conducted in late 2000 as a collaborative venture between the MCIT, the World Bank and the University of Oxford. The sample was drawn by unstratified random sampling across the six largest regions and the seven largest production sectors. The FACS survey excluded firms which, either at the time of the preceding census or at the time of

---

17 Original quote: “. . . nous avons relevé qu’une grande partie des SA, de type familial essentiellement, se sont transformées en SARL avec la mise en application de la loi 17-95 sur la société anonyme (SA).”
18 For this reason, my estimation strategy identifies the effect of changing legal status for those firms changing status after the universal application of the new legal regime in 2001.
19 This kind of migration is not unique to Morocco, either; a similar change is reported to have occurred in Spain as part of that country’s reforms in 1989 to the Sociedad Anónima (SA) and Sociedad de Responsabilidad Limitada (SRL) legal forms.
the FACS survey itself, had fewer than 10 employees. A total of 859 firms were interviewed. The ICA (‘Investment Climate Assessment’) Survey was conducted in 2004. It was designed as a follow-up to the FACS survey and was again a collaborative project of the MCIT, the World Bank and the University of Oxford. Significant portions of the FACS survey — including the entire accounts section — were replicated in the ICA survey. A total of 746 firms was interviewed, 546 of which had been interviewed for the FACS survey. The data analysed here is the subset of firms in the FACS-ICA panel that were recorded as being either SA or SARL form in both the FACS and ICA surveys. This leaves a panel of 512 firms. Of these, I drop 24 firms where key variables were either missing or implausible; this leaves a balanced panel of 488 firms.

Figure 4: Firm locations by province, FACS

The firms under consideration are, therefore, medium and large enterprises; the distribution of permanent employees is approximately log-normal (truncated below at 10, as noted) with a median of 60. Figure 4 shows the distribution of panel firms by province; Table 4 shows the range of firm sectors by region.
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Table 4: Region and sector for panel firms, FACS

<table>
<thead>
<tr>
<th></th>
<th>Casablanca</th>
<th>Fès</th>
<th>Nador</th>
<th>Rabat</th>
<th>Settat</th>
<th>Tanger</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals</td>
<td>32</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Electrical</td>
<td>14</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Food</td>
<td>12</td>
<td>9</td>
<td>9</td>
<td>0</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Garment</td>
<td>99</td>
<td>29</td>
<td>0</td>
<td>15</td>
<td>0</td>
<td>27</td>
</tr>
<tr>
<td>Leather</td>
<td>27</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Plastics</td>
<td>29</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Textile</td>
<td>71</td>
<td>7</td>
<td>3</td>
<td>8</td>
<td>7</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>284</td>
<td>57</td>
<td>15</td>
<td>33</td>
<td>30</td>
<td>69</td>
</tr>
</tbody>
</table>

Figure 5 shows the date of first interview for firms in the FACS survey, where recorded (only 405 of the 488 firms had plausible recorded dates). The figure confirms that the vast majority of first-round interviews occurred prior to the introduction of the new legal regime. Only 17 firms are recorded as having been interviewed after that date (as shown). One option would be to drop these firms; however, I choose not to do this. As the previous discussion and census data showed, many firms were still in the process of compliance with the new statutes after the 1 January implementation; thus, while the law formally applied from that date, it is not reasonable to treat the date as providing an absolutely sharp discontinuity. Rather, the point is that the first round was conducted during the reform process, so that the ICA survey then provides a point of comparison as to how market participants ultimately reacted.
Key explanatory variables. The FACS-ICA panel provides a wide range of potential explanatory variables. In due course, I will use two primary categories of explanatory variables: ‘basic characteristics’ (firm age and age squared, the Director-General’s experience and the Director-General’s gender (all taken from the FACS survey) and the number of permanent employees) and ‘firm assets’ (value of machines and equipment and value of land and buildings), as well as controls for region, bank, sector and time. Together, these two categories will form my preferred regression specifications. However, I will also add ‘firm liabilities’ (that is, measures of long-term debt) as a robustness check. I do not prefer this specification, because I am concerned that long-term debt is likely to be co-determined by a firm’s principal bank along with the decision on overdraft provision; that is, a bank holding a low estimation of a firm is presumably not likely either to extend either a generous loan or a generous overdraft limit. Table 5 summarises the key variables that this analysis will use. Further, I will control throughout for bank effects, sector effects and region effects.
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Table 5: Key explanatory variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>S.Dev.</th>
<th>1st Q.</th>
<th>Median</th>
<th>3rd Q.</th>
<th>Min.</th>
<th>Max.</th>
<th>Zeroes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (permanent employees)</td>
<td>976</td>
<td>4.2</td>
<td>1.1</td>
<td>3.3</td>
<td>4.1</td>
<td>5.0</td>
<td>2.3</td>
<td>8.0</td>
<td>0</td>
</tr>
<tr>
<td>Age (years) ( (t = 0) )</td>
<td>976</td>
<td>16.2</td>
<td>12.4</td>
<td>7.0</td>
<td>13.0</td>
<td>21.0</td>
<td>1.0</td>
<td>76.0</td>
<td>0</td>
</tr>
<tr>
<td>Years with principal bank ( (t = 0) )</td>
<td>976</td>
<td>11.8</td>
<td>9.4</td>
<td>5.0</td>
<td>10.0</td>
<td>16.0</td>
<td>0.0</td>
<td>66.0</td>
<td>2</td>
</tr>
<tr>
<td>Director-General’s years directing ( (t = 0) )</td>
<td>976</td>
<td>11.1</td>
<td>8.3</td>
<td>5.0</td>
<td>10.0</td>
<td>15.0</td>
<td>0.0</td>
<td>50.0</td>
<td>4</td>
</tr>
<tr>
<td>Dummy: Female Director-General ( (t = 0) )</td>
<td>976</td>
<td>0.04</td>
<td>0.19</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.0</td>
<td>938</td>
</tr>
<tr>
<td>Log (machines and equipment)</td>
<td>976</td>
<td>15.0</td>
<td>1.8</td>
<td>13.8</td>
<td>14.9</td>
<td>16.1</td>
<td>6.9</td>
<td>20.7</td>
<td>0</td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>976</td>
<td>14.4</td>
<td>3.5</td>
<td>14.0</td>
<td>15.0</td>
<td>15.8</td>
<td>0.0</td>
<td>21.4</td>
<td>46</td>
</tr>
<tr>
<td>Log (long-term debt)</td>
<td>976</td>
<td>13.1</td>
<td>5.6</td>
<td>13.4</td>
<td>15.0</td>
<td>16.2</td>
<td>0.0</td>
<td>20.3</td>
<td>144</td>
</tr>
</tbody>
</table>

These explanatory variables assist in describing the difference between the SA and SARL form. Table 6 compares each of the key explanatory variables between the SA and SARL firms; to avoid confounding the description with the results of the policy change, this summary is reported only for the first round (the FACS survey). The table shows that, at that time, SA firms were, on average, significantly larger, older, wealthier, had been longer with their principal bank and had more experienced Directors-General.
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Table 6: Comparison of SA and SARL: FACS survey

<table>
<thead>
<tr>
<th></th>
<th>SA (181 firms)</th>
<th>SARL (307 firms)</th>
<th>Diff. (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (permanent employees)</td>
<td>4.66 ± 1.10</td>
<td>3.96 ± 1.02</td>
<td>0.000***</td>
</tr>
<tr>
<td>Age (years)</td>
<td>21.03 ± 15.07</td>
<td>13.41 ± 9.41</td>
<td>0.000***</td>
</tr>
<tr>
<td>Years with principal bank</td>
<td>15.22 ± 11.24</td>
<td>9.80 ± 7.36</td>
<td>0.000***</td>
</tr>
<tr>
<td>Director-General’s years directing</td>
<td>12.45 ± 8.92</td>
<td>10.24 ± 7.78</td>
<td>0.006***</td>
</tr>
<tr>
<td>Dummy: Female Director-General</td>
<td>0.039 ± 0.014</td>
<td>0.039 ± 0.011</td>
<td>0.982</td>
</tr>
<tr>
<td>Log (machines and equipment)</td>
<td>15.64 ± 1.78</td>
<td>14.55 ± 1.50</td>
<td>0.000***</td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>15.40 ± 2.04</td>
<td>14.38 ± 1.99</td>
<td>0.000***</td>
</tr>
<tr>
<td>Log (long-term debt)</td>
<td>14.95 ± 4.29</td>
<td>13.34 ± 4.46</td>
<td>0.000***</td>
</tr>
</tbody>
</table>

Confidence: *** ↔ 99%, ** ↔ 95%, * ↔ 90%.
(For each variable, the p-value for mean equality is derived from a t-test allowing unequal variances.)

4.3 Firm-bank relations

Table 7 shows the distribution of primary bank by region from the FACS survey (bank names were not recorded in the ICA survey). The table shows that there is a wide range of banks involved, primarily French (or subsidiaries thereof). In due course, this information will provide a basis for controlling for bank fixed effects.20

---

20 This approach clearly requires the simplifying assumption that no firm changed bank between survey rounds. The ICA survey has a question about the number of years that the firm has banked with its principal bank; 23 firms in the panel responded that they had been with their bank for fewer than four years. Thus, while the assumption that firms did not change bank seems not to be literally true, it seems reasonable — particularly given that the role of bank fixed effects is simply as a control, rather than a variable of direct interest.
I am concerned in this paper to evaluate the impact of legal status upon banks’ willingness to provide overdraft facilities (and, indeed, the size of the overdraft limit provided). I choose overdraft provision as the dependent variable — rather than, for example, provision of bank loans — because the overdraft limit provides the clearest indication of a bank’s overall assessment of a firm. Many successful firms in the data do not have bank loans; it is very plausible — indeed, very likely — that this is not because banks would be unwilling to lend but because such firms have no borrowing need. This kind of endogeneity is much less likely in the case of overdraft provision; it can reasonably be assumed that every firm would like an overdraft facility if possible (after all, there is no requirement that it use the overdraft facility), so a bank’s decision about whether to provide an overdraft provides a reasonable measure of a bank’s confidence in a firm’s ability to repay. Further, the overdraft appears generally to be an important source of
short-term finance; of the 388 firms having an overdraft in the FACS survey, 330 were using some part of it, of which 44 were at (or beyond) their overdraft limit.

Table 8: Summary statistics for overdraft limits

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>S.Dev.</th>
<th>1st Q.</th>
<th>Median</th>
<th>3rd Q.</th>
<th>Min.</th>
<th>Max.</th>
<th>Zeroes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FACS (t = 0)</strong></td>
<td>488</td>
<td>10.9</td>
<td>5.7</td>
<td>11.5</td>
<td>13.1</td>
<td>14.5</td>
<td>0.0</td>
<td>18.6</td>
<td>100</td>
</tr>
<tr>
<td><strong>ICA (t = 1)</strong></td>
<td>488</td>
<td>10.3</td>
<td>6.2</td>
<td>0.0</td>
<td>13.1</td>
<td>14.5</td>
<td>0.0</td>
<td>18.4</td>
<td>125</td>
</tr>
</tbody>
</table>

Table 8 summarises firm overdraft limits in each round of the panel. It shows that, from the first round to the second round, there was a reduction in the mean (log) overdraft, though not the median; moreover, it shows that there were substantially more firms with no overdraft facility in the second period. Figure 6 shows the distribution of overdraft limits between the two rounds of the survey (with a 45-degree line superimposed); it augments Table 8 by showing that many of the firms with no overdraft facility in the second period did have such a facility in the first period, and vice versa.
Table 9 summarises overdraft provision and legal status for each round of the survey. It shows that SA firms were substantially more likely to have an overdraft and to have higher overdrafts. Table 10 suggests that the change in legal status may also have been relevant in banks’ overdraft decision; for example, it shows that an SA firm switching to SARL status was approximately twice as likely to lose its overdraft facility than a firm remaining as SA. This suggestion demands a more rigorous analysis.

### Table 9: Legal status and overdraft provision

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Number with overdraft</th>
<th>Average positive overdraft (’000 MAD)</th>
<th>Average overdraft (’000 MAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>FACS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SARL</td>
<td>307</td>
<td>225 (73.3%)</td>
<td>1 183 (US$111 200)</td>
<td>867 (US$ 81 500)</td>
</tr>
<tr>
<td>SA</td>
<td>181</td>
<td>163 (90.1%)</td>
<td>5 891 (US$553 600)</td>
<td>5 305 (US$498 500)</td>
</tr>
<tr>
<td>Combined</td>
<td>488</td>
<td>388 (80.0%)</td>
<td>3 161 (US$297 100)</td>
<td>2 513 (US$236 200)</td>
</tr>
<tr>
<td><strong>ICA</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SARL</td>
<td>394</td>
<td>282 (71.6%)</td>
<td>1 606 (US$179 100)</td>
<td>1 150 (US$128 300)</td>
</tr>
<tr>
<td>SA</td>
<td>94</td>
<td>81 (86.2%)</td>
<td>9 747 (US$1 087 200)</td>
<td>8 399 (US$936 800)</td>
</tr>
<tr>
<td>Combined</td>
<td>488</td>
<td>363 (74.4%)</td>
<td>3 423 (US$381 800)</td>
<td>2 546 (US$284 000)</td>
</tr>
</tbody>
</table>

(Currency conversions use the average daily interbank rate for 2000 and 2004 respectively; rates are drawn from www.oanda.com.)
Table 10: Change in legal status and change in overdraft provision

<table>
<thead>
<tr>
<th></th>
<th>Change in overdraft status</th>
<th>Average change (’000 MAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Retained</td>
<td>Gained</td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SA → SA:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>86</td>
<td>70</td>
<td>5</td>
</tr>
<tr>
<td>SA → SARL:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>95</td>
<td>70</td>
<td>1</td>
</tr>
<tr>
<td>SARL → SA:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>SARL → SARL:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>299</td>
<td>172</td>
<td>39</td>
</tr>
<tr>
<td>All:</td>
<td>488</td>
<td>318</td>
</tr>
</tbody>
</table>

(Figures are converted to USD equivalents using the average daily interbank rate for 2004; rates are drawn from www.oanda.com.)

5 Results

Table 12 reports the main results from estimation on equation 1. Column 1 shows the OLS estimates. I obtain \( \hat{\gamma}_{OLS} = 0.095 \), which is significant at the 99% confidence level. The Mundlak test — reported at the bottom of column 2 — does not reject a null hypothesis that the condition in equation 2 holds. I therefore conclude that the average effect of choosing the SA status rather than the SARL status is an increase of approximately 10 percentage points in the probability of having an overdraft facility.

Column 2 shows the fixed-effect estimates, implemented using the Mundlak methodology discussed earlier. I obtain \( \hat{\gamma}_{FE} = 0.104 \), significant at the 95% con-
fidence level. I therefore conclude that, among the firms changing legal status between periods, the average effect of choosing the SA status rather than the SARL status is, again, an increase of approximately 10 percentage points in the probability of having an overdraft facility. This implies that firms switching from SA to SARL after the universal application of the new regime were, on average, approximately 10 percentage points less likely to have an overdraft facility in 2004 than in 2000.

This is confirmed by column 2 of Table 13. In that table, I estimate directly the consequences of changing legal status. I find that switching from SA to SARL caused a reduction in the probability of having a bank overdraft of approximately 13.5 percentage points, relative to firms remaining in the SA status. As discussed earlier, the SA status was substantially more onerous as a result of the reforms, while there was effectively no change to the SARL. One would expect, therefore, that firms switching from SA to SARL would suffer a significant reduction in their probability of having an overdraft; column 2 of Table 13 reflects this. In short, the reform induced a substantial movement from the SA status to SARL, and firms doing so were punished by banks relative to firms remaining in the SA status.

6 Robustness

6.1 Choice of explanatory variables

In Table 14, I test the robustness of the results in Table 13 to the choice of explanatory variables. Columns 1 and 2 of Table 14 repeat the estimations of Table 12 where the only controls are dummies for time, region, bank and sector. Columns 3 and 4 of Table 14 repeat the estimations of Table 12 including mea-
sures of long-term debt. Neither specification substantially changes the results or the conclusions from Table 12.

6.2 A discontinuity design

I explained earlier that firms created after January 1997 were required to comply with the new *SA* and *SARL* statutes, whereas firms created before that time were allowed to remain under the old regime until January 2001. Given that the FACS survey was completed in late 2001, this corresponds to a discontinuity at approximately age four. As a robustness check on the results in Table 12, I estimate using a discontinuity design around age four.

Table 11: Age and legal status for firms aged between two and seven years

<table>
<thead>
<tr>
<th></th>
<th>2 ≤ Age (<em>t</em> = 0) ≤ 4</th>
<th>5 ≤ Age (<em>t</em> = 0) ≤ 7</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>FACS (2000)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>SARL</em></td>
<td>18</td>
<td>12</td>
<td>19</td>
</tr>
<tr>
<td><em>SA</em></td>
<td>1</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td><strong>ICA (2004)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><em>SARL</em></td>
<td>18</td>
<td>14</td>
<td>23</td>
</tr>
<tr>
<td><em>SA</em></td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 11 summarises the number of firms and legal status for a three-year block either side of that discontinuity. Were there more firms lying near the discontinuity, I could use a non-parametric estimator to fit the relationship very flexibly (that is, for example, a kernel estimator). However, with relatively so few firms near the discontinuity (as shown, only 112 between age 2 and age 7 inclusive), I choose instead simply to incorporate the discontinuity into a regression framework.
I report the results in Table 15. I report three estimations, each varying in the way that age enters; in column 1, I do not control for firm age, in column 2, I control for age with a linear specification and in column 3 I use a quadratic.

Because I am interested to compare firms under the new SA regime in 2000 with firms under the old SA regime in the same year, I run the regression only using the first period of the panel; as explained, firms aged 4 and younger in 2000 were required to comply with the new legal regime, whereas older firms were entitled to remain under the previous regime until 2001. I therefore interpret the difference between coefficients on the SA dummy for each age category as an estimate of the relative effect of the new SA regime against the old SA regime.

Column 2 of Table 13 shows that, between firms changing their status, this difference is not found to be positive (that is, firms remaining in the SA status after the reform did not receive a significantly higher probability of having an overdraft after the reform than before). The discontinuity design does not challenge this conclusion. Figure 7 illustrates the quadratic specification of OLS.3 and this resulting point estimate.

### 6.3 Sample selection

In principle, the panel component of the ICA survey was a random subset of the FACS survey. Since the FACS survey was representative of the population at the time, analysis of the panel should provide consistent estimates for the population. However, there is still a practical possibility of endogenous attrition — in particular, attrition caused by firms dropping below 10 permanent employees (and therefore not being included in the second round) and attrition caused by my dropping 24 firms with missing or implausible values for key variables. As a ro-
business check, I deal with this by an Inverse Probability Weighting methodology (see Wooldridge (2002a, 587–590), Wooldridge (2002b) and Moffitt, Fitzgerald, and Gottschalk (1999)). This method requires the assumption of ‘selection on observables’, which I identify by a probit specification: \[21\]

\[
\Pr(p_{it} = 1 \mid y_{it}, x_{1it}, x_{2it}, z_{i0}) = \Pr(p_{it} = 1 \mid z_{i0}) = \Phi(\psi z_{i0}),
\]

(4)

where \(p_{it}\) is an indicator for the appearance of firm \(i\) in the panel (so, in this context, \(p_{i0} = p_{i1} \forall i\)) and \(z_{i0}\) is some vector of firm-specific variables observed in the FACS survey. I estimate equation 4, then weight every observation in subsequent analysis by a firm-specific predicted inverse probability: \((\Phi(\hat{\psi} z_{i0}))^{-1}\).

For the vector \(z_{i0}\), I use all of the subsequent explanatory variables and both out-

\[21\] This refers, of course, to selection into sample, not selection into legal status; the latter is obviously allowed to depend upon fixed effects. Note that I could relax this assumption given that the ICA survey is itself (approximately) a representative cross-section of manufacturing firms in 2004; I could therefore use the methodology of Bhattacharya (2008) to allow for sample selection on unobservables. But the IPW method is simpler and more direct; moreover, issues of selection correction are not a primary focus of this paper.
come variables (that is, whether a firm has an overdraft facility and what overdraft facility it has) from the FACS survey. The results are reported in Table 16. Figure 8 shows the consequent inverse probability weights.\(^{22}\)

![Distribution of Inverse Probability Weights](image)

For brevity, I do not report the weighted estimations (they are available on request). However, those estimations support all of the same conclusions as the unweighted results reported.

\(^{22}\) Recall that the FACS survey involved 859 firms. Of these, 63 reported a legal status other than SA or SARL; I exclude these on the basis that they fall outside the population of interest. This leaves 793 firms, of which 783 have sufficient data in order to fit the selection probit (the 488 firms retained for the panel are then a strict subset of the 783 firms). I am required, therefore, to assume that the additional 10 omitted firms are omitted ‘at random’. (This assumption is not necessarily accurate, of course, but it is necessary — and, after all, it amounts to only about 1% of the relevant firms.)
7 Extensions

7.1 Heterogeneous effects

I am also interested to understand how the ‘legal status effect’ varies across different firms. For this, I use a standard linear regression specification, in which I interact legal status with several variables of interest.

First, I consider heterogeneous effects across firm size, firm age and the duration of a firm’s relationship with its bank; Table 17 reports the results. As discussed earlier, the SA form was primarily designed for larger and more formal firms; the reform process only further emphasised this characterisation. For this reason, one might expect that larger firms would enjoy a larger relative benefit from the SA status. However, the estimation does not produce a significant positive coefficient, either in OLS or fixed-effect specifications (columns 1 and 2 respectively).

One possible explanation for this result not being strongly positive is that smaller firms may enjoy a substantial signalling benefit from choosing the SA status: that is, banks may be particularly responsive to the status among smaller firms because banks might otherwise expect management quality among those firms to be particularly poor.

It is instructive, then, to consider the legal status effect by number of years with the principal bank (at the FACS survey); this is suggestive of the importance of information asymmetry to the relationship (see Banerjee and Duflo (2001, p.17)). I find a significant negative relationship under fixed effects estimation (column 4 of Table 17). Consideration of heterogeneity by age adds some further support to the information-asymmetry interpretation: a linear specification in age is also significantly negative under fixed effects (column 6); this, too, may be explained
by some firms (in this case, younger firms) using the SA status to signal strong management quality.

The final heterogeneous-effect specification considers heterogeneity by principal bank; the results are reported in Table 18. That table reports OLS and fixed-effect estimations with legal status interacted with principal bank, controlling for firm size, firm assets and other dummies. As shown, the joint test of equality of the legal status effect across banks rejects at the 95% confidence level. Figure 9 shows the reported point estimates with 95% confidence intervals. It shows the sets of banks that pass an homogenous-effect test: in order, banks from Wafabank to BCP may be pooled; alternatively, one may pool banks from CDM to SGMB.

Figure 9: Heterogeneous legal status effect by bank

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23 Seventy-three firms changed bank between survey rounds; these observations are dropped from these estimations.
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7.2 Legal status and the level of overdraft limit

To this point, the outcome variable has been a dummy for whether a firm has an overdraft facility. In this extension, I consider the separate question of the size of the overdraft provided, conditional upon a firm having an overdraft.

For this specification, I use the same explanatory variables (with equivalent parameters denoted in upper case) and use the log overdraft as the dependent variable:

\[
\log(\text{overdraft limit}_{it}) = \begin{cases} 
\Gamma^* \cdot S_{it} + \Delta^*_1 \cdot x_{1it} + \Delta^*_2 \cdot x_{2i} + \mu^*_{it} & \text{if overdraft limit}_{it} > 0; \\
\text{missing} & \text{if overdraft limit}_{it} = 0.
\end{cases}
\]  

(5)

This creates a selection problem: I need to estimate the effect of legal status on the (log) level of overdraft limit conditional on an overdraft being provided. I therefore need to allow for cross-equation error correlation (see Hay and Olsen (1984)). I do so using the standard specification of Heckman (1979) in order to estimate an ‘Adjusted Tobit’ model (see, for example, Van de Ven and Van Praag (1981)). This requires the assumption of bivariate normality between the error terms. I therefore estimate the first stage, in this context, using a probit; the assumption of bivariate normality implies that the marginal distribution on the first-stage error is normal.\(^{24}\) For the purposes of the secondary specification, I therefore need the following first stage:

\[
\Pr(\text{overdraft limit}_{it} > 0) = \Pr(\Gamma \cdot S_{it} + \Delta_1 \cdot x_{1it} + \Delta_2 \cdot x_{2i} + \mu_{it} > 0) = \Phi(\Gamma \cdot S_{it} + \Delta_1 \cdot x_{1it} + \Delta_2 \cdot x_{2i}),
\]  

(6)

\(^{24}\) I am unable to find an instrument for selection into having a positive overdraft. I am therefore required to use the same explanatory variables for the first-stage and second-stage estimations; I identify cross-equation correlation by the assumption on functional form alone.
where $\phi(\cdot)$ and $\Phi(\cdot)$ are respectively the pdf and cdf of the normal distribution.

I can therefore specify a bivariate normal distribution for $\mu_{it}$ and $\mu^*_{it}$,

$$
\begin{pmatrix}
\mu_{it} \\
\mu^*_{it}
\end{pmatrix}
\sim
\mathcal{N}
\left(
\begin{pmatrix}
0 \\
0
\end{pmatrix},
\begin{pmatrix}
1 & \rho \cdot \sigma_{\mu^*} \\
\rho \cdot \sigma_{\mu^*} & \sigma^2_{\mu^*}
\end{pmatrix}
\right),
$$

(7)

so that

$$
\mathbb{E}(\log(\text{overdraft limit}_{it}) \mid \text{overdraft limit}_{it} > 0)
\quad
= \Gamma^* \cdot S_{it} + \Delta^*_1 \cdot x_{1it} + \Delta^*_2 \cdot x_{2i} + \mathbb{E}(\mu^*_{it} \mid \text{overdraft limit}_{it} > 0)
\quad
= \Gamma^* \cdot S_{it} + \Delta^*_1 \cdot x_{1it} + \Delta^*_2 \cdot x_{2i} + \rho \cdot \frac{\phi\left(\Gamma \cdot S_{it} + \hat{\Delta}_1 \cdot x_{1it} + \hat{\Delta}_2 \cdot x_{2i}\right)}{\Phi\left(\Gamma \cdot S_{it} + \hat{\Delta}_1 \cdot x_{1it} + \hat{\Delta}_2 \cdot x_{2i}\right)}.
$$

(8)

The conditions for consistent estimation of the secondary specification are clearly stronger than for the primary specification of equation 1. I have just shown that I need a very specific assumption about the structure of the error terms in order to justify the Adjusted Tobit estimator. Further, unlike the primary specification, I am unable to control for fixed effects in either stage of this secondary specification.

Table 19 reports the results. Column 1 reports an OLS regression on the 751 observations with positive overdraft limits; column 2 reports the ‘second stage’ of equivalent estimations using the selection-correction (with estimation done using Full Information Maximum Likelihood). Under both specifications, I find legal status to be significant at the 99% confidence level; under both specifications,

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25 See Manning, Duan, and Rogers (1987) for Monte Carlo evidence on the use of a more flexible ‘two-part’ model for estimating in this context.

26 I do not report the first stage probit results; they are available on request.

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I estimate that choosing \( SA \) rather than \( SARL \) causes an increase of approximately 45% in the overdraft limit provided.\(^{27}\)

This result must be treated cautiously, for two reasons. First, as explained earlier, I cannot control for fixed effects here because of the need to implement the Heckman selection-correction. Second, unlike the results in Table 12, this estimation is relatively sensitive to the choice of explanatory variables.\(^{28}\) Subject to these caveats, I conclude that, conditional upon receiving an overdraft, choosing \( SA \) rather than \( SARL \) caused the overdraft limit to be approximately 45% higher than it otherwise would have been.

\(^{27}\) That is, \( \exp(0.347) - 1 \approx 0.41 \) and \( \exp(0.382) - 1 \approx 0.47 \).

\(^{28}\) Further estimations are available on request.
8 Conclusion

Summary. In this paper, I have used a representative two-period panel of Moroccan manufacturing firms to evaluate the consequences of a radical new company law regime. I showed that the reform induced a substantial proportion of manufacturing firms to switch from the more onerous \emph{SA} status to the less onerous \emph{SARL}; banks punished such firms for doing so, relative to firms remaining in the \emph{SA} status. I argued that this result was robust to the choice of explanatory variables, to the potential for endogenous sample attrition and to a regression discontinuity specification.

I extended this result to consider heterogeneity across firms. I showed that the relative benefit of the \emph{SA} status was greater for younger firms and for firms having been with their bank for a shorter time. I showed that there was significant heterogeneity between different banks’ responses to the \emph{SA} status. Finally, I considered the effect of legal status upon the level of overdraft limit provided; I estimated that, conditional upon having an overdraft, the \emph{SA} status caused an increase in the overdraft limit of approximately 45%.

Mechanisms and implications. The estimation strategy used in this paper is \emph{not} able to identify the \emph{mechanism} by which \emph{SA} firms were more likely to receive overdraft facilities than \emph{SARL} firms — and, therefore, the mechanism by which firms switching from \emph{SA} to \emph{SARL} were more likely to lose their overdraft. One explanation is that banks used legal status as an important proxy for firm characteristics that they could not observe (such as management quality). Under this interpretation, banks withdrew overdraft facilities from firms switching status because banks interpreted the change in status as indicating a decline in firm management; in effect, because banks continued to use outdated ‘rules of thumb’ to assess firms’ creditworthiness rather than adopting new assessments in
light of the new institutional regime. This explanation can be described as implying that legal status had a substantial ‘information effect’.

However, there is another explanation. It may be that banks understood the new legal regime well, but assessed that moving to the SARL status was likely to cause a decline in firm management and performance. Under this interpretation, the withdrawal of overdraft facilities was a rational and pragmatic response to the lesser legal obligations that many firms now faced. This explanation can be described as implying that legal status had a substantial ‘incentive effect’.

In Quinn (2010), I use a structural model in order to differentiate between information and incentive effects. However, regardless of which explanation is accepted, the consequences of Morocco’s company law reform in 2001 may be understood as illustrating the risks of an emerging economy failing to provide adequate supporting infrastructure for an ambitious modern statute. Specifically, the fact that a reform designed to improve standards of corporate governance would encourage a substantial share of firms to migrate to a less onerous legal form — and that, relative to firms not migrating, those firms would lose access to overdraft facilities — suggests that, in some respects at least, the reform may have been counter-productive. The World Bank’s (2003) ‘Legal and Judicial Sector Assessment’ of Morocco suggested that the reform may have been “too technical given both Morocco’s economic structures and the proficiency of the legal profession” (p.13). This paper provides some support for such a claim.
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Appendix: Further details of the legal reform

Comparison between the modern \textit{SA} and the modern \textit{SARL}

Table 1 (page 9) provides a brief summary of the differences between the modern \textit{SA} and the modern \textit{SARL} forms. This section provides further details of those differences.

\textbf{Capital.} In line with French law, both the \textit{SA} and \textit{SARL} forms have a minimum capital requirement. However, that requirement is much higher in the case of the \textit{SA}; an \textit{SARL} must have registered capital of at least 100,000 dirhams,\textsuperscript{29} while an \textit{SA} must have a minimum capital of 300,000 dirhams, or 3 million dirhams if the company issues a prospectus.\textsuperscript{30} Similarly, there are significant differences in the transferability of capital: shares in an \textit{SA} are freely transferrable (Martin 1999, p.163), whereas shares in an \textit{SARL} generally may not be transferred to third parties without the consent of a majority of shareholders representing three-quarters of the shares (Martin 1999, pp.157–158).\textsuperscript{31} Finally, the \textit{SA} has much wider scope for dealing in its capital: unlike the \textit{SA}, the \textit{SARL} may not grant security over moveable interests,\textsuperscript{32} nor represent any of its registered capital by negotiable instruments.\textsuperscript{33}

\textbf{Governance.} The \textit{SA} and \textit{SARL} forms exhibit clearly different governance structures. An \textit{SA} may choose between two governance structures, both dualist and both quite onerous: a Board of Directors with an ‘Administrator’ or a Board of Directors overseen by a Board of Trustees.\textsuperscript{34} The significance of this will be discussed in comparing the new regime with the old; for now, note merely that both structures provide significant division of management powers. In contrast, an \textit{SARL} has a unitary structure: it is governed merely by one or more designated managers,\textsuperscript{35} empowered (in the absence of specific limitation in the company’s own statutes) to take all necessary action in the interests of the company.\textsuperscript{36}

\textbf{Transparency.} Improvements in corporate transparency are an important part of the legal reforms, as subsequent discussion will show. However, understandably, there remain significant differences in transparency standards between the \textit{SA} and \textit{SARL} form under the new law. An \textit{SA} must have at least one designated auditor at all times (and at least two, if it issues a prospectus).\textsuperscript{37} In contrast, while an \textit{SARL} may appoint an auditor, appointment is obligatory only in limited circumstances.\textsuperscript{38}

\textsuperscript{29} Article 46, law 05-06.
\textsuperscript{30} Article 6, law 17-95.
\textsuperscript{31} Articles 58 and 60, law 05-96.
\textsuperscript{32} Article 54, law 17-95.
\textsuperscript{33} Article 55, law 17-95.
\textsuperscript{34} See Chapter 1 and Chapter 2 respectively, Titre 3, law 17-95.
\textsuperscript{35} Article 62, law 05-96.
\textsuperscript{36} Article 63, law 05-96.
\textsuperscript{37} Article 159, law 17-95.
\textsuperscript{38} Specifically, there are two cases where appointment is obligatory: (i) where the company has a sales turnover of at least 50 million dirhams (net of taxes), and (ii) where the appointment is granted by a judge on the application of one or more shareholders representing at least a quarter of the registered capital (Martin 1999, 162).\textsuperscript{39} In the event that an \textit{SARL} does appoint an auditor, the auditor has the same powers, obligations and responsibilities as in the \textit{SA} case: Articles 13 and 83, law 05-96.
Reforms to the *SA* form

Table 2 (page 10) provides a brief summary of the reforms to the *SA* form. This section provides further details of the substance of the reform.

**Governance.** The previous law reflected a disturbing dichotomy: by providing little guidance on the role of corporate directors, the law allowed *both* for some directors to have nearly unchecked power while other directors were permitted to play almost no part in the running of the company. Thus, in advocating the reform, the Minister for Privatisation (Abderrahmane Saaïdi) argued at the time, “We do not know who does what. The president is often invested a function of prestige without assuming any real responsibilities.”

(Mossadaq and Oudghiri 1995) Concomitantly, many expressed concern that “administrators were reduced by the old law to ratifying decisions taken by the ‘all-powerful’ president” with the only real limitation upon the president’s power being the corporate object itself (Shamamba 1995).

Law 17-95 brought radical change in this area. As explained earlier, the law introduces a ‘dualist’ system of governance, “inspired by German law, and well known to French corporate law”, involving either a Board of Directors and President or a Board of Directors and Board of Trustees; in both cases, providing new checks upon management authority (Oudghiri 1998). In doing so, the law seeks to enliven so-called ‘sleeping directors’ to be more involved in corporate management; it “encourages creation within the Board of Directors of technical committees charged to study particular questions and to formulate opinions and recommendations.”

(Alami 1999)

**Transparency.** A significant complaint under the old regime was the untrustworthiness of corporate accounts. Moroccan firms, it was often claimed, would routinely produce three sets of records: one for the manager, one for the tax authorities and one for the bank.

Law 17-95 is one important component of reforms to counter this problem. The new conditions regarding auditors have already been outlined; it bears noting that, under the previous regime, auditors did not have a “permanent presence” in the company’s operations (Oudghiri 1998). Thus, the reform also strengthens the effect of the minimum capital requirement; it “puts an end to practices of posting a fictitious capital having no relationship with the capital actually contributed.”

(L’Economiste 1998b)

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40 Original quote: “Nous ne savons pas qui fait quoi. Le président est souvent investi d’une fonction de prestige sans pour autant assumer de véritables responsabilités”.

41 Original quote: “Les administrateurs en étaient réduits à entériner des décisions prises par le “tout puissant” président.” The article was paraphrasing comments by Abdelaziz Squalli, Professor at the Faculté de Droit de Fès.

42 Original quote: “A ce jour, la seule limite connue au pouvoir du PDG était l’objet social.”

43 Original quote: “Notion nouvelle inspirée du droit allemand et bien connue du droit français des affaires”.

44 Original quote: “En effet, l’article 51 encourage la création au sein du conseil d’administration de comités techniques chargés d’étudier des questions précises et de formuler des avis et recommandations.” See Article 51, Law 17-95.

45 One businessman interviewed for this research wryly suggested a fourth set: for the manager’s wife.

46 There are other components here, too; for example, an initiative of the Central Bank of Morocco (Bank Al-Maghrib) that requires every credit file to have attached a copy of the financial statements that were filed with the Commerce Tribunal.

47 Original quote: “...la présence permanente des commissaires aux comptes...”.

48 Original quote: “Cette disposition met ainsi fin aux pratiques qui consistaient à afficher un capital fictif sans rapport avec celui réellement apporté.”

I-44

*Simon Quinn*
This relates closely to new rights to information: unlike the previous regime, law 17-95 “envisages the shareholder having access to all documents necessary to understand the management of the company. It also involves obligations to provide information in the management report relevant to subsidiary companies, significant shareholdings and takeovers.”\(^{49}\) (Oudghiri 1998) Finally, the transparency requirements bring new standards regarding conflicts of interest. “In effect, under the former law, a director was able to make contracts with the company, to borrow money from the company and to guarantee his or her own debts by the company, all without the shareholders being informed of these actions.”\(^{50}\) (Alami 1999) These practices are ended by the new statute, which requires such contracts to be subject to prior authorisation by the Board of Directors, to be the subject of an auditor’s report and to be approved by the next ordinary general meeting.\(^{51}\)

**Minority rights.** The previous law did not grant genuine rights to minority shareholders; thus, it was said of those **S.A** companies not bringing their statutes into early compliance with the new law that “they may continue to exercise on minorities the dictatorship of the majority.”\(^{52}\) (Oudghiri and Ikram 1999) The new law redresses this. Aside from the right to information and the expanded role for auditors just outlined, the legislation allows individuals or groups of shareholders holding ten percent of the capital to demand the convening of a general meeting.\(^{53}\)

**Third parties.** Under the old law, third parties had little legal reassurance in dealing with a company; because the relations between companies and their ‘administrators’ were merely contractual, a third party always bore the risk that a company administrator having ostensible authority was not actually authorised to deal on the company’s behalf. The new law rectifies this, by implementing what is sometimes known elsewhere as the ‘indoor management rule’: “[a]ny limitation of the powers of the capacities of administrators may not be contested as against third parties. Third parties may, consequently, act with reassurance: the law protects them.”\(^{54}\) (Alami 1999) Further, whereas the old law allowed the company to take legal personality from the time of the constitutive meeting, the new law requires registration on the Register of Commerce as a precondition; this provides reassurance to third parties, particularly those needing to deal with ‘founders’ of the company prior to registration, by making the corporate status of the company a matter of public record (Oudghiri 1998).

\(^{49}\)*Original quote: “La loi prévoit la mise à la disposition des actionnaires de tout document nécessaire à l’appréciation de la gestion de la société. Il est également fait obligation de mentionner les informations relatives aux filiales, participations et prises de contrôle dans le rapport de gestion.”* See *Titre V*, law 17-95.

\(^{50}\)*Original quote: “En effet, sous l’emprise de l’ancienne loi, un administrateur pouvait passer des contrats avec la société, emprunter de l’argent à la société, faire cautionner par la société ses engagements personnels, sans même que les actionnaires soient informés de ces opérations.”

\(^{51}\)*Articles 56–62, Law 17-95.

\(^{52}\)*Original quote: “Ils pourront continuer à exercer sur les minoritaires la dictature des majoritaires.”

\(^{53}\)*Article 116, law 17-95.

\(^{54}\)*Original quote: “Toute limitation des pouvoirs des administrateurs est inopposable aux tiers. Ces derniers peuvent par conséquent agir en toute quiétude: la loi les protège.”
Table 12: Explaining whether an overdraft is provided (Linear Probability Model specification)

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>FE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Dependent variable:</td>
<td>1(overdraft limit &gt; 0)</td>
<td></td>
</tr>
<tr>
<td><strong>Legal status:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: SA (0 = SARL)</td>
<td>0.095</td>
<td>0.104</td>
</tr>
<tr>
<td></td>
<td>(0.033)**</td>
<td>(0.053)**</td>
</tr>
<tr>
<td><strong>Basic characteristics:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (permanent employees)</td>
<td>0.046</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(0.022)**</td>
<td>(0.05)</td>
</tr>
<tr>
<td>Age ((t = 0))</td>
<td>0.003</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Age(^2/100) ((t = 0))</td>
<td>-0.004</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Director-General’s years directing ((t = 0))</td>
<td>0.006</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.003)**</td>
<td>(0.003)**</td>
</tr>
<tr>
<td>Dummy: Female Director-General ((t = 0))</td>
<td>-1.18</td>
<td>-1.23</td>
</tr>
<tr>
<td></td>
<td>(0.045)**</td>
<td>(0.042)**</td>
</tr>
<tr>
<td><strong>Firm assets:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (machines and equipment)</td>
<td>-0.003</td>
<td>-0.010</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>0.001</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Dummy: Land &amp; buildings missing or zero</td>
<td>-0.027</td>
<td>-0.257</td>
</tr>
<tr>
<td></td>
<td>(0.177)</td>
<td>(0.126)**</td>
</tr>
<tr>
<td><strong>Other:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time dummy ((t))</td>
<td>-0.032</td>
<td>-0.020</td>
</tr>
<tr>
<td></td>
<td>(0.019)**</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Region dummies ((t = 0))</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Bank dummies ((t = 0))</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sector dummies ((t = 0))</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Mundlak firm means</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Obs.</td>
<td>976</td>
<td>976</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.098</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\(H_0: \text{Mundlak means} = 0; \text{All} (p\text{-value}) = 0.618\)

**Confidence:** ***** → 99%, ** → 95%, * → 90%.

Parentheses show robust standard errors, clustered by region × sector.

Information on firm Directors-General is drawn only from the first round \((t = 0)\), and is therefore treated as time-invariant. Firm age is entered as firm age at \(t = 0\), and is therefore time-invariant by construction. The ‘FE’ estimation is run using the Mundlak (1978) methodology, and therefore produces estimates even for time-invariant explanatory variables.
Table 13: Heterogeneous effects by time (Linear Probability Model specification)

<table>
<thead>
<tr>
<th></th>
<th>OLS (1)</th>
<th>FE (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td><strong>(overdraft limit &gt; 0)</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Legal status:**

| (Dummy: SA) × (Dummy: t = 0) | 0.106  | -0.061 |
| (Dummy: SA → SARL) × (Dummy: t = 1) | -0.104 | -0.135 |
| (Dummy: SARL → SA) × (Dummy: t = 1) | -0.175 | -0.162 |
| (Dummy: SARL → SARL) × (Dummy: t = 1) | -0.088 | 0.075 |

**Basic characteristics:**

| Log (permanent employees) | 0.047  | 0.026  |
| Age (t = 0)               | 0.003  | 0.002  |
| Age²/100 (t = 0)          | -0.005 | -0.004 |
| Director-General’s years directing (t = 0) | 0.006 | 0.006 |
| Dummy: Female Director-General (t = 0) | -0.115 | -0.123 |

**Firm assets:**

| Log (machines and equipment) | -0.003 | -0.010 |
| Log (land and buildings)     | 0.001  | -0.012 |
| Dummy: Land & buildings missing or zero | -0.032 | -0.273 |

**Other:**

| Time dummy (t) | 0.068 | -0.078 |
| Region dummies (t = 0) | ✓ | ✓ |
| Bank dummies (t = 0) | ✓ | ✓ |
| Sector dummies (t = 0) | ✓ | ✓ |
| Mundlak firm means | ✓ | ✓ |

| Obs. | 976 | 976 |
| $R^2$ | 0.1 | 0.103 |

$H_0$: Mundlak means = 0; All (p-value) 0.026**

**Confidence:** *** ↔ 99%, ** ↔ 95%, * ↔ 90%.

Parentheses show robust standard errors, clustered by region × sector.

Information on firm Directors-General is drawn only from the first round (t = 0), and is therefore treated as time-invariant. Firm age is entered as firm age at t = 0, and is therefore time-invariant by construction. The ‘FE’ estimation is run using the Mundlak (1978) methodology, and therefore produces estimates even for time-invariant explanatory variables.
Table 14: Robustness to choice of explanatory variables

<table>
<thead>
<tr>
<th></th>
<th>OLS  (1)</th>
<th>FE   (2)</th>
<th>OLS  (3)</th>
<th>FE   (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: 1 (overdraft limit &gt; 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Legal status:**

<table>
<thead>
<tr>
<th>Dummy: SA (0 = SARL)</th>
<th>0.145 (0.033)**</th>
<th>0.098 (0.054)*</th>
<th>0.087 (0.031)**</th>
<th>0.093 (0.051)*</th>
</tr>
</thead>
</table>

**Basic characteristics:**

Log (permanent employees) | 0.021 (0.022) | 0.017 (0.051) |
Age (t = 0) | 0.002 (0.006) | 0.002 (0.006) |
Age$^2$/100 (t = 0) | -0.005 (0.01) | -0.005 (0.01) |
Director-General’s years directing (t = 0) | 0.005 (0.003)* | 0.005 (0.003)* |
Dummy: Female Director-General (t = 0) | -1.24 (0.047)** | -1.27 (0.043)** |

**Firm assets:**

Log (machines and equipment) | -0.16 (0.01) | -0.11 (0.011) |
Log (land and buildings) | -0.03 (0.01) | -0.13 (0.007)** |
Dummy: Land & buildings missing or zero | -0.98 (0.174) | -2.84 (0.123)** |

**Firm liabilities:**

Log (long-term debt) | 0.046 (0.018)** | 0.05 (0.02)** |
Dummy: long-term debt < 0 | 0.619 (0.306)** | 0.73 (0.375)* |
Dummy: long-term debt missing or zero | 0.625 (0.279)** | 0.733 (0.304)** |

**Other:**

Time dummy (t) | -0.025 (0.02) | -0.034 (0.022) | -0.03 (0.021) | -0.03 (0.023) |
Region dummies (t = 0) | ✓ | ✓ | ✓ | ✓ |
Bank dummies (t = 0) | ✓ | ✓ | ✓ | ✓ |
Sector dummies (t = 0) | ✓ | ✓ | ✓ | ✓ |
Mundlak firm means | ✓ | ✓ |          |          |

Obs. | 976 | 976 | 976 | 976 |
$R^2$ | 0.07 | 0.07 | 0.111 | 0.113 |

$H_0$: Mundlak means = 0: All (p-value) 0.902

Confidence: *** ↔ 99%, ** ↔ 95%, * ↔ 90%.

Parentheses show robust standard errors, clustered by region × sector.

Information on firm Directors-General is drawn only from the first round (t = 0), and is therefore treated as time-invariant. Firm age is entered as firm age at t = 0, and is therefore time-invariant by construction. The ‘FE’ estimations are run using the Mundlak (1978) methodology, and therefore produces estimates even for time-invariant explanatory variables.
Table 15: Discontinuity design

<table>
<thead>
<tr>
<th></th>
<th>OLS (1)</th>
<th>OLS (2)</th>
<th>OLS (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong></td>
<td>1(overdraft limit &gt; 0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Legal status:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: SA × (1 ≤ Age (t = 0) ≤ 4)</td>
<td>0.242 (0.109)**</td>
<td>-0.026 (0.572)</td>
<td>0.078 (1.215)</td>
</tr>
<tr>
<td>Dummy: SA × (5 ≤ Age (t = 0) ≤ 7)</td>
<td>0.22 (0.085)**</td>
<td>-0.267 (0.86)</td>
<td>-0.155 (1.523)</td>
</tr>
<tr>
<td>Age (t = 0) × (Dummy: SA)</td>
<td>0.081 (0.145)</td>
<td>0.033 (0.47)</td>
<td></td>
</tr>
<tr>
<td>Age²/100 (t = 0) × (Dummy: SA)</td>
<td>0.493 (3.706)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Basic characteristics:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: (5 ≤ Age (t = 0) ≤ 7)</td>
<td>0.133 (0.239)</td>
<td>0.179 (0.307)</td>
<td>0.18 (0.308)</td>
</tr>
<tr>
<td>Log (permenant employees)</td>
<td>-0.16 (0.05)</td>
<td>-0.023 (0.051)</td>
<td>-0.022 (0.052)</td>
</tr>
<tr>
<td>Age (t = 0)</td>
<td>0.004 (0.146)</td>
<td>0.012 (0.135)</td>
<td>0.018 (0.163)</td>
</tr>
<tr>
<td>Age²/100 (t = 0)</td>
<td>0.153 (1.100)</td>
<td>-0.166 (0.902)</td>
<td>-0.253 (1.199)</td>
</tr>
<tr>
<td>Director-General’s years directing (t = 0)</td>
<td>-0.008 (0.038)</td>
<td>-0.002 (0.032)</td>
<td>-0.0007 (0.033)</td>
</tr>
<tr>
<td>Dummy: Female Director-General (t = 0)</td>
<td>-0.079 (0.09)</td>
<td>-0.082 (0.085)</td>
<td>-0.078 (0.08)</td>
</tr>
<tr>
<td><strong>Firm assets:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (machines and equipment)</td>
<td>0.031 (0.034)</td>
<td>0.031 (0.035)</td>
<td>0.031 (0.035)</td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>-0.20 (0.031)</td>
<td>-0.018 (0.029)</td>
<td>-0.018 (0.029)</td>
</tr>
<tr>
<td>Dummy: Land &amp; buildings missing or zero</td>
<td>-0.039 (0.5)</td>
<td>-0.022 (0.479)</td>
<td>-0.019 (0.481)</td>
</tr>
<tr>
<td><strong>Other:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region dummies (t = 0)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Bank dummies (t = 0)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sector dummies (t = 0)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td><strong>Obs.</strong></td>
<td>122</td>
<td>122</td>
<td>122</td>
</tr>
<tr>
<td><strong>R²</strong></td>
<td>0.309</td>
<td>0.312</td>
<td>0.313</td>
</tr>
<tr>
<td><strong>Estimated difference in legal status effects:</strong></td>
<td>0.022 (p = 0.867)</td>
<td>0.241 (p = 0.427)</td>
<td>0.233 (p = 0.490)</td>
</tr>
</tbody>
</table>

**Confidence:** *** ↔ 99%, ** ↔ 95%, * ↔ 90%.

Parentheses show robust standard errors, clustered by region × sector.

Information on firm Directors-General is drawn only from the first round (t = 0), and is therefore treated as time-invariant. Firm age is entered as firm age at t = 0, and is therefore time-invariant by construction.

This regression is run only for t = 0.
Table 16: Probability of survey retention

<table>
<thead>
<tr>
<th>Eventual outcome variables:</th>
<th>Probit Marginal effects</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable: Dummy variable: firm retained</td>
<td>(1)</td>
<td>(2)</td>
<td></td>
</tr>
<tr>
<td>Eventual outcome variables:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (overdraft limit + 1)</td>
<td>0.085</td>
<td>0.032</td>
<td></td>
</tr>
<tr>
<td>(0.041)**</td>
<td>(0.015)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: overdraft limit &gt; 0</td>
<td>-1.104</td>
<td>-3.44</td>
<td></td>
</tr>
<tr>
<td>(0.352)**</td>
<td>(0.13)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Legal status:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: SA (0 = SARL)</td>
<td>-0.311</td>
<td>-0.118</td>
<td></td>
</tr>
<tr>
<td>(0.112)**</td>
<td>(0.043)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic characteristics:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (permanent employees)</td>
<td>0.195</td>
<td>0.073</td>
<td></td>
</tr>
<tr>
<td>(0.063)**</td>
<td>(0.024)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age (t = 0)</td>
<td>0.029</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>(0.013)**</td>
<td>(0.005)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age²/100 (t = 0)</td>
<td>-0.032</td>
<td>-0.012</td>
<td></td>
</tr>
<tr>
<td>(0.021)</td>
<td>(0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Director-General’s years directing (t = 0)</td>
<td>-0.003</td>
<td>-0.001</td>
<td></td>
</tr>
<tr>
<td>(0.007)</td>
<td>(0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: Female Director-General (t = 0)</td>
<td>-0.266</td>
<td>-0.103</td>
<td></td>
</tr>
<tr>
<td>(0.221)</td>
<td>(0.088)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm assets:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (machines and equipment)</td>
<td>0.022</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>(0.044)</td>
<td>(0.017)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>0.004</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>(0.044)</td>
<td>(0.016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: Land &amp; buildings missing or zero</td>
<td>0.223</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>(0.849)</td>
<td>(0.289)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Firm liabilities:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (long-term debt)</td>
<td>-0.154</td>
<td>-0.058</td>
<td></td>
</tr>
<tr>
<td>(0.03)**</td>
<td>(0.011)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: long-term debt &lt; 0</td>
<td>-1.230</td>
<td>-0.450</td>
<td></td>
</tr>
<tr>
<td>(0.313)**</td>
<td>(0.089)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: long-term debt missing or zero</td>
<td>-2.625</td>
<td>-0.695</td>
<td></td>
</tr>
<tr>
<td>(0.426)**</td>
<td>(0.035)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Other:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Region dummies (t = 0)</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Bank dummies (t = 0)</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Sector dummies (t = 0)</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>783</td>
<td></td>
<td></td>
</tr>
<tr>
<td>McFadden’s Pseudo—$R^2$</td>
<td>0.092</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion correctly predicted</td>
<td>66.8%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Confidence: *** → 99%, ** → 95%, * → 90%.

Parentheses show robust (unclustered) standard errors.
Table 17: Heterogeneous effects by firm size, firm age and years with principal bank

<table>
<thead>
<tr>
<th></th>
<th>OLS (1)</th>
<th>FE (2)</th>
<th>OLS (3)</th>
<th>FE (4)</th>
<th>OLS (5)</th>
<th>FE (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variable:</strong></td>
<td>1 (overdraft limit &gt; 0)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Legal status:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: SA (0 = SARL)</td>
<td>0.223 ** (0.129)</td>
<td>0.073 (0.144)</td>
<td>0.15 ** ** (0.039)</td>
<td>0.186 *** (0.066)</td>
<td>0.175 *** (0.047)</td>
<td>0.18 *** (0.075)</td>
</tr>
<tr>
<td>(Dummy: SA) × Log (permanent employees)</td>
<td>-0.029 (0.026)</td>
<td>0.007 (0.033)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Dummy: SA) × Years with bank (t = 0)</td>
<td></td>
<td>-0.004 (0.003)</td>
<td>-0.006 (0.003)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Dummy: SA) × Age (t = 0)</td>
<td></td>
<td>-0.005 (0.003)</td>
<td>-0.004 (0.003)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Basic characteristics:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (permanent employees)</td>
<td>0.054 ** (0.022)</td>
<td>0.025 (0.049)</td>
<td>0.046 ** (0.022) **</td>
<td>0.026 (0.048)</td>
<td>0.046 ** (0.022) **</td>
<td>0.027 (0.048)</td>
</tr>
<tr>
<td>Years with bank (t = 0)</td>
<td>0.0009 (0.002)</td>
<td>0.0007 (0.002)</td>
<td>0.003 (0.003)</td>
<td>0.003 (0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age (t = 0)</td>
<td>-0.0003 (0.002)</td>
<td>-0.0003 (0.002)</td>
<td></td>
<td></td>
<td>0.003 (0.003)</td>
<td>0.003 (0.003)</td>
</tr>
<tr>
<td>Director-General’s years directing (t = 0)</td>
<td>0.006 ** ** (0.002) ** **</td>
<td>0.006 (0.002) ** **</td>
<td>0.006 (0.002) ** **</td>
<td>0.006 (0.002) ** **</td>
<td>0.005 (0.002) ** **</td>
<td>0.005 (0.002) ** **</td>
</tr>
<tr>
<td>Dummy: Female Director-General (t = 0)</td>
<td>-1.16 (0.043) ** **</td>
<td>-1.12 (0.041) ** **</td>
<td>-1.12 (0.041) ** **</td>
<td>-1.17 (0.043) ** **</td>
<td>-1.17 (0.043) ** **</td>
<td>-1.12 (0.041) ** **</td>
</tr>
<tr>
<td><strong>Firm assets:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (machines and equipment)</td>
<td>-0.002 (0.009)</td>
<td>-0.10 (0.01)</td>
<td>-0.03 (0.009)</td>
<td>-0.11 (0.01)</td>
<td>-0.02 (0.009)</td>
<td>-0.10 (0.01)</td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>0.002 (0.01)</td>
<td>-0.12 (0.008)</td>
<td>0.001 (0.01)</td>
<td>-0.11 (0.007)</td>
<td>0.002 (0.009)</td>
<td>-0.11 (0.008)</td>
</tr>
<tr>
<td>Dummy: Land &amp; buildings missing or zero</td>
<td>-0.19 (0.178)</td>
<td>-2.59 (0.125) **</td>
<td>-0.22 (0.177)</td>
<td>-2.50 (0.124) **</td>
<td>-0.24 (0.174)</td>
<td>-2.51 (0.126) **</td>
</tr>
<tr>
<td><strong>Other:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time dummy (t)</td>
<td>-0.032 (0.019)</td>
<td>-0.20 (0.023)</td>
<td>-0.032 (0.019)</td>
<td>-0.20 (0.023)</td>
<td>-0.031 (0.019)</td>
<td>-0.20 (0.023)</td>
</tr>
<tr>
<td>Region dummies (t = 0)</td>
<td>✓ ✓ ✓ ✓ ✓ ✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bank dummies (t = 0)</td>
<td>✓ ✓ ✓ ✓ ✓ ✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sector dummies (t = 0)</td>
<td>✓ ✓ ✓ ✓ ✓ ✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mundlak firm means</td>
<td>✓ ✓ ✓</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Obs. 976 976 976 976 976 976
R² 0.099 0.102 0.1 0.102 0.101 0.103
H₀: Mundlak means = 0; All (p-value) 0.349 0.709 0.692

Confidence: *** ↔ 99%, ** ↔ 95%, * ↔ 90%.
Parentheses show robust standard errors, clustered by region × sector.
Information on firm Directors-General is drawn only from the first round (t = 0), and is therefore treated as time-invariant. Firm age is entered as firm age at t = 0, and is therefore time-invariant by construction. The ‘FE’ estimations are run using the Mundlak (1978) methodology, and therefore produces estimates even for time-invariant explanatory variables.
Table 18: **Disaggregation by bank**

<table>
<thead>
<tr>
<th>Legal status:</th>
<th>OLS</th>
<th>FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Dummy: SA) × (Dummy: BCM)</td>
<td>0.216</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.062)**</td>
<td>(0.088)</td>
</tr>
<tr>
<td>(Dummy: SA) × (Dummy: BCP)</td>
<td>0.006</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.105)</td>
</tr>
<tr>
<td>(Dummy: SA) × (Dummy: BMCE)</td>
<td>0.088</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>(Dummy: SA) × (Dummy: BMCI)</td>
<td>0.05</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>(0.1)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>(Dummy: SA) × (Dummy: CDM)</td>
<td>0.202</td>
<td>0.277</td>
</tr>
<tr>
<td></td>
<td>(0.055)**</td>
<td>(0.195)</td>
</tr>
<tr>
<td>(Dummy: SA) × (Dummy: SGMB)</td>
<td>0.028</td>
<td>-0.059</td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>(Dummy: SA) × (Dummy: Wafabank)</td>
<td>0.116</td>
<td>0.374</td>
</tr>
<tr>
<td></td>
<td>(0.052)**</td>
<td>(0.162)**</td>
</tr>
<tr>
<td>(Dummy: SA) × (Dummy: Other)</td>
<td>0.08</td>
<td>0.189</td>
</tr>
<tr>
<td></td>
<td>(0.093)</td>
<td>(0.091)**</td>
</tr>
</tbody>
</table>

**Basic characteristics:**

| Log (permanent employees)          | 0.04         | 0.022       |
|                                    | (0.024)*     | (0.053)     |

**Firm assets:**

| Log (machines and equipment)       | -.004        | -.011       |
|                                    | (0.01)       | (0.01)      |
| Log (land and buildings)           | 0.003        | -0.015      |
|                                    | (0.011)      | (0.01)      |
| Dummy: Land & buildings missing or zero | -.008    | -2.78       |
|                                    | (0.208)      | (0.149)*    |

**Other:**

| Time dummy (t)                     | -0.14        | -0.05       |
|                                    | (0.021)      | (0.023)     |
| Region dummies (t = 0)             | ✓            | ✓           |
| Bank dummies (t = 0)               | ✓            | ✓           |
| Sector dummies (t = 0)             | ✓            | ✓           |
| Mundlak firm means                 | ✓            |             |

| Obs.                                | 930          | 930         |
|                                     | H²           | 0.09        | 0.099       |

_H0_: Mundlak means = 0: All (p-value)  
1.46e-06***

_H0_: Equality of legal status effects (p-value)  
0.081*  0.003***

**Confidence:**  *** 99%, ** 95%, * 90%.

Parentheses show robust standard errors, clustered by region × sector. The ‘FE’ estimation is run using the Mundlak (1978) methodology, and therefore produces estimates even for time-invariant explanatory variables.
Table 19: Explaining overdraft limits in logs

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>Heckman</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Dependent variable:</td>
<td>Log(overdraft limit)</td>
<td></td>
</tr>
<tr>
<td>Legal status:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy: SA (0 = SARL)</td>
<td>0.347</td>
<td>0.382</td>
</tr>
<tr>
<td></td>
<td>(0.112)***</td>
<td>(0.12)***</td>
</tr>
<tr>
<td>Basic characteristics:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (permanent employees)</td>
<td>0.344</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>(0.075)***</td>
<td>(0.076)***</td>
</tr>
<tr>
<td>Age (t = 0)</td>
<td>0.008</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Age²/100 (t = 0)</td>
<td>0.015</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Director-General’s years directing (t = 0)</td>
<td>-0.004</td>
<td>-0.002</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Dummy: Female Director-General (t = 0)</td>
<td>-0.235</td>
<td>-0.272</td>
</tr>
<tr>
<td></td>
<td>(0.242)</td>
<td>(0.246)</td>
</tr>
<tr>
<td>Firm assets:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (machines and equipment)</td>
<td>0.235</td>
<td>0.234</td>
</tr>
<tr>
<td></td>
<td>(0.027)***</td>
<td>(0.026)***</td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>0.112</td>
<td>0.113</td>
</tr>
<tr>
<td></td>
<td>(0.033)***</td>
<td>(0.031)***</td>
</tr>
<tr>
<td>Dummy: Land &amp; buildings missing or zero</td>
<td>1.645</td>
<td>1.637</td>
</tr>
<tr>
<td></td>
<td>(0.54)***</td>
<td>(0.512)***</td>
</tr>
<tr>
<td>Other:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Time dummy (t)</td>
<td>0.072</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>(0.073)</td>
<td>(0.077)</td>
</tr>
<tr>
<td>Region dummies (t = 0)</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Bank dummies (t = 0)</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Sector dummies (t = 0)</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Inverse Mills’ Ratio</td>
<td></td>
<td>0.245</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.165)</td>
</tr>
<tr>
<td>Obs.</td>
<td>751</td>
<td>976</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.446</td>
<td></td>
</tr>
</tbody>
</table>

$H_0$: Independent equations (p-value) 0.142

Confidence: *** $\rightarrow$ 99%, ** $\rightarrow$ 95%, * $\rightarrow$ 90%.

Parentheses show robust standard errors, clustered by region $\times$ sector.

Information on firm Directors-General is drawn only from the first round (t = 0), and is therefore treated as time-invariant. Firm age is entered as firm age at $t = 0$, and is therefore time-invariant by construction.

The Heckman estimations in column 2 is implemented by Full-Information Maximum Likelihood.
Chapter II: Binary Signalling

Binary Signalling

Simon Quinn*

April 25, 2010

Abstract

An agent signals by using a variable that may take one of only two values: a ‘binary signal’. The agent’s unobservable type is drawn from a continuous distribution. The binary signal therefore provides only ‘coarse information’. I show that this information coarseness introduces several considerations that are not generally emphasised in the signalling literature. Specifically, I show that a principal’s risk aversion can affect the equilibrium outcome. Further, I show that the information coarseness implies an important role for other observable characteristics of the agent (‘indices’). Finally, I consider the consequences of allowing the principal a continuous and unbounded response variable; I show that this extension can change substantially the conditions under which different forms of equilibrium exist.

JEL codes: D82, D86.

*D.Phil student: Department of Economics, Centre for the Study of African Economies (‘CSAE’) and All Souls College, University of Oxford (simon.quinn@economics.ox.ac.uk; http://www.allsouls.ox.ac.uk/people.php?personid=54). This paper forms part of my D.Phil thesis, which is supervised by Professor Marcel Fafchamps; the work would not have been possible without his very generous assistance. A number of others have provided very useful comments on aspects of the paper; without implicating them in the shortcomings of the work, I thank Paul Beaudry, Daniel Clarke, Thomas Hosking, Meg Meyer and John Vickers.
Chapter II: Binary Signalling

1 Introduction

Many economic decisions require a choice between just two competing alternatives. In particular, many institutional contexts provide for formal registration, and registration is often a binary event that either does or does not occur. For example, firms may choose whether or not to incorporate in a particular legal status (Quinn (2010)) or whether or not to register for taxes (McKenzie and Seynabou Sakho (2009)), land owners may choose whether or not to formalise their property title (Field (2007)) and unemployed workers may choose whether or not to complete the requirements of a training program (Ashenfelter (1978)).

Binary choices — like other types of choice — can act as informative signals of unobservable variables. It is well recognised, for example, that the choice of duration of education (a decision on a continuous variable) can signal a worker’s unobserved ‘ability’; there is no reason why a binary choice (for example, whether or not to obtain a training program certificate) cannot play a similar role. Indeed, this has been recognised since Spence (1973, see pp.367–368).

In some respects, such ‘binary signalling’ is merely a special case of a more general signalling problem. However, there is a fundamental distinction between binary signalling and signalling on a continuous variable: in the case where an agent’s unobservable type is drawn from a continuous distribution, a binary signal at best provides ‘coarse information’ about the agent. That is, the signal may allow the principal to infer some information about the agent’s type, but does not generally allow the principal to infer the agent’s type exactly: see Meyer (1991).

In this paper, I consider the consequences of that coarseness, in a model in which both agent and principal can only take binary decisions. I argue that the information coarseness inherent in a binary signal introduces several considerations that are not generally emphasised in the signalling literature.
Motivating example: Binary signalling in education. Consider a student who has just completed secondary education. The student must decide whether or not to complete a university degree. The student faces a single employer, who may either hire or not hire her, on the basis of whether or not she has a degree. In this way, the student must make a binary choice — whether or not to obtain the degree — and the employer must make a binary response — whether or not to employ her.

1.1 Outline

The paper begins by specifying in general terms a simple model in which an informed agent (‘Student’) makes a binary signalling choice and an uninformed principal (‘Employer’) makes a binary response. The paper is then concerned with the following key questions about the model and its application; the answers are summarised here by way of outline.

(i) Does an equilibrium always exist in pure strategies? When is there a unique equilibrium in pure strategies? I show that an equilibrium always exists in pure strategies. If it is profitable for the principal to contract even if the agent chooses not to signal, there may be both pooling and separating equilibria — however, there is never more than one separating equilibrium or one pooling equilibrium in pure strategies.\(^1\) I show this in section 2.3 (starting on page 13).

(ii) Are there equilibria in mixed strategies? I consider mixed-strategy equilibria under the further assumption of principal risk-neutrality. Depending on parameter values, there may be zero, one or two equilibria in mixed strategies. I show this in section 2.4.2 (starting on page 25).

\(^1\) I define pooling equilibria and separating equilibria in this context shortly.
(iii) **How does the principal’s risk aversion affect the equilibrium outcome?**

Because a binary signal reveals coarse information about an agent’s type, a principal’s risk aversion is important for determining the principal’s payoff from contracting. I show that, for some parameter values, an increase in the principal’s risk aversion can cause a profitable potential contract to become unprofitable; under binary signalling, risk aversion itself can be a reason for market failure. I show this in section 2.5 (starting on page 31).

(iv) **Can mutually-observed agent characteristics (‘indices’) be used to mitigate — or even to overcome — the information asymmetry? Do the equilibria from the simple model continue to hold with informative indices?** I show that indices may mitigate the asymmetry, and that the equilibria from the simple model hold under indices. I show this in section 3 (page 37).

(v) **How does the outcome change if the principal’s choice variable is continuous?** I show that the basic structure of the result is unchanged; however, the solution changes substantially. In particular, the distribution from which the agent’s type is drawn becomes critical to determining the sufficient conditions for unique equilibrium. I show this in section 4 (starting on page 44).

### 1.2 Related literature

The literature on signalling games is extensive, following Spence’s (1973) early work on signalling through education. However, there has been little emphasis in that literature on cases in which the institutional context constrains a signal to be binary (or, indeed, discrete).

*Cheap talk games* are one important area of signalling research involving discrete signals. Equilibrium signalling in cheap talk games involves an informed
agent partitioning the support of its private information and then reporting only in which element of the partition the true value lies: see Crawford and Sobel (1982) and Green and Stokey (2007). Cheap talk games show that “exogenous differential signaling costs are not always needed for informative signaling”; in cheap talk games, the principal’s response creates signaling costs endogenously (Crawford and Sobel (1982, p.1434); see also Farrell and Rabin (1996)). Cheap talk games are, therefore, a very important instance of signalling games involving coarse information; however, the motivating example here does not describe a cheap talk game, precisely because the signal (education) does have “exogenously given differential signaling costs” (Crawford and Sobel (1982, p.1434)).

Insofar as achieving a tertiary education may be considered as ‘verifying’ a student’s ability, the motivating example here might also be compared to literature on costly state verification. This literature has primarily been developed in the context of warranties and corporate disclosure: for example, see Grossman and Hart (1980), Grossman (1981) and Utaka (2006). More recent work has emphasised the role of ‘certification intermediaries’ (Albano and Lizzeri (2001)), and this can produce equilibrium results broadly akin to binary signalling: for example, Lizzeri (1999, p.214) shows, in a model of certification intermediaries, that “[i]n a class of environments, the optimal choice for a monopoly intermediary is to reveal only whether quality is above some minimal standard”. However, the present context should not be viewed as a model of costly state verification. The disclosure literature concerns information “that can be certified or authenticated once disclosed” (Bolton and Dewatripont (2005, p.171)); in the present model, however, choosing a tertiary education does not disclose student ability.

For these reasons, it is more appropriate to consider the present model within a traditional signalling framework; the binary nature of the signal should be seen as
extending that framework, rather than placing the context in a different class of model (Bolton and Dewatripont (2005, pp.172–173)). In that way, the research closest to the present model is the literature on strategic manipulation of coarse information. Meyer (1991) considers a situation in which a decision-maker (for example, a firm) can observe only ordinal information about the performance of agents (for example, firm employees). Meyer assumes that the decision-maker may adjust the ‘critical cutoffs’ for reporting, and shows that it may be optimal for a principal to induce biased reporting (specifically, in the form of a biased competition between employees).

In that way, Meyer (1991) concerns a situation of coarse information in which the principal may adjust the critical cutoff for the purposes of information revelation (see also Sah and Stiglitz (1986)). In the present context, this would be analogous to a prospective employer being able to manipulate the cost of education in order to learn about a student’s ability. One might imagine this kind of behaviour in some circumstances — for example, a case in which an employer provides a scholarship for a prospective employee’s tertiary education — but this not a circumstance captured by the present model, in which the student is assumed to make his or her education decision before the employer acts. Conversely, some literature concerns situations in which a group of agents may adjust a critical cutoff for their own strategic purposes. For example, Shaked and Sutton (1981) consider a case in which a professional association may adjust its own minimum standards for practice. In effect, this amounts to a group of agents being able to adjust strategically their own critical cutoff. In the education context, this would amount to a student (or group of students) being able to decide strategically the cost of a university degree. As much as this idea has obvious appeal in the context

---

2 I will discuss, in the context of mixed strategies, the way that an employer might use mixing to change the relative benefits of education. However, this would be a screening model, because the employer would need a way of credibly pre-committing to the mixed strategy; nothing in the present model allows for such pre-commitment, so this is instead merely a signalling model.
Chapter II: **Binary Signalling**

of specific professional education — for example, medical specialists raising the cost of specialist training in their discipline — it is not a general characteristic of tertiary education.

In this paper, I study a model that is, in many respects, substantially simpler than much related work — including research on cheap talk, on costly state verification and on strategic manipulation of information revelation. Specifically, I study a case in which signalling costs (and, therefore, the ‘critical cutoff’ for binary signalling) are given exogenously, and therefore are not amenable to strategic manipulation by *either* agent (student) or principal (employer).³

One might imagine that this produces a game whose solution is a trivial special case of more general signalling models. However, in this paper, I show that even a very simple binary signalling game with exogenous signalling costs prompts a wide range of considerations for determining equilibria. Among these considerations, I emphasise particularly the role of the principal’s risk aversion. I then argue that, because binary signalling only reveals coarse information, it is a context in which observable ‘indices’ are particularly relevant. I show that it can be very important, in understanding the overall form of equilibrium, to consider the cost of signal acquisition in terms of unobserved agent ability *relative to observed agent quality*; this is a point that is not often emphasised in the literature. Finally, I extend the model to allow the principal to choose its response from a continuous and unbounded variable. I show that this extension draws specific attention to the precise form of the distribution of agent ability; this is another point not often

³Troya Martinez (2008, pp.44–58) presents a model in which an agent makes a binary choice about the *variance* with which its continuous type is reported to a principal. This is the only other research that I have been able to find in which an agent with a continuous type makes a binary choice and in which the cutoff for that choice is given exogenously. The model in Troya Martinez (2008) is important for considering merger policy, but is not applicable to the present context. That model involves an agent passing a signal to a principal, in which the expected value of that signal must be the agent’s true type; this does not capture the notion of signalling in education.
emphasised by more general signalling models.

2 A binary signal with a binary response

2.1 Game structure

I use a standard signalling structure, in which the informed agent (in this case, Student) makes a binary choice in anticipation of the response of to the un-informed principal (in this case, Employer). I allow Student to make a binary choice on the ‘signal’ variable $S \in \{0, 1\}$; $S = 1$ refers to completing a university degree. I then allow Employer to choose a binary response on the ‘response’ variable $R \in \{0, 1\}$; $R = 1$ refers to hiring Student. $S$ will have a signalling role; therefore, when I refer to Student’s decision to ‘invest in the signal’, I refer to Student choosing $S = 1$ rather than $S = 0$. I allow for both Student and Employer to adopt mixed strategies.

Players. Players are Nature, Student and Employer.

Structure and actions.

(i) Nature chooses a continuous (univariate) ‘quality’ $Q$ for Student from some strictly increasing cumulative distribution $F(\cdot)$ that is atom-less and has full support on $[Q, \overline{Q}]$:

$$\Pr(Q \leq q) = F_{Q \in [Q, \overline{Q}]}(q).$$

The cumulative density function $F(\cdot)$ (and, therefore, its probability density $f(\cdot)$) is common knowledge among players.

(ii) Student observes $Q$ and chooses a conditional probability of investing
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in the signal:

\[ s(Q) = \Pr(S = 1 \mid Q). \]  \hspace{1cm} (2)

(iii) **Employer** observes \( S \) and chooses a conditional probability of making the binary response:

\[ r(S) = \Pr(R = 1 \mid S). \]  \hspace{1cm} (3)

**Payoffs.** Payoffs exist as von Neumann-Morgenstern utility functions.

Payoffs for **Student** of type \( Q \) arise from two sources:

- **Student**’s benefit from employment: \( B(Q) > 0 \);
- **Student**’s cost of choosing \( S = 1 \) rather than \( S = 0 \): \( C(Q) > 0 \).

**Assumption 1** *The difference between benefit of employment and cost of education is increasing in Student quality:*

\[ B'(Q) - C'(Q) > 0. \]  \hspace{1cm} (4)

This restriction captures the the intuition that, as the quality of **Student** increases, it is relatively more profitable to choose \( S = 1 \) in order to induce **Employer** to hire; it implements the ‘single-crossing property’.

Payoffs for **Employer** accrue from two sources. First, **Employer** obtains profit from hiring **Student** of type \( Q \): \( P(Q) \).

**Assumption 2** *Employer profit from contracting with Student is increas-
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ing in Student quality:

\[ P'(Q) > 0. \]  \hspace{1cm} (5)

Second, additionally, Employer may gain an additional ‘human capital benefit’ from hiring a Student having completed a degree \((S = 1)\). This ‘human capital benefit’ is denoted \(H(Q)\), and is assumed not to accrue in the case that \(S = 0\).

**Assumption 3** The ‘human capital effect’ is a weakly positive and weakly increasing function of Student quality:

\[ H(Q) \geq 0, H'(Q) \geq 0. \]  \hspace{1cm} (6)

Figure 1 illustrates the structure and payoffs, where \(\pi_S\) and \(\pi_E\) denote Student and Employer payoffs respectively.

**Figure 1:** The signalling game in extensive form

Employer beliefs are formalised by the cumulative density \(G(\cdot)\) (with proba-
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...probability density \( g(\cdot) \), such that Employer believes that:

\[
\Pr(Q \leq q) = G(q). \tag{7}
\]

I consider restrictions on beliefs shortly.

2.2 Solution concept

The relevant solution concept is Perfect Bayesian Equilibrium (‘PBE’); I apply the Cho-Kreps Intuitive Criterion from the outset. I define two types of equilibrium in this context.

**Definition 1** Pooling and separating equilibria in the univariate continuous case.

- A *pooling equilibrium* is an equilibrium in which Student chooses the same \( S \) irrespective of \( Q \).
- A *separating equilibrium* is an equilibrium in which Student conditions its choice of \( S \) upon \( Q \).

Denote \( s^\ast(Q) \) as the value of \( s \) forming Student’s strategy in equilibrium as a function of \( Q \). Denote \( r^\ast(S) \) as the value of \( r \) forming Employer’s strategy in equilibrium as a function of observed \( S \). In this context, a solution is a combination of strategies \( s^\ast(Q) \) and \( r^\ast(S) \), as well as beliefs \( G(\cdot) \), such that the following holds.

(i) Student’s choice of \( s \) maximises expected Student payoffs given \( Q \)

\footnote{Note that some authors term this a ‘semi-separating equilibrium’ rather than a ‘separating equilibrium’ because “some but not all information about the principal’s type is obtained from the observation of the signal” (Bolton and Dewatripont 2005, p.104). It will soon become clear that, except in pathological cases, the binary signal can only ever convey limited information. The pathological cases — using concepts and notation to be defined and developed shortly — are where \( Q^* = \overline{Q} \) and \( S = 1 \) and where \( Q^* = \underline{Q} \) and \( S = 0 \). The equilibria outlined shortly extend to these cases.}
and \( r^*(S) \). That is,

\[
s^*(Q) \in \arg \max_s \{ B(Q) \cdot [s \cdot r^*(1) + (1-s) \cdot r^*(0)] - C(Q) \cdot s \}. \tag{8}
\]

(ii) Employer’s choice of \( r \) maximises expected Employer payoffs given \( S \) and \( G(\cdot) \). That is,

\[
r^*(S) \in \arg \max_r \{ r \cdot \mathbb{E}_g [P(Q) + S \cdot H(Q) \mid S] \}, \tag{9}
\]

where \( \mathbb{E}_g [P(Q) + S \cdot H(Q) \mid S] \equiv \int_Q [P(q) + S \cdot H(Q)] \cdot g(q \mid S) \, dq \).

(iii) [Beliefs in separating equilibrium:] If \( S = 0 \) and \( S = 1 \) are both observed with some positive probability on the equilibrium path, then Employer beliefs are formed according to Bayes’ Rule; that is,

\[
g(Q \mid S = 0) \equiv (1 - s(Q)) \cdot f(Q) \cdot \left\{ \int_Q (1 - s(q)) \cdot f(q) \, dq \right\}^{-1} ; \tag{10}
\]

\[
g(Q \mid S = 1) \equiv s(Q) \cdot f(Q) \cdot \left\{ \int_Q s(q) \cdot f(q) \, dq \right\}^{-1}. \tag{11}
\]

(iv) [Beliefs in pooling equilibrium:] If no variation in \( S \) is observed on the equilibrium path, Employer believes that any deviation by Student reveals Student to be of a type for which deviation would be profitable in a separating equilibrium (the Cho-Kreps Intuitive Criterion: Cho and Kreps (1987)). That is, if only Student of type \( Q \in X \) would, in a separating equilibrium, choose \( S = S^* \), and \( S = S^* \) is observed in a pooling
equilibrium on \( S \neq S^* \),

\[
g(Q | S = S^*) \equiv 1(Q \in X) \cdot f(Q) \cdot \left\{ \int_Q 1(q \in X) \cdot f(q) \, dq \right\}^{-1}.
\]

(12)

If no variation in \( S \) is observed on the equilibrium path and there is no deviation, Employer believes that the distribution of unobservable types is the true distribution:

\[
g(Q) \equiv f(Q).
\]

(13)

(Equations 12 and 13 both relate to beliefs under pooling equilibrium; equation 12 relates to beliefs off the equilibrium path while equation 13 relates to beliefs on the equilibrium path.)

I solve first under the assumption of a separating equilibrium in pure strategies, derive from that result the restriction on Employer beliefs, then solve for pooling equilibria in pure strategies. I then restrict attention to the case of a risk-neutral Employer in order to summarise the pure-strategy results and to solve for mixed-strategy equilibria.

### 2.3 Pure strategy equilibria

#### 2.3.1 Pure strategies: separating equilibria

I begin by solving for the restrictions necessary to support a separating equilibrium. I begin by restricting players to pure strategy equilibria; for clarity, I therefore discuss the choice of \( S^*(Q) \) and \( R^*(S) \) directly.\(^5\)

**Proposition 1** For any separating equilibrium, \( R^*(S) = S \).

\(^5\) That is, rather than denoting \( s = 0, s = 1, r = 0, r = 1 \).
Proof: Proofs are in the Appendix.

Student’s best response follows directly from Employer’s strategy.

Definition 2 The cut-off value in Q, $Q^*$.

• $Q^*$ is defined such that $B(Q^*) = C(Q^*)$.

At this stage, I assume the existence of $Q^* \in (Q, \overline{Q}]$; that is, I assume that some types of Student can afford education (if education induces Employer to hire), but not all types. This will be relaxed in due course.

Proposition 2 For any separating equilibrium, Student’s unique best response to Nature is to follow a ‘cut-off strategy’: choose $S^* = 1(Q \geq Q^*)$.\(^6\)

Summary. Student’s best response to Nature can, therefore, be simply stated: if there is a separating equilibrium, choose $S^*(Q) = 1(Q \geq Q^*)$. Employer’s best response to Student is similarly straightforward: if there is a separating equilibrium, choose $R^*(S) = S$.

To close the solution for the separating equilibrium case, I need to solve for restrictions upon Employer’s incentive compatibility condition.

Proposition 3 Employer’s incentive compatibility conditions to support a separating equilibrium are:

\[
\int_{Q^*}^{\overline{Q}} (P(q) + H(Q)) \cdot f(q) \, dq \geq 0 \tag{14}
\]

and \[
\int_{Q}^{Q^*} P(q) \cdot f(q) \, dq < 0. \tag{15}
\]

\(^6\) It is immediately obvious that this strategy is not, literally, a unique best response: an equally good response would be to make the inequality strict. In this and every subsequent case, discussion of ‘unique’ best responses must be construed accordingly.

\(^7\) I use $1(\cdot)$ to refer throughout to refer to the indicator function.
This shows that, in order for Employer to choose \( R^*(S) = S \), it must be profitable for Employer to hire Student of type \( Q \geq Q^* \) (including the additional ‘human capital effect’), but not profitable to lend to Student of type \( Q < Q^* \) (not including the ‘human capital effect’).

The necessary conditions for the existence of a separating equilibrium are therefore:

(i) that there exists some \( Q^* \in (Q, \overline{Q}) \) such that \( B(Q) \geq C(Q) \forall Q \geq Q^* \), and

(ii) that the incentive compatibility conditions in equations 14 and 15 are met.

Figures 2 and 3 illustrate.

Figure 2: **Student uses a cut-off strategy in a separating equilibrium**

\[
\begin{align*}
S &= 0 \\
S &= 1 \\
Q^* &
\end{align*}
\]

\[
\begin{align*}
0 &
\end{align*}
\]

\[
\begin{align*}
Q &
\end{align*}
\]

\[
\begin{align*}
\overline{Q} &
\end{align*}
\]

\[
\begin{align*}
B(Q) - C(Q) &
\end{align*}
\]

\[
\begin{align*}
B(Q^*) - C(Q^*) &\equiv 0
\end{align*}
\]
2.3.2 Pure strategies: Pooling on $S = 1$

Solving for pooling equilibria in this model is less straightforward than solving for separating equilibria. This because I need to consider a variety of candidate pooling equilibria, defined both by (i) whether pooling is on $S = 0$ or $S = 1$ or (ii) whether $B(Q) > C(Q)$ for all, some or no values of $Q$. I begin by considering pooling on $S = 1$. I consider first the strategy that Employer must adopt to support such an equilibrium.

**Proposition 4** For any pooling equilibrium on $S = 1$, Employer chooses $R^*(S) = S$.

Consider, then, the restriction necessary to ensure that every type of Student chooses $S = 1$.

**Proposition 5** If there is a pooling equilibrium on $S = 1$, $B(Q) > C(Q) \forall Q \in [Q, \overline{Q}]$; that is, there exists no $Q^* \in [Q, \overline{Q}]$ such that $B(Q^*) = C(Q^*)$. 

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**Proposition 6** In a pooling equilibrium on $S = 1$, Employer’s incentive compatibility conditions are:

\[
\int_{Q}^{Q} (P(q) + H(Q)) \cdot f(q) \, dq \geq 0 \tag{16}
\]

and

\[
\int_{Q}^{Q} P(q) \cdot f(q) \, dq < 0. \tag{17}
\]

**Remark.** This shows that pooling on $S = 1$ can only be supported if every type of Student can ‘afford’ to invest in the signal, and if it is profitable for Employer to choose $R = 1$ only because of the ‘human capital benefit’ ($H(Q)$).

This makes intuitive sense; for a pooling equilibrium on $S = 1$, $S$ is not an informative signal, precisely because every type of Student must be able to afford it. Thus, the choice of $S = 1$ must be justified, if at all, by the human capital benefits that the education choice confers upon Employer. Figure 4 illustrates.
2.3.3 Pure strategies: Pooling on $S = 0$ where $B(Q) \geq C(Q)$ for some but not all $Q$

I just showed that, in the case of pooling on $S = 1$, there must be no $Q^* \in [Q, \overline{Q}]$. However, in the case of pooling on $S = 0$, three scenarios are possible: the scenario where there is some $Q^* \in [Q, \overline{Q}]$ (that is, where some types of Student would choose $S = 1$ if Employer would consequently choose $R = 1$), and two scenarios where there is no $Q^* \in [Q, \overline{Q}]$ (scenarios where (i) $B(Q) > C(Q) \forall Q$ and (ii) where $B(Q) < C(Q) \forall Q$). I consider each scenario in turn; in this section, I begin with the case in which $B(Q) \geq C(Q)$ for some (but not all) values of $Q$.

**Proposition 7** For any pooling equilibrium on $S = 0$ with the existence of some $Q^* \in [Q, \overline{Q}]$, Employer will either choose $R^*(S) = 0$ (irrespective of $S$) or $R^*(S) = 1$ (again, irrespective of $S$). For the former case, Employer’s incentive compatibility condition is:

$$\int_{Q^{*}}^{\overline{Q}} (P(q) + H(Q)) \cdot f(q) \, dq < 0. \tag{18}$$

For the latter case, the incentive compatibility condition is:

$$\int_{Q}^{\overline{Q}} P(q) \cdot f(q) \, dq \geq 0. \tag{19}$$

**Remark.** Therefore, there are two cases in which there will be a pooling equilibrium on $S = 0$ despite $B(Q) \geq C(Q)$ for some $Q$: (i) the case in which the conditional distribution of $Q$ is so poor that even signalling $S = 1$ is not enough to induce $R = 1$, and (ii) the case in which the unconditional distribution is so good that Employer will choose $R = 1$ irrespective of $S$. 

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2.3.4 Pure strategies: Pooling on $S = 0$ where $B(Q) > C(Q)$ for all values of $Q$

Consider a situation in which it is profitable for all types of Student to choose $S^*(Q) = 1$, if that is required to induce Employer to choose $R = 1$.

**Proposition 8** For any pooling equilibrium on $S = 0$ with $B(Q) > C(Q)$ for all $Q$, Employer will again choose either $R^*(S) = 0$ or $R^*(S) = 1$. For the former case, Employer’s incentive compatibility condition is:

$$\int_Q^{\bar{Q}} (P(q) + H(Q)) \cdot f(q) \ dq < 0. \tag{20}$$

For the latter case, the incentive compatibility condition is:

$$\int_Q^{\bar{Q}} P(q) \cdot f(q) \ dq \geq 0. \tag{21}$$

2.3.5 Pure strategies: Pooling on $S = 0$ where $B(Q) < C(Q)$ for all values of $Q$

Finally, I consider consider the case in which it is never profitable for any type of Student to choose $S = 1$, even if Employer will consequently choose $R = 1$.

**Proposition 9** If $B(Q) < C(Q) \ \forall \ Q \in [Q, \bar{Q}]$, there exists no $Q$ such that Student of type $Q$ will deviate to $S = 1$. Employer will choose either $R^*(S) = 0$ or $R^*(S) = 1$, determined by the following incentive compatibility requirement:

$$R^*(S) = 1 \left( \int_Q^{\bar{Q}} P(q) \cdot f(q) \ dq \geq 0 \right). \tag{22}$$
2.4 Risk neutrality, and a simpler nomenclature

To this point, I have a disparate set of conditions for various forms of equilibria. I now collate and summarise the equilibria. In order to do this, I make a further assumption.

**Assumption 4** Employer is risk neutral, so that $E(P(Q)) = P(E(Q))$ and $E(H(Q)) = H(E(Q))$.

**Remark.** This assumption was not necessary for the preceding solution conditions. However, it is a valuable simplifying assumption for collating equilibrium conditions, because it allows the previous conditions to be reframed in terms of a comparison between the conditional expectation of $Q$ and Employer indifference values for $Q$. I maintain the assumption in order to collate equilibria here, and to consider mixed strategies. In section 2.5 (starting on page 31), I take a special case of the model and relax the risk-neutrality assumption, in order to explore the effect of risk-aversion on market failure.

**Definition 3** The Employer indifference point in $Q$ with human capital benefit, $Q^+$. 

- $Q^+$ is defined such that $P(Q^+) = 0$. As for $Q^+$, if $P(Q^+) > 0 \forall Q \in [\underline{Q}, \bar{Q}]$, $Q^+$ is treated as $-\infty$; if $P(Q^+) < 0 \forall Q \in [\underline{Q}, \bar{Q}]$, $Q^+$ is treated as $\infty$. 

**Definition 4** The Employer indifference point in $Q$ without human capital benefit, $Q^\times$. 

- $Q^\times$ is defined such that $P(Q^\times) + H(Q^\times) = 0$. If $P(Q^\times) + H(Q^\times) > 0 \forall Q \in [\underline{Q}, \bar{Q}]$, $Q^\times$ is treated as $-\infty$; if $P(Q^\times) + H(Q^\times) < 0 \forall Q \in [\underline{Q}, \bar{Q}]$, $Q^\times$ is treated as $\infty$. 

Remark. Given the restrictions already placed on \( P(\cdot) \) and \( H(\cdot) \), it is trivial that \( Q^+ \geq Q^* \).

Since \( P'(Q) > 0 \) and \( H'(Q) \geq 0 \), and given the risk-neutrality assumption, I can rewrite the conditions on Employer payoffs in terms of a comparison of expected \( Q \) and the critical values \( Q^+ \) and \( Q^* \):

\[
\int_{Q^*}^{\bar{Q}} (P(q) + H(Q)) \cdot f(q) \, dq \geq 0 \iff E(Q \mid Q \geq Q^*) \geq Q^*; \quad (23)
\]

\[
\int_{\bar{Q}}^{Q^+} P(q) \cdot f(q) \, dq < 0 \iff E(Q \mid Q < Q^*) < Q^+. \quad (24)
\]

Similarly, I can write:

\[
\int_{Q^*}^{\bar{Q}} P(q) \cdot f(q) \, dq < 0 \ (\geq 0) \iff E(Q) < Q^+ (\geq Q^+); \quad (25)
\]

\[
\int_{Q}^{\bar{Q}} (P(q) + H(Q)) \cdot f(q) \, dq < 0 \ (\geq 0) \iff E(Q) < Q^* (\geq Q^*). \quad (26)
\]

2.4.1 Risk-neutrality: Summary of pure strategy equilibria

Case 1: Where \( B(Q) < C(Q) \forall Q \in [Q, \bar{Q}] \). In this case, there cannot be a separating equilibrium, since there is no \( Q^* \in [Q, \bar{Q}] \). However, there is a pooling equilibrium, on \( S = 0 \) and \( R = 1 \) (\( E(Q) \geq Q^+ \)). Table 1 shows the two possibilities.

<table>
<thead>
<tr>
<th>( Q )</th>
<th>( \mathbb{E}(Q) )</th>
<th>( \bar{Q} )</th>
<th>Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q^+ )</td>
<td>( Q^+ )</td>
<td>( Q^+ )</td>
<td>Pooling (( S = 0; R = 1 ))</td>
</tr>
<tr>
<td>( Q^+ )</td>
<td>( Q^+ )</td>
<td>( Q^+ )</td>
<td>Pooling (( S = 0; R = 0 ))</td>
</tr>
</tbody>
</table>

Table 1: Summary of equilibria with \( B(Q) < C(Q) \forall Q \in [Q, \bar{Q}] \)
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**Case 2:** Where $B(Q) > C(Q) \forall Q \in [Q, \overline{Q}]$. In this case, there also cannot be a separating equilibrium; again, there is no $Q^* \in [Q, \overline{Q}]$.

There will be a pooling equilibrium on $S = 1$ if $E(Q) \geq Q^x$ and $E(Q) < Q^+$.

That is, Employer must prefer $R = 1$ if $S = 1$, but $R = 0$ if $S = 0$. If this condition does not hold, there can be no pooling equilibrium on $S = 1$. There may also be a pooling equilibrium on $S = 0$, if $E(Q) < Q^x$ or if $E(Q) \geq Q^+$.

Given that $Q^x \leq Q^+$, this presents three mutually exclusive equilibria for the case where $B(Q) > C(Q) \forall Q \in [Q, \overline{Q}]$. Table 2 illustrates.

<table>
<thead>
<tr>
<th>$Q$</th>
<th>$E(Q)$</th>
<th>$\overline{Q}$</th>
<th>Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^x, Q^+$</td>
<td>$E(Q)$</td>
<td>$\overline{Q}$</td>
<td>Pooling ($S = 0; R = 0$)</td>
</tr>
<tr>
<td>$Q^x$</td>
<td>$Q^+$</td>
<td>$\overline{Q}$</td>
<td>Pooling ($S = 1; R = 1$)</td>
</tr>
<tr>
<td>$Q^x$</td>
<td>$Q^+$</td>
<td>$\overline{Q}$</td>
<td>Pooling ($S = 0; R = 1$)</td>
</tr>
</tbody>
</table>

**Case 3:** Where $B(Q) \geq C(Q)$ for some but not all $Q \in [Q, \overline{Q}]$. It has already been shown that this implies the existence of some $Q^* \in [Q, \overline{Q}]$ such that $B(Q^*) = C(Q^*)$, $B(Q) > C(Q) \forall Q > Q^*$ and $B(Q) < C(Q) \forall Q < Q^*$.

I must consider both the possibility of a pooling equilibrium and of a separating equilibrium.

*Pooling equilibrium:* It has already been shown that there are two candidate pooling equilibria in this case, both where $S = 0$. In the first case, $S = 0$ and $R = 0$; this requires that $E(Q \mid Q \geq Q^*) < Q^x$. In the second case, $S = 0$ and $R = 1$; this requires that $E(Q) \geq Q^+$. 

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Separating equilibrium: It has already been shown that a separating equilibrium implies \( S^*(Q) = 1(Q \geq Q^*) \) and \( R^*(S) = S \), and that this requires \( \mathbb{E}(Q \mid Q \geq Q^*) \geq Q^* \) and \( \mathbb{E}(Q \mid Q < Q^*) < Q^+ \).

Therefore, the existence and form of equilibrium depends upon the relationship between three critical expectations determined by \( Q^* \) — that is, \( \mathbb{E}(Q \mid Q_u < Q^*) \), \( \mathbb{E}(Q) \) and \( \mathbb{E}(Q \mid Q \geq Q^*) \) — and two critical values for Employer payoffs — that is, \( Q^x \) and \( Q^+ \). Given the restriction that \( Q^+ \geq Q^x \), there are 10 possible combinations of regions in which \( Q^x \) and \( Q^+ \) may lie. Table 3 shows those regions, applying the rules just set out to determine the resulting equilibria.

<table>
<thead>
<tr>
<th>( Q )</th>
<th>( \mathbb{E}(Q \mid Q &lt; Q^*) )</th>
<th>( \mathbb{E}(Q) )</th>
<th>( \mathbb{E}(Q \mid Q \geq Q^*) )</th>
<th>( \bar{Q} )</th>
<th>Equilibria</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q^x, Q^+ )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Pooling ((S = 0; R = 1))</td>
</tr>
<tr>
<td>( Q^x )</td>
<td>( Q^+ )</td>
<td></td>
<td></td>
<td></td>
<td>Pooling ((S = 0; R = 1)) &amp; separating</td>
</tr>
<tr>
<td>( Q^x )</td>
<td></td>
<td>( Q^+ )</td>
<td></td>
<td></td>
<td>Separating</td>
</tr>
<tr>
<td>( Q^x )</td>
<td>( Q^x, Q^+ )</td>
<td></td>
<td></td>
<td></td>
<td>Pooling ((S = 0; R = 1)) &amp; separating</td>
</tr>
<tr>
<td>( Q^x )</td>
<td>( Q^+ )</td>
<td></td>
<td></td>
<td></td>
<td>Separating</td>
</tr>
<tr>
<td>( Q^x )</td>
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<td>( Q^+ )</td>
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<td></td>
<td>Separating</td>
</tr>
<tr>
<td>( Q^x, Q^+ )</td>
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<td>Separating</td>
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<td>( Q^x )</td>
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<td>Separating</td>
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<td>( Q^x )</td>
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<td>Separating</td>
</tr>
<tr>
<td>( Q^x, Q^+ )</td>
<td></td>
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<td></td>
<td></td>
<td>Separating</td>
</tr>
</tbody>
</table>

Discussion: Pure strategies and risk neutrality. Even limiting attention to pure strategies and to a risk-neutral principal, the model has produced a relatively intricate solution structure: this section has showed that there are 15 cases that require consideration, of which two cases each produce two equilibria.

The collation of equilibria in this risk-neutral case already emphasises the two
fundamental considerations for solving models of binary signalling. First, one must consider the relative costs and benefits from the perspective of the agent (Student): the agent may either (i) never afford the signal, (ii) always afford the signal or (iii) sometimes afford the signal. Second, one must then consider the principal’s payoffs, to be integrated over some appropriate range of agent types. Together, these considerations determine both the agent’s signal and the principal’s response.

The first two cases (just summarised in Tables 1 and 2 respectively) are substantially less interesting than the third case (Table 3), and necessarily imply a pooling equilibrium (either on $S = 0$ or, in one case, on $S = 1$). However, even if only some types of agent can afford the signal, there may still be pooling equilibria. This may occur because the signal, if obtained, still does not indicate adequate agent quality for the principal; this is shown in the last row in Table 3. Alternatively, it may also occur because the overall distribution of agent quality is so good that the principal will still be willing to contract without any signalling (the other three pooling equilibria in Table 3).

In some respects, it seems strange that a high overall quality of student ability may be a reason for students not to obtain a tertiary education. In part, this is driven by the constraint that the employer may either hire or not hire; the employer is prevented, in this model, from paying higher wages to more educated students. In this way, the simplicity of the model imposes an inefficiency on the players: no matter how much the employer benefits from the student’s extra human capital (that is, no matter how large is the human capital effect, $H(Q)$), the employer is not able to pay more educated students more: the only way in this model that an employer is able to reward more educated students is by hiring them and not hiring the less educated. One could imagine an extension of this
model, then, in which the employer could offer a more attractive contract to stu-
dents with a tertiary degree than those without.

Similarly, one might imagine an extension incorporating competition among mul-
tiple prospective employers, and/or competitive forces in the provision of educa-
tion. Nothing in the current model, for example, imposes that Employer makes
zero profit in expectation; however, this is an outcome that one might expect if
there were free entry and exit among multiple employers. This is also an exten-
sion that could be considered in future work.

I choose not to pursue these extension in the present model: my emphasis here
in on the consequences of binary signalling for information coarseness, and the
present model is a simple way of exploring that issue. I consider an extension
in section 4 (starting on page 44) in which the employer responds by choosing
from a continuous variable (for example, a continuous wage). That extension
allows the employer to reward tertiary education with a more generous contract;
I explore the implications of that (in particular, the implications for equilibrium
conditions) in that section.

2.4.2 Risk-neutrality: Mixed strategy equilibria

I defined the game earlier in terms of mixed strategies \( s(Q) \) and \( r(S) \). However, I
then limited consideration to the pure-strategy cases. I now relax that restriction.
I begin by considering mixed strategies in separating equilibria; I then turn to
consider pooling equilibria.

I imposed the ‘single-crossing property’ in the pure strategy case (Assumption
1). However, for the mixing case, I need to make an additional assumption about
the cost of education.\(^8\)

**Assumption 5** The cost of education is decreasing in Student quality:

\[
C'(Q) < 0. \tag{27}
\]

**Student mixing in separating equilibrium.** In the pure-strategy case, Student payoff is a linear combination of payoff from \(S = 0\) and \(S = 1\). Therefore, Student will mix only if indifferent between the two available pure strategies. This can be framed in terms of a modified cut-off function.

**Definition 5** The cut-off value in \(Q\) for mixed strategies, \(Q^*(r^*(1) - r^*(0))\).

- \(Q^*(r^*(1) - r^*(0))\) is defined such that, if Employer mixes using \(r^*(1)\) and \(r^*(0)\), Student of type \(Q^*\) will be indifferent between \(S = 1\) and \(S = 0\). That is,

\[
B(Q^*) \cdot r^*(1) - C(Q^*) = B(Q^*) \cdot r^*(0) \tag{28}
\]

\[
\iff B(Q^*) \cdot [r^*(1) - r^*(0)] = C(Q^*). \tag{29}
\]

Since \(B'(Q) - C'(Q) > 0\) and \(C'(Q) < 0\), \(Q^*\) is a monotonically decreasing function. That is, as \(r^*(1) - r^*(0)\) increases (i.e. as Employer’s relative response to \(S = 1\) increases), a greater proportion of Student types will invest in the signal. Additionally, note that \(Q^* = Q^*(1)\).

---

\(^8\) The reason for this further assumption is shown shortly in Equation 29. In the mixing case, I need to be able to speak about the slope of \(B(Q)\) and \(C(Q)\) separately (because \(r^*(1) - r^*(0) \in [0, 1]\)); in the pure-strategy case, it was enough to speak about the slope of \(B(Q) - C(Q)\) (because, implicitly, \(r^*(1) - r^*(0) = 1\) under a pure-strategy separating equilibrium).
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It follows, therefore, that

\[
\begin{align*}
s^*(Q) &= 1 \quad \text{if } Q > Q^*(r^*(1) - r^*(0)); \\
&= 0 \quad \text{if } Q < Q^*(r^*(1) - r^*(0)); \\
&\text{may take any value in } [0, 1] \quad \text{if } Q = Q^*(r^*(1) - r^*(0)).
\end{align*}
\]

(30)

Note additionally that, since \( B(Q) > 0 \) and \( C(Q) > 0 \), equation 29 shows that there can be no separating equilibrium in mixed strategies unless \( r^*(1) > r^*(0) \). For the separating case, I may therefore restrict attention — without loss of generality — to \( (r^*(1) - r^*(0)) \in [0, 1] \). Figure 5 illustrates.

Figure 5: Student mixing

Employer mixing in separating equilibrium. Conditional on Student having chosen \( S \), Employer payoff is also a linear combination of payoff from \( R = 1 \) and \( R = 0 \); therefore, like Student, Employer will mix only if indifferent between the two pure strategies. As in the pure-strategy case, this can be understood in terms of payoff cutoffs.
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Definition 6  Mixing gaps to induce indifference under $S = 0$ and under $S = 1$.

- $r^+$ is defined such that $\mathbb{E}(Q \mid Q < Q^*(r^+)) = Q^+$;
- $r^\times$ is defined such that $\mathbb{E}(Q \mid Q \geq Q^*(r^\times)) = Q^\times$.

It is straightforward, then, to state separately Employer’s optimal decision for $r(0)$ and $r(1)$ in separating equilibrium:

\[
\begin{align*}
  r^*(0) &= \begin{cases} 
    1 & \text{if } \mathbb{E}(Q < Q^*(r^*(1) - r^*(0))) > Q^+; \\
    0 & \text{if } \mathbb{E}(Q < Q^*(r^*(1) - r^*(0))) < Q^+; \\
    \text{may take any value in } [0, 1] & \text{if } \mathbb{E}(Q < Q^*(r^*(1) - r^*(0))) = Q^+; 
  \end{cases} \\
  r^*(1) &= \begin{cases} 
    1 & \text{if } \mathbb{E}(Q \geq Q^*(r^*(1) - r^*(0))) > Q^\times; \\
    0 & \text{if } \mathbb{E}(Q \geq Q^*(r^*(1) - r^*(0))) < Q^\times; \\
    \text{may take any value in } [0, 1] & \text{if } \mathbb{E}(Q \geq Q^*(r^*(1) - r^*(0))) = Q^\times. 
  \end{cases}
\end{align*}
\]

Though these separate conditions must hold, it is not sufficient to solve $r^*(0)$ and $r^*(1)$ separately; the structure of the decision rules requires them to be solved jointly. First, consider the case in which $r^\times > r^+$.

Proposition 10  If $r^\times > r^+$, there are at most two separating equilibria in mixed strategies: one possible equilibrium in which $(r^*(0), r^*(1)) = (1 - r^+, 1)$ and one in which $(r^*(0), r^*(1)) = (0, r^\times)$.

Figure 6 illustrates.

It is not necessary, however, that $r^\times > r^+$.

Proposition 11  If $r^+ > r^\times$, there are no mixed-strategy equilibria.

Figure 7 illustrates.
Figure 6: **Employer mixing: Mixed-strategy equilibria**

\[
\begin{align*}
    r^*(0) &= 1 - r^+; \\
    r^*(0) &= 0; \\
    r^*(1) &= 1 \\
    r^*(1) &= r^x
\end{align*}
\]

Figure 7: **Employer mixing: No mixed-strategy equilibria**
**Employer mixing in separating equilibrium: A commitment problem.** Note that the preceding solution does not generally obtain from an optimisation of this form:

\[
(r^*(0), r^*(1)) = \arg \max_{(r(0), r(1))} r(0) \cdot \mathbb{E}(P(Q) \mid Q < Q^*(r(1) - r(0))) \\
+ r(1) \cdot \mathbb{E}(P(Q) + H(Q) \mid Q \geq Q^*(r(1) - r(0))).
\]

(33)

It may be tempting to frame the problem in these terms; however, the crucial difference to this approach is that the approach in equation 33 implicitly assumes that Employer has some credible additional mechanism by which to commit to \(r^*(0)\) and \(r^*(1)\). That is, equation 33 holds only if Employer may commit to \(r^*(0)\) and \(r^*(1)\) as a screening mechanism.

It is immediately obvious that Employer may enjoy some welfare gain from having such a position. For example, consider a circumstance where, under pure strategies, \(\mathbb{E}(Q \mid Q \geq Q^*) = Q^*\) (that is, a situation in a separating equilibrium in which Employer gains precisely nothing from contracting with Student of type \(Q \geq Q^*\)); in that case, Employer may move from a zero-profit to positive-profit outcome if it could credibly announce \(r^*(1) < 1\). That is, Employer would prefer to make \(S = 1\) less profitable for Student in order to increase \(\mathbb{E}(Q \mid S = 1)\). This is not a viable strategy under the current model: if Student were to respond in this way, \(\mathbb{E}(Q \mid S = 1) > Q^*\) and Employer has incentive to renege on its commitment, by choosing \(r^*(1) = 1\). This would, then, be a relatively straightforward screening game; I note the commitment problem but do not pursue it here. The assumption that Student moves first, so that Employer cannot credibly commit to any strategy in advance, is therefore an important one in the mixed-strategy case.
Mixing in pooling equilibrium. See appendix.

Discussion: Mixed strategies and risk neutrality. I have showed in this section that there may not be any mixed-strategy equilibria (see Figure 7). Moreover, both of the possible mixed-strategy separating equilibria (in Figure 6) are, in practical terms, tenuous. The case in which $r^*(0) = 0$ and $r^*(1) = r^\times$ (the case on the right in Figure 6) provides no improvement for Employer on the case of $R_*(S) = 0$. The case in which $r^*(0) = 1 - r^+$ and $r^*(1) = 1$ (the case on the left in Figure 6) does provide an improvement for Employer on the case of $R_*(S)$: in effect, Employer may turn a no-profit situation into a profitable one by a credible promise to mix in response to $S = 0$. However, this is tenuous in the sense that, once Student plays $S^*(Q) = 1(Q \geq Q^*(r^+))$, Employer is indifferent between any values in $[0, 1]$ for $r^*(0)$; choosing $r^*(0) = 1 - r^+$ is therefore a best response, but not a unique best response. Alternatively expressed, Student may signal in anticipation of $r^*(0) = 1 - r^+$, but Employer then has no positive incentive to respond with that strategy.

2.5 Risk aversion and market failure

In the previous section, I imposed Employer risk-neutrality. This was important in order to allow a collation of a variety of disparate equilibrium conditions into a combined summary; crucially, it allowed a straightforward expression for Employer’s response in terms of critical payoff values ($Q^\times$ and $Q^+$).

In this section, I relax that assumption. I consider a special case of the original model in which the principal is allowed to have a constant absolute risk aversion. I use this special case in order to illustrate an important characteristic of binary signalling games: because a binary signal can only disclose coarse information about an agent, a principal contracting with that agent necessarily must
bear risk as to the true state of the agent’s unobservable. In the current context, Employer cannot be certain of Student’s true ability; hiring Student therefore necessarily involves gambling on that ability. The combination of signal coarseness and a principal’s risk-aversion may, therefore, be a reason for a no-contract equilibrium (that is, ‘market failure’). To my knowledge, the existing literature on information coarseness has not previously emphasised this point.

Specifically, I explore the implications of risk-aversion within a modified ‘exponential-normal’ model. The ‘exponential-normal’ model is a standard and useful method for considering risk-aversion in a principal-agent framework (for example, see Ederer, Holden, and Meyer (2009)). I implement that model by making two further assumptions. First, I impose a modified exponential utility function on Employer.

**Assumption 6** Employer has exponential payoffs with coefficient of absolute risk aversion $r$ (where $r > 0$ and $r = 0$ is a limiting case of risk neutrality), normalised so that $\pi_E(Q = 0, S = 0) = 0$:

$$P(Q) \equiv -\exp(-rQ) + 1$$

$$H(Q) \equiv \exp(4) - 1 \approx 53.6$$

$$\therefore \pi_E(Q, S) = -\exp(-rQ) + 1 + S \cdot (\exp(4) - 1).$$

(The choice of $H(Q)$ may appear quite arbitrary; it is made for the convenience of the subsequent illustration.)

Second, I impose a distributional assumption on Student ability, $Q$.

**Assumption 7** Student ability has a normal distribution with mean $\mu$ and
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variance $\sigma^2$:

\[ Q \sim N(\mu, \sigma^2). \] (37)

I limit attention to pure strategies. Note immediately that this is merely a special case of the model studied in section 2.3 (in which $Q$ is treated as $\infty$ and $Q$ is treated as $-\infty$).

**Proposition 12** Under an ‘exponential-normal’ model with $\pi(x) = -\exp(-rx)$ and $x \sim N(\mu, \sigma^2)$ but truncated below at some $z$, expected utility is:

\[ E(\pi | x \geq z) = -\exp \left( \frac{1}{2} \cdot \sigma^2 r^2 - \mu r \right) \cdot \left[ \frac{1 - \Phi (a + \sigma r)}{1 - \Phi (a)} \right], \] (38)

where $a = \frac{z - \mu}{\sigma}$. For truncation above at $z$,

\[ E(\pi | x < z) = -\exp \left( \frac{1}{2} \cdot \sigma^2 r^2 - \mu r \right) \cdot \left[ \frac{\Phi (a + \sigma r)}{\Phi (a)} \right]. \] (39)

This proposition generalises a common result for exponential-normal models to the case of truncation.\(^9\) The proposition allows the implementation of the earlier equilibrium conditions in the case of this particular model.\(^10\) I then solve for the case in which $\mu = 1$ and $\sigma = 1$.

---

\(^9\) It is well known that, where $x$ is drawn from the entire real line, the certainty equivalent is $\pi = \mu - \frac{1}{2} \cdot r\sigma^2$ (for example, see Ederer, Holden, and Meyer (2009, p.9)). Proposition 12 implies that, for truncation below at $z$, $\pi = \mu - \frac{1}{2} \cdot r\sigma^2 - \frac{1}{2} \cdot \ln [1 - \Phi (a + \sigma r)] \cdot [1 - \Phi (a)]^{-1}$; the result is symmetric for truncation above. I show this in the proof of Proposition 12.

\(^10\) Specifically, Proposition 12 implies, in the case of this particular model, that $E(\pi_E | Q \geq Q^*, S = 1) = 0 \iff -\exp \left( \frac{1}{2} \cdot \sigma^2 r^2 - \mu r \right) \cdot [1 - \Phi (a + \sigma r)] \cdot [1 - \Phi (a)]^{-1} + \exp(4) = 0$, where $z = Q^*$. This relationship describes the boundary between the condition in equation 14 and the condition in equation 18. Similarly, Proposition 12 also implies that $E(\pi_E | Q < Q^*, S = 0) = 0 \iff -\exp \left( \frac{1}{2} \cdot \sigma^2 r^2 - \mu r \right) \cdot [\Phi (a + \sigma r)] \cdot [\Phi (a)]^{-1} + 1 = 0$; this describes the boundary of the condition in equation 15. These conditions may be solved numerically.
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I begin by considering the case in which $B(Q) \geq C(Q)$ for $Q \geq Q^* \in \mathbb{R}$; that is, the case in which some types of student can afford to signal, but not all. In that case, there are five solution regions, shown by Figure 8. In Region A, there is a pooling equilibrium: $S^*(Q) = 0; R^*(S) = 0$ (that is, Region A represents the no-contract region). In Region B and Region C, there is a separating equilibrium: $S^*(Q) = 1(Q \geq Q^*); R^*(S) = S$. In Region E, there is a pooling equilibrium: $S^*(Q) = 0; R^*(S) = 1$. In Region D, there is a separating equilibrium of the form in Region B and Region C; however, there is also a pooling equilibrium of the form in Region E.

Figure 8: Solution regions for an exponential-normal model ($\mu = 1, \sigma = 1$) with some $Q^*$
Discussion: Risk aversion. The signalling literature generally concerns outcomes in which the principal may learn precisely the agent’s type. For this reason, the relationship between risk aversion and equilibrium outcome is not generally of primary interest. However, the model studied here involves a binary signal of a continuous type; for that reason, equilibria reveal only limited information about the agent’s type, so that the principal’s risk aversion is important in determining the equilibrium outcome.

In this section, I have used a special case of the exponential-normal model to illustrate some consequences of risk aversion in a binary signalling game. That example has showed that, irrespective of the agent’s cutoff value, a change in the principal’s risk aversion may change the equilibrium outcome. Importantly, the example showed that, if the signal is sufficiently cheap, an increase in the principal’s risk aversion may shift the equilibrium from a contracting equilibrium to a no-contract equilibrium (for example, in Figure 8, increasing \( r \) may shift the equilibrium from a separating equilibrium in region C or region D to a no-contract equilibrium in region A). Similarly, the example showed that, depending on the value of the signalling cutoff, this may not necessarily be the case: in much of region B, for example, no increase in \( r \) will shift the result away from a separating equilibrium. It has long been recognised that information asymmetry may be a reason for market failure (for example, Akerlof (1970)); in cases of partial information revelation, a principal’s risk-aversion may be an important cause for such an outcome.

The case studied here — the exponential-normal model — is a very restrictive one. Specifically, the assumption of constant absolute risk aversion — while allowing a closed-form derivation of conditional expected utility — is often not viewed as reasonable. It would be a valuable extension to generalise the present
result to a wider class of utility functions (for example, to the case of constant relative risk aversion) and to other distribution functions.

**Two other cases.** Risk aversion is also important in the case in which no type of Student can afford to signal (that is, the case in which $B(Q) < C(Q)$ for all $Q$). Figure 9 shows the three solution regions (in the left panel). In all three regions, $S^*(Q) = 0$. In region F and region G, $R^*(S) = 0$; in region H, $R^*(S) = 1$ (see equation 22).

Similarly, risk aversion also matters where every type of Student can afford to signal; where $B(Q) > C(Q)$ for all $Q$. This is shown in the right panel of Figure 9. In Region I, $S^*(Q) = 0$ and $R^*(S) = 0$; see equation 20. In Region J, $S^*(Q) = 1$ and $R^*(S) = 1$; see equations 16 and 17. In Region K, $S^*(Q) = 1$ and $R^*(S) = 0$; see equation 21.

Figure 9: Solution regions for an exponential-normal model ($\mu = 1, \sigma = 1$) with no $Q^*$
3 Extension: Indices

This section introduces indices. I define indices in the same way that Spence did: as characteristics of the agent that are unalterable and observed by all players (1973, pp.368–396). In the context of a student/employer relationship, one might readily think of variables like student age or past performance. However, one might also think of variables which, though alterable, could not profitably be changed for signalling purposes — for example, potentially, one might also consider student wealth or residential location.\footnote{This immediately prompts a potential extension to the model: it may be valuable to know the conditions under which a second alterable variable (such as student wealth) could profitably be used for signalling purposes alongside a binary variable. However, this is left for future work.}

Indices are of particular interest in the context of binary signal because, as shown, a binary signal reveals only limited information. It is therefore important to understand the way in which other variables might be used to mitigate that information coarseness.

I allow, then, that Student is endowed with a vector of unalterable characteristics \( x \), perfectly observable to all players. I allow \( x \) to be informative about \( Q \), so that Nature now chooses continuous univariate \( Q \) from some cumulative conditional distribution \( F_{Q|x}(\cdot) \) over a support that may vary with \( x \): \( Q \in [Q(x), \overline{Q(x)}] \). I maintain the assumption that the distribution is non-degenerate and, critically, I maintain this assumption for \textit{all} values of \( x \). That is, I assume that there is \textit{no} value of \( x \) such that \( x \) itself is perfectly informative of \( Q \). Similarly, I modify all payoff functions to be functions of \( x \): I now allow for \( B(Q, x) \), \( C(Q, x) \), \( P(Q, x) \) and \( H(Q, x) \). For simplicity, I maintain throughout the assumption of Employer risk-neutrality.
**Proposition 13** For any given $x$, the game is equivalent to the game studied earlier.

For clarity, I define three forms of equilibrium combinations in the multiple-indices case.

**Definition 7** Pooling, completely-separating and partially-separating equilibria with indices.

- With indices, a *pooling equilibrium* is an equilibrium configuration in which Student chooses the same $S$ irrespective of $Q$ and $x$.

- With indices, a *completely-separating equilibrium* is an equilibrium configuration in which Student conditions its choice of $S$ upon $Q$ for *all possible values* of $x$.

- With indices, a *partially-separating equilibrium* is an equilibrium configuration in which Student conditions its choice of $S$ upon $Q$ for *some but not all possible values* of $x$.

Figure 10 provides a simple illustration of a completely-separating equilibrium for a two-indices case (in which, for simplicity, I assume $H(Q,x) \equiv 0$, so that $Q^+(x) = Q^x(x)$).

The multiple-indices case is less interesting and less informative than a simpler single-index case. That is, one can imagine a single variable that acts as an ‘index of observable student ability’; I term such an index $I$. Figure 11 illustrates the way in which that index may inform the conditional expectation of $Q$: that expectation is now allowed to vary *both* with $S$ and with $I$. 

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Figure 10: **Completely-separating equilibrium with two indices**

![Diagram showing completely-separating equilibrium with two indices](image)

- $S = 1$
- $S = 0$
- $x_1$
- $x_2$
- $Q^*(x_1, x_2)$
- $Q^*_{\times}(x_1, x_2)$

Figure 11: **Conditional inference with a single index**

![Diagram showing conditional inference with a single index](image)

- $Q^*(I)$
- $\mathbb{E}(Q | I, S = 1)$
- $\mathbb{E}(Q | I)$
- $\mathbb{E}(Q | I, S = 0)$
Figure 12 adds an Employer indifference curve, $Q^\times$. (For simplicity, I now assume that $H(Q) = 0$, so that $Q^\times(I) = Q^+(I)$.) In this case, $Q^\times(I)$ is steeper than $Q^*(I)$. Therefore, the equilibrium is ‘partially-separating’: there are two pooling regions (in each case, pooling on $S = 0$) and a separating region.

This structure can be used to think about Employer’s loss from information asymmetry. In this model, Employer will suffer two types of loss — that is, situations of *ex post* Employer regret — (i) cases in which Employer regrets hiring Student (Student is ‘wrongly employed’) and (ii) cases in which Employer regrets not hiring Student (Student is ‘wrongly unemployed’). Figure 13 illustrates regions of Employer loss for the case in Figure 12.

The situation in Figures 12 and 13 seems unsatisfactory as a depiction of the role of tertiary education as a binary signal, for two reasons. First, one should be concerned by the two ‘pooling’ regions; intuitively, it is difficult to accept that any student would have such good observable characteristics that those characteristics...
alone would justify not needing a tertiary degree. Second, even in the separating region, the outcome lacks intuitive appeal: it is students with lower $I$ who are wrongly employed, but students with higher $I$ who are wrongly unemployed.

The key lies in the relative steepness of Employer’s indifference curve ($Q^\star$) and of Student’s indifference curve ($Q^\ast$). Figures 12 and 13 show the case in which the principal’s indifference curve is steeper than the agent’s. This corresponds to a situation in which the signal is relatively uninformative of $Q$; the Student indifference curve is relatively flat because Student finds it relatively cheap, in terms of $Q$ relative to $I$, to invest in the signal. That is, the informativeness of the signal depends upon changes in the cost of the signal in terms of the unobservable dimension, relative to changes in the cost across the observable dimension.

One may, however, be able to think of some analogous cases: in years past, for example, the best Oxford undergraduates used to proceed to academic careers without needing to complete doctorates; their observable characteristics (undergraduate results, scholarships, etc) were good enough not to require the additional signal.
This explains why Figures 12 and 13 are not appropriate for analysing the education/employment interaction: they describe a situation where the signal under consideration is relatively very uninformative. By way of illustrative example, consider instead if the signal observed were not completion of a tertiary degree (signalling workplace ‘ability’) but were observable attendance by a firm manager at a religious service; imagine, also, that the principal were a bank manager choosing whether or not to lend to the firm. For example, consider the situation in which “debtors may...seek to artificially differentiate themselves by acquiring a costly signal...they may join a religious group simply to persuade creditors that they have a heightened moral sense and business probity” (Fafchamps 1996, p.431). Religious worship may be taken as some indication of unobservable moral sense, but is surely a very weak indication — precisely because it is relatively a very cheap signal. In that case, Figures 12 and 13 seem quite apposite. Firms with the best and the worst unobservables do not need to bother with such religious attendance, because the signal is so weak as to be useless in those cases. Among the separating cases, it is the firms with lesser observables that are better able to exploit the signalling opportunity.

I conclude, therefore, that the case illustrated in Figures 12 and 13, while valuable for completeness of the exposition, is not a realistic representation of the present scenario. Instead, I consider the case where the signal is relatively informative of $Q$: this justifies making the Student indifference curve steeper than the Employer indifference curve. This is illustrated in Figure 14. This restriction eliminates the pooling regions, and shows that — as one would intuitively expect — it is students of lesser observable quality that are more likely to be wrongly unemployed.
In this model, then, the principal suffers welfare loss because, under imperfect information, agents with which the principal could profitably contract are not always able to distinguish themselves from unprofitable agents; the reason that they cannot always do so is that the informativeness of the available signal, conditional upon observed characteristics, does not match precisely the principal’s own preferences.

Critically, this means that welfare loss does not arise from the information asymmetry per se: one could imagine a scenario in which $Q^*(I) = Q^{*\times}(I) \forall I$, in which Employer welfare loss would be zero. In that case, Employer would still not be able to recover a perfect signal of $Q$ — that is, the signal would still only reveal limited information — but could obtain sufficient information to make a first-best contracting decision. In that way, depending on signalling costs, coarse information may be as valuable to a principal as cardinal information; this point has already been noted by Meyer (1991, pp.28–32).
4 Extension: An unbounded continuous response

The emphasis of this paper is on binary signalling. However, to this point, I have also limited consideration to a binary response. In this section, I relax that assumption and allow a continuous response. The fundamental distinction between a binary response and a continuous response is that, in the latter case, the principal can indicate not merely whether the agent is assessed profitable for contracting but also how profitable. This differential response changes the incentives faced by the agent and introduces new considerations to the solution.

4.1 Modifying the game

I maintain the same players, game structure and solution concept, and I limit consideration to pure strategies. I assume that Student is endowed with a single, continuous index observable to all players, I. I now allow Employer to choose any response lying on the real line: $R \in \mathbb{R}$. Further, for simplicity, I allow $Q$ to lie anywhere on the real line.

I place more structure on the payoff functions in the continuous-response case. Payoffs for Student of type $Q$ accrue from the value of the Employer response (normalised by a coefficient of unity), less the cost of signalling:

$$\pi_S = R - C(Q) \cdot S,$$

(40)
where I impose that $C(I, Q)$ is a linear decreasing function of $I$ and $Q$:\(^{13}\)

$$C(I, Q) = \gamma_c - \gamma_i \cdot I - \gamma_q \cdot Q;$$  \hspace{1cm} (41)

$$\gamma_i, \gamma_q > 0.$$  \hspace{1cm} (42)

For Employer payoffs, I assume

$$\pi_E = (\beta_c + \beta_i \cdot I + \beta_q \cdot Q + \beta_s \cdot S) \cdot \ln(R) - R.$$  \hspace{1cm} (43)

Critically, this structure provides a linear first-order condition with respect to the choice variable $R$:

$$\arg \max_{R \in \mathbb{R}} \pi_E(I, Q, S) = \beta_c + \beta_i \cdot I + \beta_q \cdot Q + \beta_s \cdot S.$$  \hspace{1cm} (44)

Note that this functional form maintains the earlier assumption that Employer is risk-neutral.

### 4.2 Equilibrium

The solution concept remains Perfect Bayesian Equilibrium with the Cho-Kreps Intuitive Criterion imposed. I maintain the earlier definitions of a separating and pooling equilibria. I begin by considering a separating equilibrium.

**Proposition 14** If a separating equilibrium exists, Student uses a cutoff function $S^*(Q) = 1(Q \geq Q^*)$ in which

$$Q^*(I) = -\left(\frac{\beta_q \cdot \varphi(Q^*(I)) + \beta_s - \gamma_c + \gamma_i \cdot I}{\gamma_q}\right),$$  \hspace{1cm} (45)

\(^{13}\) Note that this specification allows, for some values of $Q$, that $C(Q)$ is negative; nothing turns upon allowing $C < 0$ in this context. Crucially, the assumption that $\gamma_i, \gamma_q > 0$ still maintains the single-crossing property.
where

$$\varphi(Q^*(I)) \equiv \mathbb{E}(Q \mid I, Q \geq Q^*(I)) - \mathbb{E}(Q \mid I, Q < Q^*(I)).$$  \hspace{1cm} (46)$$

4.2.1 Sufficient conditions for unique equilibrium under the standard normal

Equation 45 defines a separating equilibrium in the continuous-response context. Given the generality with which the problem is framed, it appears that is not possible to derive necessary conditions for the existence of a unique equilibrium, beyond the requirement that equation 45 has a unique solution. However, it is possible to derive sufficient conditions, given the distribution function for $Q \mid I$; that is, one may derive conditions under which a unique equilibrium is guaranteed to exist, without being able to rule out unique equilibrium in other cases. I begin with the standard normal.

**Assumption 8** Assume, for present purposes, that the distribution of $Q$ conditional on $I$ is standard normal: \hspace{1cm} (47)

$$Q \mid I \sim \mathcal{N}(0, 1).$$

**Proposition 15** It is sufficient for the existence of a unique equilibrium that a given increase in the unobservable $Q$ has, in absolute value, a larger impact on the agent’s cost of signalling than it does upon the principal’s best response:

$$\left| \gamma_q \right| > 1.$$ \hspace{1cm} (48)

---

14 It remains to be seen whether the problem could be approached using techniques of optimal control theory, along the lines of Armstrong and Vickers (2010).

15 That is, assume that the distribution of $Q$ conditional on $I$ is standard normal and does not depend upon $I$. 

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Figure 15 illustrates the problem. In doing so, it shows the intuition that drives the solution: an equilibrium exists at the intersection of the function \( \varphi(x) \) with the straight-line function; if the absolute value of the slope of that line is greater than unity, a unique intersection is guaranteed to exist.

Figure 15: **Sufficient conditions for a unique equilibrium with** \( Q \mid I \sim \mathcal{N}(0, 1) \)

\[ y = -(\beta_s - \gamma_c + \gamma_i \cdot I) \cdot \beta_q^{-1} - \gamma_q \cdot \beta_q^{-1} \cdot x; \quad \beta_q > 0 \]

\[ y = -(\beta_s - \gamma_c + \gamma_i \cdot I) \cdot \beta_q^{-1} - \gamma_q \cdot \beta_q^{-1} \cdot x; \quad \beta_q < 0 \]

4.2.2 **Sufficient conditions for unique equilibrium under other distributions**

I am unable to find a characterisation of the sufficient conditions for unique equilibrium that generalises across different distributions. I therefore merely present conjectures for two other distribution functions, based upon numerical evaluation of the function \( \varphi(x) \). The numerical evaluations suggest that the general condition just presented for the standard normal case is not robust to different conditional distributions.
Conjecture 1 If $Q | I \sim \text{Standard Gumbel (Maximum)}$, the general condition for a unique equilibrium is $\beta_q \geq 0$.

Figure 16 shows the numerical evaluation of $\varphi(x)$ under the standard Gumbel (maximum) distribution; the suggestion is that, under the standard Gumbel (maximum), $\varphi(x)$ increases monotonically from a slope of zero as $x \to -\infty$ to an infinite slope as $x \to \infty$. If true, this would imply — using the earlier reasoning — that the earlier condition would not be sufficient for a unique solution. Instead, a sufficient condition would be $\beta_q > 0$.

Conjecture 2 If $Q | I \sim \text{Logistic}(0, 1)$, the general sufficient condition for a unique equilibrium is $\beta_q = 0$.

Figure 17 illustrates the numerical evaluation under the logistic case. The suggestion is that $\varphi(x)$ has a similar shape as under the standard normal, but that the slope changes from $-\infty$ as $x \to -\infty$ to $\infty$ as $x \to \infty$.\(^{16}\) If true, this suggests

\(^{16}\)This suggestion is clearer when the numerical evaluation is displayed across a wider range of $x$; however, I maintain the narrower range for comparability with earlier figures.
that the general sufficient condition for a unique equilibrium is only $\beta_q = 0$; that is, that any relevant information asymmetry implies some parameter space in which there is not a single unique equilibrium.

**Figure 17: Sufficient conditions with $Q|I \sim \text{Logistic}(0, 1)$**

\[
y = - (\beta_s - \gamma_c + \gamma_i \cdot I) \cdot \beta_q^{-1} - \gamma_q \cdot \beta_q^{-1} \cdot x; \quad \beta_q > 0
\]

\[
y = - (\beta_s - \gamma_c + \gamma_i \cdot I) \cdot \beta_q^{-1} - \gamma_q \cdot \beta_q^{-1} \cdot x; \quad \beta_q < 0
\]

**Discussion: Allowing an unbounded continuous response.** One might initially think that allowing an unbounded response from the principal is an innocuous extension to the binary-response model studied earlier. However, this section has showed that the extension introduces important new considerations. Specifically, the section has showed that it becomes important how the agent’s cost of the signal (in terms of its unobservable) compares to the principal’s response to that signal. This is broadly analogous to the point made earlier in considering indices: in section 3, I showed that the form of equilibrium depends on the relative cost of the signal to the agent and the relative benefit of the signal to the principal.
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The reason for this consideration was foreshadowed in discussion earlier. The earlier model — with a binary response — did not allow the principal to reward an agent for signalling (except by contracting only if the agent chose to signal). However, the current extension — by allowing a continuous response — requires the principal to reflect precisely its assessment of the agent’s quality in its decision. Implicitly, this introduces a coordination aspect, even in separating equilibrium (for example, if $\gamma_q = 0$ in the normal-distribution case, there may be either no separating equilibria, one, or two such equilibria). This was not an aspect of the binary-response case. In that case, multiple equilibria arose only by the possibility of having both pooling equilibria and separating equilibria; here, there is the possibility of multiple separating equilibria.

In turn, this issue depends upon the specific functional form for the distribution from which the agent’s type is drawn. I have showed that, for the standard normal, the sufficient condition depends both upon the principal’s response and the agent’s cost. However, I have conjectured, using numerical results, that this is not a general result: for both the standard Gumbel (maximum) distribution and the logistic distribution, the general sufficient condition appears to depend only upon the principal’s response ($\beta_q$).

Irrespective of the distributional assumption taken, it is possible to characterise in more detail the cut-off function used in a unique separating equilibrium (if, indeed, such a unique equilibrium exists).

**Proposition 16** The cut-off point $Q^*$ is a continuous decreasing function of $I$.

**Proposition 17** The cut-off is bounded above (below) for $\beta_q > 0$ ($< 0$) by the $\beta_q = 0$ solution.
**Proposition 18** There is no separating equilibrium that does not involve a cut-off signalling strategy.

Finally, I consider pooling equilibria.

**Proposition 19** Given the Cho-Kreps Intuitive Criterion, there are no pooling equilibria.

## 5 Conclusion

In this paper, I have considered the problem of binary signalling. I have assumed throughout that the agent (a student) has unobserved type with continuous distribution. For this reason, the agent’s choice of signal never reveals precisely the agent’s type; the paper, therefore, has explored the consequences of information coarseness for equilibrium outcomes in a signalling game.

In doing so, the paper has made several points. First, I showed that even a simple binary signalling game produces an intricate set of potential equilibrium outcomes. Second, I showed that, though mixed-strategy equilibria may exist, they are not practically very important for analysing the game. Third, I considered a special case of risk aversion; I showed that, because of the information coarseness, the principal’s risk aversion is an important determinant of the form of equilibrium (so that, for example, a risk-neutral principal may contract in circumstances where a risk-averse principal may not). Fourth, I considered the role of indices; I argued that, because binary signals may only confer coarse information, it is particularly important in this case to consider the potential for indices to mitigate further the information asymmetry. I showed that the ‘form’ of equilibrium — in the sense of thinking about how the equilibrium changes with an observable index of agent quality — depends crucially upon the cost and benefit
of the signal in terms of the unobservable compares to the cost in terms of the observable index, both for the agent and for the principal. Finally, I considered an alternative model in which the principal could respond by choice of a continuous variable (for example, an employer choosing a wage, rather than merely choosing whether or not to hire). I showed that this extension introduces additional considerations, particularly concerning the specific form of the distribution from which the agent’s type is drawn.

Of course, the considerations outlined here are not limited to binary signals. Binary signalling is merely the most extreme form of what might be termed ‘discrete signalling’: signalling by the use of a variable constrained to a finite set. This paper began with several examples of binary signalling. One could also easily imagine a wide variety of other circumstances in which a signal, though not binary, is nonetheless discrete. For example, in the education context, an employer might care only about which specific qualifications a student has, rather than the precise total amount of time spent in education. In those circumstances, too, a principal would only be able to infer the agent’s type up to a coarse partition. The difference between a continuous signal and a discrete signal is not, to my knowledge, one that has attracted significant attention. This paper suggests that important new insights may lie in further consideration of that distinction.
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References


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Appendix

Proof of Proposition 1. By contradiction. Suppose that Employer were to choose the same action irrespective of S. Then Student choosing S = 1 could profitably deviate by choosing S = 0, since C(Q) > 0. Alternatively, suppose that Employer were to choose R = 1 for S = 0 and R = 0 for S = 1. Then Student choosing S = 1 could profitably deviate to S = 0, since B(Q) > −C(Q).

Proof of Proposition 2. Following Proposition 1, Student chooses S = 1 if and only if

\[ \pi_S(R = 1, S = 1) \geq \pi_S(R = 0, S = 0) \iff B(Q) - C(Q) \geq 0. \]

From the earlier restriction that \( B(Q) - C(Q) \) is strictly increasing in Q, it follows that there exists a unique \( Q^* \), and that \( B(Q) - C(Q) \geq 0 \forall Q \geq Q^* \), and vice versa for \( Q < Q^* \).

Proof of Proposition 3. It has been shown that, under a separating equilibrium, Employer will choose \( R^*(S) = S \). For this to be an optimal response for Employer, it must be that:

\[ \pi_E(R = 1, S = 1) \geq \pi_E(R = 0, S = 1), \]

\[ \pi_E(R = 1, S = 0) < \pi_E(R = 0, S = 0). \]

Employer's expected unconditional payoffs are simply its expectations of conditional payoffs (as outlined earlier) across the range of Q under consideration. Given that Student has a unique best response to adopt a cut-off strategy, Employer correctly infers — in the separating equilibrium case — that \( S = 1 \) implies \( Q \geq Q^* \) while \( S = 0 \) implies \( Q < Q^* \). This follows from Bayes' Rule. If there is a separating equilibrium, Perfect Bayesian Equilibrium requires — as outlined earlier — that Employer form its beliefs consistently with that equilibrium. That is, if there is a separating equilibrium, Employer forms its belief of the distribution of Q by using the observed signal to condition upon whether \( Q \geq Q^* \) or \( Q < Q^* \).

Therefore, in the case of a separating equilibrium,

\[
g(Q | S = 1) = \begin{cases} 
  f(Q) \cdot \left[ \int_{Q^*}^{Q} f(q) \, dq \right]^{-1} = f(Q) \cdot (1 - F(Q^*))^{-1} & \text{if } Q \geq Q^* \\
  0 & \text{if } Q < Q^*;
\end{cases} \]

\[
g(Q | S = 0) = \begin{cases} 
  0 & \text{if } Q \geq Q^* \\
  f(Q) \cdot \left[ \int_{Q}^{Q^*} f(q) \, dq \right]^{-1} = f(Q) \cdot F(Q^*)^{-1} & \text{if } Q < Q^*.
\end{cases}
\]
Therefore, Employer’s payoffs in a separating equilibrium are:

\[
\pi_E(R = 1, S = 1) = \int_{Q^*} (P(q) + H(Q)) \cdot g(Q \mid S = 1) \, dq
\]

\[
= \int_{Q^*} (P(q) + H(Q)) \cdot f(q) \cdot g(Q \mid S = 1) \cdot (1 - F(Q^*))^{-1}
\]

\[
\pi_E(R = 1, S = 0) = \int_{Q} P(q) \cdot g(Q \mid S = 0) \, dq
\]

\[
= \int_{Q} P(q) \cdot f(q) \, dq \cdot F(Q^*)^{-1}
\]

\[
\pi_E(R = 0, S = 1) = 0
\]

\[
\pi_E(R = 0, S = 0) = 0
\]

Therefore, Employer’s incentive compatibility conditions to support a separating equilibrium are:

\[
\int_{Q^*} (P(q) + H(Q)) \cdot f(q) \, dq \geq 0 \quad (14)
\]

\[
\int_{Q} P(q) \cdot f(q) \, dq < 0 \quad (15)
\]

**Proof of Proposition 4.** See the proof of Proposition 1. □

**Proof of Proposition 5.** By contradiction. Proposition 4 showed that Employer chooses \( R^*(S) = S \); thus, if \( B(Q) < C(Q) \) for some \( Q \in [Q, \bar{Q}] \), Student of type \( Q \) may profitably deviate to \( S = 0 \). In that case, there could be no pooling equilibrium on \( S = 1 \).

**Proof of Proposition 6.** In a candidate pooling equilibrium on \( S = 1 \), the observation that \( S = 0 \) allows no restriction on Employer beliefs since, as will be shown, a pooling equilibrium on \( S = 1 \) implies that it is never profitable for Student of any type to deviate. That is, the result following will imply that, for a candidate pooling equilibrium on \( S = 1 \):

\[
g(Q \mid S = 0) = f(Q) \quad (53)
\]

\[
g(Q \mid S = 1) = f(Q). \quad (54)
\]

In order for there to be a pooling equilibrium on \( S = 1 \), Employer must choose \( R = S \) (that is, Employer must not only choose \( R = 1 \) on the equilibrium path, but must choose \( R = 0 \) if Student deviates to \( S = 0 \)). For this to be incentive compatible for Employer, it must be that:

\[
\pi_E(R = 1, S = 1) \geq \pi_E(R = 0, S = 1) \quad (55)
\]

\[
\text{and} \quad \pi_E(R = 1, S = 0) < \pi_E(R = 0, S = 0). \quad (56)
\]

It was shown earlier that there must exist no \( Q^* \in [Q, \bar{Q}] \) such that \( B(Q^*) = C(Q^*) \). Therefore, as foreshadowed, Employer’s incentive compatibility condition to support
a pooling equilibrium is taken over the entire support \( Q \in [Q, \overline{Q}] \):

\[
\int_{Q}^{\overline{Q}} (P(q) + H(Q)) \cdot f(q) \, dq \geq 0
\]

(57)

and

\[
\int_{Q}^{\overline{Q}} P(q) \cdot f(q) \, dq < 0.
\]

(58)

**Proof of Proposition 7.** I start by considering the restriction on Employer beliefs. It was shown earlier that, in a separating equilibrium with \( Q^* \in [Q, \overline{Q}] \), Student chooses \( S = 1 \) only if \( Q \geq Q^* \). Thus, Student with \( Q < Q^* \) never has an incentive to choose \( S = 1 \), whereas Student with \( Q \geq Q^* \) has that incentive if it (wrongly) anticipates a separating equilibrium. Thus, in a candidate pooling equilibrium on \( S = 0 \), the Intuitive Criterion imposes that

\[
g(Q | S = 0) = f(Q) \quad (59)
\]

\[
g(Q | S = 1) = \begin{cases} 
  f(Q) \cdot (1 - F(Q^*))^{-1} & \text{if } Q \geq Q^* \\
  0 & \text{if } Q < Q^*.
\end{cases}
\]

(60)

I must then consider four cases. First, consider the case in which Employer chooses \( R^*(0) = 0; R^*(1) = 1 \). In that case, Student of type \( Q \geq Q^* \) will profitably deviate to \( S = 1 \); there cannot then be a pooling equilibrium on \( S = 0 \). Second, consider the case in which Employer chooses \( R^*(0) = 1; R^*(1) = 0 \). Employer’s incentive compatibility requirement in this case implies that

\[
R^*(0) = 1 \left( \int_{Q}^{\overline{Q}} P(q) \cdot (f) \, dq \geq 0 \right).
\]

(61)

However, given the condition in equation 61, and the restrictions that \( P'(Q) > 0 \), \( H'(Q) \geq 0 \) and \( H(Q) \geq 0 \), it follows that:

\[
\int_{Q^*}^{\overline{Q}} P(q) \cdot f(q) \, dq < 0
\]

(62)

\[
\therefore \int_{Q}^{\overline{Q}} P(q) \cdot f(q) \, dq < 0.
\]

(63)

Therefore, the only optimal response to \( S = 0 \) must be \( R = 0 \); therefore, I can write \( R^*(0) = 0 \), which provides a contradiction to this second case.

Third, consider the case in which \( R^*(S) = 0 \) (that is, irrespective of \( S \)). In that case, there is no incentive for Student of any type to deviate from \( S = 0 \). This is incentive compatible for Employer if and only if it is unprofitable to deviate to \( R = 1 \) for \( S = 1 \):

\[
\int_{Q^*}^{\overline{Q}} (P(q) + H(Q)) \cdot f(q) \, dq < 0.
\]

(64)

This third case is illustrated in Figure 18.
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Figure 18: Employer incentive compatibility for pooling on \( S = 0 \) with some \( B(Q) \geq C(Q) \)

\[
\int_{Q}^{Q^*} (P(q) + H(Q)) \cdot f(q) \, dq < 0
\]

Fourth, consider the case in which \( R^*(S) = 1 \) (again, irrespective of \( S \)). In that case, there is also no incentive for Student of any type to deviate. Employer’s incentive compatibility condition is:

\[
\int_{Q}^{Q^*} P(q) \cdot f(q) \, dq \geq 0. \quad (65)
\]

Figure 19 illustrates this fourth case.

Figure 19: Employer incentive compatibility for pooling on \( S = 0 \) with some \( B(Q) \geq C(Q) \)

\[
\int_{Q}^{Q^*} P(q) \cdot f(q) \, dq \geq 0
\]

**Proof of Proposition 8.** Consider the same four cases again. First, consider \( R^*(0) = 0; R^*(1) = 1 \). It is trivial that \( S^*(Q) = 1 \) in that case, so this is a contradiction. Second, consider \( R^*(0) = 1; R^*(1) = 0 \). In this case, it is straightforward that, if some type of Student (wrongly) chooses \( S = 1 \), the choice will be uninformative of \( Q \); that is, 
\[
g(Q \mid S = 0) = g(Q \mid S = 1) = f(Q).
\]
This allows Employer’s incentive compatibility
condition to be derived. The incentive compatibility conditions in this case are:

\[
\int_Q (P(q) + H(Q)) \cdot f(q) \, dq < 0; \tag{66}
\]

\[
\int_Q P(q) \cdot f(q) \, dq \geq 0. \tag{67}
\]

Given the restriction that \(H(Q) \geq 0\), these last conditions are contradictory. Therefore, I can rule out this second case.

Third, consider \(R^*(S) = 0\) (irrespective of \(S\)). The incentive compatibility condition to support this response is:

\[
\int_Q (P(q) + H(Q)) \cdot f(q) \, dq < 0. \tag{68}
\]

This condition is plausible and is illustrated in Figure 20.

Figure 20: Employer incentive compatibility for pooling on \(S = 0\) with \(B(Q) > C(Q)\) for all \(Q\)

Fourth, consider \(R^*(S) = 1\) (irrespective of \(S\)). The incentive compatibility condition in this case is:

\[
\int_Q P(q) \cdot f(q) \, dq \geq 0. \tag{69}
\]

This condition is plausible, and has already been illustrated in Figure 19 (though note that Figure 19 shows \(Q^*\), whereas such cutoff does not exist in this case, because \(B(Q) > C(Q)\) for all \(Q\).

**Proof of Proposition 9.** Irrespective of \(R\), it will never be profitable for Student to deviate given \(B(Q) < C(Q)\), i.e. \(B(Q) < C(Q) \iff \pi_S(S = 1, R, Q) < \pi_S(S = 0, R, Q)\). This is sufficient for the proof. However, it is also convenient to discuss here Employer’s incentive compatibility requirement for pooling on \(S = 0\) with \(B(Q) < C(Q)\) \(\forall Q \in [Q, Q]\). Proposition 9 implies that there is no binding incentive compatibility requirement for this case, because Proposition 9 holds irrespective of Employer’s response.
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In this case, there is no type of Student for which it pays to choose $S = 1$. Employer’s optimal decision is therefore straightforward:

$$R = 1(\pi_E(R = 1, S = 0) \geq 0) = 1 \left( \int_{Q}^{Q} P(q) \cdot f(q) dq \geq 0 \right). \quad (70)$$

**Proof of Proposition 10.** Since $Q^* (r^*(1) - r^*(0))$ is strictly decreasing in $r^*(1) - r^*(0)$, and both $\mathbb{E}(Q \mid Q \geq Q^*)$ and $\mathbb{E}(Q \mid Q \geq Q^*)$ must be weakly increasing in $Q^*$, then $r^* > r^+$ implies that $\mathbb{E}(Q \mid Q \geq Q^*(r^+)) > Q^+$ and $\mathbb{E}(Q \mid Q \leq Q^*(r^+)) < Q^+$. This implies that there can be at most one $r^* \in [0, 1]$ and at most one $r^+ \in [0, 1]$. Further, note that there may be as many as two separating equilibria in mixed strategies: each of the two posited separating equilibria both meet the two separate criteria in equations 31 and 32, while also maintaining $r^*(1) - r^*(0) = r^+$ and $r^*(1) - r^*(0) = r^*$ respectively. ■

**Proof of Proposition 11.** Posit a separating equilibrium in mixed strategies. For the reasons set out in the preceding proof, $r^+ > r^*$ implies $\mathbb{E}(Q \mid Q \geq Q^*(r^+)) < Q^+$; therefore, by equation 32, $r^*(1) = 0$. Since this rules out $r^*(1) > r^*(0)$, this rules out a separating equilibrium in mixed strategies on $r^+$. Similarly, consider $r^*$. The earlier reasoning implies that $\mathbb{E}(Q \mid Q < Q^*(r^*)) < Q^+$. Equation 31 implies that $r^*(0) = 1$, but this also rules out $r^*(1) > r^*(0)$. ■

**Mixing in pooling equilibrium.** It is trivial that there can be no Student mixing in pooling equilibrium; by definition, a pooling equilibrium in this context is an equilibrium in which Student always chooses the same $S$ irrespective of $Q$, which immediately precludes mixing.

In a pooling equilibrium, Employer can infer nothing further about the conditional distribution of $Q$ from observing $S$. Therefore, Employer’s best response — allowing for mixed strategies — can be written as a modified version of equations 31 and 32:

$$r^*(0) \begin{cases} 
= 1 & \text{if } \mathbb{E}(Q) > Q^+; \\
= 0 & \text{if } \mathbb{E}(Q) < Q^+; \\
\text{may take any value in } [0, 1] & \text{if } \mathbb{E}(Q) = Q^+;
\end{cases} \quad (71)$$

$$r^*(1) \begin{cases} 
= 1 & \text{if } \mathbb{E}(Q) > Q^*; \\
= 0 & \text{if } \mathbb{E}(Q) < Q^*; \\
\text{may take any value in } [0, 1] & \text{if } \mathbb{E}(Q) = Q^*.
\end{cases} \quad (72)$$

Given that (in the pooling case) Employer mixing does not affect Student’s choice of $S$, allowing for mixing does not provide any interesting extension to the pure-strategy case.
Proof of Proposition 12. Expected utility, conditional on the truncation, is:

\[ E(\pi_E(Q) \mid Q \geq z) \]

\[ = - \int_{z}^{\infty} \frac{\pi_E(q) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp \left( -\frac{1}{2\sigma^2} \cdot (q - \mu)^2 \right) dq}{1 - \Phi \left( \frac{z - \mu}{\sigma} \right)} \]

\[ = - \int_{z}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp \left( -\frac{1}{2\sigma^2} \cdot ((q - \mu) + 2\sigma^2r)^2 \right) \cdot \frac{1}{1 - \Phi \left( \frac{z - \mu}{\sigma} \right)} dq \]

\[ = - \exp \left( \frac{1}{2} \cdot \sigma^2r^2 - \mu r \right) \cdot \frac{1}{1 - \Phi \left( \frac{z - \mu + \sigma^2r}{\sigma} \right)} \cdot \ln \left[ 1 - \Phi \left( \frac{z - \mu}{\sigma} \right) \right] \]

As in the main text, I denote the certainty equivalent \( \pi \). Then I can substitute for the utility from the certainty equivalent and, therefore, solve for the form of that equivalent:

\[ E(\pi_E(Q) \mid Q \geq z) \equiv -\exp(-r\pi) \]

\[ = - \exp \left( \frac{1}{2} \cdot \sigma^2r^2 - \mu r \right) \cdot \frac{1}{1 - \Phi \left( \frac{z - \mu + \sigma^2r}{\sigma} \right)} \cdot \ln \left[ 1 - \Phi \left( \frac{z - \mu}{\sigma} \right) \right] \]

\[ \therefore r\pi = -\mu r + \frac{1}{2} \cdot \sigma^2r^2 + \ln \left[ 1 - \Phi \left( \frac{z - \mu + \sigma^2r}{\sigma} \right) \right] \]

\[ \therefore \pi = \mu - \frac{1}{2} \cdot \sigma^2r^2 - \frac{1}{r} \cdot \ln \left[ 1 - \Phi \left( \frac{z - \mu + \sigma^2r}{\sigma} \right) \right] \]

For truncation above, expected utility is:

\[ E(\pi_E(Q) \mid Q < z) \]

\[ = - \int_{-\infty}^{z} \pi_E(q) \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \exp \left( -\frac{1}{2\sigma^2} \cdot (q - \mu)^2 \right) dq \]

The remainder of the derivation is then symmetric to the derivation for the case of truncation below. □

Proof of Proposition 13. This simply requires restating the solution concept conditional upon \( x \), to show that it has the same form as earlier. The modified solution criteria are as follows.

(i) Student’s choice of \( s \) maximises expected Student payoffs given \( Q, r^*(S) \)
and \( x \):

\[ s^*(Q, x) \in \arg \max_s \{ B(Q, x) \cdot [s \cdot r^*(1, x) + (1 - s) \cdot r^*(0, x)] - C(Q, x) \cdot s \}. \]

(ii) Employer’s choice of \( r \) maximises expected Employer payoffs given \( S, G(\cdot) \) and \( x \):

\[ r^*(S, x) \in \arg \max_r \{ r \cdot E_g [P(Q, x) + S \cdot H(Q, x) | S] \}, \]

where \( E_g [P(Q, x) + S \cdot H(Q, x) | S, x] \)

\[ \equiv \int_{Q(x)} \{ P(q, x) + S \cdot H(Q, x) | g(q | S, x) \} \, dq. \]

(iii) [Beliefs in separating equilibrium:] If \( S = 0 \) and \( S = 1 \) are both observed with some positive probability on the equilibrium path, then Employer beliefs are formed according to Bayes’ Rule, including conditioning on \( x \):

\[
g(Q | S = 0, x) \equiv (1 - s(Q, x)) \cdot f(Q, x) \cdot \left\{ \int_{Q(x)} (1 - s(q, x)) \cdot f(q, x) \, dq \right\}^{-1}; \quad (75)
\]

\[
g(Q | S = 1, x) \equiv s(Q, x) \cdot f(Q, x) \cdot \left\{ \int_{Q(x)} s(q, x) \cdot f(q, x) \, dq \right\}^{-1}. \quad (76)
\]

(iv) [Beliefs in pooling equilibrium:] If no variation in \( S \) is observed on the equilibrium path, Employer believes that any deviation by Student reveals Student to be of a type for which deviation would be profitable in a separating equilibrium (the Cho-Kreps Intuitive Criterion: Cho and Kreps (1987)). That is, if only Student of type \( Q \in X \) would, in a separating equilibrium, choose \( S = S^* \), and \( S = S^* \) is observed in a pooling equilibrium on \( S \neq S^* \),

\[
g(Q | S = S^*, x) \equiv 1(Q \in X) \cdot f(Q, x) \cdot \left\{ \int_{Q(x)} 1(g \in X) \cdot f(q, x) \, dq \right\}^{-1}. \quad (77)
\]

If no variation in \( S \) is observed on the equilibrium path and there is no deviation, Employer believes that the distribution of unobservable types is the true distribution:

\[
g(Q | x) \equiv f(Q | x). \quad (78)
\]

To frame the problem in these terms is to solve it; for any given \( x \), the game is equivalent to the game studied earlier (in which, of course, one could reframe all cutoff values as functions of \( x \) — for example, \( Q^*(x) \), \( Q^+(x) \) and \( Q^-(x) \)). One might be concerned that introducing indices may allow Employer to overcome the information asymmetry; this is ruled out by the assumption that \( Q | x \) is non-degenerate for all \( x \).  

**Proof of Proposition 14.** I solve by backward induction. By the assumption of risk-
neutrality, it follows:

$$E(\pi_E(I, Q, S) \mid I, S) = \pi_E(I, E(Q \mid I, S), S)$$

(79)

$$\therefore \arg \max_{R \in \mathbb{R}} E(\pi_E(I, Q, S)) = \beta_c + \beta_i \cdot I + \beta_q \cdot E(Q \mid I, S) + \beta_s \cdot S$$

(80)

$$= R^*(I, S).$$

(81)

Using this form for Employer’s best-response function, I solve for Student’s optimal choice of $S$:

$$S^*(I, Q) = 1 \left( R^*(I, S = 1) - C(I, Q) \geq R^*(I, S = 0) \right)$$

(82)

$$= 1 \left( \beta_q \cdot (E(Q \mid S = 1) - E(Q \mid S = 0)) \right)$$

(83)

$$+ \beta_s - \gamma_c + \gamma_i \cdot I + \gamma_q \cdot Q \geq 0$$

(84)

where $Q^*(I)$ is defined as the value of $Q$ for which, given a particular $I$, Student is indifferent between $S = 0$ and $S = 1$:

$$Q^*(I) = - \left( \frac{\beta_q \cdot E(Q \mid S = 1) - E(Q \mid S = 0)}{\gamma_q} + \frac{\beta_s - \gamma_c + \gamma_i \cdot I}{\gamma_q} \right).$$

(85)

For convenience, I refer to the difference in conditional expectations for some cut-off $Q^*(I)$ by the function $\varphi(\cdot)$:

$$\varphi(Q^*(I)) = E(Q \mid I, S = 1) - E(Q \mid I, S = 0)$$

(86)

$$= E(Q \mid I, Q \geq Q^*(I)) - E(Q \mid I, Q < Q^*(I)).$$

(87)

Using this terminology, I can rewrite the cutoff function in a way that emphasises its self-referential definition:

$$Q^*(I) = - \left( \frac{\beta_q \cdot \varphi(Q^*(I)) + \beta_s - \gamma_c + \gamma_i \cdot I}{\gamma_q} \right).$$

(45)

(88)

**Proof of Proposition 15.** I begin by considering an arbitrary truncation of $Q$.

**Definition 8** Define $\theta(x)$ as the conditional expectation of $Q$ truncated above at $x$:

$$\theta(x) = E(Q \mid Q < x) = \int_{-\infty}^{x} z \cdot f(z) dz \cdot F(x)^{-1}.$$  

(89)

It is well known that, in the case where $Q$ has a standard normal distribution, $\theta(x)$ is an Inverse Mills Ratio:

$$\theta(x) = - \frac{\phi(x)}{\Phi(x)},$$

(90)

where $\phi(\cdot)$ is the pdf and $\Phi(\cdot)$ the cdf of the standard normal.

It is well known (for example, see Lee (1992, pp.113–114) and Patel and Read (1996, p.56)) that:

$$\frac{x}{x^2 + 1} < \frac{1}{\theta(x)} < \frac{1}{x},$$

(91)
so that $\theta(x)/x \to 1$ as $x \to -\infty$. From this is follows that
\begin{align*}
\lim_{x \to -\infty} \theta(x) &= -\infty; \quad (92) \\
\lim_{x \to -\infty} \frac{d\theta(x)}{dx} &= 1. \quad (93)
\end{align*}

Now, given that the expectation of the standard normal is 0, and that the distribution is symmetric about this point, it follows that:
\begin{align*}
\lim_{x \to -\infty} \theta(x) &= 0; \quad (94) \\
\varphi(x) &= -\theta(-x) - \theta(x), \quad (95)
\end{align*}

so that
\begin{align*}
\lim_{x \to -\infty} \varphi(x) &= \infty; \quad (96) \\
\lim_{x \to -\infty} \frac{d\varphi(x)}{dx} &= 1; \quad (97) \\
\lim_{x \to -\infty} \varphi(x) &= \infty; \quad (98) \\
\lim_{x \to -\infty} \frac{d\varphi(x)}{dx} &= -1. \quad (99)
\end{align*}

I turn, then, to the sufficient conditions for a unique solution for any given $I$. For convenience (and without loss of generality) I treat $I$ as fixed and I use $x \equiv Q^*(I)$. Therefore, I am required to prove that there exists one unique solution $x$ to
\[ \beta_q \cdot \varphi(x) + \beta_s - \gamma_c + \gamma_i \cdot I + \gamma_q \cdot x = 0. \]

I consider three mutually exclusive and collectively exhaustive cases.

**First, $\beta_q = 0$.** The result is trivial, given $\gamma_q > 0$: $x = -\left(\beta_s - \gamma_c + \gamma_i \cdot I\right) \cdot \gamma_q^{-1}$.

**Second, $\beta_q > 0$.**

I assume always that $\gamma_q > 0$. I impose the condition that I claim is sufficient for a unique equilibrium: $\left| \frac{\gamma_q}{\beta_q} \right| > 1$. $\beta_q > 0$ implies, then, that $\frac{\gamma_q}{\beta_q} > 1$.

Define, then, that
\[ f(x) = \varphi(x) + \frac{\beta_s - \gamma_c + \gamma_i \cdot I}{\beta_q} + \frac{\gamma_q}{\beta_q} \cdot x. \]

Then the claim amounts to there being a unique solution to $f(x) = 0$.

Now, following the earlier limit results, it follows that:
\begin{align*}
\lim_{x \to \infty} f(x) &= \left(1 + \frac{\gamma_q}{\beta_q}\right) \cdot x + C = \infty \\
\lim_{x \to -\infty} f(x) &= \left(-1 + \frac{\gamma_q}{\beta_q}\right) \cdot x + C = -\infty
\end{align*}

Thus, it is sufficient for a unique solution $x$ to show that $f(x)$ is monotonic $\forall \ x \in \mathbb{R}$.
Chapter II: Binary Signalling

That is, given the limits just derived, I am required to show that:

\[ f'(x) = \varphi'(x) + \frac{\gamma_q}{|\beta_q|} > 0 \quad \forall x \in \mathbb{R} \]

Given the earlier restriction on \( \frac{\gamma_q}{|\beta_q|} \), I will be done if I can show that \( \varphi'(x) \geq -1 \quad \forall x \in \mathbb{R} \).

Given the earlier results on the limits of \( \varphi'(x) \), I will be done if I can show that \( \varphi(x) \) is (weakly) convex. Now, if the Inverse Mills Ratio is a convex function it follows that, for \( a, b \in \mathbb{R}, a \neq b, \lambda \in (0, 1) \):

\[
\begin{align*}
\lambda \cdot \frac{\phi(a)}{\Phi(a)} + (1 - \lambda) \cdot \frac{\phi(b)}{\Phi(b)} &> \frac{\phi(\lambda a + (1 - \lambda)b)}{\Phi(\lambda a + (1 - \lambda)b)} \\
\lambda \cdot \frac{\phi(-a)}{\Phi(-a)} + (1 - \lambda) \cdot \frac{\phi(-b)}{\Phi(-b)} &> \frac{\phi(-\lambda a + (1 - \lambda)b)}{\Phi(-\lambda a + (1 - \lambda)b)} \\
\therefore \lambda \cdot \varphi(a) + (1 - \lambda) \cdot \varphi(b) &> \varphi(\lambda a + (1 - \lambda)b).
\end{align*}
\]

Thus, if I can show that the inverse Mills’ Ratio is convex, I will be done.

It was noted earlier that \( \frac{\phi(x)}{\Phi(x)} > -x \quad \forall x \). Then it follows:

\[
\begin{align*}
&-x \cdot \frac{\phi(x)}{\Phi(x)} < \left( \frac{\phi(x)}{\Phi(x)} \right)^2 \\
\therefore &\quad \frac{d}{dx} \left( \frac{\phi(x)}{\Phi(x)} \right) = -x \cdot \frac{\phi(x)}{\Phi(x)} - \left( \frac{\phi(x)}{\Phi(x)} \right)^2 < 0 \\
\therefore &\quad - \left( 1 + \frac{d}{dx} \left( \frac{\phi(x)}{\Phi(x)} \right) \right) > -1 \\
\therefore &\quad - \frac{\phi(x)}{\Phi(x)} \cdot \left( 1 + \frac{d}{dx} \left( \frac{\phi(x)}{\Phi(x)} \right) \right) > \left[ \frac{d}{dx} \left( \frac{\phi(x)}{\Phi(x)} \right) \right] \cdot \left( x + \frac{\phi(x)}{\Phi(x)} \right) \\
\therefore &\quad \frac{d^2}{dx^2} \left( \frac{\phi(x)}{\Phi(x)} \right) = - \frac{\phi(x)}{\Phi(x)} \cdot \left( 1 + \frac{d}{dx} \left( \frac{\phi(x)}{\Phi(x)} \right) \right) - \left[ \frac{d}{dx} \left( \frac{\phi(x)}{\Phi(x)} \right) \right] \cdot \left( x + \frac{\phi(x)}{\Phi(x)} \right) > 0
\end{align*}
\]

Thus, since the inverse Mills’ Ratio is convex, so is \( \varphi(x) \). There is a unique solution.

Third, \( \beta_q < 0 \).

The result goes through symmetrically, with appropriate sign changes as required.

**Proof of Proposition 16.** I showed earlier that for any given \( I \) there exists a single unique solution. Denote the function mapping from \( I \) into the solution \( x \) as \( x^*(I) \). Recall that \( x^*(I) \) is defined such that

\[ \beta_q \cdot f(x^*(I)) = \beta_q \cdot \varphi(x^*(I)) + \beta_s - \gamma_c + \gamma_i \cdot I + \gamma_q \cdot x^*(I) = 0. \]

Note immediately that \( x^*(I) \) must be continuous, since \( \varphi(\cdot) \) is continuous. Consider again the three cases on \( \beta_q \).
First, for $\beta_q = 0$, it is trivial that
\[ \frac{dx^*(I)}{dI} = -\frac{\gamma_i}{\gamma_q} < 0 \]
For $\beta_q \neq 0$, I can write:
\[ f(x) = \varphi(x) + \frac{\beta_s - \gamma_c + \gamma_i \cdot I}{\beta_q} + \frac{\gamma_q \cdot \gamma_i}{\beta_q} \cdot x \]
\[ \Leftrightarrow \left( \varphi'(x) + \frac{\gamma_q}{\beta_q} \right) \frac{dx^*}{dI} = \frac{\gamma_i}{\beta_q} dI \]
\[ \therefore \frac{dx^*(I)}{dI} = \left( \varphi'(x) \cdot \frac{\beta_q}{\gamma_i} - \frac{\gamma_q}{\gamma_i} \right)^{-1} \]

Second, then, consider $\beta_q > 0$. I maintain that $\gamma_q > 1$, $\gamma_i > 0$, so $\beta_q > 0$ implies
\[ \frac{\gamma_q}{\beta_q} > 1 \]
\[ \therefore \frac{\beta_q}{\gamma_i} < \frac{\gamma_q}{\gamma_i} \]
I showed earlier that $\varphi'(x) < 1$, so
\[ \varphi'(x) \cdot \frac{\beta_q}{\gamma_i} < \frac{\gamma_q}{\gamma_i} \]
\[ \Leftrightarrow \frac{dx^*(I)}{dI} < 0. \]

Third, consider $\beta_2 < 0$. It follows:
\[ \frac{\gamma_q}{\beta_q} < -1. \]
I also showed earlier that $\varphi'(x) > -1$, so
\[ \varphi'(x) > \frac{\gamma_q}{\beta_q} \]
\[ \varphi'(x) \cdot \frac{\beta_q}{\gamma_i} < \frac{\gamma_q}{\gamma_i} \]
\[ \Leftrightarrow \frac{dx^*(I)}{dI} < 0. \]
Thus, given the assumptions made, the solution $x^*(I)$ decreases continuously in $I$. □

Proof of Proposition 17. This is evident simply from observing the earlier figure. For $\beta_q < 0$, the relevant intersection is between the function $\varphi(x)$ and the downward-sloping line (which shows the function $-(\beta_s - \gamma_c + \gamma_i I + \gamma_q x) \cdot \beta_q$). Denote as $y^*$ the intersection of the latter function with the downward-sloping 45-degree line; clearly, that intersection
is:
\[- \left( \frac{\beta_c - \gamma_c + \gamma_i I + \gamma_q y^*}{\beta_q} \right) = -y^* \]
\[\beta_s - \gamma_c + \gamma_i I + \gamma_q y^* = \beta_q y^* \]
\[\therefore y^* = - \left( \frac{\beta_s - \gamma_c + \gamma_i I}{\gamma_q + \beta_q} \right).\]

It is intuitively obvious from the figure that for any shifts in the relevant parameters (and, therefore, changes in \(y^*\)), it remains always the case that \(x^* < y^*\). This has a strong intuitive reasoning: as the information asymmetry becomes more important (that is, \(\beta_q\) increases), the cut-off function becomes more curved downwards from the full-information case; that is, Student is more likely to invest in the signal as the signal’s information value increases.

The reasoning applies symmetrically in the opposite direction for \(\beta_q < 0\). ■

**Proof of Proposition 18.** In such an equilibrium, there must be some \(Q^+ > Q^*\) such that Student of type \((I, Q^+)\) chooses \(S = 0\) but Student of type \((I, Q^*)\) chooses \(S = 1\). But since the signalling cost decreases monotonically in \(Q\) (that is, \(\gamma_q > 0\)), it cannot be optimal for Student of type \((I, Q^+)\) to choose \(S = 0\) unless Student of type \((I, Q^*)\) also chooses \(S = 0\). ■

**Proof of Proposition 19.** In any pooling equilibrium, the difference between \(R^*(I, 1)\) and \(R^*(I, 0)\) must be finite; all of the components of \(R^*(I, 1) - R^*(I, 0)\) and all of the parameters are finite. But \(Q\) is unbounded on the real line; thus, for any finite value \(R^*(I, 1) - R^*(I, 0)\), there is some positive probability that \(Q \geq Q^*(I)\) (and, therefore, \(S^* = 1\)) and some positive probability that \(Q < Q^*(I)\) (and, therefore, \(S^* = 0\)). That is, given the Cho-Kreps Intuitive Criterion, the unboundedness of \(Q\) prevents a pooling equilibrium. ■
Chapter III: Identifying information and incentives under binary signalling

IDENTIFYING INFORMATION AND INCENTIVES UNDER BINARY SIGNALLING: MOROCCAN MANUFACTURING FIRMS’ ACCESS TO BANK CREDIT

SIMON QUINN*

April 25, 2010

Abstract

I develop a new structural methodology for the separate identification of information and incentive effects. The method applies where there is a single variable that may be used for signalling, and where that variable may take one of only two values (‘binary signalling’). I apply the method to analyse the relationship between medium- and large-scale Moroccan manufacturing firms and their banks, using firms’ choice of legal status as the relevant binary signal. I estimate using the two-period FACS-ICA panel, on the subset of firms having an overdraft facility in both periods (approximately two-thirds of the total sample). I find that, among that limited sample, there is no relevant information asymmetry. I estimate the potential welfare loss and conclude that, in the 95% confidence region of potential information effects and incentive effects, the maximum median welfare loss from information asymmetry is equivalent to approximately only 3% of the median bank overdraft limit. For the sample of firms having an overdraft facility in both periods, this challenges the common narrative that information asymmetry is an important problem for bank credit market failure in Morocco.

JEL codes: O12, C72, G21.

*D.Phil student: Department of Economics, Centre for the Study of African Economies (‘CSAE’) and All Souls College, University of Oxford (simon.quinn@economics.ox.ac.uk; http://www.allsouls.ox.ac.uk/people.php?personid=54). This paper forms part of my D.Phil thesis, which is supervised by Professor Marcel Fafchamps; the work would not have been possible without his very generous assistance. Several others have provided very useful comments on aspects of the paper; without implicating them in the shortcomings of the work, I thank Paul Beaudry, Clare Leaver, Kerry Papps and Francis Teal. I have presented the paper at the Labour and Applied Microeconomics Seminar at the University of Oxford and at the 7th EUDN PhD Seminar; I thank the seminar participants.

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Simon Quinn
Chapter III: **Identifying information and incentives under binary signalling**

### 1 Introduction

While financial statements in Morocco generally include most of the information necessary to analyze the financial status of a company, certain lesser disclosure standards as compared to the [International Accounting Standards] reduce the usefulness and transparency of Moroccan financial reports. Moreover, financial information is not readily available, as the mechanisms to publish statements are inefficient.


**HOW IMPORTANT ARE INFORMATION ASYMMETRIES** for credit markets in developing economies? In many developing economies, including Morocco, information asymmetries are a common justification for banks and other lenders not providing more liberal access to credit; the earlier quote is one example of this.\(^1\) There is now a very large theoretical literature showing the potential for information asymmetries to cause substantial impediments to credit market efficiency, particularly in developing and emerging markets (for example, see Stiglitz and Weiss (1981), Bardhan and Udry (1999, Ch. 7) and Banerjee and Newman (1993)). A growing empirical literature also considers information problems, though the empirical identification of information asymmetry has lagged far behind theoretical developments in the area (Chiappori and Salanié (2000, pp.56–57)). As I will discuss shortly, there exists no general method for testing for information asymmetry. Instead, the empirical literature has developed a variety of approaches, each suited to a different context.

In this paper, I develop a new structural methodology to test for information asymmetry. The methodology applies to the limited class of circumstances in which there is a single variable that may be used for signalling, and where that variable may take one of only two values; I term this situation ‘binary signalling’

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\(^1\) In 2006, I travelled to Morocco to speak with members of the business and banking communities; problems of information were often mentioned when discussing reasons for limited credit provision.
(see Quinn (2010a)). I argue that this method is useful for understanding information asymmetries between Moroccan manufacturing firms and their banks. In Morocco, manufacturing firms generally choose to incorporate under one of two legal statuses: ‘SA’ and ‘SARL’. In Quinn (2010b), I show that firms incorporating in the more onerous SA status are more likely to receive bank overdraft facilities than firms incorporating as SARL and, conditional upon receiving a facility, are likely to receive a significantly higher overdraft limit.

This result may be driven by one or both of two mechanisms. First, firms incorporating as SA rather than SARL may receive more generous overdraft facilities because, as SA firms, they are required to abide by more onerous corporate governance standards; this may directly improve their operations and their performance. That is, incorporation in the SA status may have a valuable incentive effect. Second, firms incorporating as SA rather than SARL may, by doing so, be signalling the quality of their management. A firm’s ‘management quality’ is an important aspect of firm performance, but it is not an aspect that a firm may credibly certify to a bank (if asked, for example, every firm would claim to have excellent management). However, if firms with good management find it relatively less onerous to incorporate in the SA status than firms with poor management, incorporating in the SA form may be a credible way to indicate to banks and others that the firm is well-managed. That is, incorporation in the SA status may also have a valuable information effect.

There are several reasons that it is important to understand the extent to which legal status acts through an information effect rather than an incentive effect. First, the distinction is important for understanding the effect of Moroccan company law. In Quinn (2010b), I show that Morocco substantially reformed its company law in 2001, and that this reform failed in its stated objective of improv-
ing lenders’ confidence in firms. Specifically, I show that the reform induced a substantial migration from the SA form to the less onerous SARL form; firms migrating were then punished by banks with a withdrawal of overdraft facilities, whereas firms remaining the SA status did not receive any improvement in overdraft provision. This result can be explained either in terms of legal status having an information effect or having an incentive effect, or both. Suppose that legal status operates only as an incentive effect. This implies that, insofar as firm unobservables determine the decision to incorporate in the SA form, those unobservables are not of interest to banks. Under this interpretation, the legal reform discussed in Quinn (2010b) made the SA status more costly for firms, but did not then substantially improve firm performance; this explains how firms could be punished for shifting to the SARL status but not rewarded for remaining in the SA status.

Alternatively, suppose that legal status operates only as an information effect. This would explain why SA firms receive better overdraft facilities than SARL firms; by choosing SA, firms are signalling a better ‘management quality’, and this is driving banks’ overdraft decision. Further, this would explain the result that firms migrating from SA to SARL status were less likely to receive overdraft facilities than they had been previously; by switching from SA to SARL status, a firm would be revealing that its management quality was not as good as its bank had believed. This interpretation would also imply that banks remaining in the SA status would have an increase in overdraft facilities; by remaining in the status even under the more costly legal regime, those firms would be revealing their management quality to be better than their banks had anticipated. I do not find evidence of such an increase in Quinn (2010b); firms remaining in the SA status are neither significantly more likely nor significantly less likely to have an overdraft facility. This is suggestive that the SA status does not have a strong in-
formation effect; however, as a non-result, this is not itself persuasive. A formal methodology for separately identifying information and incentive effects would be useful for understanding the reasons that Moroccan banks have reacted to the SA/SARL distinction — and its reform — in the way that they have.

Access to credit is an important determinant of firm expansion among Moroccan manufacturing firms, particularly for relatively smaller firms (Fafchamps and Schündeln (2010)). Understanding the extent to which legal status acts through an information effect rather than an incentive effect has more general implications for understanding the operation of the Moroccan credit market — and, therefore, potential impediments to firm expansion. If one is willing to accept that Moroccan manufacturing firms and banks behave rationally, those banks and firms will use legal status as a signalling device if it signals valuable information that banks do not otherwise know. Suppose, for example, that a formal methodology reveals that legal status has an important role as a signalling device; this lends support to claims that information asymmetries are important for the Moroccan credit market. Conversely, if legal status is found to have a role only for its incentive effects, this lends support to claims that Moroccan banks can use a variety of other sources to infer the quality of firms’ management; in effect, such a result would imply that information asymmetries may not be an important feature of the Moroccan credit market.

In this paper, I develop a structural model of firm-bank contracting in which legal status serves as an informative binary signal; the model builds upon the reduced-form results in Quinn (2010b) and the theoretical models in Quinn (2010a). The identification strategy rests upon a single insight: if legal status has an important signalling role, the choice to invest in that signal will be relatively most valuable for the richest, largest firms and for the smallest, poorest firms. That is, the bene-
fit of legal status will be non-monotonic across other firm characteristics, if legal status has a signalling role. The intuition and formal derivation of this result will be presented shortly. Having derived the result formally, I am able to include each bank’s conditional expectation of firm management quality directly as a term in an estimation, in order to test the extent to which banks use such conditional expectations in their assessment of creditworthiness; this amounts, therefore, to a direct test of the importance of information asymmetry. By this method, I am able to estimate separately the information effect and the incentive effect of choosing the SA status rather than SARL.

Contrary to the general narrative — for example, the suggestion in the introductory quote — I find no significant evidence of information asymmetry: I obtain a point estimate of zero on the conditional expectation of firm quality. Further, by performing a welfare analysis using the estimated parameters, I find that potential welfare loss from information asymmetry is small: in the 95% confidence region of possible information and incentive effects, the maximum median welfare loss is only about 3% of the value of the median overdraft limit. This, however, is not a general finding applicable to all Moroccan manufacturing firms; the maximum-likelihood framework that I use requires that I limit estimation to the subset of firms having a positive overdraft in both periods. This paper leaves open the possibility that there is a substantial information asymmetry among those firms which, in either 2000 or 2004 (the years of the available data), had no overdraft facility; estimation on the full sample will require a more complicated model and a more flexible estimation strategy, and is left for further research.

In this way, the paper seeks to make both a methodological contribution and an empirical contribution. Empirically, the paper provides evidence that — at least among manufacturing firms having a positive overdraft in both 2000 and 2004 —
information asymmetry was *not* an important impediment to credit market operation in Morocco. In saying this, the paper does *not* claim that banks find it equally easy to access credible information about firms in Morocco as about firms in the United States or the United Kingdom; it may indeed be substantially more difficult, and banks may suffer substantial additional general operational costs as a result. Rather, the claim is that such information problems do not substantially impede the *efficient operation* of the credit market (at least for firms having an overdraft in both periods of the survey); it implies that banks are able to overcome any inconvenience by drawing reasonable inference from the other information that they do have available. This suggests that further legal reforms to improve information availability — as in the case of the reform analysed in Quinn (2010b) — may not induce banks to increase credit provision.

Methodologically, the paper presents a new approach for the separate identification of information and incentive effects: the paper argues that, if an empirical context is amenable to a formal principal-agent model of contracting under information asymmetry, such a model will yield a formal expression of the principal’s conditional expectation of the relevant unobservable (in this case, for example, the model provides a formal expression of a bank’s conditional expectation of firm management quality). This conditional expectation can then be entered *directly* in the estimation of the principal’s response (in this case, the estimation of a bank’s decision on overdraft limit), in order to test for information asymmetry. The present model extends only to the relatively simple case of a single, binary signal; however, the methodology may be extended to cover more complicated scenarios to which existing reduced-form methods may not apply. This paper also shows that, by considering the information/incentive problem using a structural model, a researcher may obtain an estimate not only of the *significance* of information asymmetry but also of the potential *welfare loss* from that asymmetry.
Chapter III: **Identifying information and incentives under binary signalling**

Estimating the potential welfare loss from information asymmetry is important for understanding how substantial the problem may be and the extent to which policy intervention may be justified. However, of the limited empirical literature seeking to identify information asymmetry, almost none has used a methodology that allows for an estimation of consequent welfare loss (see Einav, Finkelstein, and Schirmpf (forthcoming, p.1)). In this way, the present methodology suggests a new approach for estimating both the significance and the importance of information asymmetry.

### 1.1 Institutional background

Moroccan manufacturing firms generally choose one of two legal statuses: ‘SA’ (*Société Anonyme*, the more onerous status) or ‘SARL’ (*Société À Responsibilité Limitée*). In Quinn (2010b), I undertake a detailed consideration of the differences between these legal forms and outline important recent reforms; Table 1 summarises the key differences.²

<table>
<thead>
<tr>
<th><strong>Table 1: SA and SARL: Key legal differences</strong></th>
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<tr>
<td><strong>Capital</strong></td>
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<tr>
<td>Minimum capital</td>
</tr>
<tr>
<td>Share transferability</td>
</tr>
<tr>
<td>Granting security</td>
</tr>
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<td><strong>Governance</strong></td>
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<tr>
<td>Governance structure</td>
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<tr>
<td><strong>Transparency</strong></td>
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<tr>
<td>Appointment of auditors</td>
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In short, the legal obligations of the *SA* form are designed to ensure substantially

² In Quinn (2010b), I also explain the substantial reform to the *SA* form that occurred between the two rounds of the panel survey used here, though that reform is not a focus of this paper.
higher standards of corporate governance, both as a means of improving firm performance and for reassuring third parties (including banks). However, the SA form is also more costly in terms of the obligations that it imposes; as explained, this higher cost means that choosing the SA form rather than SARL may be a mechanism for signalling high management quality.

1.2 Related literature

Neither this paper nor the literature preceding it suggest a universal methodology for empirically distinguishing information and incentive effects; instead, different methods must be used for different circumstances. In some contexts, experimental and quasi-experimental methods have been used. Thus, for example, both Ausubel (1999) and Karlan and Zinman (2009) report the results of randomised experiments in credit markets; the identification strategy relies upon the different times at which incentive and information effects operate (information effects operate at the time of contract, incentive effects operate post-contract). Similarly, Luoto, McIntosh, and Wydick (2007) identify information asymmetries in Guatemalan credit markets through a before/after comparison of the introduction of a new credit information system.

Other research compares terms of trade — whether by price or by quality — of different classes of contracts, where different contractual classes are inherently more or less susceptible to information problems. For example, Greenwald and Glasspiegel (1983) compare relative prices for slaves from different regions of origin (see also Pritchett and Chamberlain (1993) and Chezum and Wimmer (1997)). Dionne and St-Michel (1991) compare the effects of an increase in insurance coverage on claims for ‘back-related’ injuries (which are difficult to diagnose) and other injuries (which are easier to diagnose); similarly, Anagol (2009) compares trade in milking cows (whose fertility can readily be tested) and dry
cows (whose fertility cannot) in order to infer adverse selection. Related papers rely upon the presence of what I will term ‘focal events’: events which occur after the time of contract and whose outcome is central to the original contract. Such events can be used to infer unobserved ‘quality’, which can be compared across relevant contractual classes. For example, Bond (1982) uses a truck’s requirement of maintenance as a ‘focal event’ to compare the markets for new and used trucks (see also Genesove (1993) and Sultan (2008)). Similarly, Chiappori and Salanié (2000) use automobile accidents in analysing the market for automobile insurance, Edelberg (2004) uses late loan payment in a market for consumer lending and Finkelstein and Poterba (2004) use mortality in the market for annuities (see also Finkelstein and Poterba (2006)).

The existing methods are not suitable for the present context. First, there is no obvious information distinction here between different classes of firms, or different classes of firm-bank contracts. The most obvious difference, of course, would be the binary signal itself; that is, the distinction between firms incorporated as *SA* and those incorporated as *SARL*. However, as the introductory discussion notes, differences in outcome between *SA* firms and *SARL* firms may be explained either by information effects and/or by incentive effects. Second, there is no ‘focal event’ in the present context.

I therefore develop a new methodology to test for information and incentive effects. I will shortly develop a simple example and a full structural model to do this. I first summarise the data that I will use for estimation; this motivates the development of the model and explains the sample restriction necessary for estimation.
Chapter III: Identifying information and incentives under binary signalling

2 Data

The data is explained and summarised more comprehensively in Quinn (2010b). For now, I note merely that I have a balanced panel of 488 Moroccan manufacturing firms (each having at least 10 permanent employees), drawn from the six major regions and seven major manufacturing sectors, observed in 2000 (the ‘FACS’ survey) and 2004 (the ‘ICA’ survey). The panel is representative, therefore, of the population of Moroccan manufacturing firms (in the selected regions and sectors) which, in 2000, had at least 10 permanent employees and had either SA or SARL status.3

For reasons explained in Quinn (2010b), I treat a firm’s overdraft limit as the most appropriate available measure of a bank’s assessment of the firm (in short, because — unlike other measures, such as bank lending — it is more reasonable to treat firm demand for an overdraft limit as inelastic). Figure 1 shows a scatter-plot of overdraft limits in each period (with 45-degree line).

I move shortly to develop a signalling model of firm-bank interaction. That model is essentially agnostic about the variables used to fit it; it is framed in terms of ‘observable firm quality’, without requiring any particular variables to constitute that quality. In due course, I will estimate under a variety of specifications drawing upon the primary measures of firm characteristics available. These are the same variables used as controls in Quinn (2010b); they are summarised here in the Data Appendix.

Because I choose to use a maximum likelihood estimator, I will be required to as-

3 I run all of the estimation without weights. In Quinn (2010b), I show that results on this panel are robust to whether or not weights are used. The primary reason for not using weights in this paper is that the analysis here relies upon a maximum likelihood estimator. If weights are used, this estimator can only be interpreted as maximising a ‘log-pseudolikelihood’, which is then not amenable to the standard likelihood ratio tests.
assume a distribution for the unobservable component of the overdraft limit. I will adopt a simple approach and assume that this distribution is normal; however, this assumption will be testable (I will use a standard skewness-kurtosis test on the estimation residuals). In practice, this requires that I cannot use my model to evaluate both the decision of whether an overdraft is provided and the conditional decision as to how much positive overdraft is provided.  

Therefore, I am required to restrict attention to those firms having a positive overdraft limit in both periods; further I am required to drop an additional seven outlier firms. Together, these limitations are necessary in order for the residuals to have an approximately normal distribution. This leaves a balanced panel of 311 firms, which I refer to as the ‘limited sample’.

Unfortunately, it is not uncommon for structural models of information asymme-

---

4 For example, if I specify the dependent variable using the common transformation $f(x) = \ln(x + 1)$, the resulting residual distribution is not remotely close to a normal, and has no reasonable prospect of passing the normality test.
try to be limited to estimation on a smaller subset for the purposes of reasonably fitting the model, and such restrictions necessarily imply a loss of generality. For example, Cardon and Hendel (2001) estimate a structural model of information asymmetry in the market for health insurance, and are required to limit consideration to single respondents of working age who are employed. The authors say (p.418):

As a consequence, our findings on the presence of adverse selection are restricted to the population studied. Restricting the sample to a subset of individuals involves the risk of making the results nonrepresentative of the population as a whole, and it has the drawback that interesting questions about household behavior are left unanswered.

The same can be said here: the resulting estimates are valid for the population of Moroccan manufacturing firms (in the selected regions and sectors) which, in 2000, had at least 10 permanent employees and had either SA or SARL status — and which had a positive overdraft limit in both 2000 and 2004. It remains valuable to know whether asymmetric information is important for that population, particularly if one is seeking general insight into the operation of credit markets in emerging economies. However, the results cannot be interpreted, in any specific sense, as valid for the general population of Moroccan manufacturing firms.5

Figure 2 shows a scatterplot of overdraft limits in each period for the limited sample. The Data Appendix provides a further summary of the overdraft and the explanatory variables, both for the full sample and for the limited sample.

5 This problem might be overcome by using a substantially more intricate model, in which banks are allowed to decide separately (i) whether to provide and overdraft and (ii) if so, how large the overdraft limit should be. This model would likely be much more difficult to solve and to estimate; it may be necessary, for example, to abandon the closed-form framework in favour of a simulation-based estimator. This is left for future research.
3 Motivating example

Consider a simple example. Suppose that a firm has three characteristics: (i) a firm size (measured in terms of asset ownership), (ii) a given quality of management and (iii) a legal status (either *SA* or *SARL*). Suppose that the firm is matched with a bank, and that the bank can observe only the firm’s size and its choice of legal status; the bank cannot observe the firm’s management quality. (For example, this may be because the firm provides audited accounts showing firm assets and legal status, but cannot credibly reveal its management quality; if asked, the firm would claim — along with every other firm — that the quality of its management is outstanding.) For simplicity, assume that the firm cannot choose its firm size (this is equivalent to assuming that the firm finds it too costly to change its asset holding simply to impress its bank). However, assume that the firm can choose its legal status, and suppose that the relative cost of choosing *SA* rather than *SARL* is decreasing in both firm size and firm management quality. That is, assume that larger firms find it relatively less costly to choose the
SA status than do smaller firms (for example, because larger firms may find the governance structure imposed by the SA form to be relatively more useful for managing a larger pool of assets); similarly, assume that better-managed firms find it relatively less costly to choose SA than worse-managed firms (for example, because better-managed firms find it relatively less onerous to have the more intricate ‘dualist’ management structure, and because better-managed firms find it relatively less onerous to have a permanent auditor).

Finally, assume that the bank assigns an overdraft limit to the firm. Suppose that, if the bank could observe management quality directly, it would assign the overdraft limit using a linear function of firm size, legal status and management quality:

\[
\log(\text{overdraft limit}) = \xi_0 + \xi_1 \cdot \text{firm size} + \xi_2 \cdot \text{dummy: } \text{SA status} + \xi_3 \cdot \text{management quality}. \tag{1}
\]

Assume that the bank prefers to assign a larger overdraft limit to firms that are larger than firms that are smaller — for example, because larger firms may be more likely to be able to repay an overdraft; that is, \(\xi_1 > 0\). Assume that, if the bank cares directly about legal status, it prefers to assign a larger overdraft limit to firms that are SA than to firms that are SARL; that is \(\xi_2 \geq 0\). (For example, the bank may prefer SA firms because it prefers the more onerous management structure or the stricter auditing requirements that the SA form imposes.) Finally, assume that if the bank could observe management quality, it would prefer to assign a larger overdraft limit to firms that are better-managed than firms that are worse-managed (for example, because better-managed firms may be more likely to be able to repay); \(\xi_3 \geq 0\).
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The bank cannot observe management quality. However, the bank can form a conditional expectation of management quality on the basis of observing firm size and legal status. Assume, then, that the bank instead assigns overdraft limit by the following function:

$$\log(\text{overdraft limit}) = \xi_0 + \xi_1 \cdot \text{firm size} + \xi_2 \cdot \text{dummy: SA status}$$

$$+ \xi_3 \cdot \mathbb{E}(\text{management quality} | \text{firm size, dummy: SA status}).$$

(2)

That is, the bank now draws a conditional expectation of management quality on the basis of observed firm size and legal status. Suppose that the survey data included a measure of the bank’s conditional expectation of firm management quality (that is, suppose that I could somehow observe \( \mathbb{E}(\cdot) \)); in that case, I could regress overdraft limit on firm size, the legal status dummy and the conditional expectation term (that is, I could add an ‘error term’ to equation 2 and use the equation for estimation). I could then interpret \( \hat{\xi}_2 \) as an estimate of the effect of the direct effect of legal status; that is, a measure of the incentive effect of legal status. I could interpret \( \hat{\xi}_3 \) as an estimate of the importance of management quality. Since the bank cannot observe management quality, I could also interpret \( \hat{\xi}_3 \) as an estimate of the importance of asymmetric information — and, therefore, as an estimate of the signalling effect of legal status and firm size.

I cannot observe the conditional expectation. The rest of the paper is concerned with showing that a relationship similar to that of equation 2 can be specified and estimated by a structural model; in effect, the rest of the paper is devoted to deriving a form for the conditional expectation and showing how that term may be incorporated into an estimation. However, before using a structural model to

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6 Equation 1 implies that the structure in equation 2 will be optimal if the bank is risk-neutral. However, I leave illustration of this point to the formal derivation of the model.
derive a specific form for the conditional expectation, I consider a simple test suggested by equation 2.

### 3.1 A simple test of the motivating example

Consider a relationship of the form specified in equation 2, and consider several potential cases on $\xi_2$ and $\xi_3$.

**Case 1: The bank cares neither about legal status nor about management quality: $\xi_2 = \xi_3 = 0$.** In this case, equation 2 implies that there will be no relationship between legal status and overdraft limit. First, $\xi_2 = 0$ implies that legal status has no *direct* effect upon overdraft limit. Second, $\xi_3 = 0$ implies that legal status cannot even affect overdraft limit by changing the bank’s conditional expectation of management quality.

Equation 2 implies that, for a given firm size, the difference in $\log(\text{overdraft limit})$ between an *SA* firm and an *SARL* firm is:

\[
\text{legal status effect} = \xi_2 + \xi_3 \cdot [\mathbb{E}(\text{management quality} \mid \text{firm size}, \text{SA status}) - \mathbb{E}(\text{management quality} \mid \text{firm size}, \text{SARL status})].
\]

(3)

Legal status may, therefore, have a positive effect by either of two mechanisms: by its direct *incentive effect* (through $\xi_2 > 0$) and/or by its indirect *information effect* (through $\xi_3 > 0$).

Therefore, this case provides a testable implication; if legal status is significant in explaining overdraft limits, it cannot be that $\xi_2 = \xi_3 = 0$. In Quinn (2010b), I show that legal status is significant in explaining overdraft limits (a result that holds when controlling for a wide range of firm characteristics, including firm
fixed-effects). Therefore, I can reject this case immediately; I know that legal status has some effect upon overdraft limits, though the effect could either be a pure incentive effect ($\xi_2 > 0, \xi_3 = 0$), a pure information effect ($\xi_2 = 0, \xi_3 > 0$) or both ($\xi_2 > 0, \xi_3 > 0$).

Case 2: The bank does not care about legal status as such: $\xi_2 = 0$. However, it does care about management quality: $\xi_3 > 0$. This case captures the key insight of the paper and provides the basis for the subsequent structural model. Assume that management quality is continuously distributed on the real line according to some distribution whose density falls as one moves away from the median — for example, assume a normal distribution (though the distribution could just as well be a $t$-distribution, a LaPlace distribution, etc).

Consider a very small firm. I assumed earlier that the $SA$ status is relatively more costly for smaller firms than larger firms. The data supports this assumption; for example, Figure 3 shows the predicted probability of choosing $SA$ rather than $SARL$ by total firm size in the initial period (with actual choice of legal status imposed as a scatter plot).\footnote{The predicted probability is obtained from a probit of legal status on the log of total assets — where total assets is defined as the sum of the value of machines and equipment and the value of land and buildings — for the initial period and the limited sample. The coefficient on $\log(\text{total assets})$ is 0.311 and is significant at the 99.9% confidence level; estimation results are available on request.}

Therefore, if a bank could observe only that a firm was very small, it would anticipate that the firm would choose the $SARL$ status. A very small firm that chose the $SARL$ status would not surprise the bank with its choice; both the simple model and the data show that most small firms choose the $SARL$ status. However, a very small firm that chose the $SA$ status would surprise the bank: assuming rational behaviour, this this choice could only be explained by the firm having outstanding management quality. Therefore, if the bank cares about man-
management quality, it will substantially reward a very small firm choosing the SA status.

Conversely, consider a very large firm. By symmetric reasoning, a bank would anticipate — if it could observe only firm size — that the firm would choose the SA status. A very large firm choosing SA would not surprise the bank; however, a very large firm choosing the SARL status would surprise the bank: this choice could only be explained by the firm having very poor management quality. Therefore, if the bank cares about management quality, it will substantially punish a very large firm choosing the SARL status.

In both the case of the very small firm and the case of the very large firm there is a substantial difference between the overdraft limit that would be provided if the firm chose the SA status in comparison to the case if the firm chose the SARL status.
Consider, however, the intermediate case: a ‘medium’-sized firm. In that case, as Figure 3 shows, firm size alone would not lead bank to anticipate either incorporation in the \( SA \) status or incorporation in the \( SARL \) status; if bank could observe only firm size, it would anticipate an approximately equal probability of either status. If the firm were to choose the \( SA \) status, this would indicate above-median management quality — but this would not be an unlikely choice of status given the firm size. Similarly, if the firm were to choose the \( SARL \) status, this would indicate below-median management quality — but this choice, too, would not be unlikely given the firm size. Therefore, for a ‘medium’-sized firm, there would be a relatively small difference in overdraft limit between \( SA \) and \( SARL \) firms, compared to the difference for firms at either end of the distribution. Figure 4 summarises.

**Figure 4: Expected and unexpected choice of legal status**

<table>
<thead>
<tr>
<th>Firm size:</th>
<th>SMALL</th>
<th>MEDIUM</th>
<th>LARGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected legal status:</td>
<td>( SARL )</td>
<td>( • )</td>
<td>( SA )</td>
</tr>
<tr>
<td>Response to observing ( SA ):</td>
<td>SURPRISED</td>
<td>( • )</td>
<td>UNSURPRISED</td>
</tr>
<tr>
<td>Response to observing ( SARL ):</td>
<td>UNSURPRISED</td>
<td>( • )</td>
<td>SURPRISED</td>
</tr>
<tr>
<td>Difference in response:</td>
<td>LARGE</td>
<td>SMALL</td>
<td>LARGE</td>
</tr>
</tbody>
</table>

**Case 3: The bank does not care about management quality: \( \xi_3 = 0 \).**

**However, it does care about legal status: \( \xi_2 > 0 \).** The previous case illustrated that the bank’s relative surprise at observing legal status varies across the distribution of firm size. This is driven by the differential cost of firm signalling. Therefore, the preceding reasoning about bank surprise holds even if the bank does not care about management quality (\( \xi_3 = 0 \)). However, if the bank does not care about management quality, the differential surprise will not produce a differential response across the distribution of firm size. Formally, equation 3
implies that, if bank does not care about management quality, the difference between overdraft limit for \( SA \) and \( SARL \) firms will be simply \( \xi_2 \) — and this will therefore be constant across the range of firm size.\(^8\)

**Case 4: The bank cares both about legal status and management quality:** \( \xi_2 > 0; \xi_3 > 0 \). This case combines the intuition from Case 2 and Case 3. If the bank cares both about legal status and management quality, its overdraft decisions will have the non-monotonic shape summarised in Figure 4, for the reasons set out in Case 2. However, if bank also cares directly about legal status, the ‘legal status effect’ will be greater still. This is shown by equation 3: \( \xi_3 > 0 \) implies the non-monotonic relationship discussed in Case 2, and \( \xi_2 > 0 \) further increases the legal status effect.

**A simple test.** This simple example illustrates an important aspect of signalling that, to my knowledge, has not previously been noted: in the case of a binary signal, the ‘information value’ of the signal (in this case, legal status) will be strongest for extreme values of an accompanying ‘index’ variable (in this case, firm size). This immediately suggests a simple test in the present panel context: look at firms switching from \( SA \) to \( SARL \) status and compare their change in overdraft limit to their initial firm assets.\(^9\) If banks care about unobservable ‘management quality’ — that is, if there is a relevant information asymmetry — one

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\(^8\) A more complicated model than that of equation 2 may allow the effect of legal status to vary monotonically with firm size (for example, by an interaction term); for example, one might imagine that the direct benefit of the \( SA \) status is increasing in firm size. However, it is difficult to imagine that the direct effect of legal status is non-monotonic across firm size.

\(^9\) Of course, to limit consideration to firms switching status — in effect, to take equation 3 in differences, rather than levels — is not an innocuous extension. Of the 311 firms in the limited sample, 70 switched from \( SA \) to \( SARL \) between rounds of the survey. I discuss the cause for this migration in Quinn (2010b) (namely, the introduction of a new company law regime); as I note in that paper, there may be a number of reasons that firms switching status are not generally representative of the entire sample. Instead of taking equation 3 in differences, I could use a more complicated model to estimate in levels separately for \( SA \) firms and for \( SARL \) — for example, by fitting a kernel regression separately for \( SA \) firms and for \( SARL \), then taking the difference. However, the purpose of the ‘simple test’ outlined here is to motivate and explain the subsequent structural model; for this reason, I choose the simpler approach of using differences.
should expect legal status to have a non-monotonic effect across the range of firm size. Specifically, I would expect that, for firms switching from SA to SARL, the consequent reduction in overdraft limit would be largest for the firms that, in the initial period, were the wealthiest and were the poorest; I would expect firms in the middle of the distribution to have the smallest decline in overdraft limit.

At this stage, I have not specified a distribution for management quality, except to assume that it is continuously distributed such that probability density falls as one moves away from the median. Without assuming any particular distribution for management quality, it is not possible to solve for the form of the conditional expectation term; I therefore cannot enter that term directly in any estimation. However, I can perform a non-parametric estimation to test whether the legal status effect varies non-monotonically across the distribution of firm size. The preceding reasoning shows that I can interpret a non-monotonic relationship between change in log(overdraft limit) and initial firm size as evidence of legal status having an information effect.

Figure 5 shows a kernel regression of change in log(overdraft limit) against total assets in the initial period, for firms switching from SA to SARL. The kernel is indeed non-monotonic; it shows that the greatest reduction in log(overdraft limit) was for firms that were, in the initial period, at either end of the size distribution. However, the test ultimately is inconclusive; the 95% confidence intervals (also shown in Figure 5) do not rule out that the effect is constant across size distribution.

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10 It should now be clear why this assumption is necessary. If management quality were, for example, to have a uniform distribution on some interval, the final term of equation 3 (that is, \(E(\text{management quality} \mid \text{firm size, SA status}) - E(\text{management quality} \mid \text{firm size, SARL status})\)) would be constant across the distribution of firm size. The non-monoticity that I have just outlined would be lost, and there would be no prospect of separately identifying \(\xi_2\) and \(\xi_3\).
Figure 5: **Kernel regression: change in overdraft against initial firm size**

The figure shows the point estimate and 95% confidence intervals for an Epanechnikov kernel with a bandwidth of 1.5 and trimming of 0.05.

**The need for a more formal approach.** I interpret the estimation in Figure 5 as suggestive of information asymmetry, but inconclusive. There are several fundamental weaknesses with the simple approach.

- *The approach has low power.* Because the simple approach is agnostic as to the distribution of management quality — and, therefore, the form of the conditional expectation — it provides a very inefficient way of testing for information asymmetry.

- *The approach does not control for other factors.* A number of relevant factors may be driving the behaviour in Figure 5. This simple example has used total firm assets as the only ‘index’ of observable firm quality; however, Moroccan banks presumably use a much wider range of firm characteristics in their assessments. I would like to control both for other observable factors and for unobservable firm fixed effects.
• The approach tests for information effects, but does not separately identify incentive effects. The simple example showed that an information effect ($\xi_3 > 0$) implies a non-monotonic relationship between firm size and $\log(\text{overdraft limit})$; however, this relationship was predicted both by Case 2 (in which legal status had no direct effect: $\xi_2 = 0$) and by Case 4 (in which legal status had both an information effect and a direct incentive effect: $\xi_2 > 0$, $\xi_3 > 0$). Therefore, this approach can test whether $\xi_3 = 0$ (the information effect), but cannot test whether $\xi_2 = 0$ (the direct effect).

• The approach says nothing about bank or firm welfare. The simple approach was motivated by the linear structure in equation 1. However, the approach says nothing about what drives this structure. Further, the simple example relies upon legal status operating as a signal; however, it does not consider (let alone test) the circumstances under which that signal will be informative. (For example, suppose that the signal is extremely cheap for firms to acquire; in that case, relying upon legal status as a signal of management quality would be as useless to banks as simply asking firms about their management quality directly.) For these reasons, even if the simple approach did provide conclusions about whether the estimated information asymmetry loss were significant, this would say nothing about whether or not it is substantial. That is, the simple approach provides no way of estimating the welfare loss from information asymmetry.

For these reasons, I argue that the present problem requires a structural model. In the rest of the paper, I build and estimate such a model. In the present context, a structural approach has the following advantages over the simple non-parametric approach just considered.

• The structural model retains the key insight from the simple model, but it estimates efficiently. The simple example showed that information asym-
metry could be identified by seeking a non-monotonic relationship across the distribution of observable firm quality; the structural model identifies asymmetry in exactly that way. However, it does so while allowing for a vector of observables and while controlling for firm fixed-effects. I then estimate efficiently by using maximum likelihood.

- The structural model allows for separate identification of information and incentive effects. The structural model makes a specific assumption about the distribution of management quality — and, therefore, about the functional form of the conditional expectation. This allows for direct estimation of both information and incentive effects.

- The structural model is grounded in assumptions about bank and firm welfare. This allows a formal consideration of firms’ and banks’ incentives to use legal status as an informative signal (indeed, it allows a formal test of whether legal status is sufficiently costly to justify its use as an informative signal). This also allows estimation of potential welfare loss from information asymmetry: the structural approach estimates not only whether information asymmetry is significant but also whether it is substantial.

4 A model of legal status as signal

4.1 Specifying the model

In this section, I develop a formal model in which a firm’s binary choice of legal status may act as an informative signal for obtaining bank finances. The model is motivated by the simple example just considered.

Players. There are three players: Nature, Firm and Bank.
**Actions.** Firm is endowed with some fixed characteristic $Q_o \in \mathbb{R}$, observable to all players. This is equivalent to firm size in the simple example, though it is treated here as an ‘index of observable firm quality’; in due course, I will allow it to be a linear combination of a variety of different observable characteristics. Nature chooses a characteristic $Q_u$ according to a conditional standard normal distribution:

$$Q_u | Q_o \sim N(0, 1).$$

(4)

$Q_u$ can be understood as ‘management quality’ (as in the simple example); however, it can also be understood as an index of all relevant firm characteristics that are unobservable to the bank (for example, it could include the ‘ability’ of a firm’s employees). More specifically, $Q_u$ should be understood as the component of firm unobservables not linearly predicted by $Q_o$; for this reason, the conditional expectation of $Q_u$ is zero for all values of $Q_o$.

Nature reveals $Q_u$ to Firm but not to Bank. For convenience, I may then refer to Firm characteristics by the double $Q \equiv (Q_o, Q_u)$, where the subscripts may be taken to stand for ‘observable’ and ‘unobservable’ respectively.

Firm then chooses whether or not to ‘invest in the signal’: to register as $SA$ ($S = 1$) rather than as $SARL$ ($S = 0$):

$$S \in \{0, 1\}.$$  

(5)

Note that this is implied, for example, by the case that $Q_u \sim N(0, 1)$ and $Q_u$ and $Q_o$ are independent, and in the case that $\begin{pmatrix} Q_o \\ Q_u \end{pmatrix} \sim N \left( \begin{pmatrix} \sigma^2_o \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right)$. 

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Chapter III: Identifying information and incentives under binary signalling

After observing $S$ (and $Q_o$), Bank chooses some overdraft limit:\textsuperscript{12}

$$L \in \mathbb{R}.$$ \hfill (6)

Figure 6 summarises the structure described.

**Figure 6: The signalling game in extensive form**

Information. The vector of relevant econometric parameters — to be outlined shortly — is generally referred to throughout as $\theta$. The time dummy is referred to as $t$. Other variables and parameters (in particular, $\epsilon$ and $\rho$) will also be outlined in due course.\textsuperscript{13} With appropriate subscripts, the three players’ information sets

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\textsuperscript{12} Of course, no firm in reality receives a negative overdraft limit. However, I allow $L$ to lie anywhere on the real line for the purposes of model simplicity. By restricting consideration to firms with positive overdraft limits, I avoid having to model a corner solution; were I to specify a more limited range for $L$ (for example, the positive real line), I would need to deal in the model with the issue of corner solutions. I will explain shortly how a distributional assumption requires me to restrict estimation to the ‘limited sample’ of firms which have a positive overdraft in both periods (as foreshadowed earlier). Figure 2 shows that, among that sample, no firm is even close to having a zero overdraft (the minimum overdraft limit lies almost seven standard deviations above zero); in practical terms, I therefore lose nothing by allowing the limit to lie anywhere on the real line.

\textsuperscript{13} In brief, $\epsilon$ will represent Bank’s idiosyncratic perception (‘perception error’) in assigning the overdraft limit; $\rho$ will represent the population correlation between $Q_u$ in each of two periods of a repeated game.
are, therefore,

\[ \Omega_N = (Q_o, \rho); \]  
\[ \Omega_F = (\theta, t, Q_o, S, Q_u); \]  
\[ \Omega_B = (\theta, t, Q_o, S, \epsilon). \]

The structure of the information sets is common knowledge (which, of course, implies mutual knowledge), as is the distribution of \( Q_u \).

**Payoffs.** Bank may choose to provide an overdraft to Firm in order to facilitate some project, in whose profits Bank may then share. The overdraft limit is understood to enter both Bank and Firm utility respectively by a log transformation. This is essentially a pragmatic assumption, made in order to fit the data more closely; as will become clear, it accommodates the stylised fact that the overdraft limit, where positive, has an approximately log-normal distribution. I therefore let the expected return of the facilitated project, in utility terms, be an increasing concave function of the log overdraft limit. Specifically, I use a double-log specification; this has the advantage that, in due course, it will allow \( \log(\text{overdraft limit}) \) to be explained by a linear combination of other factors (analogous to equations 1 and 2 in the simple example). Therefore, I specify:

\[ \pi_P \equiv A(Q, t, \epsilon) \cdot \ln[\ln(L)]. \]  

\( \pi_P \) is therefore understood to represent the expected economic profit of the project, including expected repayment of the overdraft; the expectation is understood to incorporate the risk that the project may perform so poorly that Bank cannot recoup its overdraft. I assume that \( \pi_P \) is wholly captured by Bank. This captures the intuition that Moroccan banks are more likely to earn economic profit than manufacturing firms and the intuition that, ultimately, it is the bank that must
carry the risk of overdraft default. More fundamentally, though, this is a pragmatic assumption that allows me to abstract away from considerations as to how Bank and Firm might bargain to divide the economic profit; bargaining would obviously introduce additional complexity without obvious additional value for estimation. With that assumption, the utility of Bank becomes the expected return from the project less the ‘utility value’ of the overdraft limit (that is, the value of the overdraft limit in log transformation):

\[ \pi_B \equiv \pi_P - \ln(L). \] (11)

Similarly, I assume that the utility of Firm is the utility value of the overdraft limit (again, \( \ln(L) \)) less the cost of investing in the legal status:

\[ \pi_F \equiv \ln(L) - C(Q, t) \cdot S. \] (12)

I consider the different components of these utility functions in turn.

**Project ‘technology’,** \( A(Q, t, \epsilon) \). The technology is assumed to comprise three components: a component related to Firm characteristics, \( Q \), a component unrelated to those characteristics, \( \epsilon \), and a time effect. As noted, I assume that \( \epsilon \) is unknowable to Firm. Further, I assume that \( \epsilon \) is independent of all other variables, such that \( \mathbb{E}(\epsilon) = \mathbb{E}(\epsilon | \Omega_F) = 0.14 \) In effect, \( \epsilon \) may be interpreted as Bank’s idiosyncratic perception, even a ‘perception error’. Alternatively, \( \epsilon \) may be interpreted as an idiosyncratic shock occurring after Firm’s choice of signal that affects the payoff from the project (and, therefore, Bank’s response). Because I assume that the realisation of \( \epsilon \) is independent of Firm’s choice of signal, \( \epsilon \) will not be relevant to Firm’s choice of signal; this will emerge shortly as a

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14 When I turn to the econometric implementation, I will relax this assumption to allow for firm fixed-effects. I discuss this issue in much more detail in Section 5.2.
property of the solution (see equation 25, forthcoming).

It is convenient to impose that project technology is a linear combination of relevant variables; in due course, this will imply a linear relationship between log overdraft limit and relevant variables. I therefore impose:

$$A(Q, t, \epsilon) \equiv \beta_c + \beta_o Q_o + \beta_u Q_u + \beta_s S + \beta_t t + \epsilon.$$ (13)

**The cost of signalling.** I specify the cost of signalling, $C(Q)$, as linear and decreasing separately in $Q_o$ and $Q_u$. To simplify subsequent notation, I allow the cost to relate both to a constant term (normalised to unity) and to a time effect ($\gamma_t$):

$$C(Q) \equiv 1 + \gamma_t t - \gamma_o Q_o - \gamma_u Q_u;$$ (14)

$\gamma_o, \gamma_u > 0$.

The assumption that the constant term is normalised to unity is not without loss of generality. However, the assumption will be useful for interpreting the results of the estimation; if I were to allow the constant to take on any value (say, $\gamma_c$), there would be no implications for the estimation but I would be unable either to (i) test for a single equilibrium or (ii) estimate the deadweight loss from information asymmetry. The normalisation is therefore pragmatic. However, this does not mean that a researcher could choose any arbitrary value for the constant. As subsequent discussion will show, the choice of constant has implications for the single-equation test; some values of the constant would cause this test — and, therefore, the model as a whole — to be rejected.

Therefore, to summarise, I impose the following payoff functions.
**Assumption 1** The form of Bank’s payoff function is:

\[ \pi_B = (\beta_c + \beta_o Q_o + \beta_u Q_u + \beta_s S + \beta_t t + \epsilon) \cdot \ln[\ln(L)] - \ln(L). \]  

Figure 7 illustrates this functional form. Note that, as in the simple example discussed earlier, the functional form here immediately allows for legal status to have both an ‘incentive effect’ (a linear effect, operating through \( \beta_s \)) and an ‘information effect’ (by the conditional expectation of \( Q_u \), relevant through \( \beta_u \)). Therefore, \( \beta_s \) and \( \beta_u \) are analogous to \( \xi_2 \) and \( \xi_3 \) in the simple model presented earlier.

**Assumption 2** The form of Firm’s payoff function is:

\[ \pi_F = \ln(L) - S \cdot (1 + \gamma_t t - \gamma_o Q_o - \gamma_u Q_u). \]
4.2 Solving the model

This is a signalling model; the relevant solution concept is Perfect Bayesian Equilibrium, and I impose the Cho-Kreps Intuitive Criterion. In this context, this requires the following (where I denote the cumulative distribution of Bank beliefs about the distribution of $Q_u$ by $G(\cdot)$, with pdf $g(\cdot)$).

(i) Firm’s choice of $S, S^*$, maximises its expected payoff given $\Omega_F$ and Bank’s best response, $L^*(S)$:

$$S^*(Q) = \arg \max_{S \in \{0, 1\}} \left[ \ln(L^*(S), Q_o) - S \cdot (1 + \gamma t - \gamma o Q_o - \gamma u Q_u) \right].$$

(17)

(ii) Bank’s best-response function, $L^*(S)$, maximises its expected payoff given $\Omega_B, S$ and $G(\cdot)$, where expected payoffs are determined by the belief function $G(\cdot)$:

$$L^*(S, Q_o) = \arg \max_L \mathbb{E}_{G}(A(Q, t, \epsilon) | Q_o, S, t) \cdot \ln[\ln(L)] - \ln(L).$$

(18)

(iii) [Beliefs in separating equilibrium:] If $S = 0$ and $S = 1$ are both observed with some positive probability on the equilibrium path, Bank beliefs are formed according to Bayes’ Rule:

$$g(Q_u | Q_o, t, S = 0) = \phi(Q_u) \cdot \left[ \int_{-\infty}^{\infty} \phi(q_u) \cdot (1 - S^*(Q_o, t, q_u)) \, dq_u \right]^{-1};$$

(19)

$$g(Q_u | Q_o, t, S = 1) = \phi(Q_u) \cdot \left[ \int_{-\infty}^{\infty} \phi(q_u) \cdot S^*(Q_o, t, q_u) \, dq_u \right]^{-1},$$

(20)

where $\phi(\cdot)$ refers to the pdf of the standard normal. I noted earlier that one
of the weaknesses of the simple model is that it does not require any specific functional form for the conditional expectation term; the assumption that Bank beliefs are formed by Bayes’ Rule is the critical aspect of this formal model that implies that form. This will be shown shortly.

(iv) **Beliefs in pooling equilibrium:** If no variation in $S$ is observed on the equilibrium path for given $Q_o$, Bank believes that any deviation by Firm reveals Firm to be of a type for which deviation *would* be profitable in a separating equilibrium (the Cho-Kreps Intuitive Criterion: Cho and Kreps (1987)). That is, if only Firm of type $Q_u \in X$ would, in a separating equilibrium, choose $S = S^*$ (for a given $Q_o$), and $S = S^*$ is observed in a pooling equilibrium on $S \neq S^*$ for that $Q_o$.

$$g(Q_u | Q_o, t, S^*) \equiv 1(Q_u \in X) \cdot \phi(Q_u) \cdot \left\{ \int_{-\infty}^{\infty} \phi(q_u) \cdot 1(q \in X) \, dq \right\}^{-1}. \tag{21}$$

If no variation in $S$ is observed on the equilibrium path and there is no deviation, Employer believes that the distribution of unobservable types is the true distribution:

$$g(Q_u) \equiv \phi(Q_u). \tag{22}$$

(Equations 21 and 22 both relate to beliefs under pooling equilibrium; equation 21 relates to beliefs off the equilibrium path while equation 22 relates to beliefs on the equilibrium path.)

From this definition, I may solve for the best-response functions. I solve by backward induction, considering Bank’s problem first.
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**Bank’s best-response function.** Bank’s problem is straightforward:

\[
\frac{\partial}{\partial \ln(L)} \ln(L) \bigg|_{L=L^*} = \beta_c + \beta_o Q_o + \beta_u \cdot E(Q_u | Q_o, S) + \beta_s \cdot S + \beta_t \cdot t + \epsilon - 1
\]

\[
\therefore \ln(L^*(Q_o, S)) = \beta_c + \beta_o Q_o + \beta_u E(Q_u | Q_o, S) + \beta_s S + \beta_t t + \epsilon.
\]

(23)

**Firm’s best-response function.** Anticipating the form of \(L^*(Q_o, S)\), Firm’s best response to Nature is:

\[
S^* = 1 \left[ \ln(L^*(Q_o, 1)) - 1 - \gamma t + \gamma_o Q_o + \gamma_u Q_u \geq \ln(L^*(Q_o, 0)) \right]
\]

\[
= 1 \left[ \beta_u \cdot (E(Q_u | Q_o, t, S = 1) - E(Q_u | Q_o, t, S = 0)) + \beta_s - 1 - \gamma t + \gamma_o Q_o + \gamma_u Q_u \geq 0 \right]
\]

\[
= 1 \left[ Q_u \geq Q_u^*(Q_o) \right],
\]

(24)

where I define \(Q_u^*(Q_o)\) as the value of \(Q_u\) for which, given a particular \(Q_o\), Firm is indifferent:

\[
Q_u^*(Q_o, t) = \left( \frac{\beta_u \cdot [E(Q_u | Q_o, t, S = 1) - E(Q_u | Q_o, t, S = 0)] + \beta_s - 1 - \gamma t + \gamma_o Q_o}{\gamma_u} \right).
\]

(25)

Clearly, I must now consider the particular type of equilibrium in order to solve for the difference in conditional expectations.

**Definition 1** A separating equilibrium. A separating equilibrium is as an equilibrium in which, for all values of \(Q_o\), there is some value \(Q_u\) for which Firm chooses \(S = 1\) and some value \(Q_u\) for which Firm chooses \(S = 0\).

**Definition 2** A pooling equilibrium. A pooling equilibrium is the logical complement to a separating equilibrium: an equilibrium in which, for some value
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$Q_o$, Firm either chooses $S = 1$ for all $Q_u$ or Firm chooses $S = 0$ for all $Q_u$.

**Definition 3** A **cut-off signalling strategy**. A cut-off signalling strategy is a signalling strategy such that, for any $Q_o$, there exists some value $Q_u^\ast(Q_o)$ such that Firm optimally chooses $S = 1(Q_u \geq Q_u^\ast(Q_o))$.$^{15}$

The best-response functions outlined may define a unique separating equilibrium with cut-off signalling. One further assumption is sufficient for this to hold.$^{16}$

**Assumption 3** A given increase in the unobservable $Q_u$ has a larger effect on Firm's cost of signalling than it does upon Bank's best response:

$$\gamma_u > \beta_u.$$ 

Having made this assumption, several key results obtain.

**Proposition 1** There is a unique separating equilibrium in pure strategies with cut-off signalling.

**Proposition 2** The cut-off point $Q_u^\ast$ is a continuous decreasing function of $Q_o$.

**Proposition 3** The cut-off is bounded above (below) for $\beta_u > 0$ ($< 0$) by the $\beta_u = 0$ solution.

**Proposition 4** There is no separating equilibrium that does not involve a cut-off signalling strategy.

**Proposition 5** Given the Cho-Kreps Intuitive Criterion, there are no pooling equilibria.

$^{15}$ Further, I assume throughout that, if Firm is indifferent between choosing $S = 0$ and $S = 1$, Firm chooses $S = 1$.

$^{16}$ In Quinn (2010a), the inequality is expressed in terms of absolute values. In this application, however, I restrict $\gamma_u > 0$ and $\beta_u \geq 0$, so the present form is more appropriate.
Proofs for Propositions 1 – 5: See Quinn (2010a).

**Bank inference given Firm’s best-response function:** Having solved for $S^*(Q)$, I can close the solution by providing a form for Bank’s conditional expectation of $Q_u$. It is common knowledge between Firm and Bank that $Q_u$ has a standard normal distribution conditional on $Q_o$. Given that Firm follows a cut-off function on $Q_u^*(Q_o, t)$, it may be shown (following a derivation closely related to that of the Tobit estimator) that:

$$E(Q_u|Q_o, S) = -\frac{\phi(Q_u^*(Q_o, t))}{\Phi(Q_u^*(Q_o, t))} + S \cdot \frac{\phi(Q_u^*(Q_o, t))}{\Phi(Q_u^*(Q_o, t)) \cdot (1 - \Phi(Q_u^*(Q_o, t)))},$$

(27)

so that

$$Q_u^*(Q_o, t) = -\left(\beta_u \cdot \frac{\phi(Q_u^*(Q_o, t))}{\Phi(Q_u^*(Q_o, t)) \cdot (1 - \Phi(Q_u^*(Q_o, t)))} + \beta_s - 1 - \gamma_t t + \gamma_o Q_o\right).$$

(28)

### 4.3 A solution in multiple periods

I have just showed there exists a unique solution in pure strategies for a single-period game. However, I want to estimate the model using panel data. It is important to know, therefore, whether the form of solution for the single-period game may extend into a repeated game. The literature shows that a repeated game with such ‘coarse information’ does not necessarily generalise from the single-period to multi-period context. For example, Meyer (1991) studies a repeated game in which an employer may bias a contest between employees; in that model, the employer rationally biases the second-period contest against the loser of the first-period contest in order to elicit more information about unobservable (and time-invariant) employee types. I now consider a multiple-period extension of
the present game and show a condition under which the single-period solution structure may generalise.

**Assumption 4** Players, actions and information are the same as in the single-period game. However, players may condition on past actions (that is, players are allowed a 'memory' for previous play).

**Assumption 5** In any period \( t \), the unobservable characteristic \( Q_{u,t} \) is (as before) drawn so that \( Q_{ut} \mid Q_{ot} \sim \mathcal{N}(0, 1) \). However, \( Q_{ut} \) is allowed to follow a Markov process such that \( Q_{u0} \) and \( Q_{u1} \) have a bivariate normal distribution with a correlation of \( \rho \):

\[
\begin{pmatrix}
Q_{u0} \\
Q_{u1}
\end{pmatrix}
\sim \mathcal{N}
\begin{pmatrix}
\begin{pmatrix}
0 \\
0
\end{pmatrix},
\begin{pmatrix}
1 & \rho \\
\rho & 1
\end{pmatrix}
\end{pmatrix}.
\] (29)

**Assumption 6** Even though \( \text{Firm} \) and \( \text{Bank} \) have a 'memory' for previous play, \( \text{Firm} \) payoff and \( \text{Bank} \) payoff in each period are functions only of types and actions in that period.

This assumption is required as a matter of practical necessity; the alternative is allow for (potentially multiple) equilibria in which \( \text{Firm} \) and \( \text{Bank} \) condition upon lagged types and actions. Moreover, the two periods under consideration lie four years apart; I argue that this is a sufficiently long period that it is reasonably not to include a (four-year) lag in the payoff function. From this assumption, it follows that — so long as \( \rho^2 < 1 \) — an equilibrium exists in which expectations are formed conditional only on present-period actions and types. If \( \rho = 1 \) — that is, \( Q_{u,0} \equiv Q_{u,1} \) — then differences between \( S_t \) and \( S_{t-1} \) cannot be attributed to changes in \( Q_u \); instead, as in Meyer (1991), repeated observations of a coarse
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signal can be strategically manipulated for information-revelation purposes.\(^\text{17}\)

5 Identifying information and incentive effects

Several refinements are necessary in order to specify the model in estimable form.

5.1 Further identifying assumptions

The index of observable quality. To this point, I have expressed quality in terms of the observable index, \(Q_o\), and its effect upon project productivity, \(\beta_o\). \(\beta_o\) has been included for symmetry to other components of project productivity; however, since \(Q_o\) has no meaning independent of its role in the model, I can restrict \(\beta_o = 1\) without loss of generality. I then allow that the observable index, \(Q_o\), is a linear combination of observable firm characteristics, \(x_o\):

\[
Q_o = \delta_o x_o. \tag{30}
\]

A linear approximation for \(Q_u^*(Q_o, t)\). It is unnecessarily complicated to implement equation (25) in a maximum-likelihood context: this would require the solution of a non-linear system for every evaluation of the likelihood function, with some parameter vectors not generating any solution. I therefore define a function \(\overline{Q}_u^*(Q_o, t)\), being a first-order Taylor approximation of \(Q_u^*(Q_o, t)\).

For tractability, I take the approximation around the point \(Q_o^+\), defined so that

\(^{17}\) In effect, one may conceptualise the behaviour described in Meyer (1991) as a consequence of the agent’s type being over-identified to the principal (in the sense that there are repeated observations on a single random variable, albeit ‘coarse’ observations). Where \(\rho^2 < 1\), there are as many observations as random variables (two); Firm’s type is just-identified to Bank. So long as some variation occurs in \(Q_u\) over time, the two-period game can be solved as two single-shot interactions.
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\[ Q_u^*(Q_o^+, t = 0) = 0. \]

Denote

\[ \varphi(x) \equiv \frac{\phi(x)}{\Phi(x) \cdot (1 - \Phi(x))} \quad (31) \]

\[ \therefore \varphi(0) = \frac{2\sqrt{2}}{\sqrt{\pi}}, \quad (32) \]

where \( \pi \) here denotes the irrational constant, rather than a payoff function. For convenience, and without further loss of generality, I denote \( Q_u^*(Q_o^+, t = 1) \equiv \gamma_t/\gamma_u. \)

From this, I can take a first-order approximation:

\[ Q_u^*(Q_o, t) = -\left( \frac{\beta_u \cdot \varphi(Q_u^*(Q_o), t) + \beta_s - 1 - \gamma_t \cdot t + \gamma_o \cdot Q_o}{\gamma_u} \right), \]

\[ \therefore Q_u^*(Q_o, t) \approx Q_u^*(Q_o, t) \equiv Q_u^*(Q_o^+, t) + \frac{dQ_u^*(Q_o^+)}{dQ_o} \cdot (Q_o - Q_o^+) \]

\[ = \frac{\gamma_t \cdot t}{\gamma_u} + \frac{dQ_u^*(Q_o^+)}{dQ_o} \cdot (Q_o - Q_o^+) \]

\[ = \frac{\gamma_t \cdot t}{\gamma_u} - \frac{\gamma_o}{\gamma_u} \cdot Q_o + \frac{\gamma_o}{\gamma_u} \cdot Q_o^+. \quad (33) \]

Now I can solve for \( Q_o^+ \) by noting that \( Q_u^*(Q_o^+, t = 0) = 0 \) by definition:

\[ Q_u^*(Q_o^+, t = 0) = 0 \]

\[ \iff \beta_u \cdot \varphi(0) + \beta_s - 1 + \gamma_o \cdot Q_o^+ = 0 \]

\[ \therefore \frac{\gamma_o}{\gamma_u} \cdot Q_o^+ = \frac{1 - \beta_u \cdot \frac{2\sqrt{2}}{\sqrt{\pi}} - \beta_s}{\gamma_u}. \quad (34) \]

Therefore, I can write:

\[ \overline{Q_u}(Q_o, t) = \frac{1 + \gamma_t \cdot t - \beta_u \cdot \frac{2\sqrt{2}}{\sqrt{\pi}} - \beta_s}{\gamma_u} - \frac{\gamma_o}{\gamma_u} \cdot Q_o \quad (35) \]

Now, when I estimate, I will explain the cutoff function by a constant term (\( \alpha_0 \)).
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a slope with respect to $Q_o (\alpha_1)$ and a time dummy ($\alpha_t$):

$$Q_u^*(Q_o) = \alpha_0 + \alpha_1 Q_o + \alpha_t \cdot t.$$  \hspace{1cm} (36)

The preceding reasoning implies that these parameters may be interpreted as follows:

$$\alpha_0 = \left( \frac{1 - \beta_u \cdot \frac{2\sqrt{\beta}}{\sqrt{\pi}} - \beta_s}{\gamma_u} \right);$$  \hspace{1cm} (37)

$$\alpha_1 = -\left( \frac{\gamma_o}{\gamma_u} \right);$$  \hspace{1cm} (38)

$$\alpha_t = \frac{\gamma_t}{\gamma_u}. \hspace{1cm} (39)$$

Given the earlier restrictions that $\gamma_o, \gamma_u > 0$, equation 38 implies that $\alpha_1 < 0$; the cutoff function slopes downwards in $(Q_o, Q_u)$ space. Equation 37 shows that $\alpha_0$ is decreasing in both $\beta_u$ and $\beta_s$; as the relative benefits from signalling increase (whether information benefits or incentive benefits), the intercept term for the cutoff decreases, so that a greater proportion of firms choose to invest in the signal.

Finally, note that I can obtain estimates of $\gamma_u$ and $\gamma_o$ as nonlinear functions of the estimated values of $\alpha_0$, $\alpha_1$, $\beta_u$ and $\beta_s$ (this will be important for testing the single-solution assumption and for estimating deadweight loss):

$$\hat{\gamma}_u = \frac{1 - \hat{\beta}_u \cdot \frac{2\sqrt{\hat{\beta}}}{\sqrt{\pi}} - \hat{\beta}_s}{\hat{\alpha}_0};$$  \hspace{1cm} (40)

$$\hat{\gamma}_o = -\hat{\alpha}_1 \cdot \hat{\gamma}_u;$$  \hspace{1cm} (41)

$$\hat{\gamma}_t = \hat{\alpha}_t \cdot \hat{\gamma}_u.$$  \hspace{1cm} (42)

I can therefore rewrite Firm’s and Bank’s best-response functions respectively,
as though both players use the function $Q_u^*(Q_o)$ instead of $Q_u^*(Q_o)$:

$$S^* = 1 [Q_u \geq Q_u^*(Q_o, t)] ,$$  \hspace{1cm} (43)

$$\therefore \ln (L^*(Q_o, S)) = \beta_c + \beta_o Q_o + \beta_u E(Q_o, S, t) + \beta_s S + \beta_t + \epsilon ,$$  \hspace{1cm} (44)

where the function $E(\cdot)$ is the expectation of $Q_u$ conditional upon $(Q_o, S, t)$ and assuming the function $S^*$ rather than $S^*$ (that is, $E(\cdot)$ implements the Taylor approximation):

$$E(Q_o, S, t) \equiv -\frac{\phi[Q_u^*(Q_o, t)]}{\Phi[Q_u^*(Q_o, t)]} + S \cdot \frac{\phi[Q_u^*(Q_o, t)]}{\Phi[Q_u^*(Q_o, t)] \cdot (1 - \Phi[Q_u^*(Q_o, t)])} ,$$  \hspace{1cm} (45)

Note that, for the special case that $\beta_u = 0$, the linear approximation is equivalent to the true functional forms predicted by the model; this is because, as equation 28 shows, $Q_u^*(Q_o, t)$ is then a linear function of $Q_o$.

Figure 8: **Linearised signalling cutoff and conditional expectations**

Figure 8 illustrates the linearised cutoff $(Q_u^*(Q_o, t))$ in $(Q_o, Q_u)$ space, with the consequent choice of $S^*$ and conditional expectations, $E(\cdot)$; in doing so, it illustrates the non-linearity at the core of the identification strategy.
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**Firm and Bank preferences over** $Q_u$. My model assumes that $\gamma_o, \gamma_u > 0$; the signal is costly both in terms of $Q_o$ and $Q_u$. This, in effect, is an aspect of the definition of $Q_u$. $Q_u$ is understood to represent the relevant unobserved component of Firm quality; however, without the restriction $\gamma_u > 0$, the model is uninformative as to whether $Q_u$ is framed as a ‘good’ or as a ‘bad’.\(^{18}\) The restriction that $\gamma_o, \gamma_u > 0$ implies, as shown earlier, that $\alpha_1 < 0$. I therefore restrict $\alpha_1 \leq 0$ as an (untestable) identifying assumption.\(^{19}\) In order to impose that Bank also views $Q_u$ as a ‘good’, I also restrict $\beta_u \geq 0$; this will be a testable assumption.

### 5.2 Exploiting the panel structure

**Efficiency.** I am seeking a maximum-likelihood estimator; I therefore need to specify a distribution for $\epsilon_{it}$, the component of project technology unrelated to firm characteristics. When I specified the model earlier, I assumed that $\epsilon$ is independent of all other variables, such that $E(\epsilon) = E(\epsilon | \Omega_F) = 0$. Taken literally, this assumption would rule out the possibility that there might be a relevant determinant of the overdraft limit observed by Firm and Bank but not by the researcher. To identify the model, I now relax that assumption, insofar as it relates to time-invariant firm-specific characteristics (which I term $\psi_i$).

Specifically, in order to exploit the panel structure to allow serial correlation in unobservables for each firm, I allow a standard random-effects error structure

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\(^{18}\) For example, suppose that $Q_u$ is understood solely as representing ‘management quality’; without the restriction that $\gamma_u > 0$, increasing $Q_u$ could either stand for ‘good management quality’ (so that greater $Q_u$ is more costly for Firm and more profitable for Bank) or for ‘bad management quality’ (so that greater $Q_u$ is less costly for Firm and less profitable for Bank). This problem does not arise with $Q_o$: note that the very definition of project technology $A(Q, t, \epsilon)$ defines that $A$ increases with $Q_o$.

\(^{19}\) I can test the assumption that $\alpha_1 \neq 0$, as I will discuss shortly. However, I cannot test the assumption that $\alpha_1 \leq 0$: it is an identifying assumption, necessary for the very definition of $Q_u$. That is, for any $\ell_{i,t, S_i, \pi_{oi, ci}}(\theta)$, one can reverse the signs of $\alpha_0$, $\alpha_1$, and $\beta_u$ and obtain exactly the same log-likelihood.
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(where I specify the distribution of $\psi_i$ and $\eta_{it}$ shortly):

$$\epsilon_{it} = \psi_i + \eta_{it}.$$  \hfill (46)

This is clearly consistent with the model insofar as $\psi_i$ does not correlate with the mean of $S$ or $E(\cdot)$; in that case, allowing for $\psi_i$ may improve the efficiency of estimation *without* changing the form of the conditional expectation. I now consider a method for testing that correlation.

**Firm fixed effects.** I am yet to consider the traditional endogeneity concern: that unobservables (in this case, unobservables *other than* $Q_u$) may confound estimation. In the present framework, one might readily consider this in terms of the effect of a variable that is observed to Bank and Firm but not observed by a researcher; for example, a manager’s fluency in French (not recorded in the data) may separately cause a firm to choose $SA$ status and may induce a bank to provide a higher overdraft. Insofar as such an unobservable is time-invariant, this would be captured by the term $\psi_i$.

Some creativity is required in order to control and to test for such fixed effects in the present context; given that I am using a maximum-likelihood estimator, the problem is certainly not as straightforward, for example, as adding firm dummies to the estimation (for example, see Lancaster (2000)). Moreover, given the structural assumptions made, the problem is not as simple as merely *controlling* linearly for fixed effects; instead, I must *test* for their importance and seek a specification in which they do not affect the estimates of $\beta_u$ and $\beta_s$.

Note that, with the additional identifying assumptions, I now have the following
form for Bank’s best response:

\[
\ln(L_{it}) = \beta_c + \delta_o x_{o,it} + \beta_s S_{it} + \beta_u E(\delta_{oit} x_{oit}, S_{it}, t) + \beta_t t + \psi_i + \eta_{it}. \tag{47}
\]

The model depends upon a specific functional form for \(E(\cdot)\). If the true Bank conditional inference depends upon the fixed effect, there is no way of controlling for it — for example, if the true conditional expectation is \(E(Q_o, S, t, \psi)\) rather than simply \(E(Q_o, S, t)\). However, while I cannot control for the fixed effect in the event that conditional inference depends upon it, I can certainly test for the importance of fixed effects. I do this using the method of Mundlak (1978). In the present context, this requires specifying the fixed effect as a linear combination of time averages of the other variables:

\[
\psi_i = \mu_i + \kappa_o x_{oi} + \kappa_s S_{i} + \kappa_u E(\delta_{oit} \cdot x_{oit}, S_{it}, t). \tag{48}
\]

Mundlak (1978) and Wooldridge (2009) show that, in the context of an additive fixed-effect structure (as here), this assumption produces a fixed-effect estimator identically equivalent to the usual fixed-effect specification.\(^{20}\) Therefore, equation 48 should be understood not as a restrictive identifying assumption but as an empirical specification to control and test for the role of fixed effects; it is made without further loss of generality to equation 47. I will be interested, then, to test whether — after controlling for \(x_o\) — the fixed effect is linearly independent of

\(^{20}\) In Quinn (2010b), I use this method in a reduced-form context to conduct fixed-effect estimation while testing for the importance of fixed effects. Note that there is an important distinction between the use of this method in that paper and in the present paper. In Quinn (2010b), I use the method to control for fixed effects; in that paper, therefore, the effect of firm legal status is identified solely by those firms that switch status. However, in the present paper, I use the method to test for the correlation of the fixed effect with the means of \(S\) and \(E(\cdot)\) respectively, but do not control completely for fixed effects; this is because, as the main text will shortly explain, my primary specification relies upon \(\kappa_s = \kappa_u = 0\). As the main text explained, the reason for this limitation is that I cannot derive a form of the conditional expectation that depends upon the fixed effect (that is, \(E(Q_o, S, t, \psi)\)). Therefore, the relevant parameters in this paper — unlike those in Quinn (2010b) — are identified from cross-sectional variation, including variation among firms that do not change legal status. This is consistent with the model, which is presented as a repeated interaction in which players’ memory does not matter, and which does not consider reasons that firms might change status over time.
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S and $E(\cdot)$: that is, whether $\mathbb{E}(\psi_i | x_{oit}, t, S_{it}, E(x_{oit}, S_{it}, t)) = \mathbb{E}(\psi_i | x_{oit}, t))$. If this is the case, I conclude that the estimates of $\beta_u$ and $\beta_s$ are not driven by the fixed effect; moreover, I conclude that the true conditional expectation does not depend upon the fixed effect. Wooldridge (2009) shows that this is equivalent to testing the joint hypothesis $H_o : \kappa_s = \kappa_u = 0$. I therefore (i) include firm means $x_{oi}$ throughout, in order to control for possible correlation between $x_{oi}$ and the fixed effect, and (ii) test the joint hypothesis just stated to confirm that fixed effects do not confound estimates of $\beta_u$ and $\beta_s$.\(^{21}\)

Therefore, I exploit the panel structure — both for efficiency and for fixed effects — by specifying:

$$
\epsilon_{it} = \kappa_o x_{oi} + \mu_i + \eta_{it}; \\
\mu_i \sim \mathcal{N}(0, \sigma^2_\mu); \\
\eta_{it} \sim \mathcal{N}(0, \sigma^2_\eta).
$$

Finally, I assume that $\mu_i$ and $\eta_{it}$ are each i.i.d. and independent of all other variables; this is the final assumption necessary to justify the log-likelihood function.

### 5.3 The log-likelihood function

For the model to be identified in a maximum-likelihood framework, I must be able to write an identified likelihood function for the following object of interest:

$$
\theta = (\beta_c, \beta_u, \beta_s, \beta_t, \sigma_\mu, \sigma_\eta, \kappa_o, \kappa_s, \kappa_u, \delta_o, \alpha_0, \alpha_1, \alpha_t, \rho).
$$

First, for convenience, I partition $\theta$ into vectors $\theta_F$ and $\theta_B$ — being, respectively, the parameters determining the response of Firm to Nature and the additional

\(^{21}\)Wooldridge (2009) suggests using a Wald test; however, given the maximum likelihood framework here, I use a likelihood ratio test.
parameters necessary to determine the response of Bank to Firm:

\[ \theta_F \equiv (\delta_o, \alpha_0, \alpha_1, \alpha_t)' \]

\[ \theta_B \equiv (\beta_c, \beta_u, \beta_s, \sigma, \kappa_o, \kappa_s, \kappa_u)' \]

\[ \therefore \theta \equiv (\theta_B', \theta_F', \rho)' \]

There are two outcomes of interest in each time period — Bank’s choice of signal and Firm’s choice of response — and, given the assumptions made, each is a random event with estimable probability. Consider first the response of Firm to Nature. For given Firm \( i \), the log-likelihood of choosing outcomes \( S_{i0} \) and \( S_{i1} \) is given by a bivariate probit:

\[
\ell_{S_{i0}, S_{i1}}|x_{o,i0}, x_{o,i1} (\theta_F) = \ln \Phi_2[(2S_{i0} - 1) \cdot (\alpha_0 + \alpha_1 \delta_o \cdot x_{o,i0}), (2S_{i1} - 1) \cdot (\alpha_0 + \alpha_1 \delta_o \cdot x_{o,i1} + \alpha_t) \cdot \rho \cdot (2S_{i0} - 1) \cdot (2S_{i1} - 1)],
\]

where \( \Phi_2(\cdot) \) is the cumulative density for a standard bivariate normal.\(^{22}\)

Second, consider the response of Bank to Firm. For given Firm \( i \), the log-likelihood for choosing outcomes \( \ln(L_{i0}) \) and \( \ln(L_{i1}) \) is given by a random-

\(^{22}\) That is, if \( \begin{pmatrix} X \\ Y \end{pmatrix} \sim \mathcal{N} \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \right) \), \( \Phi_2(x, y, \rho) \equiv \Pr(X \leq x \text{ and } Y \leq y) \). Note that, for \( \rho = 0 \), this collapses to the sum of a probit for \( S_{i0} \) and a probit for \( S_{i1} \): for \( \rho = 0 \), there is no efficiency gain by exploiting the panel structure to explain \( S \).
effects maximum-likelihood specification:\(^23\)

\[
\ell_{L_{i0}, L_{i1} | x_{i0}, x_{i1}, S_{i0}, S_{i1}}(\theta) = \frac{1}{2} \left\{ \frac{z_{i0}^2 + z_{i1}^2}{\sigma_\eta^2} - \frac{a_i(z_{i0} + z_{i1})^2}{\sigma_\eta^2} + \ln \left( \frac{2\sigma_\mu^2}{\sigma_\eta^2} + 1 \right) + 2 \ln \left( 2\pi\sigma_\eta^2 \right) \right\},
\]

(54)

where \( z_{it} = \ln(L_{it}) - (\beta_c + \delta_o x_{o,it} + \beta_s S_{it} + \beta_u E(\delta_{oit} x_{oit}, S_{it}, t) + \beta_t \cdot t) \)

and

\[ a_i = \frac{\sigma_\mu^2}{2\sigma_\mu^2 + \sigma_\eta^2}. \]

These log-likelihoods respectively represent the marginal likelihood and the conditional likelihood; therefore, the log-likelihood for the joint outcome is simply their sum:\(^24\)

\[
\ell_{L_{i0}, L_{i1}, S_{i0}, S_{i1} | x_{i0}, x_{i1}}(\theta) = \ell_{L_{i0}, L_{i1} | x_{i0}, x_{i1}}(\theta) + \ell_{S_{i0}, S_{i1} | x_{i0}, x_{i1}}(\theta_F).
\]

(55)

Thus the model is locally identified if \( \ell_{L_{i0}, L_{i1}, S_{i0}, S_{i1} | x_{i0}, x_{i1}}(\theta) \) has a unique maximum \( \hat{\theta} \). It does; the Mathematical Appendix illustrates this.

### 5.4 Simulation results

Table 2 reports results from six estimations on simulated data of approximately the same size as the dataset used here; as the table shows, the simulations differ in the true values used for \( \beta_s \) and \( \beta_u \). The table shows that the estimator performed well on the six specifications used (in the sense of producing relatively tight estimates around the true values), particularly in cases where \( \beta_u = 0 \) (columns 1, 2

\(^23\) Note that, for \( \sigma_\mu^2 = 0 \), this collapses to the sum of an OLS-log-likelihood for \( \ln(L_{i0}) \) and \( \ln(L_{i1}) \); for \( \sigma_\mu^2 = 0 \), there is no efficiency gain by exploiting the panel structure to explain \( \ln(L) \).

\(^24\) Though they did not explain it in these terms, Belzil and Hansen (2002, p.2080) relied upon the same principle for obtaining the joint likelihood in their model of returns to schooling.
Table 2: Simulation results

<table>
<thead>
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<th>(5)</th>
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<tbody>
<tr>
<td>True value for $\beta_s$</td>
<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
<td>1.0</td>
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<td>0.0</td>
<td>0.3</td>
<td>0.0</td>
<td>1.0</td>
<td>1.0</td>
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</table>

<p>| | | | | | | |</p>
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<tbody>
<tr>
<td>Estimates:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>-0.024</td>
<td>0.276</td>
<td>0.184</td>
<td>0.976</td>
<td>0.253</td>
<td>1.253</td>
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<tr>
<td></td>
<td>(0.094)</td>
<td>(0.094)**</td>
<td>(0.615)</td>
<td>(0.094)**</td>
<td>(0.639)</td>
<td>(0.639)*</td>
</tr>
<tr>
<td>$\beta_u$</td>
<td>0.000</td>
<td>0.000</td>
<td>0.174</td>
<td>0.000</td>
<td>0.831</td>
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<tr>
<td></td>
<td>0.008</td>
<td>0.008</td>
<td>0.366</td>
<td>0.007</td>
<td>(0.382)**</td>
<td>(0.382)**</td>
</tr>
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<td>$\alpha_0$</td>
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<td>-0.063</td>
<td>-0.063</td>
<td>-0.063</td>
<td>-0.057</td>
<td>-0.057</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.084)</td>
<td>(0.085)</td>
<td>(0.085)</td>
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<td>(0.084)</td>
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<tr>
<td>$\alpha_1$</td>
<td>1.022</td>
<td>1.022</td>
<td>1.085</td>
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<td>(0.095)**</td>
<td>(0.095)**</td>
<td>(0.230)**</td>
<td>(0.095)**</td>
<td>(0.252)**</td>
<td>(0.252)**</td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>0.098</td>
<td>0.098</td>
<td>0.100</td>
<td>0.098</td>
<td>0.101</td>
<td>0.101</td>
</tr>
<tr>
<td></td>
<td>(0.119)</td>
<td>(0.119)</td>
<td>(0.119)</td>
<td>(0.119)</td>
<td>(0.115)</td>
<td>(0.115)</td>
</tr>
</tbody>
</table>

| Obs. | 300 | 300 | 300 | 300 | 300 | 300 |
| Log-likelihood | -1123.130 | -1123.130 | -1123.070 | -1123.130 | -1123.047 | -1123.047 |

Confidence: **∗∗∗ 99%, ** ∗∗ 95%, ∗ ∗ 90%.

Results are from a single simulation for each parameter set, using the same seed for the pseudorandom-number generator. For all simulations, the true value of $(\alpha_0, \alpha_1, \alpha_t)$ is $(0, 1, 0)$. For each simulation, there are 600 observations, forming 300 pairs on $t = 0$ and $t = 1$. True values throughout for unreported parameters are: $\beta_c = 0$, $\beta_t = 0$, $\sigma_\mu = 0$, $\sigma_\eta = 1$, $\rho = 0$. 

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6 Estimation

In order to estimate, I must choose a set of explanatory variables to together constitute ‘observable firm quality’; that is, I must specify the vector $x_o$. My preferred specification explains firm quality by the number of permanent employees, the value of machines and equipment, the value of land and buildings (all taken in logs), firm age and sector dummies. I estimate under this specification and report the results in Table 9. Column 1 reports the full estimation.

The estimation returns $\hat{\beta}_s = 0.234$ and $\hat{\beta}_u = 0$, with relatively tight confidence intervals. This implies that legal status has only a direct incentive effect and no indirect signalling effect. That is, the estimator has detected no non-monotonic behaviour of conditional expectation with respect to $Q_o$. In effect, the estimation is bound by the constraint $\beta_u \geq 0$; a likelihood-ratio test of this constraint (reported at the bottom of Table 9) shows that $H_0: \beta_u \geq 0$ is not rejected by the data.

Specifically, the results imply that a firm switching from SARL to SA will enjoy an increase in overdraft limit of approximately 25%, and that this is wholly explained as an incentive effect. In short, Table 9 implies that, for the population represented by the limited sample, there is no relevant information asymmetry between firms and their banks.

Figures 9 and 10 show the estimated cutoff function and conditional expectations from this specification. The figures are equivalent to Figure 8 in the theory section earlier, and show the curvature of the conditional expectation that is central to the separate identification of information and incentive effects. They also show a clear upward shift in the cutoff function; this reflects the policy change between survey rounds (discussed in Quinn (2010b)) that made the SA form relatively more expensive and induced a substantial migration to the SARL.
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I therefore conclude, using my preferred specification, that the effect of legal status is an incentive effect, not an information effect. I will shortly consider the robustness of this result, including the robustness to alternative specifications. However, before turning to alternative specifications, I first consider the implications of the result for welfare loss.

**Figure 9: Cutoff and conditional expectations, \( t = 0 \)**

**Figure 10: Cutoff and conditional expectations, \( t = 1 \)**
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7 Welfare loss from information asymmetry

I just estimated that there is no relevant information asymmetry; this implies that the welfare loss from information asymmetry is zero. However, this result may be of relatively little policy use, because it says nothing about the potential welfare loss. In this section, I show how the present methodology allows estimation of bounds on the welfare loss from information asymmetry. The literature on the empirical identification of information asymmetry is small, but the literature on welfare loss from information asymmetry is much smaller still; almost no previous research has considered the issue (see Einav, Finkelstein, and Schrimpf (forthcoming, p.1)).

By substituting into the original payoff functions the actual overdraft limit ($L^*$) and actual legal status ($S^*$), I obtain the following expressions for actual Firm payoff, Bank payoff and total payoff:

\[
\pi_F = \ln(L^*) - S^* \cdot C(Q,t); \quad (56)
\]

\[
\pi_B = \ln(L^*) \cdot \ln(\ln(L^*)) - \ln(L^*); \quad (57)
\]

\[
\therefore \pi_T = \ln(L^*) \cdot \ln(\ln(L^*)) - S^* \cdot C(Q,t). \quad (58)
\]

I contrast this to a perfect-information counter-factual, in which $Q_u$ is known perfectly to Bank; I denote the perfect-information total utility as:

\[
\tilde{\pi}_T = \ln(\tilde{L}) \cdot \ln(\ln(\tilde{L})) - \tilde{S} \cdot \tilde{C}(Q). \quad (59)
\]

There are two reasons that information asymmetry will cause $\pi_T^* < \tilde{\pi}_T$:

(i) Bank will suffer welfare loss directly through its uncertainty about $Q_u$; despite being risk-neutral, the curvature of $\pi_B$ implies a welfare loss from having to replace $Q_u$ with $\mathbb{E}(Q_u \mid Q_o, S, t)$; and
(ii) A firm will suffer welfare loss from (sometimes) having to choose \( S = 1 \) purely for signalling purposes.

The structural framework allows approximations for these two forms of loss; the derivation of these approximations is in the Mathematical Appendix.

I want to interpret the loss in currency units, rather than in log units. I therefore define a *compensating monetary value*, \( C \), such that:

\[
\ln(L^* + C) = \ln(L^*) + D(Q_o, S),
\]

(60)

where \( D(Q_o, S) \) represents ‘expected deadweight loss’:

\[
D(Q_o, S) = \mathbb{E}(\tilde{\pi}_T - \pi^*_T | Q_o, S).
\]

Intuitively, one can consider the compensating monetary value as *the increase in overdraft limit that would be necessary to compensate a firm for its unnecessary signalling and to compensate for the bank’s uncertainty in assessing it.* Therefore, for a firm with given \( Q_o \) and \( S \),

\[
C(Q_o, S; \theta) = \exp [\ln(L^*) + D(Q_o, S)] - L^*.
\]

(61)

The point estimate from the earlier estimation is \( \hat{\beta}_u = 0 \). This implies perfect information — so, as one would expect, \( D(Q_o, S, t) = 0 \) and \( C(Q_o, S; \theta) = 0 \).

However, I also seek the upper bound of a 95% confidence interval on \( C(Q_o, S; \theta) \). Since \( C(Q_o, S; \theta) \) will, like \( L \), have an approximately log-normal distribution, it is more meaningful to consider the median compensating monetary value than the total (or mean).

The two most important parameters for determining the deadweight loss from
information asymmetry are clearly $\beta_s$ and $\beta_u$. I therefore consider the maximum median compensating monetary value, taken in the 95% confidence region of those two parameters. That is, I solve for

$$(\text{median } C(Q_o, S; \theta))^\text{upper} = \arg \max_{\beta_s, \beta_u} \text{median } C(x_o, S; \theta^*(\beta_s, \beta_u))$$

subject to

$$2 \left[ \ell(\theta^*) - \ell(\hat{\theta}_{ML}) \right] \leq (\chi^2)^{-1}(2, 0.95),$$

(62)

where $(\chi^2)^{-1}$ refers to the inverse-$\chi^2$ distribution and $\theta^*(\beta_s, \beta_u)$ refers to the vector $\hat{\theta}_{ML}$ with the elements $\hat{\beta}_s$ and $\hat{\beta}_u$ replaced by $\beta_s$ and $\beta_u$ respectively. The intuition is straightforward: I seek the maximum median compensating value within the 95% confidence region for $\beta_s$ and $\beta_u$, given that all other parameters are restricted to their maximum-likelihood estimates.

In the preferred estimation, I obtained a log-likelihood of $\ell(\hat{\theta}_{ML}) = -1152.865$. Therefore, I use $\theta_{ML}$ from that estimation and restrict that

$$2|\ell(\theta) - \ell(\hat{\theta}_{ML})| \leq (\chi^2)^{-1}(2, 0.95) = 5.9915$$

$$\Leftrightarrow \ell(\theta) \leq -1155.861.$$  

(63)

(64)

This problem can be solved numerically by a simple grid search. Figure 11 shows the intuition and the result: I estimate a maximum median compensating value of approximately 28 645 Moroccan dirhams, or approximately US$3 200. The Data Appendix shows that the median overdraft limit for the ‘limited’ sample is 1 000 000 Moroccan dirhams (approximately US$110 000). Therefore, the 95% upper bound on median compensating value is only about 3% of the median overdraft limit for the sample.

One might worry that the specification of the model somehow implicitly pre-
sumes a relatively small welfare loss. However, this is not so. When I estimated the median compensating value for the simulation reported in column 5 of Table 2 (that is, a simulation for $\beta_s = 0$ and $\beta_u = 1$), I obtained a 95% upper bound of almost 830 000 Moroccan dirhams: about 80% of the median overdraft limit.\(^{25}\) The model is therefore very capable of returning large welfare losses for reasonable potential parameter values; however, the present estimated parameter values imply a negligible welfare loss, even at the 95% upper bound.

\(^{25}\)Results are available on request.
8 Robustness

8.1 Robustness to unrestricted fixed effects

I have already outlined one important robustness check: robustness to unrestricted firm fixed effects. Column 2 of Table 9 includes the time-mean for legal status and the time-mean for the conditional expectation. I explained earlier how a test of the joint significance of these two variables is a test for robustness to fixed effects; that is, a test of whether time-invariant firm-specific fixed effects are correlated with the means of $S$ and $E(\cdot)$. Column 1 and column 2 may be compared by a likelihood-ratio test with two degrees of freedom. The test returns a statistic of 0.810, implying a $p$-value of 0.667. I therefore do not reject a null hypothesis that the preferred specification is robust to fixed effects in this way.

8.2 Mutual knowledge of signalling parameters

Mutual knowledge of game structure is an important assumption of many signalling models, including this one. For example, I assumed earlier that Bank not only knows its own payoff structure, but also knows the payoff structure and signalling parameters for Firm; this is fundamental to allowing Bank to draw inference from Firm’s choice of legal status.

To my knowledge, previous literature in the empirical analysis of contracts has not sought to test this assumption. However, the present structural methodology allows for this. I showed earlier that the joint log-likelihood (for Firm signal and Bank response) is simply the sum of the log-likelihood for Firm signal and the log-likelihood for Bank response. Because of the assumption of sequential play — which implies that Bank may condition its response on actual Firm behaviour but Firm may only anticipate Bank response — I can also maximise the log-likelihood for Firm signal and for Bank response separately (because
ln(L) is excluded from $\ell_s$. I can therefore test, for example, whether Firm and Bank share mutual knowledge about the effect upon the signalling cost of the characteristics $x_o$.

Therefore, define $\hat{\theta}_{ML}$ as maximising the joint likelihood, and define $\hat{\theta}^*$ and $\hat{\theta}^*_{F}$ as separately maximising $\ell_{L_i,0,L_i,1,|x_{o,i,0},x_{o,i,1},S_i,0,S_i,1}(\theta)$ and $\ell_{S_i,0,S_i,1,|x_{o,i,0},x_{o,i,1}}(\theta_F)$ respectively. Then $\hat{\theta}^*$ and $\hat{\theta}^*_{F}$ will each include estimates for the parameters of $\theta_F$; the vector $\theta_F$ will be overidentified. However, it will not be overidentified in the simple sense that each element of $\theta_F$ will be overidentified. Rather, as the appendix illustrates, the separate maximisation of $\ell_{L_i,0,L_i,1,|x_{o,i,0},x_{o,i,1},S_i,0,S_i,1}(\theta)$ and $\ell_{S_i,0,S_i,1,|x_{o,i,0},x_{o,i,1}}(\theta_F)$ will allow for two separate estimates for $\alpha_0$, $\alpha_t$ and the combined parameter vector $\alpha_1\delta_o$.

For this reason, estimating $\ell_{L_i,0,L_i,1,|x_{o,i,0},x_{o,i,1},S_i,0,S_i,1}(\theta)$ and $\ell_{S_i,0,S_i,1,|x_{o,i,0},x_{o,i,1}}(\theta_F)$ jointly to obtain $\hat{\theta}_{ML}$ involves making a total of $(\dim[\theta_F] - 1)$ restrictions; therefore, mutual knowledge of $\theta_F$ can be assessed by the following likelihood-ratio test:

$$2 \cdot \left[\ell_{L_i,0,L_i,1,|x_{o,i,0},x_{o,i,1},S_i,0,S_i,1}(\hat{\theta}^*) + \ell_{S_i,0,S_i,1,|x_{o,i,0},x_{o,i,1}}(\hat{\theta}^*_{F})\right] - \ell_{L_i,0,L_i,1,|x_{o,i,0},x_{o,i,1},S_i,0,S_i,1}(\hat{\theta}) \sim \chi^2(\dim[\theta_F] - 1).$$

(65)

Figure 12 illustrates the intuition justifying this test. The model implies two distinct ways of estimating the vector $\theta_F$: both by estimating directly from Firm’s signalling decision and by observing Bank’s response. For example, suppose that Firm’s age is an important determinant of the signalling decision; this can be observed directly by the biprobit of legal status on firm age. However, the model then implies that Bank knows this, so that Firm’s age enters Bank’s conditional expectation term. This response can be inferred by Bank’s response.
Figure 12: Two ways of observing $\theta_F$ if $\beta_u > 0$

Firm signals, using $\theta_F$. \[ \Rightarrow \] I observe $\theta_F$ directly.

Bank responds, using its knowledge of $\theta_F$. \[ \Rightarrow \] I observe $\theta_F$ via Bank’s response

(iif Bank cares about $Q_u$, i.e. iif $\beta_u > 0$).

As both Figure 12 and the Mathematical Appendix illustrate, this test will *not* be applicable in the special case that $\beta_u = 0$. In that case, the parameters $\alpha_0$, $\alpha_1$ and $\alpha_t$ are identified by the biprobit component of $\ell_{L_{it}, L_{i1}}$ alone, because the curvature of $E(\cdot)$ is then not relevant for the observed outcome $L_{it}$. That is, as the Mathematical Appendix shows, the vector $\theta_F$ will *not* be over-identified in the special case that $\beta_u = 0$. This makes intuitive sense: if Bank is not using the conditional expectation of $Q_u$ in its overdraft decision, there is no way for a researcher to observe Bank’s knowledge of $\theta_F$ by observing Bank’s overdraft decision.

I do not, therefore, use this test on the estimation results just reported; I estimated there that $\beta_u = 0$. However, the test will be useful in considering an alternative specification shortly. The insight may also be useful for other applications in the empirical analysis of contracts.

### 8.3 Existence and uniqueness of equilibrium

I showed earlier that a sufficient condition for the existence of a unique equilibrium is $\gamma_u > \beta_u$. I also showed that $\hat{\gamma}_u$ may be recovered as a nonlinear function of $\hat{\beta}_u$, $\hat{\beta}_s$ and $\hat{\alpha}_0$. It is straightforward to use the delta method to test
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\[ H_0 : \gamma_u = \beta_u. \]  

If \( H_0 \) rejects in favour of \( \gamma_u > \beta_u \), one can be confident that — assuming the truth of other aspects of the model — a solution exists and is unique. If \( H_0 \) does not reject, it is nonetheless still reasonable to assume \( \gamma_u > \beta_u \), implying the existence of a unique solution. The model rejects, therefore, only if \( H_0 \) rejects in favour of \( \gamma_u < \beta_u \).

In the case of the preferred specification, I obtain \( \hat{\gamma}_u = 0.160 \) (and \( \hat{\beta}_u = 0 \)), with a standard error on \( (\gamma_u - \beta_u) \) of 0.032. I therefore strongly reject \( H_0 : \gamma_u - \beta_u = 0 \) in favour of \( \gamma_u > \beta_u \); under the preferred specification, the assumption sufficient to guarantee a unique solution is reasonable.

To my knowledge, previous literature on the empirical analysis of contracts has not sought to test either necessary or sufficient conditions for the existence of the equilibrium upon which the estimation relies. It is an important aspect of the structural methodology that this assumption can be tested. This insight, too, may prove useful in other applications in the empirical analysis of contracts.

### 8.4 Error normality

I assumed earlier that, for any given observation, the difference between actual log overdraft limit and predicted log overdraft limit is:

\[
\ln(L_{it}) - \ln(\hat{L}_{it}) = \mu_i + \eta_{it},
\]

(66)

where \( \mu_i \) and \( \eta_{it} \) are each normally distributed. Therefore, \( \ln(L_{it}) - \ln(\hat{L}_{it}) \) will also be normally distributed. This is testable; I use Stata’s `sktest` command, which implements the test of D’Agostino, Belanger, and D’Agostino Jr (1990), as amended by Royston (1991). Under the preferred specification, I obtain a test
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statistic of 2.20, implying a \( p \)-value of 0.334 on the null hypothesis of normality; the preferred specification passes.

8.5 Robustness to alternative specifications

I estimate under a variety of alternative specifications, each differing in the choice of explanatory variables used to constitute ‘observable firm quality’, \( Q_o \). I collate a summary of the robustness checks on each specification in Table 8.

I begin with the most parsimonious specification: explaining firm quality only in terms of firm age and permanent employees. This is reported in Table 10. The estimation returns \( \hat{\beta}_s = 0.533 \) and \( \hat{\beta}_u = 0 \); this is similar to the preferred specification (in particular, it also returns an estimate of no information asymmetry). However, as Table 8 shows, the specification is not robust to firm fixed effects; it is too parsimonious. I then add sector dummies; see Table 11. This estimation also returns \( \hat{\beta}_u = 0 \). However, again, Table 8 shows that the specification is not robust to firm fixed effects. Further, the specification in Table 11 also rejects the null hypothesis that \( \beta_u \geq 0 \) (reported at the bottom of that table).

Alternatively, I consider a specification more extensive than the preferred specification reported in Table 9: I include not only firm assets but also firm liabilities (in the form of long-term debt). The key conclusion from the preferred specification — that there is no information asymmetry — is robust to this specification. Note however that, intuitively, one might think this is a poor specification, since firm liabilities are likely to be substantially determined by the firm’s bank alongside the decision of overdraft limit; that is, one might think that firm liabilities are more reasonable as an alternative dependent variable than as an explanatory variable. There are two ways that one might expect the robustness checks to identify such a problem. First, this might have implications for the assumption of mutual
knowledge. Recall that the assumption of mutual knowledge presupposes a strict order of play; \textit{Firm} moves first and \textit{Bank} responds. If $Q_o$ is fit with an \textit{outcome} variable — a variable that \textit{Firm} cannot know when choosing its status — this may cause the test of mutual knowledge to reject. However, that test is not applicable in this case, because the estimation returns $\hat{\beta}_u = 0$. Second, as here, the problem might cause the model to be rejected for not having normally-distributed errors (see Table 8).

I therefore conclude that, for the limited sample, the key result from my preferred specification is quite robust. First, it is robust to several checks implied by the model itself: concerning fixed effects, the uniqueness of the asserted equilibrium and the normality of the error. Second, it is robust to several alternative specifications, even though each of those specifications is itself rejected by the checks implied by the model.

### 8.6 Robustness to sample selection

All of these alternative specifications concern the ‘limited sample’: the sample of firms having a positive overdraft in both periods. I explained earlier that I cannot formally extend the analysis beyond this sample, for two reasons. First, the decision as to whether an overdraft is provided should be modelled, if at all, as a distinct decision; this would introduce additional complexity unhelpful for the present exposition. Second, if the zero-overdraft cases were included by some \textit{ad hoc} transformation (for example, by using the $f(x) = \ln(x + 1)$ transformation) this would cause the model to be rejected on the assumption of a normally-distributed error.

However, I can get some sense of the extent to which the current results are robust to sample selection. First, I include a time-variant inverse Mills’ Ratio term as
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a control in estimation on the limited sample: this is reported in Table 13. This term is formed as a standard inverse Mills’ Ratio for each period separately in which, for each period, the dependent variable is whether the firm is included in the limited sample and the explanatory variables are the variables comprising $x_o$ for that period. This does not wholly control for selection bias because the selection rule is a function of the outcome in both periods, whereas this approach can only control for potential selection endogeneity in each period separately. Nonetheless, this provides some insight into the extent to which the conclusions from the limited sample may generalise to the entire sample. Table 13 shows that the inverse Mills’ Ratio term is not significant. There is almost no change whatsoever in the estimates of $\hat{\beta}_s$ and $\hat{\beta}_h$. This suggests that the results from the limited sample may generalise, in the hypothetical case that the excluded firms were to have a positive overdraft in each period.

Second, I consider an alternative specification in which even firms with no overdraft facility are specified as implicitly having an overdraft of 10 000 MAD (approximately US$1100). This has some practical appeal; after all, even firms without a formal overdraft facility may implicitly be allowed to overdraw their account by some small amount. This change allows for estimation on the full sample of 488 firms, rather than on the limited sample of 311. The results are reported in Table 14 (in which I use firm age, employees and assets) and Table 15 (in which I add firm liabilities). The results imply that the story may be quite different when extending the model to consider whether an overdraft is provided, rather than merely how large such an overdraft should be; both estimations, for example, suggest a substantial positive information asymmetry (though neither specification shows either the information effect or the incentive effect to be sep-

27 However, this specification still does not overcome the objection that the theoretical model should not conflate Bank’s decision on whether to provide an overdraft with its decision on how large such an overdraft should be.
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However the model is really not suitable for extension in this way. Both extensions are strongly rejected on the assumption of a normally-distributed error term (see Table 8); the consequent error term has a bimodal shape, caused by the distinction between firms with a reported positive overdraft and firms with no reported overdraft. Further, the second extension is also rejected for a failure of mutual knowledge. I noted earlier that the inclusion of firm liabilities — a variable better considered as co-determined with overdraft limit, rather than a valid explanatory variable — might be rejected as implying a failure of mutual knowledge. The earlier example estimated $\hat{\beta}_u = 0$, so the mutual knowledge test was not applicable. However, in this case, $\hat{\beta}_u > 0$, so the test of mutual knowledge is valid. That the test rejects is not very useful in this particular context; this model had already been rejected for a non-normal error term, for example. However, this suggests that the test may be very useful for verification for other structural analyses of contracting in other contexts.

I conclude, therefore, that sample selection does not appear to cause inconsistent estimation in the limited sample. That is, it appears that, if the other firms were to have had a positive overdraft in both periods, the results estimated in the preferred specification would still hold. However, I also conclude that the present methodology, designed to model the decision of how large an overdraft limit should be, is not suitable for extension to the separate decision of whether an overdraft should be provided. This is left for future research.
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9 Conclusion

This paper has developed a new structural methodology for the separate identification of information and incentive effects under binary signalling. I have applied the model to analyse the relationship between Moroccan manufacturing firms and their banks, using legal status as the binary signal in question. I have found that, among firms having a positive overdraft limit in both periods of the available panel data, there is no relevant information asymmetry. Further, I estimated that, among the 95% confidence region for \((\beta_s, \beta_u)\), the maximum median deadweight loss from information asymmetry is only approximately 3% of the median overdraft limit.

This paper began with the claim — relatively common in Morocco — that information asymmetry is an important reason for credit market failure; I conclude that, for the firms in the ‘limited sample’ (representing approximately two-thirds of the data), this narrative appears to be incorrect. This does not imply that Moroccan financial statements are perfectly informative, nor that there is no room for improvement; however, it does suggest that banks may compensate adequately for inadequacies in formal reports by drawing inference on the basis of other available information. This may invite some caution, at least, to enthusiastic calls for costly policy reforms in order to improve information availability.

Of course, the conclusions drawn here are heavily dependent upon the assumptions of the model, and there are several ways in which information asymmetry may be a problem in the Moroccan credit market despite the results reported here. For example, it may be that legal status is a very weak signal of the unobservables that concern banks. For example, it may be that the signal is costly in terms of management quality but not in terms of firms’ expectations of future demand; if banks are concerned about firms’ future demand rather than firms’ management,
legal status will not be informative. Additionally, it may be that information asymmetries affect banks’ decisions about whether to provide an overdraft, rather than about how large that overdraft should be; in that case, the full-information result would be explained by the limitation of the sample to firms having an overdraft in both survey periods.

In Quinn (2010b), I showed that the introduction of a new company law regime in Morocco in 2001 induced a substantial proportion of \(SA\) firms to migrate to the \(SARL\) status, for which banks punished them (relative to firms remaining in the \(SA\) status) by withdrawing overdraft facilities; firms remaining in the \(SA\) status did not, however, receive more generous overdraft facilities. The results in this paper provide some basis for understanding that behaviour. If one accepts that information problems are generally insubstantial in the Moroccan credit market, the behaviour identified in Quinn (2010b) must be explained by incentive effects. That is, the results in Quinn (2010b) can be understood if one believes that the reform process made the \(SA\) status substantially more costly but did not substantially improve the performance of \(SA\) firms or allow those firms to signal a higher quality of management. Of course, as noted earlier, any such interpretation requires generalising the present result to the entire range of Moroccan firms, rather than merely the limited sample studied here. The extent to which the present result holds across that broader range of firms — or, indeed, in other developing and emerging economies — remains to be learned.

The present methodology may suggest further development and further application. The methodology here is specifically tailored to the relatively straightforward case of a single, binary signal. However, there may be scope for extending and applying the method in other contexts involving discrete signals (whether binary or not) and, potentially, to contexts involving multiple simultaneous signals.
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The key suggestion of this paper is that *asymmetric information may sometimes be identified by directly including a conditional expectation term in the estimation*, if the contractual context is amenable to a game-theoretic derivation of the form of that expectation; there may, therefore, be any number of circumstances in which the methods of this paper would prove useful. In developing this approach, it may be particularly useful to extend the current methodology in order to solve recursively for the principal’s conditional expectation and the agent’s decision rule; this would remove the need to implement the agent’s decision rule by a linear approximation of the true game-theoretic solution (as in this paper). Further, it may be useful to estimate by using a method other than maximum likelihood; this would improve the flexibility of the method for awkward distributions of the outcome variable (for example, in the present case, an alternative method may allow estimation on the entire sample, rather than merely on the ‘limited’ sample). Simulation-based methods may prove to be useful *both* for a recursive solution *and* for a more flexible outcome variable. These and other issues remain to be explored.
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**References**


Simon Quinn
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Data Appendix

Table 3 shows the firms’ sector and region.

Table 3: **Region and activity for panel firms, FACS**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Casablanca</th>
<th>Fès</th>
<th>Nador</th>
<th>Rabat</th>
<th>Settat</th>
<th>Tanger</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals</td>
<td>32</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Electrical</td>
<td>14</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Food</td>
<td>12</td>
<td>9</td>
<td>9</td>
<td>0</td>
<td>9</td>
<td>6</td>
</tr>
<tr>
<td>Garment</td>
<td>99</td>
<td>29</td>
<td>0</td>
<td>15</td>
<td>0</td>
<td>27</td>
</tr>
<tr>
<td>Leather</td>
<td>27</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Plastics</td>
<td>29</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Textile</td>
<td>71</td>
<td>7</td>
<td>3</td>
<td>8</td>
<td>7</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 4 summarises the overdraft limits for each period and the relationship between this and changes in overdraft provision; Table 5 shows the same summary for the limited sample.

Table 4: **Summary statistics for overdraft limits, all observations (’000 dirhams)**

<table>
<thead>
<tr>
<th>Overdraft limit</th>
<th>N</th>
<th>Mean</th>
<th>S.Dev.</th>
<th>1st Q.</th>
<th>Median</th>
<th>3rd Q.</th>
<th>Min.</th>
<th>Max.</th>
<th>Zeroes</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 0</td>
<td>488</td>
<td>2513.1</td>
<td>8483.6</td>
<td>100.0</td>
<td>500.0</td>
<td>2000.0</td>
<td>0.0</td>
<td>115000.0</td>
<td>100</td>
</tr>
<tr>
<td>t = 1</td>
<td>488</td>
<td>2546.0</td>
<td>7768.6</td>
<td>0.0</td>
<td>500.0</td>
<td>2000.0</td>
<td>0.0</td>
<td>100000.0</td>
<td>125</td>
</tr>
</tbody>
</table>

Table 5: **Summary statistics for overdraft limits, limited sample (’000 dirhams)**

<table>
<thead>
<tr>
<th>Overdraft limit</th>
<th>N</th>
<th>Mean</th>
<th>S.Dev.</th>
<th>1st Q.</th>
<th>Median</th>
<th>3rd Q.</th>
<th>Min.</th>
<th>Max.</th>
<th>Zeroes</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 0</td>
<td>311</td>
<td>3529.8</td>
<td>10169.8</td>
<td>500.0</td>
<td>1000.0</td>
<td>2500.0</td>
<td>30.0</td>
<td>115000.0</td>
<td>0</td>
</tr>
<tr>
<td>t = 1</td>
<td>311</td>
<td>3729.0</td>
<td>9366.9</td>
<td>400.0</td>
<td>1000.0</td>
<td>2500.0</td>
<td>20.0</td>
<td>100000.0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 6 summarises the key variables used for the full sample; Table 7 provides the same summary for the limited sample.

Table 6: Summary statistics for explanatory variables, all observations

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>S.Dev.</th>
<th>1st Q.</th>
<th>Median</th>
<th>3rd Q.</th>
<th>Min.</th>
<th>Max.</th>
<th>Zeroes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (years) ((t = 0))</td>
<td>976</td>
<td>16.2</td>
<td>12.4</td>
<td>7.0</td>
<td>13.0</td>
<td>21.0</td>
<td>1.0</td>
<td>76.0</td>
<td>0</td>
</tr>
<tr>
<td>Log (permanent employees)</td>
<td>976</td>
<td>4.2</td>
<td>1.1</td>
<td>3.3</td>
<td>4.1</td>
<td>5.0</td>
<td>2.3</td>
<td>8.0</td>
<td>0</td>
</tr>
<tr>
<td>Log (machines and equipment)</td>
<td>976</td>
<td>15.0</td>
<td>1.8</td>
<td>13.8</td>
<td>14.9</td>
<td>16.1</td>
<td>6.9</td>
<td>20.7</td>
<td>0</td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>976</td>
<td>14.4</td>
<td>3.5</td>
<td>14.0</td>
<td>15.0</td>
<td>15.8</td>
<td>0.0</td>
<td>21.4</td>
<td>46</td>
</tr>
<tr>
<td>Log (long-term debt)</td>
<td>976</td>
<td>13.1</td>
<td>5.6</td>
<td>13.4</td>
<td>15.0</td>
<td>16.2</td>
<td>0.0</td>
<td>20.3</td>
<td>144</td>
</tr>
</tbody>
</table>

Table 7: Summary statistics for explanatory variables, limited sample

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>S.Dev.</th>
<th>1st Q.</th>
<th>Median</th>
<th>3rd Q.</th>
<th>Min.</th>
<th>Max.</th>
<th>Zeroes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (years) ((t = 0))</td>
<td>622</td>
<td>17.6</td>
<td>13.0</td>
<td>9.0</td>
<td>14.0</td>
<td>23.0</td>
<td>1.0</td>
<td>76.0</td>
<td>0</td>
</tr>
<tr>
<td>Log (permanent employees)</td>
<td>622</td>
<td>4.4</td>
<td>1.1</td>
<td>3.5</td>
<td>4.3</td>
<td>5.1</td>
<td>2.3</td>
<td>8.0</td>
<td>0</td>
</tr>
<tr>
<td>Log (machines and equipment)</td>
<td>622</td>
<td>15.2</td>
<td>1.8</td>
<td>14.0</td>
<td>15.1</td>
<td>16.4</td>
<td>6.9</td>
<td>20.7</td>
<td>0</td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>622</td>
<td>14.6</td>
<td>3.4</td>
<td>14.2</td>
<td>15.1</td>
<td>15.9</td>
<td>0.0</td>
<td>21.4</td>
<td>26</td>
</tr>
<tr>
<td>Log (long-term debt)</td>
<td>622</td>
<td>13.6</td>
<td>5.4</td>
<td>14.1</td>
<td>15.2</td>
<td>16.5</td>
<td>0.0</td>
<td>20.3</td>
<td>80</td>
</tr>
</tbody>
</table>

I provide a more detailed discussion of the FACS and ICA surveys in Quinn (2010b).
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Mathematical Appendix

Illustrating identification

In this appendix, I illustrate the conditions under which the curvature of \( E(\cdot) \) identifies \( \beta_u \). As explained earlier, the full model exploits the panel structure to allow for correlation over time between unobservables. For simplicity, I suppress that refinement here; instead, I restrict \( \rho = 0 \) and \( \mu = 0 \) for simplicity of exposition (and I drop those variables from consideration of \( \theta \) as well as, for simplicity, \( \sigma_{\eta_i}, \kappa_o, \kappa_s \) and \( \kappa_\mu \)). Therefore, I have:

\[
\ell_{L, S, t | x, oit} (\theta) = \ell_{L, t | x, oit} (\theta) + \ell_{S, t | x, oit} (\theta_F). \tag{67}
\]

On its own, \( \ell_{S, t} \) simply specifies a probit; it is straightforward, then, that \( \ell_{S, t} \) is just-identified so long as there is sufficient variation in the \( S_{it} \) to support that probit (for example, so that no explanatory variable perfectly predicts \( S_{it} \)). This must be assumed.

The just-identification of \( \ell_{L, t} \), however, is less clear. For simplicity of notation, allow for present purposes that \( x_{oit} \) (and, therefore, \( \delta_o \)) is a scalar; so long as the vector equivalents do not include collinear variables, the result here will hold in the more general case.

Given that I restrict \( \rho = 0 \) and \( \mu = 0 \) here for simplicity, I can consider the log-likelihood for the \( i \)th firm in period \( t \) (whereas, in reality, I am required to write a log-likelihood merely for the \( i \)th firm, as explained earlier). In that case, I have the following for the \( i \)th firm in period \( t \) for explaining the overdraft limit:

\[
\ell_{L, it | x, oit, S_{it}} (\theta) = \ln \left[ \phi \left( \frac{\ln(L_{it}) - (\beta_c + \beta_u E(x_{oit}, S_{it}, t) + \beta_s S_{it} + \beta_t t + \delta_o x_{oit})}{\sigma_\eta} \right) \right]. \tag{68}
\]

From this, I can write the Jacobian and Hessian for the determination of the overdraft limit:

\[
J(\theta) = \frac{\partial \ell_{L, it | x, oit, S_{it}} (\theta)}{\partial \theta} = -\eta_{it} \cdot \begin{pmatrix}
\frac{\partial \eta_{it}}{\partial \beta_c} \\
\frac{\partial \eta_{it}}{\partial \beta_u} \\
\frac{\partial \eta_{it}}{\partial \beta_s} \\
\frac{\partial \eta_{it}}{\partial \beta_t} \\
\frac{\partial \eta_{it}}{\partial \delta_o} \\
\frac{\partial \eta_{it}}{\partial \delta_s} \\
\frac{\partial \eta_{it}}{\partial \delta_t} \\
\frac{\partial \eta_{it}}{\partial \kappa_o} \\
\frac{\partial \eta_{it}}{\partial \kappa_s} \\
\frac{\partial \eta_{it}}{\partial \kappa_\mu}
\end{pmatrix} = \eta_{it} \cdot \begin{pmatrix}
1 \\
E(\delta_o x_{oit}, S_{it}, t) \\
t \\
x_{oit} + \beta_u \left( \frac{\partial E_{it}}{\partial \beta_u} \right) \\
\beta_u \left( \frac{\partial E_{it}}{\partial \delta_o} \right) \\
\beta_u \left( \frac{\partial E_{it}}{\partial \delta_s} \right) \\
\beta_u \left( \frac{\partial E_{it}}{\partial \delta_t} \right)
\end{pmatrix}. \tag{69}
\]
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In the context of binary signalling, it follows that:

\[ H(\theta) = \eta_t. \]

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\begin{pmatrix}
\frac{\partial E_{it}}{\partial \theta_0} \\
\frac{\partial E_{it}}{\partial \theta_0} \\
\frac{\partial E_{it}}{\partial \theta_0} \\
\frac{\partial E_{it}}{\partial \theta_0} \\
\frac{\partial E_{it}}{\partial \theta_0} \\
\end{pmatrix}
= \eta_t.
\]

Now, to consider the population problem, I need only consider \( E(\mathbf{H}(\theta)) \); since \( \eta_t \) is assumed independent of all other variables, it follows that:

\[
E(\mathbf{H}(\theta)) = - \begin{pmatrix}
\frac{1}{S_{it}} & 0 \\
\frac{1}{S_{it}} & 0 \\
\frac{1}{S_{it}} & 0 \\
\frac{1}{S_{it}} & 0 \\
\frac{1}{S_{it}} & 0 \\
\end{pmatrix}
\begin{pmatrix}
E(\delta_0 x_{oit}, S_{it}, t) \\
E(\delta_0 x_{oit}, S_{it}, t) \\
E(\delta_0 x_{oit}, S_{it}, t) \\
E(\delta_0 x_{oit}, S_{it}, t) \\
E(\delta_0 x_{oit}, S_{it}, t) \\
\end{pmatrix}
- \begin{pmatrix}
\frac{1}{S_{it}} & 0 \\
\frac{1}{S_{it}} & 0 \\
\frac{1}{S_{it}} & 0 \\
\frac{1}{S_{it}} & 0 \\
\frac{1}{S_{it}} & 0 \\
\end{pmatrix}
\begin{pmatrix}
E(\delta_0 x_{oit}, S_{it}, t) \\
E(\delta_0 x_{oit}, S_{it}, t) \\
E(\delta_0 x_{oit}, S_{it}, t) \\
E(\delta_0 x_{oit}, S_{it}, t) \\
E(\delta_0 x_{oit}, S_{it}, t) \\
\end{pmatrix}
\]

The expected Hessian has the same form as an expected Hessian of an OLS regression on 1,
\( E(\delta_0 x_{oit}, S_{it}, t) \), \( S_{it} \), \( x_{oit} + \beta_u \cdot \frac{\partial E_{it}}{\partial \theta_0} \), \( \beta_u \cdot \frac{\partial E_{it}}{\partial \theta_0} \), \( \beta_u \cdot \frac{\partial E_{it}}{\partial \theta_0} \), and \( \beta_u \cdot \frac{\partial E_{it}}{\partial \theta_0} \). The parameter vector will therefore be identified from \( \ell_{it} \), alone if those variables meet the requirements of identification in an OLS context; namely, that none of the variables are explicable by a linear combination of the other variables.

This requires consideration of the latter terms; recall, then:

\[
E(\delta_0 x_{oit}, S_{it}, t) \equiv -(1 - S_{it}) \cdot \frac{\phi[Q_u^c(\delta_0 x_{oit}, t)]}{\Phi[Q_u^c(\delta_0 x_{oit}, t)]} + S_{it} \cdot \frac{\phi[-Q_u^c(\delta_0 x_{oit}, t)]}{\Phi[-Q_u^c(\delta_0 x_{oit}, t)]}
\]

\[
\frac{\partial E(\delta_0 x_{oit}, S_{it}, t)}{\partial Q_u^c} = (1 - S_{it}) \cdot \frac{\phi[Q_u^c(\delta_0 x_{oit}, t)]}{\Phi[Q_u^c(\delta_0 x_{oit}, t)]} \left( \frac{\phi[Q_u^c(\delta_0 x_{oit}, t)]}{\Phi[Q_u^c(\delta_0 x_{oit}, t)]} - \frac{\phi[-Q_u^c(\delta_0 x_{oit}, t)]}{\Phi[-Q_u^c(\delta_0 x_{oit}, t)]} \right)
+ S_{it} \cdot \frac{\phi[-Q_u^c(\delta_0 x_{oit}, t)]}{\Phi[-Q_u^c(\delta_0 x_{oit}, t)]} \left( \frac{\phi[-Q_u^c(\delta_0 x_{oit}, t)]}{\Phi[-Q_u^c(\delta_0 x_{oit}, t)]} - \frac{\phi[Q_u^c(\delta_0 x_{oit}, t)]}{\Phi[Q_u^c(\delta_0 x_{oit}, t)]} \right)
\]
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\[ Q^*_u(\delta_o \delta_{x_{it}}, t) = \alpha_0 + \alpha_1 \delta_o + \alpha_t t \]

(74)

\[ \frac{\partial Q^*_u(\delta_o \delta_{x_{it}}, t)}{\partial \delta_o} = \alpha_1 \]

(75)

\[ \frac{\partial Q^*_u(\delta_o \delta_{x_{it}}, t)}{\partial \alpha_0} = 1 \]

(76)

\[ \frac{\partial Q^*_u(\delta_o \delta_{x_{it}}, t)}{\partial \alpha_1} = \delta_o x_o \]

(77)

\[ \frac{\partial Q^*_u(\delta_o \delta_{x_{it}}, t)}{\partial \beta_o} = t \]

(78)

\[ \therefore \frac{\partial E(\delta_o \delta_{x_{it}}, S_{it}, t)}{\partial \beta_o} = \frac{\partial E(\delta_o \delta_{x_{it}}, S_{it}, t)}{\partial Q^*_u} \cdot \alpha_1 \]

(79)

\[ \frac{\partial E(\delta_o \delta_{x_{it}}, S_{it}, t)}{\partial \alpha_0} = \frac{\partial E(\delta_o \delta_{x_{it}}, S_{it}, t)}{\partial Q^*_u} \cdot \delta_o x_o \]

(80)

\[ \frac{\partial E(\delta_o \delta_{x_{it}}, S_{it}, t)}{\partial \alpha_1} = \frac{\partial E(\delta_o \delta_{x_{it}}, S_{it}, t)}{\partial Q^*_u} \cdot t. \]

(81)

I explained earlier that I must restrict \( \alpha_1 \leq 0 \). Leaving aside this restriction, it is clear that there are only two cases under which just-identification is not obtained from \( \ell_{L_{it}} \) alone. First, if \( \beta_u = 0 \), \( \ell_{L_{it}} \) does not permit identification of the signalling parameters \((\alpha_0, \alpha_1, \alpha_t)\); equation 71 shows that the last three elements of each multiplying vector are all zero, so that the Hessian can no longer be negative definite. Second, if \( \alpha_1 = 0 \), \( E(\delta_o \delta_{x_{it}}, S_{it}, t) \) takes only two values for a given \( t \), depending on whether \( S_{it} = 0 \) or \( S_{it} = 1 \). That is, if \( \alpha_1 = 0 \), the fundamental non-linearity at the heart of the identification strategy is lost, so that the conditional expectation function \( E(\cdot) \) is collinear with \( \delta_o \delta_{x_{it}} \) for given \( t \). Therefore, \( \frac{\partial E(\delta_o \delta_{x_{it}}, S_{it}, t)}{\partial Q^*_u} = 0 \) and equations 79–82 all equal zero. Therefore the last three elements of each multiplying vector in equation 71 are again all zero, so that the Hessian is again no longer negative definite.

However, so long as \( \alpha_1 < 0 \) and \( \beta_u \neq 0 \) (indeed, I restrict that \( \beta_u \geq 0 \), so \( \beta_u \neq 0 \iff \beta_u > 0 \)), the parameters are just-identified from \( \ell_{L_{it}} \) alone; in that case, the joint likelihood \( \ell_{L_{it}, S_{it}} \) allows over-identification of those parameters appearing in both \( \ell_{L_{it}} \) and \( \ell_{S_{it}} \). These are the parameters \( \alpha_0, \alpha_t \) and the parameter combination \( \alpha_1 \delta_o \). This overidentification forms the basis for the ‘test of mutual knowledge’, discussed in the main text.
Chapter III: Identifying information and incentives under binary signalling

An approximation for deadweight loss

First, by using a second-order Taylor approximation of \((\bar{\pi}_B - \pi^*_B) / \bar{\pi}_B\) around \(\mathbb{E}(Q_u \mid Q_o, S, t)\), I obtain the following estimate of_bank’s welfare loss:

\[
\frac{\bar{\pi}_B - \pi^*_B}{\bar{\pi}_B} \approx \frac{\beta_u^2}{(\ln(L^*)^2 \cdot (\ln(\ln(L^*)) - 1)} \cdot \text{Var}(Q_u \mid Q_o, S, t)
\]

\[
\therefore \bar{\pi}_B - \pi^*_B \approx \beta_u^2 \cdot \text{Var}(Q_u \mid Q_o, S, t),
\]

where it can be shown that

\[
\text{Var}(Q_u \mid Q_o, S, t) = 1 + Q^*_u(Q_o, t) \cdot \mathbb{E}(Q_u \mid Q_o, S, t) - \mathbb{E}(Q_u \mid Q_o, S, t)^2.
\]

Second, by using the linearised cutoff function \(\bar{Q}_u(Q_o, t)\), I can approximate the deadweight loss from unnecessary \(\bar{\pi}_B\) signalling; using this and the previous approximation, I can obtain an expression for ‘expected deadweight loss’. The derivation of approximate deadweight loss

\[
\mathbb{E}\left( S^* \cdot C^*(Q_o, Q_u, t) - \bar{S} \cdot \bar{C}(Q_o, Q_u, t) \mid Q_o, S = 1 \right)
\]

\[
= [1 - \Phi(Q^*_u(Q_o, t))]^{-1} \int_{Q^*_u(Q_o, t)}^{Q^*_u(Q_o, t) + \frac{\beta_u}{\gamma_u} \cdot \frac{2\sqrt{2}}{\sqrt{\pi}}} C(Q_o, q_u, t) \cdot \phi(q_u) \, dq_u
\]

\[
= [1 - \Phi(Q^*_u(Q_o, t))]^{-1} \int_{Q^*_u(Q_o, t)}^{Q^*_u(Q_o, t) + \frac{\beta_u}{\gamma_u} \cdot \frac{2\sqrt{2}}{\sqrt{\pi}}} (1 + \gamma t - \gamma_o \cdot Q_o - \gamma_u \cdot q_u) \cdot \phi(q_u) \, dq_u
\]

\[
= [1 - \Phi(Q^*_u(Q_o, t))]^{-1} \left\{ (1 + \gamma t - \gamma_o \cdot Q_o) \cdot \left[ \Phi \left( Q^*_u(Q_o, t) + \frac{\beta_u}{\gamma_u} \cdot \frac{2\sqrt{2}}{\sqrt{\pi}} \right) - \Phi (Q^*_u(Q_o, t)) \right] + \gamma_u \cdot \left[ \phi \left( Q^*_u(Q_o, t) + \frac{\beta_u}{\gamma_u} \cdot \frac{2\sqrt{2}}{\sqrt{\pi}} \right) - \phi (Q^*_u(Q_o, t)) \right]\right\}.
\]

Therefore, using these approximations, I derive an approximation for ‘expected deadweight loss’:

\[
D(Q_o, S) = \mathbb{E} \left[ \bar{\pi}_B - \pi^*_B \mid Q_o, S \right]
\]

\[
= \mathbb{E} \left[ \bar{\pi}_B - \pi^*_B \mid \bar{S} \cdot \bar{C}(Q_o, Q_u) - S^* \cdot C^*(Q_o, Q_u) \right] \mid Q_o, S\]

\[
\approx \beta_u^2 \cdot \text{Var}(Q_u \mid Q_o, S, t) + S \cdot [1 - \Phi(Q^*_u(Q_o, t))]^{-1}
\]

\[
\times \left\{ (1 + \gamma t - \gamma_o \cdot Q_o) \cdot \left[ \Phi \left( Q^*_u(Q_o, t) + \frac{\beta_u}{\gamma_u} \cdot \frac{2\sqrt{2}}{\sqrt{\pi}} \right) - \Phi (Q^*_u(Q_o, t)) \right] + \gamma_u \cdot \left[ \phi \left( Q^*_u(Q_o, t) + \frac{\beta_u}{\gamma_u} \cdot \frac{2\sqrt{2}}{\sqrt{\pi}} \right) - \phi (Q^*_u(Q_o, t)) \right]\right\}.
\]
Table 8: Collated summary of estimations

<table>
<thead>
<tr>
<th>Variables</th>
<th>Sample</th>
<th>Report</th>
<th>Normality of $\epsilon$</th>
<th>Robust under fixed effects</th>
<th>Mutual knowledge</th>
<th>Single equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Statistic</td>
<td>$p$</td>
<td>Result</td>
<td>Statistic</td>
</tr>
<tr>
<td>Age, size, assets &amp; sector</td>
<td>Limited</td>
<td>Table 9</td>
<td>2.20</td>
<td>0.334</td>
<td>✓</td>
<td>0.810</td>
</tr>
<tr>
<td>Age &amp; size</td>
<td>Limited</td>
<td>Table 10</td>
<td>0.09</td>
<td>0.956</td>
<td>✓</td>
<td>8.602</td>
</tr>
<tr>
<td>Age, size &amp; sector</td>
<td>Limited</td>
<td>Table 11</td>
<td>1.32</td>
<td>0.518</td>
<td>✓</td>
<td>5.450</td>
</tr>
<tr>
<td>Age, size, assets &amp; liabilities &amp; sector</td>
<td>Limited</td>
<td>Table 12</td>
<td>6.21</td>
<td>0.045**</td>
<td>X</td>
<td>0.434</td>
</tr>
<tr>
<td>Age, size, assets &amp; sector</td>
<td>Limited</td>
<td>Table 13</td>
<td>2.28</td>
<td>0.320</td>
<td>✓</td>
<td>3.320</td>
</tr>
<tr>
<td>Age, size &amp; assets</td>
<td>All</td>
<td>Table 14</td>
<td>65.22</td>
<td>0.000***</td>
<td>X</td>
<td>1.196</td>
</tr>
<tr>
<td>Age, size, assets &amp; liabilities &amp; liabilities</td>
<td>All</td>
<td>Table 15</td>
<td>48.61</td>
<td>0.000***</td>
<td>X</td>
<td>0.824</td>
</tr>
</tbody>
</table>

Confidence: *** ← 99%, ** ← 95%, * ← 90%.

The 'limited' sample refers to the strongly balanced sample of 311 firms having a positive overdraft in both periods.

The overidentification tests are implemented as follows.

**Normality of $\epsilon$:** This applies Stata's `sktest` command, which implements the test of D'Agostino, Belanger, and D'Agostino Jr (1990) as amended by Royston (1991). The test statistic has an adjusted $\chi^2(2)$ distribution under the null hypothesis that $\epsilon$ is drawn from a normal distribution.

**Robustness to fixed effects:** This applies a likelihood ratio test for the restriction that $\kappa = \kappa_u = 0$, as explained in the text. The test statistic has a $\chi^2(2)$ distribution under the null hypothesis that $E(\psi_i | x_{o, it}, t, S_{it}, E(x_{o, it}, S_{it}, t)) = E(\psi_i | x_{o, it}, t)$.

**Mutual knowledge:** This applies a likelihood ratio test for the restriction that Bank and Firm share mutual knowledge about $\theta_F$. The test statistic has a $\chi^2(\dim(\theta_F) - 1)$ distribution under the null hypothesis of mutual knowledge.

**Single-equilibrium condition:** This uses Stata's `nlcom` command, which implements the delta method to test the null hypothesis $\gamma_u - \beta_u = 0$; the $p$-value is the reported $p$ from a two-tailed test. The 'Result' column shows '✓' if the hypothesis does not reject (in which case the assumption is 'not unreasonable'); the column shows '✓ ✓' if the hypothesis rejects in favour of $\gamma_u > \beta_u$ (in which case the assumption is confirmed). There is no case in which the assumption is confirmed as unreasonable.
Table 9: Structural estimates: Explaining firm quality by firm age, size, assets and sector (limited sample)

<table>
<thead>
<tr>
<th>Dependent variables</th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(L)</td>
<td>√</td>
<td>√</td>
<td>√</td>
<td>X</td>
</tr>
<tr>
<td>S</td>
<td>√</td>
<td>√</td>
<td>X</td>
<td>√</td>
</tr>
</tbody>
</table>

**Signal effects:**

\[ \beta_s: \text{‘Incentive effect’} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.234</td>
<td>0.188</td>
<td>-0.477</td>
<td></td>
</tr>
<tr>
<td>(0.101)**</td>
<td>(0.157) (0.505)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \beta_u: \text{‘Information effect’} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.061</td>
<td></td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.003) (0.15)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Signal costs:**

\[ \alpha_0: \text{Constant signalling cost} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.774</td>
<td>4.918</td>
<td>2.822</td>
<td>4.960</td>
</tr>
<tr>
<td>(0.717)**</td>
<td>(0.735)** (17.755) (0.759)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \alpha_1: \text{Ratio of signalling costs, } -\left(\gamma_0/\gamma_u\right) \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.791</td>
<td>-2.238</td>
<td>-5.169</td>
<td></td>
</tr>
<tr>
<td>(0.521)**</td>
<td>(1.026)** (12.834)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \alpha_t: \text{Time dummy } (0 = \text{FACS}) \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.775</td>
<td>0.777</td>
<td>-1.590</td>
<td>0.773</td>
</tr>
<tr>
<td>(0.097)**</td>
<td>(0.097)** (5.183) (0.098)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Observable quality, \(Q_u\):**

**Basic characteristics:**

\[ \text{Age (years)} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.012</td>
<td>0.009</td>
<td>0.015</td>
<td>0.021</td>
</tr>
<tr>
<td>(0.003)**</td>
<td>(0.005)* (0.004)** (0.006)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{Log (permanent employees)} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.207</td>
<td>0.168</td>
<td>0.221</td>
<td>0.365</td>
</tr>
<tr>
<td>(0.073)**</td>
<td>(0.084)** (0.157) (0.081)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{Sector dummies} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Firm assets:**

\[ \text{Log (machines and equipment)} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.075</td>
<td>0.062</td>
<td>0.102</td>
<td>0.128</td>
</tr>
<tr>
<td>(0.03)**</td>
<td>(0.033)* (0.059)* (0.049)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{Log (land and buildings)} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.021</td>
<td>0.019</td>
<td>-0.008</td>
<td>0.059</td>
</tr>
<tr>
<td>(0.024)</td>
<td>(0.02) (0.058) (0.049)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{Dummy: Land & buildings missing or zero} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.262</td>
<td>0.238</td>
<td>0.252</td>
<td>0.578</td>
</tr>
<tr>
<td>(0.396)</td>
<td>(0.34) (0.997) (0.838)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Other:**

\[ \beta_c: \text{Overdraft constant} \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.642</td>
<td>5.462</td>
<td>4.920</td>
<td></td>
</tr>
<tr>
<td>(0.628)**</td>
<td>(1.201)** (0.863)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \beta_t: \text{Time dummy } (0 = \text{FACS}) \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.023</td>
<td>0.016</td>
<td>0.08</td>
<td></td>
</tr>
<tr>
<td>(0.075)</td>
<td>(0.079) (0.13)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mundlak firm means

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>All but (S_i) &amp; (E_i)</td>
<td>All but (S_i) &amp; (E_i)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \sigma_\eta \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.873</td>
<td>0.873</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>(0.035)**</td>
<td>(0.035)** (0.035)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \sigma_\mu \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.481</td>
<td>0.48</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>(0.06)**</td>
<td>(0.06)** (0.062)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \rho \]

<table>
<thead>
<tr>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.710</td>
<td>0.708</td>
<td>0.704</td>
<td></td>
</tr>
<tr>
<td>(0.075)**</td>
<td>(0.075)** (0.076)**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Confidence:** *** ↔ 99%, ** ↔ 95%, * ↔ 90%.

Parentheses show standard errors. For consistency, the confidence report is determined throughout by standard two-tailed Wald tests; note, however, that the structural model restricts \(\beta_u \geq 0\) and \(\alpha_1 \leq 0\) (and \(\sigma_\eta, \sigma_\mu \geq 0\)). No weights are applied, and the errors are not clustered in any way.

However, note that each recorded ‘observation’ is firm-level pair, for \(t = 0\) and \(t = 1\); therefore, the estimation is robust to firm-level serial correlation in errors and in signalled quality \(Q_u\).
Table 10: Structural estimates: Explaining firm quality by firm age and size (limited sample)

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprob: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variables: log(L)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>log(S)</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>✓</td>
</tr>
</tbody>
</table>

Signal effects:

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_s ): ‘Incentive effect’</td>
<td>0.533 (0.115)**</td>
<td>0.229 (0.156)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_u ): ‘Information effect’</td>
<td>0.000 (0.004)</td>
<td>0.000 (0.004)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Signal costs:

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 ): Constant signalling cost</td>
<td>2.143 (0.296)**</td>
<td>2.227 (0.298)**</td>
<td>-2.185 (0.299)**</td>
<td></td>
</tr>
<tr>
<td>( \alpha_1 ): Ratio of signalling costs, ( - (\gamma_o / \gamma_u) )</td>
<td>-1.144 (0.249)**</td>
<td>-1.998 (1.362)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \alpha_t ): Time dummy (0 = FACS)</td>
<td>0.686 (0.086)**</td>
<td>0.682 (0.086)**</td>
<td>-6.88 (0.086)**</td>
<td></td>
</tr>
</tbody>
</table>

Observable quality, \( Q_u \):

Basic characteristics:

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (years)</td>
<td>0.027 (0.004)**</td>
<td>0.014 (0.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log (permanent employees)</td>
<td>0.298 (0.072)**</td>
<td>0.182 (0.121)</td>
<td></td>
<td>0.355 (0.062)**</td>
</tr>
</tbody>
</table>

Other:

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_c ): Overdraft constant</td>
<td>11.116 (0.263)**</td>
<td>11.618 (0.461)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_t ): Time dummy (0 = FACS)</td>
<td>0.092 (0.075)</td>
<td>0.033 (0.077)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mundlak firm means

All but \( S_i \) & \( E_i \) All All but \( S_i \) & \( E_i \)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_\eta )</td>
<td>0.882 (0.036)**</td>
<td>0.876 (0.035)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_\mu )</td>
<td>0.828 (0.056)**</td>
<td>0.822 (0.055)**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.749 (0.063)**</td>
<td>0.746 (0.063)**</td>
<td>0.747 (0.063)**</td>
<td></td>
</tr>
</tbody>
</table>

Obs. 311 311 311 311
Log-likelihood -1265.317 -1261.016 No convergence -302.886

\( H_0: \beta_u \geq 0 \) (p-value, LR test) 0.241

Confidence: *** \( \rightarrow \) 99\%, ** \( \rightarrow \) 95\%, * \( \rightarrow \) 90\%.

Parentheses show standard errors. For consistency, the confidence report is determined throughout by standard two-tailed Wald tests; note, however, that the structural model restricts \( \beta_u \geq 0 \) and \( \alpha_1 \leq 0 \) (and \( \sigma_\eta, \sigma_\mu \geq 0 \)). No weights are applied, and the errors are not clustered in any way. However, note that each recorded ‘observation’ is firm-level pair, for \( t = 0 \) and \( t = 1 \); therefore, the estimation is robust to firm-level serial correlation in errors and in signalled quality (\( Q_u \)).
Table 11: Structural estimates: Explaining firm quality by firm age, size and sector (limited sample)

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variables:</strong> log((L))</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
</tr>
<tr>
<td>S</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Signal effects:**

\(\beta_s\): ‘Incentive effect’

\[
\begin{array}{c}
\beta_s: 0.368 \\
(0.11)^{***}
\end{array}
\]

\(\beta_u\): ‘Information effect’

\[
\begin{array}{c}
\beta_u: 0.000 \\
(0.02)\ 
\end{array}
\]

**Signal costs:**

\(\alpha_0\): Constant signalling cost

\[
\begin{array}{c}
\alpha_0: 2.649 \\
(0.37)^{***}
\end{array}
\]

\(\alpha_1\): Ratio of signalling costs, \((-\gamma_o/\gamma_u)\)

\[
\begin{array}{c}
\alpha_1: -1.591 \\
(0.439)^{***}\ 
\end{array}
\]

\(\alpha_t\): Time dummy (0 = FACS)

\[
\begin{array}{c}
\alpha_t: 0.755 \\
(0.094)^{***}\ 
\end{array}
\]

**Observable quality, \(Q_o\):**

**Basic characteristics:**

- Age (years): 0.015 (0.004)***
- Log (permanent employees): 0.306 (0.086)***
- Sector dummies: ✓ ✓ ✓

**Other:**

- \(\beta_c\): Overdraft constant

\[
\begin{array}{c}
\beta_c: 10.595 \\
(0.279)^{***}\ 
\end{array}
\]

- \(\beta_t\): Time dummy (0 = FACS)

\[
\begin{array}{c}
\beta_t: 0.058 \\
(0.074)\ 
\end{array}
\]

**Mundlak firm means**

- All but \(\widehat{S}_i\) & \(\widehat{E}_i\)
- All but \(\widehat{S}_i\) & \(\widehat{E}_i\)

\(\sigma_{\eta}\)

\[
\begin{array}{c}
\sigma_{\eta}: 0.878 \\
(0.035)^{***}\ 
\end{array}
\]

\(\sigma_{\mu}\)

\[
\begin{array}{c}
\sigma_{\mu}: 0.68 \\
(0.055)^{***}\ 
\end{array}
\]

\(\rho\)

\[
\begin{array}{c}
\rho: 0.696 \\
(0.074)^{***}\ 
\end{array}
\]

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs.</td>
<td>311</td>
<td>311</td>
<td>311</td>
<td>311</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-1212.442</td>
<td>-1209.717</td>
<td><strong>No convergence</strong></td>
<td>-288.15</td>
</tr>
<tr>
<td>(H_0: \beta_u \geq 0) (p-value; LR test)</td>
<td>0.018**</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Confidence: *** ➞ 99%, ** ➞ 95%, * ➞ 90%.

Parentheses show standard errors. For consistency, the confidence report is determined throughout by standard two-tailed Wald tests; note, however, that the structural model restricts \(\beta_u \geq 0\) and \(\alpha_1 \leq 0\) (and \(\sigma_\eta, \sigma_\mu \geq 0\)). No weights are applied, and the errors are not clustered in any way.

However, note that each recorded ‘observation’ is firm-level pair for \(t = 0\) and \(t = 1\); therefore, the estimation is robust to firm-level serial correlation in errors and in signalled quality \((Q_u)\).
Table 12: Structural estimates: Explaining firm quality by firm age, size, assets, liabilities and sector (limited sample)

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent variables: log(L)</strong></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td><strong>S</strong></td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Signal effects:**

\( \beta_s \): ‘Incentive effect’

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.184</td>
<td>0.162</td>
<td>-4.699</td>
<td></td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.098)</td>
<td>(0.157)</td>
<td>(3.628)</td>
<td></td>
</tr>
</tbody>
</table>

\( \beta_u \): ‘Information effect’

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.000</td>
<td>0.000</td>
<td>0.165</td>
<td></td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.011)</td>
<td>(0.004)</td>
<td>(1.317)</td>
<td></td>
</tr>
</tbody>
</table>

**Signal costs:**

\( \alpha_0 \): Constant signalling cost

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.688</td>
<td>5.600</td>
<td>37.639</td>
<td>5.889</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.893)***</td>
<td>(0.898)***</td>
<td>(299.776)</td>
<td>(0.946)***</td>
</tr>
</tbody>
</table>

\( \alpha_1 \): Ratio of signalling costs, \(- \gamma_o / \gamma_u \)

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1.734</td>
<td>-1.502</td>
<td>-2.724</td>
<td></td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.576)***</td>
<td>(0.525)***</td>
<td>(21.716)</td>
<td></td>
</tr>
</tbody>
</table>

\( \alpha_t \): Time dummy (0 = FACS)

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.756</td>
<td>0.753</td>
<td>-1.634</td>
<td>0.771</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.098)***</td>
<td>(0.098)***</td>
<td>(13.066)</td>
<td>(0.101)***</td>
</tr>
</tbody>
</table>

**Observable quality, \( Q_o \):**

**Basic characteristics:**

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (years)</td>
<td>0.011</td>
<td>0.013</td>
<td>0.013</td>
<td>0.02</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.003)***</td>
<td>(0.005)**</td>
<td>(0.005)***</td>
<td>(0.006)***</td>
</tr>
</tbody>
</table>

|               | Joint | Joint: FE | Overdraft | Biprobit: Signal |
|               | 0.178 | 0.2        | 0.127     | 0.303            |
| Standard errors | (0.073)*** | (0.08)** | (0.153) | (0.088)*** |

|               | Joint | Joint: FE | Overdraft | Biprobit: Signal |
|               | ✓     | ✓         | ✓         | ✓                |

**Firm assets:**

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (machines and equipment)</td>
<td>0.063</td>
<td>0.072</td>
<td>0.102</td>
<td>0.094</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.031)***</td>
<td>(0.035)**</td>
<td>(0.051)**</td>
<td>(0.052)**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (land and buildings)</td>
<td>0.018</td>
<td>0.02</td>
<td>-0.57</td>
<td>0.052</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.024)</td>
<td>(0.027)</td>
<td>(0.057)</td>
<td>(0.049)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dummy: Land &amp; buildings missing or zero</td>
<td>0.209</td>
<td>0.229</td>
<td>-0.803</td>
<td>0.437</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.403)</td>
<td>(0.443)</td>
<td>(0.955)</td>
<td>(0.837)</td>
</tr>
</tbody>
</table>

**Firm liabilities:**

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (long-term debt)</td>
<td>0.064</td>
<td>0.074</td>
<td>0.052</td>
<td>0.12</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.045)</td>
<td>(0.049)</td>
<td>(0.1)</td>
<td>(0.071)**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dummy: Long-term debt missing, zero or negative</td>
<td>0.851</td>
<td>0.961</td>
<td>0.418</td>
<td>1.738</td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.655)</td>
<td>(0.73)</td>
<td>(1.533)</td>
<td>(1.123)</td>
</tr>
</tbody>
</table>

**Other:**

\( \beta_c \): Overdraft constant

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.415</td>
<td>1.657</td>
<td>1.408</td>
<td></td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.68)***</td>
<td>(1.350)</td>
<td>(0.786)*</td>
<td></td>
</tr>
</tbody>
</table>

\( \beta_t \): Time dummy (0 = FACS)

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.019</td>
<td>0.014</td>
<td>0.167</td>
<td></td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.075)</td>
<td>(0.079)</td>
<td>(0.101)*</td>
<td></td>
</tr>
</tbody>
</table>

**Mundlak firm means**

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All but ( S_i ) &amp; ( E_i )</td>
<td>All but ( S_i ) &amp; ( E_i )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \sigma_\eta )</td>
<td>0.868</td>
<td>0.868</td>
<td>0.851</td>
<td></td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.033)***</td>
<td>(0.033)***</td>
<td>(0.034)**</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_\mu )</td>
<td>0.396</td>
<td>0.396</td>
<td>0.408</td>
<td></td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.066)***</td>
<td>(0.066)***</td>
<td>(0.064)**</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>0.723</td>
<td>0.724</td>
<td>0.714</td>
<td></td>
</tr>
<tr>
<td>Standard errors</td>
<td>(0.073)***</td>
<td>(0.073)***</td>
<td>(0.075)***</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs.</td>
<td>311</td>
<td>311</td>
<td>311</td>
<td>311</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-1128.342</td>
<td>-1128.125</td>
<td>-841.132</td>
<td>-278.762</td>
</tr>
</tbody>
</table>

\( H_0: \beta_u \geq 0 \) (p-value, LR test) 0.801

Confidence: *** → 99%, ** → 95%, * → 90%.

Parentheses show standard errors. For consistency, the confidence report is determined throughout by standard two-tailed Wald tests; note, however, that the structural model restricts \( \beta_u \geq 0 \) and \( \alpha_1 \leq 0 \) (and \( \sigma_\eta, \sigma_\mu \geq 0 \)). No weights are applied, and the errors are not clustered in any way. However, note that each recorded ‘observation’ is firm-level pair, for \( t = 0 \) and \( t = 1 \); therefore, the estimation is robust to firm-level serial correlation in errors and in signalled quality (\( Q_u \)).
Table 13: Structural estimates: Explaining firm quality by firm age, size, assets and sector (limited sample with inverse Mills’ Ratio)

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variables:</td>
<td>log(L)</td>
<td>S</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>X</td>
</tr>
</tbody>
</table>

**Signal effects:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1 (1)</th>
<th>Model 2 (2)</th>
<th>Model 3 (3)</th>
<th>Model 4 (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_s$: 'Incentive effect'</td>
<td>0.251 (0.101)**</td>
<td>0.178 (0.157)</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>$\beta_u$: 'Information effect'</td>
<td>0.000 (0.002)</td>
<td>0.000 (0.003)</td>
<td>.</td>
<td>.</td>
</tr>
</tbody>
</table>

**Signal costs:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1 (1)</th>
<th>Model 2 (2)</th>
<th>Model 3 (3)</th>
<th>Model 4 (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$: Constant signalling cost</td>
<td>-4.516 (0.722)**</td>
<td>-4.746 (0.708)**</td>
<td>-4.960 (0.759)**</td>
<td>.</td>
</tr>
<tr>
<td>$\alpha_1$: Ratio of signalling costs, $-(\gamma_0/\gamma_u)$</td>
<td>-1.306 (0.399)**</td>
<td>-1.680 (0.661)**</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>$\alpha_t$: Time dummy (0 = FACS)</td>
<td>-0.771 (0.096)**</td>
<td>-0.770 (0.096)**</td>
<td>-0.773 (0.098)**</td>
<td>.</td>
</tr>
</tbody>
</table>

**Observable quality, $Q_u$:**

**Basic characteristics:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1 (1)</th>
<th>Model 2 (2)</th>
<th>Model 3 (3)</th>
<th>Model 4 (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (years)</td>
<td>0.019 (0.007)**</td>
<td>0.013 (0.006)**</td>
<td>.</td>
<td>0.021 (0.006)**</td>
</tr>
<tr>
<td>Log (permanent employees)</td>
<td>0.279 (0.095)**</td>
<td>0.215 (0.096)**</td>
<td>.</td>
<td>0.365 (0.081)**</td>
</tr>
</tbody>
</table>

**Firm assets:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1 (1)</th>
<th>Model 2 (2)</th>
<th>Model 3 (3)</th>
<th>Model 4 (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (machines and equipment)</td>
<td>0.095 (0.035)**</td>
<td>0.08 (0.036)**</td>
<td>.</td>
<td>0.128 (0.049)**</td>
</tr>
<tr>
<td>Log (land and buildings)</td>
<td>0.022 (0.029)</td>
<td>0.021 (0.025)</td>
<td>.</td>
<td>0.059 (0.049)</td>
</tr>
<tr>
<td>Dummy: Land &amp; buildings missing or zero</td>
<td>0.314 (0.483)</td>
<td>0.302 (0.411)</td>
<td>.</td>
<td>0.578 (0.838)</td>
</tr>
</tbody>
</table>

**Other:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 1 (1)</th>
<th>Model 2 (2)</th>
<th>Model 3 (3)</th>
<th>Model 4 (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse Mills’ Ratio</td>
<td>1.532 (1.474)</td>
<td>1.279 (1.479)</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>$\beta_c$: Overdraft constant</td>
<td>3.305 (1.463)**</td>
<td>4.094 (1.589)**</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>$\beta_t$: Time dummy (0 = FACS)</td>
<td>0.02 (0.075)</td>
<td>0.009 (0.079)</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>Mundlak firm means</td>
<td>All but $\bar{S}_i$ &amp; $\bar{E}_i$</td>
<td>All</td>
<td>All but $\bar{S}_i$ &amp; $\bar{E}_i$</td>
<td>.</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>0.874 (0.035)**</td>
<td>0.873 (0.035)**</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>0.471 (0.061)**</td>
<td>0.467 (0.061)**</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.714 (0.074)**</td>
<td>0.713 (0.074)**</td>
<td>.</td>
<td>0.704 (0.076)**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Obs.</th>
<th>Log-likelihood</th>
<th>H0: $\beta_u \geq 0$ (p-value, LR test)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>311</td>
<td>-1151.339</td>
<td>0.153</td>
</tr>
<tr>
<td></td>
<td>311</td>
<td>-1149.679</td>
<td>No convergence</td>
</tr>
<tr>
<td></td>
<td>311</td>
<td>-280.553</td>
<td>.</td>
</tr>
</tbody>
</table>

**Confidence:** *** $\leftrightarrow$ 99%, ** $\leftrightarrow$ 95%, * $\leftrightarrow$ 90%.

Parentheses show standard errors. For consistency, the confidence report is determined throughout by standard two-tailed Wald tests; note, however, that the structural model restricts $\beta_u \geq 0$ and $\alpha_1 \leq 0$ (and $\sigma_\eta, \sigma_\mu \geq 0$). No weights are applied, and the errors are not clustered in any way. However, note that each recorded ‘observation’ is firm-level pair, for $t = 0$ and $t = 1$; therefore, the estimation is robust to firm-level serial correlation in errors and in signalled quality ($Q_u$).
### Table 14: Structural estimates: Explaining firm quality by firm age, size and assets (full sample)

<table>
<thead>
<tr>
<th></th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprobit: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variables: log(L)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>S</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
</tr>
</tbody>
</table>

#### Signal effects:

**\(\beta_s\): ‘Incentive effect’**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_s)</td>
<td>0.219</td>
<td>-0.208</td>
<td>-4.761</td>
<td></td>
</tr>
<tr>
<td>(SE)</td>
<td>(1.078)</td>
<td>(1.170)</td>
<td>(14.736)</td>
<td></td>
</tr>
</tbody>
</table>

**\(\beta_u\): ‘Information effect’**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_u)</td>
<td>0.357</td>
<td>0.522</td>
<td>3.381</td>
<td></td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.633)</td>
<td>(0.688)</td>
<td>(9.226)</td>
<td></td>
</tr>
</tbody>
</table>

#### Signal costs:

**\(\alpha_0\): Constant signalling cost**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_0)</td>
<td>4.867</td>
<td>4.980</td>
<td>1.736</td>
<td>5.132</td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.575)**</td>
<td>(0.589)**</td>
<td>(2.811)</td>
<td>(0.582)**</td>
</tr>
</tbody>
</table>

**\(\alpha_1\): Ratio of signalling costs, \(-\gamma_o/\gamma_u\)**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_1)</td>
<td>-1.331</td>
<td>-1.557</td>
<td>-0.384</td>
<td></td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.648)**</td>
<td>(0.879)*</td>
<td>(0.905)</td>
<td></td>
</tr>
</tbody>
</table>

**\(\alpha_t\): Time dummy (0 = FACS)**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_t)</td>
<td>0.717</td>
<td>0.717</td>
<td>0.254</td>
<td>0.728</td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.078)**</td>
<td>(0.079)**</td>
<td>(0.401)</td>
<td>(0.079)**</td>
</tr>
</tbody>
</table>

#### Observable quality, \(Q_o\):

**Basic characteristics:**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (years)</td>
<td>0.024</td>
<td>0.02</td>
<td>0.052</td>
<td>0.03</td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.012)**</td>
<td>(0.011)*</td>
<td>(0.063)</td>
<td>(0.005)**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (permanent employees)</td>
<td>0.21</td>
<td>0.175</td>
<td>1.449</td>
<td>0.261</td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.115)*</td>
<td>(0.111)</td>
<td>(1.718)</td>
<td>(0.054)**</td>
</tr>
</tbody>
</table>

**Firm assets:**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (machines and equipment)</td>
<td>0.083</td>
<td>0.074</td>
<td>-1.199</td>
<td>0.118</td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.044)**</td>
<td>(0.044)*</td>
<td>(0.347)</td>
<td>(0.037)**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Log (land and buildings)</td>
<td>0.059</td>
<td>0.054</td>
<td>0.038</td>
<td>0.096</td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.037)</td>
<td>(0.036)</td>
<td>(0.177)</td>
<td>(0.04)**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Dummy: Land &amp; buildings missing or zero</td>
<td>0.583</td>
<td>0.526</td>
<td>-0.275</td>
<td>1.007</td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.511)</td>
<td>(0.473)</td>
<td>(2.679)</td>
<td>(0.685)</td>
</tr>
</tbody>
</table>

**Other:**

**\(\beta_c\): Overdraft constant**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_c)</td>
<td>3.655</td>
<td>4.388</td>
<td>3.607</td>
<td></td>
</tr>
<tr>
<td>(SE)</td>
<td>(1.346)**</td>
<td>(1.604)**</td>
<td>(3.785)</td>
<td></td>
</tr>
</tbody>
</table>

**\(\beta_t\): Time dummy (0 = FACS)**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta_t)</td>
<td>-1.43</td>
<td>-2.17</td>
<td>-0.466</td>
<td></td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.224)</td>
<td>(0.236)</td>
<td>(0.961)</td>
<td></td>
</tr>
</tbody>
</table>

**Mundlak firm means**

- All but \(S_i\) & \(E_i\)
- All
- All but \(S_i\) & \(E_i\)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_\eta)</td>
<td>1.609</td>
<td>1.608</td>
<td>1.605</td>
<td></td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.052)**</td>
<td>(0.052)**</td>
<td>(0.052)**</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_\mu)</td>
<td>1.268</td>
<td>1.269</td>
<td>1.258</td>
<td></td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.08)**</td>
<td>(0.08)**</td>
<td>(0.08)**</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho)</td>
<td>0.751</td>
<td>0.749</td>
<td>0.746</td>
<td></td>
</tr>
<tr>
<td>(SE)</td>
<td>(0.056)**</td>
<td>(0.056)**</td>
<td>(0.056)**</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs.</td>
<td>488</td>
<td>488</td>
<td>488</td>
<td>488</td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-2456.052</td>
<td>-2455.454</td>
<td>-2042.408</td>
<td>-409.532</td>
</tr>
</tbody>
</table>

**\(H_0: \beta_u \geq 0\) (p-value; LR test)**: 1.000

### Confidence

- *** → 99%, ** → 95%, * → 90%.

Parentheses show standard errors. For consistency, the confidence report is determined throughout by standard two-tailed Wald tests; note, however, that the structural model restricts \(\beta_u \geq 0\) and \(\alpha_1 \leq 0\) (and \(\sigma_\eta, \sigma_\mu \geq 0\)). No weights are applied, and the errors are not clustered in any way. However, note that each recorded ‘observation’ is firm-level pair, for \(t = 0\) and \(t = 1\); therefore, the estimation is robust to firm-level serial correlation in errors and in signalled quality (\(Q_u\)).
Table 15: Structural estimates: Explaining firm quality by firm age, size, assets and liabilities (full sample)

<table>
<thead>
<tr>
<th>Dependent variables: log(L)</th>
<th>Joint</th>
<th>Joint: FE</th>
<th>Overdraft</th>
<th>Biprob: Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(L)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>S</td>
<td>✓</td>
<td>✓</td>
<td>✗</td>
<td>✓</td>
</tr>
</tbody>
</table>

**Signal effects:**

- \(\beta_s\): ‘Incentive effect’
  - \(\beta_s = -0.452\) (1.031) (76.451)
- \(\beta_u\): ‘Information effect’
  - \(\beta_u = 0.690\) (0.604) (47.899)

**Signal costs:**

- \(\alpha_0\): Constant signalling cost
  - \(\alpha_0 = 6.185\) (0.673)*** (0.694)***
- \(\alpha_1\): Ratio of signalling costs, \(- (\gamma_o / \gamma_u)\)
  - \(\alpha_1 = -1.103\) (0.467)***
- \(\alpha_t\): Time dummy (0 = FACS)
  - \(\alpha_t = 0.707\) (0.081)***

**Observable quality, \(Q_o\):**

**Basic characteristics:**

- Age (years)
  - \(0.023\) (0.009)***
- Log (permanent employees)
  - \(0.187\) (0.094)***

**Firm assets:**

- Log (machines and equipment)
  - \(0.053\) (0.038)
- Log (land and buildings)
  - \(0.056\) (0.039)
- Dummy: Land & buildings missing or zero
  - \(0.426\) (0.574)

**Firm liabilities:**

- Log (long-term debt)
  - \(0.165\) (0.085)**
- Dummy: Long-term debt missing, zero or negative
  - \(2.378\) (1.233)**

**Other:**

- \(\beta_c\): Overdraft constant
  - \(0.712\) (1.637)
- \(\beta_t\): Time dummy (0 = FACS)
  - \(-0.269\) (0.214)

- Mundlak firm means
  - \(\bar{S}_t, \bar{E}_t\)

- \(\sigma_{\eta}\)
  - \(1.601\) (0.051)***
- \(\sigma_{\mu}\)
  - \(1.193\) (0.083)***
- \(\rho\)
  - \(0.751\) (0.056)***

**Log-likelihood**

- Joint: \(-2430.917\)
- Joint: FE: \(-2430.505\)
- Overdraft: \(-2017.295\)
- Biprob: Signal: \(-404.014\)

**Confidence:** *** ↔ 99%, ** ↔ 95%, * ↔ 90%.

Parentheses show standard errors. For consistency, the confidence report is determined throughout by standard two-tailed Wald tests; note, however, that the structural model restricts \(\beta_u \geq 0\) and \(\alpha_1 \leq 0\) (and \(\sigma_{\eta}, \sigma_{\mu} \geq 0\)). No weights are applied, and the errors are not clustered in any way. However, note that each recorded ‘observation’ is firm-level pair, for \(t = 0\) and \(t = 1\); therefore, the estimation is robust to firm-level serial correlation in errors and in signalled quality (\(Q_u\)).
CONCLUSION

Summary. This thesis has studied the relationship between Moroccan manufacturing firms’ choice of legal status and their access to bank overdraft facilities; in doing so, the research has challenged some commonly-held views on credit access in emerging economies. Chapter One showed that the introduction of modern standards of corporate governance may induce some firms to choose a less onerous legal status, and that this may have negative relative effects upon credit provision. Chapter Two showed that, where a signal is constrained to be binary, different types of agent must pool together; even in a separating equilibrium, the principal cannot know the agent’s type precisely. As a result, the equilibrium outcome can depend upon the principal’s risk aversion and on the role of observable ‘indices’; these determinants are generally overlooked by signalling models, because they are not directly relevant where a signal is continuous. Chapter Three used a structural model to estimate that (for firms having a positive overdraft in both periods of the FACS/ICA panel) banks acted as if having full information about their clients.

Future directions. These results prompt several questions for further research. One explanation for the results of Chapter One is that banks did not update ‘rules of thumb’ for assessing firms’ creditworthiness to reflect the new legal regime. By the time of the ICA survey (2004), three years had already passed since the reform; however, it is possible that banks have changed their assessment since that time. This raises an important question about adaptation: did banks subsequently change their behaviour?
towards firms that had switched from SA to SARL? The World Bank conducted a third round of the FACS/ICA panel in 2006; the release of that data could assist in answering this question.

The results of Chapter One may also be useful for understanding the effect of overdraft facilities on firm performance. Chapter One showed that firms’ change of legal status induced a withdrawal of overdraft facilities from some firms. This could provide a natural experiment for evaluating the effect of bank overdraft facilities on firm profits and production. The identification strategy for such an evaluation would need to be more intricate than a standard instrumental variable approach, because the putative instrument (legal status) may have affected profits and production directly. However, a matching estimator might allow comparison between firms that lost their overdraft and those that retained it.

The theoretical model of Chapter Two could be extended in several directions. In its current form, the model is framed in terms of a single principal and a single agent; the introduction of market forces — for example, a zero-profit condition for the principal — may be a useful refinement. More fundamentally, a generalisation of the continuous-response version of the model would be valuable for understanding many circumstances in which a binary variable may be used as an informative signal. Similarly, both the binary-response and continuous-response models could be further generalised to allow for a ‘discrete signal’: a variable drawn from a finite set. Optimal control theory may assist several of these extensions.

Chapter Three suggests both empirical and methodological extensions. The chapter found that — for firms with a positive overdraft in both survey periods — banks act
Conclusion

as if having full information. This stands in contrast to general narratives about credit markets in emerging economies. However, there are several reasons that information asymmetry may be a substantial problem despite the results of Chapter Three. First, it may be that the model’s rationality assumption is unreasonable. Even though the specification passed a series of robustness checks, it remains possible that Moroccan banks behave in a very different way to that postulated; for example, banks may act as if they believe they have full information, but this belief may be wrong. Second, as noted in Chapter Three, the legal status signal may be orthogonal (or, at least, not strongly correlated) to the unobservables that concern banks. Third, information asymmetry may be more important for the decision whether to provide an overdraft than for the decision of how large the overdraft limit should be. At the end of Chapter Three, I estimated on the full sample and still did not find significant evidence of information asymmetry; however, as I noted in the chapter, this specification violates the model’s distributional assumption. All of these objections suggest further research.

Empirically, it may be valuable to use more data and a different identification strategy to consider the issue of information asymmetry. For example, a randomised experiment could involve auditing some firms’ accounts and business plans and revealing the results to their banks. Similarly, Morocco is in the process of introducing a national credit information bureau; I intend to research this reform in order to use it as a potential natural experiment.

Methodologically, there may be several extensions that could improve the robustness of the structural model. The most prominent limitation of the model is its inability to deal with the zero-overdraft case. An extension of the model might provide explicitly for a two-stage decision process, in which banks first decide whether to provide an
overdraft and, if so, then decide _how large_ that overdraft limit should be. Such an extension would pose formidable modelling challenges, particularly if the model were to remain robust to firm fixed effects (for example, the extension would pose difficulties analogous to those preventing the development of a ‘fixed-effect tobit’ estimator).

There may also be other valuable extensions; for example, the current model uses a linearised cut-off function, but this might be relaxed if an estimation algorithm were developed to solve iteratively for the equilibrium at each evaluation of the log-likelihood.

The relationship between Moroccan manufacturing firms and their banks provides a valuable example to inform policy. Most directly, this research is relevant for improving reforms of commercial law in emerging economies; the results of Chapter One show that such reforms may prove counterproductive if they induce a substantial share of firms to choose a less onerous legal status. More generally, the work suggests that information asymmetry may not pose a substantial problem for credit markets in emerging economies; it may be more useful if policy efforts to improve credit access were to focus on other reasons for credit constraints (including, for example, issues of lender illiquidity and of the efficacy of enforcement on collateral). Of course, these issues, too, remain to be explored further. This thesis has challenged several commonly-held views about credit access in emerging economies and, in doing so, prompts many avenues for future research.

\[ x \]

_Simon Quinn_