



**DEPARTMENT OF ECONOMICS
DISCUSSION PAPER SERIES**

**DEBT STABILIZATION IN A NON-RICARDIAN
ECONOMY**

Campbell Leith, Ioana Moldovan and Simon Wren-Lewis

Number 542
March 2011

Manor Road Building, Oxford OX1 3UQ

Debt Stabilization in a Non-Ricardian Economy*

Campbell Leith^a, Ioana Moldovan^a and Simon Wren-Lewis^b

^a University of Glasgow

^b University of Oxford

This version, February 2011

Abstract

In models with a representative infinitely lived household, modern versions of tax smoothing imply that the steady-state of government debt should follow a random walk. This is unlikely to be the case in OLG economies, where the equilibrium interest rate may differ from the policy-maker's rate of time preference such that it may be optimal to reduce debt today to reduce distortionary taxation in the future. Moreover, the level of the capital stock (and therefore output and consumption) in these economies is likely to be sub-optimally low, and reducing government debt will 'crowd in' additional capital. Using an elaborated version of the model of perpetual youth developed by Blanchard (1985) and Yaari (1985), we derive the optimal steady state level of government assets. We show how and why this level of government assets falls short of the level of debt that achieves the optimal capital stock and the level that eliminates income taxes.

- JEL Codes: E21, E32, E63
- Keywords: Non-Ricardian consumers, macroeconomic stability, distortionary taxes.

*Our thanks to Balazs Parkanyi, Matteo Salto, Alfred Greiner, Patrick Minford, Mike Wickens, Charles Brendom, Tom Holden and participants in seminars in Brussels, Cardiff and Oxford for helpful comments, but all responsibility remains ours.

1 Introduction

The problems caused by excessive levels of public debt do not need enumerating. As governments around the world try to bring deficits under control, and subsequently to reduce levels of debt in relation to GDP, a natural question to ask is how far debt levels should be reduced, and how quickly, once any immediate crisis caused by large default risk premia has diminished. In other words, what should be the ultimate target for the debt to GDP ratio, and how quickly should we get there? Until now, most analysis of this question has been undertaken using models in which consumers in effect live forever, by appropriately internalising the utility of their children (see Schmitt-Grohe and Uribe (2004) for example). The implications of the benchmark result in such models is striking: once fears of default have receded, the optimum level of debt is the historic debt level. A simple application of tax smoothing suggests that attempts to reduce the extent of distortionary taxation in the long run will require short run increases in these taxes whose cost outweighs the eventual gain. This is sometimes called the random walk steady state debt result.

In overlapping generations economies where agents do not care about their children (or do not care about them enough), we get a rather different answer to the question about optimal debt to GDP targets. There are two reasons why the random walk steady state debt result no longer holds in these Non-Ricardian economies. First, if the economy is not dynamically inefficient, then the real interest rate is likely to exceed the rate of time preference, which means that from a Ramsey planner's point of view it may be worth sacrificing some current utility in order to achieve a steady state where distortionary taxes are lower than they currently are (even if the current generation may lose out as a result). Second, the level of the capital stock (and therefore output and consumption) in these economies is likely to be sub-optimally low, and reducing government debt will 'crowd in' additional capital.

This raises an immediate question: will the debt target in such models be the debt level that eliminates the need for distortionary taxes, or will it be the level that achieves the optimal capital stock? This is one of the issues we examine in this paper. Using an elaborate version of the model of perpetual youth developed by Blanchard (1985) and Yaari (1985), we derive the optimal steady state level of government assets. We show how and why this level of government assets falls short of the level of debt that achieves the optimal capital stock and the level that eliminates income taxes.

Section 2 contrasts, in a highly simplified way, the steady state random walk debt result with the outcome when the rate of interest is above the rate of time preference, and where capital is below its optimal level. Section 3 outlines a quite rich version of the model of perpetual youth. In this version of the paper we work with a somewhat simplified version of this model, a summary of which is presented in section 4. In section 5, we discuss social welfare, the model's calibration, and our numerical results so far. A

final section concludes.

This version of the paper is preliminary. We only analyse a simplified version of our model, with a single tax and a monopoly distortion. In addition, we only present an analysis of the speed at which the optimal debt target should be reached using a second-order approximation to the non-linear model, while we hope later to present paths using a fully non-linear version of the model.

2 Optimal Debt and Optimal Capital

The benchmark model for optimal debt implies that there is no optimal level of debt. This benchmark assumes that individuals are effectively infinitely lived, and ignores the possibility of default. Taxation is distortionary, so if we could choose the level of government debt we inherit, it would be negative, and the interest payments on these government assets would pay for any government consumption. In the discussion below, we call this the zero-tax level of government assets, or A^T . Of course, without recourse to default or some equivalent expropriation mechanism, a government cannot choose the level of debt it inherits. (A Ramsey planner could in theory expropriate sufficient capital using a capital tax, and then commit to setting capital taxes to zero, but this commitment would not be credible.)

Suppose we inherit a level of assets different from A^T , the zero tax level of assets. In the absence of any other means of reducing debt except higher taxes or lower spending, then we have a choice between high taxes (or lower spending) now to reduce debt towards the optimal level, or accepting permanently positive taxes (or lower than optimal government spending) that will finance the interest payments on the inherited debt level, and so leave debt unchanged. If the costs of distortionary taxes or lower than optimal public spending are increasing at the margin, then we get a classic tax smoothing result, which is that it is optimal to keep the inherited level of debt.

However implicit in this argument is that the real rate of interest is equal to the rate at which we discount the future. We can show this formally as follows. Suppose social welfare can be represented as

$$W_t = - \sum_{i=0}^{\infty} \beta^i T_{t+i}^2 \quad (1)$$

where T is the level of distortionary taxes and β is the discount factor. The budget constraint is

$$A_t = (1 + r)A_{t-1} + T_t - G \quad (2)$$

where A are government assets. The government inherits a debt level $B_{-1} > 0$, such that government assets are negative $A_{-1} = -B_{-1}$.

The Lagrangian is

$$L = \sum_{i=0}^{\infty} \beta^i [T_{t+i}^2 + 2\lambda_{t+i}(A_{t+i} - (1+r)A_{t+i-1} - T_{t+i})] \quad (3)$$

The first order condition for taxes is

$$T_{t+i} - \lambda_{t+i} = 0 \quad (4)$$

and for debt

$$\lambda_{t+i} - \beta(1+r)\lambda_{t+i+1} = 0 \quad (5)$$

Combining gives

$$\beta(1+r)T_{t+i+1} - T_{t+i} = 0 \quad (6)$$

If $\beta(1+r) = 1$, then the FOC for debt implies the Lagrange multiplier is constant, which in turn implies constant taxes. Taxes can only be constant if they are sufficient to satisfy the budget constraint if A is constant at $-B_{-1}$, which is the tax smoothing or random walk steady state debt result. (See, for example, Schmitt-Grohe and Uribe (2004a) and Benigno and Woodford (2003)). However, if $\beta(1+r) \neq 1$, then a steady state is possible only if taxes are zero. If $\beta(1+r) < 1$ debt will decline towards this value. In this situation it is always better to reduce debt each year, because the discounted benefits of lower future taxes exceed the cost of higher taxes today. The cost of permanently positive taxes will always outweigh the cost of reducing debt, because we discount at less than the rate of interest, and so we head towards the zero tax level of government assets. If $\beta(1+r) > 1$ then debt will follow an explosive path. For an example of this, see Kirsanova, Leith, and Wren-Lewis (2007).

In this simple model, when $\beta(1+r) = 1$, the tax smoothing result is time consistent. There is no reason to deviate from the inherited level of debt at any time. This result will not be robust to two natural extensions of the model: introducing nominal debt, or staying with real debt but allowing for sticky prices. If inflation is determined by a New Keynesian Phillips Curve, then Leith and Wren-Lewis (2007) show that there is a first period incentive to reduce inherited debt somewhat (but not completely). However, this incentive recurs as we move to the next period, and so the random walk result is not just modified, but is also time inconsistent. They also show that the time consistent policy involves a very rapid reduction in debt to its initial level following a positive shock. (This is true for a closed economy, or simple open economy with a flexible exchange rate, but rates of adjustment are slower under EMU - see Leith and Wren-Lewis (2010).) However, there are reasons for wanting to focus on the time inconsistent case. In reality governments do not rapidly correct any debt disequilibrium. This may be because the costs of doing so are not only high, but they are also short term, so an

impatient government would have an incentive to stick to the time inconsistent plan.

When it comes to thinking about the optimal capital stock, we have another benchmark result, which is that if consumers are effectively infinitely lived, then in the absence of the taxation and other distortions, the level of the capital stock that would be chosen by a social planner would be the same as that produced by a market equilibrium. Because Ricardian Equivalence holds, any increase in government debt leads to a matching increase in private saving, with no impact on this capital stock. In short, debt does not crowd out capital. However, in an OLG economy, the economy will not in general generate an optimal capital stock, and government debt will crowd out capital.

The assumption that individuals leave bequests because they internalise the utility of the next generation (although with discounting), so that they are effectively infinitely lived, is a useful benchmark, but it may be at the extreme end of plausible degrees of inter-generational altruism. Even if generations want to act altruistically, factors like estate taxes on death may interfere with this. At the opposite extreme we have overlapping generation (OLG) models, which generally assume completely selfish generations that leave no intentional bequests. In the model of Perpetual Youth developed by Blanchard and Yaari, if income does not decline with age (and there is no retirement), the real rate of interest will be above the rate of time preference, and so the level of the capital stock is likely to be suboptimally low. In addition, higher government debt will crowd out private capital in OLG models, because Ricardian Equivalence no longer holds. Agents accumulate assets because it is optimal for them to do so as individuals, with no thought for the utility of future generations. It is a model of this kind we develop in the next section. Although the model we develop below is quite rich, the essence of the implications for government debt for the real interest rate can be understood by considering some key equations from a simplified version of the model. Ignoring the households' cash holdings and the tax on consumption, logarithmic utility implies that the aggregate consumption function is a linear function of human and financial wealth,

$$c_t = (1 - \gamma\beta)(lw_t + \frac{W_t}{P_t}) \quad (7)$$

where c is consumption, lw is human wealth, W/P financial assets, γ is the survival probability and β is the households' subjective discount factor. Human capital is given by

$$lw_t = (1 - \tau_t^w)w_t l_t + OI_t + \gamma \frac{\pi_{t+1}}{R_t} lw_{t+1} \quad (8)$$

where $(1 - \tau_t^w)w_t l_t$ is post-tax labour income, OI_t are other (exogenous to the household) sources of income detailed in the model section and $\frac{R_t}{\pi_{t+1}}$ is the real interest rate. Agents hold portfolios of financial assets such that they effectively receive an additional return $1/\gamma$ on their assets, conditional on their surviving. The dynamics of aggregate financial

wealth is given by

$$\frac{W_{t+1}}{P_{t+1}} \frac{\pi_{t+1}}{R_t} = \frac{W_t}{P_t} + (1 - \tau_t^w) w_t l_t + OI_t - C_t \quad (9)$$

Combining these equations implies

$$c_t = \frac{c_{t+1} \pi_{t+1}}{\beta R_t} + \frac{(1 - \gamma \beta)(1 - \gamma)}{\gamma \beta} \frac{W_{t+1}}{P_{t+1}} \frac{\pi_{t+1}}{R_t} \quad (10)$$

Ignoring capital adjustment costs such that Tobin's q is always 1, when the only asset is capital, $\frac{W_{t+1}}{P_{t+1}} \frac{\pi_{t+1}}{R_t} = K_t$, then this equation clearly implies that in steady state $r > 1/\beta$. Individual agents are always saving, but the aggregate level of assets can be constant because those who die have positive assets and the newborn have none. This is the first important implication of allowing for finite lives with no bequests: the real rate of interest can differ from the rate of time preference even in steady state. (This implications of this point are discussed in Erosa and Gervais (2001).)

The second important difference an OLG model makes is that government debt can crowd out capital. Let $\frac{W_{t+1}}{P_{t+1}} \frac{\pi_{t+1}}{R_t} = K_t + \frac{B_t}{P_t}$, where B_t is government debt as before. In steady state, if consumption and real interest rates were unchanged, government debt would crowd out private capital one for one. In fact consumption is likely to fall if capital falls, increasing the extent of crowding out. However, a reduction in the capital stock will also raise real interest rates, which for given consumption levels will raise the overall level of aggregate assets, which moderates the degree of crowding out of capital. (In the infinite life case, which we approach as γ tends to one, any increase in government debt leads to an equal increase in savings, so there is no crowding out.)

Just as government debt crowds out capital, if the government holds assets ($B < 0$), capital will be crowded in. If, when $A = B = 0$, capital is sub-optimal, then government assets can be used to move to the optimal level of capital. We could define the level of government assets that achieve this optimum capital stock as the 'optimum capital' level of assets, or A^K . Unless the economy with $A = B = 0$ is dynamically inefficient¹, such a move would not represent a Pareto improvement, because the higher taxes that the government would require to accumulate assets would hit the current generation. However, as any debt policy is almost certain to disadvantage some generation, this should not prevent us considering using debt as a means of moving towards A^K .

Defining what is optimal in an OLG model of course involves deciding how to compare different generations. Since we are interested in formulating optimal policy for our economy populated with overlapping generations of finitely lived consumers we must face the tricky issue of constructing a welfare metric. We discuss the issues involved in defining a social welfare function below. However, we essentially follow Calvo and Ob-

¹In this model of perpetual youth, $r > \theta$, so the economy is never dynamically inefficient. However introducing either government assets, or allowing income to decline with age, can allow the possibility that $r < \theta$, as we note below.

stfeld (1988) by splitting the problem into an intratemporal problem of how to allocate consumption across generations at a given point in time, and an intertemporal problem, of how to stabilise debt over time. Since we are primarily interested in the latter aspect of the problem, we abstract from the first by assuming that the policy maker ignores the intratemporal problem and only considers per capita variables when defining social welfare in an environment where government debt can crowd out private capital. In doing so we assume that the policy maker discounts welfare between generations at the same rate as household discount their own utility.

If the only implication of moving to an OLG framework was that there was some optimal capital stock, then we could simply calculate A^K , and this could become our long run debt 'target'. Indeed, if lump sum taxes were available, we could in theory immediately move to A^K : the additional tax payments would be exactly offset by interest payments on this debt. However, in the absence of lump sum taxes, any change in government assets/debt will, by changing capital, also change the real interest rate. This means that the level of government assets that would eliminate distortionary taxes (A^T) also becomes a potential 'target' for long run government debt.

In the case of the Ricardian model, the zero-tax level of assets A^T was irrelevant because of tax smoothing, as the real rate of interest was equal to the rate of time preference. However in general this condition will not hold in an OLG model. We can examine the implications of this for steady state debt in a highly oversimplified fashion as follows. Suppose social welfare can now be represented as

$$W_t = - \sum_{i=0}^{\infty} \beta^i \left[T_{t+i}^2 + \alpha (A_{t+i} - A^K)^2 \right] \quad (11)$$

where A are government assets. Welfare is negative for two reasons (which for simplicity we assume are separable): taxes are distortionary, but also capital is away from its optimal level whenever government assets are different from A^K . We still have the budget constraint

$$A_t = (1 + r)A_{t-1} + T_t - G \quad (12)$$

where we now allow the real interest rate to depend on government assets in the following simple way:

$$r_t = r_0 - \gamma A_{t-1}/2$$

which captures the idea that as government assets rise, the capital this crowds in reduces the real interest rate. As before, the government inherits a debt level $B_{-1} > 0$. We define

$$A^T = G/r$$

The Lagrangian is

$$L = \sum_{i=0}^{\infty} \beta^i \left[T_{t+i}^2 + \alpha (A_{t+i} - A^K)^2 + 2\lambda_{t+i} (A_{t+i} - (1 + r_0 - \gamma A_{t+i-1}/2)A_{t+i-1} - T_{t+i}) \right] \quad (13)$$

The FOC for taxes is

$$T_{t+i} - \lambda_{t+i} = 0 \quad (14)$$

and for debt

$$\alpha(A_{t+i} - A^K) + \lambda_{t+i} - \beta(1 + r_0 - \gamma A_{t+i})\lambda_{t+i+1} = 0 \quad (15)$$

Combining gives

$$\beta(1 + r_0 - \gamma(A_{t+i} - A^K) - \gamma A^K)T_{t+i+1} - T_{t+i} = \alpha(A_{t+i} - A^K) \quad (16)$$

which can be rewritten as

$$\beta(1 + \tilde{r})T_{t+i+1} - T_{t+i} = (\alpha + \beta\gamma T_{t+i+1})(A_{t+i} - A^K) \quad (17)$$

where $1 + \tilde{r} = 1 + r_0 + \gamma B^K$.

Consider the case where $\beta(1 + \tilde{r}) = 1$ first. Whatever the level of steady state taxes, government assets will end up at the level that achieves the optimal capital stock i.e. $A^* = A^K$ where A^* is the steady state level of assets (and $B^* = -A^*$ the steady-state level of debt). Taxes will be given by

$$T^* = G + rB^*$$

We can think of this in the following way. The case where $\beta(1 + \tilde{r}) = 1$ is akin to tax smoothing, so A^* is not attracted to A^T . However, we do not get random walk steady state debt, because reducing debt has the benefit of increasing capital and therefore output.

If $\beta(1 + \tilde{r}) > 1$, we already know that tax smoothing does not apply, and there will be some history-independent debt target. There are two possibilities. First, taxes are positive in steady state, and so steady state government assets exceed the level required to obtain the optimal capital stock. We therefore have $A^T > A^* > A^K$. Second, taxes become negative in steady state, but despite this government assets are insufficient to achieve the optimal capital stock (providing $\alpha + \beta\gamma T_{t+i+1} > 0$). In this second case, it must be that $A^K > A^* > A^T$. In both cases, we can think about optimal government assets as being a compromise between the zero-tax level and the optimal capital level. The former matters, because tax smoothing does not apply.

Is $\beta(1 + \tilde{r}) < 1$ interesting? If the term multiplying the deviation of debt from the

optimal capital level happened to be zero it would not be, because there would be no incentive to stabilise debt. However, as we have seen with the case of $\beta(1 + \tilde{r}) = 1$, the additional incentive to move debt towards the level that maximises the capital stock means that government assets can converge to this level. So providing $\beta(1 + \tilde{r})$ is not too far below one, a steady state is still possible. If it does exist, then if taxes are positive we will have $A^* < A^K$ and $A^* < A^T$. The reason is that if debt reached the optimal capital level, then there would be a tendency for debt to explode. The economy therefore stabilises when this incentive is exactly offset by the incentive to get capital a little higher. Another possibility is that $A^* > A^K$ and taxes are negative, implying $A^T < A^K < A^*$.

To sum up, in an OLG model the 'target' or 'steady state optimal' level of government assets A^* will depend on both the level of assets that delivers the optimum capital stock (A^K) and the level of assets that eliminate distortionary taxes (A^T) in ways that are likely to depend on the detailed structure and parameterisation of the model. If A^* is associated with a real interest rate below the rate of time preference, then it may be the case that A^* will not lie in between A^T and A^K . The next section sets out the model we will investigate, where we find this is indeed the case.²

3 The Complete Model

In this section we outline our model. We first consider an elaborate model, that includes money, investment adjustment costs, three different forms of distortionary taxation and government debt that can be in the form of one period bonds or perpetuities. In the current version of the paper we simplify this model, by allowing the economy to approach its cashless limit, assuming that all government debt is issued in the form of one period bonds, eliminating capital adjustment costs and by only considering a single distortionary tax, namely a labour income tax. The next section presents a summary of that simplified model. Later versions of the paper will look at the implications of moving to the more elaborate model.

Our economy is populated by overlapping generations of consumers who face a constant probability of death, such that, even if taxes were lump-sum, Ricardian Equivalence would not hold in our model.³ These consumers supply labour to imperfectly competitive firms, who combine this labour with capital rented from a representative capital rental firm, to produce a differentiated product. The accumulation of capital by the capital rental firm is subject to capital adjustment costs. The firms producing these differentiated products are also subject to the constraints implied by Rotemberg (1982) quadratic adjustment costs. Consumers' labour income is taxed, and they also

²However, equation (10) will hold, so whether $A^* < > K$ in steady state will be directly related to whether the real rate of interest at the optimal level of debt will be greater or less than the rate of time preference.

³For recent analysis that investigates further the short term role that fiscal policy can play in this class of model, see Devereau (2010).

pay consumption taxes. The profits of the capital rental firm and the final goods firms are also taxed.

3.1 Consumers' Behaviour

Here we introduce the main departure from the canonical New-Keynesian model. While there is abundant evidence of a strong interaction among fiscal impulses and output (see, for example, Blanchard and Perotti (2002) or Fatas and Mihov (1998), standard dynamic general equilibrium models downplay the role of demand. The importance of the demand side of the economy is partially restored when there is slow adjustment in nominal and real variables, but still intertemporal substitution mechanisms and Ricardian equivalence leave consumption largely unresponsive to a fiscal stimulus. Introducing a probability of death implies that consumers discount their future disposable income more heavily, such that the usual Ricardian experiment of a deficit-financed lump-sum tax cut now increases consumption.

Households face a constant probability of death $(1 - \gamma)$. As this is a constant exogenous probability, and there is a continuum of households, they imply there is no aggregate uncertainty in our economy. This implies that a consumer born at time i , who is still alive at time t receives utility from consuming a basket of consumer goods at time t ,

$$c_t^i = \left[\int_0^1 c_t^i(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (18)$$

and holding real money balances, M_t^i/P_t , and suffers disutility from supplying labour to imperfectly competitive firms, l_t^i . We can write this household's expected utility function as,

$$\sum_{t=0}^{\infty} (\beta\gamma)^t \left[\ln c_t^i + \chi \ln \frac{M_t^i}{P_t} + \vartheta \ln g_t^c + \varkappa \ln(1 - l_t^i) \right] \quad (19)$$

By reducing the household's discount factor by the survival probability γ we are implicitly conditioning on the survival of this particular household (otherwise there would be double-counting of the probability of death).

Due to the difficulties in conceptualising complete financial contracts amongst markets participants some of whom are as yet unborn, we assume that financial markets are incomplete, but in an economy without aggregate uncertainty. Instead we assume that households can hold risk-free nominal one period government bonds which pay a gross interest rate of R_t regardless of the state of nature (including the survival of the bond holder), perpetuities which pay a real coupon of ρ , and non-interest bearing money. They can also enter into survival-contingent contracts with other households, which pay an agreed sum to other households in the event of the individual's death, but entitle the individual to similar payments from deceased households should the individual survive. The individual will construct a portfolio of money, bonds and survival-contingent

contracts such that the payoff from that portfolio should the individual die is zero. However, if household i is lucky enough to survive their combined return from risk-free bonds and survival-contingent contracts written against those bonds will be $B_{t-1}^i R_{t-1}/\gamma$, for perpetuities is, $P_t(q_t + \rho)D_{t-1}^i/\gamma$, where q_t is the (real) price of those perpetuities at time t , and $P_t\rho$ is the nominal coupon received, while the return to holding money is M_{t-1}^i/γ . This is simply an alternative means of capturing the insurance contracts usually undertaken within the Blanchard-Yaari set-up. Households also buy shares, V_t^i in capital rental firms for a real price q_t^v which pay out their net cash flows as dividends, d_t .

Consumers seek to maximise utility subject to the demand schedule for their labour services and their budget constraint, which in nominal terms can be written as

$$\begin{aligned} & M_t^i + B_t^i + P_t q_t D_t^i + P_t q_t^v V_t^i + P_t(1 + \tau_t^c)c_t^i \\ = & P_t(1 - \tau_t^w)w_t l_t^i + \frac{R_{t-1}B_{t-1}^i}{\gamma} + \frac{M_{t-1}^i}{\gamma} + \frac{P_t(q_t + \rho)D_{t-1}^i}{\gamma} + \frac{P_t(q_t^v + d_t)V_{t-1}^i}{\gamma} \\ & + P_t s_t^i + (1 - \gamma) P_t \left(1 - \tau_t^k\right) \int_0^1 \Omega_{jt} dj \end{aligned} \quad (20)$$

Here consumers earn after-tax income from their labour services $P_t(1 - \tau_t^w)w_t l_t^i$, and receive their share of the post-tax profits of final goods producers, $(1 - \gamma) P_t \left(1 - \tau_t^k\right) \int_0^1 \Omega_{jt} dj$, as well as household specific public transfers, $P_t s_t^i$. Consumption purchases are taxed at the rate, τ_t^c .

Let us define

$$H_t^i \equiv \left[(1 - \tau_t^w)w_t l_t^i + s_t^i + (1 - \gamma) \left(1 - \tau_t^k\right) \int_0^1 \Omega_{jt} dj \right] \quad (21)$$

and

$$W_t^i \equiv \frac{1}{\gamma} M_{t-1}^i + \frac{R_{t-1}}{\gamma} B_{t-1}^i + \frac{P_t(q_t + \rho)}{\gamma} D_{t-1}^i + \frac{P_t(q_t^v + d_t)}{\gamma} V_{t-1}^i \quad (22)$$

as the non-financial and financial income of generation i households in period t . Then, the budget constraint can be written as

$$M_t^i \left(\frac{R_t - 1}{R_t} \right) + Q_{t,t+1} W_{t+1}^i + \left(P_t q_t - Q_{t,t+1} \frac{P_{t+1}(q_{t+1} + \rho)}{\gamma} \right) D_t^i + P_t(1 + \tau_t^c)c_t^i = P_t H_t^i + W_t^i \quad (23)$$

W_t^i represents the payoff from the household's portfolio in all states of nature, but conditional on the household surviving, and $Q_{t,t+1} = \gamma R_t^{-1}$ is the price of receiving one unit of that payoff. Note that should the household not survive, the payoff from the portfolio is zero, such that the expected payoff from one unit of the portfolio across all states of nature, including the survival/non-survival of the household, is the risk free rate of interest R_t .

Maximising household utility subject to the budget constraint yields the consumption

Euler equation,

$$Q_{t,t+1} = \gamma\beta \left\{ \frac{c_t^i(1 + \tau_t^c)P_t}{c_{t+1}^i(1 + \tau_{t+1}^c)P_{t+1}} \right\} \quad (24)$$

or equivalently,

$$1 = R_t\beta \frac{c_t^i(1 + \tau_t^c)P_t}{c_{t+1}^i(1 + \tau_{t+1}^c)P_{t+1}}, \quad (25)$$

a demand for money equation (where $m_t^i \equiv \frac{M_t^i}{P_t}$),

$$m_t^i = \chi \frac{R_t}{R_t - 1} (1 + \tau_t^c) c_t^i, \quad (26)$$

a labour supply condition,

$$(1 - \tau_t^w)w_t(1 - l_t^i) = \varkappa(1 + \tau_t^c)c_t^i, \quad (27)$$

and the no-arbitrage condition for perpetuities,

$$q_t = \frac{\pi_{t+1}}{R_t}(q_{t+1} + \rho). \quad (28)$$

and similarly for equities,

$$q_t^v = \frac{\pi_{t+1}}{R_t}(q_{t+1}^v + d_{t+1}). \quad (29)$$

Using the household budget constraint, together with the money-demand equation, the Euler equation, and the no-arbitrage condition for perpetuities, we obtain the consumer's consumption function,

$$(1 + \tau_t^c)c_t^i = \frac{1 - \gamma\beta}{1 + \chi} \left[\frac{W_t^i}{P_t} + \sum_{s=0}^{\infty} (\gamma)^s \left(\prod_{i=0}^{s-1} \frac{\pi_{t+i+1}}{R_{t+i}} \right) H_{t+s}^i \right] \quad (30)$$

where the household discounts future labour and profit income more heavily than its straight rate of time preference, as it will not receive that income should it die, but expectations are taken over all states of nature, other than the survival/non-survival of the household. We can further write this as,

$$(1 + \tau_t^c)c_t^i = \frac{1 - \gamma\beta}{1 + \chi} \left[\frac{W_t^i}{P_t} + lw_t^i \right]$$

where lw_t^i represents the generation i 's human wealth, given as the discounted value of labor income and profits, where the effective discount factor accounts for the probability of survival,

$$lw_t^i \equiv H_t^i + \sum_{s=1}^{\infty} (\gamma)^s \left(\prod_{i=0}^{s-1} \frac{\pi_{t+i+1}}{R_{t+i}} \right) H_{t+s}^i = H_t^i + \gamma \left(\frac{\pi_{t+1}}{R_t} \right) lw_{t+1}^i \quad (31)$$

3.2 Aggregating across Consumers and Consumption Dynamics.

If the size of each cohort when born is 1, then the size of a cohort i at time t is given by, γ^{t-i} . Therefore the total size of the population is given by⁴,

$$\sum_{i=-\infty}^t \gamma^{t-i} = \frac{1}{1-\gamma}. \quad (32)$$

Aggregate variables are defined as, $x_t = \sum_{i=-\infty}^t \gamma^{t-i} x_t^i$. Aggregating consumers' labour supply yields,

$$(1 - \tau_t^w)w_t \left(\frac{1}{1-\gamma} - l_t \right) = \varkappa(1 + \tau_t^c)c_t \quad (33)$$

The aggregate demand for money is given by,

$$m_t = \chi \frac{R_t}{R_t - 1} (1 + \tau_t^c)c_t \quad (34)$$

It is similarly possible to aggregate across consumers from different generations to generate an aggregate consumption function,

$$(1 + \tau_t^c)c_t = \frac{1 - \gamma\beta}{1 + \chi} \left[\frac{W_t}{P_t} + lw_t \right] \quad (35)$$

and aggregate human wealth is given by,

$$lw_t = H_t + \gamma \frac{\pi_{t+1}}{R_t} lw_{t+1}$$

where

$$H_t \equiv \left[(1 - \tau_t^w)w_t l_t + s_t + (1 - \tau_t^k) \int_0^1 \Omega_{jt} dj \right] \quad (36)$$

It should be noted that the aggregate of financial wealth, $W_t = M_{t-1} + R_{t-1}B_{t-1} + P_t(q_t + \rho)D_{t-1} + P_t(q_t^v + d_t)V_{t-1}$, takes account of the fact that not all households will have survived from last period into the current one, implying that the households' aggregate budget constraint is given by,

$$\begin{aligned} M_t + B_t + P_t q_t D_t + P_t q_t^v V_t + P_t(1 + \tau_t^c)c_t &= P_t(1 - \tau_t^w)w_t l_t + R_{t-1}B_{t-1} + M_{t-1} + P_t(q_t + \rho)D_{t-1} \\ &\quad + P_t(q_t^v + d_t)V_{t-1} + P_t s_t + P_t \left(1 - \tau_t^k \right) \int_0^1 \Omega_{jt} dj \end{aligned} \quad (37)$$

3.3 The Capital Rental Firm's Behaviour

We assume that there is a single representative firm accumulating capital for rental to the final goods firms. This firm seeks to maximise the discounted value of its cashflows.

⁴Note that this implies that an infinitesimally small number of consumers will live-forever. This is why this means of introducing non-Ricardian behaviour is sometimes called the 'perpetual youth model'.

This objective function is consistent with maximising the value of the households' equity. Therefore the firm's objective function is to maximise the following expression,

$$P_t(q_t^v + d_t)V_{t-1} = (1 - \tau_t^k)p_t^k k_t - e_t + \left\{ \sum_{z=1}^{\infty} \left(\prod_{i=0}^{z-1} R_{t+i}^{-1} \right) \frac{P_{t+z}}{P_t} \left[(1 - \tau_{t+z}^k)p_{t+z}^k k_{t+z} - e_{t+z} \right] \right\} \quad (38)$$

where p_t^k is the real rental cost of capital, k_t is the capital stock, e_t is real investment expenditure, and τ_t^k is the rate of taxation on the income from renting capital. Because of capital adjustment costs, a part of investment, $\Phi\left(\frac{e_t}{k_t}\right)k_t = \frac{\mu}{2}\left(\frac{e_t}{k_t} - \delta\right)^2 k_t$, is lost when converting investment into capital which also depreciates at rate δ . The equation of motion of the capital stock is then given by,

$$k_{t+1} = e_t - \Phi\left(\frac{e_t}{k_t}\right)k_t + (1 - \delta)k_t. \quad (39)$$

The first order condition for investment is given by,

$$\lambda_t^k \left(1 - \Phi'\left(\frac{e_t}{k_t}\right) \right) = 1 \quad (40)$$

where λ_t^k is the Lagrange multiplier associated with the equation of motion for the capital stock. Given the homogeneity of our profit function, this is equivalent to Tobin's q . Also, differentiating the Lagrangian with respect to k_{t+1} gives the equation of motion for Tobin's q ,

$$\lambda_t^k = \frac{\pi_{t+1}}{R_t} \left((1 - \tau_{t+1}^k)p_{t+1}^k + \left(-\Phi\left(\frac{e_{t+1}}{k_{t+1}}\right) + \Phi'\left(\frac{e_{t+1}}{k_{t+1}}\right)\frac{e_{t+1}}{k_{t+1}} + (1 - \delta) \right) \lambda_{t+1}^k \right) \quad (41)$$

The capital accumulated by this sector is then rented out to the imperfectly competitive firms producing final goods for consumers, as described below.

This marginal q can be related to average q (and therefore the value of household's equity) as follows. Firstly, use the equation of motion of the capital stock to rewrite as,

$$\lambda_t^k k_{t+1} = \frac{\pi_{t+1}}{R_t} \left((1 - \tau_{t+1}^k)p_{t+1}^k k_{t+1} + \left(k_{t+2} - e_{t+1} + \Phi'\left(\frac{e_{t+1}}{k_{t+1}}\right)e_{t+1} \right) \lambda_{t+1}^k \right). \quad (42)$$

Then, using the first order condition for investment, we obtain

$$\lambda_t^k k_{t+1} = \frac{\pi_{t+1}}{R_t} \left((1 - \tau_{t+1}^k)p_{t+1}^k k_{t+1} - e_{t+1} + k_{t+2}\lambda_{t+1}^k \right) \quad (43)$$

which implies that,

$$\lambda_t^k k_{t+1} + (1 - \tau_t^k)p_t^k k_t - e_t = (q_t^v + d_t)V_{t-1}$$

so we can define non-human wealth as,

$$W_t = M_{t-1} + R_{t-1}B_{t-1} + P_t(q_t + \rho)D_{t-1} + P_t\lambda_t^k k_{t+1} + (1 - \tau_t^k)P_t p_t^k k_t - P_t e_t$$

3.4 Capital and Labour Demand: Cost Minimization

The optimal combination of capital and labour employed in the production of final goods, is obtained from the cost minimization problem of the firm, given the production function it faces,

$$y_{jt} = A_t k_{jt}^\alpha l_{jt}^{1-\alpha} (g_t^p)^\theta. \quad (44)$$

This implies the following cost minimising combinations of labour and capital,

$$w_t = mc_t(1 - \alpha)A_t k_{jt}^\alpha l_{jt}^{1-\alpha} (g_t^p)^\theta$$

$$p_t^k = mc_t \alpha A_t k_{jt}^{\alpha-1} l_{jt}^{1-\alpha} (g_t^p)^\theta$$

where mc_t represents the real marginal cost, which is common across all firms,

$$mc_t = \left(\frac{p_t^k}{\alpha}\right)^\alpha \left(\frac{w_t}{1 - \alpha}\right)^{1-\alpha} A_t^{-1}, \quad (45)$$

w_t is the real wage and p_t^k the rental cost of capital. Since all firms are identical, these can be related to aggregate variables and we have:

$$y_t = A_t k_t^\alpha l_t^{1-\alpha} (g_t^p)^\theta$$

$$w_t = mc_t \left[(1 - \alpha) \frac{y_t}{l_t} \right]$$

$$p_t^k = mc_t \left[\alpha \frac{y_t}{k_t} \right]$$

3.5 Price Setting of Final Goods Firms

We define Rotemberg price adjustment costs as,

$$\frac{\phi}{2} \left(\frac{p_t(j)}{\pi^* p_{t-1}(j)} - 1 \right)^2 P_t y_t \quad (46)$$

where π^* is the steady-state inflation rate. The problem facing firm j is to maximise the discounted value of after-tax profits,

$$\max_{p_t(j)} \left[\left(1 - \tau_t^k\right) \Pi_t(j) + \sum_{z=1}^{\infty} \left(\prod_{i=0}^{z-1} R_{t+i}^{-1} \right) \left(1 - \tau_{t+z}^k\right) \Pi_{t+z}(j) \right]$$

where given the demand curve, $y_t(j) = (p_t(j)/P_t)^{-\varepsilon} y_t$, nominal profits are defined as,

$$\begin{aligned}\Pi_t(j) &\equiv p_t(j)y_t(j) - mc_t y_t(j)P_t - \frac{\phi}{2} \left(\frac{p_t(j)}{\pi^* p_{t-1}(j)} - 1 \right)^2 P_t y_t \\ &= p_t(j)^{1-\varepsilon} P_t^\varepsilon y_t - mc_t p_t(j)^{-\varepsilon} P_t^{1+\varepsilon} y_t - \frac{\phi}{2} \left(\frac{p_t(j)}{\pi^* p_{t-1}(j)} - 1 \right)^2 P_t y_t\end{aligned}\quad (47)$$

So that, in a symmetric equilibrium where $p_t(j) = P_t$, the first order conditions are given by,

$$(1 - \varepsilon) + \varepsilon mc_t - \phi \frac{\pi_t}{\pi^*} \left(\frac{\pi_t}{\pi^*} - 1 \right) + \phi \frac{\pi_{t+1}}{R_t} \frac{\pi_{t+1}}{\pi^*} \frac{y_{t+1}}{y_t} \frac{(1 - \tau_{t+1}^k)}{(1 - \tau_t^k)} \left(\frac{\pi_{t+1}}{\pi^*} - 1 \right) = 0 \quad (48)$$

which is the Rotemberg version of the Phillips curve relationship. Equilibrium real profits of all final goods producers are then given as,

$$\int_0^1 \Omega_{jt} dj \equiv P_t^{-1} \left(\int_0^1 \Pi_t(j) dj \right) = y_t \left[1 - mc_t - \frac{\phi}{2} \left(\frac{\pi_t}{\pi^*} - 1 \right)^2 \right]$$

That completes our derivation of the model, which is summarised in Appendix 1.

4 A Simplified Model in Summary

In the numerical results presented below, we simplify the economy in several respects. Specifically, we allow the economy to approach its cashless limit, we assume that all government debt is issued in the form of one period bonds, we eliminate capital adjustment costs and we only consider a single distortionary tax, namely a labour income tax. This implies that our model reduces to the following set of equations. (This can be compared with a summary version of the complete model given in the Appendix.)

The aggregate consumption function,

$$c_t = (1 - \gamma\beta) \left[\frac{W_t}{P_t} + lw_t \right] \quad (49)$$

where aggregate financial wealth in real terms is (with $b_t \equiv B_t/P_t$)

$$\frac{W_t}{P_t} = \frac{R_{t-1}}{\pi_t} b_{t-1} + k_{t+1} + p_t^k k_t - e_t \quad (50)$$

and the aggregate human wealth is

$$lw_t = H_t + \gamma \frac{\pi_{t+1}}{R_t} lw_{t+1} \quad (51)$$

with

$$H_t \equiv (1 - \tau_t^w) w_t l_t + s_t + \int_0^1 \Omega_{jt} dj \quad (52)$$

The government budget constraint is given by

$$g_t^c + g_t^p + s_t = \tau_t^w w_t l_t + b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} \quad (53)$$

The definition of profits (in real terms)

$$\int_0^1 \Omega_{jt} dj = y_t \left[1 - mc_t - \frac{\phi}{2} \left(\frac{\pi_t}{\pi^*} - 1 \right)^2 \right]$$

Combine the households' aggregate resource constraint with the government budget constraint and the definition of profits to obtain the aggregate resource constraint

$$g_t^c + g_t^p + c_t + e_t + \frac{\phi}{2} \left(\frac{\pi_t}{\pi^*} - 1 \right)^2 y_t = y_t \quad (54)$$

Labour supply is

$$(1 - \tau_t^w) w_t \left(\frac{1}{1 - \gamma} - l_t \right) = \varkappa c_t \quad (55)$$

The equation of motion of the capital stock is given by,

$$k_{t+1} = e_t + (1 - \delta) k_t \quad (56)$$

and the first order condition for investment is given by,

$$1 = \frac{\pi_{t+1}}{R_t} \left[p_{t+1}^k + (1 - \delta) \right] \quad (57)$$

Inflation is described by,

$$(1 - \varepsilon) + \varepsilon mc_t - \phi \frac{\pi_t}{\pi^*} \left(\frac{\pi_t}{\pi^*} - 1 \right) + \phi \frac{\pi_{t+1}}{R_t} \frac{y_{t+1}}{y_t} \frac{\pi_{t+1}}{\pi^*} \left(\frac{\pi_{t+1}}{\pi^*} - 1 \right) = 0 \quad (58)$$

Technology,

$$y_t = A_t k_t^\alpha l_t^{1-\alpha} (g_t^p)^\theta \quad (59)$$

Cost minimisation implies,

$$w_t = mc_t \left[(1 - \alpha) \frac{y_t}{l_t} \right] \quad (60)$$

$$p_t^k = mc_t \left[\alpha \frac{y_t}{k_t} \right] \quad (61)$$

where,

$$mc_t = \left(\frac{p_t^k}{\alpha} \right)^\alpha \left(\frac{w_t}{1 - \alpha} \right)^{1-\alpha} A_t^{-1}. \quad (62)$$

4.1 Steady-State Simplified Version

The aggregate consumption function,

$$c = (1 - \gamma\beta) \left[\frac{W}{P} + lw \right] \quad (63)$$

where aggregate financial wealth in real terms is (with $b \equiv B/P$)

$$\frac{W}{P} = \frac{R}{\pi^*} b + k + p^k k - e \quad (64)$$

and the aggregate human wealth is

$$lw = H + \gamma \frac{\pi}{R} lw \quad (65)$$

with

$$H \equiv (1 - \tau^w)wl + s + \int_0^1 \Omega_j dj \quad (66)$$

The government budget constraint is given by

$$g^c + g^p + s = \tau^w wl + b - \frac{R}{\pi} b \quad (67)$$

The definition of profits (in real terms)

$$\int_0^1 \Omega_j dj = y(1 - mc)$$

The aggregate resource constraint

$$g^c + g^p + c + e = y \quad (68)$$

Labour supply is

$$(1 - \tau^w)w \left(\frac{1}{1 - \gamma} - l \right) = \varkappa c \quad (69)$$

The equation of motion of the capital stock is given by,

$$\delta k = e \quad (70)$$

and the first order condition for investment is given by,

$$1 = \frac{\pi}{R} \left[p^k + (1 - \delta) \right] \quad (71)$$

Inflation is described by,

$$(1 - \varepsilon) + \varepsilon mc = 0 \quad (72)$$

Technology,

$$y = Ak^\alpha l^{1-\alpha} (g^p)^\theta \quad (73)$$

Cost minimisation implies,

$$w = mc \left[(1 - \alpha) \frac{y}{l} \right] \quad (74)$$

$$p^k = mc \left[\alpha \frac{y}{k} \right] \quad (75)$$

where,

$$mc = \left(\frac{p^k}{\alpha} \right)^\alpha \left(\frac{w}{1 - \alpha} \right)^{1-\alpha} A^{-1}. \quad (76)$$

5 Social Welfare

Defining what is optimal in an OLG model involves deciding how to compare different generations. Since we are interested in formulating optimal policy for our economy populated with overlapping generations of finitely lived consumers we must face the tricky issue of constructing a welfare metric. Calvo and Obstfeld (1988) define the social welfare function at time 0 as,

$$W_0 = \sum_{s=0}^{\infty} \left[\sum_{t=s}^{\infty} u(s, t) (\gamma\beta)^{t-s} \right] \rho^s + \sum_{s=-\infty}^0 \left[\sum_{t=0}^{\infty} u(s, t) (\gamma\beta)^{t-s} \right] \rho^s \quad (77)$$

where $u(s, t) = \ln c_t^s + \chi \ln \frac{M_t^s}{P_t} + \vartheta \ln g_t^c + \varkappa \ln(1 - l_t^s)$ is the utility at time t of a household born at time s . The first summation is the utility of representative agents of generations yet to be born, discounted at the policy-maker's discount factor, ρ . The second is the expected utility of households currently alive. These utilities are discounted back to the birthdate of the currently living generations, rather than the current period. Calvo and Obstfeld (1988) note that this is necessary to avoid the time inconsistency in preferences that would otherwise emerge by treating generations asymmetrically. In other words, if the policy maker did not discount utilities back to birthdates, then he would wish to change the consumption plans he put in place for currently unborn generations the moment they are born.

By changing the order of summation the welfare function can be rewritten as,

$$W_0 = \sum_{t=0}^{\infty} \left[\sum_{s=-\infty}^t u(s, t) \left(\frac{\gamma\beta}{\rho} \right)^{t-s} \right] \rho^t \quad (78)$$

so that the instantaneous flow utility to the policy maker is given by the summation over generations of their instantaneous utility discounted by the private discount factor and adjusted by the public discount factor. These are then discounted over time using the policy maker's discount factor, ρ . This can be further rewritten as,

$$W_0 = \sum_{t=0}^{\infty} \left[\sum_{z=0}^{\infty} u(t-z, t) \left(\frac{\gamma\beta}{\rho} \right)^z \right] \rho^t \quad (79)$$

which allows us to decompose the policy-maker's problem into two parts. The first part involves the policy maker's optimal allocation of consumption and labour supply across households. The second relates to the intertemporal aspects of the problem. Since we are only interested in the macroeconomic effects of fiscal adjustment in an environment where government debt can potentially crowd-out private capital, we abstract from the intratemporal intergenerational problem and focus on the intertemporal problem, such

that the social welfare function is given by,

$$W_0 = \sum_{t=0}^{\infty} \rho^t [\ln c_t + \chi \ln \frac{M_t}{P_t} + \vartheta \ln g_t^c + \varkappa \ln(1 - l_t)] \quad (80)$$

where we assume that $\rho = \beta$ such that the policy maker discounts the future at the same rate as households, but without accounting for the probability of death. In solving its intertemporal problem the policy maker ignores the distribution of variables across generations at a given point in time by focusing on per capita variables.

In order to capture the cross-sectional spread of utility across generations we need to be able to track the financial wealth of households. To do so we use the households' labour supply decision to write human wealth as,

$$lw_t^i \equiv H_t^i + \sum_{s=1}^{\infty} (\gamma)^s \left(\prod_{j=0}^{s-1} \frac{\pi_{t+j+1}}{R_{t+j}} \right) H_{t+s}^i \quad (81)$$

where

$$H_t^i \equiv z_t - \varkappa(1 + \tau_t^c)c_t^i \quad (82)$$

where $z_t = (1 - \tau_t^w)w_t + (1 - \gamma) \int_0^1 \Omega_{jt} dj$. Using the consumption Euler equation for the household, (25), allows us to further rewrite household human wealth as,

$$lw_t^i \equiv Z_t - \frac{\varkappa(1 + \tau_t^c)c_t^i}{1 - \gamma\beta} \quad (83)$$

where $Z_t = z_t + \sum_{s=1}^{\infty} (\gamma)^s \left(\prod_{j=0}^{s-1} \frac{\pi_{t+j+1}}{R_{t+j}} \right) z_{t+s}$ which is exogenous from the household's perspective. This expression can be used to rewrite the household's consumption function as,

$$(1 + \tau_t^c)c_t^i = \left(\frac{1 - \gamma\beta}{1 + \varkappa + \chi} \right) \left(\frac{W_t^i}{P_t} + Z_t \right) \quad (84)$$

which can be used to solve for any generation's consumption given their levels of financial wealth. Moreover, assuming an initial distribution of financial wealth would enable us to construct a corresponding distribution of consumption, money demand and labour supply with which to evaluate the cross-sectional distribution of utility. Further, using the household budget constraint would allow us to track the evolution of financial wealth across existing households across time. Since the movement in financial wealth is an affine transformation of the initial distribution it should be possible to tractably track that distribution over time. Therefore it is possible to derive a social welfare measure where the policy maker does not care about future generations only the generations currently alive as he commits to the optimal policy plan. If we were to attempt to monitor all social welfare, including future generations then we have the difficult problem of combining the existing distribution of financial wealth with that of each new generation

as it is born. This mixing distribution is not tractably trackable. However, we could consider a policy implemented by a policy maker who cares only about the welfare of a new born generation as they are born, for Rawlsian reasons.

5.1 Optimal Monetary and Fiscal Policy

Given the social welfare function, the optimal policy problem can be set up in terms of a Lagrangian as,

$$L_0 = \max_{\mathbf{y}_t} E_0 \sum_{t=0}^{\infty} \beta^t [U(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) - \boldsymbol{\lambda}_t f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t)]$$

where \mathbf{y}_t and \mathbf{u}_t are vectors of the model's endogenous and exogenous variables, respectively, $U(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) = \ln c_t + \chi \ln \frac{M_t}{P_t} + \vartheta \ln g_t^c + \varkappa \ln(1-l_t)$, $f(\mathbf{y}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t-1}, \mathbf{u}_t) = 0$ are the model's equilibrium conditions, and $\boldsymbol{\lambda}_t$ is a vector of Lagrange multipliers associated with these constraints.

The optimisation implies the following first order conditions,

$$\left[\frac{\partial U(\cdot)}{\partial \mathbf{y}_t} + \beta F \frac{\partial U(\cdot)}{\partial \mathbf{y}_{t-1}} + \beta^{-1} \boldsymbol{\lambda}_{t-1} F^{-1} \frac{\partial f(\cdot)}{\partial \mathbf{y}_{t+1}} + \boldsymbol{\lambda}_t \frac{\partial f(\cdot)}{\partial \mathbf{y}_t} + \beta \boldsymbol{\lambda}_{t+1} F \frac{\partial f(\cdot)}{\partial \mathbf{y}_{t-1}} \right] = 0 \quad (85)$$

where F is the lead operator, such that F^{-1} is a one-period lag. We can then solve these first order conditions in combination with the non-linear equilibrium conditions of the model, $f(\mathbf{y}_{s+1}, \mathbf{y}_s, \mathbf{y}_{s-1}, \mathbf{u}_s) = 0$. We do this fully non-linearly to obtain the steady-state of the policy makers problem. We also explore the dynamics using the perturbation methods of Schmitt-Grohe and Uribe (2004b).

We can also consider the allocation that would be achieved by a social planner who simply told households how to behave. The social planner's problem is given by,

$$L_0 = \sum_{t=0}^{\infty} \beta^t [\ln c_t + \vartheta \ln g_t^c + \varkappa \ln(1-l_t)] \quad (86)$$

subject to,

$$y_t = A_t k_t^\alpha l_t^{1-\alpha} \quad (87)$$

$$k_{t+1} = e_t + (1-\delta)k_t \quad (88)$$

and

$$y_t = c_t + g_t + e_t \quad (89)$$

Note that government debt does not exist in the social planner's problem, so the constraints involved in inheriting a positive debt level disappear.

β	γ	χ	ϑ	\varkappa	α	θ	δ	μ	ε	ϕ	A
0.9926	0.995	0	0.5	1.28	0.40	0	0.021	0	6.0	100	1

Table 1: Calibration of baseline model

5.2 Calibration

In order to analyse the main implications of our model, we have obtained a numerical solution of the steady state as well as of the log-linearised system. Table 1 summarises the values of the calibrated baseline parameters. In this preliminary version of the paper we have made a number of simplifying assumptions by assuming the economy approaches its cashless limit, $\chi \rightarrow 0$, there are no capital adjustment costs, $\mu = 0$, and government debt is only issued in the form of one period bonds. We shall reintroduce these features at a later date. For the remaining parameters, the assumed data period for the calibration is quarterly. Most values are taken from Andrés and Doménech (2005) and are similar to other DGE models as, for example, the parameters of the production function or the Phillips curve. Although most of these parameters refer to the EMU, in some cases, when no evidence exists for European countries, it is assumed that they are similar to the values habitually used for the United States. Thus, the discount factor (β) is 0.9926. The elasticity of output with respect to private capital (α) is 0.4, as in Cooley and Prescott (1995) and the output elasticity to public capital (θ) is set to 0. The depreciation rate (δ) is equal to 0.021, as estimated by Christiano and Eichenbaum (1992). Following Christiano, Eichenbaum and Evans (1997), the elasticity of demand with respect to price (ε) is set to 6, consistent with a steady-state mark-up, $\varepsilon/(\varepsilon - 1)$, equal to 1.2. The price adjustment cost parameter of $\phi = 100$ is standard and is set to ensure the log-linearised NKPC matches that obtained under Calvo (1983) pricing with empirically estimated contract duration probabilities such as those in Leith and Malley (2006). Parameter \varkappa , measuring the weight on leisure in utility, was set to 1.28, which is generally consistent with households allocating about a third of their time to market activities. While the weight given to government consumption in utility, $\vartheta = 0.5$, implies that the policy maker would ensure that government consumption as a share of GDP is in line with the empirical evidence in Gali (1994). Finally, the survival probability $\gamma = 0.995$, implying an expected adult life of 50 years.⁵

⁵We focus on economically active individuals (from 15 to 64 years old). 50 years is then a compromise between the years that Europeans are active, which is the reference variable for labour, and life expectancy which is probably a more relevant variable for consumption. We also set “economic” life expectancy equal to 50 years as a way of having a lower discount rate and, therefore, higher non-Ricardian effects. Nevertheless, in sensitivity analysis, we also consider the consequences of having a lower probability of death.

5.3 The Optimal Debt Target

In this section we examine the optimal level of steady state government assets implied by the simplified version of our model, using the calibration set out above. In addition we set inflation equal to zero. This solution is obtained by solving the non-linear equations of the model together with the first order conditions (85). This is a 'real' model, where the only distortions are monopolistic competition and income taxes, which are the only taxes available to the government.

Before doing this, consider first the steady state associated with zero government assets/debt. Government spending continues to be set at its optimal level, conditional on zero debt. This is the second column in Table 2 labelled 'Zero Debt'. The period of the model is one quarter, so the zero debt steady state corresponds to an annual real interest rate of 2.9%, and an annual capital output ratio of just over 2.8. (Recall that there is no technical progress or population growth in this model.) Public consumption is a bit over a quarter of the level of private consumption.

Variable	Zero Debt	$MC = 0$	+Lump Sum	Optimal	+Lump Sum	$MC = 0$	$\vartheta = 0.55$	Social Planner
Government Assets	0	0	0	25.93	184	71.28	25.00	n.a.
Capital	17.1	27.9	26.2	21.90	38.74	40.89	21.63	40.89
Real Interest rate	0.0086	0.0091	0.0153	0.0072	0.0024	0.0075	0.0072	n.a.
Output	1.52	2.10	2.37	1.85	2.72	2.91	1.83	2.91
Consumption	0.90	1.08	1.21	0.97	1.23	1.37	0.93	1.37
Government spending	0.26	0.44	0.61	0.43	0.68	0.68	0.44	0.68
Income tax rate	0.34	0.35	0	0.26	0	0	0.29	n.a.
Hours	0.30	0.37	0.48	0.36	0.46	0.5	0.35	0.5
Marginal Costs	0.83	1.00	1.00	0.83	0.83	1.00	0.83	n.a.

Table 2: Steady state levels

The next two columns look at the impact of the two distortions on the model, if government debt remains zero. Removing the monopoly distortion raises the level of all variables, although the impact on capital is greatest. If we in addition allow lump sum taxes, so that the income tax rate is zero, then we substitute labour for capital, and as a result there is a substantial increase in the real interest rate. However capital is still sub-optimally low in this economy. This can be seen by comparing these numbers with the final column of the table, which gives the allocation that would be chosen by

a social planner who fixed capital, output, consumption and labour supply irrespective of the government's budget constraint.

The fifth column, labelled 'Optimal', restores both distortions, but sets government assets to their optimal level, assuming a discount rate equal to the rate of time preference. As expected, the optimal steady state debt target is negative. In fact, it is optimal for government assets to exceed the level of the capital stock. A direct implication (see equation (10)), is that the steady state real interest rate is slightly below the rate of time preference. (The level of quarterly real interest rates that would equate the two is approximately 0.0075). Compared to the case where debt was zero, output is over 20% higher and private consumption 8% higher. Government consumption increases by 65%, as the interest from government assets pays for a good proportion of this expenditure. However the income tax rate remains positive, so we are clearly well below the zero tax level of government assets.

This last finding enables us to interpret our results in terms of the analysis in section 2. We noted that if the real interest rate in steady state was below the rate of time preference, then it was possible that $A^* < A^K < A^T$. From the above we can see that $A^* < A^T$, although A^K is unobservable. However, if $A^* > A^K$ (government assets were greater than that required to achieve the optimum level of capital), then this would not be a steady state, because there would be an incentive to cut taxes and raise debt because the real interest rate was below the discount rate. We could observe A^K directly by eliminating distortionary taxes. This is done in the next column. The optimal level of government assets rises substantially, with a more moderate increase in capital, output and consumption. This clearly illustrates that A^K is greater than the A^* in the previous column. The next column also eliminates the monopoly distortion, which allows us to achieve the social planner's allocation (shown in the final column).

The penultimate column increases the preferences for public goods, ϑ , relative to the fifth column, which raises steady state government spending by a small amount. Additional spending raises the income tax rate, which will necessarily raise the level of government assets required to eliminate taxes (A^T). However, A^* does not rise, because when real interest rates are below the discount rate the optimal debt level is not a compromise between A^T and A^K . Reproducing equation (17) from section 2 for a steady state, we have

$$A^* - A^K = T(\beta(1 + \tilde{r}) - 1)/(\alpha + \beta\gamma T) \quad (90)$$

While we cannot be sure from this equation that higher taxes will reduce the optimal level of government assets, it is clearly a possibility when $\beta(1 + \tilde{r}) < 1$, and this seems to be the case in the model.

We noted in section 2 that the arguments for discounting the utility of future generations by the rate of time preference were not compelling. If we discounted utility at a

lower rate, then clearly the optimal level of government assets would rise. In the most extreme case, where no discounting took place, the optimal level of capital would be the golden rule level that maximised steady state consumption. In this model with no technical progress or population growth, this would be associated with a zero real interest rate. Whether this could be achieved with a sufficiently large level of government assets is unclear, but column 6 suggests that values of government assets more than 10 times the level of GDP continue to raise steady state consumption, and produce a positive real interest rate. In that sense, the levels of A^* presented in Table 2, although historically unprecedented, are not the upper bound of what might be socially optimal in this simple OLG economy.

5.4 Transition paths

In this section we present a brief analysis of the optimal transition path to this steady state, using a linearised version of the model in which inflation is endogenised. Figure 1 plots, in years, paths for the key variables in the model. We start from a level of government assets which is not too far from the steady state (15.93, compared to the steady state of 25.93), because we are using a linear approximation to the model, and this may be misleading for initial values a long way from the steady state, and particularly where paths cross the zero debt position. We combine this starting point for debt with a starting point for capital which is derived from the steady state solution when the initial level of debt is imposed.

Figure 1 shows that dynamic adjustment is very drawn out over time. Although 50% of the adjustment in terms of debt is completed within about 75 years, complete adjustment takes around 300 years. This very long adjustment period is not surprising for two reasons. First, while complete tax smoothing no longer applies, the Blanchard-Yaari framework with realistic values for the probability of death gives only quantitatively minor deviations from Ricardian Equivalence, and so a large smoothing element is retained. Second, earlier analysis using models of this type suggest very long drawn out dynamics (e.g. Leith and Wren-Lewis (2000)). The result that debt adjustment should be very slow appears fairly robust (see Marcat and Scott (2008) for example).

Consumption declines over the first 70 odd years, which clearly shows why moving to the optimal level of debt is not a Pareto improvement. The current generation will be worse off as a result of raising the level of government assets. Most of the adjustment in debt is achieved through fiscal variables. In the very short term nominal and real interest rates are reduced, but this is not significant in terms of the overall adjustment. The zero lower bound constraint for nominal interest rates is not violated on the transition path. (We start from a value of 1.0034 for gross quarterly nominal interest rates.)

Figure 2 shows the adjustment path for debt for a variant of the model where we reduce the probability of death from 2% per annum to 1.6% per annum. The optimal steady state level of debt hardly changes, but the adjustment path is significantly slower,

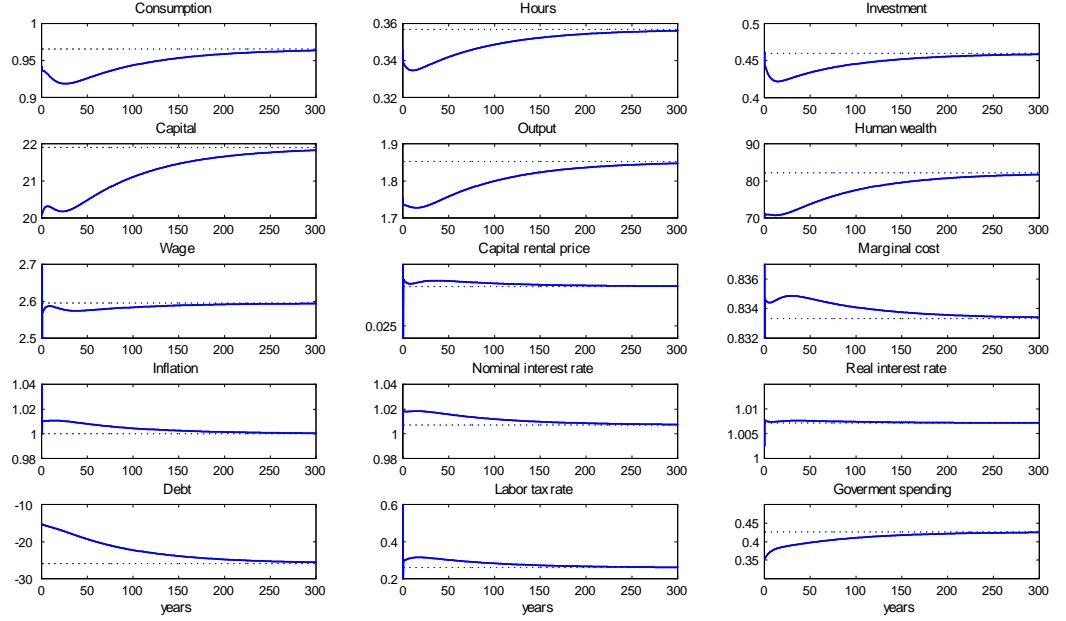


Figure 1: Transition paths to steady state.

with it taking more than 100 years to achieve half of the adjustment. This illustrates a key point about the special case where the probability of death is zero (i.e. a conventional model where agents are infinitely lived). Here the random walk steady state debt result represents a limiting case in terms of adjustment speeds, and not in terms of the long run debt target. As the probability of death shrinks to zero we take longer and longer to get to the long run debt target, so that in the limit we do not begin to reach it. However it is not the case that as the probability of death falls the long run target itself moves towards the historic level of debt.

Although the speed of adjustment is very slow, the size of adjustment required from current levels of debt is also very large. As a result, the implications for debt reduction today will still be significant. We should also note, however, that the starting point for adjustment does not involve interest rates at the zero lower bound and a large recession, so our analysis has no immediate implications for the current 'stimulus versus austerity' debate.

6 Conclusions

In models without default where agents are effectively infinitely lived, there is no optimal debt target because the costs of reducing debt are always higher than the cost of accommodating the existing level of debt. In OLG models this is no longer true for two

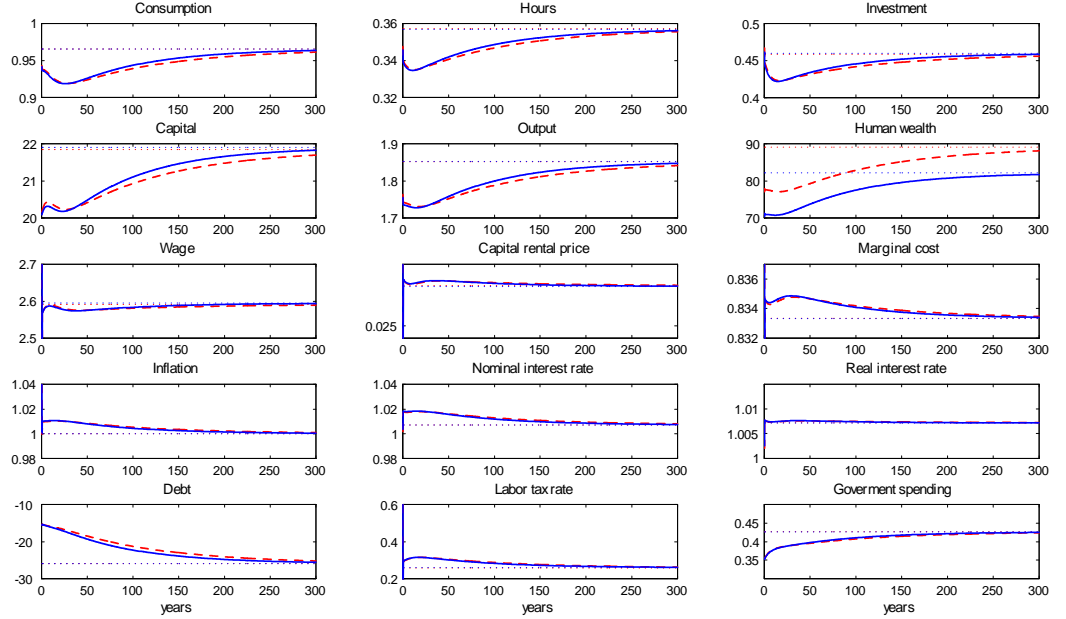


Figure 2: Transition paths to steady state, under different death probability rates: default 2% per annum (solid lines) versus 1.6% per annum (dash lines).

reasons. First, the real rate of interest is likely to be above the rate of time preference, so the benefits of future reductions in debt now outweigh the current costs of achieving lower debt. Second, the level of the capital stock is likely to be below the socially optimal level, and reductions in debt will crowd in capital.

In this paper we examine the optimal level of debt in one particular OLG model, the model of perpetual youth. We show that the optimal debt target in a calibrated version of this model involves positive government assets (i.e. a negative debt target), but these assets are below both the level required to eliminate distortionary taxes, and the level required to achieve the optimum capital stock. This is because, when the economy is distorted by monopolistic competition and income taxes, as debt declines the real rate of interest falls below the rate of time preference before the economy reaches the optimal capital stock. The optimal transition path towards this steady state is very drawn out, involving hundreds of years, but as the steady state involves historically unprecedented levels of government assets, the implications for debt adjustment in the short term may still be quantitatively significant.

There are a large number of issues to consider for future research. In this paper we have only computed results for a simplified version of our more general model. In particular, it will be interesting to consider the impact of government investment and capital. It would also be important to see how results were influenced by looking at an economy where there was (exogenous) technical progress, given the long adjustment

paths involved.

References

- [1] Agresti, A. M. and B. Mojon (2001): "Some Stylised Facts on the Euro Area Business Cycle". ECB Working Paper No. 95.
- [2] Andrés, J. and R. Doménech (2005): "Automatic Stabilizers, Fiscal Rules and Macroeconomic Stability". forthcoming, *European Economic Review*.
- [3] Ascari, G. and N. Rankin (2007), "Perpetual Youth and Endogenous Labour Supply: A Problem and Possible Solution", *Journal of Macroeconomics*, 29(4), pp 708-723.
- [4] Benassy, J-P. (2003): "Fiscal Policy and Optimal Monetary Policy Rules in a non-Ricardian Economy". *Review of Economic Dynamics*, 6, pp 498-512.
- [5] Benigno, P. and M. Woodford (2003): "Optimal Targeting Rules for Monetary and Fiscal Policy". Mimeo. New York University.
- [6] Blanchard, O. (1985). "Debt, deficits and finite horizons." *Journal of Political Economy*, vol. 93 (April), pp. 223-47.
- [7] Blanchard, O. and R. Perotti (2002): "An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output". *Quarterly Journal of Economics*, , 1329-68.
- [8] Calvo, G. (1983): "Staggered Prices in a Utility Maximizing Framework". *Journal of Monetary Economics*, 12(3), 383-98.
- [9] Calvo, G. and M. Obstfeld (1988), "Optimal Time-Consistent Fiscal Policy with Finite Lifetimes", *Econometrica*, Vol. 56, No.2, pp 411-432.
- [10] Clarida, R., J.Galí and M. Gertler (1999): "The Science of Monetary Policy: A New-Keynesian Perspective". *Journal of Economic Literature*, 37, 1661-1707.
- [11] Devereau, M (2010) 'Fiscal Deficits, Debt, and Monetary Policy in a Liquidity Trap', Central Bank of Chile Working Papers N° 581
- [12] Erosa, A and Gervais, M (2001): "Optimal Taxation in Infinitely-Lived Agent and Overlapping Generations Models: A Review" *Economic Quarterly*, pages 23-44
- [13] Fatas, A., and I.Mihov (1998): "Measuring the Effects of Fiscal Policy". Mimeo, INSEAD.
- [14] Galí, J. (1994): "Government Size and Macroeconomic Stability". *European Economic Review*, 38(1), 117-132.

- [15] Geanakoplos, J. (2008), "Overlapping Generations Models of General Equilibrium", Cowles Foundation Discussion Paper No. 1663.
- [16] Kirsanova, T., Leith, C. and S.Wren-Lewis (2007), "Optimal Debt Policy, and an Institutional Proposal to help in its Implementation", in J. Ayuso-i-Casals, S. Deroose, E. Flores, and L. Moulin (eds) "The role of fiscal rules and institutions in shaping budgetary outcomes", European Economy Economic Papers 275, April.
- [17] Leeper, E. (1991): "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies". *Journal of Monetary Economics*, 27, 129-147.
- [18] Leith, C. and J. Malley (2005), "Estimated General Equilibrium Models for the Evaluation of Monetary Policy in the US and Europe", *European Economic Review*, forthcoming.
- [19] Leith, C. and S. Wren-Lewis (2000): "Interactions between Monetary and Fiscal Policy Rules". *The Economic Journal*, 110, 93-108.
- [20] Leith, C and Wren-Lewis, S (2007), "Fiscal Sustainability in a New Keynesian Model", Oxford University Discussion Paper No. 310
- [21] Leith, C and Wren-Lewis, S (2010), "Discretionary Policy in a Monetary Union with Sovereign Debt" mimeo
- [22] Marcet, A. and A. Scott (2008). Debt, deficits and the structure of bond markets. *Journal of Economic Theory* 144 (2), 473—501.
- [23] Schmitt-Grohé, S. and M. Uribe (1997): "Balanced-Budget Rules, Distortionary Taxes and Aggregate Instability". *Journal of Political Economy*, 105 (5), 976-1000.
- [24] Schmitt-Grohe, S. and M. Uribe (2004a), "Optimal Fiscal and Monetary Policy Under Sticky Prices", *Journal of Economic Theory*, vol 114 pp 198-230.
- [25] Schmitt-Grohe, S. and M. Uribe (2004b), "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function", *Journal of Economic Dynamics and Control*, vol 27 pp 755-775.
- [26] Woodford, M. (1996): "Control of the Public Debt: A Requirement for Price Stability?". NBER Working Paper no. 5684.
- [27] Woodford, M.. (2003), "Interest and Prices", Pub. Princeton University Press.
- [28] Yaari, M. E. (1985). "The uncertain lifetime, life insurance and the theory of the consumer." *Review of Economic Studies*, vol. 32 (April), pp. 137-50.

A Appendix – Summary of Aggregate Model

The aggregate demand for money is given by,

$$m_t = \chi \frac{R_t}{R_t - 1} (1 + \tau_t^c) c_t \quad (91)$$

where all variables are now in per capita terms.

The aggregate consumption function is

$$(1 + \tau_t^c) c_t = \frac{1 - \gamma\beta}{1 + \chi} \left[\frac{W_t}{P_t} + lw_t \right] \quad (92)$$

where aggregate financial wealth in real terms is

$$\frac{W_t}{P_t} = m_{t-1} \pi_t^{-1} + \frac{R_{t-1}}{\pi_t} b_{t-1} + (q_t + \rho) D_{t-1} + \lambda_t^k k_{t+1} + (1 - \tau_t^k) p_t^k k_t - c_t \quad (93)$$

where $m_t \equiv M_t/P_t$ and $b_t \equiv B_t/P_t$, and the aggregate human wealth is

$$lw_t = H_t + \gamma \frac{\pi_{t+1}}{R_t} lw_{t+1} \quad (94)$$

with

$$H_t \equiv (1 - \tau_t^w) w_t l_t + s_t + (1 - \tau_t^k) \int_0^1 \Omega_{jt} dj \quad (95)$$

The government budget constraint is given by

$$g_t^c + g_t^p + s_t = \left[\tau_t^c c_t + \tau_t^w w_t l_t + \tau_t^k \left(\int_0^1 \Omega_{jt} dj + p_t^k k_t \right) \right] + b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} + m_t - \frac{m_{t-1}}{\pi_t} + q_t D_t - (q_t + \rho) D_{t-1} \quad (96)$$

The definition of profits (in real terms)

$$\int_0^1 \Omega_{jt} dj = y_t \left[1 - mc_t - \frac{\phi}{2} \left(\frac{\pi_t}{\pi^*} - 1 \right)^2 \right]$$

Combine the households' aggregate resource constraint with the government budget constraint and the definition of profits to obtain the aggregate resource constraint

$$g_t^c + g_t^p + c_t + e_t + \frac{\phi}{2} \left(\frac{\pi_t}{\pi^*} - 1 \right)^2 y_t = y_t \quad (97)$$

The real price of perpetuities is given by,

$$q_t = \frac{\pi_{t+1}}{R_t} (q_{t+1} + \rho) \quad (98)$$

Labour supply is

$$(1 - \tau_t^w) w_t \left(\frac{1}{1 - \gamma} - l_t \right) = \varkappa (1 + \tau_t^c) c_t \quad (99)$$

The equation of motion of the capital stock is given by,

$$k_{t+1} = e_t - \Phi\left(\frac{e_t}{k_t}\right)k_t + (1 - \delta)k_t \quad (100)$$

and the first order condition for investment is given by,

$$\lambda_t^k \left(1 - \Phi'\left(\frac{e_t}{k_t}\right)\right) = 1 \quad (101)$$

where the equation of motion for Tobin's q is,

$$\lambda_t^k = \frac{\pi_{t+1}}{R_t} \left((1 - \tau_{t+1}^k) p_{t+1}^k + \left(-\Phi\left(\frac{e_{t+1}}{k_{t+1}}\right) + \Phi'\left(\frac{e_{t+1}}{k_{t+1}}\right) \frac{e_{t+1}}{k_{t+1}} + (1 - \delta) \right) \lambda_{t+1}^k \right) \quad (102)$$

Inflation is described by,

$$(1 - \varepsilon) + \varepsilon m c_t - \phi \frac{\pi_t}{\pi^*} \left(\frac{\pi_t}{\pi^*} - 1 \right) + \phi \frac{\pi_t}{R_t} \frac{(1 - \tau_{t+1}^k) y_{t+1}}{(1 - \tau_t^k) y_t} \frac{\pi_{t+1}}{\pi^*} \left(\frac{\pi_{t+1}}{\pi^*} - 1 \right) = 0 \quad (103)$$

Technology,

$$y_t = A_t k_t^\alpha l_t^{1-\alpha} (g_t^p)^\theta \quad (104)$$

Cost minimisation implies,

$$w_t = m c_t \left[(1 - \alpha) \frac{y_t}{l_t} \right] \quad (105)$$

$$p_t^k = m c_t \left[\alpha \frac{y_t}{k_t} \right] \quad (106)$$

where,

$$m c_t = \left(\frac{p_t^k}{\alpha} \right)^\alpha \left(\frac{w_t}{1 - \alpha} \right)^{1-\alpha} A_t^{-1} \quad (107)$$

B Steady-State

The aggregate demand for money is given by,

$$m = \chi \frac{R}{R - 1} (1 + \tau^c) c \quad (108)$$

The aggregate consumption function,

$$(1 + \tau^c) c = \frac{1 - \gamma\beta}{1 + \chi} \left[\frac{W}{P} + l w \right] \quad (109)$$

aggregate of financial wealth,

$$\frac{W}{P} = \frac{m}{\pi^*} + \frac{R}{\pi^*} b + (q + \rho) D + \lambda^k k + (1 - \tau^k) p^k k - e \quad (110)$$

Aggregate human wealth is given by,

$$lw \left(1 - \frac{\gamma\pi^*}{R} \right) = H \quad (111)$$

where

$$H \equiv (1 - \tau^w)wl + s + (1 - \tau^k) \int_0^1 \Omega_j dj \quad (112)$$

The government budget constraint is given by

$$g^c + g^p + s = \left[\tau^c c + \tau^w wl + \tau^k \left(\int_0^1 \Omega_j dj + p^k k \right) \right] + b \left(1 - \frac{R}{\pi^*} \right) + m \left(1 - \frac{1}{\pi^*} \right) - D\rho$$

The definition of profits (in real terms)

$$\int_0^1 \Omega_t dj = y(1 - mc)$$

The aggregate resource constraint

$$g^c + g^p + c + e = y \quad (113)$$

The price of perpetuities is given by,

$$q = R^{-1}\pi^*(q + \rho) \quad (114)$$

Labour supply is

$$(1 - \tau^w)w \left(\frac{1}{1 - \gamma} - l \right) = \varkappa(1 + \tau^c)c \quad (115)$$

The equation of motion of the capital stock is given by,

$$e = \delta k \quad (116)$$

and the first order condition for investment is given by,

$$\lambda^k = 1 \quad (117)$$

where the equation of motion for Tobin's q is,

$$1 = \frac{\pi^*}{R} \left[(1 - \tau^k)p^k + (1 - \delta) \right] \quad (118)$$

Inflation is described by,

$$(1 - \varepsilon) + \varepsilon mc = 0 \quad (119)$$

Aggregating across the j firms,

$$y = Ak^\alpha l^{1-\alpha} (g^p)^\theta \quad (120)$$

Cost minimisation implies,

$$w = mc \left[(1 - \alpha) \frac{y}{l} \right] \quad (121)$$

$$p^k = mc \left[\alpha \frac{y}{k} \right] \quad (122)$$

where,

$$mc = \left(\frac{p^k}{\alpha} \right)^\alpha \left(\frac{w}{1 - \alpha} \right)^{1-\alpha} A^{-1} \quad (123)$$