



5d superconformal field theories and graphs

Fabio Apruzzi^a, Craig Lawrie^b, Ling Lin^{b,*}, Sakura Schäfer-Nameki^a, Yi-Nan Wang^a

^a Mathematical Institute, University of Oxford, Woodstock Road, Oxford, OX2 6GG, United Kingdom

^b Department of Physics and Astronomy, University of Pennsylvania, Philadelphia, PA 19104, USA

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ABSTRACT

We propose graphs, the Combined Fiber Diagrams (CFD), to characterize all 5d superconformal field theories (SCFTs) that arise as S^1 -reductions of 6d SCFTs. Transitions between CFDs encode mass deformations that trigger RG-flows between SCFTs. They provide a combinatorial classification of all such 5d SCFTs and encode physical information about the strongly coupled theories, like the superconformal flavor symmetry and BPS states. We consistently reproduce known results, but more importantly predict new theories and strong coupling effects in 5d SCFTs.

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1. Introduction

5d $\mathcal{N} = 1$ SCFTs are intrinsically non-perturbative quantum field theories. At low energies these can have effective descriptions in terms of weakly coupled gauge theories, however to interpolate between the infrared (IR) and ultraviolet (UV) fixed points requires methods beyond ordinary field theory, motivating a string theoretic approach. 5d theories have been engineered in string theory by (p, q) -fivebrane webs [1], or M-theory on non-compact Calabi–Yau threefolds with canonical singularities [2,3]. In the latter approach, there is a particularly elegant correspondence between geometry and physics, whereby the resolution of the singularity may be identified with a renormalization group (RG)-flow from the UV to an effective IR description.

In this letter, we show how this approach comprehensively surveys 5d SCFTs and their salient physical properties. In essence, singularities in the M-theory realization, where complex surfaces have collapsed to points, correspond to SCFTs. In the smooth phase, when these surfaces have finite volume, their geometry determines the low-energy gauge theory descriptions for the SCFT, if one exists. Complex curves inside these surfaces determine the spectrum of matter hypermultiplets, as well as additional non-perturbative states, all of which become part of the BPS spectrum in the SCFT limit, where the surfaces collapse.

Recent progress in identifying M-theory geometries related to 5d SCFTs has been made in [4–10]. The approach in this letter

is fundamentally different, as it intrinsically captures some of the strongly coupled physics and gives an efficient way of mapping out the landscape of 5d SCFTs.

We define for each SCFT a graph, the *combined fiber diagram* (CFD), which encodes key properties of the geometry. Each such graph corresponds to an equivalence class of surface configurations inside a Calabi–Yau threefold, whose singular limit defines the same SCFT. The vertices of each graph correspond to curves contained within the surfaces, and give rise to BPS states in the UV.

Transitions between CFDs encode flows between SCFTs. These reflect geometric transitions that modify the curve configuration on the surfaces, such that their collapse generates a different singularity. The graph theoretic description gives an efficient method to map out all SCFTs obtained by mass deformations from a given, starting point SCFT.

An intrinsically strongly coupled characteristic of a 5d SCFT is its flavor symmetry G_F , which generally is larger than that of its low-energy description [11]. Determining this flavor enhancement is notoriously difficult. While techniques such as the superconformal index require an effective gauge description [12], these approaches are inapplicable for examples without such a description. However, the CFD manifestly encodes the Dynkin diagram of G_F in terms of a marked subgraph. The CFD-transitions correspond to precise rules how vertices are removed and unmarked. Finally we can compute the representations of BPS states under G_F , knowing the CFD.

Our approach is rooted in the duality between M- and F-theory on a singular, elliptically fibered Calabi–Yau threefold, Y . F-theory

* Corresponding author.

E-mail address: lling@physics.upenn.edu (L. Lin).

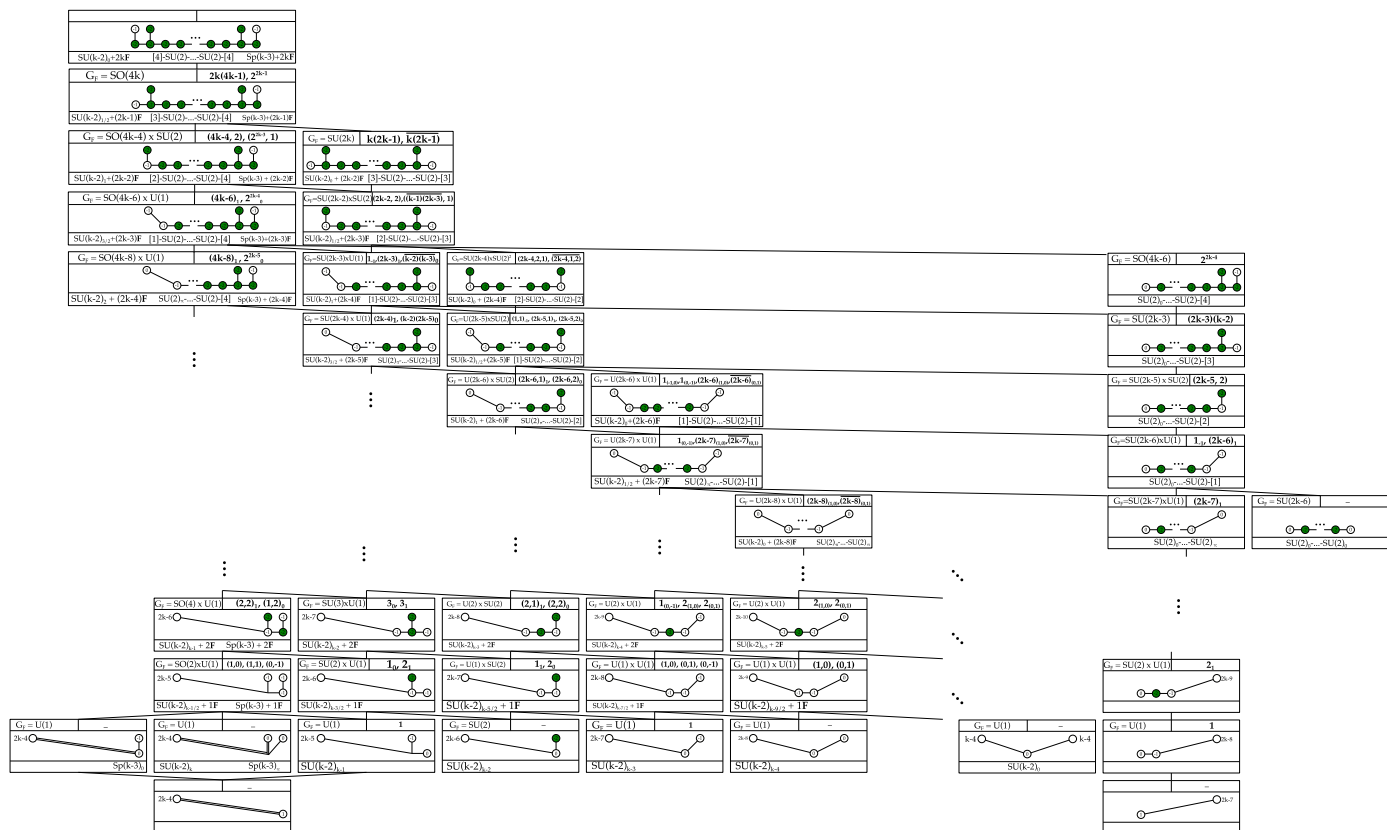


Fig. 2. CFDs for all 5d SCFTs descending from 6d (D_K, D_K) CM. Each box contains the 5d superconformal flavor symmetry, G_F , and the G_F representations of the spin 0 BPS states (right upper corner). In cases when there is a weakly coupled gauge theory description, this is noted at the bottom of each box. Connecting lines between boxes indicate transitions.

4. 5d SCFTs from (D_k, D_k) CM

Next we consider theories of arbitrary rank, descending from 6d (D_k, D_k) minimal CM, whose marginal CFD is

(4)

$$\begin{aligned}
m=4: & \quad SO(4k-6) \\
m=3: & \quad SU(2k-3) \\
m=2: & \quad SU(2k-5) \times SU(2) \\
m=1: & \quad SU(2k-6) \times U(1)
\end{aligned} \tag{8}$$

$$m=0, \theta=0: \quad SU(2k-6)$$

$$m=0, \theta=\pi: \quad SU(2k-7) \times U(1).$$

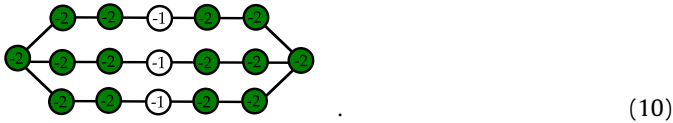
Our approach using CFDs not only determines these flavor symmetries much more efficiently and purely combinatorially than approaches using a gauge description, we can even determine the flavor symmetry in cases when such a weakly coupled description is absent. In the present case, there are $2k-6$ SCFTs that do not have any known gauge description, but we determine their superconformal flavor symmetry to be

$$U(2k-7-i), \quad i=0, \dots, 2k-7. \tag{9}$$

These CFDs and their associated geometries [14] are evidence that such non-trivial 5d UV fixed points exist; these have been observed for rank two, $k=5$, in [6,14,23].

5. 5d SCFTs from (E_6, E_6) CM

Another class of higher rank theories, that have thus far not been studied in generality, are the rank five SCFTs descending from (E_6, E_6) minimal CM. The marginal CFD is



CFD-transitions applied to this yield 93 descendant CFDs/SCFTs, included in the supplemental material. This predicts a large class of new 5d SCFTs. The only known weakly coupled description of the marginal theory is the quiver [18]

$$\begin{array}{c}
[2] \\
| \\
SU(2) \\
| \\
[2] - SU(2) - SU(3)_0 - SU(2) - [2].
\end{array} \tag{11}$$

Decoupling the flavor hypermultiplets of each $SU(2)$, step-by-step, yields descendants with quiver descriptions. Denote these by a triple (q_1, q_2, q_3) , where the q_i is either the number of fundamentals under, or the theta angle of, each of the three $SU(2)$ factors. For these quivers we find the following superconformal flavor symmetries:

$$\begin{aligned}
(1\mathbf{F}, 2\mathbf{F}, 2\mathbf{F}) &: E_6 \times E_6 \\
(0, 2\mathbf{F}, 2\mathbf{F}), (\pi, 2\mathbf{F}, 2\mathbf{F}) &: E_6 \times SU(6) \\
(1\mathbf{F}, 1\mathbf{F}, 2\mathbf{F}) &: SO(10)^2 \times U(1) \\
(0, 1\mathbf{F}, 2\mathbf{F}), (\pi, 1\mathbf{F}, 2\mathbf{F}) &: SO(10) \times SU(5) \times U(1) \\
(1\mathbf{F}, 1\mathbf{F}, 1\mathbf{F}) &: SO(8)^2 \times U(1)^2 \\
(0, 0, 2\mathbf{F}), (\pi, \pi, 2\mathbf{F}) &: SO(10) \times SU(4) \times U(1) \\
(0, \pi, 2\mathbf{F}) &: SU(5)^2 \times U(1) \\
(0, 1\mathbf{F}, 1\mathbf{F}), (\pi, 1\mathbf{F}, 1\mathbf{F}) &: SO(8) \times SU(4) \times U(1)^2 \\
(0, 0, 1\mathbf{F}), (\pi, \pi, 1\mathbf{F}) &: SO(8) \times SU(3) \times U(1)^2
\end{aligned} \tag{12}$$

$$(0, \pi, 1\mathbf{F}) : SU(4)^2 \times U(1)^2$$

$$(0, 0, 0), (\pi, \pi, \pi) : SO(8) \times SU(2) \times U(1)^2$$

$$(0, 0, \pi), (\pi, \pi, 0) : SU(4) \times SU(3) \times U(1)^2.$$

This populates only a small subtree of twelve elements in the CFD tree. Note that the CFDs are sensitive to the number of independent discrete parameters; they capture dualities between theories with different theta angles [24,25]. It would be interesting to determine the gauge theory descriptions, where they exist, for the remaining 81 CFD/SCFTs.

6. BPS states

BPS states, Φ_C , arise in M-theory from wrapped M2-branes on holomorphic curves C in \mathcal{S} . In CFD-terms, $(n, g) = (-1, 0)$ -vertices for instance correspond to spin 0 states under the 5d massive little group $SO(4)$ [26,27]. More generally, C can be a non-negative linear combination of vertices in the CFD, $C = \sum_i q_i C_i$, $q_i \geq 0$, where the q_i constrained by the decorations (n, g) , which are recursively determined

$$\begin{aligned}
n &= (C_1 + C_2)^2 = C_1 \cdot C_1 + C_2 \cdot C_2 + 2C_1 \cdot C_2, \\
g(C_1 + C_2) &= g(C_1) + g(C_2) + C_1 \cdot C_2 - 1.
\end{aligned} \tag{13}$$

Each C is associated to a weight of a representation of G_F , where the highest weights under the non-abelian subalgebra, $G_{F,na}$, are determined through the intersection numbers between C and the marked curves, F_i in the CFD

$$C \cdot F_i \geq 0, \quad (i=1, \dots, \text{rk}(G_{F,na})). \tag{14}$$

Charges under the abelian subalgebra are determined through intersection with specific combinations of unmarked vertices orthogonal to $G_{F,na}$, the $U(1)$ generators. Applying this to rank one theories reproduces the spin 0 BPS states in [28]. For the (D_k, D_k) descendants, Fig. 2 contains the predictions for spin 0 BPS states in these 5d strongly coupled SCFTs.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at <https://doi.org/10.1016/j.physletb.2019.135077>.

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