

Symmetry Breaking – A Classic Example of Quantum Interference Captured by Mixed Quantum/Classical Theory

Kayla Imanzi,^{1†} Dulat Bostan,^{1†} Max McCrea,² Josh Featherstone,² Mark Brouard,² and Dmitri Babikov^{1*}

¹*Chemistry Department, Marquette University, Milwaukee, Wisconsin 53201-1881, USA.*

²*Chemistry Research Laboratory, Department of Chemistry, University of Oxford, 12 Mansfield Road, OXFORD, OX1 3TA, U.K.*

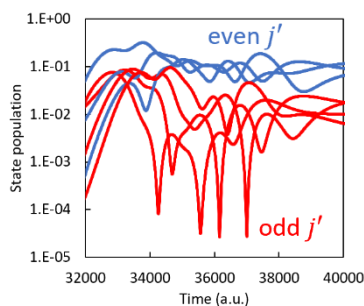
* Corresponding author: dmitri.babikov@mu.edu

† K.I. and D.B. contributed equally to this work.

Abstract:

The phenomena of propensity and inverse propensity are explored using a time-dependent mixed quantum classical theory, MQCT, in which the rotational motion of the molecule is treated quantum mechanically, whereas the scattering process is described classically. Good agreement with results of accurate full-quantum calculations is reported for a closed shell approximation to the NO + Ar system. It is shown that MQCT reproduces both phenomena in a broad range of the final states of the molecule, and for various initial rotational states, offering a unique time-dependent insight. It permits to see that both propensity and inverse propensity occur due to efficient depopulation of some states at the early post-collisional stage of the scattering process, when the molecule exists in a coherent superposition of many excited states that span a very broad range of angular momentum quantum numbers, populated by an efficient stepladder process of many consecutive transitions with small Δj .

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One of the fundamental consequences of a quantum mechanical description of inelastic molecular collisions is the presence of allowed and forbidden state-to-state transitions.¹ One classic example of this sort is a collision of a closed-shell diatomic molecule with an atom. If the molecule is homonuclear, transitions are allowed only between the pairs of rotational states that are characterized by *even* values of Δj , where j is the angular momentum quantum number associated with molecular rotation.¹ For example, if the molecule is initially in the ground state $j = 0$, then only the states with $j' = 2, 4, 6, \text{etc.}$ can be excited in a collision event (we use prime to indicate the final states). Other state-to-state transitions (with *odd* Δj values) are strictly forbidden. This quantum effect is rooted in the symmetries of rotational wavefunctions^{2,3} and therefore cannot be described by the popular method of classical trajectories (often called a quasi-classical trajectory method)^{4,5} which erroneously predicts that a molecule-atom collision leads to a continuous and smooth distribution of rotational angular momentum states,⁶⁻⁸ with no difference between even and odd values of j .

In order to mimic the quantization of states, classical trajectories are binned into “boxes” associated with different values of j and, in order to mimic the quantum-mechanical selection rules, one popular trick is to make these boxes twice as wide in the case of homonuclear diatomic molecules (to associate all trajectories with allowed transitions only, those with even Δj). While this simple fix may look appropriate for perfectly symmetric diatomic molecules, its deficiency becomes obvious in several important cases of symmetry breaking.⁹⁻¹² Namely, if the molecule is heteronuclear, the transitions with odd Δj values become allowed, but usually they remain weaker than those with even Δj . One example of this sort is brought about by isotopic substitutions,^{13,14} when the molecule-atom interaction potential remains symmetric but the center of mass of the molecule is slightly shifted. Since isotopes have very similar masses, this shift is typically small and the transitions with odd Δj remain relatively weak. Another example is when the two atoms in the molecule have similar masses, such as in CO¹⁵⁻¹⁷ or NO diatomics.^{5,18,19} In all these cases there is a strong *propensity* towards even Δj values, often called a weak selection rule, but the amount of this propensity differs from one molecule to another and is hard to predict *a priori*, making the binning of classical trajectories into boxes unreliable.

Moreover, as it was demonstrated in a classic paper by McCurdy and Miller,⁹ for the molecule-atom interaction potentials that possess certain properties, the usual propensity towards

even Δj values (seen at smaller values of Δj) may turn into an *inverse propensity* towards odd values of Δj in the range of larger Δj . Clearly, this phenomenon would not be possible to describe by the binning of classical trajectories into any kind of boxes. Importantly, McCurdy and Miller demonstrated that the propensity itself, and the phenomenon of propensity inversion, both originate in the quantum interference effect.^{9,20,21} Using the method of classical S -matrix theory they showed that the interference between different scattering paths with appropriately computed phases leads to constructive (in the case of even Δj) and destructive (in the case of odd Δj) interferences, yielding a strict selection rule in the case of homonuclear molecules, and a weak selection rule in the case of slightly asymmetric molecules, when the destructive interference is not perfect.

More recently, the approach of McCurdy and Miller was employed to explain quantum interference effects observed in the experimentally measured differential cross sections.^{18,22} The beauty of their approach is that it allows the rationalization of observations using the interference between a small number of scattering paths (in fact, just four). Unfortunately, this simple framework does not permit the reproduction of the absolute values of inelastic scattering cross sections. Moreover, the classical S -matrix theory of Miller involves an expensive root search, which becomes numerically challenging in the case of molecule-molecule scattering where the number of rotational degrees of freedom becomes large. Of course, full quantum scattering calculations are expected to reproduce the interference effects discussed above, but the issue of computational cost would be even more acute. Namely, for the molecule-molecule systems, such as diatom + symmetric top, or diatom + asymmetric top, the cost of full-quantum calculations is very substantial, particularly at higher collision energies and when the diatom is heavier than H_2 .^{23–}

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In this paper we explore an alternative approach for the description of quantum effects in molecular collisions, named the mixed quantum/classical theory (MQCT).^{28–35} In our method the scattering process of collision partners (their translational motion) is described approximately using the mean-field Ehrenfest method,^{36–39} while the rotational motion of collision partners is described rigorously by solving the time-dependent Schrodinger equation, which incorporates all quantum phenomena associated with their internal states. During the last decade MQCT was applied to several molecular systems and was found to give results often comparable to the full-quantum results.^{32,40,41} One important advantage of MQCT is its affordable numerical cost. At

present time this is probably the only method capable of treating the most complex case of asymmetric-top-rotor + asymmetric-top-rotor inelastic collision, such as $\text{H}_2\text{O} + \text{H}_2\text{O}$.^{42–44} In what follows we demonstrate that MQCT captures rather accurately the weak selection rules discussed above and gives a reliable description of the propensity inversion – the classic effect of quantum interference described by McCurdy and Miller.

In this study we focus on the NO + Ar system described in a simplified way, with fine structure of the molecule neglected, the rotational states labelled by integer values of j (as if it would be a closed-shell system) and one potential energy surface employed to describe the interaction of NO with Ar atom (half-sum of the A' and A'' states, see Ref. [19]). The propensity properties of this system are well documented.^{21,45} This PES describes NO as a rigid rotor, and we follow the same approximation in the calculation of collision dynamics, keeping the bond length fixed at $r = 2.84914$ a.u. This approximation is expected to work well for moderate rotational excitations and collision energies considered in this work.

In MQCT calculations, rotational wavefunction of the molecule is represented by the expansion over the basis set of rotational eigenstates. For NO we used a basis set of 20 rotational levels ($0 \leq j \leq 19$) with all associated projection states ($-j \leq m \leq +j$), or 400 states overall in the rotational basis set. The rotational constant of the NO molecule is $B_e = 1.69611$ cm^{-1} . Matrix elements for state-to-state transitions were computed on a grid of 81 equally spaced points that covers the molecule-atom distances in the range $4 \leq R \leq 20$ Bohr. Integration over the polar angle was done using Gauss-Legendre quadrature with 40 points. The maximum impact parameter $b_{\text{max}} = 20$ Bohr was chosen that corresponds to the molecule-atom orbital angular momentum quantum number in the range $262 \leq \ell_{\text{max}} \leq 273$ for the collision energies in the range $600 \leq E_{\text{coll}} \leq 651$ cm^{-1} considered here. One trajectory was initiated for each set of the initial quantum numbers j , $|m|$ and ℓ , or about 270 trajectories for each initial quantum state j_m of NO. The equations of motion were propagated using 4th-order Runge-Kutta method with constant step-size $\Delta t = 10$ a.u. Other details of MQCT theory, including all equations, can be found in our recent papers.^{23,29,33,46}

In order to gauge the accuracy of MQCT we also carried out a set of benchmark full-quantum calculations using HIBRIDON package⁴⁷ with the same simplifying assumptions about the NO + Ar potential energy surface and the same rotational basis set, at one value of total energy

$E_{tot} = 651 \text{ cm}^{-1}$ with maximum value of $J_{tot} \leq 190$. A comparison of cross sections obtained by two methods is presented in Fig. 1 for six different initial states of the NO molecule with corresponding collision energies. From this figure one can see that agreement between the two methods is very good, without any empirical adjustments. For the initial state $j = 0$ one can see a very pronounced weak selection rule in the range of small Δj values, up to the final $j' = 7$. Namely the values of cross sections for even Δj transitions ($j' = 2, 4, 6$) are much larger than those for odd Δj transitions ($j' = 1, 3, 5, 7$), producing “zigzag” structure in the dependence of cross section (i.e., oscillations in integral cross sections versus j'). Similar zigzags are seen in all frames of Fig.1, although for the excited rotational states the range of Δj becomes somewhat shorter (i.e., only two zigzags for the initial $j = 4$ and $j = 5$ versus three zigzags for the initial $j = 0$ and $j = 1$) and appear on both sides of the elastic channel (i.e., not only for the excitation with positive Δj , but also on the quenching side with negative Δj values). This is seen very clear in the case of the initial state $j = 5$. All these properties are reliably reproduced by MQCT calculations, as gauged by comparison with HIBRIDON. On average, over all inelastic transitions included in Fig. 1, MQCT cross sections are 16% smaller than those from the full quantum calculations, with absolute RMSD of only $\sim 0.47 \text{ \AA}^2$.

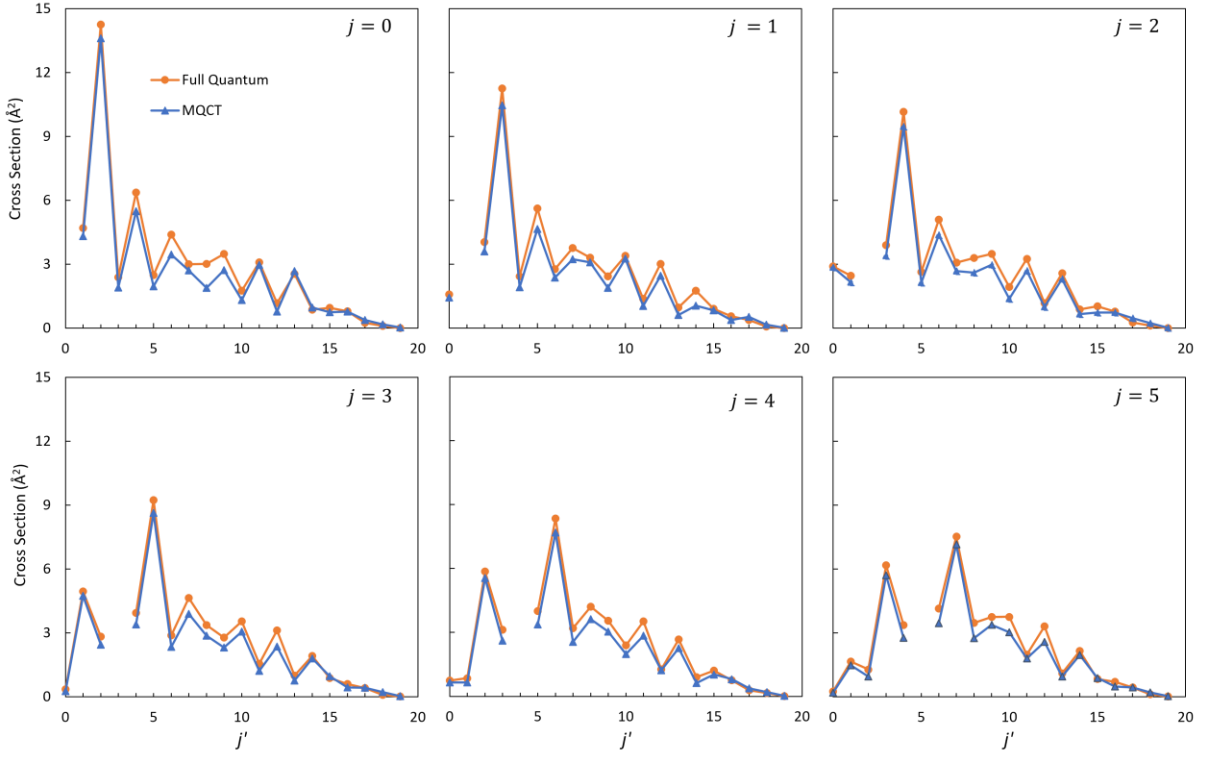


Figure 1: A comparison of inelastic state-to-state transition cross sections for NO + Ar computed by two different methods: MQCT data are blue, HIBRIDON data are orange. Different frames correspond to different initial states of NO, as indicated in each frame. Various final states are indicated along the horizontal axis. The values of elastic scattering cross sections are much larger and therefore are not included here.

One can also notice that the normal propensity towards even Δj transitions weakens as Δj is raised (which is seen as a reduction of zigzag amplitude) and eventually vanishes at some point. Namely, for the initial states $j = 0, 1, 2$ the propensity vanishes near the final $j' = 8$, for the initial states $j = 3, 4$ it vanishes near the final $j' = 9$, and for the initial state $j = 5$ it vanishes near the final $j' = 10$, yielding to the *inverse propensity* at higher values of j' . This inverse propensity is very clear in all frames of Fig. 1, with *larger* cross sections observed for *odd* Δj and *smaller* cross sections for *even* Δj transitions in the intermediate range of Δj values. Finally, the inverse propensity also weakens and vanishes at some point (near the final $j' = 15$ or 16 , depending on the initial state). Again, all these properties are accurately captured by MQCT.

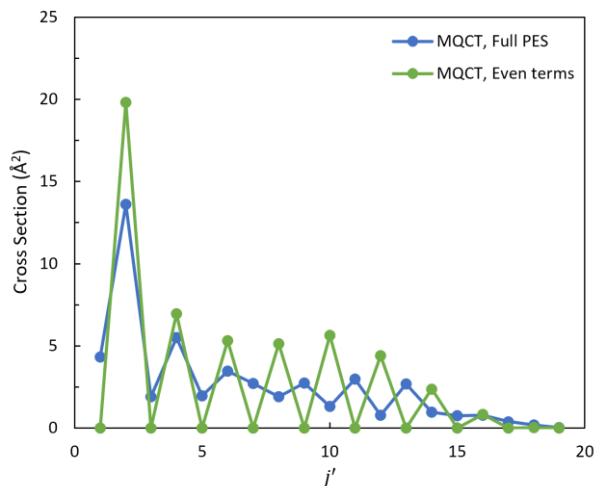


Figure 2: The effect of odd terms of PES expansion on inelastic state-to-state transition cross sections obtained by MQCT calculations for NO + Ar. The data obtained with both even and odd terms included in the PES expansion are shown in blue (full PES, the same data as in Fig. 1). The data obtained without odd terms (even terms only) are shown in green. MQCT method is used in both cases. The initial state was $j = 0$. Various final states are indicated along the horizontal axis.

McCurdy and Miller⁹ explained this behavior by the interplay between dipole ($\lambda = 1$) and quadrupole ($\lambda = 2$) terms of the molecule + atom interaction potential. Our results are entirely consistent with their explanation, except that in our case an accurate potential energy surface is employed, represented by a nine-term expansion up to $\lambda = 8$. As a numerical experiment, we tried to repeat MQCT calculations for NO + Ar with all odd λ terms removed, keeping only even expansion terms $\lambda = 0, 2, 6$ and 8 . This, effectively, makes the potential energy surface symmetric, as if the NO were a homonuclear molecule. The result is presented in Fig. 2 for the initial $j = 0$, where we see that propensity (a weak selection rule) is now replaced by a strict selection rule with cross sections being exactly zero for all transitions with odd Δj , just like in the case of a homonuclear diatomic molecule, such as O₂ or N₂. This behavior persists through a wide range of Δj values, without inverse propensity coming into play at all, until the point where cross sections become small (near $j' = 15$).

The explanation of the oscillations in integral cross section with Δj offered by McCurdy and Miller involved quantum interference between trajectories scattered by different molecular orientations.⁹ It is important to note that our approach does not allow for a similar interpretation,

simply because in the MQCT method different orientations of the molecule with respect to the impinging atomic projectile are described by a wavefunction that captures all possible orientations at once. Therefore, the afore-mentioned interference happens between different parts of the MQCT wavefunction within a matrix element, rather than between different trajectories. Consequently, in MQCT the effect of propensity comes from the matrix elements of potential coupling for state-to-state transitions, computed using the wavefunctions of the individual quantum states. In Fig. 3 we present MQCT matrix elements for the excitation of various final rotational states of the NO molecule starting from the initial state $j = 0$. Looking at these pictures one should keep in mind that at the collision energy considered here ($E_{coll} \sim 600 \text{ cm}^{-1}$) scattering partners can explore only the range of $R > 6$ bohr. This follows from the diagonal matrix element $0 \rightarrow 0$ shown in red in Fig. 3. From Fig. 3 we can clearly see that, through the entire range of molecule-atom distances, the matrix element responsible for $0 \rightarrow 2$ transition is much larger than those for $0 \rightarrow 1$ and $0 \rightarrow 3$ transitions, which explains the first (largest) zigzag in Fig. 2. Next, we can see that in the long-range part of interaction (lower frame of Fig. 3) the matrix element for $0 \rightarrow 4$ transition is larger than that for $0 \rightarrow 3$, which explains the second (smaller) zigzag in Fig. 2. But it is hard to claim **that all matrix elements** for transitions with even Δj are larger than those for transitions with odd Δj . In particular, the matrix element for $0 \rightarrow 6$ transition is smaller than those for $0 \rightarrow 5$ and $0 \rightarrow 7$, which means that, in order to explain the oscillations beyond $j' = 4$ we must invoke something else.

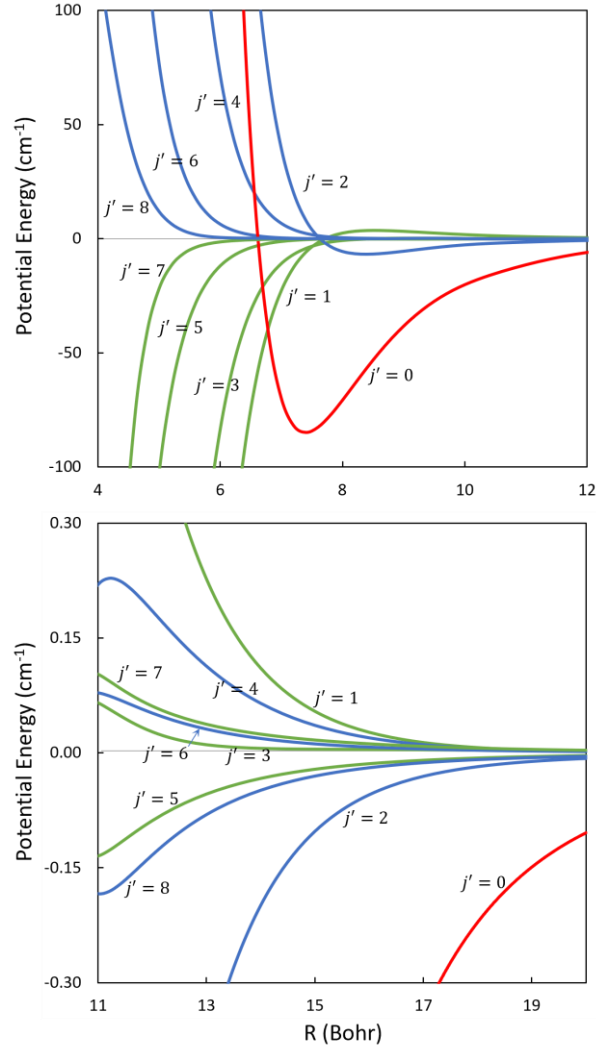


Figure 3: Radial dependence of state-to-state transition matrix elements in MQCT method, for rotational excitations in NO + Ar collision, starting from the ground state $j = 0$. Upper frame emphasizes a short-range part of the interaction potential with stronger interaction. Lower frame is focused on a long-range weak-interaction part of the process. The diagonal matrix element responsible for elastic scattering is shown in red, while blue and green colors are used for even and odd Δj transitions.

Before we discuss this “something else”, it is instructive to analyze the contributions of individual partial waves to the state-to-state transition process. In MQCT partial waves correspond to different values of molecule-atom orbital angular momentum ℓ that correlates with collision impact parameter b . In Fig. 4 we present the dependence of state-to-state transition probabilities on collision impact parameter up to $b = 15$ Bohr, starting from the initial state $j = 0$. Excitation

of the first eight states is shown in the upper frame, while the excitation of states with $j' = 9$ and above is shown in the lower frame of Fig. 4. We see, first of all, that excitations of the lower j' states occur in a way rather different from those for upper j' states. Namely, for excitation of $j' = 1, 2$ and 3 the collisions with large values of impact parameter, $b \sim 8$ Bohr and above, make very significant contributions (see upper frame of Fig. 4). Such collisions correspond to *glancing* trajectories that pass through the asymptotic range of the molecule-atom PES, without a strong encounter. These glancing trajectories also contribute some to the excitation of $j' = 4, 5$ and 6 , but can be barely identified in the dependencies for $j' = 7$ and 8 and are entirely absent in the excitation of $j' = 9$ and higher states (shown in the lower frame of Fig. 4).

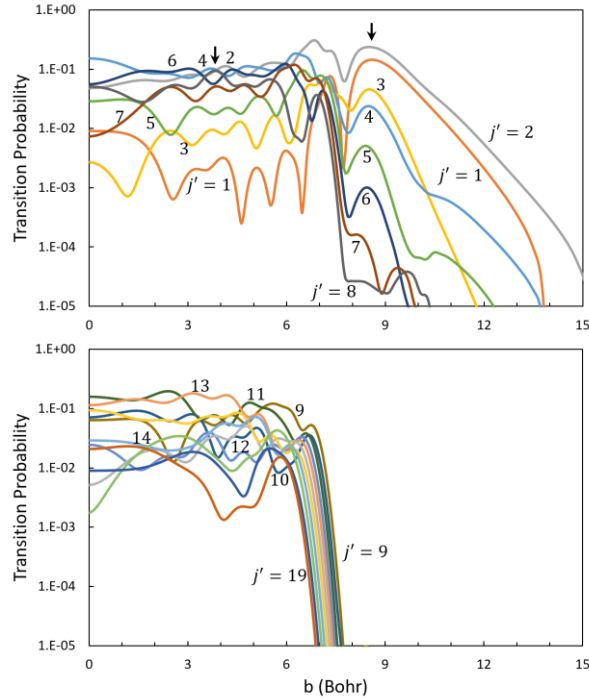


Figure 4: Dependence of state-to-state transition probabilities on collision impact parameter in MQCT calculations for rotational excitations in NO + Ar collision. Upper frame emphasizes excitation of the first eight states. Lower frame is for higher energy states. The initial state is $j = 0$. Final states are labeled by numbers and are color coded for convenience. Small arrows indicate two values of impact parameter considered in Fig. 5 below.

In contrast, for the excitations of $j' \geq 9$ only the trajectories with small impact parameters $b < 8$ Bohr are found to contribute (see lower frame in Fig. 4). These *impulsive* trajectories correspond to strong encounters. Indeed, from Fig. 4 we see that in the range $b < 8$ Bohr all

transition probabilities slightly oscillate but remain relatively high, in the range of values determined by a head on collision with $b = 0$. The conclusion is that glancing trajectories are efficient only for transitions with low Δj , while the impulsive trajectories can drive all transitions, including those with high Δj .

Manifestation of the weak selection rule (the normal propensity) is very clear in the upper frame of Fig. 4. In the inner range of impact parameter ($b < 8$ Bohr) it shows that transition probabilities for excitation of $j' = 2, 4$ and 6 are systematically larger than those for excitation of $j' = 1, 3, 5$ and 7 , which corresponds to three zigzags in Fig. 2. In the outer range ($b > 8$ Bohr) it shows that the transition probably for $j' = 2$ dominates over those for excitation of $j' = 1$ and 3 , but for $j' = 4$ and higher it decreases monotonically, without any obvious propensity. Therefore, we conclude that the major source of propensity is in the impulsive trajectories with small impact parameters.^{48,49} Glancing trajectories make significant contributions to the first zigzag in Fig. 2 (small Δj values) but they have minor influence on the following zigzags (second, third, etc.), and no influence at all on the inverse propensity at higher Δj . The inverse propensity manifests clearly in the lower frame of Fig. 4, where we see that in the inner range of impact parameter ($b < 8$ Bohr) transition probabilities for excitation of odd $j' = 9, 11$ and 13 are systematically larger than those for excitation of $j' = 10, 12$ and 14 , which is consistent with three inverse zigzags in the range $9 \leq j' \leq 14$ in Fig. 2 (and similar in all frames of Fig. 1).

At this point one should raise the question how the states in the range $j' = 9$ and above are excited (starting from $j = 0$). Matrix elements for the excitation of these states directly from the ground state are negligibly small, but the values of cross sections in the range $9 \leq j' \leq 14$ in Fig. 2 are comparable to those in the range $3 \leq j' \leq 8$. This means that the mechanism of transitions with large Δj is efficient and is different from transitions occurring directly from the ground state $j = 0$.

In order to obtain more insight into the excitation of higher j' states, we plotted the time dependence of the state populations for two typical MQCT trajectories, one with impact parameter $b = 3.7$ Bohr (small value in the inner range) and the other with $b = 8.3$ Bohr (large value in the outer range). These can be considered as “slices” through Fig. 4, indicated by two small arrows in the upper frame of that figure. Also, in these calculations for two individual trajectories, we neglected the Coriolis coupling terms that tend to “smooth” some features of the potential coupling

discussed below (although the Coriolis coupling by itself is not responsible for any critical effect). The results are presented in Fig. 5. Note that in MQCT each trajectory, in principle, can contribute to transitions into all states included in the rotational basis set, but the result of collision depends on the collision impact parameter.

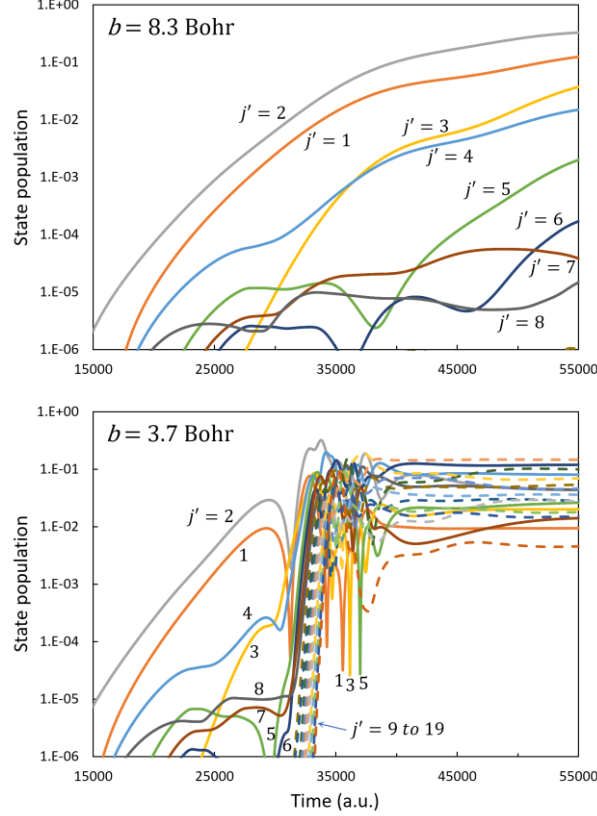


Figure 5: Time dependence of state populations for two typical MQCT trajectories with indicated values of impact parameter, during NO + Ar collision. The initial state is $j = 0$. Final states are labeled by numbers and are color coded for convenience. Dashed lines are used for rotational states with $j' \geq 9$.

The upper frame of Fig. 5 corresponds to a glancing trajectory. Populations of states with $1 \leq j' \leq 8$ grow rather monotonically during a relatively long interval of time (which, in fact, extends beyond the figure frame), without a well-defined instant of strong encounter. As discussed above, this trajectory makes some contribution to the first zigzag in Fig. 2. Indeed, in the upper frame of Fig. 5 we see clearly that the population of state $j' = 2$ exceeds those of states $j' = 1$ and 3 at all times. However, the populations of other even states (e.g., $j' = 4$ and 6) remain relatively

small and therefore cannot contribute much to the propensity observed in the total cross sections. This, again, agrees with the results presented in Fig. 4 above.

In contrast, the lower frame of Fig. 5 clearly shows the process of a short and strong encounter between the collision partners, that happens near $t \sim 35000$ a.u. At the pre-collisional stage, $t < 30000$ a.u., the time evolution of state populations is similar to that of a glancing trajectory (in the upper frame of Fig. 5). Namely, populations of states $1 \leq j' \leq 8$ increase rather monotonically, with $j' = 2$ and 1 dominating over other states. But at the beginning of the encounter, near $t \sim 31000$ a.u., the populations of states $j' = 2$ and 1 drop sharply, transferring their populations to another pair of states ($j' = 4$ and 3) that grow very quickly and synchronously with the decrease of state populations in $j' = 2$ and 1. Soon after that, the next pair of states starts gaining populations quickly ($j' = 6$ and 5) and then the next pair ($j' = 8$ and 7). As for selection rules, we can see that although the state $j' = 2$ clearly dominates in this process, the propensity for $j' = 4$ and 6 is not at all clear during the pre-collisional stage. The populations of $j' = 4$ and 6 change over time, and sometimes are larger but sometimes are smaller than the populations of their neighbors with odd $j' = 3, 5$ and 7. However, it becomes rather clear that the rotational excitation process works in a *stepladder* fashion that consists of many consecutive transitions with small values of Δj , rather than one direct process of excitation of all states, including those with large Δj values. The ability to observe this phenomenon is a very important advantage of our MQCT method which, being a time-dependent approach, permits us to see the time evolution of state populations during collision process.

Going back to the lower frame of Fig. 5, we see that as the collision progresses, more and more upper states get involved. Thus, the states with $9 \leq j' \leq 19$ (that do not have any direct transitions from $j = 0$ due to the PES expansion cut-off, see above) start receiving populations from their neighbors and come into play quickly during a short time interval between $t \sim 32000$ and 34000 a.u. Near the middle of the collision event, we observe a superposition of all states included in the basis, with populations oscillating between 0.005 and 0.1, with no obvious propensity. However, a very interesting phenomenon can be observed right after that. Namely, at the early post-collisional stage, between $t \sim 35000$ and 36000 a.u., the populations of odd states $j' = 3, 5$ and 7 drop sharply by several orders of magnitude (down to $\sim 10^{-5}$, see lower frame of Fig. 5, and the TOC image) and, although these states partially recover their populations later, they

remain small compared to populations of their neighbors with even values of j' , and this is exactly what gives rise to the propensity (and the weak selection rule). Moreover, a careful inspection of time dependencies in the lower frame of Fig. 5 indicates that the inverse propensity for the states in the range $9 \leq j' \leq 14$ is also established during the early post-collisional stage by more efficient depopulation of states $j' = 10, 12$ and 14 .

To conclude, it appears that the origin of propensity and inverse propensity is not in an efficient population of some states in the molecule, with some other states remaining unpopulated spectators. Instead, both propensity and inverse propensity occur due to efficient depopulation of some states right after the collision, when the molecule exists in a coherent superposition of many excited states that span a very broad range of angular momentum quantum numbers (basically, all states accessible at given collision energy). The ensemble of these states is populated by an efficient stepladder process of consecutive transitions with small values of Δj that takes place along the impulsive trajectories launched with small impact parameters and thus able to penetrate deeper into the interaction region.^{48,49}

This interesting story can be revealed only by a time-dependent scattering method, such as our mixed quantum-classical theory, MQCT. It is shown that MQCT reproduces both propensity and inverse propensity in a broad range of the final states of the molecule, and for various initial rotational states, with the accuracy of predicted state-to-state cross sections on the order of 15% (relative to accurate full-quantum time-independent calculations). This theory method remains computationally affordable for larger molecules, which opens new opportunities for exploration of more complicated molecular systems. **Moreover, since MQCT carries phase information and describes coherent superposition of rotational states, it has potential for the prediction of stereo-dynamical properties, such as polarization dependent differential cross sections and alignment moments.**⁵⁰⁻⁵²

CONFLICTS OF INTEREST

Authors declare no conflict of interests.

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