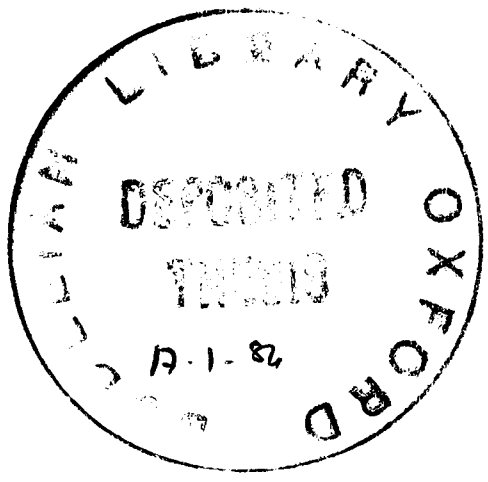
Low-energy N=1 Supergravity and N=2 Supersymmetry models



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ABSTRACT

Firstly, scalar quark mass matrices are calculated for a non-minimal SU_5 N=1 supergravity theory with realistic fermion masses. The squark mixing matrices, analogues of the Kobayashi-Maskawa matrices for quarks, are also calculated and have significant off-diagonal entries. Thus, there are non-zero flavour-changing gaugino interactions between 'up' quarks and 'up' squarks in the non-minimal model. This is in contrast to the case for minimal SU_5. It is shown that in the non-minimal model flavour-changing gluino interactions contribute to the proton decay modes $p \to \mu^+K^0$, $\bar{\nu}_\mu K^+$ at about the same rate as the mode $p \to \bar{\nu}_\mu K^+$ mediated by Wino exchange. Contributions to the K_L-K_S mass difference from flavour-changing gluino and wino interactions are small.

Secondly, for a finite N=2 globally supersymmetric theory it is shown that the set of finiteness-preserving soft operators previously derived by Parkes and West is incomplete. The complete set of 1-loop finite operators is derived by a graphical analysis, and it is shown that most of these preserve finiteness to all orders. The low-energy N=2 model of Del Aguila *et al.* is reviewed, and it is shown that new constraints on the τ-neutrino mass require an unnaturally high scale of supersymmetry breaking in the model.

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CHAPTER NOUGHT

SUPERSYMMETRY

§0.a Introduction

A supersymmetry is an invariance under transformations which mix fermions with bosons, and *vice versa*. This symmetry is not observed in the elementary particles at low energy, so it is worth asking why supersymmetric theories have aroused so much interest since the first four-dimensional supersymmetric Lagrangian was written down[1]. The reason is really two-fold : firstly, the presence of a supersymmetry softens many of the divergences that appear in a conventional theory. Exact supersymmetric theories usually have only logarithmic divergences and the superpotential is not renormalised (ignoring non-perturbative effects). A consequence of this is that large corrections to m_W of order αM_X^2 from the X boson in a supersymmetric Grand Unified theory are cancelled by the mass renormalisation $-\alpha M_X^2$ from the supersymmetric fermion partner $\tilde{X}$ of X. Thus to maintain naturally a small weak scale, one requires that $\alpha(M_{\tilde{X}}^2 - M_X^2) \lesssim (1\ \text{TeV})^2$, or in other words that there is one unbroken supersymmetry at energies above about 1 TeV.

Secondly, greater constraints are obtained by imposing extra symmetries (in this case, supersymmetries) on a Lagrangian field theory. For example, in a theory with

one supersymmetry there is the same number of fermions as bosons; even greater constraints are obtained with further supersymmetries - imposing two supersymmetries fixes the Yukawa couplings, which are free parameters of the N=1 theory.

On the other hand, to break supersymmetry at low energies may well require new adjustable parameters not present in conventional gauge theories. Some models with one global supersymmetry require an intermediate scale ($\approx 10^{10}$ Gev) of supersymmetry breaking communicated to the observable particles by small, arbitrary Yukawa couplings (this will be discussed briefly in §1.0). These models are more natural when thay are based on invariance under local, rather than global supersymmetry : when the local supersymmetry is spontaneously broken in a 'hidden' sector (which interacts with the observable fields only through gravitational couplings), the massive gauge fermion (the gravitino) which arises through the supersymmetric analogue of the Higgs effect, introduces explicit soft (i.e. dimension ≤ 3) supersymmetry breaking terms in the observable sector at about the weak scale - the complete effect of the spontaneous breakdown is carried by the mass $m_{3/2}$ of the gravitino and by another parameter A, which is of order unity.[4,5].

When a Lagrangian has two independent supersymmetries, there are greater problems in constructing realistic models. Not only does each fermion of the standard model have a scalar partner (as in N=1 theories), but each of these two has its N=2 'mirror', doubling once

again the particle content of the theory. However, for choices of the matter representations the N=2 theory is finite to all orders of perturbation theory; moreover, there are some soft terms which explicitly break the N=2 supersymmetry without destroying its finiteness. It remains to be seen whether these can be used to construct a finite N=2 model with realistic low-energy predictions.

In this thesis some aspects of N=1 and N=2 supersymmetric theories are studied. The plan of the thesis is as follows. Chapter One concerns N=1 supergravity theories. In §1.0 a brief review is given of some of the properties of N=1 locally supersymmetric models. The models considered are of the type described above, with supersymmetry spontaneously broken at about 10^{11} GeV in the hidden sector. In §1.1 the scalar quark mass matrices are calculated for a non-minimal SU_5 supergravity theory with realistic fermion masses. The squark mixing matrices, analogues of the Kobayashi-Maskawa matrix for quarks, are calculated and are shown to have significant off-diagonal entries. As a result, there are non-zero flavour-changing gaugino interactions between 'up' quarks and 'up' squarks in the non-minimal model. This is in contrast to previous results for minimal SU_5 models. In §1.2 it is shown that new supersymmetric proton decay modes may arise as a consequence of the flavour-changing gluino interactions, and in the final section, §1.3, a short discussion is given on contributions to the K_L-K_S mass difference in the non-minimal model. It is shown that these contributions are small and result in no new bounds on the gaugino masses.

Chapter Two is about N=2 theories. In §2.0 a short derivation of the N=2 multiplets and invariant action is given. In §2.1 it is shown that the set, derived in refs.[6-8], of soft supersymmetry breaking operators which are finite at one loop is incomplete; the complete set is derived by graphical analysis of the one-loop divergences, and a subset of these is shown to preserve finiteness at all orders. In §2.2 the low-energy N=2 model of Del Aguila *et al.*[11], based on the gauge group $SU_3 \times SU_4 \times U_1$ is reviewed, and two problems in it are identified : firstly, a Goldstone boson arises due to the spontaneous breakdown of a global U_1 symmetry. One possible means to avoid this problem is suggested, but is shown not to work in the model considered; secondly, it is shown that to satisfy new bounds on the ν_τ mass ($m_{\nu\tau} < 100$ eV) requires a high scale of supersymmetry breaking ($\Lambda \sim 10^9$ GeV), and it is shown finally that this introduces a hierarchy problem in the $SU_3 \times SU_4 \times U_1$ model.

In the rest of this introductory chapter is a brief review of superspace, superfields and supergraphs, and in the last section the spurion technique is introduced in order to take into account supersymmetry breaking effects when using supergraphs.

§0.b The Super-Poincaré Group

Invariance under a supersymmetry is really invariance under a larger 'space-time' group. The extra 'superspace' coordinates θ^i (i = 1..N) are anti-commuting Majorana 4-spinors which may be written in the two-spinor notation of ref.[12] as

$$\theta^i = \begin{pmatrix} \theta^i{}_\alpha \\ \bar{\theta}_i{}^{\dot{\alpha}} \end{pmatrix} \qquad (\alpha, \dot{\alpha} = 1, 2); \tag{0.1}$$

each point in superspace is labelled by a coordinate

$$z^A = (x^\mu, \theta^1 \ldots \theta^N) \tag{0.2}$$

A supersymmetric field theory is invariant under translations

$$\delta z^A = (a^\mu, \epsilon^i) \qquad i = 1..N \tag{0.3a}$$

and rotations

$$\delta z^A = i(\omega^{\mu\nu} x_\nu, d^{ij} \theta_j) \tag{0.3b}$$

of this space. The parameters

$$(a^\mu, \epsilon^i, \omega^{\mu\nu}, d^{ij})$$

of these transformations form a Grassmann manifold, so the transformations themselves form a Z_2-graded Lie group; its algebra is a graded Lie algebra A, with the usual odd-even decomposition

$$A = O \oplus E \tag{0.4}$$

and a product [, } satisfying $\{O,O\} \subset E$, etc.

It has been shown[13] that the most general supersymmetry group consistent with Poincaré invariance has the algebra generated by $\{P_\mu, J_{\mu\nu}, T_{ij}, Z_{ij}, q_i\}$ where

(i) P_μ, $J_{\mu\nu}$ are even elements and generate the Poincaré transformations

$$\delta x^\mu = a^\mu$$
$$\delta x^\mu = \tfrac{i}{2} (\omega^{\rho\sigma} J_{\rho\sigma})^\mu{}_\nu \, x^\nu \tag{0.5a}$$

(ii) $T_{ij} = -T_{ji}$ are even elements and generate the superspace rotations

$$\delta\theta^i = \tfrac{i}{2} (\alpha^{kl} T_{kl})^i{}_j \, \theta^j \tag{0.5b}$$

(iii) Z_{ij} are commuting 'central' charges and do not affect the superspace coordinates

(iv) q_i are odd elements and generate the translations

$$\delta\theta^i = \epsilon^i \tag{0.5c}$$

The 4-spinor q_i is composed of the 2-spinors Q_i, $\bar{Q}^i$. The non-vanishing products in this algebra are

$$[T_{ij}, T_{kl}] = g_{jk} T_{il} + g_{il} T_{jk} - g_{jl} T_{ik} - g_{ik} T_{jl} \tag{a}$$
$$[Q_{i\alpha}, J_{\mu\nu}] = \sigma_{\mu\nu\alpha}{}^\beta Q_{i\beta} \tag{b}$$
$$[Q_{i\alpha}, T_{jk}] = g_{ij} Q_{k\alpha} - g_{ik} Q_{j\alpha} \tag{c}$$
$$\{Q_{i\alpha}, \bar{Q}^j{}_{\dot\alpha}\} = 2\sigma^\mu{}_{\alpha\dot\alpha} p_\mu \delta^j{}_i \tag{d}$$

$$\{ Q_{i\alpha} , Q_{j\beta} \} = \epsilon_{\alpha\beta} Z_{ij} \qquad \text{(e)}$$

(0.6a-e)

together with the Poincaré algebra for P_μ, $J_{\mu\nu}$, and the conjugates of (0.6b,c,e). T_{ij} generate the isometry group of the θ-space metric g, which is taken as the unit matrix - thus T_{ij} form the algebra su(N).

The most useful representation of the algebra (0.6) is that carried by the superfields, i.e. by functions of superspace $\{z^A\}$. If $\Phi(z)$ is such a field, then (0.6) may be represented on it by taking

$$P_\mu = i\partial_\mu \qquad \text{(a)}$$

$$J_{\mu\nu} = i (x_\mu \partial_\nu - x_\nu \partial_\mu) \qquad \text{(b)}$$

$$Q_{i\alpha} = \partial_{i\alpha} - i \bar\theta_i^{\dot\alpha} \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \qquad \text{(c)}$$

$$T_{ij} = g_{ik} \theta^{k\alpha} \partial_{j\alpha} - g_{jk} \theta^{k\alpha} \partial_{i\alpha} \qquad \text{(d)}$$

$$+ g_{jk} \bar\theta_{i\dot\alpha} \bar\partial^{k\dot\alpha} - g_{ik} \bar\theta_{j\dot\alpha} \bar\partial^{k\dot\alpha} \qquad \text{(0.7a-d)}$$

where $\partial_{i\alpha} \equiv \partial/\partial\theta^{i\alpha}$ is a spinor derivative. Unlike the space-time derivative ∂_μ, $\partial_{i\alpha}$ is not covariant even in flat space, since $\{\partial_{i\alpha}, Q_{j\beta}\} \neq 0$ implies that $\partial_{i\alpha}\Phi$ is not a spinor under the representation (0.7). For this reason, the covariant spinor derivative

$$D_{i\alpha} = \partial_{i\alpha} + i \bar\theta_i^{\dot\alpha} \sigma_{\mu\alpha\dot\alpha} \partial_\mu \qquad \text{(0.8)}$$

is defined. This has the properties

$$\{ D_{i\alpha} , Q_{j\beta} \} = 0 \qquad \text{(a)}$$

$$\{ D_{i\alpha} , \bar D^j_{\dot\alpha} \} = -2i \sigma^\mu_{\alpha\dot\alpha} \partial_\mu \qquad \text{(b)}$$

$$[D_{i\alpha}\,,\,\bar{D}^{j\,2}] \;=\; -4i\sigma^{\mu}_{\alpha\dot{\alpha}}\,\bar{D}_i^{\dot{\alpha}}\,\partial_\mu\,\delta^j{}_i \qquad \text{(c)}$$

$$[D_i^{\,2}\,,\,\bar{D}^{j\,2}] \;=\; (-4iD_k\sigma^{\mu}\bar{D}^{k}\,\partial_\mu \qquad \text{(d)}$$

$$-16\,\Box\,)\,\delta^j{}_i \qquad \text{(0.9a-d)}$$

§0.c Summary of notation and results for N=1 superfields

When N=1, the anti-symmetric generators T_{ij} vanish and the superfield structure is especially simple. Some results are summarised below (all 2-spinor notation is that of Wess & Bagger[12])

(i) a chiral superfield, satisfying $\bar{D}_{\dot{\alpha}}\Phi = 0$, is written

$$\bar{\Phi}(z) \;=\; \varphi(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y) \qquad \text{(0.10a)}$$

where $y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$. Its dynamical components are a scalar,

$$\varphi(x) \;=\; \bar{\Phi}(z)\,\Big|_{\theta=0} \qquad \text{(0.10b)}$$

and a Weyl spinor

$$\psi^{\alpha}(x) \;=\; \frac{1}{\sqrt{2}}\,D^{\alpha}\,\bar{\Phi}(z)\,\Big|_{\theta=0} \qquad \text{(0.10c)}$$

If Φ_1, Φ_2, Φ_3 are chiral superfields, then from $\{D^\alpha, D^\beta\} = 0$ it follows that

$$0 \;=\; D^{\alpha}D^{2}(\bar{\Phi}_1\bar{\Phi}_2\bar{\Phi}_3) \;=\; (D^{\alpha}\bar{\Phi}_1)(D^{\beta}\bar{\Phi}_2)(D_{\beta}\bar{\Phi}_3)$$

$$+\,(D^{\alpha}\bar{\Phi}_2)(D^{\beta}\bar{\Phi}_3)(D_{\beta}\bar{\Phi}_1) + (D^{\alpha}\bar{\Phi}_3)(D^{\beta}\bar{\Phi}_2)(D_{\beta}\bar{\Phi}_1) \qquad \text{(0.11)}$$

hence, using (0.10b), the Fierz identity[14]

$$\psi^{\alpha}_{1}(\psi_2\psi_3) + \psi^{\alpha}_{2}(\psi_3\psi_1) + \psi^{\alpha}_{3}(\psi_1\psi_2) = 0 \qquad (0.12)$$

(ii) a vector superfield $V = V^{\dagger}$ is written, in the Wess-Zumino gauge[15], as

$$V(z) = -\theta\sigma^{\mu}\bar{\theta}A_{\mu}(x) + i\theta^2\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}^2\theta\lambda(x) + \frac{1}{2}D(x)\theta^2\bar{\theta}^2 \qquad (0.13)$$

where the Majorana 4-spinor formed from λ and $\bar{\lambda}$ is the gaugino, and A_μ is the gauge field.

(iii) A local gauge transformation belonging to the group G is parametrised by a Lie algebra-valued chiral superfield

$$\Lambda(z) = \Lambda^{a}(z)\, e_a$$

where $\{e_a\}$ is a basis of the gauge algebra (a = 1.. dimG). If a set $\{\Phi_\rho\}$ of chiral superfields carries the representation ρ of G then under the transformation $\Lambda(z)$,

$$\Phi_{\rho}(z) \rightarrow \rho(e^{i\Lambda(z)})\,\Phi_{\rho}(z) \qquad (0.14)$$

Invariance under these transformations is maintained by a Lie algebra-valued vector field

$$V = V^{a} e_a \qquad (0.15)$$

which transforms like

$$e^{gV} \rightarrow e^{i\Lambda^{\dagger}} e^{gV} e^{-i\Lambda} \qquad (0.16)$$

The chiral gauge field strength is defined as

$$W^{\alpha}(z) = \bar{D}^2 e^{gV} D^{\alpha} e^{-gV} \qquad (0.17)$$

where g is the gauge coupling.

§0.d The global N=1 supersymmetric action

The action S has the following three properties :

(i) S is invariant under supersymmetry transformations (0.5)

(ii) S is invariant under gauge transformations (0.14) and (0.15)

(iii) S is renormalisable.

The first requirement is met by taking the integral over all superspace of a real superfield L(z). The two translations (0.5a) and (0.5c) are just equivalent to a change in origin for the integral and therefore do not affect S. It is also left unchanged under the rotations (0.5a) and (0.5b) provided only Lorentz and SU_N invariant combinations are used for L. For N=1, only Lorentz invariance is relevant. L may be written as

$$L(z) = K(z) + P(z)\,\bar{\theta}^2 + P^{\dagger}(z)\,\theta^2 \tag{0.18}$$

where K is a real superfield and P is a chiral superfield (P is the superpotential). From the Grassmann integration rule[16]

$$\int d^8 z\, L(z) = \int d^4 x \cdot \frac{1}{16} D^2 \bar{D}^2 L(z)\,\Big|_{\theta=0} \tag{0.19}$$

the action is

$$S = \int d^4 x \left[\mathcal{D}(x) + \mathcal{F}(x) + \mathcal{F}^*(x) \right] \tag{0.20a}$$

where

$$\mathcal{D}(x) = \frac{1}{16} D^2 \bar{D}^2 K(z)\,\Big|_{\theta=0} \tag{0.20b}$$

and

$$\mathcal{F}(x) = \frac{-1}{4} D^2 P(z)\,\Big|_{\theta=0} \tag{0.20c}$$

The parts of $\mathcal{D}$ and $\mathcal{F}$ which involve only the scalar components ϕ_ρ can be written

$$-V = -\tfrac{1}{2} D^2 - |F|^2 \equiv -\tfrac{1}{2} \sum_a D^a D^a - \sum_\rho F^*_\rho F_\rho \qquad (0.21a)$$

where

$$D^a = \sum_\rho \varphi_\rho{}^\dagger \rho'(e_a) \varphi_\rho \qquad (0.21b)$$

and

$$F^*_\rho = -\frac{\partial P}{\partial \Phi_\rho}\Big|_{\theta=0} \qquad (0.21c)$$

V is the scalar potential generated by (0.20a).

The second requirement is met by forming K and P from gauge invariant combinations of the superfields. The available combinations of the gauge field V and chiral fields Φ_ρ (ρ denotes an irreducible representation of G) are

(i) $\Phi_\rho{}^\dagger \rho(e^{gV})\Phi_\rho$ for any ρ. The D-term of this (see 0.20b) gives kinetic energy terms for the scalar and spinor components of Φ_ρ.

(ii) $\Phi_\rho\Phi_\sigma$, $\Phi_\rho\Phi_\sigma\Phi_\tau$ ... etc. where ρ and σ, or ρ, σ and τ can form a gauge invariant combination. Combinations including both chiral and antichiral fields are not gauge-invariant (for any non-singlet representation) unless they are of the form (i).

The final requirement is renormalisability. In terms of the component fields, no term may have dimension greater than four. Since the dimension of $\int d^4\theta$ is 2, and that of $\bar{\theta}^2$ is -1, this means that K and P must have dimension $\leq$ 2 and 3, respectively. From (i) and (ii) above the most general gauge invariant Lagrangian is therefore defined by[17]

$$K(z) = \sum_{\rho} \bar{\Phi}^{\dagger}_{\rho} \rho(e^{gV}) \Phi_{\rho} \qquad (0.22a)$$

and

$$P(z) = \sum_{\rho\sigma} m_{\rho\sigma} \Phi_{\rho} \Phi_{\sigma} + \sum_{\rho\sigma\tau} \lambda_{\rho\sigma\tau} \Phi_{\rho} \Phi_{\sigma} \Phi_{\tau} \qquad (0.22b)$$

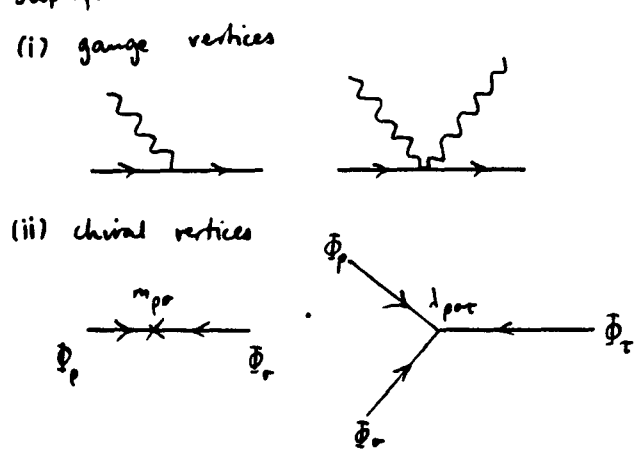
where $m_{\rho\sigma}$, $\lambda_{\rho\sigma\tau}$ vanish unless $\Phi_\rho\Phi_\sigma$ or $\Phi_\rho\Phi_\sigma\Phi_\tau$ is gauge invariant. In addition, if there are singlet fields then these may appear linearly in P :

$$P(z) \ni \sum_{\substack{\text{singlets} \\ \sigma}} a_{\sigma} \Phi_{\sigma} \qquad (0.22c)$$

§0.e N=1 supergraphs and the non-renormalisation theorem.

The most convenient Feynman rules to use for the Lagrangian (0.22) are the 'improved' Grisaru, Rocek and Siegel rules[18] in the Fermi-Feynman gauge. The vertices arising from (0.22a & b) are shown in fig 0.1. Treating

fig 0.1 Vertices generated by (0.22). An arrow entering (leaving) a vertex indicates a chiral (anti-chiral) superfield



(0.22a) as the free Lagrangian, the only non-zero free propagators are

$$\langle \bar{\Phi}^{\dagger}_{\rho}(z) \Phi_{\rho}(z') \rangle = -\Box^{-1} \delta(z-z') \qquad (0.23a)$$

and

$$\langle V(z) V(z') \rangle = \Box^{-1} \delta(z-z') \qquad (0.23b)$$

where

$$\delta(z-z') = \delta^4(x-x')\,\delta^2(\theta-\theta')\,\delta^2(\bar\theta-\bar\theta') \tag{0.24}$$

The chiral vertices (appearing in P) are converted into real vertices by using the identity

$$\int d^4x \int d^2\theta\left(-\tfrac{1}{4}\bar D^2 F\right) = \int d^8z\, F(z) \tag{0.25}$$

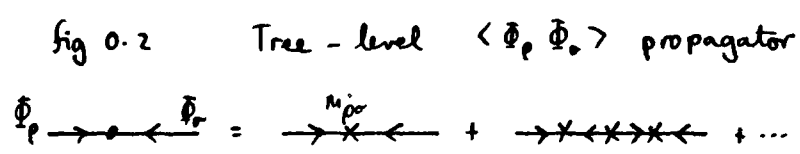
so that, from

$$\frac{\delta}{\delta\bar\Phi(z')}\bar\Phi(z) = -\tfrac{1}{4}\bar D^2\,\delta(z-z') \tag{0.26}$$

it follows that all vertices have a factor $-\frac{1}{4}\bar D^2$ ($-\frac{1}{4}D^2$) for each incoming (outgoing) chiral line, but if the vertex is chiral, one of them is omitted. Then each vertex is integrated over all superspace.

For example, the mass interaction in (0.22b) leads to massive chiral propagators at tree-level, as shown in fig 0.2.

fig 0.2 Tree-level $\langle\Phi_\rho\bar\Phi_\sigma\rangle$ propagator



Adding up these infinite series gives

$$\langle\Phi_\rho\,\bar\Phi_\sigma\rangle = \frac{-\frac{1}{4}m_{\rho\sigma}\bar D^2}{\Box(\Box-m^2_{\rho\sigma})}\,\delta(z-z') \tag{0.27a}$$

and

$$\langle\Phi_\rho^\dagger\,\Phi_\rho\rangle = \frac{-1}{\Box-\sum_\sigma m^2_{\rho\sigma}}\,\delta(z-z') \tag{0.27b}$$

The fact that all vertices are integrated over the full superspace leads directly to the non-renormalisation theorem[8] in N=1 supersymmetry. For any given diagram, all

but one of the $\int d^4\theta$ integrations can be performed by integrating by parts to remove the D's and $\bar{D}$'s from the $\delta^4(\theta-\theta')$ (occurring in each propagator). The next-to-last of these integrations is necessarily of the form

$$\int d^4x \ldots d^4y \int d^4\theta\, d^4\theta' \text{ "}D\bar{\Phi}_{\rho_1}\text{" } \cdots \text{ "}D\Phi_{\rho_m}\text{" } \delta(\theta-\theta')\left[D\cdots\bar{D}\cdots\delta(\theta-\theta')\right] \cdot A(x\ldots y) \tag{0.28}$$

for a graph with only chiral external fields $\Phi_{\rho 1}$, ... $\Phi_{\rho m}$, where "DΦ" means either Φ or $D^\alpha\Phi$ or $D^2\Phi$. By using the commutation relations (0.9), the D's and $\bar{D}$'s in the square bracket [...] can be reduced to at most a product of four; the only non-zero contribution is then from

$$\delta(\theta-\theta')\left[D^2\bar{D}^2\delta(\theta-\theta')\right] = 16\,\delta(\theta-\theta') \tag{0.29}$$

so (0.27) becomes

$$\int d^4x \ldots d^4y \int d^4\theta \text{ "}D\Phi_{\rho_1}\text{" } \cdots \text{ "}D\bar{\Phi}_{\rho_m}\text{" } A'(x,\ldots y)$$
$$= \int d^4x \ldots d^4y\, \frac{1}{16} D^2\bar{D}^2 \text{ "}D\bar{\Phi}_{\rho_1}\text{" } \cdots \text{ "}D\bar{\Phi}_{\rho_m}\text{" }\Big|_{\theta=0} A'(x,\ldots y) \tag{0.30}$$

which vanishes because all the Φ's are chiral - in other words none of the terms $m_{\rho\sigma}\Phi_\rho\Phi_\sigma$, $\lambda_{\rho\sigma\tau}\Phi_\rho\Phi_\sigma\Phi_\tau$ in the superpotential receives any renormalisation, nor is any other chiral vertex generated at any order of perturbation theory. For this reason, it is now usually thought that any Grand Unified theory must have a supersymmetry unbroken above a scale of about 1 Tev. If this is not the case, then interactions involving heavy particles with masses at the unification scale or higher will force scalar masses up to this scale. For example, in an SU_5 theory the X-boson mass contributes a large mass renormalisation to the Higgs scalar masses, see fig 0.3. There must, however, be light scalar Higgs in order to

acquire v.e.v.s at the weak scale (this is the technical side

fig 0.3 Scalar mass renormalisation graph from X-boson mass

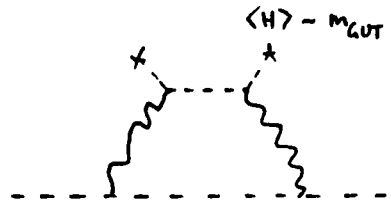
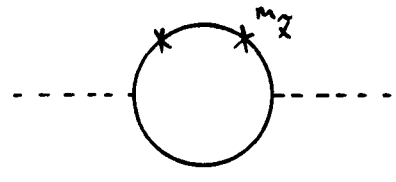


fig 0.4 Contribution from $\tilde{X}$-gaugino which (almost) cancels fig 0.3



of the gauge hierarchy problem[19]). Supersymmetry provides the answer to this problem, for the X-gaugino also has a large mass,

$$m^2_{\tilde{X}} \sim m^2_X + (1\ TeV)^2$$

and almost cancels the contribution of the X-boson (see fig 0.4).

§0.f Spontaneous supersymmetry breaking and spurion fields

From the supersymmetry algebra (0.6)

$$\{ Q_\alpha , \bar{Q}_{\dot\alpha} \} = 2\sigma^\mu_{\alpha\dot\alpha} p_\mu$$

it follows that

$$p_0 = \tfrac{1}{4} (\bar{Q}_i Q_1 + Q_1 \bar{Q}_i + \bar{Q}_i Q_2 + Q_2 \bar{Q}_i)$$

so supersymmetry is only spontaneously broken,

$$Q_\alpha |0\rangle \neq 0$$

if the vacuum energy $p_0|0\rangle$ is non-zero. That is, the scalar potential (0.21a)

$$V = \tfrac{1}{2} D^2 + |F|^2$$

has a non-zero minimum. The resulting non-zero v.e.v.s $\langle D^a \rangle$ and $\langle F_\rho \rangle$ produce scalar masses through terms in (0.22) such as

$$\sum_{\rho\sigma\tau} \lambda_{\rho\sigma\tau} \varphi_\rho \varphi_\sigma \langle F_\tau \rangle , \quad \varphi^\dagger_\rho \rho'(t_a) \varphi_\rho \langle D^a \rangle , \ etc. \qquad (0.31)$$

and thereby remove the fermion-boson mass degeneracy. This

Fig 0·5 Supergraph contributing to mass renormalisation

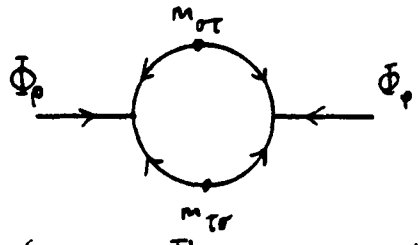
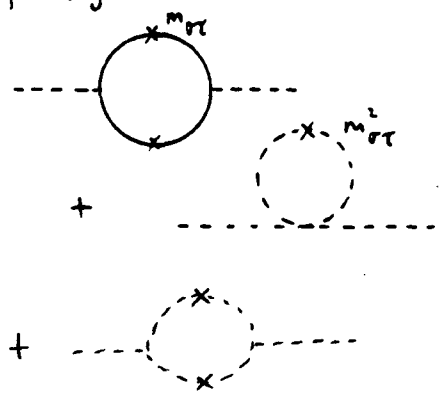


Fig 0·6 The component graphs of fig 0·5



mass difference, in turn, invalidates the results of the non-renormalisation theorem. The supergraph fig 0.5, which in the absence of supersymmetry breaking is the one-loop contribution to mass renormalisation, vanishes due to the non-renormalisation theorem; this can also be seen from the component graphs fig 0.6;

the p^2-independent parts cancel. When the masses in the two graphs are different, however, this cancellation no longer occurs.

In order to continue to use the supergraph formalism, the supersymmetry-breaking v.e.v.s can be taken into account by treating them as x-independent 'spurion' superfields

$$N_\rho(\theta) = \langle F_\rho \rangle \theta^2 \equiv n_\rho \theta^2 \qquad (0.32a)$$

and

$$U(\theta) = \sum_{\hat{}} \langle D^{\hat{}} \rangle \theta^2 \bar{\theta}^2 \equiv u \theta^2 \bar{\theta}^2 \qquad (0.32b)$$

represented in a supergraph by dotted lines, as in fig 0.7. They contribute to the scalar mass renormalisation by graphs such as the one in fig 0.8. Thus, the complete effect of the supersymmetry breaking on (say) the chiral propagator $\langle \Phi_\rho^\dagger \Phi_\rho \rangle$ is to change it to the series shown in fig 0.9; in momentum space the series is

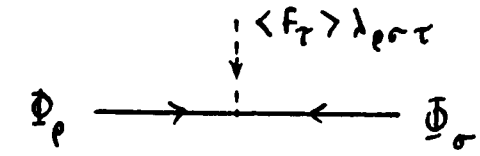
$$\langle \bar{\Phi}^\dagger(-p,\theta_1)\,\bar{\Phi}(q,\theta_2)\rangle = \tfrac{1}{2}\left[\frac{1}{p^2+m^2-u-n} + \frac{1}{p^2+m^2-u+n}\right]$$

$$(2\pi)^4\,\delta(p-q)\int d^4\theta\left[\frac{\bar{D}^2}{-4}\,\delta^4(\theta_1-\theta)\right]\delta^4(\theta)\left[\frac{D^2}{-4}\,\delta^4(\theta-\theta_2)\right] \qquad (0.33)$$

where m is a supersymmetric mass term.

fig 0·7 Spurion Interactions

(i) chiral spurion insertion



(ii) real spurion insertion

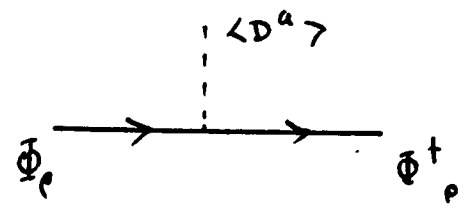


fig 0·8 A spurion contribution to mass renormalisation

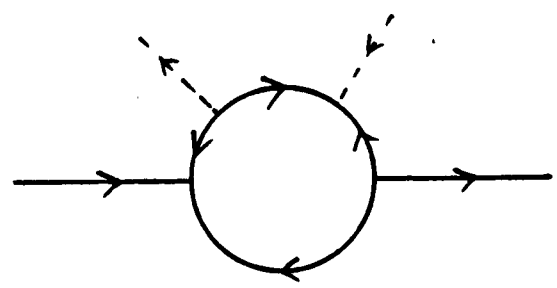
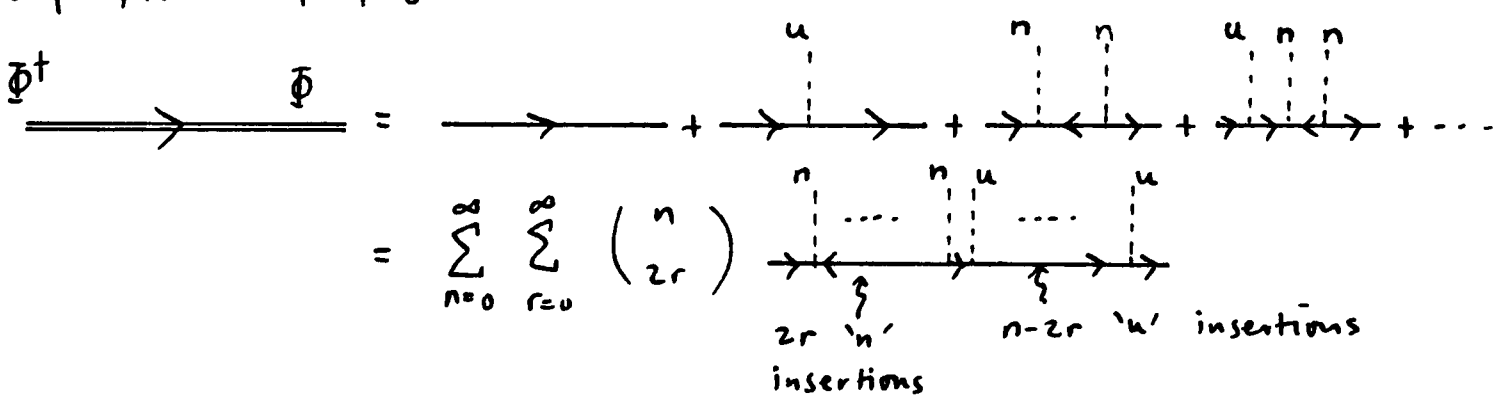


fig 0·9 All spurion scalar mass insertions in a superfield propagator $\langle \bar{\Phi}^\dagger \Phi\rangle$



This technique may be extended to explicit supersymmetry breaking terms as well; for example, a gaugino mass (which can also arise radiatively through spontaneous breaking[20]) may be written as

$$\frac{m_\lambda}{4}\,\mathrm{tr}\,\lambda^2 = \frac{-m_\lambda}{64\,g^2}\int d^2\theta\left[\theta^2\,\mathrm{tr}\,W^\alpha W_\alpha\right] \qquad (0.34)$$

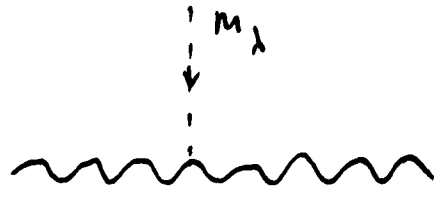
which can be drawn as a spurion insertion (fig 0.10i). The sum of any odd number of these (fig 0.10ii) forms a parity-changing massive gaugino propagator,

$$\langle W^\alpha(p,\theta_1)\,W_\beta(q,\theta_2)\rangle = \frac{\frac{1}{8}m_\lambda}{p^2(p^2+m_\lambda^2)}\int d^4\theta\left[D^\gamma\,\delta^4(\theta_1-\theta)\right].$$

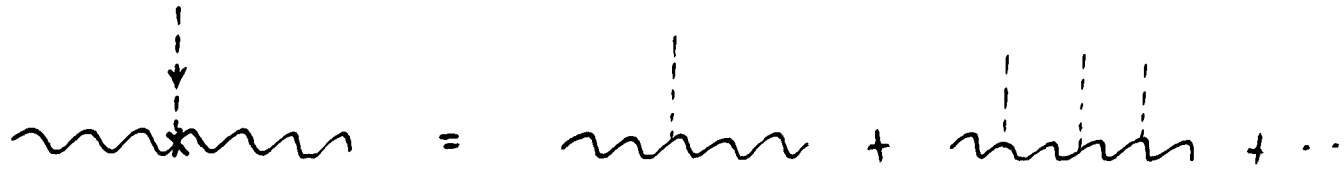
$$\theta^2\left[\bar{D}^2 D_\gamma\,\delta^4(\theta-\theta_2)\right](2\pi)^4\,\delta(p-q)\,\delta^\alpha{}_\beta \qquad (0.35)$$

fig. 0.10 massive gaugino lines

(i) a single insertion



(ii) the full massive line



CHAPTER ONE

N=1 SUPERGRAVITY MODELS

SECTION 1.0 N=1 supergravity

In this chapter scalar masses and proton decay modes are calculated for Grand Unified theories with a spontaneously broken local supersymmetry. These theories have several advantages over globally supersymmetric models : most obviously, they incorporate gravitational interactions (this will be explained further in §1.0.a); they can also accomodate a zero cosmological constant, and the spontaneous symmetry breakdown produces naturally a good low-energy mass spectrum, and can also be responsible for SU_{2L} breaking at the weak scale.

Why are these features not also present in global supersymmetry models? Firstly, the spontaneous breaking of global supersymmetry arises as a result of non-zero vacuum energy (as mentioned in §0.f) and the vacuum energy is proportional to the cosmological constant. Secondly, although realistic low-energy masses can be generated, it requires the introduction of extra fields and more couplings between them. The reason for this is the tree-level mass formula[21,22]

$$\mathrm{Str}\, M^2 \equiv \sum_J (-1)^{2J}(2J+1)\, m_J^2 = \frac{g_1^2}{2} \langle D \rangle \mathrm{Tr}\, Q \qquad (1.0.1)$$

where m_J is the mass matrix for the spin J particles, $\langle D \rangle$ is

a supersymmetry-breaking D-term derived from U_1 factors in the gauge group, and Q the corresponding charge matrix. Theories with a non-zero TrQ and $\langle D \rangle$ have U_1 anomalies unless a complicated multiplet structure is used[22,23,24]. Furthermore, such theories do not have a semi-simple group - extra U_1 factors are required[25]. If, on the other hand, most of the supersymmetry breaking arises from $\langle F \rangle$ terms then the supertrace (1.0.1) vanishes - or in other words the average fermion mass equals the average boson mass. In a realistic model (where all scalars have a mass greater than about 20 GeV) large radiative corrections must therefore be found to escape from this tree-level relation.

In practice, this implies postulating a set of 'hidden' superfields coupled weakly to the observable fields[30,31], with a large supersymmetry breaking scale ($\langle F \rangle$ ~ 10^{10} GeV) to give a sufficiently large mass to the scalars. An example of this sort of scheme is provided by the gauge non-singlet O'Raifeartaigh mechanism[26] with chiral fields X (a gauge singlet) and Y (an adjoint of SU_5) coupled to the SU_5-breaking Higgs Σ by the superpotential[27]

$$\lambda_1 \mathrm{tr}\, Y\Sigma^2 + \lambda_2 (\mathrm{tr}\,\Sigma^2 - \Lambda^2) X \qquad (1.0.2)$$

The minimum of the scalar potential generated by this breaks both the gauge symmetry :

$$\langle \Sigma \rangle = \begin{pmatrix} -2 & & & & \\ & -2 & & & \\ & & -2 & & \\ & & & 3 & \\ & & & & 3 \end{pmatrix} \qquad (1.0.3)$$

and the supersymmetry

$$-\langle F_Y^* \rangle = \lambda_1 \left[\langle \Sigma \rangle^2 - \tfrac{1}{5} \mathrm{tr} \langle \Sigma \rangle^2 \right]$$

$$-\langle F_x^* \rangle = \lambda_2 \left[\kappa \langle \Sigma \rangle^2 - \Lambda^2 \right] \qquad (1.0.4)$$

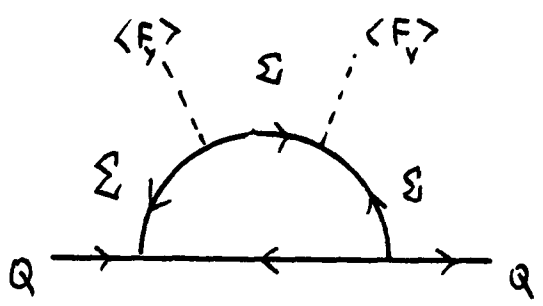


fig 1.0.1. Radiative scalar mass generated by $\langle F \rangle \neq 0$

and allows the scalars to develop radiative masses through graphs like the one in fig 1.0.1. However, because of the large mass m_G in (1.0.4), the supersymmetry scale Λ must be at least 10^9 GeV in order to generate sufficiently large scalar masses.

For a theory with spontaneously broken local supersymmetry, this sort of mass generating scheme appears much more naturally - the hidden sector fields couple only through the gravitational interaction, and give a common tree-level mass to all scalars[28], of between 200 and 300 GeV; furthermore, this explicit supersymmetry-breaking mass may give rise, at the same time, to weak breaking at about the same scale. The rest of this section is a review of how these properties come about.

§1.0.a Local supersymmetry[29]

The most general gauge invariant renormalisable Lagrangian with a global supersymmetry for the set $\{X^i\}$ of chiral N=1 superfields is (0.22)

$$\int d^4\theta \left[X_i^\dagger (e^{gV})^i{}_j X^j + P\bar{\theta}^2 + P^\dagger \theta^2 \right] \qquad (1.0.5a)$$

where the superpotential P is defined as

$$P = a_i X^i + m_{ij} X^i X^j + \lambda_{ijk} X^i X^j X^k \qquad (1.0.5b)$$

This was constructed to be invariant under global transformations $\exp(\epsilon Q + \bar{\epsilon}\bar{Q})$ of the N=1 superPoincaré group, but (as usual) when the paramter ϵ of the transformation is allowed to depend on position, the kinetic part

$$\int d^4\theta \, x^\dagger{}_i \, (e^{gV})^i{}_j \, x^j$$

picks up terms involving the derivatives $\partial_\mu \epsilon(x)$. To restore invariance under local transformations, an extra gauge field must be introduced, and as the theory is supersymmetric, this gauge field contains both boson and fermion components. Now, the even part of the superPoincaré group contains the Poincaré group, so one would expect that the local supersymmetry group contains in its even part the group of local Poincaré transformations, i.e. general coordinate transformations. This is the symmetry group of general relativity, so it is not surprising that the boson part of the extra gauge field can be identified as the graviton $g_{\mu\nu}$. Its spin 3/2 partner ψ_μ is the gravitino.

§1.0.b The scalar potential in supergravity

The locally supersymmetric action which generalises (1.0.5) is now obtained by adding to it terms involving $g_{\mu\nu}$ and ψ_μ to cancel any change under local supersymmetry transformations. The form of these additions is complicated, and the complete supergravity action $\mathcal{L}_{sg}$ is not renormalisable, although this is not surprising as the Einstein action on its own is non-renormalisable.

To be more specific, the most general scalar

potential of $\mathcal{L}_{sg}$ is

$$V = e^{-G}\left[G_i (G^{-1})^i{}_j G^j - 3\right] + \tfrac{1}{2} f^{-1}_{ab} D^a D^b \qquad (1.0.6)$$

where G is the Kähler potential, a real gauge invariant function of the scalar components $x^i = X^i|_{\theta=0}$ and of their conjugates. It can be written

$$G = 3 \log(-\tfrac{1}{3}\phi) - \log \tilde{P}^\dagger \tilde{P} \qquad (1.0.7)$$

where

$$\tilde{P} = P\big|_{\theta=0} \qquad (1.0.8)$$

and the kinetic term for the scalar fields is

$$\int d^4\theta\, \phi\left(X_i^\dagger (e^{gV})^i{}_j X^j\right) \qquad (1.0.9)$$

The affixes i,j denote differentiation,

$$G_i = \frac{\partial G}{\partial x^i}$$

$$G^i{}_j = \frac{\partial^2 G}{\partial x^j \partial x^*{}_i} \qquad (1.0.10)$$

so it follows from (1.0.9) that the kinetic term for the scalar fields is

$$D_\mu x^*_i G^i{}_j D^\mu x^j \qquad (1.0.11)$$

The remaining term in (1.0.6) is the analogue of the D-term in global supersymmetry.

A significant feature of the potential (1.0.6) is that, unlike the global supersymmetry potential, there may exist minima where some of the scalars acquire v.e.v.s and yet the vacuum energy - and hence the cosmological constant - vanishes : for example, in models with minimal

kinetic coupling ($G^i{}_j = \delta^i{}_j$, $f^{-1}{}_{ab} = \delta_{ab}$), the potential (1.0.6) at its minimum is

$$V_{min} = \exp\left(\sum |v_i|^2/M^2\right)\left\{ \left| \frac{\partial \tilde{P}_0}{\partial v_j} + \frac{v_j^* \tilde{P}_0}{M^2} \right|^2 - \frac{3|\tilde{P}_0|^2}{M^2} \right\} + \frac{1}{2} D_0^2 \qquad (1.0.12)$$

where $M = M_{pl}/\sqrt{8\pi}$, and $v^j = \langle x^j \rangle$. The term $-3|\tilde{P}_0|^2/M^2$ may cancel the positive F- and D-terms.

§1.0.c The hidden sector

Low-energy supersymmetry models with only F-type spontaneous supersymmetry breaking satisfy, at tree-level, the mass formula (1.0.1)

$$Str\, M^2 = 0$$

attempts to avoid this restriction require raising the supersymmetry breaking scale so that radiative corrections to it become significant; then to maintain the gauge hierarchy, as discussed in the introduction this breaking must take place in a sector very weakly coupled to the observable fields. Such a coupling is naturally provided by supergravity theories, for if no Yukawa interactions take place between a 'hidden' sector and the matter fields then their only interaction is by gravity, and therefore suppressed at low energy by $1/m_{pl}$. Thus, the Lagrangian at low energies contains effectively no interactions with the hidden sector, for these are negligible; however, it may contain residual effects from any large v.e.v.s in the 'hidden' scalars. With this idea, it is possible to construct a hidden sector such that the effective Lagrangian for the observable fields takes a particularly useful form : it has one part which has global supersymmetry and is

independent of the hidden fields, and another part containing explicit supersymmetry breaking terms induced by large v.e.v.s in the hidden sector.

For example, suppose the superpotential (1.0.5b) takes the form

$$P' = Q(z^p) + P(x^i) \tag{1.0.13}$$

where Q depends only on the superfields Z^p (p = 1,2 ...) belonging to the hidden sector, and P is the usual superpotential for the observable fields. If Q is such that the scalar components z^p acquire v.e.v.s[30,32]

$$\langle z^p \rangle = a^p M$$

with

$$\langle Q \rangle = m M^2$$

$$\langle \frac{\partial Q}{\partial z^p} \rangle = a^*_p m M \tag{1.0.14}$$

then the scalar potential (1.0.6) becomes[30], neglecting any terms from P which are suppressed by powers of $1/m_{pl}$,

$$V_{eff} = \sum_i \left\{ \left| \frac{\partial \tilde{P}}{\partial x^i} \right|^2 + m_{3/2}^2 |x^i|^2 + m_{3/2} Re\left[x^i \frac{\partial \tilde{P}}{\partial x^i} + (A-3)\tilde{P} \right] \right\} \tag{1.0.15a}$$

where $m_{3/2}^2 = m \exp\left(\frac{1}{2} \sum_p |b_p|^2 \right)$

and

$$A = \sum_p b^*_p (a^p + b^p) \tag{1.0.15b}$$

The non-scalar part of the effective Lagrangian is (with the minimal kinetic coupling $G^i{}_j = \delta^i{}_j$) just the supersymmetric action (1.0.5a) for the observable fields[4,5,31]. Thus, the gravity interactions are equivalent to a chiral spurion insertion $N(\theta) = Am_g\theta^2$, with the coupling

$$\int d^2\theta\ P(\theta)\ N(\theta) \qquad (1.0.16a)$$

and a scalar mass insertion $U(\theta) = m_g{}^2\theta^2\bar{\theta}^2$ occurring in the term

$$\int d^4\theta\ X^\dagger{}_i\ (e^{gV})^i{}_j\ X^j\ U(\theta). \qquad (1.0.16b)$$

The gravitini couple to the scalars through the term

$$e^{-G/2}\ \bar{\psi}_\mu\ \gamma^{\mu\nu}\ \psi_\nu$$

so that the dominant part of the gravitino mass is just the expression

$$m_{3/2} = m \exp\left(\tfrac{1}{2}\textstyle\sum |b_p|^2\right);$$

this mass arises by the analogue of the Higgs effect : the massless Goldstone fermions of the spontaneous symmetry breakdown are eaten by the gravitino to give it a mass[33,34].

In summary, a hidden sector with v.e.v.s of the form (1.0.14) produces an effective Lagrangian at energies below the Planck scale consisting of globally supersymmetric gauge and Yukawa terms for the observable fields, together with explicit supersymmetry breaking terms : a common mass insertion $m^2{}_{3/2}$ for the scalars, and cubic scalar interactions parametrised by A (1.0.15). These two parameters are the only visible trace of the hidden sector structure.

§1.0.d Interpreting the effective Lagrangian

The scalar potential (1.0.15) and the remaining supersymmetric part of the Lagrangian were derived by neglecting the observable field interactions that were

suppressed by $1/m_{pl}$, with the assumption that all mass scales in the observable sector are much less than m_{pl}. It is hoped that by doing this all the effect of the gravity interactions, which become strong at the Planck scale, and of the interactions with the hidden sector at lower energies is transmitted by the constants $m_{3/2}$ and A (some further justification for this hope is given below). Quantum corrections in such a theory come from integrating over all momenta up to the Planck scale, so that the logarithmic divergences in (1.0.15) and (1.0.6) produce a power series in $\alpha \log(m^2_{pl}/\mu^2)$ for corrections to the renormalised parameters at the scale μ^2 : this implies that all parameters renormalised at the Planck scale take approximately their tree-level values, for the corrections are small at $\mu^2 = m^2_{pl}$. By taking into account the variation with scale of the masses and Yukawa couplings in (1.0.6) and (1.0.15). the effective N=1 supergravity Lagrangian renormalised at a scale $\mu^2 < m^2_{pl}$ takes the form[4]

$$\begin{aligned} \mathcal{L}_{eff} = & \int d^4\theta \{ X_i^\dagger (e^{gV})^i{}_j X^j + \bar{\theta}^2 P + \theta^2 P^\dagger \} \\ & + m_{3/2} \int d^2\theta \left[\sum_i A_i a_i X^i + \sum_{ij} A_{ij} m_{ij} X^i X^j + \sum_{ijk} A_{ijk} \lambda_{ijk} X^i X^j X^k \right] \theta^2 \\ & - \tfrac{1}{2} \sum_i \tilde{m}_i^2 |x^i|^2 - \tfrac{1}{2} \tilde{m}_\lambda \lambda^2 + \text{h.c.} \end{aligned} \qquad (1.0.17)$$

where at $\mu^2 = m^2_{pl}$ the renormalised parameters take the values

$$A_i = A_{ij} = A_{ijk} = A \qquad (1.0.18a)$$

$$\tilde{m}_i^2 = m^2_{3/2} \qquad (1.0.18b)$$

$$\tilde{m}_\lambda = 0 \qquad (1.0.18c)$$

It should be noted that it is not entirely clear that loop corrections from supergravity to the effective

potential can be ignored. The dominant part of such a correction[4] comes from the region above the Planck mass where gravity is strong, and the interactions of the graviton and gravitino are independent of the matter fields on which they act - they respect a U_n symmetry, where n is the number of chiral fields. Their contributions to the effective action are therefore the most general non-renormalisable terms invariant under this symmetry. At low energy one may neglect any derivative terms, as external momenta are less than m_{pl}; the most general scalar potential which arises like this is[4]

$$V = \mu_1^2 \sum_i |x^i|^2 + \mu_2 \sum_i x^i \frac{\partial \tilde{P}}{\partial x^i} + \mu_3 \tilde{P} + h.c. \qquad (1.0.19)$$

where μ_i are masses of order $m_{3/2}$ - that is, the effect of the quantum gravitational couplings is to alter the coefficients of (1.0.17), without changing its form. For this reason the mass parameter $m_{3/2}$ is written instead

$$m_{3/2} = a\, m_g \qquad (1.0.20)$$

where m_g is a mass of the same order. It is the similarity between (1.0.19) and (1.0.17) which justifies neglecting gravitational interactions at low energy.

§1.0.e The renormalisation group equations and mass effects.

Renormalisation group invariance can be used to find the renormalised parameters of the supergravity Lagrangian (1.0.17) at low energy from their tree-level values (1.0.18), interpreted here as values at the Planck scale. For

example, consider the supergravity standard model Lagrangian[5]

$$\mathcal{L} = \text{kinetic terms} + U^c_i \lambda^i_j Q^j H + D^c_i K^i_j Q^j H' + m HH'$$
$$+ m_g \int d^2\theta \left[\sum_{ij} U^c_i \lambda^i_j A^i_j Q^j H + D^c_i K^i_j B^i_j Q^j H' \right] \theta^2 + h.c.$$
$$-\tfrac{1}{2} m_g^2 \int d^4\theta \left[U^c_i U^{c\dagger i} + D^c_i D^{c\dagger i} + Q^\dagger_i Q^i + H^\dagger H + H'H'^\dagger \right] \delta(\theta) \quad (1.0.21)$$

where U^c_i, D^c_i are the charge conjugate 'up' and 'down' quarks (i = 1..3), Q^i is the quark doublet, and H, H' are Higgs superfields.

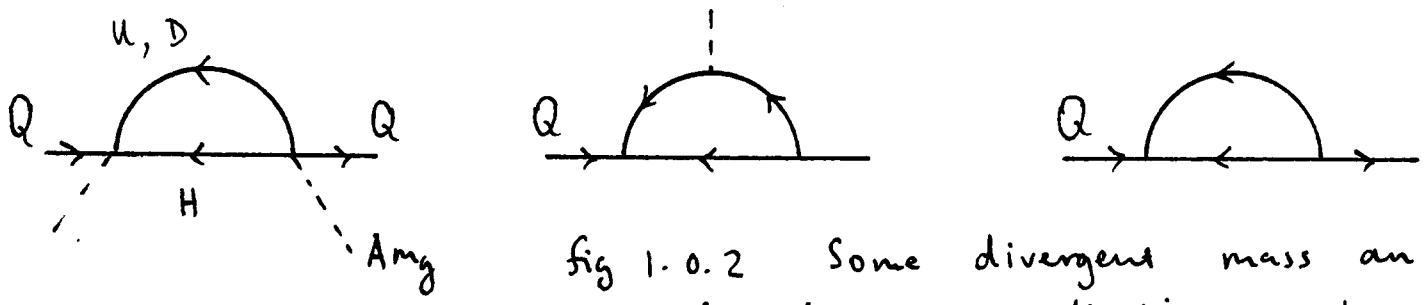


fig 1.0.2 Some divergent mass and wave-function renormalisation graphs for standard model

Some of the divergent graphs of this theory are shown in fig 1.0.2. Their contribution to the effective action is of the form

$$-\tfrac{1}{2} \partial_\mu \tilde{q}^\dagger_i (\delta z)^i_j \partial^\mu \tilde{q}^j - \tfrac{1}{2} \tilde{q}^\dagger_i (\delta m^2)^i_j \tilde{q}^j$$

where δz and δm^2 can be written

$$\delta z = \frac{a}{\epsilon} + \delta z_{fin}$$
$$\delta m^2 = b/\epsilon + \delta m^2_{fin}$$

in terms of the dimensional regularisation parameter $\epsilon = 2 - d/2$.

When the Lagrangian is renormalised to remove these divergences it can be rewritten in terms of the renormalised fields $q_r = (1-\delta z)^{1/2} q$, and it includes a mass term

$$\tilde{q}^\dagger_{r\,i} m^2_{r\,j}{}^i(\mu) \tilde{q}_r{}^j$$

In the minimal subtraction scheme[35], the bare mass m^2_b is therefore related to m^2_r by

$$m^2_b = (1-\delta z)^{-1/2}\, m^2_r(\mu)\,(1-\delta z)^{-1/2} + \delta m^2$$

$$= m^2_r(\mu) - \frac{1}{2\epsilon}\{a, m^2_r(\mu)\} + \frac{b}{\epsilon} + \frac{1}{\epsilon^2} \cdots \qquad (1.0.22)$$

As the bare mass is independent of the renormalisation scale μ, comparing inverse powers of ϵ in this equation gives, after differentiating with respect to μ, the relation

$$\mu \frac{\partial}{\partial \mu} m^2_r \Big|_{\epsilon=0} = \lambda^i{}_j \frac{\partial}{\partial \lambda^i{}_j} b + \frac{1}{2}\left\{\lambda^i{}_j \frac{\partial a}{\partial \lambda^i{}_j}, m^2_r\right\} \qquad (1.0.23)$$

If one were to try to find $m^2_r(\mu)$ without using this equation, that is by calculating the individual diagrams (to a given order) with the external momentum $p^2 = \mu^2$, then one would find corrections as a power series in a term of the form

$$\frac{\lambda^2}{8\pi^2}\int_0^1 dx \log \frac{m^2(\Lambda) - p^2 x(1-x)}{4\pi\Lambda^2} \qquad (1.0.24)$$

where M is roughly the largest mass in the loop(s) renormalised at scale Λ. When $\mu^2 \gg M^2$, this is a scale dependent contribution; the RGE resums the power series in (1.0.24), which may be a large number. However, if the renormalisation scale is taken below M^2, then (1.0.24) ceases to depend on $\mu^2 = p^2$ (to order m^2/M^2) and the effect of the graphs which contributed to (1.0.24) must be dropped from the Renormalisation Group equations. This is an important consideration in Grand Unified theories, for the heavy gauge bosons and Higgs triplets all have a mass of order the unification scale (see §1.1.e), and hence do not contribute to the RGEs below that scale.

§1.0.f SU_2 breaking in supergravity.

A fortunate property of the supersymmetric standard model (1.0.21) is that SU_2 breaking can be incorporated in it naturally. Early supergravity models introduced for this purpose another small mass scale μ of order m_W *via* a term $-\mu^2 h^\dagger h$ to allow the Higgs' to acquire a v.e.v. However, there is already present in all supergravity models a mass parameter of this order, viz. m_g. It can be shown[32] that a requirement for having a tree-level Higgs v.e.v. proportional to m_g is that the supergravity parameter A (1.0.15b) satisfy

$$A \gtrsim 3 \qquad (1.0.25)$$

Since, at the Planck scale, Am_g is the strength of *all* the trilinear scalar couplings, such a large value for A may also drive v.e.v.s for the scalar partners of matter fields. For example, the condition that the selectron not acquire a v.e.v. through this means[36] (and therefore break U_1 spontaneously) is just the opposite of (1.0.25), i.e. $A \lesssim 3$.

The resolution of these difficulties is for the Higgs v.e.v. to arise through radiative corrections. The RGE (1.1.23) for the Higgs mass shows that it decreases as the renormalisation scale is lowered from the Planck mass towards the weak scale, so if the radiative corrections are sufficient to make it negative, the Higgs field will be driven to acquire a v.e.v. Furthermore, because of the ratio of coefficients 1 : 2 : 2 : 3 : 3 in (1.1.23a-e), the Higgs masses are the first to become negative. Models have

been constructed[5,37,38] in which the correct SU_2 breaking takes place *via* this mechanism. For such models, the supersymmetric mass for the Higgs doublets must be kept small (i.e. less than m_W) for a large mass will prevent the doublets from acquiring v.e.v.s.

§1.0.g Flavour-changing gluino interactions.

The discussion up to now has been concerned with the construction of supergravity models which are consistent with certain gross features of the standard model, like the particle mass spectrum, the gauge symmetry breaking, etc. The rest of this chapter deals with more detailed predictions of specific models : calculations are made of the contributions from flavour-changing gaugino interactions, which (as will be shown) are not negligible in realistic SU_5 models, to proton decay and the K_L-K_S mass difference.

These interactions occur in the supersymmetric gauge vertices

$$\int d^4\theta\, Q_i^\dagger\, e^{gV}\, Q^i \quad , \ldots \text{ etc.}$$

of the standard model (1.0.21). Since each Q^i ($i = 1..3$) carries the same $SU_3 \times SU_2 \times U_1$ quantum numbers, these couplings are clearly flavour diagonal : for example, there is no $\bar{d}\tilde{g}c$ gluon vertex. When the mass eigenstates are different from the current eigenstates, however, the gaugino interaction can change flavour; in terms of the current states, the gaugino vertex is

$$i g \left(\bar{q}_i \lambda \tilde{q}^i - \tilde{q}^\dagger_i \lambda q^i \right) \qquad (1.0.26)$$

where q^i is a quark of the i-th generation, $\tilde{q}^i$ its scalar partner and λ a gaugino. Clearly if the quarks and squarks cannot be simultaneously diagonalised, then this vertex will change flavour.

left-handed fermion field	symbol	scalar partner
quark doublet $\begin{pmatrix} u & c & t \\ d & s & b \end{pmatrix}_L$	q^i	$\tilde{q}^i_L$
charge conjugate 'up' $(u^c, c^c, t^c)_L$	$u^c{}_i$	$\tilde{q}^\dagger_{R\,i}$
and 'down' $(d^c, s^c, b^c)_L$	$d^c{}_i$	$\tilde{d}^\dagger_{R\,i}$
lepton doublet $\begin{pmatrix} e & \mu & \tau \\ \nu_e & \nu_\mu & \nu_\tau \end{pmatrix}_L$	l^i	$\tilde{l}^i_L$
anti-leptons $(e^c, \mu^c, \tau^c)_L$	$e^c{}_i$	$\tilde{e}^\dagger_{R\,i}$

TABLE 1.1

To be precise, denote by q^i, $u^c{}_i$, ... the fields listed in table 1.1. The general mass term for the fermions f = u, d or e is

$$f^c{}_i M_f{}^i{}_j f^j + \bar{f}_i M^\dagger_f{}^i{}_j \bar{f}^c{}^j \qquad (1.0.27)$$

so the mass eigenstates are defined by

$$f_{(m)}{}^i = L_f{}^i{}_j f^j \qquad (1.0.28a)$$

and

$$f^c_{(m)\,i} = f^c{}_j R^\dagger_f{}^j{}_i \qquad (1.0.28b)$$

provided

is diagonal. Note that only if $M_f = M_f^\dagger$ can (1.0.27) be written in terms of the four-spinor

$$\psi^i_f = \begin{pmatrix} f^i \\ \bar{f}^i \end{pmatrix}$$

as

$$\bar{\psi}_{fi} M_f{}^i{}_j \psi_f{}^j$$

In this case $L_f = R_f$ and parity is conserved. Similarly, the mass term for the scalar partners $\tilde{f}^i{}_L$ and $\tilde{f}_{iR}{}^\dagger$ is, in general,

$$\tilde{f}^\dagger_{Li} M^2_{\tilde{f}L}{}^i{}_j \tilde{f}_L{}^j + \tilde{f}^\dagger_{Ri} M^2_{\tilde{f}R}{}^i{}_j \tilde{f}_R{}^j$$

so their mass eigenstates are defined as

$$\tilde{f}^i_{(m)\,L} = \tilde{L}^i_{f\,j} \tilde{f}^j_L \qquad (1.0.29a)$$

and

$$\tilde{f}^i_{(m)\,R} = \tilde{R}^i_{f\,j} \tilde{f}^j_R \qquad (1.0.29b)$$

where both $\tilde{L}_f M^2_{\tilde{f}L} \tilde{L}^\dagger$ and $\tilde{R}_f m^2_{\tilde{f}R} \tilde{R}^\dagger$ are diagonal.

These four mixing matrices L_f, R_f, $\tilde{L}_f$, $\tilde{R}_f$ parametrise all the flavour-changing processes generated by the Lagrangian. For example, the Yukawa interaction generating the up-quark masses is

$$u^c M_u q h / v$$

where h is a scalar Higgs. In terms of the mass states, this becomes

$$u^c_{(m)} R_u M_u L_d{}^\dagger q_{(m)} h / v$$

$$= \sum_{i=1}^{3} u^c_{(m)\,i} m^{(u)}_i (K d_{(m)})^i h / v$$

where $m^{(u)}{}_i = (m_u, m_c, m_t)$ are the up-quark masses, and K is the Kobayashi-Maskawa matrix[39],

$$K = L_u L_d{}^\dagger \qquad (1.0.30)$$

Similarly, the gaugino vertex (1.0.26) is, in terms of the mass states,

$$ig\left(\bar{\lambda}\,\bar{f}_{(m)}\,L_f\,\tilde{L}^{\dagger}_f\,\bar{\tilde{f}}_{(m)L} - \tilde{f}^{\dagger}_{(m)L}\,\tilde{L}_f\,L^{\dagger}_f\,f_{(m)}\,\lambda\right)$$

so these vertices are flavour-conserving if and only if the 'Super-Kobayashi-Maskawa' matrices

$$\tilde{K}_{fL,R} = L_f\tilde{L}_f^{\dagger} \qquad (R_f\tilde{R}_f^{\dagger}) \tag{1.0.31}$$

are equal to the unit matrix.

SECTION 1.1 Squark masses in low-energy supergravity GUTs

§1.1.a The minimal SU_5 model

The fermions of the standard model are accomodated in two left-handed multiplets of SU_5 for each generation[40] : the $\bar{5}$

$$\psi_{F\alpha i} = (d^c{}_i , e^c{}_i , \nu^i)_L$$

and the 10

$$\psi_T^{\alpha\beta i} = \begin{bmatrix} 0 & u^c{}_i & -u^c{}_i & -u^i & -d^i \\ -u^c{}_i & 0 & u^c{}_i & -u^i & -d^i \\ u^c{}_i & -u^c{}_i & 0 & -u^i & -d^i \\ u^i & u^i & u^i & 0 & -e^c{}_i \\ d^i & d^i & d^i & e^c{}_i & 0 \end{bmatrix} \qquad (1.1.1)$$

where α, β = 1..5 are group indices and i = 1..3 is a generation index, so (for instance) $u^c{}_i = (u^c, c^c, t^c)$. In the minimal supersymmetric Grand Unified model, these multiplets ψ_F, ψ_T are the fermion components of the N=1 superfields $F_{\alpha i}$, $T^{\alpha\beta i}$, whose Yukawa interactions are specified by the superpotential

$$P = \frac{1}{v_H} F_{\alpha i} M_d{}^i{}_j T^{\alpha\beta j} H_\beta + \frac{1}{4v_K} \epsilon_{\alpha\beta\gamma\delta\epsilon} T^{\alpha\beta}_i M_u{}^i{}_j T^{\gamma\delta j} K^\epsilon \qquad (1.1.2)$$

H and K are Higgs superfields, belonging to the $\bar{5}$ and 5 representations of SU_5, whose v.e.v.s v_H and v_K break SU_2. It is clear that the u and d quark mass matrices which follow at tree-level from this superpotential are just m_u and m_d. When the supersymmetry of the model is broken by soft, explicit terms generated by the hidden sector v.e.v.s, these terms take the form (1.0.17)

$$\mathcal{L}_{soft} = m_g \int [F_i A_d^{\ i}{}_j T^j H + T_i A_u^{\ i}{}_j T^j K] \theta^2 d^2\theta$$
$$-\tfrac{1}{2} m_y^2 \sum_{\substack{au \\ s/fields\ x}} \int d^4\theta\, x^\dagger e^{gV} x\, \delta^4(\theta) \qquad (1.1.3)$$

where A_d, A_u are adjustable parameters - though in principle they are fixed by the hidden sector - and m_g is a mass close to the gravitino mass $m_{3/2}$.

§1.1.b Squark masses in the minimal model

Following the discussion of §1.0.d, the complete low-energy supergravity Lagrangian (which includes gauge terms for all superfields and a superpotential for the Higgs multiplets, as well as 1.1.2 and 1.1.3) is taken to be an effective Lagrangian renormalised at the Planck scale m_{pl}. When renormalised at that scale, therefore, the $\tilde{u}$ and $\tilde{d}$ squarks have mass matrices

$$m^2_{\tilde{u}L} = m_1^2 + m_u^\dagger m_u$$
$$m^2_{\tilde{u}R} = m_2^2 + m_u m_u^\dagger$$
$$m^2_{\tilde{d}L} = m_3^2 + m_d^\dagger m_d$$
$$m^2_{\tilde{d}R} = m_4^2 + m_d m_d^\dagger \qquad (1.1.4)$$

where m_1^2, ... m_4^2 are terms arising from $\mathcal{L}_{soft}$ and from the D-term of the scalar potential generated by (1.1.2). Gauge interactions for the current eigenstates $\tilde{u}$, $\tilde{d}$ are flavour-diagonal, so it follows from (1.1.4) that all the matrices $L_q m^2_{\tilde{q}L} L_q^\dagger$ and $R_q m^2_{\tilde{q}R} R_q^\dagger$ are diagonal, whence $\tilde{L}_q = L_q$ and $\tilde{R}_q = R_q$. That is, the matrices $\tilde{K}_{uL,R}$ and $\tilde{K}_{dL,R}$ defined in (1.0.31) are equal to the unit matrix and gaugino interactions are flavour-conserving at the Planck mass.

This result, however, is clearly not stable under

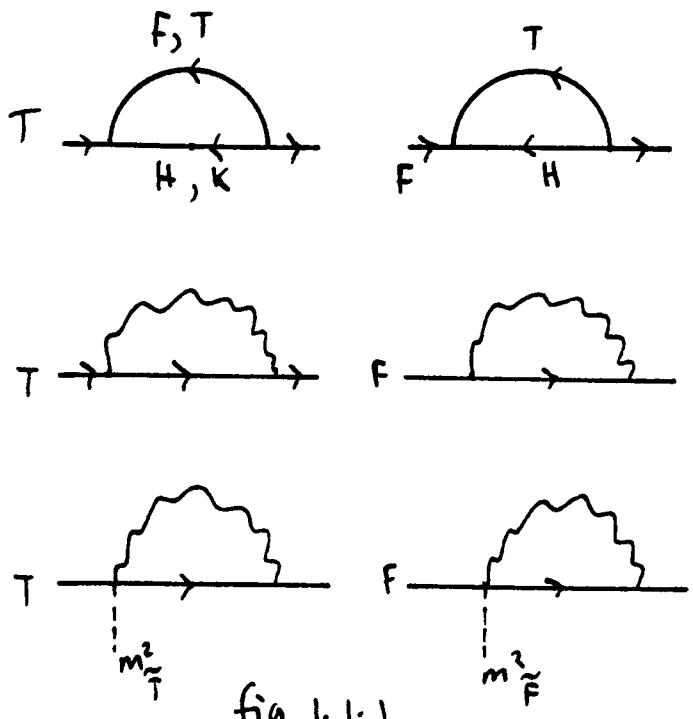


fig 1.1.1

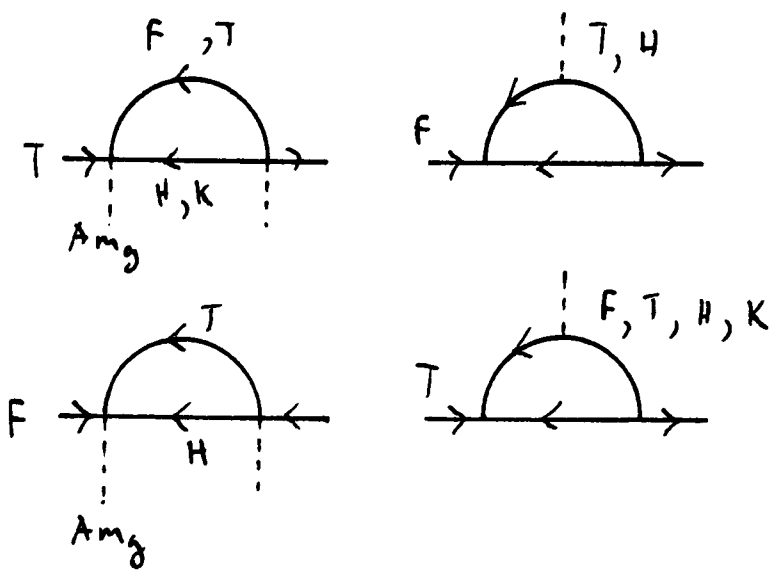


fig 1.1.2

quantum corrections. The gauge corrections of fig 1.1.1 contribute a wave-function renormalisation but this is irrelevant for the mixing matrix calculation as, again, it is proportional to the unit matrix. The mass renormalisations shown in fig 1.1.2, on the other hand, involve the Yukawa couplings m_u and m_d, so that at some renormalisation scale μ, (1.1.4) is altered to read

$$
\begin{aligned}
m^2_{\tilde{u}_L}(\mu) &= m^2_{\tilde{u}_L}(m_{pl}) + a^L_u(\mu)\, m_u^\dagger m_u + a'^L_u(\mu)\, m_d^\dagger m_d \\
m^2_{\tilde{u}_R}(\mu) &= m^2_{\tilde{u}_R}(m_{pl}) + a^R_u(\mu)\, m_u m_u^\dagger + a'^R_u(\mu)\, m_d m_d^\dagger \\
m^2_{\tilde{d}_L}(\mu) &= m^2_{\tilde{d}_L}(m_{pl}) + a^L_d(\mu)\, m_u^\dagger m_u + a'^L_d(\mu)\, m_d^\dagger m_d \\
m^2_{\tilde{d}_R}(\mu) &= m^2_{\tilde{d}_R}(m_{pl}) + a'^R_d(\mu)\, m_d m_d^\dagger
\end{aligned}
\qquad (1.1.5)
$$

Calculation[41] shows the coefficients $a_{u,d}^{L,R}$ and $a'^{L,R}_{u,d}$ all to be of order unity when evaluated at the weak scale. Furthermore, the matrix $m_d^\dagger m_d$ can be neglected[41,42] when compared with $m_u^\dagger m_u$, so that the scalar masses of (1.1.5) are diagonalised approximately by the matrices

$$
\begin{aligned}
\tilde{L}_u &\sim L_u \\
\tilde{R}_u &\sim R_u = L_u \\
\tilde{L}_d &\sim L_u \\
\tilde{R}_d &\sim R_d
\end{aligned}
\qquad (1.1.6)
$$

whence it follows that all gaugino interactions are flavour diagonal except those involving the $\tilde{d}_L$ squark[41,42,44],

which are parametrised in generation space by the matrix

$$\tilde{K}_{dL} \sim K$$

where K is the Kobayashi-Maskawa matrix (1.0.30)

§1.1.c The non-minimal SU_5 model

The fermion mas spectrum of the minimal SU_5 model described in §1.1.a shows several discrepancies with experiment. In the minimal model, the tree-level 'down' quark masses are equal to the charged lepton masses, generation by generation, but this is incompatible with the pseudo-scalar meson spectrum. To rectify this situation, Georgi and Jarlskog[43] have proposed a fermion mass generating scheme which has the tree-level relations

$$m_d = 3m_e \;,\quad m_s = \tfrac{1}{3} m_\mu \;,\quad m_b = m_\tau \qquad (1.1.7)$$

These masses scale to give good low-energy quark and lepton masses (as will be shown in the following section). To satisfy (1.1.7), Georgi and Jarlskog introduced an extra Higgs 45 in the down sector, as well as an extra 5 to give mass to the up quarks. This scheme, however, gives too large a value for the Cabibbo-Kobayashi-Maskawa mixing between the 2nd and 3rd generations[45], and to rectify this problem, a third Higgs 5 in the up sector is introduced[46].

The complete model considered here contains the Higgs multiplets listed in table 1.2, which are coupled to the matter fields F and T by the superpotential

$$P = F_i \left[\phi_d{}^i{}_j \frac{H}{v_H} + q_d{}^i{}_j \frac{L}{v_L} \right] T^j + T_i \sum_{r=1}^{3} \phi_u{}^r{}^i{}_j \frac{K_r}{v_r} T^j \qquad (1.1.8)$$

where the Yukawa matrices are

$$\phi_d = \begin{pmatrix} 0 & A & 0 \\ A & 0 & 0 \\ 0 & 0 & B \end{pmatrix}, \quad q_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \phi_u{}^1 = \begin{pmatrix} 0 & D & 0 \\ D & 0 & 0 \\ 0 & 0 & E \end{pmatrix}$$

$$\phi_u{}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & F \\ 0 & F & 0 \end{pmatrix}, \quad \phi_u{}^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & G & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad (1.1.9)$$

Higgs	Representation of SU_5	v.e.v.
K_r $(r = 1..3)$	5	v_r
H	$\bar{5}$	v_H
L	45	v_L

TABLE 1.2. Higgs structure in non-minimal model

The principal features of th is model, whose derivation follows, are that :

(i) the tree-level 'd' quark masses satisfy the Georgi-Jarlskog relation (1.1.7)

(ii) the Kobayashi-Maskawa matrix has the value

$$K = \begin{pmatrix} 1 & -\sin\theta_c & \sin\eta \sqrt{\frac{m_u}{m_c}} \\ \sin\theta_c & \cos\eta & -\sin\eta \\ \sin\eta \frac{m_c}{m_t} \sqrt{\frac{m_u}{m_c}} & \sin\eta & \cos\eta \end{pmatrix} \qquad (1.1.10)$$

where $\sin\theta_C \sim \sqrt{m_d/m_s}$, and $\sin\eta \sim m_c/m_t$. These angles agree with experiment[47], except for the CP-violating phase which has been set to zero. (1.1.11)

(iii) the scalar quark mixing matrices have the values[50]

$$(\tilde{L}, \tilde{R})_{u,d} = \begin{pmatrix} 1 & -\beta_1{}^{L,R}_{u,d} \sqrt{\frac{m_u}{m_c}} & \beta_2{}^{L,R}_{u,d} \frac{m_c}{m_t} \sqrt{\frac{m_u}{m_c}} \\ \beta_1{}^{L,R}_{u,d} \sqrt{\frac{m_u}{m_c}} & 1 & -\frac{m_c}{m_t} \beta_3{}^{L,R}_{u,d} \\ \beta_3{}^{L,R}_{u,d} \frac{m_c^2}{m_t^2} \sqrt{\frac{m_u}{m_c}} & \frac{m_c}{m_t} \beta_3{}^{L,R}_{u,d} & 1 \end{pmatrix}$$

where the coefficient functions $\beta_{u,d}^{L,R}$ are small numbers (<½) for typical values of model parameters. It follows from this result that the gaugino interaction matrices $\tilde{K}$ have significant off-diagonal entries. As will be discussed below, this has an important bearing on the proton decay in the non-minimal model.

§1.1.d Fermion masses in the non-minimal model

The tree-level fermion masses which follow from (1.1.8) are the matrices

$$M_u = \beta_u^1 + \beta_u^2 + \beta_u^3$$

$$M_d = \beta_d + q_d$$

$$M_e = \beta_d - 3 q_d \qquad (1.1.12)$$

for charge $^2/_3$ quarks, charge $^{-1}/_3$ quarks and charged leptons, respectively. Finding the eigenvalues of these matrices gives the tree-level masses

$$m_u \sim D^2/G \;,\quad m_c \sim G \;,\quad m_t \sim E$$

$$m_d \sim A^2/C \;,\quad m_s \sim C \;,\quad m_b \sim B$$

$$m_e \sim A^2/3C \;,\quad m_\mu \sim 3C \;,\quad m_\tau \sim B$$

provided $E \gg G \sim F \gg D$ and $B \gg C \gg A$; from here the Georgi-Jarlskog relation (1.1.7) follows.

Quantum corrections to (1.1.12) are due, at one-loop level, to the graphs

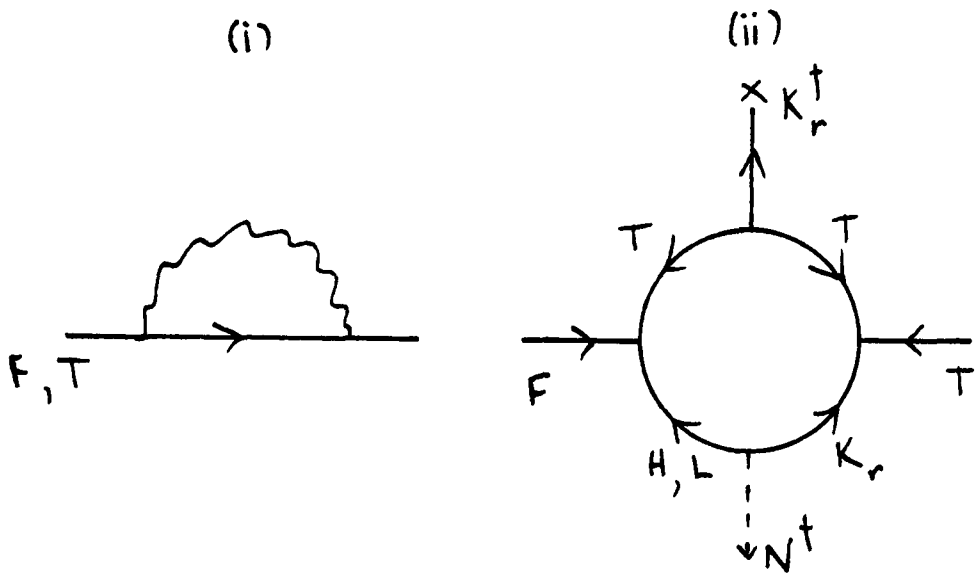


fig 1.1.3. One-loop propagators

shown in fig 1.1.3. The first set of these, fig 1.1.3i, involves gauge interactions and their contribution is, as usual, proportional to the unit matrix. Thus their effect will be to change the renormalised masses without affecting the mixing matrices. The graphs of fig 1.1.3ii, on the other hand, make contributions proportional to the Yukawa couplings so that in principle they will produce a scale dependence in the Kobayashi-Maskawa matrix K. However, their effect is in fact small : the spurion insertions $N^\dagger$ in fig 1.1.3ii arise from the explicit symmetry breaking terms which, as explained earlier, take the form (see 1.0.16)

$$\mathcal{L}_{soft} = \int d^2\theta \Big\{ F_i \Big(A_d \, p_d{}^i{}_j \frac{H}{v_H} + B_d \, q_d{}^i{}_j \frac{L}{v_L} \Big) T^j + T_i \sum_r A_u^r \, p_u^{ri}{}_j \frac{K_r}{v_r} T^j \Big\} N(\theta) - \sum_{\text{all s/fields, x}} \int d^4\theta \, x^\dagger x \, U(\theta) \qquad (1.1.13)$$

The graphs in question are clearly finite, so their dominant contribution is from scales $< m_g^2$, and their Yukawa couplings are small; thus the change they produce in the tree-level form (1.1.12) is small, and the mixing matrices are essentially scale-independent. Diagonalisation of (1.1.12) gives the results

$$L_u \sim \begin{pmatrix} 1 & -\sqrt{\frac{m_u}{m_c}} & \sin\eta \sqrt{\frac{m_u}{m_c}} \\ \sqrt{\frac{m_u}{m_c}} & \cos\eta & -\sin\eta \\ \sin\eta \frac{m_c^2}{m_t^2} \sqrt{\frac{m_u}{m_c}} & \sin\eta & \cos\eta \end{pmatrix} \qquad (1.1.14)$$

$$L_d \sim \begin{pmatrix} 1 & -\sqrt{\frac{m_d}{m_s}} & 0 \\ \sqrt{\frac{m_d}{m_s}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (1.1.15)$$

$$L_e \sim \begin{pmatrix} 1 & -\sqrt{\frac{m_e}{m_\mu}} & 0 \\ \sqrt{\frac{m_e}{m_\mu}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (1.1.16)$$

The effect of the gauge interactions fig 1.1.3i is to scale the masses themselves to give the good low-energy relations[43]

$$m_e \sim \frac{1}{10} m_d \;, \quad m_\mu \sim m_s \;, \quad m_\tau \sim \frac{1}{3} m_b$$

It should be noted that a general property of models with more than one Higgs giving mass to each sort of quark is the freedom with which their v.e.v.s may be chosen; the only conditions they should satisfy is that they give the correct W mass,

$$\frac{g_2^2}{2}\left(v_H^2 + 9 v_L^2 + \sum_r v_r^2 \right) = m_W^2 \qquad (1.1.17)$$

and that the Yukawa couplings p_d/v_H remain perturbative (i.e. < 1). There is clearly the possibilty that one v.e.v. may be small, and the corresponding Yukawa coupling quite large (i.e. close to 1). In this case, the argument used above to show that the Kobayashi-Maskawa matrix is scale-independent can no longer be used; it is an assumption in the model considered here that all the Higgs v.e.v.s are of roughly the same size.

§1.1.e Squark masses in the non-minimal model.

At tree-level, the squark masses arise from the superpotential (1.1.8), the soft supersymmetry breaking Lagrangian (1.1.13) and the gauge kinetic terms for F and T are

$$m^2_{\tilde{u}_L}(m_{pl}) = a m^2_{\frac{3}{2}} + M_u^\dagger M_u - \frac{g^2}{4}\left(\sum_r |v_r|^2 - |v_H|^2 + 9|v_L|^2\right)$$

$$m^2_{\tilde{u}_R}(m_{pl}) = a m^2_{\frac{3}{2}} + M_u M_u^\dagger$$

$$m^2_{\tilde{d}_L}(m_{pl}) = a m^2_{\frac{3}{2}} + M_d^\dagger M_d + \frac{g^2}{4}\left(\sum_r |v_r|^2 - |v_H|^2 + 9|v_L|^2\right)$$

$$m^2_{\tilde{d}_R}(m_{pl}) = a m^2_{\frac{3}{2}} + M_d M_d^\dagger \qquad (1.1.18)$$

where M_u, M_d are the quark masses (1.1.12), and a is a number of order one depending on the supergravity parameter A.

Again, the quantum corrections to this formula will determine the squark mass matrices at low energy; but the situation is different from the minimal model (where it was possible to conclude that the $\tilde{u}$-u gaugino interactions are flavour-conserving), for two reasons. Firstly, the mass effects discussed in §1.0.e change the form of the Renormalisation Group equation at scales below the Higgs triplet mass. With a non-minimal Higgs structure there is freedom in choosing which combinations of doublets remain light below the GUT scale, and these combinations affect the form of the quantum corrections (in generation space) at low energy. Secondly, in the minimal model the one-loop corrections were of the same matrix form (in the approximation $m_u^\dagger m_u \propto m_d^\dagger m_d$) as the tree-level terms; it was from this property that flavour-conservation followed. When there is more than one Higgs in the 'up' sector, this no longer remains true. Corrections to the tree-level masses

(1.1.18) above the GUT scale come from graphs (fig 1.1.4) involving only one Higgs at a time, so for example the dominant corrections to m^2_{qL} are of the form

$$\alpha\, p'^{\dagger}_{u}\, p'_{u} + \beta\, p''^{\dagger}_{u}\, p''_{u} + \gamma\, p'''^{\dagger}_{u}\, p'''_{u} \qquad (1.1.19)$$

and in general this is not proportional to $M_u^{\dagger}M_u$. Both these points are discussed in turn, below.

§1.1.f The low-energy Higgs structure

One of the roles of the Higgs multiplets H, L, K_1, K_2, K_3 is to break SU_2. Because of the radiative SU_2 breaking in this supergravity theory, discussed in §1.0.f, there must therefore be at least one Higgs doublet in both the 'up' and 'down' sectors which is kept massless at tree-level, in order to acquire an SU_2-breaking v.e.v. However, in the non-minimal model it is also possible that two or more multiplets remain massless in each sector. There are two considerations that may dictate the number of light Higgs doublets.

Firstly, extra light doublets may affect the validity of the perturbation expansion in the region between the GUT scale m_G and the Planck mass. The 1-loop β-function for the SU_5 coupling $\alpha = g^2/4\pi$ is[48]

$$\beta(g) = \frac{g^3}{32\pi^2}\left(4n_g + n_5 + 10\,n_{24} + 24\,n_{45} - 30\right)$$

where n_g is the number of generations and n_d the number of d-dimensional Higgs multiplets. For the non-minimal model it follows that

$$\bar{\alpha}^{-1}(m_{pl}) - \bar{\alpha}^{-1}(m_G) = -\frac{5}{\pi} \log(m_{pl}/m_G)$$

When the number of light doublets n_ℓ is two[49,51], $\alpha^{-1}(m_G) \sim 20$ and $m_G \sim 6 \times 10^{15}$ GeV, so $\alpha^{-1}(m_{pl}) \sim 10$; for $n_\ell = 4$, the gauge coupling becomes infinite below the Planck scale and to continue to use perturbation theory it is necessary to choose $n_\ell = 2$.

Similarly, the Yukawa couplings could evolve to be greater than 1 at m_{pl}; for example, the coupling constants $\lambda_d = p_d/v_H$ satisfy the 1-loop RGE

$$32\pi^2 \mu \frac{\partial}{\partial \mu} \lambda_d = 5 \{ \lambda_d, (1 + \tfrac{1}{4} + n)(\lambda_d^\dagger \lambda_d + \tfrac{9}{2} \kappa_d^\dagger \kappa_d + \tfrac{3}{4} \lambda_u^{r\dagger} \lambda_u^r) \} - 24 g^2 \lambda_d$$

where $\kappa_d = q_d/v_L$ and $\lambda_u^r = p_u^r/v_r$, but as all the couplings on the right-hand side are taken to be small, there is negligible change up to the Planck mass.

Secondly, the prediction for $\sin^2\theta_W$ at low energies depends sensitively on n_ℓ[51]. When $n_\ell = 2$, $\sin^2\theta_W = 0.237$ whereas for $n_\ell = 4$, $\sin^2\theta_W = 0.262$. Comparison with the experimental value[53] suggests only a total of two light doublets are allowed.

For this choice, the only possible light combination of Higgs is when they are added together in proportion to their v.e.v.s, i.e. the light Higgs doublet in the 'down' sector is

$$v_G G = v_H H + v_L L \qquad (1.1.20)$$

where only the doublet components appear on the right-hand side, and the light doublet in the 'up' sector is

$$v_K K = v_1 K_1 + v_2 K_2 + v_3 K_3 \qquad (1.1.21)$$

To see this, suppose that there are n doublets h^a, each with a v.e.v. v^a, giving mass to (say) the charge $-1/3$ quarks. If the mass term for these fields is $h^\dagger_a m^{2a}_b h^b$, then the mass eigenstates are

$$g^a = U^a{}_b h^b \qquad (1.1.22)$$

where $Um^2U^\dagger$ is diagonal. When only one of these, say g^1, is massless, the only non-zero v.e.v. is $\langle g^1 \rangle$. From (1.1.22) it follows that

$$\langle g^1 \rangle = U^1{}_b v^b$$

or in other words

$$g^1 = \frac{1}{\langle g^1 \rangle} \sum v^a h^a \quad , \quad \text{q.e.d.}$$

§1.1.g The Renormalisation group equations

The form of the RGEs, which will 'improve' the tree-level mass formulæ (1.1.18), is derived at any given scale by considering graphs mediated only by particles lighter than that scale (see §1.0.e). As will be shown in detail below (§1.2.e), the triplet parts of Higgs multiplets mediate proton decay; in order to suppress this process, they must be given a large tree-level mass M_H, of order the Grand Unification scale (models where this is not the case will be discussed briefly below). Following the discussion above, the only Higgs fields which contribute to the RGEs below m_H are therefore the light doublets (1.1.20) and (1.1.21). Calculating the relevant graphs (fig 1.1.5) shows that for renormalisation scales $\mu < M_H$, the appropriate RGEs

Fig 1.1.4. *Graphs contributing to scalar masses in the unbroken SU_5 region.*

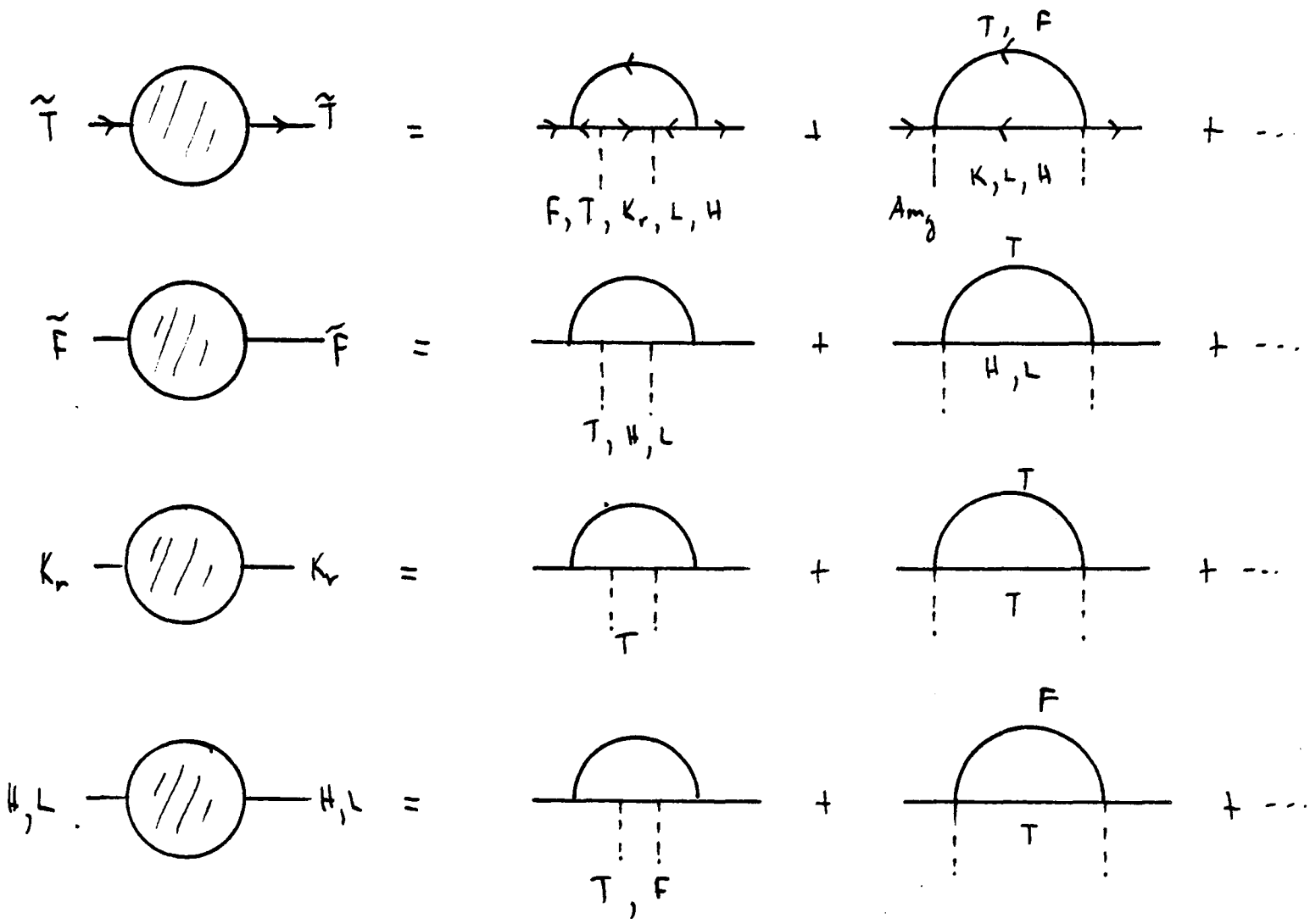
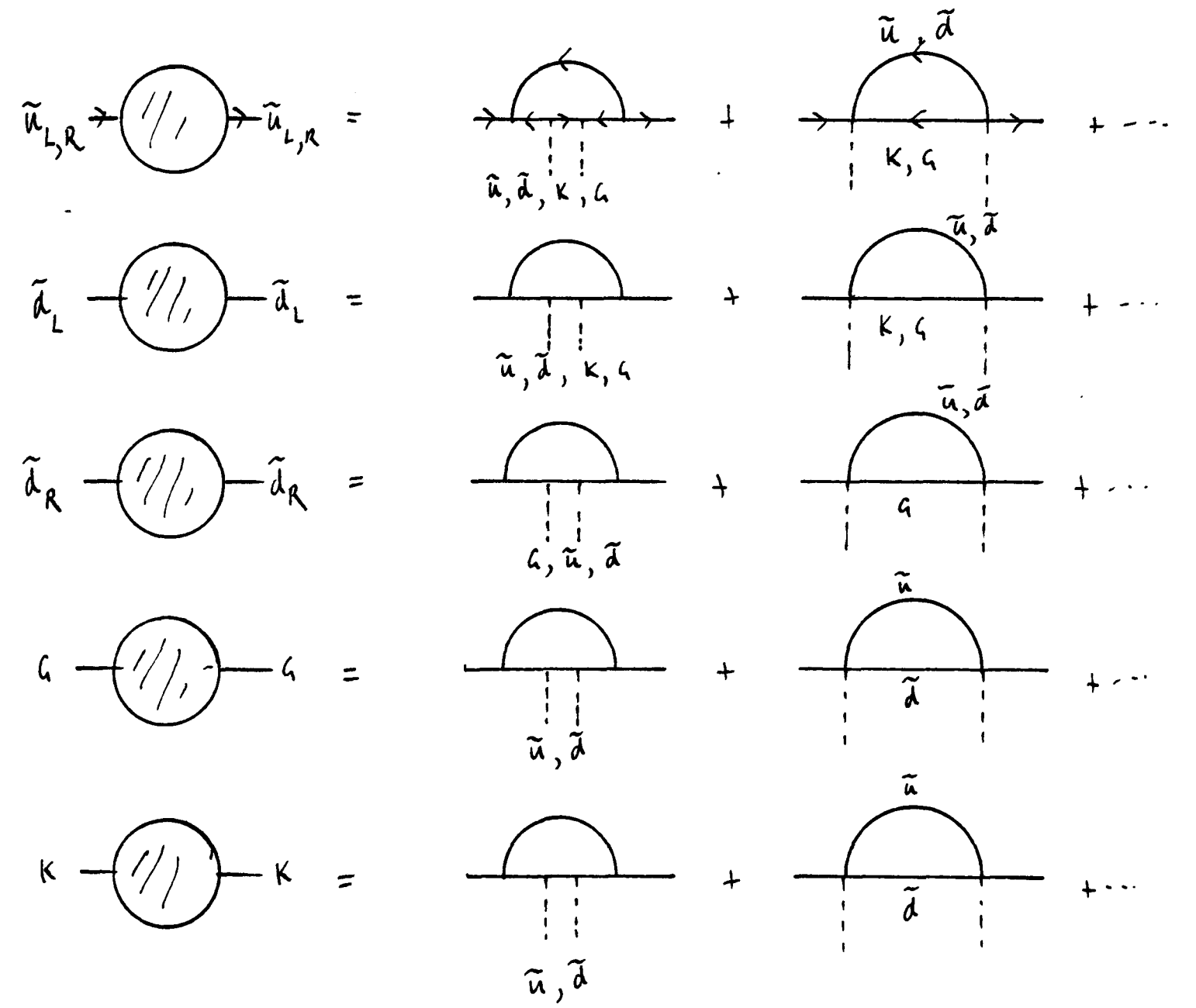


Fig 1.1.5 *Graphs contributing to scalar masses below M_H*



are, according to (1.0.23),

$$\mu \frac{\partial}{\partial \mu} m^2_{\tilde{q}L} = \frac{1}{8\pi^2}\left[\frac{1}{2}\left\{m^2_{\tilde{u}L}, \lambda^\dagger\lambda + \kappa^\dagger\kappa\right\}\right.$$

$$\left. + \lambda^\dagger(m^2_{\tilde{\kappa}} + m^2_{\tilde{u}R} + m^2_g A_L)\lambda + \kappa^\dagger(m^2_{\tilde{G}} + m^2_{\tilde{d}R} + m^2_g B_L)\kappa\right] \quad (a)$$

$$\mu \frac{\partial}{\partial \mu} m^2_{\tilde{u}R} = \frac{2}{8\pi^2}\left[\frac{1}{2}\left\{m^2_{\tilde{u}R}, \lambda\lambda^\dagger + \kappa\kappa^\dagger\right\}\right. \quad (b)$$

$$\left. + \lambda(m^2_{\tilde{\kappa}}{}^\dagger + m^2_{\tilde{q}L} + m^2_g A_L)\lambda^\dagger + \kappa(m^2_{\tilde{G}}{}^\dagger + m^2_{\tilde{q}L} + m^2_g B_L)\kappa^\dagger\right]$$

$$\mu \frac{\partial}{\partial \mu} m^2_{\tilde{d}R} = \frac{2}{8\pi^2}\left[\frac{1}{2}\left\{m^2_{\tilde{d}R}, \kappa\kappa^\dagger\right\} + \kappa(m^2_{\tilde{G}}{}^\dagger + m^2_{\tilde{q}L} + m^2_g B_L)\kappa^\dagger\right] \quad (c)$$

$$\mu \frac{\partial}{\partial \mu} m^2_{\tilde{\kappa}} = \frac{3}{8\pi^2} + \lambda^\dagger(m^2_{\tilde{u}R} + m^2_{\tilde{q}L} + m^2_{\tilde{\kappa}} + m^2_g A_L)\lambda \quad (d)$$

$$\mu \frac{\partial}{\partial \mu} m^2_{\tilde{G}} = \frac{3}{8\pi^2} + \kappa^\dagger(m^2_{\tilde{d}R} + m^2_{\tilde{q}L} + m^2_{\tilde{\kappa}} + m^2_g B_L)\kappa \quad (e)$$

(1.1.23a-e)

where λ and κ are the couplings for the light fields K and G to the quarks,

$$d = \frac{1}{v_K}\sum_r v_r d_u{}^r$$

$$K = \frac{1}{v_G}(v_H d_d + v_L K_d)$$

and the remaining quantities are defined by

$$A_L = \sum_r v_r \Delta_u{}^r / v_K$$

$$B_L = (v_H A_d + v_L B_d)/v_G$$

and $$v_K{}^2 = \sum_r |v_r|^2$$

$$v_G{}^2 = |v_H|^2 + |v_L|^2 \quad (1.1.24)$$

When μ is greater than M_H, the full SU_5 Higgs fields can contribute to the RGEs, Evaluating the infinite parts of the graphs shown in fig 1.1.4 leads to the following RGEs applicable at energies between M_H and m_{pl}

$$\mu \frac{\partial}{\partial \mu} m^2_{\tilde{T}} = \frac{1}{8\pi^2}\Big[\frac{1}{2}\Big\{m^2_{\tilde{T}}, 3\sum_{r=1}^{3}\lambda_u^{r\dagger}\lambda_u^r + \lambda_d^\dagger\lambda_d + \frac{9}{2}K_d^\dagger K_d\Big\} + 3\sum_r \lambda_u^{r\dagger}\big(m^2_{\tilde{K}r} + m^2_{\tilde{T}^\dagger} + m^2_g A_u^r\big)\lambda_u^r + \lambda_d^\dagger\big(m^2_{\tilde{F}^\dagger} + m^2_{\tilde{H}} + m^2_g A_d\big)\lambda_d + \frac{9}{2}K_d^\dagger\big(m^2_{\tilde{F}^\dagger} + m^2_{\tilde{L}} + m^2_g B_d\big)K_d\Big] \quad (a)$$

$$\mu \frac{\partial}{\partial \mu} m^2_{\tilde{F}} = \frac{1}{8\pi^2}\Big[\frac{1}{2}\Big\{m^2_{\tilde{F}}, 4\lambda_d^\dagger\lambda_d + 9K_d^\dagger K_d\Big\} + 4\lambda_d^\dagger\big(m^2_{\tilde{T}^\dagger} + m^2_{\tilde{H}} + m^2_g A_d\big)\lambda_d + 9K_d^\dagger\big(m^2_{\tilde{T}^\dagger} + m^2_{\tilde{L}} + m^2_g B_d\big)K_d \quad (b)$$

$$\mu \partial_\mu m^2_{\tilde{K}_r} = \frac{3}{4\pi^2}\,\mathrm{tr}\,\lambda_u^{r\dagger}\big(m^2_{\tilde{K}r} + m^2_{\tilde{T}} + m^2_{\tilde{T}^\dagger} + m^2_g A_u^r\big)\lambda_u^r \quad (c)$$

$$\mu \frac{\partial}{\partial \mu} m^2_{\tilde{H}} = \frac{1}{2\pi^2}\,\mathrm{tr}\,\lambda_d^\dagger\big(m^2_{\tilde{H}} + m^2_{\tilde{T}^\dagger} + m^2_{\tilde{P}} + m^2_g A_d\big)\lambda_d \quad (d)$$

$$\mu \frac{\partial}{\partial \mu} m^2_{\tilde{L}} = \frac{1}{8\pi^2}\,\mathrm{tr}\,K_d^\dagger\big(m^2_{\tilde{L}} + m^2_{\tilde{F}} + m^2_{\tilde{T}^\dagger} + m^2_g B_d\big)K_d \quad (e)$$

(1.1.25a-e)

Note that gauge interactions have been neglected for, as usual, their contribution is proportional to the unit matrix.

§1.1.h The (approximate) scalar masses and their mixing matrices

Solving the RGEs (1.1.23) and (1.1.25), together with the associated equations for the Yukawa couplings involved, would resum the large logarithms ($\log^2 m_{Pl}/m_G$ etc.) which appear in the higher loop graphs. However, keeping only the one-loop corrections gives a qualitative idea of the solution, for these corrections are not too large, as will be shown below. The two equations, as already noted, give corrections of a different form in generation space; as a result, the 1-loop corrected squark masses are given by (at the weak scale)[50]

$$m^2_{\tilde{u}L} = m^2_{\tilde{u}L}(m_{pl}) + a_u^L M_u^\dagger M_u + \sum_r b^L_{ru} p_u^{r\dagger} p_u^r$$

$$m^2_{\tilde{u}R} = m^2_{\tilde{u}R}(m_{pl}) + a_u^R M_u M_u^\dagger + \sum_r b^R_{ru} p_u^r p_u^{r\dagger}$$

$$m^2_{\tilde{d}L} = m^2_{\tilde{d}L}(m_{pl}) + a_d^L M_u^\dagger M_u + \sum_r b^L_{rd} p_u^{r\dagger} p_u^r$$

$$m^2_{\tilde{d}R} = m^2_{\tilde{d}R}(m_{pl}) + a_d^R M_u M_u^\dagger + b_d^R p_d p_d^\dagger + b'^R_d q_d q_d^\dagger \qquad (1.1.26)$$

where the coefficients take the values

$$a^L_u = a^L_d = \tfrac{1}{2} a_u^R = \left(\frac{m_g}{v_K}\right)^2 \frac{1}{8\pi^2}(3 + A_L) \log \frac{M_W}{m_H}$$

$$a_d^R = 0$$

$$c_r \equiv b^R_{rd} = b^L_{ru} = b^L_{rd} = \left(\frac{m_g}{r_r}\right)^2 \frac{3}{8\pi^2}(3 + A_u^r) \log \frac{m_H}{m_{pl}}$$

$$b_d^R = \left(\frac{m_g}{v_H}\right)^2 \frac{4}{8\pi^2}(3 + A_d) \log \frac{m_H}{m_{pl}} \qquad (1.1.27)$$

$$b'^R_d = \left(\frac{m_g}{v_L}\right)^2 \frac{9}{8\pi^2}(3 + B_d) \log \frac{m_H}{m_{pl}}$$

Diagonalising the matrices (1.1.26) gives the squark mixing matrices $\tilde{L}_{u,d}$, $\tilde{R}_{u,d}$ quoted earlier (1.1.11), where the coefficient functions β take the values

$$\beta_1{}^{L,R}_{u,d} \sim \frac{x(x + c_1)}{x^2 + x(2c_1 + c_2 + c_3) + c_1(c_2 + c_3)}$$

$$\beta_2{}^{L,R}_{u,d} \sim \frac{x^2}{x^2 + x(2c_1 + c_2 + c_3) + c_1(c_2 + c_3)}$$

$$\beta_3{}^{L,R}_{u,d} \sim \frac{x}{x + c_1} \qquad (1.1.28)$$

and for β_u^L or β_u^R, $x = a_u^{L,R} + 1$

for $\beta_d{}^L$, $x = a_d{}^L$

for $\beta_d{}^R$, $x = 0$

The numbers c_i are defined in (1.1.27). In the limit $c_i = 0$, which is the limit in which non-minimal effects vanish, the matrices (1.1.11) reduce to the value they had in the minimal case, (1.1.6).

The size of the corrections (1.1.26), and hence the

validity of the one-loop results, depends on several unknowns, viz.

(i) the supergravity parameters A_u^r, A_d and B_d. These are taken to be equal at the Planck scale, and if their common value A is too large, the fields of the trilinear couplings it multiplies may be driven to acquire v.e.v.s. The upper bound this imposes on A is about 3[52] (see §1.0.f)

(ii) the vacuum expectation values v_r, v_H and $3v_L$. Although in principle some of these may be small, it is assumed here that they all have the same size. This size is fixed by (1.1.17) at about 250 GeV.

(iii) the Higgs triplet mass M_H. In models where dimension 5 operators contribute to proton decay, this is bound below[52]: $M_H \gtrsim 10^{16}$ GeV.

With the values

$$A = 3\,, \quad v_r = v_H = 3v_L = 250 \ \text{GeV} \tag{1.1.29}$$

and taking $M_H = 10^{10}$ GeV (which maximises 1.1.27), the largest of the corrections is about $0.2m_g^2$, and the least of them is negligible. Because these corrections are small relative to the tree-level masses^2, the 1-loop calculations should represent a good solution of the RGEs (1.1.23&25); thus, only the one-loop parts of these equations have been used. In the limit that the non-minimal effects vanish, they are consistent with the Renormalisation Group solution of ref.[41].

§1.1.i Conclusions

Substituting the values (1.1.29) into (1.1.28) shows $\beta_1 \sim \beta_2 \sim {}^1/_5$, and $\beta_3 \sim {}^1/_2$ for $M_H = 10^{16}$ GeV, while in models where the Higgs triplets have mass $M_H \sim 10^{10}$ GeV, $\beta_i \lesssim {}^1/_{10}$ for each i. Because these numbers are small, the resulting squark-quark mixing matrices $\tilde{K}$ show significant discrepancies from the minimal case, and therefore in this non-minimal model gaugino interactions cannot be taken to be flavour-diagonal. Moreover, although this result has been derived within the context of a specific model, the general form of the squark mass corrections (1.1.26) holds for any model with a non-minimal Higgs structure. For example, if there are more than two light doublets (this is not ruled out in models with $M_H \sim 10^{10}$ GeV), then the radiative corrections below M_H will also be of the form (1.1.19) and the β-coefficients will be even smaller. Only in the special case where there are only two Higgs multiplets giving mass to the u and d quarks do the gaugino $\tilde{u}$-u interactions necessarily conserve flavour.

SECTION 1.2 Proton decay in supergravity theories

The conclusion of the previous section, that flavour-changing effects in gaugino interactions are non-vanishing, has a significant bearing on proton decay through dimension-five operators in supersymmetric Grand Unified theories, as will be shown in this section. The reason is that supersymmetric baryon-number violating operators contribute to proton decay when dressed by both SU_3 (gluino) exchanges and SU_2 gaugino exchanges, but the former are non-zero only when two conditions are satisfied simultaneously : firstly, the squark masses must not all be degenerate; and secondly the gluino vertex must not conserve flavour. Since both these conditions are realised in the non-minimal model described above (the stop mass $m_{\tilde{t}}$ is significantly less than the mass of the other squarks, due to the relatively large top coupling, $\lambda_{tt} \sim 1/6$), it is important to determine the size of the gluino contributions, and the modes through which gluino-dressed proton decays may proceed. It will be shown that gluino-mediated decay modes in the non-minimal SU_5 model are at least as important as wino-mediated decays $p \rightarrow \nu_\mu K^+$; gluino-mediated decays (which are absent in minimal SU_5 models) may in addition proceed through the reaction $p \rightarrow \mu^+ K^0$.

§1.2.a Proton decay in the supersymmetric standard model.

Unlike the conventional standard model, the supersymmetric standard model Lagrangian can include

dimension-four baryon number-violating F-terms, for example (in the notation of 1.0.18)

$$\int d^2\theta \; u^c_i \, D^c_j \, D^c_k \quad , \quad \int d^2\theta \; L^i \, Q^j \, D^c_k \tag{1.2.1}$$

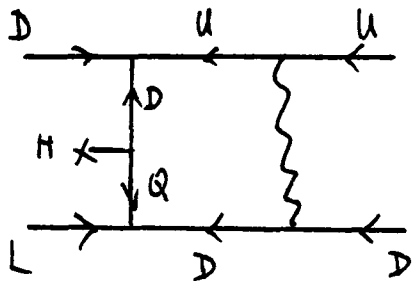


Fig 1.2.1. Proton decay from dimension 4 graph

which may combine to form a proton decay graph, e.g. fig 1.2.1, with a comparable size to other baryon decay processes[54,22]. These dangerous operators must be prohibited by imposing an 'R' parity invariance, where the R operation is defined as

$$u^c_i \to -u^c_i \quad , \quad Q^i \to -Q^i \quad , \ldots ; \quad H \to H \quad , \quad H' \to H' \tag{1.2.2}$$

The 'minimal' $SU_3 \times SU_2 \times U_1$ model (1.0.18) has this invariance, so none of the operators (1.2.1) can be generated radiatively; furthermore, (1.0.18) includes all renormalisable F-terms consistent with (1.2.2) - in this sense the supersymmetric standard model is a 'natural' R-invariant model.

§1.2.b Higgs-mediated proton decay in supersymmetric GUTs

In supersymmetric Grand Unified theories, baryon number-violating operators of dimension greater than four can be generated radiatively. Since these operators do not occur in the supersymmetric standard model, they must appear suppressed by powers of a large mass scale : candidate masses are m_{Pl} (in a supergravity theory), m_G (in a Grand Unified theory) or some intermediate scale m_I, if one exists.

To illustrate this explicitly, consider the dimension-five operator

$$\int d^2\theta \; T\,T\,T\,F \tag{1.2.3}$$

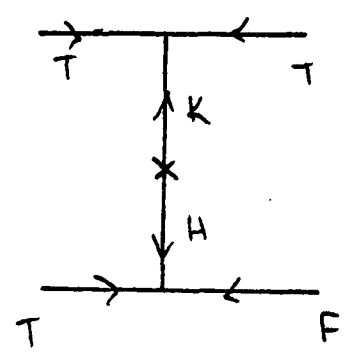


fig 1.2.2.

which is generated in a minimal SU_5 model by Higgs exchange, as shown in fig 1.2.2. The parts of it which violate baryon number are the two operators

$$\int d^2\theta \; L\,Q\,Q\,Q \tag{1.2.4a}$$

and

$$\int d^2\theta \; D^c\,U^c\,U^c\,E^c \tag{1.2.4b}$$

and it is easy to see that these are mediated by only the triplet parts of the Higgs H and K; their strength depends, therefore, on the triplet mass term

$$\int d^2\theta \; m_{HK} \sum_{a=1}^{3} H_a\,K^a \tag{1.2.5}$$

The bound on m_{HK} from requiring that proton decay not proceed too quickly is[52,64]

$$M_{Hk} \gtrsim 10^{16} \text{ GeV} \tag{1.2.6}$$

Such a mass can be obtained from the Grand Unification scale m_G; a superpotential is given below (1.2.24 & 27) which satisfies (1.2.6), but it is worth noting that not all models have such a large triplet mass : for example, some cosmological models[46] which generate the observed baryon asymmetry require $m_{HK} \lesssim 10^{12}$ GeV and of course proton decay in such a model proceeds far too quickly through the

dimension-five operator (1.2.3). Extra global symmetries must therefore be imposed[46] to forbid the dimension-five contributions to proton decay. In this case, the dominant contributions are from dimension-six operators, and are suppressed by a further factor of m_{HK}^{-1}.

§1.2.c Dominant decay modes in minimal supersymmetric SU_5 theories

The conventional decays $p \to \pi^0 e^+,\ \pi^+ \nu_e$ which occur through the graphs of fig 1.2.3 still arise, of course, in the supersymmetric theory, and since (in conventional SU_5) they produce a proton decay rate close to the experimental limit[55,61]

$$\tau_p \gtrsim 8 \times 10^{30} \text{ yrs}$$

they might also be expected to dominate supersymmetric modes. However, the unification mass m_G (which is also the mass of the intermediate X-boson) is determined by the evolution of the renormalised gauge couplings, and extra contributions to this from 'super-partners' have the effect of raising the value of m_G in supersymmetric theories[51],

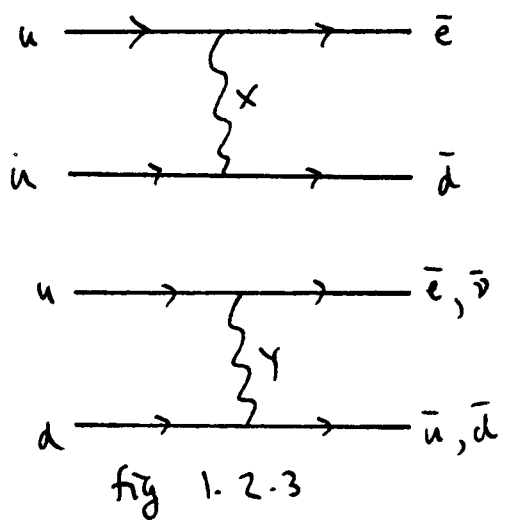


fig 1.2.3

$$M_G \sim 10^{16} \text{ GeV} \qquad (1.2.7)$$

in minimal SU_5, and therefore of suppressing the amplitude from fig 1.2.3, which is proportional to m_G^{-2}.

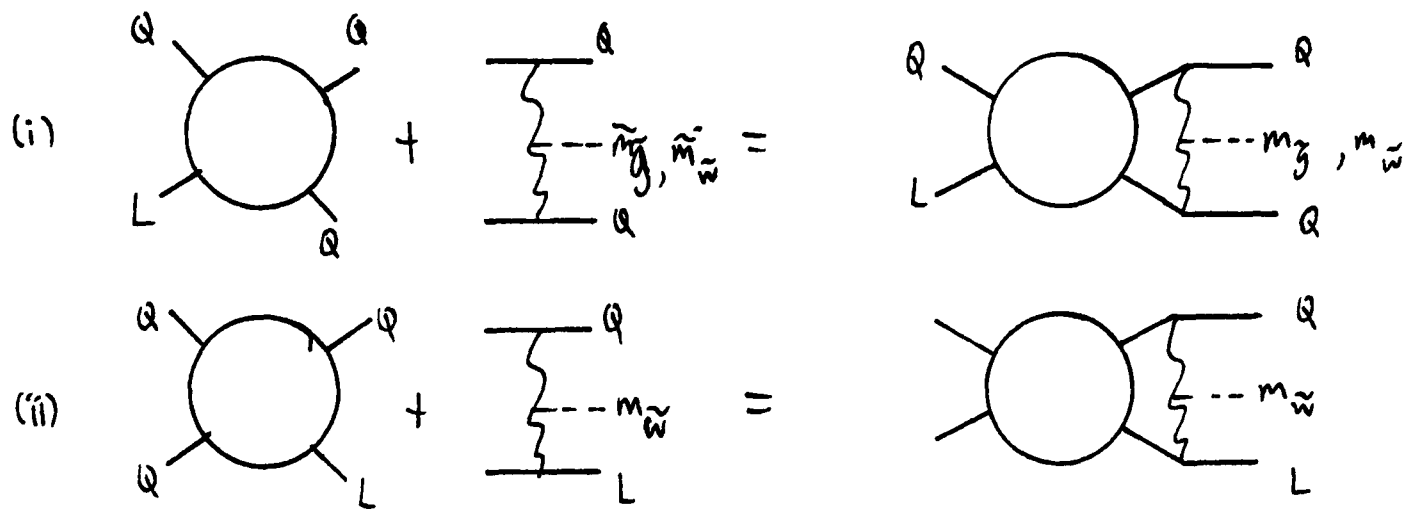


fig 1.2.4. Gaugino-dressed proton decay graphs.

On the other hand, natural values for m_{HK} give a proton lifetime of[56]

$$\tau_p \sim 10^{31} \text{ yrs} \tag{1.2.8}$$

which again is very close to the experimental limit. These contributions come from four-fermion operators obtained by dressing (1.2.3) with the gauge-mediated operators

$$\int d^4\theta \; T^\dagger T \; T^\dagger T \quad , \quad \int d^4\theta \; T^\dagger T \; F^\dagger F \tag{1.2.9}$$

as shown in fig 1.2.4. Since, by the non-renormalisation theorem (§0.e), any radiatively generated four-fermion operators

$$\int d^2\theta \; Q L \; D^\alpha Q \; D_\alpha Q \quad , \; etc \tag{1.2.10}$$

vanish identically, the gauge line must contain a supersymmetry-violating gaugino mass insertion (0.34), $m_{\tilde{g}}$, $m_{\tilde{w}}$ or $m_{\tilde{B}}$, and the resultant amplitude is therefore proportional to one of these masses. In supergravity models with minimal kinetic coupling, the gaugino masses arise only radiatively[36,57] and so one expects

$$\begin{aligned} m_{\tilde{g}} &\lesssim \alpha_3 \, m_{\tilde{g}} \\ m_{\tilde{w}} &\lesssim \alpha_2 \, m_{\tilde{g}} \\ m_{\tilde{B}} &\lesssim \alpha_1 \, m_{\tilde{g}} \end{aligned} \tag{1.2.11}$$

with the result that, other things being equal,

gluino-mediated graphs should dominate the proton decay amplitude. In fact, however, the gluino contribution to decays is greatly suppressed[58] in the minimal model, because there are no flavour-changing gauge interactions.

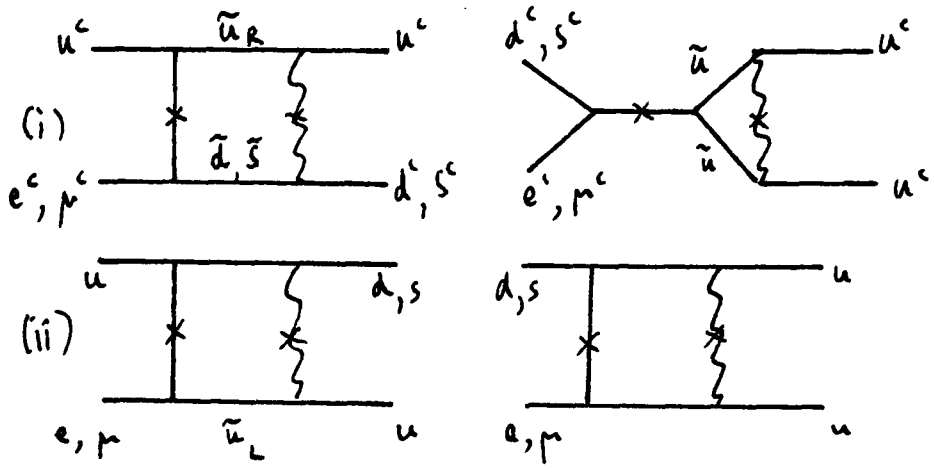
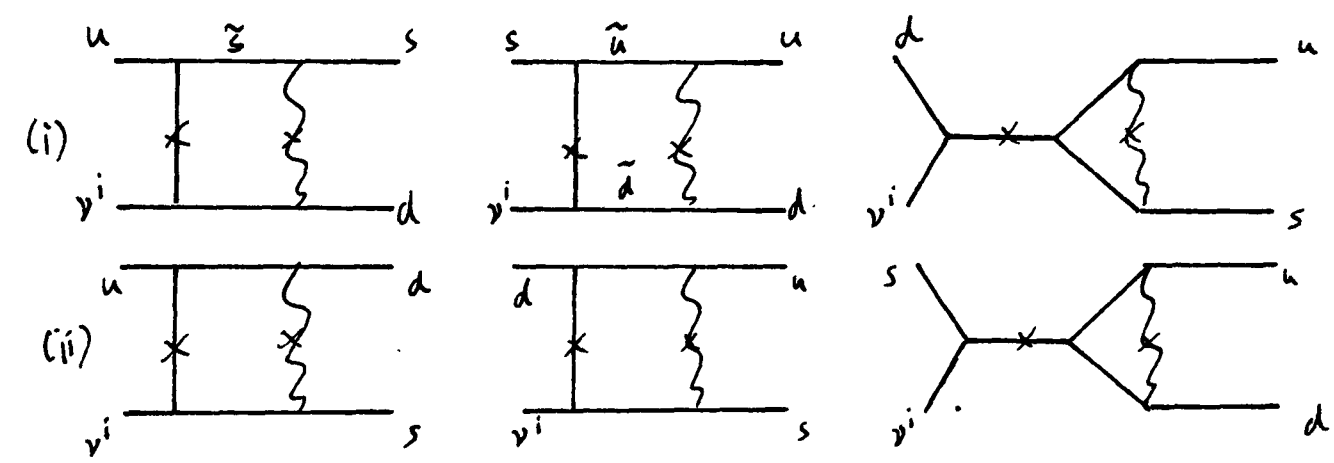


fig 1·2·5. p-decays to (i) right-handed leptons (ii) left-handed leptons

fig 1·2·6. p-decays to the neutrino $\nu^i = (\nu_e, \nu_\mu, \nu_\tau)$



To see this, consider the graphs relevant for proton decay, which are shown in figs 1.2.5&6. If all the scalar quarks have the same mass, then the loop part of all diagrams in figs 1.2.5&6 is the same (for each of the four sets 1.2.5i&ii, 1.2.6i&ii) since the Yukawa couplings are the same within each set. The sum of the graphs in fig 1.2.6i (say) can therefore be written

$$L\left[(\nu^i u)(ds) + (\nu^i s)(ud) + (\nu^i d)(su) \right] \qquad (1.2.12)$$

and by the Fierz identity (0.12), this expression vanishes, as does (by the same argument) the sum of all the remaining diagrams. In the minimal model, all the squarks except the stop $\tilde{t}$ have very nearly the same mass, as can be seen from (1.1.5), for the 1-loop corrections are proportional to small

Yukawa couplings. Thus, (as $\tilde{t}$ cannot appear in figs 1.2.5&6), none of the gluino-mediated graphs make a significant contribution to proton decay[58].

The dominant contribution which remains in the minimal SU_5 model is that from the Wino-mediated graphs of fig 1.2.4ii; because the Wino is charged, the graphs with the external states given in (1.1.12) (for instance) do not each involve the same Yukawa couplings, and the cancellation does not occur.

§1.2.d The proton decay amplitude in non-minimal SU_5

In calculating the gluino-mediated graphs of fig 1.2.4i in a model with the Higgs content of table 1.2 one encounters two important differences with the corresponding minimal model calculation. : flavour-changing effects at the gluino vertex have to be taken into account, for without these (as discussed in the previous section) there is no mixing to the $\tilde{t}$ squark, and the diagrams all sum to zero. Also, the masses for each of the Higgs triplet components of the SU_5 fields have to be found. In this section is presented the calculation of the proton decay amplitude for arbitrary Higgs masses; in the following section a superpotential P is given which generates masses of the appropriate size for each triplet. For a range of values of the parameter M in P, the dominant decay amplitude is particularly easy to find, and it is shown, finally, that in the non-minimal model the reaction $p \to \mu^+ K^0$ is of comparable size to the dominant reaction $p \to \nu_\mu K^+$ from Wino-mediated

graphs.

(i) Decays involving the left-handed lepton doublet

The relevant graphs are shown in fig 1.2.7. The mediating Higgs are the triplet parts $K_r{}^a$ (a = 1..3) of the Higgs 5s (which give mass to the up quarks) and the anti-triplet parts H_a and

$$H'_a = \sum_{b=1}^{3} L^b_{ab} \qquad (1.2.13)$$

of the $\bar{5}$ and 45 of the down sector. To evaluate these graphs, taking into account the massive gluino line (0.35), squark mass insertions must be made into each of the double lines (as in 0.33) which represent scalars when the external states are fermions. These massive propagators then introduce the generalised Kobayashi-Maskawa matrices $\tilde{K}$, as promised in §1.0.g. The complete four-fermion amplitude from fig 1.2.7i, in the approximation that the squark masses are much heavier than the quark masses, is

$$A_1 = -\sum_{r=1}^{3}\sum_{w=1}^{2} (d^j \ell_h)(u_i q^k) \frac{m_{\tilde{g}}}{m_{r,w}} \frac{g_3^2\, T(R_3)}{12\pi^2} \sum_{m,n=1}^{3} (\tilde{K}_{uL})^i{}_m (\tilde{K}^\dagger_{uL} L_u \lambda_u^{(r)} L_d^\dagger)^m_j$$

$$\cdot (L_d \lambda_d^{(w)} L_q^\dagger \tilde{K}_{qL})^h{}_n (\tilde{K}^\dagger_{qL})^n{}_k f(m^2_{\tilde{u}(m)L}, m^2_{\tilde{g}}, m^2_{\tilde{q}(n)L}) + (u^j \ell_h)(q^k d_j) \times \text{similar factor} \qquad (1.2.14)$$

where : $\lambda_u^{(r)} = \phi_u^r / v_r$ (see eq. 1-1-9 for definitions

$$\lambda_d^{(w)} = \begin{cases} \phi_d / v_d & \text{for } w=1 \\ \phi_d / v_{H'} & \text{for } w=2 \end{cases}$$

$m_{r,1}$ is the triplet mass for the F-term

$m_{r,2}$ is the triplet mass for the F-term

$m^2\tilde{q}(m)L$ is the mass^2 for the m-th $\tilde{q}_L$ squark,

and the fermion components d^j, ℓ_h etc. are in their mass eigenstates. The function f arises from the loop integral,

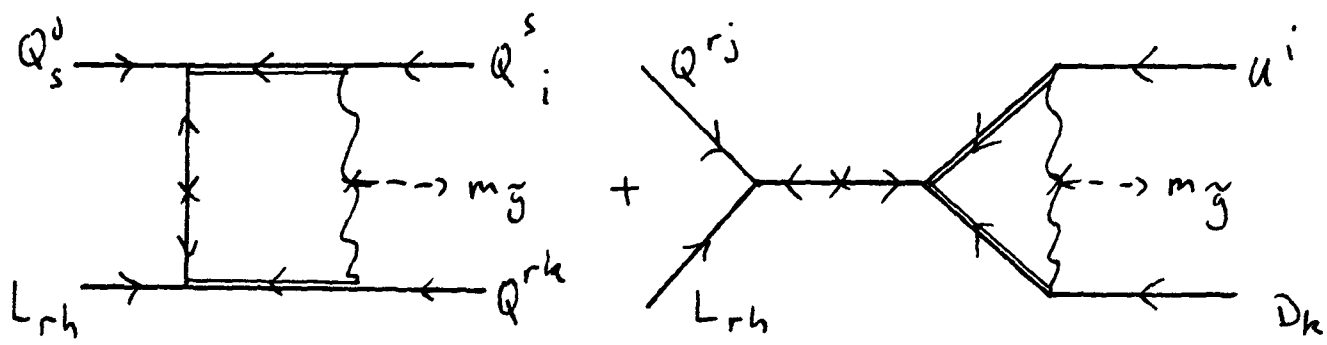


fig 1.2.7. Graphs involving the lepton doublet as a decay product. The indices r, s are SU_2 indices, and are raised and lowered with the antisymmetric ϵ_{rs}. The other indices h, i, j, k are generation indices.

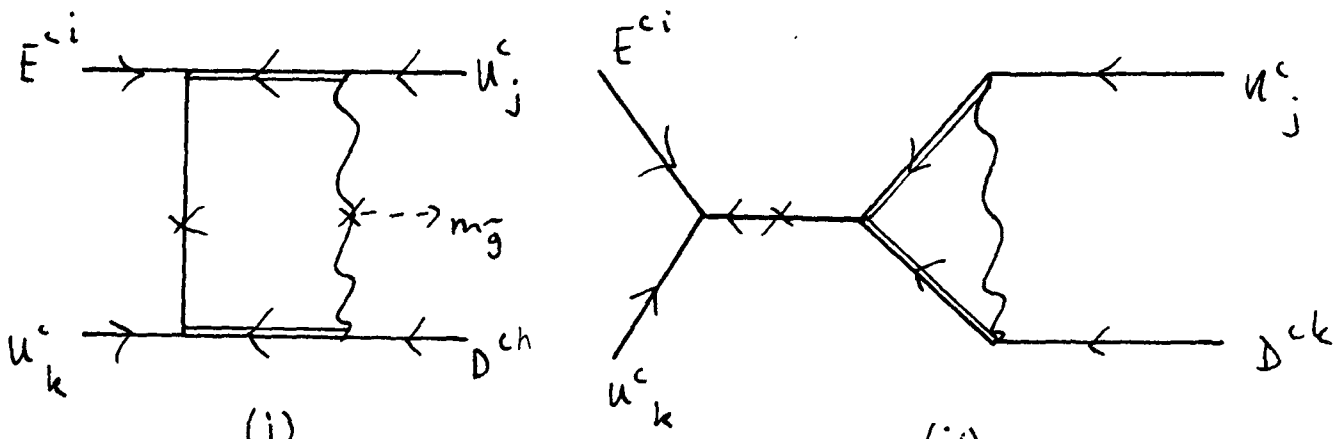


fig 1.2.8. Graphs involving the right-handed lepton.

$$f(m_1^2, m_2^2, m_3^2) = \int_0^1 dx_1\, dx_2\, dx_3\, \frac{\delta(1-x_1-x_2-x_3)}{x_1 m_1^2 + x_2 m_2^2 + m_3^2 x_3}$$

$$= \frac{1}{m_2^2 - m_3^2}\left[\frac{m_2^2}{m_1^2-m_2^2}\log\frac{m_1^2}{m_2^2} - \frac{m_3^2}{m_1^2-m_3^2}\log\frac{m_1^2}{m_3^2}\right] \qquad (1.2.15)$$

In the approximation that all the left- and right-handed squarks have a common mass $m_{\tilde{u}} = m_{\tilde{d}}$ (except the stop, $\tilde{t}$, $\wedge$ (1.2.14) reduces to

$$A_1 \approx -(d^j \ell_h)(u_i q^k) \sum_{r=1}^{3}\sum_{w=1}^{2} \frac{m_{\tilde{g}}}{m_{r,w}} \frac{g_3^2\, T(R_3)}{12\pi^2} \sum_{m=1}^{3} \tilde{K}^{i}_{uL\,3}\, \tilde{K}^{\dagger\,3}_{uL\,m}$$
$$\cdot (L_u \lambda_u^{(r)} L_d^\dagger)^m{}_j\, (L_\ell \lambda_d^{(w)} L_q^\dagger)^h{}_k\, f_{wt} \qquad (1.2.16a)$$

where

$$f_{wt} = f(m_{\tilde{t}}^2, m_{\tilde{g}}^2, m_{\tilde{d}}^2) - f(m_{\tilde{u}}^2, m_{\tilde{g}}^2, m_{\tilde{d}}^2) \qquad (1.2.16b)$$

The Fierz identity (0.12) has been used to eliminate one of the terms in (1.2.15). Note also that $m^2_{\tilde{u}}$ and $m^2_{\tilde{d}}$ refer to the first generation squark masses, not to the complete mass matrices.

The second graph, fig 1.2.7ii, gives the amplitude (in the same approximation)

$$A_2 \approx -(q^j \ell_h)(u_i d^k) \sum_{r=1}^{3}\sum_{w=1}^{2} \frac{m_{\tilde{g}}}{m_{r,w}} \frac{g_3^2\, T(R_3)}{12\pi^2} (L_\ell \lambda_d^{(w)} L_q^\dagger)^h{}_j$$
$$\cdot \sum_{m=1}^{3} \tilde{K}^{i}_{uL\,3}\, \tilde{K}^{\dagger\,3}_{uL\,m} (L_u \lambda_u^{(r)} L_d^\dagger)^m{}_k\, f_{wt} \qquad (1.2.17)$$

(ii) Decays involving the right-handed leptons.

The graphs are shown in fig 1.2.8. The amplitude of the first is

$$B_1 \approx -\sum_{r=1}^{3}\sum_{w=1}^{2} (e^{ci}\, d^{ch}\, u^c_j\, u^c_k) \frac{m_{\tilde{g}}}{m_{r,w}} \frac{g_3^2\, T(R_3)}{12\pi^2} \sum_{m=1}^{3} (\tilde{K}_{uR})^j{}_i\, (\tilde{K}^\dagger_{uR})^3{}_m$$
$$\cdot (R_u \lambda_u^{(r)} R_e^\dagger)^m{}_i\, (R_u \lambda_d^{(w)} R_d^\dagger)^k{}_h\, f_{wt} \qquad (1.2.18)$$

and that of the second is

$$B_2 \approx -\sum_{r=1}^{3}\sum_{w=1}^{2}(e^{ci}\,d^{ch}\,u^{c}_{j}\,u^{c}_{k})\frac{m_{\tilde{g}}}{m_{r,w}}\,\frac{g_3^2\,T(R_3)}{12\pi^2}\sum_{m=1}^{3}(\tilde{K}_{uR})^{j}_{\ 3}$$

$$\cdot\,\tilde{K}^{\dagger\ 3}_{uR\ \ m}\,(R_u\,\lambda_d^{(w)}\,R_d^{\dagger})^{m}_{\ \ h}\,(R_d\,\lambda_u^{(r)}\,R_e^{\dagger})^{k}_{\ \ i}\,f_{wt} \qquad (1.2.19)$$

§1.2.e Higgs triplet mass generation

The general amplitudes of (1.2.16-19) show a complicated pat tern of decay modes involving all the allowed external states e, μ, d, s, u and ν^i. This pattern simplifies considerably with the Higgs superpotential described below, and shows the dominant proton decay mode in this model to be $p \to \mu^+K^0$.

The conclusion from §§1.0.f, 1.1.e & 1.2.b was that all the Higgs triplets should have large masses (1.2.6) while, of the doublets, only the combinations

$$G = \frac{1}{v_G}(v_H H + v_{H'} H') \qquad (1.2.20a)$$

and

$$K = \frac{1}{v_K}(v_1 K_1 + v_2 K_2 + v_3 K_3) \qquad (1.2.20b)$$

should remain massless. This can be achieved by two separate superpotentials : P_1, which gives a large supersymmetric mass to both G and K; and P_2, which gives a large mass to the triplet part only of the combinations (1.2.20), by the 'missing multiplet' mechanism[59,60].

For such a mechanism (as will be described below in detail), the SU_5 symmetry must be spontaneously broken with a Higgs Σ belonging to the 75 representation of SU_5. The v.e.v. which breaks SU_5 to $SU_3 \times SU_2 \times U_1$ is

$$\langle \Sigma^{\alpha\beta}_{\gamma\delta} \rangle = M_G \left(\delta_3{}^{\alpha\beta}_{\gamma\delta} + 2\delta_2{}^{\alpha\beta}_{\gamma\delta} - \tfrac{1}{2}\delta^{\alpha\beta}_{\gamma\delta} \right). \tag{1.2.21}$$

where

$$\delta^{\alpha\beta}_{\gamma\delta} = \delta^{\alpha}{}_{\gamma}\,\delta^{\beta}{}_{\delta} - \delta^{\alpha}{}_{\delta}\,\delta^{\beta}{}_{\gamma}$$

$$\delta_2{}^{\alpha\beta}_{\gamma\delta} = \begin{cases} \delta^{\alpha\beta}_{\gamma\delta} & \text{if } \alpha,\ \beta,\ \gamma,\ \delta \text{ all take the values 4 or 5} \\ 0 & \text{otherwise} \end{cases}$$

$$\delta_3{}^{\alpha\beta}_{\gamma\delta} = \begin{cases} \delta^{\alpha\beta}_{\gamma\delta} & \text{if } \alpha,\ \beta,\ \gamma,\ \delta \text{ all take the values 1,2 or 3} \\ 0 & \text{otherwise} \end{cases} \tag{1.2.22}$$

and this v.e.v. provides a natural projection from the 45 representation into $(\bar{3},1,-2/3) \oplus (1,2,1)$ of $SU_3 \times SU_2 \times U_1$ by the definition

$$H'_\alpha = \frac{1}{M_G} L^{\delta}_{\beta\gamma} \langle \Sigma^{\beta\gamma}_{\alpha\delta} \rangle \tag{1.2.23}$$

which is equivalent to (1.2.13), for the triplet part.

Then, the superpotential P_1 can be taken as

$$P_1 = M(\alpha_1 H + \frac{\beta_1}{M_G} L\Sigma) V + M(K^1_{he}, K^2_{he}) \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \tag{1.2.24}$$

where V, U_1 and U_2 belong to the 5, $\bar{5}$, and $\bar{5}$ representations of SU_5 (they only appear in P_1, as 'auxiliary' multiplets); this gives a mass M to both the doublet and triplet parts of the combinations

$$\alpha_1 H_1 + \beta_1 H'$$

$$K^1_{he} = \gamma_1 K_1 + \gamma_2 K_2 + \gamma_3 K_3$$

and

$$K^2_{he} = \delta_1 K_1 + \delta_2 K_2 + \delta_3 K_3 \tag{1.2.25}$$

for some numbers α, β, γ and δ.

Finally, the remaining light triplets are made heavy by combining them with the multiplets θ_1 and θ_2 belonging to the $\bar{5}\bar{0}$ and **50** representations.* The term $H\theta_1\Sigma$ gives a mass (when Σ develops a v.e.v.) only to the triplet component of H :

$$H\,\theta_1\langle\Sigma\rangle = M_G \sum_{r,s=4,5} H_a\,\theta_1{}^{a\,rs}_{\;\;rs} \qquad (a = 1..3) \tag{1.2.26}$$

(because $\theta_1{}^{\alpha\beta\gamma}_{\;\;\delta\epsilon}$ is antisymmetric in its upper indices, $\theta^{\alpha\,rs}_{\;\;rs}$ vanishes for $\alpha = 4, 5$). Using similar terms for the triplet components of L and K_r, the 'triplet only' superpotential is

$$P_2 = (\alpha_2 H + \beta_2 L)\theta_1\Sigma + \sum_{r=1}^{3} \epsilon_r K_r\,\theta_2\Sigma - M\theta_1\theta_2 \tag{1.2.27}$$

§1.2.f The dominant decay amplitude.

P_1 and P_2 define a mass matrix for the components of H, L, K_r, U, V, θ_1 and θ_2 which is to be diagonalised to find the mass eigenstates. When P_1 (which gives mass to both doublet and triplet components) is written in terms of the mass eigenstates, only combinations of H, L and K_r which are orthogonal to the light doublets may appear. Thus, in P_2, the heavy triplet eigenstates, orthogonal to those of P_1, are in the same combination of H, L and K_r as are the light doublets (1.2.20).

The mass matrix in terms of the triplet components from P_2 is therefore

* Note that in these models the gauge coupling in general becomes infinite below the Planck scale as the extra Higgs representations make a large contribution to the β-function (cf §1.1.f)

$$(G \quad \theta_2)\begin{pmatrix} 0 & M_G \\ M_G & -M \end{pmatrix}\begin{pmatrix} K \\ \theta_1 \end{pmatrix} \qquad (1.2.28)$$

where G and K are defined by (1.1.20) and (1.1.21); if M can be taken to be a little larger than M_G (say $M \gtrsim 3M_G$) then the lighter eigenstate of (1.2.28), which is roughly the combination

$$G' = (1 - M_G^2/2M^2)\,G - M_G/M\;\theta_2 \qquad (1.2.29a)$$

$$K' = (1 - M_G^2/2M^2)\,K - M_G/M\;\theta_1 \qquad (1.2.29b)$$

has mass M_G^2/M and is therefore the lightest of all the Higgs triplets (those in P_1 have mass of order M) : its contribution to the decay amplitudes is therefore the largest. Setting

$$m_{r,1} = \frac{M' v_r v_H}{v_K v_G}$$

$$m_{r,2} = \frac{M' v_r v_{H'}}{v_K v_G} \qquad (1.2.30)$$

in (1.2.16-19) gives the contribution from these eigenstates ($M' = \frac{M_G^2}{M}(1-\frac{M_G^2}{2M^2})$ is the mass found by diagonalising P_1 and P_2). Evaluating the sum $\sum_{r=1}^{3}\sum_{w=1}^{2}$ in (1.2.16-19) leaves simply the mass matrices (1.1.12)

$$M^u = p_u^1 + p_u^2 + p_u^3$$

$$M^d = p_d + q_d$$

divided by v_K and v_G; the dominant contribution from figs 1.2.7&8 is therefore proportional to $m_s m_t$ (which are the largest masses that can occur). The sum of these contributions is[62]

$$-\frac{g_3^2}{24\pi^2}\frac{m_g\, m_s\, m_t}{M' v_a v_K} f_{ut}\, K'_3 \{ [(\nu_\mu d)_L (us)_L - (\nu_\mu s)_L (du)_L]\cdot$$
$$K'_3(1-\beta_{2u}^L) + (\mu u)_L (us)_L \sin\theta_c\, K^3{}_2\,(1-\beta_{2u}^L)$$
$$-2(\mu u)_R (us)_R \sin\theta_c\, K^3{}_2\,(1-\beta_2{}^R{}_u)\} \qquad (1.2.31)$$

where K is the Kobayashi-Maskawa matrix (1.1.10) and $\beta_{2U}{}^{L,R}$

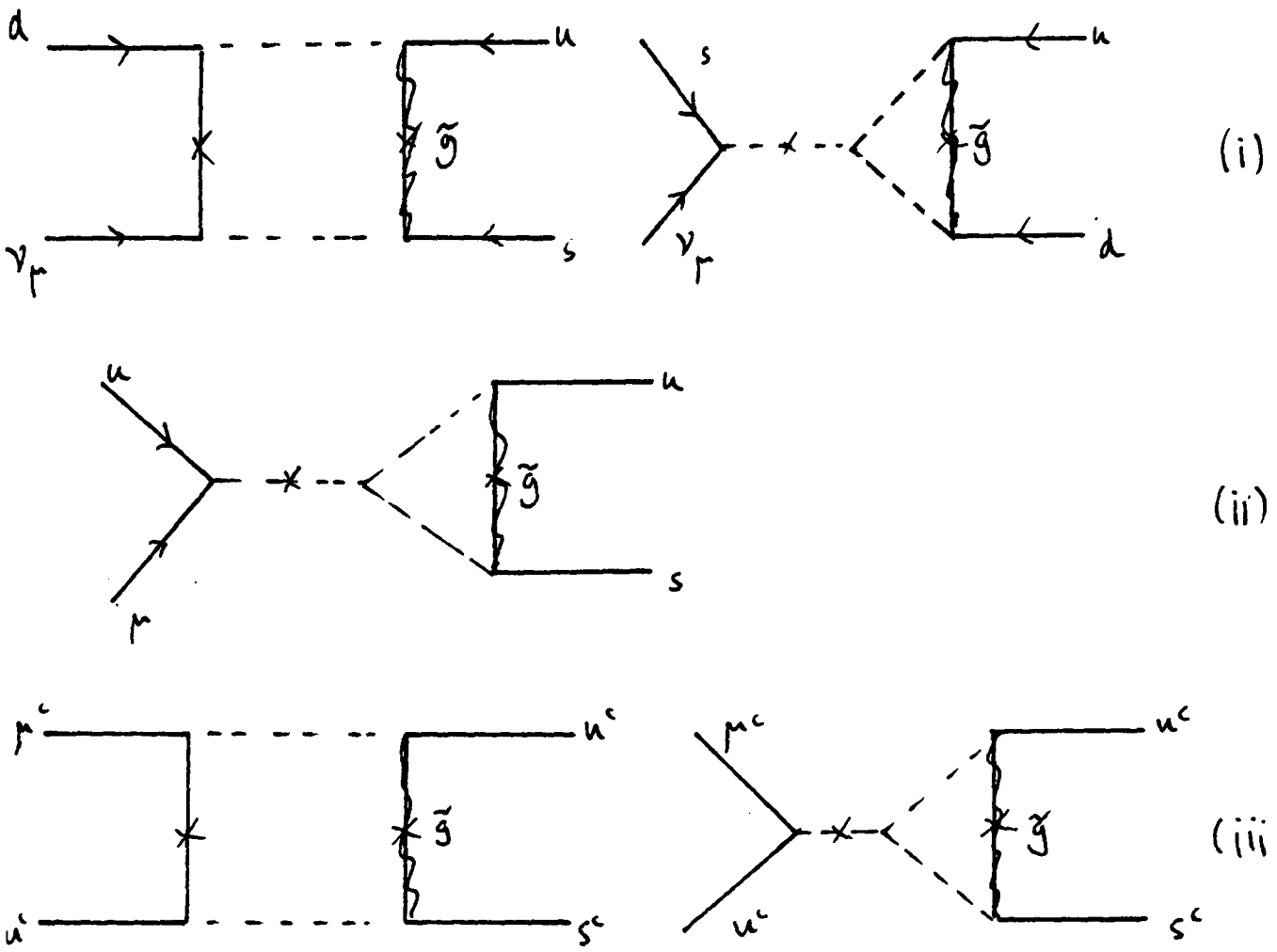


fig 1·2·9. Dominant graphs (in component form) with Higgs masses given by (1·2·24, 27)

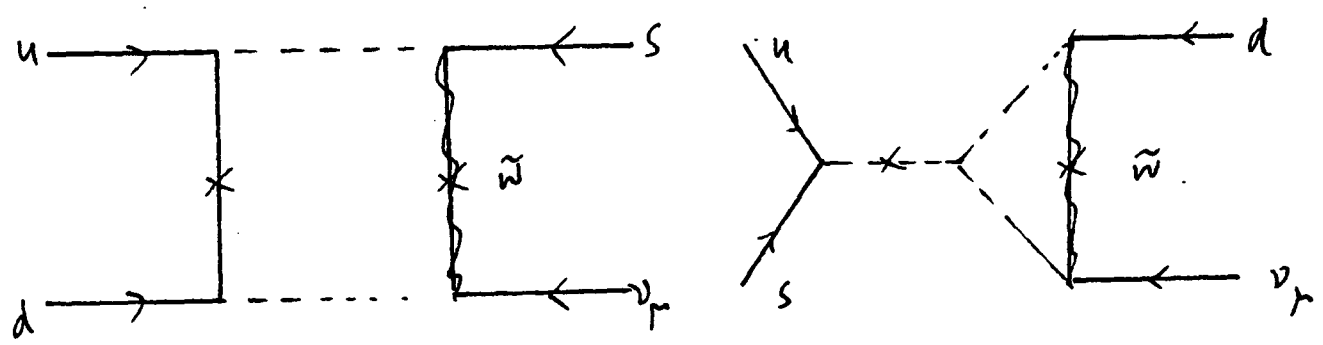


fig 1·2·10. Dominant graphs for wino-mediated p-decay

are the parameters of the gluino interaction defined by (1.1.28). The graphs which contribute to (1.2.31) are shown in fig 1.2.9. The presence in (1.2.31) of the factor f_{ut} reflects the earlier conclusion that gluino contributions vanish if the scalar masses are degenerate. Also if $f_{ut} \neq 0$ but $\beta_{2u} = 1$ (i.e. flavour-diagonal gluino couplings) then (1.2.31) vanishes - but smaller contributions (neglected here) to the ν-channels do not.

§1.2.g Comparison with wino-mediated decays

As mentioned above, the dominant contribution from wino-mediated decays arises from flavour diagonal wino interactions; even though (it is assumed) $m_{\tilde{g}}$ is greater than $m_{\tilde{w}}$, the wino amplitude is not suppressed by a further mixing angle $\tilde{K}$ at the gaugino vertex. The dominant mode is from the graphs of fig 1.2.10[63,64], which contribute the amplitude

$$\frac{-g_2^2}{24\pi^2}\,\frac{m_{\tilde{w}}\, m_s\, m_c}{M' v_4 v_K}\, f(m^2_{\tilde{d}}, m^2_{\tilde{w}}, m^2_{\tilde{\nu}}) \sin\theta_c \left[(\nu_\mu d)(s u) - (\nu_\mu s)(u d)\right] \qquad (1.2.32)$$

To compare this to the principal gluino-induced mode, which from (1.2.31) is $p \to \mu^+_R K^0$, the amplitudes must be converted into decay rates; and also estimates for the gluino masses must be used. If it is assumed that the gluino masses are less than the squark masses, then from (1.2.15) it follows approximately that

$$f_{ut} \approx \frac{m_{\tilde{t}}^2 - m_{\tilde{u}}^2}{m_{\tilde{u}}^4} \qquad (1.2.33a)$$

$$f(m^2_{\tilde{d}}, m^2_{\tilde{w}}, m^2_{\tilde{\nu}}) \approx \frac{1}{m^2_{\tilde{u}}} \qquad (1.2.33b)$$

where in the last equation it is assumed $m^2\tilde{\nu} \sim m^2\tilde{d} \sim m^2\tilde{u}$.

In ref.[65] the amplitudes (1.2.31) and (1.2.32) are converted into decay rates by changing them into equivalent phenomenological chiral Lagrangians. The result of this is

$$\frac{\Gamma^{\text{gluino}}(p \to \mu^+_R K^0)}{\Gamma^{\text{wino}}(p \to \bar{\nu}_\mu K^+)} \sim \left[\frac{m_t K^3_2 K^1_3 (1-\beta^R_{2u}) g_3^2 m_{\tilde{g}} (m^2_{\tilde{u}} - m^2_{\tilde{t}})}{m_c K^2_1 g^2_2 m_{\tilde{w}} m^2_{\tilde{u}}}\right] \times 0.06 \qquad (1.2.34)$$

Taking (as in 1.2.11)

$$m_{\tilde{g}}/m_{\tilde{w}} \approx g^2_3/g^2_2 \qquad (1.2.35)$$

and

$$\frac{m^2_{\tilde{u}} - m^2_{\tilde{t}}}{m^2_{\tilde{u}}} \sim 1 \qquad (1.2.36)$$

which follows from (1.1.18) and (1.1.27) when $m_t \gtrsim 40$ GeV, the ratio (1.2.34) can be written

$$\frac{\Gamma^{\text{gluino}}(p \to \mu^+_R K^0)}{\Gamma^{\text{wino}}(p \to \bar{\nu}_\mu K^+)} \sim 0.06 \times \left[\frac{m_t \sin^4\theta_c \, g_3^4}{m_c g_2^4}\right]^2 \qquad (1.2.37)$$

which is $\gtrsim 1$ for $m_t \gtrsim 40$ GeV, $\alpha_2 = 1/30$, $\alpha_3 = 1/10$.

Thus, in the non-minimal SU_5 model considered in this chapter, gluino-mediated proton decays are expected to be at least as important, for $m_t \gtrsim 40$ GeV, as those which proceed through wino exchange. The latter involve only left-handed operators, and the dominant mode is $p \to \nu_\mu K^+$, whereas gluino-mediated decays can produce the modes $p \to \mu^+_{L,R} K^0$, $\nu_\mu K^+$. With the Higgs masses specified here, the reaction $p \to \mu^+_R K^0$ is faster by a factor of about 2 than the other two modes.

SECTION 1.3 Contributions to the K_L-K_S mass difference

Strangeness changing neutral currents, which are naturally suppressed in the standard model because gauge boson interactions are always flavour diagonal[67], do arise in supersymmetric theories if there are flavour-changing gaugino interactions; for example, the graphs of fig 1.3.1 contribute to K_0-$\bar{K}_0$ mixing terms to the effective action, and the experimental K_L-K_S mass difference, which can be measured very accurately, could, in principle, provide bounds on model parameters such as the gaugino mass[66,68,69].

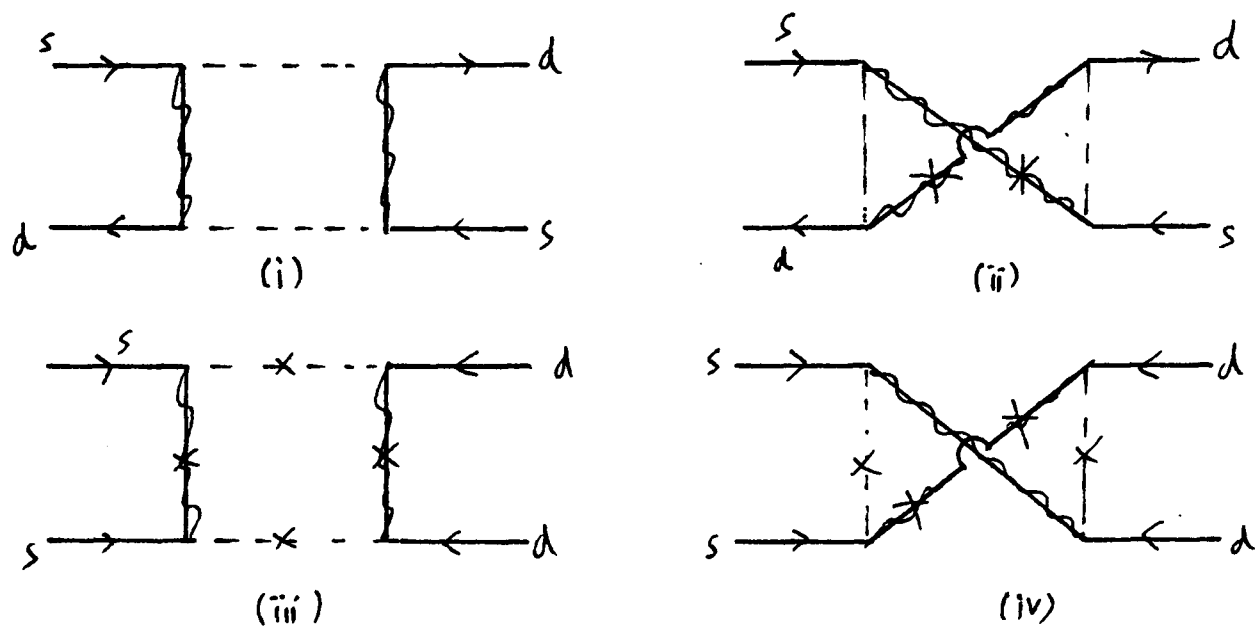


fig 1.3.1 $s\bar{d} \to \bar{s}d$ transitions which contribute a $K_0 - \bar{K}_0$ mixing term.

In this section, I will briefly discuss the reasons why there are, in fact, no such bounds arising from the contributions if fig 1.3.1 in either the minimal or the non-minimal SU_5 theory (despite the presence of flavour-changing gaugino vertices). Firstly, consider the graphs of fig 1.3.1 which are mediated by neutral gauginos - of these, clearly the gluino contributions are the largest. The internal scalar lines represent $\tilde{d}$, $\tilde{s}$ and $\tilde{b}$ squarks (see fig 1.3.2), and in the limit of degenerate 'down' squark

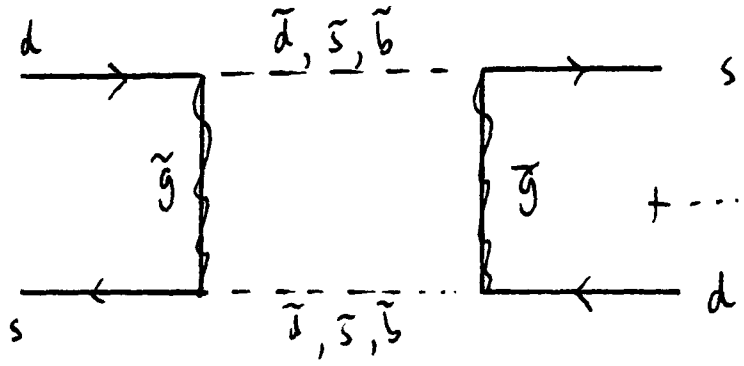


fig 1.3.2. Gluino - mediated $\Delta s = 2$ graph(s)

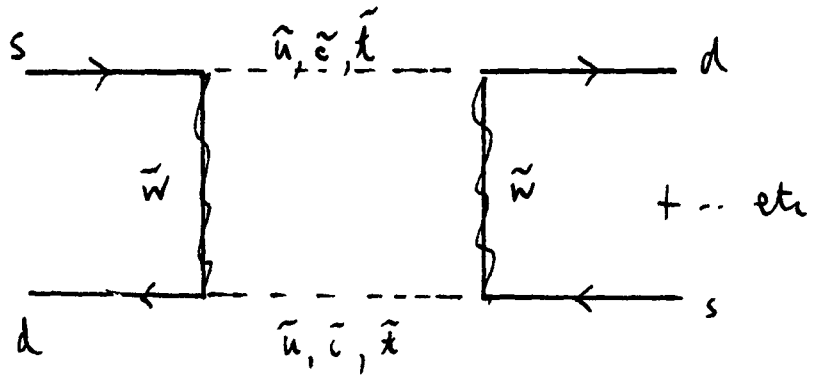


fig 1.3.3. Wino - mediated $\Delta s = 2$ graphs.

masses a 'super-GIM' mechanism[66,70] operates and these graphs (summed over the internal states) vanish : the $\tilde{d}$ squark propagator can be written

$$\langle \tilde{d}^i \, \tilde{d}^\dagger{}_k \rangle \sim \sum_{j=1}^{3} \tilde{K}_d{}^i{}_j \, \frac{1}{p^2 + m^2_{\tilde{d}(j)}} \, \tilde{K}^{\dagger j}_{d \quad k} \qquad (1.3.1)$$

so that if $m^2_{\tilde{d}(j)}$ is independent of j, this propagator is proportional to the unit matrix in generation space and cannot change flavour. In both minimal and non-minimal SU_5, the mass difference between down squarks is small (cf §1.2.c). Bouquet and others[42] have calculated the resulting amplitude using Renormalisation Group improved squark masses, and find that for all values of the gluino mass, the contribution from fig 1.3.2 does not contradict the experimental K_L-K_S mass difference. The same result is found using the 1-loop $m^2_{\tilde{d}L}$ matrix of (1.1.26).

Secondly, when the $s\bar{d} \to \bar{s}d$ transition is mediated by wino exchange (as in fig 1.3.3), the internal squarks are $\tilde{u}$, $\tilde{c}$ and $\tilde{t}$. Ellis and Nanopoulos have analysed this case[68] and they find that the K_L-K_S mass difference translates into the relation

$$\frac{1}{m_{\tilde{w}}^2}\,\frac{\alpha_2^2}{16}\,\frac{(m^2_{\tilde{u}(i)}-m^2_{\tilde{u}})(m^2_{\tilde{u}(j)}-m^2_{\tilde{u}})}{m^4_{\tilde{u}}}\,\tilde{K}^{i}_{u1}\,\tilde{K}^{i}_{u2}\,\tilde{K}^{j}_{u1}\,\tilde{K}^{j}_{u2} < \tfrac{1}{2}\times 10^{-12}\,4e\tilde{V}^2 \qquad (1.3.2)$$

assuming $m_{\tilde{w}}^2 \gg m_{\tilde{u}}^2$. Using the relation (1.2.36) and the mixing angles (1.1.11) it is easy to see that this inequality is satisfied. If, on the other hand, $m_{\tilde{w}}^2 \ll m_{\tilde{u}}^2$, then (1.3.2) becomes a bound on the squark masses (with the replacement $m_{\tilde{w}} \to m_{\tilde{u}}$) which is also satisfied by the 1-loop squark masses in both the minimal and non-minimal SU_5 models.

CHAPTER TWO

N=2 SUPERSYMMETRY

SECTION 2.0 Introduction

§2.0.a Why N=2 supersymmetry?

There are two principal reasons for studying theories with two supersymmetries : firstly, there are no arbitrary Yukawa couplings in the N=2 Lagrangian (this will be shown below) - all Yukawa interactions are fixed by the gauge coupling constant; secondly, with an appropriate choice of matter representations (see eq 2.1.5, below) the exact N=2 Lagrangian is ultraviolet finite to all orders of perturbation theory[75-78]. Since there are also explicit supersymmetry breaking terms which can be added to the Lagrangian without destroying this finiteness[6,7,8,71], the N=2 theories offer the possibility of constructing a finite model of particle interactions.

However, there are many symmetries in the exact Lagrangian which are not observed at low energy. The task in constructing a finite particle model will be to break these symmetries without destroying the finiteness properties of the Lagrangian. The most obvious problem is the further doubling of the states of an N=1 theory caused by the introduction of a further supersymmetry. One consequence of this is that to each left-handed fermion there corresponds a right-handed fermion in the same representation; but these

cannot be the left- and right-handed states of the same particle, or there could be no chiral gauge interactions.

In this chapter, some aspects of N=2 theories are studied which are relevant for the problem of constructing a finite, realistic N=2 model. In §2.1, the general set of finiteness-preserving soft operators is derived by explicitly evaluating the 1-loop infinities produced by adding gauge invariant soft terms to the N=2 theory. In the final section, §2.2, the possibility that finiteness-preserving terms might produce a realistic model below the Grand Unification scale is discussed. Two problems in the N=2 model proposed by Del Aguila *et al.*[11] are investigated : firstly, there is a Goldstone boson arising from the spontaneous breakdown of a global U_1 symmetry. One possible means of removing this massless mode is discussed, but is shown not to be a viable suggestion. Secondly, neutrinos in the model naturally acquire a large Dirac mass, because the right-handed neutrinos form SU_{2R} doublets with the right-handed charged leptons. To satisfy a new bound on the mass of the ν_τ,

it is shown that the two supersymmetries of the model have to be broken at greatly differing scales : one near the weak scale to protect the gauge hierarchy, and one at about 10^9 GeV to break SU_{2R}. It is shown, finally, that such a situation is not natural in the N=2 model considered here.

In the rest of this section, the N=2 multiplet structure and invariant action (without central charges) are

derived[72,73].

§2.0.b. The N=2 multiplets.

The constraints on the N=2 superfield which restrict the general representation (0.7) to an irreducible representation of the N=2 algebra (0.6) are not as easily solved as they are for N=1 superfields (cf eqs 0.10a and 0.13); although irreducible superfields can be defined when N=2[74], it is easier in practice to use the component N=1 superfields. The irreducible representations of (0.6) can be built by considering the individual N=1 multiplets of each supersymmetry generator Q_i. For example, the change induced by Q_1 in the scalar component of an N=1 chiral superfield Φ is, as usual[12],

$$\delta^1_\epsilon \varphi \equiv (\epsilon Q_1 + \bar{\epsilon} \bar{Q}^1)\varphi = \sqrt{2}\,\epsilon\,\psi_\phi \qquad 2.0.1$$

and (ϕ, ψ_ϕ) is an N=1 chiral multiplet under the transformations generated by Q_1. Similarly, if under transformations generated by Q_2 the change in ϕ is

$$\delta^2_\epsilon \varphi = \epsilon Q_2 \varphi = \epsilon \lambda \qquad 2.0.2$$

then $(\phi, \frac{\lambda}{\sqrt{2}})$ is also an N=1 chiral mutiplet.

Without the central charges Z_{ij}, only one further independent field is generated by more supersymmetry transformations : from (0.6) it follows that, provided all fields satisfy their equations of motion,

$$\delta^1_\epsilon \psi_\phi = \bar{\epsilon}\bar{Q}^1\psi_\sigma = i\sqrt{2}\,\bar{\epsilon}\sigma^\mu\partial_\mu\varphi \qquad 2.0.3a$$

$$\delta^2_\epsilon \lambda = \bar{\epsilon}\bar{Q}^2\lambda = \delta^1_\epsilon \psi_\phi \qquad 2.0.3b$$

$$\delta^1_\epsilon \lambda = -\delta^2_\epsilon \psi_\sigma = 2\sigma^{\mu\nu}\epsilon A_{\mu,\nu} \qquad 2.0.3c$$

where A_μ is a spin-1 vector field. Thus, (λ, A_μ) is an N=1 vector multiplet under Q_1 and (ψ_ϕ, A_μ) is a vector multiplet under Q_2. This is summarised in fig 2.0.1. Further Q_1 or Q_2 transformations do not generate any new on-shell fields, so the set $(\phi, \psi_\phi, \lambda, A_\mu)$ is an N=2 multiplet. It includes the gauge field A_μ so it is called the Yang-Mills multiplet, and it can be represented on N=1 superspace by the superfields (Φ, V) where Φ is a chiral field with components

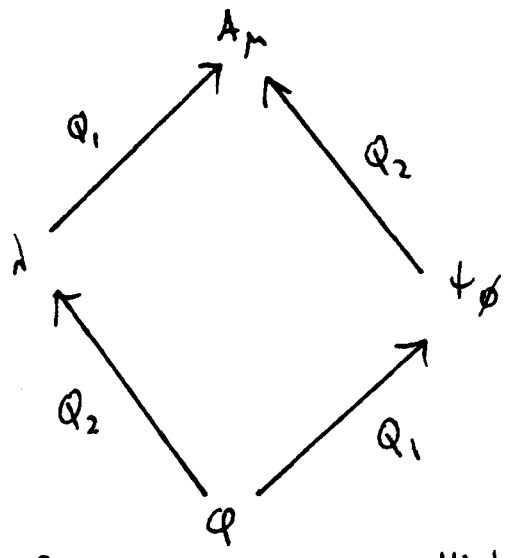


fig 2.0.1 . N=2 YM multiplet.

$$\Phi(z) = \varphi(y) + \sqrt{2}\,\theta\psi_\phi(y) + \theta^2 F_\varphi(y) \qquad 2.0.4a$$

and V is a vector field defined in the Wess-Zumino gauge by

$$V(z) = i\bar{\theta}^2\theta\lambda - i\theta^2\bar{\theta}\bar{\lambda} + \theta\sigma^\mu\bar{\theta}A_\mu + \tfrac{1}{2}\theta^2\bar{\theta}^2 D \qquad 2.0.4b$$

The N=2 hypermulitplet is constructed by considering a scalar field x as belonging both to a chiral multiplet

$$\delta^1_\epsilon x = \sqrt{2}\,\epsilon\psi_x \qquad 2.0.5$$

and to an anti-chiral multiplet

$$\delta^2_\epsilon x = \sqrt{2}\,\bar{\epsilon}\bar{\psi}_y \qquad 2.0.6$$

Then, from the algebra (0.6) it follows that

$$\delta^1_\epsilon \bar{\psi}_y = -\delta^2_\epsilon \psi_x = -i\sqrt{2}\,\bar{\epsilon}\sigma^\mu\partial_\mu y^\dagger \qquad 2.0.7$$

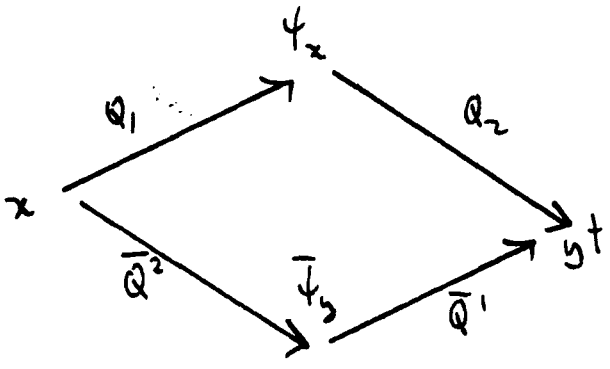


fig 2-0-2. N=2 hypermultiplet.

and once again the set of fields $(x, y^\dagger, \psi_x, \psi_y)$ form a closed multiplet under the N=2 algebra, as illustrated in fig 2.0.2. The fields (x, ψ_x) and (y, ψ_y) both form chiral N=1 multiplets, and may be considered components of the N=1 superfields X and Y respectively.

These two N=2 multiplets, the Yang-Mills Multiplet (Φ, V) and the 'matter' multiplet (X,Y) are the multiplets which contain only particles of spin ≤ 1, so these are used to construct an N=2 gauge theory.

§2.0.c The N=2 Gauge-Invariant Action

The N=2 action must be constructed to fulfill the same criteria as the N=1 action (§0.d), viz.

(i) invariance under (N=2) superPoincaré transformations

(ii) invariance under local gauge transformations, and

(iii) renormalisability.

The first of these requirements is really three conditions : Poincaré invariance, θ-space translational invariance, and θ-space rotational invariance. Of these, the first two are easily satisfied by using N=1 supersymmetric combinations of the component superfields V, Φ, X and Y. The SU_2 rotations

$$\theta^i \rightarrow \frac{i}{2} \left(d^{kj} T_{kj} \right)^i{}_l \, \theta^l \qquad 2.0.8$$

generated by T_{ij} are represented on the component fields by the induced rotation (from 0.7c)

$$Q_i \rightarrow \frac{i}{2} Q_j \left(d^{kl} T_{kl} \right)^j{}_i \qquad 2.0.9$$

so that, as

$$\left(\psi_\phi , \frac{1}{\sqrt{2}} \right) = \frac{1}{\sqrt{2}} \left(\phi_1 , \phi_2 \varphi \right) \qquad 2.0.10$$

(from 2.0.1&2) then these fields form a doublet

$$\chi = \left(\psi_\phi , \frac{A}{\sqrt{2}} \right) \qquad 2.0.11$$

under SU_2. The normalisation factor of $\sqrt{2}$ is required for the kinetic term, see eq 2.0.18, below. The remaining fields ϕ, A_μ of the vector multiplet are SU_2 invariants.

Similarly, from (2.0.5) and (2.0.7) it follows that

$$\begin{aligned} Q_{1\alpha} \bar{\psi}_{y\dot{\alpha}} &= -i\sqrt{2}\, \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu y^\dagger \\ Q_{2\alpha} \bar{\psi}_{y\dot{\alpha}} &= -i\sqrt{2}\, \sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu x \end{aligned} \qquad 2.0.12$$

so that x and $y^\dagger$ also form an SU_2 doublet,

$$z = \left(y^\dagger \quad x \right) \qquad 2.0.13$$

Thus the SU_2 invariance is maintained by taking only inert combinations of the doublets χ and z.

Gauge invariance for the N=1 superfields is ensured by using only the invariant F- and D-terms listed in (0.22); thus the kinetic terms possible are*

* note $tr \equiv \frac{\text{Trace}}{C_2(a)}$ in all this chapter.

$$\frac{tr}{64 g^2} \int d^2\theta \, W^\alpha W_\alpha + h.c. \qquad 2.0.14$$

for the gauge field V (which is in the adjoint representation), and

$$\int d^4\theta \, \Phi^\dagger_\rho (e^{gV}) \Phi \qquad 2.0.15$$

for each of the chiral fields Φ. From (2.0.11), it follows that to maintain SU_2 invariance, the 'chiral vector' field Φ must also belong to the adjoint representation; similarly, if one of the N=1 matter fields X_ρ belongs to the representation ρ of the gauge group G, then from (2.0.13) it follows that $Y_\rho^\dagger$, the N=2 'partner' of X_ρ, must belong to the conjugate representation $\bar{\rho}$. It is this property which destroys chirality in an N=2 theory : for each left-handed fermion $\psi_{x\rho}$ in the representation ρ there is a corresponding right-handed fermion $\psi_{y\rho}$ in the same representation.

Bearing these considerations in mind, the only renormalisable gauge and global supersymmetry invariant combinations are

$$\int d^4x \int d^4\theta \, tr \, \Phi^\dagger e^{gV} \Phi e^{-gV}$$

$$\int d^4x \int d^4\theta \sum_\rho [X^\dagger_\rho \rho(e^{gV}) X_\rho + Y_\rho \rho(e^{gV}) Y^\dagger_\rho] \qquad 2.0.16a, 2.0.16b$$

$$\int d^4x \int d^2\theta \sum_\rho [\lambda_\rho Y_\rho \Phi X_\rho + m_\rho Y_\rho X_\rho] \qquad 2.0.16c$$

$$\int d^4x \int d^2\theta \, [\lambda \, tr \, \Phi^3 + m \, tr \, \Phi^2] \qquad 2.0.16d$$

in addition to (2.0.14). Not all of these terms are invariant under the SU_2 transformation defined by

$$X \rightarrow X a^\dagger$$

$$Z \rightarrow Z a^\dagger \qquad 2.0.17$$

(where all the other fields remain inert). To see this, one

can expand (2.0.16) in terms of component fields (in the Wess-Zumino gauge), keeping only the parts which are not invariant under SU_2 :

$$\frac{1}{64g^2}\int d^2\theta\, \mathrm{tr}\, W^2 + \mathrm{h.c.} = -\frac{i}{2}\,\mathrm{tr}\,\bar\lambda \not\partial \lambda + \cdots \tag{a}$$

$$\int d^4\theta\, \chi_\rho^\dagger \rho(e^{gV})\chi_\rho = \chi_\rho^\dagger \Box \chi_\rho + ig\,\chi_\rho^\dagger \rho(\not A)\chi_\rho + \frac{ig}{\sqrt{2}}\left(\chi_\rho^\dagger \rho(\lambda)\psi_{\chi\rho} - \bar\psi_{\chi\rho}\rho(\bar\lambda)\chi_\rho\right) + \cdots \tag{b}$$

$$\int d^4\theta\, Y_\rho \rho(e^{gV}) Y_\rho^\dagger = y_\rho \Box y_\rho^\dagger + ig\, y_\rho \rho(\not A) y_\rho^\dagger + \frac{ig}{\sqrt{2}}\left(y_\rho \rho(\lambda)\bar\psi_{y\rho}^{(c)}\right) + \cdots \tag{c}$$

$$\int d^4\theta\, \mathrm{tr}\,\Phi^\dagger e^{gV}\Phi e^{-gV} = \mathrm{tr}\left[i\bar\psi_\phi \not\partial \psi_\phi + \frac{g}{2}\bar\psi_\phi \not A \psi_\phi + \frac{ig}{\sqrt{2}}(\varphi^\dagger\lambda\psi_\phi^{(c)} + \cdots) + \cdots\right. \tag{d}$$

$$\int d^2\theta\left[\lambda_\rho Y_\rho \Phi \chi_\rho + m_\rho Y_\rho \chi_\rho + \frac{\lambda}{3}\mathrm{tr}\,\Phi^3 + \frac{m}{2}\mathrm{tr}\,\Phi^2\right] + \mathrm{h.c.} = -\lambda\lambda_\rho^* \chi_\rho^\dagger \varphi^2 y_\rho^\dagger$$
$$-m\lambda_\rho^* \chi_\rho^\dagger \varphi\, y_\rho^\dagger - \lambda_\rho y_\rho \psi_\phi \psi_{\chi\rho} - \frac{m}{2}\mathrm{tr}\,\psi_\phi^2 - \lambda^*\lambda_\rho y_\rho \varphi^\dagger{}^2 \chi_\rho \tag{e}$$
$$-\lambda_\rho \psi_{y\rho}\psi_\phi \chi_\rho - \lambda\varphi\,\mathrm{tr}\,\psi_\phi^2 + \cdots \qquad 2.0.18\text{a-e}$$

From the scalar potential terms in (d), it is clear that

$$m = \lambda = 0 \tag{2.0.19}$$

is required by SU_2 invariance. Eq (2.0.18a) forms the invariant

$$-i\,\bar\chi \not\partial \chi \tag{2.0.20}$$

with (2.0.18d), while (2.0.18b) can only form an invariant

$$\frac{ig}{\sqrt{2}}\left(z^\dagger \chi \psi_{\chi\rho} - \bar\psi_{\chi\rho}\bar\chi z\right) \tag{2.0.21}$$

with the fermion terms in (2.0.18e) provided

$$\lambda_\rho = -\lambda_\rho^* = -ig \tag{2.0.22}$$

for each representation ρ. If (2.0.22) holds, then the remaining terms are also SU_2 inert.

Thus, the general N=2 gauge invariant action for a set of massless hypermultiplets (X_ρ, Y_ρ) labelled by the representation ρ of gauge group to which they belong, is

$$\mathcal{L}_{N=2} = \int d^4\theta \left[\sum_\rho X_\rho^\dagger e^{gV} X_\rho + \sum_\rho Y_\rho e^{gV} Y_\rho^\dagger + \mathrm{tr}\, \Phi^\dagger e^{gV} \Phi e^{-gV} \right]$$
$$+ \int d^2\theta \left[\frac{1}{64 g^2} \mathrm{tr}\, W^2 - ig \sum_\rho Y_\rho \Phi X_\rho \right] + \text{h.c.} \qquad 2.0.23$$

In addition, the mass term

$$\int d^2\theta \sum_\rho m_\rho Y_\rho X_\rho + \text{h.c.} \qquad 2.0.24$$

may also be included.

SECTION 2.1 Finiteness in explicitly broken N=2 supersymmetry

The constraints from imposing one or more supersymmetries on a gauge-invariant Lagrangian are strong enough, in some cases, to ensure finiteness at one or more loops in perturbation theory. For example, there exist N=1 supersymmetric grand unified theories which are finite up to two[82] or three[83] loops; it has been shown[79,80] that any N=1 theory is two-loop finite if it is one-loop finite, and that in general an n-loop finite theory has vanishing β-function at n+1 loops[81]. However, there are no known examples of a chiral N=1 GUT which are finite beyond 3 loops[83].

As more supersymmetries are imposed, finiteness becomes more and more natural; the maximally supersymmetric Yang-Mills theory, which has N=4 supersymmetries, is finite to all orders of perturbation theory[78]; the N=2 Yang-Mills theory coupled to matter hypermultiplets is also finite provided the hypermultiplet representations satisfy one condition (2.1.5). More importantly, it has been shown that there is a set of explicit supersymmetry breaking operators which leave both the N=4[81,94,95] and N=2[6-8,71] theories finite at one loop of the perturbation expansion, and there exist arguments[88,71,6] to show that most of these operators preserve finiteness at all orders of perturbation theory. In ref.[6], this set was derived by evaluating explicitly the infinities produced at one loop by the soft

operators, and these results are given below. In addition, a gauge invariant insertion not considered there is included in the analysis, the resulting generalisation of the finite set is calculated, and shown to be a global SU_2 rotation of the original set. Finally, arguments are given to show that a certain subset of the general 1-loop finite Lagrangian does indeed preserve finiteness at all orders, but it is not possible to conclude on the basis of the graphical analysis carried out here that the *complete* 1-loop finite Lagrangian is finite to all orders.

§2.1.a The Non-renormalisation Theorem

In N=1 supersymmetry, the non-renormalisation theorem ensures that there are no F-terms generated beyond tree-level. When there are two supersymmetries an extension of this theorem imposes even stronger constraints on non-tree level graphs.

The N=2 non-renormalisation theorem states that all N=2 supergraphs contribute to the effective action terms which can be written as integrals over full N=2 superspace[75]. To see the implications of this, consider the lowest dimension graph in N=2 Yang-Mills, which has two external fields

$$\Gamma_\alpha = D_\alpha V + O(V^2) \qquad 2.1.1$$

The term which this graph (fig 2.1.1) conributes to the effective action is of the form

$$C = \int \frac{d\Lambda}{\Lambda^{1-p}} \int d^4x \int d^8\theta \, \Gamma^\alpha \, \mathcal{R} \, \Gamma_\alpha \qquad 2.1.2$$

where Ω represents the interaction and Λ is some momentum cut-off. Ω is constructed from dimensionless terms, so it follows that

$$0 = \dim C = p - 4 + 4 + \tfrac{1}{2} + 0 + \tfrac{1}{2} \qquad 2.1.3$$

As the theory is renormalisable, it is clear from (2.1.2) that C is a convergent term.

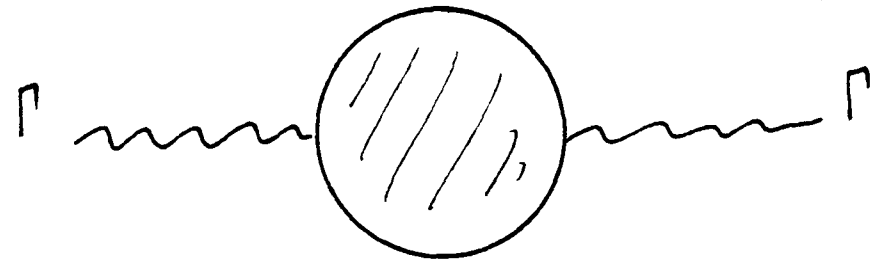


fig 2.1.1 Lowest dimension graph in N=2 Yang-Mills.

In ref.[75] it is shown that there are an infinite number of chiral ghosts in the N=2 Yang-Mills theory, and thereforeN=2 supergraphs can only be used at two or more loops. From this, because fig 2.1.1 is the lowest dimension gauge invariant graph with external gauge fields, it follows that N=2 Yang-Mills theories are finite at the two-loop and higher level. A similar argument can be used[74] to show that N=2 Yang-Mills theories coupled to matter hypermultiplets are also finite above two loops; this was first shown explicitly for the special case of N=4 Yang-Mills up to three-loop order, both in component form[84] and using N=1 superfields[85].

§2.1.b One-Loop Finiteness in the Unbroken Theory.

The N=2 non-renormalisation theorem does not apply to one-loop graphs with external Yang-Mills fields, and there are indeed such graphs which are ultraviolet divergent. The $\langle VV \rangle$ and $\langle \Phi^\dagger \Phi \rangle$ propagators at one loop (shown in fig 2.1.2) for the N=2 Lagrangian (2.0.23) both have an infinite part proportional to

$$g^2 \left(\sum_\sigma T(R_\sigma) - C_2(G) \right) \log \Lambda \qquad 2.1.4$$

where σ labels the matter representations. Similarly, the remaining one-loop divergent graphs (shown in fig 2.1.3) contribute an effective action which is also proportional to (2.1.4). It is interesting to note that the one-loop hypermultiplet propagators $\langle Y_\sigma Y_\sigma^\dagger \rangle$ and $\langle X_\sigma^\dagger X_\sigma \rangle$ vanish automatically. This is because the two contributions (fig 2.1.4) differ only by the sign of the ⟶ and ∿∿∿ lines (see 0.23); the cancellation is not surprising because chiral ghosts do not appear in the hypermultiplet graphs, so the N=2 non-renormalisation theorem can be applied even at one loop.

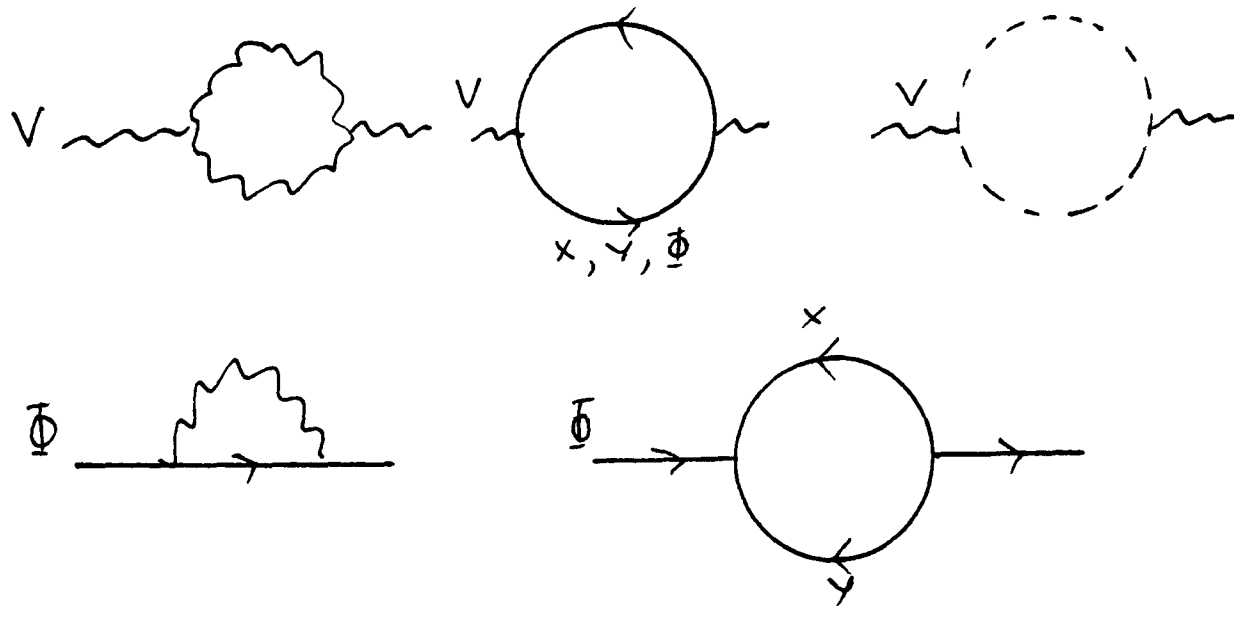


fig 2.1.2. One-loop Yang-Mills (V, $\bar{\Phi}$) propagators

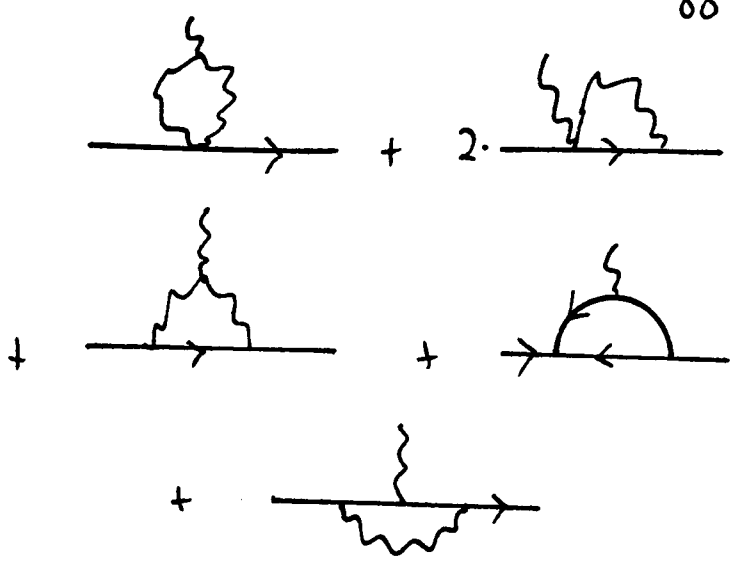


Fig.2.1.3. Divergent 3-point graphs

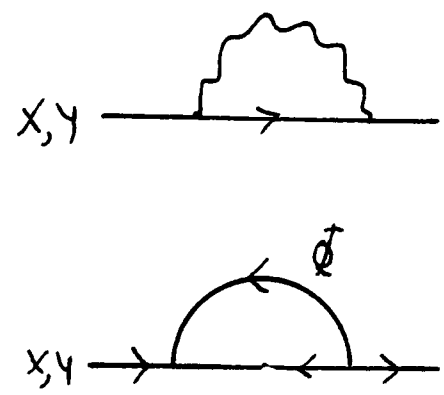


Fig 2.1.4. One-loop hypermultiplet propagator.

From (2.1.4) it now follows that the N=2 lagrangian (2.0.23) is finite to all orders provided the gauge group and matter representations are chosen to satisfy

$$\sum_r T(R_r) = C_2(G) \qquad 2.1.5$$

§2.1.c Soft Insertions

When N=2 supersymmetry is broken, the extended non-renormalisation theorem no longer applies and there may be divergent graphs of more than one loop. Nonetheless (as discussed in the introduction to this section) it has been shown that a subset of the soft (i.e. dimension ≤ 3) insertions that may be made into the exact Lagrangian do indeed preserve finiteness. To be precise, Parkes and West have shown[6,86] that no divergences are generated at one loop by the Lagrangian

$$\mathcal{L}^{(1)}_{finite} = \mathcal{L}_{N=2} + \mathcal{L}^{(1)}_{soft} \qquad 2.1.6a$$

where

$$\mathcal{L}^{(1)}_{soft} = \int d^2\theta \left\{ -\frac{\mu_0}{2} \mathrm{tr}(\Phi')^2 - \frac{\mu_1}{2} \mathrm{tr}\,\Phi^2 - \sum_r m_{2r} Y_r X_r \right\}$$
$$+ \sum_r \left\{ u_{xr} X_r^\dagger X_r - (u_\varphi + u_{xr}) Y_r Y_r^\dagger + l_r Y_r X_r + l_\varphi \mathrm{tr}\,\varphi^2 \right\}$$
$$+ u_\varphi \mathrm{tr}\,\varphi^2 \qquad 2.1.6b$$

and

$$\Phi = \Phi \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

is the SU_2 rotation of Φ defined by eq 2.0.17. The scalar components x_σ, y_σ, ϕ are defined in (2.1.16). It is easy to see that $\mathcal{L}^{(1)}_{soft}$ is not the most general set of finiteness-preserving soft operators : $\mathcal{L}_{N=2}$ is, of course, invariant under the global SU_2 rotation, but such a rotation applied to $\mathcal{L}^{(1)}_{soft}$ generates new terms not present in (2.1.6b). In particular, the gaugino-chiral gauge fermion mixing term

$$m \bar{\lambda} \psi_\phi$$

may appear (see 2.1.18 for definitions), and also the relationship between coefficients in (2.1.6b) may be altered. Thus, one can see that the set defined by a global SU_2 rotation of those found in ref.[6],

$$\mathcal{L}'_{soft} = u\left(\mathcal{L}^{(1)}_{soft}\right) \qquad 2.1.7$$

is, for any element u of SU_2, another set of finiteness preserving operators.

It will be shown in §2.1.e that the set of all soft Lagrangians (2.1.7) is in fact the most general set of 1-loop finiteness-preserving operators (with the possible exception of some cubic scalar terms which may occur for specific representations, cf §2.1.e). The reason for this is that the soft insertions considered in ref[6] are not the most general possible (see §2.1.e for details). The complete set can, however, be obtained by an SU_2 rotation of those considered by Parkes and West; one expects therefore that (2.1.7) will turn out to be the general set of

finiteness-preserving operators. This is checked by explicit calculation of the possible one-loop divergences in §2.1.e.

§2.1.d The Spurion Method

The method used to find the infinities produced by the soft insertions is the spurion method[87]. As discussed in §0.f, F- and D-type supersymmetry-breaking insertions may be written as superspace integrals,

$$\int d^4x \int d^2\theta \; N(\theta)\; T\,(V,\, \Phi,\, \chi_\sigma,\, \gamma_\sigma,\, \ldots) + h.c. \qquad 2.1.8a$$

and

$$\int d^4x \int d^4\theta \; U(\theta)\, S\,(V,\, \ldots) \qquad 2.1.8b$$

respectively, where the 'spurion' superfields N and U are x-independent F- and D-terms,

$$N(\theta) = n\,\theta^2 \;, \qquad U(\theta) = u\,\theta^2\bar{\theta}^2 \qquad 2.1.9a,b$$

which break supersymmetry. The advantages of writing the supersymmetry breaking terms in this way are that the supergraph Feynman rules can be used to evaluate their contribution to the effective action, and that dimensional analysis can be used to show, without calculation, whether a given graph may be divergent. For example, a graph with one vertex of the form (2.1.8a), where the function T has dimension 2, contributes to the effective action a term of the form

$$\int \frac{d\Lambda}{\Lambda^{1-p}} \int d^4x \int d^4\theta \; N(\theta) \, f_1(V, \Phi, \ldots) \qquad 2.1.10$$

where Λ is a momentum cut-off. Since (2.1.8a) is dimensionless, then dim(N) = 1; equation (2.1.10) is also dimensionless, so it follows that

$$p - 4 + 2 + 1 = -\dim f_1 . \qquad 2.1.11a$$

f_1 is a gauge-invariant function, so (as $tr\Phi = 0$) $\dim(f_1) \geq 2$; therefore from (2.1.11a)

$$p \leq -1 \qquad 2.1.11b$$

whence (2.1.10) converges.

Furthermore, since $N(\theta)N(\theta) = 0$, any non-zero graph containing s of the insertions (2.1.18a) and t of its conjugate must contribute a term

$$\int \frac{d\Lambda}{\Lambda^{1-q}} \int d^4x \int d^4\theta \; N(\theta) \bar{N}(\bar{\theta}) (D^2 N)^{s-1} (\bar{D}^2 \bar{N})^{t-1} \, f_2(V, \ldots) \qquad 2.1.12$$

and similar analysis shows that, for dim(T) = 2,

$$q \leq -2(t+s) + 2 \qquad (s, t \geq 1). \qquad 2.1.13$$

Thus, any graph containing any number of the dimension two insertions (2.1.18a) is convergent (provided the gauge condition 2.1.5 is satisfied). The only gauge invariant chiral functions of dimension two are

$$T_\sigma = Y_\sigma X_\sigma \qquad 2.1.14a$$

and

$$T_\varphi = \frac{1}{2} \Phi^2 \qquad 2.1.14b$$

so an allowed soft term is

$$\mathcal{L}^{(4)}_{soft} = \sum_\sigma l_\sigma y_\sigma x_\sigma + l_\varphi \hbar \varphi^2 \qquad 2.1.15$$

written in terms of the components

$$x_\sigma = X_\sigma|_{\theta=0} \quad , \quad y_\sigma = Y_\sigma|_{\theta=0} \, ,$$

$$\varphi = \Phi|_{\theta=0} \, . \qquad 2.1.16$$

The parameters l_σ, l_ϕ in (2.1.15) clearly have dimension 2.

The insertions (2.1.15) are the only spurion terms which, on dimensional grounds alone, cannot produce infinites at any order. Dimensional analysis and simple graph topology can, however, be used to show[86] that when only soft insertions are made (i.e. for dimT, dimS $\leq$ 3), the only possibly divergent graphs are those with two or three external legs, and one or two spurion vertices. In the next section all the one-loop divergent graphs of this type are evaluated, considering all the gauge invariant soft insertions which may be made in a general N=2 theory.

§2.1.e One-loop infinities in the softly broken N=2 theory.

In addition to (2.1.15), the following gauge invariant soft combinations of the N=2 fields may be formed:

$$\sum_\sigma u_{x\sigma} \int d^4\theta \, (X_\sigma^\dagger e^{gV} X_\sigma \theta^2 \bar\theta^2) = \sum_\sigma u_{x\sigma} x_\sigma^\dagger x_\sigma \qquad (a)$$

$$\sum_\sigma u_{y\sigma} \int d^4\theta \, (Y_\sigma e^{gV} Y_\sigma^\dagger \theta^2 \bar\theta^2) = \sum_\sigma u_{y\sigma} y_\sigma y_\sigma^\dagger \qquad (b)$$

$$u_\varphi \hbar \int d^4\theta \, (\Phi^\dagger e^{gV} \Phi e^{-gV} \bar\theta^2 \theta^2) = u_\varphi \hbar \varphi^\dagger \varphi \qquad (c)$$

$$\frac{m_0}{64g^2}\,tr\int d^2\theta\,(W^\alpha W_\alpha\,\theta^2)+h.c. \;=\; \frac{-m_0}{4}\,tr\,\lambda^2+h.c. \qquad (d)$$

$$\frac{m_1}{4}\,tr\int d^4\theta\,(\mathcal{D}^\alpha\bar\Phi\,\mathcal{D}_\alpha\Phi\,\theta^2\bar\theta^2)+h.c. \;=\; \frac{m_1}{2}\,tr\,\psi_\phi^2+h.c \qquad (e)$$

$$\sum_\sigma \tfrac{1}{2}m_{2\sigma}\int d^4\theta\,(\mathcal{D}^\alpha Y_\sigma\,\mathcal{D}_\alpha X_\sigma\,\theta^2\bar\theta^2)+h.c. \;=\; \sum_\sigma m_{2\sigma}\psi_{y\sigma}\psi_{x\sigma}+h.c \qquad (f)$$

$$g\sum_\sigma n_{0\sigma}\int d^2\theta\,(Y_\sigma\,\Phi\,X_\sigma\,\theta^2)+h.c. \;=\; g\sum_\sigma n_{0\sigma}\,y_\sigma\,\varphi\,x_\sigma+h.c. \qquad (g)$$

$$g\sum_\sigma n_{1\sigma}\int d^4\theta\,(Y_\sigma\,e^{-gV}\bar\Phi^t e^{gV}X_\sigma\,\theta^2\bar\theta^2)+h.c. \qquad (h)$$
$$= g\sum_\sigma n_{1\sigma}\,y_\sigma\,\varphi^t\,x_\sigma$$

$$g\sum_\sigma n_{x\sigma}\int d^4\theta\,(X_\sigma^t\,e^{gV}\bar\Phi\,X_\sigma\,\theta^2\bar\theta^2)+h.c. \;=\; g\sum_\sigma n_{x\sigma}\,x_\sigma^t\,\varphi\,x_\sigma+h.c. \qquad (i)$$

$$g\sum_\sigma n_{y\sigma}\int d^4\theta\,(Y_\sigma\,e^{-gV}\bar\Phi\,Y_\sigma^t\,\theta^2\bar\theta^2)+h.c. \;=\; g\sum_\sigma n_{y\sigma}\,y_\sigma\,\varphi\,y_\sigma^t \qquad (j)$$

(2.1.17a-j)

These are considered in ref.[6]; in addition the term

$$\frac{m_3}{8g}\,tr\int d^4\theta\,(W^\alpha\,\mathcal{D}_\alpha\bar\Phi\,\theta^2\bar\theta^2)+h.c. \;=\; \frac{im_3}{\sqrt2}\,tr\,\lambda\,\psi_\phi+h.c$$

2.1.17k

is gauge invariant and should be included.[102] The expressions on the right-hand side of eqs(2.1.17) are equal to the left-hand side in the Wess-Zumino gauge; the components are defined by (2.1.16), and by

$$\lambda^\alpha = \frac{-i}{4}W^\alpha\,(\theta=0)$$
$$\psi_\phi^\alpha = \frac{1}{\sqrt2}D^\alpha\bar\Phi\,(\theta=0)$$
$$\psi_{x\sigma}^\alpha = \frac{1}{\sqrt2}D^\alpha X_\sigma\,(\theta=0)$$
$$\psi_{y\sigma}^\alpha = \frac{1}{\sqrt2}D^\alpha Y_\sigma\,(\theta=0) \qquad 2.1.18$$

The only terms omitted from the above list are cubic scalar terms like $x_\rho x_\sigma x_\tau$ which may occur if ρ,σ and τ can form a gauge singlet; the infinities they produce can be cancelled[11] by a $tr\phi^3$ term. This last term produces

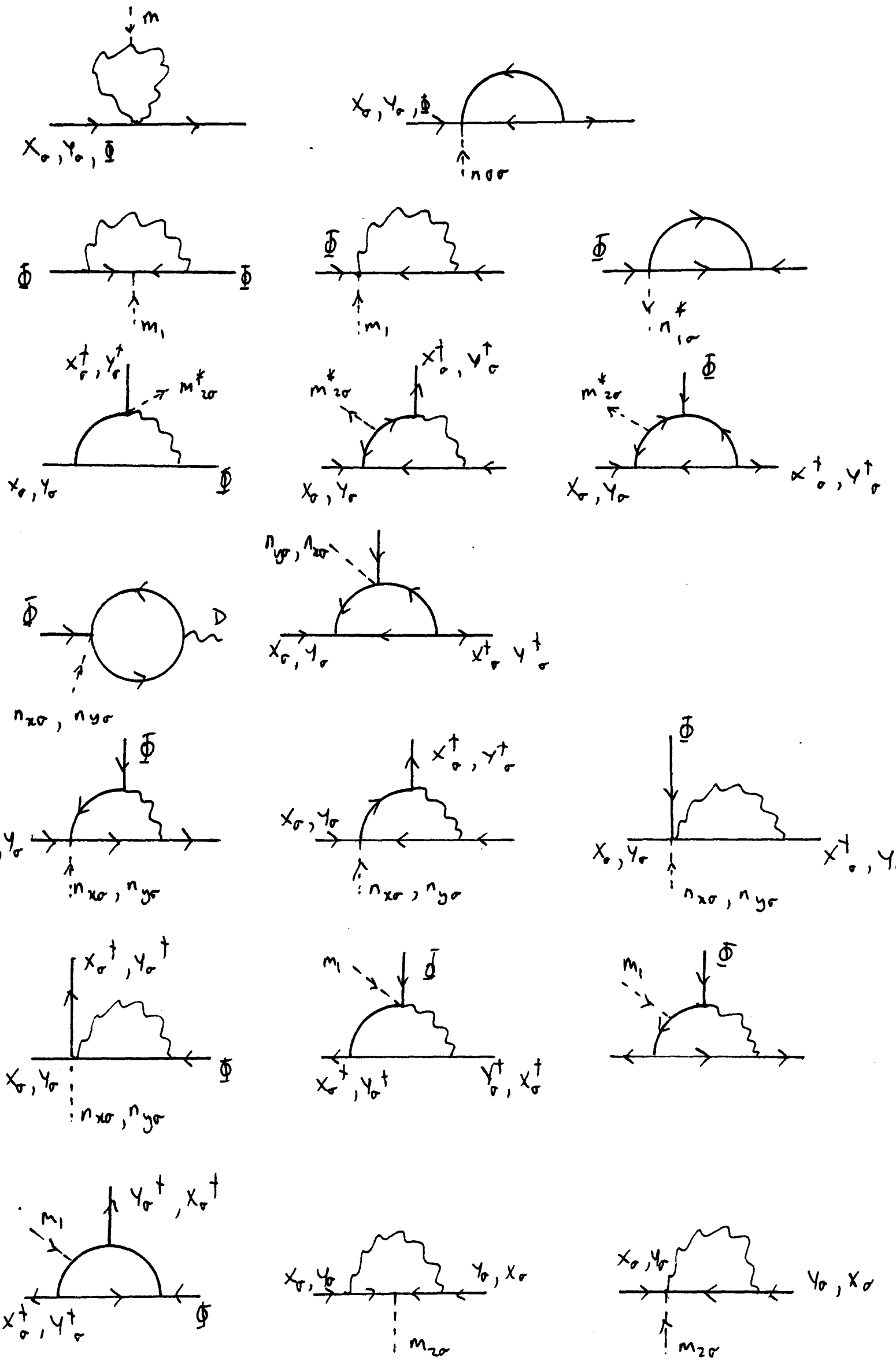


Fig 2.1.5. Graphs giving rise to cubic scalar infinities in the effective action.

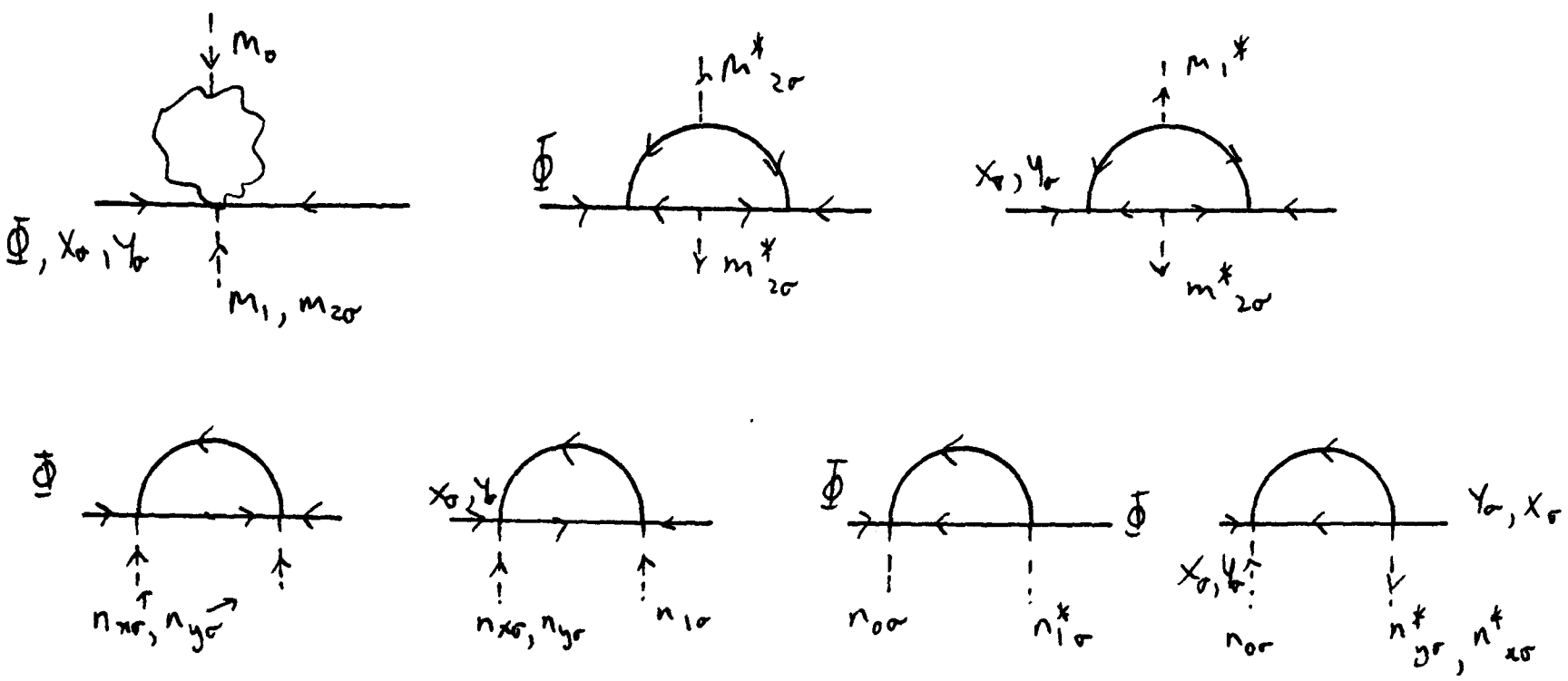


fig 2·1·6 Graphs giving rise to $A^2 - B^2$ mass terms

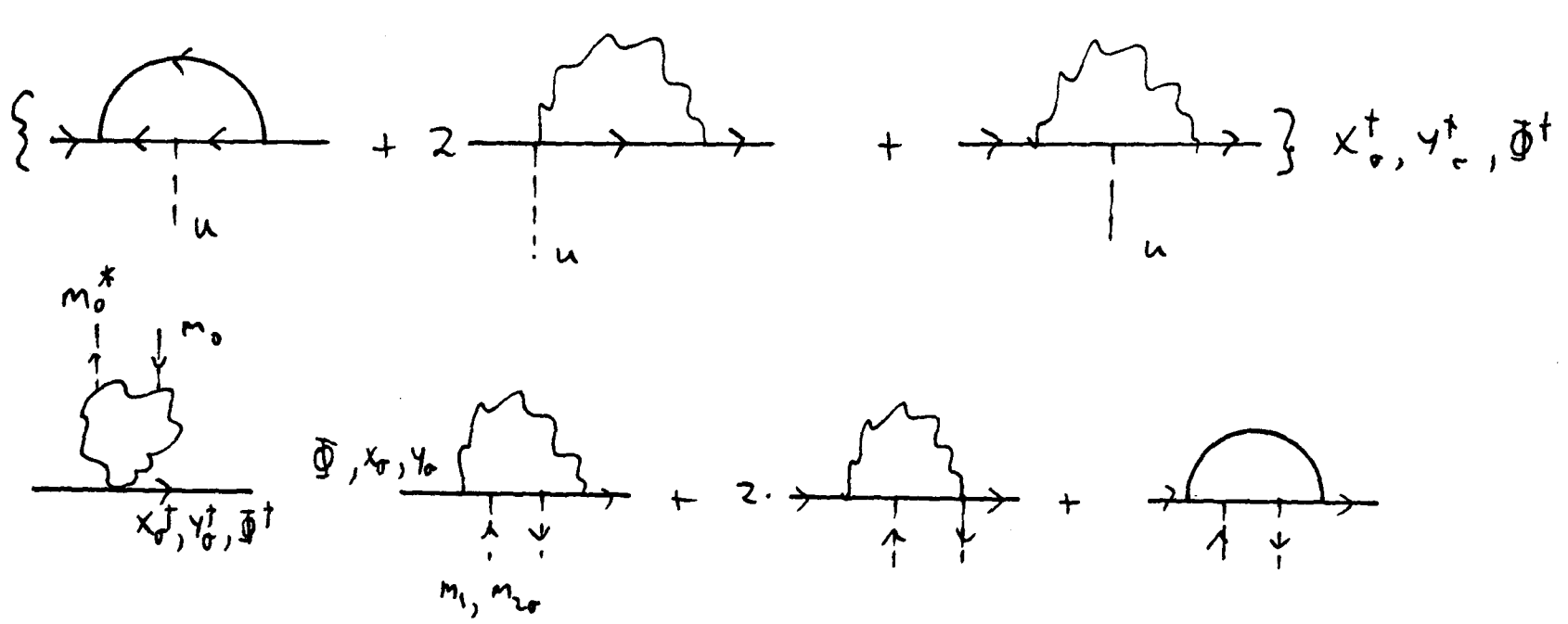
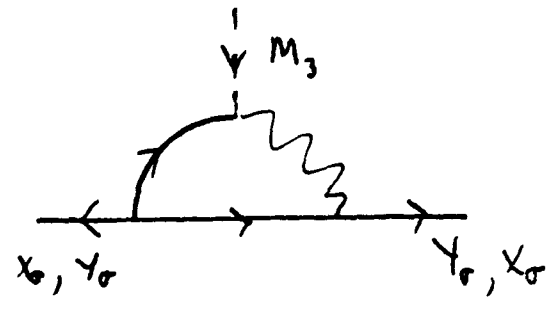
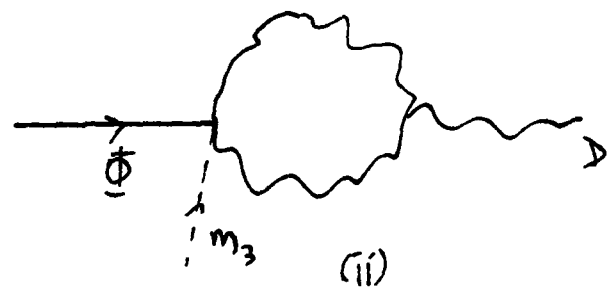


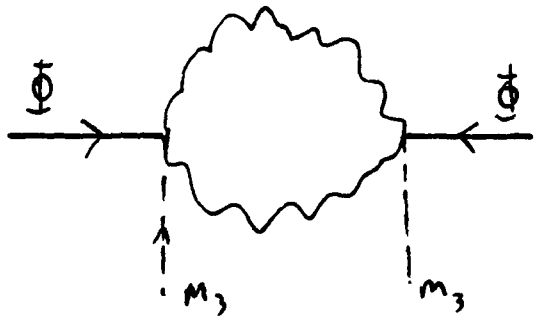
fig 2·1·7 Graphs giving rise to $A^2 + B^2$ mass terms



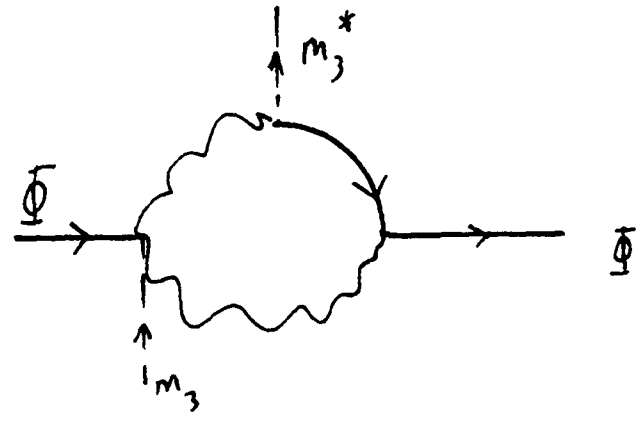
(i)



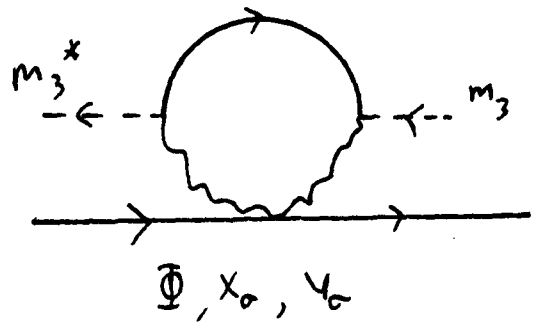
(ii)

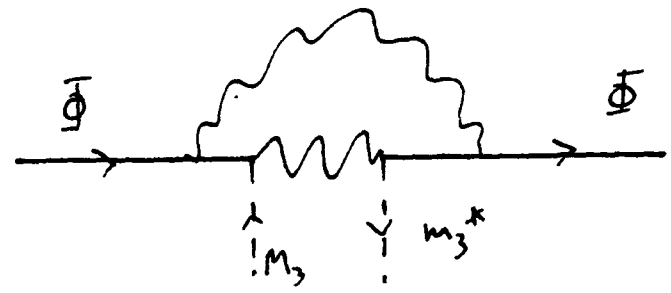


(iii)



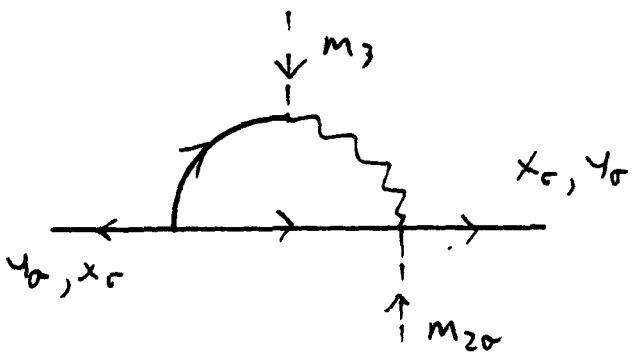
(iv)



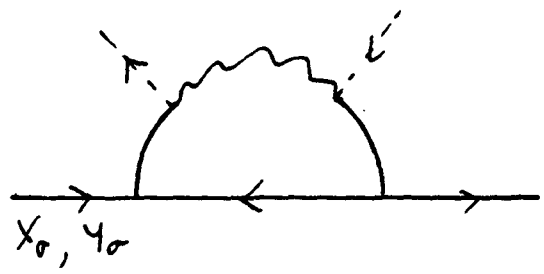


(v)

(vi)



(vii)



(viii)

fig 2·1·8 Divergent graphs generated by (2·1·17k)

divergences which can only be cancelled by the special terms $x_\rho x_\sigma x_\tau$ etc., so it is not considered here.

The insertions (2.1.17) generate logarithmically divergent terms in the effective action of three sorts : those cancelled by cubic scalar counterterms, those cancelled by scalar 'A^2-B^2' mass counterterms (these are the $tr\phi^2$ and $y_\sigma x_\sigma$ terms; they are so named because if ϕ, say, is written A + iB, then $tr\phi^2$ + h.c. = $2tr[A^2-B^2]$), and those cancelled by scalar A^2+B^2 mass counterterms (i.e. $tr\phi^\dagger\phi$, $x_\sigma^\dagger x_\sigma$, $y_\sigma y_\sigma^\dagger$). The divergent graphs of these three sorts[6] that arise from the insertions (2.1.17a-j) are shown in figs 2.1.5, 2.1.6, and 2.1.7. The extra graphs that arise from (2.1.17k) are shown in fig 2.1.8. Their contribution to the effective action is :

from i and ii

$$g^3 \sum_\sigma x_\sigma^\dagger \varphi x_\sigma \left[-2m_3 C_2(R_\sigma) - m_3 T(R_3)\right] + (x_\sigma \leftrightarrow y_\sigma^\dagger) \qquad 2.1.19a$$

from iii

$$-g^2 tr\varphi^2 \cdot m_3^2 \qquad 2.1.19b$$

from iv, v and vi

$$g^2 tr\varphi^\dagger\varphi \left[-4|m_3|^2 - 2|m_3|^2 + 2|m_3|^2\right] C_2(q) \qquad 2.1.19c$$

from vii, v and viii

$$g^2(x_\sigma^\dagger x_\sigma + y_\sigma y_\sigma^\dagger)(4\,Re\, m_3 m_{2\sigma} - 2|m_3|^2 - 2|m_3|^2) C_2(R_\sigma) \qquad 2.1.19d$$

where in each equation an $\int d^4x$ and a logarithmically divergent factor have been omitted (the factor in question is

equal to the divergent part of the integral $\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4}$).

Adding (2.1.19) to the terms already calculated by Parkes and West[6] gives the total divergent one-loop contribution to the effective action arising from (2.1.17a-k). This is, again, in three parts :

terms cancelled by cubic scalar counterterms

$$g^3 \sum_\sigma y_\sigma \varphi x_\sigma \{ [C_2(4) + 2C_2(R^\sigma)] m_0 - 2 C_2(R^\sigma) n_{0\sigma} - \sum_\rho T(R^\rho) n_{0\rho} \}$$

$$+ g^3 \sum x^\dagger_\sigma \varphi y^\dagger_\sigma \{ [C_2(4) + 2C_2(R^\sigma)] m_1 - 2C_2(R^\sigma) n^*_{1\sigma} - \sum_\rho T(R^\rho) n^*_{1\rho} \}$$

$$+ g^3 \sum_\sigma x^\dagger_\sigma \varphi x_\sigma \{ C_2(R^\sigma) [-4m^*_{2\sigma} - 3n_{x\sigma} - n_{y\sigma} - 2m_3]$$

$$+ C_2(4) [-m^*_{2\sigma} + \tfrac{1}{2} n_{x\sigma} + \tfrac{1}{2} n_{y\sigma} - m_3]$$

$$- \tfrac{1}{2} \sum_\rho T(R^\rho) (n_{x\rho} - n_{y\rho}) \} + (x_\sigma \leftrightarrow y_\sigma)$$

$$+ h.c. \qquad 2.1.20a$$

terms cancelled by A^2-B^2 scalar masses :

$$g^2 \operatorname{tr} \varphi^2 \left[(-m_1 m_0 - m_3^2) C_2(4) + \sum_\sigma T(R^\sigma) \left[-(m^*_{2\sigma})^2 \right.\right.$$

$$\left.\left. + n^*_{1\sigma} n_{0\sigma} + \tfrac{1}{2} n^2_{x\sigma} + \tfrac{1}{2} n^2_{y\sigma} \right] \right]$$

$$+ g^2 \sum_\sigma y_\sigma x_\sigma C_2(R^\sigma) [-2m_{2\sigma} m_0 - 2 m_1^* m^*_{2\sigma} + n_{0\sigma} (n^*_{x\sigma} + n^*_{y\sigma})$$

$$+ n_{1\sigma} (n_{x\sigma} + n_{y\sigma})]$$

$$+ h.c. \qquad 2.1.20b$$

terms cancelled by A^2+B^2 scalar masses :

$$g^2 \operatorname{tr} \varphi^\dagger \varphi \{ C_2(4) (-u_\varphi - 2|m_0|^2 - 2|m_1|^2 - 4|m_3|^2)$$

$$+ \sum_\sigma T(R^\sigma) (-u_{x\sigma} - u_{y\sigma} - 4|m_{2\sigma}|^2 + |n_{0\sigma}|^2 + |n_{1\sigma}|^2$$

$$+ |n_{x\sigma}|^2 + |n_{y\sigma}|^2) \}$$

$$+ g^2 \sum_\sigma x^\dagger_\sigma x_\sigma C_2(R^\sigma) \{ -u_{x\sigma} - u_{y\sigma} - u_\varphi - 2|m_0|^2 - 2|m_1|^2$$

$$- 4|m_{2\sigma}|^2 - 4|m_3|^2 + 4 \operatorname{Re}(m_3 m_{2\sigma}) + |n_{0\sigma}|^2 + |n_{1\sigma}|^2 + 2|n_{x\sigma}|^2 \}$$

$$+ (x_\sigma \leftrightarrow y_\sigma)$$

(2.1.20c)

§2.1.f The one-loop finiteness conditions

To establish the general form of the finiteness-preserving soft operators $\mathcal{L}_{soft}$ consists of two stages : firstly, it is a necessary condition on $\mathcal{L}_{soft}$ that it produce no 1-loop divergences - so the coefficients u_ϕ, $u_{x\phi}$ ... must be chosen so that (2.1.20) vanishes identically; secondly, a further argument will be given to show that at least some of this subset of the soft terms produce no infinities at the two-loop or higher level.

To begin with, the one-loop finiteness conditions must be found. Clearly a prerequisite for finiteness is that the gauge condition (2.1.5) be satisfied; with this precondition (2.1.20a) vanishes identically provided

$$n_{0\sigma} = m_0 \qquad \text{(a)}$$

$$n^*_{1\sigma} = m_1 \qquad \text{(b)}$$

$$m^*_{2\sigma} - n_{x\sigma} = m_3 \qquad \text{(c)}$$

$$n_{x\sigma} - n_{y\sigma} = -2m_3 \qquad \text{(d)}$$

(2.1.21a-d)

Then (2.1.20b) vanishes automatically, and finally (2.1.20c) vanishes if

$$u_{x\sigma} + u_{y\sigma} + u_\varphi + |m_0|^2 + |m_1|^2 + 2|m_{2\sigma}|^2 + 2|m_3|^2 = 0 \qquad (2.1.21e)$$

The finiteness conditions given by Parkes and West are given by (2.1.21) with m_3 set to zero; but if their finite Lagrangian is rotated by the SU_2 transformation

$$u = \begin{pmatrix} \bar{\alpha} & -\beta \\ \bar{\beta} & \alpha \end{pmatrix}, \quad |\alpha|^2 + |\beta|^2 = 1 \qquad (2.1.22)$$

as in (2.0.17) then it is easy to show that a Lagrangian of the form described by (2.1.21) results, where

$$m_3 = i\,(m_0\,\bar{\alpha}\beta + \alpha\bar{\beta}\,m_1) \qquad (2.1.23)a$$

provided the original parameters are replaced by their 'rotated' values

$$m_0 \to m_0\bar{\alpha}^2 - m_1\bar{\beta}^2 \qquad (b)$$

$$m_1 \to -\beta^2 m_0 + \bar{\alpha}^2 m_1 \qquad (c)$$

$$u_{x\sigma} \to |\alpha|^2 u_{x\sigma} + |\beta|^2 u_{y\sigma} \qquad (d)$$

$$u_{y\sigma} \to |\beta|^2 u_{x\sigma} + |\alpha|^2 u_{y\sigma} \qquad (e)$$

(2.1.23b-e)

Thus, the most general one-loop finite Lagrangian can always be expressed as a global SU_2 rotation of that found by Parkes and West.

By solving the equations of motion for the auxiliary fields, the soft insertions (2.1.27), subject to the finiteness conditions (2.1.21), may be written as a set of scalar masses and 'supersymmetric' masses,

$$-\mathcal{L}^{(1)}_{soft} = m^2_\varphi\,\mathrm{tr}\,\varphi^\dagger\varphi + \sum_\sigma (m^2_{x\sigma}\,x^\dagger_\sigma x_\sigma + m^2_{y\sigma}\,y_\sigma y_\sigma{}^\dagger)$$

$$+ \tfrac{1}{2}\int d^2\theta\,[\,\mu_0(\underline{\Phi}a^\dagger)^2 + \mu_1\,\mathrm{tr}\,\bar{\Phi}^2 + 2\sum_\sigma m_{2\sigma}\,Y_\sigma X_\sigma + 2m_3\,\mathrm{tr}(\Phi b^\dagger) \qquad (2.1.24)$$

where $\Phi u^\dagger$ is the rotated 'chiral vector' field,

$$(\Phi u^\dagger)\theta = \varphi + \sqrt{2}\,\theta(\alpha\psi_\phi - \bar{\beta}\tfrac{\lambda}{\sqrt{2}}) + \theta^2(F_\varphi u^\dagger) \qquad (2.1.25)$$

and

$$a = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad b = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad (2.1.26)$$

provided that the new parameters $m^2_{x\sigma}$, $m^2_{y\sigma}$, m^2_ϕ, μ_0 and μ_1 satisfy

$$m^2_{x\sigma} + m^2_{y\sigma} + m^2_\varphi - 2\,\mathrm{Re}(\mu_0^* + \mu_1^*)\,m_3 = 0 \qquad (2.1.27)$$

Bearing in mind the A^2–B^2 masses (2.1.15) which maintain finiteness to *all* orders, the 1-loop finite Lagrangian is

$$\mathcal{L}^{(1)}_{soft} = (2.1.15) + (2.1.24) \qquad (2.1.28)$$

subject to the constraint (2.1.27)

§2.1.g Finiteness at all orders

The terms in (2.1.24) proportional to μ_1 and $m_{2\sigma}$ are true N=1 supersymmetric masses and have been shown[86,88] not to produce divergences at any order of perturbation theory. The argument is essentially the same as the one used in §2.1.d to show that (2.1.25) is finiteness-preserving : from the N=1 non-renormalisation theorem, n insertions into a supergraph of the mass term

$$\tfrac{1}{2}\mu\,\mathrm{tr}\,\Phi^2\,\big|_F \qquad (2.1.29)$$

contribute to the effective action an integral of the form

$$\int \frac{d\Lambda}{\Lambda^{1-p}} \int d^4x \int d^4\theta \, \mu^n f\,(v, \bar{\Phi} \ldots) \qquad (2.1.30)$$

Because the gauge invariant function f has dimension at least 2, adding up the mass dimensions in (2.1.30) gives

$$p \leq -n \qquad (2.1.31)$$

so all insertions of this sort leave the theory finite.

It has been shown[6] that the A^2+B^2 scalar mass terms

$$m^2_{x\sigma}\, x^\dagger_\sigma x_\sigma + m^2_{y\sigma}\, y_\sigma y_\sigma^\dagger + m^2_\varphi\, \varphi^\dagger \varphi \qquad (2.1.32)$$

maintain finiteness to all orders : on dimensional grounds only diagrams with one A^2+B^2 insertion may be divergent, and by considering the result of placing this insertion at a vertex, or in the middle of a propagator, Parkes and West show that all possible infinities cancel if the rest of the theory (i.e. the complete Lagrangian except 2.1.32) is also finite.

There remain to be considered the 'rotated' supersymmetric masses

$$\int d^2\theta \left[\mu_0 \, tr\,(\bar{\Phi} a^\dagger)^2 + 2 m_3 \, tr\,(\bar{\Phi} b^\dagger)^2 \right] \qquad (2.1.33)$$

Parkes and West claim[6] that the 1-loop soft Lagrangian (2.1.24), with $m_3 = 0$, can be obtained from an SU_2 rotation of the soft Lagrangian which preserves one supersymmetry, *viz.*

$$-\mathcal{L}^{N=1}_{soft} = \int d^2\theta \left[\tfrac{1}{2} j \, tr\, \Phi^2 + \sum_\sigma m_{2\sigma}\, y_{2\sigma}\, x_{2\sigma} \right] \qquad (2.1.34)$$

which has just been shown, on dimensional grounds, to preserve finiteness; clearly if this were the case then it would be established that the soft Lagrangian (2.1.24) with

$m_3 = 0$ preserves finiteness also. However, from the the transformations (2.1.33) the most general soft Lagrangian obtainable by an SU_2 rotation of (2.1.34) is

$$-\tfrac{1}{2}\int d^2\theta \left[\mu_0' \,\mathrm{tr}(\Phi a^\dagger)^2 + \mu_1' \,\mathrm{tr}\,\Phi^2 + 2m_3' \,\mathrm{tr}(\Phi b^\dagger)^2 + 2\sum_\sigma M_{2\sigma} Y_\sigma X_\sigma \right] \tag{2.1.35}$$

where

$$\mu_0' = \alpha^2 \mu \tag{a}$$

$$\mu_1' = -\bar{\beta}^2 \mu \tag{b}$$

$$m_3' = \alpha\bar{\beta}\,\mu \tag{c}$$

$$\text{and} \quad |\alpha|^2 + |\beta|^2 = 1 \tag{d}$$

(2.1.36a-d)

The set of equations (2.1.36) does not have a solution for all values of μ_0', μ_1', m_3' (for example, one cannot have μ_0', μ_1' non-zero with $m_3 = 0$, as claimed in ref.[6]) and so it is not possible to establish by this means the finiteness at all orders of the full 1-loop finite soft Lagrangian (2.1.28), but only of the subclass of those Lagrangians (2.1.28) for which the relations (2.1.36) hold. Frere *et al.*[7,8] derive by a non-graphical method the same set of 1-loop finite soft operators as do Parkes and West, and state without proof that these all maintain finiteness to all orders. However, the only result which is proved by the graphical analysis is that (2.1.28) maintains finiteness to all orders, provided the one-loop constraint (2.1.27) and the all-orders constraint (2.1.36) are satisfied.

SECTION 2.2 Low-Energy N=2 Models

§2.2.a The problems of building a finite N=2 theory.

Although it allows the possibility of a finite theory, the unbroken massless N=2 Lagrangian (2.0.23)

$$\mathcal{L}_{N=2} = \int d^4\theta \Big[\sum_\rho X^\dagger_\rho \rho(e^{gV}) X_\rho + \sum_\rho Y_\rho \rho(e^{gV}) Y_\rho^\dagger + \mathrm{tr}\, \bar{\Phi}^\dagger e^{gV} \Phi e^{-gV} \Big] + \int d^2\theta \Big[\frac{1}{64g^2} \mathrm{tr} W^2 - ig \sum_\rho Y_\rho \Phi X_\rho \Big] + \text{h.c.} \quad (2.2.1)$$

has symmetries which are not observed at low energies; the essential problem in building a realistic, finite model is therefore to find a mechanism which can break all the extra symmetries of (2.2.1) at low energy to reproduce the standard model phenomenology, and yet not destroy the finiteness of the unbroken theory. The unwanted symmetries are :

(i) the extra gauge symmetry of the Grand Unified group G. Both G and the matter (and Higgs) content (X_ρ, Y_ρ) must be chosen to satisfy the finiteness condition (2.1.5). This means, for example, that in a finite E_6 theory there must be four hypermultiplets in the 27 representation. Also, G can not have a U_{1em} factor, as the β-function of such a factor is non-zero and cannot be cancelled by the rest of the group

(ii) the N=2 superalgebra (0.6) generates two supersymmetries and an SU_2 symmetry, all of which must be broken. As a result of the SU_2 invariance, for instance, there corresponds to each left-handed fermion in a given representation a right-handed 'mirror' fermion in the same

representation. This is perhaps the most obvious difficulty in constructing N=2 models - the mirror fermions must be made unobservable

(iii) if the matter content is in n generations $(X_\rho{}^i, Y_{\rho i})$ of the same representation ρ, then the Lagrangian (2.2.1) has a global U_n symmetry. this is because the only Yukawa interaction $igY_{\rho i}\Phi X_\rho{}^i$ is independent of i = 1..n. This symmetry may be broken explicitly by the N=2 mass terms (2.0.24) but in practice this is not helpful as it preserves the SU_2 invariance. Even when the U_n symmetry is explicitly broken, there remains a global U_1 symmetry for each hypermultiplet, viz.

$$X_\rho \to e^{i\theta} X_\rho \quad , \quad Y_\rho \to e^{-i\theta} Y_\rho \qquad (2.2.2)$$

The spontaneous breakdown of this symmetry which occurs when (X_ρ, Y_ρ) is a Higgs hypermultiplet may produce a Goldstone boson; this is a problem in the $SU_3 \times SU_4 \times U_1$ model considered below.

Of all these symmetries the low-energy Lagrangian must retain only an $SU_3 \times U_1$ gauge symmetry. The U_1 global symmetries mentioned in (iii) are not a problem unless spontaneously broken, or if they prevent non-chiral mass terms (as in §2.2.c)

§2.2.b The possibility of a low-energy model

Ultimately, one would hope that the source of the symmetry breaking in a realistic, finite N=2 model would be

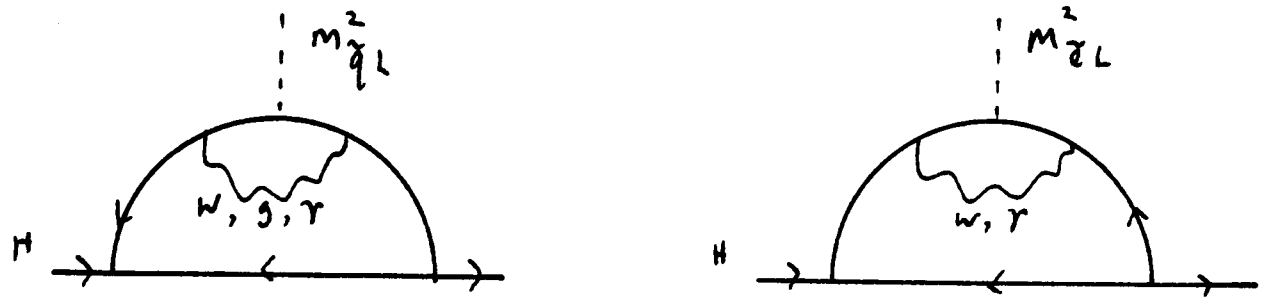


fig 2·2·0 The scalar mass insertions receive a different renormalisation according to their $SU_3 \times SU_2 \times U_1$ quantum numbers

from N=2 supergravity. If the N=2 supergravity Lagrangian[10] produces similar low-energy terms to those of N=1 supergravity, then a finite effective Lagrangian of such a theory below the Planck scale will consist of 'hard' terms in the N=2 globally supersymmetric part of the Lagrangian, together with soft, finiteness-preserving terms of the form (2.1.28). Below the unification scale, the finiteness relations (2.1.21) between the coefficients of these terms will be destroyed by renormalisation effects. For example, if the gauge group is spontaneously broken to $SU_3 \times SU_2 \times U_1$ at some scale M then the effective theory below M is not, in general, finite. The spurion insertions in fig 2.2.0, which above M will be part of a finiteness-preserving set of operators, receive different renormalisations according to their $SU_3 \times SU_2 \times U_1$ quantum numbers, and the finiteness relations are no longer satisfied.

This is the starting point for investigating N=2 models in this section : a low-energy gauge group and low-energy multiplet structure are chosen, and to the globally supersymmetric Lagrangian for these are added soft, explicit supersymmetry breaking terms of the sort listed in (2.1.17), without stipulating that their coefficients satisfy the finiteness conditions (2.1.21), with the aim of producing a realistic model which is (possibly) the low-energy remnant

of a finite, explicitly broken, N=2 Grand Unified theory.

What prospect is there that such a low-energy model can be incorporated in a finite GUT? Several authors[89-93] have investigated this possibility : clearly the most restrictive prerequisite is that the Grand Unified group satisfy the finiteness condition (2.1.5), with at least three generations of quarks and leptons. A list of possible groups is given in refs.[89,92]. Kalara *et al.*[92] conclude that requiring massive mirror fermions in a finite N=2 GUT with a weak scale of order 1 TeV makes it difficult to construct a symmetry breaking pattern which includes the standard group $SU_3 \times SU_2 \times U_1$; Ferrara and Roncadelli[93] have shown that there are further constraints on the pattern of supersymmetry breaking which arise from requiring that coupling constants remain perturbative up to the Planck scale. In fact, there have been few positive results so far in the search for a finite N=2 GUT.

The results for the low-energy models are more promising, however. In the following two sections are presented two N=2 models proposed by Del Aguila *et al*[11] based on the gauge groups $SU_3 \times SU_2 \times U_1$ and $SU_3 \times SU_4 \times U_1$. The former model has problems with its vacuum structure, as will be shown; but the latter can give good low-energy quarks and charged lepton masses, and has naturally heavy mirror fermions.

§2.2.c The $SU_3 \times SU_2 \times U_1$ model.

An important class[11] of finiteness-preserving soft operators omitted from (2.1.17), because they do not occur in a general theory, is that of generalised gauge invariant cubic scalar couplings, e.g. $x_\rho x_\sigma x_\tau$, $x_\rho x_\sigma y_\tau$ etc. which may appear if there is a gauge invariant coupling between the representations ρ,σ and τ (using the notation of §2.1, cf 2.1.16). It is found that these may occur in the combinations

$$k_1 (x_\sigma x_\rho x_\tau + y^\dagger_\sigma y^\dagger_\rho y^\dagger_{\bar\tau}) + h.c. \qquad 2.2.3$$

and

$$k_2 (x_\sigma x_\rho y_{\bar\tau} + y^\dagger_\sigma y^\dagger_\rho x^\dagger_{\bar\tau}) + h.c. \qquad 2.2.4$$

provided the infinities they produce (see fig 2.2.1) are cancelled by further scalar masses $x_\rho^\dagger x_\rho$, $y_\rho y_\rho^\dagger$, $tr\phi^\dagger\phi$, as well as a cubic scalar term $tr\phi^3$; the combination of such terms that is required is irrelevant for this discussion, because at low energies any such relation would be destroyed, as was discussed in the previous section.

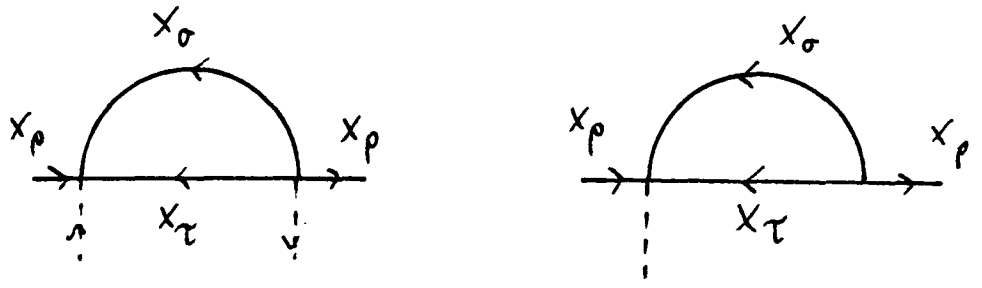


fig 2.2.1. Divergent graphs containing the generalised cubic scalar insertions

This class of soft insertions is important because it allows charge conjugate quarks to have a non-zero mass. However, such cubic scalar interactions must not be large in any workable theory because they destabilise the vacuum. Not only could non-neutral fields acquire a v.e.v. through

them (as in §1.0.f) but the N=2 scalar potential has flat directions in which the positive quartic terms vanish - in these directions the potential is therefore unbounded below if the cubic terms are also present. It is this contradiction which rules out the $SU_3 \times SU_2 \times U_1$ model. To see this, denote the supersymmetric standard model multiplets by Q^i, $U^c{}_i$, $D^c{}_i$, L^i and $E^c{}_i$ as in (1.0.18). The natural N=2 extension of this model is obtained by taking as matter hypermultiplets (X_σ, Y_σ) the pairs

$$(Q^i, Q'^i) \quad , \quad (u^c{}_i, u'^c{}_i) \quad , \ldots \text{ etc}$$

where the primed fields Q'^i, $U^{c\prime}{}_i$,... etc. are the mirror quarks and leptons which transform by the representation conjugate to that of their N=2 partners. The unbroken Lagrangian (2.2.1) for these fields now has a chiral symmetry

$$u^c{}_i \rightarrow e^{i\theta_i} u^c{}_i$$

$$u'^c{}_i \rightarrow e^{-i\theta_i} u'^c{}_i \,, \qquad 2.2.5$$

and so on for the other fields, which must prevent the $u^c{}_i$ fermions from gaining a mass : $u^c m u$ is not invariant under (2.2.5). None of the terms in (2.1.17) break this symmetry, therefore one of the insertions (2.2.4) must be used to give mass to the quarks and leptons (see fig 2.2.2); as mentioned above this eliminates the $SU_3 \times SU_2 \times U_1$ model.

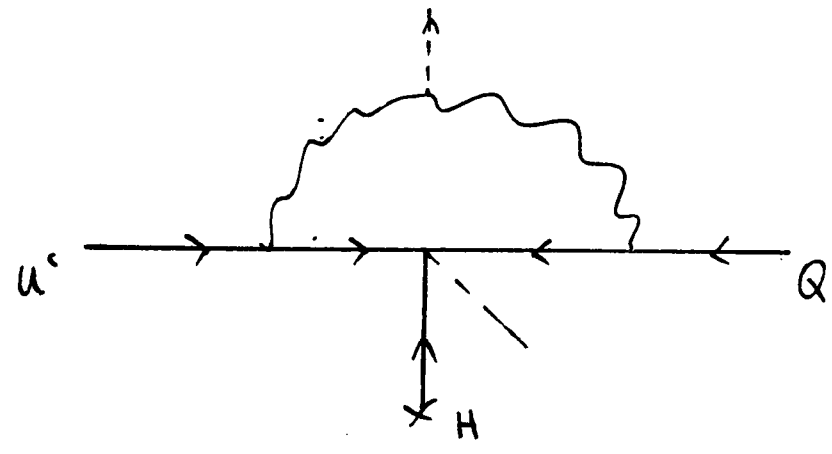


fig 2.2.2 A diagram which may generate quark masses in the $SU_3 \times SU_2 \times U_1$ model.

§2.2.d The $SU_3 \times SU_4 \times U_1$ model.

One way to ensure that the chiral symmetry (2.2.2) does not prohibit fermion (and, for that matter, mirror fermion) masses is to put the left- and right-handed states into the same hypermultiplet. This is not possible in the $SU_3 \times SU_2 \times U_1$ model because the right-handed fermions are required to be singlets under SU_{2L}; the only possible assignment would be

$$(\chi, \gamma) = (Q^i, Q^c{}_i)$$

but then $u^c{}_i$ and $d^c{}_i$ also form a doublet of SU_2.

The minimal extension of the standard gauge group which allows both fermion masses and chiral weak interactions is to suppose that the grand unified group of these theories is broken, penultimately, to $SU_3 \times SU_4 \times U_1$, and that matter is accomodated in the hypermultiplets

$$\chi^i{}_L = \begin{pmatrix} L^i \\ L^{c\prime i} \end{pmatrix} \in (1, 4, -\tfrac{1}{2}) \qquad \text{(a)}$$

$$\gamma_{Li} = \begin{pmatrix} L'_i \\ L^c_i \end{pmatrix} \in (1, \bar{4}, \tfrac{1}{2}) \qquad \text{(b)}$$

$$\chi^i_Q = \begin{pmatrix} Q^i \\ Q^{ci} \end{pmatrix} \in (3, 4, \tfrac{1}{6}) \qquad \text{(c)}$$

$$\gamma_{Qi} = \begin{pmatrix} Q'_i \\ Q^c{}_i \end{pmatrix} \in (\bar{3}, \bar{4}, -\tfrac{1}{6}) \qquad \text{(d)}$$

2.2.6a-d

whose X-components belong to the indicated representations of $SU_3 \times SU_4 \times U_1$[11]. As usual, i = 1..3 is the generation index. SU_4 contains a maximal subgroup $SU_{2L} \times SU_{2R}$ and with the assignment (2.2.6) the left-handed fermions q^i, ℓ^i form

doublets under SU_{2L}, while the right-handed (u^c_i, d^c_i) and (e^c_i, ν^c_i) form a doublet under SU_{2R}. Thus to ensure that only the left-handed fermions interact with the W_L and Z_L bosons, SU_{2R} must be broken at a higher energy than SU_{2L} (so that W_R, Z_R are heavier than W_L, Z_L). Denoting by Y the generator of the U_1 part, and by $T^a_{L,R}$ the generators of the $SU_{2L,R}$ parts of the gauge group, it follows from (2.2.6) that the charge is

$$Q = T_{3L} + T_{3R} + Y \qquad 2.2.7$$

and that, below the breaking scale m_4, the massless gauge field is

$$\partial_\mu \left(W^{\pm}_{L\mu} T^{\pm}_{L} + \tan\theta_W A_\mu Q \right)$$

where

$$\sin^2\theta_W = \frac{1}{2 + g_4^2/g_1^2} \qquad 2.2.8$$

and g_1 and g_4 are the U_1 and SU_4 gauge couplings.

§2.2.e Tree-level mirror fermion masses

It is possible to break SU_4 down to SU_{2L} at a (high) scale m_4, and with the same mechanism to give a tree-level mass of order m_W to the mirror fermions only, thus making these particles unobservable[11]. The Higgs multiplets required for such a scheme are the two hypermultiplets (X_1, Y_1) and (X_2, Y_2) whose X components belong to the $(1,4,-\frac{1}{2})$ and $(1,4,\frac{1}{2})$ representations, respectively. Thus, the N=2 superymmetric part of the Lagrangian for this theory is

$$\mathcal{L} = \text{kinetic term} + i\left[g_3 Y_Q \Phi_3 X_Q + g_4 \left(Y_Q \Phi_4 X_Q + Y_L \Phi_4 X_L + Y_1 \Phi_4 X_1 + Y_2 \Phi_4 X_2 \right) + g_1 \left(\tfrac{1}{6} Y_Q \Phi_1 X_Q - \tfrac{1}{2} Y_L \Phi_1 X_L - \tfrac{1}{2} Y_1 \Phi_1 X_1 - \tfrac{1}{2} Y_1 \Phi_1 X_1 \right) \right] + \text{h.c.} \qquad 2.2.9$$

where the subscripts 1,3,4 pertain to the U_1, SU_3 or SU_4 part of the gauge group. To (2.2.9) must be added soft terms to break the unwanted symmetries. Note that the 'chiral vector' multiplet Φ_4 can be used as a Higgs field to break SU_4 - any $-m^2 tr\phi^\dagger\phi$ term in the scalar potential will drive a v.e.v. in its scalar components - and this is the only Higgs multiplet which can generate fermion masses at tree level. In particular, if Φ_4 acquires a v.e.v. of the form

$$\langle \Phi_4 \rangle = \begin{pmatrix} 0 & W \\ 0 & 0 \end{pmatrix} \qquad 2.2.10$$

in 2 x 2 block form then the mirror quarks acquire a mass through

$$ig\, Y_{Q\,i} \langle \Phi_4 \rangle X^{i}_{\alpha} = ig\, Q'^{c}_{\ i}\, W\, Q'^{i} \qquad 2.2.11$$

and similarly for the mirror leptons.

Equation (2.2.10) is the desired form for $\langle \Phi_4 \rangle$ (the other Φ multiplets cannot acquire v.e.v.s as $SU_3 \times U_1$ must remain unbroken); to achieve it, soft terms are added to the scalar potential of the following type :

(i) negative mass-squared terms

$$-m_x^2 \left(x_1^\dagger x_1 + x_2^\dagger x_2 \right) - m_y^2 \left(y_1 y_1^\dagger + y_2 y_2^\dagger \right) \qquad 2.2.12$$

These may be adjusted to give v.e.v.s of any size for the x_r and y_r scalars (r = 1,2). They are chosen to give

$$\langle x_r \rangle^2 = u^2$$

$$\langle y_r \rangle^2 = v^2 \quad \forall r \quad m_W \qquad 2.2.13$$

where u » v

(ii) cubic scalar operators

$$m_1 y_1 \phi_4 x_1 + m_2 y_2 \phi_4 x_2 \qquad 2.2.14$$

The full scalar potential for the x_r, y_r and ϕ_4 can now be written

$$V(x_r, y_r, \phi) = V_4 + V_1 + V_{soft} \qquad 2.2.15$$

where V_4 and V_1 are the scalar potentials coming from the F- and D-terms of (2.2.9); they are

$$V_4 = \tfrac{1}{2} g_4^2 \{ 2 + [\phi_4, \phi_4^\dagger]^2 + 2 \sum_r (x_r^\dagger \{ \phi_4, \phi_4^\dagger \} x_r + y_r \{ \phi_4, \phi_4^\dagger \} y_r^\dagger) + \sum_{r,s} (\tfrac{1}{2} | x_r^\dagger x_s + y_r y_s^\dagger |^2 + (y_r y_s^\dagger)(x_s^\dagger x_r) - (y_r x_s)(x_s^\dagger y_r) - \tfrac{1}{2} (y_r x_r)(x_s^\dagger y_s^\dagger) - \tfrac{1}{8} (\sum_r |x_r|^2 - \sum_r |y_r|^2)^2 \} \qquad 2.2.16a$$

and

$$V_1 = \tfrac{1}{2} g_1^2 \{ [\sum_r Z_r (|x_r|^2 + |y_r|^2)]^2 + 4 \sum_{r,s} Z_r Z_s [(y_r x_s)(x_s^\dagger y_s^\dagger) - |x_r|^2 |y_s|^2 \} \qquad 2.2.16b$$

The minimum of the full potential (2.2.15) is found by using the hierarchy of v.e.v.s u » v, with the following steps :

(i) The only terms in (2.2.15) which drive a v.e.v. for ϕ_4 are the $m_r y_r \phi_4 x_r$ terms (2.2.14) - for when y_r and x_r have v.e.v.s this is a linear term in ϕ_4. It is easy to show that m_r may be chosen to give

$$\forall r \quad \langle \phi_4 \rangle^2 = w^2$$

where $u \gg w \gg v$. Since $\langle \phi_4 \rangle$ also breaks SU_4, w must be of order the weak scale.

(ii) With this hierarchy, the dominant term in (2.2.15) affecting the alignment of the v.e.v.s is $\frac{1}{2}g_4^2 |x_1^\dagger x_2|^2$ so these two v.e.v.s will be orthogonal. The largest v.e.v.s must be used to break SU_{2R}, so these are chosen as

$$\langle x_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ u \\ 0 \end{pmatrix} \quad \text{and} \quad \langle x_2 \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ u \end{pmatrix} \qquad 2.2.17$$

(iii) The next largest terms are

$$g_u^2 \left(x_1^\dagger \{ \varphi_u^\dagger, \varphi_4 \} x_1 + x_2^\dagger \{ \varphi_u^\dagger, \varphi_4 \} x_2 \right) + \sum_r m_r y_r \varphi_u x_r + h.c. \qquad 2.2.18$$

Assuming (2.2.17), the last term can be written

$$u \left(m_1 y_{1\alpha} \varphi^\alpha{}_3 + m_2 y_{2\beta} \varphi^\beta{}_4 \right)$$

which tends to drive v.e.v.s in $y_{1\alpha}$, $\phi^\alpha{}_3$, $y_{2\beta}$ and $\phi^\beta{}_4$. Furthermore, it is easy to show that the first term of (2.2.18) - which dominates - favours

$$\langle \varphi^3{}_3 \rangle = \langle \varphi^3{}_4 \rangle = \langle \varphi^4{}_3 \rangle = \langle \varphi^4{}_4 \rangle = 0$$

so that the $\langle y_R \rangle$ v.e.v.s break only SU_{2L}, as anticipated in (2.2.13).

(iv) Finally, $\langle y_1 \rangle$ may be chosen to be the charge-conserving

$$\langle y_1 \rangle = \begin{pmatrix} 0 \\ v \\ 0 \\ 0 \end{pmatrix} \qquad 2.2.19a$$

and then the $g_4^2 |y_1^\dagger y_2|^2$ then shows that $\langle y_2 \rangle$ also conserves U_1,

$$\langle y_2 \rangle = \begin{pmatrix} v \\ 0 \\ 0 \\ 0 \end{pmatrix} . \qquad (2.2.19b)$$

This choice of soft terms then ensures, by the argument in (iii), that the v.e.v. of ϕ_4 is

$$\langle \varphi_4 \rangle = \begin{bmatrix} 0 & 0 & 0 & w_2 \\ 0 & 0 & w_1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad 2.2.20$$

where

$$w_1^2 + w_2^2 = w^2 \approx \frac{2 m_W^2}{g^2 4}$$

gives tree-level mirror fermion masses of order m_W.

§2.2.f The Axion problem

The Higgs structure and scalar potential of §2.2.e achieve the 'first' aim in building an N=2 model, *viz.* to break the global SU_2 invariance of the exact theory which does not allow chiral gauge fermion interactions. There still remain, however, many unwanted symmetries. For example, none of the family U_3 symmetries discussed in §2.2.a have been broken; but all of the scalar soft terms can do this explicitly. Thus, generation dependence is put in by hand in this model (as in most models) - fermion masses arise at one loop through the graphs of fig 2.2.3. Although some bounds on the soft parameters are given by these graphs, it is found that a realistic pattern of quark and lepton masses can be generated at one loop[11]. The SU_4 breaking scale $m_4 = g_4 u/\sqrt{2}$ must be about 2 TeV to give a sufficiently heavy top quark.

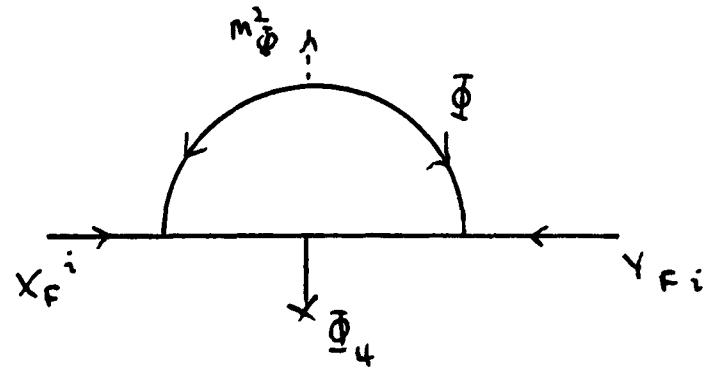
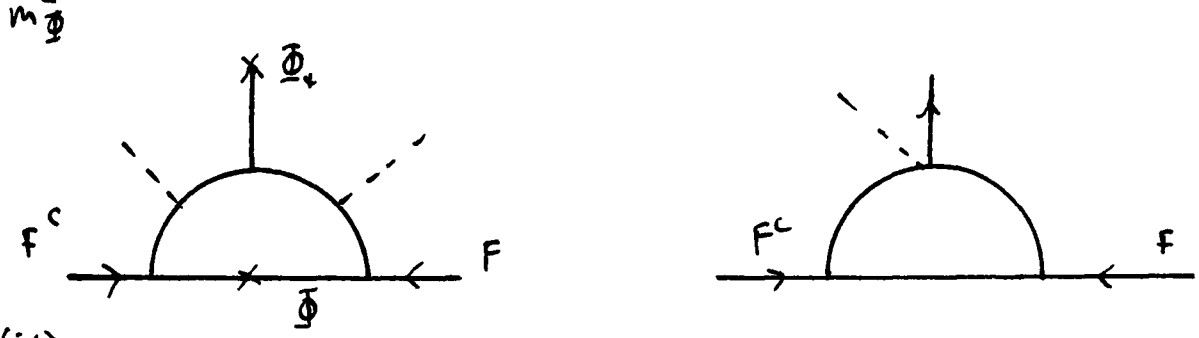


fig 2.2.3 (i) generation independent graph; such terms cannot be larger than ½ MeV so there are bounds on m^2_{Φ}



(ii) Generation dependent graphs. The dependence comes in the first graph from the scalar mass insertions, and from the second from the cubic scalar coupling.

The U_1 symmetries carried by the Higgs multiplets (X_r, Y_r) are more of a problem. Of the four massless modes which are generated by the Higgs' v.e.v.s only three acquire mass, leaving one Goldstone boson. In ref.[11], Del Aguila et al. break this global symmetry explicitly by the addition of an extra Higgs multiplet coupling to X_r and Y_r. A more elegant means to eliminate this remaining massless particle is to use only one Higgs hypermultiplet to break the SU_4 group. Since such a procedure results in a more economical theory, it is a possibility worth investigating.

Firstly, where does the Goldstone Boson come from? To see this, note that the U_1 symmetries in question are given by

$$X_r \rightarrow e^{i\theta_r} X_r$$

$$Y_r \rightarrow e^{-i\theta_r} Y_r \qquad (2.2.21)$$

The dangerous parts of this symmetry are carried by the scalar Higgs doublets ξ_r and η_r, defined by

$$\chi_r = \begin{pmatrix} \xi'_r \\ \xi_r \end{pmatrix} \quad , \quad y_r = \begin{pmatrix} \eta_r \\ \eta'_r \end{pmatrix} \tag{2.2.22}$$

since only these acquire v.e.v.s; of the four modes α_r and β_r defined by setting

$$\xi_r = (a_r + u)\, e^{i\alpha_r}$$

$$\eta_r = (b_r + u)\, e^{i\beta_r}$$

which are left massless by the scalar mass terms $m_r^2 x_r^\dagger x_r$ + $m_r'^2 y_r y_r^\dagger$, two acquire masses through the cubic scalar terms (2.2.14), while one is 'eaten' by the T-boson, where T is the SU_4 generator

$$T = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.2.23}$$

since the symmetry (2.2.21) acting on ξ_r and η_r is the same as the gauge transformation

$$\chi_r \rightarrow e^{-i\theta T} \chi_r$$

$$y_r \rightarrow e^{i\theta T} y_r \quad \text{for } \theta_1 = \theta_2 = -\theta \tag{2.2.24}$$

The combination of α_r, β_r orthogonal to those given mass by (2.2.14) and (2.2.23) remains massless.

It is clear from the above argument that no massless modes survive if there is only one Higgs multiplet : (2.2.24) remains a symmetry and provided there is a cubic scalar term like (2.2.14) then all the Higgs modes have masses greater than m_W. However, it is impossible to break SU_{2L} at the weak scale and at the same time not to break U_{1em}, for the following reason : suppose the Higgs multiplet with the desired property is (X,Y), where X belongs

to a d-dimensional irreducible representation of SU_4. Its v.e.v. $x = \langle X \rangle$ must satisfy

$$T_{1L}\, x = T_{2L}\, x = T_{3L}\, x = 0 \qquad (2.2.25a)$$

$$Q\, x = (T_{3L} + T_{3R} + z)\, x = 0 \qquad (2.2.25b)$$

where z is the hypercharge of x, and

$$T_a\, x \neq 0 \qquad (2.2.25c)$$

for all generators T_a of SU_4 except those of SU_{2L}. From (2.2.25), it follows that

$$z\, x = -T_{3R}\, x . \qquad (2.2.26)$$

It is easy to see that (2.2.25c) and (2.2.26) are inconsistent. x is a point in a d-dimensional vector space, with coordinates x^I (I = 1..d). Choosing a basis such that T_{3R} is diagonal in this representation,

$$T^I_{3RJ} = T^{(I)}\, \delta^I{}_J$$

then (2.2.26) is

$$z\, x^I = -T^{(I)}\, x^I$$

Because T_{3R} is traceless ($\Sigma T^{(I)} = 0$), this equation has a solution only when exactly one of the x^I's is non-zero (say $x^1 \neq 0$). It is clearly always possible to find a generator T_a which annihilates this vector, i.e.

$$T_a{}^I{}_1 = 0 \qquad \forall\ I$$

so there is no possibilty of breaking $SU_4 \to SU_{2L}$ using only a single Higgs representation.

Alternative mechanisms for avoiding the axion problem are

(i) explicitly breaking the global U_1 symmetry, as

mentioned above. One way to do this is given in ref.[11].

(ii) making it 'invisible' by reducing its coupling to matter[96,97]. This possibility has not been investigated.

§2.2.g The τ-neutrino mass problem

Sarkar and Cooper[98] have recently pointed out that cosmological arguments, together with laboratory bounds on the τ-neutrino lifetime, give a much stronger bound on the ν_τ mass, $m_{\nu\tau} < 100$ eV. This follows since an unstable ν_τ must decay after nucleosynthesis, and the normal decay modes $\nu_\tau \to e^+e^-\nu$, $\gamma\nu$ would give an unacceptable nucleo-dissociation. Then, either the ν_τ is stable, or it decays through a mode which does not affect the nucleosynthesis - for example, it may decay through an 'invisible' Majoron[99], $\nu_\tau \to J\nu$, which couples very weakly to matter. In this section, the implications of the former possibility on the $SU_3 \times SU_4 \times U_1$ model are considered.

If the ν_τ is stable, then bounds on the cosmological energy density[101] imply that $m_{\nu\tau}$ is either less than 100 eV or greater than 1 GeV; the experimental upper bound[100] of $m_{\nu\tau} < 164$ MeV rules out the second possibility, so one is left with

$$m_{\nu_\tau} < 100 \text{ eV} \tag{2.2.27}$$

It will be shown in this section that to accomodate such a small τ-neutrino mass in the $SU_3 \times SU_4 \times U_1$ model of ref.[11]

requires an unnaturally high scale of supersymmetry breaking.

The problem arises because the right-handed neutrinos in the $SU_3 \times SU_4 \times U_1$ model form SU_{2L} doublets with the right-handed leptons, and thus the neutrinos acquire a Dirac mass of the same size as that of the corresponding charged lepton, from the graphs of fig 2.2.3. To remove this degeneracy, Del Aguila *et al.*[11] introduce a Majorana mass term $M(\nu^c)^2$ for the right-handed neutrinos, so that the mass matrix for the left- and right-handed states becomes

$$(\nu \quad \nu^c) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix} + h.c. \tag{2.2.28}$$

where m is the Dirac mass term. If $M \gg m$, then the lighter eigenstate of this matrix is approximately

$$\nu_{light} \simeq \nu - \frac{m}{M} \nu^c \tag{2.2.29a}$$

which has a mass

$$m_\nu \approx m^2/M \tag{2.2.29b}$$

The Majorana mass term $M(\nu^c)^2$ is generated by introducing a family of gauge singlet hypermultiplets

$$\{ (x^i_M, y_{Mi}), \; i = 1 \ldots 3 \} \tag{2.2.30}$$

together with the soft terms

$$\mathcal{L}^M_{soft} = \sum_{i=1}^{3} \{ \mu_x y_{Li} x^i_M x_1 + (\mu'_x)^2 (x^i_M)^2 + (x_M \hookrightarrow y_M) \} \tag{2.2.31}$$

These generate a Majorana mass through the graph of fig

2.2.4, of order

$$M \sim \frac{g_1^2}{16\pi^2} m_{\tilde{\lambda}_1} \qquad (2.2.32)$$

Even without the new mass bound (2.2.27), to satisfy existing experimental bounds on the light neutrinos Del Aguila *et al.* were forced to raise the supersymmetry breaking scale to about 100 TeV. If, however, the ν_τ mass must be as light as 100 eV, then the supersymmetry scale must be increased again : from (2.2.29b) (with $m = m_\tau \sim 2$ GeV) it follows that

$$m_{\tilde{\lambda}_1} \gtrsim 10^9 \text{ GeV}. \qquad (2.2.33)$$

The problem with this is now apparent : $\tilde{\lambda}_1$ couples to the scalars in the theory through the graph in fig 2.2.5, and therefore the natural size of m_W^2 is of order $\alpha_1 m_{\tilde{\chi}_1}^2$. To put it another way, $m_{\tilde{\chi}_1}$ is a supersymmetry-violating mass, and since the supersymmetry scale must be so high (2.2.33) in this model, the weak scale is no longer protected from large radiative corrections.

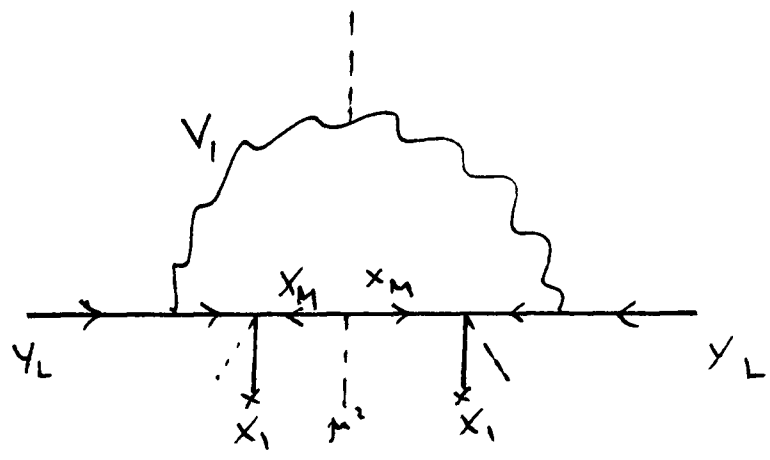


fig 2.2.4. Majorana mass for the right-handed neutrino.

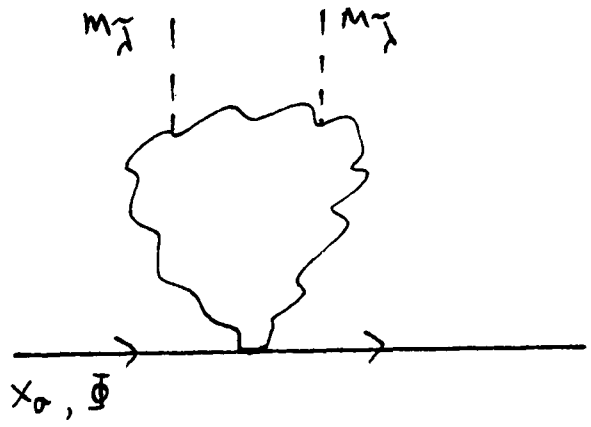


fig 2.2.5. The gaugino mass produces a large mass renormalisation.

§2.2.h The hierarchy problem in N=2 theories

An interesting possibilty in N=2 theories to avoid the consequences of (2.2.33), which has not previously been considered in detail, is to cancel the large radiative corrections of $m_{\tilde{\chi}_1}^2$ (or of any other large supersymmetry breaking term) by maintaining one supersymmetry unbroken at a low energy (say 1 TeV) to protect the weak scale, while allowing the other supersymmetry to be broken at a much larger scale. In this section it will be shown that this cannot be achieved within the context of the $SU_3 \times SU_4 \times U_1$ model because radiative corrections from the large scale 'drag up' the lower scale. Necessary (but not sufficient) conditions will also be given for maintaining a hierarchy of supersymmetry scales within a more general N=2 model.

I will show this by considering the one-loop corrections to the ϕ_4 scalar, whose v.e.v. breaks SU_2. It is a necessary condition for maintaining naturally a weak scale that these corrections be less than about 1 TeV. From (2.1.20c), the general expression for the infinite part of the corrections is

$$g_4^2 \, \mathrm{tr} \, |\varphi_4|^2 \left\{ C_2(4) \left[m_{\varphi_4}^2 - 2|m_0|^2 - 2|m_1|^2 - 4|m_3|^2 \right] \right.$$
$$\left. + \sum_r T(R_\sigma) \left[m_{x\sigma}^2 + m_{y\sigma}^2 - 4|m_{2\sigma}|^2 + |n_{0\sigma}|^2 + |n_{1\sigma}|^2 \right.\right.$$
$$\left.\left. + |n_{x\sigma}|^2 + |n_{y\sigma}|^2 \right] \right\} \qquad (2.2.34a)$$

which can be rewritten in words as

$$g_4^2 \, \mathrm{tr} \, |\varphi_4|^2 \left\{ C_2(4) \left[m_{\varphi_4}^2 - |\text{gaugino masses}|^2 \right] \right.$$
$$\left. + \sum_r T(R^\sigma) \left[|\text{scalar mass insertions} - |\text{fermion mass}|^2 \right.\right.$$
$$\left.\left. + |\text{cubic scalar terms}|^2 \right] \right\} \qquad (2.2.34b)$$

If one of the terms in this expression breaks a supersymmetry at a large scale Λ, then it is not sufficient simply to cancel its large contribution to the ϕ_4 mass with another term, for this will require an unnatural fine-tuning of the parameters. The only natural way this can be achieved is by collecting the terms of (2.2.34) into supersymmetric mass terms - this has already been done in (2.1.24), from which it follows that the most general soft insertion which preserves one supersymmetry is

$$\mathcal{L}_{soft} = \int d^2\theta \left[\mu\, tr\,(\Phi_u^+)^2 + \sum_\sigma m_{2\sigma} \gamma_\sigma \chi_\sigma \right] \qquad (2.2.35)$$

for some element u of SU_2. If the remaining supersymmetry is broken at a high scale Λ, then one of the parameters μ or $m_{2\sigma}$ must be of order Λ. In the $SU_3 \times SU_4 \times U_1$ model considered, it is clear that μ must be small, for μ^2 is a tree-level mass term for ϕ_4. On the other hand, the masses $m_{2\sigma}$ preserve not just one, but both supersymmetries, so nothing is gained by taking large values for $m_{2\sigma}$.

Another possibility is that a supersymmetric mass term be formed from just one component of the hypermultiplet, say

$$m \chi_\sigma^2 \Big|_F$$

Of course, this is only possible if σ is a real representation, and there are no real multiplets in the $SU_3 \times SU_4 \times U_1$ model. Finally, if one of the hypermultiplets belongs to the adjoint representation of the gauge group, then its scalar component may form an N=1 multiplet with the spinor component of Φ or V; again, this possibilty is not relevant for the $SU_3 \times SU_4 \times U_1$ model.

§2.2.i Conclusions.

The $SU_3 \times SU_4 \times U_1$ model considered here can give good quark and charged lepton masses, while scalars and mirror fermions have masses of order m_W. Without the addition of explicit U_1 breaking terms, the spontaneous breakdown of a chiral U_1 symmetry produces a massless scalar; one possible means of avoiding this, by using a single Higgs hypermultiplet to break the gauge symmetry, was investigated but was shown not to be a viable alternative.

Secondly, if the ν_τ lifietime measurements imply that the ν_τ is stable, then the resulting stringent mass bound (2.2.27) is a serious problem for the model. I have shown that to satisfy (2.2.27) using the Majorana mass mechanism requires a dramatic increase in the supersymmetry breaking scale (cf 2.2.33), and that this is unnatural within the context of the model considered, because large radiative corrections from the high scale 'drag up' the weak scale : a hierarchy of supersymmetry breaking scales cannot be maintained. An alternative to this scheme, which has not been considered here, is to allow the ν_τ to decay through an invisible axion J which couples weakly to matter : in this way the decay of a long-lived ν_τ will not dissociate primordially synthesised He^4 and deuterium, so the mass bound (2.2.27) need not apply. If the 'Majoron' J can be identified with the Goldstone boson of §2.2.f, then both of the problems identified here in the $SU_3 \times SU_4 \times U_1$ model will be solved.

REFERENCES

1. J.Wess, B.Zumino; Nuc Phys. B70, 39 (1970).
2. M.B.Green, J.H.Schwarz; Phys Lett. 149B, 117 (1985).
3. D.Gross, J.Harvey, E.Martinec, R.Rohm; Nuc Phys. B256, 253 (1985).
4. L.Hall, J.Lykken, S.Weinberg; Phys Rev. D27, 2359 (1983).
5. L.Alvarez-Gaume, J.Polchinski, M.B.Wise; Nuc Phys. B221, 495 (1983).
6. A.Parkes, P.West; Phys Lett. 127B, 353 (1983).
7. J.M.Frere, L.Mezincescu,Y.P.Yao, Phys Rev. D29, 1196 (1984)
8. Same authors, Phys Rev. D30, 2238 (1984).
9. S.Kalara, D.Chang, R.Mohapatra, A.Gangopadhyaya; Phys Lett. 145B, 323 (1984).
10. E.Cremmer, C.Kounnas, A.Van Proeyen, J.P.Derendinger, S.Ferrara, B.de Wit, L.Girardello; Nuc Phys. B250, 385 (1985).
11. F.Del Aguila, M.Dugan, B.Grinstein, L.Hall, G.G.Ross, P.West; Nuc Phys. B250, 225 (1985).
12. J.Wess, J.Bagger; "Supersymmetry and Supergravity", Princeton U.P. (1983).
13. R.Hung, J.Lopuszanski, M.Sohnius; Nuc Phys. B88, 257 (1975).
14. M.Fierz; Helv Phys Acta. 33 855 (1960).
15. J.Wess, B.Zumino; Nuc Phys. B78, 1 (1974).
16. F.Berezin, G.Kac; Ann Math Soc Transl. 11, 311 (1971).
17. S.Ferrara, B.Zumino; Nuc Phys. B79, 413 (1974).
18. M.Grisaru, M.Rocek, W.Siegel; Nuc Phys. B159, 429 (1979).
19. eg. G.G.Ross; "Grand Unified Theories", Benjamin-Cummings (1984).

20. P.Fayet, J.Iliopoulos; Phys Lett. 51B 461 (1974).

21. S.Ferrara, L.Girardello, F.Palumbo, Phys Rev. D20, 403 (1979).

22. S.Weinberg; Phys Rev D26 287 (1982).

23. A.Lahanus, T.R.Taylor; Phys Lett. 114B, 132 (1982).

24. R.Barbieri, S.Ferrara, D.Nanopoulos, Zeit Phys. C13, 267 (1982).

25. P.Fayet; Nuc Phys. B90, 104 (1975).

26. L.O'Raifeartaigh; Nuc Phys B96, 331 (1975).

27. J.Ellis,L.Ibanez,G.G.Ross; Phys Lett 113B, 283 (1982)
Nuc Phys B221, 29 (1983)
E.Witten; Phys Lett 105B, 207 (1981).

28. H.P.Nilles; Phys Lett. 115B, 193 (1982).

29. H.P.Nilles; Phys Rep. 110, 1 (1984).

30. R.Barbieri, S.Ferrara, C.Savoy, Phys Lett. 119B, 343 (1982).

31. A.Chamseddine, R.Arnowitt, P.Nath; Phys Rev Lett. 49, 970 (1982).

32. H.P.Nilles, M.Srednicki, D.Wyler; Phys Lett 120B, 346 (1983).

33. E.Cremmer, B.Julia, J.Scherk, P.van Niewenhuizen, S.Ferrara, L.Girardello; Nuc Phys B147, 105 (1979).

34. S.Deser, B.Zumino; Phys Rev Lett. 38, 1433 (1975).

35. G.'t Hooft; Nuc Phys B61, 455 (1973).
S.Weinberg; Phys Rev. D8, 3497 (1973)

36. J.M.Frere, D.R.T.Jones, S.Raby; Nuc Phys B222, 11 (1983).

37. L.Ibanez, C.Lopez, Phys Lett 126B, 54 (1983).

38. J.Ellis, J.Hagelin, D.Nanopoulos, K.Tamvakis; Phys Lett. 125B, 275 (1983).

39. M.Kobayashi, T.Maskawa; Prog Th Phys. (Kyoto) 49, 652

(1973)

40. H.Georgi, S.L.Glashow; Phys Rev Lett. 32, 438 (1974).
41. J.Donaghue, H.Nilles, D.Wyler; Phys Lett. 128B, 55 (1983).
42. A.Bouquet, J.Kaplan, C.A.Savoy; Phys Lett. 148B, 69 (1985).
43. H.Georgi, C.Jarlskog; Phys Lett. 86B, 297 (1979).
44. M.J.Duncan; Nuc Phys. B241, 21 (1983).
45. L.Maiani; Lectures at TASI-84, Ann Arbor (1984).
46. G.D.Coughlan, R.Holman, P.Ramond, G.G.Ross, M.Ruiz-Altaba, J.Valle; Phys Lett. 148B, 401 (1985).
47. L.Maiani; Lectures at TASI-84, Ann Arbor (1984).
48. D.R.T.Jones; Nuc Phys. B87, 127 (1975).
49. D.Nanopoulos, K.Tamvakis; Phys Lett. 113B, 151 (1982).
50. G.G.Ross, S.K.Wilkinson; Oxford Preprint 50/85.
51. M.Einhorn, D.R.T.Jones; Nuc Phys. B196, 475 (1982).
52. J.Ellis, D.Nanopoulos, S.Rudaz; Nuc Phys. B202,43 (1982).
53. W.Marciano, Z.Parsa; Proceedings of Particles and Fields Summer Study on Elementary Particle Physics and Future Facilities (Snowmass,CO,1982), (AIP, New York 1983) 155.
54. N.Sakai, T.Yanagida; Nuc Phys. B197, 533 (1982).
55. M.Krishnaswamy et al.; Phys Lett. 115B, 349 (1982).
56. S.Dimopoulos, S.Raby, F.Wilczek; Phys Lett.112B, 133 (1982).
57. R.Barbieri, L.Girardello, A.Masiero; Phys Lett. 127B, 429 (1983).
58. V.M.Belyaev, M.Vysotsky; Phys Lett.127B, 215 (1983).
59. A.Masiero, D.Nanopoulos, K.Tamvakis, Y.Yanagida; Phys Lett. 115B, 380 (1982).
60. B.Grinstein; Nuc Phys. B206, 387 (1982).
61. G.Battistoni et al.; Phys Lett. 118B, 461 (1982).

62. S.Chadha, G.D.Coughlan, M.Daniel, G.G.Ross; Phys Lett. 149B, 477 (1984).

63. S.Chadha, M.Daniel; Phys Lett.137B, 374 (1984).

64. J.Ellis, J.Hagelin, D.Nanopoulos, K.Tamvakis; Phys Lett. 124B, 484 (1983).

65. S.Chadha, M.Daniel; Nuc Phys. B229, 105 (1983).

66. T.Inami, C.Lim; Nuc Phys B207 533 (1982)

67. S.Glashow, J.Iliopoulos, L.Maiani; Phys Rev. D2, 1285 (1970).

68. J.Ellis, D.Nanopoulos; Phys Lett. 110B, 44 (1982).

69. R.L.Barbieri, R.Gath, Phys Lett. 110B, 211 (1982).

70. B.A.Campbell; Phys Rev. D28, 209 (1983).

71. B.Sazdovic, O.Tavasov; Nuc Phys. B250, 39 (1983).

72. M.F. Sohnius; Nuc Phys: B138, 109 (1978).

73. P.Fayet; Nuc Phys. B113, 135 (1976).

74. P.S.Howe, K.S.Stelle, P.K.Townsend; Nuc Phys B214, 519 (1983).

75. M.Grisaru, W.Siegel; Nuc Phys. B201, 292 (1982).

76. D.R.T.Jones; Nuc Phys. B87, 127 (1975).

77. P.S.Howe, P.C.West; Nuc Phys. B242, 364 (1984).

78. P.S.Howe, K.Stelle, P.West; Phys Lett. 124B, 55 (1983).

79. D.R.T.Jones, L.Mezincescu; Phys Lett. 138B, 293 (1984).

80. A.Parkes, P.West; Phys Lett 138B, 99 (1984).

81. A.Parkes, P.West; Nuc Phys. B256, 340 (1985).

82. S.Hanindi, J.Patera, J.H.Schwarz; Phys Lett 141B, 349 (1984).

83. D.R.T.Jones, A.Parkes; "Search for a 3-Loop finite chiral theory", Liverpool preprint LTH-131 (1985).

84. S.Ferrara, B.Zumino; Nuc Phys. B79, 413 (1974).
D.R.T.Jones; Phys Lett. 72B, 199 (1977).
E.Poggio, H.Pendleton; Phys Lett 72B, 200 (1977).
O.Tavasov, A.Vladiminov, A.Yu; Phys Lett 93B, 429 (1980).

85. M.Grisaru, M.Rocek, W.Siegel; Phys Rev Lett. 45, 1063 (1980).

86. A.Parkes, P.West; Nuc Phys. B222, 269 (1983).

87. M.Grisaru,W.Siegel; Nuc Phys. B194, 65 (1982).

88. A.Parkes, P.West; Phys Lett. 122B, 365 (1983).

89. I.Koh,S.Rajpoot; Phys Lett 135B, 397 (1984).

90. F.Dong, T.Tu, R.Xue, X.Zhon; Phys Lett. 140B, 333 (1984).

91. J.P.Derendinger, S.Ferrara,A.Masiero; Phys Lett. 143B, 133 (1984).

92. S.Kalava, D.Chang, R.Mohapatra, A.Gangopudhaya; Phys Lett. 145B, 323 (1984).

94. D.Namazie, A.Salam, J.Strathdee; Phys Rev D28, 1481 (1984).

95. J.Van der Bij, Y.P.Yao; Phys Lett. 125B, 171 (1983).

96. M.Dine, W.Fischler, M.Srednicki; Phys Lett. 104B, 199 (1981).

97. M.Wise, H.Georgi, S.Glashow; Phys Rev Lett. 47, 402 (1981).

98. S.Sarkar, A.Cooper; Phys Lett. 148B, 347 (1984).

99. Y.Chikashige, R.Mohapatra, R.D.Peccei; Phys Lett 98B, 265 (1981).

100 C. Matteuzzi et al. (SLAC); Phys Rev Lett 52, 1869 (1984).

101 B.Lee, S.Weinberg; Phys Rev Lett 39, 165 (1977).

102. D.R.T. Jones, L. Mezincescu & Y-P. Yao, phys. Lett. 148B, 317 (1984)