STATIC HEDGING UNDER CEV MODEL WITH DRIFT

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Abstract

We consider the problem of finding a static hedge for the Up-and-Out-Put option where the underlying asset follows the CEV model with drift ($\mu \neq 0$). The main results of this MSc thesis are the derivation of the analytic formula of the static hedge which dismantles into vanilla European options and a numerical proof of better performance of the hedge than the static hedge which is provided under the assumption that the underlying asset follows the Black-Scholes model.

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I would also like to thank her for her smile...

Introduction

Exotic options are quite complex derivatives and the techniques that are used to hedge and value these are also advanced. There are various types of exotic options like the barrier options, the lookback options, the Asian options, the Peroni options etc. Among the exotic options, these which have the greatest demand are the barrier options. The barrier options where first introduced in the Over-The-Counter markets of the United States prior to the vanilla options and nowadays are traded on both stock exchanges and OTC markets. Barrier options are traded on stocks, stock indices, currencies, commodities and interest rates. There is a main reason that barrier options are prefered to vanilla options. That is, if the investor has a very specific "certainty" of how the underlying asset will behave in the future excluding the cases he feels are unlikely to happen, then investing in barrier options he reduces the cost of the option. The financial reason that the cost of the option is reduced is that when the buyer invests in a barrier option he also has to meet an additional condition (hit or not hit the barrier) [1].

There are various types of barrier options. An up-and-out option, written on the underlying S, is the one that will pay the payoff $F(S_T)$ at maturity T in case the underlying has not hit the upper barrier U. According to what a payoff is, a barrier option that is cancelled when hits the barrier can be an up-and-out put, an up-and-out call or an up-and-out digital option. In an analogous way the up-and-in and the down-and-in(out) options are defined.

The *delta* of the option is defined to be the slope of the price of the option with respect to the price of the underlying asset $\Delta = \frac{\vartheta C}{\vartheta S}$, where C is the price of the option and S is the price of the underlying asset. *Delta-hedging* is an example of *dynamic-hedging*. In delta-hedging delta is used in order to find

the position in the asset one should hold in order to "cancel" the delta of the portfolio [2]. Because of the fact that *delta* changes over time, the investor has to rebalance periodically. On the other hand, an investor instead of using *dynamic-hedging*, can use *static-hedging*. That is, create the hedge initially when setting the portfolio and then never re-adjust. These kinds of hedging schemes are also called *hedge-and-forget* schemes [2]. Static replication strategies seem to have some advantages over the dynamic replication strategies, mainly because of the existence of the transaction costs. Since dynamic hedging needs constant rebalancing, this could lead to really bad effects on the portfolio because of the transaction costs. This can be mitigated in case the investor trades periodically which results in a small approximation error when the *gamma* of the underlying is small. However, barrier options have regions where *gamma* is relatively big which leads to greater approximation errors than the desirable [1]. On the other hand, if someone constructs a fixed portfolio with liquid derivatives of which the price coincides with the given barrier option until hitting the barrier and then hedge *shorting* in the corresponding liquid derivatives, then the afore-mentioned problem does not exist anymore. Thus, static hedging does not face tansaction costs problems since the investor will trade when the barrier is first hit.

The payoff function at the time of maturity T for the case of an UOP option is:

$$I_{sup_{t\in[0,T]}S_t<U} \cdot (K - S_T)^+,$$

where $K < U$. The static hadge in that case is defined by a function

$$G : \mathbb{R}_+ \to \mathbb{R},$$

where the European option with payoff $G(S_T)$ has the same price as the UOP option until hitting the barrier. Thus, we dismantle the barrier options into vanilla European options, all with the same maturity T.

Since the major breakthrough of Black and Scholes, a great amount of literature make the assumption that the price of the underlying asset follows the geometric Brownian motion with constant volatility. This assumption

leads to the conclusion that the underlying asset follows the lognormal distribution:

$$lnS_T \sim \mathcal{N}[lnS_0 + (\mu - \frac{\sigma^2}{2})T, \sigma\sqrt{T}].$$

In the case of the Black-Scholes model the static hedges are given in a closed form which consist of vanilla options. Hedging can be considered as the 'mirror' problem of pricing, since if the investor knows the price of the option, he also knows how much he has to short in order to hedge. Thus, in the case of Black-Scholes model, the problem of static hedging is the same to the pricing problem. The static hedge of an UOP option is given by the formula

$$G(x) = (K - x)^+ - \frac{K}{U}(\frac{x}{U})^{\frac{2\mu}{\sigma^2}}(x - \frac{U^2}{K})^+.$$

where K is the strike price of the option, U the barrier and $\mu = r - q$.

On the other hand, if the assumption that Black and Scholes did, held then the Black-Scholes formula would be able to determine the volatility of the underlying simply equating the model price of the option with its market price and we would obtain a constant volatility. However, the implied volatilities with respect to the logarithm of the strike prices K are not constant but they "draw" the well known implied volatility smile instead. Since this is the case, in this thesis we are trying to extend the Black and Scholes model using the *Constant Elasticity of Variance (CEV)* model in order to partially capture the volatility smile that is observed in the market data and more specifically in the FX market.

As it is shown in [1] the prices of the barrier options and their hedge ratios may diverge quite a lot from the corresponding lognormal values.

In the CEV model the underlying, under the pricing measure $\mathbb{Q}$, satisfies the following SDE:

$$dS_t = \mu S_t dt + \delta S_t^{\beta+1} dB_t$$

As one can see in [1] and [4], the closed-form formula for a call option of which the underlying asset is under the CEV model, where $\beta < 0$ is,

$$C(S; K, T) = e^{-qt} S Q(y_0; n, \zeta) - e^{-rT} K(1 - Q(\zeta; n - 2, y_0))$$

where

$$n = 2 + \frac{1}{|\beta|},$$

$$\zeta = \frac{2\mu S^{-2\beta}}{\delta^2 \beta (e^{2\mu\beta T} - 1)},$$

$$y_0 = \frac{2\mu K^{-2\beta}}{\delta^2 \beta (1 - e^{-2\mu\beta T})},$$

Q(x;u,v) is the complementary non-central chi-square distribution function with u degrees of freedom and v is the noncentrality parameter. Finally, K is the strike price of the option.

The aim of this thesis is to find an analytic formula of the static hedge of an up-and-out put option under the Constant Elasticity of Variance (CEV) model, with drift ($\mu \neq 0$), which is time-homogeneous diffusion model, following and extending the work of the "*Static Hedging Under Time-Homogeneous Diffusions*" by Peter Carr and Sergey Nadtochiy [3]. What is more, it provides numerical results which prove that this model is more suitable than the Black-Scholes model for the case of static hedging.

The main idea of this thesis is that the corresponding European put payoff (long), with strike price K, minus (short) a function g with support in $[U, \infty]$, which will be a number (η) times the European call payoffs, with strike price K^*, comprise the payoff function G we are looking for. This, can be explained financially since the price of the barrier UOP option will be equal to the price of the European put option minus the price of some European call options which is a "reward" for taking the risk of hitting the barrier investing on a barrier option. Thus, the function g and more specifically η and K^* will be found when we equate the price of the put option with the price of the call options when the underlying is at the barrier, where the call options cancel out the price of the put option (the price of the option should be zero when the underlying hits the barrier).

In order to do so we divide this MSc. thesis into two parts: In the first chapter we provide the model and the analytical formula of the static hedge

for an up-and-out-put option while in the second chapter we show through numerical results that our model 'beats' the Black-Scholes model.

Chapter 1

The model

The problem of static hedging (and its short-maturity approximation) of an UOP option under the CEV model, that is, a model in which, under the pricing measure $\mathbb{Q}$, the underlying satisfies

$$dS_t = \mu S_t dt + \delta S_t^{\beta+1} dB_t$$

under the risk-neutral measure, has been solved for the case of $\mu = 0$. However it is not always very realistic to assume that the drift is zero. More specifically, the FX index usually has a drift of the form $\mu = r - q$, where r is the domestic and q the foreign interest rate. If the maturity is small then the effect of the drift is, in principle small as well, but, the lack of it may produce an error of the same order as the static hedge approximation itself. Thus, it is important to derive a short maturity single-call approximation of a static hedge in the CEV model with drift ($\mu \neq 0$).

Introducing $M_t = e^{-\mu t} S_t$ and using Ito's formula we get the following Stochastic Differential Equation satisfied by M:

$$dM_t = \delta S_t^{\beta} M_t dB_t \Leftrightarrow dM_t = \delta e^{\mu\beta t} M_t^{\beta+1} dB_t.$$

According to the Dubins-Schwarz theorem every continuous local martingale $M = (M_s)_{s\geq 0}$ can be written as a time-changed Brownian motion $(B_{[M]_s})_{s\geq 0}$, where $[M] = ([M]_s)_{s\geq 0}$ is the (continuous) quadratic variation of M. Therefore, the differential of a local martingale $\delta e^{\beta\mu t} dB_t$ can be written as a differential of a time-changed Brownian motion $d(\tilde{B}_{\frac{\delta^2}{2\beta\mu}(e^{2\beta\mu t}-1)})$, since

$[\int_0^T \delta e^{\beta\mu s} dB]_s = \int_0^T \delta^2 e^{2\beta\mu s} ds = \frac{\delta^2}{2\beta\mu}(e^{2\mu\beta T} - 1)$. Consequently, since $M_T = \int_0^T \delta e^{\mu\beta t} M_t^{\beta+1} dB_t \Leftrightarrow M_T = \int_0^T M^{\beta+1} dB_{f(t)}$, where $f(t) = \frac{\delta}{2\beta\mu}(e^{2\mu\beta t} - 1)$, we get the following:

$$\begin{aligned}
\tilde{M}_T &= M_{f^{-1}(T)} \\
&= \int_0^{f^{-1}(T)} M^{\beta+1}_{f^{-1}(f(t))} d(B_{f(t)}) \\
&= \int_0^T M^{\beta+1}_{f^{-1}(t)} dB_t \\
&= \int_0^T \tilde{M}_t^{\beta+1} dB_t.
\end{aligned}$$

Thus, we conclude that, the CEV process with drift can be written as an exponential multiplied by a time-changed CEV process with zero drift, $d\tilde{M}_t = \tilde{M}_t^{\beta+1} dB_t$, where $M_t = \tilde{M}_{\frac{\delta^2}{2\beta\mu}(e^{2\beta\mu t}-1)}$.

Keeping in mind that $M_t = e^{-\mu t} S_t$, the price of a call option model 1 is

$$\begin{aligned}
C(S_0, 0; T, K) &= e^{-rT} E^Q [S_T - K]^+ \\
&= e^{(-r+\mu)T} E^Q [e^{-\mu T} S_T - e^{-\mu T} K]^+ \\
&= e^{(-r+\mu)T} E^Q [M_T - \tilde{K}]^+ \\
&= e^{(-r+\mu)T} E^Q [\tilde{M}_{f(T)} - \tilde{K}]^+,
\end{aligned}$$

where $f(T) = \frac{\delta^2}{2\mu\beta}(e^{2\mu\beta T} - 1)$

In order to proceed we have to make clear the notification we are about to use at the following lines:

The time t price of a call option with maturity T and strike K in a model where the underlying is a CEV with drift ($\mu \neq 0$) in which the interest rate is r, given that the underlying is at U is denoted

$$C(U,t;T,K) = e^{-r(T-t)}\mathbb{E}_{t,U}(S_T-K)^+ = e^{-r(T-t)}\mathbb{E}\left((S_T-K)^+\Big|\, S_t = U\right),$$

The time t price of a call option with maturity T and strike $\tilde{K}$ in a model where the underlying is a driftless CEV ($\mu = 0$) with zero interest rate ($r = 0$) and $\delta = 1$, given that the underlying is at $\tilde{U}$, is denoted

$$\tilde{C}(\tilde{U},t;T,\tilde{K}) = e^{-r(T-t)}\mathbb{E}_{t,\tilde{U}}(\tilde{M}_T-K)^+ = e^{-r(T-t)}\mathbb{E}\left((\tilde{M}_T-\tilde{K})^+\Big|\, \tilde{M}_t = \tilde{U}\right),$$

Therefore, starting with the call price in a model with drift:

$$\begin{aligned} C(U,t;T,K) &= e^{-r(T-t)}E_{t,U}[S_T-K]^+ \\ &= e^{\mu T}e^{-r(T-t)}E_{f(t),Ue^{-\mu t}}[\tilde{M}_{f(T)}-e^{-\mu T}K]^+ \\ &= e^{\mu T}e^{-r(T-t)}\tilde{C}(Ue^{-\mu t},f(t);f(T),e^{-\mu T}K) \\ &= e^{\mu T}e^{-r(T-t)}\tilde{C}(Ue^{-\mu t},f(T)-f(t),e^{-\mu T}K) \end{aligned}$$

where the last line is true because of time-homogeneity. Thus, without loss of generality and avoiding ambiguity we will denote

$$\tilde{C}(\tilde{U},T-t,\tilde{K}) := \tilde{C}(\tilde{U},t;T,\tilde{K})$$

The same arguments hold in the case of a put option.

Our main aim is to find η and κ^* such that

$$\eta C(U,t;T,K^*) \approx P(U,t;T,K),$$

with a maximal possible precision, asymptotically, as $t \to T$.

We are going to do so through the driftless ($\mu = 0$, $r = 0$) CEV-model. The above is equivalent to

$$\eta\tilde{C}(Ue^{-\mu t},f(T)-f(t),e^{-\mu T}K^*) \approx \tilde{P}(Ue^{-\mu t},f(T)-f(t),e^{-\mu T}K)$$

or

$$\eta\tilde{C}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K}^*)\approx\tilde{P}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K})\ \text{ as }\ \tau\to 0,$$

where

$$\begin{aligned}
&\tilde{K}^*=e^{-\mu T}\kappa^*,\quad \tilde{K}=e^{-\mu T}K,\quad \tilde{U}=Ue^{-\mu T},\\
&f(T)-f(t)=\tau,\quad t=g(f(T)-\tau),\\
&g=f^{-1},\quad h(\tau):=T-t=T-g(f(T)-\tau).
\end{aligned}$$

$$h'(0)=g'(f(T))=\frac{1}{f'(T)}=\frac{e^{-2\mu\beta T}}{\delta^2},$$

Proposition 1. *As $\tau\to 0$, we have*

$$\begin{aligned}
&\frac{d}{d\tau}\left(\tilde{P}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K})\right)=\\
&\frac{|\beta|}{2\sqrt{2\pi}}\left(\tilde{K}\tilde{U}\right)^{(1+\beta)/2}\frac{1}{\sqrt{\tau}}\exp\left(-\frac{1}{2\beta^2\tau}\left(\tilde{U}^{-\beta}-\tilde{K}^{-\beta}-\tilde{U}^{-\beta}\beta\mu h'(0)\tau\right)^2\right)\left(1+\underline{\underline{O}}(\tau)\right),\\
&\frac{d}{d\tau}\left(\tilde{C}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K}^*)\right)=\\
&\frac{|\beta|}{2\sqrt{2\pi}}\left(\tilde{K}^*\tilde{U}\right)^{(1+\beta)/2}\frac{1}{\sqrt{\tau}}\exp\left(-\frac{1}{2\beta^2\tau}\left(\tilde{K}^{*-\beta}-\tilde{U}^{-\beta}+\tilde{U}^{-\beta}\beta\mu h'(0)\tau\right)^2\right)\left(1+\underline{\underline{O}}(\tau)\right)
\end{aligned}$$

Proof. First, using the results of [3] we derive

$$\begin{aligned}
&\partial_\tau\tilde{P}(\tilde{U},\tau,\tilde{K})=\\
&\frac{|\beta|}{2\sqrt{2\pi}}\left(\tilde{K}\tilde{U}\right)^{(1+\beta)/2}\frac{1}{\sqrt{\tau}}\exp\left(-\frac{1}{2\beta^2\tau}\left(\tilde{U}^{-\beta}-\tilde{K}^{-\beta}\right)^2\right)\left(1+\underline{\underline{O}}(\tau)\right),\\
&\partial_\tau\tilde{C}(\tilde{U},\tau,\tilde{K}^*)=\\
&\frac{|\beta|}{2\sqrt{2\pi}}\left(\tilde{K}^*\tilde{U}\right)^{(1+\beta)/2}\frac{1}{\sqrt{\tau}}\exp\left(-\frac{1}{2\beta^2\tau}\left(\tilde{K}^{*-\beta}-\tilde{U}^{-\beta}\right)^2\right)\left(1+\underline{\underline{O}}(\tau)\right),
\end{aligned}$$

where the residual terms are estimated uniformly over $\tilde{U}$ changing on a small enough interval. This means that we can plug $\tilde{U}e^{\mu h(\tau)}$ instead of $\tilde{U}$ in the above formulas.

Now, in a similar manner, we deduce

$$\partial_{\tilde{U}}\tilde{P}(\tilde{U},\tau,\tilde{K}) = -\frac{1}{2\sqrt{2\pi}}\left(\tilde{K}\tilde{U}\right)^{(1+\beta)/2}\left(\tilde{U}^{-\beta}-\tilde{K}^{-\beta}\right)\tilde{U}^{-\beta-1}\cdot$$
$$\int_0^\tau s^{-3/2}\exp\left(-\frac{1}{2\beta^2 s}\left(\tilde{U}^{-\beta}-\tilde{K}^{-\beta}\right)^2\right)ds\left(1+\underline{\underline{O}}(\tau)\right),$$
$$\partial_{\tilde{U}}\tilde{C}(\tilde{U},\tau,\tilde{K}^*) = -\frac{1}{2\sqrt{2\pi}}\left(\tilde{U}\tilde{K}^*\right)^{(1+\beta)/2}\left(\tilde{K}^{*-\beta}-\tilde{U}^{-\beta}\right)\tilde{K}^{*-\beta-1}\cdot$$
$$\int_0^\tau s^{-3/2}\exp\left(-\frac{1}{2\beta^2 s}\left(\tilde{K}^{*-\beta}-\tilde{U}^{-\beta}\right)^2\right)ds\left(1+\underline{\underline{O}}(\tau)\right),$$

where, again, the residual terms are estimated uniformly over $\tilde{U}$ changing on a small enough interval, and we plug $\tilde{U}e^{\mu h(\tau)}$ instead of $\tilde{U}$.

Now applying L'Hopital's rule we conclude that

$$\frac{\partial_{\tilde{U}}\tilde{P}(\tilde{U},\tau,\tilde{K})}{\tau\partial_\tau\tilde{P}(\tilde{U},\tau,\tilde{K})},$$
$$\frac{\partial_{\tilde{U}}\tilde{C}(\tilde{U},\tau,\tilde{K}^*)}{\tau\partial_\tau\tilde{C}(\tilde{U},\tau,\tilde{K}^*)}$$

are bounded as $\tau\to 0$, uniformly over $\tilde{U}$ changing on a small enough interval.

Recall that

$$\frac{d}{d\tau}\left(\tilde{P}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K})\right) = \partial_\tau\tilde{P}(\tilde{U},\tau,\tilde{K}) + \mu h'(\tau)\tilde{U}e^{\mu h(\tau)}\partial_{\tilde{U}}\tilde{P}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K}),$$
$$\frac{d}{d\tau}\left(\tilde{C}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K}^*)\right) = \partial_\tau\tilde{C}(\tilde{U},\tau,\tilde{K}^*) + \mu h'(\tau)\tilde{U}e^{\mu h(\tau)}\partial_{\tilde{U}}\tilde{C}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K}^*),$$

and collecting the above results and plugging $\tilde{U}e^{\mu h(\tau)}$ instead of $\tilde{U}$, we obtain the statement of the proposition.

□ □

Proposition 2. *With the choice*

$$\tilde{K}^* = \left(2\tilde{U}^{-\beta}-\tilde{K}^{-\beta}\right)^{-1/\beta},$$
$$\eta = \left(\frac{\tilde{K}}{\tilde{K}^*}\right)^{(1+\beta)/2}\exp\left(\frac{\mu h'(0)}{\beta}\tilde{U}^{-\beta}\left(\tilde{K}^{*-\beta}-\tilde{K}^{-\beta}\right)\right),$$

as $\tau \to 0$, *we have*

$$\frac{d}{d\tau}\left(\tilde{P}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K})\right) - \eta\frac{d}{d\tau}\left(\tilde{C}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K}^*)\right) = \frac{d}{d\tau}\left(\tilde{P}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K})\right)\underline{\underline{O}}(\tau)$$

Proof. From Proposition 1 we get the following:

$$\begin{aligned}&\frac{d}{d\tau}(\tilde{P}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K})) = \\ &c\frac{1}{\sqrt{\tau}}exp(-\frac{1}{2\beta^2\tau}(\tilde{U}^{-\beta}-\tilde{K}^{-\beta}-\tilde{U}^{-\beta}\beta\mu h'(\tau))^2)(1+\underline{\underline{O}}(\tau)) = \\ &c\frac{1}{\sqrt{\tau}}exp(-\frac{1}{2\beta^2\tau}[(\tilde{U}^{-\beta}-\tilde{K}^{-\beta})^2 \\ &+\tilde{U}^{-2\beta}\beta^2\mu^2h'(0)^2\tau^2 - 2(\tilde{U}^{-\beta}-\tilde{K}^{-\beta}(U^{-\beta}\beta\mu h'(0)\tau))])(1+\underline{\underline{O}}(\tau)).\end{aligned}$$

and

$$\begin{aligned}&\frac{d}{d\tau}(\tilde{C}(\tilde{U}e^{\mu h(\tau)},\tau,\tilde{K})) = \\ &c'\frac{1}{\sqrt{\tau}}exp(-\frac{1}{2\beta^2\tau}[(\tilde{K^*}-\tilde{U}^{-\beta})^2 + U^{-2\beta}\beta^2\mu^2h'(0)^2\tau^2 - 2(\tilde{U}^{-\beta}-K^{-\beta})(\tilde{U}^{-\beta}\beta'(0)\tau)]),\end{aligned}$$

where $c = \frac{|\beta|}{2\sqrt{2\pi}}(\tilde{K}\tilde{U})^{(1+\beta)/2}$ and $c' = \frac{|\beta|}{2\sqrt{2\pi}}(\tilde{K^*}\tilde{U})^{(1+\beta)/2}$

now since

$$\eta = \left(\frac{\tilde{K}}{\tilde{K}^*}\right)^{(1+\beta)/2}\exp\left(\frac{\mu h'(0)}{\beta}\tilde{U}^{-\beta}\left(\tilde{K}^{*-\beta}-\tilde{K}^{-\beta}\right)\right) = (\frac{\tilde{K}}{K^*})^{\frac{1+\beta}{2}}exp(\frac{\mu h'(0)}{\beta}U^{-\beta}(2\tilde{U}^{-\beta}-2\tilde{K}^{-\beta})))$$

it is easy to see the statement of the proposition. □

Proposition 3. *With the choice of $\tilde{K}^*$ and η as in Proposition 2, as $\tau \to 0$, we have*

$$\tilde{P}(\tilde{U}e^{\mu h(\tau)}, \tau, \tilde{K}) - \eta\tilde{C}(\tilde{U}e^{\mu h(\tau)}, \tau, \tilde{K}^*) = \tilde{P}(\tilde{U}e^{\mu h(\tau)}, \tau, \tilde{K})\underline{\underline{O}}(\tau)$$

Proof. In order to proove the above proposition we define:

$$f(\tau) = \tilde{P}(\tilde{U}e^{\mu h(\tau)}, \tau, \tilde{K}) - \eta\tilde{C}(\tilde{U}e^{\mu h(\tau)}, \tau, \tilde{K}^*)$$

and

$$g(\tau) = \tilde{P}(\tilde{U}e^{\mu h(\tau)}, \tau, \tilde{K})\underline{\underline{O}}(\tau).$$

Therefore from proposition 2 we have that:

$$f'(\tau) = g'(\tau)\underline{\underline{O}}(\tau).$$

Consequently since g' is always positive we get the following:

$$\begin{aligned} f(\tau) = \int_0^\tau f'(s)ds &= \int_0^\tau |g'(s)\underline{\underline{O}}(s)|ds \\ &= \int_0^\tau g'(s)|\underline{\underline{O}}(s)|ds \\ &\leq c\int_0^\tau g'(s)sds \\ &= c(g(\tau)\tau - g(\tau)) \\ &= g(\tau)(c\tau - 1) \end{aligned}$$

Therefore, $f(\tau) = g(\tau)\underline{\underline{O}}(\tau)$ from which we get the proposition. □

Finally, let us compute the hedge parameters in the original variables:

$$\begin{aligned} \tilde{K}^* &= (2\tilde{U}^{-\beta} - \tilde{K})^{-\frac{1}{\beta}} \\ &= (2(e^{-\mu\tau}U)^{-\beta} - (e^{-\mu\tau}K)^{-\beta})^{-\frac{1}{\beta}} \\ &= e^{-\mu\tau}(2U^{-\beta} - K^{-\beta})^{-\frac{1}{\beta}}) \end{aligned}$$

Therefore,

$$K^* = (2U^{-\beta} - K^{-\beta})^{-\frac{1}{\beta}}.$$

while the same way we get

$$\eta = \left(\frac{K}{K^*}\right)^{(1+\beta)/2} \exp\left(\frac{\mu}{\beta\delta^2} U^{-\beta}\left(K^{*-\beta} - K^{-\beta}\right)\right).$$

Chapter 2

Numerical results

2.1 Description of the database

As mentioned previously the second part of the aim of this thesis is to show how to derive static hedge of an up and out put option under the CEV model. Consequently, the first step in order to derive numerical results that show the appropriateness of the model was to find a database with barrier options. A database with barrier options would be really helpful since it would provide the market prices of the barrier options and the prices of their corresponding underlying assets. This would help us to compare the market prices to the static hedge we derive. This is possible and the advisable way to do things since hedging is considered to be the "mirror" problem of pricing. Consequently we would have a "good" measure of how well our model fits to the market prices of the barrier options and thus how good and accurate is our static hedge. However, after a thorough research we were not able to find such a database.

Because of that finally we devised another way to implement the project using a database where we found European options and their corresponding implied volatilities. The data we used are from *Wharton Research Data Services (WRDS)* and more specifically we chose *PHLX, Implied Volatility*. This database is about options on *foreign exchange market (FX)*. In the FX market there are two parties involved; the first party buys an amount of a currency paying an amount of another currency. In our case we used

Australian dollar to USD, while the years available for the specific database are: 1995, 1996 and 1997.

The variables that were available at the *PHLX, Implied Volatility* and used by us were:

1. Volatility at the Money Ask Price
2. Volatility at the Money Bid Price
3. Volatility of Out of the Money Call Ask
4. Volatility of Out of the Money Call Bid
5. Out of the Money Call Strike Price
6. Expiration Date
7. Volatility of Out of the Money Put Ask
8. Volatility of Out of the Money Put Bid
9. Out of the Money Put Strike Price
10. At the Money Strike Price
11. Spot Price

Finally, another problem that appeared using this specific database was that there were cases where for the same dates (quote and expiration date) there were different data. Thus, before proceeding with the implementation of the experiment we had to clear the database of the additional data.

2.2 Description of the experiment

The experiment is divided into two main steps. The first step takes place at t_1, which is 15 days before the maturity T of each option, and concerns the calibration of β. The reason that we chose the interval $[t_1, T]$ to be at the maximum 15 days was that we are concerned about short maturity options. The second step takes place from time t_2 until T and concerns the computation of the hedge.

2.2.1 Calibration of β

As mentioned earlier the CEV model is described by the following SDE:

$$dS_t = \mu S_t dt + \delta S_t^{\beta+1} dB_t.$$

As one can see the CEV model has three parameters: μ, δ and σ. In our case, since we use the FX index, the drift is equal to the domestic minus the foreign interest rate. From the famous 'interest rate parity relationship' [2] we have that

$$K = S_t e^{(r-q)(T-t)}$$

from which we conclude that $r - q = \mu = \frac{1}{T} log \frac{K}{S_t}$.

Concerning the second parameter, δ, we decided to fix it doing the following assumption; the Black-Scholes model is described by the following SDE:

$$dS_t = \mu S_t dt + \sigma S_t dB_t.$$

Although it is not correct in general, that the equation

$$dS_t = \mu S_t dt + \Sigma(t, S_t) S_t dB_t,$$

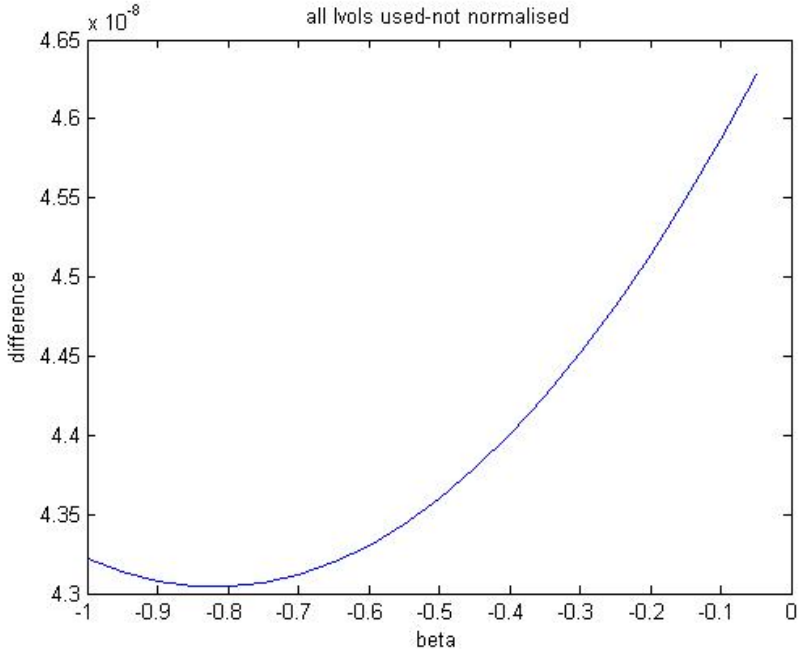
where $\Sigma(t, \cdot)$ the implied volatility, provides us the option prices, in our case it is approximately correct. That is because when the time to maturity is short and we are at-the-money, then both implied volatility and local volatility are located next to the spot volatility (see *"http://users.ox.ac.uk/ oxma0016/"*). Consequently, it is reasonable to suppose the following;

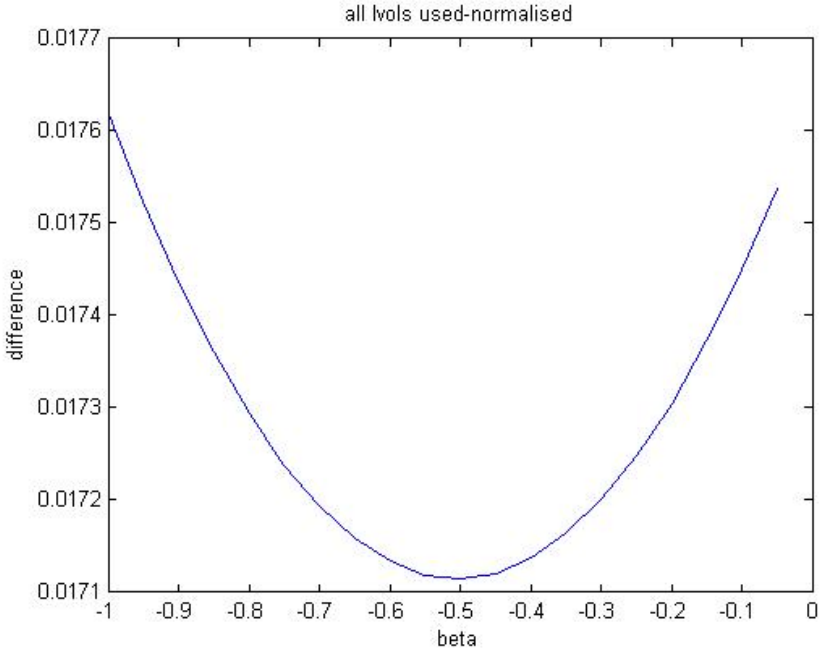
$$\Sigma_0 = \delta S_t^{\beta} \Rightarrow \delta = \Sigma_0 S^{-\beta},$$

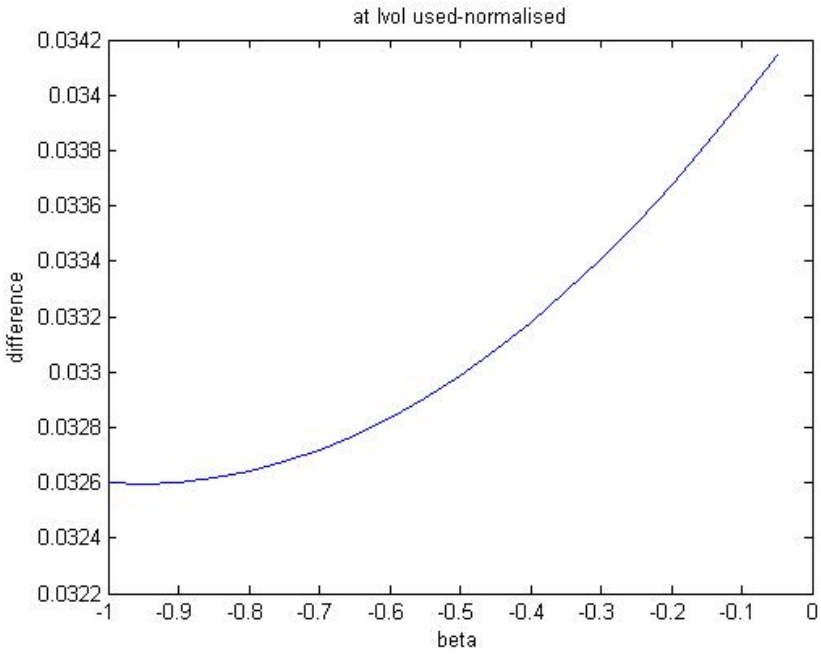
where Σ_0 the implied volatility given by the database.

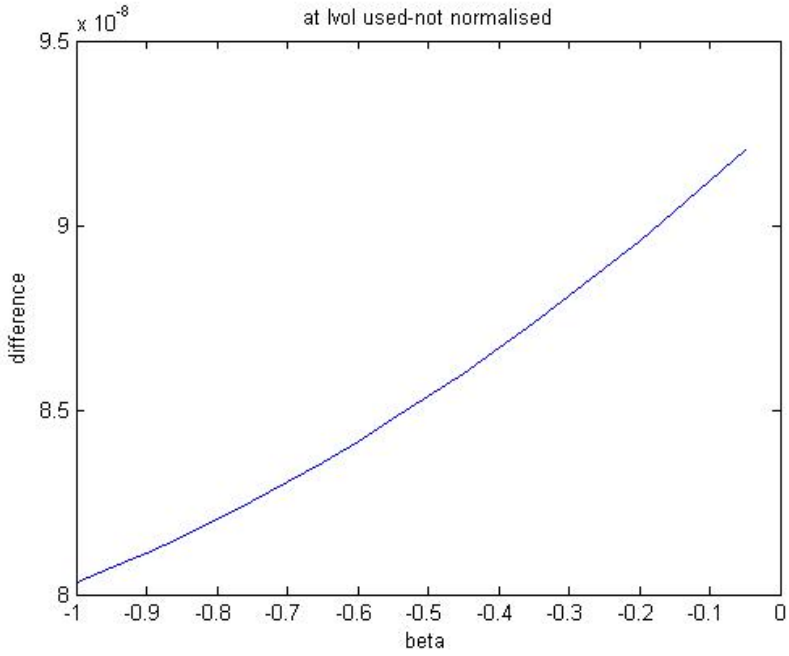
After these assumptions the only parameter of the model left to estimate is β. According to the definition of implied volatility the market prices of the options are given by the Black-Scholes formula plugging in the given implied volatility instead of the constant (spot) volatility. Therefore, in order to calibrate β we calculated the options' prices using implied volatility, which give the market prices of the options and we also calculated the prices of

the options which are predicted by the CEV-model (see Introduction), for betas taking prices in the interval $[-1, -0.05]$. Finally, the prices of beta that the predictions of the CEV-model were closer to the market prices of the options were chosen for each case. As it is shown earlier the PHLX database provides 3 different kinds of implied volatilities. These are the at-the-money implied volatility (bid, ask), the out-of-the money volatility (bid, ask) and finally the in-the-money volatility (bid, ask). Therefore for this procedure of minimisation we had to decide whether we would use the at-the-money volatility alone or all the 3 different kinds of volatility since there are antipathic views concerning the specific subject. What is more, there was another dilemma that we had to overcome. That was whether we should "normalise" the differences between the market prices and the prices that were produced by the model. By "normalisation" we mean the division of the afore-mentioned differences by the market spread, that is the difference between the bid and the ask price of the option, before we square them and sum them up. There is the view that after that kind of normalisation we would force the model to concentrate on approximating the options with smaller spreads, which are more liquid, more precisely than the other options whose spreads are large and therefore we do not trust the market prices that much. The following figures illustrate the "behaviour" of β's with respect to the differences of the market and the model prices for one specific date (16/05/97). In the following plots the calibration of β is $\beta = -0.8000$, $\beta = -0.5000$, $\beta = -0.9500$ and $\beta = -1$ respectively.


all Ivols used-not normalised
x 10^-8
4.65
4.6
4.55
4.5
4.45
4.4
4.35
4.3
difference
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
beta


all Ivols used-normalised
0.0177
0.0176
0.0175
0.0174
0.0173
0.0172
0.0171
difference
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
beta


at Ivol used-normalised
0.0342
0.034
0.0338
0.0336
0.0334
0.0332
0.033
0.0328
0.0326
0.0324
0.0322
difference
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
beta


x 10^-8
at Ivol used-not normalised
9.5
9
8.5
8
difference
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0
beta

2.2.2 Calculation of the static hedge

The second step of the experiment which produces the main results of this thesis is the calculation of the static hedge. As mentioned earlier in order to replicate the Up and Out Put barrier option we long a European put option and we short η European call options with strike price K^*. Consequently, if this was the case the model would provide us the "perfect" hedge with some approximating error. Therefore the second step of the experiment includes the creation (formation) of the static hedge by the time the underlying asset hits the barrier and the observation of the behaviour of the static hedge at time of maturity T.

Since the database we used provides data about European options (bid, ask implied volatilities and in, at, out of the money strike prices) we had to mimic the barrier options and more specifically its barriers. In order to proceed with that we found the maximum value of the underlying of each option lying between t_1 and the maturity day T, excluding of course the maturity day, and set the barrier to be the price among the given which was closest to $S_{t_1} + \frac{1}{2}(S_{max} - S_{t_1})$, where S_{t_1} is the price of the underlying asset at the first day of the option and S_{max} the maximum value that the asset price attained throughout the life of the option. The reason we did not set the barrier simply to be the maximum price of the underlying asset is that in many cases an asset attains a very high or very low price because of an extreme event. This could possibly affect the experiment. Consequently we wanted to rule out such cases. Therefore, at the time of hitting the barrier, t_2, we set 2 hedging portfolios. For both of them we used *Out of the Money Put Strike Price* ($\tilde{K} = e^{-\mu t}K_-$) as the strike price of the put option and for the first portfolio (*hedge1*) we used the *At the Money Strike Price* ($\tilde{K_0} = e^{-\mu t}K_0$) as the strike price of the call option, while for the second one (*hedge2*) we used the *Out of the Money Call Strike Price* ($\tilde{K_+} = e^{-\mu t}K_+$) all multiplied by the factor $e^{-\mu t}$ as the theory necessitates. Finally, we set as time to maturity of the European options to be $\tau' = T - t_2$. Therefore, we expect two cases to come up:

1. K^* lies in between the *At the Money Strike Price* and the *Out of the Money Call Strike Price*. In that case according to the theory *hedge1* should be negative (under-replicate) while *hedge2* should be positive (super-replicate).

2. K^* lies after the two strike prices where both of the portfolios should be negative.

Finally, we compare the performance of the hedge to the Black-Scholes case, that is when the underlying asset is driven by the SDE:

$$dS_t = \mu S_t dt + \sigma S_t dB_t.$$

2.3 Numerical results

As we have already mentioned this experiment is divided into two main steps, the calibration of β and the calculation of the static hedge. In this section we show the outcome of the numerical procedure.

Concerning the calibration of β we examined four different cases:

1. At-the-money implied volatility (bid-ask) normalised by the spread

2. At-the-money implied volatility (bid-ask) not normalised by the spread

3. All 3 different kinds of implied volatility (bid-ask) normalised by the spread

4. All 3 different kinds of implied volatility (bid-ask) not normalised by the spread

As it is shown in table 2.1 although the case where all 3 kinds of implied volatilities have been used without normalisation of the spreads seems to provide the best calibration of betas, the differences among the rest of the cases are not that obvious. Consequently, we decided to proceed the experiment using all four types of calibration and produce 4 different numerical results.

As far as the performance of the static hedge is concerned we provide four tables (see 2.3, 2.5, 2.7, 2.9) one for each of the cases for the calibration of β. In the tables there are 11 columns:

i) The first column gives the expiry date of each option (one for each month).

ii) The second column gives the values of β.

iii) The third column gives the value of the at-the-money strike price (K_0) as is given by the database.

iv) The fourth column gives the value of the out-of-the money (K_-) strike price.

v) The fifth column gives the value of the in-the-money (K_+) strike price.

vi) The sixth is about the values of K^* computed by us.

vii) The seventh column is about the values of the first static hedge, the one computed using K_0 as strike price of the call options which we short.

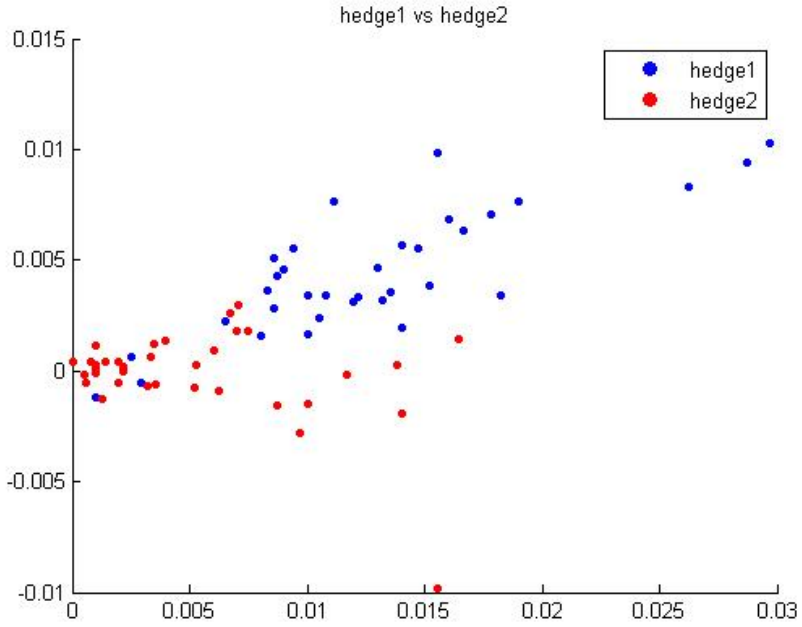
viii) The eigth column is about the values of the second static hedge, the one computed using K_+ as the strike price of the call options we short.

and finally the last three columns are the corresponding values of K^*, hedge1 and hedge2 for the Black-Scholes case ($\beta = 0$).

In theory, in case one of the 2 strike prices K_-, K_+ coincided with the theoretical strike price then we would obtain the perfect hedge. Therefore, since we short calls, whenever K^* lies in between the other two strike prices then hedge1 should be negative (sub-replicate) while hedge2 should be positive (super-replicate), while when the case is that K^* is greater than K_- and K_+ both of the hedges should be negative. As it is indicated in the tables most of the times our expectations are affirmed. Furthermore, one would expect that when the ablolute value of the difference is big then the performance of the two hedges becomes better especially as far as the "sign" of the hedge is concerned. When the difference $|K^* - K_-|$ is big then the error is relatively big and hence the sign should not be affected as shown in the following scatter plots. In the diagrams below we plot the negative difference of hedge1 to the theoretical hedge (the one where the calls we short with strike price K^*) as a function of the absolute difference $|K_- - K^*|$ and

the positive difference of hedge2 to the theoretical hedge as a function of the absolute difference $|K_+ - K^*|$. We do the same for the case of the hedges that the Black-Scholes model predict. What we expect to see from these plots is a "cloud" that

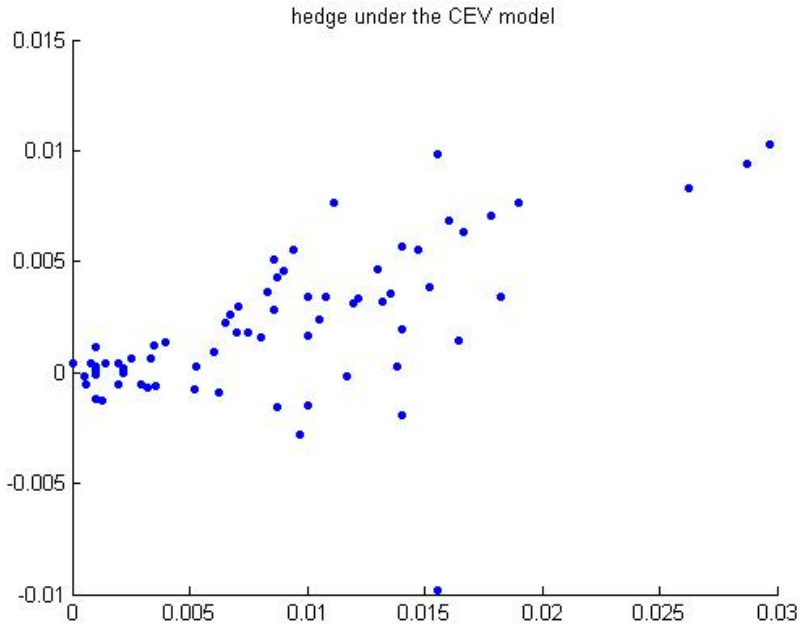
1. has a downwards slope as we go towards 0
2. convergent to 0
3. and non-negative.

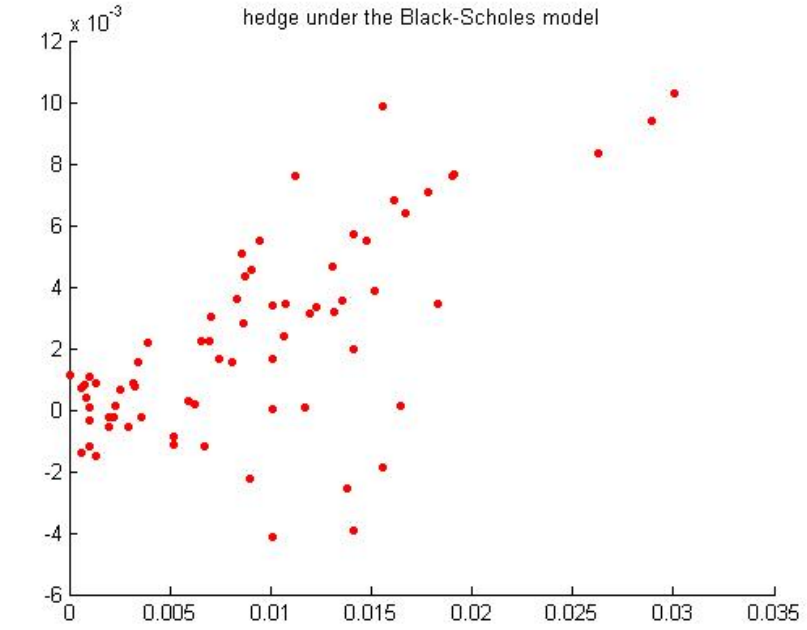


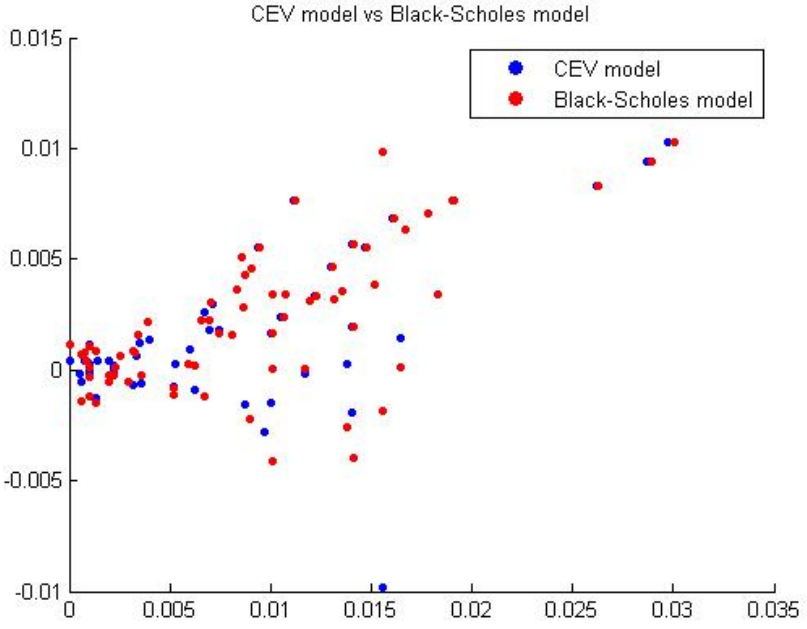
In the above scatter plot we observe that hedge1 has better performance than hedge2. This is because, as we can see in the tables (see 2.3, 2.5, 2.7, 2.9), in most of the cases K^* "moves" towards K_+. Therefore, the distance between $|K^* - K_-|$ is greater than the distance $|K^* - K_+|$. Apparently, as the distance of the strike prices becomes smaller then the theoretical hedge is closer to the real one which means that, due to numerical reasons, the possibility of a faulty sign is bigger, which apparently produces a better approximation of *hedge1*. Thus, the values that correspond to *hedge1* are

seldomly negative while the slope is also better. Nonetheless, both hedges have nice performance.

At the following diagrams we plot the performance of the hedges under the CEV and the Black-Scholes model severally and then together in order to compare them. One can notice that in the CEV model as the distance of the strike prices approaches 0 the error of the hedge approaches 0 as well, at least in a better way than in the Black-Scholes model. What is more, the diagramms of the CEV model are less scattered than the corresponding of the Black-Scholes model. Finally, the number of the observations that are negative are less in the case of our model and whenever we have negative values are not that much below zero as they are in the Black-Scholes case. Therefore, according to the diagrams below the selection of the CEV model is more appropriate for the creation of the static hedge than the Black-Scholes model.




hedge under the Black-Scholes model
x 10^-3
12
10
8
6
4
2
0
-2
-4
-6
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035


CEV model vs Black-Scholes model
CEV model
Black-Scholes model
0.015
0.01
0.005
0
-0.005
-0.01
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035

2.4 The code

In this section we describe the way one can 'run' the code that we implemented and is shown in the Appendix. Initially, in order to run the code one has to download the data and store them in an excel sheet from *Wharton Research Data Services (WRDS)*, *PHLX, Implied Volatility*. After doing so, she should select from the database page the choices we described in section 2.1 and erase the first column of the excel sheet that the database outputs ('CURCD'). The dataset we selected is 'Australian dollar currency'.

The main code that we implemented is the function 'final_code.m'. The 'final_code.m' provides the following:

a) beta (β).

b) Kzero_n, which is the At-the-money strike price corresponding to the date where the underlying hits the barrier.

c) Kminus_n, Out-of-the-money strike price of the aforementioned date.

d) Kplus_n, In-the-money strike price.

e) Kbar_n, which is K^*.

f) hedge1.

g) hedge2.

h) Kbar_BS, which is K^* for the Black-Scholes model.

i) hedge1_BS.

j) hedge2_BS.

In order to run the code one has to input the number of the row in the excel sheet that corresponds to the expiry date of the option she chooses. In the code there are comments describing how the user can change from one type of the calibration of β to another. In case someone wants to implement the different types of calibration severally, she can do so using one of the following functions;

a) minimiser_Ivolall_not_divided.m

b) minimiser_Ivolall_divided.m

c) minimiser_Ivol_At_not_divided.m

d) minimiser_Ivol_At_divided.m

where the name of each function corresponds to the type of calibration.

In order to implement the 'final_code.m' for the total of the dataset one has to run the script named 'dataset.m'. This script outputs the matrix 'A' which encompasses the aforementioned data that the 'final_code.m' outputs for all the options of the dataset.

Finally, we have to note that the function 'final_code.m' uses the functions 'phi.m', 'eurocall.m' implemented by professor Mikes Giles and the function 'CEV_CallnPutPrice_n.m' implemented by Dr. Sergey Nadtochiy.

Table 2.1: Calibration of β

expiry date	All-normalised	All-not-normalised	At-normalised	At-not-normalised
10-Feb-95	-0.05	-0.05	-0.05	-0.05
10-Mar-95	-0.05	-0.05	-0.05	-0.05
13-Apr-95	-0.05	-0.3	-0.05	-0.05
12-May-95	-0.05	-0.05	-0.05	-0.05
16-Jun-95	-0.05	-0.05	-0.05	-0.05
14-Jul-95	-0.05	-0.05	-0.05	-0.05
11-Aug-95	-1	-1	-1	-1
15-Sep-95	-1	-1	-1	-1
13-Oct-95	-0.05	-0.05	-0.05	-0.05
10-Nov-95	-1	-1	-1	-1
15-Dec-95	-0.9	-0.75	-0.9	-1
12-Jan-96	-0.05	-0.05	-0.05	-0.05
16-Feb-96	-1	-0.7	-1	-0.05
15-Mar-96	-0.05	-0.05	-0.05	-0.05
12-Apr-96	-1	-1	-1	-1
10-May-96	-1	-1	-1	-1
14-Jun-96	-0.7	-0.3	-0.7	-1
12-Jul-96	-0.05	-0.05	-0.05	-1
16-Aug-96	-0.05	-0.05	-0.05	-0.05
13-Sep-96	-1	-1	-1	-1
11-Oct-96	-0.05	-0.05	-0.05	-0.05
15-Nov-96	-0.05	-0.05	-0.05	-0.05
13-Dec-96	-1	-1	-1	-1
10-Jan-97	-0.05	-0.05	-0.05	-0.35
14-Feb-97	-0.65	-0.1	-0.65	-0.95
14-Mar-97	-0.05	-1	-0.05	-0.05
11-Apr-97	-0.4	-0.05	-0.4	-1
16-May-97	-0.95	-0.5	-0.95	-1
13-Jun-97	-0.55	-0.05	-0.55	-0.85
11-Jul-97	-1	-0.2	-1	-1
15-Aug-97	-1	-1	-1	-0.05
12-Sep-97	-0.05	-0.05	-0.05	-0.1
10-Oct-97	-0.05	-0.15	-0.05	-0.05
14-Nov-97	-0.05	-0.05	-0.05	-0.05
12-Dec-97	-0.85	-0.8	-0.85	-0.35

Table 2.3: All Ivols used - not normalised

expiry date	β	$K0$	$K-$	$K+$	K^*	hedge1	hedge2	BS^*_K	BS-hedge1	BS-hedge2
10-Feb-95	-0.05	1.5500	1.5500	1.5500	1.5655	-0.0098	-0.0098	1.5655	-0.0018	-0.0019
10-Mar-95	-0.05	1.6250	1.6000	1.6499	1.6432	-0.0034	0.0026	1.6433	-0.0117	-0.0012
13-Apr-95	-0.15	1.6250	1.6000	1.6500	1.6362	-0.0076	0.0003	1.6362	-0.0137	-0.0026
12-May-95	-0.05	1.6250	1.6000	1.6500	1.6356	-0.0051	0.0014	1.6335	-0.0087	0.0001
16-Jun-95	-0.05	1.6000	1.5900	1.6100	1.6025	-0.0007	0.0018	1.6025	-0.0010	0.0016
14-Jul-95	-0.05	1.5900	1.5800	1.6100	1.6090	-0.0076	-9.9059e-05	1.6090	-0.0087	3.4631e-04
11-Aug-95	-1	1.6000	1.5900	1.6200	1.6140	-0.0057	9.4236e-04	1.6141	-0.0082	2.6494e-04
15-Sep-95	-1	1.5600	1.5500	1.5700	1.5629	-5.4465e-04	0.0029	1.5629	8.4311e-04	0.0030
13-Oct-95	-0.05	1.5800	1.5700	1.6000	1.5978	-0.0071	2.9680e-05	1.5978	-0.0078	2.5255e-04
10-Nov-95	-1	1.5800	1.5700	1.5900	1.5922	-0.0033	2.1104e-04	1.5923	-0.0035	1.3857e-04
15-Dec-95	-0.80	1.5300	1.5200	1.5500	1.5490	-0.0076	1.5478e-04	1.5491	-0.0067	4.1627e-04
12-Jan-96	-0.05	1.5500	1.5400	1.5600	1.5620	-0.0031	4.2354e-04	1.5620	-0.0048	2.4217e-04
16-Feb-96	-0.05	1.5400	1.5300	1.5500	1.5481	-0.0016	5.3946e-04	1.5480	-0.0016	-5.6712e-04
15-Mar-96	-0.05	1.5300	1.5200	1.5400	1.5408	-0.0034	4.2219e-04	1.5408	-0.0025	7.9589e-04
12-Apr-96	-1	1.5200	1.5100	1.5200	1.5300	-0.0017	-0.0015	1.5301	-9.7296e-05	1.9341-e05
10-May-96	-1	1.5200	1.5100	1.5200	1.5340	-0.0020	-0.0019	1.5341	-0.0040	-0.0039
14-Jun-96	-0.85	1.5400	1.5300	1.5600	1.5560	-0.0068	0.0013	1.5561	-0.0023	0.0022
12-Jul-96	-0.05	1.5500	1.5400	1.5600	1.5632	-0.0032	6.8866e-04	1.5632	3.7666e-04	8.8824e-04
16-Aug-96	-0.05	1.5500	1.5500	1.5500	1.5490	0.0012	0.0011	1.5490	0.0011	0.0011
13-Sep-96	-1	1.5600	1.5600	1.5700	1.5694	-0.0055	-5.5770e-04	1.5694	-0.0079	-0.0014
11-Oct-96	-0.4	1.5600	1.5600	1.5800	1.5683	-0.0036	-1.8958e-04	1.5683	-0.0029	8.2251e-05
15-Nov-96	-0.05	1.6600	1.6600	1.6700	1.6687	-0.0043	-0.0013	1.6687	-0.0047	-0.0015
13-Dec-96	-1	1.6800	1.6700	1.7000	1.6930	-0.0047	0.0018	1.6931	-0.0033	0.0022
10-Jan-97	-0.05	1.6900	1.6800	1.7000	1.6965	-0.0022	0.0012	1.6965	-0.0015	0.0016
14-Feb-97	-0.2	1.6300	1.6200	1.6500	1.6467	-0.0064	6.3923e-04	1.6467	-0.0059	7.5107e-04
14-Mar-97	-0.4	1.6200	1.6000	1.6400	1.6347	-0.0055	2.8207e-04	1.6358	-0.0084	-8.5893e-04
11-Apr-97	-0.05	1.6400	1.6300	1.6500	1.6501	-0.0034	4.1495e-04	1.6501	-0.0020	0.0011
16-May-97	-0.8	1.6400	1.6300	1.6500	1.6486	-0.0028	4.4032e-04	1.6487	-0.0018	8.5781e-04
13-Jun-97	-0.05	1.6400	1.6200	1.6500	1.6536	-0.0036	-6.4139e-04	1.6536	-0.0030	-2.4492e-04
11-Jul-97	-1	1.6900	1.6700	1.7000	1.7005	-0.0024	-1.5277e-04	1.701	-8.5172e-04	7.3479e-04
15-Aug-97	-1	1.6300	1.6100	1.6500	1.6597	-0.0103	-0.0027	1.6601	-0.0133	-0.0041
12-Sep-97	-0.05	1.5900	1.5700	1.6100	1.6162	-0.0083	-9.1614e-04	1.6162	-0.0057	1.6542e-04
10-Oct-97	-0.05	1.6200	1.6200	1.6300	1.6290	-0.0046	2.4399e-04	1.6290	-0.0050	9.40334e-05
14-Nov-97	-0.05	1.7000	1.6800	1.7100	1.7152	-0.0039	-7.6662e-04	1.7152	-0.0046	-0.0011
12-Dec-97	-0.65	1.6800	1.6600	1.7000	1.7087	-0.0094	-0.00156	1.7090	-0.01090	-0.0022

Table 2.5: All Ivols used - normalised

expiry date	β	$K0$	$K-$	$K+$	K^*	hedge1	hedge2	BS^*_K	BS-hedge1	BS-hedge2
10-Feb-95	-0.05	1.5500	1.5500	1.5500	1.5655	-0.0099	-0.0098	1.5655	-0.0019	-0.0019
10-Mar-95	-0.05	1.6250	1.6000	1.6500	1.6433	-0.0034	0.0026	1.6433	-0.0117	-0.0012
13-Apr-95	-0.30	1.6250	1.6000	1.6500	1.6361	-0.0077	2.7398e-04	1.6362	-0.0137	-0.0026
12-May-95	-0.05	1.6250	1.6000	1.6500	1.6336	-0.0051	0.0014	1.6336	-0.0087	1.4186e-04
16-Jun-95	-0.05	1.6000	1.5900	1.6100	1.6025	-6.6567e-04	0.0018	1.6025	-9.9395e-04	0.0016
14-Jul-95	-0.05	1.5900	1.5800	1.6100	1.6090	-0.0076	-9.906e-05	1.6090	-0.0087	-3.4632e-04
11-Aug-95	-1.00	1.6000	1.5900	1.6200	1.6140	-0.0057	9.4236e-04	1.6141	-0.0082	2.6494e-04
15-Sep-95	-1.00	1.5600	1.5500	1.5700	1.5629	0.00054	0.0030	1.5629	8.4312e-04	0.0030
13-Oct-95	-0.05	1.5800	1.5700	1.6000	1.5978	-0.0071	-2.9681e-05	1.5978	-0.0078	-2.5255e-04
10-Nov-95	-1.00	1.5800	1.5700	1.5900	1.5922	-0.0033	2.1104e-04	1.5923	-0.0035	1.3857e-04
15-Dec-95	-0.75	1.5300	1.5200	1.5500	1.5490	-0.0076	1.5619e-04	1.5491	-0.0067	4.1628e-04
12-Jan-96	-0.05	1.5500	1.5400	1.5600	1.5620	-0.0031	4.2354e-04	1.5620	-0.0049	-2.4218e-04
16-Feb-96	-0.70	1.5400	1.5300	1.5500	1.5480	-0.0016	-5.4587e-04	1.5481	-0.0016	-5.6713e-04
15-Mar-96	-0.05	1.5300	1.5200	1.5400	1.5408	-0.0034	4.2219e-04	1.5408	-0.0025	7.9589e-04
12-Apr-96	-1.00	1.5200	1.5100	1.5200	1.5300	-0.0017	-0.0015	1.5301	-0.9730e-04	1.934e-05
10-May-96	-1.00	1.5200	1.5100	1.5200	1.5340	-0.0020	-0.0019	1.5341	-0.0040	-0.0039
14-Jun-96	-0.30	1.5400	1.5300	1.5600	1.5561	-0.0068	0.0014	1.5561	-0.0023	0.0022
12-Jul-96	-0.05	1.5500	1.5400	1.5600	1.5632	-0.0032	-6.8866e-04	1.5632	-3.7666e-04	8.8824e-04
16-Aug-96	-0.05	1.5500	1.5500	1.5500	1.5490	0.0012	0.0011	1.5490	0.0011	0.0011
13-Sep-96	-1.00	1.5600	1.5600	1.5700	1.5694	-0.0055	-5.5770e-04	1.5694	-0.0079	-0.0014
11-Oct-96	-0.05	1.5600	1.5600	1.5800	1.5683	-0.0037	-1.8777e-04	1.5683	-0.0029	8.225e-05
15-Nov-96	-0.05	1.6600	1.6600	1.6700	1.6687	-0.0043	-0.0013	1.6687	-0.0047	-0.0015
13-Dec-96	-1.00	1.6800	1.6700	1.7000	1.6930	-0.0047	0.0018	1.6931	-0.0033	0.0022
10-Jan-97	-0.05	1.6900	1.6800	1.7000	1.6965	-0.0022	0.0012	1.6965	-0.0015	0.0016
14-Feb-97	-0.10	1.6300	1.6200	1.6500	1.6467	-0.0064	6.4090e-03	1.6467	-0.0059	7.5105e-04
14-Mar-97	-1.00	1.6200	1.6000	1.6400	1.6346	-0.0056	2.5624e-03	1.6348	-0.0084	-8.5893e-04
11-Apr-97	-0.05	1.6400	1.6300	1.6500	1.6501	-0.0034	4.1495e-03	1.6501	-0.0020	0.0011
16-May-97	-0.50	1.6400	1.6300	1.6500	1.6486	-0.0028	0.00044	1.6487	-0.0018	8.5781e-04
13-Jun-97	-0.05	1.6400	1.6200	1.6500	1.6536	-0.0036	-6.4134e-03	1.6536	-0.0030	-2.4493e-04
11-Jul-97	-0.20	1.6900	1.6700	1.7000	1.7006	-0.0024	-1.3268e-03	1.7006	-8.5172e-04	7.3480e-04
15-Aug-97	-1.00	1.6300	1.6100	1.6500	1.6597	-0.0103	-0.0028	1.6601	-0.0133	-0.0041
12-Sep-97	-0.05	1.5900	1.5700	1.6100	1.6162	-0.0083	-9.1614e-04	1.6162	-0.0057	1.6543e-04
10-Oct-97	-0.15	1.6200	1.6200	1.6300	1.6290	-0.0046	2.4326e-04	1.6290	-0.0050	9.403e-05
14-Nov-97	-0.05	1.7000	1.6800	1.7100	1.7152	-0.0039	-7.6662e-04	1.7152	-0.0046	-0.0011
12-Dec-97	-0.80	1.6800	1.6600	1.7000	1.7087	-0.0094	-0.0016	1.7090	-0.0109	-0.0022

Table 2.7: At Ivols - not normalised

expiry date	β	$K0$	$K-$	$K+$	K^*	hedge1	hedge2	BS^*_K	BS-hedge1	BS-hedge2
10-Feb-95	-0.05	1.5500	1.5500	1.5500	1.5655	-0.0099	-0.0098	1.5655	-0.0019	-0.0019
10-Mar-95	-0.05	1.6250	1.6000	1.6500	1.6433	-0.0034	0.0026	1.6433	-0.0117	-0.0012
13-Apr-95	-0.05	1.6250	1.6000	1.6500	1.6362	-0.0076	2.9536e-04	1.6362	-0.0137	-0.0026
12-May-95	-0.05	1.6250	1.6000	1.6500	1.6336	-0.0051	0.0014	1.6336	-0.0087	1.4186e-04
16-Jun-95	-0.05	1.6000	1.5900	1.6100	1.6025	-6.6567e-04	0.0018	1.6025	-9.9395e-04	0.0016
14-Jul-95	-0.05	1.5900	1.5800	1.6100	1.6090	-0.0076	-9.906e-05	1.6090	-0.0087	-3.4632e-04
11-Aug-95	-1.00	1.6000	1.5900	1.6200	1.6140	-0.0057	9.4236e-04	1.6141	-0.0082	2.6494e-04
15-Sep-95	-1.00	1.5600	1.5500	1.5700	1.5629	5.4000e-04	0.0030	1.5629	8.4312e-04	0.0030
13-Oct-95	-0.05	1.5800	1.5700	1.6000	1.5978	-0.0071	-2.968e-05	1.5978	-0.0078	-2.5255e-05
10-Nov-95	-1.00	1.5800	1.5700	1.5900	1.5922	-0.0033	2.1104e-04	1.5923	-0.0035	1.3857e-04
15-Dec-95	-1.00	1.5300	1.5200	1.5500	1.5490	-0.0077	1.4914e-04	1.5491	-0.0067	4.1628e-04
12-Jan-96	-0.05	1.5500	1.5400	1.5600	1.5620	-0.0031	4.2354e-04	1.5620	-0.0049	-2.4218e-04
16-Feb-96	-0.05	1.5400	1.5300	1.5500	1.5481	-0.0016	-5.3946e-04	1.5481	-0.0016	-5.6713e-04
15-Mar-96	-0.05	1.5300	1.5200	1.5400	1.5408	-0.0034	4.2219e-04	1.5408	-0.0025	7.9589e-04
12-Apr-96	-1.00	1.5200	1.5100	1.5200	1.5300	-0.0017	-0.0015	1.5301	-9.730e-05	0.1934e-04
10-May-96	-1.00	1.5200	1.5100	1.5200	1.5340	-0.0020	-0.0019	1.5341	-0.0040	-0.0039
14-Jun-96	-1.00	1.5400	1.5300	1.5600	1.5560	-0.0068	0.0013	1.5561	-0.0023	0.0022
12-Jul-96	-1.00	1.5500	1.5400	1.5600	1.5631	-0.0032	-0.00071	1.5632	-3.7666e-04	8.8824e-04
16-Aug-96	-0.05	1.5500	1.5500	1.5500	1.5490	0.00118	0.0011	1.5490	0.0011	0.0011
13-Sep-96	-1.00	1.5600	1.5600	1.5700	1.5694	-0.0055	-5.5770e-04	1.5694	-0.0079	-0.0014
11-Oct-96	-0.05	1.5600	1.5600	1.5800	1.5683	-0.0036	-1.8777e-04	1.5683	-0.0029	0.8225e-04
15-Nov-96	-0.05	1.6600	1.6600	1.6700	1.6687	-0.0043	-0.0013	1.6687	-0.0047	-0.0015
13-Dec-96	-1.00	1.6800	1.6700	1.7000	1.6930	-0.0047	0.0018	1.6931	-0.0033	0.0022
10-Jan-97	-0.35	1.6900	1.6800	1.7000	1.6965	-0.0022	0.0012	1.6965	-0.0015	0.0016
14-Feb-97	-0.95	1.6300	1.6200	1.6500	1.6466	-0.0064	6.2673e-04	1.6467	-0.0059	7.5105e-04
14-Mar-97	-0.05	1.6200	1.6000	1.6400	1.6348	-0.0055	2.9713e-04	1.6348	-0.0084	-8.5893e-04
11-Apr-97	-1.00	1.6400	1.6300	1.6500	1.6500	-0.0034	3.9200e-04	1.6501	-0.0020	0.0011
16-May-97	-1.00	1.6400	1.6300	1.6500	1.6486	-0.0028	4.3795e-04	1.6487	-0.0018	8.5781e-04
13-Jun-97	-0.85	1.6400	1.6200	1.6500	1.6534	-0.0036	-6.8705e-04	1.6536	-0.0030	-2.4493e-04
11-Jul-97	-1.00	1.6900	1.6700	1.7000	1.7005	-0.0024	-1.5277e-04	1.7006	-8.5172e-04	7.3480e-04
15-Aug-97	-0.05	1.6300	1.6100	1.6500	1.6601	-0.0101	-0.0027	1.6601	-0.0133	-0.0041
12-Sep-97	-0.10	1.5900	1.5700	1.6100	1.6162	-0.0083	-9.1969e-04	1.6162	-0.0057	1.6543e-04
10-Oct-97	-0.05	1.6200	1.6200	1.6300	1.6290	-0.0046	2.4399e-04	1.6290	-0.0050	0.9403e-04
14-Nov-97	-0.05	1.7000	1.6800	1.7100	1.7152	-0.0039	-7.6662e-04	1.7152	-0.0046	-0.0011
12-Dec-97	-0.35	1.6800	1.6600	1.7000	1.7088	-0.0093	-0.0016	1.7090	-0.0109	-0.0022

Table 2.9: At Ivols - normalised

expiry date	β	$K0$	$K-$	$K+$	K^*	hedge1	hedge2	BS^*_K	BS-hedge1	BS-hedge2
10-Feb-95	-0.05	1.5500	1.5500	1.5500	1.5655	-0.0099	-0.0098	1.5655	-0.0019	-0.0019
10-Mar-95	-0.05	1.6250	1.6000	1.6500	1.6433	-0.0034	0.0026	1.6433	-0.012	-0.0012
13-Apr-95	-0.05	1.6250	1.6000	1.6500	1.6362	-0.0076	2.9536e-04	1.6362	-0.014	-0.0026
12-May-95	-0.05	1.6250	1.6000	1.6500	1.6336	-0.0051	0.0014	1.6336	-0.0087	1.4186e-04
16-Jun-95	-0.05	1.6000	1.5900	1.6100	1.6025	-6.6567e-04	0.0018	1.6025	-9.9395e-04	0.0016
14-Jul-95	-0.05	1.5900	1.5800	1.6100	1.6090	-0.0076	-9.906e-05	1.6090	-0.0087	-3.4632e-04
11-Aug-95	-1.00	1.6000	1.5900	1.6200	1.6140	-0.0057	9.4236e-04	1.6141	-0.0082	2.6494e-04
15-Sep-95	-1.00	1.5600	1.5500	1.5700	1.5629	5.4465e-04	0.0030	1.5629	8.4312e-04	0.0030
13-Oct-95	-0.05	1.5800	1.5700	1.6000	1.5978	-0.0071	-0.2968e-04	1.5978	-0.0078	-2.5255e-04
10-Nov-95	-1.00	1.5800	1.5700	1.5900	1.5922	-0.0033	2.1104e-04	1.5923	-0.0035	1.3857e-04
15-Dec-95	-0.90	1.5300	1.5200	1.5500	1.5490	-0.0077	1.5196e-04	1.5491	-0.0067	4.1628e-04
12-Jan-96	-0.05	1.5500	1.5400	1.5600	1.5620	-0.0031	4.2354e-04	1.5620	-0.0049	-2.4218e-04
16-Feb-96	-1.00	1.5400	1.5300	1.5500	1.5480	-0.0016	-5.4883e-04	1.5481	-0.0016	-5.6713e-04
15-Mar-96	-0.05	1.5300	1.5200	1.5400	1.5408	-0.0034	4.2219e-04	1.5408	-0.0025	7.9589e-04
12-Apr-96	-1.00	1.5200	1.5100	1.5200	1.5300	-0.0017	-0.0015	1.5301	-9.730e-05	1.934e-05
10-May-96	-1.00	1.5200	1.5100	1.5200	1.5340	-0.0020	-0.0019	1.5341	-0.0040	-0.0040
14-Jun-96	-0.70	1.5400	1.5300	1.5600	1.5560	-0.0068	0.0013	1.5561	-0.0023	0.0022
12-Jul-96	-0.05	1.5500	1.5400	1.5600	1.5632	-0.0032	-6.8866e-04	1.5632	-3.7666e-04	8.8824e-04
16-Aug-96	-0.05	1.5500	1.5500	1.5500	1.5490	0.0012	0.0011	1.5490	0.0011	0.0011
13-Sep-96	-1.00	1.5600	1.5600	1.5700	1.5694	-0.0055	-5.5770e-04	1.5694	-0.0079	-0.0014
11-Oct-96	-0.05	1.5600	1.5600	1.5800	1.5683	-0.0036	-1.8777e-04	1.5683	-0.0029	8.225e-05
15-Nov-96	-0.05	1.6600	1.6600	1.6700	1.6687	-0.0043	-0.0013	1.6687	-0.0047	-0.0015
13-Dec-96	-1.00	1.6800	1.6700	1.7000	1.6930	-0.0047	0.0018	1.6931	-0.0033	0.0022
10-Jan-97	-0.05	1.6900	1.6800	1.7000	1.6965	-0.0022	0.0012	1.6965	-0.0015	0.0016
14-Feb-97	-0.65	1.6300	1.6200	1.6500	1.6466	-0.0064	6.3173e-04	1.6467	-0.0059	7.5105e-04
14-Mar-97	-0.40	1.6400	1.6300	1.6500	1.6500	-0.0034	0.0004	1.6501	-0.0020	0.0011
11-Apr-97	-0.95	1.6400	1.6300	1.6500	1.6486	-0.0028	4.3854e-04	1.6487	-0.0018	8.5781e-04
16-May-97	-0.55	1.6400	1.6200	1.6500	1.6535	-0.0036	-6.6991e-04	1.6536	-0.0030	-2.4493e-04
13-Jun-97	-1.00	1.6900	1.6700	1.7000	1.7005	-0.0024	-1.5277e-04	1.7006	-8.5172e-04	7.3480e-04
11-Jul-97	-1.00	1.6300	1.6100	1.6500	1.6597	-0.0103	-0.0028	1.6601	-0.0139	-0.0041
15-Aug-97	-0.05	1.5900	1.5700	1.6100	1.6162	-0.0083	-9.1614e-04	1.6162	-0.0057	1.6543e-04
12-Sep-97	-0.05	1.6100	1.5900	1.6200	1.6059	-0.0023	6.7243e-04	1.6059	-0.0031	7.6453e-04
10-Oct-97	-0.05	1.6200	1.6200	1.6300	1.6290	-0.0046	2.4399e-04	1.6290	-0.0050	9.403e-05
14-Nov-97	-0.05	1.7000	1.6800	1.7100	1.7152	-0.0039	-7.6662e-04	1.7152	-0.0046	-0.0011
12-Dec-97	-0.85	1.6800	1.6600	1.7000	1.7087	-0.0094	-0.0016	1.7090	-0.0109	-0.0022

Chapter 3

Conclusion and Future Research

In order to hedge barrier options one can use either dynamic hedging or static hedging. Dynamic hedging can be really expensive due to the transaction costs. On the contrary, static hedging does not confront such kind of problem since the strategy is to create the portfolio and wait. Until now there have been either analytical or numerical developments of static hedging. In this thesis we provide the analytical formula of the static hedge of an Up-and-Out-Put option under the Constant Elasticity of Variance with drift ($\mu \neq 0$) model. What is more, we test numerically our model and we compare it to the Black-Scholes model, showing that our model provides a better approximation of the static hedge. However there could be some improvements of the numerical method we used to test the static hedge and more specifically for the calibration of the parameters of the CEV model. One might notice that the values for β "jump" quite a lot and many times they obtain the extreme values {-1,-0.05}. In order to calibrate β we considered that δ is fixed and equal to $\Sigma_0 S^{-\beta}$ where Σ_0 is the implied volatility and then we used an optimisation technique in order to calibrate β. Instead of this approach one should optimise over the pair (δ,β). An even better improvement would be to use the SABR model instead of the CEV model. It is well known that the CEV model does not represent completely the volatility smile that the FX market appears. Therefore, in order to 'flip' the right wing of the volatility smile one should make δ stochastic and consequently get the SABR

model.Finally, we had to deal with the problem of the data deficit. Since we were not able to find a database to provide barrier options we did not have the prices of the options which would provide a much better overview of how good our static hedge is.

Appendix

```
C(1) = 2;          %feb
C(2) = 79;         %mar
C(3) = 229;        %apr
C(4) = 343;        %may
C(5) = 475;        %jun
C(6) = 607;        %jul
C(7) = 721;        %aug
C(8) = 866;        %sept
C(9) = 979;        %oct
C(10) = 1111;      %nov
C(11) = 1249;      %dec
C(12) = 1356;      %jan
C(13) = 1476;      %feb
C(14) = 1590;      %mar
C(15) = 1722;      %apr
C(16) = 1842;      %may
C(17) = 1974;      %jun
C(18) = 2094;      %jul
C(19) = 2219;      %aug
C(20) = 2339;      %sep
C(21) = 2452;      %oct
C(22) = 2611;      %nov
C(23) = 2703;      %dec
C(24) = 2832;      %jan
C(25) = 2955;      %feb
C(26) = 3074;      %mar
C(27) = 3193;      %apr
C(28) = 3319;      %may
C(29) = 3457;      %jun
C(30) = 3591;      %jul
C(31) = 3693;      %aug
C(32) = 3813;      %sep
C(33) = 3935;      %oct
C(34) = 4082;      %nov
C(35) = 4211;      %dec
```

```
A=zeros(35,10);

for i = 1:35

    [beta,Kzero_n,Kminus_n,Kplus_n,Kbar_n,hedge1,hedge2,
     Kbar_BS,hedge1_BS,hedge2_BS]=final_code(C(i));


    A(i,:)=[beta,Kzero_n,Kminus_n,Kplus_n,Kbar_n,hedge1,
     hedge2,Kbar_BS,hedge1_BS,hedge2_BS];

end

for i=1:35
dzero(i)=abs(A(i,2)-A(i,5));
dplus(i)=abs(A(i,4)-A(i,5));
end


m1(:,1)=abs(A(:,2)-A(:,5));
m1(:,2)=-A(:,6);
m1(:,3)=abs(A(:,4)-A(:,5));
m1(:,4)=A(:,7);
m1(:,5)=abs(A(:,2)-A(:,8));
m1(:,6)=-A(:,6);
m1(:,7)=abs(A(:,4)-A(:,8));
m1(:,8)=A(:,10);

% scatter(m1(:,1),m1(:,2),20,'b','filled'), hold on
% scatter(m1(:,3),m1(:,4),20,'b','filled'), hold on
%
% scatter(m1(:,5),m1(:,6),20,'r','filled'), hold on
% scatter(m1(:,7),m1(:,8),20,'r','filled'), hold on
```

```
function x=phi(z)
x=1/2+1/2*erf(z/sqrt(2));
```

```
function [beta]=minimiserIvolAtdivided(index)

%Kbar_BS_test,Kbar_BS_theoretic,

[numeric,txt,raw]=xlsread('full_database');

raw2 = raw(:,1:2);

rraw = raw2(2:size(raw2,1),:);

col1 = datenum(rraw(:,1), 'dd/mm/yyyy');

```

```
col2 = datenum(rraw(:,2), 'dd/mm/yyyy');
gio = horzcat(col1,col2, numeric);


day2=gio(index-1,2);  %the day of MATURITY which you are
  looking for the options

a=[];

for i=1:length(col2)
    if find(col2(i)==day2)  %finds the rows which have
      second day which is the same to day2 at the excel
      sheet
     a=[a;i];
    end
end



date1=zeros(length(a),1);
date2=zeros(length(a),1);
diff=zeros(length(a),1);


for j=1:length(a)
   date1(j)=gio(a(j),1);         %stores the quote days of
     the option that expire at day2
   date2(j)=gio(a(j),2);         %is supposed to be the same
     (no practical use-just check)+i use it in the diff
end

diff = date2-date1;


date1new=zeros(length(a),1);
date2new=zeros(length(a),1);

k=1;
for j=1:length(a)
   if (diff(j)<=15)
       date1new(k)=date1(j);     %stores the quote days that
          are less than 15 days away from the expiry day
       date2new(k)=date2(j);     %supposed to be the same
         and same with date2 but with less inputs
       k=k+1;
   end
end

date1new = date1new(find(date1new~=0));
```

```
55 date2new = date2new(find(date2new~=0));
56
57 dif1 = date2new-date1new;          %this is the difference
     between the quote days of the option and the expiry day -
     should be <=15
58
59 x=zeros(length(date1new),2);
60 x(:,1) = date1new;
61 x(:,2) = date2new;
62
63   for i=1:length(x)
64       y(i) = find(gio(:,1)==x(i,1) & gio(:,2)==x(i,2));
65   end
66
67   matrix=zeros(length(y),12);    %matrix->1col:qdate  2col:
       exp_date  3col:spot  4col:Strike_At  5col:VolAtAsk  6col
       :Vol_At_Bid
68
69                                  %7col:strike_in
70
71   for i=1:length(y)              %matrix->8col:Vol_In_Ask  9
       col:Vol_In_Bid  10col:Strike_Out  11col:Vol_Out_Ask  12
       col:Vol_Out_Bid
72   matrix(i,:)=gio(y(i),:);
73   end
74
75                                %15-day option found
76  %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
77
78   St1=matrix(1,3);               %the first day of the
       option
79
80   Kminus=matrix(1,10);
81   Kzero=matrix(1,4);
82   Kplus=matrix(1,7);
83
84   IVIA=matrix(1,8);%i.vol in the money ask
85
86   IVIB=matrix(1,9);%i.vol in the money bid
87
88   IVOA=matrix(1,11);%i.vol out-ask
89
90   IVOB=matrix(1,12);%i.vol out-bid
91
92   IVAA=matrix(1,5);%i.vol at-ask
93   IVAB=matrix(1,6);%i.vol at the money-bid
94
95   t_calendar=(matrix(1,2)-matrix(1,1)); %the time to
       maturity = T(maturity) - t1
```

```
96
97    if (t_calendar>=0 && t_calendar<=4)
98          tau=t_calendar;
99    end
100   if (t_calendar>=7 && t_calendar<=11)
101         tau=t_calendar-2;
102   end
103   if (t_calendar==14 || t_calendar==15)
104         tau=t_calendar-4;
105   end
106
107   tau=tau/252;
108
109   mu=1/tau*log(Kzero/St1);
110
111   Ktildeminus=Kminus*exp(-mu*tau);
112   Ktildezero=Kzero*exp(-mu*tau);
113   Ktildeplus=Kplus*exp(-mu*tau);
114
115   callAm=eurocall(St1,Ktildeminus,IVOA,tau); %BS-model
116   callBm=eurocall(St1,Ktildeminus,IVOB,tau);
117   callm=0.5*(callAm+callBm);
118
119   callAz=eurocall(St1,Ktildezero,IVAA,tau);
120   callBz=eurocall(St1,Ktildezero,IVAB,tau);
121   callz=0.5*(callAz+callBz);
122
123   callAp=eurocall(St1,Ktildeplus,IVIA,tau);
124   callBp=eurocall(St1,Ktildeplus,IVIB,tau);
125   callp=0.5*(callAp+callBp);
126
127
128   beta=[-1.0:0.05:-0.05];
129   cevcm=zeros(length(beta),1);
130   cevcz=zeros(length(beta),1);
131   cevcp=zeros(length(beta),1);
132
133   diff=zeros(length(beta),1);
134
135 if (callAm-callBm==0 || callAz-callBz==0 || callAp-callBp
      ==0)
136     %did this in order to check the BS model
137
138     for i=1:length(beta)
139     sigma=(IVAA+IVAB)*(1.0/2)*S^(-beta(i));
140
141
142     T_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu*beta(i
          )*T)-1);
```

```
143
144     cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildeminus,1.0);
145     cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildezero,1.0);
146     cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildeplus,1.0);
147
148     dif3(i)=((cevcm(i)-callm))^2+((cevcz(i)-callz))^2+((
          cevcp(i)-callp))^2;
149     end
150
151 else
152     for i=1:length(beta)
153          sigma=(IVAA+IVAB)*(1.0/2)*St1^(-beta(i));
154
155          %sigma=1.0;
156          tau_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu
               *beta(i)*tau)-1);
157          %T_cur = T*0.01;
158          cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildeminus,1.0);%CEV_model
159          cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildezero,1.0);
160          cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildeplus,1.0);
161
162          diff(i)=((cevcm(i)-callm)/(callAm-callBm))^2+((
               cevcz(i)-callz)/(callAz-callBz))^2+((cevcp(i)-
               callp)/(callAp-callBp))^2;
163
164
165     end
166 end
167
168
169
170  plot(beta,diff),title('at Ivol used-normalised'),xlabel('
       beta'),ylabel('difference')
171  [dmin,i]=min(diff);
172  beta=beta(i);
```

```
1 function [beta]=minimiserIvolAtnotdivided(index)
2
3
4
5 [numeric,txt,raw]=xlsread('full_database');
6
7 raw2 = raw(:,1:2);
```

```
8
9  rraw = raw2(2:size(raw2,1),:);
10
11 col1 = datenum(rraw(:,1), 'dd/mm/yyyy');
12
13 col2 = datenum(rraw(:,2), 'dd/mm/yyyy');
14 gio = horzcat(col1,col2, numeric);
15
16 %day2=gio(1876-1,2);          %the day of MATURITY which i am
      looking for the options
17 day2=gio(index-1,2);
18
19 a=[];
20
21 for i=1:length(col2)
22     if find(col2(i)==day2)  %finds the rows which have
         second day which is the same to day2 at the excel
         sheet
23      a=[a;i];
24     end
25 end
26
27
28
29 date1=zeros(length(a),1);
30 date2=zeros(length(a),1);
31 diff=zeros(length(a),1);
32
33
34 for j=1:length(a)
35    date1(j)=gio(a(j),1);          %stores the quote days of
        the option that expire at day2
36    date2(j)=gio(a(j),2);          %is supposed to be the same
        (no practical use-just check)+i use it in the diff
37 end
38
39 diff = date2-date1;
40
41
42 date1new=zeros(length(a),1);
43 date2new=zeros(length(a),1);
44
45 k=1;
46 for j=1:length(a)
47    if (diff(j)<=15)
48        date1new(k)=date1(j);     %stores the quote days that
            are less than 15 days away from the expiry day
49        date2new(k)=date2(j);     %supposed to be the same
            and same with date2 but with less inputs
```

```
50          k=k+1;
51      end
52  end
53
54  date1new = date1new(find(date1new~=0));
55  date2new = date2new(find(date2new~=0));
56
57  dif1 = date2new-date1new;           %this is the difference
      between the quote days of the option and the expiry day -
      should be <=15
58
59  x=zeros(length(date1new),2);
60  x(:,1) = date1new;
61  x(:,2) = date2new;
62
63    for i=1:length(x)
64        y(i) = find(gio(:,1)==x(i,1) & gio(:,2)==x(i,2));
65    end
66
67    matrix=zeros(length(y),12);    %matrix->1col:qdate  2col:
        exp_date  3col:spot  4col:Strike_At  5col:VolAtAsk  6col
        :Vol_At_Bid
68
69                                   %7col:strike_in
70
71    for i=1:length(y)              %matrix->8col:Vol_In_Ask  9
        col:Vol_In_Bid  10col:Strike_Out  11col:Vol_Out_Ask  12
        col:Vol_Out_Bid
72    matrix(i,:)=gio(y(i),:);
73    end
74
75                                   %15-day option found
76  %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
77
78    St1=matrix(1,3);               %the first day of the
        option
79
80    Kminus=matrix(1,10);
81    Kzero=matrix(1,4);
82    Kplus=matrix(1,7);
83
84    IVIA=matrix(1,8);%i.vol in the money ask
85
86    IVIB=matrix(1,9);%i.vol in the money bid
87
88    IVOA=matrix(1,11);%i.vol out-ask
89
90    IVOB=matrix(1,12);%i.vol out-bid
91
```

```
92    IVAA=matrix(1,5);%i.vol at-ask
93    IVAB=matrix(1,6);%i.vol at the money-bid
94
95    t_calendar=(matrix(1,2)-matrix(1,1)); %the time to
        maturity = T(maturity) - t1
96
97    if (t_calendar>=0 && t_calendar<=4)
98          tau=t_calendar;
99    end
100   if (t_calendar>=7 && t_calendar<=11)
101         tau=t_calendar-2;
102   end
103   if (t_calendar==14 || t_calendar==15)
104         tau=t_calendar-4;
105   end
106
107   tau=tau/252;
108
109   mu=1/tau*log(Kzero/St1);
110
111
112   Ktildeminus=Kminus*exp(-mu*tau);
113   Ktildezero=Kzero*exp(-mu*tau);
114   Ktildeplus=Kplus*exp(-mu*tau);
115
116   callAm=eurocall(St1,Ktildeminus,IVOA,tau); %BS-model
117   callBm=eurocall(St1,Ktildeminus,IVOB,tau);
118   callm=0.5*(callAm+callBm);
119
120   callAz=eurocall(St1,Ktildezero,IVAA,tau);
121   callBz=eurocall(St1,Ktildezero,IVAB,tau);
122   callz=0.5*(callAz+callBz);
123
124   callAp=eurocall(St1,Ktildeplus,IVIA,tau);
125   callBp=eurocall(St1,Ktildeplus,IVIB,tau);
126   callp=0.5*(callAp+callBp);
127
128
129   beta=[-1.0:0.05:-0.05];
130   cevcm=zeros(length(beta),1);
131   cevcz=zeros(length(beta),1);
132   cevcp=zeros(length(beta),1);
133
134   diff=zeros(length(beta),1);
135
136 if (callAm-callBm==0 || callAz-callBz==0 || callAp-callBp
      ==0)
137     %did this in order to check the BS model
138
```

```
139     for i=1:length(beta)
140     sigma=(IVAA+IVAB)*(1.0/2)*S^(-beta(i));
141     %sigma=(IVAA+IVAB+IVOA+IVOB+IVIA+IVIB)*(1.0/6)*S^(-beta
          (i));
142
143     T_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu*beta(i
          )*T)-1);
144
145     cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildeminus,1.0);
146     cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildezero,1.0);
147     cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildeplus,1.0);
148
149     dif3(i)=((cevcm(i)-callm))^2+((cevcz(i)-callz))^2+((
          cevcp(i)-callp))^2;
150     end
151
152 else
153     for i=1:length(beta)
154          sigma=(IVAA+IVAB)*(1.0/2)*St1^(-beta(i));
155          %sigma=(IVAA+IVAB+IVOA+IVOB+IVIA+IVIB)*(1.0/6)*S
               ^(-beta(i));
156          %sigma=1.0;
157          tau_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu
               *beta(i)*tau)-1);
158          %T_cur = T*0.01;
159          cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildeminus,1.0);%CEV_model
160          cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildezero,1.0);
161          cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildeplus,1.0);
162
163         %diff(i)=((cevcm(i)-callm)/(callAm-callBm))^2+
164         %((cevcz(i)-callz)/(callAz-callBz))^2
165         %+((cevcp(i)-callp)/(callAp-callBp))^2;
166         diff(i)=((cevcm(i)-callm))^2+((cevcz(i)-callz))
              ^2+((cevcp(i)-callp))^2;
167
168     end
169 end
170
171
172
173 plot(beta,diff),title('at Ivol used-not normalised'),xlabel
      ('beta'),ylabel('difference')
174 [dmin,i]=min(diff);
```

```
beta=beta(i);
```

```
function [beta]=minimiserIvolalldivided(index)



[numeric,txt,raw]=xlsread('full_database');

raw2 = raw(:,1:2);

rraw = raw2(2:size(raw2,1),:);

col1 = datenum(rraw(:,1), 'dd/mm/yyyy');

col2 = datenum(rraw(:,2), 'dd/mm/yyyy');
gio = horzcat(col1,col2, numeric);

%day2=gio(1876-1,2);          %the day of MATURITY which i am
   looking for the options
day2=gio(index-1,2);

a=[];

for i=1:length(col2)
    if find(col2(i)==day2)  %finds the rows which have
      second day which is the same to day2 at the excel
      sheet
     a=[a;i];
    end
end



date1=zeros(length(a),1);
date2=zeros(length(a),1);
diff=zeros(length(a),1);


for j=1:length(a)
   date1(j)=gio(a(j),1);          %stores the quote days of
     the option that expire at day2
   date2(j)=gio(a(j),2);          %is supposed to be the same
      (no practical use-just check)+i use it in the diff
end

diff = date2-date1;


date1new=zeros(length(a),1);
```

```
date2new=zeros(length(a),1);

k=1;
for j=1:length(a)
   if (diff(j)<=15)
       date1new(k)=date1(j);     %stores the quote days that
          are less than 15 days away from the expiry day
       date2new(k)=date2(j);     %supposed to be the same
         and same with date2 but with less inputs
       k=k+1;
   end
end

date1new = date1new(find(date1new~=0));
date2new = date2new(find(date2new~=0));

dif1 = date2new-date1new;          %this is the difference
  between the quote days of the option and the expiry day -
  should be <=15

x=zeros(length(date1new),2);
x(:,1) = date1new;
x(:,2) = date2new;

  for i=1:length(x)
      y(i) = find(gio(:,1)==x(i,1) & gio(:,2)==x(i,2));  w
  end

  matrix=zeros(length(y),12);   %matrix->1col:qdate  2col:
    exp_date  3col:spot  4col:Strike_At  5col:VolAtAsk  6col
    :Vol_At_Bid

                                %7col:strike_in

  for i=1:length(y)             %matrix->8col:Vol_In_Ask  9
    col:Vol_In_Bid  10col:Strike_Out  11col:Vol_Out_Ask  12
    col:Vol_Out_Bid
  matrix(i,:)=gio(y(i),:);
  end

                              %15-day option found
 %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%

  St1=matrix(1,3);              %the first day of the
    option

  Kminus=matrix(1,10);
  Kzero=matrix(1,4);
  Kplus=matrix(1,7);
```

```
83
84  IVIA=matrix(1,8);%i.vol in the money ask
85
86  IVIB=matrix(1,9);%i.vol in the money bid
87
88  IVOA=matrix(1,11);%i.vol out-ask
89
90  IVOB=matrix(1,12);%i.vol out-bid
91
92  IVAA=matrix(1,5);%i.vol at-ask
93  IVAB=matrix(1,6);%i.vol at the money-bid
94
95  t_calendar=(matrix(1,2)-matrix(1,1)); %the time to
      maturity = T(maturity) - t1
96
97  if (t_calendar>=0 && t_calendar<=4)
98          tau=t_calendar;
99  end
100 if (t_calendar>=7 && t_calendar<=11)
101         tau=t_calendar-2;
102 end
103 if (t_calendar==14 || t_calendar==15)
104         tau=t_calendar-4;
105 end
106
107 tau=tau/252;
108
109 mu=1/tau*log(Kzero/St1);
110
111
112 Ktildeminus=Kminus*exp(-mu*tau);
113 Ktildezero=Kzero*exp(-mu*tau);
114 Ktildeplus=Kplus*exp(-mu*tau);
115
116 callAm=eurocall(St1,Ktildeminus,IVOA,tau); %BS-model
117 callBm=eurocall(St1,Ktildeminus,IVOB,tau);
118 callm=0.5*(callAm+callBm);
119
120 callAz=eurocall(St1,Ktildezero,IVAA,tau);
121 callBz=eurocall(St1,Ktildezero,IVAB,tau);
122 callz=0.5*(callAz+callBz);
123
124 callAp=eurocall(St1,Ktildeplus,IVIA,tau);
125 callBp=eurocall(St1,Ktildeplus,IVIB,tau);
126 callp=0.5*(callAp+callBp);
127
128
129 beta=[-1.0:0.05:-0.05];
130 cevcm=zeros(length(beta),1);
```

```
131    cevcz=zeros(length(beta),1);
132    cevcp=zeros(length(beta),1);
133
134    diff=zeros(length(beta),1);
135
136  if (callAm-callBm==0 || callAz-callBz==0 || callAp-callBp
       ==0)
137      %did this in order to check the BS model
138
139      for i=1:length(beta)
140      %sigma=(IVAA+IVAB)*(1.0/2)*S^(-beta(i));
141      sigma=(IVAA+IVAB+IVOA+IVOB+IVIA+IVIB)*(1.0/6)*S^(-beta(
           i));
142      %sigma=1.0;
143      T_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu*beta(i
           )*T)-1);
144
145      cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
           Ktildeminus,1.0);
146      cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
           Ktildezero,1.0);
147      cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
           Ktildeplus,1.0);
148
149      dif3(i)=((cevcm(i)-callm))^2+((cevcz(i)-callz))^2+((
           cevcp(i)-callp))^2;
150      end
151
152  else
153      for i=1:length(beta)
154           %sigma=(IVAA+IVAB)*(1.0/2)*St1^(-beta(i));
155           sigma=(IVAA+IVAB+IVOA+IVOB+IVIA+IVIB)*(1.0/6)*St1
               ^(-beta(i));
156
157           tau_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu
               *beta(i)*tau)-1);
158
159           cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildeminus,1.0);%CEV_model
160           cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildezero,1.0);
161           cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildeplus,1.0);
162
163          diff(i)=((cevcm(i)-callm)/(callAm-callBm))^2+((
              cevcz(i)-callz)/(callAz-callBz))^2+((cevcp(i)-
              callp)/(callAp-callBp))^2;
164          %diff(i)=((cevcm(i)-callm))^2+((cevcz(i)-callz))
              ^2+((cevcp(i)-callp))^2;
```

```
165
166     end
167 end
168
169
170
171 plot(beta,diff),title('all Ivols used-normalised'),xlabel('
    beta'),ylabel('difference')
172 [dmin,i]=min(diff);
173 beta=beta(i);
```

```
1  function [beta]=minimiserIvolallnotdivided(index)
2
3
4
5  [numeric,txt,raw]=xlsread('full_database');
6
7  raw2 = raw(:,1:2);
8
9  rraw = raw2(2:size(raw2,1),:);
10
11 col1 = datenum(rraw(:,1), 'dd/mm/yyyy');
12
13 col2 = datenum(rraw(:,2), 'dd/mm/yyyy');
14 gio = horzcat(col1,col2, numeric);
15
16 %day2=gio(1876-1,2);          %the day of MATURITY which i am
       looking for the options
17 day2=gio(index-1,2);
18
19 a=[];
20
21 for i=1:length(col2)
22     if find(col2(i)==day2)  %finds the rows which have
         second day which is the same to day2 at the excel
         sheet
23      a=[a;i];
24     end
25 end
26
27
28
29 date1=zeros(length(a),1);
30 date2=zeros(length(a),1);
31 diff=zeros(length(a),1);
32
33
34 for j=1:length(a)
```

```
35      date1(j)=gio(a(j),1);          %stores the quote days of
          the option that expire at day2
36      date2(j)=gio(a(j),2);          %is supposed to be the same
          (no practical use-just check)+i use it in the diff
37  end
38
39  diff = date2-date1;
40
41
42  date1new=zeros(length(a),1);
43  date2new=zeros(length(a),1);
44
45  k=1;
46  for j=1:length(a)
47      if (diff(j)<=15)
48          date1new(k)=date1(j);      %stores the quote days that
              are less than 15 days away from the expiry day
49          date2new(k)=date2(j);      %supposed to be the same
            and same with date2 but with less inputs
50          k=k+1;
51      end
52  end
53
54  date1new = date1new(find(date1new~=0));
55  date2new = date2new(find(date2new~=0));
56
57  dif1 = date2new-date1new;            %this is the difference
      between the quote days of the option and the expiry day -
      should be <=15
58
59  x=zeros(length(date1new),2);
60  x(:,1) = date1new;
61  x(:,2) = date2new;
62
63    for i=1:length(x)
64        y(i) = find(gio(:,1)==x(i,1) & gio(:,2)==x(i,2));
65    end
66
67    matrix=zeros(length(y),12);    %matrix->1col:qdate  2col:
        exp_date  3col:spot  4col:Strike_At  5col:VolAtAsk  6col
        :Vol_At_Bid
68
69                                   %7col:strike_in
70
71    for i=1:length(y)              %matrix->8col:Vol_In_Ask  9
        col:Vol_In_Bid  10col:Strike_Out  11col:Vol_Out_Ask  12
        col:Vol_Out_Bid
72    matrix(i,:)=gio(y(i),:);
73    end
```

```
74
75                                  %15-day option found
76  %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
77
78   St1=matrix(1,3);                %the first day of the
       option
79
80   Kminus=matrix(1,10);
81   Kzero=matrix(1,4);
82   Kplus=matrix(1,7);
83
84   IVIA=matrix(1,8);%i.vol in the money ask
85
86   IVIB=matrix(1,9);%i.vol in the money bid
87
88   IVOA=matrix(1,11);%i.vol out-ask
89
90   IVOB=matrix(1,12);%i.vol out-bid
91
92   IVAA=matrix(1,5);%i.vol at-ask
93   IVAB=matrix(1,6);%i.vol at the money-bid
94
95   t_calendar=(matrix(1,2)-matrix(1,1)); %the time to
       maturity = T(maturity) - t1
96
97   if (t_calendar>=0 && t_calendar<=4)
98          tau=t_calendar;
99   end
100  if (t_calendar>=7 && t_calendar<=11)
101         tau=t_calendar-2;
102  end
103  if (t_calendar==14 || t_calendar==15)
104         tau=t_calendar-4;
105  end
106
107  tau=tau/252;
108
109  mu=1/tau*log(Kzero/St1);
110
111
112  Ktildeminus=Kminus*exp(-mu*tau);
113  Ktildezero=Kzero*exp(-mu*tau);
114  Ktildeplus=Kplus*exp(-mu*tau);
115
116  callAm=eurocall(St1,Ktildeminus,IVOA,tau); %BS-model
117  callBm=eurocall(St1,Ktildeminus,IVOB,tau);
118  callm=0.5*(callAm+callBm);
119
120  callAz=eurocall(St1,Ktildezero,IVAA,tau);
```

```
121   callBz=eurocall(St1,Ktildezero,IVAB,tau);
122   callz=0.5*(callAz+callBz);
123
124   callAp=eurocall(St1,Ktildeplus,IVIA,tau);
125   callBp=eurocall(St1,Ktildeplus,IVIB,tau);
126   callp=0.5*(callAp+callBp);
127
128
129   beta=[-1.0:0.05:-0.05];
130   cevcm=zeros(length(beta),1);
131   cevcz=zeros(length(beta),1);
132   cevcp=zeros(length(beta),1);
133
134   diff=zeros(length(beta),1);
135
136 if (callAm-callBm==0 || callAz-callBz==0 || callAp-callBp
      ==0)
137
138     %BSmodel (all IV's=0.1)
139     for i=1:length(beta)
140     %sigma=(IVAA+IVAB)*(1.0/2)*S^(-beta(i));
141     sigma=(IVAA+IVAB+IVOA+IVOB+IVIA+IVIB)*(1.0/6)*S^(-beta(
          i));
142
143     T_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu*beta(i
          )*T)-1);
144
145     cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildeminus,1.0);
146     cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildezero,1.0);
147     cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildeplus,1.0);
148
149     dif3(i)=((cevcm(i)-callm))^2+((cevcz(i)-callz))^2+((
          cevcp(i)-callp))^2;
150     end
151
152 else
153     for i=1:length(beta)
154          %sigma=(IVAA+IVAB)*(1.0/2)*St1^(-beta(i));
155          sigma=(IVAA+IVAB+IVOA+IVOB+IVIA+IVIB)*(1.0/6)*St1
               ^(-beta(i));
156
157          tau_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu
               *beta(i)*tau)-1);
158
159          cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildeminus,1.0);%CEV_model
```

```
160           cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
                Ktildezero,1.0);
161           cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
                Ktildeplus,1.0);
162
163          %diff(i)=((cevcm(i)-callm)/(callAm-callBm))^2+((
               cevcz(i)-callz)/(callAz-callBz))^2+((cevcp(i)-
               callp)/(callAp-callBp))^2;
164          diff(i)=((cevcm(i)-callm))^2+((cevcz(i)-callz))
               ^2+((cevcp(i)-callp))^2;
165
166      end
167  end
168
169
170
171  plot(beta,diff),title('all Ivols used-not normalised'),
       xlabel('beta'),ylabel('difference')
172
173  [dmin,i]=min(diff);
174  beta=beta(i);
```

```
1   function [ret_call]=CEVCallnPutPricen(beta,S,T,K,sigma)
2
3
4
5
6
7
8   r=0.0;
9   a=0.0;
10
11  if (S==0)
12      ret_call=0.0;
13
14  else
15
16
17  kstar=1.0/((sigma*sigma*T)*(beta)*(beta));
18  kbar=kstar;
19
20
21  x=kstar*((S)^(-2.0*beta));
22
23  y=kbar*(K^(-2.0*beta));
24  z=2.0+(1.0/abs(beta));
25
26  sum1=1.0-ncx2cdf(y,z,x);
27  %estimates the probability that the sum of the
```

```
%squares of Gaussian random variables with non-zero means
  and the same
%variance sigma^2 is less than some value


sum2=ncx2cdf(x,z-2,y);                          %P = NCX2CDF(X(at
  the values in X)
                                                %,V(degrees of
                                                  freedom),
                                                %DELTA(
                                                  noncentrality
                                                  parameter))


ret_call= S*sum1 - K*sum2;

end


end
```

```
function [beta,Kzero_n,Kminus_n,Kplus_n,Kbar_n,hedge1,
  hedge2,Kbar_BS,hedge1_BS,hedge2_BS]=finalcode(index)

%Kbar_BS_test,Kbar_BS_theoretic,

[numeric,txt,raw]=xlsread('full_database');

raw2 = raw(:,1:2);

rraw = raw2(2:size(raw2,1),:);

col1 = datenum(rraw(:,1), 'dd/mm/yyyy');

col2 = datenum(rraw(:,2), 'dd/mm/yyyy');
gio = horzcat(col1,col2, numeric);

day2=gio(index-1,2); %the day of MATURITY which we are
  looking for the options

a=[];

for i=1:length(col2)
    if find(col2(i)==day2)  %finds the rows which have
      second day which is the same to day2 at the excel
      sheet
     a=[a;i];
    end
end
```

```
25
26
27
28 date1=zeros(length(a),1);
29 date2=zeros(length(a),1);
30 diff=zeros(length(a),1);
31
32
33 for j=1:length(a)
34    date1(j)=gio(a(j),1);          %stores the quote days of
        the option that expire at day2
35    date2(j)=gio(a(j),2);
36 end
37
38 diff = date2-date1;
39
40
41 date1new=zeros(length(a),1);
42 date2new=zeros(length(a),1);
43
44 k=1;
45 for j=1:length(a)
46    if (diff(j)<=15)
47        date1new(k)=date1(j);     %stores the quote days that
            are less than 15 days away from the expiry day
48        date2new(k)=date2(j);
49        k=k+1;
50    end
51 end
52
53 date1new = date1new(find(date1new~=0));
54 date2new = date2new(find(date2new~=0));
55
56 dif1 = date2new-date1new           %this is the difference
     between the quote days of the option and the expiry day -
     should be <=15
57
58 x=zeros(length(date1new),2);
59 x(:,1) = date1new;
60 x(:,2) = date2new;
61
62   for i=1:length(x)
63       y(i) = find(gio(:,1)==x(i,1) & gio(:,2)==x(i,2));
64   end
65
66   matrix=zeros(length(y),12);    %matrix->1col:qdate  2col:
       exp_date  3col:spot  4col:Strike_At  5col:VolAtAsk  6col
       :Vol_At_Bid
67
```

```
68                                  %7col:strike_in
69
70  for i=1:length(y)               %matrix->8col:Vol_In_Ask  9
      col:Vol_In_Bid  10col:Strike_Out  11col:Vol_Out_Ask  12
      col:Vol_Out_Bid
71  matrix(i,:)=gio(y(i),:);
72  end
73
74                                                      %15-day
                                                          option
                                                          found
75  %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
76
77  St1=matrix(1,3);                %the first day of
78                                  %the option
79
80  Kminus=matrix(1,10);
81  Kzero=matrix(1,4);
82  Kplus=matrix(1,7);
83
84  IVIA=matrix(1,8);%i.vol in the money ask
85
86  IVIB=matrix(1,9);%i.vol in the money bid
87
88  IVOA=matrix(1,11);%i.vol out-ask
89
90  IVOB=matrix(1,12);%i.vol out-bid
91
92  IVAA=matrix(1,5);%i.vol at-ask
93  IVAB=matrix(1,6);%i.vol at the money-bid
94
95  t_calendar=(matrix(1,2)-matrix(1,1)); %the time to
      maturity = T(maturity) - t1
96
97  if (t_calendar>=0 && t_calendar<=4)
98          tau=t_calendar;
99  end
100 if (t_calendar>=7 && t_calendar<=11)
101         tau=t_calendar-2;
102 end
103 if (t_calendar==14 || t_calendar==15)
104         tau=t_calendar-4;
105 end
106
107 tau=tau/252;
108
109 mu=1/tau*log(Kzero/St1);
110 %mu=0;
111
```

```
112   Ktildeminus=Kminus*exp(-mu*tau);
113   Ktildezero=Kzero*exp(-mu*tau);
114   Ktildeplus=Kplus*exp(-mu*tau);
115
116   callAm=eurocall(St1,Ktildeminus,IVOA,tau); %BS-model
117   callBm=eurocall(St1,Ktildeminus,IVOB,tau);
118   callm=0.5*(callAm+callBm);
119
120   callAz=eurocall(St1,Ktildezero,IVAA,tau);
121   callBz=eurocall(St1,Ktildezero,IVAB,tau);
122   callz=0.5*(callAz+callBz);
123
124   callAp=eurocall(St1,Ktildeplus,IVIA,tau);
125   callBp=eurocall(St1,Ktildeplus,IVIB,tau);
126   callp=0.5*(callAp+callBp);
127
128
129   beta=[-1.0:0.05:-0.05];
130   cevcm=zeros(length(beta),1);
131   cevcz=zeros(length(beta),1);
132   cevcp=zeros(length(beta),1);
133
134   diff=zeros(length(beta),1);
135
136 if (callAm-callBm==0 || callAz-callBz==0 || callAp-callBp
      ==0)   %did this in order to check the BS model
137 %BSmodel (all IV's=0.1)
138     for i=1:length(beta)
139     %sigma=(IVAA+IVAB)*(1.0/2)*S^(-beta(i));
140     sigma=(IVAA+IVAB+IVOA+IVOB+IVIA+IVIB)*(1.0/6)*St1^(-
          beta(i));
141     %sigma=1.0;
142     T_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu*beta(i
          )*T)-1);
143     %T_cur = T*0.01;
144     cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildeminus,1.0);
145     cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildezero,1.0);
146     cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,T_cur,
          Ktildeplus,1.0);
147
148     %dif3(i)=((cevcm(i)-callm))^2+((cevcz(i)-callz))^2+((
          cevcp(i)-callp))^2
149     %; At implied volatilites-not normalised
150     diff3(i)=((cevcm(i)-callm)/(callAm-callBm))^2+((cevcz(i
          )-callz)/(callAz-callBz))^2+((cevcp(i)-callp)/(callAp-
          callBp))^2; %At implied volatilities-normalised
151     end
```

```
152
153 else
154     for i=1:length(beta)
155          %sigma=(IVAA+IVAB)*(1.0/2)*St1^(-beta(i));
156          sigma=(IVAA+IVAB+IVOA+IVOB+IVIA+IVIB)*(1.0/6)*St1
               ^(-beta(i));
157
158          tau_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu
               *beta(i)*tau)-1);
159
160          cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildeminus,1.0);%CEV_model
161          cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildezero,1.0);
162          cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
               Ktildeplus,1.0);
163
164         %diff(i)=((cevcm(i)-callm)/(callAm-callBm))^2+((
              cevcz(i)-callz)/(callAz-callBz))^2+((cevcp(i)-
              callp)/(callAp-callBp))^2;
165         diff(i)=((cevcm(i)-callm))^2+((cevcz(i)-callz))
              ^2+((cevcp(i)-callp))^2;
166
167     end
168 end
169
170
171
172 %plot(beta,diff) plot the performance of beta
173 [dmin,i]=min(diff);
174 beta(i);
175
176
177 %sigma=(IVAA+IVAB)*(1.0/2)*St1^(-beta(i));
178 sigma=(IVAA+IVAB+IVOA+IVOB+IVIA+IVIB)*(1.0/6)*St1^(-beta(i)
      );
179 tau_cur=(sigma*sigma/(2.0*mu*beta(i)))*(exp(2.0*mu*beta(i)*
      tau)-1);
180 cevcm(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,
      Ktildeminus,1.0)%CEV_model
181 cevcz(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,Ktildezero
      ,1.0);
182 cevcp(i)=CEV_CallnPutPrice_n(beta(i),St1,tau_cur,Ktildeplus
      ,1.0);
183
184 Bidminus=cevcm(i)-callBm          %we check these differences
       in order to see
185 Askminus=callAm - cevcm(i)       %if the model value is
      between the spread of the market values
```

```
186                                     %it is a measure of how
                                          well the model fits
187
188 Bidzero=cevcz(i)-callBz
189 Askzero=callAz - cevcz(i)
190
191 Bidplus=cevcp(i)-callBp
192 Askplus=callAp - cevcp(i)
193
194
195
196
197 if (beta(i)~=-1.0 && beta(i)~=-0.05)
198     disp('ok')
199 else
200     disp('beta is not ok')
201 end
202
203 beta=beta(i);
204
205
206                                     %beta_calibrated found
207 %~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~%
208
209 dim=size(matrix,1)-2;                          %that is in
      order to exclude the last two days
210
211
212 Smax=max(matrix(1:size(matrix,1)-2,3));     %finds the
      maximum value, which will be used to find the barrier-date
213
214
215 Saprox=(Smax-St1)*1.0/2 + St1;
216
217
218 ind=find(matrix(:,3)==Smax);                   %finds the index
       of the maximum date
219
220
221
222 b = abs(matrix(ind+1:size(matrix,1)-1,3)-Saprox);
223
224
225 c=zeros(length(b),2);
226
227
228 c(:,1) = b;
229
230 k=1;
```

```
231 for i=1:length(b)
232 c(i,2) = ind + k;
233 k=k+1;
234 end
235
236 d=min(c(:,1));
237 e=find(c(:,1)==d);
238 f=ind+e;
239
240 U=matrix(f,3);                          %this is the
241                                         %barrier
242 St2=matrix(f,3);                        %this is the price
    of the
243                                         %asset one day
                                              after the barrier
244
245
246
247  Kminus_n=matrix(f,10);
248  Kzero_n=matrix(f,4);
249  Kplus_n=matrix(f,7);
250
251
252  IVIA_n=matrix(f,8);                    %i.vol in the money
       ask
253
254  IVIB_n=matrix(f,9);                    %i.vol in the money
       bid
255
256  IVOA_n=matrix(f,11);                   %i.vol out-ask
257
258  IVOB_n=matrix(f,12);                   %i.vol out-bid
259
260  IVAA_n=matrix(f,5);                    %i.vol at-ask
261  IVAB_n=matrix(f,6);                    %i.vol at the money
       -bid
262
263
264
265 t_calendar_n=(matrix(1,2)-matrix(f,1));
266
267
268  if (t_calendar_n>=0 && t_calendar_n<=6)
269          tau_n=t_calendar_n;
270  end
271  if (t_calendar_n>=7 && t_calendar_n<=11)
272          tau_n=t_calendar_n-2;
273  end
274  if (t_calendar_n==14 || t_calendar_n==15)
```

```
275             tau_n=t_calendar_n-4;
276      end
277
278      tau_n=tau_n/252;
279
280      Ktildeminus_n=Kminus_n*exp(-mu*tau_n);
281      Ktildezero_n=Kzero_n*exp(-mu*tau_n);
282      Ktildeplus_n=Kplus_n*exp(-mu*tau_n);
283
284     pcall1=eurocall(St2,Ktildeminus_n,IVOA_n,tau_n);
285     pcall2=eurocall(St2,Ktildeminus_n,IVOB_n,tau_n);
286
287     pcall=0.5*(pcall1+pcall2);
288     put=pcall-St2+Ktildeminus_n;                              %
          put with i.vol A and strike Kzero_n
289
290     callA0=eurocall(St2,Ktildezero_n,IVAA_n,tau_n);           %
          call with i.vol A and strike Kzero_n
291
292
293     callB0=eurocall(St2,Ktildezero_n,IVAB_n,tau_n);           %
          call with i.vol B and strike Kzero_n
294
295
296     call0=0.5*(callA0+callB0);                                %
          average call with strike Kzero_n
297
298
299
300     callA1=eurocall(St2,Ktildeplus_n,IVIA_n,tau_n);           %
          call with i.vol B and strike Kplus_n
301
302     callB1=eurocall(St2,Ktildeplus_n,IVIB_n,tau_n);           %
          call with i.vol B and strike Kplus_n
303
304     call1=0.5*(callA1+callB1);                                %
          average call with strike Kzero_n
305
306     IVA=(IVAA+IVAB)/2;
307
308
309    Kbar_BS=U^2/Kminus_n;
310    eta_BS=(Kminus_n/U)^(1-(2.0*mu/(IVA*IVA)));
311    %eta_BS=(Kminus_n/U)* (1+2*mu/sigma^2 )
312
313    hedge1_BS=put-eta_BS*call0;
314    hedge2_BS=put-eta_BS*call1;
315
316
```

```
317
318
319
320 delta=IVA*St2^(-beta);
321
322 Kbar_n=(2.0*(U)^(-beta)-Kminus_n^(-beta))^(-1.0./beta);  %
    this is kstar use the smallest K - for now St=ST
323
324
325 A=(Kminus_n/Kbar_n)^((1+beta)/2);
326 B=mu/(beta*delta*delta)*U^(-beta)*(Kbar_n^(-beta)-Kminus_n
    ^(-beta));
327 eta=A*exp(B);
328
329 hedge1=put-eta*call0;
330 hedge2=put-eta*call1;
```

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