

Macroeconomic Models of the Japanese Crisis

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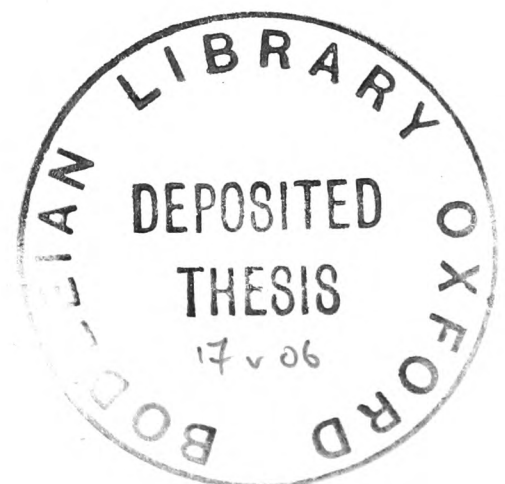
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Abstract

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Japan has experienced a prolonged stagnation since bursting the asset market bubble early in the 1990's. It is very important to understand the underlying problems in order to find a remedy to escape this stagnation. This thesis aims to theoretically analyse the current Japanese economy, especially from the viewpoint of multiple equilibria. According to this view, the same fundamentals can yield a multiple outcome depending on history or expectations. This thesis argues that Japan's situation can be regarded as a bad equilibrium which has been provoked by wide-spread pessimism and a bubble collapse.

Three chapters independently attempt to construct theoretical models describing the current Japanese situation. Chapter 2 demonstrates that demand externalities yield multiple equilibria. In a bad equilibrium, firms dare not participate in trade, which causes aggregate demand and welfare to decrease. A global games approach then illustrates how equilibrium is selected. Chapter 3, with the objective of seeing if Japan's depression was provoked by the misconduct of monetary policy, investigates the relation between indeterminacy and a monetary policy rule using a sticky price and firm-specific investment model. The standard Taylor principle is shown to be almost sufficient to eliminate indeterminacy, which suggests that the Bank of Japan did *not* exacerbate the economy while interest rate rules functioned, that is, until 1999. Chapter 4 focuses on a zero nominal interest rate bound, which has been observed since 1999. The ineffectiveness of the monetary policy yields a bad short-run outcome where real economic activity and asset prices become lower. There are long-run multiple equilibria in this story, and that is our explanation for the problem. Within this model, however, our analysis does *not* justify a claim that a zero bound for the interest rate causes a long-run equilibrium to be a bad one.

Contents

1	Introduction	1
1.1	Overview of the Japanese Economy	1
1.2	Possible Reasons for Japan’s Stagnation	3
1.2.1	Decline in Technology and Population Growth	3
1.2.2	Slow Recovery from a Bubble Collapse	6
1.3	Japanese Stagnation as a Bad Equilibrium	8
1.4	Structure of the Thesis	9
1.5	Limitations of the Thesis	10
1.6	References	11
2	Demand Externality and Multiple Equilibria	14
2.1	Introduction	15
2.2	Matsuyama Model	18
2.2.1	Basic Model	18
2.2.2	Horizontal Complementarities	22
2.2.3	Vertical Complementarities	23
2.2.4	Modified Basic Model and Multiple Equilibria	24

2.3	Extending the Matsuyama Model: Expanding the Possibilities of Multiple Equilibria	30
2.3.1	Horizontal Complementarities	30
2.3.2	Vertical Complementarities	32
2.3.3	More General Model of Multiple Equilibria	35
2.4	Equilibrium Selection	43
2.4.1	Bank Runs Model	44
2.4.2	Matsuyama Model	48
2.4.3	More General Multiple Equilibria Model	51
2.5	Bankruptcy and Equilibrium Selection	53
2.5.1	Model with a Bankruptcy Risk	54
2.5.2	Effect of Uncertainty	60
2.6	Concluding Remarks	63
2.7	References	66
2.A	Appendix	68
2.A.1	Derivation of the Utility Maximisation Problem	68
2.A.2	Firms' Profits in the Model of Vertical Complementarities	72
2.A.3	Proof of the Uniqueness of Equilibrium	73
2.A.4	Effect of Uncertainty on a Critical Value in the Bank Runs Model	74
2.A.5	Distribution of the Proportion of Inactive Firms l	75
3	Reexamination of the Taylor Principle: Firm-Specific Investment, Sticky Prices, and Indeterminacy	77
3.1	Introduction	78
3.1.1	Taylor Principle and Indeterminacy	78
3.1.2	Importance of Capital Accumulation	80

3.1.3	Contribution of this Chapter	82
3.2	Standard Model with Sticky Prices, Adjustment Costs and Firm-Specific Investment	87
3.2.1	Model Setup	87
3.2.2	Log-Linearisation	92
3.2.3	Stability Analysis	95
3.3	Robustness Check	107
3.4	Expansion of a Stability Boundary in Richer Models	110
3.4.1	Inflation Persistence	112
3.4.2	Two Sectors Model with Flexible and Sticky Price Firms	119
3.4.3	Sticky Wages	124
3.5	Concluding Remarks	128
3.6	References	129
3.A	Appendix	133
3.A.1	Inflation Dynamics	133
3.A.2	Optimal Reset Prices by Rational Firms When Irrational Firms Exist	137
4	Zero Nominal Interest Rate Bound and Multiple Equilibria	142
4.1	Introduction	143
4.1.1	Liquidity Trap	143
4.1.2	Literature Review	144
4.1.3	Methods and Results	146
4.2	Krugman Model	149
4.3	Semi-Sensible Krugman Model	150
4.3.1	Semi-Micro-Founded Model	150
4.3.2	Demand Shock Response: Elimination of Multiple Equilibria	157

4.3.3	Relaxing the Assumption of the Keynesian Consumption Function . .	158
4.3.4	Relaxing the Assumption of Fixed Price Expectations	162
4.4	Ramsey Model with Perfectly Flexible Prices	163
4.5	Variations upon the Ramsey Model with Not Perfectly Flexible Prices	167
4.5.1	Dynamics under Sticky Price Expectations	168
4.5.2	Dynamics under Sticky Prices: New-Keynesian Approach	178
4.5.3	Comparison between Two Not Perfectly Flexible Price Models	191
4.5.4	Comparison with the Krugman Model	192
4.6	Kiyotaki Model: Necessary Conditions for Understanding the Japanese Economy	195
4.6.1	With Perfectly Flexible Prices: Emergence of Multiple Equilibria . . .	196
4.7	Variations upon the Kiyotaki Model with Not Perfectly Flexible Prices	205
4.7.1	Dynamics under Sticky Price Expectations	206
4.7.2	Dynamics under Sticky Prices: New-Keynesian Approach	208
4.7.3	Effect of a Zero Nominal Interest Rate Bound	214
4.8	Concluding Remarks	215
4.9	References	220
4.A	Appendix	223
4.A.1	Dynamics of the Semi-Sensible Krugman Model	223
4.A.2	Semi-Sensible Krugman Model with the Euler Consumption Function	225
4.A.3	Kiyotaki Model Dynamics with Perfectly Flexible Prices	228
4.A.4	Slight Modification of the Utility Function in the Kiyotaki Model with Not Perfectly Flexible Prices	232
5	Concluding Remarks	233
5.1	Summary of the Findings in Each Chapter	233
5.2	Toward a Better Model to Explain Japan's Stagnation	235

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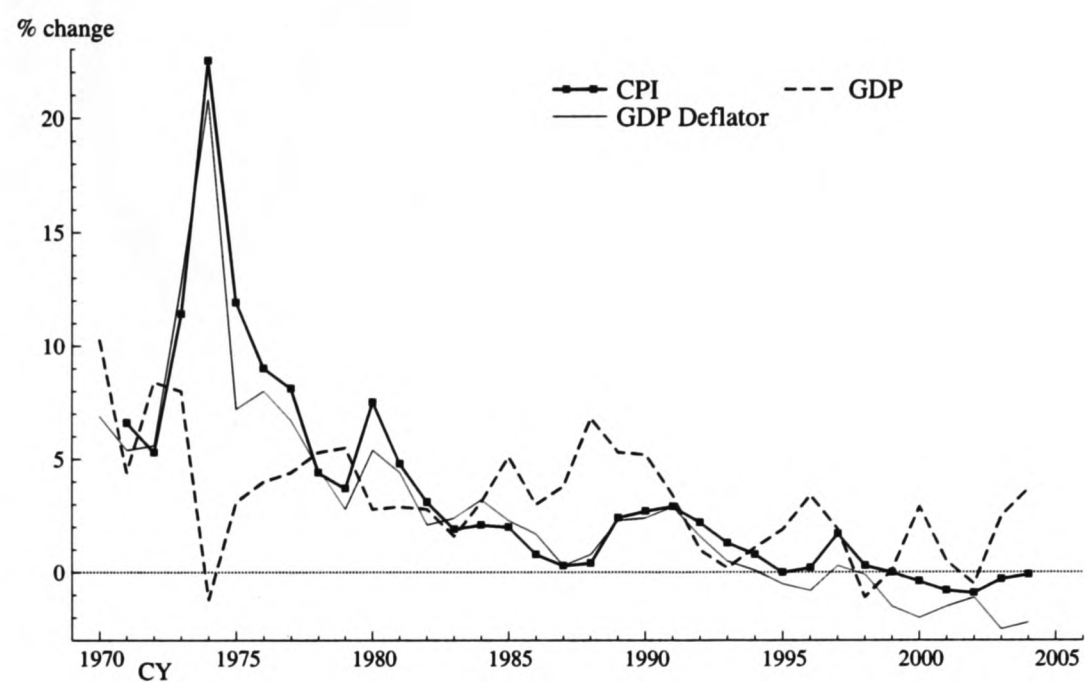
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Chapter 1

Introduction

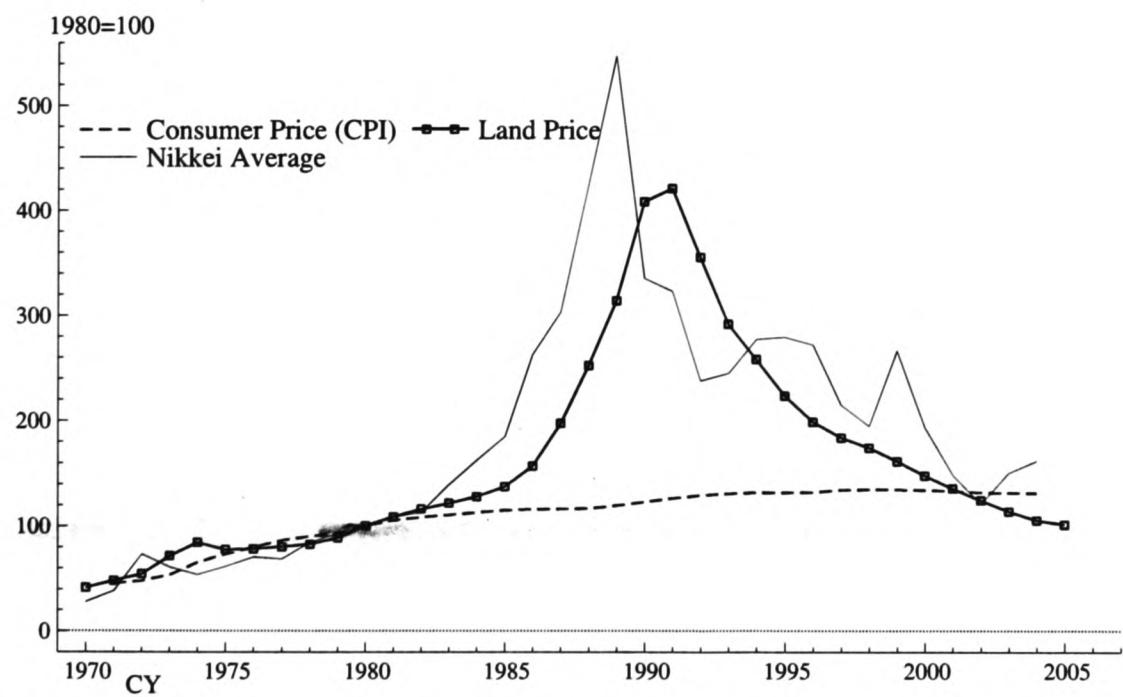
1.1 Overview of the Japanese Economy

Despite Japan's rapid growth dating from 1950's, the country has been struggling in a prolonged depression ever since the asset market bubble burst in 1991 (Figure 1.1.1). The average GDP growth rate was 3.6% before the burst (1983-1991), when it then dropped to 0.5% (1991-2000). Land and stock prices decreased by more than half (Figure 1.1.2), many firms went bankrupt due to heavy non-performing loans and a lack of demand, and many employees lost their jobs. The government tried to revitalise the Japanese economy by increasing public investment, but fiscal deficits and debts increased. The Bank of Japan, the country's central bank, has continuously reduced the short-term interest rate, which has fallen to almost zero since 1999 (Figure 1.1.3). Then, in March 2001, the Bank introduced a process of quantitative monetary easing which adopted the outstanding balance of current accounts held by financial institutions at the Bank as a new policy target. Many firms have, however, faced difficulty in repaying their debts, and the money supply has increased very little. Overall prices (e.g. the consumer price index) have been decreasing since 1999 (Figure 1.1.1). This situation is known as 'Japan's lost decade.'



Sources: Cabinet Office, Ministry of Internal Affairs and Communications

Figure 1.1.1: GDP and Goods Prices



Sources: Ministry of Internal Affairs and Communications, Nihon Keizai Shinbun, and JREI

Figure 1.1.2: Assets and Goods Prices

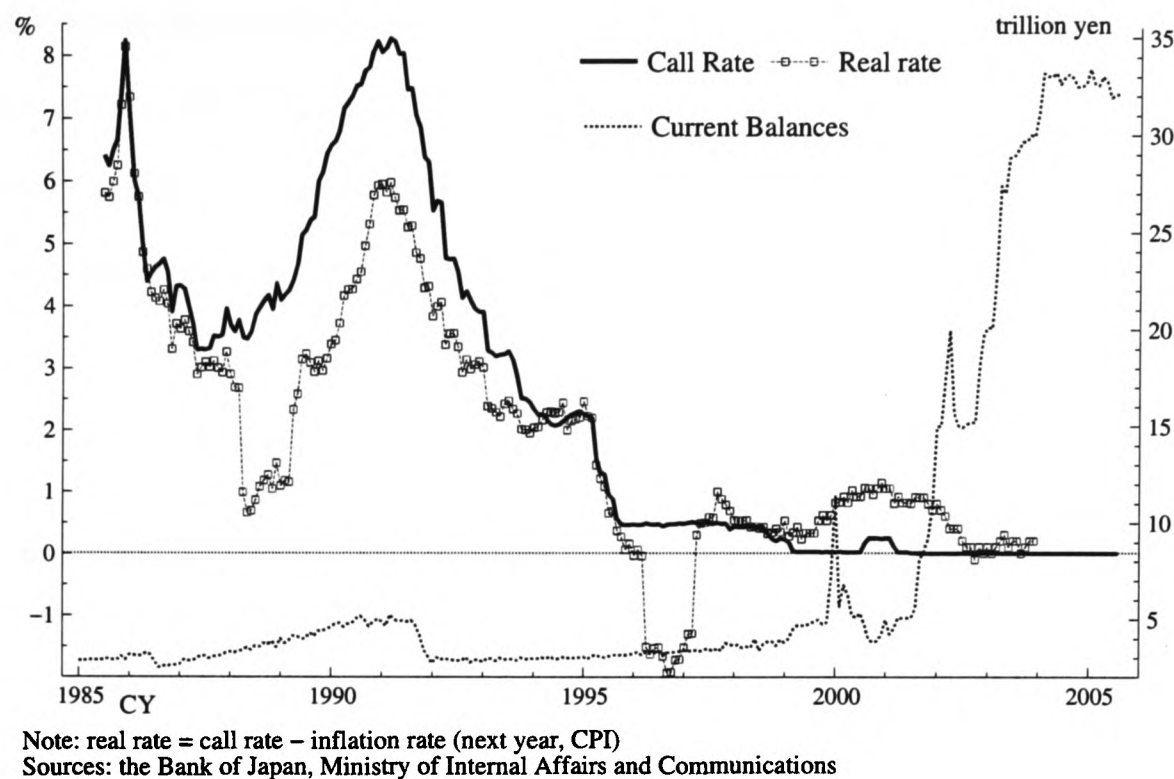


Figure 1.1.3: Monetary Policy Instruments

1.2 Possible Reasons for Japan's Stagnation

Studies of Japan's failure to emerge from this slump have focused on several reasons. Although their arguments have been widely discussed, it seems to me that they have many shortcomings.

1.2.1 Decline in Technology and Population Growth

According to the Solow or Ramsey model, there are only two factors which determine growth: population and technology. Firstly, regarding population, there is no doubt that its growth rate has been decreasing in Japan. A total population growth rate decreased from the peak of 2.3% in 1972 to 0.2% in 2000 and, at last, it seems that population started to decrease for the first time in 2005. Since a long-run growth rate increases with a population growth rate in the Solow or Ramsey model, this factor appears to dampen economic growth to some extent.

Secondly, regarding technology growth, Hayashi and Prescott (2002) argued that total factor productivity (TFP) dropped after the asset market bubble burst using the method of simple growth accounting (see the following table). They then attributed the reason for the Japanese depression to a decline in productivity.

Growth Rates of GDP and TFP (annual base, %)

	Before the burst	After the burst
GDP Growth (Hayashi and Prescott: 2002)	3.6 (83-91)	0.5 (91-00)
TFP (Hayashi and Prescott: 2002)	3.7 (83-91)	0.3 (91-00)
TFP (Kawamoto: 2004)	2.3 (80-90)	2.1 (90-98)
TFP (Jorgenson and Motohashi: 2003)	1.0 (75-90)	{ 0.7 (90-95) 1.1 (95-00)
TFP (Kasuya et al.: 2002)	-1.6 (85-89)	{ -0.9 (90-94) 0.1 (95-99)
TFP (Fukao et al.: 2003)	0.4 (83-91)	0.2 (91-98)

Sample periods reported in brackets.

Admitting that a TFP growth rate declined in this decade, several reasons for this can be considered. Firstly, technological innovation may have been hampered by the structural inefficiency of the Japanese economic system. More than a decade ago the Japanese system, which incorporates seniority-order wage allocation and a close relationship between firms and main banks, was greatly acclaimed as a sophisticated method to enable success (for example, see Hoshi, Kashyap, and Scharfstein 1991). Nowadays, however, these systems are regarded as old-fashioned, inflexible and inefficient. Workers cannot change their jobs freely; firms suffer from the burden of high labour costs due to an increase in senior workers; and firms find difficulty in borrowing money from other than main banks. These inefficiencies may have

dampened technological growth.

Secondly, an unhealthy credit system may have prevented technological innovation. Many economists blame deflation and heavy non-performing loans, emphasising a financial accelerator effect. The theoretical underpinning for this effect is supported by, for example, the work of Bernanke and Blinder (1988), Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). Non-performing loans damage the solvency of firms and prevent them from borrowing money from banks which, in turn, leads to a reduction in investment. Thus, aggregate demand decreases and, as a result, prices also decrease. The decline in prices decreases the value of collateral and increases bad loans even more. This argument has been widely emphasised, especially since deflation and the bankruptcy of some private banks have been observed.

*

However, these explanations from the viewpoint of the Solow or Ramsey model are not convincing. It is true that population growth has been decreasing, but this alone cannot address the reason why Japan has fallen into a depression since the early 1990's. Since a population change is slow and predictable, this cannot explain the bust from the early 1990's compared with the boom in the 1980's.

Regarding technology growth, the measurement method by Hayashi and Prescott (2002) casts doubts. Their method is simple growth accounting which, as many people have pointed out, has a lot of problems. One example is labour and capital utilisation. During a depression, firms reduce the working hours of employees and the utilisation of capital. If this reduction is not correctly measured, production inputs, namely labour and capital, are overestimated. This lowers the measured value of TFP growth. As the above table shows, Kawamoto (2004) controlled this problem and obtained an almost unchanged TFP growth rate before and after the bubble burst¹. Other authors, Jorgenson and Motohashi (2003) and Kasuya, Nakajima,

¹Kawamoto's method is criticised by Tomura (2005). He argued that Kawamoto overlooked a decrease in

Saita, and Tanemura (2002), used price data in order to more accurately measure quality changes in labour and capital utilisation. They obtained the results that TFP growth rates were almost steady, and that even in the aftermath of the bubble burst, the decline of TFP was only small. Finally, Fukao, Inui, Kawai and Miyagawa (2003) measured TFP growth using the DIP Database which was compiled to take account of the quality of capital and labour utilisation. Their result also denied a large drop in the TFP growth rate before and after the burst.

Thus the Solow or Ramsey model perspective is not satisfactory as a reason for the Japanese crisis.

1.2.2 Slow Recovery from a Bubble Collapse

Since the Solow model holds true only in the long-run, the short-run TFP and GDP growth may exhibit a different movement. Of course, according to a Real Business Cycle framework, even the short-run TFP and GDP move together. This is because deep parameters such as households' preferences are very stable, and only a change in technology becomes sufficient to cause economic fluctuations. However, for several reasons, a temporary not permanent TFP shock which was not measured empirically or excessive capital due to the bubble economy may have caused the more than a decade-long stagnation. Japan's recovery process may have been simply too slow since the bubble collapse.

As a reason for slow recovery, firstly, we can point out persistence thanks to price persistence and investment adjustment costs. When a TFP shock occurs, economies cannot instantaneously reach an equilibrium; it takes some time to absorb the shock by gradually adjusting capital and prices. Thus, in the short-run, the movements of TFP and GDP are not necessarily the same. The Japanese economy since the early 1990's can be regarded as

statutory workweek in the 1990's. Incorporating this effect works to increase a cyclical labour input and, in turn, decrease TFP growth.

an adjustment process from a bubble collapse.

Secondly, as was discussed above, a heavy burden of bad loans may have prevented recovery. An unhealthy credit system has the effect of amplifying economic busts, which may have also slowed down recovery.

Thirdly, government policies may not have performed well, partly because of ineffective public expenditure and partly because of accumulated debts. Public investment tends to help inefficient sectors such as construction firms in rural areas which have a large voting power. Such excessive public investment resulted in the accumulation of government debts. Consequently, a large number of Ricardian people feared the future risk of tax increases or, at worst, a government default, and tried to consume less for their uncertain future. Irresponsibility of fiscal policy may have prevented Japan's revitalisation.

Fourthly, monetary policy may have been too little, too late. For example, McCallum (2001) pointed out that the Bank of Japan had been too tight since the middle of the 1990's. Furthermore, the short-term interest rate has fallen to almost zero since 1999, which has made monetary policy almost ineffective. In order to escape such a situation, called a liquidity trap, Krugman (1998, 2000) suggested that the Bank of Japan should try anything in order to change people's expectations.

*

However, several questions are posed with this argument. Firstly, it is uncertain whether it really takes more than a decade for the economy to recover. If so, irrespective of the size of a shock, an economy must always have a similar period of a depression. However, in the post-war period, the previous recessions seem to have ended much more quickly than this time. The country's economy mostly escaped from depression in a few years. The huge accumulation of government debts and the incoming of the zero bound are rare events, but

these did not happen at the beginning of the depression. Soon after the burst of the bubble, the government implemented a large scale of fiscal expansion, and the Bank of Japan lowered the short-term interest rate continuously. One would have expected that the use of both fiscal and monetary policy would have been effective enough to end the stagnation in a few years. Furthermore, compared with other countries, Japan's depression seems very long. The four points made above do not seem sufficient to explain what has been going on.

Moreover, the above argument implies that, in the long-run, the economy necessarily goes back to a unique equilibrium. All we are left with is the idea that the country will not be able to achieve as high an economic growth as before due to a demographic change. If that were true, we would not need to be too concerned about Japan's lost decade. The current depression would be interpreted as just a recovery process.

In my view, it is unsatisfactory to explain the Japanese stagnation from the perspective of recovery sluggishness. It is natural to think that even a temporary economic downturn must be harmful not only in the short-run but also in the long-run. To investigate whether that is true is the purpose of this thesis.

1.3 Japanese Stagnation as a Bad Equilibrium

Bearing this in mind, the thesis focuses on the possibility of multiple equilibria. If there are multiple equilibria, despite no or only a tiny change in technology, the economic situation may vary significantly. Economies may either stay at a good equilibrium or be trapped in a bad one. Even a temporary shock may bring a big and long-lasting change if this is big enough to shift an equilibrium to another. This idea can help explain the difference between the TFP and GDP growth rates in the above table.

Furthermore, under the existence of multiple equilibria, expectations become a key el-

ement to determine which equilibrium is selected. A pessimistic view, even if it is formed without a strong reason, can become so self-fulfilling that it shifts an economy to a bad equilibrium. Supposing inertia in terms of expectations and fundamentals, a bad equilibrium at present could continue for a long period. It will be very difficult to escape from such a depression. In my view, such gloomy features seem to have been a wide-spread situation in Japan for more than a decade. I will be seeking an explanation for this fact.

It is common to have only a unique equilibrium in standard macroeconomic models. It will thus be very important to justify my story of multiple equilibria with sound micro-founded models. This thesis aims to construct multiple equilibria models from a theoretical point of view so that they can well depict what is happening to the Japanese economy. My aim is also to develop models sufficiently such that, alongside this multiple equilibria story, I will be able to analyse the effects of monetary and fiscal policy. This will enable us to discuss policy implications, in order to find possible remedies to revitalise the economy.

1.4 Structure of the Thesis

This thesis consists of three main chapters.

The first chapter will focus on demand externalities to demonstrate the existence of multiple equilibria. In a bad equilibrium, firms hesitate to participate in trade, which lowers aggregate demand. Circumstances become so bad that it is not worth firms participating in trade, which justifies the initial hesitation. This equilibrium is considered to correspond to the current Japanese economy. As a way to revitalise the economy, the role of "irresponsible" fiscal policy is examined. Despite Ricardian households, a huge enough fiscal expenditure is shown to be effective to shift a bad equilibrium to a good one. This suggests that expansionary fiscal policy may be helpful. Further, I will investigate how equilibrium is selected

using a global games approach. Studying this will help to understand how Japan fell into a depression and is stuck there.

The next two chapters will investigate monetary policy, and examine whether Japan's stagnation was provoked by the misconduct of the Bank of Japan.

Firstly, I will investigate whether its policy caused any economic disturbances when it was able to adjust a short-term interest rate in both directions. The effect of interest rate rules on indeterminacy will be studied. Indeterminacy, a special kind of multiplicity, is problematic in that a change in expectations becomes self-fulfilling without a change in fundamentals, and that can yield economic fluctuations. How indeterminacy can be achieved and avoided is the subject of this chapter. I incorporate sticky prices and firm-specific investment. Doing this, I will show that the Taylor principle is almost sufficient to achieve determinacy. Thus, from the perspective of determinacy, the Bank of Japan seems to have acted properly. This chapter ends up by simply disproving one possible story about Japan's collapse.

The final part of this thesis will study the era after monetary policy in a narrow sense stopped functioning. In other words, it will focus on a zero nominal interest rate bound, which has been seen in Japan since 1999. Whether the ineffectiveness of monetary policy produces an additional bad equilibrium, which pushes investment and asset prices down, will be examined. It will be demonstrated that the zero bound yields a huge depression in the short-run, but that this zero bound cannot be responsible for a bad long-run outcome of the kind that I am seeking for my story.

1.5 Limitations of the Thesis

This thesis, from a couple of perspectives, would seem to succeed in explaining the Japanese crisis. However, an even better model is required to integrate all the independent chapters. Such a synthesis would provide a much more sophisticated and integrated story which would

embed all of a firm's trade participation, capital investment, fiscal policy and monetary policy. Furthermore, this could well magnify the possibility of multiple equilibria. Such an extension will be our future task.

Although I denied several widespread arguments about Japan's lost decade, in fact, I certainly admit the importance of credit channels. The heavy burden of bad loans is one of the main characteristics in this Japanese stagnation, which may thus have damaged the economy and delayed recovery more seriously than before.

However, I am also sceptical about the argument that simply resolving bad loans can revitalise the economy. Even if banks and firms became free from bad loans, a credit market would not be active under a bad equilibrium. There, demand is so low that firms do not want to borrow money for investment even if banks are willing to lend. To further address this argument, causality may become opposite. In other words, resolving a credit system does not increase demand, but increasing demand resolves a credit system. This is because escaping from a bad equilibrium can increase demand, which increases firms' profits. Thus, this may help reduce bad loans and resolve a banking crisis. The last argument may be rather extreme, but the main message of this thesis is that the economy must escape from a bad equilibrium in order to make full use of a healthy credit system.

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Chapter 2

Demand Externality and Multiple Equilibria

JEL classification: C72, E12, E62, O11

Keywords: business connections, relative prices, equilibrium selection, global games, bankruptcy

Abstract

Based on Matsuyama's (1995) monopolistic competition models, this chapter shows there can exist multiple equilibria arising from demand externalities. In a bad equilibrium, firms dare not participate in trade, which causes aggregate demand and welfare to decrease. The government plays an important role in shifting the economy. It is demonstrated that the possibility of multiple equilibria increases as business connections become more complex. In a more general model than Matsuyama, a difference in relative prices yields similar multiple equilibria, among which a bad equilibrium entails a low level of price levels. This corresponds to the current Japanese economy. The next part of this chapter studies why Japan has fallen into a bad equilibrium. In order to answer this question, the thesis applies the approach of global games to the above monopolistic competition models. The main conclusion is that fundamentals affect expectations and, in turn, the equilibrium selection. This suggests that bursting the asset market bubble in the early 1990's could have been the direct cause of the depression.

2.1 Introduction

This chapter aims to explain the reason for Japan's depression from the viewpoint of demand (pecuniary) externalities and multiple equilibria. The theory of demand externalities addresses the fact that demand in one sector produces more demand in other sectors, which becomes the engine of an economic boom. On the other hand, low demand decreases demand even further, which makes depression more serious.

The demand externalities become the source of multiple equilibria. For example, an increase in production by only one firm may not be enough to lift an economy because this hardly raises aggregate demand. Thus, production by no firm secures an equilibrium. In

contrast, if all firms increase their production, aggregate demand can increase sufficiently to cover the costs of their production. Thus, production by all firms becomes another equilibrium. This idea, which originated from Rosenstein-Rodan (1943), was elegantly formalised by Murphy, Shleifer and Vishny (1989). Looking at Japan since the early 1990's, one can see that the economy has been grim, and people have become pessimistic. Firms may have not dared to participate in trade although they would have had a chance to make a profit if they had traded simultaneously. It seems to me that this story is a very good candidate for explaining Japan's depression since the 1990's.

Furthermore, the possibility of such multiple equilibria would probably be reinforced in a developed economy like that of Japan. Firms make a complex business connection with many other firms and affect each other through the trade. Taking the example of car manufacturing, each part - tyres, seats, engines, and bodies - is produced by different firms. The firm which produces bodies sells them to car assembly firms and purchases necessary parts such as steel from upstream firms. In such a trade, an increase in the demand for certain goods causes an increase in profits of relevant firms and an increase in aggregate income. This leads to an increase in the demand for other goods, and activates trade participation. In this sense, a cobweb economy may yield a higher degree of demand externalities, which enhances the possibility of multiple equilibria.

This chapter aims to examine this conjecture with sound micro-foundation. The model by Murphy, Shleifer and Vishny (1989) successfully formalised multiple equilibria, but seems too stylised to assess the effect of business connections. Therefore, instead, this chapter uses the monopolistic competition model by Matsuyama (1995). As will be explained later, his paper is composed mainly of four models: a basic model with monopolistic competition, a horizontal complementarity model, a vertical complementarity model and a modified basic model which is constructed to yield multiple equilibria. Although the first three models do not

have multiple equilibria, the models of horizontal and vertical complementarities can describe business connections very well. The last modified basic model with multiple equilibria takes us back to outcomes which resemble those of Rosenstein-Rodan (1943) and Murphy, Shleifer and Vishny (1989) in many respects. But more than in those models, we can study the properties of multiple equilibria and give good reasons for those outcomes. Thus, combining these four models enables us to investigate whether a high degree of business connections enhance the possibility of multiple equilibria.

The Matsuyama model is based on some strong assumptions, however. One of them is that the elasticity of substitution between goods is one. This is a convenient way to make a model simple, but there is no guarantee for this assumption. In addition, a rather strong assumption is imposed onto fixed costs in order to yield multiple equilibria. The following part of this chapter will construct a more general model by relaxing these assumptions, and aim to see if there are multiple equilibria. It will become clear that similar multiple equilibria can be produced, but that a mechanism to yield multiple equilibria becomes different. In this new model, price distortion as well as a demand externality plays a key role in the occurrence of multiple equilibria. Interestingly, we can find that the result of this model is quite similar to that of Ball and Romer (1991) which incorporates menu costs. Furthermore, it will be discovered that no firm participation entails deflation, which is the case in Japan.

Another question which arises here is that of exactly why Japan might have fallen into the bad equilibrium described by this model. The second part of this chapter investigates how an equilibrium is selected using the idea of global games by Morris and Shin (2001). The main implication is that the existence of a small uncertainty makes multiple equilibria unique. Every agent chooses a strategy simply depending on the observation of a certain fundamental. Following the discussion of equilibrium selection with a simple model, namely the bank runs model of Diamond and Dybvig (1983), I will apply their approach to the Matsuyama models.

For Japan, we can conjecture that bursting the asset market bubble in the early 1990's may be a reason why a bad equilibrium was chosen. In other words, the bubble burst decreased the fundamentals such as aggregate demand and the profitability of firms, which resulted in the selection of passive strategies.

The final part of this chapter studies the effect of bankruptcy and its uncertainty on equilibrium selection. The above bank runs model and the global game approach are silent on this matter; uncertainty may encourage firms' participation as well as discourage it. However, it is natural to think that uncertainty would make economies inactive. This chapter, by incorporating bankruptcy, will illustrate that such uncertainty really does increase the probability that a bad equilibrium is selected.

This study originally arose from the experience in Japan, but it can hold in all countries because the models here do not impose any special assumptions applying only to Japan. Furthermore, the idea might help to explain not only a depression but also a poverty trap in developing countries.

2.2 Matsuyama Model

This section introduces the several models of Matsuyama (1995)¹. One of his models produces multiple equilibria, which seems to explain Japan's situation.

2.2.1 Basic Model

The basic model is constructed as follows.

<Household>

¹Mankiw (1988) also exploits a similar model.

There is a representative consumer. We assume a Cobb-Douglas utility function:

$$\alpha \int_0^1 \ln c(z) dz + (1 - \alpha) \ln(N), \quad (2.2.1)$$

$$\text{s.t. } \int_0^1 p(z)c(z)dz + N \leq L + \Pi - T, \quad (2.2.2)$$

where $z \in [0, 1]$ is the variety of a product. $c(z)$ and $p(z)$ represent the consumption and the price of a product z respectively. N is leisure and L is the total amount of time, so working hours are $L - N$. Π is a profit, and T is a lump-sum tax. $\alpha \in (0, 1)$ is the coefficient which determines the allocation of consumption and leisure. Here, leisure is set as a numeraire (i.e. wages are equal to 1).

A representative consumer chooses the optimal level of consumption and hours of labour/leisure. The solution of this utility maximization problem is given by

$$c(z) = \frac{\alpha(L + \Pi - T)}{p(z)} \quad (2.2.3)$$

(see Appendix 2.A.1 for the derivation). When the government spends G on each goods regardless of its price, the total demand for each goods, $q(z)$, becomes

$$q(z) = \frac{\alpha(L + \Pi - T) + G}{p(z)}. \quad (2.2.4)$$

The government budget constraint is $T = G + L'$, where L' is the labour of the government sector.

<Firm>

There are two types of firm. One is a competitive fringe firm whose technology exhibits constant returns to scale. It produces one unit of goods by using one unit of labour input. In other words, this firm produces goods for the price of one because the labour is a numeraire. The other type is a monopolistic firm with the technology of increasing returns to scale. This firm produces q units of goods by using $(1 - \mu)q + F$ units of labour input where $\mu \in [0, 1]$. Since the monopolistic firm faces a downward sloping demand with an elasticity of one and the competitive fringe sells the goods for the price of one, the monopolistic firm sets the price

as $p(z) = 1$. Thus, μ can be interpreted as mark-up. Each monopolistic firm receives the profits of

$$\begin{aligned}\pi(z) &= \mu q(z) - F \\ &= \mu\{\alpha(L + \Pi - T) + G\} - F.\end{aligned}\tag{2.2.5}$$

The profits are the same among all the monopolistic firms, so the aggregate profit for $z \in [0, 1]$ is also

$$\Pi = \mu\{\alpha(L + \Pi - T) + G\} - F.$$

Using the above formulation, the aggregate profit Π can be calculated as

$$\Pi = \frac{A^0}{1 - \mu\alpha},\tag{2.2.6}$$

where

$$A^0 = \mu\{\alpha(L - T) + G\} - F.\tag{2.2.7}$$

The aggregate demand, Q , becomes

$$Q = \int_0^1 q(z)dz = \alpha(L + \Pi - T) + G\tag{2.2.8}$$

$$= \frac{\alpha(L - F) + G - \alpha T}{1 - \mu\alpha}.\tag{2.2.9}$$

Although workers are fully employed in this model, the above equation implies something like the multiplier in the Keynesian sense (Keynes, 1936). In other words, a unit increase in government spending balanced with an increase in lump-sum tax raises aggregate demand by $(1 - \alpha)/(1 - \mu\alpha)$. The reasoning is as follows. A unit increase in government spending directly raises the demand by one. At the same time, a unit increase in tax reduces the net income by one and the demand by α . Hence, the net increase in demand becomes $(1 - \alpha)$. The increase in the demand raises the firm's profits by $\mu(1 - \alpha)$, which leads to an increase in the consumer's income by the same amount. The demand increases by $\mu(1 - \alpha)$ multiplied

by α . This cumulative process repeats such that

$$\begin{aligned}\Delta Q &= (1 - \alpha) + \mu\alpha(1 - \alpha) + (\mu\alpha)^2(1 - \alpha) + \dots \\ &= \frac{1 - \alpha}{1 - \mu\alpha}.\end{aligned}\tag{2.2.10}$$

In short, a demand externality yields a multiplier effect. This multiplier effect is larger as the mark-up, μ , is larger.

Next, let us consider the effect of government expenditure on welfare. Consumption and leisure are obtained as follows:

$$c(z) = \alpha(L + \Pi - T),\tag{2.2.11}$$

$$N = (1 - \alpha)(L + \Pi - T),\tag{2.2.12}$$

$$\text{where } L + \Pi - T = \frac{L - F + \mu G - T}{1 - \mu\alpha}.\tag{2.2.13}$$

Substituting into (2.2.1) yields the utility function of

$$\alpha \ln(\alpha) + (1 - \alpha) \ln(1 - \alpha) + \ln(L + \Pi - T)\tag{2.2.14}$$

$$= \alpha \ln(\alpha) + (1 - \alpha) \ln(1 - \alpha) + \ln\left(\frac{L - F + \mu G - T}{1 - \mu\alpha}\right).\tag{2.2.15}$$

A balanced-budget increase in government spending entails an increase in tax by the same amount. Since $\mu \leq 1$, this decreases consumption and leisure, which results in a decrease in welfare. This feature contradicts Keynes's insight. This is because the Matsuyama model assumes perfect price/wage flexibility and no involuntary unemployment.

However, there are some special cases where the Matsuyama model yields the same result as the Keynesian model. First, when the monopolistic firms have full mark-up; $\mu = 1$, government spending financed by tax has no impact on the welfare. Secondly, suppose that government expenditure is covered not by tax (an increase in T) but by the lay-off of civil servants (a decrease in L'). Since L' does not enter the utility function, government expenditure can raise welfare. Finally, this model assumes that government spending does not directly affect utility. It is natural, however, to suppose that the utility is a function of government

expenditure and its partial derivative is positive. In this sense, this analysis is limited in scope, and the Matsuyama model could become closer to the standard Keynesian view.

2.2.2 Horizontal Complementarities

Under a perfectly competitive market, the economy is zero-sum, that is, some sectors cannot expand themselves without the sacrifice of other sectors. However, in a monopolistic competitive economy, it is possible that all sectors grow simultaneously. A positive sector shock, for example, increases this sector's profits and then the aggregate income, followed by increases in the demand for each goods and the profits of other sectors. This feature, as one kind of business connection, appears to increase the demand externalities.

Suppose there are two sector groups ($i = 1, 2$). Under the same assumptions as before, the profit of each firm is given by

$$\begin{aligned}\Pi_i &= \mu_i Q_i - n_i F \\ &= \mu_i [\alpha_i (L + \Pi - T) + G_i] - n_i F\end{aligned}\tag{2.2.16}$$

where α_i is the budget share, n_i represents the size of sector i , and $\Pi = \Pi_1 + \Pi_2$. This can be simplified as

$$\Pi_1 = A_1 + \mu_1 \alpha_1 (\Pi_1 + \Pi_2),\tag{2.2.17}$$

$$\Pi_2 = A_2 + \mu_2 \alpha_2 (\Pi_1 + \Pi_2),\tag{2.2.18}$$

$$\text{where } A_i \equiv \mu_i [\alpha_i (L - T) + G_i] - n_i F.\tag{2.2.19}$$

Therefore,

$$\begin{pmatrix} \Pi_1 \\ \Pi_2 \end{pmatrix} = \frac{1}{1 - \mu_1 \alpha_1 - \mu_2 \alpha_2} \begin{pmatrix} 1 - \mu_2 \alpha_2 & \mu_1 \alpha_1 \\ \mu_2 \alpha_2 & 1 - \mu_1 \alpha_1 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}\tag{2.2.20}$$

An increase in A_1 raises the profits of sector 2 as well as sector 1. In this way, government spending in one sector can increase the profits of all sectors.

2.2.3 Vertical Complementarities

In the process of manufacturing, quite a few goods are produced by divided and collaborated work among multiple firms. Consider the case of Toyota's car manufacturing. Most parts of cars - tyres, seats, and bodies - are produced not by Toyota but by many specialising firms. These firms do not necessarily produce all these parts. They purchase even simpler goods such as rubber, screws, and steel from other firms. Such a trade continues from downstream to upstream firms. In this way, firms construct a vertical relationship with each other, which yields complementarities and amplifies the multiplier effect.

Assume goods are produced in S stages, and each stage has a continuum of varieties, $z_j \in [0, 1]$, ($j = 1, 2, \dots, S$). The goods of $j = 1$ denote the final consumption goods, and the goods of $j = 2$ denote the input goods required for the production of the goods of $j = 1$. The other assumptions are the same as before. For $1 \leq j \leq S - 1$, the competitive fringe produces goods as follows:

$$q(z_j) = \exp \left[\int_0^1 \ln q(z_{j+1}) dz_{j+1} \right] \quad (2.2.21)$$

In order to understand this equation intuitively, assume that all varieties z_j are uniformly distributed between 0 and 1. We can then simplify this to

$$q_j = q_{j+1}. \quad (2.2.22)$$

In other words, competitive fringes produce one unit of output j from one unit of input $j + 1$.

On the other hand, the monopolistic firm produces goods as

$$q(z_j) = \mu_j q(z_j) - F_j + \exp \left[\int_0^1 \ln q(z_{j+1}) dz_{j+1} \right]. \quad (2.2.23)$$

Similarly, in a special case, we can simplify this equation to

$$q_j = \mu_j q_j - F_j + q_{j+1}. \quad (2.2.24)$$

Output of goods j (i.e. q_j) can be decomposed into profits (i.e. $\mu_j q_j - F_j$) and input of the goods $j + 1$ (i.e. q_{j+1}).

For $j = S$, while the competitive fringe converts one unit of labour into one unit of output, the monopolistic firm uses the input of $(1 - \mu_S)q(z_S) + F_S$. Due to the competitive fringe and the unit elasticity of demand, all the prices are $p(z_j) = 1$ for $1 \leq j \leq S$. Therefore, using equation (2.2.8), the demand for the consumption goods, Q_1 , is given by

$$\begin{aligned} Q_1 &= \int_0^1 q(z_1) dz_1 \\ &= \alpha \left(L + \sum_{j=1}^S \Pi_j - T \right) + G. \end{aligned} \quad (2.2.25)$$

Equation (2.2.23) leads to

$$Q_{j+1} = (1 - \mu_j)Q_j + F_j. \quad (2.2.26)$$

Combine this with $\Pi_j = \mu_j Q_j - F_j$, and recursive calculation yields

$$Q_1 = \frac{\alpha B + G - \alpha T}{1 - \mu\alpha}, \quad (2.2.27)$$

$$\text{where } B = L - \sum_{j=1}^S \left[F_j \prod_{k=j+1}^S (1 - \mu_k) \right], \quad (2.2.28)$$

$$\mu = 1 - \prod_{j=1}^S (1 - \mu_j). \quad (2.2.29)$$

This result shows that even a small mark-up in each stage of production can yield a high (close to one) mark-up overall. For example, assuming $\mu_j = 0.3$ for all j , the overall mark-up μ becomes 0.76 for $S = 4$ and 0.94 for $S = 8$. Demand externalities of outputs and profits are amplified through vertical production. Since a large mark-up leads to the high multiplier, the long chain of production heightens the effectiveness of government policies. Furthermore, as will be shown later, this increases the possibility of multiple equilibria.

2.2.4 Modified Basic Model and Multiple Equilibria

On the part of economic development, Matsuyama (1995) showed an example of multiple equilibria. There are two possibilities such that (i) all firms participate in trade and (ii) no firm participates in trade. The basic idea resembles that of Murphy, Shleifer and Vishny (1989). The common key assumption for multiple equilibria is that firms behave monopolistically and

there exist demand externalities. Results are quite similar, too. There are two equilibria such that all firms are active or inactive, and the role of the government is justified. However, the model of Murphy, Shleifer and Vishny differs in that they assumed a wage premium instead of fixed costs as goods, which will be explained below.

Matsuyama modified the basic model in the following two ways. Firstly, suppose that there are two stages $t = 1$ and 2. At $t = 1$, monopolistic firms decide whether to participate in trade. In contrast, a competitive fringe always stays in the market. Secondly, so far, I have assumed that fixed costs F are used for labour. Instead, assume that they are used for goods:

$$F = \exp \left[\int_0^1 \ln f(z) dz \right], \quad (2.2.30)$$

where $f(z)$ is the investment spent on the variety z .

Hence, demand for goods increases if other firms participate in trade and pay fixed costs. The profit function, equation (2.2.5), is transformed to

$$\begin{aligned} \pi &= \pi(z) = \mu q(z) - F \\ &= \mu \{ \alpha(L + \Pi - T) + sF + G \} - F, \end{aligned} \quad (2.2.31)$$

where $s \in [0, 1]$ represents the proportion of monopolistic firms that participate in trade.

Since the total profits are $\Pi = s\pi$,

$$\pi(s) = \frac{\mu \{ \alpha(L - T) + G \} - (1 - \mu s)F}{1 - \mu \alpha s}. \quad (2.2.32)$$

The profits positively depend on s , thus the decisions of trade participation also depend on s . There are three cases.

1. All firms participate in trade if $\pi(s = 0) > 0$ or

$$G + \alpha(L - T) > \frac{1}{\mu} F. \quad (2.2.33)$$

2. No firms participate in trade if $\pi(s = 1) < 0$ or

$$G + \alpha(L - T) < \frac{1 - \mu}{\mu} F. \quad (2.2.34)$$

3. Multiple equilibria arise if both $\pi(s = 0) < 0$ and $\pi(s = 1) > 0$ are satisfied, that is,

$$\frac{1 - \mu}{\mu} F < G + \alpha(L - T) < \frac{1}{\mu} F. \quad (2.2.35)$$

In other words, there is a good equilibrium where all firms participate in trade and make profits, while there is a bad equilibrium where no firms participate because, if they do, they suffer losses.

I call the former equilibrium good because its welfare is higher. For, as shown in equation (2.2.14), the consumer's utility depends only on his disposal income $L + \Pi - T$ and Π is clearly higher in the former equilibrium.

One thing is worth noting here. Our model does not have any involuntary unemployment even in the bad equilibrium. This is unsatisfactory in order to explain a situation like a poverty trap.

Discussion

The above condition suggests low (high) fixed costs lead to eliminating the bad (good) equilibrium. This is because low fixed costs enable firms to make profits even under low demand. On the other hand, high fixed costs reduce profitability, which discourages firms from participation. It is true that fixed costs are used for goods, so they contribute to an increase in demand, but the effect is not enough for each firm to offset an increase in production costs.

It is difficult to judge whether the fixed costs in Japan have increased or not in the last decade, but I suspect that they have increased. Firstly, real fixed costs, as the ratio of nominal fixed costs to output prices, seem to have increased because some fixed costs are quite inflexible compared with the output prices. One example is entry costs such as

registration fees. In general, the fees decided by the government are inflexible because their change entails tediously long procedures. Secondly, looking at interest rates, a continuous reduction of them seems to have functioned to decrease the costs of borrowing. However, there is a zero interest rate bound. Therefore, combined with deflation, the real interest rates are considered to have increased. The final example is related to bad loans. The weakening of the solvency of both banks and firms has made it more difficult for most firms to borrow money from banks, except for a few good firms which can raise money from a stock market. Hence, even though the policy interest rates have decreased to almost zero, the interest rates for private loans have decreased less. These factors seem to have contributed to an increase in fixed costs, which has resulted in the prolonged stagnation in Japan.

At first glance, it may seem that it is easy to choose a good equilibrium because everyone likes a good equilibrium so much that they coordinate with one another. The economy is, in fact, too complex and too large to coordinate with all the people and firms simultaneously. Unless a sufficient number of agents behave in the same way, the action entails more private costs than private benefits, in spite of the fact that it may be socially beneficial. In this way, it can be difficult to achieve a good equilibrium. Such a difficulty would be more serious when shifting one equilibrium to another than when selecting a equilibrium because a good (or bad) history is likely to affect expectations positively (or negatively).

An additional figure, Figure 2.2.1, shows the relationship between the profits π and the government spending G . Regarding how these two equilibria arise depending on G , three cases are observed. Firstly, when G is small, the aggregate demand is too low to produce positive profits, hence only a bad equilibrium survives. Secondly, as an opposite case, when G is sufficiently large, the aggregate demand is so high that a firm is better off from participation irrespective of the other sector's action. Thus there is only a good equilibrium. Finally, when the amplitude of G is in the middle range, there exist multiple equilibria. The thick

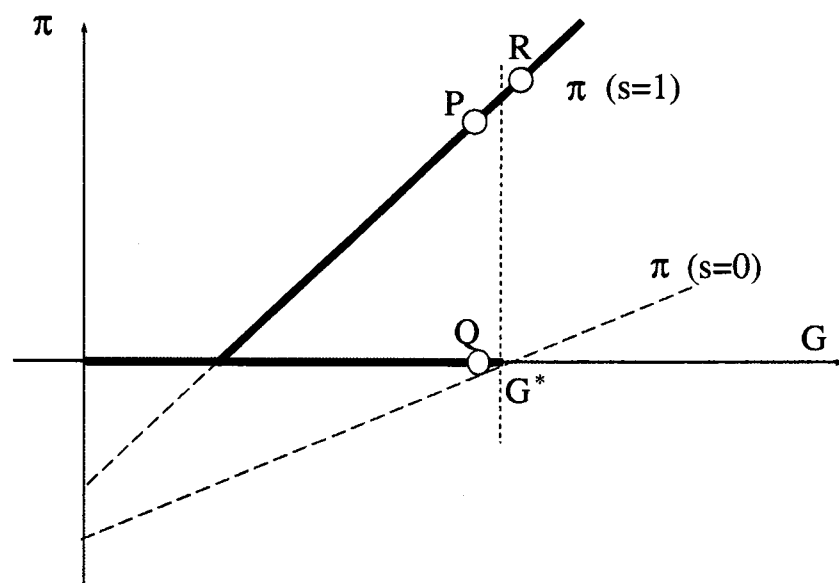


Figure 2.2.1: Government Spending and Profits

lines in this figure represent the possible equilibria. These things suggest that government expenditure can play a significant role in deciding equilibrium².

Turning our attention to welfare, we can obtain a Keynesian-like implication. As discussed in Section 2.2.1, in the basic model, government spending reduces the consumer's utility, and this is because no unemployment exists and the consumer's utility does not depend on government expenditure. This contradicts the view of Keynes. However, if the economy is trapped in a bad equilibrium, government spending to help escape from the equilibrium could increase utility. To see this in Figure 2.2.1, compare the utility at points P , Q , and R . For a sufficiently close P and R , the utility satisfies $u_P > u_Q$ and $u_P \sim u_R = u_P - \varepsilon$ (ε is small). Hence utility increases; $u_R > u_Q$, owing to an increase in government expenditure. Such a feature, i.e. that government expenditure can raise welfare comes much closer to the Keynesian view than that obtained in Section 2.2.1.

We can thus claim that, if the economy is stuck at the bad equilibrium, the optimal level of government spending should be G^* in Figure 2.2.1. If the economy is at the good equilibrium and if we can be sure that the economy does not fall into the bad equilibrium,

²The reason for different slopes between $\pi(s = 1)$ and $\pi(s = 0)$ is because the multiplier of government expenditure increases due to demand externalities.

then government expenditure should be at a minimum level to allow the good equilibrium to exist. In this figure, this level is expressed as the intersection point of two lines $\pi(s = 0)$ and $\pi(s = 1)$.

According to the above formalisation, the current situation of Japan can be understood as a bad equilibrium. Indeed, since the bubble burst in 1991, the strategy of firms has become extremely passive. They have hesitated to start up business. Although there has been some recovery, especially in corporate earnings on several occasions since 1991, such recovery has not been large or sustainable enough to boost the economy enabling Japan to escape from the prolonged slump. All the time, the recovery of employment and investment was smaller than expected from recovered corporate earnings. Instead, if all the firms had simultaneously increased their demand at that time, they could have made higher profits and Japan may not have fallen into a prolonged depression.

Sufficiently large government spending can, in principle, shift the economy from a bad equilibrium to a good one. However, in reality, this role of the government may be put in doubt because it is quite difficult to raise as much tax as is needed. A heavy burden of tax will distort the efficient resource allocation as well as the vertical equality between poor and rich. Actually, this reason is very weak. In the short-term, government does not need to collect the necessary money from tax. Irresponsible fiscal policy which seems not to be sustainable can shift the equilibrium from bad to good at any rate.

2.3 Extending the Matsuyama Model: Expanding the Possibilities of Multiple Equilibria

2.3.1 Horizontal Complementarities

This section and the next modify the above Matsuyama model in order to see how the possibilities of multiple equilibria vary with a degree of business connections. Firstly, in order to study the effect of the number of sectors, his one-sector model is modified to a multiple-sectors model by incorporating horizontal complementarities.

Suppose that there are N sectors ($i = 1, 2, \dots, N$) and two stages ($t = 1, 2$). At $t = 1$, firms in each sector decide whether to participate in trade. A sector i consists of a number of firms which produce the goods whose varieties are $z_i \in [0, 1]$. Each sector produces a different kind of goods (i.e. $z_i \neq z_j$). At $t = 2$, the active firms which participate in trade start their business and make a certain amount of profits from business. The other inactive firms produce nothing and their profits are zero. Instead, in inactive sectors, goods are produced by a competitive fringe. Assume that all firms in the same sector behave in the same way and that fixed costs are spent on goods (not on labour), then equation (2.2.16) is transformed as

$$\Pi_i(n, N) = \mu_i \left[\alpha_i (L + \Pi - T) + \sum_{i=1}^n n_i F + G_i \right] - n_i F, \quad (2.3.1)$$

for $i = 1, 2, \dots, n$, where n is the number of active sectors. For simplicity, assume the following symmetry:

$$\mu_i \equiv \mu, \quad \alpha_i \equiv \alpha/N, \quad n_i \equiv 1/N, \quad G_i \equiv G/N.$$

Let me explain the idea behind these equations. Firstly, the second equation is not α/n but α/N , which assumes that, irrespective of the number of active sectors, a consumer spends equal proportions of α/N from his income on the goods made in each sector. That is, even though other sectors are inactive, one sector cannot occupy all the consumer's demand. Instead, competitive fringes in the inactive sectors produce the corresponding goods. This

assumption is crucial to yield multiple equilibria and plausible as long as the goods are different in each sector. Secondly, as the last three equations ensure, the total size of the economy does not differ from the case of one sector. That is, $\sum \alpha_i = \alpha$, $\sum n_i = 1$, and $\sum G_i = G$. Finally, both tax T and labour L are kept fixed.

The profit equation is rearranged as follows.

$$\Pi_i(n, N) = \mu [\alpha/N (L + n\Pi_i(n, N) - T) + (nF/N)/N + G/N] - F/N. \quad (2.3.2)$$

Hence,

$$\Pi_i(n, N) = \frac{\mu [\alpha (L - T) + nF/N + G] / N - F/N}{1 - \mu\alpha \cdot n/N}. \quad (2.3.3)$$

The sum of the profits in all the sectors becomes

$$\begin{aligned} \Pi(n, N) &= \sum_{i=1}^n \Pi_i(n, N) + \sum_{j=n+1}^N 0 \\ &= \frac{\{\mu [\alpha (L - T) + nF/N + G] - F\} \cdot n/N}{1 - \mu\alpha \cdot n/N}. \end{aligned} \quad (2.3.4)$$

If $\Pi_i(N, N) > 0$, there is a good equilibrium where N sectors participate in trade. The condition is rewritten as

$$0 < \Pi(N, N) = \frac{\mu [\alpha (L - T) + F + G] - F}{1 - \mu\alpha}. \quad (2.3.5)$$

On the other hand, if $\Pi_i(1, N) < 0$, no sector participates in trade. The condition is

$$0 > \Pi(1, N) = \frac{\{\mu [\alpha (L - T) + F/N + G] - F\}/N}{1 - \mu\alpha/N}. \quad (2.3.6)$$

After all, if these two conditions, that is

$$\frac{1 - \mu}{\mu} F < [\alpha (L - T) + G] < \frac{1 - \mu/N}{\mu} F, \quad (2.3.7)$$

are satisfied, then multiple equilibria coexist³. As the number of sectors becomes larger, the condition becomes more likely to be satisfied. The region of a good equilibrium shrinks. This is because, as N increases, each sector loses its own market share and, in turn, a private benefit from participating in trade becomes smaller.

This result implies that, as the economy grows and more varieties of goods are produced,

³This inequality coincides with equation (2.2.35) in the limit as $N \rightarrow \infty$.

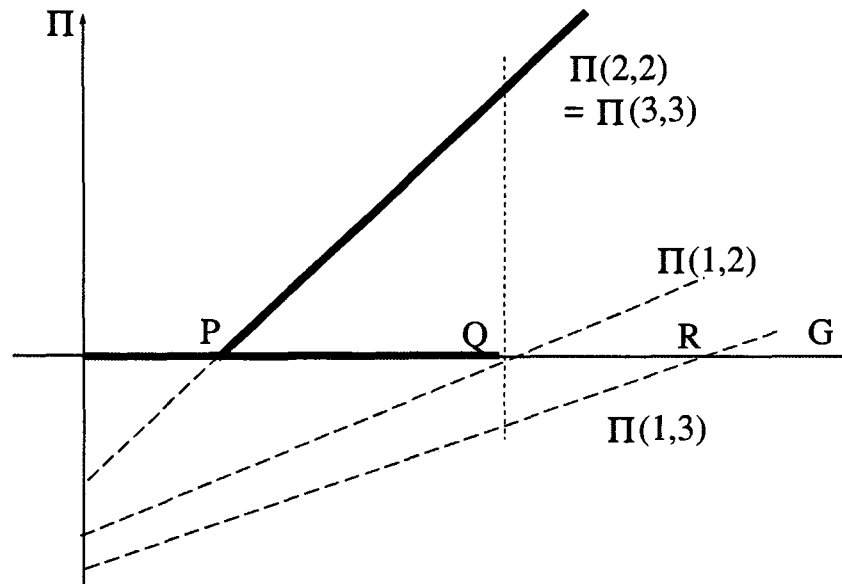


Figure 2.3.1: Comparison of Firms' Profits when $N = 2$ and $N = 3$.

the possibility of multiple equilibria increases. Furthermore, the necessary government expenditure G^* to shift the economy from a bad to a good equilibrium increases. In the special case of $N = 2$ and 3, Figure 2.3.1 shows how the profit line moves depending on a change in N . In the case of $N = 2$, there are multiple equilibria if government expenditure is between P and Q . In the case of $N = 3$, while the line $\Pi(N, N)$ does not move, the line $\Pi(1, N)$ shifts to the right. Therefore, the intersection point of $\Pi(1, N) = 0$ moves to the right from Q to R , and the range of government expenditure which can produce multiple equilibria is widened.

2.3.2 Vertical Complementarities

This section investigates multiple equilibria with the model of vertical complementarities. In addition to the above horizontal complementarities, these vertical complementarities increase the degree of business connections, and in turn, the possibility of multiple equilibria. The model consists of two periods $t = 1$ and 2 and, at the beginning of $t = 1$, there are potentially S monopolistic firms ($j = 1, 2, \dots, S$) which build up the supply chain of goods. There also exist S firms (competitive fringe) with the technology of constant returns to scale. At $t = 1$, each monopolistic firm decides whether to participate in trade or not. If not, the goods are

produced by the competitive fringe. For simplicity, suppose all the properties of mark-up and fixed costs are the same, that is $\mu_j \equiv \mu_0$ and $F_j \equiv F$ for all j .

Firstly, assume that fixed costs are used for labour. It will soon be shown that there are no multiple equilibria. As Appendix 2.A.2 shows, if participating in trade, the profit of monopolistic firm j can be calculated as

$$\Pi_1 = \frac{\mu_0 [\alpha(L - T) + G] - F}{1 - \mu\alpha} \quad (2.3.8)$$

$$\Pi_j = (1 - \mu_0)^j \Pi_1 \text{ for } 2 \leq j \leq S, \quad (2.3.9)$$

$$\text{where } \mu = 1 - (1 - \mu_0)^S. \quad (2.3.10)$$

Normally, the mark-up μ_0 lies between 0 and 1, so $\Pi_j < \Pi_{j-1}$ from equation (2.3.9). In other words, downstream firms which are close to consumers make higher profits than upstream firms. This asymmetry causes technical complexity for a further analysis, in contrast to the model of horizontal complementarities.

Note this equation is not quite correct unless all S firms participate in trade. To be more accurate, depending on which firm is active, Π_j becomes

$$\Pi_j = (1 - \mu_0)^k \Pi_1 = (1 - \mu_0)^k \frac{\mu_0 [\alpha(L - T) + G] - F}{1 - \mu_n \alpha}, \quad (2.3.11)$$

where μ is replaced by μ_n defined by the number of active sectors n instead of S , that is $\mu_n = 1 - (1 - \mu_0)^n$, and k represents the order of firm j counted from the downstream firms among active firms. For instance, if the firms $j = 1, 2, 5, 6$ are active, then $k = 4$ for the firm $j = 6$. However, in this special case of symmetric parameters, profits are always larger in downstream firms than in upstream firms, so if firm j can make profits from trade, then all firms with $i < j$ should participate in trade. Hence, k always coincides with j .

Equation (2.3.9) suggests that, under the assumption that firms have the same parameters, if one of the S firms makes positive (or negative) profits from trade, the other firms also make positive (or negative) profits. The action of the other firms changes only the magnitude of

the firm's profit through a change in μ and does not change the sign of the profit at all. Thus, the choice of a firm's participation in trade is independent of that of the other firms, and multiple equilibria do not arise.

Alternatively, assume that all the fixed costs are spent for the final goods instead of labour. Then, multiple equilibria arise. Equation (2.2.25) is redefined as

$$Q_1 = \alpha \left(L + \sum_{j=1}^S \Pi_j - T \right) + G + \sum_{j=1}^S F_j. \quad (2.3.12)$$

Then a similar calculation yields the following result:

$$\Pi_j = (1 - \mu_0)^j \frac{\mu_0 [\alpha(L - T) + G + nF] - F}{1 - \mu_n \alpha}. \quad (2.3.13)$$

Again, n is the number of active sectors. Multiple equilibria arise if the following conditions are satisfied for all j :

$$(A) \Pi_j = (1 - \mu_0)^j \frac{\mu_0 [\alpha(L - T) + G + SF] - F}{1 - \mu \alpha} > 0 \text{ for } n = S \quad (2.3.14)$$

$$(B) \Pi_j = (1 - \mu_0)^j \frac{\mu_0 [\alpha(L - T) + G + F] - F}{1 - \mu_0 \alpha} < 0 \text{ for } n = 1. \quad (2.3.15)$$

Condition (A) means that, provided all firms participate, they can make positive profits, and condition (B) means that, provided all other firms are inactive, a firm suffers a loss from trade. These conditions are rewritten as follows:

$$\frac{1 - S\mu_0}{\mu_0} F < [\alpha(L - T) + G] < \frac{1 - \mu_0}{\mu_0} F. \quad (2.3.16)$$

The region of multiple equilibria positively depends on S . The higher S , the higher the possibility of multiple equilibria. In particular, an increase in S raises the region of the good equilibrium while that of the bad equilibrium does not change. This suggests that, once an economy is stuck in the bad equilibrium, the necessary government expenditure G^* to shift the economy to the good equilibrium does not change with S . This implication is different from that of the previous horizontal complementarity model.

It is known that Japanese manufacturing companies have been making very complex

products by constructing a long supply chain with trade partners. This suggests that, in an economy like that of Japan, S is high, and multiple equilibria are more likely to occur.

2.3.3 More General Model of Multiple Equilibria

The previous multiple equilibria model imposed a rather strong assumption. As was expressed in equation (2.2.1), the model assumed that the cross-partial elasticity of substitution between goods was one. This assumption conveniently restricts the demand elasticity to one, which leads monopolistic firms to always set the same prices as their competitive fringes. However, such an assumption can be too restrictive. Besides, the model assumed that the fixed costs are used not for labour but only for goods, which was also a crucial element in yielding multiple equilibria.

This section constructs a more general model by relaxing these assumptions. As readers will soon see, similar multiple equilibria can be produced from the new model as well. However, the mechanism to yield multiple equilibria is different. In this new model, price distortion as well as demand externalities plays a key role. Interestingly, we find that the result of this model is very similar to that of Ball and Romer (1991), which incorporates menu costs. Furthermore, it will be shown that no firm participation entails deflation, which is the case in Japan.

Model Setup

Unless noted otherwise, the notation is the same as that in Section 2.2.1.

<Household>

The utility of a representative consumer is described as

$$\alpha \ln C + (1 - \alpha) \ln(N), \tag{2.3.17}$$

where C is the composite consumption. Using Dixit and Stiglitz (1977), C is defined as

$$C = \left[\int_0^1 c(z)^{\frac{1}{1+\theta_1}} dz \right]^{1+\theta_1} \quad (\theta_1 > 0), \quad (2.3.18)$$

where θ_1 represents mark-up; the cross-partial elasticity of substitution can thus be represented by $\sigma_1 = 1 + 1/\theta_1$. A larger σ_1 means that the varieties can be highly substituted for each other. Note that θ_1 is different from the previously appearing μ , although both are related to mark-up. Whilst the former is derived from the elasticity of substitution as will be shown later, the latter is derived from the technology of firms. As $\theta_1 \rightarrow \infty$ or $\sigma_1 \rightarrow 1$, the composite consumption converges to $\int_0^1 \ln c(z) dz$. Hence, the utility function in Section 2.2.1 corresponds to a special case. In addition, suppose that there are other dimensional varieties of goods. In other words, suppose that a variety $z \in [0, 1]$ is composed of two varieties of z_1 and z_2 , and that these are not perfect substitutes. The former is produced by a monopolistic firm, while the latter is produced by a competitive fringe. $c(z)$ is defined as

$$c(z) = 2^{-\theta_2} \left(c(z_1)^{\frac{1}{1+\theta_2}} + c(z_2)^{\frac{1}{1+\theta_2}} \right)^{1+\theta_2} \quad (\theta_2 > 0). \quad (2.3.19)$$

Similarly, θ_2 is mark-up. The coefficient $2^{-\theta_2}$ is derived so that, when the price of each goods is the same, the price index can coincide with it. This will be shown later. If a monopolistic firm does not participate in trade, then only z_2 is produced. A representative consumer faces the following budget constraint:

$$\int_0^1 \{p(z_1)c(z_1)dz_1 + p(z_2)c(z_2)dz_2\} + N \leq L + \Pi - T. \quad (2.3.20)$$

Again, leisure is set as a numeraire (i.e. wages are equal to 1).

The consumer chooses the optimal level of consumption and hours of labour/leisure. As

Appendix 2.A.1 shows, the solution of this utility maximization problem can be obtained as

$$c(z) = \left(\frac{p(z)}{P} \right)^{-\frac{1+\theta_1}{\theta_1}} C \quad (2.3.21)$$

$$c(z_i) = \left(\frac{p(z_i)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2} c(z) \quad (i = 1, 2) \quad (2.3.22)$$

$$N = (1 - \alpha)(L + \Pi - T) \quad (2.3.23)$$

$$\begin{aligned} PC &= \int_0^1 p(z) c(z) dz \\ &= \int_0^1 \{p(z_1) c(z_1) + p(z_2) c(z_2)\} dz \\ &= \alpha(L + \Pi - T) \end{aligned} \quad (2.3.24)$$

$$P = \left[\int_0^1 p(z)^{-\frac{1}{\theta_1}} dz \right]^{-\theta_1} \quad (2.3.25)$$

$$p(z) = 2^{\theta_2} \left(p(z_1)^{-\frac{1}{\theta_2}} + p(z_2)^{-\frac{1}{\theta_2}} \right)^{-\theta_2} \quad (2.3.26)$$

As noted earlier, the last equation suggests that, when $p(z_1) = p(z_2)$, $p(z) = p(z_1) = p(z_2)$.

Assume that the government spends a nominal amount of money G on goods with the same elasticity as consumption so that it does not distort the allocation efficiency. The demand for each goods, $q(z)$ and $q(z_i)$, then becomes

$$\begin{aligned} q(z) &= c(z) + g(z) \\ &= \left(\frac{p(z)}{P} \right)^{-\frac{1+\theta_1}{\theta_1}} C + \left(\frac{p(z)}{P} \right)^{-\frac{1+\theta_1}{\theta_1}} \frac{G}{P} \\ &= \left(\frac{p(z)}{P} \right)^{-\frac{1+\theta_1}{\theta_1}} \cdot \frac{\alpha(L + \Pi - T) + G}{P}, \end{aligned} \quad (2.3.27)$$

$$\begin{aligned} q(z_i) &= c(z_i) + g(z_i) \\ &= \left(\frac{p(z_i)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2} c(z) + \left(\frac{p(z_i)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2} g(z) \\ &= \left(\frac{p(z_i)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2} (c(z) + g(z)) \end{aligned} \quad (2.3.28)$$

$$= \left(\frac{p(z_i)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2} \left(\frac{p(z)}{P} \right)^{-\frac{1+\theta_1}{\theta_1}} \cdot \frac{\alpha(L + \Pi - T) + G}{P}. \quad (2.3.29)$$

<Firm>

Assume that fixed costs are used not for goods but for labour. In Section 2.2.4, the opposite assumption was necessary to yield multiple equilibria, but in this new model, this is no longer required.

There exists a competitive fringe which produces 1 unit of z_2 goods with 1 unit of labour, so it always sets a price of 1. A monopolistic firm that produces a variety z_1 has a technology of increasing returns to scale, and its profit is given by

$$\pi(z_1) = p(z_1)q(z_1) - mc \cdot q(z_1) - F, \quad (2.3.30)$$

where mc is the marginal cost of production which is set as constant. The firm faces downward sloping demand described in equation (2.3.29):

$$q(z_1) = \left(\frac{p(z_1)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2} \left(\frac{p(z)}{P} \right)^{-\frac{1+\theta_1}{\theta_1}} \cdot \frac{\alpha(L + \Pi - T) + G}{P}.$$

The firm takes P as given because there are a large number of firms that produce different varieties. However, since there are only two firms in the same variety z , the monopolistic firm can influence $p(z)$ by setting $p(z_1)$. This results in a variable price elasticity of demand, so the calculation of the optimal $p(z_i)$ becomes a little cumbersome. Thus, for simplicity, we assume $\theta = \theta_1 = \theta_2$ in the following discussion. Then, the demand curve is transformed to

$$q(z_1) = \left(\frac{p(z_1)}{P} \right)^{-\frac{1+\theta}{\theta}} \frac{1}{2} \frac{\alpha(L + \Pi - T) + G}{P}. \quad (2.3.31)$$

Therefore, the monopolistic firm sets the price p_1 as follows:

$$p_1 \equiv p(z_1) = (1 + \theta)mc. \quad (2.3.32)$$

Here, θ can be interpreted as mark-up. The price may be higher or lower than 1. Since the monopolistic firm and the competitive fringe produce different varieties of goods, the former

can set a price higher than 1. The composite price index of a variety z , p_m , thus becomes

$$\begin{aligned} p_m \equiv p(z) &= 2^\theta \left(p_1^{-\frac{1}{\theta}} + 1 \right)^{-\theta} \\ &= \left(\frac{2}{(1/p_1)^{\frac{1}{\theta}} + 1} \right)^\theta. \end{aligned} \quad (2.3.33)$$

Consider the following game with two stages, $t = 1$ and 2. Monopolistic firms first decide whether to participate in trade at $t = 1$. If they participate in trade, they produce the goods of z_1 by setting p_1 at $t = 2$. If not, the corresponding varieties z_1 are not produced and the composite price index of a variety z , $p(z)$, becomes 1⁴. Suppose that a fraction s ($0 \leq s \leq 1$) of monopolistic firms participate in trade. Then, the price index becomes

$$\begin{aligned} P &= \left[\int_0^1 p(z)^{-\frac{1}{\theta}} dz \right]^{-\theta} \\ &= \left[1 - s + s p_m^{-\frac{1}{\theta}} \right]^{-\theta} \end{aligned} \quad (2.3.34)$$

from the definition of P . Active monopolistic firms make profits given by

$$\begin{aligned} \pi(z_1) &= p(z_1)q(z_1) - mc \cdot q(z_1) - F \\ &= (p_1 - mc)q(z_1) - F = (p_1 - mc) \left(\frac{p_1}{P} \right)^{-(1+\frac{1}{\theta})} \frac{1}{2} \frac{\alpha(L + \Pi - T) + G}{P} - F \\ &= \left(\frac{p_1}{P} \right)^{-\frac{1}{\theta}} \frac{1}{2} \mu \{ \alpha(L + \Pi - T) + G \} - F. \end{aligned} \quad (2.3.35)$$

The last equation is obtained because

$$p_1 - mc = (1 + \theta)mc - mc = \theta mc = \frac{\theta}{1 + \theta} p_1 \quad (2.3.36)$$

and μ is defined by

$$\mu \equiv \frac{\theta}{1 + \theta} (< 1).$$

Using equation (2.3.34), the profits can be rewritten as

$$\pi(z_1) = \frac{p_1^{-\frac{1}{\theta}}/2}{1 - s + s p_m^{-\frac{1}{\theta}}} \cdot \mu \{ \alpha(L + \Pi - T) + G \} - F. \quad (2.3.37)$$

⁴In other words, we assume that if a monopolistic firm does not participate in trade, then $p(z) = p(z_2)$ and $c(z) = c(z_2)$.

Since aggregate profits become $\Pi = s\pi(z_1)$, we obtain

$$\pi(z_1) = \frac{p_1^{-\frac{1}{\theta}}/2}{1-s+sp_m^{-\frac{1}{\theta}}} \cdot \mu\{\alpha(L+s\pi(z_1)-T)+G\} - F. \quad (2.3.38)$$

Therefore, a monopolistic firm's profits are given by

$$\begin{aligned} \left(1 - s\mu\alpha \frac{p_1^{-\frac{1}{\theta}}/2}{1-s+sp_m^{-\frac{1}{\theta}}}\right) \cdot \pi(z_1) &= \frac{p_1^{-\frac{1}{\theta}}/2}{1-s+sp_m^{-\frac{1}{\theta}}} \cdot \mu\{\alpha(L-T)+G\} - F \\ \pi(z_1) &= \frac{\frac{p_1^{-\frac{1}{\theta}}/2}{1-s+sp_m^{-\frac{1}{\theta}}} \cdot \mu\{\alpha(L-T)+G\} - F}{1 - s\mu\alpha \frac{p_1^{-\frac{1}{\theta}}/2}{1-s+sp_m^{-\frac{1}{\theta}}}}. \end{aligned} \quad (2.3.39)$$

Multiple Equilibria

If $s = 1$, that is, all firms participate in trade, then a monopolistic firm's profits become

$$\pi(z_1) = \frac{\frac{p_1^{-\frac{1}{\theta}}/2}{p_m^{-\frac{1}{\theta}}} \cdot \mu\{\alpha(L-T)+G\} - F}{1 - \mu\alpha \frac{p_1^{-\frac{1}{\theta}}/2}{p_m^{-\frac{1}{\theta}}}}. \quad (2.3.40)$$

In contrast, if $s = 0$, that is, no other firms participate in trade, then its profits are

$$\pi(z_1) = p_1^{-\frac{1}{\theta}}/2 \cdot \mu\{\alpha(L-T)+G\} - F. \quad (2.3.41)$$

Provided that the former and the latter profits are positive and negative respectively, there arise multiple equilibria. The condition can be written as

$$\frac{2p_m^{-\frac{1}{\theta}}F}{\mu p_1^{-\frac{1}{\theta}}} < \alpha(L-T)+G < \frac{2F}{\mu p_1^{-\frac{1}{\theta}}}. \quad (2.3.42)$$

In order to satisfy this condition, $p_m > 1$ is necessary. From equation (2.3.33), we can see that it is equivalent to

$$p_m = \left(\frac{2}{(1/p_1)^{\frac{1}{\theta}} + 1} \right)^{\theta} > 1 \quad (2.3.43)$$

$$\Leftrightarrow p_1 > 1 \quad (2.3.44)$$

$$\Leftrightarrow mc > \frac{1}{1+\theta}. \quad (2.3.45)$$

Monopolistic firms set prices higher than one when the marginal cost of production mc or

mark-up θ is high. For example, if mark-up θ is equal to 0.2, mc needs to be higher than 0.83. A lower θ or higher σ requires marginal costs mc to be closer to 1.

One reason for multiple equilibria again lies in the demand externalities, but there is another important reason: price distortion. Suppose that only one monopolistic firm participates in trade and sets its price p_1 higher than one. Since all other firms are inactive, the price index P is approximately one. Thus, the relative price of p_1 over P becomes high, which reduces the demand for the goods. The firm therefore cannot make a profit in this case. On the other hand, suppose that all firms participate in trade. Compared with the previous case, the price index P increases, and the relative price of p_1 decreases. This results in a relatively higher demand and profits. In the Matsuyama model, the cross-partial elasticity of substitution was one and monopolistic firms set the same price as the competitive fringe. Therefore, the above channel through relative prices does not exist, and multiple equilibria cannot arise without a further assumption such that fixed costs are used not for labour but for goods. In this new model, price distortion plays a crucial role in yielding multiple equilibria.

Next, I will compare welfare in the two equilibria. Since composite consumption and leisure are proportional to $(L + \Pi - T)/P$, the utility of a representative consumer is higher as $(L + \Pi - T)/P$ is larger. This term in each equilibrium is given by

$$\bullet L - T \quad (\text{when } s = 0) \quad (2.3.46)$$

$$\bullet (L + \Pi - T)/P = \frac{L + \left(\frac{p_1^{-\frac{1}{\theta}}/2}{p_m^{-\frac{1}{\theta}}} \cdot \mu \{ \alpha(L - T) + G \} - F \right) \left(1 - \mu \alpha \frac{p_1^{-\frac{1}{\theta}}/2}{p_m^{-\frac{1}{\theta}}} \right)^{-1} - T}{p_m} \quad (\text{when } s = 1). \quad (2.3.47)$$

It is not clear which of the two is larger. There are two opposite effects. As $p_m > 1$, a higher p_m reduces real income and, in turn, welfare. On the other hand, a higher p_m increases monopolistic firms' profits, by which more money is distributed to households. Depending

CHAPTER 2. DEMAND EXTERNALITY AND MULTIPLE EQUILIBRIA

on the parameter, the former can exceed the latter, and vice versa. The equilibrium of no participation is not necessarily bad in this model.

Interestingly, such a feature is quite similar to that of Ball and Romer (1991). They also use a monopolistic competition model and show that there arise multiple equilibria. Which equilibrium is better is not determinate. Differences between the two models can be found in firms' actions and necessary assumptions to yield multiple equilibria. Their model does not assume the existence of competitive fringe. Instead, it assumes menu costs, which prevent firms from changing their prices. Firms' action is not whether they participate in trade, but whether they change their prices or not. In other words, one equilibrium corresponds to the case where all firms change prices fully, and the other corresponds to the complete non-adjustment of prices. Depending on the value of menu costs, the former equilibrium may be better or worse than the latter.

Our model can help explain Japan's situation as being somewhat like a deflation spiral. As was already shown in the figure in the Introduction, overall prices have been decreasing since 1999. Compared with other countries, this is one main characteristic peculiar to Japan. According to this model, when the aggregate price index is low, there is little incentive for monopolistic firms to participate in trade by setting higher prices. Thus, they dare not participate in trade, which reduces the aggregate price even further. The economy gets trapped in the equilibrium where no monopolistic firms participate in trade; the (nominal) income of households becomes low; and overall prices are also low. These features describe Japan's current situation very well. True, this model addresses that such a situation may not be bad because households may have more purchasing power, but considering many other costs caused by deflation such as the accumulation of bad loans, this equilibrium with no participation and low prices does not seem to be desirable.

2.4 Equilibrium Selection

So far, this chapter has discussed business connections using the monopolistic competition models and showed the possibility of multiple equilibria. Japan's current economy can be considered to be in a bad equilibrium along this formulation. However, there remains an unanswered question regarding why Japan has fallen into the bad equilibrium in the first place.

This chapter proposes chiefly two answers to the question above. The first answer is that some kind of large shock eliminated the multiple equilibria. Suppose that the economy is initially at a good equilibrium, but a large negative demand shock arises⁵. Even though it is temporary, this shock may cause only the bad equilibrium to survive, which traps the economy in the bad equilibrium in the long-run.

The second answer is that expectations affected equilibrium selection. It is hard to see how expectations are formed, but Morris and Shin (2001) demonstrated an excellent idea on this question. This idea originated from Carlsson and van Damme (1993)'s "Global games", that uncertainty could remove the multiplicity in a game with multiple equilibria. Morris and Shin extended and showed how an equilibrium is selected. According to them, the determinants of equilibrium are certain fundamentals that affect the expectations of agents. A fundamental change does not need to be large enough to eliminate the multiple equilibria which may have arisen under no uncertainty.

While the first answer is intuitively very simple, the second is a little hard to understand, and moreover, we cannot be entirely sure whether the approach of global games can be applied until it is proved. This section introduces Morris and Shin's approach, and investigates how an equilibrium is selected in the simple bank runs model by Diamond and Dybvig (1983), which will be discussed in Section 2.4.1. The following two sections, Section 2.4.2 and 2.4.3,

⁵In the Matsuyama model, you may suppose a rise in fixed costs.

apply their approach to the previous Matsuyama models.

2.4.1 Bank Runs Model

This section overviews the theory of equilibrium selection by Morris and Shin (2001) using the bank runs model by Diamond and Dybvig (1983).

Model Setup

There are three stages: $t = 0, 1, 2$, and continuous consumers. The consumers deposit the endowment of unit 1 to a bank at $t = 0$. At $t = 1$, they decide whether to withdraw their deposits or not. If they withdraw, they receive unit 1, giving a utility of 0 at $t = 2$, i.e. $u(1) = 0$. On the other hand, if they do not withdraw, they get a utility of $r - l$ at $t = 2$. Here, r is the rate of return and $l \in [0, 1]$ is the proportion of consumers who withdraw their deposits at $t = 1$. This reflects the costs of premature liquidation.

Assuming r is between 0 and 1, we can find multiple equilibria. To be more concrete, if all consumers are patient, their utility becomes r because l is 0. Since this utility r is greater than the zero utility which is obtained from withdrawal at $t = 1$, this strategy is justified as a pure equilibrium. In contrast, if a patient consumer believes that all other consumers are impatient (i.e. $l = 1$), his utility will be $r - 1$ which is smaller than 0. Hence, one also withdraws the money at $t = 1$, and another equilibrium arises such that all consumers withdraw their deposits. The former equilibrium is considered a good one (their utility is $r > 0$), and the latter a bad one (their utility is 0).

Incomplete Information and Equilibrium Selection

Assume that there is uncertainty about the return r , and r has a normal distribution with mean $\bar{r} \in (0, 1)$ and precision α (i.e. variance $1/\alpha$). Consumers cannot observe r directly but

can indirectly infer it from the signal they receive:

$$x_i = r + \epsilon_i, \quad (2.4.1)$$

where ϵ_i is normally distributed with mean 0 and precision β , and independent across consumers.

A depositor i receives the signal x_i , and updates one's belief about r as

$$\begin{aligned} \rho \equiv E(r|x_i) &= \bar{r} + \frac{1/\alpha}{1/\alpha + 1/\beta}(x_i - \bar{r}) \\ &= \frac{\alpha\bar{r} + \beta x_i}{\alpha + \beta}. \end{aligned} \quad (2.4.2)$$

Denote the standard normal distribution function as $\Phi()$ and define γ as

$$\gamma \equiv \frac{\alpha^2(\alpha + \beta)}{\beta(\alpha + 2\beta)}. \quad (2.4.3)$$

Then, Morris and Shin obtain the following theorem.

Theorem 1 *Provided that $\gamma \leq 2\pi$, there is a unique equilibrium. In this equilibrium, every patient consumer withdraws if and only if $\rho < \rho^*$, where ρ^* is the unique solution to*

$$\rho^* = \Phi(\sqrt{\gamma}(\rho^* - \bar{r})). \quad (2.4.4)$$

In the limit as γ tends to zero, ρ^ tends to $1/2$.*

In other words, as long as depositors receive sufficiently precise signals (i.e. β is high relative to α), they adopt a unique switching strategy with the switching point ρ^* . As Figure 2.4.1 shows, the switching point is obtained from the intersection point of the 45° line (the left-hand side of equation (2.4.4)) and the normal distribution curve (the right-hand side of equation (2.4.4)). The higher the fundamentals $\bar{r}$ or the higher the signal x_i they observe, the less likely they are to withdraw their deposits. Fundamentals affect their beliefs which become self-fulfilling.

The proof runs as follows. Since an impatient consumer gets a zero utility from withdrawal, at the switching signal ρ^* , the expected utility should also be zero from leaving the

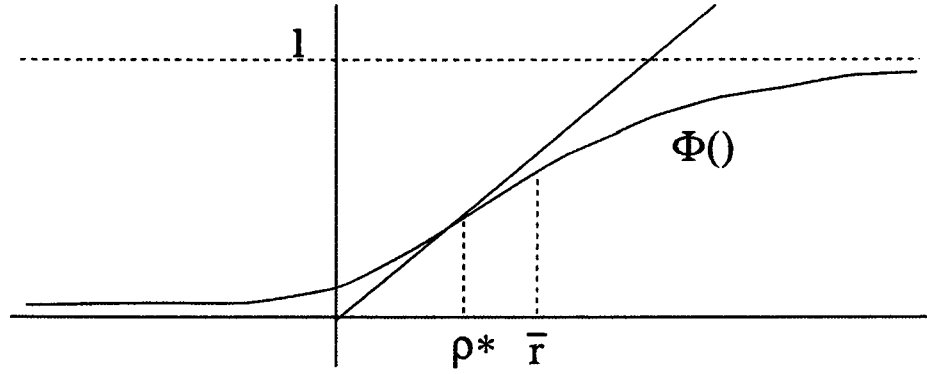


Figure 2.4.1: Switching Point ρ^*

money:

$$0 = E(r - l|\rho^*). \quad (2.4.5)$$

This is rearranged as

$$\begin{aligned} 0 &= E(r|\rho^*) - E(l|\rho^*) \\ &= \rho^* - \Pr(\rho_j < \rho^*|\rho^*), \end{aligned} \quad (2.4.6)$$

because the consumer whose signal ρ_j is less than ρ^* withdraws the money, which accounts for the proportion l of all the consumers. Conditional on the observed signal ρ_i ,

$$\begin{aligned} \rho_j < \rho_i &\Leftrightarrow \frac{\alpha\bar{r} + \beta x_j}{\alpha + \beta} < \rho_i \\ &\Leftrightarrow x_j < \rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}). \end{aligned} \quad (2.4.7)$$

Return r has a normal distribution with mean ρ_i and precision $\alpha + \beta$ because

$$\begin{aligned} \text{Var}(r|x) &= \frac{1}{\alpha}(1 - \text{Cor}(r, x)) \\ &= \frac{1}{\alpha} \left(1 - \frac{(1/\alpha)^2}{(1/\alpha + 1/\beta) \cdot 1/\alpha} \right) \\ &= \frac{1}{\alpha + \beta}. \end{aligned} \quad (2.4.8)$$

From $x_j = r + \epsilon_j$, the distribution of x_j given ρ_i is normal with mean ρ_i and precision:

$$\frac{1}{\frac{1}{\alpha+\beta} + \frac{1}{\beta}} = \frac{\beta(\alpha + \beta)}{\alpha + 2\beta}. \quad (2.4.9)$$

Hence,

$$\begin{aligned}
 \Pr(\rho_j < \rho_i | \rho_i) &= \Pr\left(x_j < \rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}) | \rho_i\right) \\
 &= \Phi\left(\sqrt{\frac{\beta(\alpha + \beta)}{\alpha + 2\beta}}\left(\rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}) - \rho_i\right)\right) \\
 &= \Phi(\sqrt{\gamma}(\rho_i - \bar{r})),
 \end{aligned} \tag{2.4.10}$$

and equation (2.4.6) becomes

$$0 = \rho^* - \Phi(\sqrt{\gamma}(\rho^* - \bar{r})). \tag{2.4.11}$$

This coincides with the equation in Theorem 1.

If γ is small enough, then the slope of $\Phi(\sqrt{\gamma}(\rho^* - \bar{r}))$ becomes less than one, so the number of intersection points becomes at most one. Note that the derivative of the distribution function, i.e. the density function, has the maximum slope in the centre whose value is $\sqrt{\gamma/2\pi}$. Hence, provided $\gamma \leq 2\pi$, there is at most one intersection point.

In order to complete the proof, we next need to show that this is a unique equilibrium. This can be done by the procedure of iterated dominance. For a detailed proof, refer to Appendix 2.A.3. *Q.E.D.*

Effect of Uncertainty

The most crucial effect of uncertainty is in making multiple equilibria unique. Here, I would like to investigate another effect of uncertainty on equilibrium selection. Assume that $\bar{r} > 1/2$. For $\rho_0 = 1/2$, $\Phi(\sqrt{\gamma}(\rho_0 - \bar{r})) < 1/2 = \rho_0$. Thus, Figure 2.4.1 suggests that, if there is only one intersection, inequality becomes as $\rho^* < \rho_0 = 1/2 < \bar{r}$. The smaller the noise becomes (i.e. $\gamma \rightarrow 0$, or β is high relative to α), the flatter the $\Phi()$ curve becomes. Therefore, a decrease in uncertainty increases ρ^* up to $1/2$. A high ρ^* is equivalent to a high threshold to participate in trade, so a small uncertainty discourages participation in trade.

Moreover, for the same value of ρ^* , the observation x^* also increases. This leads to a

further discouragement of trade. In order to verify this, note

$$\rho^* = \frac{\alpha \bar{r} + \beta x^*}{\alpha + \beta}.$$

This means that ρ^* is the weighted average of x^* and $\bar{r}$, and an increase in β makes ρ^* closer to x^* . If $\bar{r} > 1/2$, then $x^* < \rho^* < \bar{r}$. Therefore, a decrease in uncertainty increases x^* toward ρ^* for the same value of ρ^* .

To summarise, if uncertainty decreases, these above two channels, that is, an increase in ρ^* and an increase in x^* for a given ρ^* , make the condition of deposit withdrawals less stringent, leading to a bad equilibrium. Appendix 2.A.4 illustrates this result in a special case using numerical calculation.

Intuitively, the reason for this can be understood in the following way. When uncertainty is small, a low observation x compared with $\bar{r}$ induces a consumer to think that the true r is low, which discourages him from keeping his deposit. On the other hand, when uncertainty is large, he just reckons his low observation as noise. Hence, as long as $\bar{r}$ is high, he thinks that the real r will be high, so he will keep his deposit.

This result is opposite to our intuition. In reality, high uncertainty usually discourages active preferable actions. It is true that, if $\bar{r}$ is smaller than $1/2$, a decrease in uncertainty reduces x^* , which does not contradict our intuition. However, in the case where $\bar{r} > 1/2$, there still remains the contradiction. This brings us to Section 2.5, where we will discuss the effect of uncertainty on the choice of equilibrium by incorporating a bankruptcy risk.

2.4.2 Matsuyama Model

Applying the theory by Morris and Shin, this section investigates how an equilibrium is selected in the Matsuyama model of Section 2.2.4. Recall that a firm's profits are given by

equation (2.2.32)⁶:

$$\pi = \frac{\mu\{\alpha_0(L - T) + G\} - (1 - \mu s)F}{1 - \mu\alpha_0 s}, \quad (2.4.12)$$

where s is the proportion of active firms, so $s = 1 - l$ using the previous notation. If both $\pi(s = 0) < 0$ and $\pi(s = 1) > 0$ are satisfied, that is,

$$\begin{aligned} \frac{1 - \mu}{\mu}F &< G + \alpha_0(L - T) < \frac{1}{\mu}F \\ &\Leftrightarrow \mu\{G + \alpha_0(L - T)\} < F < \frac{\mu\{G + \alpha_0(L - T)\}}{1 - \mu} \\ &\Leftrightarrow F_{\min} < F < F_{\max}, \end{aligned} \quad (2.4.13)$$

then multiple equilibria arise. This is the case when there is no uncertainty.

Assume then that the fixed cost F is unknown, and we can obtain the similar implication. In other words, multiple equilibria become unique under a small uncertainty and the monopolistic firm participates in trade if and only if its fixed cost $F < F^*$.

In the following, I will show how to obtain the switching point F^* . The setup is similar to before. Assume that F has a normal distribution with mean $\bar{F}$ and precision α (i.e. variance $1/\alpha$). Firms cannot observe F directly but can indirectly infer it from the signal:

$$x_i = F + \epsilon_i, \quad (2.4.14)$$

where ϵ_i is normally distributed with mean 0 and precision β , and independent across consumers. A firm i receives the signal x_i , and updates its belief about F as

$$\zeta \equiv E(F|x_i) = \frac{\alpha\bar{F} + \beta x_i}{\alpha + \beta}. \quad (2.4.15)$$

The parameter γ is defined in the same way as in equation (2.4.3):

$$\gamma \equiv \frac{\alpha^2(\alpha + \beta)}{\beta(\alpha + 2\beta)},$$

To write the result first,

Proposition 2 *There is a unique equilibrium when γ is small. In this equilibrium, every*

⁶In order to avoid confusion with the Bank Run model, I redefine the parameter α in the utility function as α_0 .

firm participates in trade if and only if $\zeta < \zeta^$, where ζ^* is the unique solution to*

$$\mu \{ \alpha_0 (L - T) + G \} = \{ 1 - \mu \Phi(\sqrt{\gamma}(\zeta^* - \bar{F})) \} \zeta^*. \quad (2.4.16)$$

The proof of this proposition is briefly sketched as follows. Suppose that all the firms obey the above strategy. Then, the proportion s is given by

$$E(s|\zeta^*) = \Pr(\zeta_j < \zeta^*|\zeta^*).$$

Note, in this model, large fixed costs ζ discourage the participation in trade and reduce s . It can be rearranged as

$$\begin{aligned} E(s|\zeta^*) &= \Pr \left(x_j < \zeta^* + \frac{\alpha}{\beta}(\zeta^* - \bar{F}) | \zeta^* \right) \\ &= \Phi \left(\sqrt{\gamma}(\zeta^* - \bar{F}) \right), \end{aligned}$$

in a similar way to the bank runs model. Since the switching point of ζ^* satisfies

$$\begin{aligned} E(\pi|\zeta^*) &= E \left[\frac{\mu \{ \alpha_0 (L - T) + G \} - (1 - \mu s)F}{1 - \mu \alpha_0 s} | \zeta^* \right] \\ &= 0, \end{aligned}$$

we obtain

$$E [\mu \{ \alpha_0 (L - T) + G \} - (1 - \mu s)F | \zeta^*] = 0. \quad (2.4.17)$$

Substituting $E(F|\zeta^*)$ and $E(s|\zeta^*)$, we can obtain the equation in the above proposition.

Actually, the last argument is not exactly true because

$$\begin{aligned} E \left[\frac{(1 - \mu s)F}{1 - \mu \alpha_0 s} | \zeta^* \right] &= \frac{E[(1 - \mu s)F | \zeta^*]}{E[1 - \mu \alpha_0 s | \zeta^*]} + Cov \left[(1 - \mu s)F, \frac{1}{1 - \mu \alpha_0 s} | \zeta^* \right] \\ &\neq \frac{E[(1 - \mu s)F | \zeta^*]}{E[1 - \mu \alpha_0 s | \zeta^*]}, \\ E[(1 - \mu s)F | \zeta^*] &= E[(1 - \mu s) | \zeta^*] \cdot E[F | \zeta^*] + Cov[(1 - \mu s), F | \zeta^*] \\ &\neq E[(1 - \mu s) | \zeta^*] \cdot E[F | \zeta^*] \end{aligned}$$

This is a problem which does not arise in the simple bank run model. Nevertheless, as this chapter independently showed in Appendix 2.A.5, provided that the number of potential

agents is sufficiently large, the variance of s ($= 1 - l$) given ζ^* becomes so small that the covariance term can be neglected⁷.

We cannot tell how many solutions equation (2.4.16) has. However, in the special case where γ is small, there is a unique solution. In equation (2.4.16), the left-hand side is constant with ζ^* , while it is undetermined whether the right-hand side is increasing or decreasing with ζ^* because the term in the bracket is decreasing. However, when γ is sufficiently small, the slope of this term becomes flat, which makes the right-hand side increase with ζ^* . Therefore, its solution always exists, and it becomes unique.

Due to the uniqueness of the solution, we can prove the existence of a unique equilibrium by applying the same procedure of iterated dominance as discussed in Appendix 2.A.3. In this model, the procedure of iterated dominance starts from $F_{\min}$ and $F_{\max}$. *Q.E.D.*

One example is shown in Figure 2.4.2. The parameters are given in the following table. The increasing curve represents the right-hand side of equation (2.4.16). The horizontal axis is ζ^* , and the intersection point of two curves corresponds to the switching point.

α	0.05	μ	0.2	$\bar{F}$	10	L	100
β	0.02	α_0	0.3	G	20	T	15

Here, I assumed that the fixed costs F are unknown, but we can derive a similar result by assuming mark-up μ or government expenditure G is unknown instead.

2.4.3 More General Multiple Equilibria Model

In this section, I will briefly discuss how an equilibrium can be selected in the monopolistic competition model discussed in Section 2.3.3. Rewrite equation (2.3.39):

$$\pi(z_1) = \frac{\frac{p_1^{1-\theta}/2}{1-s+sp_m^{1-\theta}} \cdot \mu\{\alpha(L-T) + G\} - F}{1 - s\mu\alpha \frac{p_1^{1-\theta}/2}{1-s+sp_m^{1-\theta}}}.$$

⁷The derivation is one of our contribution. Because it was unnecessary in the simple Bank Run model, the distribution of s or l was not discussed in Morris and Shin.

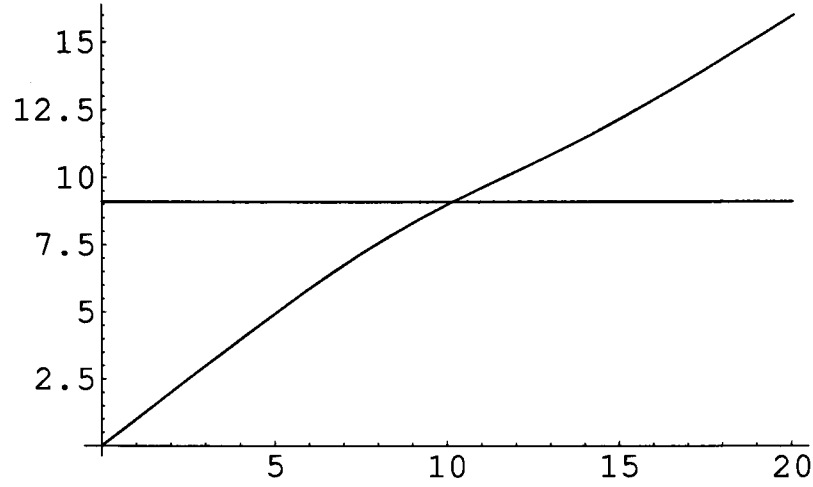


Figure 2.4.2: Switching Point ζ^*

In order to limit our attention to the case where there are multiple equilibria, let us assume that $p_1 > 1$. The sign (positive or negative) of profits in order to consider the participation decisions of firms become

$$\begin{aligned} \text{sign}(\pi(z_1)) &= \text{sign} \left\{ \frac{p_1^{1-\theta}/2}{1-s+sp_m^{1-\theta}} \cdot \mu\{\alpha_0(L-T)+G\} - F \right\} \\ &= \text{sign} \left[p_1^{1-\theta}/2 \cdot \mu\{\alpha_0(L-T)+G\} - F(1-s+sp_m^{1-\theta}) \right]. \end{aligned} \quad (2.4.18)$$

Again, assume that the fixed costs F are unknown and define ζ as the expected value of F . Then, we can derive a similar proposition.

Proposition 3 *There is a unique equilibrium if γ is small. In this equilibrium, every monopolistic firm participates in trade if and only if $\zeta < \zeta^*$, where ζ^* is the unique solution to*

$$p_1^{1-\theta}/2 \cdot \mu\{\alpha_0(L-T)+G\} = \zeta^* \left\{ 1 - \Phi(\sqrt{\gamma}(\zeta^* - \bar{F})) + \Phi(\sqrt{\gamma}(\zeta^* - \bar{F})) p_m^{1-\theta} \right\}. \quad (2.4.19)$$

The proof of this proposition is very similar to that discussed before. The proportion of active firms, s , is described as

$$\begin{aligned} E(s|\zeta^*) &= \Pr(\zeta_j < \zeta^*|\zeta^*) \\ &= \Phi(\sqrt{\gamma}(\zeta^* - \bar{F})). \end{aligned} \quad (2.4.20)$$

At the switching point ζ^* , equation (2.4.18) becomes zero, so the equation in the proposition can be derived. Looking at the left-hand side of this equation, it is constant with respect to ζ^* . In the right-hand side, the term $\{1 - \Phi() + \Phi()p_m^{1-\theta}\}$ is the weighted average of $p_m^{1-\theta} (< 1)$ and 1, and it is decreasing from 1 to $p_m^{1-\theta}$ as $\Phi()$ is increasing from 0 to 1. This $\Phi()$ is increasing with ζ^* , so the term $\{1 - \Phi() + \Phi()p_m^{1-\theta}\}$ is decreasing with ζ^* . As ζ^* in the right-hand side is increasing, it is not clear how many solutions there are. However, if γ is small, $\Phi()$ becomes flat, so the right-hand side becomes increasing with ζ^* . This results in a unique solution. As discussed in Section 2.3.3, there are maximum and minimum values of fixed costs which yield multiple equilibria. Starting from these points, we can prove the uniqueness.

This analysis suggests that the reason why Japan has fallen into a bad equilibrium was a certain kind of bad fundamentals which are, in our example, fixed costs. They made people's expectations pessimistic, which led to the self-fulfilling selection of a bad equilibrium.

Looking back at the beginning of this depression, I suspect that the asset market bubble burst in 1991 was a main cause. The burst heavily damaged the asset value of firms and the value of banks' collateral, which caused many firms to face tightened financial constraints. It became extremely difficult for firms to get finance for new business. Some firms were forced to repay their borrowings or to liquidate by sacrificing their bright business opportunities. Subsequently, fixed costs must have increased. Therefore, the bubble burst can be considered as a main candidate for the trigger of Japan's prolonged depression.

2.5 Bankruptcy and Equilibrium Selection

This section investigates the effect of bankruptcy and its uncertainty on equilibrium selection. An increase in bankruptcy has become a serious problem in Japan since the burst of the asset

market bubble. A large number of small firms as well as some big firms went bankrupt due to a tightened financial condition and low profits. This seems to have worsened Japan's depression.

Intuitively, our model is motivated by the following idea. Firstly, in trade, the medium of payment is usually not cash but bills such as accounts payable and receivable. If one of the firms goes bankrupt and cannot pay its accounts payable, all other firms will suffer a loss. Hence, the high risk of bankruptcy can provide one reason for the selection of a bad equilibrium. In order to activate trade participation among many firms, firms need to believe that other firms are solvent enough to afford to pay their bills. An increase in bankruptcy reduces the expected profits of firms. This discourages firms from participation in trade, and reduces aggregate demand, which in turn, raises the bankruptcy risk further. Secondly, in order to avoid the risk of bankruptcy, firms need to take account of the worst firm which is most likely to go bankrupt. From the viewpoint of a firm A, if uncertainty about other firms increases, then the worst firm will indeed be considered insolvent, so the risk of bankruptcy will increase. At the same time, the best firm can be regarded as even more solvent, but this has no positive effect on firm A's profit because the probability of bankruptcy does not change. Hence, uncertainty has an asymmetric effect, and discourages trade participation.

2.5.1 Model with a Bankruptcy Risk

I modify the basic bank runs model so that it can incorporate bankruptcy. In order to avoid the confusion of terms between bank runs and bankruptcy, readers may need to suppose that, in this model, agents are neither consumers nor banks. Instead, assume agents are firms. It is not a bank but an ordinary firm which may go bankrupt.

Consider three cases concerning firms' profits. Denote the proportion of inactive firms as l . Firstly, if firms participate in trade and no bankruptcy occurs in all the firms, they

receive profits of $1 - l$. Secondly, if they participate in trade and at least one active firm goes bankrupt, their revenues become zero and their profits fall to $-l$. In other words, once a trade partner goes bankrupt, firms cannot collect any revenues by liquidating their bills. Thirdly, if firms do not participate in trade, their profits become zero. Furthermore, if these profits are not enough to cover their debt burden, they go bankrupt and their utility becomes $-\infty$. Assume the following condition of bankruptcy:

$$\text{A firm } j \text{ goes bankrupt if and only if } \Pi_j < d_j. \quad (2.5.1)$$

The value d_j represents the insolvency of the firm and can take both positive and negative signs. The lower d_j , the more solvent the firm, in the sense that the firm can survive in the market even though its profit is low. In particular, if d_j is negative, the firm does not always go bankrupt, even though it suffers a loss as long as the loss is not sufficiently large. On the other hand, if d_j is positive, the firm has to make positive profits to maintain its business. The insolvency d_j is thought to depend on their assets, their debts, the relationship with their main bank, their history, laws, macroeconomic situations, and so on. To summarise, firms' profits are given by

$$\Pi_j = \begin{cases} 1 - l & \text{if participation in trade and no bankruptcy in all the firms} \\ -l & \text{if participation in trade and bankruptcy in some firms} \\ 0 & \text{if no participation in trade,} \end{cases} \quad (2.5.2)$$

and their utility reads:

$$U_j = \begin{cases} \Pi_j & \text{if } \Pi_j > d_j \\ -\infty & \text{if } \Pi_j < d_j. \end{cases} \quad (2.5.3)$$

As a further assumption, suppose that the insolvent firm whose d_i is positive always participates in trade. This assumption comes from the idea that firms extremely dislike bankruptcy and there may be a chance to avoid bankruptcy if they participate in trade.

Complete information and Multiple Equilibria

Suppose that each firm can observe the solvency, d_j , of all the firms. Consider two possible equilibria.

- (i) All firms participate in trade.

As $l = 0$, $\Pi_j = 1$ if no bankruptcy occurs. The condition of no bankruptcy is $1 > d_j$ for $\forall j$.

- (ii) No firms participate in trade except for insolvent firms.

Let the proportion of insolvent firms be $1 - l_{is}$ ($0 < l_{is} < 1$). This equilibrium suggests that the proportion of inactive firms l equal l_{is} . In order that this becomes an equilibrium, it requires that there is no incentive for solvent firms to participate in trade. If they participate in trade and no bankruptcy occurs, they can always make a positive profit of $1 - l_{is}$. Hence, to discourage trade participation, there needs to be bankruptcy for some firms. The condition for this is given by $1 - l_{is} < d_j$ for $\exists j$.

To sum up, there exist multiple equilibria if $1 - l_{is} < \max(d_j) < 1$.

Incomplete information and Equilibrium Selection

Next, consider the case of incomplete information. Suppose a firm j knows his own solvency d_j but cannot observe d_i of other firms. However, it knows that the realisation of d_i obeys

$$d_i = d + \epsilon_i, \tag{2.5.4}$$

where d and ϵ_i are normally distributed with mean $\bar{d}$ and 0 and precision α and β respectively.

The expected profit of a firm j from trade is given by

$$\begin{aligned} E(\Pi_j) &= P(1 - E(l)) + (1 - P)(-E(l)) \\ &= P - E(l), \end{aligned} \quad (2.5.5)$$

where P is the probability that bankruptcy does *not* occur. As seen in Section 2.4.1, denote the standard normal distribution function as $\Phi()$ and define γ and κ as

$$\gamma \equiv \frac{\alpha^2(\alpha + \beta)}{\beta(\alpha + 2\beta)}, \quad \kappa \equiv \frac{\beta(\alpha + \beta)}{\alpha + 2\beta}. \quad (2.5.6)$$

Furthermore, define the number of firms (sufficiently large) as N , and δ as

$$\delta \equiv \delta(d) \equiv \frac{\alpha \bar{d} + \beta d}{\alpha + \beta}. \quad (2.5.7)$$

Set $\delta^0 \equiv \delta(0)$ and $\delta^* \equiv \delta(d^*)$. Using these variables, we can obtain the following result.

Proposition 1 *There is an equilibrium where firms do not participate in trade if and only if $\delta^* < \delta < \delta^0$, only when δ^* is the unique solution to*

$$P = E(l|\delta^*), \quad (2.5.8)$$

$$\text{where } P = \Phi \left(\sqrt{\kappa}^{-1/2} \{1 - E(l|\delta^*) - \delta^*\} \right)^{N-1}, \quad (2.5.9)$$

$$E(l|\delta^*) = \Phi \left(\sqrt{\kappa} \left\{ \delta^0 + \frac{\alpha}{\beta}(\delta^* - \bar{d}) - \delta^* \right\} \right) - \Phi \left(\sqrt{\gamma}(\delta^* - \bar{d}) \right). \quad (2.5.10)$$

Figure 2.5.1 shows the two curves, P and $E(l|\delta)$, and how to obtain the switching point δ^* . Here, the horizontal axis is δ , and the switching point δ^* is obtained as the intersection point of two curves (i.e. $\delta^* = -10.4$). In drawing this diagram, I used the following parameters:

α	0.01	$\bar{d}$	-5
β	0.1	N	100

Equation (2.5.7) suggests that the equivalent d^* to this δ^* is -11.0. To put it differently, the firm i whose d_i is smaller than -11.0 or larger than 0 chooses to participate in trade, while the firm i whose d_i is between -11.0 and 0 chooses not to participate.

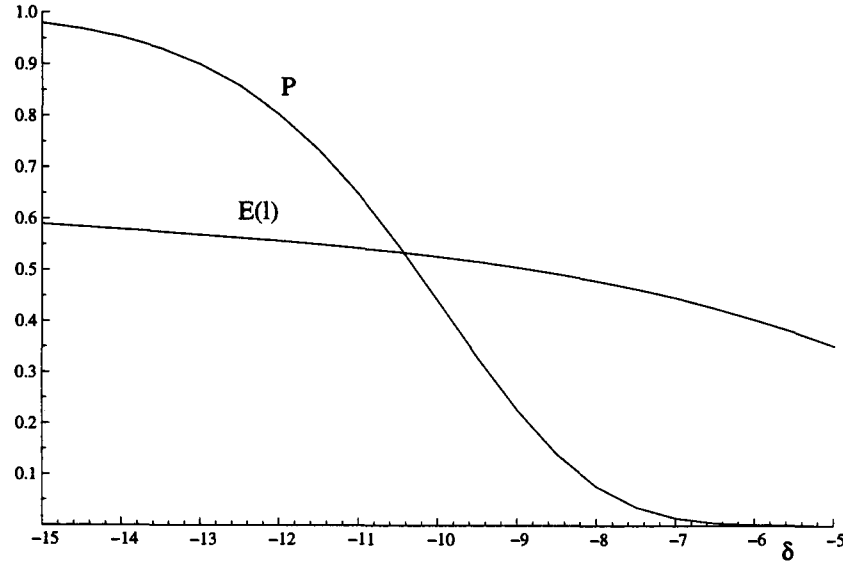


Figure 2.5.1: Switching Point δ^*

The proposition can be proved as follows. Equation (2.5.5) suggests that the switching point δ^* satisfies:

$$E(\Pi_i) = 0 \quad \therefore P = E(l|\delta^*), \quad (2.5.11)$$

because, if firms do not participate in trade, their profits become zero. We need to calculate both sides of the equation.

First, I calculate the right-hand side, $E(l|\delta^*)$. The insolvent firm whose d_i is positive inevitably participates in trade, which contributes to the reduction of the proportion of inactive firms, l . In contrast, a sufficiently solvent firm (i.e. d_i is sufficiently low) reckons that other firms may also be solvent and hence participate in trade due to little fear of bankruptcy. Assuming that the above proposition holds, the expected value of l is given by

$$\begin{aligned} E(l|\delta_i) &\equiv E(l|d_i) = \Pr [d^* < d_j < 0 | d_i] \\ &= \Pr [\delta^* < \delta_j < \delta^0 | \delta_i] \end{aligned} \quad (2.5.12)$$

from the viewpoint of firm i . To be more accurate, slightly solvent firms ($-1 < d_j < 0$) never participate in trade because there is a non-zero possibility that they go bankrupt and they greatly dislike to go bankrupt. However, it does not matter as long as the calculated critical value d^* becomes less than -1 . As shown in Section 2.4.1, the distribution of d_j is described

as

$$d_j | \delta_i \sim N(\delta_i, 1/\kappa). \quad (2.5.13)$$

Hence, equation (2.5.12) can be transformed as

$$E(l | \delta_i) = \Phi \left(\sqrt{\kappa} \left\{ \delta^0 + \frac{\alpha}{\beta}(\delta_i - \bar{d}) - \delta_i \right\} \right) - \Phi \left(\sqrt{\kappa} \left\{ \delta^* + \frac{\alpha}{\beta}(\delta_i - \bar{d}) - \delta_i \right\} \right). \quad (2.5.14)$$

The next step is the calculation of the probability P that no bankruptcy occurs. As long as the profit Π_j of a firm j exceeds d_j , this firm does not go bankrupt, so P is given by

$$P = \Pr [\Pi_j - d_j > 0 \mid \delta_i] \quad \text{for all } j \in \text{active firms} \quad (2.5.15)$$

$$= \Pr [1 - l - d_j > 0 \mid \delta_i] \quad \text{for all } j \in \text{active firms}, \quad (2.5.16)$$

from the perspective of a firm i (see Figure 2.5.2). The firm which causes bankruptcy is always insolvent (i.e. $d_j > 0$), and an inactive firm is always solvent (i.e. $d_j < 0$). Thus, in the situation where no bankruptcy happened, even if another inactive firm had participated in trade, bankruptcy would not have occurred. In other words, if bankruptcy does not happen among a certain set of active firms, it still holds for all the firms:

$$P = \Pr [1 - l - d_j > 0 \mid \delta_i] \quad \text{for all } j = 1, 2, \dots, N \text{ except for } i \quad (2.5.17)$$

$$= (\Pr [1 - l - d_j > 0 \mid \delta_i])^{N-1}. \quad (2.5.18)$$

The last equation is derived from the fact that each firm is independent of each other, conditional on δ_i . The further calculation is quite complex, because both d_j and l have some distributions and are correlated with each other. However, provided N is sufficiently large, l tends to have small variance, and converges to a certain number as shown in Appendix 2.A.5. Combined with the distribution of d_i shown in equation (2.5.13), the distribution of $l + d_i$ can be approximated as

$$l + d_j \mid \delta_i \sim N \left(E(l | \delta_i) + \delta_i, \frac{1}{\kappa} \right), \quad (2.5.19)$$

when N is large, and equation (2.5.18) becomes:

$$P = \Phi \left(\sqrt{\kappa} \{1 - E(l | \delta^*) - \delta^*\} \right)^{N-1} \quad (2.5.20)$$

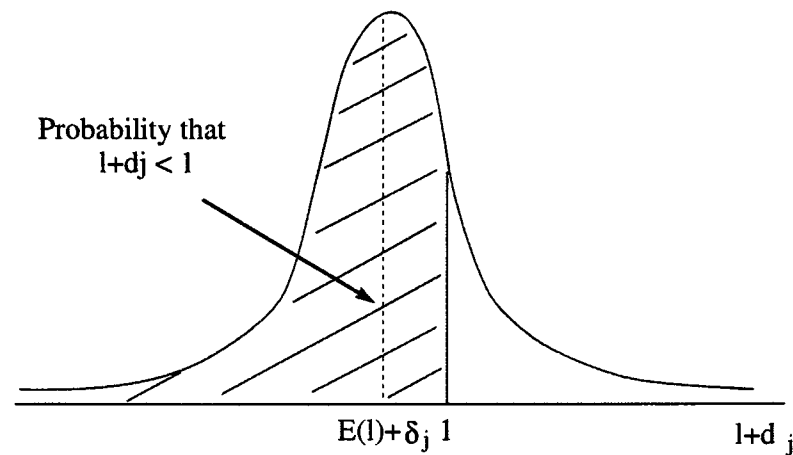


Figure 2.5.2: Probability of no Bankruptcy

This is the same as equation (2.5.9), and the proof is completed. *Q.E.D.*

However, comparing with the bank runs model, this proposition does not contain one important point, the uniqueness of strategies. In the bank runs model, the proof could be made by the procedure of iterated dominance. However, in this model, the proof is much more complex because a clear threshold does not exist. In the bank runs model, if r is larger than 1 (or smaller than 0), then participation (or not-participation) becomes a dominant strategy. Thus, there are clear threshold points of r , namely, 0 and 1. However, in this model of bankruptcy, except for $d = 0$, such a threshold point does not exist. Even if a firm has very low d_i , its profits may become negative if another firm is so insolvent that it goes bankrupt. Therefore, the procedure of iterated dominance may not be applied in this case. Taking account of such complexity, it can be assumed that, on the contrary, the uniqueness does not hold. Future research is needed to answer this question.

2.5.2 Effect of Uncertainty

As we saw above, the existence of uncertainty produces the above strategy of Proposition 4. This section discusses how the switching point is affected by uncertainty. Firstly, let us look at the numerical result presented in Figure 2.5.3. The parameters are given in the table below. In comparison with those of Figure 2.5.1, all the variables are the same, except for

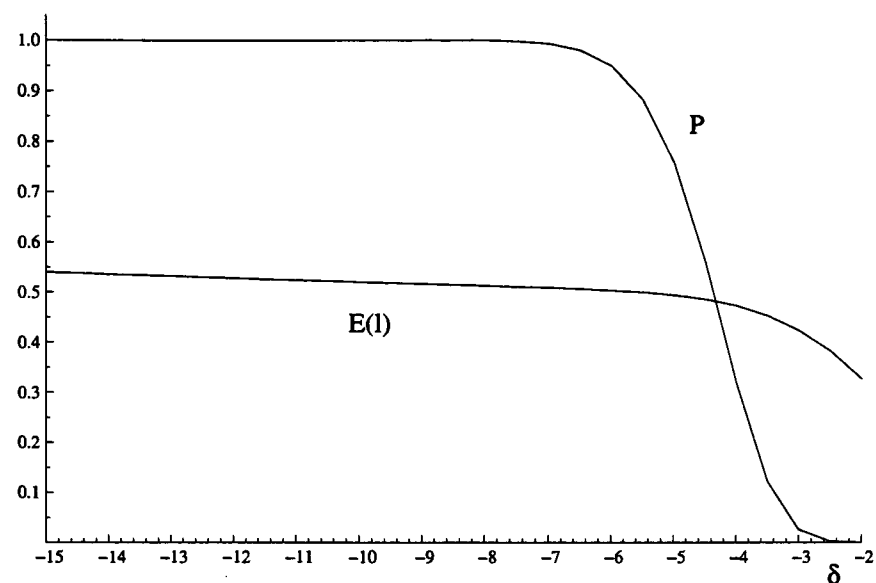


Figure 2.5.3: Effect of a Decrease in Uncertainty

the increase in β from 0.1 to 0.5 (i.e. a decrease in uncertainty). As these figures show, due to the increase in β , the intersection point δ^* rises from -10.4 to -4.3. From equation (2.5.7), it implies that the equivalent d^* rises from -11.0 to -4.3. In other words, the decrease in uncertainty raises the switching point, relaxes the condition of participation in trade, and activates trade participation.

α	0.01	$\bar{d}$	-5
β	0.5	N	100

The reason for the shift of δ^* can be explained as follows. Firstly, a decrease in uncertainty reduces the bankruptcy risk. In other words, it increases the probability that bankruptcy does not occur, P . As Figure 2.5.4 shows, the decrease in uncertainty makes the distribution converge to the mean $E(l) + \delta_j$, which increases the shadow area, that is, P . It is possible to explain the reason intuitively. When uncertainty is small, the solvency (d_i) of other firms is likely to be similar to that of one's own firm. When uncertainty is large, the solvency of other firms varies largely from good to bad. Hence, from the viewpoint of one's own firm, the worst firm is regarded as more likely to go bankrupt. In contrast, the best firm is already solvent enough to stand the risk of bankruptcy, so the decrease in uncertainty does not decrease the

probability of bankruptcy. For this asymmetry, uncertainty raises the bankruptcy risk and discourages firms from participating in trade.

The second channel is the shift of $E(l|\delta^*)$, which is a little ambiguous. I would like to evaluate the effect of uncertainty when uncertainty is small. Writing down equation (2.5.10) again:

$$E(l|\delta^*) = \Phi \left(\sqrt{\kappa} \left\{ \delta^0 + \frac{\alpha}{\beta}(\delta^* - \bar{d}) - \delta^* \right\} \right) - \Phi \left(\sqrt{\gamma}(\delta^* - \bar{d}) \right).$$

With a decrease in uncertainty to zero (i.e. an increase in β to infinity), the above equation becomes

$$\begin{aligned} E(l|\delta^*) &\sim \Phi \left(\sqrt{\kappa} \{ \delta^0 - \delta^* \} \right) - \Phi \left(\sqrt{\gamma}(\delta^* - \bar{d}) \right) \\ &\sim \Phi(\infty) - \Phi(0) \\ &\sim 1/2, \end{aligned}$$

because $\kappa \rightarrow \infty$ and $\gamma \rightarrow 0$. The curve $E(l|\delta^*)$ becomes flatter and takes the value of about a half. This can be clearly observed by comparing Figure 2.5.1 with 2.5.3. The curve $E(l|\delta^*)$ in the latter figure is surely flatter than that in the former figure. At least in this figure, it results in an increase in δ^* , although there remains a doubt as to whether the last argument always holds true in all situations.

Nevertheless, comparing these two channels, it seems that the first one through P dominates the second one through $E(l|\delta^*)$, and that uncertainty always reduces the switching point.

Such an implication that uncertainty discourages trade participation may sound only natural but, at the same time, very interesting if we consider the results from the bank runs model and other models. As explained before, in the bank runs model, the effect of uncertainty on equilibrium selection can contradict our intuition. That is, there is a case where an increase in uncertainty encourages consumers to keep their deposits, which results in a good equilibrium. The standard neoclassical model of investment (e.g. Jorgenson,

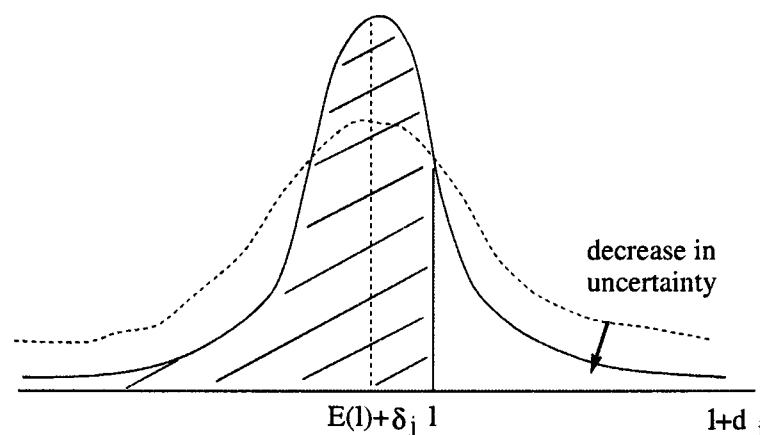


Figure 2.5.4: Change in a Bankruptcy Risk due to a Decrease in Uncertainty

1971) also yields the contradictory effect of uncertainty on investment: uncertainty increases investment. In order to obtain a plausible result, it is necessary to impose further assumptions such as the monopoly facing a downward sloping demand or the risk averse utility of firms. Our result uses a different assumption instead, a bankruptcy risk. It produces an asymmetry in the sense that an increase in uncertainty heightens the risk of bankruptcy for bad firms, but the improvement of solvency for good firms does not contribute to reducing the bankruptcy risk. Therefore, an increase in uncertainty discourages participation.

2.6 Concluding Remarks

This chapter began by discussing the current Japanese situation using Matsuyama's multiple equilibria model regarding the trade participation decisions of firms. Japan's present situation may well be regarded as being that of a bad equilibrium in this model, where no firm dares to participate in trade and hence the utility of a representative consumer is low. The sections which follow the initial section demonstrated that complex business connections, a larger number of sectors or a longer supply chain, all increase the possibility of multiple equilibria. This suggests that Japan, which is composed of a complex economy, is prone to the multiplicity of equilibrium. Then, Section 2.3.3 relaxed two important assumptions in the

model that the elasticity of substitution is one and fixed costs are demanded as goods, and confirmed that multiple equilibria still arise. In the new model, the dependence of relative prices on the behaviours of other firms plays a significant role in firms' participation decision. This model can further help to explain the situation of deflation which is peculiar to Japan. One drawback in the model is that a seemingly bad equilibrium with no monopolistic firm's trade participation is not necessarily bad. However, for many other reasons which do not incorporate in the model, both no participation and deflation are considered to be very harmful to the economy, and it is natural to regard such an equilibrium as bad.

The second part of this chapter studied how an equilibrium is selected, which provides a clue as to why Japan has fallen into a depression. The idea of global games by Morris and Shin helps answer this question. If there is a small uncertainty, agents choose a unique strategy determined by certain fundamentals. Sections 2.4.2 and 2.4.3 applied their approach to the Matsuyama models and successfully demonstrated that there exists a unique equilibrium under a small uncertainty. According to this idea, the cause of an equilibrium shift is a fundamental change, not just a random sunspot movement from one equilibrium to another. This idea may suggest that the asset market bubble burst in 1991 may have caused the economy to fall into a bad equilibrium.

Section 2.5 showed that bankruptcy affects multiple equilibria, in particular that uncertainty not only determines which equilibrium is chosen but also prevents firms from participating in trade. Widespread uncertainty combined with bankruptcy in Japan appears to be one of the causes of prolonged stagnation. This result might be considered trivial, but it makes a clear contrast with that of Morris and Shin where uncertainty can encourage participation in trade.

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However, the global games approach in the second section potentially has a serious contradiction with the discussion about government intervention in the first section. Firstly, in the global games approach, since multiple equilibria are eliminated, there is no hysteresis effect. In order to revitalise the economy, a temporary government expenditure is not helpful. Once the government shrinks its spending to the original level, the economy goes back to the original equilibrium. Thus, only permanent measures are sufficient to solve this problem.

There is an alternative approach which was implicitly assumed in the first section. Assume explicitly that there is some sort of inertia in an economy. The economy becomes highly history dependent. Then, even if temporary, sufficiently big government expenditure may enable the economy to shift from a bad equilibrium to a good one. This is because, once the economy moves to a good equilibrium, it is likely to stay at the good equilibrium in the future even after the fiscal expansion finishes. Although this is only a suggestion, and a more rigorous analysis is needed to justify this argument, this approach is quite different from the global games approach.

It is unsatisfactory, but this chapter does not present a definite answer as to which approach is more plausible or as to whether there is any possible compromise. It seems sensible to me that we keep both of the approaches in mind.

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Finally, let me state some of the policy recommendations which come from this story. Firstly, government intervention should have a permanent effect. Temporary big public spending may enable the economy to shift from a bad equilibrium to a good one, but without any long-lasting improvement of fundamentals, the effect of public spending may not last long. Secondly, to be more concrete, government expenditure becomes more effective if it can create new demand. For example, the government should spend money on the sectors which

have high spill-over effects. If goods are produced in a long supply chain, then the multiplier becomes large. Such a sector is probably manufacturing, especially firms which produces complex final goods. Thirdly, a policy which reduces bankruptcy and uncertainty would be effective. This not only raises the multiplier but also affects the equilibrium selection toward a good equilibrium. This may be possible in two extreme ways: (1) the policy may save insolvent firms or (2) the policy may force insolvent firms to go bankrupt and expel them from the market. It is more practically plausible to reduce the damage which bankruptcy causes to other firms. This can be achieved by promoting the system where non-market (negotiation market) loans or assets can be easily securitised and liquidated in the public market (e.g. Asset Backed Security (ABS)).

So far, I have focused on the role of the government. However, considering that the Japanese economy heavily depends on exports, a more feasible scenario of Japan's recovery may be seen through a rise in exports. The economies of the United States and China have been doing extremely well. Let us suppose that somehow the yen does not appreciate much against the dollar. Then, this model suggests that Japan may be able to achieve a fundamental recovery. Of course, to study that possibility fully, we would need an open economy model, which I have not provided here.

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2.A Appendix

2.A.1 Derivation of the Utility Maximisation Problem

Basic Model

This appendix shows how to derive equation (2.2.3) from equations (2.2.1) and (2.2.2). First, construct Lagrangian L as

$$L = \alpha \int \ln c(z) dz + (1 - \alpha) \ln(N) + \lambda \left\{ L + \Pi - T - N - \int_0^1 p(z)c(z) dz \right\}. \quad (2.A.1)$$

The first-order conditions with respect to $c(z)$ and N are

$$\frac{\partial L}{\partial c(z)} = 0 = \frac{\alpha}{c(z)} - \lambda p(z) \quad (2.A.2)$$

$$\text{and } \frac{\partial L}{\partial N} = 0 = \frac{1 - \alpha}{N} - \lambda. \quad (2.A.3)$$

Deleting λ from the two equations yields

$$p(z)c(z) = \frac{\alpha}{1 - \alpha} N. \quad (2.A.4)$$

Integration over z yields

$$\int_0^1 p(z)c(z) dz = \frac{\alpha}{1 - \alpha} N. \quad (2.A.5)$$

The left-hand side is equal to $L + \Pi - T - N$ from the budget constraint. Therefore, leisure can be calculated as

$$N = (1 - \alpha)(L + \Pi - T), \quad (2.A.6)$$

and from equation (2.A.4), the consumption of variety z becomes

$$c(z) = \frac{\alpha(L + \Pi - T)}{p(z)}. \quad (2.A.7)$$

More General Multiple Equilibria Model

The utility maximisation problem in Section 2.3.3 can be solved by division into two stages.

First Dimension z Construct Lagrangian L as

$$\begin{aligned} L &= \alpha \ln C + (1 - \alpha) \ln(N) + \lambda \left\{ L + \Pi - T - N - \int_0^1 \{p(z_1)c(z_1)dz_1 + p(z_2)c(z_2)dz_2\} \right\} \\ &= \alpha \ln \left[\int_0^1 c(z)^{\frac{1}{1+\theta_1}} dz \right]^{1+\theta_1} + (1 - \alpha) \ln(N) + \lambda \left\{ L + \Pi - T - N - \int_0^1 p(z)c(z)dz \right\}. \end{aligned} \quad (2.A.8)$$

In the last equation, I defined $p(z)$ as

$$p(z)c(z) \equiv p(z_1)c(z_1) + p(z_2)c(z_2). \quad (2.A.9)$$

The first-order conditions with respect to $c(z)$ and N are

$$\begin{aligned} \frac{\partial L}{\partial c(z)} &= 0 = \alpha(1 + \theta_1) \frac{\partial \ln \left[\int_0^1 c(z)^{\frac{1}{1+\theta_1}} dz \right]}{\partial c(z)} - \lambda p(z) \\ &= \alpha(1 + \theta_1) \frac{\frac{1}{1+\theta_1} c(z)^{\frac{1}{1+\theta_1}-1}}{\left[\int_0^1 c(z)^{\frac{1}{1+\theta_1}} dz \right]} - \lambda p(z) \\ &= \alpha \frac{c(z)^{-\frac{\theta_1}{1+\theta_1}}}{C^{\frac{1}{1+\theta_1}}} - \lambda p(z), \end{aligned} \quad (2.A.10)$$

$$\frac{\partial L}{\partial N} = 0 = \frac{1 - \alpha}{N} - \lambda. \quad (2.A.11)$$

Using the first equation, replace $p(z)$ in $\int_0^1 p(z)c(z)dz$:

$$\begin{aligned} \int_0^1 p(z)c(z)dz &= \int_0^1 \alpha \frac{c(z)^{-\frac{\theta_1}{1+\theta_1}}}{\lambda C^{\frac{1}{1+\theta_1}}} c(z)dz \\ &= \alpha/\lambda. \end{aligned} \quad (2.A.12)$$

The budget constraint leads to

$$\begin{aligned} \int_0^1 p(z)c(z)dz + N &= L + \Pi - T, \\ \frac{\alpha}{\lambda} + \frac{1 - \alpha}{\lambda} &= L + \Pi - T \\ \lambda &= 1/(L + \Pi - T). \end{aligned} \quad (2.A.13)$$

Therefore, we obtain

$$N = (1 - \alpha)(L + \Pi - T) \quad (2.A.14)$$

$$\int_0^1 p(z)c(z)dz = \alpha(L + \Pi - T). \quad (2.A.15)$$

Define the price index of the composite consumption P as

$$PC \equiv \alpha(L + \Pi - T), \quad (2.A.16)$$

$$\text{or equivalently, } PC \equiv \int_0^1 p(z)c(z)dz. \quad (2.A.17)$$

Substituting λ into equation (2.A.10) yields

$$\begin{aligned} c(z)^{\frac{\theta_1}{1+\theta_1}} &= \frac{\alpha(L + \Pi - T)}{p(z)} C^{-\frac{1}{1+\theta_1}} = \frac{PC}{p(z)} C^{-\frac{1}{1+\theta_1}} \\ \therefore c(z) &= \left(\frac{p(z)}{P} \right)^{-\frac{1+\theta_1}{\theta_1}} C. \end{aligned} \quad (2.A.18)$$

In order to derive P , substitute $c(z)$ into C :

$$\begin{aligned} C &= \left[\int_0^1 \left(\left(\frac{p(z)}{P} \right)^{-\frac{1+\theta_1}{\theta_1}} C \right)^{\frac{1}{1+\theta_1}} dz \right]^{1+\theta_1} \\ &= P^{\frac{1+\theta_1}{\theta_1}} C \left[\int_0^1 p(z)^{-\frac{1}{\theta_1}} dz \right]^{1+\theta_1} \end{aligned}$$

Therefore, P is given by

$$P = \left[\int_0^1 p(z)^{-\frac{1}{\theta_1}} dz \right]^{-\theta_1} \quad (2.A.19)$$

Second Dimension z_1 and z_2 The above calculation shows how a representative consumer allocates consumption C and leisure N given the following budget constraint:

$$\int_0^1 p(z)c(z)dz + N = L + \Pi - T. \quad (2.A.20)$$

As a second-stage optimisation problem, I calculate here how the consumer maximises consumption $c(z)$ given another budget constraint:

$$p(z_1)c(z_1) + p(z_2)c(z_2) = p(z)c(z). \quad (2.A.21)$$

Lagrangian is constructed as

$$\begin{aligned} L &= \ln c(z) + \lambda \{p(z)c(z) - p(z_1)c(z_1) - p(z_2)c(z_2)\} \\ &= \ln \left[2^{-\theta_2} \left(c(z_1)^{\frac{1}{1+\theta_2}} + c(z_2)^{\frac{1}{1+\theta_2}} \right)^{1+\theta_2} \right] + \lambda \{p(z)c(z) - p(z_1)c(z_1) - p(z_2)c(z_2)\}. \end{aligned} \quad (2.A.22)$$

Differentiate it with respect to $c(z_i)$ for $i = 1, 2$:

$$\frac{\partial L}{\partial c(z_i)} = 0 = (1 + \theta_2) \frac{\frac{1}{1+\theta_2} c(z_i)^{\frac{1}{1+\theta_2}-1}}{c(z_1)^{\frac{1}{1+\theta_2}} + c(z_2)^{\frac{1}{1+\theta_2}}} - \lambda p(z_i) \quad (2.A.23)$$

$$\therefore p(z_i) = \frac{c(z_i)^{\frac{1}{1+\theta_2}-1}}{\lambda \left(c(z_1)^{\frac{1}{1+\theta_2}} + c(z_2)^{\frac{1}{1+\theta_2}} \right)}. \quad (2.A.24)$$

Substitute this into equation (2.A.21), and λ can be obtained:

$$p(z)c(z) = \frac{c(z_1)^{\frac{1}{1+\theta_2}}}{\lambda \left(c(z_1)^{\frac{1}{1+\theta_2}} + c(z_2)^{\frac{1}{1+\theta_2}} \right)} + \frac{c(z_2)^{\frac{1}{1+\theta_2}}}{\lambda \left(c(z_1)^{\frac{1}{1+\theta_2}} + c(z_2)^{\frac{1}{1+\theta_2}} \right)} = 1/\lambda. \quad (2.A.25)$$

Hence, equation (2.A.24) becomes

$$\begin{aligned} p(z_i) &= p(z)c(z) \frac{c(z_i)^{\frac{1}{1+\theta_2}-1}}{c(z_1)^{\frac{1}{1+\theta_2}} + c(z_2)^{\frac{1}{1+\theta_2}}} \\ &= p(z)c(z)c(z_i)^{\frac{1}{1+\theta_2}-1} c(z)^{-\frac{1}{1+\theta_2}} 2^{-\frac{\theta_2}{1+\theta_2}}. \end{aligned} \quad (2.A.26)$$

$$\therefore c(z_i) = \left(\frac{p(z_i)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2} c(z). \quad (2.A.27)$$

Finally, from the budget constraint, we can calculate $p(z)$ as follows. Substitute the above $c(z_i)$:

$$\begin{aligned} p(z)c(z) &= p(z_1)c(z_1) + p(z_2)c(z_2) \\ &= p(z_1) \left(\frac{p(z_1)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2} c(z) + p(z_2) \left(\frac{p(z_2)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2} c(z). \end{aligned} \quad (2.A.28)$$

Dividing $c(z)$ from the both sides and rearranging it, we can obtain

$$p(z) = 2^{\theta_2} \{p(z_1)^{-\frac{1}{\theta_2}} + p(z_2)^{-\frac{1}{\theta_2}}\}^{-\theta_2}. \quad (2.A.29)$$

Government Spending The total government spending in a nominal base, G , coincides with $\int_0^1 p(z)g(z)dz$ because

$$\begin{aligned} \int_0^1 p(z)g(z)dz &= \int_0^1 p(z) \left(\frac{p(z)}{P} \right)^{-\frac{1+\theta_1}{\theta_1}} \frac{G}{P} dz = GP^{\frac{1}{\theta_1}} \left[\int_0^1 p(z)^{-\frac{1}{\theta_1}} dz \right] \\ &= G. \end{aligned} \quad (2.A.30)$$

Similarly, in the lower dimension,

$$\begin{aligned} p(z_1)g(z_1) + p(z_2)g(z_2) &= p(z_1) \left(\frac{p(z_1)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2}g(z) + p(z_2) \left(\frac{p(z_2)}{p(z)} \right)^{-\frac{1+\theta_2}{\theta_2}} \frac{1}{2}g(z) \\ &= p(z)^{\frac{1+\theta_2}{\theta_2}} \frac{1}{2} \left(p(z_1)^{-\frac{1}{\theta_2}} + p(z_2)^{-\frac{1}{\theta_2}} \right) g(z) = p(z)^{\frac{1+\theta_2}{\theta_2}} p(z)^{-\frac{1}{\theta_2}} g(z) \\ &= p(z)g(z). \end{aligned} \quad (2.A.31)$$

2.A.2 Firms' Profits in the Model of Vertical Complementarities

Matsuyama finished his study of vertical complementarities by deriving the model of Section 2.2.3. This appendix, for the special case where all the properties such as mark-up and fixed costs are identical among monopolistic firms, calculates the profits which these firms make. This calculation continues from Section 2.2.3. Let $\mu_j \equiv \mu_0$ and $F_j \equiv F$ for all j , and then equations (2.2.28) and (2.2.29) become

$$\begin{aligned} B &= L - \sum_{j=1}^S \left[F_j \prod_{k=j+1}^S (1 - \mu_k) \right] = L - \sum_{j=1}^S F (1 - \mu_0)^{S-j} \\ &= L - F \frac{1 - (1 - \mu_0)^S}{1 - (1 - \mu_0)} = L - F \frac{1 - (1 - \mu_0)^S}{\mu_0} \\ &= L - \frac{\mu F}{\mu_0}, \end{aligned} \quad (2.A.32)$$

$$\begin{aligned} \text{where } \mu &= 1 - \prod_{j=1}^S (1 - \mu_j) \\ &= 1 - (1 - \mu_0)^S. \end{aligned} \quad (2.A.33)$$

Since, from equation (2.2.27), the output of the final goods is given by

$$Q_1 = \frac{\alpha B + G - \alpha T}{1 - \mu \alpha},$$

the profit of the firm $j = 1$ which produces the final goods can be described as

$$\begin{aligned}
 \Pi_1 &= \mu_0 Q_1 - F_0 \\
 &= \frac{\mu_0(\alpha B + G - \alpha T) - (1 - \mu\alpha)F_0}{1 - \mu\alpha} \\
 &= \frac{\mu_0\{\alpha(L - \frac{\mu F_0}{\mu_0}) + G - \alpha T\} - (1 - \mu\alpha)F_0}{1 - \mu\alpha} \\
 &= \frac{\mu_0\{\alpha(L - T) + G\} - F_0}{1 - \mu\alpha}
 \end{aligned} \tag{2.A.34}$$

The profits of the other firms are

$$\begin{aligned}
 \Pi_j &= \mu_0 Q_j - F \\
 &= \mu_0\{(1 - \mu_0)Q_{j-1} + F\} - F \\
 &= (1 - \mu_0)(\mu_0 Q_{j-1} - F) = (1 - \mu_0)\Pi_{j-1} \\
 &= (1 - \mu_0)^j \Pi_1,
 \end{aligned} \tag{2.A.35}$$

for $2 \leq j \leq S$. The second equation is derived from equation (2.2.26).

2.A.3 Proof of the Uniqueness of Equilibrium

This appendix shows that if there is a unique symmetric equilibrium in switching strategies, no other equilibrium exists, which is made by Morris and Shin (2001). Provided that all other consumers follow the strategy with the switching point $\hat{\rho}$, the expected utility of a consumer who observes the posterior signal ρ is

$$\begin{aligned}
 u(\rho, \hat{\rho}) &= \rho - \Phi \left(\sqrt{\frac{\beta(\alpha + \beta)}{\alpha + 2\beta}} \left(\hat{\rho} + \frac{\alpha}{\beta}(\hat{\rho} - \bar{r}) - \rho \right) \right) \\
 &= \rho - \Phi \left(\sqrt{\gamma} \left(\hat{\rho} - \bar{r} + \frac{\beta}{\alpha}(\hat{\rho} - \rho) \right) \right).
 \end{aligned} \tag{2.A.36}$$

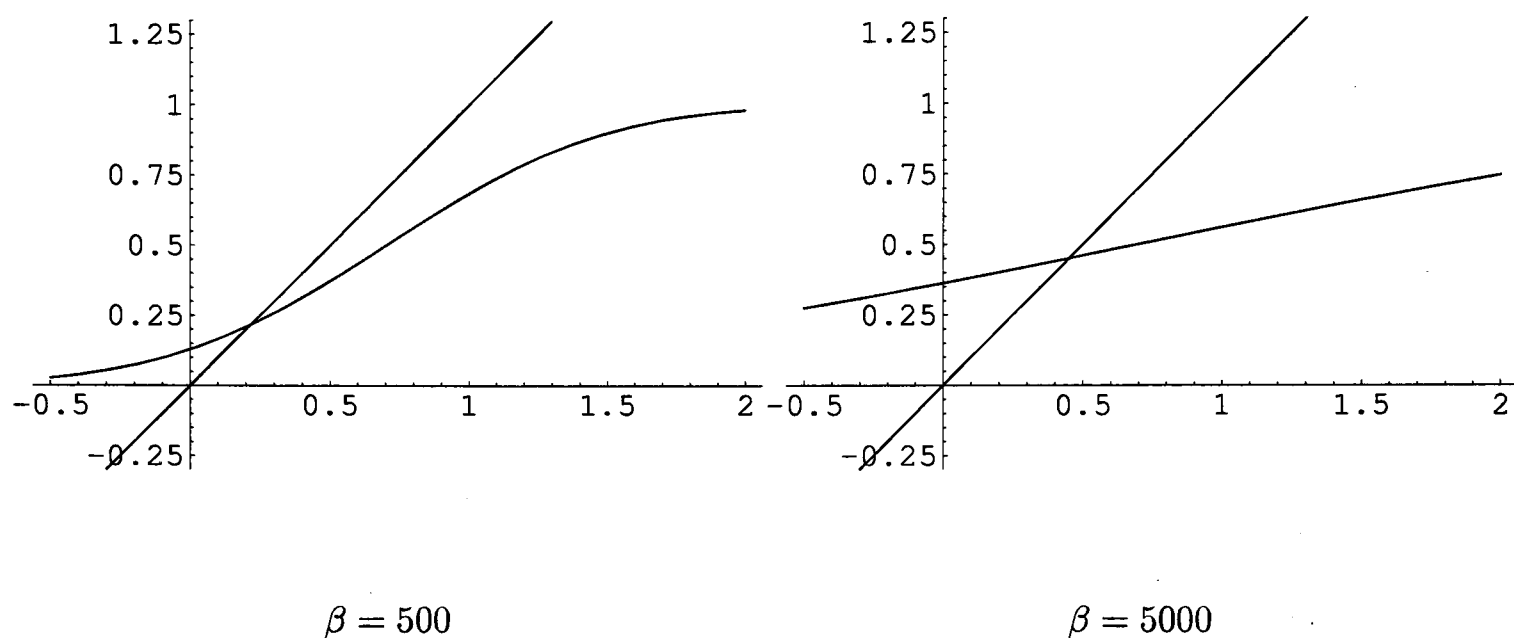
The second term of the right-hand side, or the proportion of consumers who withdraw their deposits, is equal to the probability that another consumer receives a signal lower than $\hat{\rho}$ given a posterior ρ . If r is negative, it is better to withdraw their deposits irrespective of

the actions of other consumers. Hence a sufficiently low ρ makes withdrawal the dominant strategy. Denote $\underline{\rho}_1$ as its threshold value. Withdrawing money becomes the dominant action, not only if one's signal is lower than $\underline{\rho}_1$ but also if it is lower than $\underline{\rho}_2$, where $\underline{\rho}_2$ satisfies $u(\underline{\rho}_2, \underline{\rho}_1) = 0$. This $\underline{\rho}_2$ is shown to be larger than $\underline{\rho}_1$ from the initial assumption that there is a unique solution for $u(\rho, \rho) = 0$. Furthermore, for other consumers, if their signals are lower than $\underline{\rho}_2$, then leaving money in the bank cannot be an optimal strategy. Therefore, if a signal is lower than $\underline{\rho}_2$, then withdrawal becomes the dominant strategy. In this way, we can generate the increasing sequence of critical values: $\underline{\rho}_1 < \underline{\rho}_2 < \dots < \underline{\rho}_k < \dots$. For instance, in the k rounds, when the signal is $\rho < \underline{\rho}_k$, withdrawal becomes the dominant strategy. This procedure of iterated dominance can be continued until we reach $u(\underline{\rho}, \underline{\rho}) = 0$. There might be multiple solutions of ρ which satisfy this equation, and $\underline{\rho}$ is the solution with the lowest value. In a similar way, from a sufficiently high ρ , we can obtain other sequences of dominant strategies and the highest $\bar{\rho}$ which solves $u(\bar{\rho}, \bar{\rho}) = 0$. However, if this equation $u(\rho, \rho) = 0$ yields a unique solution, then $\underline{\rho}$ and $\bar{\rho}$ should coincide. This means that only one strategy remains after the procedure of iterated dominance and this strategy is the only equilibrium strategy.

2.A.4 Effect of Uncertainty on a Critical Value in the Bank Runs Model

The main chapter discussed how to obtain the switching point, ρ^* , and how uncertainty changes it analytically. This appendix will show these arguments more explicitly by using numerical calculation. The parameter values are chosen as $\alpha = 50$, $\beta = 500$ or 5000 , and $\bar{r} = 0.7$. Depending on the different values of precision factor β , we can obtain two figures as below. The horizontal axis represents ρ . As we saw in Figure 2.4.1, the switching point ρ^* is the intersection point between the 45° line (the left-hand side of equation (2.4.4)) and the normal distribution curve (the right-hand side of equation (2.4.4)). As uncertainty becomes

smaller (larger β), the normal distribution curve becomes flatter, and ρ^* becomes larger. This increases the possibility of bank runs. This result coincides with the prediction in the main chapter.



2.A.5 Distribution of the Proportion of Inactive Firms l

In Morris and Shin's paper, what was not discussed was how the proportion of inactive firms is distributed. However, it is very important to investigate this when we treat more complex models than the bank runs model. This appendix calculates its exact and asymptotic distribution. All the models in this chapter treat the binomial choice of firms, participation or non-participation. Under the switching strategy, the choice is determined by whether their signals are larger or smaller than a certain switching point. From the viewpoint of one firm, other firms seem to choose participation and non-participation with a certain probability such as $1 - x$ and x respectively. This means that their choice is obedient to the binomial distribution. Denote n and $N - n$ as the number of active and inactive sectors respectively, so that l is written as $l = (N - n)/N$. Assuming that x is the probability of non-participation,

n obeys the following binomial distribution:

$$f_{N-n}^N(x) = {}_N C_{N-n} x^{N-n} (1-x)^n. \quad (2.A.37)$$

Using the formula of binomial distribution, the moments of l are given by

$$E(l) = \sum_{n=0}^N \frac{N-n}{N} f_{N-n}^N(x) = x \quad (2.A.38)$$

$$\begin{aligned} E(l^2) &= \sum_{n=0}^N \left(\frac{N-n}{N} \right)^2 f_{N-n}^N(x) \\ &= \frac{x(1-x)}{N} + x^2 \end{aligned} \quad (2.A.39)$$

$$Var(l) = \frac{x(1-x)}{N} = \frac{E(l)\{1-E(l)\}}{N}. \quad (2.A.40)$$

Therefore, l has the distribution of mean $E(l)$ and variance $E(l)\{1-E(l)\}/N$. As N becomes larger, the variance of l becomes smaller. Therefore, we can reckon l as a certain number $E(l)$, and neglect the correlation with other variables. More correctly, l converges to the normal distribution from the Central Limit Theorem:

$$\sqrt{N}(l - E(l)) \xrightarrow{D} N(0, E(l)\{1-E(l)\}). \quad (2.A.41)$$

Chapter 3

Reexamination of the Taylor

Principle: Firm-Specific

Investment, Sticky Prices, and

Indeterminacy

JEL Classification: C62, E22, E31, E52, E61, E62

Keywords: Monetary policy, interest rate rules, inflation persistence, heterogeneity, sticky wages

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

Abstract

This chapter aims to check whether Japan's prolonged downturn was provoked by monetary policy. In particular, it investigates under what condition interest rate rules by central banks make equilibrium determinate. Several studies, especially Sveen and Weinke (2004), have claimed that once taking account of capital, an active monetary policy rule is not enough to achieve determinacy, and that a sufficient response to real economic activity is needed. This chapter points out that the reason for this indeterminacy is that there is too much forward-lookingness in these models. We construct richer models with some persistence and heterogeneities so that they can better describe the real economic world, and demonstrate that determinacy becomes more likely to occur even without an aggressive policy rule responding to real economic activity. These results suggest that, from the viewpoint of determinacy, we do not need to be too much worried about the design of a monetary policy rule, and that Japanese monetary policy did not act improperly.

3.1 Introduction

3.1.1 Taylor Principle and Indeterminacy

This chapter aims to see whether Japan's prolonged downturn was provoked by monetary policy. In particular, it begins with the discussion of the Taylor principle. This principle states that a passive rule, which reacts to inflation less than one for one, causes indeterminacy (for example, Kerr and King : 1996, Rotemberg and Woodford : 1997, 1999, Clarida, Gali and Gertler: 2000, and Bullard and Mitra: 2002). Suppose people expect deflation. This raises the real interest rate combined with the passive rule. Output decreases which, in turn, leads to deflation. Hence, the expectation of deflation (inflation) becomes self-fulfilling. An active policy rule, which reacts to inflation more than one-for-one, appears to be essential to avoid

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

such indeterminacy.

Indeterminacy is problematic in several respects¹. Under indeterminacy, extrinsic uncertainty begins to matter. In other words, even though there is no change in fundamentals, an infinite number of dynamic paths possibly exist. A change in expectations becomes self-fulfilling, and produces economic booms and bursts. Many people (for example, Woodford: 1991, 2003) argued that such a possibility had already been pointed out by Keynes (1936) with the term "animal spirits." They seem to render investment movement volatile. These authors believe that the Great Depression might have been caused or, at least amplified, by prevailing pessimism.

The problem is not only depression but also instability or an economic fluctuation. An economic fluctuation is harmful in that people begin to face more uncertainty. When analysing welfare, it is common to assume that stabilising economies is a main policy objective. Indeterminacy brings a fluctuation without a change in fundamental, so it is clearly problematic.

Moreover, because of indeterminacy, policy analyses become much more difficult. A comparative statics is no longer well-defined. If there are multiple scenarios, one policy which is optimal under one situation may become destructive in another situation. It is true that we may reduce multiplicity by choosing a certain selection criterion, but what a good criterion is needs to be carefully considered and is not a easy question.

There has been wide-spread suspicion that the Bank of Japan may not have acted properly for this decade. For example, McCallum (2000, 2001a) argues that monetary policy was too tight in the 1990's. However, according to several estimates of Japan's policy response, it seems that the Bank of Japan at least followed the Taylor principle. For instance, Clarida, Gali and Gertler (1998) measured Japan's monetary policy reaction from 1979 to 1994 and obtained a higher coefficient value than one, 2.04, as an inflation term, as well as 0.08 as an

¹See Woodford (1991, 2003) for a discussion on the problem of indeterminacy.

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

output gap term. Jinushi, Kuroki and Miyao (2000) compared the monetary policy responses before and after 1987, when an asset market bubble started. They obtained 1.1 for an inflation term and 0.13 for an output gap term in the former period and 2.1 for an inflation term and -0.12 for an output gap term in the latter (until 1995). All these estimations suggest that the Bank of Japan did indeed follow the Taylor principle.

However, theoretically speaking, the orthodox Taylor principle is not sufficient for determinacy. Most previous models were based on very simple macroeconomic models with only three equations: an IS curve, an inflation adjustment process (i.e. a Phillips curve) and a Taylor rule for the interest rate. Such simple models sit oddly with the much richer macroeconomic models which are now increasingly in use.

3.1.2 Importance of Capital Accumulation

It is therefore important to have a good understanding about the condition for determinacy if we are to understand whether the Bank of Japan carried out its work well during the period from 1990 to the present.

The aim of this chapter is to reexamine the Taylor principle. In particular, we reexamine this principle in one of the much richer macroeconomic models, in which the economy is engaged in a growth process and on which the capital accumulation process is carefully modelled. Also, rather than making investment the residual after consumption has been determined, as in the standard Ramsey model, investment is modelled as being undertaken by forward-looking firms, whose investment process is subject to adjustment costs. Those adjustment costs will make little difference to the nature of a long-run equilibrium, but they will constrain the process of adjustment back to that equilibrium. Under sticky prices, forward-looking firms optimise their reset prices looking at the future level of real marginal costs. Since they are affected by the future capital stock and the latter is affected by their

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

present reset prices, rational firms have to take account of all these effects. As will be shown in the main text, this channel can make the expectation of an investment boom self-fulfilling even under active monetary policy. In other words, we will see that the Taylor principle can become insufficient for determinacy in such circumstances.

It was Dupor (2001) who was the first to find that the Taylor principle was subject to change once we take account of capital accumulation. He argued that the generally accepted results went upside down. In other words, he argued that the active rule made equilibrium indeterminate, and the passive rule achieved determinacy. This is a very very striking claim. It is important either to understand it in detail, or to show that it is wrong. Obviously, I prefer the second of these two paths.

Dupor's model was very misleading in two points. Firstly, it did not take account of the adjustment costs of investment. This is a huge problem in that Dupor completely neglects an important adjustment process of an economy. The existence of adjustment costs prevents capital from adjusting instantaneously, which provides us with a much richer view of a dynamic process. A second problem is that Dupor assumed the existence of a capital market. Not firms but households assemble capital, and the capital is rented to firms. There is only one kind of capital. Such a model setup was widely used probably due to tractability and convention, but it does not look plausible.

In his sequential paper, Dupor (2002) fixed the first problem and found that, if capital cannot move discretely, passive rules no longer cause determinacy. Active rules still cause indeterminacy, but if adjustment costs are sufficiently high, then determinacy can be achieved. However, he also claimed that the required value of adjustment costs for determinacy lay outside the empirically plausible region, so that the economy was still likely to be indeterminate. Furthermore, the second problem with his analysis is still not resolved. There thus appeared to be much work still to do even after Dupor's second paper.

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

Sveen and Weinke (2005) resolved both problems. They investigated determinacy by incorporating not only adjustment costs but also firm-specific investment. Their model is based on Woodford (2003, 2004, 2005) and Sveen and Weinke (2004). A capital market does not exist any longer, and capital is uniquely customised in each firm, and a firm cannot borrow capital from the market or other firms. Each firm takes charge of investment, and constructs its own capital. Woodford argues that this heterogeneity yields a higher degree of strategic complementarity of the pricing decisions among firms, and decreases the speed of adjustment toward a steady state. Modelling firm-specific investment, Sveen and Weinke (2005) found the following results. A passive rule continues to make equilibrium indeterminate. Even an active rule cannot make equilibrium determinate. In order to make equilibrium determinate, a policy rule must respond sufficiently to real economic activity as well as inflation. This claim of Sveen and Weinke is, although less disturbing than that of Dupor, still a very radical one. It argues that, to achieve satisfactory performance, a central bank must respond to output in its Taylor rule, possibly by a great deal. The question remains: is this strong claim true?

3.1.3 Contribution of this Chapter

Analytical approach

The Sveen and Weinke (2005) model does *not* seem to be rich enough for us to be sure if we have grasped robust insights into reality. This model can be improved in two respects. Firstly, introducing some more persistence makes the model closer to reality. Although some persistence has already been incorporated through sticky prices and the adjustment costs of capital, the real economy has more factors which are persistent. One is the persistence of not only prices but also their changes (inflation). The movement of an inflation rate is known to be highly persistent. Another persistence is found in wages. They are clearly very sticky. By

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

incorporating these persistence, our model can become more convincing.

Secondly, introducing some heterogeneities helps to improve the model. It is true that two heterogeneities have already been taken account of. The first one is a capital heterogeneity. The Sveen and Weinke model correctly supposes firm-specific investment, so capital is considered to be different across firms. Secondly, the timing of price resetting is heterogeneous. In the Calvo-type (1985) sticky price model, some firms are lucky enough to change their prices, but others are not. The randomness of the lottery outcome yields a heterogeneity regarding price resetting. These points have already been considered, but regarding price resetting, there seems to be a more fundamental heterogeneity. In reality, without relying on luck, some firms can systematically adjust much more frequently than others. For example, on the one hand, some firms such as supermarkets reset their prices almost every day. On the other hand, there are firms which face tight regulations, which restricts the flexibility of price changes. Such a difference is not caused by luck in the Calvo sense. As another example, a fraction of firms may not be perfectly rational while there are still perfectly rational firms. This also yields a heterogeneity regarding price resetting.

Incorporating these persistence and heterogeneities will make our model much richer than Sveen and Weinke (2005). This chapter attempts to construct such models with micro-foundations. In particular, it firstly incorporates inflation persistence. This requires us to treat the price resetting heterogeneity as well. Some firms are rational and forward-looking. Other firms reset their prices in the backward-looking way. To deal with this heterogeneity, the Sveen and Weinke model will be modified so that the former firms decide their reset prices and investment fully taking account of the latter firms. The second part introduces a similar but different type of heterogeneity. Supermarkets being an example, there are a large number of firms which change prices almost every day. We thus assume that a fraction of firms can reset their prices freely at any time. The other firms cannot reset their prices

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

freely, and their resetting chance obeys a Calvo-type lottery. The third part takes account of sticky wages as well as sticky prices.

With these models, we examine how a stability boundary changes, especially whether a monetary rule should really respond to real economic activity.

Our Result: a Reestablishment of the Taylor Principle

In addition to such analytical reasoning, practically speaking, constructing a richer model is also very important. According to Sveen and Weinke (2005), it is claimed that central banks must respond sufficiently to real economic activity. However, such a policy may be problematic. Orphanides (2004) argues that being too responsive to economic activity is harmful because of a high measurement error. Real-time data are not the same as final revised data. He measures the coefficients of the interest rate rule in the US, and compared them before and after Paul Volcker's appointment in 1979. A main difference was found not in the coefficient of inflation but in that of real economic activity. While the former coefficient increased only slightly from 1.6 to 1.8, the latter decreased significantly from 0.6 to 0.2. McCallum (2001b) also cast doubts on the role of an output response because of the unobservability of the "natural rate" level of output. Conceptually, the "natural rate" may not be the only one which should be used. There can be, for example, "potential," "capacity," "trend," "NAIRU," "market clearing," and "flexible price" output levels. These insights imply that monetary authorities should not react to economic activity or at most should react moderately because it is so difficult to measure the activity. However, such policy may lead to indeterminacy according to Sveen and Weinke (2005).

It is thus very important in the practical sense for us to investigate more deeply what kind of monetary policy is necessary to prevent indeterminacy. Improving the analysis by Sveen and Weinke in the above manner enables us to better understand if monetary policy

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

really does need to respond to real economic activity in the way which Sveen and Weinke suggest, even though Orphanides and McCallum have suggested that this will be so difficult.

As will become clear later, each of these three modifications results in relaxing the necessity of responding to output. This is mainly because indeterminacy in the Sveen and Weinke model arises mainly from there being too much forward-lookingness of price resetting. Both inflation persistence and the existence of flexible price firms reduce the extent of forward-lookingness. We will show that introducing these features helps achieve determinacy more easily. Our finding suggests that simply following the Taylor principle by actively responding to inflation is sufficient for determinacy. This is a very important criticism of Sveen and Weinke's work. It is good that they have been critical of Dupor's weird results. We will show that they have not been critical enough. When one *is* properly critical, we are able to return right back to our belief in the Taylor principle as the criterion for good monetary policy.

Analytical Contribution: the Usefulness of a Continuous-time Framework

Another lesser important contribution of this chapter is that it demonstrates the value of using a continuous-time framework.

The need to do this was prompted by the misleading work of Carlstrom and Fuerst (2005). These authors compared a discrete-time framework with a continuous-time one, and argued that the latter model yielded a totally different result. This result was the same as Dupor (2001) namely that a passive rule makes equilibrium determinate while an active rule makes it indeterminate. They attributed the reason for this to an extra restriction in a continuous-time framework such that today's marginal costs equalled a real interest rate. This led them to ridicule the use of continuous-time models for the discussion of this issue. They wanted to criticise Dupor but found that the continuous-time models ended up supporting him. They therefore recommended that continuous-time analyses be abandoned.

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

I show that this work is extremely misleading. I discard their unrealistic assumption that there is a market for universal capital. Since firms cannot instantaneously adjust their capital stock by purchasing it from such a capital market, marginal costs do *not* need to equal a real interest rate. I thus demonstrate that introducing firm-specific investment removes any possible difficulty of this kind. When I do this, I show that the continuous-time analysis yields a similar result to that obtained using a discrete-time framework. This renders the contribution of Carlstrom and Fuerst completely unimportant; their work can be safely ignored. Their work, despite how they present it, does *not* constitute any grounds for abandoning the Taylor principle.

*

The structure of this chapter is as follows. In Section 3.2, we construct a model with the adjustment costs of investment, firm-specific investment and sticky prices. This model is similar to that of Sveen and Weinke (2005) except that we use a continuous-time framework. Then, we investigate the stability boundary. The next section checks the robustness of results by changing some of parameters' values. Section 3.4 attempts to improve the previous model by incorporating some more persistence and heterogeneities. It will become clear that a stability boundary is expanded in such richer models. This is why we are able to remain (to a very large extent) in the position of a defender of the Taylor principle.

3.2 Standard Model with Sticky Prices, Adjustment Costs and Firm-Specific Investment

3.2.1 Model Setup

The following model is almost the same as Sveen and Weinke (2005). The only difference is that we implement a continuous-time framework instead of a discrete-time framework.

This chapter follows Calvo (1983) as the mechanism of price stickiness. Assume that the prices of consumption and investment goods are sticky. In order to incorporate price stickiness, monopolistic competition is introduced.

<Households>

A representative household maximises one's utility

$$\max \int_t^\infty u(\bar{C}, \bar{L}) e^{-\rho(s-t)} ds \quad (3.2.1)$$

subject to

$$\int_t^\infty \bar{P}\bar{C} e^{-R(s-t)} ds \leq A(t) + \int_t^\infty (W\bar{L} + \Pi) e^{-R(s-t)} ds. \quad (3.2.2)$$

$\bar{C}$, $\bar{L}$ and $A(t)$ are respectively aggregated consumption, labour supply, and wealth at time t . A bar $\bar{X}$ represents an aggregated or average value. W is a wage, ρ is a discount factor, and R is the average nominal interest rate given by

$$R(s) = \frac{\int_t^s i(t') dt'}{s - t}, \quad (3.2.3)$$

using the nominal interest rate $i(t')$ at each time t' . No-Ponzi game version of the budget constraint is described as

$$\lim_{t \rightarrow \infty} e^{-R(t-s)} A(t) \geq 0. \quad (3.2.4)$$

A Dixit-Stiglitz aggregate price index $\bar{P}$ is given by

$$\bar{P} = m^\theta \left(\sum_j^m P_j^{-1/\theta} \right)^{-\theta}, \quad (3.2.5)$$

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

where P_j is the price of a good j ($j = 1, 2, \dots, m$) and $\theta(> 0)$ represents mark-up². Similarly, consumption is aggregated as

$$\bar{C} = m^{-\theta} \left(\sum_j^m C_j^{\frac{1}{1+\theta}} \right)^{1+\theta}, \quad (3.2.6)$$

using consumption of each good C_j . From this formalisation, a demand function for each good becomes

$$\frac{C_j}{\bar{C}} = \left(\frac{P_j}{\bar{P}} \right)^{-\frac{1+\theta}{\theta}} \quad (3.2.7)$$

First-order conditions are given by

$$\begin{cases} u_C(\bar{C}, \bar{L}) = \lambda e^{(\rho-i)t} \bar{P} \\ -u_L(\bar{C}, \bar{L}) = \lambda e^{(\rho-i)t} W, \end{cases} \quad (3.2.8)$$

using Lagrange multiplier λ . In particular, this chapter assumes the following utility function which is separable with respect to consumption and labour such that

$$u(\bar{C}, \bar{L}) = \frac{\bar{C}^{1-\sigma_1} - 1}{1-\sigma_1} + \gamma \frac{(1-\bar{L})^{1-\sigma_2} - 1}{1-\sigma_2}, \quad (3.2.9)$$

where $\sigma_1(> 0)$ is the degree of risk aversion of consumption, $\sigma_2(> 0)$ corresponds to this with respect to leisure $(1-\bar{L})$, and γ is a parameter to determine the weight between consumption and labour. When $\sigma_1 = \sigma_2 = 1$, the utility function becomes

$$u(\bar{C}, \bar{L}) = \log(\bar{C}) + \gamma \log(1 - \bar{L}).$$

<Firms>

Monopolistic firms produce goods which are used for consumption and investment. Note capital cannot move discretely. Given a price, firms adjust their labour so that they are able to produce as many goods as demanded. Therefore, current real marginal costs are directly affected not by marginal of product of capital but by the marginal product of labour. The latter then determines inflation dynamics, which will be later derived.

This model becomes complex for the following reason. The level of capital affects the marginal product of labour and, in turn, an optimal price. The future level of capital is

² $(1 + \theta)/\theta$ represents the elasticity of substitution between different goods.

determined by their choice of prices at the current period. Therefore, firms need to select their present optimal prices considering the effect of their decision on the future capital path. Furthermore, since firms reset their prices in a heterogeneous manner and there is no capital market, the level of capital may differ across firms off equilibrium. Firms have to take account of relative capital when deciding their reset prices.

Firms can adjust their prices at idiosyncratic random intervals, which are determined by the Poisson rate $\lambda(> 0)$. A high λ means high price flexibility. The probability that the next timing of price revision is during the interval from s to $s + ds$ provided the last price is set at or before t is described as $e^{-\lambda(s-t)}\lambda ds$.

Firms maximise their present-valued profits V_j as follows:

$$V_j \left(\frac{K_j(t)}{\bar{K}(t)}, t \right) = \max E_t \int_t^\infty e^{-\lambda(s-t)} \lambda ds \cdot \left\{ \int_t^s e^{-(i-\pi)(s'-t)} \Pi \left(\frac{P_j^*(t_0)}{\bar{P}(s')}, \frac{K_j(s')}{\bar{K}(s')}, s' \right) ds' + e^{-(i-\pi)(s-t)} V_j^* \left(\frac{K_j(s)}{\bar{K}(s)}, s \right) \right\} \quad (3.2.10)$$

subject to

$$\Pi \left(\frac{P_j(t)}{\bar{P}}, \frac{K_j}{\bar{K}}, s \right) = \frac{P_j(t)}{\bar{P}} Y_j - \frac{W}{\bar{P}} L_j - \{I_j + J(I_j, K_j)\} \quad (3.2.11)$$

$$\dot{K}_j = I_j - \delta K_j \quad (3.2.12)$$

$$\frac{Y_j}{\bar{Y}} = \left(\frac{P_j}{\bar{P}} \right)^{-\frac{1+\theta}{\theta}} \quad (3.2.13)$$

$$Y_j = h K_j^\alpha L_j^{1+\beta-\alpha} \quad (3.2.14)$$

$$J(I_j, K_j) = \Psi(I_j/K_j) K_j. \quad (3.2.15)$$

P_j , Y_j , Π_j , K_j , I_j and $J(I_j, K_j)$ are a firm j 's price, output, real profits, capital, investment and adjustment costs respectively. Produced goods are used for investment as well as consumption. Investment goods including adjustment costs are purchased for the aggregate price index of $\bar{P}$. For simplicity, I assume their composition is the same as that of consumption:

$$I_j + J(I_j, K_j) = m^{-\theta} \left(\sum_{k=1}^m \{I_{jk} + J(I_{jk}, K_j)\}^{\frac{1}{1+\theta}} \right)^{1+\theta}, \quad (3.2.16)$$

so equation (3.2.7) can be extended to aggregate demand $\bar{Y}$. A production function can incorporate arbitrary returns to scale by means of β . A special case $\beta = 0$ represents constant returns to scale. An optimal price and a present-valued profit are given by

$$P_j^*(t) \equiv \begin{cases} P_j^*(t) & \text{if the price is reset at } t \\ P_j^*(t_0) & (t_0 < t) \text{ otherwise} \end{cases}$$

$$V_j \left(\frac{K_j(t)}{K(t)}, t \right) \equiv V_j^* \left(\frac{K_j(t)}{K(t)}, t \right) \text{ if the price is reset at } t.$$

V_j^* represents a firm j 's present-valued profit when it can reset its price. In calculating V_j , I change the order of integration between s and s' . That is, I integrate a function with respect to s before s' , and the range of s and s' becomes $[s', \infty]$ and $[t, \infty]$ respectively. Then, we can simplify V_j as

$$\begin{aligned} V_j(t) &= \max E_t \int_t^\infty e^{-(i-\pi)(s'-t)} \Pi_j(s') ds' \int_{s'}^\infty e^{-\lambda(s-t)} \lambda ds + \int_t^\infty e^{-(i-\pi+\lambda)(s-t)} \lambda V_j^*(s) \\ &= \max E_t \int_t^\infty e^{-(i-\pi+\lambda)(s-t)} \{ \Pi_j(s) + \lambda V_j^*(s) \} ds. \end{aligned} \quad (3.2.17)$$

Define a new variable X as

$$X \equiv I/K \quad (3.2.18)$$

and Hamiltonian as

$$\begin{aligned} H_j(L_j(s), X_j(s), K_j(s), P_j(t)) \\ = \Pi_j(s) + \lambda V_j^*(s) + Q_j(X_j(s) - \delta)K_j(s) - \nu \left\{ \frac{Y_j(s)}{Y(s)} - \left(\frac{P_j(t)}{P(s)} \right)^{-\frac{1+\theta}{\theta}} \right\}, \end{aligned}$$

where $t < s$. Q_j is a co-state variable of K_j and ν is a Lagrange multiplier. A reset price P_j cannot be a control variable unless firm j wins the lottery to reset its price. First-order

conditions become

$$\frac{\partial H_j(s)}{\partial L_j(s)} = 0 = \frac{P_j}{\bar{P}} Y_{jL} - \frac{W}{\bar{P}} - \frac{\nu}{\bar{Y}} Y_{jL} \quad (3.2.19)$$

$$\frac{\partial H_j(s)}{\partial X_j(s)} = 0 = -(1 + \Psi'(X_j)) K_j - Q_j K_j \quad (3.2.20)$$

$$\begin{aligned} \frac{\partial H_j(s)}{\partial K_j(s)} &= (i - \pi + \lambda) Q_j - \dot{Q}_j \\ &= \frac{P_j}{\bar{P}} Y_{jK} - (X_j + \Psi(X_j)) + \nu Y_{jK} + \lambda V_{jK}^* + Q_j (X_j - \delta) - \frac{\nu}{\bar{Y}} Y_{jK} \end{aligned} \quad (3.2.21)$$

Deleting ν and Q , we can rewrite them as

$$\Psi''(X_j) \dot{X}_j = (i - \pi + \lambda - X_j + \delta)(1 + \Psi'(X_j)) - \frac{W}{\bar{P}} \frac{Y_{jK}}{Y_{jL}} + X_j + \Psi(X_j) - \lambda V_{jK}^*.$$

Note $V_{jK}^* = Q_j = 1 + \Psi'(X_j)$ because of the envelope theorem³. Therefore, we can obtain the following investment schedule:

$$\Psi''(X_j) \dot{X}_j = (i - \pi - X_j + \delta)(1 + \Psi'(X_j)) - \frac{W}{\bar{P}} \frac{Y_{jK}}{Y_{jL}} + X_j + \Psi(X_j). \quad (3.2.22)$$

Next consider a firm's reset price. If a firm can reset its price at t and more importantly if this price did not affect the future profits, then the reset price $P_j(t)$ would satisfy

$$\begin{aligned} \frac{\partial H_j(t)}{\partial P_j(t)} = 0 &= \frac{Y_j}{\bar{P}} - \frac{1 + \theta}{\theta} \frac{\nu}{\bar{Y}} \frac{Y_j}{P_j} \\ &= \frac{Y_j}{\bar{P}} - \frac{1 + \theta}{\theta} \left(\frac{P_j}{\bar{P}} - \frac{W}{\bar{P} Y_{jL}} \right) \frac{Y_j}{P_j}. \end{aligned}$$

Let us call this a desired price $P_j^\#$, and it is given by the current real marginal costs S_j times

³Put differently, first-order approximation yields

$$\begin{aligned} V_j^*(t) &= \max E_t [\Pi_j(t) dt + \lambda V_j^*(t) dt \\ &\quad + (1 - (i - \pi + \lambda) dt) \int_{t+dt}^{\infty} e^{-(i-\pi+\lambda)(s-t)} \{ \Pi_j(s) + \lambda V_j^*(s) \} ds] \\ &= \max E_t \left[\Pi_j(t) dt + \lambda V_j^*(t) dt + (1 - (i - \pi + \lambda) dt) (V_j^*(t) + V_{jK}^*(t) \dot{K}_j dt) \right] \\ (i - \pi) V_j^*(t) &= \max E_t [\Pi_j(t) + V_{jK}^*(t) \dot{K}_j]. \end{aligned}$$

The right-hand side of the equation should correspond to Hamiltonian, so V_{jK}^* is equal to the shadow price Q_j .

a markup $1 + \theta$:

$$\frac{P_j^\#}{\bar{P}} = (1 + \theta)S_j \quad (3.2.23)$$

$$\text{where } S_j \equiv \frac{W}{\bar{P}Y_{jL}}. \quad (3.2.24)$$

However, an optimal reset price is not the desired price because firms cannot reset their prices at any periods. When a firm wins a lottery to reset its price, it maximises $V_j^*(t)$ so that

$$\begin{aligned} \frac{\partial V_j^*(t)}{\partial P_j(t)} = 0 &= \frac{\partial}{\partial P_j(t)} \int_t^\infty e^{-(i-\pi+\lambda)(s-t)} H_j(s) ds \\ &= \int_t^\infty e^{-(i-\pi+\lambda)(s-t)} \frac{1}{\theta} \frac{Y_j(s)}{\bar{P}(s)} \left\{ -1 + (1 + \theta)S_j(s) \frac{\bar{P}(s)}{P_j(t)} \right\} ds. \end{aligned} \quad (3.2.25)$$

<Market clearing>

A goods market is cleared provided

$$\bar{Y} = \bar{C} + \bar{I} + J(\bar{I}, \bar{K}). \quad (3.2.26)$$

<Policy rule>

Assume a Taylor rule using a nominal interest rate as an instrument. A mathematical form is shown later.

3.2.2 Log-Linearisation

Around a steady state, we now derive the log-linearised form of the model which we will use. In the following, a subscript of zero represents the value of a steady state, and small letters ($c, w, \bar{p}, l$ etc) represent logarithm deviation around a steady state.

<Households>

In equilibrium,

$$i_0 - \pi_0 = \rho. \quad (3.2.27)$$

We can also obtain the following Euler equation and labour supply function:

$$\bar{c} = \frac{\Delta i - \pi}{\sigma_1} \quad (3.2.28)$$

$$w - \bar{p} = \sigma_1 \bar{c} + \sigma_2 \frac{L_0}{1 - L_0} \bar{l}, \quad (3.2.29)$$

where $\Delta i - \pi$ is the deviation of a real interest rate in a level.

<Firms>

Log-linearising a production function yields

$$y_j = \alpha k_j + (1 + \beta - \alpha) l_j. \quad (3.2.30)$$

In equilibrium,

$$X_j^0 = I_j^0 / K_j^0 = \delta,$$

and the law of motion with respect to capital becomes

$$\dot{k}_j = \delta x_j. \quad (3.2.31)$$

Regarding X , rewrite its law of motion:

$$\Psi''(X_j) \dot{X}_j = (i - \pi - X_j + \delta)(1 + \Psi'(X_j)) - \frac{W}{\bar{P}} \frac{Y_{jK}}{Y_{jL}} + X_j + \Psi(X_j).$$

In equilibrium,

$$0 = \rho(1 + \Psi'(\delta)) - \frac{W_0}{\bar{P}_0} \frac{Y_{jK0}}{Y_{jL0}} + \delta + \Psi(\delta),$$

and the law of motion can be transformed to

$$\begin{aligned} \dot{x}_j = & \frac{1 + \Psi'(\delta)}{\delta \Psi''(\delta)} (\Delta i - \pi) + \rho x_j \\ & - \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \left\{ w - \bar{p} + \frac{1}{1 + \beta - \alpha} y_j - \frac{1 + \beta}{1 + \beta - \alpha} k_j \right\}. \end{aligned} \quad (3.2.32)$$

Since they are in a linear form, we can aggregate simply by summing them:

$$\bar{y} = \alpha \bar{k} + (1 + \beta - \alpha) \bar{l} \quad (3.2.33)$$

$$\bar{\dot{k}} = \delta \bar{x} \quad (3.2.34)$$

$$\begin{aligned} \bar{x} = & \frac{1 + \Psi'(\delta)}{\delta \Psi''(\delta)} (\Delta i - \pi) + \rho \bar{x} \\ & - \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \left\{ w - \bar{p} + \frac{1}{1 + \beta - \alpha} \bar{y} - \frac{1 + \beta}{1 + \beta - \alpha} \bar{k} \right\}. \end{aligned} \quad (3.2.35)$$

Regarding reset prices, equation (3.2.25) can be log-linearised as

$$0 = \int_t^\infty e^{-(\rho+\lambda)(s-t)} \{ \tilde{p}_j^*(s) - s_j(s) \} ds, \quad (3.2.36)$$

because, in equilibrium, $i - \pi = \rho$, $P_j = \bar{P}$ and $S_j = 1/(1 + \theta)$. A tilda ($\sim$) represents a relative value to its average, that is:

$$\tilde{p}_j(s) \equiv p_j(s) - \bar{p}(s), \quad (3.2.37)$$

and, with asterisk, $p_j^*(t)$ represents an optimal reset price. What makes this model complex is that each firm's p_j^* is not always equal to an average optimal price p^* . Reset prices are influenced by firms' capital which cannot be purchased instantly through a capital market, so there arises heterogeneity about the reset prices among firms. Firms' investment and pricing decision can be decomposed into aggregate and relative terms. Woodford (2005) wisely derived p_j^* and p^* by assuming the specific forms such as

$$p_j^* = p^* - \tau_1 \tilde{k}_j \quad (3.2.38)$$

$$\tilde{x}_j = -\tau_2 \tilde{k}_j - \tau_3 \tilde{p}_j, \quad (3.2.39)$$

where $\tilde{k}_j = k_j - \bar{k}$ and $\tilde{x}_j = x_j - \bar{x}$. As explained in Appendix 3.A.1, the parameters τ_1, τ_2 , and τ_3 are obtained numerically from three equations. Also, Appendix 3.A.1 shows that

inflation dynamics becomes

$$\dot{\pi} = \rho\pi - \frac{\lambda(\rho + \lambda)}{A}s, \quad (3.2.40)$$

$$\text{where } A \equiv 1 + \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} - \frac{\alpha}{1 + \beta - \alpha} \frac{\delta\tau_3}{\rho + \lambda + \delta\tau_2}$$

$$s = w - \bar{p} - (\bar{y} - \bar{l}). \quad (3.2.41)$$

<Market clearing>

$$Y_0\bar{y} = C_0\bar{c} + \delta(1 + \Psi'(\delta))K_0\bar{x} + (\delta + \Psi(\delta))K_0\bar{k} \quad (3.2.42)$$

$$Y_0\bar{\dot{y}} = C_0\bar{\dot{c}} + \delta(1 + \Psi'(\delta))K_0\bar{\dot{x}} + (\delta + \Psi(\delta))K_0\bar{\dot{k}} \quad (3.2.43)$$

<Policy rule>

Assume the following monetary policy rule:

$$\Delta i = \phi_y\bar{y} + \phi_\pi\pi, \quad (3.2.44)$$

then we can write

$$\Delta i - \pi = \phi_y\bar{y} + (\phi_\pi - 1)\pi. \quad (3.2.45)$$

$\Delta i - \pi$ is the level deviation of a real interest rate. Call a rule active if $\phi_\pi > 1$, and passive if $\phi_\pi < 1$.

3.2.3 Stability Analysis

To put the above equations together, especially (3.2.28), (3.2.34), (3.2.35) and (3.2.40), we can construct the system of four differential equations with respect to $z = \{\pi, \bar{y}, \bar{x}, \bar{k}\}$ such as

$$\dot{z} = Bz. \quad (3.2.46)$$

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

B is defined as

$$B = \begin{pmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{pmatrix},$$

where

$$\left\{ \begin{array}{l} B_{11} = \rho \\ B_{12} = -\frac{\lambda(\rho+\lambda)}{A} \left(\frac{\sigma_1 Y_0}{C_0} + \frac{\sigma_2 \frac{L_0}{1-L_0} + 1}{1+\beta-\alpha} - 1 \right) \\ B_{13} = \frac{\lambda(\rho+\lambda)}{A} \frac{\sigma_1 \delta (1+\Psi'(\delta)) K_0}{C_0} \\ B_{14} = \frac{\lambda(\rho+\lambda)}{A} \left(\frac{\sigma_1 (\delta + \Psi(\delta)) K_0}{C_0} + \alpha \frac{\sigma_2 \frac{L_0}{1-L_0} + 1}{1+\beta-\alpha} \right) \\ B_{21} = \frac{C_0}{Y_0} \frac{\phi_\pi - 1}{\sigma_1} + \frac{\delta (1+\Psi'(\delta)) K_0}{Y_0} B_{31} \\ B_{22} = \frac{C_0}{Y_0} \frac{\phi_y}{\sigma_1} + \frac{\delta (1+\Psi'(\delta)) K_0}{Y_0} B_{32} \\ B_{23} = \frac{\delta (1+\Psi'(\delta)) K_0}{Y_0} B_{33} + \frac{(\delta + \Psi(\delta)) K_0}{Y_0} B_{43} \\ B_{24} = \frac{\delta (1+\Psi'(\delta)) K_0}{Y_0} B_{34} \\ B_{31} = \frac{1+\Psi'(\delta)}{\delta \Psi''(\delta)} (\phi_\pi - 1) \\ B_{32} = \frac{1+\Psi'(\delta)}{\delta \Psi''(\delta)} \phi_y - \frac{\rho(1+\Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \left(\frac{\sigma_1 Y_0}{C_0} + \frac{\sigma_2 \frac{L_0}{1-L_0} + 1}{1+\beta-\alpha} \right) \\ B_{33} = \rho + \frac{\rho(1+\Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \frac{\sigma_1 \delta (1+\Psi'(\delta)) K_0}{C_0} \\ B_{34} = \frac{\rho(1+\Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \left(\frac{\sigma_1 (\delta + \Psi(\delta)) K_0}{C_0} + \frac{\alpha \sigma_2 \frac{L_0}{1-L_0} + 1 + \beta}{1+\beta-\alpha} \right) \\ B_{41} = 0 \\ B_{42} = 0 \\ B_{43} = \delta \\ B_{44} = 0 \end{array} \right. \quad (3.2.47)$$

There are three jump variables except for $\bar{k}$. Hence, if the number of positive eigenvalues is three, then equilibrium is determinate. If it exceeds three, equilibrium is unstable. If it is less than three, equilibrium is indeterminate.

In order to check the eigenvalues of the system, I use the following parameters' values:

$\alpha = 0.3$	$\beta = 0$	$\delta = 0.1$	$h = 1$	$\rho = 0.02$	$\theta = 0.2$	$\phi_\pi = 1.5$
$\sigma_1 = 1$	$\sigma_2 = 1$	$\gamma = 1$	$\phi_y = 0$	$a = 3$	$\lambda = 1$	

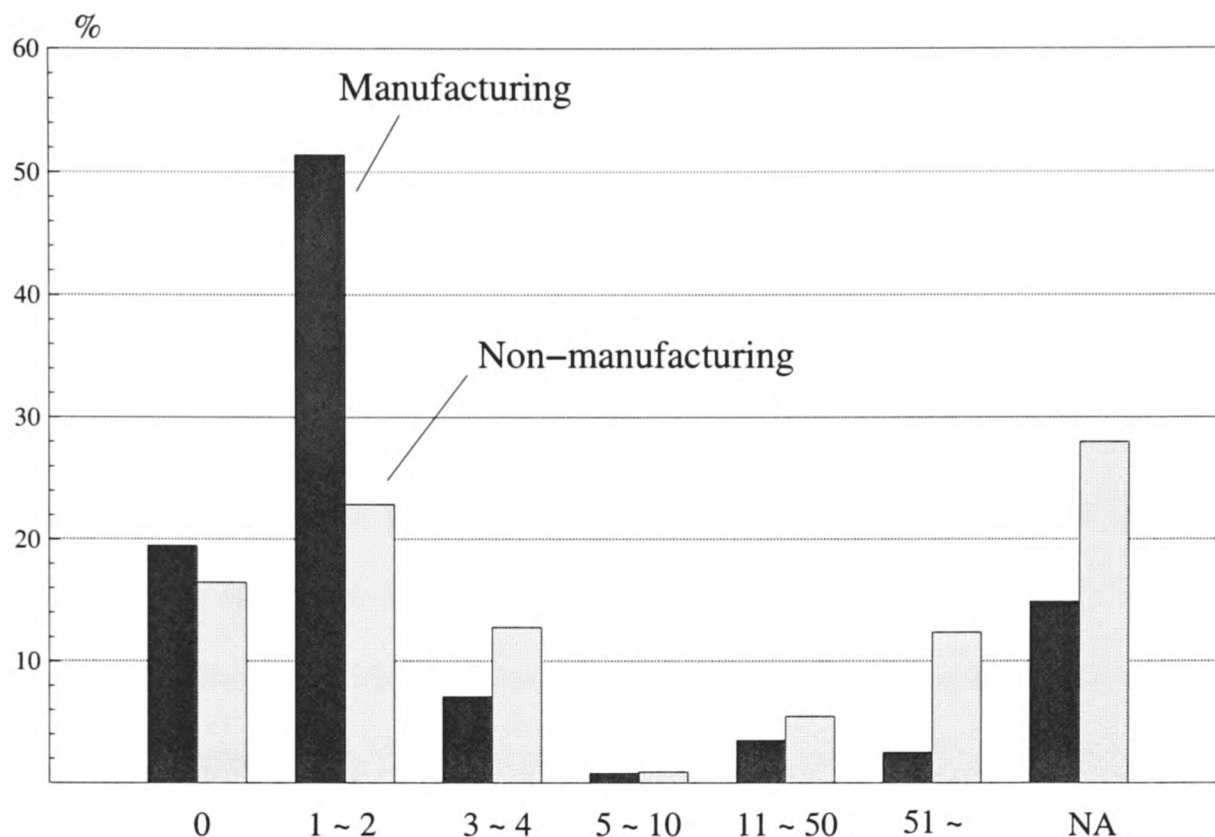


Figure 3.2.1: Number of Price Changes in the Past Year (the Bank of Japan 2000)

To match the data of Japan, a capital share α , a depreciation rate δ , and mark-up θ are chosen to be 0.3, 0.1 and 0.2 respectively⁴. Taking account of depression in the 1990's, as a discount factor, this chapter sets a slightly lower value of ρ than that of the US. The coefficients of labour supply σ_2 and γ are selected so that equilibrium labour supply is approximately half of the maximum, but there can be many other possibilities including the degree of risk aversion σ_1 . A sensitivity to this will be checked in the next section. Adjustment costs are given by

$$\Psi(X) = \frac{a}{2}(X - \delta)^2, \quad (3.2.48)$$

where a is the parameter to show the degree of adjustment costs. This chapter chooses $a = 3$ because, according to Woodford (2003, 2004), this value seems to be the most plausible empirically⁵. The appropriateness of $a = 3$ is not obvious, however, so I will examine how

⁴In their macroeconomic model, Fujiwara et al. (2004) used values such as $\alpha = 0.37$, $\delta = 0.06$, $\theta = 0.25$ from the data of System of National Account.

⁵In order to understand its degree, compare the amplitude of adjustment costs in equilibrium with the case where there is no investment. When $X = 0$, total investment costs become $I + J = (X + \Psi(X))K = a\delta^2 K/2 = 0.015K$. In equilibrium at $X = \delta$, they amount to $I + J = \delta K = 0.1K$. These differences do not look so unrealistic.

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

a different a affects our result later. Price flexibility is set at $\lambda = 1$ unless noted otherwise, which corresponds to the case where prices are reset once a year. According to the survey by the Bank of Japan (2000), companies are the most likely to reset their prices once or twice a year, which accounts for about fifty percent in manufacturing industries and about twenty percent in non-manufacturing industries (see Figure 3.2.1)⁶. There have been a number of estimations on the parameters of a Taylor rule. For example, Taylor (1993) measured the values of $\phi_\pi \sim 1.5$ and $\phi_y \sim 0.5$. According to Clarida, Gali and Gertler (2000), the US monetary policy has values of $\phi_\pi \sim 2$ and $\phi_y \sim 1$.

This system is too complex to investigate analytically, but computational calculation yields the following results:

- Suppose $\phi_y = 0$. Under a passive rule, equilibrium is always indeterminate. A slightly active rule makes equilibrium determinate. However, a moderately or very active rule causes indeterminacy.

$\phi_\pi > \phi_\pi^* > 1$	$1 < \phi_\pi < \phi_\pi^*$	$\phi_\pi < 1$
− − − + (indeterminate)	− + + + (determinate)	− − + + (indeterminate)

This is shown in Figure 3.2.2. At $\lambda = 1$, $\phi_\pi^* = 1.011$. Therefore, for the price flexibility λ around 1 and the rule $\phi_\pi = 1.5$ which was suggested by Taylor (1983), equilibrium is likely to be indeterminate.

- If a policy rule depends not only on inflation but also on real economic activity, then determinacy can be achieved more easily. For example, at $\phi_\pi = 1.5$, a stronger response to output than $\phi_y^* = 0.12$ can rule out the possibility of indeterminacy. Even a passive rule can lead to determinacy if the response is sufficiently close to that of an active rule.

⁶In the UK, Hall, Walsh and Yates (1997) reported similar frequencies of price revision.

To put it differently, even for $\lambda > \lambda^*$, given a small $\varepsilon > 0$,

	$\phi_\pi > 1 - \varepsilon$
$\phi_y < \phi_y^*$	---+ or --++ (indeterminate)
$\phi_y > \phi_y^*$	-+++ (determinate)

This feature is graphically shown in Figures 3.2.3 and 3.2.4. Note, in the latter figure, the scales of horizontal axes are largely different.

- When the adjustment costs are sufficiently large (high a), irrespective of the degree of price flexibility λ , determinacy becomes possible if only a Taylor rule is active. For example, at the parameters such that $\phi_\pi = 1.5$ and $\phi_y = 0$, a threshold a is about 30. This result is consistent with that of the model without capital. If capital is regarded as fixed because of high enough adjustment costs, then we can neglect the effect of capital and focus only on labour as a production factor. In this case, active rules always yield determinacy as Clarida, Gali and Gertler (2000) and Bullard and Mitra (2002) argue.

The dynamics of this model can be demonstrated using the Blanchard and Kahn (1980) method⁷. Figure 3.2.5 shows the dynamics under determinacy, where $\phi_\pi = 1.01$ and $\phi_y = 0$, supposing capital is initially lower than its equilibrium level ($k = -1$). A horizontal axis is time in a year scale in all figures except for the bottom-left one. All variables converge to zero which is an equilibrium value. The last figure shows the relevant saddle path in two dimensions. The saddle path is actually a line in four-dimensional space, but leaving out the other two dimensions does not change the saddle path. This is because there is only one state variable which is capital. The other three variables (investment, consumption and inflation) are uniquely determined by capital, so the omitted other dimensions (consumption and inflation) do not affect the saddle path regarding capital and investment.

A lower level of capital induces high investment. Demand increases, and firms employ

⁷I would like to thank Tatiana Kirsanova for the relevant code.

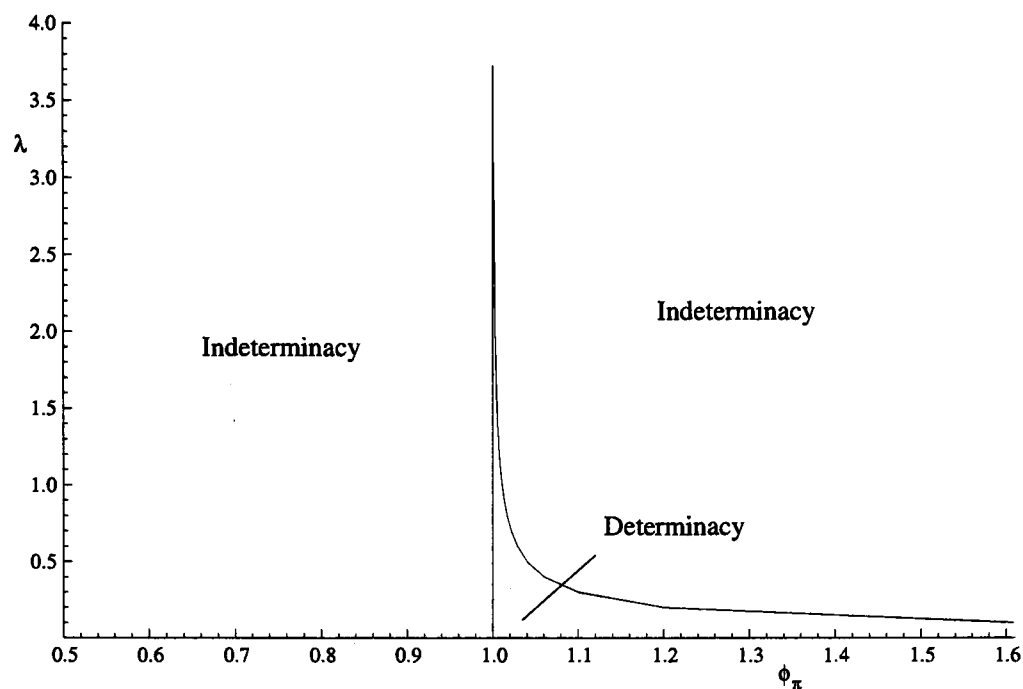


Figure 3.2.2: Stability Bound

more labour in order to produce more goods. Real wages increase, which increases the inflation rate. A central bank raises a nominal interest rate more than one-for-one, which increases the real interest rate. Owing to intertemporal optimisation, households decrease their consumption. These features match this figure.

Sveen and Weinke (2005) examined determinacy using almost the same model, except for a discrete-time framework, and obtained very similar results. The only difference is that, in their model, a sufficiently large ϕ_π leads to determinacy even under highly flexible prices, the reason for which will be investigated later. In the other respect, our model yields the same results.

There is thus no big difference between discrete-time and continuous-time frameworks like the one pointed out by Carlstrom and Fuerst (2005). As noted at the beginning of this chapter, these authors argued that a continuous-time framework yielded the opposite result: a passive monetary policy rule made equilibrium determinate and an active rule made it indeterminate. They attributed the reason for this to an extra restriction in a continuous-time framework such that today's marginal costs equal a real interest rate. This led them to

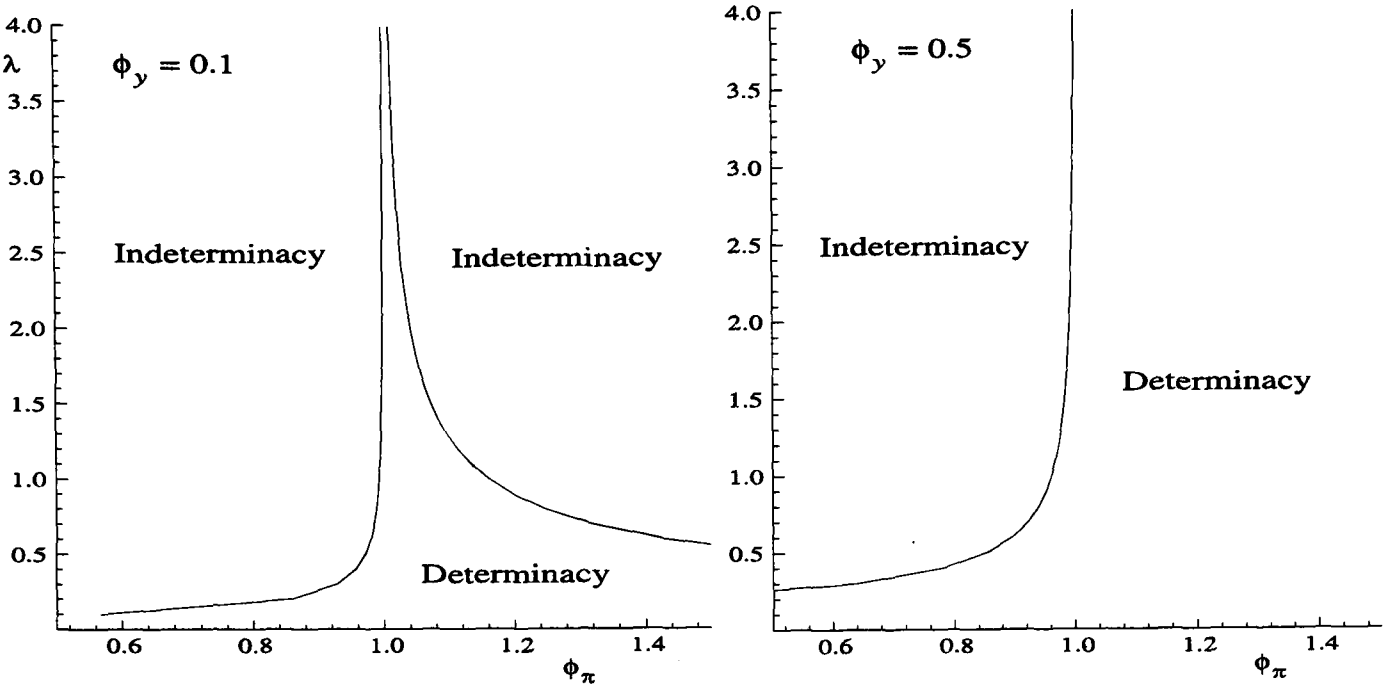


Figure 3.2.3: Stability Bound Change by Responding to Real Economic Activity

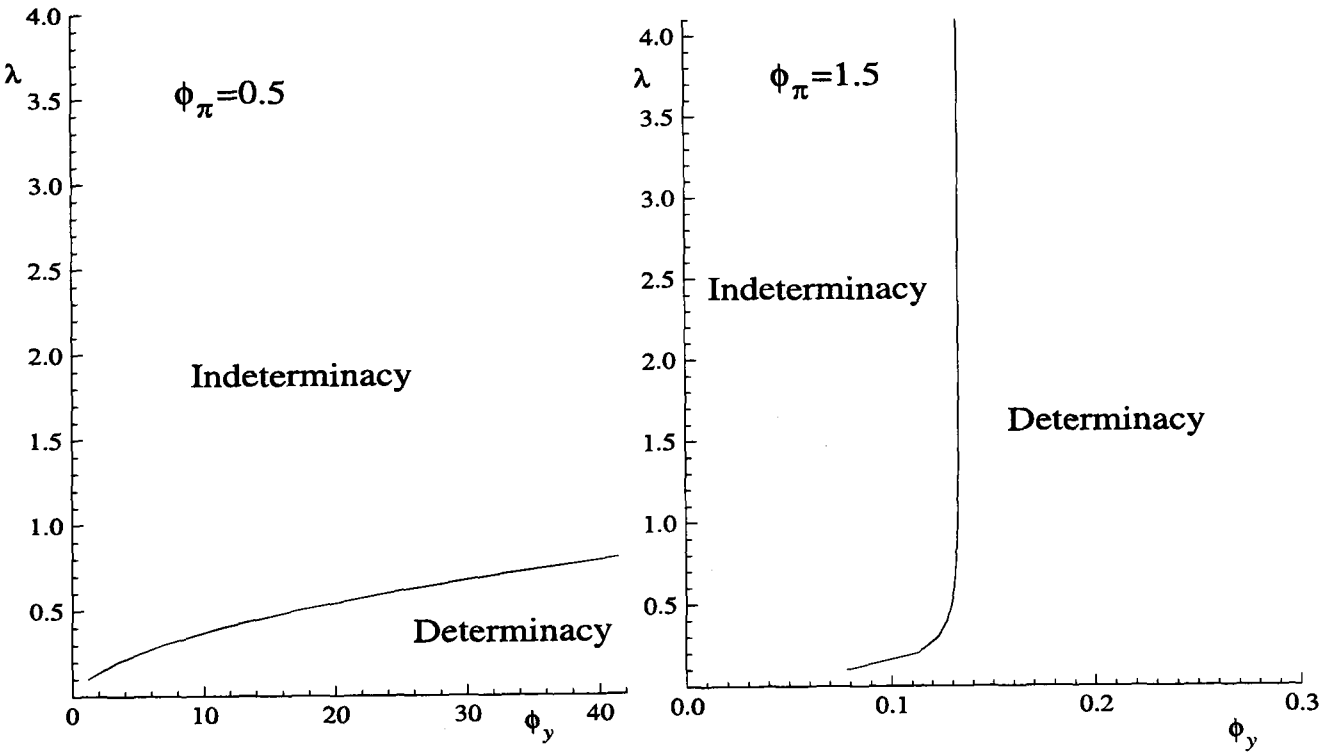


Figure 3.2.4: Stability Bound Change by Responding to Real Economic Activity

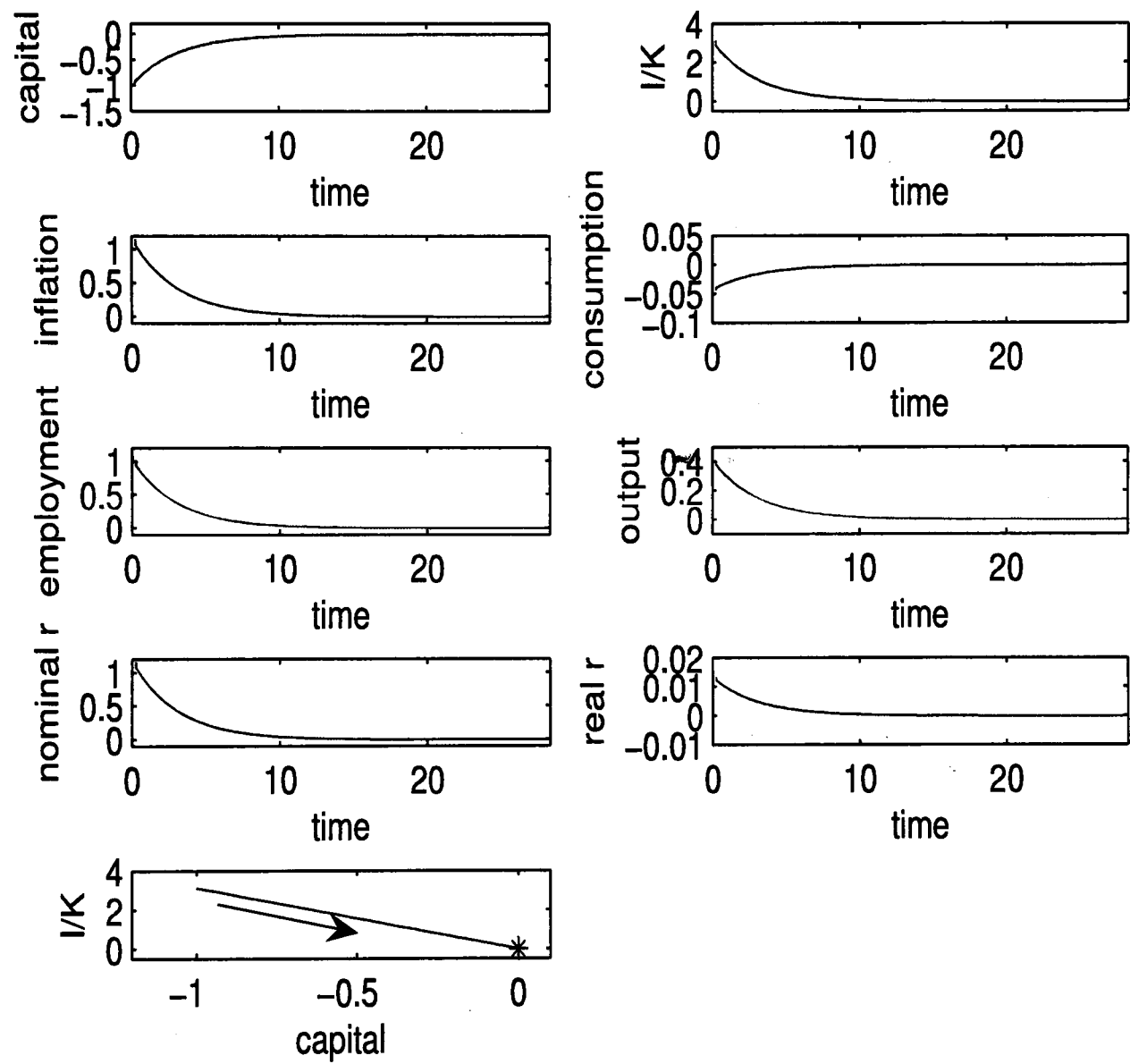


Figure 3.2.5: Dynamics under Determinacy

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

ridicule the use of continuous-time models for the discussion of this issue. They wanted to criticise Dupor but found that the continuous-time models ended up supporting him. They therefore recommended that continuous-time analyses be abandoned.

Our result show that this work is extremely misleading. I discard their unrealistic assumption that there is a market for universal capital and replace this with firm-specific investment. Since firms cannot instantaneously adjust their capital stock by purchasing it from such a capital market, marginal costs do *not* need to equal to a real interest rate. The continuous-time analysis then yields a similar result to that obtained using a discrete-time framework. We thus think that their initially striking claim is due to a misunderstanding of the appropriate restrictions to producing a model in which capital is endogenous. Their work, despite how they present it, does *not* constitute any grounds for abandoning the Taylor principle.

Reasons for Indeterminacy Under an active policy rule, why does equilibrium become indeterminate with a degree of two⁸? The intuition runs as follows. Suppose that people expect an investment boom. It has three effects.

1. An investment boom raises the current demand, which increases the current marginal costs. This jumps inflation up and, given an active rule, the real interest rate as well. Hence, investment shrinks. This is the "normal" outcome achieved by an active rule, which is stabilising.
2. Also a rise in future capital caused by a rise in investment decreases the future marginal product of capital. This discourages current investment, which is stabilising as well.
3. However, the third effect, which comes from the future, is destabilising. A rise in investment raises the future level of capital, which decreases the future marginal costs.

Thus, the optimal level of prices reset in the Calvo process decreases. There arises

⁸In other words, there are two more negative eigenvalues than there should be.

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

deflation and, provided that monetary policy is active, real interest rates decrease.

This rationalises the expectation of a rise in investment.

If the third effect dominates the others, the expectation is self-fulfilling. In other words, equilibrium becomes indeterminate. This is mostly the case in our calibrated model. Recall that indeterminacy has a degree of two. This means that there is another factor of self-fulfilling expectation. This is a consumption boom. The mechanism of this is exactly the same. A consumption boom raises demand, which increases investment. This can lead to a reduction in a real interest rate and, in turn, a rise in consumption in the end.

It is forward-lookingness of price setting that increases the third effect. Hence, it is natural to conjecture that non-forward-looking pricing can enhance the possibility of determinacy. To put it differently, inflation persistence can help in achieving determinacy. Due to persistence, the effect of present inflation lasts longer. This increases the first effect relative to the third one and, in turn, the possibility of determinacy. This conjecture will be verified in the next section.

Next, consider the reason for indeterminacy with a degree of one under a passive policy rule⁹. Suppose an increase in an expected inflation rate. Because of ineffective policy, a real interest rate decreases. Demand, both investment and consumption, increases, which results in a rise in the inflation rate. Thus, such a belief of inflation becomes self-fulfilling.

It is a little counter-intuitive that a slightly active policy rule can cause determinacy but a moderately or strongly active rule causes indeterminacy. This is because, irrespective of the policy rule, the above second effect continues functioning. The second effect exceeds the third one, and equilibrium becomes determinate under a slightly active rule. However, if the rule is more aggressive, the third effect comes to dominate the second. Since the third effect seems to dominate the first under our calibration, such a policy rule causes indeterminacy.

⁹In other words, there is one more negative eigenvalue than there should be.

The reason why equilibrium becomes determinate under highly sticky prices ($\lambda \sim 0.1$) can be understood for the same reason. Since actual prices do not change much, the influence of monetary policy which reacts to inflation becomes relatively weak. Hence, similar to the case of a low ϕ_π , the second effect above comes to exceed the other effects, and determinacy can be achieved.

Reasons for the Difference from Sveen and Weinke (2005) As noted above, our model yields almost the same result as that of Sveen and Weinke (2005). The only difference is that, in their model, a sufficiently large ϕ_π leads to determinacy even under highly flexible prices. The reason for this seems to be understood by the forward-lookingness of central banks. Depending on whether central banks respond to past or future expected inflation, equilibrium can become determinate or indeterminate. With a discrete-time framework, Carlstrom and Fuerst (2000, 2005) reported that an interest rate rule dependent on backward data tends to make the condition of determinacy less stringent. On the other hand, a forward-looking response raises the possibility of indeterminacy. A detailed analysis is actually limited in this chapter because of the continuous-time framework but, thanks to the previous discussion, it is easy to infer the reason for this. The third effect above, which induces indeterminacy, is caused by future inflation. Thus, the possibility of indeterminacy is amplified as a policy rule becomes more and more forward-looking. Sveen and Weinke (2005) used almost the same model as ours except for a discrete-time framework, and showed that a sufficiently large ϕ_π leads to determinacy even under highly flexible prices. They assumed a rule which depends on current inflation. Other papers, that of Dupor (2002) and Gali, Lopez-Salido, and Valles (2004), which did not assume firm-specific investment but the existence of a capital market, reported that a too active monetary policy causes indeterminacy. This is the same result as mine. They assumed the policy rule which depends on expected

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

inflation. In this sense, our continuous-time framework seems to be closer to the forward-looking monetary policy. Of course, in order for us to make sure, this needs to be proved with an appropriate discrete-time model.

Necessity of Responding to Real Economic Activity The second bullet point above, the effects of which are demonstrated in Figure 3.2.3 and 3.2.4, may be preferable because it makes the condition of determinacy less stringent. If a monetary policy rule responds not only to inflation but also to output, then equilibrium can become determinate under plausible conditions. The reason runs as follows. Suppose an investment boom again. Demand increases and, owing to a policy rule responding to output, a real interest rate increases. This decreases investment and makes the expectation of the investment boom inconsistent. As noted above, there are other channels which do or do not rationalise the investment boom. However, at any rate, as ϕ_y becomes larger, this new channel becomes dominant, and makes the boom not self-fulfilling. Then, equilibrium becomes determinate.

It is often explained that, since the deviation of output¹⁰ from equilibrium becomes an indicator for future inflation, responding to output enables the achieving of determinacy. However, we need to consider this a little more carefully. It is true that the deviation of output can predict future inflation, but responding to future inflation does not decrease but increases the possibility of indeterminacy as explained above. However, if pricing decisions are not highly forward-looking or inflation becomes persistent, then the first effect becomes larger and the third becomes smaller. Thus, the information of the future inflation, if not of the too distant future, can become useful in order to achieve determinacy.

¹⁰More correctly real marginal costs

3.3 Robustness Check

For the plausible parameters such as $\lambda = 1$ and $\phi_\pi = 1.5$, equilibrium is likely to be indeterminate. It is true that a Taylor rule which reacts to output as well as inflation can exclude the possibility of indeterminacy, but there are a couple of problems that react to economic activity. Firstly, as Orphanides (2004) argues, too high ϕ_y is problematic because of a measurement error. Real-time data are not the same as final revised data. He measured the coefficients of the interest rate rule in the US, and compared them before and after Paul Volcker's appointment in 1979. A main difference was found not in the coefficient of inflation but in that of real economic activity. While the former coefficient increased only slightly from 1.6 to 1.8, the latter decreased greatly from 0.6 to 0.2. This suggests that the stability of the US economy is attributed not to tough reaction to inflation but to little response to output. Secondly, as McCallum (2001b) argues, there is the unobservability of the "natural rate" level of output. Conceptually, the "natural rate" may not be the only one which should be used. There can be, for example, "potential," "capacity," "trend," "NAIRU," "market clearing," and "flexible price" output levels. These papers imply that an inactive or moderate policy rule against economic activity is preferable.

In terms of determinacy, in the previous case, $\phi_y^* = 0.12$ was the critical value, so $\phi_y = 0.2$ which was obtained in Orphanides can still achieve determinacy. However, taking account of the inaccuracy of several parameters, $\phi_y = 0.2$ seems to be a very narrow margin for determinacy. It is thus important to check how much this critical value changes depending on several parameter values. Although there has been some discussion on the robustness (for example, Dupor: 2002), it does not seem enough and, moreover, as we pointed out in the Introduction, the setup of Dupor is wrong. This section investigates robustness more thoroughly with our better model.

The critical value ϕ_y^* which divides determinacy and indeterminacy depends on many

parameters such as σ_1 , σ_2 , γ , and adjustment costs. These parameters are all very difficult to measure, and have a wide deviation. Dupor (2002) already noticed the importance of the effect of labour supply elasticity and adjustment costs. However, in addition, the risk aversion of consumers (σ_1) is important to determine stability. Dupor and the above calculation assumed its value was one but, as the famous 'equity premium puzzle'¹¹, suggests, the degree of risk aversion may be higher. A high degree of risk aversion (high σ_1) weakens the effect of the current real interest rate on consumption, and increases the threshold ϕ_y^* . In fact, as Figure 3.3.1 shows, if σ_1 and σ_2 become ten times larger than the above parameters, i.e. equal to 10, then ϕ_y^* rises as much as 1.5 from 0.1.

Even though risk aversion is held constant, an increase in equilibrium labour supply due to a decrease in γ has a similar effect on ϕ_y^* . If the level of equilibrium labour supply rises from 0.4 to 0.9, then ϕ_y^* rises from 0.1 to 0.8.

A high degree of adjustment costs expand the region of determinacy. As a limit, if adjustment costs are infinite, capital can be regarded as constant. This is equivalent to a model with only labour as a production input. In such a model, without responding to output, only an active policy rule is sufficient to achieve determinacy. On the other hand, lower adjustment costs increase the possibility of indeterminacy. The effect of adjustment costs on determinacy is shown in Figure 3.3.2. For example, a decrease of a from 3 to 0.3 raises ϕ_y^* approximately to 0.3.

Summing up, the threshold $\phi_y^* = 0.12$ calculated in our standard model cannot be a definitive level. In order to avoid indeterminacy with confidence, central banks may have to react to output much more aggressively than $\phi_y^* = 0.12$. However, this produces the other problems that Orphanides (2004) and McCallum (2001b) pointed out.

¹¹A good survey is that of Kocherlakota (1996).

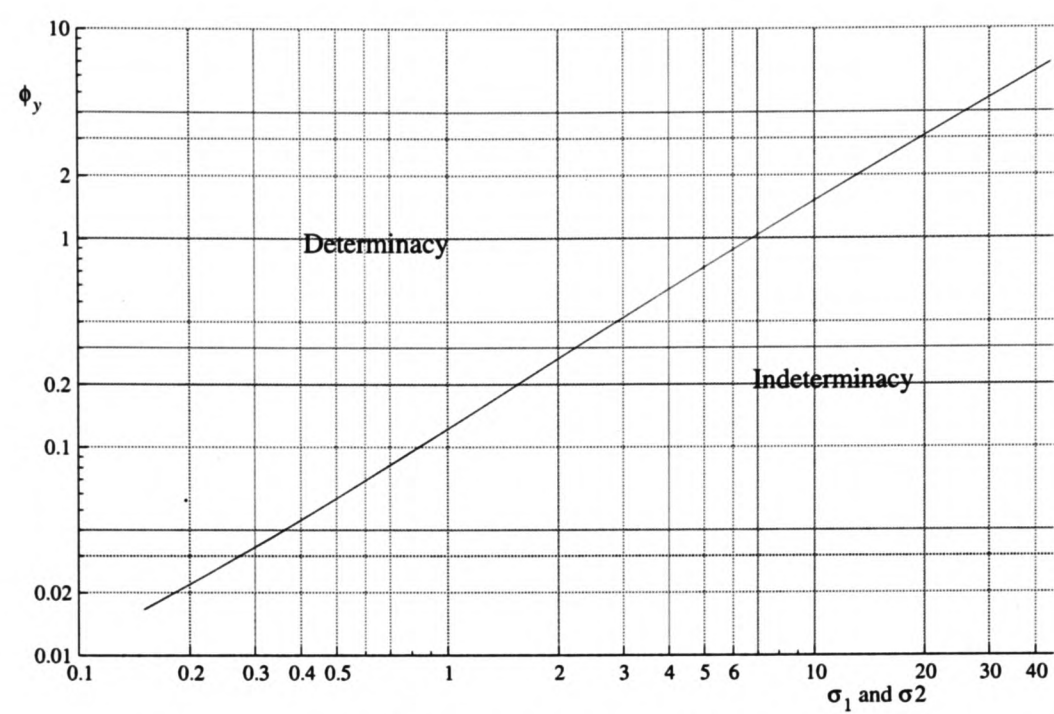


Figure 3.3.1: Change of Households' Parameters

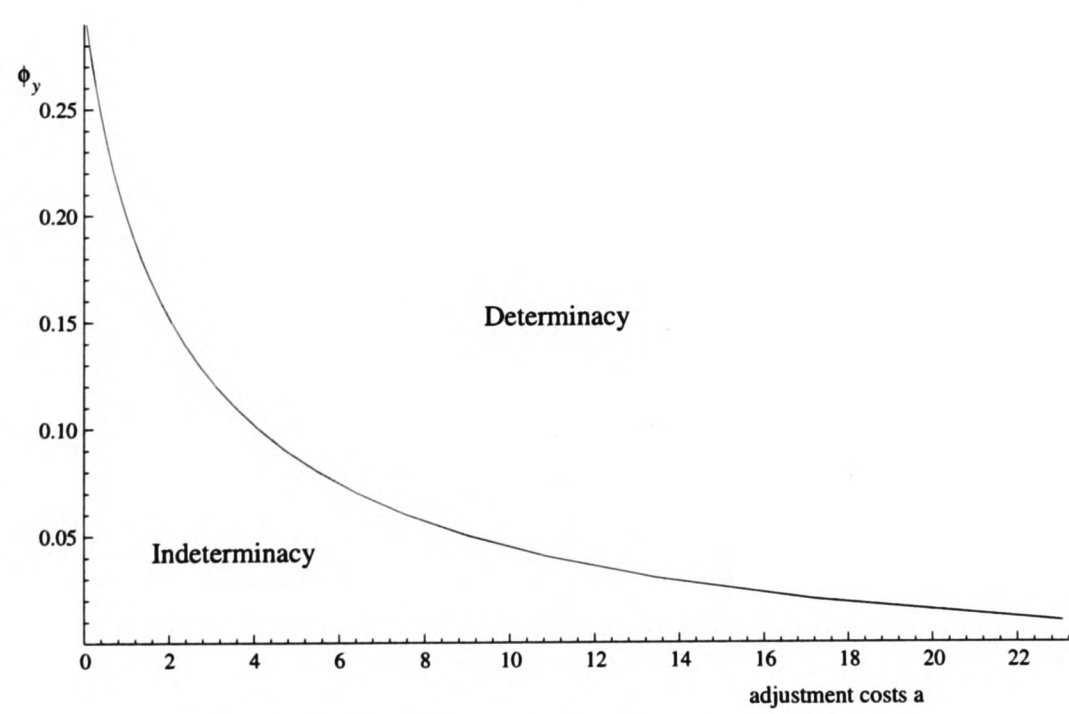


Figure 3.3.2: Change of Adjustment Costs

3.4 Expansion of a Stability Boundary in Richer Models

We examined stability by introducing the Ramsey growth model with sticky prices, the adjustment costs of capital and firm-specific investment. Apart from our continuous-time framework, this model followed the same approach as that of Sveen and Weinke (2005). It became clear that, in order to achieve determinacy, a monetary rule should obey not only inflation but also real economic activity.

However, in several respects, this model is not rich enough for us to be sure if we have grasped robust insights into reality. This section attempts to improve the model by incorporating the following two more realistic features.

Firstly, introducing persistence helps improve our model. Although we incorporated some persistence through sticky prices and the adjustment costs of capital, the real economy has more factors which are persistent. One is the persistence of not only prices but also their changes (inflation). The movement of an inflation rate is known to be highly persistent. High inflation tends to be followed by high inflation in the following period. Low inflation is followed by low inflation. In our previous models, however, an inflation rate is regarded as a jump variable. Another persistence is found in wages. They are clearly very sticky. By incorporating these persistence, our model can become more convincing.

Secondly, heterogeneities are also important. It is true that we have already taken account of two heterogeneities. The first one is a capital heterogeneity. Our model correctly suppose firm-specific investment, so capital is considered to be different across firms. Secondly, the timing of price resetting has been heterogeneous. In the Calvo-type sticky price model, some firms are lucky enough to change their prices, but others are not. Whether a firm can win a lottery or not yields a heterogeneity regarding price resetting. These points have been already considered, but regarding price resetting, there seems to be a more fundamental heterogeneity. In reality, without relying on luck, some firms can systematically adjust much

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

more frequently than others. For example, on the one hand, some firms such as supermarkets reset their prices almost every day. On the other hand, there are firms which face tight regulations, which restricts the flexibility of price changes. Such a difference is not caused by a luck in the Calvo sense. With the definition in the Calvo model, each firm seems to have different price flexibility λ . As another example, a fraction of firms may not be perfectly rational while other firms are perfectly rational. This also yields a heterogeneity regarding price resetting.

Incorporating these persistence and heterogeneities make our model much richer than Sveen and Weinke (2005). This section attempts to construct such models with micro-foundations. However, because of analytical complexity, I will not integrate all these. Instead, this section explores three models separately. Firstly, it incorporates inflation persistence. This also requires us to treat the price resetting heterogeneity. Some firms are perfectly rational, and forward-looking, but other firms reset their prices in the backward-looking way. The second part of this section introduces a similar but different type of heterogeneity. It assumes that a fraction of firms can reset their prices freely at any time. The other firms cannot reset their prices, and its chance obeys a Calvo-type lottery. The last part of this section takes account of sticky wages as well as sticky prices. However, persistent inflation and heterogeneities are not considered.

With these models, we examine how a stability boundary changes, especially, if a monetary rule should really obey real economic activity. As will become clear, all three modifications expand a stability boundary. Central banks no longer need to aggressively respond to output to achieve determinacy.

The reason for this result will be explained later in detail but, regarding the first two modifications, this has already been mentioned. It is forward-lookingness of price setting which causes indeterminacy despite an active policy rule. On the one hand, an investment

boom increases the current marginal costs, which leads to a rise in optimal reset prices. On the other hand, an increase in capital reduces the future marginal costs, which makes forward-looking firms lower their optimal reset prices. Comparing these two effects, firms reset optimal prices. If the predicted optimal prices increase, a real interest rate increases, and the investment boom does not occur. If the prices decrease, the investment boom becomes self-fulfilling. The above second effect arises from the forward-lookingness of firms. Therefore, we can easily conjecture that, as firms become less forward-looking, determinacy becomes more likely to be achieved. Our first two modifications, inflation persistence and the existence of flexible price firms, both decrease the degree of forward-lookingness of price setting, which expands the stability boundary.

3.4.1 Inflation Persistence

As our first refinement, this section introduces inflation persistence. There are two primarily relevant papers on inflation persistence, those of Galí and Gertler (1999) and Christiano, Eichenbaum, and Evans (2001). They examined inflation persistence by introducing a fraction of rule-of-thumb price setters. These firms reset their prices looking at the past price. The difference between Galí and Gertler and Christiano et al. is that a referred past price is equal to an optimal price determined by rational firms in the former paper and to an irrational firm's own price in the latter. In both of the models, adding such irrational firms yields inflation persistence, which makes theoretical models closer to empirical results. However, the interest of these authors did not lie in determinacy. This section aims to examine the effect of backward-looking pricing firms on determinacy.

Rule-of-thumb price setting firms are irrational because they do not optimise their behaviour in a forward-looking way. Furthermore, we assume that these irrational firms think that there exist only rule-of-thumb price setting firms. Of course, this belief is wrong, but this

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

is more consistent with their non-optimal pricing actions than assuming that they correctly perceive the situation.

In addition to such amendment, introduce inflation indexation. Suppose that a firm which cannot revise its price can still change its price smoothly. Such a treatment is very common in sticky price models. Denote the degree of price indexation as ϖ_1 ($0 < \varpi_1 < 1$). For the unlucky firm, its price changes as

$$p(t + dt) = p(t) + \varpi_1 \pi dt. \quad (3.4.1)$$

Clearly, if there is no indexation ($\varpi_1 = 0$), the price of the unlucky firm does not change. Denote the proportion of irrational firms as ϖ_2 ($0 < \varpi_2 < 1$).

Irrational Firms

Firstly, we construct the model of irrational rule-of-thumb price setters. One difficulty is in how to assume the pricing decision of irrational firms. In a continuous-time framework, we cannot simply assume that irrational firms reset prices in a backward-looking way because past prices cannot be disconnected with current prices.

The best we can do, to avoid great complexity, is to proceed as follows. At the very end of this section, I explain in two paragraphs what additional (difficult) things could be done better than this. But I have not done these things.

We assume that irrational firms reset their prices p_I^* obeying the following pricing rule-of-thumb:

$$\dot{\pi}_I^* = \kappa y_I + \dot{\pi} \quad (3.4.2)$$

where $\pi_I^* = \dot{p}_I^*$ and $\kappa > 0$. A subscript I represents irrational firms. A positive response to output, $\kappa > 0$, implies that this inflation rate is a state variable, that is, persistent. For instance, an increase in y_I makes $\dot{\pi}_I^*$ positive. Since π_I^* is sticky, it increases gradually and its level becomes positive. The second term suggests that firms reset their prices referring

**CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC
INVESTMENT, STICKY PRICES, AND INDETERMINACY**

to aggregate inflation. This equation is interpreted as a Phillips curve provided persistent inflation.

Irrational firms decide semi-optimal investment given their pricing rule. It is not optimal because they wrongly assume that all other firms obey this pricing rule-of-thumb. Despite irrationality, investment is determined in a forward-looking way, and long-run equilibrium is coherent with that of rational firms. Rational and irrational firms have the same economic levels in the end after a shock.

Irrational firms' behaviour is described as follows:

$$\sigma_1 \dot{c}_I = \Delta i_I - \pi_I \quad (3.4.3)$$

$$w_I - \bar{p}_I = \sigma_1 c_I + \sigma_2 \frac{L_0}{1 - L_0} l_I \quad (3.4.4)$$

$$\begin{aligned} \dot{x}_I = & \frac{1 + \Psi'(\delta)}{\delta \Psi''(\delta)} (\Delta i - \pi) + \rho x_I \\ & - \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \left\{ w - \bar{p} + \frac{1}{1 + \beta - \alpha} y_I - \frac{1 + \beta}{1 + \beta - \alpha} k_I \right\} \end{aligned} \quad (3.4.5)$$

$$\dot{k}_I = \delta x_I \quad (3.4.6)$$

$$\tilde{\pi}_I^* \equiv \dot{\pi}_I^* - \dot{\pi}^* = \kappa y_I \quad (3.4.7)$$

$$\tilde{p}_I^* \equiv \dot{p}_I^* - \dot{p}^* = \tilde{\pi}_I^* \quad (3.4.8)$$

$$\pi = \frac{\lambda}{1 - \varpi_1} \tilde{p}_I^* \quad (3.4.9)$$

$$y_I = \alpha k_I + (1 + \beta - \alpha) l_I \quad (3.4.10)$$

$$Y_0 y_I = C_0 c_I + \delta(1 + \Psi'(\delta)) K_0 x_I + (\delta + \Psi(\delta)) K_0 k_I. \quad (3.4.11)$$

The first two equations come from households' optimisation. The third and fourth equations represent the law of motion with respect to investment and capital. They are the same as those in the previous standard model. The fifth equation represents rule-of-thumb pricing. From our assumption that irrational firms think that there are only rule-of-thumb price setters, the seventh equation is derived. The derivation of this follows a similar way to that

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

of equations (3.A.1) or (3.A.16) in the Appendix. Similarly, from the assumption, the last two equations, a production function and a market clearing condition, are obtained. These aggregate variables (such as consumption and inflation) are wrongly calculated by irrational firms.

These equations construct the system of five differential equations, namely with respect to y_I , x_I , k_I , $\tilde{p}_I^*$, and $\tilde{p}_I^* = \tilde{\pi}_I^*$. Because of inflation persistence, the last three are state variables.

Rational Firms

From the viewpoint of rational firms, their optimal pricing becomes much more complex to calculate. Rational firms need to take account not only of irrational firms' pricing decision but also of a relative capital level of rational firms compared with irrational firms. Leaving technical deviation in Appendix 3.A.2, I write down differential equations:

$$\dot{\tilde{p}}_R^* = (\rho + \lambda\varpi_2)\tilde{p}_R^* + \delta\tau_1\tau_3\tilde{p}_R - \lambda\varpi_2\tilde{p}_I^* - \frac{\rho + \lambda}{A}s + \frac{\alpha}{1 + \beta - \alpha}\frac{\rho + \lambda}{A}\tilde{k}_R \quad (3.4.12)$$

$$Y_0\dot{\bar{y}} = C_0\bar{c} + \delta(1 + \Psi'(\delta))K_0\bar{x} + (\delta + \Psi(\delta))K_0\bar{k} \quad (3.4.13)$$

$$\dot{y}_I = \bar{y} - \lambda(y_I - \bar{y}) - \frac{1 + \theta}{\theta}\lambda s_I + \frac{1 + \theta}{\theta}(1 - \varpi_1)\pi \quad (3.4.14)$$

$$\dot{k}_R = \delta x_R \quad (3.4.15)$$

$$\begin{aligned} \dot{x}_R = & \frac{1 + \Psi'(\delta)}{\delta\Psi''(\delta)}(\Delta i - \pi) + \rho x_R \\ & - \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta\Psi''(\delta)} \left\{ w - \bar{p} + \frac{1}{1 + \beta - \alpha}y_R - \frac{1 + \beta}{1 + \beta - \alpha}k_R \right\}. \end{aligned} \quad (3.4.16)$$

The subscript R represents the value of rational firms. The first equation is inflation dynamics. Refer Appendix 3.A.2 to the deviation. When no irrational firm exists ($\varpi_2 = 0$), it is reduced to the ordinary inflation dynamics equation (3.2.40). The coefficient of $\tilde{p}_R^*$ increases by $\lambda\varpi_2$. This makes equilibrium more likely to be determinate as we will see later. The second one is the market clearing condition, and each term of the right-hand side can be obtained by the Euler equation or the weighted sum of rational and irrational firms' investment functions.

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

The third equation describes the law of motion with respect to demand for irrational firms' goods. This is derived in Appendix 3.A.2. The fourth and fifth equations are the same as before.

The system of rational firms' equations yields five more differential equations, namely with respect to $\tilde{p}_R^*$, $\bar{y}$, x_R , k_R , and y_I . Note that the variable y_I has a true value, which is different from that calculated by irrational firms, and that y_I cannot move discretely because of price stickiness. Among these variables, there are two state variables: k_R and y_I . To sum up, the number of differential equations becomes ten, of which jump and state variables are each five. Hence, if the number of negative eigenvalues is five, then equilibrium becomes determinate. If it is higher than five, equilibrium becomes indeterminate.

Results

Parameter values for the calibration are the same as before. As well, assume $\kappa = 1$ but, according to our numerical calculation, the amplitude of this value does not affect a determinacy condition as long as it is positive. Numerical results are as follows:

1. When both ϖ_1 and ϖ_2 are zero, that is, when there is neither inflation indexation nor irrational firms, the number of negative eigenvalues is seven, so equilibrium is indeterminate with a degree of two. Needless to say, this is the same result as in Section 3.2.
2. If the fraction of flexible price firms is high (high ϖ_2), equilibrium becomes determinate (Figure 3.4.1). Its threshold ϖ_2^* can be very low. For example, for $\lambda = 1$, $\varpi_1 = 0$ and $\phi_y = 0$, ϖ_2^* is 0.14 or 14%.
3. The more highly a price is indexed to inflation (high ϖ_1), the smaller the region of determinacy becomes (again Figure 3.4.1). However, this effect looks small.

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

As conjectured above, the existence of such irrational firms who reset their prices in a non forward-looking manner helps achieve determinacy. If there are more than 14 percent of these firms, responding to output becomes unnecessary. These results look desirable. Without responding aggressively to real economic activity, equilibrium can easily become determinate. This enables us to avoid indeterminacy and the problems posed by Orphanides (2004) and McCallum (2001b) both at the same time.

In this model with a continuous-time framework, inflation indexation has the same effect as increasing price flexibility λ , as well represented in equation (3.4.9). Thus, inflation indexation shrinks the stability boundary (see Figures 3.2.2 and 3.2.3). The effect of inflation indexation in this continuous-time framework seems to be very different from that in a discrete-time framework. In the latter, by indexing prices to the past inflation rate, an inflation rate comes to have more persistence. This can work not to shrink but to expand the stability boundary. However, in order to check this statement, we need to construct an equivalent model in a discrete-time framework.

*

In trying to make our model perfectly consistent with that of Gali and Gertler (1999) and Christiano, Eichenbaum, and Evans (2001), we would encounter two difficulties. Firstly, as noted above, in a continuous-time framework, it seems difficult for us to make an appropriate form because the past and the current situations are not clearly divided. The analysis may be possible by using an integral over the past periods. But this looks very difficult and I have not pursued this option. Secondly, we have to note that capital investment, combined with adjustment costs, is determined in a forward-looking way. When they optimise their investment, they have to consider the future path of real interest rates, which requires the future path of inflation as well. However, according to the framework by these authors, irrational firms cannot or should not forecast the future prices because they reset prices with

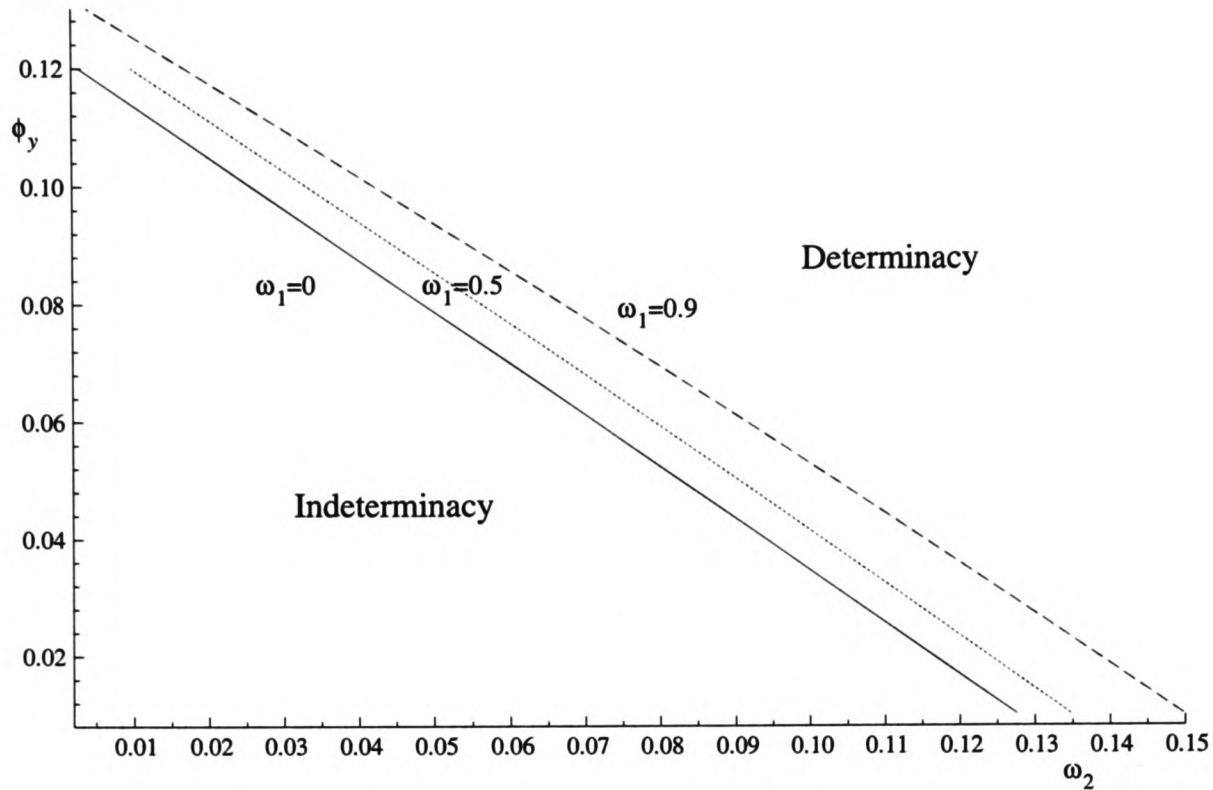


Figure 3.4.1: Effect of Inflation Persistence

a backward-pricing rule. These firms may assume that the future inflation rate is constant at a current level or at an equilibrium level, but the former assumption does not enable their capital to converge to an equilibrium level after a certain shock. We would thus need to assume that irrational firms believe that the future inflation rate is always at an equilibrium level. But it is not certain whether this assumption is valid or not. So even doing such work in more detail would possibly run into trouble.

I have tried to go a little way in this direction. My attempt to make a slightly closer assumption to that of Gali and Gertler (1999) and Christiano, Eichenbaum, and Evans (2001) was as follows. I analyse the stability bound by assuming that irrational firms' reset prices as $P_I^* = \bar{P}$. This can be transformed to $\tilde{p}_I^* = 0$ and $\tilde{\pi}_I^* = 0$. By calculation, I show that this modification has almost no effect on the threshold ϖ_2^* , which remains at 14%.

3.4.2 Two Sectors Model with Flexible and Sticky Price Firms

This section introduces a heterogeneity regarding price stickiness. In particular, it assumes that there are two types of firm. One type faces ordinary price stickiness, and the other can reset their prices freely at any time.

The existence of the latter firms is justified empirically, for example by looking at supermarkets which reset their prices almost every day. This also seems to be supported by the survey conducted by the Bank of Japan (2000). Figure 3.2.1 demonstrated how many times firms changed their prices in the past year. The largest answer lies from 1 to 2 times. This result is consistent with our previous choice of $\lambda = 1$. What is worth noting is that there are about 5 percent of manufacturing companies and 20 percent of non-manufacturing companies which answered that they had changed more than 10 times that year. They seem to reset their prices almost freely. Considering that there are respectively about 15 and 30 percent of non-available (NA) answers, the proportion of such flexible price firms can be even higher.

This survey asked another question as to how important each factor was for price-setting. The result is shown below. True, the second most important factor suggests that firms are forward-looking. However, the first as well as the fourth factor suggests the importance of the current condition. These factors emerge if prices are perfectly flexible. If firms can adjust their prices at any time, they do not need to take account of the optimal prices in the future. They simply reset prices so that they are equal to current marginal costs plus fixed mark-up. In the table, the fourth factor clearly corresponds to this. Moreover, since a current market condition influences the current marginal costs under flexible prices, the first factor is important, too.

For these reasons, this section introduces a heterogeneity regarding price stickiness such that there are a fraction of flexible price firms¹².

¹²Actually, this second survey by the Bank of Japan may simply suggest that some firms wrongly reset prices as if they could reset prices at any time although actual prices are not perfectly flexible. I constructed

Factors Determining Price-Setting

current market condition (demand & supply)	35.6%
market share and future profits	22.2
determined by consumers	18.9
fixed mark-up plus costs	18.4
regulations	-70.9

Notes: Diffusion index (percent points), all industries

Source: the Bank of Japan (2000)

Denote the proportion of flexible price firms as ϖ_2 ($0<\varpi_2<1$). Note that both sectors are perfectly rational. Following the notation in the previous section, subscripts R and I respectively represent the value of sticky and flexible price firms.

This model with flexible price firms as well as sticky price firms can be obtained in a quite similar way to the previous one with inflation persistence. Sticky price firms reset their prices taking account of the actions of flexible price firms as Appendix 3.A.2 described.

Both of the firms are perfectly rational. Therefore, although flexible price firms do not need to be forward-looking when deciding their prices, when deciding their investment, they have to consider the future paths of all relevant variables including sticky price firms' prices and capital. The model can be constructed in the following way.

Flexible Price Firms

Flexible price firms reset prices as

$$p_I^\# - \bar{p} = w - \bar{p} - (y_I - l_I). \tag{3.4.17}$$

Since firms can adjust prices at any time, they choose their prices $p^\#$ to maximise their current profits. Therefore, the ratio of a reset price to an aggregate price is equal to the ratio of real

such a model and investigated the stability for this, and my results are almost the same as those which will soon appear.

wages to the marginal product of labour multiplied by constant mark-up. Furthermore, the law of motion with respect to capital and investment becomes

$$\dot{k}_I = \delta x_I \quad (3.4.18)$$

$$\dot{x}_I = \frac{1 + \Psi'(\delta)}{\delta \Psi''(\delta)} (\Delta i - \pi) + \rho x_I - \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \{y_I - k_I\}. \quad (3.4.19)$$

Sticky Price Firms

Sticky price firms take account of the actions of flexible price firms in a very similar way to that of rational firms to irrational firms as in the previous section. From Appendix 3.A.2, we obtain

$$\widetilde{p}_R^* = (\rho + \lambda \varpi_2) \widetilde{p}_R^* + \delta \tau_1 \tau_3 \widetilde{p}_R - \lambda \varpi_2 s_I - \frac{\rho + \lambda}{A} s + \frac{\alpha}{1 + \beta - \alpha} \frac{\rho + \lambda}{A} \widetilde{k}_R \quad (3.4.20)$$

$$\dot{k}_R = \delta x_R \quad (3.4.21)$$

$$\begin{aligned} \dot{x}_R = & \frac{1 + \Psi'(\delta)}{\delta \Psi''(\delta)} (\Delta i - \pi) + \rho x_R \\ & - \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \left\{ w - \bar{p} + \frac{1}{1 + \beta - \alpha} y_R - \frac{1 + \beta}{1 + \beta - \alpha} k_R \right\}. \end{aligned} \quad (3.4.22)$$

In addition, we need

$$Y_0 \bar{y} = C_0 \bar{c} + \delta(1 + \Psi'(\delta)) K_0 \bar{x} + (\delta + \Psi(\delta)) K_0 \bar{k} \quad (3.4.23)$$

$$\dot{y}_I = \bar{y} - \lambda(y_I - \bar{y}) - \frac{1 + \theta}{\theta} \lambda \widetilde{s}_I + \frac{1 + \theta}{\theta} (1 - \varpi_1) \pi. \quad (3.4.24)$$

Summing up the equations of both flexible and sticky price firms, we can construct the system of seven differential equations with respect to k_I , x_I , $\widetilde{p}_R^*$, k_R , x_R , $\bar{y}$, and y_I . While aggregate output $\bar{y}$ can jump because consumption is a jump variable, sticky price firms' output y_R cannot move discretely because of price stickiness. Thus, the residual y_I of $\bar{y}$ from y_R is regarded as a state variable, too. Hence, k_R , k_I , and y_I are state variables. Looking at the sign of eigenvalues, if there are three negative and four positive signs, then equilibrium becomes determinate. Instead, if there are more than three negative signs, equilibrium

becomes indeterminate. Otherwise, it is unstable.

Results

Our numerical results are very similar to those which were obtained in the previous section.

1. When both ϖ_1 and ϖ_2 are zero, that is, when there is neither inflation indexation nor flexible price firms, the number of negative eigenvalues is five, so equilibrium is indeterminate with a degree of two. Needless to say, this is the same result as in Section 3.2.
2. Suppose $\lambda = 1$. If the fraction of flexible price firms is high (high ϖ_2), equilibrium becomes determinate (Figure 3.4.2). Its threshold ϖ_2^* is 0.23 or 23% for $\varpi_1 = 0$ and $\phi_y = 0$. A stability bound expands.
3. However, as prices become stickier in sticky price firms (i.e. λ becomes lower), a stability bound shrinks (Figure 3.4.3).
4. The more highly a price is indexed to inflation (high ϖ_1), the larger the region of determinacy becomes (again Figures 3.4.2 and 3.4.3).

Regarding the final point, the effect of inflation indexation is opposite to that in the previous inflation persistence model. It reduces ϕ_y^* , and this effect is larger than that in the previous model. The reason for this is not clear, but I suspect that inflation indexation lowers the necessity for sticky price firms to consider the future, which expands the stability boundary.

It is shown that this heterogeneity expands a stability boundary. In other words, as the proportion of flexible price firms increases, forward-lookingness of sticky price firms' price resetting comes to have less destabilising effect on the whole economy, which leads to

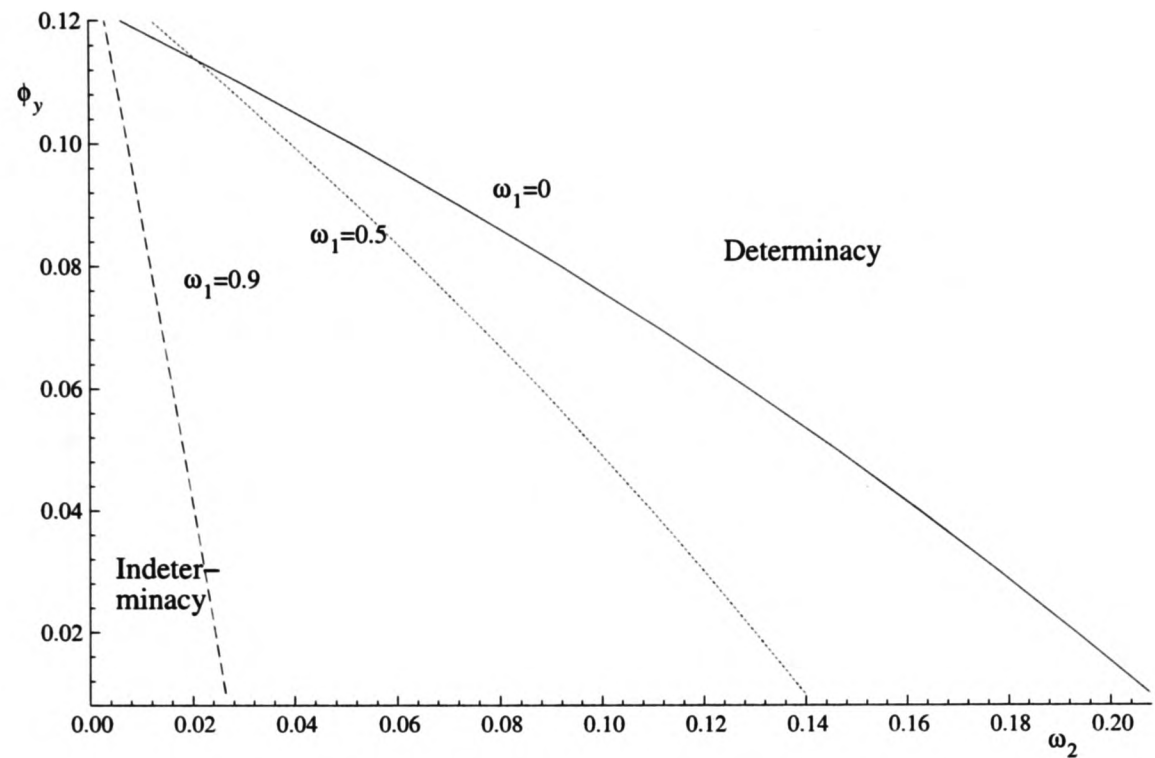


Figure 3.4.2: Effect of Flexible Price Firms

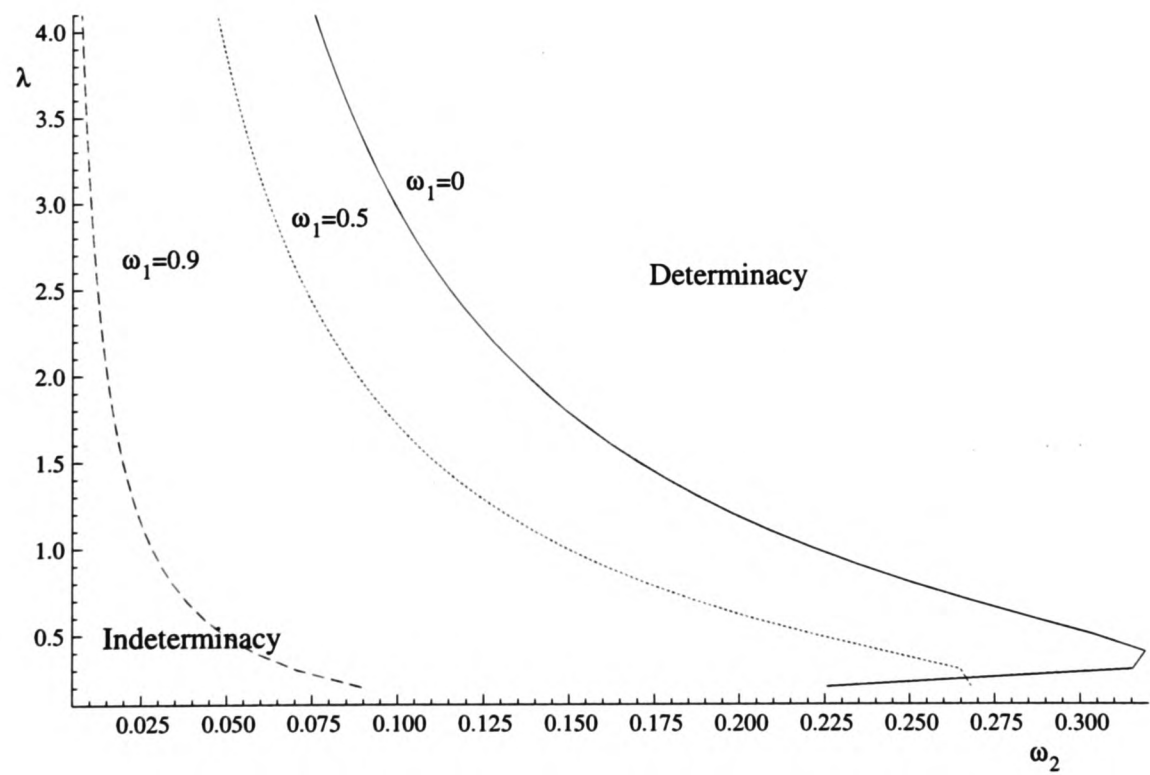


Figure 3.4.3: Effect of Flexible Price Firms

determinacy even without aggressively responding to real economic activity. If there exists more than 23 percent of flexible price firms, responding to real economic activity becomes completely unnecessary.

Actually, I discussed the final conclusion taking the same price flexibility $\lambda = 1$ as before for sticky price firms. It may seem a little unfair to introduce flexible price firms keeping this value unchanged, and it might be better to increase price stickiness so that the overall price flexibility may stay the same as before. As Figure 3.4.3 demonstrates, this has an effect to increase forward-lookingness of price resetting, and to destabilise the economy. If we decrease λ from 1 to 0.5 (i.e. price stickiness increases), then the threshold ϖ_2^* increases from 0.23 to 0.30. More flexible price firms are needed for determinacy.

However, in order to thoroughly resolve the above concern, we need to know actual firms' distribution with respect to price stickiness. Furthermore, in order to incorporate this distribution, we need to modify our two sector model to a multiple (or infinite) sector model with different λ 's. This problem will become extremely difficult, and I will not tackle this in this chapter.

3.4.3 Sticky Wages

As the final refinement, this section introduces wage stickiness as well as price stickiness. As will be shown below, the introduction of wage stickiness also lowers the critical value ϕ_y^* .

The reason can be understood in the following way. Again assume an investment boom. Aggregate demand increases, and the level of employment increases. This raises wages, which enhances the relative profitability of investment to employment, and leads to the realisation of the investment boom. As wages become stickier, the rise in wages is dampened. Thus, wage stickiness helps achieve determinacy.

Following Erceg, Henderson and Levin (2000), this section includes wage stickiness and

modifies to continuous-time framework. It is a simple analogy of the sticky price model. Denote the wage flexibility as λ_w . A value $\lambda_w = 1$ implies that a wage is revised roughly every year.

Assume a number of monopolistically competitive households which provide their labour to firms. The aggregate labour index $\bar{L}$ and subsequently the wage index $\bar{W}$ are given by

$$\bar{L} = m_w^{-\theta_w} \left(\sum_j m_w L_j^{\frac{1}{1+\theta_w}} \right)^{1+\theta_w} \quad (3.4.25)$$

$$\bar{W} = m_w^{\theta_w} \left(\sum_j m_w W_j^{-1/\theta_w} \right)^{-\theta_w}, \quad (3.4.26)$$

where $\theta_w (> 0)$ represents mark-up. From this formalisation, a labour demand function is calculated as

$$\frac{L_j}{\bar{L}} = \left(\frac{W_j}{\bar{W}} \right)^{-\frac{1+\theta_w}{\theta_w}} \quad (3.4.27)$$

A household j maximises

$$\max \int_t^\infty u(\bar{C}, L_j) e^{-\rho(s-t)} ds \quad (3.4.28)$$

subject to

$$\int_t^\infty \bar{P} \bar{C} e^{-R(s-t)} ds \leq A(t) + \int_t^\infty (W_j L_j + \Pi_j) e^{-R(s-t)} ds. \quad (3.4.29)$$

Firstly, consider the case of flexible wages. The optimal wage is obtained as

$$\frac{W}{\bar{P}} = -(1 + \theta_w) \frac{u_L}{u_C}. \quad (3.4.30)$$

Apart from a mark-up term, this equation simply means that real wages are equal to a marginal rate of substitution of consumption for leisure (MRS)¹³. When wages are sticky, such wages are no longer optimal. Similar to equation (3.2.36) as a firms' optimal price equation, the optimal wage is given by

$$0 = \int_t^\infty e^{-(\rho+\lambda_w)(s-t)} \{w_j^*(t) - \bar{p}(s) - mrs_j(s)\} ds \quad (3.4.31)$$

¹³Erceg et al. (2000) assumes a government subsidy to wages in order to equalise real wages with MRS . However, I think this assumption causes a problem. Households lose the incentive to supply as much labour as demand if the real wage becomes lower than MRS , which causes goods to become in a short supply.

after log-linearisation. The deviation of MRS_j , mrs_j , is transformed to

$$\begin{aligned} mrs_j &= \sigma_1 \bar{c} + \sigma_2 \frac{L_0}{1 - L_0} l_j \\ &= mrs + \sigma_2 \frac{L_0}{1 - L_0} (l_j - \bar{l}) \\ &= mrs - \frac{1 + \theta_w}{\theta_w} \sigma_2 \frac{L_0}{1 - L_0} (w_j - \bar{w}). \end{aligned} \quad (3.4.32)$$

The last equation is derived from a labour demand function: equation (3.4.27). Differentiating equation (3.4.31) with respect to t yields

$$\begin{aligned} 0 &= \frac{1}{\rho + \lambda_w} \left(1 + \frac{1 + \theta_w}{\theta_w} \sigma_2 \frac{L_0}{1 - L_0} \right) \dot{w}_j^* \\ &\quad - \left(1 + \frac{1 + \theta_w}{\theta_w} \sigma_2 \frac{L_0}{1 - L_0} \right) (w_j^* - \bar{w}) - (\bar{w} - \bar{p} - mrs). \end{aligned} \quad (3.4.33)$$

Using

$$\bar{w} \equiv \pi_w = \lambda_w (w_j^* - \bar{w}) \quad (3.4.34)$$

which is similar to equation (3.A.1), the above equation can be rearranged as

$$\dot{\pi}_w = \rho \pi_w + \frac{\lambda_w (\rho + \lambda_w)}{1 + \frac{1 + \theta_w}{\theta_w} \sigma_2 \frac{L_0}{1 - L_0}} (\bar{w} - \bar{p} - mrs). \quad (3.4.35)$$

This is wage inflation dynamics. Another differential equation is given by

$$\dot{\bar{w}} - \dot{\bar{p}} = \pi_w - \pi. \quad (3.4.36)$$

Combining other differential equations (3.2.34), (3.2.35), (3.2.40), and (3.2.43):

$$\begin{aligned} \dot{\pi} &= \rho \pi - \frac{\lambda(\rho + \lambda)}{A} s \\ \dot{\bar{k}} &= \delta \bar{x} \\ \dot{\bar{x}} &= \frac{1 + \Psi'(\delta)}{\delta \Psi''(\delta)} (\Delta i - \pi) + \rho \bar{x} \\ &\quad - \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \left\{ w - \bar{p} + \frac{1}{1 + \beta - \alpha} \bar{y} - \frac{1 + \beta}{1 + \beta - \alpha} \bar{k} \right\} \\ Y_0 \dot{\bar{y}} &= C_0 \bar{c} + \delta(1 + \Psi'(\delta)) K_0 \bar{x} + (\delta + \Psi(\delta)) K_0 \bar{k}, \end{aligned}$$

we can construct the system of six differential equations with respect to $(\pi, \bar{y}, \bar{x}, \bar{k}, \pi_w, \bar{w} - \bar{p})$.

In the last equation, consumption is derived from the Euler equation (3.2.28). Among them,

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

four variables except for $\bar{k}$ and $\bar{w} - \bar{p}$ are jump variables. Therefore, if the number of positive eigenvalues is four, an equilibrium becomes determinate.

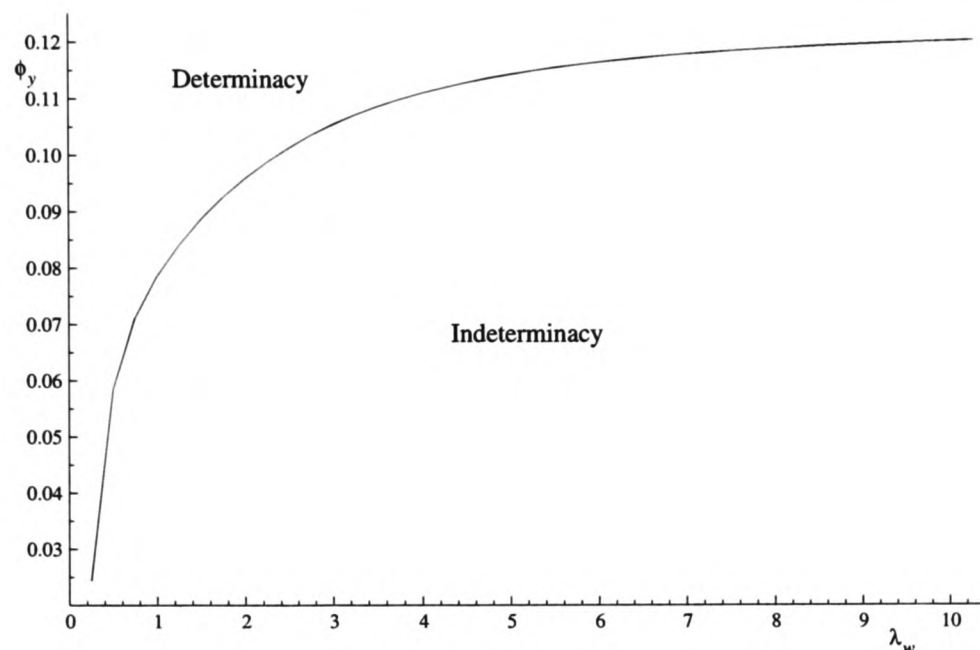


Figure 3.4.4: Effect of Sticky Wages

For numerical calculation, I use the parameter value of $\theta_w = \theta$ as mark-up. Figure 3.4.4 shows the relationship between wage flexibility λ_w and the critical value of real economic activity response ϕ_y^* . When λ_w is sufficiently large, the critical value converges to $\phi_y^* = 0.12$, which corresponds to that under perfect wage flexibility in Section 3.2. As wages become sticky, the critical value decreases. For example, if wage stickiness is the same as price stickiness, that is $\lambda_w = \lambda = 1$, then the critical value becomes 0.08. A decrease in ϕ_y^* makes it easier to achieve determinacy.

The reason has already been explained at the beginning of this section. Suppose the expectation of an investment boom. Aggregate demand increases, and so does the level of employment. This raises wages, and the amplitude of this increase becomes larger as wages become more flexible. Since an increase in wages encourages investment in order to substitute a production input from labour to capital, wage flexibility functions to increase investment. This leads to the self-fulfilling expectation of the investment boom. In contrast, as wages

become stickier, the investment boom comes less likely to be rationalised. This expands the stability boundary.

3.5 Concluding Remarks

In this chapter, we have aimed to see whether Japan's stagnation was provoked by the misconduct of the Bank of Japan. This has required us to theoretically investigate what kind of interest rate rules make equilibrium determinate. We have done this in a much more satisfactory model than that used by the initial protagonists of the Taylor principle. We have thus carried out our work using a model with adjustment costs of investment, firm-specific investment and Calvo-type sticky prices. Sveen and Weinke (2005) had already demonstrated that, whilst passive rules make equilibrium indeterminate, active rules can make it either indeterminate or determinate. But, they said that, by responding to real economic activity, equilibrium can become determinate. They therefore recommended that a response to output (possibly a strong one) be included in the required rules in order to make monetary policy be good policy. This is a strong claim. However, their model is not rich enough for us to grasp robust insights into reality.

This chapter aims to improve the Sveen and Weinke model by introducing some persistence and heterogeneities into the model. In particular, we incorporate inflation persistence, the existence of some flexible price firms, and sticky wages. All these modifications expand a stability boundary, which makes the necessity of responding to real economic activity much less. Indeed, we end up defending something very like the Taylor principle. This helps to avoid the problem regarding output gap measurement for which we were alerted by McCallum (2001b) and Orphanides (2004).

To summarise, in reality we do not need to be too much concerned about the validity

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

of the Taylor principle. The Bank of Japan did indeed follow that principle (Clarida, Gali and Gertler: 1998, Jinushi, Kuroki and Miyao: 2000). This suggests that, at least from the perspective of determinacy, the Bank of Japan had not worsened the Japanese downturn.

Of course, since 1999, Japan's monetary policy has become clearly passive because of the incoming of a zero nominal interest rate. That appears to have caused indeterminacy. We would think that, under the circumstances, the expectation of deflation might become self-fulfilling. In fact, actual prices (CPI) have been decreasing in Japan since then. Furthermore, apart from indeterminacy, such a zero bound may have worsened Japan's depression in several respects. These important questions are all taken up in the next chapter.

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**CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC
INVESTMENT, STICKY PRICES, AND INDETERMINACY**

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INVESTMENT, STICKY PRICES, AND INDETERMINACY

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3.A Appendix

3.A.1 Inflation Dynamics

This Appendix aims to derive inflation dynamics. Modifying Woodford (2003, 2004, 2005), we analyse this in a continuous-time framework.

In a sufficiently short interval of dt , a proportion of λdt firms can revise their prices. Log-linearising the form of aggregate price index

$$\bar{P} = m^\theta \left(\sum_j^m P_j^{-1/\theta} \right)^{-\theta},$$

we can obtain

$$\begin{aligned} \overline{p(t+dt)} &= (1 - \lambda dt) \cdot \overline{p(t)} + \lambda dt \cdot p^*(t) \\ \overline{p(t)} + \bar{p}dt &= (1 - \lambda dt) \cdot \overline{p(t)} + \lambda dt \cdot p^*(t) \\ \therefore \pi &= \lambda \tilde{p}^*, \end{aligned} \tag{3.A.1}$$

where p^* is the aggregated optimal reset price.

In a similar way to Woodford, regarding optimal prices and investment, conjecture the following form:

$$p_j^* = p^* - \tau_1 \tilde{k}_j \tag{3.A.2}$$

$$\tilde{x}_j = -\tau_2 \tilde{k}_j - \tau_3 \tilde{p}_j. \tag{3.A.3}$$

Rewrite equation (3.2.36):

$$0 = \int_t^\infty e^{-(\rho+\lambda)(s-t)} \{\tilde{p}_j^*(s) - s_j(s)\} ds. \quad (3.A.4)$$

From equation (3.2.24), real marginal costs are log-linearised as

$$\begin{aligned} s_j &= w - \bar{p} - (y_j - l_j) \\ &= s - (\tilde{y}_j - \tilde{l}_j) \\ &= s - \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} \tilde{p}_j^* - \frac{\alpha}{1 + \beta - \alpha} \tilde{k}_j, \end{aligned}$$

because of the demand function that monopolistic firms face:

$$\tilde{y}_j = -\frac{1 + \theta}{\theta} \tilde{p}_j.$$

Noting that $\tilde{p}_j(s) \equiv p_j(t) - \bar{p}(s)$ depends on t , we differentiate equation (3.A.4) with respect to t :

$$\begin{aligned} 0 &= - \left(\tilde{p}_j^*(t) - s + \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} \tilde{p}_j^*(t) + \frac{\alpha}{1 + \beta - \alpha} \tilde{k}_j(t) \right) \\ &\quad + \int_t^\infty e^{-(\rho+\lambda)(s-t)} \left\{ \frac{d\tilde{p}_j^*(s)}{dt} \left(1 + \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} \right) + \frac{\alpha}{1 + \beta - \alpha} \frac{d\tilde{k}_j(s)}{dt} \right\} ds. \end{aligned} \quad (3.A.5)$$

Here

$$\begin{aligned} \frac{d\tilde{p}_j(s)}{dt} &= \frac{d}{dt} (p_j(t) - \bar{p}(s)) = \frac{d}{dt} p_j(t) = \frac{d}{dt} (p_j(t) - \bar{p}(t) + \bar{p}(t)) \\ &= \dot{p}_j(t) + \pi(t). \end{aligned}$$

$d\tilde{k}_j(s)/dt$ is derived as follows. From the law of motion with respect to capital and the conjectured form of investment $\tilde{x}_j = -\tau_2 \tilde{k}_j - \tau_3 \tilde{p}_j$,

$$\dot{\tilde{k}}_j(s) = \delta \tilde{x}_j(s) = -\delta \tau_2 \tilde{k}_j(s) - \delta \tau_3 \tilde{p}_j(s),$$

which can be solved as

$$\tilde{k}_j(s) = \tilde{k}_j(t) e^{-\delta \tau_2 (s-t)} - \int_t^s ds' e^{-\delta \tau_2 (s-s')} \delta \tau_3 \tilde{p}_j(s').$$

Therefore, for $s > t$,

$$\begin{aligned}\frac{d\tilde{k}_j(s)}{dt} &= - \int_t^s ds' e^{-\delta\tau_2(s-s')} \delta\tau_3 \frac{d\tilde{p}_j(s')}{dt} \\ &= - \frac{\delta\tau_3}{\delta\tau_2} \left(\tilde{p}_j(t) + \pi(t) \right) \left(1 - e^{-\delta\tau_2(s-t)} \right).\end{aligned}$$

Substituting them into equation (3.A.5) yields

$$\begin{aligned}0 &= - \left(\tilde{p}_j^*(t) \left(1 + \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} \right) - s + \frac{\alpha}{1 + \beta - \alpha} \tilde{k}_j(t) \right) \\ &\quad + \left(1 + \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} \right) \frac{\tilde{p}_j^*(t) + \pi(t)}{\rho + \lambda} - \frac{\alpha\delta\tau_3}{1 + \beta - \alpha} \frac{\tilde{p}_j^*(t) + \pi(t)}{(\rho + \lambda)(\rho + \lambda + \delta\tau_2)}.\end{aligned}$$

This can be transformed to

$$\tilde{p}_j^* = -\pi + B(\rho + \lambda)/A\tilde{p}_j^* - (\rho + \lambda)/As + \frac{\alpha}{1 + \beta - \alpha}(\rho + \lambda)/A\tilde{k}_j,$$

where

$$B = 1 + \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} \tag{3.A.6}$$

$$A = B - \frac{\alpha}{1 + \beta - \alpha} \frac{\delta\tau_3}{\rho + \lambda + \delta\tau_2}. \tag{3.A.7}$$

Using the conjectured forms, the left-hand side can be rewritten as

$$\begin{aligned}\tilde{p}_j^* &= \tilde{p}^* - \tau_1 \tilde{k}_j = \tilde{p}^* - \tau_1 \delta \tilde{x}_j \\ &= \tilde{p}^* - \tau_1 \delta (-\tau_2 \tilde{k}_j - \tau_3 \tilde{p}_j^*) \\ &= \tilde{p}^* - \tau_1 \delta \{ -\tau_2 \tilde{k}_j - \tau_3 (\tilde{p}^* - \tau_1 \tilde{k}_j) \}.\end{aligned}$$

Similarly, the right-hand side can be rewritten, and we can obtain the form of

$$\tilde{p}^* = -\pi + (B(\rho + \lambda)/A - \delta\tau_1\tau_3)\tilde{p}^* - (\rho + \lambda)/As + C\tilde{k}_j. \tag{3.A.8}$$

Since this is the equation of averaged values, the final term should vanish. Writing this explicitly:

$$\begin{aligned}0 &= C \\ &= -\tau_1 \delta (\tau_2 - \tau_1 \tau_3) - \tau_1 B(\rho + \lambda)/A + \frac{\alpha}{1 + \beta - \alpha} (\rho + \lambda)/A.\end{aligned} \tag{3.A.9}$$

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

Going back to inflation dynamics, using equation (3.A.1), equation (3.A.8) becomes

$$\dot{\pi} = (B(\rho + \lambda)/A - \delta\tau_1\tau_3 - \lambda)\pi - \lambda(\rho + \lambda)/As.$$

The coefficient of π looks complex but, using the condition of τ'_j s (equation (3.A.9)), we can verify this is equal to ρ . Thus we obtain

$$\dot{\pi} = \rho\pi - \frac{\lambda(\rho + \lambda)}{A}s. \quad (3.A.10)$$

This is inflation dynamics.

Parameter τ'_j s

τ'_j s are not still determined, however. Their conditions are derived from the law of motion with respect to investment. Equation (3.2.32) implies

$$\begin{aligned} \tilde{x}_j &= \rho\tilde{x}_j - \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta\Psi''(\delta)} \left\{ \frac{1}{1 + \beta - \alpha}\tilde{y}_j - \frac{1 + \beta}{1 + \beta - \alpha}\tilde{k}_j \right\} \\ &= \rho\tilde{x}_j + \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta\Psi''(\delta)} \left\{ \frac{1}{1 + \beta - \alpha}\frac{1 + \theta}{\theta}\tilde{p}_j + \frac{1 + \beta}{1 + \beta - \alpha}\tilde{k}_j \right\}. \end{aligned}$$

Substituting equations (3.A.2) and (3.A.3) yields the relationship with respect to $\tilde{p}_j$ and $\tilde{k}_j$.

Note that $\tilde{p}_j$ is given by

$$\begin{aligned} \tilde{p}_j(t + dt) &= (1 - \lambda dt)(\tilde{p}_j(t) - \pi dt) + \lambda dt \tilde{p}_j^*(t) \\ \tilde{p}_j &= -\lambda\tilde{p}_j - \pi + \lambda\tilde{p}_j^* \\ &= -\lambda\tilde{p}_j + \lambda(\tilde{p}_j^* - \tilde{p}^*) \\ &= -\lambda\tilde{p}_j - \lambda\tau_1\tilde{k}_j, \end{aligned} \quad (3.A.11)$$

and the comparison of the coefficients of $\tilde{p}_j$ and $\tilde{k}_j$ yields

$$\tilde{p}_j : \delta\tau_2\tau_3 + \lambda\tau_3 = -\rho\tau_3 + \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta\Psi''(\delta)} \frac{1}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} \quad (3.A.12)$$

$$\tilde{k}_j : \delta\tau_2^2 + \lambda\tau_1\tau_3 = -\rho\tau_2 + \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta\Psi''(\delta)} \frac{1 + \beta}{1 + \beta - \alpha}. \quad (3.A.13)$$

We can numerically solve τ_1 , τ_2 , and τ_3 from three equations (3.A.9), (3.A.12) and (3.A.13).

Moreover, for stability, τ'_j s need to satisfy the following condition. Consider the dynamics of

$\tilde{p}_j$ and $\tilde{k}_j$. They are described as

$$\begin{aligned}\dot{\tilde{p}}_j &= -\lambda\tilde{p}_j - \lambda\tau_1\tilde{k}_j \\ \dot{\tilde{k}}_j &= \delta\tilde{x}_j = -\delta\tau_2\tilde{k}_j - \delta\tau_3\tilde{p}_j.\end{aligned}$$

The corresponding characteristic equation becomes

$$t^2 + (\lambda + \delta\tau_2)t + \lambda\delta(\tau_2 - \tau_1\tau_3) = 0.$$

Since $\tilde{p}_j$ and $\tilde{k}_j$ are not jump variables, both of the roots have to be negative. Hence, the following condition is imposed.

$$\lambda + \delta\tau_2 > 0 \tag{3.A.14}$$

$$\lambda\delta(\tau_2 - \tau_1\tau_3) > 0. \tag{3.A.15}$$

3.A.2 Optimal Reset Prices by Rational Firms When Irrational Firms Exist

We amend the inflation equation which was derived in the previous appendix by adding two more realistic factors. Firstly, we incorporate inflation indexation. Assume that a firm j which cannot revise one's price can still change the price in proportion to aggregate inflation. Let ϖ_1 ($0 < \varpi_1 < 1$) be the degree of adjustment. Secondly, assume that a proportion of ϖ_2 firms ($0 < \varpi_2 < 1$) is irrational.

Aggregate Price Similar to equation (3.A.1), an aggregate price $\bar{p}$ and its rate of change π are given by

$$\begin{aligned}\overline{p(t+dt)} &= (1 - \lambda dt)\overline{p(t)} + \varpi_1\pi dt + \lambda dt\{\varpi_2 p_I^*(t) + (1 - \varpi_2)p_R^*(t)\} \\ \therefore \pi &= \frac{\lambda}{1 - \varpi_1}(\varpi_2 \tilde{p}_I^* + (1 - \varpi_2)\tilde{p}_R^*).\end{aligned} \tag{3.A.16}$$

The subscript I and R represents the value of irrational and rational firms respectively. Unless there is a subscript of j , for example p_R^* not p_{Rj}^* , the variable represents the average value

CHAPTER 3. REEXAMINATION OF THE TAYLOR PRINCIPLE: FIRM-SPECIFIC INVESTMENT, STICKY PRICES, AND INDETERMINACY

among irrational or rational firms. Denote a reset price of irrational and rational firms as p_I^* and p_R^* respectively. A tilda ($\sim$) represents a relative value to its average. Because of inflation indexation, the aggregate inflation rate is multiplied by $1/(1 - \varpi_1)$.

Real Side Regarding a real side, the law of motion can be written as:

$$\dot{k}_R = \delta x_R \quad (3.A.17)$$

$$\begin{aligned} \dot{x}_R = & \frac{1 + \Psi'(\delta)}{\delta \Psi''(\delta)} (\Delta i - \pi) + \rho x_R \\ & - \frac{\rho(1 + \Psi'(\delta)) + \delta + \Psi(\delta)}{\delta \Psi''(\delta)} \left\{ w - \bar{p} + \frac{1}{1 + \beta - \alpha} y_R - \frac{1 + \beta}{1 + \beta - \alpha} k_R \right\}. \end{aligned} \quad (3.A.18)$$

In aggregation, we can obtain

$$\bar{u} = \varpi_2 u_I + (1 - \varpi_2) u_R, \quad (3.A.19)$$

where $u = y, k$ and x , supposing $U_0 = U_{0I} = U_{0R}$ in equilibrium for $U_0 = Y_0, K_0$ and X_0 .

The proof of this is straightforward even with respect to x :

$$\begin{aligned} \bar{X} &= \frac{\bar{I}}{\bar{K}} = \frac{\varpi_2 X_I K_I + (1 - \varpi_2) X_R K_R}{\varpi_2 K_I + (1 - \varpi_2) K_R} \\ \bar{X}_0(1 + \bar{x}) &= \frac{\bar{I}_0}{\bar{K}_0} \frac{\varpi_2(1 + k_I + x_I) + (1 - \varpi_2)(1 + k_R + x_R)}{\varpi_2(1 + k_I) + (1 - \varpi_2)(1 + k_R)} \\ &= \bar{X}_0(1 + \varpi_2 x_I + (1 - \varpi_2) x_R) \\ \therefore \bar{x} &= \varpi_2 x_I + (1 - \varpi_2) x_R. \end{aligned}$$

Moreover, from a downward sloping demand curve, we obtain dynamics with respect to y_I .

$$\begin{aligned} y_I - \bar{y} &= -\frac{1 + \theta}{\theta} (p_I - \bar{p}) \\ \dot{y}_I &= \bar{\dot{y}} - \frac{1 + \theta}{\theta} (\dot{p}_I - \pi) \\ &= \bar{\dot{y}} - \frac{1 + \theta}{\theta} (\lambda(p_I^* - p_I) + \varpi_1 \pi - \pi) \\ &= \bar{\dot{y}} - \lambda(y_I - \bar{y}) - \frac{1 + \theta}{\theta} \lambda \tilde{p}_I^* + \frac{1 + \theta}{\theta} (1 - \varpi_1) \pi, \end{aligned} \quad (3.A.20)$$

where $\dot{p}_I$ can be derived in a similar way to that to derive equation (3.A.16). The behaviour of households and the market clearing condition do not change.

Optimal Prices The pricing decision problem by rational firms becomes more complex than before. Capital, as a state variable, affects firms' profits, so optimal prices depend on the relative value of capital. What makes it complex is that the relative value is measured not only among rational firms but also between rational and irrational firms. Rational firms need to take account of the dynamics of capital both in rational and irrational firms. Modifying Woodford (2005), assume the following forms:

$$\widetilde{p}_j^* = \widetilde{p}_R^* - \tau_1 \widetilde{k}_{Rj} \quad (3.A.21)$$

$$\widetilde{x}_{Rj} = -\tau_2 \widetilde{k}_{Rj} - \tau_3 \widetilde{p}_{Rj}, \quad (3.A.22)$$

where an asterisk (*) represents an optimal price and a tilde (~) with a subscript Rj represents a firm j 's relative value to rational firms' average, that is,

$$\widetilde{u}_{Rj} \equiv u_j - u_R = (u_j - \bar{u}) - (u_R - \bar{u}) \quad (3.A.23)$$

$$= \widetilde{u}_j - \widetilde{u}_R, \quad (3.A.24)$$

for $u = p, y, k$ and x . Similar to Appendix 3.A.1, equation (3.2.36) is transformed to

$$\begin{aligned} 0 &= \int_t^\infty e^{-(\rho+\lambda)(s-t)} \{ \widetilde{p}_j^*(s) - s_j(s) \} ds \\ &= \int_t^\infty e^{-(\rho+\lambda)(s-t)} \left\{ \widetilde{p}_j^*(s) - s + \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} \widetilde{p}_j^*(s) + \frac{\alpha}{1 + \beta - \alpha} \widetilde{k}_j(s) \right\} ds, \end{aligned}$$

The relative price changes over time as

$$\widetilde{p}_j(s) \equiv p_j(t) + \int_t^s \varpi_1 \pi(s') ds' - \bar{p}(s). \quad (3.A.25)$$

Only this variable depends on t directly. Differentiate it with respect to t , and using

$$\begin{aligned} \frac{d\widetilde{p}_j(s)}{dt} &= \frac{d\widetilde{p}_{Rj}(s)}{dt} = \widetilde{p}_j(t) + (1 - \varpi_1)\pi(t) \\ \frac{d\widetilde{k}_j(s)}{dt} &= \frac{d\widetilde{k}_{Rj}(s)}{dt} = -\frac{\delta\tau_3}{\delta\tau_2} \left(\widetilde{p}_j(t) + (1 - \varpi_1)\pi(t) \right) \left(1 - e^{-\delta\tau_2(s-t)} \right), \end{aligned}$$

we obtain

$$\widetilde{p}_j^* = -(1 - \varpi_1)\pi + B(\rho + \lambda)/A\widetilde{p}_j^* - (\rho + \lambda)/As + \frac{\alpha}{1 + \beta - \alpha}(\rho + \lambda)/A\widetilde{k}_j, \quad (3.A.26)$$

where

$$B = 1 + \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} \quad (3.A.27)$$

$$A = B - \frac{\alpha}{1 + \beta - \alpha} \frac{\delta \tau_3}{\rho + \lambda + \delta \tau_2}. \quad (3.A.28)$$

This dynamics equation is the same as that in the standard model except that the coefficient of π is multiplied by $(1 - \varpi_1)$. The left-hand side of the equation is rewritten as

$$\begin{aligned} \widetilde{\dot{p}}_j^* &= \widetilde{\dot{p}}_R^* - \tau_1 \widetilde{\dot{k}}_{Rj} = \widetilde{\dot{p}}_R^* - \delta \tau_1 (-\tau_2 \widetilde{k}_{Rj} - \tau_3 \widetilde{p}_{Rj}^*) \\ &= \widetilde{\dot{p}}_R^* - \delta \tau_1 \{-\tau_2 \widetilde{k}_{Rj} - \tau_3 (\widetilde{p}_j^* - \widetilde{p}_R)\} \\ &= \widetilde{\dot{p}}_R^* - \delta \tau_1 \{-\tau_2 \widetilde{k}_{Rj} - \tau_3 (\widetilde{p}_R^* - \tau_1 \widetilde{k}_{Rj} - \widetilde{p}_R)\}. \end{aligned}$$

Similarly, $\widetilde{p}_j^*$ and $\widetilde{k}_j$ on the right-hand side can be given by $\widetilde{p}_j^* = \widetilde{p}_R^* - \tau_1 \widetilde{k}_{Rj}$ and $\widetilde{k}_j = \widetilde{k}_{Rj} + \widetilde{k}_R$.

Then we obtain the equation for an averaged value. In order that this does not include the relative term of $\widetilde{k}_{Rj}$, the following condition must be satisfied:

$$0 = -\tau_1 \delta (\tau_2 - \tau_1 \tau_3) - \tau_1 B (\rho + \lambda) / A + \frac{\alpha}{1 + \beta - \alpha} (\rho + \lambda) / A. \quad (3.A.29)$$

Inflation dynamics can be written as

$$\begin{aligned} \widetilde{\dot{p}}_R^* &= \delta \tau_1 \tau_3 \widetilde{p}_R^* - (1 - \varpi_1) \pi + (B (\rho + \lambda) / A - \delta \tau_1 \tau_3) \widetilde{p}_R^* \\ &\quad - (\rho + \lambda) / A s + \frac{\alpha}{1 + \beta - \alpha} (\rho + \lambda) / A \widetilde{k}_R. \end{aligned} \quad (3.A.30)$$

Finally, from equation (3.A.16) and (3.A.29), we can obtain the following inflation dynamics equation:

$$\widetilde{\dot{p}}_R^* = (\rho + \lambda \varpi_2) \widetilde{p}_R^* + \delta \tau_1 \tau_3 \widetilde{p}_R^* - \lambda \varpi_2 \widetilde{p}_I^* - \frac{\rho + \lambda}{A} s + \frac{\alpha}{1 + \beta - \alpha} \frac{\rho + \lambda}{A} \widetilde{k}_R, \quad (3.A.31)$$

where

$$\begin{aligned} s &= w - \bar{p} - (\bar{y} - \bar{l}) \\ \widetilde{p}_R &= -\frac{\theta}{1 + \theta} \widetilde{y}_R = -\frac{\theta}{1 + \theta} \frac{\bar{y} - \varpi_2 y_I}{1 - \varpi_2} \\ \widetilde{k}_R &= k_R - (\varpi_2 k_I + (1 - \varpi_2) k_R) = \varpi_2 (k_R - k_I). \end{aligned}$$

Here, the inflation rate of aggregated prices is described as

$$\begin{aligned} \dot{\pi} = & (\rho + \lambda\varpi_2)\pi + \frac{\lambda\varpi_2}{1 - \varpi_1} \{\tilde{p}_I^* - (\rho + \lambda)s_I\} \\ & + \frac{\lambda(1 - \varpi_2)}{1 - \varpi_1} \left[\delta\tau_1\tau_3\tilde{p}_R - \frac{\rho + \lambda}{A}s + \frac{\alpha}{1 + \beta - \alpha} \frac{\rho + \lambda}{A} \tilde{k}_R \right]. \end{aligned} \quad (3.A.32)$$

The condition for τ_1, τ_2 , and τ_3 , equation (3.A.29), is the same as equation (3.A.9). The other conditions are also found to be exactly the same as equations (3.A.12) and (3.A.13). Clearly when there is no irrational firm, that is $\varpi_2 = 0$, this equation reproduces the ordinary inflation dynamics equation (3.2.40).

Application to Section 3.4.2 This model can be applied to the model where there are a fraction of firms which can reset their prices freely. The other types of firm, sticky price firms, optimise their reset prices taking account of the former flexible price firms. p_I^* in inflation dynamics is simply replaced by $s_I = w - \bar{p} - (y_I - l_I)$. This is because flexible price firms reset their prices at a desired price $P_I^\#$ which maximises their current profits, and the log-linearised form of this is equal to log-linearised real marginal costs, that is, $p_I^\# - \bar{p} = s_I$.

Chapter 4

Zero Nominal Interest Rate Bound and Multiple Equilibria

JEL Classification: E22, E31, E43, E52

Keywords: Liquidity trap, monetary policy, sticky prices, capital investment, non-linear dynamics, excess demand

Abstract

This chapter aims to examine whether Japan's downturn was exacerbated by a liquidity trap using Krugman's (2003) model of multiple equilibria. I need a longer-run Ramsey growth model with refined micro-foundations to show that Krugman's model is valid only under the assumptions of fixed price expectations and the Keynesian consumption function. Once these assumptions are relaxed, Krugman's multiple equilibria disappear. Although an economy experiences bad short-run outcomes similar to Krugman's bad equilibrium due to a liquidity trap, this cannot be the fundamental cause of a bad outcome in the long-run. We then build a model containing Kiyotaki's (1988) long-run multiple equilibria. Again, our analyses do not justify a claim that the zero bound causes a bad long-run equilibrium. However, in the conclusion to the chapter, I discuss that, from the perspective of equilibrium selection, there might be a possibility that Japan's prolonged stagnation was induced by the ineffectiveness of the monetary policy.

4.1 Introduction

4.1.1 Liquidity Trap

This chapter aims to examine whether Japan's downturn was exacerbated by a liquidity trap. A liquidity trap occurs when monetary policy becomes ineffective because a nominal interest rate cannot be below zero. In Japan, the short-term nominal interest rate has been almost zero since 1999, and conventional monetary control through interest rates has stopped functioning. Despite alarms provided by Keynes (1936), such a situation was treated only from a theoretical curiosity. However, observing Japan's experience, people have begun to realise the liquidity trap as a possible phenomenon.

There has been a great deal of research into the liquidity trap, which can be divided

mainly into two categories: the effects of the liquidity trap and the remedies to escape from it. Most studies focus on the latter. This is quite understandable considering the practical need to do so. Besides, it seems rather obvious that the ineffectiveness of monetary policy is not desirable.

However, it is extremely important to deeply understand what happens under the liquidity trap. Such an understanding would help find underlying problems and remedies. Bearing such a motivation in mind, this chapter aims to see how the existence of the zero nominal interest rate bound influences the economy, especially investment and asset prices.

4.1.2 Literature Review

I would like to introduce several analyses concerning the effect of the liquidity trap. A basic feature was addressed by Keynes (1936) and Hicks (1937). Under the liquidity trap, the interest elasticity of money demand becomes near-infinite, or an LM curve becomes flat. Thus, an increase in money supply cannot shift the LM curve. To put it differently, bonds and money become equivalent assets, so monetary policy by means of the exchange of money with bonds has no influence on output.

Among recent papers, one of the most influential is that of Krugman (1998), which demonstrated the existence of a liquidity trap. This model can be simplified as follows. Assume an endowment economy. We can thus focus only on the nominal side of the economy. Moreover, assume that prices are so sticky that the current price (P) cannot move, but that the government can hold the future price (P^*) at a constant target level. Households optimise their consumption, from which the Euler equation is derived: $1 + i = (P^*/P)(1 + \rho)$, where i , ρ , and P are a nominal interest rate, an equilibrium real interest rate and a current price level respectively. Since all the variables in the right-hand side of the equation are predetermined or fixed, the nominal interest rate is determined from the Euler equation. This equation

suggests that the nominal interest rate reaches a zero bound in the following cases. The first case is when a current price level (P) happens to be high. Combined with a constant target level of the future price (P^*), this causes deflation, which makes i become zero. However, that current prices are high does not seem to describe the feature of the liquidity trap well. The second case holds even though a current price level is stable. If the equilibrium real interest rate ρ becomes negative, then the nominal interest rate can become zero. This second case looks more realistic than the first one, and Krugman argued that this could apply to Japan.

In the above papers, the liquidity trap was regarded as a unique equilibrium. In contrast, Benhabib, Schmitt-Grohe and Uribe (2001, 2002) demonstrated the model of multiple equilibria. They showed that the existence of a zero bound yields a self-fulfilling deflationary equilibrium in addition to a normal equilibrium. The mechanism is very simple. Again, assume an endowment economy. Thus, we can focus only on the nominal side of economy. In addition, for simplicity, assume that prices are perfectly flexible. Consider two equations with respect to a nominal interest rate and an inflation rate: the Taylor rule and the Fischer rule. Monetary policy follows the active Taylor rule, $i = i(\pi)$, where π is an inflation rate. The Fischer rule satisfies $\pi = \pi(i) = i - \rho$. Under normal circumstances, the slope of the active Taylor-rule curve is higher than that of the Fischer-rule curve as Figure 4.1.1 shows, so they should intersect only once at $\pi = \pi^*$. However, because of a nominal interest rate bound, the Taylor-rule curve makes a kink at $i = 0$, which yields another equilibria at $\pi = -\rho$. This is called a deflationary equilibrium.

These papers succeeded in explaining Japan's situation to some extent, but some of their features are by no means satisfactory. In Krugman (1998), the assumption of a negative equilibrium real interest rate is highly doubtful. Households' discount rate should not be negative. A decline in population growth may have had the effect of decreasing the equilibrium real interest rate to some extent. It seems that this country's population started to decline

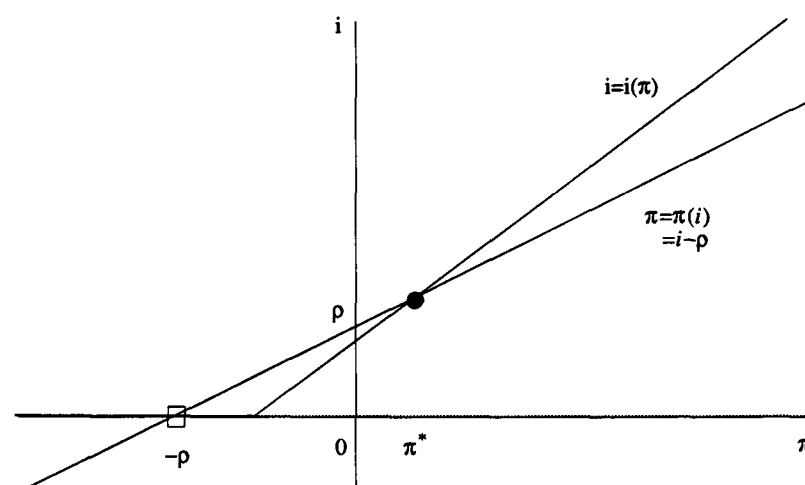


Figure 4.1.1: Liquidity Trap by Benhabib, Schmitt-Grohe and Uribe (2001a, 2002)

in 2005. However, the decline appears to be too small to make the equilibrium real interest rate negative. As another problem, the model of the endowment economy seems to be too simplified; this neglects investment and capital accumulation, which can explain more sophisticated features such as the swing of asset prices and volatile economic fluctuations, although Krugman noticed the importance of these things in his paper. The model used by Benhabib, Schmitt-Grohe and Uribe (2001, 2002), unlike Krugman's (1998) model, does not use strange assumptions such as a negative equilibrium real rate. However, while the normal equilibrium is determinate in their model, the deflationary outcome is indeterminate. As I argued in the previous chapter, such indeterminate equilibrium is unsatisfactory. There thus remains a question as to why the deflationary equilibrium can be actually selected over the other good one. Moreover, as already noted, this model also overlooks investment. There is no analysis as to what happens if we take account of investment.

4.1.3 Methods and Results

This dissatisfaction becomes the motivation for this chapter. It aims to provide a more convincing story about the liquidity trap, and to help in understanding what happens under the liquidity trap.

It is Krugman (2003) whose insight provides the basic motivation of this chapter. Krugman proceeded to demonstrate the model of multiple equilibria caused by a zero nominal interest rate bound^{1,2}. This model incorporated investment and asset prices very clearly. In a good equilibrium, investment is stabilised by means of active monetary policy. But, in a bad equilibrium, a collapse of asset prices gives rise to a depression. Investment cannot be stimulated to take up the slack because of low demand and the ineffectiveness of monetary policy caused by the zero bound. This model is extremely simple, and does not need to assume a negative equilibrium real interest rate as Krugman's (1998).

However, his model lacks a micro-foundation, so we cannot tell how plausible his model is and what assumption is needed for the multiple equilibria which he obtains. In order to compensate for this problem, this chapter investigates his model more deeply with micro-founded models. It begins with the simple Krugman model, and extends it in the direction of a Ramsey model. In other words, agents become more rational, and the assumption of prices is modified from fixed to flexible.

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Let me state the results of this chapter in advance. The Krugman model can be replicated with a moderate micro-foundation. In order to replicate his model, I show that we need two very strong assumptions: fixed price expectations and a Keynesian consumption function. These assumptions might be valid in the short-run, but definitely not in the long-run. Replacing the Keynesian consumption function with intertemporally optimised consumption (Euler equation) is the first step in making the Krugman model closer to the Ramsey model. This results in eliminating one of the equilibria: the bad determinate equilibrium. Furthermore, relaxing the fixed price expectations while maintaining the Keynesian consumption

¹Krugman (2000) also mentioned similar multiple equilibria, but this model is even more ad-hoc than that of Krugman (2003).

²He called this a fourth-generation model as the model of a financial crisis.

function leads to the complete elimination of multiple equilibria. In addition, the further relaxation of *both* of the assumptions also yields unique equilibrium, which is a natural outcome in the Ramsey model. In an intermediate region between the Krugman and the Ramsey models, that is, with sticky prices (i.e. prices which are neither fixed nor fully flexible but change *gradually*) and with intertemporal consumption optimisation, the short-run adjustment process comes to resemble that of the Krugman model. In other words, subject to a sufficiently large negative shock to cause a nominal interest rate to reach zero, consumption drops with income. However, there is no multiplicity in terms of dynamic paths or final outcomes, so Krugman's story of multiple equilibria does not seem to hold in the medium to long-run. Krugman's multiple equilibria would not affect the equilibrium of the economy, and a zero bound cannot be the cause of a bad equilibrium. It can only influence the adjustment path. However, if there are multiple equilibria even in the long-run, then the story might be different. As Kiyotaki (1988) demonstrated, assuming increasing returns to scale technology can produce long-run multiple equilibria. The economic features of long-run multiple equilibria are quite similar to those in the Krugman short-run multiple equilibria. Moreover, they can be well compared to the Japanese depression. However, again we cannot find a strong analytical support that the zero bound induces a bad outcome in the long-run. Although the zero bound can be observed in the process of adjustment towards such a bad equilibrium (as well as on the path to a good equilibrium), this is not a cause but only a consequence. All it does is make the adjustment path more unpleasant. It does not affect the destination of the path. Nevertheless, there is one final possibility that the zero bound causes the bad long-run equilibrium as the economy's final destination. This might happen because of equilibrium selection. It is just possible that a bad short-run outcome due to a zero bound to the nominal interest rate might cause the economy to select the bad long-run equilibrium as the one to which the economy converges. I have not done proper work on this problem, but I conclude

with some remarks about this possibility.

*

The structure of this chapter is as follows. After Section 4.2 introduces the Krugman model, Section 4.3 attempts to refine his model with a moderate micro-foundation and to make it clear what are important assumptions. Sections 4.4 and 4.5 relax all of his crucial assumptions, and examine how results will change and how they are connected with the Krugman model. Section 4.6 and its subsequent section apply Kiyotaki's (1988) model, where there are long-run multiple equilibria.

4.2 Krugman Model

Krugman's (2003) model is summarised as follows. Assume nominal stickiness. The economy is demand-side driven. Tobin's Q determines investment and, through a multiplier effect, output y .

$$y = y(Q). \tag{4.2.1}$$

Tobin's Q is increasing in y and decreasing in an interest rate i .

$$Q = Q(y, i). \tag{4.2.2}$$

A central bank responds to y such that

$$i = i(y). \tag{4.2.3}$$

Combining the last two equations, we can deduce a curve of Q given y . If monetary policy is effective enough, then Q becomes decreasing in y , as shown on the left of Figure 4.2.1. When the output is very low, an ideal nominal interest rate becomes negative but, in reality, it cannot be achieved. An interest rate is stuck at zero, and then Tobin's Q becomes increasing in y , and multiple equilibria can arise as in the right figure. Two points, left and right,

are stable, and the lower one corresponds to a bad equilibrium which is bound at the zero nominal interest rate. In the right figure, the slope of the curve $y = y(Q)$ becomes steeper when output is low. This is because there is a certain lower bound to investment.

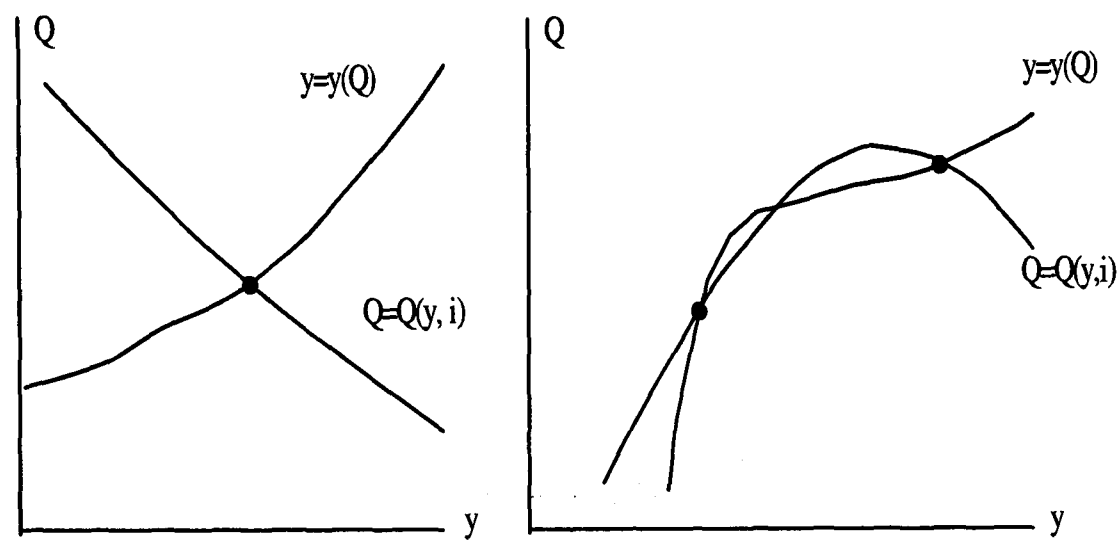


Figure 4.2.1: Krugman's Multiple Equilibria

4.3 Semi-Sensible Krugman Model

4.3.1 Semi-Micro-Founded Model

I now refine this Krugman model with a micro-foundation in order to see the appropriateness of underlying assumptions. To state a result first, the Krugman model requires the following three assumptions: (1) fixed price expectations, (2) a Keynesian consumption function, and (3) a zero long-run growth rate. In particular, the first two assumptions are extremely important to yield multiple equilibria. I begin with the first assumption.

Assumption 1. Households regard prices as constant or an expected inflation rate as zero:

$$\pi_e = 0.$$

However, actual prices as well as wages can freely vary. The prices households perceive

P_e may differ from the actual prices firms perceive P . This assumption is very important. An equilibrium real interest rate can deviate from a representative household's discount rate. Thus, the marginal product of capital which should be strongly related to the real interest rate can have multiple values.

<Households>

Based on a Keynesian framework, assume that households spend a fixed proportion of current income on consumption.

Assumption 2. Consumption C is described as

$$C = cY, \tag{4.3.1}$$

where Y is aggregate output and c represents a propensity to consume ($0 < c < 1$).

In addition, a labour supply function, such as equation (4.5.3) which will be later derived, determines the level of nominal wages, but this is not important for the main result of this model. This is because the above Keynesian consumption function determines consumption irrespective of real wages.

<Firms>

Firms maximise their profits. Their decision of investment and employment is rational under fixed price expectations. A maximisation problem is given by

$$\int_t^\infty e^{-R(s-t)} \Pi(s) ds, \tag{4.3.2}$$

subject to a production function, the law of motion with respect to capital, the following form of adjustment costs:

$$Y = hK^\alpha L^{1-\alpha} \tag{4.3.3}$$

$$\dot{K} = -\delta K + I \tag{4.3.4}$$

$$J(I, K) = \Psi(I/K)K. \tag{4.3.5}$$

A transversality condition is

$$\lim_{t \rightarrow \infty} e^{-R(t-s)} Q(t) P(t) K(t) = 0 \quad (4.3.6)$$

Profits in each period are given by

$$\Pi = PY - P(I + J(I, K)) - WL, \quad (4.3.7)$$

and R is the average nominal interest rate:

$$R(s) = \frac{\int_t^s i(t') dt'}{s - t}, \quad (4.3.8)$$

using the nominal interest rate $i(t')$ at each time t' . P is the price of both consumption and investment goods, and W is a wage. A production function of output Y has constant returns to scale (CRS) technology, where K is capital, L is labour and α ($0 < \alpha < 1$) is a capital share. I is investment, δ is the depreciation rate of capital, and $J(I, K)$ is adjustment costs which exhibit CRS in K and I . $\Psi(x)$ is increasing and convex such that

$$\Psi' \geq 0, \quad \Psi'' > 0. \quad (4.3.9)$$

This chapter, in particular, assumes that adjustment costs are zero when investment only covers capital depreciation:

$$\Psi(\delta) = \Psi'(\delta) = 0. \quad (4.3.10)$$

Define Hamiltonian as

$$H = \Pi + QP(I - \delta K). \quad (4.3.11)$$

Its first-order conditions read

$$\frac{\partial H}{\partial L} = PY_L - W = 0 \quad (4.3.12)$$

$$\frac{\partial H}{\partial I} = -P\{1 + \Psi'(I/K)\} + QP = 0 \quad (4.3.13)$$

$$\begin{aligned} \frac{\partial H}{\partial K} &= iQP - (\dot{Q}P + Q\dot{P}) \\ &= PY_K - P\{\Psi(I/K) - I/K \cdot \Psi'(I/K)\} - \delta QP. \end{aligned} \quad (4.3.14)$$

The first equation states that the marginal product of labour is equal to real wages. This is not

important for our analysis because, as explained above, real wages do not affect consumption.

The second equation is written as

$$Q = 1 + \Psi'(X) \quad \text{or} \quad X = f(Q), \quad (4.3.15)$$

where

$$X \equiv I/K. \quad (4.3.16)$$

This equation implies investment is a function of Tobin's Q . Furthermore, the third equation yields

$$\dot{Q} = (i - \pi_e + \delta)Q - Y_K + \Psi(X) - X \cdot \Psi'(X), \quad (4.3.17)$$

where π_e is expected inflation, $\dot{P}/P$. This is zero due to the assumption of fixed price expectations. As a result, a real interest rate becomes the same as a nominal interest rate. If a real interest rate became negative, this could prevent investment from plummeting. However, because of a zero bound, a real interest rate cannot be negative. This ineffectiveness of monetary policy produces a bad equilibrium.

<Market clearing>

A market is cleared as

$$Y = C + I + J(I, K). \quad (4.3.18)$$

<Long-run equilibrium>

Assumption 3. The economy exhibits no long-run growth under a normal circumstance (or at a good equilibrium in the following definition). However, subject to a shock, the growth rate may deviate from zero.

This assumption is consistent with a standard Ramsey model with no population or technology growth. In a Ramsey model, such an assumption is not needed. No growth rate is an outcome of the model rather than the assumption. However, in the model of this

section, without this condition, the equilibrium growth rate cannot be determined, and this assumption is required.

Let us focus on equilibrium. Assume that output, capital and investment grow at the same speed g . Its corollary is that the level of these variables can be arbitrary. From Assumption 3, in a good equilibrium, g is zero. However, it can be non-zero if there are other equilibria. The law of motion with respect to capital implies

$$\begin{aligned}\dot{K} &= -\delta K + I \\ &= gK,\end{aligned}$$

so

$$X = g + \delta. \quad (4.3.19)$$

Tobin's Q is constant, and equation (4.3.17) becomes

$$Q = \frac{Y_K - \Psi(g + \delta) + (g + \delta) \cdot \Psi'(g + \delta)}{i + \delta}. \quad (4.3.20)$$

The marginal product of capital is given by g using the market clearing condition:

$$\begin{aligned}Y &= cY + I + J(I, K) \\ Y_K &= \alpha \frac{Y}{K} = \alpha \frac{g + \delta + \Psi(g + \delta)}{1 - c}.\end{aligned} \quad (4.3.21)$$

Substituting this into the above equation (4.3.20) yields

$$Q = Q(g, i) = \frac{\alpha \frac{g + \delta + \Psi(g + \delta)}{1 - c} - \Psi(g + \delta) + (g + \delta) \cdot \Psi'(g + \delta)}{i + \delta}. \quad (4.3.22)$$

With equation (4.3.15):

$$Q = 1 + \Psi'(g + \delta), \quad (4.3.23)$$

we can draw two curves of Q as a function of g as shown in Figure 4.3.1. The second equation should be interpreted in the opposite direction; g is determined by Q , so in the figure, I denote it as $g = g(Q)$ ³. Regarding $Q = Q(g, i)$, we need to consider the movement of interest rates

³For example, provided the standard quadratic form of adjustment costs $\Psi(X) = a(X - \delta)^2/2$, $Q = 1 + ag$, so $g = g(Q)$ becomes linear in Q , and $Q = 1$ at $g = 0$.

i. Assume the monetary policy rule such as

$$i = i(g), \quad i' \geq 0, \quad (4.3.24)$$

and then strong control by a monetary authority through interest rates i makes the slope of $Q = Q(g, i)$ negative. Conversely, weak or no control makes it positive.

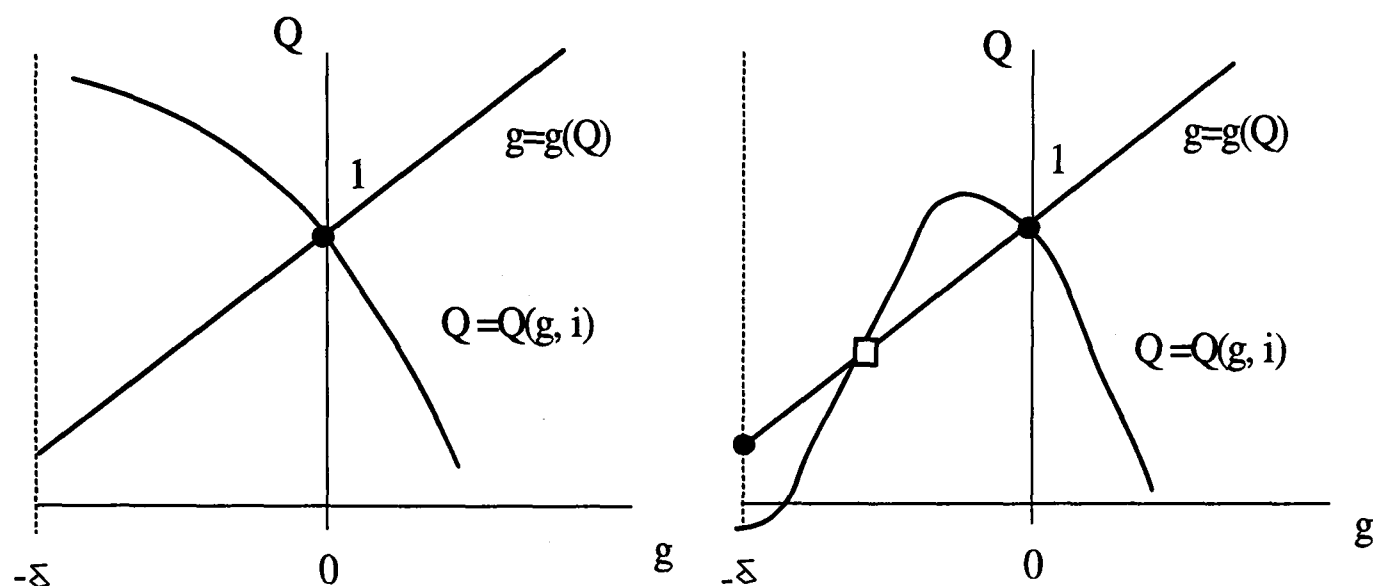


Figure 4.3.1: Multiple Equilibria in the Semi-Sensible Krugman Model

The left figure in Figure 4.3.1 represents the case where there is no nominal interest rate bound. The stronger influence it has, the more downward the slope of $Q = Q(g, i)$ becomes. Two curves intersect only once. This chapter calls this point a (Krugman or short-run) good equilibrium. In the equilibrium, g is equal to zero from Assumption 3, which corresponds to the case where Tobin's Q is one; investment only covers depreciation; and capital stays constant.

The right figure is the one which incorporates a zero nominal interest rate bound. When g is lower than a certain level, a corresponding nominal interest rate reaches zero, and monetary policy becomes ineffective. Thus, in this region, the curve $Q = Q(g, i)$ becomes increasing with g . Assuming non-negative investment, $I = 0$ or $g = -\delta$ becomes one of stable equilibrium. Capital decreases at the depreciation rate, which is considered as a bad equilibrium.

From equations (4.3.22) and (4.3.23), a condition for the existence of each equilibrium is written as follows. Allow the deviation of a good equilibrium from $g = 0$ when a certain shock occurs. In order to have a good equilibrium around $g = 0$, the following condition is needed:

$$\begin{aligned} Q(g, i) &= 1 + \Psi'(g + \delta) \quad \text{at } g = 0 \text{ for } i \geq 0 \\ i &= \delta \left(\frac{\alpha}{1 - c} - 1 \right) \geq 0 \\ &\leftrightarrow c \geq 1 - \alpha. \end{aligned} \tag{4.3.25}$$

In order to have a good equilibrium, the marginal product of capital needs to cover the cost of capital depreciation. The former increases as demand increases, which can be induced by a high propensity to consume c . A high capital share α also increases the profitability from capital investment. Therefore, a high c or α yields a good equilibrium. Note that this condition is not strictly correct because the good equilibrium may have $g \neq 0$. In order to derive an accurate form, we have to assume a specific policy reaction function $i = i(g)$.

The bad equilibrium exists if

$$\begin{aligned} Q(g, i) &\leq 1 + \Psi'(g + \delta) \quad \text{at } g = -\delta \\ \frac{\alpha}{1 - c} - 1 &\leq \delta \frac{1 + \Psi'(0)}{\Psi(0)}. \end{aligned} \tag{4.3.26}$$

This condition suggests that the low propensity to consume, a high depreciation rate or small adjustment costs for null investment functions to produce the bad equilibrium.

As Appendix 4.A.1 shows, the good and the bad equilibria are both determinate. The intermediate one is stable but indeterminate. In the figure, a squared mark and a filled circle represent an equilibrium which is indeterminate and determinate respectively. Mathematically, the system of this model composes of two differential equations with respect to K and Q (or $X = I/K$). Under determinacy, the eigenvalues of K and Q are non-positive and positive respectively. In particular, the stock of capital can become arbitrary. This is because

of the lack of price adjustment mechanisms. Therefore, the eigenvalue of K becomes not negative but zero. This implies that, for any values, capital always stays at equilibrium and that an economy appears to converge to equilibrium instantaneously despite the existence of investment adjustment costs.

4.3.2 Demand Shock Response: Elimination of Multiple Equilibria

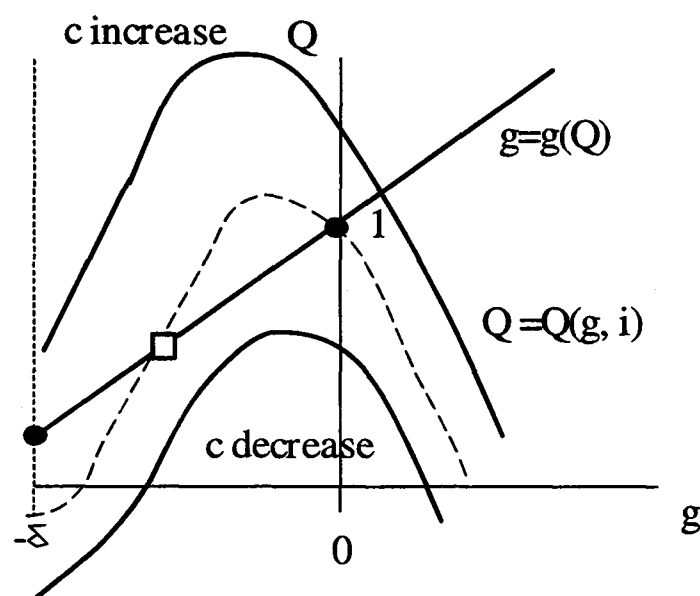


Figure 4.3.2: Response of a Demand Shock

I now consider how multiple equilibria can be eliminated. In particular, I focus on the propensity to consume c . This variable may be changed by an unknown demand shock, but can be partly controlled, for instance, through the government. A demand shock shifts Tobin's Q as Figure 4.3.2. From equation (4.3.22), a negative demand shock (a decrease in c) shifts the curve $Q = Q(g, i)$ downward. A sufficiently large negative shock leads to the elimination of the good equilibrium. Only the bad equilibrium survives, so the economy drops into a poverty trap. On the other hand, a big positive shock can eliminate the bad equilibrium, which leads to the good equilibrium.

In this respect, government expenditure can be a useful measure for an economic recovery.

A demand increase can shift the bad equilibrium to good provided its scale is sufficiently large.

*

In this way, we succeeded in reproducing the Krugman model with more micro-foundations. There are three important assumptions, that is, fixed price expectations, the Keynesian consumption function and no long-run growth. I will relax the first two assumptions later. This will make the model closer to the Ramsey model which is based on a perfect rational behaviour and perfectly flexible prices.

The third assumption is not needed once we relax the first assumption. As will be discussed later, under a flexible price model, a long-run growth rate is uniquely determined to be zero if there is neither population nor technology growth. In other words, the third assumption is regarded not as an assumption but as an outcome; it is just a subset of the first assumption. Therefore, we do not examine the importance of the third assumption separately.

4.3.3 Relaxing the Assumption of the Keynesian Consumption Function

We now relax the assumption of the Keynesian consumption function and replace this with the Euler equation. Then, as we will soon see, Krugman's multiple equilibria do not arise. Instead, another type of multiple equilibria can be observed, but they will probably not be observed in reality because a bad equilibrium is indeterminate.

By means of the Euler equation, we can model rational and forward-looking households. They optimise their future consumption as well as the present one intertemporally. As a result, an important determinant for their consumption becomes an interest rate because it represents a price of future goods. In contrast, current income does not much affect current consumption. A fall in current income does not necessarily lead to a fall in consumption. This can work to eliminate the bad equilibrium in the Krugman model. Even if investment

and output decrease, robust consumption can help supporting the economy if households are forward-looking.

Thanks to the Euler equation, consumption comes to have a dynamic process. This makes our model more complex. Together with other dynamic processes with respect to capital and investment, there arise three differential equations. It thus becomes more difficult to investigate the existence of multiple equilibria and the stability of each equilibrium.

The main part of this thesis, in order to grasp the gist of this model, considers a simplified model by regarding capital as fixed. This simplification reduces the number of dynamic processes from three to one (consumption), and it becomes extremely easy for us to investigate the properties of the model. Furthermore, the main features are not much different from the one which incorporates capital. An entire model and its dynamics are studied in Appendix 4.A.2.

This modified model can be solved as follows. The Euler equation, such as equation (4.5.2), states that consumption growth comes to depend on a nominal interest rate:

$$g_C = \dot{C}/C = (i - \pi_e - \rho)/\sigma, \quad (4.3.27)$$

where $\pi_e = 0$, ρ is a discount rate and σ is an intertemporal elasticity of substitution. Conversely, a nominal interest rate is determined by the growth rate of output by monetary policy:

$$i = i(g). \quad (4.3.28)$$

Since we neglect capital, $Y = C$. In other words, the growth rates of consumption and output are the same: $g = g_C$.

As Figure 4.3.3 shows, these two relationships between the growth rate and the interest rate produce two roots at $i = 0$ and $i > 0$. The better equilibrium has $g = 0$, and a corresponding interest rate is equal to the discount rate ρ . In the worse equilibrium, the economy continues to collapse at the speed of $g = -\rho/\sigma$. The nominal interest rate reaches

zero, which prevents monetary policy from functioning to recover the economy. Under a normal circumstance, the monetary policy rule needs to have a higher slope than the Euler equation. In other words,

$$i'(g) > \sigma. \quad (4.3.29)$$

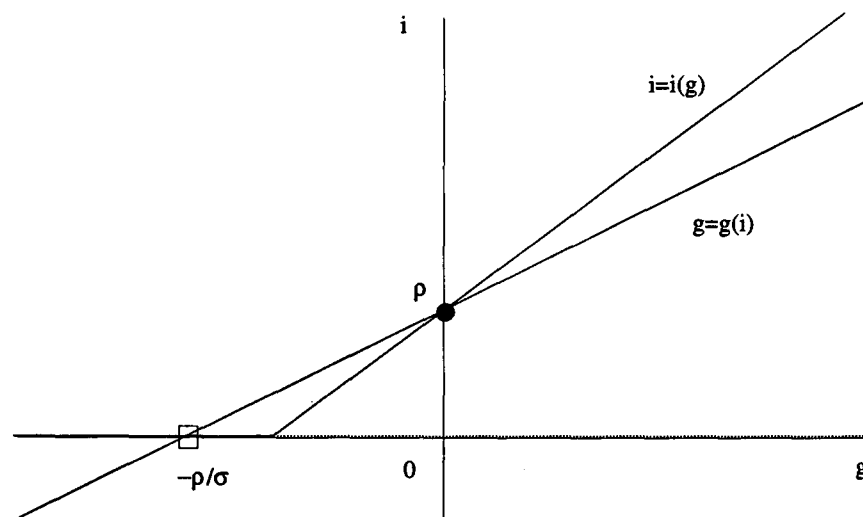


Figure 4.3.3: Multiple Equilibria under Fixed Price Expectations when Households Obey the Euler Equation

It can be demonstrated that the better and the worse equilibria are respectively determinate and indeterminate. Consider the following thought experiment. Suppose that people expect a decrease in consumption. If monetary policy is effective, this leads to a decrease in the interest rate, which increases consumption from the Euler equation. Thus, under the better equilibrium, such an expectation is not self-fulfilling, which suggests determinacy. On the other hand, if monetary policy is not effective, a decrease in consumption does not result in a reduction of the interest rate. This allows the expectation of a consumption to be self-fulfilling. The worse equilibrium becomes indeterminate.

Despite multiple equilibria, the worse equilibrium may well be eliminated. This is because the better equilibrium exhibits determinacy and the worse equilibrium exhibits indeterminacy. Thus, the latter equilibrium may well not be chosen over the former. There is another

reason. Figure 4.3.3 is very similar to Figure 4.1.1 by Benhabib, Schmitt-Grohe and Uribe (2001, 2002) which was introduced at the beginning of this chapter. Regarding their multiple equilibria, Evans and Honkapohja (2003) reported that the worse equilibrium does not satisfy the E-stability. In other words, under imperfect information, agents cannot correctly learn the economic properties through adaptive learning, and the economy becomes unstable. This instability simply depends on which slope is higher between two curves in the figure. Hence, the worse equilibrium in Figure 4.3.3 is also considered as unstable.

Intuitively, the reason why the bad and determinate equilibrium disappears in this modified model is as follows. Owing to fixed price expectations, a real interest rate is always the same as a nominal interest rate. Hence, a nominal interest rate bound makes real interest rates zero. From the Euler equation, this functions to prevent consumption from dropping although households' incomes decrease. A decrease in demand becomes milder, which leads to eliminating the Krugman bad equilibrium.

It is interesting to point out the difference between Krugman's two papers. As explained in the Introduction, Krugman (1998), in his first paper on the liquidity trap, used not the Keynesian consumption function but the intertemporal consumption function. In contrast, as this thesis has made clear, his later paper (2003) is based on the Keynesian consumption function. It is not clear whether Krugman was aware of this need for a shift of assumptions, but this is a very crucial assumption shift which yields multiple equilibria.

Even more curiously, once we introduce the model of not perfectly flexible prices (one variation of a Ramsey model), the necessity of the Keynesian consumption function partly disappears. Without the assumption, consumption comes to behave as if it obeys the Keynesian consumption function. The details will be discussed in the next section. However, this does not mean that the importance of the assumption of the Keynesian consumption function is entirely lost. The modified model to the Euler equation cannot produce multiple

equilibria as did Krugman's.

4.3.4 Relaxing the Assumption of Fixed Price Expectations

What happens if we relax the assumption of fixed price expectations while maintaining the Keynesian consumption function? This section will not construct an analytical model to verify my following claims. However, by referring to the following sections, the reasons for my claims will become clear.

Firstly, begin with the case where price expectations as well as actual prices are perfectly flexible. In this case, an economy satisfies classical dichotomy. Nominal changes can never influence the real side of the economy. In particular, monetary policy by means of the control of nominal interest rates becomes totally ineffective. A change in nominal interest rates is perfectly cancelled by a change in an inflation rate, which keeps real interest rates unchanged. Therefore, the existence of a zero nominal interest rate bound has no effect on the real side of the economy; the Krugman multiple equilibria do not arise.

Secondly, consider the case where price expectations are not perfectly flexible. The above result seems to hold true. In other words, as long as price expectations are partially flexible, the zero bound does not yield multiple equilibria. This is because, in the long run, economies should converge to the equilibrium which is the same as that under perfectly flexible prices. In the equilibrium, the dichotomy holds, and monetary policy seems to have no effect on the real side of the economy, at least in the long-run.

However, even though there are no multiple equilibria, there might exist multiple dynamic paths. In an adjustment process toward the equilibrium, we cannot deny the possibility that there are multiple paths, and they may be compared with the Krugman multiple equilibria. This possibility will be examined in the next section.

The reason why we do not write down and solve a model in this framework is as follows.

First and foremost, its result is expected to be almost the same as that of the Ramsey model which will be exploited in the next section. The only difference between the two models is the setup of consumption: either the Keynesian consumption function or the Euler equation. However, as we recently discussed, under not perfectly flexible prices, this difference does not matter much. Even with the Euler equation, consumption appears to obey the Keynesian consumption function. Secondly, intertemporal consumption by means of the Euler equation is far more widely accepted than the Keynesian consumption function. While the latter lacks a micro-foundation, the former approach has a strong theoretical basis with rational and forward-looking households. For these reasons, a detailed analysis will be left to the next section.

4.4 Ramsey Model with Perfectly Flexible Prices

We saw that there are two important assumptions to yield multiple equilibria. One is the assumption of fixed price expectations. This assumption breaks classical dichotomy. Monetary policy, in particular, the existence of a zero nominal interest rate bound, comes to have a real effect on the economy. The other assumption is that of the Keynesian consumption function. A consumption collapse with a decrease in income induces a catastrophic equilibrium. The relaxation of either leads to the elimination of the Krugman bad equilibrium. There is also a third assumption, no long-run growth. But, as will soon become clear, this is automatically satisfied once we relax the first assumption and allow expected prices to change. In the Ramsey model, equilibrium is uniquely determined and, without technology and population growth which I assume here, an equilibrium growth rate becomes zero.

This section now relaxes both the two important assumptions in the Krugman model. Then, our framework becomes a standard general equilibrium model: the Ramsey model.

This is constructed assuming perfectly rational expectations and behaviour. People have correct expectations about the economy including prices. Households maximise their utility intertemporally instead of following the Keynesian type consumption function. Firms maximise their profits, and optimise their investment decision. Furthermore, we advance to assume perfectly flexible prices. A relevant paper is, for example, that of Abel and Blanchard (1983). Such a model is exactly the opposite to the previous Krugman model: the Krugman and the Ramsey models can be interpreted as short-run and long-run respectively.

Our aim in this section is to confirm that Krugman's multiple equilibria disappear in the Ramsey growth model with sound micro-foundations. Three strong assumptions in the Krugman model are all relaxed. In addition, this study is important for us to proceed to the models of the next section where prices are neither fixed nor fully flexible. Regarding the assumption of price stickiness, these models are the closest to reality. Furthermore, as will be shown, they produce a similar outcome to Krugman's bad equilibrium. These models with not perfectly flexible prices are built onto the Ramsey model, so constructing the Ramsey model is an essential step.

Hereafter, assume monopolistic competition. All goods are differentiated, and each firm has a monopolistic power to decide its price. This assumption is not necessary here, but becomes important when sticky prices are incorporated. This is because, without monopolistic competition, while a firm cannot reset its price, the other firms can adjust the price of the identical good, which makes economy no different from that under flexible prices. Investment and consumption goods are not separated; all goods are used both for consumption and investment.

A model is now constructed as follows.

Model Setup

<Households>

A representative household maximises one's utility

$$\max \int_t^\infty u(\bar{C}, L) e^{-\rho(s-t)} ds \quad (4.4.1)$$

subject to

$$\int_t^\infty \bar{P}\bar{C} e^{-R(s-t)} ds \leq A(t) + \int_t^\infty (WL + \Pi) e^{-R(s-t)} ds. \quad (4.4.2)$$

A bar ($\bar{X}$) represents an aggregated or average value. No-Ponzi game version of the budget constraint is described as

$$\lim_{t \rightarrow \infty} e^{-R(t-s)} A(t) \geq 0. \quad (4.4.3)$$

Consumption is aggregated by each good j :

$$\bar{C} = m^{-\theta} \left(\sum_j^m C_j^{\frac{1}{1+\theta}} \right)^{1+\theta}, \quad (4.4.4)$$

where m is the number of goods. The parameter $\theta(> 0)$ represents mark-up, or $(1 + \theta)/\theta$ represents the elasticity of substitution between different goods. $\bar{P}$ is an aggregate price index, and is given by

$$\bar{P} = m^\theta \left(\sum_j^m P_j^{-1/\theta} \right)^{-\theta}, \quad (4.4.5)$$

using the price of each good P_j . Define a utility function as

$$u(\bar{C}, L) = \frac{\bar{C}^{1-\sigma_1} - 1}{1 - \sigma_1} + \gamma_L \frac{(1 - L)^{1-\sigma_2} - 1}{1 - \sigma_2}, \quad (4.4.6)$$

where $\sigma_1(> 0)$ is the degree of risk aversion of consumption, $\sigma_2(> 0)$ corresponds to that with respect to leisure $(1 - \bar{L})$, and γ_L is a parameter to determine the weight between consumption and labour. This setup is the same as that in Chapter 3. First-order conditions lead to the following equations:

$$\sigma_1 \frac{\bar{C}}{\bar{C}} = i - \pi - \rho \quad (4.4.7)$$

$$\frac{W}{\bar{P}} = \gamma_L \frac{\bar{C}^{\sigma_1}}{(1 - L)^{\sigma_2}}. \quad (4.4.8)$$

<Firms>

There are m firms, each of which produces differentiated goods as a monopoly. A firm j maximises its present-valued profits:

$$\max \int_t^\infty e^{-R(s-t)} [P_j Y_j - W L_j - \bar{P}(I_j + J(I_j, K_j))] ds.$$

For simplicity, assume that produced goods are used for investment as well as consumption.

Investment goods can be aggregated in the same way as consumption:

$$I_j + J(I_j, K_j) = m^{-\theta} \left(\sum_k^m \{I_{jk} + J(I_{jk}, K_j)\}^{\frac{1}{1+\theta}} \right)^{1+\theta}. \quad (4.4.9)$$

From the above definition of aggregate consumption and a price index, we obtain a firm j 's demand curve as

$$Y_j^d = \left(\frac{P_j}{\bar{P}} \right)^{-\frac{1+\theta}{\theta}} \bar{Y}. \quad (4.4.10)$$

A production function is

$$Y_j = h K_j^\alpha L_j^{1+\beta-\alpha}. \quad (4.4.11)$$

Capital evolves as

$$\dot{K}_j = I_j - \delta K_j. \quad (4.4.12)$$

Define Hamiltonian as

$$H = P_j Y_j - W L_j - \bar{P}(I_j + J(I_j, K_j)) + \bar{P}Q(I_j - \delta K_j),$$

and its first-order conditions with respect to I_j , K_j , and L_j are transformed to

$$Q = 1 + \Psi'(I/K) \quad (4.4.13)$$

$$\dot{Q} = (i - \pi + \delta)Q - Y_K + \Psi(I/K) - I/K \Psi'(I/K) \quad (4.4.14)$$

$$\bar{P}Y_L = (1 + \theta)W. \quad (4.4.15)$$

Here, owing to homogeneity across firms, I omit the subscript j . The first two equations yield the law of motion with respect to investment:

$$\Psi''(X)\dot{X} = (i - \pi + \delta)(1 + \Psi'(X)) - Y_K + \Psi(X) - X\Psi'(X), \quad (4.4.16)$$

using $X = I/K$. Capital evolves as

$$\dot{K} = (X - \delta)K. \quad (4.4.17)$$

<Market clearing condition>

A goods market is cleared as

$$\bar{Y} = \bar{C} + \bar{I} + J(\bar{I}, \bar{K}). \quad (4.4.18)$$

Under perfectly flexible prices, we do not need to assume a monetary policy rule. A classical dichotomy holds true. Monetary policy whose instrument is a nominal interest rate only determine an inflation rate. A real interest rate, which affects the real side of the economy, is independent of monetary policy rules. Hence, the existence of a zero bound has no effect on the real side of the economy. The Krugman model cannot be applied.

Furthermore, as Abel and Blanchard (1983) demonstrated, such a Ramsey framework has uniqueness. Multiple equilibria disappear. An equilibrium growth rate is zero, given no population and technology growth. Variables' levels are also uniquely determined except for a price level. This is nothing more than Arrow-Debreu equilibrium. Thus, the Krugman story is not appropriate to address multiplicity in the long-run.

4.5 Variations upon the Ramsey Model with Not Perfectly Flexible Prices

The Krugman and the Ramsey models have a different time horizon, short-run and long-run. In order to examine a connection between the two models, say in the medium run, let us extend the Ramsey model from flexible to not perfectly flexible prices. In contrast to the

case of perfectly flexible prices, a nominal change cannot be instantaneously adjusted, which causes some real changes. Monetary policy comes to affect not only the nominal side but the real side of the economy. The existence of a zero bound may become harmful and, as Krugman argued, it may produce a disastrous outcome.

Interestingly, under not perfectly flexible prices, one of the properties of the Krugman model re-emerges. Consumption behaves as if it obeys a Keynesian consumption function even if it is the case that the structural equation is, in fact, an Euler equation. The logic of this runs as follows. When demand is low, a nominal interest rate reaches zero. If prices are fixed, a real interest rate is zero, and consumption does not drop. However, under partially flexible prices, a demand decrease causes deflation. The combination of the zero bound and the deflation renders the real interest rate positive, which results in a decrease in consumption. In this way, consumption moves in the same direction with output like the Keynesian consumption function.

A further decrease in demand caused by a decrease in consumption may yield a bad outcome as the Krugman model predicted in addition to a normal outcome. The final conclusion is not certain but, if this is the case, we do not need to use the above controversial assumptions in the way that Krugman did. This section examines the dynamics of consumption in more detail, and checks if there is multiplicity.

4.5.1 Dynamics under Sticky Price Expectations

In this section, in particular, we introduce a model of sticky price expectations. *Actual* prices and wages are perfectly flexible, however. The way to formalise sticky price expectations follows the idea of Friedman (1968). This section aims to examine how such a sticky price model can be related to the previous Krugman model. By incorporating such a model, this section aims to see if the Krugman story addresses anything about an adjustment mechanism

toward a long-run equilibrium.

The following model is not based on rational expectations, and may lack a micro-foundation. Furthermore, this model does not address an important element in sticky prices; a change in an expected price level does not have any effects on the economy. This is because immediately after its change, actual prices and wages are perfectly adjusted.

Despite this demerit, there is a tractable advantage over currently popular new-Keynesian approaches. In the latter approach, combined with non-linearity, the possibility of excess demand can arise. Under sticky prices, the ratio of the marginal product of labour to real wages does not stay constant. Thus, when a large shock happens, the marginal product of labour can become lower than real wages. This discourages an incentive for firms to produce as many goods as are demanded; demand comes to be constrained by limited supply. In order to continue calculation, an additional assumption is needed, but it is very difficult to reach an agreement as to what assumption should be imposed. The details will be discussed later.

This chapter does not log-linearise models. This is firstly because our focus is on multiple equilibria. A narrow view around an equilibrium is not enough to think of a shift between multiple equilibria. Secondly, economies exhibit a kink before and after reaching a zero bound. A non-linear analysis enables us to examine the effect of the zero bound profoundly. For these reasons, I keep models non-linear, and analyse their dynamics numerically.

Model Setup

Model setup is very similar to the above Ramsey model, so some of the equations are not rewritten in this section. One difference is that households have sticky price expectations, and believe that prices are $\bar{P}_e$, which does not necessarily coincide with actual prices.

<Households>

A representative household maximises one's utility

$$\max \int_t^\infty u(\bar{C}, L) e^{-\rho(s-t)} ds. \quad (4.5.1)$$

First-order conditions lead to the following equations:

$$\sigma_1 \frac{\dot{\bar{C}}}{\bar{C}} = i - \pi_e - \rho \quad (4.5.2)$$

$$\frac{W}{\bar{P}e} = \gamma_L \frac{\bar{C}^{\sigma_1}}{(1-L)^{\sigma_2}}. \quad (4.5.3)$$

The second equation gives a nominal wage. The wage should not be sticky since the right-hand side can jump.

Following Friedman (1969), assume households form expected prices in an adaptive way:

$$\dot{\bar{P}e} = \kappa(\bar{P} - \bar{P}e). \quad (4.5.4)$$

This can be related to modern learning ideas. It can be transformed to $\dot{\pi}e = \kappa(\pi - \pi e)$ using an inflation rate π .

<Firms>

There are m firms, each of which produces differentiated goods as a monopoly. A firm j maximises its present-valued profits:

$$\max \int_t^\infty e^{-R(s-t)} [P_j Y_j - W L_j - \bar{P}(I_j + J(I_j, K_j))] ds.$$

Capital evolves as

$$\dot{K} = I - \delta K. \quad (4.5.5)$$

Constructing the same Hamiltonian as before yields

$$\bar{P}Y_L = (1 + \theta)W. \quad (4.5.6)$$

Real wages are equal to the marginal product of labour divided by mark-up. The law of motion with respect to investment is described as

$$\Psi''(X)\dot{X} = (i - \pi + \delta)(1 + \Psi'(X)) - Y_K + \Psi(X) - X\Psi'(X), \quad (4.5.7)$$

using $X = I/K$. Capital evolves as

$$\dot{K} = (X - \delta)K. \quad (4.5.8)$$

We assume that firms form the expectation of an inflation rate π_e in the same manner as households. Investment dynamics is given by

$$\Psi''(X)\dot{X} = (i - \pi_e + \delta)(1 + \Psi'(X)) - Y_K + \Psi(X) - X\Psi'(X). \quad (4.5.9)$$

<Monetary Policy Rule>

A monetary policy rule is considered as

$$i = \max \left\{ \phi_y \left(\frac{\bar{Y}}{Y^*} - 1 \right) + \phi_\pi (\pi_e - \pi^*) + \rho + \pi^*, 0 \right\} \quad (4.5.10)$$

with a zero bound. Y^* and π^* represent equilibrium values.

<Market clearing condition>

A goods market is cleared as

$$\bar{Y} = \bar{C} + \bar{I} + J(\bar{I}, \bar{K}). \quad (4.5.11)$$

System of Differential Equations

Combining the above equations, especially (4.5.2), (4.5.4), (4.5.8) and (4.5.9), we can construct the system of differential equations where the number of variables is four (i.e. capital K , investment $X = I/K$, consumption C , and an expected price level Pe). There are two jump variables which are X and C , and the others are state variables.

Actually, the eigenvalue of Pe is not as negative but as zero. Its level can be arbitrary; $Pe = 1$ instead of $Pe = 10$ has no real effect on the economy. This is because actual nominal prices and wages are perfectly flexible. No matter how much Pe changes, both actual prices and wages change instantaneously so that they can offset the change in Pe . Such a feature clearly misses an important element under sticky prices: the effect of nominal shocks on

the real side of the economy⁴. This treatment comes as a compromise in order to avoid an excess demand problem, which inevitably appears with the model of actual price stickiness in Section 4.5.2. As a result, we can treat the economy as if it has only one state variable K and two jump variables C and X .

Method of Numerical Calculation

Modifying the code by Brunner and Strulik (2002), this chapter implements non-linear numerical calculation. A dynamic path is calculated in the backward manner from a long-run unique equilibrium. This approach is useful only when there is only one predetermined state variable. Otherwise, subject to the starting point of numerical calculation, which is slightly deviated from the long-run equilibrium, hugely different dynamic paths can be obtained. In this model, thanks to the above discussion, we can neglect the level of Pe and consider there to be only one predetermined state variable K . As long as there is determinacy, the level of capital uniquely determines the other variables of X and C . Therefore, irrespective of the choice of a starting point, we can obtain nearly the same saddle-path. In order to check the validity of this method, I linearised this model and numerically calculated its dynamics using the Blanchard and Kahn (1980) method. I then obtained an identical result in the neighbourhood of equilibrium⁵.

There is a reason why this thesis adopts the Brunner and Strulik approach over the Blanchard and Kahn one. It is true that the latter is much more widely used. However, this is applicable only in a linearised model, which is not a sufficient tool to provide answers to our several questions. For instance, if there is multiplicity, we would like to investigate the dynamics between multiple equilibria beyond the neighbourhood of each equilibrium. We need to draw a global dynamic picture. Furthermore, before and after reaching a zero bound,

⁴Nevertheless, a shock in either actual prices or wages can surely affect the real side of the economy because expected prices cannot adjust quickly enough.

⁵I would like to thank Tatiana Kirsanova for the relevant code.

there is surely a kink (non-linearity) where an economy’s adjustment process changes. We would like to know what exactly happens after the coming of the zero bound, but linearised models cannot incorporate such a kink. To answer these questions, we need to keep non-linearity, and the Brunner and Strulik approach is very useful.

Numerical Results

For calibration, adjustment costs are given by

$$\Psi(X) = \frac{a}{2}(X - \delta)^2. \tag{4.5.12}$$

This chapter uses the following parameter values.

$\alpha = 0.3$	$\beta = 0$	$\delta = 0.1$	$h = 1$	$\rho = 0.02$	$\theta = 0.2$	$\pi^* = 0.02$
$\sigma_1 = 1$	$\sigma_2 = 1$	$\gamma_L = 1$	$\phi_\pi = 1.5$	$\phi_y = 0.5$	$a = 3$	$\kappa = 1$

Except for the flexibility of price expectations κ , all variables are chosen to fit the Japanese economy. See Chapter 3 for more discussion. From equation (4.5.4), a value $\kappa = 1$ implies that it takes roughly one year for people to adjust their price expectations. In the simulation, non-negative investment is assumed.

Figures 4.5.1 and 4.5.2 show the movements of some of the important variables obtained by numerical calculation. The left and right figures represent the dynamics starting from high and low levels of capital respectively. In equilibrium, all variables are normalised to be one, except that an inflation rate and a nominal interest rate converge to π^* and $\pi^* + \rho$ respectively. As the second figures from the top demonstrate, in the left (right) figure, investment is lower (higher) than depreciation, which results in a decrease (increase) in capital toward equilibrium. Since the only state variable whose eigenvalue is negative is capital, corresponding to a certain capital level, we can determine the values of other variables such as investment, consumption, and an inflation rate. According to this calculation, equilibrium is determinate, so the choice of such variables is unique.

A single saddle-path can be drawn in a phase diagram as Figure 4.5.3. A dotted line shows the saddle-path when there is no zero bound, and an asterisk point represents equilibrium.

Figure 4.5.2 demonstrates the dynamics when we assume that there is no zero bound. The most notable difference from Figure 4.5.1 is found in the graphs of consumption and a nominal interest rate. Focus on the case where capital is initially excessive, or look at the left figures. As Figure 4.5.1 shows, in the case with a zero bound, an inflation rate decreases, and a nominal interest rate reaches zero, which raises the real interest rate. Hence, households reduces their consumption and, with investment, aggregate demand plummets which, in turn, causes a drop in households' income. In contrast, if there were no zero bound, as Figure 4.5.2 shows, a nominal interest rate can become low enough to keep the real interest rate negative. This raises the level of households' consumption although their income is not high, and the level of demand does not drop much. Thus, a disastrous situation such that consumption and total output plummet can be avoided. The possibility of reaching a zero bound increases as an equilibrium inflation rate π^* decreases from 0.02 or a discount factor ρ decreases from 0.02.

As Figure 4.5.1 demonstrates, the zero bound of a nominal interest rate seems to hasten a recovery. A capital stock changes more rapidly when the economy faces the zero bound than when it does not. The time constant which is the time for capital to converge half the way to equilibrium is roughly 3 and 5 years respectively. The reason for this is because, under the zero bound, investment plummets more, which demolishes an excess capacity faster.

According to our calculation, the existence of a zero bound does not yield multiple dynamic paths. Actually, I expected to see multiplicity in the region where a zero bound comes to constrain. My original conjecture came from the fact that a passive rule tends to yield indeterminacy. However, our result is not contradictory. Determinacy can be examined by looking at the sign of eigenvalues in a matrix of the system of differential equations, but

this must be done only using the values of equilibrium. A zero bound does not occur at equilibrium. So a passive monetary policy off equilibrium does not mean indeterminacy. Of course, this reasoning cannot deny the possibility of indeterminacy. Furthermore, our program algorithm is a backward calculation from an equilibrium, so we cannot utterly exclude the possibility that another path exists. However, since a small change of a starting point from the equilibrium or from the kinked point does not make any difference to the obtained dynamic path in my calculation, it is very likely that there is no multiplicity.

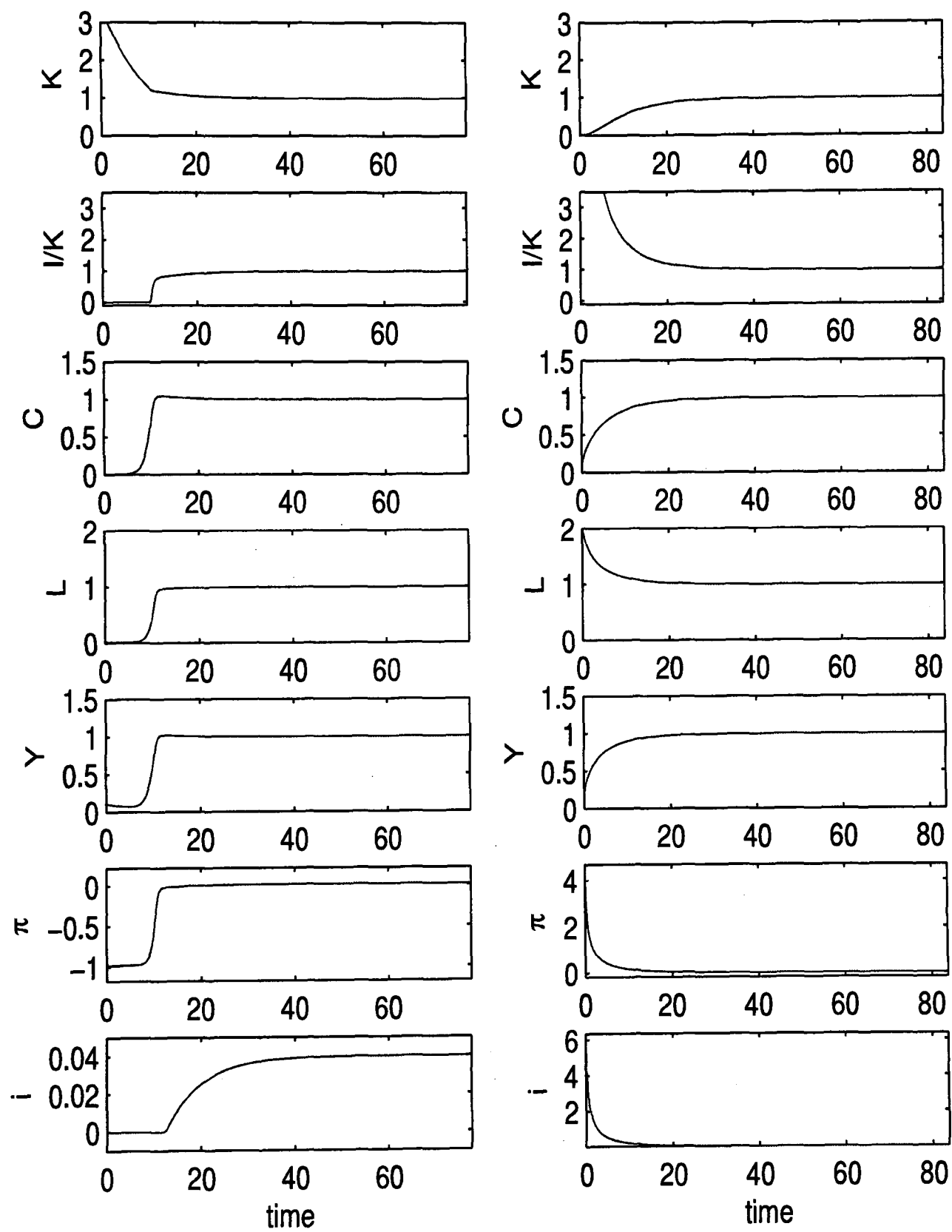


Figure 4.5.1: Dynamics under Sticky Price Expectations with a Zero Bound: left from a high level of capital, right from a low level of capital

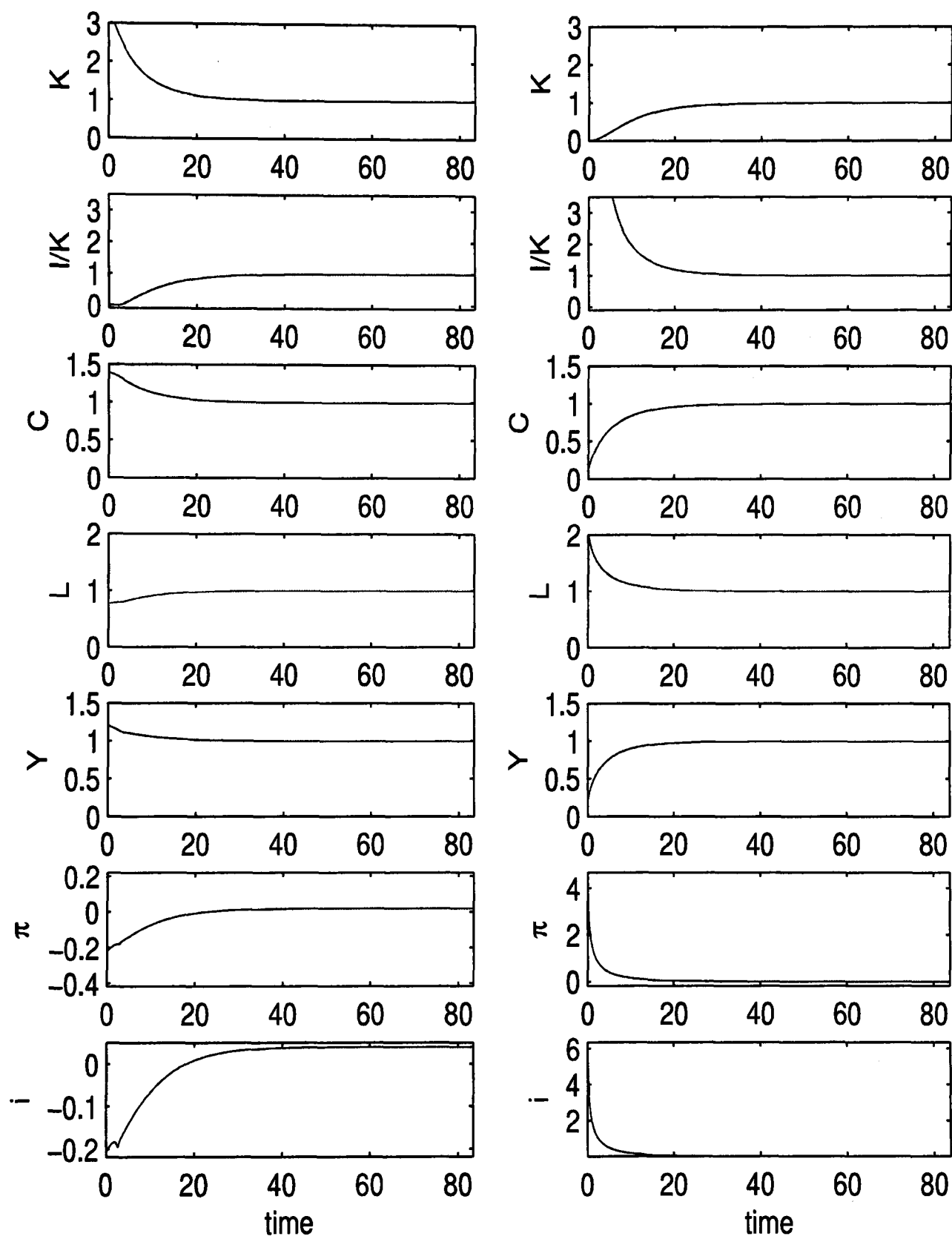


Figure 4.5.2: Dynamics under Sticky Price Expectations without a Zero Bound

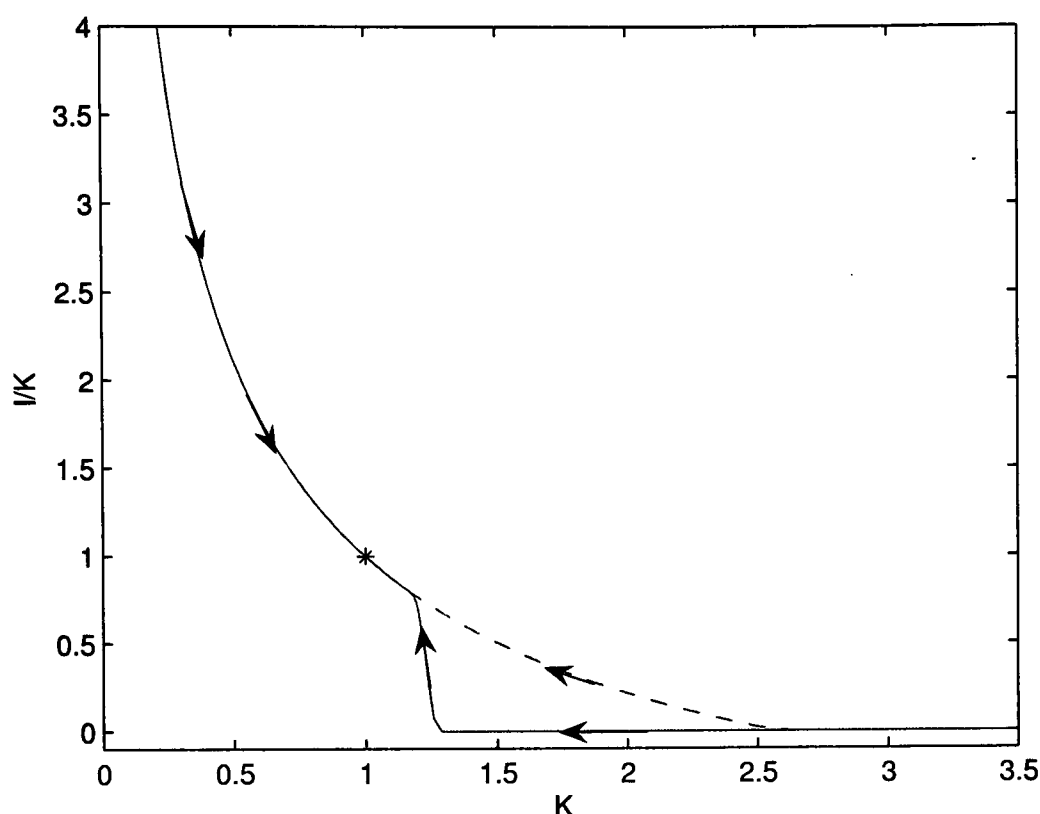


Figure 4.5.3: Saddle-Path under Sticky Price Expectations: a dotted line shows the case where there is no zero bound

4.5.2 Dynamics under Sticky Prices: New-Keynesian Approach

As another approach to make a bridge between the Krugman and the Ramsey model, this section constructs a fully micro-founded model of sticky prices using a new-Keynesian approach. In contrast to Section 4.5.1, price dynamics is derived from optimisation under perfectly rational expectations. This model has the following characteristics: (1) firm-specific investment, (2) Rotemberg-type (1982) price stickiness, (3) monopolistic competition, (4) non-linearity, and (5) a zero nominal interest rate bound. As shown in Chapter 3, the first characteristic makes calculation very complex even with the first-order approximation under Calvo-type (1983) price stickiness. This is because a heterogeneity arises among firms. In this part of the chapter, alternatively, I will use Rotemberg-type (1982) price stickiness. This is purely for tractability. As a source of price stickiness, assume firms pay the money in order to adjust prices. The costs are the same among firms, so all firms become homogenous, which makes it

much easier to implement not only first-order but also non-linear analyses. Rotemberg-type price stickiness may look less realistic than Calvo's; when firms change price, the degree of a price change would not matter. However, as a matter of fact, the features of these two models do not greatly differ⁶. It turns out that a Rotemberg method is very tractable. The third characteristic, monopolistic competition, is needed to incorporate sticky prices. Regarding monetary policy, two cases will be compared: with and without a zero nominal interest rate bound.

Non-linear analysis enables us to investigate dynamics under the existence of multiple equilibria. Moreover, it highlights one feature which has been neglected in most of the existing literature: excess demand. Under the new-Keynesian framework, equation (4.4.15) no longer holds true. Real wages can deviate from the marginal product of labour divided by mark-up. A large shock can make real wages higher than the marginal product of labour, under which situation firms lose an incentive to supply as many goods as are demanded. Such a phenomenon can be widely observed in reality. The recent boom of iPod demonstrates this example. Consumers could not purchase the product although they were eager to buy it. Even though some of them were willing to pay more money, they had no such power to modify the retail price. Meanwhile, its producer, Apple Computer, did not raise the price.

Such a problem of excess demand has been completely overlooked in the past studies. True, as long as a shock is small, this will not happen. This is because around equilibrium real wages are lower than the marginal product of labour, thanks to the positive mark-up. However, we cannot be certain how much shock is tolerable to avoid this problem. Lower mark-up or higher competition increases the possibility of excess demand. Furthermore, past welfare analyses may be wrong. Most papers modify a standard sticky price model in order to eliminate welfare distortion caused by monopoly and to make an equilibrium the first best

⁶Dupor (2001) analyses indeterminacy using a Rotemberg model, and his result is qualitatively the same as that derived from a Calvo model.

point. This modification is done by introducing fiscal subsidies, but a problem is that this treatment is equivalent to assuming zero mark-up θ although firms can still make profits from producing and selling goods. This causes excess demand to arise even for a tiny shock. Firms cease to produce as many goods as are demanded because, if they supplied goods more, their profits would decrease.

However, this phenomenon has little importance to explain Japan's crisis which we are concerned. We construct a non-linear model in order to deal with a zero bound on nominal interest rates, and this discovers the excess demand problem just as a by-product.

Furthermore, there remains a big question as to what happens under excess demand or how limited supply should be distributed among households and firms. Owing to this unanswered question, the new-Keynesian approach is not satisfactory. The previous model of sticky price expectations is still important since this model does not encounter such a problem. Using both approaches looks like a sensible compromise.

Model Setup

The model setup overlaps with the previous one to a large extent so, unless needed, I will omit the deviation.

<Households>

Households' optimisation yields

$$\sigma_1 \frac{\bar{C}}{\bar{C}} = i - \pi - \rho \quad (4.5.13)$$

$$\frac{W}{\bar{P}} = \gamma_L \frac{\bar{C}^{\sigma_1}}{(1-L)^{\sigma_2}}. \quad (4.5.14)$$

<Firms>

Introduce price stickiness as Rotemberg (1982). A firm j maximises its present-valued

profits:

$$\max \int_t^\infty e^{-R(s-t)} \left[P_j Y_j - W L_j - \bar{P}(I_j + J(I_j, K_j)) - \bar{P} \frac{\gamma}{2} \left(\frac{\dot{P}_j}{P_j} - \pi^* \right)^2 \right] ds, \quad (4.5.15)$$

where

$$Y_j^d = \left(\frac{P_j}{\bar{P}} \right)^{-\frac{1+\theta}{\theta}} \bar{Y}.$$

The last term is newly added as a quadratic cost of price adjustment. γ , and π^* represent its degree and an equilibrium inflation rate respectively. Define Hamiltonian as

$$\begin{aligned} H = & P_j Y_j - W L_j - \bar{P}(I_j + J(I_j, K_j)) - \bar{P} \frac{\gamma}{2} \left(\frac{\mu_j}{P_j} - \pi^* \right)^2 \\ & + \bar{P} Q(I_j - \delta K_j) + Q_\mu \mu_j + v_1(Y_j^d - Y_j), \end{aligned}$$

where $\mu_j = \dot{P}_j$. Its first-order conditions with respect to I_j , K_j , μ_j , P_j , and L_j are transformed to

$$Q = 1 + \Psi'(I/K) \quad (4.5.16)$$

$$\dot{Q} = (i - \pi + \delta)Q - \frac{W}{\bar{P}} \frac{Y_K}{Y_L} + \Psi(I/K) - I/K \Psi'(I/K) \quad (4.5.17)$$

$$Q_\mu = \gamma(\pi - \pi^*) \quad (4.5.18)$$

$$iQ_\mu - \dot{Q}_\mu = \bar{Y} + \gamma\pi(\pi - \pi^*) - \left(\bar{P} - \frac{W}{Y_L} \right) \frac{1+\theta}{\theta} \frac{\bar{Y}}{\bar{P}} \quad (4.5.19)$$

$$0 = P Y_L - W - v_1 Y_L. \quad (4.5.20)$$

Here, owing to a homogeneity across firms, I omit the subscript j . The first two equations yield the law of motion with respect to investment:

$$\Psi''(X) \dot{X} = (i - \pi + \delta)(1 + \Psi'(X)) - \frac{\alpha}{1 + \beta - \alpha} \frac{W L}{\bar{P} K} + \Psi(X) - X \Psi'(X), \quad (4.5.21)$$

using $X = I/K$. Capital evolves as

$$\dot{K} = (X - \delta)K. \quad (4.5.22)$$

The third and fourth equations are transformed as:

$$\dot{\pi} = (i - \pi)(\pi - \pi^*) + \frac{\bar{Y}}{\theta \gamma} \left(1 - \frac{1 + \theta}{1 + \beta - \alpha} \frac{W L}{\bar{P} Y} \right). \quad (4.5.23)$$

This equation represents inflation dynamics under Rotemberg price stickiness. The second term in the right-hand side corresponds to real marginal costs. The deviation of this inflation dynamics turns out to be much easier than that in Chapter 3 which assumes Calvo-type price stickiness. This equation looks very similar to the log-linearised form of inflation dynamics: equation (3.2.40) in Chapter 3,

$$\dot{\pi} = \rho\pi - \frac{\lambda(\rho + \lambda)}{A}(w - \bar{p} - (\bar{y} - \bar{l})), \quad (4.5.24)$$

where $A \equiv 1 + \frac{\alpha - \beta}{1 + \beta - \alpha} \frac{1 + \theta}{\theta} - \frac{\alpha}{1 + \beta - \alpha} \frac{\delta\tau_3}{\rho + \lambda + \delta\tau_2}$.

Therefore, we do not lose much information by choosing the Rotemberg price stickiness model over the Calvo one. On the contrary, there is a merit in that we can implement a non-linear analysis. The only demerit is that we cannot know a plausible parameter value of γ . In order for this, we need to use inflation dynamics in the Calvo model.

<Monetary Policy Rule>

A monetary policy rule is considered as

$$i = \max \left\{ \phi_y \left(\frac{\bar{Y}}{Y^*} - 1 \right) + \phi_\pi (\pi - \pi^*) + \rho + \pi^*, 0 \right\} \quad (4.5.25)$$

with a zero bound.

<Market clearing condition>

A goods market is cleared as

$$\bar{Y} = \bar{C} + \bar{I} + J(\bar{I}, \bar{K}). \quad (4.5.26)$$

Excess Demand

Actually, this calculation has overlooked one thing: excess demand. With this formalisation, the marginal product of labour may become lower than real wages but, in this case, firms do

not produce as many goods as people demand. Thus, an additional condition is needed:

$$\frac{W}{\bar{P}} \leq MPL = (1 + \beta - \alpha) \frac{Y}{L}. \quad (4.5.27)$$

If this becomes equality, supply becomes lower than demand; households' consumption and firms' investment are constrained by supply. In equilibrium, equation (4.5.23) implies

$$(1 + \theta) \frac{W_0}{\bar{P}_0} = MPL_0,$$

so we do not need to consider the effect of excess demand in its neighbourhood. This is why conventional linearised new-Keynesian approaches are justified without taking account of this excess demand problem. However, once the economy goes far from the equilibrium, demand-supply imbalance happens. This imposes another constraint on the consumption and investment decision.

Regarding households, consumption begins to be constrained by a limited supply:

$$\bar{C} = \bar{C}^C. \quad (4.5.28)$$

A superscript C represents constrained demand (consumption and investment) when supply is lower than demand. Substitution between consumption and labour supply implies

$$\frac{W}{\bar{P}} = \gamma_L \frac{(\bar{C}^C)^{\sigma_1}}{(1 - L)^{\sigma_2}}. \quad (4.5.29)$$

Regarding firms, construct new Hamiltonian:

$$\begin{aligned} H = & PY - WL - \bar{P}(I + J(I, K)) - \bar{P}\bar{Y} \frac{\gamma}{2} \left(\frac{\mu}{\bar{P}} - \pi^* \right)^2 \\ & + \bar{P}Q(I - \delta K) + Q_\mu \mu + \nu_1(Y^d - Y) + \nu_2(I^C - I). \end{aligned}$$

Independent variables are I , K , μ , P , and L . In the case where supply is lower than demand, $Y_d > Y_j$ and $I^C = I$. In other words, $\nu_1 = 0$ and $\nu_2 > 0$. First-order conditions with respect

to P , μ , and L are given by

$$H_P = \bar{Y} + \gamma\pi(\pi - \pi^*) = iQ_\mu - \dot{Q}_\mu$$

$$H_\mu = 0 = -\gamma(\pi - \pi^*) + Q_\mu$$

$$H_L = 0 = P \cdot MPL - W.$$

The first and second equations are simplified to

$$\dot{\pi} = (i - \pi)(\pi - \pi^*) - \frac{\bar{Y}}{\gamma}. \quad (4.5.30)$$

This is new inflation dynamics when supply is lower than optimal demand. The deviation of real marginal costs from equilibrium is positive. Thus, restricted consumption leads to a rise in a price level, which prevents a reduction of supply to some extent. The third first-order condition above means that the marginal product of labour is equal to real wages, and this is transformed to

$$\begin{aligned} (1 + \beta - \alpha) \frac{Y}{L} &= \frac{W}{P} \\ &= \gamma_L \frac{(\bar{C}^C)^{\sigma_1}}{(1 - L)^{\sigma_2}}. \end{aligned} \quad (4.5.31)$$

Finally, a market clearing condition is given by

$$Y = \bar{C}^C + I^C + J(I^C, K). \quad (4.5.32)$$

Among the three equations, (4.5.31), (4.5.32) and the production function, unknown jump variables are $\bar{C}^C$, I^C , Y and L . Therefore, there is one degree of freedom. In other words, the way to allocate the limited supply to households and firms becomes arbitrary. In this sticky price and monopolistic competition model as in other relevant models, only firms can set prices. Purchasers cannot set prices, so they cannot offer higher prices to buy more goods to satisfy their demand. Thus, a limited supply must be distributed among households and firms without price differentiation. If there is no investment, it becomes easier to answer as to how the goods are distributed among them. Although it may be doubtful whether limited

goods are equally supplied to all households, it would be natural to think that a representative household consumes as much as this limited supply. However, in the model of investment, some of the limited supply is shared with firms for investment as well as households for consumption. The way of allocation between them becomes a very difficult question for us to answer.

In this chapter, as one of many possibilities, I assume that limited supply is allocated to households firstly, and then to firms' investment. Hence, households' Euler equation holds true. The law of motion with respect to investment can be obtained as follows. Differentiating equation (4.5.31) yields

$$\left\{ \frac{\beta - \alpha}{L} - \frac{\sigma_2}{1 - L} \right\} \dot{L} = \sigma_1 \frac{\dot{\bar{C}}}{\bar{C}} - \alpha \frac{\dot{K}}{K}.$$

Differentiating equation (4.5.32) becomes

$$\begin{aligned} \dot{Y} &= \bar{Y} \left\{ \alpha \frac{\dot{K}}{K} + (1 + \beta - \alpha) \frac{\dot{L}}{L} \right\} \\ &= \bar{C} + (1 + \Psi') K \dot{X} + (X + \Psi) \dot{K}. \end{aligned} \tag{4.5.33}$$

From these equations, $\dot{X}$ can be obtained.

System of Differential Equations

I now combine the above equations, especially (4.5.13), (4.5.21), (4.5.22) and (4.5.23) in a normal case, or (4.5.13), (4.5.22), (4.5.30) and (4.5.33) in an excess demand case. Then we can construct the system of differential equations where the number of variables is four (i.e. K , $X = I/K$, C , and π). Among them, jump variables are all variables except for capital.

Numerical Results

Parameters are the same as before. In addition, the parameter of price stickiness γ is chosen to be 10. This is computed by comparing the Rotemberg inflation dynamics with Calvo's as

follows. Log-linearising the former (equation (4.5.23)) reads

$$\dot{\pi} = \rho\pi - \frac{\bar{Y}}{\theta\gamma}(w - \bar{p} - (\bar{y} - \bar{l})). \quad (4.5.34)$$

Comparing this with equation (4.5.24)

$$\dot{\pi} = \rho\pi - \frac{\lambda(\rho + \lambda)}{A}s, \quad (4.5.35)$$

we obtain the following relationship:

$$\frac{\bar{Y}}{\theta\gamma} = \frac{\lambda(\rho + \lambda)}{A}, \quad (4.5.36)$$

from which we can obtain γ given the price flexibility parameter λ in Calvo-type price stickiness. The frequency of price revision is set as once a year, $\lambda = 1$, which becomes equivalent to $\gamma = 10$.

Figures 4.5.4 and 4.5.5 show the results by numerical calculation⁷. These are very similar to the previous figures which were obtained using the assumption of sticky price expectations: Figures 4.5.1 and 4.5.2. Thus, we can deduce the same implications as before regarding the effect of a zero bound and the relation with the Krugman multiple equilibria. The time constant is roughly 3 years when the economy faces the zero bound. This length is shorter than that when the economy does not face the zero bound, which is about 5 years. Figure 4.5.6 demonstrates a saddle-path which converges to equilibrium. A dotted line represents the path when there is no zero bound.

As a notable difference, excess demand occurs when the level of initial capital is very low: $K < 0.5$ ⁸. Corresponding to the right case in Figure 4.5.4, Figure 4.5.7 demonstrates the movement of real wages (a solid line) and the marginal product of labour (MPL: a dashed line) in two kind of horizontal scales: time and capital. A dotted line represents the equilibrium values of real wages and MPL in the case when the level of capital in each period is in

⁷In addition, I implemented numerical calculation neglecting the effect of excess demand. In other words, firms are always willing to hire enough people to produce goods. In this case, the dynamics exhibited explosion. This is because labour exceeded one and the utility function came to make no sense.

⁸In this figure, this is equivalent to $t < 35$, but what is important is not the timing but the capital level. The figure shows the dynamics in which capital starts from around zero, though it does not need to be this.

equilibrium. In other words, they express the equilibrium values at that capital level before a shock. As shown in the figure, at $K < 0.5$, firms stop hiring a sufficiently large number of people because otherwise MPL would be lower than real wages. This renders MPL equal to real wages, and demand is constrained by a limited supply.

Such a situation, for example, can be caused by a large positive technology shock which increases an equilibrium capital level. In reality, this case applies to the recent iPod boom. The invention of the music gadget stimulated consumers, but its manufacturing capacity was considered to be much too short. This resulted in restraining demand. Until supply came to meet the demand, an increase in capacity would be needed, and this would take time. There were, of course, alternative options for adjusting the demand-supply imbalance such as raising its price or by hiring a sufficiently large number of people to produce the goods. However, this did not happen. This event seems to be very good evidence for this model. Our study reflects the important limitation of conventional sticky price approaches (both Calvo and Rotemberg) as a device for describing nominal inertia.

However, we do not argue that this is important in order to understand the Japanese economy. The situation is rather opposite in that demand has been too low.

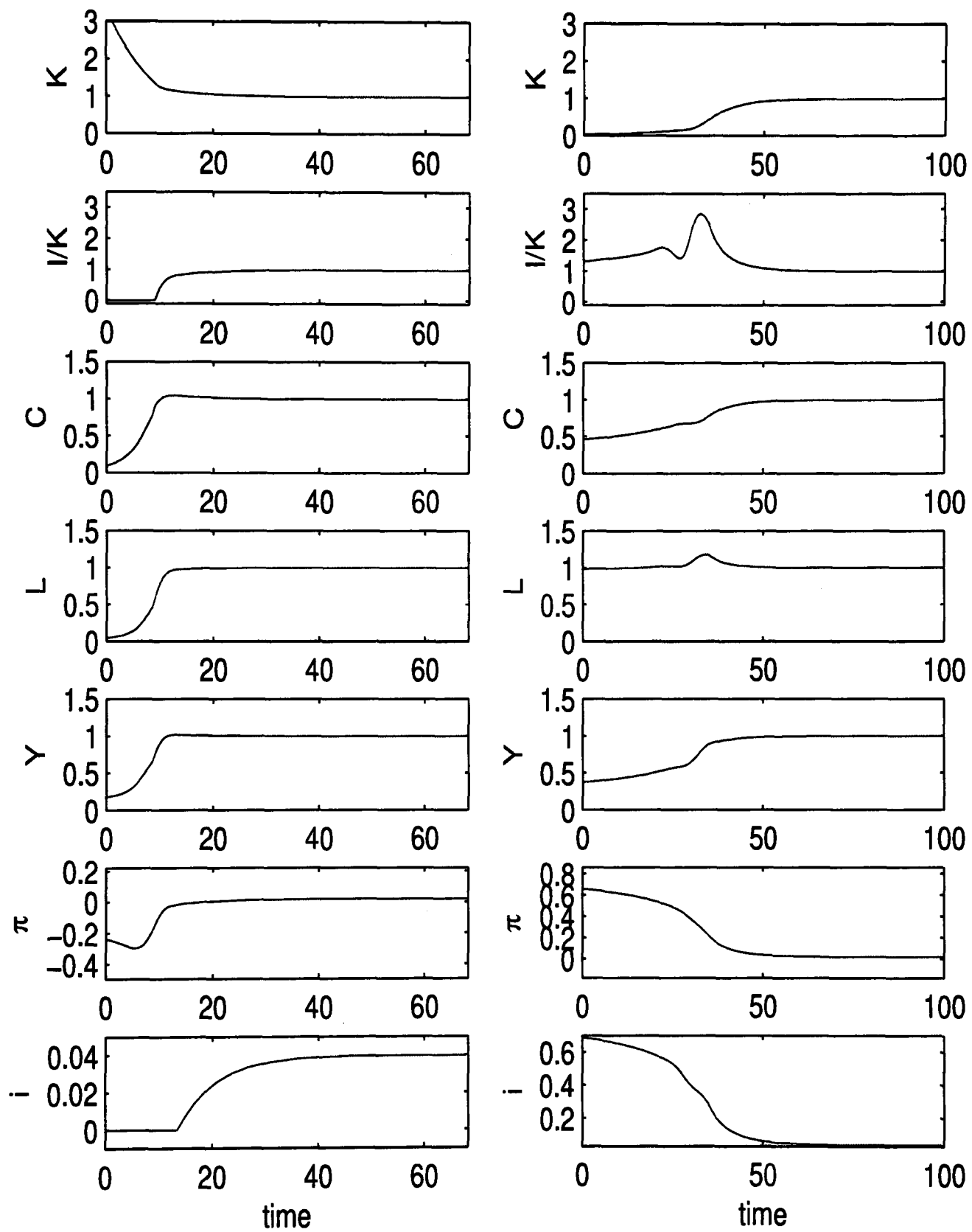


Figure 4.5.4: Dynamics under Sticky Prices with a Zero Bound

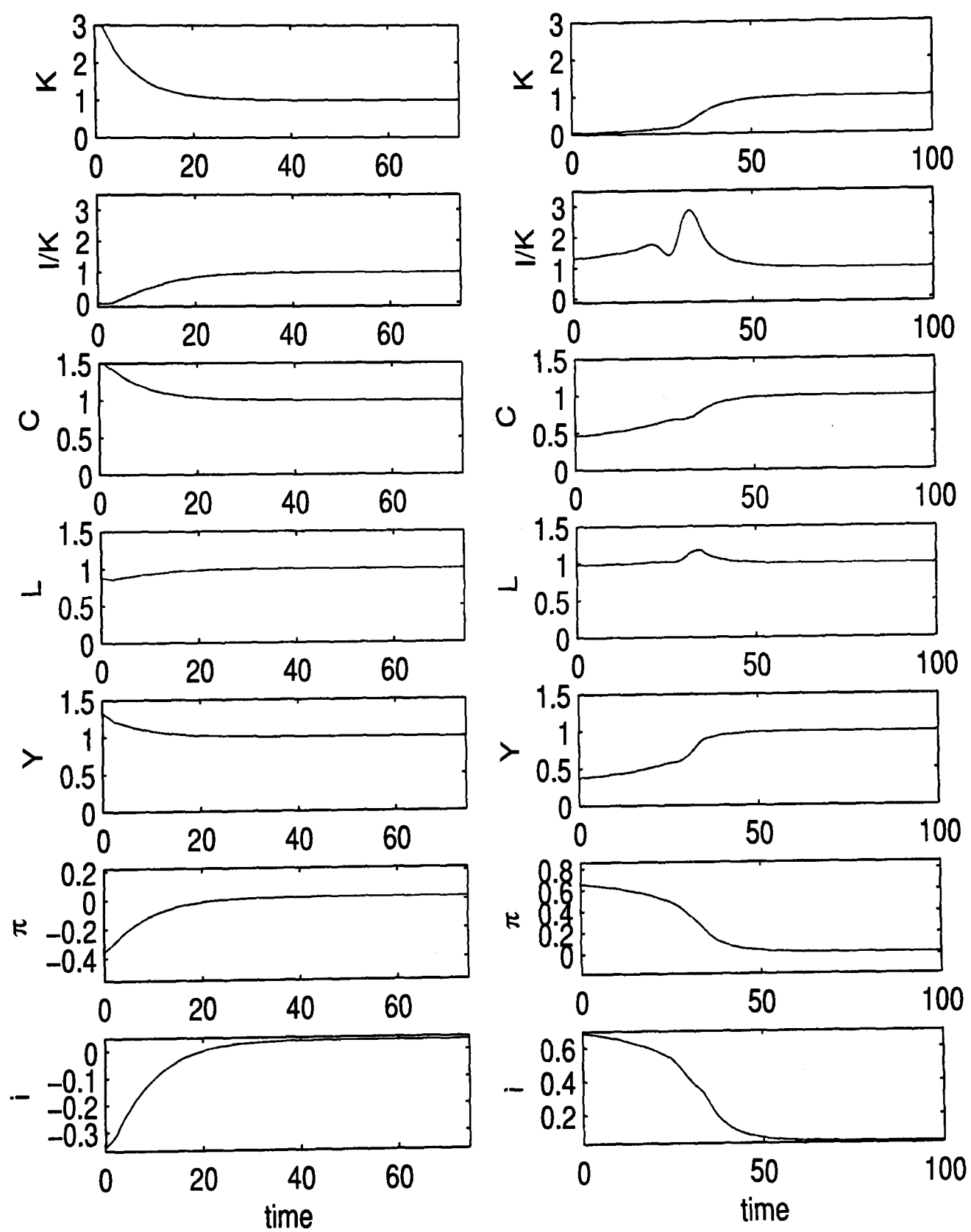


Figure 4.5.5: Dynamics under Sticky Prices without a Zero Bound

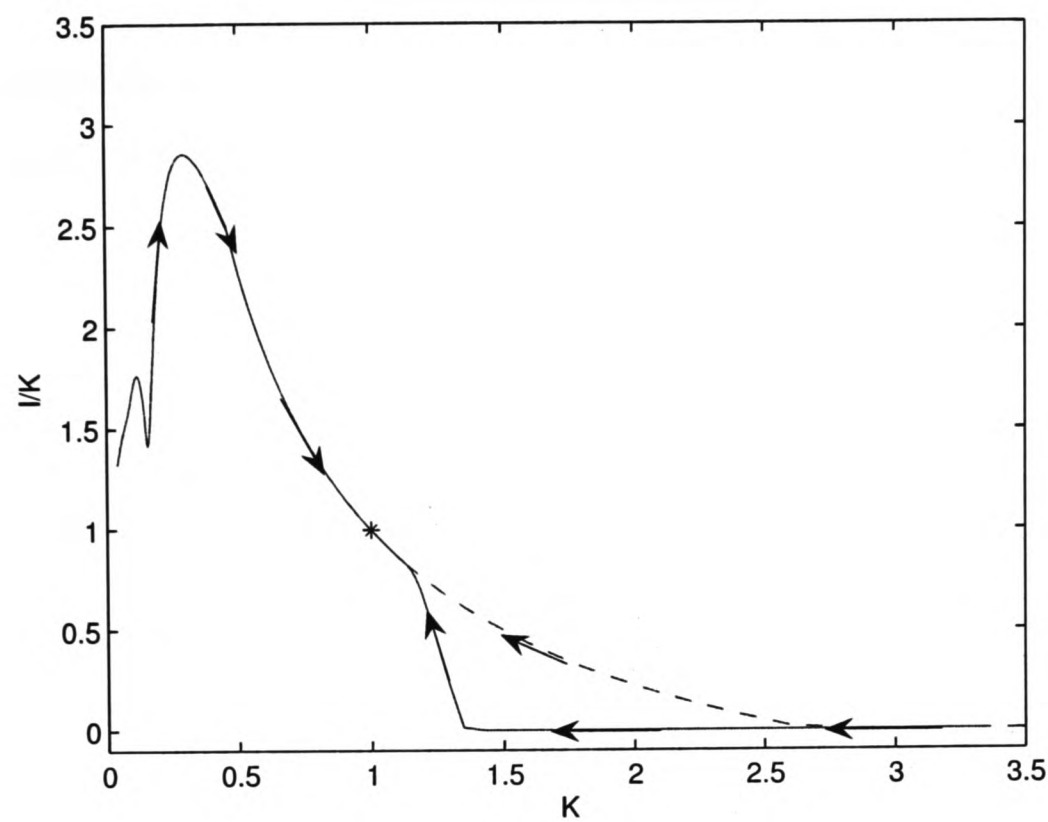


Figure 4.5.6: Saddle-Path under Sticky Prices

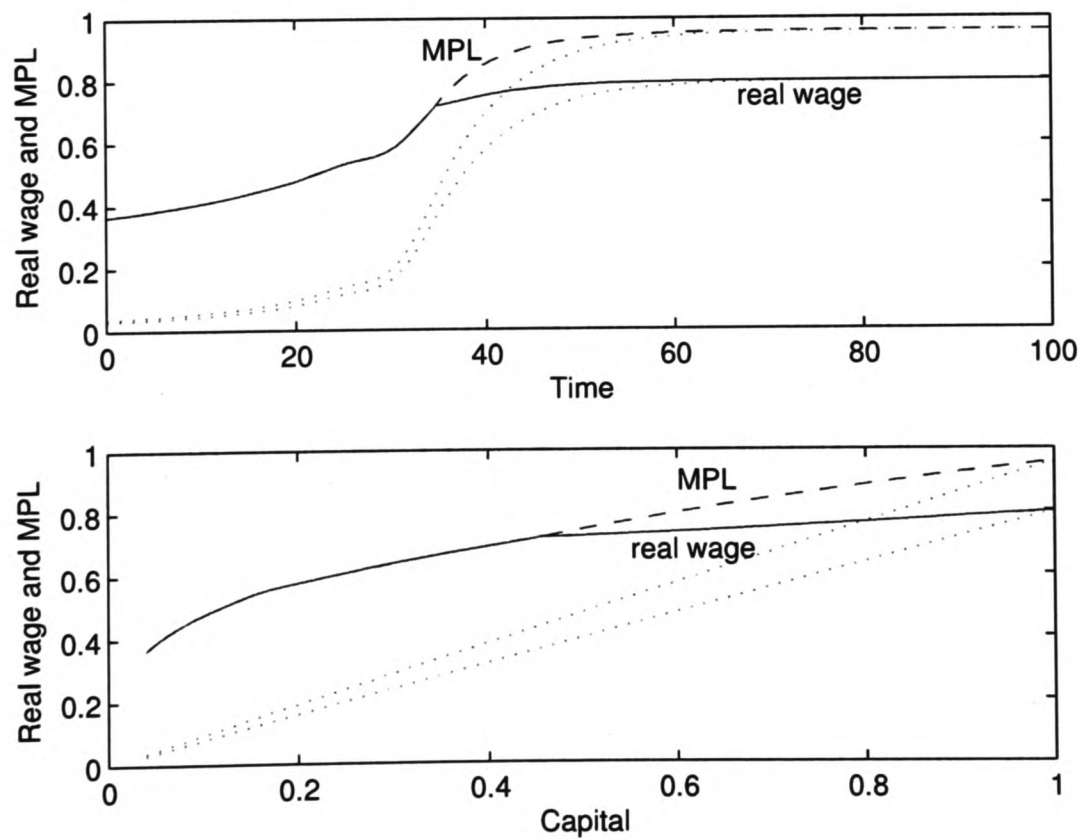


Figure 4.5.7: Real Wages and the Marginal Product of Labour

4.5.3 Comparison between Two Not Perfectly Flexible Price Models

This chapter presented two models with not perfectly flexible prices, sticky price expectations and sticky prices. Which model is the better one? Personally, I am inclined to the sticky price model.

In order to explain why, let me summarise the merits and the demerits of each model:

	Merits	Demerits
Sticky price expectation model	· not ambiguous non-linearity	· lack of a micro-foundation and rationality · perfectly flexible actual prices/wages · no real effect of a change in a price expectation level
Sticky price model (Rotemberg)	· sound micro-foundation and rationality · incorporate excess demand	· ambiguous non-linearity caused by excess demand

The sticky price model has many advantages over the sticky price expectation model. The former is well mico-founded. All agents are perfectly rational, which is a good basic framework and helps avoid an ad-hoc assumption about expectations. A price level change clearly has an effect on the real side of the economy through price stickiness. When demand becomes high, real wages come to exceed the marginal product of labour. Then, firms lose the incentive to hire more people, which lowers a good supply compared to demand. Such a situation can happen in the real world, but this has been overlooked in existing literature. In this respect, incorporating this is a very important contribution made by this chapter.

However, a problem is that we cannot tell what exactly will happen under excess demand. The way of distribution of goods to demanders becomes arbitrary, which yields an ambiguous non-linearity in the model. We have to rely on an ad-hoc assumption to proceed with the

calculation.

Nevertheless, in normal circumstances, our analyses from the model are valid. Unless there is a huge shock, real wages are lower than the marginal product of labour because of firms' monopolistic power, which does not lead to excess demand. In this sense, our dissatisfaction about ambiguous non-linearity has only second-order importance.

In contrast, the sticky price expectation model can play the role only of compensating this ambiguous non-linearity because excess demand does not arise. In other respects, this model does not look to outperform the sticky price model. There are many demerits, as are given in the above table.

For these reasons, I would choose the sticky price model over the sticky price expectation model. In fact, except for the situation of excess demand, the results of these models are qualitatively quite similar, so my choice may not matter much. However, if there occurs any significant difference between the two, this chapter would like to adopt the sticky price model.

4.5.4 Comparison with the Krugman Model

The last section compared the properties of the two not perfectly flexible price models. Instead, this section puts these two models together, and compares them with the Krugman model.

In terms of the movement of consumption, these two sticky price models look very similar to the Krugman model. In these models, under the existence of a zero bound, consumption and income appear to simultaneously decrease to zero. This can be observed in the left of Figures 4.5.1 and 4.5.4 where a capital level is initially excessive. Alongside, investment also plummets, and capital gradually decreases. This is exactly what Krugman's short-run equilibrium addresses. Recall that, in the Krugman model, we assumed the Keynesian consumption function, $C = cY$. This additional assumption was necessary to yield multiple

CHAPTER 4. ZERO NOMINAL INTEREST RATE BOUND AND MULTIPLE EQUILIBRIA

equilibria. However, in these sticky price models, we do not need to assume the Keynesian consumption function. Without this assumption, consumption can become low in proportion to low income. The mechanism was discussed at the beginning of this Section 4.4. When capital is excessive, investment decreases. This decreases demand, which causes deflation. A central bank lowers a nominal interest rate, which eventually reaches zero. The combination of deflation and the zero nominal interest rate leads to a positive real interest rate, which dampens consumption from the Euler equation. Hence, as a result, consumption appears to decrease with output. In contrast, recall our previous result that the model of the Euler equation and the fixed price expectations did not produce such a property. Since the model neglects deflation, a real interest rate does not become positive. Hence, consumption does not decrease, and behaviour like that of the Keynesian consumption function is not observed.

Also in the case with no zero bound, the Krugman model and these not perfectly flexible price models would share a similar feature. In the former model, Krugman's bad equilibrium would vanish. Monetary policy would be always effective, which could prevent asset prices from collapsing. Then, investment and income would not drop and, from the Keynesian consumption function, neither would consumption. Similarly, in the latter not perfectly flexible price models, without the zero bound, consumption would not decrease. An unlimited decrease in a nominal interest rate could render a real interest rate negative which, from the Euler equation, would help stop a consumption drop.

Judging from these similarities, what we thought was needed to yield the Krugman multiple equilibria, the Keynesian consumption function and the fixed price expectations, seems actually unnecessary. True, as we saw, the relaxation of *either* of two assumptions leads to the elimination of the Krugman bad equilibrium. However, the relaxation of *both* assumptions reproduces some of the characteristics of the Krugman bad equilibrium.

*

However, there are two important differences between these two models. Firstly, in the Krugman model, multiple equilibria exist. On the other hand, in these not perfectly flexible price expectation models, there is only one equilibrium. Even in an adjustment process, the dynamics has only a unique saddle-path. Such uniqueness implies that Krugman's multiple equilibria do not affect either the long-run outcome or a dynamic path. The behaviour under a zero nominal interest rate just happens to look similar between the two models.

Secondly, the existence of a zero bound to a nominal interest rate plays a totally different role. In the Krugman model, a zero bound is the indispensable cause of the bad short-run equilibrium. The ineffectiveness of monetary policy raises real interest rates, which yields Krugman's bad equilibrium. In contrast, in the sticky price models, the zero bound is not a cause but just a consequence of excessive capital. This situation occurs in the adjustment process where an economy shrinks to an equilibrium by demolishing excessive capital. A decrease in demand causes deflation and, in turn, results in the zero bound only as a consequence. It is true that, in the short-run, the existence of a zero bound functions to further worsen the economy. The zero bound leads to a rise in real interest rates, which discourages investment and consumption even more. However, a fundamental reason for this depression is not the zero bound but excessive capital which may be partly caused by a negative technology shock.

These differences may lead to the following rather extreme implication: a zero bound is not a problem. The economy can converge to the unique equilibrium sooner or later although, in the short-run, the economy becomes disastrous. On the contrary, it might be better to reach a zero bound because its bound seems to hasten a recovery. In both models with not perfectly flexible prices, a time constant decreases from 5 to 3 years after the economy reaches the zero bound. This is because investment plummets, which demolishes an excess capacity

faster. Recessions can finish quickly⁹.

*

Such an implication is hard to accept. The existence of a zero bound may well cause a loss even in the long-run. As our models demonstrated, the standard Ramsey model and its variations denied this possibility, but the Krugman model still seems to be very important.

Is there any other way to support the original Krugman idea? Can we assure that a zero bound is a cause of the bad equilibrium? One possible idea is to maintain the assumption of fixed prices, separating them not only from flexible prices but also from some degree of price stickiness, and to insist on the importance of the Krugman model. This approach seems to be shared among some Keynesian economists, especially Hicks (1965) and Malinvaud (1977, 1984). Prices move slowly, so households and firms may regard prices as being completely fixed. However, there is no strong justification to stick to the fixed-price assumption, so currently dominant economics will not adopt such a view. Rational agents may well not be bothered by Krugman's multiple equilibria model.

As the second approach, the next section introduces a multiple equilibria model by Kiyotaki (1988).

4.6 Kiyotaki Model: Necessary Conditions for Understanding the Japanese Economy

The previous models of partially sticky prices do not have any multiplicity. Although its short-run dynamics has a similar property such that consumption decreases with income, this result suggests that Krugman's multiple equilibria should not affect a long-run outcome.

⁹Of course, it is necessary that this be checked by a welfare analysis.

However, things may change if there are multiple equilibria in the long-run. Short-run multiple equilibria may influence an outcome in the long-run.

Moreover, looking at the Japanese economy, a depression longer than a decade may well not be regarded as temporary. Such a situation seems to be at one equilibrium which differs from the one before 1990. However, the above Ramsey model has only a unique equilibrium, so cannot well explain this prolonged depression. The model of long-run multiple equilibria can be a good tool to understand the country's experience.

This section proposes such a possibility using Kiyotaki (1988), which produces the model of multiple equilibria with a general equilibrium framework by assuming the technology of increasing returns to scale (IRS). In a good equilibrium, a representative household supplies all its available time to labour, and the level of capital and welfare becomes high. In a bad equilibrium, labour supply, capital and welfare are all low.

This chapter modifies his model in several respects in order to examine dynamics explicitly. The original Kiyotaki model consisted of two-periods. There were no adjustment costs, and dynamic behaviour was implicit and limited in an intuitive conjecture. In this chapter, I instead use an infinite time horizon and introduce the adjustment costs of investment¹⁰. Moreover, I use more general assumptions regarding labour elasticity than Kiyotaki although he mentions this in a footnote to his paper¹¹.

4.6.1 With Perfectly Flexible Prices: Emergence of Multiple Equilibria

Model Setup

Firstly, assume that prices are perfectly flexible. Model setup is as follows:

<Households>

¹⁰This extension seems to have been previously conducted by Mohamad Hammour. However, this has not been published and, despite my several inquiries, I have not yet received the paper.

¹¹Footnote 3 in page 699.

A representative household supplies one's labour L up to N . Utility is given by the form of $u(\bar{C} - vL^{1+\frac{1}{\varepsilon}})$, where v is a parameter and ε is the elasticity of labour supply. This form differs from that in the previous Ramsey models. Such modification is necessary to yield multiple equilibria as will soon be seen¹². Furthermore, we also need to assume a relatively high ε . Utility maximisation is described as

$$\max \int_t^\infty u(\bar{C} - vL^{1+\frac{1}{\varepsilon}}) e^{-\rho(s-t)} ds$$

subject to

$$\int_t^\infty \bar{P}\bar{C}e^{-R(s-t)} ds \leq A(t) + \int_t^\infty (WL + \Pi)e^{-R(s-t)} ds.$$

An aggregated consumption index is given by

$$\bar{C} = m^{-\theta} \left(\sum_j^m C_j^{\frac{1}{1+\theta}} \right)^{1+\theta} \quad (4.6.1)$$

Denote the degree of risk aversion σ and

$$u(\bar{C} - vL^{1+\frac{1}{\varepsilon}}) = \frac{(\bar{C} - vL^{1+\frac{1}{\varepsilon}})^{1-\sigma} - 1}{1-\sigma}.$$

Its first-order conditions become

$$0 = u' - \lambda \bar{P} e^{(\rho-i)(s-t)} \quad (4.6.2)$$

$$0 = -v(1 + \frac{1}{\varepsilon})u' + \lambda W e^{(\rho-i)(s-t)}. \quad (4.6.3)$$

Hence, regarding labour supply $L^s = L^s(W/\bar{P})$, we obtain the following result:

$$\begin{cases} W/\bar{P} = (1 + \frac{1}{\varepsilon})vL^{\frac{1}{\varepsilon}}, & \text{then } L < N \\ W/\bar{P} > (1 + \frac{1}{\varepsilon})vN^{\frac{1}{\varepsilon}} & \text{then } L = N. \end{cases} \quad (4.6.4)$$

Finally no-Ponzi game version of the budget constraint is

$$\lim_{t \rightarrow \infty} e^{-R(t-s)} A(t) \geq 0. \quad (4.6.5)$$

<Firms>

¹²In order that this chapter be consistent, it may be better to use this new utility form throughout. However, the former form is consistent with that in Chapter 3.

A demand curve is written as

$$Y_j = \left(\frac{P_j}{\bar{P}} \right)^{-\frac{1+\theta}{\theta}} \bar{Y}. \quad (4.6.6)$$

A production function is

$$Y_j = hK_j^\alpha L_j^{1+\beta-\alpha}, \quad (4.6.7)$$

where β is positive if technology exhibits increasing returns to scale (IRS). Firms maximise the discounted value of their profits:

$$\max \int_t^\infty e^{-R(s-t)} [P_j Y_j - W L_j - \bar{P}(I_j + J(I_j, K_j))] ds,$$

subject to

$$\dot{K}_j = -\delta K_j + I_j. \quad (4.6.8)$$

Define Hamiltonian as

$$H = P_j Y_j - W L_j - \bar{P}(I_j + J(I_j, K_j)) + \bar{P}Q(I_j - \delta K_j),$$

and first-order conditions become

$$\frac{1}{P_j} \frac{\partial H}{\partial L_j} = 0 = \frac{1+\beta-\alpha}{1+\theta} h K_j^\alpha L_j^{\beta-\alpha} - \frac{W}{P_j} \quad (4.6.9)$$

$$\frac{1}{\bar{P}} \frac{\partial H}{\partial I_j} = 0 = -(1 + J_I) + Q$$

$$\therefore Q = 1 + \Psi'(I_j/K_j) \quad (4.6.10)$$

$$\begin{aligned} \frac{\partial H}{\partial K_j} &= iQ\bar{P} - (\dot{Q}\bar{P} + Q\dot{\bar{P}}) \\ &= \frac{\alpha}{1+\theta} P_j h K_j^{\alpha-1} L_j^{1+\beta-\alpha} - \bar{P}J_K - \bar{P}Q\delta \\ \therefore \dot{Q} &= (i - \pi + \delta)Q - \frac{\alpha}{1+\theta} \frac{P_j}{\bar{P}} h K_j^{\alpha-1} L_j^{1+\beta-\alpha} + \Psi(I_j/K_j) - I_j/K_j \Psi'(I_j/K_j). \end{aligned} \quad (4.6.11)$$

A transversality condition is

$$\lim_{t \rightarrow \infty} e^{-R(t-s)} Q(t) \bar{P}(t) K(t) = 0. \quad (4.6.12)$$

<Market clearing>

$$\bar{Y} = \bar{C} + \bar{I} + J(\bar{I}, \bar{K}). \quad (4.6.13)$$

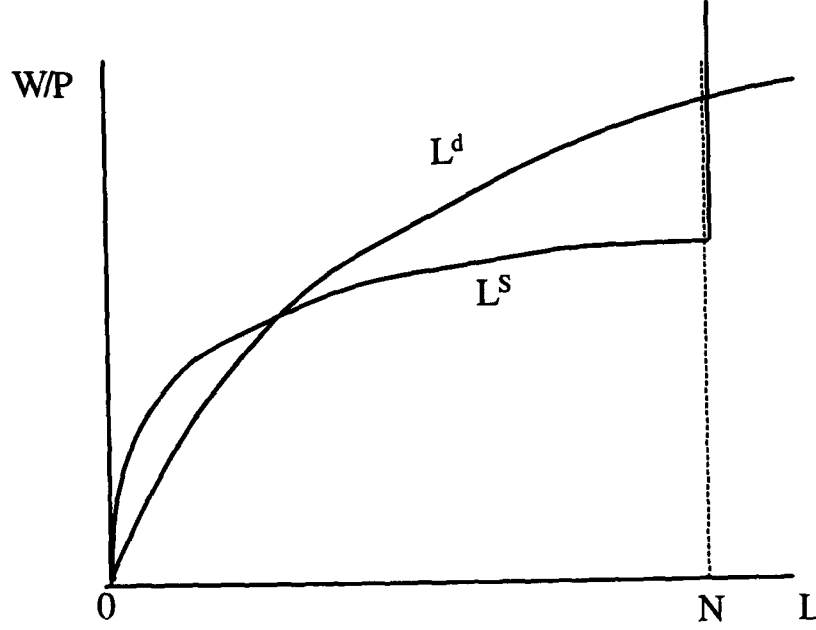


Figure 4.6.1: Multiple Equilibria in the Kiyotaki Model

Multiple Equilibria

In equilibrium, $\dot{Q} = 0$ and $P_j = \bar{P}$. Moreover, as we will see, labour is bounded either at $L = 0$ or N . From equations (4.6.10) and (4.6.11), we can obtain

$$(i - \pi + \delta)(1 + \Psi'(\delta)) = \frac{\alpha}{1 + \theta} h K^{\alpha-1} L^{1+\beta-\alpha} - \Psi(\delta) + \delta \Psi'(\delta).$$

With equation (4.6.9), deleting K yields the labour demand function $L^d = L^d(W/\bar{P})$:

$$\begin{aligned} \frac{W}{\bar{P}} &= \frac{h(1 + \beta - \alpha)}{1 + \theta} \left[\left\{ (i - \pi + \delta)(1 + \Psi'(\delta)) + \Psi(\delta) - \delta \Psi'(\delta) \right\} \frac{1 + \theta}{h\alpha} L^{-(1+\beta-\alpha)} \right]^{\frac{\alpha}{\alpha-1}} L^{\beta-\alpha}. \\ &= (1 + \beta - \alpha) \left(\frac{h}{1 + \theta} \right)^{\frac{1}{1-\alpha}} \left[(i - \pi + \delta)(1 + \Psi'(\delta)) + \Psi(\delta) - \delta \Psi'(\delta) \right]^{\frac{\alpha}{\alpha-1}} L^{\frac{\beta}{1-\alpha}}. \end{aligned} \quad (4.6.14)$$

Equations (4.6.4) and (4.6.14) are drawn as in Figure 4.6.1, which is the case of multiple equilibria. Stable equilibria are at $L = 0$ and N .

There are two necessary conditions for multiple equilibria. The first one is for the stability of the bad equilibrium at $L = 0$. The slope of the labour supply curve has to be higher than

that of the labour demand curve. The reason is as follows. If this condition is satisfied, as in Figure 4.6.1, at a certain low level of real wages (but not zero), labour demand becomes higher than labour supply. Thus, firms can hire only a limited number of employees. The decrease in employment is also followed by a decrease in capital and, owing to IRS technology, the marginal product of labour decreases. This decreases real wages, and forces firms to contract employment even further. Mathematically, the condition is written as follows.

Condition 1 for multiple equilibria:

$$\frac{1}{\varepsilon} < \frac{\beta}{1 - \alpha}. \quad (4.6.15)$$

This condition is satisfied if either the elasticity of labour supply, ε , or the scale of increasing returns, β , is high. It is clear that we require $\beta > 0$ or increasing returns to scale.

Actually, it is not certain whether both conditions are satisfied in reality. However, it is also difficult to assert that this condition is violated. Although an estimated β is usually not far from 0, there are not a few empirical results which support that β is positive. It seems much more difficult to estimate ε than β , and Japan's elasticity may be lower than those in the U.S. and Britain considering Japan's inflexible labour mobility. However, these things do not deny the possibility of multiple equilibria, and we may argue that all the countries have multiple equilibria by satisfying the above condition.

The good equilibrium at $L = N$ is stable. This is because L is bounded at N , and the marginal product of capital becomes decreasing with capital. Thus, an incremental increase in capital from the equilibrium lowers the marginal product of capital, which works to decrease investment and in turn capital. Hence, destabilising expectations about investment cannot be self-fulfilling.

As the other condition, L^d needs to intersect L^s at $L = N$, which is satisfied if $L^d(N)^{-1} > L^s(N)^{-1}$, that is,

Condition 2 for multiple equilibria:

$$(1 + \beta - \alpha) \left(\frac{h}{1 + \theta} \right)^{\frac{1}{1-\alpha}} [(i - \pi + \delta)(1 + \Psi'(\delta)) + \Psi(\delta) - \delta\Psi'(\delta)]^{\frac{\alpha}{\alpha-1}} N^{\frac{\beta}{1-\alpha}} > (1 + \frac{1}{\varepsilon})vN^{1+\frac{1}{\varepsilon}}. \quad (4.6.16)$$

Then a stable equilibrium exists at $L = N$. The higher the scale of increasing returns, the more likely multiple equilibria are to arise. High productivity h or low real interest rates also enable the yielding of this equilibrium by raising labour demand.

At first glance, it may seem that a bad equilibrium should not be chosen because it is not desirable. However, there is the problem of coordination failure. As in Chapter 2, although a household is representative, because of monopolistic competition, there are a large number of firms. Therefore, it becomes very difficult for these firms to coordinate their actions. If everyone else chooses not to invest, one firm suffers a loss from investment because aggregate demand is extremely low¹³. For these reasons, a bad equilibrium becomes a stable equilibrium as well as a good equilibrium. In order to find how an equilibrium is selected, the approach of global games may be applied as was discussed in Chapter 2, but this is not an objective of this chapter.

System of Differential Equations

As an advantage over Kiyotaki (1988), who had only two periods and no investment adjustment costs, we can explicitly examine a capital adjustment process. Firstly, find the dynamics in an analytical form. Summing up the above equations, non-linear dynamics can

¹³In this sense, a demand externality is an important element for multiple equilibria.

be described as follows.

$$\dot{K} = (X - \delta)K \quad (4.6.17)$$

$$\frac{d}{dt}\Psi'(X) = (i - \pi + \delta)(1 + \Psi'(X)) - \frac{\alpha}{1 + \theta}hK^{\alpha-1}L^{1+\beta-\alpha} + \Psi(X) - X\Psi'(X) \quad (4.6.18)$$

$$i - \pi = \rho + \sigma \frac{\dot{C} - (1 + \frac{1}{\epsilon})vL^{\frac{1}{\epsilon}}\dot{L}}{C - vL^{1+\frac{1}{\epsilon}}}, \quad (4.6.19)$$

where $L \leq N$. The first two differential equations represent the laws of motion with respect to capital and investment. The third one is the Euler equation.

It looks as if there are three independent differential equations, but actually, independent ones number two. Either consumption or investment becomes an independent variable. This is because of perfectly flexible prices. Real wages and real interest rates, which are independent of nominal changes, mutually connect consumption and investment. Clearly, real interest rates bridge the above second and third equations. Regarding real wages, labour supply and demand functions connect consumption and investment. When $L < N$, we obtain

$$\begin{aligned} \frac{W}{P} &= \frac{1 + \beta - \alpha}{1 + \theta}hK^{\alpha}L^{\beta-\alpha} \\ &= \left(1 + \frac{1}{\epsilon}\right)vL^{\frac{1}{\epsilon}} \end{aligned} \quad (4.6.20)$$

$$L = \left[\frac{h(1 + \beta - \alpha)}{(1 + \theta)\left(1 + \frac{1}{\epsilon}\right)v}K^{\alpha} \right]^{\frac{1}{\frac{1}{\epsilon} + \alpha - \beta}}. \quad (4.6.21)$$

Here, we used a market clearing condition:

$$Y = hK^{\alpha}L^{1+\beta-\alpha} = C + XK + \Psi(X)K. \quad (4.6.22)$$

The other case is $L = N$. These conditions make C become a function of K and X . Hence, we can derive two independent differential equations with respect to K and X .

The deviation of X 's differential equation is a little cumbersome, so I show only the result.

Assume the following form of adjustment costs:

$$\Psi(X) = \frac{a}{2}(X - \delta)^2.$$

When $L = N$, differentiating the above market clearing condition and substituting this into

the law of motion with respect to investment yield

$$\dot{X} = \left[a + \sigma \frac{\{1 + a(X - \delta)\}^2}{C - vN^{1+\frac{1}{\epsilon}}} K \right]^{-1} \left[(\rho + \delta)\{1 + a(X - \delta)\} - \frac{\alpha}{1 + \theta} \frac{Y}{K} - \frac{1}{2}a(X - \delta)(X + \delta) + (\{1 + a(X - \delta)\}) \frac{\sigma}{C - vN^{1+\frac{1}{\epsilon}}} \left(\alpha \frac{Y}{K} - \left\{ X + \frac{a}{2}(X - \delta)^2 \right\} \dot{K} \right) \right]. \quad (4.6.23)$$

The condition for $L = N$ is described as

$$N < \left[\frac{h(1 + \beta - \alpha)}{(1 + \theta)(1 + \frac{1}{\epsilon})v} K^\alpha \right]^{\frac{1}{\frac{1}{\epsilon} + \alpha - \beta}} \quad (4.6.24)$$

When $L < N$, using equation (4.6.21) in addition, we obtain

$$\begin{aligned} \dot{X} = & \left[a + \sigma \frac{\{1 + a(X - \delta)\}^2}{C - vL^{1+\frac{1}{\epsilon}}} K \right]^{-1} \left[(\rho + \delta)\{1 + a(X - \delta)\} - \frac{\alpha}{1 + \theta} \frac{Y}{K} \right. \\ & - \frac{1}{2}a(X - \delta)(X + \delta) + (\{1 + a(X - \delta)\}) \frac{\sigma}{C - vL^{1+\frac{1}{\epsilon}}} \\ & \cdot \left(\alpha \frac{Y}{K} + (1 + \beta - \alpha) \frac{Y}{L} \left[\frac{h(1 + \beta - \alpha)}{(1 + \theta)(1 + 1/\epsilon)v} \right]^{\frac{1}{1/\epsilon + \alpha - \beta}} \frac{\alpha}{1/\epsilon + \alpha - \beta} K^{\frac{\alpha}{1/\epsilon + \alpha - \beta} - 1} \right. \\ & \left. \left. - \left\{ X + \frac{a}{2}(X - \delta)^2 \right\} - v(1 + 1/\epsilon)L^{1/\epsilon} \left[\frac{h(1 + \beta - \alpha)}{(1 + \theta)(1 + 1/\epsilon)v} \right]^{\frac{1}{1/\epsilon + \alpha - \beta}} \frac{\alpha}{1/\epsilon + \alpha - \beta} K^{\frac{\alpha}{1/\epsilon + \alpha - \beta} - 1} \right) \dot{K} \right]. \end{aligned} \quad (4.6.25)$$

The levels of other variables (e.g. C , L and Y) can be easily derived from the above equations.

Appendix 4.A.3 analytically demonstrates that the good equilibrium is stable and determinate and that the intermediate one is unstable. The bad one is not well-defined. Since equilibrium capital stock is zero, $X = I/K$ and Tobin's Q which is given by $Q = 1 + \Psi'(X)$ cannot be calculable. Nevertheless, with the help of numerical simulation, we can check its stability; it seems to be stable and indeterminate as we will soon see.

Numerical Calculation

Numerically, this section draws non-linear dynamic paths in the whole region without first-order approximation¹⁴. Here, non-negative investment is assumed, that is, $X \geq 0$. Parame-

¹⁴I modified the Brunner and Strulik code (2002).

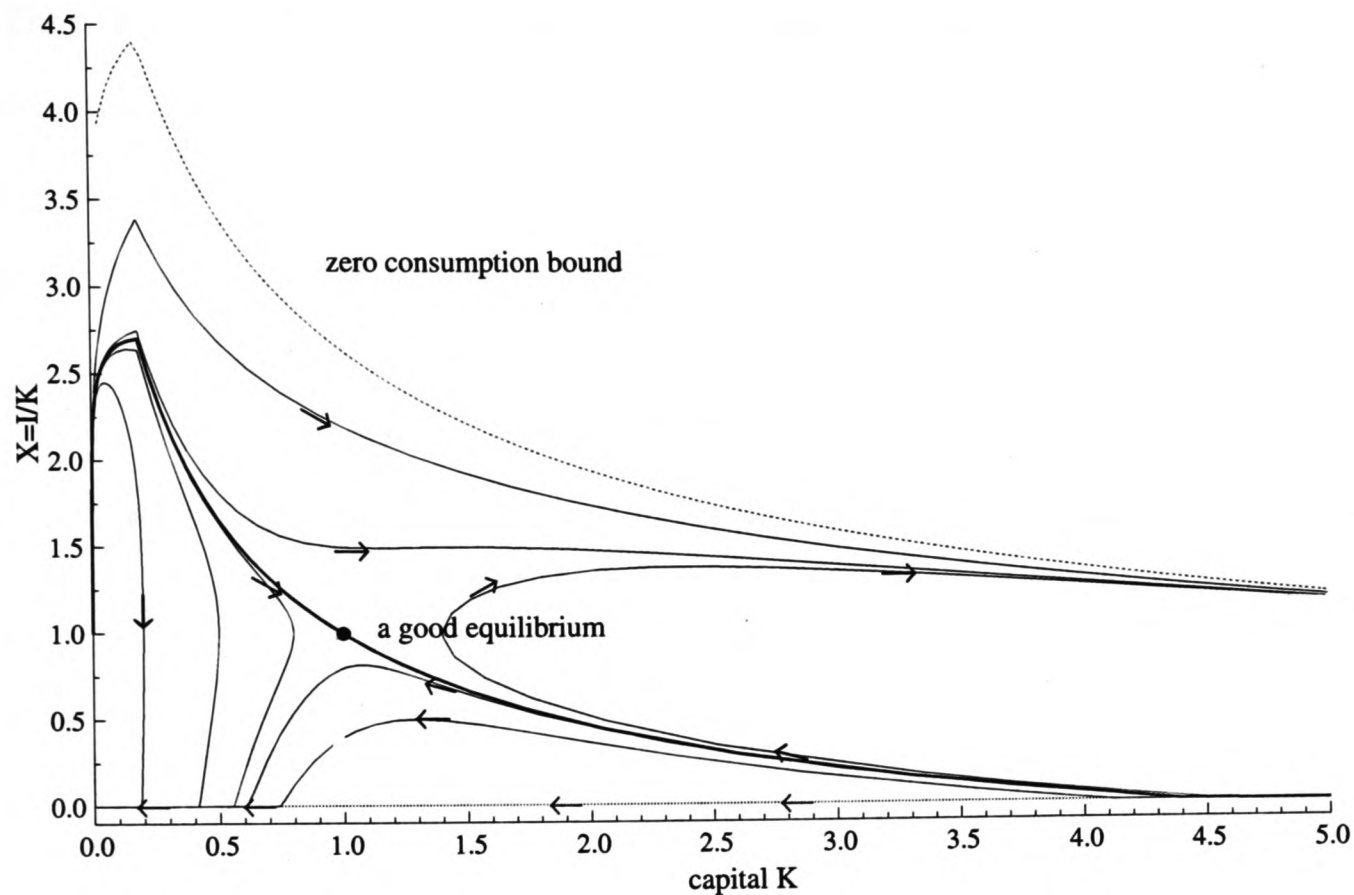


Figure 4.6.2: Kiyotaki Model Dynamics under Flexible Prices

ters' values are the same as before except for

$\beta = 0.1$	$\varepsilon = 10$	$v = 0.5$
$N = 1$	$\sigma = 1$	

Figure 4.6.2 demonstrates dynamics paths. A horizontal and a vertical axis respectively represents the ratio of K and $X = I/K$ to the values at a good equilibrium. A bad equilibrium is at $K = 0$. Regarding the paths toward the bad equilibrium, $X = 0$ seems to be a steady path but, as easily verified from analytical forms, this is not true. There is no equilibrium X , which stays constant. In other words, at null capital, an equilibrium X is not well-defined.

Around $K > 4.5$, investment reaches zero. This implies that, when an initial capital level is high enough, only the good equilibrium survives. The reason is not clear, but I suspect as follows. The constraint that investment is non-negative prevents aggregate demand from decreasing infinitely. Hence, firms can avoid suffering too much loss, which results in eliminating the possibility that excessive capital ends up with the bad equilibrium.

There are diverging paths to higher capital. However, these violate the transversality condition, so we can reject them. The dotted curve shows a bound to prevent consumption from being negative. An economy cannot go beyond the curve. A kink around $K = 0.2$ suggests that labour reaches maximum ($L = N$). At lower capital than 0.2, labour is not bounded by N . This labour constraint renders an adjustment mechanism non-linear.

There is indeterminacy, or there are an infinite number of paths toward the bad equilibrium. The reason is easily understood by the following thought experiment. Assume the expectation of an investment downturn. This decreases demand which, in turn, decreases employment. Also a decline in investment gradually decreases capital. Combined with increasing returns to scale technology, the decreases in employment and capital lower the marginal product of capital, which decreases investment. The expectation of the investment downturn thus becomes self-fulfilling. On the other hand, toward the good equilibrium, a single saddle-path exists, that is, the equilibrium is determinate. Around the good equilibrium, labour is bounded by N , which makes the technology decreasing returns to scale. Thus, a decrease in capital caused by the expectation of an investment downturn does not decrease the marginal product of capital. This denies the expectation of the investment fluctuation, and makes the good equilibrium determinate.

4.7 Variations upon the Kiyotaki Model with Not Perfectly Flexible Prices

This section, instead of perfectly flexible prices, allows prices to be partially flexible but not completely fixed. In a similar method to that in Sections 4.5.1 and 4.5.2, we can numerically investigate a propagation mechanism under not perfectly flexible prices. Since a labour supply function changes between $L < N$ and $L \geq N$, there arises another non-linearity. In this sense,

the model becomes more complex.

There is one technical trick for the analysis of this model, which is discussed in Appendix 4.A.4. In short, this section allows a slight temporary increase in labour supply over N as employees work overtime. By paying extra compensation, firms can increase employment when demand increases. To match this amendment, a consistent utility function will be introduced.

4.7.1 Dynamics under Sticky Price Expectations

Firstly, assume sticky price expectations as before such that people gradually adjust their expected prices. We can again construct four independent differential equations with respect to K , C , X , and P_e . The number of jump and state variables are the same as before. Jump variables are C and X . The state variable whose eigenvalue is negative is only one, K . Thus, the backward calculation method is valid to find a saddle-path.

Figure 4.7.1 demonstrates the movements of several variables over time. The values at Kiyotaki's good equilibrium are normalised to be one except for an inflation rate and a nominal interest rate. In each figure, left and right represent the case where an initial capital level is high and low respectively. As before, when an initial capital level becomes sufficiently high, a nominal interest rate reaches zero, which raises a real interest rate and causes consumption to drop. This feature resembles the consumption movement derived from the Keynesian consumption function in the Krugman model.

Figure 4.7.2 shows various dynamic paths as well as a saddle-path toward the good equilibrium which is shown as a thick solid line. Two points with an asterisk represent equilibria. The dynamic path which converges to Kiyotaki's good equilibrium is unique. On the other hand, similar to the case of flexible prices, there are an infinite number of dynamic paths toward the bad equilibrium. There are diverging paths as well, but these can be rejected

because they violate the transversality condition. If we do not limit our attention only to the saddle-path toward the good equilibrium, given a certain capital level, not only investment but also consumption can have arbitrary values. Thus, in these two-dimensional X and K graphs, one more variable, C , is free, and dynamic paths can intersect each other.

This two-dimensional figure implies the followings. When an initial capital level is low enough, the good equilibrium cannot be achieved. On the other hand, when an initial capital level is high, there seems to be no dynamic path which makes the economy fall into the bad equilibrium¹⁵. Finally, in the intermediate region of capital, the economy can proceed both to good and bad equilibria.

Consider why there is a lower bound of capital which allows an economy to converge to the good equilibrium. The lower bound can be observed around $K = 0.6$ ¹⁶. In the region $K > 0.6$, an economy's adjustment process looks quite normal. Start from a level of capital which is lower than that at equilibrium. This may be caused by earthquake destruction or a technology innovation. Investment then increases, which increases demand and, in turn, employment. Inflation occurs, and a central bank makes real interest rates increase. This causes consumption to decrease. These movements are quite natural. Then, what happens around $K = 0.6$? It seems households' substitution between consumption and labour comes to play an important role. Again, starting from low capital, an increase in investment causes employment to increase. The substitution between consumption and labour functions to increase consumption. As labour increases, this effect becomes larger and, in the end, consumption increases despite a rise in real interest rates. Aggregate demand increases

¹⁵On this matter, however, our two-dimensional K and X figure cannot give a definitive answer. This is because, besides K and X , there is the other jump variable C . Therefore, although the figure does not find a path toward the bad equilibrium from excessive capital, for the same K and X , a different C may yield the path. However, according to numerical calculation, changing an initial C as well as X from a high enough capital level always results in either converging to the good equilibrium or exploding and then violating the transversality condition.

¹⁶The level of a lower bound of capital is not important because we chose some arbitrary parameters such as ϵ .

further, which induces higher inflation and, in turn, higher real interest rates. The rise in real interest rates discourages investment, which prevents an economy from converging to a good equilibrium.

4.7.2 Dynamics under Sticky Prices: New-Keynesian Approach

Like in the Ramsey model, assuming Rotemberg price stickiness enables us to investigate dynamics under sticky prices. There are four independent differential equations with respect to K , C , X , and π . The number of jump and state variables is the same as before. Except for K , all are jump variables. Thus, the backward calculation method is valid to find a saddle-path. Again, there can arise excess demand. In such a region, this chapter calculates the model assuming limited goods are distributed to households prior to firms.

Figures 4.7.3 and 4.7.4 respectively demonstrate the movements of several variables over time and various dynamic paths including a saddle-path with respect to K and X . Qualitatively, these figures look very like the previous figures under sticky price expectations, except for non-linearity caused by excess demand. Therefore, most of the previous implications hold true in this model.

However, there is another big qualitative difference. There is no higher bound of capital. According to numerical calculations, starting from excessive capital, an economy can converge not only to the good equilibrium but also to the bad equilibrium depending upon the values of jump variables C and X . However, the reason is unclear. This result contrasts not only with the sticky price expectation model but also with the perfectly flexible price model.

What is happening at the lower bound of capital for the good equilibrium? The bound is observed around $K = 0.05$. At this point, almost all the values are very stable, and seem to be in equilibrium. X is at the equilibrium rate δ , where investment just covers capital depreciations. Capital thus stays almost constant at $K = 0.05$. The real interest

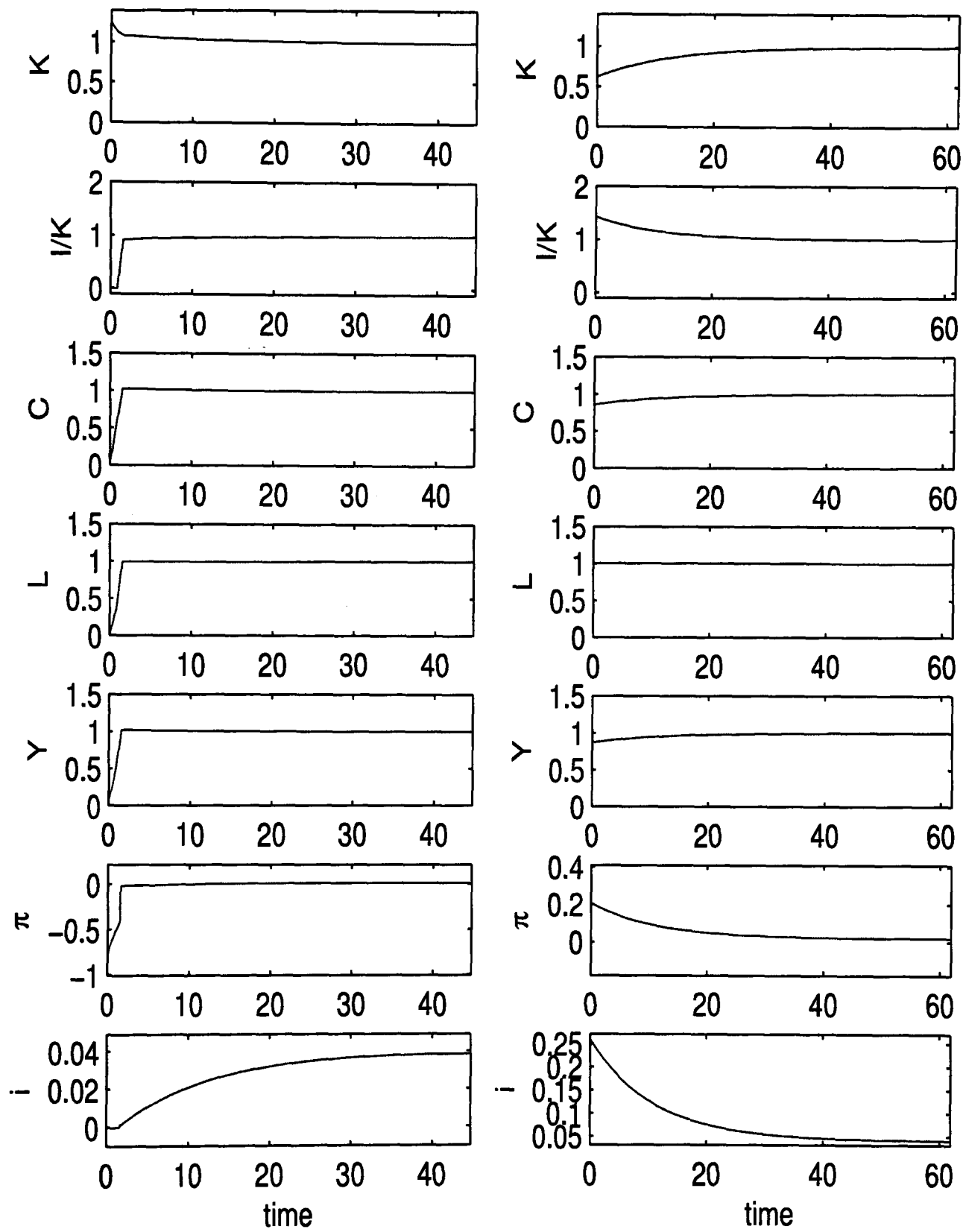


Figure 4.7.1: Dynamics toward Kiyotaki's Good Equilibrium under Sticky Price Expectations

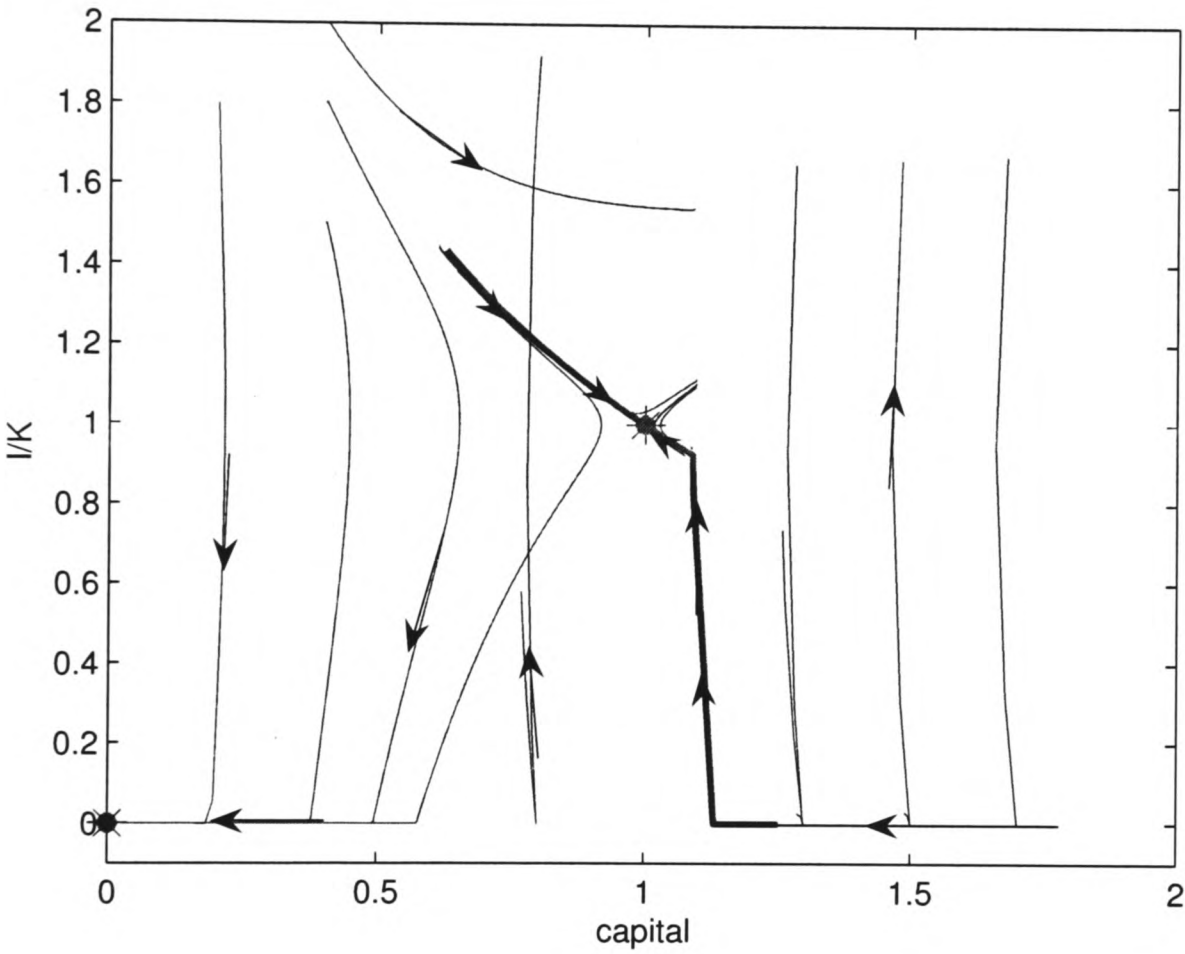


Figure 4.7.2: Several Dynamic Paths under Sticky Price Expectations

rate is almost equal to households' discount rate, which renders consumption stable from the Euler equation. However, inflation dynamics does not exhibit stability. There exists the excess demand problem, and real marginal costs deviate from their equilibrium such as $W/P = MPL$ but not $(1 + \theta)W/P = MPL$. This causes an inflation rate to keep going up.

Intuitively, the reason for the lower bound of capital can be explained as follows. Suppose that an initial capital level is very low. In order to move to the good equilibrium, a high degree of investment is required. However, because low capital reduces MPL , MPL becomes lower than W/P , and demand is constrained by the limited supply of goods. Hence, firms cannot do as much investing as they would like to do. If this effect becomes so large, then investment becomes lower than capital depreciation, which results in a decrease in capital. An economy thus converges only to the bad equilibrium.

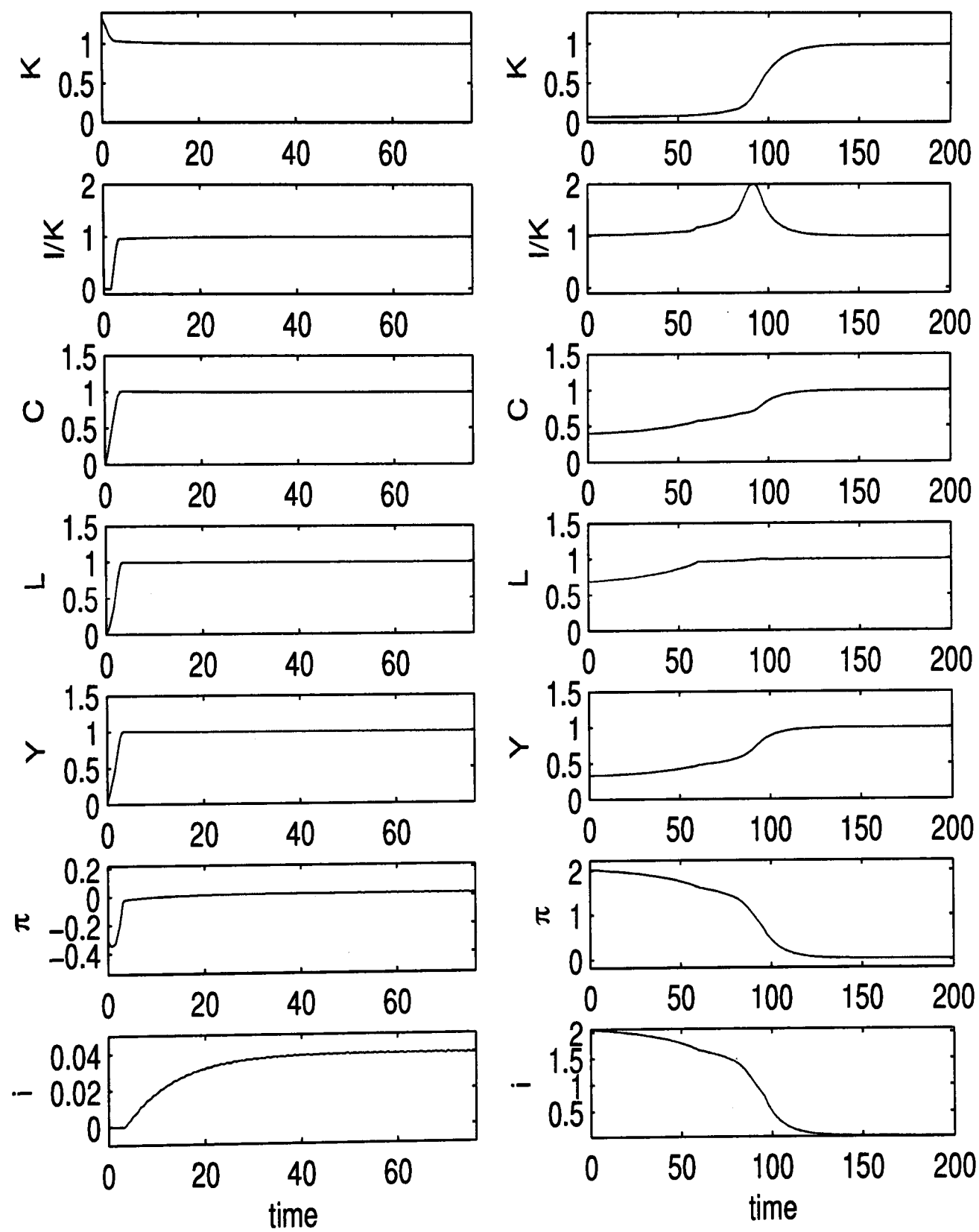


Figure 4.7.3: Dynamics toward Kiyotaki's Good Equilibrium under Sticky Prices

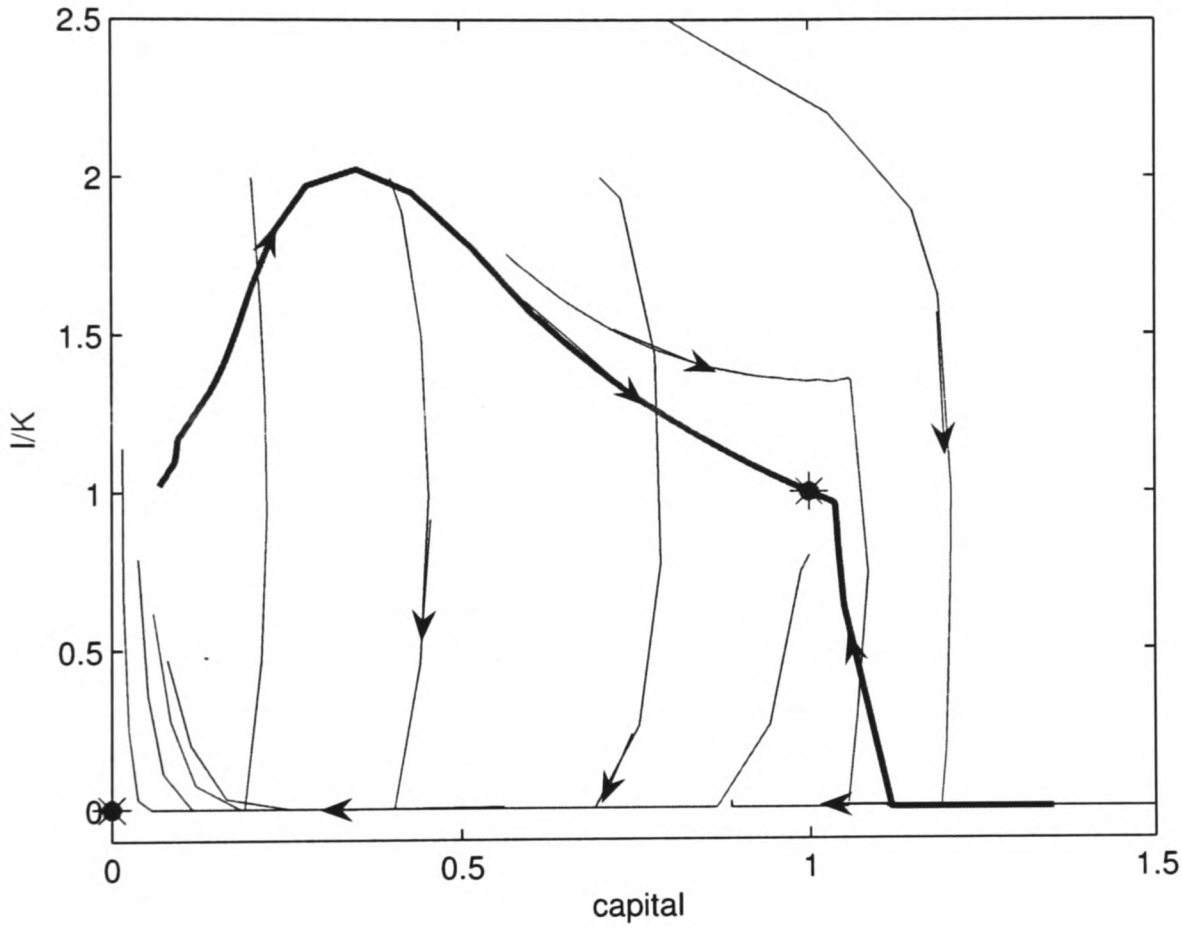


Figure 4.7.4: Several Dynamic Paths under Sticky Prices

4.7.3 Effect of a Zero Nominal Interest Rate Bound

Our biggest question is whether and how the Kiyotaki model is related to Krugman's short-run multiple equilibria. The properties of two models look similar. In a good equilibrium, investment and asset prices (Tobin's Q) are high. In a bad equilibrium, both are low. Investment cannot cover the depreciation of capital, which results in a depletion of capital.

However, the role of a zero nominal interest rate bound differs hugely. On the one hand, the Krugman model requires the existence of a zero nominal interest rate bound as a key element. The ineffectiveness of monetary policy increases a real interest rate and yields a disastrous equilibrium with low investment and asset prices (Tobin's Q).

On the other hand, in the Kiyotaki model, the zero bound is not a cause of multiplicity. Important conditions for multiplicity are firms' increasing returns to scale technology and the high elasticity of labour supply. The zero bound is regarded as a consequence in the adjustment process toward one of equilibria.

In fact, comparing the Kiyotaki model with the previous CRS technology models in Section 4.4, there is a feature which becomes closer to that of the Krugman model. The zero bound can be observed not only in the good long-run equilibrium but also in the bad long-run equilibrium. In the CRS technology models, a long-run equilibrium is unique. The zero bound is observed only when capital is initially excessive. Thus, the zero bound is by no means harmful in the long-run. In contrast, in the Kiyotaki model, the zero bound begins to constrain in every region. According to numerical calculation, in order to converge to the good equilibrium, a zero bound occurs only when capital is excessive. However, even when capital is not excessive, a nominal interest rate can decrease to zero. As an economy collapses to the bad equilibrium, output declines and a central bank lowers a nominal interest rate. Then, it eventually reaches zero. In this respect, the zero bound may be accompanied with a long-run disaster.

However, again this does not mean that the zero bound is the cause of the bad equilibrium. Note that, in the phase diagram, the left end of the saddle-path toward the good equilibrium, in other words, a lower bound of capital, is determined independently of the zero bound. Along the path, the economy can enjoy a good period with a high level of output and inflation, so a nominal interest rate is far beyond zero. The existence of the zero bound does not matter to the lower bound of capital. Therefore, even if the zero bound were eliminated, the region of capital which leads to the good equilibrium would not increase.

On the contrary, because of the zero bound, the region of capital which allows the bad equilibrium seems to decrease. As Figures 4.5.3 and 4.5.6 demonstrated, the zero bound shifts the saddle-path's intersecting point of capital with $X = 0$ leftward. These figures were obtained with Ramsey's single equilibrium framework, but the same shift can be confirmed with the Kiyotaki model. This shift implies that, thanks to the zero bound, the possibility to converge to the bad equilibrium may be reduced.

Summing up, from our micro-founded models, we can derive the following implications:

- In the short-run, the zero nominal interest rate bound causes a disaster by causing consumption and investment to plummet.
- In the long-run, the zero bound does not cause a bad equilibrium.
- The zero bound can be observed in the adjustment process toward the bad long-run equilibrium as well as the good one. However, this is just a consequence.

4.8 Concluding Remarks

This chapter aims to examine whether Japan's downturn was exacerbated by a liquidity trap. It refined Krugman's (2003) model by constructing a multiple-equilibria model with a sound

micro-foundation. We showed one of the most important assumptions to be fixed price expectations. The other key assumption is that households' consumption is proportional to current income, which is a standard assumption in elementary Keynesian economics. Needless to say, in reality price expectations are not fixed and consumption is optimised intertemporally. Once these assumptions are relaxed, the multiplicity of equilibria disappears although an economy experiences bad short-run outcomes similar to Krugman's bad equilibrium due to a liquidity trap. This implies that Krugman's multiple equilibria cannot influence long-run outcomes and therefore that the zero bound cannot be a fundamental cause of a disaster in the long-run. This implication still holds true even if there are multiple equilibria in the long-run using the Kiyotaki model. As long as we simply interpret the analytical result, our micro-founded models do not justify Krugman's idea that a zero bound can be the cause of a bad equilibrium, where by this we mean cannot be the cause of a bad *long-run* equilibrium.

*

Now, how do we explain the Japanese prolonged stagnation? In particular, consider the following questions: what happened at the beginning of the depression in the early 1990's?; what was a dynamic adjustment mechanism like?; how did the zero bound affect the country? With the help of our most sophisticated micro-founded model, the Kiyotaki model with not perfectly flexible prices, we attempt to answer these questions.

Firstly, suppose that a capital level was too low at the beginning of the Japanese depression. As Figures 4.7.2 and 4.7.4 demonstrated, too low capital leads necessarily to the bad long-run equilibrium. Therefore, if this supposition is right, we can explain why the country fell into prolonged stagnation.

However, it is hard to accept that a capital level was initially too low in the early 1990's. First and foremost, looking back to the late 1980's, the level of capital was excessive. The

economy in the late 1980's seems to have been a bubble where the asset market was overheated. Stock and land prices more than tripled. Superfluous money was wasted on golf courses, art and development of the land. It is natural to argue that huge investment in that period resulted in excessive capital. Secondly, no shock can be found as a reason for low capital. Low capital can be induced by an increase in technology or population growth, but this is very unlikely. On the contrary, Japan's ageing problem is a reflection of a reduced rate of population growth. As was discussed in the Introduction to this thesis, total factor productivity did not change much. We could possibly attribute a change in households' preference as a reason for a low capital level. However, there is no evidence whatsoever that this happened. For these reasons, it seems sensible to assume that an initial capital level was not too low.

*

Provided that capital stock was initially at an intermediate or excessive level, what can we say? In such a case, an economy can end up with both good and bad equilibria in the long-run¹⁷.

Between the two equilibria, Japan seems to have shifted toward the bad equilibrium. Of course, some may argue that Japan had been moving to the good equilibrium, and that the depression was merely an adjustment process from excessive capital. However, the span of this Japanese depression cannot be regarded as short-run; it has lasted more than a decade. Such a span is significantly longer than that of the country's previous recessions. The economy mostly escaped from depression in a few years. Our models also address that the economy should have recovered in a shorter time than a decade. As Figures 4.7.1 and 4.7.3 show,

¹⁷Actually, this is not true in the sticky price expectation model. According to our numerical calculation, there exists a upper bound of capital, above which an economy converges only to the good equilibrium. However, as discussed in Section 4.5.3, this thesis would like to adopt the sticky price model and to consider that, even from a position of excessive capital, the economy can go to both the good and the bad equilibria.

according to our numerical calculation, the time constant which is the time for capital to converge half the way to equilibrium is only about 1 year when the zero bound of a nominal interest rate constrains¹⁸. In this respect, the country's depression seems to have been too long to argue that the economy was moving toward the good equilibrium.

In the process toward the bad equilibrium, the economy experiences a depression. Investment becomes low, and aggregate demand decreases. This causes disinflation, and worse, deflation. A nominal interest rate reaches zero, which raises the real interest rate. This, in turn, decreases consumption. Such a catastrophe can be well compared with the Krugman bad equilibrium, where not only investment but also consumption decreases with income. This feature seems to very well describe the country's stagnation.

If there were no zero bound, the depression would be milder. An infinite decrease in nominal interest rates could make real interest rates negative, which would help increase consumption. A collapse in demand would be softened by the increase in consumption. Nevertheless, non-existence of zero bound cannot prevent the economy from falling into the bad equilibrium in the end.

*

The toughest question arising here asks why Japan moved not to the good but to the bad equilibrium. A simple answer is that there was coordination failure. Although the good equilibrium is desirable, this cannot always be achieved. Since there are quite a number of monopolistic firms, they cannot all communicate and coordinate. If all other firms are so pessimistic that they do not invest, then neither should our own firm invest. Otherwise, this firm suffers a loss due to the lack of demand. This answer can address why the bad equilibrium can be chosen, but not why it was chosen in Japan.

¹⁸However, if the zero bound does not constrain, the time constant comes to increase to roughly 7 years.

CHAPTER 4. ZERO NOMINAL INTEREST RATE BOUND AND MULTIPLE EQUILIBRIA

It is almost impossible to assert how equilibrium is selected. There is no consensus on this matter, although several approaches are presented including the global game approach discussed in Chapter 2. The difficulty is mainly because expectations, which are extremely important in deciding an equilibrium, are hard to model. Admitting the difficulty, this thesis dares to present a couple of very bold answers as to why Japan has fallen into the bad equilibrium.

One possible answer is that the burst of the asset market bubble in the early 1990's brought pessimism to the Japanese. A sudden drop in asset prices may have quickly changed people's expectations. A widespread pessimism dampened investment. This caused a decrease in aggregate demand and deflation, which lowered profitability from investment. This caused people's pessimism to become self-fulfilling.

As another explanation, the zero bound may be blamed. Assume that a capital level was initially excessive from its good equilibrium level and that there is some kind of inertia regarding people's actions. An excessive position of capital can be explained especially by the over-investment in the late 1980's. Starting from excessive capital, irrespective of whether the economy moves to the good or the bad equilibria, capital must decrease. Following the dynamic process which we have discussed several times, a low level of investment can cause a nominal interest rate to reach zero and consumption to decrease. Aggregate demand decreases further, which reduces the profitability of capital investment and makes the short-run position of economy a disastrous one. In contrast, if there were no zero bound, the real interest rate could be negative and consumption would not decrease. Thus, the great depression would be softened. In this respect, the existence of the zero bound seems to increase the momentum toward the bad long-run equilibrium. If people believe that the good equilibrium is achieved in the end, at some point in an adjustment process, they should gradually begin investing. However, past disaster caused by the zero bound may worsen people's expectations, which

may discourage investment and consumption. In other words, we may argue that, if there is a certain kind of inertia in people's actions, the zero bound amplifies the possibility that the bad long-run equilibrium is selected.

This final point might be connected with the idea of Krugman's multiple equilibria. We may proceed to argue that Krugman's short-run multiple equilibria might function as a selection device for Kiyotaki's long-run multiple equilibria. In Krugman's multiple equilibria, a demand shock is affected by a change in a proportion to consume, and is very important to determine the existence of each equilibrium. Such a demand shock can also be incorporated in our richest model, namely in the Kiyotaki model with not perfectly flexible prices. In this model, a demand shock possibly arises from a change in households' preferences. Moreover, the demand shock, if negative, is amplified by the zero bound. In a similar way that a large negative demand shock eliminates Krugman's good equilibrium, the negative demand shock which is produced and amplified in the above way might increase the possibility that the economy moves towards the bad long-run equilibrium. This conjecture suggests that the ineffectiveness of monetary policy due to the zero interest rate bound might be one of the culprits for the selection of the bad long-run equilibrium.

However, these three explanations are only conjecture. This thesis cannot yet tell in detail how these mechanisms work. It is thus essential to provide a much more convincing model on equilibrium selection, for example, by using the global games approach. This is a task for the future.

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CHAPTER 4. ZERO NOMINAL INTEREST RATE BOUND AND MULTIPLE
EQUILIBRIA

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4.A Appendix

4.A.1 Dynamics of the Semi-Sensible Krugman Model

This appendix analyses the dynamics of the semi-sensible Krugman model. From equation (4.3.15), Tobin's Q is written as

$$Q - 1 = \Psi'(I/K) = \Psi'(X). \quad (4.A.1)$$

Equation (4.3.17) is transformed to

$$(1 + \dot{\Psi}'(X)) = (i + \delta)(1 + \Psi'(X)) - Y_K(X) + \Psi(X) - X\Psi'(X), \quad (4.A.2)$$

where Y_K can be written as a function of X because of equation (4.3.21):

$$Y_K(X) = \alpha \frac{X + \Psi(X)}{1 - c}.$$

Log-linearise X as $X = X_0 e^x = (g_0 + \delta)(1 + x)$. x is the logarithmic deviation of X around a steady state $g_0 + \delta$. In the following, small letters (k , x , etc.) stand for the deviation of variables. An equilibrium g_0 satisfies

$$0 = (i_0 + \delta)(1 + \Psi'(g_0 + \delta)) - Y_K(g_0 + \delta) + \Psi(g_0 + \delta) - (g_0 + \delta)\Psi'(g_0 + \delta), \quad (4.A.3)$$

where i_0 is an equilibrium interest rate. Equation (4.A.2) can be transformed to

$$\begin{aligned} (g_0 + \delta)\Psi''\dot{x} &= (i_0 + \delta)(g_0 + \delta)\Psi''x - Y'_K(g_0 + \delta)x + i'(g_0 + \delta)(1 + \Psi')x \\ &\quad + (g_0 + \delta)\Psi'x - (g_0 + \delta)\Psi'x - (g_0 + \delta)^2\Psi''x \\ \dot{x} &= \left(i_0 - g_0 - \frac{Y'_K}{\Psi''} + i' \frac{1 + \Psi'}{\Psi''} \right) x. \end{aligned} \quad (4.A.4)$$

Log-linearise capital as $K = K_0 e^{g_0 t + k}$. The law of motion with respect to capital is

$$\dot{K} = I - \delta K = (X - \delta)K, \quad (4.A.5)$$

which can be transformed to

$$\begin{aligned}
 (\dot{k} + g)K_0 e^{g_0 t + k} &= \{(g_0 + \delta)e^x - \delta\} K_0 e^{g_0 t + k} \\
 \dot{k} + g_0 &\simeq (g_0 + \delta)(1 + x) - \delta \\
 &\simeq g_0 + (g_0 + \delta)x \\
 \dot{k} &= (g_0 + \delta)x.
 \end{aligned} \tag{4.A.6}$$

To sum up, we can obtain

$$\begin{pmatrix} \dot{x} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} i_0 - g_0 - \frac{Y'_K}{\Psi''} + i' \frac{1 + \Psi'}{\Psi''} & 0 \\ g_0 + \delta & 0 \end{pmatrix} \begin{pmatrix} x \\ k \end{pmatrix}.$$

Eigenvalues of this matrix are the roots of

$$t \left(t - \left(i_0 - g_0 - \frac{Y'_K}{\Psi''} + i' \frac{1 + \Psi'}{\Psi''} \right) \right) = 0.$$

Recalling that Q (that is, x) is a jump variable, we can deduce that the equilibrium becomes saddle-path stable if

$$i_0 - g_0 - \frac{Y'_K}{\Psi''} + i' \frac{1 + \Psi'}{\Psi''} > 0. \tag{4.A.7}$$

If the left-hand side is negative, equilibrium becomes indeterminate. This condition implies that, if the slope of $Q = Q(g, i)$ is higher than that of $g = g(Q)$, then equilibrium becomes indeterminate. On the other hand, if the former is lower, equilibrium becomes determinate.

This can be proved in the following way.

$$\begin{aligned}
 \frac{d}{dX} Q(g, i) |_{at X=g_0+\delta} &= \frac{d}{dX} \frac{Y_K(X) - \Psi(X) + X\Psi'(X)}{i(X) + \delta} \Big|_{at X=g_0+\delta} \\
 &= \frac{Y'_K + (g_0 + \delta)\Psi''}{i_0 + \delta} - \frac{\{Y_K - \Psi + (g_0 + \delta)\Psi'\}i'}{(i_0 + \delta)^2} \\
 &= \frac{Y'_K + (g_0 + \delta)\Psi''}{i_0 + \delta} - \frac{(1 + \Psi')i'}{i_0 + \delta} \\
 \frac{d}{dX} g^{-1}(Q) |_{at X=g_0+\delta} &= \Psi'',
 \end{aligned}$$

thus

$$\frac{d}{dX} Q(g, i) - \frac{d}{dX} g^{-1}(Q) |_{at X=g_0+\delta} = -\frac{\Psi''}{i_0 + \delta} \left(i_0 - g_0 - \frac{Y'_K}{\Psi''} + i' \frac{1 + \Psi'}{\Psi''} \right).$$

The term inside the bracket is the same as the left-hand side of equation (4.A.7). Hence, by comparing the slope of two curves, we can judge whether equilibrium is determinate or not.

4.A.2 Semi-Sensible Krugman Model with the Euler Consumption Function

This appendix replaces the Keynesian consumption function by the Euler equation, and examines how the properties of the semi-sensible Krugman model will change. The assumption of $\pi e = 0$ is maintained. A model is constructed as follows.

$$g_C = \frac{\dot{C}}{C} = \frac{i - \rho}{\sigma} \quad (4.A.8)$$

$$(1 + \dot{\Psi}'(X)) = (i + \delta)(1 + \Psi'(X)) - Y_K + \Psi(X) - X\Psi'(X) \quad (4.A.9)$$

$$\dot{K} = I - \delta K = (X - \delta)K. \quad (4.A.10)$$

The first equation represents the Euler equation for households. The second equation is the law of motion with respect to investment $X = I/K$, which was the same as equation (4.A.2). The last one represents capital accumulation. A market clearing condition is given by

$$Y = C + I + J = C + XK + \Psi(X)K. \quad (4.A.11)$$

Finally, assume the following monetary policy rule:

$$i = i(X), \quad (4.A.12)$$

where $i \geq 0$ and $i' \geq 0$. An interest rate increases as capital evolves at a high speed.

Equilibrium

Let an economy's equilibrium growth rate be g_0 . From the above five equations, it satisfies

$$g_0 = \frac{i_0 - \rho}{\sigma} \quad (4.A.13)$$

$$0 = (i_0 + \delta)(1 + \Psi'(X_0)) - Y_{K0} + \Psi(X_0) - X_0\Psi'(X_0) \quad (4.A.14)$$

$$g_0 = X_0 - \delta \quad (4.A.15)$$

$$Y_0 = C_0 + X_0K_0 + \Psi(X_0)K_0 \quad (4.A.16)$$

$$i_0 = i(X_0), \quad (4.A.17)$$

where a variable with a subscript 0 represents an equilibrium value. The equilibrium marginal product of capital Y_{K0} is written as $Y_{K0} = \alpha Y_0 / K_0$ using a capital share α in a production function.

In particular, suppose that $g_0 = 0$ at the better equilibrium. Then, we obtain $i_0 = \rho$. In the worse equilibrium, from $i_0 = 0$, a growth rate is $g_0 = -\rho/\sigma$. These values are the same as those in the main chapter which were obtained assuming fixed capital.

Log-Linearisation

I now log-linearise the dynamics around equilibrium. Define x and k as $X = X_0 e^x$, $K = K_0 e^{g_0 t + k}$, and so on. After some algebra, we can obtain

$$\dot{c} = i' / \sigma \quad (4.A.18)$$

$$\dot{x} = \left(i_0 - g_0 + i' \frac{1 + \Psi'}{\Psi''} \right) x - \frac{Y_{K0}}{(g_0 + \delta)\Psi''} (y - k) \quad (4.A.19)$$

$$\dot{k} = (g_0 + \delta)x \quad (4.A.20)$$

$$Y_0 \dot{y} = C_0 i' / \sigma (g_0 + \delta)x + (g_0 + \delta + \Psi)K_0 \dot{k} + (g_0 + \delta)(1 + \Psi')K_0 \dot{x}. \quad (4.A.21)$$

The last equation is derived from the differentiation of the market clearing condition. Summing up, the following system of differential equations is obtained:

$$\begin{pmatrix} \dot{y} - \dot{k} \\ \dot{x} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} A & B & 0 \\ C & D & 0 \\ 0 & g_0 + \delta & 0 \end{pmatrix} \begin{pmatrix} y - k \\ x \\ k \end{pmatrix}. \quad (4.A.22)$$

Stability

Hence, a characteristic equation is given by

$$t(t^2 - (A + D)t + AD - BC) = 0.$$

Clearly, the corresponding eigenvalue to capital is zero. This implies that predetermined capital can have an arbitrary value. Instead, its growth rate is determined.

The other variables ($y - k$ and x) can jump. Thus, the other roots should be positive under determinacy.

The better equilibrium Firstly, in the better equilibrium, we examine its roots. In the equilibrium, $g_0 = 0$ and $i_0 = \rho$. Recalling that we assumed $\Psi(\delta) = \Psi'(\delta) = 0$ for simplicity, the characteristic equation is simplified as

$$t^2 - \frac{\rho\delta\Psi'' + i'\delta - \delta\alpha}{\delta\Psi''}t + \frac{Y_{K0}}{\Psi''} \left(1 - \frac{\delta K_0}{Y_0}\right) \left(\frac{i'}{\sigma} - 1\right) = 0.$$

The last term becomes positive if and only if

$$i' > \sigma. \quad (4.A.23)$$

Furthermore, if this condition is satisfied, it is very likely that the coefficient of t^1 becomes negative. This suggests that both of roots are positive, which makes equilibrium determinate. To be contrary, if the condition $i' > \sigma$ is violated, then one root is positive and the other is negative, which makes equilibrium indeterminate. Actually, the same condition was derived before as in equation (4.3.29) in the main section where capital is regarded as fixed.

This condition may well be satisfied if monetary policy is effective enough. Thus, under a normal circumstance, the better equilibrium is determinate.

The worse equilibrium Secondly, check the stability of the worse equilibrium, where $i = 0$, $i' = 0$ and $g_0 = -\rho/\sigma$. The characteristic equation becomes as

$$t^2 - \left(\alpha \frac{1 + \Psi'}{\Psi''} + g_0 \right) t + \frac{\alpha g_0 - (1 - \alpha)(\delta + \Psi) + g_0 \Psi'}{\Psi''} = 0.$$

Let us focus on the last term. Since ρ is as small as 0.01, the term which includes g_0 can be neglected compared with the second term in the numerator. Thus, the coefficient of a t^0 term is negative, which means that two eigenvalues are positive and negative. The equilibrium is indeterminate.

4.A.3 Kiyotaki Model Dynamics with Perfectly Flexible Prices

(1) A good equilibrium First, we investigate the dynamic behaviour around a good equilibrium where $L = N$. Rewrite equation (4.6.10) and (4.6.11):

$$Q = 1 + \Psi'(I/K)$$

$$\dot{Q} = (i - \pi + \delta)Q - \frac{\alpha}{1 + \theta} \frac{P}{\bar{P}} h K^{\alpha-1} L^{1+\beta-\alpha} + \Psi(I/K) - I/K \Psi'(I/K).$$

We can log-linearise these equations in the very similar way to Appendix 4.A.1, except for mainly three differences. The first difference is that, because this model is a general equilibrium, the term of interest rates is determined by households' sector. In equilibrium, $i - \pi = \rho$, but off equilibrium, real interest rates are affected by consumption growth (or vice versa). Secondly, since labour supply is fixed at N , the marginal product of capital is not constant but decreasing in capital. Thirdly, there is no growth in this model, that is, $g = 0$. Denote

the deviation of real interest rates from equilibrium as $\Delta i - \pi$, we can obtain

$$\begin{aligned}
 \delta \Psi''(\delta) \dot{x} &= (\Delta i - \pi)(1 + \Psi'(\delta)) + (\rho + \delta) \delta \Psi''(\delta) x \\
 &\quad - \frac{\alpha}{1 + \theta} h K_0^{\alpha-1} N^{1+\beta-\alpha} \{p - \bar{p} + (\alpha - 1)k\} + \delta \Psi'(\delta) x - \delta \Psi'(\delta) x - \delta^2 \Psi''(\delta) x \\
 &= (\Delta i - \pi)(1 + \Psi'(\delta)) + \rho \delta \Psi''(\delta) x \\
 &\quad - \{(\rho + \delta)(1 + \Psi'(\delta)) + \Psi(\delta) - \delta \Psi'(\delta)\}(\alpha - 1)k.
 \end{aligned} \tag{4.A.24}$$

The last equation is obtained because, in equilibrium, $\dot{Q} = 0$ and $Q = 1 + \Psi'(\delta)$. Under flexible prices, $p = \bar{p}$. The law of motion with respect to capital is the same as equation (4.A.6):

$$\dot{k} = \delta x. \tag{4.A.25}$$

Rewrite the Euler equation (equation (4.6.2)) in households' sector:

$$\begin{aligned}
 0 &= u' - \lambda \bar{P} e^{(\rho-i)(s-t)} \\
 &= (C - v N^{1+\frac{1}{\epsilon}})^{-\sigma} - \lambda \bar{P} e^{(\rho-i)(s-t)},
 \end{aligned}$$

which is log-linearised as

$$\begin{aligned}
 -\sigma \frac{\dot{C}}{C_0 - v N^{1+\frac{1}{\epsilon}}} &= \pi + \rho - i, \\
 \text{so } \Delta i - \pi &= \sigma \frac{C_0}{C_0 - v N^{1+\frac{1}{\epsilon}}} \dot{c}
 \end{aligned} \tag{4.A.26}$$

The consumption deviation c can be described by x and k using the market clearing condition:

$$\begin{aligned}
 Y &= C + I + J(I, K) \\
 Y_0 \alpha k &= C_0 c + \delta K_0 (x + k) + \Psi(\delta) K_0 k + \delta \Psi'(\delta) K_0 x \\
 c &= \left(\alpha \frac{Y_0}{C_0} - (\delta + \Psi(\delta)) \frac{K_0}{C_0} \right) k - \delta (1 + \Psi'(\delta)) \frac{K_0}{C_0} x.
 \end{aligned} \tag{4.A.27}$$

Summing up equations (4.A.24), (4.A.26) and (4.A.27),

$$\begin{aligned}
 & \delta\Psi''(\delta)\dot{x} \\
 &= (1 + \Psi'(\delta))\sigma \frac{C_0}{C_0 - vN^{1+\frac{1}{\epsilon}}} \left\{ \left(\alpha \frac{Y_0}{C_0} - (\delta + \Psi(\delta)) \frac{K_0}{C_0} \right) \dot{k} - \delta(1 + \Psi'(\delta)) \frac{K_0}{C_0} \dot{x} \right\} \\
 &+ \rho\delta\Psi''(\delta)x - \{(\rho + \delta)(1 + \Psi'(\delta)) + \Psi(\delta) - \delta\Psi'(\delta)\}(\alpha - 1)k. \\
 &\left\{ \delta\Psi''(\delta) + (1 + \Psi'(\delta))\sigma \frac{C_0}{C_0 - vN^{1+\frac{1}{\epsilon}}} \delta(1 + \Psi'(\delta)) \frac{K_0}{C_0} \right\} \dot{x} \\
 &= (1 + \Psi'(\delta))\sigma \frac{C_0}{C_0 - vN^{1+\frac{1}{\epsilon}}} \left(\alpha \frac{Y_0}{C_0} - (\delta + \Psi(\delta)) \frac{K_0}{C_0} \right) \delta x \\
 &+ \rho\delta\Psi''(\delta)x + \{(\rho + \delta)(1 + \Psi'(\delta)) + \Psi(\delta) - \delta\Psi'(\delta)\}(1 - \alpha)k. \tag{4.A.28}
 \end{aligned}$$

Therefore,

$$\begin{pmatrix} \dot{x} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} A & B \\ \delta & 0 \end{pmatrix} \begin{pmatrix} x \\ k \end{pmatrix},$$

and B is positive. Its characteristic equation is given by

$$t^2 - At - \delta B = 0,$$

so one root is positive and the other is negative. This means that the good equilibrium is saddle-path stable.

(2) An intermediate equilibrium The deviation is very similar to that before, but a bit more complex because not only capital but also labour deviates from the equilibrium. The law of motion with respect to $X = I/K$ can be rewritten as

$$\begin{aligned}
 \delta\Psi''(\delta)\dot{x} &= (\Delta i - \pi)(1 + \Psi(\delta)) + \rho\delta\Psi''(\delta)x \\
 &- \{(\rho + \delta)(1 + \Psi'(\delta)) + \Psi(\delta) - \delta\Psi'(\delta)\}\{(\alpha - 1)k + (1 + \beta - \alpha)l\}. \tag{4.A.29}
 \end{aligned}$$

The Euler equation (equation (4.6.2)) reads

$$\Delta i - \pi = \sigma \frac{C_0 \dot{c} - (1 + \frac{1}{\epsilon}) v L_0^{1+\frac{1}{\epsilon}} \dot{l}}{C_0 - v L_0^{1+\frac{1}{\epsilon}}}, \tag{4.A.30}$$

and $\dot{c}$ and $\dot{l}$ are obtained from the market clearing condition and from the later equation (4.A.31) respectively. From equation (4.6.4) and (4.6.9), real wages are given by

$$\frac{W}{P} = \frac{1 + \beta - \alpha}{1 + \theta} h K^\alpha L^{\beta - \alpha} = \left(1 + \frac{1}{\epsilon}\right) v L^{\frac{1}{\epsilon}}.$$

Log-linearising this yields l :

$$\begin{aligned} \alpha k + (\beta - \alpha)l &= \frac{1}{\epsilon}l. \\ \therefore l &= \frac{\alpha}{\frac{1}{\epsilon} + \alpha - \beta}k. \end{aligned} \tag{4.A.31}$$

Substituting $\Delta i - \pi$ and l into equation (4.A.29), we can obtain the differential equation $\dot{x}$ as a function of x and k . The coefficient of x and k is each positive and negative. Especially, regarding k , it is verified because the last term of equation (4.A.29) is

$$-\{(\alpha - 1)k + (1 + \beta - \alpha)l\} = \frac{\frac{1}{\epsilon}(1 - \alpha) - \beta}{\frac{1}{\epsilon} + \alpha - \beta}k,$$

and its coefficient is negative from equation (4.6.15) as a condition for multiple equilibria.

Therefore, the system of differential equations has the form of

$$\begin{pmatrix} \dot{x} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} A & -B \\ \delta & 0 \end{pmatrix} \begin{pmatrix} x \\ k \end{pmatrix},$$

where A and B are positive. Its characteristic equation becomes

$$t^2 - At + \delta B = 0,$$

so, for both of the roots, their real parts are positive. This suggests that the intermediate equilibrium is unstable.

(3) A bad equilibrium Since capital becomes zero, some variables such as Tobin's Q cannot be well-defined. According to our numerical calculation, there are multiple paths to converge to this equilibrium as shown in Figure 4.6.2.

4.A.4 Slight Modification of the Utility Function in the Kiyotaki Model with Not Perfectly Flexible Prices

When we incorporate not perfectly flexible prices, we slightly modify the Kiyotaki model. We allow more flexible labour supply over the bound N . The reason for this is because it is natural to assume some flexibility of labour supply in the short or medium run. Employees work overtime by receiving an extra payment. Thus, when demand increases, an employment level can temporarily exceed N .

We assume that households may supply labour more than N , but they demand a high compensation, instead. In Figure 4.6.1, this modification is graphically represented as an upward sloping labour supply curve in the region of $L > N$. To be more concrete, assume the following labour supply curve:

$$W/\bar{P} = \begin{cases} \mu(L - N) + (1 + \frac{1}{\epsilon}) v N^{\frac{1}{\epsilon}} & (L \geq N) \\ (1 + \frac{1}{\epsilon}) v L^{\frac{1}{\epsilon}} & (L < N), \end{cases} \quad (4.A.32)$$

where μ represents the slope of a labour supply curve. A high μ prevents households from supplying more labour. In the Kiyotaki model, this is assumed to be infinite. I set this 10 in numerical calculation. In order to make a model consistent, a household's utility function needs to be transformed to

$$u(\bar{C}, L) = u \left\{ \bar{C} + \frac{1}{\epsilon} v N^{1+\frac{1}{\epsilon}} - \frac{\mu}{2} (L - N)^2 - \left(1 + \frac{1}{\epsilon}\right) v N^{\frac{1}{\epsilon}} L \right\} \quad (4.A.33)$$

in $L > N$. This slightly changes the Euler equation.

Chapter 5

Concluding Remarks

5.1 Summary of the Findings in Each Chapter

This thesis has provided several theoretical models which explain the Japanese stagnation from early in the 1990's. Chapter 2 focused on business connections, and demonstrated that there are multiple equilibria regarding firms' decision of trade participation. In a good equilibrium, all firms participate in trade, but in a bad equilibrium, no firm dares to participate. Since there are a large number of firms, their coordination is almost impossible, and a good equilibrium cannot always be achieved. For instance, if all other firms are too pessimistic to take part in trade, an aggregated economy becomes inactive. Thus, even if one firm is eager to participate in trade, it cannot sell enough goods to make a profit. Furthermore, this chapter has demonstrated that the possibility of multiple equilibria increases as business connections become more complex and that a bad equilibrium entails deflation. Such a situation seems to apply to the Japanese experience from the early 1990's.

Chapter 3 examined whether following the simple Taylor rule exacerbates poor performance in the economy. In general, the Taylor principle states that a central bank should adjust a nominal interest rate by more than one-for-one responding to inflation. This principle is

required in order to avoid indeterminacy which becomes a source of undesirable fluctuations. However, recent studies, especially Sveen and Weinke (2004), have claimed, that once we take account of a capital accumulation process the Taylor principle is not enough; responding to output is also necessary. The chapter has pointed out that too much forward-lookingness in their models is the cause of this indeterminacy. It has become clear that our richer models with some persistence and heterogeneities expand the stability boundary, which rescues the view that following a simple Taylor principle is not bad policy. To put it differently, our study turns out to be unhelpful in understanding the Japanese stagnation, because the Bank of Japan seems to have followed the Taylor principle, at least until 1999, when a zero nominal interest rate bound was reached.

Chapter 4 focused on a zero nominal interest rate bound, and aimed to understand what happens in a liquidity trap. Following Krugman (2003), it investigated the effect of the zero bound on the real side of the economy. According to Krugman, the ineffectiveness of monetary policy yields a bad equilibrium where asset prices and investment are low and an economy continues to collapse. However, his model lacks a micro-foundation. Using a sound micro-foundation, this chapter aimed to answer the question as to whether the existence of the zero bound can cause a bad equilibrium even in the long-run. Our models have made it clear that Krugman's model is based on two important assumptions: fixed price expectations and the Keynesian consumption function. Unless both of these are satisfied, the zero bound cannot be the cause of the bad long-run equilibrium although it induces a bad short-run outcome with a collapse in investment and consumption. This result still holds even though there are multiple equilibria in the long-run. Nevertheless, the concluding remarks in this chapter suggest that, from the perspective of equilibrium selection, the zero bound might be able to induce the economy to move towards a bad long-run equilibrium.

5.2 Toward a Better Model to Explain Japan's Stagnation

I have argued that Chapter 3 cannot help in an understanding of Japan's prolonged stagnation. There is thus a question as to which of Chapter 2 or 4 can help us to better understand this. And the answer of this thesis is both. Each model has its strengths and weaknesses, and they are complementary to each other rather than substituting for the other. Put differently, the combination of the effects described in these two models seems to have aggravated the Japanese economy.

Chapter 2 can be applied throughout the depression period, i.e. from 1990 to the present. Since there is no condition for its validity such as the zero bound, the model can address the depression from its start in the early 1990's. This chapter also successfully applied the global games approach in order to consider how an equilibrium is selected. However, this model overlooks monetary policy. Furthermore, the model is static; it incorporates only the participation decision; and it does not tell about important dynamic features with respect to prices, consumption and investment.

On the other hand, the story of Chapter 4 is considered to have relevance to the later period of the depression. A short-term nominal interest rate was not zero before 1999. It is true that some people may have expected the coming of the zero bound long before 1999, but the liquidity trap should not have been a problem at least at the beginning of the depression in the early 1990's. However, this model can present an important dynamic process. It is very useful to investigate the movement of consumption, investment, inflation and interest rates over time.

A better model should be the one which integrates these chapters. A synthesis of these two models would provide a much more sophisticated study regarding Japan's trap, and probably magnify the possibility of multiple equilibria. I would describe Japanese stagnation as a bad equilibrium where market participation is inactive, investment is low, and firms

make no profits, opposed to a good equilibrium where all firms participate in trade, spend a large amount of money on capital investment and make profits.

Moreover, in order to make a better model, considering equilibrium selection is extremely important. We need to construct convincing reasoning as to why Japan has fallen into a bad equilibrium and how Japan can go back to a good equilibrium. As Chapters 2 and 4 discussed, the effectiveness of monetary and fiscal policy can greatly differ depending on how equilibrium is selected. In order to find a remedy to revitalise the economy, this study is essential.

There are many other aspects which this thesis has not considered. In particular, the heavy burden of bad loans for both banks and corporations has surely resulted in the loss of opportunities and prevented a recovery. However, the main thrust of this thesis is that simply resolving bad loans cannot lead to revitalising the economy. As long as the country is stuck at a bad equilibrium, demand stays so low that few firms want to borrow money. Therefore, even though banks become healthy, they can hardly find borrowers. Nevertheless, incorporating the bad loan problem with our models is extremely important. The complementarity of each model helps deepen our understanding of, and find a remedy for, Japan's prolonged stagnation.

