

# Is Identity Non-Contingent?

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## Abstract

I present a novel argument against the non-contingency of identity. I first argue that the necessity of distinctness is intimately connected with numerous paradoxes of recombination. In particular, I argue that those who reject the necessity of distinctness have natural solutions to various paradoxes of recombination which have plagued the metaphysics of modality. Moreover, I argue that adding the necessity of distinctness to modest, paradox-free assumptions is sufficient to reinstate the paradoxes. Given that identity is non-contingent only if distinctness is necessary, I suggest this constitutes new evidence against the non-contingency of identity.

If all identical things are necessarily identical, *identity is necessary*. If all distinct things are necessarily distinct, *distinctness is necessary*. If both identity and distinctness are necessary, *identity is non-contingent*.

Is identity non-contingent? I will argue that the necessity of distinctness is intimately connected with the paradoxes of recombination in ways that have not yet been appreciated. I will begin by highlighting that various paradoxes of recombination involve resources from which one can recover the necessity of distinctness. I will then argue that if distinctness is not necessary, these paradoxes all have natural solutions. Thus, insofar as one views such paradoxes as unresolved, there is reason to view the necessity of distinctness—and thus the non-contingency of identity—with suspicion. I shall also argue that the necessity of distinctness reinstates the paradoxes of recombination when added to modest, paradox-free assumptions. These two observations, I suggest, constitute new evidence against the non-contingency of identity.

I will first survey the central motivations for the non-contingency of identity (§1), and informally outline several paradoxes of recombination (§2). To highlight the connections between the principle and paradoxes,

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I will then formulate the paradoxes more carefully and at a higher level of generality (§3). Doing so will help to identify exactly how extant paradoxes involve resources from which one can recover the necessity of distinctness. It will also help to establish that with only slightly ‘weaker’ resources, from which one cannot recover the necessity of distinctness, the resultant paradoxes have natural solutions (§4). I will then show that adding the necessity of distinctness to these ‘weaker’ resources reinstates the paradoxes, which further strengthens the case against the principle. Finally, I will identify a class of paradoxes of recombination which do not require the necessity of distinctness, and argue that these paradoxes ought to be separated from the initial paradoxes of interest (§5).

## 1 The Necessity of Distinctness

A famous argument shows that the necessity of identity follows from extremely weak assumptions.<sup>1</sup> The argument is simple. If  $x$  is identical with  $y$ , then whichever properties  $x$  has  $y$  has too. Yet  $x$  has the property of being necessarily identical with  $x$ . Thus  $y$  has this property, so necessarily  $x$  is identical with  $y$ .

Proceeding more carefully, the argument can be stated in any language with an identity predicate  $=$ , the standard logical connectives, a necessity operator  $\Box$  (with  $\Diamond$  introduced by the usual abbreviation), and a variable-binding device  $\lambda$  used to form complex predicates. Given any such language, the argument can be understood as showing that the necessity of identity is a theorem of any system which includes the following logical assumptions along with the standard rules and axioms of classical logic.

**Reflexivity**  $v = v$

**Leibniz’s Law**  $v_1 = v_2 \rightarrow (Fv_1 \rightarrow Fv_2)$

**Necessitation** If  $A$  is a theorem, so is  $\Box A$

**Extensional  $\beta$**   $(\lambda v_1 \dots v_n. \phi) a_1 \dots a_n \leftrightarrow \phi[a_i/v_i]$ , where  $v_1, \dots, v_n$  are any distinct variables and  $\phi[a_i/v_i]$  ( $1 \leq i \leq n$ ) results from substituting each free occurrence of  $v_i$  in  $\phi$  for  $a_i$  successively, re-lettering bound variables so that no free variables in any  $a_i$  become bound

Put informally, the first three assumptions require that identity is a reflexive relation subject to Leibniz’s Law, and that the theorems of the system are closed under necessitation. The final assumption is the modest idea that the result of applying a predicate formed by  $\lambda$  to a sequence of terms (a  $\beta$ -*redex*) is materially equivalent to the result of substituting in  $\phi$  those terms in the correct order for the variables bound by  $\lambda$  (the corresponding  $\beta$ -*contractum*). To take an example, according to this assumption Jack is such that he is happy iff Jack is happy. The necessity of identity may then be argued for as follows.

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1.  $\Box x = x$  Refl, Nec  
<sup>1</sup>The argument is associated with both Ruth Barcan Marcus and Saul Kripke, although there is some evidence that the argument originated with Quine after he refereed Barcan Marcus (1947), which contains a more complex argument for the principle. See Burgess (2014).

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|--|------------|
| 2. $(\lambda z. \Box x = z)x$  | 1, $\beta$ |
| 3. $x = y \rightarrow ((\lambda z. \Box x = z)x \rightarrow (\lambda z. \Box x = z)y)$ | LL         |
| 4. $x = y \rightarrow (\lambda z. \Box x = z)y$  | 2, 3       |
| 5. $x = y \rightarrow \Box x = y$  | 4, $\beta$ |

Since this argument uses such modest assumptions, it is a powerful way of motivating the necessity of identity.<sup>2</sup>

Is distinctness necessary too? Although one might think the necessity of identity and distinctness stand in parity, the modest assumptions above are perfectly consistent with the failure of the necessity of distinctness. Indeed, the assumptions are even consistent with the bold claim that any infinity of things could have all been identical. Famously, however, Arthur Prior (1955) observed that our modest assumptions can be supplemented with two natural principles from which the necessity of distinctness follows: that what is necessary is closed under modus ponens, and that what is the case is necessarily possible.<sup>3</sup> These principles are known as the K and B axiom schemas of modal logic respectively.

$$\mathbf{K} \quad \Box(A \rightarrow B) \rightarrow \Box A \rightarrow \Box B$$

$$\mathbf{B} \quad A \rightarrow \Box \Diamond A$$

As before, the argument is straightforward. Since our initial assumptions establish the necessity of identity, we grant that if it is possible that  $x$  is distinct from  $y$  then  $x$  is distinct from  $y$ . Indeed, by Necessitation this must be necessary, so by the K axiom if it is necessarily possible that  $x$  is distinct from  $y$  then  $x$  is distinct from  $y$  as a matter of necessity. Yet by the B axiom if  $x$  is distinct from  $y$  then that is a necessary possibility. Thus, if  $x$  is distinct from  $y$  then  $x$  is distinct from  $y$  as a matter of necessity. Formally:

- |   |           |
|---|-----------|
| 1. $x = y \rightarrow \Box x = y$                     | NI        |
| 2. $\Diamond x \neq y \rightarrow x \neq y$           | 1         |
| 3. $\Box \Diamond x \neq y \rightarrow \Box x \neq y$ | 2, Nec, K |
| 4. $x \neq y \rightarrow \Box \Diamond x \neq y$      | B         |
| 5. $x \neq y \rightarrow \Box x \neq y$               | 3, 4      |

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<sup>2</sup>Certain advocates of *first-order contingentism*—the view that it is contingent which individuals exist—will not grant this argument's assumptions. For example, contingentists like Stalnaker (1994; 2003) maintain that standing in a relation requires existence, i.e. the 'Being Constraint' from Williamson (2013, chap. 4). Yet, as Williamson (2013, pp. 154) notes, such contingentists will have to reject Reflexivity lest they lapse into necessitism. Nonetheless, they may still accept an existentially qualified version of Reflexivity:

$$\exists \text{Reflexivity} \quad \exists x x = v \rightarrow v = v$$

Yet from this qualified principle, one may derive an existentially qualified version of the necessity of identity using an argument similar to the above:

$$\exists \text{NI} \quad \forall x \forall y (x = y \rightarrow \Box (\exists v v = x \rightarrow x = y))$$

Relatedly, according to the particular version of counterpart theory developed by David Lewis (1968), identity is not necessary. On one reconstruction of Lewis's particular theory, this requires failures of Extensional  $\beta$ ; see Dorrr et. al. (forthcoming, chap. 10) for discussion of this point.

<sup>3</sup>Prior's own argument assumes that necessity obeys S5, however Kripke (1972) observed that only these two additional principles are actually needed.

Consequently, from this still sparse collection of assumptions one can produce a deductive argument for the non-contingency of identity.

Nevertheless, the B axiom schema is much more contentious than any of the previous assumptions. Although philosophical folklore has it that metaphysical necessity obeys the modal logic S5, which includes all instances of B, the extant arguments for the schema are not particularly persuasive; they are also few and far between. For example, Timothy Williamson (2013, chap. 3, sect. 3.3) argues that the logic of metaphysical necessity is S5 on the grounds that the theorems of S5 are exactly those formulas with the special status of ‘metaphysical universality’.<sup>4</sup> However, as Williamson makes explicit, his argument just assumes that all instances of B are themselves metaphysically universal. There is also a style of picture-thinking according to which metaphysical necessity in some sense concerns *all* possible worlds and therefore obeys S5.<sup>5</sup> But rather than motivating the B axiom, this picture-thinking tends to rely on some less obvious version of it, for instance the principle that each world is necessarily a *possible* world. Moreover, there is a noteworthy disparity between the somewhat underwhelming motivation for B and the compelling motivation for the axioms of the system S4, which are motivated by the guiding idea that metaphysical necessity is either the broadest species of necessity, or perhaps the broadest species of ‘objective’ or ‘real’ necessity.<sup>6</sup> (Although I shall remain neutral about which of these modalities is metaphysical necessity, I assume that one of them is; furthermore, if these species of necessity are distinct, my argument will apply to both of them anyway.)

Since I shall contend that different resources required by different paradoxes of recombination result in the necessity of distinctness, I shall not assume the B axiom. Alongside this, my core modal assumptions will not include the closely related Barcan formula, which states that if it is possible that some individual is a certain way, there is an individual which is possibly that way. (Intuitively, one may think of the Barcan formula as stating that it is not possible for there to be new individuals.) It is well known that adding the B axiom schema to a natural system of quantified S4 results in all instances of the Barcan formula (although not vice-versa).<sup>7</sup> Nonetheless, without the B axiom the motivation for the Barcan formula is not particularly clear, even in settings where it is often just tacitly assumed (see §3.1).

With these preliminaries in place, the main theses of the article can be outlined more carefully:

<sup>4</sup>As defined by Williamson, a formula *A* of the usual propositional modal language is *metaphysically universal* just if the formula which results from uniformly substituting distinct propositional variables for distinct proposition letters in *A* and then adding a prefix of propositional quantifiers binding each of the new propositional variables is true on the intended interpretation of this extended quantificational language.

<sup>5</sup>See Hale (2020, pp. 142-143) for a recent version of this argument. This way of motivating S5 is often associated with Lewis (1986), whose concrete modal realism is hospitable to the style of argument.

<sup>6</sup>For relevant background see Bacon (2018), who characterises what it is to be the broadest species of necessity in a certain coarse-grained setting, and Bacon & Zeng (forthcoming) who adapt this characterisation to settings which involve weaker assumptions about granularity; see also Dorr (2016, pp. 68-70). Following Williamson (2016), Roberts (MS) characterises what it is to be the broadest species of ‘objective’ necessity and suggests that it is different from the broadest species of necessity *simpliciter*. According to each characterisation of these (perhaps different) species of necessity, S4 is a lower bound of the logic of the necessity in question. Nonetheless, each characterisation can be supplemented with further, less obvious assumptions which secure S5. Relatedly, Ditter (2020) argues that if necessity is reduced to essence, then it only obeys S4 and not S5.

<sup>7</sup>For this result, the underlying quantification theory of the system must include universal generalisation and the standard schema of universal instantiation. Moreover, it is crucial that open formulas are counted as theorems of the system, for as Kripke (1963) observes, there is a system of quantified S5 that is axiomatised only in terms of closed formulas which lacks the Barcan formula and its converse.

**Requirement** The paradoxes of recombination require the necessity of distinctness insofar as:

- (a) they all use resources from which one can recover the necessity of distinctness (without the B axiom), and
- (b) with slightly weaker resources, from which one cannot recover the necessity of distinctness, the paradoxes of recombination have natural solutions.

**Sufficiency** In the presence of the weaker assumptions, accepting the necessity of distinctness reinstates the original paradoxes.

Thus, by Requirement, those who reject the necessity of distinctness will be able to handle the paradoxes of recombination naturally. Moreover, since the B axiom implies the necessity of distinctness in the presence of our initial assumptions, Sufficiency may be considered as new evidence against the B axiom itself.

To close these preliminaries, let me be clear that I am not offering a refutation of the necessity of distinctness, or anything nearly as strong as that. As we shall see, there are other arguments which motivate the principle which are harder, although not impossible, to resist. Nevertheless, Requirement and Sufficiency create significant theoretical pressure in the other direction. At the very least, *pace* Williamson (1996), they may lead one to doubt that the underderivability of the necessity of distinctness in certain systems is merely an artefact of their expressive limitations. Indeed, together they isolate how a local modal asymmetry between identity and distinctness may generate a more globally elegant conception of metaphysical necessity.

## 2 Paradoxes of Modal Recombination

In this section, I will informally outline several paradoxes of recombination. In the following section, I will formulate these paradoxes much more carefully and at a higher level of generality. However, for now the aim will be to appreciate what motivates the puzzles. One of the main points to keep in mind is that I shall formulate the paradoxes without assuming the B axiom, which constitutes a small departure from the way in which their authors present them.

### 2.1 Class Puzzle

The first recombinatorial puzzle is a variant of a puzzle due to Peter Fritz (2017).<sup>8</sup> It can be reconstructed with two natural assumptions about classes and their modal profiles. The first assumption is a modest comprehension principle for classes.<sup>9</sup>

**Modest Class Comprehension** Necessarily, for any physical property there is a class of exactly

<sup>8</sup>Fritz's puzzle is related to that of Fine (2003, pp. 223-224). However, Fine's puzzle is based around cardinality considerations, whereas Fritz's is not.

<sup>9</sup>If one understands Modest Class Comprehension as a schema whose instances concern only physical predicates then, to adapt a notion from Uzquiano (2015), the schema is not only predicative but *ultrapredicative*: its instances 'comprehend' on predicates which contain no class constants or variables whatsoever. It is also worth noting that a recombinatorial puzzle could be generated by weakening Modest Class Comprehension by deleting 'exactly' from it and the principles of recombination considered below.

everything which has it.

The second principle intuitively requires that the members of a class stay fixed across possibilities. More carefully, it requires that class membership is *modally constant* in the sense that (i) whatever is a member of a given class is necessarily a member of that class, (ii) whatever is not a member of a given class is necessarily not a member of that class, and (iii) that class membership is also modally non-increasing.<sup>10</sup> Although the necessity of class membership implies that class membership is modally non-decreasing, without the Barcan formula the necessity of class non-membership does *not* imply that class membership is modally non-increasing, hence the explicit inclusion of that requirement. The issue is that without the Barcan formula it is possible for there to be new existents which might be members of the class in question, even if no actual non-member becomes one of the classes's members. Moreover, without the necessity of distinctness the fact that class membership is modally non-increasing does *not* imply that whatever is not a member of a given class is necessarily not a member of that class. For the class might not acquire any new members at some possibility, but some individual which is not actually a member of the class might become identical with one of its members at the possibility in question. Thus, it is important to separate these two conditions in the current setting and characterise modal constancy in terms of both of them for it to work in the intended way.

**Class Constancy** Necessarily, class membership is modally constant.

The puzzle itself centres around two principles of recombination which are motivated in extremely similar ways. Strictly speaking, the principles may be understood schematically, as concerning the possible patterns of instantiation which certain intrinsic qualitative properties may exhibit. However, to fix ideas, I shall state the principles in terms of a representative property: being an electron. On this understanding, the first principle states that no matter what electrons there are, it is metaphysically possible for them all to be electrons along with some other electron. This principle is motivated by an aversion to positing arbitrary divisions between what is and is not metaphysically possible. After all it seems arbitrary to deny, for example, that necessarily whatever electrons there are, it is possible for them all to be electrons along with some duplicate of one of them which is located in a different region to any of the other electrons; were there to be no such possibility, the idea that metaphysical possibility has an extremely liberal extent would seem to be undermined. Indeed, this seems particularly plausible if the Barcan formula fails.

The second principle of recombination is motivated in a similar way. The intuitive idea behind the principle is that it ought to be the case that all possible electrons could be electrons together in a single possibility. Again, if this were not metaphysically possible there would be a concern of modal arbitrariness.

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<sup>10</sup>This notion of modal constancy is based on Linnebo's (2013) notion of extensional definiteness, which he characterises in a B-free setting. See §3.2 for a closely related formal definition. (Note also that some contingentists may prefer to formulate condition (i) as conditional on the existence of certain entities as follows: whatever is a member of a given class is necessarily a member of that class if both it and the class exist. Nevertheless, as I explain in §3, the more formal discussion of the paradoxes shall take place under the assumption of necessitism, which can be treated as a purely simplifying assumption for the purposes of the core argument.)

For if it is not possible that every possible electron is in fact an electron, what is the obstacle to their compossibility?<sup>11</sup> It is difficult to be comfortable with the idea that there are some particular possible electrons such that it is possible that every possible electron is in fact an electron *except those particular possible electrons*.

To reduce complexity, I shall follow Agustín Rayo’s (2020, pp. 10-11) streamlined versions of these two thoughts, which only partly capture the two thoughts but still suffice to generate the puzzle.

(C1) Necessarily, whichever class comprises exactly the electrons, possibly there is an electron which is necessarily not a member of it.

(C2) Possibly, some class comprises exactly the electrons and is such that necessarily every electron is possibly a member of it.

It is crucial to appreciate that these principles only capture the intended recombinatorial ideas due to Modal Class Comprehension and Class Constancy. Modest Class Comprehension ensures that necessarily there is a class of electrons, so (C1) is not trivialised and (C2) has an appropriate witness. Moreover, Class Constancy ensures that the modal facts about the (non-)membership of a class reflect non-modal facts about which things are (non-)members of it. Nonetheless, although both recombinatorial principles are motivated by the same aversion to arbitrariness, they are incompatible in the presence of these two assumptions about classes. By (C2) and Modest Class Comprehension, it is possible that there is a class of electrons such that necessarily every electron is possibly a member of it. Thus, by Class Constancy, every such possible electron is in fact a member of the class at the initial possibility. But this contradicts (C1), for at the possibility in question it is now impossible for there to be an electron which necessarily not a member of that class.

To be clear, the source of puzzlement is not that (C1) and (C2) seem like unassailable modal principles. Rather, the puzzle is that both principles are motivated by exactly the same aversion to arbitrariness which has been deemed integral to the theoretical role of metaphysical necessity. Yet given their similar motivation, there is no obvious symmetry breaker between the two principles.<sup>12</sup> It thus strikes one as unprincipled to reject one of the principles over the other. Moreover, given their theoretical basis, to jettison both principles without adequate reason seems like an impatient reaction to the puzzle. With due patience, the puzzle highlights a tension in the idea that metaphysical possibility—indeed *any* species of possibility—can have the type of liberal recombinatorial profile that metaphysical possibility is often unreflectively assumed to

<sup>11</sup>Certain concrete modal realists like Lewis (1986) might reject such claims because they postulate limitations on the size of possible spacetimes. Nevertheless, as Hawthorne & Uzquiano (2011) note, the argument can be stated in terms of intrinsic qualitative properties which admit the possibility of co-location amongst their instances.

<sup>12</sup>Regarding a closely related argument (see f.n. 8 above), Fine (2003, p. 224) suggests that it would be ‘most natural’ to reject a principle like (C2) over (C1) on the basis of a ‘limitation of size’ principle. On this view, there would in Fine’s words be a ‘potential infinity of possible objects’ which each possibility is ‘incapable of accommodating’. However, without an argument that there cannot be more than set-many things, it is not clear what is supposed to motivate a ‘limitation of size’ principle of that character. Another ‘potentialist’ view, which is quite dissimilar to Fine’s, is explored by Fritz (2017), Roberts (2019) and Rayo (2020). This alternative potentialist view, however, is not motivated by a limitation of size principle. Moreover, Fritz and Roberts do not posit a significant asymmetry between (C1) and (C2); rather, each principle is seen as expressing a true proposition relative to different respective ‘contexts’ (Fritz) or ‘interpretations’ (Roberts). In contrast, Rayo rejects (C2) on the basis of general considerations about the ‘open-endedness’ of modal space.

have.

## 2.2 Plurals Puzzle

When the class-based puzzle was presented, it might have been noticeable that the principles (C1) and (C2) were motivated without even invoking the ideology of classes. Instead, the motivation used plural locutions like ‘the electrons’. Peter Fritz (2017) has stressed this point, and has formulated a ‘pure’ version of the class puzzle using plural quantification. Fritz’s plural puzzle is pure in the sense that it does not involve any extraneous objects to the electrons themselves, such as classes thereof.

Fritz (2017, Appendix B) uses a standard unrestricted modal plural comprehension principle to formulate his puzzle, but like previously the puzzle only requires a certain modest comprehension principle.

**Modest Plural Comprehension** Necessarily, for any physical property there are some things which  
comprise exactly everything which has it.

As before, the puzzle also exploits the modal constancy of plural membership. Although Fritz does not note this explicitly, it is a standard assumption of modal plural logic and is validated by his semantics.<sup>13</sup>

**Plural Constancy** Necessarily, plural membership is modally constant.

Fritz’s puzzle then centres around plural analogues of (C1) and (C2), which are motivated in completely parallel fashion.

**(P1)** Necessarily, whichever things comprise exactly the electrons, it is possible that there is an electron  
which is necessarily not one of those things.

**(P2)** It is possible that there are some things, the electrons, and it is necessary that every electron is  
possibly one of those things.

As before, (P1) and (P2) capture the intended thoughts only due to the two assumptions about plurals. Yet, once again, (P1) and (P2) are incompatible in the presence of these two assumptions. By (P2) possibly there are some things (the electrons) which comprise exactly every electron, and necessarily every electron is possibly one of them. So by Plural Constancy, every such possible electron is in fact one of the electrons at the initial possibility. But, as before, that is incompatible with (P1).

## 2.3 Actuality Puzzle

Another similar puzzle due to Fritz uses devices of modal anaphora instead of plural quantification. One can think of devices of modal anaphora as generalised actuality operators. For example, consider the sentence ‘it is possible that everyone who is actually rich is poor’. One might want to say something more general

<sup>13</sup>See Fritz (2017, p. 559). For useful background on modal plural logic, see Linnebo (2013, 2016) and Williamson (2016b).



than this, namely that it is not just a fact about actuality but as a matter of necessity it is true of everyone who is rich. Devices of modal anaphora, like the pair of expressions ‘as matters stand’ and ‘in fact’, are designed to express such generalisations. Put metaphorically, one may think of ‘as matters stand’ as taking a portrait of how things stand. This portrait can then be carried to other possibilities and retrieved by ‘in fact’ as a reminder of how matters stood, even when such matters are no longer possible (due the failure of the B axiom). With this in mind, we can then use these operators to express the more general claim with the sentence ‘necessarily *as matters stand* it is possible that everyone who was *in fact* rich is poor’. These informal devices of modal anaphora are typically formalised by so-called ‘Vlach operators’  $\uparrow$  (as matters stand) and  $\downarrow$  (in fact), which will receive a proper treatment later on.<sup>14</sup>

Fritz uses such devices to articulate another ‘pure’ recombinatorial puzzle. Before presenting the puzzle, however, it is important to appreciate two constraints to which the devices are subject in order to meet their intended design. The first constraint guarantees that the devices of anaphora are *non-forgetful* in the sense that whatever is the case must remain in fact the case as a matter of necessity: ‘in fact’ never forgets how matters stand. In terms of the portrait metaphor, this requirement amounts to the idea that portraits never lose information when carried to another possibility. The second constraint guarantees that the devices of anaphora are *non-enriching* in the sense that if something is possibly in fact the case, it is the case: ‘in fact’ never enriches how matters stand. Again in terms of the portrait metaphor, this requirement amounts to the idea that no extra information is added to a portrait when it is carried to different possibilities. Together, these constraints ensure that how matters stand is never distorted by the devices as a matter of necessity.

**No Forgetting** Necessarily as matters stand, whatever is the case is necessarily in fact the case.

**No Enriching** Necessarily as matters stand, whatever is possibly in fact the case, is the case.

Although Fritz does not consider these two constraints explicitly, formal versions of them are validated by the standard semantics for the Vlach operators which he uses.

With these two constraints in mind, Fritz’s second puzzle can be articulated. Once again, it centres around two principles which are motivated by the aversion to arbitrariness from before.

(A1) Necessarily as matters stand it is possible for there to be an electron which is necessarily not in fact an electron.

(A2) Possibly as matters stand necessarily every electron is possibly in fact an electron.

However, these two recombinatorial principles are incompatible in the presence of our assumptions about the devices of anaphora. To merely sketch the argument for now, by (A2) it is possible for one to take a portrait of how things stand such that any possible electron is possibly an electron according to that portrait. Thus,

<sup>14</sup>For relevant background see Vlach (1973). In order to achieve further expressive power, some, such as Correia (2007), have explored introducing infinitely many pairs of such operators. Such further expressive power is not needed here, although it could easily be captured by the means introduced used below.

by No Enriching, every such possible electron is an electron at the initial possibility. But then, in contrast to (A1), when one takes a portrait at the initial possibility it is impossible for there to be an electron which is necessarily not an electron according to that portrait. For we just argued that at the initial possibility the electrons are just all the possible electrons, and by No Forgetting whatever is an electron at the initial possibility must remain an electron according to the portrait of that possibility.

### 3 Behind the Paradoxes

Although it was helpful to present the paradoxes informally, it is important not to leave the presentation at an informal level. One of the principal concerns is that each paradox used quantified modal reasoning that tends to rely on assumptions which are hidden from plain view, or even invoke assumptions which are not strictly required. Since my contention about the necessity of distinctness pertains to exactly which assumptions each paradox requires, it is thus crucial that the paradoxes are formulated much more carefully. Indeed, that is the purpose of this section. The strategy will also be to formulate the paradoxes at a higher level of generality, so that they can all be presented within a single system.

#### 3.1 Background Logic

Each of the paradoxes involved distinctive resources, such as plural quantification, generalised actuality operators, or a modest theory of classes. Although one may think that these different resources are completely separate pieces of ideology, they can all be captured by using a general type of instrument for metaphysical theorising: a higher-order language. Indeed, one might even take the view that some of these seemingly distinct pieces of ideology are just different ways of articulating one and the same higher-order resource. However, that view will not be mandatory in what follows: the central aim will be to develop a general framework in which each of the paradoxes can be articulated, which can then be probed to see which assumptions the paradoxes share and require. This approach will have the benefit of allowing one to isolate how the necessity of distinctness plays a structurally similar role in the plural, class-based and actuality-based versions of the paradoxes. Moreover, it will be clear how to adapt these observations to the versions of the paradoxes which explicitly use either plural quantification, quantification over classes, or Vlach operators.

One can think of a higher-order language as a function from a vocabulary to a collection of terms, which includes expressions like formulas and predicates built syntactically from the particular vocabulary in question. In higher-order languages, each term (and piece of vocabulary) has a *type* which determines its syntactic profile—which expressions it can be combined with to produce which terms of different types. In the particular typing system I shall use, there is a single ‘base’ type  $e$  (the type of individual terms). The ‘complex’ types, such as that of predicates of individual terms, are provided by the rule that whenever  $\tau_1, \dots, \tau_n$  are types,  $\langle \tau_1, \dots, \tau_n \rangle$  is a type. Thus, for example, unary predicates of individual terms are of type  $\langle e \rangle$  and formulas are of type  $\langle \rangle$ . Languages based on such typing systems are called *relationally typed*.

Without labouring through the syntactic details, I shall be using what Dorr (2016, Appendix A2) calls a  $\lambda K$ -language based on a relationally typed signature (intuitively, a signature is a vocabulary whose members have already been assigned types). In fact, the vocabulary on which our particular higher-order language will be based contains the usual stock of logical constants and the two expressions ‘ $\Box$ ’ and ‘ $E$ ’, which express metaphysical necessity and the property of being an electron on their intended readings respectively. The expression ‘ $\Box$ ’ is a sentential operator, so it has type  $\langle\langle\rangle\rangle$ , meaning that when it is combined with a formula it produces a formula expression. The expression ‘ $E$ ’ is a predicate of individual terms, so it has type  $\langle e \rangle$ . Other terms of the language will include predicates of individuals like ‘ $\lambda x^e. \exists X^{\langle e \rangle} \Box Xx$ ’ (being an individual which has least one property necessarily), sentences like ‘ $\Box \forall x^{\langle \rangle} (\Box x \rightarrow x)$ ’ (metaphysical necessity is necessarily truth-implying), and predicates of propositional operators like ‘ $\lambda X^{\langle \langle \rangle \rangle} . \forall W^{\langle \langle \rangle \rangle} (\exists Z^{\langle \langle \rangle \rangle} W(Z) \rightarrow \forall Y^{\langle \langle \rangle \rangle} (W(Y) \rightarrow \Box \forall x^{\langle \rangle} (Xx \rightarrow Yx)))$ ’ (strictly implying every instance of every non-empty property of propositional operators). I shall often omit type superscripts when they are obvious from the context, and write  $\alpha : \sigma$  to abbreviate the claim that expression  $\alpha$  is of type  $\sigma$ . I shall also adopt a number of stylistic conventions: I use lowercase variables  $x, y, z$  for type  $e$  variables, lowercase variables  $p, q$  for type  $\langle \rangle$  variables, and notate ‘ $\forall_\sigma (\lambda x^\sigma . \phi)$ ’ using the more familiar ‘ $\forall x^\sigma \phi$ ’. Finally, by ‘an entity of type  $\sigma$ ’, I intend the more long-winded ‘an entity which is a possible semantic value of an expression of type  $\sigma$ ’.

The *background logic* to the entire discussion will be a fairly weak system called **H4**. In effect, it is just a minimal higher-order logic with the postulates of the modal system **S4**. In addition to these postulates, **H4** comprises a classical quantification theory (for quantifiers of each type) and the principle Extensional  $\beta$ .

**PL** All instances of propositional tautologies.

**MP** From  $A$  and  $A \rightarrow B$  infer  $B$

**Gen** From  $A \rightarrow B$  infer  $A \rightarrow \forall_\sigma x B$  when  $x$  does not occur free in  $A$

**UI**  $\forall_\sigma F \rightarrow Ft$ , where  $F : \langle \sigma \rangle$  and  $t : \sigma$

**Extensional  $\beta$**   $(\lambda v_1 \dots v_n . \phi) a_1 \dots a_n \leftrightarrow \phi[a_i/v_i]$ , where  $v_1, \dots, v_n$  are any distinct variables,  $a_1, \dots, a_n$  are any terms such that  $v_i, a_i : \tau_i$  for all  $1 \leq i \leq n$ , and  $\phi[a_i/v_i]$  results from substituting each free occurrence of  $v_i$  in  $\phi$  for  $a_i$  successively, re-lettering bound variables so that no free variables in any  $a_i$  become bound

**K**  $\Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$

**T**  $\Box p \rightarrow p$

**4**  $\Box p \rightarrow \Box \Box p$

**Nec** From  $A$  infer  $\Box A$

As is standard, identity is treated as higher-order indiscernibility. More formally, when  $x$  and  $y$  are variables of the same type  $\sigma$ , the string  $x = y$  abbreviates the formula  $\forall X (Xx \leftrightarrow Xy)$ , where  $X$  is of type  $\langle \sigma \rangle$ . As a

result, there are two easy theorem schemas of **H4** which correspond to the initial assumptions of Reflexivity and Leibniz’s Law (I reuse the labels for convenience).

**Reflexivity**  $v = v$ , where  $v$  is a variable of any type

**Leibniz’s Law**  $v_1 = v_2 \rightarrow \forall X(Xv_1 \rightarrow Xv_2)$ , where  $v_1, v_2 : \sigma$  and  $X : \langle \sigma \rangle$

It is also straightforward to show that identity at a given type is an equivalence relation—indeed, the most stringent equivalence relation—on the entities of that type. Nonetheless, besides these rudimentary facts about identity, **H4** is fairly ‘grain-neutral’. For all that has been said, higher-order entities might be individuated extensionally, or by metaphysically necessary equivalence, or by some even finer grained criterion. Indeed, since **H4** only includes Extensional  $\beta$  there is no guarantee that a given  $\beta$ -redex is even identical with its corresponding  $\beta$ -contractum. With that being said, due to the Russell-Myhill paradox, **H4** is flatly inconsistent with at least one natural version of the structured theory of propositions.<sup>15</sup>

A less obvious theorem schema is that of *necessitism*, the thesis that necessarily whichever entities (of any type) exist necessarily exist.

**Necessitism**  $\Box \forall x \Box \exists y (y = x)$ , where  $x, y : \sigma$  for any type  $\sigma$

Although necessitism is a controversial thesis, it follows from the classical quantification theory and our rule of necessitation for metaphysical necessity, both of which are highly plausible assumptions. Moreover, as Fritz (2016, Appendix A) shows, our paradoxes still arise within the presence of ‘free’ quantification theory, which is usually adopted to block the derivation of necessitism.<sup>16</sup> Thus it will be harmless to work under the assumption of necessitism for current purposes. Supplementing this, for essentially the same reasons, **H4** proves all instances of another controversial modal thesis: the converse Barcan formula.

**CBF**  $\Box \forall X (\Box \forall x Xx \rightarrow \forall x \Box Xx)$ , where  $x : \sigma$  for any type  $\sigma$

Intuitively, one can think of CBF as stating that existence (at any type) is non-decreasing: no entity of any type can fail to exist. This is of course importantly different from the Barcan formula, which is not a theorem of **H4** due to the absence of the B axiom (recall the discussion from §1). Thus, without B, necessitists may maintain that it is possible for there to be new individuals, even though no individual can fail to exist.

Given that **H4** includes versions of the four key assumptions used in the initial derivation of the necessity of identity, it too is a theorem of the system. In fact, as one may have already anticipated, the necessity of identity holds with respect to identity at all types.

**NI**  $x = y \rightarrow \Box x = y$ , where  $x, y : \sigma$  for any type  $\sigma$

Nonetheless, not all instances of the necessity of distinctness (at arbitrary types) are theorems of **H4**. Thus,

<sup>15</sup>See Dorr (2016, pp. 63-64) and Goodman (2017).

<sup>16</sup>For the relevant background, see Stalnaker (1994, 2003) and Williamson (2013, chap. 4).

not only is there no guarantee that distinct individuals are necessarily distinct, but one also has to keep in mind that distinct higher-order entities of the same type, such as propositions, properties of individuals, propositional operators, and so on, may be possibly identical. This feature will become relevant at various points.

It is also helpful to observe that **H4** proves a modal comprehension schema. Intuitively, this schema is an existence postulate for higher-order entities. Put roughly, it states that for any complex type there is some entity of that type such that necessarily it is had by the things which meet condition  $\phi$ .<sup>17</sup>

**Comp**  $\exists X \Box \forall x_1 \dots x_n (X x_1 \dots x_n \leftrightarrow \phi)$ , where  $X : \langle \tau_1, \dots, \tau_n \rangle$  and  $x_1 : \tau_1, \dots, x_n : \tau_n$

In the presence of **Comp**, we can assure ourselves of the existence of various higher-order entities in **H4**. To take one example from before, **Comp** secures the existence of a property which, as a matter of necessity, is had by exactly those individuals which have some property necessarily:  $\exists X \Box \forall x (Xx \leftrightarrow \exists Y \Box Yx)$ . Indeed, in **H4** everything has such a property, since everything has the property of self-identity necessarily (by Reflexivity,  $\beta$  and Nec). To take another more *recherché* example from before, **Comp** also secures the existence of a property which, as a matter of necessity, is had by exactly those propositional operators which strictly imply every instance of every non-empty property of propositional operators:  $\exists W \Box \forall X (W(X) \leftrightarrow \forall U (\exists Z U(Z) \rightarrow \forall Y (U(Y) \rightarrow \Box \forall p (Xp \rightarrow Yp))))$ , where  $W, U : \langle \langle \langle \rangle \rangle \rangle$  and  $X, Y, Z : \langle \langle \rangle \rangle$ . Interestingly, in **H4** this property is non-empty since by **Comp** there is a propositional operator which necessarily applies to no proposition truly ( $\lambda p. p \rightarrow \neg p$ ). Such examples provide a glimpse of the vast existential commitments of **H4**, despite the theory's modesty in other respects. These commitments make **H4** particularly useful for characterising the different resources used by the various paradoxes of recombination.

### 3.2 Classes and Plurals

The first task is to reformulate the class and plurals based puzzles. Although classes of individuals and plural quantification over individuals resemble properties of individuals and quantification over such properties respectively, there are important modal differences. Most notably, the instances of a property may vary modally, whereas class and plural membership are modally constant. So, class- and plural-talk must be simulated by higher-order quantification over *modally constant* entities. However within **H4** we cannot yet guarantee that there is a sufficient portfolio of such entities (in effect, this was established in Gallin 1975).

Thus, in order to reformulate the two puzzles, an extension to **H4** is required.

<sup>17</sup>The derivation of **Comp** may be less familiar, so it is worth presenting explicitly. Primarily, the argument exploits instances of Extensional  $\beta$  which involve a  $\beta$ -redex in which the  $\lambda$ -abstract is applied to the same variables which the  $\lambda$  device binds. (I shall use 'PC' to indicate that an inference relies on some combination of PL and the quantification theory.)

- |  |                         |
|--|-------------------------|
| 1. $(\lambda x_1 \dots x_n. \phi) x_1 \dots x_n \leftrightarrow \phi[x_i/x_i]$                         | $\beta$                 |
| 2. $\forall x_1 \dots x_n ((\lambda x_1 \dots x_n. \phi) x_1 \dots x_n \leftrightarrow \phi[x_i/x_i])$ | 1, PC                   |
| 3. $\forall x_1 \dots x_n ((\lambda x_1 \dots x_n. \phi) x_1 \dots x_n \leftrightarrow \phi)$          | 2, Def. $\phi[x_i/x_i]$ |
| 4. $\Box \forall x_1 \dots x_n ((\lambda x_1 \dots x_n. \phi) x_1 \dots x_n \leftrightarrow \phi)$     | 3, Nec                  |
| 5. $\exists X \Box \forall x_1 \dots x_n (X x_1 \dots x_n \leftrightarrow \phi)$                       | 4, PC                   |

To present the extension, I shall define the notion of modal constancy in higher-order terms.<sup>18</sup> As before, the intuitive idea is that a higher-order entity is modally constant exactly when its (non-)instances are necessarily its (non-)instances, and its instances are modally non-increasing. In H4, this latter condition can be characterised by the higher-order entity in question admitting a restricted version of the Barcan formula. (In the next three definitions,  $\sigma = \langle \sigma_1, \dots, \sigma_n \rangle$  is any complex type and  $x$  is a sequence of variables  $x_1, \dots, x_n$  such that  $x_1 : \sigma_1, \dots, x_n : \sigma_n$ .)

$$\begin{aligned} \textbf{Constancy } \textit{Const}^{(\sigma)} &:= \lambda X^\sigma (\Box \forall x (Xx \rightarrow \Box Xx) \wedge \Box \forall x (\neg Xx \rightarrow \Box \neg Xx) \wedge \\ &\quad \Box \forall Y^\sigma (\forall x (Xx \rightarrow \Box Yx) \rightarrow \Box \forall x (Xx \rightarrow Yx))) \end{aligned}$$

The theory H4 is then extended to the theory H4C with a *constant comprehension principle* which states that necessarily every property or relation is coextensive with a constant entity.<sup>19</sup> To reduce notational clutter, it will help to introduce another abbreviation before stating this principle.

$$\textbf{Coextension} \equiv^{(\sigma, \sigma)} := \lambda X^\sigma Y^\sigma \forall x (Xx \leftrightarrow Yx)$$

$$\textbf{CComp} \quad \Box \forall X^\sigma \exists Y^\sigma (\textit{Const}(Y) \wedge X \equiv Y), \text{ where } \sigma \text{ is a complex type}$$

In the case where  $\sigma = \langle e \rangle$ , CComp states that every property of individuals is coextensive with some modally constant property of individuals. Such properties of individuals will be used to simulate classes of individuals, and quantification over such properties will be used to simulate plural quantification over individuals. Indeed, in H4C we can simulate classes of classes of individuals and plu-plural quantification and so on.

The principles involved in both the class and plurals based paradoxes have analogues in H4C. Class (Plural) Constancy is captured by the use of modally constant higher-order entities. Moreover, Modest Class (Plural) Comprehension is captured by the combination of Comp and CComp, which together ensure that every physical property of individuals which can be expressed in the language is coextensive with some constant property of individuals. Obviously, the wider the range of physical predicates in the signature, the greater the extent to which Modest Class (Plural) Comprehension is captured in the language, but we shall only be interested in the physical property of being an electron.

Having established these preliminaries, the two paradoxes can be reformulated. The new statement of the paradoxes centres around two principles corresponding to (C1)/(P1) and (C2)/(P2) respectively. The first principle states that necessarily there is some constant property whose instances are exactly the electrons, and possibly there is an electron that necessarily does not have it. The second principle states that it is possible for there to be some constant property whose instances are exactly the electrons such that necessarily every electron possibly has it.

<sup>18</sup>This definition of modal constancy is the higher-order analogue of Linnebo's (2013) notion of extensional definiteness from §2.1; it is used in recent work by Dorr et. al. (forthcoming) and Bacon & Zeng (forthcoming).

<sup>19</sup>Compare Gallin's (1975, chap. 3) principle of extensional comprehension. Note also that each extension of H4 is to be understood as closed under the original rules of H4.

$$(\mathbf{CP1}) \quad \Box \exists X (Const(X) \wedge X \equiv E \wedge \Diamond \exists y (Ey \wedge \Box \neg Xy))$$

$$(\mathbf{CP2}) \quad \Diamond \exists X (Const(X) \wedge X \equiv E \wedge \Box \forall y (Ey \rightarrow \Diamond Xy))$$

At this point it is helpful to make two observations. First, notice that (CP1) and (CP2) only capture the intended recombinatorial ideas due to the definition of modal constancy. Second, notice that when  $X$  is a constant property, the fact that  $y$  does not have  $X$  implies that  $y$  lacks  $X$  as a matter of necessity, and the fact that  $y$  possibly has  $X$  implies that  $y$  does in fact have  $X$ . Thus, given T, the principle (CP1) is H4C-equivalent to the principle which results from deleting its rightmost necessity operator:

$$(\mathbf{CP1'}) \quad \Box \exists X (Const(X) \wedge X \equiv E \wedge \Diamond \exists y (Ey \wedge \neg Xy))$$

Similarly, the principle (CP2) is H4C-equivalent to the principle which results from deleting its rightmost possibility operator:

$$(\mathbf{CP2'}) \quad \Diamond \exists X (Const(X) \wedge X \equiv E \wedge \Box \forall y (Ey \rightarrow Xy))$$

This issue is explored further in §4.1, in which I consider a system in which similar equivalences break down. For now, however, the key point is that in H4C (CP1) and (CP2) imply one another's negations, so at least one of them must be rejected in the context of that system. This should dispel any lingering concerns that the paradoxes require assumptions beyond those of H4C, such as the B axiom or the Barcan formula—a point which was much harder to appreciate in the informal presentation of the paradoxes.

### 3.3 Actualities

To reformulate Fritz's second puzzle involving generalised actuality operators, such operators must be characterised within H4C. As it turns out, they can be characterised neatly as certain kinds of modally constant propositional operators.<sup>20</sup> To anticipate what is to come, this constancy requirement will ensure that the operators are non-forgetful and non-enriching in the senses required for Fritz's puzzle.

To begin with, we characterise the notion of an *actuality*. An actuality is a constant propositional operator which is coextensive with truth.

$$\mathbf{Actuality} \quad Act^{\langle\langle\langle\rangle\rangle\rangle} := \lambda X^{\langle\langle\rangle\rangle} (Const(X) \wedge \forall p (Xp \leftrightarrow p))$$

Due to CComp, it is straightforward to show that in H4C it is necessary that there is an actuality: it is just the constant property coextensive with truth.

$$\mathbf{Necessary Actualities} \quad \Box \exists X Act(X)$$

Call the theory which results from adding Necessary Actualities to the background logic H4A. Given the definition of actualities, H4A is a subsystem of H4C. As before, it might help to think of an actuality as a

<sup>20</sup>The general technique of characterising related notions, such as the actual 'world-proposition', using CComp dates back to Daniel Gallin (1975, sect. 11). In what follows, I adapt Dorr et. al.'s (forthcoming, chap. 1) method of characterising an actuality operator in theories like H4C. (Dorr et. al. work in a theory strictly weaker than H4C which includes what I call below 'Rigid Comprehension' instead of CComp.)

type of portrait of how things stand. The requirement that actualities be constant means that such portraits never distort how matters stood: they apply to exactly the same propositions as a matter of necessity. One can then take different portraits of how things stand in different possibilities and even compare them at further possibilities.

Helpfully, H4C proves that our portraits are non-forgetful and non-enriching in the required senses:

**No Forgetting**  $\Box\forall X(Act(X) \rightarrow \forall p(p \rightarrow \Box Xp))$

**No Enriching**  $\Box\forall X(Act(X) \rightarrow \forall p(\Diamond Xp \rightarrow p))$

Indeed, respectively these assumptions correspond to the constraints that the instances of actualities are necessarily their instances, and that the non-instances of actualities are necessarily their non-instances. With this in mind, we can now reformulate the two recombinatorial premises of Fritz’s second puzzle.

(CA1)  $\Box\exists X(Act(X) \wedge \Diamond\exists x(Ex \wedge \Box\neg XEx))$

(CA2)  $\Diamond\exists X(Act(X) \wedge \Box\forall x(Ex \rightarrow \Diamond XEx))$

Notice that (CA1) and (CA2) only capture the intended recombinatorial ideas due to No Forgetting and No Enriching. However, (CA1) and (CA2) are equivalent to one another’s negations in H4C. Thus, once again, a recombinatorial puzzle can be generated by appealing to necessary existence of modally constant properties.

## 4 Against the Necessity of Distinctness

It is now clear that the reasoning behind each paradox can be presented as a formal derivation in H4C. Having established this, I now turn to arguing that the paradoxes require the necessity of distinctness in the sense at issue, and that, in the presence of paradox-free assumptions, accepting the necessity of distinctness reinstates the paradoxes.

### 4.1 Requirement

It is easy to see that the necessity of distinctness (with respect to all types) is a theorem of H4C.

**ND**  $x \neq y \rightarrow \Box x \neq y$ , where  $x, y : \sigma$  for any type  $\sigma$

The argument is straightforward.<sup>21</sup> Suppose that  $x$  and  $y$  are different entities of arbitrary type  $\sigma$ . Then  $x$  has the property of being identical with  $x$  and  $y$  lacks this property. By CComp, there is a modally constant property coextensive with this property. As a matter of necessity, this constant property applies only to  $x$  and not to  $y$ . Thus necessarily there is at least one property over which  $x$  and  $y$  disagree, hence by the necessitation of Leibniz’s Law, necessarily  $x$  is distinct from  $y$ .

<sup>21</sup>Versions of this argument may be found in Linnebo (2013) and Dorr et. al. (forthcoming).



1. $x \neq y \rightarrow ((\lambda z.z = x)x \wedge \neg(\lambda z.z = x)y)$	RefI, PC, $\beta$
2. $\exists X(\text{Const}(X) \wedge X \equiv (\lambda z.z = x))$	CComp
3. $\exists X(\text{Const}(X) \wedge (x \neq y \rightarrow (Xx \wedge \neg Xy)))$	1, 2
4. $\exists X(\text{Const}(X) \wedge (x \neq y \rightarrow (\Box Xx \wedge \Box \neg Xy)))$	3, Def. <i>Const</i>
5. $\exists X(\text{Const}(X) \wedge (x \neq y \rightarrow \Box(Xx \wedge \neg Xy)))$	4, K, Nec, PC
6. $x \neq y \rightarrow \Box \exists X(Xx \wedge \neg Xy)$	5, PC
7. $x \neq y \rightarrow \Box x \neq y$	6, Nec, LL

Clearly, this style of reasoning can be adapted to any theory of classes or plurals according to which they are modally constant. Therefore, each of the class- and plural-based paradoxes uses assumptions from which one can recover the necessity of distinctness for individuals at the very least.

There is also a direct argument for ND within H4A. The argument is a version of Williamson's (1996) argument for the necessity of distinctness (of individuals) from a standard logic of actuality. Again, the basic idea is straightforward. Suppose that  $x$  and  $y$  are different entities of arbitrary type  $\sigma$ . Then  $x$  has the property of being actually identical with  $x$  and  $y$  lacks this property. Yet, as a matter of necessity, this constant property applies only to  $x$  and does not apply to  $y$ , so necessarily  $x$  is not  $y$ .

1. $x \neq y \rightarrow ((\lambda z.z = x)x \wedge \neg(\lambda z.z = x)y)$	RefI, PC, $\beta$
2. $\forall X(\text{Act}(X) \rightarrow \forall p(Xp \leftrightarrow p))$	Def. <i>Act</i>
3. $\exists X(\text{Act}(X) \wedge (x \neq y \rightarrow (X((\lambda z.z = x)x) \wedge \neg X((\lambda z.z = x)y))))$	1, 2, Necessary Actualities
4. $\exists X(\text{Act}(X) \wedge (x \neq y \rightarrow (\Box X(x = x) \wedge \Box \neg X(y = x))))$	3, $\beta$ , Def. <i>Const</i>
5. $\exists X(\text{Act}(X) \wedge (x \neq y \rightarrow (\Box(\lambda z.Xz = x)x \wedge \Box \neg(\lambda z.Xz = x)y)))$	4, $\beta$ , K, Nec, PC
6. $x \neq y \rightarrow \Box \exists X(Xx \wedge \neg Xy)$	5, K, Nec, PC
7. $x \neq y \rightarrow \Box x \neq y$	6, Nec, LL

Since Fritz's actuality puzzle uses constant actualities, this constitutes a direct argument for ND from the assumptions involved in that puzzle.

Although this establishes that each of the paradoxes *uses* resources from which one can recover ND, it does not show that each of the paradoxes *requires* ND in our initial sense. However, there is a convincing case to be made for this claim too. As I shall argue, with only slightly 'weaker' resources from which one cannot recover ND, the paradoxes of recombination have natural solutions. Consequently, it may be seen that ND is a key assumption of the paradoxes.

Recall that modal constancy is a demanding condition, which consists of three components. A modally constant entity is one whose: instances are necessarily its instances, non-instances are necessarily its non-instances, and instances are modally non-increasing. Yet, as I emphasised initially, without the B axiom none of these conditions is equivalent to the others. Thus, one can weaken the demands to produce a less stringent condition of properties and relations. To this end, call an entity of a complex type *rigid* exactly when its instances are necessarily its instances and its instances are non-increasing. (In the next two

definitions,  $\sigma = \langle \sigma_1, \dots, \sigma_n \rangle$  is any complex type and  $x$  is a sequence of variables  $x_1, \dots, x_n$  such that  $x_1 : \sigma_1, \dots, x_n : \sigma_n$ .)

$$\mathbf{Rigidity} \quad \mathit{Rig}^{(\sigma)} := \lambda X^\sigma (\Box \forall x (Xx \rightarrow \Box Xx) \wedge \Box \forall Y^\sigma (\forall x (Xx \rightarrow \Box Yx) \rightarrow \Box \forall x (Xx \rightarrow Yx)))$$

Intuitively, rigid properties do not lose their instances, however non-instances can become instances *only if* they become identical with something which already was an instance of the property. To take an example, suppose that necessarily whatever is an angel is necessarily an angel and that necessarily the angels are non-increasing. If some non-angel is possibly identical with one of the initial angels, it would follow that angelhood is rigid but not constant. Clearly, then, rigidity is very similar to constancy. However, crucially, from a principle of *rigid comprehension* one cannot establish ND. To appreciate this, let H4R be the theory which results from adding all instances of the following schema to the background logic.

$$\mathbf{RComp} \quad \Box \forall X^\sigma \exists Y^\sigma (\mathit{Rig}(Y) \wedge X \equiv Y), \text{ for each complex type } \sigma$$

Since every constant property is rigid but not vice-versa, H4R is a proper subsystem of H4C. Yet H4R is consistent with widespread failures of ND. This is a consequence of the model in the appendix, but to get an intuitive sense of why it is the case it helps to revisit the argument for ND within H4C. In that argument, there was a crucial step (from line 3 to line 4) which relied on the fact that the non-instances of the modally constant property which is coextensive with *being identical with x* are necessarily its non-instances. Yet if there is merely a rigid property coextensive with *being identical with x*, one cannot rule out that one of its non-instances possibly has it: to do so one would have to assume the relevant instance of ND. Thus the argument for ND in H4C breaks down in H4R.

The premises of each paradox can now be recast in terms of rigid entities. For example, consider versions of the premises used in the reformulation of the class- and plural-based paradoxes.

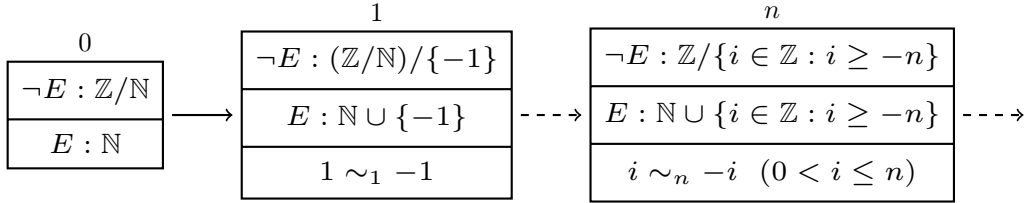
$$(\mathbf{RP1}) \quad \Box \exists X (\mathit{Rig}(X) \wedge X \equiv E \wedge \Diamond \exists y (Ey \wedge \Box \neg Xy))$$

$$(\mathbf{RP2}) \quad \Diamond \exists X (\mathit{Rig}(X) \wedge X \equiv E \wedge \Box \forall y (Ey \rightarrow \Diamond Xy))$$

Although superficially similar to the versions of the premises involving constant properties, (CP1) and (CP2), (RP1) and (RP2) are radically different claims. According to (RP1), necessarily there is a rigid property had exactly by the electrons despite it being possible for an electron not to have it as a matter of necessity. But surely that is false: since we are not assuming ND, why would it be impossible for the new electron in question to become identical with one of the initial electrons? Indeed, when ND is not assumed, the overarching aversion to modal arbitrariness creates pressure to think that such an identity *should* be possible. Thus, strikingly, accepting (RP1) runs afoul of this aversion whereas rejecting (RP1) respects it. Moreover, as before, (RP2) is equivalent to the negation of (RP1) in H4R, so the aversion to arbitrariness thereby creates pressure to endorse (RP2). Indeed, to endorse (RP2) one only need posit *some* possibility,

perhaps at which there is an absolute infinity of electrons, such that any possible electron must possibly be identical with one of the initial electrons there. The moral is that recasting the premises in terms of rigidity completely inverts the situation: whereas before the aversion to modal arbitrariness generated a paradox, it now prevents one from taking grip. In other words, the aversion to arbitrariness itself now acts as the symmetry breaker between the principles which was previously lacking.<sup>22</sup>

To aid this point, it is important to check that (CP2) is not implied by (RP2) in H4R; it is also instructive to observe why the implication fails. I establish this model-theoretically in the appendix, but the model can be sketched without too much detail. In the model, the worlds are the natural numbers, with the accessibility relation as their usual partial ordering, and the individual domain of each world contains just the integers. (For convenience, I shall speak as if the natural numbers are the non-negative integers when describing the model.) The model interprets type  $e$  identity by a function from worlds  $w$  to a congruence relation  $\sim_w$  between the integers. At the initial world the extension of ‘E’ is just the set of non-negative integers. Moreover, at each successor world  $n$  the extension of ‘E’ is just the set of natural numbers and the negative integers greater than or equal to  $-n$  and each such negative integer  $-i$  is interpreted as ‘identical’ with  $i$  there. Finally, the type  $\langle e \rangle$  domain of each world contains all functions from natural numbers (i.e. worlds) to sets of integers that are closed under the interpretation of ‘=’ at that world.



In this model, ‘E’ is assigned an intension which is rigid but not constant. It is rigid because the extension of ‘E’ at a world is included in its extension at all accessible worlds, and, at a given world, if there is some accessible world at which a member of the extension of ‘E’ satisfies a certain condition, then at the initial world there is some member of the extension of ‘E’ there which satisfies that condition at the accessible world. Moreover, the only way for something which is not in the extension of ‘E’ at a world to belong to its extension at some accessible world is to be interpreted as ‘identical’ at that world with one of the members of the extension of ‘E’ at the initial world. As a result, (RP2) is true. However, ‘E’ is not assigned

<sup>22</sup>One might worry that insofar as one ought to be taken in by the second informal recombinatorial principle (about collecting all possible electrons together in a single possibility), one should also have been taken in by the idea that this could occur without any bizarre collapses of identity. However, (RP2) does not capture the intuitive idea that all possible electrons could be electrons together whilst ‘remaining distinct’, so in endorsing (RP2) one is not fully speaking to the motivation for the second informal recombination principle. Nevertheless, the problem with this concern is that, without ND, it is not clear how to express the intuitive idea that all possible electrons could be electrons together whilst ‘remaining distinct’ via an attractive principle of recombination. For example, one crude attempt to capture the idea would be to enrich (RP2) with an assertion of the possibility of the necessity of distinctness for electrons:

$$(\text{NDRP2}) \quad \Diamond(\forall x \forall y (Ex \wedge Ey \rightarrow (x \neq y \rightarrow \Box x \neq y)) \wedge \exists X (Rig(X) \wedge X \equiv E \wedge \Box \forall y (Ey \rightarrow \Diamond Xy)))$$

Yet if ND fails in general, it is difficult to see the attraction towards a principle like (NDRP2). This suggests that, without ND, the considerations which motivate the second informal recombinatorial principle do *not* motivate a strengthened version of it with an added ‘remaining distinct’ condition. In other words, those who deny ND have a principled explanation of why they are taken in by the second informal recombinatorial principle but not by the idea that this type of recombination could occur without any bizarre collapses of identity. Thanks to Peter Fritz and Timothy Williamson for discussion about these issues.

a constant intension, for at any world some individual which does not belong to the extension of ‘E’ at that world belongs to its extension at some accessible world. Indeed, at no world is there a constant intension coextensive with ‘E’, for at every world the extension of every intension in the model is closed under the interpretation of ‘identity’ at that world. Thus, (CP2) is not true. This shows that it is perfectly stable to maintain (RP2) and reject (CP2) on the basis that there are no constant properties to witness the latter’s truth. Indeed, (CP1) and (CP2) are no longer equivalent to one another’s negations in H4R, so they can both be rejected due to the lack of constant properties (they are in fact both false in the model in the appendix).

To complement these observations, this treatment of the paradoxes has an important feature worth highlighting. To introduce it, consider again the recombination principles in terms of constant properties:

$$(\mathbf{CP1}) \quad \Box \exists X (Const(X) \wedge X \equiv E \wedge \Diamond \exists y (Ey \wedge \Box \neg Xy))$$

$$(\mathbf{CP2}) \quad \Diamond \exists X (Const(X) \wedge X \equiv E \wedge \Box \forall y (Ey \rightarrow \Diamond Xy))$$

Previously I highlighted that in H4C (CP1) is equivalent to the following ‘demodalized’ principle (CP1’), and (CP2) is equivalent to the following ‘demodalized’ principle (CP2’).

$$(\mathbf{CP1'}) \quad \Box \exists X (Const(X) \wedge X \equiv E \wedge \Diamond \exists y (Ey \wedge \neg Xy))$$

$$(\mathbf{CP2'}) \quad \Diamond \exists X (Const(X) \wedge X \equiv E \wedge \Box \forall y (Ey \rightarrow Xy))$$

These equivalences hold, essentially, because the following is a theorem of H4C:

$$\Box \forall X (Const(X) \rightarrow \Box \forall x (\neg Xx \leftrightarrow \Box \neg Xx))$$

Nevertheless, the analogues of these equivalences break down with respect to the recombination principles in terms of rigid properties. In particular, consider the following two principles. The first is the above principle (RP1), and the second is the claim that it is necessary that there is some rigid property coextensive with *being an electron* such that it is possible for there to be an electron which does not have it.

$$(\mathbf{RP1}) \quad \Box \exists X (Rig(X) \wedge X \equiv E \wedge \Diamond \exists y (Ey \wedge \Box \neg Xy))$$

$$(\mathbf{RP1'}) \quad \Box \exists X (Rig(X) \wedge X \equiv E \wedge \Diamond \exists y (Ey \wedge \neg Xy))$$

In H4R, (RP1) implies (RP1’) but not vice-versa. Thus, even if one rejects (RP1), one can accept the strictly weaker (RP1’). Moreover, interestingly, (RP1’) is consistent with (RP2).<sup>23</sup> According to the picture on which both hold, whatever electrons there are it is possible for there to be a new electron, but there is at

<sup>23</sup>See the final result in the appendix. To get a sense of why they are consistent, one only needs to consider a tweak to the model sketched above. The idea behind the tweak is to add a ‘buffer’ world  $m'$  between each world  $m$  and its successor  $n$  from the original model. In this new world  $m'$ ,  $-n$  (an element in the first-order domain) is in the extension of ‘E’ but not yet ‘identical’ with  $n$ . Nevertheless, as in the original model,  $-n$  becomes ‘identical’ with  $n$  at world  $n$ . This generates model of (RP2) intuitively because if it is possible for there to be a new electron, it is always possible for that new electron to become identical with one of the initial electrons. It also generates a model of (RP1’) intuitively because it is always possible for there to be a genuinely new electron (before its identity collapses into one of the initial electrons at the next world).

least one possibility at which it is necessary that any new electron is possibly identical with one of the initial electrons. Those who endorse this picture can thus speak to the initial motivation for (RP1), which was that it should always be possible for all electrons to be electrons along with a duplicate of one of them which is located in a different region to any of the others. To accommodate this, one just has to recognise that, at some possibility, any possible duplicate of an electron is *possibly* identical with one of the initial electrons, which is an entirely reasonable idea. Indeed, if ND fails, this idea is motivated by the anti-arbitrariness considerations which drove the initial paradoxes. Finally, although (RP1') is consistent with (RP2), it is inconsistent with the following strengthening of (RP2):

$$(\mathbf{RP2'}) \quad \Diamond \exists X (Rig(X) \wedge X \equiv E \wedge \Box \forall y (Ey \rightarrow Xy))$$

Put informally, (RP2') states that it is possible for there to be some rigid property coextensive with *being an electron* such that necessarily every electron has it. But one can hardly be accused of positing modal arbitrariness if one rejects (RP2') but endorses the slightly weaker (RP2). After all, in doing so, one is maintaining that it is possible for there to be some electrons such that every possible electron is *possibly* one of them, even though every possible electron might not *automatically* be one of them. If ND fails, then, a natural picture of recombination becomes available. One can endorse (RP2), perhaps in part on the basis that it is possible for there to be an absolute infinity of electrons, and also accept (RP1') to recognise that even at that possibility there could be a new electron. However, in asserting (RP2) one is also recognising that that possible new electron must become identical with one of the initial electrons at some further possibility. This picture offers an elegant solution to the paradoxes by speaking to the initial motivation for the recombination principles but not running afoul of anti-arbitrariness considerations. Moreover, crucially, it is not available to those who accept the necessity of distinctness; it is in this sense that I claim the paradoxes of recombination require the necessity of distinctness.

To summarise, without ND one may reject both (CP1) and (CP2) on the basis that there are no constant properties of electrons to witness their claims. It was therefore mistaken to assume that an aversion to positing arbitrary divisions between what is and is not metaphysically possible solely motivated (CP1) and (CP2). For without an abundant supply of constant properties, that aversion does not motivate the particular recombinatorial possibilities such principles posit. So in that setting both principles can be rejected without incurring a penalty of arbitrariness. Additionally, without ND there is clear theoretical pressure to endorse (RP2) and reject (RP1), for the former merely requires there to be some possibility at which any possible electron must tolerate the possibility of a collapse of identity. Moreover, as we saw, one can even accept a slight weakening of (RP1) to ameliorate its denial. Crucially, each of these points can be adapted to the initial plural and class-based versions of the paradox too. Those who reject ND should reject both Class Constancy and Plural Constancy, and consider endorsing the weaker claims that class membership and plural membership are merely modally rigid. If they do so, their rejection of ND will create pressure to endorse (C2) and (P2) and reject both (C1) and (P1); in other words, they have an

analogous solution to the initial plural and class-based versions of the paradox.

## 4.2 Actualities Revisited

My suggested treatment of the paradox involving actualities is similar. H4R only guarantees that there is a rigid actuality:

$$\textbf{Rigid Actuality } RAct^{\langle\langle\langle\rangle\rangle\rangle} : \lambda X^{\langle\langle\rangle\rangle} (Rig(X) \wedge \forall p (Xp \leftrightarrow p))$$

$$\textbf{Necessary RActualities } \Box \exists X RAct(X)$$

However, in H4R rigid actualities must only be non-forgetful whereas they need not be non-enriching. As one may recall, the condition of being non-forgetful corresponded to the idea that whichever propositions a given actuality applies to, it must necessarily apply to them. In contrast, the condition of being non-enriching corresponded to the idea that whichever propositions a given actuality does not apply to, it must necessarily not apply to them. But only the former idea and not the latter is required of rigid actualities: it might be that some actuality does not apply to a particular proposition but that proposition is possibly identical with some proposition to which it does apply. In other words, rigid actualities may enrich how things stand because there may be failures of the necessity of distinctness at the level of propositions.

This changes matters considerably, for the new analogue of (CA2) becomes the claim that possibly there is some rigid actuality such that necessarily, for any electron, it is possible that the rigid actuality in question applies to the proposition that it is an electron.

$$\textbf{(RA2)} \quad \Diamond \exists X (RAct(X) \wedge \Box \forall x (Ex \rightarrow \Diamond XEx))$$

In less stilted terms: possibly, as matters stand necessarily any electron is possibly in fact an electron. Yet, as with (RP2), this should strike one as extremely plausible. For given the failure of ND, what would prevent there being some possibility at which any possibly new electron is possibly identical with one of the initial electrons? But were such a collapse of identity to occur, the proposition that the possibly ‘new’ electron is an electron would just *become* the proposition that the initial electron is an electron. More carefully, it is a theorem of H4R that even if two individuals are distinct, if they become identical then the respective propositions that each individual is an electron also become identical:<sup>24</sup>

$$\textbf{Propositional Collapse } \forall x \forall y (x \neq y \rightarrow \Box (x = y \rightarrow Ex = Ey))$$

Thus, due to the possible collapse between two once distinct propositions, the rigid actuality in question possibly distorts how matters once stood. Indeed, rigid actualities are designed to accommodate such mild

<sup>24</sup>The argument for Propositional Collapse just exploits the following instance of Leibniz’s Law and the Reflexivity of propositional identity, both of which are theorems of H4R:

$$\begin{aligned} x = y &\rightarrow ((\lambda z. Ez = Ex)x \rightarrow (\lambda z. Ez = Ex)y) \\ Ex &= Ey \end{aligned}$$

distortions.

Complementing this, the new analogue of (CA1) becomes the claim that necessarily there is a rigid actuality such that possibly there is an electron for which, as a matter of necessity, the actuality does not apply to the proposition that it is an electron.

$$(RA1) \quad \Box \exists X (RAct(X) \wedge \Diamond \exists x (Ex \wedge \Box \neg XEx))$$

In less stilted terms: necessarily, as matters stand, it is possible for there to be an electron which is necessarily not in fact an electron. Yet, as with (RP1), this should strike one as an ill-motivated claim: as just seen, given the failure of ND there is no reason to think that at no possibility any possibly new electrons could not become identical with one of the initial electrons there. Indeed, (RA1) is just equivalent to the negation of (RA2) in H4R.

At this point, it is instructive to reconsider the Williamsonian argument for ND using actualities in H4A. In that argument, there was a crucial step from the claim that it is not actually the case that  $y$  is identical with  $x$  to the claim that necessarily it is not actually the case that  $y$  is identical with  $x$  (for since it is necessary that  $x$  is actually self-identical, it followed that  $x$  and  $y$  are necessarily distinct). But one can now appreciate that this crucial step is just an instance of No Enriching. However, unlike constant actualities, merely rigid actualities *are* permitted to enrich how matters stand, so the Williamsonian argument breaks down in H4R.<sup>25</sup>

More generally, the logic of rigid actualities in H4R differs from the logic of constant actualities in H4A. Intuitively, the main difference is that constant actualities must remain ‘consistent’ properties of propositions as a matter of necessity, whereas rigid actualities need not do so. In particular, it is possible for rigid actualities to apply to both a proposition and its negation. To see why, suppose there is an electron  $x$  and an individual  $y$  which is not an electron, but that it is possible that  $x$  is  $y$ . The propositions that  $x$  is an electron and that  $y$  is an electron are distinct, since they are not even materially equivalent. Thus, by the definition of rigid actualities, if  $X$  is a rigid actuality it applies to the proposition that  $x$  is an electron and the proposition that  $y$  is not an electron. However, since  $x$  is possibly  $y$ , by the principle of Propositional Collapse it is possible that the proposition that  $x$  is an electron just is the proposition that  $y$  is an electron. As a result, standard modal reasoning shows that it is possible that  $X$  applies to the proposition that  $y$  is an electron. But since  $X$  is rigid and applies to the proposition that  $y$  is not an electron, it is therefore possible that  $X$  applies to a proposition and its negation.

(One might object to this consequence on the grounds that an actuality operator is meant to be in some sense ‘equivalent’ to an expression of the form ‘in  $w$ ’ for a particular world  $w$ . Thus, as a matter of necessity, any adequate actuality operator must at least respect the ‘consistency’ of worlds. However, this objection fails to appreciate that without the B axiom there is no guarantee that the consistency of a world is a

<sup>25</sup>See Bacon (2018, sect. 5.4) and Dorr et. al. (forthcoming, chap. 4, sect. 2) for similar reactions to Williamson’s actuality argument. Bacon and Dorr et. al. endorse the considerations about the alternative logic of actualities in the following paragraph too.

non-contingent matter: each world need not necessarily be a *possible* world. Consequently, rigid actualities may indeed be ‘equivalent’ in the operative sense to expressions of the form ‘in  $w$ ’ for a particular world  $w$ . Such expressions, however, will just have a unfamiliar logical character in a B-free setting.)

To summarise the central thread, I have discussed how ‘weakening’ the resources of constant properties to rigid properties leads to a slightly more modest metaphysics. In effect, one retains as much of the role of constant properties as one can do without committing to ND (see §4.3 for further discussion). But once this modest weakening is adopted, a natural picture of recombination becomes available that allows one to handle the paradoxes. The initial recombinatorial claims stated in terms of constant properties become eminently deniable. Moreover, there is always at least one recombinatorial claim stated in terms of rigid properties which involves considerable modal arbitrariness, so the paradoxes do not recur. This, I suggest, constitutes evidence that the paradoxes of recombination *require* the necessity of distinctness in the sense at issue. Of course, I do not claim that the necessity of distinctness is the only assumption the paradoxes require.<sup>26</sup> Nonetheless, it is one such assumption whose status was antecedently controversial, and is only more so in light of the above.

### 4.3 Sufficiency

The case against the necessity of distinctness can be strengthened even further. In addition to requiring the necessity of distinctness, there is also a sense in which the necessity of distinctness is *sufficient* to generate the paradoxes. To be exact, accepting the necessity of distinctness in the presence of the paradox-free theory H4R reinstates the initial puzzles.

The clearest way to appreciate this point is to revisit the definition of rigidity. For any rigid property, its instances are necessarily its instances, and its instances are non-increasing. Moreover, there is no requirement that entities which lack the property necessarily lack it. However, an entity which lacks the property possibly has it only if that entity becomes identical with one of the property’s initial instances. Now, if ND is true that necessary condition is never satisfied. If one entity lacks a rigid property and another has it, then they are distinct by Leibniz’s Law. Thus by ND they are necessarily distinct, and so the former entity cannot become identical with the latter entity. As a consequence, necessarily any entity which lacks a rigid property must lack it as a matter of necessity. More generally, it is necessary that every rigid property is modally constant if ND is true (see Dorr et. al. (forthcoming, Proposition C.3)). These informal considerations can of course be turned into a derivation of CComp within the theory that results from adding ND to H4R.

<sup>26</sup>One can of course identify other assumptions which satisfy theses similar to Requirement. For example, consider the view (in the context of H4) that every property is coextensive with an *inattentive* property: a property whose instances are non-increasing and whose non-instances are necessarily its non-instances, but which may lose instances from possibility to possibility. In principle one could ‘weaken’ the resources of constant properties to inattentive properties whilst maintaining ND, in order to prevent the paradoxes of recombination from being generated. Relative to that set of resources, the claim that every inattentive property is modally non-decreasing would satisfy a thesis similar to Requirement. However, this view is much less motivated than H4R, for RComp is an extremely useful principle. After all, it allows one to vindicate familiar and useful ideas in metaphysics, for example that ‘world-talk’ is in good standing (see Dorr et. al. (forthcoming); c.f. Gallin (1975, sect. 11)) and that every entity has a haecceity. But an analogous principle of ‘inattentive comprehension’ would provide no such benefits. The point, therefore, is that the resources of H4R are particularly well motivated, so it is more pertinent that the paradoxes cannot be generated in H4R.



Indeed, that theory just is H4C.

The philosophical upshot is that endorsing ND in the context of H4R results in CComp and thereby reinstates the recombinatorial puzzles. It is in this sense that ND suffices to generate the paradoxes of recombination: in accepting it alongside certain paradox-free assumptions, one lapses back into paradox.

## 5 Kaplanian Paradoxes

The considerations above generalise to other well-known paradoxes of recombination. For example, Ted Sider (2009a) presents two paradoxes of recombination based on a puzzle from Peter Forrest and David Armstrong (1984) which has also been explored by Daniel Nolan (1996). Sider’s paradoxes suggest that in the context of certain set-theoretic assumptions, necessitism places severe constraints on recombination. However, as Sider makes explicit, each paradox requires the now suspect assumption that set-membership is modally constant. More generally, since Fritz’s puzzles require such minimal assumptions, the considerations above can be adapted to various paradoxes of recombination—many of which involve additional assumptions beyond those made by Fritz.<sup>27</sup> Nevertheless, I now wish to consider a family of paradoxes involving the recombination of propositions, properties and relations which involve even fewer assumptions. The most famous member of this family is due to David Kaplan (1995) and is known as Kaplan’s Paradox.

Kaplan’s Paradox centres around the recombinatorial idea that for any proposition it should be possible that it and only it is queried. After all, for each proposition it certainly seems possible that there could be just one agent who entertains exactly that proposition. Since ‘is queried’ is just a propositional operator, this idea can be formalised in our higher-order language as follows.

**Possible Queries**  $\exists X^{(\langle \rangle)} \forall p \Diamond (Xp \wedge \forall q (Xq \leftrightarrow p = q))$

Usually, the paradox is presented as an argument that Possible Queries is incompatible with a thought suggested by the framework of possible worlds semantics: that there are more propositions than there are worlds. As the thought goes, in possible worlds semantics propositions are just treated as sets of worlds, so by Cantor’s Theorem there are more propositions than worlds. Yet Possible Queries states that for each proposition there is some world in which it is uniquely queried, which contradicts the claim that there are more propositions than worlds.<sup>28</sup> However, so far I have avoided purely model-theoretic arguments for modal conclusions. Thankfully, though, a recourse to possible worlds semantics is not even required for, surprisingly, Possible Queries is just inconsistent even in the modest background logic H4. Philosophically this is extremely pertinent because ND is *not* a theorem schema of H4.

<sup>27</sup>Other such examples include the set- and plural-based formulations of the Forrest-Armstrong paradox due to Jeffrey Sanford Russell & John Hawthorne (forthcoming).

<sup>28</sup>Kaplan himself took this phenomenon to indicate that there is a foundational problem with using possible worlds semantics to study intensional languages with propositional quantifiers. However, Ding & Holliday (2020) highlight that Kaplan’s phenomenon generalises to a certain algebraic semantics for such languages. As Ding & Holliday show, Kaplan’s sentence is unsatisfiable over a class of algebraic frames in which the underlying Boolean algebra is complete but may fail to be atomic. Intuitively, atomicity corresponds to the assumption that there are worlds. (See Fine (1970) and Holliday (2017) for useful background.)

Before considering the philosophical ramifications of this observation, it is important to appreciate how far it generalises. For not only might one be tempted by Possible Queries, one might also think that any entity of any complex type is possibly the only entity of that type which is queried. Schematically:

**Generalised Possible Queries**  $\exists X^{(\sigma)} \forall x^\sigma \Diamond (Xx \wedge \forall y^\sigma (Xy \leftrightarrow x = y))$ , for any complex type  $\sigma$

Nevertheless, every instance of this schema is also inconsistent in H4. We are thus presented with a family of Kaplanian paradoxes of recombination, none of which requires ND.<sup>29</sup>

Upon seeing this, one might be tempted to think that rejecting ND is not a sufficiently general response to the paradoxes of recombination, for, the thought runs, the correct solution to each of the initial paradoxes must apply in equal measure to the Kaplanian paradoxes. Yet there is one overriding reason why this temptation should be resisted: the Kaplanian paradoxes should be classified as *sui generis* paradoxes of recombination. My argument for this classificatory claim pertains to why Possible Queries (and each instance of Generalised Possible Queries) is inconsistent in H4. The reason it is inconsistent is due to a result now known as Prior's Theorem, which concerns the non-modal subsystem that results from removing all of the modal principles (K, T, 4 and Nec) from H4. Surprisingly, this rather weak system yields the result that there is a particular proposition which is never queried alone: the proposition that everything which is queried is false. For if that proposition is queried, it cannot be true (since it would then be false). Thus if it is queried, it is false—in which case something queried is true. But since no proposition is both true and false, there must be at least two propositions which are queried. Indeed, this argument made no assumptions about querying other than that it is a propositional attitude, so Prior's Theorem applies to any propositional operator whatsoever.

**Prior's Theorem**  $\forall X^{(\Diamond)} (X \forall p (Xp \rightarrow \neg p) \rightarrow \exists p (Xp \wedge p) \wedge \exists p (Xp \wedge \neg p))$

Given this, with just the K and Nec principles of H4 one can use Prior's Theorem to derive the negation of Possible Queries.<sup>30</sup>

**Impossible Queries**  $\forall X^{(\Diamond)} \exists p \Box (Xp \rightarrow \exists q (p \neq q \wedge Xq))$

Similarly, this non-modal subsystem of H4 includes a generalisation of Prior's Theorem. To see this generalisation, say that an entity  $\Phi$  of a complex type  $\sigma = \langle \sigma_1, \dots, \sigma_n \rangle$  is *instantiated* just in case there are some things of types  $\sigma_1, \dots, \sigma_n$  respectively which have it.

**Instantiation**  $I^{(\sigma)}(\Phi) := \exists y_1^{\sigma_1} \dots \exists y_n^{\sigma_n} \Phi y_1 \dots y_n$ , where  $y_1 \dots y_n$  are the first variables of the relevant

<sup>29</sup>The phenomenon generalises even further. For example, Bacon & Uzquiano (2018, pp. 993-994) and Uzquiano (forthcoming) show, in effect, that the inconsistency of Possible Queries in H4 is merely the unary instance of the more general phenomenon that there are  $n \geq 1$  propositions such that one cannot query all  $n$  of them without querying some distinct proposition. Moreover, Uzquiano (forthcoming) develops a general heuristic, based on the connection between Kaplan's paradox and Cantor's theorem, for identifying further 'elusive propositions' which cannot be uniquely queried.

<sup>30</sup>In fact, since the consequent of Prior's Theorem states that some queried proposition is true and some queried proposition is false, one derive the stronger claim that there is some proposition whose querying necessitates the querying of a proposition which is not materially equivalent to it.

types not to occur freely in  $\Phi$  given some fixed ordering

(Notice that when  $\sigma = \langle \rangle$ , instantiation just amounts to being the case.) The generalisation of Prior's Theorem then states that for any complex type  $\sigma$ , there is some type  $\sigma$  entity which is not the only type  $\sigma$  entity which is queried. The type  $\sigma$  entity in question is *being such that every type  $\sigma$  entity which is queried is not instantiated*. For if that entity is queried, it is not instantiated—in which case there is some type  $\sigma$  entity which is queried and *is* instantiated. Thus since nothing is both instantiated and not instantiated, there must be at least two type  $\sigma$  entities which are queried.<sup>31</sup>

**Generalised Prior's Theorem** Where  $\sigma = \langle \sigma_1, \dots, \sigma_n \rangle$ , and  $x_1 : \sigma_1, \dots, x_n : \sigma_n$ ,

$$\forall Z^{(\sigma)} (Z\lambda x_1 \dots x_n. (\forall X (ZX \rightarrow \neg I(X))) \rightarrow \exists X (ZX \wedge I(X)) \wedge \exists X (ZX \wedge \neg I(X)))$$

As before, with only the K and Nec principles of H4 one can use Generalised Prior's Theorem to derive the negation of each instance of Generalised Possible Queries.

**Generalised Impossible Queries**  $\forall X^{(\sigma)} \exists x^\sigma \Box (Xx \rightarrow \exists y^\sigma (x \neq y \wedge Xy))$ , for any complex type  $\sigma$

Thus the Kaplanian paradoxes are intimately connected with Prior's surprising result.

In light of the significant role which Generalised Prior's Theorem plays in the inconsistency of Generalised Possible Queries, it is natural to view the Kaplanian paradoxes as not rooted in the same phenomena as our initial recombinatorial puzzles. There is of course no doubt that the Kaplanian paradoxes do constitute puzzles: but the puzzlement is already apparent with respect to the non-modal Generalised Prior's Theorem, and the Kaplanian paradoxes merely draw a routine modal consequence from that result. It would seem unfair, therefore, to demand that a solution to our initial paradoxes of recombination generalise to the Kaplanian paradoxes.<sup>32</sup> The solution to the latter must involve either accepting Generalised Impossible Queries or rejecting some non-modal principle from which it follows, but neither such option seems relevant to the initial paradoxes of recombination. In particular, the most natural way of rejecting Generalised

<sup>31</sup>Let H be the theory which results from removing K, T, 4 and Nec from H4. It is easy to see the following lemma:  $\vdash_H I(\lambda x_1 \dots x_n. \phi) \leftrightarrow \phi$ , whenever each  $x_i$  ( $1 \leq i \leq n$ ) does not occur free in  $\phi$  (the argument relies on vacuous instances of Extensional  $\beta$ ). The derivation of Generalised Prior's Theorem then runs as follows. (In this derivation,  $\sigma = \langle \sigma_1, \dots, \sigma_n \rangle$  is an arbitrary complex type, and  $X : \sigma$ ,  $x_1 : \sigma_1, \dots, x_n : \sigma_n$ , and  $O : \langle \langle \sigma \rangle \rangle$  are variables, and thus do not occur as constituents of one another; see appendix.)

$\vdash_H$  **Generalised Prior's Theorem**

- |  |              |
|--|--------------|
| 1. $\forall X (OX \rightarrow \neg I(X)) \rightarrow (O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \rightarrow \neg I(\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))))$                      | PC           |
| 2. $\neg I(\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X)))) \leftrightarrow \neg \forall X (OX \rightarrow \neg I(X))$   | Lemma        |
| 3. $\forall X (OX \rightarrow \neg I(X)) \rightarrow (O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \rightarrow \neg \forall X (OX \rightarrow \neg I(X)))$   | 1, 2         |
| 4. $O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \rightarrow (\forall X (OX \rightarrow \neg I(X)) \rightarrow \neg \forall X (OX \rightarrow \neg I(X)))$   | 3, PC        |
| 5. $O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \rightarrow \neg \forall X (OX \rightarrow \neg I(X))$  | 4, PC        |
| 6. $O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \rightarrow \exists X (OX \wedge I(X))$   | 5, PC        |
| 7. $O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \rightarrow (O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \wedge \neg \forall X (OX \rightarrow \neg I(X)))$                            | 5, PC        |
| 8. $O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \rightarrow (O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \wedge \neg I(\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))))$ | 7, Lemma, PC |
| 9. $O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \rightarrow \exists X (O(X) \wedge \neg I(X))$  | 8, PC        |
| 10. $O\lambda x_1 \dots x_n. (\forall X (OX \rightarrow \neg I(X))) \rightarrow \exists X (OX \wedge I(X)) \wedge \exists X (O(X) \wedge \neg I(X))$   | 6, 9, PC     |

A similar 'predicational' version of Prior's theorem, which may be derived without the above lemma, is also used by Cian Dorr (MS) to argue for his thesis of 'plural signification'.

<sup>32</sup>A similar sentiment is endorsed by Uzquiano (2015, pp. 12-13), who argues that no solution to Kaplan's Paradox helps with Uzquiano's novel paradox of recombination.

Impossible Queries is to weaken universal instantiation.<sup>33</sup> But our initial recombinatorial puzzles did not require universal instantiation. First, one may recall the observation from Fritz (mentioned in §3.1) that his puzzles still arise within the presence of ‘free’ quantification theory. This can be appreciated more directly by recognising that, provided one endorsed CComp, the principles (CP1) and (CP2) (and also (CA1) and (CA2)) are equivalent with the negations of one another even in proper subsystems of H4 which include merely the weakening of universal instantiation in question. On reflection, then, that the Kaplanian paradoxes do not require ND does *not* undermine the hypothesis that ND is one of the driving assumptions behind our initial paradoxes of recombination.

## 6 Conclusion

The connections between the necessity of distinctness and the paradoxes of recombination constitute new evidence against the non-contingency of identity. If distinctness is not necessary, resources which the paradoxes require become unavailable. Moreover, if distinctness is necessary, paradox-free assumptions are plunged back into inconsistency. Of course, there is no doubt that the necessity of distinctness belongs to an abductively attractive conception of metaphysical necessity, as Williamson (1996) emphasises. But the abductive approach to theory choice demands a thorough investigation of the alternatives. If, as is arguably the case, there are competing conceptions of metaphysical necessity which lack the necessity of distinctness, their inbuilt capacity to handle the paradoxes of recombination can only enhance their standing. Thus, although my considerations are not decisive against the non-contingency of identity, they do give one pause for thought: is identity non-contingent?

## Appendix

I now establish several consistency results to which I appealed in the main body, namely that in H4R: ND is not a theorem schema; (RP1) does not imply (CP1); (CP1) is not equivalent to the negation of (CP2), and (RP1') is consistent with (RP2). (It is a corollary of the first that H4R is a proper subsystem of H4C.) Since I shall only be interested in consistency, the models will make some simplifying assumptions; in particular, they equate higher-order identity with necessary equivalence. Interestingly, the models will also validate the Barcan formula at all types, which has the benefit of demonstrating that one may consistently hold (RP2) in its presence even without committing to (CP2). The basic idea of the model theory is to interpret type  $\sigma$  identity in the object-language with a function from worlds to congruence relations which need not be the identity relation amongst elements of the model.

As in the main body, I shall use what is known as a relationally typed  $\lambda K$ -language with the base type  $e$ .

<sup>33</sup>Kaplan himself favoured this response, which is explored critically by Bacon et. al (2017). It is also worth noting that those who wish to extend the ‘potentialist’ solution discussed in f.n. 12 to Kaplan’s Paradox will also have to reject universal instantiation, since they do not reject K or Nec for metaphysical necessity.

Recall that the set of types  $Typ$  is defined recursively by the rules that  $e \in Typ$ , and whenever  $\tau_1, \dots, \tau_n \in Typ$  (for  $n \geq 0$ ),  $\langle \tau_1, \dots, \tau_n \rangle \in Typ$ . Throughout, I use  $\bar{\tau}$  to abbreviate  $\tau_1, \dots, \tau_n$ , with  $0 \leq i \leq n$  given by the context. Finally, by the *transitive closure* of a complex type  $\langle \bar{\tau} \rangle$  I mean the set whose elements are exactly those things which bear the transitive closure of  $\in$  to  $\langle \bar{\tau} \rangle$ .

**Definition 1.** A *signature*  $\Sigma$  for the simply typed  $\lambda$ -calculus is a set of constants  $\mathbf{const}$  and a total function  $\mathbf{type}_\Sigma$  from  $\mathbf{const}$  to  $Typ$ . Given an infinite set of variables  $\mathbf{var}$  disjoint from  $\mathbf{const}$ ,  $\mathbf{type}_v$  is a total surjection from  $\mathbf{var}$  to  $Typ$  such that  $|\mathbf{type}_v^{-1}(\sigma)| = \aleph_0$  and  $\mathbf{type}_v^{-1}(\sigma) \cap \mathbf{type}_v^{-1}(\tau) = \emptyset$  whenever  $\sigma \neq \tau$ . (No variable can be a string which includes multiple other variables as constituents.) The *type assignment function*  $\mathbf{type}$  is then defined recursively as follows (where the variables  $\bar{v}$  are all distinct).

$$\mathbf{type}(c) = \mathbf{type}_\Sigma(c), \text{ whenever } c \in \mathbf{const}$$

$$\mathbf{type}(v) = \mathbf{type}_v(v), \text{ whenever } v \in \mathbf{var}$$

$$\mathbf{type}(\alpha\bar{\beta}) = \langle \rangle, \text{ whenever } \mathbf{type}(\alpha) = \langle \tau_1, \dots, \tau_n \rangle \text{ and } \mathbf{type}(\beta_i) = \tau_i$$

$$\mathbf{type}(\lambda\bar{v}.\alpha) = \langle \tau_1, \dots, \tau_n \rangle, \text{ whenever } \mathbf{type}(\alpha) = \langle \rangle \text{ and } \mathbf{type}(v_i) = \tau_i$$

A *term* of the simply typed  $\lambda$ -calculus of signature  $\Sigma$  ( $\mathcal{L}_\Sigma$ ) is any element of  $\mathbf{dom}(\mathbf{type})$ . The signature  $\Sigma$  consists of the set of constants  $\{\rightarrow\} \cup \{\neg\} \cup \{\forall_\sigma : \sigma \in Typ\} \cup \{\square\} \cup \{E\}$ , and the type assignment function  $\mathbf{type}_\Sigma$  defined as follows:

$$\begin{aligned} \mathbf{type}_\Sigma(\rightarrow) &= \langle \langle \rangle, \langle \rangle \rangle & \mathbf{type}_\Sigma(\square) &= \langle \langle \rangle \rangle \\ \mathbf{type}_\Sigma(\neg) &= \langle \langle \rangle \rangle & \mathbf{type}_\Sigma(E) &= \langle e \rangle \\ \mathbf{type}_\Sigma(\forall_\sigma) &= \langle \langle \sigma \rangle \rangle \end{aligned}$$

Following the convention from the main body, I write  $\xi : \sigma$  to mean that  $\mathbf{type}(\xi) = \sigma$ .

**Definition 2.** A *frame* is a triple  $\mathfrak{A} = \langle W, R, |\mathfrak{A}| \rangle$  where  $W$  is a non-empty set,  $R$  is a transitive reflexive relation over  $W$ , and  $|\mathfrak{A}|$  is a non-empty set. When  $\mathfrak{A} = \langle W, R, |\mathfrak{A}| \rangle$  is a frame and  $\tau$  is a type, the function  $\mathfrak{A}(\cdot)$  is defined inductively on  $Typ$  as follows.

$$\mathfrak{A}(\tau) = |\mathfrak{A}|, \text{ when } \tau = e$$

$$\mathfrak{A}(\tau) = (\mathcal{P}(\Pi_{i \leq n} \mathfrak{A}(\tau_i)))^W, \text{ when } \tau = \langle \tau_1, \dots, \tau_n \rangle$$

**Definition 3.** Let  $\tau = \langle \tau_1, \dots, \tau_n \rangle$  and  $\sigma$  be an arbitrary type. A *congruence frame* is a quadruple  $\mathfrak{A} = \langle W, R, |\mathfrak{A}|, \sim \rangle$ , where  $\mathfrak{A} = \langle W, R, |\mathfrak{A}| \rangle$  is a frame and  $\sim$  is a function from each  $w \in W$  to an equivalence relation on  $\mathfrak{A}(e)$  such that  $Rwv \Rightarrow \sim_w \subseteq \sim_v$ . We then define the function  $\sim^\tau$  from  $w \in W$  to  $\mathfrak{A}(\tau) \times \mathfrak{A}(\tau)$  inductively as follows:

$$f \sim_w^\tau g \text{ iff for all } v \in W \text{ such that } R w v, \langle a_1, \dots, a_n \rangle \in f(v) \text{ iff } \langle b_1, \dots, b_n \rangle \in g(v), \text{ where } a_i \sim_v^{\tau_i} b_i$$

It is helpful to note the following special case:

$$p \sim_w^\langle \rangle q \text{ iff } p|R(w) = q|R(w)$$

Finally,  $f \in \mathfrak{A}(\tau)$  is  $\sim$ -closed iff for all  $v \in W$ , if  $\langle a_1, \dots, a_n \rangle \in f(v)$  and each  $a_i \sim_v^{\tau_i} b_i$  then  $\langle b_1, \dots, b_n \rangle \in f(v)$ .

When  $\mathfrak{A}$  is a congruence frame, the function  $\mathfrak{A}^\sim(\cdot)$  is defined inductively on  $Typ$  as follows:

$$\mathfrak{A}^\sim(e) = \mathfrak{A}(e)$$

$$\mathfrak{A}^\sim(\tau) = \{f \in \mathfrak{A}(\tau) : f \text{ is } \sim\text{-closed}\}$$

Call a type  $\tau$  *pure* when it is a complex type whose transitive closure does not include  $e$ . One can check that  $\mathfrak{A}^\sim(\tau) = \mathfrak{A}(\tau)$  when  $\tau$  is pure.

**Definition 4.** When  $\langle W, R, |\mathfrak{A}|, \sim \rangle$  is a congruence frame, an *assignment* is a function from the variables of each type  $\tau$  to  $\mathfrak{A}^\sim(\tau)$ . When  $g$  and  $g'$  are assignments and  $v$  is a variable,  $g \approx_v g'$  iff  $g'$  possibly differs from  $g$  only on  $v$ . We write  $g \approx_{\bar{v}} g'$  to abbreviate the claim that there are  $g_1, \dots, g_n = g'$  such that  $g \approx_{v_1} g_1, \dots, g_{n-1} \approx_{v_n} g_n$ .

**Definition 5.** A *model* is a quintuple  $\mathfrak{A} = \langle W, R, |\mathfrak{A}|, \sim, [\cdot] \rangle$ , where  $\langle W, R, |\mathfrak{A}|, \sim \rangle$  is a congruence frame and  $[\cdot]$  is a function from each term  $\xi : \tau$  in  $\mathcal{L}_S$  and assignment  $g$  to  $\mathfrak{A}^\sim(\tau)$  such that:

$$[c]^g \in \mathfrak{A}^\sim(\tau), \text{ whenever } c : \tau \text{ and } c \in \mathbf{const}$$

$$[v]^g \in g(v), \text{ whenever } v \in \mathbf{var}$$

$$[\alpha \bar{\beta}]^g(u) = \{ \langle \rangle : \langle [\beta_i]^g : i \leq n \rangle \in [\alpha]^g(u) \}, \text{ when } \alpha : \langle \tau_1, \dots, \tau_n \rangle \text{ and } \beta_i : \tau_i$$

$$[\lambda v_1 \dots v_n. \alpha]^g(u) = \{ \langle \bar{s} \rangle : s_i \in \mathfrak{A}^\sim(\tau_i) \text{ s.t. there is some } \langle \bar{o} \rangle \text{ s.t. } o_i \in \mathfrak{A}^\sim(\tau_i) \text{ and } s_i \sim_u^{\tau_i} o_i \text{ and } \langle \rangle \in [\alpha]^{g'}(u), \text{ where } g \approx_{\bar{v}} g' \text{ and } g'(v_i) = o_i \}, \text{ where } v_i : \tau_i$$

A model  $\mathfrak{A} = \langle W, R, |\mathfrak{A}|, \sim, [\cdot] \rangle$  is *logical* exactly when for each  $u \in W$ :

$$[\neg](u) = \{p \in \mathfrak{A}(\langle \rangle) : \langle \rangle \notin p(u)\}$$

$$[\rightarrow](u) = \{ \langle p, q \rangle \in \mathfrak{A}(\langle \rangle) \times \mathfrak{A}(\langle \rangle) : \langle \rangle \in ((\{\langle \rangle\} - p(u)) \cup q(u)) \}$$

$$[\Box](u) = \{p \in \mathfrak{A}(\langle \rangle) : \text{for all } u' \text{ s.t. } R u u', \langle \rangle \in p(u')\}$$

$$[\forall_\sigma](u) = \{S \in \mathfrak{A}^\sim(\langle \sigma \rangle) : S(u) = \mathfrak{A}^\sim(\sigma)\}$$

This can be put in more familiar terms with some derived semantic clauses, for arbitrary assignment  $g$ :

$$[\neg\phi]^g(u) = \{\langle \rangle\} - [\phi]^g(u)$$

$$[\phi \rightarrow \psi]^g(u) = (\{\langle \rangle\} - [\phi]^g(u)) \cup [\psi]^g(u)$$

$$[\Box\phi]^g(u) = \bigcap_{u' \text{ s.t. } R u u'} [\phi]^g(u')$$

$$[\forall_\sigma \Phi]^g(u) = \bigcap_{g \approx_v g'} [\Phi v]^{g'}(u), \text{ where } v : \sigma, \Phi : \langle \sigma \rangle$$

A formula  $\phi$  is *true at*  $\mathfrak{A}, w, g$  for  $w \in W$  and  $g$  a assignment (abbreviated:  $\mathfrak{A}, w, g \models \phi$ ) iff  $\langle \rangle \in [\phi]^g(w)$ . Validity in a model is defined as truth at all worlds on all assignments.

**Proposition.** *The theorems of H4R are valid in every logical model.*

*Proof.* The main base case is RComp. So take an arbitrary logical model  $\mathfrak{A} = \langle W, R, |\mathfrak{A}|, \sim, [\cdot] \rangle$  and  $w \in W$ . When  $\tau = \langle \tau_1, \dots, \tau_n \rangle$  is an arbitrary complex type with  $0 \leq i \leq n$ , take any  $f \in \mathfrak{A}^\sim(\tau)$  and define the function  $f^R$  as follows:

$$f^R(u) = \begin{cases} \{ \langle \bar{s} \rangle \in \Pi_{i \leq n} \mathfrak{A}^\sim(\tau_i) : \text{there is some } \langle \bar{o} \rangle \in f(w) \text{ s.t. } s_i \sim_u^{\tau_i} o_i \}, & \text{when } Rwu \\ \emptyset, & \text{otherwise} \end{cases}$$

Clearly,  $f^R \in \mathfrak{A}^\sim(\tau)$ . Thus since  $f \in \mathfrak{A}^\sim(\tau)$  too,  $f(w) = f^R(w)$ . Moreover, by construction,  $[Rig(X)]^g(w)$  when  $g(X) = f^R$ , so  $\langle \rangle \in [RComp]^g(w)$  for each assignment  $g$  and  $w \in W$ . It is straightforward to check that H4R's rules of proof preserve validity over logical models, so the proposition holds by induction on the length of proofs.  $\square$

**Theorem.** *H4R has neither ND,  $(RP2) \rightarrow (CP2)$ , nor  $(CP1) \leftrightarrow \neg(CP2)$  as theorems.*

*Proof.* Let  $\mathfrak{A} = \langle \mathbb{N}, \leq, \mathbb{Z}, \sim, [\cdot] \rangle$  be a model such that (with  $[\neg]$ ,  $[\rightarrow]$ ,  $[\Box]$  and  $[\forall_\sigma]$  defined in the manner above):

$$i \sim_n j \text{ iff either } i = j \text{ or } |j| = |i| \leq n$$

$$[E] = \{ \langle n, S \rangle : S = (\mathbb{N} \cup \{i \in \mathbb{Z}/\mathbb{N} : i \geq -n\}) \}$$

One can verify that  $\mathfrak{A}$  is a logical model of H4R since  $[E]$  is  $\sim$ -closed. However, ND is not true in  $\mathfrak{A}$  since when  $m < n \leq r$ ,  $n \not\sim_m -n$  yet  $n \sim_r -n$ . It is thus not a model of H4C. Moreover, (RP2) is valid in  $\mathfrak{A}$  since  $[Rig(E)](n) = \{ \langle \rangle \}$  for all  $n \in \mathbb{N}$  (the assignment does not matter). Nonetheless, neither (CP1) nor (CP2) are valid in  $\mathfrak{A}$ ; indeed, at arbitrary  $n \in \mathbb{N}$ , there is no  $f \in \mathfrak{A}^\sim(\langle e \rangle)$  such that  $f(n) = [E](n)$  and  $f(n) = f(m)$  for all  $m \geq n$ . Thus,  $[Const(X) \wedge X \equiv E]^g(n) = \{ \}$  for all  $n \in \mathbb{N}$  and arbitrary  $g$ .  $\square$

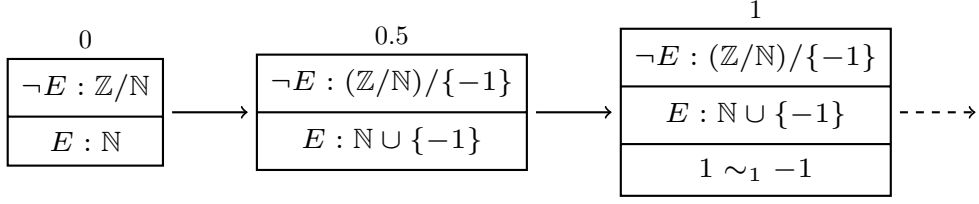
**Theorem.**  $\not\models_{H4R} \neg((RP2) \wedge (RP1'))$

*Proof.* Let  $\mathbb{N}' = \{n' : \text{either } n' \in \mathbb{N} \text{ or } n' = n.5 \text{ for some } n \in \mathbb{N}\}$ , and for  $n' \in \mathbb{N}'$  write  $\lceil n' \rceil$  to denote the least natural number greater than or equal to  $n'$ . Let  $\mathfrak{A} = \langle \mathbb{N}', \leq, \mathbb{Z}, \sim, [\cdot] \rangle$  be the following model (with  $[\neg]$ ,  $[\rightarrow]$ ,  $[\Box]$  and  $[\forall_\sigma]$  defined in the manner above):

$$i \sim_n j \text{ iff either } i = j \text{ or } |j| = |i| \leq n$$

$$[E](n) = (\mathbb{N} \cup \{i \in \mathbb{Z}/\mathbb{N} : i \geq -\lceil n \rceil\})$$

The model can be depicted by the following diagram (using the diagram conventions from the main body):



One can verify that  $\mathfrak{A}$  is a logical model of H4R because  $[E]$  is  $\sim$ -closed. Now consider the following function in  $\mathfrak{A}^{\sim}(\langle e \rangle)$ :

$$f(n) = \{i \in \mathbb{Z} : \text{there is some } m \in \mathbb{N} \text{ s.t. } i \sim_n m\}$$

Clearly when  $g(X) = f$ ,  $[Rig(X) \wedge X \equiv E]^g(0) = \{\langle \rangle\}$ . Moreover, since for every  $i \in \mathbb{Z}$  there is some  $n \in \mathbb{N}$  such that  $i \sim_n n$ ,  $[\Box \forall y (Ey \rightarrow \Diamond Xy)]^g(0) = \{\langle \rangle\}$  too. Thus  $\mathfrak{A}, 0, g \models (\text{RP2})$ , for arbitrary  $g$ . Next, for each  $k \in \mathbb{N}'$  consider the following function in  $\mathfrak{A}^{\sim}(\langle e \rangle)$ :

$$f_k(n) = \{i \in \mathbb{Z} : \text{there is some } m \in [E](k) \text{ s.t. } i \sim_n m\}$$

For each  $k \in \mathbb{N}'$ , when  $g(X) = f_k$ ,  $[X \equiv E]^g(k) = \{\langle \rangle\}$  since  $[E]$  is  $\sim$ -closed. Moreover, one can check that  $[Rig(X)]^g(k) = \{\langle \rangle\}$  due to the properties of  $\sim$ . Nonetheless, for each  $k \in \mathbb{N}'$  let  $k' = [k].5$ . Observe that  $f_k(k') \neq [E](k')$  because there is some  $i \in [E](k')$  for which there is no  $m \in [E](k)$  s.t.  $i \sim_{k'} m$ . Hence  $[\Diamond \exists y (Ey \wedge \neg Xy)]^g(k) = \{\langle \rangle\}$  when  $g(X) = f_k$ . Consequently, we may conclude that  $\mathfrak{A}, 0, g \models (\text{RP1}')$ , for arbitrary  $g$ .  $\square$

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