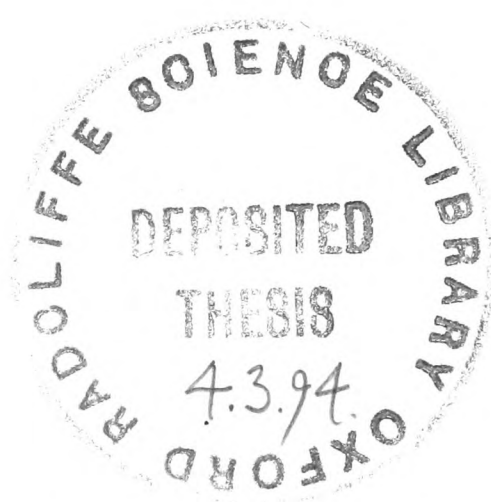


Mixing in Multiphase Jet Flow: Experimental Comparison with a Computational Model

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Submitted for the Degree of Doctor of Philosophy
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Abstract

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A series of experiments has been conducted for comparison with the results of a computer code called CHYMES. It is intended to calculate the coarse mixing of molten metal with water by solving the equations of the Separated Flow Model. These are derived by volume averaging and the terms that relate them to the particular case of particulate flow are discussed.

An experimental apparatus that is compatible with CHYMES and coarse mixing has been constructed which projects a jet of ball bearings into a thin tank of water. Experiments over a wide range of conditions were conducted at room temperature. Owing to practical difficulties only one, poorly controlled experiment with hot ball bearings was performed.

Under nearly all sets of conditions an arrow-shaped plume was obtained. The speed of penetration of the plume varied little with changes in experimental conditions. The width of the plume was most strongly influenced by the widths of the tank and the jet. The individual paths of some particles were followed; it appeared that their motion was mostly dependent on their position in the plume. A model of the plume is proposed, based upon its front being impermeable to water in the vertical direction.

Much of the detail of the experimental plumes was not present in the computational results and they responded differently to changes in conditions. It is proposed that this is a result of the different forms of the two sets of plumes. To rectify this an experimental plume was volume averaged. A method to determine a suitable averaging volume size is described. The process results in a plume similar to the computational ones. The length scales required for volume averaging to be successful are discussed and the possibility that this method is inappropriate for describing coarse mixing is admitted.

To Iris and Ian Gilbertson

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Nomenclature

Symbols

a_f, b_f Defined by (3.19)

a_n, b_n The boundary into and out of the n th particle

A Plane area

A_{ijkl} tensor defined by (3.3)

A_L Defined in (3.27)

$\mathbf{b}$ Mobility of a phase

C_{Avm} Coefficient for added mass term for a single particle with a low Reynolds number

C_D Drag coefficient of a particle in a multiphase flow

$C_{D\infty}$ Drag coefficient for an isolated particle

C_H Coefficient for the Basset force on a single particle with a high Reynolds number

C_{vm} Coefficient for added mass term for a single particle with a high Reynolds number

d Diameter or thickness

d_{ij} Deviatoric stress tensor

e Specific internal energy

e_{kk} Rate of expansion tensor for a fluid

$F_{\dot{m}}$ Momentum transferred between phases owing to evaporation

g Gravitational acceleration

h Enthalpy

H Heaviside step function

k Constant in (3.31)

K Kinetic energy density

l_v Size of averaging volume

- L** Length scale over which the flow properties change significantly, constraint of Hamilton’s equation
- $\dot{m}$** Rate of evaporation per unit volume
- M** Rate of momentum transfer owing to relative motion between the two phases per unit volume
- $M^\dagger$** Rate of momentum transfer per unit volume
- n** Exponent of equating terminal speed of an isolated particle with that of one in a multiphase flow
- $\underline{n}$** Unit normal to an interface
- n_p** Number of particles per unit volume
- p** Pressure
- q** Rate of heat transfer between phases
- r** Radial distance from a point
- Re** Reynolds number
- S** Surface area, displacement
- S_p** Particle spacing
- t** time
- T^+** Dimensionaless time defined by (1.1)
- u, U** Speed of a phase or a particle
- U_e** Speed of a dynamic wave
- U_α** Speed of a continuity wave
- V** Volume
- w** Speed of an interface between two phases
- x** Distance along one axis
- α** Volume fraction

γ Function indicating the presence of a phase at a point

δ Delta function

δ_{ij} Kronecker delta function

ε Dummy time

θ Rate of production of a conserved quantity

λ Additional virtual mass coefficient

μ Dynamic viscosity

μ'', e_{ij} Defined in (3.4)

ν Kinematic viscosity

ξ Local coordinate system

ρ Density

σ stress

ϕ Non-convective flux of a quantity

Φ Lagrange multiplier

ψ Any flow variable

ω Outcome of a measurement

Ω Class of all measurements

Subscripts

0 Original

1,2,3 Indices identifying the axes of a coordinate system

f Continuous fluid

i, j Indices in Section 2.2

j, k Phases

m Melt

- n** nth experiment
- p** Particle
- r** Relative
- s** Steam
- t** Tank
- w** water
- y** Lateral direction

Superscripts

- *** Random variation from the averaged variable
- T** Turbulent

Operations

- $\langle\langle\rangle\rangle$ Volume averaging defined by (2.2)
- $\langle\rangle$ Phase volume averaging defined by (2.3)
- $\hat{}$ Area average defined by (2.13)
- $|||$ Time average defined by (2.14)
- $\{\}$ ensemble average defined by (2.15)

- $\underline{}$ Vector

- ∇ Space derivative defined by (2.22). Sometimes accompanied by a subscript denoting the coordinate system that it is with respect to.

$\frac{D}{Dt}$, $\underline{D}_{kp}$, $\underline{a}_{jk}$ Operations defined on page 50

Chapter 1

Introduction

Nuclear power stations have at their heart a reactor where heat is generated by the fission of the nuclei of uranium atoms. It is this which differentiates nuclear power stations from other power stations. The reactor is cooled by passing a fluid through it. The heat transferred can be used for electricity generation, usually via a heat exchanger acting as a boiler in a conventional power generation circuit. A disadvantage of this is that great pains must be taken to ensure that the fission products, and the ionizing radiation they produce, are isolated from the outside environment; failure may be dangerous to life, both immediately and over many years, and it would be difficult and expensive to remedy its effects. Because the hazards of radioactive material are imperceptible and long lasting, nuclear reactors are strictly regulated; more so than nearly any other sort of plant, even though the consequences of an accident might be comparable e.g. the Bhopal disaster in 1984.

Until now all commercial reactors in the U.K. have used gas as a coolant. The first one to use water is being built now at Sizewell in Suffolk and will employ the Pressurised Water Reactor(P.W.R.) design shown schematically in Figure 1.1. The structure of the reactor core is shown in Figure 1.2. A public inquiry under Sir Frank Layfield was conducted into the introduction of this type of reactor (F.Layfield [1]). A major part of the inquiry concerned the risk of release of radioactive material into the environment. The likelihood of this happening may be estimated using probabilistic safety analysis (The Uranium Institute [2]). This is used to evaluate a wide variety of safety risks. In its simplest form a probability is assigned to each of the events which might lead to a certain conclusion. Event trees are then constructed and the overall probability of an event calculated. A refinement of this model is that probability distributions rather than definite figures are assigned to each event; these are then sampled numerically to produce the overall probability e.g. by a Monte-Carlo method (T.G.Theofanous

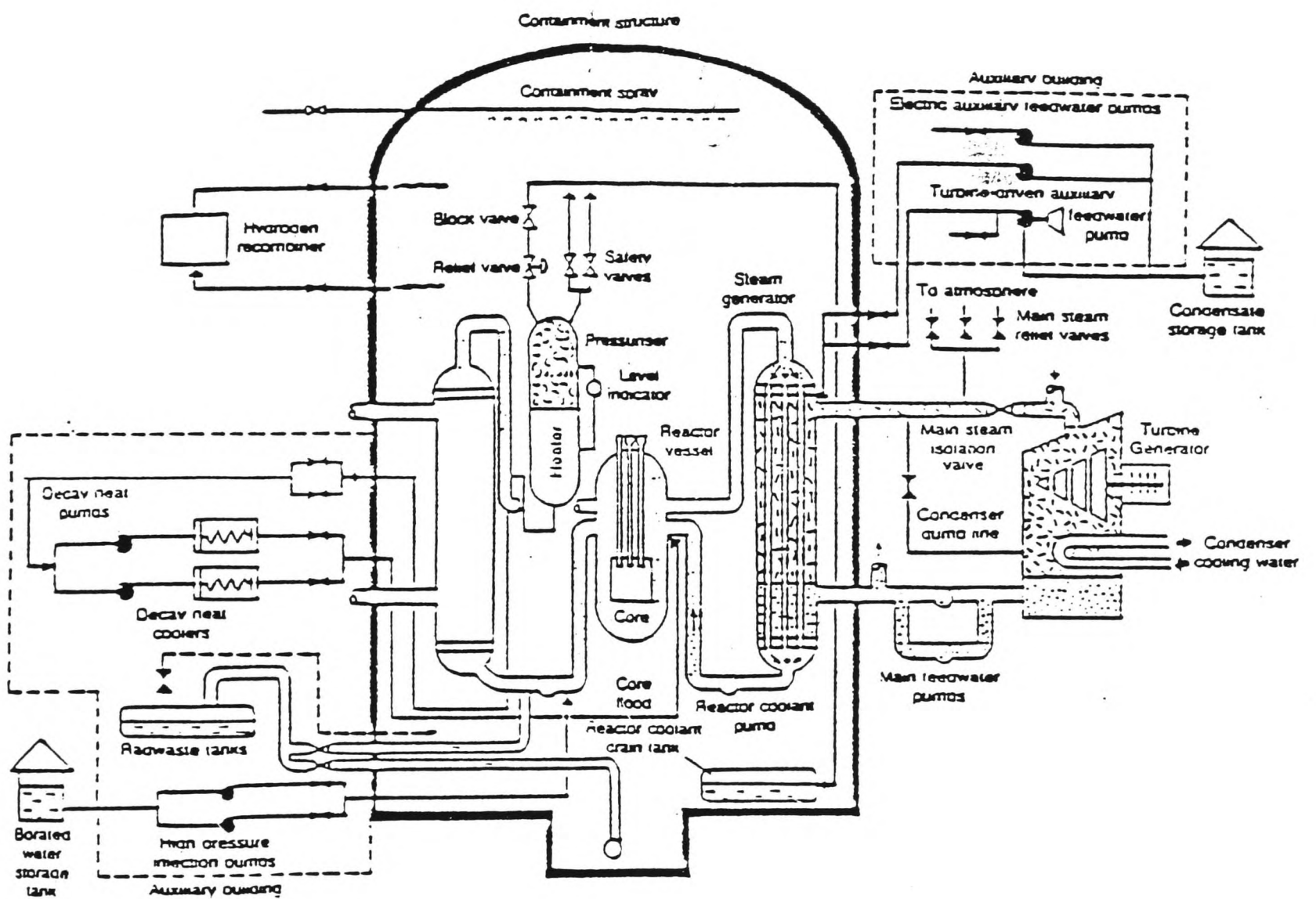


Figure 1.1: The Pressurised Water Reactor (The Uranium Institute [22])

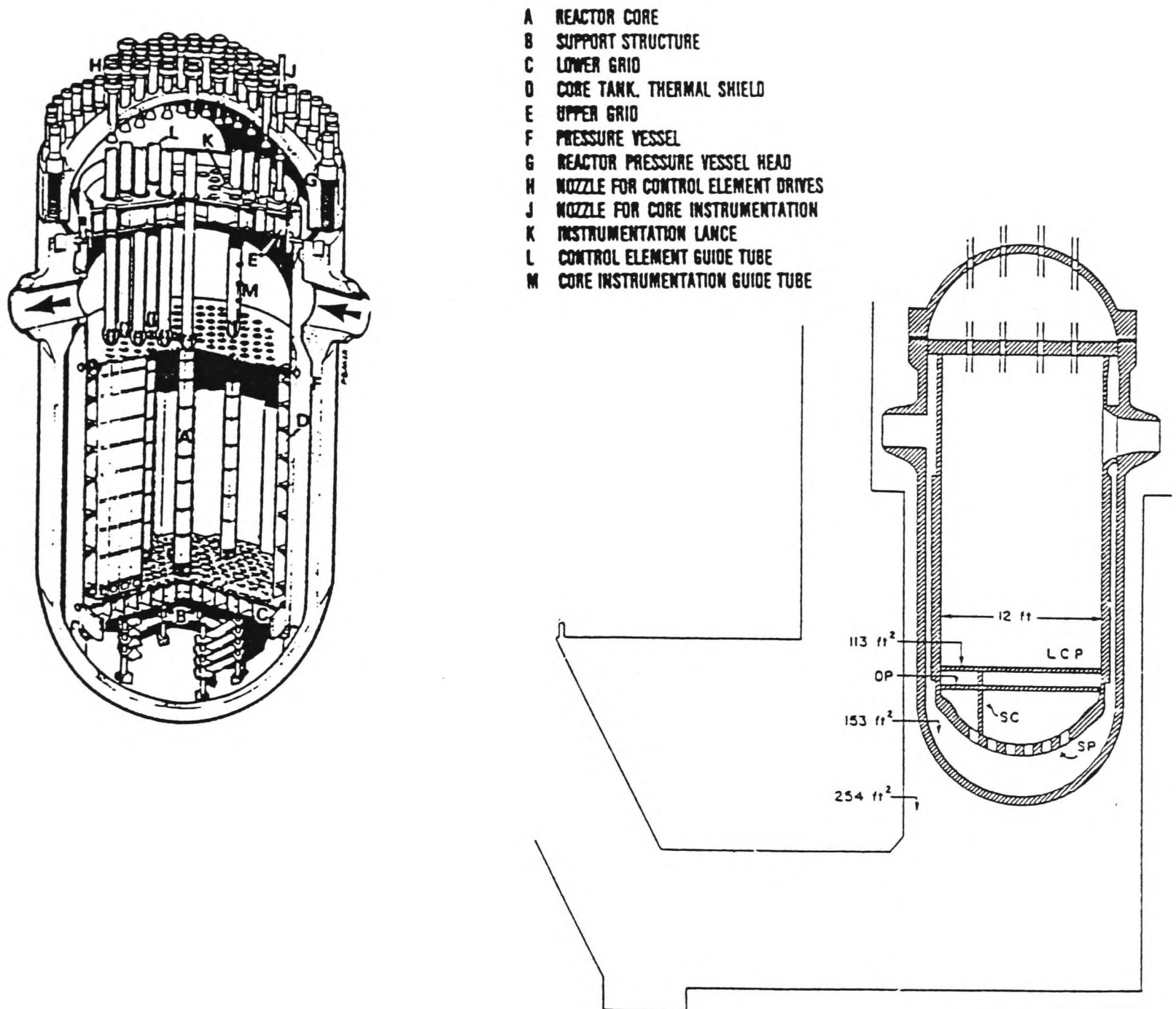


Figure 1.2: The Structure of the Reactor Core: left- LCP-lower core plate, DP-diffuser plate, SC-support columns, SP-support plate (T.G.Theofanous and M.Saito [5]), right-(M.L.Corradini [6])

et al [3]; I.Cook [4]). This is not an absolute probability, but a relative one that allows the comparison of the likelihood of different trains of events (American Physics Society [7]) . A reactor is judged to be sufficiently safe if the expected frequency of a release of radioactive material is less than once every ten million years [1]. The difficulties in this method of analysis are determining which events might lead to a release, and what their probability distributions are. Events of particular concern are those following the melting of the reactor core.

1.1 Core Melting Accidents

A reactor core will melt if it is not cooled. This problem is particularly severe in water cooled reactors because if the water is replaced by a gas the rate of cooling of the reactor is reduced by several orders of magnitude. This might happen through the draining of the water from the cooling circuit, or the vaporization of the water in the core if it stops circulating; causes of these include a breach in the primary circuit (e.g. because of failure owing to crack propagation), equipment failure, fire, malicious damage, or operator error or perversity. If cooling should cease the nuclear reaction is stopped by inserting neutron-absorbing control rods fully into the core and introducing borated water into the primary circuit to prevent the generation of more heat.

Even after the reaction has stopped the core still requires cooling because unstable reaction products generate some heat, called decay heat. This cooling may be provided by the emergency core cooling system which consists of accumulators, low pressure pumps, and high pressure pumps. Despite the diversity of safety systems available it is possible, though unlikely, for them to fail or be ineffective e.g. a station blackout would stop the pumps working, or if there is a large breach in the primary circuit the accumulator water might merely drain out of the primary circuit. If all should fail the temperature of the core will rise. The reactor is designed so that natural convection by the coolant will remove significant quantities of heat; this will be very much less effective if the coolant in the core is replaced by vapour through evaporation or drainage (R.J.Dallman and R.B.Duffey [8]). If the core temperature exceeds 800°C convection is further restricted owing to the zircaloy cans surrounding the fuel swelling and buckling, obstructing the coolant channels. Once their temperature is about 900°C the cans will react with steam to produce zirconium oxide, hydrogen, and heat. If the core temperature exceeds 1200°C then this reaction becomes violent [7]. The heat from the reaction exacerbates the situation and the hydrogen produced is a notable hazard in its own right.

Once the core reaches a temperature of 1800°C the fuel begins to melt. The melted portion of the core will coincide with the boundary between the liquid coolant and its vapour so that

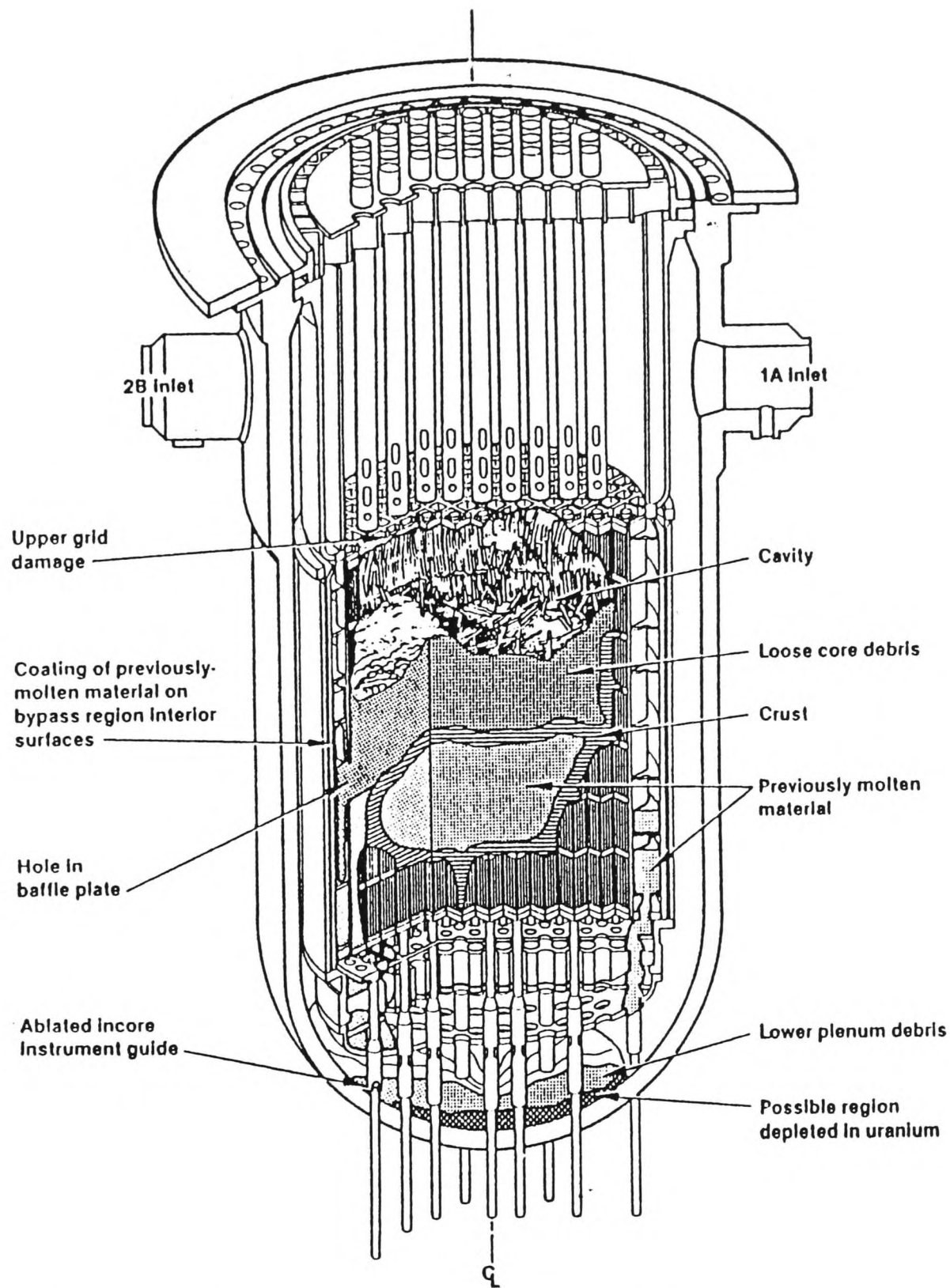


Figure 1.3: The Final State of the Three Mile Island Number 2 Reactor [8]

a crucible shaped region of melt is formed suspended by a frozen crust cooled by conduction and convection. Natural convection within the melt causes the slopping of materials over the side of the 'crucible' and thence to the lower plenum [8]. Much of the above was deduced from the final state of the Three Mile Island Reactor which is shown in Figure 1.3. Eventually the melted region will reach the core support plate. It is thought that this will fail locally, between the pillars supporting the reactor core (see Figure 1.4) (M.A.Abolfadl and T.G.Theofanous [9]). The melt will then drain under a gravity head of about 1.5 Bar [9] into the lower plenum. This is likely to contain a large pool of saturated water. It also contains several horizontal, perforated plates through which the melt will pass before arriving at the lower head of the reactor vessel. This is penetrated by many instrument and control tubes. If the melt is not sufficiently cooled

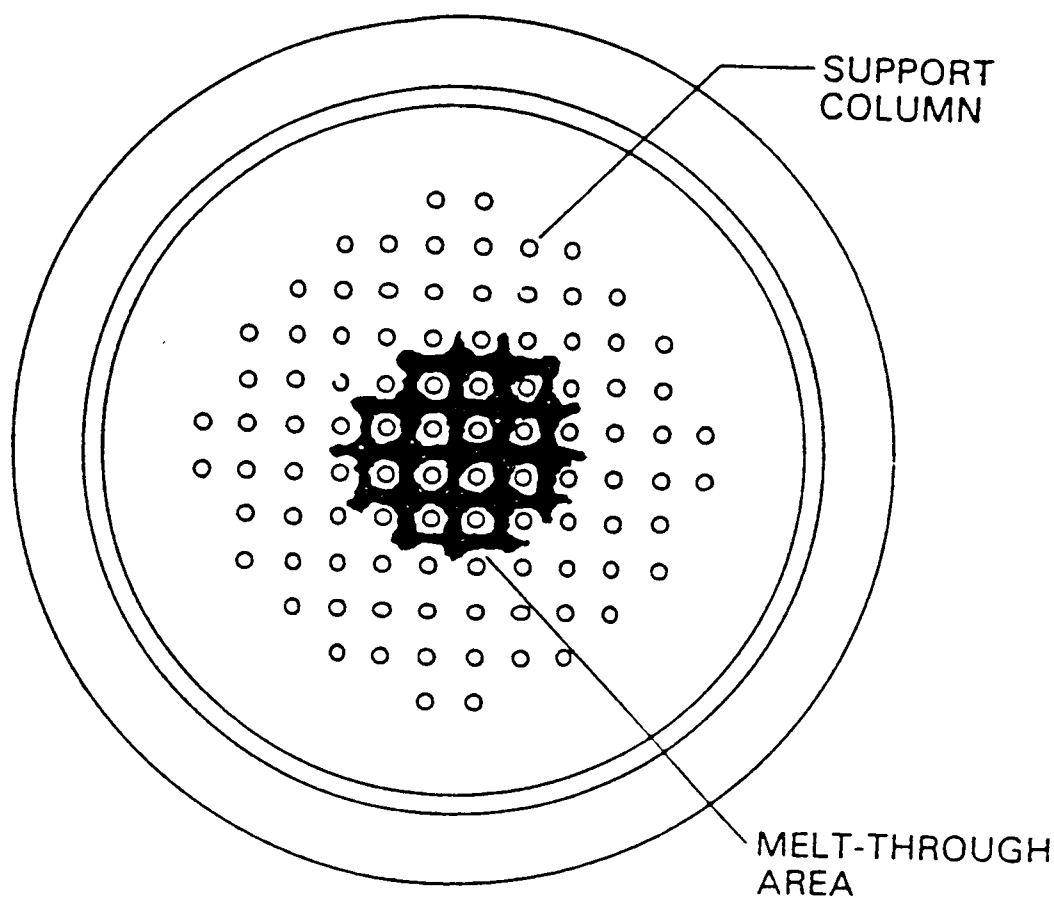


Figure 1.4: Schematic Diagram of the Melt-Through of the Lower Core Plate [9]

it will melt through the lower head, probably near these tubes as the head is thinnest there [8]. It will then fall onto the floor of the reactor containment and attack the concrete floor.

It takes about half an hour from the start of an accident for the core to start melting and then about another half hour for most of the core to melt [7]. Even at that stage there is no danger to anyone outside the reactor containment; unfortunately there is a phenomena which might threaten this state. It is known by many names (steam explosion, rapid phase transformation, and, inaccurately, fuel-coolant interaction) but I will call it the vapour explosion.

1.2 The Vapour Explosion

The vapour explosion can occur when a very hot liquid comes in contact with a cooler, volatile one. It is a wholly physical phenomenon caused by the rapid vaporization of the cooler liquid that leaves the chemical composition of the ingredients unchanged. Normally heat transfer from a body at a high temperature immersed in a liquid is by film boiling¹. The rate of heat transfer from the body is small owing to the low thermal conductivity of the vapour. A vapour explosion requires a high heat transfer rate, typically one thousand times that of film boiling; therefore the hot liquid must fragment rapidly to increase the area of contact between the liquids, and to allow them to come into direct contact with each other. Vapour explosions have occurred in

¹i.e. When sufficient vapour is produced through boiling to envelop the body in a film of vapour.

many situations: metal casting, paper making, and in experimental nuclear reactors (L.C.Witte et al [10]). They have also taken place when water and liquified natural gas have mixed, though the mechanism of these is thought to be different from that in other situations (R.C.Reid [11]).

A vapour explosion is dangerous in a P.W.R. as it might provide enough energy to form a missile from the degraded core and breach the reactor containment.

1.2.1 Experimental Investigations of Vapour Explosions

Accidents in aluminium foundries instigated the first serious investigation into vapour explosions (R.Long [12]). This consisted of an exhaustive series of experiments (880) involving the pouring of molten aluminium into a container of water at room temperature. The conclusion of these was that three conditions had to be fulfilled for a vapour explosion to occur:

- Molten metal must penetrate down to the bottom of the vessel: no explosions occurred if the pour diameter was too small or if the melt was broken up on entry either because of a large drop height, or because a grill had been placed over the water container;
- Some sort of trigger must occur when the molten metal is covering the bottom: explosions were suppressed if the container floor was coated with grease or paint;
- The melt temperature at the container floor has to exceed a certain value: in the experiments the deeper the water pool was, the hotter the melt had to be to precipitate an explosion.

It was postulated that vapour explosions were caused by the entrapment of water between the melt and the container floor, whose vaporization caused the jet of melt to fragment so rapidly that vapour blankets did not form over the resulting drops of melt, so allowing very high rates of heat transfer. Support for this came from experiments where only a small amount of water was present (water depth less than two inches): no explosion occurred, but molten aluminium was projected throughout the experiment's surroundings.

That this is the sole cause of vapour explosions has been disproved by two sets of small scale experiments. In the first (R.H.Bradley and L.C.Witte [13]) small amounts of various metals were injected through a one sixteenth of an inch hole into water with a temperature of 24°C. The metals used were Mercury, Arsorcolo-158², a lead-tin alloy, and tin. All four metals violently fragmented at high temperatures; at low temperatures they were quenched. There was also an intermediate state where the metal expanded into a porous mass. A sinuous instability could

² An alloy with a melting point of 70°C which does not contract a great deal on freezing.

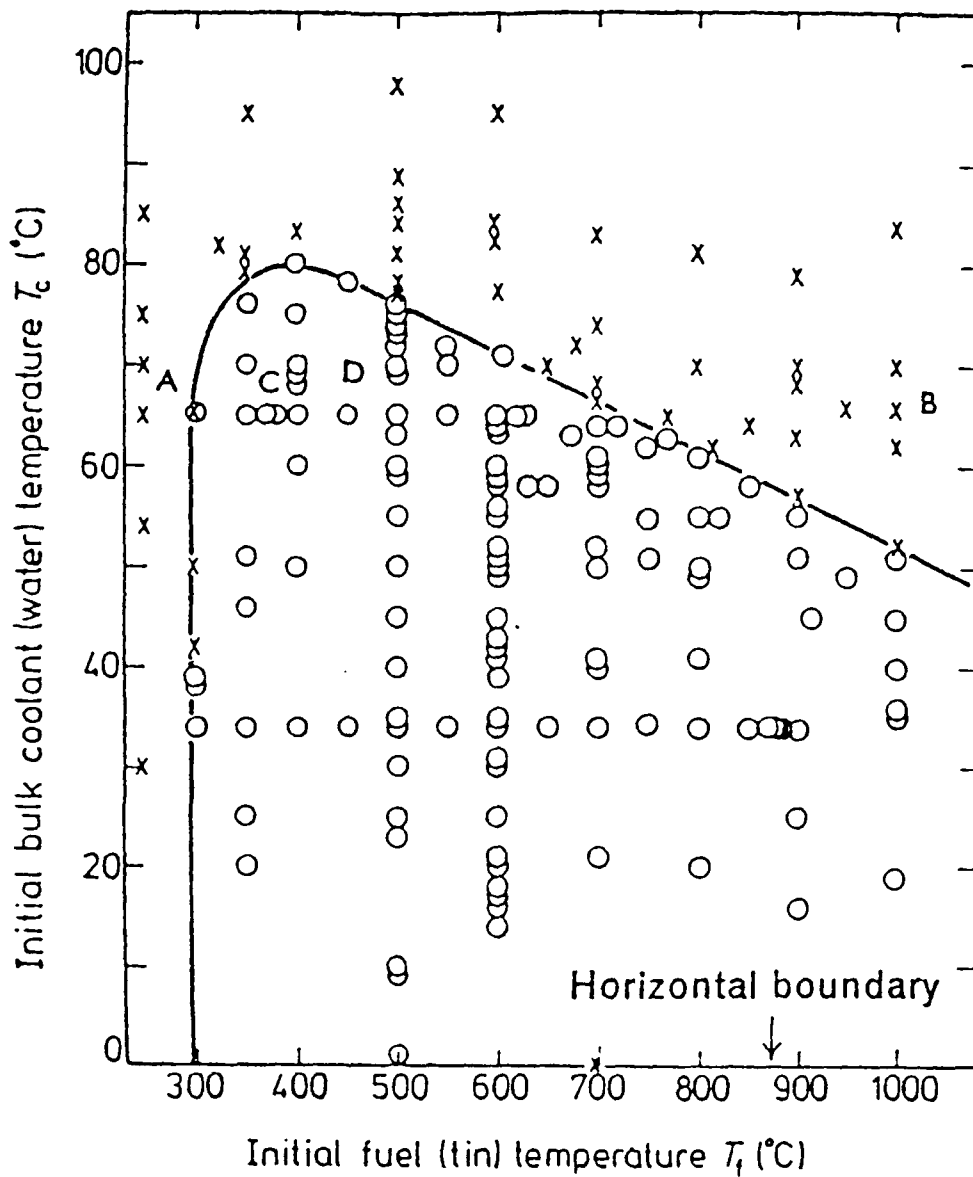


Figure 1.5: The Temperature Interaction Zone: $\circ$ indicates an interaction, $\times$ indicates no interaction [16].

be seen on the jet, but no extensive hydrodynamic break-up was observed. The front of the jet was shrouded in vapour and explosions often occurred at the point where its apex met the jet's neck. Explosions occurred without the jet being anywhere near a wall.

The second set of small experiments consisted of the pouring of a small amount of tin (twelve grammes) into water (F.A.Dullforce et al [16]). Ciné photography showed that there was a dwell time of 10–200ms before an interaction would occur which repeated itself three or four times with a delay of a few milliseconds between each cycle. An interaction would only occur spontaneously when the melt and coolant had certain temperatures, as shown in Figure 1.5. This regime was called the Temperature Interaction Zone (T.I.Z.). Using the percentage of material fragmented as a criteria, the most violent explosions took place close to the boundary of the T.I.Z., to the right of the apex of the regime shown in Figure 1.5. The same distinction between violent and non-violent explosions as in [13] could be made.

There have been larger scale experiments involving kilogrammes of corium³. These include the SUW and the WUMT series at what was AEE Winfrith (see D.F. Fletcher [14]), and the

³Corium is the name given to an alloy containing those metals found in a melted core.

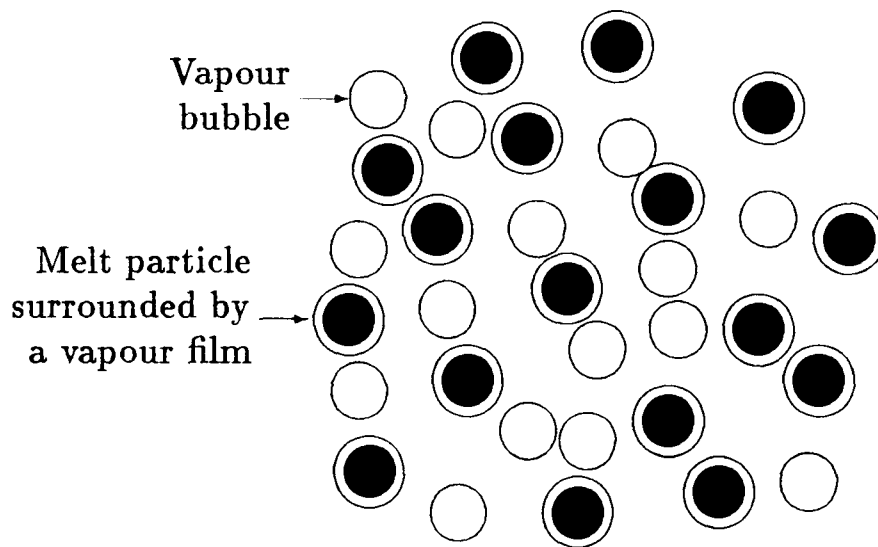


Figure 1.6: Proposed Configuration of Phases in Coarse Mixing.

experiments performed in the FITS vessel at the Sandia National Laboratory (M.L.Corradini [17], [6] and see F.Young [18]). Explosions in these experiments were triggered by the impact of the melt with the water pool's floor or surface (or sometimes by both). They were occasionally triggered at other times as well. Again an intermediate interaction was observed where the fuel-coolant interactions were isolated and no shock waves or fine fragmentation were observed: rather than a large, sharp pressure peak, a much flatter profile is produced. This sort of interaction has been called a steam spike or an incoherent event as opposed to a steam explosion or coherent event.

1.2.2 The Progress of a Vapour Explosion

In light of the above results, and unpublished information, a model for the vapour explosion in a P.W.R. has been widely adopted e.g. D.F.Fletcher [19]. This has four stages:

1. *Coarse Mixing*: Sometimes called premixing. At this stage the hot and cold liquids mix for a time of the order of one second. Vapour is produced by film boiling around the edge of the hot liquid. This allows only low heat transfer rates so there is a negligible pressure rise. It has been suggested that the typical configuration of the phases at this stage is that shown in Figure 1.6. During this stage the melt is fragmented into droplets with a diameter of 5–10 mm, each of which is surrounded by a stable vapour film.
2. *Triggering*: This is some sort of event that causes the local fragmentation of the melt. If the melt and coolant have suitable temperatures then an explosion will be spontaneously triggered owing to the film boiling around a liquid sphere being inherently unstable, especially if it is close to the minimum film boiling temperature (S.J.Corrrell [20]). Alternatively an external trigger might be provided by the melt striking a surface, or by a pressure pulse in the coolant from a small chemical explosion or another vapour explosion,

or from some mechanical blow;

3. *Propagation*: Once triggered the rapid heat transfer caused by the fragmentation of melt particles enables a detonation wave to propagate through the melt-coolant mixture. It is sustained by the extra heat transfer caused by the hydrodynamic fragmentation of particles in its path [17]. It may be at this stage that the steam spike differs from the vapour explosion: if there is too little melt or too much vapour close to the trigger the detonation wave will not be sustained. In a vapour explosion the thermal energy of the melt is liberated over a period of a few milliseconds;
4. *Expansion*: Finally the high pressure vapour resulting from the explosion will do work against the mixture's confinement.

1.2.3 The Vapour Explosion and Nuclear Reactors

When considering the danger of vapour explosions in nuclear reactors it is important to distinguish between core melt accidents, as described above, and core disruptive accidents. The latter was a major event in the Chernobyl accident, and also occurred in the destructive tests of some experimental reactors, such as SPERT-1 [10]. At Chernobyl, owing to the behaviour of the operators, the nuclear reaction was allowed to get out of control, causing a large and sudden increase in the power produced by the reactor. In that reactor the fuel rods were encased in ceramic and lay within the coolant channels. The power surge disintegrated the ceramic and projected it into the coolant (J.Gittus [21]). The effect of this was similar to that when molten metal fragments: the rate of heat transfer suddenly greatly increased and a vapour explosion occurred. The effects of this were compounded by a hydrogen and a graphite fire. Such an accident cannot happen in a P.W.R. as similar power surges are not possible and the geometry of the core is completely different [21].

Whether it is possible for a core melt accident to cause a vapour explosion, and whether it matters if it does, is a matter of some controversy. The only serious core melt accident involving a large reactor occurred at Three Mile Island. In this, again mainly through operator behaviour⁴ the core melted and relocated to the bottom of the reactor vessel. No fuel-coolant interaction occurred.

Some people think that it is impossible for major damage to result from a vapour explosion (B.Pershagen [23]), while others (e.g.[1]) do not find the evidence for this convincing. The

⁴The only equipment failure was the sticking of one valve (The Uranium Institute [22]).

major problem is that no one can predict with any degree of certainty what happens during a core melting accident. Even in simple conditions the mechanism of the vapour explosion is far from clear and this is compounded by ignorance about how the melt, the coolant, and the structures within the reactor core will behave in a core melting accident. It is unlikely that a satisfactory, mechanistic model will be formulated in the near future.

1.2.4 The Importance of Coarse Mixing

Because of the incomprehension described above, attempts to evaluate the risk to the containment of a reactor from a vapour explosion have revolved around energy considerations. If the entire core melted there would be about 140GJ of thermal energy available. From small scale experiments, such as the FITS series, the efficiency of a steam explosion is 1-2% [17]. The maximum explosion efficiency is thought to be 4% [1]; therefore 1.4–5.6 GJ is available if the entire mass of metal took place in the explosion. It is estimated that about 1.5GJ of mechanical energy is required to breach the reactor containment; therefore, even if the explosion efficiency was definitely correct, it would be uncertain whether it was possible for the reactor containment to be breached. However it is far from assured that the efficiencies from such experiments can be applied directly to a full-scale situation and it is not known how much of the melt participates in the explosion.

To resolve the latter problem work has mostly concentrated on coarse mixing. This is the least opaque stage in a vapour explosion because its time scale is long, making experiments easier, and it is controlled by the processes of fluid dynamics and heat transfer, both of which are mature disciplines. If coarse mixing can be adequately modelled then the maximum amount of melt that participates in an explosion can be determined, provided that suitable triggering and propagation models are available to estimate the proportions of melt, liquid coolant, and vapour necessary to support an explosion. The mass, or a probability distribution of the mass, available to participate in an explosion can then be calculated and hence the amount of thermal energy available. From this it should then be possible to construct a conservative probability distribution of the energy available to propel a missile at the reactor containment.

Unfortunately for this approach, despite the claims of some [9], the modelling of coarse mixing has proved to be far from easy.

1.3 The Modelling of Coarse Mixing

1.3.1 Experiments

Two clouds of ignorance hamper the modelling of coarse mixing. The first is uncertainty of the situation that is to be modelled; for example, the form, and the quantity of melt that might enter a water pool can only be speculated on.

The second is what processes are operating during coarse mixing. Little is known about how a heavy, immiscible jet behaves, and even less if the jet is very hot as well. It was reasoned that it should be possible to model such jets using the extensive body of knowledge concerning hydrodynamic and thermodynamic processes.

A model of coarse mixing is required to estimate the distribution of the phases involved and their length scales. Direct experimentation is difficult owing to expense, complexity, and difficulty in instrumentation. There is also the danger from vapour explosions. The rôle of experimentation is, therefore, to provide information about the processes that influence coarse mixing, and to check the validity of theoretical models.

There is a dearth of data for the transient behaviour of jets of a hot, heavy liquid entering another liquid. A widely used source is the FITS series of experiments [18], [17], [6]. However in these experiments because the interior of the jet was shrouded by steam and peripheral melt drops, no detailed information could be obtained; instead overall parameters such as mixture depth, mixture volume, and vessel pressure were measured. On leaving its original container the melt broke into particles with a diameter of 10–20 mm. It was not possible to estimate how this size changed owing to detail within the jet not being visible, and other problems such as the particles' luminosity. However inspection of the debris left after the experiments suggested that the final average drop size was $10\text{mm} \pm 5\text{mm}$ if a vapour explosion did not happen. This agreed with estimations made of the minimum particle size based on the size at which particles would be swept from the vessel by the vapour. A vapour explosion reduced the final particle diameter by about ten times.

An impression of the mixing of the three phases is shown in Figure 1.7. The results from the experiments were correlated using a non-dimensional time T^+ , based on the inertia of the liquids, defined as:

$$T^+ = \frac{U_t}{d_{mo}} t \left(\frac{\rho_w}{\rho_m} \right)^{1/2} \quad (1.1)$$

where the subscript m refers to the melt, subscript w to the water, d_{mo} is the diameter of the melt drops when they enter the water pool and U_t is the terminal velocity of the drop. This

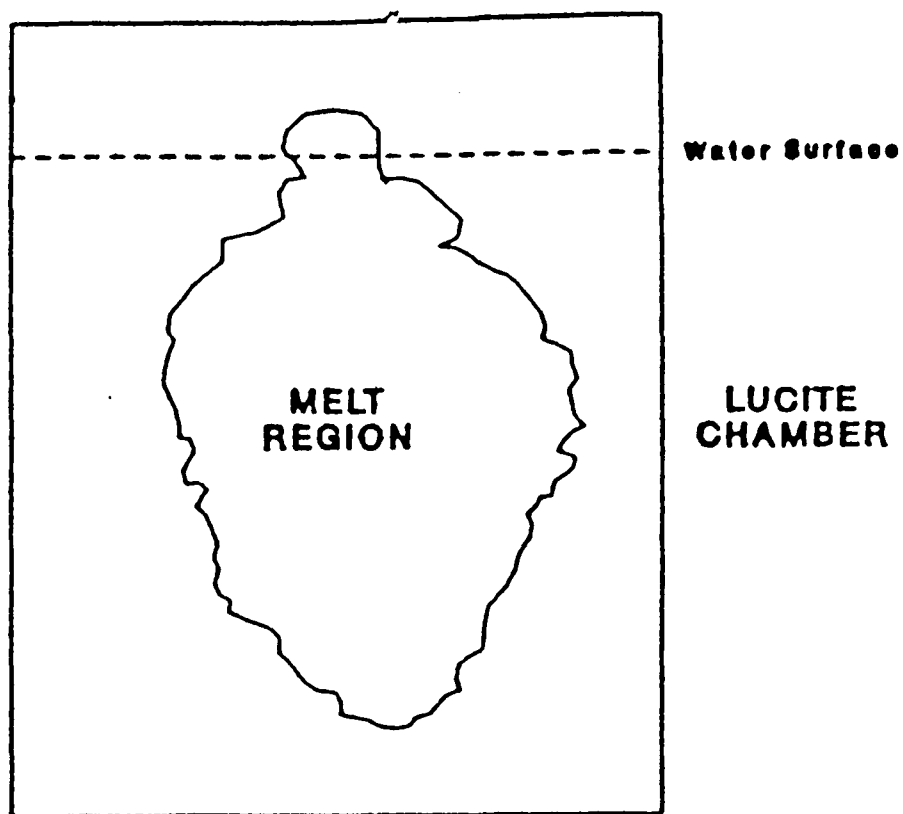


Figure 1.7: Impression of the Melt Outline in the FITS Experiments [18]

worked well for mixture depth, but was not so effective for mixture volume. However that may be because of the difficulty in defining, let alone measuring, such a volume. It was stated that less mixing was observed than in experiments where the material simulating the melt and the water were both at room temperature ('isothermal experiments').

More recently the MIX series of experiments have been performed at Winfrith (M.K.Denham et al [24]). In these about three kilogrammes of corium with a temperature of 3600K were released into a pool of saturated water. The melt was broken up into drops with a diameter between 1 and 10mm by passing it through a grid of carbon rods. The width of the jet entering the water pool was varied by changing the length of a skirt through which the melt passed. Observations of the melt front location and level swell are shown in Figure 1.8, and measurements of melt front penetration are shown in Figure 1.9. In MIXA01 and, to a lesser extent, MIXA04, the jet was wide enough for the drops to occupy the entire width of the tank and to be greatly retarded by the upflow of the steam generated. After some time Rayleigh-Taylor instabilities developed and the melt front accelerated. The profile of the leading edge of the narrowest jet (MIXA06) was similar to those observed during experiments in the FITS vessel. No vapour explosions occurred.

The mixing of molten metal and water was also examined at the Argonne National Laboratory in the United States (S.K.Wang et al, [25]). Wood's metal was used with a temperature of 600°C–750°C, which is high enough to cause film boiling. The experiments were performed to test a theoretical model of jet break-up. The jet took the shape shown in Figure 1.10 where a 'head' is formed at the stagnation point, and is followed by a column of material. Break-up

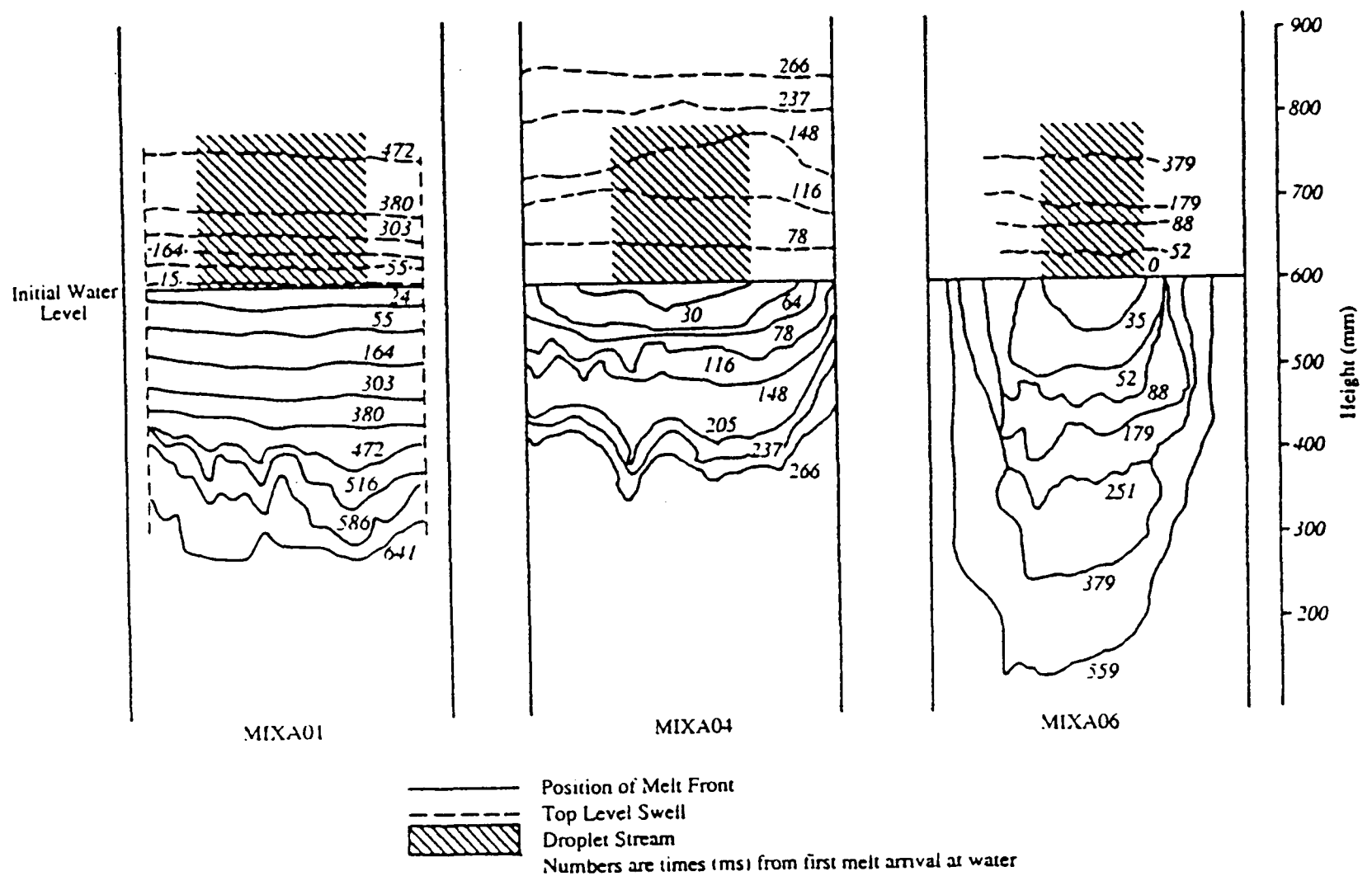


Figure 1.8: Melt Outline and Level Swell in MIX Experiments [24]

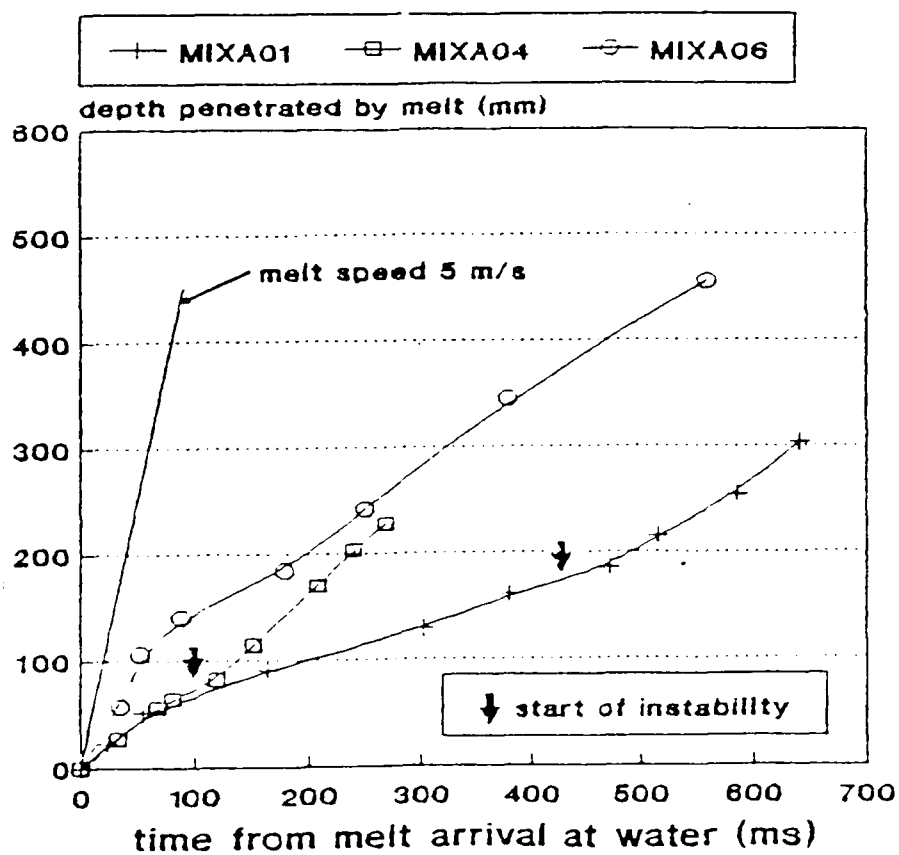


Figure 1.9: Penetration of Melt Jets in MIX Experiments [24]

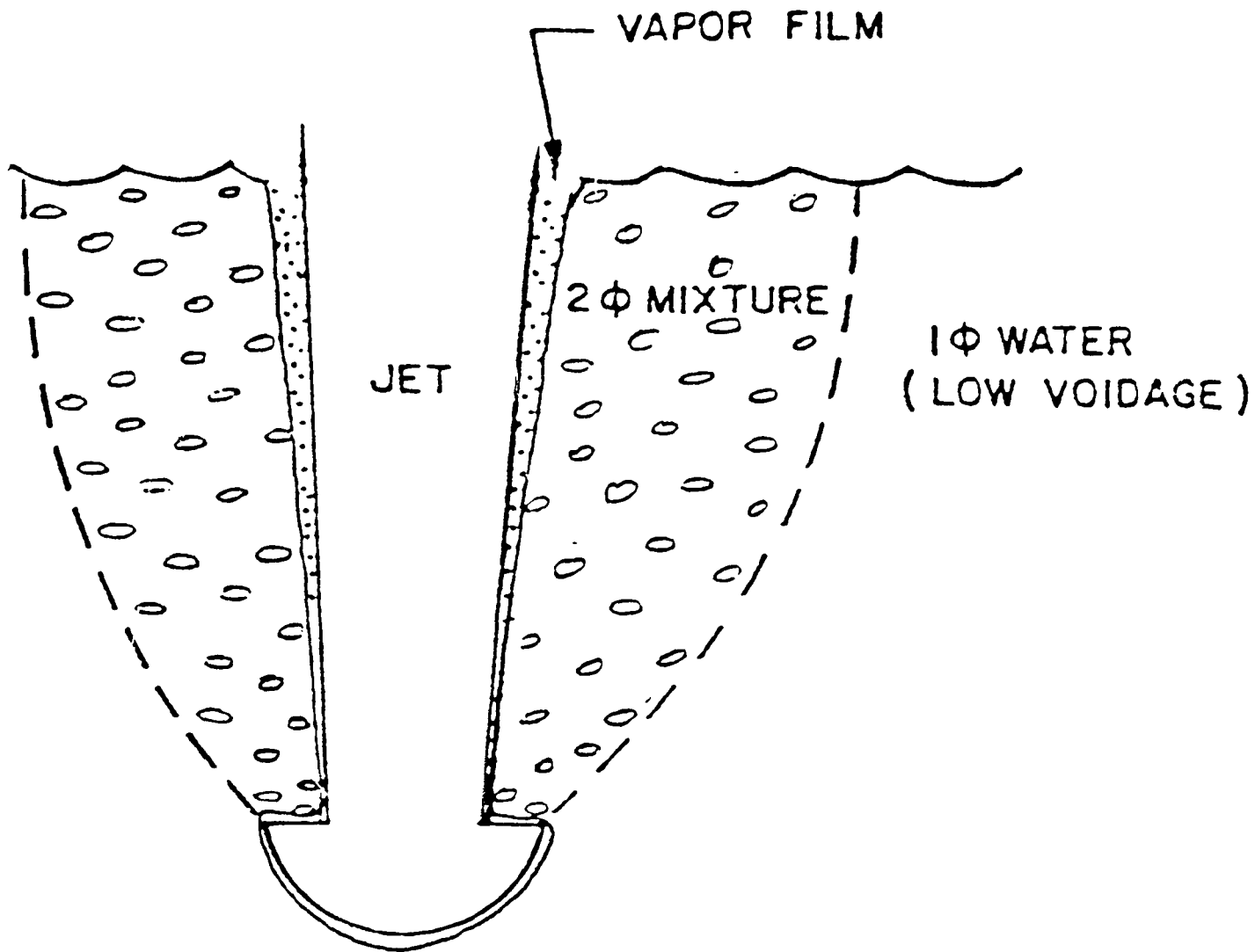


Figure 1.10: The Shape of a Jet of Molten Wood's Metal in Water [25]

occurs through the Kelvin-Helmholtz instability acting between the column and its vapour film, and by boundary layer stripping of the head by the flow of the vapour. As these mechanisms are controlled by vapour production they would be greatly influenced by a fuel-coolant interaction somewhere on the jet. Provided the metal temperature exceeded 600°C the experiments and the theory agreed quite well.

The difference between the behaviour of the melt in the FITS and MIX series of experiments and those in [25] may have two explanations: in the Argonne experiments the metal was melted using direct heat, while in the FITS experiments the melt was generated by a thermite reaction. There is evidence that this can pressurize the melt, causing it to fragment on leaving the nozzle of its container. In the MIX experiments the melt was forced to fragment. In addition molten corium is much hotter than Wood's Metal so the rate of production of vapour will be much larger and encourage the break-up of the melt jet.

Mercury jets with the same temperature as the water pool into which they drop have been examined (M.Gilbertson et al [26]), resulting in pictures such as those shown in Figure 1.11. The shape of the jet is similar to that of the jets in the Argonne experiments. Other notable features are the sinuous instability of the column at later times, and the widespread dispersion of mercury drops in the lower part of the tank once the head of the jet has broken up.

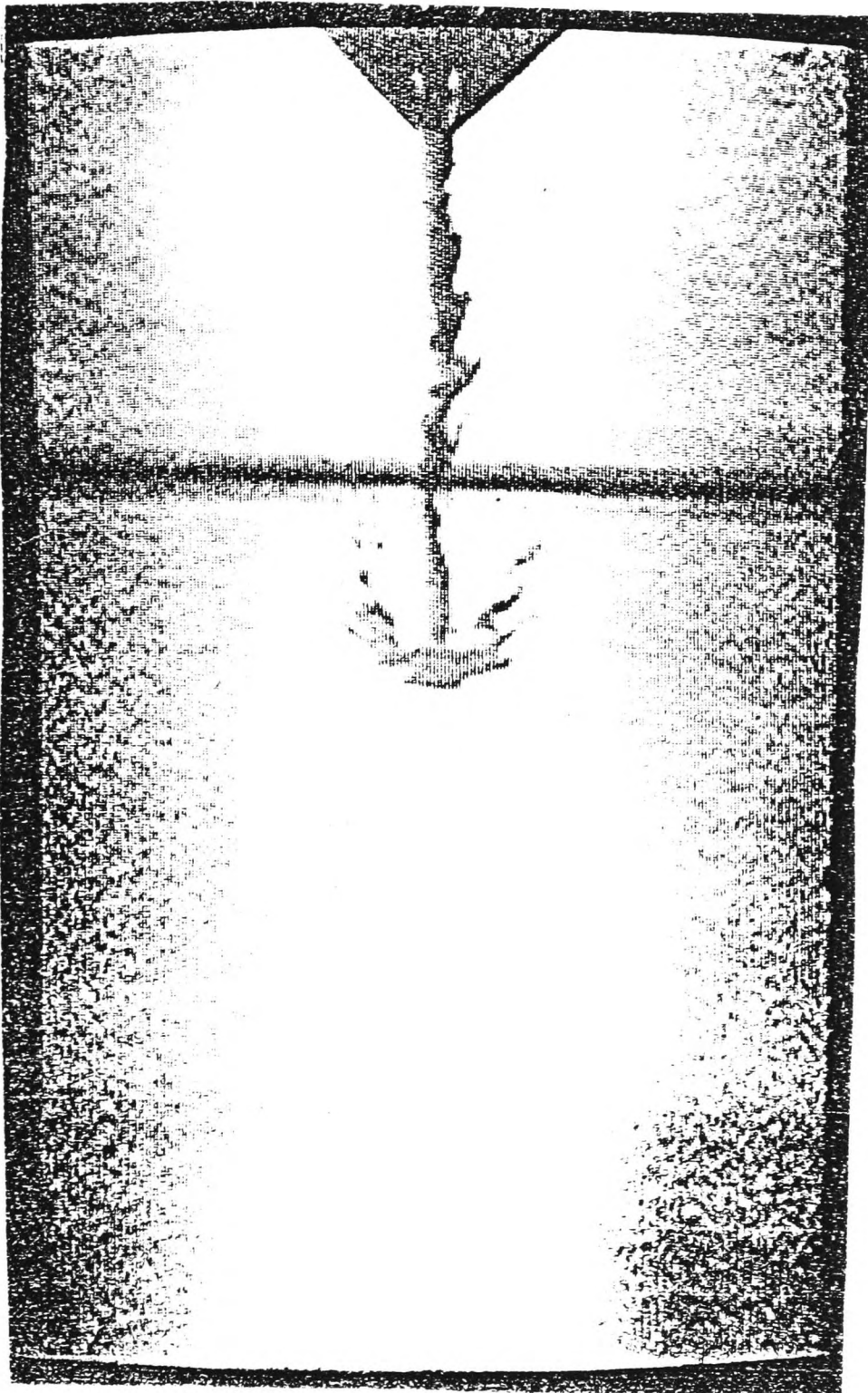


Figure 1.11: A Mercury Jet in Water [26]

From the above it is apparent that a variety of break-up mechanisms operate on a liquid jet. Many of these are poorly understood, as are the circumstances under which each operate; therefore it is not possible to construct a good model of jet break-up. This is bypassed by assuming that the melt is dispersed into individual, spherical drops before entering the water pool, which is what happened in the FITS experiments. Further hydrodynamic fragmentation of these drops could be modelled using a simple model derived from the FITS experiments:

$$\frac{d_m}{d_{mo}} = \exp^{-T+} \quad (1.2)$$

An alternative approach would be to use a Weber number based model.

1.3.2 Theoretical Models

Most work on modelling coarse mixing has been on the separated flow model. This describes the behaviour of each component (or phase⁵) of a multiphase flow separately using conservation equations for mass, momentum, and, sometimes, energy similar to those used for single-phase flow. In these, terms describing the distribution of mass, momentum, and energy within each phase and their transfer between phases have to be determined; these are known as constitutive relations. Various numerical codes have been formulated to solve these equations. Little progress has been made in improving the equations and effort has instead been concentrated on increasing the sophistication of the codes used to solve them. A steady, one-dimensional model and then an unsteady, one-dimensional model were solved first (S.G.Bankoff and S.H.Han [15]). Later a two-dimensional, unsteady model was developed using the PHOENICS solver (see [19]). A similar model which used a different solver is found in [9]. The problem with these is that they cannot allow the water and the steam to slip relative to each other i.e. they must travel with the same velocity. When slip was permitted in one-dimensional models, the results were very different from those when it was not [14]. This led to the current generation of models which are two-dimensional, unsteady, three-fluid models. Its main members are the Advanced Fluid Dynamics Model (A.D.F.M.) (W.R.Bohl et al [27]), the Integrated Fuel-Coolant Interaction (I.F.C.I.) model [18], the PM-ALPHA code (W.H.Amorasooriya and T.G.Theofanous [28]), and the CHYMES code (A.Thyagaraja and D.F.Fletcher [29]).

An alternative approach was that of the WISCI model, based on the FITS vessel experi-

⁵Used here this does not necessarily refer to the physical phase of a component of a flow. Indeed sometimes two different phases can be composed of the same substance in the same thermodynamic state; an example of this is annular flow in a pipe where the film of liquid around the walls and the droplets in the middle of the pipe may be treated as separate phases.

ments. It used a Lagrangian approach where a packet of melt which has been dropped into a water pool was followed. The character of the packet was determined using correlations from experiments. Not surprisingly the model agreed well with some of the experimental data, but not with others, and has since been discarded. It had two main failings: experimental correlations were used to estimate the quantity of each phase present, though it was impossible to estimate these with any accuracy from three-dimensional experiments; secondly a conical geometry was imposed on the path of the melt, but a glance at Figure 1.7 will demonstrate the folly of this.

1.3.3 The Validation of Codes

The improvement of the codes solving the equations of the separated fluids model has allowed them to conform to most of the gross physical requirements of the physical situation. By comparison of the results of one model with another for fixed test cases⁶ the accuracy of the numerical method used to solve the equations can be tested. However there are still uncertainties about how well this model conforms to the physical process of coarse mixing. These include:

- whether the equations adequately describe the processes in coarse mixing;
- the form of the constitutive relations;
- the implications of the assumptions made in the derivation of the equations of the separated flow model.

The importance of constitutive relations is shown in (D.F.Fletcher [30]) where the results of the CHYMES and PM-ALPHA codes are compared for various constitutive relations. These codes were developed separately but produce very similar results when the same constitutive relations are applied. If the constitutive relations are changed then very different results are produced.

To clarify these uncertainties it is necessary to appeal to experimental studies. This process is called the validation of the numerical codes. While full-scale experiments are not practically possible, and it would be difficult to acquire detailed results from them anyway, experiments which have some of the characteristics of coarse mixing are performed instead. Provided that they do not have any features that contravene the physical assumptions made in formulating the codes it should be possible to use the codes to model the results from such experiments. The comparison of the experimental results with those of a code can help to improve its equations,

⁶A process called verification. The test cases are theoretically based.

and show what conditions must be fulfilled for it to be applicable to a problem. Care has to be taken that the presentation of the experimental data is similar to that of the codes. Also the experimental data should not be used to tune the numerical codes i.e. any refinements should not apply peculiarly to specific experimental conditions.

The experiments described in this thesis were performed to contribute to the validation of the CHYMES model, developed at the Culham Laboratory of AEA Technology (once the United Kingdom Atomic Energy Authority). The organisation of the thesis is as follows: Chapter 2 will describe the derivation of the equations of the separated flow model; Chapter 3 will describe the constitutive relations employed; Chapter 4 will describe the experimental arrangement and programme; Chapters 5, 6, and 7 the experimental results; these are discussed in Chapter 8; the results of the experiments are compared with those of CHYMES in Chapters 9 and 10; Chapter 11 will discuss the use of the separated flow model to model coarse mixing and conclude the thesis.

Chapter 2

The Separated Flow Model

Numerical models of coarse mixing solve the partial differential equations of the separated flow model. ‘Separated flow’ in this case means that equations are written for each phase rather than for a mixture of phases; it should not be confused with a physically ‘separated flow’ such as annular flow in a pipe, though the model can be applied to such a situation. The model treats each of the phases of a multiphase flow as a continuum, and conservation equations are written to describe their motion. To call a field a continuum is to say that its properties vary continuously through space, and that the properties of the field may be assumed to be uniform throughout the smallest volume that can (or need) be resolved. If an instrument is used to measure a property of a flow, the above volume would be that which contains just enough of the mixture for the instrument to make a measurement. This assumption is analogous to that made in single-phase mechanics: a real fluid is composed of discrete molecules and at a molecular level is anything but a continuum; but, for most applications, the smallest length over which variations in properties need be perceived is larger, by several orders of magnitude, than the spacing between molecules and the fluid may be treated as a continuum [28].

It is also assumed that the phases are interpenetrating. This means that any volume in a flow will contain all the phases and that the proportion of each phase is independent, within limits, of the size of the volume. The amount of a flow occupied by a phase, k , is determined by the volume fraction, α_k . This may be defined as the volume of phase k enclosed by some region in the flow, divided by the total volume, V , of that region,

$$\alpha_k = \frac{V_k}{V}. \quad (2.1)$$

How the phases interact with each other, and how they behave internally, is described by the constitutive relations. These are the only way information relating to the real, discontinuous nature of a flow may be included in the model.

Because each phase is treated separately the model can be applied to a flow of any number of phases with boundaries of any shape using identical equations to those in this chapter. Most of the rest of this chapter will refer to a dispersed¹, two-phase flow for ease of description.

It is possible to postulate the equations used by the separated flow model from the Navier-Stokes equations by using, for example, an effective density $\alpha_k \rho_k$ [29]. This is not a useful method : it does not expose the limitations of the separated flow model, nor does it yield any information about the necessary form of the constitutive relations. Instead a more fundamental approach is described below.

2.1 Averaging

Averaging is the process by which the irregular nature of multiphase flow may be presented as that of a continuum. The concept of averaging is familiar from the treatment of experimental results where a result is found from the mean of several trials. This may be called the *ensemble average*. Other averages may be found by averaging over space (averaging over three spatial dimensions is called *volume averaging* and over two *area averaging*) or over time (*time averaging*).

2.1.1 The Volume Average

The most important average that is used is the volume average. It may be defined as:

$$\langle\langle\psi_k\rangle\rangle = \frac{1}{V} \int_{V_k} \psi_k dV. \quad (2.2)$$

i.e. a quantity, ψ_k , associated with phase k is integrated over that portion of a defined volume of mixture, V, (called the *averaging volume*) containing phase k, and divided by V. Note that if $\psi_k = 1$ then $\langle\langle\psi_k\rangle\rangle = \alpha_k$. But, as the model describes each phase separately, one is interested in just phase k; therefore the local volume average is employed. This is defined as

$$\langle\psi_k\rangle = \frac{1}{V_k} \int_{V_k} \psi_k dV. \quad (2.3)$$

The two averages may be related using (2.1) so that

$$\langle\langle\psi_k\rangle\rangle = \alpha_k \langle\psi_k\rangle \quad (2.4)$$

Three of the length scales that may be defined are: S_p , the separation of regions of the dispersed phase in the averaging volume; l_v , the size of the averaging volume; L, the length scale over

¹i.e. one where a phase is in the form of discrete particles within another phase

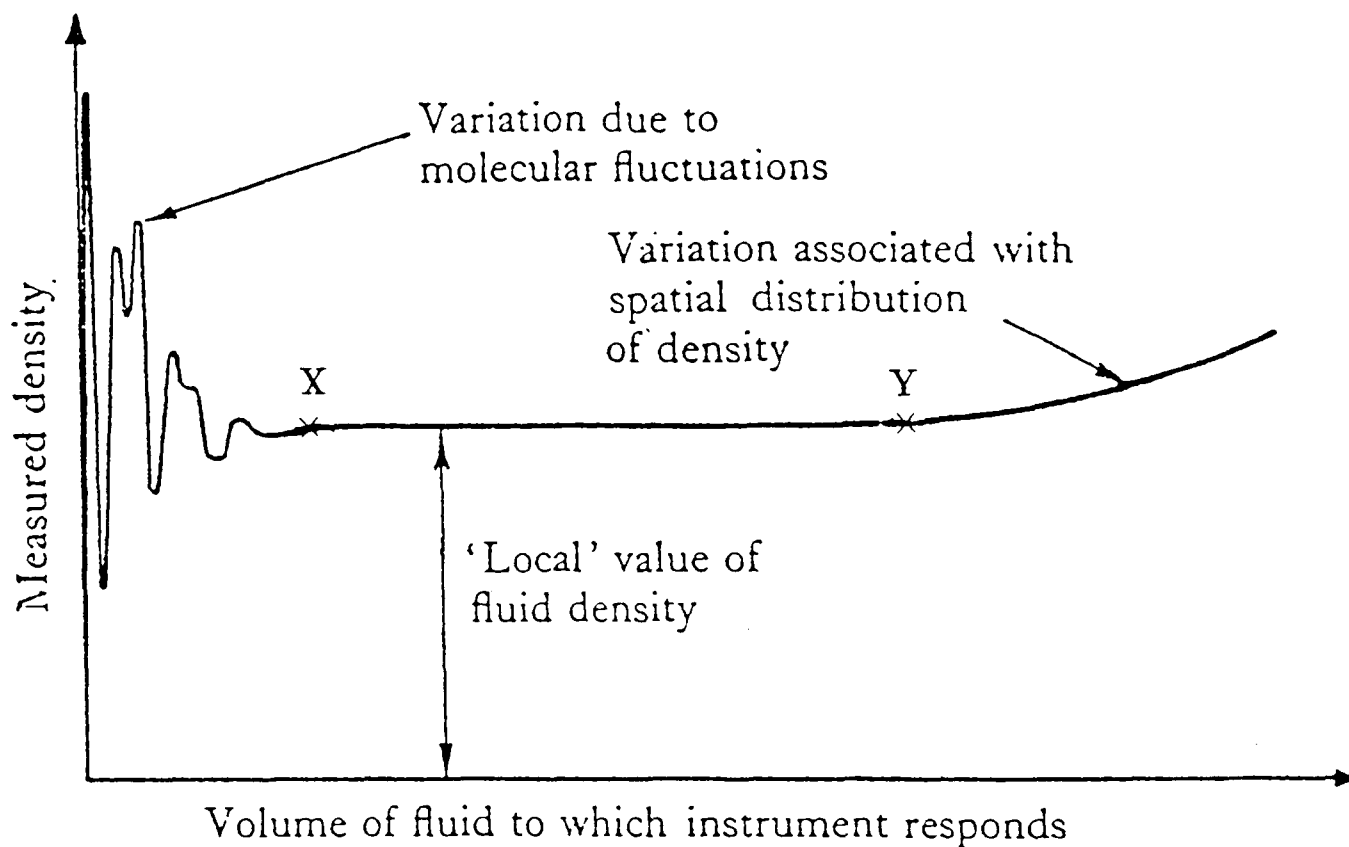


Figure 2.1: The Effect of the Averaging Volume Length Scale upon the Measured Value of Density in a Single-Phase Fluid [31]. In a Multiphase Mixture the Particles will be Analogous to the Molecules in a Single Phase Fluid.

which it is necessary to perceive variations in the flow properties. Volume averaging is subject to two limitations on these length scales.

The first has already been touched upon and is a result of the requirement that the phases should be treated as continua. To appreciate this, imagine an averaging volume in a flow consisting of phases j and k . Let this volume be infinitesimal initially, and then increase in size. If it is not situated within the phase being averaged, k , then any averaged quantity will be zero at first. As it grows it will encounter regions of phase k and the averaged quantity will acquire some value. At first it will fluctuate violently with increasing V as new particles are encountered by the averaging volume and the number of particles partially within the averaging volume is significant compared with the small number wholly enclosed by it. As V increases further the effect of the partially enclosed particles decreases until it appears that the averaged quantity is changing smoothly with averaging volume size. It is desirable that the separated flow equations are generally applicable to a flow; the averaged quantity must be independent of local variations in the spatial distribution of particles. For this to be true then the particle spacing length scale must be small compared to the averaging volume length scale i.e.

$$S_p \ll l_v. \quad (2.5)$$

The situation is analogous to the definition of a continuum in a fluid. There the particles are

molecules and their properties, such as their mass, are averaged within a volume to give overall properties such as density. Density represents the same thing as particle volume fraction in a multiphase mixture. The effect the size of the averaging volume has upon the value of the averaged property is shown in Figure 2.1. If (2.5) is satisfied the averaging volume will be large enough to lie to the right of point X; if not then the averaging volume will correspond to a point to the left of X. Note that in air at standard pressure and temperature a control volume that lies between points X and Y contains around 3×10^{10} molecules (G.K.Batchelor [31]).

The second limitation is that significant variations of averaged quantities should not be eliminated. In other words the averaged quantity should be independent of small variations in the size of the averaging volume and its contents approximately uniform. If this is so then an averaged property should be independent of further averaging [29] i.e.

$$\langle\langle\langle\langle\psi\rangle\rangle\rangle\rangle = \langle\langle\psi\rangle\rangle. \quad (2.6)$$

where the parentheses indicate that two averaging operations are carried out. Now imagine an averaging volume within a dispersed, two-phase mixture whose centroid coincides with the origin of a coordinate system $x_i, i = 1, 2, 3$. If $\langle\langle\psi\rangle\rangle$ is expanded as a Taylor series, using the repeated index notation²,

$$\langle\langle\psi\rangle\rangle = \langle\langle\psi\rangle\rangle_0 + x_i \left(\frac{\partial \langle\langle\psi\rangle\rangle}{\partial x_i} \right)_0 + \frac{x_i x_j}{2!} \left(\frac{\partial^2 \langle\langle\psi\rangle\rangle}{\partial x_i \partial x_j} \right)_0 + \dots \quad (2.7)$$

where the subscript 0 means ‘evaluated at $x_i = 0$ ’ i.e. at the centre of the averaging volume. Averaging (2.7) results in [29]

$$\langle\langle\langle\langle\psi\rangle\rangle\rangle\rangle = \langle\langle\psi\rangle\rangle_0 + \frac{1}{2!} \left(\frac{\partial^2 \langle\langle\psi\rangle\rangle}{\partial x_i \partial x_j} \right)_0 \frac{1}{V} \int_{V_k} x_i x_j dV + \dots \quad (2.8)$$

The second term of (2.7) disappears because of the coincidence of the origin of the coordinate system and the centroid of the averaging volume. If the approximations

$$\frac{1}{V} \int_{V_k} x_i x_j dV = O(l_v^2) \quad (2.9)$$

and

$$\left(\frac{\partial^2 \langle\langle\psi\rangle\rangle}{\partial x_i \partial x_j} \right)_0 = O \left(\frac{\langle\langle\psi\rangle\rangle}{L^2} \right) \quad (2.10)$$

are used then

$$\langle\langle\langle\langle\psi\rangle\rangle\rangle\rangle \approx \langle\langle\psi\rangle\rangle + O \left(\frac{l_v^2 \langle\langle\psi\rangle\rangle}{L^2} \right) \quad (2.11)$$

²i.e. all terms with a subscript represent the addition of that term over the system’s axes.

therefore for (2.6) to hold

$$l_v \ll L. \quad (2.12)$$

In Figure 2.1 if this condition is not satisfied the averaging volume will correspond to a point to the right of Y.

The average defined by (2.3) is derived from (2.4); therefore condition (2.12) will apply to it as well. For the averaging process to be useful the averaging volume should be of a size so that it lies between points X and Y on Figure 2.1.

2.1.2 The Area Average

An area average is defined as [29]

$$\hat{\psi}_{(2,3)} = \frac{1}{A_{(2,3)}} \int_{A_k} \psi_k dA \quad (2.13)$$

where the subscript (2,3) refers to the (2,3) plane in an orthogonal (1,2,3) space. Usually it is used to replace the volume average in flows whose properties depend upon only one spatial coordinate, but it has also been used for the derivation of equations of flows with more dimensions by combination with the time average.

2.1.3 The Time Average

Multiphase flows are often treated in a similar manner as turbulent flows by being time averaged. The time average is defined as (E.J.Hinze [33])

$$\| \psi \| (x, t) = \frac{1}{t_0} \int_t^{t+t_0} \psi(x, t + \varepsilon) d\varepsilon \quad (2.14)$$

Like the size of an averaging volume, the value of t_0 is constrained: it must be large compared to the time that the phase being averaged is not present at a point, and it must be small compared to the time scale over which the overall behaviour of the flow changes significantly.

R.Panton [34] used the time average in conjunction with the area average for a pipe flow. It was commented that the combination of averages places a new constraint on the flow that stipulates that the distribution of particles over the section of the pipe must be uniform. This is to be expected because time averaging an area average effectively converts it to a volume average: the averaging volume will be a slice taken through the pipe with a thickness equal to the phase's velocity multiplied by the averaging time. To comply with (2.12) the distribution of particles within this slice must be sufficiently uniform to avoid masking variations in the averaged quantities.

The time average is most useful on its own for flows where the value of averaged properties do not vary over space and, in combination with the area average, for flows where the flow properties vary significantly only in one direction.

S.Bannerjee and A.M.C.Chan [35] considered it necessary to both volume and time average to eliminate discontinuities in time. This operation is not necessary. The reason given for it is that whenever a particle enters an averaging volume a discontinuity is introduced which must be eliminated; but the reason for volume averaging (or area averaging in one-dimensional flows) is to ensure that the effect of these discontinuities, is negligible compared with that of the average value; therefore after volume averaging the flow should appear to be continuous. If a discontinuity arrives at a point or area at a certain time, this discontinuity must have travelled through preceding points at earlier times i.e. if a discontinuity is present within the flow at any time it will be visible in space; it will not suddenly appear. As volume averaging eliminates discontinuities in space then none can exist in time either; therefore, if volume averaging has been performed time averaging is unnecessary for the formulation of the equations for the separated flow model. The converse is also true.

2.1.4 Ensemble Averaging

A quantity measured in an experiment will often appear to have significantly different values during different trials under the same conditions. In the absence of a systematic error the mean value of this quantity will converge to a constant over many trials. The mean, otherwise known as the ensemble average, may be defined for some quantity ψ as [29]:

$$\{\psi\}(\mathbf{x}, t) = \frac{1}{N} \sum_{n=1}^N \psi_n(\mathbf{x}, t) \quad (2.15)$$

or

$$\{\psi\}(\mathbf{x}, t) = \int_{\Omega} \psi(\mathbf{x}, t : \omega) d\mu(\omega). \quad (2.16)$$

where Ω denotes the set of all possible outcomes of an experiment, ω is a particular outcome, and $\mu(\omega)$ is its probability distribution.

The ensemble average has sometimes been used to derive averaged equations for the separated flow model e.g.[35]. This is not a satisfactory method as it does not relate the averaged terms in an equation to the physical behaviour of a flow, nor give any indication of what the constitutive relations should be. Ensemble averaging also suffers from the same limitations as volume averaging. For example, the separated flow model equations were derived using ensem-

ble averages in [36] (D.D.Joseph and T.S.Lundgren). In this an indicator function³, $\gamma_k(\underline{x}, t)$, is defined as

$$\gamma_k(\underline{x}, t) = \begin{cases} 0 & \text{if } (\underline{x}, t) \text{ lies within a particle} \\ 1 & \text{if } (\underline{x}, t) \text{ lies within the fluid.} \end{cases} \quad (2.17)$$

Phase k is defined as a continuous fluid, and $\{\gamma_k\}$ is identified with α_w and $\{1 - \gamma_k\}$ with α_m . This identification is incorrect; for example, if a perfectly repeatable experiment with a single sphere moving in a fluid was performed, then at any point along its path at the appropriate time $\{\gamma_k\}$ would equal one. Clearly the particle volume fraction can never be this.

The form of the equations obtained by ensemble averaging is the same as that which results from volume averaging where a weighting function is applied (R.Jackson and D.A.Saville [37], T.B.Anderson and R.Jackson [38]). The weighting function, $g(r)$, where r is the radial distance from a point at which the average is evaluated, must have the following properties [38]:

- $g(r)$ is positive;
- $g(r)$ decreases monotonically as r increases;
- $g(r)$ tends to zero as r tends to infinity;
- $g(r)$ is normalised such that $4\pi \int_0^\infty g(r)r^2 dr = 1$.

The Gaussian distribution function satisfies the above conditions when the averaged quantity varies randomly from some mean value in different flows under the same conditions. The use of the weighted average is subject to the same length scale restrictions as the unweighted volume average [37], [38]. This suggests that ensemble averaging is a way of weighting volume averaging and suffers from the same restrictions on length scales as volume averaging. If a flow was observed several times, subject to the same nominal conditions, it would be expected that the value of volume averaged quantities would vary randomly about some mean. This would allow the identification of an ensemble average with a volume average and conforms with the usual intention when ensemble averaging is employed: to obtain experimental results that may be compared with those of a theoretical model.

2.1.5 Averaging and Coarse Mixing

Any of the averages described above might be applied to a general conservation equation to obtain an expression that describes the distribution of some quantity associated with a phase, though some are more suitable than others.

³This is described more fully in Section 2.2.

The flows in coarse mixing will be at least two-dimensional if they act anything like liquid jets; therefore area averaging is not suitable, either on its own, or in conjunction with the time average because it would not be possible to define an area over which the properties of the jet do not vary a great deal.

Ensemble averaging is not very useful because of the difficulty of relating averaged quantities to a physical basis unless the distribution of particles is wholly random or volume averaging is performed first.

The time average and the volume average may be interchangeable. The time average is most suitable for steady flows where there is a significant spatial variation in properties, and the volume average for the opposite situation. Most numerical codes, including CHYMES, are Eulerian; therefore the flow in most places will be unsteady as the plume of particles passes them. Volume averaging is then the most suitable method for modelling coarse mixing.

2.2 The Volume Averaging of Derivatives

Averaged equations should be composed of averaged variables if they are to be useful rather than averaged functions, such as derivatives, of variables. It is then necessary to find a method to allow the conversion of the latter to functions of averaged variables. The derivatives that will be present in the separated flow model equations are those of space and time. The discussion below follows the method found in W.G.Gray and P.C.Y.Lee [39]. There are alternative methods e.g. J.C.Slattery [40], but all have a similar result.

Generalised Functions The separated flow model is based on the assumption that any quantity associated with a phase (even a particulate phase) forms a field throughout the whole flow. In other words the value of a property at any time of a small portion of a flow is solely determined by its position, not on the presence of that certain portion of material; that just provides material with which the field is associated. A discontinuous distribution of a property is converted to a continuous one by averaging. The averaging process depends only on the phase that is being averaged; it is independent of the contents of the intervening space, several other phases could occupy that space, or just one. In the following discussion the subscript k denotes the phase that is being averaged and the subscript j denotes whatever might occupy the surrounding medium. This situation is ideal for the application of *generalised functions* and the method described in [39] relies upon their use. Detailed information about them may be found in Lighthill [41]. One such function is the Heaviside step function, defined

as

$$H(x_1 - a) = \begin{cases} 0, & x_1 < a \\ 1, & x_1 \geq a \end{cases} \quad (2.18)$$

in an orthogonal coordinate system (x_1, x_2, x_3) . Its derivative is

$$\frac{dH(x_1 - a)}{dx_1} = \delta(x_1 - a) \quad (2.19)$$

where $\delta(x)$ represents the dirac delta function evaluated at x . If a section is taken through a one-dimensional multiphase mixture an indicator function may be defined

$$\gamma_k(x_1) = H(x_1 - a_0) - H(x_1 - b_0) + H(x_1 - a_1) - H(x_1 - b_1) + \dots \quad (2.20)$$

where a refers to a transition from phase j to phase k and b refers to the opposite. This is exactly the same as the indicator function defined by (2.17).

After differentiation

$$\frac{d\gamma_k(x_1)}{dx_1} = \delta(x_1 - a_0) - \delta(x_1 - b_0) + \delta(x_1 - a_1) - \delta(x_1 - b_1) + \dots \quad (2.21)$$

In a three dimensional mixture where $\underline{n}_k$ is the unit normal at a boundary pointing from phase k to phase j and

$$\underline{\nabla}_x = \underline{i} \frac{\partial}{\partial x_1} + \underline{j} \frac{\partial}{\partial x_2} + \underline{k} \frac{\partial}{\partial x_3}, \quad (2.22)$$

if similar expressions in the x_2 and x_3 directions to (2.21) are used

$$\underline{\nabla}_x \gamma_k(\underline{x}) = - \sum_{m=0}^n \underline{n}_k \delta(\underline{x} - \underline{x}_{jk}) \quad (2.23)$$

where $\underline{x}_{jk}$ is the coordinate of the boundary between the phases.

Now consider the coordinate system shown in Figure 2.2. An averaging volume is located at $\underline{x}$ with respect to some fixed coordinate system. $\underline{x}$ acts as the origin of a local coordinate system, $\underline{\xi}$, which is used to locate points within the averaging volume. The size and shape of the averaging volume is assumed to be independent of time and space. For such a system (2.4) may be written as

$$\langle\langle\psi_k\rangle\rangle = \frac{1}{V} \int_V \gamma_k(\underline{x} + \underline{\xi}, t) \psi_k(\underline{x} + \underline{\xi}, t) dV_{\xi} \quad (2.24)$$

2.2.1 The Average of a Space Derivative

Using the method in [39], from (2.24)

$$\langle\langle\nabla\psi_k\rangle\rangle = \frac{1}{V} \int_V \gamma_k[\nabla\psi] dV_{\xi}. \quad (2.25)$$

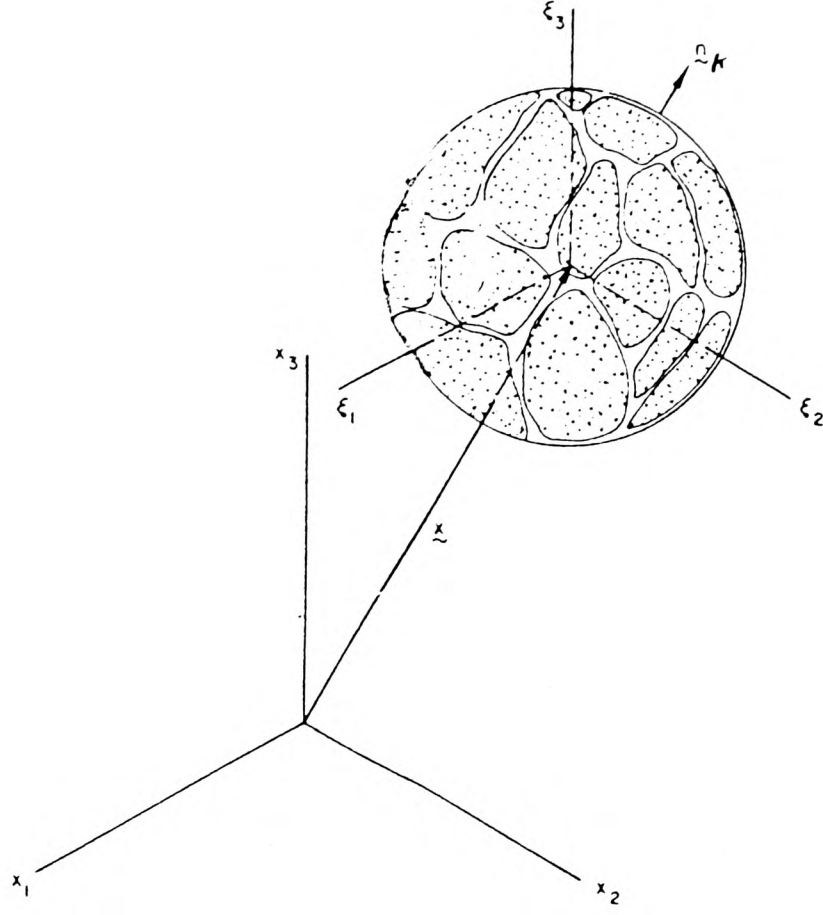


Figure 2.2: The Coordinate System Used in Section 2.2 [39]

Using the chain rule,(2.23), and the properties of the delta function⁴ (2.25) may be re-written as

$$\langle\langle \underline{\nabla} \psi_k \rangle\rangle = \frac{1}{V} \int_V \underline{\nabla} [\gamma_k \psi_k] dV_\xi + \frac{1}{V} \int_{A_{jk}} \psi_k \underline{n}_k dA. \quad (2.26)$$

where A_{jk} is the surface of the interface between j and k .

If the $\underline{\nabla}$ is considered to be $\underline{\nabla}_x$ it may be removed from inside the integral sign as the size and shape of the averaging volume is assumed to be independent of position; hence, using (2.24),

$$\langle\langle \underline{\nabla}_x \psi_k \rangle\rangle = \underline{\nabla}_x \langle\langle \psi_k \rangle\rangle + \frac{1}{V} \int_{A_{jk}} \psi_k \underline{n}_k dA. \quad (2.27)$$

Thus the volume average of the space derivative of a variable can be related to the space derivative of the volume averaged variable.

A form of Gauss's theorem for multiphase flow was also derived in [39] which relates gradients of volume averaged quantities to their values on the interface between j and k . If the $\underline{\nabla}$ in (2.26) is assumed to be $\underline{\nabla}_\xi$ then using Gauss's theorem

$$\langle\langle \underline{\nabla}_\xi \psi_k \rangle\rangle = \frac{1}{V} \int_S \underline{n}_k \psi_k \gamma_k dS_\xi + \frac{1}{V} \int_{A_{jk}} \psi_k \underline{n}_k dA \quad (2.28)$$

where S is the area of the surface of the averaging volume. When evaluating the first integral only the area where phase k is present is integrated so if the right hand sides of (2.27) and

⁴Specifically, if the product of a delta function and some ordinary function is integrated the result will be the ordinary function evaluated at the point where the delta function is non-zero.

(2.28) are equated,

$$\underline{\nabla}_\xi \langle \langle \psi_k \rangle \rangle = \frac{1}{V} \int_{S_k} \underline{n}_k \psi_k dS_\xi. \quad (2.29)$$

is obtained. $S_k + A_{jk}$ forms a closed surface; therefore

$$\int_{S_k} \psi_k \underline{n}_k dS + \int_{A_{jk}} \psi_k \underline{n}_k dA = 0, \quad (2.30)$$

whence

$$\underline{\nabla}_\xi \langle \langle \psi_k \rangle \rangle = -\frac{1}{V} \int_{A_{jk}} \psi_k \underline{n}_k dA. \quad (2.31)$$

2.2.2 The Average of a Time Derivative

Again, as in [39], from (2.24)

$$\left\langle \left\langle \frac{\partial \psi_k}{\partial t} \right\rangle \right\rangle = \frac{1}{V} \int_V \frac{\partial \psi_k}{\partial t}(\underline{x} + \underline{\xi}, t) \gamma_k(\underline{x} + \underline{\xi}, t) dV_\xi. \quad (2.32)$$

Using the chain rule, (2.23), and assuming that the size and shape of the averaging volume are independent of time,

$$\left\langle \left\langle \frac{\partial \psi_k}{\partial t} \right\rangle \right\rangle = \frac{\partial}{\partial t} \langle \langle \psi_k \rangle \rangle - \frac{1}{V} \int_V \psi_k \frac{\partial \gamma_k}{\partial t} dV. \quad (2.33)$$

The total derivative of γ_k with respect to time is

$$\begin{aligned} \frac{d\gamma_k}{dt} &= \frac{\partial \gamma_k}{\partial t} + \frac{dx_1}{dt} \frac{\partial \gamma_k}{\partial x_1} + \frac{dx_2}{dt} \frac{\partial \gamma_k}{\partial x_2} + \frac{dx_3}{dt} \frac{\partial \gamma_k}{\partial x_3} \\ &= \frac{\partial \gamma_k}{\partial t} + \underline{w}_{jk} \cdot \underline{\nabla} \gamma_k. \end{aligned} \quad (2.34)$$

where $\underline{w}_{jk}$ is the velocity of the interface between the phases. $\underline{\nabla} \gamma_k$ only exists at a phase boundary so the integral with respect to volume in (2.32) may be transformed to one with respect to surface area. If an observer was positioned on the boundary no change with time would be discernable; therefore

$$\frac{d\gamma_k}{dt} = 0, \quad (2.35)$$

Substituting (2.35) into (2.33)

$$\left\langle \left\langle \frac{\partial \psi_k}{\partial t} \right\rangle \right\rangle = \frac{\partial \langle \langle \psi_k \rangle \rangle}{\partial t} - \frac{1}{V} \int_{A_{jk}} \psi_k \underline{w} \cdot \underline{n}_k dA. \quad (2.36)$$

2.3 The Averaging of Conservation Equations

The conservation equation of some property ψ_k is

$$\frac{\partial \rho_k \psi_k}{\partial t} + \underline{\nabla} \cdot \rho_k \psi_k \underline{u}_k = \underline{\nabla} \cdot \underline{\phi}_k + \rho_k \theta_k \quad (2.37)$$

where $\underline{\phi}_k$ is the non-convective part of the flux of ψ_k , and θ_k is its specific rate of production.

If the equation is volume averaged using (2.3) or (2.24),

$$\left\langle \left\langle \frac{\partial \rho_k \psi_k}{\partial t} \right\rangle \right\rangle + \langle \langle \nabla \cdot \rho_k \psi_k \underline{u}_k \rangle \rangle = \langle \langle \nabla \cdot \underline{\phi}_k \rangle \rangle + \langle \langle \rho_k \theta_k \rangle \rangle. \quad (2.38)$$

To solve this equation it is necessary to express it in terms of individual averaged variables rather than as averages of products or derivatives of variables. Applying (2.27) and (2.36)

$$\begin{aligned} \frac{\partial}{\partial t} \langle \langle \rho_k \psi_k \rangle \rangle + \nabla \cdot \langle \langle \rho_k \phi_k \underline{u}_k \rangle \rangle &= \nabla \cdot \langle \langle \underline{\phi}_k \rangle \rangle + \langle \langle \rho_k \theta_k \rangle \rangle \\ + \frac{1}{V} \int_{A_{jk}} \rho_k \psi_k (\underline{w}_{jk} - \underline{u}_k) \cdot \underline{n} dA &+ \frac{1}{V} \int_{A_{jk}} \rho_k \underline{\phi}_k \cdot \underline{n}_k dA \end{aligned} \quad (2.39)$$

Converting volume averages to local volume averages using (2.2)

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_k \langle \rho_k \psi_k \rangle + \nabla \cdot \alpha_k \langle \rho_k \psi_k \underline{u}_k \rangle &= \nabla \cdot \alpha_k \langle \underline{\phi}_k \rangle + \alpha_k \langle \rho_k \theta_k \rangle \\ + \frac{1}{V} \int_{A_{jk}} \rho_k \phi_k (\underline{w}_{jk} - \underline{u}_k) \cdot \underline{n}_k dA &+ \frac{1}{V} \int_{A_{jk}} \rho_k \underline{\phi}_k \cdot \underline{n}_k dA. \end{aligned} \quad (2.40)$$

This is still not in a useful form because of the products of quantities enclosed by averaging brackets. As in single phase fluid mechanics further simplification may be made if it is assumed that all the phases are incompressible; this eliminates all the products except for that in the second term of (2.40), $\psi_k \underline{u}_k$. A result of the continuum assumption is that the value of ψ_k is nearly uniform in the averaging volume. Any property ψ_k may then be defined as

$$\psi_k = \langle \psi_k \rangle + \psi_k^* \quad (2.41)$$

where ψ_k^* is the variation of ψ_k from $\langle \psi_k \rangle$ over space and where, by definition, $\langle \psi_k^* \rangle = 0$. It can then be shown that [34]

$$\langle \psi_k \underline{u}_k \rangle = \langle \psi_k \rangle \langle \underline{u}_k \rangle + \langle \psi_k^* \underline{u}_k^* \rangle; \quad (2.42)$$

hence (2.40) may be written in terms of averaged variables so that

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_k \rho_k \langle \psi_k \rangle + \nabla \cdot \alpha_k \rho_k (\langle \psi_k \rangle \langle \underline{u}_k \rangle + \langle \psi_k^* \underline{u}_k^* \rangle) &= \nabla \cdot \alpha_k \langle \underline{\phi}_k \rangle + \\ \alpha_k \rho_k \langle \theta_k \rangle + \frac{\rho_k}{V} \int_{A_{jk}} \phi_k (\underline{w}_{jk} - \underline{u}_k) \cdot \underline{n}_k dA &+ \frac{\rho_k}{V} \int_{A_{jk}} \underline{\phi}_k \cdot \underline{n}_k dA \end{aligned} \quad (2.43)$$

2.3.1 The Conservation of Mass

For the conservation of mass $\psi_k = 1$, $\theta_k = \phi_k = 0$; therefore (2.43) becomes

$$\frac{\partial \alpha_k}{\partial t} + \nabla \cdot \alpha_k \langle \underline{u}_k \rangle = \frac{1}{V} \int_{A_{jk}} (\underline{w}_{jk} - \underline{u}_k) \cdot \underline{n}_k dA \quad (2.44)$$

because $\langle \psi_k^* \rangle = 0$. Note that

$$\dot{m}_{jk} = \frac{\rho_k}{V} \int_{A_{jk}} (\underline{w}_{jk} - \underline{u}_k) \cdot \underline{n}_k dA \quad (2.45)$$

where $\dot{m}_{jk}$ is the rate of conversion of mass from phase j into phase k per unit volume of mixture; therefore, if the three phases present in coarse mixing are denoted by m (melt), w (water), and s (steam), the equations for the conservation of mass are:

$$\frac{\partial \alpha_m}{\partial t} + \nabla \cdot (\alpha_m \langle \underline{u}_m \rangle) = 0 \quad (2.46)$$

$$\frac{\partial \alpha_w}{\partial t} + \nabla \cdot (\alpha_w \langle \underline{u}_w \rangle) = -\frac{\dot{m}_{ws}}{\rho_w} \quad (2.47)$$

$$\frac{\partial \alpha_s}{\partial t} + \nabla \cdot (\alpha_s \langle \underline{u}_s \rangle) = \frac{\dot{m}_{ws}}{\rho_s} \quad (2.48)$$

2.3.2 The Conservation of Momentum

For the conservation of momentum $\psi_k = \underline{u}_k$ and $\theta_k = \underline{g}$. $\underline{\phi}_k$ is the stress tensor for phase k, $\underline{\sigma}_k$; its definition is problematic and controversial, and is dealt with in Section 3.1.

From (2.43)

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_k \rho_k \langle \underline{u}_k \rangle + \nabla \cdot \alpha_k \rho_k (\langle \underline{u}_k \rangle \langle \underline{u}_k \rangle + \langle \underline{u}_k^* \underline{u}_k^* \rangle) = & -\alpha_k \nabla \langle \underline{\sigma}_k \rangle + \\ & \alpha_k \rho_k \underline{g} + \frac{\rho_k}{V} \int_{A_{jk}} \underline{u}_k (\underline{w}_{jk} - \underline{u}_k) \cdot \underline{n}_k dA - \frac{\rho_k}{V} \int_{A_{jk}} \underline{\sigma}_k \cdot \underline{n}_k dA. \end{aligned} \quad (2.49)$$

Setting

$$\underline{F}_{mjk} = \frac{\rho_k}{V} \int_{A_{jk}} \underline{u}_k (\underline{w}_{jk} - \underline{u}_k) \cdot \underline{n}_k dA, \quad (2.50)$$

i.e. the force per unit volume of mixture owing to the conversion of phase j to phase k, and

$$\underline{M}_{jk}^\dagger = \frac{1}{V} \int_{A_{jk}} \underline{\sigma}_k \underline{n}_k dA, \quad (2.51)$$

i.e. the momentum transferred from phase j to phase k, the resulting equations are:

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_m \rho_m \langle \underline{u}_m \rangle + \nabla \cdot \alpha_m \rho_m (\langle \underline{u}_m \rangle \langle \underline{u}_m \rangle + \langle \underline{u}_m^* \underline{u}_m^* \rangle) \\ = -\nabla \alpha_m \langle \underline{\sigma}_m \rangle + \alpha_m \rho_m \underline{g} + \underline{M}_{wm}^\dagger + \underline{M}_{sm}^\dagger \end{aligned} \quad (2.52)$$

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_w \rho_w \langle \underline{u}_w \rangle + \nabla \cdot \alpha_w \rho_w (\langle \underline{u}_w \rangle \langle \underline{u}_w \rangle + \langle \underline{u}_w^* \underline{u}_w^* \rangle) \\ = -\nabla \alpha_w \langle \underline{\sigma}_w \rangle + \alpha_w \rho_w \underline{g} - \underline{M}_{wm}^\dagger - \underline{M}_{ws}^\dagger - \underline{F}_{mws} \end{aligned} \quad (2.53)$$

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_s \rho_s \langle \underline{u}_s \rangle + \nabla \cdot \alpha_s \rho_s (\langle \underline{u}_s \rangle \langle \underline{u}_s \rangle + \langle \underline{u}_s^* \underline{u}_s^* \rangle) \\ = -\nabla \alpha_s \langle \underline{\sigma}_s \rangle + \alpha_s \rho_s \underline{g} - \underline{M}_{sm}^\dagger + \underline{M}_{ws}^\dagger + \underline{F}_{mws} \end{aligned} \quad (2.54)$$

2.3.3 The Conservation of Energy

The steam and water fields are assumed to be isothermal and saturated; therefore provided the pressure field does not vary a great deal and velocities are not very high, these do not require conservation of energy equations. In the case of the melt if mechanical processes are negligible compared with thermal ones $\psi_m = e_m$, $\underline{\phi}_k = \underline{q}_k$, and $\theta_k = 0$; thus, from (2.43),

$$\begin{aligned} \frac{\partial}{\partial t} \alpha_m \rho_m \langle e_m \rangle + \underline{\nabla} \cdot \alpha_m \rho_m (\langle e_m \rangle \langle \underline{u}_m \rangle + \langle e_m^* \underline{u}_m^* \rangle) = \\ \underline{\nabla} \cdot \alpha_m \langle \underline{q}_m \rangle + \frac{\rho_m}{V} \int_{A_{mw}} \underline{q}_m \cdot \underline{n}_m dA. \end{aligned} \quad (2.55)$$

Because the initial melt temperature is very high in coarse mixing (about 3000K), its change in temperature while mixing should be small compared with the initial temperature; therefore because the fluids are assumed to be saturated $\langle \underline{q}_m \rangle$ is a constant. Hence, as

$$\frac{\rho_k}{V} \int_{A_{wm}} \underline{q}_m \cdot \underline{n}_m g dA = \dot{m}_{ws} h_{fg} \quad (2.56)$$

for saturated fluids, the melt energy conservation equation is

$$\frac{\partial \alpha_m \rho_m \langle e_m \rangle}{\partial t} + \underline{\nabla} \cdot \alpha_m \rho_m (\langle e_m \rangle \langle \underline{u}_m \rangle + \langle e_m^* \underline{u}_m^* \rangle) = \dot{m}_{ws} h_{fg}. \quad (2.57)$$

Chapter 3

Constitutive Relations

The equations (2.46)–(2.48), (2.52)–(2.54), and (2.57) and the constitutive relations $\underline{\sigma}_{jk}$, $\underline{M}_{jk}$, $\underline{F}_{\dot{m}_{ws}}$, and $\dot{m}_{ws}$ constitute the separated fluids model used by numerical codes such as CHYMES. The constitutive relations introduce the detail of a system into the model. For example in an annular flow of water and air, the gas core would be one phase and the water cylinder enclosing it another. To apply the separated flow model it would be necessary to define terms for the stress distribution in each phase and the momentum exchange between them at their interface. The flow of particles in a fluid has a complicated nature so simplification is necessary to provide workable relations. In coarse mixing it is possible that the flow might consist of melt particles in a churn flow there is a comparable amount of water and vapour present, or of all three phases in similar amounts. In such cases the flow is practically indescribable at present. The discussion will be therefore restricted mostly to a situation where the particles of one phase are moving within another single, continuous phase.

The separated flow model demands the satisfaction of the length scale inequalities (2.5) and (2.12) so that the size of the averaging volume is large compared with the particle spacing, but small compared with the length over which averaged variables change significantly. The actual value of a variable at any point may differ from the averaged one owing to local conditions, but these should average out over the averaging volume; therefore the constitutive relations may be evaluated by considering a single particle in a fluid, both of which have the average characteristics of the material in the averaging volume and multiplying the result by the number of particles. The number of particles in an averaging volume is equal to the volume occupied by the particles divided by the volume of a single particle so that the number per unit volume of mixture is

$$n_p = \frac{6\alpha_p}{\pi d_p^3} \quad (3.1)$$

3.1 The Stress Field

The definition of the stress fields and their relation to $\underline{M}_{jk}$ is controversial. This discussion will begin with a review of the definition and evaluation of the stress field within a single phase flow, taken from the description in [31].

3.1.1 Stress in a Single Phase Flow

Stress in a fluid is a result of short range forces between molecules. In this case ‘short’ means small compared with the distance over which changes in the flow properties can be discerned. It can be shown that stress may be expressed as a symmetrical tensor, the diagonal terms of which represent the stresses normal to the surface of an element of fluid and the off-diagonal terms the tangential stresses; hence stress can be written, in index notation, as

$$\sigma_{ij} = -p\delta_{ij} + d_{ij} \quad (3.2)$$

where d_{ij} is called the deviatoric stress tensor and δ_{ij} is the Kronecker delta function¹. p is defined as the mean normal stress with the sign reversed; under certain circumstances it can be identified with the thermodynamic property, pressure, but here only the mechanical definition will be used. After consideration of transport theory for simple shear it is assumed that

$$d_{ij} = A_{ijkl} \frac{\partial u_k}{\partial x_l} \quad (3.3)$$

where A_{ijkl} is a tensor dependant on the local state of the fluid and symmetrical about i and j . If the fluid is isotropic² it can be shown that

$$d_{ij} = 2\mu e_{ij} + \mu'' \Delta \delta_{ij} \quad (3.4)$$

where μ , μ'' are scalar constants, e_{ij} is the symmetrical part of $\partial u_i / \partial x_j$, and Δ is the rate of expansion of the fluid $e_{kk} = \underline{\nabla} \cdot \underline{U}$. For an incompressible fluid $e_{kk} = 0$; therefore at any point

$$\sigma_{ij} = -p\delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (3.5)$$

3.1.2 Stress in the Continuous Phase

(3.5) can be applied directly at any point in the continuous phase of a multiphase flow. The deviatoric stress tensor is difficult to handle because it is dissipative rather than conservative

¹ $\delta_{ij}=1$ if $i=j$, and $=0$ otherwise.

²i.e. the deviatoric stress tensor generated in an element of fluid by a given velocity gradient is independent of the orientation of the element.

in nature; hence Gauss's theorem cannot be applied because what is happening at the edges of the averaging volume may bear no resemblance to what is happening inside it; therefore shear stresses are often neglected. This is equivalent to assuming that the continuous fluid is potential flow, though effects caused by viscosity at the phase boundaries, such as separation from the particles, may be incorporated through M_{jk} provided the relative velocity between the particles and the surrounding fluid is large, and the boundary layers present within the flow are small compared with the particle spacing. Nevertheless attempts have been made to devise terms to represent shear stress. For example, in [42](M.Ishii) the shear stress in the continuous fluid is given as

$$\underline{\tau} = \mu_f(\nabla\langle\underline{U}_f\rangle + (\nabla\langle\underline{U}_f\rangle)^t) + \mu_f \frac{b(\alpha_f)}{\alpha_f}(\nabla\alpha_p\langle\underline{U}_r\rangle + \langle\underline{U}_r\rangle\nabla\alpha_p) \quad (3.6)$$

where $\langle\underline{U}_r\rangle = \langle\underline{U}_p\rangle - \langle\underline{U}_f\rangle$ and b is called the mobility of the continuous phase and tends to zero with α_p and is equal to one if the interface is regular and not greatly affected by phase change. There is no experimental evidence for (3.6) and it greatly complicates the model; therefore it is neglected.

If the continuous fluid is incompressible and viscosity is neglected the averaged stress field is

$$\langle\underline{\sigma}_f\rangle = \alpha_f\langle p_f\rangle\underline{I}. \quad (3.7)$$

The gradient of this stress is

$$\nabla\langle\underline{\sigma}_f\rangle = \alpha_f\nabla\langle p_f\rangle + \langle p_f\rangle\nabla\alpha_f \quad (3.8)$$

As discussed in the next section on page 39 the second term of the above expression is often neglected by taking into account the effect of the pressure field on the particles so that

$$\nabla\langle\sigma_f\rangle = \alpha_f\nabla\langle p_f\rangle \quad (3.9)$$

A similar result can be obtained by lumping the term dependent on the gradient of α_f in with the momentum exchange term $M_{pf}^\dagger$ (D.A.Drew [49]); but if this is done the expression used to evaluate the latter term must be able to take into account this force.

3.1.3 Stress in the Dispersed Phase

Modelling stress in the dispersed phase is less straightforward. In some models exactly the same approach as for the continuous phase is taken e.g. [42]. The simplest definition of pressure in the dispersed phase is that in a bubbly or droplet flow the pressure within the bubble is

approximately the same as the pressure without provided the surface tension is low [42]. The stress in the dispersed phase is similar to (3.8) so that

$$\nabla \langle \underline{\sigma}_p \rangle = \alpha_p \nabla \langle p_f \rangle + \langle p_f \rangle \nabla \alpha_p. \quad (3.10)$$

It has been a matter of controversy whether there is a force dependent on the volume fraction gradient (i.e. whether the last term of (3.10) exists) (A.Properetti and A.V.Jones [43]) or even whether a fraction of it should be included (W.T.Sha and S.L.Soo [44]). Arguments for each case have been obscured by attempts to force the separated fluids model into specific situations, and by the importance of the term to the stability of numerical codes. A result of such ad hoc approaches is that terms may be neglected if they are not obvious, or the same elements may appear in different terms and be counted twice.

There is a fundamental difficulty in applying the model of stress in a single phase fluid to the dispersed phase of a multiphase flow: the particles do not really form a continuum they just act like one; the pressure within one particle should not directly affect the motion of other particles. A set of hollow spheres might be imagined. Their motion, and that of the surrounding fluid, would be the same if the interior of all the spheres contained fluid with the same pressure or if each contained fluid with different pressures in different spheres; there must be some mechanism that allows the transmission of stress throughout the particulate phase if stress is to have any meaning. Some have denied that any transmission occurs and have left any sort of stress term out of the momentum equation for the particulate phase (J.D.Murray [45]). This is not quite correct as there are two mechanisms that might allow the transmission of stress: collisions between particles, and transmission through the continuous phase.

The Effect of Particle Collisions If the particles are not rigid if they collide they will merge with each other and act as a continuum. Similarly such particles can break up as well. If agglomeration occurs the change in size and shape of the components of the flow will affect the constitutive relations.

If the particles are rigid collisions will involve the transfer of momentum. The estimation of this is difficult as it requires the determination of the relative velocity between particles at a given time and place. The transfer would not be significant unless there are large random components in the particles' velocity or the particles are close together. The momentum transfer owing to these has sometimes been modelled as the turbulent dissipation term $\langle \underline{U}_p^* \underline{U}_p^* \rangle$. An obvious origin for fluctuations is a turbulence field in the continuous fluid whose length scale is larger than the particle diameter, but this will only be important for small particles with a low

inertia. $\langle \underline{U}_p^* \underline{U}_p^* \rangle$ is thought to be zero in a sedimenting flow; this is discussed further on page 40 and in Section 3.1.4.

Transmission of Stress through the Continuous Fluid In the absence of collisions particles can only influence each other through the continuous fluid lying between them. This will manifest itself in the stress at the surface of each particle which is modelled by the term $M_{jk}^\dagger$ in the momentum equations. It is possible to demonstrate the origin of the term containing pressure, $\alpha_p \nabla p_f$, in most sets of momentum equations derived for the separated flow model, including those solved by CHYMES.

Let the stress around the surface of a particle be divided into two parts:

$\underline{\sigma}_a$ - equal to the stress variation within the bulk fluid and which does not reflect the presence of individual particles;

$\underline{\sigma}_b$ - the perturbation in the bulk fluid stress field near the particles owing to their presence and movement;

therefore

$$\underline{M}_{pf}^\dagger = \frac{1}{V} \int_{A_{pf}} \underline{\sigma}_b + \underline{\sigma}_a dA. \quad (3.11)$$

Defining

$$\underline{M}_{pf} = \frac{1}{V} \int_{A_{pf}} \underline{\sigma}_b dA \quad (3.12)$$

and neglecting shear stress in the bulk fluid,

$$\underline{M}_{pf}^\dagger = \underline{M}_{pf} - \frac{1}{V} \int_{A_{pf}} p_f \underline{n}_p dA. \quad (3.13)$$

The sign convention is that normals which point into the particles and momentum transferred from the particles to the continuous fluid are positive. When considering σ_a the particles may be replaced by spheres of fluid because its value is independent of the presence of the particles; therefore

$$\frac{1}{V} \int_{A_{pf}} p_f \underline{n}_p dA = \frac{1}{V} \sum_{i=1}^n \oint_{A_i} p_f \underline{n}_p dA. \quad (3.14)$$

For each of these spheres because p_f changes slowly in space

$$p_f \approx \langle p_f \rangle(\underline{x}_i) + (\underline{x}_s - \underline{x}_i) \cdot \nabla \langle p_f \rangle \quad (3.15)$$

where $\underline{x}_i$ is the position of the centre of the i th sphere of fluid, and $\underline{x}_s$ is the coordinate of a point on its surface. If the identity

$$\oint_{A_i} (\underline{x}_s - \underline{x}_i) \cdot \underline{n}_p dA = V_b \quad (3.16)$$

is used [43] then

$$\underline{M}_{pf}^\dagger = \underline{M}_{pf} - \frac{1}{V} \sum_{i=1}^n V_b \nabla \langle p_f \rangle. \quad (3.17)$$

Assuming that ∇p_f is nearly the same for all the particles within the averaging volume as required by (2.12)

$$\underline{M}_{pf}^\dagger = \underline{M}_{pf} - \alpha_p \nabla \langle p_f \rangle. \quad (3.18)$$

which, if $\langle \underline{\sigma}_p \rangle$ is assumed to be zero as in [45], are the terms commonly found in the separated flow model equations.

Similar steps can be followed in the continuous fluid. If the overall stress field in the liquid is independent of the presence of the particles then the stress should be averaged over the entire averaging volume, not just over the fluid so that as in [45]

$$\langle \underline{\sigma}_f \rangle = \langle p_f \rangle \underline{I}. \quad (3.19)$$

By Newton's Third Law $\underline{M}_{fp}^\dagger = -\underline{M}_{pf}^\dagger$ so that in the fluid's momentum equation using (3.19) and (3.18)

$$\begin{aligned} -\langle \nabla \sigma_f \rangle + \underline{M}_{fp}^\dagger &= -\nabla \langle p_f \rangle - \underline{M}_{pf} + \alpha_p \nabla \langle p_f \rangle. \\ &= -\alpha_f \nabla \langle p_f \rangle - \underline{M}_{pf}. \end{aligned} \quad (3.20)$$

The pressure field in the continuous fluid will only be independent of the presence of the particles if they are very small and dilute because the fluid pressure will depend on the fluid motion and if that is significantly affected by the particles then so will be the pressure field. In such a case α_f will be nearly equal one anyway.

In [43] the particles dissected by the boundary of the averaging volume were considered very carefully. A result of this was the inclusion of an extra term

$$-\nabla(\alpha_p \langle \bar{p}_w \rangle)$$

where $\bar{p}_w$ is the average pressure on the particles' surfaces. Unless the surface pressure is very different from the volume averaged pressure (for example, if the particles' size is changing, or the velocity between the bubbles and the liquid is very large) this term will be small.

Alternative Definitions of Stress in the Particulate Phase The description of stress in the particulate phase has suffered from a confusion in the definition of 'pressure'. For example, in [46] (R.C.Givler) p_p was taken to be equal to the pressure disturbances caused by the motion

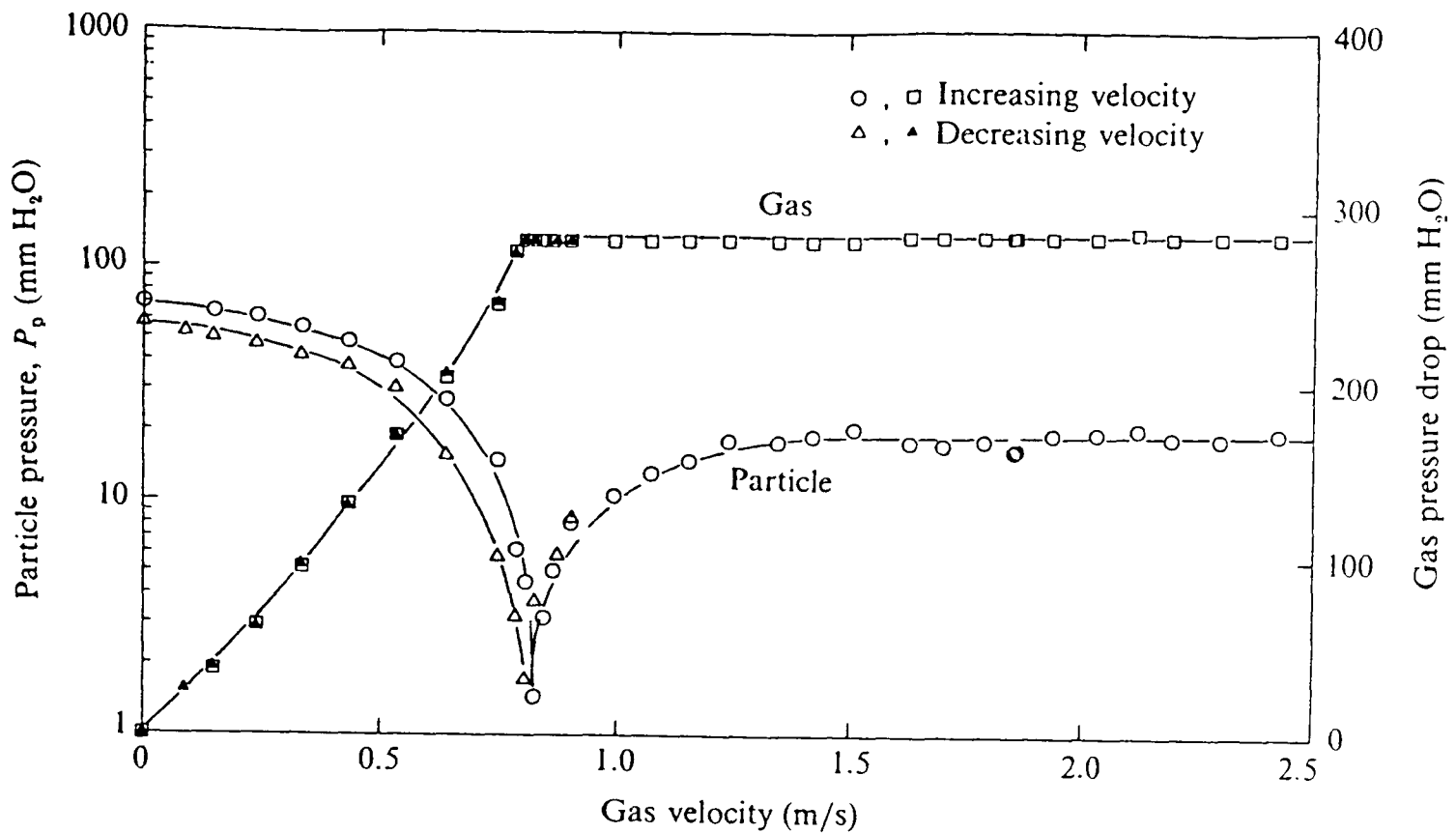


Figure 3.1: Measurements of the Relationship between Particle Pressure and Fluid Velocity in a Gas- Fluidised Bed [48]

of a sphere. From viscous flow theory it was estimated that for an isolated sphere with a very low Reynolds number (less than about 0.5)

$$p_s = p_f + \frac{9}{32} \rho_f \langle \underline{U}_r \rangle^2. \quad (3.21)$$

It is claimed that the right hand term represents the force necessary to overcome the inertia of the fluid displaced by the particle, a task usually undertaken by an added mass term. It is misleading to call this ‘pressure’ as it is not concerned with the transmission of stress through the particulate phase.

An alternative definition of particle pressure is presented in [47](S.L.Passman). It is argued that the definition of particle pressure in (3.18) causes the equations to be ‘ill-behaved’. To prevent this the flow field’s properties should be incorporated into the definition of particle pressure, but this is infeasible owing to its complicated nature; therefore, as in the kinetic theory of gases, particle pressure is defined as the momentum flux of the particles crossing a plane. This idea is taken further in [44] where a particle temperature is defined which is then used to define various properties of the particulate phase such as pressure and viscosity. In [48] (C.S.Campbell and D.G.Wang) particle pressure, when defined in this way, was measured in a gas fluidised bed. An example of the pressures measured is shown in Figure 3.1. The point of minimum pressure corresponds with that of minimum fluidisation, which is equivalent to steady sedimentation. The subsequent rise in pressure is caused by large bubbles forming in the bed. The minimum pressure was thought to be actually zero though it was not measured as this

owing to measurement difficulties³. This definition of particle pressure is not connected with the stress field and is actually about estimating the effect of particle fluctuations (see page 37); in the absence of these the above definition would eliminate particle pressure from the separated flow model equations.

3.1.4 Stresses Caused by Turbulence

The stress caused by turbulence (also known as the Reynolds stress) is represented by the term $\langle \underline{U}_k^* \underline{U}_k^* \rangle$ in the momentum equations. The traditional way of modelling Reynolds stress in a single phase fluid (e.g. see [32]) is to define an apparent viscosity, μ_f^T , such that

$$\rho_f \langle \underline{U}_f^* \underline{U}_f^* \rangle = -\mu_f^T (\nabla \langle \underline{U}_f \rangle + (\nabla \langle \underline{U}_f \rangle)^t). \quad (3.22)$$

This model can be extended to multiphase flows e.g from [49]

$$\rho_f \langle \underline{U}_f^* \underline{U}_f^* \rangle = \mu_f^T (\nabla \langle \underline{U}_f \rangle + (\nabla \langle \underline{U}_f \rangle)^t) + a_f \underline{I} + b_f \langle \underline{U}_r \rangle \langle \underline{U}_r \rangle \quad (3.23)$$

where a_f is the induced eddy ‘pressure’ and b_f is associated with the passage of particles. From a cell model⁴ it can be estimated that

$$a_f = \frac{1}{6} \rho_f |\underline{U}_r|^2 \quad (3.24)$$

$$b_f = -\frac{\rho_f}{2} \quad (3.25)$$

and

$$\langle \underline{U}_p^* \underline{U}_p^* \rangle = O \left(\frac{\rho_p}{\rho_f} \langle \underline{U}_f^* \underline{U}_f^* \rangle \right). \quad (3.26)$$

This model suffers from the same problems as the expression for shear stress in that it causes complication but it has not been verified by any experimental data.

An attempt has also been made to derive a term for Reynolds stress using the cell model (G.A.Arnold, D.A.Drew, and R.T.Lahey [51]), but results in a very long and practically useless expression.

In [50](C.Pauchon and P.Smerka) it was postulated that

$$\langle \underline{U}_p^* \underline{U}_p^* \rangle = 0 \quad (3.27)$$

and from the calculation of potential flow around a sphere in a current with a velocity gradient,

$$\langle \underline{U}_f^* \underline{U}_f^* \rangle = -\frac{1}{4} \langle \underline{U}_r \rangle^2. \quad (3.28)$$

³The point of minimum fluidisation varied with position in the bed and its height was significantly smaller than the size of the diaphragm used to measure particle pressure.

⁴This is where a representative particle is surrounded by a sphere of fluid and the flow in the sphere is ensemble averaged over all possible positions of the particle within the sphere.

These are not useful because the fluctuations here are caused by the deviation of the fluid velocity from its average value owing to the presence of the particles, not by turbulence; (3.28) is analogous to the second term of (3.21).

3.2 Momentum Transfer

Of all constitutive relations it is those which govern momentum transfer that have had the most attention. Unfortunately this involves a large amount of complicated behaviour because momentum transfer is a result of the stress fields in the flow which vary a great deal and are established by a number of ‘microscopic’ processes which cannot be calculated exactly.

3.2.1 The Motion of A Single Particle

A common approach to the modelling of momentum transfer is to select terms that are analogous to those in the equation of motion of a single particle; various modifications are then made to allow for the real conditions within a multi-phase flow. The equation of motion of a single, small, rigid sphere falling through a viscous, incompressible, quiescent fluid may be written as (R.Clift et al [52], M.R.Maxey and J.J.Riley [53])

$$\frac{\pi D^3}{6} \rho_p \frac{dU_p}{dt} = (\rho_p - \rho_f) \frac{\pi D^3}{6} g - 3\pi D \rho_f \nu U_p - C_{Avm} \rho_f \frac{dU_p}{dt} - \frac{3}{2} D^2 \rho_f \sqrt{\pi \nu} \int_{t_0}^t \frac{\frac{dU_p}{d\epsilon}}{\sqrt{t - \epsilon}} d\epsilon. \quad (3.29)$$

This only applies to particles whose Reynolds number is less than one and can be solved exactly [53]. It is thought that the form for higher Reynolds numbers is similar (T.S.Tsuji et al [56]) so that

$$\frac{\pi D^3}{6} \rho_p \frac{dU_p}{dt} = (\rho_p - \rho_f) \frac{\pi D^3}{6} g - C_{D\infty} \frac{1}{2} \rho_f U_r^2 - C_{vm} \rho_f \frac{dU_p}{dt} - C_H D^2 \rho_f \sqrt{\pi \nu} \int_{t_0}^t \frac{\frac{dU_p}{d\epsilon}}{\sqrt{t - \epsilon}} d\epsilon \quad (3.30)$$

where the coefficients $C_{D\infty}$, C_{vm} , and C_H are found empirically.

The various terms on the right hand sides of (3.29) and (3.30) can be identified with certain forces. The first term is buoyancy⁵, and the second term is the drag on the sphere. In (3.29) the exact form for a particle with a very small Reynolds number (< 1) is shown in (3.30) the term is in a more general form where $C_{D\infty}$ is the drag coefficient.

The third and fourth terms account for the forces owing to the transient motion of the particle, virtual mass and the Basset force respectively. Virtual mass⁶ is the force required to

⁵In this case, and in the future, the term ‘buoyancy’ means ‘the difference between the weight of the particle and the water it displaces.’

⁶Otherwise known as added mass.

accelerate the fluid displaced by the particle as it moves. In an inviscid flow for an isolated sphere C_{Avm} is equal to 1/2 (T.B.Benjamin [54]). The Basset force takes into account the history of the sphere as it falls through the fluid (A.B.Basset [55]). Its form in (3.29) is exact. At high Reynolds numbers the magnitude of these forces can be significantly affected by the behaviour of a particle's wake. From dimensional analysis it can be shown (T.S.Temkin and S.S.Kim [57]) that C_{vm} and C_H depend on the density ratio ρ_p/ρ_w and the acceleration number

$$A_c = \frac{d_p}{U_r^2} \frac{dU_r}{dt}. \quad (3.31)$$

However they also depend on the nature of the acceleration because of the interaction between the flow and vortices shed from the wake: in an oscillating flow the acceleration of the particle increases the overall force on it while in a non-oscillating flow the opposite happens [56].

Experiments where $\rho_p/\rho_f = 7.72$ indicate that provided the flow was not oscillating a simple model may be used to model the transient effects such that (P.R.Schöneborn [58]):

- $C_{D\infty}$ equals that in a steady flow with the same Reynolds number at any time;
- $C_{vm}=0.5$;
- $C_H=0$.

These will not be accurate in an oscillating flow [58] and there is no indication of the range of A_c to which they might be applied.

3.2.2 The Form of $\underline{M}_{jk}$ used in Numerical Models

(3.29) and (3.30) apply to a single, rigid particle falling through a quiescent fluid. If the fluid is not quiescent their form is much more complicated, even at low Reynolds numbers [53]. In CHYMES the virtual mass force is neglected as well as the Basset force to simplify the coding. If metal particles are falling through water a 6% error is introduced to the calculation of a particle's acceleration.

The melt particles in coarse mixing are likely to have Reynolds numbers much greater than one because of their density and size; therefore the inertial forces on a particle will dominate its motion and the drag will be independent of its Reynolds number. The drag coefficient formulation⁷ will then be suitable for expressing the drag force.

The form of the drag term used in numerical models is often arbitrary and not connected with that for a single, isolated sphere; an example of such an expression is (F.H.Harlow and

⁷Which is effectively the ratio of the drag force to the inertia of the fluid moved by the particle.

A.A.Amsden [63]):

$$\underline{M}_{jk} = \alpha_j \alpha_k \frac{3}{4} C_{D\infty} \frac{\rho_j \rho_k L_j L_k}{\rho_j L_k + \rho_k L_j} \left(\frac{1}{L_k} + \frac{1}{L_j} \right)^2 |U_j - U_k| (U_j - U_k) \quad (3.32)$$

where L_j and L_k are the length scales of phases j and k . One of the reasons this equation is liked by numerical analysts is that it gives the ‘correct behaviour’ where $\underline{M}_{jk}$ tends to zero, and the influence of each term evaluated as either phase becomes dilute. For many flows this does not happen; an example is if the particles are rigid α_f cannot be less than its value when they are tightly packed. If the particles are not rigid their shape and size would change to such a degree that the assumptions of the separated flow model would be breached. There is, therefore, nothing to be gained in having this limit. No physical evidence is quoted to support the form of (3.32).

(3.32) is not now commonly used and a more physical approach is taken instead. The drag on a single, isolated particle is well known and given by

$$\frac{1}{2} \rho_f \frac{\pi D^2}{4} C_{D\infty} |\underline{U}_r| \underline{U}_r.$$

If this is applied to the average particle discussed at the beginning of this chapter and the result multiplied by the number of particles per unit volume, n_p ⁸, then

$$\underline{M}_{mw} = \frac{3}{4} \frac{C_{D\infty}}{d_p} \rho_w \alpha_m \alpha_w |\langle \underline{U}_r \rangle| \langle \underline{U}_r \rangle. \quad (3.33)$$

The α_w is included when three phases are present to prevent the drag between the particle and the water being calculated if the melt collects in a pool.

3.2.3 The Effect of Flow Conditions

(3.33) may be applied in a model with $C_{D\infty}$ equal to that for an isolated sphere dropping through a quiescent, boundless fluid with a given Reynolds number (Figure 3.2). For Reynolds numbers of between about one hundred and one hundred thousand the drag coefficient is 0.44 [52]. In a real multiphase flow there will be a number of other influences on the drag experienced by individual particles. ‘Drag’ here means the overall force experienced by a particle owing to the stress distribution over its boundary caused by its motion relative to the surrounding fluid. In particular two influences may be important: that of the tank walls, and that of other particles.

The Effect of the Tank Walls It is well known that the boundary of even a very large vessel can affect the motion of a particle falling within it (J.R.Strom and R.C.Kintner [61]). Only for

⁸Given by (3.1)

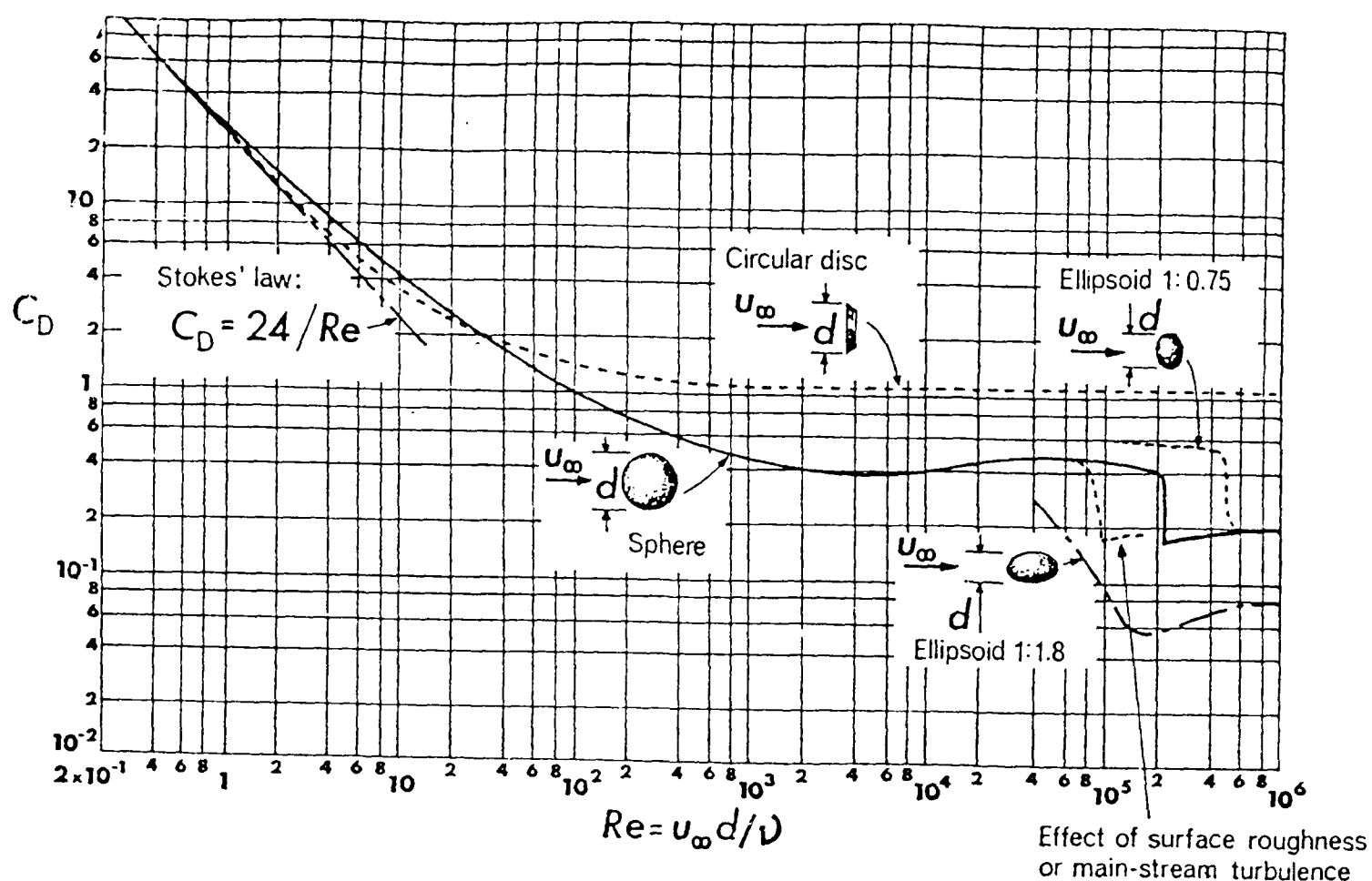


Figure 3.2: The Relation of the Drag Coefficient of an Isolated Particle to its Reynolds Number (B.S.Massey [59]).

very low Reynolds numbers can suitable modifications to $C_{D\infty}$ be found analytically because of the importance of wakes if they are higher (S.Uno and R.C.Kintner [62]). Further uncertainty exists if there is a group of particles whether the walls interact only with the particles closest to them, or with the whole group; therefore the effects of the sides of the tank are ignored though they may be significant.

The Effect of Other Particles The motion of a particle in a multiphase flow will be affected by the influence the presence and the motion of other particles has on the flow. Much of the data that allows the behaviour of particles in a group to be evaluated has been provided by experiments on fluidised beds; fortunately it has been shown that the behaviour of beds near the point of minimum fluidisation⁹ is the same as that for a sedimenting bed (J.F.Richardson and W.N.Zaki [64]).

It is assumed that the drag coefficient formulation can be used for a particle within a uniform, sedimenting bed. If this is compared with the formulation for a free falling, isolated particle then

$$C_D = C_{D\infty} \left(\frac{U_t}{U_r} \right)^2 \quad (3.34)$$

⁹i.e. when the force exerted on the particles by the fluid nearly matches their weight. If the fluid velocity is increased beyond this point the particles will start to move within the bed

where $\underline{U}_t$ is the terminal velocity of an isolated particle, and $C_{D\infty}$ is its drag coefficient. The velocity ratio in (3.34) can be found exactly for particles with small Reynolds numbers. One approach is to treat a cloud of particles as a porous bed (H.C.Brinkman [65]), and another is to treat the cloud as a liquid with a high viscosity (P.G.W.Hawksley [66]). One more method is to postulate that a particle's velocity is a function of the average velocity gradient, and a characteristic length scale and then find the velocity ratio from a force balance (H.H.Steinour [67]). This was verified by the results of experiments on particles with a very low Reynolds number (about 2.5×10^{-3}). These are not very useful in coarse mixing because they cannot be applied to particles with a large Reynolds number.

An approach which can be related to the motion of larger particles may be found in [68] (J.M.Burgers) and, in an improved manner, in [69] (G.K.Batchelor) for particles with a low Reynolds number. The modification to the velocity of a sphere owing to the presence of a given configuration of particles is evaluated and integrated over the domain of all possible positions to give the ensemble averaged velocity. The domain must contain many particles, and the mean difference between the individual velocity of each particle and the average velocity must equal zero [69]; in other words the length constraints (2.5) and (2.12) must be obeyed. The result of this is

$$\frac{U_r}{U_t} = 1 - k\alpha_k \quad (3.35)$$

where $k=6.88$ [68], or 6.55 [69] to an accuracy of order α_k^2 . It is assumed that the fluid flow satisfies $\nabla^4 \underline{U}_w = 0$, that the cloud of particles is homogenous, and that the positions of the particles are wholly random. The assumption that particles are randomly positioned may not be true in real flows, even if the particles are very small. It was suggested that this was the reason why k was measured experimentally as 3.77 (J.M.Han and G.M.Hansley [70]).

It is not possible to calculate $\underline{U}_t/\underline{U}_r$ directly for larger spheres (i.e. where the Reynolds number is greater than one) as the introduction of inertia into the equations makes their solution very hard, and such particles will have wakes whose structure is complicated and unpredictable; therefore experimental data must be relied upon. In [64] from dimensional analysis and experimental data it was found that

$$\frac{U_r}{U_t} = (1 - \alpha_p)^{n-1} \quad (3.36)$$

where $n=4.65$ if $Re_t < 0.2$, or 2.39 if $Re_t > 500$, or is given by various correlations for intermediate Reynolds numbers.

In [71] (A.D.Maude and R.L.Whitmore) (3.36) is derived by considering the increase in velocity of the water flow through a suspension owing to the conservation of mass. n accounts

for the uneven distribution of the increase in flow around sedimenting particles and is dependent on the drag coefficient and Reynolds number of an unhindered particle falling. This would support the theory that the formation of any flow structure will affect the distribution of the increased flow. After comparison with correlations for a single body it was postulated that n for high Reynolds numbers should be half that for low ones. The theory agreed well with Richardson and Zaki's measurements.

An accurate correlation for n was deduced from the amalgamation of a large number of experimental results taken from several papers (J.Garside and M.R.Al-Dibouni [72]); it is

$$\frac{5.09 - n}{n - 2.73} = 0.104 Re_t^{0.877}. \quad (3.37)$$

The drag coefficient can then be found from the combination of (3.34) and (3.36) so that

$$C_D = C_{D\infty}(1 - \alpha_p)^{-2(n-1)}. \quad (3.38)$$

This may then be substituted into (3.33) to evaluate the drag force between the particles and the surrounding fluid.

Note that if (3.36) is expanded as a Taylor series and $n=k+1$ then, to the $O(\alpha_p)$, (3.35), for small Reynolds numbers, and (3.36), for all Reynolds numbers, are similar.

It has been shown for two stationary cylinders standing in a flow strong forces exist between them if they are within about five diameters of each other (see Figure 3.3, and M.M.Zdravkovich [73]). Experiments with spheres show that they experience similar forces ([56], K.C.Lee [74], P.N.Rowe and G.A.Herwood [75]). If a cylinder, or sphere, lies in front of another, the range of its influence is large. It is difficult to be more accurate than this owing to the very complicated nature of the wake.

The melt particles are likely to have Reynolds numbers between one thousand and ten thousand so their wakes will be open and turbulent. Each wake will consist of two regions: the primary wake, where vorticity is generated and which will follow the particle closely, and a turbulent secondary wake that will be left far behind the particle (L.Fan and K.Tsuchiya [76]). The detailed structure of the wake is complicated and not very predictable. The best thing that might be said is that a particle will leave a region of turbulence behind it whose mean velocity will be in the same direction as that of the particle, but less than it. It is this that causes the reduction in the drag of a particle when it is following another.

A result of the strong forces between particles with high Reynolds numbers is that they can give rise to highly ordered motion; for example two particles can have a regular cyclic motion as

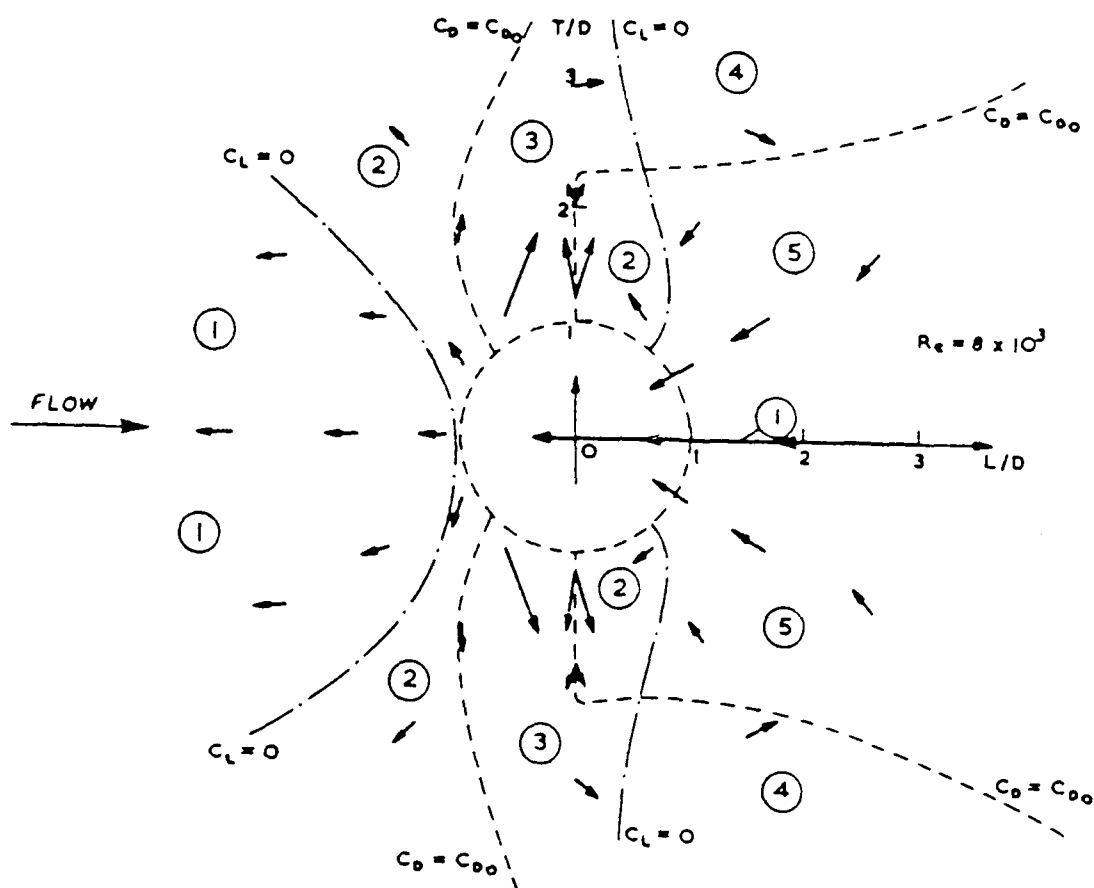


Figure 3.3: The Direction of the Force Experienced by a Cylinder if placed near the One Shown [73]

they fall if one is following the other, and very stable structures can form, such as a line abreast of particles (A.F.Fortes et al [77]). The formation of such structures may affect the value of n .

3.2.4 The Effect of Film Boiling on Drag

When a very hot particle drops through water it will be enclosed by a vapour film. Over the front portion of the particle the vapour film is thin and might be assumed to be part of the body of the melt particle. Even though the steam film will move relative to the particle's surface, the momentum exchanged should be small because of the low density of the vapour; therefore, provided that the cavity of steam following the particle is not very large, the particle and steam film may be modelled as a single entity behind which vapour bubbles (i.e. particles) are forming. The drag on this entity should be slightly reduced because the vapour layer will shield the sphere's surface from the liquid and reduce surface friction. In addition as the sphere's temperature increases the vapour layer becomes thicker and increases the apparent buoyancy of the particle.

$\underline{M}_{wm}$ has to be modified to account for these. Experiments indicate that the drag coefficient is reduced by about 7% for a sphere free falling in saturated water with a temperature of about 500°C (D.H.Cho and G.A.Lambert [78], Y.Zvirin et al [79]). A similar reduction occurs in forced convection, except at higher relative velocities where it is larger [79]. A reasonably accurate theory based upon laminar flow is presented in [80] (W.S.Bradfield et al). A dissenting set of

results are reported in [81] (F.S.Gunnerson and P.R.Chappidi) where the drag is reduced by about thirty per cent when a sphere has a temperature of 300°C and then steadily increases with temperature.

The above discussion applies only to particles that are isolated and on which there is film boiling. It is not known what the effect of other melt particles and vapour particles will have on the film. Also a particle's motion will be greatly affected if it is subject to boiling regimes other than stable, film boiling [79], [80].

3.3 Other Derivations of the Momentum Transfer Terms

There have been a number of attempts to derive the momentum transfer terms in the conservation equations rigorously. Some of these have already been touched upon; for example in [50] $\underline{M}_{jk}$ was derived from various pressure differences (see page 35). This results in equations which contain terms which are not complementary in form. There is no reason why this should not be so provided the total momentum transferred from one phase to another balances that transferred in the other direction, if there is no dissipation, so that Newton's third law is satisfied. The terms in equations (2.52)-(2.54) are complementary only because the stress model chosen, and the assumptions made in their derivation, force them to be like that. In [50] it is attempted to derive constitutive relations for averaged equations rigorously. If the higher order terms are neglected and potential flow used, in one dimension x

$$M_{pf} = \frac{\alpha_p \rho_f}{2} \left(\frac{\partial}{\partial t} \langle U_r \rangle + \frac{\partial}{\partial x} \langle U_p \rangle \langle U_r \rangle \right) + \frac{\alpha_p}{4} \frac{\partial}{\partial z} \rho_f \langle U_r \rangle^2 \quad (3.39)$$

and

$$M_{fp} = -\frac{\alpha_p \rho_f}{2} \left(\frac{\partial}{\partial t} \langle U_r \rangle + \frac{\partial}{\partial x} \langle U_p \rangle \langle U_r \rangle \right) - \frac{1}{4} \rho_f \langle U_r \rangle^2 \frac{\partial \alpha_f}{\partial x}. \quad (3.40)$$

$\underline{M}_{pf}$ and the first term of $\underline{M}_{pf}$ are added mass terms in form. The second term of $\underline{M}_{pf}$ is (3.28) and discussed in Section 3.1.4.

Multiphase flow is very complicated and $\underline{M}_{jk}$ cannot be solved exactly; therefore a number of assumptions must be made to make the problem tractable; these often ignore important aspects of real flows and do not improve upon the approach based on the equation of a single particle. For example in [50] assumptions have to be made so that:

- the particle properties are uniform within the averaging volume (i.e. (2.12) is satisfied);
- the mixture is dilute;
- the flow round the particles is potential flow.

A consequence of the last assumption is that drag cannot be accounted for.

An alternative approach that also suffers from these faults may be found in [82] (D.A.Drew and R.T.Lahey) and [49] (D.A.Drew et al). No assumptions are made initially about the physics behind the constitutive relations and the method followed may be applied to all continua. In a flow with no heat or mass transfer M_{jk} will depend on the volume fractions and the velocities of the phases, and their derivatives. There are a number of conditions that must be fulfilled for constitutive relations to be wholly reliable. These are:

- **Coordinate Invariance** The constitutive relations should not be dependent on the coordinate system that has been selected;
- **Equipresence** If a relation depends upon a certain field, then all the other relations should depend on that field to ensure that all dependencies have been accounted for;
- **Objectivity** The variables used in constitutive relations should not depend upon the coordinate frame in which the variables are expressed;
- **Homogeneity** The constitutive relations can only apply to the material for which they were derived;
- **Isotropy**;
- **Just Setting** The equations of motion have unique solutions for given constitutive relations, initial and boundary conditions;
- **Dimensional Invariance** i.e. the constitutive relations should be dimensionally correct;
- **Correct Low Concentration Limits** As α_p tends to zero the equation of motion should tend to that for a single particle. There is not actually a reason for adopting this for averaged equations: if the flow becomes very dilute then volume averaged equations cannot be used as either of the constraints (2.5) or (2.12) will cease to apply.

One of the more important of the above conditions is objectivity. This can be tested for a variable $\psi(\underline{x})$ by applying the Galilean transformation $\underline{x}' = Q(t)\underline{x} + \underline{b}(t)$, where $Q(t)$ and $\underline{b}(t)$ are functions of time only; if $\psi(\underline{x}') = \psi(\underline{x})$ then ψ is objective. This results in the following variables for the constitutive relations:

$$\alpha_k, \frac{D_k \alpha_k}{Dt}, \nabla \alpha_k, \underline{U}_r, \underline{a}_{jk}, \underline{D}_{jp}, \underline{D}_{kp}, \nabla \underline{U}_r$$

and the properties of the phases j and k , where

$$\frac{D_j \alpha_k}{Dt} = \frac{\partial \alpha_k}{\partial t} + \underline{U}_j \cdot \nabla \alpha_k,$$

$$\underline{D}_{kp} = \frac{1}{2} \left[\nabla \langle \underline{U}_k \rangle + (\nabla \langle \underline{U}_k \rangle)^T \right],$$

and

$$\underline{a}_{jk} = \frac{D_j \langle \underline{U}_k \rangle}{Dt} - \frac{D_k \langle \underline{U}_j \rangle}{Dt}.$$

If the principle of equipresence was to fully implemented then a very long expression would result that consisted of terms containing every possible combination of the above variables. This is infeasible so intuition, reasoning, experience, and knowledge of the physical situation are used to postulate which of the variables, and which of their combinations, are likely to be significant. In the case of momentum exchange in a particulate flow the result must be a vector and equal zero if $\underline{U}_r$ is zero. This results in [49]

$$\underline{M}_{pf} = A_1 \underline{U}_r + A_2 \underline{a}_{pf} + A_3 \underline{U}_r \cdot \underline{D}_{pp} + A_4 \underline{U}_r \cdot \underline{D}_{fp} + A_5 \underline{U}_r \cdot \nabla \underline{U}_r + A_6 \underline{U}_r \cdot [\nabla \underline{U}_r]^T. \quad (3.41)$$

The same blend of talents must then be used to determine what the coefficients $A_1 - A_6$ represent.

$A_1 \underline{U}_r$ is a drag term and equated to (3.33) except the α_w is dropped.

$A_2 \underline{a}_{jk}$ represents virtual mass so

$$A_2 = \alpha_p \rho_f C_{vm}. \quad (3.42)$$

A simple expression used to describe C_{vm} in multi-phase flow is

$$C_{vm} = 0.5 + B \alpha_p \quad (3.43)$$

Virtual mass results from the displacement of water by the particle. The force owing to this on a particle is calculated in [84] (N.Zuber) where the virtual mass of a sphere contained within a moving sphere of fluid is found, giving B a value of 1.39. This is improved upon in [85] (C.van Wijngaarden) where potential flow and ensemble averaging, as in [69], gives B a value of 1.5. An empirical correlation results in $B=2.1$ (Y.Mokeyev [86]). However in [49] it was stated that experimental evidence indicates that C_{vm} should be less than 0.5 in multi-phase flow, between 0.1 and 0.5.

For virtual mass to be objective it must include the term associated with A_5 (D.Drew et al [83]). A_5 may then be expressed as expressed as

$$A_5 = \alpha_p \rho_w C_{vm} (1 - \lambda(\alpha_p)). \quad (3.44)$$

The form of λ is not known, but it has the limits $\lim_{\alpha_p \rightarrow 1} \lambda = 2$ and $\lim_{\alpha_p \rightarrow 0} \lambda = 0$ [83] and changing its value does not seem to affect predictions of bubbly flows [49].

The term $A_3 \underline{U}_r \cdot \underline{D}_{pp}$ has been shown to be a lift force (i.e. a force whose direction is perpendicular to the direction of $\underline{U}_r$) in a slow, viscous flow (P.G.Saffman [109]). There is no evidence that this force occurs in other sorts of flow.

The terms associated with A_4 and A_6 do not have analogous terms in the equation of motion of single particles, and are neglected.

Even though the derivation above starts from general principles, large assumptions have to be made at an early stage for constitutive relations to be found. Some forces which might be present are left out, such as the Basset force, and even those terms that remain cannot be adequately explained. One benefit of this approach is that it indicates the correct form of variables in constitutive relations.

3.4 Alternative Methods Used to Derive Momentum Equations

There are a number of methods for deriving momentum conservation equations for multi-phase flow that do not involve averaging. One method uses the statistical methods used in turbulent flows (Y.A.Buyevich [87], M.W.Reeks [88], and see D.C.Leslie [89] for an introduction to these methods). This method is lengthy and difficult to understand and interpret physically. In addition nothing is said about the form of the ‘aerodynamic force’ (equivalent to drag) acting upon particles.

A second method models the particles by using Boltzmann’s equation (G.Wang and J.Ni [90], [91]) which is a conservation equation in terms of a velocity distribution function. The main difficulty of this method is determining that function. To do this the following assumptions were made:

- no collisions between particles;
- the velocity field of the continuous phase is specified;
- the forces on each particle are buoyancy and drag (i.e. the Basset force and virtual mass are neglected);
- the concentration of particles is described by number concentration, which implies that length constraints (2.5) and (2.12) apply.

The results compared well with experimental measurements of a steady flow of 1mm diameter particles in a 4×6 cm rectangular pipe. The major problem in the application of this model is that the continuous phase's velocity field must be known, which it is not in coarse mixing.

A third method is to use variational methods [50]. In this the principle

$$\delta \int \int L dV dt = 0 \quad (3.45)$$

is applied to a control volume where

$$L = K + L_1 + \dots + L_n, \quad (3.46)$$

K is the kinetic energy density, and $L_1, \dots, L_n$ are the constraints on the flow which are multiplied by Lagrange multipliers $\Phi_1, \dots, \Phi_n$. It is assumed that (2.41)¹⁰ applies to the continuous phase,

$$\langle \underline{U}_p^* \underline{U}_p^* \rangle = 0 \quad (3.47)$$

and

$$\langle U_f^* U_f^* \rangle = m(\alpha_p) \langle U_r \rangle^2 \quad (3.48)$$

where $m(\alpha_p)$ depends on the local flow configuration. Hence

$$K = \frac{1}{2} \alpha_f \rho_f \langle U_f \rangle^2 + \frac{1}{2} \alpha_f \rho_f m(\alpha_p) \langle U_r \rangle^2 + \frac{1}{2} \alpha_p \rho_p \langle U_p \rangle^2. \quad (3.49)$$

There are two constraints on the flow: the conservation of mass of each phase, and that the sum of their volume fractions must equal one i.e.

$$L_1 = \Phi_f \rho_f \left(\frac{\partial \alpha_f}{\partial t} + \nabla \cdot \alpha_f \langle U_f \rangle \right) + \Phi_p \rho_p \left(\frac{\partial \alpha_p}{\partial t} + \nabla \cdot \alpha_p \langle U_p \rangle \right) \quad (3.50)$$

and

$$L_2 = \Phi_\alpha (\alpha_p + \alpha_f). \quad (3.51)$$

If Φ_α is identified with $-\langle p_f \rangle$, the resulting momentum equations are:

$$\begin{aligned} & \frac{\partial}{\partial t} \alpha_f \rho_f \langle U_f \rangle + \nabla \cdot \alpha_f \rho_f \langle U_f \rangle \langle U_f \rangle + \rho_f \nabla \alpha_f m(\alpha_p) \langle U_r \rangle^2 \\ & + \frac{1}{2} \alpha_f \rho_f \langle m'(\alpha_p) \rangle \langle U_r \rangle^2 \nabla \alpha_f = -\alpha_f \nabla \langle p_f \rangle + \rho_f \left(\frac{\partial}{\partial t} \alpha_f m(\alpha_p) \langle U_r \rangle \right. \\ & \quad \left. + \nabla \cdot \alpha_f m(\alpha_p) \langle U_r \rangle \langle U_r \rangle + \alpha_p m(\alpha_p) \langle U_r \rangle \nabla \langle U_r \rangle \right) \end{aligned} \quad (3.52)$$

$$\begin{aligned} & \frac{\partial}{\partial t} \alpha_p \rho_p \langle U_p \rangle + \nabla \cdot \alpha_p \rho_p \langle U_p \rangle \langle U_p \rangle + \frac{1}{2} \alpha_p \nabla \alpha_f \rho_f m'(\alpha_p) \langle U_r \rangle^2 \\ & = -\alpha_p \nabla \langle p_f \rangle - \rho_f \left(\frac{\partial}{\partial t} \alpha_f m(\alpha_p) \langle U_r \rangle + \nabla \cdot \alpha_f m(\alpha_p) \langle U_r \rangle \langle U_r \rangle \right. \\ & \quad \left. + \alpha_p m(\alpha_p) \langle U_r \rangle \nabla \langle U_r \rangle \right) \end{aligned} \quad (3.53)$$

¹⁰i.e. $\underline{U}_k = \langle \underline{U}_k \rangle + \underline{U}_k^*$.

where $m'(\alpha_p) = \partial m(\alpha_p)/\partial \alpha_p$.

When the dilute limit is taken ($\alpha_p \rightarrow 0$), according to potential flow theory $m(\alpha_p) \rightarrow \alpha_p/2$, and (3.52) and (3.53) take the same form as the equations found using volume averaging and potential flow theory in the same paper (see above, (3.39) and (3.40)). This suggests that the variational method may be applied to non-dilute flows but volume averaging cannot. A problem with the method is determining $m(\alpha_p)$ which suffers from the same problems as the constitutive relations in averaged equations.

The above methods show some promise. In the second case if the continuous fluid's flow field can be specified then its results compares well with those of experiments. Overall these methods share many of the assumptions made for the volume averaged equations and suffer from the problem of very complicated small scale behaviour in the flow.

3.5 Heat and Mass Transfer

The constitutive relations concerned with heat and mass transfer, $\dot{m}_{ws}$ and $\underline{F}_{\dot{m}_{ws}}$, will not be important in the rest of this thesis and will not be described in detail.

Briefly, when determining $\dot{m}_{ws}$ the idea of an 'average' particle is used, as in the transfer of momentum. No super-heating or condensation is allowed. The heat transfer is modelled in the standard way with a heat transfer coefficient. This has two components, one accounting for radiation in the usual manner, and one for convection. The latter is determined from the results of film boiling experiments in [93] (V.K.Dhir and G.P.Parohit) for natural convection and [94] (L.C.Witte) for forced convection.

It is assumed that all the momentum transferred through evaporation is transferred from the liquid to the vapour so that

$$\underline{F}_{\dot{m}_{ws}} = \dot{m}_{ws} \langle \underline{U}_w \rangle. \quad (3.54)$$

3.6 Summary

If there is no heat of mass transfer between phases CHYMES solves the volume averaged equations (2.46)–(2.48), (2.52)–(2.54), (3.18), (3.19) - which implies $\underline{\sigma}_p = 0$, (3.20), and (3.33) with various relations for $C_{D\infty}$. Other models use very similar equations.

Chapter 4

The Experimental Apparatus

The validation of CHYMES requires experiments that simulate the processes involved in coarse mixing, and the environment for which CHYMES has been developed. This environment will be described first before the apparatus and the recording system are described. The experimental programme will then be stated.

4.1 Features Required for the Experiments

CHYMES models the mixing of a jet of molten metal within a pool of saturated, quiescent water in a tank. The following features of CHYMES must be accommodated by the apparatus:

- the jet enters the water pool under gravity;
- the jet and the tank are symmetrical about the tank's centre line and so are two-dimensional;
- the jet is narrower than the tank and consists of melt already fragmented into spherical particles of a uniform size;
- CHYMES calculates the volume fraction and the velocity of each phase throughout the domain over which calculations are performed and over time.

There are several reasons why molten corium, or other molten metals, are not used in these experiments as the melt: the practical problems associated with handling and containing liquid metal with a very high temperature; the difficulty of fragmenting a molten jet into well defined droplets; the difficulty of modelling the coalescence and fragmentation of particles; and the danger of a vapour explosion. To overcome these problems the melt particles are simulated by

metal ball bearings. The regularity of these is a closer approximation to the particles modelled by CHYMES than molten metal particles because of their regular, well defined nature.

This is an important feature of validation experiments: it is often more revealing to reflect the code's characteristics than those of the situation it models, which are often poorly defined. A code is formulated for an ideal situation; when the model is applied to a real situation, such as the mixing of molten corium and water, a number of restrictions apply if the real situation is to be identified with the idealized one. There are then two questions to be answered when validating a code:

- does the model, as solved by the code, adequately describe the behaviour of the idealized system?;
- do the assumptions made to convert the real situation to the idealized one significantly distort the results?

Not all the processes present in a real situation need be modelled; indeed the neglect of ill defined processes, such as particle fragmentation, may make fundamental features clearer. However it is also important that processes not included within the model do not happen in the experiments. Examples of these are collisions between particles in the jet entering the tank, and phenomena owing to some feature of the experiments not being in the regime modelled by the code e.g. nucleate boiling, rather than film boiling, occurring on the particles. The following requirements have then to be met by the experimental apparatus:

- the size of the apparatus should be of the same order as that of the reactor vessel to avoid the effects of scale on mixing processes. These vessels have a radius of about two metres;
- the transfer of momentum between the particles and the continuous fluid is dependent on Reynolds number; therefore the particles in the experiments should have a similar Reynolds number to those in coarse mixing. Furthermore relative length scales are important in the separated flow model; hence the particles in the experiments should be of a similar size to those in coarse mixing;
- the particles in the experiments should be in the correct boiling regime - stable film boiling - otherwise both heat transfer and momentum transfer will be wrongly modelled. This means that the particles' temperature should lie within the appropriate region (SF) in Figure 4.1. It is impractical for the particles in the experiments to have a similar temperature to molten corium, so the heat transfer owing to radiation in the experiments

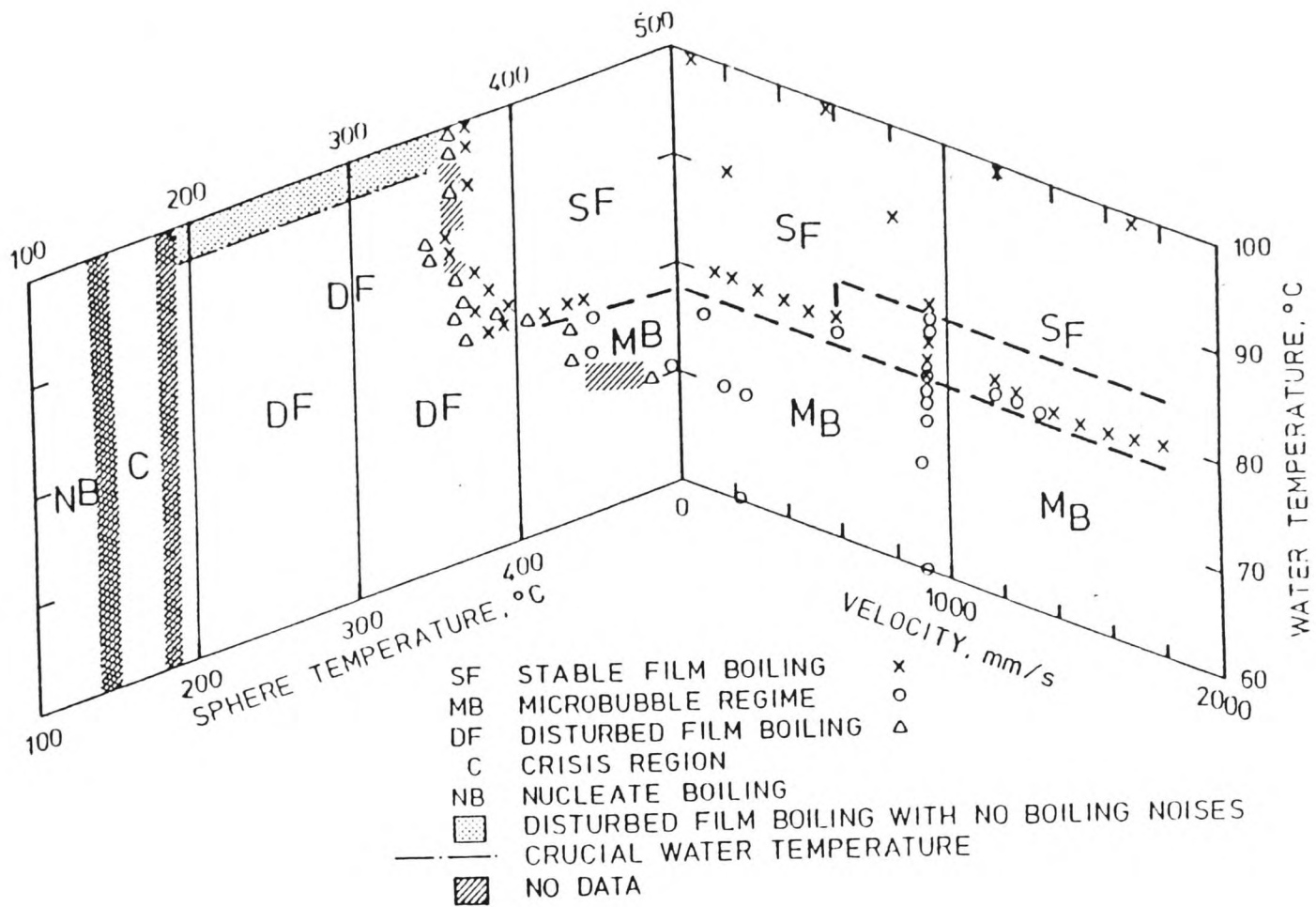


Figure 4.1: The Temperature Regime that the Particles Must Occupy for Stable Film Boiling to Occur [95]

will be much less than in the mixing of corium (by about 57 times). This does not matter greatly in the validation of CHYMES, but the experiments cannot be used as a direct simulation of coarse mixing;

- because CHYMES is a two-dimensional code the particle jet can only be planar¹ or cylindrical.

For the comparison of the experimental and computational results the distribution of the phases has to be clearly visible: this was not possible in previous, three dimensional mixing experiments, such as those in the FITS vessel; therefore the experiments were performed in a plane tank just larger than the particle diameter. A disadvantage of this is that the walls affect the motion of the particles.

4.2 Description of the Experimental System

The apparatus required for the experiments has three elements:

- a projection mechanism that will discharge particles in a controlled and known manner;

¹i.e. the motion of the particles is restrained to two dimensions in a cartesian space.

- a tank into which the particles enter, and which restrains their motion to two dimensions;
- heaters that warm the water in the tank to saturation, and the particles to a sufficiently high temperature to cause film boiling.

Before describing the apparatus in detail, the target parameters used in its design will be discussed.

4.2.1 Design Parameters

When designing the apparatus a number of approximate parameters within which to work have to be defined. These are: the size of the particles, the initial particle volume fraction, the initial speed of the particles, the duration of the jet, and the width of the jet.

In the MIX series of experiments with molten corium, that are also intended to contribute towards the validation of CHYMES, the molten jet was split into drops of diameters between 1 and 10 millimetres[24]. This accords with the final drop size in the FIT series of experiments if no explosion occurred [6] and in another set of experiments with corium where the largest particles were spherical with a diameter of 5–6mm (M.S.El Genk et al [96]). A particle size of 6mm was then chosen so that results of previous tests of ball bearings in plane tanks could be used (F.Lyell et al [97]).

The maximum volume fraction of particles in a three-dimensional square array is 0.40. The maximum volume fraction in the experiments was limited by practical constraints on the minimum spacing between particles in the projection mechanism. This spacing was set at 9mm, which corresponds to a volume fraction of 0.18 on release of the particles with a 6mm diameter in a tank of width 7.8mm.

In coarse mixing if a particle of melt falls more than about 50mm in a gas its speed will exceed its terminal speed in water. To ensure that this is also true for particles in the experiments a projection speed of 1m/s was selected which is similar to their terminal speed.

The duration of the particle jet depends upon the size of the water tank. Dimensions of 1.0m tall and 0.5m wide were selected as these are of a similar order to those of a reactor vessel. If they were bigger practical problems would ensue for little benefit. In a tank of this depth a stainless steel particle travelling at its terminal velocity would take about one second to travel from the top to the bottom of the tank; therefore the particle jet should run for at least one second. In fact, for safety, a time of 1.25 seconds was used. This entails the delivery of a number of rows of particles equivalent to a length of 1.25m.

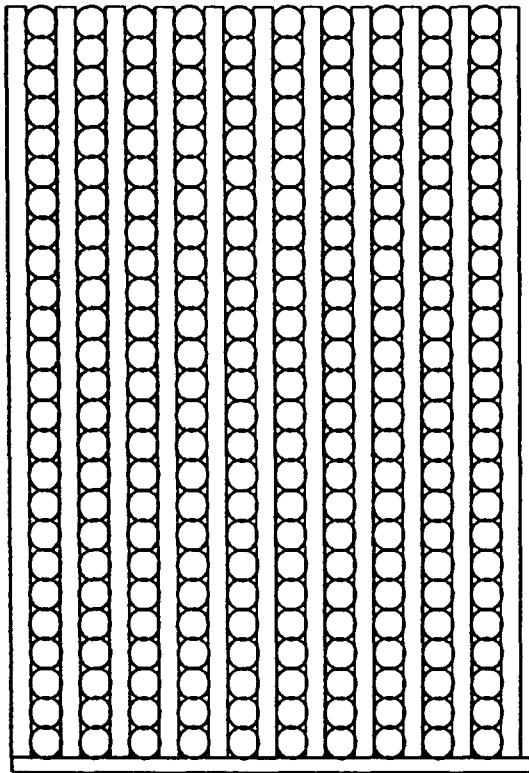


Figure 4.2: A Possible Experimental Arrangement

Finally the width of the particle jet was set at ten columns of particles. The greater the jet width the more complex the apparatus must be. This number of columns ensures that the width of the jet is similar in order to the diameter of the jet in coarse mixing, and allows scope for it to be varied.

4.2.2 The Projection Mechanism

The major problem in the design of the projection mechanism is to devise a method for releasing the particles cleanly with a known velocity and concentration. The most obvious method of projection is to allow particles to fall freely through a funnel into the tank. This is unsuitable because of the lack of control that it affords: the concentration of particles would be difficult to determine, and could not be changed without changing the velocity of the particles on entry to the water pool. Funnels also impart spurious velocities to the particles through collisions with the funnel and each other [97] and the characteristics of the jet may change with time. For example a possible arrangement is shown in Figure 4.2 where columns of particles are held in columns and released by the removal of a plate at the bottom. The particles at the top of each column will drop a distance to the water's surface that is greater, by the length of the column, than that dropped by the particles at the bottom; therefore their velocity will be much higher.

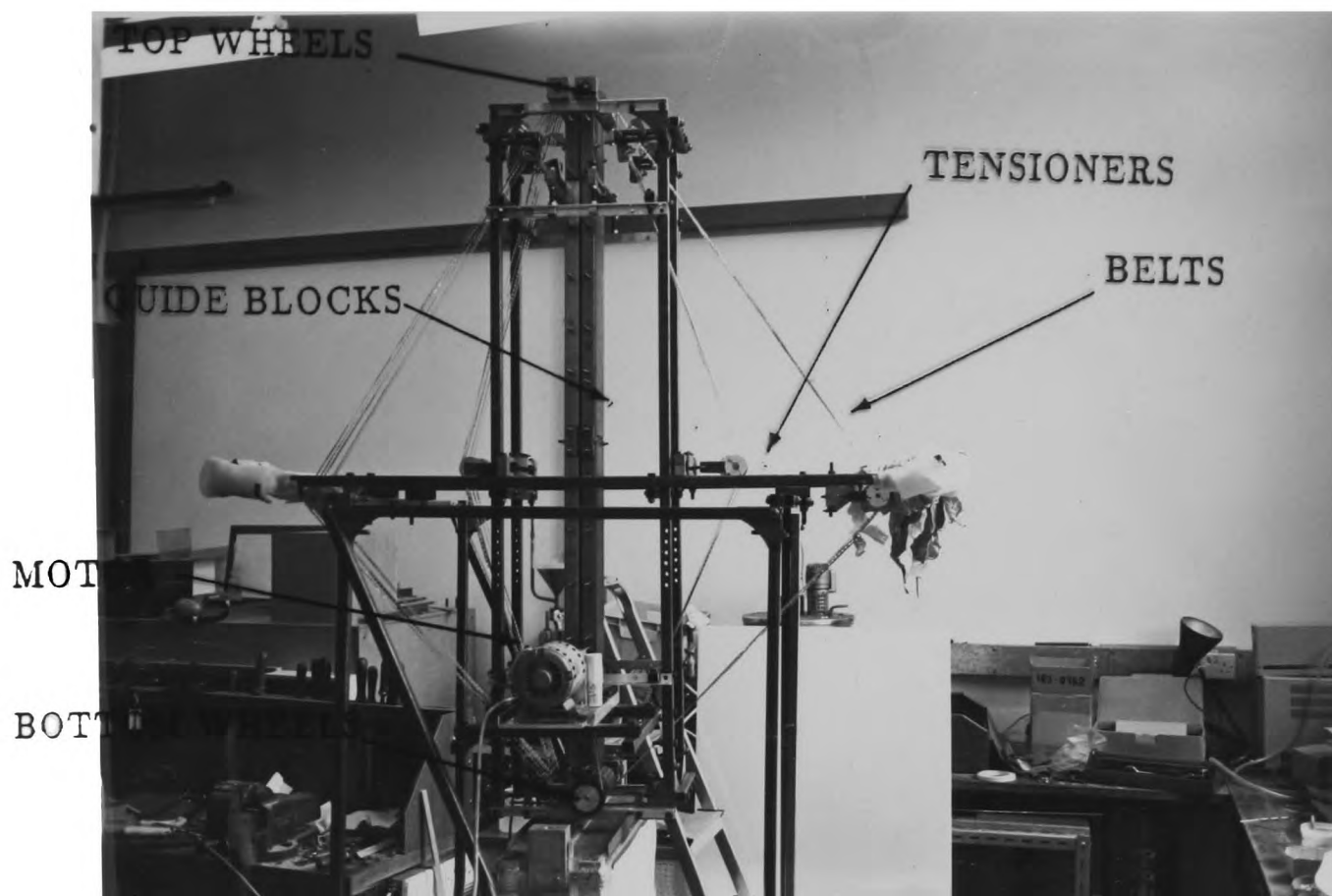


Figure 4.3: The Projection Mechanism

If the plate had a periodic motion that allowed only one particle to drop at a time this problem would be eliminated, but it would be difficult to arrange owing to the high frequency required; in addition the particles' velocity could not be altered, except by varying the distance through which the particles fall before entering the pool.

There are a number of approaches that might avoid these problems. However the apparatus must be practical, and allow the particles to drop straight down. The latter requirement eliminates all machines that rely on particles held at the edge of a disk: a tangential velocity will be induced on each ball unless the disk is brought to a stop before each particle is released. The method that was finally selected is now described.

Description of the Projection Mechanism The projection mechanism is shown in Figure 4.3. To eliminate collisions between particles, and to control the particles, it was decided that each particle should be individually supported within the mechanism. The support was provided by opposing perforations in two parallel belts into which each ball fitted as shown in Figure 4.4. The belts were made from stainless steel strip of thickness 0.005 inches. This material is strong, flexible, and allows accurate holes to be made in it. It withstands well the temperatures that the particles are heated to except from suffering a loss of elasticity if locally deformed. The holes in the belt were made by a punch and their spacing determined with a jig.

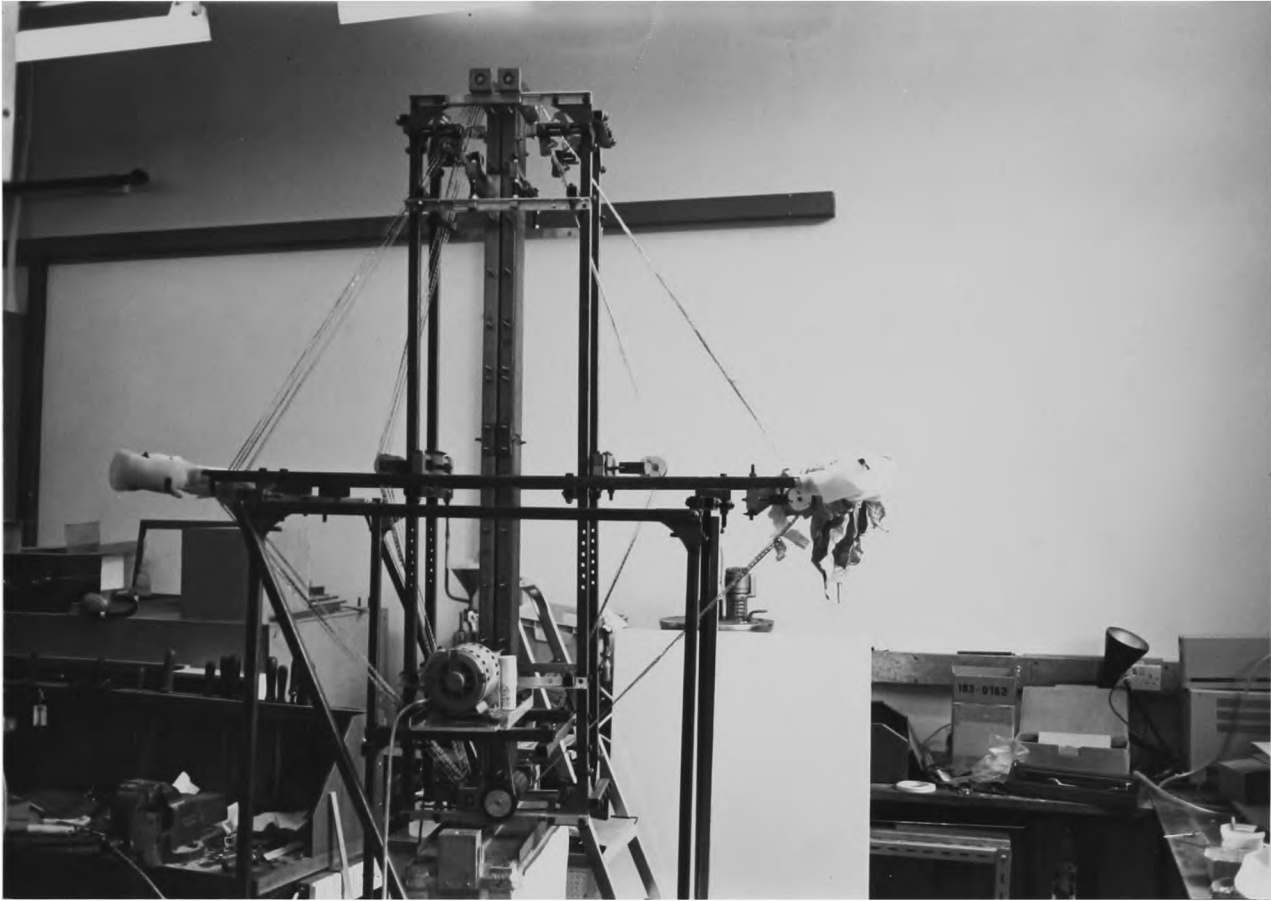


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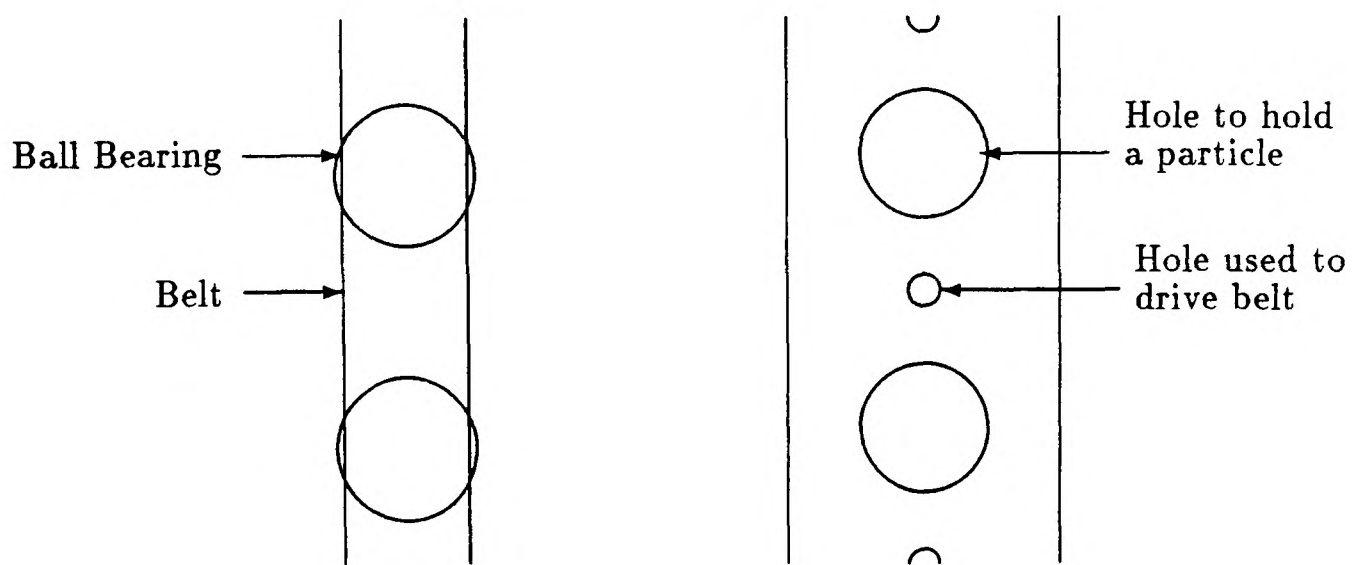


Figure 4.4: Diagram Showing how each Particle is Supported



Figure 4.5: The Wheels at the Bottom of the Projection Mechanism

The balls are released by passing the belts round wheels at the bottom of the mechanism (see Figure 4.5). These are grooved to accommodate the portions of the particles that project through the belts. This mechanism released the particles quite cleanly, but some had small lateral velocity components that were significant enough to prevent them from entering the tank. These were eliminated by passing the particles through the guide shown in Figure 4.6. The holes which guide the particles are reamed to minimise any retardation to a particle to a minimum. Pins could be inserted in the holes at either end of the guide to ensure that the tank is aligned correctly with the jet of particles.

The belts are driven by pegs that project from the wheels and mesh with a second, smaller set of holes in the belt (see Figure 4.4). The wheels are keyed onto shafts whose ends run through ball bearing races. The separation between the shafts could be adjusted accurately



Figure 4.6: The Guide for the Balls

through the turning of a screw. At one of their ends the shafts are keyed onto plates on which gears linking the two sets of wheels are mounted. The plates allow the position of one set of wheels to be adjusted relative to the other.

The Drive of the Mechanism At the other end of one of the shafts to that where the plates and gears are mounted, there is a timing pulley driven by the power train through a timing belt. The power train consists of a 100W motor² and an electromagnetic clutch³ as shown in Figure 4.7. The speed of the belts could be altered by changing the relative sizes of the timing pulleys. The volume fraction could be modified independently of the projection speed by omitting balls in the array held within the projection mechanism.

The Arrangement and Tensioning of the Belts Owing to the high speed of the belts, and the required rate of supply of particles, it would have been impractical for the particles to have been placed within the holes while the apparatus was running; instead they have to be inserted within the machine and held there until the mechanism is started. This entailed a column of particles being held by each pair of belts between the bottom set of wheels and a top set of wheels. Those were grooved circumferentially so that particles could be loaded from the top of the machine.

The belts were tensioned by simple, spring loaded pulleys as shown in Figure 4.8. The pulley wheels ran round oilite bearings. Individual pulleys were provided for each belt because

²A type BCP1508B, single phase, A.C. motor made by the F.H.P. division of G.E.C. Machines Ltd.

³A type 51-05-BB-24V(d.c.)-10W-029 clutch manufactured by Stephenson-Gobin Ltd.

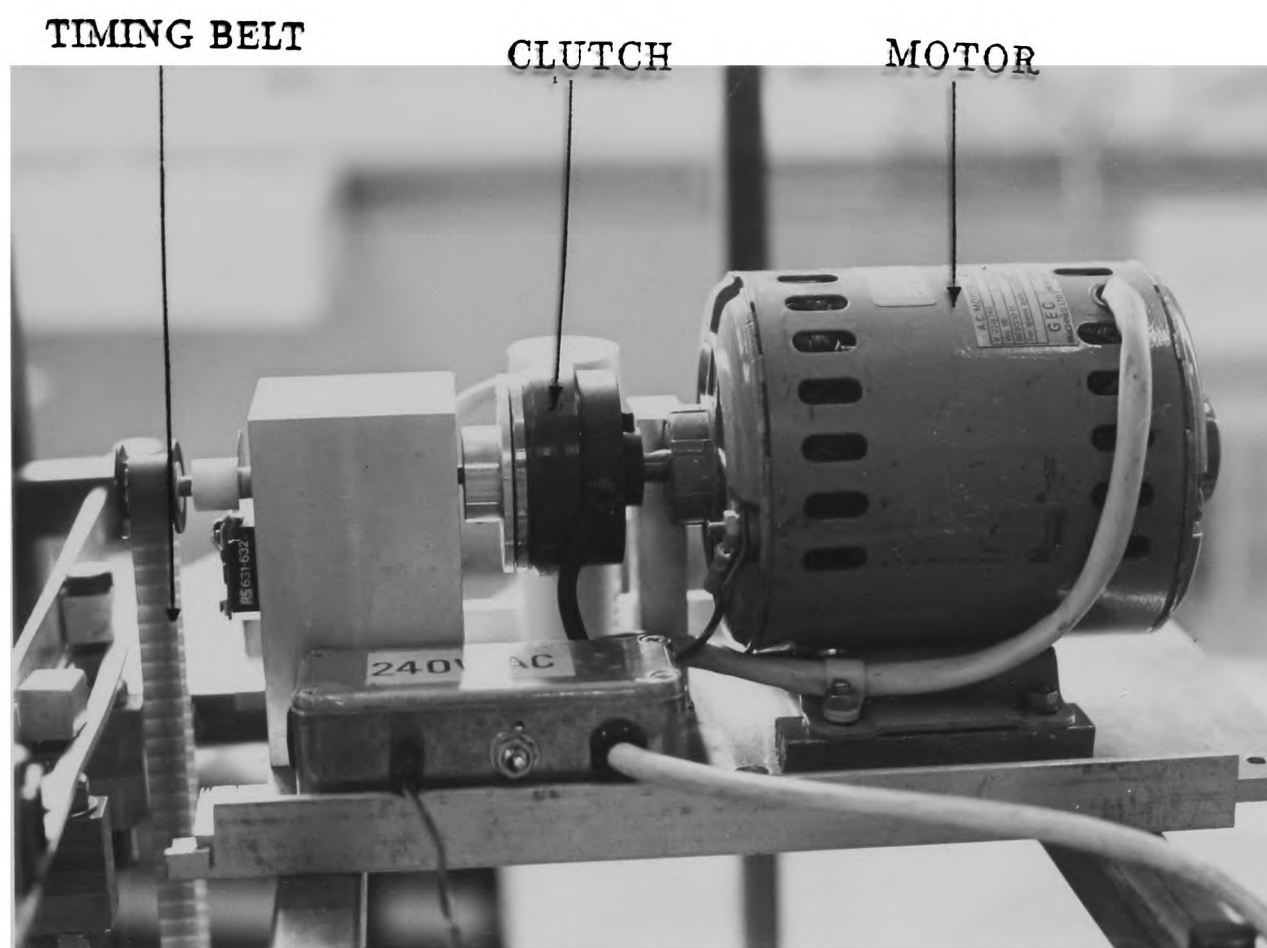


Figure 4.7: The Power Train

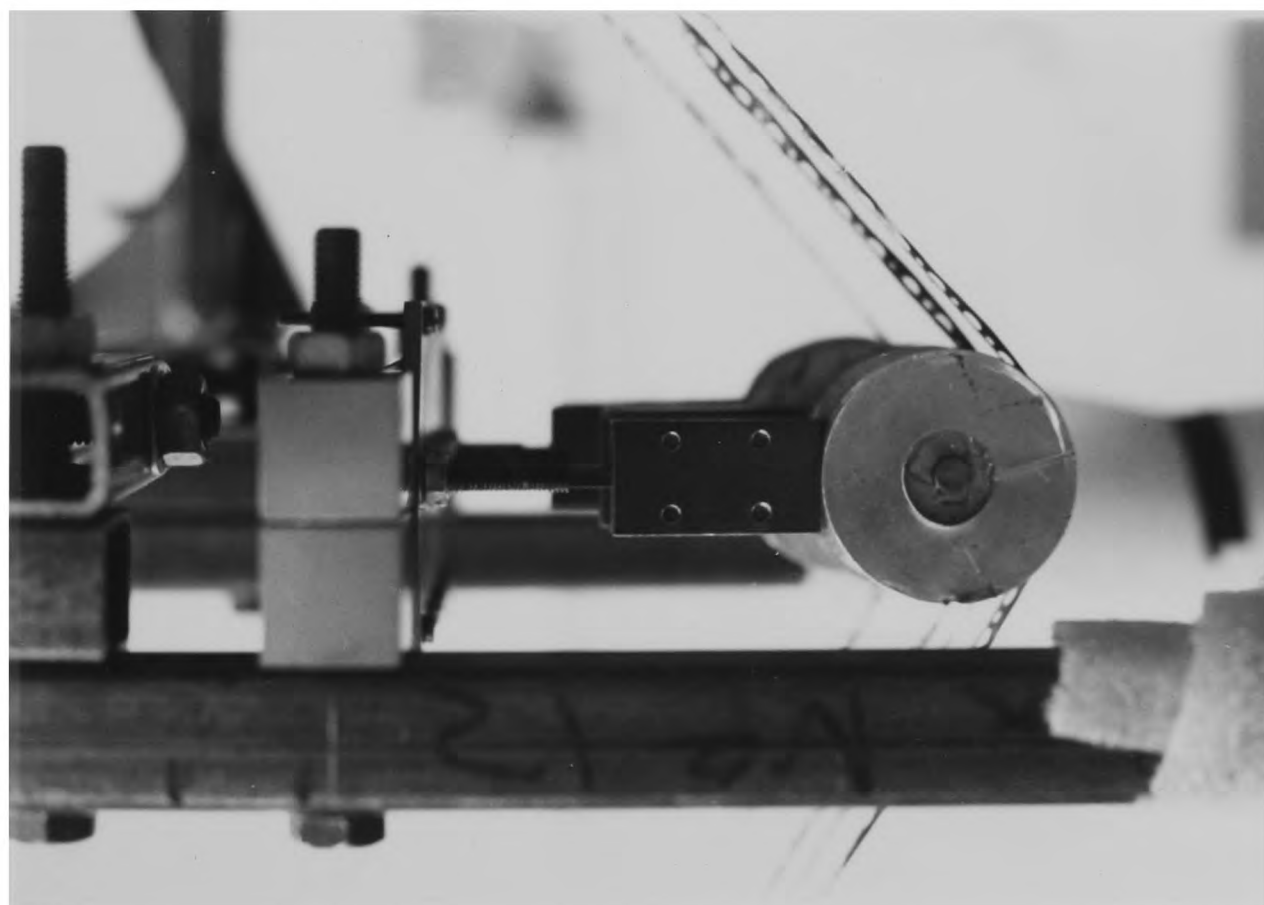


Figure 4.8: A Tensioning Pulley



Figure 4.7: The Power Train

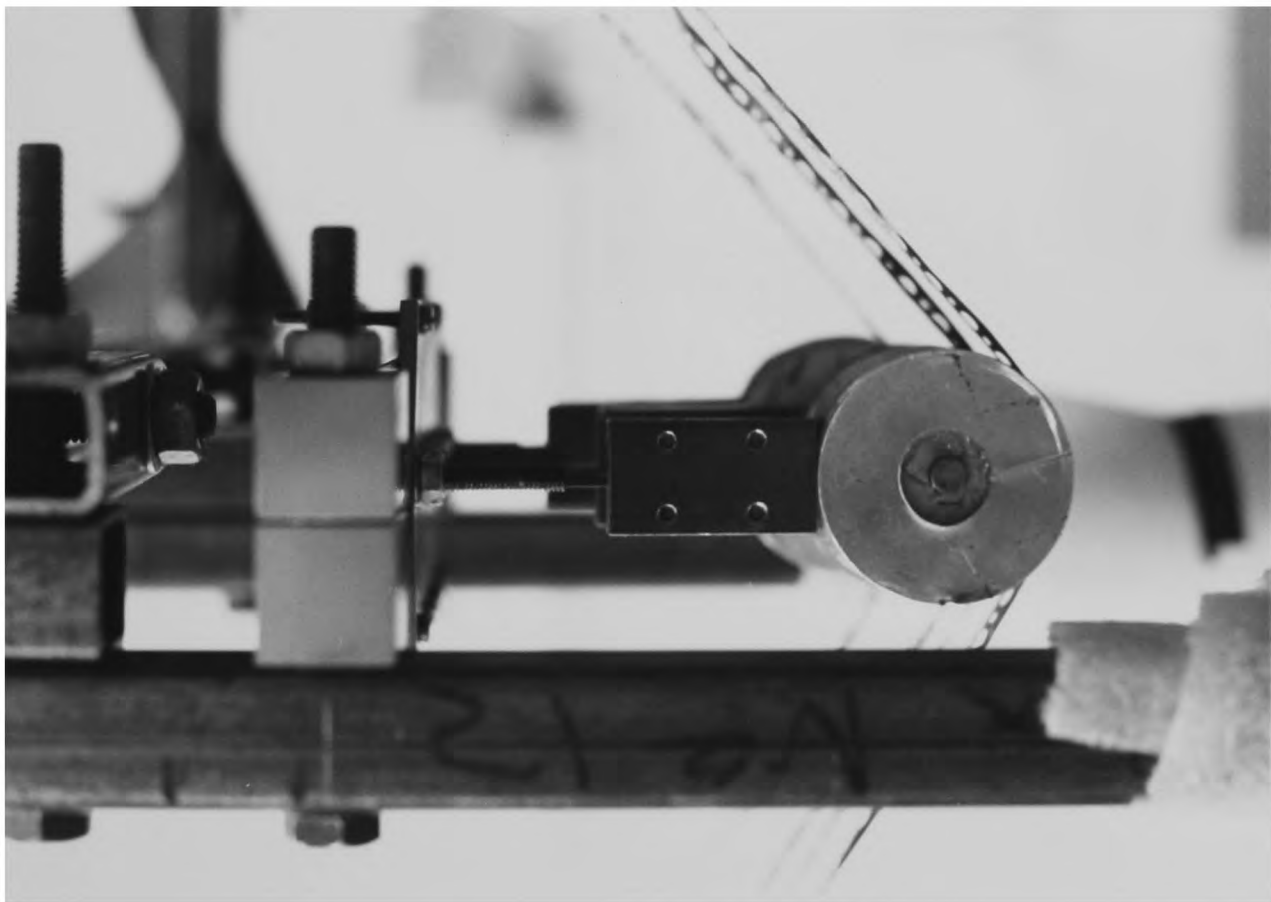


Figure 4.8: A Tensioning Pulley



Figure 4.9: The First Set of Guides

the position of the pulley was strongly dependent on the length of the belt. Because of size constraints two rows of pulleys were necessary for each set of belts. There was a tendency for belts to work themselves off their pulleys which was remedied by gluing thin plates onto the pulley wheels.

All the components of the projection mechanism that were not bought in were made from brass, apart from the shafts which were made from stainless steel. Brass was selected because it performs well at high temperatures and does not suffer badly from corrosion.

The Guide Blocks The centre of mass of each particle does not lie in the same plane as the belts; therefore there will be a lateral force pushing the belts out sideways. There will be little resistance to this owing to the slightness of the belts so some sort of guide is required to maintain the separation of the belts. The guides used in the initial, unheated experiments are shown in Figure 4.9. They consist of a sandwich of forged components held together by several lengths of stud. This was necessary as it is not possible to machine flat most materials over a large length because of distortion owing to stress relief. It was thought that the distortion might be sufficient (0.9mm on each guide) to leave some particles unsupported.

The spacing between the guide blocks was in fact more critical than anticipated. Even if their distortion was much less than 0.9mm the belts could twist so that their perforations no longer supported the particles. The best method of fixing the spacing would have been to bolt

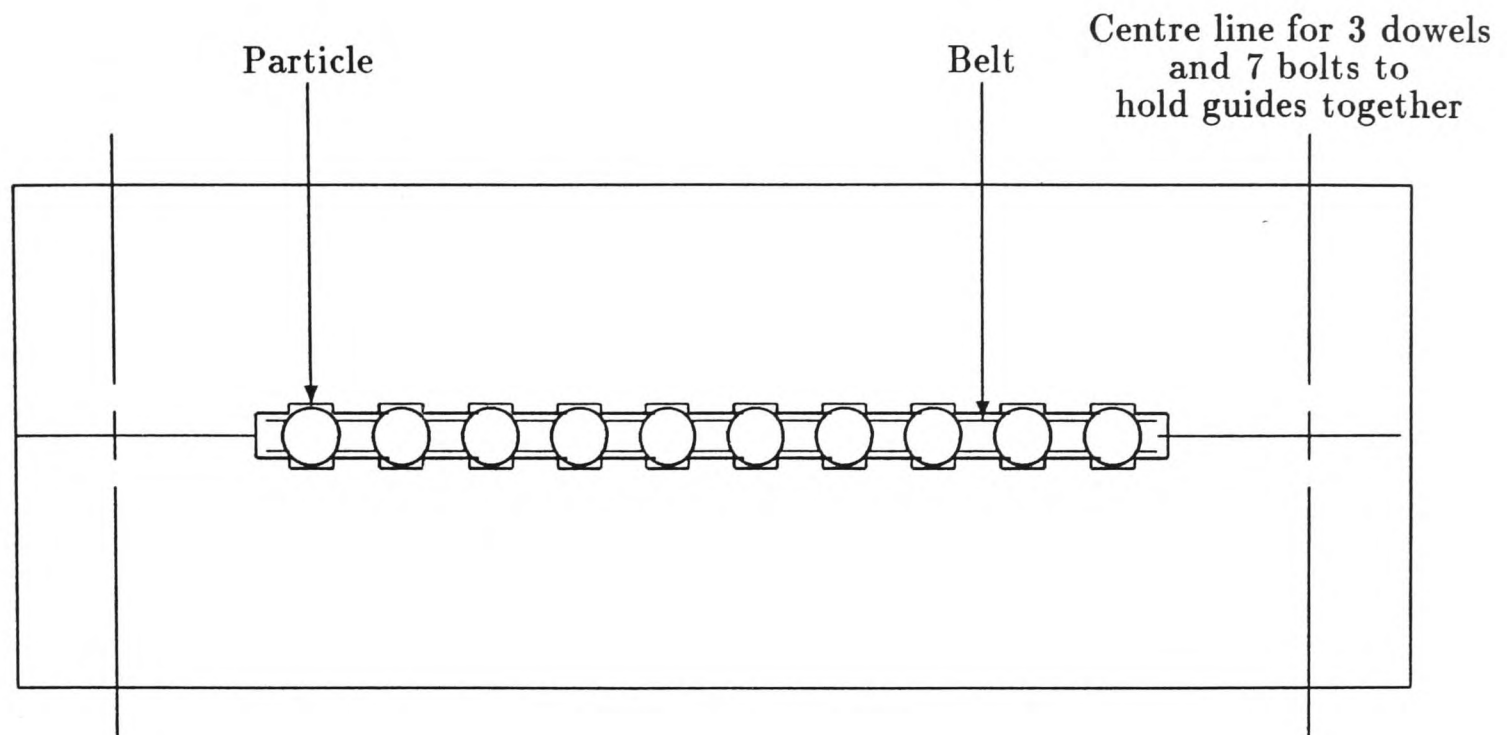


Figure 4.10: Top View of the Guide Blocks Used in the Heated Experiments

the guides onto spacers of the correct thickness. The problem with this was that the actual gap required proved to be larger than expected if the balls were not to be greatly retarded. This is probably a result of the holes in the belts not opposing each other exactly. The separation of the guides was then determined empirically in the unheated experiments. The resulting gap was then measured and appropriate spacers made for the hot experiments.

The repeated channels down the guides accommodate the portions of the particles that project through the belt. To ensure that the edges of all the channels lay in the same plane the blocks were assembled upon a marking table. Steel was the only material available in bars of the required size and form for the guides, so tool steel was used for the edges, and carbon steel for the central portions of the guides.

The guide blocks had to be replaced for the heated experiments as the original ones distorted badly when heated. The new guides are shown in Figure 4.10. These were made from cast iron billets, which have a high degree of dimensional stability that allows grooves to be machined over each guide's entire length. The advantages of these guides over the old ones were: they could be bolted together to discourage distortion; they could be accurately aligned using dowels; and any slippage between individual components, that might have happened in the old guides, was eliminated.

Unfortunately, despite these precautions, when the balls and belts were heated there was a large resistance to their movement. This was not apparent at room temperature but any possible movement ceased if the temperature exceeded roughly 300°C. When the apparatus

was allowed to cool the particles were not released and could only be removed by separating the guide blocks. If the apparatus was loaded again then the belts and balls would move freely until they were heated. If the belts were removed and the channels in the guide block filled as shown in Figure 4.2, apart from one instance, most of the particles were retained by the guide blocks when they were heated and could only be removed when cool again.

It was possible that the heat was distorting the guides. This was discounted for the following reasons:

- the most probable form of distortion is the bowing outwards of the guide blocks; this would disrupt the balls held in the belts, but it would not affect the freely stacked columns of particles;
- particles easily dropped through unfilled channels in the heated guide blocks when the belts had been removed;
- the insertion of one or 0.01 inches thick shims between the guide blocks made no difference to the resistance to the balls;
- the distortion of the block could either be elastic, in which case the particles would be released when they had cooled, or plastic, in which case the mechanism could not be moved again. Neither pattern of behaviour was observed.

Another explanation is that the stainless steel particles and belts are adhering to the cast iron guides though the mechanism behind this is obscure. The cast iron billets oxidised only a little, acquiring a thin layer of fine powder; careful cleaning of the components made very little difference to the performance of the apparatus. The resistance to the movement of the belts could be reduced by coating the belts and guides with penetrating oil before heating them; when hot the belts could be turned by hand, but the resistance was still too large for them to be moved by the motor.

The Heating of the Particles For stable film boiling to occur the particles must be heated to a temperature of several hundred degrees Celcius. To allow a safety margin a design temperature of six hundred degrees was adopted. It was not possible to heat the particles to such a temperature once they were in motion; therefore a system was devised to heat the particles within the guides beforehand.

A furnace, without a top or bottom, was built around the guide blocks as shown in Figure 4.11. The two walls of the furnace parallel to the plane within which the balls lay consisted

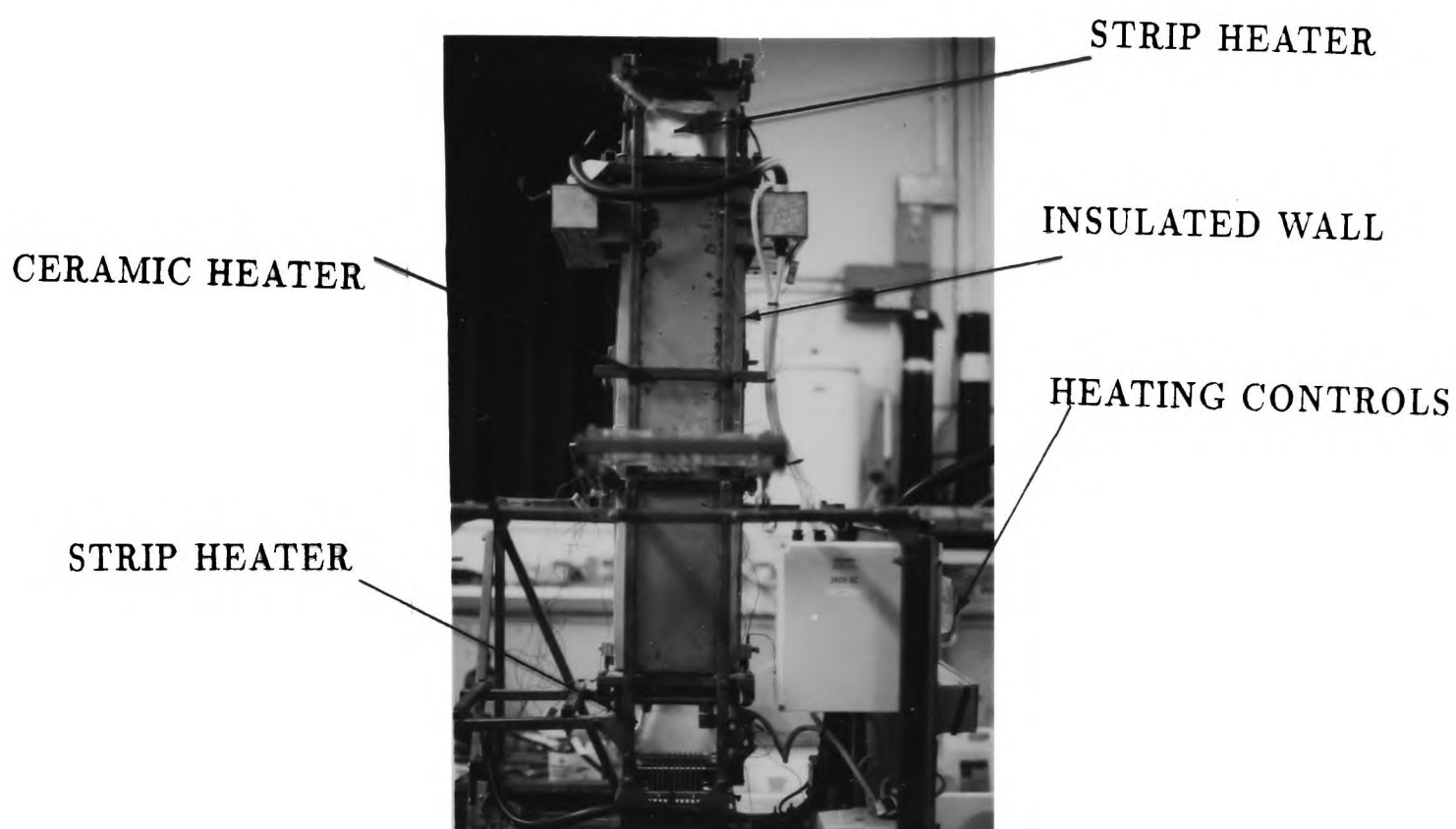


Figure 4.11: The Heating System for the Particles from the Front of the Apparatus.

of radiant heaters⁴ embedded in ceramic insulation. Heaters that were sufficiently long to heat the entire guide block could not be obtained so four extra heaters⁵ were fitted, two above and two below the main ones. All the heaters were backed with insulation⁶.

The portions of the unheated walls that corresponded with the main heaters were made of 24mm thick insulation hung from 1/4 inch thick steel plate. The strip heaters, and the insulation that covered the rest of the unheated walls, were hung from studs anchored to these plates.

The larger heaters were controlled by a Watlow series 985 controller, and each set of strip heaters by a Watlow series 100 controller. The heaters were controlled by thermocouples positioned so that their ends were close to each heater's surface. It was assumed that the balls and the guide blocks would be in equilibrium with each other; therefore the balls' temperature was measured by thermocouples inserted inside holes drilled in the guide blocks at several places along the blocks. When the power was switched on, it was increased gradually and produced temperatures profiles like that shown in Figure 4.12. Convection from the top surface of the guides is not strong enough to explain the large temperature difference between the top half

⁴Watlow VE06A36S ceramic fibre heaters, rated at 2500W each.

⁵Watlow mineral insulated strip heaters, rated at 500W each.

⁶Walker's Millboard No.261 which has a thermal conductivity of 0.12 W/mK.



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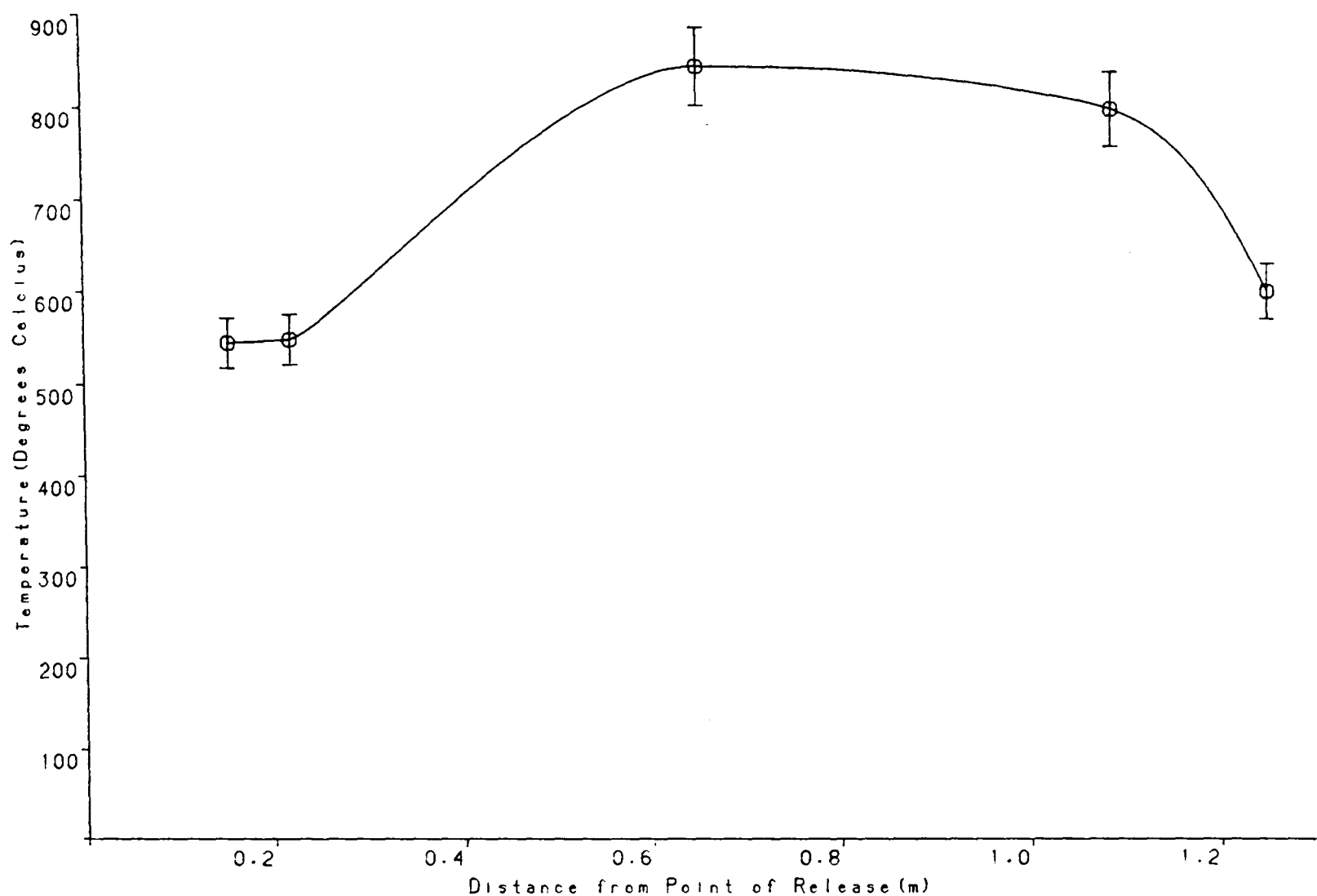


Figure 4.12: Typical Temperature Distribution Within the Guide Blocks

and the bottom half of the guides and no other heat sinks are evident. It might be caused by some internal feature within the larger heaters.

4.2.3 The Tank

Aside of the obvious requirement that a tank has to hold water, the most important problem in its design is to minimise the interference of the restraining walls with the particles. Tests had been made of this elsewhere [97]; it was found that if the spacing between the walls varied slightly from a critical value the velocity of a particle could be affected in terms of both direction and magnitude through collisions with the walls. For a ball of diameter 6mm this critical distance was found to be 7.8mm. These tests also indicated that it was important for the gap between the walls to be as uniform as possible. To ensure this a tank was constructed where an inner tank, into which the balls are projected, sits inside an outer tank. Both tanks are filled with water so that the walls of the inner tank are not bowed by hydrostatic pressure. This structure may be seen in Figure 4.13. The inner tank consisted of two restraining walls clamped at the edges on to 5/16 inch bars to separate them as shown in Figure 4.13. In the experiments where the particles were unheated the restraining walls were single panes of 6mm float glass. These would have been destroyed in the hot experiments and were replaced by sheets of a clear

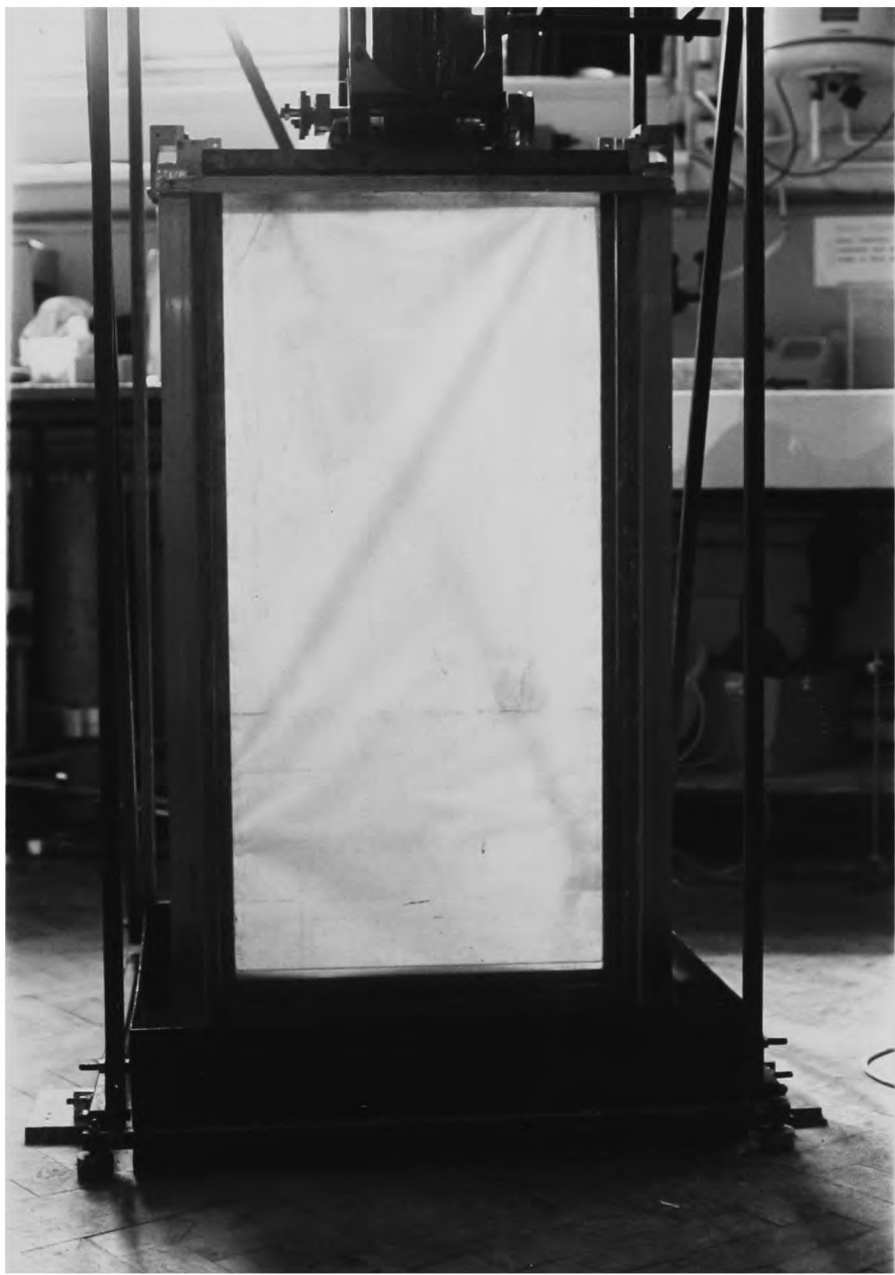


Figure 4.13: The Construction of the Tank

ceramic⁷. These are not supplied in large panes and so each wall consisted of three panes glued onto aluminium bars where they met⁸.

To provide the solid boundaries specified by CHYMES, and to allow the removal of the particles after an experiment a U-shaped assembly of bars was inserted into the inner tank. All the metal parts of the tank were made from dural as this is light and not badly corroded by water. An exception was the frame for retrieving the particles which was made of dural in the unheated experiments and brass in the heated ones. In addition this had to be isolated by spacers from the bottom and sides of the tank in the heated experiments by to prevent the heat from the balls damaging the tank.

The outer tank was made from a rigid frame of welded dural components onto which 12mm thick panes of glass were glued.

The water was heated with two 1500W cable heaters⁹ wound around a brass frame and inserted, either side of the inner tank. Provided equilibrium is reached the temperature of the water in the inner tank should not be significantly different from that in the outer tank. The heaters were controlled by a variac, and the water temperature measured by a thermocouple mounted on a rod.

4.2.4 The Recording of the Experiments

The experiments were recorded using the high speed video-recording systems manufactured by NAC Ltd. These consist of a recorder, a camera, and a stroboscope. The stroboscope was controlled automatically by the recorder and produced light pulses $10\mu\text{s}$ long. The recordings were made on ordinary video tape. Two versions of the system were used, one older than the other. Their main characteristics were the same except one could record at four hundred frames a second and two hundred frames a second while the other could only record at the latter speed; all the experiments were recorded in black and white at a rate of two hundred frames a second.

The camera was of the plumbicon type. It records a picture by scanning it line by line, 250 lines making up the picture. This places a limit on the resolution of the picture parallel to the shorter sides of the picture.

⁷CERAM, supplied by J.B.Treasure & Co. of Liverpool.

⁸By Pacer Glass/Metal kit adhesive.

⁹Philips Thermocoax heaters

4.3 Measurements of the Performance of the Apparatus

Two aspects of the experimental apparatus need to be evaluated: the influence of the restraining walls of the inner tank on the motion of the particles, and the time taken for the projection mechanism to attain a constant speed.

4.3.1 The Influence of the Restraining Walls

The restraining walls of the inner tank will retard a particle falling between them because of viscous effects, the constriction by the walls of the flow around a particle, and interference with the particle's wake. The rate of fall of an isolated particle is determined only by its buoyancy, Reynolds number, and the spacing between the restraining walls. If it has a high Reynolds number its drag coefficient is independent of speed so if the separation between the walls of a tank is uniform, the modification to the particle's terminal velocity will be constant. This may be expressed as a modification to the drag coefficient, as in [98]¹⁰(G.Volpicelli et al).

To determine the effect of the walls single stainless steel particles were projected into the tank at three different initial velocities, and aluminium particles at the middle of the speeds. The motion of each particle was recorded on videotape, and its position at a given time estimated using a small image processor; the speed of each particle could then be found. The results are shown in Figure 4.14 where the speeds recorded for each particle have been normalised with respect to the particle's estimated terminal speed in an unbounded fluid. The times have been normalised using the terminal speed and the diameter of the particles. The resolution of the processor was quite coarse, and is responsible for an error of about ± 0.02 in the estimation of the normalised speed. In addition because of the small size of the image of each particle compared with that of the cross hairs used to locate them a further uncertainty of ± 0.04 is introduced. Some particles experienced a large reduction of speed for short (about 0.05s) periods, which may be caused by a collision with a restraining wall. Only one particle, out of the thirty nine tested, persisted in this state which indicates a series of collisions. The data for times less than 40 are neglected because the particles are decelerating from their free fall velocities to their terminal velocities at these times. The average terminal velocity of a particle in the tank compared with that in an unbounded fluid was 0.754. This was found using linear regression and has an upper limit of 0.760 and a lower one of 0.748 with 95% confidence. This can be related to a particle's drag coefficient using (3.34). For an isolated particle in an unbounded

¹⁰There the particles had a diameter of 3.18mm in a tank of width 3.55mm ($d_p/d_t=0.90$ compared with 0.77 in these experiments) and their terminal velocity was 0.70 times that of a particle in an unbounded fluid.

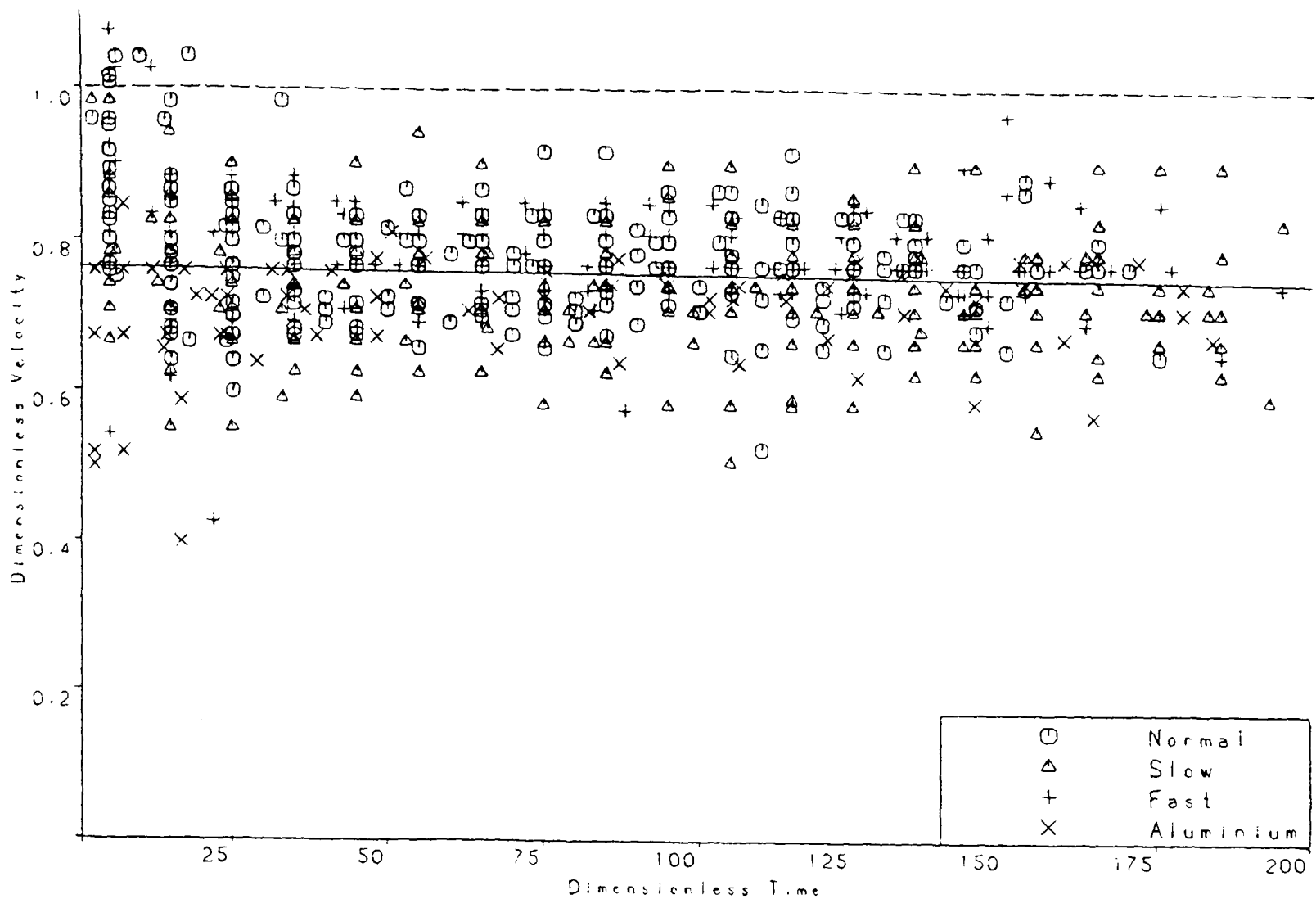


Figure 4.14: The Terminal Velocities of Particles within the Tank.

tank with a high Reynolds numbers $C_{D\infty} = 0.44$ [59]. The resulting effective drag coefficient for a single particle moving with a constant speed within the tank is then 0.76.

4.3.2 The Transient Behaviour of the Motor

The results of the experiments are for comparison with those of a numerical model. The boundary conditions of a numerical model are well defined; therefore it is important that the behaviour of the particles on release from the projection mechanism is well known as well. In an experiment when the clutch is closed there will be a period of time over which the belts accelerate to a constant velocity. This might have been avoided by leaving the first few rows of holes above the bottom wheels empty of balls; this was not done because unless the belts were pushed against the bottom wheels by particles they could bow and allow the balls to shift from their holes, or ride off the wheels.

To measure the transient an optical shaft encoder¹¹, consisting of a slotted disk and an optical sensor, was attached to the shaft between the timing pulley and the clutch. The output signal of the encoder was converted to a square wave which was sampled¹² so that the sampling

¹¹Radio Spares type RS 631-632

¹²This was with a Metrabyte Dash 16 card mounted in an IBM XT computer

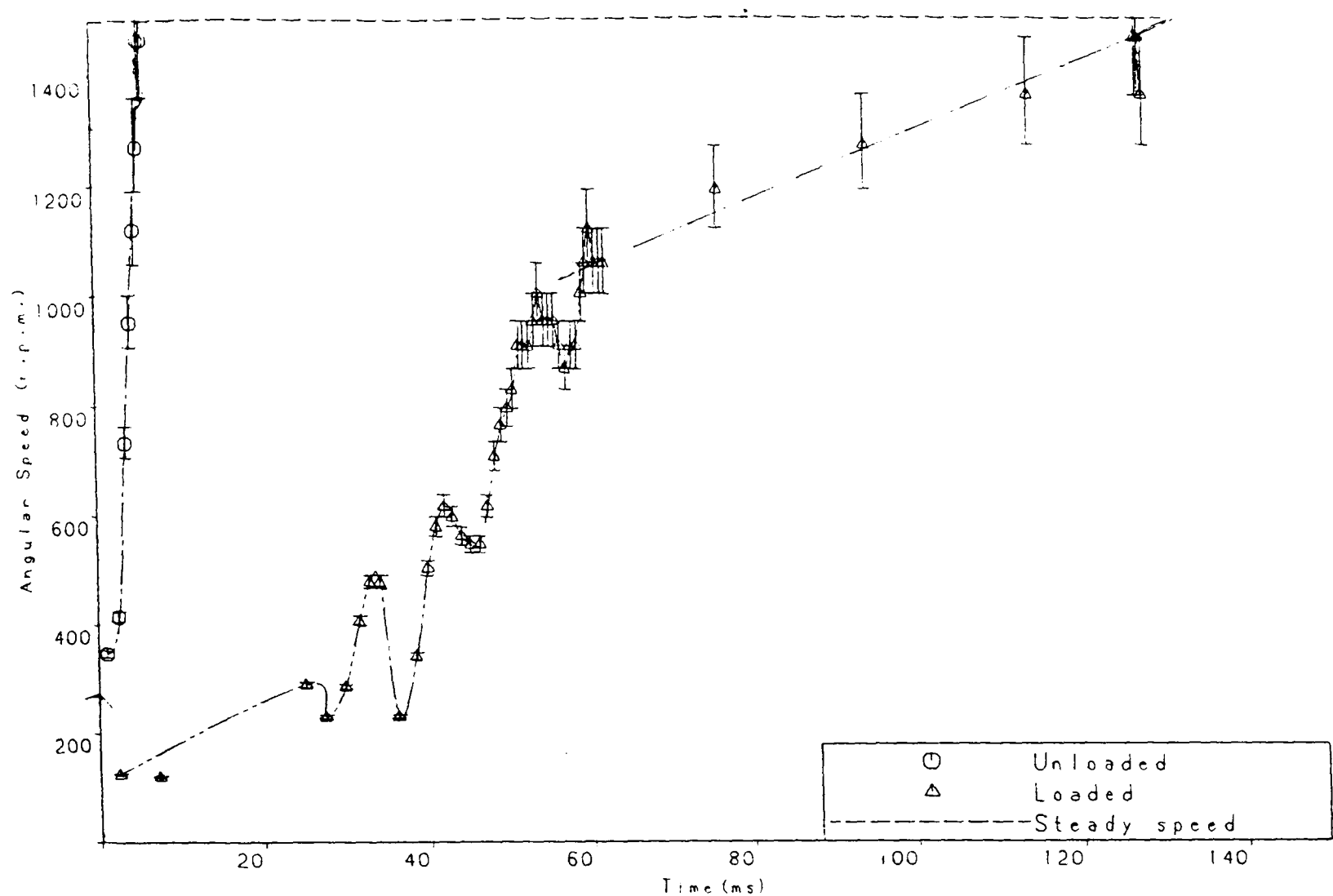


Figure 4.15: The Transient Behavior of the Motor Immediately after the Clutch is Closed

time was less than the duration of the wave peak caused by the passing of the slot. The angular speed of the shaft could then be estimated. The result is shown in Figure 4.15.

4.4 The Experimental Programme

Two parts to the experimental programme were envisaged: in the first the particles and water would be at room temperature, and in the second they would to be heated. Isothermal experiments were performed for a wide variety of initial conditions. These are detailed in Table 4.1. The 'Label' is a letter by which each set of conditions will be referred to in the future. Because of the large number of references made to them, the plumes labelled 'a' will be called 'normal plumes'. A specific experiment will be referred to by a letter and a number. Experiments a5 and a6 are plumes where only the top portion and the central portion of the tank were recorded respectively.

Owing to the problems with the apparatus for the heated experiments only one result was obtained which is described separately from the isothermal plumes in Chapter 6.

Label	Number of Recordings	Particle Spacing (mm)	Particle Velocity (m/s)	Number of Particles in a Row	Particle Density (kg/m ³)	Tank Width (m)	Offset of Jet Centre line from the Tank's Centreline(m)
a	6	9	0.95	10	7800	0.475	0.0
b	2	9	1.16	10	7800	0.475	0.0
c	2	9	0.68	10	7800	0.475	0.0
d	2	9	0.95	6	7800	0.475	0.0
e	1	9	0.95	10	2300	0.475	0.0
f	1	9	0.95	10	7800	0.215	0.0
g	1	9	0.95	6	7800	0.215	0.0
h	2	18	0.95	5	7800	0.475	0.0
i	1	9	0.95	10	7800	0.280	0.098
j	1	9	0.95	5	7800	0.280	0.118
k	2	9	0.95	10	7800	0.345	0.065
l	1	9	0.95	7	7800	0.345	0.065
m	1	9	1.16	10	7800	0.345	0.065

Table 4.1: Conditions Under which the Isothermal Experiments were Performed

Chapter 5

The Behaviour of the Experimental Particle Jets

In this and the following two chapters the overall characteristics of the jets of particles and the motion of individual balls within them will be described.

Under most conditions the behaviour of the plumes in successive experiments was similar. In Figure 5.1 various features that will be commonly referred to in the future are labelled. The word ‘jet’ will refer to the particles about to enter the water pool, and ‘plume’ to the collective motion of the particles in the tank.

The plumes may be divided into three classes: the majority of plumes that share a common development (sets of conditions a–g in Table 4.1); plumes with a low initial particle volume fraction (set of conditions h); and plumes where the centerline of the jet is not coincident with that of the tank (sets of conditions i–m). The history of the first class will be described here and that of the others separately in the next chapter (Section 6.3).

The measurement of the overall characteristics of plumes was made from video recordings of the whole tank. The individual motion of many of the particles could not be followed from these because they could not be discerned clearly owing to poor contrast with the background (e.g. those in the region close to the top of Figure 5.1). In addition there is the problem of identifying a single particle on succeeding pictures and its position would be difficult to determine accurately owing to the resolution of the video camera in the lateral direction. This was of the order of 3.5mm compared with the particle diameter of 6mm, which was also of the same order as the distance moved by a particle between frames. To examine the individual motion of particles, recordings close to the top and the centre of the tank were made (experiments a5 and a6 respectively); these are described in Chapter 7.

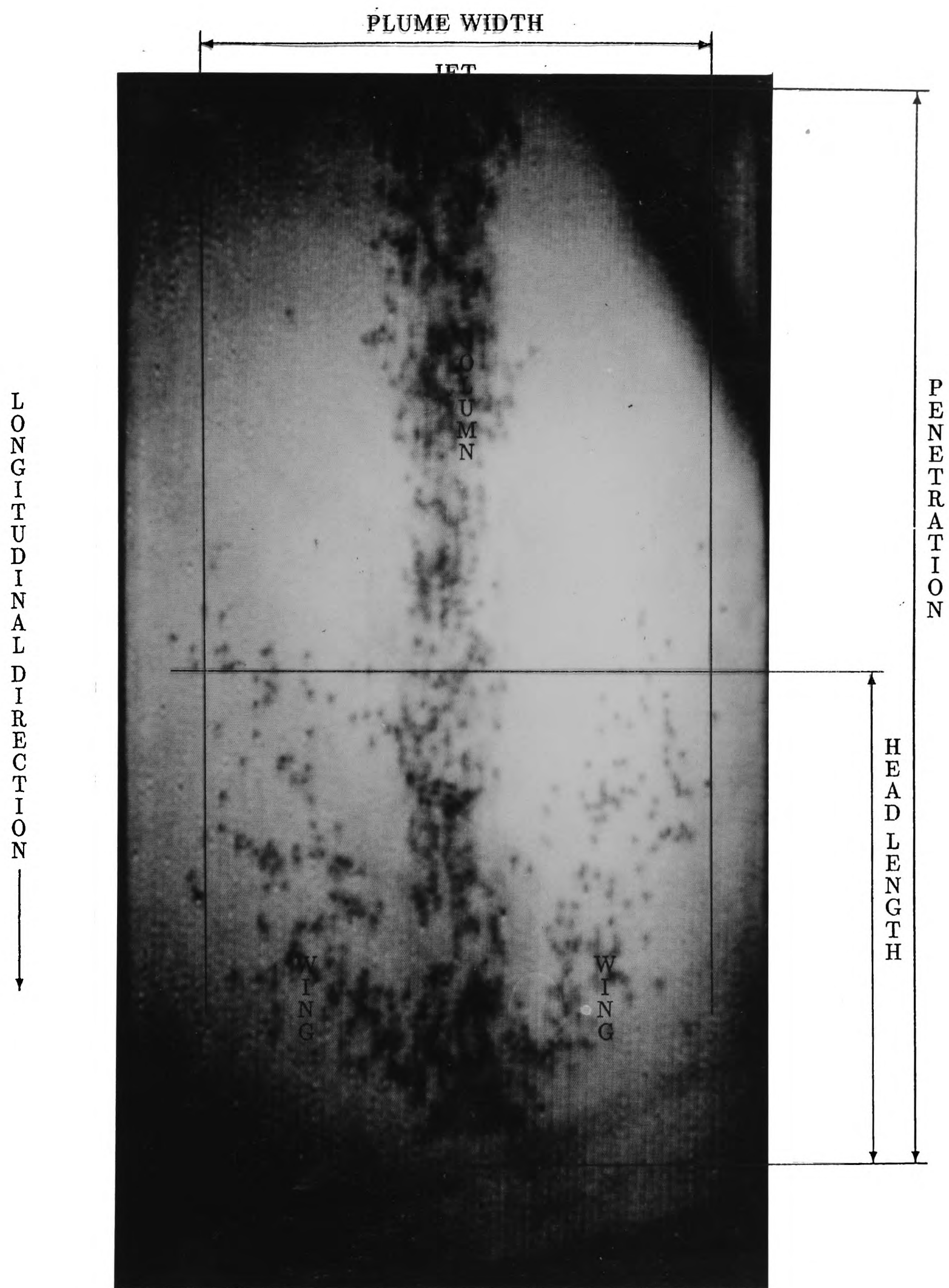


Figure 5.1: The Form of a Typical Plume of Particles. The Picture is of experiment a2.



Figure 5.1: The Form of a Typical Plume of Particles. The Picture is of experiment a2.



Figure 5.2: A Close up of a Normal Plume 0.02s after entering the Water Pool taken from Experiment a5.

5.1 The Appearance of the Plumes

The form and growth of plumes appear to be similar over a range of initial velocities, void fractions, particle densities, and jet widths provided the initial volume fraction exceeds some minimum value and the centrelines of the tank and the jet coincide.

On entering the water the jet of particles has the square front and sides shown in Figure 5.2. Note the air cavities attached to many of the particles. In Figure 5.3 and Figure 5.4 it can be seen that the rectangular shape of the plume is maintained until about 0.05s. From this time the particles at the front of the plume start to move laterally, producing the 'ears' either side of the front of the plume seen in Figure 5.5 and Figure 5.6.

The particles close to the front of the plume have now lost their air cavities and no free air bubbles are present in the front of the plume, unlike in the following column of particles. These differences are maintained as the plume falls through the tank. As an example Figure 5.7 shows the plume about half way down the tank: there is a clear contrast between the column where air is entrained and the head where it is absent. It is probable from this that the bubbles are stripped from the head owing to a larger relative velocity between the water and the particles there than in the column.

A crater in the water surface has been formed by the jet. Its depth has a maximum of 70mm, after 0.13s. This is maintained for a further 65ms before it starts filling. This is probably caused by the reaction of the water to the momentum imparted to it by incoming particles. When

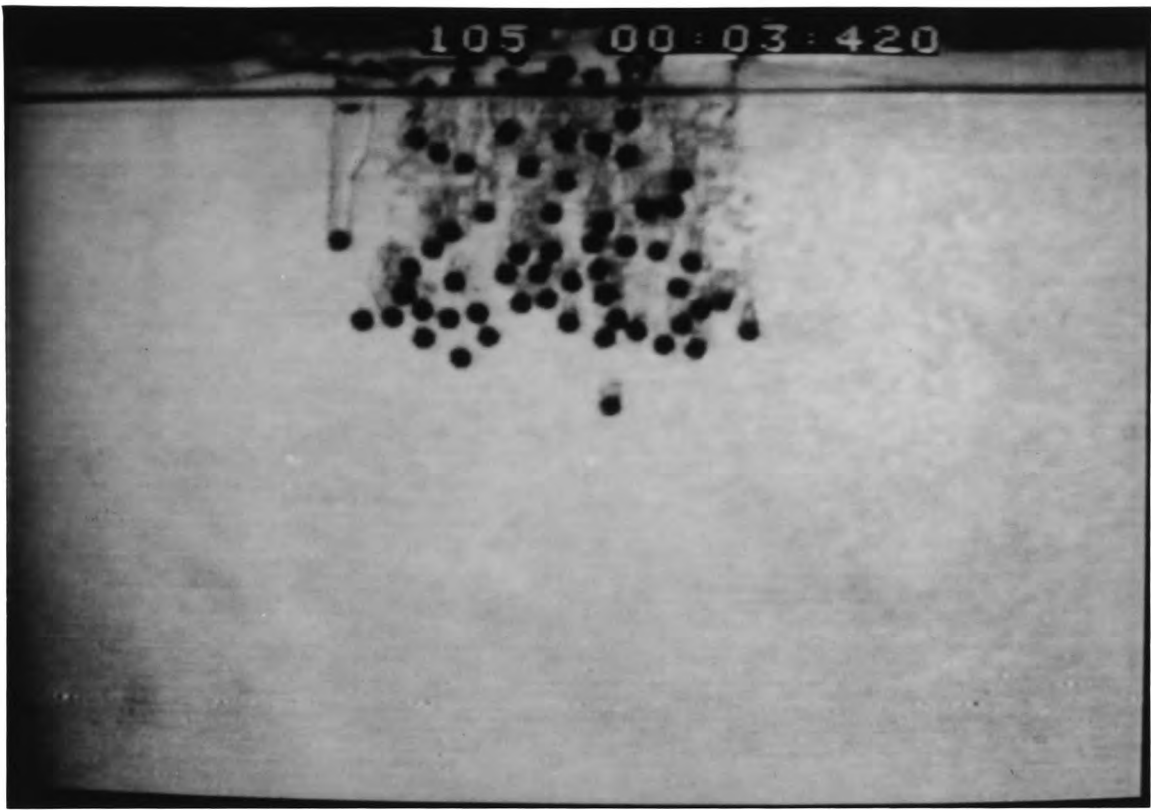


Figure 5.3: A Close up of a Normal Plume 0.05s after Entering the Water Pool taken from Experiment a5.



Figure 5.4: A Normal Plume 0.05s after entering the Water Pool, taken from Experiment a2.



Figure 5.5: A Normal Plume 0.10s after entering the Water Pool, taken from Experiment a2.

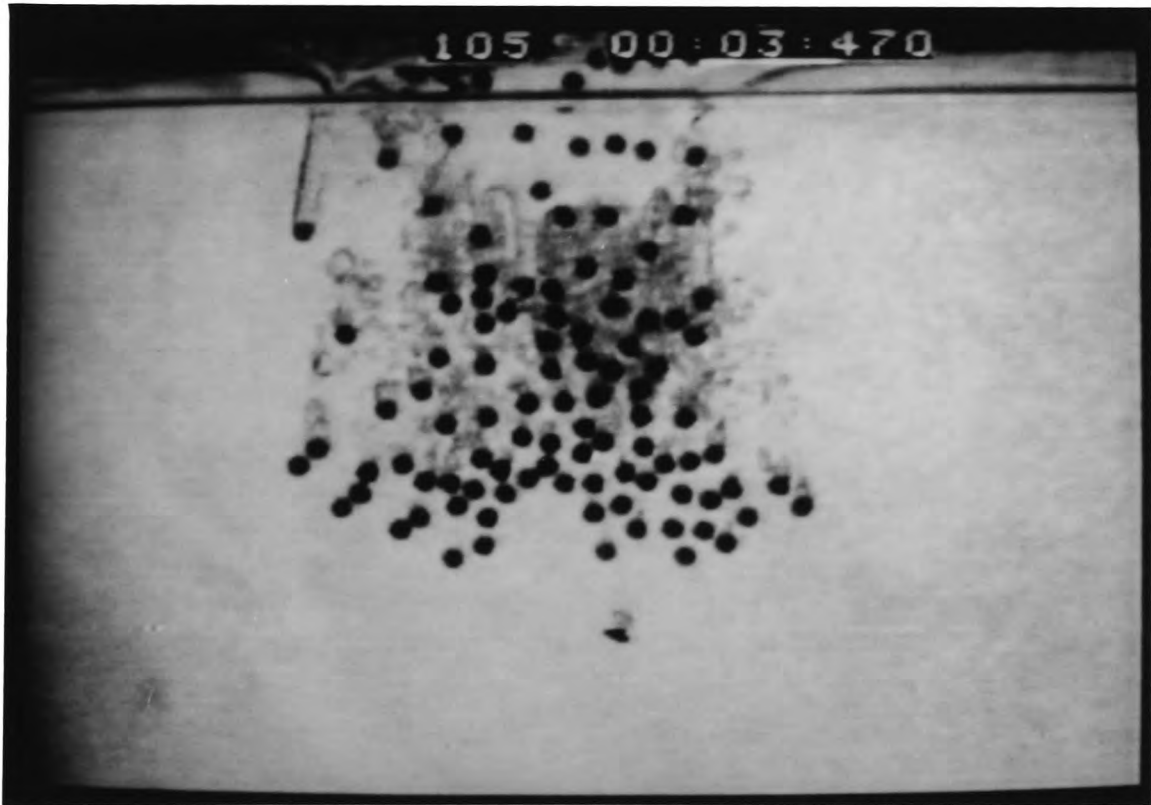


Figure 5.6: A Close up of a Normal Plume 0.10s after entering the Water Pool taken from Experiment a5.



Figure 5.7: A Close up of a Normal Plume taken from Experiment a6

the water is stagnant this can be provided by a static pressure gradient, hence the depression. After 0.445s the depression has filled, perhaps because the water close to the surface now has a motion downwards so less momentum is imparted to it; the energy transferred by particles entering the pool at this stage must be just sufficient to maintain this motion once dissipation has been accounted for.

Figure 5.8 shows the plume after 0.155s; the head has grown larger and denser. Note the bent air cavities of some of the particles on the left edge of the column which shows there is a significant flow of water into it that prevents the escape of any of the particles.

The head of the plume continues to grow and takes on the crescent shape seen in Figure 5.9 and Figure 5.10. At the centre of the head the particles are close together, but the mean spacing between them increases with distance along the wings. The head grows in length and thickness¹ with penetration. The plumes may be broadly divided into three regions: the head of the plume as defined in Figure 5.1; the wings of the plume which are the regions to the left and right of the plume; and the column of particles following the head (i.e. above it in Figure 5.1).

If the column is defined as the envelope bounded by the paths of the most extreme particles, it is straight and has a constant width similar to that of the jet. The distribution of particles within the column is not at all uniform with conglomerations of particles and voids forming. Some of these can be quite large, for example see the void half way down the column in Figure 5.13 and the conglomeration about half way between it and the top of the tank or near

¹Defined, in Figure 5.1 as the longitudinal distance from the front of the plume to where the column joins the head.

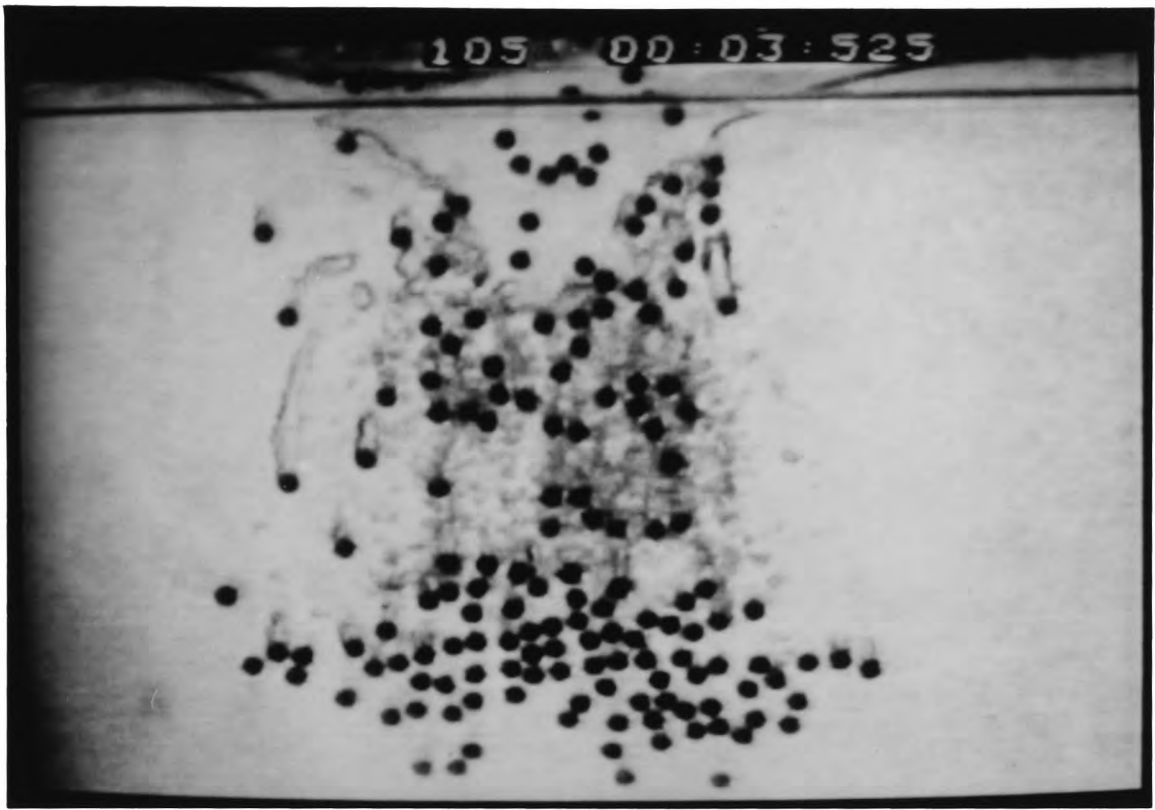


Figure 5.8: A Close up of a Normal Plume 0.3s after Entering the Water Pool taken from Experiment a5.



Figure 5.9: A Normal Plume 0.25s after entering the Water Pool taken from Experiment a2.



Figure 5.10: A Normal Plume 0.45s after entering the Water Pool taken from Experiment a2.

the bottom of the column. The non-uniform distribution of the particles on a smaller scale can be seen in Figure 5.11. The presence of these features does not appear to depend on position relative to the rest of the tank or the plume except near the surface at early times (Figure 5.12) where there is a lower number density of particles than later (compare with Figure 5.11) owing to the crater in the water surface which allows the particles to fall freely further, increasing the separation between them. Note that nearly all the free air bubbles remain within the column.

Figures 5.14 and 5.15 show the plume after it has reached the bottom of the tank; this does not seem to affect the column and wings. The dark shadow at the bottom of each tank walls is algae growing on the tank's floor and pushed sideways and upwards by the water motion caused by the plume.

Photographs of the sets of experiments b–g are shown in figures 5.16, 5.17, 5.18, 5.19, 5.20, and 5.21. On the whole the general features of the normal plumes are replicated; the head looks similar for each plume, but there is some variation in the shape of the column. For example in Figure 5.16, which shows the plume when the projection speed is higher than normal, the conglomerations of particles within the column are large enough for some to protrude from the side of the column. There is a particularly pronounced protrusion in the plume of aluminium particles (Figure 5.19) which is large enough to form a second head similar to the one at the bottom, halfway along the column. However the apparatus often jammed when aluminium particles were used and it is not clear whether this accumulation of particles is natural or caused by a short interruption of the proper working of the apparatus.

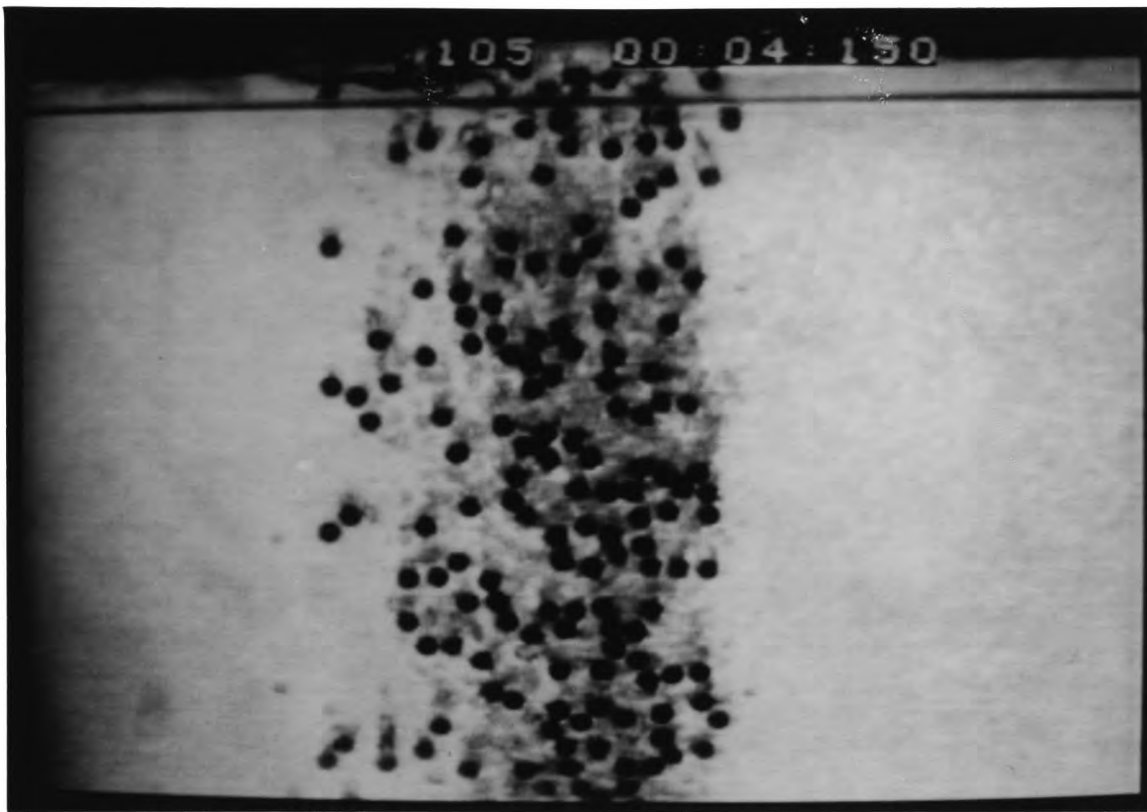


Figure 5.11: A Close-up of the Particle Column taken from Experiment a5 0.78s after the Jet entered the Water Pool.

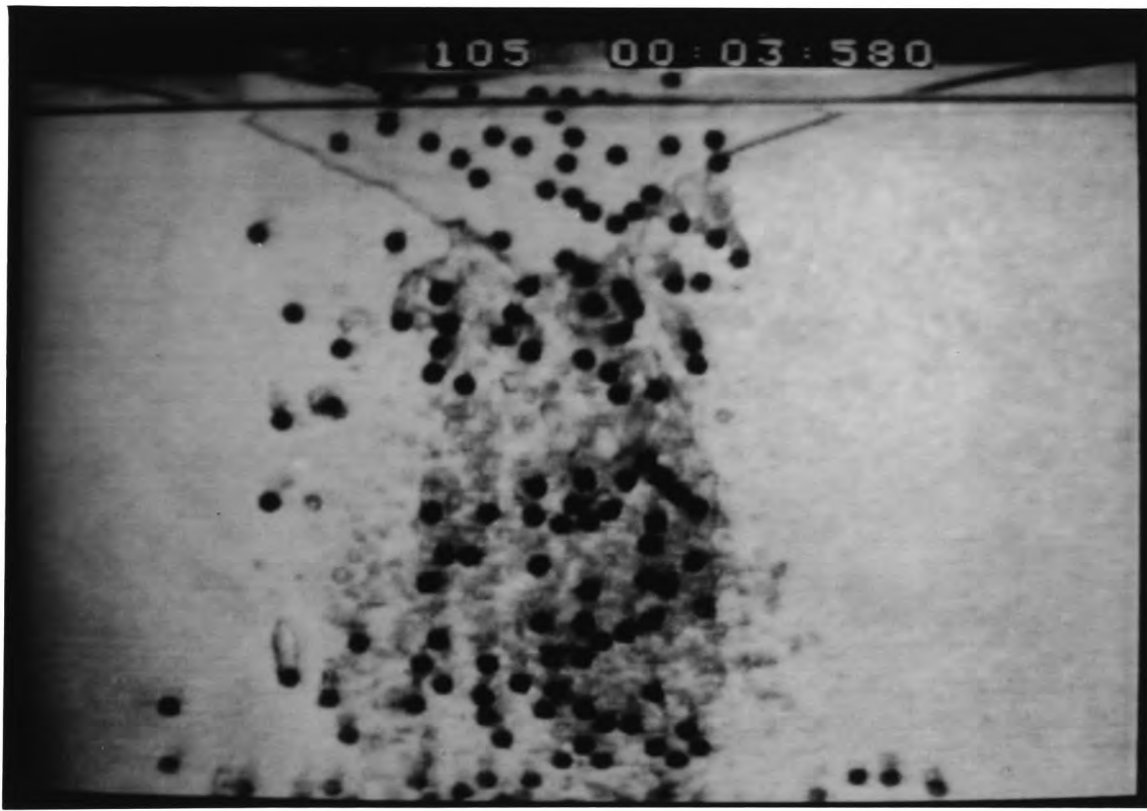


Figure 5.12: A Close-up of the Particle Column taken from Experiment a5 0.21s after the Jet entered the Water Pool. The Head of the Plume is just out of the Frame.

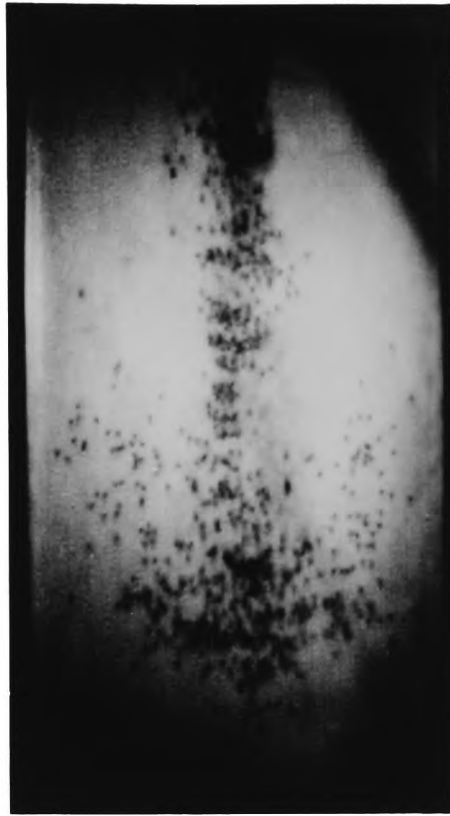


Figure 5.13: A Normal Plume 0.75s after entering the Water Pool taken from Experiment a2.



Figure 5.14: A Normal Plume 0.95s after entering the Water Pool taken from Experiment a2.

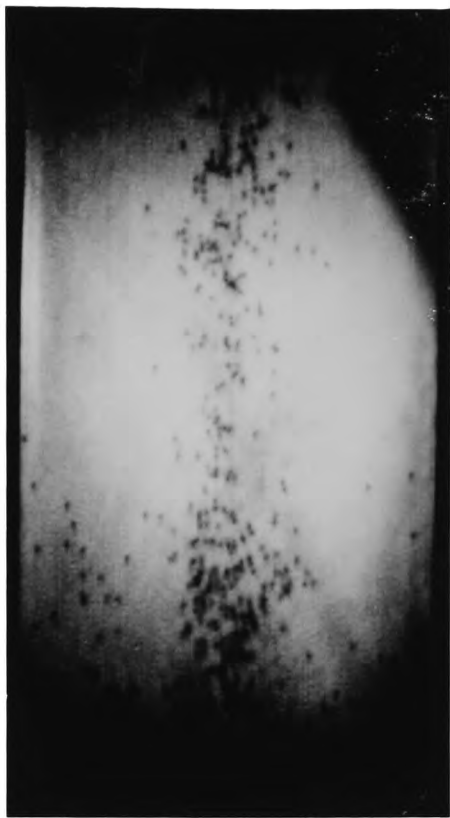


Figure 5.15: A Normal Plume 1.30s after entering the Water Pool taken from Experiment a2.

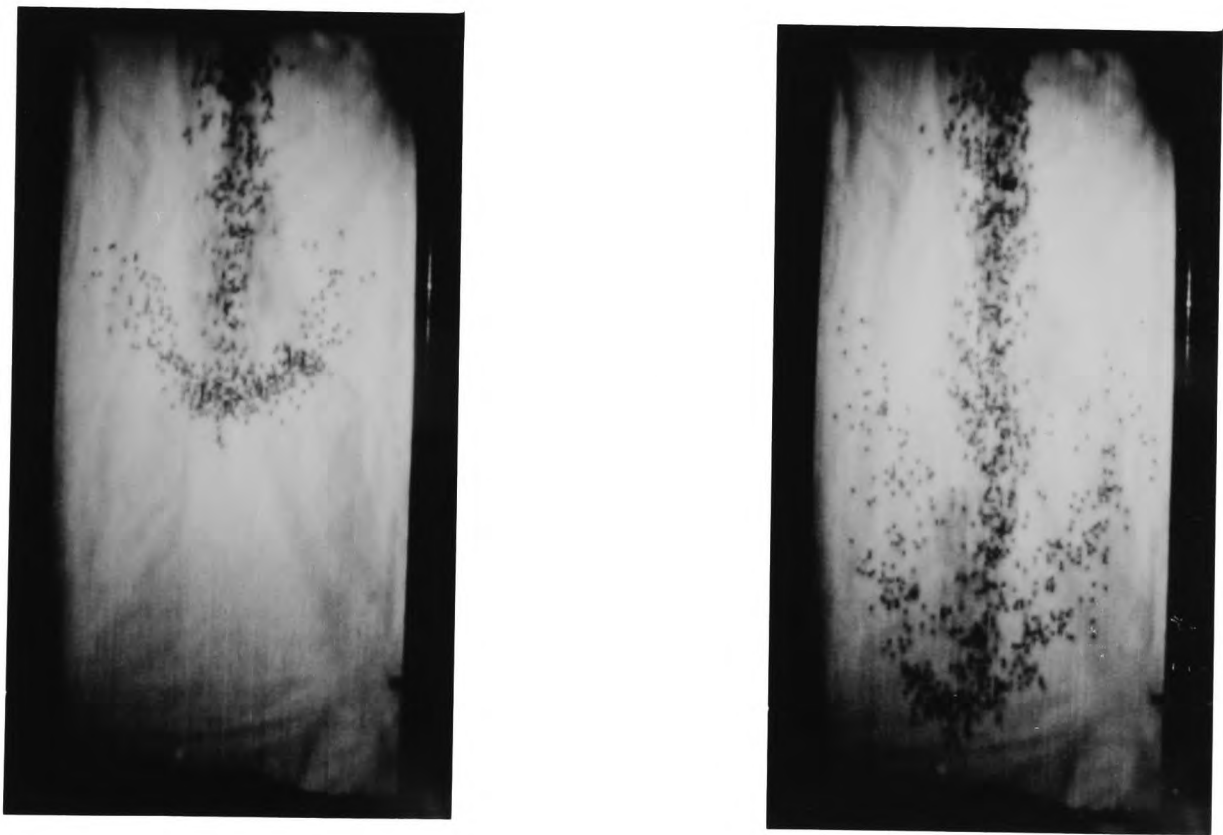


Figure 5.16: Experiment b2 where Particles which have a Higher Projection Speed than those in a Normal Plume after 0.25s and 0.75s



Figure 5.17: Experiment c1 where Particles which have a Lower Projection Speed than those in a Normal Plume after 0.25s and 0.75s

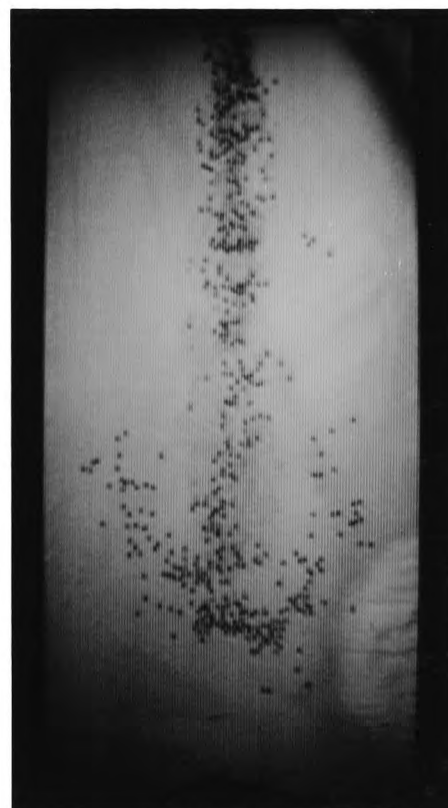


Figure 5.18: Experiment d2 where Particles consisting of Six Columns of Particles after 0.40s and 1.20s

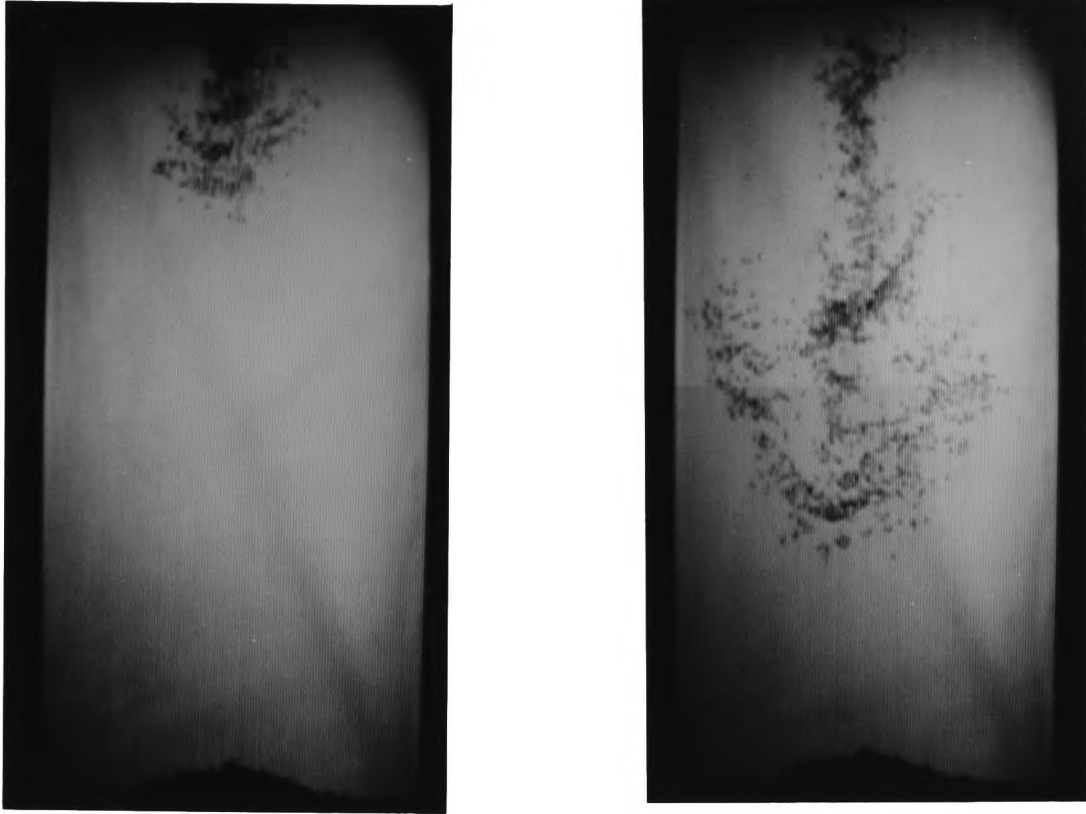


Figure 5.19: Experiment e1 where the Particles are Aluminium, after 1.265s and 2.065s

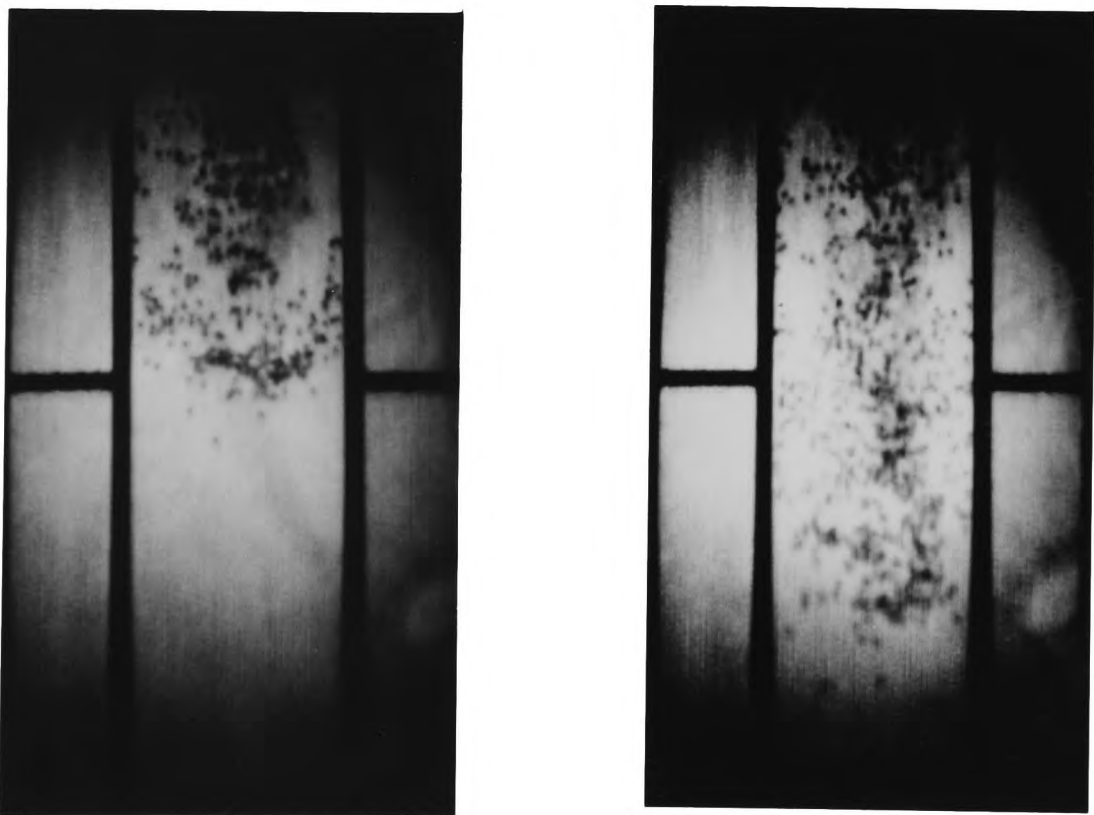


Figure 5.20: Experiment f1 where the Plume is within a Narrow Tank and Ten Columns Wide, after 0.25s and 0.75s



Figure 5.21: Experiment g1 where the Plume is within a Narrow Tank and Six Columns Wide, after 0.25s and 0.75s

5.2 The Jet before entering the Water

After a ball has been released from the projection mechanism it will accelerate under gravity until it enters the water pool. If the projection mechanism is running at a constant speed the spacing between subsequent particles will increase as they fall owing to their acceleration. This can be seen in Figure 5.22 which shows the change in the separation of two successive rows of particles as they fall. The motion of the particles was determined by integrating a simple equation where acceleration is equal to the difference between drag and buoyancy i.e.

$$\frac{\pi}{6}d_p^3\rho_p\frac{du_p}{dt} = \frac{\pi}{6}d_p^3(\rho_p - \rho_w)g - \frac{\pi}{8}C_Dd_p^2\rho_wu_p^2. \quad (5.1)$$

The influence of surrounding particles was neglected.

At early times the projection mechanism is accelerating and the difference between the speed of the first few rows of particles when they are released is sufficient for one row to catch up another. To test the importance of this the paths of the particles were calculated as they fell through the air. The equations of motion of a body under a constant acceleration used. The result is shown in Figure 5.23 The initial speed of the particles was taken from the measurements of the transient behaviour of the motor (Section 4.3.2). A collision occurs when the separation between particles is less than their diameter (6mm in this case). It was assumed to be elastic so that the front particle acquires all the excess momentum possessed by the particle that has hit it and kinetic energy is conserved.

The transient motion might have two unfortunate consequences:

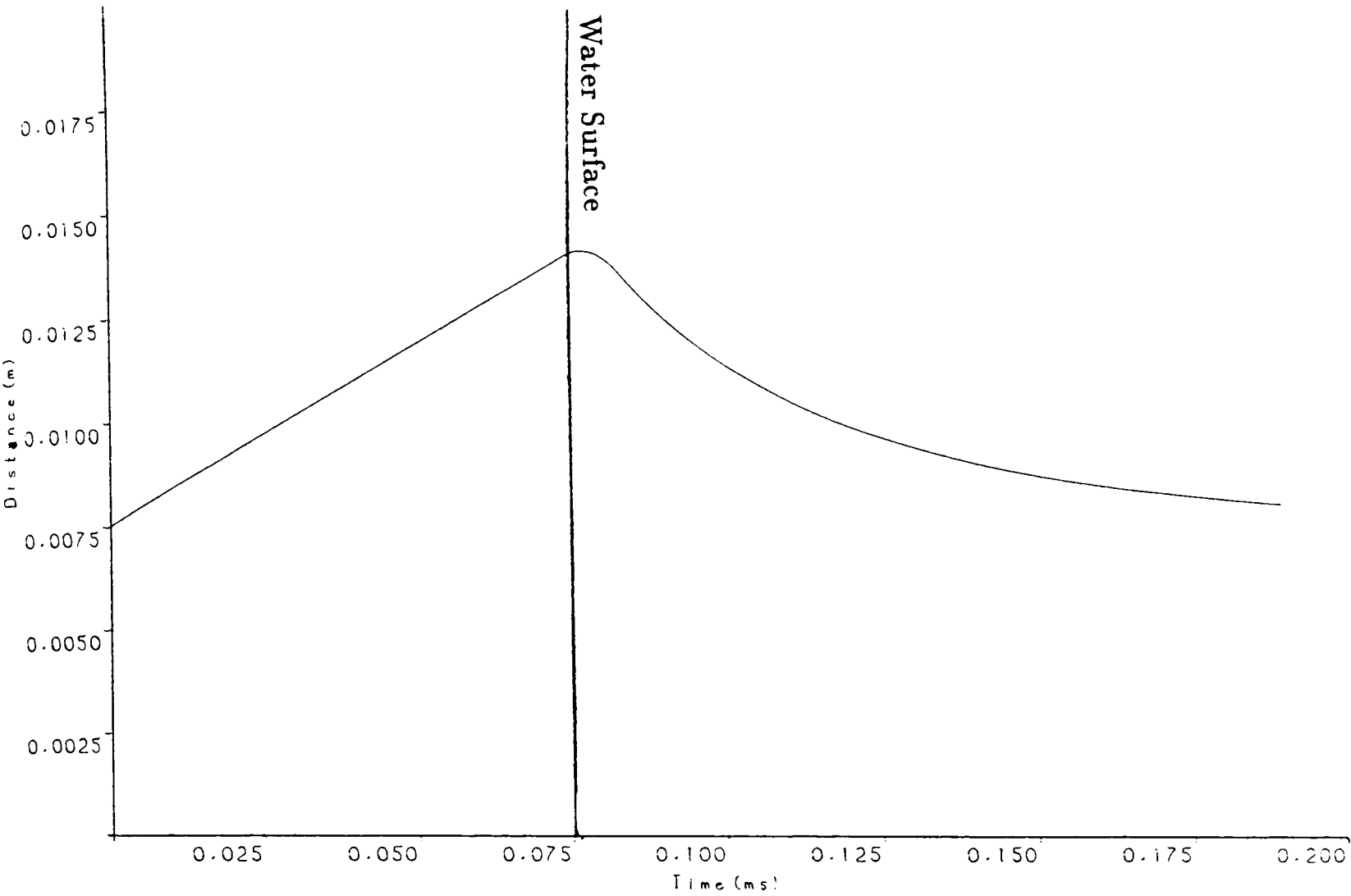


Figure 5.22: Prediction from Equation 5.1 of Separation between Two Adjacent Rows of Particles in a Normal Plume.

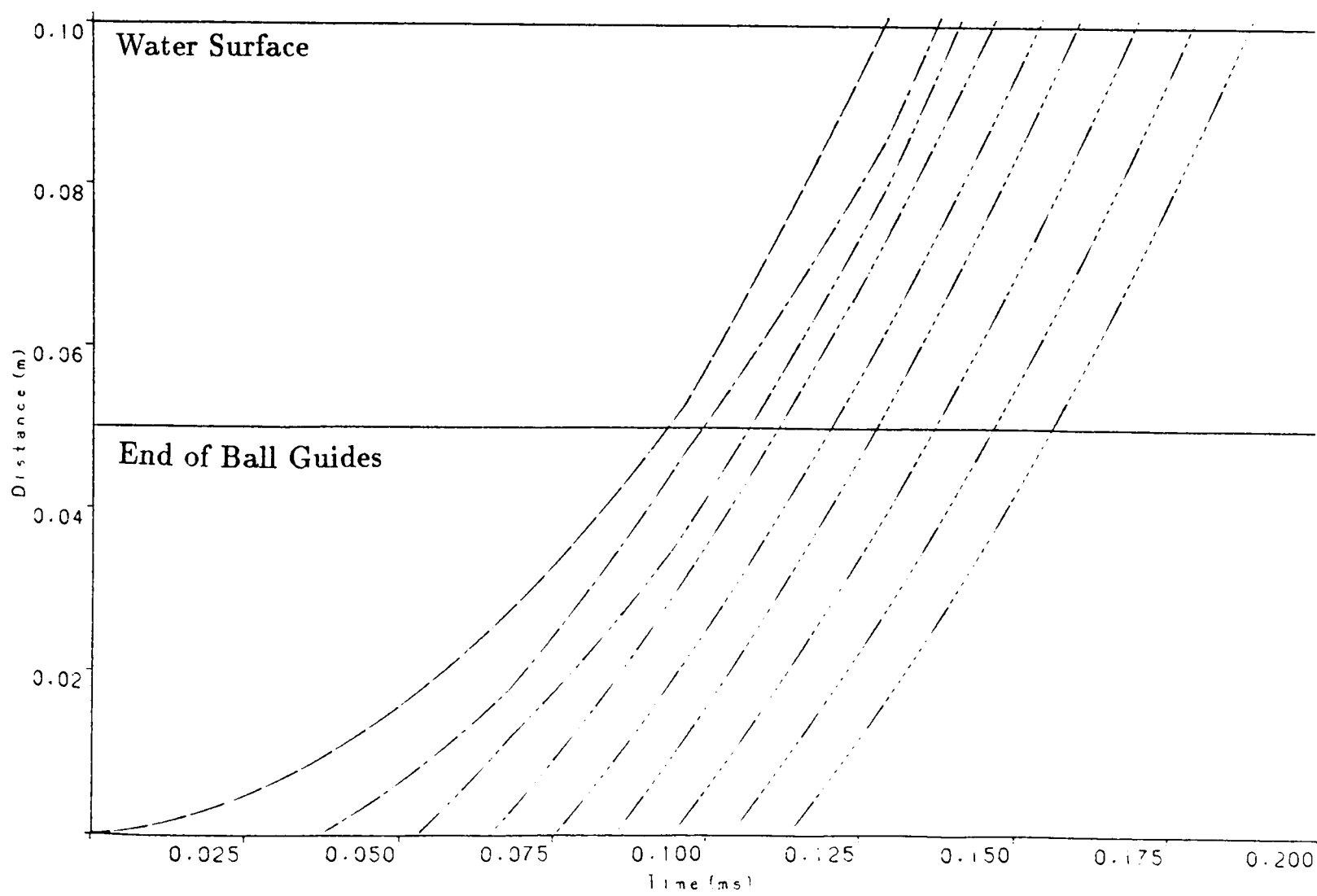


Figure 5.23: The Predicted Motion of the First Rows of Particles in a Normal Plume at Early Times

- particles may collide with each other causing their paths to deviate sideways;
- the particles may be artificially concentrated at the front of the plume.

The first consequence does not appear to be a major problem. Only the first four rows of the particles are involved in collisions. All but one of the collisions take place within the guides which prevent the particles moving laterally. In addition the paths of the early particles observed in the experiments are not grossly non-vertical and the early plume has a square outline (see Figures 5.3 and 5.4).

The second consequence is unlikely to cause any new processes. The spacing at the water surface between the first six rows of particles is reduced owing to the transient behaviour of the motor. The spacing is, from the front of the plume, 14.5, 6.2, 8.4, 12.2, and 10.2mm compared with 15.5mm when the projection mechanism is running steadily. If the particles are projected with a steady speed then from the solution of (5.1) the separation between successive rows is 7.98mm 120ms after entering the water pool. The separation changes little with projection speed: if the speed is increased to the value given in row b of Table 4.1, the separation after the same time is 8.05mm; if it is decreased to the value given in row c the separation is 7.95mm. Only one of the spacings between rows is smaller than this; therefore though the bunching of the first few rows of particles is only hastening the time at which they reach their equilibrium spacing. In fact the particles are unlikely to achieve this spacing because, as has been shown in experiments with cylinders [73] and spheres [74], [75] if it has a high Reynolds number, a particle within the wake of another experiences much less drag than if it were outside it. This will likely to dominate the motion of any group of particles once they have entered the water if they are reasonably close together. For these reasons effect of the transient behaviour of the motor on the overall behaviour of the plume is negligible.

Chapter 6

The Measurement of the Overall Characteristics of the Plumes

This chapter describes the measurement of the overall characteristics of the plumes and the plumes whose development is different from that commonly seen.

6.1 The Method of Measurement

The measurement of the overall behaviour of the plume was made using the MOVIAS system, manufactured by NAC, in conjunction with the high speed video recorder. The positions of any number of points on successive frames of a video recording, separated by some given time interval, are defined and stored in a computer by an operator. MOVIAS can then plot the particle positions in various ways and calculate quantities derived from them, such as the distance between two points. The system determines the velocity of a point by adding the distance moved by the point since the last frame examined, to the distance that it will move before the next, and dividing by the time between these two frames. To assign positions to the points selected the system effectively divides the video frame into an array of pigeon holes: the position of any feature or particle is recorded as the coordinates of the pigeon hole in which it lies. The error in measurement will then equal the size of the pigeon holes. There are 640 divisions along the longer side of the frame and 512 along the other which results in a resolution error, in this case, of $\pm 1.0\text{mm}$ longitudinally and $\pm 0.9\text{mm}$ laterally. Because of the camera resolution the actual total resolution error in the lateral direction is $\pm 2.6\text{mm}$.

To measure the overall characteristics of a plume points were allocated to prominent positions at its edge. Three quantities were measured:

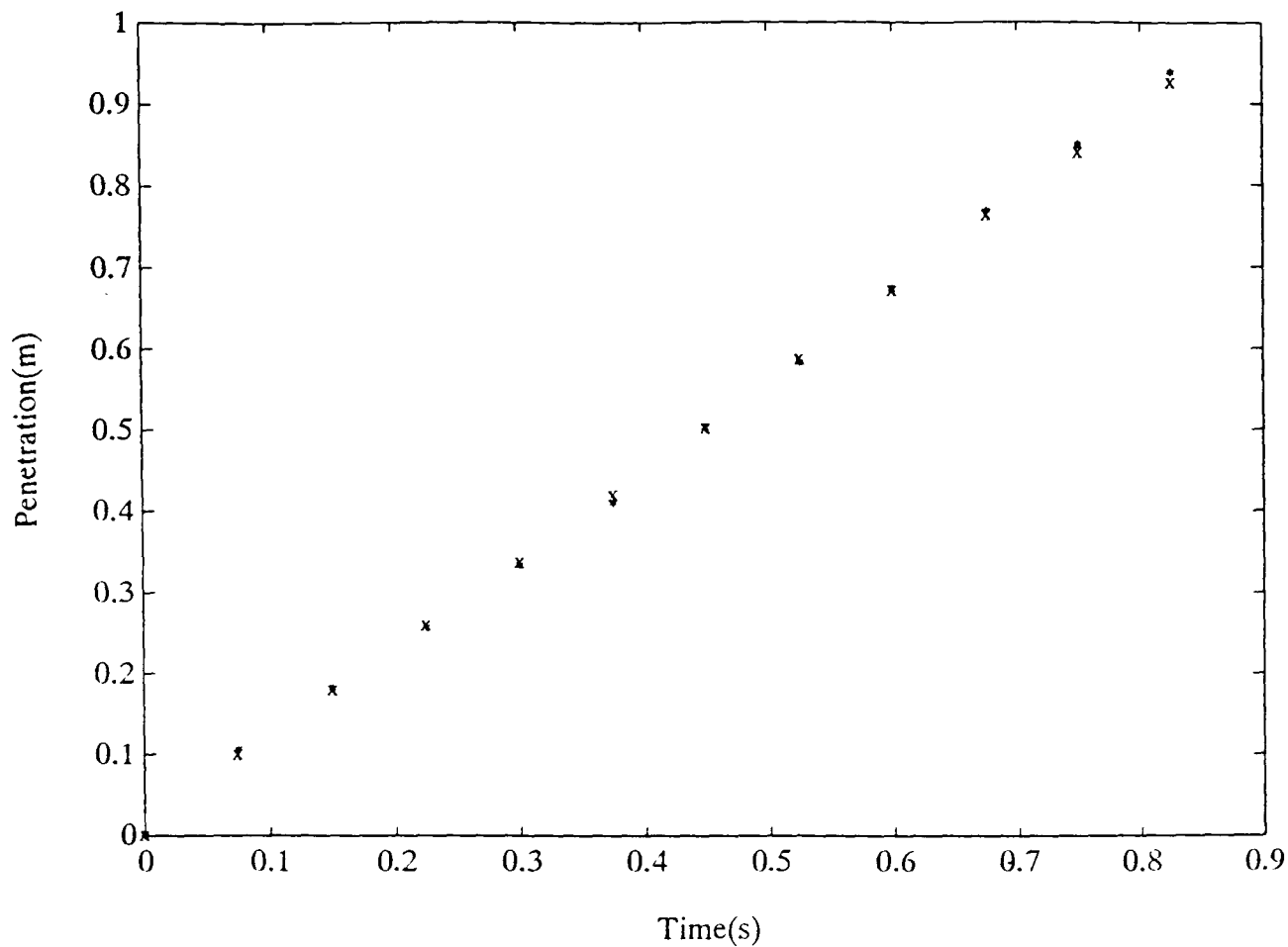


Figure 6.1: Repeatability of the Estimation of the Penetration of the Plume in Experiment a3.

- the plume's penetration (the longitudinal distance from the top of the tank to the leading edge of the plume.),
- the plume's width (the lateral distance between its extremities),
- the head length (the longitudinal distance from the leading edge of the plume to the mean position of the top of the wings).

A problem was that the location of the edge of the plume depended somewhat on the judgement of the person who was doing it: the extent of the plume can only be judged from the distribution of the particles; it does not present a continuous, well defined edge, particularly in areas where there is large spacing between particles. In addition a judgement had to be made when single particles, or groups of them, moved independently of the main plume. To evaluate this error the measurement of a plume was repeated. The results are shown in figures 6.1, 6.2, and 6.3; the resultant variation (which includes the resolution error) depends upon the quantity that is being measured. In the case of penetration the maximum error was 12mm and therefore negligible compared with the magnitude of the penetration which is of the order of hundreds of millimetres. The edges of the plume were more difficult to fix firmly owing to the smaller concentration of particles at the sides but the maximum error of the width of the plume was 22mm which is only 6% and still acceptable. The problem of defining the edges of the head

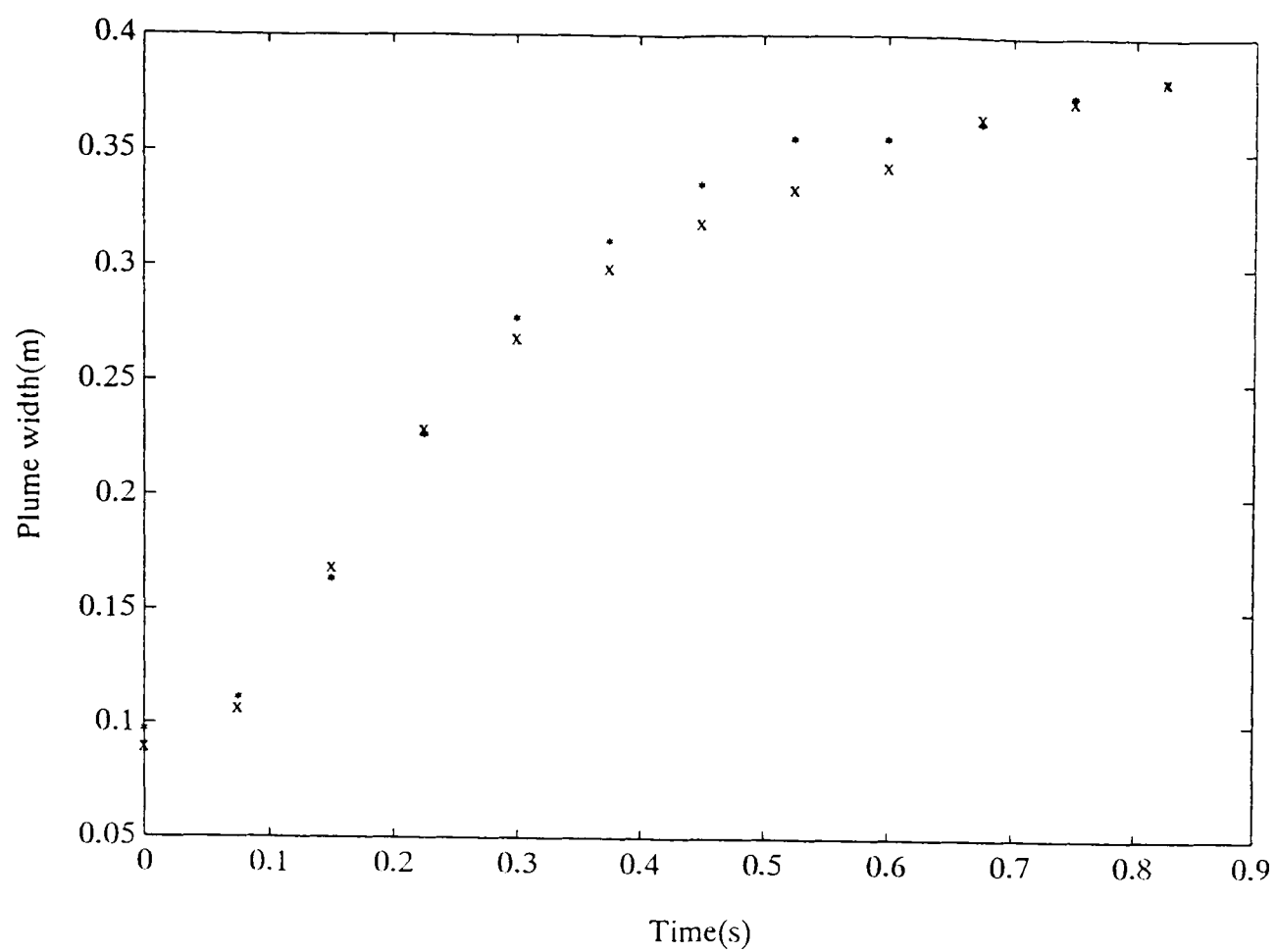


Figure 6.2: Repeatability of the Estimation of the Width of the Plume in Experiment a3.

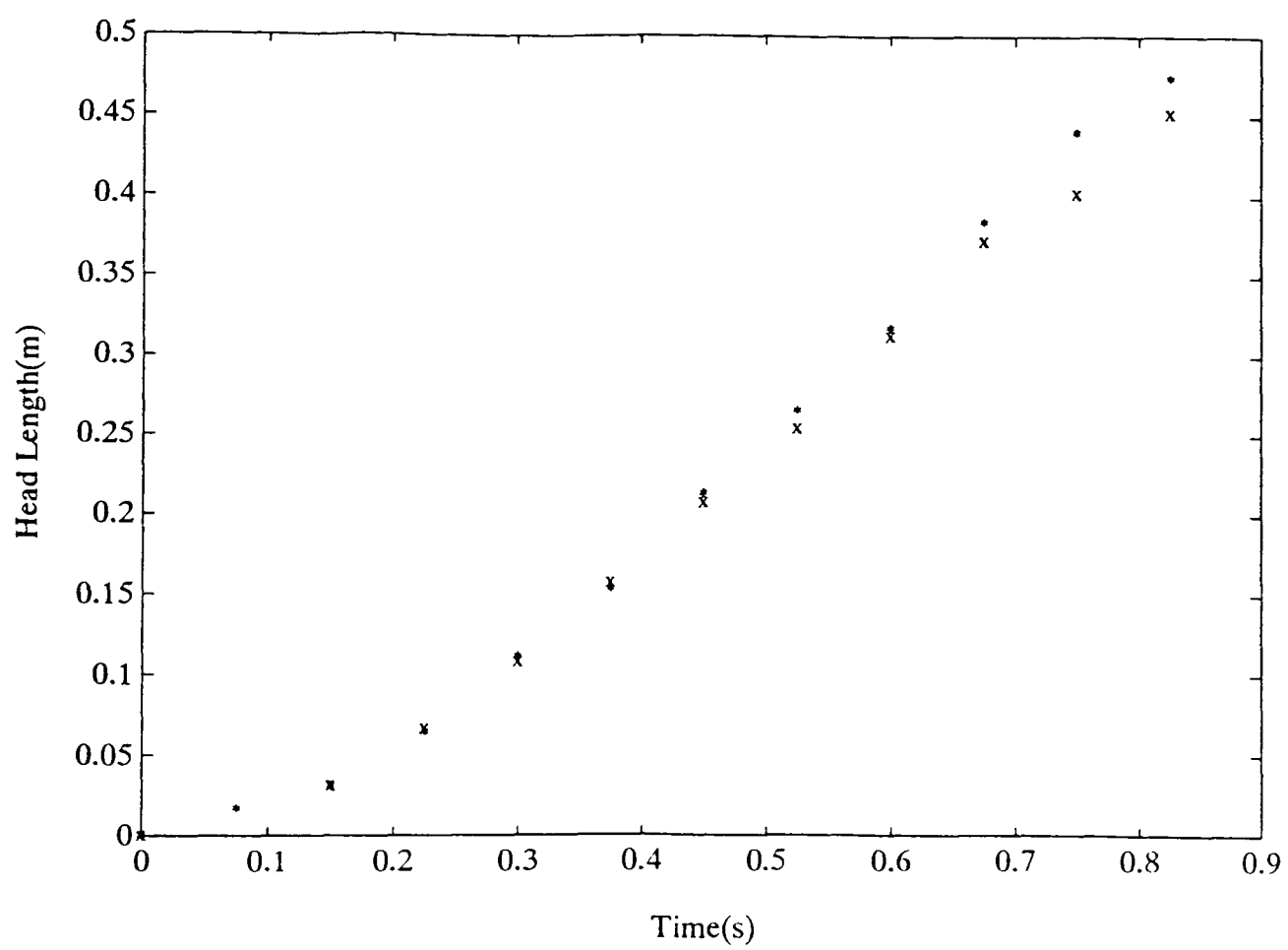


Figure 6.3: Repeatability of the Estimation of the Length of the Head of the Plume in Experiment a3.

was worse when measuring its length, particularly at later times, because of the low number density of particles within the wings. The largest error was 38mm or 10% .

6.1.1 Water Velocity Measurement

It would have been desirable to measure the water velocity in the tank; unfortunately this was not possible. Point measurements, with a laser doppler anemometer for example, would not be useful because the flow at any one point will be unsteady, and no one place is representative of others in the tank. To overcome this difficulty, and to indicate the flow over an area, some form of flow tracer might have been used such as hydrogen bubbles generated locally by electrolysis, or neutrally buoyant particles. However tracer particles have to be very small (less than a millimetre in diameter) otherwise they will not follow the local movement of the water owing to their inertia, and will significantly affect the flow that they are being used to measure. If this restriction is satisfied and the tracing particles are to be visible then only a small area of the plume can be filmed and the situation would not be greatly improved from that if point measurements were taken. An experiment, under normal conditions, was conducted when a flow indicator¹ was mixed with the water in the tank. The indicator works through its optical properties changing if the fluid it is mixed with moves. It proved to be useful for indicating the presence of turbulence, but that swamped the indicator and no other water movement could be discerned.

6.1.2 The Repeatability of the Experiments

A separate matter from the repeatability of the measurement of the plume is the repeatability of its features in different plumes.

Photographs of four normal plumes are shown in Figure 6.4, the penetration of the front of the normal plumes against time in Figure 6.5, the variation in width with time in Figure 6.6, and the variation of the head length with time in Figure 6.7.

The greatest scatter of measured penetration was $\pm 49\text{mm}$ (or 6%), of width $\pm 16\text{mm}$ (or 7%), and of head length $\pm 30\text{mm}$ (or 10%). These are small and the overall appearance of different plumes is very similar. Some variation of the overall characteristics of the plumes in different experiments might be expected because the overall character of the plume is a measure of the overall behaviour of the particles which constituted it; their motion is somewhat random and therefore their collective behaviour will be as well. If particular regions within the plumes shown

¹This is called Mearlmaid and is produced by the Cornelius Chemical Company of Romford, Essex.



Figure 6.4: Photographs showing the Four Normal Plumes after 0.45s. Moving clockwise from the Top Left the Experiments are a1, a2, a4, and a3.

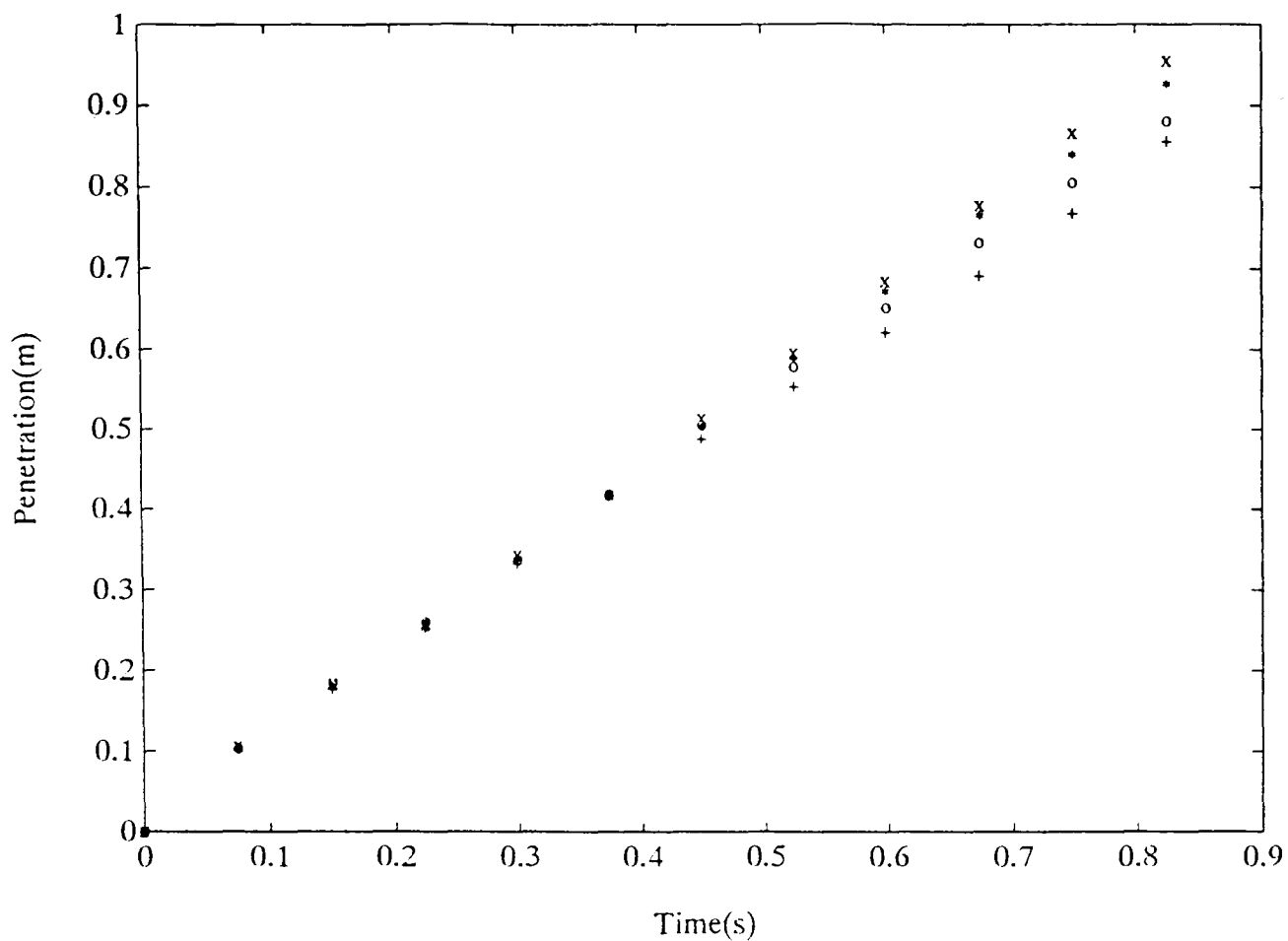


Figure 6.5: The Variation of Jet Penetration with Time for the Four Normal Plumes. x is Experiment a1, o is Experiment a2, * is Experiment a3, and + is Experiment a4.

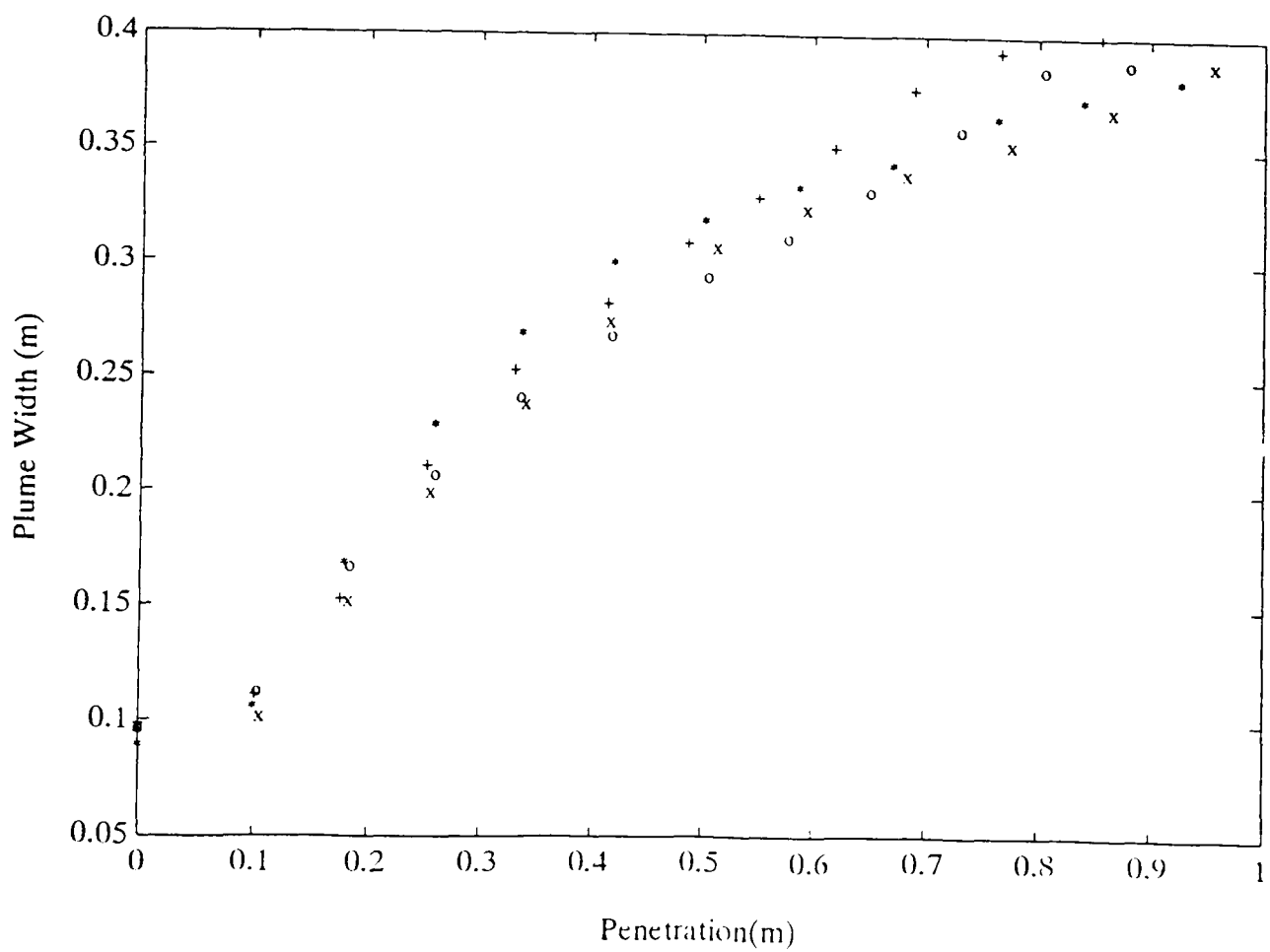


Figure 6.6: The Variation of the Overall Plume Width with Time for the Four Normal Plumes. x is Experiment a1, o is Experiment a2, * is Experiment a3, and + is Experiment a4.

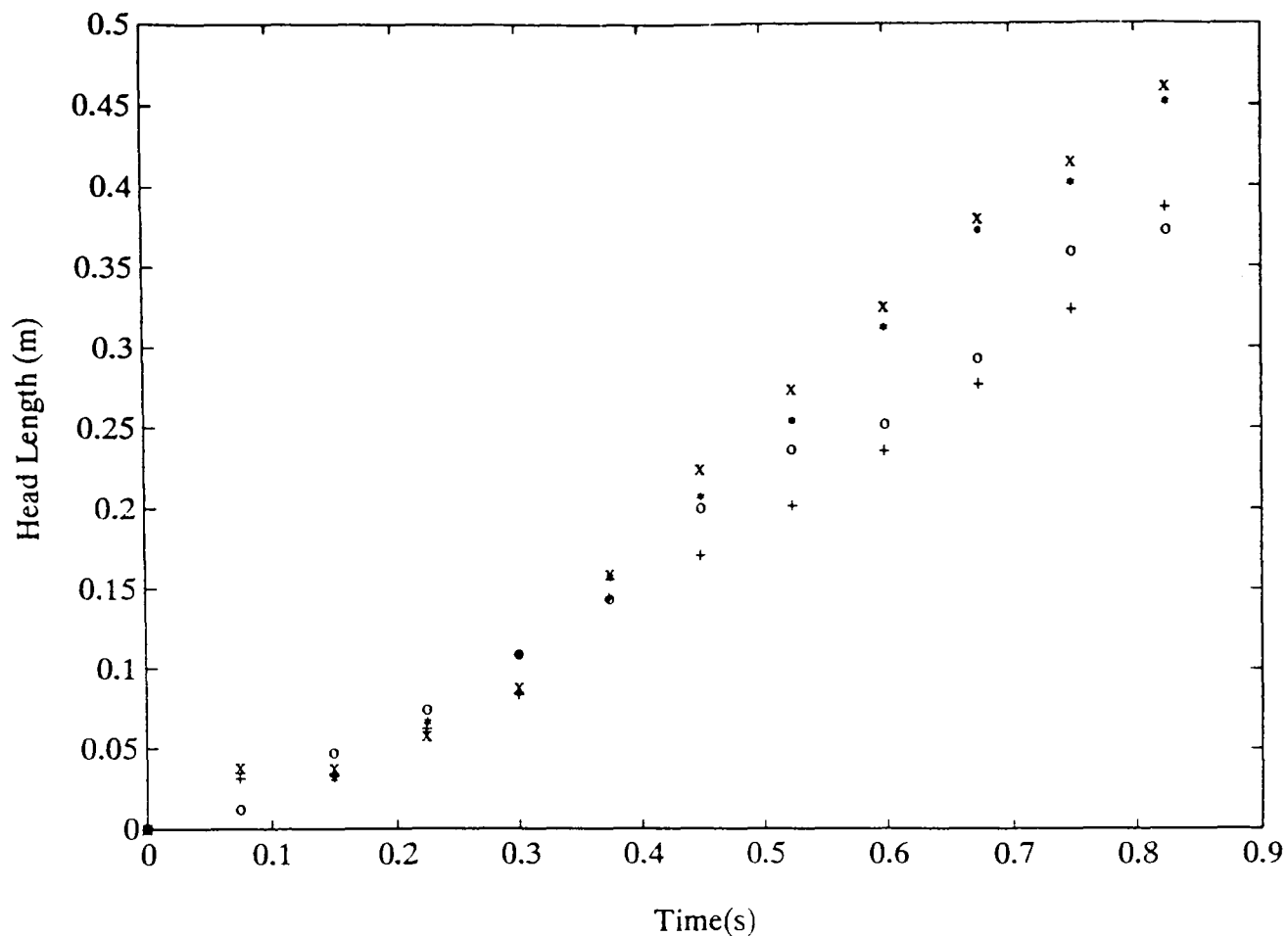


Figure 6.7: The Variation of the Head Length with Time for the Four Normal Plumes. x is Experiment a1, o is Experiment a2, * is Experiment a3, and + is Experiment a4.

in Figure 6.4 are compared, the distribution of particles is different in different plumes. The situation is analogous to the distribution of gas molecules within a sealed box: the direction of travel of each molecule within the box is random. There is an extremely small probability that at any time the particles' motion will be such that they all accumulate at one end of the box; but in fact in nearly all cases the distribution of molecules will be indistinguishable in terms of their collective characteristics. Similarly, of the possible configurations that the particle plumes may take there is one which will be more probable than the others. The arrangement of the particles within a plume is not as an extreme case as that of the gas molecules, and sometimes different configurations of particles can be obtained; for an example compare the shape of the two plumes where the initial jet width was reduced (set of conditions d) shown in Figure 6.8. An alternative explanation might be that there are several different, distinct states that the plume can have, rather than variations from a most probable one. However overall the plumes took the form of the right hand plume shown in Figure 6.8 and the scatter of their measured characteristics was small so if different, distinct states can exist they are not often achieved.



Figure 6.8: Photographs after 0.75s of the Two Plumes where the Initial Jet Width was Reduced, Experiments d1 and d2, showing the Different Configurations Possible.

6.1.3 Dimensional Analysis

Three dimensions of the plume are measured. If it is assumed that the air bubbles in the plume do not affect these lengths and their properties are neglected, the variables that the measurements might depend upon are:

U_j - the speed of the particle jet at the water surface;

ρ_p - the density of the particles;

d_p - the diameter of the particles;

ρ_w - the density of the fluid in the tank;

μ_w - the viscosity of the fluid;

t - time;

g - acceleration owing to gravity;

n - the number of particles in a row;

x - the separation of the particles in a row;

y - the separation of rows;

d_t - the width of the tank;

l_t - the depth of the tank.

The subscript 'j' denotes a condition at the water surface. Because the experiments are for comparison with CHYMES, it would be convenient to express the characteristics of the jet in the same manner as it. The variables CHYMES specifies which are different from those above are jet width, which may be expressed as

$$d_j = nx, \quad (6.1)$$

and particle volume fraction, which may be expressed as

$$\alpha_{p,j} = \frac{\pi d^3/6}{xy\tau} \quad (6.2)$$

where τ is the thickness of the tank. If one of the balls is moving in water then provided it is moving with a speed relative to the surrounding fluid greater than about 90mm/s, the drag exerted on it will be independent of the water's viscosity. When falling steadily isolated particles have a speed about 0.8m/s relative to the surrounding water; therefore it is assumed that a particle's motion is independent of the water's viscosity. Because the motion of the plume is determined by the collective motion of the particles, it follows that it is also independent of fluid viscosity.

If 'l' is one of the measured dimensions of the plume, then

$$l = f(U_j, \rho_p, \rho_w, t, g, d_j, \alpha_{p,j}, d_t, d_p). \quad (6.3)$$

If the dimension of mass is eliminated then

$$l = f(U_j, \rho_p/\rho_w, t, g, d_j, \alpha_{p,j}, d_t, d_p). \quad (6.4)$$

The dimension of time can be eliminated by forming one, dimensionless group, tg/U_j ; or two of the three groups $U_j t$, gt^2 , and U_j^2/g all of which have the dimension of length; that may be eliminated using any one of the lengths detailed in (6.4).

The selection of the scales to eliminate time and length has to be made by using physical or theoretical evidence. In addition dimensional analysis cannot indicate the functional relationship between the variables for which again additional evidence must be used.

6.2 The Measurements

6.2.1 Penetration

The penetration of the plumes with time is shown in Figure 6.9. Straight lines may be fitted

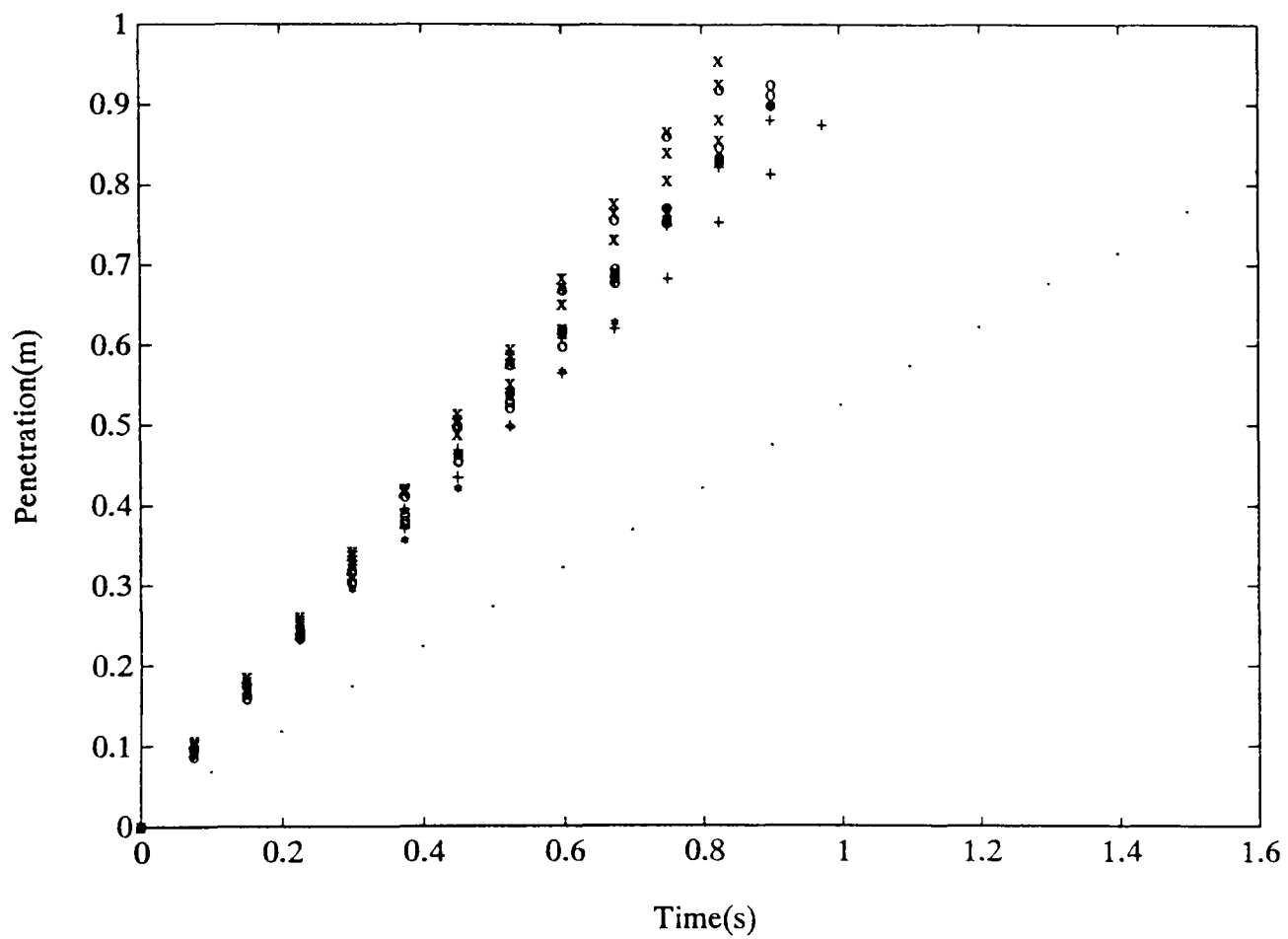


Figure 6.9: The Penetration of Plumes plotted against Time: x=normal plumes (experiment set a); o=plumes with a different projection speed (experiment sets b and c); +=thin plumes (experiment set d); ·=aluminium particle plume (experiment set e); *=plumes in a narrow tank (experiment sets f and g).

to each plume's line of data points with a high degree of confidence. The gradient of each of these lines will be equal to the speed of descent of the plume, which means that they move steadily. The speeds of descent are detailed in Table 6.1 with the maximum deviations within 95% confidence limits. With the exception of the plume of aluminium particles the penetration velocity appears to be not strongly dependent on any of the initial or boundary conditions; there may be a weak dependency on the jet width.

If a body, such as the plume, is falling steadily through stagnant water, the buoyancy of the body is balanced by the drag on it² i.e. if quantities associated with the body are denoted by the subscript 'b':

$$(\rho_b - \rho_w)V_b g = \frac{1}{2}\rho_w A_b C_D U_b^2 \quad (6.5)$$

where V_b is the volume of the body and A_b is its projected area. Let $V_b/A_b = kH$ where H is a measure of the depth of the body and k is a geometrical factor. Hence

$$U_b^2 = 2g \left(\frac{\rho_b}{\rho_w} - 1 \right) \frac{kH}{C_D}. \quad (6.6)$$

The group of variables kH/C_D depend upon the geometry of the body.

If the body is now specified to be the plume so that all quantities with the subscript 'b' are equivalent to the appropriate quantities associated with the plume and denoted by the subscript 'pl', because the speed of the plume is constant

$$U_{pl} = \frac{x_{pl}}{t} \quad (6.7)$$

where x_{pl} is the distance penetrated by the plume. Hence, if the geometry of the body is assumed to be unchanging, if (6.7) is combined with (6.6)

$$x_{pl} \propto (t\sqrt{g}) \sqrt{\frac{\rho_p}{\rho_w} - 1} \quad (6.8)$$

is obtained. This is plotted in Figure 6.10 for a normal plume and one composed of aluminium particles (condition set e); the agreement between the two is very good. If a single line is fitted to all the data then the downwards speed of any plume of stainless steel particles is 1.00 ± 0.02 m/s with 95% confidence limits. This compares with the terminal speed of an isolated particle of 0.84m/s.

(6.8) may be converted to an equation of dimensionless groups by dividing both sides by an appropriate length scale. For the plumes described here it does not matter which of the length scales is used because it can be cancelled entirely from (6.8). However changing a length scale

²In other words the body forces are balanced by the surface forces

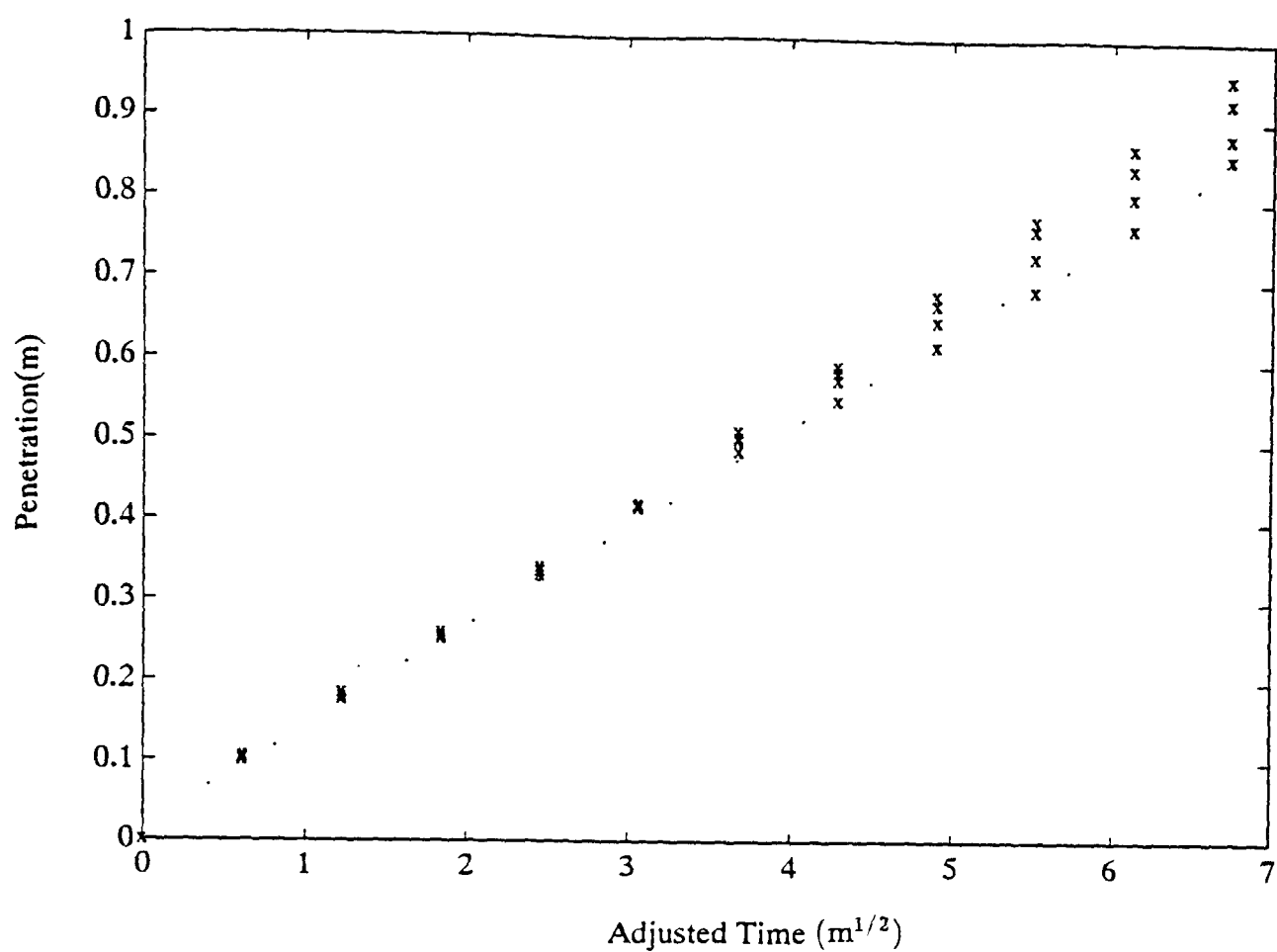


Figure 6.10: The Penetration of Plumes plotted against Time adjusted as in (6.8): x=normal plumes (experiment set a) and . =aluminium particle plume (experiment set e).

Set of Conditions	Velocity at the Water Surface (m/s)	α_p at the Water Surface	Jet Width (m)	Tank Width (m)	Particle Density (kg/m ³)	Average Velocity (m/s)	Maximum Variation from the Average Velocity with 95% Confidence(%)
a	1.70	0.10	0.081	0.475	7800	1.08	3
b	1.00	0.08	0.081	0.475	7800	1.01	1
c	1.96	0.16	0.081	0.475	7800	1.06	5
d	1.70	0.10	0.054	0.475	7800	0.93	6
e	1.70	0.10	0.081	0.475	2700	0.99	1
f	1.70	0.10	0.081	0.215	7800	0.98	8
g	1.70	0.10	0.054	0.215	7800	0.98	3

Table 6.1: The Speed of Penetration of the Plumes. The Speed of the Plume in Experiment Set e has been Corrected for Density as in (6.8).

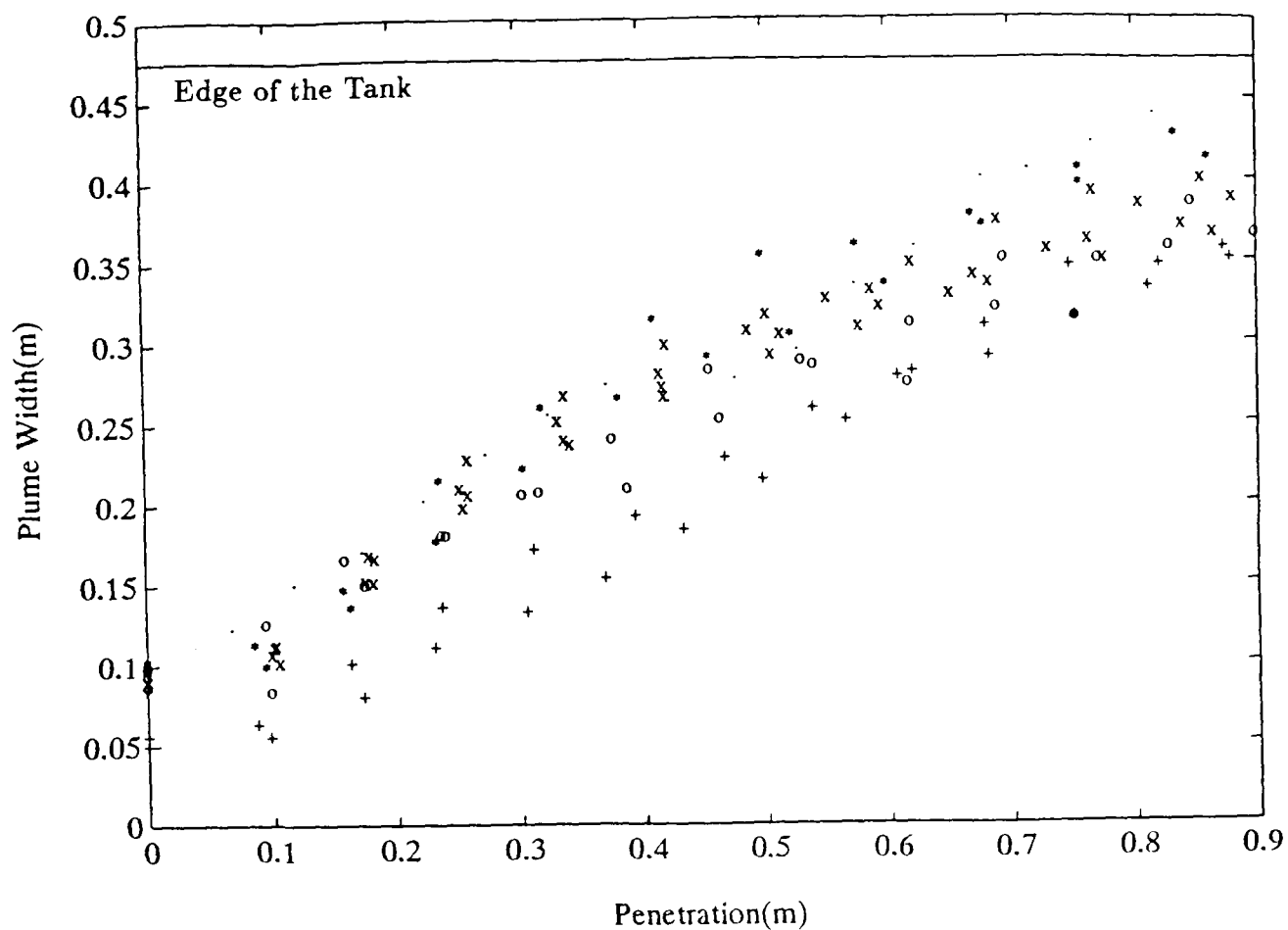


Figure 6.11: The Variation of Overall Plume Width with Distance Penetrated by the Plume: x=normal plumes (experiment set a); o=plumes with a higher projection speed (experiment set b); *=plumes with a lower projection speed (experiment set c); +=thin plumes (experiment set d); ·=aluminium particle plume (experiment set e).

by an amount that exceeds the conditions described here, or changing the particle diameter at all, might influence the value of the geometrical variables. In such a case the length scales would be more important.

6.2.2 The Width of the Plume

The maximum instantaneous width of the plumes is plotted against penetration in Figure 6.11, in the case of plumes where the whole width of the tank was used (experiments sets a–e), and Figure 6.12, for plumes where the tank width is reduced (experiment sets f and g). Two regimes might be discerned: one over about the first 0.1s where the rate of spread is small, and a second, after 0.1m, where the plume grows until it tends to some steady value as it approaches the tank wall. Of the conditions tested, the widths of the jet and (experiment sets d, f, and g) had the most effect, both restricting the rate of growth of the plumes. No clear trends are apparent from the changes in other parameters. More experiments would be necessary to say anything explicit about the effect of these parameters except that a large change in the particle density had little effect on the spreading of the plume.

6.2.3 The Length of the Head

The change in the length of the head of the plume is shown in Figures 6.13 and 6.14. Again

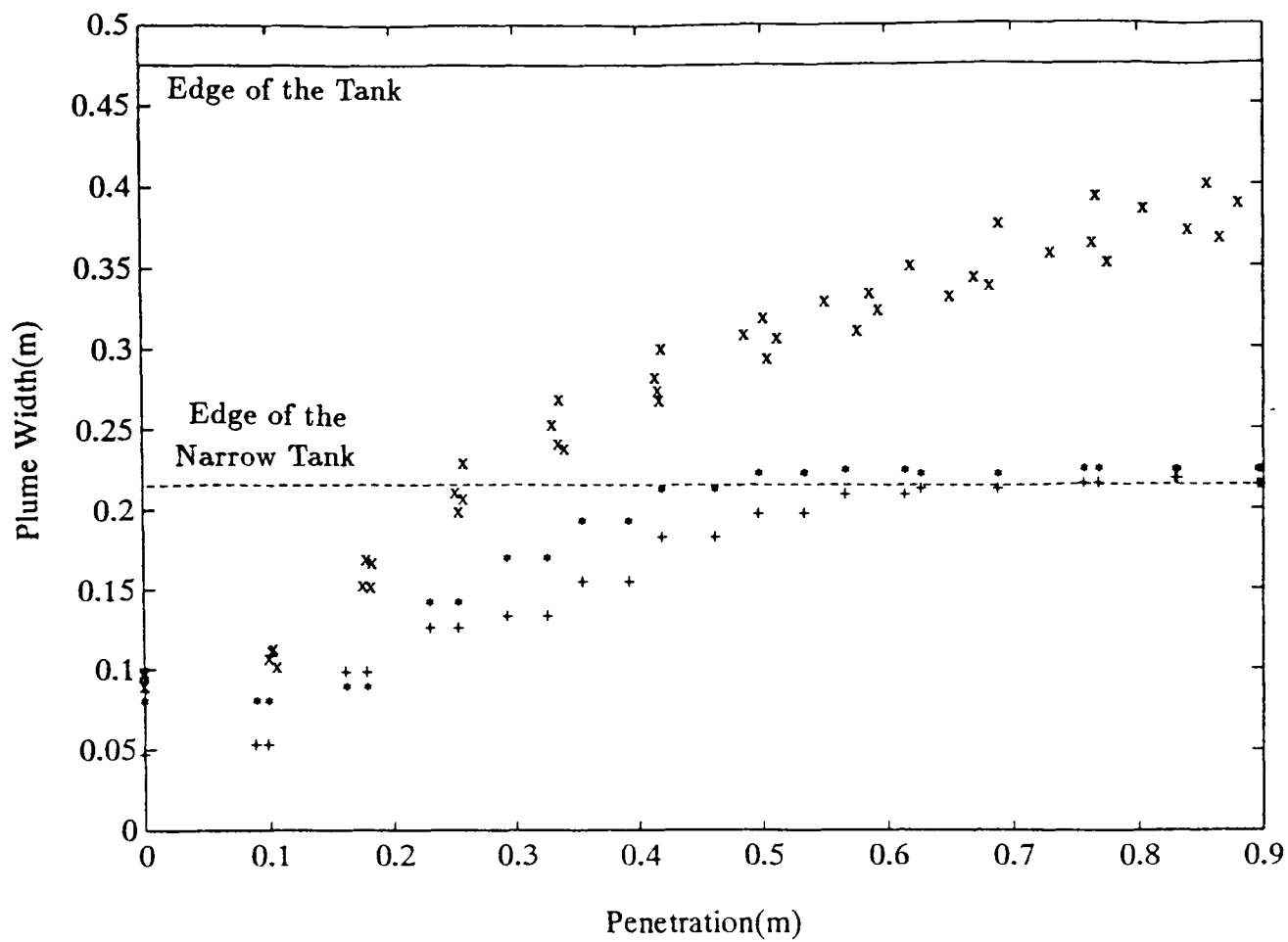


Figure 6.12: The Variation of the Plume Width with Penetration: x=normal plumes (experiment set a); *=thick plume in a narrow tank (experiment f); +=thin plume in a narrow tank (experiment g).

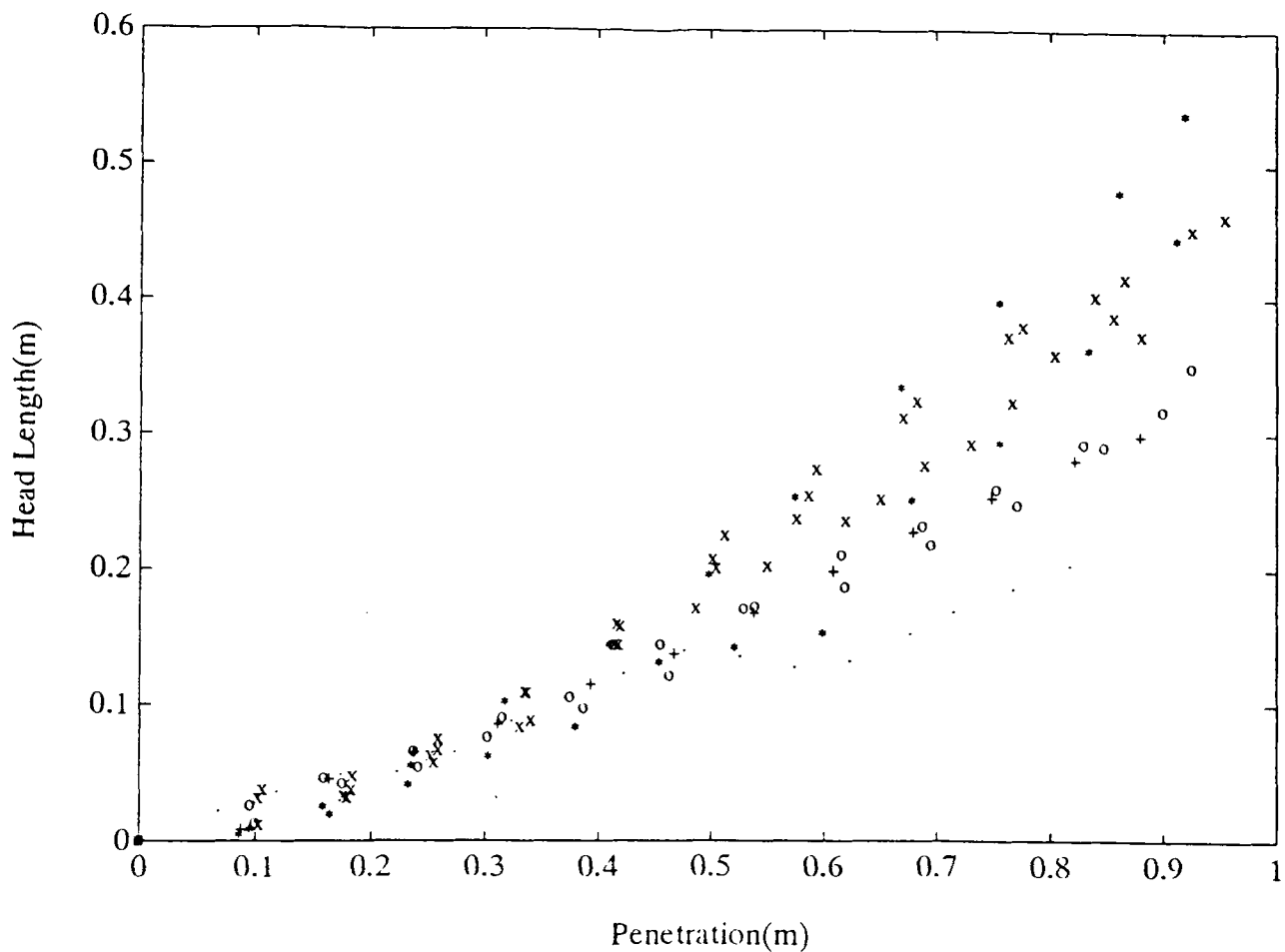


Figure 6.13: The Variation of Head Length with Penetration. x=normal plumes (experiment set a); *=plumes with a higher projection speed (experiment sets b); o=plumes with a lower projection speed (experiment set c); +=thin plumes (experiment set d); ·=aluminium particle plume (experiment e).

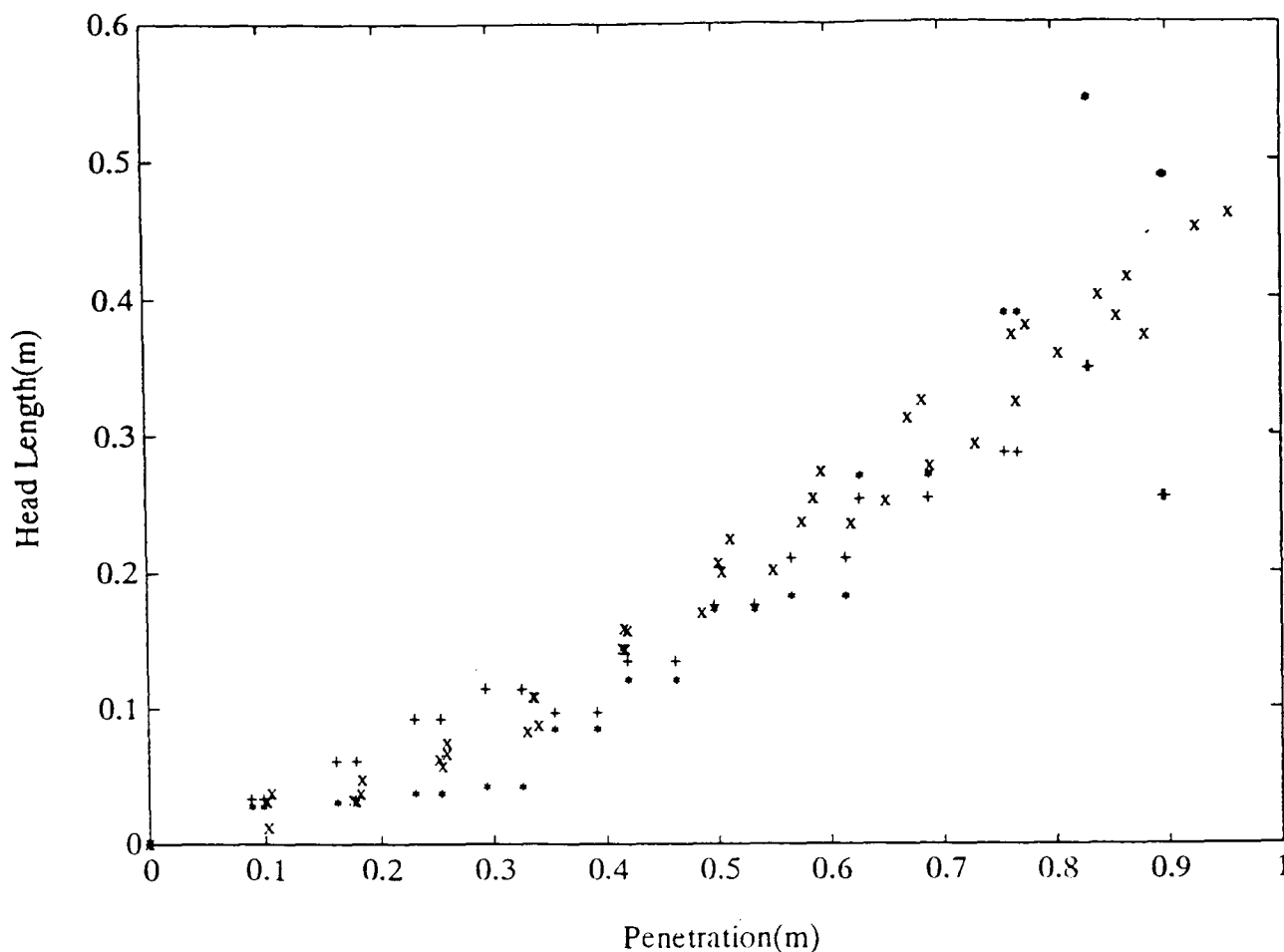


Figure 6.14: The Variation of Head Length with Penetration of the Normal Plumes (x, experiment set a), a Ten-Column Plume within a Narrow Tank (*, experiment g), and a Six-Column Plume Within a Narrow Tank (+, experiment set h).

there is not enough data to make any firm comments, but of the conditions tested the growth of the head is encouraged by a high projection speed and a large jet width. It is not clear why there is a step after 0.5m of the growth of the head of the aluminium particles, though it might be because of the growth of a second head in the particle column in that experiment.

6.3 Plumes under Unusual Conditions

6.3.1 Jets adjacent to a Wall

The plumes described above have a form symmetrical about the centreline of the tank. This is assumed by CHYMES and other computer models; because inviscid equations are solved, the tank walls are modelled using the boundary condition that the normal velocity of each phase at a wall is zero. To save computing time the motion of only half of a jet is modelled and the above boundary condition is imposed on its centreline. To test this two experiments were performed where the edge of the jet was adjacent to the wall: one where the jet was ten particle columns wide (experiment set i) and one where it was only five columns wide (experiment set j). Pictures of the two plumes are shown in Figures 6.15 and 6.16. Their qualitative behaviour was similar to that of the plumes described above. If the movement of groups of air bubbles are followed on the video recording a vortex can be seen behind the wing of the plume, which is strong enough in the wider jet to entrain particles from the column. The vortex persists after

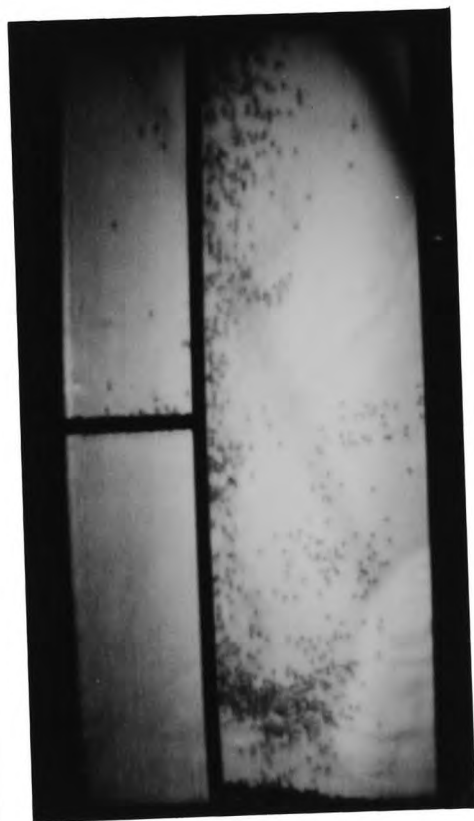


Figure 6.15: Sequence of Pictures showing the Development of a Ten Column Plume adjacent to a Tank Wall, experiment i1, at 0.10, 0.25, 0.75, and 0.95s.

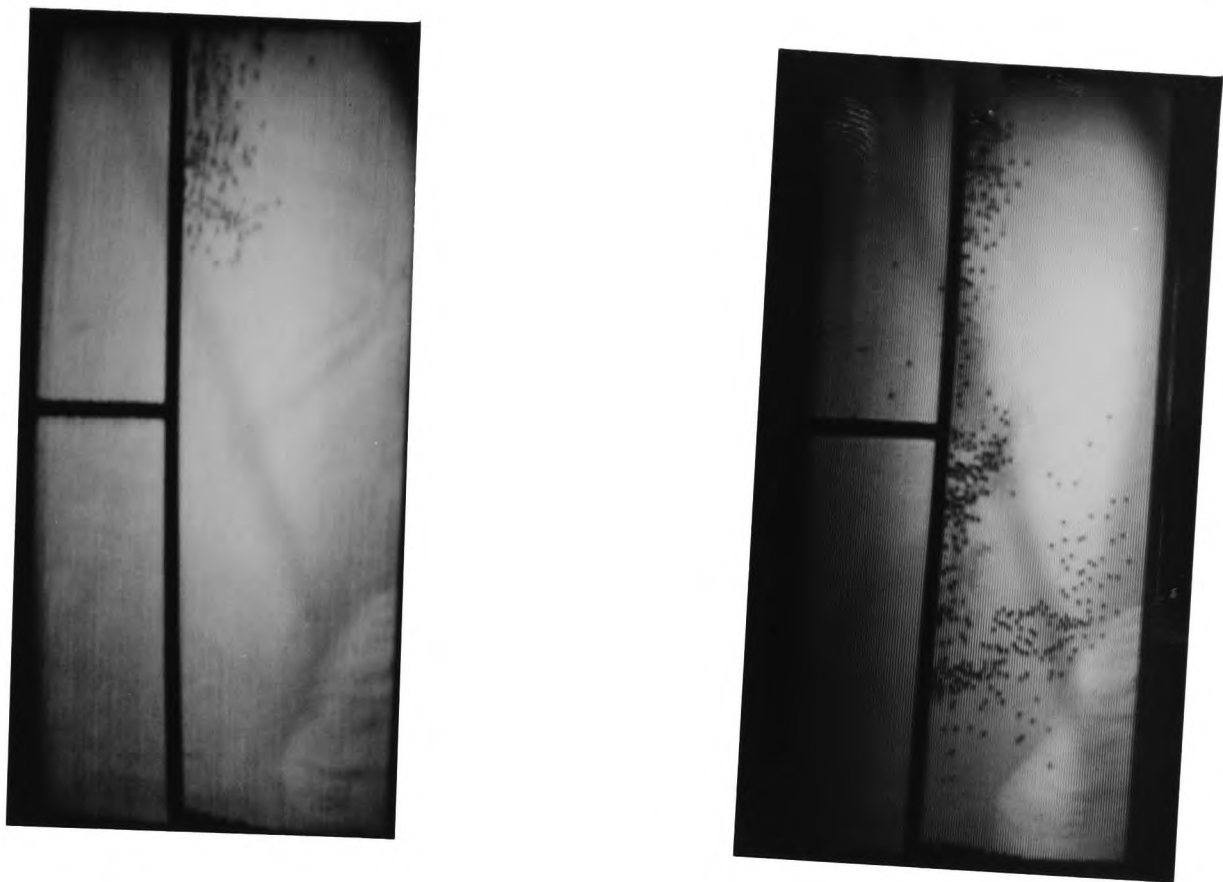


Figure 6.16: A Picture of a Five Column Plume adjacent to a Tank Wall, experiment j1, at 0.25 and 0.75s.

the head has hit the tank floor. The width of the column behind the head decreases with time, leaving a cloud of air bubbles (the grey cloud half way along the column in the bottom left hand picture of Figure 6.15). An obvious difficulty with these plumes is that a boundary layer will form at the wall and the particles in the column will be slower than in a free plume. However if the width of a turbulent boundary layer on a flat plate is calculated³ if the free stream velocity is 2.0m/s the boundary layer is only 19mm after a length of 1m.

The penetration of the thick and the thin plumes were similar to the penetration of the full plumes, as shown in Figure 6.17. The speed downwards of the thicker plume was $1.05 \pm 0.01\text{m/s}$ and of the thinner $0.92 \pm 0.03\text{m/s}$ with 95% confidence limits. The growth in the width of the plumes is shown in Figure 6.18 where it has been doubled to allow direct comparison with the other plumes. Their width was larger than that of the normal plumes at any given time. If the difference between the plume width and the jet width of the two plumes was plotted, as in Figure 6.19, the data fitted a single line, suggesting that it is the tank width, or possibly the difference between the tank and jet width that influences the spreading of the plume rather than the width of the jet alone. The growth in the length of the plumes' heads, shown in Figure 6.20, was comparable to that of the normal plumes, except at later times (after 0.6s) when the head length of the thinner jet was less than the others.

³Thickness= $0.37x(U_{\infty}\rho_f x/\mu_f)^{-1/5}$ (H.Schlichting [98]) where x is the distance along the plate and U_{∞} is the free stream velocity.

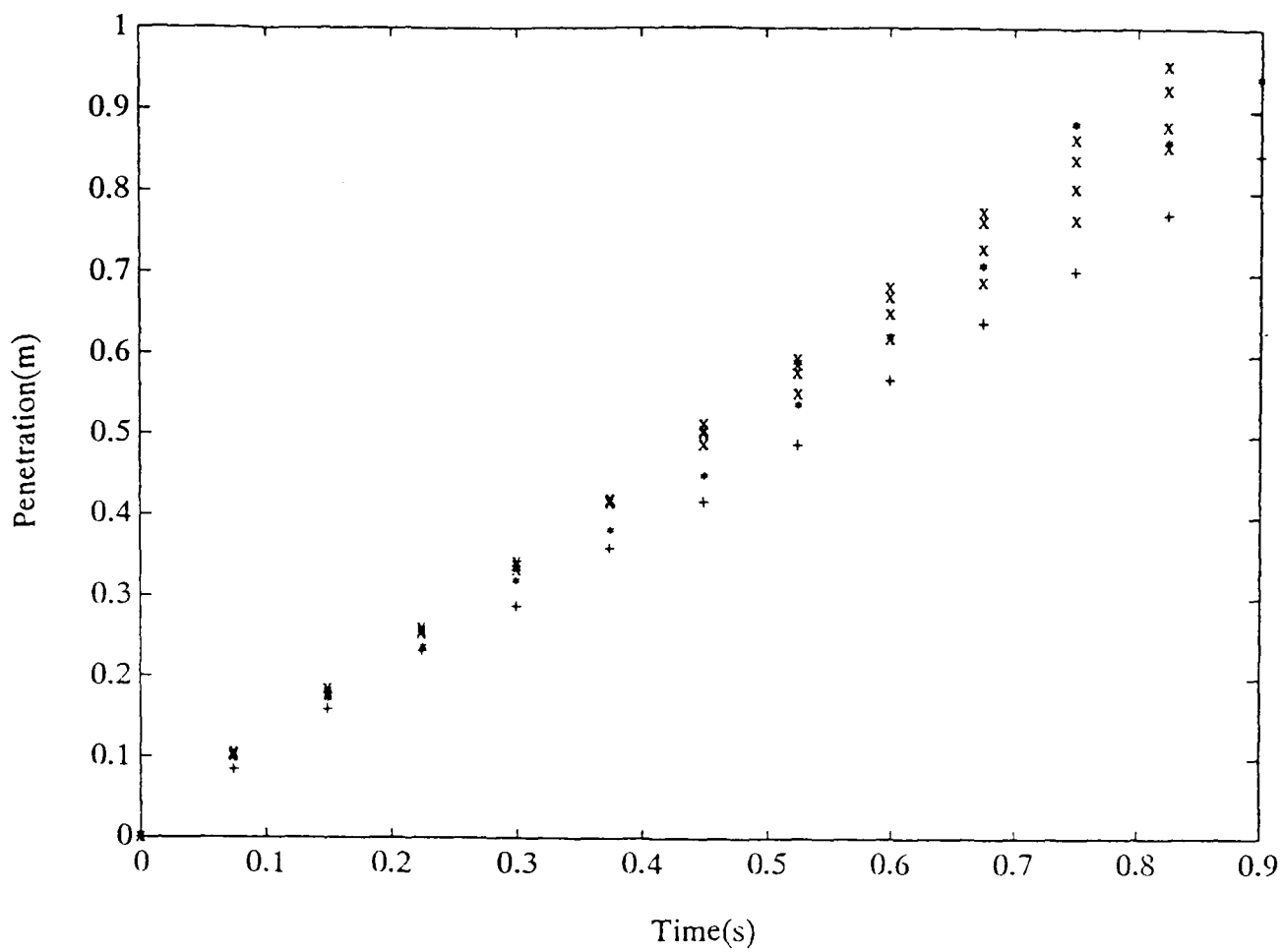


Figure 6.17: The Penetration of the Plumes adjacent to a Tank Wall (*=ten columns, experiment i1; +=five columns, experiment j1) compared with that of Normal Plumes (x, experiment set a).

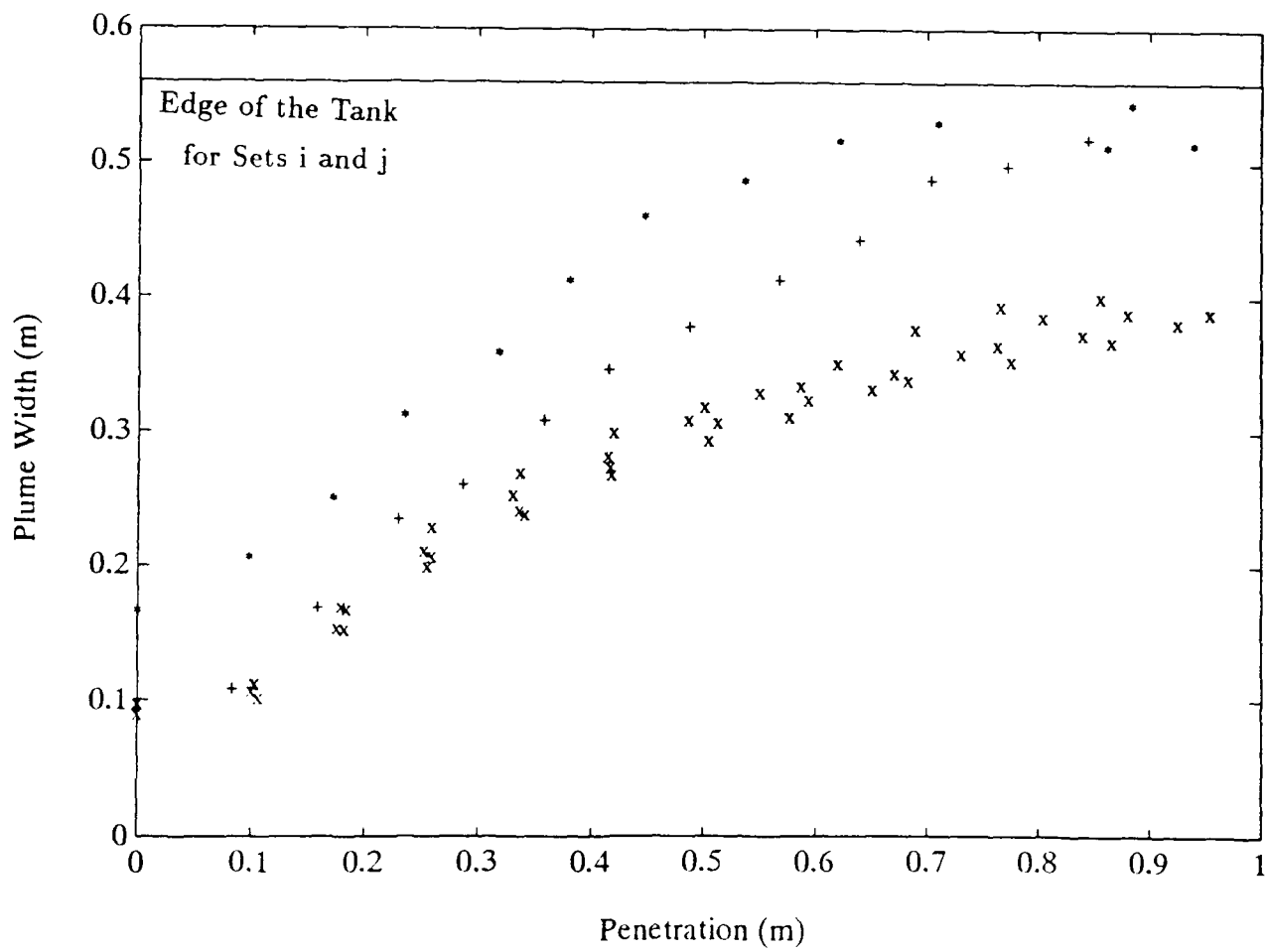


Figure 6.18: The Width of the Plume for the Plumes adjacent to a Tank Wall (*=ten columns, experiment i1; +=five columns, experiment j1) compared with that of Normal Plumes (x, experiment set a).

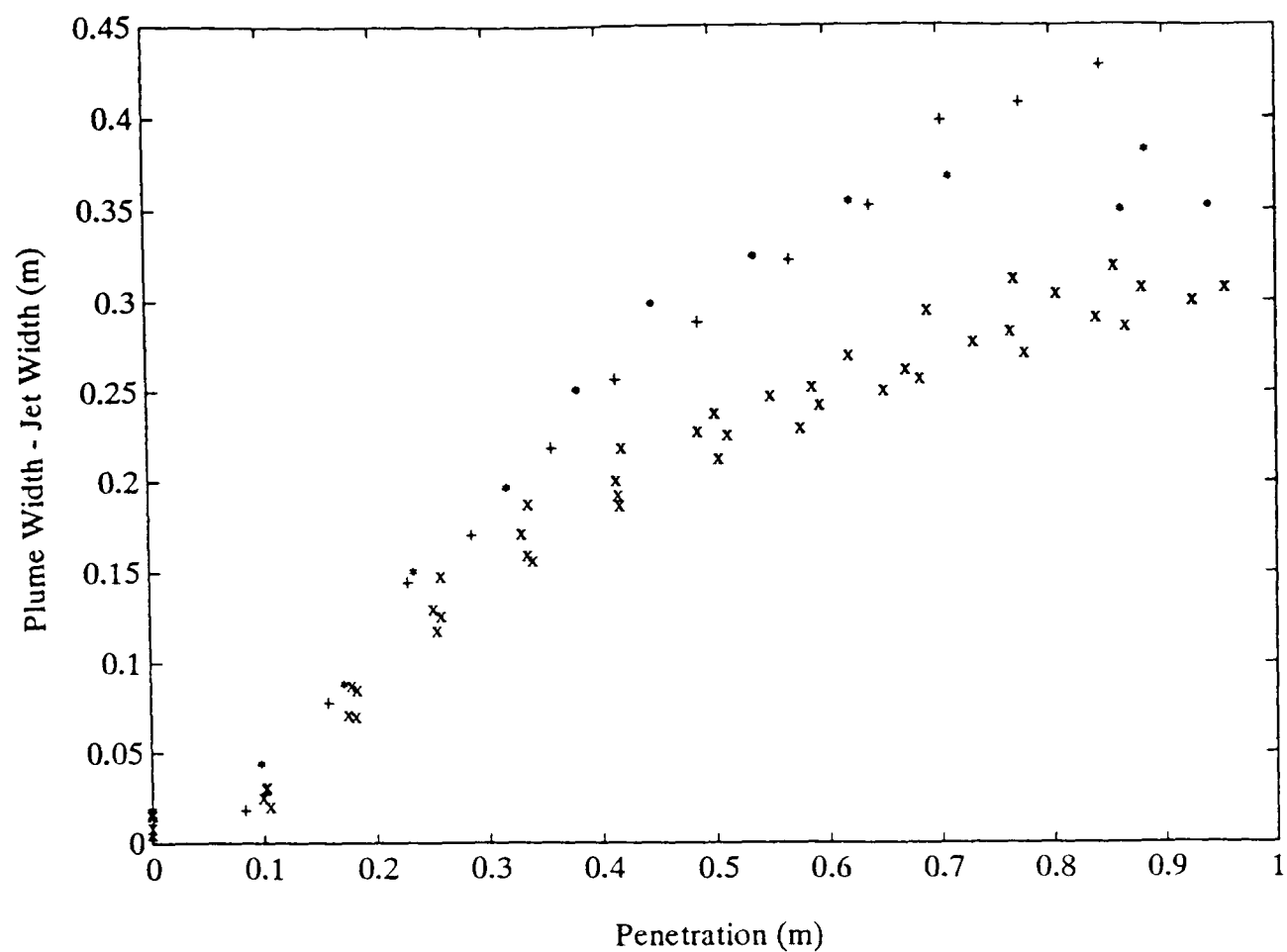


Figure 6.19: The Difference between the Plume Width and the Jet Width of the Plume for the Plumes adjacent to a Tank Wall (*=ten columns, experiment i1; +=five columns, experiment j1).

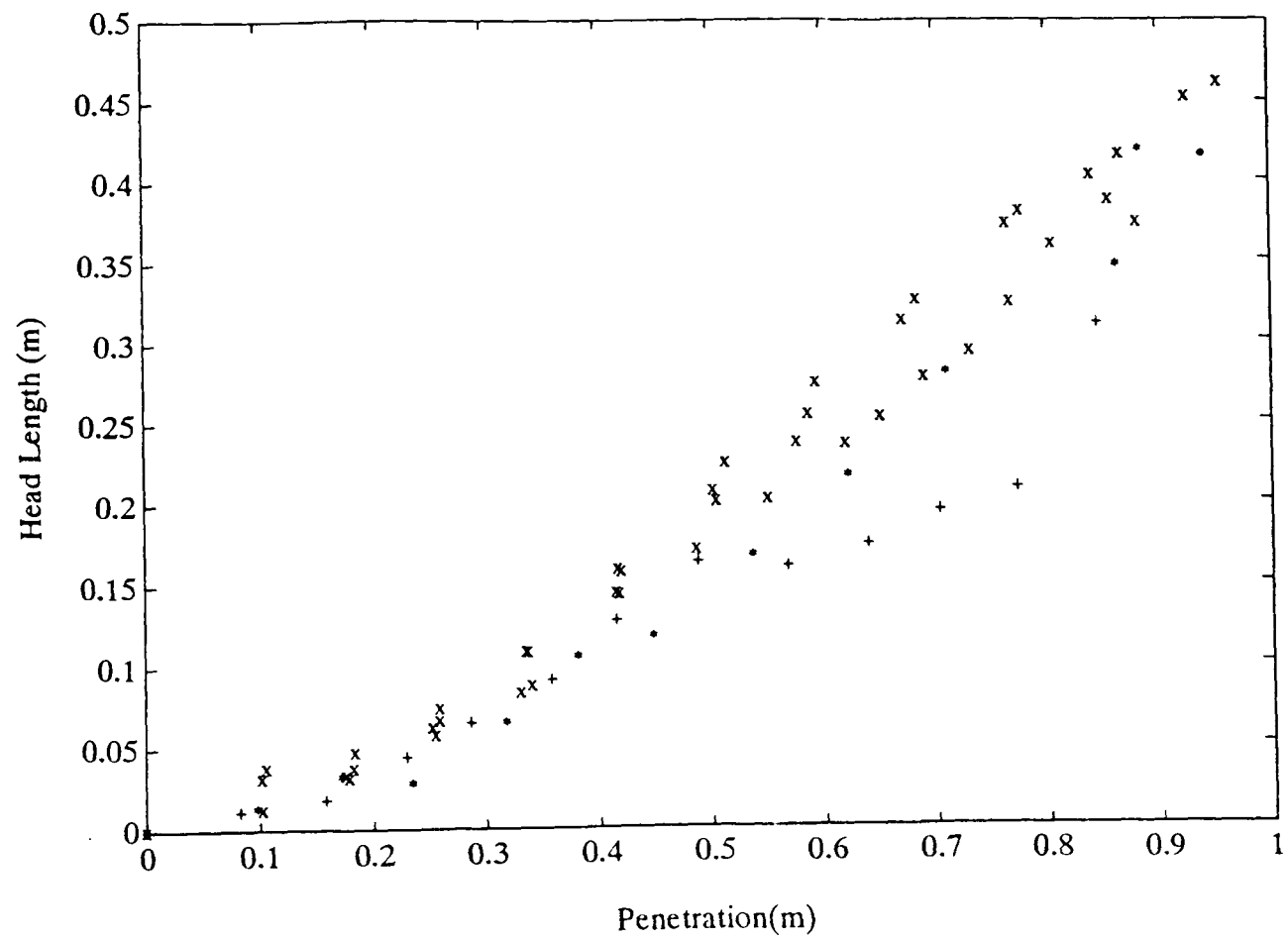


Figure 6.20: The Head Length of the Plumes adjacent to a Tank Wall (*=ten columns, experiment i1; +=five columns, experiment j1) compared with that of Normal Plumes (x, experiment set a).

6.3.2 Plumes with a Low Initial Volume Fraction

The volume fraction of the plume could be changed independently of the speed of the plume at the water surface by omitting particles in the array held by the projection mechanism. Two experiments⁴ were performed where every other particle was left out, resulting in a particle volume fraction at the water surface of 2.6%; all other conditions were as for the normal plumes. Figure 6.21 shows one of them. Some aspects of the behaviour of these plumes were different from those of the others; most noticeably the spreading of the particles was suppressed and the shape of the head of the plume was different, not at all crescent shaped.

The penetration of the sparse plumes compared with that of the normal ones is shown in Figure 6.22 and the spreading in Figure 6.23. The rate of penetration was steady, as for the other plumes, and was $0.91\text{m/s} \pm 0.03\text{m/s}$ with 95% confidence limits. In both experiments the spreading of the plumes was substantially less (by at least a quarter) than that of the normal plumes. The column of the plumes had a similar character to that of the other plumes, though less dense, with a constant width and shifting regions of particle concentration.

6.3.3 Plumes Offset from the Tank Centreline

Four experiments - k1, k2, l1, and m1 - were conducted where the centreline of the jet was offset from the centreline of the tank as shown in Figure 6.24. Two of these were normal plumes, one consisted of seven columns, and one of ten columns projected with the same- higher than normal- speed as in experiment set b. Pictures of these are shown in figure 6.25, 6.26, and 6.27.

These plumes are unusual because, though many of the features of the other plumes are replicated, the path of the plume to the bottom of the tank is not straight, but curved: the plume is bent towards the nearest wall and touches it before swinging around, moving diagonally, and striking the bottom of the tank. Because of this it is not possible to present penetration or spreading data in a manner comparable to that of the other plumes. In addition, because the plume is asymmetrical, it cannot be modelled by CHYMES. The point of detachment of the plume from the wall moves downward with time, the column below it becoming ever more horizontal. The development of the head of the plume is similar to that in other plumes. The vortices following the wings of the plume (shown on the overlay to Figure 6.24), indicated by the motion of the particles and the air bubbles, were very evident. The vortex between the column and the nearest tank wall created a region empty of particles in the bottom right hand corner of the tank even after the head had reached the bottom of the tank. At earlier times

⁴Experiment set h.

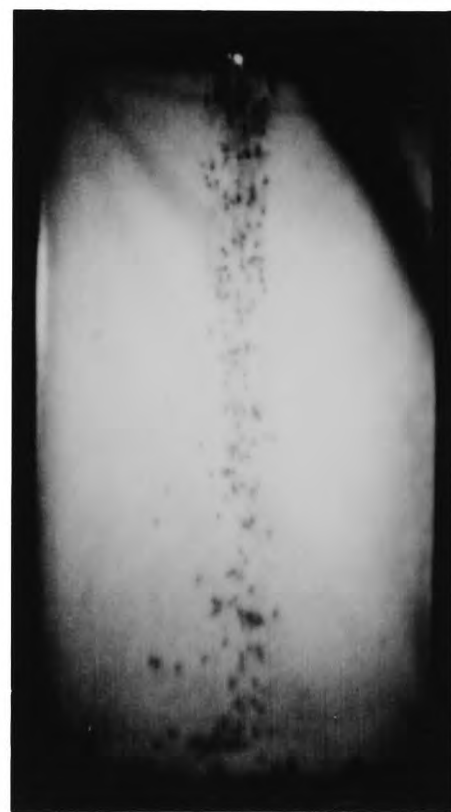


Figure 6.21: The Development of a Sparse Plume, experiment h2, after 0.10, 0.45, 0.75, and 0.95s.

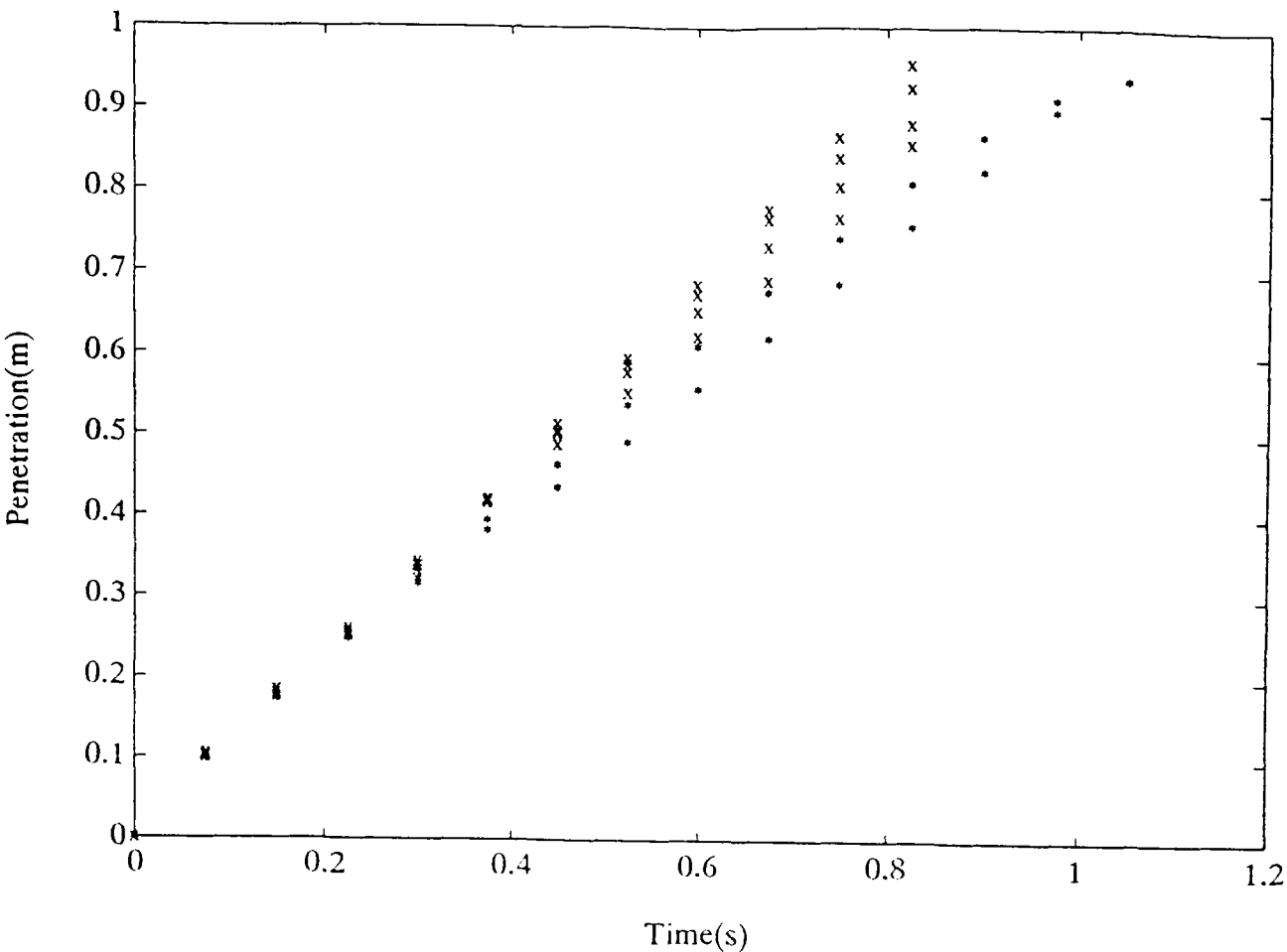


Figure 6.22: The Penetration of the Sparse Plume (*, experiment set h) compared with that of the Normal Plumes (x, experiment set a).

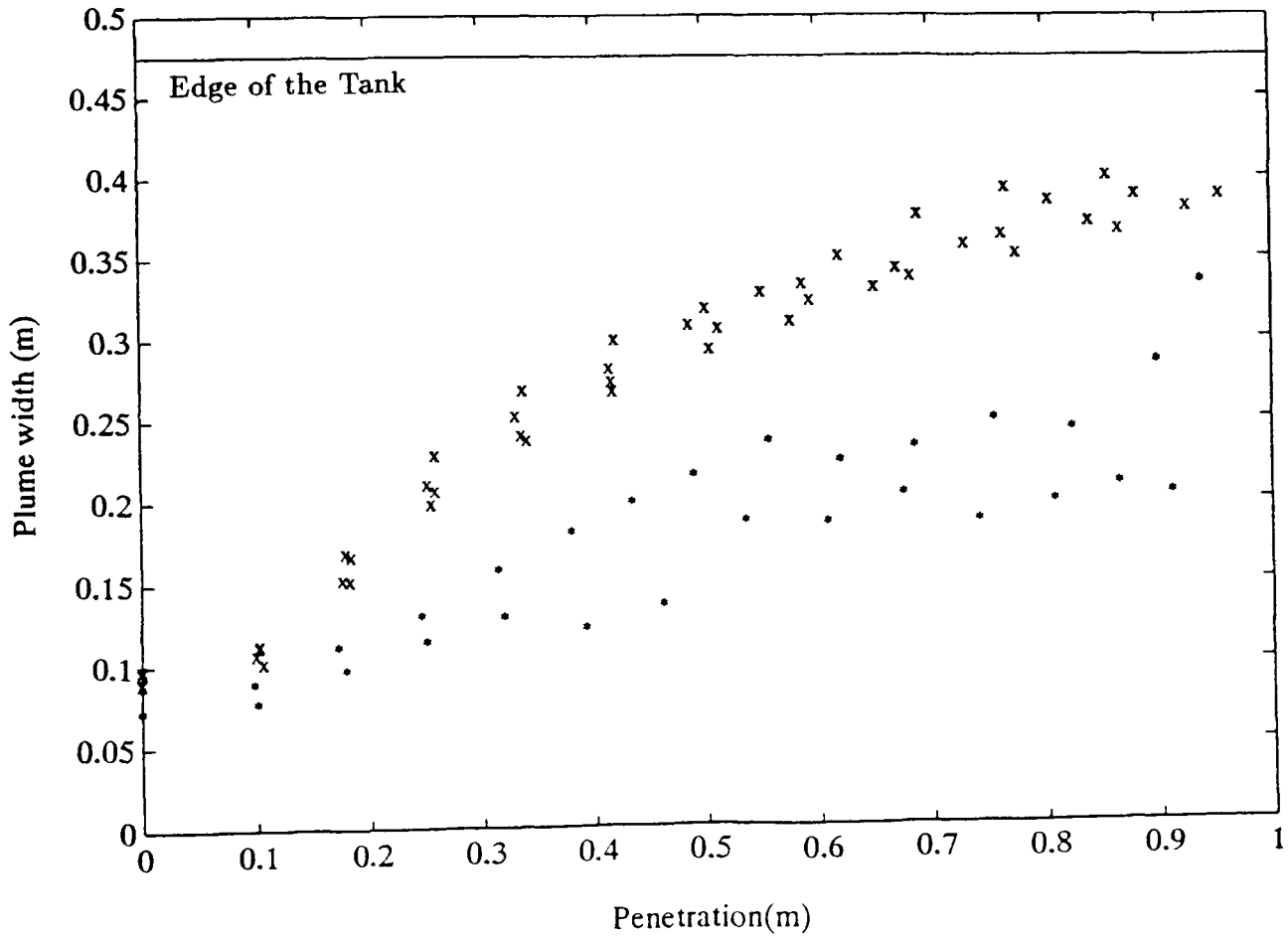


Figure 6.23: The Variation of the Width of the Plume with Time for the Sparse Plumes (*, experiment set h) and the Normal Plumes (x, experiment set a).

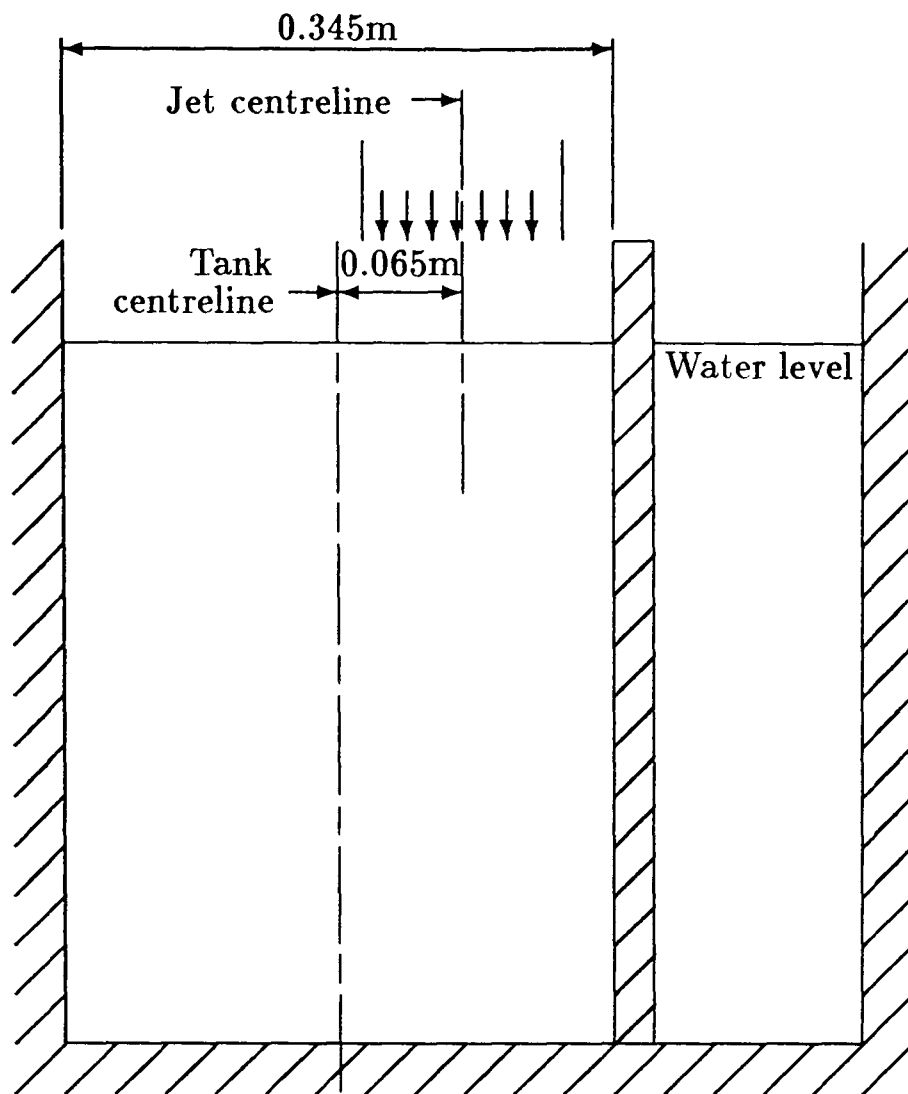


Figure 6.24: The Geometry of the Offset Jets.

it caused a stationary wedge of balls between the tank wall and the column to be suspended, particularly when the particle injection velocity was high. The plume resulting from the jet with the higher injection velocity was similar to the others. The thinner plume displayed similar characteristics to the normal ones as well but not as clearly: the edges of the plume appeared to be more ragged.

6.4 The Effect of Boiling

Owing to the problems encountered in making the apparatus for projecting hot particles work, only one plume of hot particles was produced which is shown in Figure 6.28. The plume consisted of columns of particles falling freely into the tank under gravity in the manner shown in Figure 4.2. The supply of particles was irregular owing to some occasionally sticking within the apparatus. The uncontrolled nature of the plume is probably the reason for it spreading widely. Because of this the plume is not analogous to the others described in this chapter and cannot be compared with a CHYMES calculation, but it does allow the small scale behaviour of boiling particles in a plume to be seen. This behaviour conforms to the assumptions used in the formulation of CHYMES: the particles and their films fall as a single, composite sphere



Figure 6.25: The Development of a Normal, Offset Plume, Experiment k1, after 0.10, 0.24, 0.75, and 0.95s.



Figure 6.26: Pictures of an Offset Plume Seven Columns Wide, Experiment 11, after 0.25 and 0.95s.



Figure 6.27: Pictures of an Offset Plume with a High Injection Speed, Experiment m1, after 0.25 and 0.95s.

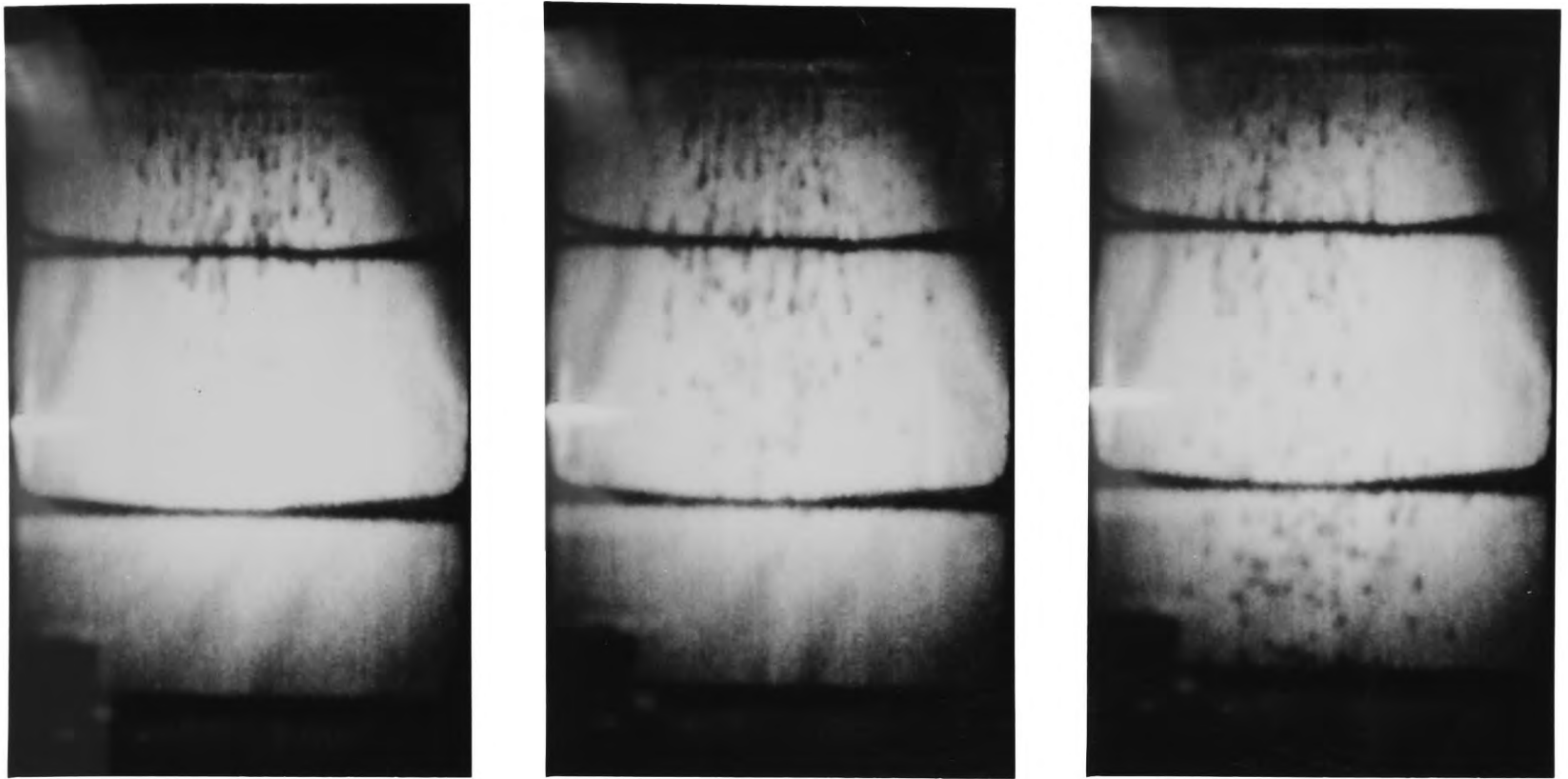


Figure 6.28: The Plume of Hot Particles after 0.25, 0.45, and 0.75s. The Dark, Horizontal Lines are the Joins between the Panes of Ceramic Material that make up the walls of the Inner Tank in this Experiment.

from which vapour bubbles are shed. The bubbles do not seem to affect the paths of following particles.

Once the particles have accumulated at the bottom of the tank the situation changes: large vapour bubbles form within the bed of particles on the tank floor and violent boiling takes place. The flow caused by the rising bubbles is strong enough to prevent the particles entering the tank from dropping; instead they accumulate close to the surface and are swept sideways by the recirculation flow in the water caused by the vapour flow. This can be seen in Figure 6.29 where the majority of particles entering the tank are excluded from its central portion and forced around the sides. CHYMES is not intended to model this situation when a particle bed forms.

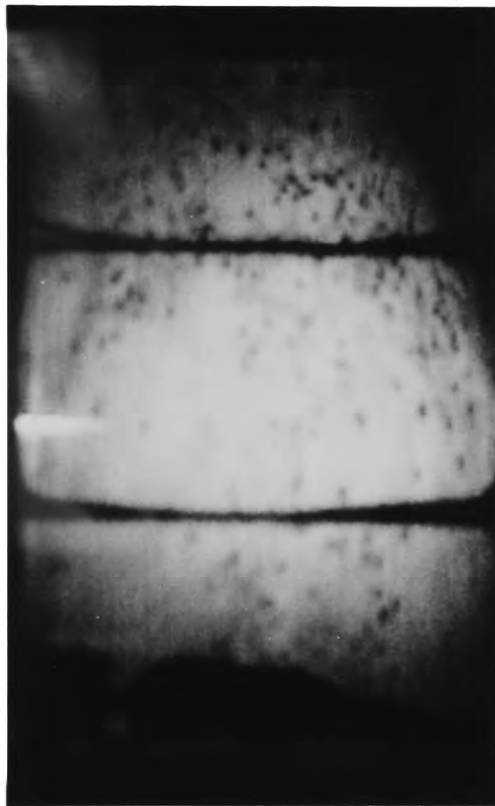


Figure 6.29: Boiling within the Tank after the Accumulation of Particles on the Floor. Particles are still entering the tank.

Chapter 7

The Measurement of the Motion of Individual Particles

The movements of individual balls were followed on the close-up video recordings using the MOVIAS system¹ to record the positions of particles. Their paths could then be displayed, and their positions relative to any point and velocities calculated.

Two close-up recordings were made, both of normal plumes: one of the top of the tank in experiment a5, and one of the plume after it is about a third of the way down the tank in experiment a6. The resolution errors are $\pm 0.75\text{mm}$ and $\pm 0.3\text{mm}$ for a5 and $\pm 0.4\text{mm}$ and $\pm 1.0\text{mm}$ for a6, in the longitudinal and lateral directions respectively. The errors in the measured speed if it is 1m/s are $\pm 4\%$, $\pm 3\%$, $\pm 0.5\%$, and $\pm 0.5\%$ in the same order as above, and if it is 0.25m/s $\pm 15\%$, $\pm 11\%$, $\pm 2\%$, and $\pm 3\%$.

The paths of twenty particles in the head and twenty in the column of the plume in each recording were followed, divided into four sets of ten particles denoted by the letters X and Y if the particles are in the head, and A and B if the particles are in the column. Even though each plume consists of hundreds of particles, the motion of a few of them is assumed to be typical of the others because the overall behaviour of a plume is determined by the collective motion of individual particles; if the collective motion is repeatable then to some extent the motion of individual particles must be as well. If their motion was perfectly repeatable identical then at a given instant a particle with a certain position will have the same motion as a particle with the same position in another plume. However this is unlikely and the motion of a particle may deviate from that expected because of the influence of the flow around other, nearby particles, even though its overall behaviour may be consistent with the plume's. In other words the effect

¹See Section 6.1.

of the local distribution of the particles may sometimes mask or distort the motion caused by the processes fashioning the overall shape of the plume.

MOVIAS can calculate the components of the accelerations of the particles from their velocity components in a similar manner to the calculation of the velocity components from the particles' positions. This was not used because of the uncertainty introduced by:

- the deviation of a particle's velocity from that expected owing to local conditions;
- the doubling of the error owing to the uncertainty of a particle's position;
- the increase in the error owing to the approximation of the unsteady movement of a particle to a steady one over four time steps, rather than over two as in the case of velocities.

Instead the components of acceleration of the particles were estimated by drawing straight lines through suitable portions of graphs of velocity against time and measuring their gradients. Many particles had nearly constant accelerations for some time and this method enables their magnitudes to be estimated.

7.1 The Motion of Particles in the Column of the Plume

Four sets of particles in the column of the plume were followed:

- set a5A which was at the top of the water pool at earlier times;
- set a5B which was from the column at the top of the tank at later times, well away from the head;
- set a6A which approached the back of the head in the central portion of the tank;
- set a6B which is well behind the head in the central portion of the tank.

The paths of the particles in these sets are presented in Figure 7.1–Figure 7.4. The time interval between marks is 10ms for a5 and 25ms for a6.

A common feature of all four sets of particles is that many of the particles do not fall straight down the tank but their paths converge or diverge. For an example see the paths of particles 2–6 in Figure 7.3. These do not reflect a change in the overall flow field - the column remains straight and its width² does not change - and so must reflect the local flow conditions such as the influence of the wakes of other particles.

²This is defined as the envelope of the paths of the most extreme particles.

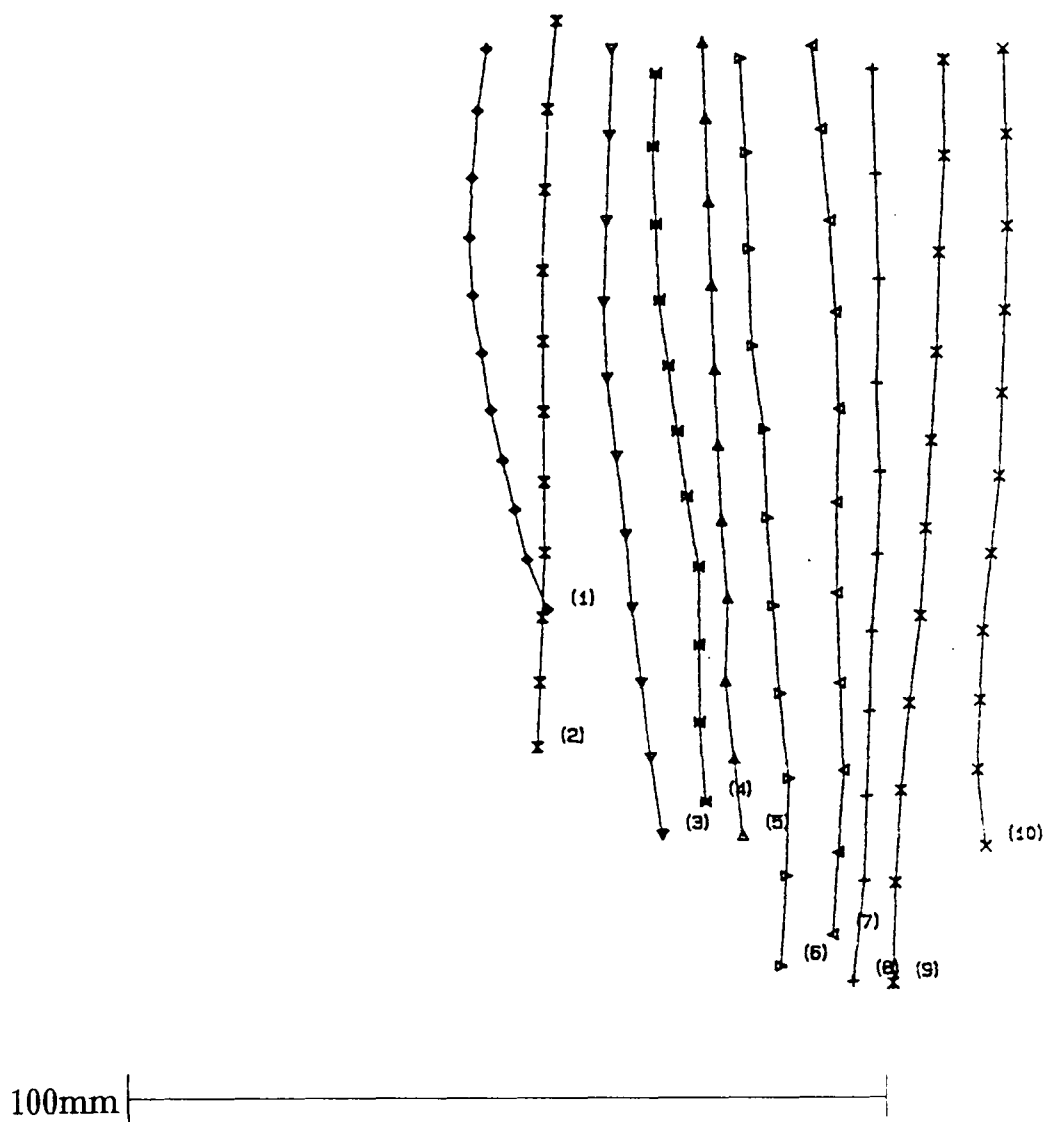


Figure 7.1: The Paths of the Particles of Particle Set a5A within the Column.

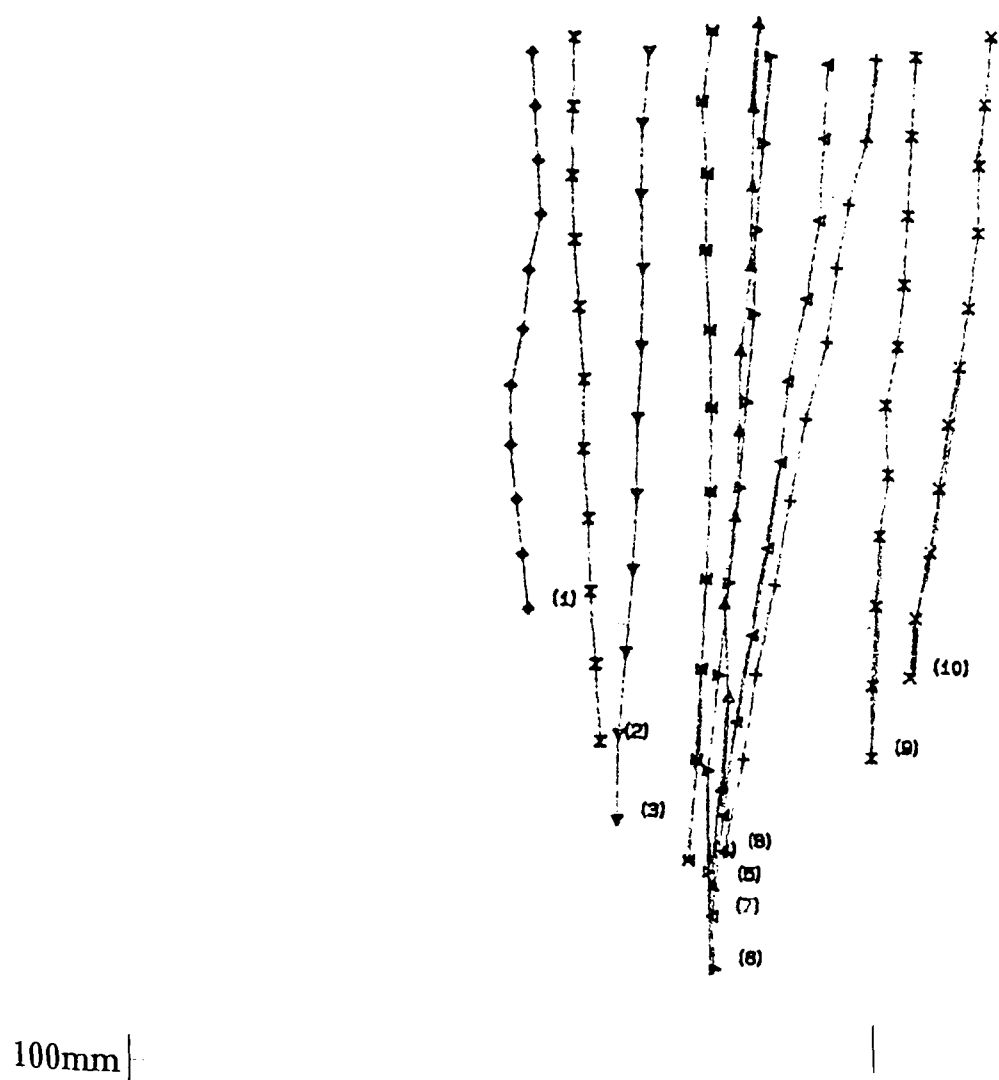


Figure 7.2: The Paths of the Particles of Particle Set a5B within the Column.

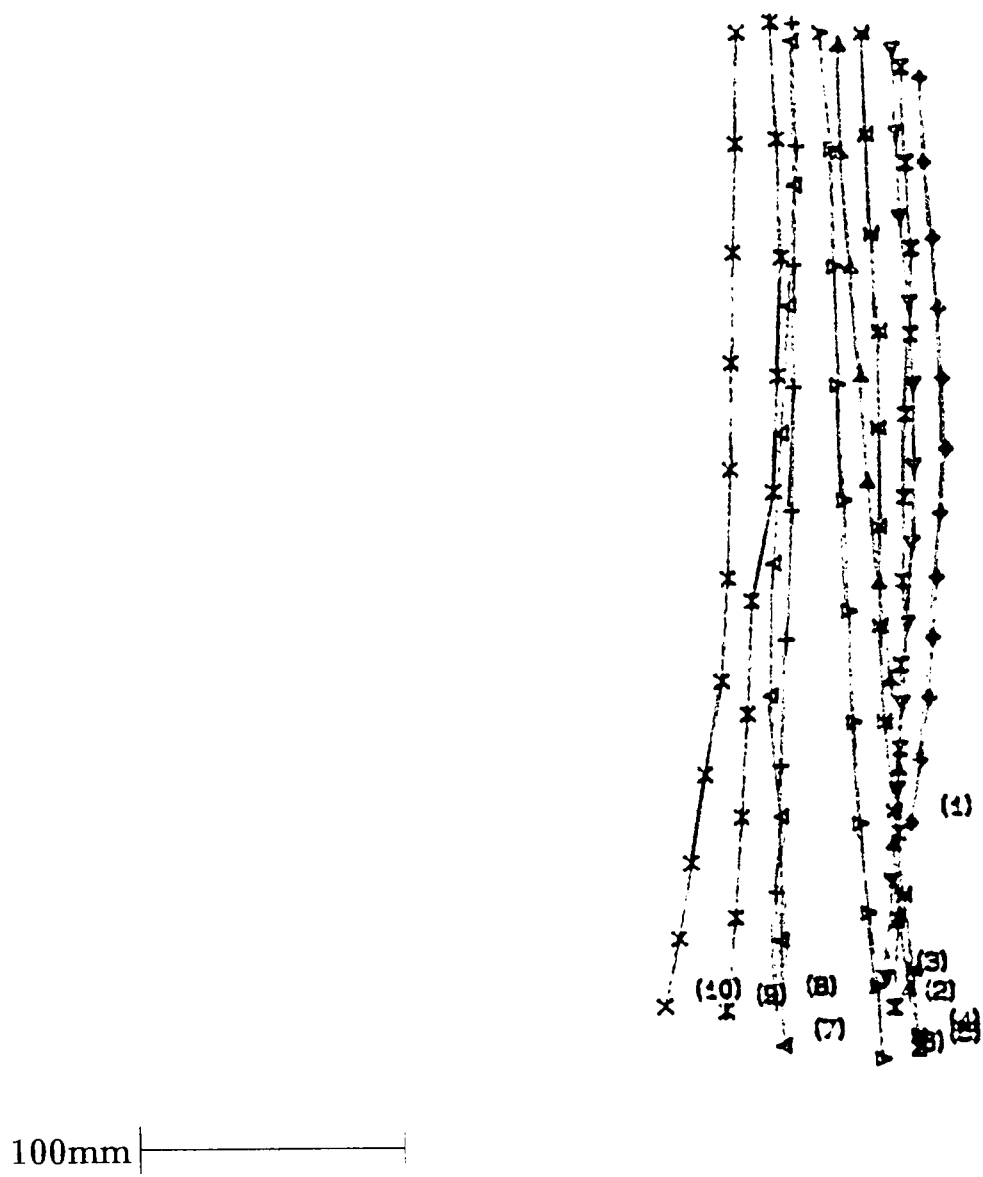


Figure 7.3: The Paths of the Particles of Particle Set a6A within the Column.

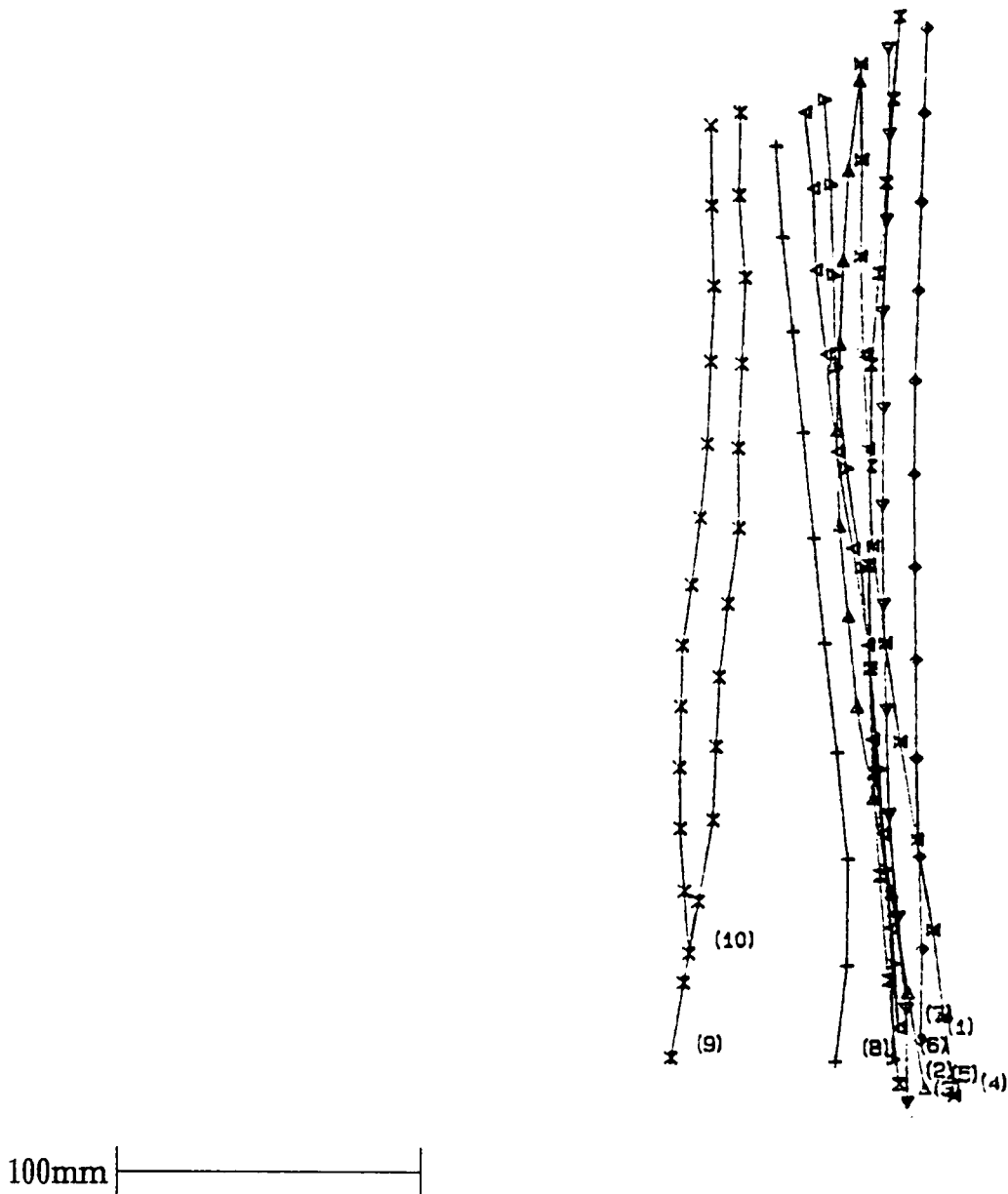


Figure 7.4: The Paths of the Particles of Particle Set a6B within the Column.

The variations of the downwards (longitudinal) component of velocity with time are shown for the four sets of particles in Figure 7.5–Figure 7.8. The downwards speeds of a large number of particles appear to be affected by their proximity to the water surface or the head of the plume. For example, the particles of set a5a have just entered the water pool and are moving faster than their terminal speeds. The particles towards the centre of the jet have particularly large speeds because they can fall freely longer than the others owing to the depression in the water surface. The particles (except 5 and 7 which move at roughly constant speeds of 1.5 and 1.7m/s respectively) decelerate at a rate up to 13.5m/s^2 (particle 8) for the first 0.05s. At the other end of the column the head of the plume travels move slowly than the particles approaching it; therefore the particles should slow as they enter the head. This is what happens to the particles of set a6A (except 2 and 3 which accelerate at later times) which decelerate at rates of up to 5.9m/s^2 . The different behaviour of particles 2 and 3 might be because they are within a bunching of particles (see Figure 7.3) so their motion is greatly affected by local conditions: on page 47 it was shown that particles can strongly interact with each other, particularly if one lies in the wake of another. In addition because the particles are moving laterally, their changing position in the flow may affect their motion. Particles 4, 5, and 7 of set a5B also all

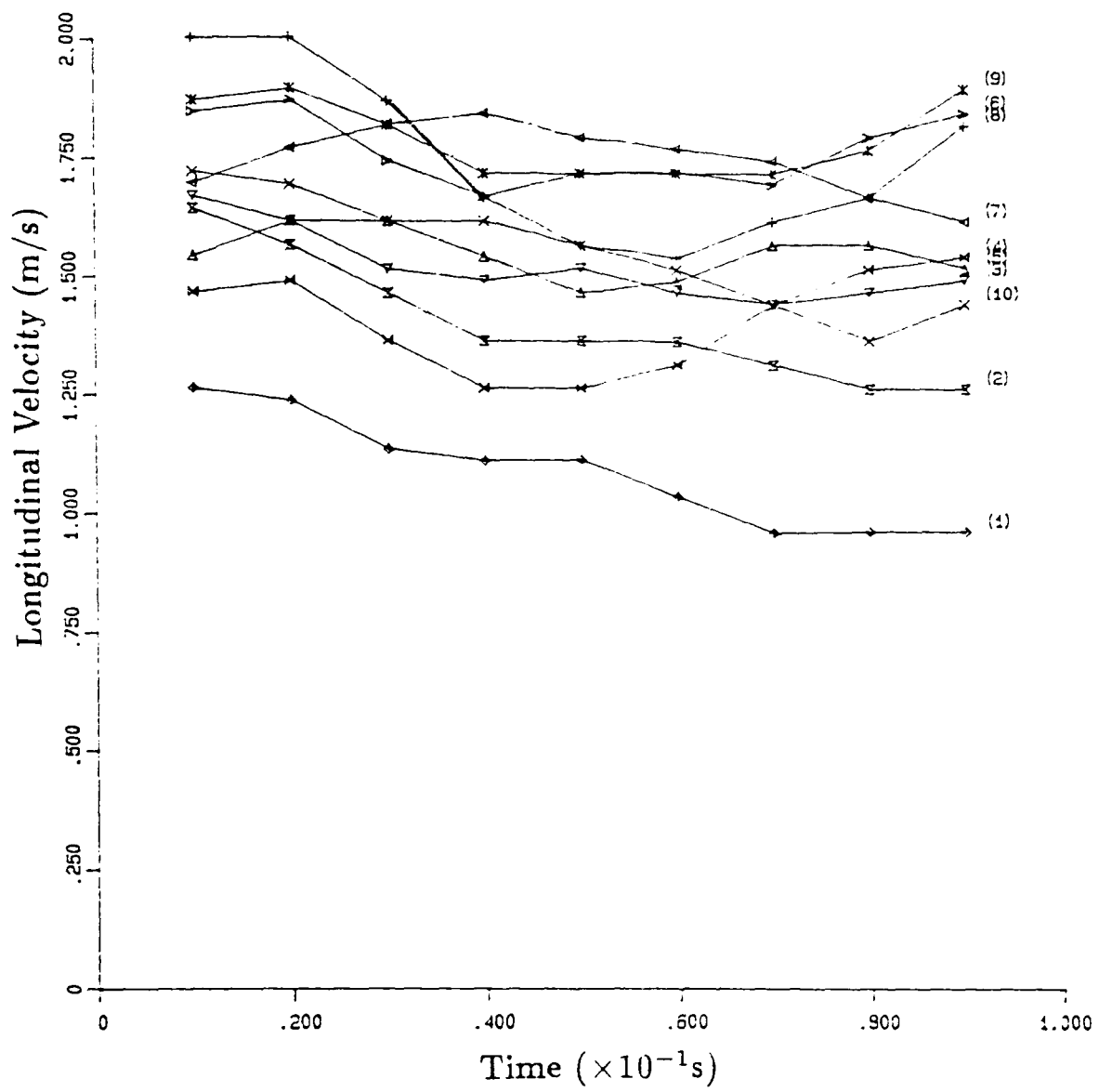


Figure 7.5: The Variation of the Longitudinal Component of Velocity with Time of the Particles of Set a5A.

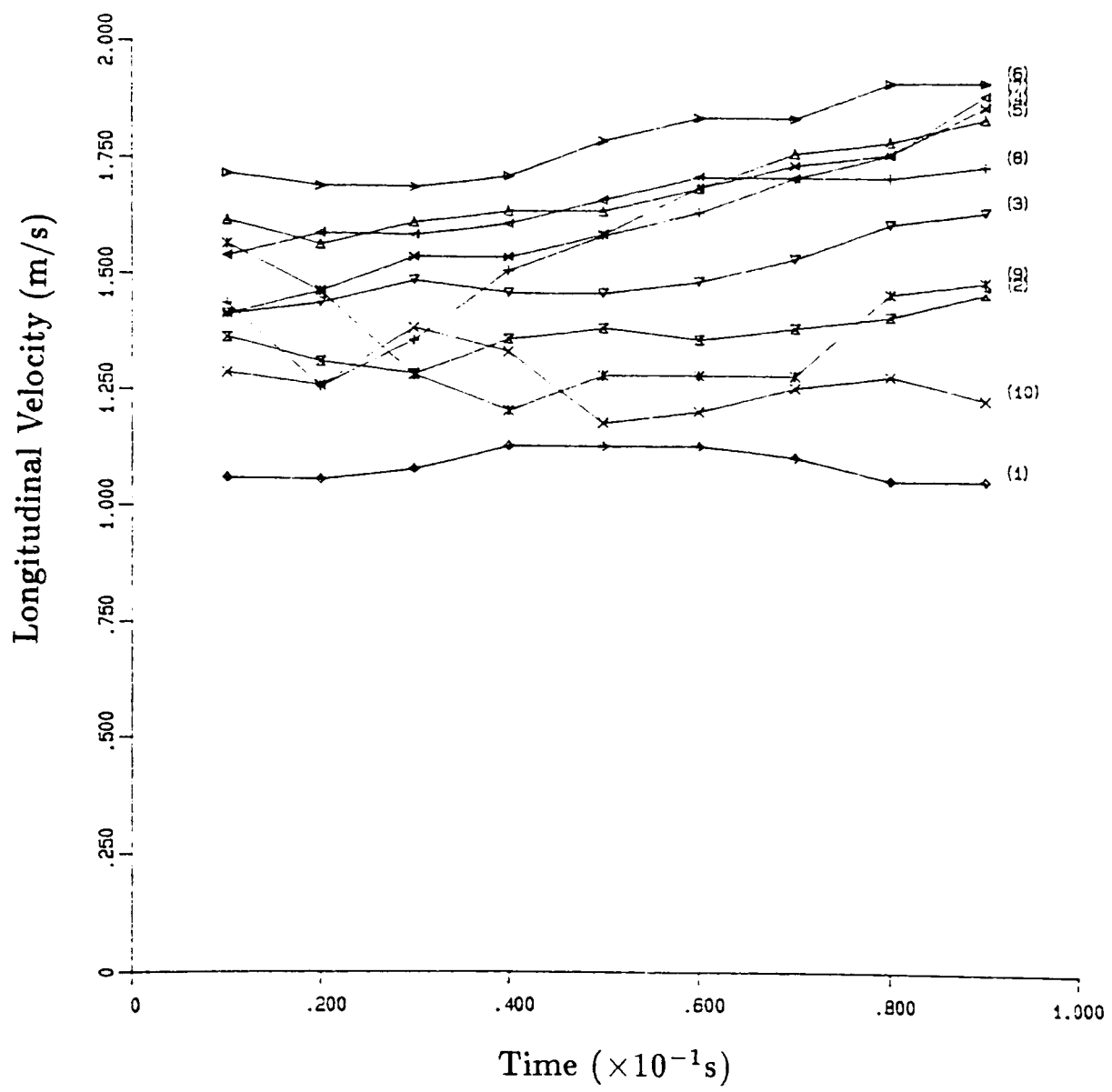


Figure 7.6: The Variation of the Longitudinal Component of Velocity with Time of the Particles of Set a5B.

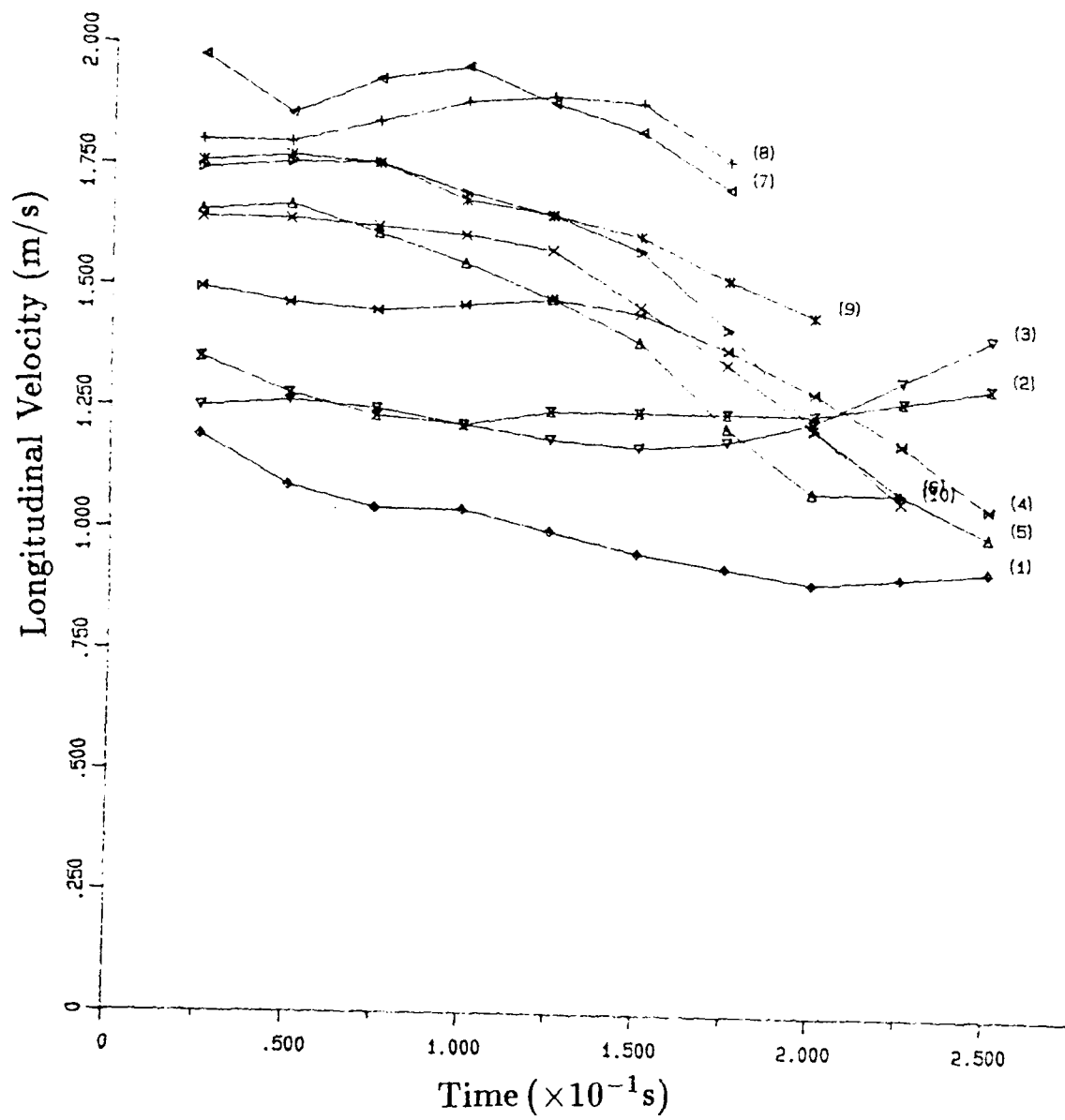


Figure 7.7: The Variation of the Longitudinal Component of Velocity with Time of the Particles of Set a6A.

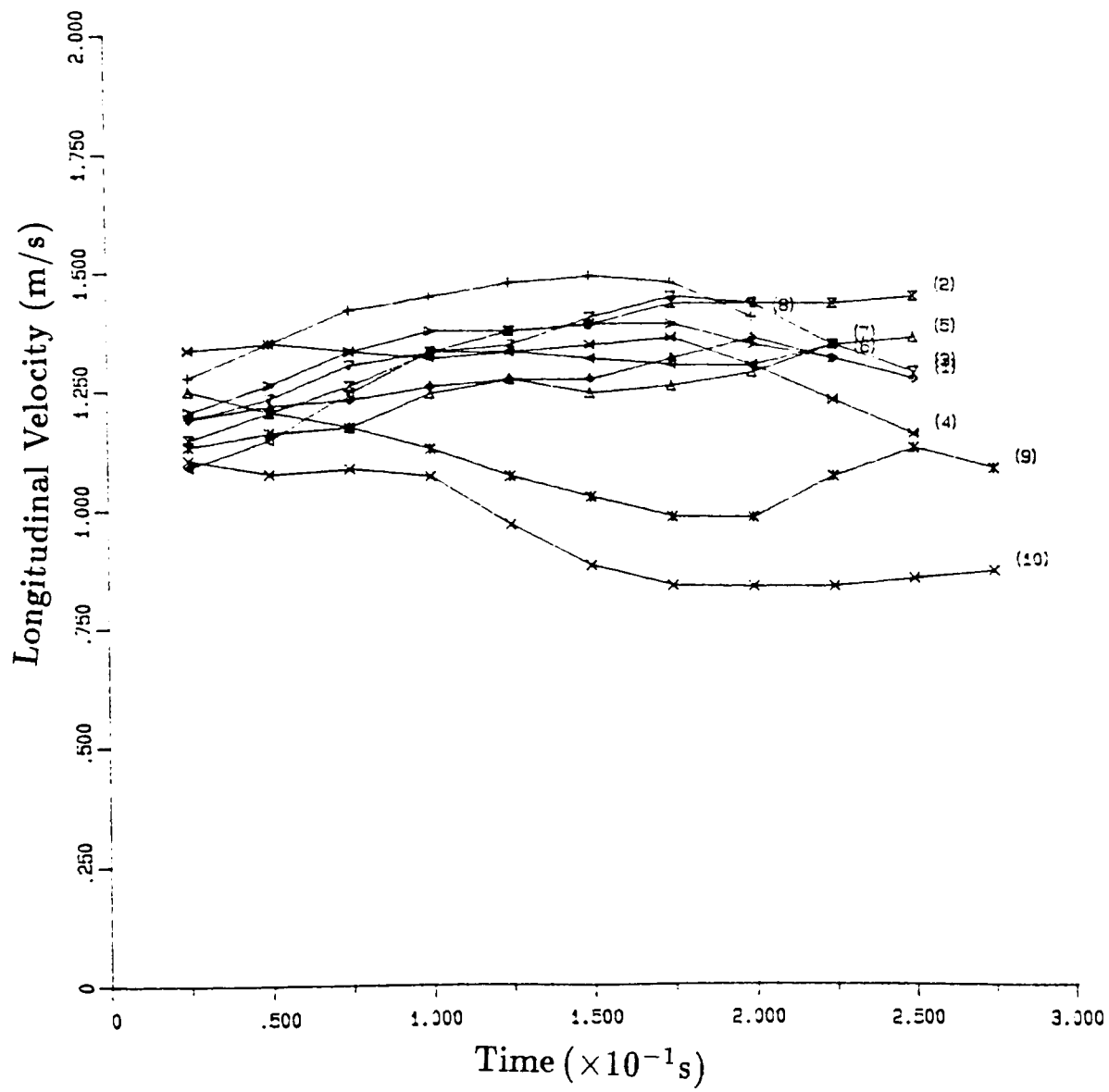


Figure 7.8: The Variation of the Longitudinal Component of Velocity with Time of the Particles of Set a6B.

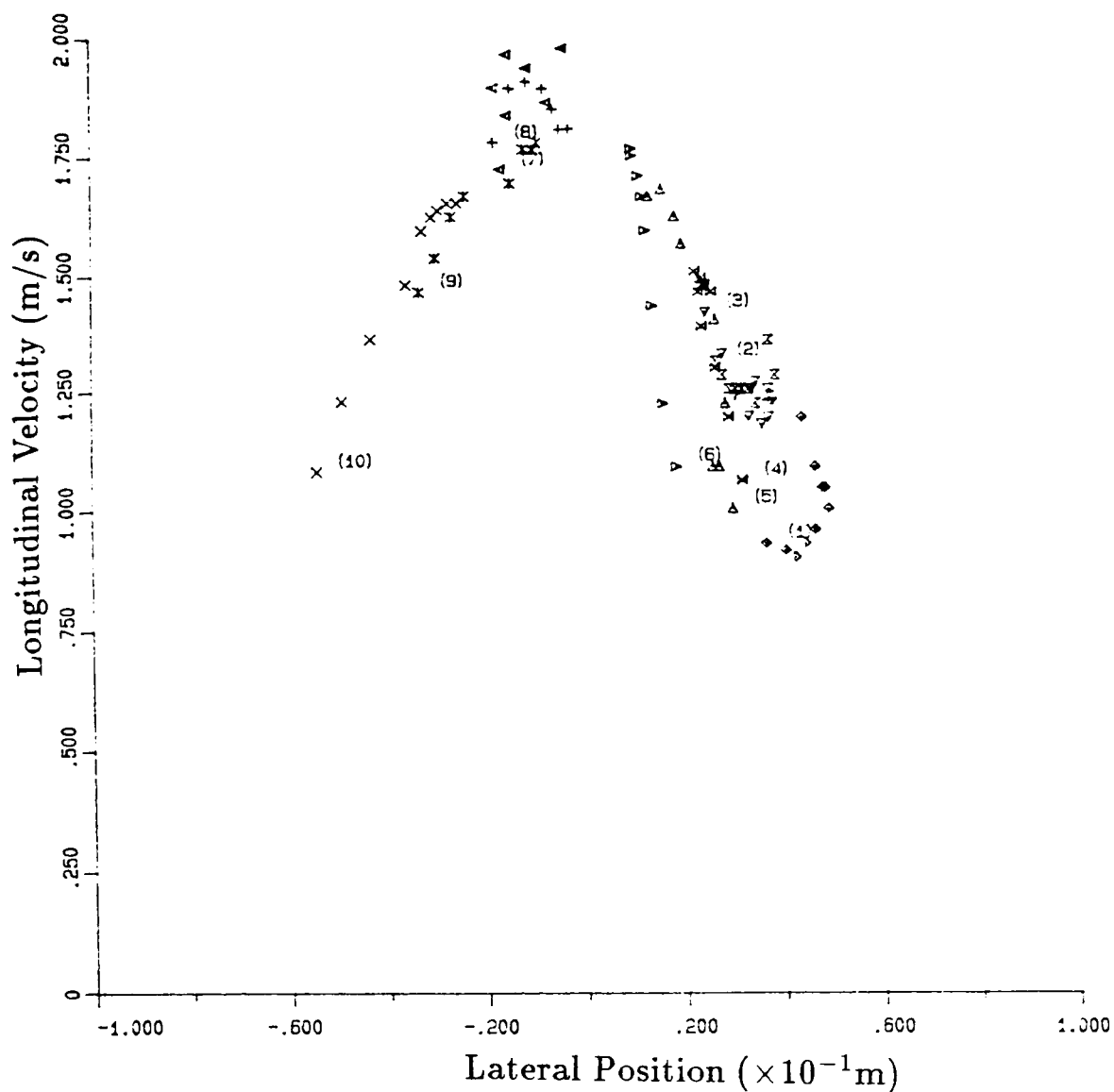


Figure 7.9: The Variation of the Longitudinal Component of Speed of the Particles in Set a6A with their Lateral Position.

accelerate when bunching together.

All the particles, aside from a couple at the edge of the column³, move with speeds greater than the terminal speed of an isolated particle, up to nearly 2m/s or two and a half times more (particle 7 of set a6A). The speed of a particle appears to be dependent on its lateral position; in all four sets of particles those towards the centre of the column move faster than those at the edges; for example see Figure 7.9 which shows the variation of the longitudinal velocity with the lateral position for particle set a6A. In other sets of particles the same general characteristic is apparent for most of the particles, but the relationship can be distorted by local flow conditions; for example see Figure 7.10 in which downwards speed is plotted against lateral position for set a6B; the data for particles 2, 5, 6, and 7 is scattered and these particles also converge in on one another.

The range of speeds of particle set a6B is substantially less than for the other sets (1.1–1.4m/s against 0.95–2.0m/s). The concentration of particles also appears to be less in this set than in the others.

³Particle 1 of set a6A and particle 10 of set a6B.

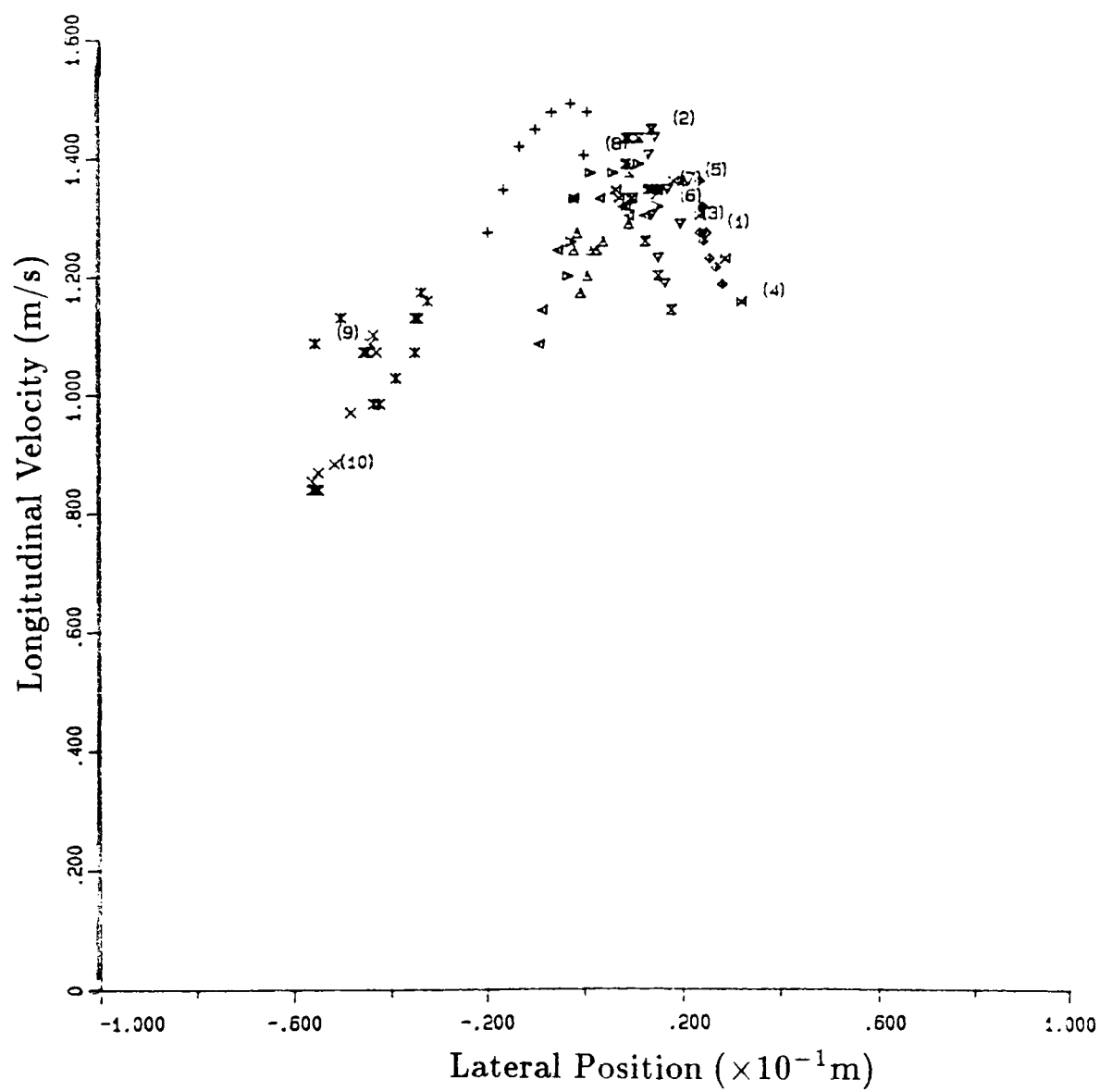


Figure 7.10: The Variation of the Longitudinal Component of Speed of the Particles in Set a6B with their Lateral Position.

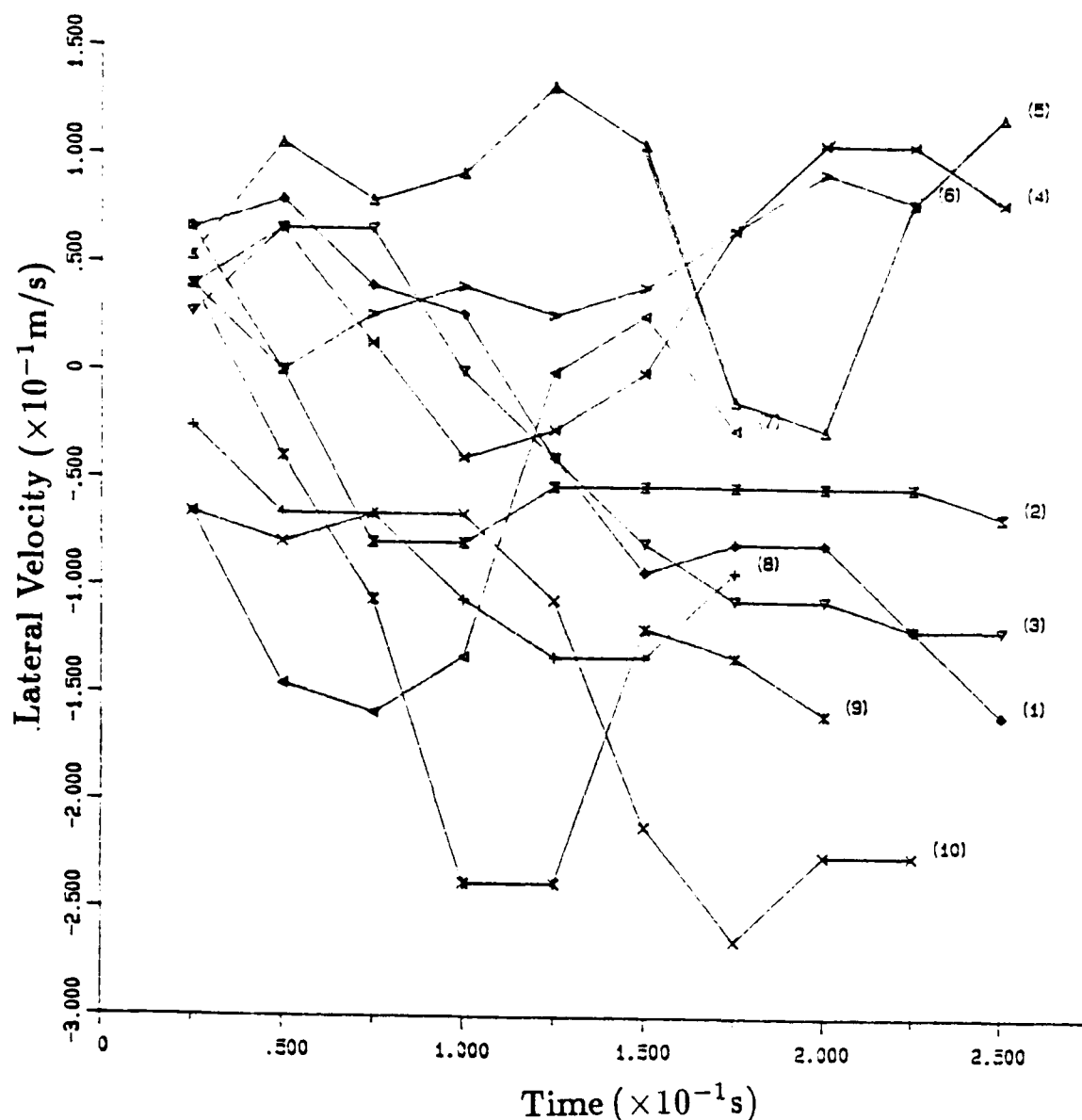


Figure 7.11: The Variation of the Lateral Component of Velocity with Time for Particle Set a6A.

An example of the lateral speed against time is shown in Figure 7.11. The magnitude of the speeds is much less than the longitudinal speed and no systematic variation can be seen, suggesting that it is dominated by the local flow conditions. The other sets of particles behave in a similar way.

7.2 The Motion of Particles in the Head of the Plume

It has been shown in Section 6.2.1 that the plume moves downwards with a constant speed. Deviations of individual particles from this movement will be more easily identifiable if the plume's speed is subtracted from them.

stream.

The paths of the particles in the head of the plume once transformed in this manner are shown in Figure 7.12–Figure 7.15, along with photographs that show their original positions.

Again four sets of particles were followed:

- set a5X which consisted of particles at the very front of the plume;

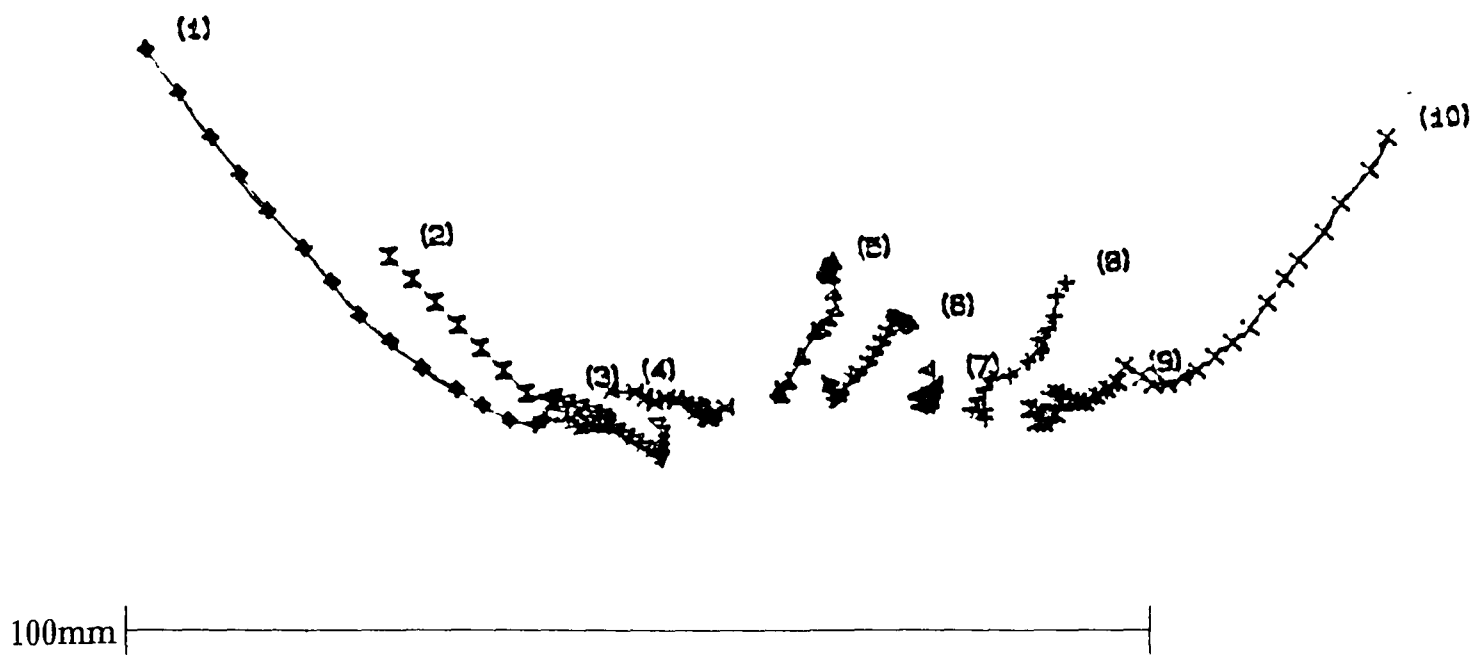


Figure 7.12: The Motion of Particles shown relative to the Front of the Plume for Particle Set a5X.

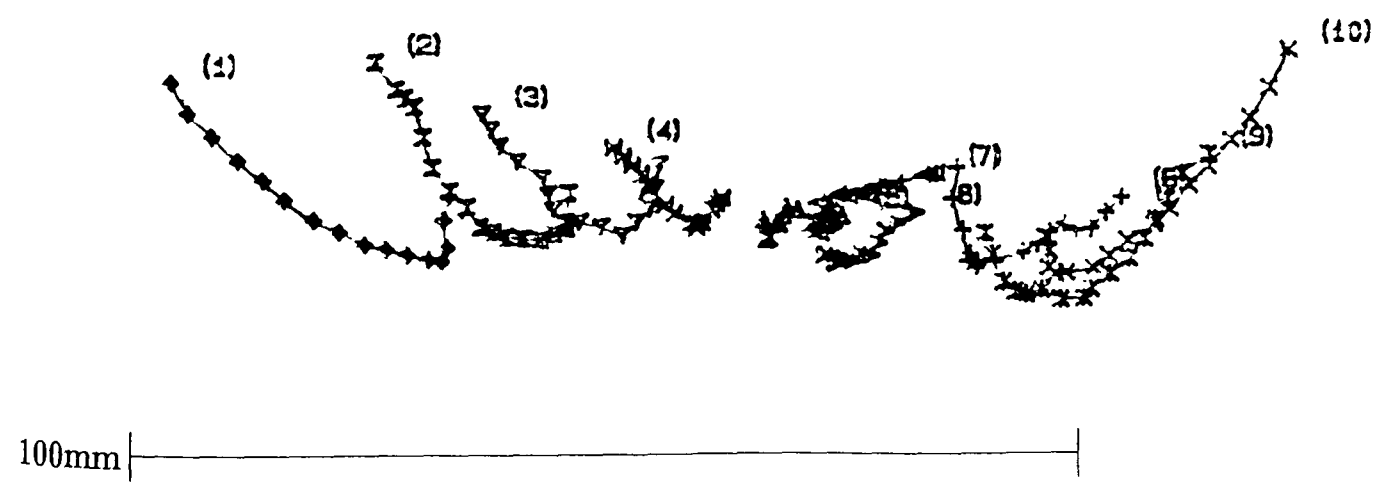


Figure 7.13: The Motion of Particles shown relative to the Front of the Plume for Particle Set a5Y.

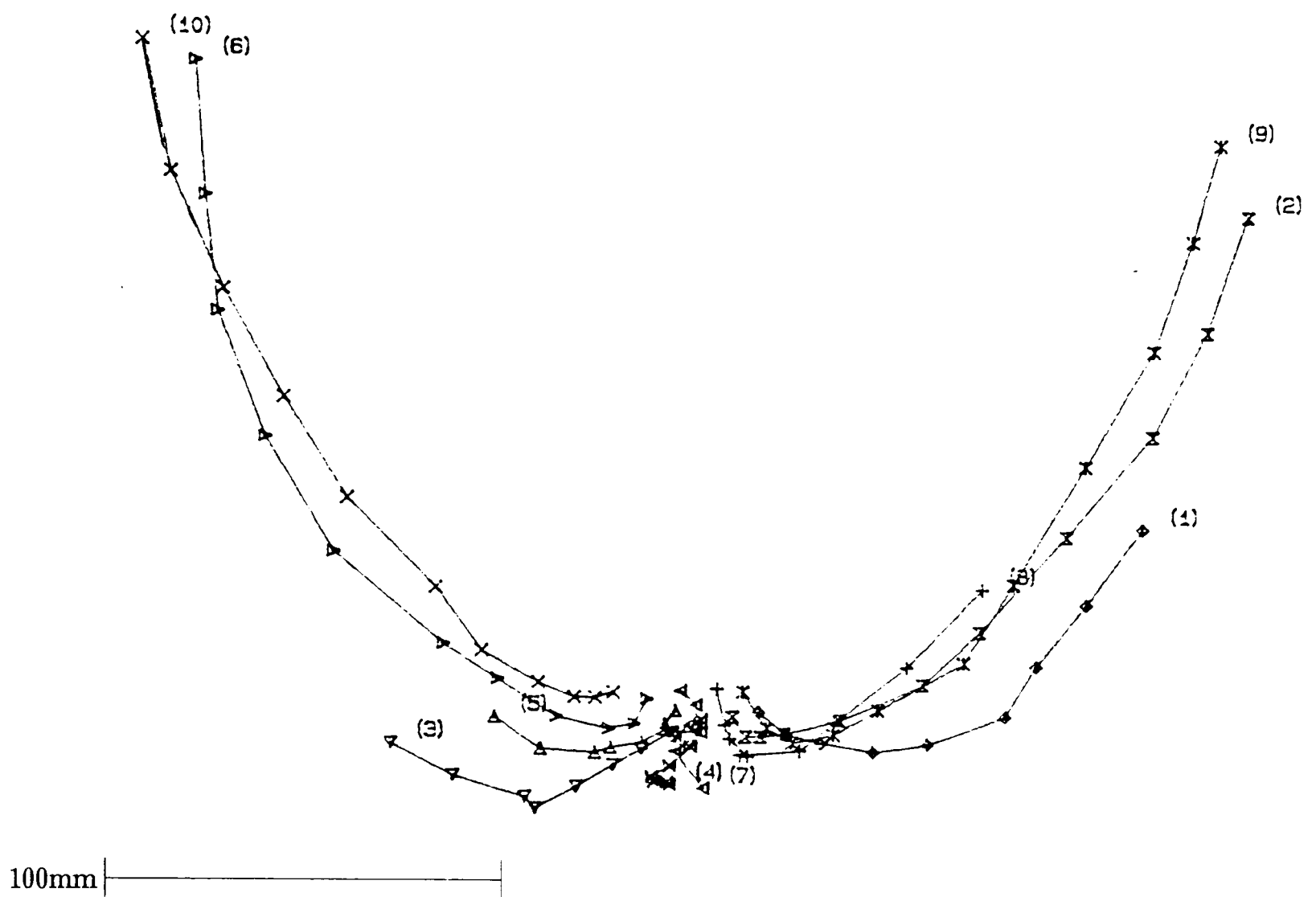


Figure 7.14: The Motion of Particles shown relative to the Front of the Plume for Particle Set a6X.

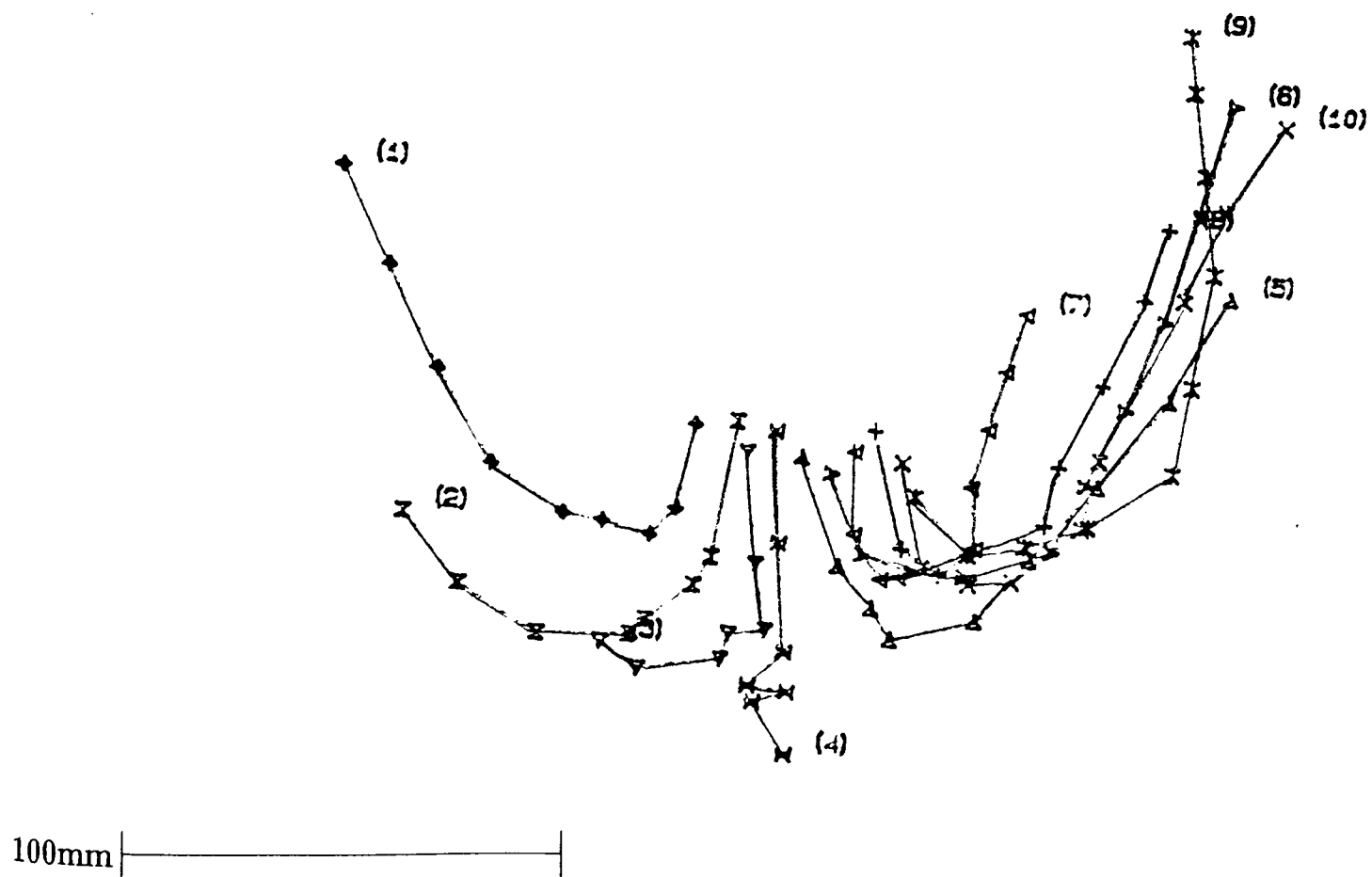


Figure 7.15: The Motion of Particles shown relative to the Front of the Plume for Particle Set a6Y.

- set a5Y which was a little way (about 20mm) behind set a5X;
- set a6X which consisted of particles at the front of the plume in experiment a6;
- set a6Y of particles close to the back of the head of the plume in experiment a6.

The paths of the particles are reasonably regular, particularly in the wings of the plumes. In addition particles could move substantially sideways without coming into contact with another. An exception is in the wing of particle set a6Y that contains particles 5, 6, 9, and 10 which are influenced by each other and other nearby particles.

Three regions of particles within the head may be identified:

1. particles near the centre of the plume that are almost static relative to the its front e.g. particles 5 and 7 of particle set a5X and particles 4 and 7 in set a6X. The membership of this class changes slowly, but particles at the edges can start to move outwards persistently, e.g. particle 5 in set a5Y, and new particles sometimes join the region from the column e.g. particle 4 in set a6Y.
2. Particles at the edge of the above region, but close to the front of the plume. These move downwards with a speed similar to that of the plume, but move sideways as well e.g. particles 3, 4, and 9 in particle set a5X. Their movement is slow, but persistent, and can result in particles leaving this region and becoming part of the plume's wings e.g. particle 2 in particle set a5Y. This region is most prominent in experiment a5 when the plume is close to the surface of the water pool and is not present in experiment a6 when it is further down the tank.
3. Particles towards the edges of the head e.g. particles 1, and 10 in all four sets of particles, or approaching the back of the head e.g. all the particles but particle 4 in set a6Y. These move towards the front of the plume before they are swept sideways and backwards into its wings.

These regions will be referred to by their number in the following discussion.

An extension of the identification of these regions is the hypothesis that the the entire behaviour of the plume is steady i.e. once the plume has become established the particles' velocities and accelerations are functions of their position relative to the front of the plume only; in other words, if a particle is selected at an instant and has a certain position relative to the leading point of the plume, the motion of another particle at that position at any other time will be exactly the same. This might be tested by plotting the velocities and accelerations

of particles against their positions to test for a unique relationship. This was quite a successful treatment for the velocities in experiment a6 but not for experiment a5, as will be seen, This and the fact that region 2 is present in a5 and not in a6 suggests that transitional behaviour takes place in a5 as the plume is established and it is not typical of the development of the plume.

The same method was not successfully applied to the accelerations of the particles; the result was irregular and not very systematic. This may be partly because the instantaneous accelerations will be affected more by local conditions than velocities and partly because the acceleration gradients could be quite large so that a small error in the position of a particle caused a large error in the acceleration assigned to that position. Despite this there was some consistency in accelerations over different regions of the plume and these will be used for estimating the forces on particles in the next chapter.

7.2.1 The Movement of Particles in the Lateral Direction

Figure 7.16 shows the lateral velocity component of the particles plotted against their lateral position for particle set a6X; the linear relationship between the two quantities over the central 140mm of the plume is striking. The gradient of the line is 6.0s^{-1} . Many of the particles (1, 5, 6, 9, 10) have similar accelerations away from the plume's centreline, lying between 1.2 and 1.4m/s^2 . Particles 2, 3, and 8 have higher accelerations, up to 2.7m/s^2 for particle 2. Outside this region the lateral speed of the particles drops towards zero with decelerations between 1.15m/s^2 for particle 10 and 2.47m/s^2 for particle 6.

A similar relationship results from particle set a6Y, also with a gradient of 6.0s^{-1} . This is shown in Figure 7.17. Note that the linear portion of the relationship is smaller (about 100mm) and so, therefore, are the maximum lateral speeds of the particles. The accelerations of the particles away from the centreline covered a similar range to those of set X, between 1.2m/s^2 (particle 10) and 2.7m/s^2 (particle 6), but were more evenly distributed within it. The decelerations of the extreme particles were less than for those of set X, mostly (four particles) between 0.20m/s^2 (particle 6) and 0.53m/s^2 (particle 1), though the deceleration of particles 2 (1.31m/s^2) and 10 (1.24m/s^2) was greater.

If lateral acceleration is a function of lateral position only then

$$\frac{dU_{py}}{dt} = U_{py} \frac{dU_{py}}{dS_y} \quad (7.1)$$

where the subscript 'y' refers to the lateral direction, U is speed, and S displacement. From Figure 7.16 the maximum acceleration is 2.4m/s^2 , which compares well with the maximum

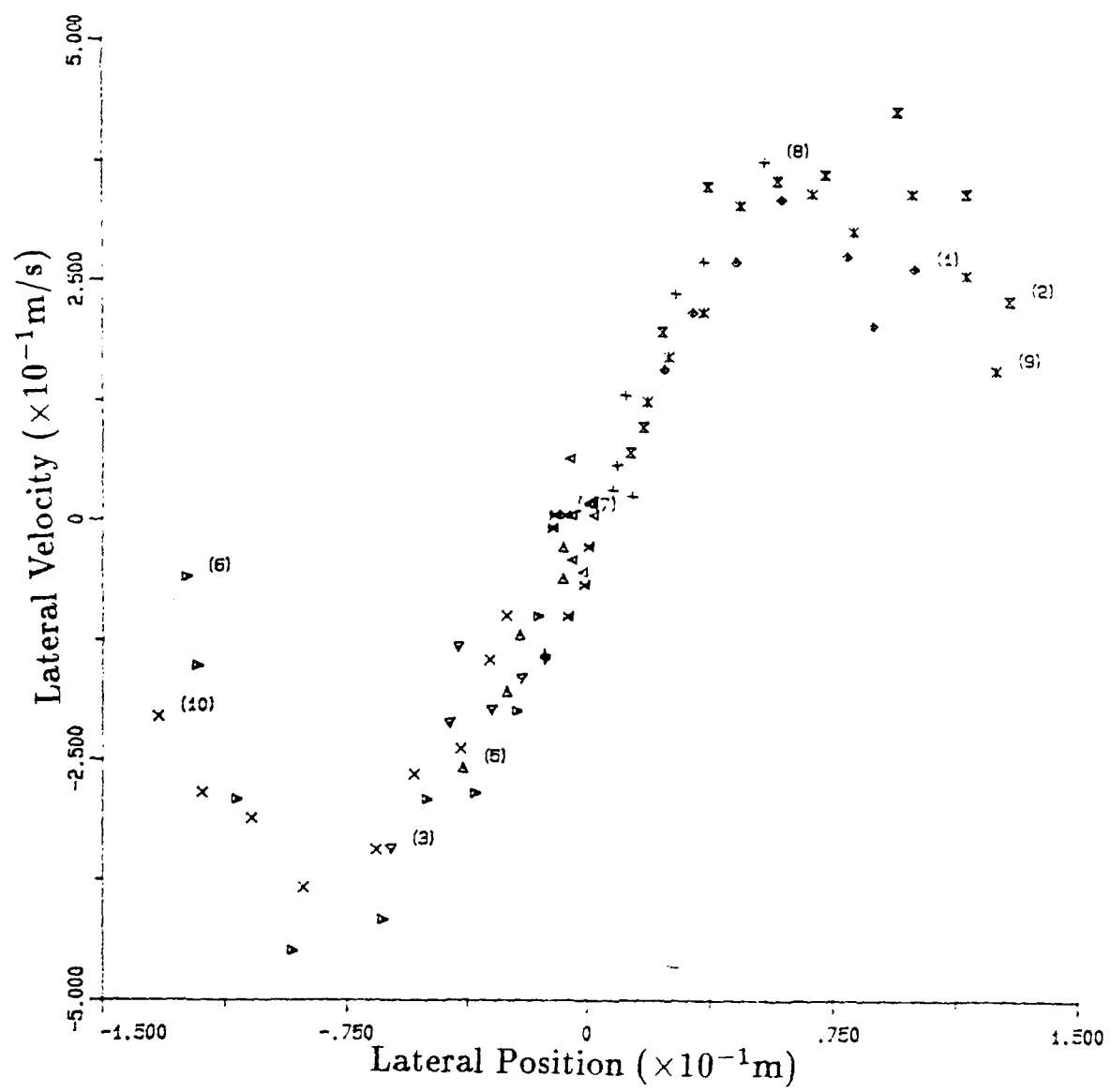


Figure 7.16: The Lateral Velocity Component of the Particles in Set a6X plotted against their Lateral Position

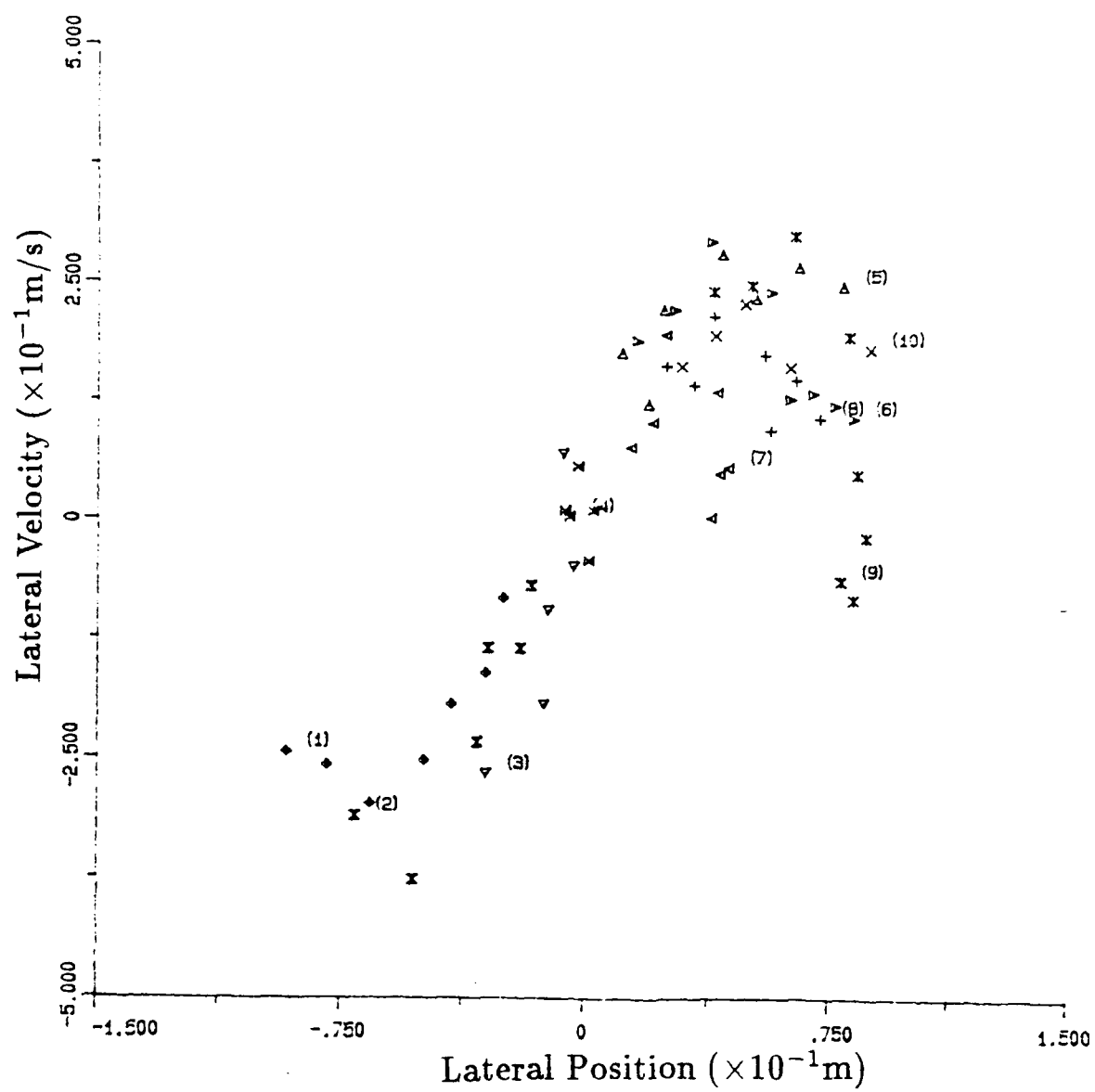


Figure 7.17: The Lateral Velocity Component of the Particles in Set a6Y plotted against their Lateral Position

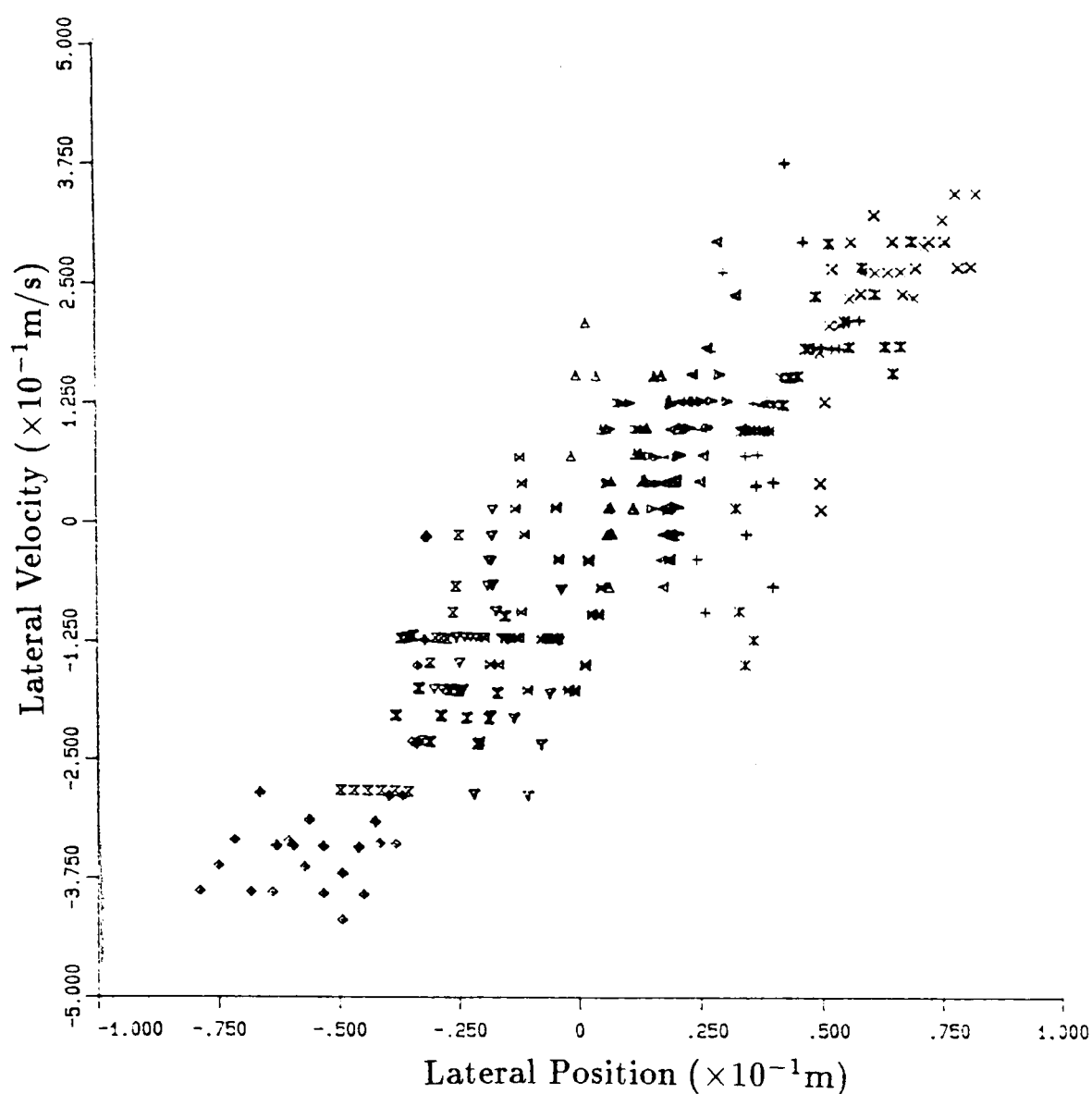


Figure 7.18: The Lateral Velocity Component of the Particles in both Sets of Particles in Experiment a5 plotted against their Lateral Position

measured acceleration.

Experiment a5 has a similar relationship between lateral velocity and position to that in experiment a6 (Figure 7.18) with a gradient of 5.4 s^{-1} . The data is more scattered, mostly because the plume has not settled down to a steady state. In addition some of the particles, such as those in region 2, move only a little between time steps; therefore the error of the measurement of their change in position is large and so is the error in the calculation of the particle velocities and accelerations. The measured accelerations varied from 0 to 12.5 m/s^2 for particle set X and 0 to 16 m/s^2 for particle set Y. These do not compare well with the result of (7.1) because the motion of an individual particle is dominated by its longitudinal position rather than its lateral one. The particles that are far from the centre of the plume (particles 1, 2, and 10 of set X and 1, and 10 of set Y) accelerate laterally until a steady speed is attained, between 0.20 m/s (for particle Y10) and 0.38 m/s (for particle X1) in magnitude.

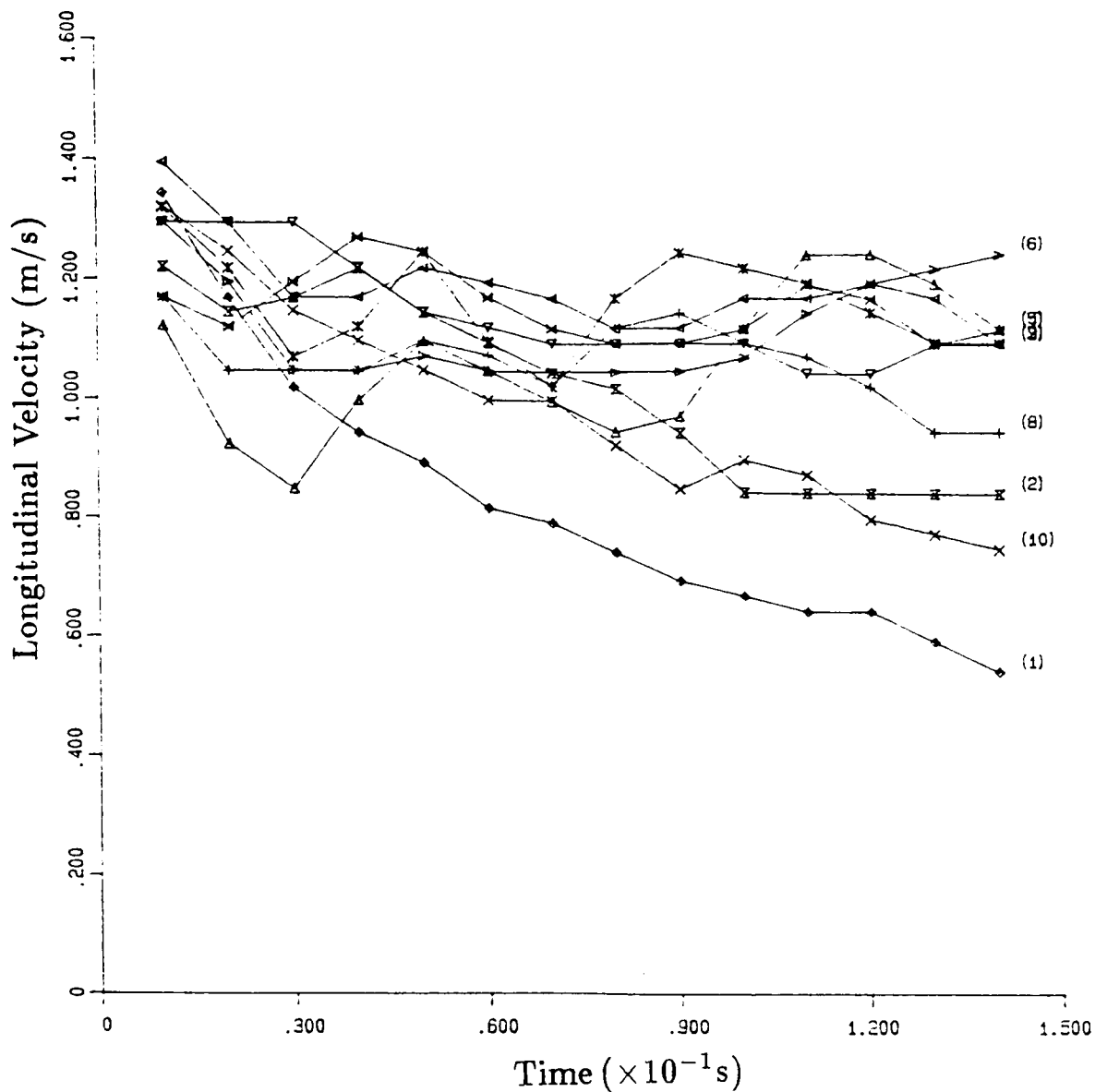


Figure 7.19: The Variation of the Longitudinal Velocity of the Particles of Set a5X with Time

7.2.2 The Movement of the Particles in the Longitudinal Direction

Proceeding in the same manner for the longitudinal position is more difficult. The particles in experiment a5 are not suitable for this treatment because they all have similar longitudinal positions and move unsteadily near the water surface. Instead the variation of particle velocity with time is shown in Figure 7.19 for particle set 5X and Figure 7.20 for particle set 5Y. All the particles in experiment a5 initially have an initial deceleration of up to 18m/s^2 . This is the same as that felt by a single, isolated particle owing to drag when it is moving with a speed of 1.5m/s relative to the surrounding water, if the local flow conditions and transient forces are ignored. This is similar to the initial speed of the particles in a5, though care should be taken because the calculated velocity of the particles relative to the fluid is not very sensitive to their acceleration. The length of time of this deceleration depends upon a particle's position in the plume; most decelerate until they near the front of the plume. The particles at the extreme edges of the plume (particles 1 and 10 of set X and 9 and 10 of set Y) continue to decelerate as they move away from the head of the plume, but at a slower rate varying from 3.1m/s^2 for particle Y1 to 4.6m/s^2 for particle X2. If this deceleration is wholly attributed to drag the relative velocity between the particles and the water is within 10% of the downward speed of

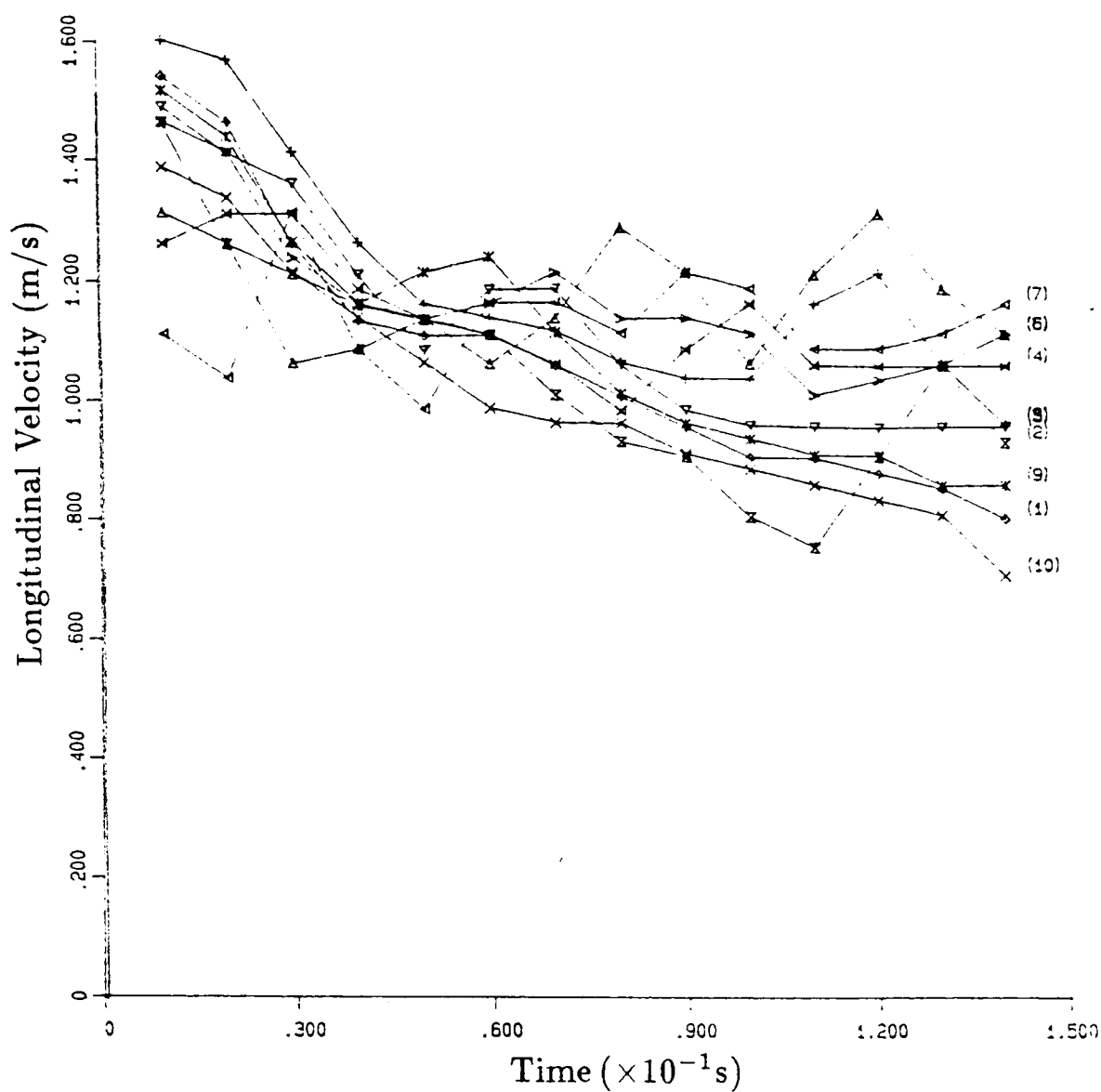


Figure 7.20: The Variation of the Longitudinal Velocity of the Particles of Set a5Y with Time the plume. A possible conclusion is that the water is nearly stagnant though this must be tentative because again the relative velocity is not very sensitive to acceleration.

The influence of the lateral position on the longitudinal component of velocity is prominent at early times in particle set a6Y, because its members have just left the column of the plume. This is shown in Figure 7.21 and causes some scatter if longitudinal speed is plotted against longitudinal position, as in Figure 7.22. Both quantities are relative to the front of the plume. Further scatter is generated by the interactions between particles in the right hand wing of the plume. Despite these there is some correlation between the two quantities. The deceleration of the particles before they are brought to rest with respect to the front of the plume varies from 3.1m/s^2 for particle 6 to 5.9m/s^2 for particle 5, which are similar to the deceleration of particles in the column as they enter the head (particle set a6A, page 122). Particles 3 and 4 remain close to the front of the plume, but the other particles move backwards with respect to it with accelerations between 2.2m/s^2 for particle 8 and 2.9m/s^2 for particle 2.

The dependency of longitudinal velocity on lateral position is much weaker for particle set a6X because its members start close to the front of the plume where they are nearly stationary with respect to it. Longitudinal speed is plotted against longitudinal position in Figure 7.23.

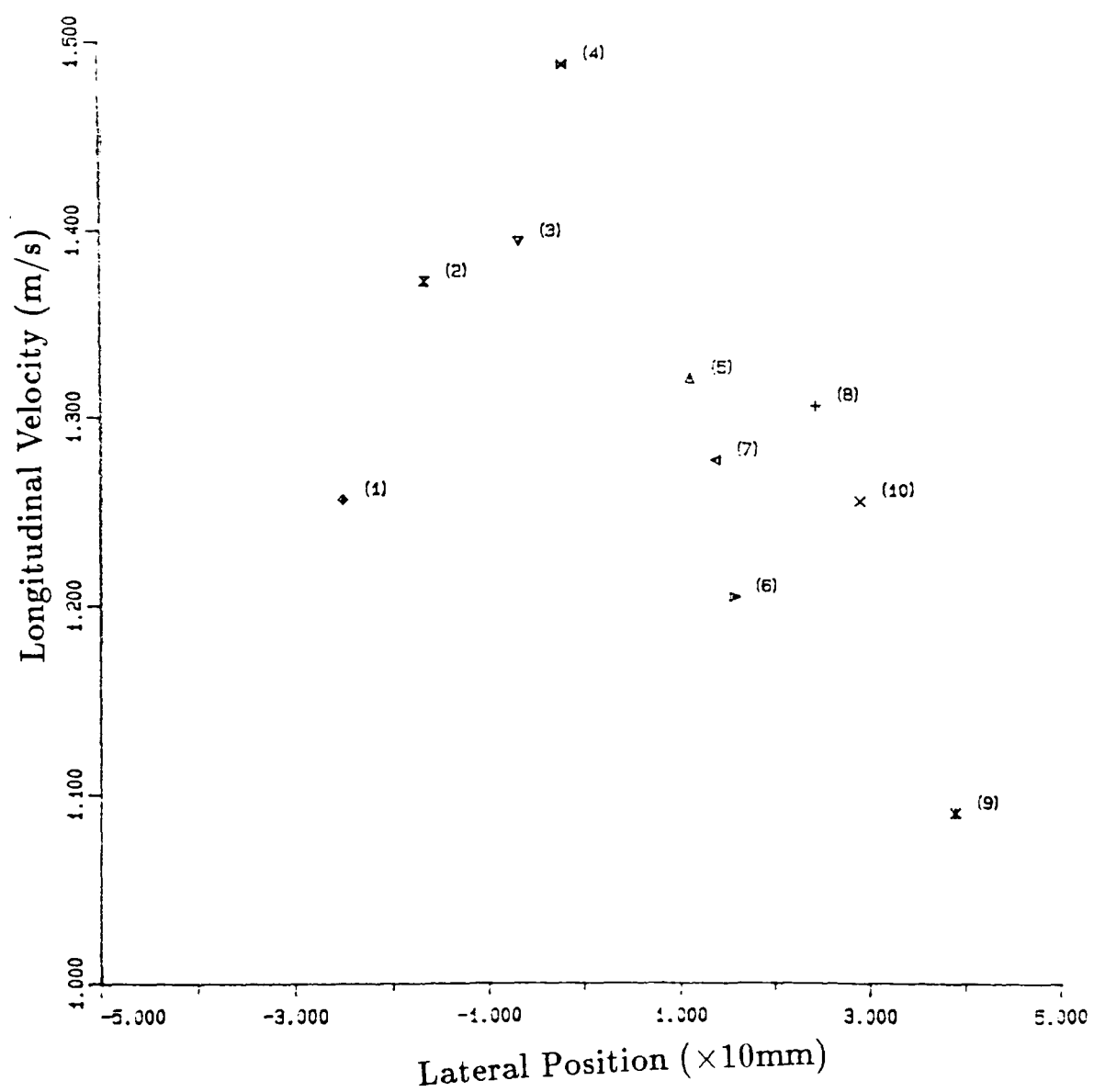


Figure 7.21: The Variation of Longitudinal Speed with Lateral Position on Entry of the Particles into the Column for Particle set a6Y.

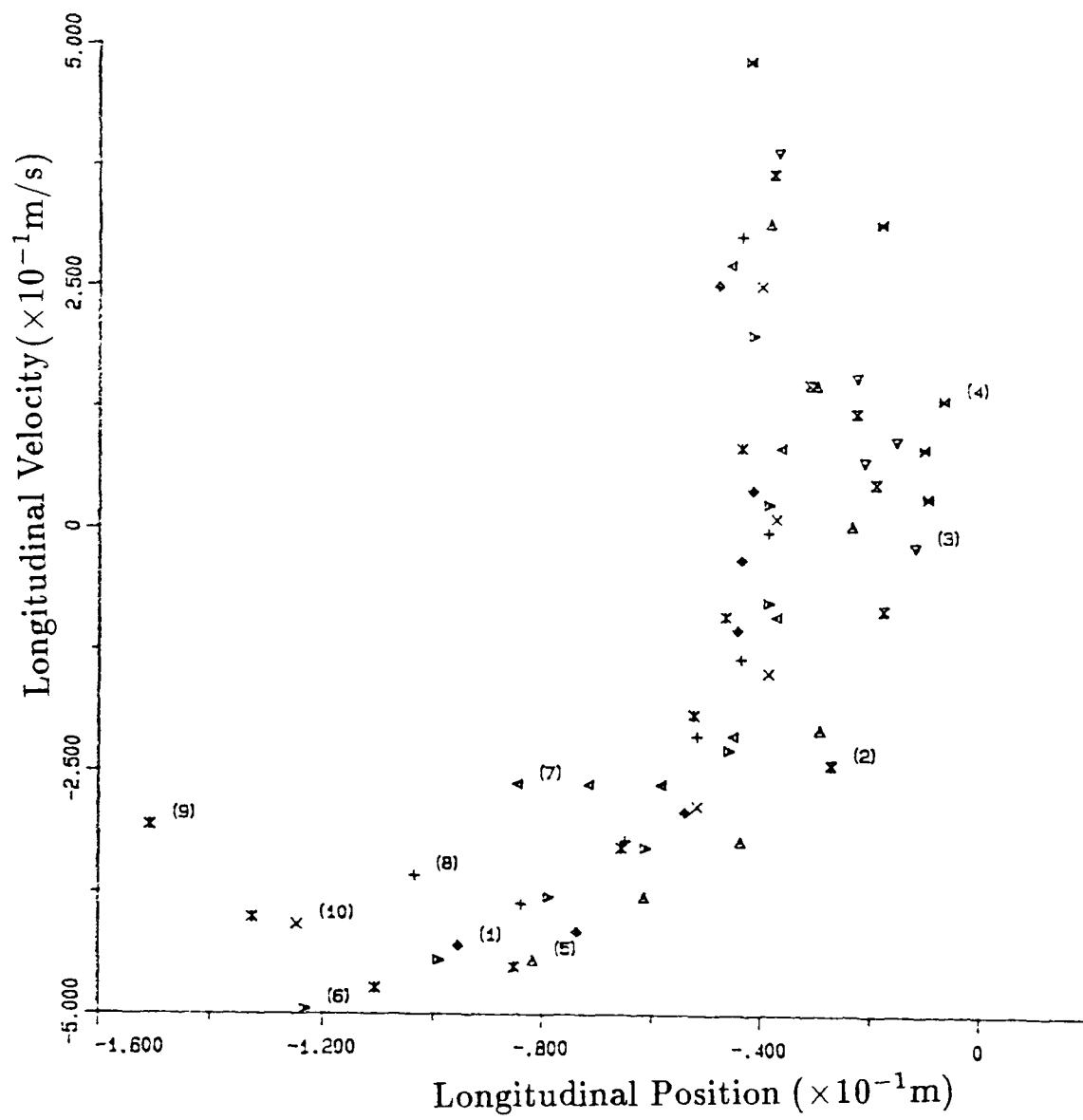


Figure 7.22: The Longitudinal Velocity Component of the Particles in Set a6Y plotted against their Longitudinal Position relative to the Front of the Plume.

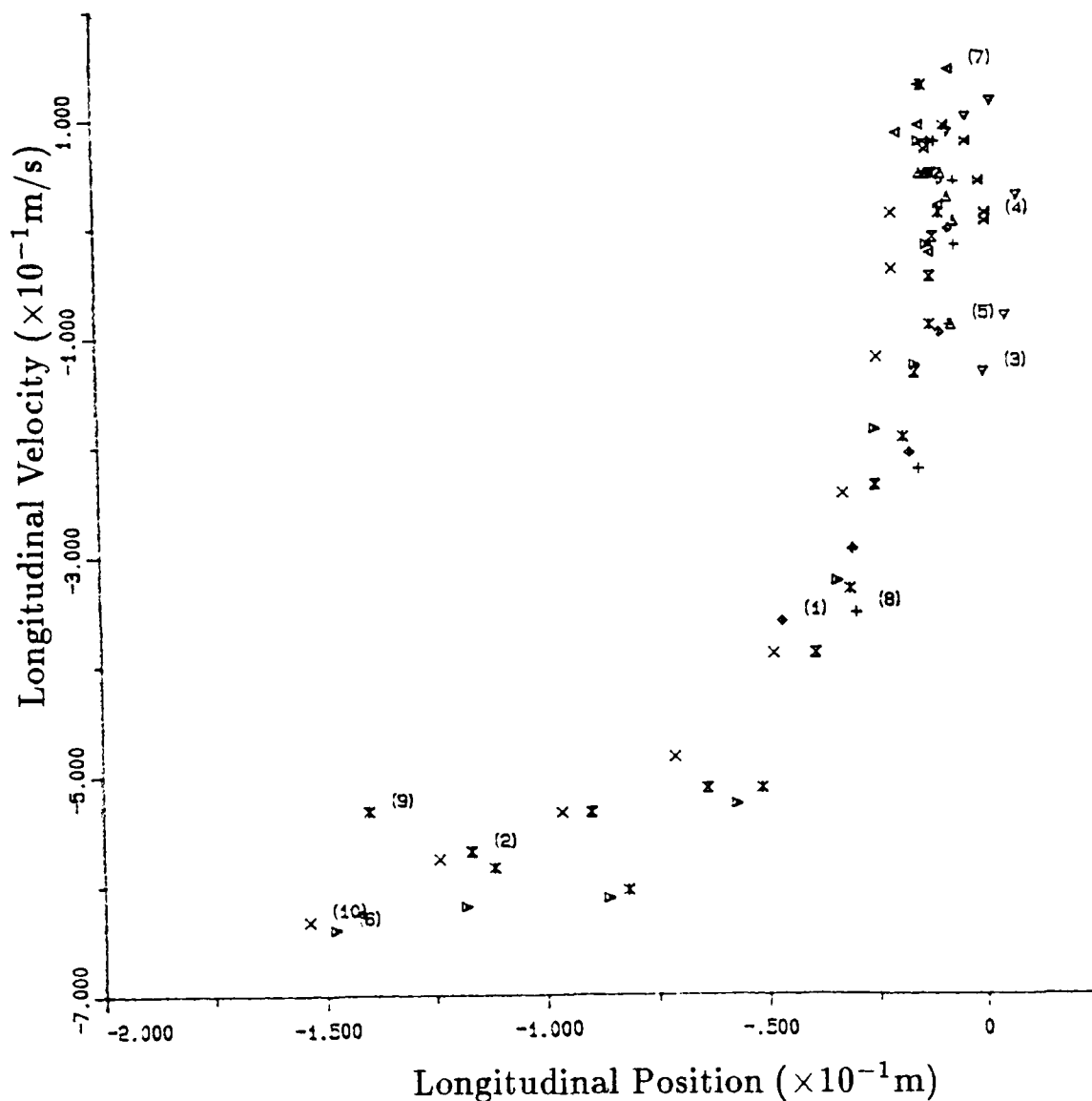


Figure 7.23: The Longitudinal Velocity Component of the Particles in Set a6X plotted against their Longitudinal Position relative to the Front of the Plume.

A unique line is defined, suggesting again that once the particles start to move away from the front of the plume the effects of unsteadiness and the velocity distribution in the column have been eliminated and the longitudinal speed of particles is mostly a function of longitudinal position. With the exception of particles 4 and 7, which remain in the central part of the front of the plume, the particles accelerate away from the front of the plume at rates between 2.1m/s^2 (particle 3) and 3.8m/s^2 (particle 8).

The particles in the wings of the plume in a6 tend to a steady longitudinal speed. In particle set a6X their speed (relative to a fixed origin) lies between 0.35m/s (particle 6) and 0.45m/s (particle 9). In particle set a6Y it lies between 0.47m/s (particle 6) and 0.69m/s (particle 9). Because the speeds of these particles are not changing very much the speed of the water surrounding them should not either. The slip between the particles and water in a steady flow is fixed; therefore it is possible to estimate the longitudinal component of velocity of the water flow along the sides of the tank. If the particles are not interacting with each other the slip will be equal to 0.84m/s , the terminal speed of an isolated particle, then for particle set a6X the approximate speed of the upwards flow is 0.43m/s and for particle set a6Y is 0.30m/s .

Chapter 8

Discussion of Experimental Results

This chapter will discuss the results of experiments and offer possible explanations for the behaviour observed.

If the transient terms of equation (3.29)¹ are neglected two forces remain: buoyancy and drag. Buoyancy is constant in magnitude and direction; the drag will change with the state of the fluid at the interface between the two phases; therefore a particle will only deviate from a downward path and its terminal speed through the influence of water currents. This will also be true for a particle in a multiphase flow if collisions between particles are neglected.

Two consequences of the size and mass of the particles in the experiments (and in coarse mixing) are that their inertia is important, and their motion affects that of the water.

The motion of a particle moving in the lateral direction may be crudely described by the equation

$$\frac{\pi}{6}d_p^3\rho_p\frac{dU_p}{dt} = \frac{1}{2}\rho_w\frac{\pi}{4}d_p^2(U_w - U_p)^2C_D. \quad (8.1)$$

If this is integrated it can be shown that if a particle such as those in the experiments² were placed in a water stream it would take 1.3s to accelerate from rest to 90% of the stream's velocity. This is not so true for aluminium particles which would take 0.4s to reach 90% of a water stream's speed and are in the tank for a longer time because of their lower terminal velocity. These times are for a steady current; as the particles move they will come under the influence of different currents so that they may never accommodate one current before entering another.

A result of this behaviour is that the particle paths are streak lines, not the water's streamlines: in a steady flow the paths of light particles will be the same as the paths of elements of

¹The equation of motion of a single, isolated particle.

²Stainless steel spheres with a 6mm diameter.

the surrounding fluid; if the particles are heavy they will have a significant relaxation time and will not reflect the fluid element paths; the paths of heavy particles depends on their history as well as the current state of the fluid. Unless the particles are moving steadily it will not be possible to deduce any but the most general features of the water's motion from that of the particles.

If the particles are moving in the longitudinal direction the effect of inertia is less extreme because the movement of a particle is driven by its buoyancy. For example if the buoyancy force $\frac{\pi}{6}d_p^3g(\rho_p - \rho_w)$ is included in (8.1) if solved the time for the speed of an isolated particle to drop from 1.7m/s³ to within 10% of its terminal speed is 96mm. The longitudinal speed of the particle will never be that of the surrounding liquid because buoyancy must be balanced by drag: drag is dependent on the relative velocity between the particle and the surrounding liquid so that if the buoyancy force is significant, so must the relative velocity.

Another consequence of the size and mass of the particles is that the motion of the water and that of the particles are not independent. The particles will affect the water owing to the distortion of the flow as it passes round them and through the transfer of momentum to the fluid when they have a large size or concentration. The change in the flow will affect the distribution of particles which in turn will affect the particle distribution and this will modify the flow, and so on.

8.1 The Column of the Plume

A description of the column of particles may be found on page 80. The motion of individual particles within the column is described in Section 7.1, starting on page 119.

The particle distribution in the column is disorganized and groups of particles continually grow and dissolve. The particles (the paths of which are shown in Figure 7.1–Figure 7.4) can move in the lateral as well as the longitudinal directions, though not in a discernibly systematic manner.

It has been shown that small groups of particles can move in a regular manner with respect to each other, and form stable structures [77]. Large groups of particles (large enough to satisfy the length constraints (2.5) and (2.12); in a fluidised bed for example) under certain circumstances can form stable structures, but under others the arrangement appears to be unstable [99]. A reason for this behaviour is the interaction of two sorts of concentration waves that can pass through a particulate flow.

³The speed of the particles in a normal plume when entering the water pool.

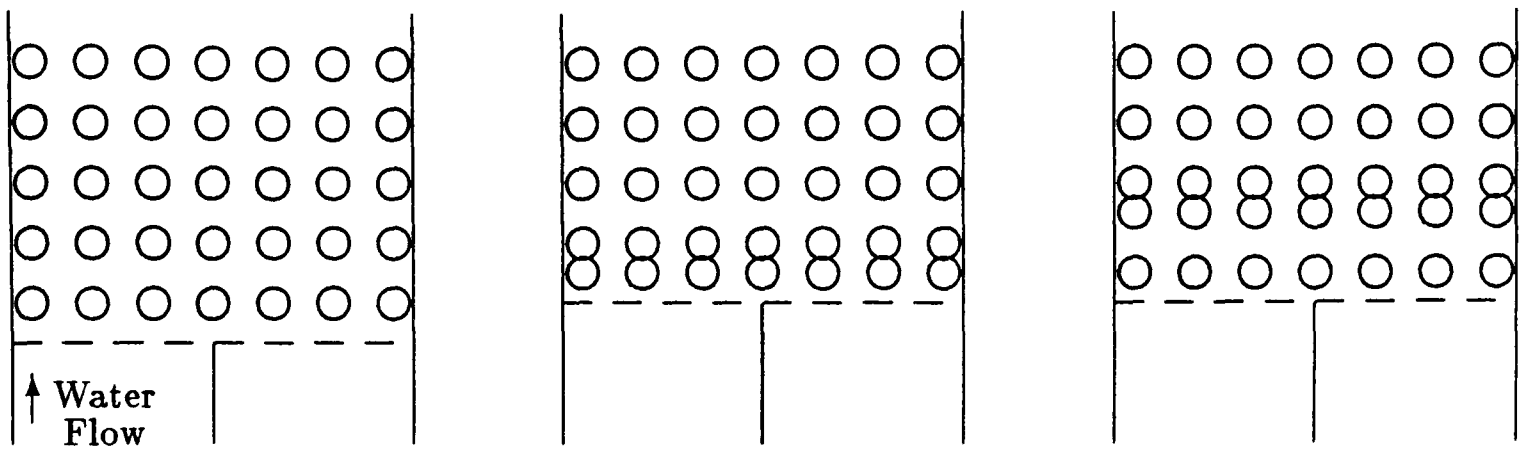


Figure 8.1: Diagram Illustrating the Origins of Dynamic Waves.

The first is the dynamic wave which occurs because the drag on a particle, as discussed in Section 3.2.3, increases with particle volume fraction (i.e. a function of the proximity of neighboring particles); if some particles in a stable array are perturbed the mixture will try to regain its equilibrium through the propagation of a concentration wave. For example consider the situation in Figure 8.1 where there is a stable, fluidised bed in a cylinder with a perforated piston at the bottom. If the piston is raised the bottom layer of particles approaches the one above it. The latter will be repelled because of the approach of the nearby particles, bringing it close to the next layer. The process is repeated and a wave of high particle concentration passes the bed.

The wave is analogous to an elastic wave in a single-phase fluid, opposing the expansion or compression of a particle arrangement. The speed of such a wave is given by

$$U_e = \sqrt{\frac{\partial p}{\partial \rho}}. \quad (8.2)$$

In a vertical, one-dimensional particulate flow the difference between the drag force and the buoyancy is analogous to pressure in a single phase flow and the mass of particles per unit volume ($\alpha_p \rho_p$) is analogous to density. An expression has been found for U_e using these ideas [100] (P.U.Foscolo and L.G.Gibilaro). The drag on a single particle is evaluated using the Richardson and Zaki equation, (3.36). The result is [100]

$$U_e = \sqrt{3.2gd_p\alpha_p \left(1 - \frac{\rho_w}{\rho_p}\right)}. \quad (8.3)$$

The other type of wave is the continuity wave. If a stable group of particles are falling steadily their velocity will be determined by the force necessary to balance their buoyancy. This force is dependent on the particle volume fraction and velocity so if the former changes the latter must adjust to maintain equilibrium; therefore a perturbation in the particle velocity

in one area of a mixture will propagate a wave of changed particle volume fraction through the rest of it.

The speed of a continuity wave after an infinitesimal variation is [100]

$$U_{\alpha} = \alpha_p \frac{dU_r}{d\alpha_w}, \quad (8.4)$$

where U_r is the relative velocity between the particles and the surrounding liquid. If the Richardson and Zaki expression for U_r is used [100]

$$U_{\alpha} = nU_t\alpha_p(1 - \alpha_p)^{n-1}. \quad (8.5)$$

The interaction between these waves determines the stability of a particulate flow. There are three possible states: a stable arrangement of the particles; an unstable arrangement, where bubbles form within the bed and move through it creating regions of high and low particle concentration; and an intermediate, transitional state where areas of high voidage appear, but persist. The state of a fluidised bed (and therefore a sedimenting system as well) depends on the relative magnitudes of the two wave speeds such that [101] (L. Gibilaro et al)

$$U_e - U_{\alpha} = \begin{cases} +ve & : \text{system stable (homogenous)} \\ 0 & : \text{stability limit} \\ -ve & : \text{system unstable (bubbling)} \end{cases} \quad (8.6)$$

In the case of stainless steel or aluminium particles with a diameter of six millimetres the system is always unstable except at low concentrations⁴ so that a sedimenting group of these particles nearly always consists of shifting voids and concentrations of particles except at very low concentrations. The model is supported by experimental evidence [101].

The bunching of particles in the plume will affect their speed locally because the drag force on a particle will change with α_p and it will accelerate. In addition particles in a group do not move independently of each other: momentum is transferred to the water in the same direction as the particles; this allows the group to act more like a solid body, diverting water around it and reducing the drag on the particles, even if they are very small [102] (E.F. Hinch). This may be the reason for different sets of particles having different ranges in their downwards speed.

Nearly all the longitudinal velocities of the particles exceeded that of an isolated particle⁵. From the results of the model described on page 90 the effect of a high entry speed into the pool should have been eliminated in about the first 0.1s; therefore away from the surface the

⁴2% for stainless steel particles and 9% for aluminium ones in a cylindrical system. In the plane tank these thresholds may be different.

⁵An exception is particle 10 of particle set a6B.

particles can only move faster than their terminal velocity if the water through which they are falling is also moving downwards. The only source of momentum for this movement, away from the head of the plume where there may be a wake, is the transfer of momentum from the particles to the water. If the particles are moving steadily the speed of the water may be estimated by subtracting the slip velocity from the particles' velocity. This can only be a rough estimation because the slip velocity will vary with the particle concentration; but if the slip is as for an isolated particle then the water velocity varies from 0.15 to 1.24m/s.

If the slip is assumed to be reasonably uniform across a section of the column then because the profile of the particle velocity in the lateral direction has the appearance of a gaussian distribution (Figure 7.9–Figure 7.10), so will the water velocity profile. This is consistent with the velocity profile of a steady, buoyancy driven fluid ('buoyant') jet in stagnant surroundings described in [103](R.S.Turner). It is also apparent from the paths of particles (such as those marked by particles' cavities described on page 80) that is entrained from the surroundings into the column as in a buoyant fluid. However the effect of the entrainment is different: in buoyant jets it allows the velocity profile to flatten and broaden; in particulate jets it maintains the profile. This is because the particles which are the source of the momentum driving the water do not move outwards because of the inertia of the particles and the entrainment flow into the column prevent.

At early times a depression in the surface of the water pool forms owing to the momentum transferred from the particles to the water in excess of that required to overcome dissipation. Later on the depression disappears; therefore from the Bernoulli equation the downwards velocity of the water close to the top of the pool is about 1.2m/s. This view is supported by the movement of particles in the column close to the surface: most particles of set a5A decelerate (see Figure 7.5) but those of set a5B actually accelerate (see Figure 7.6). A sketch of the possible water flows outside the head of the plume is shown in Figure 8.2.

8.2 The Head of the Plume

Three areas of interest may be identified concerning the head of the plume:

- the speed of penetration of the plume;
- the motion of the particles in the wings of the plume;
- the origin of the lateral movement of the particles.

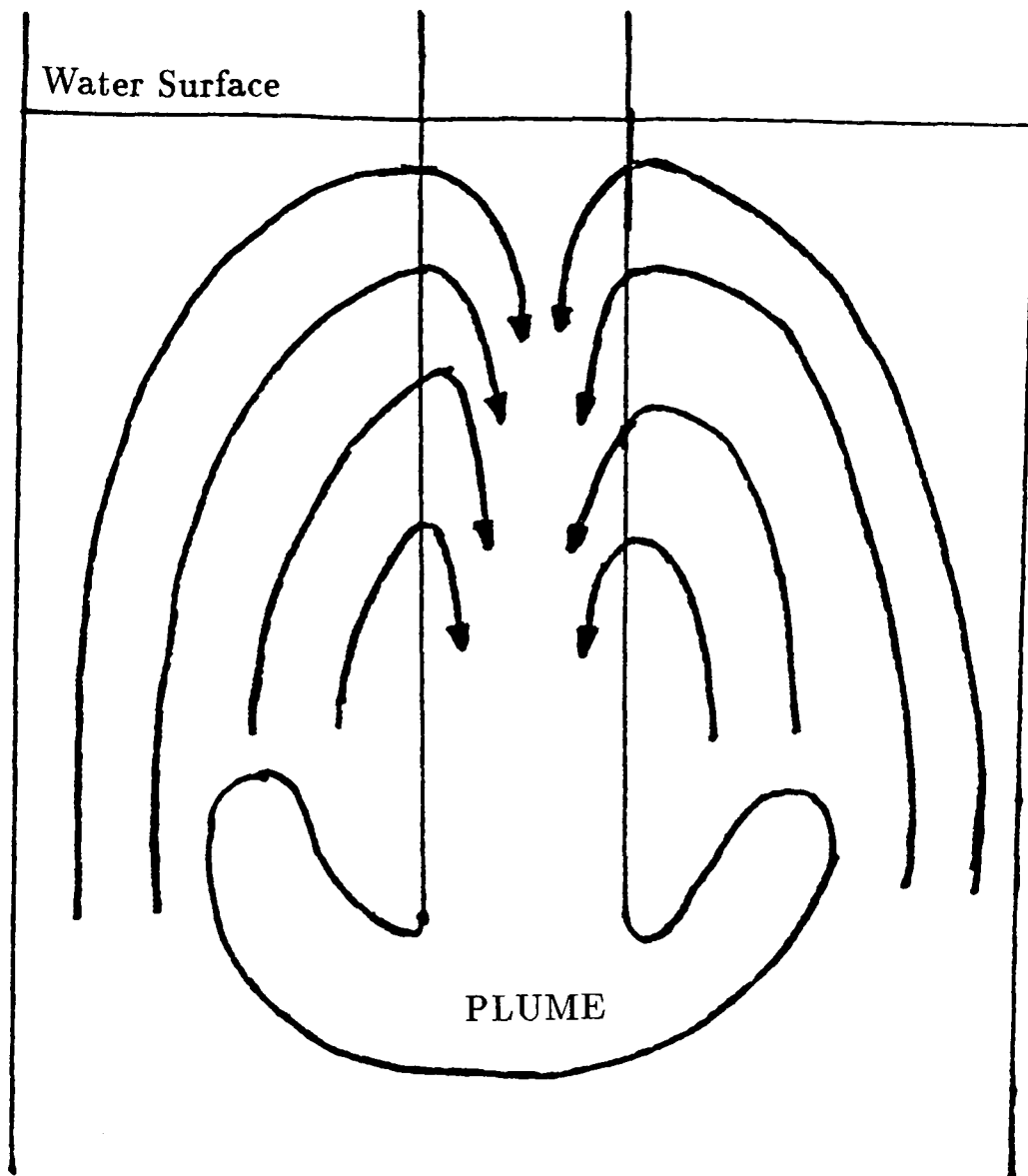


Figure 8.2: A Sketch of the Water Flows outside the Head of the Plume.

8.2.1 The Penetration of the Plume

The penetration speed of the plume always exceeds the terminal speed of an isolated, single particle. It was demonstrated in Table 6.1 and on pages 107 and 110 that the rate of penetration of the plume was 1.0m/s for the initial and boundary conditions in the experiments with the possible exceptions of the sparse plumes (experiment set h) and the narrower plumes (experiment sets d and j) where the rate is reduced by up to 10%. Apart from these, the 'body' at the front of the plume acts independently of the projection velocity of the particles (between 0.68m/s and 1.16m/s, which correspond to 1.0 and 2.0m/s at the water surface) and tank width (between 0.275m and 0.475m).

The front of the plume acts like a single body so that the total drag on the particles is reduced. In Section 5.2 it was shown that the experimental results are consistent with motion dominated by buoyancy and drag forces. Apart from density, from page 99, the speed of a body falling freely depends on V_b/A_b ⁶ and the drag coefficient C_D , both of which depend on the geometry of the body.

⁶ V_b is the volume of the body and A_b its projected area.

The insensitivity to speed might be expected because, using the model described on page 90, it⁷ has little effect on the separation between subsequent rows, especially when real effects, such as wakes, are taken into account. In addition the terminal speed of a particle is independent of its initial speed.

It is difficult to delimit the ‘body’ formed at the front of the plume. Some liquid must be diverted around the front of the plume, and the question is where it passes through the head of the plume. A possible place is near the edge of the column of the plume: the particles in the column are decelerating with respect to the front of the plume, transferring momentum to the water and opposing the fluid trying to enter the front of the head, forcing it sideways; hence it is possible that the structure of the ‘body’, and so the penetration speed, will depend on the supply of momentum from particles entering from the column. This will probably be independent of the initial particle speed and the tank width but not of the width of the jet.

In sparse plumes α_p is much reduced so that the amount of momentum per unit volume transferred to the water is also reduced and water may penetrate between particles or groups of particles increasing the amount of drag per particle, reducing the plume’s penetration velocity.

8.2.2 The Wings of the Plume

The flow in the wings of the plume appear to be similar to that in the plume column with the formation and dissolution of groups of particles⁸ and the path and velocity of many particles appear to be affected by their local surroundings (e.g. see the paths of the particles in set a6Y (Figure 7.15) that enter the right hand wing of the plume). The lateral deceleration of the particles will determine the overall width of the plume. From Figure 7.16 and Figure 7.17, which shows the lateral speeds of individual particles, and Figure 6.11, which show the growth of the overall width of the plume, when the particles enter the plume’s wings they are decelerated in the lateral direction. From Figure 7.21 and Figure 7.22, which show the longitudinal speeds of particles, and Figure 6.13, which shows the growth in the length of the head of the plume, they are also decelerated in the longitudinal direction and move away from the front of the plume; they acquire a steady downwards speed.

From photographs of plumes it is apparent that the particle stream in a wing wheels round from the centre of the jet rather than the distance from the plume’s centreline governing the

⁷And so the speed of particles and the particle volume fraction at the water surface within the range given in Table 6.1.

⁸For example compare the distribution of the particles in the wings of the plume shown at different times in Figure 5.10 and Figure 5.13. In both cases the distribution is non-uniform and is different in form at each time.

individual motion of the particles. For example in Figure 5.13, a photograph of a normal plume, the width of the wing is not substantially smaller away from the central region of the head than when close to it. Also, in Figure 7.14 and Figure 7.15, diagrams of the paths of the particles in the head, the radius of curvature of the particle paths increases the closer they are to the plume's centreline. Over the range of conditions tested, apart from the sparse plumes⁹, the greatest influence on the width of the plume was exerted by the width of the tank compared with that of the jet¹⁰. Much of the effect of the jet width may be eliminated if the difference between the edge of the plume and the edge of the column is taken instead of the distance to the plume's centreline as shown in Figure 6.19, though more data is required to confirm this generally.

In the longitudinal direction when the particles leave the central portion of the plume they leave a region where the water is moving downwards and enter one where it is, at best, stagnant; thus they are retarded and move away from the front of the plume. In fact, because the extreme particles fall downwards steadily at later times with a speed less than that of an isolated particle, the water may be flowing upwards with a velocity that was estimated- on page 141- to be between 0.30 and 0.43m/s. Some of this will be because of water displaced around the impermeable front of the plume: by continuity if the central 120mm of the plume is impermeable and moving downwards at 1m/s the average upwards velocity of the water in the rest of the tank is 0.34m/s. This is within the estimated range of the speed of the flow upwards, but cannot be the only influence because otherwise the rate of growth in the length of the head would be larger if the tank was narrower; Figure 6.14 contradicts this.

It is difficult to find out exactly what is happening in the wings of the plume because several different mechanisms may be operating, the motion of particles will modify the flow, and the particles will not exactly follow a flow because of their large inertia.

The form of the particulate plumes is similar to that of a jet of liquid falling into another, less dense liquid, such as that in Figure 1.11. These can be modelled well using free streamline theory if the effects of instability are ignored (D.F.Hopkins and J.M.Robertson [104]). In a particulate jet it might be postulated that the water set in motion by particles in the column acts like a buoyant jet and sweeps the particles sideways in its flow. This is, in fact, unlikely because the liquid jet has to have a velocity towards the front of the plume i.e it must have an

⁹Experiment set e.

¹⁰Compare the effect on the growth in the width of the plumes in the narrow tank (experiment sets f and g, Figure 6.12) and if one side of the plume coincides with the side of the tank (experiment sets i and j, Figure 6.18) compared with the effect changes in initial conditions have on the growth of plumes (Figure 6.11).

absolute downwards speed of more than 1.0m/s. On page 146 it was estimated that the water velocity had a maximum of 1.24m/s and that would be found only sometimes for particles towards the centre of the column; the water speed is likely to be less than 1.0m/s in most of the column and will be left behind by the front of the plume; therefore some water may enter the head, but only over a small part of the column, perhaps only at certain times. In addition, if the plume was acting like a liquid jet the water's longitudinal velocity (and therefore the particles') would decrease as material approached the front of the plume to compensate for the increase in its lateral velocity. Though the members of particle set a6A do decelerate as they approach the head, this is not accompanied by a simultaneous increase in their lateral velocity, as shown in Figure 7.3. The energy lost by the particles must then be dissipated, perhaps in the two large vortices that have been observed behind the head of the plume (see page 105 for an example).

8.2.3 The Spreading of the Plume

The spreading of the plume close to the front of the plume is its most remarkable feature. It was shown in Figure 7.16–Figure 7.18 that over about the central 120mm of the head of the plume the lateral velocity of particles is proportional to their distance from its centreline and outside this region their lateral speed tends to zero. The lateral motion of the particles must be the result of the motion of the water. This is governed by the structure and movement of the particles towards the centre of the plume. The penetration speed of the plumes is not influenced by the width of the tank, suggesting that this does not affect the structure of the plume so it is possible that the lateral acceleration of the central particles is independent of the projection speed of the particles and tank width.

Two forces may be exerted on the particles by the water: a static force owing to its stress field, and a dynamic force owing to it dragging particles with it when in motion. The two forces are inseparable because a water flow will result from a pressure gradient; however it is possible to estimate their relative magnitudes.

The 'body' that is impermeable in the longitudinal direction might be identified with the central 120mm of the plume where they are accelerated outwards¹¹. Because the 'body' is largely made up of water, the pressure gradient outside of it may be imposed inside. At the centreline of the plume stagnation may be assumed. At the edges of the 'body', from page 141,

¹¹In fact because the inertia of the particles will encourage them to overshoot it probably should be smaller than this.

the longitudinal velocity of the water is 0.3m/s upwards. If it is assumed that the water has a negligible lateral velocity at this place and the pressure distribution is linear, from the Bernoulli equation the pressure gradient is 7600Pa/m. Because the water is likely to have a lateral velocity component at the edges, this is a maximum gradient. The force on a particle owing to a pressure gradient is given by

$$\text{Volume of particle} \times \text{Pressure gradient} ;$$

hence the acceleration of a particle owing to the pressure gradient is:

$$\frac{dU_p}{dt} = \frac{1}{\rho_p} \frac{dp}{dx} = 0.97\text{m/s}^2. \quad (8.7)$$

From page 133 the particle accelerations vary from 1.2 to 2.7 m/s², with a bias towards the lower end of the range. The balance of the acceleration may be attributed to drag from water that has penetrated partway through the head or come from the column of the plume.

To summarize, a possible model of the front of the plume is that there is a central region which is impermeable to water in the longitudinal direction owing to the momentum of material entering the head from the column of the plume. This allows the plume to act like a solid body, thus reducing the drag on the particles so that they fall faster than an isolated particle. The water in front of the plume is forced sideways and a pressure gradient is imposed on the material within the ‘body’. This propels the particles sideways and accounts for a large part of the observed lateral acceleration of the particles, the balance of which may be accounted for by the flow of water also driven by the pressure gradient. The behaviour of the ‘body’ is consistent with what might be expected when the initial and boundary conditions are changed. Once the particles are outside the central region of the plume the flow is diverted upwards by the walls of the tank and the influence of the upwards flow resulting from the continuity of mass. Water is entrained by the column to replace that dragged down with the particles so two large recirculation cells are formed. The sparse plumes are quite permeable so most of the water can travel through the plume rather than laterally.

8.3 Turbulence

Turbulence may be defined as ‘an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned [33].’ It has been suggested that the lateral motion of the particles can be mostly attributed to diffusion caused by turbulence (R.W.Hall and D.F.Fletcher [105]). if turbulence is present within a liquid its vortices may capture small particles and move them in

a random direction [33]; some of the particles will be swept outwards from the plume, making it wider. The paths of particles will only be affected by turbulence if its smallest wavelength is much larger than the particle's diameter[33], and has a strength such that the lateral speed of fluid past a particle is of the same order as its downward speed. If either of these conditions are not met then it is likely that the particle will not reside in an eddy long enough for its path to be affected. In addition if the vortices are smaller than the particle it will come under the influence of several of them whose effects will cancel out one another and the particle will follow the slow, large scale movement of the liquid. Because of the inertia and size of the particles it is unlikely that their motion will be affected by turbulence.

Three possible sources of turbulence within the fluid are: the wakes of particles and groups of particles; the interface between a moving jet of liquid and the surrounding stagnant fluid; the degeneration of a laminar jet owing to the presence of walls.

There is no experimental evidence that turbulent diffusion is important. Pictures from an experiment where the water was mixed with Mearlmaid show that the water through which the particles have fallen is dominated by persistent small scale eddies produced in the wake of particles in the head of the plume (Figure 8.3). The jet probably acquires a turbulent velocity profile from nearly the first instant it forms; therefore once the jet profile has been established there is no change on the diffusive forces acting on the particles. That the widening of the jet is concentrated in a small region at the front of the plume indicates that turbulence within the jet has little effect on the spreading of particles, otherwise spreading would occur along the entire length of the plume, mirroring the conical spread of the fluid jet predicted for turbulent, buoyant fluid plumes¹². As this is not observed, even for air bubbles, the turbulence generated is not strong enough to have a significant effect on any particles within the plume. In addition from Figure 8.3 it can be seen that some of the particles in the wings of the plume are falling with clear water either side of them i.e. they have not encountered any turbulence, and yet have moved a great deal laterally.

¹²For an example see [103].

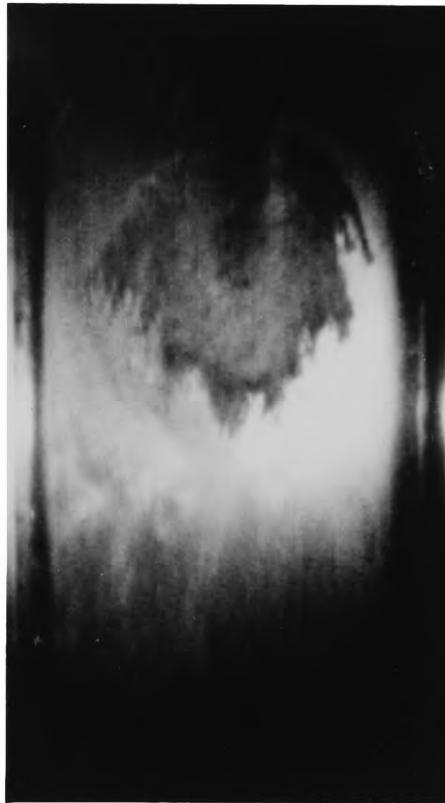


Figure 8.3: Picture of a Normal Plume Falling through Water containing Mearlmaid.

Chapter 9

The Interpretation of Numerical and Experimental Results

This chapter is devoted to the comparison of the results of a numerical code with those of the experiments. The next chapter will describe the steps necessary for the comparison to be fair and the implications this has for the application of the model.

9.1 The CHYMES Code

The experimental plumes are for comparison with the results of the CHYMES code developed by AEA Technology. This has been designed specifically for the modelling of coarse mixing and is documented in [29]. All the calculations represented here were performed by Dr D.F.Fletcher and Mr G.P.Dobson of AEA Technology. The code is based on the separated flow model described in Chapters 2 and 3. It is a finite difference code which means that the domain over which the equations are solved is divided into cells that are small enough for the variation of any quantity to be assumed linear.

The Main Features of CHYMES

- The hydrodynamic motion of all the phases is described by the separated flow model using equations 2.46–2.48, 2.52–2.54, and 2.57. An implication of this is that the length constraints 2.5 and 2.12 are satisfied.
- Motion is modelled in two dimensions using either cartesian or cylindrical coordinates.
- There are three components to the flow: hot melt, cold liquid, and the vapour phase of the cold liquid.

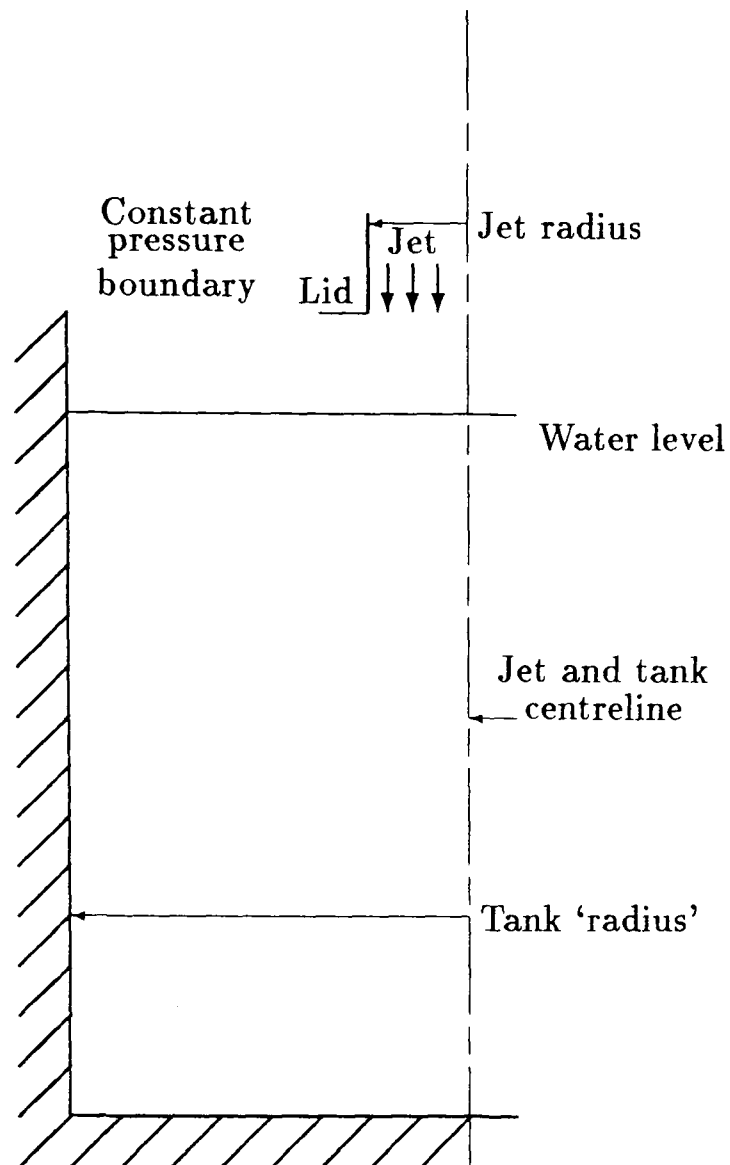


Figure 9.1: The Geometry of the Computational Domain of CHYMES

- The geometry of the field used for computation in CHYMES is shown in Figure 9.1.

These features are not unique to CHYMES, but are shared with other coarse mixing models such as PM-ALPHA which is detailed in [27]. The codes differ in the constitutive relations and the method of computation employed. If the same constitutive relations are used it has been shown that the PM-ALPHA and the CHYMES codes produce very similar results[76], indicating that the results of CHYMES are independent of their method of calculation.

The constitutive relations used are those detailed in Section 3.6, the summary of Chapter 3: if there is no heat or mass transfer these are (3.18), (3.19), (3.20), and (3.33). The drag coefficient is usually fixed, but it can be allowed to vary according to local conditions. The constitutive relations affect the value of several terms in the equations used by the code; they do not affect the basic assumptions of the separated flow model, or the form in which the results are presented.

The Boundary Conditions in CHYMES As discussed in Chapter 7¹, it is assumed that the plume is symmetrical; so, only half of it is modelled with a false wall at its centre line. The

¹Page 105

boundary condition at a wall is that adjacent to it the normal velocity components are zero.

The top of the tank consists of the three regions shown in Figure 9.1: one at the centre of the tank where the jet enters; one at the edge where material may escape from the tank and a constant pressure is maintained; and an intermediate region - the 'lid' - that separates the other two regions through which nothing can enter or leave the tank. In the calculations presented here, the last region has a thickness of just one cell, to separate the other two areas.

The Form of Presentation of the Results of CHYMES CHYMES calculates the volume fraction of each phase within a cell and their velocity at a cell's boundary. It can present other quantities, such as the temperature of the melt, when required. The volume fraction data is presented in a contour plots with an interval of 0.02 between lines. The velocity of each phase is represented as arrows whose length are proportional to the magnitude of the velocity. This is not very revealing because of the smallness of the arrows compared with the plume makes it difficult to discern both their size and direction. The data was not available in any other form; for this reason this work will concentrate on the volume fraction data.

9.1.1 The Parameters of CHYMES

CHYMES requires a number of parameters to be set before a calculation is made. These may be divided into four classes: the dimensions of the tank and the water pool within it; the properties of the phases; the melt jet characteristics; and the solution conditions.

The Size of the Tank and the Water Pool The tank in the calculations is assumed to extend up to the point of release of the particles. The top 0.1m of the codes's tank contains air. Apart from the calculation of the normal plume the tank depth was set to 0.5m rather than that of the tank because of the long time necessary for a computation². To check that the behaviour of the plume was independent of the depth of the water a normal plume was calculated in a 0.5m deep tank, and in a 1.035m deep tank, and the results compared: they were almost identical, as may be seen in Figure 9.2 which shows the two plumes after 0.4s.

²Over twelve hours for a 0.5m deep tank.

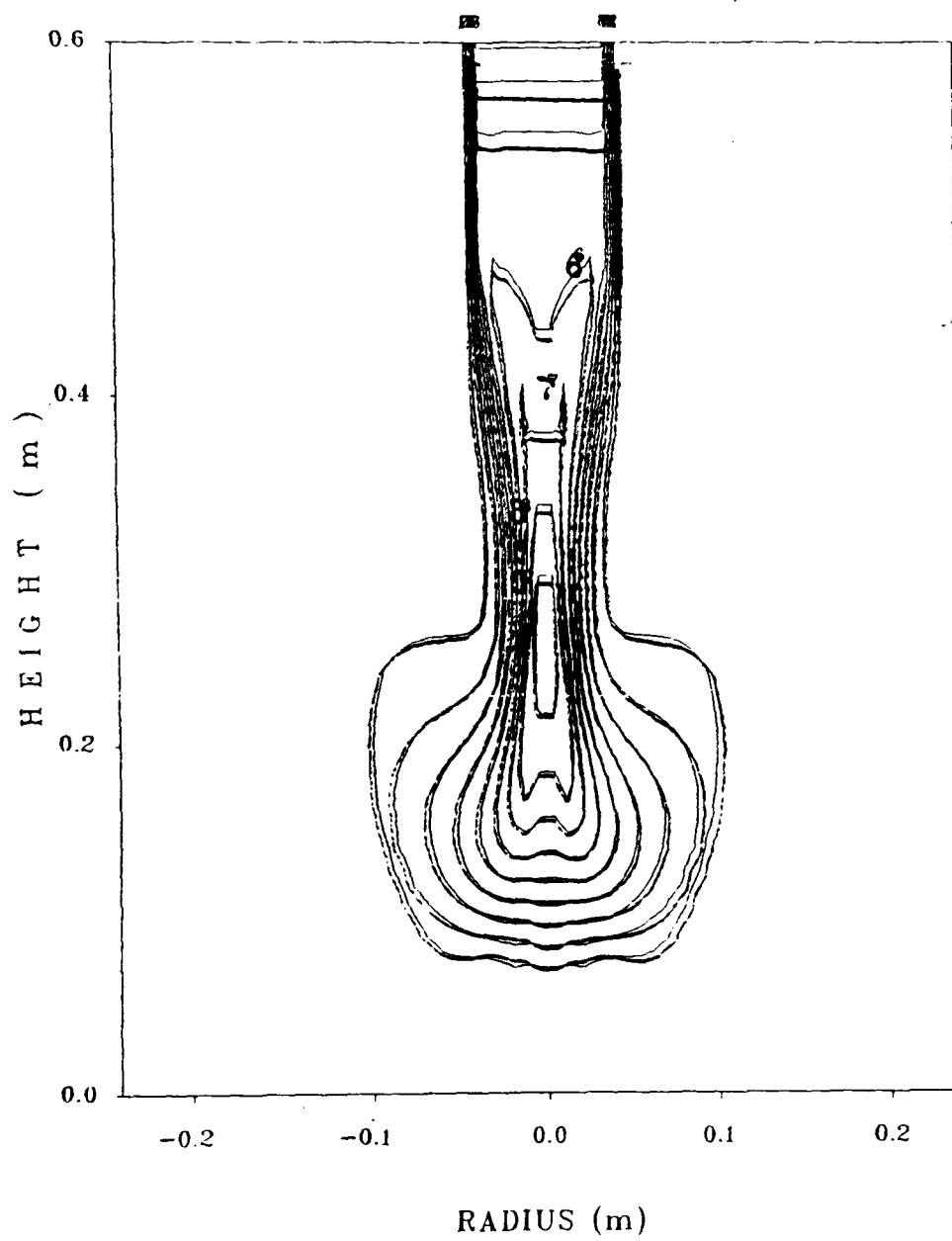


Figure 9.2: A Normal Plume falling in a 0.5m Tank Compared with that, on the Overlay, in a 0.935m Deep Tank.

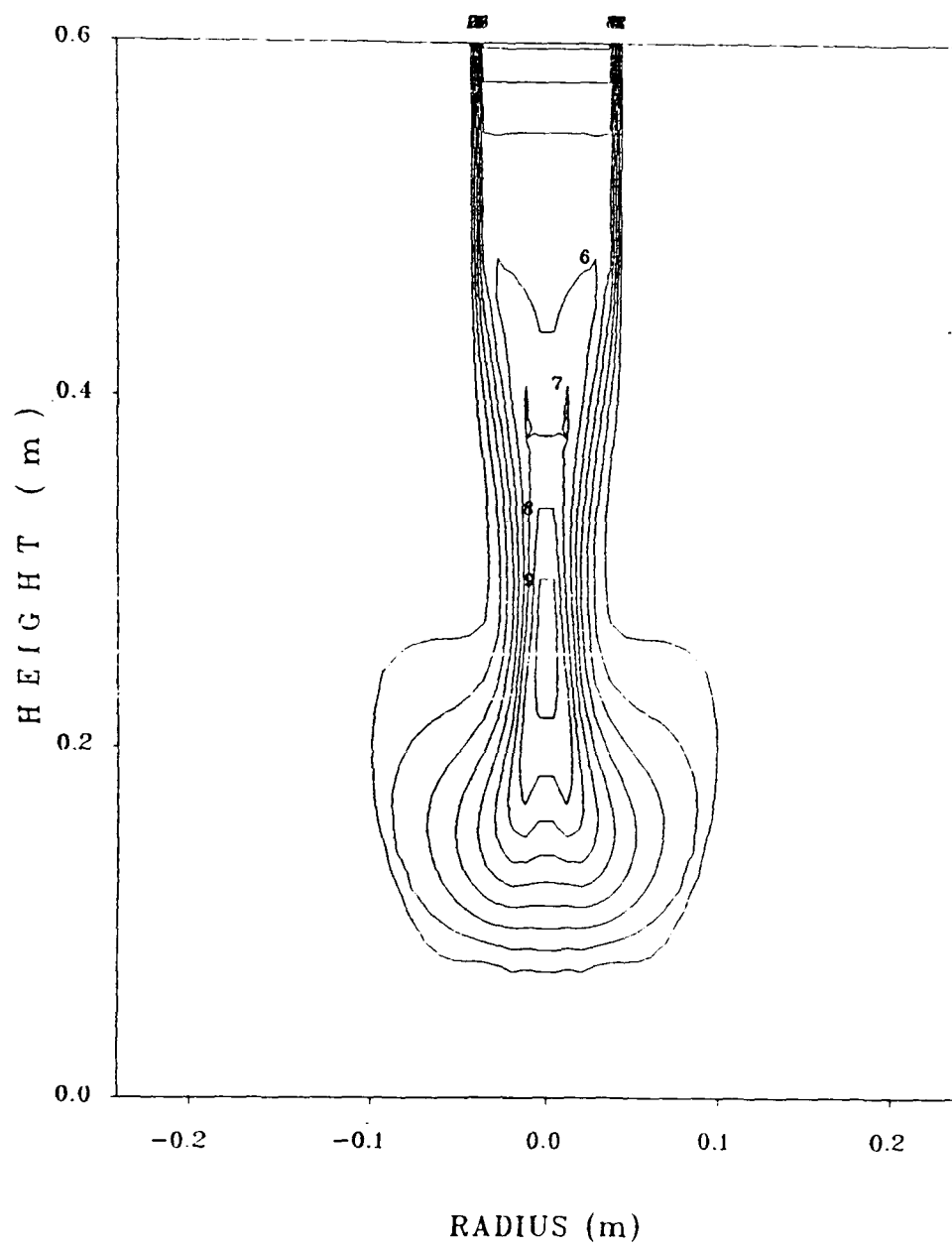


Figure 9.2: A Normal Plume falling in a 0.5m Tank Compared with that, on the Overlay, in a 0.935m Deep Tank.

The Properties of the Phases These are the length scales, and properties of the phases³, all easily determined from tables.

The Characteristics of the Melt Jet These are principally the volume fraction, velocity, and width of the melt jet.

CHYMES requires the velocity to ramp up over any time desired from zero to the set constant value. This prevents difficulties in calculating the flow when the jet meets the water surface owing to it having a shock like character (D.F.Fletcher [106]).

The definition of the width of the jet provides an intimation of the problem of applying a continuum model to a particulate jet. The experimental jets consist of between five and ten columns of particles; it may be defined as:

- the distance between the centrelines of the outside columns of the particle jet;
- the distance between the outer edges of the column;
- ten times the spacing of the particles.

These different definitions arise because the particle spacing is not insignificant compared with the width of the jet. Any one of them could be justified in some way, but the first was adopted for the calculations. If the second definition had been used the overall particle volume fraction of the jet would not have equalled the true proportion of the jet occupied by the particles. There is little fundamental difference between the first and the third definitions; the first does not include all the 'melt' in the jet and no 'melt' is present at the edges of the jet if it is defined the third way.

The Solution Parameters These are required by the method of solution used by the code, the most important of which are the time and space steps in the calculations. These should be small enough for the results of the model to be independent of their size⁴, but should not be so small that the code runs inefficiently or becomes unstable. Another way of putting it is that the size of the steps should be small enough for the change in a calculated quantity to be just discernible over one of them. This is the definition of the length scale L in the inequality (2.12).

³Namely density, initial temperature, heat capacity, latent heat of freezing, solidus temperature, and emissivity of the melt; the density, temperature, latent heat of vaporization, surface tension, and viscosity of the liquid; the density, conductivity, heat capacity, and viscosity of the vapour. Apart from the densities these are not required in the calculations presented in this and the next chapters.

⁴i.e. small enough for the variation of flow variables in a cell to be approximated to a straight line.

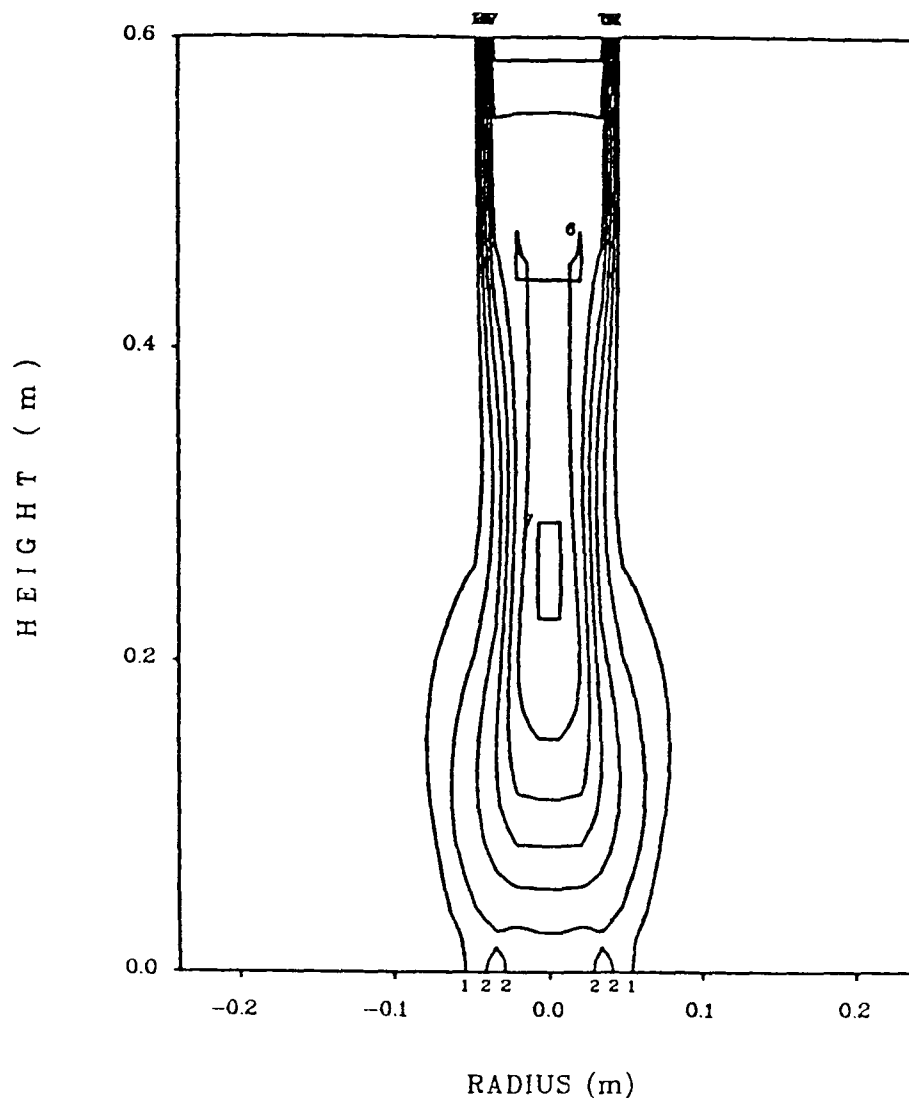


Figure 9.3: A Calculated Normal Plume after 0.4s where the Size of a Grid Cell is $13.3 \times 20\text{mm}$

A grid sensitivity study was carried out by Dr D.F.Fletcher. The time step was set at $25\mu\text{s}$ which is adequate for the solution to converge well, and to be stable. The most suitable grid cell size was found to be 8mm laterally and 10mm longitudinally. These dimensions were used in all of the calculations not involved in the sensitivity study. The drag coefficient was fixed at 0.55.

If the grid was coarsened so that the dimensions of each cell were $13.3 \times 20\text{mm}$ the result is shown in Figure 9.3 was obtained. A comparable plume calculated using an $8 \times 10\text{mm}$ grid is placed alongside. Compared with the finer grid the greatest width of the plume is slightly reduced. In the longitudinal direction the plume is elongated; the point where the column of the plume broadens has a similar elevation in both plumes, but the contours of the coarser grid plume below this point are further away. The 0.16 contour disappears completely, as does the notch at the bottom of the 0.14 contour if the larger cell size is used.

Figure 9.4 shows the result of elongating the cells to 13.3mm in the lateral direction only. The overall shape and the longitudinal position of the contours are similar to those of the finer grid plume except the detail within the plume is slightly different. There was, therefore, little benefit in reducing the lateral dimension of the cell to less than 8mm.

The result shown in Figure 9.5 is for a $13.3 \times 6.67\text{mm}$ so that the cells were stretched laterally

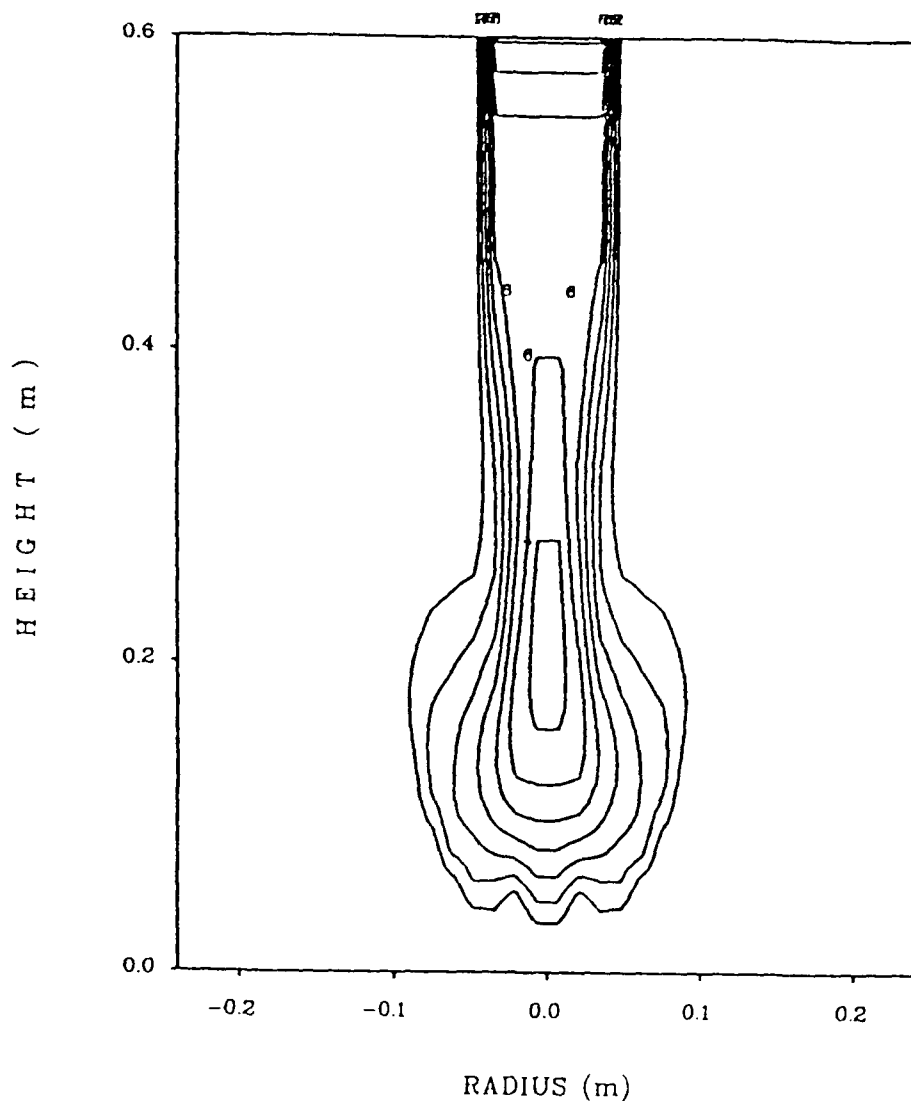


Figure 9.4: A Calculated Normal Plume after 0.4s when the Size of a Grid Cell is $13.3 \times 10\text{mm}$ and shrunk longitudinally. Apart from the creation of a notch straddling the plume's centre line, and the enlargement of the 0.14 contour, the plume is similar to that for a $8 \times 10\text{mm}$ grid. This suggests that there would be little benefit in reducing the longitudinal dimension of the cells to less than 10mm.

9.2 The Comparison of CHYMES Calculations with Experimental Results

Care had been taken to ensure that the initial conditions of the experiments were controlled and known so that the results could be compared directly with those of numerical calculations.

Figure 9.6 shows the development of a computed normal plume. Unless otherwise stated the drag coefficient in this plume, and in all subsequent ones, was fixed at 0.76, the drag coefficient determined in Section 4.3.1 for an isolated particle in the tank. Initially a head to the plume forms that is quite rectangular in shape. The largest value of α_p is found towards the back of the head in the centre of the plume; it gradually decreases outwards from this point to the edges of the plume. After 0.6s the largest value of α_p has increased to a maximum of 0.22 and has moved closer to the front of the plume, increasing its gradient. Material away from the

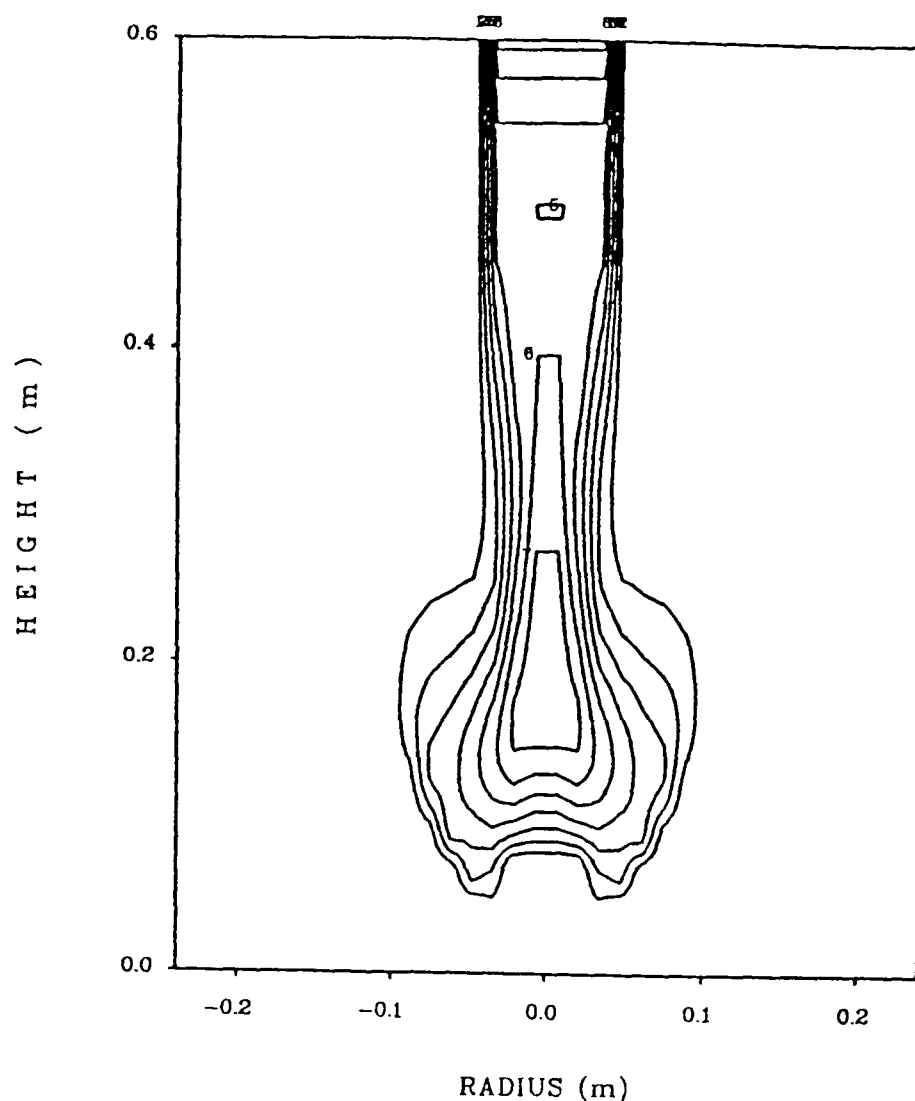


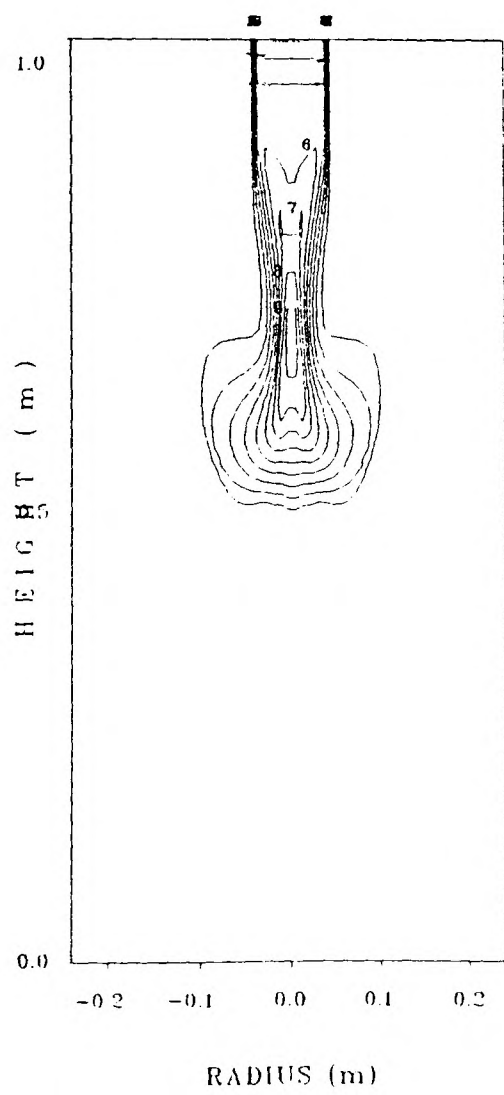
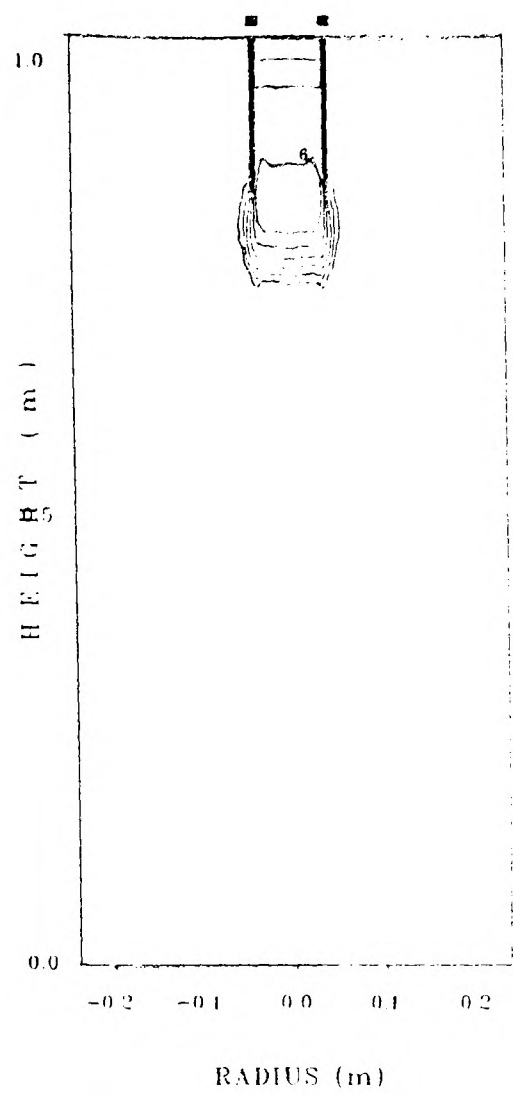
Figure 9.5: A Calculated Normal Plume after 0.4s when the Size of a Grid Cell is $13.3 \times 6.67\text{mm}$

central portion of the plume is convected upwards with respect to its front, giving it an arrow like shape. This becomes more pronounced until when the plume has reached the tank floor it is tulip shaped. The reason for this is that the material in the column of the plume accelerates as it falls down the tank up to a maximum speed of 3.5m/s . If after the plume entered the water its acceleration was constant, its magnitude would be 2.0m/s^2 .

There are a number of dissimilarities between the computational and experimental plumes:

- the head of the computational plume becomes crescent shaped after 0.5s, much later than in the experimental plume where it has this shape after 0.1s ⁵;
- the particle volume fraction in the calculated plumes varies gradually from the interior to the edges while an experimental plume can have quite well defined edges, at its leading edge for example;
- the distribution of particles within the two sorts of plume is different: in the computed plume there is only one concentration of particles, towards the back of the head of the plume; in the experimental plumes this is not apparent and there is a high concentration of particles at the front of the head, and large variations in particle concentration within

⁵see figures 5.5 and 5.6.



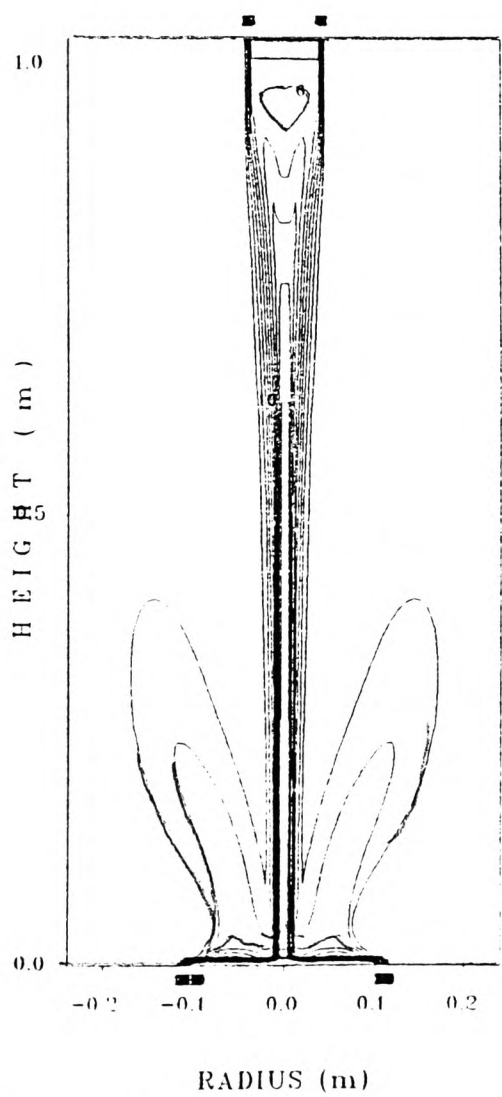
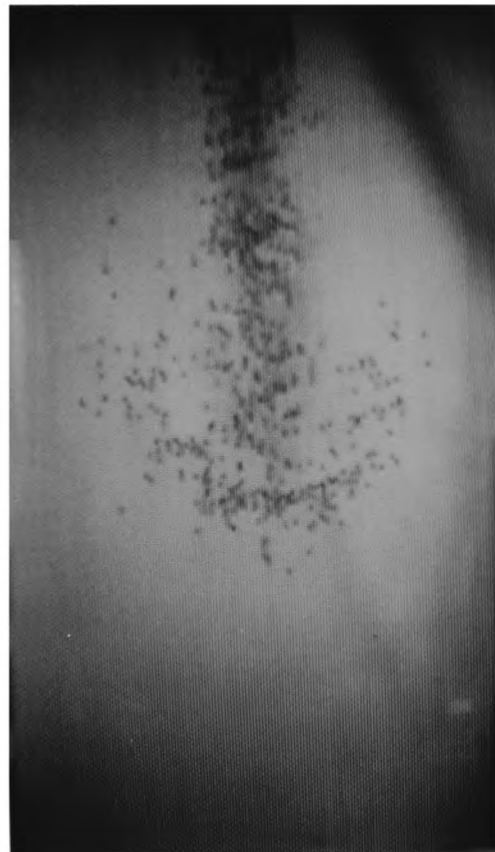
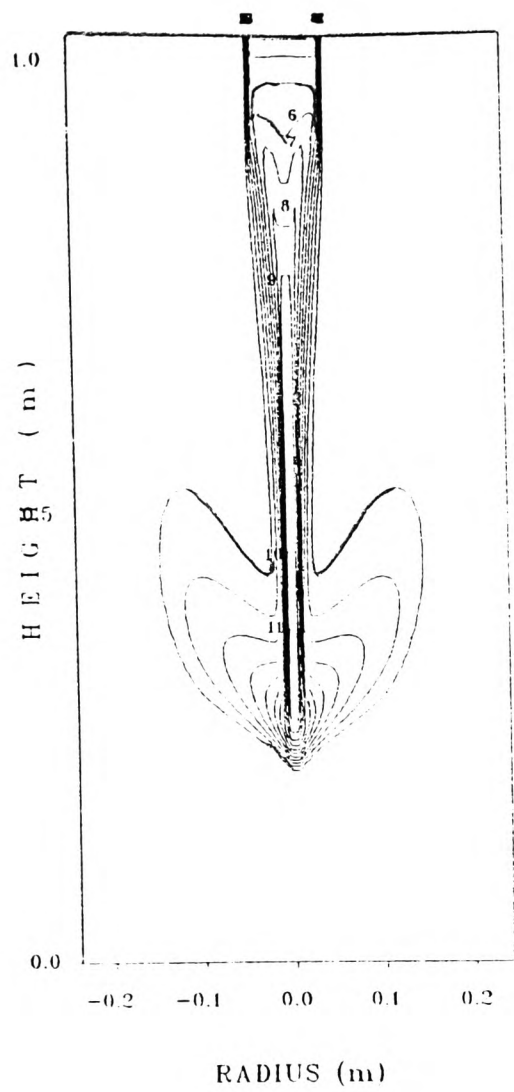


Figure 9.6: The Calculated Development of the Particle Volume Fraction in a Normal Plume after 0.2, 0.4, 0.6, and 0.8s compared with an Experimental Plume.

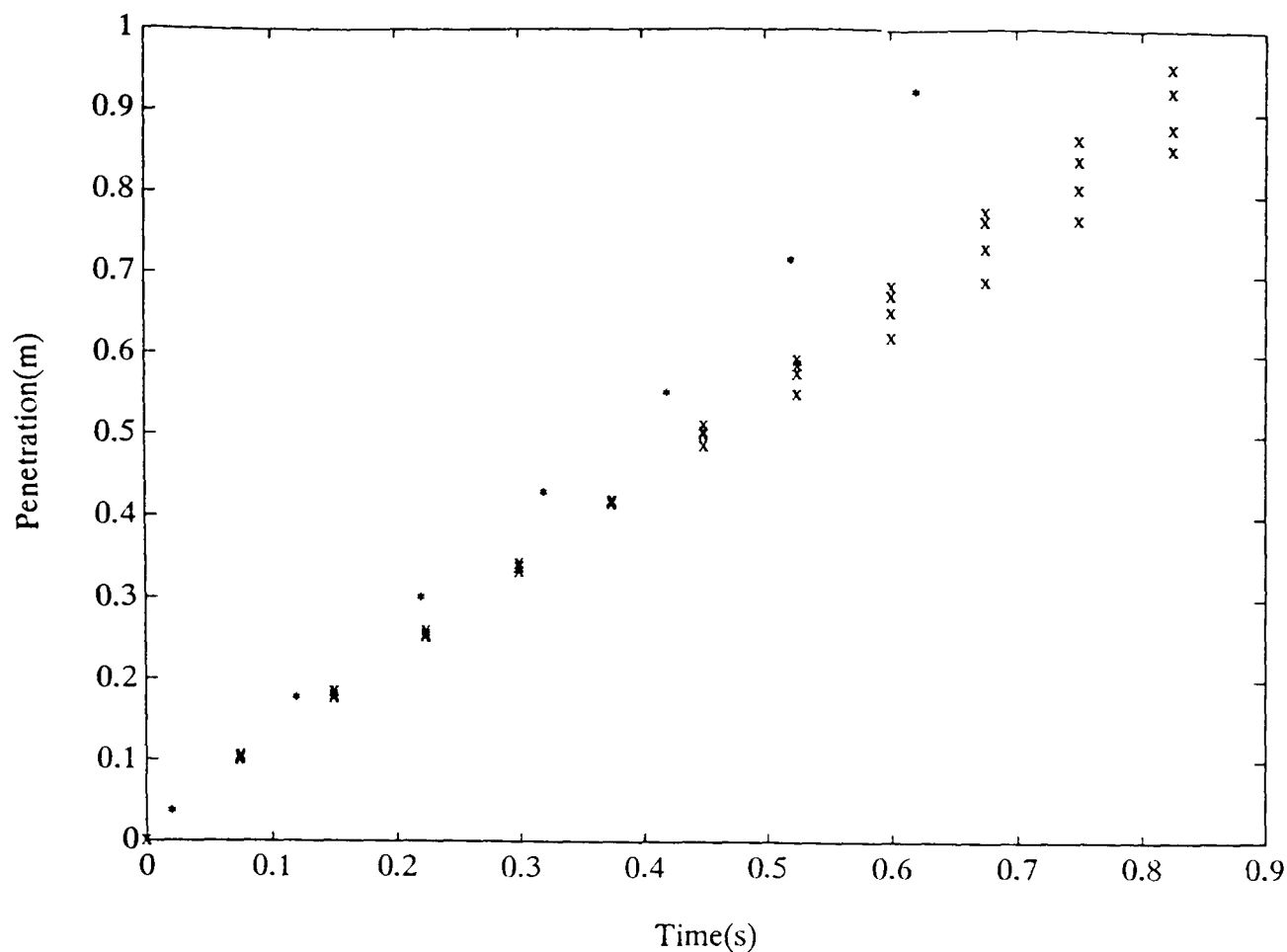


Figure 9.7: Penetration plotted against Time for the Normal Computed Plume (*) and the Normal Experimental Plumes (×).

the any region of the plume.

If the edge of the plume is defined as coincident with the 0.02 contour the penetration speed of the computational plume (the gradient of the line of penetration plotted against time in Figure 9.7) is constantly 1.3m/s for the first 0.4s after which it accelerates to 2.1m/s. These are larger than the velocity of the experimental plumes (about 1.0m/s) and they did not change their speed half way down the tank. The width of the computational plume- shown in Figure 9.8- initially increases more slowly than in the experiments and its rate does not decrease at later times as that of the experimental plumes does. The largest particle velocities, 3m/s, are found in the column behind the head.

The particle and the water velocity plots for a normal plume after 0.6s are shown in Figures 9.9 and 9.10 respectively. The greatest water velocities appear to be in part of the particle column following the head. There is a region of dead water either side of the column, behind the head. The highest water velocity is very large, 3.0m/s. This compares with the largest estimated water speed in the experiments of 1.2m/s (see page 146).

The Effect of a Transient Projection Speed In the experiments the projection mechanism took about 0.1s to accelerate to its steady speed (see Section 4.3.2). The projection speed of the particles in the computations had to be ramped up to avoid calculation problems; usually the duration of this ramp was set at 0.01s, but for one calculation it was increased to 0.1s to test whether this significantly affected the plume. Figure 9.11 shows this plume after 0.4s,

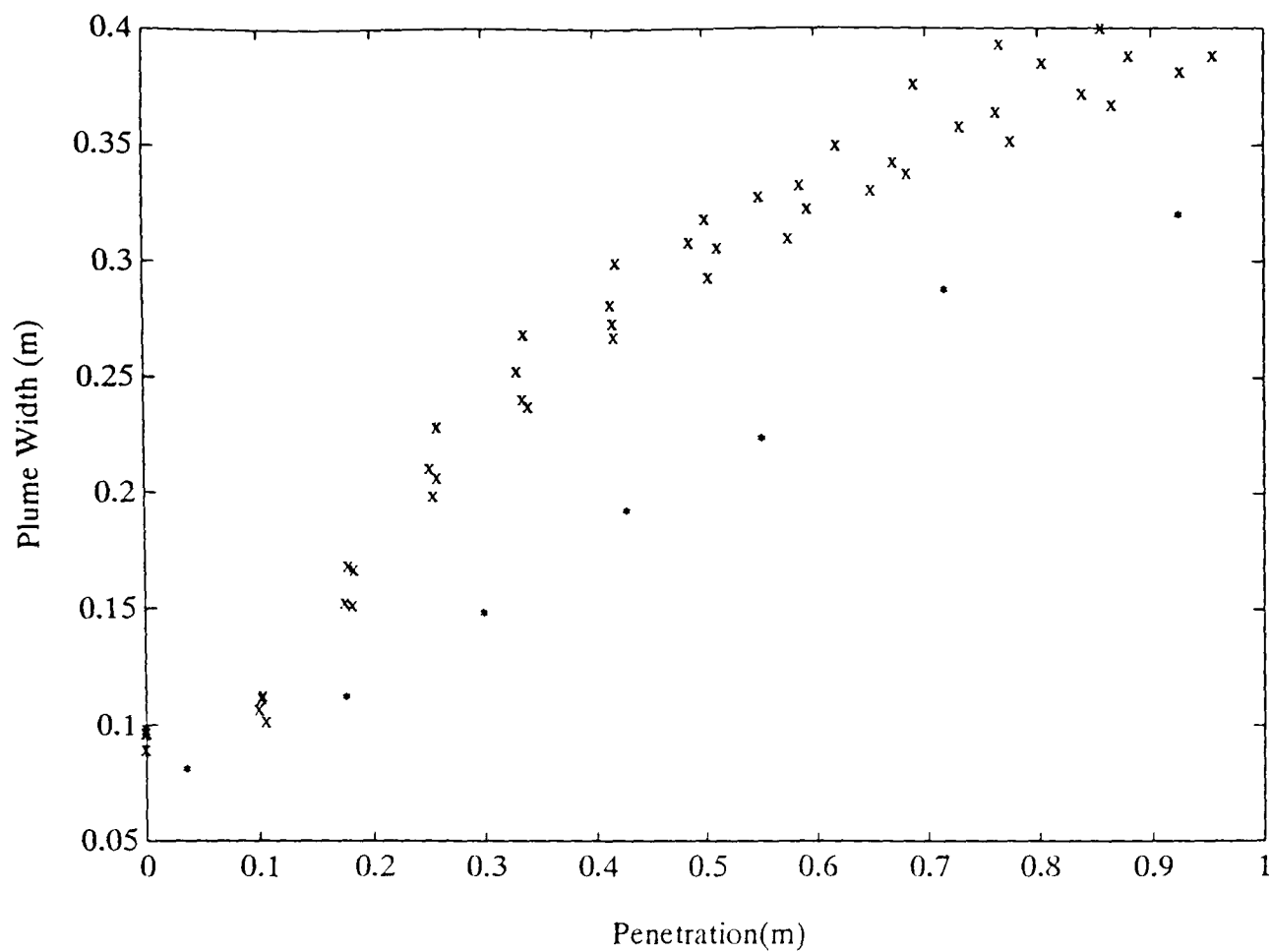


Figure 9.8: The Width of the Plume plotted against Penetration for the Normal Computed Plume (*) and the Normal Experimental Plumes (x).

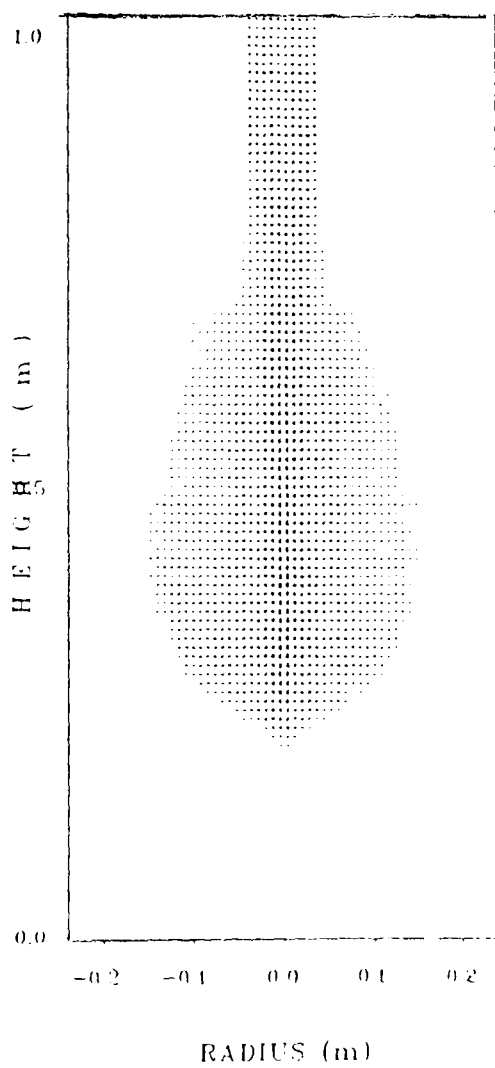


Figure 9.9: The Velocity Distribution of the Particles in a Normal Plume after 0.6s.

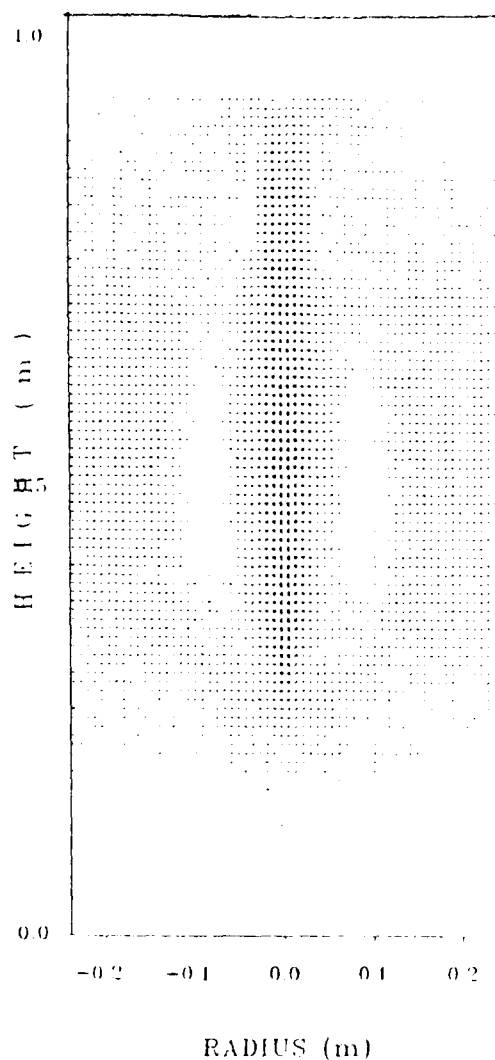


Figure 9.10: The Velocity Distribution of the Liquid in a pool into which a Normal Plume has entered after 0.6s.

Figure 9.12 shows its penetration against time, and Figure 9.13 the growth of its width against penetration, all compared with a normal plume with a smaller ramp time. The development of the plume does not appear to be greatly affected, just delayed by about 45ms.

The Effect of the Boundary Conditions The results of CHYMES for the other experimental conditions are shown in Figures 9.14–9.20 for the sets of conditions a–f, h, and j of Table 4.1 after 0.4s⁶. It was not possible to simulate a plume whose centreline is offset from that of the tank owing to the imposition of symmetry upon the system; an exception is when the edge of a jet is coincident with a tank wall which can be taken to be the centre line of a jet in a CHYMES calculation. The results of the thin jet (experiment set d) may not be very accurate owing to the low number of cells occupied by the jet in the lateral direction. The rate of penetration and the head width as a proportion of that of the normal jet of each plume after 0.4s, and the maximum particle volume fraction are shown in Table 9.1 where the edge of the plume is assumed to be coincident with the 0.02 particle volume fraction contour. From this table the speed of the experimental plumes after they had been corrected for density all lay

⁶The latest time available before the plumes reached the bottom of the tank in the calculations.

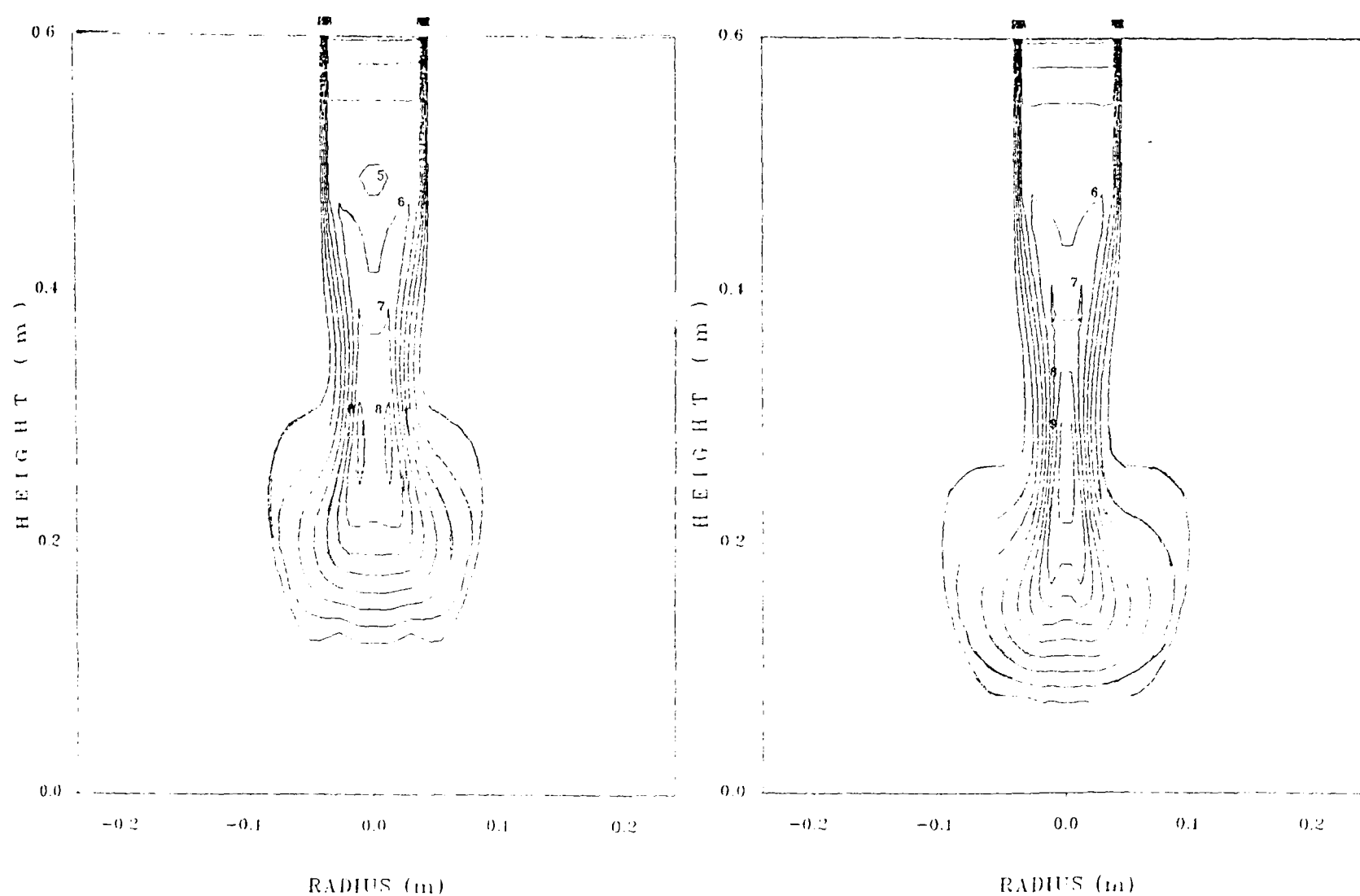


Figure 9.11: The Normal Plumes calculated after 0.4s if the Ramp Time is 0.1s (left) and 0.01s (right).

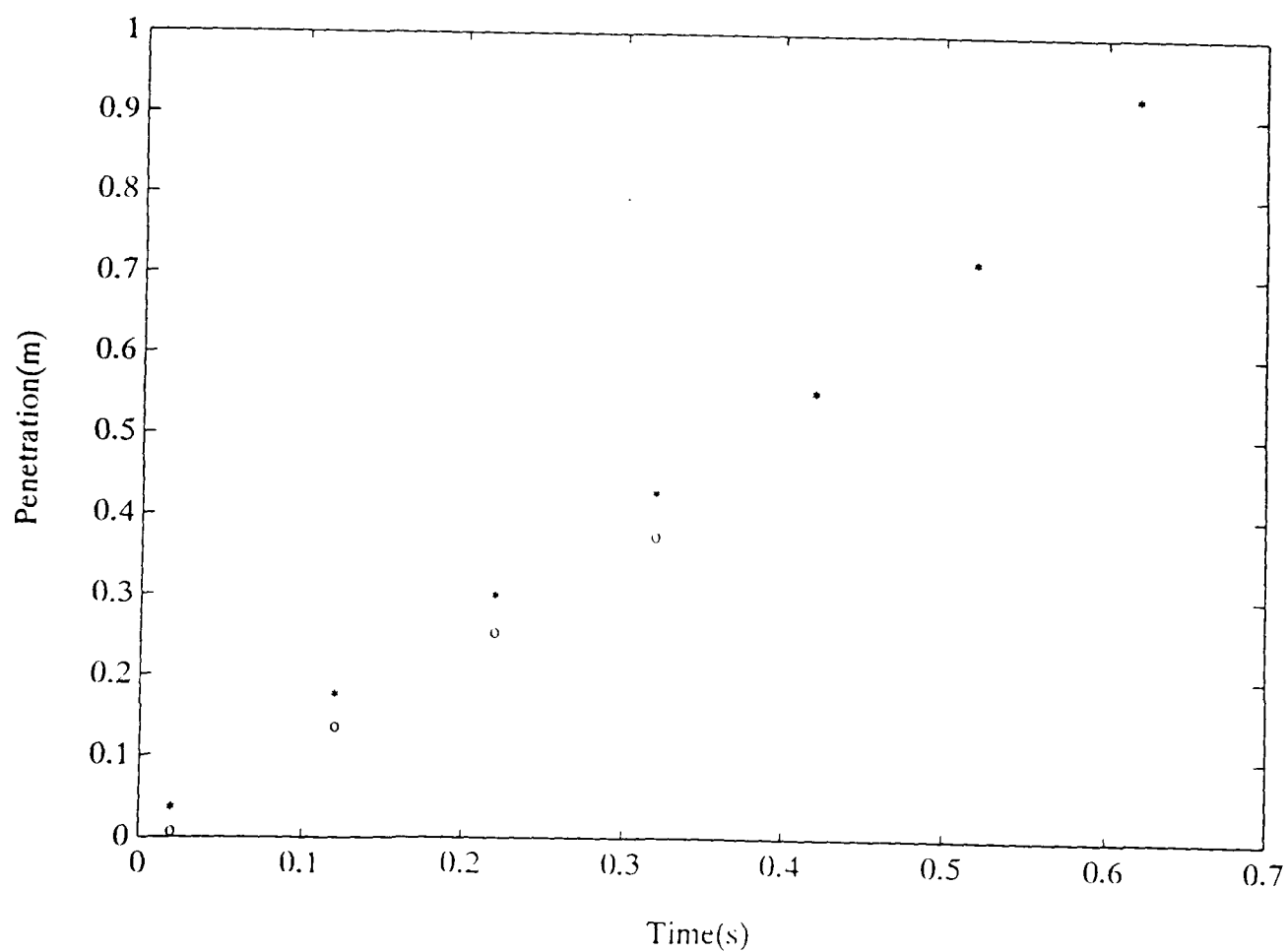


Figure 9.12: The Penetration of a Normal Plume with a Ramp Time of 0.1s (o) compared against that with a Ramp time of 0.01s (*)

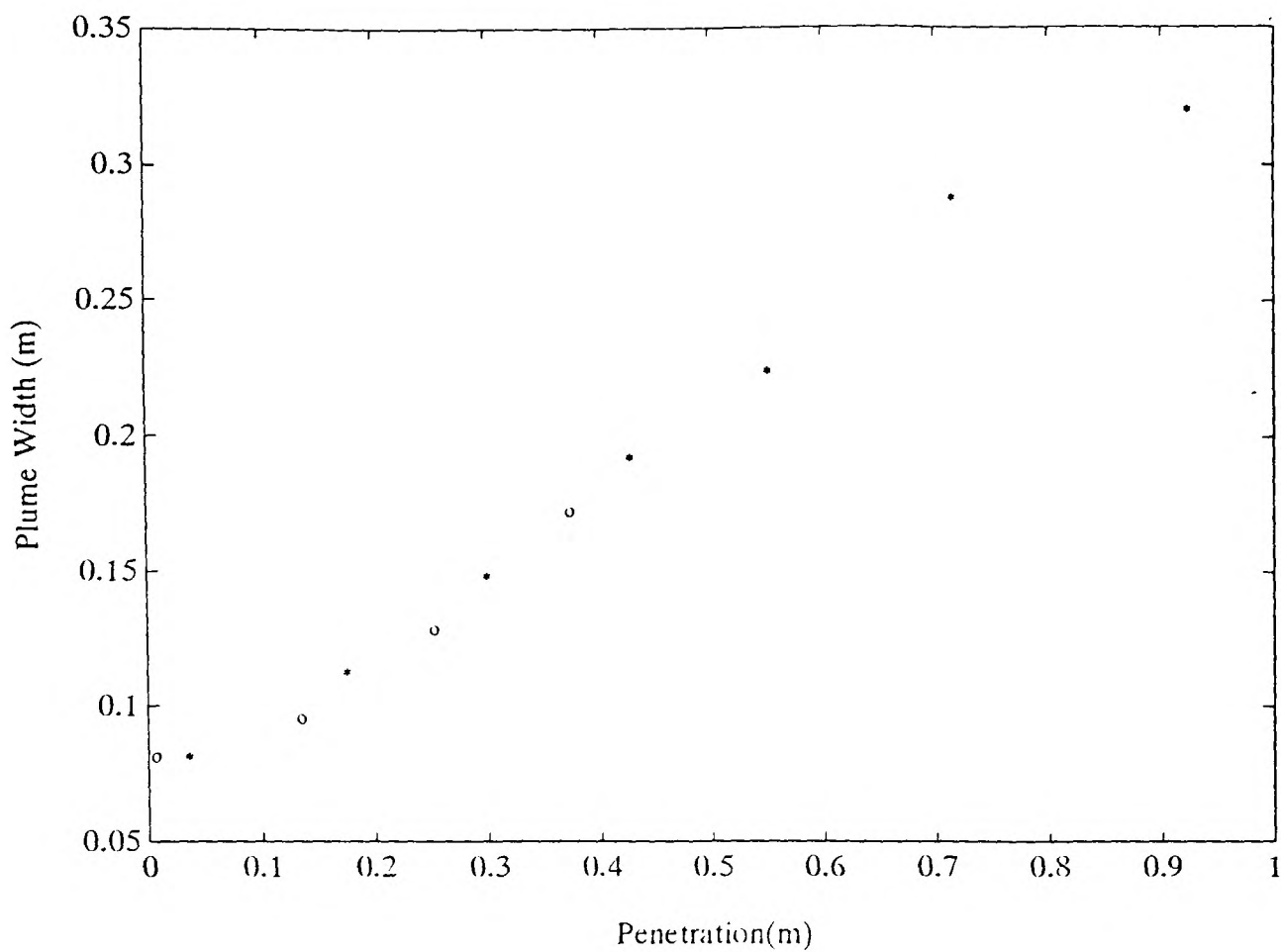


Figure 9.13: The Growth in the Width of a Normal Plume with a Ramp Time of 0.1s (o) compared against that with a Ramp time of 0.01s (*).

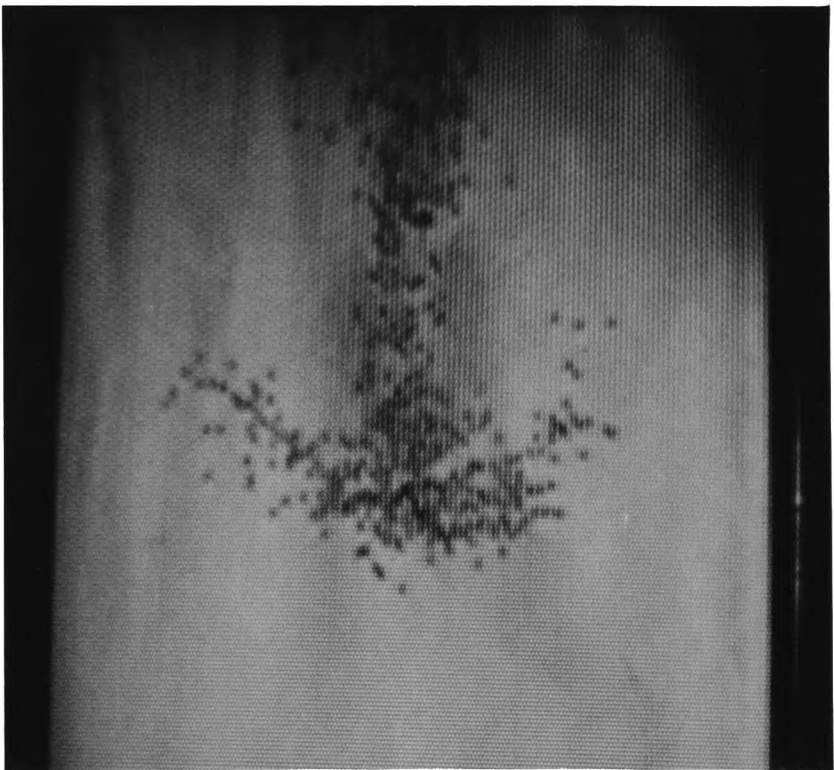
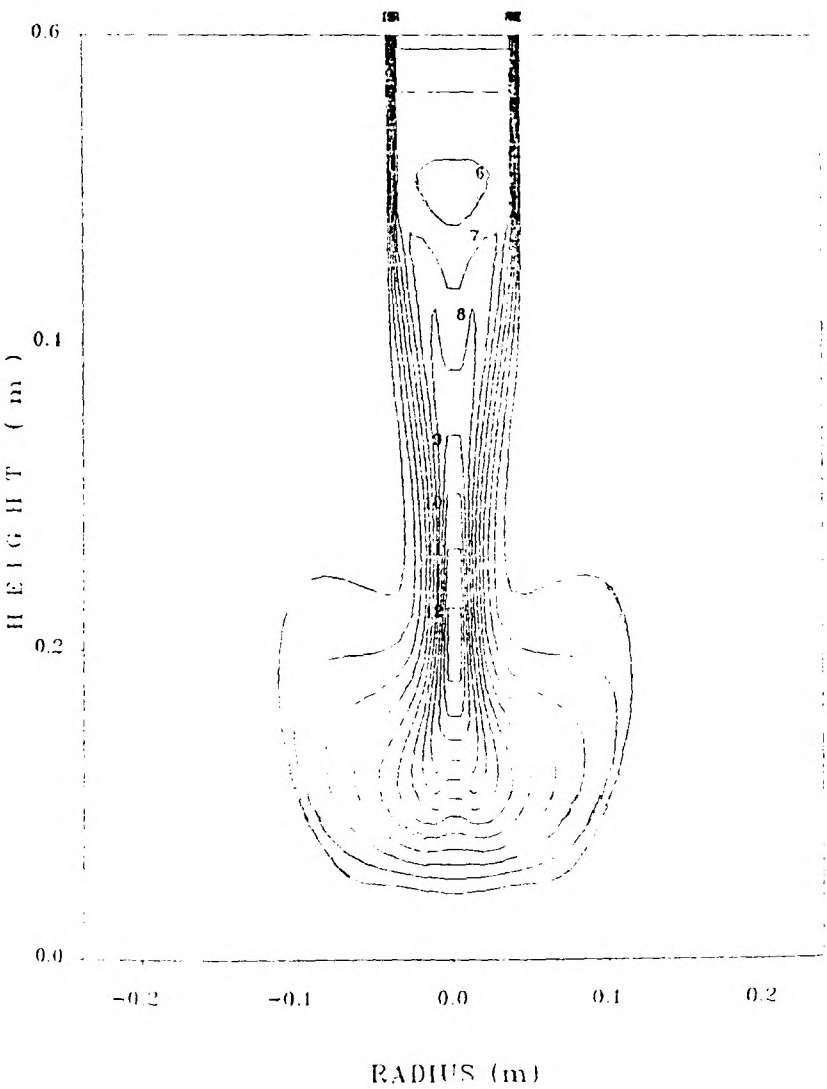


Figure 9.14: The Result of a CHYMES Calculation after 0.4s for Experiment Set b where the Projection Speed is greater than for the Normal Plumes compared with an Experimental Plume.

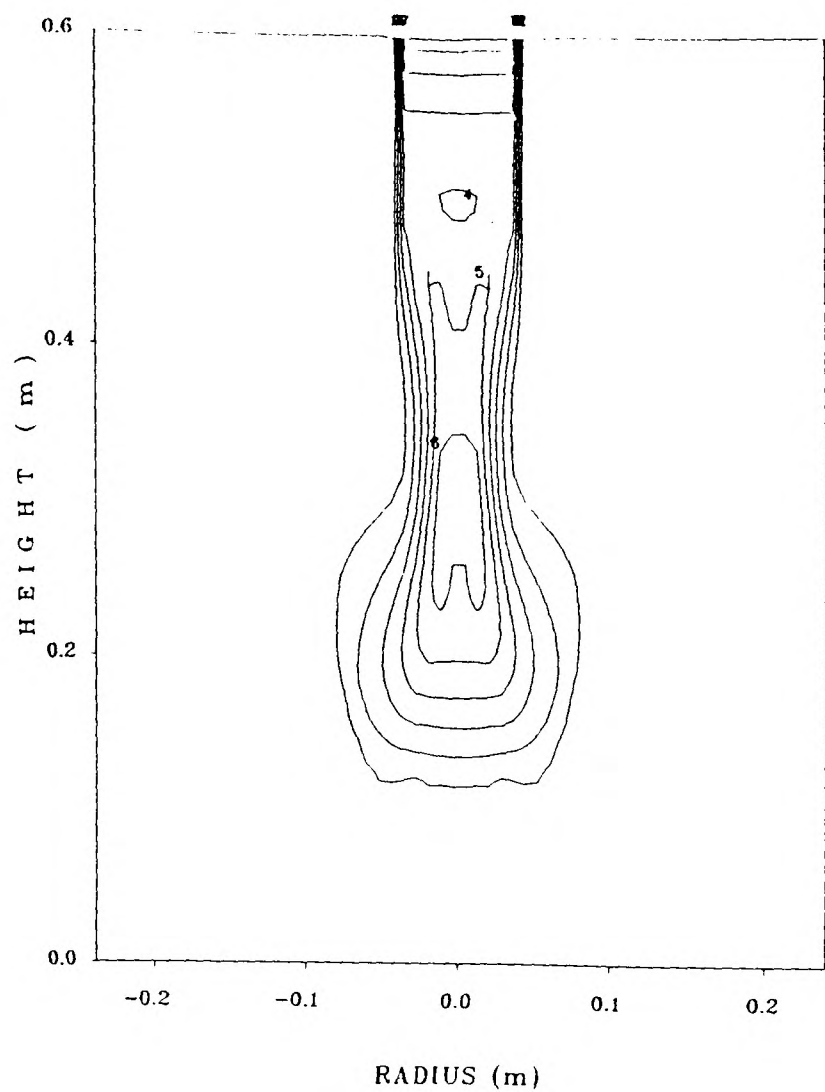


Figure 9.15: The Result of a CHYMES Calculation after 0.4s for Experiment Set c where the Projection Speed is Less than for the Normal Plumes compared with an Experimental Plume.

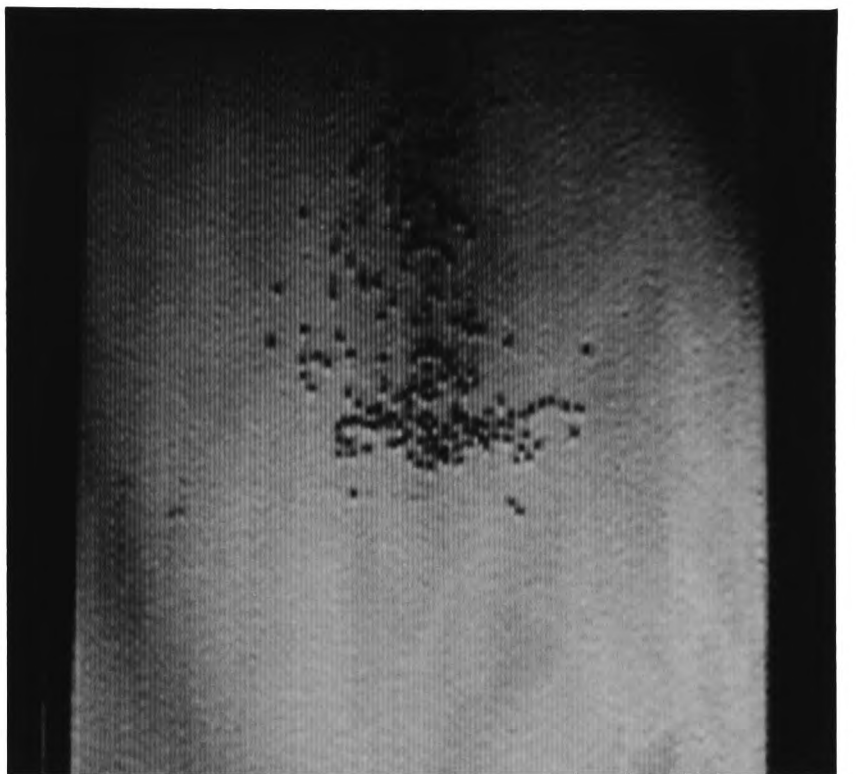
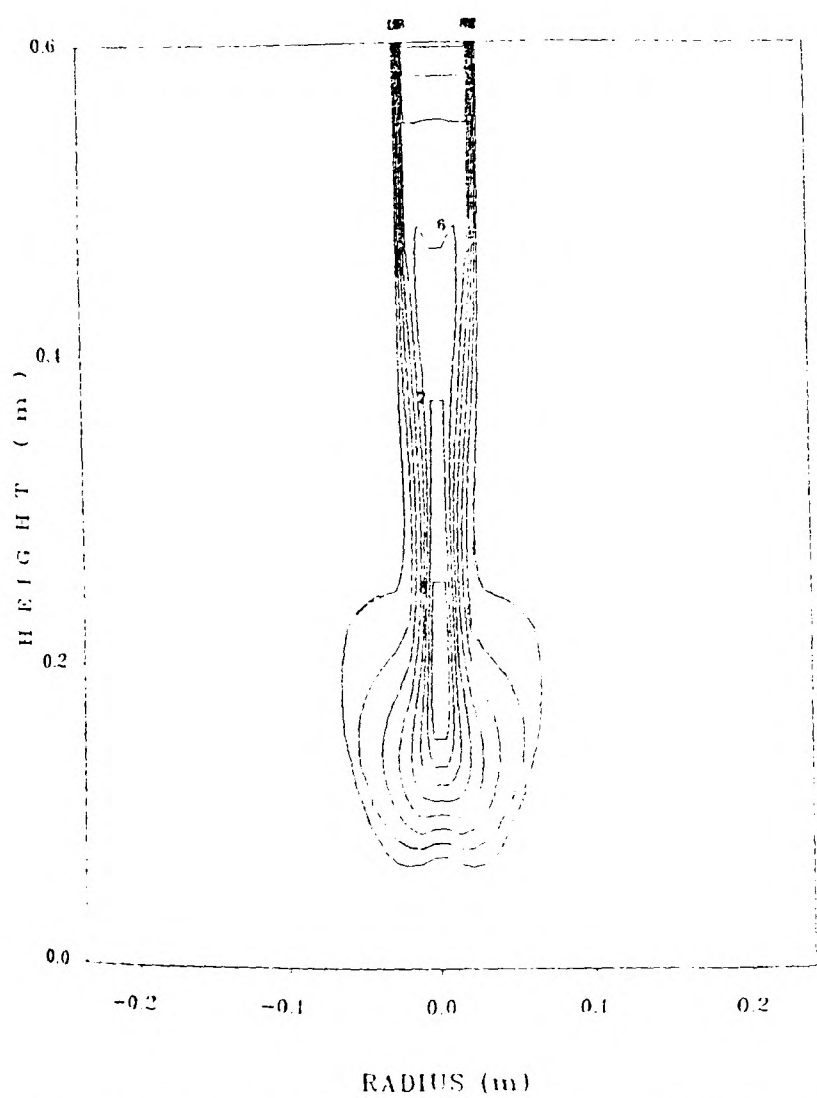


Figure 9.16: The Result of a CHYMES Calculation after 0.4s for Experiment Set d where the Jet Width is Reduced compared with an Experimental Plume.

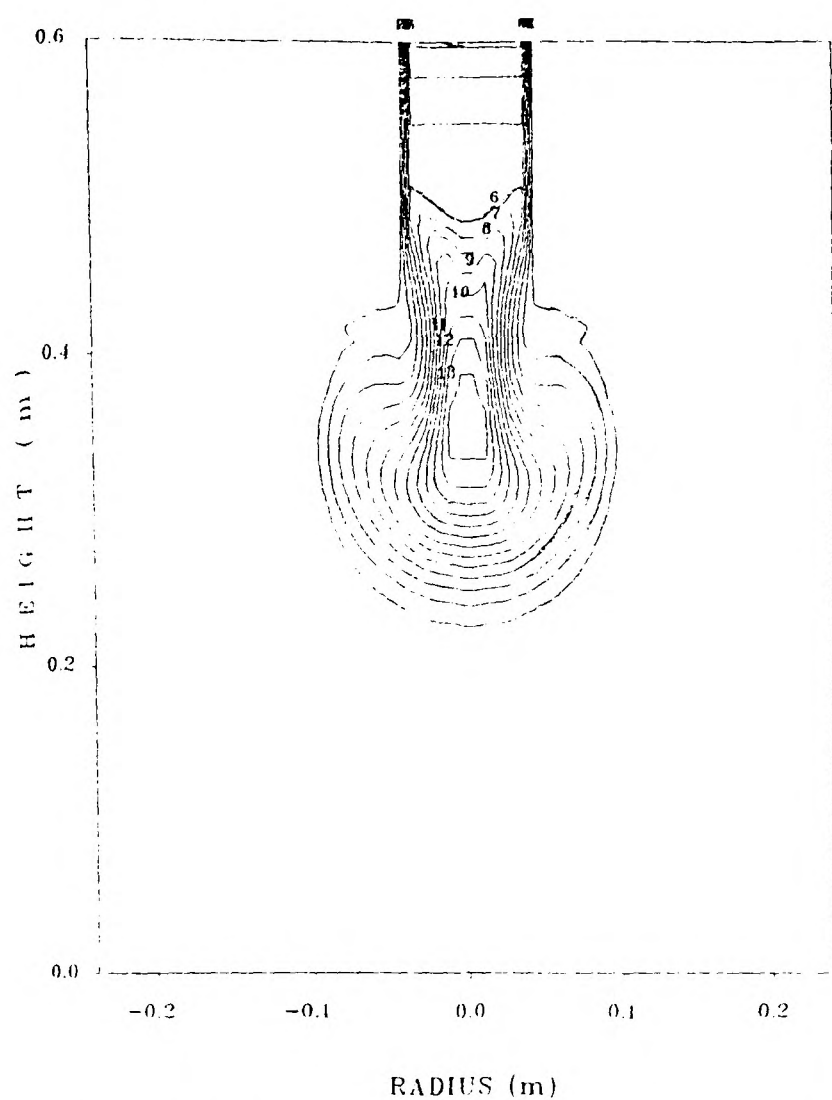


Figure 9.17: The Result of a CHYMES Calculation after 0.4s for Experiment Set e where the 'Melt' Particles are Aluminium compared with an Experimental Plume.

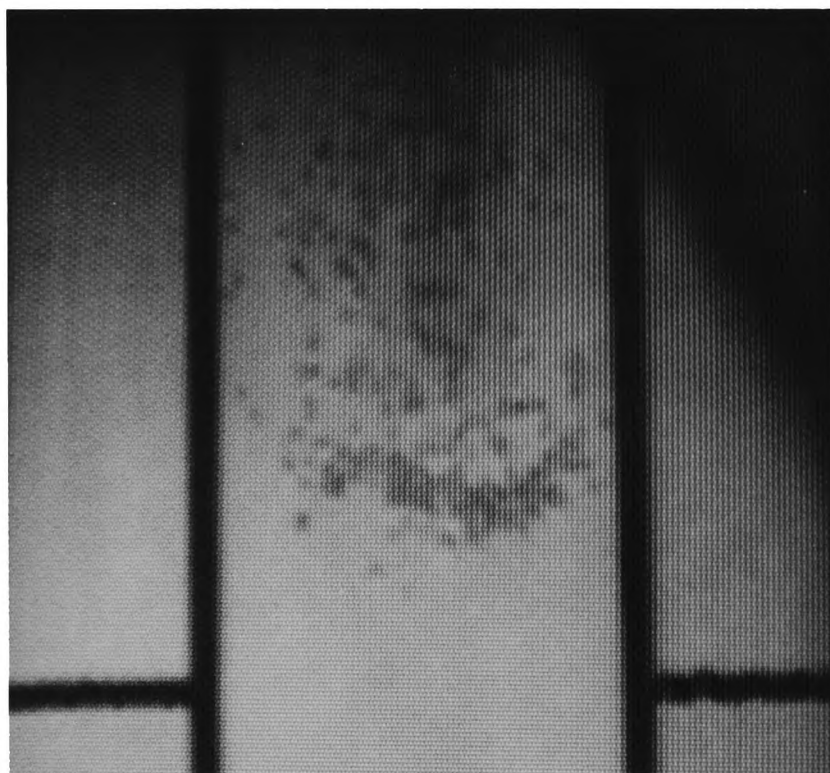
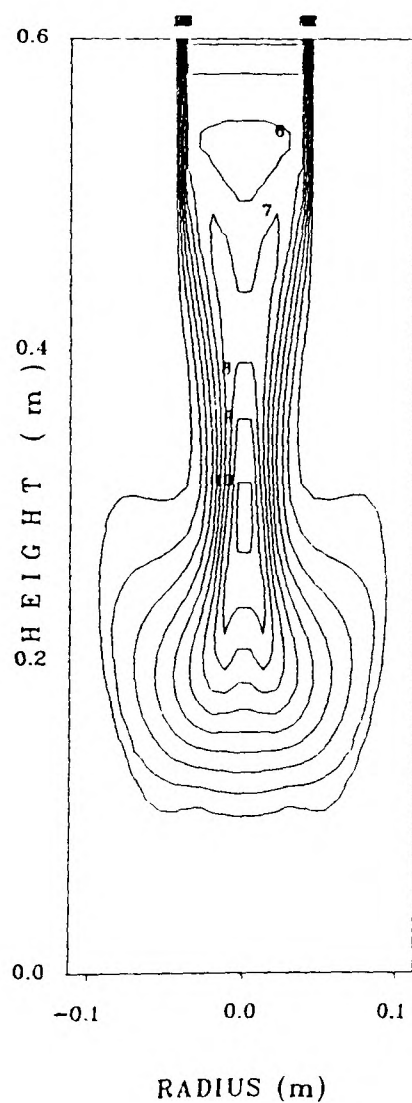


Figure 9.18: The Result of a CHYMES Calculation after 0.4s for Experiment Set f where the Tank Width is Reduced.

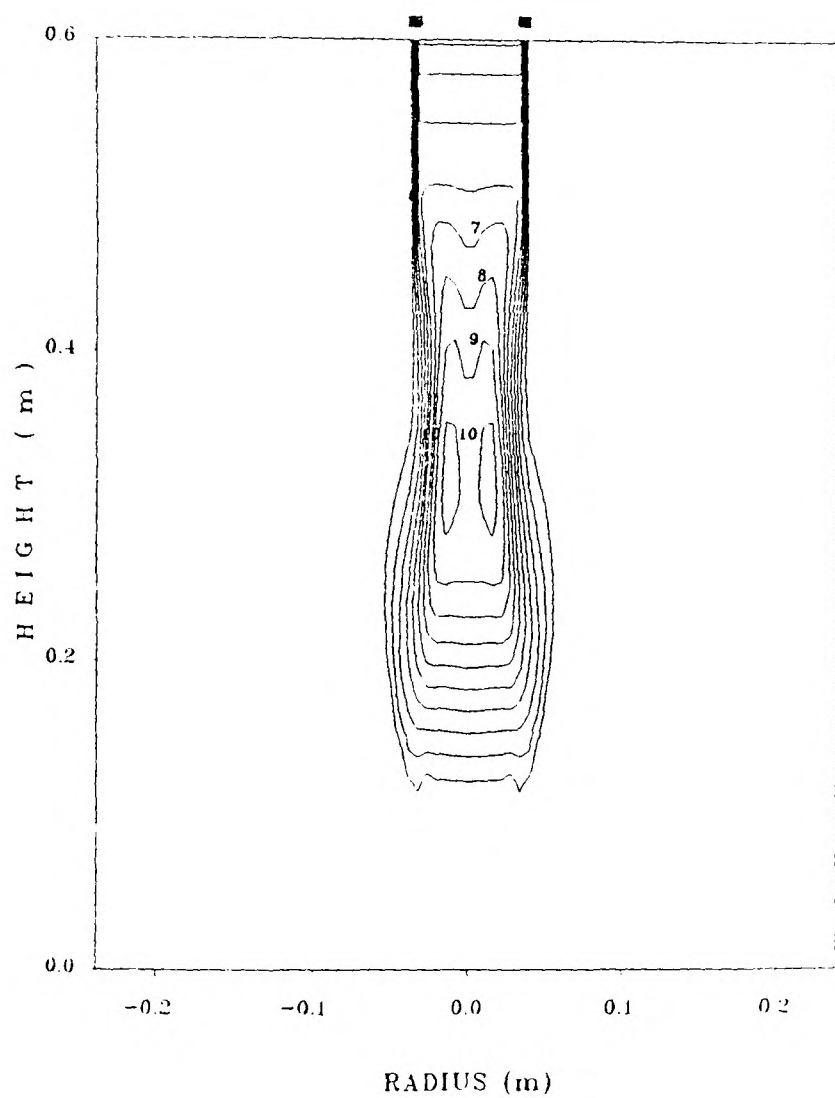


Figure 9.19: The Result of a CHYMES Calculation after 0.4s for Experiment Set h where the Initial Particle Volume Fraction is Reduced compared with an Experimental Plume. The Interval between Contours is 0.005.

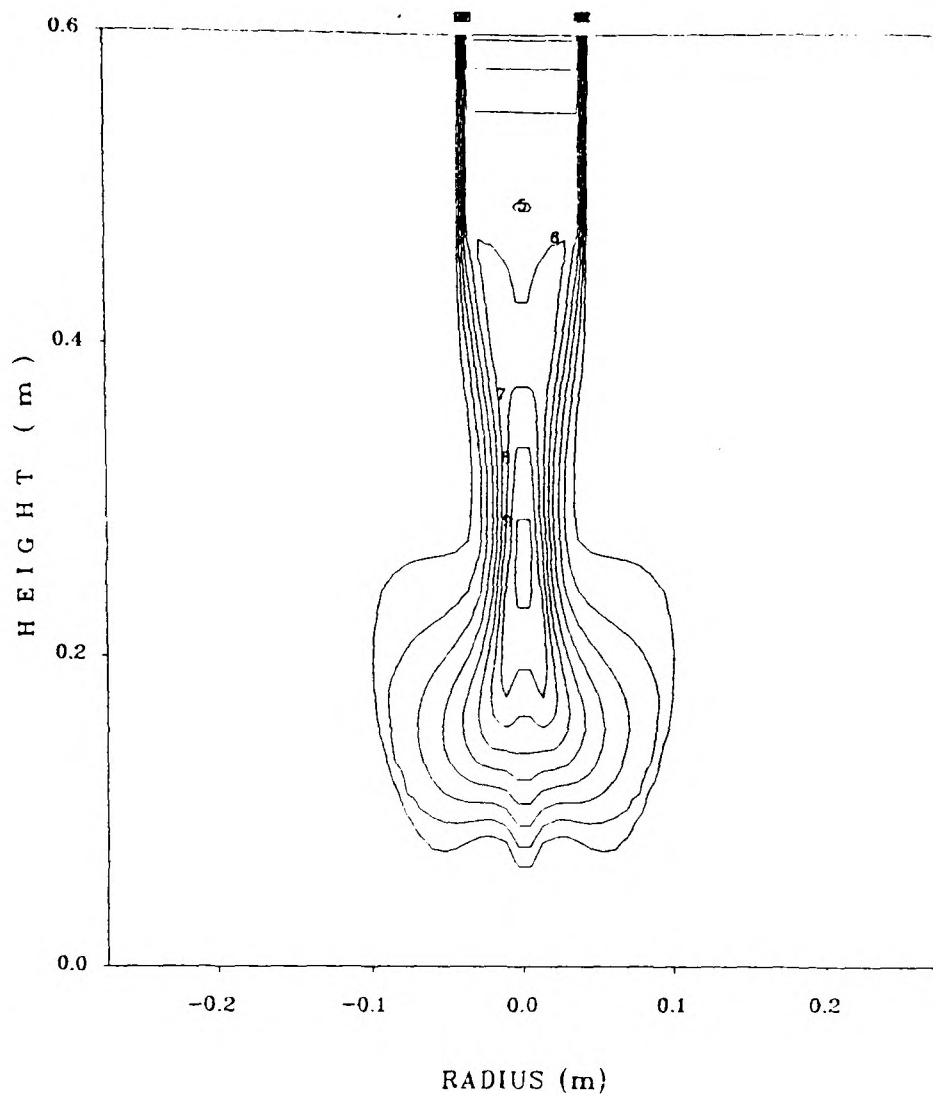


Figure 9.20: The Result of a CHYMES Calculation after 0.4s for Experiment Set j where One Side of the Plume coincided with the Tank Wall compared with an Experimental Plume.

within 10% of 1.0m/s with exception of the plume of aluminium particles which moved much faster. The penetration speed of the normal, calculated plume was 1.3m/s, the head width 0.18m and the maximum particle volume fraction 0.18 after 0.4s. The plume width could be affected by the initial conditions, particularly by the projection speed. The maximum value of α_p increased if the projection speed increased, or the particle density decreased.

The effect of the initial jet conditions on the calculated plumes is not consistent with that on the experimental plumes. For example, the code predicts that the width of the tank does not greatly affect the width of the plume, but the experiments indicate the opposite⁷. The width of the plumes vary quite a large amount with jet conditions, unlike those measured in the experiment. In addition the spreading of the experimental plumes over the first 0.4s is much greater than those produced by CHYMES, and the rate of penetration less. The jet characteristics did not greatly affect the penetration of the computed plumes with the exception of the aluminium plume whose penetration speed, corrected for density, is much greater than that of the normal plumes. Note the very rounded shape of this computed plume compared with the others.

⁷see figures 6.11, 6.12, and 6.18

Set of Conditions	Penetration (%)	Head Width (%)	Maximum α_p
b (Fast)	108	119	0.24
c (Slow)	101	84	0.12
d (Thin)	100	47	0.16
e (Aluminium)	126	108	0.26
f (Narrow Tank)	103	95	0.20
h (Sparse)	97	55	0.05
i (Offset)	116	103	0.18

Table 9.1: The Measured Characteristics of Calculated Plumes Compared with those of a Normal Plume. The Penetration of the Plume of Aluminium Particles has been corrected for Buoyancy.

Clearly the simple comparison of the calculated plumes and the experimental plumes is not very satisfactory; as things stand it would not be possible to infer much that is correct about a real plume from the results of the computational model. A possible reason for the discrepancies is that the physical modelling is not satisfactory; for example, the drag coefficient used in the calculations is usually that of an isolated, free falling particle (0.76 in this case). An improvement might result from allowing the drag coefficient to vary according to the local particle concentration to take into account the effect of neighbouring particles. The Richardson and Zaki equation, (3.38), may be used to modify the drag coefficient to allow for this; the result is shown in Figure 9.21. This differs little in form from a plume if a fixed drag coefficient is employed. The plume was retarded by 12% and spreading was increased by 8% compared with a similar plume with a fixed drag coefficient.

As discussed in Section 8.3, it has been suggested [105] that the effective drag on particles is much larger than expected owing to turbulence. This is not modelled explicitly by CHYMES or included in the values usually assigned to drag coefficients. One reason for this suggestion is that if the drag coefficient (again fixed) is increased to 2.0, the computed plume superficially looks more like those seen in the experiments with a crescent shaped head at early times; such a plume is shown in Figure 9.22. The penetration of the plume and the growth in its width are shown in Figure 9.23 and Figure 9.24 respectively; its development appears to be much closer to that of the experimental plumes. Any further increase in the drag coefficient results in the

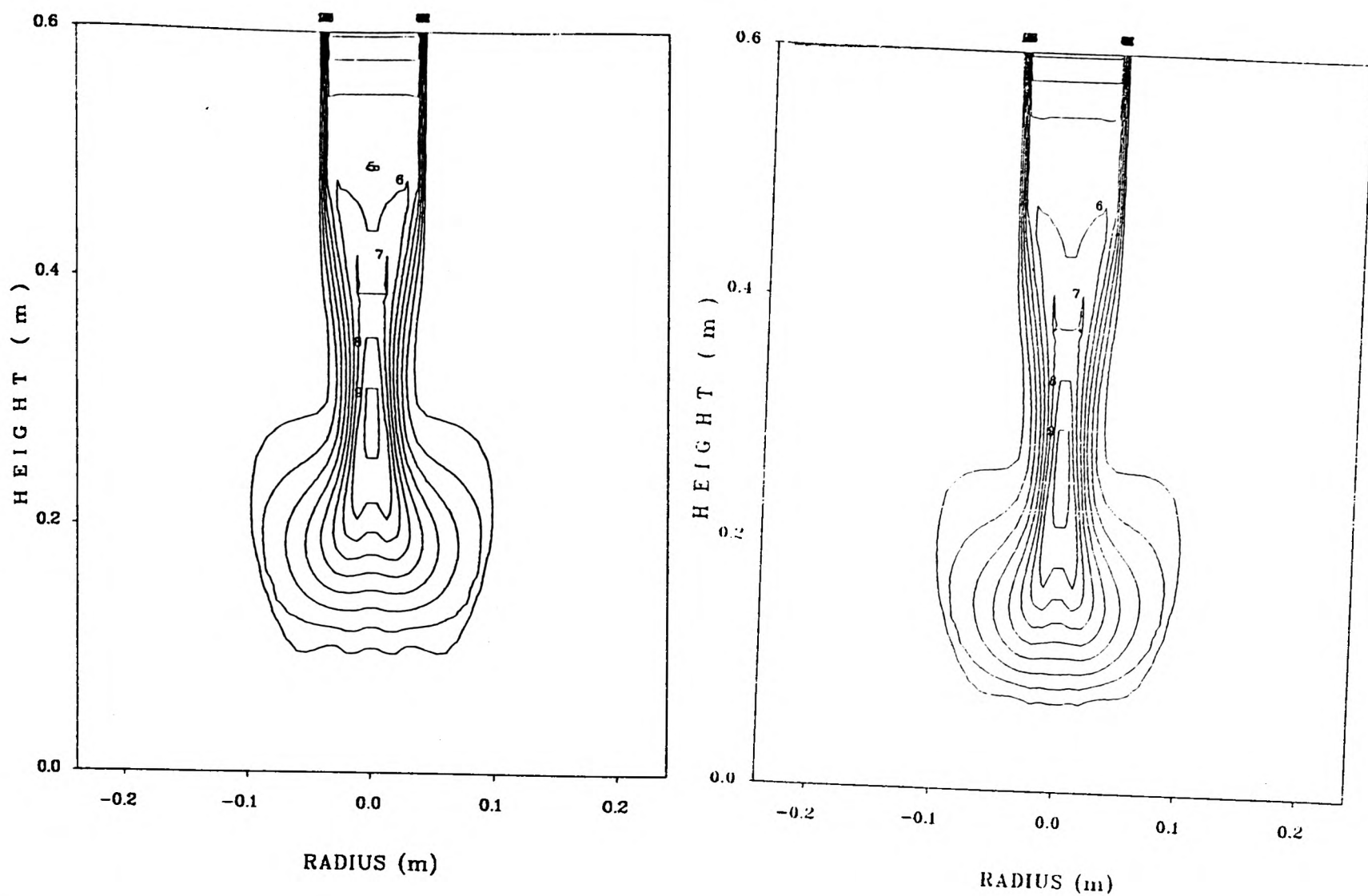


Figure 9.21: A Normal Plume after 0.4s when Equation 3.38 is used to Determine the Drag Coefficient compared with a Normal Plume when a Fixed Drag Coefficient is Used(Right).

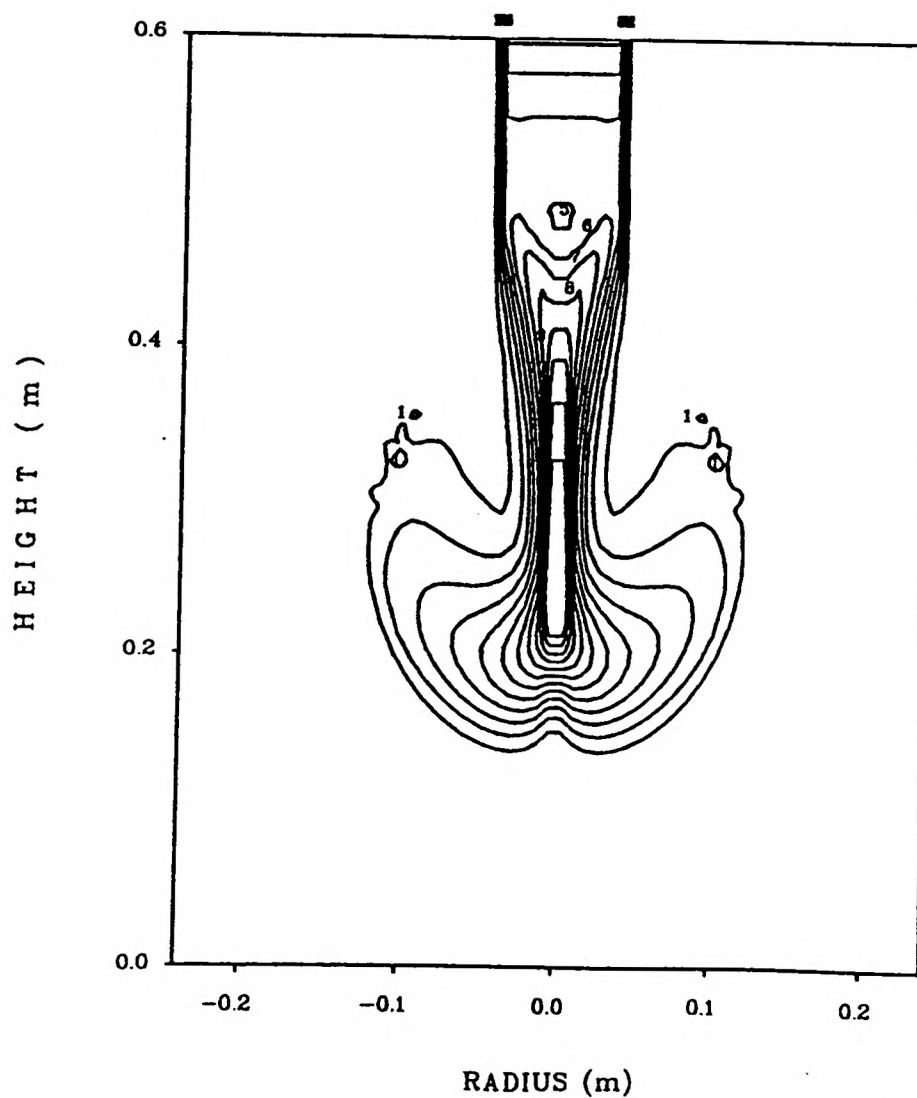


Figure 9.22: A Normal Plume after 0.4s if the Drag Coefficient in the Model is fixed at 2.0

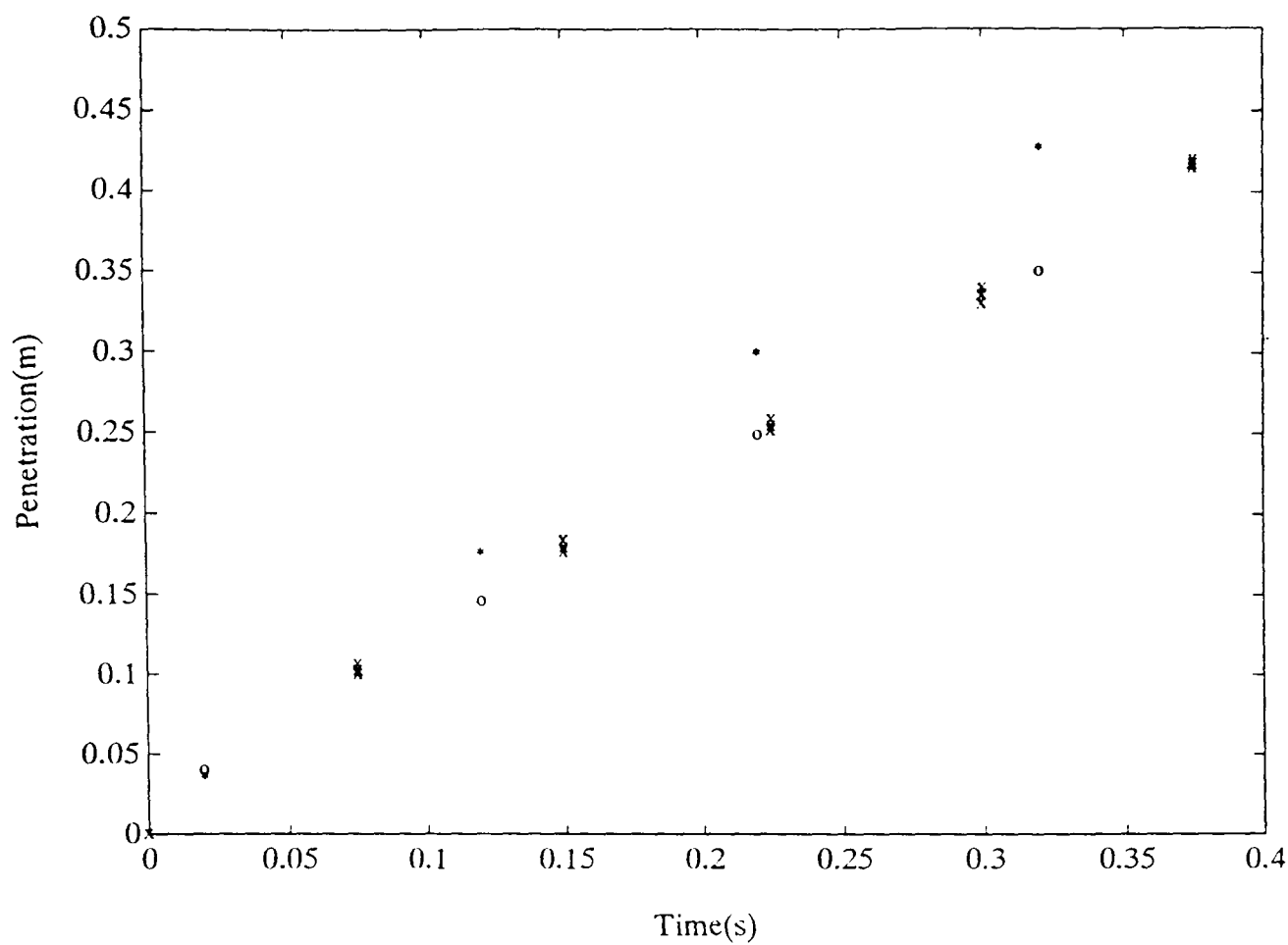


Figure 9.23: The Penetration of a Computed Normal Plume where the Drag Coefficient is Equal to 2.0 (o) compared with that where the Drag Coefficient is 0.76 (*) and the Experimental Normal Plumes (x).

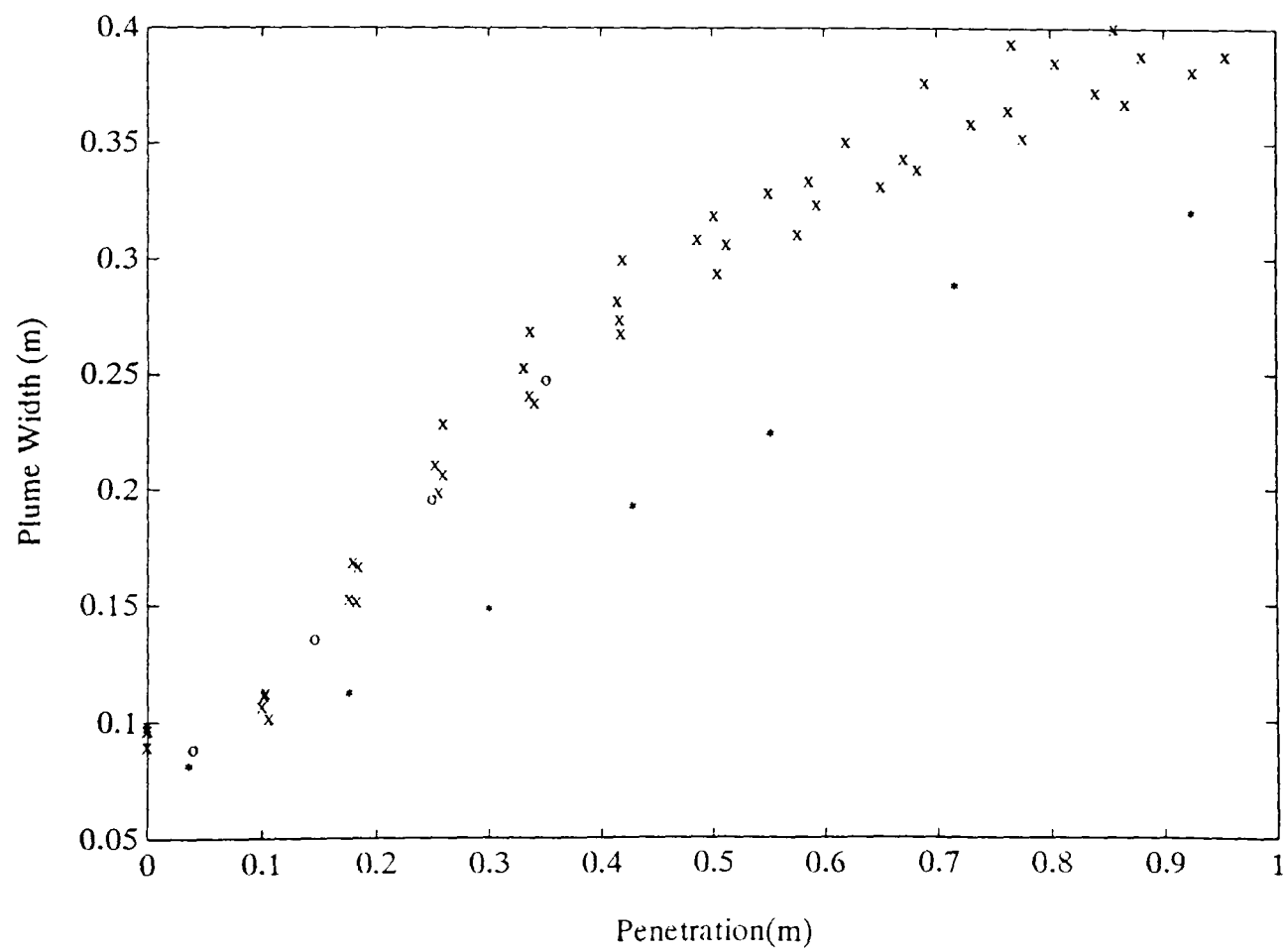


Figure 9.24: The Growth in Width of a Computed Normal Plume where the Drag Coefficient is equal to 2.0 (o) compared with that where the Drag Coefficient is 0.76 (*) and the Experimental Normal Plumes (x).

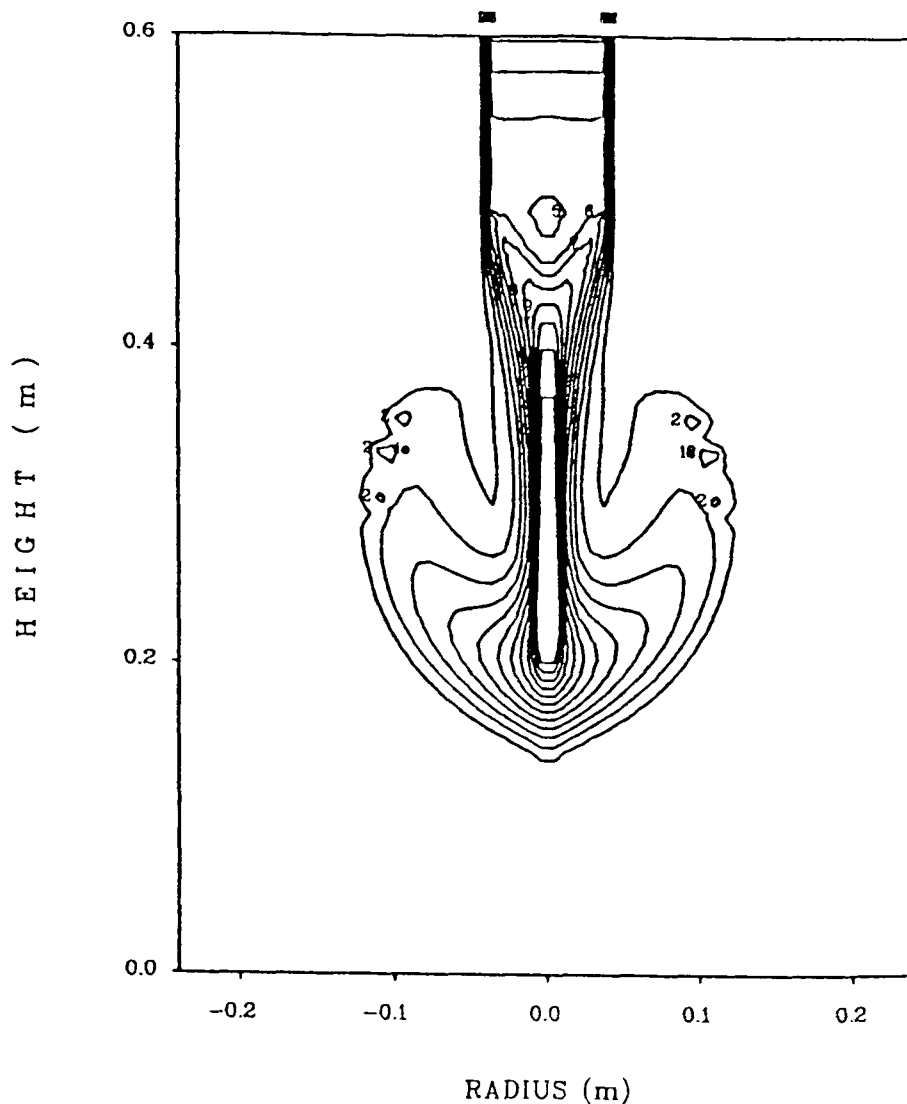


Figure 9.25: A Normal Plume after 0.4s if the Drag Coefficient in the Model is fixed at 3.0

plume shown in Figure 9.25: the penetration and spreading of the plume are similar to those if the drag coefficient is 2.0, but the wings are swept back further.

The drag coefficient was allowed to vary according to local conditions from that for an isolated particle (0.76) when the particle volume fraction was less than 0.02 to 2.0 if the particle volume fraction was equal to 0.12, the particle volume fraction of the core of the plume. Figure 9.26 shows a plume where the drag coefficient varies linearly between the two fixed points and Figure 9.27 shows a plume where the drag coefficient changes according to the Richardson and Zaki equation with an exponent modified to accommodate the fixed points. These adjustments make little difference to the results of the calculations.

It is difficult to account for nearly a three fold increase in the drag coefficient; from the discussion in Section 8.3 it is unlikely that turbulence is responsible and though the inclusion of transient forces may improve matters they are unlikely to be large enough. However a more fundamental question is whether the increase in drag coefficient actually does improve the form of the plume. The overall shape of the plume appears to be more like that in the experiments, but a number of dissimilarities remain: the numerical plume still has diffuse edges; there is a concentration of particles at the back of the head, and no concentrations anywhere else. The selection of the 0.02 contour as the edge of the computational plumes was arbitrary; the 0.01

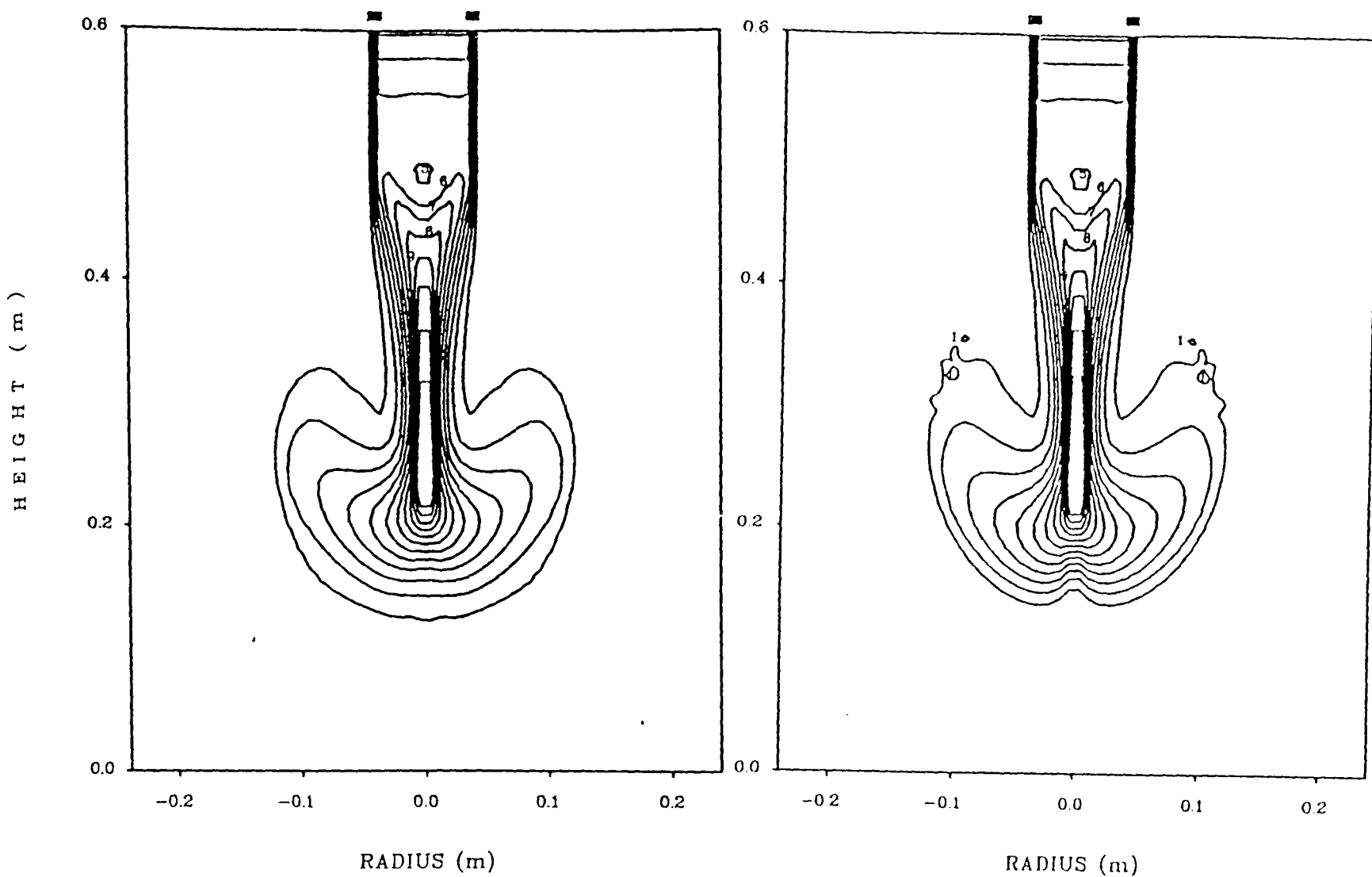


Figure 9.26: A Normal Plume after 0.4s if the Drag Coefficient in the Model is allowed to vary Linearly (Left) compared with a Fixed Drag Coefficient (Right).

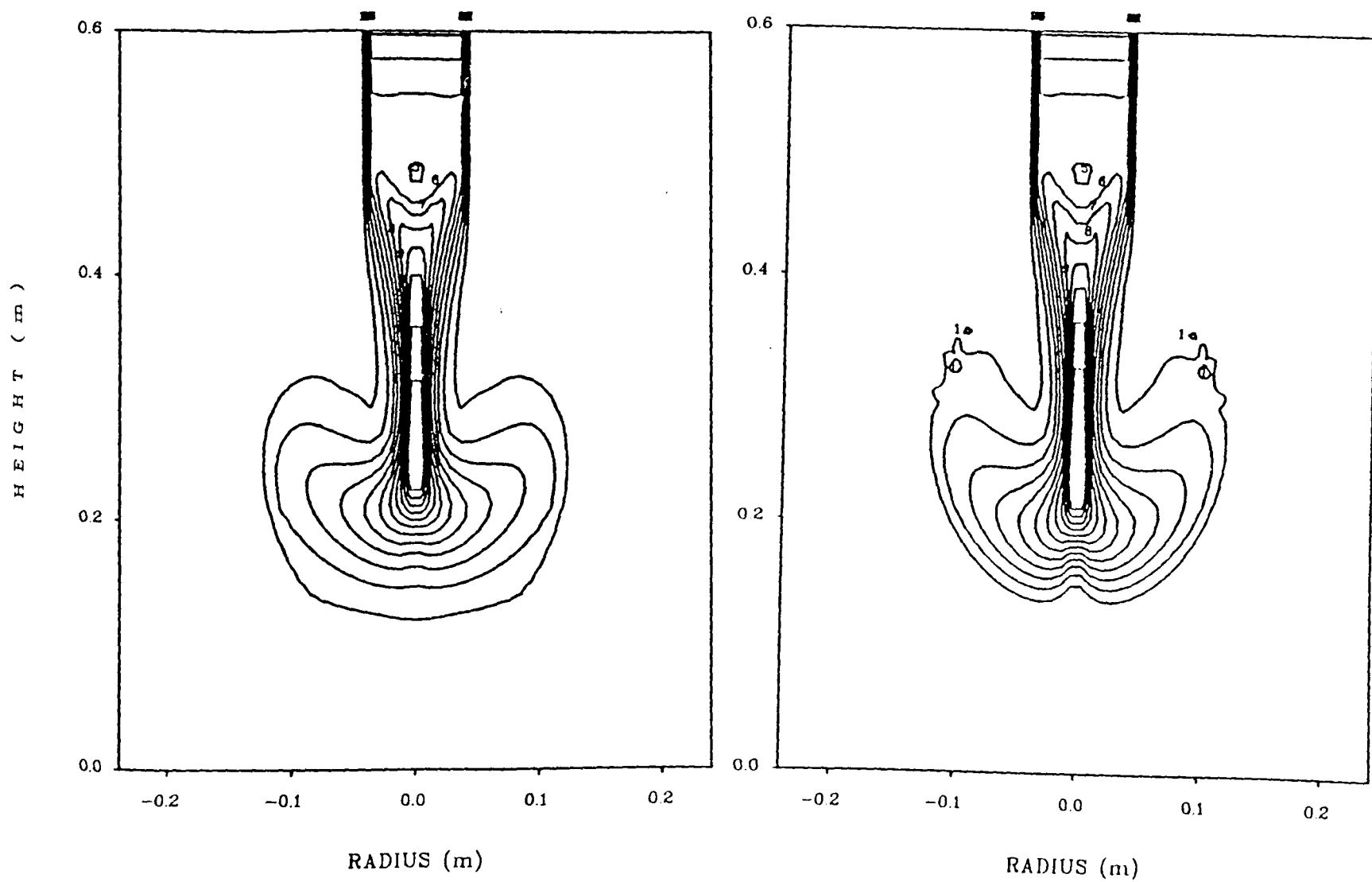


Figure 9.27: A Normal Plume after 0.4s if the Drag Coefficient in the Model is allowed to vary According to a Modified Richardson and Zaki Equation (Left) compared with a Fixed Drag Coefficient (Right).

contour line might have been plotted and used instead, as might have any other contour line. In the experiments the 0.02 contour line is equivalent to a particle separation of 16mm if they are in a square array. The selection of this contour may not be compatible with the line judged to be the edge of the experimental plume. In addition that judgement was made by eye and is subject to some subjectivity and inexactitude. For a proper, accurate comparison to be made the results of CHYMES and those of the experiments should be rendered into the same form.

Chapter 10

The Representation of a Distribution of Particles

The root of the problem of the comparison of the numerical and the experimental plumes is that the contour plots represent a continuous function, the particle volume fraction α_p , while in the experiments the distribution of lots of discrete points is seen; the experimental plumes are in a non-averaged form and the computational plumes in an averaged one. For a proper comparison between the two sorts of plume it is necessary to process them so that they are represented in the same manner, either by converting computational results into the form of experimental ones, or vice versa.

10.1 The Conversion of the Numerical Plumes to the Form of the Experimental Plumes

The purpose of this process is to represent the particle volume fraction contours as discrete particles. This can be done by drawing particles to scale, separated by a distance determined by the value of the nearest contour; an example of this is in Figure 10.1. This treatment helps to make the contour plots more comprehensible; for example the concentration of particles at the back of the head is very apparent. However it is not accurate because of the difficulty in exactly varying the particle separation in two dimensions with particle volume fraction. The operation does not aid the comparison of the two forms of results because in the experiments the particles are irregularly spaced, and the exact arrangement of particles differs in each plume; it is not possible to recover an non-averaged distribution from an averaged one. In addition the process does not resolve the problems of defining the edge of the plume.

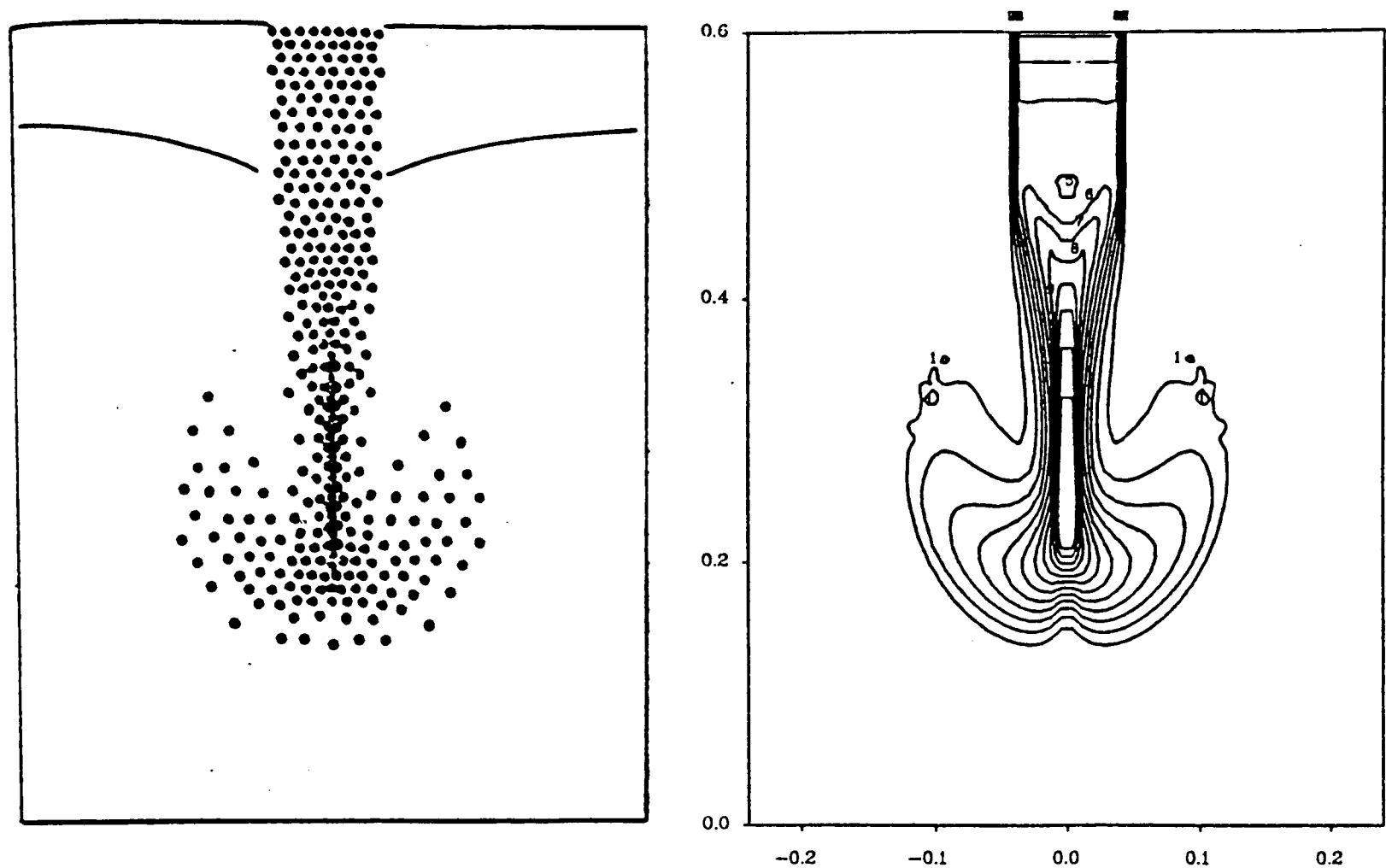


Figure 10.1: A Normal Plume after 0.4s when the Drag Coefficient is fixed at 2.0 and converted into a Discrete Form (D.B.R.Kenning [107]) (Left) compared with the Original Plot (right).

To compare the two sorts of plume it is necessary for them to have the same character: averaged or non-averaged. This is not the same as them both having a discrete or a continuous form. The process described above converts the presentation of results from a continuous form to a discrete one; but the results are still in an *averaged* form. The point can be illustrated by considering an election. Each member of an electorate will have a unique set of political views and when he or she is choosing a candidate for an office it is unlikely that there will be one whose manifesto exactly reflects them; instead he or she must decide which one most nearly matches the overall shape of his or her opinions. The election result can be determined if the detailed beliefs of all the electorate were known; it is impossible to determine the views of an elector from the result, even if it was known which candidate he or she voted for. Similarly, if the positions of the particles in an experimental plume are known their overall characteristics may be found by averaging, but the process cannot be reversed.

The distribution of particles is described by the particle volume fraction which, from equation (2.1), is calculated by defining an averaging volume and calculating the proportion of it occupied by particles. For this to be done it is necessary to estimate the positions of the particles in a real plume.



Figure 10.2: The Plume Processed. It is a Normal Plume after 0.60s.

10.2 The Location of Particles in an Experimental Plume

A plume may consist of several hundred particles. If the whole plume is to be processed it is then highly desirable to automate any procedure that deals with individual particles. This requires the image of a plume on video tape to be converted into digital form. The plume shown in Figure 10.2- which is a normal plume after 0.6s- will be used to illustrate this process and subsequent ones. The picture was converted to a digital form with an Oxford Framestore Applications Image Analyser. This is based on a PC type computer, and allows a frame of the video recording to be 'grabbed' and converted to digital form. Each side of the image is divided into five hundred and twelve divisions; the cells- called pixels, from picture elements- thus created are then assigned a number- called the grey level- between 0 and 255 corresponding to the darkness of their contents: the darker the cell the smaller the grey level. A grey level of 0 corresponds to black and one of 255 to white. The dimensions of the cells for the example plume were 2.4mm longitudinally (parallel to the direction of gravity) and 1.5mm laterally (perpendicular to the direction of gravity). These compare with a particle diameter of 6mm. The grey level of every cell making up an image could be recorded sequentially, row by row, onto a five inch floppy disk.

The particles may be identified if they have a grey level different from that of their surroundings. Unfortunately in the digitised plume the darkness of a particle and the contrast with its surroundings depends on its position in the tank owing to variations in the strength of the illumination and the effect of air entrained by the column of particles. The plume needs to

be processed to correct this defect.

The images were transferred to half inch magnetic tape and thence onto a Gemstone Campus Image Processing Computer. Many of the effects of non-uniform illumination could be eliminated by subtracting an image of the tank before the plume had entered it from one of the plume. The result could then be *density sliced*. This is where a threshold is selected such that the pixels with a smaller grey level are assigned one grey level (say 0, or black), and all those with a higher one another (say 255, or white); thus the particles represented by groups of black pixels on a white background. To take into account the variation of darkness in space the image was divided into regions which were treated separately. For example, the plume in Figure 10.2 was divided into three areas:

the top 250mm of the column; This region was neglected because the contrast was so slight that it was not possible to distinguish between the particles and their surroundings.

the rest of the column; The contrast between the particles and the background and the level of illumination were lower here than in the rest of the plume owing to the air entrained by the column.

the head and wings of the plume. The contrast in this area was generally quite good though some of the outlying particles are faint.

The plume after image processing is shown in Figure 10.3. Individual particles can often not be distinguished from other, close-by particles. This is partly because of the problems of contrast and illumination outlined above; however in addition unless there is an empty, or nearly empty, pixel between the edges of two particles they cannot be distinguished from one another; particle centres must then be separated by at least 7.5mm in the lateral direction and 8.4mm in the longitudinal one if they are to be seen individually; as in previous analysis the large difference between the size of the particles and that of the plume causes inaccuracy. Assumptions must then be made about the distribution of particles and the plume will not be exactly reproduced. For the purposes of averaging this should not matter because this process removes the effect of local particle distribution; therefore, provided that the gross structure of the plume is correct, assumptions about the local particle distribution should not affect the final, averaged result.

For individual particles to be assigned positions the edges of the groups of black pixels must be located before they are divided into individual particles. These tasks were performed by the computer programs described in detail in Appendix A. Black pixels not in contact with any other were discarded as spurious. If a group was larger than one particle, the particles

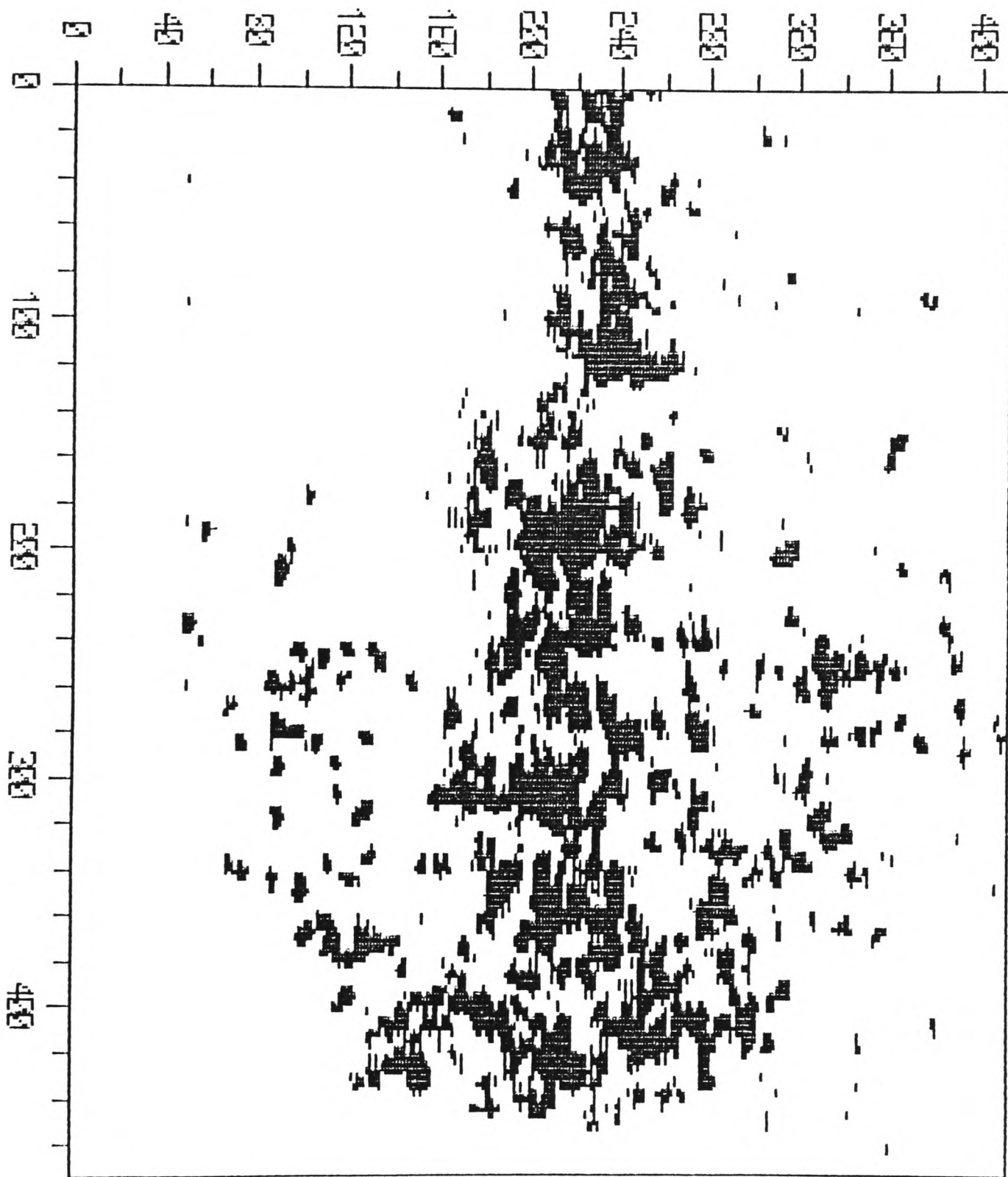


Figure 10.3: The Example Plume after Image Processing. The Scales on the Axes are in Millimetres.

within it were assumed to form a rectangular array. The spacing between the particles in such a group could be any distance from 6mm, the minimum possible, to 7.5mm in the lateral direction and 8.4mm in the longitudinal, the maximum separation between particles for them to be indistinguishable. To determine the best separation the number of particles identified in the plume was compared with the number, 480, expected from the jet characteristics¹. When the particles in the groups were closely packed 634 particles were identified; when the separation was the maximum expected 470 particles were identified; therefore the latter particle spacing was selected. The size of the group was rounded to the nearest particle diameter.

It is difficult to determine how exactly the assumed particle distribution reproduces the real one. On one hand the resolution error of the particle separation may be larger than estimated if there is a lot of air present or some other local, darkening effect; on the other the particles may have an arrangement that allows a higher maximum particle volume fraction than the simple, square one. The former discrepancy will cause the particle concentration to be overestimated, and the latter the opposite.

The result of the operations on the example plume may be seen in Figure 10.4. The size of the blobs is approximately to scale. The positions of the centres of the particles are stored in a computer file to allow further processing, such as averaging.

10.3 Averaging

The averaged variable denoting particle distribution is the particle volume fraction, α_p . It will be recalled from (2.1) that this is defined as:

$$\alpha_p = V_p/V \quad (10.1)$$

where V is the size of the averaging volume and V_p the volume of particles within it. To calculate α_p at any point the size and shape of the averaging volume, and the volume of particles within it have to be known.

The purpose of averaging is to eliminate the effect of the local distribution of the particles so that only the volume present is important. For the process to be successful length scale constraints (2.5) and (2.12)² must be satisfied. (2.12) implies that α_p is uniform, or at least

¹While the apparatus is getting up to speed seven rows, or seventy particles, enter the water in 100ms. After this the speed of projection of the balls and their spacing is known so the number rate of particles can be easily found. No particles may be seen in the top 250mm of the column and are discounted.

²i.e. the particle spacing must be much smaller than the size of the averaging volume which must be much smaller than the distance over which the flow properties vary significantly.

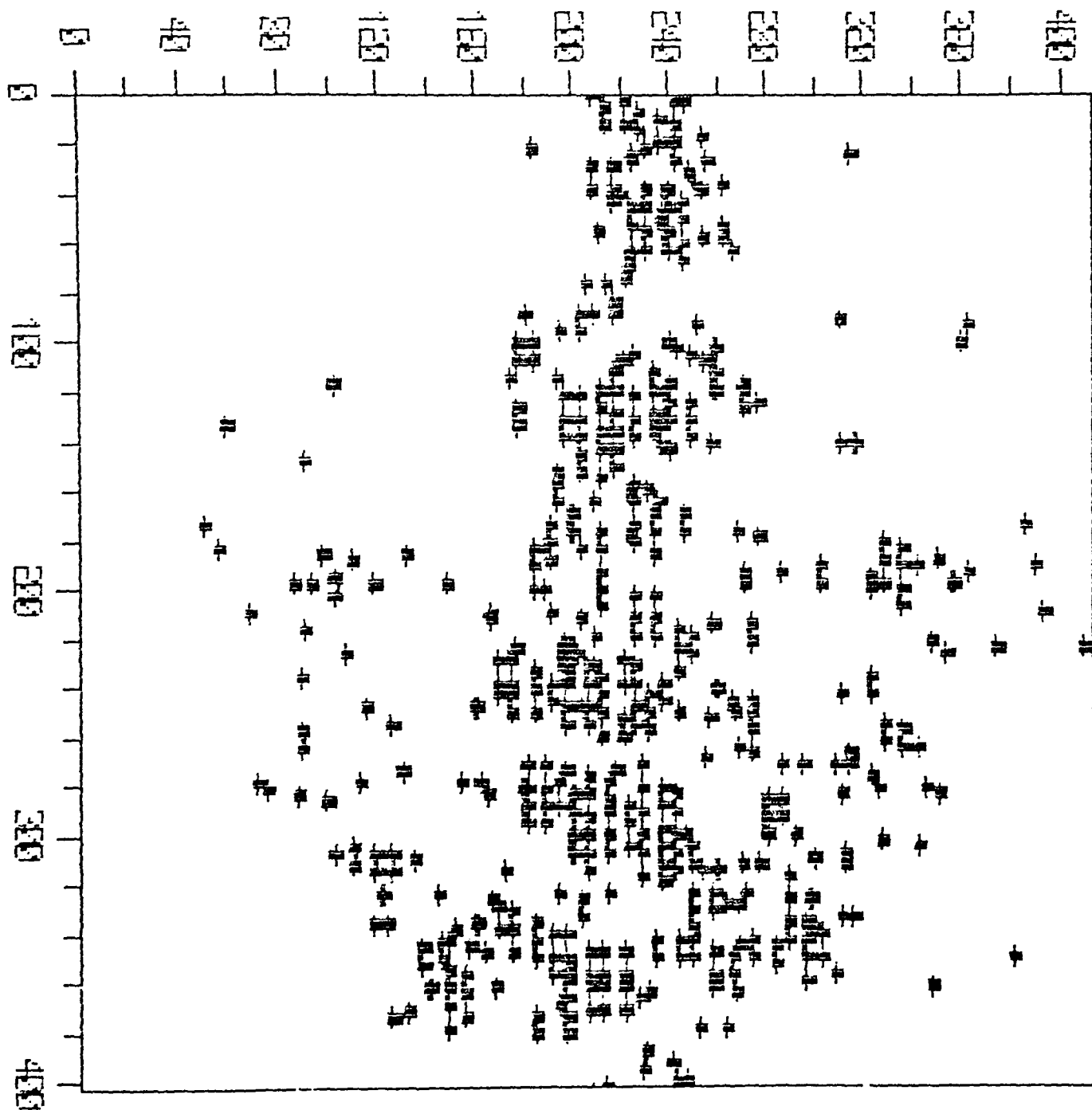


Figure 10.4: The Estimated Positions of the Particles in the Example Plume. The Scales on the Axes are in Millimetres.

nearly uniform, in the averaging volume; this sets a maximum to the size of the averaging volume. (2.5) must be satisfied for the average to be independent of the local particle distribution, and the size and shape of the averaging volume; this defines a minimum size for the averaging volume. The question that has to be answered is how much larger than the particle spacing does the averaging volume have to be?

The computational grid and the experiments are based on a rectangular geometry so it would be sensible for the averaging volume to be a cuboid. The thickness of the volume was set to be equal to that of the tank. This allows the plume to be treated as being perfectly two dimensional so that the distribution of the ‘melt’ only varies in two dimensions; in a three dimensional flow the extra dimension will introduce more irregularity into the distribution of the ‘melt’.

The volume of particles within a plane averaging volume, and so α_p , can be calculated exactly by the program AVVOL.FOR described in Appendix B.

α_p is a dimensionless quantity that depends on four lengths: the tank thickness, d_t ; the particle diameter, d_p ; the particle spacing, S_p ; the size of the averaging volume, l_v . In the experimental plumes d_t was set to 7.8mm and d_p to 6mm. The ‘true’ value of α_p in a uniform distribution is $V_p/(S_p^2 d_t)$ if V_p is the volume of a particle. $V_p = \pi d_p^3/6$ if the particles are spherical so a likely set of length ratios whose combination would reproduce the ‘true’ value of α_p is then d_p/S_p , l_v/S_p , and d_t/S_p . If this is so there is no benefit in presenting the results of calculations in non-dimensional form because any change in S_p will affect all three non-dimensionalised variables. Calculations will then be made using the three variables α_p , S_p , and l_v .

10.3.1 Uniform Distributions

If the averaging volume is allowed to grow from a small size in a uniform distribution the calculated value of α_p will fluctuate with an ever smaller magnitude about its ‘true’ value. An example is shown in Figure 10.5 where the centre of the averaging volume coincides with that of a particle in a square, two-dimensional distribution of spheres in a tank of the same thickness as that in the experiments. This is similar to Figure 2.1.

The sizes of the undulations in α_p will vary with the position of the centre of the averaging volume; for example, if it is offset by a quarter of S_p in each direction α_p varies very little from its ‘true’ value, as can be seen in Figure 10.6. To find the value of l_v at which the calculated value of α_p becomes independent of the position of the averaging volume, as well as its size, α_p

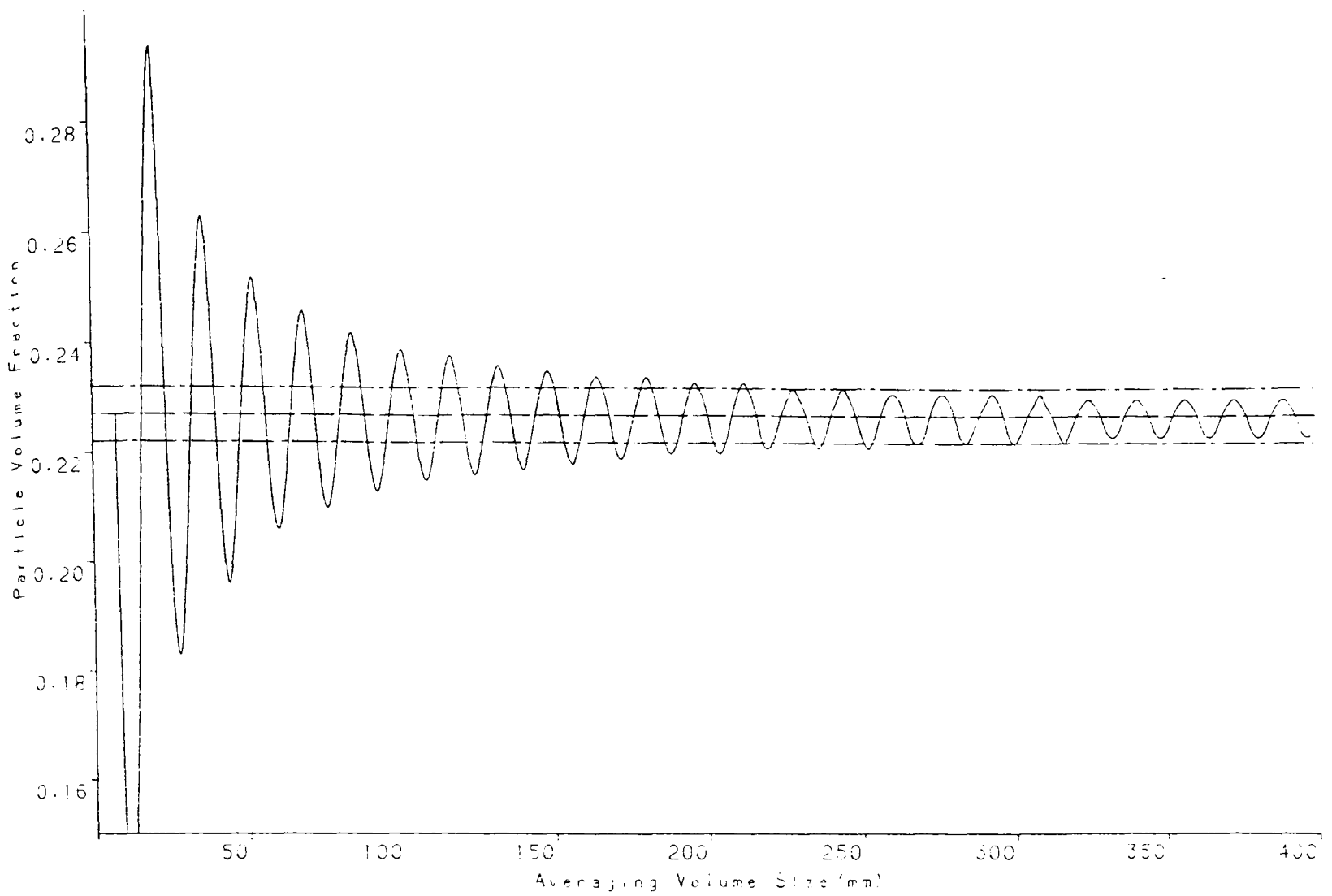


Figure 10.5: The Variation of α_p with l_v when the Centre of the Averaging Volume coincides with that of a Particle in a Uniform, Square Array. $S_p=8\text{mm}$.

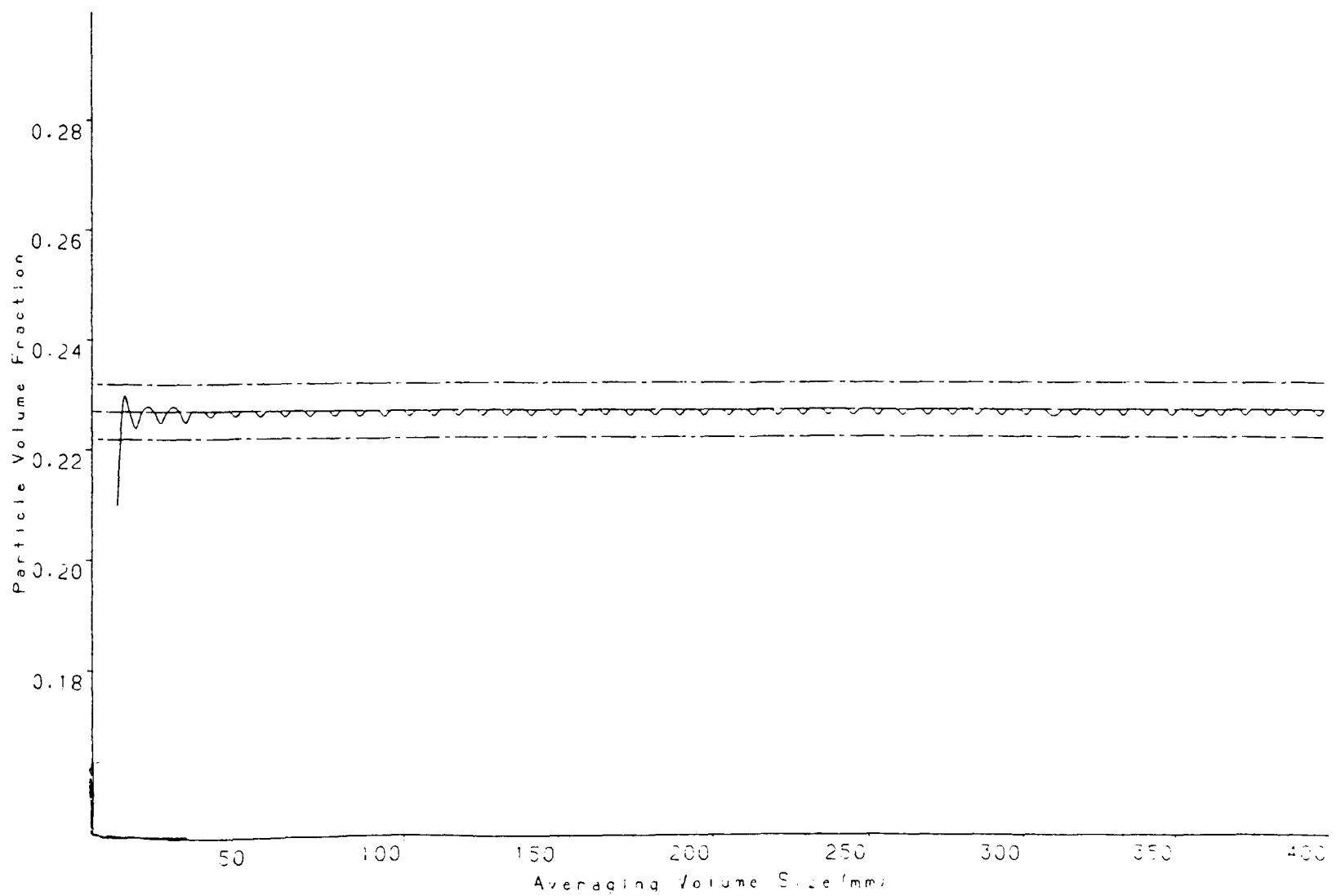


Figure 10.6: The Variation of α_p with l_v when the Centre of the Averaging Volume is $S_p/4$ from the nearest Particle in both Directions, $S_p=8\text{mm}$.

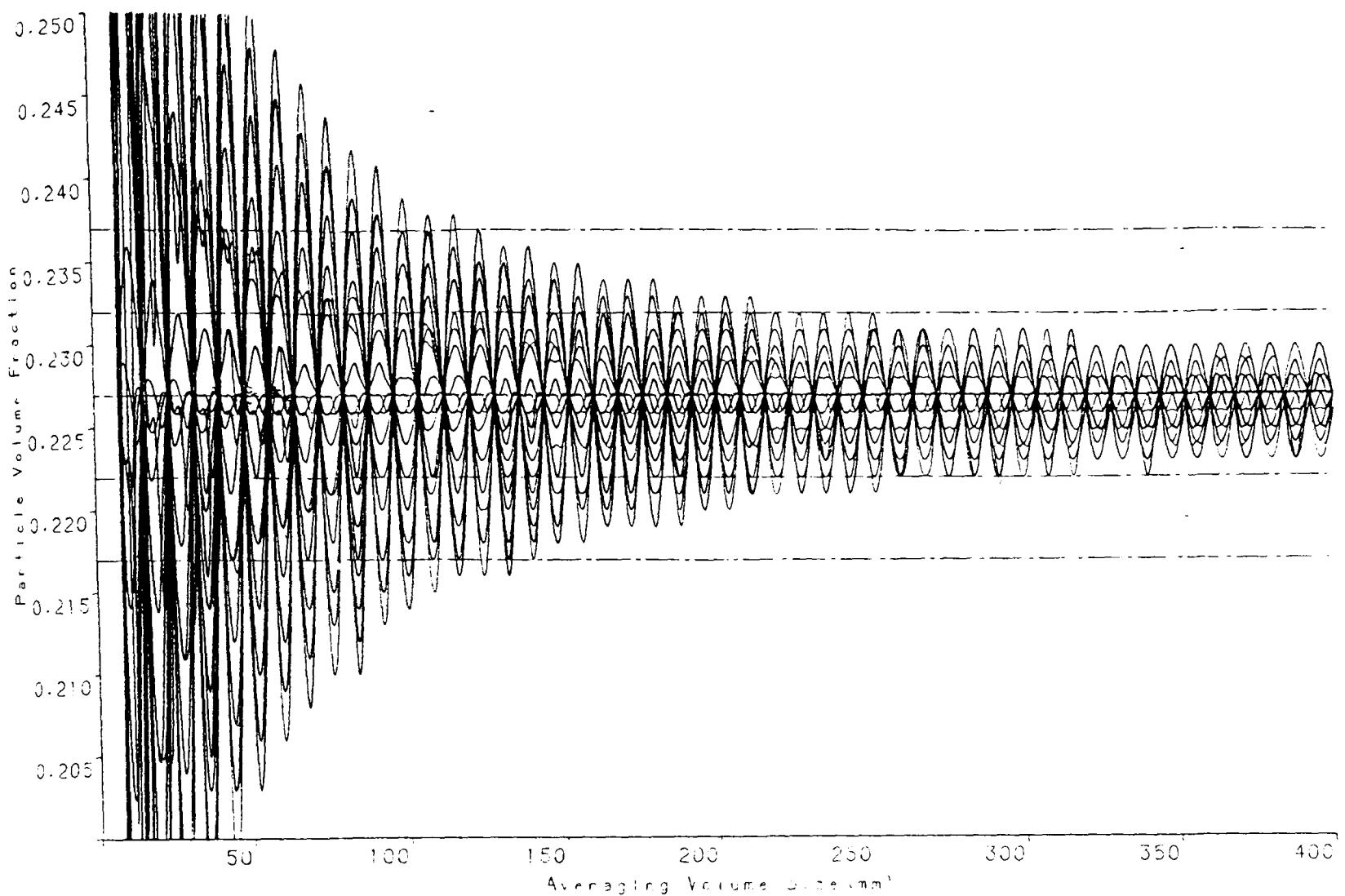


Figure 10.7: The Variation of α_p with l_v and the Position of the Centre of the Averaging Volume. $S_p = 8\text{mm}$.

must be plotted against averaging volume size for a range of positions relative to a particle in the array, as in Figure 10.7.

The size of the undulations of α_p tend asymptotically to zero with averaging volume size; therefore a threshold value for the magnitude of the undulations (i.e. the accuracy of the calculation) must be specified. The results for square arrays of several spacings for a number of thresholds are shown in Table 10.1. The thresholds are absolute figures, not a proportion of the 'true' value of α_p , because the accuracy of plots depend on absolute values rather than relative ones.

The calculated value of α_p depends on the 'true' value of α_p , l_v , and S_p ; therefore the results are quite complicated. l_v divided by S_p does not change a great deal unless the flow is dilute, as in the case if the particle centres are separated by 15mm; then, because the true value of α_p is very low, the deviation of its calculated value from its 'true' value is also small in absolute terms.

Similar tests have been made on truly three dimensional, regular arrays of particles using a spherical averaging volume (A.Celmiņš [108]). The size of averaging volume necessary for the calculated α_p to be within 0.005 of its true value was much smaller than those in Table 10.1,

S_p	True α_p	Threshold	l/S_p	l (mm)	Number of Particles in Volume
6	0.403	0.0050	36	216	1369
		0.0075	21	125	484
		0.010	16	96	289
8	0.226	0.0050	32	256	1089
		0.0075	24	192	625
		0.010	17	135	324
10	0.145	0.0050	29	291	900
		0.0075	21	125	484
		0.010	16	96	289
15	0.064	0.0050	16	240	289
		0.0075	10	152	121
		0.010	8	123	81

Table 10.1: The Effect of Particle Spacing and the Threshold of Variations from the True α_p on the Necessary Size of an Averaging Volume.

less than eight times S_p , partly as a result of the use of a spherical averaging volume. No details were given of the calculations so it is difficult to account fully for the difference.

10.3.2 Non-Uniform Distributions

It has been shown that for uniform particle distributions very large averaging volumes may be necessary for α_p , and the other averaged quantities, to be independent of the averaging volume size and position, and the local particle distribution.

The purpose of averaging is to eliminate the variations of an averaged property from its 'true' value owing to local conditions and to reveal their overall, large scale variation; for example in a gas it is not the spacing between individual molecules that matters, but the average spacing in a region, a measure of which is the gas's density. Within the averaging volume the averaged property should be approximately constant. A test for this- discussed in Section 2.1.1- is to average the averaged property again and to see if a different result is arrived at; if the averaging process has been successful then this should not happen i.e. for the particle volume fraction $\langle \alpha_p \rangle = \alpha_p$. It was shown that for this to be true l_v has to be much smaller than L^3 so that the distribution of an averaged quantity could be approximated to a straight line over l_v . If this is not true then many steps in the development of the model of a particulate flow can not be applied; for example equation (2.41), $\psi_k = \langle \psi_k \rangle + \psi_k^*$. The model is then inappropriate for the situation and the calculated averaged variables are never independent of the averaging volume size, l_v : once l_v is large enough for the undulations in its value to be small the calculated value of an averaged quantity continues to vary, reflecting overall changes. An example is shown in Figure 10.8 where the spacing from one particle centre, positioned a distance x from one axis, to the next in the x -direction is evaluated by using the formula $S_p(x) = 6(1 + 7 \times 10^{-6}x^2)\text{mm}$.

The situation is analogous to the measurement of the density of a gas around a point whose temperature is different from that of the bulk of the gas. A small control volume may be defined that is sufficiently large for the gas within it to be treated as a continuum rather than as a collection of molecules. If the density of the gas is measured as the volume is increased in size, it will change owing to the effect the temperature gradient on the mean separation of the molecules.

The most extreme form of the non-uniformity is a discontinuity or an edge to the distribution. A continuum model cannot model a discontinuity and so it is smeared out and lost. An example of the effect of an edge is shown in Figure 10.9 where there are no particles with posi-

³The length over which the averaged quantity changes significantly.

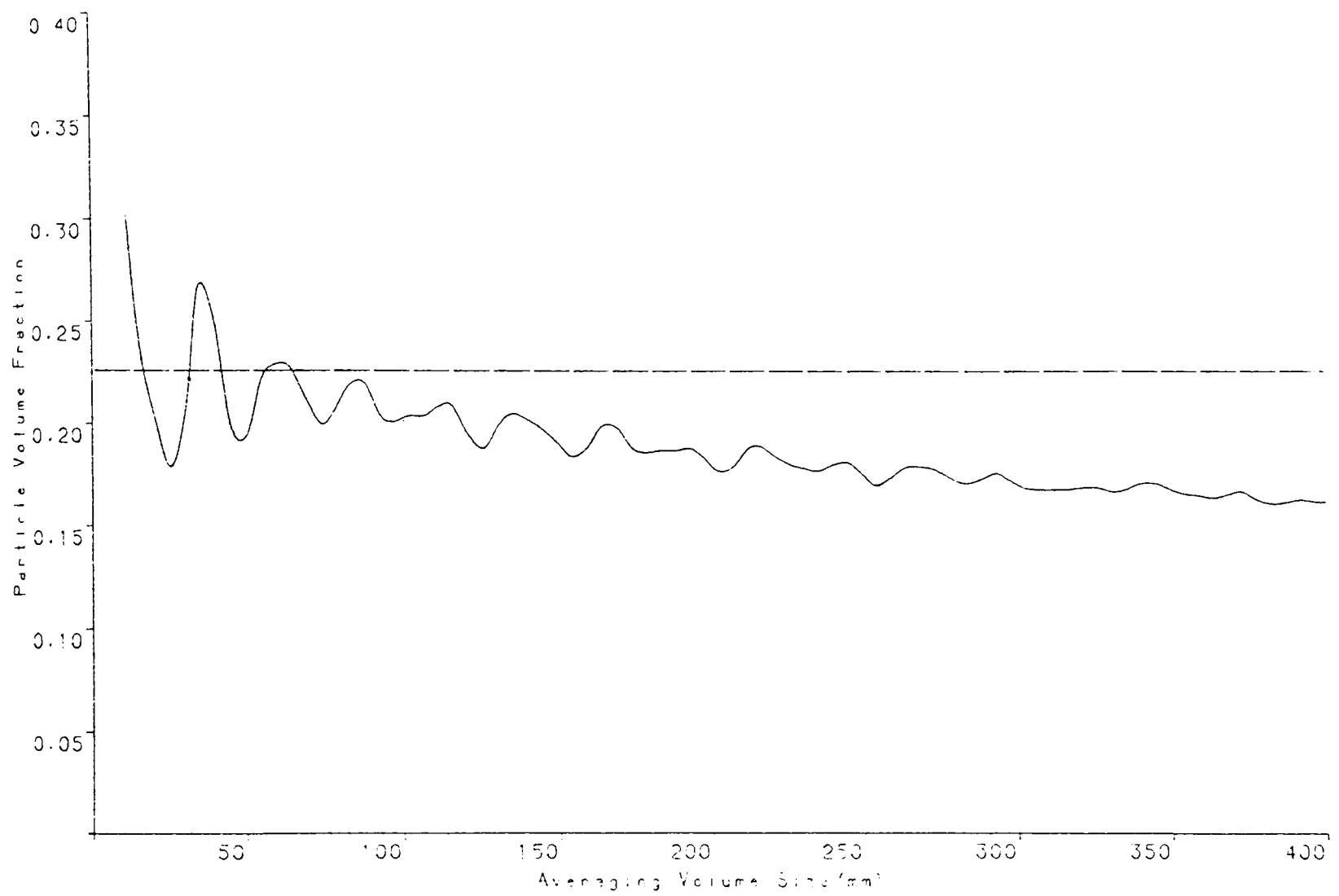


Figure 10.8: The Variation in the Calculated Value of α_p where the Particle Spacing is a Function of Position in One Direction

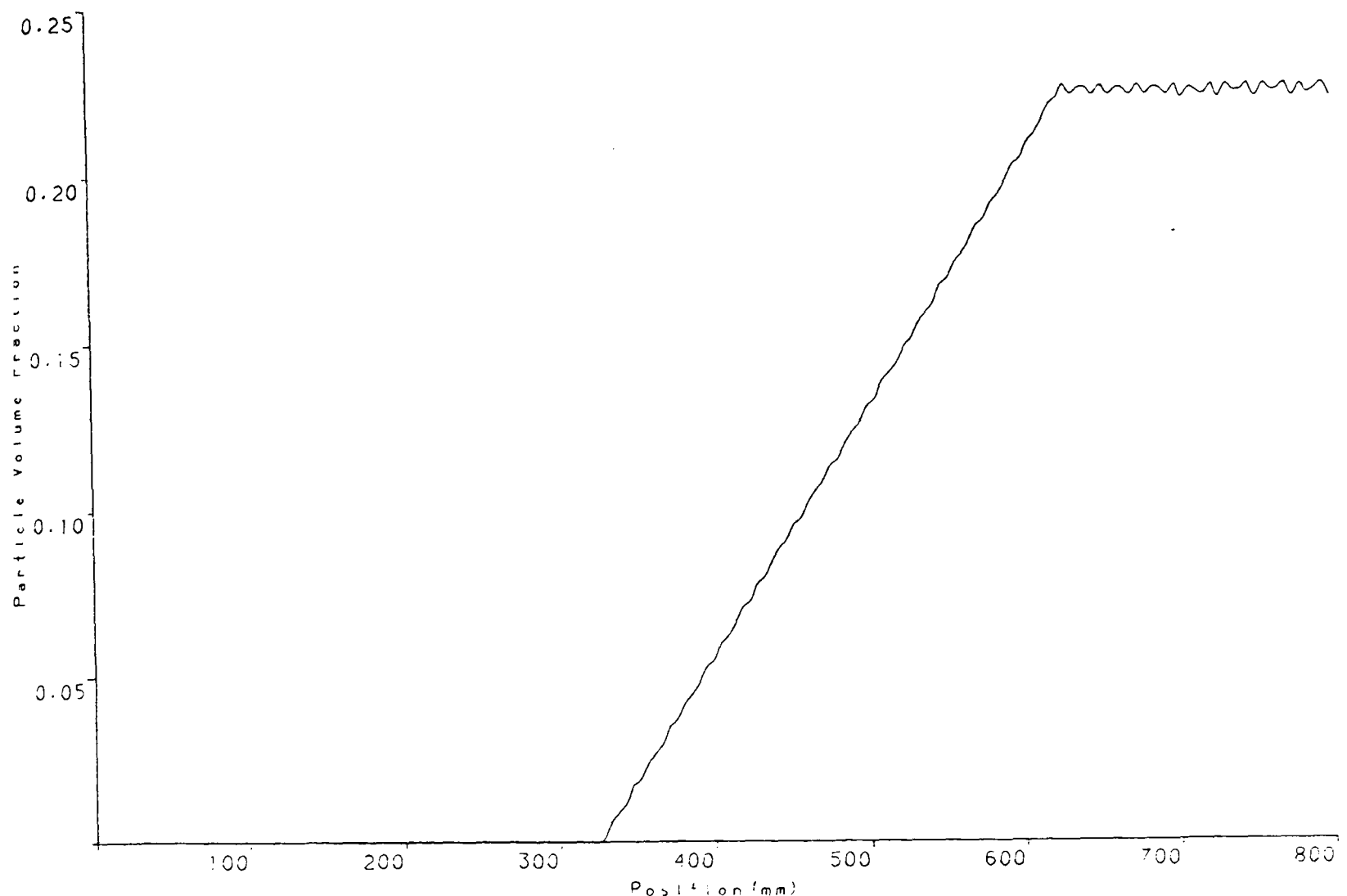


Figure 10.9: The Effect of an Edge to a Particle Distribution on the Calculated Values of α_p if the Averaging Volume is Large enough for α_p to be within 0.01 of its ‘true’ value in a Uniform Distribution.

tions less than 480mm and an uniform particle distribution⁴at positions greater than this. This distribution is transected by an averaging volume just large enough (length of side 135mm) for the calculated values of α_p to be accurate to ± 0.01 in an uniform distribution. If the result was converted into contours a series of evenly spaced lines would be seen, rather than the ‘correct’ result where they would all be superimposed at 480mm.

10.4 The Averaging of Experimental Plumes

The computer program AVVOL.FOR can be used to average experimental plumes. The example plume is shown in Figure 10.10 after it has been averaged using an averaging volume of dimensions equal to those of the computational grid; it is little different from the figure showing the estimated particle distribution in the plume⁵. It is clear that the averaging process has not been successful: the value of α_p at any point depends on the local distribution of particles rather than the point’s position in the plume. For example in the groups of particles in the

⁴The particle diameter is 6mm, and their spacing 8mm so the ‘true’ value of α_p is 0.227.

⁵Figure 10.4

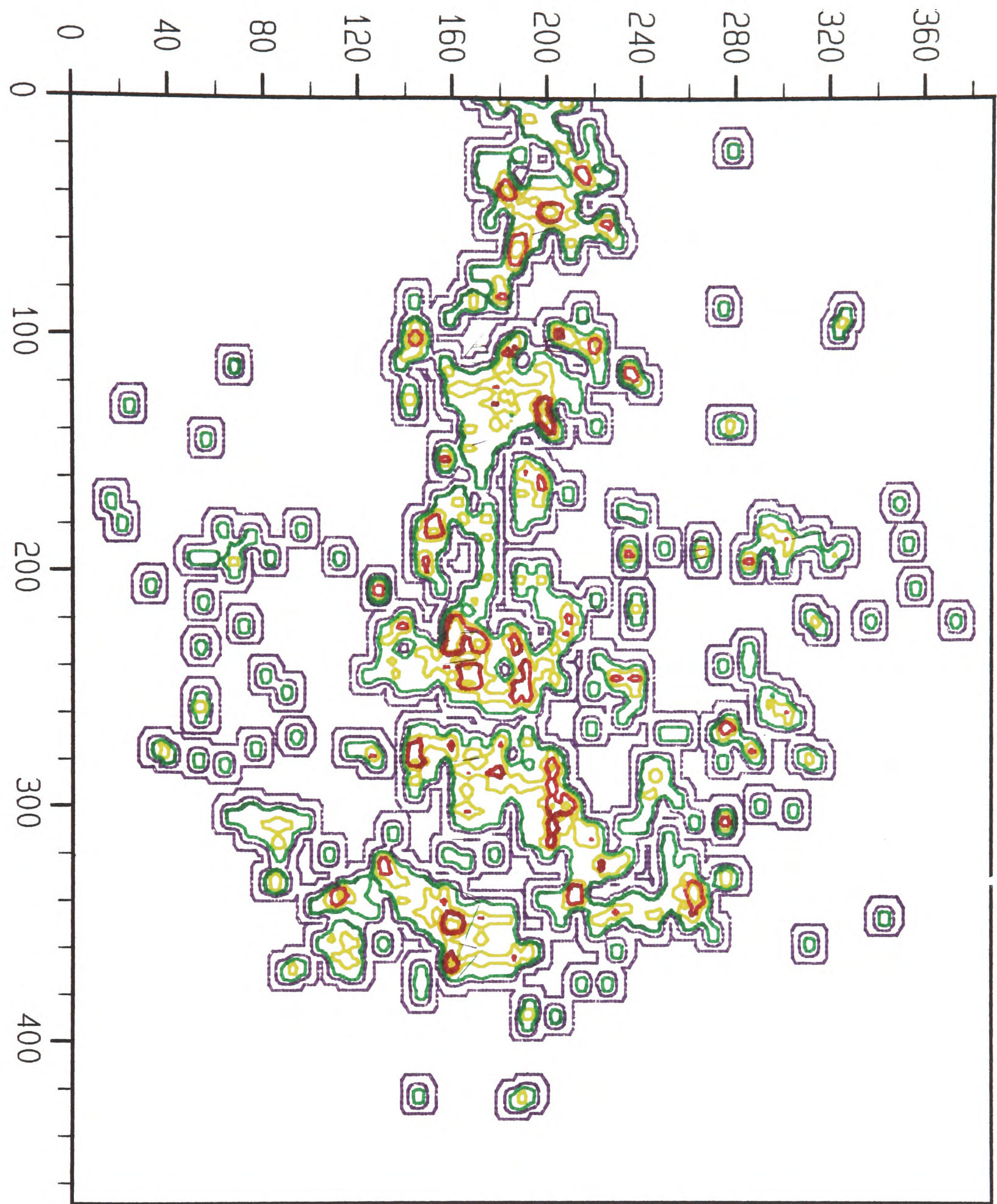


Figure 10.10: The Experimental Plume after averaging using an 8×10 mm Averaging Volume.

The Axes are in Units of mm.

column that are larger than the averaging volume the particle distribution is assumed to be uniform and, therefore, so should α_p ; in fact it varies with the location of the averaging volume in these groups, as might be expected from the results of the previous section.

The results of a computational model will be identical if the boundary and initial conditions are the same no matter how many times it is run. From page 97 the local distribution of particles in the experiments differs in plumes with identical initial conditions at a given time; therefore if a normal experimental plume after averaging reflects the local particle distribution the same result would not be obtained from a different normal experimental plume and the result would not be comparable to the computational plume. In other words after averaging the distribution of α_p should be identical in experiments a1, a2, a3, and a4. The same could be said for any set of experiments.

If an averaging volume with a length of side double that of the computational grid (so its volume quadruples) or four times that of the computational grid (so its volume increases sixteen times) is used the pattern of contours still depends upon the local particle distribution rather than the plume's characteristics as shown in Figures 10.11 and 10.12; some of the contours reflect the shape of individual particles (such as the square contour towards the top left hand corner) or groups of particles (such as the contours of the group towards the front of the plume).

The dimensions of the averaging volume for α_p to be within 0.01 of its 'true' value in the uniform distribution assumed in the groups of particles are 136×121 mm. If an averaging volume of this size is applied to the experimental the effect of the local particle distribution appears to be eliminated as shown in Figure 10.13. This has the character of the CHYMES plots with a gradual monotonic decrease in α_p from the central core to the edges of the plume.

If the size of the averaging volume is further increased to 200mm square then, as may be seen from Figure 10.14, all dependence on local detail is lost, but so is most of the overall structure of the plume. Any further increase in size will merely cause the particulate mass to be spread more thinly.

The averaging volume in Figure 10.13 is large enough for an uniform distribution of particles, but the experimental plume is clearly not uniform. To test whether a larger averaging volume is required the program FLEXVOL.FOR- detailed in Appendix B- may be used. This is the same as AVVOL.FOR except that the size of the averaging volume is allowed to increase until one of two criteria are satisfied. The first is that the particle distribution is even enough for the averaging process to be successful so that the undulations in the calculated value of α_p converge onto its 'true' value; when their magnitude becomes less than 0.02, the spacing between the

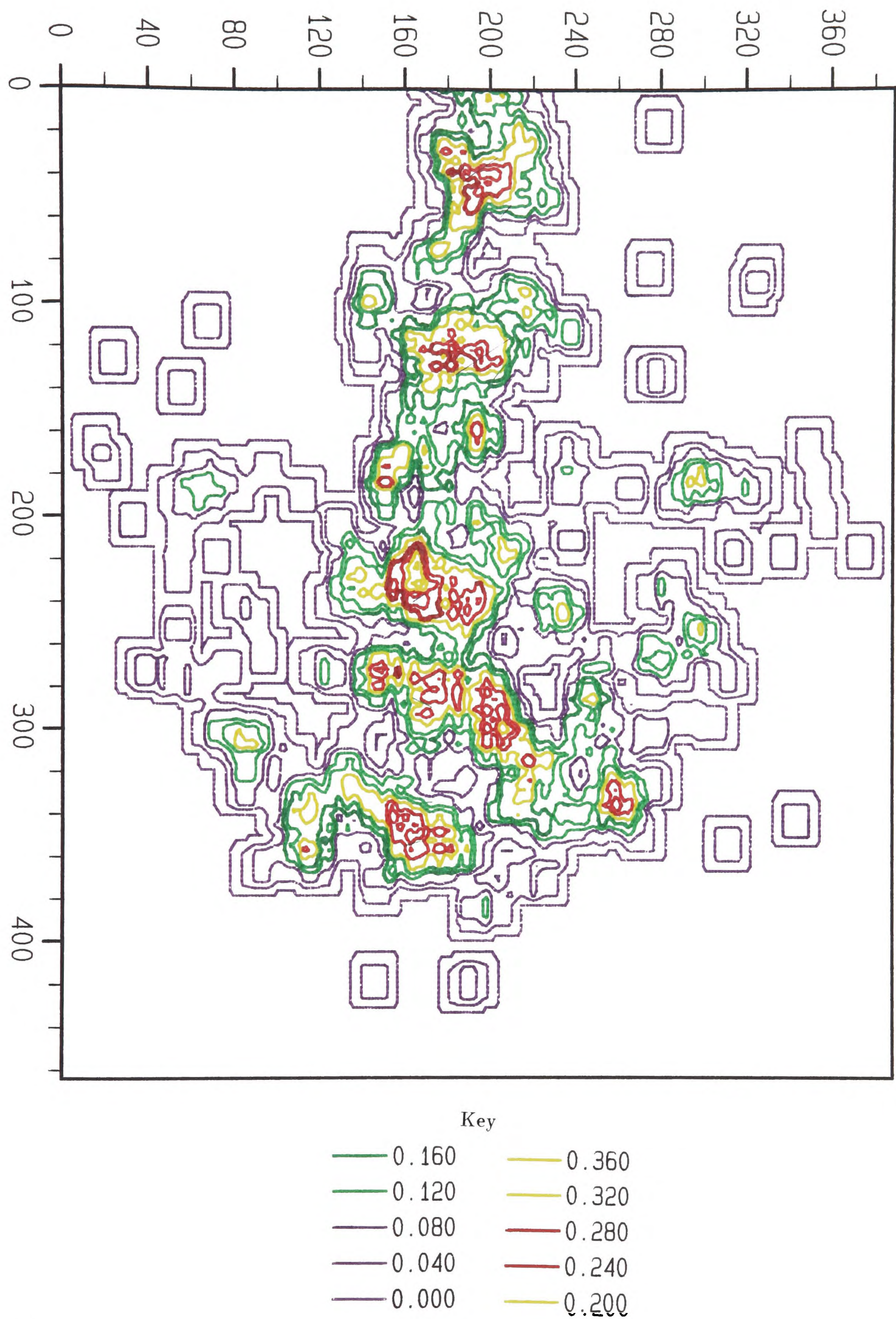


Figure 10.11: The Experimental Plume after averaging using an 16×20 mm Averaging Volume. The Axes are in Units of mm.

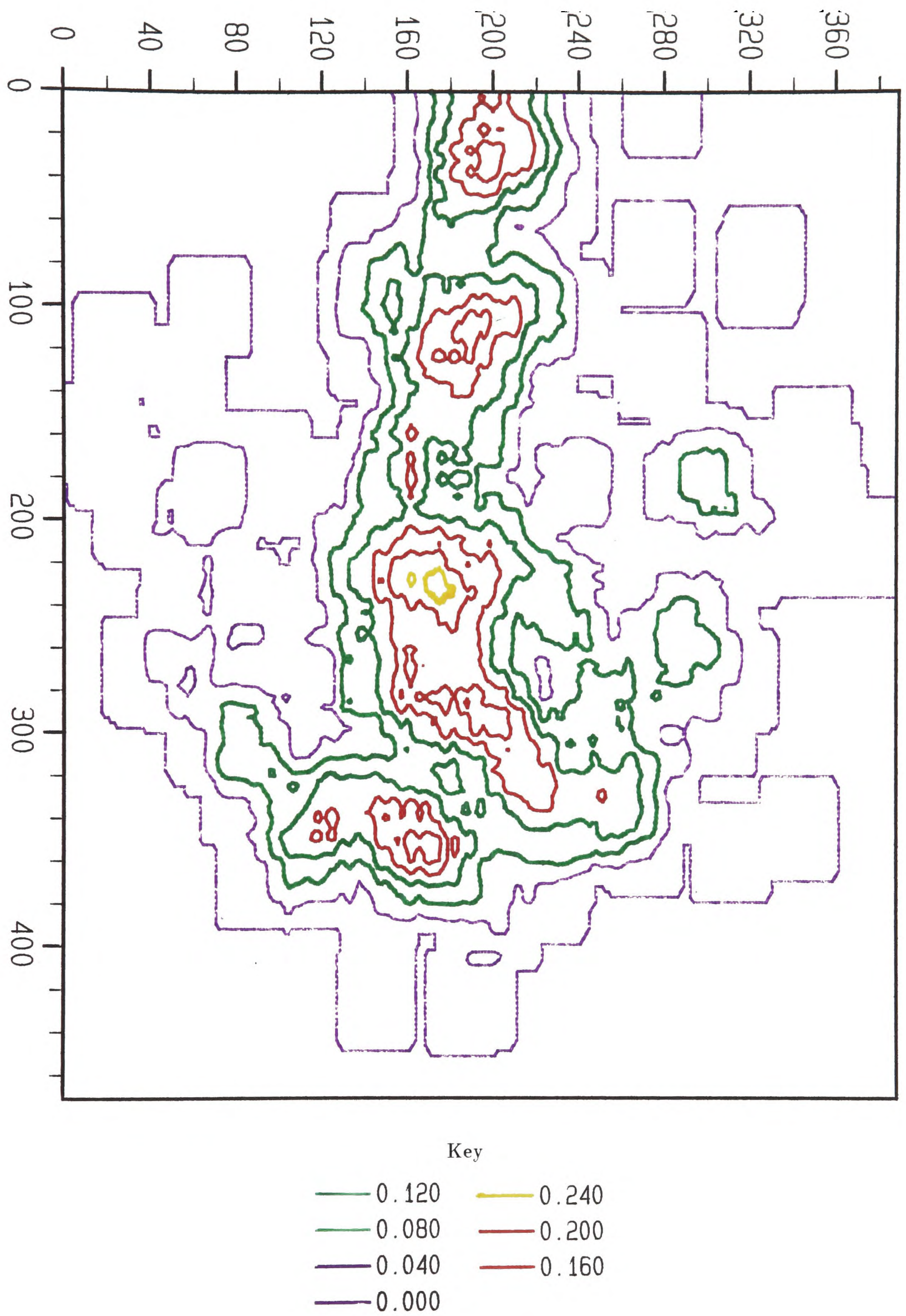


Figure 10.12: The Experimental Plume after averaging using an 32×40mm Averaging Volume.
The Axes are in Units of mm.

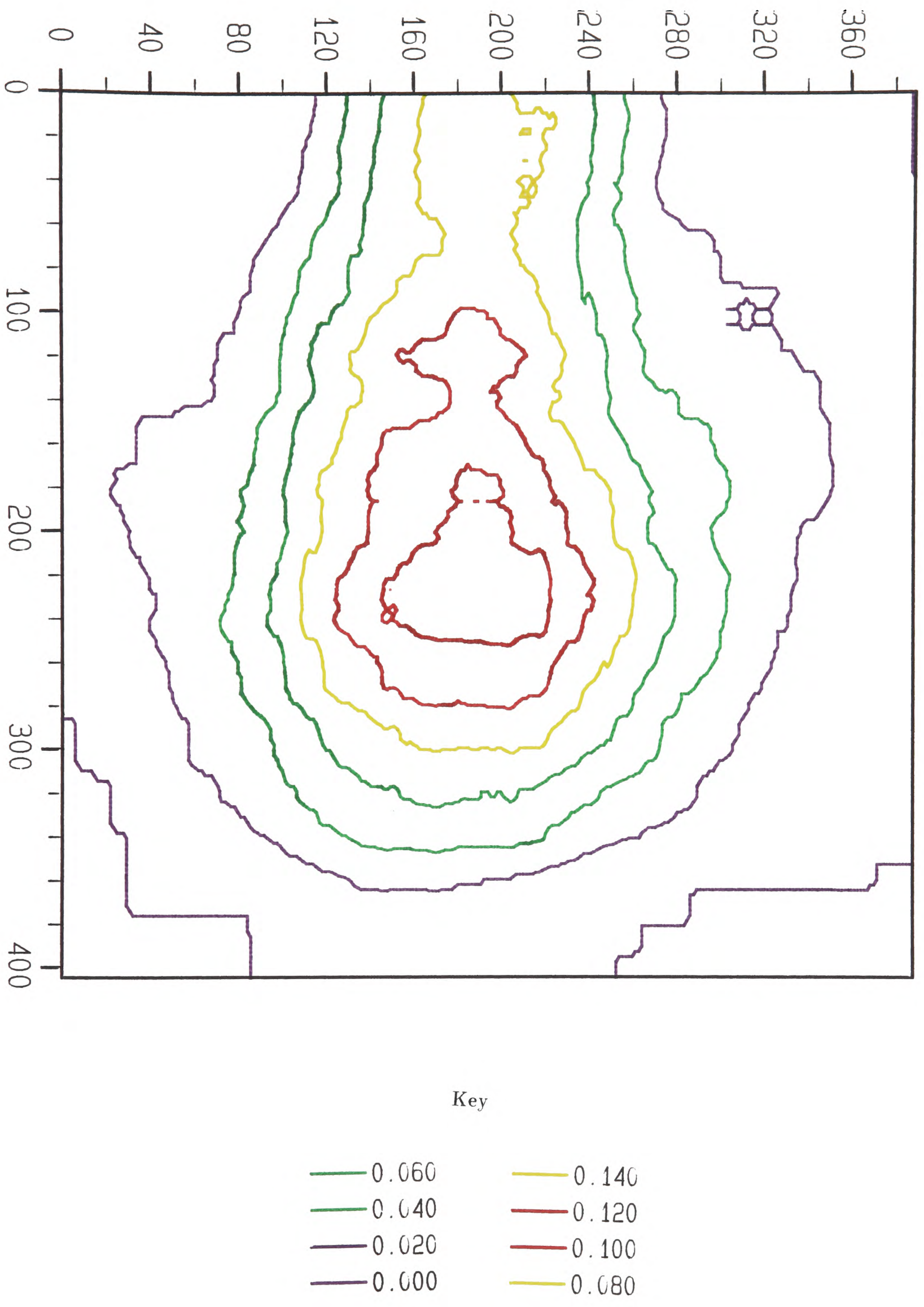


Figure 10.13: The Experimental Plume after averaging using an 136x121mm Averaging Volume. The Axes are in Units of mm.

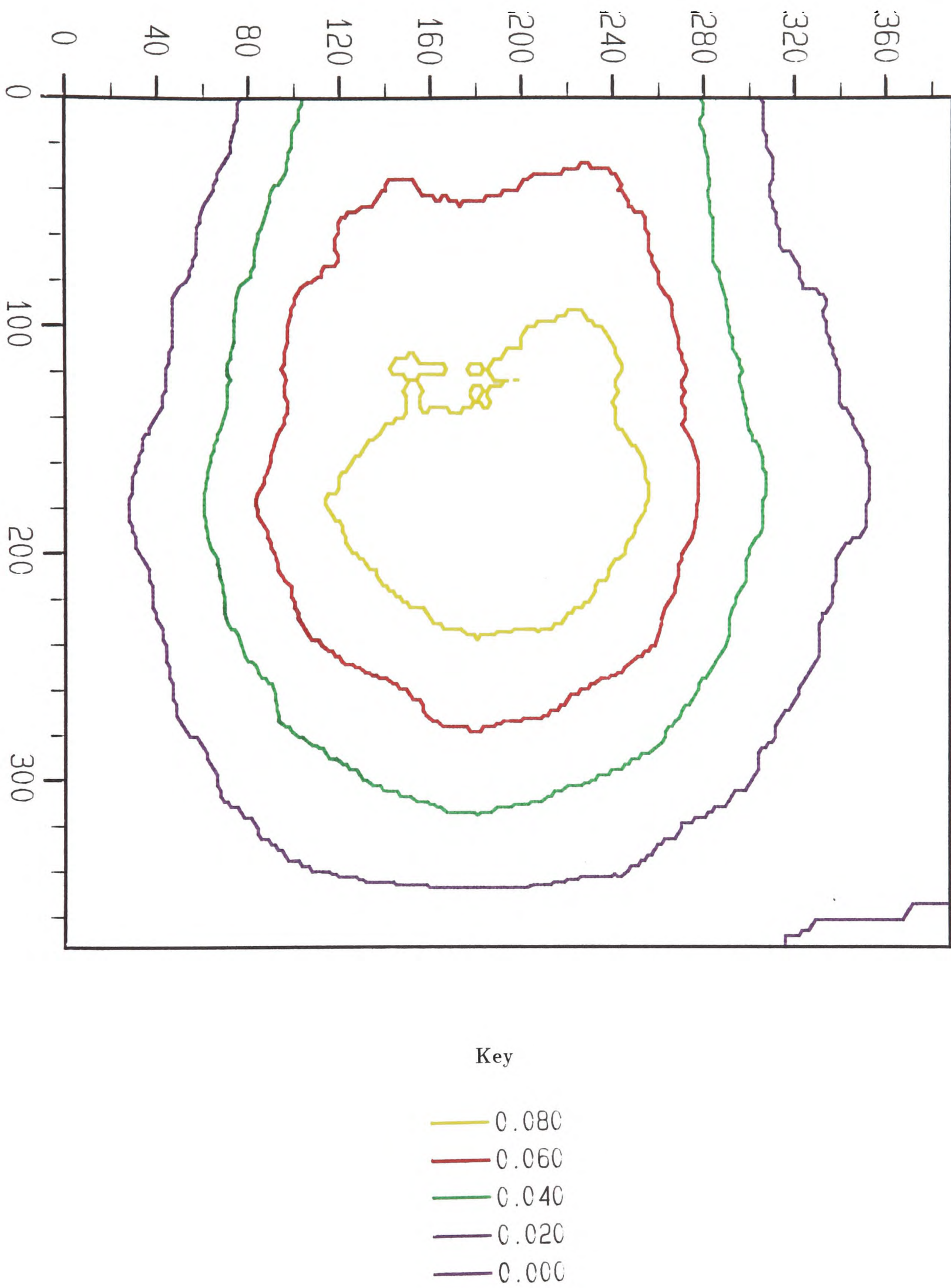


Figure 10.14: The Experimental Plume after averaging using an 200x200mm Averaging Volume. The Axes are in Units of mm.

contours, the process is stopped and α_p recorded. The other criterion is needed if the averaging process is unsuccessful and the 'true' value of α_p always varies with l_v . The incrementation of l_v ceases when α_p starts to vary monotonically with l_v . At this point the magnitude of the undulations of α_p will be small. The minimum size of the averaging volume should not be less than for a regular array. From Table 10.1 the minimum size of the volume for averaging to be successful⁶ falls with the 'true' value of α_p so that if S_p is greater than 10mm it is smaller than when the array is closely packed. The range over which the criteria are tested should be twice the size of the maximum particle spacing as this will be the largest wavelength of the undulations; therefore it is 20mm.

The result of the application of this program to the test plume is shown in Figure 10.15, which is very similar to Figure 10.13, the plume after it has been averaged with a fixed averaging volume suitable for an uniform particle distribution. It is not directly comparable to CHYMES results because when deriving the equations of the separated flow model the size and shape of the averaging volume must be independent of its position⁷; however it confirms that the non-uniform distribution of particles does not significantly distort the plume if a fixed averaging volume suitable for a uniform distribution is used. Figure 10.13 and Figure 10.15 have many features that are similar to those of the CHYMES results, namely:

- the particle concentration at the edges of the plume changes gradually;
- a dense core to the plume is formed;
- the core is set back from its leading edge (which roughly coincides with the 0.02 contour);
- the highest value of α_p is less in the averaged plot than the numerical one, but in both cases it is much less than that of a closely packed array.

These features, which are not readily apparent from the original pictures of the experiments, may be attributed to the difficulties the separated flow model has modelling non-uniform particle distributions owing to the continuum assumption. An anomaly is that the plume calculated by CHYMES after 0.6s has a crescent shaped head and the averaged experimental plume does not.

The 0.02 contour of α_p of the averaged experimental plume has penetrated the tank further (670mm against 620mm) and is wider (340mm against 290mm) than that produced by

⁶i.e. for the fluctuations in the calculated value of α_p from its 'true' value to be always less than a given threshold.

⁷Page 27

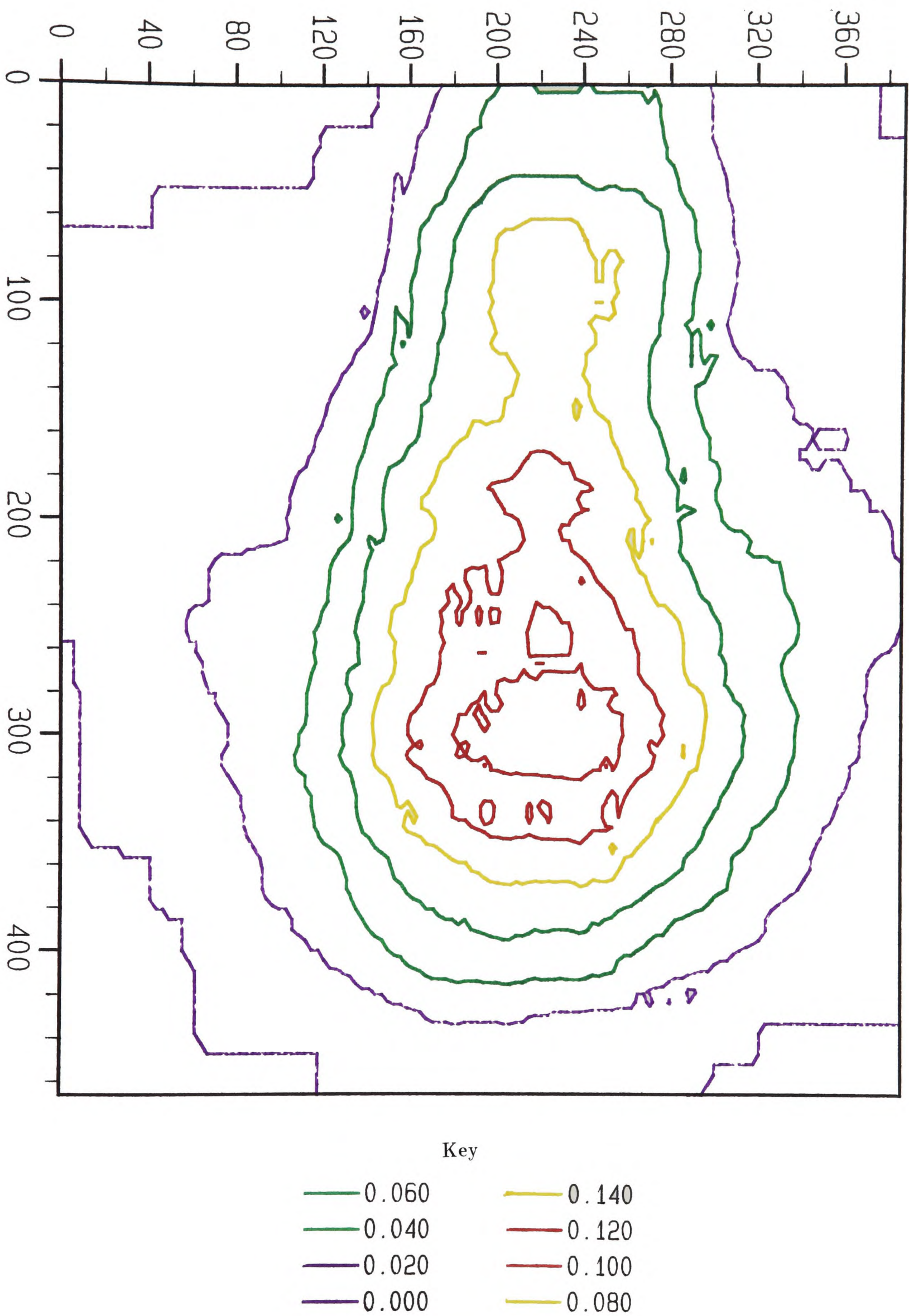


Figure 10.15: The Experimental Plume after Averaging with an Averaging Volume of Flexible Dimensions.

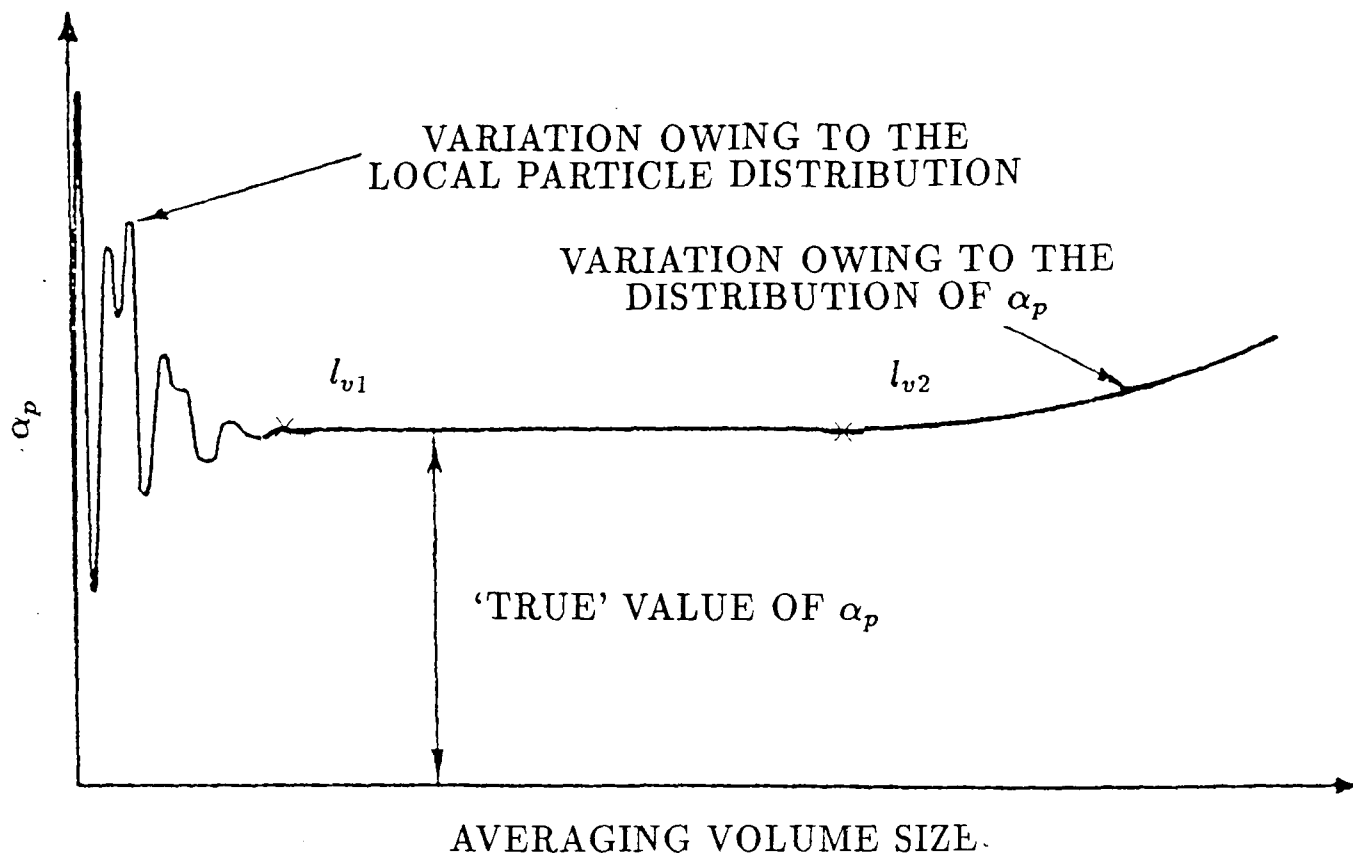


Figure 10.16: Diagram Showing the Dependence of the Averaging Process on the Relative Sizes of the Length Scales Used. It is based on a Diagram in [31].

CHYMES. In addition the maximum value of α_p is less, 0.12 against 0.22.

The averaging process eliminates the concentration of the solid phase⁸ into discrete particles by smearing the solid out evenly according to its proportion in that area of the plume. The reason for the dissimilarity between the computational model and what is seen in real plumes lies in the relative sizes of the length scales involved in averaging: S_p , l_v , and L . l_v has to exceed S_p by a certain amount for an averaged quantity to be insensitive to the size, shape, and centre position of the averaging volume. This is denoted by l_{v1} in Figure 10.16. l_v has also be small enough for the overall variation in the averaged quantity to be insensitive to its l_v ; this is denoted by l_{v2} in Figure 10.16. This is the criterion used to determine the size of a computational grid; therefore l_{v2} may be identified with the grid cell dimensions. If l_{v2} is greater than l_{v1} the averaging process will be successful; if it is not a suitable value for l_v cannot be defined and volume averaging is an inappropriate method to describe the system; any non-linear variation in a quantity is approximated to a linear one over such long lengths that detail is eliminated. This detail cannot be recovered from the plots of averaged quantities, such as α_p , as discussed on page 179. For example in the experimental plume displayed in Figure 10.15 the width of the column is similar to the size of the averaging volume and so acquires a one-dimensional character with no variation in the particle distribution visible in the lateral direction but the smearing out of its edges.

In the CHYMES computations the length scale inequality $S_p \ll l_v$ is automatically imposed

⁸The 'melt' in coarse mixing.

by the use of averaged variables but this violates the other inequality $l_v \ll L$ and $l_{v1} > l_{v2}$; hence the details of the flow are eliminated and features not apparent in an unaveraged, experimental plume, are introduced. The averaged experimental plume and the numerical plume are not identical: apart from the overall head shape there are other differences in their form. These may be caused by poor constitutive relations and might be eliminated by better physical modelling though the results of CHYMES do not appear to be very sensitive to changes in the term governing the transfer of momentum between the phases. However, even if the constitutive relations perfectly mimicked the physical processes there is evidence that it might be difficult to ever produce a recognisibly realistic picture of an experimental plume using the separated flow model.

Chapter 11

The Use of the Separated Flow Model

The separated flow model is a set of partial differential conservation equations that describe the motion of each phase in a multiphase flow. Computer codes such as CHYMES solve these equations. This thesis is concerned with how appropriate the separated flow model is to coarse mixing. There are two elements to the process:

- are all the important physical processes included and adequately described in the equations?
- are the assumptions underlying the separated flow model satisfied in coarse mixing?

Provided enough is known about the processes, faults originating from the first element can be rectified; this is not true for those from the second.

11.1 The Physical Processes

In a particulate flow where there is no heat or mass transfer apart from convection and buoyancy there are two terms in the momentum conservation equation:

- a term depending on the stress field in the continuous liquid;
- a term depending on the relative velocity between the two phases.

The modelling of stress in the two fluids has often been confused. In the continuous phase the stress in the bulk of the liquid can be modelled just like any other variable, resulting in (3.20). Stress (called the Reynolds stress) also results from turbulence and is represented by the

term $\langle \underline{U}_f^* \underline{U}_f^* \rangle$, which is neglected in CHYMES. From the picture of a plume when Mearlmaid is mixed with the water (Figure 8.3), it is clear that there is widespread turbulence throughout the tank. This will not affect the particles directly because of their size and mass, but it may affect them indirectly through modifying the flow of the continuous fluid. Much of the turbulence is generated in the wakes of particles; an improvement to the code may then be the incorporation of a suitable turbulence model into the code with a turbulence length scale consistent with that produced in the particles' wakes.

Stress in the particulate phase is more problematical. Particle stress, σ_p^1 , can only have some meaning through the collisions between particles. These may be systematically caused by unsteadiness so, for example, some particles slow down and are run into by those following them. In the experimental plumes this only takes place when the particles are entering the pool because at other times particle motion appears to be steady. The effect of the collisions would be to diffuse the particles because of the transfer of the excess momentum from the faster, following particles to the slower, leading ones through elastic conditions. From Figure 5.6, which is a close-up of a plume after 0.05s, this does not seem to have a significant effect: the particles bunch closely at the front of the plume.

Collisions between particles might also occur through their random movement owing to local flow conditions. This is equivalent to a Reynolds stress and is modelled by the term $\langle \underline{U}_p^* \underline{U}_p^* \rangle$, also currently set to zero in CHYMES. The paths of particles do not appear to significantly suffer from random lateral deviations, suggesting that the neglect of this term is correct. This accords with measurements made in fluidised beds (page 37).

A stress term may appear in the momentum equations to take into account the effect of the pressure field in the continuous fluid on the particles, as described on page 39, resulting in equations (3.18) and (3.20). From the estimation on page 151 of the acceleration owing to this field, it can have a significant effect on the motion of the particles.

The transfer of momentum between phases is modelled using the drag coefficient formulation shown in equation (3.33). This, on its own, does not take into account transient forces, the effect of the tank walls, or the effect of other particles.

The most important transient forces are thought to be virtual mass and the Basset force. In a non-oscillating flow for an isolated particle it is thought that the Basset force may be neglected² and the virtual mass coefficient, C_{vm} , set to 0.5. However this may not be true for

¹i.e. the term describing the distribution of stress within the particulate phase.

²Which is fortunate as it would be very difficult to model.

some acceleration numbers³ or types of flow (page 52). In a particulate flow the value of C_{vm} may be affected by other particles, though there is contradictory evidence for what this effect may be (see page 51). In addition in a volume averaged flow an extra term associated with the coefficient λ is required (see page 51), though it may not be large enough to be significant. There is clearly much uncertainty and scope for careful experiments for evaluating the transient forces. However the calculated acceleration of a particle if C_{vm} is set to 0.5, and the Basset force and λ neglected, differs by only 6% from that if all the transient forces were neglected; therefore these assumptions would have to introduce a very large error into the estimation of the transient forces to significantly affect the overall result for a particle. Transient forces may probably then be adequately described if the term

$$0.5\alpha_p\rho_f\mathbf{a}_{pf}$$

is included in the particle momentum equation and its negative in the fluid, where $\mathbf{a}_{pf}$ is defined on page 50.

The walls of the tank may retard the particles and affect the flow through the effects of viscosity, principally the growth of boundary layers. If the side walls of the tank are considered, not much can be said with certainty about whether they retard particles within the plume and the plume itself. However it is likely that the behaviour of individual particles is dominated by local flow conditions rather than the presence of the walls, which may affect only the gross flow of water, which should be modelled by CHYMES. It is unlikely that the boundary layers on the side walls will be important: if the free stream velocity of the water is 0.3m/s, as estimated on page 141, then using the method on page 107 the boundary layer will only be 30mm thick after 1m.

In the CHYMES calculations the retardation of the particles by the clear, restraining walls was incorporated into the drag coefficient. This has the fault that if the water is moving faster than the particle the presence of the walls encourage the acceleration of the particle. A better method would have been to include it as a separate, dissipative term in the equations.

The effect of other particles on the drag coefficient can be estimated by using the well established modification of the drag coefficient defined by equation (3.38). In the densest parts of the plume this doubles the drag on the particulate phase, but makes little difference to the form of the computed plume (see Figure 9.21). This suggests that the code is quite tolerant to inaccuracies and uncertainties in the physical relations and only the largest errors greatly distort the result.

³Defined by (3.31).

If the drag coefficient was increased to 2.0, the contour plot of α_p (Figure 9.22) appeared to be more similar to the experimental plume than if the expected, lower drag coefficient was used. However there were doubts whether this actually was an improvement because a number of differences in the character of the two sorts of plume remained, and whether the contour plots are directly comparable to the pictures of the experiments. In Chapter 10 an experimental plume was converted to a contour plot which seemed to support these doubts. From the discussion in this section there is no evidence that the increase in the drag coefficient (nearly a three fold increase) can be attributed to any physical process; there is then no reason to favour this value of drag coefficient over any other.

11.2 The Assumptions of the Separated Flow Model

The basis of the separated flow model is that the overall behaviour of a plume of ‘melt’ particles is repeatable once the small scale behaviour of the particles owing to local conditions in the plume has been averaged out. In sections 7.2.1 and 7.2.2 evidence was presented that the motion of the particles is steady i.e. the velocity of a particle depended principally on its position, though its motion could sometimes deviate from that expected owing to local conditions. This supports the Separated Flow Model, but the two plumes are very different in character and in their overall behaviour. This is because the Separated Flow Model treats the particle plume as a liquid once it has been averaged; the averaged equations of motion effectively describe a dense jet falling into water through which it can diffuse and with which the exchange of momentum is allowed.

The transformation of the particles into a liquid is effected by volume averaging. For this to be successful the length scale inequalities $S_p \ll l_v$ and $l_v \ll L$ have to be satisfied⁴. For the experiments described in this thesis, from the discussion in Section 10.4, it is possible that both of these inequalities cannot be satisfied⁵. These lie at the heart of the model; if they are not satisfied the separated flow model is inappropriate for the situation in question. In such a case the particles would be acting as particles over a scale comparable to that over which the behaviour of the flow changes significantly; on average the motion of the particles in the same place in a different plumes would be similar, but they would not be acting as a fluid. As well as introducing the problems of presentation and interpretation of results discussed in

⁴Where S_p is the particle spacing, l_v is the size of the averaging volume, and L is the length over which the flow characteristics change significantly.

⁵The size of l_v has only been examined for the experiments performed. A more extensive and rigorous study for determining l_v for different sizes and shapes of particles in different arrangements would be beneficial.

Chapter 10, the modelling the motion of the plume is incorrect: in reality the mass of the melt is concentrated in the particles and cannot be assumed to be smeared out.

For the separated flow model to be applied to a particulate flow either L has to be much larger (over ten times) than it has been here, or the minimum necessary size of L_v has to be much smaller. It may not be possible to increase the minimum value of L unless the size of the particles is also changed because it is sometimes set by the behaviour of individual particles rather than the size of the jet or the tank. For example the bunching of particles at the front of the plume, modelled in Section 5.2, is independent of the size of the jet and the tank; if wakes are important to the longitudinal motion of the particles, they are also independent of tank size; the stability of the particle array in the plume (determined by the relative magnitudes of (8.3) and (8.4)) depends on the particles, not on the size of the system.

L_v depends on S_p and d_p ; therefore it can only be reduced if the size of the particles is also reduced. As the particles become smaller their ability to move independently of the fluid motion also diminishes until the particles act as flow tracing particles, their motion mimicking that of the liquid's.

A reason for assuming that the large scale motion of the particles is similar to that of a liquid is that it is then possible to model it numerically; modelling a plume made of individual particles is difficult because of the complicated small scale phenomena, particularly that in their wakes, associated with their motion. The modelling of the isothermal plumes described here is made a little simpler by their steady motion.

11.3 The Comparability of the Experiments with Coarse Mixing

CHYMES has been compared with the results of experiments which are themselves models of coarse mixing. The experiments should not have been significantly affected by any phenomena not present or taken account of by CHYMES. Care was taken that they were carefully controlled so that the results could be compared directly with the computational results. However there are features in coarse mixing which will be significantly different from those of the experiments.

The first difference is the geometry of the system. The diameter of the bottom of reaction vessels is about 4m, roughly eight times the width of the experimental tank and twice its height. The width of the tank appeared to make little difference to the character of the plume, but

did affect its spreading⁶. As the size of the tank increases, the influence of the walls on the fluid surrounding the plume should lessen and the plume will tend to a maximum size.

There are a number of extra structures within a reactor core which have not been modelled. CHYMES should be able to model them provided any asymmetry introduced can be accommodated, they do not introduce any large scale viscous effects such as wakes, and melt cannot collect on top of them.

Little is known about the size of the jets of melt. Evidence from the Three Mile Island accident suggests that the melt pours through local failures in plates and, in particular, penetrations through them[8]. There is no indication of a catastrophic failure. The melt jets are then likely to be of the same order of size as the jets in the experiments. Local failures introduce the possibility of multiple, interacting jets of melt. The size of the particles in the experiments was comparable to that found in the debris after coarse mixing experiments.

The particles in the experiments were solid, but in coarse mixing they are liquid. These will not be any different from solid particles when they are stable and spherical; but they will be able to break-up or agglomerate. Models are available for the break-up of particles, such as the empirical equation (1.2) from the FITS series of experiments, or by using Weber numbers. Provided that these are successful in tracking the particle diameter unless the particles become very small (less than 1mm in diameter) their physical modelling will not be affected. Note that the particle diameter will be like any other flow variable so that in any one place it can only have a single, mean value, rather than a range. No equivalent model exists for the agglomeration of particles, though one could be formulated which allows the diameter of the particles to grow over certain ranges of velocity gradients and melt volume fractions. If the agglomerations become too large the phases cease to be interpenetrating and fall outside the scope of the model.

Finally the temperature of the melt in coarse mixing (about 3000K) is far higher than that in the experiments (at most about 1000K). CHYMES predicts that the plume is little affected by vapour production at the lower temperature (see Figure 11.1). However at high temperatures the vapour production is so high that a very different plume is predicted (see Figure 11.2). What the effect of increased heat transfer on a real plume, and whether the existing constitutive relations are adequate, is unclear. There is an opportunity for careful experiments with melt at this temperature where the behaviour of individual particles can be seen and measured, though these will be practically difficult.

⁶See figures 6.12, and 6.18.

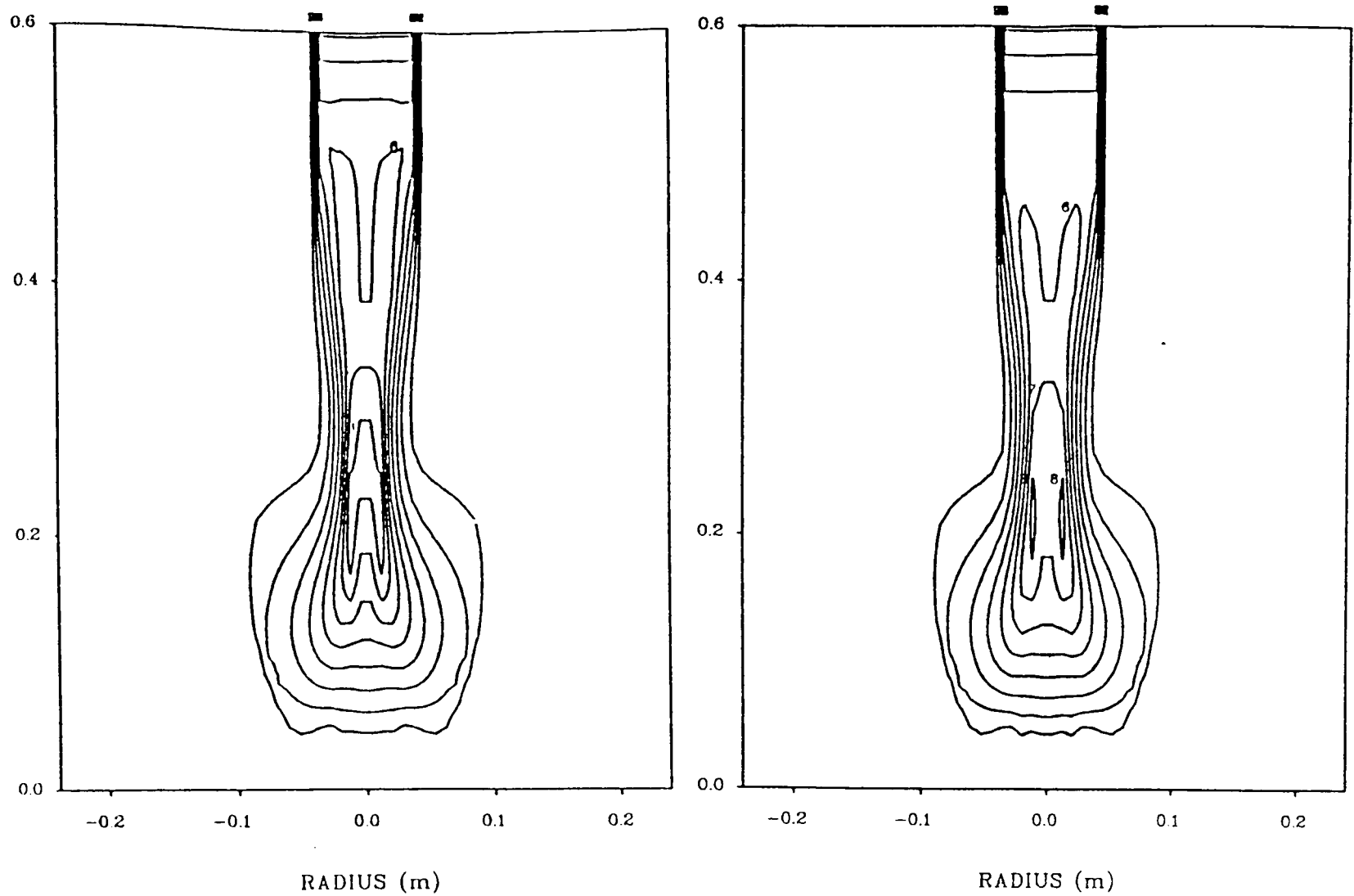


Figure 11.1: Figure comparing the Computed Plume if the Melt Temperature is 1000K (left) with that produced if the Melt Temperature is 300K (right).

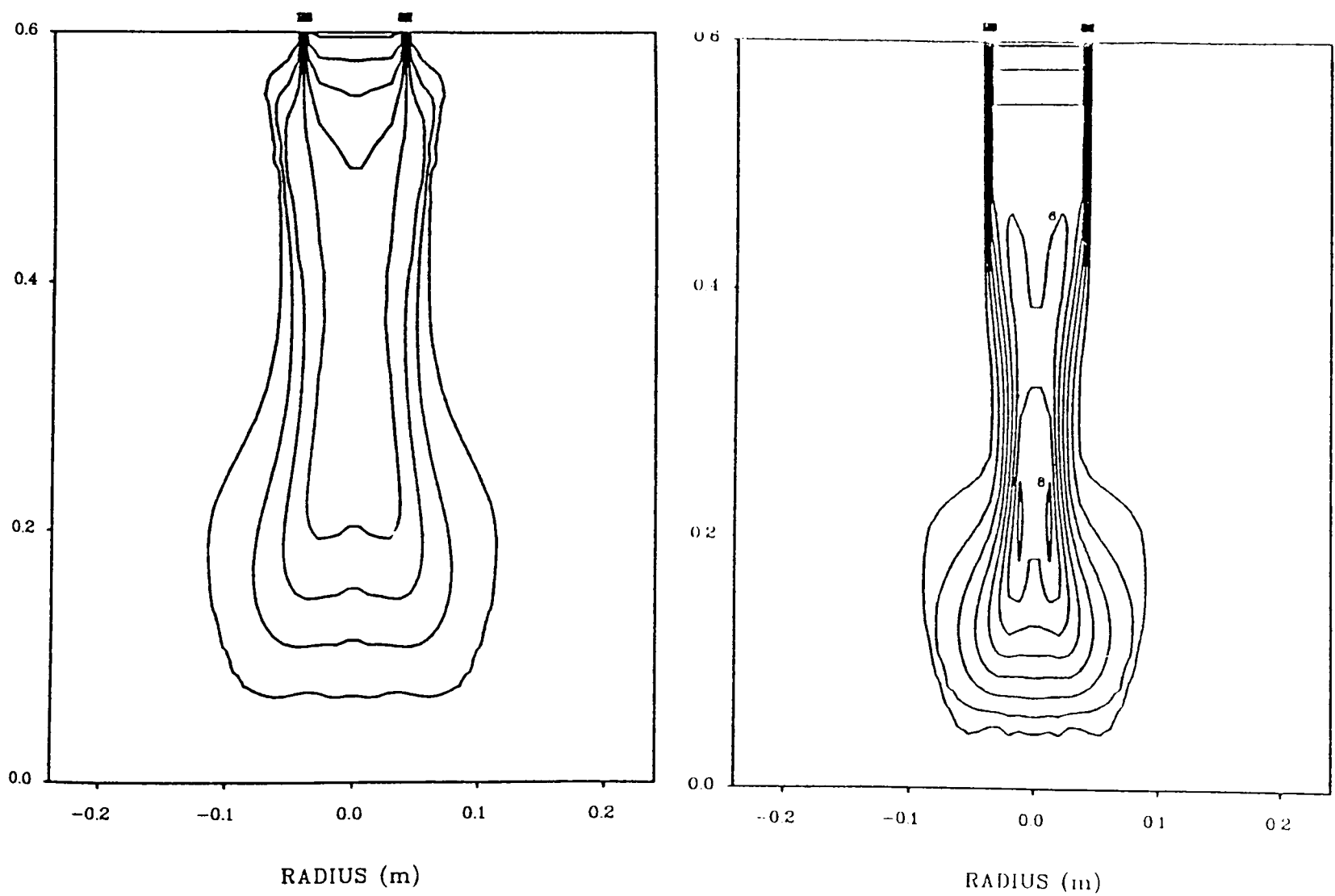


Figure 11.2: Figure comparing the Computed Plume if the Melt Temperature is 3000K (left) with that produced if the Melt Temperature is 300K (right).

11.4 Conclusions

1. This thesis describes the comparison of the results of an experimental programme with those of a computer code called CHYMES.
2. CHYMES is intended to model the coarse mixing of molten metal particles with water. It does this by solving the equations of the Separated Flow Model of multiphase flow.
3. The Separated Flow Model and the constitutive relations, which relate the model to a specific system, have been described.
4. The equations of the separated flow model are generated by averaging variables. The most suitable form of averaging for coarse mixing was discussed and volume averaging selected.
5. Three length scales are involved in volume averaging:
 - the particle spacing S_p ;
 - the size of the averaging volume l_v ;
 - the length scale over which the flow variables change significantly L .

An alternative definition of L is the length scale over which the variation of a variable in space may be approximated to a straight line. It may be identified with the cell size of the grid that the computational domain is divided into.

For the averaging process to be successful the length scale inequalities

$$S_p \ll l_v$$

and

$$l_v \ll L$$

have to be satisfied.

6. An experimental apparatus was constructed which allowed the control of the jet conditions and which was compatible with coarse mixing and CHYMES.
7. Plumes were produced over a wide range of conditions when the particles were unheated. These were repeatable and consistent and took the form of an arrow shaped head with a straight following shaft unless the plume was sparse, in which case its spreading was reduced. The distribution of the particles within the plume could be uneven and unstable.

8. Only one poorly controlled plume was produced with heated particles owing to practical problems. The assumption that the plume can be modelled as consisting of melt particles from which independent vapour particles are produced appeared to be true.
9. The rate of penetration of the plume was steady and exceeded the terminal speed of an isolated particle. It changed very little with experimental conditions if buoyancy was taken account of. A small systematic reduction (less than 10%) in the penetration speed might result from reducing the width of the jet or by thinning it out a great deal.
10. The spreading of the plume appeared to be most strongly affected by the width of the tank compared with that of the jet. No systematic dependence on other conditions could be seen unless the jet was very sparse.
11. No systematic dependence of the length of the head of the plume on the jet conditions could be perceived.
12. If individual particles in the column of the plume were followed they could travel at over double the terminal speed of an isolated particle. Their speed downwards appeared to be a function of their lateral position. They could move laterally, but in a non-systematic fashion.
13. If particles in the head of the plume were followed their motion appeared to be steady and to depend principally on their position relative to the front of the plume. Local conditions could cause the motion of some of the particles to deviate from that expected. This pattern of behaviour accords with that assumed by CHYMES.
14. The lateral velocity of the particles in the central region of the plume was proportional to the distance from the plume's centreline. Outside this region their lateral velocity tended to zero.
15. The particles in the wings of the plume accelerated longitudinally away from the front of the plume until a steady speed was achieved.
16. The motion of the particles seemed to be consistent with the influence of the motion of the surrounding fluid. It did not appear to be influenced by diffusive processes such as collisions between particles and fluid turbulence.
17. A model of the plume has been proposed based on the formation by the particles of a 'body' in the front, central portion of the plume which is impermeable to water in the

longitudinal direction.

18. The experimental results were compared with those of CHYMES, but were different in form. Changing the jet conditions or the tank dimensions had different effects on the numerical and the experimental plumes. Much of the detail of the distribution of the particles was missing in the computational plumes and the distribution was smoother than in the experimental ones.
19. The results of CHYMES did not appear to be very sensitive to the form of the term describing the exchange of momentum between phases.
20. Many of the features of the computational plumes were reproduced when an experimental result was processed so that it had the same form as them. It is not possible to process the computational results so that the detail of the experimental plumes is recovered.
21. A method was proposed to determine the minimum size of l_v necessary for volume averaging to be successfully applied.
22. It was demonstrated that because of the length scales involved it was possible that both of the length scale inequalities cannot be satisfied in the experiments. If this is true the presentation of the results and the physical modelling employed is inappropriate for the plumes described here.
23. It was suggested that the model might benefit from the introduction of an added mass term and one accounting for fluid turbulence, though these are likely to only have a minor effect on the results.
24. For the above reasons it is possible that the separated flow model is an inappropriate method for describing the experiments.
25. With the exception of the degree of steam production, it is thought that the experiments were an adequate model of coarse mixing.

Appendix A

Computer Programs used for the Location the of Particles

A.1 GROUP.FOR

A.1.1 Description

Purpose To locate the edges of the black areas representing groups of particles in the image processed data.

Input FINAL.IMG, which, after a text header, contains the grey levels of the pixels of the processed picture of the plume listed line by line in the longitudinal direction. Only the area containing nearly all the particles is read e.g. for the plume in Figure 10.2 rows 25 to 295 in the lateral direction and columns 104 to 388 in the longitudinal direction.

Output GRPOS.DAT in which the data of each group of pixels representing particles is recorded. Using the terminology defined in Figure A.1, the lateral position of each column of pixels is recorded and then the beginning and the ends of each line of pixels in that column in two successive lists. Up to nine lines per row may be recorded; if there are less than nine the redundant positions are filled with zeroes.

Main Routine After some preliminary statements the program calls a subroutine **REED** that reads the data from the input file into the array **A**. It then steps through each column of the image in **A** looking for black pixels¹. When one is found if it is isolated it is eliminated, otherwise **ENDS** is called to locate the positions of the peripheral pixels of the group of

¹i.e. when the value held in **A** is zero.

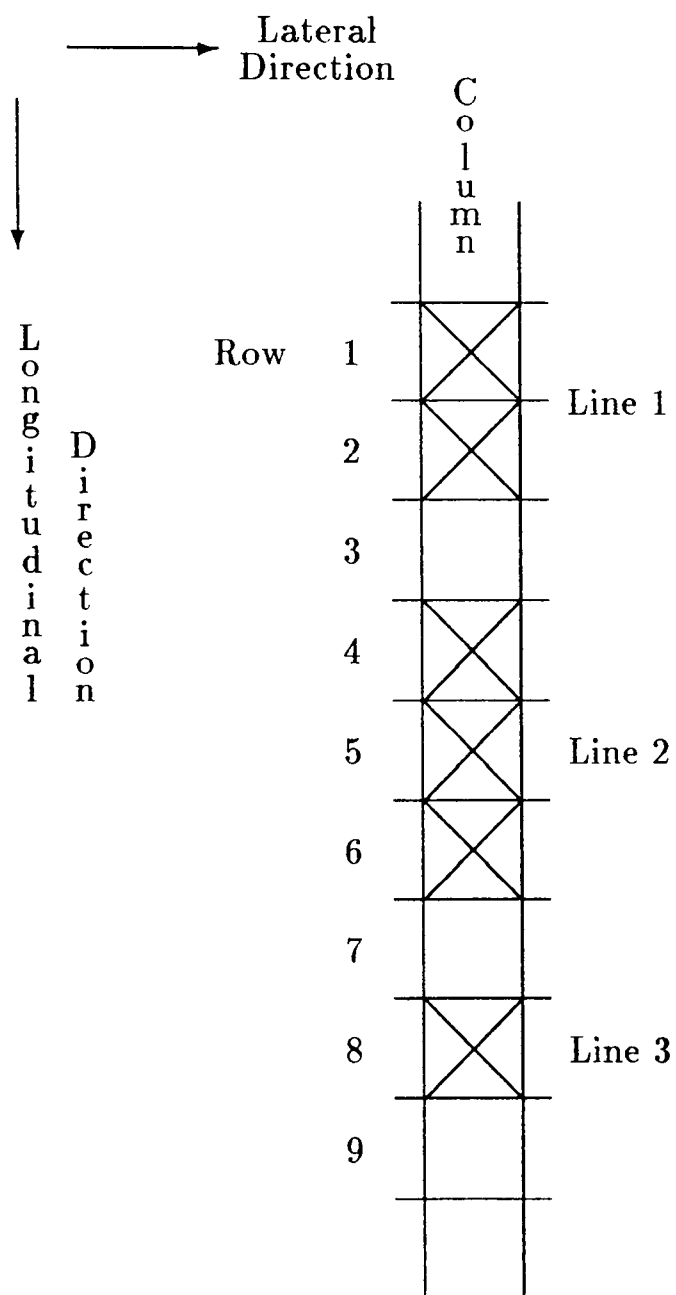


Figure A.1: Diagram showing the Terminology used with Respect to Groups of Particles.

dark pixels. **FINISH** is called to record their positions in the output file; it also eliminates the group from **A** so that it is not counted again. Once the whole of **A** has been examined the program ends.

REED This reads in the grey levels (0 for black, 255 for white) of the processed image of the plume into an array **A**. A text header is discarded before each row of pixels in the image is read from a file, converted from an ASCII data type to integers, and stored in **A**. Most of this routine was written by a member of the staff of Oxford University Computer Services.

ENDS This routine locates the pixels at the edge of a group by moving a search pixel clockwise around the edge of the group, and their positions are stored in the array **XE**. **IF1** denotes the direction of the last movement of the search pixel. If the search pixel last moved in the lateral direction the darkness of the pixels either side of the search pixel in the longitudinal direction is tested. If the last move was in the longitudinal direction then the next pixel in the direction of travel is examined; if it is white the search pixel is at the edge of the group and its position recorded by calling **RECORD**. The sign of the movement of the search pixel (i.e. whether it was towards or away from an axis) is recorded by the value of **INC**.

The pixels adjacent to the search pixel are examined in an anticlockwise direction from the one next in order from that which the search pixel has just moved. When a black pixel has been found the search pixel moves there, and appropriate values set for **IF1** and **INC**.

Eventually the search pixel returns to its original position, the top, left hand pixel. If the row is only one pixel wide, or its end has already been visited the routine passes control back to the main loop, otherwise the end is found and its position recorded first.

RESULT In this routine the extremity of a line of pixels in a group is stored within **XF**. The value of the first dimension of **XE** is determined by the number of columns between the first column of the group and the one being examined; the second is the number of the lines from the origin to the pixel; the third parameter is 1 if the extremity is the beginning of the line, and 2 if it is the end of a line. Later analysis is made easier if the values of each list ascend numerically so the list of the positions of the ends of the lines are sorted each time an end is found.

FINISH This writes the positions of the peripheral pixels of a group of particles to the output file. It first checks that each line has a beginning and an end: a few configurations of pixels can cause only one of these to be recorded. If a row has only one recorded end, the routine counts

back in **A** from this end to the unknown one. After **XE** has been recorded the group of pixels is obliterated from **A** so that it is not recorded again.

Variables

A Array into which the processed image is stored.

B Vector into which the values of a row of pixels is read

I,J Counters in the longitudinal and lateral directions respectively when **A** is searched.

IEND When the extremity of a line has been found this indicates whether it is the beginning or the end.

I1, I2, I3 Counters used when transferring the contents of **XE** to the output file.

I4, J2 Counters used when eliminating a group from **A** after it has been processed.

IF1 Records the direction of the last move by the search pixel. Equals zero if it was in the lateral direction, one if in the longitudinal direction.

IFLAG Indicates whether the search pixel has yet moved from the first pixel of a group. Equals one if it has not, two if it has.

INC Indicates the direction of travel of the search pixel. Equals 1 if it travelled away from the system's axes, -1 if it moved towards them.

J1 Equals **J** less one.

XE Array in which the positions of the peripheral pixels of a group are stored.

A.1.2 Listing

```
integer A(285,317),XE(200,9,2)
C OPEN OUTPUT FILE.
open (unit=2,name='grpos5a.dat',type='new',carriagecontrol='list')
C READ IN IMAGE OF PLUME AND STORE IN A.
call REED(A)
C START SEARCHING A FOR GROUPS OF PARTICLES. I IS IN THE LATERAL
C DIRECTION AND J IN THE LONGITUDIAL.
do J=2,317
  do I=2,285
C type *,I,J
    if (A(I,J).eq.0) then
      if ((A(I-1,J).ne.0).and.(A(I+1,J).ne.0).and.
1(A(I,J-1).ne.0).and.(A(I,J+1).ne.0)) then
        A(I,J)=255
        goto 10
      endif
```

```

C LOCATE PIXELS AT THE EDGE OF THE GROUP.
c type *,I,J,A(I,J)
    call ENDS(I,J,A,XE)
C STORE INFORMATION AND REMOVE GROUP FROM A.
    call FINISH(J,XE,A)
    endif
10 continue
    enddo
enddo
11 format(I3)
close (unit=2)
stop
end

C SUBROUTINE TO READ THE IMAGE DATA
subroutine REED(A)
integer A(285,317)
BYTE B(388)
CHARACTER*80 TITLE
C OPEN FILE CONTAINING IMAGE
OPEN (UNIT=1,NAME='FINAL5A.IMG',TYPE='OLD')
C READ IN TEXT HEADER
READ(1,11) TITLE
11 FORMAT(8X,80///)
C READ THE DATA IN ROW BY ROW
DO J=1,317
C STORE EACH ROW IN B
    READ(1,40) (B(I),I=1,388)
    DO I=103,388
C STORE THE ROW IN A IN THE CORRECT FORM
        A(I,J)=ZEXT(ZEXT(B(I)))
    ENDDO
40 FORMAT(388A1)
ENDDO
CLOSE(UNIT=1)
return
end

C SUBROUTINE TO LOCATE THE EDGE OF A GROUP OF OCCUPIED PIXELS
subroutine ENDS(I,J,A,XE)
integer A(285,317),XE(200,9,2)
IFLAG=1
IF1=0
INC=-1
C CLEAR XE
do K1=1,200
    do K2=1,9
        do K3=1,2
            XE(K1,K2,K3)=0
        enddo
    enddo
enddo
IY=J
IX=I
15 continue
C HAS THE SEARCH PIXEL RETURNED TO ITS ORIGINAL POSITION?
if ((IX.eq.I).and.(IY.eq.J).and.(IFLAG.ne.1)) then
35 continue
C IF AT THE END OF THE TOP ROW RECORD THIS AND RETURN TO THE MAIN
C ROUTINE.
    if (IF1.eq.0) return
    if (A(IX+1,IY).ne.0) then
        IEND=2
        call RECORD(IX,IY,IEND,XE,J)
        return
    else

```



```

C IF NOT AT THE END OF THE TOP ROW MOVE ALONG IT UNTIL THE END IS
C REACHED.
    IX=IX+1
    goto 35
endif
return
endif
C SET THE FLAG SHOWING THAT THE SEARCH PIXEL HAS MOVED FROM ITS ORIGINAL
C POSITION.
if (IFLAG.eq.1) IFLAG=2
C IF THE LAST MOVEMENT OF THE SEARCH PIXEL WAS LONGITUDINALLY THEN...
if (IF1.eq.0) then
C CALCULATE WHICH END OF A LINE OF PIXELS IS EXPECTED
    IEND=(3+INC)/2
C IF AT THE END OF A LINE THEN RECORD THE POSITION OF THE SEARCH PIXEL
    if (A(IX+INC,IY).ne.0) call RECORD(IX,IY,IEND,XE,J)
    if (A(IX,IY+INC).eq.0) then
C IF THE FIRST PIXEL IN A LATERAL DIRECTION IS OCCUPIED THEN MOVE IN
C ITS DIRECTION
        IY=IY+INC
        IF1=1
        goto 15
    endif
    if (A(IX+INC,IY).eq.0) then
C IF THE FIRST PIXEL IN A LONGITUDINALL DIRECTION IS OCCUPIED THEN MOVE IN
C ITS DIRECTION
        IX=IX+INC
        goto 15
    endif
C IF THE SECOND PIXEL IN A LATERAL DIRECTION IS OCCUPIED THEN MOVE IN
C ITS DIRECTION
    if (A(IX,IY-INC).eq.0) then
        IY=IY-INC
        INC=-INC
        IF1=1
        goto 15
    endif
C IF ALL THREE PIXELS UNOCCUPIED THEN MOVE THE SEARCH PIXEL BACK TO ITS
C LAST POSITION
    IX=IX-INC
    INC=-INC
    goto 15
C IF THE LAST MOVEMENT OF THE SEARCH PIXEL WAS IN THE LATERAL DIRECTION
C THEN....
else
C IF THE PIXEL EITHER SIDE OF THE SEARCH PIXEL IS UNOCCUPIED THEN RECORD
C THE END OF A LINE
    if (A(IX+1,IY).ne.0) then
        IEND=2
        call RECORD(IX,IY,IEND,XE,J)
    endif
    if (A(IX-1,IY).ne.0) then
        IEND=1
        call RECORD(IX,IY,IEND,XE,J)
    endif
    if (A(IX-INC,IY).eq.0) then
C IF THE FIRST PIXEL IN A LONGITUDINAL DIRECTION IS OCCUPIED THEN MOVE IN
C ITS DIRECTION
        IX=IX-INC
        IF1=0
        INC=-INC
        goto 15
    endif
    if (A(IX,IY+INC).eq.0) then

```



```

C IF THE FIRST PIXEL IN A LATERAL DIRECTION IS OCCUPIED THEN MOVE IN
C ITS DIRECTION
    IY=IY+INC
    goto 15
endif
    if (A(IX+INC,IY).eq.0) then
C IF THE SECOND PIXEL IN A LONGITUDINAL DIRECTION IS OCCUPIED THEN MOVE IN
C ITS DIRECTION
        IX=IX+INC
        IF1=0
        goto 15
    endif
C IF ALL THREE PIXELS UNOCCUPIED THEN MOVE THE SEARCH PIXEL BACK TO ITS
C LAST POSITION
    IY=IY-INC
    INC=-INC
    goto 15
endif
end

C SUBROUTINE TO RECORD THE END OF A LINE OF PIXELS IN ITS APPROPRIATE
C POSITION IN XE.
subroutine RECORD(IX,IY,IEND,XE,J)
integer XE(200,9,2)
K=IY-J+1
IC=0
10 continue
    IC=IC+1
C IF LONGITUDINAL POSITION OF THE END IS THE LARGEST YET FOUND WITH THIS
C LATERAL POSITION THEN RECORD IT
    if (XE(K,IC,IEND).eq.0) goto 20
C IF LARGER THEN AN END ALREADY RECORDED THEN EXAMINE THE NEXT POSITION
C IN XE
    if (XE(K,IC,IEND).lt.IX) goto 10
C IF END ALREADY RECORDED THEN RETURN
    if (XE(K,IC,IEND).eq.IX) return
C IF ENDS WITH A LARGE POSITION ALREADY FOUND THEN MOVE THEM UP IN XE TO
C MAKE ROOM FOR THE PRESENT ONE
    do I1=8,IC,-1
        XE(K,I1+1,IEND)=XE(K,I1,IEND)
    enddo
20 continue
C RECORD THE END OF THE LINE OF PIXELS
    XE(K,IC,IEND)=IX
    return
end

C SUBROUTINE TO RECORD THE POSITION OF THE EDGE OF A GROUP OF PIXELS IN
C THE OUTPUT FILE
subroutine FINISH(J,XE,A)
integer A(285,317),XE(200,9,2),XST
J1=J-1
C WRITE POSITIONS OF PERIPHERAL PIXELS TO OUTPUT FILE.
do I1=1,200
    do J2=1,9
C CHECK IF NO MORE LINES OF PIXELS IN GROUP
        if (XE(I1,1,1).eq.0) goto 50
C CHECK IF ANYMORE LINES OF PIXELS ON THIS ROW
        if ((XE(I1,J2,1).eq.0).and.(XE(I1,J2,2).eq.0)) goto 41
C IF ONE END OF A LINE OF PIXELS IS MISSING THEN COUNT FROM THE KNOWN END
C UNTIL THE OTHER ONE IS FOUND
        if ((XE(I1,J2,1).eq.0).or.(XE(I1,J2,2).eq.0)) then
            if (XE(I1,J2,1).eq.0) then
                INC1=-1
                XST=XE(I1,J2,2)

```

```

        IE=1
    else
        INC1=1
        XST=XE(I1,J2,1)
        IE=2
    endif
45    continue
    XST=XST+INC1
    if (A(XST,J1+I1).eq.255) then
        XE(I1,J2,IE)=XST-INC1
    else
        goto 45
    endif
endif
enddo
41    continue
C    WRITE THE POSITIONS OF THE PIXELS AT THE END OF LINES TO THE OUTPUT
C    FILE
    do I3=1,2
        write (2,100) (J1+I1,((XE(I1,I2,I3),I2=1,9)))
    enddo
enddo
50    continue
    write (2,200)
C    CLEAR THE GROUP FROM A SO THAT IT IS NOT COUNTED AGAIN.
    do J2=1,200
        if (XE(J2,1,1).eq.0) goto 60
        do I5=1,9
            if ((XE(J2,I5,1).eq.0)) goto 55
            do I4=XE(J2,I5,1),XE(J2,I5,2)
                A(I4,(J1+J2))=255
            enddo
        enddo
    enddo
55    continue
    enddo
60    continue
    return
100 format (10I3)
200 format (/)
end

```

A.2 RBPOS.FOR

A.2.1 Description

Purpose To use the data generated by GROUP.FOR to estimate the positions of the particles in the plumes.

Input GRPOS.DAT as described above.

Output RBPOS.DAT in which the real longitudinal and lateral positions of the centres of the particles in the plume are stored in millimetres.

Description After some preliminaries the positions of the ends of the first five lines of pixels of the first group are read in. Five lines of pixels are equivalent to one line of particles. Any lines that are only one pixel long are eliminated to remove the shading caused by entrained air in the column. The lateral position of the centreline of the particle row is calculated. If the number of lines of data is less than or equal to the equivalent of a particle radius they are ignored and the processing of the next group of pixels started. The beginning of the longitudinal row is assumed to be at the most extreme beginning of the five lines of pixels. If the overall width of the group is less than one particle diameter it is increased to this. The positions of the centres of particles for that row are recorded by starting at the centre of the first particle in the row and the positions every 8.4mm (or whatever particle spacing has been set) along it are recorded until the end of the line is reached. The next line of particles in that column (if any) is then adopted and the possible particles centres on it recorded. If there are no more particle rows XE is emptied and the next group examined. When all the groups have been processed the program stops.

Variables

DX, DY The real size of a pixel, in the longitudinal and lateral directions respectively.

I0 Counter of the row of pixels when a particle row is being read from the input file.

ISP Counter of the number of blank lines read since the end of the last group.

IX0, IY0 Position of the pixel at the top, left hand corner of each group.

IX1 Longitudinal position of the pixel at the beginning of a line of pixels.

IX2 Longitudinal position of the pixel at the end of a line of pixels.

IX22 Longitudinal position of the pixel at the end of the next line of pixels.

IYI Lateral position of the top of a particle row.

IYM Lateral position of the bottom of a particle row.

IYN Lateral position of a line of pixels.

J1, J2, J3 Counters used when emptying XE.

XE Array in which the information read from GROUP.FOR is stored.

XI Real longitudinal position of the centre of a particle.

XTA Longitudinal position of the start of a row of particles.

YM Real lateral position of the centre of a particle.

A.2.2 Listing

```

      integer XE(4,9,2),XTA
C   OPEN FILE CONTAINING DATA FROM GROUP.FOR.
      data XE,DX,DY,ISP,IF1/72*0,2.38,1.48,0,2/
      open (unit=1,name='grpos5a.dat',type='old')
C   OPEN FILE TO CONTAIN BALL POSITIONS.
      open (unit=2,name='rbpos91.dat',type='new',
            lcarriagecontrol='list')
C   READ IN DATA FROM GROUP.FOR.
10  continue
      read (1,400) IY0,IX0
      ISP=ISP+1
C   CHECK IF ALL THE GROUPS OF PIXELS HAVE BEEN EXAMINED.
      if (ISP.gt.50) goto 50
      if (IY0.eq.0) goto 10
      XTA=500
      ISP=0
c   type *,IY0,IX0
      backspace 1
C   READ A NUMBER OF ROWS OF PIXELS EQUIVALENT IN DEPTH TO A PARTICLE
C   DIAMETER.
      do IO=1,5
C   READ IN THE POSITIONS OF ONE END OF THE LINES OF PIXELS WITH THE LATERAL
C   POSITION IYN.
          read (1,100) IYN,(XE(IO,I1,1),I1=1,9)
C   READ IN THE POSITIONS OF THE OTHER END OF THE LINES OF PIXELS WITH THE LATERAL
C   POSITION IYN.
          read (1,100) IYN,(XE(IO,I1,2),I1=1,9)
          if (IO.eq.1) IYI=IYN
          if (IYN.ne.0) IYM=IYN

```

```

C IF THE REMAINING PORTION OF THE GROUP LESS THAN HALF THE DIAMETER OF A
C PARTICLE IN THICKNESS THEN IGNORE IT.
    if (XE(IO,1,1).eq.0) then
        if (IO.lt.3) goto 10
        goto 20
    endif
C FIND THE BEGINNING OF THE ROW OF PARTICLES.
    if ((XE(IO,1,1).ne.0).and.(XE(IO,1,1).lt.XTA))
1    XTA=XE(IO,1,1)
    enddo
20 continue
C IF THE WHOLE GROUP IS ONLY ONE PIXEL WIDE INCREASE ITS WIDTH
do IO=1,5
    if (XE(IO,1,2).ne.XE(IO,1,1)) IF1=1
enddo
if (IF1.eq.2) XE(1,1,2)=XE(1,1,2)+3
IF1=2
C ELIMINATE LINES OF PIXELS ONLY ONE PIXEL LONG.
do IO=1,5
    do JO=1,9
        if (XE(IO,JO,1).ne.0) then
            if (XE(IO,JO,1).eq.XE(IO,JO,2)) then
                XE(IO,JO,1)=XE(IO,JO+1,1)
                XE(IO,JO,2)=XE(IO,JO+1,2)
                if ((XE(IO,JO+2,1).eq.0).or.(JO.eq.8)) then
                    XE(IO,JO+1,1)=0
                    XE(IO,JO+1,2)=0
                endif
            else
                goto 15
            endif
        endif
    enddo
15 continue
enddo
C FIND THE LATERAL POSITION OF THE MIDDLE OF THE PARTICLE ROW.
YM=0.5*real(IYM+IYI)
YM=DY*YM
IX1=XTA
IX2=XTA
IF=1
C FIND THE END OF THE PARTICLE ROW.
do I2=1,5
    if (XE(I2,1,2).gt.IX2) IX2=XE(I2,1,2)
enddo
40 continue
C MOVE FROM THE CENTRE OF THE PARTICLE AT THE BEGINNING OF THE CURRENT
C LINE TO THE CENTRE OF THE PARTICLE AT THE END OF IT IN 6MM STEPS.
do XI=(real(IX1)-0.5)*DX+3.,(real(IX2)+0.5)*DX-3.,8.4
C RECORD PARTICLE POSITIONS.
write (2,200) XI,YM,2.
enddo
C INCREMENT LINE COUNTER.
IF=IF+1
IX1=500
IX22=IX2
C STEP THROUGH PIXEL ROWS TO FIND THE NEXT LINE OF PARTICLES.
do I5=1,4
C IF THE BEGINNING OF THE NEXT LINE IS LESS THAN ITS CURRENT VALUE AND
C GREATER THAN THE END OF THE LAST LINE THEN SET IX1 TO THIS.
    if ((XE(I5,IF,1).ne.0).and.(XE(I5,IF,1).lt.IX1)
1 .and.(XE(I5,IF,1).gt.IX2)) IX1=XE(I5,IF,1)
C IF THE END OF THE NEXT LINE IS GREATER THAN ITS CURRENT VALUE THEN SET
C IX22 TO THIS.
    if (XE(I5,IF,2).gt.IX22) IX22=XE(I5,IF,2)
enddo

```

```

C  IF THERE IS NOT ANOTHER LINE THEN CLEAR XE FOR THE NEXT PARTICLE ROW.
  if (IX22.eq.IX2) then
    do J1=1,5
      do J2=1,9
        do J3=1,2
          XE(J1,J2,J3)=0
        enddo
      enddo
    enddo
    enddo
    ISP=0
C  READ THE NEXT PARTICLE ROW.
    goto 10
  endif
  IX2=IX22
C  READ THE NEXT PARTICLE LINE.
  goto 40
50 continue
  write (2,300)
  close (unit=1)
  close (unit=2)
100 format (10I3)
200 format (3F4.0)
C  WRITE A BLANK LINE AT THE BOTTOM OF THE FILE TO SHOW IT IS THERE.
300 format(/)
400 format (2I3)
  stop
  end

```

Appendix B

Averaging Programs

B.1 AVVOL.FOR

B.1.1 Description

Purpose To volume average a given two-dimensional distribution of particles using an averaging volume of fixed size.

Input A file containing the coordinates of the centres of particles; in this case RBPOS.DAT.

Output A file, AVVOL.DAT, containing the particle volume fraction as calculated at each pixel.

Description The program works by moving an averaging volume from pixel to pixel in an array representing the image of the plume and containing the position of each of the particles. As it moves the particle volume fraction is calculated by adding all the particles, or parts of particles, that lie within the averaging volume. These may be divided into the three classes shown in Figure B.1:

1. those whose centres are more than one particle radius (i.e. wholly) within the averaging volume;
2. those who lie within a radius, inside or outside, of one boundary so they are dissected by it;
3. those who lie within one radius of two boundaries and are therefore dissected by both.

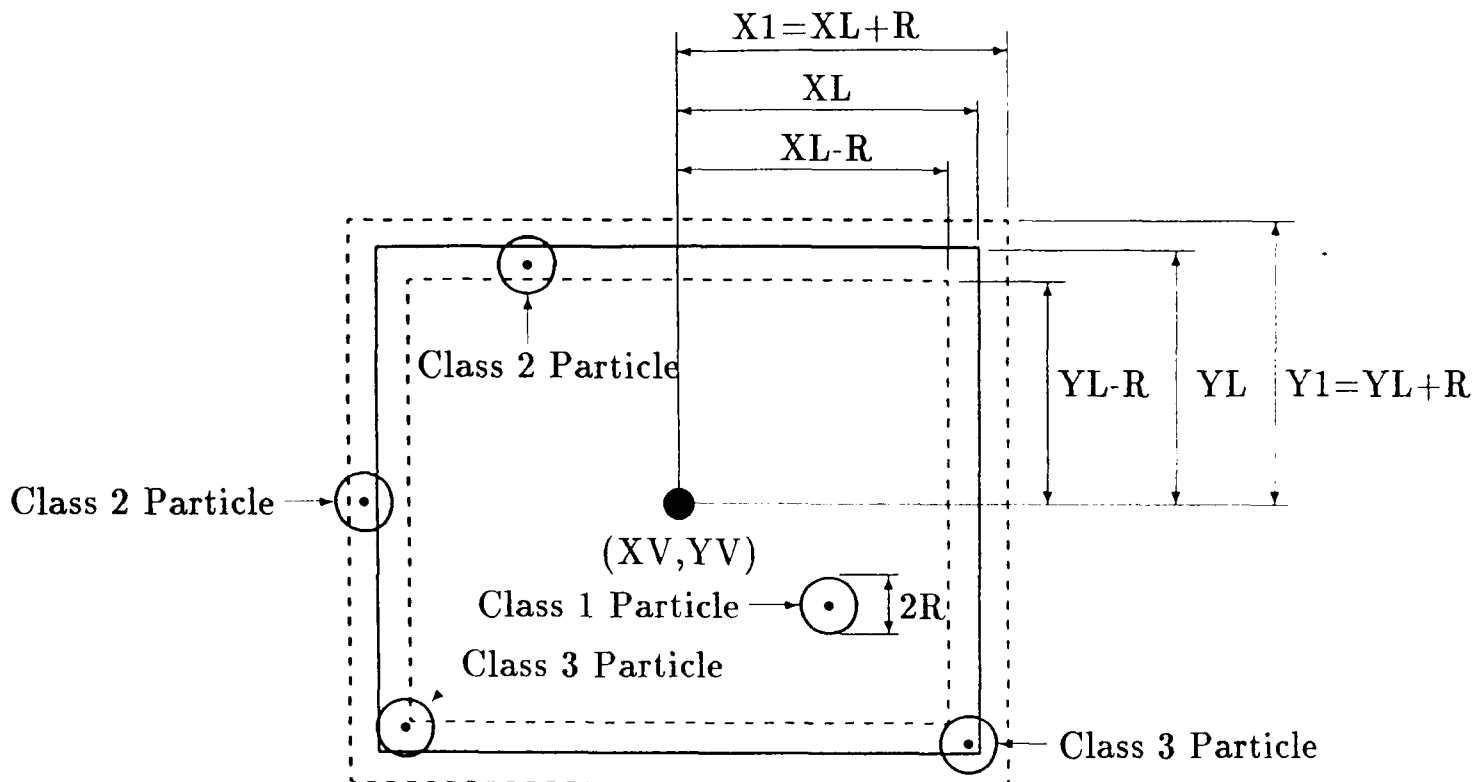


Figure B.1: Diagram of an Averaging Volume showing the Classes of Particle and the Lengths involved in Averaging.

The particles of the first class are wholly immersed in the averaging volume. The volume of the portion of a particle of the second class can be calculated exactly by the function **VOLUME**. If the distance from the centre of the particle is x , that from the centre of the particle to the edge of the averaging volume is X , and the radius of the particle is R then

$$\begin{aligned} \text{VOLUME} &= \int_X^R \pi(R-x)^2 dx \\ &= \frac{2}{3}\pi R^3 - \pi \left(X R^2 - \frac{X^3}{3} \right). \end{aligned} \quad (\text{B.1})$$

The volume of the portion of a particle of the third class of particles can be found using **VOLUME** as well, provided the corner of the averaging volume does not lie within the particle, as in the case of the particle in the bottom, left hand corner of Figure B.1. If it does then the portion of the particle within the averaging volume has to be integrated numerically by the routine **PARVOL**, at the heart of which lies a NAG routine. **PVOL** is the function that is numerically integrated by **PARVOL**.

The program starts, after some initial statements, by reading the file containing the coordinates of the particles and storing them in appropriate positions in an array. The cells of this array are analogous to the pixels making up an image. The centre of the averaging volume moves through each of these, calculating the particle volume fraction. The contents of all the pixels which might affect α_p are examined. When a particle is found it is classified and the

volume within the averaging volume calculated and added to a running total. When all the pixels have been searched the particle volume fraction is calculated using this total, and stored in the output file. The running volume total is set to zero before the centre of the averaging volume moves to the next cell.

Variables The letter 'X' in a variable names means it refers to the longitudinal direction, and the letter 'Y' means it refers to the lateral direction. All lengths are in millimetres.

ALPHA The particle volume fraction.

BX,BY The distance of the pixel under examination to the boundaries of the averaging volume.

CENT The array within which the coordinates of the centres of particles are stored.

DX, DY The size of a pixel in millimetres.

I1, I2 Counters used in the lateral and longitudinal directions respectively when searching the contents of pixels.

I3, I4 Counters used in the lateral and longitudinal directions when setting the centre of the averaging volume.

IOX, IOY If any portion or the whole of a particle infringes the averaging volume, the distance from its centre to that of the averaging volume will be less than these.

IX,IY Coordinates of the pixel within in which the coordinates of the centre of a particle, (X, Y), are stored.

PI π

PSV The running total of the volume of particles within an averaging volume.

PV(1) Volume of a particle within the averaging volume.

PV(2) Volume of a particle protruding from a second dissecting averaging volume boundary, where applicable.

PVF Volume of a particle within an averaging volume returned by **PARVOL**.

R Radius of the particles.

TAU Thickness of the tank in the experiments in mm.

VS Volume of one particle.

X, Y Coordinates of the centre of a particle in mm.

X1, Y1 Distance from the centre of a particle under examination to the centre of the averaging volume in the longitudinal and lateral directions respectively.

XL, YL Half the dimensions of the averaging volume in mm.

XV,YV IX and IY in mm.

B.1.2 Listing

```
dimension CENT(300,463,2),PV(2)
data CENT,XL,YL,DX,DY,R,PI,ALPHA,TAU,IC
1/105200*0.,30.,30.,1.,1.,1.5,3.14159,0.,7.8,0/
C OPEN INPUT FILE.
open (unit=1,name='rbpos5a.dat',type='old')
C OPEN OUTPUT FILE
open (unit=2,name='avpos.dat',type='new',
lcarriagecontrol='list')
C CALCULATE THE VOLUME OF ONE PARTICLE.
VS=(4.*PI*R**3.)/3.
C CALCULATE THE DIMENSIONS OF THE AREA IF IN WHICH THE CENTRE OF A
C PARTICLE LIES IT WILL INFRINGE THE AVERAGING VOLUME
IOX=int((XL+R)/DX+0.5)
IOY=int((YL+R)/DY+0.5)
type *,IOX,IOY
10 continue
C READ POSITIONS OF PARTICLES FROM THE INPUT FILE.
read (1,100) X,Y
if (X.ne.0) then
C CALULATE THE COORDINATES OF THE PIXEL IN WHICH A PARTICLE LIES.
IX=int(X/DX)-104
IY=int(Y/DY)+75
C STORE THE POSITION OF THE CENTRE OF A PARTICLE IN THE APPROPRIATE PIXEL.
CENT(IX,IY,1)=X
CENT(IX,IY,2)=Y
goto 10
endif
C START STEPPING THROUGH THE PIXELS IN THE LATERAL AND THEN IN THE
C LONGITUDINAL DIRECTIONS AND CALCULATING THEIR REAL POSITIONS.
do I3=101,363
YV=real(I3-75)*DY
do I4=IOX+1,200
XV=real(I4+104)*DX
PSV=0.
C STEP THROUGH ALL THE PIXELS WHICH MIGHT CONTAIN THE CENTRE OF A
C PARTICLE WHICH INFRINGES THE AVERAGING VOLUME.
do I1=I3-IOY,I3+IOY
do I2=I4-IOX,I4+IOX
C IF THE CURRENT PIXEL DOES NOT CONTAIN THE CENTRE OF A PARTICLE THEN
C MOVE TO THE NEXT ONE.
if (CENT(I2,I1,1).eq.0.) goto 20
C STORE THE PARTICLE'S CENTRE COORDINATES IN AN ARRAY.
X=CENT(I2,I1,1)
Y=CENT(I2,I1,2)
```

```

C  CALCULATE THE DISTANCE FROM THE PIXEL UNDER EXAMINATION TO THE CENTRE      ENTRE
C  OF THE AVERAGING VOLUME.
X1=abs(XV-X)
Y1=abs(YV-Y)
C  CALCULATE THE DISTANCE FROM THE PIXEL UNDER EXAMINATION TO THE EDGE      DGE
C  OF THE AVERAGING VOLUME.
BX=abs(XL-X1)
BY=abs(YL-Y1)
C  IF THE PARTICLE DOES NOT INFRINGE THE AVERAGING VOLUME THAN IGNORE IT.    RE IT.
if ((X1.ge.XL+R).or.(Y1.ge.YL+R)) goto 20
C  IF THE PARTICLE IS WHOLLY CONTAINED BY THE AVERAGING VOLUME THEN ADD      ADD
C  ITS ENTIRE VOLUME TO THE RUNNING TOTAL.
if ((X1.le.XL-R).and.(Y1.le.YL-R)) then
    PX(1)=VS
    goto 30
endif
C  CHECK FOR INFINGEMENT OF LATERAL BOUNDARY.
if (Y1.le.YL-R) then
    VX=0.
C  IF THIS IS THE ONLY INFRINGED BOUNDARY THEN CALCULATE THE VOLUME
C  ENCLOSED BY IT.
    if (X1.lt.XL) VX=VS
    PV(1)=VOLUME(BX,VX,R,PI)
    goto 30
endif
C  CHECK FOR INFINGEMENT OF LONGITUDINAL BOUNDARY.
if (X1.le.XL-R) then
    VX=0.
C  IF THIS IS THE ONLY INFRINGED BOUNDARY THEN CALCULATE THE VOLUME
C  ENCLOSED BY IT.
    if (Y1.lt.YL) VX=VS
    PV(1)=VOLUME(BY,VX,R,PI)
    goto 30
endif
C  IF THIS LINE REACHED THEN PARTICLE MUST BE NEAR A CORNER OF THE
C  AVERAGING VOLUME AND INFRINGING BOTH BOUNDARIES.
VX=0.
PV(1)=VOLUME(BX,VX,R,PI)
PV(2)=VOLUME(BY,VX,R,PI)
if ((PV(1).eq.0.).or.(PV(2).eq.0)) then
    if (PV(1).eq.0) PV(1)=PV(2)
    goto 30
endif
C  IF THE CORNER OF THE AVERAGING VOLUME IS NOT WITHIN THE PARTICLE
C  CALCULATE THE VOLUME OF THE PARTICLE IN THE AVERAGING VOLUME.
if (((X1-XL)**2+(Y1-YL)**2).ge.R**2) then
    if ((X1.lt.XL).and.(Y1.lt.YL)) PV(1)=VS-PV(1)-PV(2)
    if ((X1.gt.XL).and.(Y1.gt.YL)) PV(1)=0.
    if ((X1.lt.XL).and.(Y1.gt.YL)) PV(1)=PV(2)
    if (PV(1).lt.0) then
        type *, 'Added volumes too large', BX, PVQ, BY, PV(2), PV(1)
        stop
    endif
else
C  IF THE CORNER OF THE AVERAGING VOLUME IS WITHIN THE PARTICLE
C  CALCULATE THE VOLUME OF THE PARTICLE IN THE AVERAGING VOLUME.
    call PARVOL(BX,BY,PVF,R,PV)
    if ((PVF.gt.PV(1)).or.(PVF.gt.PV(2))) then
        type *, 'Integrated volume too big', PVF, BX, PV(1), BY, PV(2)
        stop
    endif
    if (PVF.le.0) then
        type*, 'PVF less than 0', PVF
        stop
    endif
endif

```

```

      if ((X1.lt.XL).and.(Y1.lt.YL)) PV(1)=VS-PV(1)-PV(2)+PVF
      if ((X1.ge.XL).and.(Y1.ge.YL)) PV(1)=PVF
      if ((X1.gt.XL).and.(Y1.lt.YL)) PV(1)=PV(1)-PVF
      if ((X1.lt.XL).and.(Y1.gt.YL)) PV(1)=PV(2)-PVF
    endif
30 continue
C INCREMENT THE RUNNING TOTAL OF PARTICLE VOLUME ENCLOSED BY THE
C AVERAGING VOLUME.
  PSV=PSV+PV(1)
20 continue
      enddo
    enddo
  if (PSV.lt.0.) then
    type *, 'Negative volume', PSV
    stop
  endif
C CALCULATE THE PARTICLE VOLUME FRACTION.
  ALPHA=PSV/(4.*XL*YL*TAU)
  if (ALPHA.gt.1.) then
    type *, 'Unfeasibly large volume fraction', ALPHA
    stop
  endif
  IC=IC+1
C WRITE THE PARTICLE VOLUME FRACTION IN THE OUTPUT FILE.
  write (2,200) ALPHA
enddo
enddo
  write (2,*) 'Averaging complete', I3, I4, IC, IOX, IOY
  close(unit=1)
  close(unit=2)
  stop
100 format(2f5.1)
200 format(f6.3)
300 format (2I3)
end

C SUBROUTINE TO CALCULATE THE VOLUME OF A PARTICLE WITHIN THE AVERAGING
C VOLUME IF IT IS DISSECTED BY THE BOUNDARY.
function VOLUME(X,V,R,PI)
C THIS FORMULA FOUND BY INTEGRATION.
  VOLUME=abs(V-(2.*PI*R**3./3.-PI*(X*R**2.-X**3./3.)))
  return
end

C SUBROUTINE TO CALCULATE THE VOLUME OF A PARTICLE WITHIN THE AVERAGING
C VOLUME IF IT IS DISSECTED BY TWO BOUNDARIES. THE NUMERICAL INTEGRATION
C ROUTINE D01ATF PRODUCED BY NAG LTD IS USED.
subroutine PARVOL(XA,YA,PVF,R,PV)
real PVF
double precision A,B,EPSABS,EPSREL,RESULT,ABSERR
1 ,XB,W(800),RD
integer LW,IW(200),LIW,IFAIL
data EPSABS,EPSREL,LW,LIW,IFAIL/0.01,0.001,800,200,-1/
external PVOL,D01ATF
dimension PV(2)
common /SQUID/RD,XB
RD=dblE(R)
XB=dblE(XA)
A=dblE(YA)
B=dsqrt(RD**2-XB**2)
call D01ATF(PVOL,A,B,EPSABS,EPSREL,RESULT,ABSERR,W,LW
1 ,IW,LIW,IFAIL)
if (IFAIL.ne.0) then
  type *, 'Integration error', RESULT, ABSERR, IFAIL

```

```

    stop
endif
if (ABSERR.gt.RESULT*0.001) then
    type *, 'Large integration error', RESULT, ABSERR
    stop
endif
PVF=real(RESULT)
return
end

subroutine PVOL(X,FV,N)
integer N
double precision FV(N),X(N),RD,XB
common /SQUID/RD,XB
do I=1,N
    FV(I)=(RD**2-X(I)**2)*dacos(XB/dsqrt(RD**2-X(I)**2))
1      -XB*dsqrt(RD**2-X(I)**2-XB**2)
enddo
return
end

```

B.2 FLEXVOL.FOR

B.2.1 Description

Purpose To estimate the size of the averaging volume necessary for the particles it contains to be treated as a continuum, and to calculate and record the particle volume fraction.

Input A file containing the coordinates of the centres of particles; in this case RBPOS.DAT.

Output A file, FLEXPOS.DAT, to contain the particle volume fraction and any other desired information.

Description This program is based on AVVOL.FOR: the calculation of the particle volume fraction is almost exactly the same; the only difference is that the particle volume fraction from the previous calculation is stored. The computation of particle volume fraction is contained within a loop that determines the final size of the averaging volume. This (the top of which is at label 15) repeats the calculation of α_p , incrementing the averaging volume until either of two criteria described in Section 10.4 are fulfilled over a test range of magnitude **ISM** pixels. The direction of change of the calculated value of α_p is denoted by **ISGN** and the size of its undulations by **ADIFF**. A counter (**ISIZ**) counts the number of increments for which either of these criteria have been satisfied; if at any point they are not it, and **ADIFF**, are reset to their original values and the process starts again. When **ISIZ** reaches **ISM** a flag, **IF**, is set and the size of the averaging volume decreased by **ISM** increments to the size when the criteria were first satisfied. The new value of **IF** triggers the recording of the result of the subsequent calculation before the centre of the averaging volume moves to the next pixel. It is possible to set the lateral size (**YL**) of the averaging volume to be some proportion of the longitudinal (**XL**).

Variables Only those variables different or additional to those used in AVVOL.FOR are listed here.

ADIFF The difference between **AMAX** and **AMIN**.

ALPHA A vector containing particle volume fractions: **ALPHA**(1) contains the particle volume fraction calculated for the current size of averaging volume, and **ALPHA**(2)

- contains the calculated value of the particle volume fraction before the last increment.
- AMIN** The smallest particle volume fraction since **ISIZ** was last equal to zero.
- AMAX** The largest particle volume fraction since **ISIZ** was last equal to zero.
- AS** The difference between **ALPHA(1)** and **ALPHA(2)**. If it is positive the calculated value of the particle volume fraction is increasing, if it is negative the value of the particle volume fraction is decreasing.
- IF** A flag which, when equal to one, allows the particle volume fraction, and any other desired information, to be written in the output file.
- ISGN** Integer denoting the sign of **AS** for the operation before the current one. If **AS** is positive it is equal to 1; if **AS** is negative, it is equal to -1; if **AS** is zero, it is equal to 0.
- ISIG** The same as **ISGN** except that it is for the current operation.
- ISIZ** Counter of the number of increments for which the two criteria have been satisfied.
- ISM** The number of times the criteria must be satisfied successively before the incrementation of the averaging volume ceases.
- SCL** Increment of the size of the averaging volume.

B.2.2 Listing

```

dimension CENT(200,263,2),PV(2),ALPHA(2)
data CENT,XL,YL,DX,DY,R,PI,ALPHA,TAU,ISIZ,ISIG,
1 ISM,SCL,IF,NB/105200*0.,2.,2.,2.38,1.48,3.,3.14159,2*0.
2,10.,1,0,15,1.,0,0/
open (unit=1,name='rbpos5a.dat',type='old')
open (unit=2,name='flexpos.dat',type='new',
1carriagecontrol='list')
open (unit=3,name='errpos.dat',type='new',
1carriagecontrol='list')
VS=(4.*PI*R**3.)/3.
10 continue
  read (1,100) X,Y
  if (X.ne.0.) then
    IX=int(X/DX)
    IY=int(Y/DY)
    CENT(IX,IY,1)=X
    CENT(IX,IY,2)=Y
c    type*,IX,X,IY,Y
    goto 10
  endif
do I3=1,263,2
  YV=real(I3)*DY
  do I4=1,200,2

```



```

        XV=real(I4)*DX
        AMIN=1.
        ADIFF=1.
        AMAX=0.
15      continue
        YL=XL
        PSV=0.
        IOX=int((XL+R)/DX+0.5)
        IOY=int((YL+R)/DY+0.5)
        ALPHA(2)=ALPHA(1)
        ALPHA(1)=0.
        do I1=I3-IOY,I3+IOY
            do I2=I4-IOX,I4+IOX
if ((I1.lt.1).or.(I2.lt.1).or.(I1.gt.462).or.(I2.gt.399))then
            write (3,300) I1,I2,I3,I4,XL
            goto 40
endif
        PV(1)=0.
        PV(2)=0.
c      type *,I2,I1
        if (CENT(I2,I1,1).eq.0.) goto 20
        if (IF.eq.1) NB=NB+1
        X=CENT(I2,I1,1)
        Y=CENT(I2,I1,2)
        X1=abs(XV-X)
        Y1=abs(YV-Y)
        BX=abs(XL-X1)
        BY=abs(YL-Y1)
c      type *,X,Y,X1,Y1
        if ((X1.ge.XL+R).or.(Y1.ge.YL+R)) goto 20
        if ((X1.le.XL-R).and.(Y1.le.YL-R)) then
            PSV=PSV+VS
            goto 20
        endif
        if (Y1.le.YL-R) then
            VX=0.
            if (X1.lt.XL) VX=VS
            PV(1)=VOLUME(BX,VX,R,PI)
            goto 30
        endif
        if (X1.le.XL-R) then
            VX=0.
            if (Y1.lt.YL) VX=VS
            PV(1)=VOLUME(BY,VX,R,PI)
            goto 30
        endif
        VX=0.
        PV(1)=VOLUME(BX,VX,R,PI)
        PV(2)=VOLUME(BY,VX,R,PI)
        if ((PV(1).eq.0.).or.(PV(2).eq.0)) then
            if (PV(1).eq.0) PV(1)=PV(2)
            goto 30
        endif
        if (((X1-XL)**2.+(Y1-YL)**2.).ge.R**2.) then
            if ((X1.lt.XL).and.(Y1.lt.YL)) PV(1)=VS-PV(1)-PV(2)
            if ((X1.gt.XL).and.(Y1.gt.YL)) PV(1)=0.
            if ((X1.lt.XL).and.(Y1.gt.YL)) PV(1)=PV(2)
            if (PV(1).lt.0.) then
                type *,'Added volumes too large',I4,I3,BX,BY,PV(2),PV(1)
                stop
            endif
        else
            call PARVOL(BX,BY,PVF,R,PV)

```



```

    if ((PVF.gt.PV(1)).or.(PVF.gt.PV(2))) then
        type *, 'Integrated volume too big', PVF, BX, PV(1), BY, PV(2)
        stop
    endif
    if (PVF.le.0.) then
        type *, 'PVF less than 0', PVF
        stop
    endif
    if ((X1.lt.XL).and.(Y1.lt.YL)) PV(1)=VS-PV(1)-PV(2)+PVF
    if ((X1.ge.XL).and.(Y1.ge.YL)) PV(1)=PVF
    if ((X1.gt.XL).and.(Y1.lt.YL)) PV(1)=PV(1)-PVF
    if ((X1.lt.XL).and.(Y1.gt.YL)) PV(1)=PV(2)-PVF
endif
30 continue
c type *, PSV, PV
PSV=PSV+PV(1)
20 continue
        enddo
        enddo
c type *, I1, I2, I3, I4, ISIZ, PSV
if (PSV.lt.0.) then
    type *, 'Negative volume', PSV
    stop
endif
ALPHA(1)=PSV/(4.*XL*YL*TAU)
if (ALPHA(1).gt.1.) then
    type *, 'Unfeasibly large volume fraction', ALPHA(1), PSV
    1, XL, YL, ISIZ
    stop
endif
if (ALPHA(1).lt.AMIN) AMIN=ALPHA(1)
if (ALPHA(1).gt.AMAX) AMAX=ALPHA(1)
ADIFF=AMAX-AMIN
AS=ALPHA(1)-ALPHA(2)
if (AS.lt.0.) ISGN=-1
if (AS.gt.0.) ISGN=1
if (AS.eq.0.) ISGN=0
if (IF.eq.1) then
    write (2,200) XV,YV,ALPHA(1),XL
    IF=0
    XL=2.
    ISIZ=0
    NB=0
else
    if ((ISGN.eq.ISIG).or.(ADIFF.lt.0.005)) then
        if (ISIZ.eq.ISM) then
            XL=XL-SCL*real(ISM+1)
            IF=1
            ISIZ=ISIZ-ISM+1
        endif
        if (ISIZ.gt.ISM) then
            type *, 'ISIZ too large'
            stop
        endif
        ISIZ=ISIZ+1
    else
        ISIZ=0
        ISIG=ISGN
        AMIN=1.0
        AMAX=0.
        ADIFF=1.0
    endif
    XL=XL+SCL

```

```

        goto 15
    endif
40 continue
    enddo
enddo
write (2,*) 'Averaging complete',I3,I4,IOX,IOY
close(unit=1)
close(unit=2)
close(unit=3)
stop
100 format(2f5.1)
200 format(2f4.0,f5.3,f4.0)
300 format (4I3,f4.0)
end

function VOLUME(X,V,R,PI)
VOLUME=abs(V-(2.*PI*R**3./3.-PI*(X*R**2.-X**3./3.)))
return
end

subroutine PARVOL(XA,YA,PVF,R,PV)
real PVF
double precision A,B,EPSABS,EPSREL,RESULT,ABSERR
1  ,XB,W(800),RD
integer LW,IW(200),LIW,IFAIL
data EPSABS,EPSREL,LW,LIW,IFAIL/0.01,0.001,800,200,-1/
external PVOL,D01ATF
dimension PV(2)
common /SQUID/RD,XB
RD=dblE(R)
XB=dblE(XA)
A=dblE(YA)
B=dsqrt(RD**2-XB**2)
call D01ATF(PVOL,A,B,EPSABS,EPSREL,RESULT,ABSERR,W,LW
1 ,IW,LIW,IFAIL)
if (IFAIL.ne.0) then
    type *, 'Integration error',RESULT,ABSERR,IFAIL
    stop
endif
if (ABSERR.gt.RESULT*0.001) then
    type *, 'Large integration error',RESULT,ABSERR
    stop
endif
PVF=real(RESULT)
return
end

subroutine PVOL(X,FV,N)
integer N
double precision FV(N),X(N),RD,XB
common /SQUID/RD,XB
do I=1,N
    FV(I)=(RD**2-X(I)**2)*dacos(XB/dsqrt(RD**2-X(I)**2))
1    -XB*dsqrt(RD**2-X(I)**2-XB**2)
enddo
return
end

```

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