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South African maths teachers' perceptions about estimation, practice, and praxis



A dissertation submitted in partial fulfilment of the requirements of the Master of Science in Education (Comparative and International Education)

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Abstract

This research project explores how the often-under-taught mathematical skill of estimation can be a useful tool in helping young South Africans grow into 'future leaders'. The paper argues that in a socio-economic landscape which is defined by inequality and under-education, the skill of estimation, when taught well, has the potential to greatly benefit the lives of young and old. Serving as foundational research for further intervention studies, this research project explores nine maths teachers' perceptions about the skill of estimation and their adapted teaching practice and praxis according to these perceptions. Inductive analysis of data from interviews, a WhatsApp group, and a focus group reveal that estimation is an undeniably important mathematical skill. Taught in maths classrooms, estimation is perceived to be a transferable life skill which can assist with problem solving and contextualisation of information. Findings from this research project further reveal that despite teachers' uncertainty about estimation, they find it important to adapt their teaching practices to better integrate the skill into their classrooms. Teachers' adaptive praxis is aligned with critical maths education literature and provides preliminary data for widespread teaching interventions in South Africa. These conclusions contribute to global dialogues about the importance of estimation and provide reasons to believe that South African teachers are willing to change their teaching praxis to better serve the 'future leaders' they teach. This research project aims to lay the foundation for further research and resultant interventions in South African maths classrooms.

Key words: *estimation, mathematics, high school, South Africa*

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This dissertation is written for the children of South Africa who, for too long, have suffered. You have been my inspiration from the beginning, and it is for you that I hope to keep living my life.

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Abbreviations

ANA	Annual National Assessments (for Grade 9s)
CAPS	Curriculum and Assessment Policy Statement
EMIS	Education management information systems
FET	Further Education and Training (Grade 10-12)
GET	General Education and Training (Grade 7-9)
IEB	Independent Examinations Board (Grade 12 school leavers' examinations for some private schools)
ICT	Information communication technology
KZN	KwaZulu-Natal
NSC	National Senior Certificate (Grade 12 school leavers' examination for all public schools and some private schools)
RQ	Research Question
SES	Socio-economic status
SE	Socio-economic

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Introduction

Estimation is important. According to instrumental policy documents which shaped maths curricula globally (Cockcroft, 1982; NCTM, n.d.), estimation is one of the most important mathematical and life skills that can be taught to children. The skill, which has not yet been authoritatively researched in the South African context, has proven its daily worth for people worldwide (Hanson & Hogan, 2000). Although most of the research surrounding the skill's place in classrooms was conducted in the late 20th Century, this research project reimagines how estimation can be used to critically empower citizens of South Africa, where maths achievement levels are infamously low (Macupe, 2020). In response to this context, and a personal ambition to make positive impacts on mathematics education in education, this research project was born. As an exploratory mixed-methods research project which heeds the complexities of the South African context, this project strives to lay the groundwork for future maths teaching interventions. Leaning on critical theories of place and pedagogies, the project attempts to understand and reframe power imbalances present in South African society by empowering participants to adapt their teaching praxis (Aslan Tutak et al., 2011). The findings of this research project reveal a deep willingness of South African teachers to adapt to the needs of their learners and the world, and a keenness to continue learning how to do this effectively.

This dissertation has been written in the midst of the Covid-19 pandemic and while South Africa faces unparalleled political instability ('A Moment of Rupture: Editorial', 2021; Wicks, 2021). The critical lens of this dissertation contextualises how research can pave the way towards educating children, their teachers, and – by extension – other adults, to be more reflexive about their thoughts and decisions, not only in the maths classroom. Esteemed economist, Daniel Kahneman, suggests in his best-seller, *Thinking, Fast and Slow*, "questioning what we believe is difficult at the best of times… but we can benefit from the informed opinions of others" (2011, p. 3). This research project attempts to do this, to determine why, for South Africans who have been betrayed by systematic repression for years (Sguazzin, 2021), estimation may seem like an unusable skill which is reserved for working out how much money is needed for the next grocery haul. Like Kahneman (2011) suggests, whether appreciated or not, estimation is an intuitive skill which requires practice and contextual understanding. To elucidate this, the findings of this research project are

telling: learning happens by engaging with others; estimation is intuitive, but takes practice and requires and develops contextual knowledge; and maths education is not only about learning to add, subtract, and draw graphs. In fact, Kahneman's (2011) contention that we all need to think both fast and slowly is affirmed by how nine South African teachers approach the teaching of maths.

Without including a section on positionality, this research project weaves personal reflections, aims, values, and connections to the South African context into all sections of the paper; positionality is not defined but rather exudes itself through the authorship, as is illustrated by the previous paragraph.

Currently, and for many years, South Africa has faced an educational crisis (Gustafsson & Nuga, 2020; Roodt, 2018; Spaull, 2019a) which is linked to a democratic crisis (*Where Does South Africa Go from Here?*, 2021); this continues to impact the lives of millions and the future of the country. Such a critical reflection on the state of the country contextualises the need to solve problems facing the country, one of which stems from mathematics education and burgeons into everyday societal interactions. In response, this research project proposes a way forward: curriculum change through researched interventions which can help learners become more mathematically skilled and critically aware. Following a simple mixed method design, which is based on pragmatism and ethical awareness, this research project rigorously analyses collected data to answer the following research questions:

RQ 1. What are South African Mathematics and Mathematical Literacy teachers' perceptions about the importance of estimation in the classroom and beyond?

RQ 2. How do teachers' classroom practices / praxis change through participation in exploratory research projects?

- **a.** What pointers should be kept in mind when designing a teaching intervention for high school maths teachers?

Outline of the study

The Introduction and Conclusion of this research project bookend five chapters comprising of a literature review, research questions, research methodology, findings, and

discussion. These should be read in conjunction with the attached Appendices. Before continuing, I draw attention to the use of the word "maths" throughout this dissertation: this term is used to refer to all forms of mathematical thought and curricula and is used to avoid confusion with Mathematics and Mathematical Literacy, which are chosen as part of the compulsory subjects to be completed in Grade 12 school leavers' examinations.[1]

The Literature Review, comprising of four parts, contextualises the South African education system, addresses some interventions currently operating in the country, locates the project in critical theory literature, and addresses issues specific to maths education and the skill of estimation. The review of relevant literature reveals that education is influential over, and influenced by, other parts of everyday life and therefore is only one part of the broader system of societal organisation. As a result, while it may seem 'education specific', this research project addresses many other societal complexities through its critical theory approach.

The Research Questions and Methodology appear next. Together, these chapters direct the research project by asking questions and detailing how the project was designed to answer them. The research questions' purpose and context are explained according to the needs of this study. The Methodology rationalises the project, specifies data collection and analysis tools and methods, argues for the project's validity and rigor, and reflects on limitations of the current research design.

Chapters 4 and 5 house the Findings and Discussion respectively. The Findings address research questions by illuminating key themes; the Discussion connects findings from this research project with relevant literature and deliberates over its implications for further research in South African mathematics education. These two chapters, together,

[1] In South Africa, Grades R-9 are compulsory. Grade 9, the final year of the General Education and Training (GET) phase of education, is also the time when learners are required to choose at least seven subjects to study for their final three years of schooling in the Further Education and Training (FET) phase. Each child is required to study two languages, Life Orientation, Mathematics or Mathematical Literacy, and then three other subjects. These are the subjects tested in the final Grade 12 year.

Public (state-funded) schools write the National Senior Certificate (NSC) examinations and follow the Curriculum and Assessment Policy Statements (CAPS). Although some private schools do the same, many follow the Independent Examinations Board (IEB) assessment structure. IEB schools cover a very similar maths syllabus to that in NSC schools and it is for this reason that I have not differentiated between these two syllabi in this dissertation.

reveal South African maths teachers' eagerness to continue learning, reflecting, and changing their practices.

The Conclusion briefly summarises the outputs of this research project and provides suggestions for further research. Knowledge presented in this dissertation sets the foundation for further research and interventions that can enhance South Africa's democratic future.

Chapter 1: Literature Review

By locating this research project in the global and local literature, this Literature Review introduces its relevant context and aims. The review of the state of South African education, with specific reference to maths and accompanying intervention strategies, provides the impetus for conducting this research. After this conceptualisation, the Literature Review discusses the role played by Critical Theories in shaping this research project and ends by addressing the topic of this project: mathematical estimation. These sections, together, perform the task of relating global issues to local contexts, broad questions to researchable topics, and locating the literature gap which guided this project.

1.1. The state of South African education

It can be argued that education is just one of the socio-economic issues impeding South Africans' social, economic, and personal progress (Bush et al., 2010; Oosthuizen & Bhorat, 2006; Venkat, 2018). Despite recent improvements, the education offered to the majority of South African children remains below the standards of neighbouring and other middle-income countries (Spaull, 2013). This is despite the Constitutional mandate to provide all children with a basic education (Government of South Africa, 1996), and curriculum documents affirming the aims that education should "improve the quality of life of all citizens and free the potential of each person" (Department of Basic Education, 2011b, p. 1). Although the notion of 'good quality education' is subjective, commentators suggest that the majority of school-going South African children do not receive good quality education (Amnesty International, 2020; Spaull, 2019a). Reasons for this lack of quality education range from ineffective national service delivery (Gumede & Biyase, 2016; Oosthuizen & Bhorat, 2006) to inefficient and impractical classroom practices and pedagogies (Adler, 2002; Venkat & Spaull, 2015). These factors impact children's lived experiences of the education system.

What follows is a brief overview of the South African education system which is grounded in Critical Theory and therefore presents critiques of current systems of power relations.

1.1.1. A brief history of South African education to the present

The democratically elected government of 1994 inherited large-scale educational inequality that characterised the racial segregation of apartheid (van der Berg, 2007). Bantu education, implemented in 1953, was designed to stunt Black children's aspirations and offer a sub-standard education compared to their White counterparts (Glaser, 2018). In response, various curriculum changes have altered the educational landscape in the country, many of which have been designed to improve under-resourced schools which teach predominantly Black children (Pienaar & McKay, 2014).[2] However, in order to fully understand socio-economic status and the hierarchical social structures prevailing in South Africa, one cannot really untangle race and class (Taylor & Yu, 2009). These layers of intersectional identity have resulted in children experiencing various forms of discrimination despite national policies implemented to prevent this from happening.

One of the ways that the South African government has attempted to address these inequalities is through pro-poor funding strategies for educational institutions. The South African Schools Act (1996) developed a ranking system of schools' needs, according to the socio-economic status (SES) of the communities they served, and instituted national state funding for schools according to the Quintile System (Amended National Norms and Standards for School Funding, 2012). Surprisingly, it is difficult to find public documentation of racial demographics according to school quintiles, yet it is widely accepted that no-fee schools are attended by those living in poorer areas, who happen to fall within non-White racial categories (Amnesty International, 2020; Pienaar & McKay, 2014; Spaull, 2013). In more recent years, low-cost private schools have bourgeoned in both urban and rural areas,

[2] To understand the context of this study, note that in South Africa, racial categorisations are part of the vernacular. The categories *Black*, *White*, *Indian*, *Coloured*, and *Asian* appear on most documentation as a means of categorising and identifying individuals as part of racial groupings. Thus, when this research report speaks about Black children, it is not meant in a derogatory manner, but instead refers to a group of people Bantu Steven Biko worked to liberate at the height of the Black man's strife during Apartheid (Biko, 2005). His discourse presented 'Black' as a term of power and identity, not one that merely refers to skin colour. Moreover, the term 'Black' is meant to refer to people with African heritage only and does not include other races (like the newly popularised terms, 'BAME', and 'BIPOC' (Saeed et al., 2019)). The premise for using the term 'Black' (and other race-based terminology common in South Africa) in this dissertation follows the norms of racial categorisation set by policy documents and local academic trends (Adler, 2002; Department of Basic Education, 2020b; Graven, 2002; National Planning Commission, 2012).

offering children from poor backgrounds an alternative to state-funded education (England, 2012).

1.1.2. Education quality: definition and debates

Since the introduction of the Sustainable Development Goals in 2012 (United Nations, 2018), there has been a global paradigm shift: education equity is no longer only about who has access to education, but also relates to the quality of education that is delivered. However, education quality is arguably subjective and context-specific (Sahlberg, 2015; UNESCO & Education for All, 2005; UNESCO: Learning Portal, 2021). In South Africa, the 'quality' of education is a topic of discussion in academia (See Gustafsson & Nuga, 2020; Spaull, 2013), policy-making (Department of Basic Education, 2020b), and public discourses (Maarman, 2021). Without a shared understanding of what it means in South Africa, quality education is difficult to achieve; how can one aim for a goal that is undefined? Thus, when the Department of Education states their aim is to offer "a *quality* and inclusive, safe and healthy basic education system" (Department of Basic Education, 2020b, p. 11) which would "improve the *quality* of life of all citizens and free the potential of each person" (Department of Basic Education, 2011b, p. 1), one wonders what exactly this means.

To better understand education quality in the South African context, the Annual Performance Plan for the Department of Basic Education (2020, p. 23), proposes a "Theory of Change for Quality Basic Education" which designates responsibilities to various educational practitioners. It is suggested that these stakeholders can improve learners' opportunities "through the acquisition of foundational language and numeracy competencies, and beyond that, specific subject content knowledge and life skills" (Department of Basic Education, 2020b, p. 23). However, there is no indication how this can be operationalised by the nine provincial education departments. The document merely concludes that "A national strategy to bring about quality basic education is needed… [which] must be a living one, characterised by a lively and constructive policy discourse, informed by reliable information produced by government and non-government players" (p. 25). This national strategy does not yet exist and thus the country has continued struggling educate her children through the Covid-19 Pandemic.

In recent years, the Department of Basic Education (2018) has reported qualitative improvements in education delivery and outcomes. Motala and Carel (2019) show more equitable resource distribution among schools, which is aligned with the 'pro-poor' funding and resource allocation strategies. However, despite these apparent improvements, millions of children continue to receive poor quality education (Spaull, 2019a). Although not all children are affected by the various factors impeding education quality, many have been left with no choice but to adapt to the systemic, institutional, and classroom-specific issues which hinder their academic progress. Discussed in Chapter 1.1.4. in terms of 'cognitive poverty' (Crouch & Gustafsson, 2018), this academic strife is a common occurrence in all levels of schooling in South Africa and impacts children for the rest of their lives.

Although the compounding injustices of education inequality are visible throughout South Africa's education system, they are most clearly exemplified by national school leavers' results. Last year's National Senior Certificate (NSC) results indicated a drop in the national Grade 12 pass rate to 76,2% with only 53,8% and 80,8% of learners passing Mathematics and Mathematical Literacy respectively (Department of Basic Education, 2021), this is despite the pass mark for these subjects being 30%. To illustrate the dire straits of the national education system, Appendix A briefly reviews South Africa's recent performance in national and international benchmarking tests.

1.1.3. Constitutional mandates, policy documents, and curriculum aims

Recent musings on the South African philosophy of education have resulted in my own confusion about the fundamental mandate of the education system I grew up in. The following table offers a brief overview of the mandated educational rights and aims, which are contrasted by the vast and deep inequalities between the types and quality of education experienced by children in the country (Spaull, 2019a):

***Table 1**: Ideological pressures on education in South Africa*

Constitution	**National Curriculum Statements**	***Action Plan to 2024: Towards the realisation of schooling 2030***
〉 All children have the right to education and are mandated to attend school until completing Grade 9 or reaching 15 years of age 〉 *(Government of South Africa, 1996; South African Schools Act, 1996)*	〉 The education system is designed to "improve the quality of life of all citizens and free the potential of each person" 〉 *(Department of Basic Education, 2011b, p. 11)*	〉 Document provides an illustration of the national trend to shift educational curricula and practices to "groom an entrepreneurial mindset" 〉 (Department of Basic Education, 2020a, p. 143)

This summary illustrates how the national constitution and curricula put pressure on teachers and education stakeholders to not only provide an *academic* education to children, but also to *enrich their lives* through character development and entrepreneurial training. Nevertheless, various barriers stand in the way of education being able to achieve these aims for the majority of South Africans (Bush et al., 2010; Oosthuizen & Bhorat, 2006). The distance between national curriculum documents and the lived experiences of children is vast, and, to add to this discrepancy, there remains a tension between the ideals set out by policymakers and those trying to enact these policies. Thus, barriers to good quality education range from national policy challenges (such as the language of teaching and learning (Feza, 2016)) to localised classroom dynamics (Venkat & Spaull, 2015), and leave many children to experience 'cognitive poverty' due to their educational experiences from early childhood (Crouch & Gustafsson, 2018). These barriers are addressed in the next section of this chapter.

The national curriculum is influential in determining children's experiences at school and the quantity and quality of what they learn. Amendments to these national curricula have been traced by many authors (Graven, 2002; Gumede & Biyase, 2016; Vithal & Volmink, 2005) who report how these adjustments continue adding content and strategies to redress the injustices of Apartheid (Gumede & Biyase, 2016). Setting out what children need to learn is a major political exercise (Malcolm, 1999), and, as such, it is difficult to combine all of the aforementioned educational aims. References to children's *capabilities* by

the National Planning Commission (2012) may give the indication that the country's curriculum is designed around Sen's (1999) Capabilities Approach to education. However, this is not reflective of the lived reality of millions of children in the country, who do not have the means to turn experiences into opportunities, objects into tools, and daily life into personal development. Because, as Spaull (2013) reflects, personal development is near impossible without "increased access to education, and an improvement in the quality thereof" (p. 10).

To further understand the educational experiences of children in South African schools, one can look at the use of a single word in curriculum documents: 'learner'. In policy documents, reports, and classrooms, *learners* are in attendance. This word choice is significant because it indicates how children are perceived by the country's education system. Biesta's (2010) reflections on the use of this word are relevant to the politics of this research: in order for there to be learners, there must be *something to learn* and someone to teach them these things. In this construction of the learner, "the learner is not simply lacking what it is that needs to be learned; here the learner is lacking the very capacity to learn without the intervention of the educator" (Biesta, 2010b, p. 542). This idea is evocative of Freire's (1970) banking concept of education, in which learners are understood as empty vats to be filled with knowledge from others; again, this envisages no co-construction of knowledge by the learners themselves. Because the South African trend of teacher training tends towards this definition of learners, children at schools are more likely to be *subjects of education* and not *collaborators in the process of education*. This is the impetus for my writing of this dissertation: to try and re-imagine how one mathematical skill can change the way that children at schools can digest and construct knowledge as parts of the education system and not just products of it. These ideas are further discussed in Chapter 1.3.

1.1.4. Current factors hampering progress in education

The current education landscape in South Africa is marred by various forms of inequality. Issues of access, survival, learner characteristics and context, enabling inputs, teaching and learning quality, educational outputs, and outcomes, are detailed in Appendix B, which further justifies why there is a dire need to amend the South African educational

landscape (Farrell, 2012; UNESCO: Learning Portal, 2021). However, the recent Covid-19 pandemic is said to have exacerbated educational inequalities even further (Allais, 2021; Francis et al., 2020).

The 'cognitive poverty' (Crouch & Gustafsson, 2018) experienced by millions of children in South Africa is worth noting in the context of this research. To illustrate this effectively, I lean on Gilman's (1900) corset analogy, which emphasises how internalised limitations are often normalised by subjugated members of society. Predominantly Black children have internalised a negative self-worth which hinders their academic careers (Das-Munchi et al., 2016; Lacour & Tissington, 2011); their literacy and mathematical underachievement are due not only to educational inputs (including resources (Naude & Meier, 2019), and teaching styles (Chetty, 2019)), but also because of systemic injustices which they have internalised. Thus, when children believe that they (a), will not succeed in mathematics, (b), are working towards achieving an education that will not elevate them out of poverty, and (c), their changes of 'success' in life are low, they become even more cognitively deprived. To change this trajectory, mathematics classrooms which allow and encourage children to question norms and 'play' with numbers are more likely to be helpful: I use Critical Theories to further this idea and justify this research project (Chapter 1.3).

With children having lost school days in both 2020 and 2021, evidence indicates that it may take children years to catch up on this lost time (Andrabi et al., 2020). Although commentators have suggested that the pandemic has provided an opportunity for a reboot of the current system (through highly refined curricula implementation and teaching strategies), this seems unlikely when looking at current progress (IPASA, 2020; 'Organise or Die: Editorial', 2021; Pournara & Bowie, 2020). Despite efforts from the private sector, NGOs, and the Department of Basic Education, many children lost innumerable hours of support from schools and social, academic, and skills development. Additionally, the Covid-19 pandemic has emphasised the weaknesses of information communication technologies (ICTs) and education management information systems (EMIS) at schools (Department of Basic Education, 2020b). The priorities outlined in the Department of Basic Education's Annual Performance Plan for 2020/2021 (2020b) are predicted to be achieved on delayed timescale due to the pandemic, but "the overall goal of the various actors in the basic

education sector must remain to improve the quality of learning outcomes, and reduce educational inequalities" (p. 20).

Although the Covid-19 pandemic is blamed for the most recent Grade 12 pass rate decline, this does not do justice to the extent of the problems that the country's education system faces (Department of Basic Education, 2021). This is why, especially after educational inequalities have been exacerbated by the Covid-19 pandemic, this research project focuses on a skill that is arguably useful in all contexts and can empower children to make more informed choices about the information they receive and process. This research project deals with the skill of estimation, a skill which has not yet been addressed through other intervention strategies and can be contextually incorporated across the Mathematics and Mathematical Literacy curricula. Before addressing estimation directly, the next section of this literature review highlights various intervention strategies which have been implemented to improve maths skills in South Africa.

1.2. Intervention strategies

To preface this research project's broader aim (which is to lay the groundwork for further research and interventions in curriculum and pedagogical change), this section briefly outlines some interventions that have attempted to change the trajectory of South Africa's education system. Mostly, these have been accuracy-based in nature, which is inherently contradictory to the aims of estimation.

1.2.1 Mathematics and Mathematical Literacy

In 2006, the Department of Basic Education introduced Mathematical Literacy (Maths Lit) in the Further Education and Training (FET) Phase, as the alternative subject to Mathematics (Graven & Buytenhuys, 2011). Maths Lit was designed to provide learners with more practical maths by contextualising otherwise abstract content (Department of Basic Education, 2003). By the time that Maths Lit was introduced, only 50% of the Grade 10 cohort was choosing to study Mathematics until their final school year. The introduction of the subject has meant that all children in the FET phase are now forced to complete some form of mathematical skill learning (Christiansen, 2006).

Although fair critiques have been made by educationalists about the true agenda of the subject (like by Christiansen (2006), who believes that the subject provides only superficial coverage of complex mathematical applications), others argue that the subject offers children a unique opportunity to learn about the everyday application of maths (Jojo, 2020). Currently, the Maths Lit curriculum is said to encompass five elements: elementary mathematical content, authentic real-life contexts, problem-solving, decision-making and communication, and the integration of content and/or skills in solving problems (Department of Basic Education, 2011c). At the time of its introduction into an outcomes-based education system, the Department of Education (2003) introduced Maths Lit to teachers and schools as a subject that

> provides learners with an awareness and understanding of the role that mathematics plays in the modern world….driven by life-related applications of mathematics. It enables learners to develop the ability and confidence to think numerically and spatially in order to interpret and critically analyse everyday situations and to solve problems. (p. 9)

According to Graven and Buytenhuys (2011), the introduction of Maths Lit was politically motivated, which is worth addressing. As previously mentioned, the South African education system is somewhat designed in accordance with the ideals of the Capabilities Approach to education which suggests that education should offer "the freedom and opportunity for an individual to convert whatever resources... into achievements or outcomes of different kinds" (Tikly & Barrett, 2011, p. 7). This design is, in itself, political: it attempts to empower South African children to find identities and achievements for themselves in a way that was previously almost impossible (for the majority) during apartheid. Maths Lit attempts to counter the narrative (and reality) of the restrictive access to Mathematics which has been perpetuated since South Africa's democratic birth. As part of the national curriculum standards, which cite "social transformation; outcomes-based education; high knowledge and high skills; integration and applied competence; progression; articulation and portability; human rights, inclusivity, environmental and social justice; valuing indigenous knowledge systems; and credibility, quality and efficiency" (Department of Basic Education, 2003, p. *v*) as its aims, subject-specific objectives of Maths

Lit are, arguably, designed as vehicles for social transformation. An extract from the current Maths Lit curriculum document highlights these:

> Mathematical Literacy allow[s] individuals to make sense of, *participate in and contribute to the twenty-first century world* — a world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways. Such competencies include the *ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology*… The subject Mathematical Literacy should enable the learner to become a self-managing person, *a contributing worker and a participating citizen in a developing democracy*... [emphasis added] (Department of Basic Education, 2011c, p. 8)

This quote illustrates how Maths Lit has been positioned as a subject that prepares children for the current and future challenges they may face. Jojo's (2020) opinion is that the government portrays the current Maths Lit curriculum as one that enables people to "make informed decisions and choices … by comparing, conjecturing, calculating and problem solving through the use of numbers and by using and applying technology to assist them" (p. 6). However, is this really happening? Although the Maths Lit pass rate is high (80,8%), this is thanks to the increasing number of learners who could handle the pressures of Mathematics 'Core' but have chosen what is regarded as the 'easier option', Maths Lit (Department of Basic Education, 2021; le Cordeur, 2020). With the syllabus still relying heavily on numerical examples and problem solving (which requires a nuanced understanding of language), the Maths Lit curriculum has not stood up to its expectations; it has not yet achieved its utilitarian objective and instead remains somewhat abstract.

Although Maths Lit is not the sole interest of this research project, it provides vital reference points for the philosophical framework influencing the nature of the research.

1.2.2. Other intervention strategies

To address the inequalities of the South African education system and improve teaching and learning experiences, many public, private, and NGO-based interventions have been attempted. Yielding various successes, these interventions have targeted a range of

inputs and outputs within the education sector (See CASME, 2021; Cotton On Foundation, 2021; Numeric, 2021; Zenex Foundation, 2021). However, only those interventions that are supported by the provincial and national Departments of Education are likely to be rolled out on a large scale, and why academics work tirelessly with school teachers to improve maths skills and teaching methodologies (Graven, 2002, 2004, 2017; Venkat, 2018; Venkat & Mathews, 2019; Venkat & Spaull, 2015). However, because most of this research is conducted in more affluent classroom environments (Venkat & Mathews, 2019) – a theme that is also visible in this research project, as addressed in Chapter 3 – this research project aims to answer research questions that relate to children's experiences across the range of South African schools.

With consideration to these intervention and research trends in South Africa, this project aims to offer an alternate narrative to maths education in the country: one that translates maths into everyday language for all children such that the subject is no longer exclusionary. Interventions, including those not specifically related to school-level challenges, have been negatively affected by the Covid-19 pandemic school closures, and thus it is hoped that *interventions* are no longer relied upon to rectify the state of education in South Africa. Instead, the hope is that national education systems begin operating successfully without the crutch of external interventions.

1.3. Critical Theory

Because of personal reflections and further research into the dire state of education in South Africa, I have begun to ask the question: "H*ow can formal education teach children in South Africa to be more critical about the information they are given, with specific emphasis on the broader maths curricula?*" This fundamentally critical question introduces the theoretical frameworks guiding this research project: Critical Theory and Critical Pedagogies. Additionally, the research is guided by an opinion of what education should *not* be: deposits of information into children's empty minds (Freire, 1970); alternatively, if teachers cultivate learning environments that encourage curiosity, creativity, and critical thinking, perhaps children will be more aware (and critical) of the information that is being offered to them. It could be argued that this would make these children more democratically active (Carr, 2011). Various theorists have proposed pedagogical approaches

which apply Freire's (1970) 'Education for Liberation' strategies to classrooms; this research project pays particular attention to those related to maths education.

Before delving into the specifics, I would like to take note of the following equation presented by Williams (2001): "people + place = politics" (p. 3). Not only does the mathematical layout of this suggestion suit the theme of this research project, but it also represents the complexities of education. Especially in a context like South Africa, where both people and places are politicised by law and culture, education provides an opportunity for complexities to be combined and further politicised in the classroom. The classroom is a political space: Gruenewald (2003) argues that educational institutions need to be understood in terms of a critical pedagogy of place by combining theories surrounding critical pedagogy and place-based education. His argument is that "place-based pedagogies are needed so that the education of citizens might have some direct bearing on the wellbeing of the social and ecological places people actually inhabit" (Gruenewald, 2003, p. 3). It is because of this argument, and the fact that teachers and learners have an opportunity to "reinhabit their places… to pursue the kind of social action that improves the social and ecological life of places" (Gruenewald, 2003, p. 6), that my research is based on what happens in classrooms.

To explore this idea of critical maths education, I lean on Frankenstein's (1983) work, which suggests teaching practices that can root maths education within critical epistemologies. In her seminal contribution to the *Journal of Education* in 1983, Marilyn Frankenstein (1983) suggests that "mathematical literacy is vital for liberatory social change in our advanced technological society… Knowledge of basic [maths] and statistics is an important part of gaining real popular, democratic control over the economic, political, and social structures of our society" (p. 315). Like Fanon's (1968) reflection on the post-colonial experiences of the 'wretched of the earth' (which goes some way to depict the current status of South African masses), Frankenstein (1983) argues that education can liberate the masses. However, to do this successfully, both the content *and* methods of education need to be adapted (Frankenstein, 1983). One way of achieving this critical consciousness is proposed by Freire (1970), who suggests approaching maths teaching and learning both objectivity and subjectivity. This is done through asking how numbers fit into context, what they say and mean, and why this is important.

Fundamentally, critical maths education challenges the assumption that maths, and its teaching and learning, are neutral. If pursued with this in mind, critical maths education can contribute towards democracy and social justice (Ernest & Sriraman, 2016). Frankenstein's (1983) thesis explains this using the following example: "a mathematically illiterate populace can be convinced, for example, that social welfare programmes are responsible for their declining standard of living, because they will not research the numbers" (p. 327). Following this argument, mathematics that is taught to expand learners' conceptualisations of concepts beyond their 'face value' may enable more effective reflectivity and critique of information fed to them. In line with this, Aslan Tutak et al. (2011) suggest that critical reflexivity, which is termed as 'praxis', can lead to critical consciousness, enabling people to challenge well-established ways of thinking. For both teachers and learners, a habit of critical reflexivity may enhance their democratic awareness and participation. However, this conceptualisation of *learners* contradicts that presented in policy and curriculum documents, as discussed in Chapter 1.1.3.

Despite their philosophical applicability, these theories must still be grounded in relatable contexts. Like Freire's (1970) problem-posing education, in which learners generate and collaboratively solve problems which relate to their experience, the South African national Curriculum and Assessment Policy Statements ((CAPS) Department of Basic Education, 2011c) aims to produce critical leaners, as indicated in Appendix C. Although the aims stated in these documents suggest that learners can, and should, be taught in a way that encourages creativity and criticism, this is difficult to achieve. Vithal's (2003) research suggests that it is almost impossible to articulate one pedagogical style or practice that will address inequalities in South Africa, and that the best option is to encourage questioning in classrooms. Purposefully problematising knowledge in order to transform it, as suggested by Skovsmose and Nielsen (1996), is one way to constantly force learners to question, critique, and re-formulate numbers in such a way that makes sense to them. However, this is difficult when the syllabi for both Mathematics and Maths Lit rely on rule-based thinking. Added to this, Brodie & Pournara (2005) raise concerns that classrooms dominated by critical pedagogies may lose the maths (or numeracy) within broader social and political issues, and struggle to cover the whole syllabus.

Because of these concerns, and the fact that teachers who are currently employed to teach mathematical subjects may not feel equipped to teach employ such critical pedagogical stances (Graven, 2017), this research project focuses on one part of the current Mathematics and Maths Lit curriculum: estimation.

1.4. Estimation

Attempting to account for, respond to, and adress the challenges facing maths education in South Africa, this research project focuses on a skill that can arguably be beneficial to all people: estimation. Globally, the term 'estimation' became synonymous with maths teaching after a defining document, known as the "Cockcroft Report", was released in 1982, which outlined why and how maths should be taught, and was influential in determining what children should learn in maths classrooms around the world. This report explained how numeric skills are a necessity for all adults:

> whilst realising that there are some who will not achieve all of them, we would include among the mathematical needs of adult life the ability to read numbers and to count, to tell the time, to pay for purchases and to give change, to weigh and measure, to understand straightforward timetables and simple graphs and charts, and to carry out any necessary calculations associated with these… We believe that… it is important to have the feeling for number which permits sensible estimation and approximation … which enables straightforward mental calculation to be accomplished. Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed, whether this be little or much. (Cockcroft, 1982, p. 11)

Not only did the Cockcroft Report set the stage for prolific research on the skill of estimation in the 1980s and 1990s, but this document has continued to influence maths teaching to this day. The report (1982) suggests two important aspects of the skill: the ability to retrospectively judge the reasonableness of the result of a calculation or measurement, and the ability to make subjective judgements about measures which cannot easily be achieved with accuracy. Estimation is therefore ubiquitous, not only in terms of when it is useful, but also its problem-solving capacity. According to the report (1982), industry and commerce

rely extensively on the ability to estimate, which is often experienced as a 'feeling' of correctness and is therefore highly contextual.

Since the Cockcroft Report, many other researchers have weighed in on the importance of estimation and how the skill can be improved (LeFevre et al., 1993; Levin, 1981; B. J. Reys et al., 1991; R. E. Reys et al., 1991). After the initial wave of research about the skill, which has since not been revived, Hanson and Hogan (2000) published a study which positively correlated good estimation skills with high SAT test scores; however, there is very little literature available about the skill of estimation and other cognitive abilities. This research project does not draw attention to estimation in relation to academic success (although this would be a good subject to research further), but rather attempts to link school-based estimation to numeric life skills which are needed outside of the classroom. Estimation, taught both formally (in classrooms) and informally (in any other settings), is a possible enabler of critical and creative thought in everyday life.

Estimation has proven difficult to define: Berry et al. (2014) suggest that estimation requires learners to conceptualise and manipulate numbers mentally; Hanson and Hogan (2000) break estimation into three categories: numerosity (an informed guess about the number of dots or objects in a sequence), measurement (the practical, everyday uses of estimation of different tangible measures we encounter), and computational estimation (the use of mental arithmetic skills and number concepts to achieve answers that are reasonably close to a calculated result). This research project proposes a new definition which takes the lead from R. E. Reys (1984), Segovia et al. (1989) and Senol et al., (2015), who suggest 6 characteristics of the mathematical skill including: its use by determining the value of an arithmetic operation or quantity, its occurrence in mental maths, the speed with which it is performed, and the fact that answers may not precise. Based on these characteristics and with reference to other scholars' work (Berry et al., 2014; Hanson & Hogan, 2000; LeFevre et al., 1993; R. E. Reys et al., 1982), I propose the following definition, which also makes reference to the graphical part of the skill (something which has not yet been documented): *estimation is the skill used to reasonably accurately and rapidly predict outcomes to numeric problems (which includes graphic, measurement, and pure number examples), and being able to assess the accuracy of such answers, thus answering the questions: "What answer am I predicting, and does this answer look right?"*. However, to

estimate accurately, a fluency with numbers and representation of numbers is necessary. This is why Van de Walle (2007) suggests that estimation is a higher order skill which builds on number sense and numbers' flexibility. Number sense relates to a set of key mathematical abilities which summarise instructive understanding of numbers such that they can be constructed and deconstructed to suit the necessary use (NCTM, n.d.).

1.4.1. Estimation in Curriculum Assessment Policy Statements

Currently, estimation is included in many CAPS documents and is referred to as 'approximation', 'guessing', and 'rounding' at different stages of the curricula. The skill is first mentioned in the Foundation Phase, where learners use estimation for grouping and measurement (Department of Basic Education, 2011a). In the Intermediate Phase, the skill is developed further in both Mathematics (as a lower-order thinking skill), and in Natural Science and Technology (as a foundational skill needed for more complex operations) (Department of Basic Education, 2011e, 2011f). In the General Education and Training (GET) Phase, estimation appears in both the Mathematics and Social Science curricula, but it is presented (in both cases) as part of more general measurement skills (Department of Basic Education, 2011d, 2011f). Lastly, in the FET Phase (which is taught by teachers who participated in this research project), estimation is regarded as a simple, routine procedure in both Mathematics and Maths Lit, but is only addressed as its own topic in the latter (Department of Basic Education, 2011d, 2011c).

Appendix D provides further information about how estimation is presented in South African curricula and illustrates that although estimation is present in all phases of children's education in South Africa, it varies in importance per subject and grade. In response to this, and with the conclusions from my research project, I hope to provide teachers with tools for integrating estimation into their classrooms more frequently.

1.5. Summary

The South African experience of estimation in maths is yet to be explored. However, in conjunction with critical theories on education and a clear understanding of the South African context, this research project aims to make amends to this. In response to the

literature reviewed, this research project was designed to empower teachers and learners to be more critical of information they access and are ‘fed’. The next chapter justifies and defines the research questions which guided the design of the project, and which best illuminate a path to improving maths education in South Africa.

Chapter 2: Research Questions

This research project is the result of almost five years of teaching Mathematics at a High School level and a curiosity about what fundamental skills should be included in maths curricula. My research questions have been guided by this curiosity, awareness of the logistical limitations of Covid-19, and the need to design a project that can answer questions well. In thinking about this, I turned to Cai and Mamlok-Naaman's (2020) work about how best to formulate and ask researchable questions relating to Mathematics education. Their suggestion is that questions should arise from contextually significant links to prior research and lead to extension of fields of knowledge. Additionally, Schoenfield (2020) calls for research questions that are meaningful (to practice or theory) and generative (thus providing valuable insights and a foundation for further research). These guidelines have directed my broad existential questions into more researchable questions for the context of this project.

Having considered the literature, my experiences, and with respect to the limitations of Covid-19, I formulated the following research questions (RQs), which guided my research design:

RQ 1. What are South African Mathematics and Mathematical Literacy teachers' perceptions about the importance of estimation in the classroom and beyond?

RQ 2. How do teachers' classroom practices / praxis change through participation in exploratory research projects?

- **a.** What pointers should be kept in mind when designing a teaching intervention for high school maths teachers?

RQ1: Perceptions and importance

Because the aim of my project is to lay the groundwork for interventions (which can later become common practice and thus will no longer be called 'interventions'), I wanted to gauge teachers' interest in the skill of estimation through their *perceptions*. I chose to use the word *perceptions* because this word houses two definitions: (1) "The ability to see, hear, or become aware of something through the senses", and (2) "The way in which something is regarded, understood, or interpreted" (Lexico.com, 2021). Thus, RQ1 investigates two

interlinked ideas: (a), how participants expressed their feelings, experiences, and connections to estimation, and (b), how participants defined or understood estimation. My reliance on this definitional clarity improved the rigor of my research, and moves the research away from previous studies' lack of clarity when using this term (for example, Finster & Milanowski, 2018; Lai & Waltman, 2008; Prewett et al., 2019; Tiedemann, 2002). Additionally, for the first RQ, *importance* remained undefined. This was done to allow answers to be inductively revealed from the data instead of being imposed as expected themes.

RQ2: Practices / praxis

RQ2 was asked with the aim of laying the groundwork for intervention development. Understanding how teachers changed their teaching practices helps to determine their level of reflexivity. *Practice / praxis* has been used on purpose. For this research project, I rely on Freire's (1970) conceptualisation of praxis, which is one that combines action and reflection. This form of intentional reflexivity develops as teachers become more morally informed and committed to their profession (Kemmis, 2010; Kemmis & Smith, 2008). Praxis therefore refers to an intentional practice, an intentional pedagogy. This RQ therefore asks: how do teachers think their classroom teaching styles and practices changed as a result of being part of this research project, and how have they reflected on these changes? This question is therefore asking about, (1) teachers' perceived change of practice, and (2) their critical reflections about why their practice has changed.

RQ2.a: Interventions

RQ2.a serves as an extension of RQ2; it uses knowledge from RQ2 to delve deeper into other considerations that need to be made when designing interventions. Fundamentally, the question asks what stressors in the classroom could hinder teachers' ability to fulfil the requirements of an intervention project. Asking about teachers' classroom experiences also gives some insight into their learners' experiences (Dewaele et al., 2017), and therefore provides reason to design an intervention which accounts for limiting factors which frustrate and limit teachers and learners.

Summary and research objective

Led by these research questions, this research project was designed to achieve one large objective: improving mathematics education in South Africa.

Chapter 3: Methodology

This research project design developed in response to a personal quest to improve maths teaching and learning in South Africa, which has remained a central theme throughout the implementation process. The following section of this dissertation explains the methodology of the research project, which became an exercise in translating my personal moral considerations and research ideas into practical strategies for collecting and analysing data.

3.1. Rationale

I approached this research project with a reluctance to identify with any particular paradigm and instead tried to adhere to participatory research design norms, but this was less effective than I had hoped. However, I kept in mind the critical theory upon which I based my overarching research question: '*How can formal education teach children in South Africa to be more critical about the information they are given, with specific emphasis on the broader Maths curricula?'*[3]. Now, as I reflect on what really guided my research design and implementation, it is clear that I was influenced by (1) pragmatism: *What can I do to answer my research questions?* and (2) personal morals: *How can I ensure that participants can comfortably and clearly participate in this project?* Thus, my project was implicitly guided by both the pragmatic paradigm, and a feminist ethics of care.

3.1.1. Pragmatism

Pragmatism is a practical and applied research philosophy which supports mixed methods social science inquiry. Developed by John Dewey, pragmatism serves as a "doctrine of meaning, a theory of truth … the focus is on the consequences and meanings of an action or event in a social situation" (Denzin & Lincoln, 2018, p. 555). Consensus in the mixed methods field of research suggest that mixed methods studies are pragmatic in nature because they follow a logic of inquiry (Biesta, 2010a). Johnson and Onwuegbuzie (2004) argue that mixed methods research is driven by the very questions researchers attempt to

[3] Refer to Chapter 1.3.

answer, and that we should therefore “choose the combination or mixture of methods and procedures that works best for answering [the] research questions” (p. 17). It was because of this emphasis on research questions that I spent such time defining and refining mine. To answer my exploratory research questions, a mainly qualitative mixed-methods approach was used as it “value[s] participants as the chief sources of information” (Creswell & Plano Clark, 2018, p. 14) and allows the question, ‘*why?*’ to be asked many times over. Although my project relies mostly on this type of qualitative data, it too has been quantified (Tashakkori & Teddlie, 1998) through data analysis, by analysing the frequency of words, and by creating links between codes.

3.1.2. Feminist Pragmatism and Ethics of Care

Feminist pragmatism has been portrayed most prominently by Charlene Haddock Seigfried (1996) as the most effective means of conceptualising mixed methods research. Her influential work illustrates how early pragmatists’ theories about experience and consensual constructivism are aligned with feminists’ close negotiation between theory and contextually specific praxis. Added to this, Keith's (1999) interpretation of Seigfried’s (1996) work further emphasises how pragmatism values inclusiveness and community, something that this research project has worked to achieve. Fundamentally, the Feminist Ethics of Care epitomises what all good research should account for, and formalises actions which I have internalised as my own moral standards; following these research ethics therefore felt like ‘second nature’.

The Feminist Ethics of Care goes above and beyond normative considerations of care, including those regarded as fundamentals (including confidentiality and anonymity, non-maleficence, and beneficence (Cohen et al., 2018)), and those relating specifically to power within systems (Held, 2006; Tronto, 1993). My project demanded an acknowledgement of power within education systems and society, and within the context of data collection. The Feminist Ethics of Care provides ethical considerations to counteract normative power imbalances which could otherwise have negatively influenced my participants and the longer-term impacts of my project. Thus, just as Gilligan (1982) anecdotally describes this ethical stance as “seeing a world comprised of relationships rather than of people standing alone, a world that coheres through human connection

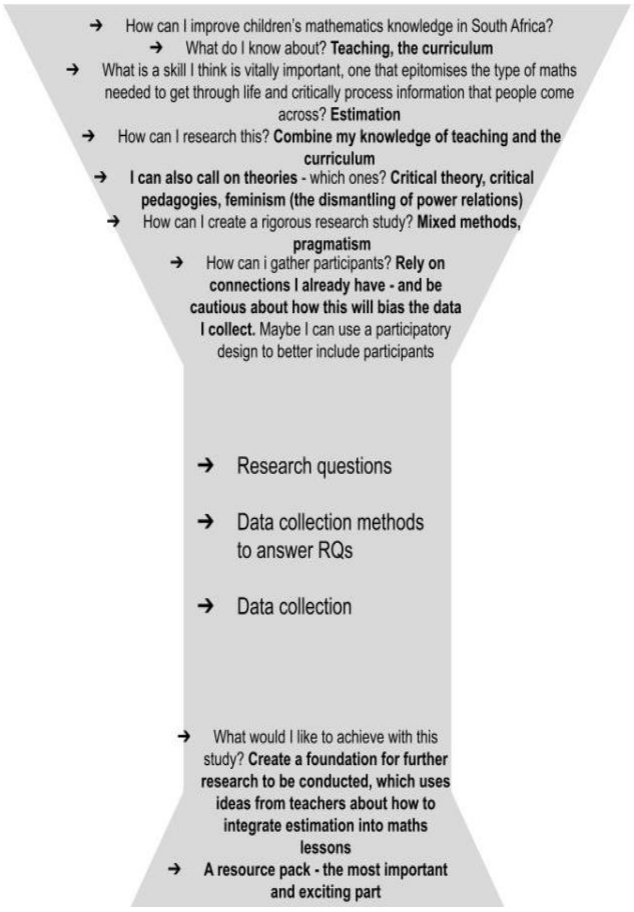
rather than through systems of rules" (p. 27), this research project attempted to be considerate of participants' needs and intentions for their contributions to the project. To do this, and to thank my participants for their contributions, I will create an Estimation Teaching Guide for high school Mathematics and Mathematical Literacy teachers based on the findings of this project. This aims to include examples from this research project, and others, about how the estimation can be integrated into classrooms. Some other practical examples of ethical considerations are listed in Appendix E, alongside the formalised University of Oxford ethical clearance through CUREC.

3.2. Designing the research project

Following the pragmatist epistemology, accompanying Feminist Ethics of Care, and critical theories of education, it was necessary to think about the accompanying ontological and axiological approaches to the research project. According to Lincoln and Guba's (1985) seminal work, my research is inclined towards relativism and subjectivity. This has meant that: (a), participants' contributions were understood as their version of reality which subjectively combines meaning-making from their entire lives and responses to social, political, economic, and other stressors over time; and (b), participants' contributions were valued as instrumental – yet subjective – contributions to emancipating both the participants themselves, and the broader society, through education. These considerations played a part in designing and implementing a research project that fits together with ease, is representative of my own values, and can be regarded as rigorous.

To perform this research project, I found it beneficial to visualise systems of thinking. The following diagram illustrates my flow of ideas and rationale when designing the research project. Throughout the process, I kept re-drawing diagrams like this and have thus summarised some important points to capture the essence of my design and implementation process:

***Figure 1**: Research design process and explanation*



I had to direct my many thoughts, knowledge, and experience to create a manageable project.
To do this, I asked and answered many questions, some of which are indicated in the diagram. Answers, in short, are in **bold**.

Then, I designed my research methodology to answer research questions.

The most exciting part of the project is finding answers, and thinking about what I can do with these.

3.3. Research Questions

Having considered the questions and thoughts at the top of the funnel (*Figure 1*), I crafted the following research questions (RQs):

RQ 1. What are South African Mathematics and Mathematical Literacy teachers' perceptions about the importance of estimation in the classroom and beyond?

RQ 2. How do teachers' classroom practice / praxis change through involvement with research studies?

a. What are some pointers to keep in mind when designing an intervention for high school mathematics teachers?

To answer these questions, I designed and implemented data collection methods which could best suit the RQs. This was done with my ideal, and actual, sample in mind.

3.4.1. Data collection: Sample

Despite these expected limitations due to Covid-19, which have been discussed by many authors (for example, Campbell, 2021; Dodds & Hess, 2020; Fine & Abramson, 2020), recruiting participants was more challenging than anticipated. Being aware of the vast differences between schools in South Africa and having taught maths at a range of schools in the country, I felt it important to include teachers who could represent schools located in, and serving, different communities. To do this, I relied on networks of teachers I had developed through my teaching experience. This purposive and convenience recruitment was done to garner the support of teachers who (a), fit the standardising characteristic of currently teaching Grade 10 Mathematics or Maths Lit, (b), were from schools across the SES spectrum in the South African education landscape, and (c), who I could develop easy working rapports with due to our professional connections (Cohen et al., 2018).

Despite these plans, I struggled to make the plan come into fruition, partly due to the online nature of the data collection. Colleagues at low SES schools were initially keen to join the project but later lost interest. This has meant that although my final sample covers a relatively broad SES spectrum, there are no participants teaching at schools which serve severely underprivileged children. When reflecting on my sample size and research methodology, I have found it difficult to reconcile my ideal sample (which could be more representative of teachers from across the SES spectrum), and the final group of nine participants with whom I was able to work. In retrospect, I am almost certain that if I had been in South Africa to conduct my data collection in-person, I could have been able to work with more schools. Administrative requirements of the study impacted 'poorer' schools' abilities to join the study: many more of them may have been able to join if the process had happened in-person. Thus, the virtual environment has biased my sample in favour of the voices of people who (a), have enough data to communicate online, and (b), who work at schools that support their voices being heard.

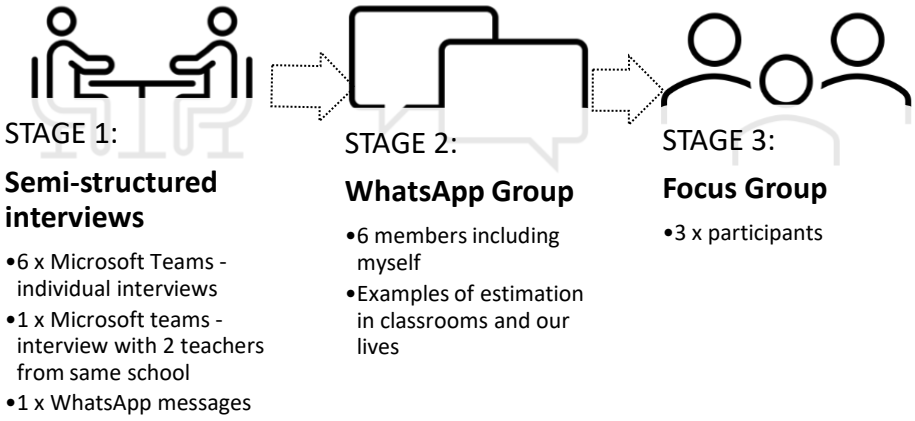
My final sample nine participants: two each from four schools and one other. Because some of the schools are private, it is not possible to declare their quintile ranking but an estimated ranking of their learnerships' SES status is as follows: one prestigious private girls' school; one less prestigious private school; one peri-urban school based in a township which caters for lower-middle-class Black girls; and two middle-class public schools based in

suburbia. Of the participants, one identified as male and all others as females, all but two had already taught for more than 3 years (in high schools or other grades), six participants teach or have previously taught other school subjects, and four had worked in other industries before becoming teachers. Appendix F provides further information about participants' teaching careers and participation in the research project. Despite some participants agreeing for their names to be published, I have maintained continuity by giving each participant a pseudonym which is somewhat aligned to their associated gender and mother-tongue.

3.4.2. Data collection: Methods and tools

The data collection for this research project involved 3 stages, as shown below:

***Figure 2**: Data collection process*



The three stages of data collection were designed to elicit inter-connected information about all three research questions; Stage 2 was designed to provide additional examples of estimation in classrooms and act as an example of interventional participatory research, and Stage 3 delved deeper into topics covered in both prior stages. Although Stage 1 began early in the year, the last two interviews were conducted the same week as the WhatsApp group began. The WhatsApp group then ran for 4 weeks and was followed immediately afterwards by the focus group.[4]

[4] For more details, Appendix G provides a summary of intended and expected data collection strategies compared how the process panned out. It shows how some of the data was collected in unexpected ways, and how this influenced the type of data that was eventually analysed. Proposed interview schedules, WhatsApp group information, and focus group guiding questions are attached in Appendix H.

Stage 1: Semi-structured interviews

Semi-structured interviews were chosen as the first form of data collection as they provided a platform from which to learn about participants, discuss topics for the research project, and elicit their responses about further research. The nature of semi-structured interviews is often informal and relies on interviewers and interviewees sharing a good working rapport (Cohen et al., 2018); the semi-structured format of the interviews allowed participants be part of deciding how the interviews developed. Interviews allowed me to get to know participants, become aware of their interest in the research project and subject matter, and develop a deeper understanding of topics I was researching. As Guest et al. (2017) suggest, semi-structured interviews can provide participants with the freedom to discuss a range of topics without feeling judged. This is particularly relevant in the case of this research project, in which I constantly reminded participants that there were no 'right' answers and that the project was exploratory in nature: I was trying to better understand participants' own experiences and beliefs.

The eight semi-structured interviews were conducted over 3 months on Microsoft Teams. It took a while for participants to complete the required ethical clearance and thus the interviews did not happen in quick succession. This provided time for me to check and correct the automated transcriptions of the interviews between each one, and thus better prepare for the upcoming interviews with an awareness of emerging themes. One interview included two participants the same school which meant that the teachers could discuss topics without needing my intervention; it was hoped that more interviews would take this shape but, because teachers struggled to find suitable time for this to occur. All other interviews were conducted one-on-one. Additionally, one participant chose to answer the interview schedule's questions via WhatsApp messages and thus, even though her answers were succinct, it was not possible to ask her to elaborate on ideas. My interview schedule (see Appendix H) was designed as a template which could be edited and adjusted as the project progressed; it served as a guide, not a script. Each interview began with informal conversation which developed interviewer-interviewee rapport (Cohen et al., 2018), and included me re-checking participants' consent agreements before beginning the recording. Interviews lasted between 40 and 60 minutes and concluded with discussing the idea of a collaborative WhatsApp group, and assessing participants' willingness to be part of this.

Stage 2: WhatsApp group

Initially, I had planned to have a virtual Group Meeting, where all participants could meet and decide how they proceed from interviews to the WhatsApp group. This intention was in line with my ideal of implementing a participatory research project, but it did not happen. I realised that teachers were happy to be told what to do (specifically relating to the WhatsApp group) and therefore they sacrificed meeting other participants. The WhatsApp group was not only set up to explicitly collect data, but it also served another purpose relating to RQ2: it acted as a reminder to participants to continue looking for examples of estimation in their classroom and daily lives. Like Colom's (2021) research indicates, WhatsApp groups can be useful tools for garnering and maintaining support for participatory research, especially as participants are usually reliant on this as a means of communication. Ours was designed to be a collaborative effort for teachers to share, and implement, new ideas for using estimation in their classroom.

Despite all indicating a willingness to be part of the WhatsApp group, only five participants joined the group. The group was much less active than I had anticipated, and in four weeks, only eleven examples of estimation were offered by participants: seven relating to Maths Lit and four relating to life outside the classroom. Although I was disappointed by the activity on the group, there is something to be learnt from how participants contributed to the shared virtual space, which is be discussed in Chapter 4.2.1. Without overloading the WhatsApp group, I sent reminder messages to the group and included examples of estimation in my own life. Further information about the WhatsApp group is included in Appendix H.

Stage 3: Focus group

The focus group was composed of three participants (Mandla, Jennifer, and Tammy). These participants were chosen because they work at the schools at the extremes (and average) of the SES spectrum represented by the sample. Additionally, these participants had showed particular interest in the research project and had made references to empowerment through education. Because this is in line with the critical theories upon which this research project is based, it felt necessary to continue the research with these participants, particularly for RQ2 and RQ2.a.

Focus groups, according to Guest et al. (2017), provide participants with an opportunity to reflect and discuss issues of interest. This is what happened with the focus group in this project: we spent 15 minutes getting to know about each other (as participants had not yet met), after which I co-constructed the flow of the discussion with participants. The participants happily followed the strategic plan which had been designed for the focus group (see Appendix H) and which I systematically worked through with them. I also made use of a vignette (in the form of a quote from another participant) to elicit responses to a central theme: reasons for teaching maths. The vignette, which was designed to align with research aims, methodologies, and intended outcomes, aimed to validate data collected in the interviews (Skilling & Stylianides, 2020). One of the most rewarding parts about the focus group was participants' appreciation for their participation in the project and their willingness to offer up, and learn from, suggestions about how to improve their teaching practices.

3.5. Data analysis

The data analysis process was iterative and continuous: it can be tracked through the various steps I took to continually analyse data as the project progressed. By analysing data for many months, I became well-versed in the themes that kept appearing and thus I could make links between themes early in the project. Because I was analysing data while reading for my Literature Review, my interpretations of the data were being constantly verified and further explored by readings; I believe this also assisted in ensuring the data's validity and credibility (Cohen et al., 2018). The following paragraphs detail my data analysis technique and illustrate one practical example of how this was done.

I took notes during my Stage 1 interviews, kept reflective journals about how my interview style adapted to each participant, and made notes on my interview schedule to indicate how I had asked the questions differently for each interview.

After this first step of analysis, I used Microsoft Stream's auto-transcription service, and checked all interviews by listening to them in full and correcting any errors. While doing this, I made notes of potential themes that arose from the data, and whether these were similar to those in other interviews.

After about half of the interviews, I began coding the completed transcriptions in NVivo. I re-read each interview again and coded the data according to nodes which I had pre-entered according to themes which I had noted during interviews and proofing of transcriptions. These were descriptive nodes (Saldaña, 2015) and I found myself continually adding new nodes and coding a high percentage of each interview. At this stage, most of the nodes were independent, aside from one parent-node, 'estimation', which held many child-nodes. When I added new nodes, I would go back to previously-coded interviews to re-code sections that may fall into these new nodes. By doing a continuous back-and-forth checking of my coding, I was building up a familiarity with the data and one which has, ultimately, provided me with a deep understanding of what participants have said.

I coded all interviews, the text response, and contributions to the WhatsApp group before conducting the focus group. Some quotes were coded to multiple nodes, which made the coding matrix (Appendix I) useful when interrogating at coding- and thematic-overlaps. This helped me prepare a vignette for the focus group and allowed me to reflect on the themes that needed to be revisited in the focus group. The focus group therefore acted as a means to justify and deepen my understanding of certain perceptions and allowed for critical reflection on teaching practices. I coded the focus group according to the codes that had emerged from the previous analysis steps, and only created 2 new codes: 'vignette', 'focus group', which accounted for participants' responses to these parts of the project.

The penultimate analysis step involved me aggregating nodes together under parent nodes and running a coding matrix (Appendix I) to check that my assumption about major themes was certified by the quantified data.

The final step of analysis involved the creation of a new node: 'summary quotes'. To add to this, I re-read quotes from every parent-node and copied across quotes that epitomised what was being said. This enabled me to summarise all themes into one node, which was in proportion to the number of coded quotes in each node (see Appendix J).

Although I kept my RQs in mind while analysing the data, I did not try to answer them until the emergent themes had been documented. Once this had been done, I felt more confident to draw links between themes, literature, and reflections of the research

project. When drafting my Finding and Discussion chapters, I used quotes from my 'summary quotes' node as these best exemplified emergent themes.

To illustrate this data analysis, I provide an example from Marlise: "We do estimation with Maths Lit in the Grade 10 where… you do a sum quickly in your head, estimate what the answer would be and do your calculations and see if you in that area".

- This was initially coded under the parent node 'estimation', and child node 'estimation - eg classroom'. When other participants gave more specific examples, I clarified this quote by making it part of a child-node: 'estimation – maths lit'.
- This parent-child set was coded under 'estimation – context' after all interviews had been coded, and nodes aggregated. The final coding is thus estimation/estimation-context/estimation-egclassroom/estimation-mathslit.
- When doing my final read-through of data, I placed quotes like this under the newest parent-node, 'summary quotes'.
- When examining the code book (Appendix J), it is possible to see how these levels of child-nodes have been classified into larger coding groups which have all been aggregated.
- The above quote, although not used in this final dissertation, is useful when answering RQ1. It adds to the depth of knowledge about how teachers perceive estimation to play a role in answering questions in the classroom, when estimation is used (in comparison to the calculation of exact answers), and can be compared to other Maths Lit examples provided in the WhatsApp and focus groups.

The inductive data analysis used throughout this research project has enabled themes to emerge continually. This data has also been quantified, which is has proven to be another way of validating the credibility of the emergent themes and results (Spencer et al., 2003).

3.6. Validity and rigor

Validity in mixed methods data encompasses aspects from both qualitative and quantitative research, which Lincoln & Guba (1985) suggest can be summarised into 4 key criteria: credibility (the internal validity of relationships between themes), transferability

(generalising results beyond the scope of the project), dependability (consistency of results), and confirmability (objective analysis of data). Every effort has been made in my data analysis to continually check coding patterns, the relationships between themes, and whether my data collection tools have been able to answer the RQs. Although the sample size is small, the qualitative data collected goes some way to represent teachers throughout South Africa because those in the project have taught in different contexts and can speak from years of experience which have been shaped by the broader educational landscape of the country. The data from all three stages of data collection show consistent themes which have been checked multiple times. These forms of validity were evidenced by not having to introduce new nodes for coding after the 4th interview (except for those specific to data collection later in the process), and instead only needing to aggregate child-nodes into levels within parent-nodes.

Rigor in mixed methods is defined by Teddlie & Tashakkori (2010) as the "degree of consistency of conclusions within the project, consistency with the state of knowledge… and the degree of correspondence between the conclusions and the research questions" (p. 33). By consistently re-reading and re-analysing the data collected in this project, and by using the focus group as a form of member-checking certain themes, this project is rigorous as possible. The upcoming chapters on Findings and the Discussion provide examples of how there is a definitive link between research questions and answers, thus emphasising the fluidity and effectiveness of the pragmatist research design.

3.7. Limitations

Despite the successes, this research project faced many challenges. Many of the limitations of the project have already been mentioned in the above sections, but in retrospect, I believe that I consistently made decisions which benefited my participants and ensured the rigour of my project. However, there are some broader issues which I would have liked to address. Firstly, I believe a larger sample would have made this project able to draw comparisons between cases and views. Secondly, the project could have answered a broader question about the use of estimation in maths classrooms from other educational stakeholders. This could have been done through collecting interview and picture data from learners in a range of schools in the country, and may be an idea for further research. One

of the most relevant criticisms of my project is my inexperience in conducting interviews. This meant that, for both the interviews and focus group, I had to be very aware of my questioning techniques and found myself asking leading questions or offering my critical hypothesis of estimation's use (as referred to in the Literature Review). I noticed this tendency after conducting the first interview and made a concerted effort to change this habit thereafter, but it seems that it continued to happen at least twice more (as noticed in my interview transcriptions), which could have altered participants' perceptions and therefore impacted on the collected data.

Although these limitations play some role in weakening my data, I believe that the rigor with which my data were analysed still make my findings relevant and useful for future intervention research in this area.

3.8. Summary

Following a pragmatist approach to asking and answering research questions, I conducted this research project with the intention to better understand maths teachers' perceptions and practices. The project has been successful in achieving this intention because it revealed genuine representations of teachers' experiences and views. The depth with which teachers answered questions assisted in my ability to analyse data and articulate emergent themes. In the following two chapters, this research design proves its worth by discussing its findings.

Chapter 4: Findings

In answering the research questions (Chapter 2), this pragmatic research project fulfils its aims. The following systematic analysis of all collected data, which ensures its credibility, provides holistic answers to the RQs and thus all answers form part of a collective solution to problems illuminated by the literature review. Although the findings do not attempt to be fully transferable to other contexts, their representation of nine Grade 10 maths teachers in the eThekwini area of KwaZulu-Natal is compelling. This chapter presents the findings according to the research questions by drawing on emergent themes from the data, which include estimation's links to context and problem solving, teachers' belief in the critical importance of the skill, and how best to integrate the teaching of the skill into everyday maths classes. Without referring to literature, this chapter relies solely on collected data; broader social implications are discussed in the following chapter with reference to pertinent literature. Although the contributions of Mandla, Jennifer, and Tammy seem to dominate the following section, this is only because of their involvement in the focus group which resulted in their contributions being more explanatory than the other six participants.

4.1. RQ1 What are South African Mathematics and Mathematical Literacy teachers' perceptions about the importance of estimation in the classroom and beyond?

Estimation is important within and beyond the classroom. Despite many uncertainties about estimation, participants indicated a great appreciation of the skill. This perceived importance is related, according to participants, to context and problem solving. The transferability of the skill of estimation provides teachers with an ability to expand learners' understanding of maths and other 'real life' contexts; additionally, estimation's transferability to problem solving skills provides teachers with further reasons to integrate the skill into their classrooms. Linked to this, however, is the fundamental belief about who teachers think they are teaching: are children really just the *learners* portrayed by Biesta (2010b) and national curricula? Teachers answered this with a resounding 'no!', an idea which relates to RQ2.

The following three sections detail three themes which combine to answer RQ1: uncertainty, contextualising with (and for) estimation, and the link between problem solving and estimation.

4.1.1. Uncertainty

Participants' uncertainty about estimation ranged from definitions to examples, and from how to when estimation is used. Despite this, a common story was told: estimation may be underdefined in the syllabus, but it is important and must be taught because it relates to problem solving and contextual knowledge. Estimation was therefore seen as a "life skill" (Tammy) which is pervasive throughout maths learning.

This project's definition of estimation (offered in the Literature Review) took some time to develop and it was therefore no surprise that participants struggled to offer succinct definitions when asked. In most cases, participants developed definitions as they continued speaking to me. For example, Mandla offered a variety of conceptualisations of the term: "it's a very broad concept but… is really the act of making a judgement on a dimension or magnitude of a set parameter"; later in the same interview, Mandla then referred to graphical estimation: "instead of just looking at the numbers… I could graph that information so I can have a graph which gives me an idea of the shape of the population"; his contribution to the Focus group later reiterated a further differentiation: "some estimates are for the purpose of verifying, but some estimates are really for the purpose of trying to gather understanding". Lastly, Mandla also developed his reflection on the term as follows: "whatever you're dealing with is guided by your initial understanding of whatever you want to compute… it seems like in the absence of being able to estimate, we run the risk of not being able to solve the problem". Mandla's evolution of definitions is a representation of other participants' confusion: "I was gonna go with guessing but it can't be guessing. It's more than guessing" (Emma), and others who had no way of describing the skill except for providing examples:

> Estimation is a life skill. Estimation [uh], you're trying to, to… Estimation. So I go to the shop with a R100 bill. Can I buy two cheeses that cost R48,75? Can I buy 2 blocks? So I'm trying to make sense, make numbers sensible to me. You know, so that I can do quick calculations with them. (Jacqui)

Like with the definition, when participants were asked for examples of using estimation in their classrooms, their first responses indicated their uncertainty. Participants spoke of the use of the skill in terms of incidental ("How long do you think it will take you to complete this assignment?" – Tammy) and intentional usage ("If that [*gesturing at graph]* is 5, what do you think the *x* of this turning-point is going to be?" – Mandla). While speaking about these examples – and others – participants grew increasingly confident in offering examples of using the skill in the classroom, as shown by Skye "Covid graphs… are things that you know these girls will have to grapple with one day... and possibly even things that they read in the news". The perceived importance of estimation is contradicted by the curriculum's underrepresentation and dysconnectivity of the skill. This is noticeably problematic when teachers depend on the curriculum and textbook to know what and how to teach (as was illustrated by newly qualitied teachers, Skye and Lindokuhle). Additionally, Maths Lit examples outnumbered those from Mathematics 'Core' in all areas of data collection (21 examples coded as "Maths Lit" and only 7 as "Mathematics 'Core'"), which indicates that Maths Lit offers a more contextual approach to maths teaching and learning.

Participants also offered mixed opinions about when and how estimation can be implemented. Discussions about this centred around whether participants thought estimation happened before or after attempting a calculation or solving a problem. There were three opinions about this: 1. estimation happens after completing a calculation as a way of checking an answer; 2. estimation provides guidance at the beginning *and* offers an opportunity to check the calculation at the end; 3. estimation is performed independently of calculations but followed similar rules. With most participants justifying the last two opinions, estimation can be understood to be used both intentionally and as part of subconscious processes which happen continually.

Teachers' uncertainty underlies an important theme: estimation is adaptable and contextual. Trends in the data are indecisive and indicative of teachers' inexperience with estimation. They *feel* uncertain, but they, and their learners, know the *feeling* of getting a right or wrong answer when handling maths problems. This is indicated in Appendix K, the word trees generated by NVivo to illustrate how both participants, and their learners' experienced visceral responses to the skill of estimation and how it can be used to judge answers' accuracy. Some examples include: "I try to develop in my students an intrinsic

feeling that you get: something doesn't make sense" (Marlise), "when you're going the wrong way, you must almost *feel* that it's wrong, and [the learners] sometimes do. They will say to me, Ma'am, this *feels* wrong and then we look at and say yes, it is". By encouraging learners to suggest answers, even if they are 'wrong', teachers continue developing conducive learning environments which can further empower learners to learn and critique information they create and discover.

4.1.2. Contextualising information with, and for, estimation

The most frequently coded theme was that of context. Including examples of estimation used in classrooms and in 'real life', context accounted for 261 first-step quotes and 94 final-step quotes (see Appendix J), participants constantly reiterated that when teaching maths, "you've gotta teach within the context, because otherwise all you're doing is [teaching] a foreign language." (Emma). However, within this theme were various nuanced discussions about why estimation specifically is such a contextual skill: the argument for estimation's context specificity is well-articulated by Tammy: "it's your life experience that will give you different skills that you develop, but then you'll develop skills that are relevant to your life", and supported by Jacqui's opinion that in order to learn maths skills, "[learners] have to be able to… make a connection with something in their brain which allows them to understand the maths". These quotes, combined, illustrate how maths skills are developed as people progress through life, and are eased by making connections to versions of truth that they have already internalised. Jennifer (and others) reflected on how contextual knowledge helps learners either believe or disregard answers: "the calculator could tell you any rubbish… you've got to be so careful that the context and the numbers fit together and make sense", to which Tammy added, "it doesn't matter where you are in the world. If you know your own weight, you will use that to judge somebody else's", and "I always say to [learners] that you don't have to be a painter… to say that you can't use a million litres of paint for a bookshelf" – some contexts are universal and thus some estimations may be universally contextual.

4.1.3. Estimation and problem solving

Additionally, participants argued for the transferability of the estimation to another maths skillset: problem solving. Out of nine participants, all three in the Focus group, and one other, likened the skill of estimation to problem solving. Mandla puts this eloquently: "In the absence of being able to estimate, you run the risk of not being able to solve the problem because if you lack the skill of estimation… you may just be lucky to get the answer." This theme is complementary to the prominent idea that estimation happens before or before-and-after a calculation; estimation enables learners (and teachers) to have a "general idea of where the question needs to go" (Jennifer). This starting block is not only foundational when attempting the highest cognitive level questions in assessments (which amass to 15% of all assessment requirements - Department of Basic Education, 2011a, 2011b), but is also a way for teachers to determine which specific mathematical skills children lack. Jennifer provides an example of this: "what I'm trying to gather here is to verify [the learners'] understanding over and above the estimates: their understanding of what the square root is".

Themes and quotes from the collected data indicate that problem solving and estimation are inseparable: part of the same skill, they rely on each other to begin, develop, and complete solutions to problems. Both estimation and problem solving incorporate a variety of different skills; estimation provides the beginnings of problem-solving techniques and answers to solved problems can be checked by the internalised *feeling* of whether the answer is right or wrong. Although not explicitly mentioned, this problem-solving ability was linked to 'real life' examples of estimation which could give participants and their learners an ability to know where to start and what methods to use, when solving problems (according to Jennifer and Jacqui, respectively). Thanks to Tammy's perception of estimation as a 'life skill' like problem solving, I was able to relate the skill to the critical theories which were used to justify this research project, and which will be further discussed in the following chapter.

4.1.4. Summary

No matter the teaching context, teachers who participated in this research project displayed a notable awareness of the importance of estimation both within their classroom

(and in the context of helping their learners achieve highly in maths assessments), and as part of a set of life skills or "tools" (Mandla). With estimation as part of children's toolboxes, they are more able to decipher meanings behind, critique, and substantiate information they create and discover. It is for this reason that estimation is understood to be an important skill for all people to have.

4.2. RQ2 How do teachers' classroom practices / praxis change through participation in exploratory research projects?

Teachers are forced to adapt their pedagogies to accommodate the myriad challenges they face when trying to share knowledge with their learners. The data collected in this research project reveals that teachers can and will change their perceptions and practices in response to their involvement in an exploratory research project. As a negotiation between external and internal pressures, participants found this research project to be an opportunity to

> sit down and think. This could be the only new thing in your lesson compared to the, I don't know, last three years that you've been doing the same thing. You get to you add this component of estimation. I think that's where we can actually get better. (Mandla)

Although I did not attempt to monitor how teachers changed their teaching practice (which could be done through observations or interviews with learners), it was clear that teachers felt the need to reassess their classroom habits and were, especially because of the WhatsApp group, encouraged to keep looking for examples of estimation. Participants' initial confusion about estimation later developed into a willingness to learn more about the skill that I was researching: they embraced their roles as participants and changed their praxis according to how they perceived their learners and the skill.

At first glance, teachers were uncertain about how estimation fitted into their teaching practice. Focus group members (like others) hardly contributed to the WhatsApp group and noted that they had struggled to isolate incidents of using estimation in their classrooms. However, as they continued discussing the skill within the focus group, they continued the trend that had been set by all other interviewees: the more they discussed

and were reminded of the topic, the more examples they could give – relating to both in-class and 'real world' contexts.

By answering RQ2, the following three sections delve deeper into how reminders to participate and perceptions about learners encouraged teachers to adapt their teaching practice / praxis.

4.2.1. Reminders through participation

The WhatsApp group served as a reminder to participants to keep engaging with the topic of estimation (both in their classrooms and beyond those four walls). Both Mandla and Tammy indicated some frustration at not contributing more to the group, but suggested that they were not "coming across examples" (Tammy). Even though they tried and "kept thinking, 'oh I wanna put something on the group'… [they] would think 'oh there was nothing'" (Tammy), teachers believed that it was not because they were too busy to contribute to the group, but rather that… "it's just that it's not something that's actually coming up very often" (Tammy). However, after the ensuing discussion, participants agreed that the quietness of the WhatsApp group was, instead, due to teachers not recognising when they were using estimation. Mandla suggested: "I can guarantee that probably more than 60% of your lessons will have estimation" but that it goes unnoticed because of the pressure of the curriculum and uncertainty relating to its definition. Even though Jennifer did not join the WhatsApp group, her contribution to the focus group followed the same conclusions developed by Tammy and Mandla: estimation is happening all the time, but we are unaware of it. Because we had spent time pondering over the topic, and "given it a name now" (Jennifer), participants were able to acknowledge that estimation permeates many parts of our lives (as was jokingly reflected by Tammy: "I will estimate how long before I can put my kettle on"), and happens without much effort in lessons. Thus, instead of needing to plan estimation into lessons, the mere awareness of the topic makes teachers see that they are "using it consistently and continuously during our lessons" (Mandla).

Participants noted that by participating, whether in the WhatsApp group or not, they felt more inclined to plan their lessons more carefully to include elements of estimation, and to reflect on their teaching practices. Not only did participants notice changes in their teaching practices, they also rationalised their adaptations according to how important they

perceived the skill to be for mathematical achievement and life skills. Mandla noted “we're using [estimation] consistently and continuously during our lessons”; while Jennifer noted: “it’s something I'm going to try and use a lot more in my lessons”. Mandla furthered this discussion about the use of estimation: “[these discussions] are very helpful… our calling is not really just to teach the content, but we're trying to develop someone who's going to be able to think, we’re developing future leaders”, and Tammy responded with: “you're not just trying to teach them how to find the derivative of something. You train them to be able to make logical decisions and judgments about the information that is fed to [them]”.

4.2.2. Believing in “future leaders”

The idea of adapting to suit learners’ needs is linked to teachers’ belief in learners’ capabilities. Illustrated by Mandla’s response to the vignette, learners are, in fact, “future leaders”. This term altered the tone and attention of the focus group, which delved into a discussion about how teachers perceived the learners they were teaching, and why they believed it was important to teach mathematics. Participants’ responses to each other about the vignette and my follow-up questions elicited a common theme: teachers are not trying to teach maths for the sake of passing on skills relating specific to the curriculum. Instead, they are teaching children skills that will assist them outside the classroom. Each focus group participant told a slightly different story: Mandla linked maths to leadership and individual success outside the classroom in his interview, and continued this discourse in the focus group; Jennifer initially linked estimation to problem solving, and then agreed with Mandla that all teaching should extend beyond the classroom to give children problem solving skills that are not only related to maths; and Tammy presented another narrative: the teaching of maths is contextual (in terms of the subject matter and classroom dynamics), and classrooms should be the place where children learn skills that can be transferred to other environments outside the classroom. These three opinions converge on one point: maths teaching is not, and should not, remain classroom-specific: teachers are responsible for developing leaders of thought and action.

All participants reflected on their new-found appreciation for the skill of estimation, as is articulated by Jennifer: “[estimation] is a really, really good skill that the children should develop”. Moreover, participants appreciated that their involvement in the research

project enabled them to think about their teaching practice differently, like Mandla, who said “I would just like to thank you for the opportunity to talk about this... it just gives me a different perspective”. Mandla also detailed how he would be incorporating estimation in his upcoming university-preparation classes. Because of this appreciation for the skill, focus group members were able to reflect on their changed classroom practices:

> it definitely has given me a sense of awareness now because [estimation is] not something I ever thought about until you asked us to think about it. And then as I was teaching, I was like, oh, I'm not using estimation. I'm not using estimation. And then, oh my word, I just used estimation! (Tammy)

4.2.3. Teachers’ adaptations

Teachers’ practices and praxis have become more critical in response to this exploratory research project. They have become more aware of, and reflective about, their lesson plans, pedagogical approaches, and specific methods relating to certain mathematical skills. These changes are not only aligned with estimation specifically, but also include problem solving, and innovative approaches to using estimation to teach sections. This is illustrated by Jennifer’s response to Mandla’s plans for teaching Grade 12 finance: “we're going to be starting financial maths and I'm going to use that. Yes I will. I've never thought about it like that before, but that's actually a brilliant suggestion. Thank you [Mandla]”. Some teachers who were not able to give feedback on being part of the research project (as they were not part of the focus group) indicated that they would try to teach or use estimation more in their classrooms, as is indicated by Eva:

> So, sometimes it is good to go back to the basics and hone those skills so… now just chatting to you and then seeing and thinking about estimation, and just trying to see how, in the classroom, are we actually using it? Because it's not something that you actually actively think about. That's why it's been very, very interesting.

4.2.4. Summary

Fundamentally, teachers' practices and praxis become more critical through their participation in exploratory research projects. Participants indicated a willingness to change their habits to empower children to solve 'real world' problems with more contextual awareness and transferable skills.

4.3. RQ2.a What pointers should be kept in mind when designing a teaching intervention for high school maths teachers?

This research project has provided ample opportunities to imagine what type of intervention can be implemented in South African classrooms. However, by understanding the external and internal stressors impacting on teachers' classroom practices it is possible to determine some pointers for designing such an intervention. By considering teachers' challenges in- and outside their classrooms, answers to the final research question suggest strategies for developing and implementing sustainable teacher interventions in South Africa.

4.3.1.1. Teachers' struggles in delivering lessons: learners

Participants mentioned a variety of struggles they faced while teaching; none related to the availability of resources, infrastructure, or school management, which is possibly because (a), all participants were from schools that had sufficient resources to operate functional classrooms, and (b), the question of resources and support was never explicitly asked. Instead, participants made passing comments about how their teaching practices have been limited through a variety of factors, which are thus discussed.

Having the potential to engage willingly and constructively in classrooms, 'learners' hold power. While Jacqui and Emma discussed the lacklustre attitude of many of their learners, they joined four other participants in arguing that it is possible to develop a positive milieu in classrooms such that children are empowered to make meaningful contributions. Despite this positivity, it was clear that many participants had to work hard to address questions regarding the usefulness of maths, and estimation specifically (as mentioned by Skye). When children are not provided with good enough responses to these

cries for meaning, usefulness, and purpose, it seems that they are less likely to enjoy doing maths. Moreover, learners' actions and engagement are reflective of the safety and comfort they feel in their classrooms; Tammy suggests that when children first arrive in her classroom, many are "very uncomfortable with guessing", but that they become more engaged as they grow accustomed to the safe space of the classroom. In the same school, Skye defines how she tries to make learners "feel like they all… can ask questions, but often some of them don't … It's also a social thing… you try pull them out of". These reflections highlight the need to improve learners' classroom experiences and attitude towards maths for them to engage more constructively with the subject matter. One of the ways this can be done is by addressing the 'elephant in every classroom': *why are we studying this*? Mandla addressed this directly: "[maths] is not really about the content. We're here to translate the ways of thinking, and then [the learners] can create their own ways of thinking as well" – Mandla calls these ways of thinking "tools" which fit into our "toolbox" for ease of access whenever necessary.

4.3.1.2. Teachers' struggles in delivering lessons: external

Aside from the learners themselves, teachers deal with a range of external pressures when deciding what and how to teach. Jacqui and Emma spoke of parents' misaligned expectations for their children, often encouraging their children to take the easiest route to achieve their Grade 12 qualification; five of the nine participants bemoaned learners' reliance on calculators, which they thought to be counterproductive when teaching children to compute simple calculations mentally ("I think they still prefer not to do the work if necessary" – Skye; "the calculator's only ever as good as its operator" – Eva); according to Marlise and Jennifer, learners, parents, school administrators, and management are inclined to emphasise high marks for standardised assessments over any other forms of assessment; and the ubiquitous power of the curriculum ensures that teachers are both held accountable for, and restricted in, what they teach. These factors influence teachers' teaching methodologies, which are also impacted by the environment of the school (as indicated by the four participants who had also taught at a primary school level, and one teacher who had taught abroad). These external expectations weigh down on teachers' abilities to teach maths in interesting and fun ways; more relevantly, it influences their

ability to spend time teaching estimation skills when most external expectations do not value this skill.

4.3.2. Strategies for sustainable teaching interventions

In response to these stressors and participants' contributions, teaching interventions should:

1. be adaptable contexts across resource and infrastructural spectra,
2. educate teachers on the topic at hand,
3. not require too much extra work, and
4. be enticing for teachers.

These considerations set the stage for an intervention that can be adapted to suit the needs of any teacher, and one which is sustainable and implementable for teachers across both GET and FET phases. Interventions can be sustainable by demanding minimal external support. This is the imagined future of the Estimation Teaching Guide.

To make interventions adaptable, they must include globally relevant examples which introduce teachers to the idea that estimation is for use by everyone. An example of this was provided by Jennifer: "It doesn't matter where you are in the world, if you know your own weight, you will use that to judge somebody else". Additionally, the intervention should not cost any extra resources or time to implement. This way, it can be shared amongst all teachers and thus it is hoped that teachers throughout the country can benefit. Other examples in the Teaching Guide can provide ways of asking questions in class that encourage learners to turn every calculation into an opportunity to estimate. Some more specific examples can be included too – this will need to be grade- and topic-specific to be most useful.

For teachers to use estimation, they need to be educated in the skill. This was achieved through discussions with participants in this research project, where teachers slowly became more familiar with the topic after thinking about it, asking me questions, and being asked leading questions about how certain concepts are taught. For future interventions, this can be achieved through providing definitions, examples, and reflections. Through this education, teachers can make their own decisions about how best to use their

new knowledge and skills, and can use intervention guides to help them apply these adaptations to their classrooms.

One of the ways that teaching interventions can be made more enticing is by ensuring that teachers do not feel the need to work significantly harder. Mandla spoke about this in the focus group, suggesting that instead of re-planning lessons to better integrate estimation into each one, it would be easier (and as productive) to learn more about estimation and set up mental reminders to use the skill. Additionally, if teachers realise the benefits of implementing such an intervention, they may be more inclined to participate in an intervention. This was illustrated by Eva (who was the first to be interviewed) who showed immense interest in the WhatsApp group and who made the most contributions to the group. Her interest in the topic increased as her interview progressed and exemplifies how teachers can be receptive to new ideas even after many years of teaching. The teachers in my research project realised the critical need for estimation in the syllabus and it is hoped that their interest in the topic continues to change their, and their colleagues', teaching practices for years to come.

4.4. Summary of findings

According to participants, the transferability of estimation makes it a skill worth teaching. Additionally, it serves teachers' aims to educate learners to be "future leaders" who can solve real-world problems by contextualising numbers. To do this successfully, teachers who have been part of the research project saw it fitting to adapt their teaching praxis to suit the needs of the children that they teach. By reflecting on the stressors they face when delivering lessons, teachers provided indicators for the development of a sustainable teaching intervention which could lead to wide-scale development across the South African educational landscape.

Chapter 5: Discussion: “Can the general populace make sense of it?”

This penultimate chapter connects research findings to the relevant fields of literature, policy, and a broader societal importance. By addressing each of these fields systematically, this discussion positions this research project in current educational dialogues in South Africa. It summarises the key issues of estimation in South African and global literature, how research can and should change national policy, and the social implications of the research conducted for this project. In conclusion, it argues for this research’s importance in answering the question, “Can the general populace make sense of it?” with reference to how numbers fit into everyday life. This quote, from Jacqui, is at the root of what this research project has been about: can everyday people make sense of the information they process daily?

By detailing the project’s contributions to literature on estimation and South African policy and curricula, the following interlinked sections argue that this research project has important social implications for the country. The penultimate section of this chapter provides author reflections on the project and its outcomes.

5.1. Literature

This research project has illustrated the importance of teaching estimation in South African maths classrooms because of two related characteristics: the skill enhances (and relies upon) contextual knowledge, and it is likened to problem solving. These two characteristics begin a dialogue about estimation in South African maths classrooms and forge a path towards including the skill in further discussions about curricula. In congruence with critical theories of praxis (Chapter 1.3.), estimation can be argued to empower people to make more informed decisions.

The contextualised knowledge needed to solve some estimation problems is related to the idea of ‘number sense’ which forms the principal concern of many primary school teachers (Castronovo, 2012; Mildenhall, 2011; Senol et al., 2015). Added to this maths-based context, learners become more aware of the world around them and can make comparisons between known and unknown contexts as they progress into high school; Mandla argues this articulately: “the spirit of being able to communicate your

understanding and make judgments in light of information: this is when [learners] apply mathematics… without even realising it". Researchers have agreed that skills must be taught through association with already-known contexts or skills (Adler, 2002; Restani et al., 2019). To do this successfully, teachers could use examples of estimation to link known maths content to contextual knowledge and skills. This relates to Lerman's (2000) thesis that for knowledge to be internalised and useful, it *must* be accompanied by contextual references between people and places.

In the context of South Africa, where lived experiences vary dramatically between SES and locations, it is difficult to make maths contextually relevant on a national scale. Instead, like some Tammy suggests: "you don't have to be a painter… to say that you can't use a million litres of paint for a bookshelf"), there are examples of estimation that are globally relevant which could, and should, be incorporated into classrooms. With an improved knowledge and understanding of estimation, teachers may be more inclined to diversify their use of the skill and create more contextually specific examples to use in classrooms. This research project has provided evidence for how teachers may be willing to change their teaching practice in response to their own education and interest in maths concepts which can assist their learners' success; Mandla illustrates this: he believes his teaching role encompasses "assisting [learners] to think in a logical manner… so therefore I'm under pressure to make sure that [they] are able to think logically." For teachers to become more educated in the uses and benefits of estimation, it would be beneficial to include the skill more prominently in the curriculum, as this is what many teachers base their everyday practices on (Adu & Ngibe, 2014).

The problem-solving characteristic of estimation is also vitally important in maths classrooms and beyond. This has been verified by Desli and Lioliou's (2020) research which suggests that children and adults who use estimation effectively begin solving problems with more confidence and arrive at answers faster than those with less skill competence. Additionally, research into the use of arithmetic approximation suggests that estimation is a vital skill for both children and adults when addressing daily numeric problems (Ganor-Stern, 2016; Sayers et al., 2020). Thus, estimation goes above and beyond what the Cockcroft Report (1982) suggests (as discussed in the Literature Review, Chapter 1.4.): instead of merely retrospectively judging answers' correctness and measures, estimation

also occurs before calculations begin (to predict the ballpark of an answer), which can direct problem-solving strategies. Teachers (and their learners) perceive estimation viscerally and in terms of its use. Their *feelings* of 'right' or 'wrong' indicate contextual knowledge of the problem and may influence how they continue solving the problem.

Problem-solving has been included in maths curricula for many years because of its potential to prepare learners for modern challenges (Leblanc, 1997). However, as Koichu (2020) argues, *problem-posing* education is still relegated to the fringes of maths classrooms, something which inhibits Freire's (1970) liberatory approach to education. When learners are asked to pose problems for their peers, they are requested to ask questions which they deem important and answerable. Their contributions transform classrooms into learning environments that are aligned with pedagogies of empowerment, and learners into co-constructors of knowledge (Crespo, 2015). To pose these problems, learners need to have a grasp of the mathematical and grander context of their questions, something which Emma noted: "You've gotta think within the context, because otherwise all you're doing is spewing out something that is like a foreign language".

Contextualisation, problem-solving, and problem-posing are fundamental maths skills that are needed to liberate a population (see also Literature Review, Chapter 1.3. and Frankenstein, 1983; Freire, 1970). By exploring the possibilities of using maths in social sciences and statistics, Frankenstein (1983) indicates that "mathematical literacy is vital in the struggle for liberatory social change in our advanced technological society" (p. 315). Regarding her thesis, Frankenstein (1983) has argued for maths, and therefore estimation, to play an empowering role in civil society, something which may be difficult if teachers continue perceiving those they teach according to Biesta's (2010) definition of 'learners'.

Teachers in this research project have provided reason to believe that they are instead teaching "future leaders" who have a role to play in society. One of the reasons that teachers thought they should increase their use of estimation and adapt their practice and praxis was for the service of the learners themselves. Jennifer argued that "you're not just trying to teach them how to find the derivative of something. You actually want to teach them to be thinking, operating adults" – she perceives her learners as important role-players in society and her classroom. Thus, despite the various restrictions teachers face in the political context of classrooms, and education more broadly, teachers who believe in the

agency of their learners are willing to adapt to suit learners' needs (Long et al., 2017). This can be linked to similar conclusions reached by recent research globally (Rapanta et al., 2020; Wolff & De Costa, 2017), which argues that teachers are more willing to adapt to the needs of learners who have proven their desire to act with intent in the wider world.

To change the common perception of 'learners' into 'future leaders' in South Africa will take some time. However, this change is necessary if the country wants democracy to rely on quality education, instead of the undereducation which epitomises current systems (Phatlane, 2020). This fundamental issue of democracy is one which this research project aims to address: if we teach maths (and other subjects) more effectively, perhaps the general populace will make better sense of the information they co-construct and are given. To do this, policy needs to be reconsidered.

5.2. Policy

Relating to critical literature, maths learning, and intervention studies, this research argues for more effective translation of policy into practice. South African policy documents assert that estimation should be taught in schools (as discussed in Chapter 1.1.3., 1.4.1.), however, this research project has argued that this is not being achieved. This failure is not an issue unique to South Africa, as is seen by similar problems documented in the British educational system (Sayers et al., 2020).

The Mathematical Literacy curriculum is currently more suited to the teaching of estimation, as was argued by participants in this study. However, according to Machaba (2017), Maths Lit has still not achieved the aims of giving learners the "ability to reason, make decisions, solve problems, manage resources, interpret information, schedule events and use and apply technology" (Department of Basic Education, 2011c, p. 8) through the contextualisation of maths skills. Additionally, despite estimation playing a role in each of the subject's five elements (see Chapter 1.2.1.), some Maths Lit teachers have not yet integrated the skill into their classrooms effectively.

This research project has illuminated the gaps between policy and practice: what is officially documented is not yet being brought to fruition in classrooms. Although there could be myriad reasons for this, this project has highlighted one in particular: some

important skills are not explicitly emphasised within the curriculum and thus teachers have not integrated them into their daily practice. Additionally, because of the segmentation of the maths curricula, policy does not encourage teachers to emphasise the skills-overlap which estimation relies on. For example, estimation in the GET Mathematics curricula (and textbooks) is presented as an isolated skill which relates specifically to the rounding of numbers; but this does not tell the whole story of estimation's use. Younger teachers like Lindokuhle and Skye, who rely on these documents to guide their teaching, spoke about their inadequate preparedness to teach the skill in the meaningful ways that it could be. As a skill that requires contextual knowledge and maths understanding, estimation must be taught across a variety of topics and skills to be operationalised more habitually. It is hoped that the Estimation Teaching Guide will provide teachers with part of the resources and impetus needed to begin incorporating the skill into their classrooms more effectively and sustainably.

To change current teaching practice/praxis, teachers may require the help of interventions to guide their adaptations to curricula and provide effective mechanisms for integrating estimation into their classrooms. Sathorar and Geduld (2018) have argued that teachers' continual adaptations to curricular needs and other external pressures are already exhausting, and thus adding to this load without careful consideration may not yield positive outcomes. However, according to Parker et al. (2020), if teachers are engaged in participatory research (and interventions), they are more likely to make lasting changes to their teaching praxis. With South African curricula already including estimation briefly, the topic is not a complete anomaly to teachers. It is therefore hoped that this research project can provide sufficient evidence for the development of a participatory teaching intervention to integrate this skill more effectively into classrooms. This intervention can be designed sustainably according to the external and internal stressors impacting on teachers' classroom practices (see Answers to RQ2.a, Higgins, 2019). Although teachers make their own decisions about how to use new knowledge and skills, intervention guides can help them apply these adaptations to their classrooms effectively (Murray et al., 2012). It is therefore hoped that the findings of this dissertation provide reasons for better interventions to be designed and adjusted to become national policies and practices instead of isolated attempts at improvements.

5.3. Social implications

Although there is very literature to support claims that estimation skills lead to feelings of civic responsibility and democratic thought, this research project has begun a discussion around these topics. Instead of understanding estimation as a maths-only skill, this project positions the skill in relation to two others: contextualisation and problem solving. It can be argued that, together, these three skills empower people to make clearer decisions about information.

The ability to contextualise information abides by critical theories' suggestion that contextually significant education empowers children to make radical social change (Freire, 1970). As discussed in relation to Critical Theories (Chapter 1.3.), "people + place = politics" (Williams, 2001, p. 3), thus classroom spaces are fertile grounds for children to learn and unlearn their role in society. Furthermore, critical pedagogies of place (Gruenewald, 2003; Williams, 2001) highlight the need for teachers to first contextualise the power vested in their classrooms before they can expect to empower children within these spaces. Thus, as teachers become more accustomed to understanding – and undermining – the power imbalances in their classrooms, they are more likely to empower children to do the same in broader society. Teachers can ask more questions (which was indicated by participants to assist with bringing estimation to the forefront of classrooms), request more critique, and allow for more mistakes by their learners, thus habituating them to critically engage with society. As teachers learn to centre their lessons around the needs of the children that they teach, these children may embrace this power and learn to question not only others' actions, but also their own. Using estimation to problem-solve relies on the contextualisation of information and reminds that the problem solver to think about broad perspectives. Laouris and Michaelides (2017) have argued that as children become accustomed to using unfamiliar contexts and skills to solve problems, they tend to engage more freely in democratic dialogue. Their study in Cyprus resonates with the complexities of the South African landscape and provides reason to believe that problem solving skills, such as estimation, can be useful at all levels of societal engagement with democratic processes. Through this, South Africans who have developed critical awareness of information may be more able to make decisions about their futures and political endeavours, thus advancing South Africa's democracy towards one that relies on education.

5.4. Reflections on research outcomes

The instability in South Africa at the time of writing this dissertation has reiterated the fact that change is needed soonest. To be part of facilitating this change, interventions heeding to the above recommendations, and following promising examples like those happening already (for example, CASME (2021) and Numeric (2021)) can be implemented. Challenges facing these interventions have been well-documented (Amnesty International, 2020; Oosthuizen & Bhorat, 2006; Spaull, 2019a), and it is with reference to these that this project offers new perspectives on intervention designs. Having realised the importance of classroom as spaces for critical reflexivity, this research project has highlighted the need to use my relative privilege to continue working with teachers to adapt their teaching practices, which can perhaps begin with a focus on mathematical estimation. This dissertation contributes to current research interventions in South African maths classrooms and aims to encourage researchers to continue working closely with their participants to elicit the most relevant and contextually significant information. With the country currently leaving masses of children critically undereducated, true democracy (which has been argued by Glaeser et al. (2007) to rely on education) can only be achieved by improving civic engagement which begins at schools.

5.5. Summary of research outcomes

By conceptualising estimation as a critical element of social change, this research project emphasises the need for maths – specifically estimation – to be taught to school children. Using the formal education sector to impart this transferable skillset enables children to better understand and contextualise knowledge and solve problems. However, for this to be achieved, teachers need to be trained. This may improve their lessons, empower their learners, and educate the next generation of leaders. This research project has provided a rigorous argument for estimation to be more intentionally included in maths classrooms such that the critique and contextualisation of numbers become a national norm – by doing this, the general populace will ‘make sense of [information]’ just as Jacqui suggested.

Conclusion

Estimation is important, important enough for teachers to want to change their teaching praxis for the "future leaders" they are teaching. This exploratory research project has achieved its aims of laying the groundwork for further interventions in maths classrooms, relating specifically to estimation. It has documented how estimation is perceived as important – yet complex – by South African teachers by making specific reference to problem solving and the contextualisation of a range of mathematical concepts. The skill of estimation is one of the ways that teachers can link the abstract concepts of the curriculum to daily life, therefore enhancing learners' ability to understand and critique similar information. With the belief that they are teaching future leaders, teachers can use estimation to further their own, and their learners', conceptualisation of tricky mathematical concepts and better equip them with the necessary skills to solve real-world problems. Moreover, this project advances critical literature on how maths can be used to empower children to make contributions to democracy by processing information critically and contextually. The findings of this research project reveal a deep willingness of South African teachers to adapt to the needs of their learners and the world, and a keenness to continue learning how to do this effectively. As noted in the Introduction, estimation is a skill that uses both fast and slow thinking (Kahneman, 2011): quick maths can provide almost-immediate answers, but slower thinking enables users to critically engage with and question how contextually accurate those answers may be. It is for this reason that estimation is arguably one of the most important skills that can be taught in schools and used for lifetimes, and one which can work to secure South Africa's democratic future.

Areas for further research

The conclusions drawn from the available data contribute to the literature surrounding the complexities of the South African education landscape and provide a ray of light to direct future studies about teachers' adaptations of practice and praxis. Conclusions reached about the ubiquity of estimation, and why it should be better integrated into classrooms, can direct future studies that try to improve maths education outputs and outcomes in the country. Further research into estimation-specific interventions for

teachers will go a long way in ensuring that more children and adults are able to use estimation to assess the information they process daily.

Based on the limitations of this research project, future research would benefit from incorporating a larger sample size which is more representative of the range of schooling contexts in the country. Longitudinal studies may allow for effective monitoring of teachers' practical adaptations and learners' benefits from interventions which arise from research. Additionally, research into the links between estimation and problem-solving will allow for more detailed analysis of how the two concepts are related and can be combined in classrooms.

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Appendices

Appendix A: National and international benchmark test ranking

National and International benchmark test results reveal a similar story to that of the latest NSC results. By 2014, the average Grade 9 Annual National Assessment (ANA) Mathematics mark was 11% (Department of Basic Education, 2014). These results not only represent the systemic issues in the education sector and go some way to explaining why the country performs poorly in international tests, but also indicate how fragile the system has become to accept 30% as a pass mark for Mathematics and Mathematical Literacy (Macupe, 2020). Despite some improvements since 2011, South Africa scored poorly in the Trends in International Mathematics and Science Study (TIMSS) in 2015, ranking second-last overall in the Grade 8 test (which was taken by grade 9 South African learners) (Mullis et al., 2016). Further analysis of the TIMSS (2015) assessment revealed that there was a 73-point mean score difference between no-fee (Quintile 1-3) and fee-paying (Quintile 4-5) Grade 9s, reflecting how socio-economic status of learners influences their educational outcomes (Isdale et al., 2017). In KwaZulu-Natal, the province home to the participants in this research study, almost 70% of Grade 5 learners who participated in the 2015 TIMSS study did not score the necessary 400 points to achieve even the lowest ranked benchmark for the test (Isdale et al., 2017). Although this poor performance may not be the case across the spectrum of all South African learners, it is clear that Mathematics education is in dire need of change.

Appendix B: Current factors hampering progress in education

Although not experienced by all, the problems facing some South African learners are seemingly endless: teachers have low content knowledge (Rollnick et al., 2008), some schools do not have sufficient textbooks for their classes (Mutasa, 2012), school buildings are dilapidated (Amnesty International, 2020), girls face a high possibility of gender-based violence at schools (Bhana & Mayeza, 2021), and children from different provinces and race groups have varied experiences of education (Spaull, 2019a).

Using a combination of Farrell's (2012) and UNESCO's Learning Portal (2021) criteria for analysing educational quality, the following section briefly reviews some of the current challenges facing the South African education system.

Access

Access refers to "the probabilities of children from different social groupings getting into the school system, or some particular level or portion of it" (Farrell, 2012, p. 157). Barriers limiting children's access to education, and, more specifically, to education of a high quality, in South Africa include language, location, disability, age, school acceptance rates and criteria, and cultural pressures. Relating specifically to maths education, Feza (2016) has documented how language skills and the teaching of maths in second languages affect early-age numeracy; this has compounding negative effects as children progress through the school system. Details about other aspects of educational environments hindering children's ease of access to educational facilities and experiences have been well documented by Nic Spaull's (2013) Centre for Development and Expertise, based in the Western Cape. Many of the challenges have not yet been addressed, as seen by Spaull's more recent reflections on the state of inequity in education in the country (Oosthuizen & Bhorat, 2006; Spaull, 2019a).

Survival

The *survival rate* of children refers to the probability that they remain within the schooling system in some way (Farrell, 2012). Researchers at the Research on Socioeconomic Policy at the University of Stellenbosch in South Africa have presented various answers to the question, '*How many children drop out of school before completing*

their studies?' because of the difficulties of capturing and measuring data (Pretorius, 2019). One of the researchers in the department, Nic Spaull (2019b), released this statement on his personal blog, which reflects work done using the EMIS system in South Africa, SA-SAMS, and is based on the 2018 Grade 12 cohort:

> of 100 children that started Grade 1 in 2007, only 51 made it to matric, 40 passed and 17 got bachelor's passes. So, the pass mark of 78% is probably more like 40% if you take into account the 400,000 kids that drop out the system before matric".

Reflections of this type demonstrate the sheer volume of drop-outs that face the education sector of the country. Many children do not survive the education system and therefore do not fully access the education that is granted to them.

Learner characteristics and context

According to UNESCO's (2021) criteria, *learning characteristics* refer to the historical and present barriers and enablers which are inherent in learners. Broader than learner characteristics, *context* refers to "public resources for education, parental support, national standards, labour market demands, socio-cultural and religious factors, peer effects, and time available for schooling and homework" (UNESCO: Learning Portal, 2021). Layered together, these represent factors that directly affect children's everyday experiences of learning. In South Africa, it has been proven that social and economic status (SES) and race are the major determinants of where children can attend school, and how well they will do (Ogbonnaya & Awuah, 2019; Spaull & Kotze, 2015). The intersectionality of learner characteristics and contexts creates layers of complex identities which define children's access to, and engagement with, school. Spaull and Kotze (2015) provide evidence for how children who face both individual and systemic barriers to educational gains perpetually lag their classmates; "starting behind and staying behind" (p. 13), these children are the most likely to become a burden on the state after they leave the formal education sector. It is therefore in the state's best interest to improve the quality of their lives and experiences to avoid such a tragedy. Education quality can be measured by how well schools negotiate the identity politics and individual needs of children in classrooms; do children feel engaged and able to learn and contribute according to their lived experiences?

Enabling inputs and teaching and learning

Criteria that classify as *enabling inputs* are those that are offered by the school, including infrastructure, human and other resources, teaching and learning materials, and school management. The quality of these inputs shape learners' experiences at school. Oosthuizen and Bhorat (2006) provide an analysis of some of the struggles facing school infrastructure which has been revisited by the national *Action Plan to 2024* in the National Development Plan for 2030 (Department of Basic Education, 2020a). Currently, the country is working on improving the facilities and services offered by schools, but this mammoth task will take time to sufficiently address. Added to this, *teaching and learning* relates specifically to classroom-based experiences: it includes teaching and learning time, assessment types and frequency, and class size. The South African teaching landscape is diverse: some teachers who taught in Bantu schools during Apartheid are still in classrooms having taught many new national curricula; some teachers have all the resources and support they need to plan and deliver 'successful' lessons; some classrooms are too small to accommodate the correct number of desks and chairs. These conditions differ from school to school and classroom to classroom, and additionally, teachers' skills are vastly varied (Spaull & Kotze, 2015). Combined, these barriers influence the possible education quality which can be offered to children.

To measure how these barriers have affected education at a school level in South Africa, it is possible to look at two measures: outputs and outcomes, both of which relate to quality and equality.

Output

Output relates to whether children from various contexts are learning the same things at the same depth as each other. This has become an area of prolific research since South Africa became officially desegregated (Case & Deaton, 1999; Gustafsson, n.d.; Macha, 2017). Graven's (2017) research, which primarily focuses on mathematics education, has shown that even in the post-apartheid context, opportunity and performance gaps persist mainly along racial lines and thus education outputs remain racially differentiated. Education in South Africa lacks a standardised quality, and is highly inequitable: the NSC

results released each year show that previously marginalised provinces (like the Eastern Cape, and to some extent, KZN) continue to underperform in comparison to others (Department of Basic Education, 2011i, 2021).

Outcomes

Farrell (2012) defines *outcomes* as

> the probabilities that children from various social groupings will live relatively similar lives subsequent to and as a result of schooling (have equal incomes, have jobs of roughly the same status, have equal access to sites of political power, etc.). (p. 157)

Belling (2020) and Spaull and Kotze (2015) have proven that South Africa rates relatively poorly according to this factor: those children who attend schools with favourable access to resources and other educational inputs are more likely to find employment or higher education opportunities after completing Grade 12. Children's futures are most often dictated to them by the educational inputs of their schools. However, Statistics South Africa (2020) has reported that more recently, the younger Millennial Generation have made more substantial strides in educational outcomes compared to their Gen X compatriots. This may be related to the more stable political environment in which Millennials grew up. It is hoped that as the nation develops its democratic identity, this trend can continue and thus create a more equal society.

Appendix C: Aims of the national curricula

Quote from a Curriculum and Assessment Policy Statement Department of Basic Education, 2011c, p. 5:

> The National Curriculum Statement Grades R-12 aims to produce learners that are able to:
>
> - identify and solve problems and make decisions using critical and creative thinking;
> - work effectively as individuals and with others as members of a team;
> - organise and manage themselves and their activities responsibly and effectively;
> - collect, analyse, organise and critically evaluate information;
> - communicate effectively using visual, symbolic and/or language skills in various modes;
> - use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
> - demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

Appendix D: Estimation in the CAPS curriculum

Phase (Grades)	Subject	Estimation in the curriculum
Foundation (R-3)	Mathematics	By the end of the phase: estimation in measurement, grouping. "Checking results of calculations [and] judging reasonableness of solutions [enables] learners… to judge the reasonableness of solutions" (Department of Basic Education, 2011a, p. 349)
Intermediate (4-6)	Mathematics	Estimation is mentioned frequently – relating to measurements, number operations, and number relationships. Mentioned as part of the lower order thinking skills (Department of Basic Education, 2011e)
	Natural Science and Technology	Skill mentioned as part of foundational skills needed for more complex operations (Department of Basic Education, 2011f)
GET (7-9)	Mathematics	Estimation is used to check solutions, perform mental calculations and measurements (Department of Basic Education, 2011g)
	Social Science (History and Geography)	Estimation in Geography as part of measurement section. Middle order cognitive skill required to show comprehension / understanding (Department of Basic Education, 2011h)
FET (10-12	Mathematics	Estimation is regarded as a routine procedure (one cognitive level above the lowest) (Department of Basic Education, 2011d). Estimation is referred to using terms such as "rounding" and "approximation". The first time that estimation is mentioned with reference to assessment is regarding to Grade 12 Project, which requires learners to perform a variety of mathematical processes, including estimation and graphing estimates (which is also the first reference to graphical representation of estimation in all CAPS documents)
	Mathematical Literacy	Estimation is used for anticipating and checking the reasonableness of answers. Grade 10: a whole section dedicated to estimation, which is contextually specific: "It is expected that learners will be shown methods for estimating answers for every type of calculation that they are exposed to" (Department of Basic Education, 2011c, p. 29) Estimation is regarded as a routine procedure and appears in textbooks as a standalone skill (Veremeulen et al., 2019)

Appendix E: Ethical considerations

From the beginning of the project, I decided that fulfilling CUREC expectations were the least I could do to perform my research ethically and with care. It was through this that I decided to follow a Feminist Ethics of Care approach, which is discussed in the Methodology.

Examples of practical ethical considerations

Below is a non-exhaustive list of ethical considerations that were enacted during the research project:

- I offered to pay for participants' data / internet needed to participate in the project.
- I adjusted research collection methods and encouraged participants to participate in any way that suited their schedule – this meant that one participant answered questions as WhatsApp texts instead of on a video call. This also meant that there were only a few participants who joined the WhatsApp group.
- I kept track of whose voices were dominating interviews (especially when more than one was interviewed at once), and worked to adjust this in real-time by reverting to less engaged speakers to offer opinions if desired – this is certified by the evenly weighted speaker coding in NVivo.
- Although some participants agreed for their names to be used in the research project, I have anonymised all participants by assigning pseudonyms to them (according to their self-identifying gender and mother-tongue). This was done to ensure contributions could be published at a later stage.

CENTRAL UNIVERSITY
RESEARCH ETHICS COMMITTEE (CUREC)

CUREC 1A Checklist
for the Social Sciences and Humanities

University of Oxford places a high value on the knowledge, expertise, and integrity of its members and their ability to conduct research to high standards of scholarship and ethics. The research ethics clearance procedures have been established to ensure the University is meeting its obligations as a responsible institution. They start from the presumption that all members of the University take their responsibilities and obligations seriously and will ensure that their research involving human participants is conducted according to the established principles and good practice in their fields and in accordance, where appropriate, with legal requirements. Since the requirements of research ethics review will vary from field to field and from project to project, the University accepts that different guidelines and procedures will be appropriate.

Please refer to Where and how to apply for ethical review and the CUREC flowchart first to see if you need ethics approval.

Please complete this form using a word processor and email it, together with your supporting documents, to your Departmental Research Ethics Committee (DREC) (if applicable). If you don't have a DREC please email this form to ethics@socsci.ox.ac.uk using your official ox.ac.uk email address. Only type-written, emailed applications will be accepted.

SECTION A: Filter for CUREC 2 application

This section determines whether your study raises more complex issues requiring the completion of a full application for ethical review, known as the CUREC 2 application.

(Please mark 'X' in the Yes/ No column.)

	Yes	No
1. Are research participants classed as people whose ability to give free and informed consent is in question? (This may include under 18s (although see competent youths), prisoners, or adults at risk.) Your attention is drawn to the University's Safeguarding Code of Practice and its implications for researchers involving children or adults at risk. This includes the need for the work to be risk assessed and for researchers to undertake related training. (**Note**: If any of your participants are aged 16 or under, answer 'Yes' here **and** also answer question 5 below.)	☐	☒
2. By taking part in the research, will participants be at risk of criminal prosecution (e.g. by providing information on drug abuse or child abuse)?	☐	☒
3. Does the research involve the deception of participants?	☐	☒
4. Does your research raise issues relevant to the Counter-Terrorism and Security Act (the Prevent duty), which seeks to prevent people from being drawn into terrorism? Please see advice on this on our Best Practice Guidance web page.	☐	☒

If you answered 'No' to **all** the questions above, go to **Section B**.

If you answered 'Yes' to **any** question above, continue to question 5 below.

	Yes	No
5. Is your project covered by a CUREC Approved Procedure (formerly known as "CUREC Protocols")?	☐	☐

If yes, give the specific Approved Procedure number(s):	

If you answered '**Yes'** to **ANY** of questions **1-4**, **and** answered '**No'** to question **5**, **stop completing this checklist and do not submit it for ethical review**. Instead, complete the CUREC 2 application form from the CUREC website, then submit that for ethical review.

If you answered '**Yes'** to ANY of questions 1-3, **and** answered '**Yes'** to question 5, go on to **Section B**.

SECTION B: Contact details and project description

Contact details:	
1. Principal Investigator **or** supervisor (if student research) (give title and full name)	[redacted]
2. Name of student (if student research)	[redacted]
3. Degree programme (if student research), e.g. BA, BSc, MSc, MPhil, DPhil	MSc
4. Department or Institute name	Education
5. Address for correspondence (if different from above)	
6. University (not private) e-mail address and telephone number	[redacted]
7. Name and status of others taking part in the project (e.g. third year undergraduate; postdoctoral research assistant)	

Project description:	
8. Title of research project	South African high school teachers' perceptions about the usefulness of estimation in Mathematics and Mathematics Literacy classrooms and beyond.
9. List of location(s) where project will be conducted	South Africa – but all interaction with research participants will be virtual.
10. If your research involves overseas fieldwork or travel and your department requires a travel risk assessment, will you have completed and returned a risk assessment form beforehand? (This must be approved by your department before you travel. If you are travelling overseas, you are strongly advised to take out University travel insurance.) Please also address any physical or psychological risks for Oxford researchers and local fieldworkers in question 16 below and discuss with your Safety Officer.	Yes ☐ No ☐ Not required in this instance ☒
11. Anticipated duration of overall research project	3 months or years (maximum 5)

12. Anticipated start and end dates of the part of the research project involving human participants and/or personal data	From: (15/02/21) To: (15/08/2020) Note: You will need ethics approval before you start your research. CUREC 1As may take up to **30 days** to process. Retrospective ethics approval **cannot** be granted.
In the case of international or collaborative research, will you submit or have you submitted this project for ethical review or consideration elsewhere (e.g. collaborator's/ local ethics committee, or other local approval)?	Yes ☒ No ☐ If 'Yes', please attach ethics or other approvals and give more details below. If 'No', please explain your reasons below. Please also refer to the Best Practice Guidance on Ethical Review of social-sciences based research conducted outside the UK (BPG 16), which includes an Ethics Issues Checklist for International Research (Appendix A)
Research ethics applications have been sent to the South African Department of Education and the KwaZulu-Natal Department of Education for review.	
13. External organisation funding the research (if applicable)	
14. Brief description of research (about 150 words) in lay language. a) When describing the research, include your methodology, how you are applying professional guidelines, and the use to which results/ data will be put. Please also declare any conflicts of interest here.	
My proposed study will research teachers' perceptions about the skill of estimation and its use in the mathematics classroom and beyond. The work is built upon my belief that estimation is a fundamental skill for all who wish to be able to quickly analyse, critique, and interact with everyday quantification, numerical, visualisation, and measurement problems. We may be able to avoid certain mathematically challenging parts of the world by way of our life design, but we will not be able to avoid mathematics entirely, therefore having the skills to quickly engage with everyday mathematics problems through	

accurate estimation techniques will stand people in good stead. This is particularly important in South Africa, where mathematics education is failing the nation.

I plan to use a participatory methodology which encourages research participants to engage with the research on their own terms and get out of it whatever they wish. It is hoped that the design of the research will benefit the participants by way of their mere participation.

My research will be guided by social sciences and specifically educational research guidelines, as indicated in Section D of this form.

The results will be used as part of my Masters Dissertation and will hopefully form the foundation of forthcoming intervention studies developed to assist teachers with teaching skills of estimation in their classrooms. If given the opportunity, I would like to publish a summary of my findings in an academic journal.

b) Description of participants and how you will obtain informed consent to take part in the research.

i. Description of participants and your criteria for inclusion/ exclusion.

My research will involve interaction with a maximum of eight teachers who teach Grade 10 Maths or Maths Literacy in South African schools (and who may also teach other subjects and grades). These teachers will be representative of four types of schools: one Quintile 5 school, one Quintile 1 school, one 'rich' private school, and one low-cost private school.

The South African schooling system is diverse; state schools are divided according to Quintiles (with Quintile 5 schools being state and private funded; and Quintile 1 schools being funded only by the state and requiring no fee payments by parents), and private schools ranging in cost. Finding participants to represent the extremes of both the state and private education sector, I hope to develop a full understanding of the diversity in experiences and perceptions across the field.

ii. Your method(s) of recruitment.

I have worked closely with a three of the four schools which will be targeted for this study. The fourth school is a suggestion made by an ex-colleague who is interested in being part of the study. She has met and developed working relationships with staff members at this school through shared professional development and administrative workshops and hopes that these staff members would also be interested in participating in the study.

iii. Your processes for obtaining informed consent from participants.

The school's principals will be contacted about their willingness to participate in my study. I will enquire about their school's research and consent procedures for involving the teachers in research while teaching. The following are the minimum steps I propose, which will be adjusted to account for any school-specific requirements.

1. Apply for consent to research in schools through the Department of Basic Education and the Department of Education for KwaZulu-Natal.
2. Contact school principals (via email? With the request to principals letter)to ask for their consent for teaches to be involved in the research study. They should feel comfortable opting out of the research process if they do not feel comfortable.
3. The school can suggest teachers for the study, or they could put me in direct contact with the Maths / Maths Lit department to ask for participants in the study. If , so this should also be included in your letter to the principal – and what you expect the school to do in terms of recruitment and how your will access potential participants (or perhaps they will appoint a liaison person – maybe the Head of maths?)
4. I will ask for two teachers per school to participate. Each school or set of colleagues can decide who will take up the offer.
5. Each participant who agrees to participate must know that they can remove themselves from the study at any time, but is encouraged to make themselves available for at least the first 2 stages of the data collection as these will not be time consuming and do not influence classroom practice. You should set an end date for withdrawal (this will protect your data collection) The data collection stages are outlined at the end of Question 14. b) below.

Please attach separate supporting documents (in Word), if appropriate, for your research (English language versions only). Tick those you are submitting below. If appropriate supporting documents are not submitted, you will be asked to provide these separately, which may delay the ethical review process.

Recruitment and advertisement material (e.g. a poster, social media recruitment text, or brief invitation letter/ email).	☐
Information for participants to read (or hear) before they agree to take part (e.g. written information or, if applicable, an outline oral information script).	☒

A document to record informed consent. Templates for written consent forms and/ or oral information scripts (in case of an oral consent process) are available from the CUREC website.	☒
Questions to be asked of participants (e.g. interview questions, or a preliminary scope of questions, or a sample questionnaire).	☒
(If applicable) debriefing document after participants have taken part.	☐

Please add any further details. If you feel the above approaches are not appropriate for your study, provide details on how you will obtain consent from participants. Please answer question 15 if you cannot obtain informed consent.

The data collection phase of the study will occur in four stages, as outlined below:

1. Interviews (on Teams) per school. Each will be attended by the two participants of each school and myself. Details about privacy, confidentiality, and any other pertinent information will be discussed in this meeting too. The participants will be able to ask any questions or make any clarifications they need to in this virtual space.
2. Group meeting (on Teams) for all participants. This will be designed for information giving (by myself about the design of the study going forward) and sharing (by the participants about how they envisage the study may proceed).
3. Whatsapp group (which is automatically end-to-end encrypted) for all participants as a means of communication about practices in their classrooms, thoughts as the study develops, and whatever else they had decided in the Group Meeting (step 2).
4. Focus group meeting (on Teams) including up to four participants from the main study who will be asked to discuss their experiences and any changes they have made to their teaching praxis as a result of the study.

15. If you cannot obtain informed consent from participants according to CUREC guidelines and good practice in your discipline, please give a brief explanation and justification of this decision below.

16. What are the ethical issues connected with your research and what steps have you taken to address them? Please do not answer 'none'. We need to see that you have identified potential ethical issues with respect to your research and have taken steps to address them. If applicable, please address:

a) Participant burdens and/or risks

All participants in the study will be active teachers who already have a heavy workload. This study is therefore mindful of time pressures and, certain burdens and/or risks have been identified and will be addressed in the following ways:

- All information needed for the initial part of the study (Steps 1 and 2) will be captured during online meetings and will therefore not require teachers' extra time, for example filling in survey forms.

- Teachers will be contacted about their preferred online meeting platform and how best to make the virtual meetings as pain-free as possible. It is expected that some of the participants have practice using online meeting platforms but may not have used Teams before. Others may not have met online before and may therefore require some coaching before they feel comfortable speaking on this platform. It is hoped that Microsoft Teams will be used (for privacy reasons) and thus I may need to purchase data for the downloading and streaming on Teams for the meetings, and for the ongoing Whatsapp group. It is very important for me that those who have not used online meeting platforms do not feel anxious or overwhelmed by the need to meet online (with myself and later with participants from other schools) and therefore hope that those who offer to be part of the study will be willing to navigate this new technology. However, I will also convey my willingness and ability to help them feel comfortable in this online space as much as possible; they should know that all online meeting spaces are safe spaces where there is no class / experience which should make some participants feel inferior to others.
- Because of the diversity of participants, English will be used as the language of communication, but this may not be the mother-tongue of all participants. One of the participants who has informally agreed to be part of the study (as we were colleagues before I came to study at the University of Oxford) can translate any parts of speech that other isiZulu native speakers may struggle to say confidently in English.
- As a young, White, middle-class researcher from a prestigious institution, I need to act carefully when handling all parts of the data collection. It is imperative that this study does not become about researching "the other". Instead, if all participants buy into the idea of working together, we may be able to make a carefully situated social space that allows for beneficial interaction between diverse people instead of emphasising why these differences could separate our experiences.

It is difficult to predict the direction that any interview may take, and as a participatory research study, both the researcher and the participants should feel comfortable to either continue speaking or cut short conversation about a particular topic. I need to remind myself and the participants that this research will have the participants' best interests at heart and will therefore act in ways to protect them from any emotional (or other) harm which they could be exposed to as part of the study.

My last consideration is to impress upon the participants that this research provides a space for classroom praxis (theory and practice) changes but does not require participants to change. They are more than welcome to be part of the study and continue just as they were; this does not diminish their contribution to the study.

b) Your own physical and psychological safety as a researcher or of fieldworkers you may employ (see the University's and Social Science Division's Safety in Fieldwork guidance)

My physical and psychological safety should not be at risk during this research process. I also aim to be in constant contact with my supervisor and will speak to her about any concerns I have for the research as

it progresses. This stands for concerns relating to my safety, that of my participants, and the general direction of the study.

c) Data protection/ confidentiality (also see Section 18).

All schools will remain anonymous but some of their demographic statistics may make them easier to identify. Teachers can choose whether they would prefer their names to be used (as a form of giving them a voice) or whether to be referred to using pseudonyms. These names can be changed at any stage the data collection and processing and will be used throughout the transcription process in order to maintain a constant degree of anonymity.

Microsoft Teams will be used to conduct and record the interviews, group meeting, and focus group as this provides sufficient data protection. Whatsapp's end-to-end encryption will enable data shared in the group chat to be kept confidential. All data collected and processed will be uploaded to the OneDrive associated with my university account and protected by a password.

Participants will be asked to participate in all virtual meetings in private to secure their confidentiality, however it is difficult to guarantee the privacy of all participants due to unexpected events (like if another participant has a family member walk into the room while on a call). These privacy issues will be discussed with the participants as part of the informed consent practice and a decision can be made with the participants concerned if there are any possible breaches of privacy. You should mention here what will be recorded and in what form

All quotes and input which I would like to use for the Focus Group and later in the presentation of my dissertation will be given to the participants to check whether they feel comfortable with their contributions being shared. This will serve as a form of member checking of data before it is used in the public domain. You should mention here what will be recorded and in what form

Any other ethical issues. For more guidance on ethical issues, please see http://researchsupport.admin.ox.ac.uk/governance/ethics/resources

17. Will your research involve discussing sensitive issues? This could be information relating to race or ethnic origin, political opinions, religious beliefs, physical/ mental health, trade union membership, sexual life or criminal activities. If you answered 'Yes', make sure you include some supporting information (as directed in **section 14b** above, showing the range of questions covering these issues.	Yes ☐	No ☒

Please provide further details:

Management and handling of personal and other research data
All information provided by participants is considered research data for the purpose of this form. Any research data from which participants can be identified is known as personal data; any personal data which is sensitive is considered special category data. Management of personal data, either directly or via a third party, must comply with the requirements of the General Data Protection Regulation (GDPR) and the Data Protection Act 2018, as set out in the University's Guidance on Data Protection and Research. In answering the questions below, please also consider the points raised in the Data Protection Checklist and whether, for higher-risk data processing, a separate Data Protection Impact Assessment (DPIA) may also be required for the research. Advice on research data management and security is available from Research Data Oxford and your local IT department. Advice on data protection is available from information.compliance@admin.ox.ac.uk. a) Please mark 'X' against the data you will collect for your research

Consent records (written consent forms, audio-recorded consent, assent forms (for research involving minors) including participant name)	☒
Online consent (may be anonymous)	☐
Opt-out forms	☐
Contact details for research purposes only (destroyed when no longer needed for this research)	☐
Contact details kept for future studies	☒
Audio recordings (preferably using PIN-protected audio recorder and stored on device's hard drive)	☐
Video recordings why do you need to video record? audio would be sufficient for the interviews and focus groups?	☒

Transcript of audio/ video recordings	☒
Photographs	☒
Task results (e.g. paper/ online tasks, diary completion)	☐
Questionnaire answers	☐
Field notes is it possible you may make your own notes about situations? Even though your field is virtual some aspects of the field are still relevent?	☐
Other (please specify below)	☐
Whatsapp text messages	

b) For each of the types of data selected above, state how this will be physically transferred from where it is collected to a local secure storage site (and backed up as necessary). This includes paper records and data captured electronically.

All data received for this study will be placed into a password protected OneDrive folder associated with my university account.

Consent forms: These will be scanned or photographed and emailed to my university email address and uploaded into the password-controlled data folder on my OneDrive account.

Contact details: These will be saved directly onto an Excel sheet in the protected folder.

Video recordings: These will be recorded on the Microsoft Teams platform and then downloaded into the protected folder. Once downloaded, the original recordings will be deleted from the Teams platform. If some participants cannot connect via Teams, I will Whatsapp call them and record the conversation via a voice recording software. Each of these recordings will be downloaded into the protected folder and deleted off the recording device.

Transcripts: Transcripts will be created by Teams and downloaded into the protected folder. These will be checked by myself and then the original video or audio files will be deleted.

Photographs: Photographs shared on the Whatsapp group will be downloaded directly into the OneDrive folder and not onto my cellphone.

Whatsapp text messages: These messages will automatically be backed up onto the Whatsapp server. They will not be deleted until this part of the data collection and the Focus Group meeting has been completed. Once the group becomes inactive, it will be deleted and all messages will be saved in the OneDrive folder. Participants in this group will be reminded in the informed consent process that they are free to remove themselves from the Whatsapp group if they feel uncomfortable in this virtual space.

c) How and where will each type of data be stored during the research (until the end of all participant involvement)? Describe the arrangements for ensuring confidentiality, i.e. location of storage (e.g. Nexus 365 OneDrive for Business, SharePoint), security arrangements and de-identification of such data. Do not store unencrypted data in freely available cloud services or unprotected USB drives.

For all of the above-mentioned data, copies will be stored in a password protected OneDrive folder on my password-protected computer. This folder is also available online but will require two different passwords (one to enter OneDrive and another for the folder) and can therefore be regarded as secure.

d) Will you use a unique participant number on research data instead of a participant name? If yes, state whether or not you will retain a list of participant names against numbers (i.e. pseudonymisation via a linkage list). Where will the list be stored, and when will it be destroyed?

As previously mentioned, if the participants want their names to be used, they can ask for this. They may change their minds about this decision at any stage before the dissertation is printed. For all others, pseudonyms will be used (for example, Teacher A, Teacher B). These pseudonyms will be documented alongside a list of participants' first names in a separate Excel sheet in the protected OneDrive folder which will be destroyed once the dissertation has been written.

e) Who will have access to the research data?

As the researcher, I will have access to the research data. If I would like to use any quotes or data shared in the WhatsApp group in the Focus Group meeting, I will first get permission from the participants who contributed those pieces of data.

In the informed consent process, I will also alert participants to the possibility that their quotes may be used in the presentation of the data in the dissertation. I will therefore create a document of all of the pieces of direct data from participants that I wish to use in the presentation of the results and check with

each participant that they are still comfortable with these pieces of data being used. If not, these pieces of data may be anonymised or removed completely (according to the wishes of the participants.

At the end of the research study, I hope to create a resource pack for each participating school entitled, Estimation for the Maths Masses. This will include teaching prompts for High School teachers for how and when to include estimation in the classroom, based on inputs from my research. When creating the "Estimation for the Maths Masses", I may use participants' contributions to the study and will therefore reference them as active contributors. Again, if they choose for their work to not be included, they may do so at any stage. This is particularly important as I would like to make this an Open-Source contribution to the Maths teaching sphere and could therefore be seen by many different organisations and people.

f) If research data is to be shared with another organisation, how will it be transferred/ disclosed securely?

g) When and how will identifiable data (including audio/ video recordings & photos) be destroyed or deleted?

Note: Records of consent should be retained for a minimum of three years after publication or public release. Some funders may require longer periods. If you wish to retain contact details in order to re-approach participants about future studies, you must detail this in information provided to them and obtain specific consent for this.

There is a possibility that I could continue this research as a piloted intervention study which trains teachers to implement new estimation teaching strategies in their classrooms. I will need to retain this study's research participants' contact details for this as I would like to work with teachers with whom I already have a good rapport and who represent a wide range of socio-economic levels in society. I will confirm that this is my intention when gathering the data and consent at the early stages of this study.

Records of consent will be kept in the protected folder and on a protected personal Google Drive folder once this research study has been completed (as the OneDrive folder may be automatically be deleted when I leave this university). If data does need to be copied to a Google Drive folder, it will need to be done with care to delete any other versions of the data stored as temporary files.

All other records of data will be deleted once the dissertation has been marked. Data filed will need to be deleted from any temporary holding places too (like a "Recycling Bin" on a computer or online).

h) Please confirm that you will store other research data safely for at least 3 years after final publication or public release and adhere to any additional	Yes ☒	No ☐

research funder policies. For more information about the University policies, please see the University's web pages on research data management.		
If 'Yes', please give details of who will store the data and on storage format, location and security. Note that open science is encouraged. If 'No', please provide further details below.		
Any extra data which must be kept (as per advice from my supervisor) will be kept in the protected folder and on a protected personal Google Drive folder once this research study has been completed (as the OneDrive folder may be automatically be deleted when I leave this university). If data does need to be copied to a Google Drive folder, it will need to be done with care to delete any other versions of the data stored as temporary files.		
i) Does your research involve the use of secondary (i.e. previously collected) data? Common sources of secondary data include censuses, information collected by government departments, organisational records and data that was originally collected for other research purposes. (If "No", please go to question 19.)	Yes ☐	No ☒
j) Do you have data access agreements for the use of this secondary data? (If so, please attach these.)	Yes ☐	No ☐
k) Is your use of this secondary data compatible with what the data subjects/ participants agreed that their data should be used for?	Yes ☐	No ☐
l) Could this data be linked back to an individual or individuals? If yes, address how securely any personally identifiable data will be transferred to you, and where and for how long it will be stored during or after the research. Who will have access to it?	Yes ☐	No ☐

19. Publication and dissemination of research data and outputs. How will you disseminate and feedback project outcomes at the end of the research?
I will create a useful guide for teachers, "Estimation for the maths masses", which will be given to all participants and will be created as an Open-Source document to be shared freely. Participants can do with this as they wish. I will also offer to send a copy of my dissertation to each participating school once it has been completed.

SECTION C: Methods and procedures to be used	
1. Please indicate the methods to be used:	**Mark 'X'**
a) Analysis of existing records	☐
b) Snowball sampling (recruiting through contacts of existing participants)	☒
c) Use of casual or local workers e.g. interpreters	☐
d) Participant observation	☐
e) Covert observation	☐
f) Observation of specific organisational practices	☐
g) Participant completes questionnaire in hard copy	☐
h) Participant completes online questionnaire or other online task	☐
i) Using social media	☒
j) Participant performs paper and pencil task	☐
k) Participant performs verbal or aural task (e.g. for linguistic study)	☐
l) Focus group	☒
m) Interview	☒
n) Audio recording of participant ?? (you will generally need specific consent from participants for this)	☐
o) Video recording of participant (you will generally need specific consent from participants for this)	☒
p) Photography of participant (you will generally need specific consent from participants for this)	☐
q) Others (please specify below)	☒
Please ensure you have addressed any potential ethical issues related to these methods in question B14 and in your Participant Information Sheet.	
The study will include capturing pictures of notes sent by participants to a shared WhatsApp group. These pictures may be of hand-written notes or something similar, but will not include pictures of people.	

SECTION D: Professional guidelines and training		
In this section, please mark 'X' against at least one of the following professional guidelines you aim to adhere to. You should use the principles listed in your chosen guideline(s) in conducting your own research. This is not an exhaustive list.		
Research specialism/ methodology	**Association and guidance document**	**Please mark 'X'**
Anthropology	Association of Social Anthropologists of the UK and Commonwealth	☐
Computer Sciences	ACM Code of Ethics and Professional Conduct	☐
Criminology	British Society of Criminology	☐
Education	British Educational Research Association Ethical Guidelines for Educational Research	☒
Geography	Association of American Geographers Statement on Professional Ethics	☐
History	Oral History Society of the UK Ethical Guidelines	☐
Internet-based Research	British Psychological Society: Conducting Research on the Internet Association of Internet Researchers Ethics Guide ACM Code of Ethics and Professional Conduct Association of Internet Researchers (AoIR) Also see CUREC's Best Practice Guidance on internet-based research	☐
Law (Socio-Legal)	Socio-Legal Studies Association: Statement of Principles of Ethical Research	☐
Management	Academy of Management's Professional Code of Ethics	☐
Political Science	American Political Science Association (APSA) Guide to Professional Ethics in Political Science	☐
Politics	Political Studies Association. Guidelines for Good Professional Conduct	☐
Psychology	British Psychological Society Code of Ethics and Conduct	☐
Social Research	Social Research Association: Ethical Guidelines	☒
Sociology	The British Sociological Association: Statement of Ethical Practice	☐

Visual Research	ESRC National Centre for Research Methods Review Paper: Visual Ethics: Ethical Issues in Visual Research	☐
Other professional guidelines. Please specify the other guidelines used here:		☐

Please indicate what training in research ethics (or research methodology) the researchers involved with this study have received, e.g. the title of the course and date completed (online training available at http://researchsupport.admin.ox.ac.uk/support/training/ethics), or discussions between researchers and supervisors, if applicable.

I have received research and ethics training throughout the Foundations of Educational Research module as part of my MSc course. Many classroom discussions have included those surrounding morals and ethics, and one week was specifically dedicated to this topic.

I have also had regular meetings with my supervisor, and we have discussed various ethical considerations of the study.

SECTION E: Signatures or email endorsements

The SSH IDREC Secretariat accepts either option below. If you have a DREC, check which option it prefers.

Option 1: Email confirmation from a University of Oxford email address can be accepted. Separate emails should come from each of the relevant signatories as outlined below, indicating acceptance of the relevant responsibilities. Pasted images of signatures cannot be accepted.

Option 2: Handwritten signatures. Please scan them and the rest of the checklist pages to create a single PDF document and email to us.

Please ensure this checklist is signed by:

For staff research:

1. Principal Investigator
2. Head of Department (or nominee)

For student research:

1. Principal Investigator (project supervisor)
2. Head of Department (or nominee)
3. Student researcher

Principal Investigator signature/ supervisor signature (if student research)

I understand my responsibilities as Principal Investigator as outlined in the CUREC glossary and guidance on the CUREC website. I declare that the answers above accurately describe the research as presently designed, and that a new checklist will be submitted should the research design change in a way which would alter any of the above responses so as to require completion of CUREC 2 (involving full scrutiny by an IDREC). I will inform the relevant IDREC if I cease to be the principal investigator on this project and supply the name and contact details of my successor if appropriate.

Signature (or email endorsement using the above declaration): [redacted]

Print name: [redacted] **Date**: _____

Departmental endorsement signature

I have read the research project application named above. On the basis of the information available to me, I:

- consider the PI and student researcher (if applicable) to be aware of their ethical responsibilities in regard to the ethical issues associated with this research;
- am satisfied that: the proposed project design and scientific methodology are sound; the project has been subject to appropriate peer review; and is likely to contribute to existing knowledge and/or to the education and training of the researcher(s) and that it is in the public interest.

Signed by Head of Department or nominee (example nominees for student research include the Director of Graduate Studies/ Director of Undergraduate Studies):

Signature (or email endorsement using the above declaration): _____

Print name: [redacted] **Date**: _____

Student signature (if student research)

I understand the questions and answers that have been entered above describing the research, and I will ensure that my practice in this research complies with these answers, subject to any modifications made by the principal investigator properly authorised by the CUREC system.

Signature by student (or email endorsement using the above declaration): [redacted]

Print name: [redacted] **Date**: _12 Jan 2021_____

CUREC Application approval email

Sunday, July 25, 2021 at 18:54:39 British Summer Time

Subject: CUREC Approval [redacted]
Date: Wednesday, 3 February 2021 at 14:37:30 Greenwich Mean Time
From: [redacted]
To: [redacted]
CC: Student CUREC
Attachments: Outlook-baw3oixa.png

Dear [redacted]

'South African high school teachers' perceptions about the usefulness of estimation in Mathematics and Mathematics Literacy classrooms and beyond.'

The above application has been considered on behalf of the Departmental Research Ethics Committee (DREC) in accordance with the procedures laid down by the University for ethical approval of all research involving human participants.

I am pleased to inform you that, on the basis of the information provided to DREC, the proposed research has been judged as meeting appropriate ethical standards, and accordingly, approval has been granted.

Please continue to follow all current guidance issued by CUREC during the pandemic, notably COVID-19: CUREC guidance on research involving human participants, https://researchsupport.admin.ox.ac.uk/governance/ethics/coronavirus

If relevant please also check the CUREC website for their best practice research guides, https://researchsupport.admin.ox.ac.uk/governance/ethics/resources/bpg

Stay safe,

Yours sincerely,

[redacted]

[redacted]
Researcher in Child Development and Learning
Departmental Research Ethics Committee Member
15 Norham Gardens, Oxford OX2 6PY
Email: pinar.kolancali@education.ox.ac.uk

Data protection assessment form



Privacy by Design

Reference number:

Project Name/ Processing Activity/Business Practice:

South African high school teachers' perceptions about the usefulness of estimation in Mathematics and Mathematics Literacy classrooms and beyond

Screening assessment for Data Protection Impact Assessment

Description of personal data to be processed and the purpose of the processing:
Video and audio recording of Microsoft Teams interviews for my MSc Dissertation project.

Part 1

Certain processing automatically requires a DPIA because it is considered to be high risk to the individuals whose data is involved. This processing has been identified by the ICO.

Complete the checklist below to help you identify if the processing you are intending to carry out requires a DPIA **before** you begin processing.

Examples are given to help you identify the type of processing in each category, the lists of examples are not exhaustive.

Answer Yes or No for each category of processing operation.

Are you intending to carry out processing which involves the following:	**Yes**	**No**

1. new technologies? examples: • machine learning and deep learning • connected and autonomous vehicles • intelligent transport systems • smart technologies (including wearables) • market research involving neuro-measurement (emotional response analysis and brain activity)		X
2. existing technology used in novel way?		X
3. AI?		X
4. decisions about an individual's access to a product, service or opportunity based in any way on automated decision- making (including profiling)? examples: • credit checks • mortgage or insurance applications • pre-check processes related to contracts (smartphones)		X
5. decisions about an individual's access to a product, service or opportunity which involves special category data *?		X
6. any profiling of individuals on a large scale +? examples; • data processed by Smart meters or IoT applications • hardware/software offering lifestyle/fitness monitoring • social media networks • application of AI to an existing process		X
7. any processing of biometric data? examples: • facial recognition systems • workplace access systems/identity verification • access control/identity verification for hardware/applications (including voice recognition/fingerprint/facial recognition)		X
8. any processing of genetic data? examples: • medical diagnosis • DNA testing • medical research		X
9. combining, comparing or matching personal data from multiple sources?		X

examples: • fraud prevention • direct marketing • monitoring personal use/uptake of statutory services or benefits • federated identity assurance services		
10. personal data not obtained direct from the individuals and the right of those individuals to be informed would be impossible to achieve or involve disproportionate effort? examples: • list brokering • direct marketing • online tracking by third parties • online advertising • data aggregation/data aggregation platforms • re-use of publically available data		X
11. tracking an individual's geolocation or behaviour online or otherwise? examples: • social networks, software applications • hardware/software offering lifestyle/health/fitness monitoring • IoT devices, applications and platforms • Online advertising • web and cross-device tracking • data aggregation/data aggregation platforms • eye tracking • data processing at the workplace • data processing in the context of home and remote working • processing location data of employees • loyalty schemes • tracing services (tele- appending, tele-matching) • wealth profiling- identification of high net-worth individuals for the purposes of direct marketing		X
12. personal data of children or other vulnerable ** individuals for marketing purposes? examples: • connected toys • social networks		X

13. personal data of children or other vulnerable individuals for profiling? examples: • connected toys • social networks		X
14. personal data of children or other vulnerable individuals for automated decision- making? examples: • connected toys • social networks		X
15. personal data of children with the intention of offering online services directly to children? examples: • connected toys • social networks		X
16. a risk that a personal data breach could jeopardise the physical health or safety of individuals? examples: • whistleblowing procedures • complaint procedures • social care records • criminal convictions and offences		X
17. systematic and extensive profiling with significant effects***? examples: • credit checks • mortgage/ loan applications • fraud prevention • insurance underwriting • application of AI		X
18. special category data on a large scale? examples: • political parties membership data • trade union membership data • health records processed by hospitals/health clinics/gym chains • social care records • research (including medical research) • fraud prevention • application of AI		X

• dating websites/applications		
19. systematic++ monitoring of publicly accessible places on a large scale? examples: • automatic number plate recognition • intelligent transport systems • traffic management systems involving monitoring of vehicle/driver behaviour • Wi-Fi/Bluetooth/RFID tracking • audio/video surveillance of public areas • application of AI		X

* *Special category* data is information about an individual's:

race, ethnic origin, politics, religion, trade union membership, genetics, biometrics (where used for ID purposes), health, sex life or sexual orientation.

Criminal offence data is not included in the definition of special category data but there are separate safeguards for personal data relating to criminal convictions and offences or related security measures.

+ *large scale* means considering:

the number of individuals concerned, the volume of data, the variety of data, the duration of processing and the geographical extent of processing

** *vulnerable* means where there is a power imbalance between individuals and the controller so that the individuals are unable to easily consent or object to the processing of their data or exercise their rights e.g. children, employees, segments of the population requiring special protection such as the mentally ill. asylum seekers, the elderly, patients and individuals in any relationship where an imbalance in the relationship is identified.

****significant effect* means something that has noticeable impact on an individual that can affect their circumstances, behaviour or choices in a significant way

++ *systematic* means according to a system, is pre-arranged, organised, methodical, occurs as a result of a general plan for data collection or is carried out as part of a strategy

If you have answered Yes to <u>any</u> of the above questions you are required to carry out a DPIA before you begin processing. Complete Part 3 and then please refer to our DPIA template.

<u>Part 2</u>

This part applies where your answers to <u>all</u> the questions in Part 1 are No.

The GDPR states that a DPIA must be carried out where processing **is likely to result in a high risk** to the interests of individuals. This means <u>any</u> interests, it is not limited to privacy rights. You must consider the **potential for harm**, which includes significant economic or social disadvantage, to individuals or society at large, arising from the processing. Assessing the **level of risk** means considering **both the likelihood and severity of any potential harm to individuals**.

"High risk" means:

- harm is more likely or
- the potential harm is more severe or
- both

Complete the checklist below to help you identify if the processing you are intending to carry out is likely to result in a high risk to the interests of individuals.

Are you intending to carry out processing which involves the following?	**Yes**	**Likelihood of potential harm to individuals (unlikely, likely, highly likely)**	**Severity of potential harm to individuals (minor, moderate, severe)**	**No**
Evaluation				X

Scoring				X
automated decision- making which has legal or similar significant effect				X
Systematic monitoring				X
sensitive data				X
data of a highly personal nature				X
data processed on a large scale				X
matching datasets				X
combining datasets				X
data concerning vulnerable individuals				X
innovative use of new technological or organisational solutions				X
applying new technological or organisational solutions				X
preventing individuals from exercising a right				X
preventing individuals from using a service				X
preventing individuals from using a contract				X

A **Yes** answer where the likelihood of potential harm is **likely** or **highly likely** and/or the potential harm is **moderate** or **severe** indicates the processing is likely to result in a high risk to the rights and freedoms of individuals and a DPIA is needed. **Complete Part 3 and then please refer to our DPIA template.**

Part 3

If your screening assessment indicates no DPIA is required, you must keep this under **review**. If there is any proposed change to the processing you are intending to carry out

or after processing has begun, you must carry out another screening assessment before the change to processing takes place.

Retain your screening assessment documentation in accordance with your retention schedule.

To help you ensure the processing you are going to carry out has data privacy included in the project plan **carry out a Privacy by Design Assessment** of your processing. **Please refer to our DPA template.**

Screening Assessment Outcome:	No DPIA form required
Reasons:	Answered 'no' to all questions
Carried out by:	[redacted]
Role:	MSc student
Date:	12 03 2021
Approved by:	
Role:	
Date:	
First review Date:	

Email to potential participants

Department of Education
University of Oxford
15 Norham Gardens
Oxford
Ox2 6DQ
+44 1865 274024

http://www.education.ox.ac.uk

[redacted]

06 Feb 2021

Invitation to participate in research study: "South African high school teachers' perceptions about the usefulness of estimation in Mathematics and Mathematics Literacy classrooms and beyond."

Dear [redacted],

I am writing to you to enquire about conducting some research at your school. I am studying my Masters in Education at the University of Oxford in the United Kingdom under my research supervisor, [redacted]. In my research study, "South African high school teachers' perceptions about the usefulness of estimation in Mathematics and Mathematics Literacy classrooms and beyond", I aim to understand how Grade 10 Mathematics and Mathematics Literacy teachers perceive and teach the skill of estimation in their classrooms.

The research will take place with eight teachers in four different schools in the greater eThekwini area of KwaZulu-Natal. Each participant will be involved with one interview, one group meeting, and added to a WhatsApp group for the sharing of study-based information regarding the study. Certain participants may be invited to participate in a Focus Group towards the end of the research process. All communication will happen virtually due to the Covid-19 pandemic and participants will be guided through the use of Microsoft Teams as the preferred online meeting platform. The study will progress over a maximum of three months, but participants can expect to be involved in online meetings for a maximum of three hours and part of the WhatsApp group for four teaching weeks.

I am writing to request that you allow two of your Mathematics / Mathematical Literacy teachers who are currently teaching Grade 10 (but who may be involved with teaching other subjects and grades) to participate in this study. By participating in this study, you would allow staff members to contribute to

research which will be used to better understand teaching praxis (theory and practice) in South African classrooms.

The University of Oxford has strict ethical procedures which must be followed by all researchers associated with their name. They take data protection, privacy, confidentiality, and ethical standards very seriously and my research proposal has therefore already been ethically approved through their Central University Research Ethics Committee. In line with this, you, as the principal of XXX, is my first port of call. If you agree to allow two staff members to participate in the study, I will then contact them. They will be given information about the study's design and progression, and how they can be involved. They will need to sign a consent form which details all information of theirs which may be used, and it is made clear that they may choose to remove themselves from the study at any point. As the researcher, I make myself available to answer any questions or respond to any concerns that any of the participants may have.

The schools' names will not be used in the study and all participants have an option of their first names being used or being completely anonymised. No sensitive data will be collected or discussed. The data collected for the study will be kept strictly confidential, available only to my supervisor and myself and not used beyond each participants' wills as indicated on their consent form. All recordings will be destroyed at the end of the research period and kept in password protected folders until them.

If you would like your school to participate in this study, or you would like more information, please do not hesitate to contact me by replying to this email. I have attached another document which you should complete if you grant approval to participate in this study. Please complete it and return to me at this email address.

Thank you for your time and attention. I look forward to hearing from you at your soonest convenience.

Yours sincerely,

[redacted]

Department of Education
University of Oxford
15 Norham Gardens
Oxford
Ox2 6DQ
+44 1865 274024

http://www.education.ox.ac.uk

[redacted]

PARTICIPANT INFORMATION SHEET

Central University Research Ethics Committee (CUREC) Approval Reference: [redacted]

South African high school teachers' perceptions of the usefulness of estimation in Mathematics and Mathematics Literacy classrooms and beyond.

Purpose of the study

This research study aims to understand teachers' perceptions about the usefulness of the skill of estimation within and outside Mathematics and Mathematical Literacy classrooms. I hope to use this information to collaboratively create more awareness about the uses of the skill of estimation while investigating how it is currently used by participating teachers in their classrooms.

I am interested in the topic of estimation because it is a skill that people use every day, even when they do not think that they are. I am hoping to understand how much this skill is taught and used in Mathematics and Mathematical Literacy classrooms in a range of schools in South Africa. As a group of participants, we will also address how estimation is used outside of the classroom and whether we, as teachers, are preparing for these uses.

Participants

I have selected four types of schools in the country to represent the variety South Africa has to offer. I have contacted your school principal to ask for permission to speak with two members of staff who teach Grade 10 Mathematics or Mathematical Literacy (and who may teach other subjects). You are being asked to participate in this study because *(your principal has suggested that you may have some interest in this field of work / we have already worked together and thus our professional rapport may make your participation in this study less taxing on your work)*.

Consent

You do not have to participate in this study and can indicate your consent, confidentiality, and privacy agreements on the attached consent form. If you choose to participate and later wish to withdraw during the study, it is asked that you inform the researcher as soon as possible, at which stage you can decide how any data you have given should be used or discarded of. It is requested that if you wish to withdraw, you do so before the 30th of June 2021 and we will make arrangements for what data of yours you would like removed from the study.

Research process

The research process will follow four steps, detailed below. All participants are asked to participate in the first three steps, and up to four members of the participating group will be asked to participate in the fourth step. These four members will be chosen according to how they have participated in the study – I hope to further my research with participants who have made interesting or outlying contributions to the study. These participants may not have contributed the most to any part of the study, but if their case studies will provide more information for answering my overarching research questions, they will be asked to pursue continued participation in the study. These participants are by no means superior to any others.

For privacy and data protection, the University of Oxford advises the use of the Microsoft Teams app for video and audio calls. It is for this reason that all interviews and meetings will be held on these platforms for the duration of the research study. Step 3 requires the use of a WhatsApp group chat which is automatically end-to-end encrypted. If you foresee any issues with using either of these two applications or would like assistance downloading them or buying data for their usage, please make a note *at the end of the consent form* so that the researcher can contact you directly about this.

1. Interviews with each school's participants.
   These interviews will involve yourself and another Grade 10 Maths / Maths Lit teacher from your school, and the researcher. This semi-structured interview will start by offering any clarifications or answering any questions that may have been asked to the researcher up until this time. Thereafter, the researcher will use an interview schedule to guide her questions to you about your experiences of teaching estimation and how important you think it is as a skill in the classroom and outside it.
2. Group meeting
   This group meeting will include all participating members of this study. It will start with an introduction of the aims of the meeting and then a chance for all participants to introduce themselves. I will then make some proposals about different ways for the study to proceed, and get feedback and other suggestions from participants. As a group, we will then decide on what works best for everyone, taking into consideration the time and effort needed by each participant, how best to contribute and share information, and what each participant wants to get out of their participation.
3. WhatsApp group chat

This will be set up as a platform for the sharing of information by all participants. It is hoped that participants will feel comfortable sharing information about how they have used estimation in their classrooms or when they have found themselves using the skill. This will run for 4 teaching weeks and it is hoped that all participants will willingly engage with others on this platform. If participants are not comfortable participating in this WhatsApp group they are asked to keep a journal of how and when they use estimation in their classroom (or any other ideas stemming from the Group Meeting) and submit this to me. Their participation in this part of the study is optional but preferred.

4. Focus group

Up to four participants will be asked to participate in this final data collection stage of the research study. It will be an opportunity for participants to ask each other questions and, together with the research, unpack how and why certain changes or non-changes have occurred in participants perceptions and beliefs throughout the study. The participants may be asked detailed questions about their own experiences of the study.

Risks of involvement

I cannot foresee any risks to participants and researchers while participating in this research. However, I must acknowledge that interviews and group meetings that involve dynamic discussions and could create uneasy situations. It is because of this that as the researcher, I am committed to developing trusting relationships with each of the participants during the first interviews, introducing the collaborative working environment envisaged in the Group meeting, and closely monitoring the information shared in the WhatsApp group. If you do, at any stage, feel uncomfortable, please do not hesitate to contact me and I will address any issues as soon as possible.

Research outputs and what you will get out of participating

The research will be used as part of the data collection for my dissertation. This is one of the requirements for the completion of my MSc in Comparative and International Education at the University of Oxford in the UK. I may also use this data to write a submission for publication in an academic journal.

There is a possibility that I will further this research in the next three years by designing an implementing intervention strategies for improved teaching methodologies for estimation as part of a D.Phil and request consent to keep your contact details for the next three years in case this comes to fruition – this is also indicated on the attached consent form.

As a participant, there will be no direct or personal benefit for you by participating in this research but it is hoped that it is an enjoyable and interesting experience for you. Once the main research has been completed, I hope to create an Open-Source *Estimation for the Maths Masses* guide for how teachers can include more estimation examples in their classrooms, and this information will come from ideas that we, as a research group, have created or shared together. You will be sent a copy of this and you are more than welcome to share this resource with as many colleagues as you wish. You will be credited as contributors to this resource.

What happens to the data you have provided?

The data you provide during the study is called *research data*. Any data which makes you identifiable is called *personal data*.

Research data (including consent forms) will be stored securely for at least three years after the submission of the dissertation.

The researcher ([redacted]) and [redacted] supervisor ([redacted]) will have access to the *research data*.

The consent form allows you to decide how you want to be identified / anonymised in the research study and whether or not direct quotes from your participation can be included in the research outputs.

Personal data will be stored securely in a password-protected folder until the dissertation has been marked and will then be deleted.

The University of Oxford is classed as the data controller with respect to your *personal data*, and as such will determine how your personal data is used in the study. For further information about your rights with respect to your personal data, please see http://www.admin.ox.ac.uk/councilsec/compliance/gdpr/individualrights/.

Research publications

The University of Oxford commits to the dissemination of its research for the benefit of society ad the economy, and in support of this commitment, has established an online archive of research produced in its name. The archive includes theses (dissertations) submitted as part of a postgraduate degree at the university and available for researchers (in full-text format) to use freely.

Upon successful submission of the dissertation emanating from this research study, it may be stored both online and as a hard-copy in the university archives. If so, this research will be freely accessible.

This research may also be published in an academic journal and used as foundational research for further studies by the same researcher.

Ethical approval

This study has been reviewed and granted ethical approval by the University of Oxford Central University Research Ethics Committee ([redacted]). Furthermore, the KwaZulu-Natal Department of Education has been contacted requesting approval to conduct research in their schools.

Contact details

If you have any queries or concerns, or if you would like to learn more about the research design and process, please do not hesitate to contact the researcher, who will happily also put you in contact with her research supervisor if needed.

[redacted]

Phone number: [redacted] *(this number can be used for contact via WhatsApp too)*

Email: [redacted]

The research is supervised by [redacted], who can be contacted by email:

[redacted]

If you have any ethical concerns about this study, please contact the Chair of the Ethics Committee directly at student.curec@education.ox.ac.uk.

Thank you

Thanks for reading through all of this information and please do not hesitate to contact me about any queries and concerns. The attached consent form can be filled in and returned to me via email, after which we can begin the study.

Department of Education
University of Oxford
15 Norham Gardens
Oxford
Ox2 6DQ
+44 1865 274024

http://www.education.ox.ac.uk

[redacted]

PARTICIPANT CONSENT FORM

Central University Research Ethics Committee (CUREC) Approval Reference: [redacted]

South African high school teachers' perceptions of the usefulness of estimation in Mathematics and Mathematics Literacy classrooms and beyond.

I am researching teachers' perceptions about the usefulness of the skill of estimation within and outside Mathematics and Mathematical Literacy classrooms. I hope to use this information to collaboratively create more awareness about the uses of the skill of estimation while investigating how it is currently used by participating teachers in their classrooms.

By completing this form, you are agreeing to work as an active participant in this research study. Please read each statement carefully and initial those you consent to. You may withdraw your partial or full consent to each statement during the research process by contacting me in writing.

Please read this form in conjunction with the Information Sheet attached, which provides further information about the procedures to be taken in this study and how you may be involved.

1. I have read and understood the Information Sheet provided and have had the opportunity to consider the information give, ask any questions, and have all clarifications addressed satisfactorily. ☐

2. I understand that my participation is voluntary and that I may ask to withdraw from the study at any time, without needing to give reason, and without any adverse effects. ☐

3. I understand that research data collected in this study will be used for the creation of a dissertation. ☐

4. I understand that data collected may be used for publication in an academic journal. ☐

5. I consent for my contact details may be kept by the researcher for a period of three (3) years after the study is concluded, and that I may be contacted in this time in the case that further research on a similar topic is conducted by this researcher. ☐

6. I understand who will have access to personal data provided, how the data will be stored, and what will happen to this data after the study. ☐

7. I understand how to raise a concern or make a complaint. ☐

8. I consent to being audio recorded. ☐

9. I consent to being video recorded. ☐

10. I consent to my contributions in a WhatsApp group format being used for the purposes of this study (including texts, voice recordings, and photographs). ☐

11. I understand how audio / video / WhatsApp contributions will be used in this research. ☐

12. **a)** I consent for my first name to be used in data analysis and research outputs. ☐

OR

b) I consent for my name to be pseudonymised in data analysis and research outputs. ☐

OR

c) I do not wish to be directly quoted in any form. ☐

13. **I agree to participate in this study.** ☐

Please note any other issues of consent, confidentiality, and privacy below which you wish to address with the researcher. Please also indicate if you foresee any challenges downloading and using the WhatsApp and Microsoft Teams applications for use during the research study.

_____

_____

_____

_____

_____	_____	_____
Name of participant	Date (dd/mm/yyyy)	Signature of participant

Department of Education
University of Oxford
15 Norham Gardens
Oxford
Ox2 6DQ
+44 1865 274024

http://www.education.ox.ac.uk

[redacted]

PRINCIPAL PERMISSION FORM

Approval for participation in the research study entitled "South African high school teachers' perceptions about the usefulness of estimation in Mathematics and Mathematics Literacy classrooms and beyond."

I, _____ (name), give my full approval for my school, _____

_____ (school name) to participate in the aforementioned study conducted by [redacted]

[redacted] from the University of Oxford.

School details (including address and contact details):

_____

_____

_____

_____

Signed: _____ on _____ (date).

Research approval: KwaZulu-Natal Department of Education

KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

OFFICE OF THE HEAD OF DEPARTMENT

Private Bag X9137, PIETERMARITZBURG, 3200
Anton Lembede Building, 247 Burger Street, Pietermaritzburg, 3201
Tel: 033 3921063 / 033-3921051
Email: Phindile.duma@kzndoe.gov.za
Buyi.ntuli@kzndoe.gov.za

Enquiries: Phindile Duma/Buyi Ntuli **Ref.**:2/4/8/9004

[redacted]

Dear [redacted]

PERMISSION TO CONDUCT RESEARCH IN THE KZN DoE INSTITUTIONS

Your application to conduct research entitled: **"SOUTH AFRICAN HIGH SCHOOL TEACHERS' PERCEPTIONS OF USEFULNESS OF ESTIMATION IN MATHEMATICS AND MATHEMATICS LITERACY CLASSROOMS AND BEYOND"**, in the KwaZulu-Natal Department of Education Institutions has been approved. The conditions of the approval are as follows:

1. The researcher will make all the arrangements concerning the research and interviews.
2. The researcher must ensure that Educator and learning programmes are not interrupted.
3. Interviews are not conducted during the time of writing examinations in schools.
4. Learners, Educators, Schools and Institutions are not identifiable in any way from the results of the research.
5. A copy of this letter is submitted to District Managers, Principals and Heads of Institutions where the Intended research and interviews are to be conducted.
6. The period of investigation is limited to the period from 15 February 2021 to 15 December 2023.
7. Your research and interviews will be limited to the schools you have proposed and approved by the Head of Department. Please note that Principals, Educators, Departmental Officials and Learners are under no obligation to participate or assist you in your investigation.
8. Should you wish to extend the period of your survey at the school(s), please contact Miss Phindile Duma/Mrs Buyi Ntuli at the contact numbers above.
9. Upon completion of the research, a brief summary of the findings, recommendations or a full report/dissertation/thesis must be submitted to the research office of the Department. Please address it to The Office of the HOD, Private Bag X9137, Pietermaritzburg, 3200.
10. Please note that your research and interviews will be limited to schools and institutions in KwaZulu-Natal Department of Education.

UMLAZI DISTRICT **PINTOWN DISTRICT**

Dr. EV Nzama
Head of Department: Education
Date: 24 February 2021

Appendix F: Participant details

	Pseudonym	School	Interview	WhatsApp	Focus Group	Teaching subjects - present	Teaching Subjects - past	Teaching experience	Teaches or taught other subject(s)	Taught Primary School Maths	Worked in other industry	Taught abroad
1	Mandla	Peri-urban township	Yes	Yes	Yes	Maths FET	Maths FET/GET	Long Experience	Yes	No	Yes	No
2	Jennifer	Middle-class public	Yes	No	Yes	Maths FET/GET	Maths FET/GET	Long Experience	No	No	Yes	Yes
3	Tammy	Prestigious private girls'	Yes	Yes	Yes	Maths GET/Maths Lit	Maths FET/GET/Maths Lit	Long Experience	Yes	Yes	No	No
4	Marlise	Middle-class public	Yes	Yes	No	Maths Lit	Maths FET/GET/Maths Lit	Long Experience	Yes	Yes	No	No
5	Jacqui	Less prestigious private girls'	Yes – with Emma	No	No	Maths FET	Maths FET/GET/Maths Lit	Long Experience	No	Yes	Yes	No
6	Emma	Less prestigious private girls'	Yes – with Jacqui	No	No	Maths Lit	Maths GET/Maths Lit	Long Experience	Yes	No	No	No
7	Skye	Prestigious private girls'	Yes	Yes	No	Maths FET/GET/Maths Lit	Maths FET/GET/Maths Lit	Newly Qualified	No	No	No	No
8	Lindokuhle	Peri-urban township	No – text responses	No	No	Maths GET/Maths Lit	Maths GET/Maths Lit	Newly Qualified	Yes	No	Yes	No
9	Eva	Middle-class public	Yes	Yes	No	Maths GET/Maths Lit	Maths FET/GET/Maths Lit	Long Experience	Yes	Yes	No	No

Notes:

- Teaching subjects ('present' and 'past') only relate to mathematics subjects at different phases in High Schools
- Teaching experience: 'Newly Qualified': qualified in the last 3 years; Long Experience': everyone else

Appendix G: Data collection: intended and expected data collection strategies

	Semi-structured interviews	**WhatsApp group**	**Focus group**
Intention – what do I want to get out of this?	〉 Rich, deep data representative of teachers' perceptions of the skill of estimation in and outside their classrooms 〉 Data from a variety of schools – representing a large range of perspectives	〉 Examples of how estimation is used in classrooms – with details from both Mathematics and Mathematical Literacy classes 〉 Collaboration between teachers and use of others' resources to implement change in teaching practice	〉 Rich, deep data about how teachers' perceptions about estimation have changed over the course of the study 〉 Reflections on the use of the WhatsApp group
What I thought the process would look like	〉 Semi-structured interviews with 2 members of each school simultaneously, therefore being able to discuss and develop ideas together	〉 Participation by more of the participants 〉 Teachers' regular posting of pictures or worked examples from lessons at least once a week 〉 Teachers to design the group to suit their availability and needs 〉 My intermittent messages to the group about how I have used estimation in my daily life 〉 3 weeks in operation	〉 Online discussion with 3 participants, chosen for their eagerness to participate and their different teaching experiences – therefore bringing together possibly opposing perceptions and practices

What ended up happening	〉 6 semi-structured interviews with one teacher per school, except for: 〉 1 semi-structured interview with 2 teachers from the same school 〉 1 response to interview's guiding questions via WhatsApp 〉 School selection and sampling meant that the range of schools represented was smaller than hoped	〉 Very few participants joined 〉 A total of 11 examples were presented: 7 specifically relating to Mathematical Literacy, and 4 to estimation in 'normal' life 〉 No discussion between participants 〉 1 photo and 2 audio file from participants 〉 A total of 29 contributions, including my 15 messages to the group 〉 4 weeks in operation	〉 Online discussion with selected participants 〉 Interest in sharing ideas about teaching and estimation 〉 Discussion not relying on my direction, but following participants' lead 〉 Reflections on both the WhatsApp group and teaching practices
What the change meant for collected data	〉 Data remained as rich, but mostly individualised (therefore not achieving intended collaboration between teachers within the same school) 〉 Data representative of a smaller range of schools and teachers – only one male, and one person of colour	〉 Less data to be collected than anticipated 〉 No examples of GET or FET Mathematics using estimation 〉 So little data collected that there is very little quantitative analysis to do, thus making the study less mixed methods in nature	〉 No real change to expected data from this section, but useful reflections on the use of the WhatsApp group, and development of ideas about estimation in classrooms since initial interviews

Appendix H: Data collection materials

Stage 1: Semi-structured interviews

These interviews were guided by the following interview questions.

Good day… I am [redacted], currently based in Oxford in the United Kingdom as I read for my MSc in Education. Thank you so much for joining me today to discuss some of your experiences, opinions, and perceptions about teaching Maths and the skill of estimation in South Africa. The structure of this interview is designed to be flexible and thus you are more than welcome to add in ideas, ask questions, ask for more time, and offer clarifications. Any form of participation is welcomed! I would just like to start by checking with you both that it is okay to record this session. Please indicate if you are still in agreement with this. Remember that you may withdraw from the research study any time before the end of June this year. You have both submitted your consent forms, but these are subject to your continual interest in participation and can be withdrawn at any time. I just want to check the recording device is working okay…

This session we are just aiming to get to know each other a little and then speak a little about teaching experiences.

Before we begin, do either of you have any questions you'd like to ask, or comments to make? I will happily turn off the recording for these if you would prefer.

1. Please tell me a little about your teaching experience in the past, and what and who you teach now.
2. This study aims to understand a little more about the concept of estimation. What do you understand this word to mean? You can include examples to further explain your definition.
3. Relating to this definition, is there a specific type of estimation that you teach or use in your classroom, specifically relating to Grade 10 but also for other years.
4. Can you think of any ways that estimation is used outside the classroom - and give some examples?

5. Do these practical ideas ever get brought into your classroom and practiced there?
6. The CAPS curriculum requires estimation to be taught or used in a range of subjects from the youngest (Foundation) phase. Can you comment on your learners' estimation abilities in class? Do they ever use the concept without being asked to do so? Are your learners well practised at this skill?
7. Thank you and information about the WhatsApp group.

Stage 2: WhatsApp Group

This is the emailed information sent to each participant about joining the WhatsApp group and its intended purpose:

I am hoping to get the next step of the research study under way for the month of May. At the end of our interviews I spoke to each of you about the possibility of creating a shared WhatsApp group with other participants in this study to collate pieces of information about how you do or don't use estimation in your classroom, and all of you who I am now emailing have said that you would be happy to be part of this stage of research. So, some ideas to proceed:

I'd like to create a WhatsApp group for us all (at the moment, there will be 6 of us, from [redacted], [redacted], and [redacted] – I think that [redacted] and [redacted] may still join us for this part of the research). I think I have most of your phone numbers so that shouldn't be too hard to do.

Ideally the group would be active for 3-4 weeks, and I am thinking of starting it next week (3rd May).

The group's aim is to provide a platform for teachers to document when and how they have used estimation in their classroom, for whatever grades they teach. I am aware that some of you observe others' lessons and teach other subjects, but ask that the documented use of estimation is recorded only for when you teach your own Maths or Maths Lit classes. I don't mind what format you use to record your use of estimation – I think if I was asked to do this I may just jot down information in my diary as the lesson goes on, and then send a picture of that every afternoon when the day is done. Others of you may want to do it at the end of

every week… I really don't mind but just ask that you do try to provide some insight into your use of estimation where possible.

To keep the conversation going, I will try to ask probing questions and reflect on ways that I have used estimation in my daily life (you are welcome to do the same)

I won't control what happens on the group, so ask that you would be respectful of others. Feel free to use the group as you wish, and if you would feel more comfortable to message me directly and not on the group, that would be fine too.

I hope that as the group gets busier with information, that we all become more and more aware of how much estimation is used… and that it creates an opportunity for us to share and discuss ideas more freely.

Again, please feel free to let the group organically develop into something that is useful and worthwhile – you are more than welcome to make suggestions about how to make it work differently / to suit you better.

I will go ahead and create the group on Sunday, 2nd May. Because you have all previously said you would be happy to be part of this part of the research I take it as a given that it is okay for me to add you to it. However, if you would like to change that, and would prefer to be left out… please email me to let me know. Remember, at any stage, you are welcome to leave the WhatsApp group and/or research study – please do just tell me 😊

This is the information I sent to those on the WhatsApp group:

This is our group to start... hopefully we'll add some more members as time goes on. I hope you've had a good weekend and feel ready for the week of work ahead.

Stage 3: Focus Group

The following are the guiding questions and quote used as a vignette for the Focus Group:

1. Thanks for participation again; aims of the session: try to further understand estimation usage in classroom.
2. Relating to your initial comment that …, has your view of estimation changed since the start of this study?

- [redacted]: estimation: has to be a precursor – then this is also used to reassess answers (analytical skill)
- [redacted]: making a judgement on a set of parameters or content; 2 options: all of the information (and so an accurate calculation can be made), or some of the information and thus no accurate answer could ever be arrived at
- [redacted]: Having an idea of where the problem needs to go – how to start and a predicted answer

3. Change of ideas since the interview / awareness in the classroom?
4. Reasons for participating in the WhatsApp group
5. Has your participation in the WhatsApp group or any others' contributions to the group changed your perceptions about estimation and actions in the classroom?
6. Were you surprised by any of your perceptions and actions during this study? Have you reflected further on these?
7. Quote from another participant:
8. "you've also got to realize that people don't think the same way. So you gotta be able to teach estimating skills in in many ways. … Are you not just teaching thought? Is is estimation not teaching … thought of the process rather than learning the process. You actually have to become involved in the process and and create your own knowledge. I mean because Maths is amazing because you can give the answer in many ways you know. So that requires thought is that that which would be trying to do is that not what our curriculum tells us to do. We are supposed to be creating Individual thinkers but, but we're not because we are imparting their knowledge in the same way to all kids."
9. Thoughts? Response? How important is estimation?
10. Thank you

Appendix I: Coding matrix

Colour have been assigned according to the following rules: x>0: yellow; x>5: green; x>10: teal; x>100: dark blue; x=cross reference: red.

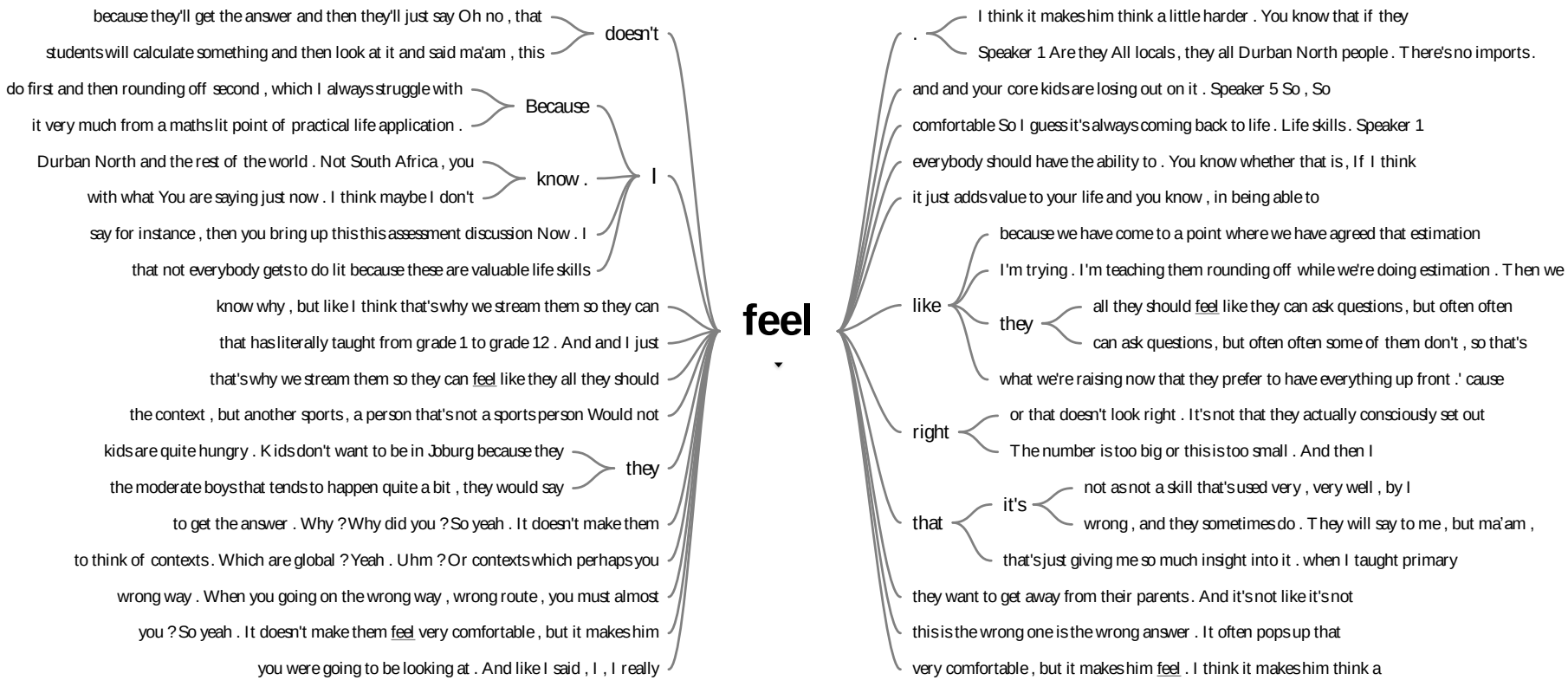
	A : Academic trajectory	B : Estimation	C : Estimation - accuracy and use	D : Classroom rapport	E : Estimation - confidence use	F : Estimation - assessment	G : Estimation - context	H : Estimation - eg classroom school	I : Estimation - maths core	J : FET WhatsApp	K : GET WhatsApp	L : Estimation - maths lit	M : FET WhatsApp	N : Estimation - textbook	O : Estimation - eg life	P : WhatsApp	Q : Estimation - general thoughts	R : Estimation - definition	S : Estimation - in practice	T : Focus Group - use	U : History - current teaching	V : Maths skills learning	W : Critical thought leade rship	X : Estimation	Y : Maths vs Maths Lit classes	Z : Social Impact	AA : WhatsApp Group
1 : Academic trajectory	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0	0	0	0	0
2 : Estimation	0	277	50	8	24	8	185	126	7	0	0	21	7	11	42	4	81	31	38	0	1	41	0	8	2	8	0
3 : Estimation - accuracy and use	0	50	50	8	24	0	22	14	1	0	0	1	0	1	4	0	11	3	9	0	0	11	0	2	0	3	0
4 : Classroom rapport	0	8	8	8	3	0	3	1	0	0	0	0	0	0	0	0	3	0	0	0	0	3	0	0	0	1	0
5 : Estimation - confidence use	0	24	24	3	24	0	8	4	1	0	0	1	0	0	1	0	3	1	2	0	0	4	0	0	0	1	0
6 : Estimation - assessment	0	8	0	0	0	8	4	4	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7 : Estimation - context	0	185	22	3	8	4	185	126	7	0	0	21	7	11	42	4	32	12	16	0	1	30	0	6	2	5	0
8 : Estimation - eg classroom school	0	126	14	1	4	4	126	126	7	0	0	21	7	11	2	0	18	6	14	0	0	25	0	6	2	3	0
9 : Estimation - maths core	0	7	1	0	1	0	7	7	7	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0
10 : FET WhatsApp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11 : GET WhatsApp	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12 : Estimation - maths lit	0	21	1	0	1	1	21	21	1	0	0	21	7	0	0	0	1	0	1	0	0	2	0	0	1	0	0
13 : FET WhatsApp	0	7	0	0	0	0	7	7	0	0	0	7	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14 : Estimation - textbook	0	11	1	0	0	0	11	11	0	0	0	0	0	11	0	0	2	0	0	0	0	1	0	0	0	0	0
15 : Estimation - eg life	0	42	4	0	1	0	42	2	0	0	0	0	0	0	42	4	10	4	0	0	0	1	0	0	0	1	0
16 : WhatsApp	0	4	0	0	0	0	4	0	0	0	0	0	0	0	4	4	0	0	0	0	0	0	0	0	0	0	0
17 : Estimation - general thoughts	0	81	11	3	3	0	32	18	0	0	0	1	0	2	10	0	81	31	9	0	0	18	0	0	0	3	0
18 : Estimation - definition	0	31	3	0	1	0	12	6	0	0	0	0	0	0	4	0	31	31	1	0	0	4	0	0	0	1	0
19 : Estimation - in practice	0	38	9	0	2	0	16	14	1	0	0	1	0	0	0	0	9	1	38	0	0	5	0	1	1	0	0
20 : Focus Group - use	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	0	0	0	0	0	0
21 : History - current teaching	2	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	31	0	0	0	0	0	0
22 : Maths skills learning	1	41	11	3	4	0	30	25	1	0	0	2	0	1	1	0	18	4	5	0	0	66	4	11	0	7	0
23 : Critical thought leadership	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	4	2	0	0	0
24 : Estimation	0	8	2	0	0	0	6	6	0	0	0	0	0	0	0	0	0	0	1	0	0	11	2	11	0	3	0
25 : Maths vs Maths Lit classes	0	2	0	0	0	0	2	2	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	8	0	0
26 : Social Impact	0	8	3	1	1	0	5	3	0	0	0	0	0	0	1	0	3	1	0	0	0	7	0	3	0	14	0
27 : WhatsApp Group	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2

Appendix J: Code book

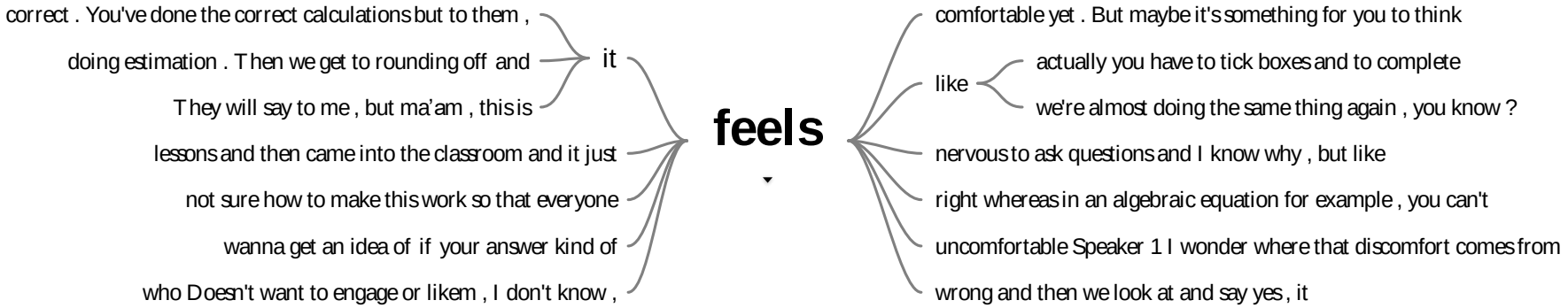
Colours are assigned according to child- and parent-nodes: the darker the colour, the more aggregated the data within that node is.

Node Name	Description	Files	References	Code Matrix: Summary Quotes
Academic trajectory	How learners progress through schooling	4	5	0
Estimation	Broadest code definition; aggregation of all ‘estimation’ codes	10	418	126
Estimation\Estimation - accuracy and use	How accurate answers to estimation are, and how teachers use the skill in classrooms; aggregated codes	9	61	23
Estimation\Estimation - accuracy and use\Classroom rapport	Classroom atmosphere, milieu, teachers’ rapport with children which impact on learners’ interactions	3	8	5
Estimation\Estimation - accuracy and use\Estimation - confidence use	How confidently learners use estimation’ whether they use it independently of being asked to do so	8	24	12
Estimation\Estimation - assessment	The use of estimation in assessments, including in an ad-hoc way in discussions, and in formal assessments	4	8	5
Estimation\Estimation - context	How mathematical knowledge, and estimation, is contextual; the context in which mathematics and estimation are used; aggregated codes	10	261	94
Estimation\Estimation - context\Estimation - eg classroom school	Examples of estimation from classrooms; aggregated codes	10	131	64
Estimation\Estimation - context\Estimation - eg classroom school\Estimation - maths core	Examples of estimation in mathematics ‘core’ classrooms; aggregated codes	3	7	7
Estimation\Estimation - context\Estimation - eg classroom school\Estimation - maths core\FET WhatsApp	Examples of estimation in FET mathematics ‘core’ classrooms in WhatsApp group	0	0	0
Estimation\Estimation - context\Estimation - eg classroom school\Estimation - maths core\GET WhatsApp	Examples of estimation in GET mathematics ‘core’ classrooms in WhatsApp group	0	0	0
Estimation\Estimation - context\Estimation - eg classroom school\Estimation - maths lit	Examples of estimation in mathematics literacy classrooms; aggregated codes	7	21	8
Estimation\Estimation - context\Estimation - eg classroom school\Estimation - maths lit\FET WhatsApp	Examples of estimation in mathematics literacy classrooms from WhatsApp group	1	7	0
Estimation\Estimation - context\Estimation - eg classroom school\Estimation - maths lit\FET WhatsApp	Estimation as presented in textbooks	2	11	3
Estimation\Estimation - context\Estimation - eg classroom school\Estimation - textbook	Examples of estimation from outside classrooms; aggregated codes	10	42	16
Estimation\Estimation - context\Estimation - eg life	Examples of estimation from outside mathematics classrooms from WhatsApp group	1	4	0
Estimation\Estimation - context\Estimation - eg life\WhatsApp	General ideas of how estimation is used, both in the classroom and outside; includes whether estimation is a conscious / subconscious process	8	38	13
Estimation\Estimation - context\Estimation - in practice	General comments on estimation; includes whether estimation happens pre- or post-calculation; aggregated codes	9	88	52
Estimation\Estimation - general thoughts	Participants’ explicit definitions of estimation; references to what participants thought estimation was in different contexts	9	31	19
Estimation\Estimation - general thoughts\Estimation - definition	Participants’ reflections on the use and usefulness of the Focus Group and research project	1	6	2
Focus Group - use	Participants’ career trajectories	8	31	0
History - current teaching	How, when and why maths skills are taught and learnt; aggregated codes	9	87	66
Maths skills learning	Mathematics and teaching’s ability to develop critical thought and leadership in learners; aggregated codes	4	18	14
Maths skills learning\Critical thought leadership	The social impacts of teaching and learning	4	14	11
Maths skills learning\Critical thought leadership\Social Impact	How estimation specifically is taught and learnt	2	11	7
Maths skills learning\Estimation	Responses to vignette in Focus Group	1	4	6
Maths skills learning\Vignette	Number of children taking Mathematics vs those doing Mathematics Literacy	3	8	0
Maths vs Maths Lit classes	Final coding selection; quotes for use in dissertation report	9	150	NA
Summary quotes	Participants’ contributions to WhatsApp group	1	2	0
WhatsApp Group				

Appendix K: Word trees (generated by NVivo)






correct . You've done the correct calculations but to them ,
doing estimation . Then we get to rounding off and
it
They will say to me , but ma'am , this is
lessons and then came into the classroom and it just
not sure how to make this work so that everyone
wanna get an idea of if your answer kind of
who Doesn't want to engage or likem , I don't know ,
feels
comfortable yet . But maybe it's something for you to think
like
actually you have to tick boxes and to complete
we're almost doing the same thing again , you know ?
nervous to ask questions and I know why , but like
right whereas in an algebraic equation for example , you can't
uncomfortable Speaker 1 I wonder where that discomfort comes from
wrong and then we look at and say yes , it