

Weighing Supermassive Black Holes



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Abstract

Supermassive black holes (SMBHs) drastically influence the evolution of their host galaxies, as evident in the tight correlations between SMBH mass and galaxy properties. Yet, many fundamental questions about how SMBHs form, grow, and co-evolve with galaxies are unanswered, due to uncertainties in local SMBH mass measurements and the lack of direct dynamical measurements of SMBH masses beyond the local Universe.

In this thesis, I first present three metrics that evaluate the precision and accuracy of SMBH mass measurements with molecular gas or megamaser dynamics. Applying them to all existing SMBH mass measurements using the two methods, I demonstrate the importance of high angular (and thus spatial) resolution observations to reduce SMBH mass uncertainties and show that Atacama Large Millimeter/sub-millimeter Array (ALMA) observations can provide the most precise and accurate SMBH mass measurements across large samples of galaxies.

Then, I present measurements of the SMBH masses of the local lenticular galaxies NGC 383 and NGC 1574 using high-resolution ALMA observations of the CO(2-1) emission line. Both measurements improved the precision and accuracy of the SMBH masses compared to lower-resolution measurements. In particular, the NGC 383 measurement is the highest-resolution and thus the most accurate SMBH mass measurement using the molecular gas method to date.

Next, I discuss the prospects for the first cold-gas dynamical SMBH mass measurements at high redshifts, by utilising strong gravitational lensing as a magnifying glass. I also present candidate targets for such measurements, along with their available ALMA data. Additionally, I address a challenge for such measurements – accurately modelling the stellar mass distribution in dusty galaxies – by developing a neural network that efficiently deprojects stellar mass profiles.

Finally, I present an upper limit on the SMBH mass in the lensed sub-millimetre galaxy ID141 at redshift $z = 4.24$, demonstrating the potential of gas-dynamical modelling of strongly lensed galaxies to directly constrain the masses of high-redshift SMBHs and their correlations with galaxy properties.

Overall, this thesis establishes high-resolution observations as the key to reduce the uncertainties of local SMBH mass measurements with ALMA. It also establishes the method to obtain the first high-redshift dynamical measurements of SMBH masses from cold gas kinematics.

Statement on the use of generative artificial intelligence

I declare that any use of generative artificial intelligence (AI) during the preparation of this thesis complies with University, divisional and departmental guidance. I acknowledge the use of ChatGPT5, published by OpenAI, to assist with literature searches, code debugging, and formatting the LaTeX thesis template, but not for generating or editing text or figures in this thesis.

Statement of originality

I carried out the work presented in this thesis as a DPhil student at the Sub-department of Astrophysics, Department of Physics, University of Oxford, between October 2022 and July 2026 under the supervision of Prof. Martin Bureau. My research was supported by a Science and Technology Facilities Council (STFC) DPhil studentship under grant ST/X508664/1 and the Balliol College J. T. Hamilton Scholarship in physics. I also conducted research as a visiting student in the Department of Physics at the University of Montréal and the MILA institute, from October 2024 to December 2024, supported by long-term attachment funding from STFC. I declare that no part of this thesis has been submitted in support of another degree, diploma or other qualification at the University of Oxford or any other higher education institute. Except where otherwise stated or where reference is made, the work in this thesis is entirely my own.

The work presented in Chapter 2 has been published in [Zhang et al. \(2024\)](#). It built upon Dr. Mark Smith’s unfinished research, presented in Chapter 6.3 of [Smith \(2020\)](#). As the first author of the paper, I introduced two additional figures of merit, compiled all SMBH mass measurements using molecular gas or megamasers as of April 2024, developed the method to measure $R_{\min}$ and $V_{\max}$ for molecular gas measurements, and performed all comparisons between molecular gas and masers.

The work presented in Chapter 3 has been published in [Zhang et al. \(2025a\)](#). As the first author of the paper, I conducted all steps of the ALMA data reduction (except for the self-calibration performed by Dr. Ilaria Ruffa), performed all data analyses, and wrote the manuscript. The ALMA data used in this work were obtained from ALMA projects led by two co-authors: Prof. Martin Bureau and Dr. Timothy A. Davis.

The work presented in Chapter 4 has been published in [Zhang et al. \(2025b\)](#). As the first author, I performed the ALMA data reduction, analysed the data, and wrote the manuscript. The ALMA data used in this work were obtained from ALMA projects led by Prof. Martin Bureau and Dr. Lijie Liu.

The work presented in Chapter 5 is entirely my original research. The principal investigators of the ALMA data presented are stated in Chapter 5.2.2.

The work presented in Chapter 6 is based on a conference paper ([Yantovski-Barth et al., 2025](#)) to which the first author (Misha Yantovski-Barth) and I contributed equally. My main contributions to the work are: bringing forward the idea of deprojection with a multi-Gaussian expansion, generating the neural network training set, designing the neural network

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Chapter 1

Introduction

1.1 Black holes

Black holes, dark objects with extremely high mass density, have been a mystery of the Universe for centuries. In 1784, John Michell postulated the existence of "dark stars", objects so massive that everything, including light, would fall back onto them due to gravity ([Michell, 1784](#)). Later, in 1796, Pierre-Simon Laplace independently introduced the concept of such objects, followed by a mathematical "proof" in 1779 ([Laplace, 1799](#)). Although their postulations are based on the classical theories of light and gravity, it is commonly believed that they marked the beginning of the theory of black holes (e.g. [Montgomery et al., 2009](#)).

We now know that black holes should be described in the framework of Einstein's general theory of relativity ([Einstein, 1916](#)). In this framework, Karl Schwarzschild found a metric that describes the spacetime around a non-rotating, chargeless, spherically symmetric object ([Schwarzschild, 1916](#)):

$$ds^2 = - \left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 , \quad (1.1)$$

where s is a line element, t is time, r , θ and ϕ are the usual spherical coordinates, G is the gravitational constant, c is the speed of light and M is the mass of the object. It was immediately noticed that the Schwarzschild metric has a singularity at the so-called "Schwarzschild radius"

$$R_{\text{Schw}} \equiv \frac{2GM}{c^2} . \quad (1.2)$$

This singularity cannot occur in extended astrophysical objects such as stars, since the Schwarzschild radius is much smaller than the objects' sizes. However, it can become a problem for sufficiently compact astrophysical bodies. Many attempts were made to interpret or avoid the singularity, such as by applying a coordinate transformation (e.g. [Schwarzschild, 1916](#); [Eddington, 1924](#); [Lemaître, 1933](#)). Eventually, [Finkelstein \(1958\)](#) showed that the singularity at the Schwarzschild radius forms an "event horizon", through which causality (as does light and everything else) can cross only in one direction. This formed the last major building block of the modern concept of black holes. The Schwarzschild metric was later generalised to metrics that allow non-zero charge and angular momentum, and event horizons have been shown to emerge in all of them (e.g. [Reissner, 1916](#); [Weyl, 1917](#); [Nordström, 1918](#); [Jeffery, 1921](#); [Kerr, 1963](#); [Newman & Janis, 1965](#); [Newman et al., 1965](#)).

In parallel with the thriving theories of black holes, numerous studies in the early 1900s also demonstrated that black holes are more than just mathematical concepts. They found that the gravitational collapse of a star after it exhausts its fuel can lead to a compact object supported by electron degeneracy pressure (a white dwarf; e.g. [Fowler 1926](#); [Chandrasekhar 1931, 1935](#)) or, for more massive objects, by neutron degeneracy pressure and the strong nuclear force (a neutron star; e.g. [Baade & Zwicky 1934](#); [Oppenheimer & Volkoff 1939](#); [Harrison et al. 1965](#)). However, if the mass of the neutron star exceeds $2 - 4 M_{\odot}$ (e.g. [Rhoades & Ruffini, 1974](#); [Lattimer & Prakash, 2001](#); [Özel & Freire, 2016](#)), no force will be able to support it against gravity, and it will inevitably collapse further into a black hole (e.g. [Oppenheimer & Snyder, 1939](#); [Penrose, 1965](#)).

Despite the theoretical consensus that black holes should be abundant in the Universe, they are intrinsically difficult to detect because of their darkness and the small sizes of their Schwarzschild radii. Until recently, black holes could only be detected indirectly through the luminous signatures of accretion onto them or their gravitational influence on neighbouring objects. The first observational evidence of stellar-mass black holes came from monitoring X-ray binaries with only one visible component. Through this method, [Bolton \(1972, 1975\)](#) and [Webster & Murdin \(1972\)](#) found that the light curve and the line-of-sight velocity variations of HD 226868 are consistent with a dark companion in the mass range of a black hole. In parallel, the discovery of quasars (e.g. [Schmidt, 1963](#); [Matthews & Sandage, 1963](#)), with

luminosities that indicate highly energetic processes, led to the postulation that they are powered by accreting supermassive black holes (SMBHs; [Lynden-Bell 1969](#); [Lynden-Bell & Rees 1971](#)). That postulation was soon supported by dynamical studies of nearby galaxies (e.g. [Sargent et al., 1978](#); [Tonry, 1984](#); [Dressler & Richstone, 1988](#)), which indicated a central concentration of mass unlikely to be explained by the observed stellar population. The first clear detection of a SMBH came from observing individual stars orbiting the Milky Way centre, Sgr A* ([Schödel et al., 2002](#); [Ghez et al., 2005](#)), through which the mass of the black hole could be directly measured. I will introduce further methods to dynamically measure the masses of SMBHs in Section 1.3.

The past decade marked the beginning of direct observations of black holes. They can be revealed by detecting the gravitational waves emitted by merging stellar-mass black holes through observatories such as the Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo (e.g. [Abbott et al., 2016a,b, 2017](#)), as well as the gravitational-wave background produced by a cosmic population of SMBHs ([Agazie et al., 2023](#); [Xu et al., 2023](#); [EPTA Collaboration et al., 2023](#); [Reardon et al., 2023](#)). It is also now possible to literally see SMBHs through horizon-scale images produced by the Event Horizon Telescope (EHT; [Event Horizon Telescope Collaboration et al. 2019a, 2022a](#)). These multi-messenger observations have provided the most unambiguous evidence for the existence of black holes and enabled the most direct studies of black hole physics.

1.2 Supermassive black holes and their host galaxies

SMBHs are black holes with masses greater than $\approx 10^5 M_{\odot}$. It is now believed that every massive galaxy hosts a SMBH at its centre (e.g. [Kormendy & Ho, 2013](#)). Actively accreting SMBHs can shine in the form of active galactic nuclei (AGN) across the electromagnetic spectrum, e.g. in the radio, optical and X-ray bands (see e.g. [Padovani et al. 2017](#) for a review). By contrast, inactive SMBHs do not produce luminous radiation and can be detected only through their gravitational influence. To date, active SMBHs have been detected as early in cosmic time as redshift $z \approx 10$ ([Napolitano et al., 2025](#)), whereas inactive SMBHs, due to the challenges to confirm them dynamically (see Section 1.3 for details), have only been detected at $z \lesssim 2$ ([Newman et al., 2025](#)).

Two main unresolved puzzles concerning SMBHs are their formation and growth mechanisms, as well as their roles in galaxy evolution. In this section, I thus introduce our current knowledge as well as unanswered questions regarding both puzzles.

1.2.1 Origin of supermassive black holes

Unlike stellar-mass ($5 - 100 M_{\odot}$) black holes, whose formation pathways are relatively well understood, the origin of SMBHs remains a mystery. Many have postulated an evolutionary link between SMBHs and stellar-mass black holes, suggesting that SMBHs grew from light stellar-mass seeds through accretion and mergers (e.g. [Madau & Rees, 2001](#); [Volonteri, 2010](#)). However, this origin struggles to produce the massive ($\gtrsim 10^9 M_{\odot}$) SMBHs observed in the early Universe (e.g. [Wu et al., 2015](#); [Bosman et al., 2024](#)) within such a short time (e.g. [Inayoshi et al., 2020](#)), assuming that the accretion rate is limited by the Eddington limit ([Eddington, 1926](#)). The rapid cosmic emergence of SMBHs thus requires episodes of super-Eddington accretion and/or massive seeds (e.g. [Hu et al., 2025](#); [Mehta et al., 2026](#); [Bhowmick et al., 2026](#)). Many physical mechanisms have been proposed to enable and sustain super-Eddington accretion (e.g. [Abramowicz et al., 1988](#); [Davies et al., 2011](#); [Massonneau et al., 2023](#)). In parallel, numerous theories for the origin of massive SMBH seeds have also been introduced, such as direct collapse of pristine gas (e.g. [Begelman et al., 2006](#)), runaway stellar collisions in dense star clusters (e.g. [Portegies Zwart et al., 2004](#)), collapse of supermassive stars (e.g. [Woods et al., 2019](#)) and primordial black holes (e.g. [Ziparo et al., 2025](#)). However, current observations are still unable to provide conclusive evidence to distinguish between light and heavy seeds. Optimistic simulations predict that detecting a population of $10^{5-6} M_{\odot}$ black holes at $z \gtrsim 8$ can provide key diagnostics of the seed mass ([Hu et al., 2025](#)), while others predict that this requires SMBH detections at $z \gtrsim 15$ ([Jeon et al., 2025](#)).

Regardless of the seeding scenario, all current theories predict a population of intermediate-mass ($10^{2-5} M_{\odot}$) black holes (IMBHs). However, direct observational evidence of IMBHs is relatively scarce (see e.g. [Greene et al. 2020](#) for a review), except for recent strong signs in the globular cluster ω Centauri ([Häberle et al., 2024](#)) and gravitational wave events for which the remnant is more than $100 M_{\odot}$ (e.g. [Abbott et al., 2020](#)). This is because IMBHs dynamically influence only a small region around them, making it challenging to detect them with current

instruments. A larger number of IMBH detections, especially off-nuclear IMBHs, would place constraints on the origin of SMBHs (e.g. [Greene et al., 2020](#)).

1.2.2 Co-evolution of supermassive black holes and galaxies

Although SMBHs dynamically influence only a small region of their host galaxies, they are not passive actors of galaxy evolution. Instead, SMBHs actively affect the star formation and baryon cycle within and around galaxies. In other words, SMBHs and galaxies co-evolve with each other.

1.2.2.1 Correlations between SMBH mass and galaxy properties

The strongest observational evidence for the co-evolution of SMBHs and galaxies is the tight correlations between SMBH mass (M_{BH}) and multiple host galaxy properties. Studies of these correlations began with [Kormendy & Richstone \(1995\)](#), who argued that the ratio between SMBH mass and the mass of the stellar bulge is nearly constant. [Magorrian et al. \(1998\)](#) then established this scaling relation between SMBH mass and bulge mass with a quantitative fit. Later works refined this relation and investigated its dependence on other galaxy properties (e.g. [McConnell & Ma, 2013](#); [Graham & Sahu, 2023](#)).

The tightest correlation between SMBHs and galaxies is the famous " $M - \sigma$ " relation between the SMBH mass M_{BH} and the stellar velocity dispersion within one effective radius σ_e . It was first discovered by [Ferrarese & Merritt \(2000\)](#) and [Gebhardt et al. \(2000\)](#) and later updated by numerous studies (e.g. [Gültekin et al., 2009](#); [Beifiori et al., 2012](#); [McConnell & Ma, 2013](#); [van den Bosch, 2016](#); [Saglia et al., 2016](#); [Sahu et al., 2019](#)). The intrinsic scatter of this relation is only 0.42 dex, and the observed scatter is only 0.46 dex ([Saglia et al., 2016](#)), allowing one to use it to conveniently estimate a SMBH mass from the kinematics of the large-scale galaxy stellar population. Despite the tightness of the relation, there is growing evidence that it depends on galaxy properties such as morphology and stellar mass (e.g. [McConnell & Ma, 2013](#); [Saglia et al., 2016](#); [Krajinović et al., 2018](#)). However, the limited quantity, precision and accuracy of current SMBH mass measurements preclude conclusive inferences (e.g. [McConnell & Ma, 2013](#); [Kormendy & Ho, 2013](#); [Saglia et al., 2016](#)).

Another widely studied SMBH – galaxy scaling relation is that with the host galaxy's

total stellar mass (M_* ; e.g. [Reines & Volonteri 2015](#); [van den Bosch 2016](#)). Although the relation has a much larger scatter of 0.84 dex and depends on galaxy morphology ([van den Bosch, 2016](#)), it is relatively easy to measure, making it a convenient tool to evaluate whether the relationship between SMBHs and galaxies differs between low and high redshifts. Recent *James Webb Space Telescope* (JWST) observations of high-redshift galaxies revealed a population of SMBHs that are overmassive relative to the local SMBH mass – stellar mass relation (e.g. [Pacucci et al., 2023](#); [Herrera-Camus et al., 2026](#)), suggesting that SMBH growth precedes galaxy growth at early cosmic times. The most puzzling population of early SMBHs is the little red dots (LRDs; e.g. [Matthee et al. 2024](#); [Akins et al. 2025](#); [Inayoshi & Ho 2025](#)), with some studies suggesting SMBH-to-stellar mass ratios of $\sim 0.01 - 1$ or even no detectable stellar counterpart ([Furtak et al., 2024](#); [Durodola et al., 2025](#); [Juodžbalis et al., 2025](#)), while others indicate much lower ratios ([Naidu et al., 2025](#); [Rusakov et al., 2026](#)). The controversy originates from debates on whether the SMBH mass determinations at high redshifts are reliable ([Lupi et al. 2024](#); [Bertemes et al. 2025](#); see Section 1.3 for a more detailed discussion).

Apart from the SMBH – galaxy scaling relations discussed above, additional correlations between SMBH mass and galaxy structural parameters, such as the Sérsic index n (e.g. [Graham & Driver, 2007](#); [Savorgnan et al., 2013](#)) and the break radius of the core-Sérsic profile r_b (e.g. [Rusli et al., 2013](#); [Dullo & Graham, 2014](#)), have also been explored. Notably, while the $M_{\text{BH}} - n$ relation is considered less fundamental than the $M_{\text{BH}} - \sigma$ relation (e.g. [Shankar et al., 2017](#)) due to its larger scatter, the $M_{\text{BH}} - r_b$ relation can have less scatter than the $M_{\text{BH}} - \sigma$ relation for ultramassive ($\gtrsim 10^{10} M_\odot$) black holes ([Dullo et al., 2021](#)).

1.2.2.2 SMBH feedback mechanisms

The numerous tight correlations between SMBH mass and galaxy properties indicate that SMBHs must have played a crucial role in galaxy evolution. During the active phase of a SMBH, it not only grows rapidly from the fuel provided by the host galaxy, but it also exerts "SMBH feedback" on the galaxy, mainly by injecting energy and momentum into the interstellar medium (ISM) via the central AGN. SMBH feedback can alter the amount and the physical properties of star-forming gas by ejecting it, disturbing it and/or preventing it from

cooling, leading to reduced or completely quenched star formation (e.g. [Kormendy & Ho, 2013](#); [Heckman & Best, 2014](#); [Harrison, 2017](#); [Harrison et al., 2018](#)). Occasionally, SMBH feedback can also boost the star-formation rate by compressing the gas reservoir via shocks (e.g. [Rees, 1989](#); [Salomé et al., 2015](#); [Cresci et al., 2015](#)).

There are two main modes of AGN feedback. Radiative (i.e. quasar, wind) mode feedback occurs when the SMBH accretes efficiently at a high Eddington ratio. The AGN produces intense radiation that couples to the ISM by producing shocks and driving outflows (e.g. [Silk & Rees, 1998](#); [Di Matteo et al., 2005](#)). In contrast, kinematic (i.e. radio, jet) mode feedback is associated with radiatively inefficient accretion at a low Eddington rate. In this mode, the energy output of the AGN is dominated by powerful radio jets, whose mechanical energy can heat up the interstellar medium and prevent it from cooling (e.g. [Churazov et al., 2005](#); [Best et al., 2006](#)). Cosmological simulations have demonstrated that both feedback modes are essential for reproducing the local scaling relations between SMBHs and galaxies, as well as other observational constraints (e.g. [Weinberger et al., 2018](#); [Davé et al., 2019](#)).

Many observations have provided evidence for both modes of AGN feedback in the local Universe (e.g. [Cicone et al., 2014](#); [Ishibashi et al., 2018](#); [Nesvadba et al., 2021](#); [Saito et al., 2022](#)). However, direct observations of AGN feedback, especially radiative mode feedback, are scarce at high redshifts (e.g. [Stacey et al., 2022](#); [Bischetti et al., 2024](#); [Spilker et al., 2025](#)). This is because radiation pressure operates most efficiently on the scale of the innermost gas clump that encompasses the AGN ($\approx 0.05 - 1.5$ kpc; [Bieri et al. 2017](#)), so the resulting outflow is usually too compact to be resolved by current instruments at high redshifts.

1.3 Weighing supermassive black holes

Understanding the origin and growth of SMBHs, and constraining the physical mechanisms governing SMBH – galaxy co-evolution, both require a large number of precise and accurate SMBH mass measurements. Moreover, measuring both the SMBH mass and multi-wavelength AGN luminosities can reveal fundamental correlations closely related to the physics of black hole accretion, such as the fundamental plane of black hole activity ([Merloni et al., 2003](#); [Falcke et al., 2004](#)) and the mm-fundamental plane of black hole accretion ([Ruffa et al., 2024](#); [Elford et al., 2026](#)). In this section, I will present an overview of the major

techniques to estimate SMBH masses, from the local Universe to high redshifts, and briefly discuss their advantages and limitations.

The methods to infer SMBH masses can broadly be divided into two categories: dynamical methods and non-dynamical methods. For dynamical methods, the mass of the SMBH is measured by observing and modelling its dynamical influence on the motions of nearby objects, such as stars, masers and gas. Such influence is easily detectable only within the region where the SMBH dominates the galaxy gravitationally. This region is the SMBH's gravitational sphere of influence (SoI), formally defined as the spherical region where the SMBH mass exceeds the total enclosed mass of other galaxy components, predominantly stars. A common approximation for the radius of the SoI is

$$R_{\text{SoI}} \equiv \frac{GM_{\text{BH}}}{\sigma_e^2} . \quad (1.3)$$

Measuring SMBH masses dynamically thus requires spatially resolving the SoI. Otherwise, it is nearly impossible to disentangle the SMBH's contribution to the observed kinematics from the contributions of other galaxy components. [Davis \(2014\)](#) showed that the minimum spatial resolution typically required to dynamically estimate a SMBH mass is $2 R_{\text{SoI}}$.

Due to the strict requirement on spatial resolution, dynamical methods to weigh SMBHs are not always feasible. Alternatively, SMBH masses can be estimated using indirect, non-dynamical methods that are calibrated using the scaling relations between SMBHs and galaxies established via dynamical measurements. To date, almost all determinations of SMBH masses beyond the local Universe are non-dynamical.

1.3.1 In the local Universe

1.3.1.1 Direct imaging

Direct imaging of SMBHs with the EHT also enables direct measurements of SMBH masses by constraining the physical size of the black hole "shadow" in the image, after correcting for general relativistic effects ([Event Horizon Telescope Collaboration et al., 2019c, 2022b](#)). This method does not fit well into either of the two categories defined above, since it is direct but non-dynamical. Such measurements, although conceptually straightforward, are not

free of systematic uncertainties, including those arising from intrinsic source variability and uncertainties in interferometric imaging (e.g. [Lockhart & Gralla, 2022](#); [Chang et al., 2025](#)). Additionally, given the current capabilities of EHT ([Event Horizon Telescope Collaboration et al., 2019b](#)), the targets for direct imaging at the horizon scale are currently limited to Sgr A* and the nearby elliptical galaxy M87. Nevertheless, future extensions to the EHT, such as the next-generation EHT (ngEHT; [Johnson et al. 2023](#)) and the Black Hole Explorer (BHET; [Johnson et al. 2024](#)), will be able to expand such studies to a much larger number of galaxies (e.g. [Zhang et al., 2025c](#)).

1.3.1.2 Stellar dynamics

The measurement of the SMBH mass in our own Milky Way ([Schödel et al., 2002](#); [Ghez et al., 2005](#)) is the most well-known example of the stellar dynamics method. However, the Milky Way is the only giant galaxy for which we can spatially resolve individual stars at the galactic centre. More commonly, stellar kinematics is extracted from the integrated spectra of unresolved stars, providing line-of-sight velocity, velocity dispersion and higher-order moment maps of the stellar population, which are then used to derive the SMBH mass.

It is generally recognised that the first stellar-dynamical measurement of a SMBH mass was performed by [Sargent et al. \(1978\)](#) and [Young et al. \(1978\)](#), using one of the earliest photon-counting spectrographs. Since then, advances in the spectral and spatial resolutions, sensitivity, and wavelength coverage of spectroscopic instruments, from the *Hubble Space Telescope* (HST) Faint Object Spectrograph (FOS) and Space Telescope Imaging Spectrograph (STIS), to adaptive optics-assisted near-infrared integral-field spectrographs [IFSs; e.g. Spectrograph for INtegral Field Observation in the Near-Infrared (SINFONI), Gemini Near-infrared Integral Field Spectrograph (NIFS), OH-suppressing Infrared Integral Field Spectrograph ([OSIRIS](#))], to wide-field optical IFSs [e.g. Multi Unit Spectroscopic Explorer (MUSE)], and most recently to sensitive near-infrared space spectrographs [e.g. *James Webb Space Telescope* (JWST) Near-Infrared Spectrograph (NIRSpec)], have all enabled SMBH mass measurements in progressively further, fainter and more dust-obscured galaxies (e.g. [Cappellari et al., 2002](#); [Drehmer et al., 2015](#); [Simon et al., 2024](#); [Nguyen et al., 2026c](#)).

Typically, stellar-dynamical measurements of SMBH masses model the kinematics us-

ing either the Schwarzschild orbit-superposition method (Schwarzschild, 1979) or the Jeans Anisotropic Modelling (JAM; Cappellari 2008, 2020) method. The Schwarzschild method models the full kinematics of stars by computing a large number of orbits in a given gravitational potential and then superposing these orbits, with the weights of the superposition optimised to reproduce the observed properties. The JAM method, on the other hand, models only the second moment of the line-of-sight velocity distribution (LOSVD) $V_{\text{rms}} \equiv \sqrt{V^2 + \sigma^2}$ (where V is the luminosity-weighted mean line-of-sight velocity and σ is the luminosity-weighted line-of-sight velocity dispersion) by solving the Jeans equations (Jeans, 1915, 1922), assuming axisymmetry and constant anisotropy in the meridional plane. The JAM method is computationally more efficient than the Schwarzschild method, but it can also produce biased results if the underlying assumptions are inaccurate (e.g. Lablanche et al., 2012; Li et al., 2016).

The strength of the stellar dynamics method is that, in principle, it applies to all galaxies regardless of morphological type or SMBH activity. In practice, however, the method currently has a selection bias towards early-type galaxies (ETGs; Kormendy & Ho, 2013; van den Bosch, 2016; Shankar et al., 2017) because they have relatively massive SMBHs (hence larger SoI radii), less complex dynamical structures and less dust obscuration, which simplify both the extraction of the kinematics from the observed spectra and the subsequent dynamical modelling.

1.3.1.3 Megamaser dynamics

H₂O megamaser emission arises in the dense, warm molecular gas of AGN circumnuclear discs, where collisional pumping creates a population inversion that amplifies microwave radiation by stimulated emission. Such systems are preferentially detected in obscured (Type 2) AGN (see e.g. Lo 2005 for a review) due to their favourable edge-on geometry and high gas column densities. Megamasers are bright and typically only $\approx 0.05 - 5$ pc away from the galaxy centre, making them excellent tracers of the steep gravitational potential near the central SMBH. They are usually probed by very-long-baseline interferometry (VLBI) observations with sub-microarcsecond angular resolution. Such observations typically resolve the SoI by a factor of > 10 (Zhang et al., 2024), making megamaser dynamics the "gold

standard" of SMBH mass measurements in terms of both precision and accuracy (Kormendy & Ho, 2013). So far, the highest-resolution and most accurate extragalactic SMBH mass measurement is that of the galaxy NGC 4258 using megamasers (Herrnstein et al., 2005).

Aside from the high physical resolution, another advantage of the megamaser method is that the masers within the circumnuclear disc of a galaxy are usually in near-circular rotation. This substantially simplifies dynamical modelling as, under spherical symmetry, the circular velocity V_c at a radius R is determined only by the total mass enclosed within that radius $M(< R)$:

$$V_c = \sqrt{\frac{GM(< R)}{R}} . \quad (1.4)$$

Because $R \ll R_{\text{SoI}}$ due to the high resolution of the method, $M(< R)$ is dominated by the SMBH mass, further simplifying the equation to

$$V_c = \sqrt{\frac{GM_{\text{BH}}}{R}} . \quad (1.5)$$

A rotation curve that follows this relation, with a characteristic behaviour $V \propto R^{-1/2}$, is called a "Keplerian" rotation curve. It allows one to explicitly compute M_{BH} from the measured radius and rotation velocity of a maser, with minimum systematic uncertainties. Occasionally, the rotation curve traced by megamasers can also be sub-Keplerian due to the non-negligible self-gravity of the megamaser disc (e.g. Lodato & Bertin, 2003; Mamyoda et al., 2009) and/or warps in the disc. Yet, precise SMBH masses can still be obtained in these cases.

Another benefit of megamasers is that they can provide independent distance measurements. This is because long-term monitoring of megamaser discs yields the acceleration a of masers, allowing the radius R of the maser to be computed independently of the distance:

$$R = \frac{V_c^2}{a} . \quad (1.6)$$

By then comparing the physical size with the observed angular size of the radius, one can derive a precise distance for the megamaser galaxy. For this reason, when long-term monitoring observations are available, SMBH mass measurements using megamasers are independent of the assumed distance, unlike most other methods. These distance measurements also make

megamasers crucial anchors of the extragalactic distance ladder and an independent method of measuring the Hubble constant (Zhao et al., 2018; Reid et al., 2019; Pesce et al., 2020a,b). However, only 6 of the 26 galaxies with a SMBH mass measurement derived using megamasers have such geometric distance measurements, owing to the rarity of suitably edge-on, stable maser discs and the need for many years of high-precision monitoring.

Despite the remarkable precision, accuracy and simplicity of the megamaser dynamics method, it applies only to a small number of nearby Type 2 AGN that host megamasers. The SMBH masses of these AGN are limited to $10^{6-8} M_{\odot}$. As a result, the maser method only produces a small fraction of all dynamical SMBH mass measurements (van den Bosch, 2016), and complementary methods are needed to probe a wider range of SMBH masses in a more diverse sample of galaxies.

1.3.1.4 Ionised gas dynamics

Following the initial success of the stellar-dynamical method to measure SMBH masses, Harms et al. (1994) performed the first SMBH mass measurement using ionised gas kinematics, by observing the ionised gas disc of M87 with *HST* FOS. Like megamasers, the ionised gas motion can be almost circular, making dynamical modelling relatively straightforward. Additionally, the ionised gas method applies to a wider range of galaxies than the megamaser method. Thanks to *HST*, the number of ionised gas measurements of SMBH masses surged in the late 1990s and 2000s (e.g. Ferrarese et al., 1996; van der Marel & van den Bosch, 1998; Sarzi et al., 2001; de Francesco et al., 2006; Neumayer et al., 2007).

Nevertheless, a major challenge of this method is that the kinematics of ionised gas is susceptible to non-gravitational forces, such as those due to shocks and radiation pressure, which can induce significant turbulence and non-circular motions that in turn bias dynamical modelling and SMBH mass measurements (e.g. Cappellari, 2002a; Shapiro et al., 2006; Osorno et al., 2023). In those cases, an accurate dynamical model can still be obtained by explicitly modelling the hydrodynamics of the ionised gas, including its pressure-dominated velocity dispersion (e.g. Neumayer et al., 2007; Jeter et al., 2019). However, the modelling process is then extremely complex and vulnerable to other systematic uncertainties. Alternatively, SMBH masses derived using dispersion-dominated ionised gas kinematics are often

modelled using the Jeans equations and can be treated as upper limits, and such upper limits have been derived for a large sample of nearby galaxies (e.g. [Sarzi et al., 2002](#); [Beifiori et al., 2009](#)).

1.3.1.5 Molecular gas dynamics

Over the past decade, a new method to measure SMBH masses, cold molecular gas dynamics, has emerged thanks to the extraordinary sensitivity and angular resolution of the latest generation of sub-millimetre interferometers, such as the Atacama Large Millimeter/sub-millimeter Array (ALMA). Following a proof-of-concept measurement in the galaxy NGC 4526 ([Davis et al., 2013b](#)), the mm-Wave Interferometric Survey of Dark Object Masses (WISDOM) collaboration has performed molecular gas dynamical measurements of SMBH masses in 11 massive ETGs ([Davis et al., 2013b](#); [Onishi et al., 2017](#); [Davis et al., 2017, 2018](#); [Smith et al., 2019](#); [North et al., 2019](#); [Smith et al., 2021](#); [Ruffa et al., 2023](#); [Dominiak et al., 2025](#); [Zhang et al., 2025a,b](#)), a dwarf elliptical galaxy ([Davis et al., 2020](#)) and a peculiar luminous infrared galaxy with central spiral arms and a Seyfert nucleus ([Lelli et al., 2022](#)). Other researchers have also published molecular gas dynamical SMBH mass measurements of 16 additional ETGs ([Barth et al., 2016](#); [Boizelle et al., 2019](#); [Nagai et al., 2019](#); [Ruffa et al., 2019b](#); [Boizelle et al., 2021](#); [Cohn et al., 2021](#); [Kabasares et al., 2022](#); [Nguyen et al., 2022](#); [Cohn et al., 2023, 2024](#); [Dominiak et al., 2024](#); [Kabasares et al., 2024](#); [Nguyen et al., 2026b](#)) and three barred spiral galaxies ([Onishi et al., 2015](#); [Nguyen et al., 2020, 2021](#)).

Similarly to masers, cold molecular gas often moves on nearly circular orbits, making its kinematics at any given radius R largely independent of the large-scale mass distribution beyond that radius. Molecular gas is also dynamically colder and less vulnerable to non-gravitational forces than ionised gas (e.g. [Boizelle et al., 2021](#); [Smith et al., 2021](#)). Molecular gas is therefore an excellent tracer of the gravitational potential near a SMBH. Although ETGs have historically been believed to contain little or no molecular gas due to their inactive star formation, recent observations have revealed significant amounts of cold molecular gas in the central regions of local ETGs (see e.g. [Ruffa & Davis 2024](#) for a review). The molecular gas method is thus also applicable to a broad range of galaxies. Furthermore, by utilising the highest angular resolutions afforded by ALMA, the method can resolve SMBH SoIs by

factors > 10 , comparable to the megamaser method, enabling a large number of SMBH mass measurements as precise and accurate as the current "gold standard" (Zhang et al., 2024).

Nevertheless, the method has not realised its full potential. The effective resolutions of most existing measurements using this method are considerably lower than the highest achievable by ALMA and only marginally resolve the SoI (Zhang et al., 2024), for three separate reasons. First, high-resolution ALMA observations are time-consuming and challenging to obtain during the initial proof-of-concept stage of the method. Second, no consensus has been reached on whether and by how much observations that do not highly resolve a SoI can bias the SMBH mass determined via molecular gas kinematics. Finally, the commonly adopted cold molecular gas tracer, $^{12}\text{CO}(2-1)$, can sometimes have a central hole in its distribution, which precludes probing the kinematics deep within the SoI, regardless of the angular resolution of the observations. Therefore, it is imperative to obtain the first molecular gas dynamical measurements of SMBH masses that highly resolve SoI, in galaxies without central molecular gas holes, to demonstrate the importance of resolution on the precision and accuracy of SMBH masses derived with this method.

1.3.1.6 Cross-checks between different methods

Because local SMBH mass measurements have been derived using a variety of methods, each having different systematics and biases, it is essential to cross-check the methods by applying more than one of them to measure the SMBH mass in a given galaxy. However, only very few galaxies have had such a cross-check. These include comparisons between (1) stellar and ionised gas measurements in IC 1459 (Cappellari et al., 2002), M81 (Devereux et al., 2003; Nguyen et al., 2026a), Centaurus A (Neumayer et al., 2007; Cappellari et al., 2009), NGC 3227 (Davies et al., 2006; Hicks & Malkan, 2008), NGC 3379 (Shapiro et al., 2006), NGC 3998 (de Francesco et al., 2006; Walsh et al., 2012), NGC 4151 (Onken et al., 2007; Hicks & Malkan, 2008) and NGC 4335 (Verdoes Kleijn et al., 2002); (2) stars, ionised gas, and direct imaging in M87 (e.g. Gebhardt & Thomas, 2009; Walsh et al., 2013; Event Horizon Telescope Collaboration et al., 2019c; Jeter et al., 2019; Jeter & Broderick, 2021; Liepold et al., 2023; Simon et al., 2024); (3) stars, masers and ionised gas in NGC 4258 (Herrnstein et al., 2005; Pastorini et al., 2007; Siopis et al., 2009; Drehmer et al., 2015; Nguyen et al.,

2026c); (4) ionised gas and molecular gas in NGC 4261 (Ferrarese et al., 1996; Boizelle et al., 2021; Ruffa et al., 2023) and NGC 7052 (van der Marel & van den Bosch, 1998; Smith et al., 2021); and (5) stars and molecular gas in NGC 404 (Nguyen et al., 2017; Davis et al., 2020), NGC 524 (Krajnović et al., 2009; Smith et al., 2019), NGC 1332 (Rusli et al., 2011; Barth et al., 2016), NGC 4697 (Schulze & Gebhardt, 2011; Davis et al., 2017), NGC 4751 (Rusli et al., 2013; Dominiak et al., 2025) and NGC 6861 (Rusli et al., 2013; Kabasares et al., 2022). This list is based on the lists of Liang et al. (2024) and Dominiak et al. (2025), updated to March 2026.

Among SMBH mass measurement methods, the ionised gas method disagrees most often with other methods, mainly due to the prominent non-gravitational motions of ionised gas. In contrast, molecular gas kinematics generally agrees with other methods, except in NGC 1332.

1.3.2 Beyond the local Universe

Traditional dynamical methods to determine SMBH masses are generally impractical beyond $z \approx 0.1$, as they are unable to resolve the SoI of more distant galaxies. Until very recently, all SMBH masses beyond the local Universe had been determined from indirect methods: reverberation mapping (RM) and single-epoch estimators.

1.3.2.1 Reverberation mapping

In general, RM refers to the method of mapping the structure of the broad-line region (BLR) of an AGN by measuring the time lag Δt between variations of continuum emission from the AGN accretion disc and the corresponding variations of line emission from the BLR surrounding the accretion disc. This time lag is the time it takes for light from the accretion disc to reach the BLR. Thus, the radius of the BLR is given by

$$R_{\text{BLR}} = c\Delta t . \quad (1.7)$$

This approach allows one to study the structure of the BLR without spatially resolving it. Furthermore, the rotation velocity of BLR clouds, ΔV , can be measured from the width of the emission line. Combining position and velocity information, one can then compute the

SMBH mass using

$$M_{\text{BH}} = f \frac{R_{\text{BLR}} \Delta V^2}{G}, \quad (1.8)$$

where f is the virial factor (also called the "form factor"), that depends on the (typically unknown) three-dimensional (3D) geometry of the BLR.

RM probes emission much closer to the SMBH than the previously introduced dynamical methods, so it can safely ignore the dynamical contributions of other galaxy components. However, the lack of spatially resolved information makes this method highly sensitive to the uncertain value of f . One can calibrate the average virial factor $\langle f \rangle$ so that the population of RM-inferred SMBH masses matches the local $M_{\text{BH}} - \sigma$ relation, but the calibrated factors range from 2.8 (Graham et al., 2011) to 5.5 (Onken et al., 2004). In addition, the intrinsic scatter of f of individual targets around $\langle f \rangle$ is another factor of 2 (Shen et al., 2024). One can also directly measure f by constructing a full 3D model of the BLR (e.g. Bentz et al., 2010; Pancoast et al., 2014), given good velocity resolution, sufficient long-term monitoring and a high signal-to-noise ratio (S/N). When f is determined this way, RM can also be considered a direct method of measuring SMBH masses (e.g. Bentz, 2016; Cackett et al., 2021). Nevertheless, the resulting SMBH masses are still highly dependent on the assumed models of the BLR geometry, kinematics and responsivity (e.g. Grier et al., 2017; Li & Wang, 2025).

Despite its calibration uncertainty, RM has successfully produced a large number of SMBH mass determinations from the local Universe to $z \approx 4.5$ (e.g. Blandford & McKee, 1982; Cao et al., 2022; Shen et al., 2024).

1.3.2.2 Single-epoch estimators

At redshifts $z \gtrsim 4$, even RM becomes challenging due to fainter emission lines and cosmological time dilation. Instead, the BLR radius is estimated from the luminosity L of emission lines (e.g. $\text{H}\alpha$, $\text{H}\beta$ and C IV) using empirical radius – luminosity ($R - L$) relations (e.g. Koratkar & Gaskell, 1991; Bentz et al., 2013), which are calibrated via RM. An $R - L$ relation enables a SMBH mass estimate from a single observation of an AGN, efficiently providing a large number of SMBH masses at high redshifts (e.g. Wu & Shen, 2022; Pacucci et al., 2023). Such estimates are thus referred to as "single-epoch" estimates. The intrinsic scatter

of single-epoch masses around RM masses is another factor of ≈ 3 (Shen et al., 2024). SMBH masses inferred with this method are therefore order-of-magnitude estimates.

Both RM and single-epoch estimates apply only to AGN. At high redshifts, they were limited to the brightest quasars (Kormendy & Ho, 2013) until the recent launch of *JWST* (e.g. Harikane et al., 2023; Juodžbalis et al., 2026), but selection effects still limit our ability to interpret the population of early SMBHs (e.g. Silverman et al., 2025; Li et al., 2025). Moreover, both methods rely on the assumption that broad emission lines originate from rotation around a SMBH. However, alternative explanations, such as resonant scattering (Naidu et al., 2025) and electron scattering (Rusakov et al., 2026), have also been shown to agree with at least a subset of high-redshift AGN, leading to debates about the reliability of SMBH mass estimates in these systems.

1.3.2.3 Dynamical methods

Over the past two years, the first dynamical SMBH mass measurements beyond the local Universe have been performed using various methods.

The first method is spatially resolving the BLR of an AGN using GRAVITY+ infrared interferometry (Gravity+ Collaboration et al., 2022). The milliarcsecond resolution of the instrument (and its precursor GRAVITY) enables to probe the innermost dust continuum structure and ionised gas kinematics within AGN (e.g. GRAVITY Collaboration et al., 2024b), providing direct SMBH mass measurements (e.g. GRAVITY Collaboration et al., 2021, 2024a). Two measurements have been performed beyond the local Universe so far, one at $z \approx 2$ (Abuter et al., 2024), the other at $z \approx 4$ (Gravity+ Collaboration et al., 2026). Such observations also improve the calibrations of single-epoch masses (e.g. Li & Wang, 2025). However, the main challenge for this method, similarly to the traditional ionised gas method, is the prominent outflows powered by the central AGN (Gravity+ Collaboration et al., 2026). Additionally, the method is currently biased towards the brightest quasars.

Another approach to measure high-redshift SMBH masses with ionised gas kinematics is high- S/N *JWST* observations combined with spectroastrometry. Spectroastrometry is the technique of measuring positions more precisely than the nominal angular resolution given a high S/N and an accurate model of the point spread function (e.g. Beckers, 1982; Gnerucci

et al., 2011). Using this technique, Juodžbalis et al. (2025) measured the SMBH in a strongly gravitationally lensed LRD at $z = 7.04$ using *JWST* NIRSpec IFS observations of its $H\alpha$ kinematics. However, this LRD is ideal for such a measurement, with an extreme SMBH-to-stellar mass ratio ($M_{\text{BH}}/M_* > 2$), lens magnification and a kinematic major axis that aligns with the direction of the IFS slices. It is still unclear whether this method can be applied to a large sample of galaxies (private communication).

Finally, strong gravitational lensing also enabled the dynamical mass measurement of an inactive black hole of a $z = 1.95$ quiescent galaxy using stellar dynamics (Newman et al., 2025). Due to the high lens magnification factor of $\mu = 29$, the measurement robustly resolved the SoI by a factor of 1.8. The only caveat is that the measurement was performed assuming a fixed lens model, so the contribution of the lens model uncertainties to the uncertainty of the SMBH mass was not quantified. Newman et al. (2025) stated that the systematic uncertainty of the total magnification is $\approx 40\%$, which would propagate to an $\approx 20\%$ systematic uncertainty of the SMBH mass.

In summary, dynamical measurements of SMBH masses beyond the local Universe are scarce and mostly limited to luminous quasars or the most highly magnified strongly lensed galaxies. It is thus essential to develop a method that applies to diverse systems across a wide range of redshifts. Such a method could provide the first precise constraint on the evolution of SMBH – galaxy scaling relations across cosmic time and shed light on the origin and growth of the first SMBHs.

1.4 Structure of this thesis

This thesis presents my research to address two previously discussed issues with dynamical SMBH mass measurements. First, the lack of high-resolution molecular gas dynamical SMBH mass measurements in the local Universe. Second, the lack of direct dynamical SMBH mass measurements beyond the local Universe. Chapter 2 introduces three figures of merit for dynamical SMBH mass measurements using rotationally supported tracers, and applies them to compare all existing SMBH mass measurements using molecular gas or megamasers, demonstrating the importance of high-resolution measurements and the unexplored potential of the molecular gas method. Chapters 3 and 4 present two molecular gas dynamical

measurements of SMBH masses in the nearby lenticular galaxies NGC 383 and NGC 1574, respectively, using high-resolution ALMA observations. Chapter 5 shifts the focus to high redshifts by discussing the prospects of using the magnification effect of strong gravitational lensing to perform the first high-redshift dynamical SMBH mass measurements using cold gas kinematics, followed by a presentation of candidate targets for such measurements. Chapter 6 tackles a challenge for such measurements – accurately modelling the stellar mass distribution of dusty high-redshift galaxies – by developing a neural network-based method that efficiently deprojects one-dimensional (1D) parametric mass surface density profiles into axisymmetric 3D mass volume density distributions. Chapter 7 presents a dynamical upper limit on the SMBH mass of the lensed sub-millimetre galaxy ID141 at $z = 4.24$. Chapter 8 summarises the thesis and presents my ongoing and future works.

Chapter 2

Figures of merit for supermassive black hole mass measurements using molecular gas or megamaser kinematics

The material in this chapter was originally published in [Zhang et al. \(2024\)](#).

2.1 Introduction

The mass of a supermassive black hole (SMBH) can be directly measured from observations of a kinematic tracer sufficiently close to it that its contribution to the total galactic gravitational potential can be disentangled from (i.e. is comparable to) the contributions of other constituents (e.g. stars, gas, dust and/or dark matter). This is simplest when observations resolve the spatial scales on which the SMBH dominates the potential (e.g. [Boizelle et al., 2019](#); [North et al., 2019](#); [Ruffa et al., 2023](#)). However, with a reliable model of the distributions of the other mass components, the SMBH's contribution can still be discerned (with commensurately larger uncertainties) by tracing velocities higher than those expected from models with no SMBH (e.g. [Davis et al., 2018](#); [Smith et al., 2019](#)). Stars and ionised gas are common kinematic tracers and have yielded most of the published SMBH mass (M_{BH}) measurements to date. Measurements with megamasers (hereafter "masers" for short) as tracers are often thought of as the "gold standard" (e.g. [Herrnstein et al., 2005](#); [Kuo et al., 2011](#); [Gao et al., 2017](#); [Zhao et al., 2018](#); [Kuo et al., 2020](#)), as very long baseline interferometry (VLBI) allows

to resolve very small angular scales (and thus physical scales) close to the SMBHs. However, suitable masers are rare and are detected almost exclusively in galaxies with Seyfert 2 active galactic nuclei (AGN), hosting SMBHs of relatively low masses ($10^6 \lesssim M_{\text{BH}}/M_{\odot} \lesssim 10^8$; e.g. [Braatz et al. 1996](#); [Greenhill et al. 2003](#); [van den Bosch 2016](#)).

More recently, advances in millimetre interferometry have enabled **the resolution of** molecular gas on the spatial scales over which central SMBHs dominate the galactic potentials. Following an initial proof-of-concept measurement in NGC 4526 ([Davis et al., 2013c](#)), additional measurements have been performed by the mm-Wave Interferometric Survey of Dark Object Masses (WISDOM) team in ten typical early-type galaxies (ETGs; [Onishi et al. 2017](#); [Davis et al. 2017, 2018](#); [Smith et al. 2019](#); [North et al. 2019](#); [Smith et al. 2021](#); [Ruffa et al. 2023](#); [Zhang et al. 2025a,b](#); [Dominiak et al. 2025](#)), a dwarf ETG ([Davis et al., 2020](#)), and a peculiar luminous infrared galaxy (LIRG) with a Seyfert nucleus ([Lelli et al., 2022](#)). Using similar techniques, other groups have presented molecular gas SMBH mass measurements of twelve additional ETGs ([Barth et al., 2016](#); [Boizelle et al., 2019](#); [Nagai et al., 2019](#); [Ruffa et al., 2019c](#); [Boizelle et al., 2021](#); [Cohn et al., 2021](#); [Kabasares et al., 2022](#); [Nguyen et al., 2022](#); [Cohn et al., 2023](#); [Dominiak et al., 2024](#)) and three late-type galaxies (LTGs), all barred spirals ([Onishi et al., 2015](#); [Nguyen et al., 2020, 2021](#)). Although the molecular gas technique is well suited to a range of galaxies, it is more challenging to apply to LTGs because the shallower and occasionally non-axisymmetric potentials make non-circular gas motions more significant (e.g. [Combes et al., 2019](#)).

VLBI observations of masers for SMBH mass measurements resolve much smaller physical scales than molecular gas observations. However, the absolute scales do not necessarily offer a meaningful comparison, as the scale required to perform a SMBH mass measurement, the radius of the SMBH’s sphere of influence (SOI), is proportional to the SMBH mass. [Kormendy & Ho \(2013\)](#) showed that, among all SMBH measurements published by then, that of NGC 4258 by [Herrnstein et al. \(2005\)](#) using masers had the most resolution elements across the SOI and was thus the most precise extragalactic SMBH mass measurement. [North et al. \(2019\)](#) however demonstrated that, considering instead the number of spatially-resolved Schwarzschild radii, their molecular gas observations of NGC 383 resolved spatial scales comparable to those of the maser measurements of the Megamaser Cosmology Project (e.g.

Kuo et al., 2011; Gao et al., 2017).

In this chapter, we consider three separate metrics that quantify the physical scales probed by kinematic tracers in circular motions: the radius of the innermost detected dynamical tracer divided by respectively the SMBH's Schwarzschild radius R_{Schw} , SOI radius R_{SOI} (estimated using the effective stellar velocity dispersion) and equality radius R_{eq} (the radius at which the SMBH mass equals the enclosed stellar mass). We then adopt the metrics to compare the resolved spatial scales of maser and molecular gas dynamical SMBH mass measurements. In Section 2.2, we introduce the metrics by rewriting the circular velocities $V_c(R)$ of the Keplerian circular velocity curves using different M_{BH} -independent forms. In Section 2.3, we estimate and collect the parameters of all existing SMBH mass measurements to date that use molecular gas and/or maser observations. We also compare their minimum resolved spatial scales using the metrics introduced and discuss the results and their implications. We conclude in Section 2.4.

2.2 Keplerian circular velocity curves in different units

For kinematic tracers dominated by rotation (so primarily molecular gas and masers rather than ionised gas or stars), the key factor for the precision of a SMBH mass measurement (assuming a sufficient signal-to-noise ratio has been achieved) is how well the observations resolve the innermost part of the rotation curve, where the motion is predominantly Keplerian and gravitationally-dominated by the SMBH (Davis, 2014). In other words, a precise measurement requires observations with a spatial resolution sufficiently high to probe the kinematics very close to the SMBH and deep into its potential well. VLBI maser measurements are thus considered the gold standard of SMBH measurements, as they resolve spatial scales much smaller than those reached by other methods (see e.g. Table 2.1). However, as argued above, an absolute scale does not necessarily offer a meaningful metric, because the scale required to perform a SMBH mass measurement (the SOI) itself depends on M_{BH} . A better spatial resolution will only lead to a more precise M_{BH} measurement at a fixed mass. It is thus more meaningful to compare the spatial scales probed by different kinematic tracers using a set of units that eliminate the M_{BH} dependence. In this section, we thus rewrite the circular velocity $V_c(R)$ of Keplerian rotation using three different pairs of such units and introduce

three corresponding metrics to compare SMBH mass measurements that use different tracers.

2.2.1 Natural units (c and R_{Schw})

The velocity of any kinematic tracer in a circular motion due to gravity is such that the gravitational acceleration equals the centrifugal acceleration:

$$\frac{GM(R)}{R^2} = \frac{V_c^2}{R}, \quad (2.1)$$

where G is the gravitational constant, $M(R)$ the mass enclosed within a sphere of radius R and V_c the circular velocity at R . The enclosed mass $M(R)$ includes contributions from all the mass components (SMBH, stars, gas, dust, dark matter, etc). However, for the rest of the discussion, we assume that $M(R)$ in the central region of each galaxy is dominated by the SMBH and the stars, as the gas fraction and the dark matter fraction are usually low in the nuclear regions of local galaxies (e.g. [Cappellari et al., 2013](#); [Zhu et al., 2024](#)). We also note that for a non-spherically symmetric mass distribution, mass outside of the radius R also contributes to the kinematics. Those contributions however tend to cancel out, and the net effect is negligible compared to the high circular velocities in the nuclear region. Multiplying both sides of equation (2.1) by the Schwarzschild radius

$$R_{\text{Schw}} \equiv \frac{2GM_{\text{BH}}}{c^2} \quad (2.2)$$

([Schwarzschild, 1916](#)), where c is the speed of light, yields

$$\left(\frac{V_c}{c}\right)^2 = \frac{1}{2} \left(\frac{R}{R_{\text{Schw}}}\right)^{-1} \frac{M(R)}{M_{\text{BH}}}. \quad (2.3)$$

For a tracer much closer to the SMBH than the SOI radius, the BH mass dominates over the stellar mass (i.e. $M(R) \approx M_{\text{BH}}$), and

$$\left(\frac{V_c}{c}\right)^2 \approx \frac{1}{2} \left(\frac{R}{R_{\text{Schw}}}\right)^{-1}. \quad (2.4)$$

In this manner, we have rewritten the Keplerian relation between V_c and R in natural units, eliminating its explicit dependence on the SMBH mass. This approach allows a fair compar-

ison of observations of different kinematic tracers, in terms of the proximity of each tracer to the SMBH in units of the Schwarzschild radius, even when the SMBH masses are different by orders of magnitude. Observations that resolve circumnuclear disc material the fewest Schwarzschild radii away from the SMBH will thus detect the highest rotational velocities and reveal the physical processes of SMBH accretion and feedback closest to the galactic nucleus.

As a caveat, the Keplerian relation in equation (2.4) is only satisfied by kinematic tracers well within the SOI, so if an observation does not highly resolve the SOI, as is the case for many molecular gas observations (shown later in Section 2.3.2.4), the velocities of the kinematic tracer are expected to deviate from equation (2.4). By contrast, most SMBH measurement observations using masers do highly resolve the SOI. The masers are thus expected to closely follow equation (2.4). This will be discussed in detail in Section 2.3.2.4.

2.2.2 Units of σ_e and R_{SOI}

Equations (2.3) and (2.4) express V_c and R in natural units, providing a metric that quantifies how well an observation spatially resolves the material and motions in the circumnuclear disc. However, this metric is not ideal for evaluating the reliability of SMBH mass measurements, as the natural units relate only to the SMBH and contain no information on how other components of the gravitational potential compare to it. For example, at a fixed radius in natural units, a kinematic tracer in a galaxy with an overly massive central bulge (compared to its SMBH) will be more affected by stars than one in a galaxy with an under-massive bulge. Everything else being equal, the latter case generally yields a more precise SMBH mass than the former. To evaluate a SMBH mass measurement in units that reflect both the SMBH and the stars, and thus the likely precision of the measurement, we normalise R by the radius of the SMBH SOI

$$R_{\text{SOI}} \equiv \frac{GM_{\text{BH}}}{\sigma_e^2} \quad (2.5)$$

(e.g. Wolfe & Burbidge, 1970), where σ_e is the stellar velocity dispersion measured within one effective (half-light) radius R_e of the galaxy, a proxy for the depth of the galaxy potential well and thus the stellar component at large spatial scales. The relation between V_c and R

then becomes

$$\left(\frac{V_c}{\sigma_e}\right)^2 = \left(\frac{R}{R_{\text{SOI}}}\right)^{-1} \frac{M(R)}{M_{\text{BH}}}. \quad (2.6)$$

Again, for $R \ll R_{\text{SOI}}$, $M(R) \approx M_{\text{BH}}$, and

$$\left(\frac{V_c}{\sigma_e}\right)^2 \approx \left(\frac{R}{R_{\text{SOI}}}\right)^{-1}. \quad (2.7)$$

It is worth noting that when R is normalised by R_{SOI} , V_c is normalised by σ_e . This is reasonable as the SOI is the region where the influence of the SMBH potential exceeds that of the stellar potential. So, for a tracer at R_{SOI} , the SMBH's contribution to the circular velocity should approximately equal the contribution from the stars, which is of the order of σ_e . Again, observations that only marginally resolve the SOI ($R \approx R_{\text{SOI}}$) have $M(R)$ considerably larger than M_{BH} , so the velocities are expected to deviate from equation (2.7).

2.2.3 Units of V_{eq} and R_{eq}

Although σ_e is measurable for most nearby galaxies and provides a convenient way to estimate the size of the SOI, it is only a proxy for the dynamics of the stars and their underlying gravitational potential. The SOI definition in equation (2.5) is thus only an approximation to the more physically meaningful definition that the SOI is the region where the SMBH potential dominates over the stellar potential, or equivalently where the SMBH mass dominates over the stellar mass. Therefore, we consider a more formal and accurate definition of the SOI radius, the equality radius R_{eq} , such that

$$M_*(R_{\text{eq}}) = M_{\text{BH}}, \quad (2.8)$$

where $M_*(R)$ is the stellar mass enclosed within the radius R . We then define V_{eq} to be the circular velocity at R_{eq} due only to the SMBH:

$$V_{\text{eq}} \equiv \sqrt{\frac{GM_{\text{BH}}}{R_{\text{eq}}}}. \quad (2.9)$$

In these units, the relation between V_c and R becomes

$$\left(\frac{V_c}{V_{\text{eq}}}\right)^2 = \left(\frac{R}{R_{\text{eq}}}\right)^{-1} \frac{M(R)}{M_{\text{BH}}} . \quad (2.10)$$

Again, for $R \ll R_{\text{eq}}$, $M(R) \approx M_{\text{BH}}$, and

$$\left(\frac{V_c}{V_{\text{eq}}}\right)^2 \approx \left(\frac{R}{R_{\text{eq}}}\right)^{-1} . \quad (2.11)$$

Unlike the definition of R_{SOI} that approximates the comparison between the SMBH and the stellar potential with the comparison between V_c and σ_e , the definition of R_{eq} comes from an explicit comparison between the SMBH mass and the stellar mass distribution. Therefore, V_{eq} and R_{eq} are better physically-motivated and more accurate units to compare the depths of the potentials probed by different observations. Having said that, detailed modelling of the stellar mass distribution near the SMBH SOI is not available (nor feasible) for every galaxy with a dynamically measured SMBH mass. This is particularly the case for maser galaxies, as their SOI are often smaller than the angular resolutions of even the best optical telescopes (e.g. *Hubble Space Telescope*, *HST*, and *James Webb Space Telescope*; see Section 2.3.2.4). Hence, V_{eq} and R_{eq} are only available for a subset of the galaxies considered in the next section, and this will be discussed in detail.

2.3 Comparing maser and molecular gas measurements

2.3.1 Methods

In this section, we compile all molecular gas and maser SMBH mass measurements in the literature and compare them using the metrics introduced in Section 2.2. Instead of re-deriving the entire rotation curve of each galaxy in the new units, we consider only the radius R_{min} and velocity V_{max} of the innermost kinematic tracer measurement in these units. This reveals the smallest spatial scale probed by each observation and indicates the quality of the mass measurement.

For maser galaxies, we thus adopt the (redshifted or blueshifted) maser spot that has the highest velocity relative to the galaxy’s dynamical centre. The uncertainties of the maser

position and velocity and those of the dynamical centre position and velocity are taken from the original publication and are added using the standard error propagation procedures for Gaussian random variables.

By contrast, molecular gas observations yield continuous emission distributions in the radius-velocity planes, commonly shown in the literature as major-axis position-velocity diagrams (PVDs). For each galaxy, we thus visually identify the major-axis position (i.e. the radius) with the highest velocity (i.e. the velocity peak) in the central region of the PVD. If the observation resolves the SOI, this velocity traces the velocity rise due to the SMBH. If the SOI is unresolved and no velocity rise is detected, this velocity represents the innermost resolved velocity within which the observed velocity rapidly falls off to zero. In this case, a BH mass measurement is still possible if the velocity is substantially higher than that expected from models of the other mass components only.

In practice, we extract the corresponding radius and velocity using WebPlotDigitizer¹ (Rohatgi, 2022). If the redshifted and blueshifted velocity peaks look almost symmetric, we measure both peaks and adopt the average $R_{\min}$ and $V_{\max}$ of the two measurements, with half of the differences as the uncertainties. If the resulting $V_{\max}$ uncertainty is smaller than the channel width of the data cube, we inflate it to the channel width.

We note that the channel width is a conservative estimate of the typical uncertainty of velocity measurements from data cubes. If one of the peaks has a substantially lower velocity than the other or is indistinguishable from noise, we adopt the other (higher velocity) peak and estimate the position uncertainty from the spatial width of that peak and the velocity uncertainty as the channel width. We also note that $R_{\min}$ is sometimes marginally smaller than the full-width at half-maximum (FWHM) of the synthesised beam of the observations, as the position of the central unresolved emission can be measured more precisely than the beam FWHM. We show five examples of our measurements of $R_{\min}$ and $V_{\max}$ for molecular gas observations in Appendix to Zhang et al. (2024). Three examples are from observations that resolve the SOI, and two are from observations that do not. The rest of the measurements are shown in the online supplementary material (PVDs reproduced with permission). Finally, we deproject the observed maximum rotation velocity $V_{\max}$ into the maximum circular velocity assuming perfect circular motion and an infinitely thin disc, $V_c = V_{\max}/\sin(i)$, adopting the

¹<https://automeris.io/WebPlotDigitizer/index.html>

inclination and inclination uncertainty listed in each SMBH mass measurement paper.

While there may be non-circular motions or warps in some molecular gas and maser discs, in most cases the errors introduced in V_c will be small ($\lesssim 30 \text{ km s}^{-1}$; see e.g. [Herrnstein et al. 2005](#); [Nguyen et al. 2020](#)) compared to the large V_c in the nuclear regions. These errors are thus unimportant for the following discussion. More complex central non-circular motions (e.g. inflows and/or outflows) associated with bright emission could of course exist in some objects, and they could mimic Keplerian rises of rotation curves due to SMBHs. The galaxy Fairall 49 may be such an example, with faint blueshifted emission near the nucleus explainable by either a Keplerian velocity rise or non-circular motions ([Lelli et al. 2022](#) argued for a molecular gas inflow). Our measurements of V_{max} and R_{min} for this object could thus be more significantly affected.

We thus acknowledge that our measurements of R_{min} and V_{max} of molecular gas observations are only rough estimates with considerable uncertainties. For example, the identification of the velocity peaks is performed visually and may be subject to human biases and uncertainties (e.g. confirmation bias and uncertainties marking the exact location of each peak by hand). Moreover, this method slightly but systematically overestimates V_{max} , as it adopts the highest detected velocity, slightly boosted by the velocity dispersion of the gas, rather than the mean of the velocity distribution of the innermost kinematic tracer. We take these effects into account in the adopted uncertainties. For example, we repeat our measurements three times and compare the results to estimate human-related uncertainties. We also ensure that the uncertainty of V_{max} is at least as large as the gas velocity dispersion. The final adopted uncertainties are thus conservative and likely much larger than the systematic uncertainties caused by any of these effects. A more reliable method to measure the R_{min} and V_{max} of molecular gas observations could be to inspect the original data cube of each galaxy and identify the “innermost detected kinematic tracer” by applying a signal-to-noise ratio threshold. However, we have confirmed using the data cubes of the WISDOM project that these potentially more accurate measurements do not offer a substantial improvement over our more simplistic approach.

2.3.2 Results

2.3.2.1 Summary of measured and adopted quantities

Table 2.1 lists all molecular gas and maser SMBH mass measurements in the literature, as well as our own measurements of $R_{\min}$ and $V_{\max}$ obtained from those. If a galaxy has multiple SMBH mass measurements from the same method, we consider only the most recent one. All uncertainties listed are 1σ uncertainties. Some inclinations have zero associated uncertainties as they were fixed during the kinematic modelling. If a maser paper does not state an inclination, we assume that the inclination was fixed at 90° , as detection of maser emission requires an almost edge-on maser disc. We added the stellar dynamical mass measurement of Sgr A* in the Milky Way (MW) as a reference point, as the motions of individual stars can be measured. The MW $R_{\min}$ and $V_{\max}$ are taken to be those of the star S2 from GRAVITY Collaboration et al. (2018; hereafter GC18).

If uncertainties are provided, the distances listed in Table 2.1 are taken from the listed references. Otherwise, we adopt the distances listed in Saglia et al. (2016) and re-scale the SMBH masses accordingly (as dynamical mass measurements scale linearly with the assumed distance). If a galaxy is not in Saglia et al. (2016), we adopt a distance from another source, retrieved from the HyperLeda² database (Makarov et al., 2014), and assume a 10% uncertainty, consistent with the approach of van den Bosch (2016).

We also compile the stellar velocity dispersions within one effective radius σ_e of all the galaxies to compute their R_{SOL} . For the MW, we adopt $\sigma_e = 105 \pm 20 \text{ km s}^{-1}$ (Gültekin et al., 2009). For other galaxies, we prioritise the σ_e referenced by the SMBH mass measurement paper. If that paper does not list σ_e , we search the following sources (in descending order of priority):

1. Sources with well spatially-resolved stellar kinematics, such as the ATLAS^{3D} project (Cappellari et al., 2013) and the MASSIVE survey (Veale et al., 2017). For MASSIVE galaxies, if uncertainties are not provided, we assume 10 km s^{-1} .
2. van den Bosch (2016), who compiled the best σ_e measurements of all galaxies with a SMBH mass measurement before June 2016.

²<http://leda.univ-lyon1.fr/>

3. Other literature sources (e.g. [Greene et al., 2010](#)) with σ_e measurements.
4. The "central stellar velocity dispersion" listed in HyperLeda ([Makarov et al., 2014](#)), defined as the mean stellar velocity dispersion within a circular aperture of 0.595 kpc radius.
5. The "velocity dispersion" listed in the Sloan Digital Sky Survey (SDSS) data release 16 (DR16; [Ahumada et al. 2020](#)), measured within a circular aperture of 1.5 arcsec radius.

We correct the stellar velocity dispersions from HyperLeda and SDSS to σ_e using $\sigma_e/\sigma = (R/R_e)^{0.08}$ (for spiral galaxies; [Falc3n-Barroso et al. 2017](#)), where R is the radius of the aperture and σ the corresponding velocity dispersion, and R_e is computed from the stellar mass model adopted by the SMBH mass measurement paper. If the galaxy does not have a stellar mass model, we adopt the r -band Petrosian half-light radius from the SDSS DR16 ([Ahumada et al., 2020](#)). Because stellar velocity dispersion scales only slowly with aperture size ($\sigma_e/\sigma = (R/R_e)^\alpha$, where $-0.06 < \alpha < 0.08$; [Falc3n-Barroso et al. 2017](#); [Zhu et al. 2024](#)), the differences between σ and σ_e are unlikely to substantially affect our results. We are unaware of any σ measurement of Fairall 49.

To measure R_{eq} , we use the stellar mass model described in each SMBH mass measurement paper. For molecular gas measurements, this usually implies multiplying the best-fitting multi-Gaussian expansion (MGE; [Emsellem et al. 1994](#); [Cappellari 2002a](#)) model of the stellar light distribution stated, constructed using the `mge_fit_sectors` procedure of [Cappellari \(2002a\)](#), with the corresponding best-fitting mass-to-light ratio (M/L). We then use the `mge_radial_mass` procedure³ in the JAMPY package⁴ ([Cappellari, 2008, 2020](#)) to convert the two-dimensional stellar mass profile into a one-dimensional radial profile. We interpolate this radial profile to identify the radius R_{eq} at which the enclosed stellar mass equals the best-fitting SMBH mass. Finally, we estimate the uncertainty of R_{eq} using Monte Carlo methods: we recompute R_{eq} using 10^4 random realisations of M_{BH} and M/L sampled from Gaussian distributions with means and standard deviations equal to the best-fitting M_{BH} and M/L and their 1σ uncertainties, respectively. We take the standard deviation of the resultant distribution of R_{eq} as the uncertainty. The R_{eq} of the MW is calculated from the MW nuclear

³From <https://pypi.org/project/mgefit/>

⁴From <https://pypi.org/project/jampy/>

star cluster model of [Feldmeier-Krause et al. \(2017\)](#).

Our procedure to measure R_{eq} is reliable when the adopted stellar mass model comes from an overall dust-free optical or near-infrared image (usually from *HST*) with a resolution comparable to or better than R_{eq} . Otherwise, the result will depend strongly on the point-spread function (PSF) and/or the dust distribution model assumed. By checking the Mikulski Archive for Space Telescopes⁵, we confirm that no image satisfies both criteria for any of the maser galaxies in this work except NGC 4258 (whose model is detailed in [Drehmer et al. 2015](#)). The main limitation is that maser galaxies typically have much smaller SOI ($R_{\text{eq}} \sim 0''.01$) than galaxies with molecular gas measurements (see Section 2.3.2.4). Additionally, the dense molecular gas required to trigger maser emission is often associated with prominent dust, so maser galaxies are typically dusty in their cores. By contrast, all of the SMBH mass measurement papers using molecular gas contain a reliable stellar mass model.

2.3.2.2 Sample galaxies in the $M_{\text{BH}} - \sigma_e$ diagram

Figure 2.1 shows the correlation of the SMBH masses and the effective stellar velocity dispersions of all galaxies with a SMBH mass measurement using maser or molecular gas kinematics (and the MW), compared to the local $M_{\text{BH}} - \sigma_e$ relation of [van den Bosch \(2016\)](#). Maser observations span a SMBH mass range ($M_{\text{BH}} \sim 10^7 M_{\odot}$) much narrower than that of molecular gas observations. The main reason is that the required maser emission originates from a specific type of nuclear activity most frequently present in Seyfert 2 AGN (see [Lo 2005](#) for a review), with a narrow range of SMBH masses. By contrast, molecular gas observations probe a wide range of galaxy and thus SMBH masses, ranging from massive ellipticals (e.g. [Cohn et al., 2021](#); [Smith et al., 2021](#); [Ruffa et al., 2023](#)) to dwarfs (e.g. [Davis et al., 2020](#); [Nguyen et al., 2022](#)), constraining SMBH – galaxy scaling relations across the entire dynamic range.

2.3.2.3 Comparing $R_{\text{min}}/R_{\text{Schw}}$

Figure 2.2 shows the correlation of the radii and the circular velocities of the innermost kinematic tracer measurements of all galaxies with a SMBH mass measurement using maser or molecular gas kinematics (and the MW), in the units of R_{Schw} and c , respectively. Because

⁵<https://mast.stsci.edu/search/ui/#/hst>

R_{Schw} quantifies the physical scales of the motions probed and of processes such as accretion and feedback in the circumnuclear discs, observations with smaller $R_{\text{min}}/R_{\text{Schw}}$ trace these processes closer to the SMBHs. Using this metric, the best molecular gas SMBH mass measurements resolve spatial scales (i.e. numbers of Schwarzschild radii) comparable to those of the best maser SMBH mass measurements. In fact, the highest-resolution (smallest $R_{\text{min}}/R_{\text{Schw}}$) molecular gas measurement is the recent WISDOM measurement of NGC 383 (Zhang et al., 2025a), that probes material with smaller $R_{\text{min}}/R_{\text{Schw}}$ ($\approx 41,300$) and higher V_c ($\approx 1040 \text{ km s}^{-1}$) than those of all previous molecular gas *and* maser measurements. This is because, as masers are present in galaxies with SMBH masses typically two orders of magnitude smaller than those of galaxies studied with molecular gas, the poorer angular resolutions of molecular gas observations (compared to those of maser observations) are compensated by the relatively more massive SMBHs probed. Consequently, the highest resolution molecular gas observations can probe physical processes in the circumnuclear discs closer to the SMBHs than maser observations. We note again, however, that this does not guarantee more reliable SMBH mass determinations, as the $R_{\text{min}}/R_{\text{Schw}}$ metric does not take into consideration the gravitational contributions of stars.

While observations with small $R_{\text{min}}/R_{\text{Schw}}$ mostly follow the Keplerian velocity profile of equation (2.4), those with large $R_{\text{min}}/R_{\text{Schw}}$ tend to deviate from this relation, because the contributions of the stars to the velocity profiles become significant when $R \sim R_{\text{SOI}}$. To quantify the typical deviations expected, we generate velocity profiles of a mock galaxy with a total stellar mass $M_* = 10^{11} M_\odot$, a SMBH mass $M_{\text{BH}} = 5 \times 10^8 M_\odot$ and different Sérsic indices n . This stellar mass is approximately the midpoint of the total stellar mass dynamic range of the galaxies probed with the molecular gas method, and the (M_*, M_{BH}) pair adopted is consistent with the $M_* - M_{\text{BH}}$ relation of van den Bosch (2016). We note that these parameters are chosen only to illustrate reasonable qualitative trends at large radii, not to reproduce the data points in this region of the diagram. The mass surface density radial profile of the Sérsic model is

$$\Sigma(R) = \Sigma_e \exp \left\{ -b_n \left[\left(\frac{R}{R_e} \right)^{1/n} - 1 \right] \right\} . \quad (2.12)$$

Table 2.1: Molecular gas and maser SMBH mass measurements.

Galaxy	Distance (Mpc)	$R_{\min}$ (mas)	$V_{\max}$ (km s ⁻¹)	σ_e (km s ⁻¹)	Inclination (deg)	$\log(M_{\text{BH}}/M_{\odot})$	R_{SOI} (mas)	R_{eq} (mas)	Reference
Molecular gas measurements									
Fairall 49	86.7 ± 6.1	110 ± 30	208 ± 15	...	58 ± 3	8.20 ± 0.28	...	280 ± 90	Lelli et al. (2022)
NGC 315	70 ± 7.0	70 ± 30	532 ± 10	341 ± 7	74.2 ± 0.1	9.32 ± 0.05	230 ± 30	740 ± 70	Boizelle et al. (2021)
NGC 383	66.6 ± 9.9	44 ± 7	634 ± 20	239 ± 16	38 ± 1	9.55 ± 0.02	840 ± 120	830 ± 20	Zhang et al. (2025a)
NGC 404	3.1 ± 0.4	75 ± 35	61 ± 2	40 ± 3	37 ± 1	5.74 ± 0.09	100 ± 30	60 ± 10	Davis et al. (2020)
NGC 524	23.3 ± 2.3	480 ± 20	131 ± 10	220 ± 11	20 ± 5	8.60 ± 0.22	310 ± 160	280 ± 70	Smith et al. (2019)
NGC 997	90.4 ± 9.0	190 ± 30	162 ± 5	215 ± 10	35 ± 0	8.99 ± 0.25	210 ± 120	250 ± 60	Dominiak et al. (2024)
NGC 1097	14.5 ± 1.5	1450 ± 250	210 ± 10	195 ± 4	46 ± 5	8.15 ± 0.10	230 ± 50	530 ± 40	Onishi et al. (2015)
NGC 1275	71 ± 7.1	30 ± 10	420 ± 20	245 ± 28	46 ± 9	9.04 ± 0.19	230 ± 110	530 ± 120	Nagai et al. (2019)
NGC 1332	22.3 ± 1.9	210 ± 50	500 ± 20	331 ± 15	84 ± 1	8.82 ± 0.04	240 ± 30	230 ± 10	Barth et al. (2016)
NGC 1380	17.1 ± 1.7	150 ± 30	270 ± 10	215 ± 8	76.9 ± 0.1	8.17 ± 0.15	170 ± 60	230 ± 50	Kabasares et al. (2022)
NGC 1574	19.3 ± 1.9	60 ± 20	120 ± 10	216 ± 16	27 ± 2	8.00 ± 0.08	100 ± 20	190 ± 30	Ruffa et al. (2023)
NGC 1684	62.8 ± 2.3	420 ± 90	334 ± 10	262 ± 10	66.5 ± 0.4	9.16 ± 0.08	300 ± 60	350 ± 40	Dominiak et al. (2024)
NGC 3258	31.9 ± 3.9	90 ± 30	486 ± 10	260 ± 10	49.0 ± 0.1	9.35 ± 0.04	920 ± 100	900 ± 40	Boizelle et al. (2019)
NGC 3504	32.4 ± 2.1	75 ± 35	148 ± 10	119 ± 10	53 ± 1	7.19 ± 0.05	30 ± 6	58 ± 3	Nguyen et al. (2020)
NGC 3557	43.3 ± 4.3	140 ± 10	225 ± 22	282 ± 16	56 ± 1	8.85 ± 0.02	180 ± 20	620 ± 10	Ruffa et al. (2019c)
NGC 3593	7 ± 2	290 ± 50	124 ± 10	55 ± 7	75.0 ± 0.1	6.38 ± 0.08	100 ± 30	90 ± 10	Nguyen et al. (2022)
NGC 3665	34.7 ± 6.8	250 ± 50	220 ± 10	219 ± 10	69.9 ± 0.2	8.76 ± 0.03	310 ± 40	540 ± 20	Onishi et al. (2017)
NGC 4261	31.1 ± 3.1	65 ± 40	582 ± 15	263 ± 12	60.8 ± 0	9.21 ± 0.01	670 ± 60	1140 ± 10	Ruffa et al. (2023)
NGC 4429	16.5 ± 1.6	550 ± 100	170 ± 10	178 ± 8	66.8 ± 0.1	8.18 ± 0.09	260 ± 60	270 ± 20	Davis et al. (2018)
NGC 4526	16.4 ± 1.8	350 ± 30	288 ± 10	209 ± 10	79 ± 0	8.65 ± 0.14	550 ± 180	540 ± 100	Davis et al. (2013c)
NGC 4697	11.4 ± 1.1	210 ± 30	230 ± 10	169 ± 8	76 ± 1	8.10 ± 0.02	340 ± 30	410 ± 10	Davis et al. (2017)
NGC 4751	26.9 ± 2.9	210 ± 70	687 ± 30	355 ± 14	78.6 ± 0.2	9.45 ± 0.04	740 ± 90	460 ± 40	Dominiak et al. (2025)
NGC 4786	62.1 ± 9.3	170 ± 30	263 ± 20	264 ± 10	69.3 ± 0.7	8.70 ± 0.14	100 ± 30	220 ± 30	Kabasares et al. (2024)
NGC 5193	45.7 ± 3.2	170 ± 20	234 ± 10	194 ± 7	60.7 ± 0.1	8.15 ± 0.18	70 ± 30	90 ± 20	Kabasares et al. (2024)
NGC 6861	27.3 ± 4.5	1390 ± 250	468 ± 20	389 ± 3	72.7 ± 0.1	9.30 ± 0.20	430 ± 200	500 ± 140	Kabasares et al. (2022)
NGC 7052	69.3 ± 5.0	220 ± 20	382 ± 30	266 ± 13	75 ± 1	9.40 ± 0.02	450 ± 50	580 ± 10	Smith et al. (2021)
NGC 7469	68.4 ± 18.8	160 ± 40	182 ± 10	152 ± 16	53 ± 1	7.20 ± 0.97	9 ± 8	80 ± 70	Nguyen et al. (2021)
PGC 11179	89 ± 8.9	210 ± 10	391 ± 20	266 ± 9	60.0 ± 0.1	9.28 ± 0.09	270 ± 60	140 ± 20	Cohn et al. (2023)
UGC 2698	91 ± 9.1	120 ± 30	481 ± 20	304 ± 6	68 ± 1	9.39 ± 0.14	260 ± 80	170 ± 30	Cohn et al. (2021)

Galaxy	Distance (Mpc)	$R_{\min}$ (mas)	$V_{\max}$ (km s ⁻¹)	σ_e (km s ⁻¹)	Inclination (deg)	$\log(M_{\text{BH}}/M_{\odot})$	R_{SOL} (mas)	R_{eq} (mas)	Reference
Maser measurements									
CGCG 074-064	87.6 ± 7.6	0.28 ± 0.01	902 ± 3	113 ± 3	91 ± 1	7.38 ± 0.04	24 ± 3	...	Pesce et al. (2020a)
Circinus	2.8 ± 0.5	5.3 ± 0.1	260 ± 2	158 ± 18	90 ± 0	6.06 ± 0.08	15 ± 4	...	Greenhill et al. (2003)
ESO 558-G009	107.6 ± 5.9	0.44 ± 0.09	575 ± 14	170 ± 20	98 ± 1	7.23 ± 0.03	5 ± 1	...	Gao et al. (2017)
IC 1481	78.7 ± 5.5	7.3 ± 2.7	170 ± 6	95 ± 26	90 ± 0	7.11 ± 0.11	16 ± 10	...	Mamyoda et al. (2009)
IC 2560	31 ± 13	1.14 ± 0.17	325 ± 20	141 ± 10	90 ± 0	6.62 ± 0.06	6 ± 1	...	Yamauchi et al. (2012)
IRAS 08452-0011	213 ± 15	0.30 ± 0.13	832 ± 4	137 ± 11	85.2 ± 0.3	7.52 ± 0.03	7 ± 1	...	Kuo et al. (2020)
J0437+2456	65.3 ± 3.6	0.13 ± 0.08	442 ± 11	110 ± 13	81 ± 1	6.46 ± 0.05	3 ± 1	...	Gao et al. (2017)
J1346+5228	121 ± 12	0.22 ± 0.04	788 ± 8	139 ± 4	90 ± 0	7.24 ± 0.06	7 ± 1	...	Zhao et al. (2018)
Mrk 1	61.5 ± 2.3	1.48 ± 0.21	203 ± 4	86 ± 4	90 ± 2	6.51 ± 0.07	6 ± 1	...	Kuo et al. (2020)
Mrk 1029	120.8 ± 6.6	0.39 ± 0.05	163 ± 26	132 ± 15	79 ± 2	6.28 ± 0.12	0.8 ± 0.3	...	Gao et al. (2017)
NGC 1068	15.9 ± 9.4	8.0 ± 0.2	330 ± 10	151 ± 7	90 ± 0	6.95 ± 0.02	22 ± 2	...	Lodato & Bertin (2003)
NGC 1194	58.0 ± 6.3	2.17 ± 0.30	685 ± 15	148 ± 24	85 ± 0	7.85 ± 0.02	50 ± 20	...	Kuo et al. (2011)
NGC 1320	34.2 ± 1.9	0.38 ± 0.07	402 ± 11	141 ± 16	90 ± 0	6.74 ± 0.21	7 ± 4	...	Gao et al. (2017)
NGC 2273	29.5 ± 1.9	0.28 ± 0.05	958 ± 15	145 ± 17	84 ± 0	6.93 ± 0.02	12 ± 3	...	Kuo et al. (2011)
NGC 2960	67.1 ± 7.1	0.42 ± 0.06	581 ± 15	151 ± 7	89 ± 0	7.03 ± 0.02	6.2 ± 0.6	...	Kuo et al. (2011)
NGC 3079	15.9 ± 1.2	4.8 ± 0.4	145 ± 25	145 ± 7	84 ± 0	6.36 ± 0.09	6 ± 1	...	Yamauchi et al. (2004)
NGC 3393	49.2 ± 8.2	1.7 ± 0.4	575 ± 5	148 ± 10	90 ± 0	7.20 ± 0.32	13 ± 10	...	Kondratko et al. (2008)
NGC 4258	7.3 ± 0.5	4.5 ± 0.2	1001 ± 7	115 ± 11	81 ± 0	7.58 ± 0.01	350 ± 70	244 ± 5	Hernstein et al. (2005)
NGC 4388	16.5 ± 1.6	2.5 ± 0.2	405 ± 3	107 ± 7	90 ± 0	6.86 ± 0.01	34 ± 5	...	Kuo et al. (2011)
NGC 4945	3.7 ± 0.4	9.2 ± 3.8	149 ± 4	135 ± 6	90 ± 0	6.14 ± 0.18	18 ± 8	...	Greenhill et al. (1997)
NGC 5495	95.7 ± 5.3	0.37 ± 0.15	533 ± 67	166 ± 19	95 ± 1	7.04 ± 0.08	4 ± 1	...	Gao et al. (2017)
NGC 5765B	112.2 ± 5.3	0.56 ± 0.03	760 ± 3	158 ± 18	72.4 ± 0.5	7.62 ± 0.02	13 ± 3	...	Pesce et al. (2020b)
NGC 6264	132.1 ± 19.3	0.39 ± 0.01	729 ± 1	159 ± 15	91 ± 2	7.44 ± 0.06	7 ± 2	...	Pesce et al. (2020b)
NGC 6323	109.4 ± 28.8	0.27 ± 0.02	540 ± 2	158 ± 26	91.5 ± 0.3	7.01 ± 0.11	3 ± 1	...	Pesce et al. (2020b)
UGC 3789	51.5 ± 4.3	0.35 ± 0.01	847 ± 1	107 ± 12	84.9 ± 0.6	7.08 ± 0.03	18 ± 4	...	Pesce et al. (2020b)
UGC 6093	152 ± 15	0.29 ± 0.05	816 ± 13	155 ± 18	94 ± 4	7.41 ± 0.02	6 ± 2	...	Zhao et al. (2018)
Sgr A* (S2)	0.008	1400 R_{Schw}	7650	105 ± 20	...	6.61 ± 0.01	1.6 pc	3.1 pc	GC18

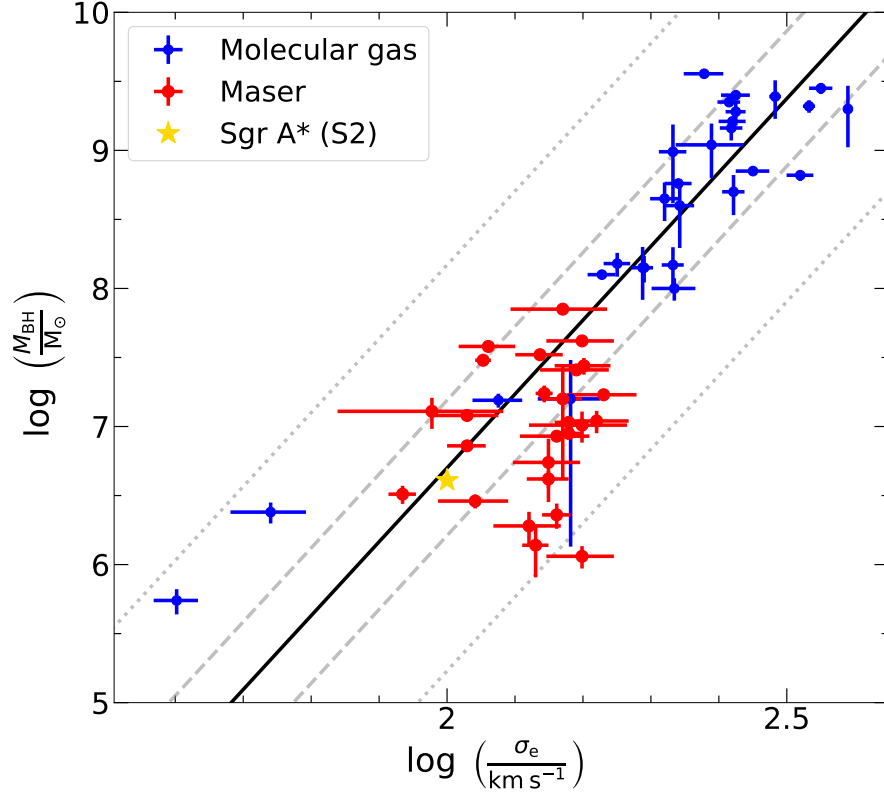


Figure 2.1: Correlation of the SMBH masses and the effective stellar velocity dispersions of all galaxies with a SMBH mass measurement using maser or molecular gas kinematics (and the MW). The solid black line shows the local $M_{\text{BH}} - \sigma_e$ relation of [van den Bosch \(2016\)](#), while the dashed and dotted grey lines show the 1σ and 3σ observed scatter of the relation, respectively. Maser observations (filled red circles) probe a narrow SMBH mass range ($M_{\text{BH}} \sim 10^7 M_\odot$), much narrower than that of molecular gas observations (filled blue circles).

We estimate the half-light radius R_e of each model using the relation of [Shen et al. \(2003\)](#):

$$\frac{R_e}{\text{kpc}} = \begin{cases} 0.10 \left(\frac{M_*}{M_\odot} \right)^{0.14} \left(1 + \frac{M_*}{3.98 \times 10^{10} M_\odot} \right)^{0.25}, & \text{if } n < 2.5 \\ 2.88 \times 10^{-6} \left(\frac{M_*}{M_\odot} \right)^{0.56}, & \text{if } n > 2.5 \end{cases} \quad (2.13)$$

We approximate b_n as

$$b_n \approx 2n - \frac{1}{3} + \frac{4}{405n} + \frac{46}{25515n^2} \quad (2.14)$$

(see [Ciotti & Bertin, 1999](#)), where n is the index of the Sérsic profile. The mass surface density at the effective radius, Σ_e , is normalised by the total stellar mass using

$$M_* = \Sigma_e \left(2\pi R_e^2 \right) \frac{n e^{b_n}}{b_n^{2n}} \Gamma(2n) \quad (2.15)$$

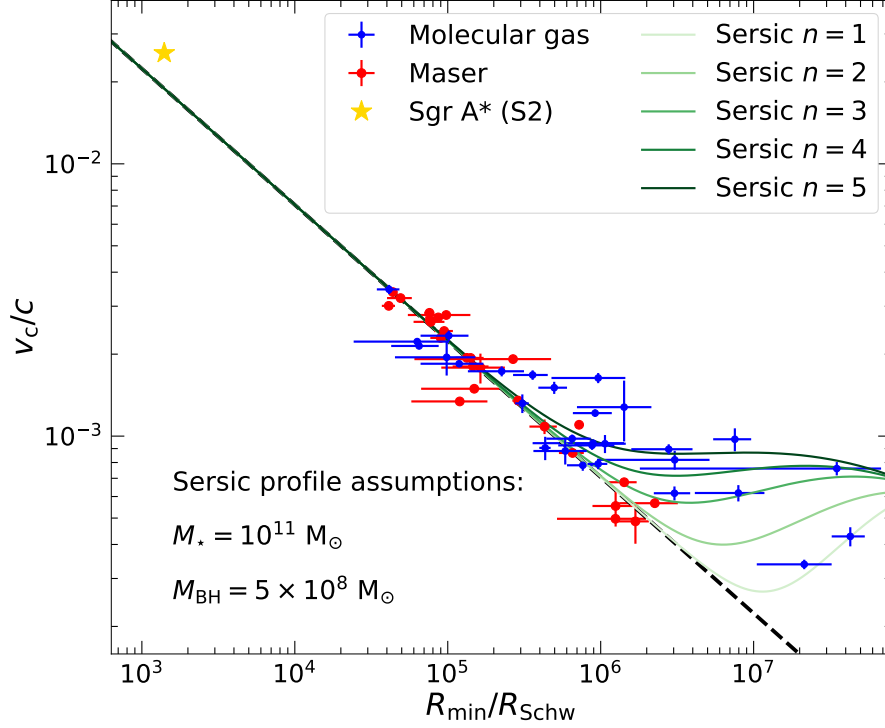


Figure 2.2: Correlation of the radii and the circular velocities of the innermost kinematic tracer measurements of all galaxies with a SMBH mass measurement using maser or molecular gas kinematics (and the MW), in units of R_{Schw} and c , respectively. The measurement of Sgr A* in the MW uses the peribothron of star S2. The dashed black line shows the Keplerian relation of equation (2.4). Observations probing fewer Schwarzschild radii from the SMBH tend to follow the Keplerian curve very well, while observations with lower resolutions (in the unit of R_{Schw}) deviate from the relation, as the contributions of stars become significant. The expected velocity profiles for a range of Sérsic indices n are shown for comparison as solid curves in different shades of green, for a galaxy with total stellar mass $M_* = 10^{11} M_\odot$ and SMBH mass $M_{\text{BH}} = 5 \times 10^8 M_\odot$. The best molecular gas SMBH mass measurements resolve spatial scales (i.e. numbers of Schwarzschild radii) comparable to those of the best maser SMBH mass measurements. In fact, the recent high-resolution WISDOM measurement of NGC 383 (leftmost blue data point; Zhang et al. 2025a) probes material with smaller $R_{\text{min}}/R_{\text{Schw}}$ and higher rotational velocities than those of all previous molecular gas *and* maser measurements.

where Γ is the Gamma function. To compute the corresponding three-dimensional mass volume density profiles, we first fit each Sérsic profile with a set of Gaussians (i.e. we perform a MGE fit) using the `mge_fit_1d` procedure of Cappellari (2002a). We then use the `mge_vcirc` procedure of the JAMPY package (Cappellari, 2008, 2020) to deproject the Gaussians, add a SMBH at the centre and compute the resulting circular velocity curve. The resulting curves are shown as solid curves with different shades of green in Figure 2.2 (with analogous curves in Figures 2.3 and 2.4). The data points at large $R_{\text{min}}/R_{\text{Schw}}$ roughly follow the coloured curves, demonstrating the robustness of our measurements of the innermost tracers.

2.3.2.4 Comparing $R_{\min}/R_{\text{SOI}}$

As argued above, detecting material that is fewer Schwarzschild radii away from the SMBH does not in itself guarantee a better SMBH mass measurement, as the measurement precision is more directly related to the factor by which the observations spatially resolve the SOI. Analogously to Figure 2.2, Figure 2.3 thus shows the correlation of the radii and the circular velocities of the innermost kinematic tracer measurements of all galaxies with a SMBH mass measurement using maser or molecular gas kinematics (and the MW), but this time in the units of R_{SOI} and σ_e , respectively.

To compute the σ_e of each Sérsic profile, we adopt the `jam_sph_proj` procedure of the JAMPY package to calculate a prediction of the second stellar velocity moment ($V_{*,\text{RMS}}$) from the MGE of the Sérsic profile. Then, we compute σ_e^2 as the luminosity-weighted sum of $V_{*,\text{RMS}}^2$ within R_e . The resulting σ_e are 110, 111, 122, 126 and 129 km s^{-1} for $n = 1, 2, 3, 4$ and 5, respectively.

Again, almost all observations that resolve the SOI ($R_{\min}/R_{\text{SOI}} < 1$) follow the Keplerian circular velocity curve, while observations with innermost tracers farther from the SMBHs than R_{SOI} are more substantially affected by the galaxies' stellar contents. SMBH mass measurements are nevertheless possible for such observations, if the stellar mass models are sufficiently accurate. In those cases the best-fitting SMBH masses usually have large uncertainties, as the measurements are sensitive to uncertainties in the stellar mass models (and unrealistically small uncertainties would almost certainly imply underestimated systematic uncertainties). So far, all SMBH mass measurements using observations with $R > R_{\text{SOI}}$ are measurements with the molecular gas method, as some of the targeted galaxies were revealed to have central holes in their circumnuclear CO(2-1) or CO(3-2) emission (e.g. Davis et al., 2018; Smith et al., 2019, 2021). These holes may imply that the inner edges of the molecular gas discs do not reach the SOI or that the molecular gas within the SOI only emits at higher CO transitions and/or only exists as higher-density molecular tracers. In these cases, the radii of the innermost kinematic tracers are greater than R_{SOI} regardless of the angular resolutions of the observations.

The maser observations generally resolve the SOI better than the molecular gas observations, in agreement with the generally better SMBH mass precision they offer. Nevertheless,

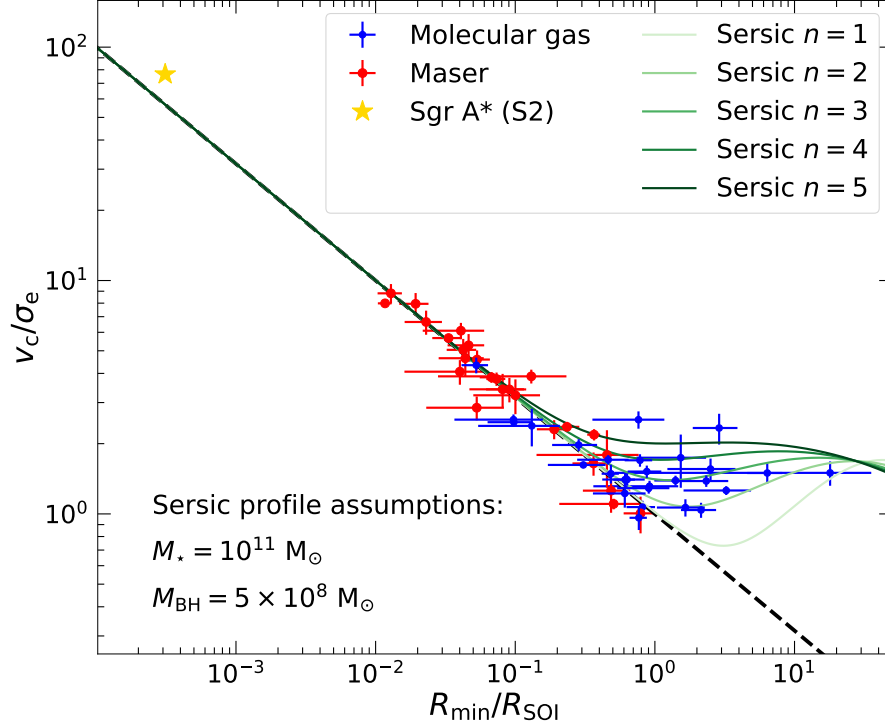


Figure 2.3: As Figure 2.2, but in the units of R_{SOI} and σ_e , respectively. The σ_e of each Sérsic profile is derived by first using the `jam_sph_proj` procedure of the `JAMPY` package to compute the second stellar velocity moment ($V_{*,\text{RMS}}$) map and then calculating the luminosity-weighted sum of $V_{*,\text{RMS}}^2$ within R_e . Maser observations generally resolve the SOI better than molecular gas observations. However, the recent high-resolution WISDOM measurement of NGC 383 (leftmost blue data point; Zhang et al. 2025a) has a resolution (in the unit of R_{SOI}) that is in the same range as that of the best maser observations.

the highest-resolution molecular gas measurement (smallest $R_{\text{min}}/R_{\text{SOI}}$), again the recent ALMA measurement of NGC 383 (Zhang et al., 2025a), has a resolution ($R_{\text{min}}/R_{\text{SOI}} = 0.05$) that is in the same range as that of the best maser observations. As ALMA’s best angular resolution at the frequency of CO(2-1) ($\approx 0''.018$) is about 40% of R_{min} of this measurement, future ALMA molecular gas observations with the most extended configurations can provide comparable $R_{\text{min}}/R_{\text{SOI}}$ and thus SMBH mass precision as the highest-resolution measurements using masers (assuming the observations can achieve sufficient signal-to-noise ratios).

Although the angular resolutions of VLBI maser observations are generally much higher than those of ALMA molecular gas observations, the angular sizes of the SOI of galaxies with maser kinematic SMBH measurements are also usually much smaller than those of galaxies with molecular gas kinematic SMBH measurements, partly because of the smaller M_{BH} of the galaxies with maser emission, and partly because maser-hosting galaxies are on average farther away due to the scarcity of masers. For these reasons, future molecular gas observations with

the longest baselines of ALMA can relatively easily achieve the same $R_{\min}/R_{\text{SOI}}$ as those of the best maser measurements so far. By contrast, megamaser observations have already exploited the highest angular resolution of current facilities (e.g. ≈ 0.35 mas at the frequency of water masers for the Very Long Baseline Array; [Napier 1995](#)). It is thus more challenging to improve the precision of SMBH measurements using masers.

2.3.2.5 Comparing $R_{\min}/R_{\text{eq}}$

Because $R_{\text{SOI}} \equiv GM_{\text{BH}}/\sigma_{\text{e}}^2$ is only an approximation to the actual SOI radius, we repeat our comparison with R_{SOI} replaced by the equality radius R_{eq} , the formal definition of the SOI radius. Figure 2.4 thus shows the correlation of the radii and the circular velocities of the innermost kinematic tracer measurements of all galaxies with a SMBH mass measurement using maser or molecular gas kinematics (and the MW), this time in the units of R_{eq} and V_{eq} , respectively. Although only one maser galaxy (NGC 4258) with a SMBH mass determination has an R_{eq} measurement, this galaxy also has the smallest $R_{\min}/R_{\text{Schw}}$ and $R_{\min}/R_{\text{SOI}}$ ratios of all maser galaxies, so the maser data point in Figure 2.4 represents the best maser observations. These observations still have more resolution elements across the SOI than the molecular gas observations (using $R_{\min}/R_{\text{eq}}$ as the metric), consistent with the results using the $R_{\min}/R_{\text{SOI}}$ metric. However, again, ALMA observations with the most extended configurations can relatively easily produce SMBH measurements with a $R_{\min}/R_{\text{eq}}$ and a precision comparable to the highest-resolution measurements using masers.

2.3.3 Other considerations

In practice, achieving a precise SMBH mass using molecular gas observations requires not only high spatial resolution but also a precise determination of the disc inclination, as the SMBH mass depends strongly on the inclination through the deprojection of the velocities: $M_{\text{BH}} \propto \sin^{-2} i$. Due to this $\sin^{-2} i$ dependence, any inclination uncertainty contributes more to the M_{BH} uncertainty budget for smaller (i.e. more face-on) inclinations. As maser discs can only be observed nearly edge-on, inclination uncertainties impact SMBH mass uncertainties less for the maser method than for the molecular gas method.

Another advantage of the maser method is that it sometimes enables an independent

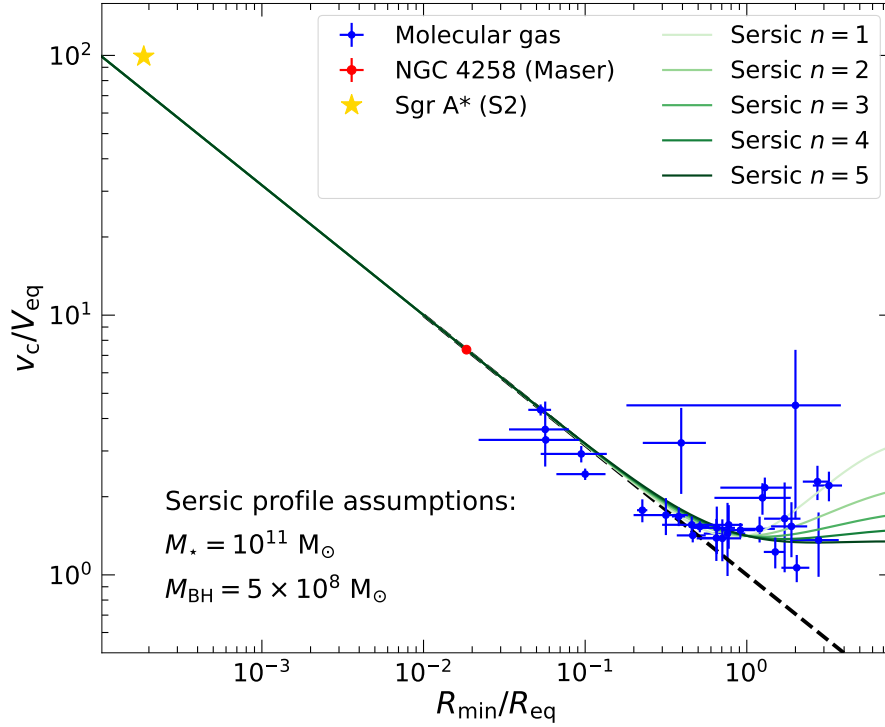


Figure 2.4: As Figure 2.2, but in the units of R_{eq} and V_{eq} , respectively. The only maser galaxy for which we can compute R_{eq} is NGC 4258, which has the largest angular SOI size and the highest resolution SMBH measurement (using either $R_{\text{min}}/R_{\text{Schw}}$ or $R_{\text{min}}/R_{\text{SOI}}$ as the metric) of all maser galaxies. So far, molecular gas observations have fewer resolution elements across R_{eq} than the maser observations of NGC 4258, consistent with the results using R_{SOI} as the metric.

galaxy distance determination (e.g. Kuo et al., 2013; Pesce et al., 2020a). As M_{BH} scales linearly with distance, the distance uncertainty often dominates the M_{BH} uncertainty budget, even though it is customary not to include nor quote the distance uncertainty in other M_{BH} measurements (as rescaling the SMBH mass to a different distance is straightforward and does not require redoing any fit). An independent distance determination using masers thus improves the true SMBH mass precision (including the distance uncertainty), if the distance obtained is more precise than other distance estimates (obtained using e.g. the Tully-Fisher relation; Tully & Fisher 1977).

On the other hand, a clear advantage of the molecular gas method is that it yields truly three-dimensional data and thus a full two-dimensional velocity map of each circumnuclear disc, while the maser method yields effectively one-dimensional kinematics, i.e. the velocities of a few maser spots along the kinematic major axis of the disc only. With many more data points sampling all azimuths, the molecular gas method can provide a reliable SMBH mass even when the data have relatively lower signal-to-noise ratios. Moreover, two-dimensional

velocity information allows to properly constrain non-circular motions (e.g. [Lelli et al., 2022](#)) and the internal structure of the disc (e.g. warps; [Nguyen et al. 2020](#); [Ruffa et al. 2023](#)), that can substantially affect the SMBH mass measurement. Modelling these effects is much more difficult using maser data.

Another advantage of the molecular gas method is that molecular gas observations help constrain the mass distribution of the molecular gas disc itself. An accurate molecular gas mass profile allows to disentangle the dynamical effects of the SMBH from those of the self-gravity of the molecular gas disc, thus reducing the systematic uncertainty of the SMBH mass measurement. This is especially crucial when the total molecular gas mass within $R_{\min}$ is a considerable fraction of the SMBH and/or stellar mass. Measurements with masers, by contrast, require many assumptions to model the disc self-gravity indirectly (e.g. [Lodato & Bertin, 2003](#)).

2.4 Conclusions

The mass of a SMBH can be measured using spectroscopic observations of kinematic tracers at sufficiently small radii, such that the mass enclosed within the tracer’s orbit is dominated by the SMBH. Such tracers then follow a Keplerian circular velocity curve, $V_c \propto R^{-1/2}$, that can be written in M_{BH} -independent forms (equations [2.4](#), [2.7](#) and [2.11](#)), affording a fair comparison between SMBH mass measurements that probe different SMBH masses and angular scales. In this chapter, we have compared SMBH mass measurements using molecular gas and maser observations, adopting as the metric the radii of their innermost kinematic tracers divided by R_{Schw} , R_{SOI} and R_{eq} , respectively. We have thus shown that the best molecular gas observations resolve material fewer Schwarzschild radii away from the SMBHs than the best maser observations, so molecular gas observations can probe motions and physical processes closer to the SMBHs than the maser method. Conversely, the best maser observations typically resolve the SMBHs’ SOI better than the best molecular gas observations, whether the SOI is defined using the effective stellar velocity dispersion σ_e or the equality radius R_{eq} . Already, molecular gas observations using the most extended configurations of ALMA can spatially resolve the SOI comparably well, leading to SMBH masses as precise as the most precise SMBH masses derived using masers.

If we accept the claim that masers offer "gold standard" measurements of SMBH masses for $M_{\text{BH}} \sim 10^7 M_{\odot}$ (as they resolve the smallest physical scales around the SMBHs), we should consider the molecular gas method capable of producing equally precise measurements across a much wider SMBH mass range. Most of these precise measurements will be towards the high-mass end, where the physical size of the SOI is the largest. Nevertheless, the method can also achieve high precision for a closer, less massive SMBH if a regular and dynamically cold molecular gas disc is present at the centre of the galaxy. Galaxies with lower-mass SMBHs are more likely to have turbulent gas kinematics due to the smaller V_c and stronger impact of stellar feedback, but regular and circularly rotating CO discs suitable for SMBH mass measurements are also present in some (e.g. [Davis et al., 2020](#)). SMBH measurements with molecular gas are thus highly complementary to existing high-precision measurements with masers.

In practice, achieving high angular and thus spatial resolutions requires long baselines and, in turn, long integration times (≈ 5 hr with ALMA). Such observations are not necessarily immediately feasible for a large sample of galaxies. However, the potential for the molecular gas method to steadily increase the number of "gold standard" mass measurements across the entire SMBH mass range is clear. Moreover, it remains to be seen whether the two methods yield consistent masses for the same galaxies, as no galaxy with an existing SMBH mass measurement from maser observations so far meets the selection criteria for a SMBH measurement using molecular gas ([Liang et al., 2024](#)).

Chapter 3

A 5% precision supermassive black hole mass measurement in the galaxy NGC 383 using molecular gas kinematics

The material in this chapter was originally published in [Zhang et al. \(2025a\)](#).

3.1 Introduction

Although the molecular gas kinematics method has produced > 30 measurements of supermassive black hole (SMBH) masses (e.g., [Davis et al., 2013b](#); [Boizelle et al., 2021](#)), none of them have achieved comparable precision and accuracy to the "gold standard" measurements using megamaser kinematics (e.g., [Herrnstein et al., 2005](#); [Pesce et al., 2020a](#)), as these measurements typically used observations that only marginally resolved the sphere of influence (SoI) of the SMBH. This chapter presents a measurement of the SMBH mass of the early-type galaxy NGC 383 using ultra-high resolution ($\approx 0''.034$) Atacama Large Millimeter/submillimeter Array (ALMA) observations of the $^{12}\text{CO}(2-1)$ emission line. A previous mm-Wave Interferometric Survey of Dark Object Masses (WISDOM) study ([North et al., 2019](#)) inferred a SMBH mass of $(4.2 \pm 0.4) \times 10^9 M_{\odot}$ using $\approx 0''.13$ ALMA observations of the same line. According to [Zhang et al. \(2024\)](#), the physical scale probed by this prior measurement is only ≈ 3 times worse than that of the best SMBH measurement using masers ([Herrnstein et al., 2005](#)), when evaluated in unit of the SMBH Schwarzschild

radius ($R_{\text{Sch}} \equiv 2GM_{\text{BH}}/c^2$, where G is the gravitational constant and c is the speed of light). With ≈ 4 times better angular resolution, this new study aims to achieve the highest spatial resolution measurement (in the unit of R_{Sch}) of all SMBH mass measurements to date, spatially resolving the central molecular gas disc with unprecedented detail to derive a much more precise SMBH mass. Section 3.2 introduces the target and presents our new ALMA observations and data reduction. We describe our molecular gas dynamical modelling technique to measure the SMBH mass in Section 3.3 and discuss the sources of uncertainties and the importance of high-resolution observations in Section 3.4. We conclude briefly in Section 3.5.

3.2 ALMA Observations

3.2.1 Target: NGC 383

NGC 383 is an unbarred dusty lenticular galaxy (de Vaucouleurs et al., 1991) and the brightest galaxy of the NGC 383 group of galaxies, consisting of 11 members (Sakai et al., 1994). It is also the host galaxy of the radio source 3C 031, an AGN with relatively low-power radio jets showing a characteristic Fanaroff-Riley type I morphology (Fanaroff & Riley, 1974) on kpc scales, with optical spectral properties typical of low-excitation radio galaxies (LERGs; see e.g. MacDonald et al. 1968; Bridle & Perley 1984; Laing & Bridle 2002; van Velzen et al. 2012). The combination of these characteristics makes 3C 031 the prototype of this class of AGN (see also Ruffa et al., 2020). We adopt a distance $D = 66.6 \pm 9.9$ Mpc derived by Freedman et al. (2001) using the Tully-Fisher relation (Tully & Fisher, 1977). At this distance, an angle of $1''$ corresponds to a spatial extent of ≈ 323 pc.

The SMBH of NGC 383 has a large SoI. Adopting $M_{\text{BH}} = (4.2 \pm 0.4) \times 10^9 M_{\odot}$ from North et al. (2019) and an effective (half-light) stellar velocity dispersion $\sigma_e = 239 \pm 16 \text{ km s}^{-1}$ from van den Bosch (2016), we expect the radius of the SoI to be $R_{\text{SoI}} \equiv GM_{\text{BH}}/\sigma_e^2 = 320$ pc or $\approx 1''.0$. Our $0''.050 \times 0''.024$ (geometric average $\approx 0''.034$) angular resolution ALMA observations presented in the next sub-section spatially resolve this SoI by a factor of ≈ 29 in radius (≈ 24 for our updated M_{BH} in Section 3.3.5), and should thus enable a precise SMBH mass determination.

The total molecular gas mass of NGC 383 is $(1.49 \pm 0.19) \times 10^9 M_{\odot}$ (Ocaña Flaquer et al., 2010), derived from single-dish observations of the $^{12}\text{CO}(1-0)$ and the $^{12}\text{CO}(2-1)$ lines with the Institut de Radioastronomie Millimétrique (IRAM) 30-m telescope. North et al. (2019) however showed that the molecular gas mass enclosed in the nuclear region is negligible compared to the enclosed SMBH and stellar masses.

3.2.2 Observations and data reduction

We observed the $^{12}\text{CO}(2-1)$ line of NGC 383 with ALMA using configuration C43-10 from 2021 September 5 to 2021 September 9, as part of project 2019.1.00582.S (PI: M. Bureau). The observations consist of three tracks with a total on-source integration time of 3.9 h. The shortest and the longest baselines are 122 and 16,196 m, respectively, providing a maximum recoverable scale (MRS) of $0''.5$ (0.16 kpc). Because the MRS is smaller than the expected R_{SoI} , we combine those data with two prior intermediate-resolution observing tracks to improve the uv -plane coverage and fully recover the SoI. One track (project 2015.1.00419.S, PI: T. Davis) was observed on 2016 June 21, using ALMA configuration C36-4 (baselines of 15 to 704 m). The other track (project 2016.1.00437.S, PI: T. Davis) was observed on 2017 August 16, using ALMA configuration C40-7 (baselines of 21 to 3637 m). These two lower-resolution tracks were used in North et al. (2019) to derive the previous SMBH mass. The MRS of the combined observing tracks is $\approx 2''.7$ (≈ 0.87 kpc), approximately the size of the molecular gas disc as estimated by North et al. (2019). The details of all observing tracks used in this study are summarised in Table 3.1.

Each of the five (three new and two previous) observing tracks has four spectral windows (SPWs). A high-resolution SPW with a bandwidth of 1.875 GHz ($\approx 2500 \text{ km s}^{-1}$) was centred on the redshifted $^{12}\text{CO}(2-1)$ line frequency (≈ 227 GHz) in ALMA band 6. The new observations used 1920 channels of 976.6 kHz ($\approx 1.3 \text{ km s}^{-1}$), whereas the old observations used 3840 channels of 488.3 kHz ($\approx 0.65 \text{ km s}^{-1}$). Three additional SPWs, each of 2 GHz with 128 channels of 15.63 MHz ($\approx 20.3 \text{ km s}^{-1}$), were employed to detect continuum emission. The properties of the spectral windows of all adopted observing tracks are listed in Table 3.2. For all observing tracks, a standard calibration strategy was adopted, using a single bright quasar as both flux and bandpass calibrator and another one (or two) as phase

calibrator. The flux and bandpass calibrator of the new high-resolution tracks presented here was J2253+1608. The phase calibrators were J0057+3021 and J0112+3208.

All the new high-resolution observing tracks were calibrated with the COMMON ASTRONOMY SOFTWARE APPLICATION (CASA) package¹ (McMullin et al., 2007), version 6.2.1.7, using the standard pipeline provided by the ALMA Science Archive². While carefully inspecting the calibrated visibilities of the three tracks using the CASA task `plotms`, prominent residual noise spikes were identified in some edge channels of the three continuum SPWs, which were flagged manually using the CASA task `flagdata`. The diagnostic weblog generated by the ALMA pipeline for the same tracks also revealed that the weather conditions at the ALMA site were very poor during the observations. Combined with the very extended configuration of the array, this resulted in substantial residual phase noise (inducing significant amplitude decorrelation) after the standard calibration. A continuum image of each new track was thus created, revealing that only the first track was severely affected by the adverse weather conditions (showing substantial signal decorrelation and image artefacts). We therefore excluded this track from the rest of the analysis and combined the remaining calibrated (two new and two previous) tracks from all three ALMA projects using the CASA task `concat`.

The analysis reported in the following was carried out using this combined dataset only, which was imaged using CASA version 6.5.2. All the data products were created with a Högbom deconvolver (Högbom, 1974), a pixel size of 0''.01 and an image size of 960×960 pixels, to reduce file sizes while sampling the synthesised beams properly.

3.2.3 Continuum emission

The continuum SPWs and the line-free channels of the line SPWs of the combined dataset were used to produce the continuum image. Various experiments with data reduction revealed that self-calibration³ could improve the quality of the data products. We thus performed two runs of phase-only self-calibration using the CASA task `gaincal`, progressively decreasing the solution interval from 5 to 1 min, leading to an increase of the dynamic range (i.e. the ratio

¹<https://casa.nrao.edu/>

²<https://almascience.eso.org/aq/>

³https://casaguides.nrao.edu/index.php/First_Look_at_Self_Calibration_CASA_6.5.4

between the peak flux and the root-mean-square noise) of the continuum image by a factor of about 4 (from ≈ 1000 to ≈ 4000). The final continuum image was created using the `tclean` task in multi-frequency synthesis mode (Rau & Cornwell, 2011), with no spectral dependence, and Briggs weighting with a robust parameter of 2 (equivalent to natural weighting). The latter choice was made to maximise the sensitivity and possibly image emission from the base of the radio jets, that are known to dominate the continuum spectrum of LERGs like NGC 383 down to mm wavelengths (see e.g. Ruffa et al., 2019a, 2020). Indeed, the galaxy’s radio-sub-mm spectral energy distribution was analysed by North et al. (2019), who reported a best-fitting power-law index $\alpha = -0.66 \pm 0.03$, consistent with synchrotron radiation from AGN radio jets (see e.g. Laing & Bridle, 2002; Ruffa et al., 2020).

The continuum image obtained has a root-mean-square (RMS) noise of $18 \mu\text{Jy beam}^{-1}$ and a synthesised beam full-width at half-maximum (FWHM) of $0''.080 \times 0''.051$ ($\approx 26 \times 16 \text{ pc}^2$) at a position angle (PA) of $14^\circ.3$. As shown in Figure 3.1, the continuum image reveals only one source near the galaxy’s kinematic centre (best-fitting SMBH position; see Section 3.3.5), with an integrated flux density of $71.0 \pm 0.6 \text{ mJy}$, consistent (within the usual $\approx 10\%$ ALMA flux calibration uncertainty) with that reported by North et al. (2019). The deconvolved size of the source, derived by fitting a two-dimensional (2D) Gaussian with the CASA task `imfit`, is consistent with it being spatially unresolved (i.e. a point source). The source remains unresolved if we image the continuum with a different robust parameter (e.g. 0.5).

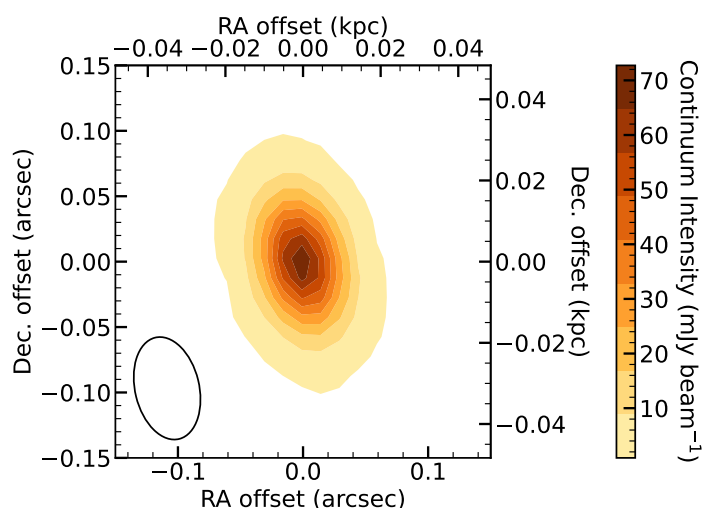


Figure 3.1: Central region ($0''.3 \times 0''.3$) of the NGC 383 1.3 mm continuum image, showing the only (point) source detected. Contour levels are equally spaced between the peak intensity of $72.7 \pm 0.6 \text{ mJy beam}^{-1}$ and 50 times the RMS noise. The synthesised beam ($0''.080 \times 0''.051$) is shown in the bottom-left corner as a black open ellipse.

Table 3.1: Properties of ALMA observing tracks.

Project code	Track	Date	Config.	Baseline range	ToS (s)	MRS (arcsec, kpc)	Calibration
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
2015.1.00419.S	uid_A002_Xb499c3_X377d	2016-06-21	C36-4	15 m – 0.7 km	121	7.9, 2.6	Pipeline, CASA 4.5.3
2016.1.00437.S	uid_A002_Xc36f2a_X46c	2017-08-16	C40-7	21 m – 3.6 km	1669	1.7, 0.5	Pipeline, CASA 4.7.2
2019.1.00582.S	uid_A002_Xf031e1_X16fe	2021-09-05	C43-10	122 m – 16.2 km	4940	0.5, 0.2	Excluded manually
	uid_A002_Xf031e1_X1dfe	2021-09-05	C43-10	122 m – 16.2 km	4757	0.5, 0.2	Pipeline & manual phase-only
	uid_A002_Xf06573_X1594	2021-09-09	C43-10	122 m – 16.2 km	4494	0.5, 0.2	self-calibration, CASA 6.2.1.7
							Pipeline & manual phase-only
							self-calibration, CASA 6.2.1.7

Notes. Columns: (1) Project code. (2) Track ID. (3) Observation date (year-month-day). (4) ALMA array. (5) Minimum and maximum baseline length. (6) Total on-source time (ToS). (7) Maximum recoverable scale (MRS), i.e. the largest angular scale that can be recovered with the given array. (8) Calibration method.

Table 3.2: Properties of spectral windows of adopted observing tracks.

Project code	Track	SPW	Bandwidth (GHz, km s^{-1})	N_{channel}	Channel width (MHz, km s^{-1})	Central frequency (GHz)	Flagged
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
2015.1.00419.S	uid_A002_Xb499c3_X377d	0	2, 2600	128	15.63, 20.3	228.6	No
		1	2, 2600	128	15.63, 20.3	241.8	No
		2	2, 2600	128	15.63, 20.3	243.8	No
		3	1.88, 2500	3840	0.49, 0.6	226.6	No
2016.1.00437.S	uid_A002_Xc36f2a_X46c	0	2, 2600	128	15.63, 20.3	228.7	No
		1	2, 2600	128	15.63, 20.3	242.0	No
		2	2, 2600	128	15.63, 20.3	243.9	No
		3	1.88, 2500	3840	0.49, 0.6	226.8	No
2019.1.00582.S	uid_A002_Xf031e1_X1dfe	0	2, 2600	128	15.63, 20.3	228.8	Yes
		1	2, 2600	128	15.63, 20.3	240.6	Yes
		2	2, 2600	128	15.63, 20.3	242.5	Yes
		3	1.88, 2500	1920	0.98, 1.3	226.8	No
	uid_A002_Xf06573_X1594	0	2, 2600	128	15.63, 20.3	228.8	No
		1	2, 2600	128	15.63, 20.3	240.6	No
		2	2, 2600	128	15.63, 20.3	242.5	No
		3	1.88, 2500	1920	0.98, 1.3	226.8	No

Notes. Columns: (1) Project code. (2) Track ID. (3) SPW ID. (4) Total bandwidth. (5) Number of channels. (6) Channel width. (7) Central frequency. (8) Whether the SPW is flagged manually.

3.2.4 Line emission

3.2.4.1 Imaging

After applying the continuum self-calibration to the line SPWs, the CO line emission was isolated in the uv -plane using the CASA task `uvcontsub`, which forms a continuum model from linear fits to line-free channels in frequency and then subtracts this model from the visibilities. We then created the CO data cube using the `tclean` task with Briggs weighting and a robust parameter of 0.5, to maximise the angular resolution while achieving sufficient sensitivity. We adopted a channel width of 20 km s^{-1} , larger than that of [North et al. \(2019; \$10 \text{ km s}^{-1}\$ \)](#) but necessary as the combined data cube used here generally has lower signal-to-noise ratios (S/N). The channel velocities were computed in the rest frame, a velocity of zero corresponding to the rest frequency of the $^{12}\text{CO}(2-1)$ line (i.e. 230.538 GHz). The continuum-subtracted dirty cube was cleaned in regions of line emission (identified interactively) to a conservative threshold of about 1.5 times the RMS noise (σ_{RMS} ; measured from line-free channels). The final, self-calibrated and cleaned $^{12}\text{CO}(2-1)$ data cube has $\sigma_{\text{RMS}} = 0.14 \text{ mJy beam}^{-1}$ and a synthesised beam FWHM of $0''.050 \times 0''.024$ ($\approx 16 \times 8 \text{ pc}^2$) with a PA of $22^\circ.5$, well resolving the SoI. The data cube dynamic range (peak S/N) is ≈ 9 .

3.2.4.2 Moment maps

We visualise the final data cube by creating zeroth-moment (integrated-intensity) and first-moment (intensity-weighted mean line-of-sight velocity) maps using a masked moment technique (e.g. [Dame, 2011](#)). The mask is generated by taking a copy of the original data cube and smoothing that copy first spatially with a uniform filter and then spectrally with a Hanning window. Due to the small synthesised beam of our data cube, the S/N (≈ 3 per synthesised beam per 20 km s^{-1} channel) are generally not as high as those of the [North et al.’s \(2019\)](#) data cube. We thus chose the size of the filters (3 times the synthesised beam FWHM spatially and 11 channels spectrally) and the clipping threshold ($0.5 \sigma_{\text{RMS}}$ of the unsmoothed data cube or $\approx 6.5 \sigma_{\text{RMS}}$ of the smoothed cube) to optimise the trade-off between flux recovery and noise reduction. As a result, we smoothed over more pixels to suppress the noise in the moment maps and reduced the clipping threshold until most of the flux was recovered. All pixels in the smoothed cube above the threshold were then selected, and the moment maps were

created from the unsmoothed cube using only those pixels. Irrespective of this procedure, we stress that the modelling described in Section 3.3 uses the final unmasked data cube, not the moment maps.

Figure 3.2 shows the resulting moment maps. The molecular gas disc extends ≈ 2 kpc in radius along its major axis, and we recover the weak spiral structure seen in CO by North et al. (2019). The slight central dip in the zeroth-moment map is likely a physical decrease of the CO surface brightness within a radius of $\approx 0''.1$, rather than an artefact of the masking procedure as suggested by North et al. (2019), as the dip remains as the clipping threshold of the mask is decreased. Dips or holes are often present at the centres of molecular gas discs observed at high resolution (e.g. Davis et al., 2018; Izumi et al., 2020; Nguyen et al., 2021; Ruffa et al., 2023). They are potentially due to the dissociation or excitation of CO caused by AGN activity (e.g. Izumi et al., 2020) and/or the suppression of molecular cloud formation due to strong tidal forces (Sarzi et al., 2005). Nevertheless, the CO central depression of NGC 383 does not impact the SMBH mass measurement, as there is sufficient emission within the fainter region to robustly trace the kinematics near the SMBH (see Section 3.4.1.3 for a zoomed-in view of the nuclear disc). The first-moment (mean velocity) map reveals a typical dynamically cold rotating disc, ideal for a SMBH mass measurement. The enhanced velocities at the centre of the velocity map clearly indicate the presence of a SMBH. In contrast to the previous observations of North et al. (2019) suggesting an unwarped disc, our high-resolution observations reveal a slight twist of the isovelocity contours within the central $\approx 0''.3$ in radius, indicative of a PA warp and/or non-circular motions. We present models of this feature in Section 3.4.1. Other features match those recovered at lower angular resolution by North et al. (2019).

3.2.4.3 Total flux

Figure 3.3 shows the integrated $^{12}\text{CO}(2-1)$ spectrum of NGC 383, extracted from the central $6'' \times 6''$ region of our high-resolution data cube, thus covering all of the detected emission. The spectrum clearly shows the typical double-horned shape of a rotating disc, also observed by Lim et al. (2000), Okuda et al. (2005) and North et al. (2019). The total $^{12}\text{CO}(2-1)$ flux in the cleaned data cube is $128.9 \pm 1.3 \text{ Jy km s}^{-1}$. This is about 50% larger than the total

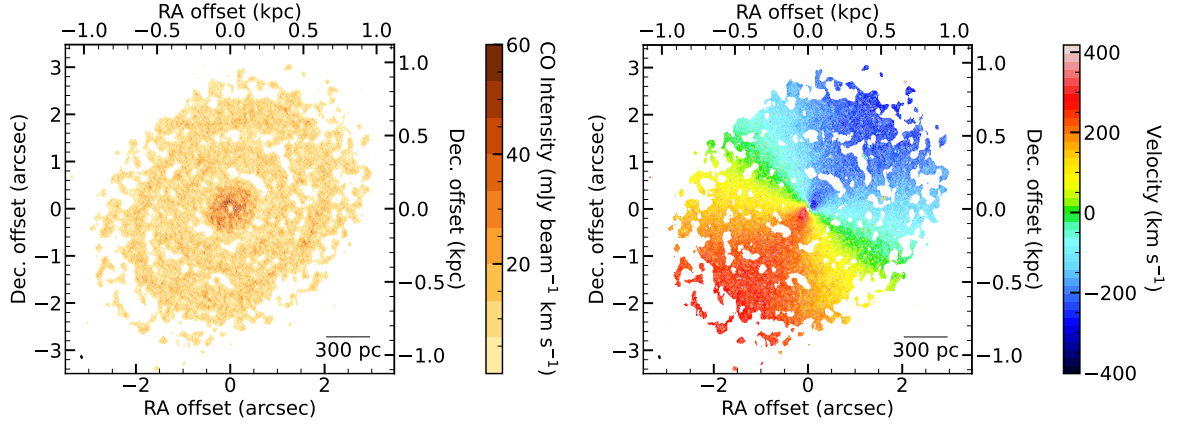


Figure 3.2: Zeroth-moment (integrated-intensity; left panel) and first-moment (intensity-weighted mean line-of-sight velocity; right panel) maps of NGC 383 created from our ALMA $^{12}\text{CO}(2-1)$ data cube. The synthesised beam ($0''.050 \times 0''.024$) is shown as a black filled ellipse in the bottom-left corner of each panel, while a 300 pc scale bar is shown in the bottom-right corner of each panel. Positions are measured relative to the best-fitting kinematic centre; velocities relative to the best-fitting systemic velocity (see Section 3.3.5).

flux of $87.1 \text{ Jy km s}^{-1}$ reported by [North et al. \(2019\)](#). The flux discrepancy becomes even larger if we perform another `tclean` run with a shallower cleaning threshold of $2 \sigma_{\text{RMS}}$, the total flux in the resultant data cube then increasing to $161.0 \pm 1.3 \text{ Jy km s}^{-1}$. This flux discrepancy is due to a known issue (e.g. [Jorsater & van Moorsel, 1995](#); [Bureau & Carignan, 2002](#)) when evaluating total fluxes of certain multi-configuration datasets. When the dirty beam has a highly irregular shape, its effective area can differ substantially from that of the (clean) Gaussian beam of equivalent angular resolution, causing the true total flux in the residual map of `tclean` (hereafter "residual flux" for short) to be lower than the measured residual flux by a factor

$$\epsilon = \frac{\text{clean beam area}}{\text{dirty beam area}} . \quad (3.1)$$

Here, ϵ is thus the correction factor required to obtain the true residual flux. The true total flux T is thus related to the total flux in the CLEAN components C and the measured residual flux R by

$$T = C + \epsilon R , \quad (3.2)$$

while the measured total flux M is

$$M = C + R . \quad (3.3)$$

For a data cube produced with a deeper CLEAN, the discrepancy between M and T is smaller, as less flux remains in the residual map. Thus M should converge asymptotically to T for a sufficiently deep cleaning threshold ($\ll 1 \sigma_{\text{RMS}}$). Such a deep CLEAN is however impractical, as it introduces numerous artefacts in the output data cube, so we instead compute T and ϵ using the two different values of C and R from our two `tclean` runs, following the prescription of [Jorsater & van Moorsel \(1995\)](#):

$$\epsilon = \frac{C_2 - C_1}{R_1 - R_2} \quad (3.4)$$

$$T = \frac{R_1 C_2 - C_1 R_2}{R_1 - R_2} . \quad (3.5)$$

Substituting $C_1 = 61.5 \text{ Jy km s}^{-1}$ and $R_1 = 67.4 \pm 1.3 \text{ Jy km s}^{-1}$ from the deeper `tclean` run, and $C_2 = 51.4 \text{ Jy km s}^{-1}$ and $R_2 = 109.5 \pm 1.3 \text{ Jy km s}^{-1}$ from the shallower `tclean` run, we obtain $\epsilon = 0.24 \pm 0.01$ and $T = 77.6 \pm 0.9 \text{ Jy km s}^{-1}$. We verified this result by creating two other data cubes cleaning down to thresholds of $1 \sigma_{\text{RMS}}$ and $0.7 \sigma_{\text{RMS}}$, respectively. The ϵ and T derived from those two additional data cubes agree well with those above.

We note that the total flux of each data cube used in this flux correction procedure is obtained by integrating over the channels of the integrated spectrum with $S/N \geq 3$. Integrating over the full velocity range of each data cube yields a consistent total flux within the statistical uncertainties. Our computed total $^{12}\text{CO}(2-1)$ flux is slightly smaller than that reported by [North et al. \(2019\)](#), but it is consistent with the single-dish $^{12}\text{CO}(2-1)$ flux of $74.4 \pm 2.8 \text{ Jy km s}^{-1}$ from [Ocaña Flaquer et al. \(2010\)](#) considering ALMA's flux calibration uncertainty ($\approx 10\%$). This implies that the fluxes reported by [North et al. \(2019\)](#) might also have been slightly overestimated, for similar reasons. In any case, our SMBH mass measurement is insensitive to this flux rescaling, so we proceed with the conservatively cleaned data cube (with a threshold of $1.5 \sigma_{\text{RMS}}$) to minimise the artefacts introduced by `tclean`.

3.2.4.4 PVD

Figure 3.4 shows the kinematic major-axis position-velocity diagram (PVD) of NGC 383, extracted along a PA of 142° (the best-fitting PA obtained in Section 3.3.5) by summing the flux within a 15-pixel wide pseudo slit. When creating PVDs, we adopt a masking procedure

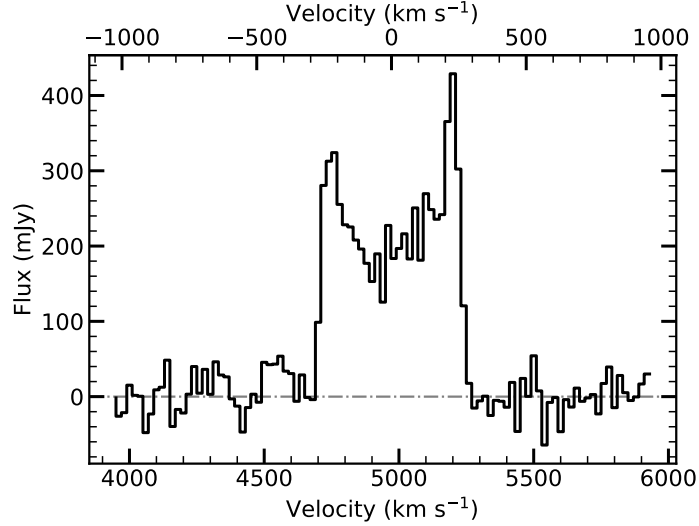


Figure 3.3: Integrated $^{12}\text{CO}(2-1)$ spectrum of NGC 383, extracted from the central $6'' \times 6''$ region of our high-resolution data cube, covering all of the detected emission. Velocities are measured relative to the best-fitting systemic velocity along the top axis (see Section 3.3.5). The dot-dashed line indicates the zero flux level. The spectrum shows the typical double-horned shape of a rotating disc.

slightly different from that used to create the moment maps, to avoid masking out the central region: we use a spatial Gaussian filter of FWHM equal to that of the synthesised beam (rather than the larger uniform filter). We then select all pixels in the smoothed cube above $0.5 \sigma_{\text{RMS}}$ of the unsmoothed data cube. Figure 3.4 shows only the central $1''.5$ on either side of the kinematic centre, revealing a sharp Keplerian rise (from the outside in) of the line-of-sight velocities within the central $\approx 0''.5$ in radius, the characteristic kinematic signature of a SMBH. As our high-resolution observations spatially resolve material ≈ 4 times closer to the SMBH than the previous observations of North et al. (2019), we detect velocities up to $\approx 635 \text{ km s}^{-1}$ in the blueshifted half of the disc, ≈ 1.8 times larger than the highest velocity previously detected ($\approx 350 \text{ km s}^{-1}$).

A Keplerian circular velocity curve $V_c(R) \propto R^{-1/2}$, where $V_c(R)$ is the circular velocity at radius R , and the four times better angular (and thus spatial) resolution, suggest we should detect a maximum velocity of $\approx 700 \text{ km s}^{-1}$ (twice the previous maximum velocity). Features up to $\approx 700 \text{ km s}^{-1}$ are indeed detected if we lower the clipping threshold of the PVD, but those features are indistinguishable from noise due to the low S/N . Hence, we do not claim detection of such high velocities.

Noticeably, our high-resolution observations reveal a mild asymmetry in the detected velocity peaks at small radii; the redshifted component of the molecular gas disc only

reaches $\approx 535 \text{ km s}^{-1}$. This asymmetry could arise from non-circular motions (e.g. gas inflows/outflows) or a deficiency of gas in the redshifted component of the nuclear disc (due to a specific gas morphology, e.g. a nuclear spiral). We discuss this further in Section 3.4.1. Another noticeable feature is that a small amount of central emission extends beyond the major-axis position of zero to the opposite side of the disc (i.e. into the "forbidden quadrants" of the PVD). This is consistent with the observed central twist of the isovelocity contours of the velocity map, again indicating a possible PA warp and/or non-circular motions. We attempt to reproduce this feature with 3D disc models in Section 3.4.1.

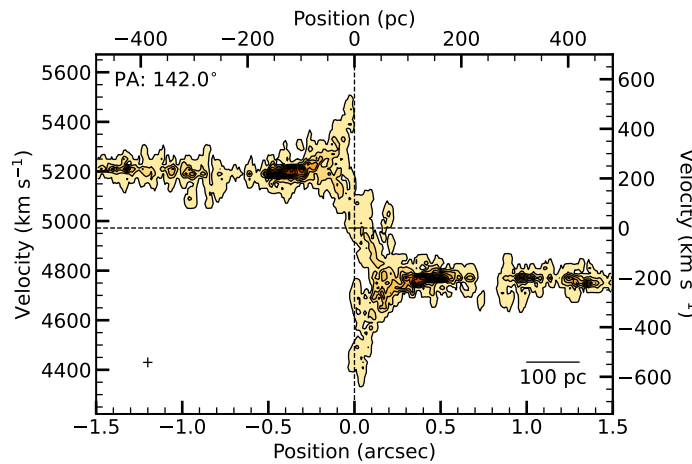


Figure 3.4: Major-axis position-velocity diagram of NGC 383, covering the central $1''.5$ on either side of the kinematic centre. The sharp increase (from the outside in) of the line-of-sight velocities within the central $\approx 0''.5$ is a clear kinematic signature of a central SMBH. Positions are measured relative to the best-fitting kinematic centre; velocities relative to the best-fitting systemic velocity along the right axis (see Section 3.3.5). The cross in the bottom-left corner shows the synthesised beam FWHM along the kinematic major axis and the channel width. Velocities up to $\approx 635 \text{ km s}^{-1}$ are detected, ≈ 1.8 times larger than the highest velocity ($\approx 350 \text{ km s}^{-1}$) probed by the earlier observations of North et al. (2019).

3.3 Dynamical Modelling

Our method to measure the SMBH mass of NGC 383 has been used and described extensively in previous WISDOM papers (e.g. Davis et al., 2017; Smith et al., 2019; North et al., 2019; Ruffa et al., 2023), but as is often the case we need to make slight adjustments to these procedures for the specific galaxy being studied. We thus summarise the method and discuss the specifics of the NGC 383 modelling in this section.

We analyse the observed gas kinematics using the publicly available PYTHON version of

the KINEMATIC MOLECULAR SIMULATION tool⁴ (KinMS; Davis et al. 2013a). This takes an input model of the gas distribution and kinematics to create a mock data cube, taking into account beam smearing, spatial and velocity binning, and line-of-sight projection effects. The simulated data cube can then be compared directly to the observed data cube to infer the best-fitting parameters of the input model as well as their uncertainties, calculated via a Markov chain Monte Carlo (MCMC) χ^2 minimisation routine.

When computing the χ^2 , we rescale the uncertainties of the data cube by a factor of $(2N)^{0.25}$, where $N = 404,722$ is the number of pixels with detected emission, defined as the pixels included in the mask of Section 3.2.4. This approach attempts to account for the potential systematic uncertainties usually dominating large data sets, leading to more realistic fit uncertainties. The idea was proposed by van den Bosch & van de Ven (2009) when using χ^2 confidence levels and was later adapted by Mitzkus et al. (2017) for Bayesian methods. We have since adopted this approach in various WISDOM papers (e.g. Smith et al., 2019; North et al., 2019). In addition, we assume a diagonal covariance matrix, ignoring the correlation between nearby pixels. Applying the full covariance matrix would be computationally intractable, and the impact would be much smaller than that of the rescaling described above (see Davis et al. 2017 for an extensive discussion of this issue).

3.3.1 Mass model

To generate a model of the gas kinematics, we assume that the gas particles are in circular motions (we explore the validity of this assumption in Section 3.4.1), with the circular velocity curve determined only by the mass distribution of the galaxy. When creating the mass model, we assume that the gravitational potential within the region of interest is dominated by the stars and the SMBH, ignoring contributions from other possible mass components (e.g. molecular gas, ionised gas, dust and dark matter). This assumption is reasonable as the molecular gas mass of NGC 383 is insignificant within the SMBH SoI (North et al., 2019), and dark matter is usually negligible in the nuclear regions of massive galaxies (e.g. Cappellari et al., 2013; Zhu et al., 2024).

We construct our model of the stellar mass distribution using an approach similar to

⁴https://github.com/TimothyADavis/KinMS_fitter
<https://kinms.space/>

Table 3.3: MGE parameterisation of the NGC 383 stellar light distribution.

I_j ($L_{\odot, F160W} \text{ pc}^{-2}$)	σ_j (arcsec)	q_j
(1)	(2)	(3)
5789.27	0.114	0.9
2359.50	0.836	0.95
6889.76	1.07	0.95
4379.08	2.34	0.95
3535.85	4.61	0.903
999.79	12.5	0.95

Notes. Parameters of the deconvolved Gaussian components. (1) Surface brightness. (2) Standard deviation (width). (3) Axial ratio.

that of [North et al. \(2019\)](#). The stellar light distribution is modelled by performing a multi-Gaussian expansion (MGE; [Emsellem et al. 1994](#)), using the Python version of the `mge_fit_sectors_regularized` procedure⁵ of [Cappellari \(2002b\)](#) on a combined *Hubble Space Telescope* (*HST*) Near Infrared Camera and Multi-Object Spectrometer F160W-filter and Two Micron All Sky Survey (2MASS) *H*-band image. We mask the *HST* image to exclude regions obscured by dust and mask the 2MASS image to remove pixels contaminated by a nearby star, using the same masks as [North et al. \(2019\)](#). The combined masked image is thus fit by a sum of 2D Gaussians that are then analytically deprojected to a three-dimensional (3D) axisymmetric light distribution (given a free inclination i). We list the parameters of each spatially-deconvolved Gaussian of the MGE model in Table 3.3.

Our MGE model yields a light profile almost identical to that used by [North et al. \(2019\)](#), except for the innermost Gaussian component. The MGE components published by [North et al. \(2019\)](#) were deconvolved by the point-spread function (PSF) of the 2MASS image, mistakenly even in regions of the *HST* image, resulting in an unrealistically narrow innermost component. By contrast, our listed MGE components have been deconvolved by the *HST* PSF calculated using the TINYTIM package⁶ ([Krist et al., 2011](#)). Nevertheless, the difference between the integrated masses of the previous and the new innermost Gaussian components is less than 20% of the SMBH mass uncertainty derived in Section 3.3.5 and less than 7% of the difference between the SMBH masses derived from the intermediate-resolution data and new high-resolution data. In the comparison of the statistical and systematic uncertainties of the two measurements presented in Section 3.4.2, this small difference is negligible.

⁵Version 5.0 from <https://pypi.org/project/mgefit/>

⁶Version 7.5 from <https://github.com/spacetelescope/tinytim/releases/tag/7.5>

The circular velocity curve corresponding to the 3D stellar light distribution is computed using the `mge_circular_velocity` procedure of the JEANS ANISOTROPIC MODELLING (JAM) package⁷ of Cappellari (2008). This procedure assumes a solar mass-to-light ratio (M/L), so we multiply the circular velocity generated with this procedure by the square root of our (free) M/L . As in North et al. (2019), we adopt a linearly varying M/L :

$$M/L(R) = M/L_{\text{inner}} + (M/L_{\text{outer}} - M/L_{\text{inner}}) \left(\frac{R}{3.5 \text{ arcsec}} \right), \quad (3.6)$$

where R is the cylindrical radius, M/L_{inner} is the M/L at $R = 0$ and M/L_{outer} is the M/L at $R = 3''.5$ (approximately the outer edge of the gas disc). The inner and the outer M/L are free parameters of our model, and the M/L is constant (at M/L_{outer}) at $R > 3''.5$.

Finally, we add a central point mass representing the SMBH and compute the circular velocity curve resulting from both the stars and the SMBH. KINMS then combines the circular velocity curve with a variable velocity dispersion to model the kinematics of the gas distribution.

3.3.2 Gas distribution

The mild spiral structure in the molecular gas disc of NGC 383 makes it inappropriate to assume a smooth axisymmetric gas distribution described by a simple parametric function (e.g. an exponential disc). Instead of attempting to construct a complicated function describing this 2D light/mass density distribution (that would necessarily have a large number of parameters), we simply adopt the observed but spatially-deconvolved molecular gas distribution as the input. Using the SKYSAMPLER tool⁸ developed by Smith et al. (2019), we thus uniformly sample the (deconvolved) CLEAN components produced by the CASA task `tclean`, and generate 4.1×10^6 gas particles that exactly replicate the observed CO surface brightness distribution when convolved by the synthesised beam. The gas particles are then passed to KINMS with the position, systemic velocity, PA and inclination of the gas disc (assumed to be equal to those of the stars) as free parameters. Our procedure is identical to that used by North et al. (2019) but uses the CLEAN components of our new combined observations

⁷Version 7.2 from <https://pypi.org/project/jampy/>

⁸<https://github.com/Mark-D-Smith/KinMS-skySampler>

rather than the old ones.

3.3.3 Nuisance parameters

Our dynamical modelling involves four nuisance parameters: the two coordinates of the kinematic centre (where the SMBH is) relative to the phase centre of the data cube, the systemic velocity of the gas disc and the integrated intensity of the CLEAN components (i.e. the scaling factor of the input gas distribution). We note that previous WISDOM papers assumed that the kinematic centre coincides with the gas disc’s morphological centre (the centre of the molecular gas emission). This assumption was usually valid as the separation between the kinematic and the morphological centres was often much smaller than the synthesised beam size. However, in our high-resolution data cube, the two positions are more than one synthesised beam apart. We thus fit the two positions separately.

As we adopt the CLEAN components as the input gas distribution model, we can directly compute the position of the morphological centre relative to the data cube’s phase centre, by calculating the intensity-weighted average position of the gas particles sampled from the CLEAN components. In this way, we confirm that this offset is less than 1% of the synthesised beam FWHM, so from now on we assume that the morphological centre coincides with the data cube’s phase centre. This leaves only the coordinates of the kinematic centre (relative to the phase centre) as free parameters, speeding up convergence.

3.3.4 Prior distributions

Although our data cube has a substantially higher angular and thus spatial resolution than the previous one from [North et al. \(2019\)](#), a MCMC fit with uninformative priors does not guarantee a more precise SMBH mass because of the lower S/N of our data cube (typically ≈ 3 per synthesised beam per 20 km s^{-1} channel). Indeed, the lower S/N worsens the constraint on the inclination, which tightly correlates with the SMBH mass (through the deprojection of the velocities, i.e. $M_{\text{BH}} \propto \sin^{-2} i$) and dominates the error budget (see Section 3.4.2.2). Hence, we place informative priors on the inclination and the M/L using constraints derived from fitting the previous intermediate-resolution data cube.

Rather than directly adopting the results of [North et al. \(2019\)](#) as priors, we re-analyse

the intermediate-resolution data cube with our modified code that does not assume an overlap of the molecular gas kinematic and morphological centres. Table 3.4 lists the best-fitting parameters and associated uncertainties resulting from fitting the intermediate-resolution data cube with our modified code. The best-fitting SMBH mass $\log(M_{\text{BH}}/M_{\odot}) = 9.59 \pm 0.05$ (1σ uncertainty here and throughout this chapter), consistent within 1σ with the SMBH mass $\log(M_{\text{BH}}/M_{\odot}) = 9.63 \pm 0.04$ of North et al. (2019) (here and throughout this chapter, when we evaluate the difference of two measurements, we adopt an uncertainty $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$, where σ_1 and σ_2 are the uncertainties of the first and the second measurement, respectively). The slight difference is statistically insignificant but could be the result of separating the kinematic and morphological centres and/or the improved MGE model. The systemic velocity of $4977 \pm 1 \text{ km s}^{-1}$ is significantly higher than that of North et al. (2019; $4925 \pm 1 \text{ km s}^{-1}$), as we have corrected a previous inaccuracy in imaging the intermediate-resolution data: when converting from frequency to velocity, the previous imaging pipeline adopted a rounded rest frequency of 230.5 GHz for the $^{12}\text{CO}(2-1)$ line, instead of the more accurate 230.538 GHz, causing a (non-physical) velocity offset of $\approx 49 \text{ km s}^{-1}$. The best-fitting systemic velocity of North et al. (2019) agrees with ours after accounting for this inaccuracy.

We then adopt the posterior distributions of the inclination and the inner and outer M/L as priors when fitting the high-resolution data cube. We use flat priors for all other parameters except the SMBH mass, for which we use a flat logarithmic prior. The ranges of the flat priors are listed in the second column of Table 3.4.

3.3.5 Results

We fit the entire high-resolution data cube using a MCMC of 10^5 steps, ensuring that the chain has converged and fully sampled the posterior distribution. The best-fitting parameters and their uncertainties are listed in the last three columns of Table 3.4. The one-dimensional marginalised posterior distribution of each non-nuisance parameter and the covariances between them are shown in Figure 3.5.

The best-fitting SMBH mass is $(3.58 \pm 0.19) \times 10^9 M_{\odot}$, in good agreement with that obtained from fitting the intermediate-resolution data cube. The SMBH mass precision has however improved from $\approx 10\%$ to $\approx 5\%$. The central panel of Figure 3.6 shows the kinematic

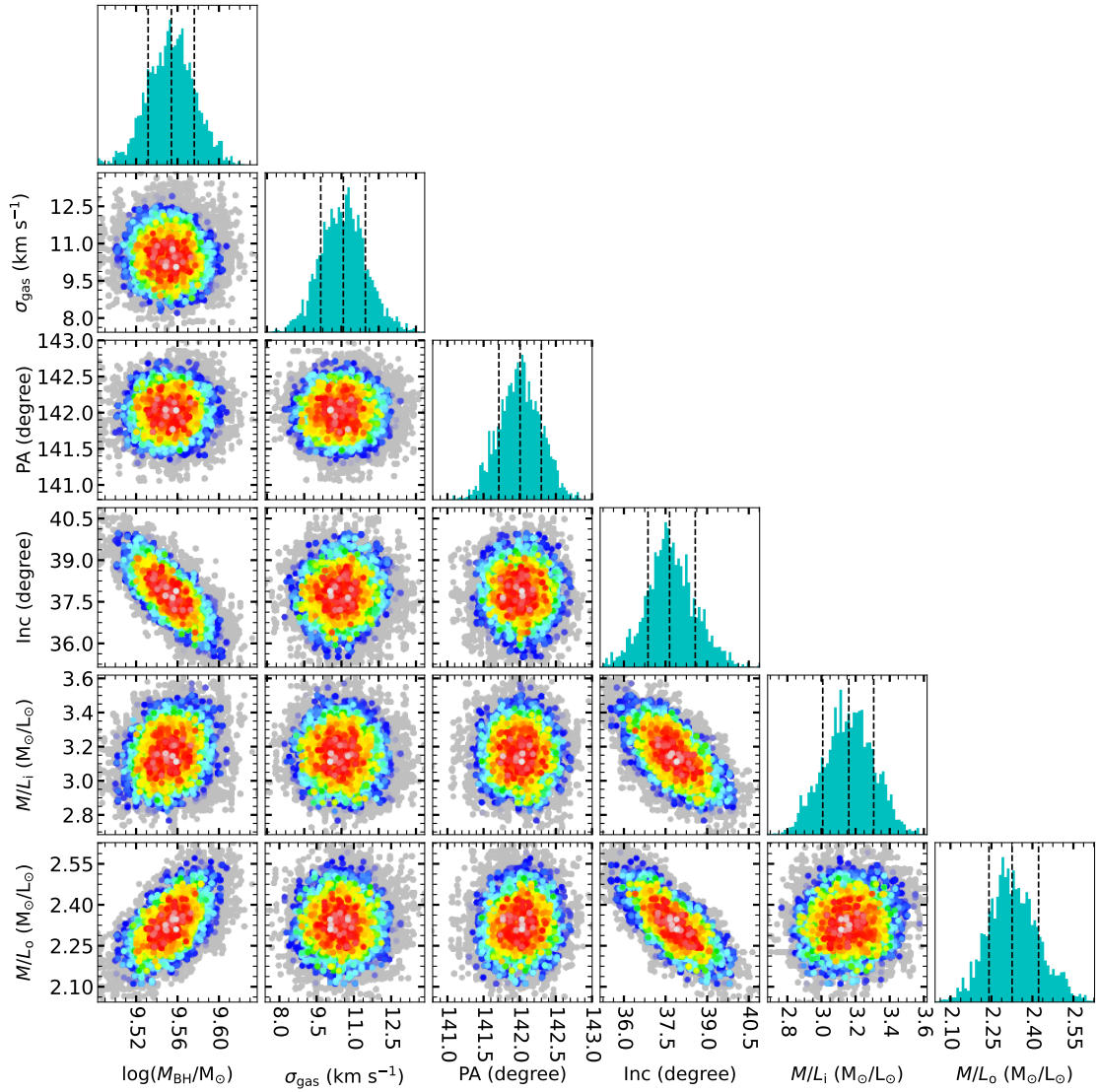


Figure 3.5: Corner plots showing the covariances between non-nuisance model parameters. The colours represent increasing confidence levels, from 68.3% (red, 1σ) to 99.7% (grey, 3σ). The histograms show the one-dimensional marginalised posterior distribution of each parameter; the dashed lines indicate the median and the 1σ confidence interval.

major-axis PVD of the best-fitting model overlaid on the observed PVD. The Keplerian rise (from the outside in) of the velocities up to $\approx 635 \text{ km s}^{-1}$ due to the SMBH is well reproduced by our model. By contrast, a model with no SMBH (left panel of Figure 3.6) fails to account for the Keplerian rise. The best-fitting model of North et al. (2019; right panel of Figure 3.6, with corrected kinematic centre and systemic velocity) reproduces the PVD nicely up to $\approx 350 \text{ km s}^{-1}$, the highest velocity detected by the intermediate-resolution observations, but it slightly overshoots the Keplerian rise beyond $\approx 350 \text{ km s}^{-1}$. Indeed, our best-fitting model has a reduced χ^2 (χ_r^2) of 1.59 for the high-resolution data cube and 1.20 for the intermediate-resolution data cube, better than the χ_r^2 of the best-fitting model of North et al. (2019), that are

1.97 for the high-resolution data cube and 1.27 for the intermediate-resolution data cube. This highlights the importance of high-resolution observations, that fully resolve the high-velocity material around the SMBH, to the precision and accuracy of SMBH mass measurements.

We note that our best-fitting model does not reproduce the extended emission in the forbidden quadrants of the PVD, as it assumes an unwarped disc and purely circular orbits. We present models of this feature and show that it does not affect the best-fitting SMBH mass in Section 3.4.1.

The stellar mass-to-light ratio of NGC 383 decreases from $3.16 \pm 0.15 \text{ M}_\odot/\text{L}_{\odot, \text{F160W}}$ at the centre of the galaxy to $2.32 \pm 0.09 \text{ M}_\odot/\text{L}_{\odot, \text{F160W}}$ at the outer edge of the molecular gas disc ($R = 3''.5$). Both the inner and the outer M/L agree with those of the intermediate-resolution fit. This stellar M/L gradient could result from a negative radial age gradient in the stellar population. This is because the luminosity L of a star scales rapidly with its mass M : $L \propto M^{1-6}$ (Kippenhahn & Weigert, 1990), causing a more massive star to have both a lower M/L and a shorter lifetime, thereby increasing the average M/L of a simple stellar population with age.

3.4 Discussion

3.4.1 Warps and/or non-circular motions

A potential source of systematic error in our model is the simplification that the molecular gas moves in circular orbits in an unwarped disc. Contrary to this assumption, the high-resolution velocity map (right panel of Figure 3.2) reveals a twist of the isovelocity contours within the central $\approx 0''.3$ in radius, suggesting a PA warp and/or non-circular motions. A distortion of the velocity field is also suggested by the forbidden quadrant emission in the kinematic major-axis PVD (Figure 3.4). We thus also created a kinematic minor-axis PVD of the data, shown in Figure 3.7. For a rotating disc with no warp nor non-circular motion, the minor-axis PVD traces the zero-velocity contour as well as some emission at non-zero velocities due to the velocity dispersion, beam smearing and the finite width of the pseudo slit, that should be perfectly symmetric in all four quadrants. By contrast, the minor-axis PVD of NGC 383 appears asymmetric, with more positive-velocity emission near $-0''.2$ and more

Table 3.4: Best-fitting parameters and associated uncertainties, from the fits to the intermediate-resolution and high-resolution data cubes.

Parameter	Search range	Intermediate resolution			High resolution		
		Best fit	1 σ uncertainty	3 σ uncertainty	Best fit	1 σ uncertainty	3 σ uncertainty
Mass model							
$\log (M_{\text{BH}}/M_{\odot})$	8.70 – 9.95	9.60	± 0.05	–0.11, +0.15	9.55	± 0.02	± 0.07
Inner stellar M/L ($M_{\odot}/L_{\odot,\text{F160W}}$)	0.01 – 10	3.04	–0.28, +0.32	–0.71, +1.1	3.16	± 0.15	–0.42, +0.41
Outer stellar M/L ($M_{\odot}/L_{\odot,\text{F160W}}$)	0.01 – 10	2.41	–0.17, +0.24	–0.39, +0.60	2.32	–0.08, +0.10	–0.25, +0.28
Molecular gas disc							
Position angle (degree)	112 – 172	142.11	–0.37, +0.34	± 1.1	142.01	–0.30, +0.29	–0.90, +0.80
Inclination (degree)	26 – 89	37.2	–2.2, +2.0	± 5.3	37.6	± 1.0	–2.3, +2.8
Velocity dispersion (km s^{-1})	0 – 25	9.3	± 1.0	–3.0, +3.4	10.6	± 0.9	–2.7, +2.8
Nuisance parameters							
Integrated intensity (Jy km s^{-1})	5 – 200	74.8	–4.8, +5.0	–13, +15	82.6	± 4.1	–12, +11
Kinematic centre X offset (arcsec)	–3.5 – 3.5	–0.12	± 0.02	± 0.05	–0.12	± 0.01	± 0.03
Kinematic centre Y offset (arcsec)	–3.5 – 3.5	0.00	± 0.02	–0.05, +0.06	0.00	± 0.01	± 0.03
Systemic velocity (km s^{-1})	4890 – 5040	4977.1	–1.3, +1.4	–4.4, +4.1	4971.9	± 1.0	–2.8, +3.0

Notes. The X and Y offsets are measured relative to the phase centre of the high-resolution data cube, $\text{RA} = 01^{\text{h}}07^{\text{m}}24^{\text{s}}96$, $\text{Dec.} = 32^{\circ}24'45''.21$ (J2000.0).

negative-velocity emissions near $+0''.2$, consistent with the shape of the isovelocity contour twist.

Despite the many features that suggest a PA warp and/or non-circular motions, the low S/N and the relatively small number of pixels in the region of the velocity twist make it challenging to model those features using the current data cube. Instead, we re-image the data with Briggs weighting and a robust parameter of 2.0 (equivalent to natural weighting), to maximize the S/N . The resulting data cube has a synthesised beam FWHM of $0''.093 \times 0''.062$ ($\approx 30 \times 20 \text{ pc}^2$) with a PA of $12^\circ.9$, sufficient to resolve the velocity twist. The twist is more prominent in the velocity map of the new data cube (see the left panels of Figure 3.8), thanks to the improved S/N (new cube dynamic range, i.e. peak S/N , of ≈ 22). We thus only use this naturally-weighted data cube for the investigations in this sub-section.

3.4.1.1 Tilted-ring models

To create a non-parametric model of a PA warp and/or non-circular motions, we first use the 3DFIT task of the ^{3D}BAROLO package⁹ (Di Teodoro & Fraternali, 2015) to fit a tilted-ring model to the data cube. We divide the central $1''.2$ in radius of the cube into 20 rings, each with a width of $0''.06$ (approximately the FWHM of the synthesised beam's minor axis), fully covering and sufficiently sampling the region of the velocity twist. By default, each ring is characterised by ten parameters: x and y coordinates of the ring's centre (x_0, y_0), systemic velocity (V_{sys}), position angle (PA), inclination (Inc), gas surface density (Σ_{gas}), vertical thickness (z_0), rotation velocity (V_{rot}), radial velocity (V_{rad}) and velocity dispersion (σ_{gas}). We normalise the gas surface density using the azimuthal average of the observed $^{12}\text{CO}(2-1)$ intensity in each ring and fix z_0 to be zero (infinitely thin disc approximation), leaving only eight free parameters. We set "TWOSTAGE=true" so that all parameters vary freely in the first fitting stage, but the geometric parameters x_0 , y_0 and V_{sys} of each ring are regularised and kept fixed (by default to the median of the 20 rings) in the second fitting stage.

As disc warps and non-circular motions are often degenerate, causing similar distortions of the isovelocity contours, we perform two separate fits. In the first fit (warp model), the PA and Inc of each ring are regularised by Bezier interpolation in the second fitting stage, allowing them to vary across rings (i.e. create a warp), but the radial velocity is fixed to

⁹Version 1.7 from <https://bbarolo.readthedocs.io/en/latest/>

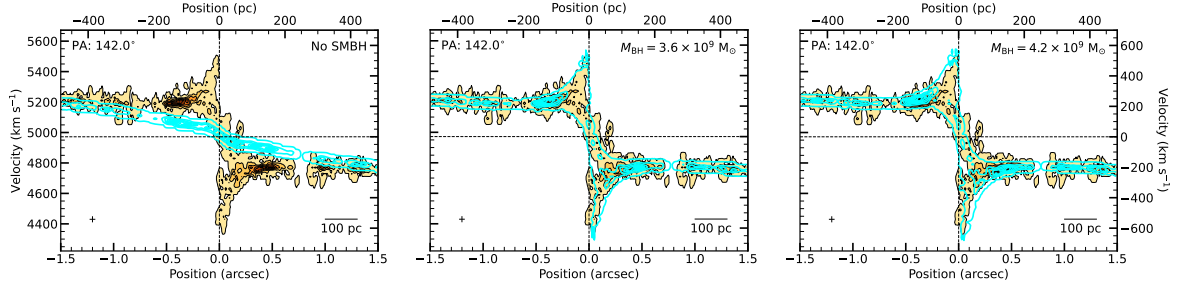


Figure 3.6: Observed kinematic major-axis position-velocity diagram of NGC 383 (orange scale with black contours), overlaid with the PVDs of different models (cyan contours): no SMBH (left), best-fitting model from this work (centre) and best-fitting model from [North et al. \(2019\)](#) (right). Positions are measured relative to the best-fitting kinematic centre; velocities relative to the best-fitting systemic velocity along the right axis (see Section 3.3.5). The cross in the bottom-left corner of each panel shows the synthesised beam FWHM along the kinematic major axis and the channel width. Our best-fitting model reproduces the material beyond $\approx 350 \text{ km s}^{-1}$ better than the best-fitting model of [North et al. \(2019\)](#), demonstrating the importance of high-resolution observations to the precision and accuracy of SMBH mass measurements.

$V_{\text{rad}} = 0$. In the second fit (radial motion model), the PA and Inc are fixed to the median across the rings in the second fitting stage, but the radial velocity V_{rad} is left free, providing a pure radial motion model. In both fits, we only consider pixels within a boolean mask, constructed similarly to the mask used to create the moment maps in Section 3.2.4, except that we clip at $1 \sigma_{\text{RMS}}$ of the naturally-weighted data cube. All other parameters of the 3DFIT task are set to their defaults.

For the warp model, the best-fitting ring parameters yielded by the second fitting stage suggest that the velocity field at the centre of NGC 383 is consistent with an abrupt change of PA from 142° in the outer region to $\approx 110^\circ$ in the inner region near $R = 0''.2$. However, the PA of the innermost region is highly uncertain and is sensitive to the initial guess. Any PA between $\approx 100^\circ$ and $\approx 130^\circ$ yields a similar RMS of $\approx 30 \text{ km s}^{-1}$ in the first-moment (i.e. intensity-weighted mean line-of-sight velocity) residuals within the central $1''.2$ in radius. For reference, a model with no PA warp nor radial motion has a RMS residual velocity of $\approx 50 \text{ km s}^{-1}$ in the same region. For the radial motion model, Figure 3.9 shows the radial profile of the radial velocities yielded by the second fitting stage. V_{rad} is inconsistent with zero (to 1σ) at $R \leq 0''.21$ only, revealing a roughly linear trend of increasing radial velocity towards the centre. The RMS of the residual velocities is $\approx 20 \text{ km s}^{-1}$. The $3\text{D}\text{BAROLO}$ tilted-ring fits therefore provide mildly more compelling evidence for radial motions than for

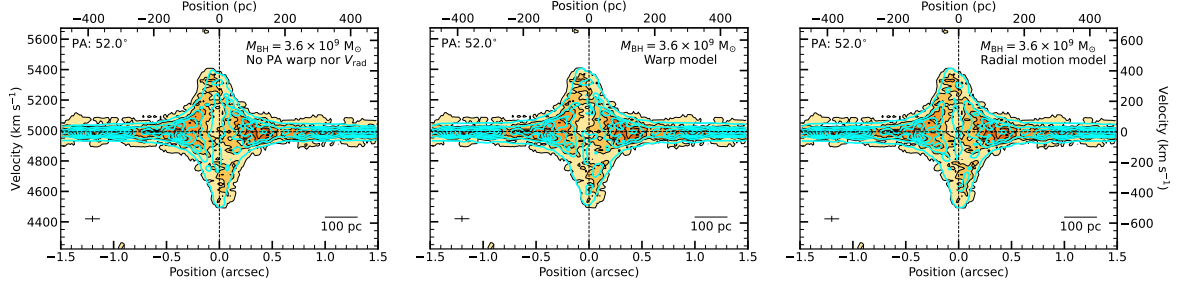


Figure 3.7: Same as Figure 3.6, but showing the kinematic minor-axis PVD of NGC 383 imaged with natural weighting, overlaid with the PVDs of the best-fitting model with no warp nor radial motion (left), a warp (centre) and radial motions (right). Although the warp model suggests a different minor-axis PA in the innermost region, we adopt the same large-scale PA as other models to compare them. The asymmetry of the observed minor-axis PVD within the central $\approx 0''.2$ in radius is consistent with the observed twist of the zero-velocity contour. Both the warp and the radial motion model partly reproduce the asymmetry, while the model with no warp nor radial motion has a symmetric minor-axis PVD by construction.

a PA warp. To determine whether the radial motions represent an inflow or an outflow, one has to know which side of the gas disc is closer to the observer. As the dust lanes in the galaxy’s *HST* image appear only on the southwestern side of the galaxy (North et al., 2019), the southwestern side is likely the near side, and the gas disc is likely rotating in a clockwise direction in the plane of the sky. The radial motions (if confirmed) therefore likely represent an outflow.

3.4.1.2 Parametric models

To obtain parametric models of the PA warp and radial motions and fully sample the posterior distributions of the parameters, we repeat the above analysis with KINMS. While ^{3D}BAROLO allows the kinematics of each ring to vary freely without considering the underlying mass distribution, the circular velocity curve of KINMS is determined by the mass profile. KINMS thus allows us to check whether the best-fitting SMBH mass changes after incorporating a PA warp or non-circular motions in the kinematic model. Motivated by the ^{3D}BAROLO results, we thus use KINMS to fit a warp involving an abrupt PA change at a particular radius but no radial motion (warp model) and to fit radial motions with a linear variation of the radial velocities with radius but no warp (radial motion model). The warp model replaces the PA in the model of Section 3.3 with three new free parameters: the PA in the inner region (PA_{inner}), the PA in the outer region (PA_{outer}) and the radius at which the PA changes abruptly (R_{change}).

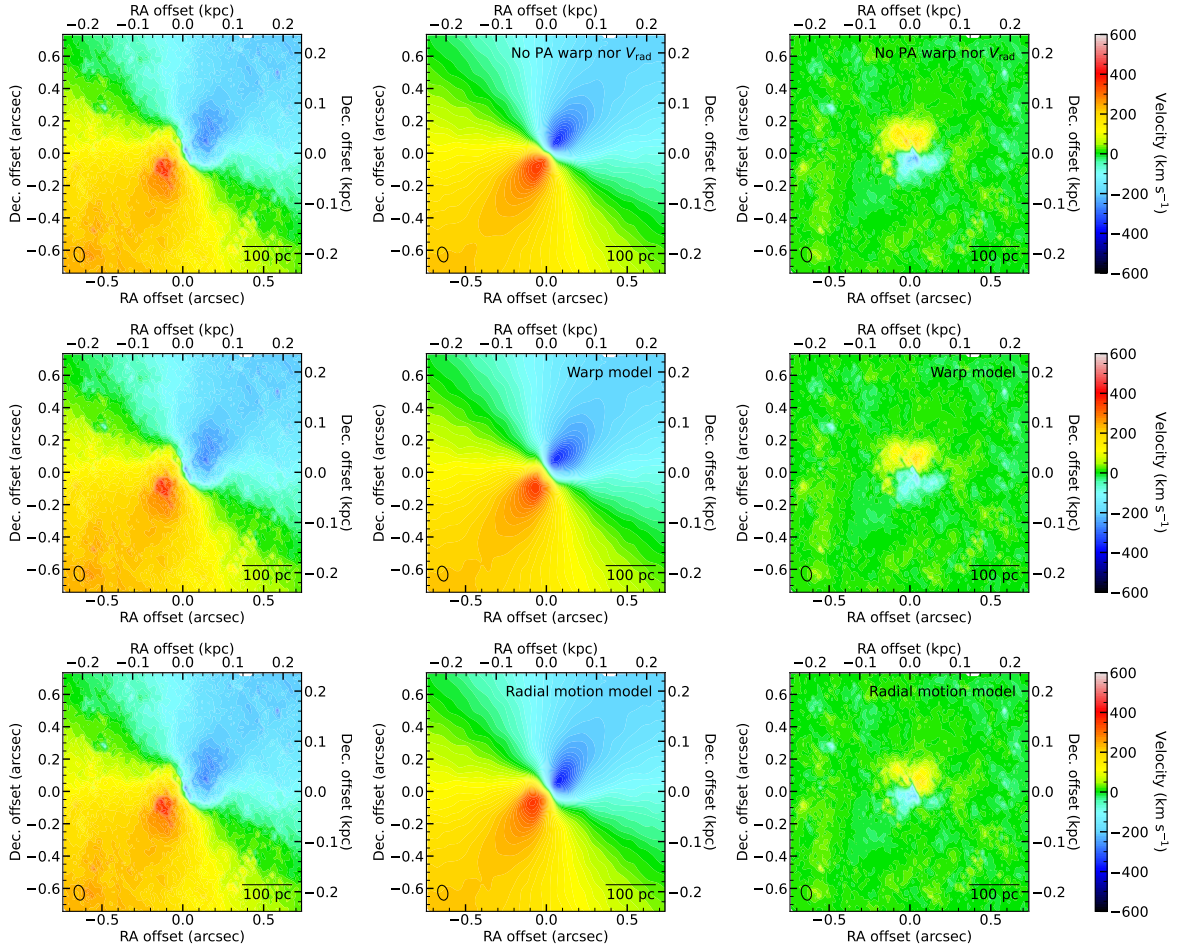


Figure 3.8: Left panels: first-moment (intensity-weighted mean line-of-sight velocity) map of the NGC 383 data cube imaged with natural weighting, showing only the central $1''.5 \times 1''.5$. The central velocity twist is more prominent as natural weighting maximises the S/N . Middle panels: first-moment maps of the best-fitting dynamical model with no PA warp nor non-circular motion (top), the best-fitting warp model (middle) and the best-fitting radial motion model (bottom). Right panels: corresponding first-moment residual maps (data – model). The synthesised beam ($0''.093 \times 0''.062$) is shown as a black open ellipse in the bottom-left corner of each panel, while a 100 pc scale bar is shown in the bottom-right corner of each panel. Positions are measured relative to the best-fitting kinematic centre; velocities relative to the best-fitting systemic velocity (see Section 3.3.5).

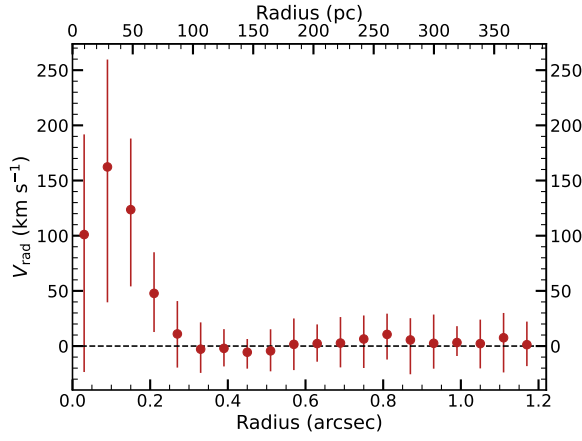


Figure 3.9: Radial profile of the best-fitting radial velocities (with 1σ error bars), from our $3D$ BAROLO radial motion model of NGC 383. The radial velocity is inconsistent with zero (to 1σ) at $R \leq 0''.21$ only, revealing evidence of radial motions in the innermost region of NGC 383.

The radial motion model has two additional parameters: the radial velocity at $R = 0$ ($V_{\text{rad},0}$) and the radius beyond which the radial velocity is 0 (R_{cutoff}). Other parameters are identical to the ones listed in Section 3.3 for the model with no warp nor radial motion (hereafter "regular disc model").

The best-fitting warp model yields an inner PA $\text{PA}_{\text{inner}} = 117^{\circ+25^{\circ}}_{-31^{\circ}}$ and an outer PA $\text{PA}_{\text{outer}} = 142.08 \pm 0.13$. The PA changes abruptly at $R_{\text{change}} = 0''.18 \pm 0''.05$. The remaining best-fitting parameters, including the SMBH mass, are almost identical to those of the regular disc model. Because the inner PA is consistent with the outer PA within 1σ , the warp model is not strongly favoured over models with no PA warp. Table 3.5 compares the statistics of the best-fitting warp model to those of the best-fitting regular disc model. Although the warp model has a slightly lower χ^2_r , it has two more free parameters. Considering instead the Bayesian information criterion (BIC), defined as $\text{BIC} \equiv k \ln N - 2 \ln P$, where k is the number of free parameters, N the number of constraints, and $-2 \ln P$ is the χ^2 defined in Section 3.3, the warp model is actually less preferred than the regular disc model. The top panels and the middle panels of Figure 3.8 show the central $1''.5 \times 1''.5$ of first-moment map of the regular disc model and of the warp model, respectively, the data and the residuals (data – model). The mask used to create the first-moment maps here is again similar to that used in Section 3.2.4, but we clip at $0.65 \sigma_{\text{RMS}}$ of the naturally-weighted data cube. As the regular disc model does not produce any velocity twist, velocity residuals up to $\pm 280 \text{ km s}^{-1}$ remain in the central region. By contrast, the warp model produces a mild velocity twist and reduces the velocity

Table 3.5: Statistics of the models discussed.

Model	Max $ V_{\text{res}} $ (km s^{-1})	RMS V_{res} (km s^{-1})	χ_r^2	k	BIC
(1)	(2)	(3)	(4)	(5)	(6)
Regular disc	281	26	2.080	10	12043
Warp	207	24	2.075	12	12063
Radial motion	224	23	2.072	12	12060

Notes. (1) Model. (2) Maximum velocity residual (central $1''.5 \times 1''.5$). (3) RMS velocity residual (central $1''.5 \times 1''.5$). (4) Reduced χ^2 statistic (entire cube). (5) Number of free parameters. (6) Bayesian information criterion.

residuals. However, it does not fully reproduce the observed twist and leaves residuals up to $\approx 210 \text{ km s}^{-1}$ at the centre. We have also tested other parametric models of a PA warp (e.g. linear warp with radius), but none yields a better BIC than the current model. This strongly suggests that a PA warp does not satisfactorily explain the twist observed in the velocity field.

It is also worth noting that if a PA warp from 142° in the outer CO disc to $\approx 110^\circ$ in the inner CO disc is present in NGC 383, then the rotation axis of the inner CO disc would align better with the radio jet (with a position angle $\text{PA}_{\text{jet}} = -19.7^\circ \pm 1.0^\circ$; Laing & Bridle 2014) than that of the outer CO disc. Although recent studies (e.g. Ruffa et al., 2020) suggest that jets and 100-pc-scale CO discs are not preferentially aligned, the relative orientations of jets and the rotation axes of sub-pc maser discs are usually within $\approx 30^\circ$ (Kamali et al., 2019). As our high-resolution observations of NGC 383 probe the same physical scale (in the unit of the Schwarzschild radius) as maser observations (Zhang et al., 2024), the PA warp could be an indication that our observations start to reveal CO gas more closely related to the central accretion disc that powers the AGN jet than earlier CO observations.

The best-fitting radial motion model suggests a radial velocity linearly decreasing from $V_{\text{rad}} = 294_{-181}^{+136} \text{ km s}^{-1}$ at $R = 0$ to $V_{\text{rad}} = 0$ at $R_{\text{cutoff}} = 0''.28_{-0.07}^{+0.05}$. These parameters are consistent with the ^{3D}BAROLO tilted-ring fits (Figure 3.9) but have large uncertainties. We thus only have tentative evidence of radial motions, with $V_{\text{rad},0}$ only $\approx 1.6\sigma$ different from zero. The remaining best-fitting parameters, including the SMBH mass, are all in agreement with those of the regular disc model. Table 3.5 lists the statistics of the radial motion model, while the bottom panels of Figure 3.8 shows the first-moment maps of the model and data, and the associated velocity residual map. The radial motion model has the smallest χ_r^2 of the three models, but it is still not preferred over the regular disc model when considering the

BIC. It yields the smallest RMS velocity residual in the central region, but large residuals up to $\approx 220 \text{ km s}^{-1}$ remain, suggesting non-circular motions more complicated than pure radial motions.

Finally, we compare the kinematic major-axis and minor-axis PVDs of the three models to those of the data in Figures 3.10 and 3.7, respectively. Although the warp model suggests a different major-axis PA in the innermost region, we adopt the same large-scale PA as other models to compare them. Both the warp and the radial motion models partly but not fully reproduce the forbidden quadrant emission of the major-axis PVD and the asymmetry of the minor-axis PVD, suggesting again that neither model fully explains the observed distortions of the velocity field.

We note that we have also tested parametric models combining a PA warp and radial motions. However, the models' PA warp and radial motion parameters could not be simultaneously constrained, leading to unconverged MCMC chains, nor could the combined models significantly reduce the velocity residuals, again because the data do not have sufficient S/N to break the degeneracy between a PA warp and a non-zero V_{rad} .

3.4.1.3 Implications for the SMBH mass

As neither a PA warp nor pure radial motion satisfactorily explains the observed velocity twist, that feature is more likely associated with more complicated non-circular motions (not necessarily pure radial inflow or outflow) that depend on the azimuthal angle and involve rotation velocities that are increased or decreased compared to those expected from pure circular motions. For example, the strong kink along the minor axis of the observed first-moment map, located at approximately $(-0''.1, 0''.2)$ from the centre, is reminiscent of those due to streaming motions along spiral arms or barred orbits. To search for such structures in the nuclear region, the central $1''.5 \times 1''.5$ of the zeroth-moment map of the data cube imaged with natural weighting is shown in the left panel of Figure 3.11. The zeroth-moment map of an axisymmetric model generated using ^{3D}BAROLO and the associated residuals (data – model) are also shown in the middle and right panels of Figure 3.11, respectively. This reveals two arm-like structures of positive residuals near $(0''.25, -0''.5)$ and $(0''.2, 0''.45)$, that may be evidence of a weak spiral structure. However, that structure is speculative and statistically

insignificant compared to the substantial random and structured noise near the centre of the zeroth-moment map. Future observations with higher sensitivity are required to confirm the nature of this feature. The large-scale spiral features observed by [North et al. \(2019\)](#); see also the left panel of Figure 3.2) are also recovered in the velocity residual map covering the entire molecular gas disc (Figure 3.12). The tentative spiral-like structure in the zoomed-in zeroth-moment map is potentially a continuation of that large-scale spiral structure and could explain the velocity twist in the nuclear region. Unfortunately, the relatively low S/N and the limited number of pixels in the region of the velocity twist make it difficult to fit more complicated non-circular motion models to the data. In any case, we note that our best-fitting SMBH mass is not sensitive to potential non-circular motions, because of the dominance of the SMBH on the kinematics in the central region. All the models discussed above yield almost identical SMBH masses $M_{\text{BH}} \approx 3.6 \times 10^9 M_{\odot}$, and the best-fitting M_{BH} remains unchanged even if we mask out the twist region during the fit. Hence, we continue to adopt the results of the regular disc model in the following discussions, as it has the smallest BIC (Table 3.5).

Despite the evidence of non-circular motions, the asymmetry between the redshifted and blueshifted central velocity peaks of NGC 383 is unlikely to be caused by non-circular motions, as this feature is well reproduced by our best-fitting regular disc model assuming pure circular motions (see Figure 3.6). This implies that the asymmetry is most likely due to a deficiency of $^{12}\text{CO}(2-1)$ emission on the redshifted side of the innermost region, that has already been incorporated into our model by construction as we use the observed (but deconvolved) intensity distribution as the input gas distribution. This deficit can have multiple origins, in particular, a non-axisymmetric gas morphology (e.g. the tentative nuclear spiral discussed above).

3.4.2 Uncertainties

The typical uncertainties associated with a SMBH mass measurement using molecular gas kinematics have been discussed extensively in previous WISDOM papers ([Onishi et al., 2017](#); [Davis et al., 2017, 2018](#); [Smith et al., 2019](#); [North et al., 2019](#); [Smith et al., 2021](#); [Lelli et al., 2022](#); [Ruffa et al., 2023](#)). In the following, we thus discuss only the dominant

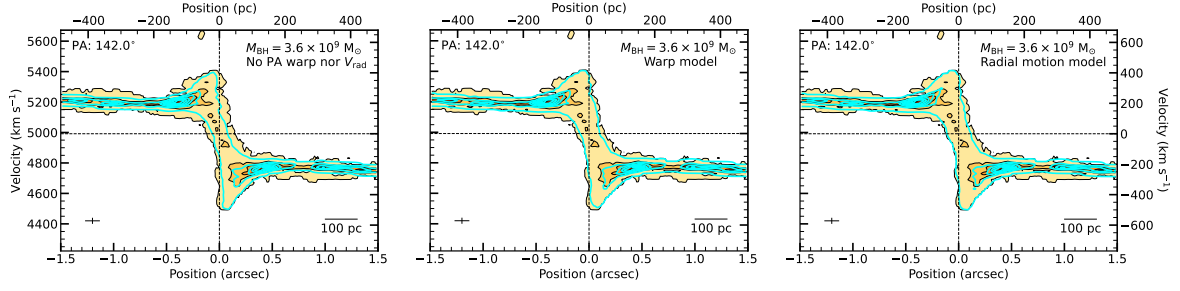


Figure 3.10: Same as Figure 3.7, but showing the kinematic major-axis PVD of NGC 383 imaged with natural weighting. Both the warp and the radial motion model reproduce the forbidden quadrant emission slightly better than the model with no warp nor radial motion (in which all forbidden quadrant emission is due to beam smearing).

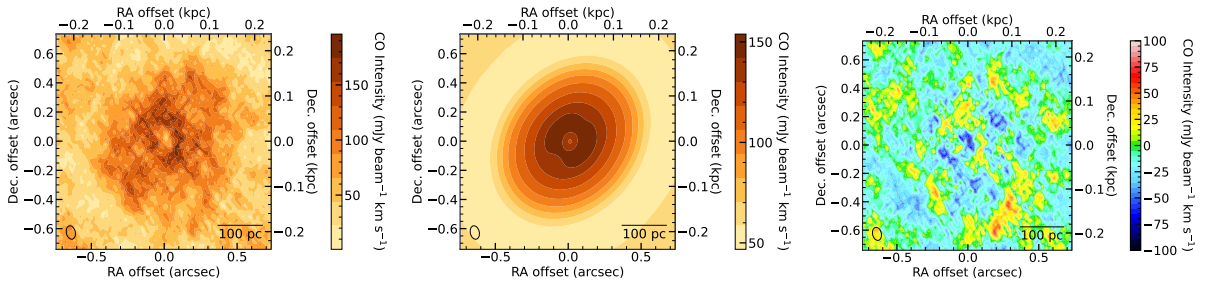


Figure 3.11: Left: zeroth-moment (integrated-intensity) map of the NGC 383 data cube imaged with natural weighting, showing only the central $1''.5 \times 1''.5$. The dip within a radius of $\approx 0''.1$ is a physical decrease of the CO surface brightness. Middle: zeroth-moment map of the best-fitting axisymmetric model generated with ^{3D}BAROLO. The innermost few contours appear slightly asymmetric due to the elongation of the synthesised beam. Right: corresponding zeroth-moment residual map (data – model). The two arm-like structures of positive residuals near $(0''.25, -0''.5)$ and $(0''.2, 0''.45)$ may be evidence of a weak spiral structure.

sources of uncertainties of the SMBH mass of NGC 383, as derived from our high-resolution observations.

3.4.2.1 χ^2 rescaling

As discussed in Section 3.3, we rescaled the uncertainties of the data cube by $(2N)^{0.25}$ to attempt to account for the potential systematic uncertainties usually dominating large data sets. Without χ^2 rescaling, the SMBH mass precision would be 0.05%, an unrealistic uncertainty that severely underestimates potential systematic effects. To verify that this procedure provides a reasonable estimate of the systematic uncertainties and to confirm that the high-resolution data truly yields a more precise SMBH mass, we perform an independent estimate of the systematic uncertainties by adopting a procedure analogous to bootstrapping. We divide the

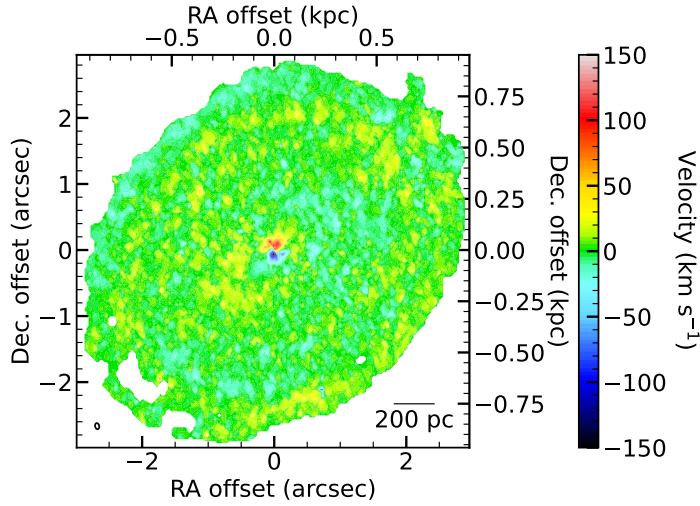


Figure 3.12: Full (i.e. entire disc) first-moment (i.e. intensity-weighted mean line-of-sight velocity) residual map of NGC 383, created by subtracting the first-moment map of the best-fitting model with no warp nor radial motion from the first-moment map of the data cube imaged with natural weighting. Apart from the large residuals at the centre, the large-scale spiral features observed by [North et al. \(2019\)](#); see also the left panel of Figure 3.2) are also recovered. This suggests non-circular motions more complicated than pure radial motions.

data cube into four sub-samples, each one a spatial quadrant of the data cube, bounded by planes of fixed position angles. We fit each sub-sample independently, keeping the kinematic centre position and the systemic velocity fixed to their best-fitting values in Table 3.4. For each model parameter, we then adopt the mean of the four best-fitting parameters as the overall best fit, and their standard deviation as the 1σ uncertainty. Table 3.6 compares the best-fitting parameters and the associated 1σ uncertainties obtained using this approach to those obtained in Section 3.3.5 using the MCMC approach with χ^2 rescaling. The best-fitting parameters obtained using the bootstrapping-like approach all agree within 1σ with those obtained using the MCMC approach and the uncertainties are also very similar. We thus confirm that the χ^2 rescaling adopted provides reasonable estimates of the systematic uncertainties of all (non-nuisance) parameters of our model.

3.4.2.2 Other uncertainties

As our observations spatially resolve the SMBH’s SoI by a factor of ≈ 24 radially (estimated using our updated SMBH mass and σ_e , but see Section 3.4.3 for a more physically-motivated calculation), they probe the innermost region of the molecular gas disc where the kinematics is dominated by the SMBH and is essentially independent of the stellar mass distribution. Consequently, the inferred SMBH mass depends only weakly on the stellar M/L . The positive

Table 3.6: Best-fitting parameters and associated 1σ uncertainties, obtained via MCMC with χ^2 rescaling and bootstrapping, respectively.

Parameter	χ^2 rescaling	Bootstrapping
Mass model		
$\log (M_{\text{BH}}/M_{\odot})$	9.554 ± 0.022	9.571 ± 0.024
Inner stellar M/L ($M_{\odot}/L_{\odot, \text{F160W}}$)	3.16 ± 0.15	3.04 ± 0.17
Outer stellar M/L ($M_{\odot}/L_{\odot, \text{F160W}}$)	2.32 ± 0.09	2.32 ± 0.10
Molecular gas disc		
Position angle (degree)	142.01 ± 0.29	142.19 ± 0.27
Inclination (degree)	37.6 ± 1.0	38.0 ± 1.0
Velocity dispersion (km s^{-1})	10.6 ± 0.9	10.2 ± 0.9

correlation between SMBH mass and outer M/L (see Figure 3.5) is likely a by-product of the SMBH mass – inclination (anti-)correlation and the M/L – inclination (anti-)correlation. We verified that a model with a spatially-constant M/L does not improve the fit quality (using the BIC as the metric) and yields a SMBH mass $\log (M_{\text{BH}}/M_{\odot}) = 9.58 \pm 0.02$ consistent with that from our radially linearly-varying M/L model.

The SMBH mass of NGC 383 is however strongly degenerate with the inclination of the molecular gas disc. As $M_{\text{BH}} \propto \sin^{-2} i$, the SMBH mass uncertainty is often dominated by the inclination uncertainty, especially when the gas disc is nearly face-on. With an inclination of only 38° , the inclination uncertainty contributes $\approx 55\%$ to the NGC 383 SMBH mass uncertainty budget. If the inclination were fixed to the best-fitting inclination, the SMBH mass precision would be $\approx 4\%$ for the high-resolution data cube fit and $\approx 9\%$ for the intermediate-resolution data cube fit.

We note that every uncertainty discussed so far is much smaller than that introduced by the $\approx 15\%$ uncertainty of the adopted distance (66.6 ± 9.9 Mpc; Freedman et al. 2001). Indeed, as the SMBH mass scales linearly with the adopted distance, the distance uncertainty can impact the SMBH mass more than all other uncertainties combined. Nevertheless, as scaling the SMBH mass to a different distance is straightforward (i.e. it does not require redoing the fit), we follow the customary practice of not including the distance uncertainty in our results.

3.4.3 Merits of high-resolution observations

Our high-resolution observations of the molecular gas disc of NGC 383 spatially resolve material with a maximum line-of-sight velocity $V_{\text{obs}} \approx 635 \text{ km s}^{-1}$, higher than the $\approx$

350 km s⁻¹ detected by the intermediate-resolution observations of [North et al. \(2019\)](#). Deprojecting this with $V_c = V_{\text{obs}}/\sin i$, where the best-fitting inclination $i = 37^\circ.6$, the highest-velocity material resolved by our observations has a circular velocity $V_c \approx 1040$ km s⁻¹. This is larger than the maximum circular velocities probed by all previous molecular gas kinematic SMBH mass measurements ([Zhang et al., 2024](#)). This largest V_c is the result of our measurement resolving material physically closest to the SMBH in terms of the number of Schwarzschild radii ($R/R_{\text{Schw}} \approx 4.1 \times 10^4$), as $(V_c/c) = \sqrt{2} (R_{\text{Schw}}/R)^{1/2}$ for a Keplerian circular velocity curve ([North et al., 2019](#)). Our measurement also has the largest R_{SoI}/R ratio ($R_{\text{SoI}}/R \approx 24$; [Zhang et al. 2024](#)), thus spatially resolving the SoI (defined using the effective stellar velocity dispersion σ_e) better than all existing molecular gas SMBH mass measurements.

We also consider an alternative definition of the SoI radius, the radius R_{eq} at which the enclosed stellar mass equals the SMBH mass:

$$M_*(R = R_{\text{eq}}) = M_{\text{BH}} . \quad (3.7)$$

This R_{eq} is a more accurate definition of the SoI radius than $R_{\text{SoI}} \equiv GM_{\text{BH}}/\sigma_e^2$, as σ_e is only a proxy of the galaxy's gravitational potential and thus the stellar mass at large spatial scales. By contrast, R_{eq} is defined from an explicit comparison of the SMBH mass and the stellar mass. Using the MGE model in Table 3.3 and the best-fitting M/L and SMBH mass, we derive $R_{\text{eq}} = 0''.88 \pm 0''.02$ (285 ± 6 pc), in good agreement with $R_{\text{SoI}} = 0''.84 \pm 0.05$. Our observations thus resolve R_{eq} by a factor of ≈ 25 , again better than all prior molecular gas SMBH measurements ([Zhang et al., 2024](#)).

In other words, using either R_{Schw} , R_{SoI} or R_{eq} as the reference spatial scale, our measurement is the highest-resolution SMBH mass measurement using molecular gas kinematics to date. We note that our observations spatially resolve material closer to the SMBH in the unit of R_{Schw} than even the best SMBH mass measurement using masers ($R/R_{\text{Schw}} \approx 4.4 \times 10^4$ for NGC 4258; [Herrnstein et al. 2005](#)).

The high spatial resolution of our observations results in a SMBH mass precision of 5% after χ^2 rescaling, consistent with the uncertainty estimated via bootstrapping. The SMBH mass uncertainty **cannot** be directly compared to those of previous SMBH mass

measurements, as each measurement adopts a different approach to incorporate systematic uncertainties. Among the measurements that use χ^2 rescaling and do not fix the inclination, our SMBH mass precision is higher than all but those of NGC 3557 (5%; [Ruffa et al. 2019b](#)) and NGC 7052 (4%; [Smith et al. 2021](#)). This demonstrates again the importance of high spatial resolution for a precise SMBH mass.

NGC 383’s SMBH mass uncertainty arises primarily from the inclination uncertainty. The mass precision would improve more substantially with spatial resolution for a less face-on disc. We note that the precision of a SMBH measurement also depends on the S/N , but at a given S/N the ALMA integration time (t_{int}) is inversely proportional to the fourth power of the synthesised beam’s FWHM (θ_{beam}): $t_{\text{int}} \propto \theta_{\text{beam}}^{-4}$. Therefore, ultra-high-resolution observations require much longer integration times that limit their schedulability, or one must make do with lower S/N . With the current high-resolution observations, the SMBH mass precision is impaired by the limited S/N . If our high-resolution observations had achieved the same S/N as the existing intermediate-resolution observations, the mass precision would have been $\approx 4\%$, estimated by re-running the MCMC assuming a lower σ_{RMS} . To summarise, higher spatial resolution observations should lead to better SMBH mass precision at a given inclination and S/N , but achieving the desired S/N is more costly.

A higher spatial resolution should improve not only the precision but also the accuracy of a SMBH mass measurement, as it can reveal features that can bias the kinematic modelling but often remain unresolved and thus undetected at lower resolutions. For example, our high-resolution observations of NGC 383 reveal an asymmetry of the central blueshifted and redshifted velocity peaks and an offset between the kinematic and morphological centres, both unseen in the intermediate-resolution observations. By incorporating both features in our model, we reduced the inaccuracy of our kinematic modelling. The best-fitting SMBH mass changed slightly from $(4.2 \pm 0.4) \times 10^9$ to $(3.6 \pm 0.2) \times 10^9 M_{\odot}$ (both 1σ uncertainties), although the two masses are statistically consistent. The high-resolution observations also unveiled a velocity twist within the central $\approx 0''.3$ in radius, thus providing evidence of non-circular motions in the circumnuclear disc. Although the best-fitting M_{BH} is unaffected in this case because the twist is limited to the central region where the SMBH dominates the kinematics, an accurate model of non-circular motions could substantially improve the SMBH

mass constraint for similar observations of other galaxies (e.g. [Lelli et al., 2022](#)).

We however caution that a higher spatial resolution does not guarantee a more precise and accurate SMBH mass measurement when there is a central dip or hole in the CO surface brightness distribution. Although the slight central dip of NGC 383 does not affect our measurement, a central hole in the CO distribution often prevents high-resolution observations from probing emission sufficiently close to the SMBH and thus prohibits a precise SMBH mass measurement (e.g. [Kabasaes et al., 2022](#); [Ruffa et al., 2023](#)). Nevertheless, one may be able to adopt a different kinematic tracer, such as [CI], which often extends closer to the SMBH than CO and does not have a central hole in its distribution (e.g. [Izumi et al., 2020](#); [Nguyen et al., 2021](#)), to obtain a precise SMBH mass measurement even in the presence of a central CO hole.

3.5 Conclusions

We presented a measurement of the SMBH mass of NGC 383, a lenticular galaxy hosting a jetted AGN. We used ALMA observations of the $^{12}\text{CO}(2-1)$ emission line with a synthesised beam FWHM of $0''.050 \times 0''.024$ ($\approx 16 \times 8 \text{ pc}^2$), ≈ 4 times better than previous observations, spatially resolving the SMBH SoI by a factor of ≈ 24 in radius. This spatial resolution, in the unit of either R_{Schw} , R_{SoI} or R_{eq} , is the highest of all SMBH mass measurements using molecular gas. The observations also spatially resolve material closer to the SMBH in terms of the number of Schwarzschild radii ($R/R_{\text{Schw}} \approx 4.1 \times 10^4$) than the best measurements using masers ($R/R_{\text{Schw}} \approx 4.4 \times 10^4$ for NGC 4258; [Herrnstein et al. 2005](#)). Our high-resolution data detect a clear Keplerian rise (from the outside in) of the (deprojected) circular velocities up to $\approx 1040 \text{ km s}^{-1}$. They also reveal features undetected in previous observations: an offset between the gas disc’s kinematic and morphological centres, a mild asymmetry in the innermost velocity peaks and a central velocity twist suggesting non-circular motions. Our best-fitting kinematic model yields a SMBH mass of $(3.58 \pm 0.19) \times 10^9 M_{\odot}$, more precise but consistent within $\approx 1.4\sigma$ with the previous SMBH mass of $(4.2 \pm 0.4) \times 10^9 M_{\odot}$ derived from intermediate-resolution data ([North et al., 2019](#)). This SMBH mass is insensitive to models of the central velocity twist. The F160W-filter stellar M/L decreases from $3.16 \pm 0.15 M_{\odot}/L_{\odot, \text{F160W}}$ at the centre to $2.32 \pm 0.09 M_{\odot}/L_{\odot, \text{F160W}}$ at the outer edge of the molecular

gas disc ($R = 3''.5$).

Despite this highest spatial resolution, this SMBH mass measurement is not the most precise because of the relatively low inclination, associated with a high inclination uncertainty, and the relatively low S/N . It nevertheless demonstrates the importance of high resolution to the precision and accuracy of SMBH mass measurements. Although high-resolution observations require relatively long ALMA integration times, they will steadily increase the number of precise and accurate SMBH mass measurements using molecular gas kinematics, and they will ultimately rival the "gold standard" measurements using masers. This will, in turn, allow constraining SMBH – galaxy scaling relations more tightly across the entire galaxy range.

Chapter 4

Improving the supermassive black hole mass measurement of NGC 1574 using high-resolution molecular gas kinematics

The material in this chapter was originally published in [Zhang et al. \(2025b\)](#)

4.1 Introduction

This chapter presents a supermassive black hole (SMBH) mass measurement in the galaxy NGC 1574 using Atacama Large Millimeter/submillimeter Array (ALMA) observations of the $^{12}\text{CO}(2-1)$ emission line with synthesised beam full-widths at half-maximum (FWHM) of $0''.078 \times 0''.070$ ($\approx 7.5 \times 6.7$ pc²; geometric mean $0''.074$ or 7.1 pc). A previous CO-dynamical SMBH mass measurement in this galaxy, performed by [Ruffa et al. \(2023\)](#), inferred a mass of $(1.0 \pm 0.2) \times 10^8 M_{\odot}$ (1σ uncertainty here and throughout this chapter) using ALMA observations of the same line at an intermediate angular resolution ($0''.202 \times 0''.138$). The intermediate-resolution data however slightly under-resolve the angular size of the sphere of influence (SoI; see Section 4.4.2). The new study presented in this chapter thus aims to improve the precision and accuracy of the existing M_{BH} measurement, using data that fully spatially resolve the SoI. This study is the second of a series of high-angular resolution SMBH mass measurements performed by the mm-Wave Interferometric Survey of Dark Object Masses (WISDOM), following the measurement of the SMBH mass of NGC 383 by

Zhang et al. (2025a). Studies like these can reveal previously spatially-unresolved features of the circumnuclear molecular gas discs, such as small-scale warps and non-circular motions (e.g. Lelli et al., 2022; Zhang et al., 2025a), which could systematically bias SMBH mass measurements using molecular gas observations that only marginally resolve (or under-resolve) the SoI. High spatial resolution observations of molecular gas can also probe the circumnuclear disks of SMBHs down to a few times $10^4 R_{\text{Schw}}$, revealing the central fuelling and feedback mechanisms of SMBHs (e.g. Zhang et al., 2025a).

This chapter is structured as follows. Section 4.2 introduces the target, describes the ALMA observations and our data reduction procedures, and presents the final data product. Our molecular gas dynamical modelling technique and its results, including the best-fitting SMBH mass, are presented in Section 4.3. Section 4.4 discusses the sources of statistical and systematic uncertainties. We conclude in Section 4.5.

4.2 ALMA observations

4.2.1 Target: NGC 1574

NGC 1574 is a relatively face-on ($i \approx 27^\circ$; Ruffa et al., 2023) lenticular galaxy, one of the 46 known members of the Dorado group (Maia et al., 1989). It contains a nuclear stellar bar (e.g. Phillips et al., 1996; Laurikainen et al., 2011; Gao et al., 2019), which might be associated with the signs of weak non-circular motions detected in previous ALMA CO observations (Ruffa et al., 2023). The luminosity distance of NGC 1574 is 19.9 ± 2.0 Mpc, derived from the surface brightness fluctuation (SBF) distance modulus reported by Tonry et al. (2001). Assuming a cold dark matter cosmology with a Hubble constant $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$, a matter density (relative to the critical density) $\Omega_M = 0.3$ and a cosmological constant (relative to the critical density) $\Omega_\Lambda = 0.7$ today, the SBF distance corresponds to a redshift $z = 0.0046$ and an angular-diameter distance of 19.7 ± 2.0 Mpc. At this distance, an angle of $1''$ corresponds to a physical separation of ≈ 95 pc. This distance is consistent with that adopted in the dynamical modelling of Ruffa et al. (2023; 19.76 Mpc). We have confirmed that the angular-diameter distance of 19.3 Mpc reported by Ruffa et al. (2023) is a typographical error.

Assuming a SMBH mass $M_{\text{BH}} = (1.0 \pm 0.2) \times 10^8 M_\odot$ (Ruffa et al. 2023) and $\sigma_e =$

$216 \pm 16 \text{ km s}^{-1}$ (Bernardi et al., 2002), the radius of the SMBH SoI (R_{SoI}) of NGC 1574, as estimated using $R_{\text{SoI}} \equiv GM_{\text{BH}}/\sigma_e^2$, is 9.2 pc or $0''.1$. Our ALMA observations (see Section 4.2.2) with a mean synthesised beam FWHM of $0''.074$ (7.1 pc) should thus be the first to spatially resolve the SMBH SoI of NGC 1574, enabling for the first time a highly reliable SMBH mass determination in this object. We provide a more accurate estimate of the size of the SoI of NGC 1574 in Section 4.4.2, using our updated SMBH mass and a more formal SoI definition.

4.2.2 Observations and data reduction

The $^{12}\text{CO}(2-1)$ emission line of NGC 1574 was observed with the ALMA 12-m array in band 6 using configuration C-8 on 2023 September 16 as part of project 2022.1.01122.S (PI: M. Bureau). The observations consist of two tracks with baseline ranges of 92 m to 8.8 km and 83 m to 8.5 km and on-source observing times of 5977 and 5984 s, respectively. We also use three prior intermediate-resolution ALMA observations of the same line to improve the uv -plane coverage. Two tracks (both part of project 2015.1.00419.S, PI: T. Davis) used the 12-m array and have baseline ranges of 15 m to 0.7 km and 80 m to 3.2 km and on-source times of 393 and 756 s, respectively. The other track (project 2016.2.00053.S, PI: L. Liu) used the 7-m Atacama Compact Array (ACA) and has baselines ranging from 9 to 49 m and a total on-source time of 2359 s. These three prior tracks were presented by Ruffa et al. (2023), who used them to derive the previous SMBH mass. The properties of all the tracks used in this study are summarised in Table 4.1.

Each of the five observing tracks has four spectral windows (SPWs). One is always centred at $\approx 229.8 \text{ GHz}$ to target the redshifted $^{12}\text{CO}(2-1)$ line (rest frequency 230.538 GHz), covering a bandwidth of 1.875 GHz ($\approx 2400 \text{ km s}^{-1}$) with 1920 channels of 976.6 kHz ($\approx 1.3 \text{ km s}^{-1}$) for the 12-m array observations and a bandwidth of 2 GHz ($\approx 2600 \text{ km s}^{-1}$) with 2048 channels of 976.6 kHz ($\approx 1.3 \text{ km s}^{-1}$) for the ACA observations. In all observations, the other three SPWs are deployed to map continuum emission. In the two most recent, high angular resolution observing tracks, these three SPWs are centred at 212.8, 214.8 and 227.8 GHz and have the same channel width and total bandwidth as the line SPW. The continuum SPWs of the three older observing tracks are centred at 231.7, 245.2 and 247.2 GHz and have 128

channels of 15.63 MHz ($\approx 20 \text{ km s}^{-1}$), covering a total bandwidth of 2 GHz.

For all the tracks, a standard calibration strategy was adopted, using a single bright quasar as both flux and bandpass calibrator and a second bright quasar as phase calibrator. The flux and bandpass calibrator of the high-resolution observing tracks is J0519-4546, and the phase calibrator is J0425-5331.

The raw data were calibrated with the COMMON ASTRONOMY SOFTWARE APPLICATION (CASA) package¹ (McMullin et al., 2007), version 6.4.1.12, using the standard script provided by the ALMA Science Archive². We combined all the calibrated observing tracks using the CASA task `concat`. The combined dataset was then imaged with CASA version 6.5.2 using the `tclean` task with a Högbom deconvolver (Högbom, 1974). For both the line and continuum data, we chose a pixel size of $0''.02$ and an image size of 200×200 pixels, sufficient to properly sample the synthesised beam and cover the full spatial extent of the emission. Further details of the line data cube and the continuum image are presented below.

4.2.3 Line emission

4.2.3.1 Imaging

We separated the $^{12}\text{CO}(2-1)$ line emission from the continuum emission in the uv -plane using the CASA task `uvcontsub`, which fits a continuum model that is linearly-varying with frequency to interactively-identified line-free channels and then subtracts the best-fitting model from the calibrated visibilities. We then imaged the continuum-subtracted visibilities in the cube mode of `tclean` using Briggs weighting with a robust parameter of 1.0. After some trials, we chose this weighting to ensure that the SMBH SoI is sufficiently spatially resolved while compensating for the data's relatively low signal-to-noise ratio (S/N). We adopted a channel width of 20 km s^{-1} , sufficient for a reliable analysis of the gas kinematics (e.g. Ruffa et al., 2023; Zhang et al., 2025a) while also achieving a reasonable S/N . The channel velocities were computed in the rest frame, with the rest frequency of the $^{12}\text{CO}(2-1)$ line (230.538 GHz) corresponding to a velocity of zero. The dirty (uncleaned) data cube was cleaned in interactively-identified regions of line emission down to a threshold of $1.5 \sigma_{\text{RMS}}$, where σ_{RMS} is the root-mean-square (RMS) noise evaluated in line-free regions of the dirty

¹<https://casa.nrao.edu/>

²<https://almascience.eso.org/aq/>

cube. The final cleaned data cube has $\sigma_{\text{RMS}} = 0.15 \text{ mJy beam}^{-1}$ and synthesised beam FWHM of $0''.078 \times 0''.070$ ($\approx 7.5 \times 6.7 \text{ pc}^2$), with a position angle (PA) of 31.5° . The data cube's average S/N in regions of line emission is ≈ 4 .

4.2.3.2 Moment maps

We adopt a masked-moment technique (Dame, 2011) to create the zeroth (integrated-intensity), first (intensity-weighted mean line-of-sight velocity) and second (intensity-weighted line-of-sight velocity dispersion) moment maps of the final data cube. To generate the mask, we take a copy of the data cube and smooth it spatially with a uniform filter of size 3 times the mean synthesised beam FWHM, and spectrally with a Hanning window of 5 channels. The mask includes all pixels of the smoothed data cube with a flux density larger than $1 \sigma_{\text{RMS}}$ of the unsmoothed data cube. The moment maps are then created using only the masked pixels of the unsmoothed data cube and are shown in Figure 4.1. The filter sizes and the flux threshold were chosen by trial and error to minimise the noise in the moment maps while recovering most of the emission. We also note that the moment maps are generated only to help visualise the data cube. Our dynamical modelling described in Section 4.3 uses the entire unmasked data cube rather than the moment maps.

The integrated-intensity map shows a molecular gas disc extending $\approx 2''.5 \times 2''.2$ ($\approx 240 \times 210 \text{ pc}^2$) in projection. The brightest CO emission is slightly offset from the geometric centre of the gas disc, by $\approx -0''.07$ ($\approx -7 \text{ pc}$) in right ascension and $\approx 0''.09$ ($\approx 9 \text{ pc}$) in declination, consistent with the offsets observed in Ruffa et al. (2023) but smaller than the synthesised beam size of the old observations. Compared to the intermediate-resolution data cube presented by Ruffa et al. (2023), our high-resolution data cube reveals more molecular gas clumps, as it spatially resolves smaller physical scales ($\approx 7 \text{ pc}$). Analysis of these molecular gas clumps is however beyond the scope of this study.

The mean line-of-sight velocity map reveals a rotating disc with a prominent twist of the isovelocity contours across the entire gas distribution. Dynamical modelling of the intermediate-resolution data cube suggests that this twist is due to a linearly-varying PA warp, from 32° (measured counterclockwise from the celestial North to the redshifted side of the kinematic major axis) at the outer edge of the disc to 342° at its centre (after going through

Table 4.1: Properties of the ALMA observing tracks analysed in this work.

Project code	Date	Array	Baseline range	T_{obs} (s)	MRS (arcsec, kpc)	FoV (arcsec, kpc)	Calibration	Reference
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
2015.1.00419.S	2016 June 12	12-m	15 m – 0.7 km	393	5.5, 0.5	24.4, 2.3	Pipeline	Ruffa et al. (2023)
2015.1.00419.S	2016 September 27	12-m	80 m – 3.2 km	756	3.3, 0.3	24.4, 2.3	Pipeline	Ruffa et al. (2023)
2016.2.00053.S	2017 July 24	7-m	9 m – 49 m	2359	30.2, 2.9	41.8, 4.0	Pipeline	Ruffa et al. (2023)
2022.1.01122.S	2023 September 16	12-m	92 m – 8.8 km	5977	1.0, 0.1	26.3, 2.5	Pipeline	This work
2022.1.01122.S	2023 September 16	12-m	83 m – 8.5 km	5984	1.0, 0.1	26.3, 2.5	Pipeline	This work

Notes. Columns: (1) Project code. (2) Observation date. (3) ALMA array. (4) Minimum and maximum baseline lengths. (5) Total on-source observing time (T_{obs}). (6) Maximum recoverable scale (MRS), i.e. largest angular scale that can be recovered with the given array configuration. (7) Field of view (FoV), i.e. primary beam FWHM. (8) Calibration method. (9) Reference in which the track data were first presented.

0°). Our higher-resolution data however indicate that the PA warp in the nuclear region is slightly larger than previously inferred (see Section 4.3.4). Crucially, the velocity map also clearly reveals an increase (from the outside-in) of the mean line-of-light velocities within the central $\approx 0''.15$ (≈ 15 pc) in radius, consistent with the expected gas motion around a central SMBH and absent from previous observations.

The intensity-weighted line-of-sight velocity dispersion map reveals a radial gradient of the CO velocity dispersion (σ_{gas}), the disc being mostly dynamically cold ($\sigma_{\text{gas}} \leq 20 \text{ km s}^{-1}$) but with a velocity dispersion enhancement in its central regions (σ_{gas} up to $\approx 80 \text{ km s}^{-1}$). It is however likely that this central enhancement is primarily caused by observational effects such as beam smearing, which often substantially increase the observed velocity dispersions in areas with large velocity gradients (such as the central regions of galaxies). The observed enhancement therefore does not necessarily imply an intrinsic velocity dispersion gradient.

4.2.3.3 Total flux

We obtain the spatially-integrated $^{12}\text{CO}(2-1)$ spectrum of NGC 1574 by summing all pixels (without applying any masks) within the central $2''.6 \times 2''.6$ region of the data cube, that covers all of the detected emission. The integrated spectrum (Figure 4.2) exhibits the typical double-horned shape of a rotating disc and only mild asymmetries. To compute the total $^{12}\text{CO}(2-1)$ flux of the data cube, we then integrate this spectrum across its full-width at zero-intensity (FWZI; the velocity range including all channels with line emission above 3σ). The total flux is $13.8 \pm 0.1 \pm 1.4 \text{ Jy km s}^{-1}$, where the second error is systematic and results from the typical $\approx 10\%$ ALMA flux calibration uncertainty.

We note, however, that the total flux measured is sensitive to the cleaning depth. When we create another data cube with the same imaging procedure but a shallower cleaning depth of $2.0 \sigma_{\text{RMS}}$, the resulting total flux is $15.6 \pm 0.1 \pm 1.6 \text{ Jy km s}^{-1}$. This is because fitting a clean Gaussian beam to the dirty beam systematically underestimates the effective area of the synthesised beam, especially for multi-configuration data, causing an overestimate of the total flux for any finite cleaning depth (see e.g. Jorsater & van Moorsel 1995, Bureau & Carignan 2002 and Zhang et al. 2025a for a detailed explanation of the issue). We thus follow the prescription of Jorsater & van Moorsel (1995) to derive the true total flux

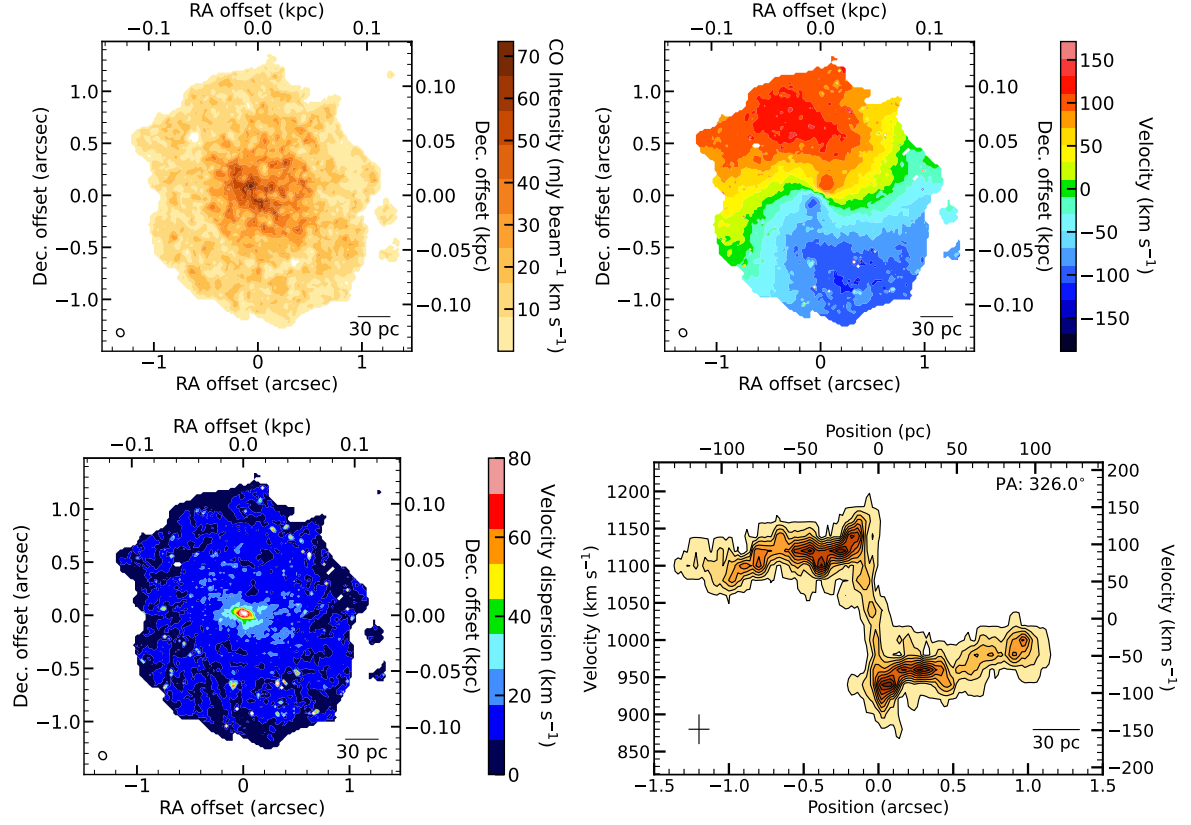


Figure 4.1: Data products of NGC 1574 created from our ALMA $^{12}\text{CO}(2-1)$ data cube. **Top-left:** Zeroth-moment (integrated-intensity) map. **Top-right:** First-moment (intensity-weighted mean line-of-sight velocity) map. **Bottom-left:** Second-moment (intensity-weighted line-of-sight velocity dispersion) map. The synthesised beam ($0''.078 \times 0''.069$) is shown as a black open ellipse in the bottom-left corner of each map. **Bottom-right:** Kinematic major-axis position-velocity diagram (PVD), covering the central $1''.5$ on either side of the kinematic centre. The cross in the bottom-left corner shows the synthesised beam FWHM along the kinematic major axis and the channel width. A 30 pc scale bar is shown in the bottom-right corner of each panel. In all panels, positions are measured relative to the best-fitting kinematic centre (see Section 4.3.4); velocities relative to the best-fitting systemic velocity along the right axis of the PVD.

(corresponding to an infinite cleaning depth) from the measured cleaned fluxes and total fluxes of the two data cubes cleaned to different depths. The corrected total flux obtained with this method is $10.1 \pm 0.1 \pm 1.0 \text{ Jy km s}^{-1}$. This is smaller than the integrated flux of $17.9 \pm 0.1 \pm 1.8 \text{ Jy km s}^{-1}$ computed from the intermediate-resolution data cube of [Ruffa et al. \(2023\)](#), but it agrees perfectly with the corrected total flux of these intermediate-resolution data (also $10.1 \pm 0.1 \pm 1.0 \text{ Jy km s}^{-1}$) after applying the same flux correction procedure. We thus adopt this corrected total flux, but stress that the total flux and associated flux correction have no impact on our derived SMBH mass. The measured FWZI ($\approx 260 \text{ km s}^{-1}$) and FWHM ($\approx 220 \text{ km s}^{-1}$) of the $^{12}\text{CO}(2-1)$ line both agree with those reported by [Ruffa et al. \(2023\)](#).

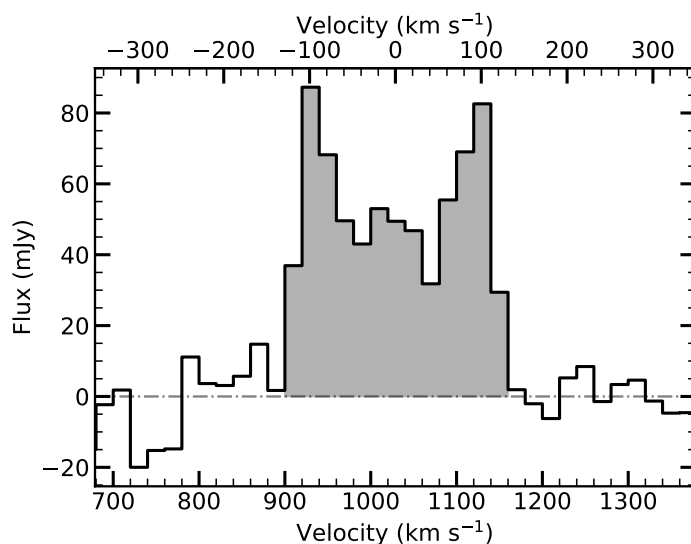


Figure 4.2: Integrated $^{12}\text{CO}(2-1)$ spectrum of NGC 1574, extracted from the central $2''.6 \times 2''.6$ region of the data cube, covering all of the detected emission. Velocities are measured relative to the best-fitting systemic velocity along the top axis (see Section 4.3.4). The dot-dashed line indicates the zero flux level. The shaded region highlights the channels within the line FWZI, used to measure the total flux. The spectrum shows the typical double-horned shape of a rotating disc.

4.2.3.4 PVD

The bottom-right panel of Figure 4.1 shows the kinematic major-axis position-velocity diagram (PVD) of NGC 1574 extracted along a PA of $326^\circ 0$ (the best-fitting innermost kinematic PA derived in Section 4.3.4) by summing all the flux within a 4-pixel-wide (approximately the synthesised beam FWHM) pseudo-slit centred on the major axis. When creating the PVD, we applied a mask similar to the one used for the moment maps, except that the data cube was

spatially smoothed using a Gaussian filter with a FWHM equal to 1.5 times that of the synthesised beam. We then selected all the pixels of the smoothed data cube with a flux density larger than $0.7 \sigma_{\text{RMS}}$ of the unsmoothed data cube. This approach avoids over-smoothing the central region, which could result in a mask that excludes faint emission in central high-velocity channels. The filter sizes and the flux threshold were again chosen by trial and error to ensure that the kinematics closest to the SMBH is clearly revealed while minimising noise. The resultant PVD reveals a prominent increase of the (projected) line-of-sight velocities toward the centre, up to $\approx 170 \text{ km s}^{-1}$, similar to the Keplerian circular velocity curve expected in the presence of a SMBH. By comparison, the intermediate-resolution observations of [Ruffa et al. \(2023\)](#) only revealed mild evidence of a central velocity enhancement, reaching a (projected) line-of-sight velocity of only $\approx 120 \text{ km s}^{-1}$ in the innermost regions due to a resolution insufficient to detect higher-velocity gas closer to the SMBH (beam smearing effects). This clearly demonstrates that the higher-resolution ALMA CO observations presented in this work are the first to spatially resolve the SMBH SoI of NGC 1574, in turn enabling a more accurate SMBH mass measurement, which we present in the following section.

4.2.4 Continuum emission

A continuum image was created using the multi-frequency synthesis mode of the CASA task `tclean` and Briggs weighting with a robust parameter of 0.5. We choose this weighting to marginally spatially resolve the source while still reaching a S/N of ≈ 200 . This results in synthesised beam FWHM of $0''.051 \times 0''.047$ ($\approx 4.9 \times 4.5 \text{ pc}^2$) at a PA of $-18^\circ.8$ and a RMS noise of $11 \mu\text{Jy beam}^{-1}$. The continuum image, shown in Figure 4.3, reveals only a point-like source spatially coincident with the galaxy’s kinematic centre (the best-fitting SMBH position in Section 4.3.4). We fit a two-dimensional (2D) Gaussian to the source with the CASA task `imfit`, revealing a marginally-resolved source with a deconvolved FWHM size of $0''.024 \times 0''.016$ ($\approx 2.3 \times 1.5 \text{ pc}^2$) and an integrated flux density of $2.37 \pm 0.03 \pm 0.24 \text{ mJy}$ (where the second uncertainty is systematic) at a central frequency of 231 GHz.

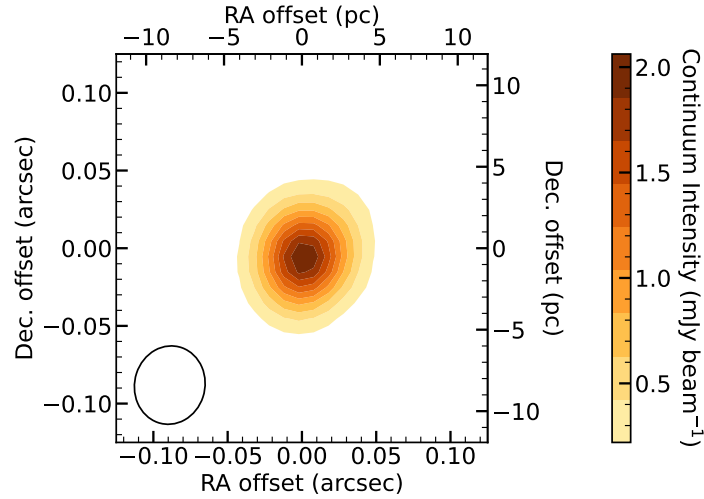


Figure 4.3: Central region ($0''.25 \times 0''.25$) of the NGC 1574 1.3-mm continuum image, showing the only source detected. Contour levels are equally spaced between the peak intensity of $2.02 \pm 0.02 \text{ mJy beam}^{-1}$ and 20 times the RMS noise. The synthesised beam ($0''.078 \times 0''.069$) is shown in the bottom-left corner as a black open ellipse. The source is only marginally spatially resolved.

4.3 Dynamical modelling

Our procedure to model the gas dynamics and infer the SMBH mass in NGC 1574 is similar to those used in previous WISDOM SMBH papers (e.g. [Davis et al., 2017, 2018](#); [Smith et al., 2019, 2021](#); [Zhang et al., 2025a](#)). In this section, we thus provide only an outline of our method and focus on the specifics of modelling NGC 1574.

We forward model the observed gas kinematics using the publicly-available PYTHON version of the KINEMATIC MOLECULAR SIMULATION³ (KINMS; [Davis et al. 2013a](#)) tool. KINMS takes an input model of a galaxy’s gas distribution and circular velocity profile to construct a simulated data cube, taking into account observational effects such as beam smearing, spatial and velocity binning, and line-of-sight projection. Then, the simulated data cube is fitted to the observed data cube using GASTIMATOR⁴, a Markov chain Monte Carlo (MCMC) Gibbs sampler, to infer the best-fitting model parameters and their posterior distributions. We define the χ^2 to be

$$\chi^2 \equiv \frac{1}{\sigma_{\text{RMS}}^2} \sum_i (\text{data}_i - \text{model}_i)^2, \quad (4.1)$$

³https://github.com/TimothyADavis/KinMS_fitter
<https://kinms.space/>

⁴<https://github.com/TimothyADavis/GAstimator>

where the summation is over all pixels of the data cube. We further re-scale the χ^2 by a factor of $1/\sqrt{2N}$, where $N = 36376$ is the number of pixels with detected emission, defined as the pixels included in the mask of Section 4.2.3.2. The χ^2 rescaling is equivalent to inflating the uncertainties by a factor of $(2N)^{1/4}$. This approach provides a realistic estimate of the systematic uncertainties, that usually dominate over the statistical uncertainties in large data sets. It was first proposed by [van den Bosch & van de Ven \(2009\)](#) when using χ^2 confidence levels and was later adapted by [Mitzkus et al. \(2017\)](#) for Bayesian methods. [Smith et al. \(2019\)](#) and [Zhang et al. \(2025a\)](#) showed that the uncertainties of the gas dynamical parameters estimated using this approach are consistent with those evaluated from bootstrapping and are thus reliable.

We also assume a diagonal covariance matrix when evaluating the χ^2 , ignoring the spatial correlations between nearby pixels introduced by the synthesised beam, as inverting the full covariance matrix would be computationally intractable. [Davis et al. \(2017\)](#) and [Davis et al. \(2018\)](#) adopted the full covariance matrix, but had to limit the fitting to a small region to reduce the number of pixels and thus compensate for the substantial increase of the computational time. As our high-resolution data cube has a huge number of pixels, limiting the number of pixels in the fitting would bias the results much more than assuming a diagonal covariance matrix. In any case, the covariance is much smaller than the χ^2 rescaling described above, and so would have a negligible impact on the best-fitting SMBH mass and its uncertainty.

4.3.1 Gas distribution

Although our high spatial resolution observations spatially resolve clumpy structures in the intensity distribution (see Figure 4.1), the azimuthally-averaged intensity profile is still consistent with an exponential profile, so we adopt the approach of [Ruffa et al. \(2023\)](#) and model the spatial distribution of the CO(2-1) emission as a simple exponential disc:

$$\Sigma_{\text{gas}}(R) \propto e^{-R/R_s} \quad , \quad (4.2)$$

where Σ_{gas} is the surface brightness of the CO gas, R the galactocentric cylindrical radius and R_s the exponential disc scale length, a free parameter of the model. We have confirmed that adopting a non-parametric gas distribution, such as using the CLEAN components generated

by `tclean` as the input cloud model (e.g. [Smith et al., 2019](#); [Zhang et al., 2025a](#)), does not affect the best-fitting parameters presented in Section 4.3.4 by more than 1σ but introduces additional systematic uncertainties due to the finite cleaning depth. We have also confirmed that the best-fitting exponential surface brightness profile agrees well with the observed deconvolved, azimuthally-averaged gas intensity profile obtained by sampling the CLEAN components.

4.3.2 Mass model

To model the mass distribution of NGC 1574, we assume that the total mass in the central $\approx 0.3 \times 0.3 \text{ kpc}^2$ region we observe is dominated by the SMBH and stellar masses, ignoring the contributions of other potential mass components such as gas and dark matter. This assumption is justified as the total molecular gas mass of NGC 1574 is less than 5% of the total SMBH and stellar masses enclosed within the extent of the gas disc ([Ruffa et al., 2023](#); much less within the SMBH SoI), while the dark matter mass is usually negligible at the centres of massive galaxies (e.g. [Cappellari et al., 2013](#); [Zhu et al., 2024](#)).

Our stellar mass distribution model is identical to that of [Ruffa et al. \(2023\)](#). A multi-Gaussian expansion (MGE; [Emsellem et al. 1994](#)) was performed on a *Hubble Space Telescope* (*HST*) Wide-Field Camera 2 F606W-band image, using the Python version of the `mge_fit_sectors_regularized` procedure⁵ of [Cappellari \(2002b\)](#), to model the projected stellar light distribution of NGC 1574. The parameters of the best-fitting two-dimensional (2D) Gaussians, spatially deconvolved from *HST*'s point-spread function (computed using the TINYTIM package⁶; [Krist et al. 2011](#)), are listed in Table C1 of [Ruffa et al. \(2023\)](#). In each forward model, the 2D Gaussians are analytically deprojected to a three-dimensional (3D) axisymmetric light distribution given the inclination i , a free parameter of the model. The 3D stellar light distribution is then multiplied by a spatially constant mass-to-light ratio (M/L), another free parameter of the model, to obtain the 3D stellar mass volume density. Here, we assume that the light in the F606W band is not contaminated by the galactic nucleus, as there is no sign of an AGN in this galaxy from any existing observation. Finally, we add a freely-varying point mass (M_{BH}) at the centre to represent the SMBH, yet another free

⁵Version 5.0 from <https://pypi.org/project/mgefit/>

⁶Version 7.5 from <https://github.com/spacetelescope/tinytim/releases/tag/7.5>

parameter of the model.

We assume that the gas particles are in circular motions when deriving the gas kinematics from the input mass distribution. This assumption is largely consistent with the observed gas kinematics of NGC 1574, despite the signs of mild non-circular motions discussed in Section 4.4.1. We compute the circular velocity curve corresponding to the combined (stellar + SMBH) mass distribution using the `mge_vcirc` procedure of the JEANS ANISOTROPIC MODELLING (JAM) package⁷ of Cappellari (2008).

Finally, we add a spatially-constant velocity dispersion (σ_{gas}) to the resultant circular velocity curve, another free parameter of the model. We assume that the gas velocity dispersion does not vary spatially because, as shown in Figure 4.1, potential variations (except the artificial central increase due to beam smearing) are much smaller than the channel width of 20 km s^{-1} and the central velocity enhancement caused by the SMBH. Models with spatially variable velocity dispersion profiles thus do not fit the data better nor noticeably change the best-fitting SMBH mass presented in Section 4.3.4.

4.3.3 Position angle warp

To explain the observed kinematic twist of the gas disc, we adopt a PA warp model in which the PA varies linearly with radius from PA_{inner} at $R = 0$ (the disc's centre) to PA_{outer} at $R = 1''.2$ (approximately the outer edge of the disc). This warp model is the same as the one adopted by Ruffa et al. (2023). Yet, the innermost PA that we can model will be closer to the true PA at $R = 0$, thanks to the higher spatial resolution of our data. This should improve the precision and accuracy of our measurements.

Our final dynamical model thus has a total of five free parameters describing the molecular gas disc (i , PA_{inner} , PA_{outer} , σ_{gas} and R_s), two parameters describing the mass distribution (M_{BH} and M/L) and four "nuisance" parameters (offsets of the kinematic centre from the observation's phase centre, systemic velocity of the galaxy and total intensity of the molecular gas disc).

⁷Version 7.2 from <https://pypi.org/project/jamspy/>

4.3.4 Results

We fit the dynamical model to the entire data cube (without applying any mask) using an MCMC chain of 2×10^5 steps. We discard the first 10^4 steps as the burn-in to ensure the chain has converged, and confirm that the posterior distribution of each parameter is fully sampled after 2×10^5 steps. Table 4.2 lists the best-fitting parameters and their 1σ (68.3% confidence level) and 3σ (99.7% confidence level) uncertainties. Figure 4.4 shows each non-nuisance parameter's one-dimensional (1D) marginalised posterior distribution and the 2D joint posterior distributions between them.

We obtain a best-fitting SMBH mass $M_{\text{BH}} = (6.2 \pm 1.2) \times 10^7 M_{\odot}$ and a F606W-band M/L of $7.13^{+0.75}_{-0.62} M_{\odot}/L_{\odot, \text{F606W}}$. The PA changes linearly from $\text{PA}_{\text{inner}} = 326.0^{+1.9}_{-1.8}$ at $R = 0$ to $\text{PA}_{\text{outer}} = 39.0 \pm 1.6$ at $R = 1''.2$ (after going through 360°). Our SMBH mass is 1.7σ smaller than but is statistically consistent with the SMBH mass of $(1.0 \pm 0.2) \times 10^8 M_{\odot}$ derived from the intermediate-resolution data of Ruffa et al. (2023), that did not spatially resolve the SoI. The M/L is high but consistent with the $B - R$ colour of the galaxy $B - R = 1.47 \pm 0.23$ (Gao et al., 2022). According to the M/L -colour relation of Bell et al. (2003) from stellar population synthesis models, the relation between $B - R$ colour and V -band M/L is $\log[(M/L_V)/(M_{\odot}/L_{\odot, V})] = 0.816(B - R) - 0.633$. Thus, the expected V -band M/L is $M/L_V = 3.7^{+2.0}_{-1.3} {}^{+1.4}_{-0.5} M_{\odot}/L_{\odot, V}$, where the first uncertainty is statistical and the second uncertainty results from the ≈ 0.1 dex scatter of the relation. Our dynamically derived M/L_{F606W} is thus consistent with the range expected from the colour of the stellar population. It also agrees nicely with $M/L_{\text{F606W}} = 7.33^{+0.92}_{-0.83} M_{\odot}/L_{\odot, \text{F606W}}$ from Ruffa et al. (2023). However, the innermost PA is smaller than the $342^\circ \pm 0^\circ.02$ of Ruffa et al. (2023) by 8σ , implying that the previous model underestimated the amplitude of the PA warp in the central region (as expected, given that it was spatially unresolved in their data). Nevertheless, the PA warp within the previously unresolved central $0''.2 \times 0''.2$ region is still part of the large-scale linear PA warp. A two-component PA warp model that allows both the PA and its radial gradient to change abruptly at $R = 0''.2$ does not result in significantly different inner and outer PAs nor improves the Bayesian information criterion (BIC; $\text{BIC} \equiv k \ln N + \chi^2$, where k is the number of model parameters). The rest of the model parameters all agree within the uncertainties with those of Ruffa et al. (2023).

Table 4.2: Best-fitting model parameters and associated uncertainties.

Parameter	Search range	Best fit	1σ uncertainty	3σ uncertainty
Mass model				
$\log (M_{\text{BH}}/M_{\odot})$	6 – 10	7.79	± 0.08	$-0.24, +0.22$
Stellar M/L ($M_{\odot}/L_{\odot, \text{F606W}}$)	1 – 15	7.1	$-0.6, +0.7$	$-1.6, +2.4$
Molecular gas disc				
Inner position angle (degree)	300 – 380	326.0	$-1.8, +1.9$	$-5.2, +5.5$
Outer position angle (degree)	0 – 70	39.0	± 1.6	$-5.0, +5.3$
Inclination (degree)	10 – 40	26.7	$-1.4, +1.3$	$-3.7, +3.5$
Velocity dispersion (km s^{-1})	5 – 40	11.3	$-1.0, +1.1$	$-3.0, +3.9$
Scale length (arcsec)	0.1 – 1.2	0.55	± 0.03	$-0.10, +0.12$
Nuisance parameters				
Disc integrated intensity (Jy km s^{-1})	5 – 30	13.1	± 0.8	$-2.5, +2.4$
Kinematic centre X offset (arcsec)	$-1.5 - 1.5$	0.00	± 0.01	± 0.02
Kinematic centre Y offset (arcsec)	$-1.5 - 1.5$	0.02	± 0.01	± 0.02
Systemic velocity (km s^{-1})	950 – 1100	1028.7	± 1.2	$-3.8, +3.5$

Notes. The X and Y offsets are measured relative to the phase centre of the data cube, $\text{RA} = 04^{\text{h}}21^{\text{m}}58^{\text{s}}.75$, $\text{Dec.} = -56^{\circ}58'28''.43$ (J2000.0).

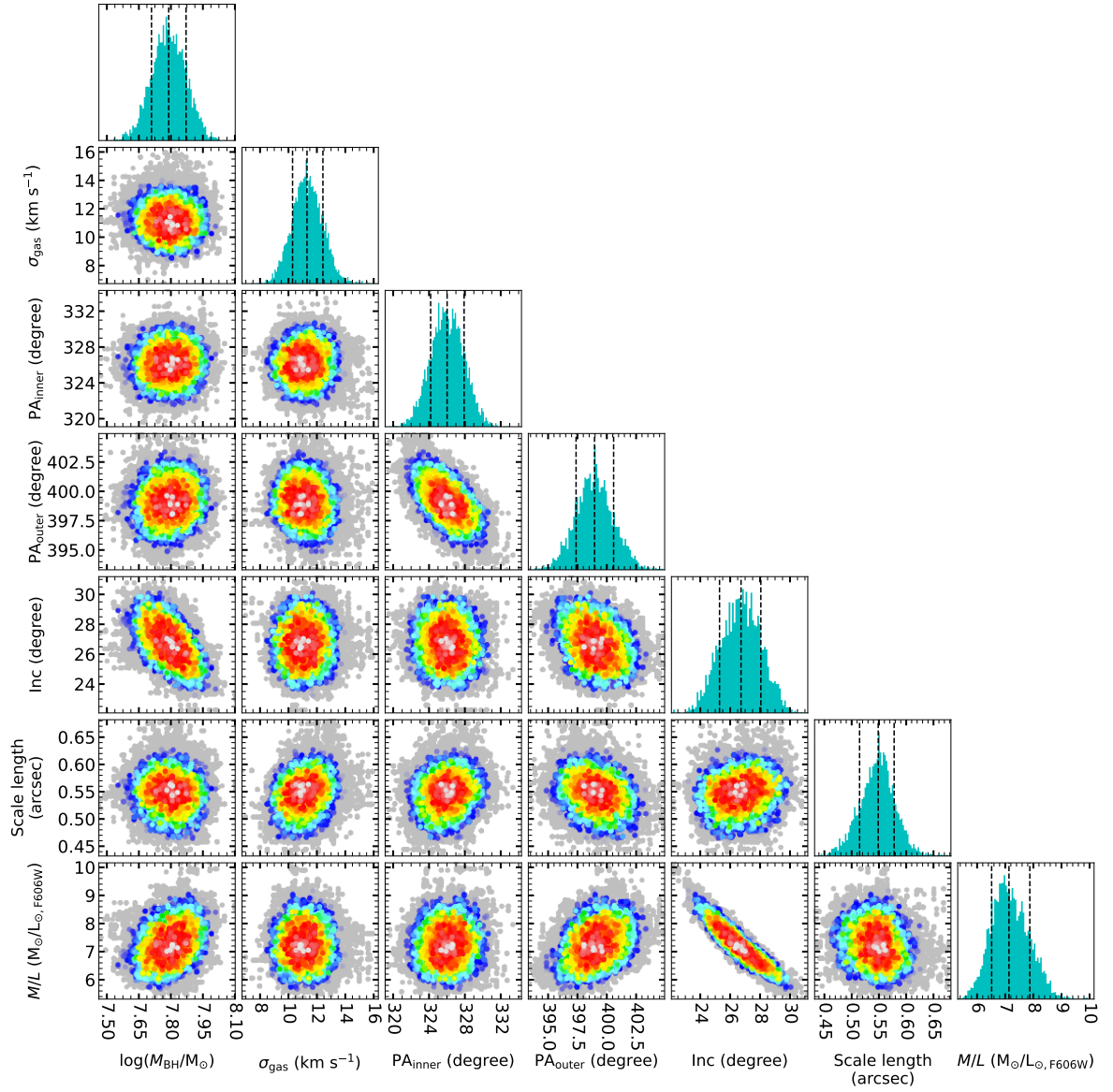


Figure 4.4: Corner plots showing the 2D joint posterior distributions between non-nuisance model parameters. The colours represent increasing confidence levels from 68.3% (red, 1σ) to 99.7% (grey, 3σ). The histograms show each parameter’s 1D marginalised posterior distribution; the dashed lines indicate the medians and the 1σ confidence intervals.

Figure 4.5 compares the (luminosity-weighted mean line-of-sight) velocity map derived from the high-resolution ALMA data of NGC 1574 (panel a) with the equivalent velocity map of the best-fitting model from this work (panel b) and that of the best-fitting model of [Ruffa et al. \(2023; panel c\)](#). Panels (d) and (e) show the corresponding velocity residual (data – model) maps of the two models. The model derived using only the intermediate-resolution data slightly underestimates the amplitude of the PA warp in the previously unresolved central $0''.2 \times 0''.2$ region, leading to larger velocity residuals.

Figure 4.6 then compares the kinematic major-axis PVD of the high-resolution data with

those of three different models: a model with no SMBH (left), the best-fitting model from this work (centre) and the best-fitting model of [Ruffa et al. \(2023\)](#) with a slightly more massive SMBH. The model with no SMBH fails to reproduce the central velocity rise, while the best-fitting model derived using only the intermediate-resolution data overshoots the velocity increase due to its slightly larger SMBH mass. By contrast, the best-fitting model from this work accurately reproduces the PVD at most positions and velocities. The slight difference between the width of the data PVD and that of the model PVD near a major-axis position of $0''.1$ is likely due to small non-circular motions, which are also noticeable in the velocity residual map (panel d of Figure 4.5; discussed further in Section 4.4.1).

We finally compare the models' reduced χ^2 (χ_r^2) statistics measured using the high-resolution data. Our new best-fitting model has $\chi_r^2 = 2.14$, while the previous model of [Ruffa et al. \(2023\)](#) has $\chi_r^2 = 2.37$ (with the same free parameters). This new measurement of NGC 1574 thus demonstrates the importance of high-resolution observations, that spatially-resolve the SMBHs' SoI, for accurate SMBH mass measurements and dynamical models.

4.4 Discussion

4.4.1 Main uncertainties

The statistical uncertainty of the SMBH mass in NGC 1574 is substantially increased by the uncertainty of the inclination. This is because M_{BH} is proportional to the square of the deprojected rotational velocity v_{rot} , $M_{\text{BH}} \propto v_{\text{rot}}^2$, and v_{rot} is related to the observed line-of-sight velocity v_{obs} by $v_{\text{rot}} = v_{\text{obs}}/\sin i$, so that $M_{\text{BH}} \propto \sin^{-2} i$. This implies that the SMBH mass uncertainty is largest when the galaxy is nearly face-on, as is the case for NGC 1574 ($i \approx 27^\circ$). The SMBH mass uncertainty would shrink from 19% to 16% with no inclination uncertainty.

Contrary to many other measurements, the SMBH mass of NGC 1574 is not anti-correlated with its stellar M/L , as our high spatial resolution observations spatially resolve the SMBH SoI and trace the inner Keplerian part of the rotation curve, which is largely dominated by the SMBH and is relatively independent of the stars. This breaks the mass degeneracy. The positive correlation between SMBH mass and stellar M/L is then a by-product of the SMBH

mass – inclination anti-correlation and the M/L – inclination anti-correlation, as a lower inclination implies higher deprojected velocities and thus both a higher SMBH mass and a higher stellar mass (Smith et al., 2019).

A potential source of systematic uncertainty in our measurement is the assumption that the gas particles move on circular orbits. Indeed, the residual (luminosity-weighted mean line-of-sight) velocity map of our model (panel d of Figure 4.5) shows spiral arm-like residuals up to $\pm 55 \text{ km s}^{-1}$ (after deprojection), suggesting mild non-circular motions and recovering the finding of Ruffa et al. (2023). This could result from the non-axisymmetric potential induced by the nuclear stellar bar of NGC 1574, potentially producing shocks along the bar edges that would lead to gas inflows/outflows down to the scale of the circumnuclear molecular gas disc observed. However, the non-circular motions are too small compared to the circular velocities detected within the SMBH SoI (maximum $\approx 380 \text{ km s}^{-1}$ after deprojection) to substantially affect the SMBH mass measurement.

Another potential source of systematic uncertainty in our SMBH mass measurements is the effect of dust extinction on the MGE fit and associated stellar mass distribution (e.g. Boizelle et al., 2021; Cohn et al., 2024), but the *HST* image of NGC 1574 does not show any obvious sign of dust obscuration.

As is customary for dynamical SMBH mass measurements, we choose not to include in the SMBH mass uncertainty budget the 10% uncertainty introduced by the distance, which is directly proportional to M_{BH} . Rescaling our results to a different distance therefore does not require redoing any dynamical modelling.

4.4.2 Resolving the SMBH sphere of influence

An empirical approximation for the radius of a SMBH’s SoI is $R_{\text{SoI}} \equiv G M_{\text{BH}} / \sigma_e^2$. Given our updated SMBH mass (see Section 4.3.4) and $\sigma_e = 216 \pm 16 \text{ km s}^{-1}$ (Bernardi et al., 2002), R_{SoI} is thus 5.7 pc or $\approx 0''.059$. The angular resolution of our observation, $0''.074$, is thus slightly worse than R_{SoI} . However, R_{SoI} is only a proxy for the more formal and physically-motivated definition of the SoI, that is the equality radius R_{eq} at which the enclosed stellar mass equals the SMBH mass:

$$M_*(R = R_{\text{eq}}) = M_{\text{BH}} . \quad (4.3)$$

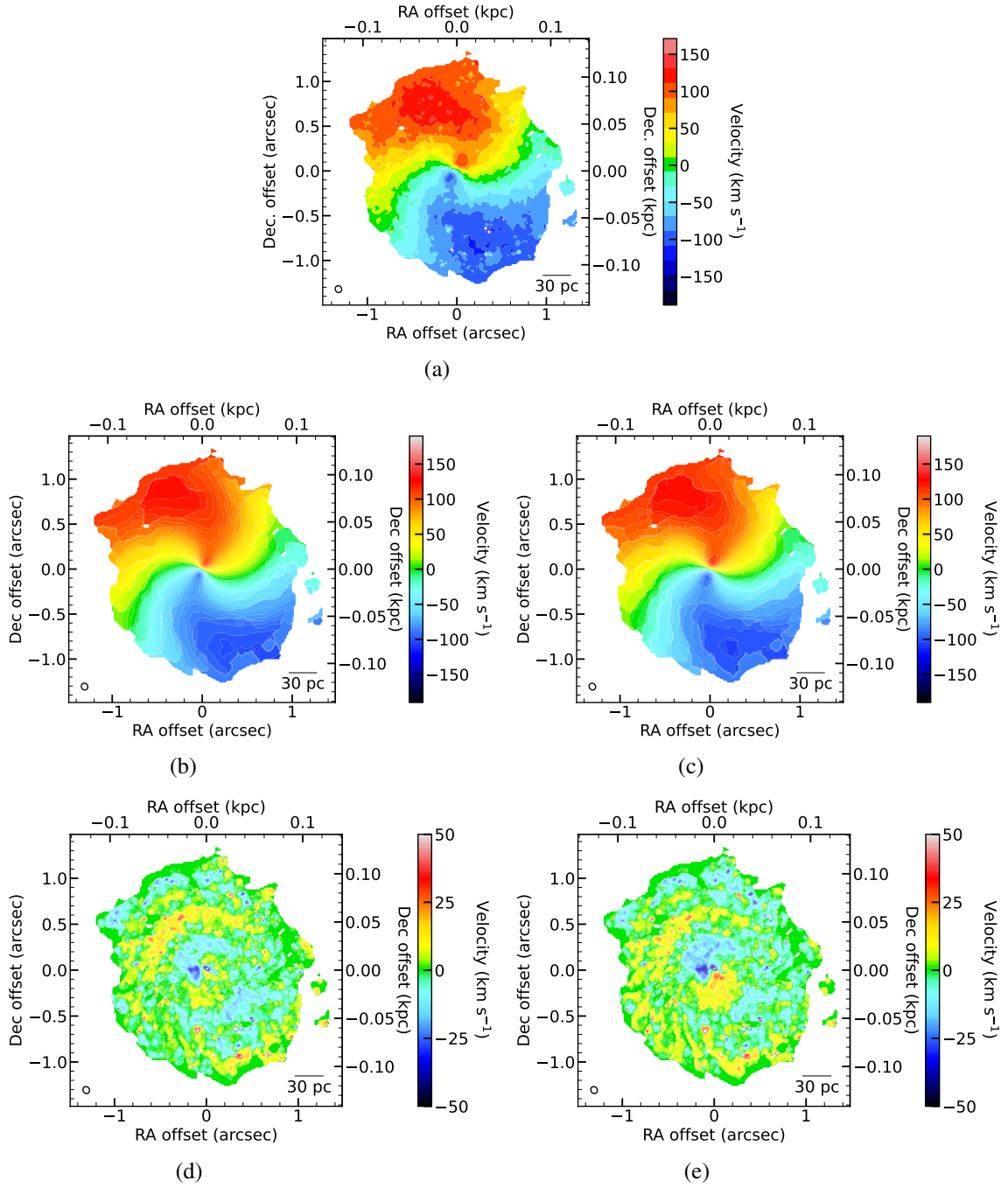


Figure 4.5: (a) First-moment (intensity-weighted mean line-of-sight velocity) map of the NGC 1574 data cube. (b) – (c) First-moment maps of the best-fitting dynamical model from (b) this work and (c) the best-fitting model of [Ruffa et al. \(2023\)](#), obtained by fitting intermediate spatial resolution observations. (d) – (e) Corresponding first-moment residual maps (data – model). The synthesised beam ($0''.078 \times 0''.069$) is shown as a black open ellipse in the bottom-left corner of each panel, while a 30 pc scale bar is shown in the bottom-right corner of each panel. Positions are measured relative to the best-fitting kinematic centre; velocities relative to the best-fitting systemic velocity. The model derived using only intermediate-resolution observations slightly underestimates the amplitude of the PA warp in the central $0''.2 \times 0''.2$ region, leading to larger velocity residuals.

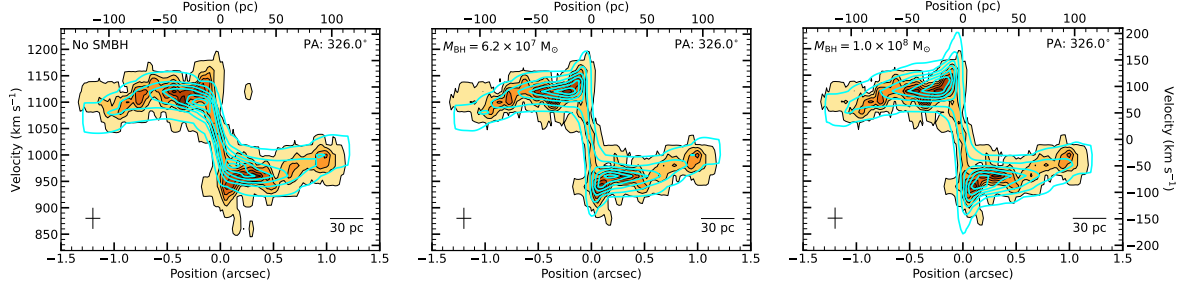


Figure 4.6: Observed kinematic major-axis position-velocity diagram of NGC 1574 (orange scale with black contours), overlaid with the PVDs of different models (cyan contours): no SMBH (left), best-fitting model from this work (centre) and best-fitting model of [Ruffa et al. \(2023\)](#) derived using only intermediate spatial resolution observations that do not spatially resolve the SMBH SoI (right). Positions are measured relative to the best-fitting kinematic centre; velocities relative to the best-fitting systemic velocity along the right axis. The cross in the bottom-left corner of each panel shows the synthesised beam FWHM along the kinematic major axis and the channel width. A scale bar is shown in the bottom-right corner of each panel. Our best-fitting model reproduces the Keplerian velocity increase toward the galaxy centre nicely, but the best-fitting model of [Ruffa et al. \(2023\)](#) overshoots the velocity rise. This demonstrates that high-resolution observations that spatially resolve the SoI are necessary for accurate SMBH mass measurements.

To compute R_{eq} , we derive the cumulative mass distribution of the stars using the MGE components of [Ruffa et al. \(2023\)](#), our best-fitting stellar M/L and the `mge_radial_mass` procedure of the JAM package. We then calculate the radius where the cumulative stellar mass equals the SMBH mass, yielding $R_{\text{eq}} = 0''.12 \pm 0''.03$ (11.0 ± 2.6 pc), where the uncertainty arises from the uncertainties of the stellar M/L and SMBH mass. Figure 4.7 compares the relative contributions of the stars and the SMBH to the total cumulative mass distribution of NGC 1574, and shows R_{eq} as a vertical black line (as well as R_{SoI} and the spatial extent of the synthesised beam, R_{beam}). The high-resolution data presented in this chapter spatially resolve R_{eq} by a factor of $R_{\text{eq}}/R_{\text{beam}} \approx 1.6$, and are thus the first to fully spatially resolve the SMBH SoI of NGC 1574. For this reason, our SMBH mass measurement is more accurate than that reported by [Ruffa et al. \(2023\)](#) using observations with an angular resolution of $0''.17$ (thus larger than both R_{SoI} and R_{eq}).

In practice, the radius of the innermost kinematic tracer that a molecular gas observation can detect is usually slightly smaller than R_{beam} , as positions can be measured more precisely than the FWHM of the synthesised beam ([Zhang et al., 2024](#)). We thus estimate the radius of the innermost detected kinematic tracer (R_{min}) of our high-resolution observations from the PVD (Figure 4.6), following the procedure outlined by [Zhang et al. \(2024\)](#). This yields

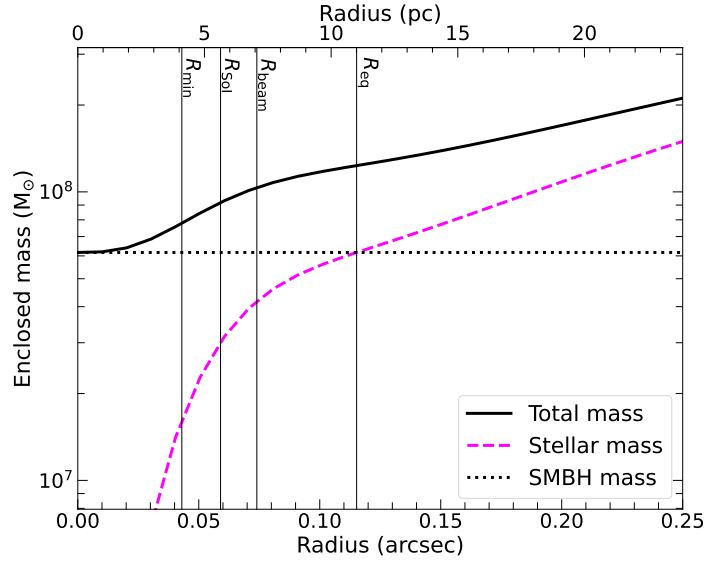


Figure 4.7: Cumulative mass distribution of NGC 1574, showing the relative contributions of stars (magenta dashed line) and the SMBH (black dotted line) to the total enclosed mass (black solid line). The four vertical black lines show the radius of the innermost detected kinematic tracer $R_{\min}$, the traditionally-defined radius of the SMBH SoI R_{SoI} , the mean spatial extent of the synthesised beam R_{beam} and the radius at which the enclosed stellar mass equals the SMBH mass R_{eq} .

$R_{\min} \approx 0''.043$ or 4.1 pc (shown by another vertical black line in Figure 4.7), smaller than both R_{SoI} ($R_{\min}/R_{\text{SoI}} = 0.72$) and R_{eq} ($R_{\min}/R_{\text{eq}} = 0.37$) and well within the region where the SMBH dominates over the stars. As a result, our observations probe deep into the region where the gravitational influence of the SMBH dominates the gas kinematics and detect a prominent Keplerian velocity increase, even though the synthesised beam only resolves R_{eq} by a factor of ≈ 2 .

Although our high-resolution measurement does not formally provide a better statistical uncertainty ($\approx 20\%$) than that of the previous intermediate-resolution measurement, this is partly because the previous measurement predicted a larger SMBH mass and was thus overconfident in the significance of the SMBH detection, resulting in an overconfident uncertainty. Other attempts to improve molecular-gas-dynamical SMBH mass measurements using observations that highly resolve the SoI (e.g. [Ruffa et al. 2023](#) for NGC 4261 and [Zhang et al. 2025a](#) for NGC 383) all led to statistical uncertainties substantially better than those of lower-resolution measurements. These high-resolution measurements also revealed that the lower-resolution measurements tend to slightly overestimate the SMBH masses, implying that measurements that **only** marginally resolve the SoI might be systematically inaccurate. Hence, it is also crucial to obtain precise and accurate SMBH mass measurements using observations

that highly resolve the SoI to identify whether the observed scatters of the SMBH–galaxy scaling relations represent intrinsic scatters caused by variations of galactic properties (e.g. morphology or mass; [McConnell & Ma, 2013](#)) or are dominated by measurement inaccuracies. Additionally, high-resolution observations of molecular gas around the SMBH can probe the dynamics within the circumnuclear disk down to a few times $10^4 R_{\text{Sch}}$, revealing physical processes directly related to the fuelling and feedback of SMBHs, such as radial motions and/or alignment between molecular gas disk and radio jet ([Zhang et al., 2025a](#)).

4.4.3 Comparison with SMBH – galaxy property scaling relations

To place our SMBH mass measurement in the context of SMBH – galaxy property scaling relations, we compare in Figure 4.8 the SMBH mass and effective stellar velocity dispersion ($\sigma_e = 216 \pm 16 \text{ km s}^{-1}$; [Bernardi et al. 2002](#)) of NGC 1574 (red data point) with the best-fitting $M_{\text{BH}} - \sigma_e$ relation of [van den Bosch \(2016\)](#); black solid line). The dashed and the dotted lines show one and three times the observed scatter of the relation, respectively. We also show other galaxies with SMBH masses derived using molecular gas kinematics (blue data points; compiled by [Zhang et al. 2024](#)) and other methods (grey data points; compiled by [van den Bosch 2016](#)). Our updated SMBH mass places NGC 1574 slightly below the observed 1σ scatter of the relation, indicating that the galaxy has one of the smallest SMBH masses among galaxies with $\sigma_e \geq 200 \text{ km s}^{-1}$. This does not make NGC 1574 a significant outlier of the $M_{\text{BH}} - \sigma_e$ relation, as it is well within the observed 3σ scatter. However, the intrinsic scatter might be substantially smaller than the observed scatter and needs to be constrained by more precise and accurate SMBH mass measurements.

Figure 4.9 similarly compares the SMBH mass and total stellar mass M_* of NGC 1574 with those of other galaxies with SMBH mass measurements and with the best-fitting $M_{\text{BH}} - M_*$ relation of [van den Bosch \(2016\)](#). The stellar masses of galaxies whose SMBHs were measured using the molecular gas method were derived from the published MGE models if available. Our dynamically derived SMBH mass of NGC 1574 is well within the observed 1σ scatter of the $M_{\text{BH}} - M_*$ relation. In other words, the SMBH of NGC 1574 is not less massive than that expected from the $M_{\text{BH}} - M_*$ relation. The slight offset of NGC 1574 from the mean $M_{\text{BH}} - \sigma_e$ relation is thus more likely the result of an overly large σ_e rather than a

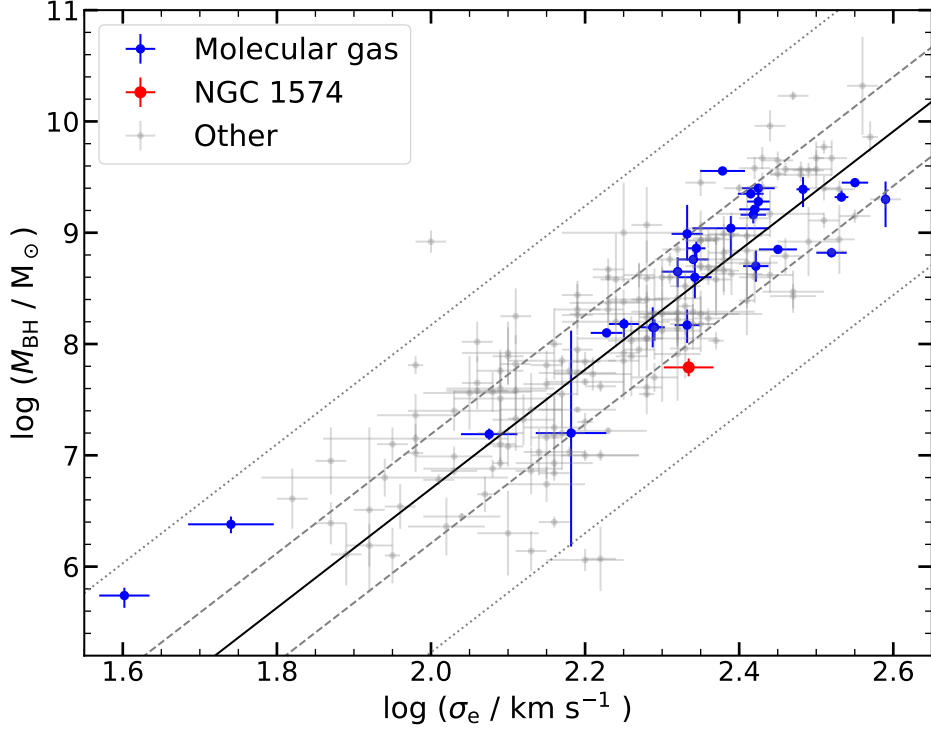


Figure 4.8: SMBH mass (M_{BH}) and effective stellar velocity dispersion (σ_e) of NGC 1574 (red data point), compared to those of other galaxies with SMBH mass measurements derived using molecular gas kinematics (blue data points) and other methods (grey data points). The best-fitting $M_{\text{BH}} - \sigma_e$ relation of [van den Bosch \(2016\)](#) is shown as a black solid line, with one and three times the observed scatter around the relation shown with dashed and dotted lines, respectively. Our SMBH mass measurement places NGC 1574 slightly below the relation’s 1σ scatter.

truly small M_{BH} . This could be caused by motions induced by the stellar bar of the galaxy.

4.5 Conclusions

We presented a measurement of the SMBH mass of NGC 1574, a lenticular galaxy hosting a nuclear stellar bar. ALMA observations of the $^{12}\text{CO}(2-1)$ emission line with synthesised beam FWHM of $0''.078 \times 0''.070$ ($\approx 7.5 \times 6.7 \text{ pc}^2$) reveal a prominent velocity increase towards the centre of the molecular gas disc, a kinematic twist associated with a PA warp throughout the disc, and evidence of mild ($< 15\%$ of the typical circular velocities) non-circular motions at the very centre, potentially induced by the nuclear bar. Our forward dynamical modelling of the data cube yields a SMBH mass $M_{\text{BH}} = (6.2 \pm 1.2) \times 10^7 M_\odot$, slightly smaller than but statistically consistent with the previous SMBH mass of $(1.0 \pm 0.2) \times 10^8 M_\odot$ derived using only intermediate-resolution data that do not spatially resolve the SMBH SoI (R_{SoI} or R_{eq} ; [Ruffa et al. 2023](#)). Our new SMBH mass places NGC 1574 slightly below the 1σ scatter

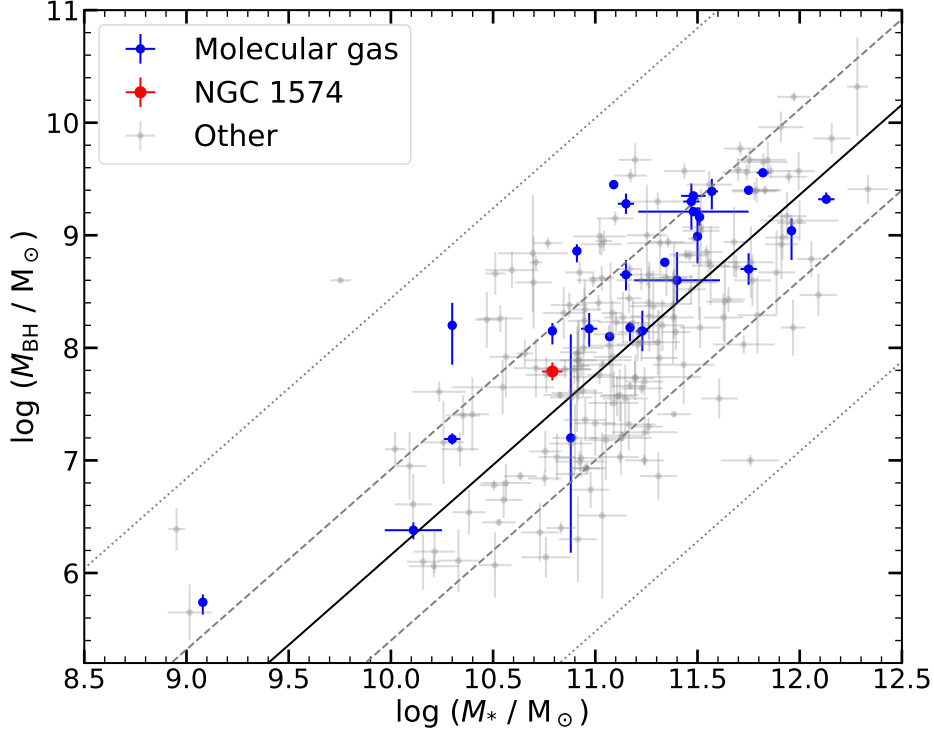


Figure 4.9: As Figure 4.8, but for the SMBH masses (M_{BH}) and total stellar masses (M_*). Our SMBH mass measurement places NGC 1574 within the 1σ scatter of the relation.

of the $M_{\text{BH}} - \sigma_e$ relation of [van den Bosch \(2016\)](#). The PA warp is most consistent with a linear radial variation from $\text{PA}_{\text{inner}} = 326.0^{+1.9}_{-1.8}$ at $R = 0$ to $\text{PA}_{\text{outer}} = 39.0 \pm 1.6$ at $R = 1''.2$, suggesting that the PA warp within the central $0''.2 \times 0''.2$ region is larger than previously thought. The F606W-band M/L is $6.88^{+0.69}_{-0.49} M_{\odot}/L_{\odot, \text{F606W}}$, agreeing with the result of [Ruffa et al. \(2023\)](#) within 1σ .

Our new high spatial resolution measurement of the SMBH mass of NGC 1574 does not improve the precision of the SMBH mass ($\approx 20\%$), but it considerably improves its accuracy and the PA warp model, as our observations are the first to spatially resolve the SMBH SoI of NGC 1574 in terms of both the synthesised beam size R_{beam} ($R_{\text{beam}}/R_{\text{eq}} = 0.64$) and the radius of the innermost detected kinematic tracer R_{min} ($R_{\text{min}}/R_{\text{eq}} = 0.37$). Our study thus again emphasises the importance of high-resolution observations, that fully spatially resolve the gravitational spheres of influence of the SMBHs, for accurate M_{BH} measurements and gas dynamical models.

Chapter 5

Prospects for the first dynamical supermassive black hole mass measurements using cold gas kinematics at high redshifts

5.1 Introduction

Although scaling relations between supermassive black hole (SMBH) mass (M_{BH}) and host galaxy properties have been extensively studied in the local universe (e.g. [Ferrarese & Merritt, 2000](#); [Gebhardt et al., 2000](#); [Beifiori et al., 2012](#); [Kormendy & Ho, 2013](#)), they remain highly uncertain beyond a redshift $z \approx 0$, due to the lack of direct dynamical measurements of SMBH masses at higher redshifts.

At intermediate redshifts ($0.1 \lesssim z \lesssim 4.5$), reverberation mapping (RM; e.g. [Blandford & McKee, 1982](#); [Shen et al., 2024](#)) provides relatively accurate SMBH mass determinations (see e.g. [Cackett et al. 2021](#) for a review). As discussed in Chapter 1, RM measures the radius of the broad-line region (BLR) of an active galactic nucleus (AGN) using the time delay between the variability of the continuum emission and that of the BLR line emission, allowing SMBH masses to be determined without spatially resolving their spheres of influence (SoI). Nevertheless, SMBH masses inferred using RM depend on a scaling factor f that accounts for the unknown geometry of the BLR. The average value of f is typically calibrated using the

local SMBH mass – effective stellar velocity dispersion ($M_{\text{BH}} - \sigma_e$) relation, ranging from 2.8 (Graham et al., 2011) to 5.5 (Onken et al., 2004), and the intrinsic scatter of f across individual targets is around a factor of 2 (Shen et al., 2024). Most SMBH masses inferred using RM thus have substantial calibration (i.e. systematic) uncertainties. At high redshifts ($z \gtrsim 4.5$), almost all SMBH mass determinations are from the "single-epoch" method (e.g. McLure & Jarvis, 2002; Wu & Shen, 2022), which relies on empirical correlations between BLR radius and emission line luminosity ($R - L$ relations), calibrated using RM measurements. The typical scatter of single-epoch masses around RM masses is another factor of ≈ 3 (Shen et al., 2024).

Using single-epoch measurements, recent *James Webb Space Telescope* (*JWST*) observations of high-redshift quasars (e.g. Harikane et al., 2023; Übler et al., 2023; Stone et al., 2024b; Maiolino et al., 2024; Juodžbalis et al., 2024, 2026) revealed a population of SMBHs that are overmassive by orders of magnitude relative to the local SMBH mass – stellar mass ($M_{\text{BH}} - M_*$) relation. These overmassive SMBHs imply rapid black hole growth in the early Universe, either from massive black hole seeds near the Eddington accretion rate or lighter black hole seeds with super-Eddington accretion (e.g. Larson et al., 2023; Kokorev et al., 2023; Bogdán et al., 2024; Suh et al., 2025; Mehta et al., 2026; Bhowmick et al., 2026). Nevertheless, there are growing concerns about the uncertainties of these indirect M_{BH} measurements (e.g. Lupi et al., 2024; Bertemes et al., 2025) and the biases of such observations towards the most luminous quasars (e.g. Silverman et al., 2025; Li et al., 2025). Direct dynamical M_{BH} measurements of both active and inactive SMBHs in the early Universe are thus imperative to verify these results.

Fewer than a handful of dynamical M_{BH} measurements have been performed beyond the local universe. These measurements use either GRAVITY+ observations that spatially resolve the ionised gas kinematics within the BLR (Abuter et al., 2024; Gravity+ Collaboration et al., 2026), or *JWST* Near Infrared Spectrograph observations that probe the stellar or ionised gas kinematics within the SoI with the aid of strong gravitational lensing and/or spectroastrometry (Newman et al., 2025; Juodžbalis et al., 2025). The current sample is small and still biased towards luminous quasars. Moreover, ionised gas kinematics is known to be susceptible to non-gravitational forces that induce significant turbulence and non-circular motions, which may hinder robust dynamical modelling and M_{BH} measurements (e.g. Cappellari et al.,

2002; Kormendy & Ho, 2013; Osorno et al., 2023), especially when the spatial resolution is relatively poor. Local studies have demonstrated that the cold molecular gas kinematics probed by the Atacama Large Millimeter/sub-millimeter Array (ALMA) is less perturbed and produces more accurate M_{BH} measurements than ionised gas kinematics (e.g. Boizelle et al., 2021; Smith et al., 2021). Hezaveh (2014) first explored the idea of using strong gravitational lensing as natural magnifying glasses to perform cold gas dynamical SMBH mass measurements at high redshifts, predicting that ALMA can resolve the SoI of $\approx 20\%$ of strongly lensed galaxies with $M_{\text{BH}} \gtrsim 10^8 M_{\odot}$, enabling a substantial number of dynamical SMBH mass measurements at high redshifts.

In this chapter, I will present a sample of high-redshift strongly lensed galaxies that are good candidates for gas-dynamical measurements of SMBH masses using ALMA. I will also show existing high-resolution ALMA observations of some of these candidates, when available. I will conclude by discussing prospects for expanding the sample size using ongoing astronomical surveys such as Euclid (Euclid Collaboration et al., 2025b) and the Legacy Survey of Space and Time (LSST; Ivezić et al., 2019), as well as future instruments including the next-generation Very Large Array (ngVLA; McKinnon et al., 2019) and the Square Kilometre Array (SKA; Braun et al., 2015).

5.2 Strongly gravitationally lensed candidates for gas-dynamical SMBH mass measurements at high redshifts using ALMA

5.2.1 Sample selection

A necessary prerequisite for reliable gas-dynamical modelling is that the gas kinematics is dynamically cold (i.e. rotation dominated), with a ratio of the maximum gas rotation velocity to its mean velocity dispersion $V_{\text{rot}}/\sigma_{\text{gas}} \gg 1$. If the gas kinematics is dispersion-dominated ($V_{\text{rot}}/\sigma_{\text{gas}} < 1$), the gas will be strongly affected by non-gravitational forces (e.g. shocks and pressure gradients), making pure dynamical modelling of the mass distribution insufficient to reproduce the observed gas kinematics. In such a case, obtaining an accurate gas-kinematic model would require full hydrodynamical modelling, which is nearly intractable due to the large number of physical processes involved. However, dynamically cold rotating discs are

rare in the early universe, and unambiguously identifying them requires spatially resolving each gas disc with > 3 resolution elements (Rizzo et al., 2022), which can be time-consuming for ALMA observations of high-redshift galaxies, even with strong lens magnifications. For this reason, the number of candidates for high-redshift M_{BH} measurements with ALMA is limited by the number of galaxies with a confirmed cold rotating disc (≈ 40 ; Rowland et al. 2024) rather than the number of galaxies whose SoI can be resolved with magnification. I thus compiled the initial sample below from all strongly lensed galaxies with archival high-resolution ($< 0''.3$) ALMA observations of their cold gas kinematics. Then, I selected galaxies whose spatially resolved cold gas kinematics are consistent with cold rotating discs, as indicated by the literature sources listed in the last column of Table 5.1.

In the next selection stage, for each galaxy I compared the highest possible source-plane physical resolution achievable with ALMA with the estimated physical size of the SMBH SoI. Specifically, I first compiled the SMBH mass estimates of all candidate galaxies, either from the literature (only available for quasars) or by estimating them myself using the stellar masses (M_*) and assuming two separate $M_{\text{BH}} - M_*$ scaling relations: the empirical local relation of van den Bosch (2016) and the high-redshift relation predicted by the analytic model of Pacucci & Loeb (2024). When a stellar mass was unavailable, I conservatively assumed that it equals the total molecular gas mass (see e.g. Figure 13 of Litke et al. 2022). Then, I estimated the effective stellar velocity dispersion (σ_e) using M_{BH} and the $M_{\text{BH}} - \sigma_e$ relation of van den Bosch (2016). The radius of the SMBH SoI is then estimated to be $R_{\text{SoI}} \equiv G M_{\text{BH}} / \sigma_e^2$, where G is the gravitational constant. The estimated M_{BH} and R_{SoI} , the underlying assumption on the $M_{\text{BH}} - M_*$ relation (local or high-redshift) and the M_* determination method (estimated from the gas mass or not) are listed in Table 5.1.

I then computed the highest possible physical resolution l_{min} using

$$\left(\frac{l_{\text{min}}}{\text{pc}} \right) = \frac{4.848}{\sqrt{\mu}} \left(\frac{\theta_{\text{min}}}{\text{arcsec}} \right) \left(\frac{D_A}{\text{Mpc}} \right), \quad (5.1)$$

where θ_{min} is the best possible angular resolution of ALMA at the observed frequency of the line, D_A is the angular-diameter distance at the source redshift, computed assuming a cold dark matter cosmology with cosmological parameters from Planck Collaboration et al. (2020), and μ is the total magnification of the line from the literature. Finally, l_{min} is compared with

the largest estimate of R_{SoI} . For quasars, this is obtained using the most recent M_{BH} inferred using the single-epoch method. For non-quasars, this is estimated using the high-redshift $M_{\text{BH}} - M_*$ relation, which yields a R_{SoI} always larger than that estimated using the local $M_{\text{BH}} - M_*$ relation. All galaxies with $l_{\text{min}} \leq 2 R_{\text{SoI}}$ are selected (i.e. all sources whose SoI diameter can be spatially resolved at the highest ALMA resolution if the SMBH is as massive as that inferred by [Pacucci & Loeb 2024](#)). This is the minimum resolution needed to infer a SMBH mass using ALMA observations of cold gas kinematics ([Davis, 2014](#)). In total, 10 galaxies passed this selection stage.

In the third and final selection stage, I investigated whether the candidate galaxies have sufficiently bright cold gas-tracing lines to be detectable with ALMA at the required angular resolution within a reasonable integration time. For this purpose, I primarily considered the $[\text{C II}]$ ($^2\text{P}_{3/2} - ^2\text{P}_{1/2}$) fine-structure line, a widely adopted tracer of the neutral atomic and dense molecular gas at high redshifts (e.g. [Stacey et al., 2010](#); [Zanella et al., 2018](#); [Madden et al., 2020](#)), as it is much brighter than other cold gas tracers (e.g. CO and $[\text{C I}]$) within ALMA’s frequency range. Although some studies (e.g. [Wilson et al., 2024](#)) indicate that the $[\text{O I}]$ line can be a brighter and equally good tracer of the neutral atomic gas in high-redshift star-forming galaxies, the lack of spatially resolved archival observations of $[\text{O I}]$ for our candidate galaxies makes it risky to target this line with high-resolution observations. I thus first collected the integrated flux densities of $[\text{C II}]$ from the literature sources listed in Table 5.1. For each galaxy, I then computed the flux density per resolution element and velocity channel by averaging its integrated flux density first over the velocity full-width at half-maximum (FWHM) of the line and then over the average (across channels) spatial extent of the $[\text{C II}]$ emission (in units of the synthesised beam area) in a single velocity channel. Here, I adopted a channel width of 30 km s^{-1} , which is required to robustly measure the gas velocity dispersion of cold rotating discs ($\sigma_{\text{gas}} \approx 30 - 50 \text{ km s}^{-1}$; [Rowland et al. 2024](#)). In principle, the channels can be further binned to improve the signal-to-noise ratio (S/N) slightly. However, because the S/N improvement is much smaller than the potential inaccuracies of other assumptions (e.g. the assumed scaling relations), I chose not to consider this possibility in the selection. The synthesised beam size assumed in this computation corresponds to the worst angular resolution that can spatially resolve the largest estimate of

the SoI diameter ($2 R_{\text{SoI}}$). Finally, I computed the integration time required to reach $S/N \geq 5$ using the ALMA sensitivity calculator¹, and excluded galaxies with total on-source times that are unlikely to be awarded (> 15 h). Only 6 galaxies passed this final selection criterion. Their names, physical properties, and the literature references used in the selection are listed in Table 5.1.

Additionally, as the selection so far adopted the largest (i.e. most optimistic) estimates of R_{SoI} , I further classified the selected candidate galaxies into three categories, based on how likely it is that each SoI will be spatially resolved:

- **Grade A:** i) SoI diameter ($2 R_{\text{SoI}}$) can be resolved based on M_{BH} inferred from the single-epoch method or from the local $M_{\text{BH}} - M_*$ relation (the most conservative estimate for non-quasars). ii) [C II] emission can be detected with $S/N \geq 5$ at this resolution. Grade A candidates are highly promising for dynamical M_{BH} measurements if the actual observations reach the desired resolutions and sensitivities.
- **Grade B:** i) SoI diameter ($2 R_{\text{SoI}}$) cannot be resolved if M_{BH} follows the local $M_{\text{BH}} - M_*$ relation, but SoI radius (R_{SoI}) can be resolved if M_{BH} follows the high-redshift $M_{\text{BH}} - M_*$ relation. ii) [C II] emission can be detected with $S/N \geq 5$ at this resolution. Grade B candidates may well yield dynamical M_{BH} measurements or at least informative M_{BH} upper limits (which would suggest that some SMBHs in the early universe are not as massive as those of the *JWST*-discovered AGN whose masses are inferred using the single-epoch method).
- **Grade C:** i) SoI diameter ($2 R_{\text{SoI}}$) cannot be resolved if M_{BH} follows the local $M_{\text{BH}} - M_*$ relation, but SoI diameter can be resolved if M_{BH} follows the high-redshift $M_{\text{BH}} - M_*$ relation. ii) [C II] emission can be detected with $S/N \geq 5$ at this resolution. Grade C candidates only satisfy the minimum requirement for dynamical M_{BH} measurements under even the most optimistic assumptions. Nevertheless, it will still be possible to measure M_{BH} if the SMBHs are among the most massive SMBHs at their redshifts, or if the data quality is much better than expected.

Following these criteria, I have identified 1 Grade A, 2 Grade B and 3 Grade C candidates.

¹<https://almascience.eso.org/proposing/sensitivity-calculator>

Table 5.1: Properties of strongly lensed candidates for high-redshift SMBH mass measurements using ALMA.

Name	z	Type	μ_{tot}	Ref.	$R_{\text{Sol, est.}}$ (pc)	$\log(M_{\text{BH, est.}}/M_{\odot})$	Method	Ref.	$I_{[\text{C II}]}$ (Jy km s ⁻¹)	Ref.	$V_{\text{rot}}/\sigma_{\text{gas}}$	Ref.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Grade A												
ID141	4.2442	DSFG	5.9	D22	48	8.85	Local	D22	86	Z26	8.0	Z26
Grade B												
SPT 0113-46	4.2334	DSFG	36	R21	45	8.80	High-z	R21	91	G15	9.2	R21
SPT 0418-47	4.2246	DSFG	30	C24	37	8.66	High-z	R20	127	G15	9.7	R20
Grade C												
SPT 0345-47	4.2958	DSFG	14	R21	24	8.37	High-z	R21	64	G15	5.6	R21
SPT 0441-46	4.4803	DSFG	10.8	R21	22	8.30	High-z	R21	43	G15	15.8	R21
SPT 2132-58	4.7678	DSFG	6.3	R21	25	8.38	High-z	R21	35	G15	7.2	R21
To be confirmed												
SPT 0346-52	5.6559	DSFG	5.6	S16	95	9.32	Gas	L22	63	G15	...	...
SPT 0532-50	3.3988	DSFG	10	D19	34	8.61	Gas	B17	113	G15	...	...
PJ0116-24	2.125	HyLIRG	17	L24	36	8.65	High-z	L24	...	...	9.4	L24

Columns: (1) Galaxy name. (2) Galaxy redshift. (3) Galaxy classification: dusty star-forming galaxy (DSFG) or hyperluminous infrared galaxy (HyLIRG). No quasar has passed all selection criteria. (4) Total lens magnification of the [C II] line. If unavailable, continuum magnification at the wavelength closest to the [C II] line. (5) Reference for the lens magnification. (6) Estimated Sol radius. (7) Estimated SMBH mass. (8) SMBH mass estimation method. "Local" means estimation using the local $M_{\text{BH}} - M_*$ relation of [van den Bosch \(2016\)](#). "High-z" means estimation using the high-redshift $M_{\text{BH}} - M_*$ relation of [Pacucci & Loeb \(2024\)](#). "Gas" means first estimating the stellar mass assuming it equals the molecular gas mass, and then estimating the SMBH mass using the high-redshift $M_{\text{BH}} - M_*$ relation of [Pacucci & Loeb \(2024\)](#). (9) Reference for the stellar mass or the molecular gas mass used in the SMBH mass estimation. (10) Total [C II] flux. (11) Reference for the total [C II] flux. (12) Ratio of the gas maximum rotation velocity V_{rot} to its mean velocity dispersion σ_{gas} . (13) Reference for $V_{\text{rot}}/\sigma_{\text{gas}}$. The abbreviations used are: D22 ([Dye et al., 2022](#)), Z26 ([Zhang et al., in prep.](#)), R21 ([Rizzo et al., 2021](#)), C24 ([Cathey et al., 2024](#)), R20 ([Rizzo et al., 2020](#)), G15 ([Gullberg et al., 2015](#)), S16 ([Spilker et al., 2016](#)), L22 ([Litke et al., 2022](#)), D19 ([Dong et al., 2019](#)), B17 ([Bothwell et al., 2017](#)) and L24 ([Liu et al., 2024](#)).

I discuss the specifics of each of these candidates and present high-resolution ALMA observations of each, when available (so far only Grade A and B targets), in the following sub-section (Section 5.2.2). I have also identified three galaxies, also listed in Table 5.1, which are likely good candidates based on their known physical properties but have not yet been confirmed due to their unknown [C II] flux and/or $V_{\text{rot}}/\sigma_{\text{gas}}$. I present their properties and discuss the observations needed to confirm whether they are good candidates in Section 5.2.3.

5.2.2 Discussion of Grade A – C candidates

5.2.2.1 ID141

ID141 is a sub-millimetre galaxy at $z = 4.24417$ (Dye et al., 2022) with an extreme star-formation rate (SFR) of $2400 \pm 500 \text{ M}_{\odot} \text{ yr}^{-1}$, and is the only Grade A candidate thus far thanks to its large stellar mass of $4.8^{+2.5}_{-2.3} \times 10^{11} \text{ M}_{\odot}$.

We observed the [C II] line of ID141 with ALMA in both Cycle 11 (Project 2024.1.01711.S, PI: H. Zhang) and Cycle 12 (Project 2025.1.00772.S; PI: H. Zhang), on 2025 May 7 in configuration C43-6, on 2025 October 20 in configuration C43-8 and on 2025 December 10 in configuration C43-5. Using COMMON ASTRONOMY SOFTWARE APPLICATIONS (CASA), the observations were calibrated, reduced, combined, continuum-subtracted and finally imaged with Briggs weighting and a robust parameter of 0.0. The resulting synthesised beam FWHM is $0''.082 \times 0''.063$ (geometric mean $0''.072$) and the root-mean-squared (RMS) noise is $0.76 \text{ mJy beam}^{-1}$ in 20 km s^{-1} channels. Figure 5.1 shows the zeroth (integrated-intensity), first (intensity-weighted mean line-of-sight velocity) and second (intensity-weighted line-of-sight velocity dispersion) moment maps of the final data cube, as well as the integrated spectrum of the [C II] emission. Here and throughout this chapter, the moment maps are generated using the masked-moment technique (e.g. Dame, 2011) described in Chapter 3, but the integrated spectrum is computed by summing all pixels without masking. A detailed analysis of the data will be presented in Chapter 7.

The angular resolution of the current data corresponds to an average physical resolution of $\approx 200 \text{ pc}$ in the source plane, after correcting for the total magnification of $\mu_{\text{tot}} = 5.9$ (Dye et al., 2022). If we instead adopt the magnification near the dynamical centre of the galaxy, $\mu \approx 3.24$, the source plane resolution near the SMBH is then $\approx 280 \text{ pc}$. This resolution is

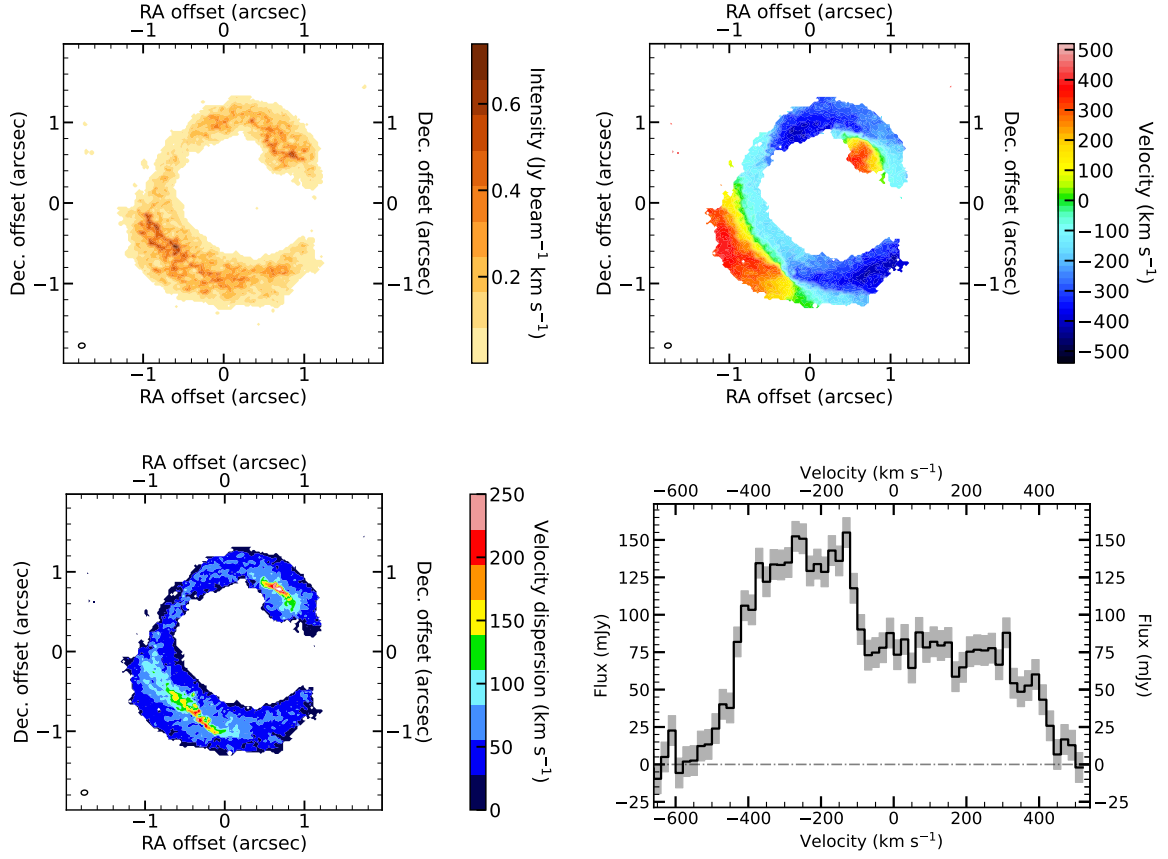


Figure 5.1: Data products of ID141 created from our ALMA [C II] data cube. **Top-left:** Zeroth-moment (integrated-intensity) map. **Top-right:** First-moment (intensity-weighted mean line-of-sight velocity) map. **Bottom-left:** Second-moment (intensity-weighted line-of-sight velocity dispersion) map. The synthesised beam ($0''.082 \times 0''.063$ FWHM) is shown as a black open ellipse in the bottom-left corner of each map. **Bottom-right:** Integrated spectrum. The shaded region indicates the 1σ statistical uncertainty. The dot-dashed line indicates the zero flux level. In all panels, positions are measured relative to the best-fitting source galaxy centre position in the reconstructed source-plane image; velocities are in the rest frame, assuming $z = 4.24417$.

sufficient to resolve the SoI diameter if the SMBH is as massive as that predicted by the high-redshift $M_{\text{BH}} - M_*$ relation of Pacucci & Loeb (2024), which yields $\log(M_{\text{BH}}/M_\odot) \approx 9.75$ and $R_{\text{SoI}} \approx 177$ pc. However, if the SMBH follows the local $M_{\text{BH}} - M_*$ relation of van den Bosch (2016), the SMBH mass would be $\log(M_{\text{BH}}/M_\odot) \approx 8.85$ and the SoI radius would be $R_{\text{SoI}} \approx 48$ pc. Resolving the SoI diameter would then require an angular resolution of at least $0''.025$ (assuming $\mu = 3.24$), and detecting the [C II] emission at this resolution with $S/N \geq 5$ requires an on-source integration time of at least 5.1 h. This can be achieved by adding long-baseline observations in configuration C43-9. Such observations took place as part of our Cycle 11 ALMA programme, but the data were unusable due to poor weather conditions during the observations. I have proposed Cycle 13 ALMA observations to re-obtain this

final building block of high-resolution data of ID141, which will likely enable the first cold gas-dynamical measurement of a SMBH mass at $z \approx 4$.

5.2.2.2 SPT 0113-46

SPT 0113-46 (hereafter SPT 0113) is a dusty star-forming galaxy (DSFG) at $z = 4.2334$ (Reuter et al., 2020). Unlike most other candidate galaxies that have extremely high intrinsic SFRs and [C II] luminosities due to our selection criteria, SPT 0113 lies on the star-forming main sequence, with a SFR of $120 \pm 30 \text{ M}_\odot \text{ yr}^{-1}$ and a stellar mass of $6_{-1}^{+2} \times 10^{10} \text{ M}_\odot$ (Rizzo et al., 2021). The bright [C II] emission of SPT 0113 is mainly the result of its high lens magnification of $\mu = 36$ (Rizzo et al., 2021).

Rizzo et al. (2021) studied the spatially resolved [C II] kinematics of SPT 0113 at an angular resolution of $0''.35 \times 0''.19$ (geometric mean $0''.26$) and an average source-plane physical resolution of $\approx 300 \text{ pc}$, revealing a dynamically cold rotating disc with $V_{\text{rot}}/\sigma_{\text{gas}} = 9.2$. However, these archival data did not resolve the estimated SoI radius (45 pc) even under the optimistic assumption that the SMBH follows the high-redshift $M_{\text{BH}} - M_*$ relation of Pacucci & Loeb (2024). We thus re-observed the [C II] emission of SPT 0113 using configuration C43-8 in ALMA Cycle 12 (Project 2025.1.01248.S, PI: J. Elford). I combined this new dataset with the archival data and imaged the combined dataset using Briggs weighting with a robust parameter of 1.0, resulting in a synthesised beam FWHM of $0''.077 \times 0''.062$ (geometric mean $0''.069$) and an average source-plane physical resolution of $\approx 80 \text{ pc}$. The moment maps and integrated spectrum created from the resulting data cube are shown in Figure 5.2. Although the desired resolution of 45 pc was not reached, as the observations were only 72% complete, the resolution achieved is already sufficient to spatially resolve the predicted SoI diameter of the SMBH and thus enable a SMBH mass measurement or at least an informative upper limit. Analysis of the data is ongoing.

5.2.2.3 SPT 0418-47

SPT 0418-47 (hereafter SPT 0418) is a DSFG at $z = 4.225$ (Reuter et al., 2020), with a SFR of $352 \pm 65 \text{ M}_\odot \text{ yr}^{-1}$ and a stellar mass of $1.2_{-0.1}^{+0.2} \times 10^{10} \text{ M}_\odot$ (Rizzo et al., 2020). SPT 0418 was initially classified as a single rotating disc (with $V_{\text{rot}}/\sigma_{\text{gas}} = 9.7$) by Rizzo et al.

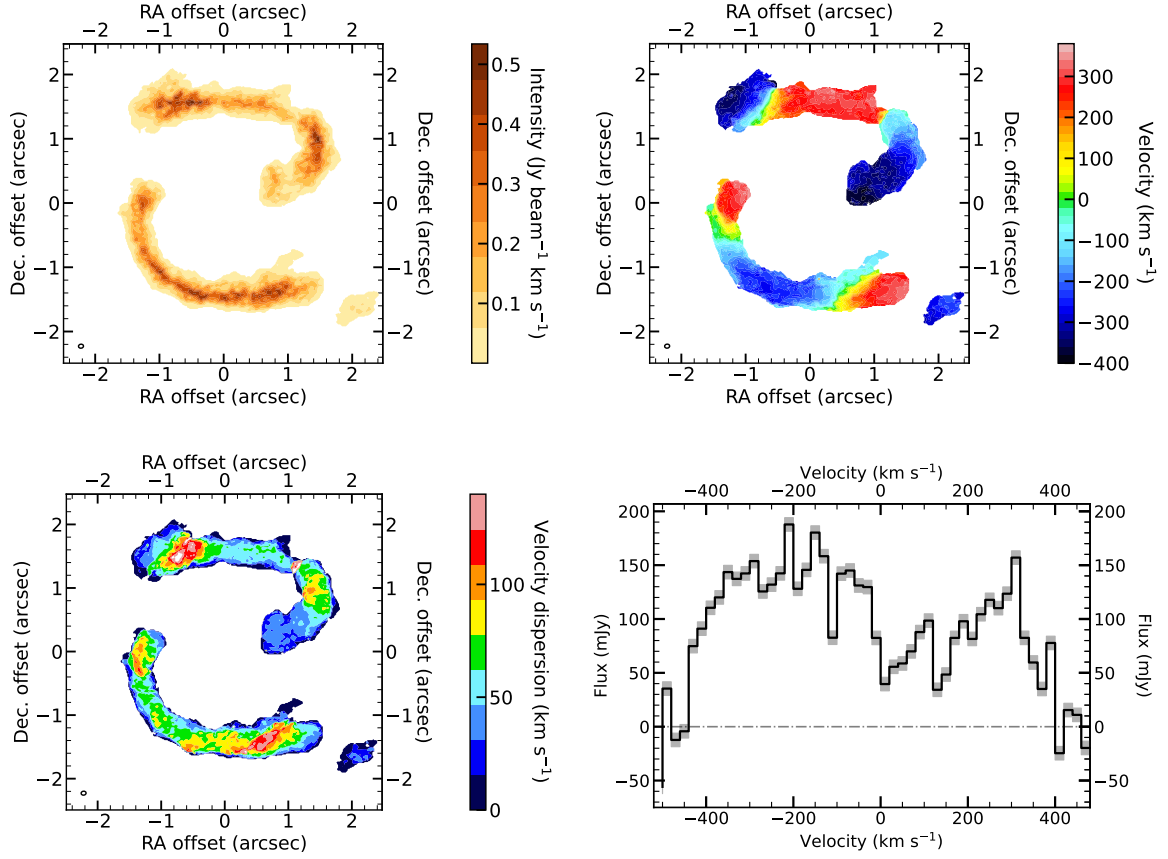


Figure 5.2: As Figure 5.1, but for SPT 0113-46.

(2020). Although higher-resolution follow-up observations using ALMA and JWST (Cathey et al., 2024) revealed that the source is in fact a pair of merging galaxies, the main galaxy (SPT 0418A) remains consistent with being a cold rotating disc.

High-resolution ALMA observations of the [C II] kinematics of SPT 0418 have already been obtained by projects 2016.1.01374.S (PI: Y. Hezaveh) and 2016.1.01499.S (PI: K. Litke). After combining and imaging the data with Briggs weighting and a robust parameter of 0.5, I obtained a data cube with a synthesised beam FWHM of $0''.032 \times 0''.025$ (geometric mean $0''.028$). The moment maps and integrated spectrum are presented in Figure 5.3. This angular resolution corresponds to an average source-plane physical resolution of 36 pc, assuming a magnification of $\mu = 29.5$ (Cathey et al., 2024). These data can already spatially resolve the estimated SoI radius of 37 pc, assuming that the SMBH follows the high-redshift $M_{\text{BH}} - M_*$ relation of Pacucci & Loeb (2024). Nevertheless, the S/N of the data is relatively low (peak $S/N \approx 3.5$, average $S/N \approx 2$), so the effective resolution (i.e. the smallest scale at which emission can be statistically significantly detected) might be lower than the synthesised beam FWHM. It is worth noting that although the synthesised beam FWHM depends on the

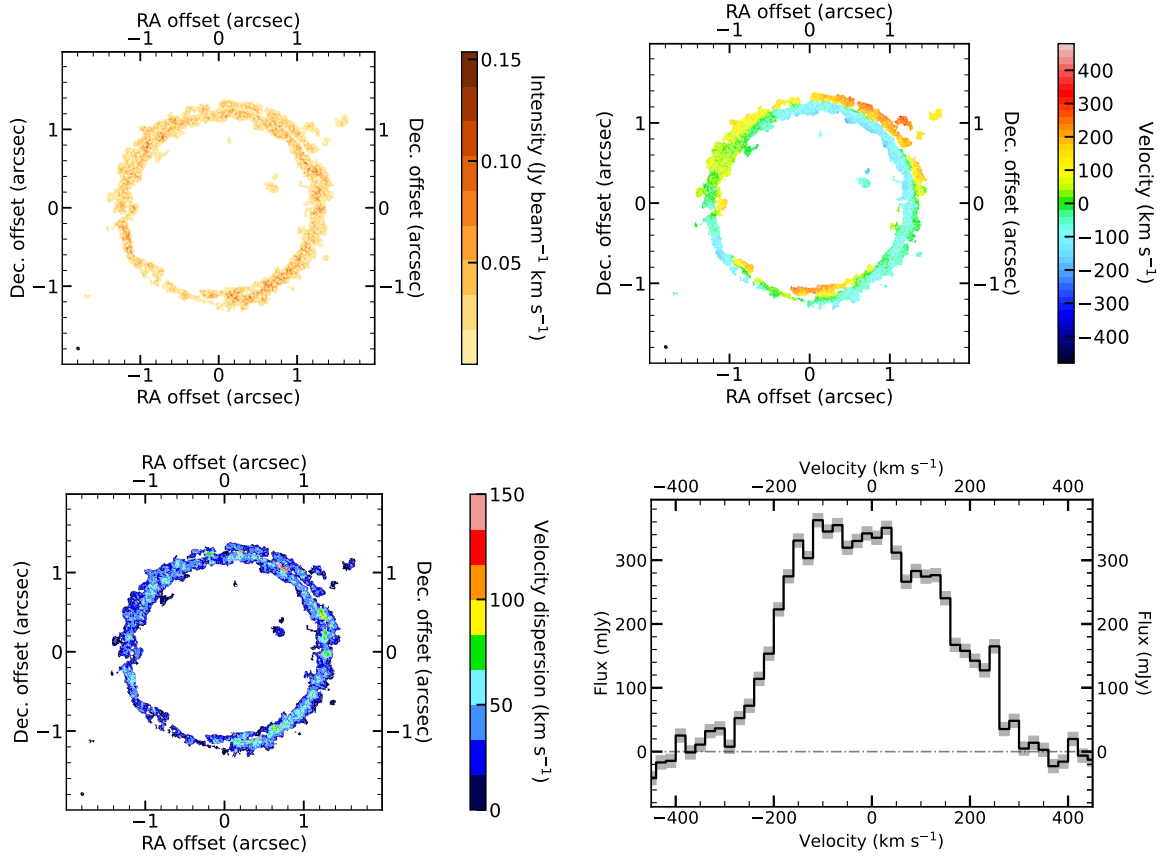


Figure 5.3: As Figure 5.1, but for SPT 0418-47.

weighting scheme adopted for imaging, the effective resolution does not, as it is determined by the uv coverage and the S/N of the visibilities. For this reason, any estimate of the angular resolution (including all estimates in this sub-section) using image-space data cubes are highly preliminary, and the true resolving power of the data should be determined by analysing the data directly in uv space (see details of our uv -space fitting in Chapter 7). We are actively working to derive dynamical SMBH mass constraints from all existing ALMA observations of SPT 0418 (Yantovski-Barth et al., in prep.).

5.2.2.4 SPT 0345-47

SPT 0345-47 (hereafter SPT 0345) is a DSFG at $z = 4.296$ (Reuter et al., 2020). Its SFR is much higher than that expected from the star-forming main sequence, with a SFR of $1300 \pm 300 \text{ M}_{\odot} \text{ yr}^{-1}$ and a stellar mass of $2.3^{+0.4}_{-0.3} \times 10^{10} \text{ M}_{\odot}$ (Rizzo et al., 2021). Despite its relatively small SoI radius of 24 pc, the diameter of the SoI can be resolved by ALMA observations with angular resolutions better than $0''.026$, thanks to the high lens magnification of $\mu = 14$ (Rizzo et al., 2021).

Rizzo et al. (2021) observed SPT 0345 in [C II] at an angular resolution of $0''.18 \times 0''.16$ (geometric mean $0''.17$) and an average source-plane physical resolution of ≈ 310 pc, revealing a dynamically cold rotating disc with $V_{\text{rot}}/\sigma_{\text{gas}} = 5.6$. However, no high-resolution [C II] observation has been obtained for this galaxy.

5.2.2.5 SPT 0441-46

SPT 0441-46 (hereafter SPT 0441), at $z = 4.4803$ (Reuter et al., 2020), is another DSFG in the Rizzo et al. (2021) sample. It has a SFR of $700 \pm 100 \text{ M}_{\odot} \text{ yr}^{-1}$, a stellar mass of $(1.8 \pm 0.2) \times 10^{10} \text{ M}_{\odot}$ and a $V_{\text{rot}}/\sigma_{\text{gas}}$ of 15.8 (Rizzo et al., 2021). The $0''.23 \times 0''.19$ (geometric mean $0''.21$; average source-plane physical resolution ≈ 430 pc) observations of Rizzo et al. (2021) revealed a steeply rising (from the outside in) rotation curve towards the centre of the galaxy, but this could be explained by a compact stellar mass distribution (effective radius $R_e = 110 \pm 20$ pc) rather than a massive SMBH. High-resolution ($\leq 0''.028$) follow-up observations are thus imperative to distinguish the relative contributions of stars and the SMBH to the centrally rising rotation velocities. However, such an observation has not been performed yet.

Although SPT 0441 is classified as a Grade C candidate based on its average lens magnification of $\mu = 10.8$ (Rizzo et al., 2021), the lens magnification near the dynamical centre of the galaxy (the presumed position of a putative SMBH) is ≈ 20 . If this magnification factor were adopted instead, SPT 0441 would be classified as a Grade B candidate whose SoI radius could be resolved using ALMA's highest possible angular resolution at the frequency of [C II], $0''.015$ (if sufficient cold gas is present at the galaxy centre).

5.2.2.6 SPT 2132-58

SPT 2132-58 (hereafter SPT 2132), also in the Rizzo et al. (2021) sample, is a DSFG at $z = 4.7678$ (Reuter et al., 2020) and is our highest-redshift candidate galaxy to date. It has a SFR of $1000 \pm 200 \text{ M}_{\odot} \text{ yr}^{-1}$, a stellar mass of $(1.9 \pm 0.3) \times 10^{10} \text{ M}_{\odot}$ and a $V_{\text{rot}}/\sigma_{\text{gas}}$ of 7.2 (Rizzo et al., 2021). Resolving the estimated SoI diameter of SPT 2132 requires an angular resolution of $\leq 0''.018$, which at the redshifted frequency of [C II] (329.508 GHz) is only achievable using the most extended ALMA configuration (C-10). Nevertheless, the region where the SMBH

dominates gravitationally is likely larger than our estimated SoI size, because the $0''.25 \times 0''.22$ (geometric mean $0''.23$; average source-plane physical resolution 610 pc) observations of [Rizzo et al. \(2021\)](#) showed that the rotation curve of SPT 2132 declines (from the outside in) towards the centre of the galaxy, implying that the total enclosed mass in the central region is relatively small. This would make the radius at which the SMBH mass equals the total enclosed mass of other components, R_{eq} , much larger than R_{SoI} , allowing the velocity increase due to the SMBH to be detectable across a much larger range of radii.

5.2.3 Confirmation of other potential candidates

In addition to performing dynamical SMBH mass measurements at high redshifts using high-resolution ALMA observations of the handful of confirmed candidates, it is crucial to expand the sample size by identifying additional candidates using intermediate-resolution (but still spatially resolved) ALMA observations. Such observations can provide robust measurements of $V_{\text{rot}}/\sigma_{\text{gas}}$ and the [C II] flux distributions of the targets, enabling to assess whether they are suitable for higher-resolution follow-up observations to measure their SMBH masses. In principle, such observations can be conducted for a large sample of highly magnified galaxies. Nevertheless, the long integration times (~ 1 h per target, even with lens magnification) required for spatially resolved observations of high-redshift galaxies make such a large survey challenging. Hence, the optimal strategy is to prioritise targets that are most likely to satisfy all selection criteria, as inferred from their known physical properties. I present three such galaxies below.

5.2.3.1 SPT-0346 52

SPT 0346-52 (hereafter SPT 0346) is a DSFG at $z = 5.6559$ ([Weiß et al., 2013](#)). It has a luminous [C II] line of integrated flux density $I_{[\text{C II}]} = 63 \text{ Jy km s}^{-1}$ and a large inferred SoI radius of 95 pc. It would be a Grade B candidate if its cold gas were confirmed to be in a dynamically cold rotating disc. However, kinematic classifications of SPT 0346 have provided ambiguous results. Although observations of the water absorption lines by [Jones et al. \(2019\)](#) imply that the lensed source is a single disturbed rotating disc, spatially resolved [C II] observations by [Litke et al. \(2019, 2022\)](#) instead support a major merger of two galaxies. The

ambiguity of previous studies results from their insufficient spatial resolutions: the effective source-plane physical resolution of ≈ 700 pc only marginally resolves the source’s major axis (Litke et al., 2022). Higher-resolution observation of the [C II] kinematics of SPT 0346 have been obtained as part of ALMA project 2023.1.01441.S (PI: A. Amvrosiadis). The observations’ angular resolution of $0''.048$ corresponds to an average source-plane physical resolution of ≈ 120 pc, which resolves the source’s major axis by > 5 beams. It is thus likely that the observations can also provide an unambiguous kinematic classification of SPT 0346. Analysis of the data is ongoing.

5.2.3.2 SPT 0532-50

SPT 0532-50 (hereafter SPT 0532) is a DSFG at $z = 3.3988$ (Weiß et al., 2013). Its estimated SoI radius of 34 pc makes it a potential Grade C candidate. It also has a high integrated [C II] flux density of 113 Jy km s^{-1} . Dong et al. (2019) observed the CO kinematics of SPT 0532 at an angular resolution of $0''.34$ (average source-plane physical resolution of ≈ 800 pc) and revealed smooth velocity gradients consistent with regular rotation. However, the observations did not have sufficient resolution to rule out a close merger. Higher-resolution follow-up observations available in the ALMA archive all failed to detect any cold gas emission, as they were too shallow. Observations of the [C II] line at $\approx 0''.06$ angular resolution would achieve a source-plane physical resolution of ≈ 140 pc, resolving the source’s major axis by > 5 beams and unambiguously identifying the nature of the system. Such observations only require ≈ 0.6 h of on-source integration to reach $S/N \geq 5$.

5.2.3.3 PJ0116-24

PJ0116-24 (hereafter PJ0116) is a hyperluminous infrared galaxy at $z = 2.125$ (Liu et al., 2024). It has a large SFR of $1490 \pm 400 \text{ M}_{\odot} \text{ yr}^{-1}$ and a dynamically cold rotating disc ($V_{\text{rot}}/\sigma_{\text{gas}} = 9.4$) confirmed by high-resolution observations of the CO(3-2) emission (Liu et al., 2024). Nevertheless, the CO lines are not sufficiently bright to be detected at the angular resolution required for a dynamical SMBH mass measurement, and the [C II] line has not been observed thus far. Assuming that PJ0116 follows the [C II] luminosity – SFR scaling relation of Schaerer et al. (2020), its total [C II] flux density would be $1148 \text{ Jy km s}^{-1}$. This estimate is

more conservative than but agrees within 20% with estimates from a standard [C II]-to-infrared luminosity ratio (10^{-3} ; [Schaerer et al. 2020](#)) or from the standard [C II]-to-CO(1-0) luminosity ratio of star-forming galaxies (≈ 5200 ; [Gullberg et al. 2015](#)). Such a high [C II] flux density, combined with the estimated SoI radius of 36 pc, would make PJ0116 a Grade C candidate. However, spatially resolved observations of the [C II] kinematics are necessary to confirm the presence of bright [C II] emission and verify that [C II] also traces dynamically cold gas. For example, $0''.05$ observations would achieve a source-plane resolution of ≈ 100 pc, resolve the galaxy's kinematic major axis by a factor of ≈ 80 and only require 0.5 h of on-source integration to reach $S/N > 5$.

5.3 Conclusions

In this chapter, I presented a selection procedure to identify strongly lensed high-redshift galaxy candidates for dynamical measurements of SMBH masses using ALMA. I applied the procedure to all high-redshift strongly lensed galaxies with confirmed dynamically cold rotating discs and identified a total of six promising candidates (1 Grade A, 2 Grade B and 3 Grade C). Then, I discussed the properties of all Grade A – C candidates and presented their existing high-resolution ALMA observations, so far only available for Grade A and B candidates. I also discussed three other galaxies that are likely good candidates but require confirmation from intermediate-resolution ALMA observations with moderate on-source integration times.

Although the number of candidates is currently limited by the number of confirmed cold rotating discs in strongly lensed galaxies, the sample is growing rapidly due to the unprecedented power of Euclid and LSST to discover new strong lenses (e.g. [Lines et al., 2025](#); [LSST Science Collaboration et al., 2009](#)) as well as the associated efforts to obtain ALMA and *JWST* follow-up observations to study their kinematics. Furthermore, the revolutionary angular resolution and sensitivity of the next generation of (sub-)millimetre and radio arrays, such as ngVLA ([McKinnon et al., 2019](#)) and SKA ([Braun et al., 2015](#)), will enable gas-dynamical SMBH mass measurements in much fainter galaxies and/or at even higher redshifts. This is because the frequency range of ngVLA is designed to cover the CO(1-0) line in both local and high-redshift galaxies ([Selina et al., 2018](#)), and the line will be within the frequency

range of SKA1 for galaxies at $z \gtrsim 6.5$ ([Braun et al. 2019](#); $z \gtrsim 3.5$ for SKA2). Hence, now is the perfect time to pursue the first cold gas-dynamical SMBH mass measurements at high redshifts. These measurements will serve as pilot studies for a large number of similar measurements in the future, that will directly trace how SMBHs grow and co-evolve with their host galaxies across cosmic time.

Chapter 6

Rapid deprojection of galaxy stellar mass surface density profiles and computation of circular velocity curves with neural networks

The research presented in this chapter was originally published in [Yantovski-Barth et al. \(2025\)](#), to which the first author and I contributed equally. Below, I focus on presenting my own contributions to the project.

6.1 Introduction

Measuring a supermassive black hole (SMBH) mass dynamically requires disentangling the contributions of the SMBH, stars and other galaxy components (e.g. gas and dark matter) to the observed galaxy kinematics. Due to the conservation of total dynamical mass, an accurate model of the stellar mass distribution is pivotal to breaking the degeneracy between these individual mass components, and in turn to reliably determine the mass of the SMBH. However, the stellar mass distributions of galaxies are diverse and often deviate from commonly assumed simple parametric (projected) mass surface density profiles (e.g. Sérsic profile; [Sersic 1968](#)) and/or spherical symmetry, leading to biased dynamical models.

The conventional approach to this problem is to perform a multi-Gaussian expansion

(MGE; [Emsellem et al. 1994](#); [Cappellari 2002a](#)) of an optical or near-infrared *Hubble Space Telescope* (*HST*) or *James Webb Space Telescope* (*JWST*) image of the galaxy. This method fits the galaxy's surface brightness distribution (i.e. its stellar light surface density projected onto the sky) $\Sigma(x', y')$ with a sum of two-dimensional (2D) Gaussians:

$$\Sigma(x', y') = \sum_{i=1}^N \frac{\Sigma_i}{2\pi \sigma_i^2 q'_i} \exp \left[-\frac{1}{2\sigma_i^2} \left(x'^2 + \frac{y'^2}{q_i'^2} \right) \right], \quad (6.1)$$

where x' and y' are coordinates in the plane of the sky (along and perpendicular to the galaxy's major axis, respectively), and Σ_i , σ_i and q'_i are the normalisation, width and observed axis ratio of the i -th Gaussian, respectively. The MGE is then multiplied by a dynamically fitted mass-to-light ratio (M/L) to derive the 2D stellar mass distribution (i.e. the stellar mass surface density projected onto the sky). Finally, assuming axisymmetry and given any inclination i (fitted dynamically), this 2D distribution is deprojected analytically into an axisymmetric three-dimensional (3D) stellar mass volume density distribution

$$\rho(x, y, z) = \sum_{i=1}^N \frac{\Sigma_i}{(\sqrt{2\pi} \sigma_i)^3 q_i} \exp \left[-\frac{1}{2\sigma_i^2} \left(x^2 + y^2 + \frac{z^2}{q_i^2} \right) \right], \quad (6.2)$$

where x and y are coordinates in the equatorial plane of the galaxy (perpendicular to the symmetry axis), z is the coordinate along the symmetry axis, and the intrinsic axis ratios q_i are also obtained analytically from the observed axis ratios q'_i and the inclination i through

$$q_i^2 = \frac{q_i'^2 - \cos^2 i}{\sin^2 i}. \quad (6.3)$$

The main advantages of the MGE approach are that it provides a much more flexible stellar mass model compared to one-dimensional (1D) parametric profiles and that it enables an efficient deprojection by analytically computing all deprojected quantities. The circular velocity curve resulting from this 3D mass distribution can also be computed efficiently using Equation 45 of [Cappellari \(2020\)](#).

However, the high-redshift galaxies selected for dynamical SMBH mass measurements with the Atacama Large Millimeter/sub-millimeter Array (ALMA) are often dusty, star-forming galaxies. Their *HST* or *JWST* images are often heavily obscured by dust in the

rest-frame optical and lack sufficient angular resolution in the rest-frame near-infrared (e.g. [Cathey et al., 2024](#)), which precludes deriving robust models of the stellar light (and thus mass) distributions from the images. Moreover, light from active galactic nuclei (AGN) can contaminate stellar light in images, leading to biased SMBH masses even in the local universe (e.g. [Lelli et al., 2022](#)). Quasars at high redshifts can also completely outshine their host galaxies, making it nearly impossible to obtain accurate stellar mass models in the central regions critical to determining the SMBH masses (e.g. [Marshall et al., 2021](#)).

For these reasons, it is common to assume simple parametric stellar mass profiles when decomposing the kinematics of high-redshift galaxies. It is also common to further assume that the deprojection of an assumed 1D parametric mass profile to 3D is spherically symmetric (e.g. [Rizzo et al., 2020, 2021](#); [Roman-Oliveira et al., 2026](#)), as computing the circular velocity curves of the more general axisymmetric mass distributions requires a computationally inefficient numerical integration. However, the circular velocity curve produced by a spherically symmetric mass distribution can deviate substantially from that produced by a flattened axisymmetric distribution with the same 1D profile (e.g. [Noordermeer, 2008](#); [Price et al., 2022](#)). The assumption of spherical symmetry can thus strongly bias the results of dynamical modelling.

In this chapter, we present a novel approach to efficiently compute the axisymmetric deprojections of any 1D analytic stellar mass surface density profile and its associated circular velocity curves, using a simple neural network (NN) that maps the parameters of the analytic profile to the parameters of the associated MGEs. This enables flexible stellar mass volume density distribution models to be constructed from simple analytic stellar mass surface density profiles, and in turn to derive accurate SMBH masses, without requiring high-resolution optical/near-infrared images.

This chapter is structured as follows. Section [6.2](#) describes the methodology of the deprojection, the architecture of the NN and the procedures for training it. Section [6.3](#) shows a test of this method using the local galaxy NGC 4697, which has an existing dynamical SMBH mass measurement that used a stellar mass model constructed from an MGE. We conclude in Section [6.4](#).

6.2 Methodology

6.2.1 Efficient deprojection of 1D parametric mass surface density profiles with MGEs

In the absence of informative optical/near-infrared images, one could fit an arbitrary MGE directly to kinematic data and constrain its parameters dynamically. However, the high flexibility of MGEs, combined with the lack of a physically motivated regularisation, would cause the MGEs to overfit the kinematic data, leaving no room for other galaxy components. For example, a MGE could infer a point-like stellar distribution with no SMBH ($M_{\text{BH}} = 0$).

Hence, we develop an approach that combines physically motivated 1D parametric stellar mass surface density profiles with the efficient analytic deprojection of MGEs. That is, we train a NN that maps the parameters of any given 1D profile $\Sigma_*(r')$ to those of a 1D MGE that closely approximates this profile with an error of $\lesssim 0.1\%$:

$$\Sigma_*(r') \approx \Sigma_{\text{MGE}}(r') = \sum_{i=1}^N \frac{\Sigma_i}{2\pi \sigma_i^2} \exp\left(-\frac{r'^2}{2\sigma_i^2}\right). \quad (6.4)$$

Here, $q'_i = 1$ is assumed for the normalisation constant of each Gaussian. The 1D Gaussians can be converted to 2D assuming a unique observed axis ratio q' (fitted dynamically), such that

$$r' = \sqrt{x'^2 + \frac{y'^2}{q'^2}}, \quad (6.5)$$

where x' and y' are, again, coordinates in the plane of the sky, and the Σ_i of each Gaussian is also rescaled by the actual q' . The 2D Gaussians can be deprojected into 3D by converting q' into the intrinsic axis ratio q using Equation 6.3. The associated 3D volume density distribution is then given by Equation 6.2 with all q_i equal to q . Finally, as stated above, the circular velocity curve of each resulting mass distribution can be computed efficiently from the deprojected Gaussians using Equation 45 of Cappellari (2020). The rapid computation of circular velocity curves enables fitting axisymmetric 3D stellar mass volume density models to the data within a tractable amount of computation time. This substantially improves the flexibility and accuracy of the stellar mass models compared to the traditional approach of assuming spherical symmetry.

We note that we are not the first to perform deprojections of parametric stellar mass surface density profiles with MGEs. Many previous studies (e.g. [Emsellem et al., 2011](#); [Pechetti et al., 2020](#)) deprojected 1D analytic profiles by fitting them with MGEs using the `mge_fit_1d` routine of the MGEFIT package¹. However, the fitting process is less computationally efficient than a NN. It is also incompatible with advanced gradient-based sampling techniques (e.g. No-U-Turn Sampler; [Hoffman & Gelman 2014](#)) that require the modelling pipeline to be automatically differentiable, while NN architectures are differentiable by design. Alternatively, the mapping between parametric profiles and MGEs can be performed using linear least-squares optimisation. We are currently working to compare this approach to the NN approach described here in terms of computational efficiency and gradient stability.

6.2.2 Nuker stellar mass profile

In this sub-section, we demonstrate our method using the Nuker stellar mass profile ([Lauer et al., 1995](#)), although the method can be generalised to any 1D profile (e.g. Sérsic and core-Sérsic).

6.2.2.1 Profile

The Nuker stellar mass surface density profile is given by

$$\Sigma_*(r') = 2^{\frac{\beta-\gamma}{\alpha}} \Sigma_b \left(\frac{r'}{R_b} \right)^{-\gamma} \left[1 + \left(\frac{r'}{R_b} \right)^{\alpha} \right]^{\frac{\gamma-\beta}{\alpha}}, \quad (6.6)$$

where γ is the inner power-law slope, β is the outer power-law slope, α is the sharpness of the transition between the inner and the outer power laws, R_b is the break radius where the transition takes place and Σ_b is the surface density at the break radius. The Nuker profile has been shown to accurately fit the central regions of nearby early-type galaxies (ETGs; e.g. [Lauer et al. 1995, 2007](#)), providing more model flexibility than the Sérsic profile in the inner few tens of parsecs, the most important region to determine SMBH masses.

¹<https://pypi.org/project/mgefit/>

6.2.2.2 Training set

The shape of the Nuker profile is determined by only three of its parameters: α , β and γ . In principle, one should only have to train the NN to learn the mapping between these three shape parameters and MGE Gaussians, setting $\Sigma_b = 1$ and $R_b = 1$, and then rescale the normalisations of the Gaussians with Σ_b and the widths of the Gaussians with R_b . However, in practice, we established that coupling the Gaussian widths to R_b can result in unphysical models. This is because any MGE is nearly flat on scales much smaller than the smallest Gaussian width. Allowing the Gaussian widths to vary with R_b gives the model unintended flexibility to produce a flat inner mass volume density profile by having an unphysically large R_b (which drives the smallest Gaussian width to be much larger than the physical scale of the data, producing an artificial core) rather than $\gamma = 0$. Therefore, we fixed the Gaussian widths σ_i on a logarithmic grid ranging from 1 pc to 10 kpc, thus sampling all physical scales of a galaxy that can be probed with realistic ALMA observations. We adopted a total of $N = 64$ Gaussians, sufficient to approximate any Nuker profile to better than 0.1% accuracy. The task of the NN is thus to learn the mapping between the Nuker parameters (α , β , γ and R_b) and the normalisations Σ_i of the 64 Gaussians.

We generated the training set for the NN by drawing 5×10^6 random samples from the priors of the Nuker parameters, listed in Table 6.1. We determined the priors from the distributions of the Nuker parameters of local ETGs (e.g. Lauer et al., 2007), adopting the widest possible range that covers all physical parameter values. We imposed bounds on $\beta - \gamma$ instead of β to be consistent with the observational constraint that the inner power-law slope is always substantially shallower than the outer power-law slope. Additionally, we drew 10^6 random samples as the validation set to evaluate whether the NN has overfitted the training set or can generalise to new inputs. We computed the stellar mass surface density profiles for each set of parameters in the training set, which is what the NN aims to reproduce during training. The details of the NN architecture and the training process are detailed in the next sub-section (Section 6.2.3).

Table 6.1: Nuker model parameters and their priors.

Parameter	Prior range
α	0.1 – 10.0
$\beta - \gamma$	0.3 – 3.0
γ	0.0 – 1.2
$\log(R_b/\text{pc})$	0.3 – 3.3
$\log(\Sigma_b/M_\odot \text{ pc}^2)$	1.0 – 5.5
q	0.1 – 1.0

6.2.3 Neural network architecture and training

As learning the mapping between two sets of parameters is straightforward for a NN, we adopt a simple architecture, shown in the flowchart of Figure 6.1. The NN has six linear layers, the widest of them having 512 parameters, connected by Scaled Exponential Linear Unit (SELU; Klambauer et al. 2017) activation functions. The last activation function that produces the output Gaussian normalisations is a symexp function, defined as

$$y = \text{symexp}(x) = 10^{-12} \text{sgn}(x) \left(10^{|x|} - 1 \right), \quad (6.7)$$

where $\text{sgn}(x)$ is the sign function. It is the inverse of the symlog function

$$x = \text{symlog}(y) = \text{sgn}(y) \log \left(1 + \frac{|y|}{10^{-12}} \right). \quad (6.8)$$

The symexp function, similar to the exponential function, is designed to expand the dynamic range of the parameters. Yet, it can have both positive and negative outputs, which is necessary to allow both positive and negative MGE Gaussians, in turn necessary to improve their performance reproducing Nuker profiles. On the other hand, the symlog function substantially compresses the dynamic range of the parameters. This allows the NN weights to remain small in the linear layers while reproducing the large dynamic range of Gaussian normalisations (from $\sim \pm 10^{-12}$ – $\sim \pm 10^3$) after symexp transformation.

The NN has two possible outputs: one during training, the other when used. The first output is the final output when the NN is integrated into our dynamical modelling pipeline, SUPERB MASSES FROM GAS KINEMATICS (SUPERMAGE). It is the components (inferred normalisations and fixed widths) of the 64 MGE Gaussians, which are then passed to the modelling pipeline to compute the resulting circular velocity curve. During training,

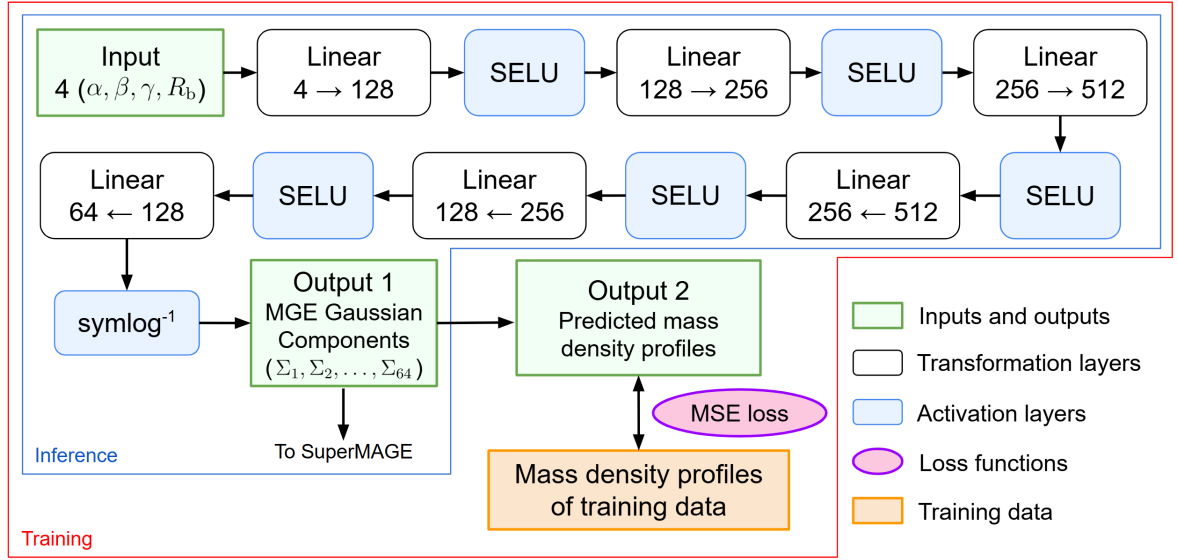


Figure 6.1: Flowchart of the NN architecture that maps Nuker profile parameters to MGE parameters.

the first output is used to compute the second output, the predicted mass surface density profile (i.e. the sum of the predicted Gaussians). We then compare this with the true Nuker mass surface density profile to evaluate the loss function, which we choose to be the mean squared error (MSE; i.e. the mean squared difference) between the true and predicted profiles.

The training process involves iteratively varying the NN weights to minimise the loss function. Specifically, we adopted the Adam optimiser (Kingma & Ba, 2014) with a learning rate of 10^{-5} . After ≈ 20 hours of training on an NVIDIA GeForce RTX 4060 Ti graphics processing unit (GPU), the NN achieves $< 1\%$ fractional error when predicting the Nuker mass profile from the input Nuker parameters. This performance is more than sufficient for our science task, given that the statistical uncertainty of the fitted dynamical mass profile is typically much larger than 1% . Further improvements may be achieved by longer training or by further tuning the optimiser’s hyperparameters.

6.3 Test on NGC 4697

To test whether the NN-based deprojection of 1D analytic profiles enables parametric stellar mass models sufficiently flexible to produce unbiased SMBH mass measurements, we performed a dynamical analysis of the well-studied nearby elliptical galaxy NGC 4697. The SMBH in NGC 4697 has previously been measured (Davis et al., 2017) using an MGE-

based stellar mass model, constructed using the standard procedure described in Section 6.1. Specifically, [Davis et al. \(2017\)](#) fitted a 2D MGE to an *HST* Advanced Camera for Surveys F850LP-filter image. They then removed the narrowest component of the MGE to correct for potential contamination from the low-luminosity AGN of the galaxy ([Wrobel et al., 2008](#)). They also tested the systematic uncertainty of their M_{BH} measurement by repeating the analysis without removing the narrowest MGE component. Here, we will re-measure the SMBH mass of NGC 4697 using a Nuker profile with NN-based deprojection as the stellar mass model. Then, we will compare our best-fitting M_{BH} with the results of [Davis et al. \(2017\)](#), considering both the model without the narrowest MGE component and the model with it.

6.3.1 ALMA observations

To ensure a fair comparison between the different modelling procedures, we used the same ALMA data as [Davis et al. \(2017\)](#). The ALMA observations are from Project 2015.1.00598.S (PI: M. Bureau) and have been calibrated using the standard pipeline of the ALMA archive². [Davis et al. \(2017\)](#) imaged the data using Briggs weighting with a robust parameter of 0.5, and the synthesised beam full-width at half-maximum (FWHM) of the data cube is $0''.54 \times 0''.52$, slightly underresolving the measured SMBH sphere of influence radius of $0''.41$ ([Zhang et al., 2024](#)). Nevertheless, a M_{BH} measurement can still be obtained due to the data’s high average signal-to-noise ratio of ≈ 6 . Moment maps and position-velocity diagrams illustrating the quality of the data are presented in [Davis et al. \(2017\)](#), so they are not reproduced in this chapter.

Beside the different stellar mass models, the only difference between our modelling approach and that of [Davis et al. \(2017\)](#) is that we fit the data directly in uv space, instead of imaging the data and fitting them in image space. This is because uv -space fitting produces more reliable uncertainties of model parameters by allowing the covariance between data points to be safely ignored, as noise is uncorrelated across visibilities in uv space. By contrast, noise is highly correlated across neighbouring pixels in image space, so one must either invert the covariance matrix when computing the chi-square (χ^2), which is time-consuming, or assume that the data points are independent, which may bias the best-fitting

²<https://almascience.nrao.edu/aq>

parameters and their uncertainties. [Davis et al. \(2017\)](#) adopted the former approach, unlike most existing ALMA M_{BH} measurements that adopt the latter approach. Statistically, the χ^2 function of [Davis et al. \(2017\)](#) is nearly identical to ours, so we can directly compare their results with ours despite fitting different data formats. The details of our uv -space data processing, gridding and fitting procedures are not part of my work and are presented in detail in [Yantovski-Barth et al. \(2025\)](#) and the documentation of our uv -space handling package `VISCUBE`³.

6.3.2 Dynamical modelling

Our dynamical model is identical to that presented in [Davis et al. \(2017\)](#), except that the stellar mass surface density model is a Nuker profile with an oblate axisymmetric deprojection in 3D, computed using our NN given an inclination i and an intrinsic axis ratio q (both are free parameters of the model). We fit the parameters of the Nuker profile purely dynamically, without using any image or other prior (besides those on the Nuker parameters themselves; see Table 6.1).

We forward-modelled the molecular gas distribution and dynamics as well as fitted the ALMA data in uv -space using the Python package `SUPERMAGE`, a dynamical and strong gravitational lensing modelling code written in an automatically differentiable framework. We sampled the posterior distribution of the model parameters using the Metropolis-adjusted Langevin algorithm (MALA; [Roberts & Tweedie 1996](#)). The strategy for tuning the algorithm’s hyperparameters is described in Appendix C of [Yantovski-Barth et al. \(2025\)](#).

6.3.3 Results and discussion

Using an axisymmetric Nuker profile as our stellar mass model, our best-fitting SMBH mass is $\log(M_{\text{BH}}/M_{\odot}) = 7.99^{+0.02}_{-0.03} (1\sigma) {}^{+0.06}_{-0.07} (3\sigma)$. Figure 6.2 compares the 1σ and 3σ confidence intervals of our best-fitting M_{BH} with those derived by [Davis et al. \(2017\)](#) using MGE-based stellar mass models, both with and without the narrowest component. Our best-fitting M_{BH} is statistically consistent with both best-fitting MGE models, lying between those of the two models. This demonstrates that a Nuker profile with an axisymmetric deprojection can

³<https://viscube.readthedocs.io/en/latest/>

produce an unbiased SMBH mass measurement in NGC 4697, without requiring any image to constrain the stellar mass distribution.

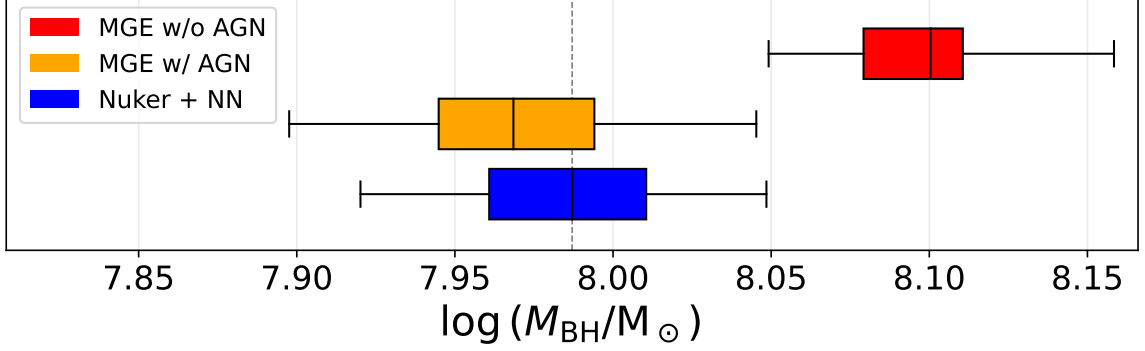


Figure 6.2: 1σ (boxes) and 3σ (whiskers) confidence intervals for the SMBH mass of NGC 4697, derived using different stellar mass models: MGE without the narrowest component (red, top), MGE with the narrowest component (middle, orange) and Nuker mass profile deprojected using a NN (bottom, blue). Without using any image as stellar mass distribution prior, the "Nuker + NN" approach yields a SMBH mass consistent with both previous measurements that instead used a MGE of an optical image.

Notably, our best-fitting M_{BH} is more consistent with the model that keeps the narrowest MGE component than with the model that removes it. This implies that if the stellar mass distribution of NGC 4697 indeed follows a Nuker profile down to the nuclear region, the light within the central point spread function of the *HST* image may actually be dominated by stars rather than the AGN. In this case, the correction applied by [Davis et al. \(2017\)](#) would be an over-correction, leading to an underestimate of the central stellar mass and thus an overestimate of the SMBH mass.

Our dynamical model is further compared to those of [Davis et al. \(2017\)](#) in Figure 6.3, where our best-fitting stellar mass surface density profile (left) and associated circular velocity curve (right) are overlaid on those derived using MGE profiles. Despite not utilising any constraint from images, the stellar mass model derived by fitting an axisymmetric Nuker profile dynamically is entirely consistent with the MGE-based stellar models derived using an *HST* image, at all radii beyond the synthesised beam of the ALMA data. The three stellar mass models diverge significantly only below the resolution of the *HST* image, where the uncertain contribution of the AGN complicates the interpretation of the MGE fit and leads to discrepancies between the two MGE models. By contrast, our Nuker model naturally extrapolates the profile below the *HST* resolution without requiring any assumption about the putative light contribution of an AGN. The best-fitting circular velocity curve of our Nuker

model also agrees well with those of the MGE models at all radii beyond the *HST* resolution. This comparison therefore shows that an axisymmetric Nuker profile deprojected using a NN can produce accurate dynamical models of NGC 4697 even in the absence of an optical image.

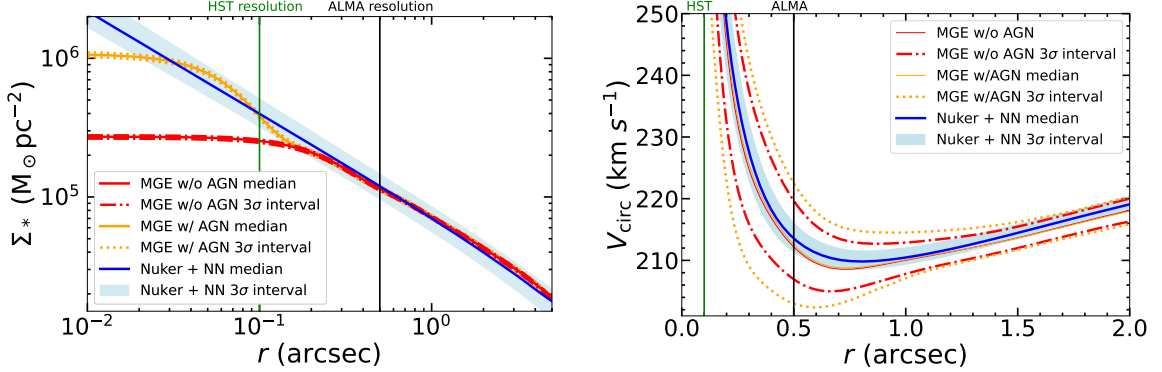


Figure 6.3: Comparison of the projected stellar mass surface density profiles (left) and circular velocity curves (right) derived using our best-fitting Nuker + NN model (blue) and the MGE models with (orange) and without (red) an AGN from Davis et al. (2017). The 3σ confidence interval of the Nuker model is shown as a shaded region, whereas those of the MGE models are shown with dashed lines (with an AGN) and dot-dashed lines (without an AGN). The angular resolution of the *HST* image used to construct the MGE model and that of the ALMA data are shown as vertical lines. Our model, fitted dynamically without using any image, agrees well with models whose stellar mass distributions are constrained by fitting MGEs to an *HST* image. The differences of the inferred SMBH masses are caused entirely by model differences below the *HST* resolution.

The best-fitting parameters and associated uncertainties of our dynamical model, including the Nuker model parameters and other parameters defined in Davis et al. (2017), are listed in Table 6.2. The posterior distributions and the covariances between all non-nuisance model parameters are shown in Figure 6.4.

6.4 Conclusions

In this chapter, we introduced a method to efficiently compute axisymmetric deprojections of 1D analytic mass surface density profiles and their associated circular velocity curves using NNs that map the parameters of the analytic profile to those of a MGE. This approach enables the construction of more flexible stellar mass distribution models from analytic profiles than the conventional approach of assuming spherical symmetry. Using the Nuker profile as an example, we demonstrated how to design and train the NN. We tested the approach by integrating it into the dynamical modelling pipeline to infer the SMBH mass of NGC 4697

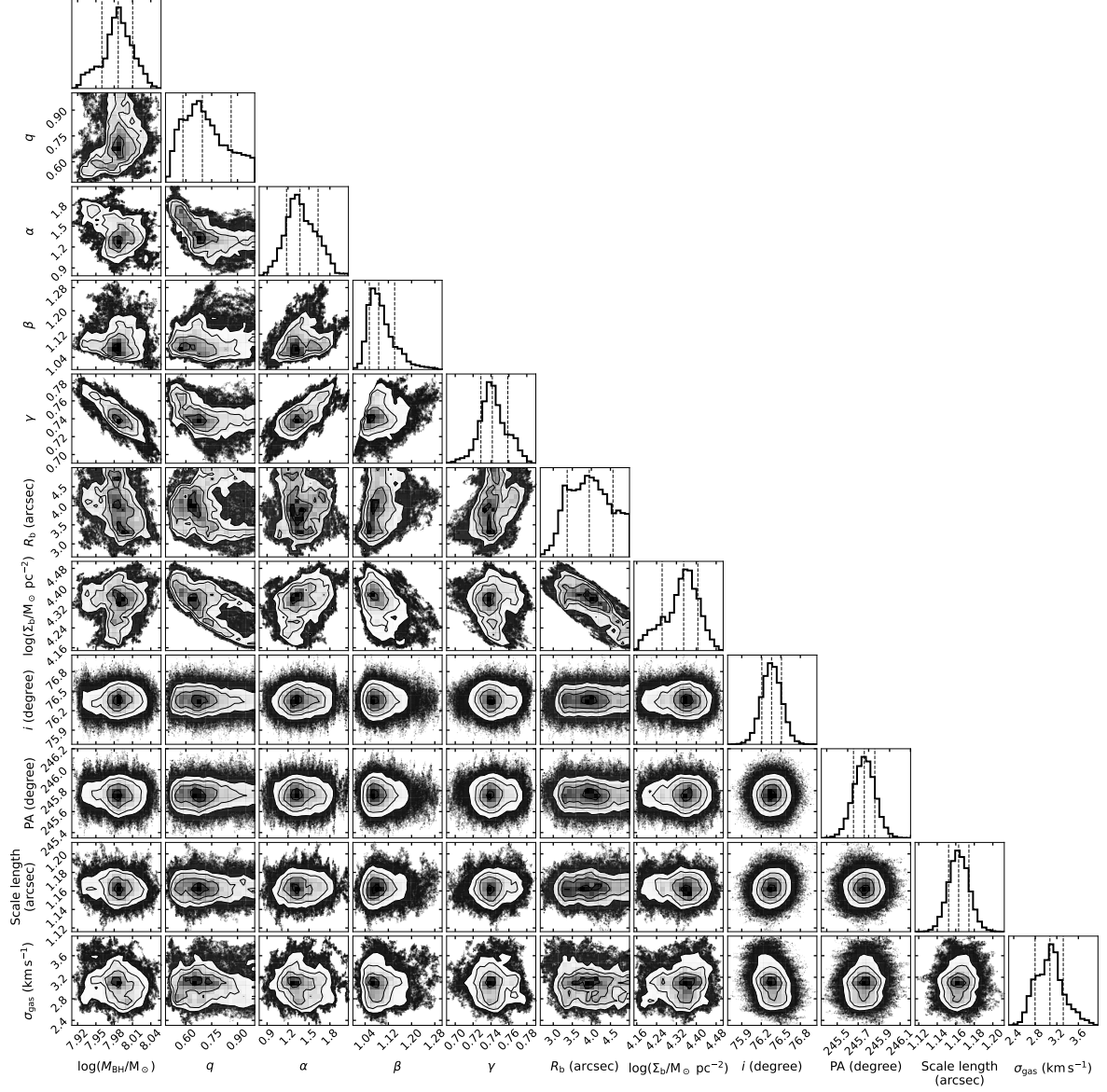


Figure 6.4: Corner plot showing the covariances between non- nuisance model parameters of our best-fitting Nuker + NN model of NGC 4697. The contours represent increasing confidence levels, from 68.3% (1σ) to 99.7% (3σ). The histograms show the 1D marginalised posterior distribution of each parameter. The dashed lines indicate the medians and 1σ confidence intervals.

Table 6.2: Best-fitting model parameters and associated uncertainties of NGC 4697.

Parameter	Best fit	1σ uncertainty	3σ uncertainty
Mass model			
$\log (M_{\text{BH}}/M_{\odot})$	7.99	$-0.03, +0.02$	$-0.07, +0.06$
α	1.37	$-0.19, +0.26$	$-0.51, +0.66$
β	1.09	$-0.03, +0.05$	$-0.08, +0.19$
γ	0.74	$-0.01, +0.02$	$-0.05, +0.04$
R_b (arcsec)	3.94	$-0.58, +0.62$	$-1.20, +1.05$
$\log (\Sigma_b/M_{\odot} \text{ pc}^2)$	4.35	$-0.09, +0.06$	$-0.18, +0.14$
q	0.70	$-0.11, +0.17$	$-0.20, +0.30$
Molecular gas disc			
Position angle (degree)	245.8	± 0.1	± 0.3
Inclination (degree)	76.36	± 0.15	± 0.46
Velocity dispersion (km s^{-1})	3.1	$-0.3, +0.2$	$-0.6, +0.8$
Scale length (arcsec)	1.16	± 0.01	$-0.03, +0.04$
Nuisance parameters			
Integrated intensity (Jy km s^{-1})	16.8	± 0.2	± 0.5
Kinematic centre X offset (arcsec)	-0.15	± 0.00	$-0.01, +0.02$
Kinematic centre Y offset (arcsec)	0.61	± 0.00	± 0.01
Central velocity offset (km s^{-1})	0.35	$-0.13, +0.14$	$-0.36, +0.43$

Notes. The X and Y offsets are measured relative to the phase centre of the observation, $\text{RA} = 12^{\text{h}}48^{\text{m}}35^{\text{s}}.907$, $\text{Dec.} = -05^{\circ}48'03''.06$ (International Celestial Reference System). The central velocity offset is measured relative to the systemic velocity of 1241 km s^{-1} .

from ALMA data, finding that an axisymmetric Nuker stellar mass model, deprojected by a NN and fitted dynamically without using an image, produces a SMBH mass and a circular velocity curve that agree well with those of MGE models constructed using a *HST* image. This demonstrates that accurate gas-dynamical measurements of SMBH masses can be obtained by assuming simple analytic stellar mass profiles, without requiring high-resolution observations of the stellar light distribution. Our approach opens the door to measure SMBH masses in a wide range of galaxies for which images are unavailable, heavily obscured by dust or contaminated by AGN. This includes strongly lensed high-redshift galaxies, which are often dusty or outshone by AGN in the optical. This work thus paves the way for the first cold gas-dynamical measurements of SMBH masses at high redshifts.

Chapter 7

A dynamical supermassive black hole mass upper limit and exceptionally fast rotation in the gravitationally lensed galaxy ID141 at redshift $z = 4.24$

7.1 Introduction

In this chapter, we derive a dynamical upper limit on the supermassive black hole (SMBH) mass of ID141, a redshift $z = 4.24$ strongly gravitationally lensed galaxy, utilising high-resolution ($0''.072$ or ≈ 200 pc on average after correcting for lens magnification) Atacama Large Millimeter/sub-millimeter Array (ALMA) observations of the [C II] emission line and a novel dynamical and strong lens forward-modelling pipeline. In Section 7.2, we present the ALMA observations, our data processing pipeline and the final data products. We introduce our dynamical and lens modelling procedure in Section 7.3, and then present the results for various model assumptions in Section 7.4. In Section 7.5, we compare the inferred dynamical mass of each component of the galaxy with literature estimates, comment on the galaxy's exceptionally high rotational velocity V_c and its ratio with the gas velocity dispersion σ_{gas} and discuss the prospects of future dynamical measurements of the SMBH mass (M_{BH}) of ID141 and other strongly lensed high-redshift galaxies. We conclude in Section 7.6.

7.2 ALMA Observations

7.2.1 Target: ID141

ID141 (H-ATLAS J142413.9+022303) is a strong gravitationally lensed sub-millimetre galaxy (SMG) at $z = 4.24417$ (Dye et al., 2022), initially discovered in the *Herschel* Astrophysical Terahertz Large-Area Survey (Eales et al., 2010). It is one of the brightest SMGs in the survey, due to both its lens magnification ($\mu \approx 6$) and large intrinsic far-infrared (FIR) luminosity ($\approx 1.5 \times 10^{13} L_{\odot}$; Dye et al. 2022). Spectral energy distribution (SED) fitting suggests that ID141 has a large stellar mass $M_* = 4.8_{-2.3}^{+2.5} \times 10^{11} M_{\odot}$ and an extreme star-formation rate (SFR) of $2400 \pm 500 M_{\odot} \text{ yr}^{-1}$ (Dye et al., 2022), making it an ideal laboratory to study the most extreme star-forming systems in the early Universe.

For this reason, ID141 has been observed with ALMA using several emission lines that trace the cold and warm phases of the interstellar medium, including CO(7 – 6), [C I] ($^3P_2 - ^3P_1$), [C I] ($^3P_1 - ^3P_0$) and H₂O (Dye et al., 2022). These observations, with angular resolutions of $0''.11$ (≈ 300 pc after correcting for lens magnification), enabled spatially resolved kinematic studies of the galaxy. Dye et al. (2022) inferred that the gas kinematics is consistent with a dynamically cold rotating disc, ideal for gas dynamical modelling and M_{BH} measurements, although the possibility of a close pair of galaxies could not be ruled out. Dye et al. (2022) also concluded that the total dynamical mass of ID141 is much lower than its gas mass inferred from standard conversion factors. Furthermore, the de-lensed position-velocity diagram (PVD) of the CO emission revealed a moderate velocity increase towards the centre of the galaxy, which could be the dynamical signature of a SMBH or a compact stellar spheroid. Resolving these puzzles requires observations with higher resolutions and sensitivities, as well as probing complementary gas tracers. Therefore, we performed the first spatially resolved observations of the $158\text{-}\mu\text{m}$ [C II] emission of this galaxy, which are detailed in the following sub-section (Section 7.2.2).

Throughout this work, we assume a cold dark matter cosmology with a Hubble constant today $H_0 = 67.4 \text{ km s}^{-1} \text{ Mpc}^{-1}$, a matter density (relative to the critical density) $\Omega_M = 0.315$ and a cosmological constant (relative to the critical density) $\Omega_{\Lambda} = 0.685$ (Planck Collaboration et al., 2020). The angular diameter distance of ID141 is thus $D_A = 1429.4 \text{ Mpc}$ and its

luminosity distance $D_L = 39310.1$ Mpc.

7.2.2 Observations and data reduction

We observed the $158\text{-}\mu\text{m}$ [C II] line of ID141 with ALMA in both Cycle 11 and Cycle 12. Part of the Cycle 11 observations was taken on 2025 May 7 in configuration C43-6, as part of Project 2024.1.01711.S (PI: H. Zhang), with a total observing time including all overheads of 91.2 min. The target was again observed on 2025 September 11 in configuration C43-9 for a total observing time of 121.0 min, but the weather conditions severely degraded after the first $\approx 25\%$ of the observations. Due to the resulting poor sensitivity, these observations were labelled "SemiPass" by ALMA's quality assurance pipeline, and we chose not to include them in our analysis following a careful manual examination of the data. The Cycle 12 observations (Project 2025.1.00772.S; PI: H. Zhang) consist of two observing tracks: one taken on 2025 October 20 in configuration C43-8, the other taken on 2025 December 10 in configuration C43-5. The total observing times including all overheads were 47.24 min and 35.77 min, respectively. Table 7.1 summarises the properties of all the observations used in this study.

For all observations, we centred one spectral window (SPW) on the redshifted frequency of the [C II] line, 362.409 GHz. This SPW covers a bandwidth of 1.875 GHz (1550 km s^{-1}), using 240 channels of 7.8125 MHz (6.5 km s^{-1}). We also placed three other SPWs at 348.608, 350.419 and 360.598 GHz to map the continuum emission. These SPWs each cover a bandwidth of 2.0 GHz (1650 km s^{-1}) with 128 channels of 15.625 MHz (12.9 km s^{-1}).

The raw data were calibrated with the COMMON ASTRONOMY SOFTWARE APPLICATION (CASA) package¹ (McMullin et al., 2007), with version numbers listed in the last column of Table 7.1, using the standard script provided by the ALMA Science Archive². Then, we combined the calibrated datasets using the CASA task `concat`. The calibrated and combined data product will be referred to as the "calibrated measurement set", or "calibrated MS", in the following sections.

¹<https://casa.nrao.edu/>

²<https://almascience.eso.org/aq/>

Table 7.1: Properties of the ALMA observations analysed in this chapter.

Project code	Date	Config.	Baseline range	T_{obs} (min)	Ang. res. (arcsec, kpc)	MRS (arcsec, kpc)	FoV (arcsec, kpc)	Calibration
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
2024.1.01711.S	2025 May 7	C43-6	15 m – 2.5 km	91.2	0.096, 0.67	1.7, 12.0	16.4, 113	CASA 6.6.1.17
2025.1.00772.S	2025 October 20	C43-8	67 m – 8.3 km	47.2	0.040, 0.28	0.6, 4.3	16.4, 113	CASA 6.6.6.17
2025.1.00772.S	2025 December 10	C43-5	15 m – 1.4 km	35.8	0.18, 1.3	2.6, 18.0	16.4, 113	CASA 6.6.6.17

Notes. Columns: (1) Project code. (2) Observation date. (3) ALMA array configuration. (4) Minimum and maximum baseline lengths. (5) Total observing time including all overheads. (6) Angular resolution, i.e. synthesised beam FWHM, for Briggs weighting with a robust parameter of 0.5. (7) Maximum recoverable scale, i.e. largest angular scale that can be recovered with the given array configuration. (8) Field of view, i.e. primary beam FWHM. (9) Calibration pipeline version.

7.2.3 *uv*-space pre-processing and gridding

7.2.3.1 Pre-processing

To fit the ALMA data directly in *uv* space (see Section 7.3.3 for details), we pre-process the visibilities in the calibrated MS. First, we separate the [C II] line emission from the continuum emission using the CASA task `uvcontsub`, which fits a continuum model that varies linearly with frequency to line-free channels (identified interactively) and then subtracts the best-fitting model from the calibrated visibilities. Then, we apply the CASA task `mstransform` to rebin the data to 60 channels of 20 km s^{-1} (24.2 MHz). This reduces the calibrated MS to a manageable size, and the channel width of 20 km s^{-1} remains sufficient to properly sample the velocity dispersion of [C II] (measured in Section 7.4). Finally, we shift the phase centres of all observations to $\text{RA} = 14^{\text{h}}24^{\text{m}}13^{\text{s}}.896$, $\text{Dec.} = 02^{\circ}23'03''.84$ (International Celestial Reference System; ICRS) using the CASA task `phaseshift`, to ensure consistency between visibilities from different observations. These preprocessing steps prepare the calibrated MS for gridding.

7.2.3.2 Gridding

We grid the calibrated, continuum-subtracted and phase-shifted ALMA data in *uv* space using the Python package `VISCUBE`. The details of the gridding procedure are described in the `VISCUBE` documentation³, so we summarise only the specifics for gridding ID141 below.

We choose the field of view in *uv* space such that the image-space pixel size is 7.77 mas, oversampling the angular scale corresponding to the longest baseline by a factor of 3. We then choose the number of pixels to be 451, such that the image-space field of view is $3''.5 \times 3''.5$, covering all the detected emission. We adopt a Kaiser-Bessel window with a shape parameter of 9.0 as the gridding kernel. We take the average of all visibilities in each *uv*-space pixel, weighted by the inverse variance of each visibility multiplied by the kernel function, as the pixel's value. We then take the standard error of this average as the uncertainty. Additionally, we mask out *uv*-space pixels with no visibility and pixels in which all visibilities are flagged by ALMA's calibration pipeline.

³<https://viscube.readthedocs.io/en/latest/intro.html>

7.2.4 Imaging

Although we model the ALMA data directly in uv space, we image both the continuum and the [C II] line emission to help visualise the data. All imaging steps used the `tclean` task in version 6.6.1.17 of CASA, which employs the CLEAN algorithm of Högbom (1974).

7.2.4.1 Line imaging

To generate a data cube of the [C II] emission of ID141, we image the calibrated, continuum-subtracted and phase-shifted MS in the cube mode of `tclean` with Briggs weighting and a robust parameter of 0.0. We first create a dirty (i.e. uncleaned) data cube and then clean it in regions of line emission (identified interactively) down to a threshold of $1.5 \sigma_{\text{RMS}}$, where σ_{RMS} is the root-mean-square (RMS) noise evaluated in line-free regions of the dirty cube. The final cleaned data cube has a σ_{RMS} of $0.76 \text{ mJy beam}^{-1}$ and a synthesised beam full-width at half-maximum (FWHM) of $0''.082 \times 0''.063$ (geometric mean $0''.072$, or $\approx 200 \text{ pc}$ on average after lens magnification) with a position angle (PA) of -83.4° .

Throughout this paper, we take $0''.072$ as the nominal angular resolution of the data. Although imaging the MS with uniform weighting would result in a much smaller synthesised beam ($0''.051$), this does not represent the true resolving power of the data because the longest baselines, which have larger weights when using uniform weighting, have signal-to-noise ratios $S/N < 1$ and hence do not provide any constraint on the lens or the dynamical model (see Section 7.3). In contrast, imaging the MS with natural weighting would result in a much larger synthesised beam FWHM ($0''.129$). Yet, this also does not represent the true resolving power of the data, as emission can be detected from much smaller spatial scales than this due to the higher S/N of the naturally weighted data cube. We thus choose Briggs weighting with a robust parameter of 0.0 for the optimal trade-off. We note that this choice affects only data visualisation and interpretations of data quality, not our data analysis, as we fit the data directly in uv space.

Figure 7.1 shows the zeroth (integrated-intensity), first (intensity-weighted mean line-of-sight velocity) and second (intensity-weighted line-of-sight velocity dispersion) moment maps of the final data cube, as well as the integrated spectrum of the [C II] emission. The moment maps are created using a masked-moment technique (Dame, 2011). We generate a

mask by first taking a copy of the data cube and smoothing it spatially with a uniform filter of size 3 times the (geometric mean of the) synthesised beam FWHM, and spectrally with a Hanning window of 5 channels. The mask includes all pixels of the smoothed data cube with a flux density larger than $0.5 \sigma_{\text{RMS}}$ of the unsmoothed data cube. We then create the moment maps using only the masked pixels of the unsmoothed data cube. The integrated spectrum is created by summing all pixels (without applying any masks) within the central $2'' \times 2''$ region of the data cube, covering all the detected emission.

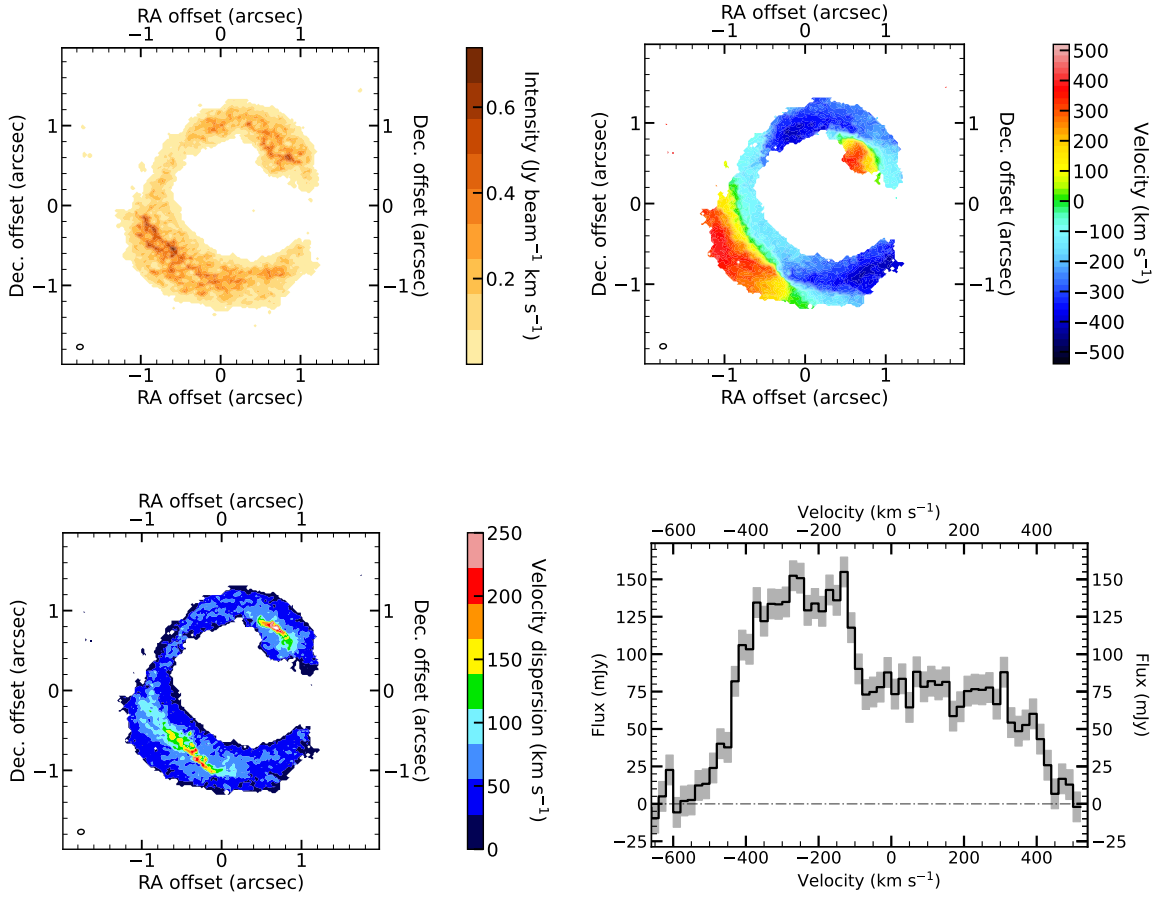


Figure 7.1: Data products of ID141 created from our ALMA [C II] data cube. **Top-left:** Zeroth-moment (integrated-intensity) map. **Top-right:** First-moment (intensity-weighted mean line-of-sight velocity) map. **Bottom-left:** Second-moment (intensity-weighted line-of-sight velocity dispersion) map. The synthesised beam ($0''.082 \times 0''.063$) is shown as a black open ellipse in the bottom-left corner of each map. **Bottom-right:** Integrated spectrum. The shaded region shows the 1σ statistical uncertainties. The dot-dashed line indicates the zero flux level. In all maps, positions are measured relative to the best-fitting galaxy centre position (see Section 7.4); velocities are in the rest frame, assuming $z = 4.24417$.

The [C II] integrated-intensity map of ID141 has a clumpy distribution, unlike the smoother intensity distributions of other gas tracers (CO, H₂O and [C I]) observed previously by [Dye](#)

et al. (2022), potentially tracing individual star-forming regions on scales of ≈ 200 pc. Notably, the peak of the [C II] emission is offset from both the northern and the southern lensed image of the dynamical centre. This offset emission has previously been observed in other gas tracers (Dye et al., 2022), and the source-plane reconstruction presented in Section 7.4.1 suggests it is produced by a disturbed gas distribution. Despite this irregular gas morphology, both the velocity map and the velocity dispersion map are in good agreement with two lensed images of a single regularly rotating disc (discussed in detail in Section 7.5.2).

The [C II] spectrum exhibits an asymmetry between positive- and negative-velocity emission, which results from differential magnification (the lens magnification is higher on the negative-velocity side of the disc), consistent with previously observed kinematic tracers (Dye et al., 2022). The total [C II] flux is $85.5 \pm 1.6 \pm 8.6 \text{ Jy km s}^{-1}$, where the second uncertainty is systematic and results from the typical $\approx 10\%$ ALMA flux calibration uncertainty. This is in statistical agreement with but slightly lower than the total flux of $107 \pm 17 \text{ Jy km s}^{-1}$ derived by Cox et al. (2011) using observations from the Atacama Pathfinder Experiment (APEX) telescope.

Figure 7.2 shows the PVD of ID141 along the lensed kinematic major axis in the south-east image. We choose to plot the PVD in the image plane instead of the source plane so that it does not depend on our model assumptions about the source-plane gas distribution. We generated this PVD using the following steps. First, we sampled the kinematic major-axis in the source plane with evenly spaced points. Second, we computed the coordinates of the corresponding image-plane points by performing forward ray-tracing using the `forward_raytrace` task of CAUSTICS (Stone et al., 2024a), assuming the best-fitting lens model derived in Section 7.4. Here, although each source-plane point has multiple images in the image plane, we only plot the PVD of the south-east image, as it has the highest magnification. Third, we connected the image-space points using linear interpolation to form a curved path that closely approximates the lensed kinematic major axis. Finally, we expanded the curved path into a pseudo-slit with a width of four pixels and extracted the PVD along this slit by summing all pixels across its width. We choose not to apply any mask to the PVD to avoid masking out emission features and to provide an accurate visualisation of the noise level. The contours are equally spaced between the 1σ noise level (measured directly from the emission-free pixels of the PVD) and

the maximum of the PVD. The positions in the PVD are measured along the curved path relative to the coordinate of the kinematic centre in the image plane. The resulting PVD shows that the rotation curve is largely flat over the spatial extent of the data, except for a slight hint of higher velocities near the galaxy centre. This could be produced by a central SMBH or a compact stellar spheroid, and we discuss this feature further in Section 7.4. The asymmetry of the PVD results from differential magnification: the blueshifted side of the disc is more magnified and thus has a larger angular size in image space than the redshifted side.

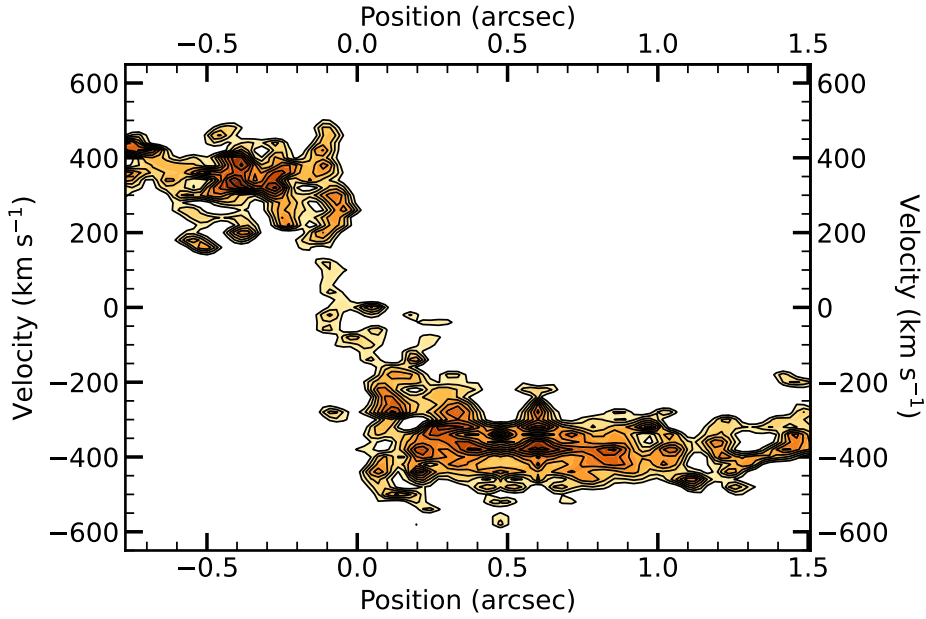


Figure 7.2: Position-velocity diagram of ID141 along a curved path that closely traces its lensed kinematic major axis in the south-east image. Positions are measured along the curved path relative to the image-plane kinematic centre. The rotation curve is largely flat except for a slight hint of higher velocities near the galaxy centre.

7.2.4.2 Continuum imaging

Although a detailed study of the continuum emission is beyond the scope of this work, we created a continuum image from our calibrated MS for use by future studies. We adopted the multi-frequency synthesis mode of `tclean`, again using Briggs weighting with a robust parameter of 0.0. We cleaned the dirty image down to 3 times the RMS noise of the continuum image $\sigma_{\text{RMS, cont}} = 51 \mu\text{Jy beam}^{-1}$. The synthesised beam FWHM is $0''.077 \times 0''.058$ (geometric mean $0''.067$, or ≈ 190 pc on average after lens magnification) with a PA of $-82^\circ.9$. The central frequency of the continuum image is 356 GHz, corresponding to an observing wavelength

of $842\ \mu\text{m}$ and a rest-frame wavelength of $161\ \mu\text{m}$. At this wavelength, the continuum is dominated by the thermal emission of dust (Dye et al., 2022).

The final continuum image is shown in Figure 7.3. The continuum emission is less spatially extended than the $[\text{C II}]$ emission, with clear emission peaks that coincide with the image positions of the galaxy dynamical centre. This reflects notable differences between the distribution of cold neutral gas and that of dust, in line with the recent discovery of extended $[\text{C II}]$ emission in other star-forming galaxies at similar redshifts (e.g. Herrera-Camus et al., 2021; Lambert et al., 2023; Ikeda et al., 2025). The continuum map also reveals several additional clumps of emission to the north-east of the southern image, which are likely associated with the offset $[\text{C II}]$ emission. The integrated continuum flux, also computed by summing all pixels within the central $2'' \times 2''$ region, is $97.8 \pm 0.8 \pm 9.8$ mJy (the second uncertainty is again systematic and associated with the ALMA flux calibration uncertainty), in reasonable agreement with the $850\ \mu\text{m}$ flux of 121 ± 8 mJy from Dye et al. (2018).

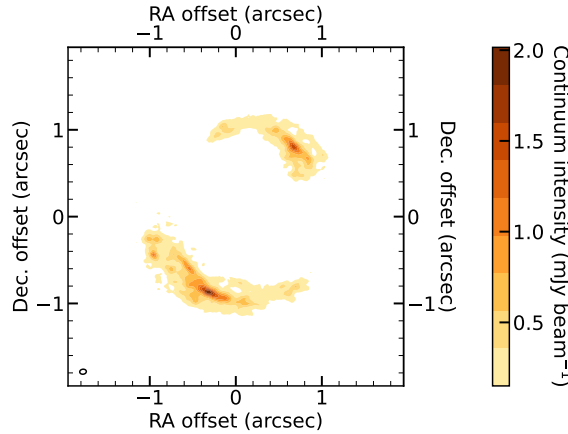


Figure 7.3: $840\text{-}\mu\text{m}$ continuum image of ID141, showing all emission above $3\ \sigma_{\text{RMS, cont.}}$. The synthesised beam ($0''.077 \times 0''.058$) is shown as a black open ellipse in the bottom-left corner.

7.3 Modelling

Unlike traditional analysis pipelines that perform lens modelling and dynamical modelling separately, we simultaneously forward model the strong lensing and $[\text{C II}]$ dynamics using the PYTHON package SUPERB MASSES FROM GAS KINEMATICS (SUPERMAGE). This allows to sample the covariance between lens and dynamical parameters, providing robust M_{BH} uncertainties. Additionally, we fit the gridded ALMA data directly in uv space. This avoids

potential measurement inaccuracies caused by the CLEAN algorithm, which may suppress information and introduce artefacts to the generated data cube (e.g. [Maresca & Dye, 2025](#)). uv -space fitting also produces more reliable model parameter uncertainties, by allowing the covariance between data points to be safely ignored ([Yantovski-Barth et al., 2025](#)). This is because noise is uncorrelated across visibilities in uv space (so the data points are independent) but highly correlated between neighbouring pixels in image space (so that assuming independent data points, as is often done, can bias the results). Moreover, SUPERMAGE is fully automatically differentiable and graphics processing unit (GPU)-accelerated, allowing orders-of-magnitude more efficient sampling of model parameters compared to traditional approaches. In this section, we present each component of our modelling and fitting procedure in detail.

7.3.1 Dynamical modelling

Our dynamical model of ID141 has three mass components: the central SMBH, stars and gas. The gas component is often neglected in local M_{BH} measurements but is crucial in high-redshift star-forming galaxies like ID141, whose gas mass dominates the total dynamical mass at large radii ([Dye et al., 2022](#)). We ignore a potential dark matter contribution, as the maximum radius probed by our data is only $\lesssim 2$ kpc and the dark matter fraction within this radius is typically $\lesssim 10\%$ for the most massive galaxies at this cosmic time (e.g. [Rizzo et al., 2021](#); [Danhaive et al., 2026](#)). Our model circular velocity curve $V_c(R)$ is thus given by

$$V_c(R) = \sqrt{V_{\text{c,BH}}^2(R) + V_{\text{c,*}}^2(R) + V_{\text{c,gas}}^2(R)} , \quad (7.1)$$

where R is the radius in polar (or cylindrical) coordinates and $V_{\text{c,BH}}$, $V_{\text{c,*}}$ and $V_{\text{c,gas}}$ are the circular velocity contribution of the SMBH, stars and gas, respectively.

We assume pure circular rotation and a geometrically thin molecular gas disc. We also add a small spatially constant gas velocity dispersion σ_{gas} (such that $V_c/\sigma_{\text{gas}} \gg 1$). We discuss the validity of these assumptions in Section 7.5.2 and describe our model for each mass component below.

7.3.1.1 Stellar distribution

As discussed in Chapter 6, for M_{BH} measurements in the local Universe, the stellar mass distribution model is typically constructed by performing a multi-Gaussian expansion (MGE; [Emsellem et al. 1994](#); [Cappellari 2002b](#)) of an optical/near-infrared *Hubble Space Telescope* (*HST*) or *James Webb Space Telescope* (*JWST*) image of the galaxy. However, ID141 has not been observed by *JWST* and the only *HST* observations of it (Proposal 12488; PI: M. Negrello) were too shallow to detect the background source galaxy. Moreover, even if we were to obtain a *JWST* image of ID141, the galaxy’s extremely dusty nature (see the spectral energy distribution in Figure A2 of [Dye et al. 2022](#)) would make any stellar mass model derived from the heavily obscured image highly dependent on the assumed dust distribution and dust extinction law, leading to orders-of-magnitude systematic uncertainties. If we were to instead fit the MGE parameters dynamically without constraints from optical images, the high flexibility of MGE and the lack of physically motivated regularisations would cause MGE to overfit the kinematic data, leaving no room for other galaxy components (e.g. it would infer a point-like stellar distribution with $M_{\text{BH}} = 0$).

For these reasons, we adopt parametric one-dimensional (1D) stellar mass surface density profiles as our stellar mass models. However, unlike the conventional approach of assuming spherical symmetry, we deproject the 1D profiles into axisymmetric three-dimensional (3D) mass distributions using the neural network approach described in Chapter 6. The intrinsic axis ratio of the deprojected 3D MGE Gaussians, q , is a free parameter of the models.

In this work, we consider three different parametric models widely adopted to fit the central regions of galaxies: the Sérsic profile ([Sérsic, 1968](#)), the Nuker profile ([Lauer et al., 1995](#)) and the core-Sérsic profile ([Trujillo et al., 2004](#)).

The Sérsic profile. The mass surface density profile of the Sérsic model is

$$\Sigma_{\text{Sérsic}}(R) = \Sigma_e \exp \left\{ -b_n \left[\left(\frac{R}{R_e} \right)^{1/n} - 1 \right] \right\}, \quad (7.2)$$

where R_e is the effective (half-mass) radius, Σ_e the surface mass density at the effective radius, n the Sérsic index and b_n is approximated to be

$$b_n \approx 2n - \frac{1}{3} + \frac{4}{405n} + \frac{46}{25515n^2} \quad (7.3)$$

(see [Ciotti & Bertin 1999](#)). We impose flat priors on each parameter based on observational constraints, encompassing the full range of values derived from existing observations of low- and high-redshift galaxies (e.g. [Caon et al., 1993](#); [Fischer et al., 2019](#); [Euclid Collaboration et al., 2025a](#)). The priors are listed in Table 7.2.

The Nuker profile. The mass surface density of the Nuker profile is

$$\Sigma_{\text{Nuker}}(R) = 2^{\frac{\beta-\gamma}{\alpha}} \Sigma_b \left(\frac{R}{R_b} \right)^{-\gamma} \left[1 + \left(\frac{R}{R_b} \right)^{\alpha} \right]^{\frac{\gamma-\beta}{\alpha}}, \quad (7.4)$$

where γ is the inner power-law slope, β the outer power-law slope, α the transition sharpness between the inner and the outer power law, R_b the break radius where the transition takes place and Σ_b the mass surface density at the break radius. The priors of each parameter are again listed in Table 7.2. We impose bounds on $\beta - \gamma$ instead of β to be consistent with the observational constraint (e.g. [Lauer et al., 2007](#)) that the inner power-law slope is always shallower than the outer power-law slope.

The Core-Sérsic profile. The mass surface density of the core-Sérsic profile is defined as

$$\Sigma_{\text{CS}}(R) = \Sigma_e \left[1 + \left(\frac{R_b}{R} \right)^{\alpha} \right]^{\gamma/\alpha} \exp \left\{ -b_n \left[\left(\frac{R^{\alpha} + R_b^{\alpha}}{R_e^{\alpha}} \right)^{1/(n\alpha)} - 1 \right] \right\}. \quad (7.5)$$

This profile combines a power law in the inner region with a Sérsic profile in the outer region to fit the stellar distribution across a wide range of physical scales. The parameters are thus defined similarly to the parameters of the Nuker and Sérsic profiles: γ is the inner power-law slope, n the Sérsic index, α the transition sharpness, R_b the break radius, R_e the effective radius and Σ_e the surface mass density at the effective radius.

Again, the priors for the core-Sérsic parameters were initially based on the range of values in the literature (e.g. [Rusli et al., 2013](#); [Dullo & Graham, 2014](#)). However, those priors are

not identical to those of the equivalent parameters of the Sérsic profile or the Nuker profile due to sample selection effects. In particular, because the core-Sérsic profile has only been fitted to early-type galaxies (ETGs) with depleted cores, no fit exists for galaxies with low n ($0.5 \lesssim n \lesssim 2$) and/or large γ ($0.8 \lesssim \gamma \lesssim 1.2$), even though such ranges are permitted physically. We thus extend the priors of these parameters to match those of the equivalent parameters of the Sérsic profile or the Nuker profile. The final adopted priors are shown in Table 7.2.

7.3.1.2 Gas distribution

Despite the clumpy and somewhat disturbed gas morphology in the image plane, the reconstructed source-plane gas distribution (see Section 7.4.1) is still consistent with a single exponential disc, except for an offset clump of emission to the north of the dynamical centre. Thus, to simplify the model, we follow the convention of assuming a thin exponential disc gas profile:

$$\Sigma_{\text{gas}}(R) \propto e^{-R/R_s}, \quad (7.6)$$

where R_s is the scale length of the exponential disc. We adopt this profile both as the model for the [C II] surface brightness distribution and as the model for the gas mass distribution. The scale lengths of the surface brightness model and the mass model are constrained to be the same, while the normalisations of the two models are independent (i.e. the mass-to-light ratio M/L and thus total mass of the gas M_{gas} is a free parameter).

7.3.1.3 SMBH

We model the SMBH as a central point mass, which produces a Keplerian circular velocity curve

$$V_{\text{c,BH}} = \sqrt{\frac{GM_{\text{BH}}}{R}}. \quad (7.7)$$

The characteristic signature of a Keplerian circular velocity curve is that the circular velocity increases with decreasing radius as $R^{-1/2}$, although a spatially-unresolved compact distribution of stars would produce a similar effect.

7.3.2 Lens modelling

Following the procedure of [Dye et al. \(2018, 2022\)](#), we model the foreground lens using an elliptical power-law (EPL) model with a dimensionless mass surface density

$$\kappa(x, y) = \frac{2-t}{2} \left(\frac{R_{\text{ein}} \sqrt{q_{\text{lens}}}}{\tilde{R}} \right)^t, \quad (7.8)$$

where $\tilde{R}$ is the elliptical radius defined as

$$\tilde{R} \equiv \sqrt{q_{\text{lens}}^2 x^2 + y^2}. \quad (7.9)$$

The free parameters of the model are the power-law index t , Einstein radius R_{ein} , lens axis ratio q_{lens} and the on-sky coordinates and position angle of the lens: $x_{0,\text{lens}}$ (east-west offset relative to the phase centre of the data), $y_{0,\text{lens}}$ (north-south offset relative to the phase centre of the data) and ϕ_{lens} (position angle measured from north through west). We impose tight priors on the lens parameters based on earlier literature studies (see Table 7.2; e.g. [Dye et al. 2022](#)).

Although the *HST* image of ID141 shows that the lens is, in fact, a pair of galaxies, a single EPL lens already provides a good fit to the data, and adding a second lens only affects the magnification by $\approx 5\%$, much smaller than the statistical uncertainties of the dynamical parameters (see Section 7.4).

Unlike the conventional procedure of determining the lens model before fitting the dynamical parameters, we forward model the gravitational strong lensing and the gas dynamics simultaneously by lensing each velocity channel of the source-plane model cube individually using CAUSTICS ([Stone et al., 2024a](#)). The resulting image-plane model cube is then Fourier-transformed into a visibility cube in uv space on the same grid as the gridded ALMA data. Finally, the model visibility cube is fitted to the data visibility cube to determine the lens and dynamical parameters using procedures detailed in the following sub-section (Section 7.3.3).

Table 7.2: Model parameters and priors.

Parameter	Prior range
SMBH mass	
$\log (M_{\text{BH}}/M_{\odot})$	0 – 11
Sérsic stellar mass profile	
n	0.5 – 10
R_e (kpc)	0.1 – 8
$\log (\Sigma_e/M_{\odot} \text{ pc}^{-2})$	1 – 5
Nuker stellar mass profile	
α	0.1 – 10
$\beta - \gamma$	0.3 – 3
γ	0.001 – 1.2
R_b (pc)	2 – 2000
$\log (\Sigma_b/M_{\odot} \text{ pc}^{-2})$	1 – 5.5
Core-Sérsic stellar mass profile	
α	1.5 – 10
γ	0.001 – 1.2
n	0.5 – 10
R_b (pc)	1 – 100
R_e (kpc)	0.1 – 10
$\log (\Sigma_e/M_{\odot} \text{ pc}^{-2})$	1 – 5.5
Gas distribution	
$\log (M_{\text{gas}}/M_{\odot})$	5.5 – 12
R_s (kpc)	0.5 – 2
Position angle (degree)	80 – 120
Inclination (degree)	23 – 57
Other dynamical quantities	
σ_{gas} (km s ⁻¹)	5 – 100
q	0.1 – 1
Lens	
$x_{0, \text{lens}}$ (arcsec)	-1 – 1
$y_{0, \text{lens}}$ (arcsec)	-1 – 1
ϕ_{lens} (degree)	140 – 230
q_{lens}	0.5 – 1
R_{ein} (arcsec)	0.5 – 1.5
t	0.5 – 1.5
Nuisance parameters	
Integrated gas intensity (Jy km s ⁻¹)	1 – 150
Kinematic centre X offset (arcsec)	-1 – 1
Kinematic centre Y offset (arcsec)	-1 – 1
Kinematic centre velocity offset (km s ⁻¹)	-50 – 50

Notes. The X and Y offsets are measured relative to the phase centre of the data cube, RA = 14^h24^m13^s.896, Dec. = 02°23′03″84 (ICRS), in the east-west and the north-south direction, respectively. The velocity offset is measured relative to the adopted galaxy redshift $z = 4.24417$.

7.3.3 uv -space fitting

We infer the best-fitting model parameters by minimising the χ^2 , defined as

$$\chi^2 = \sum_i \frac{(\text{data}_i - \text{model}_i)^2}{\sigma_i^2}, \quad (7.10)$$

where data_i and σ_i are the value and the uncertainty of the i -th uv -space pixel, respectively, and model_i is the model prediction at that pixel. The summation is over all uv -space pixels and over both real and imaginary components. The χ^2 outside of the prior range listed in Table 7.2 is defined as ∞ .

With ≈ 20 model parameters, finding the global minimum of the χ^2 can be challenging. To explore the parameter space properly and ensure that all significant local minima are identified, we adopt a swarm optimisation approach. First, by sampling the prior distributions we generate 1.2×10^5 random guesses of the model parameters and evaluate their χ^2 . Second, we select the top 0.02% (24) guesses with the lowest χ^2 and use them as the starting points of 24 separate Levenberg-Marquardt (LM; [Levenberg, 1944](#); [Marquardt, 1963](#)) optimisers. Each LM optimiser then descends to a local minimum from its starting point by using the gradients (automatically computed by PyTorch) of the χ^2 function with respect to the model parameters. Third, we compare the χ^2 of the minima found by all LM optimisers and take the local minimum with the lowest χ^2 as the global minimum. Finally, to compute the uncertainties of the model parameters, we sample the global minimum using the Metropolis-adjusted Langevin algorithm ([Roberts & Tweedie, 1996](#)), generating 2×10^5 samples and discarding the first 5% as burn-in. We take the median of the sample as the best-fitting parameters and compute the 1σ uncertainties using the 68.3% confidence intervals, as well as the 3σ uncertainties using the 99.7% confidence intervals.

7.4 Results

7.4.1 Lens model

Despite not using the continuum data, we obtained tight constraints on the strong gravitational lens model, and the best-fitting lens parameters are nearly identical across the fits with the

different stellar mass profiles. Here and throughout this chapter, we will adopt the lens parameters from the Sérsic model, as it has the lowest Bayesian information criterion (BIC; see Section 7.4.2). The Einstein radius of $0''.957 \pm 0''.003$ (1σ uncertainty here and throughout the paper) and the lens axis ratio of 0.876 ± 0.010 agree well with the $R_{\text{ein}} = 0.97 \pm 0.04$ and $q_{\text{lens}} = 0.88 \pm 0.02$ derived by Dye et al. (2022). In contrast, the lens position angle of $200.2^\circ \pm 2.5^\circ$ and the power-law index of 1.17 ± 0.05 are larger than the $\phi_{\text{lens}} = 186^\circ$ and $t = 1.01 \pm 0.03$ of Dye et al. (2022). However, these slight differences have a negligible impact on the dynamical models. For example, the lens magnification at the kinematic centre is $\mu_c = 3.24$ according to our model but $\mu_c = 3.41$ according to the model of Dye et al. (2022). This $\approx 5\%$ difference of the magnification would propagate to a SMBH mass difference of only 2%.

Using our lens model, we also derive pixelated reconstructions of the source-plane [C II] integrated-intensity map and (luminosity-weighted mean line-of-sight) velocity map, following the procedure of Dye et al. (2018). The reconstructed maps are presented in Figure 7.4. Notably, the source-plane velocity map of ID141 is consistent with a single rotating disc. In contrast, the [C II] intensity distribution appears mildly disturbed, and the peak of the [C II] intensity is slightly offset from the dynamical centre of the disc, consistent with the image-plane intensity map (Figure 7.1). Nevertheless, the [C II] distribution remains close to an exponential disc, and the disturbance is small and limited to the central region, where the gas mass does not dominate the kinematics (see Section 7.4.2). For these reasons, our assumption of an exponential gas distribution does not substantially affect our dynamical modelling results. Non-parametric gas models that directly reproduce the observed disturbance will be explored in a future work.

7.4.2 Dynamical model

7.4.2.1 Sérsic stellar mass profile

Table 7.3 lists the best-fitting parameters of the dynamical model with a Sérsic stellar mass surface density profile, along with their 1σ (68.3% confidence level) and 3σ (99.7% confidence level) uncertainties. The top panel of Figure 7.5 shows the 1D marginalised distributions of all non-nuisance model parameters and the two-dimensional (2D) covariances between

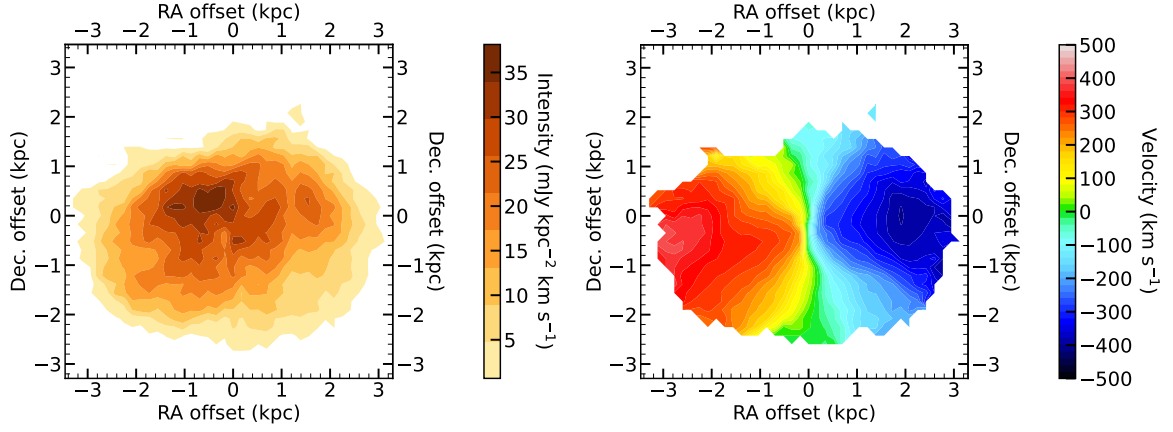


Figure 7.4: [C II] integrated-intensity map (left) and (luminosity-weighted mean line-of-sight) velocity map (right) of the reconstructed source of ID141. The velocity map is well-described by a single rotating disc, but the peak of the [C II] intensity is slightly offset from the dynamical centre of the disc.

them. The posterior distribution of the SMBH mass does not rule out $M_{\text{BH}} = 0$, suggesting that a SMBH is not necessary to reproduce the observed [C II] kinematics. Instead, the slight velocity increase towards the central region of ID141 is produced by a compact ($R_e = 1.18^{+0.46}_{-0.25}$ kpc) stellar distribution with a large Sérsic index. In fact, our best-fitting Sérsic index, $n = 9.31^{+0.51}_{-0.92}(1\sigma)^{+0.69}_{-3.36}(3\sigma)$, is upper-bounded by our prior of $n \leq 10$ rather than constraints from the data. Such a large Sérsic index is rare among high-redshift SMGs (Gillman et al., 2023, 2024), which are expected to have disc-like morphologies. However, it is possible that the Sérsic index of the mass distribution is substantially higher than that of the luminosity distribution due to radial gradients of the stellar M/L (e.g. Szomoru et al., 2013). If we impose a tighter bound of $n \leq 6$ to be more consistent with the luminosity-weighted Sérsic index distribution of SMGs (Gillman et al., 2024), we obtain a non-zero best-fitting SMBH mass $\log(M_{\text{BH}}/M_\odot) = 10.04 \pm 0.08$ but a much worse χ^2 ($\Delta\chi^2 = 88$). We choose not to do this to ensure that our dynamical model is independent of studies of other SMGs. Although the Sérsic index is somewhat extreme, our best-fitting stellar mass distribution is still physically plausible: the effective stellar mass surface density, defined as $\Sigma_{\text{eff}} \equiv M_*/(2\pi R_e^2)$, is only $\approx 9.6 \times 10^3 M_\odot \text{pc}^{-2}$, far less than the maximum observed value of $\approx 3 \times 10^5 M_\odot \text{pc}^{-2}$ across the Universe (Hopkins et al., 2010; Grudić et al., 2019). A deep high-resolution rest-frame near-infrared image of ID141 from *JWST*, appropriately corrected for dust attenuation, could provide a photometric constraint on n and substantially improve

the M_{BH} measurement, potentially ruling out $M_{\text{BH}} = 0$.

The total stellar mass, computed by integrating the Sérsic profile to $R = \infty$, is $M_* = 8.6^{+1.4}_{-1.0} \times 10^{10} M_{\odot}$. Assuming the optical stellar mass – size relation of [Miller et al. \(2025\)](#), a $z = 4$ galaxy with this stellar mass should have $R_e \approx 4.0$ kpc. Our best-fitting R_e is however smaller than this by about twice the intrinsic scatter of the relation (0.24 dex). The total gas mass (again integrated to $R = \infty$) is $M_{\text{gas}} = (1.42 \pm 0.09) \times 10^{11} M_{\odot}$, with a well-constrained scale length of 1.31 ± 0.05 kpc. The left panel of Figure 7.6 presents the circular velocity curve decomposition between SMBH, stars and gas, according to our best-fitting Sérsic model. The stars dynamically dominate over the gas within the central ≈ 1.5 kpc and are responsible for the central velocity increase, whereas the contribution from the SMBH is negligible at all radii beyond the spatial resolution limit. We provide a detailed comparison between our dynamically fitted component masses and the photometry-based masses in the literature in Section 7.5.1.

Despite the unconstrained lower limit of the SMBH mass, we can still derive an informative upper limit, as models with an overly massive SMBH would overpredict the rotation velocities in the central region of the galaxy, even with the minimum possible stellar mass. For the Sérsic model, the 3σ (i.e. 99.7%) upper limit of the SMBH mass is $\log(M_{\text{BH}}/M_{\odot}) = 9.88$ or $M_{\text{BH}} = 7.6 \times 10^9 M_{\odot}$. We compare this upper limit with the mass estimates of active galactic nuclei (AGN) at similar cosmic epochs in Section 7.5.1. The top row of Figure 7.7 compares our best-fitting Sérsic model (left) with the Sérsic model with the SMBH mass equal to the inferred upper limit (right) in term of how well the models reproduce the observed PVD. The best-fitting model, without requiring a SMBH, accurately reproduces the line-of-sight velocity distributions along the kinematic major axis in the image plane, including the slightly higher velocities near the centre, whereas the model with a $7.6 \times 10^9 M_{\odot}$ SMBH slightly overshoots the central velocities. Indeed, the best-fitting Sérsic model has $\chi^2 = 74,769$ and a reduced $\chi^2_r = 0.1468$, whereas the model with the upper-limit SMBH mass has $\chi^2 = 74,782$ and $\chi^2_r = 0.1469$. The statistics of the best-fitting Sérsic model are listed and compared to those of the other two models in Table 7.4.

Table 7.3: Best-fitting Sérsic model parameters and associated uncertainties.

Parameter	Search range	Best fit	1σ uncertainty	3σ uncertainty
SMBH and stellar mass model				
$\log(M_{\text{BH}}/M_{\odot})$	0.0 – 11.0	5.8	-4.3, +3.3	-5.8, +4.1
n	0.50 – 10.00	9.31	-0.92, +0.51	-3.4, +0.69
R_e (kpc)	0.10 – 8.00	1.18	-0.25, +0.46	-0.52, +0.98
$\log(\Sigma_e/M_{\odot} \text{ pc}^{-2})$	1.00 – 5.00	3.36	-0.21, +0.14	-0.45, +0.32
q	0.10 – 1.00	0.51	-0.31, +0.39	-0.41, +0.49
Lens				
$x_{0, \text{lens}}$ (arcsec)	-1.000 – 1.000	0.248	-0.006, +0.005	-0.019, +0.015
$y_{0, \text{lens}}$ (arcsec)	-1.000 – 1.000	0.281	± 0.005	± 0.015
ϕ_{lens} (degree)	140.0 – 230.0	200.2	-2.6, +2.4	-7.7, +6.4
q_{lens}	0.500 – 1.000	0.876	-0.009, +0.011	-0.024, +0.028
R_{ein} (arcsec)	0.500 – 1.500	0.957	± 0.003	-0.007, +0.009
t	0.50 – 1.50	1.17	± 0.05	-0.13, +0.10
Molecular gas disc				
$\log(M_{\text{gas}}/M_{\odot})$	5.50 – 12.00	11.15	± 0.03	-0.11, +0.08
R_s (kpc)	0.50 – 2.00	1.31	± 0.05	± 0.13
Position angle (degree)	80.0 – 120.0	98.5	-0.5, +0.6	-1.6, +1.7
Inclination (degree)	23.0 – 57.0	45.6	-1.3, +1.1	-3.3, +3.6
σ_{gas} (km s $^{-1}$)	5.0 – 100.0	65.8	-0.7, +0.8	-2.2, +2.7
Nuisance parameters				
Integrated gas intensity (Jy km s $^{-1}$)	1.0 – 150.0	66.8	-0.9, +1.0	-3.0, +3.5
Kinematic centre X offset (arcsec)	-1.000 – 1.000	0.065	± 0.008	-0.023, +0.025
Kinematic centre Y offset (arcsec)	-1.000 – 1.000	0.002	-0.010, +0.011	-0.022, +0.030
Kinematic centre velocity offset (km s $^{-1}$)	-50.0 – 50.0	-19.1	-1.0, +1.1	-2.6, +3.2

Notes. The X and Y offsets are measured relative to the phase centre of the data cube, RA = 14^h24^m13^s.896, Dec. = 02°23′03″.84 (ICRS), in the east-west and the north-south direction, respectively. The velocity offset is measured relative to the adopted galaxy redshift $z = 4.24417$.

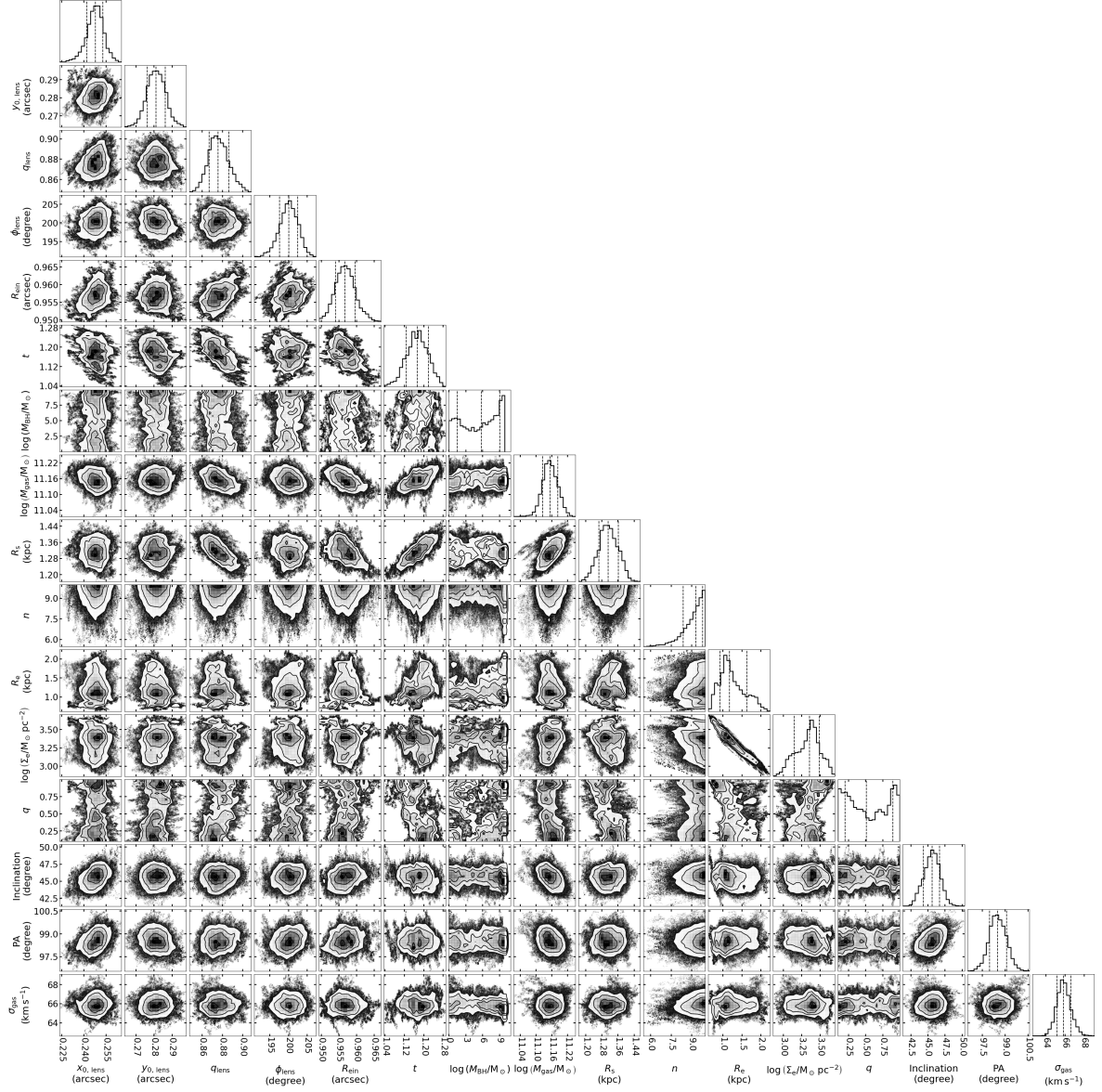


Figure 7.5: Corner plot showing the covariances between non- nuisance model parameters of our best-fitting Sérsic (top), Nuker (middle) and core-Sérsic (bottom) models. The contours represent increasing confidence levels, from 68.3% (1σ) to 99.7% (3σ). The histograms show the 1D marginalised posterior distribution of each parameter. The dashed lines indicate the medians and 1σ confidence intervals.

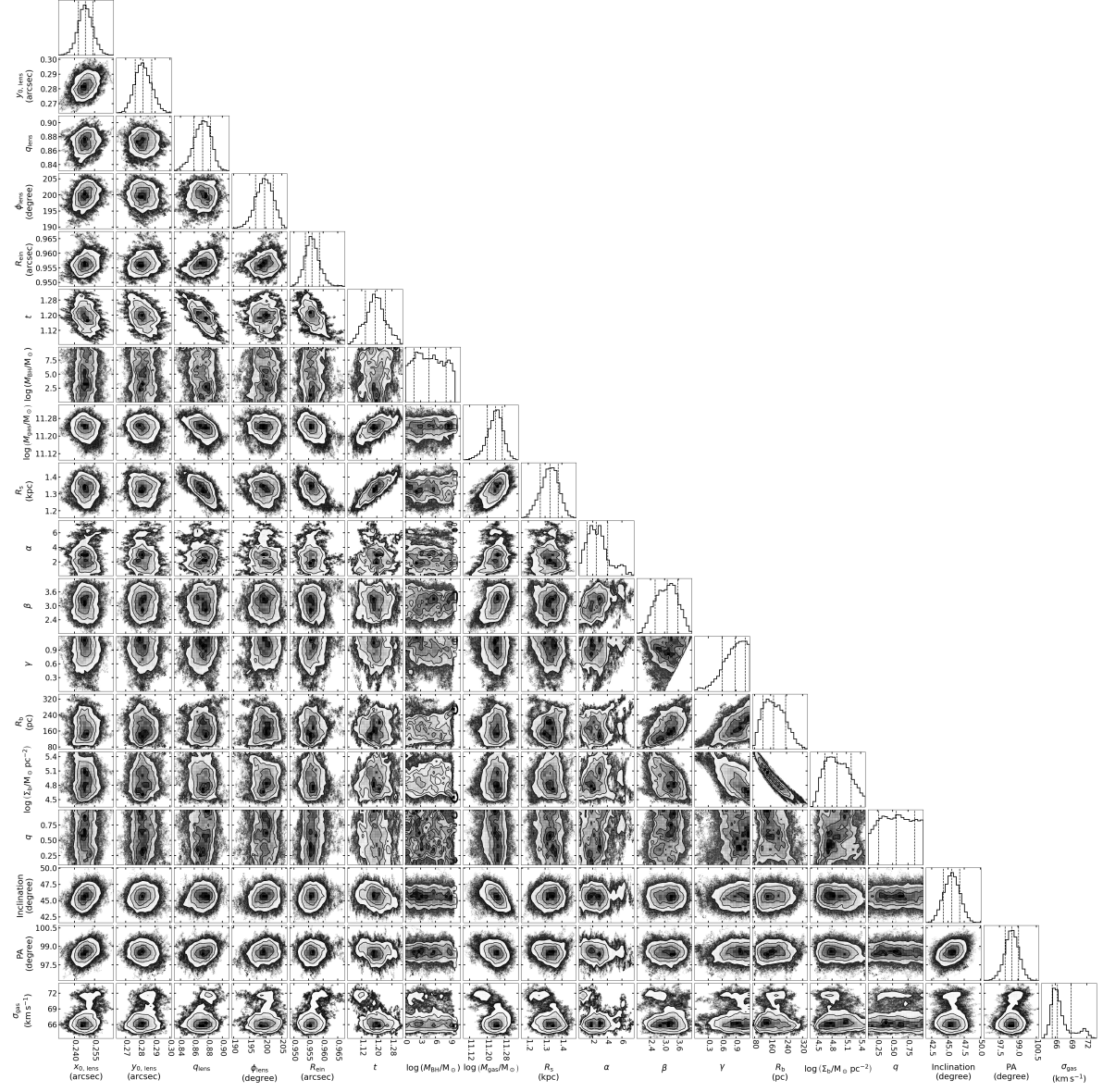


Figure 7.5: Continued.

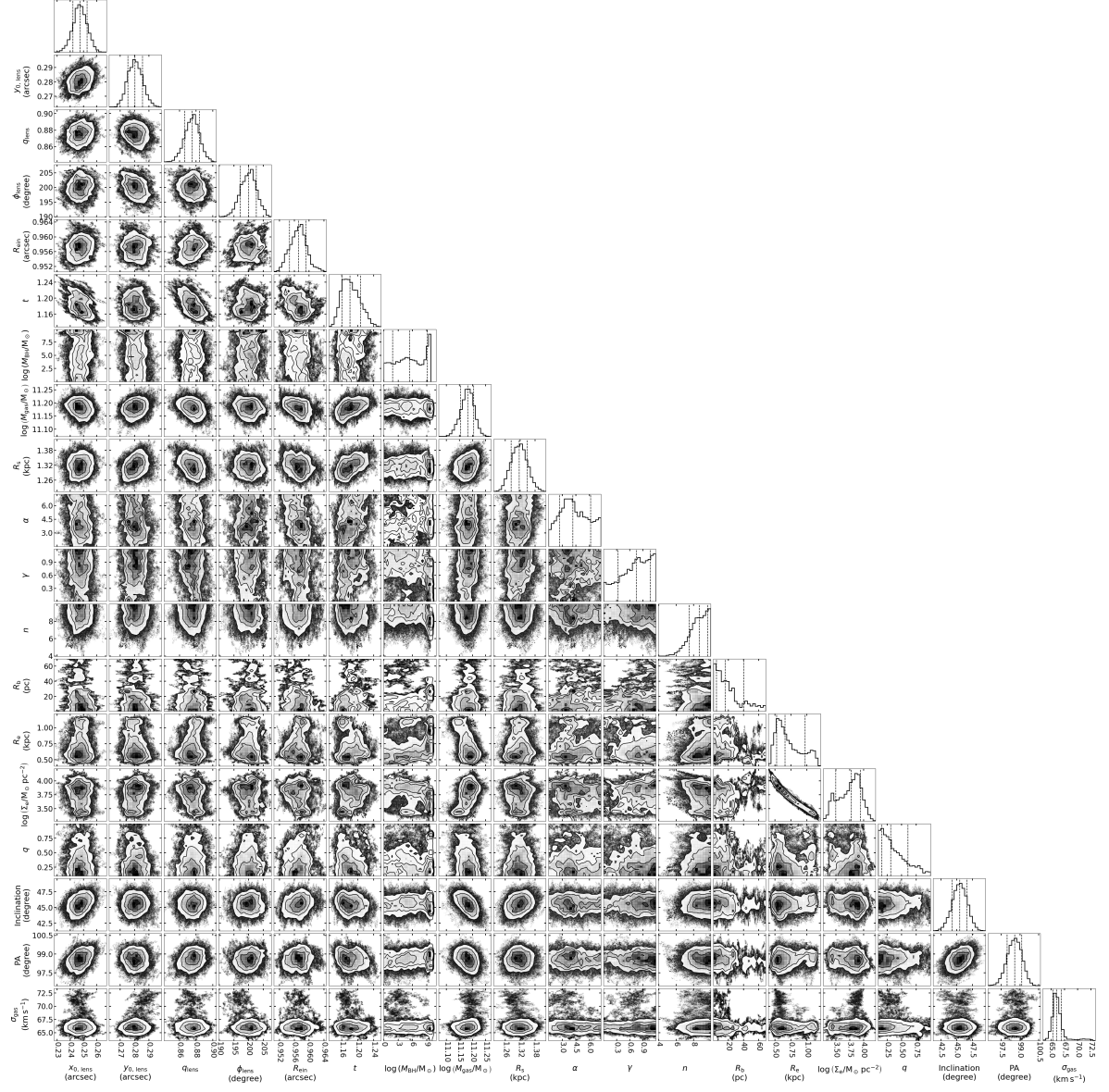


Figure 7.5: Continued.

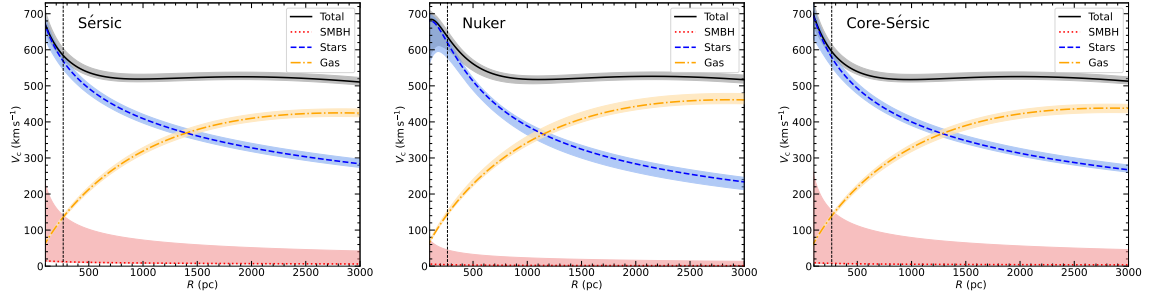


Figure 7.6: Circular velocity curve decomposition of ID141, assuming a Sérsic (left), a Nuker (centre) and a core-Sérsic (right) stellar mass surface density profile. In each panel, the curves show the relative contributions of the SMBH (red dotted line), stars (blue dashed line) and gas (orange dot-dashed line) to the total circular velocity curve (black solid line). The 1σ uncertainties of the profiles are shown as shaded regions. The vertical black line shows the physical resolution of the observation at $R = 0$ (the galaxy centre).

Table 7.4: Statistics of the dynamical models.

Model	χ^2	χ_r^2	k	BIC
(1)	(2)	(3)	(4)	(5)
Sérsic	74769	0.1468	20	75032
Nuker	74765	0.1468	22	75054
Core-Sérsic	74766	0.1468	23	75068

Notes. (1) Model. (2) χ^2 . (3) Reduced χ^2 . (4) Number of free parameters. (5) Bayesian information criterion.

7.4.2.2 Nuker stellar mass profile

The best-fitting parameters of the dynamical model with a Nuker stellar mass surface density profile, along with their 1σ and 3σ uncertainties, are listed in Table 7.5. The 1D marginalised posteriors and the 2D joint posterior distributions of the non-nuisance Nuker model parameters are shown in the middle panel of Figure 7.5. Similar to the Sérsic model, the Nuker model does not rule out $M_{\text{BH}} = 0$, favouring that the high central velocities are due to a compact stellar mass distribution with a break radius $R_b = 175^{+61}_{-52}$ pc and steep power-law indices $\gamma = 0.89^{+0.21}_{-0.28}$ and $\beta = 3.12^{+0.46}_{-0.52}$. The best-fitting R_b is smaller than the source-plane physical resolution of ≈ 280 pc at $R = 0$, but this stellar mass distribution is again not unphysical, as the stellar mass surface density at the break radius $\Sigma_b = 8.0^{+8.0}_{-3.5} \times 10^4 \text{ M}_\odot \text{ pc}^{-2}$, lower than the maximum limit of $3 \times 10^5 \text{ M}_\odot \text{ pc}^{-2}$ (Hopkins et al., 2010; Grudić et al., 2019).

If we naively integrate the best-fitting Nuker profile to $R = \infty$, the total stellar mass $M_* \approx 1.1 \times 10^{10} \text{ M}_\odot$. However, the Nuker profile typically applies only to the nuclear regions of galaxies, and extrapolations of the profile are usually unreliable (e.g. Graham et al., 2003). Indeed, our naively computed total stellar mass for the Nuker model is substantially smaller

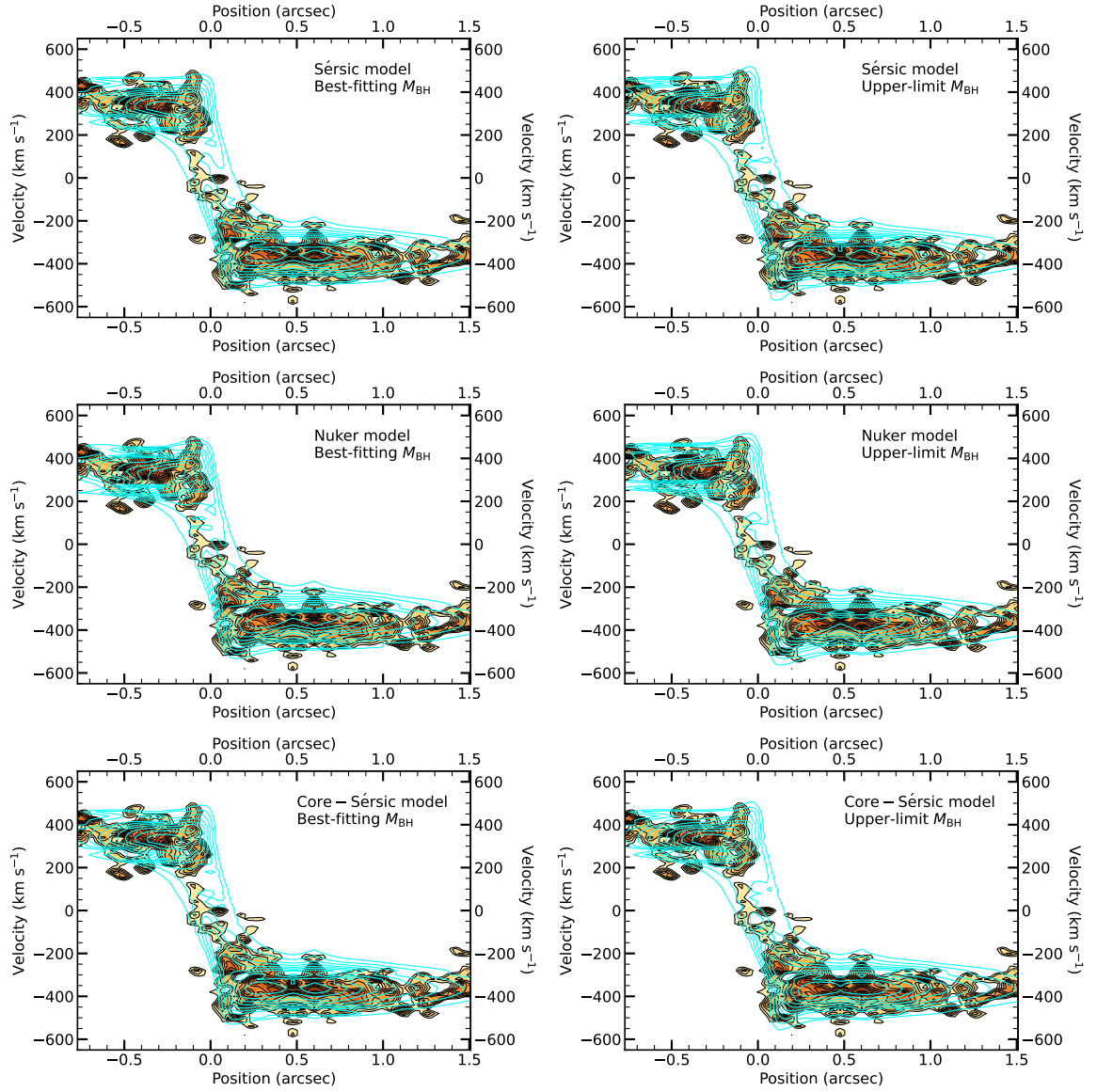


Figure 7.7: Position-velocity diagram of ID141 along a curved path that closely traces its lensed kinematic major axis in the south-east image (orange scale with black contours), overlaid with the PVD of the best-fitting model (cyan contours; left column) and the PVD of the model with its SMBH mass equal to the upper limit (cyan contours; right column) for the Sérsic model (top), the Nuker model (middle) and the core-Sérsic model (bottom). Positions are measured along the curved path relative to the image-plane kinematic centre. For all models the slightly higher velocities in the central region can be produced by a compact stellar mass distribution without a SMBH, while an overmassive SMBH overshoots the central velocities.

than that of the Sérsic model and the stellar mass derived from SED fitting ($4.8_{-2.3}^{+2.5} \times 10^{11} M_{\odot}$; Dye et al. 2022), indicating that the power-law index beyond the radial extent of the data may be flatter than the naive extrapolation. Nevertheless, the total gas mass (integrated to $R = \infty$) $M_{\text{gas}} = (1.73 \pm 0.13) \times 10^{11} M_{\odot}$ is consistent with that of the Sérsic fit within 3σ . The central panel of Figure 7.6 presents the circular velocity curve decomposition between SMBH, stars and gas for the best-fitting Nuker model.

The 3σ upper limit of the SMBH mass derived using the Nuker model is $\log (M_{\text{BH}}/M_{\odot}) = 9.80$ or $M_{\text{BH}} = 6.3 \times 10^9 M_{\odot}$, similar to that from the Sérsic model. Again, as shown in the PVD comparison of Figure 7.7, the best-fitting Nuker model (left; with a free SMBH mass) accurately reproduces the PVD, while the Nuker model with the upper-limit SMBH mass (right) slightly overshoots the central velocities. The best-fitting Nuker model has $\chi^2 = 74765$ and $\chi_r^2 = 0.1468$, while the Nuker model with the upper-limit SMBH mass has $\chi^2 = 74781$ and $\chi_r^2 = 0.1469$. Again, the statistics of the best-fitting Nuker model are listed and compared with those of the other two models in Table 7.4.

7.4.2.3 Core-Sérsic stellar mass profile

The best-fitting parameters of the dynamical model with a core-Sérsic stellar mass surface density profile, along with their 1σ and 3σ uncertainties, are listed in Table 7.6. The 1D marginalised posteriors and the 2D joint posterior distributions of the non-nuisance core-Sérsic model parameters are shown in the bottom panel of Figure 7.5. Like the other two models, the core-Sérsic model does not rule out $M_{\text{BH}} = 0$, but instead reproduces the high central velocities by a compact stellar mass distribution with a break radius $R_b = 16_{-11}^{+25}$ pc and an effective radius $R_e = 0.66_{-0.13}^{+0.32}$ kpc. This extremely small break radius is much smaller than the source-plane physical resolution of ≈ 280 pc, suggesting that the data can in fact be well-fitted by a single Sérsic profile and do not support the presence of an inner core in the stellar mass distribution (the usual reason for introducing a core into the pure Sérsic profile). Indeed, the parameters describing the shape of the inner core, γ and α , are both unconstrained by the data (i.e. they both have a flat posterior distribution). Nevertheless, the best-fitting R_e of the core-Sérsic fit is smaller than that of the pure Sérsic fit ($R_e = 1.18_{-0.25}^{+0.46}$ kpc; see Section 7.4.2.1). This is because the core-Sérsic profile requires that the stellar mass

distribution be flattened at the centre, suppressing the mass in the central region relative to a pure Sérsic profile with the same R_e . Although our best-fitting R_b is small, the central suppression extends to much larger radii (out to $\approx R_e$) due to the requirement that the inner cored profile transitions smoothly to the outer Sérsic profile (i.e. $\alpha < 10$). The model must therefore compensate for this central suppression by reducing R_e , so that sufficient mass is placed in the central region to reproduce the data.

The 3σ upper limit of the SMBH mass derived using the core-Sérsic model is $\log (M_{\text{BH}}/M_\odot) = 9.86$ or $M_{\text{BH}} = 7.2 \times 10^9 M_\odot$, consistent with the upper limits of the other two models. The bottom row of Figure 7.7 compares the data PVD, the PVD of the best-fitting core-Sérsic model (left; with a free SMBH mass) and the PVD of the best-fitting core-Sérsic model with the SMBH mass equal to the inferred upper limit (right). Again, the best-fitting core-Sérsic model fits the data remarkably well, while the best-fitting core-Sérsic model with the upper-limit SMBH mass overshoots the central velocities. The best-fitting core-Sérsic model has $\chi^2 = 74766$ and $\chi_r^2 = 0.1468$, while the core-Sérsic model with the upper limit SMBH mass has $\chi^2 = 74782$ and $\chi_r^2 = 0.1469$. The last row of Table 7.4 lists the statistics of the best-fitting core-Sérsic model and compares them with those of the other two models.

As a result of the smaller R_e , the total stellar mass of the the core-Sérsic model (integrated to $R = \infty$) $M_* = 6.5_{-0.7}^{+1.1} \times 10^{10} M_\odot$, $\approx 2\sigma$ smaller than the Sérsic model stellar mass ($M_* = 8.6_{-1.0}^{+1.4} \times 10^{10} M_\odot$; see Section 7.4.2.1). The effective stellar mass surface density is thus $\Sigma_{\text{eff}} \equiv M_*/(2\pi R_e^2) = (2.3 \pm 1.0) \times 10^4 M_\odot \text{pc}^{-2}$, consistent with the observational upper limit of $\approx 3 \times 10^5 M_\odot \text{pc}^{-2}$ across the Universe (Hopkins et al., 2010; Grudić et al., 2019). The total gas mass (again integrated to $R = \infty$) $M_{\text{gas}} = (1.52 \pm 0.09) \times 10^{11} M_\odot$ is also consistent with those of the Sérsic and Nuker models. The right panel of Figure 7.6 presents the rotation curve decomposition between SMBH, stars and gas for the best-fitting core-Sérsic model, showing that the inferred velocity contributions of individual mass components agree remarkably well across all stellar mass models considered here.

7.4.2.4 Model comparison

Comparing the three stellar mass surface density models considered in this study, although the Nuker model has the smallest χ^2 , it also has two more free model parameters than the Sérsic

Table 7.5: Best-fitting Nuker model parameters and associated uncertainties.

Parameter	Search range	Best fit	1σ uncertainty	3σ uncertainty
SMBH and stellar mass model				
$\log(M_{\text{BH}}/M_{\odot})$	0.0 – 11.0	4.7	-3.0, +3.4	-4.7, +5.1
α	0.1 – 10.0	2.5	-1.3, +1.6	-2.4, +4.9
$\beta - \gamma$	0.30 – 3.00	2.31	-0.60, +0.46	-1.5, +0.69
γ	0.01 – 1.20	0.89	-0.28, +0.21	-0.85, +0.31
R_b (pc)	2 – 2000	175	-52, +61	-99, +150
$\log(\Sigma_b/M_{\odot} \text{ pc}^{-2})$	1.00 – 5.50	4.90	-0.25, +0.30	-0.49, +0.59
q	0.10 – 1.00	0.56	± 0.29	-0.46, +0.44
Lens				
$x_{0, \text{lens}}$ (arcsec)	-1.000 – 1.000	0.248	-0.005, +0.006	-0.017, +0.016
$y_{0, \text{lens}}$ (arcsec)	-1.000 – 1.000	0.282	-0.005, +0.006	-0.015, +0.017
ϕ_{lens} (degree)	140.0 – 230.0	199.7	-2.8, +2.7	-8.8, +6.4
q_{lens}	0.500 – 1.000	0.872	-0.013, +0.011	-0.036, +0.032
R_{ein} (arcsec)	0.500 – 1.500	0.956	-0.002, +0.003	-0.006, +0.010
t	0.50 – 1.50	1.19	± 0.05	± 0.13
Molecular gas disc				
$\log(M_{\text{gas}}/M_{\odot})$	5.50 – 12.00	11.24	-0.04, +0.03	-0.12, +0.09
R_s (kpc)	0.50 – 2.00	1.33	-0.06, +0.05	-0.14, +0.13
Position angle (degree)	80.0 – 120.0	98.5	-0.5, +0.6	-1.8, +1.5
Inclination (degree)	23.0 – 57.0	45.6	± 1.2	± 3.3
σ_{gas} (km s $^{-1}$)	5.0 – 100.0	66.4	-1.0, +2.6	-2.5, +6.7
Nuisance parameters				
Integrated gas intensity (Jy km s $^{-1}$)	1.0 – 150.0	67.0	-1.0, +1.1	-3.1, +4.1
Kinematic centre X offset (arcsec)	-1.000 – 1.000	0.062	± 0.008	-0.023, +0.022
Kinematic centre Y offset (arcsec)	-1.000 – 1.000	-0.002	-0.012, +0.013	-0.034, +0.030
Kinematic centre velocity offset (km s $^{-1}$)	-50.0 – 50.0	-18.8	-1.2, +2.2	-3.8, +8.8

Notes. The X and Y offsets are measured relative to the phase centre of the data cube, $\text{RA} = 14^{\text{h}}24^{\text{m}}13^{\text{s}}.896$, $\text{Dec.} = 02^{\circ}23'03''.84$ (ICRS), in the east-west and the north-south direction, respectively. The velocity offset is measured relative to the adopted galaxy redshift $z = 4.24417$.

Table 7.6: Best-fitting core-Sérsic model parameters and associated uncertainties.

Parameter	Search range	Best fit	1 σ uncertainty	3 σ uncertainty
SMBH and stellar mass model				
$\log(M_{\text{BH}}/M_{\odot})$	0.0 – 11.0	5.4	-3.6, +3.7	-5.4, +4.4
α	0.1 – 10.0	4.2	-1.5, +2.0	-2.6, +3.1
γ	0.01 – 1.20	0.76	-0.42, +0.31	-0.75, +0.44
n	0.5 – 10.0	8.7	-1.2, +0.9	-3.8, +1.3
R_b (pc)	1 – 100	16	-11, +25	-15, +53
R_e (kpc)	0.10 – 10.00	0.66	-0.13, +0.32	-0.24, +0.51
$\log(\Sigma_e/M_{\odot} \text{ pc}^{-2})$	1.00 – 5.50	3.78	-0.28, +0.16	-0.47, +0.39
q	0.10 – 1.00	0.32	-0.15, +0.30	-0.22, +0.68
Lens				
$x_{0, \text{lens}}$ (arcsec)	-1.000 – 1.000	0.247	± 0.005	-0.016, +0.015
$y_{0, \text{lens}}$ (arcsec)	-1.000 – 1.000	0.281	± 0.005	-0.013, +0.016
ϕ_{lens} (degree)	140.0 – 230.0	200.1	-2.9, +2.6	-8.6, +6.6
q_{lens}	0.500 – 1.000	0.876	-0.009, +0.008	-0.028, +0.024
R_{ein} (arcsec)	0.500 – 1.500	0.957	± 0.002	-0.006, +0.007
t	0.50 – 1.50	1.18	-0.02, +0.03	-0.05, +0.07
Molecular gas disc				
$\log(M_{\text{gas}}/M_{\odot})$	5.50 – 12.00	11.18	-0.03, +0.02	-0.08, +0.07
R_s (kpc)	0.50 – 2.00	1.32	± 0.03	-0.08, +0.09
Position angle (degree)	80.0 – 120.0	98.6	± 0.6	-1.9, +1.5
Inclination (degree)	23.0 – 57.0	45.5	± 1.2	-3.5, +3.2
Gas velocity dispersion (km s ⁻¹)	5.0 – 100.0	66.0	-0.7, +0.9	-2.1, +6.9
Nuisance parameters				
Integrated gas intensity (Jy km s ⁻¹)	1.0 – 150.0	66.9	± 0.9	-3.0, +2.8
Kinematic centre X offset (arcsec)	-1.000 – 1.000	0.064	± 0.006	-0.017, +0.015
Kinematic centre Y offset (arcsec)	-1.000 – 1.000	-0.001	± 0.006	-0.010, +0.009
Kinematic centre velocity offset (km s ⁻¹)	-50.0 – 50.0	-18.9	-1.1, +1.3	-3.3, +9.3

Notes. The X and Y offsets are measured relative to the phase centre of the data cube, RA = 14^h24^m13^s.896, Dec. = 02°23′03″.84 (ICRS), in the east-west and the north-south direction, respectively. The velocity offset is measured relative to the adopted galaxy redshift $z = 4.24417$.

model. If we consider instead the BIC, defined as $\text{BIC} \equiv k \ln(N) + \chi^2$, where k is the number of free parameters and $N = 509162$ is the number of data points, then the Sérsic model is strongly favoured over the Nuker model and the core-Sérsic model ($\Delta\text{BIC} = -22$ and -36 , respectively), due to its simplicity. Therefore, we take the results of the Sérsic fit as our final results and the differences among the three fits as the systematic uncertainties. For example, the best-fitting gas mass (integrated to $R = \infty$) $M_{\text{gas}} = (1.42 \pm 0.09^{+0.31}_{-0.00}) \times 10^{11} \text{ M}_{\odot}$, where the first uncertainty is statistical and the second systematic. We adopt the largest SMBH mass upper limit of the three models (that of the Sérsic model) as our final SMBH mass upper limit, yielding $M_{\text{BH}} \leq 7.6 \times 10^9 \text{ M}_{\odot}$.

7.5 Discussion

7.5.1 Dynamical masses of individual components

7.5.1.1 SMBH mass

Although our SMBH mass upper limit is large relative to the population of $z \approx 4$ SMBHs (e.g. [Taylor et al., 2025](#)), it still provides a valuable constraint on the $M_{\text{BH}} - M_*$ relation at high redshifts, because ID141 also has an extremely large stellar mass $M_* = 4.8^{+2.5}_{-2.3} \times 10^{11} \text{ M}_{\odot}$ ([Dye et al., 2022](#)). Assuming this SED-based stellar mass, the BH-stellar mass ratio is $M_{\text{BH}}/M_* \leq 0.015$, lower than that of most AGN hosts at similar cosmic epochs (e.g. [Targett et al., 2012](#); [Ma et al., 2026](#)). If we instead adopt our best-fitting dynamical stellar mass (assuming a Sérsic profile) $M_* = 8.6^{+1.4}_{-1.0} \times 10^{10} \text{ M}_{\odot}$, $M_{\text{BH}}/M_* \leq 0.09$. Hereafter, we will primarily adopt the stellar mass derived from SED fitting in our discussion, as this is the stellar-mass determination method adopted by most other studies of the high-redshift $M_{\text{BH}} - M_*$ relation. This enables us to isolate the effect of the M_{BH} measurement method on the relation.

Figure 7.8 compares our dynamical SMBH mass upper limit and the SED-based stellar mass of ID141 with the SMBH and stellar mass estimates of *JWST*-discovered AGN hosts at $4 \leq z \leq 5$ ([Harikane et al., 2023](#); [Maiolino et al., 2024](#); [Chen et al., 2025](#); [Fei et al., 2025](#); [Ma et al., 2026](#)), as well as those of the two $z \approx 4$ quasars of [Targett et al. \(2012\)](#). Also shown are the local $M_{\text{BH}} - M_*$ relation of [Reines & Volonteri \(2015\)](#) and the $z = 4.5$ relation

obtained by rescaling this local relation by the redshift-dependent scaling factor derived by [Pacucci & Loeb \(2024\)](#). This $z = 4.5$ relation is nearly equivalent to a fixed SMBH-stellar mass ratio $M_{\text{BH}}/M_* = 0.01$. Although the relation of [Pacucci & Loeb \(2024\)](#) agrees well with observational results published before 2024, our up-to-date (as of March 2026) sample suggests that AGN SMBHs at $4 \leq z \leq 5$ could be even more massive than predicted by this relation. Our upper limit of the SMBH mass of ID141 lies close to this relation, implying we have $\approx 3\sigma$ evidence that the SMBH of this star-forming galaxy is not as (over)massive as that of an average $4 \leq z \leq 5$ AGN. Although this result is not statistically conclusive, it does demonstrate that gas-dynamical measurements of M_{BH} can provide the first direct constraints on the scaling relations between inactive SMBHs and galaxies at high redshifts. For example, if we could discover a statistically significant number of systems with similar M_{BH}/M_* as ID141, this would imply that either the single-epoch method (used for most high-redshift SMBH mass measurements so far) systematically overestimates SMBH masses, or *JWST*-discovered AGN are more massive than the general population of coeval SMBHs.

7.5.1.2 Stellar, gas and total dynamical masses

Using CO(7-6) and H₂O as kinematic tracers, [Dye et al. \(2022\)](#) reported that the dynamical mass of ID141 within a radius $R = 2$ kpc (the maximum spatial extent of [C II]) is $\approx 0.8 \times 10^{11} M_{\odot}$, smaller than the gas masses within $R = 2$ kpc of $3.2 - 3.8 \times 10^{11} M_{\odot}$ derived using standard conversion factors, regardless of the gas tracer used (e.g. CO, [C I], [C II] and dust). This discrepancy increases when considering the total stellar mass derived from SED fitting, $4.8 \times 10^{11} M_{\odot}$, a significant fraction of which is likely within $R = 2$ kpc. Similar discrepancies have been reported by studies of other SMGs (e.g. [Yang et al., 2017](#); [Mizukoshi et al., 2021](#); [Neeleman et al., 2020](#)). [Dye et al. \(2022\)](#) attempted to resolve this discrepancy by considering alternative chemical evolution models with different star-formation histories and initial mass functions.

Using [C II] as the kinematic tracer, our independent dynamical analysis yields a dynamical mass within $R = 2$ kpc $M_{\text{dyn}} = (1.13 \pm 0.05) \times 10^{11} M_{\odot}$ (the systematic uncertainty is much smaller than the statistical uncertainty and is thus neglected). The derived cumulative mass profile of ID141 is shown in Figure 7.9 (along with the relative contributions of the SMBH,

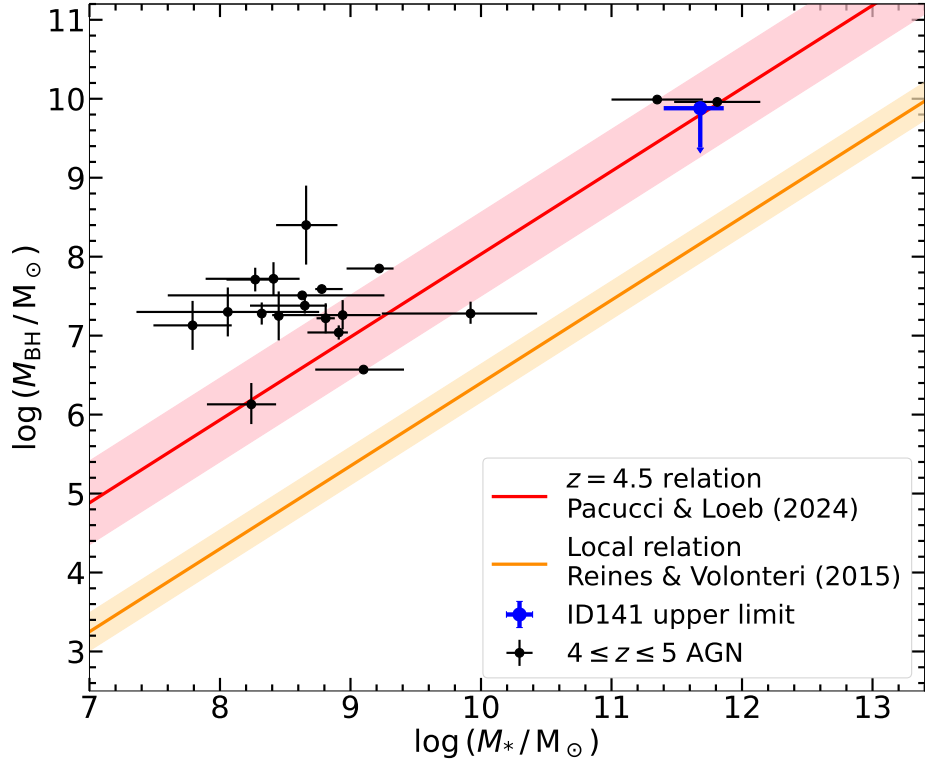


Figure 7.8: Comparison between the dynamical SMBH mass upper limit and SED-based stellar mass of ID141 (blue data points) and the SMBH and stellar mass estimates of $4 \leq z \leq 5$ AGN hosts (black data points). The local $M_{\text{BH}} - M_*$ relation of [Reines & Volonteri \(2015\)](#) is shown in orange, while the $z = 4.5$ relation of [Pacucci & Loeb \(2024\)](#) is shown in red. Our upper limit indicates that the SMBH of ID141 is likely not as (over)massive as that of an average $4 \leq z \leq 5$ AGN.

stars and gas) for each of our best-fitting models. The difference between our dynamical mass measurement and that of [Dye et al. \(2022\)](#) can be completely explained by the different, but statistically consistent, inclinations: our best-fitting inclination is $45^\circ.6 \pm 1^\circ.2$ while the inclination derived by [Dye et al. \(2022\)](#) is $54^\circ \pm 10^\circ$. Our dynamical mass agrees with the gas masses of most chemical evolution models considered by [Dye et al. \(2022\)](#). Our dynamically estimated gas mass within $R = 2$ kpc is $(6.43 \pm 0.32^{+1.24}_{-0.00}) \times 10^{10} M_\odot$ (where the second uncertainties are the systematic uncertainties estimated from the differences between the models), which agrees more closely with models that invoke a recent starburst than models that assume continuous star formation ([Dye et al., 2022](#)).

By comparing our dynamically measured gas mass with the integrated [C II] flux density $S_{[\text{C II}]} = 85.5 \pm 1.6 \pm 8.6 \text{ Jy km s}^{-1}$, we can measure the [C II] luminosity-to-mass conversion

factor $\alpha_{[\text{C II}]}$. The apparent (i.e. observed) $[\text{C II}]$ luminosity $L_{[\text{C II}],\text{obs}}$ is related to $S_{[\text{C II}]}$ by

$$\left(\frac{L_{[\text{C II}],\text{obs}}}{L_{\odot}} \right) = \frac{1.04 \times 10^{-3}}{1+z} \left(\frac{f_{\text{rest}}}{\text{GHz}} \right) \left(\frac{D_L}{\text{Mpc}} \right)^2 \left(\frac{S_{[\text{C II}]}}{\text{Jy km s}^{-1}} \right), \quad (7.11)$$

where f_{rest} is the rest frequency of the line (1900.537 GHz). For ID141, this yields $L_{[\text{C II}],\text{obs}} = (4.98 \pm 0.09 \pm 0.50) \times 10^{10} L_{\odot}$ (where the systematic uncertainty is dominated by ALMA's flux calibration uncertainty). Assuming that the magnification factor of the $[\text{C II}]$ line equals the ALMA Band 4 global magnification factor $\mu = 5.9$ (Dye et al., 2022; computed by dividing the lensed total continuum luminosity by the intrinsic total continuum luminosity), the intrinsic $[\text{C II}]$ luminosity $L_{[\text{C II}],\text{int}} = (0.84 \pm 0.02 \pm 0.08) \times 10^{10} L_{\odot}$. The $[\text{C II}]$ luminosity-to-mass conversion factor of ID141 is thus estimated to be $\alpha_{[\text{C II}]} \equiv M_{\text{gas}}/L_{[\text{C II}],\text{int}} = 7.64 \pm 0.89_{-0.71}^{+1.48} M_{\odot}/L_{\odot}$, where the upper systematic uncertainty is dominated by the systematic uncertainty of the gas mass while the lower systematic uncertainty is dominated by ALMA's flux calibration uncertainty. This conversion factor is considerably smaller than the canonical factor of $30 M_{\odot}/L_{\odot}$ (Zanella et al., 2018) and the factors predicted by most simulations (e.g. Khatri et al., 2025; Vallini et al., 2025), but it agrees well with other observations estimating gas masses dynamically (e.g. Rizzo et al., 2021).

Our estimated dynamical stellar mass within $R = 2$ kpc is $(4.9 \pm 0.4_{-1.4}^{+0.0}) \times 10^{10} M_{\odot}$ (where the systematic uncertainties are again estimated from the differences across the models), an order of magnitude smaller than the SED-based stellar mass of $4.8_{-2.3}^{+2.5} \times 10^{11} M_{\odot}$ (Dye et al., 2022). However, the spatial extent of the stellar distribution could be considerably larger than a radius of 2 kpc. If we instead integrate the best-fitting Sérsic stellar mass surface density profile to $R = \infty$, the total dynamical stellar mass is $8.6_{-1.0}^{+1.4} \times 10^{10} M_{\odot}$, more similar to (but still smaller and inconsistent to 1σ with) the SED-based stellar mass.

7.5.2 A massive cold rotating disc at $z = 4$

The dynamical analysis of Dye et al. (2022) suggested that the ID141 system is either a single rotating disc with minor disturbances or two closely separated rotating discs. Our dynamical model assumes a single dynamically cold rotating disc. To test this assumption, in Figure 7.10 we compare the observed first-moment map of the $[\text{C II}]$ emission (left) with the first-moment map of our best-fitting dynamical model using the Sérsic stellar mass surface

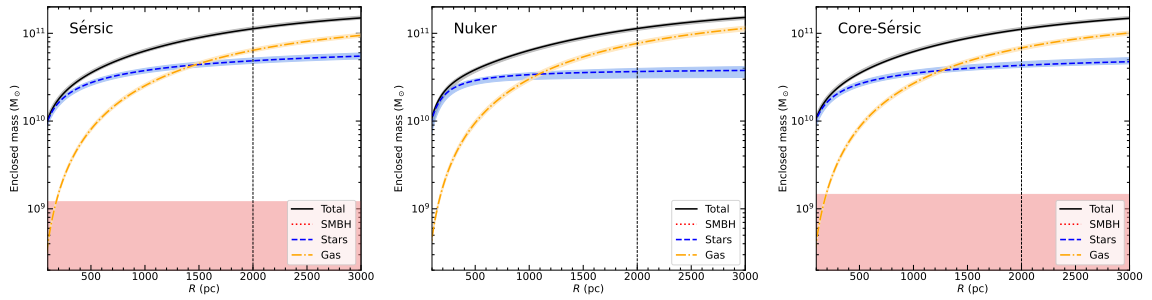


Figure 7.9: Cumulative mass distribution of ID141 assuming a Sérsic (left), a Nuker (centre) and a core-Sérsic (right) stellar mass surface density profile. In each panel, the curves show the relative contributions of the SMBH (red dotted line; below the limits of the plot), stars (blue dashed line) and gas (orange dash-dotted line) to the total cumulative mass profile (black solid line). The 1σ uncertainties of the profiles are shown as shaded regions. The vertical black line shows $R = 2$ kpc, the approximate radius of the gas disc.

density profile (middle). The first-moment maps created using the other two models are not presented, as they are visually indistinguishable from that created using the Sérsic model. We apply the moment mask method described in Section 7.2.4.1 to the model cube to ensure consistency. We also present a map of the residuals (right), obtained by subtracting the model moment map from the data moment map. Our dynamical model accurately reproduces the observed [C II] kinematics. The residuals are nearly zero across the entire map, except for small ($\approx 100 \text{ km s}^{-1}$) residuals near the kinematic minor axis, which are driven by the large velocity gradients near the minor axis and the slight asymmetry of the gas distribution. This consistency between the kinematics of the [C II] line and that of a single rotating disc was also observed in other gas tracers such as CO and H₂O (Dye et al., 2022).

The [C II] disc of ID141 is also strongly rotation-dominated. The flat outer part of the circular velocity curve has an amplitude $V_{\text{flat}} \approx 525 \text{ km s}^{-1}$, and our best-fitting gas velocity dispersion is $\sigma_{\text{gas}} = 65.8^{+0.8}_{-0.7} \text{ km s}^{-1}$. The rotation-to-random motion ratio is thus $V_{\text{flat}}/\sigma_{\text{gas}} \approx 8.0$, similar to other $z \approx 4$ starburst galaxies that have cold rotating discs (e.g. Rizzo et al., 2020, 2021), but much larger than that predicted by most theoretical models and simulations (e.g. Wisnioski et al., 2015; Pillepich et al., 2019). Nevertheless, this ratio is consistent with the latest model of Wisnioski et al. (2025), which specifically incorporates the dependence of V/σ on stellar mass and the Toomre parameter Q (Toomre, 1964).

What is more unusual than the large $V_{\text{flat}}/\sigma_{\text{gas}}$ of ID141 is the large rotation velocity V_{flat} itself. The observed $V_{\text{flat}} \approx 525 \text{ km s}^{-1}$ is independent of our dynamical model (except for the dependence on the well-constrained inclination i) and can be derived by simply identifying the

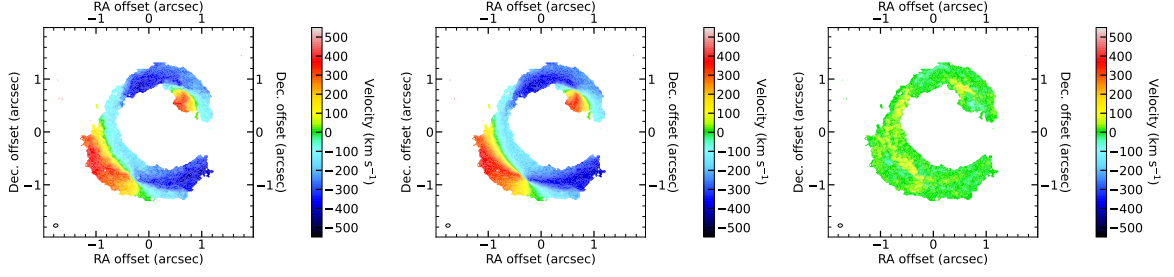


Figure 7.10: Left: first-moment (intensity-weighted mean line-of-sight velocity) map of the ID141 data cube. Centre: first-moment map of the best-fitting Sérsic dynamical model. The same mask is applied to the data and model cubes for consistency. Right: first-moment residual map (data – model). The synthesised beam ($0''.082 \times 0''.063$) is shown as a black open ellipse in the bottom-left corner of each panel. Positions are measured relative to the best-fitting kinematic centre (see Section 7.4); velocities are in the rest frame, assuming $z = 4.24417$.

largest velocity in the first-moment map (Figure 3.2) or the PVD (Figure 3.4) and dividing this velocity by $\sin i$. Before ID141, only four galaxies at $z > 4$ had been reported to have rotation velocities larger than 500 km s^{-1} : GN20 (Hodge et al., 2012), AzTEC/C159 (Fraternali et al., 2021), J1000+0234 (Fraternali et al., 2021) and SGP38326-1 (Roman-Oliveira et al., 2023), all of which are SMGs. However, contrary to ID141, none of these galaxies has $V_{\text{rot}} > 500 \text{ km s}^{-1}$ within the central 0.5 kpc. This implies that ID141 has the steepest potential well ever observed at such an early cosmic epoch. Even in the local Universe, such a steep gravitational potential is found almost exclusively in massive ETGs. This provides support to the view that $z \approx 4$ SMGs are the progenitors of local ETGs (e.g. Toft et al., 2014), although mechanisms such as dynamical heating induced by major mergers (e.g. Barnes & Hernquist, 1992; Hopkins et al., 2008) and compaction triggered by disc instability (e.g. Dekel & Burkert, 2014; Zolotov et al., 2015) are required to transform the rotation-supported gas discs of $z \approx 4$ SMGs into the dispersion-supported stellar populations of local ETGs.

7.5.3 Dynamical measurements of SMBH masses of high-redshift strongly lensed galaxies with ALMA

Although we only obtained a dynamical upper limit of the SMBH mass of ID141, our study demonstrates the potential of high-resolution ALMA observations of strongly gravitationally lensed galaxies to provide direct dynamical constraints on high-redshift SMBH masses and

their scaling relations with host galaxy properties. Our observations did not use the highest resolution of ALMA. If we can repeat the C43-9 observations that were previously affected by poor weather, we will achieve an angular resolution of $0''.025$, ≈ 3 times better than the current resolution. The innermost radius probed by these $0''.025$ observations will be ≈ 100 pc (after correcting for lens magnification), spatially resolving the SMBH sphere of influence (SoI) radius of a $\approx 10^9 M_\odot$ SMBH. Such observations will enable a precise dynamical measurement of the SMBH mass of ID141 if it is within the 1σ scatter of the [Pacucci & Loeb \(2024\)](#) relation. If the SMBH mass is instead more than 1σ smaller than that expected from the relation, we will obtain a much tighter upper limit than the current one, that will provide the first unambiguous dynamical evidence that a high-redshift inactive SMBH is not as (over)massive as AGN at the same cosmic epoch. Therefore, it is likely that future higher-resolution ALMA observations of ID141 will provide, for the first time, a strong dynamical constraint on the scaling relations between inactive SMBHs and galaxies at $z \approx 4$.

Moreover, ID141 is not the only high-redshift galaxy for which a dynamical measurement of the SMBH mass can be performed using ALMA. Several galaxies have much higher lens magnifications of $\mu > 10$ as well as cold rotating discs (e.g. [Rizzo et al., 2020, 2021](#); [Liu et al., 2024](#)), allowing us to highly spatially resolve their SMBH SoI and construct robust dynamical models. The dynamical and lens modelling framework we established in this work can thus be used to obtain the first statistically meaningful sample of direct dynamical SMBH mass measurements in the early Universe.

7.6 Conclusions

We presented a dynamically measured upper limit of the SMBH mass of ID141, a strongly gravitationally lensed SMG at $z = 4.24$, using ALMA observations of the $[\text{C II}]$ line with an angular resolution of $0''.072$ and an average source-plane physical resolution of ≈ 200 pc. We fitted the ALMA data directly in uv space using SUPERMAGE, which simultaneously forward models gas dynamics and strong gravitational lensing in an automatically differentiable and GPU-accelerated framework. After considering three different parametric profiles for the stellar mass surface density (Sérsic, Nuker and core-Sérsic), we derived a SMBH mass upper limit of $M_{\text{BH}} = 7.6 \times 10^9 M_\odot$, which suggests that the inactive SMBH of ID141 is less massive

than predicted by [Pacucci & Loeb \(2024\)](#) for an average $z \approx 4.5$ AGN host with the same stellar mass. We also dynamically inferred a stellar mass $M_* = 8.6^{+1.4}_{-1.0} \times 10^{10} M_\odot$ (integrated to $R = \infty$) and a gas mass $M_{\text{gas}} = (6.43 \pm 0.32^{+1.24}_{-0.00}) \times 10^{10} M_\odot$ within a radius of 2 kpc (the maximum extent of the [C II] emission), yielding a [C II] luminosity-to-mass conversion factor $\alpha_{[\text{C II}]} \equiv M_{\text{gas}}/L_{[\text{C II}],\text{int}} = 7.64 \pm 0.89^{+1.48}_{-0.71} M_\odot/L_\odot$. Additionally, we inferred the dynamical mass within 2 kpc to be $(1.13 \pm 0.05) \times 10^{11} M_\odot$, resolving a previously reported discrepancy between the dynamical mass and gas masses inferred from standard conversion factors ([Dye et al., 2022](#)). Furthermore, our data unambiguously show that ID141 has a dynamically cold rotating gas disc, with a large rotation-to-random motion ratio $V_{\text{flat}}/\sigma_{\text{gas}} \approx 8.0$. In particular, the rotation velocity $V_{\text{flat}} \approx 525 \text{ km s}^{-1}$ of ID141 is one of the largest among $z > 4$ galaxies, supporting the view that high-redshift SMGs are the progenitors of local ETGs.

By far, dynamical measurements of SMBH masses and SMBH – host galaxy scaling relations are restricted nearly exclusive to local galaxies. Current high-redshift studies of the co-evolution of SMBHs and galaxies are almost entirely based on indirect SMBH mass estimates and are limited to AGN hosts. Nevertheless, our study demonstrates the potential of combining the highest angular resolutions of ALMA with the magnifying power of strong gravitational lensing to spatially resolve spatial scales comparable to the SoI radii of high-redshift SMBHs. This will provide direct constraints on the SMBH – galaxy scaling relations at high redshifts, regardless of whether the SMBH is active. Our modelling approach can be used to model a considerable number of strongly lensed galaxies to produce the first statistically meaningful sample of dynamical SMBH mass measurements beyond the local Universe.

Chapter 8

Conclusions

8.1 Summary

This thesis focussed on applying high-resolution Atacama Large Millimeter/sub-millimeter Array (ALMA) observations to improve cold gas-dynamical supermassive black hole (SMBH) mass measurements in the local Universe and to perform the first such measurements at high redshifts. Chapter 2 established figures of merit for SMBH mass measurements using tracers dominated by rotation, by evaluating the minimum physical scale probed by the observations in physically motivated units. Chapters 3 and 4 then demonstrated the potential of high-resolution ALMA observations to yield highly accurate and precise SMBH mass measurements in a large sample of galaxies, carrying out two such high-resolution measurements in the local lenticular galaxies NGC 383 and NGC 1574 and achieving better precision and accuracy than previous lower-resolution measurements. In particular, the NGC 383 measurement is the highest-resolution measurement using the molecular gas method, yielding a precision of 5% on the SMBH mass including systematics. The NGC 383 observations also reach fewer Schwarzschild radii away from the SMBH than the highest-resolution megamaser observations, demonstrating ALMA's ability to probe SMBH accretion and feedback physics closer to the SMBH than very-long-baseline interferometry observations of megamasers. Such a high resolution allows to detect a kinematic twist within the central ≈ 100 pc, which may trace a position angle (PA) warp that aligns the inner molecular gas disc's angular momentum vector with the direction of the radio jet of the galaxy, as expected, or may trace an outflow within the circumnuclear disc.

Shifting the focus to high-redshift galaxies, Chapter 5 discussed the prospects of using strong gravitational lensing as natural magnifying glasses to perform the first cold gas-dynamical SMBH mass measurements beyond the local Universe, introduced our target selection strategy and presented the properties and ALMA observations of the most promising candidate galaxies. Chapter 6 then addressed a practical challenge to such measurements – accurately modelling the stellar mass distributions of dusty galaxies or galaxies dominated by active galactic nuclei (AGN) – and presented a solution based on deprojecting one-dimensional (1D) parametric stellar mass surface density profiles into axisymmetric three-dimensional (3D) stellar mass volume density distributions using neural networks. Finally, Chapter 7 presented a dynamically inferred SMBH mass upper limit in ID141, a lensed sub-millimetre galaxy at $z = 4.24$, using high-resolution ALMA observations of [C II]. The upper limit suggests that the SMBH of ID141 is not as overly massive relative to the local SMBH mass – galaxy stellar mass ($M_{\text{BH}} - M_*$) relation as typical coeval AGN. The observations also revealed that ID141 has a dynamically cold rotating disc ($V_{\text{rot}}/\sigma_{\text{gas}} = 8.0$, where V_{rot} is the maximum rotation velocity of the gas and σ_{gas} its mean velocity dispersion) and one of the highest rotation velocities of $z > 4$ galaxies.

Overall, this thesis expanded the toolbox for SMBH mass determinations across cosmic time in two major ways. First, it highlighted the importance of high-resolution observations for precise and accurate SMBH mass measurements in the local Universe and ALMA’s ability to obtain a large sample of such measurements. These measurements are crucial to constrain the intrinsic scatter of local SMBH – galaxy scaling relations and their dependence on galactic properties. For instance, there is growing evidence that the relation between SMBH mass and effective stellar velocity dispersion ($M_{\text{BH}} - \sigma_e$ relation) steepens above a stellar mass of $M_* \approx 2 \times 10^{11} M_{\odot}$, suggesting that the most massive SMBHs grow primarily through dry mergers, while lower-mass SMBHs grow mainly through secular evolution (Krajnović et al., 2018; Dullo et al., 2021). However, a larger number of precise mass measurements for high-mass SMBHs is required to confirm this trend. On the other hand, a large number of precise mass measurements for low-mass ($10^5 - 10^6 M_{\odot}$) SMBHs may help constrain the seeding mechanism of SMBHs (Bhowmick et al., 2025).

Second, this thesis developed a method to obtain dynamical SMBH mass constraints

beyond the local Universe – observing and modelling the kinematics of cold gas within the SMBH SoI of strongly gravitationally lensed galaxies. This method is highly complementary to existing SMBH mass determination methods at high redshifts, applicable to both active and inactive SMBHs. While currently small, the sample of promising targets will soon rapidly expand, by following up with ALMA new high-magnification strong lenses discovered by Euclid and the Legacy Survey of Space and Time. Such dynamical SMBH mass determinations at high redshifts will provide tight constraints on the cosmic evolution of SMBH – galaxy scaling relations for the first time, offering valuable insights into how SMBHs grow and co-evolve with their host galaxies throughout cosmic history.

Nevertheless, despite the progress made in this thesis, our understanding of the origin and growth of SMBHs and the physical mechanisms driving SMBH – galaxy coevolution remains limited by the quantity, precision and reliability of SMBH mass determinations across cosmic time. Building on this research, future projects should i) continue to refine the methods to accurately measure local SMBH masses, ii) improve constraints on existing SMBH – galaxy correlations and discover new correlations from a statistically significant sample of high-resolution dynamical measurements, and iii) obtain additional dynamical constraints on the SMBH masses of high-redshift galaxies. The final sub-sections below (Sections 8.2.1 – 8.2.3) summarise some planned and/or ongoing efforts along these lines.

8.2 Ongoing and future works

8.2.1 Testing the impact of uv -space fitting on the accuracy of dynamical SMBH mass measurements using ALMA

Dynamical modelling of ALMA data to infer SMBH masses is traditionally performed on image-space data generated using the CLEAN algorithm (Högbom, 1974) from raw visibilities. However, CLEAN is known to create artefacts and suppress information in images, biasing analyses that require sensitivity to faint signals (Maresca & Dye, 2025). Moreover, noise between nearby pixels is highly correlated in image space, so an accurate determination of parameter uncertainties requires inverting the full covariance matrix when computing the chi-squared (χ^2). This is often computationally intractable, especially for high-resolution data

cubes with large numbers of pixels. Alternatively, one can assume independent data points and separately compensate for the resulting underestimation of the statistical uncertainties, and/or estimate systematic uncertainties, for example by rescaling the χ^2 by $1/\sqrt{2N}$ (van den Bosch & van de Ven, 2009; Mitzkus et al., 2017), where N is the total number of data points (i.e. constraints), as done in many works (e.g. Smith et al., 2019; Zhang et al., 2025a) of the mm-Wave Interferometric Survey of Dark Object Masses (WISDOM). This approach has been shown to yield uncertainties similar to those evaluated via bootstrapping (Smith et al., 2019; Zhang et al., 2025a), but it provides at best a rough estimate of the true uncertainties.

By contrast, fitting ALMA data directly in uv space avoids the artefacts introduced by CLEAN and provides efficient and reliable uncertainty estimates, as noise is independent across uv -space visibilities. To test whether and by how much uv -space fitting improves the accuracy of SMBH mass constraints over image-space fitting, we compare the performance of SUPERB MASSES FROM GAS KINEMATICS (SUPERMAGE; a tool fitting in uv -space) and KINEMATIC MOLECULAR SIMULATION (KINMS; a tool fitting in image-space) when inferring the SMBH mass of the local unlensed galaxy NGC 4697. We otherwise use the same ALMA data and dynamical model.

To carry out this comparison, we adopt the dynamical model described in Davis et al. (2017), using their multi-Gaussian expansion (MGE) as the stellar light model and removing the narrowest MGE component, assuming that the central AGN dominates this component. The models of SUPERMAGE and KINMS are identical in image space, but SUPERMAGE Fourier-transforms the model into uv space, where it is fitted to the gridded and velocity-binned ALMA visibilities, while KINMS fits the image-space ALMA data cube after forward-modelling beam smearing and velocity and spatial binning. Additionally, we separately fit both the (comparatively) low-angular resolution ALMA data (Project 2015.1.00598.S; PI: M. Bureau) presented in Davis et al. (2017) and new high-resolution data (Project 2022.1.01122.S; PI: M. Bureau) to investigate the effect of resolution on the accuracy of uv -space fitting relative to image-space fitting.

Figure 8.1 compares the best-fitting M_{BH} of NGC 4697 and its 1σ (boxes) and 3σ (whiskers) uncertainties from KINMS (image-space fitting) and SUPERMAGE (uv -space fitting), as well as for both the low-resolution data and the high-resolution data described above.

For both datasets, the best-fitting M_{BH} derived using SUPERMAGE is statistically consistent with that derived using KINMS with χ^2 rescaling. However, the statistical uncertainties yielded by SUPERMAGE are substantially larger than the unrealistic $\approx 0.2\%$ uncertainties yielded by KINMS without χ^2 rescaling. This demonstrates that while image-space fitting that ignores covariances between nearby pixels (to reduce computation time) in turn requires χ^2 rescaling to compensate for the underestimated uncertainties, uv -space fitting yields realistic statistical uncertainties. Moreover, if no χ^2 rescaling is applied, the best-fitting M_{BH} inferred by SUPERMAGE, using either the low-resolution or the high-resolution data, agrees well with that inferred using KINMS and the high-resolution data but is considerably larger than that inferred using KINMS and the low-resolution data (despite an identical model). This shows that uv -space fitting is less sensitive to potential biases of low-resolution dynamical modelling, which would otherwise require high-resolution data to correct. In the future, we will repeat this test using alternative dynamical models (e.g. parametric stellar mass profiles) and other galaxies.

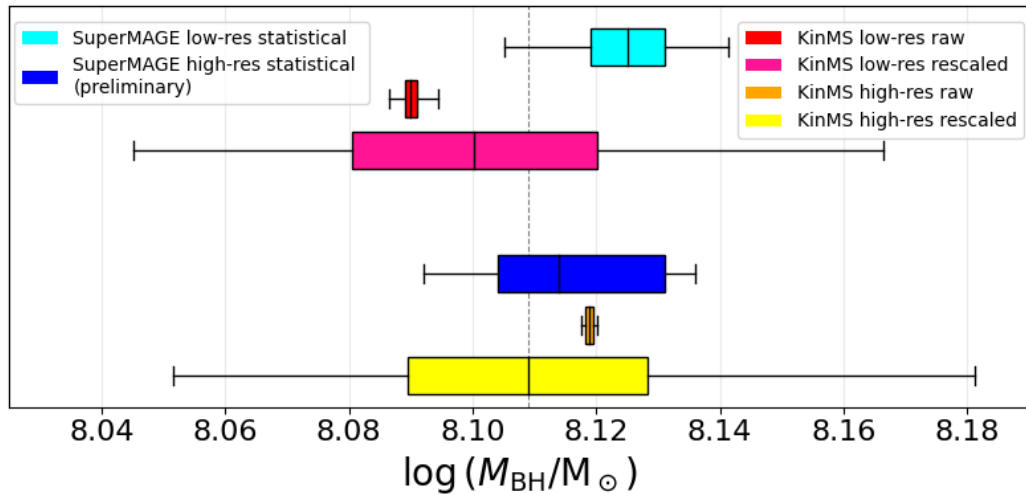


Figure 8.1: Comparison of the KINMS (image-space fitting) and SUPERMAGE (uv -space fitting) best-fitting M_{BH} of NGC 4697, derived from modelling both low-resolution and high-resolution data. Uncertainties are indicated by boxes (1σ) and whiskers (3σ).

8.2.2 Radial acceleration relation at high accelerations

The radial acceleration relation (RAR) of galaxies (McGaugh et al., 2016; Lelli et al., 2017) is a tight correlation between the observed accelerations derived from galaxy rotation curves g_{obs} and the accelerations predicted from baryonic matter g_{bar} , which deviate from the expected

one-to-one relation below a critical g_{bar} scale $a_0 \sim 10^{-10} \text{ m s}^{-2}$. The universality of this scale implies that it results from a fundamental law (Lelli et al., 2017; Stiskalek & Desmond, 2023), such as modified Newtonian dynamics (e.g. Milgrom, 1983), although it is debated whether the RAR can also emerge under the standard cold dark matter cosmology (e.g. Desmond, 2017; Di Cintio & Lelli, 2016; Li et al., 2022).

In any case, the RAR has not been explored above accelerations $g_{\text{bar}} \sim 10^{-8} \text{ m s}^{-2}$, due to the insufficient spatial resolutions of the existing primarily H I and H α data. Yet, g_{obs} is also expected to deviate from g_{bar} at the high accelerations present at galaxy centres, due to the presence of SMBHs (not accounted for in the usual baryonic budgets) and/or massive dark matter cores (predicted by certain dark matter theories; Hu et al. 2000; Goodman 2000; Berezhiani & Khoury 2015). High-resolution ALMA CO observations, obtained for gas-dynamical measurements of SMBH masses, can also be used to probe the RAR at high accelerations. If g_{obs} were to similarly deviate from g_{bar} at a universal high acceleration scale, this would imply a previously unknown correlation between SMBH mass and the mass distribution of the SMBH host galaxy, novel dark matter properties and/or the need to further modify gravity.

To investigate how g_{obs} deviates (or not) from g_{bar} at high accelerations, due to the presence of SMBHs or otherwise, I compiled all galaxies with molecular gas-dynamical SMBH mass measurements (same compilation as in Table 2.1) and compared their baryonic accelerations $g_{\text{bar}}(R)$ (where R is the radius) to their baryonic *and* SMBH accelerations $g_{\text{bar+BH}}(R)$, as predicted by the dynamical models presented in the associated SMBH mass measurement papers. This is appropriate as $g_{\text{bar+BH}}$ should equal g_{obs} if each model accurately reproduces the ALMA kinematic data. Of course, it would be preferable to derive g_{obs} directly from the data in the usual manner, by fitting the data of each galaxy to derive a 1D rotation curve $V_{\text{rot}}(R)$ and computing the acceleration using $g_{\text{obs}}(R) = V_{\text{rot}}^2(R)/R$, but this would be time-consuming as the individual papers do not derive such rotation curves, fitting instead to the entire data cubes. For the moment, using $g_{\text{bar+BH}}$ is thus sufficient, as a first proof-of-concept step (as all the required information is already available). Proper g_{obs} can be derived at a later stage if the results turn out to be promising.

The comparison of $g_{\text{bar+BH}}(R)$ to $g_{\text{bar}}(R)$ is shown in Figure 8.2 for every galaxy. As

expected from the known presence of central SMBHs, the galaxies deviate from the 1 : 1 relation at high accelerations. However, the galaxies clearly do not follow a universal RAR at high accelerations, each galaxy following its own curve. Nevertheless, the acceleration scales on which the galaxies deviate from the 1 : 1 relation (by $\gtrsim 10\%$) are roughly similar, spanning less than one order of magnitude in g_{bar} ($g_{\text{bar}} \sim 10^{-8.5} - 10^{-7.5} \text{ m s}^{-2}$), despite the ≈ 4 orders of magnitude dynamic range of the SMBH masses ($M_{\text{BH}} \approx 10^{5.7} - 10^{9.7} M_{\odot}$) and the ≈ 2.5 orders of magnitude dynamic range of the SoI radii ($R_{\text{SoI}} \approx 1 - 300 \text{ pc}$). This implies that SMBH mass somehow correlates with the distribution of baryonic mass (usually dominated by stellar mass) in the central regions of nearby galaxies. Work is ongoing to solidify this result and refine its interpretation in the context of SMBH – galaxy co-evolution. In parallel, other WISDOM members are trying to confirm this result by measuring g_{obs} directly from the data in the usual manner (Korsaga et al., in prep.). Also interestingly, if we were to find that g_{obs} is significantly different from $g_{\text{bar+BH}}$ in a large number of galaxies, this would suggest that central SMBHs are not the only reason causing the deviations of g_{obs} from g_{bar} at high accelerations, implying additional processes such as novel dark matter or gravitational physics.

8.2.3 SMBH mass measurements of ID141 and SPT 0113 at $z \approx 4$

As discussed in Section 7.5.3, follow-up observations of ID141 using the C43-9 configuration of ALMA would achieve a sufficient spatial resolution to resolve the SoI of the SMBH if it is within the 1σ scatter of the $M_{\text{BH}} - M_{\star}$ relation of Pacucci & Loeb (2024). Such observations thus promise a dynamical SMBH mass measurement or at least a highly informative upper limit. An alternative route to improve the SMBH mass constraint of ID141 is to obtain a *James Webb Space Telescope* Mid-Infrared Instrument (MIRI) image in the rest-frame near-infrared to constrain the shape of the stellar distribution. However, the challenge will remain to correct the shape parameters (e.g. Sérsic index n) for heavy dust obscuration (Dye et al., 2022). Additionally, the point-spread function full-width at half-maximum of MIRI is $\gtrsim 0''.2$ ($\approx 600 \text{ pc}$ in the source plane of ID141), an order of magnitude larger than the resolution achievable with ALMA data, making it difficult for the observations to rule out a compact stellar distribution.

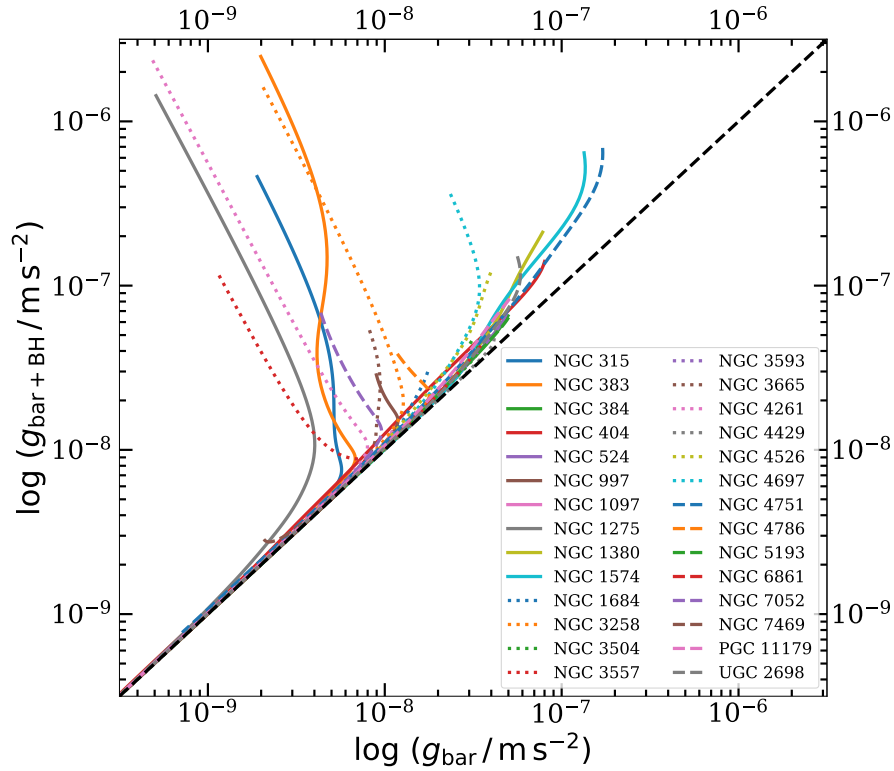


Figure 8.2: Comparison of the predicted baryonic accelerations and the predicted baryonic and SMBH accelerations, for all galaxies with a CO gas-dynamical SMBH mass measurement. Each curve is plotted between the maximum (lowest $g_{\text{bar+BH}}$) and the minimum (highest $g_{\text{bar+BH}}$) radius probed by the associated observations.

Another $z = 4$ galaxy promising for a dynamical SMBH mass measurement is SPT 0113, mainly due to its high lens magnification $\mu = 36$ (Rizzo et al., 2021). Our Cycle 12 ALMA program (Project 2025.1.01248.S, PI: J. Elford) observed the galaxy in the C43-8 configuration. Although only 72% of the observations were completed, the data reached an angular resolution of $0''.069$ and a source-plane physical resolution of ≈ 80 pc. This will resolve the diameter of the SMBH SoI of SPT 0113 if the SMBH mass is equal to or greater than that predicted by the $M_{\text{BH}} - M_*$ relation of Pacucci & Loeb (2024), $M_{\text{BH}} \approx 6.3 \times 10^8 M_{\odot}$. If the SMBH mass is instead lower, we will obtain an informative upper limit, while future observations with more extended ALMA configurations will provide tighter constraints. Analysis of the data is ongoing.

In parallel with my work on ID141 and SPT 0113, other WISDOM members are deriving a SMBH mass constraint for the remaining Grade B target (see Section 5.2), SPT 0418, using the ALMA observations from projects 2016.1.01374.S (PI: Y. Hezaveh) and 2016.1.01499.S (PI: K. Litke), which have an angular resolution of $0''.028$ and a source-plane physical resolution

of ≈ 36 pc. Additionally, we are preparing for ALMA programmes that aim to increase the number of high-redshift SMBH mass measurement candidates by obtaining intermediate-resolution [C II] observations of highly magnified galaxies. These will allow to verify whether the galaxies host cold rotating discs and sufficiently bright [C II] emission. With these efforts, we expect to obtain $\gtrsim 5$ high-redshift cold gas-dynamical SMBH mass constraints in the next two years, providing the first insights on the SMBH – galaxy scaling relations of inactive SMBHs in the early Universe.

Bibliography

- Abbott B. P., et al., 2016a, [Phys. Rev. Lett.](#), **116**, 061102
- Abbott B. P., et al., 2016b, [Phys. Rev. Lett.](#), **116**, 241103
- Abbott B. P., et al., 2017, [Phys. Rev. Lett.](#), **118**, 221101
- Abbott R., et al., 2020, [Phys. Rev. Lett.](#), **125**, 101102
- Abramowicz M. A., Czerny B., Lasota J. P., Szuszkiewicz E., 1988, [ApJ](#), **332**, 646
- Abuter R., et al., 2024, [Nature](#), **627**, 281
- Agazie G., et al., 2023, [ApJ](#), **951**, L8
- Ahumada R., et al., 2020, [ApJS](#), **249**, 3
- Akins H. B., et al., 2025, [ApJ](#), **991**, 37
- Baade W., Zwicky F., 1934, [Physical Review](#), **46**, 76
- Barnes J. E., Hernquist L., 1992, [ARA&A](#), **30**, 705
- Barth A. J., Boizelle B. D., Darling J., Baker A. J., Buote D. A., Ho L. C., Walsh J. L., 2016, [ApJ](#), **822**, L28
- Beckers J. M., 1982, [Optica Acta](#), **29**, 361
- Begelman M. C., Volonteri M., Rees M. J., 2006, [MNRAS](#), **370**, 289
- Beifiori A., Sarzi M., Corsini E. M., Dalla Bontà E., Pizzella A., Coccato L., Bertola F., 2009, [ApJ](#), **692**, 856
- Beifiori A., Courteau S., Corsini E. M., Zhu Y., 2012, [MNRAS](#), **419**, 2497

- Bell E. F., McIntosh D. H., Katz N., Weinberg M. D., 2003, [ApJS](#), **149**, 289
- Bentz M. C., 2016, in Boffin H. M. J., Hussain G., Berger J.-P., Schmidtobreick L., eds, *Astrophysics and Space Science Library Vol. 439, Astronomy at High Angular Resolution*. p. 249 ([arXiv:1505.04805](#)), [doi:10.1007/978-3-319-39739-9_13](#)
- Bentz M. C., et al., 2010, [ApJ](#), **716**, 993
- Bentz M. C., et al., 2013, [ApJ](#), **767**, 149
- Berezhiani L., Khoury J., 2015, [Phys. Rev. D](#), **92**, 103510
- Bernardi M., Alonso M. V., da Costa L. N., Willmer C. N. A., Wegner G., Pellegrini P. S., Rit   C., Maia M. A. G., 2002, [AJ](#), **123**, 2990
- Bertemes C., et al., 2025, [A&A](#), **693**, A176
- Best P. N., Kaiser C. R., Heckman T. M., Kauffmann G., 2006, [MNRAS](#), **368**, L67
- Bhowmick A. K., et al., 2025, [MNRAS](#), **538**, 518
- Bhowmick A. K., et al., 2026, [ApJ](#), **997**, 187
- Bieri R., Dubois Y., Rosdahl J., Wagner A., Silk J., Mamon G. A., 2017, [MNRAS](#), **464**, 1854
- Bischetti M., et al., 2024, [ApJ](#), **970**, 9
- Blandford R. D., McKee C. F., 1982, [ApJ](#), **255**, 419
- Bogd  n   ., et al., 2024, [Nature Astronomy](#), **8**, 126
- Boizelle B. D., Barth A. J., Walsh J. L., Buote D. A., Baker A. J., Darling J., Ho L. C., 2019, [ApJ](#), **881**, 10
- Boizelle B. D., et al., 2021, [ApJ](#), **908**, 19
- Bolton C. T., 1972, [Nature](#), **235**, 271
- Bolton C. T., 1975, [ApJ](#), **200**, 269
- Bosman S. E. I., et al., 2024, [Nature Astronomy](#), **8**, 1054

- Bothwell M. S., et al., 2017, [MNRAS](#), **466**, 2825
- Braatz J. A., Wilson A. S., Henkel C., 1996, [ApJS](#), **106**, 51
- Braun R., Bourke T., Green J. A., Keane E., Wagg J., 2015, in *Advancing Astrophysics with the Square Kilometre Array (AASKA14)*. p. 174, [doi:10.22323/1.215.0174](#)
- Braun R., Bonaldi A., Bourke T., Keane E., Wagg J., 2019, [arXiv e-prints](#), p. [arXiv:1912.12699](#)
- Bridle A. H., Perley R. A., 1984, [ARA&A](#), **22**, 319
- Bureau M., Carignan C., 2002, [AJ](#), **123**, 1316
- Cackett E. M., Bentz M. C., Kara E., 2021, [iScience](#), **24**, 102557
- Cao S., Zajaček M., Panda S., Martínez-Aldama M. L., Czerny B., Ratra B., 2022, [MNRAS](#), **516**, 1721
- Caon N., Capaccioli M., D’Onofrio M., 1993, [MNRAS](#), **265**, 1013
- Cappellari M., 2002a, [MNRAS](#), **333**, 400
- Cappellari M., 2002b, [MNRAS](#), **333**, 400
- Cappellari M., 2008, [MNRAS](#), **390**, 71
- Cappellari M., 2020, [MNRAS](#), **494**, 4819
- Cappellari M., Verolme E. K., van der Marel R. P., Verdoes Kleijn G. A., Illingworth G. D., Franx M., Carollo C. M., de Zeeuw P. T., 2002, [ApJ](#), **578**, 787
- Cappellari M., Neumayer N., Reunanen J., van der Werf P. P., de Zeeuw P. T., Rix H.-W., 2009, [MNRAS](#), **394**, 660
- Cappellari M., et al., 2013, [MNRAS](#), **432**, 1709
- Cathey J., et al., 2024, [ApJ](#), **967**, 11
- Chandrasekhar S., 1931, [MNRAS](#), **91**, 456

Chandrasekhar S., 1935, [MNRAS](#), **95**, 207

Chang D. O., Johnson M. D., Tiede P., 2025, [ApJ](#), **989**, L1

Chen C.-H., Ho L. C., Li R., Zhuang M.-Y., 2025, [ApJ](#), **983**, 60

Churazov E., Sazonov S., Sunyaev R., Forman W., Jones C., Böhringer H., 2005, [MNRAS](#), **363**, L91

Cicone C., et al., 2014, [A&A](#), **562**, A21

Ciotti L., Bertin G., 1999, [A&A](#), **352**, 447

Cohn J. H., et al., 2021, [ApJ](#), **919**, 77

Cohn J. H., et al., 2023, [ApJ](#), **958**, 186

Cohn J. H., et al., 2024, [ApJ](#), **975**, 179

Combes F., et al., 2019, [A&A](#), **623**, A79

Cox P., et al., 2011, [ApJ](#), **740**, 63

Cresci G., et al., 2015, [ApJ](#), **799**, 82

Dame T. M., 2011, [arXiv e-prints](#), p. [arXiv:1101.1499](#)

Danhaive A. L., et al., 2026, [MNRAS](#), **546**, [stag119](#)

Davé R., Anglés-Alcázar D., Narayanan D., Li Q., Rafieeantsoa M. H., Appleby S., 2019, [MNRAS](#), **486**, 2827

Davies R. I., et al., 2006, [ApJ](#), **646**, 754

Davies M. B., Miller M. C., Bellovary J. M., 2011, [ApJ](#), **740**, L42

Davis T. A., 2014, [MNRAS](#), **443**, 911

Davis T. A., et al., 2013a, [MNRAS](#), **429**, 534

Davis T. A., Bureau M., Cappellari M., Sarzi M., Blitz L., 2013b, [Nature](#), **494**, 328

Davis T. A., Bureau M., Cappellari M., Sarzi M., Blitz L., 2013c, [Nature](#), **494**, 328

Davis T. A., Bureau M., Onishi K., Cappellari M., Iguchi S., Sarzi M., 2017, [MNRAS](#), **468**, 4675

Davis T. A., et al., 2018, [MNRAS](#), **473**, 3818

Davis T. A., et al., 2020, [MNRAS](#), **496**, 4061

Dekel A., Burkert A., 2014, [MNRAS](#), **438**, 1870

Desmond H., 2017, [MNRAS](#), **464**, 4160

Devereux N., Ford H., Tsvetanov Z., Jacoby G., 2003, [AJ](#), **125**, 1226

Di Cintio A., Lelli F., 2016, [MNRAS](#), **456**, L127

Di Matteo T., Springel V., Hernquist L., 2005, [Nature](#), **433**, 604

Di Teodoro E. M., Fraternali F., 2015, [MNRAS](#), **451**, 3021

Dominiak P., Bureau M., Davis T. A., Ma C.-P., Greene J. E., Gu M., 2024, [MNRAS](#), **529**, 1597

Dominiak P., et al., 2025, [MNRAS](#), **542**, 2039

Dong C., et al., 2019, [ApJ](#), **873**, 50

Drehmer D. A., Storch-Bergmann T., Ferrari F., Cappellari M., Riffel R. A., 2015, [MNRAS](#), **450**, 128

Dressler A., Richstone D. O., 1988, [ApJ](#), **324**, 701

Dullo B. T., Graham A. W., 2014, [MNRAS](#), **444**, 2700

Dullo B. T., Gil de Paz A., Knapen J. H., 2021, [ApJ](#), **908**, 134

Durodola E., Pacucci F., Hickox R. C., 2025, [ApJ](#), **985**, 169

Dye S., et al., 2018, [MNRAS](#), **476**, 4383

Dye S., et al., 2022, [MNRAS](#), **510**, 3734

EPTA Collaboration et al., 2023, [A&A](#), **678**, A50

Eales S., et al., 2010, [PASP](#), 122, 499

Eddington A. S., 1924, [Nature](#), 113, 192

Eddington A. S., 1926, The Internal Constitution of the Stars. Cambridge University Press

Einstein A., 1916, [Annalen der Physik](#), 354, 769

Elford J. S., Ruffa I., Davis T. A., Bureau M., Fender R., Gensior J., Williams T., Zhang H., 2026, [MNRAS](#), 546, [stag037](#)

Emsellem E., Monnet G., Bacon R., 1994, [A&A](#), 285, 723

Emsellem E., et al., 2011, [MNRAS](#), 414, 888

Euclid Collaboration et al., 2025a, [arXiv e-prints](#), p. [arXiv:2503.15309](#)

Euclid Collaboration et al., 2025b, [A&A](#), 697, A1

Event Horizon Telescope Collaboration et al., 2019a, [ApJ](#), 875, L1

Event Horizon Telescope Collaboration et al., 2019b, [ApJ](#), 875, L2

Event Horizon Telescope Collaboration et al., 2019c, [ApJ](#), 875, L6

Event Horizon Telescope Collaboration et al., 2022a, [ApJ](#), 930, L12

Event Horizon Telescope Collaboration et al., 2022b, [ApJ](#), 930, L15

Falcke H., K rding E., Markoff S., 2004, [A&A](#), 414, 895

Falc n-Barroso J., et al., 2017, [A&A](#), 597, A48

Fanaroff B. L., Riley J. M., 1974, [MNRAS](#), 167, 31P

Fei Q., et al., 2025, [arXiv e-prints](#), p. [arXiv:2509.20452](#)

Feldmeier-Krause A., Zhu L., Neumayer N., van de Ven G., de Zeeuw P. T., Sch del R., 2017, [MNRAS](#), 466, 4040

Ferrarese L., Merritt D., 2000, [ApJ](#), 539, L9

Ferrarese L., Ford H. C., Jaffe W., 1996, [ApJ](#), 470, 444

Finkelstein D., 1958, [Physical Review](#), **110**, 965

Fischer J.-L., Domínguez Sánchez H., Bernardi M., 2019, [MNRAS](#), **483**, 2057

Fowler R. H., 1926, [MNRAS](#), **87**, 114

Fraternali F., Karim A., Magnelli B., Gómez-Guijarro C., Jiménez-Andrade E. F., Posses A. C., 2021, [A&A](#), **647**, A194

Freedman W. L., et al., 2001, [ApJ](#), **553**, 47

Furtak L. J., et al., 2024, [Nature](#), **628**, 57

GRAVITY Collaboration et al., 2018, [A&A](#), **615**, L15

GRAVITY Collaboration et al., 2021, [A&A](#), **654**, A85

GRAVITY Collaboration et al., 2024a, [A&A](#), **684**, A167

GRAVITY Collaboration et al., 2024b, [A&A](#), **690**, A76

Gao F., et al., 2017, [ApJ](#), **834**, 52

Gao H., Ho L. C., Barth A. J., Li Z.-Y., 2019, [ApJS](#), **244**, 34

Gao H., Ho L. C., Li Z.-Y., 2022, [ApJS](#), **262**, 54

Gebhardt K., Thomas J., 2009, [ApJ](#), **700**, 1690

Gebhardt K., et al., 2000, [ApJ](#), **539**, L13

Ghez A. M., Salim S., Hornstein S. D., Tanner A., Lu J. R., Morris M., Becklin E. E., Duchêne G., 2005, [ApJ](#), **620**, 744

Gillman S., et al., 2023, [A&A](#), **676**, A26

Gillman S., et al., 2024, [A&A](#), **691**, A299

Gnerucci A., et al., 2011, [A&A](#), **533**, A124

Goodman J., 2000, [New Astron.](#), **5**, 103

Graham A. W., Driver S. P., 2007, [ApJ](#), **655**, 77

- Graham A. W., Sahu N., 2023, [MNRAS](#), **518**, 2177
- Graham A. W., Erwin P., Trujillo I., Asensio Ramos A., 2003, [AJ](#), **125**, 2951
- Graham A. W., Onken C. A., Athanassoula E., Combes F., 2011, [MNRAS](#), **412**, 2211
- Gravity+ Collaboration et al., 2022, [The Messenger](#), **189**, 17
- Gravity+ Collaboration et al., 2026, [A&A](#), **706**, A99
- Greene J. E., et al., 2010, [ApJ](#), **721**, 26
- Greene J. E., Strader J., Ho L. C., 2020, [ARA&A](#), **58**, 257
- Greenhill L. J., Moran J. M., Herrnstein J. R., 1997, [ApJ](#), **481**, L23
- Greenhill L. J., et al., 2003, [ApJ](#), **590**, 162
- Grier C. J., Pancoast A., Barth A. J., Fausnaugh M. M., Brewer B. J., Treu T., Peterson B. M., 2017, [ApJ](#), **849**, 146
- Grudić M. Y., Hopkins P. F., Quataert E., Murray N., 2019, [MNRAS](#), **483**, 5548
- Gullberg B., et al., 2015, [MNRAS](#), **449**, 2883
- Gültekin K., et al., 2009, [ApJ](#), **698**, 198
- Häberle M., et al., 2024, [Nature](#), **631**, 285
- Harikane Y., et al., 2023, [ApJ](#), **959**, 39
- Harms R. J., et al., 1994, [ApJ](#), **435**, L35
- Harrison C. M., 2017, [Nature Astronomy](#), **1**, 0165
- Harrison B. K., Thorne K. S., Wakano M., Wheeler J. A., 1965, *Gravitation Theory and Gravitational Collapse*. University of Chicago Press
- Harrison C. M., Costa T., Tadhunter C. N., Flütsch A., Kakkad D., Perna M., Vietri G., 2018, [Nature Astronomy](#), **2**, 198
- Heckman T. M., Best P. N., 2014, [ARA&A](#), **52**, 589

Herrera-Camus R., et al., 2021, [A&A](#), **649**, A31

Herrera-Camus R., Förster Schreiber N. M., Vallini L., Bouwens R., Silverman J. D., 2026, [Nature Astronomy](#), **10**, 34

Herrnstein J. R., Moran J. M., Greenhill L. J., Trotter A. S., 2005, [ApJ](#), **629**, 719

Hezaveh Y. D., 2014, [ApJ](#), **791**, L41

Hicks E. K. S., Malkan M. A., 2008, [ApJS](#), **174**, 31

Hodge J. A., Carilli C. L., Walter F., de Blok W. J. G., Riechers D., Daddi E., Lentati L., 2012, [ApJ](#), **760**, 11

Hoffman M. D., Gelman A., 2014, [Journal of Machine Learning Research](#), **15**, 1593

Högbom J. A., 1974, [A&AS](#), **15**, 417

Hopkins P. F., Hernquist L., Cox T. J., Kereš D., 2008, [ApJS](#), **175**, 356

Hopkins P. F., Murray N., Quataert E., Thompson T. A., 2010, [MNRAS](#), **401**, L19

Hu W., Barkana R., Gruzinov A., 2000, [Phys. Rev. Lett.](#), **85**, 1158

Hu H., Inayoshi K., Haiman Z., Ho L. C., Ohsuga K., 2025, [ApJ](#), **983**, L37

Ikeda R., et al., 2025, [A&A](#), **693**, A237

Inayoshi K., Ho L. C., 2025, [arXiv e-prints](#), p. [arXiv:2512.03130](#)

Inayoshi K., Visbal E., Haiman Z., 2020, [ARA&A](#), **58**, 27

Ishibashi W., Fabian A. C., Maiolino R., 2018, [MNRAS](#), **476**, 512

Ivezić Ž., et al., 2019, [ApJ](#), **873**, 111

Izumi T., et al., 2020, [ApJ](#), **898**, 75

Jeans J. H., 1915, [MNRAS](#), **76**, 70

Jeans J. H., 1922, [MNRAS](#), **82**, 122

Jeffery G. B., 1921, [Proceedings of the Royal Society of London Series A](#), **99**, 123

- Jeon J., Liu B., Taylor A. J., Kokorev V., Chisholm J., Kocevski D. D., Finkelstein S. L., Bromm V., 2025, [ApJ](#), **988**, 110
- Jeter B., Broderick A. E., 2021, [ApJ](#), **908**, 139
- Jeter B., Broderick A. E., McNamara B. R., 2019, [ApJ](#), **882**, 82
- Johnson M. D., et al., 2023, [Galaxies](#), **11**, 61
- Johnson M. D., et al., 2024, in Coyle L. E., Matsuura S., Perrin M. D., eds, Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series Vol. 13092, Space Telescopes and Instrumentation 2024: Optical, Infrared, and Millimeter Wave. p. 130922D ([arXiv:2406.12917](#)), [doi:10.1117/12.3019835](#)
- Jones G. C., Maiolino R., Caselli P., Carniani S., 2019, [A&A](#), **632**, L7
- Jorsater S., van Moorsel G. A., 1995, [AJ](#), **110**, 2037
- Juodžbalis I., et al., 2024, [Nature](#), **636**, 594
- Juodžbalis I., et al., 2025, [arXiv e-prints](#), p. [arXiv:2508.21748](#)
- Juodžbalis I., et al., 2026, [MNRAS](#), **546**, stag086
- Kabasares K. M., et al., 2022, [ApJ](#), **934**, 162
- Kabasares K. M., et al., 2024, [ApJ](#), **966**, 132
- Kamali F., et al., 2019, [A&A](#), **624**, A42
- Kerr R. P., 1963, [Phys. Rev. Lett.](#), **11**, 237
- Khatri P., Romano-Díaz E., Porciani C., 2025, [A&A](#), **697**, A174
- Kingma D. P., Ba J., 2014, [arXiv e-prints](#), p. [arXiv:1412.6980](#)
- Kippenhahn R., Weigert A., 1990, *Stellar Structure and Evolution*. Springer
- Klambauer G., Unterthiner T., Mayr A., Hochreiter S., 2017, [arXiv e-prints](#), p. [arXiv:1706.02515](#)

- Kokorev V., et al., 2023, [ApJ](#), **957**, L7
- Kondratko P. T., Greenhill L. J., Moran J. M., 2008, [ApJ](#), **678**, 87
- Koratkar A. P., Gaskell C. M., 1991, [ApJ](#), **370**, L61
- Kormendy J., Ho L. C., 2013, [ARA&A](#), **51**, 511
- Kormendy J., Richstone D., 1995, [ARA&A](#), **33**, 581
- Krajnović D., McDermid R. M., Cappellari M., Davies R. L., 2009, [MNRAS](#), **399**, 1839
- Krajnović D., Cappellari M., McDermid R. M., 2018, [MNRAS](#), **473**, 5237
- Krist J. E., Hook R. N., Stoehr F., 2011, in Kahan M. A., ed., Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series Vol. 8127, Optical Modeling and Performance Predictions V. p. 81270J, [doi:10.1117/12.892762](#)
- Kuo C. Y., et al., 2011, [ApJ](#), **727**, 20
- Kuo C. Y., Braatz J. A., Reid M. J., Lo K. Y., Condon J. J., Impellizzeri C. M. V., Henkel C., 2013, [ApJ](#), **767**, 155
- Kuo C. Y., et al., 2020, [MNRAS](#), **498**, 1609
- LSST Science Collaboration et al., 2009, [arXiv e-prints](#), p. [arXiv:0912.0201](#)
- Lablanche P.-Y., et al., 2012, [MNRAS](#), **424**, 1495
- Laing R. A., Bridle A. H., 2002, [MNRAS](#), **336**, 1161
- Laing R. A., Bridle A. H., 2014, [MNRAS](#), **437**, 3405
- Lambert T. S., et al., 2023, [MNRAS](#), **518**, 3183
- Laplace P. S., 1799, Allgemeine Geographische Ephemeriden, **4**, 1
- Larson R. L., et al., 2023, [ApJ](#), **953**, L29
- Lattimer J. M., Prakash M., 2001, [ApJ](#), **550**, 426
- Lauer T. R., et al., 1995, [AJ](#), **110**, 2622

- Lauer T. R., et al., 2007, [ApJ](#), **664**, 226
- Laurikainen E., Salo H., Buta R., Knapen J. H., 2011, [MNRAS](#), **418**, 1452
- Lelli F., McGaugh S. S., Schombert J. M., Pawlowski M. S., 2017, [ApJ](#), **836**, 152
- Lelli F., Davis T. A., Bureau M., Cappellari M., Liu L., Ruffa I., Smith M. D., Williams T. G., 2022, [MNRAS](#), **516**, 4066
- Lemaître G., 1933, Annales de la Société Scientifique de Bruxelles, **53**, 51
- Levenberg K., 1944, Quarterly of Applied Mathematics, **2**, 164
- Li Y.-R., Wang J.-M., 2025, [ApJ](#), **979**, 126
- Li H., Li R., Mao S., Xu D., Long R. J., Emsellem E., 2016, [MNRAS](#), **455**, 3680
- Li P., McGaugh S. S., Lelli F., Tian Y., Schombert J. M., Ko C.-M., 2022, [ApJ](#), **927**, 198
- Li J., et al., 2025, [ApJ](#), **981**, 19
- Liang F.-H., et al., 2024, [MNRAS](#), **527**, 9343
- Liepold E. R., Ma C.-P., Walsh J. L., 2023, [ApJ](#), **945**, L35
- Lim J., Leon S., Combes F., Dinh-V-Trung 2000, [ApJ](#), **545**, L93
- Lines N. E. P., Li T., Collett T. E., Holloway P., Nightingale J. W., Rojas K., Verma A., Walmsley M., 2025, [Nature Astronomy](#), **9**, 1116
- Litke K. C., et al., 2019, [ApJ](#), **870**, 80
- Litke K. C., et al., 2022, [ApJ](#), **928**, 179
- Liu D., et al., 2024, [Nature Astronomy](#), **8**, 1181
- Lo K. Y., 2005, [ARA&A](#), **43**, 625
- Lockhart W., Gralla S. E., 2022, [MNRAS](#), **509**, 3643
- Lodato G., Bertin G., 2003, [A&A](#), **398**, 517
- Lupi A., Trinca A., Volonteri M., Dotti M., Mazzucchelli C., 2024, [A&A](#), **689**, A128

- Lynden-Bell D., 1969, [Nature](#), **223**, 690
- Lynden-Bell D., Rees M. J., 1971, [MNRAS](#), **152**, 461
- Ma Z., et al., 2026, [arXiv e-prints](#), p. arXiv:2601.15962
- MacDonald G. H., Kenderdine S., Neville A. C., 1968, [MNRAS](#), **138**, 259
- Madau P., Rees M. J., 2001, [ApJ](#), **551**, L27
- Madden S. C., et al., 2020, [A&A](#), **643**, A141
- Magorrian J., et al., 1998, [AJ](#), **115**, 2285
- Maia M. A. G., da Costa L. N., Latham D. W., 1989, [ApJS](#), **69**, 809
- Maiolino R., et al., 2024, [A&A](#), **691**, A145
- Makarov D., Prugniel P., Terekhova N., Courtois H., Vauglin I., 2014, [A&A](#), **570**, A13
- Mamyoda K., Nakai N., Yamauchi A., Diamond P., Huré J.-M., 2009, [PASJ](#), **61**, 1143
- Maresca J., Dye S., 2025, [MNRAS](#), **537**, 2708
- Marquardt D. W., 1963, Journal of the Society for Industrial and Applied Mathematics, **11**, 431
- Marshall M. A., Wyithe J. S. B., Windhorst R. A., Di Matteo T., Ni Y., Wilkins S., Croft R. A. C., Mechtley M., 2021, [MNRAS](#), **506**, 1209
- Massonneau W., Volonteri M., Dubois Y., Beckmann R. S., 2023, [A&A](#), **670**, A180
- Matthee J., et al., 2024, [ApJ](#), **963**, 129
- Matthews T. A., Sandage A. R., 1963, [ApJ](#), **138**, 30
- McConnell N. J., Ma C.-P., 2013, [ApJ](#), **764**, 184
- McGaugh S. S., Lelli F., Schombert J. M., 2016, [Phys. Rev. Lett.](#), **117**, 201101
- McKinnon M., Beasley A., Murphy E., Selina R., Farnsworth R., Walter A., 2019, in Bulletin of the American Astronomical Society. p. 81

- McLure R. J., Jarvis M. J., 2002, [MNRAS](#), **337**, 109
- McMullin J. P., Waters B., Schiebel D., Young W., Golap K., 2007, in Shaw R. A., Hill F., Bell D. J., eds, *Astronomical Society of the Pacific Conference Series Vol. 376, Astronomical Data Analysis Software and Systems XVI*. p. 127
- Mehta D. H., Regan J. A., Prole L., 2026, [Nature Astronomy](#), **10**, 583
- Merloni A., Heinz S., di Matteo T., 2003, [MNRAS](#), **345**, 1057
- Michell J., 1784, *Philosophical Transactions of the Royal Society of London Series I*, **74**, 35
- Milgrom M., 1983, [ApJ](#), **270**, 371
- Miller T. B., et al., 2025, [ApJ](#), **988**, 196
- Mitzkus M., Cappellari M., Walcher C. J., 2017, [MNRAS](#), **464**, 4789
- Mizukoshi S., et al., 2021, [ApJ](#), **917**, 94
- Montgomery C., Orchiston W., Whittingham I., 2009, *Journal of Astronomical History and Heritage*, **12**, 90
- Nagai H., et al., 2019, [ApJ](#), **883**, 193
- Naidu R. P., et al., 2025, [arXiv e-prints](#), p. [arXiv:2503.16596](#)
- Napier P., 1995, in Zensus J. A., Diamond P. J., Napier P. J., eds, *ASP Conf. Ser. Vol. 82, Very Long Baseline Interferometry and the VLBA*. Astron. Soc. Pac., San Francisco, p. 59
- Napolitano L., et al., 2025, [A&A](#), **693**, A50
- Neeleman M., Prochaska J. X., Kanekar N., Rafelski M., 2020, [Nature](#), **581**, 269
- Nesvadba N. P. H., et al., 2021, [A&A](#), **654**, A8
- Neumayer N., Cappellari M., Reunanen J., Rix H. W., van der Werf P. P., de Zeeuw P. T., Davies R. I., 2007, [ApJ](#), **671**, 1329
- Newman E. T., Janis A. I., 1965, [Journal of Mathematical Physics](#), **6**, 915

- Newman E. T., Couch E., Chinnapared K., Exton A., Prakash A., Torrence R., 1965, [Journal of Mathematical Physics](#), **6**, 918
- Newman A. B., et al., 2025, [arXiv e-prints](#), p. [arXiv:2503.17478](#)
- Nguyen D. D., et al., 2017, [ApJ](#), **836**, 237
- Nguyen D. D., et al., 2020, [ApJ](#), **892**, 68
- Nguyen D. D., et al., 2021, [MNRAS](#), **504**, 4123
- Nguyen D. D., et al., 2022, [MNRAS](#), **509**, 2920
- Nguyen D. D., et al., 2026a, [arXiv e-prints](#), p. [arXiv:2601.17439](#)
- Nguyen D. D., et al., 2026b, [arXiv e-prints](#), p. [arXiv:2601.19842](#)
- Nguyen D. D., et al., 2026c, [ApJ](#), **999**, 97
- Noordermeer E., 2008, [MNRAS](#), **385**, 1359
- Nordström G., 1918, Koninklijke Nederlandse Akademie van Wetenschappen Proceedings Series B Physical Sciences, **20**, 1238
- North E. V., et al., 2019, [MNRAS](#), **490**, 319
- Ocaña Flaquer B., Leon S., Combes F., Lim J., 2010, [A&A](#), **518**, A9
- Okuda T., Kohno K., Iguchi S., Nakanishi K., 2005, [ApJ](#), **620**, 673
- Onishi K., Iguchi S., Sheth K., Kohno K., 2015, [ApJ](#), **806**, 39
- Onishi K., Iguchi S., Davis T. A., Bureau M., Cappellari M., Sarzi M., Blitz L., 2017, [MNRAS](#), **468**, 4663
- Onken C. A., Ferrarese L., Merritt D., Peterson B. M., Pogge R. W., Vestergaard M., Wandel A., 2004, [ApJ](#), **615**, 645
- Onken C. A., et al., 2007, [ApJ](#), **670**, 105
- Oppenheimer J. R., Snyder H., 1939, [Physical Review](#), **56**, 455

Oppenheimer J. R., Volkoff G. M., 1939, [Physical Review](#), **55**, 374

Osorno J., Nagar N., Richtler T., Humire P., Gebhardt K., Gultekin K., 2023, [A&A](#), **679**, A37

Özel F., Freire P., 2016, [ARA&A](#), **54**, 401

Pacucci F., Loeb A., 2024, [ApJ](#), **964**, 154

Pacucci F., Nguyen B., Carniani S., Maiolino R., Fan X., 2023, [ApJ](#), **957**, L3

Padovani P., et al., 2017, [A&A Rev.](#), **25**, 2

Pancoast A., Brewer B. J., Treu T., 2014, [MNRAS](#), **445**, 3055

Pastorini G., et al., 2007, [A&A](#), **469**, 405

Pechetti R., Seth A., Neumayer N., Georgiev I., Kacharov N., den Brok M., 2020, [ApJ](#), **900**, 32

Penrose R., 1965, [Phys. Rev. Lett.](#), **14**, 57

Pesce D. W., et al., 2020a, [ApJ](#), **890**, 118

Pesce D. W., et al., 2020b, [ApJ](#), **891**, L1

Phillips A. C., Illingworth G. D., MacKenty J. W., Franx M., 1996, [AJ](#), **111**, 1566

Pillepich A., et al., 2019, [MNRAS](#), **490**, 3196

Planck Collaboration et al., 2020, [A&A](#), **641**, A6

Portegies Zwart S. F., Baumgardt H., Hut P., Makino J., McMillan S. L. W., 2004, [Nature](#), **428**, 724

Price S. H., et al., 2022, [A&A](#), **665**, A159

Rau U., Cornwell T. J., 2011, [A&A](#), **532**, A71

Reardon D. J., et al., 2023, [ApJ](#), **951**, L6

Rees M. J., 1989, [MNRAS](#), **239**, 1P

Reid M. J., Pesce D. W., Riess A. G., 2019, [ApJ](#), **886**, L27

Reines A. E., Volonteri M., 2015, [ApJ](#), 813, 82

Reissner H., 1916, [Annalen der Physik](#), 355, 106

Reuter C., et al., 2020, [ApJ](#), 902, 78

Rhoades C. E., Ruffini R., 1974, [Phys. Rev. Lett.](#), 32, 324

Rizzo F., Vegetti S., Powell D., Fraternali F., McKean J. P., Stacey H. R., White S. D. M., 2020, [Nature](#), 584, 201

Rizzo F., Vegetti S., Fraternali F., Stacey H. R., Powell D., 2021, [MNRAS](#), 507, 3952

Rizzo F., Kohandel M., Pallottini A., Zanella A., Ferrara A., Vallini L., Toft S., 2022, [A&A](#), 667, A5

Roberts G. O., Tweedie R. L., 1996, *Bernoulli*, 2, 341

Rohatgi A., 2022, Webplotdigitizer: Version 4.6, <https://automeris.io/WebPlotDigitizer>

Roman-Oliveira F., Fraternali F., Rizzo F., 2023, [MNRAS](#), 521, 1045

Roman-Oliveira F., Rizzo F., Fraternali F., 2026, [MNRAS](#), 546, stag213

Rowland L. E., et al., 2024, [MNRAS](#), 535, 2068

Ruffa I., Davis T. A., 2024, [Galaxies](#), 12, 36

Ruffa I., et al., 2019a, [MNRAS](#), 484, 4239

Ruffa I., et al., 2019b, [MNRAS](#), 489, 3739

Ruffa I., et al., 2019c, [MNRAS](#), 489, 3739

Ruffa I., Laing R. A., Prandoni I., Paladino R., Parma P., Davis T. A., Bureau M., 2020, [MNRAS](#), 499, 5719

Ruffa I., et al., 2023, [MNRAS](#), 522, 6170

Ruffa I., et al., 2024, [MNRAS](#), 528, L76

- Rusakov V., et al., 2026, [Nature](#), **649**, 574
- Rusli S. P., Thomas J., Erwin P., Saglia R. P., Nowak N., Bender R., 2011, [MNRAS](#), **410**, 1223
- Rusli S. P., et al., 2013, [AJ](#), **146**, 45
- Saglia R. P., et al., 2016, [ApJ](#), **818**, 47
- Sahu N., Graham A. W., Davis B. L., 2019, [ApJ](#), **887**, 10
- Saito T., et al., 2022, [ApJ](#), **935**, 155
- Sakai S., Giovanelli R., Wegner G., 1994, [AJ](#), **108**, 33
- Salomé Q., Salomé P., Combes F., 2015, [A&A](#), **574**, A34
- Sargent W. L. W., Young P. J., Boksenberg A., Shortridge K., Lynds C. R., Hartwick F. D. A., 1978, [ApJ](#), **221**, 731
- Sarzi M., Rix H.-W., Shields J. C., Rudnick G., Ho L. C., McIntosh D. H., Filippenko A. V., Sargent W. L. W., 2001, [ApJ](#), **550**, 65
- Sarzi M., et al., 2002, [ApJ](#), **567**, 237
- Sarzi M., Rix H.-W., Shields J. C., Ho L. C., Barth A. J., Rudnick G., Filippenko A. V., Sargent W. L. W., 2005, [ApJ](#), **628**, 169
- Savorgnan G., Graham A. W., Marconi A., Sani E., Hunt L. K., Vika M., Driver S. P., 2013, [MNRAS](#), **434**, 387
- Schaerer D., et al., 2020, [A&A](#), **643**, A3
- Schmidt M., 1963, [Nature](#), **197**, 1040
- Schödel R., et al., 2002, [Nature](#), **419**, 694
- Schulze A., Gebhardt K., 2011, [ApJ](#), **729**, 21
- Schwarzschild K., 1916, Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften, pp 189–196

Schwarzschild M., 1979, [ApJ](#), **232**, 236

Selina R. J., et al., 2018, in Murphy E., ed., Astronomical Society of the Pacific Conference Series Vol. 517, Science with a Next Generation Very Large Array. p. 15 ([arXiv:1810.08197](#)), [doi:10.48550/arXiv.1810.08197](#)

Sersic J. L., 1968, Atlas de Galaxias Australes. Observatorio Astronomico

Shankar F., Bernardi M., Sheth R. K., 2017, [MNRAS](#), **466**, 4029

Shapiro K. L., Cappellari M., de Zeeuw T., McDermid R. M., Gebhardt K., van den Bosch R. C. E., Statler T. S., 2006, [MNRAS](#), **370**, 559

Shen S., Mo H. J., White S. D. M., Blanton M. R., Kauffmann G., Voges W., Brinkmann J., Csabai I., 2003, [MNRAS](#), **343**, 978

Shen Y., et al., 2024, [ApJS](#), **272**, 26

Silk J., Rees M. J., 1998, [A&A](#), **331**, L1

Silverman J. D., et al., 2025, [ApJ](#), **995**, L67

Simon D. A., Cappellari M., Hartke J., 2024, [MNRAS](#), **527**, 2341

Siopis C., et al., 2009, [ApJ](#), **693**, 946

Smith M., 2020, PhD thesis, University of Oxford

Smith M. D., et al., 2019, [MNRAS](#), **485**, 4359

Smith M. D., et al., 2021, [MNRAS](#), **503**, 5984

Spilker J. S., et al., 2016, [ApJ](#), **826**, 112

Spilker J. S., Champagne J. B., Fan X., Fujimoto S., van der Werf P. P., Yang J., Yue M., 2025, [ApJ](#), **982**, 72

Stacey G. J., Hailey-Dunsheath S., Ferkinhoff C., Nikola T., Parshley S. C., Benford D. J., Staguhn J. G., Fiolet N., 2010, [ApJ](#), **724**, 957

Stacey H. R., Costa T., McKean J. P., Sharon C. E., Calistro Rivera G., Glikman E., van der Werf P. P., 2022, [MNRAS](#), **517**, 3377

Stiskalek R., Desmond H., 2023, [MNRAS](#), **525**, 6130

Stone C., et al., 2024a, [The Journal of Open Source Software](#), **9**, 7081

Stone M. A., Lyu J., Rieke G. H., Alberts S., Hainline K. N., 2024b, [ApJ](#), **964**, 90

Suh H., et al., 2025, [Nature Astronomy](#), **9**, 271

Szomoru D., Franx M., van Dokkum P. G., Trenti M., Illingworth G. D., Labbé I., Oesch P., 2013, [ApJ](#), **763**, 73

Targett T. A., Dunlop J. S., McLure R. J., 2012, [MNRAS](#), **420**, 3621

Taylor A. J., et al., 2025, [ApJ](#), **986**, 165

Toft S., et al., 2014, [ApJ](#), **782**, 68

Tonry J. L., 1984, [ApJ](#), **283**, L27

Tonry J. L., Dressler A., Blakeslee J. P., Ajhar E. A., Fletcher A. B., Luppino G. A., Metzger M. R., Moore C. B., 2001, [ApJ](#), **546**, 681

Toomre A., 1964, [ApJ](#), **139**, 1217

Trujillo I., Erwin P., Asensio Ramos A., Graham A. W., 2004, [AJ](#), **127**, 1917

Tully R. B., Fisher J. R., 1977, [A&A](#), **54**, 661

Übler H., et al., 2023, [A&A](#), **677**, A145

Vallini L., et al., 2025, [A&A](#), **700**, A117

Veale M., et al., 2017, [MNRAS](#), **464**, 356

Verdoes Kleijn G. A., van der Marel R. P., de Zeeuw P. T., Noel-Storr J., Baum S. A., 2002, [AJ](#), **124**, 2524

Volonteri M., 2010, [A&A Rev.](#), **18**, 279

- Walsh J. L., van den Bosch R. C. E., Barth A. J., Sarzi M., 2012, [ApJ](#), **753**, 79
- Walsh J. L., Barth A. J., Ho L. C., Sarzi M., 2013, [ApJ](#), **770**, 86
- Webster B. L., Murdin P., 1972, [Nature](#), **235**, 37
- Weinberger R., et al., 2018, [MNRAS](#), **479**, 4056
- Wei A., et al., 2013, [ApJ](#), **767**, 88
- Weyl H., 1917, [Annalen der Physik](#), **359**, 117
- Wilson S. N., et al., 2024, [A&A](#), **685**, A30
- Wisnioski E., et al., 2015, [ApJ](#), **799**, 209
- Wisnioski E., Mendel J. T., Leaman R., Tsukui T., bler H., Frster Schreiber N. M., 2025, [MNRAS](#), **544**, 2777
- Wolfe A. M., Burbidge G. R., 1970, [ApJ](#), **161**, 419
- Woods T. E., et al., 2019, [Publ. Astron. Soc. Australia](#), **36**, e027
- Wrobel J. M., Terashima Y., Ho L. C., 2008, [ApJ](#), **675**, 1041
- Wu Q., Shen Y., 2022, [ApJS](#), **263**, 42
- Wu X.-B., et al., 2015, [Nature](#), **518**, 512
- Xu H., et al., 2023, [Research in Astronomy and Astrophysics](#), **23**, 075024
- Yamauchi A., Nakai N., Sato N., Diamond P., 2004, [PASJ](#), **56**, 605
- Yamauchi A., Nakai N., Ishihara Y., Diamond P., Sato N., 2012, [PASJ](#), **64**, 103
- Yang C., et al., 2017, [A&A](#), **608**, A144
- Yantovski-Barth M. J., Zhang H., Smyth N., Stone C., Bureau M., Hezaveh Y., Perreault-Levasseur L., 2025, [arXiv e-prints](#), p. [arXiv:2511.20746](#)
- Young P. J., Westphal J. A., Kristian J., Wilson C. P., Landauer F. P., 1978, [ApJ](#), **221**, 721
- Zanella A., et al., 2018, [MNRAS](#), **481**, 1976

Zhang H., et al., 2024, [MNRAS](#), **530**, 3240

Zhang H., et al., 2025a, [MNRAS](#), **537**, 520

Zhang H., Bureau M., Ruffa I., Davis T. A., Dominiak P., Elford J. S., Lelli F., Williams T. G., 2025b, [MNRAS](#), **541**, 2540

Zhang X. A., et al., 2025c, [ApJ](#), **985**, 41

Zhao W., et al., 2018, [ApJ](#), **854**, 124

Zhu K., Lu S., Cappellari M., Li R., Mao S., Gao L., Ge J., 2024, [MNRAS](#), **527**, 706

Ziparo F., Gallerani S., Ferrara A., 2025, [J. Cosmol. Astropart. Phys.](#), **2025**, 040

Zolotov A., et al., 2015, [MNRAS](#), **450**, 2327

de Francesco G., Capetti A., Marconi A., 2006, [A&A](#), **460**, 439

de Vaucouleurs G., de Vaucouleurs A., Corwin Herold G. J., Buta R. J., Paturel G., Fouque P., 1991, Third Reference Catalogue of Bright Galaxies. Springer Nature

van Velzen S., Falcke H., Schellart P., Nierstenhöfer N., Kampert K.-H., 2012, [A&A](#), **544**, A18

van den Bosch R. C. E., 2016, [ApJ](#), **831**, 134

van den Bosch R. C. E., van de Ven G., 2009, [MNRAS](#), **398**, 1117

van der Marel R. P., van den Bosch F. C., 1998, [AJ](#), **116**, 2220