

# Supplementary: A Spatially Constrained Probabilistic Model for Robust Image Segmentation

Abhirup Banerjee and Pradipta Maji

## I. BASICS OF HMRF MODEL BASED SEGMENTATION

### A. GHMRF: Gaussian Distribution for HMRF Model Based Segmentation

In the E-step or expectation step of each iteration, the latent variables are measured, given the observed pixel intensity values and the estimates of the parameters in previous iteration, as follows:

$$\tau_{il}^{(t)} = \frac{p(l|x_{N_i}) f_G(y_i; \theta_l^{(t)})}{\sum_{m \in \mathcal{L}} p(m|x_{N_i}) f_G(y_i; \theta_m^{(t)})}, \quad (1)$$

where  $\theta^{(t)}$  is the estimate of the parameters at  $t$ th iteration. Here, the expression of  $\tau_{il}$  in (1) denotes the posterior probability that the  $i$ th pixel belongs to the  $l$ th image class  $\Omega_l$  and hence, is considered as the membership value of pixel  $i$  to  $\Omega_l$ . The membership function varies based on the selection of clique potential function

$$E_c(x_i, x_j) = -a\delta(x_i - x_j), \quad (2)$$

where  $a$  is a scaling parameter, or

$$E_c(x_i, x_j) = \frac{0.1}{d_{ij}} [1 - \delta(x_i - x_j)], \quad (3)$$

where  $d_{ij}$  is the Euclidean distance between pixels  $i$  and  $j$ , in prior  $p(l|x_{N_i})$ . In the M-step or maximization step of each iteration, optimal estimates of the parameters are obtained as

$$\hat{\mu}_l^{(t+1)} = \frac{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)} y_i}{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)}}; \quad \hat{\sigma}_l^{2(t+1)} = \frac{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)} (y_i - \mu_l^{(t+1)})^2}{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)}}. \quad (4)$$

A. Banerjee is with the Department of Engineering Science, Institute of Biomedical Engineering, University of Oxford, Oxford, United Kingdom and the Radcliffe Department of Medicine, Division of Cardiovascular Medicine, University of Oxford, Oxford, United Kingdom. E-mail: abhirup.banerjee@cardiov.ox.ac.uk.

P. Maji is with the Biomedical Imaging and Bioinformatics Lab, Machine Intelligence Unit, Indian Statistical Institute, Kolkata, India. E-mail: pmaji@isical.ac.in.

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### B. tHMRF: Student's $t$ -Distribution for HMRF Model Based Segmentation

The latent variables  $\tau_{il}$  are again estimated in the E-step of the iterative EM algorithm using the membership function (1), where the probability  $f_G(y; \theta_l)$  is substituted using the probability function of Student's  $t$ -distribution  $f_t(y; \theta_l)$ . Another latent variable  $u_{il}$  of the model, which measures the inlierness of the  $i$ th pixel to image class  $\Omega_l$ , is estimated as

$$u_{il}^{(t)} = \frac{v_l^{(t)} + 1}{v_l^{(t)} + d(y_i; \mu_l^{(t)}, \sigma_l^{(t)})}. \quad (5)$$

The optimal estimates of the parameters  $\mu_l$  and  $\sigma_l^2$  are obtained in the M-step of each iteration, as follows:

$$\hat{\mu}_l^{(t+1)} = \frac{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)} u_{il}^{(t)} y_i}{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)} u_{il}^{(t)}}; \quad \hat{\sigma}_l^{2(t+1)} = \frac{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)} u_{il}^{(t)} (y_i - \hat{\mu}_l^{(t+1)})^2}{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)} u_{il}^{(t)}}. \quad (6)$$

$\nu_l^{(t+1)}$  is estimated numerically from

$$1 + \frac{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)} (\log u_{il}^{(t)} - u_{il}^{(t)})}{\sum_{i \in \mathcal{S}} \tau_{il}^{(t)}} + \psi\left(\frac{v_l + 1}{2}\right) - \log\left(\frac{v_l + 1}{2}\right) + \log\left(\frac{\nu_l}{2}\right) - \psi\left(\frac{\nu_l}{2}\right) = 0, \quad (7)$$

where  $\psi(s) = \frac{1}{\Gamma(s)} \frac{\partial}{\partial s} \Gamma(s)$  represents the digamma function.

### C. StNHMRF: Stomped Normal Distribution for HMRF Model Based Segmentation

The latent variables  $\tau_{il}$  are estimated in E-step, as follows:

$$\tau_{il}^{(t)} = \begin{cases} 1, & \text{if } i \in \underline{A}(\Omega_l) \\ \frac{p(l|x_{N_i}) f_{StN}(y; \theta_l^{(t)})}{\sum_{m \in \mathcal{L}} p(m|x_{N_i}) f_{StN}(y; \theta_m^{(t)})}, & \text{else if } i \in B(\Omega_l) \\ 0, & \text{otherwise.} \end{cases} \quad (8)$$

The parameters  $\mu_l$  and  $\sigma_l^2$  are estimated in the M-step of each iteration, as follows:

$$\hat{\mu}_l^{(t+1)} = \alpha \mathcal{A}_l^{(t)} + (1 - \alpha) \mathcal{B}_l^{(t)}; \quad (9)$$

$$\text{where } \mathcal{A}_l^{(t)} = \frac{\sum_{i \in \underline{A}(\Omega_l)} y_i}{|\underline{A}(\Omega_l)|} \quad \text{and} \quad \mathcal{B}_l^{(t)} = \frac{\sum_{i \in B(\Omega_l)} \tau_{il}^{(t)} y_i}{\sum_{i \in B(\Omega_l)} \tau_{il}^{(t)}},$$

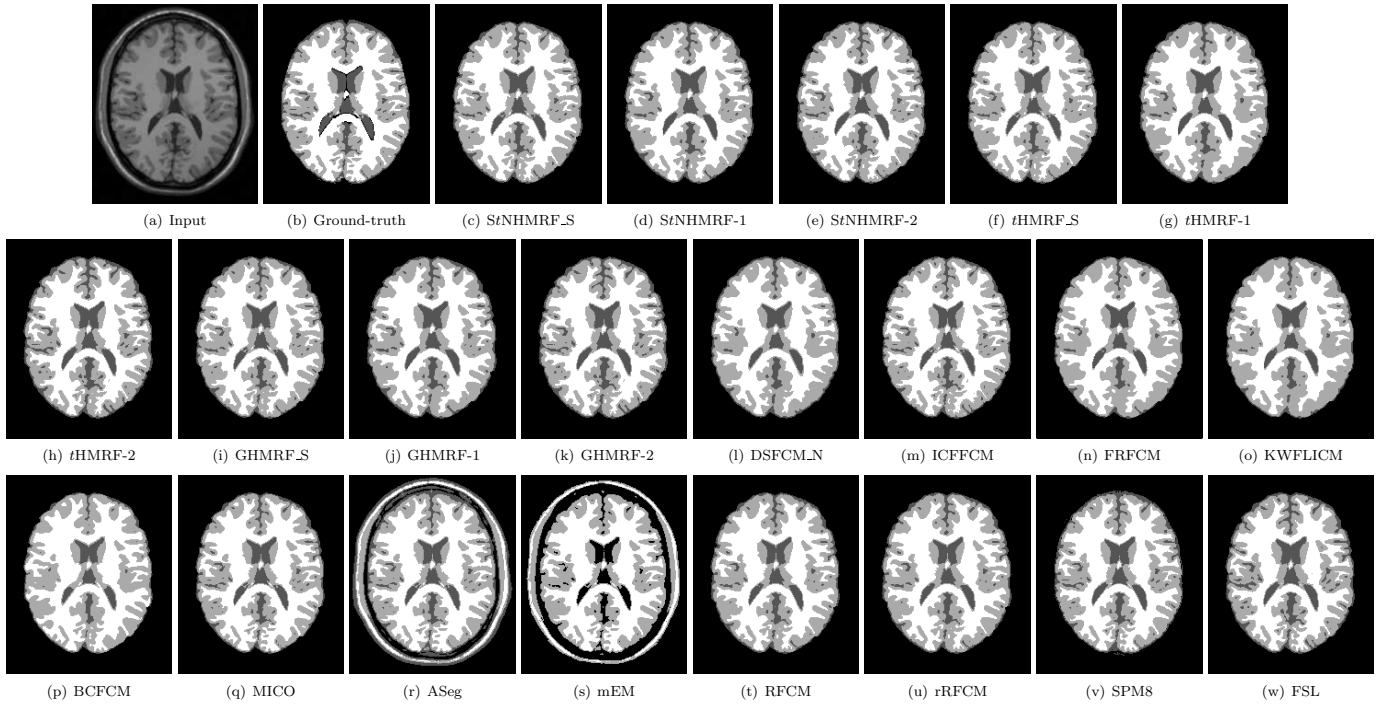


Fig. 1. Segmented images obtained by different segmentation algorithms on BrainWeb database with 1% noise and 40% bias field.

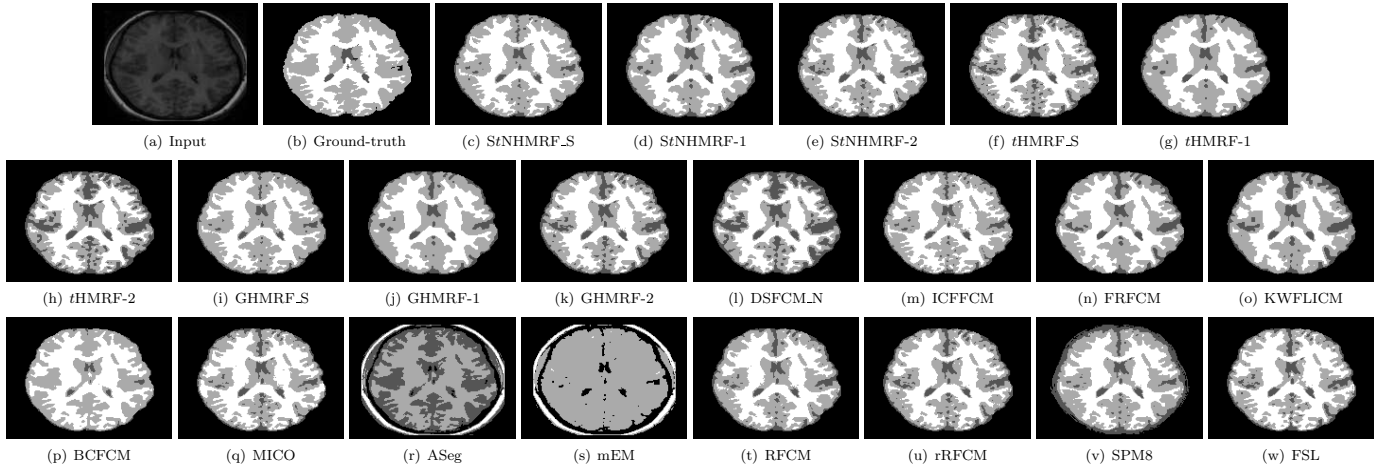


Fig. 2. Segmented images obtained by different segmentation algorithms on IBSR database volume number 3.

$\alpha$  is the relative importance of lower approximation region;

$$\hat{\sigma}_l^{(t+1)} = \frac{\sum_{i \in \underline{A}(\Omega_l)} (y_i - \hat{\mu}_l^{(t+1)})^2 + \sum_{i \in B(\Omega_l)} \tau_{il}^{(t)} (y_i - \hat{\mu}_l^{(t+1)})^2}{|\underline{A}(\Omega_l)| + \sum_{i \in B(\Omega_l)} \tau_{il}^{(t)}}. \quad (10)$$

## II. PROPOSED MODEL

### A. GHMF-S: GHMF with New Class Label Distribution

This section incorporates the concept of proposed class label distribution into Gaussian distribution and HMF model based image segmentation algorithm (GHMF) [1]. Let  $y_i$  be the intensity value of the  $i$ th pixel, where  $i \in \mathcal{S}$  and  $x_i$  denotes its corresponding label,  $x_i \in \mathcal{L} = \{1, 2, \dots, L\}$ . Hence, the

image can be represented as a mixture of finite number of Gaussian distributions, given its neighborhood configuration  $\mathcal{N}$ , as follows:

$$p(y_i | x_{\mathcal{N}_i}, \theta) = \sum_{l \in \mathcal{L}} p(y_i | X_i = l) p(l | x_{\mathcal{N}_i}) \quad \forall i \in \mathcal{S} \quad (11)$$

where  $p(y_i | l) = \frac{1}{\sqrt{2\pi}\sigma_l} \exp(-\frac{(y_i - \mu_l)^2}{2\sigma_l^2})$ .

Assuming the pixel intensities are statistically independent, the probability density of the entire image can be written as

$$p(\underline{y} | \underline{x}_{\mathcal{N}}, \theta) = \prod_{i \in \mathcal{S}} p(y_i | x_{\mathcal{N}_i}, \theta) = \prod_{i \in \mathcal{S}} \sum_{l \in \mathcal{L}} p(y_i | l) p(l | x_{\mathcal{N}_i}). \quad (12)$$

As the estimation of parameters  $\theta = \{\mu_l, \sigma_l; l \in \mathcal{L}\}$  from the above expression using either ML or MAP principles is computationally infeasible, the EM algorithm is applied to

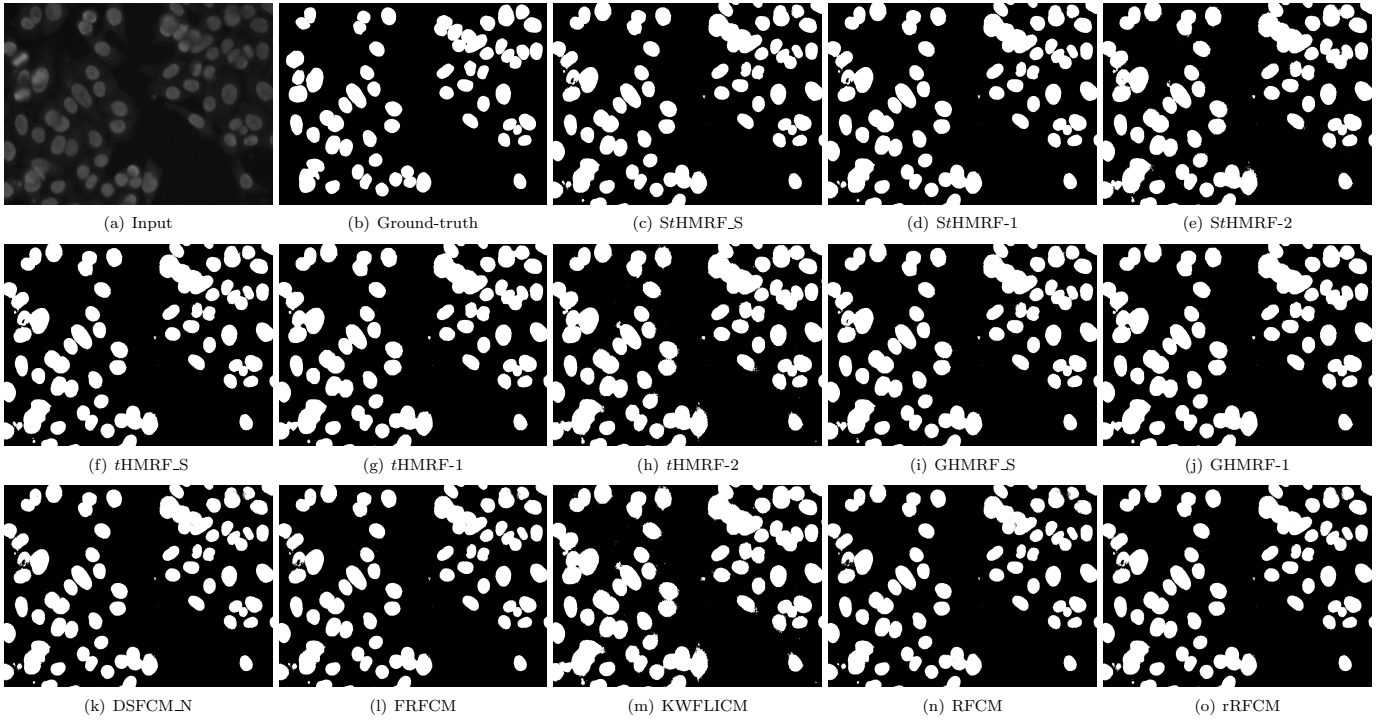


Fig. 3. Segmented images obtained by different segmentation algorithms on MIVIA dataset image number 01.

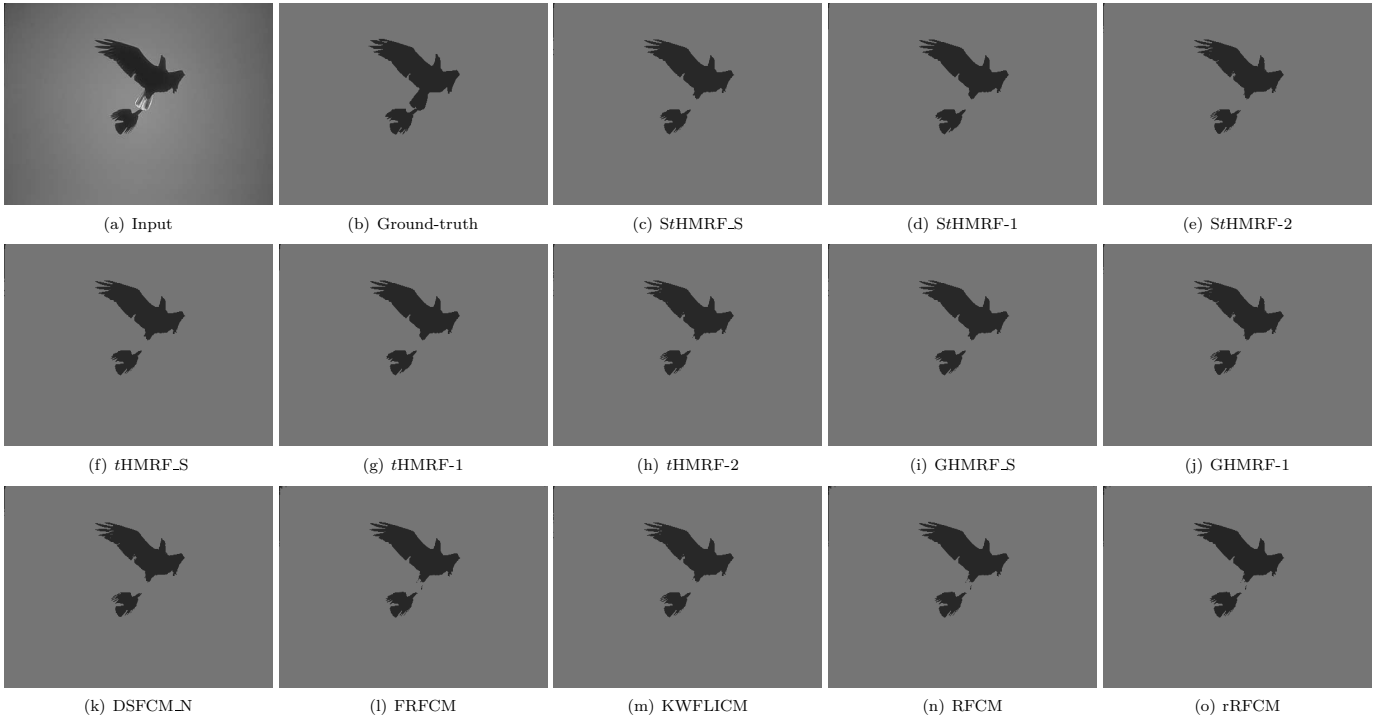


Fig. 4. Segmented images obtained by different segmentation algorithms on Berkeley Segmentation Dataset image number 135069.

solve the above problem. The standard EM algorithm has two parts: first it tries to estimate a set of latent variables based on the given data in its E-step; and then in the M-step, it tries to find the optimum estimate of the parameters of the distribution based on the original variables and the new set of latent variables. Iteratively optimizing these two steps, the EM algorithm converges to its local optimum solution. The latent

variables for the EM algorithm are defined as

$$\delta_{il} = \begin{cases} 1, & \text{if } X_i = l \\ 0, & \text{otherwise.} \end{cases}$$

In the E-step, the latent variables are estimated, given the

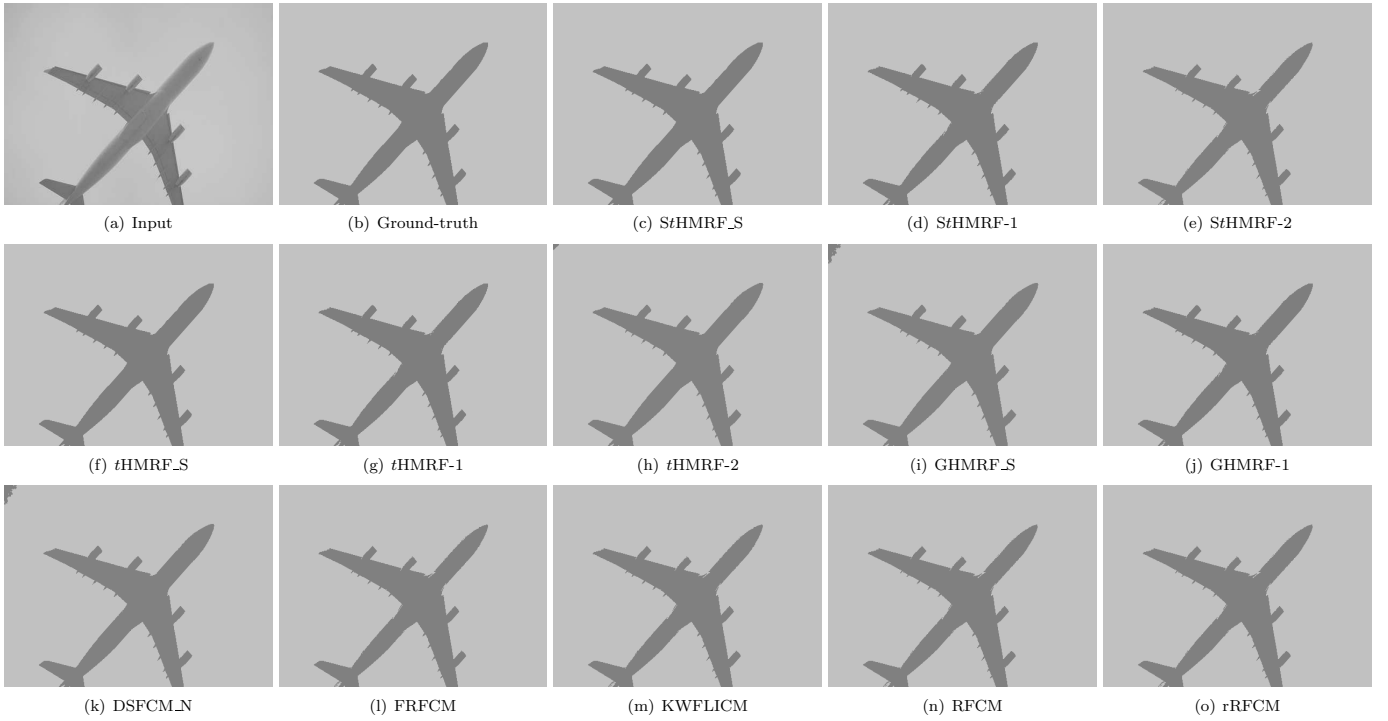


Fig. 5. Segmented images obtained by different segmentation algorithms on PASCAL VOC dataset image number 2008\_005443.

observed variables and the current estimate of the parameters: of the parameters:

$$\begin{aligned} E(\delta_{il}|y_i, \theta^{(t)}) &= p(\delta_{il} = 1|y_i, \theta^{(t)}) = p^{(t)}(l|y_i) \\ &= \frac{p^{(t)}(y_i|l) p(l|x_{\mathcal{N}_i})}{\sum_{m \in \mathcal{L}} p^{(t)}(y_i|m) p(m|x_{\mathcal{N}_i})} = \tau_{il}^{(t)} \end{aligned} \quad (13)$$

where  $\theta^{(t)}$  is the estimate of the parameters at  $t$ th iteration. The expression of  $\tau_{il}$  in (13) denotes the posterior probability that the  $i$ th pixel belongs to the  $l$ th tissue class  $\Omega_l$ , given its neighborhood configuration  $\mathcal{N}_i$  or, simply, the membership value of pixel  $i$  to class  $\Omega_l$ . Evidently, it estimates the belongingness of the  $i$ th pixel to  $\Omega_l$ . Hence, it can be considered as the membership value of pixel  $i$  to class  $\Omega_l$ , and the corresponding expression as the membership function, which calculates the membership of a pixel to a specific class. The prior probability  $p(l|x_{\mathcal{N}_i})$  is derived as

$$p(l|x_{\mathcal{N}_i}) = \frac{1}{Z} \exp \left( - \sum_{j \in \mathcal{N}_i} E_c(l, x_j) \right), \quad (14)$$

where  $Z$  is a normalizing constant and  $E_c(l, x_j)$  follows from

$$E_c(x_i, x_j) = \frac{a_i}{2|\mathcal{N}_i|} \left[ (\mu_{x_i} - \mu_{x_j})^2 \left( \frac{1}{\sigma_{x_i}^2} + \frac{1}{\sigma_{x_j}^2} \right) - 1 \right], \quad (15)$$

where  $a_i \geq 0$ .

In the M-step, the  $Q$ -function, that is, expected complete data log-likelihood, is calculated, given the current estimate

$$\begin{aligned} Q(\theta|\theta^{(t)}) &= E_X [\log p(x, y|\theta) | y, x_{\mathcal{N}}, \theta^{(t)}] \\ &= \sum_{i \in \mathcal{S}} \sum_{l \in \mathcal{L}} \tau_{il}^{(t)} [\log p^{(t)}(y_i|l) + \log p^{(t)}(l|x_{\mathcal{N}_i})] \\ &= \sum_{i \in \mathcal{S}} \sum_{l \in \mathcal{L}} \tau_{il}^{(t)} \left[ -\log \sigma_l - \frac{(y_i - \mu_l)^2}{2\sigma_l^2} + C \right. \\ &\quad \left. - \sum_{j \in \mathcal{N}_i} \frac{a_i}{2|\mathcal{N}_i|} [(\mu_l - \mu_{x_j})^2 \left( \frac{1}{\sigma_l^2} + \frac{1}{\sigma_{x_j}^2} \right) - 1] \right]. \end{aligned}$$

Optimizing the  $Q$ -function with respect to parameters, the optimal estimates of the parameters are obtained as

$$\hat{\mu}_l = \frac{\sum_{i \in \mathcal{S}} \tau_{il} \left( y_i + \frac{a_i}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \left( 1 + \frac{\sigma_l^2}{\sigma_{x_j}^2} \right) \mu_{x_j} \right)}{\sum_{i \in \mathcal{S}} \tau_{il} \left( 1 + \frac{a_i}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \left( 1 + \frac{\sigma_l^2}{\sigma_{x_j}^2} \right) \right)}; \quad (16)$$

$$\hat{\sigma}_l^2 = \frac{\sum_{i \in \mathcal{S}} \tau_{il} ((y_i - \mu_l)^2 + \frac{a_i}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (\mu_l - \mu_{x_j})^2)}{\sum_{i \in \mathcal{S}} \tau_{il}}. \quad (17)$$

The optimal labeling of the pixels of the image are estimated according to the MAP criterion:

$$\begin{aligned} \hat{x} &= \arg \max_x [p(y|x)p(x)] \\ &= \arg \min_x \sum_{i \in \mathcal{S}} \left[ \frac{(y_i - \mu_{x_i})^2}{2\sigma_{x_i}^2} + \log \sigma_{x_i} \right. \\ &\quad \left. + \frac{a_i}{2|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} [(\mu_{x_i} - \mu_{x_j})^2 \left( \frac{1}{\sigma_{x_i}^2} + \frac{1}{\sigma_{x_j}^2} \right) - 1] \right] \end{aligned} \quad (18)$$

TABLE I

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF NEW SCALING PARAMETER OVER EXISTING CLASS LABEL DISTRIBUTIONS FOR BRAIN MR IMAGE SEGMENTATION

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
StNHMF model				
Dice Coefficient				
Existing-1-ws	0.861580	0.077099	6.3E-07	1.4E-07
Existing-1	0.858091	0.078481		
Existing-2-ws	0.852395	0.078517	2.4E-08	1.0E-10
Existing-2	0.846699	0.078862		
Sensitivity				
Existing-1-ws	0.896779	0.055905	0.0337	0.0489
Existing-1	0.896170	0.055948		
Existing-2-ws	0.893181	0.056361	1.1E-04	1.4E-05
Existing-2	0.890192	0.057102		
Specificity				
Existing-1-ws	0.980827	0.006590	1.2E-04	1.0E-04
Existing-1	0.980480	0.006657		
Existing-2-ws	0.979522	0.006712	1.8E-06	8.8E-08
Existing-2	0.978564	0.006927		
tHMF model				
Dice Coefficient				
Existing-1-ws	0.855114	0.073178	1.2E-03	1.3E-03
Existing-1	0.850931	0.071611		
Existing-2-ws	0.817787	0.083471	2.3E-10	1.5E-11
Existing-2	0.784643	0.099475		
Sensitivity				
Existing-1-ws	0.884614	0.051891	0.0594	0.0695
Existing-1	0.882962	0.050941		
Existing-2-ws	0.865230	0.060702	9.2E-07	2.1E-07
Existing-2	0.847498	0.073947		
Specificity				
Existing-1-ws	0.979027	0.006009	1.8E-03	2.4E-03
Existing-1	0.978386	0.005987		
Existing-2-ws	0.973751	0.007469	1.9E-11	2.9E-11
Existing-2	0.969117	0.009649		
GHMF model				
Dice Coefficient				
Existing-1-ws	0.853900	0.073366	3.3E-07	3.5E-08
Existing-1	0.846710	0.074879		
Existing-2-ws	0.828409	0.081843	8.1E-08	1.5E-11
Existing-2	0.810108	0.091419		
Sensitivity				
Existing-1-ws	0.887709	0.054080	1.4E-03	1.4E-03
Existing-1	0.884305	0.054941		
Existing-2-ws	0.874643	0.061790	2.4E-05	2.4E-07
Existing-2	0.864982	0.070211		
Specificity				
Existing-1-ws	0.978752	0.006322	6.6E-06	4.0E-08
Existing-1	0.977571	0.006814		
Existing-2-ws	0.975221	0.007605	2.4E-09	7.3E-11
Existing-2	0.972871	0.008715		

TABLE II

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF NEW SCALING PARAMETER OVER EXISTING CLASS LABEL DISTRIBUTIONS FOR HEP-2 CELL SEGMENTATION

Algorithm	Mean	Std. Dev.	p-value	
			Paired $t$	Wilcoxon
StNHMF model				
Dice Coefficient				
Existing-1-ws	0.895576	0.031789	0.1028	0.3848
Existing-1	0.892288	0.033415		
Existing-2-ws	0.879865	0.042589	0.0628	0.0801
Existing-2	0.867441	0.059945		
Sensitivity				
Existing-1-ws	0.933717	0.052259	0.0300	0.0244
Existing-1	0.926420	0.060414		
Existing-2-ws	0.917545	0.054735	4.7E-03	4.9E-03
Existing-2	0.908669	0.049883		
Specificity				
Existing-1-ws	0.864138	0.071558	0.0204	4.9E-03
Existing-1	0.845440	0.084227		
Existing-2-ws	0.786314	0.123006	1.9E-03	9.8E-04
Existing-2	0.727188	0.166897		
tHMF model				
Dice Coefficient				
Existing-1-ws	0.883215	0.042770	0.0335	0.0244
Existing-1	0.874381	0.051621		
Existing-2-ws	0.849513	0.060905	3.0E-03	2.9E-03
Existing-2	0.824645	0.075688		
Sensitivity				
Existing-1-ws	0.926526	0.051528	0.0304	0.0244
Existing-1	0.919323	0.060256		
Existing-2-ws	0.910478	0.053975	4.8E-03	4.9E-03
Existing-2	0.901634	0.048392		
Specificity				
Existing-1-ws	0.795096	0.117400	6.7E-03	9.8E-04
Existing-1	0.761646	0.147109		
Existing-2-ws	0.666743	0.171329	8.9E-04	9.8E-04
Existing-2	0.571839	0.215553		
GHMF model				
Dice Coefficient				
Existing-1-ws	0.876072	0.050192	0.0469	0.0654
Existing-1	0.864202	0.066295		
Existing-2-ws	0.846468	0.065121	3.2E-04	9.8E-04
Existing-2	0.828734	0.065785		
Sensitivity				
Existing-1-ws	0.923112	0.053294	0.0303	0.0244
Existing-1	0.915953	0.061921		
Existing-2-ws	0.907143	0.055906	4.8E-03	4.9E-03
Existing-2	0.898332	0.050607		
Specificity				
Existing-1-ws	0.763175	0.142030	0.0114	9.8E-04
Existing-1	0.717452	0.190148		
Existing-2-ws	0.652590	0.182457	2.7E-04	9.8E-04
Existing-2	0.587728	0.183026		

Using an iterative optimization technique, termed as iterated conditional modes (ICM) algorithm [2], this optimization problem is reduced to

$$\hat{x}_i = \arg \min_{x_i} \left[ \frac{(y_i - \mu_{x_i})^2}{2\sigma_{x_i}^2} + \log \sigma_{x_i} \right] + \frac{a_i}{2|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \left[ (\mu_{x_i} - \mu_{x_j})^2 \left( \frac{1}{\sigma_{x_i}^2} + \frac{1}{\sigma_{x_j}^2} \right) - 1 \right] \quad (19)$$

#### B. StNHMF<sub>S</sub>: StNHMF with New Class Label Model

This section introduces the novel class label distribution into the rough-probabilistic clustering and HMF model based

image segmentation algorithm (StNHMF) [3]. The algorithm models the intensity distribution of the image as a mixture of finite number of stomped normal (SN) distributions, given its neighborhood configuration  $\mathcal{N}$ , as follows:

$$p(y_i | x_{\mathcal{N}_i}, \theta) = \sum_{l \in \mathcal{L}} p(y_i | l) p(l | x_{\mathcal{N}_i}) \quad \forall i \in \mathcal{S} \quad (20)$$

where  $p(y_i | l) = \frac{1}{D_l} \frac{1}{\sigma_l} \phi(z_{il})$ ,  $z_{il} = \begin{cases} k_l, & \text{if } i \in \underline{A}(\Omega_l) \\ \frac{y_i - \mu_l}{\sigma_l}, & \text{if } i \in B(\Omega_l) \end{cases}$

The prior probability  $p(l | x_{\mathcal{N}_i})$  is calculated according to equation (14) and (15). Here, each tissue class  $\Omega_l$  is represented by a crisp lower approximation  $\underline{A}(\Omega_l)$  and a probabilistic

TABLE III

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF NEW SCALING PARAMETER OVER EXISTING CLASS LABEL DISTRIBUTIONS IN “BERKELEY IMAGE SEGMENTATION DATASET”

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
StNHMRf model				
Dice Coefficient				
Existing-1-ws	0.887714	0.061679	8.2E-06	2.4E-04
Existing-1	0.885203	0.061201		
Existing-2-ws	0.883475	0.060779	3.7E-04	1.7E-03
Existing-2	0.881746	0.060383		
Sensitivity				
Existing-1-ws	0.911335	0.068022	3.2E-04	1.2E-03
Existing-1	0.909651	0.067862		
Existing-2-ws	0.909720	0.067736	<b>0.5869</b>	0.3955
Existing-2	0.909789	0.067626		
Specificity				
Existing-1-ws	0.911170	0.057926	0.0291	2.4E-04
Existing-1	0.908665	0.060913		
Existing-2-ws	0.908226	0.060861	1.5E-04	2.4E-04
Existing-2	0.907786	0.060809		
tHMRf model				
Dice Coefficient				
Existing-1-ws	0.876430	0.061682	2.0E-05	2.4E-04
Existing-1	0.869843	0.061432		
Existing-2-ws	0.861289	0.061067	4.6E-06	2.4E-04
Existing-2	0.852735	0.060942		
Sensitivity				
Existing-1-ws	0.898481	0.070257	1.2E-04	2.4E-04
Existing-1	0.893905	0.071816		
Existing-2-ws	0.892975	0.070425	0.3002	<b>0.6333</b>
Existing-2	0.892045	0.069521		
Specificity				
Existing-1-ws	0.906099	0.061679	1.2E-03	4.9E-04
Existing-1	0.904738	0.062049		
Existing-2-ws	0.903266	0.062074	6.2E-04	4.9E-04
Existing-2	0.901794	0.062121		
GHMRf model				
Dice Coefficient				
Existing-1-ws	0.872134	0.061810	1.2E-05	2.4E-04
Existing-1	0.864338	0.061984		
Existing-2-ws	0.857075	0.060994	1.3E-04	4.9E-04
Existing-2	0.849812	0.060366		
Sensitivity				
Existing-1-ws	0.896880	0.071760	1.4E-05	2.4E-04
Existing-1	0.889925	0.073970		
Existing-2-ws	0.888910	0.074855	0.2721	0.0647
Existing-2	0.887896	0.076145		
Specificity				
Existing-1-ws	0.898705	0.063991	3.9E-04	4.9E-04
Existing-1	0.893801	0.065739		
Existing-2-ws	0.893354	0.066174	0.3546	<b>0.6045</b>
Existing-2	0.892906	0.066853		

TABLE IV

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF NEW SCALING PARAMETER OVER EXISTING CLASS LABEL DISTRIBUTIONS IN “PASCAL VISUAL OBJECT CLASSES (VOC) DATASET”

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
StNHMRF model				
Dice Coefficient				
Existing-1-ws	0.855245	0.031083	6.9E-04	2.4E-04
Existing-1	0.851944	0.031651		
Existing-2-ws	0.844702	0.032346	2.5E-03	2.4E-04
Existing-2	0.837460	0.034545		
Sensitivity				
Existing-1-ws	0.899662	0.034038	2.0E-03	7.3E-04
Existing-1	0.896351	0.036026		
Existing-2-ws	0.892180	0.034548	1.1E-04	4.9E-04
Existing-2	0.888009	0.033218		
Specificity				
Existing-1-ws	0.914994	0.026409	2.7E-03	4.9E-04
Existing-1	0.907788	0.030033		
Existing-2-ws	0.886988	0.038445	2.4E-04	2.4E-04
Existing-2	0.866187	0.049891		
tHMRF model				
Dice Coefficient				
Existing-1-ws	0.848073	0.034558	3.2E-04	7.3E-04
Existing-1	0.842142	0.034931		
Existing-2-ws	0.813957	0.036663	5.4E-07	2.4E-04
Existing-2	0.785772	0.040917		
Sensitivity				
Existing-1-ws	0.890928	0.032694	4.4E-04	1.2E-03
Existing-1	0.886762	0.034452		
Existing-2-ws	0.873019	0.033380	5.6E-06	2.4E-04
Existing-2	0.859276	0.033482		
Specificity				
Existing-1-ws	0.889322	0.034558	1.0E-03	2.4E-04
Existing-1	0.877421	0.042687		
Existing-2-ws	0.842660	0.051154	5.7E-05	2.4E-04
Existing-2	0.807900	0.065304		
GHMRF model				
Dice Coefficient				
Existing-1-ws	0.842880	0.034082	8.3E-04	4.9E-04
Existing-1	0.834213	0.037241		
Existing-2-ws	0.815069	0.036524	4.3E-07	2.4E-04
Existing-2	0.795925	0.037039		
Sensitivity				
Existing-1-ws	0.888976	0.035739	2.9E-05	2.4E-04
Existing-1	0.883000	0.038025		
Existing-2-ws	0.873415	0.037124	6.6E-05	2.4E-04
Existing-2	0.863829	0.037115		
Specificity				
Existing-1-ws	0.877091	0.042732	2.0E-03	2.4E-04
Existing-1	0.860106	0.056965		
Existing-2-ws	0.836746	0.054942	2.3E-05	2.4E-04
Existing-2	0.813386	0.055722		

boundary  $B(\Omega_l)$ , as follows:

$$\underline{A}(\Omega_l) = \left\{ i \in \mathcal{S} : \left| \frac{y_i - \mu_l}{\sigma_l} \right| < k_l \right\}; \quad (21)$$

$$B(\Omega_l) = \{ i \in \mathcal{S} : i \notin \underline{A}(\Omega_p), \forall p \in \mathcal{L} \}. \quad (22)$$

The same expectation-maximization algorithm can be used, with the following modifications in the estimates of latent variables in the expectation step as

$$\tau_{il} = \begin{cases} 1, & \text{if } i \in \underline{A}(\Omega_l) \\ \frac{p(y_i|l)p(l|\mathcal{N}_i)}{\sum_{m \in \mathcal{L}} p(y_i|m)p(m|\mathcal{N}_i)}, & \text{else if } i \in B(\Omega_l) \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

and in the estimates parameters in the maximization step as

$$\hat{\mu}_l = \alpha \mathcal{A}_l + (1 - \alpha) \mathcal{B}_l; \quad (24)$$

$$\begin{aligned} \text{where } \mathcal{A}_l &= \frac{\sum_{i \in \underline{A}(\Omega_l)} \left( y_i + \frac{\alpha_i}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \left( 1 + \frac{\sigma_l^2}{\sigma_{x_j}^2} \right) \mu_{x_j} \right)}{\sum_{i \in \underline{A}(\Omega_l)} \left( 1 + \frac{\alpha_i}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \left( 1 + \frac{\sigma_l^2}{\sigma_{x_j}^2} \right) \right)}, \\ \mathcal{B}_l &= \frac{\sum_{i \in B(\Omega_l)} \tau_{il} \left( y_i + \frac{\alpha_i}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \left( 1 + \frac{\sigma_l^2}{\sigma_{x_j}^2} \right) \mu_{x_j} \right)}{\sum_{i \in B(\Omega_l)} \tau_{il} \left( 1 + \frac{\alpha_i}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} \left( 1 + \frac{\sigma_l^2}{\sigma_{x_j}^2} \right) \right)}, \end{aligned}$$

TABLE V

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF NOVEL CLASS LABEL DISTRIBUTION IN BRAIN MR IMAGE SEGMENTATION

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
StNHMRf model				
Dice Coefficient				
Proposed	0.868557	0.074774		
Without scale	0.865068	0.075861	6.3E-07	1.4E-07
Existing-1-ws	0.861580	0.077099	6.3E-07	1.4E-07
Existing-2-ws	0.852395	0.078517	3.3E-11	2.9E-11
Sensitivity				
Proposed	0.897998	0.056020		
Without scale	0.897389	0.055929	0.0337	0.0489
Existing-1-ws	0.896779	0.055905	0.0337	0.0489
Existing-2-ws	0.893181	0.056361	6.8E-07	1.0E-06
Specificity				
Proposed	0.981520	0.006572		
Without scale	0.981173	0.006562	1.2E-04	1.0E-04
Existing-1-ws	0.980827	0.006590	1.2E-04	1.0E-04
Existing-2-ws	0.979522	0.006712	2.1E-11	2.9E-11
tHMRf model				
Dice Coefficient				
Proposed	0.863481	0.078494		
Without scale	0.859297	0.075494	1.2E-03	1.3E-03
Existing-1-ws	0.855114	0.073178	1.2E-03	1.3E-03
Existing-2-ws	0.817787	0.083471	2.4E-11	1.5E-11
Sensitivity				
Proposed	0.887919	0.055847		
Without scale	0.886267	0.053547	0.0594	0.0695
Existing-1-ws	0.884614	0.051891	0.0594	0.0695
Existing-2-ws	0.865230	0.060702	1.1E-07	1.9E-07
Specificity				
Proposed	0.980310	0.006764		
Without scale	0.979668	0.006278	1.8E-03	2.4E-03
Existing-1-ws	0.979027	0.006009	1.8E-03	2.4E-03
Existing-2-ws	0.973751	0.007469	3.0E-12	2.9E-11
GHMRf model				
Dice Coefficient				
Proposed	0.868279	0.072383		
Without scale	0.861090	0.072527	3.3E-07	3.5E-08
Existing-1-ws	0.853900	0.073366	3.3E-07	3.5E-08
Existing-2-ws	0.828409	0.081843	1.8E-10	1.5E-11
Sensitivity				
Proposed	0.894519	0.054564		
Without scale	0.891114	0.053953	1.4E-03	1.4E-03
Existing-1-ws	0.887709	0.054080	1.4E-03	1.4E-03
Existing-2-ws	0.874643	0.061790	6.3E-06	1.1E-07
Specificity				
Proposed	0.981114	0.006227		
Without scale	0.979933	0.006116	6.6E-06	4.0E-08
Existing-1-ws	0.978752	0.006322	6.6E-06	4.0E-08
Existing-2-ws	0.975221	0.007605	6.9E-10	1.5E-11

TABLE VI

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF S<sub>t</sub>NHMRf MODEL IN BRAIN MR IMAGE SEGMENTATION

Algorithm	Test	p-value		
		Dice	Sensitivity	Specificity
<i>t</i> HMRf_S	Paired <i>t</i>	0.0751	2.3E-07	1.9E-03
	Wilcoxon	0.2348	7.2E-08	7.7E-04
GHMRf_S	Paired <i>t</i>	0.3780	4.0E-07	2.8E-03
	Wilcoxon	0.1691	9.2E-07	5.3E-03

TABLE VII

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF NOVEL CLASS LABEL DISTRIBUTION IN HEP-2 CELL SEGMENTATION

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
StNHMRF model				
Dice Coefficient				
Proposed	0.902152	0.033866		
Without scale	0.898864	0.031947	0.1028	0.3848
Existing-1-ws	0.895576	0.031789	0.1028	0.3848
Existing-2-ws	0.879865	0.042589	0.0750	0.0801
Sensitivity				
Proposed	0.948311	0.039805		
Without scale	0.941014	0.045194	0.0300	0.0244
Existing-1-ws	0.933717	0.052259	0.0300	0.0244
Existing-2-ws	0.917545	0.054735	2.3E-03	9.8E-04
Specificity				
Proposed	0.901536	0.069698		
Without scale	0.882837	0.066144	0.0204	4.9E-03
Existing-1-ws	0.864138	0.071558	0.0204	4.9E-03
Existing-2-ws	0.786314	0.123006	5.8E-03	9.8E-04
tHMRF model				
Dice Coefficient				
Proposed	0.900883	0.035199		
Without scale	0.892049	0.036797	0.0335	0.0244
Existing-1-ws	0.883215	0.042770	0.0335	0.0244
Existing-2-ws	0.849513	0.060905	5.4E-03	2.0E-03
Sensitivity				
Proposed	0.940933	0.037186		
Without scale	0.933730	0.043655	0.0304	0.0244
Existing-1-ws	0.926526	0.051528	0.0304	0.0244
Existing-2-ws	0.910478	0.053975	2.3E-03	9.8E-04
Specificity				
Proposed	0.861995	0.072025		
Without scale	0.828545	0.091095	6.7E-03	9.8E-04
Existing-1-ws	0.795096	0.117400	6.7E-03	9.8E-04
Existing-2-ws	0.666743	0.171329	6.0E-04	9.8E-04
GHMRF model				
Dice Coefficient				
Proposed	0.899812	0.034315		
Without scale	0.887942	0.038036	0.0469	0.0654
Existing-1-ws	0.876072	0.050192	0.0469	0.0654
Existing-2-ws	0.846468	0.065121	9.5E-03	4.9E-03
Sensitivity				
Proposed	0.937430	0.039001		
Without scale	0.930271	0.045489	0.0303	0.0244
Existing-1-ws	0.923112	0.053294	0.0303	0.0244
Existing-2-ws	0.907143	0.055906	2.3E-03	9.8E-04
Specificity				
Proposed	0.854623	0.070165		
Without scale	0.808899	0.098800	0.0114	9.8E-04
Existing-1-ws	0.763175	0.142030	0.0114	9.8E-04
Existing-2-ws	0.652590	0.182457	1.1E-03	9.8E-04

TABLE VIII

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF S<sub>t</sub>NHMRf MODEL IN HEP-2 CELL SEGMENTATION

Algorithm	Test	p-value		
		Dice	Sensitivity	Specificity
<i>t</i> HMRf_S	Paired <i>t</i>	0.4274	0.0178	0.0152
	Wilcoxon	<b>0.7217</b>	0.0244	2.0E-03
GHMRf_S	Paired <i>t</i>	0.3703	0.0176	8.1E-03
	Wilcoxon	<b>0.6523</b>	0.0244	9.8E-04

Using MAP criterion and ICM algorithm, the class label of the *i*th pixel is estimated as

$$(\hat{\sigma}_l^2) = \frac{\sum_{i \in \mathcal{S}} \tau_{il} \left( (y_i - \mu_l)^2 + \frac{\alpha_i}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} (\mu_l - \mu_{x_j})^2 \right)}{\sum_{i \in \mathcal{S}} \tau_{il}}. \quad (25)$$

$$\hat{x}_i = \arg \min_{x_i} \left[ \frac{1}{2} z_{ix_i}^2 + \log \sigma_{x_i} + \frac{\alpha_i}{2|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} [(\mu_{x_i} - \mu_{x_j})^2 \left( \frac{1}{\sigma_{x_i}^2} + \frac{1}{\sigma_{x_j}^2} \right) - 1] \right] \quad (26)$$

TABLE IX

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF NOVEL CLASS LABEL DISTRIBUTION IN “BERKELEY IMAGE SEGMENTATION DATASET”

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
StNHMRF model				
Dice Coefficient				
Proposed	0.892734	0.062692		
Without scale	0.890224	0.062176	8.2E-06	2.4E-04
Existing-1-ws	0.887714	0.061679	8.2E-06	2.4E-04
Existing-2-ws	0.883475	0.060779	5.8E-07	2.4E-04
Sensitivity				
Proposed	0.914705	0.068409		
Without scale	0.913020	0.068204	3.2E-04	1.2E-03
Existing-1-ws	0.911335	0.068022	3.2E-04	1.2E-03
Existing-2-ws	0.909720	0.067736	1.7E-04	4.9E-04
Specificity				
Proposed	0.916180	0.052406		
Without scale	0.913675	0.055082	0.0291	2.4E-04
Existing-1-ws	0.911170	0.057926	0.0291	2.4E-04
Existing-2-ws	0.908226	0.060861	0.0228	2.4E-04

<b><math>t</math>HMRf model</b>				
<b>Dice Coefficient</b>				
Proposed	0.889602	0.062759		
Without scale	0.883016	0.062126	2.0E-05	2.4E-04
Existing-1-ws	0.876430	0.061682	2.0E-05	2.4E-04
Existing-2-ws	0.861289	0.061067	8.7E-07	2.4E-04
<b>Sensitivity</b>				
Proposed	0.907632	0.067427		
Without scale	0.903056	0.068791	1.2E-04	2.4E-04
Existing-1-ws	0.898481	0.070257	1.2E-04	2.4E-04
Existing-2-ws	0.892975	0.070425	3.0E-04	2.4E-04
<b>Specificity</b>				
Proposed	0.908822	0.061005		
Without scale	0.907461	0.061331	1.2E-03	4.9E-04
Existing-1-ws	0.906099	0.061679	1.2E-03	4.9E-04
Existing-2-ws	0.903266	0.062074	3.5E-04	2.4E-04

<b>GHMRf model</b>				
<b>Dice Coefficient</b>				
Proposed	0.887728	0.062190		
Without scale	0.879931	0.061879	1.2E-05	2.4E-04
Existing-1-ws	0.872134	0.061810	1.2E-05	2.4E-04
Existing-2-ws	0.857075	0.060994	5.3E-06	2.4E-04
<b>Sensitivity</b>				
Proposed	0.910789	0.067677		
Without scale	0.903835	0.069659	1.4E-05	2.4E-04
Existing-1-ws	0.896880	0.071760	1.4E-05	2.4E-04
Existing-2-ws	0.888910	0.074855	8.9E-06	2.4E-04
<b>Specificity</b>				
Proposed	0.908512	0.061019		
Without scale	0.903609	0.062413	3.9E-04	4.9E-04
Existing-1-ws	0.898705	0.063991	3.9E-04	4.9E-04
Existing-2-ws	0.893354	0.066174	8.0E-05	2.4E-04

TABLE X

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF  $S_t$ NHMRf MODEL IN “BERKELEY IMAGE SEGMENTATION DATASET”

Algorithm	Test	p-value		
		Dice	Sensitivity	Specificity
$t$ HMRf_S	Paired $t$	3.5E-05	2.6E-08	0.0315
	Wilcoxon	4.9E-04	2.4E-04	4.9E-04
GHMRf_S	Paired $t$	1.3E-05	1.1E-04	0.0282
	Wilcoxon	2.4E-04	2.4E-04	2.4E-04

### III. EXPERIMENTAL RESULTS AND DISCUSSION

For BrainWeb database, the brain MR images are generated using an MRI simulator by varying bias field artifacts (0, 20, and 40%) and noise levels (0, 1, 3, 5, 7, and 9%) present in the image. The anatomical model is used as ground truth

TABLE XI

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF NOVEL CLASS LABEL DISTRIBUTION IN “PASCAL VISUAL OBJECT CLASSES DATASET”

Algorithm	Mean	Std. Dev.	p-value	
			Paired $t$	Wilcoxon
StNHMRf model				
Dice Coefficient				
Proposed	0.861847	0.030635		
Without scale	0.858546	0.030741	6.9E-04	2.4E-04
Existing-1-ws	0.855245	0.031083	6.9E-04	2.4E-04
Existing-2-ws	0.844702	0.032346	9.6E-04	7.3E-04
Sensitivity				
Proposed	0.906285	0.030669		
Without scale	0.902974	0.032242	2.0E-03	7.3E-04
Existing-1-ws	0.899662	0.034038	2.0E-03	7.3E-04
Existing-2-ws	0.892180	0.034548	2.7E-05	2.4E-04
Specificity				
Proposed	0.929407	0.024509		
Without scale	0.922200	0.024434	2.7E-03	4.9E-04
Existing-1-ws	0.914994	0.026409	2.7E-03	4.9E-04
Existing-2-ws	0.886988	0.038445	6.1E-04	2.4E-04

<b><math>t</math>HMRf model</b>				
<b>Dice Coefficient</b>				
Proposed	0.859935	0.035456		
Without scale	0.854004	0.034736	3.2E-04	7.3E-04
Existing-1-ws	0.848073	0.034558	3.2E-04	7.3E-04
Existing-2-ws	0.813957	0.036663	1.0E-06	2.4E-04
<b>Sensitivity</b>				
Proposed	0.899261	0.029898		
Without scale	0.895095	0.031164	4.4E-04	1.2E-03
Existing-1-ws	0.890928	0.032694	4.4E-04	1.2E-03
Existing-2-ws	0.873019	0.033380	8.4E-07	2.4E-04
<b>Specificity</b>				
Proposed	0.913124	0.024006		
Without scale	0.901223	0.027910	1.0E-03	2.4E-04
Existing-1-ws	0.889322	0.034558	1.0E-03	2.4E-04
Existing-2-ws	0.842660	0.051154	3.7E-05	2.4E-04

<b>GHMRf model</b>				
<b>Dice Coefficient</b>				
Proposed	0.860214	0.032060		
Without scale	0.851547	0.032279	8.3E-04	4.9E-04
Existing-1-ws	0.842880	0.034082	8.3E-04	4.9E-04
Existing-2-ws	0.815069	0.036524	1.8E-05	2.4E-04
<b>Sensitivity</b>				
Proposed	0.900928	0.031698		
Without scale	0.894952	0.033619	2.9E-05	2.4E-04
Existing-1-ws	0.888976	0.035739	2.9E-05	2.4E-04
Existing-2-ws	0.873415	0.037124	1.8E-06	2.4E-04
<b>Specificity</b>				
Proposed	0.911061	0.023820		
Without scale	0.894076	0.030553	2.0E-03	2.4E-04
Existing-1-ws	0.877091	0.042732	2.0E-03	2.4E-04
Existing-2-ws	0.836746	0.054942	9.0E-05	2.4E-04

TABLE XII

STATISTICAL SIGNIFICANCE ANALYSIS FOR THE IMPORTANCE OF  $S_t$ NHMRf MODEL IN “PASCAL VISUAL OBJECT CLASSES DATASET”

Algorithm	Test	p-value		
		Dice	Sensitivity	Specificity
$t$ HMRf_S	Paired $t$	0.3064	2.0E-04	9.6E-04
	Wilcoxon	<b>0.6614</b>	4.9E-04	2.4E-04
GHMRf_S	Paired $t$	0.2328	4.8E-04	3.7E-04
	Wilcoxon	0.1506	1.7E-03	2.4E-04

segmentation of the generated volumes. The “percent noise” denotes the percent ratio of the standard deviation of additive white Gaussian noise over the signal for brightest tissue. The manual segmentation for each of the 18 volumes of IBRSR database is generated by an expert supervisor, serving as the



TABLE XIII

STATISTICAL SIGNIFICANCE ANALYSIS OF THE PERFORMANCE OF StNHMRFS OVER STATE-OF-THE-ART ALGORITHMS IN BRAIN MR IMAGE SEGMENTATION

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
Dice Coefficient				
S <sub>t</sub> NHMRFS	0.868557	0.074774		
DSFCM_N	0.802091	0.118710	2.4E-06	4.4E-11
ICFFCM	0.839345	0.099948	2.4E-03	1.4E-03
FRFCM	0.821467	0.108783	8.3E-05	3.0E-08
KWFLICM	0.816302	0.114600	1.3E-04	1.6E-06
BCFCM	0.850898	0.053002	2.2E-03	1.3E-03
MICO	0.853559	0.090479	8.0E-03	0.0189
ASeg	0.663345	0.190230	5.5E-09	1.5E-11
mEM	0.604897	0.121859	1.7E-14	1.5E-11
RFCM	0.858737	0.072801	3.1E-05	5.0E-05
rRFCM	0.838089	0.092067	1.1E-05	1.5E-11
SPM8	0.829218	0.164019	0.0203	2.7E-04
FSL	0.830975	0.072109	1.2E-15	1.5E-11
Sensitivity				
S <sub>t</sub> NHMRFS	0.897998	0.056020		
DSFCM_N	0.831242	0.110633	1.7E-05	2.0E-10
ICFFCM	0.866048	0.099258	5.0E-03	5.0E-03
FRFCM	0.843112	0.104947	9.1E-05	2.5E-09
KWFLICM	0.836768	0.103975	3.3E-05	4.8E-10
BCFCM	0.850749	0.047603	4.6E-14	1.5E-11
MICO	0.885788	0.075168	0.0414	0.1651
ASeg	0.769097	0.199141	7.7E-05	1.0E-09
mEM	0.664417	0.128373	1.3E-13	1.5E-11
RFCM	0.888701	0.054362	4.0E-06	1.7E-07
rRFCM	0.880523	0.063241	1.6E-05	1.3E-09
SPM8	0.887681	0.156802	0.3046	<b>0.9558</b>
FSL	0.884582	0.054352	2.5E-06	4.5E-07
Specificity				
S <sub>t</sub> NHMRFS	0.981520	0.006572		
DSFCM_N	0.972611	0.013798	6.7E-06	1.1E-08
ICFFCM	0.977687	0.011973	2.4E-03	6.6E-03
FRFCM	0.975682	0.012833	2.2E-04	1.2E-05
KWFLICM	0.973961	0.014014	2.0E-04	8.0E-06
BCFCM	0.978647	0.006816	1.9E-04	1.9E-04
MICO	0.979337	0.009837	6.7E-03	0.0119
ASeg	0.934725	0.029774	3.6E-12	1.5E-11
mEM	0.931670	0.025061	7.4E-15	1.5E-11
RFCM	0.979618	0.006808	7.3E-05	9.7E-06
rRFCM	0.976741	0.009781	3.8E-05	1.5E-11
SPM8	0.953132	0.163535	0.1461	0.0558
FSL	0.976818	0.006379	3.6E-11	1.5E-11

gold standard for segmentation. All MR volumes of BrainWeb and IBSR are of size  $181 \times 217 \times 181$  and  $256 \times 128 \times 256$ , respectively. The middle slices of volumes are considered for both quantitative and qualitative analysis.

The indirect immunofluorescence (IIF) images from “MIVIA HEP-2 Images Dataset” [4] were acquired with the help of a fluorescence microscope coupled with a mercury vapor lamp and with a digital camera. The images have a resolution of  $1388 \times 1038$  pixels, a color depth of 24 bits, and they are stored in an uncompressed format. A single channel is sufficient to convey all the information. The “Berkeley Image Segmentation Dataset” [5] is comprised of a set of natural images along with their segmentation maps provided by different individuals. The images have a resolution of  $321 \times 481$  pixels and a color depth of 24 bits. The images in the “PASCAL Visual Object Classes (VOC) Dataset” [6] are of varying resolution, with color depth of 24 bits.

TABLE XIV

STATISTICAL SIGNIFICANCE ANALYSIS OF THE PERFORMANCE OF StNHMRFS OVER STATE-OF-THE-ART ALGORITHMS IN HEP-2 CELL SEGMENTATION

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
Dice Coefficient				
<i>St</i> NHMRF_S	0.902152	0.033866		
DSFCM_N	0.884777	0.033473	1.2E-03	2.0E-03
FRFCM	0.888642	0.044594	7.1E-03	6.8E-03
KWFLICM	0.876330	0.040623	0.0469	0.0654
RFCM	0.889359	0.039772	0.0203	0.0322
rRFCM	0.887206	0.035464	0.0977	0.1162
Sensitivity				
<i>St</i> NHMRF_S	0.948311	0.039805		
DSFCM_N	0.866357	0.063944	1.9E-04	9.8E-04
FRFCM	0.860137	0.072904	2.7E-04	9.8E-04
KWFLICM	0.878055	0.061868	5.2E-04	9.8E-04
RFCM	0.860879	0.063872	2.2E-04	9.8E-04
rRFCM	0.893911	0.052116	1.6E-03	9.8E-04
Specificity				
<i>St</i> NHMRF_S	0.901536	0.069698		
DSFCM_N	0.854213	0.073230	0.0112	2.9E-03
FRFCM	0.893964	0.053630	0.2483	0.2461
KWFLICM	0.789908	0.119459	4.5E-03	9.8E-04
RFCM	0.878767	0.061023	0.0353	0.0527
rRFCM	0.822883	0.095880	8.9E-03	9.8E-04

TABLE XV

STATISTICAL SIGNIFICANCE ANALYSIS OF THE PERFORMANCE OF StNHMRFS OVER STATE-OF-THE-ART ALGORITHMS IN “BERKELEY IMAGE SEGMENTATION DATASET”

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
Dice Coefficient				
StNHMRF_S	0.892734	0.062692		
DSFCM_N	0.839993	0.064692	1.0E-04	2.4E-04
FRFCM	0.831208	0.059810	9.7E-06	2.4E-04
KWFLICM	0.842352	0.064057	1.0E-04	2.4E-04
RFCM	0.828032	0.064878	1.3E-04	2.4E-04
rRFCM	0.848573	0.067077	3.8E-04	2.4E-04
Sensitivity				
StNHMRF_S	0.914705	0.068409		
DSFCM_N	0.887088	0.073704	4.5E-06	2.4E-04
FRFCM	0.885314	0.072856	8.6E-07	2.4E-04
KWFLICM	0.885433	0.072887	1.5E-06	2.4E-04
RFCM	0.886961	0.072888	7.5E-07	2.4E-04
rRFCM	0.887425	0.073501	2.3E-06	2.4E-04
Specificity				
StNHMRF_S	0.916180	0.052406		
DSFCM_N	0.891800	0.064189	4.8E-04	2.4E-04
FRFCM	0.895860	0.061089	2.8E-04	2.4E-04
KWFLICM	0.897873	0.061587	5.7E-04	2.4E-04
RFCM	0.898447	0.062013	4.9E-04	2.4E-04
rRFCM	0.900115	0.062393	1.1E-03	2.4E-04

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TABLE XVI  
STATISTICAL SIGNIFICANCE ANALYSIS OF THE PERFORMANCE OF  
StNHMRFS OVER STATE-OF-THE-ART ALGORITHMS IN “PASCAL  
VISUAL OBJECT CLASSES DATASET”

Algorithm	Mean	Std. Dev.	p-value	
			Paired <i>t</i>	Wilcoxon
Dice Coefficient				
<i>St</i> NHMRF_S	0.862196	0.030822		
DSFCM_N	0.801248	0.034823	1.8E-05	2.4E-04
FRFCM	0.807004	0.033026	2.0E-05	2.4E-04
KWFLICM	0.808145	0.041647	1.9E-04	2.4E-04
RFCM	0.834925	0.035625	2.4E-06	2.4E-04
rRFCM	0.830847	0.037255	1.0E-04	2.4E-04
Sensitivity				
<i>St</i> NHMRF_S	0.906129	0.030639		
DSFCM_N	0.829927	0.052994	6.3E-05	2.4E-04
FRFCM	0.831637	0.053537	7.1E-05	2.4E-04
KWFLICM	0.840494	0.052141	2.6E-04	2.4E-04
RFCM	0.867126	0.030774	1.0E-06	2.4E-04
rRFCM	0.873037	0.034119	4.4E-06	2.4E-04
Specificity				
<i>St</i> NHMRF_S	0.930950	0.024646		
DSFCM_N	0.901619	0.024687	3.2E-05	2.4E-04
FRFCM	0.917347	0.022865	5.6E-04	2.4E-04
KWFLICM	0.882689	0.036336	1.4E-04	2.4E-04
RFCM	0.917278	0.023175	2.5E-04	2.4E-04
rRFCM	0.897825	0.029227	3.4E-04	2.4E-04

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