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TRADE LIBERALIZATION AND WELFARE UNDER
OLIGOPOLY**

Dermot Leahy, J. Peter Neary

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Manor Road Building, Oxford OX1 3UQ

WHEN THE THREAT IS STRONGER THAN THE EXECUTION: TRADE LIBERALIZATION AND WELFARE UNDER OLIGOPOLY*

Dermot Leahy[†]

Maynooth University

J. Peter Neary[‡]

University of Oxford,

CEPR and CESifo

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Abstract

We compare trade liberalization under Cournot and Bertrand competition in reciprocal markets. In both cases, the critical level of trade costs below which the possibility of trade affects the domestic firm's behavior is the same; trade liberalization increases trade volume monotonically; and welfare follows a U-shaped pattern. However, welfare is usually greater under Bertrand than Cournot competition, despite the fact that for higher trade costs the volume of trade is greater under Cournot competition. In general, there exists a “Nimzowitsch Region” in parameter space, where welfare is higher under Bertrand competition even though no actual trade takes place.

Keywords: Cournot and Bertrand Competition; Cross-Hauling; Nimzowitsch Region; Oligopoly and Trade; Trade Liberalization

JEL Classification: F12, F13

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[†]Department of Economics, Finance and Accounting, National University of Ireland Maynooth, Maynooth, Co. Kildare, Ireland; e-mail: dermot.leahy@nuim.ie.

[‡]Department of Economics, University of Oxford, Manor Road, Oxford OX1 3UQ, UK; e-mail: peter.neary@economics.ox.ac.uk.

1 Introduction

The sign and magnitude of the gains from trade and from trade liberalization continue to be among the central issues in international trade. Recent work has thrown new light on the quantitative extent of these gains under perfect competition and monopolistic competition with heterogeneous firms.¹ However, far less is known about the effects of trade liberalization in oligopolistic markets, despite growing evidence that trade is dominated by large firms.² In this paper we compare trade liberalization under Cournot and Bertrand oligopoly in a unified framework that allows for product differentiation. There is a widely held perception that the results of oligopoly trade models are highly sensitive to the mode of competition. On the contrary, we show that many of the predictions are in fact qualitatively robust to whether firms compete on quantity or price. At the same time, there are important quantitative differences between the two models, reflecting the fact that firms compete more aggressively in the Bertrand than in the Cournot case.

To focus attention on the nature of competition, we consider a symmetric two-country world in which a single domestic firm faces competition in both home and foreign markets from a single foreign firm. We show that a number of the effects of trade liberalization are common irrespective of the mode of competition. In particular, trade liberalization increases trade volume monotonically; welfare as a function of trade costs follows a U-shaped pattern; and the critical level of trade costs below which the possibility of trade affects the domestic firm's behavior is the same under Cournot and Bertrand competition, and is increasing in the degree of product differentiation. On the other hand, there are important quantitative differences between the two cases. For any trade cost below the critical level, welfare is usually higher under Bertrand than under Cournot competition. This is because the pro-competitive effects of trade are always stronger under Bertrand competition. This is true even though there is a region of trade costs in which there is no actual trade under Bertrand competition

¹See, for example, Arkolakis, Costinot, and Rodríguez-Clare (2012), Melitz and Redding (2015), and Arkolakis, Costinot, Donaldson, and Rodríguez-Clare (2012).

²See, for example, Mayer and Ottaviano (2008) and Freund and Pierola (2015).

while the volume of trade is positive under Cournot. The mere threat of trade under price competition is sufficient to discipline the home firm and is more effective in reducing prices and raising welfare than actual trade is under Cournot competition. This range of trade costs is larger the more substitutable are the goods. We call the region of parameter space in which this outcome holds the “Nimzowitsch Region,” after the remark attributed to chess grandmaster Aron Nimzowitsch, “the threat is stronger than the execution.”³ When trade costs are sufficiently low, the volume of trade is always higher under Bertrand competition, and the level of welfare is usually higher too. There is also an intermediate region in which there are positive but lower imports under Bertrand. However, in almost all cases, the pro-competitive effects are stronger under price competition and welfare is higher as a result. We first use linear demands to obtain explicit solutions. We then extend the analysis to consider more general functional forms and show that many of the results are qualitatively robust.

Our results build on and extend a large literature on the welfare effects of trade liberalization under oligopoly. (For an overview, see Leahy and Neary (2011).) The topic was first studied by Brander (1981) and Brander and Krugman (1983) using a model of two-way trade in segmented markets. In this reciprocal-markets setting, they demonstrated that, under Cournot competition, intra-industry trade can occur in equilibrium even when goods are identical. Bernhofen (1999) introduced product differentiation into Cournot and Bertrand oligopoly models of intra-industry trade, focusing on the effects of trade on profits and consumer surplus. The effects of trade liberalization under Bertrand competition in a reciprocal markets setting were first derived by Clarke and Collie (2003). For related work, see Brander and Spencer (2015) and Collie and Le (2015). All of these papers considered the effects of trade costs, while the effects of quotas were explored by Harris (1985) and Krishna (1989). As we shall see, the effects of quotas serve as a useful contrast with the effects of trade liberalization under Bertrand competition.

³See http://en.wikipedia.org/wiki/Aron_Nimzowitsch.

Section 2 introduces the linear model and illustrates the effects of trade liberalization on outputs, trade volumes, profits and welfare under Cournot and Bertrand competition when trade costs are sufficiently low that imports occur in equilibrium. Section 3 turns to consider the case where imports do not occur in equilibrium when firms compete on price, but nevertheless the threat of foreign entry serves to discipline the home firm. It shows how this outcome depends on the underlying parameters, and relates the findings to some classic results in game theory. Section 4 shows that the main results continue to hold with general demands.

2 Cournot versus Bertrand Competition

2.1 The Setting

We use a common specification of preferences and technology to obtain explicit solutions and to allow us to compare the results under Cournot and Bertrand competition. On the demand side we assume a representative consumer in the home country with quasi-linear utility:

$$U = z_0 + u(x, y) \tag{1}$$

where the home market sales of the home and foreign oligopolistic firms are denoted x and y respectively. Here z_0 is the consumption of the outside good which we assume is produced under perfect competition. This is a composite commodity defined over all the other goods in the economy. Thus the sub-utility function $u(x, y)$, represents the domestic utility from consumption of the oligopolistic goods. We assume that preferences for the oligopolistic goods are quadratic; in Section 4 we show that the qualitative results continue to hold for more general specifications. The sub-utility function is represented by:

$$u(x, y) = a(x + y) - \frac{1}{2}b(x^2 + 2exy + y^2). \tag{2}$$

where e is an inverse measure of the degree of product differentiation, ranging from the case of perfect substitutes ($e = 1$) to that of independent demands ($e = 0$). We can write the identity between national expenditure and national income:

$$z_0 + px + p^*y = I + \Pi \quad (3)$$

where p and p^* are the home market prices of the goods produced by home and foreign oligopolistic firms respectively, I is factor income, and Π is the total profits of the home firm in both markets. We can make use of this in the quasi-linear utility function to write home welfare as:

$$W = U = \chi + \Pi + I \quad (4)$$

where $\chi = u(x, y) - px - p^*y$ is home consumer surplus. As is standard in these models we assume the non-numéraire sector is small in factor markets and so treat I as constant.

Maximization of (1) subject to the budget constraint yields linear inverse demand functions:

$$p = a - b(x + ey) \quad (5)$$

$$p^* = a - b(y + ex) \quad (6)$$

The parameter $b > 0$ can be interpreted as an inverse measure of market size. We assume that b is independent of e , which is the standard Bowley specification of linear demands.⁴

On the cost side, we assume that marginal costs are constant and we ignore fixed costs. Hence the home and foreign firms' operating profits in the home market are:

$$\pi = (p - c)x \quad (7)$$

⁴See, for example, Vives (1985). This specification has been criticized on the grounds that the market size increases as products become differentiated. This feature is avoided by the alternative Shubik-Levitan specification, used by Collie and Le (2015), which sets $b = \frac{\beta}{1+e}$ where $\beta > 0$.

and

$$\pi^* = (p^* - c - t)y \quad (8)$$

where c are the marginal production costs of the home and foreign firms, assumed to be constant, and t is the per-unit trade cost.

We will first examine the effects of symmetric multilateral trade liberalization under quantity competition and then compare that with what happens under price competition. We confine attention to the symmetric case, where the home and foreign firms face symmetric demands, the same production cost functions, and the same trade cost. Because of symmetry, equilibrium foreign market sales of the home and foreign firms are also equal to the home market sales y and x respectively. As the countries are mirror images of each other we need only consider the effect of a transport cost reduction on equilibrium in the home market.

2.2 Quantity Competition

Using the linear inverse demand functions (5) and (6), the firms' first-order conditions for output are $bx = p - c$ and $by = p^* - c - t$. These can then be solved for the Cournot-Nash equilibrium outputs:

$$x = \frac{1}{b(2+e)} \left[A + \frac{e}{2-e}t \right] \quad (9)$$

$$y = \frac{1}{b(2+e)} \left[A - \frac{2}{2-e}t \right] \quad (10)$$

where $A \equiv a - c$ is the difference between the maximum price consumers are willing to pay and the marginal cost of production, so can be interpreted as a measure of the competitiveness of each firm. At free trade ($t = 0$), imports y equal the home firm's sales x . There is two-way trade in the oligopolistic sector since, from the symmetry of the model, foreign market sales of the home and foreign firms are also equal to y and x respectively. As first shown by Brander (1981), this is true even when products are identical ($e = 1$), the case of cross-hauling or two-way trade in identical products. As goods become more differentiated,

e falls below one and the volume of trade rises further. As trade costs increase, two-way trade persists, though at a diminishing level: home sales rise and imports fall, reaching zero at the prohibitive level of trade costs $\widehat{t}^C$, which from (10) equals:

$$\widehat{t}^C = \frac{2-e}{2}A \quad (11)$$

We are mainly concerned with the effect of trade liberalization on welfare, but an important preliminary step, which is also of independent interest, is its implications for profits. Focusing on the home firm, its total profits equal the sum of its profits on home sales and on exports. The first are given by (7) while the second equal the foreign firm's profits in the home market (8): because of the symmetry of the model, home exports x^* equal home imports y , so the home firm's profits on its exports are $\pi^* = (p^* - c - t)x^* = (p^* - c - t)y$. Substituting in turn from the first-order conditions, these are proportional to home and export sales respectively: $\pi = bx^2$ and $\pi^* = b(x^*)^2$. Differentiating these and using the derivatives of (9) and (10), the effect of a multilateral change in trade costs on total profits can be shown to equal:

$$\frac{d(\pi + \pi^*)}{dt} = 2bx\frac{dx}{dt} + 2bx^*\frac{dx^*}{dt} = \frac{2(ex - 2x^*)}{4 - e^2} \quad \left\{ \begin{array}{l} < 0 \text{ when } t = 0 \text{ (so } x = x^*) \\ > 0 \text{ when } t = \widehat{t}^C \text{ (so } x^* = 0) \end{array} \right. \quad (12)$$

Profits are decreasing in trade costs at free trade, but increasing in them in the neighborhood of autarky. It follows that profits must be a U-shaped function of trade costs. The intuition for this is straightforward. First, starting from free trade, exports are harmed more by an increase in the firm's own costs than home sales are helped by an equal rise in its rival's costs; hence total sales and profits fall for a small increase in t at free trade. Second, starting from autarky, exports are initially zero, so a small fall in trade costs has a negligible effect on profits in the export market. By contrast, home sales are initially at the monopoly level, so a small fall in the foreign firm's trade costs has a first-order effect on home-market profits.

Hence, overall profits fall for a small reduction in t at autarky.

It is clear that trade is locally bad for profits, in the neighborhood of autarky. Must it be globally bad? More precisely, are profits lower in free trade than in autarky? Anderson, Donsimoni, and Gabszewicz (1989) showed that this must be true when goods are perfect substitutes. However, this need not be true for values of the substitution parameter below unity. Equation (12) suggests that the losses from increased foreign competition in the home market (reducing home sales as t falls) are lower as e falls and goods are less substitutable; moreover, the gains from increased export opportunities (increasing x^* as t falls) are higher. The net effect is that total profits can be higher than in autarky for sufficiently low tariffs and sufficiently low values of the substitutability parameter e . To see this, solve for the trade cost at which profits with trade equal profits in autarky:

$$t^{\Pi A} = \frac{4 - 4e - e^2}{4 + e^2} \frac{2 - e}{2} A = \frac{4 - 4e - e^2}{4 + e^2} \hat{t}^C \quad (13)$$

where $\hat{t}^C$ is the prohibitive trade cost from (11). In the case of $e = 1$ considered by Anderson, Donsimoni, and Gabszewicz (1989), $t^{\Pi A}$ is negative, equal to $-0.1A$; i.e., when goods are identical, profits are always lower with trade (whether free or not) than in autarky. However, for any value of e less than $2(\sqrt{2} - 1)$ (approximately 0.828), profits are higher in free trade than in autarky. Moreover, as e falls, the range of values of t at which profits are higher than in free trade expands rapidly. This is shown in Figure 1, which illustrates in $\{t, e\}$ space the loci corresponding to autarky, $\hat{t}^C$, the minimum value of profits, $t^{\Pi min}$, and the same level of profits as in autarky, $t^{\Pi A}$.⁵

Combining the results so far on changes in prices and profits, we can consider the full effect of changes in trade costs on welfare. Consider in turn the components of welfare in (4). Consumer surplus must rise monotonically as trade costs fall. This is because a reduction in

⁵As we have already seen, Cournot profits are always U-shaped. The level of trade costs at which profits are minimized is: $t^{\Pi min} = \frac{(2-e)^2}{4+e^2} A$, which is always strictly positive. This value is exactly the average of the prohibitive trade cost $\hat{t}^C$ and the trade cost at which profits return to their autarky level, $t^{\Pi A}$.

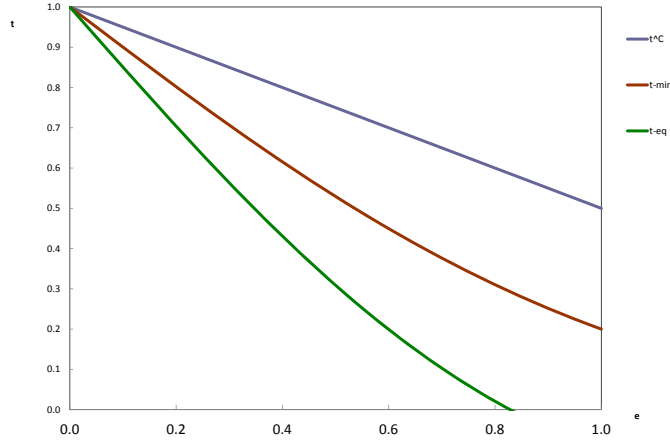


Figure 1: Loci of t and e that yield Autarky, Minimum Profits, and Autarky Profits

trade costs lowers the prices of both goods to home consumers. To this must be added the U-shaped relationship between profits and trade costs already derived. In the neighborhood of free trade, welfare is clearly falling in trade costs. All that is left is to consider the sum of consumer surplus and profits for a small fall in t starting in autarky (where $t = t^C$). Consumer surplus rises because the price falls, but profits on home sales fall both because the price falls and because sales are reduced. The price effects cancel, so the total fall in profits outweighs the rise in consumer surplus. Thus home welfare (the sum of profits and consumer surplus) is also a U-shaped function of t , reaching its maximum at free trade but its minimum below the prohibitive level of trade costs. The combinations of values of t and e at which welfare is minimized and returns to its autarky level are shown in Figure 2.

2.3 Price Competition

How do the effects of trade liberalization on trade and welfare differ if firms compete in price rather than quantity? To compare Bertrand and Cournot competition we use the same demand and cost functions. However, to solve for the Bertrand equilibrium in this case, we

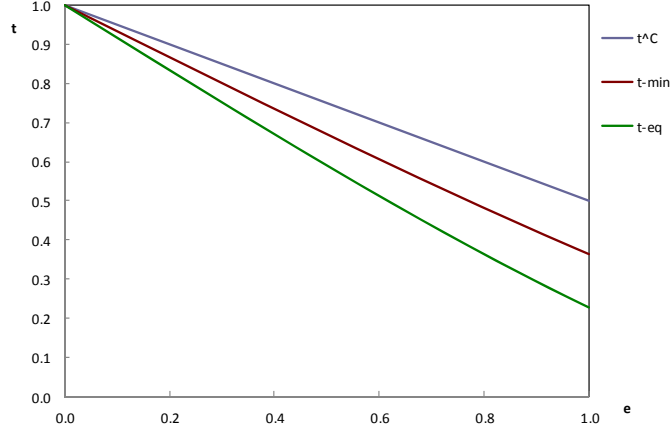


Figure 2: Loci of t and e that yield Autarky, Minimum Welfare, and Autarky Welfare

need to use the direct demand functions, which can be obtained by inverting the system in (5) and (6) to get:

$$x = \frac{1}{b(1-e^2)} [(1-e)a - (p - ep^*)] \quad (14)$$

and

$$y = \frac{1}{b(1-e^2)} [(1-e)a - (p^* - ep)] \quad (15)$$

For the moment consider only interior equilibria in which both firms export positive quantities. We will return to corner solutions later. The first-order conditions for the optimal choice of prices are $p - c = b(1 - e^2)x$ for the home firm and $p^* - c - t = b(1 - e^2)y$ for the foreign firm. These can be solved for the Bertrand-Nash equilibrium prices which in turn can be combined with the direct demand functions to obtain the equilibrium quantities under Bertrand competition:

$$x = \frac{1}{b(1+e)(2-e)} \left[A + \frac{e}{(1-e)(2+e)} t \right] \quad (16)$$

$$y = \frac{1}{b(1+e)(2-e)} \left[A - \frac{2-e^2}{(1-e)(2+e)} t \right] \quad (17)$$

As in the Cournot case, imports equal the home firm's sales at free trade and are decreasing in trade costs, falling to zero when trade costs reach the threshold level which sets (17) equal to zero. This threshold is given by:

$$\hat{t}^B = \frac{(1-e)(2+e)}{2-e^2} A \quad (18)$$

Comparing this to the corresponding threshold under quantity competition it is straightforward to show that: $\hat{t}^B < \hat{t}^C$. Profits and welfare also behave quite similarly to quantity competition for trade costs between zero and the prohibitive trade cost level $\hat{t}^B$. Using the first-order conditions for prices, maximized profits are equal to $b(1-e^2)x^2$ and $b(1-e^2)y^2$. Total profits for the home firm are then equal to the sum of these, and their behavior as trade costs change can be shown to equal:

$$\frac{d(\pi + \pi^*)}{dt} = 2b(1-e^2) \left[x \frac{dx}{dt} + x^* \frac{dx^*}{dt} \right] = \frac{ex - (2-e^2)x^*}{2-e^2} \begin{cases} < 0 \text{ when } t = 0 \text{ (so } x = x^*) \\ > 0 \text{ when } t = \hat{t}^B \text{ (so } x^* = 0) \end{cases} \quad (19)$$

Profits are decreasing in trade costs at free trade, but increasing in them in the neighborhood of the threshold trade cost $\hat{t}^B$. Hence, as in the Cournot case, it follows that profits must be a U-shaped function of trade costs. Once again, consumer surplus falls monotonically in trade costs and, as in the Cournot case, welfare is a U-shaped function of trade costs.

2.4 Trade Liberalization and the Mode of Competition

Figure 3 summarizes the results so far. (Without loss of generality we normalize autarky welfare to zero.) These results have been shown in different ways by Brander (1981), Clarke and Collie (2003), and others. Their broad outlines are familiar from Vives (1985): price competition is more competitive than quantity competition, leading to lower prices and

higher outputs. In a trade context this implies in turn that price competition also leads to higher welfare.

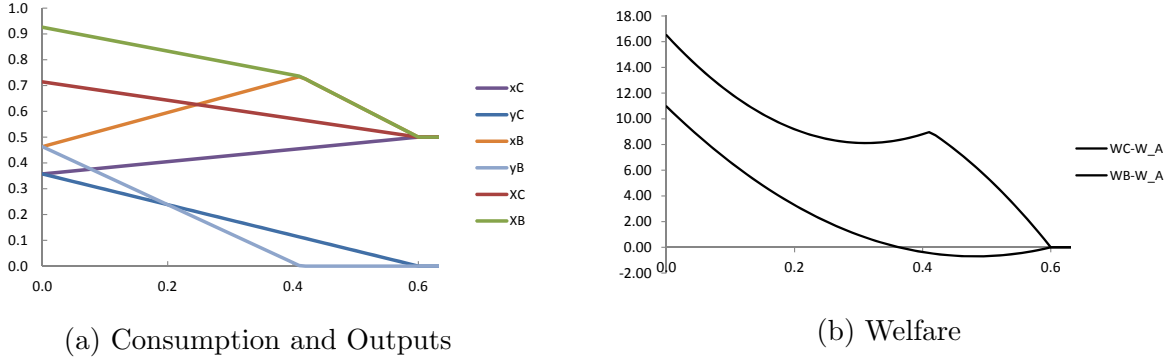


Figure 3: Effects of Trade Liberalization in Cournot and Bertrand Competition

However, we have not yet considered what happens under price competition for trade costs in the range between $\hat{t}^B$ and $\hat{t}^C$. To understand this case we have to give more careful attention to the nature of the game between firms.

3 The Nimzowitsch Region

3.1 Price Competition in the Neighborhood of Autarky

Even when trade costs are too high for exports to take place, they may not be too high to prevent the threat of exports from affecting the domestic firm's behavior. Substituting $\hat{t}^B$ into (16) shows that the home firm's output is $x = \frac{A}{b(2-e^2)}$ at this threshold level of trade costs, which is above the unconstrained monopoly output level, $x^M = \frac{A}{2b}$. The home firm does not have an incentive to lower its output and raise its price, since its rival would then make positive sales and this would lower the home firm's domestic profits. However, if trade costs are high enough the firm can behave as an unconstrained monopolist. As we will see, this requires that t reaches $\hat{t}^C$, the prohibitive threshold under quantity competition. Hence the critical level of trade costs below which the possibility of trade affects the domestic firms' behavior is the same under Cournot and Bertrand competition. At levels of trade

costs $\hat{t}^B \leq t \leq \hat{t}^C$, the home firm chooses a price at which the foreign firm is just unable to produce. In this region the home firm's output is:⁶

$$x = \frac{A - t}{be} \quad (20)$$

which clearly falls in t .

To demonstrate these results, we need to consider how the firms' best-response functions behave in the neighborhood of autarky. Consider first the foreign firm. In an interior Bertrand equilibrium (where imports are strictly positive) of the kind already considered in the last sub-section, the foreign firm's best-response function is:

$$p^* = B^*(p; t) = \operatorname{argmax}_{p^*} \pi^*(p^*, p; t) = \frac{1}{2} [(1 - e)a + c + t + ep] \quad (21)$$

However, if the home firm's price falls to a level where the zero-import constraint binds, then the foreign firm's best response is to charge a price equal to its marginal cost of serving the market, $c + t$. The zero-import constraint is $y(p^*, p) \geq 0$, which defines the maximum foreign price consistent with positive imports as a function of the home price. From (15), this is:

$$p^* = \tilde{p}^*(p) = (1 - e)a + ep \quad (22)$$

Combining these two regimes, the foreign firm's best-response function is:

$$p^* = \tilde{B}^*(p, t) = \begin{cases} c + t & \text{when } \tilde{p}^*(p) \leq B^*(p; t) \\ B^*(p; t) & \text{when } \tilde{p}^*(p) \geq B^*(p; t) \end{cases} \quad (23)$$

This locus is kinked where it intersects the zero-import locus, and is shown by the bold locus in Figure 4 (a).

⁶To see this, find the level of p that sets $y = 0$ for any given p^* . From (15) this is: $p = \frac{p^* - (1-e)a}{e}$. In this region, $(\hat{t}^B \leq t \leq \hat{t}^C)$ where the foreign firm is just kept out of the market: $p^* = c + t$. Combine these and make use of $p = a - bx$ to obtain (20).

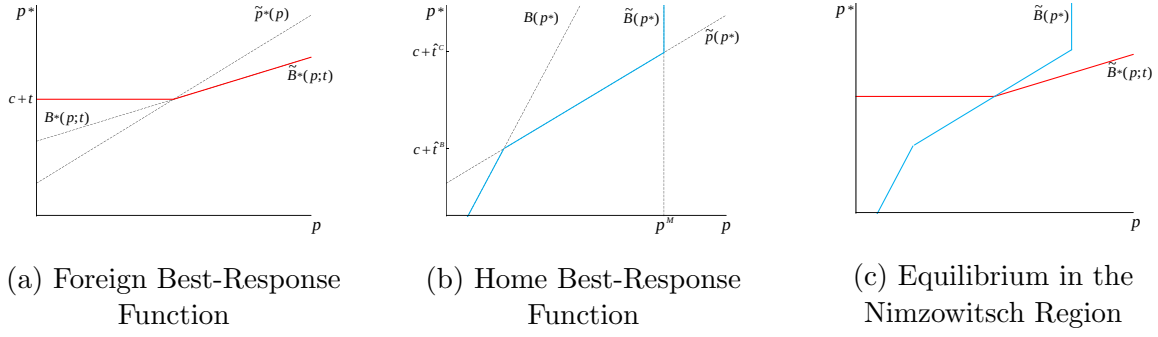


Figure 4: Strategic Interactions in the Nimzowitsch Region

In the same way, we can examine the best responses of the home firm. In this case there are three distinct regimes. First, for low import prices, the home firm's best response is also to charge a low price along its unconstrained best-response function $B(p^*)$, which is:

$$p = B(p^*) = \operatorname{argmax}_p \pi(p, p^*) = \frac{1}{2} [(1 - e)a + c + ep^*] \quad (24)$$

This case corresponds to tariffs below the threshold $\hat{t}^B$ as defined in equation (18). By contrast, for very high import prices, the home firm is an unconstrained monopolist, and so it charges the monopoly price shown by p^M . This price is the solution to the zero-import constraint (22) when the foreign firm charges a price just sufficient to choke off import demand: i.e., a price equal to its marginal cost of production plus the prohibitive tariff $\hat{t}^C$ from (11).⁷ Finally, for intermediate import prices, the home firm's best-response is to set its own price at the level such that the zero-import constraint just binds: from its perspective it operates on the inverse of (22), which we denote $\tilde{p}(p^*)$. Combining these three regimes, the home firm's best-response function is:

$$p = \tilde{B}(p^*) = \begin{cases} B(p^*) & \text{when } \tilde{p}(p^*) \leq B(p^*) \\ \tilde{p}(p^*) & \text{when } B(p^*) \leq \tilde{p}(p^*) \leq p^M \\ p^M & \text{when } p^M \leq \tilde{p}(p^*) \end{cases} \quad (25)$$

⁷Combining (22) and (11) and $\hat{p}^* = c + \hat{t}^C$ gives the monopoly price $p^M = \frac{1}{2}(a + c)$.

This function has two kinks, as shown by the bold locus in Figure 4 (b).

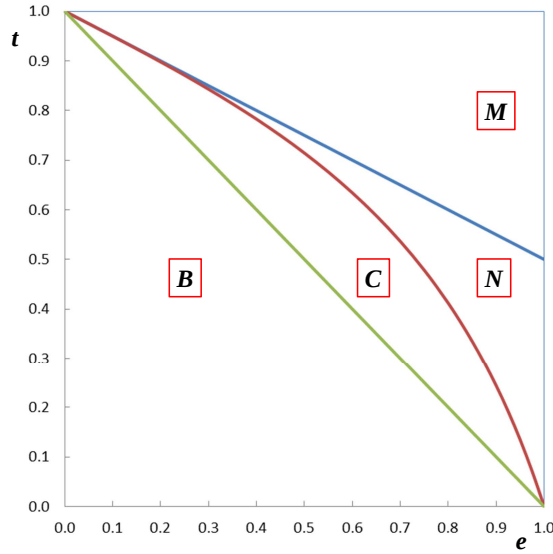


Figure 5: Regions of Trade

B: $y^B > y^C$; C: $y^C > y^B > 0$; N: The Nimzowitsch Region; M: Monopoly

Finally, we can bring together the responses of the two firms to show the full equilibrium, as in Figure 4 (c). Notice first that the home firm's best-response function (25) is not affected by the actual level of the tariff t . A change in the tariff shifts the foreign firm's best-response function only, so the equilibrium moves along the home firm's function. There are therefore three possible regions, depending on where the intersection point occurs. In the case shown, the tariff lies between $\hat{t}^B$ and $\hat{t}^C$, and so the foreign firm's best-response function intersects the home's along the zero-imports locus. This outcome therefore lies in the Nimzowitsch Region: the home firm's behavior is affected by the threat of imports, even though no actual imports take place. Figure 5 summarizes the results to date and shows how the size of the Nimzowitsch Region varies with the degree of substitutability e : it is largest when goods are close substitutes and declines as they become more differentiated.

3.2 Maximum versus Minimum Import Constraints

It is instructive to compare this outcome with the case studied by Krishna (1989), where the foreign firm faces a quantitative trade barrier, such as a quota or a voluntary export constraint. There the constraint takes the form of a maximum level of imports, whereas in our case the constraint is a minimum one: imports cannot fall below zero. This difference affects the nature of the game in important ways.⁸ In particular, it determines whether the home firm's best-response function is continuous or not.

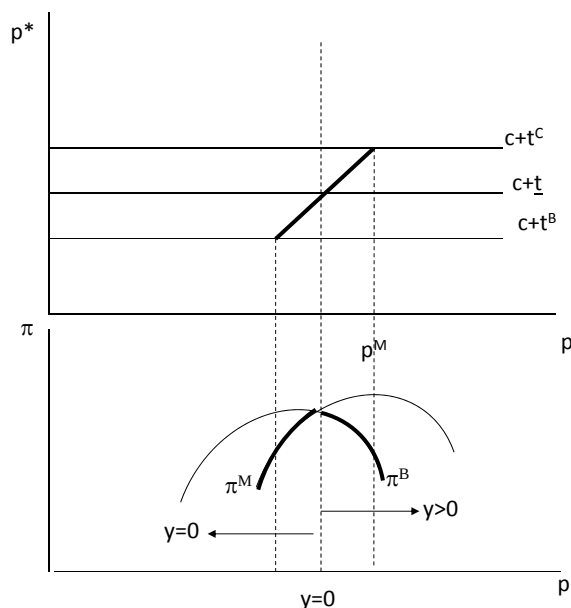


Figure 6: Price Competition: The Home Firm's Perspective

To see why this is so, note that in both cases the home firm is choosing between two options, whose implications for profits can be represented by two concave functions, as shown

⁸A further complication when a maximum import constraint binds is that, since consumers wish to purchase more than the permitted level of imports, some rationing mechanism must be adopted to allocate it. Different rationing mechanisms have different implications for the equilibrium outcome. This problem does not arise in our context, since consumers are always able to purchase the level of imports they desire at the prevailing prices.

in Figure 6. One, denoted by π^M , is the profits it would obtain as a monopolist, which is relevant only when the constraint that imports cannot be negative is binding. The other, denoted by π^B , is the profits it would obtain in a Bertrand equilibrium where both firms have positive sales; this is relevant only when the constraint on the value of imports is non-binding.

In our case with a minimum import constraint, the home firm's profit function is the lower envelope of these two concave functions, indicated by the solid line in the lower panel of Figure 6. Such a lower envelope is itself concave. In addition, an unconstrained monopoly (the case of zero imports) always yields higher profits than a Bertrand duopoly equilibrium. Hence, the home firm's best-response function is continuous, and there is a unique equilibrium in pure strategies. As the tariff changes, the monopoly profit function is unaffected, but the function representing profits in an unconstrained Bertrand equilibrium shifts.

By contrast, in the case of a maximum import constraint considered by Krishna, the home firm's profit function is the upper rather than the lower envelope of the two concave functions. As a result, the profit function of the home firm is non-concave. In addition, for a particular foreign price (which as Krishna shows is profit-maximizing for the foreign firm), the two functions yield the same profit levels for the home firm. Hence its best-response function is discontinuous and the game has no equilibrium in pure strategies. As the permitted level of imports changes, the constrained monopoly profit function shifts, but the function representing profits in an unconstrained Bertrand equilibrium is unaffected.

The difference between the two cases can also be related to two classic outcomes in game theory. The case of import quotas considered by Krishna is analogous to Edgeworth's demonstration that an equilibrium in pure strategies does not exist in a Bertrand game with capacity constraints. By contrast, the Nimzowitsch Region is analogous to the outcome in a Bertrand game with identical products, where the more efficient firm captures the entire market by charging a price equal to the marginal cost of the second most efficient firm. This is exactly what happens in the case of $e = 1$: any non-zero trade cost is prohibitive, with

only the home firm selling to consumers at a price equal to the foreign firm's marginal cost of serving the home market: its marginal production cost plus the trade cost. The novel feature of the current model is that a similar outcome arises even when products are differentiated.

3.3 Welfare in the Nimzowitsch Region

Finally, consider the level of welfare in this region of potential though not actual competition from imports. Welfare in the absence of trade is consumer surplus plus home profits. This can be written as $W = Ax - \frac{1}{2}bx$. Totally differentiate this to get:

$$dW = (A - bx)dx. \quad (26)$$

When (20) is combined with (26), it is clear that welfare is falling in trade costs in the region $t^B \leq t \leq t^C$. Hence under price competition, unlike under quantity competition, trade liberalization starting from autarky initially raises welfare: the home firm is disciplined by the threat of trade, without any trade taking place. Home welfare under Bertrand competition is shown by the curve labeled W^B in Figure 3. This figure thus allows us to compare trade liberalization under Cournot and Bertrand competition. (The figure is drawn for an intermediate value of the substitution parameter, $e = 0.8$, in this case.) For $t < t^C$ welfare is higher under Bertrand than under Cournot competition.

4 General Demands

Consider now the case of general inverse demands: $p(x, y)$ for the home good and $p^*(y, x)$ for the foreign. We assume that the demand functions are twice differentiable and strictly decreasing in own price, $p_x < 0$ and $p_y^* < 0$. We also assume that they are symmetric, in the sense that their levels and first and second derivatives are equal when evaluated at the same

point:

$$\left. \begin{aligned} p(x_0, y_0) &= p^*(y_1, x_1) \\ p_i(x_0, y_0) &= p_j^*(y_1, x_1), \quad i, j = x, y, \quad i \neq j \\ p_{ij}(x_0, y_0) &= p_{ji}^*(y_1, x_1), \quad i, j = x, y \end{aligned} \right\} \text{ when: } x_0 = y_1 \text{ and } y_0 = x_1 \quad (27)$$

Let $e(x, y) = p_y/p_x$ and $e^*(x, y) = p_x^*/p_y^*$ be inverse measures of product differentiation for the home and foreign demand functions respectively. Assume that the inverse demand system can be inverted to get the direct demands:

$$x = x(p, p^*) \quad \text{and} \quad y = y(p^*, p) \quad (28)$$

For this inversion to be possible, we require that own-price effects in demand dominate cross-price effects, in the sense that $ee^* < 1$.⁹

The home firm's profit function can be written as a function of either quantities or prices: $\pi(x, y) = [p(x, y) - c]x$ or $\pi(p, p^*) = (p - c)x(p, p^*)$. It will be convenient to assume two additional mild restrictions on the first of these profit functions, which in turn places restrictions on the inverse demand function the firm faces:

Assumption 1. $\pi_{xx} = p_x + xp_{xx} < 0$ holds everywhere.

Assumption 2. $\pi_{xx} + \pi_{xy} = 2p_x + p_y + x(p_{xx} + p_{xy}) < 0$ holds in a symmetric equilibrium (where $x = y$).

Assumption 1 ensures that the home firm's marginal revenue is always downward-sloping, and (since $p_x < 0$) it implies that the demand function can be convex but not too much so. As for Assumption 2, it implies that home marginal revenue falls following an equal increase in the outputs of both goods, so goods can be complements in demand ($p_{xy} > 0$) but not too much so. This is weaker than assuming strategic substitutes, which implies that $\pi_{xy} < 0$.

⁹Inverting the inverse demand functions to yield well-behaved direct demand functions requires that $p_x p_y^* - p_x^* p_y > 0$, which from the definitions of e and e^* implies that $(1 - ee^*)p_x p_y^* > 0$.

Turning to the production side, we continue to assume identical constant marginal production costs c and per-unit trade costs t . Hence, under monopoly, the home firm has first-order condition:

$$\pi_x = p(x^M, 0) + x^M p_x(x^M, 0) - c = 0 \quad (29)$$

where x^M is the monopoly output, and we write $p^M = p(x^M, 0)$ for the monopoly price.

Under Cournot oligopoly the home firm has first-order condition:

$$\pi_x = p(x, y) + x p_x(x, y) - c = 0 \quad (30)$$

while that for the foreign firm is:

$$\pi_y^* = p^*(y, x) + y p_y^*(y, x) - c - t = 0 \quad (31)$$

Under Bertrand oligopoly, the home firm has first-order condition:

$$\pi_p = (p - c) x_p(p, p^*) + x = 0 \quad (32)$$

with a similar expression for the foreign firm. To facilitate comparison with the corresponding Cournot first-order condition (30), it is very helpful to rewrite this as an output first-order condition. To do this, we can use footnote 9 to write $x_p = \frac{p_y^*}{p_x p_y^* - p_y p_x^*} = \frac{1}{p_x} \frac{1}{1 - e e^*}$. Substituting into (32) yields the home firm's quantity first-order condition under Bertrand competition:¹⁰

$$p(x, y) + x p_x(1 - e e^*) - c = 0 \quad (33)$$

¹⁰The same outcome would be obtained if we assumed that the home firm has "Bertrand conjectural variations": i.e., it conjectures that, if it increases its output, the foreign firm's output will change so as to exactly offset the home firm's impact on the foreign price. In effect, the home firm conjectures that its own quantity affects its own price as follows: $[dp/dx]_{\bar{p}^*} = p_x + p_y [dy/dx]_{\bar{p}^*} = p_x - p_y (p_x^*/p_y^*)$. Conjectural variations are a very unsatisfactory way of modeling strategic interaction in general, but as here they can provide a useful way of comparing Cournot and Bertrand behavior on a common basis. See, for example, Dixit (1986).

Similar derivations for the foreign firm yield:

$$p^*(y, x) + yp_y^*(1 - ee^*) - c - t = 0 \quad (34)$$

Armed with these first-order conditions in comparable form, we can now turn to consider their implications for the robustness of the results derived in previous sections for linear demands.

4.1 Zero Trade Costs

At $t = 0$ the sales of the home and foreign firm are equal. Thus under Cournot, $x^C = y^C$, and under Bertrand, $x^B = y^B$, where C and B denote the Cournot and Bertrand equilibria respectively. We can write the first-order condition for the home and foreign firms under Cournot at $t = 0$ as $\pi_x(q^C) = \pi_y^*(q^C) = 0$, where $q = x = y$ is the symmetric output which is at the level q^C in the zero trade cost Cournot equilibrium. From (34), the first-order condition under Bertrand can be written as: $\pi_x(q^B) + \beta(q^B) = 0$, where q^B represents the symmetric output at the zero trade cost Bertrand equilibrium, and where $\beta(q) = -xp_x ee^* = -yp_y^* ee^*$ is positive at any $q > 0$. Since $\beta(q)$ is positive everywhere we have: $\pi_x(q^B) < 0$ but $\pi_x(q^C) = 0$ and since $\pi_x(q)$ falls in q from Assumption 2 it follows that: $q^B > q^C$. Therefore, at zero trade costs, the volume of trade is higher under Bertrand than Cournot competition. It must also be the case that prices at free trade are lower under Bertrand than Cournot.

4.2 Prohibitive Trade Costs

We can write the Cournot equilibrium output pair as a function of the trade cost as $\{x^C(t), y^C(t)\}$. As the trade cost increases the Cournot equilibrium output of the foreign firm falls and at the threshold prohibitive trade cost, $\hat{t}^C$, it reaches zero. Let $\hat{x}^C = x^C(\hat{t}^C)$ and $\hat{y}^C = y^C(\hat{t}^C) = 0$ be the Cournot equilibrium outputs at $\hat{t}^C$. At the prohibitive trade cost the Cournot first-order conditions given in (30) and (31) can be written as: $p(\hat{x}^C, 0) + \hat{x}^C p_x(\hat{x}^C, 0) - c = 0$

and $p^*(0, \hat{x}^C) - c - \hat{t}^C = 0$ for the home and foreign firm respectively. At the prohibitive trade cost the home first-order condition takes exactly the same form as under monopoly, so, from (29), we have $\hat{x}^C = x^M$. Making use of this allows us to write: $\hat{t}^C = p^*(0, x^M) - c$. This means that, if the trade cost were at $t = \hat{t}^C$ and the home firm were to behave as an unthreatened monopolist, the foreign firm's trade-cost-inclusive price-cost margin would be zero and the foreign firm would be unable to export profitably.

Consider next the case of price competition. Let $\hat{t}^B$ be the threshold prohibitive trade cost under Bertrand. Write the Bertrand equilibrium outputs at $\hat{t}^B$ as: $\hat{x}^B = x^B(\hat{t}^B)$ and $\hat{y}^B = y^B(\hat{t}^B) = 0$. At $\hat{t}^B$ the first-order conditions under Bertrand given in (30) can be written as: $p(\hat{x}^B, 0) + \hat{x}^B p_x(\hat{x}^B, 0) - c + \beta(\hat{x}^B, 0) = 0$ for the home firm where $\beta = -xp_x e e^*$ and by $p^*(0, \hat{x}^B) - c - \hat{t}^B = 0$ for the foreign firm. Since $\beta > 0$ it follows that: $p(\hat{x}^B, 0) + \hat{x}^B p_x(\hat{x}^B, 0) - c < 0$. Making use of this and Assumption 1 we get: $\hat{x}^B > \hat{x}^C = x^M$. It then follows that $\hat{t}^B = \{p^*(0, \hat{x}^B) - c\} < \{p^*(0, \hat{x}^C) - c\} = \hat{t}^C$. The prohibitive trade cost under Bertrand is lower than that under Cournot but the output of the home firm is higher under Bertrand at the prohibitive trade cost than it is under Cournot. In fact the home output needs to be higher under Bertrand to make the lower trade cost a prohibitive one.

4.3 Comparing the Volume of Trade

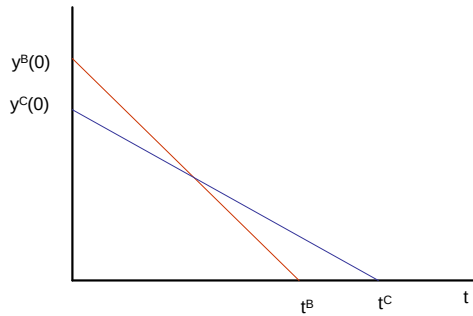


Figure 7: Volume of Trade under Quantity and Price Competition

So, at zero trade costs the volume of trade is higher under Bertrand than Cournot competition. However, the prohibitive trade cost under Bertrand is lower than that of Cournot.

So, provided only that the volume of trade is continuous in trade costs, the outcome is qualitatively as shown in Figure 7.

4.4 Welfare

We now consider welfare under general demands and discuss the extent to which the results obtained in the previous sections are dependent on the assumption of linear demands. We show that many of the results are qualitatively robust.

Welfare is given by expression (4) above. The change in consumer surplus is: $d\chi = -(x dp + y dp^*)$ and the change in the profits of the home firm is: $d\Pi = (p - c)dx + x dp + (p^* - c - t)dx^* + x^* dp^*$. Combining these and making use of the symmetry of the model which implies that home exports x^* equal home imports y , we can write the change in welfare as:

$$dW = d\chi + d\Pi = (p - c)dx + (p^* - c - t)dx^* \quad (35)$$

We first consider Cournot competition. To obtain the effect of trade costs on outputs we must differentiate (30) and (31). This yields: $dx^C/dt = -(p_y + xp_{xy})/\Delta$ and $dx^{*C}/dt = dy^C/dt = (2p_x + xp_{xx})/\Delta$. Here $\Delta \equiv \pi_{xx}\pi_{yy}^* - \pi_{xy}\pi_{yx}^*$, which must be positive to ensure that the Cournot equilibrium is stable. Hence an increase in the trade cost reduces the volume of trade ($dy/dt < 0$) and it also raises domestic sales if the goods are strategic substitutes ($p_y + xp_{xy} < 0$). Combining these results at $t = 0$ we can deduce the effect of trade costs on welfare:

$$\left. \frac{dW^C}{dt} \right|_{t=0} = (p - c) \left(\frac{dx^C}{dt} + \frac{dy^C}{dt} \right) = \frac{p - c}{\Delta} \{ (2 - e)p_x + x(p_{xx} - p_{xy}) \} < 0 \quad (36)$$

So at $t = 0$ an increase in trade costs reduces welfare. The sign of (36) follows from stability ($\Delta > 0$) and from Assumption 2.¹¹ As for welfare, at the prohibitive trade cost $\hat{t}^C$, we have

¹¹Due to symmetry at $t = 0$, it follows that $\Delta = (\pi_{xx} + \pi_{xy})(\pi_{xx} - \pi_{xy})$. Since this is positive from stability, and since $\pi_{xx} + \pi_{xy}$ is negative from Assumption 2, it follows that $\pi_{xx} - \pi_{xy} = (2 - e)p_x + x(p_{xx} - p_{xy})$ is also negative.

$y = x^* = 0$ which yields:

$$\left. \frac{dW^C}{dt} \right|_{t=\hat{t}^C} = (p - c) \frac{dx^C}{dt} = -\frac{p - c}{\Delta} (p_y + x p_{xy}) \quad (37)$$

Under quantity competition a small fall in the trade cost from the prohibitive level, $\hat{t}^C$, reduces welfare if outputs are strategic substitutes.

Next we consider price competition. First we examine the effect of trade costs on prices. From (32) and the corresponding expression for the foreign firm we obtain: $dp^C/dt = -\left(y_{p^*}/\tilde{\Delta}\right) \pi_{pp^*}$ which is positive if and only if prices are strategic complements; and $dp^{*C}/dt = \left(y_{p^*}/\tilde{\Delta}\right) \pi_{pp} > 0$. Here $\tilde{\Delta} \equiv \pi_{pp} \pi_{p^*p^*} - \pi_{pp^*} \pi_{p^*p}$ which must be positive from the stability of the Bertrand equilibrium. The price derivatives can be combined with the derivatives of (24) to obtain the effects of the trade costs on outputs: $dx^B/dt = (y_{p^*}/\tilde{\Delta}) (x_{p^*} \pi_{pp} - x_p \pi_{pp^*})$ and $dx^{*B}/dt = dy^B/dt = (y_{p^*}/\tilde{\Delta}) (y_{p^*} \pi_{pp} - y_p \pi_{pp^*})$. An increase in the trade cost reduces the volume of trade ($dy/dt < 0$) and it also raises domestic sales if $x_{p^*} \pi_{pp} - x_p \pi_{pp^*} < 0$.¹² At $t = 0$ we can make use of the symmetry of the model to obtain:

$$\left. \frac{dW^B}{dt} \right|_{t=0} = (p - c) \left(\frac{dx^B}{dt} + \frac{dy^B}{dt} \right) = \frac{p - c}{\pi_{pp} + \pi_{pp^*}} (x_p + x_{p^*}) y_{p^*} < 0. \quad (38)$$

Hence a small increase in trade costs starting at zero reduces welfare. At $t = \hat{t}^B$, the volume of trade is zero and the effect of a local change in the trade cost is:

$$\left. \frac{dW^B}{dt} \right|_{t=\hat{t}^B, -} = (p - c) \frac{dx^B}{dt} = \frac{p - c}{\tilde{\Delta}} (x_{p^*} \pi_{pp} - x_p \pi_{pp^*}) y_{p^*} \quad (39)$$

which is positive if $x_p \pi_{pp^*} - x_{p^*} \pi_{pp} > 0$. Note that this is the left-hand derivative at $t = \hat{t}^B$.

Between $\hat{t}^B$ and $\hat{t}^C$, the equilibrium is in the Nimzowitsch region and occurs along the $y = 0$ locus. Let $\hat{t}^B \leq \bar{t} \leq \hat{t}^C$. Then with $y = 0$ we have $p^*(x, 0) - c - \bar{t} = 0$. For this condition to be satisfied, x must increase in $\bar{t}$ for $\hat{t}^B \leq \bar{t} \leq \hat{t}^C$ and so in the Nimzowitsch

¹²This is guaranteed to be negative (and hence dx/dt is guaranteed to be positive) in the linear case. It must also be negative if prices are strategic substitutes.

region an increase in the trade cost reduces welfare by reducing home sales. This allows us to sign the following derivatives:

$$\left. \frac{dW^B}{dt} \right|_{t=\hat{t}^B,+} < 0 \quad \left. \frac{dW^B}{dt} \right|_{t=\hat{t}^C} < 0 \quad (40)$$

(By contrast with (39), the first term is the left-hand derivative at $t = \hat{t}^B$.) Starting at $t = \hat{t}^C$, a small reduction in trade costs leads to trade under quantity competition but only the threat of trade under price competition. However, from (37), it reduces welfare under Cournot competition provided that outputs are strategic substitutes, whereas it increases welfare under price competition. This confirms that the qualitative features of welfare as a function of trade costs are the same with general demands as in the linear case.

Finally, we can ask, is there always a Nimzowitsch Region? The answer is yes except when consumers are willing to demand some of the imported good no matter how high its price. This requires two distinct conditions. First, demand cannot have a choke price as in the case of linear demands, or of many of the other demand functions used in applied economics: consumers must have an infinite reservation price for the first item, as in the case of CES or logistic demand.¹³ Second, there must be no fixed cost of exporting by the foreign firm. If both of these assumptions hold, then there is no finite tariff high enough to cut off trade, so the assumption that $p^* = c + t$ at $y = 0$ does not hold.

5 Conclusion

In this paper we have compared trade liberalization under Cournot and Bertrand in a unified reciprocal markets framework and shown that the critical level of trade costs below which the possibility of trade affects the domestic firms' behavior is the same under Cournot and Bertrand competition. However, for any trade cost below this critical level welfare is higher

¹³Requiring that demand must have a choke price for the Nimzowitsch Region to exist is in line with Vives (1985), following Friedman (1977).

under Bertrand than under Cournot competition. The pro-competitive effects of trade are stronger under Bertrand competition despite the fact that for trade costs below but close to the critical level the volume of trade is higher under Cournot competition.

References

- ANDERSON, S. P., M.-P. DONSIMONI, AND J. J. GABSZEWICZ (1989): “Is International Trade Profitable to Oligopolistic Industries?,” *International Economic Review*, 30(4), 725–733.
- ARKOLAKIS, C., A. COSTINOT, D. DONALDSON, AND A. RODRÍGUEZ-CLARE (2012): “The Elusive Pro-Competitive Effects of Trade,” Working Paper, MIT, revised July 2014.
- ARKOLAKIS, C., A. COSTINOT, AND A. RODRÍGUEZ-CLARE (2012): “New Trade Models, Same Old Gains?,” *American Economic Review*, 102(1), 94–130.
- BERNHOFEN, D. M. (1999): “Intra-Industry Trade and Strategic Interaction: Theory and Evidence,” *Journal of International Economics*, 47(1), 225–244.
- BRANDER, J., AND P. KRUGMAN (1983): “A ‘Reciprocal Dumping’ Model of International Trade,” *Journal of International Economics*, 15(3-4), 313–321.
- BRANDER, J. A. (1981): “Intra-Industry Trade in Identical Commodities,” *Journal of International Economics*, 11(1), 1–14.
- BRANDER, J. A., AND B. J. SPENCER (2015): “Intra-Industry Trade with Bertrand and Cournot Oligopoly: The Role of Endogenous Horizontal Product Differentiation,” forthcoming in *Research in Economics*.
- CLARKE, R., AND D. R. COLLIE (2003): “Product Differentiation and the Gains from Trade under Bertrand Duopoly,” *Canadian Journal of Economics*, 36(3), 658–673.
- COLLIE, D. R., AND V. P. M. LE (2015): “Product Differentiation, the Volume of Trade and Profits under Cournot and Bertrand Duopoly,” *International Journal of the Economics of Business*, 22(1), 73–86.
- DIXIT, A. (1986): “Comparative Statics for Oligopoly,” *International Economic Review*, pp. 107–122.

- FREUND, C., AND M. D. PIEROLA (2015): “Export Superstars,” forthcoming in *Review of Economics and Statistics*.
- FRIEDMAN, J. W. (1977): *Oligopoly and the Theory of Games*. Amsterdam: North-Holland.
- HARRIS, R. (1985): “Why Voluntary Export Restraints Are ‘Voluntary’,” *Canadian Journal of Economics*, 18(4), 799–809.
- KRISHNA, K. (1989): “Trade Restrictions as Facilitating Practices,” *Journal of International Economics*, 26(34), 251–270.
- LEAHY, D., AND J. P. NEARY (2011): “Oligopoly and Trade,” in D. Bernhofen, R. Falvey, D. Greenaway and U. Kreickemeier (eds.): *Palgrave Handbook of International Trade*, Basingstoke: Palgrave Macmillan, 197-235.
- MAYER, T., AND G. I. OTTAVIANO (2008): “The Happy Few: The Internationalisation of European Firms,” *Intereconomics: Review of European Economic Policy*, 43(3), 135–148.
- MELITZ, M. J., AND S. J. REDDING (2015): “New Trade Models, New Welfare Implications,” *American Economic Review*, 105(3), 1105–46.
- VIVES, X. (1985): “On the Efficiency of Bertrand and Cournot Equilibria with Product Differentiation,” *Journal of Economic Theory*, 36(1), 166–175.