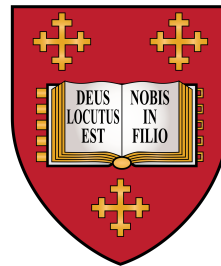


A measurement of charm mixing using
 $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays at LHCb and
the performance of a TORCH prototype

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I dedicate this thesis to Mum, Dad and Alex;
thank you for your endless love and support.

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Abstract

A measurement of the charm-mixing parameters is made using promptly produced $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays, exploiting proton-proton collision data collected by LHCb during 2015–18 and corresponding to an integrated luminosity of 5.9 fb^{-1} . The effective mixing parameters are interpreted in terms of the hadronic parameters of the D^0 decay, which are valuable for studies of CP violation. This is the first measurement of $D^* \rightarrow D^0 \pi$, $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays in bins of phase-space at a hadron collider.

A prototype Time Of internally Reflected CHerenkov light (TORCH) detector, designed to identify pions, kaons and protons in the momentum range $2\text{--}20 \text{ GeV}/c$, is employed in beam tests at the CERN PS. Measurements of the collected photon yields and single-photon time resolutions are performed for different particle entry points on the plate and compared to expectations. These studies show that the performance of the TORCH prototype approaches the design goals for the full-scale detector.

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List of Abbreviations

BDT		Boosted Decision Tree
CF		Cabibbo favoured
DCS		Doubly Cabibbo suppressed
DTF		Decay Tree Fitter
ECAL		Electronic calorimeter
GEM		Gas Electron Multiplier
GPD		General Purpose Detector
HCAL		Hadronic calorimeter
IP		Interaction point
IT		Inner Tracker
LHC		Large Hadron Collider
MC		Monte Carlo simulation
MCP-PMT		Multi-Channel Plate Photo-Multiplier Tube
MWPC		Multi Wire Proportional Chamber
OT		Outer Tracker
PGun		Particle Gun MC
PV		Primary Vertex
RICH		Ring-Imaging CHerenkov detector
RS		Right sign
SM		Standard Model
SPD		Scintillating Pad Detector
ToF		Time of flight
TT		Tracker Turicensis
TORCH		Time Of internally Reflected CHerenkov light
WS		Wrong sign
VELO		Vertex Locator

1 | Introduction

Flavour studies are a key component of the search for physics beyond the Standard Model (SM), which has so far withstood all the tests performed to break it. Precision measurements of flavour transitions test whether SM particles behave as expected, or if there are external fields that can be explained by beyond SM theories. Neutral meson mixing is a subgroup of precision measurements; in particular the mixing of charm mesons is of interest as they are the only meson that undergoes mixing due to an up-type quark. Additionally, the charm-mixing parameters are hard to predict using current theoretical techniques; in the charm sector the experimentally measured charm-mixing parameters are used to confirm predictions of the Standard Model.

Charm-meson decays are equally interesting; due to the large charm quark mass there are many possible final states. In particular, multi-body decays (to three or more particles) have a rich resonant sub-structure, and the $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ decay has one of the largest branching fractions of the D^0 meson. Experimentally, good vertexing and particle identification are required to reconstruct specific D^0 decays, along with efficient tracking to identify tracks of interest in the busy environment created by hadron colliders. With increasing numbers of particles produced in the decay of charm and beauty hadrons, the momenta of the final-state particles decreases and thus to maintain high signal purity the particle-identification performance must extend to low momenta.

The physics potential of LHCb is limited by the current lack of positive particle identification for pions, kaons and protons below 20 GeV/c. TORCH, a proposed sub-detector for a future upgrade of LHCb, will provide discrimination for these hadrons in the range 2–20 GeV/c, aiding the reconstruction of multi-body decays.

This thesis details two independent avenues of research:

- the measurement of the effective charm-mixing parameters in $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$

decays using data collected by the LHCb experiment in the years 2015–2018, and

- the timing resolution and photo-counting efficiency measured in test-beam studies of a TORCH detector prototype.

1.1 Thesis Outline

The remainder of this thesis is organised as follows:

Chapter 2 — The Standard Model of Particle Physics is introduced, and details of the theory of charm-meson mixing and $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays are given.

Chapter 3 — The CERN accelerator complex and the LHCb detector is described.

Chapter 4 — The method used to obtain a sample of $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ signal candidates is presented, with discussion about potential backgrounds.

Chapter 5 — A measurement of the signal yields is made, with considerations to the differences in the detection efficiency of $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ and $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decays.

Chapter 6 — The effective charm-mixing parameters are measured, and uncertainties assigned. The interpretation of these parameters in terms of the hadronic parameters is shown.

Chapter 7 — The TORCH detector is described, along with the prototype and testbeam facilities, and analysis of data collected is performed.

Chapter 8 — The conclusions and outlook are given.

1.2 Contributions of the author

I developed the analysis presented in Chapters 4, 5, and 6, except for the basis of the mixing fit code which was provided by a colleague and the interpretation of the mixing parameters (presented in Sec. 6.4.1) which was performed by another member of the analysis team.

The TORCH analysis presented in Chapter 7 builds upon work previously undertaken, and data previously collected, by members of the TORCH collaboration, however all analysis results are produced by the author. The work resulted in a paper published in Nuclear Instrumental Methods A (Ref. [1]) where I was the lead author.

Beyond the data analysis, I undertook shift work as Data Manager throughout the LHC Run 3 start-up, and during the 2022 TORCH testbeam campaign. I also actively participated in an LHCb outreach programme, *I'm a Scientist: CERN Zone*, inspiring high school children, and volunteered in outreach activities organised by Oxford Physics.

2 | Theoretical background

2.1 The Standard Model of Particle Physics

The Standard Model of Particle Physics (SM) describes the known fundamental particles and forces, and the interactions between them. The fundamental forces are the strong, electromagnetic and weak interactions, which couple differently to different particles. Gravity is an additional fundamental force that is not described by the SM. The SM bosons, which are integer-spin excitations of the fields, consist of the $W^\pm$ and Z particles which are massive, and the massless gluon and photon. In addition, the Higgs boson is an excitation of the Higgs field which gives the $W^\pm$ and Z their masses [2]. The fermions, with half-integer spin, make up the rest of the SM. There are three generations of each fermion group. Each group consists of two quarks, one with charge of $\frac{+2}{3}e$ and one with $\frac{-1}{3}e$ charge, and two leptons, one electron-like with charge $-1e$ and a neutrino, which is electrically neutral. Each generation has different masses. In addition the anti-particles of all fermions exist, which have opposite sign for all additive quantum numbers.

The strong interaction couples to quarks and gluons, which have a colour charge that can take one of three states (commonly denoted as red, green and blue). Observable particles are colour-neutral (either colour-anticolour or red-green-blue). The electromagnetic force couples photons to any particle with non-zero electric charge. The weak interaction couples to quarks and leptons, and $W^\pm$, Z and Higgs bosons, and is the only interaction that involves the Z boson and neutrinos. At high energies the electromagnetic and weak interactions can be unified into a single electroweak interaction framework. Figure 2.1 gives an overview of the SM particles and interactions.

Hadrons are particles that consist of more than one quark, of which there are two classes: mesons and baryons. Both have integer electric charges. Mesons

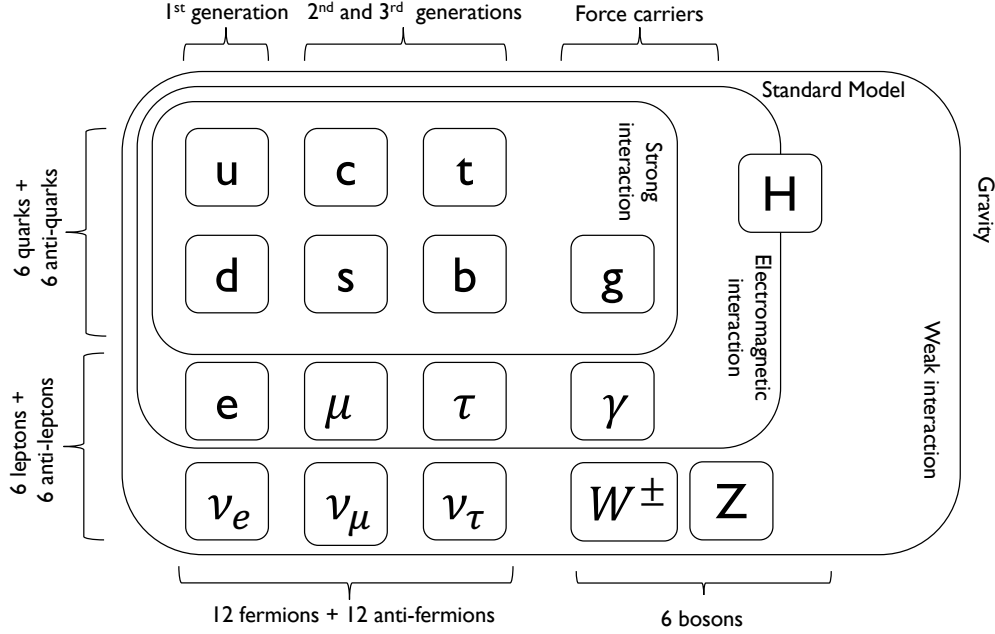


Figure 2.1: Standard Model particles and their interactions.

are bosons and most commonly consist of a quark–anti-quark pair (such as a kaon or a pion), in a colour-anticolour state. Baryons are fermions and commonly consists of three quarks (such as protons or neutrons). Recent discoveries of tetraquark [3, 4] and pentaquark [5] states have extended the hadron sector and are currently being studied at LHCb.

2.1.1 Standard Model discrete symmetries

There are three operators that can be applied to particles to indicate properties of the SM:

1. Charge-conjugation (C) transforms a particle to its anti-particle by changing the sign of all the additive quantum numbers, including the electrical charge, lepton and quark-flavour quantum numbers, and baryon number.
2. Parity (P) inverts a particle's spatial coordinates e.g. a particle at $(t, \mathbf{x})$ is transformed to $(t, -\mathbf{x})$.

3. Time reversal (T) reverses a particles temporal coordinates e.g. a particle at $(t, \mathbf{x})$ is transformed to $(-t, \mathbf{x})$.

These transformations are discrete; each operator has eigenvalues of ± 1 .

The electromagnetic and strong interactions are invariant under each of these operators. However, the weak interaction couples only to right-handed anti-fermions and left-handed fermions. Thus C and P are maximally violated by the weak interaction. However, if CP is violated then this implies T violation also, as any Lorentz-invariant quantum field theory must satisfy CPT .

2.1.2 Cabibbo-Kobayashi-Maskawa matrix

The weak interaction couples to weak eigenstates of the quarks, which are not the same as the mass eigenstates of the down-type quarks, and allows quark type to change. The down weak eigenstates (d', s', b') are a linear superposition of mass eigenstates (d, s, b) , described by the Cabibbo-Kobayashi-Maskawa (CKM) matrix [6, 7] written as

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \quad (2.1)$$

In the Standard Model the CKM matrix, V_{CKM} , is unitary as the mass eigenstates form an orthonormal basis. It can be parametrised in terms of 3 angles $(\theta_{12}, \theta_{13}, \theta_{23})$, and one phase of a complex number δ , written as:

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (2.2)$$

where $c_{ij}, s_{ij} = \cos \theta_{ij}, \sin \theta_{ij}$ [8]. The magnitude of the complex term dic-

tates the amount of CP violation within the Standard Model, as the phase will change sign when the Hermitian conjugate of the CKM matrix is applied to anti-particles.

The four free parameters $(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$ must be determined experimentally, and the mixing angles (θ_{ij}) are found to be small. The CP -violating phase is larger. This means that the CKM matrix is approximately diagonal (promoting quark transitions within the same generation), with small off-diagonal elements (the quark transitions across generations). The quark transitions within the same generation are called Cabibbo-favoured transitions whilst those across generations are known as Cabibbo-suppressed.

Furthermore, the unitarity of the CKM matrix defines a set of criteria, one of which is

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad (2.3)$$

which forms a triangle in the complex plane, shown in Fig. 2.2. The three angles of this triangle can be written as:

$$\alpha = \arg \left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right), \quad (2.4)$$

$$\beta = \arg \left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right), \quad (2.5)$$

$$\gamma = \arg \left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right). \quad (2.6)$$

Independent measurements of all three angles provide a test of the Standard Model. To measure γ directly requires the suppressed $b \rightarrow c$ and $b \rightarrow u$ transitions to be studied. These measurements are theoretically clean, as no loop-level processes contribute but are limited in their precision by the size of current data samples. Therefore, performing measurements of γ using many different final states is desired. The use of $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays in a measurement of γ will be discussed in Sec. 2.4.4. Indirect measurements of γ can be performed us-

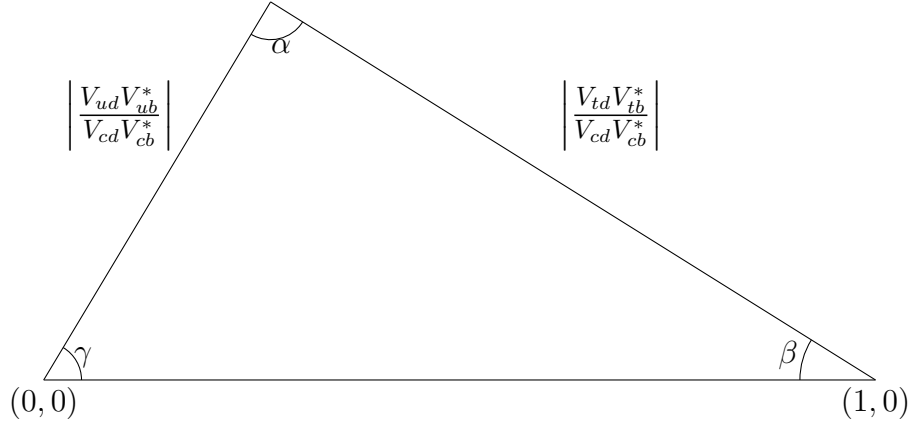


Figure 2.2: CKM Unitary Triangle.

ing global fits to the unitarity triangle, assuming that it is closed. Measurements of α, β and the lengths of the triangle sides are performed to constrain the value of γ . The decays used to measure α and β can involve amplitudes that have loop processes and thus are sensitive to high-mass Beyond-SM particles. Thus a difference in the measurement of γ from direct and indirect processes would indicate New Physics contributions. LHCb has made many measurements of γ using a variety of B^0 and $B^\pm$ decays. A recent LHCb combination, with inputs from the charm sector, yields $\gamma = (63.8^{+3.5}_{-3.7})^\circ$ [9] whilst indirect measurements yield $\gamma = (65.5^{+1.1}_{-2.7})^\circ$ [10], in good current agreement. The status in 2021 of direct and indirect γ measurements, via the CKM Unitary Triangle, is presented in Fig. 2.3.

2.2 Meson mixing

Neutral meson mixing is a phenomenon that occurs due to the fact that the mass eigenstates of the meson are linear combinations of the flavour eigenstates. The Standard Model (SM) does not permit flavour-changing neutral currents at tree-level, and thus studying these loop-level processes can allow for the probing of high mass New Physics particles that may contribute to the mixing amplitude.

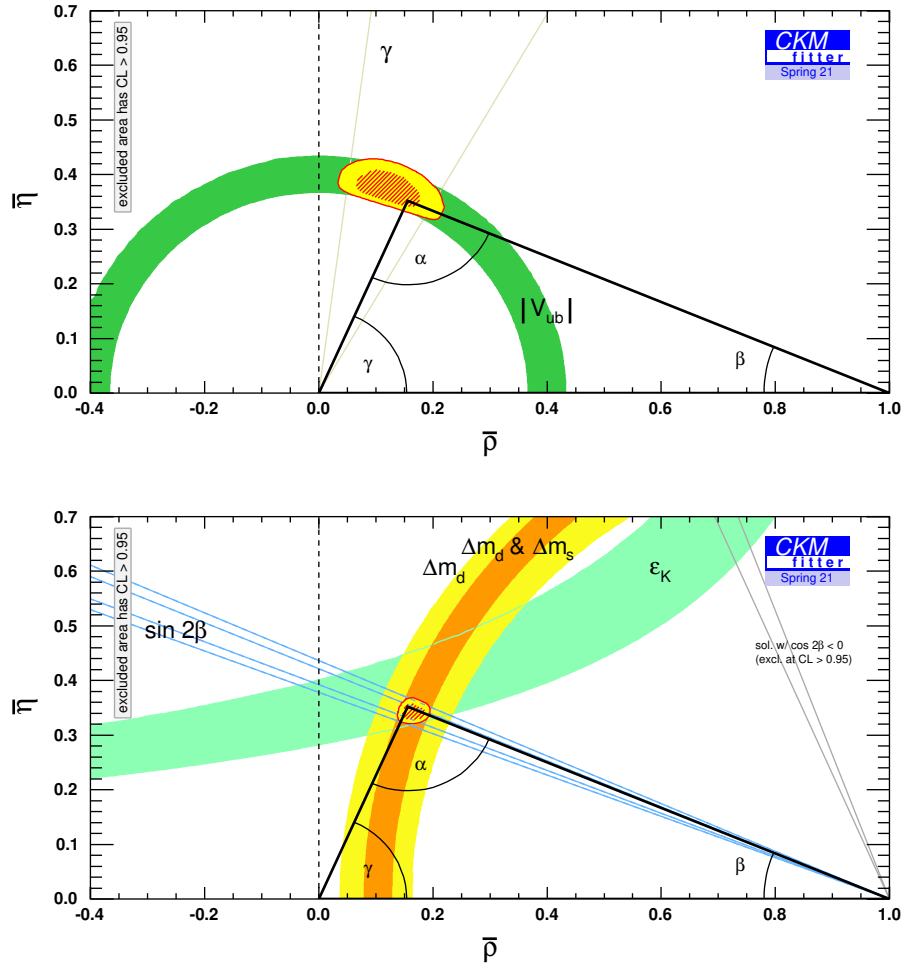
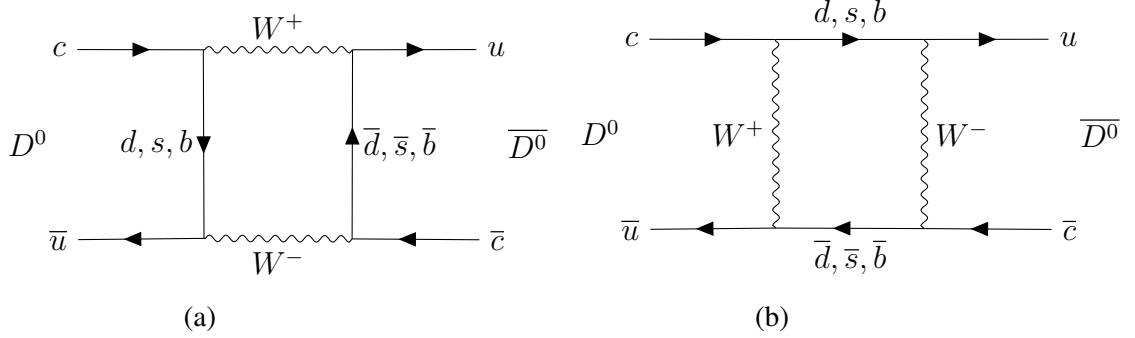
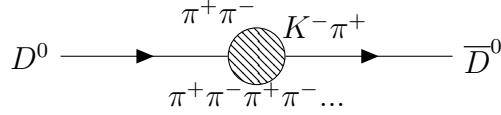


Figure 2.3: Status of (top) direct and (bottom) indirect γ measurements in 2021, from [10]. The direct measurements involve tree-level decays, whilst the indirect measurements involve loop processes.

Figure 2.4: Short distance D^0 mixing.Figure 2.5: Long distance D^0 mixing.

Mixing of neutral kaons and B -mesons has been previously observed and is well understood [11, 12].

Short-distance SM charm mixing through box diagrams such as Fig. 2.4 (with $\Delta C = 2$) is suppressed relative to the strange and beauty sectors due to the Glashow-Iliopoulos-Maiani (GIM) mechanism [13] for s and d quarks and the CKM factor for b quarks. Theoretical predictions of the contributions to charm mixing via short-distance effects fall 3–5 orders of magnitude below experimental evidence [14], but New Physics effects could enhance this amplitude. Long-distance contributions to charm mixing (involving any resonance of both D^0 and $\bar{D}^0$ as shown in Fig. 2.5) are expected to dominate but are hard to estimate as they are nonperturbative. The long-distance contributions involve two $\Delta C = 1$ transitions, and thus contribute to a difference in the decay rates of meson and anti-meson. Both mechanisms contribute to the mass difference of meson and anti-meson.

2.2.1 GIM mechanism

The GIM mechanism explains the behaviour of flavour-changing neutral currents (FCNCs). The theory was developed in 1970 to explain the low rate of $K_L^0 \rightarrow \mu^+ \mu^-$ decays (which were not yet observed). At the time, the quark model consisted only of u , d and s . The decay involves an $s \rightarrow d$ quark transition which cannot be mediated by the $W^\pm$ bosons in a tree-level process, the lowest-order process is a loop involving two W bosons. As shown in Fig. 2.6a, the amplitude of the process involving only a u quark in the loop is

$$\mathcal{A}^u(K_L^0 \rightarrow \mu^+ \mu^-) \propto \sin \theta_c \cos \theta_c, \quad (2.7)$$

where θ_c is the Cabibbo angle, which is not vanishing. Glashow, Iliopoulos and Maiani proposed a solution which required a new quark to exist, the charm quark, which would have a charge of $+\frac{2}{3}e$ like the u quark. It would couple to d and s in an orthogonal combination to that of the u , thus contributing an amplitude

$$\mathcal{A}^c(K_L^0 \rightarrow \mu^+ \mu^-) \propto -\sin \theta_c \cos \theta_c, \quad (2.8)$$

depicted in Fig. 2.6b. This leads to the cancellation of the mass-independent parts of the two amplitudes, and the decay rate would vanish except that the mass of the u and c are not the same. The total decay rate is proportional to $m_c^2 - m_u^2$, which led to a prediction of the charm quark mass, $m_c \approx 3 \text{ GeV}/c^2$.

The same mechanism is used to explain the approximate cancellation of contributions from s and d quarks to the charm-meson box-mixing diagrams, and in this case the decay rate is even smaller due to the reduced mass difference between the two down-type quarks, $m_s^2 - m_d^2$.

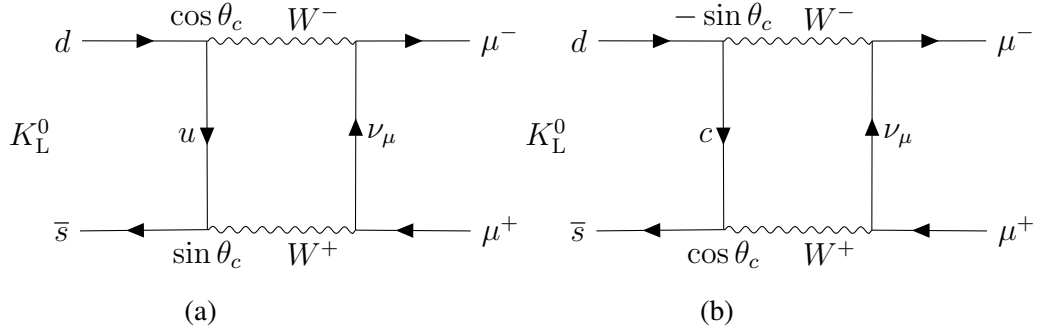


Figure 2.6: (a) $K_L^0 \rightarrow \mu^+ \mu^-$ decays before the c quark hypothesis invoked by the GIM mechanism. (b) c quark contributions to $K_L^0 \rightarrow \mu^+ \mu^-$ decays.

2.2.2 CKM factor

From knowledge of the flavour-changing charged currents described by the CKM matrix, the third contribution to the charm box-mixing diagram comes from b quarks in the loop that have a large coupling to the top quark and much smaller couplings to the u and c quarks [15]:

$$|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3},$$

$$|V_{cb}| = (40.8 \pm 1.4) \times 10^{-3}.$$

This leads to a CKM suppression relative to the diagram involving s quarks by

$$\frac{|V_{ub}V_{cb}^*|^2}{|V_{us}V_{cs}^*|^2} = \mathcal{O}(10^{-6}).$$

The mass of the b enhances the amplitude by $m_b^2/m_s^2 = \mathcal{O}(10^3)$, but it remains suppressed relative to the s and d box contributions.

2.2.3 Long-distance contributions to charm mixing

Two approaches are used to estimate long-distance contributions to charm mixing. The first is an inclusive approach using the Operator Product Expansion

(OPE) [14, 16]. This involves expanding the mixing parameters in terms of local operators in units of Λ_{QCD}/m_c (where Λ_{QCD} is the energy scale at which perturbative QCD breaks down), however $m_c \sim \Lambda_{QCD}$ so it is necessary to calculate many terms to converge. Quark-hadron duality is also assumed, requiring that many states must be summed over. This approach tends to result in SM predictions of charm mixing below experimental evidence. The second is an exclusive approach involving the sum over all intermediate hadronic states which could contribute to mixing [17]. Inputs from models or experimental data are used, but there are many possible decay modes and each of these must be known to high precision. The difference in the decay rates is generated via $SU(3)_F$ breaking when summing over states within the same $SU(3)_F$ multiplet. This approach gives results that are in line with experimental results.

The uncertainty in the SM prediction of charm mixing leaves the possibility for contributing amplitudes involving New Physics such as Z' or charged Higgs bosons [16]. But there are also large uncertainties in estimates of the contributions that these processes could make and so no conclusions have been reached yet.

2.2.4 Formalism

Charm mixing can be formalised in the following manner.

The mass eigenstates, which are superpositions of the flavour eigenstates, can be written as

$$|\psi\rangle = a(t)|D^0\rangle + b(t)|\bar{D}^0\rangle. \quad (2.9)$$

The effective Hamiltonian, $\mathcal{H}$, of the Schrödinger equation,

$$i\hbar \frac{\partial}{\partial t} \psi = \mathcal{H} \psi, \quad (2.10)$$

is given by

$$\begin{aligned}\mathcal{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} &= \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} \\ &= \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}\end{aligned}\quad (2.11)$$

with Hermitian mass matrix $\mathbf{M}$ and decay matrix $\mathbf{\Gamma}$. Thus, the eigenstates of the Hamiltonian are $|D_{1,2}\rangle$ with masses $m_{1,2}$ and widths $\Gamma_{1,2}$. The eigenvalues, describing the time evolution of the states, are $\lambda_{1,2} = m_{1,2} - \frac{i}{2}\Gamma_{1,2}$. The states evolve with time as

$$|D_{1,2}(t)\rangle = e^{-i(m_{1,2} - \frac{i}{2}\Gamma_{1,2})t}|D_{1,2}(0)\rangle. \quad (2.12)$$

The relationship between mass and flavour eigenstates can be simplified, using CPT symmetry, to

$$\begin{aligned}|D_1\rangle &= p|D^0\rangle + q|\bar{D}^0\rangle, \\ |D_2\rangle &= p|D^0\rangle - q|\bar{D}^0\rangle,\end{aligned}\quad (2.13)$$

where p and q are complex coefficients whose magnitudes sum to one and

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}. \quad (2.14)$$

Equation 2.13 can be rearranged to give $|D^0\rangle$ and $|\bar{D}^0\rangle$:

$$\begin{aligned}|D^0\rangle &= \frac{1}{2p}(|D_1\rangle + |D_2\rangle), \\ |\bar{D}^0\rangle &= \frac{1}{2q}(|D_1\rangle - |D_2\rangle).\end{aligned}\quad (2.15)$$

Substituting Eqn. 2.12 into Eqn. 2.15 and again using Eqn. 2.13 yields the time-dependence of the flavour eigenstates as

$$\begin{aligned} |D^0(t); D^0(t=0)\rangle &= g_+(t)|D^0\rangle + \frac{q}{p}g_-(t)|\bar{D}^0\rangle, \\ |\bar{D}^0(t); D^0(t=0)\rangle &= g_+(t)|\bar{D}^0\rangle + \frac{p}{q}g_-(t)|D^0\rangle, \end{aligned} \quad (2.16)$$

with

$$g_+(t) = e^{-iMt - \frac{1}{2}\Gamma t} \cos\left(\frac{1}{2}\Delta Mt - \frac{i}{4}\Delta\Gamma t\right), \quad (2.17)$$

$$g_-(t) = e^{-iMt - \frac{1}{2}\Gamma t} i \sin\left(\frac{1}{2}\Delta Mt - \frac{i}{4}\Delta\Gamma t\right). \quad (2.18)$$

Here $M = \frac{m_1+m_2}{2}$, $\Gamma = \frac{\Gamma_1+\Gamma_2}{2}$, $\Delta M = m_1 - m_2$ and $\Delta\Gamma = \Gamma_1 - \Gamma_2$.

In the limit of charge-parity (CP) symmetry, the charm-mixing parameters x and y are defined by the normalised mass and decay-rate difference of the energy eigenstates:

$$x = \frac{m_1 - m_2}{\Gamma} = \frac{\Delta M}{\Gamma}, \quad (2.19)$$

$$y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma} = \frac{\Delta\Gamma}{2\Gamma}. \quad (2.20)$$

The probability of mixing *e.g.* a D^0 to $\bar{D}^0$ occurs as

$$\begin{aligned} P(D^0(t) \rightarrow \bar{D}^0) &= |\langle \bar{D}^0 | D^0(t) \rangle|^2 \\ &= \left| \frac{q}{p} \right|^2 |g_-(t)|^2 \\ &= \frac{1}{2} \left| \frac{q}{p} \right|^2 e^{-\Gamma t} (\cosh(y\Gamma t) - \cos(x\Gamma t)). \end{aligned} \quad (2.21)$$

If either x or y is non-zero then D^0 – $\bar{D}^0$ mixing will occur.

Measurements of the charm-mixing parameters were made at BaBar [18],

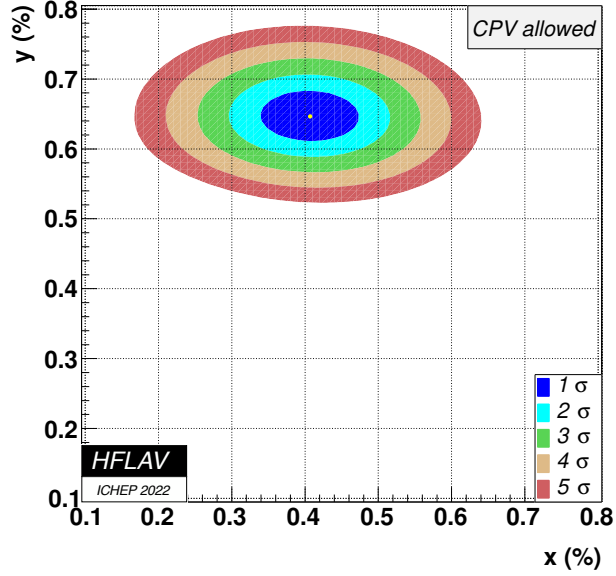


Figure 2.7: Current status of the charm-mixing parameters x and y . Taken from [25].

Belle [19] and CDF [20]. The average of results from these experiments ruled out no charm mixing at greater than 10σ , but no single measurement had a significance of greater than 5σ [21]. The first observation of $D^0-\bar{D}^0$ oscillations from a single measurement was reported by LHCb in 2012 and used $D^0 \rightarrow K^\pm \pi^\mp$ decays [22], and was later confirmed by CDF [23]. The mass difference of the charm mesons was only measured with a significance of beyond 5σ in a single measurement recently at LHCb, using $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays [24]. The current world averages of the mixing parameters are as follows [25]:

$$x = (0.407 \pm 0.044)\%,$$

$$y = (0.647 \pm 0.024)\%,$$

which are displayed in Fig. 2.7.

2.3 CP violation in charm

For completeness, the phenomenology of CP violation (CPV) in the charm sector will be briefly discussed. The small level of CPV in the charm sector allows us to treat D^0 and $\bar{D}^0$ decays to Cabibbo-favoured (or doubly Cabibbo-suppressed) charge-conjugate final states in the same manner. No measurement of the CPV parameters will be reported in this thesis.

The phenomenology of CPV, the difference in behaviour of particle and anti-particle, can be categorised into three classes. The first is CPV in decay, which occurs when the magnitude of the decay amplitudes of CP -conjugate processes are different:

$$\mathcal{A}_{D^0 \rightarrow f} \neq \mathcal{A}_{\bar{D}^0 \rightarrow \bar{f}}. \quad (2.22)$$

The second is CPV in mixing which arises when the probability of a meson oscillating to its anti-meson after time t is different to that of the anti-meson oscillating to the meson:

$$P(D^0(t) \rightarrow \bar{D}^0) \neq P(\bar{D}^0(t) \rightarrow D^0). \quad (2.23)$$

The third category is CPV in the interference of mixing and decay which can exist only for final states that can be accessed by the meson and anti-meson. It is present if the complex parameter λ_f has a non-zero imaginary part, with

$$\lambda_f = \frac{q \mathcal{A}_{\bar{D}^0 \rightarrow \bar{f}}}{p \mathcal{A}_{D^0 \rightarrow f}}. \quad (2.24)$$

CPV in the second and third categories would manifest itself in a measurement of $|q/p| \neq 1$ or $\phi = \arg(q/p) \neq 0$ (for p and q defined in Eqn. 2.13).

CPV in the charm sector is expected to be small as the CKM elements V_{cd} ,

V_{cs} , and V_{cb} are predominantly real, and because the GIM mechanism is very effective. For CPV to occur, at least two amplitudes must contribute to the decay rate. For singly Cabibbo-suppressed decays (proportional to *e.g.* $|V_{us}^* V_{cs}|^2$) such as $D^0 \rightarrow K^+ K^-$, both tree-level decays and single-loop (‘penguin’) decays contribute. CPV in decay was recently observed by the LHCb collaboration in $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$ decays and was found to be a 10^{-3} effect [26].

For Cabibbo-favoured and doubly Cabibbo-suppressed decays of neutral D^0 mesons (proportional to *e.g.* $|V_{ud}^* V_{cs}|^2$ and $|V_{cd}^* V_{cd}|^2$ respectively) the second amplitude occurs via the decay involving mixing which has a small amplitude. Thus, CPV in mixing, and in the interference of mixing and decay, is predicted in the SM to be smaller than current experimental uncertainty, and hence a measurement of non-zero CPV with existing datasets would be a sign of New Physics, contributing to the loop diagrams. The current world-average [25] of the CPV parameters in neutral charm-meson yields

$$|q/p| = 0.994_{-0.015}^{+0.016},$$

$$\phi = (-2.6 \pm 1.1)^\circ,$$

which are displayed in Fig. 2.8.

2.4 $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays

This section will focus on the differences between $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ and $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decays and why these decays are useful for measuring the charm-mixing parameters and angle γ of the CKM unitarity triangle.

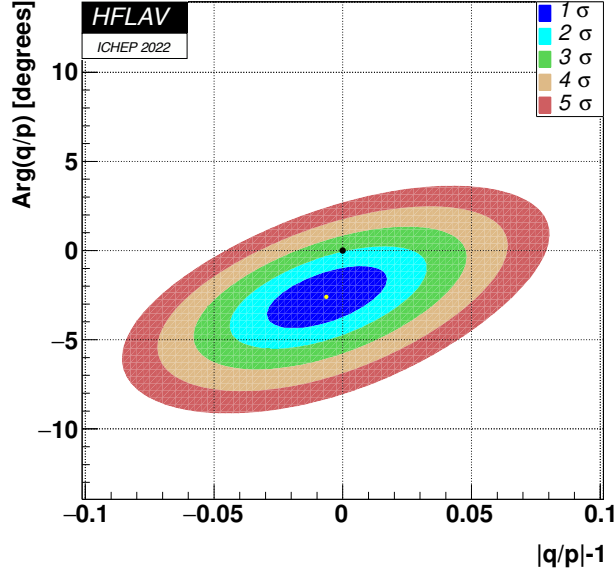
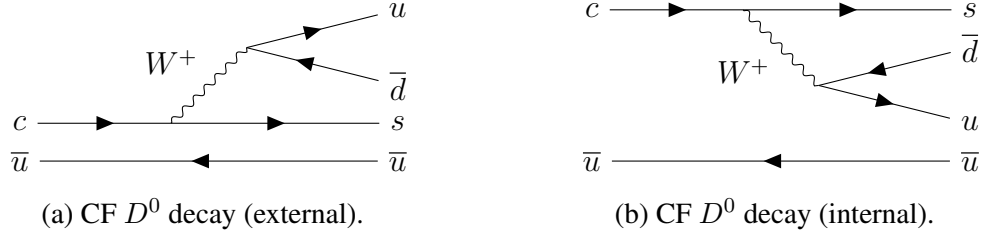


Figure 2.8: Current status of the charm CPV parameters $|q/p|$ and $\phi = \arg(q/p)$. Taken from [25].

2.4.1 Right-sign decays

The $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ decay is dominated by Cabibbo-favoured (CF) amplitudes, involving quark transitions ($c \rightarrow s$ and $d \rightarrow u$) that are within the same generation. The decay has a large branching fraction, $\text{BF}(D^0 \rightarrow K^- \pi^+ \pi^- \pi^+) = (8.22 \pm 0.14)\%$ [15], which is roughly twice as large as that of the two body $D^0 \rightarrow K^- \pi^+$ decay. This decay is referred to as the ‘right-sign’ (RS) decay because it is the dominant decay. Often in experiments the decay chain $D^{*+} \rightarrow D^0 \pi^+$ is reconstructed (with charge conjugate decay $D^{*-} \rightarrow \bar{D}^0 \pi^-$) and thus the kaon and tagging pion (from the D^*) have opposite charges.

The decay takes place through tree-level processes; example Feynman diagrams are shown in Figs. 2.9a and 2.9b, where the external decay is colour-favoured compared to the internal decay. Intermediate resonances will decay to the final state of $K^- \pi^+ \pi^- \pi^+$, either from the decay of two excited mesons to two final-state particles each, or from the decay to one excited meson and one stable meson where the excited meson decays to three final-state particles (usu-

Figure 2.9: RS D^0 decays.

ally via another resonance). For example in Fig. 2.9b, an excited neutral kaon can decay to $K^-\pi^+$, and a ρ^0 can decay to $\pi^-\pi^+$.

The relative contribution of these different processes can be assessed by fitting so-called ‘amplitude models’ to the data. The most precise model of $D^0 \rightarrow K^-\pi^+\pi^-\pi^+$ decays was constructed using a sample of 900,000 RS candidates from $B \rightarrow D^*X$, $D^* \rightarrow D^0\pi$ decays collected during LHCb Run 1 [27]. The dominant contribution to the amplitude was from the axial resonances, such as $D^0 \rightarrow a_1(1260)^+ K^-$. Here, the $a_1(1260)^+$ can decay to $\rho^0\pi^+$ and then the ρ^0 decays to $\pi^+\pi^-$, as shown in Fig. 2.10a. The dominant resonances measured in the model are presented in Table 2.1. Non-resonant decays are also possible, shown in Fig. 2.10b.

The decay rate of RS events at a point in phase space, Ω , is given by

$$\begin{aligned}
 \Gamma(D^0 \rightarrow K^-\pi^+\pi^-\pi^+)(t) &\equiv \int_{\Phi} \Gamma(D^0 \rightarrow K^-\pi^+\pi^-\pi^+)(t, \Omega) d\Omega \\
 &= \int_{\Phi} |\mathcal{A}_{RS}(\Omega)|^2 d\Omega \\
 &= \exp(-\Gamma t) |A_{CF}|^2,
 \end{aligned} \tag{2.25}$$

where $|A_{CF}|^2$ is the CF amplitude integrated over some region of phase space, Φ , and higher-order corrections due to mixing have been safely neglected.

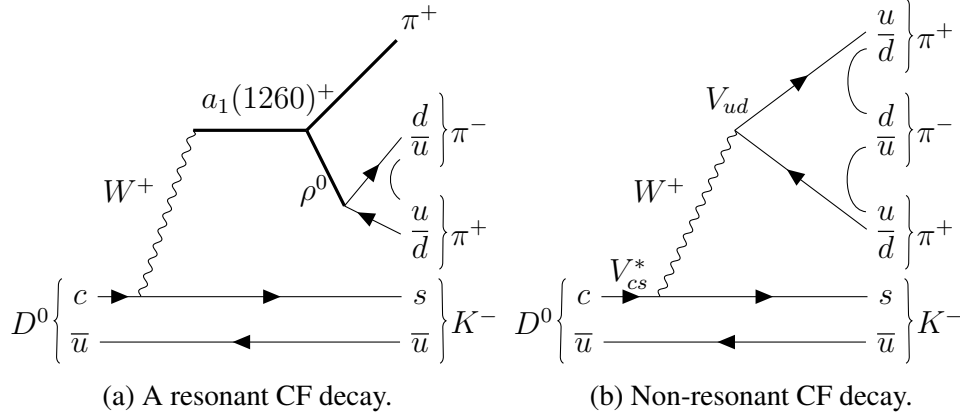


Figure 2.10: Examples of CF $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ decays.

Table 2.1: The five largest resonances (by fit fraction) of the CF decay according to the model presented in Ref. [27]. The other measured resonances have fit fractions of less than 5%. The uncertainties given are statistical and systematic respectively.

Resonance	Fit fraction (%)
$a_1(1260)^+ K^-$	$38.07 \pm 0.24 \pm 1.38$
$[K^- \pi^+]^{L=0} [\pi^+ \pi^-]^{L=0}$	$22.04 \pm 0.28 \pm 2.09$
$[\bar{K}^*(892)^0 \rho(770)^0]^{L=2}$	$8.47 \pm 0.09 \pm 0.67$
$[\bar{K}^*(892)^0 \rho(770)^0]^{L=0}$	$7.34 \pm 0.08 \pm 0.47$
$[\bar{K}^*(892)^0 \rho(770)^0]^{L=1}$	$6.03 \pm 0.05 \pm 0.25$

2.4.2 Wrong-sign decays

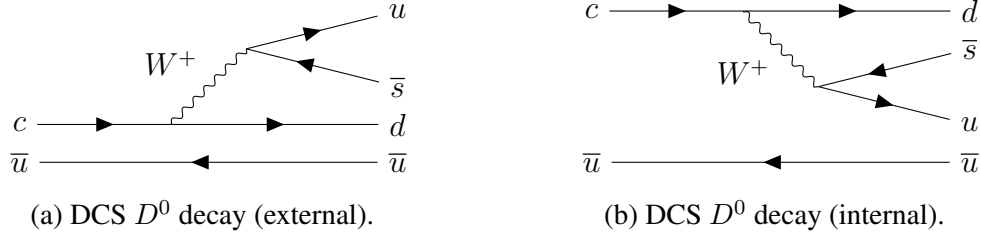
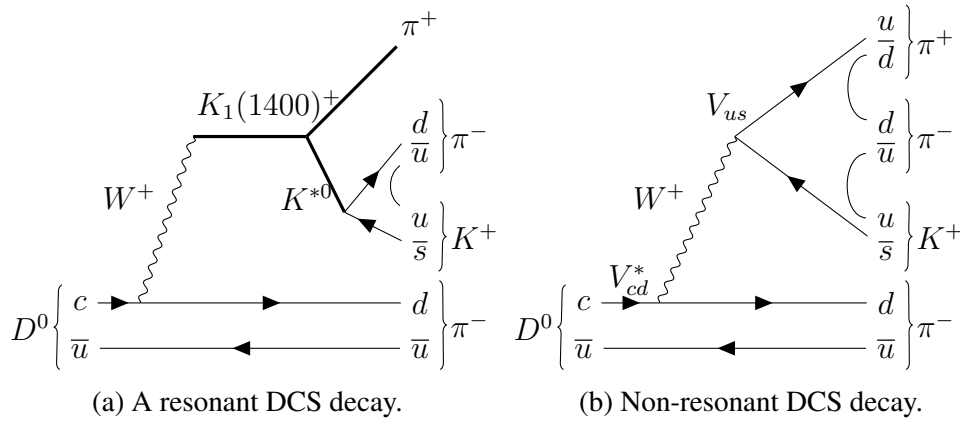
The suppressed $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decay has two sets of contributing amplitudes. As it is not possible to distinguish whether particles have undergone a DCS decay or have mixed before decaying, the combination of both amplitudes are together referred to as ‘wrong-sign’ (WS) decays. With the $D^{*+} \rightarrow D^0 \pi^+$ production mechanism, the tagging pion and the kaon have the same charge in WS D^0 decays. In RS decays, the equivalent mixing amplitude would be followed by a DCS $\bar{D}^0$ decay and thus contributions of this decay path are negligible compared to the CF amplitude.

The larger amplitude is the doubly Cabibbo-suppressed (DCS) amplitude, involving two quark transitions across generations. The decay rate of DCS decays relative to the CF decay is suppressed by the CKM factor [15],

$$\frac{|V_{cd}V_{us}|^2}{|V_{cs}V_{ud}|^2} \sim 3 \times 10^{-3}.$$

Figures 2.11a and 2.11b show example tree-level processes of DCS decays. The first amplitude model of $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decays was constructed using a sample of 3000 WS candidates from $B \rightarrow D^* X$, $D^* \rightarrow D^0 \pi$ decays collected during LHCb Run 1 [27]. The dominant contribution to the amplitude is from axial resonances, such as $D^0 \rightarrow K_1(1270/1400)^+ \pi^-$. Figures 2.12a and 2.12b show resonant and non-resonant examples of $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decays respectively. All the measured resonances in the model are presented in Table. 2.2.

The second contributing amplitude is from D^0 mesons that have undergone mixing to a $\bar{D}^0$ state. The $\bar{D}^0$ then decays via the CF amplitude to give the same decay of $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$. The magnitude of the mixing amplitude is small, but it changes as a function of time and thus can be measured relative to the DCS $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decays. Figure 2.13 shows how the mixing amplitude evolves with time for the similar $D^0 \rightarrow K^\pm \pi^\mp$ decays; in the absence of mixing

Figure 2.11: WS D^0 decays.Figure 2.12: Examples of DCS $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decays.

the DCS and CF decay rates would be time-independent, leading to a constant decay rate with time. The presence of mixing leads to an increasing WS decay rate as a function of time relative to the CF decay rate, causing the positive slope observed in the figure.

The combined contributions from the DCS and mixing amplitudes give a time-integrated WS decay rate relative to the RS decay rate which is currently measured to be [15]

$$\frac{\Gamma(\text{WS})}{\Gamma(\text{RS})} = (3.22 \pm 0.05) \times 10^{-3}.$$

As a function of phase space and time, the decay rate of $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$

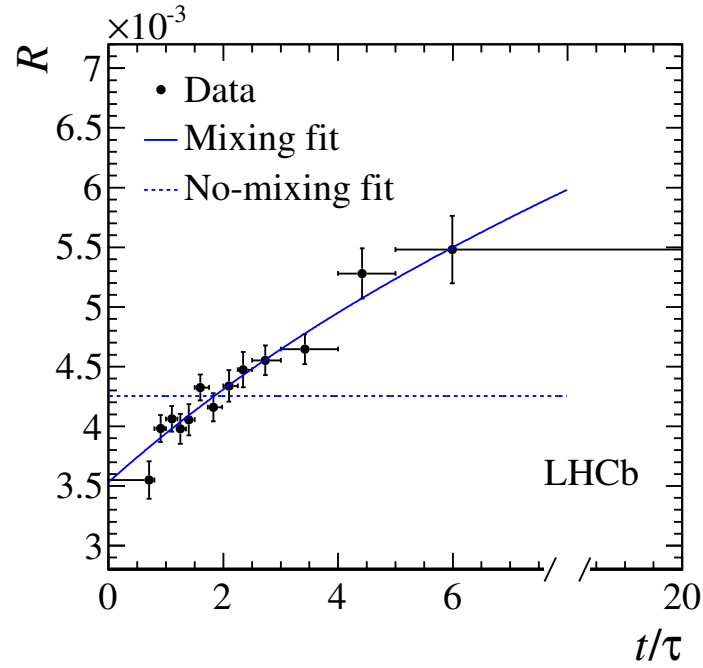


Figure 2.13: The behaviour of the WS decays (involving the DCS and mixing amplitudes, ‘Mixing fit’) relative to the DCS amplitude alone (‘No-mixing’) in $D^0 \rightarrow K^\pm \pi^\pm$ decays, using data collected at LHCb during 2011. Taken from [22].

Table 2.2: The resonances of the WS decay according to the model presented in Ref. [27]. The uncertainties given are statistical and systematic, respectively.

Resonance	Fit fraction (%)
$K_1(1400)^+[K^*(892)^0\pi^+]\pi^-$	$26.55 \pm 1.97 \pm 2.13$
$[K^+\pi^-]^{L=0}[\pi^+\pi^-]^{L=0}$	$20.90 \pm 1.30 \pm 1.50$
$K_1(1270)^+\pi^-$	$18.15 \pm 1.11 \pm 2.30$
$[K^*(892)^0\rho(770)^0]^{L=2}$	$10.19 \pm 1.03 \pm 0.79$
$[K^*(892)^0\rho(770)^0]^{L=0}$	$9.62 \pm 1.58 \pm 1.03$
$[K^*(892)^0\rho(770)^0]^{L=1}$	$8.42 \pm 0.83 \pm 0.57$
$[\rho(1450)^0K^*(892)^0]^{L=0}$	$8.16 \pm 1.24 \pm 1.69$

is given by

$$\begin{aligned}
\Gamma(D^0 \rightarrow K^+\pi^-\pi^+\pi^-)(t, \Omega) &= |\langle f | \mathcal{H} | D^0(t) \rangle|^2 \\
&= |\langle f | \mathcal{H}_{DCS} | D^0(t) \rightarrow D^0 \rangle + \langle f | \mathcal{H}_{CF} | D^0(t) \rightarrow \bar{D}^0 \rangle|^2 \\
&= \left| \mathcal{A}_{DCS}(\Omega)g_+(t) + \mathcal{A}_{CF}(\Omega)\frac{q}{p}g_-(t) \right|^2 \\
&= |\mathcal{A}_{DCS}(\Omega)|^2|g_+(t)|^2 + |\mathcal{A}_{CF}(\Omega)|^2\left|\frac{q}{p}\right|^2|g_-(t)|^2 \\
&\quad + \mathcal{A}_{DCS}(\Omega)g_+(t)\mathcal{A}_{CF}^*(\Omega)\left(\frac{q}{p}\right)^*g_-^*(t) \\
&\quad + \mathcal{A}_{DCS}^*(\Omega)g_+^*(t)\mathcal{A}_{CF}(\Omega)\left(\frac{q}{p}\right)g_-(t) \\
&\equiv |\mathcal{A}_{WS}(t, \Omega)|^2, \tag{2.26}
\end{aligned}$$

where the dependence on the position in phase space, Ω , has been specifically detailed in the amplitudes. The functions $g_{\pm}(t)$ are presented in Eqns. 2.17 and 2.18. The integral over (part of) phase space of $|\mathcal{A}_{DCS}(\Omega)|^2$ and $|\mathcal{A}_{CF}(\Omega)|^2$ yields the phase-space integrated amplitudes $|A_{DCS}|^2$ and $|A_{CF}|^2$. Integrals over phase space of the interference between the DCS and CF amplitudes are

defined to equal the interference parameter $\kappa e^{-i\delta}$,

$$\frac{\int \mathcal{A}_{DCS}(\Omega) \mathcal{A}_{CF}^*(\Omega) d\Omega}{|\mathcal{A}_{DCS}| |\mathcal{A}_{CF}|} \equiv \kappa e^{-i\delta}. \quad (2.27)$$

The magnitude of the coherence factor, κ , ranges from 0 to 1 and is a measure of the coherence between the CF and DCS decays that have different resonance structures [28]. The phase δ is the average phase-difference between CF and DCS decays, $\delta = \delta_{CF} - \delta_{DCS}$. When integrated over the entire phase space, these parameters are referred to as $\kappa_D^{K3\pi}$ and $\delta_D^{K3\pi}$. Other super- and sub-scripts are used to denote integrals over partial regions of phase space.

Thus, the WS decay rate (integrated over some phase-space region Φ) can be written as

$$\begin{aligned} \Gamma(D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-)(t) &\equiv \int_{\Phi} \Gamma(D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-)(t, \Omega) d\Omega \\ &= \int_{\Phi} |\mathcal{A}_{WS}(t, \Omega)|^2 d\Omega \\ &= \exp(-\Gamma t) \left(|A_{DCS}|^2 - \kappa y' |A_{DCS}| |A_{CF}| \Gamma t + \frac{x^2 + y^2}{4} |A_{CF}|^2 (\Gamma t)^2 \right) \end{aligned} \quad (2.28)$$

where $y' = y \cos \delta - x \sin \delta$.

2.4.3 Ratio of WS to RS decays

Of interest to experimental physicists is the ratio of $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ to $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ decays (with charge-conjugate decays implicit throughout, unless specified), as taking the ratio leads to cancellations of many systematic effects; the dominant RS decays act as a control channel. The time-dependent ratio can be constructed by dividing Eqn. 2.28 by Eqn. 2.25 and is commonly

written as (up to second-order in decay time and neglecting CPV):

$$R\left(\frac{t(D^0)}{\tau_{D^0}}\right) = \frac{\Gamma[D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-] \left(\frac{t(D^0)}{\tau_{D^0}}\right)}{\Gamma[D^0 \rightarrow K^- \pi^+ \pi^- \pi^+] \left(\frac{t(D^0)}{\tau_{D^0}}\right)} \approx r^2 - r\kappa y' \frac{t(D^0)}{\tau_{D^0}} + \frac{x^2 + y^2}{4} \left(\frac{t(D^0)}{\tau_{D^0}}\right)^2, \quad (2.29)$$

where $\Gamma[\cdot]$ is the decay rate for the given decay, $t(D^0)$ is the decay-time of the D^0 meson and τ_{D^0} is the D^0 lifetime. The parameter $r = \frac{A_{DCS}}{A_{CF}}$ is denoted $r_D^{K3\pi}$ when the decay rates are integrated over the entire phase space. This ratio of decay rates will be measured in this thesis using data collected by LHCb.

A model-independent measurement of the charm-mixing parameters and/or hadronic parameters can be made by interpreting $\kappa y'$ and $x^2 + y^2$ in terms of x and y using external knowledge of κ and δ , or in terms of κ and δ using external knowledge of x and y .

2.4.4 A measurement of γ using $B \rightarrow DK, D \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$

Knowledge of the hadronic parameters (r, κ and δ) is essential for measuring CP violation in $B \rightarrow DK$ measurements involving D decays to multi-body final states.

The $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ final state is useful in inclusive measurements of the CKM angle γ due to the particular nature of the B and D amplitude suppression factors (r_B and r_D respectively) and due to its large yield [28]. The quark transitions $b \rightarrow u$ and $b \rightarrow c$ must be accessible in a decay chain to measure γ . This is present in decays $B^\pm \rightarrow DK^\pm$ where D is a D^0 or $\bar{D}^0$. Each D meson can decay to either the RS or WS final states, giving four decays:

- $B^\pm \rightarrow [K^\mp \pi^\pm \pi^\mp \pi^\pm]_D K^\pm$: a suppressed B decay followed by a RS D decay, or a favoured B decay followed by a WS D decay (known as ‘Opposite Sign’ due to the charges of the two kaons);

- $B^\pm \rightarrow [K^\pm \pi^\mp \pi^\mp \pi^\pm]_D K^\pm$: a favoured B decay followed by a RS D decay, or a suppressed B decay followed by a WS D decay (known as ‘Same Sign’).

In Opposite Sign decays, the two decay paths have suppressed and favoured components and so both contribute at similar levels, whereas the Same Sign decays are dominated by the favoured plus favoured decay. In the Same Sign, mixing in the D decay can be neglected, whereas in Opposite Sign decays D^0 mixing should be considered. This leads to decay rates [29] of the following form

$$\Gamma(B^\pm \rightarrow [K^\pm \pi^\mp \pi^\mp \pi^\pm]_D K^\pm) \propto 1 + (r_D)^2 r_B^2 + 2r_D r_B \kappa_D \cos(\delta_B - \delta_D \pm \gamma), \quad (2.30)$$

$$\begin{aligned} \Gamma(B^\pm \rightarrow [K^\mp \pi^\pm \pi^\mp \pi^\pm]_D K^\pm) \propto & (r_D)^2 + r_B^2 + 2r_D r_B \kappa_D \cos(\delta_B + \delta_D \pm \gamma) \\ & - r_D \kappa_D (y \cos \delta_D - x \sin \delta_D) \frac{t}{\tau} + \frac{(x^2 + y^2)}{4} \frac{t^2}{\tau^2} \\ & - r_B (y \cos(\delta_B \pm \gamma) + x \sin(\delta_B \pm \gamma)). \end{aligned} \quad (2.31)$$

In Eqn. 2.31, the interference term ($2r_D r_B \kappa_D \cos(\delta_B + \delta_D \pm \gamma)$) is large (with $r_B \sim 0.1$ and $2r_D \sim 0.1$) allowing for good sensitivity to γ (provided the values of κ_D and δ_D are known). The mixing contributions (in terms involving x and y) are sub-dominant.

A measurement of these decay rates (in practice, the ratio of the Opposite Sign to Same Sign decay rates is measured) allows for a measurement of r_B (the ratio of suppressed-to-favoured B decays), δ_B (the strong-phase difference of the B decays) and γ to be performed, although there are multiple solutions for γ . The hadronic parameters of the D decay ($r_D^{K3\pi}, \kappa_D^{K3\pi}, \delta_D^{K3\pi}$) are inputs into this measurement that must be taken from external sources.

The sensitivity to γ is improved if the phase space of the $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$

decays is partitioned [30]. A split into two bins of phase space is enough to break the multiple-solution degeneracy of γ . However, partitioning into more bins is desired so as to increase the coherence in each bin to maximise the sensitivity to γ . To measure γ with data binned by phase space, knowledge of the hadronic parameters in each bin is required.

2.4.5 Current status

The Belle collaboration performed a time-integrated measurement of the WS/RS ratio $R = (0.320 \pm 0.018^{+0.018}_{-0.013})\%$ [31]. LHCb Run 1 data were used to perform a time-dependent measurement of this ratio, in which $D^0-\bar{D}^0$ mixing was observed in the $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ channel for the first time [32]. The values obtained (in a mixing-unconstrained fit) are:

$$\begin{aligned} r_D^{K3\pi} &= (5.67 \pm 0.12) \times 10^{-2}, \\ (\kappa y')_D^{K3\pi} &= (0.3 \pm 1.8) \times 10^{-3}, \\ \frac{1}{4}(x^2 + y^2) &= (4.8 \pm 1.8) \times 10^{-5}. \end{aligned}$$

In Ref. [33], a binning scheme was derived using the LHCb model that would lead to good sensitivity on the CKM angle γ in a measurement using $B \rightarrow DK, D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays. The binning scheme is based on minimising the strong-phase difference across each bin. The scheme employs four bins which increases the sensitivity to γ whilst maintaining reasonable yields in each bin. The phase-space binning will be described in more detail in Sec. 5.1.2.

Up to now, measurements of the hadronic parameters, κ and δ , have only been accessible at facilities that produce quantum-correlated $D^0-\bar{D}^0$ pairs such as CLEO-c and BESIII. The hadronic parameters (phase-space integrated, and in the four phase-space bins) are currently constrained by measurements using CLEO-c [33], BESIII [34] and Run 1 LHCb [32] data, with values given in

Table 2.3: Current constraints on the hadronic parameters, coming from BESIII and CLEO-c data, including LHCb results in the phase-space integrated values.

Phase-space division	$r (\times 10^{-2})$	κ	$\delta (^\circ)$
PS integrated	5.50 ± 0.07	$0.44^{+0.10}_{-0.09}$	161^{+28}_{-18}
PS 1	5.43 ± 0.10	$0.66^{+0.18}_{-0.21}$	117^{+14}_{-9}
PS 2	5.78 ± 0.11	$0.85^{+0.14}_{-0.21}$	145^{+23}_{-14}
PS 3	5.76 ± 0.10	$0.78^{+0.12}_{-0.12}$	160^{+19}_{-20}
PS 4	5.06 ± 0.12	$0.25^{+0.16}_{-0.25}$	228^{+15}_{-29}

Table 2.3, and visualised in Fig. 2.14 (showing the highly non-Gaussian limits).

A measurement of γ using the $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ final state, and employing the binning in phase space, was performed using LHCb Run 1 and Run 2 data [29], yielding

$$\gamma = (54.8^{+6.0+0.6+6.7}_{-5.8-0.6-4.3})^\circ$$

where the final uncertainty comes from the limited knowledge of the hadronic external inputs in each phase-space bin. As this is a dominant uncertainty, it is desirable to improve the precision of the hadronic parameters.

This thesis will present a measurement of the effective mixing parameters ($\kappa y'$ and $x^2 + y^2$) at LHCb using Run 2 data, that can be used to measure κ and δ per phase-space bin using constraints on x and y .

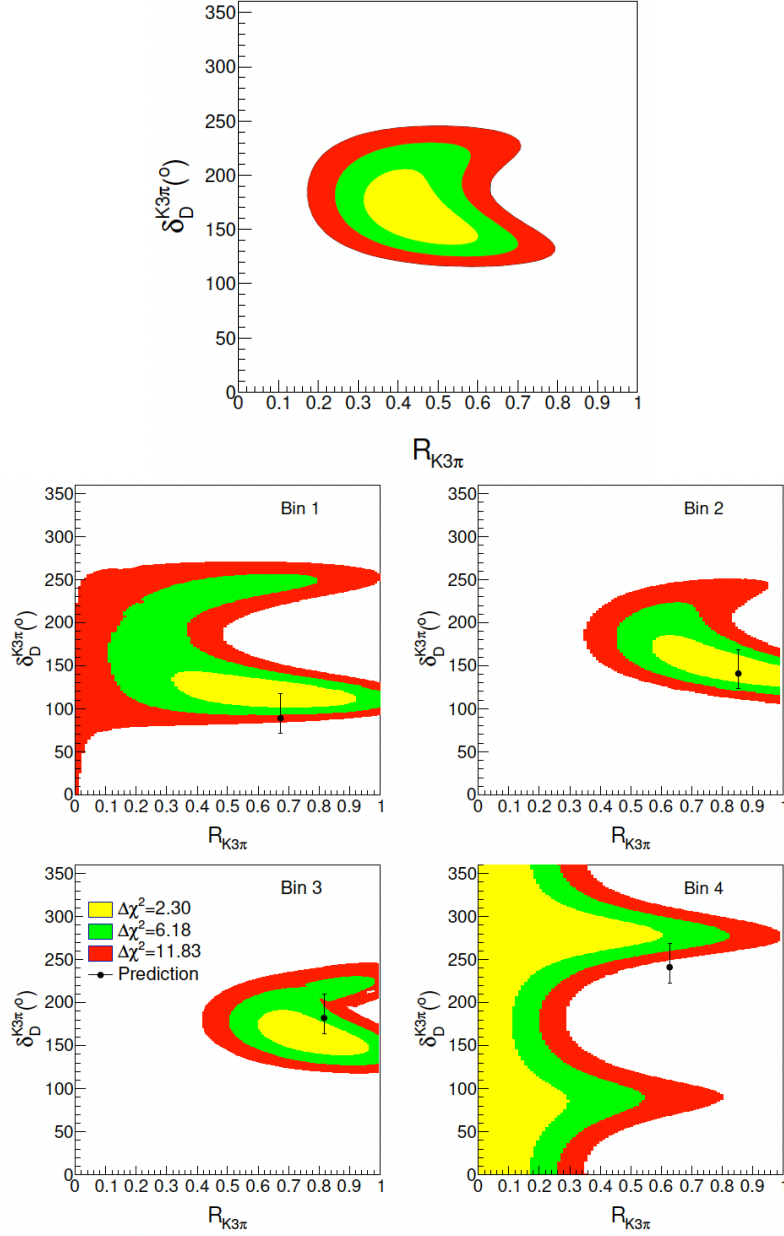


Figure 2.14: Constraints on the hadronic parameters from BESIII and CLEO-c data (and LHCb for phase-space integrated parameters (top)). Scans of $\Delta\chi^2$ with intervals $\Delta\chi^2 = 2.30, 6.18, 11.83$ (corresponding to 68.3%, 95.4% and 99.7% CLs). Black points show the predictions from the amplitude models [27], where the global offset in average strong-phase difference comes from the phase-space integrated result. Taken from [34].

3 | The LHCb detector at CERN

This chapter describes the accelerators used at CERN to achieve colliding proton beams, each with an energy of 6.5 TeV at time of writing. A description of the LHCb detector in the years 2015 – 2018 is also given.

3.1 CERN and the accelerator complex

The Large Hadron Collider (LHC) is a two-ring superconducting-hadron accelerator and collider located at CERN [35]. The tunnel in which the LHC is housed is situated between 45 m and 170 m under the surface of the earth, and has a circumference of 26.7 km. Protons, and in special fills Pb ions, are injected in bunches into the LHC at energies of 450 GeV, after being accelerated in the CERN accelerator complex shown in Fig. 3.1.

The Linear Accelerator (LINAC 2) accelerates ionised hydrogen from rest to 50 MeV using radiofrequency cavities over a length of 37 m. The protons enter the Proton Synchrotron Booster (PSB), and circulate until reaching an energy of 1.4 GeV. The Proton Synchrotron (PS) provides further acceleration up to 25 GeV, before the protons enter the Super Proton Synchrotron (SPS) which is the final acceleration stage before entering the LHC. The number of proton bunches in each stage of the accelerator complex are dictated by the requirements on the number of LHC proton bunches, namely 2808 bunches per LHC ring with $\sim 1 \times 10^{11}$ protons per bunch, and a 25 ns spacing between bunches.

Once the proton bunches have been injected into the LHC, the superconducting magnets with nominal field strength of 8.33T guide the protons in a circular path around the ring. Two-thirds of the ring consists of the dipole magnets, whilst the quadrupole magnets focus the beam.

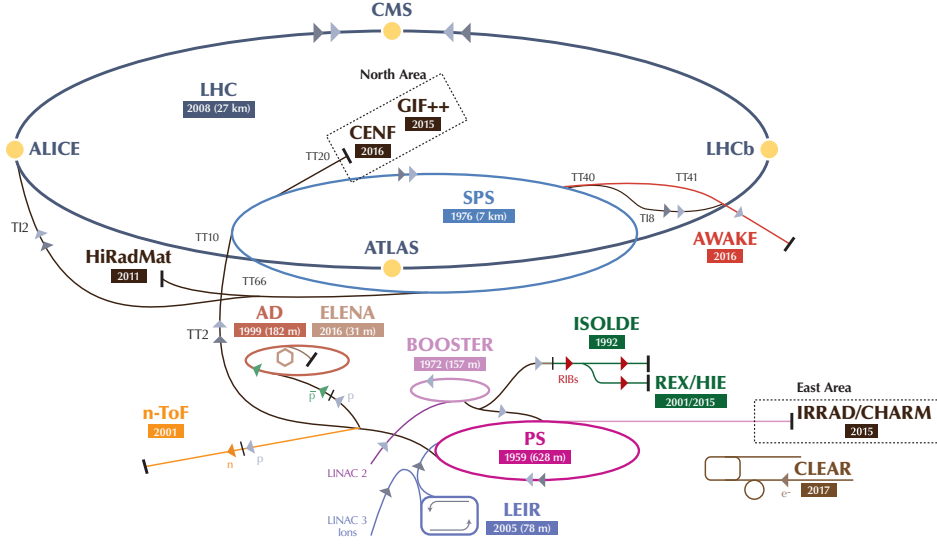


Figure 3.1: The accelerator complex at CERN. Taken from [36].

The number of events produced per second in the pp -collisions is given by

$$N_{event} = \mathcal{L} \sigma_{event} \quad (3.1)$$

where σ_{event} is the cross-section of the event process and $\mathcal{L}$ is the machine *luminosity*.

The luminosity is dependent on the parameters of the beams

$$\mathcal{L} = \frac{N_1 N_2 \sqrt{n_1 n_2} f_{rev} \gamma_r F}{4\pi \epsilon_n \beta^*}, \quad (3.2)$$

with

$$F = \left(1 + \left(\frac{\theta_c \sigma_z}{2\sigma^*} \right)^2 \right)^{-\frac{1}{2}}. \quad (3.3)$$

Here, $N_{1,2}$ are the number of particles in each bunch, $n_{1,2}$ are the number of bunches per beam, f_{rev} is the revolution frequency and γ_r is the relativistic gamma factor. The normalised transverse beam emittance, ϵ_n , is a measure of

the spread in position and momentum of the particles in the beam; this quantity is invariant under acceleration ($\epsilon_n = \gamma_r \beta \epsilon$ for emittance ϵ (in one dimension)). β^* is the value of the beta function (related to the transverse size of the beam) at the collision point [37]. The geometric luminosity reduction factor, F , is due to the crossing angle at the interaction point; θ_c is the full crossing angle, σ_z is the RMS bunch length and σ^* is the transverse RMS beam size.

LHCb operated at $\mathcal{L} = 4 \times 10^{32} \text{ cm}^2 \text{ s}^{-1}$ throughout Run 2 (2015–2018), exceeding its design peak luminosity of $\mathcal{L} = 2 \times 10^{32} \text{ cm}^2 \text{ s}^{-1}$. This luminosity is lower than that of the ATLAS and CMS experiments, a choice motivated by the challenges of performing flavour physics at a hadron collider. The luminosity at LHCb is maintained at a constant level throughout an LHC *fill*, made possible by changing the beta function and the transverse overlap of the beams.

3.2 The LHCb detector

The LHCb detector is a single-arm spectrometer, designed for the study of b and c hadrons [38]. At high energies, b - and c -hadrons are predominantly produced in a forward or backward cone. The angular coverage in the forward direction spans from 10 mrad to 300 mrad in the bending plane, and from 10 mrad to 250 mrad in the non-bending plane, providing complementary coverage to that of the LHC General Purpose Detectors (GPDs). The coordinate system in LHCb is right-handed, with the z -axis along the beam direction (with z increasing along the detector from the interaction point), the x -axis is horizontal (pointing towards the outside of the LHC ring) and the y -axis is vertically upwards.

The beam pipe in the region of the LHCb detector is designed to reduce the number of material interactions [38]. Of the 19 m long beampipe, a 12 m length is made of Beryllium as it has a high radiation length. Stainless steel is used

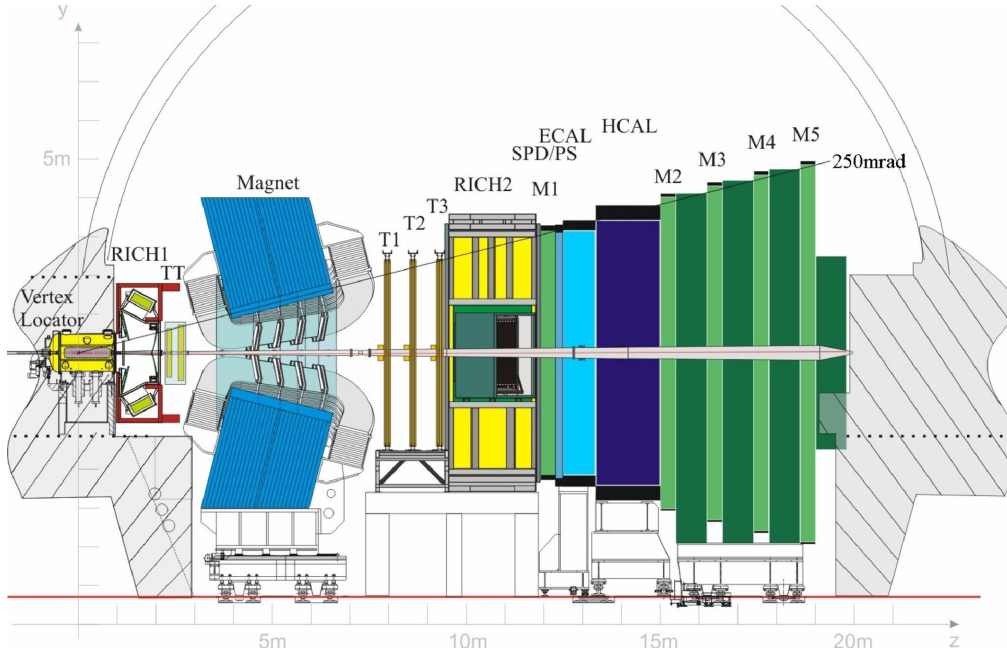


Figure 3.2: View of the LHCb detector. Taken from [38].

for the remaining length, whilst the bellows (used to compensate for thermal expansion and mechanical tolerances) are made of aluminium.

The LHCb detector consists of tracking systems to measure the momentum and trajectory of charged particles, particle-identification systems to identify charged particles, calorimeter systems to measure the energy of particles and identify neutral particles, and muon chambers to identify muons. These are shown in Fig. 3.2. The overall performance of the spectrometer is described in Ref. [39], and is summarised here with particular attention to the components important for the study of $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays.

Table 3.1 details the data-taking periods, collision energies and luminosity collected by LHCb in the years between 2011 and 2018. The analysis presented here uses data collected during Run 2.

Table 3.1: The Run 1 and Run 2 LHCb data-taking conditions including centre-of-mass (CoM) collision energy and integrated luminosity, $\mathcal{L}_{int}$.

Period	Year	CoM energy (TeV)	$\mathcal{L}_{int}$ (fb ⁻¹)
Run 1	2011	7	1.1
	2012	8	2.1
Run 2	2015	13	0.3
	2016	13	1.7
	2017	13	1.7
	2018	13	2.2

3.2.1 Tracking system

High multiplicity decays of b - and c -hadrons require high tracking efficiency and a low rate of incorrectly reconstructed tracks in order to correctly reconstruct the decay.

The tracking system of the detector comprises an inner vertex locator (VELO) which surrounds the pp interaction region and four planar tracking systems, the Tracker Turicensis (TT) and the tracking stations T1-T3. T1-T3 each consist of two types of trackers; the Inner Tracker (IT) is located in the region close to the beam pipe whilst the Outer Tracker (OT) surrounds the IT. The TT sits upstream of the dipole magnet; T1-T3 are downstream. The magnetic field bends charged particles in opposite directions depending on their charge. The momenta of the charged particles are measured with a relative uncertainty that varies from 0.5% at low momentum (5 – 20 GeV/ c) to 1.0% at 200 GeV/ c .

3.2.1.1 Vertex Locator

In order to study the lifetimes and properties of b - and c -hadrons, the vertex of the hadron decay (usually spatially separated from the primary vertex (PV)) must be identified. The VELO [40] is designed to measure both the PV and displaced vertices. The VELO is made of two symmetric halves, each with 21

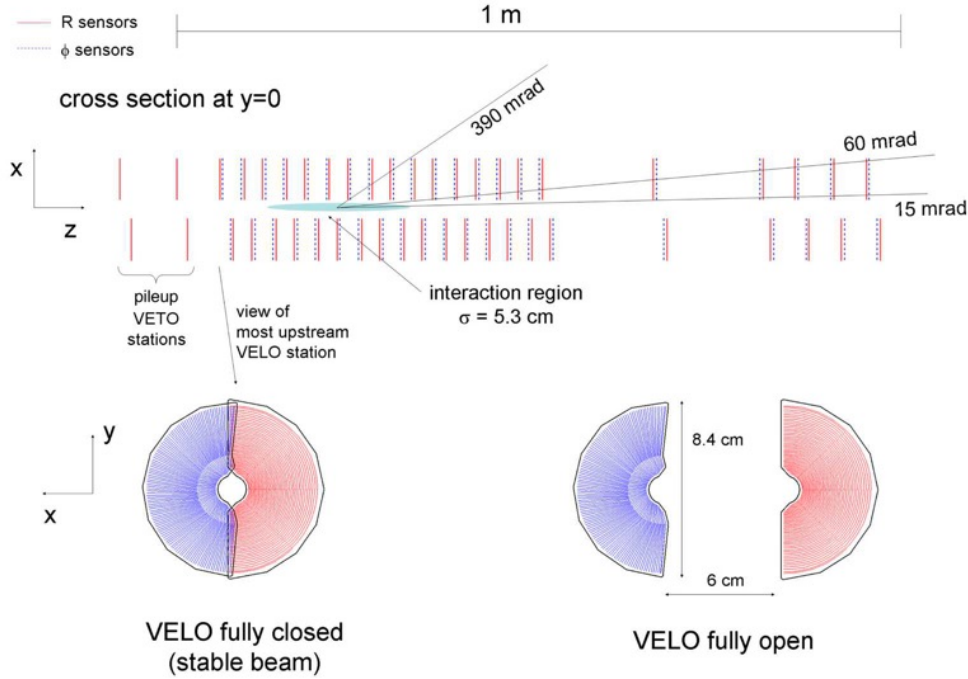


Figure 3.3: (top) A cross-section looking down on the VELO from above, showing how the layers are distributed around the beam line. (bottom) The sensors as seen looking along the beam pipe. The blue half shows a ϕ -sensor with strips oriented radially from the centre in two separate regions. The red half shows r -sensors with strips curving around the beam pipe. Taken from [38].

silicon modules that are arranged along the z -axis. The silicon modules have alternating r - and ϕ -sensors so that the three-dimensional positions of tracks and vertices can be determined. The strip pitch varies from 40 to 100 μm across the sensor. A cross-section of the VELO is shown in Fig. 3.3.

The designed operating position of the VELO has its sensors at 7 mm from the beam position. This is more precise than the knowledge of the beam position during the *unstable beams* period of the LHC energy ramping. In addition, the beam positions can vary from fill to fill, and the reconstruction algorithm requires that the VELO is centred around the collision point. For that reason, the two VELO halves move from an open, safe, position when the beams are not stable to a position centred around the collision point every fill. The closing of

the VELO is done using dedicated algorithms and is monitored to ensure safety.

A primary vertex resolution of $13\text{ }\mu\text{m}$ ($71\text{ }\mu\text{m}$) in the transverse plane (along the beam axis) is achieved for vertices with 25 tracks [40]. The impact parameter (IP), which is the minimum distance of a track to a primary vertex, has a resolution of $(15 \pm 29/p_T)\text{ }\mu\text{m}$ (p_T in GeV/c).

3.2.1.2 Magnet

The dipole magnet is used to generate integrated magnetic fields of up to 4 Tm in the y -direction, so that the charged particles are bent in the $x - z$ plane. To minimise possible systematic biases, the magnet polarity is switched frequently. The different data-taking conditions are referred to as *MagUp* and *MagDown*.

3.2.1.3 Tracker Turicensis and Inner Tracker

The TT [41] is a planar tracking station of $150\text{ cm} \times 130\text{ cm}$. The IT covers a cross-shaped area extending up to 126 cm wide and 42 cm high. Both systems employ silicon microstrip sensors with a strip pitch of $200\text{ }\mu\text{m}$ to achieve a single-hit resolution of $50\text{ }\mu\text{m}$. Each of the four tracking systems consists of four layers of sensors, with the first and fourth layers vertically aligned to measure the x -hit coordinate, and the second and third layers oriented at $\pm 5^\circ$ to the vertical to provide information about the y -hit coordinate. The geometry and orientation of the four layers of the TT are shown in Fig. 3.4. These detectors are designed for higher occupancies than the gaseous OT.

3.2.1.4 Outer Tracker

The OT [42] is a straw-tube drift-time detector and consists of three stations (T1-T3) and covers an area of $5 \times 6\text{ m}^2$. Each of the stations consists of four layers, arranged with the same orientations as the inner silicon trackers. Each of the twelve layers is made from a double layer of straw tubes.

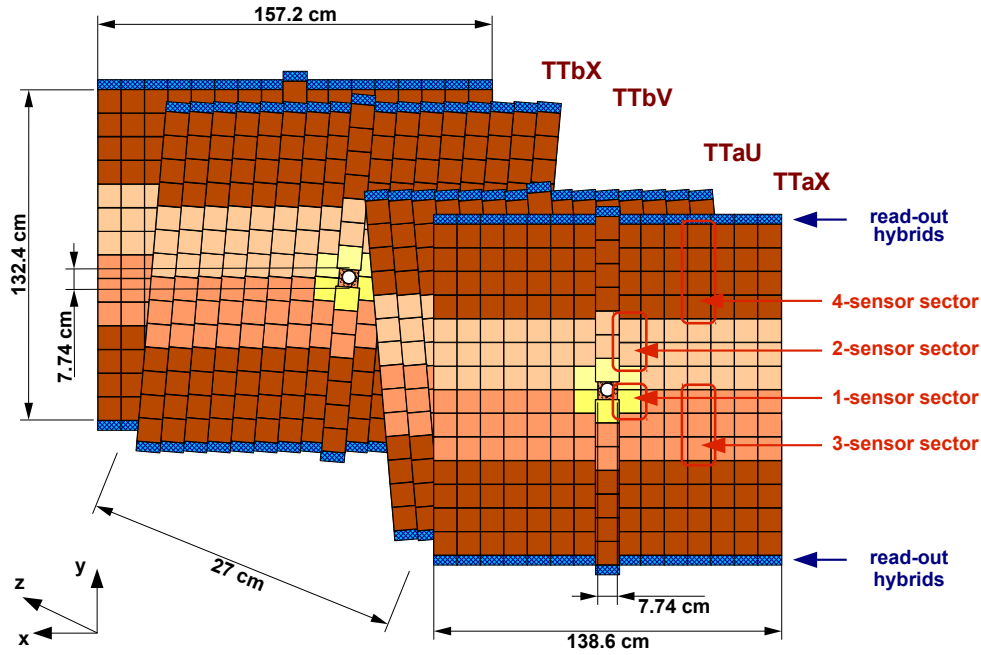


Figure 3.4: TT layout, comprising four layers of silicon strip detectors with varying angles to provide the $x - y$ coordinates of hits. Taken from [41].

As particles traverse the straw tubes filled with a mixture of Argon (70%) and CO_2 (30%)[42], the atoms are ionised and electrons are collected on the anode wire running through the centre of the straw tube. The drift-time of the charge is measured, which is maximally 35 ns (longer than the nominal Run 2 bunch spacing of 25 ns). The single hit resolution is $205 \mu\text{m}$ [39].

The performance of the Outer Tracker in Run 2 was improved compared to that of Run 1 [42] with the introduction of real-time calibration of the time and spatial alignment of the detector. This improved the drift-time and position resolution of the detector by 20%.

3.2.2 Charged particle identification

Particle identification (PID) is crucial for the study of b - and c -hadrons, as the hadronic final states often consist of numerous kaons and pions. In particular for multi-body D^0 decays, it is important to distinguish between $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$,

$D^0 \rightarrow K^+\pi^-\pi^+\pi^-$, $D^0 \rightarrow K^+K^-\pi^+\pi^-$ and $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ decays (among others) to reduce the level of misidentified background. The most abundant particles produced in LHCb are pions; the PID system must provide positive identification of the less abundant kaons and protons.

Charged hadron identification is achieved by two ring-imaging Cherenkov detectors (RICH1 and RICH2) [43] which are positioned on either side of the dipole magnet.

Charged hadrons travelling faster than the speed of light within a medium will produce Cherenkov radiation in a cone around the particle's trajectory. The angle of the cone to the trajectory, the Cherenkov angle θ_c , is related to the velocity of the particle, β (in units of c), by the relation

$$\cos \theta_c = \frac{1}{\beta n} \quad (3.4)$$

where n is the refractive index of the medium. This in turn places a lower limit on the momentum, p , of a hadron (with mass m) that will produce radiation (for a given refractive index),

$$p > \frac{m}{\sqrt{n^2 - 1}}. \quad (3.5)$$

In the RICH detectors, the volume of media that the charged particles travels through is referred to as a *radiator*. The radiator material is carefully chosen to ensure that pions, kaons and protons produce Cherenkov radiation over the momenta range of interest. Table 3.2 shows the refractive index and momentum thresholds for the radiators used in RICH1 and RICH2.

The Cherenkov radiation is reflected out of the LHCb acceptance (active area) by a spherical mirror, and then reflected on to the photon detectors by a second plane mirror. The Cherenkov cone produces a ring of hits on the detector plane; the pattern is fit to the best mass hypothesis (using external information

Table 3.2: The refractive index, n , and momentum thresholds for different particle species for the different radiators used in RICH1 and RICH2.

Detector	Radiator	n	Momentum threshold (GeV/ c)		
			π	K	p
RICH1	C ₄ F ₁₀	1.0014	2.6	9.3	17.7
RICH2	CF ₄	1.0005	4.4	15.6	29.7

of the particle’s momentum).

Hybrid Photon Detectors are used to detect the Cherenkov photons and read out the hit positions. In RICH1, the photon detectors are situated above and below the beam, whereas in RICH2 they are situated horizontally away from the beam. The RICH1 photodetectors are protected from the dipole magnetic field by an iron magnetic shield.

3.2.2.1 RICH1

RICH1 covers the full acceptance of LHCb, and performs PID of kaons and pions over the momentum range 2 – 60 GeV/ c . The detector is situated between the VELO and the TT. A cross-sectional schematic of RICH1 is shown in Fig. 3.5.

In Run 2, the only (and in Run 1, the main) radiator material is C₄F₁₀ gas at nominal pressure, contained within a light-tight and gas-tight chamber [44]. Figure 3.6 shows the reconstructed Cherenkov angles produced by different particles species in the C₄F₁₀ radiator. The momentum threshold for kaons to produce Cherenkov radiation in C₄F₁₀ is 9.3 GeV/ c , so particles with momentum below this threshold which produced Cherenkov radiation would be identified as pions, and all other charged particles with no associated ring would be assigned as kaons, known as the “veto mode”. This gives no kaon-proton separation up to this momentum.

In Run 1, an aerogel radiator was additionally used to extend the range of $\pi - K$ separation to momenta as low as 1 GeV/ c and also to provide $p - K$ separa-

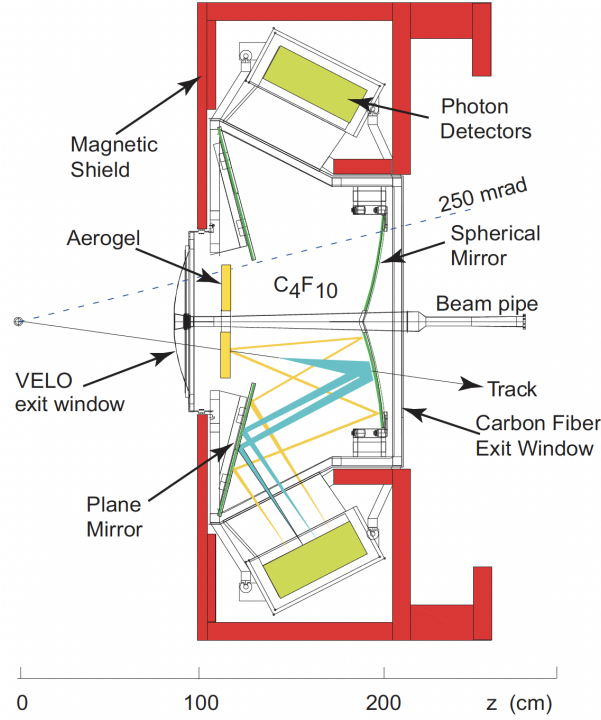


Figure 3.5: The layout of RICH1. Charged hadrons passing through the gas (and Aerogel in Run 1) produce Cherenkov radiation (yellow and blue lines) that are reflected by mirrors to photon detectors at the periphery of the detector. Taken from [38].

tion [43]. However, the aerogel was removed at the end of Run 1 as the number of Cherenkov photons produced was too low for efficient reconstruction.

During 2016–2018, the average detected Cherenkov yield per hadron was 30 ± 2 photons, which decreased slightly over Run 2 due to the expected ageing of the detectors.

3.2.2.2 RICH2

RICH2 performs PID of kaons, pions and protons over the momentum range $15 - 100 \text{ GeV}/c$. It has a reduced acceptance of $\pm 15 \text{ mrad}$ to $\pm 120(100) \text{ mrad}$ horizontally (vertically), and is situated behind the trackers T1-T3. The radiator is CF_4 gas [44]. During 2016–2018, the average detected Cherenkov yield per hadron was 18.5 ± 1.2 photons.

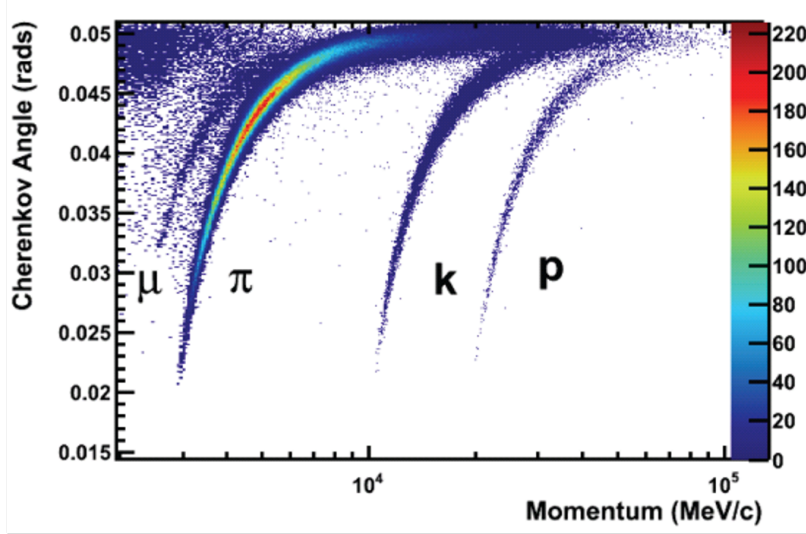


Figure 3.6: The reconstructed Cherenkov angle for different particle species in the RICH1 C_4F_{10} radiator. The low momentum thresholds for each species are clearly seen, as well as the saturation of the Cherenkov angle for all species at higher momenta. Taken from [43].

The combined $K - \pi$ separation performance of the RICH detectors in Run 2 is shown in Fig. 3.7. The kaon identification efficiency is greater than 90% across a large momentum range, and the misidentification rate of pions is very low. Kaons from $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays have momenta that peak at 20 GeV/c, where the efficiency of kaon identification is greatest.

3.2.3 Calorimeters

The calorimeter system [45] measures the energy and position of particles and provides PID for photons, electrons and hadrons, and high E_T signals in this system form part of the hardware trigger. The system comprises a Scintillating Pad Detector (SPD) which consists of rectangular scintillating pads which improve the separation of photons (from π^0 decays) and electrons, by selecting charged particles. A 15mm thick lead convertor is placed between the SPD and the Preshower which helps to distinguish the EM showers from charged pions. A shashlik-type electronic calorimeter (ECAL), made of a sampling scintilla-

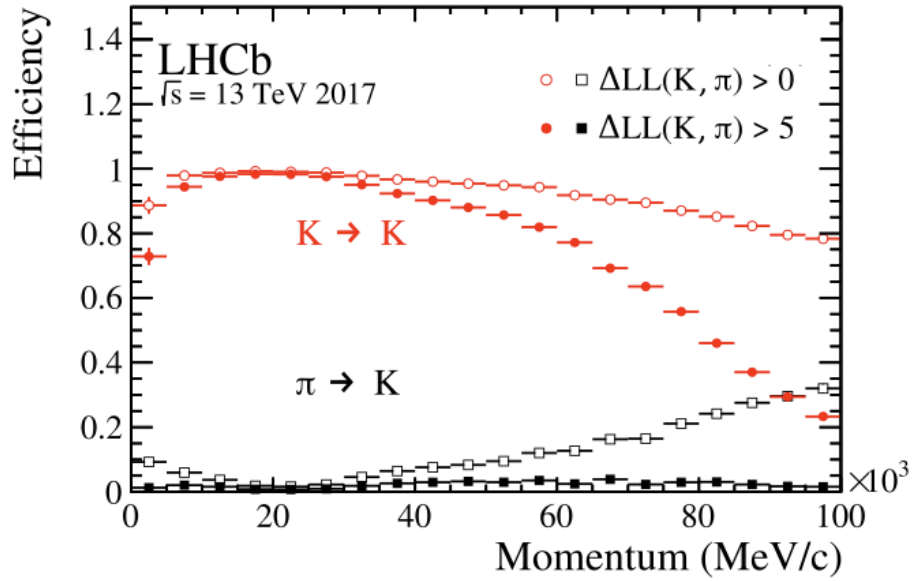


Figure 3.7: The efficiency of selecting kaons, and of mis-identifying pions, for two different $\Delta LL(K, \pi)$ selections (the difference in log-likelihood between kaon and pion mass hypotheses) as a function of momentum. The $K \rightarrow K$ efficiency below 10 GeV/c results from the veto mode. Taken from [44].

tor and lead structure, and a sampling iron and scintillator hadronic calorimeter (HCAL) complete the calorimeter system.

The working principle of the calorimeters is as follows. High-energy particles produce many particles in a *shower*, where the number of secondary particles is proportional to the energy of the initial particle. The particles in the shower produce scintillation light via the excitation and subsequent de-excitation of electrons in the scintillator material. The photons travel along wavelength-shifting (WLS) fibres to Photo-Multiplier Tubes (PMTs) located at the edge of the detector.

Each component of the calorimeter system is segmented into cells; different sizes of cells are used in different areas to account for different occupancies and the shower behaviour. Each cell in the SPD/PS has its own WLS fibre connected to a single PMT. In the ECAL and HCAL, fibre bunches from each cell are connected to individual PMTs. The ECAL and HCAL outputs used for

triggering must be calibrated; in particular, the gain of the electronic channels vary as a function of the position of the cell, with larger gains for cells closer to the beam.

The HCAL (ECAL) measures the transverse energy, E_T , of hadrons (electrons, photons, π^0) using the relation $E_T = E \sin \theta$ where θ is the angle between the beam axis and the line from the interaction point to the centre of the cell where the particle is detected. A *cluster* is defined to be a group of 2×2 cells. The E_T of a cluster is the sum of the E_T in the four contributing cells. Information from the calorimeter system is used in the trigger to select events which contain clusters with high E_T .

The ECAL has a thickness equivalent to 25 radiation lengths in order to fully contain the showers from high energy photons and electrons. This provides the best energy resolution. The HCAL resolution does not have a strong dependence on the number of interaction lengths, so the thickness is limited by space (to 5.6 interaction lengths). The ECAL resolution [46] is given by the parametrisation

$$\frac{\sigma_E}{E} = \frac{(8.5 - 9.5)\%}{\sqrt{E(\text{GeV})}} \oplus 0.8\%, \quad (3.6)$$

and for the HCAL the resolution is

$$\frac{\sigma_E}{E} = \frac{(69 \pm 5)\%}{\sqrt{E(\text{GeV})}} \oplus (9 \pm 2)\%. \quad (3.7)$$

3.2.4 Muon chambers

Muon detection uses five muon chambers (M1-M5) [47]. M1 is a triple Gas Electron Multiplier (GEM) detector positioned in front of the calorimeters that is used to improve the standalone muon p_T measurement that is important in the trigger. M2-M5 are Multi Wire Proportional Chambers (MWPCs) positioned behind the calorimeters, interleaved with 80cm thick iron absorbers. For the

GEM chambers the minimum efficiency of muon detection is 98.7% and for the MWPCs the minimum efficiency achieved is 99.3% [45].

Although the decay of interest, $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$, does not have muons in the final state, information from the muon detectors is used in the trigger and provides PID separation for muons and pions.

3.2.5 Trigger and DAQ

During Run 2, the average number of visible pp interactions per bunch crossing, μ , was equal to 1.1. During Run 1, the value of μ varied depending on the number of bunches in the beam (in 2012 $\mu = 1.8$). The rate of data produced by all channels in the sub-detectors is too large, meaning that only a fraction of events are read out to be stored. Real-time reconstruction and triggering reduces the rate of data saved by three orders of magnitude [48].

3.2.5.1 Definitions

Key terms relating to the trigger system used by LHCb are defined:

- Trigger - the act of selecting an event based on one or more of its properties; events that are not selected are discarded.
- Trigger line - a set of requirements that must be passed in order for an event to be selected.
- Trigger independent of signal (TIS) - triggering on an event where the property used is not related to the signal component (the physics of interest) of the event.
- Trigger on signal (TOS) - triggering on an event where the property used is related to the signal component of the event.

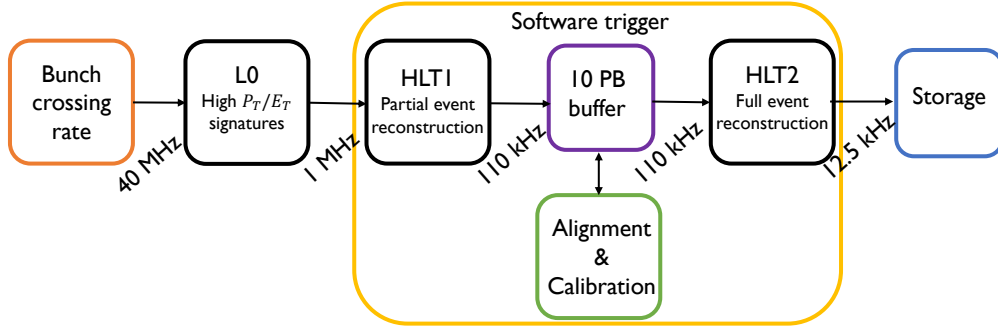


Figure 3.8: An overview of the LHCb trigger system during Run 2. Reproduced from [48].

- High level Trigger (HLT) - the software system used to perform triggering, which is executed on CPUs.

In Run 2, LHCb used a three-stage triggering system (L0, HLT1, HLT2) which is depicted in Fig. 3.8.

3.2.5.2 L0

L0 is the first stage of the trigger system. A low-level hardware trigger selects events based on high E_T clusters in the calorimeters and muon-detector information. In 2017, a HCAL cluster with $E_T \geq 3.46$ GeV led to an event being triggered, as well as an electron ECAL cluster with $E_T \geq 2.11$ GeV or a muon with $p_T \geq 1.35$ GeV. The E_T and p_T thresholds varied slightly over the years of data-taking, with thresholds generally tighter in earlier years.

3.2.5.3 HLT1

HLT1 is the second stage of the trigger system, and the first of the higher-level software triggers. Information from the VELO and tracking stations is used to reconstruct tracks of the final-state particles with $p_T > 500$ MeV/ c and to reconstruct the primary vertex (PV). First, hits in the VELO are combined to form straight lines (*VELO tracks*) that point to the beam line using a Kalman filter, which also allows for the PV to be reconstructed. VELO tracks are combined

with at least three hits in the TT (in a small window around the straight-line extrapolation of the VELO track) to form *upstream tracks*. From this a momentum and charge estimate is made which is used to reject low p_T tracks from further reconstruction at this stage. *Long tracks* are formed by combining upstream tracks with hits in the IT and OT. The search window is restricted by the charge estimation and the maximum possible deflection (for $p_T > 500 \text{ MeV}/c$). A Kalman filter is then used on all tracks to obtain the optimal parameter estimation. Muon tracks are also identified. Events with hadronic final states are selected based on the properties of one- or two-track signatures in the event, such as high p_T or good separation from the PV.

3.2.5.4 Buffer and real-time alignment (RTA)

After HLT1, events are buffered to disk storage. Alignment and calibration of the detector can be performed in real time using the data in the buffer. This information is then applied to the data. All sub-detectors undergo alignment and/or calibration that is performed at least once every fill, although the calibration constants are only updated if they differ significantly from the previously measured values. The 10 PB buffer also allows for data processing to take place when data are not been taken.

3.2.5.5 HLT2

HLT2 is the third stage of the trigger system, also a software trigger. Full event reconstruction takes place; charged particles have their tracks reconstructed, neutral particles are reconstructed and particle identification is performed. Here, the track reconstruction on HLT1 is repeated, and lower-momentum tracks are also reconstructed (with the hits already used in the first reconstruction removed). For the low momentum tracks no TT hits are required and the possible deflection from VELO to IT/OT is maximal (for $p_T > 80 \text{ MeV}/c$). In addition,

downstream tracks which have no VELO hits associated (for example if a K_S^0 decays outside the VELO) can be reconstructed. Fake tracks (random combinations of hits or track segments either side of the magnet) are rejected by requiring a good χ^2 of the Kalman filter, which is run over all tracks, and also by a multi-variate neural network trained to remove fake tracks. Clones (tracks which share hits) are also removed if they share enough hits; the track with more hits overall is retained. The RICH reconstruction is performed on a per-event basis, using information from all reconstructed tracks and all Cherenkov photons, returning a likelihood for each mass hypothesis. Photons, electrons and π^0 mesons are reconstructed in the calorimeters, using an algorithm to build clusters from the energy deposits to determine the total energy of each particle. Further cuts on properties of the final state particles are made and events that pass certain requirements are written to disk. Selections can be *inclusive*, requiring only a heavy-flavour decay signature, or *exclusive* where a specific final state is required. Around half of all triggered events are read out as part of the *full stream* where the full set of sub-detector and trigger raw banks are stored for later offline analysis.

3.2.5.6 Turbo stream

The Turbo stream is special stream of the HLT2 trigger and readout [49]. The full decay chain (including intermediate particles) is also reconstructed during HLT2. Only the signal decay of interest is written out, all other information about the underlying event is discarded to minimise the required disk space. In the case of $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays, there are only five final-state tracks (which includes the tagging pion from the $D^* \rightarrow D^0 \pi$ decay), whereas a hard proton-proton event typically produces 40 tracks within LHCb, allowing for a much reduced dataset per event. A small sample is read out as the *TurCal* stream which retains both the full and reduced data formats, which is used for

calibration.

3.2.5.7 Offline

LHCb runs the same reconstruction and selection algorithms in the offline processing as it does online during the triggering. This means that there are no differences in the offline and online selection that must be accounted for. It also means that Turbo stream data can be used directly for analysis, as the reconstruction is as good as the offline reconstruction. A dedicated application runs to transform the HLT2 output into a format ready for analysis.

Further offline data processing can take place; for example refitting tracks such that the vertex position is constrained to the PV, or a mass constraint is applied to the vertex. High-level variables such as PID likelihoods are also calculated. The data are formatted into ROOT nTuple format.

3.3 Simulation

Modelling of the physics processes and propagation of particles through the detector is an extremely important part of LHCb analyses [50]. LHCb relies upon Monte Carlo (MC) simulation to estimate efficiencies and to get an idea of backgrounds that are present. This involves repeatedly simulating both the interactions of the particles under study and the performance of the detector. However this is a computationally expensive process, and so fast simulation techniques have been developed to increase the speed of generation whilst maintaining emphasis on important detector and physics effects. The following will describe the simulation of the proton-proton collisions, the modelling of the particles' propagation through the detector and the fast simulation techniques used in the analysis that will be presented in the following chapters of this thesis.

3.3.1 Simulation of proton-proton collisions

Event generation involves three components: the primary (proton-proton) event generation, the specific particle decay, and pile-up generation.

pp -collisions at 13 TeV are simulated with PYTHIA 8 [51], a general-purpose event generator. The first step is the initialisation of the generator which sets configurables such as the beam energy and crossing angle of the beams [52]. The event loop is then called; a minimum-bias event¹ is generated and decayed to see if the desired signal particle is created (the particle at the top of the decay chain of interest). If the particle is present, the specified signal decay proceeds, else a new event is generated. Additional interactions per bunch-crossing can occur; these are modelled by adding extra minimum-bias interactions to the main event.

The event generation incorporates many physics processes including the hard process, resonant decays, hadronisation and particle decays, as well as the rest of the underlying event, initial state radiation and final state radiation. The cross-sections of hard processes are performed using Monte Carlo integration. Parton showers are generated with Markov Chain MC, and fragmentation uses models to form partons and then hadrons. Final-state radiation is implemented with PHOTOS [53]. The decay and time evolution of the particles produced are handled by EvtGen [54]. The signal decay, and model to be used, are specified by a user decay file.

At this point, cuts are made requiring that particles will travel inside the LHCb acceptance, *e.g.* having $\theta \equiv \frac{|P_{x/y}|}{P_z} < 400 \text{ mrad}$. This is to minimise computation time spent on events that will not pass selection cuts applied later. The spatial position of the PV is smeared according to Gaussian distributions in x, y and z .

¹An event with no requirement about the content.

3.3.2 Simulation of propagation through the detector

The detector simulation involves a model of the LHCb detector geometry, tracking particles as they pass through the detector, and hit creation in the sub-detectors. The same reconstruction algorithms that are used for data are used on the simulated events. Truth information (the properties of the particles' trajectories in simulation, as opposed to how the particles are reconstructed) is also stored.

The detector is modelled using GEANT4 [55, 56], which also handles particle interactions with materials. The detector model is composed of many different detector components, for each, the size, location and material properties are specified. Particles are propagated in steps throughout the detector. At each step there are many possible processes that can occur such as scattering, emitting radiation and decay. All possible processes are calculated and chosen based on likelihood. Every particle track must be propagated through the detector, including those produced in secondary interactions such as Cherenkov photons in the RICH detectors and particle showers in the calorimeters.

The detector response is modelled through the Boole [57] digitisation stage where particle interactions and energy deposits are used to create 'hits'. The triggers used in data-taking are employed by Moore [58] in the simulation stage: L0, then HLT1, then HLT2. Events are reconstructed with Brunel [59]. The nTuples are created using DaVinci [60].

The current time taken to generate one event in Run 2 conditions is $\mathcal{O}(1\text{min})$.

3.3.3 Fast simulation techniques at LHCb

The proton-proton collision produces many simulated particles, and usually only a fraction of these particles are related to the signal decay of interest. The remaining particles contribute to the detector efficiency and resolution, and so

it is useful to retain these, but to find ways to avoid re-generating these in every simulated event.

3.3.3.1 ReDecay

ReDecay [61] is a fast simulation technique where a single PYTHIA 8 event is generated (that decays to the signal particle of interest). The signal decay is then discarded apart from the kinematics of the signal particle, and the particles in the underlying event are propagated through the detector. The signal particle kinematics and origin vertex are used repeatedly to generate $\mathcal{O}(100)$ times new signal decays, with the underlying event remaining the same for all these events. Only the signal decay products are propagated through the detector, and then merged with the persisted underlying event. The total event is digitised at the same time to ensure that the effects of the underlying effect on the performance of the detector are included. This ensures that the efficiencies and resolutions of this method are identical to the full simulation.

This method reduces the computation time per event by approximately 10–20 times. However, different events that stem from the same signal decay kinematics are correlated. The effect of this correlation on the simulation sample will depend on exactly which physical quantities of the decay of interest are studied. For decays where the kinematics of the final-state particles are highly constrained by the signal kinematics, the impact of the correlations in ReDecay simulation would need to be considered. However for multi-body decays, there is a lot of freedom in the kinematics of the final-state particles (which will decay each time via a different intermediate resonance) and thus the final-state particles are largely uncorrelated for events that share the same signal decay particle.

3.3.3.2 Particle Gun

Particle Gun is a production tool, used in place of PYTHIA, that directly generates the signal particle. The user specifies the kinematics of the signal particle, which can be sampled from a histogram. The particle's initial position is located at the PV (smearing can be applied if desired) and the signal decay proceeds as for the full simulation. None of the underlying event is propagated through the detector, leading to a generation times that are up to 50 times faster. The drawbacks of this method are that the occupancy of the detector is lower, leading to an over-optimistic picture of the detector resolution and efficiency. Variables affected by local occupancy may have to be weighted to look like Full Simulation or Data. In addition, as there is no underlying event it is not possible to trigger with a TIS stream (see Sec. 3.2.5.1).

3.4 LHCb Upgrades

In the years 2019–2023 LHCb underwent an upgrade, moving to a trigger completely performed by software. This allows the experiment to operate at an increased luminosity; the average number of pp interactions per bunch crossing is increasing from one (Run 2) to five. The luminosity delivered at LHCb will be $\mathcal{L} = 2 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$. In order to handle the more challenging environment, a considerable proportion of the entire detector was replaced, and the DAQ system also underwent a significant upgrade in order to read out the detector at the LHC bunch crossing rate of 40 MHz [62]. The VELO was upgraded to a new silicon pixel detector, which has better hit resolution and simpler track reconstruction. It will also sit closer to the beam axis. The TT was replaced by the Upstream Tracker, composed of silicon strip detectors in four layers. The new design will allow hits to be recorded closer to the beam pipe than previously. The downstream tracking (IT and OT) was replaced by the Scintillating

Fibre tracker, consisting of 12 layers of plastic fibre mats covering an area of $6 \times 5 \text{ m}^2$. The RICH1 detector layout is changed to reduce the occupancy, and both RICH detectors have new photon detectors (Multianode PMTs) that can readout at 40 MHz. The front-end electronics of the calorimeters and muon chambers have been replaced. This upgrade, LHCb Upgrade I, will collect data from 2022–2032 (with a planned break in 2026–2028), to collect an anticipated integrated luminosity of 50 fb^{-1} . Commissioning of the new experiment has been underway in 2022–2023 and physics data-taking is anticipated from next year.

Future upgrades are also planned, in particular for the high luminosity phase of the LHC (HL-LHC). LHCb Upgrade II is planned to be installed in the years 2033–2034 and will take data from 2035 until the end of the LHC programme. The highest instantaneous luminosity delivered at LHCb will be $\mathcal{L} = 1.5 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$, with the aim of collecting 300 fb^{-1} , with a pile-up of 40 [63]. LHCb Upgrade II will feature a VELO with a 4D tracking system in order to separate PVs in space and time, and other tracking sub-detectors will have higher granularity to decrease occupancy. New calorimeters will be employed that are more radiation hard, have $\mathcal{O}(10 \text{ ps})$ timing resolution and smaller cell sizes to reduce overlapping showers. The RICH detectors will continue to provide charged particle PID, albeit with new photodetectors with higher granularity and fast timing. In addition, the Time Of internally Reflected CHerenkov light (TORCH) detector is a proposed solution for identifying charged hadrons at momenta less than $20 \text{ GeV}/c$ to complement the RICH reach. The TORCH detector measures time-of-flight of particles from PV to the TORCH radiator, where Cherenkov radiation is produced and propagates to photodetectors on the periphery of the detector. Similarly to the RICH, TORCH relies on the development of photodetectors with high granularity and good time resolution. TORCH will be further discussed in Chapter 7.

4 | Selection of $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ candidates

In this chapter, the analysis strategy is presented, along with the selection that isolates $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ candidates. Multi-variate analysis techniques are used, and additional requirements are placed to remove specific backgrounds.

4.1 Analysis strategy

The analysis uses data collected during proton-proton collisions at LHCb during Run 2, exploiting the *prompt* production of D^0 mesons. The D^0 mesons are prompt if they originate from the decay of a charged D^* meson, in a $D^{*+} \rightarrow D^0 \pi^+$ decay. The charge of the pion is used to tag the flavour of the D^0 meson at production (the charge conjugate decay is $D^{*-} \rightarrow \bar{D}^0 \pi^-$). Due to the low energy release of the decay, the pion is produced with low momenta in the D^* rest frame and hence is referred to as the *slow* pion, π_s , to differentiate it from the pions produced in the D^0 decay. The difference between the reconstructed masses of the D^* and the D^0 ($m(D^*)$ and $m(D^0)$ respectively) provides a discriminating variable that is used to distinguish signal candidates from background. The distribution of $\Delta m = m(D^*) - m(D^0)$ peaks at around $145 \text{ MeV}/c^2$ for true prompt decays. All Run 2 data are analysed together, as the selection and reconstruction efficiency does not vary significantly over the data-taking period.

The phase-space integrated analysis measures the WS/RS ratio, R , in time bin j (repeated from Eqn. 2.29)

$$R_j = (r_D^{K3\pi})^2 - r_D^{K3\pi} (\kappa y')_D^{K3\pi} \left\langle \frac{t_j}{\tau_{D^0}} \right\rangle + \frac{x^2 + y^2}{4} \left\langle \left(\frac{t_j}{\tau_{D^0}} \right)^2 \right\rangle, \quad (4.1)$$

where $(\kappa y')_D^{K3\pi} = \kappa_D^{K3\pi} y'_{K3\pi}$. Both $x^2 + y^2$ and $(\kappa y')_D^{K3\pi}$ are fit as single pa-

rameters to allow for different interpretations of the measurement. The notation $\left\langle \frac{t_j}{\tau_{D^0}} \right\rangle \left(\left\langle \left(\frac{t_j}{\tau_{D^0}} \right)^2 \right\rangle \right)$ denotes the average (squared) decay time (in units of D^0 lifetime) of RS events in time bin j .

The phase-space binned measurement simultaneously measures r and $\kappa y'$ in bins of phase space i

$$R_{ij} = (r_i)^2 - r_i(\kappa y')_i \left\langle \frac{t_{ij}}{\tau_{D^0}} \right\rangle + \frac{x^2 + y^2}{4} \left\langle \left(\frac{t_{ij}}{\tau_{D^0}} \right)^2 \right\rangle, \quad (4.2)$$

where t_{ij} are the decay times of RS events in phase-space bin i and time bin j . The value of $x^2 + y^2$ is shared among all bins in the fit.

4.1.1 Efficiency considerations

A measurement of the CP -averaged ratio of decay rates is performed defined as (with time dependence implicit)

$$R \equiv \sqrt{\frac{\Gamma[D^0 \rightarrow K^+\pi^-\pi^+\pi^-]\Gamma[\bar{D}^0 \rightarrow K^-\pi^+\pi^-\pi^+]}{\Gamma[D^0 \rightarrow K^-\pi^+\pi^-\pi^+]\Gamma[\bar{D}^0 \rightarrow K^+\pi^-\pi^+\pi^-]}}, \quad (4.3)$$

which allows for the cancellation of some nuisance efficiency effects.

The expected number of events measured by LHCb is not exactly proportional to the decay rate due to production asymmetries and detection efficiencies. The expected number of events, N , in each category (split by D^0 flavour at production) is given by:

$$N_{D^0}^{WS} \propto \Gamma[D^0 \rightarrow K^+\pi^-\pi^+\pi^-] \times \epsilon^{WS}(K^+\pi^-\pi^+\pi^-\pi_s^+) \times P_{D^{*+}}, \quad (4.4)$$

$$N_{D^0}^{RS} \propto \Gamma[D^0 \rightarrow K^-\pi^+\pi^-\pi^+] \times \epsilon^{RS}(K^-\pi^+\pi^-\pi^+\pi_s^+) \times P_{D^{*+}}, \quad (4.5)$$

$$N_{\bar{D}^0}^{WS} \propto \Gamma[\bar{D}^0 \rightarrow K^-\pi^+\pi^-\pi^+] \times \epsilon^{WS}(K^-\pi^+\pi^-\pi^+\pi_s^-) \times P_{D^{*-}}, \quad (4.6)$$

$$N_{\bar{D}^0}^{RS} \propto \Gamma[\bar{D}^0 \rightarrow K^+\pi^-\pi^+\pi^-] \times \epsilon^{RS}(K^+\pi^-\pi^+\pi^-\pi_s^-) \times P_{D^{*-}}, \quad (4.7)$$

where $\Gamma[D \rightarrow f]$ is the decay rate of the D meson to the final state f . The production probability of the $D^{*\pm}$ is given by $P_{D^{*\pm}}$. The detection efficiency of different D^0 decays is given by $\epsilon^{\text{mode}}(f\pi_s^\pm)$, for WS or RS modes. Although the final states are very similar, differences in the kinematics of the final-state particles due to different decay dynamics can lead to differences in efficiencies between the WS and RS final states.

The final-state kinematics are correlated to D^0 kinematics and thus the slow pion kinematics, meaning that the detection efficiency of the final-state is not independent of the efficiency for detecting the slow pion. However, in Sec. B, it is shown that the final-state efficiency is independent of the slow pion charge, up to a constant factor. This factor, associated to the slow-pion charge asymmetry, is the same for both WS and RS final states.

If experimentally the ratio of decay rates, R , is constructed in the following manner,

$$R = \sqrt{\frac{N_{D^0}^{WS} \times N_{\bar{D}^0}^{WS}}{N_{D^0}^{RS} \times N_{\bar{D}^0}^{RS}}} \times \sqrt{\frac{\epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{RS} \times \epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{RS}}{\epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{WS} \times \epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{WS}}}, \quad (4.8)$$

then the slow-pion charge asymmetry and D^* production probabilities cancel. The efficiency of the RS and WS final states is further discussed in Sec. 5.3.3.

4.2 Online selection

The data collected by LHCb are subject to online selections that classify the recorded event into different streams, such that all events satisfying certain criteria are stored together, whilst at the same time only saving events of interest. These streams, known as *triggers*, have been generally described in Sec. 3.2.5. To isolate events that are prompt $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ candidates, the following triggers are used. At L0, the event is triggered by a hadron from the D^0 decay,

or the event is triggered independent of the D^0 decay by a hadron or lepton. At HLT1, at least one of the hadron tracks from the D^0 decay must pass the one track multivariate analysis decision, or at least one pair of tracks must pass the two track multivariate analysis decision. At HLT2, RS and WS decays are separately selected (based on the relative charge of the slow pion and the kaon). Requirements are placed on the kinematics of the final-state hadrons, and on their combinations.

4.3 Offline selection

The decay chain is fitted with the `DecayTreeFitter` (DTF) [64] algorithm that fits the entire chain at once, rather than a bottom-up ‘leaf-by-leaf’ approach. In this processing, constraints are applied on the D^* candidate such that it must originate and decay at the primary vertex (PV) (which is determined from particles not associated to the signal event). This is because the excited D^* meson has a small width (83.4 ± 1.8 keV [15]), and so decays very quickly. This constraint improves the resolution of the signal variable Δm . In addition, the reconstructed D^0 mass is constrained to its PDG value [15], which improves the resolution on Δm and the kinematics of the D^0 daughters.

Further cuts are applied to reduce the background fraction in the sample of candidates. An overview of the selection is presented here, with more details and justification for the selection given in later in the chapter.

Signal candidates must satisfy

$$\Delta m = m(D^*)_{\text{reco}} - m(D^0)_{\text{reco}} \in [139, 152] \text{ MeV}/c^2$$

and

$$|m(D^0)_{\text{reco}} - m(D^0)_{\text{PDG}}| < 24 \text{ MeV}/c^2,$$

which is approximately three times the resolution on the D^0 mass. The WS and RS data after only these cuts (and the triggers) are applied is shown in Fig. 4.1. The RS data are rather pure, however the WS data are dominated by background which consists of fake D^0 and fake D^* candidates.

To reduce the presence of decays from charm mesons that originate from B meson decays (*secondary events*), the decay-time range of signal events is restricted to $[0.24, 8]\tau_{D^0}$ and a cut is applied on the D^0 impact parameter χ^2 ($\chi^2_{IP}(D^0) < 9$).

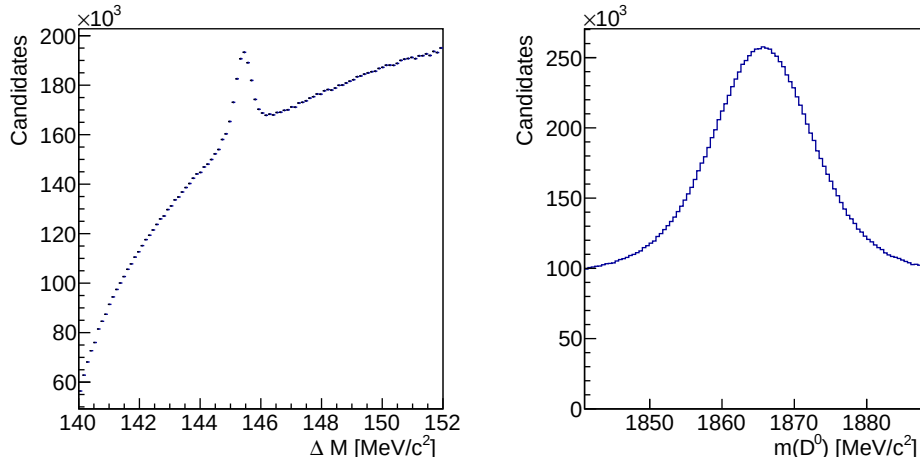
To reduce the impact of RS events that are reconstructed as WS events after the misidentification of a kaon and pion of opposite charge (known as *double mis-ID* events), requirements on the PID separation between kaons and pions for each D^0 decay-product are imposed. These requirements also help to reduce backgrounds from other D^0 decays such as $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ and $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$. The PID requirements on the D^0 decay products are:

$$\text{DLL}(K^-) > 8, \quad \text{DLL}(\pi^+) < 0, \quad \text{DLL}(\pi^-) < 5,$$

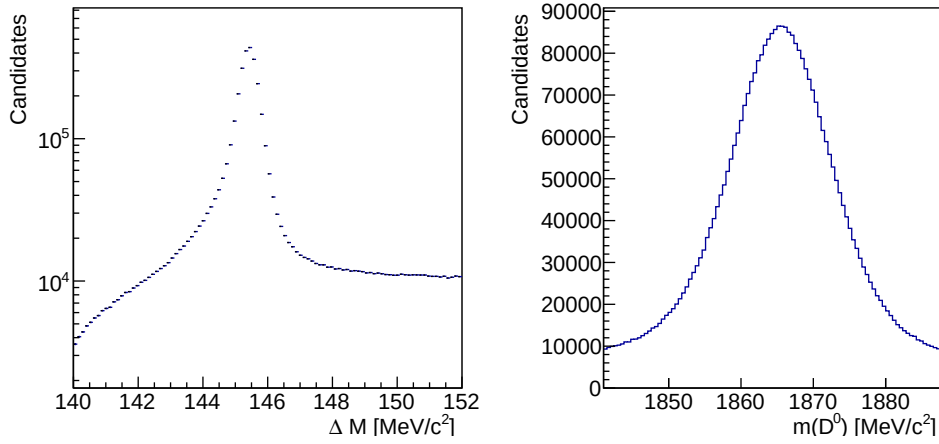
where DLL is the change in log-likelihood between kaon and pion PID hypotheses, and the charges are shown for a RS D^0 decay. The tight cuts in the first two requirements are in place to remove double mis-ID events, and the third requirement is imposed by the HLT2 line. There is no PID requirement on the slow pion.

The possibility that a reconstructed $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ event contains a track that is a *clone* (is reconstructed from hits that belong to another particle in the same event) is reduced by removing events where the track angle between any two pairs of tracks is less than 0.03° .

A background consisting of the slow pion coming from track that is a *ghost* (is reconstructed from unassociated hits from two or more particles, or background, in the upstream and downstream parts of the detector) is removed by



(a) WS data



(b) RS data

Figure 4.1: The distributions of (left) Δm and (right) $m(D^0)$ in (a) all 2018 *MagUp* WS data and (b) a sample of 2018 *MagUp* RS data after triggers and requirements on Δm and $m(D^0)$ are made.

a cut on the ghost probability of the slow-pion track ($\text{Prob}_{\text{ghost}}(\pi_s) < 0.05$), which is a high-level variable computed during reconstruction that depends on the tracking-only ghost probability and PID information.

Events with an opposite-sign dipion invariant mass, $m(\pi^+\pi^-)$, within $10 \text{ MeV}/c^2$ of the nominal K_S^0 mass are removed to minimise $D^0 \rightarrow K_S^0 K^\pm \pi^\mp$ background.

The dominant residual backgrounds consist of random slow pion background (a true D^0 matched with an unassociated pion) and combinatorial background (a fake D^0 composed of unassociated true $K^\pm \pi^\mp \pi^\pm \pi^\mp$ tracks). These backgrounds are evident as the continuum underneath the signal peak in the Δm distribution (see Fig. 4.1). A boosted decision tree is used to reduce the level of random slow pion and combinatorial backgrounds. The background level from a true D^* decay with a true $D^0 \rightarrow X$ decay, with $X \neq K^\pm \pi^\mp \pi^\pm \pi^\mp$ is small.

Multiple candidates, two or more candidates which originate from the same event (identified by the run number and event number stored in the nTuples), of the same sign are treated by randomly selecting one of the candidates. This removes 1.4% of RS candidates and 2.0% of WS candidates (in the range $140 < \Delta m < 152 \text{ MeV}/c^2$). In addition, the same event can be reconstructed as both a RS and a WS candidate. In this case, as the chance of being a true RS candidate is much more likely, the WS candidate is removed. This selection is performed after all other criteria are applied and before the mass fit. This removes 10% of WS candidates (whose distribution in Δm is background-like).

4.4 Boosted Decision Tree

A gradient boosted decision tree (BDT) [65] is employed to increase the signal purity of the WS data. Simulated events (generated using a phase-space model and accounting for different LHCb running conditions) are used as the signal proxy in the training of the BDT (further details about the simulation samples

used are given in Section 5.3.1). To generate a sample of simulation events that are as signal-like as possible the *sPlot* technique [66] is used to subtract the contribution from background events in the sample (truth-matching is not used as the efficiency in some samples is very low).

The *sPlot* technique is a method used to separate signal and background distributions of a given ‘control’ variable in a mixed data sample. Firstly a maximum likelihood fit is performed on a discriminating variable to obtain the yields of signal and background events in the data. The discriminating variable used should be one where it is easy to model the signal and background components and is uncorrelated to the control variables which are being unfolded. Here, the discriminating variable is Δm in the region $140 < \Delta m < 152 \text{ MeV}/c^2$. The fit uses signal and background models as described in Section 5.2. Next the *sWeights*, the set of covariance-weighted quantities used to weight a control variable to produce the signal distribution, are calculated. The *sWeights* are applied to the distributions of the variables used to train the BDT.

The background sample used in the BDT training comprises RS data events with $152 < \Delta m < 157 \text{ MeV}/c^2$ (away from the Δm signal and thus composed of a random slow pion) and $40 < |m(D^0) - m(D^0)_{\text{PDG}}| < 64 \text{ MeV}/c^2$ (far from $m(D^0)$ signal and thus composed of combinatorial background). Events in the low Δm sideband are not used as the yield is not large and the region is close to the signal peak. Around 160,000 signal and 160,000 background events are used to train the BDT, and the same numbers for the testing.

The discriminating variables used to train the BDT, in order of importance, are

- D^* DTF fit χ^2 ,
- $p_{\text{T}}(D^0)$,
- $\text{Prob}_{\pi}(\pi_s)$,

- D^0 decay vertex χ^2 ,
- $\chi^2_{\text{IP}}(\pi_s)$,
- D^0 origin vertex χ^2 ,

with D^* DTF fit χ^2 having more influence than the other variables which have approximately equal importance. The BDT works to keep the efficiency uniform in D^0 decay time.

The uGradientBoosting BDT and knn-Ada loss function from the hep-ml Python library [67] are employed as they are designed to produce an outcome that favours a uniform selection efficiency in a specified variable. The knn-Ada loss function accounts for the k -nearest neighbours of each event when calculating the loss gradient, thus leading to a more uniform response over all the events [68]. Figure 4.2a illustrates the relationship between the efficiency of the BDT and the lifetime of D^0 , indicating that the uniformity falls short of being entirely successful. Different numbers of nearest neighbours and other training parameters are scanned to find the optimal training setup.

The performance of the BDT is evaluated using the ROC AUC (Receiver Operating Characteristic Area Under Curve). The ROC AUCs for weighted training and testing samples are 0.91 and 0.89 respectively, where a value of 0.5 means the signal and background are indistinguishable and a value of 1 means the samples are perfectly separable. Figure 4.2b compares the BDT outcome distributions for signal and background training and testing samples. There is good separation between signal and background predictions and good agreement between testing and training samples. The ROC AUCs and BDT outcomes show that the BDT is not being overtrained and that its performance is high.

The effect of the BDT on the purity of the WS data sample is shown for different BDT cuts in Figure 4.3a. The optimisation of the BDT cut maximises the signal significance in WS data defined by $\frac{s}{\sqrt{s+b}}$ where $s(b)$ is the number of

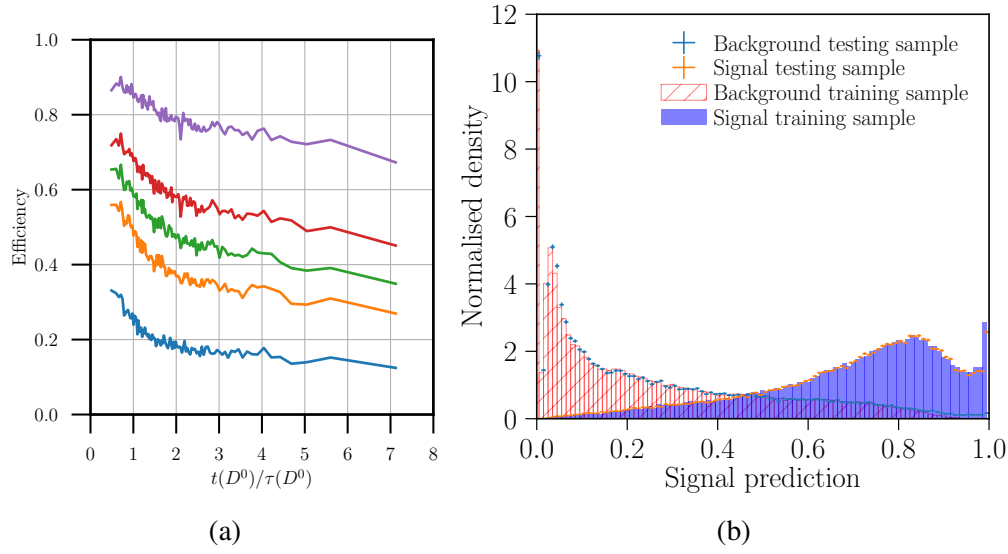


Figure 4.2: (a) The efficiency of the BDT as a function of D^0 decay time. The colours [purple, red, green, orange, blue] represent the time-integrated BDT thresholds of [0.8, 0.6, 0.5, 0.4, 0.2] respectively. (b) The BDT signal prediction values for events in the assigned signal and background regions for the training and testing data samples.

signal (background) candidates found within $\pm 0.9 \text{ MeV}/c^2$ of the signal peak in a simultaneous mass fit to the global sample (described below). The optimised value is 0.4 as shown in Figure 4.3b (using a sample of 2018 *MagUp* data), thus only events with BDT score greater than 0.4 are retained in the sample.

The WS and RS data after all signal cuts and BDT are applied are shown in Fig. 4.4, where the purity of the WS sample is clearly improved relative to Fig. 4.1. Table 4.1 gives approximate signal yields (of the 2018 *MagUp* sample) and the purity of the samples as different selections are applied.

4.5 Backgrounds

The dominant background is the random slow pion background followed by combinatorial backgrounds. The next chapter describes how a mass fit is used to separate signal and background contributions. Some backgrounds may be

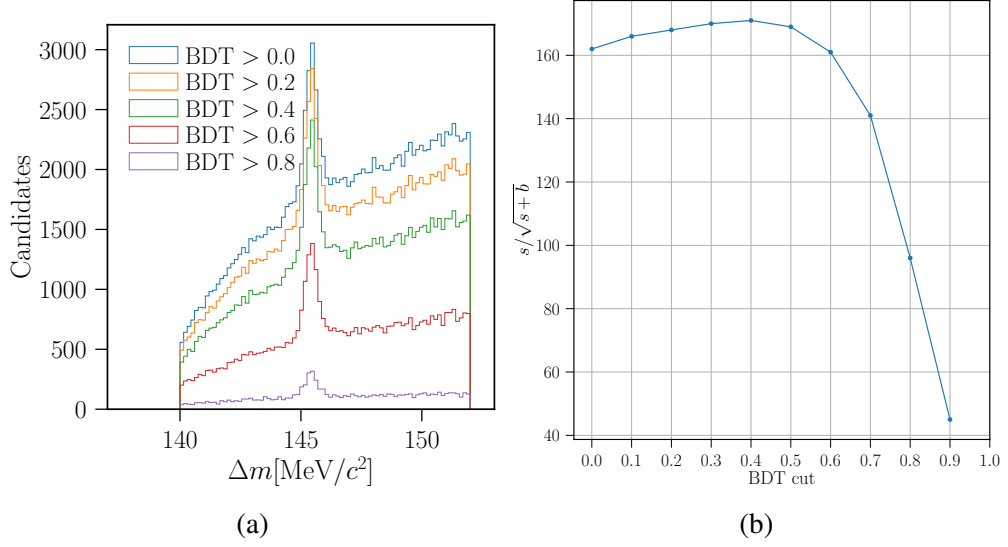


Figure 4.3: (a) The Δm distribution and (b) the signal significance in 2018 *MagUp* WS data for different BDT working points.

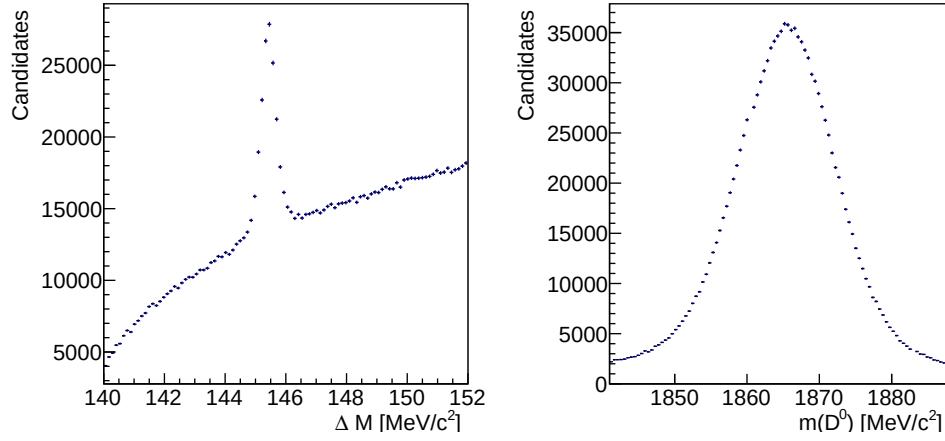
Table 4.1: The signal yields and purity of the 2018 *MagUp* samples as the selection is applied. The RS sample is dominated by signal throughout, whereas the WS sample is dominated by background. The purity of the sample is defined by the ratio of the signal yield to the total yield.

Selection	WS		RS	
	Yield	Purity (%)	Yield	Purity (%)
Triggers, Δm , and $m(D^0)$ cuts	0.26×10^6	1.7	63×10^6	79
Additional cuts (not BDT)	0.13×10^6	4.2	33×10^6	89
All selection, including BDT	0.08×10^6	5.9	23×10^6	93

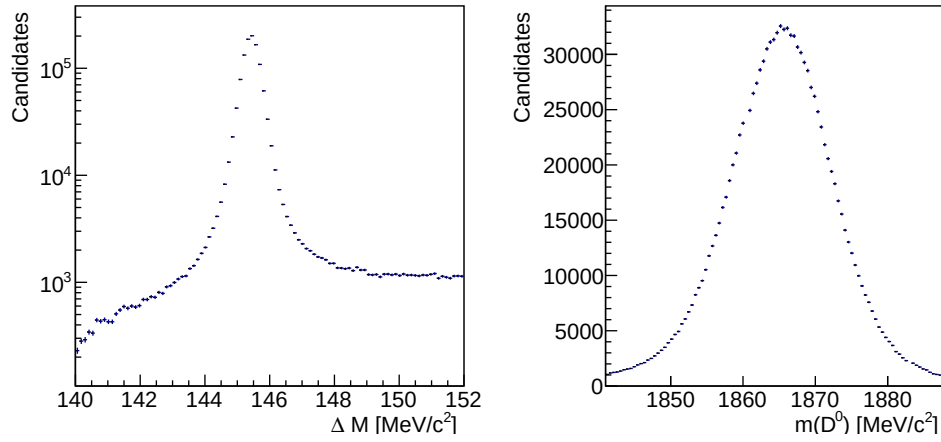
indistinguishable from signal in the mass fit and thus bias the measured signal yields. These are discussed in the following sections.

4.5.1 Secondary events

‘Secondary’ decays of the form $\bar{B} \rightarrow D^{*+}X$, $D^{*+} \rightarrow D^0\pi^+$ can bias the time-dependent measurement of the WS to RS ratio in prompt data. If secondary decays are reconstructed as prompt decays then the reconstructed D^0 decay time is systematically larger than their true decay time as this is measured from the



(a) WS data



(b) RS data

Figure 4.4: The distributions of (left) Δm and (right) $m(D^0)$ in (a) all 2018 *MagUp* WS data and (b) a sample of 2018 *MagUp* RS data, after the signal selection has been applied.

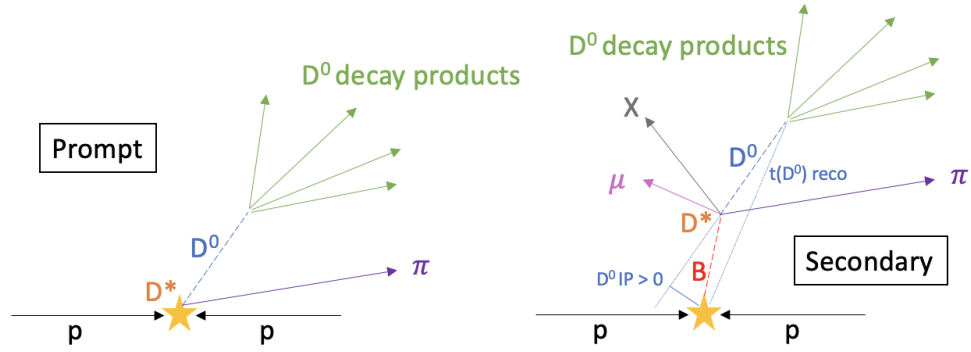


Figure 4.5: A schematic showing the prompt and secondary production channels and topologies.

flight distance from the PV (see Fig. 4.5). This not only affects the distribution of events in decay time, but also sculpts the Δm distribution. This is because the `DecayTreeFitter` forces the D^* to originate from the primary vertex and so the opening angle between the D^0 and π^+ is reduced and thus the Δm distribution is shifted to lower values.

The upper decay time limit at eight D^0 lifetimes and the requirement that $\chi_{\text{IP}}^2(D^0) < 9$ reduces the presence of this background in the sample considerably (as well as the BDT (see below)), but some secondary events still remain. It is possible to correct for the bias on the D^0 decay time if the fraction of secondary decays (and their true D^0 decay time) in each time bin is known. The distribution of $\log(\chi_{\text{IP}}^2(D^0))$ is used as a discriminating variable to measure the fraction of secondary decays in the prompt sample.

Figure 4.6 shows distributions of $\log(\chi_{\text{IP}}^2(D^0))$ for the lowest and highest time bins of a RS data sample, before and after the BDT threshold is applied. The BDT, dependent on the behaviour on D^* DTF fit χ^2 (where the DTF constrains the D^* to be produced at the PV), favours true prompt events over secondary events in the sample. However, secondary events can still pass the BDT requirement, and will have a distorted $\chi_{\text{IP}}^2(D^0)$ distribution due to the BDT.

Thus, the measurement of the secondary yields present in the prompt data

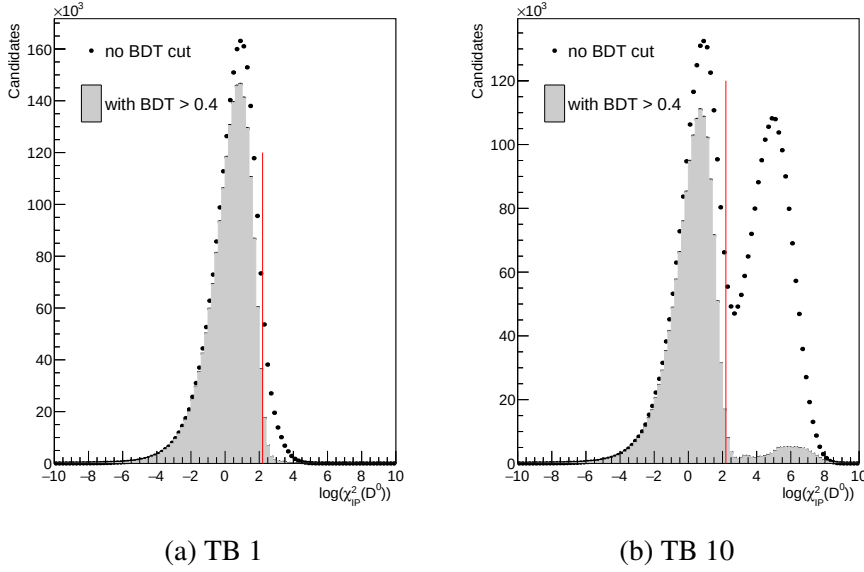


Figure 4.6: The $\log(\chi_{\text{IP}}^2(D^0))$ distribution in 1/10th Run 2 RS data before (black points) and after BDT selection (grey histogram) for (a) the lowest time bin and (b) the highest time bin. The lowest time bin shows little presence of secondary events as expected. The right-hand peak in the highest time bin shows the secondary events, which are largely removed by the BDT selection. The cut at $\chi_{\text{IP}}^2(D^0) < 9$ is indicated with the red line, showing that there is only a small contribution of secondary events to the selected prompt sample.

sample is conducted on RS data that does not have either the $\chi_{\text{IP}}^2(D^0) < 9$ or BDT cut applied. A cut around the signal peak ($142 < \Delta m < 146 \text{ MeV}/c^2$) is applied, and a sideband-subtracted distribution of $\log(\chi_{\text{IP}}^2(D^0))$ is generated by removing the measured sideband contribution from $149 < \Delta m < 152 \text{ MeV}/c^2$ (calculated from a Δm fit to the low time bins).

The $\log(\chi_{\text{IP}}^2(D^0))$ data distribution is fitted per decay time bin with a signal PDF and a secondary PDF convolved with a Gaussian (to account for mismodelling of the PV resolution in the secondary MC sample) :

- Signal PDF: an Extended Cruijff function (see Sec. 5.2) with the μ and β parameters fixed from a fit to the highest time bin, and confirmed with a fit to the lowest time bin. The two width parameters (σ_L/σ_R) and the tail

parameters (α_L/α_R) are free to float.

- Secondary PDF: a smoothed PDF derived from a template. The template is generated from Particle Gun cocktail samples of $B^\pm$ and B^0 decays. The samples are combined with weights according to the $B^\pm$ and B^0 production cross-sections, and in addition the sample is reweighted to the p_T (D^0) data distribution (to account for the L0 trigger).
- Gaussian: the mean and width are fixed from a fit to the highest time bin.

Per time bin, the yields of the Signal and Secondary PDFs are free to float. The data distributions and fitted models are shown in Fig. 4.7. The fit to the lowest decay time bin is hard to perform, so instead of assessing the secondary fraction from the fit the secondary fraction is fixed to zero. The fit with the Signal PDF alone is used to check that the signal peak in data is consistent across the decay-time range (required as the parameters of the Signal PDF are fixed from the fit to the highest time bin). After, the secondary fraction in this decay time bin is assigned a non-zero value, based on extrapolation of the signal fractions in the other decay time bins.

After the fit is performed, the secondary fraction is calculated by measuring the yield of the secondary and signal events in the region $\chi^2_{\text{IP}}(D^0) < 9$. This is plotted per time bin in Fig. 4.8a. The true decay-time distribution of secondary events is calculated from events in the secondary MC sample that satisfy $\chi^2_{\text{IP}}(D^0) < 9$. This is plotted against the mean reconstructed decay time of events in the signal sample in Fig. 4.8b (this ignores any migration across the $\chi^2_{\text{IP}}(D^0) = 9$ boundary due to the convolution).

To assess the impact of the BDT on the signal fraction, both the secondary template yields and data yields in the region $\chi^2_{\text{IP}}(D^0) < 9$ are measured before and after the BDT threshold is applied. The BDT cut reduces the yield by a similar fraction for both samples, meaning the secondary fraction remains ap-

proximately constant. As such, no further changes are made to the measured secondary fractions to account for the BDT.

With knowledge of the secondary fraction and the true decay time of the secondary events per time bin, the presence of secondary events within the prompt sample can be accounted for by altering the theoretical parametrisation that the data are fit to.

According to Eqn. 2.29, the theoretical WS/RS ratio, R , (in time bin j) of prompt events (with no biases due to secondary events) is

$$R_j = \frac{N_j^{WS,prompt}}{N_j^{RS,prompt}} = a + b\langle t_j \rangle + c\langle t_j^2 \rangle, \quad (4.9)$$

where the yield N is indicative of the decay rate after efficiency corrections have been applied. For simplicity, the factors τ_{D^0} have been removed and coefficients have been replaced by a, b, c , and the superscript ‘prompt’ is used to represent the prompt yields.

The data sample actually consists of prompt and secondary (denoted by ‘sec’) events. Provided that the efficiency of reconstructing true secondary events as prompt events is the same for WS and RS candidates, *i.e.* $\frac{N_j^{WS,sec}}{\Gamma_j^{WS,sec}} = \frac{N_j^{RS,sec}}{\Gamma_j^{RS,sec}}$, the ratio of WS/RS prompt and secondary yields is denoted by R' :

$$\begin{aligned} R'_j &= \frac{N_j^{WS,prompt} + N_j^{WS,sec}}{N_j^{RS,prompt} + N_j^{RS,sec}} \\ &= R_j(1 - f_j^{sec}) + R_j^{sec} f_j^{sec}, \end{aligned} \quad (4.10)$$

where

$$f_j^{sec} = \frac{N_j^{RS,sec}}{N_j^{RS,prompt} + N_j^{RS,sec}}, \quad (4.11)$$

and

$$R_j^{\text{sec}} = \frac{N_j^{WS,\text{sec}}}{N_j^{RS,\text{sec}}} \quad (4.12)$$

$$= a + b\langle t_j^{\text{sec}} \rangle + c\langle (t_j^{\text{sec}})^2 \rangle. \quad (4.13)$$

Hence, the data are fitted to R'_j in the mixing fit; more details are given in Sec. 6.1.1.

4.5.2 Double mis-ID

This background comes from true RS events that have been reconstructed with the pion and kaon ID swapped such that they are reconstructed as WS events. These events have a broad distribution in $m(D^0)$, but peak in Δm . The nominal cut of $|m(D^0) - m(D^0)_{\text{PDG}}| < 24 \text{ MeV}/c^2$ reduces the presence of this background but does not remove all of it.

To evaluate the size of this background the WS data are processed with the default method, but with a larger range of allowed $m(D^0)$ values ($|m(D^0) - m(D^0)_{\text{PDG}}| < 64 \text{ MeV}/c^2$), and with an extended Δm range ($140 < \Delta m < 158$). Two swapped $m(D^0)$ hypotheses ($m(D^0)_{\text{swap}}^{1,2}$), one for each of the same sign pions, are constructed where the pion and kaon ID hypotheses (masses) are switched. When calculating the value of $m(D^0)_{\text{swap}}^{1,2}$ per event, it is important to use track momenta that have not been recalculated in the kinematic fit (DTF) as this distorts the swapped mass distributions relative to the $m(D^0)$ distribution.

In addition, the Δm distribution is not used as the signal distribution because this is constructed from the assumption that the pion and kaon have been correctly identified. Instead the Δm^* distribution is used, where Δm^* is the Δm constructed from a $m(D^0)$ that uses the PDG D^0 mass instead of the energies of the final state particles that are PID-dependent. This means that the Δm^* variable is independent of the mass hypothesis assigned to the D^0 decay products. It

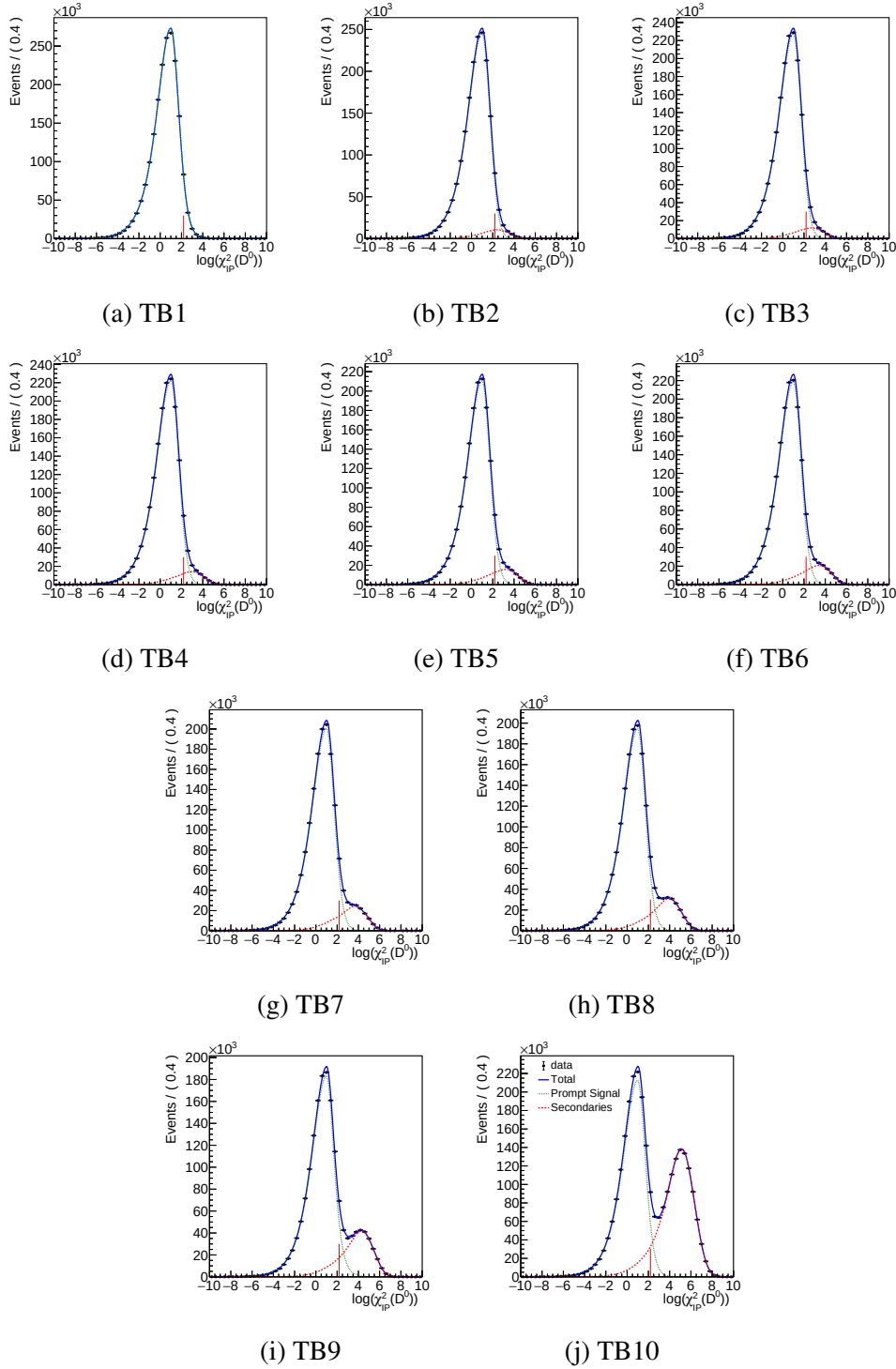


Figure 4.7: Fits per time bin to measure the secondary yield in the phase-space integrated data sample (with no BDT applied). The secondary fraction is measured in the region to the left of the vertical red line (at $\chi^2_{\text{IP}}(D^0) = 9$).

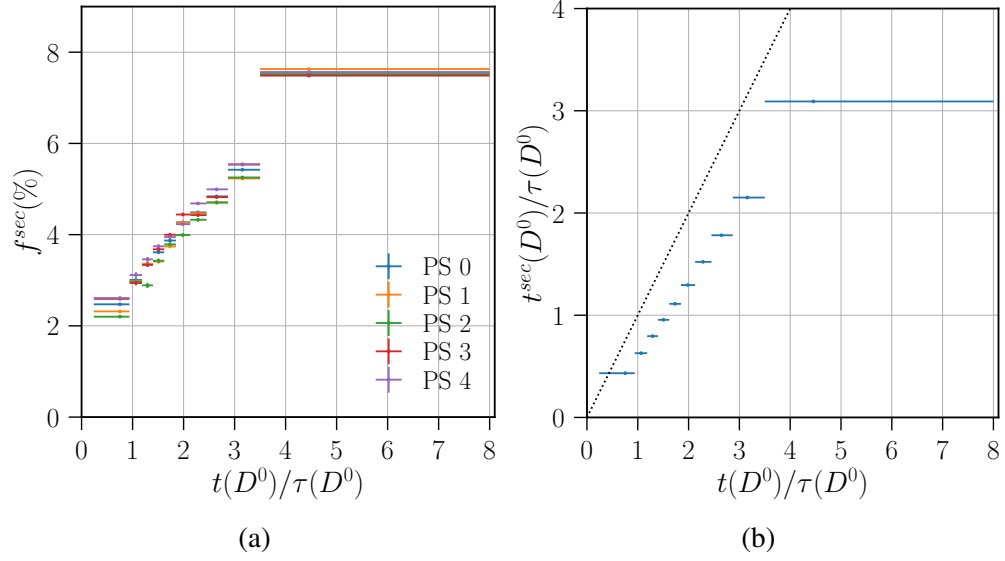


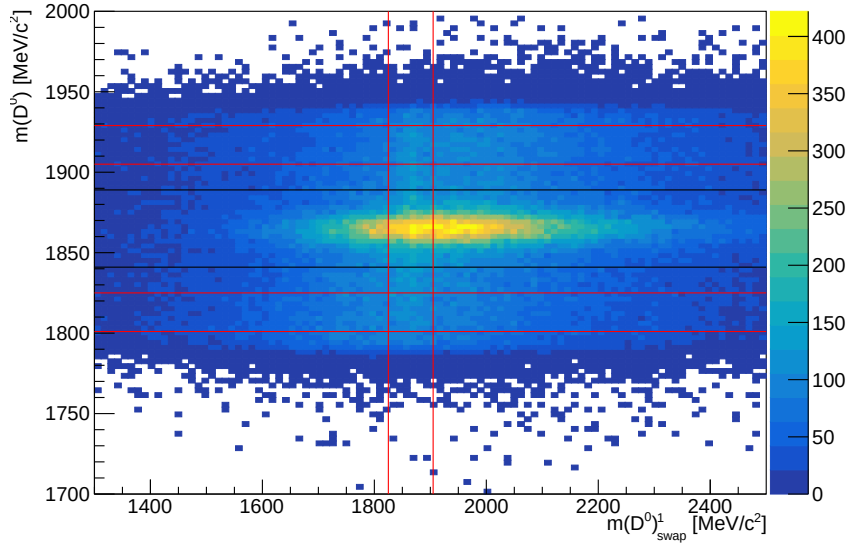
Figure 4.8: (a) Fraction of secondary decays per phase-space bin in 1/10th of Run 2 RS data (with no BDT applied), where the TB1 fraction has been extrapolated. (b) True time distribution of secondary decays in phase-space integrated sample, obtained from the secondary MC sample (with no BDT applied).

is for this reason that the Δm cut is loosened, so that the high Δm^* distribution is not distorted by event migration around the upper cut. It was checked that no events from $152 < \Delta m < 158 \text{ MeV}/c^2$ would appear in a Δm^* signal peak.

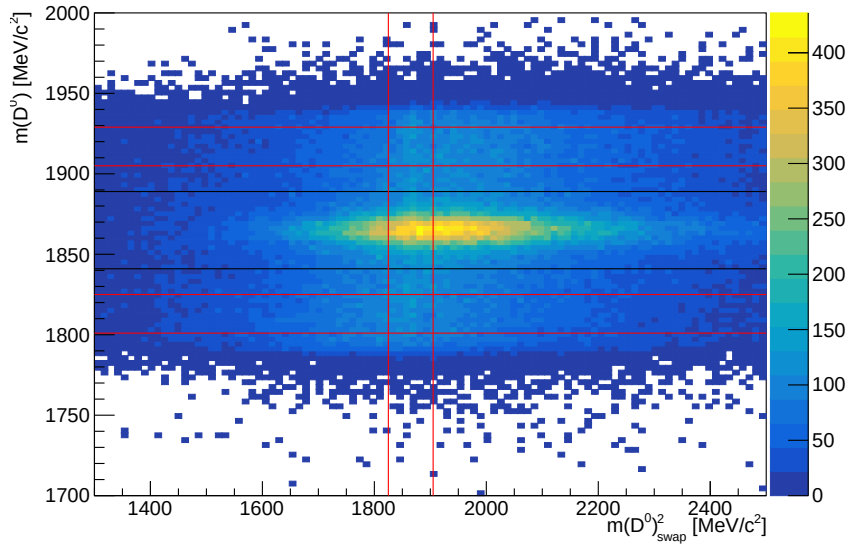
The double mis-ID background yield is measured in the $m(D^0)$ sideband region, then extrapolated into the signal region of $m(D^0)$ under the assumption that the double mis-ID yield varies uniformly over $m(D^0)$. The sideband and signal region both have a width of $48 \text{ MeV}/c^2$ so the extrapolation of yields is 1:1. Figure 4.9 shows the 2D $m(D^0) - m(D^0)_{\text{swap}}$ plane, with signal and sideband regions highlighted.

Specifically we study the Δm^* distribution of WS candidates with the following cuts applied:

- $40 < |m(D^0) - m(D^0)_{\text{PDG}}| < 64 \text{ MeV}/c^2$,
- $|m(D^0)_{\text{swap}}^{1,2} - m(D^0)_{\text{PDG}}| < 40 \text{ MeV}/c^2$,



(a) First swapped mass hypothesis



(b) Second swapped mass hypothesis

Figure 4.9: The 2D $m(D^0) - m(D^0)_{\text{swap}}$ plane, for (a) $m(D^0)^1_{\text{swap}}$ (b) $m(D^0)^2_{\text{swap}}$, in a sample of 2018 *MagUp* WS data. The region between the two vertical red lines is the region in $m(D^0)_{\text{swap}}$ used to measure double mis-ID. Vertical bands in the data here indicate the presence of double mis-ID candidates. The region between each of the sets of horizontal red lines indicates the upper and lower sidebands in $m(D^0)$, whilst the black lines enclose the nominal signal region.

- $\text{BDT} > 0.4$,
- $0 < t(D^0)/\tau(D^0) < 8$, (or split by time bin)
- $139 < \Delta m^* < 152 \text{ MeV}/c^2$ (and $139 < \Delta m < 158 \text{ MeV}/c^2$).

To accommodate both requirements in the second bullet, the entire data sample is used twice, once with the $m(D^0)_{\text{swap}}^1$ condition satisfied and once with the $m(D^0)_{\text{swap}}^2$ condition satisfied. The Δm^* distributions from both samples are combined into a single distribution before fitting.

The double mis-ID background is present if a peak in Δm^* is seen. If seen, it is fit with a Gaussian for the signal peak and the RooBkg PDF (see Sec. 5.2). The Δm^* distribution in a sample of WS data in the $m(D^0)$ sidebands is shown, with fits superimposed, for different PID cuts in Fig. 4.10. The presence of the double mis-ID background is clear, with enhanced yields in the vertical bands.

4.5.2.1 Different PID cuts

It is expected that tightening the PID requirements of the pions and kaon will decrease the double mis-ID yield, but will also reduce the signal yield. There are irreducible PID cuts within the HLT2 line (referred to as *baseline* cuts). The PID requirements are varied to see how the signal and double mis-ID background yields vary. A summary of the estimated double mis-ID contamination for different PID cuts is given in Table 4.2. Tighter cuts than those imposed by the HLT2 line are only applied to the pions of opposite sign to the kaon, as it is these that contribute to double mis-ID. From this, the PID cuts applied in the offline selection are determined (the ‘tight’ cuts are chosen). The choice is a balance between maximising signal yields and minimising the systematic uncertainty associated with the removal of these candidates, due to the uncertainty on the measured mis-ID yields.

Some double mis-ID still remains after the PID cuts are applied. The yields

per time and phase-space bin measured using this method are subtracted from the measured WS signal yield. The systematic uncertainty associated with this subtraction is discussed in Sec. 6.2.4.

Table 4.2: The double mis-ID yield, and its fraction of the total WS Run 2 signal yield, as a function of the PID requirements stated.

PID cut	DLL(K^+)	DLL($\pi_{1,2}^-$)	DLL(π^+)	Mis-ID yield	% of signal
Baseline	> 5	< 5	< 5	87721 ± 464	14.3 ± 0.1
Tight	> 8	< 0	< 5	12660 ± 299	2.7 ± 0.1
Very tight	> 10	< -1	< 5	6489 ± 301	1.5 ± 0.1

4.5.2.2 Double mis-ID yields as a function of time and phase space

We are interested in the variation of the WS double mis-ID fraction per time bin and phase-space bin. A priori, the double mis-ID yield is proportional to the RS yield, and thus will have a different time- and phase-space dependence to that of the WS sample. In addition, the PID classification can depend on the momentum of the tracks, which can be correlated to the accepted D^0 decay time. Using the full Run 2 sample, the WS double mis-ID yield per time and phase-space bin is measured using the fit method described above. In addition the sample is split by kaon charge to correct WS D^0 and $\bar{D}^0$ yields separately.

Figure 4.11a shows the flavour-integrated, phase-space integrated mis-ID yields measured per time bin. The double mis-ID yield is compared to the RS yield per time bin, shown in Fig. 4.11b. From this, it is confirmed that the double mis-ID yields have the same time dependence as the RS decays (which is as expected as they are RS events), and it does not seem that any reconstruction or selection effects are biasing this dependence. The mis-ID yield per time and phase-space bin (flavour-integrated) is shown in Fig. 4.11a. The behaviour across all the phase-space bins is found to be consistent, and all bins follow a similar trend to that seen in the phase-space integrated yields. The

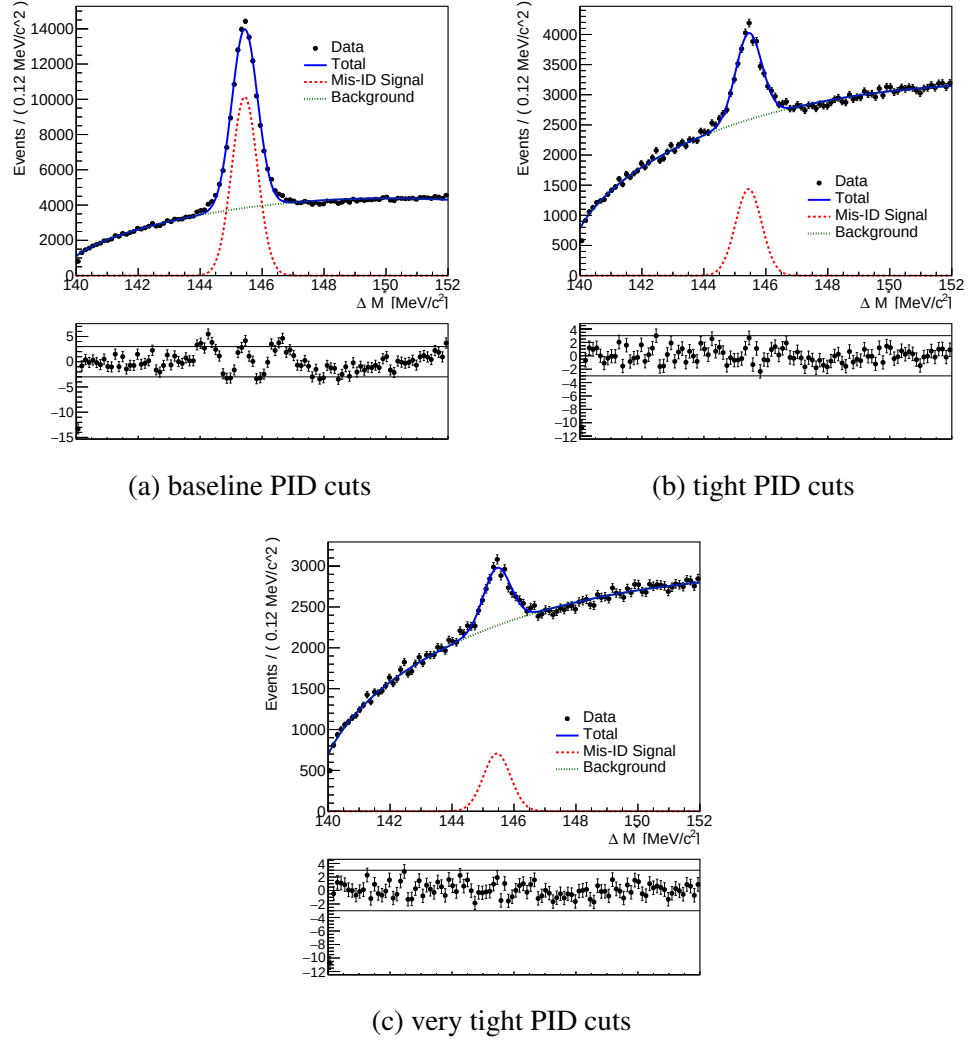


Figure 4.10: The Δm^* distribution of Run 2 WS data in the $m(D^0)$ sideband, with different PID cuts. The fit measures the double mis-ID yield.

time-integrated phase-space binned yields of the mis-ID background are compared to the RS signal yields per phase-space bin in Fig. 4.11c. This also shows that the mis-ID yields are consistent with the yields from RS decays, with no additional reconstruction or selection effects visible.

When correcting the measured WS yields to account for the double mis-ID yields, corrections are taken directly from the yields measured in fits to phase-space and time-binned data. The double mis-ID yields from (phase-space integrated) samples split by flavour are shown in Fig. 4.11d. There are no significant differences in the yields from D^0 and $\bar{D}^0$ samples.

4.5.3 Clones

A search for candidates with clone tracks (where two of the reconstructed final-state tracks are actually from the same single particle) is performed by studying the angle between tracks in the same event. Clones appear as pairs of tracks which have a very small angle between them. A peak is observed at angles very close to 0° , indicating the presence of clone tracks that must be removed. A cut has been added to the selection, such that the minimum angle between two tracks (of all final state particles) in a candidate event must be greater than 0.03° . The distribution in data of the angles between tracks and the cut imposed are shown in Fig. 4.12.

4.5.4 Slow-pion ghosts

Signal candidates comprised of a true D^0 matched with a slow-pion track which is actually a ghost track can be present in the data, formed from true clusters in the VELO and incorrectly assigned clusters in the TT. This leads to the momentum and charge assigned to the slow-pion track to be wrong, but often with the correct direction so the candidates can peak in the Δm distribution, perhaps incorrectly tagging the D^0 flavour at production. It is expected that the absolute

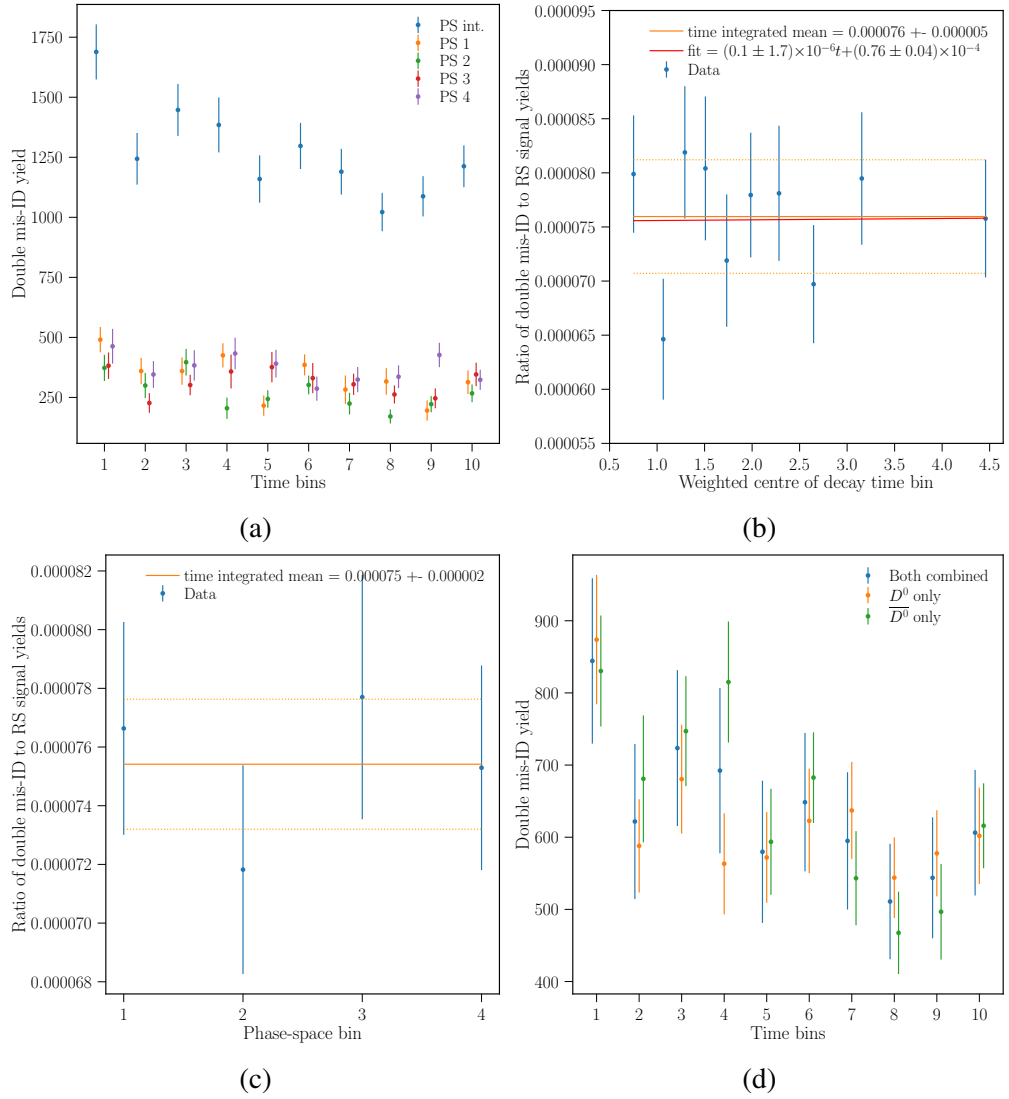


Figure 4.11: (a) The double mis-ID yield per time bin and phase-space (= PS) bin from Run 2 WS data sidebands, employing tight PID cuts (uncertainties are statistical only). (b) The double mis-ID yield divided by the measured Run 2 RS signal yield as a function of decay time. Here, the data are super-imposed with the time-integrated mean calculated from the data and a fit as a function of decay time. The fitted slope is consistent with zero and so confirms that the time dependence is like that of the RS sample. (c) The time-integrated mis-ID yield relative to the Run 2 RS signal yield per phase-space bin, with tight PID cuts. (d) The double mis-ID yield per time bin, split by flavour, from all Run 2 WS data, employing tight PID cuts (uncertainties are statistical only). Additionally, the results of the fit to the flavour-integrated sample are shown, with the yields halved to make comparison easier (the uncertainties are not halved).

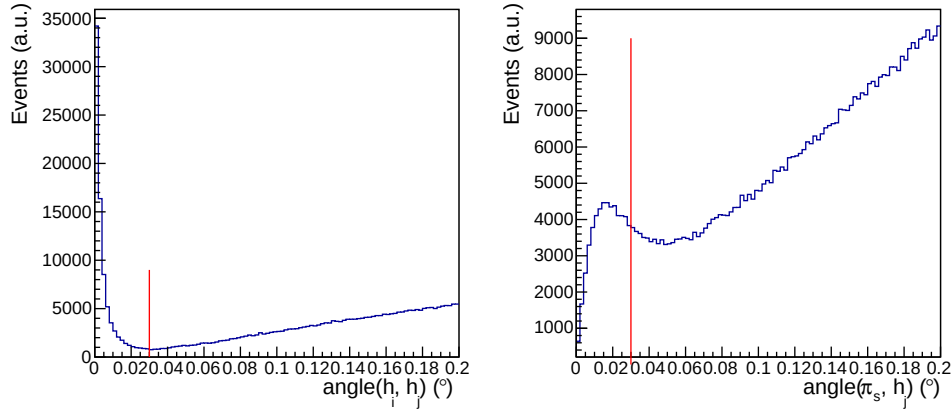


Figure 4.12: (left) The angle between tracks of particles from the D^0 decay (h_j) and (right) the angle between the slow pion track and particles from the D^0 decay in a 2018 *MagUp* RS data sample. The peak at zero indicates the presence of clones, with small angles between pairs of tracks. The red line indicates the cut in place.

yield of WS and RS candidates with slow-pion ghosts is equal (because every true candidate can be reconstructed as a ghost with either positive or negative charge), and thus slow pion ghost candidates will bias the WS/RS ratio.

There are four different categories of slow-pion ghost candidates:

1. a ghost candidate where the true same-sign candidate is in the signal sample;
2. a ghost candidate where the true opposite-sign candidate is in the signal sample;
3. a ghost candidate where the true same-sign candidate is not in the signal sample;
4. a ghost candidate where the true opposite-sign candidate is not in the signal sample.

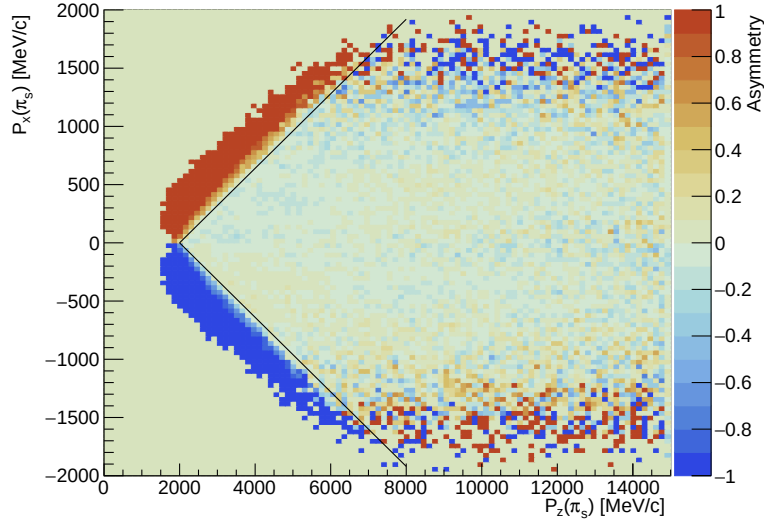
In all cases, a cut on the slow-pion ghost probability (discussed below) will reduce the fraction of ghost candidates in the sample. For category 1, the random

choice of a multiple candidate will reduce the ghost rate by half. For category 2, RS ghosts from true WS events will occur at a very low rate compared to the RS signal rate. WS ghosts from RS events have the potential to bias the WS/RS ratio and thus WS candidates originating from the same event as a RS candidate are removed. For categories 3 and 4, it is necessary to consider why the true candidate does not pass the selection; either the true slow pion does not pass the p_T requirements in the trigger, or the true pion passes out of the acceptance. The former needs no further correction as the ghost candidate that passes selection must have a pion of higher momentum than the true momentum and so the Δm will not be signal-like. Ghost candidates of the latter case can be removed by applying fiducial cuts to remove regions with high slow-pion asymmetry.

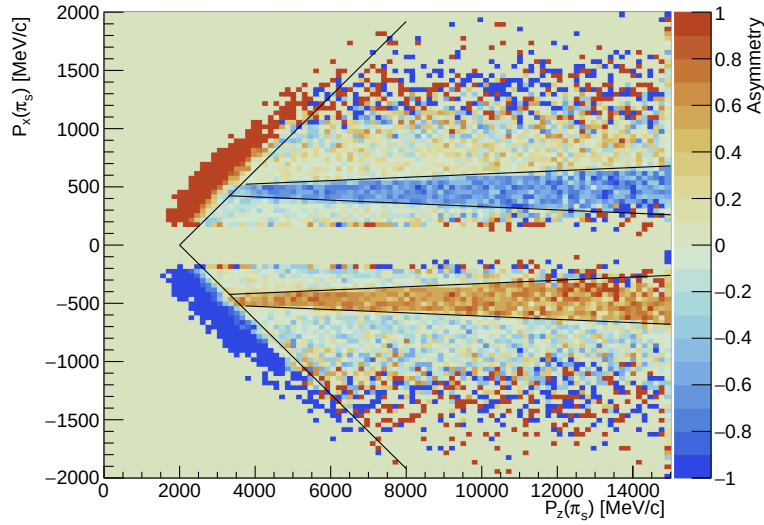
4.5.4.1 Fiducial cuts

The slow-pion asymmetry, $\frac{N(\pi_s^+) - N(\pi_s^-)}{N(\pi_s^+) + N(\pi_s^-)}$, is studied as a function of the slow-pion momentum components. Regions of phase space with low p_z are at the edge of the LHCb acceptance, and thus prone to asymmetries as particles are either swept in or swept out of the acceptance depending on their charge (due to the magnetic field). Further, particles with low p_y can be swept into the beampipe and out of the minimum angular acceptance, leaving areas of π^+ and π^- depletion on different sides of the beampipe for a certain magnet polarity. Combining data collected with opposite magnet polarity reduces the observed asymmetry, but in order to not rely on having a perfectly symmetric detector and exactly equal integrated luminosities per magnet polarity period, regions in phase space of high asymmetry are removed.

Figure 4.13 shows the asymmetry across the two-dimensional plane of slow-pion momentum components, $p_z(\pi_s) - p_x(\pi_s)$, for trajectories near the beam-pipe and away from the beam-pipe. Requirements are placed to remove regions of high asymmetry (indicated by black lines in Fig. 4.13).



(a)



(b)

Figure 4.13: The asymmetry of slow pions in $p_z(\pi_s) - p_x(\pi_s)$ in a sample of 2018 *MagUp* RS data, (a) away from the beam pipe ($|p_y| > 0.015p_z$) and (b) near to the beam pipe ($|p_y| < 0.015p_z$). Cuts (black lines) to remove regions of high asymmetry (red and blue regions) are super-imposed.

4.5.4.2 Slow-pion ghost probability

The HLT2 line imposes a selection on the track ghost probability to reduce the contamination of ghost slow pions. An additional high-level variable is calculated during reconstruction, which provides the probability that the slow pion is a ghost, $\text{Prob}_{\text{ghost}}(\pi_s)$. If ghosts are present in the sample, the WS/RS ratio as a function of $\text{Prob}_{\text{ghost}}(\pi_s)$ should increase. It is also possible that ghosts are present that are insensitive to $\text{Prob}_{\text{ghost}}(\pi_s)$.

Figure 4.14a shows the WS/RS ratio of (phase-space and time integrated) Run 2 data in 16 bins of $\text{Prob}_{\text{ghost}}(\pi_s)$, up to the cut of $\text{Prob}_{\text{ghost}}(\pi_s) = 0.05$. The data are fit twice with a straight line, once with no gradient and once with a slope. The gradient of the slope is zero within 0.5σ showing there is no bias as a function of $\text{Prob}_{\text{ghost}}(\pi_s)$. Additionally, Fig. 4.14b shows the cumulative WS/RS ratio as a function of $\text{Prob}_{\text{ghost}}(\pi_s)$, the WS/RS ratio is stable as additional data with larger $\text{Prob}_{\text{ghost}}(\pi_s)$ are added.

Thus it is deemed that the removal of candidates as described, the fiducial cuts, and the requirements on the slow pion ghost probability remove all ghosts at current sensitivity. A systematic uncertainty to account for any possible residual ghost candidates is applied and is discussed in Sec. 6.2.5.

4.5.5 $D^0 \rightarrow K_S^0 K \pi$

Another potential source of background are the decays $D^0 \rightarrow K_S^0 K^\pm \pi^\mp$, $K_S^0 \rightarrow \pi^+ \pi^-$ which have the same final state as the signal decays. However, the topology is different to the signal decays as the K_S^0 decay vertex is in general be detached from the D^0 decay vertex.

WS $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ candidates are susceptible to $D^0 \rightarrow K_S^0 K^+ \pi^-$ contamination, with branching fraction $(3.3 \pm 0.5) \times 10^{-3}$ [15]. Accounting for the branching fraction of $K_S^0 \rightarrow \pi^+ \pi^-$ $((69.20 \pm 0.05)\%$ [15]), this background is 8.6

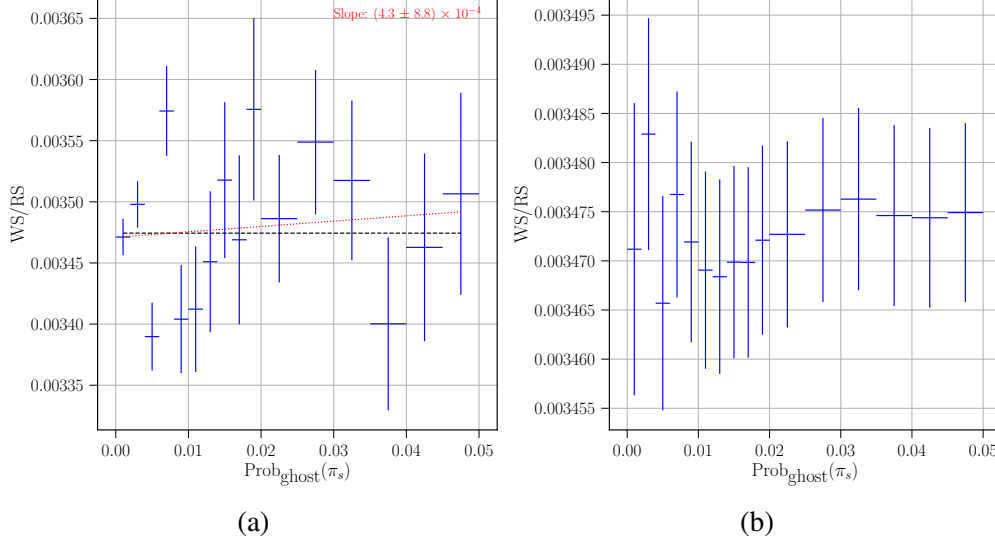


Figure 4.14: (a) The WS/RS ratio as a function of slow pion ghost probability in the region $\text{Prob}_{\text{ghost}}(\pi_s) < 0.05$. Two linear fits to all bins in the range $\text{Prob}_{\text{ghost}}(\pi_s) < 0.05$ are performed, with only a small positive slope (red) compared to when the gradient is fixed to zero (black). (b) The cumulative ratio of WS/RS signal events, so the bin centred at $\text{Prob}_{\text{ghost}}(\pi_s) = 0.0475$ includes all events with $\text{Prob}_{\text{ghost}}(\pi_s) < 0.05$.

times more abundant than that of the WS signal. RS $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ candidates are susceptible to $D^0 \rightarrow K_S^0 K^- \pi^+$ contamination, with branching fraction $(2.17 \pm 0.34) \times 10^{-3}$ [15], which is 55 times smaller than the RS signal branching fraction (when accounting for the $K_S^0 \rightarrow \pi^+ \pi^-$ decay). Thus, background from these decays are much more likely to appear in the WS sample (and are negligible in the RS sample), which would lead to a bias the WS/RS ratio.

Background events with a short K_S^0 flight distance can be present in the WS data, as well as events with a poor D^0 decay vertex fit. This background will peak in the Δm spectrum as the events originate from a true D^* and a true D^0 . If the background is present, a signal peak in the opposite-sign dipion invariant mass around the mass of the K_S^0 will be visible in the WS sample.

The dipion invariant mass is studied using variables that have not undergone the kinematic fit (DTF), as the refitting could distort the shape and position of

the K_S^0 peak. Both $\pi^+\pi^-$ hypotheses are studied. Figures 4.15a and 4.15b show a small peak in the Run 2 WS distribution for each pion-pair hypothesis respectively, which is fit with a Gaussian and a linear background.

To reduce this background in the inclusive phase-space sample, and because the phase-space binning purposefully excludes a region around the K_S^0 mass, a nominal veto of $\pm 10 \text{ MeV}/c^2$ around the PDG mass of the K_S^0 is applied. This cut reduces the presence of this background, however because the bin number is calculated using the momenta after the DTF is applied, and the DTF could distort the momenta of the pions originating from a K_S^0 , there could be some leakage of background events around the veto.

The 2017 *MagUp* WS sample is used to assess the potential leakage of this background around the veto. In order to see where the K_S^0 peak of $K_S^0 K^+ \pi^-$ background events appears in the refitted dipion invariant-mass spectrum, we use the sWeight technique. The dipion invariant-mass distribution is fit with a Gaussian and a linear background in Fig. 4.15c, which yields sWeights to isolate the $K_S^0 K^+ \pi^-$ events from the rest of the sample. The sWeights are applied to the refitted dipion invariant-mass spectrum, giving the distribution shown in Fig. 4.15d. This distribution is fit with another Gaussian in the region where the sWeighted event yield is positive. The fraction of background events that are removed by the nominal veto is 94.6%, leaving a small leakage around the veto of around 5%. This is mainly due to the fact that the mean of the mass distribution is shifted to lower values (around $495 \text{ MeV}/c^2$) compared to the PDG mass in both the refitted and un-refitted variables. This leakage can be accounted for by scaling the overall measured $K_S^0 K^+ \pi^-$ yield by 5% and comparing this to the overall WS yield measured in the simultaneous mass fit.

From the whole Run 2 WS sample (with BDT, time and Δm cuts), the measured $K_S^0 K^+ \pi^-$ yield is equal to 6312 ± 515 (measured from fits shown in Fig. 4.15). Thus, the approximate number of $K_S^0 K^+ \pi^-$ events that remain in the

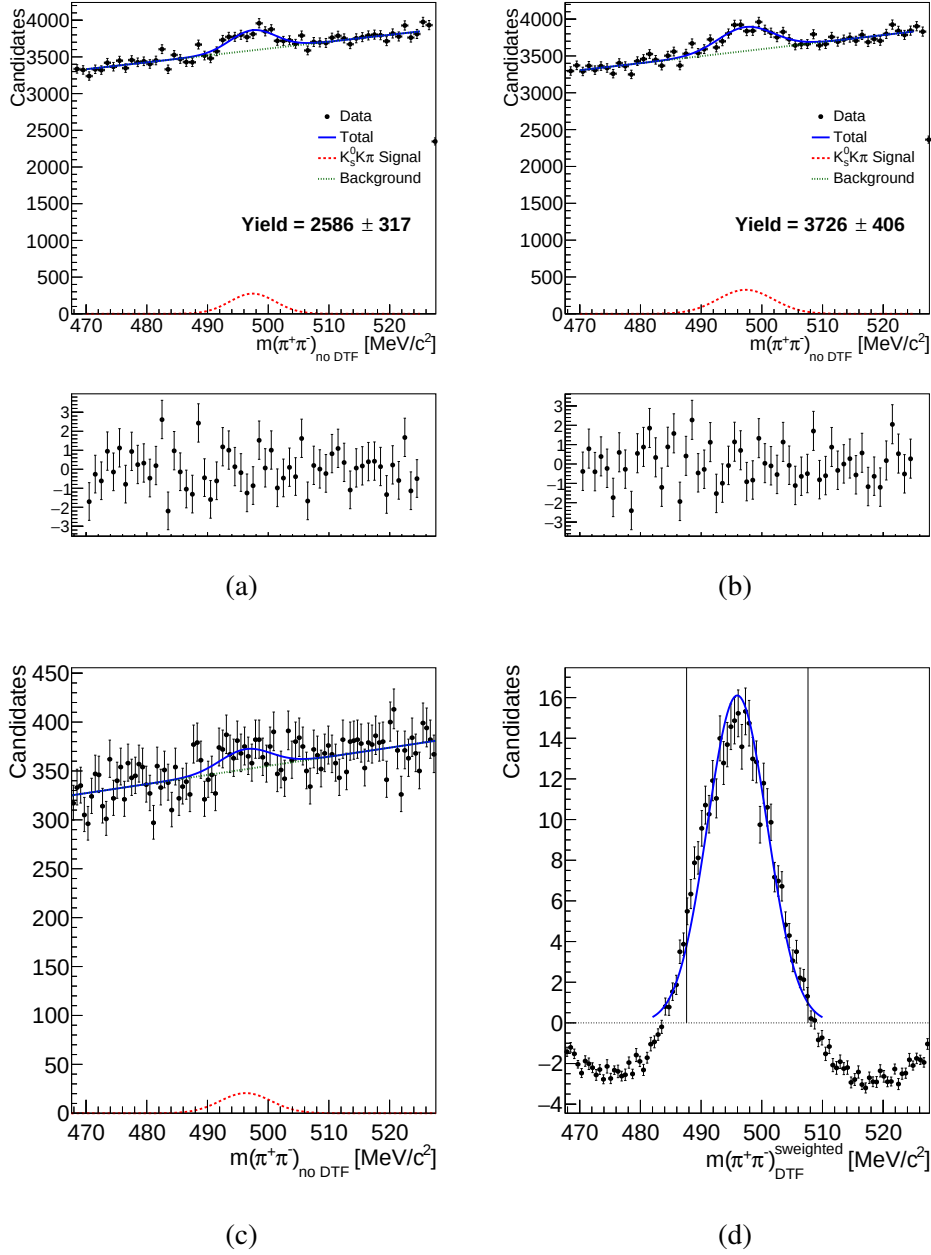


Figure 4.15: (a) First hypothesis and (b) Second hypothesis dipion invariant mass distributions of Run 2 WS data, with fits superimposed. (c) The not-refitted $m(\pi^+\pi^-)$ distribution in 2017 *MagUp* WS data around the K_S^0 mass peak, with fit superimposed. (d) The sWeighted refitted $m(\pi^+\pi^-)$ distribution in 2017 *MagUp* WS data around the K_S^0 mass peak. The vertical lines indicate the veto boundaries.

WS sample after the veto due the leakage is 316 ± 26 , which is $(0.07 \pm 0.01)\%$ relative to the Run 2 WS signal yield (442,000 events). This is very small and thus a systematic uncertainty will be applied, rather than a correction.

4.5.6 Mis-ID and partially reconstructed charm decays

Another source of background that can affect both the measured WS and RS signal yields are decays of the form $D^* \rightarrow D^0 \pi$, $D^0 \rightarrow X$, where X is any 4+ body final state of the D^0 that is not $K\pi\pi\pi$. Potential $D^0 \rightarrow X$ decays, with branching fractions (BF) as found in the PDG [15], include four-body mis-ID charm:

- $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$: BF = $(0.247 \pm 0.011)\%$;
- $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$: BF = $(0.756 \pm 0.02)\%$;

and partially reconstructed five-body final states (where one particle is not reconstructed):

- $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+ \pi^0$: BF = $(4.3 \pm 0.4)\%$;
- $D^0 \rightarrow K^- \pi^+ \eta', \eta' \rightarrow \pi^+ \pi^- \gamma$: BF = $(0.19 \pm 0.01)\%$;
- $D^0 \rightarrow K^- \pi^+ \eta, \eta \rightarrow \pi^+ \pi^- \gamma$: BF = $(0.080 \pm 0.003)\%$;
- $D^0 \rightarrow K^- \pi^+ \pi^- \mu^+ \nu$: BF < 0.13% (90% CL).

Mis-ID charm decays

The mis-ID backgrounds have branching fractions that are much smaller than that of the RS decay (BF = $(8.22 \pm 0.14)\%$), but are much larger than that of the WS decay (BF = $(0.0265 \pm 0.0006)\%$). Such decays have the potential to bias the WS/RS ratio if the WS and RS selection does not remove these backgrounds.

$D^0 \rightarrow KK\pi\pi$ and $D^0 \rightarrow \pi\pi\pi\pi$ backgrounds are ruled out from the signal sample by computing the swapped mass hypotheses of $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ candidates which would identify true $D^0 \rightarrow KK\pi\pi$ or $D^0 \rightarrow \pi\pi\pi\pi$ decays. Swapping the PID (mass) hypothesis of a pion (with opposite sign to the kaon) to a kaon hypothesis allows for the construction of $m_{1,2}^{\pi \rightarrow K}(D^0)$ which would peak at the D^0 mass if there are true $D^0 \rightarrow KK\pi\pi$ background events in the signal sample. Similarly the construction of $m^{K \rightarrow \pi}(D^0)$ probes the presence of $D^0 \rightarrow \pi\pi\pi\pi$ decays in the sample. Track kinematics without the DTF are used to avoid any distortion in the kinematics from a wrongly assigned PID hypothesis. The distribution of these masses is shown in Fig. 4.16 for 2018 *MagDown* WS and RS events. There are no signal events with swapped masses near the expected $m(D^0)$ peak (in the entire Run 2 sample) and so these backgrounds are ruled out. No systematic uncertainty is assigned for these backgrounds.

This study does not eliminate mis-ID charm decays from appearing in the sideband $m(D^0)$ regions, which are studied to identify partially reconstructed mis-ID backgrounds. However, the presence of mis-ID charm decays in the sideband regions is expected to be at the same order of magnitude as in the signal region, and so should not contribute significantly to the yields in the sideband regions.

Partially reconstructed charm decays

Charm decays to five-body final states that are reconstructed as $D^0 \rightarrow K\pi\pi\pi$ will peak in Δm but not in $m(D^0)$, so applying the signal window in $m(D^0)$ will remove most of the background. Due to the missing energy, partially reconstructed background candidates are likely to fall in the lower $m(D^0)$ sideband, with an exponential decrease as $m(D^0)$ increases.

The branching fractions of partially reconstructed backgrounds are only known for the Cabibbo-favoured decays; all are significantly smaller than that of the RS decay, except for $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+ \pi^0$ which is roughly half as abun-

dant. It is expected that the doubly Cabibbo suppressed branching fractions is suppressed by a similar factor to that of the WS/RS decays, thus the proportion of these backgrounds in the WS sample is similar to that of the RS sample. Thus, any bias caused by these events will not be a first order effect.

Events that fall in the $m(D^0)$ sidebands in data are studied to measure the fraction of mis-ID events that are present. This is done by measuring a signal yield in Δm , following a similar procedure to that used to study the double mis-ID background. Additional cuts are put in place to suppress any double mis-ID (and any same-sign $K^+ \leftrightarrow \pi^+$ mis-ID) in both the WS and RS samples. The Δm distributions in the 2017 *MagUp* WS and RS samples after these cuts are applied are shown in Fig. 4.17. Fits are performed on the Δm distribution with the same model as for the double mis-ID studies (a Gaussian signal peak and a RooBkg shape). If the source of this background is such that these yields from the sidebands are uniformly distributed in the $m(D^0)$ signal window as well, the partially reconstructed contamination would be 2.6% of the WS yield and 1.1% of the RS yield, which would induce a significant bias on the measured WS/RS ratio. Thus, it is important to try to identify the source of these events in the $m(D^0)$ sidebands. Events in the peak of the RS sample (shown in Fig. 4.17 (b)) are studied, as this is a cleaner sample than that of the WS peak, which is situated on top of a large combinatorial background.

A sample of $D^0 \rightarrow K\pi\pi\pi^0$ decays is generated using Particle Gun. The decays are reconstructed in the same way as the $D^0 \rightarrow K\pi\pi\pi$ signal decays. The value of $m(D^0)$ constructed from the kaon and three pions falls far below the signal and sideband regions of $m(D^0)$ used in this analysis (for 10,000 MC events, $m(D^0) < 1740 \text{ MeV}/c^2$). Thus this background is ruled out from contributing to both the signal and sideband yields.

A sample of 7 million $D^0 \rightarrow K\pi\eta', \eta' \rightarrow \rho^0\gamma, \rho^0 \rightarrow \pi^+\pi^-$ decays is generated using Particle Gun, and events are reconstructed as $D^0 \rightarrow K\pi\pi\pi$ decays. A

small sample of events passed the HLT1 and HLT2 triggers, of these 387 events fell in the $m(D^0)$ signal window and 1552 events fell in the low $m(D^0)$ sideband window. This results in a background yield of $(0.022 \pm 0.001)\%$ relative to the signal yield, and a yield in the low $m(D^0)$ sidebands of 30% relative to the proportion of RS signal that falls in the low $m(D^0)$ sideband. As stated previously, background events of these type are expected to contribute to both the measured WS and RS signal yields. However to assign a systematic uncertainty, it will be assumed that this background only affects the WS yield. This is a conservative approach, but is chosen as there is not a model of the resonance structure of the $D^0 \rightarrow K\pi\eta'$ decay and the strong phase of the decay is unknown. $D^0 \rightarrow K\pi\eta$ decays are expected to contribute in a similar manner, with yields obtained from the relative branching fractions.

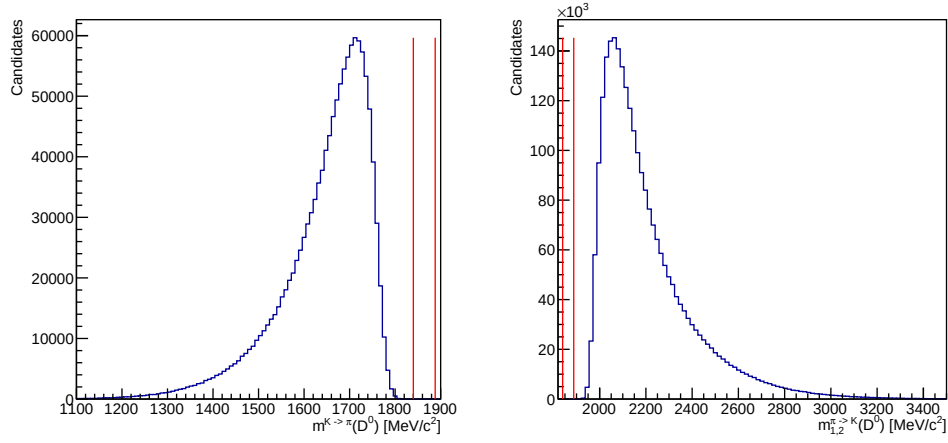
The yield in the $m(D^0)$ sideband regions can also be attributed to poorly reconstructed true $D^0 \rightarrow K\pi\pi\pi$ decays. As a function of phase space the partially reconstructed candidates (both from lower and upper sidebands) have similar distributions to the RS signal candidates (the distributions of signal and lower sideband candidates across phase space are shown in Fig. 4.18).

The conclusion is that the majority of the yield in the $m(D^0)$ sidebands is not coming from mis-ID or partially reconstructed charm decays, instead they are just poorly reconstructed $D^0 \rightarrow K\pi\pi\pi$ events. This will not lead to a bias on the WS/RS ratio. A small fraction of events come from $D^0 \rightarrow K\pi\eta^{(\prime)}$ decays. A systematic uncertainty will be assigned, but no correction to the WS/RS ratio is made.

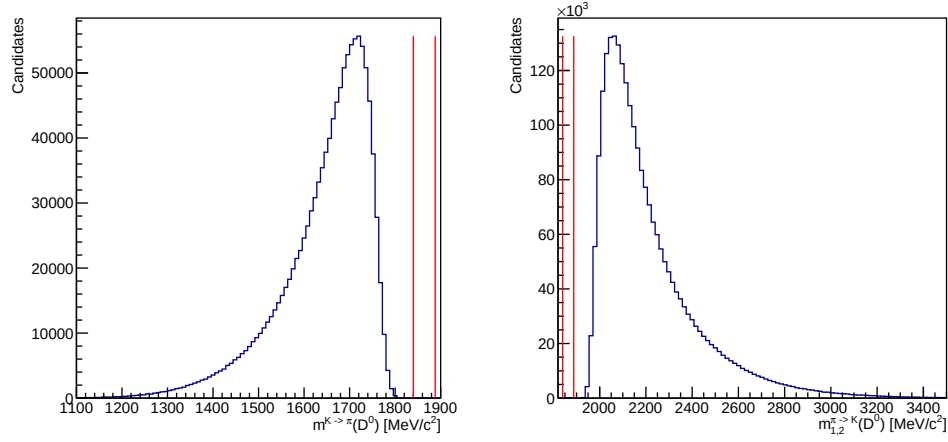
4.5.7 Summary

Table 4.3 gives a summary of the backgrounds discussed in this section, with details on which sample they affect, how large they are relative to the signal yield and how they are reduced. Details of the systematic uncertainties associated

with these backgrounds are given in Chapter 6.



(a) WS candidates



(b) RS candidates

Figure 4.16: (left) $m^{K \rightarrow \pi}(D^0)$ and (right) $m_{1,2}^{\pi \rightarrow K}(D^0)$ distributions of (a) WS and (b) RS signal candidates in a sample of 2018 *MagDown* data. The red lines indicate a $\pm 24 \text{ MeV}/c^2$ window around the true D^0 mass.

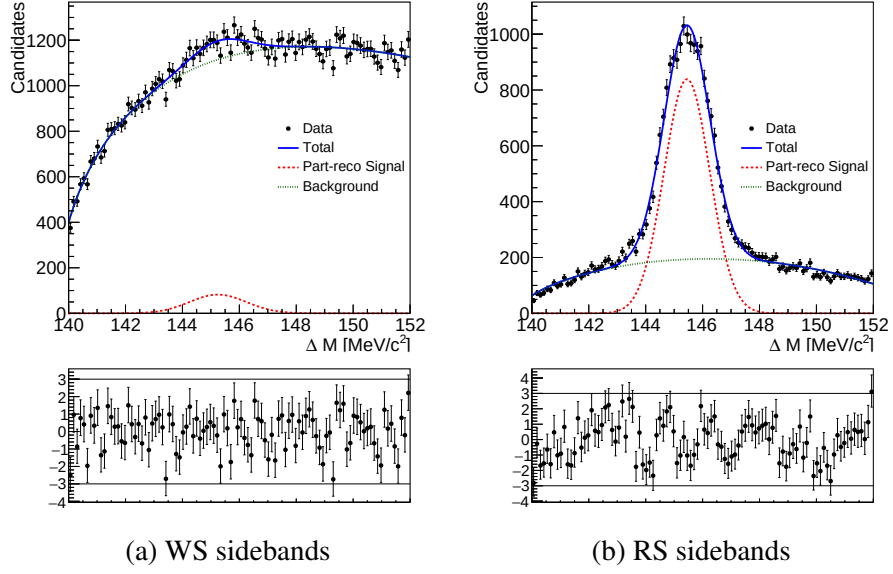


Figure 4.17: The Δm distribution in (a) WS 2017 *MagUp* and (b) a sample of RS 2017 *MagUp* data in the $m(D^0)$ sidebands.

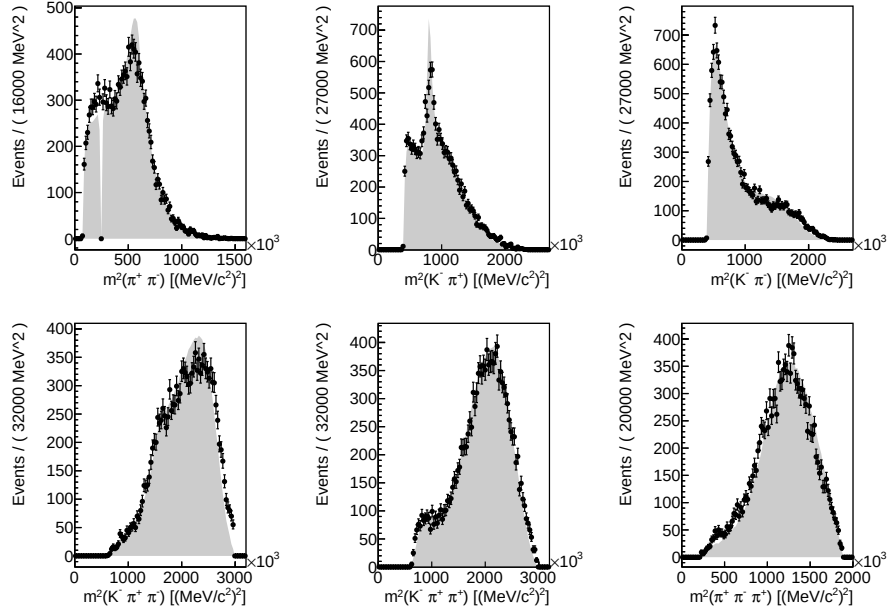


Figure 4.18: Invariant mass distributions of the RS partially reconstructed candidates in the lower $m(D^0)$ sideband. The grey histogram indicated the distribution of RS signal candidates.

Table 4.3: An overview of how different backgrounds are accounted for in the analysis.

Background	Affects sample	Relative size (before)	How treated	Relative size (after)
Secondary contamination	RS & WS	5 – 45%	Reduced by BDT, $t(D^0)$ and $\chi^2_{\text{IP}}(D^0)$ cuts. Measured fraction accounted for in fit.	3 – 8%
WS double mis-ID	WS	14.3%	Partially removed by additional PID cuts. Remaining yield deducted from WS yields.	2.7%
Clones	RS & WS	3.3%	Removed by cut on angles between tracks.	negligible
Slow pion ghosts	WS (& RS)	27% 10% 12%	Removed by cut on slow pion ghost probability, fiducial cuts and multiple candidate removal.	negligible
$D^0 \rightarrow K_S^0 K \pi$	WS (& RS)	5%	Removed by veto. Systematic assigned.	0.07%
Partially reconstructed charm	RS & WS	$< 1.1\%$	Mostly poorly reconstructed RS events in sidebands. Small contamination from $D^0 \rightarrow K \pi \eta^{(\prime)}$ decays, systematic assigned.	0.03%

5 | Measuring the signal yields

In this chapter, the signal yields are measured using a fit to the Δm distribution. To assess the evolution of the WS/RS ratio as a function of phase space and decay time, the data are binned before performing each mass fit. The binning and fit procedure are described below. Furthermore, the selection (in particular the HLT1 and HLT2 trigger selection) induces biases on the measured yields as a function of decay time that must be corrected. The second half of this chapter describes the Monte Carlo simulation samples and methods used to measure the biases.

5.1 Phase-space and time binning

5.1.1 Binning in time

Events are binned into 10 decay time bins in the range $0.24 < t(D^0)/\tau_{D^0} < 8$. The lower bound is imposed as D^0 decays with very short decay times do not pass the trigger requirements. The upper bound reduces the presence of secondary candidates in the sample. The bin boundaries are chosen to approximately equalise the WS yields per bin. The bin boundaries, in units of $t(D^0)/\tau_{D^0}$, are:

$$[0.24, 0.94, 1.185, 1.40, 1.62, 1.85, 2.13, 2.45, 2.87, 3.5, 8].$$

Figure 5.1 shows the decay-time distribution of RS events, and the WS and RS yields per bin. The method for measuring the yields is described in the following sections.

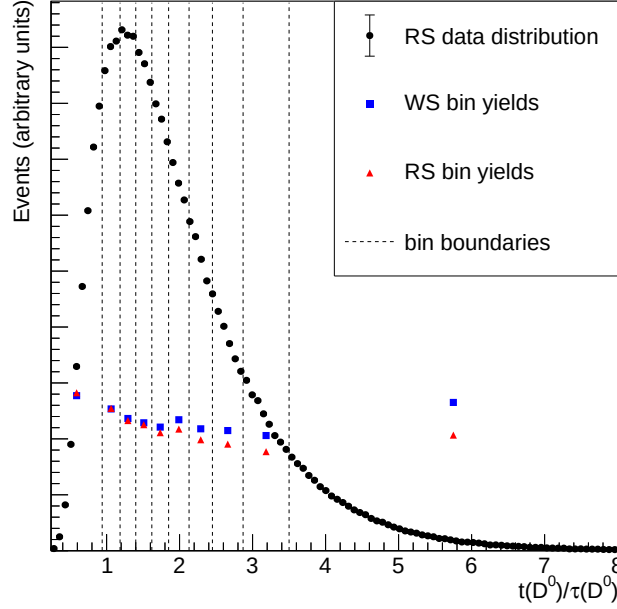


Figure 5.1: Decay time distribution of RS events, with bin boundaries and yields per bin overlaid.

5.1.2 Binning in phase space

The four-bin division of phase space proposed in Ref. [33] increases sensitivity to the CKM angle γ in a measurement using $B \rightarrow DK$, $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays. A fifth bin collects a small region of phase space in which $D^0 \rightarrow K_S^0 K^\pm \pi^\mp$, $K_S^0 \rightarrow \pi^\pm \pi^\mp$ decays are likely to fall (events with $|m(\pi^\pm \pi^\mp) - m(K_S^0)| < 10 \text{ MeV}/c^2$). These events in bin 5 are removed in both the phase-space integrated and binned analyses.

The scheme splits candidate events into bins of *normalised* strong phase difference, $\tilde{\delta}_{K3\pi}$, defined at a point in phase space, Ω , as

$$\begin{aligned} \tilde{\delta}_{K3\pi}(\Omega) = & \arg(\mathcal{A}_{\bar{D}^0 \rightarrow K^+ \pi^- \pi^+ \pi^-}(\Omega) \mathcal{A}_{D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-}^*(\Omega)) \\ & - \arg\left(\int \mathcal{A}_{\bar{D}^0 \rightarrow K^+ \pi^- \pi^+ \pi^-}(\Omega') \mathcal{A}_{D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-}^*(\Omega') d\Omega'\right). \end{aligned} \quad (5.1)$$

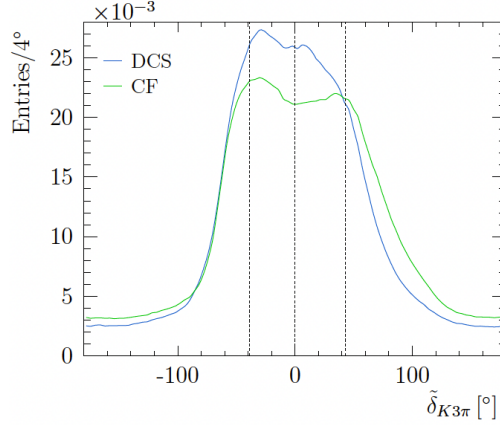


Figure 5.2: Distributions of the normalised strong phase difference for DCS and CF events. Bin boundaries are indicated by dotted lines. Each of the histograms has been normalised to unit area. Taken from Ref. [33].

The value of the $\tilde{\delta}_{K3\pi}$ per candidate can be calculated from the models in Ref. [27] using the kinematics of the particles from the D^0 decay. Figure 5.2 shows distribution of the normalised strong-phase difference for DCS and CF events.

The bins are chosen such that each bin has an equal product of the CF and DCS phase-space averaged amplitude integrals. This choice of binning gives a statistical uncertainty to γ that is only 22% larger than the uncertainty achievable by an unbinned (model-dependent) method [33].

5.2 Mass fit

The BDT and other selection presented in Chapter 4 removes specific backgrounds and some, but not all of, the combinatorial background from the samples. To measure the WS and RS signal yields as a function of decay time and phase space, a fit to the Δm distribution is performed on candidates in each of the decay-time and phase-space bins.

The PDF fit to the Δm distribution is the sum of a signal PDF and a background PDF, where the fractions of signal and background are free to float in the fit. The signal shape is modelled by an Extended Cruijff function (a modified

bifurcated Gaussian with asymmetric tails),

$$S(\Delta m; \mu, \beta, \sigma_L, \sigma_R, \alpha_L, \alpha_R) = \begin{cases} \exp\left(\frac{-(\Delta m - \mu)^2(1 + \beta(\Delta m - \mu)^2)}{2\sigma_L^2 + \alpha_L(\Delta m - \mu)^2}\right) & \text{for } \Delta m < \mu \\ \exp\left(\frac{-(\Delta m - \mu)^2(1 + \beta(\Delta m - \mu)^2)}{2\sigma_R^2 + \alpha_R(\Delta m - \mu)^2}\right) & \text{for } \Delta m \geq \mu \end{cases} \quad (5.2)$$

where μ can be interpreted as the value of Δm at the peak, σ_L and α_L (σ_R and α_R) control the width and tail to the left (right) of the peak position, and β modifies the signal shape. β is fixed per time bin to a value measured in a signal-only fit to truth-matched MC to improve the fit stability. The background is modelled by an analytic background shape (referred to throughout this text as ‘RooBkg’),

$$B(\Delta m; \Delta m_0, a, b) = (\Delta m - \Delta m_0)^{\frac{1}{2}} + a(\Delta m - \Delta m_0)^{\frac{3}{2}} + b(\Delta m - \Delta m_0)^{\frac{5}{2}} \quad (5.3)$$

for $\Delta m > \Delta m_0$, and equals zero otherwise. Δm_0 is the low mass threshold which is fixed to 140 MeV/ c^2 in the fits, and a and b float and take different values for the RS and WS fits.

The fit is a simultaneous extended maximum-likelihood binned fit for the WS and RS Δm spectrum per bin such that the signal shape is shared between the two distributions. Each has a background PDF with its own parameters. Only the low-mass threshold is shared between every time bin; all other (non-fixed) parameters float independently. To measure the phase-space binned yields per time bin, the data are first split by phase-space bin before fitting the Δm distribution. Additionally, all samples are split by kaon charge. The Δm distribution and fit in a single time bin, for K^+ candidates, are shown in Fig. 5.3. The fits in each time bin, split by kaon charge, are shown in Appendix A. The time-integrated yields are 442×10^3 for WS Run 2 data and 128×10^6 for RS Run 2 data. The RS yields per year are approximately 6×10^6 in 2015, 31×10^6 in 2016, 41×10^6 in 2017, 50×10^6 in 2018, with roughly equal yields across

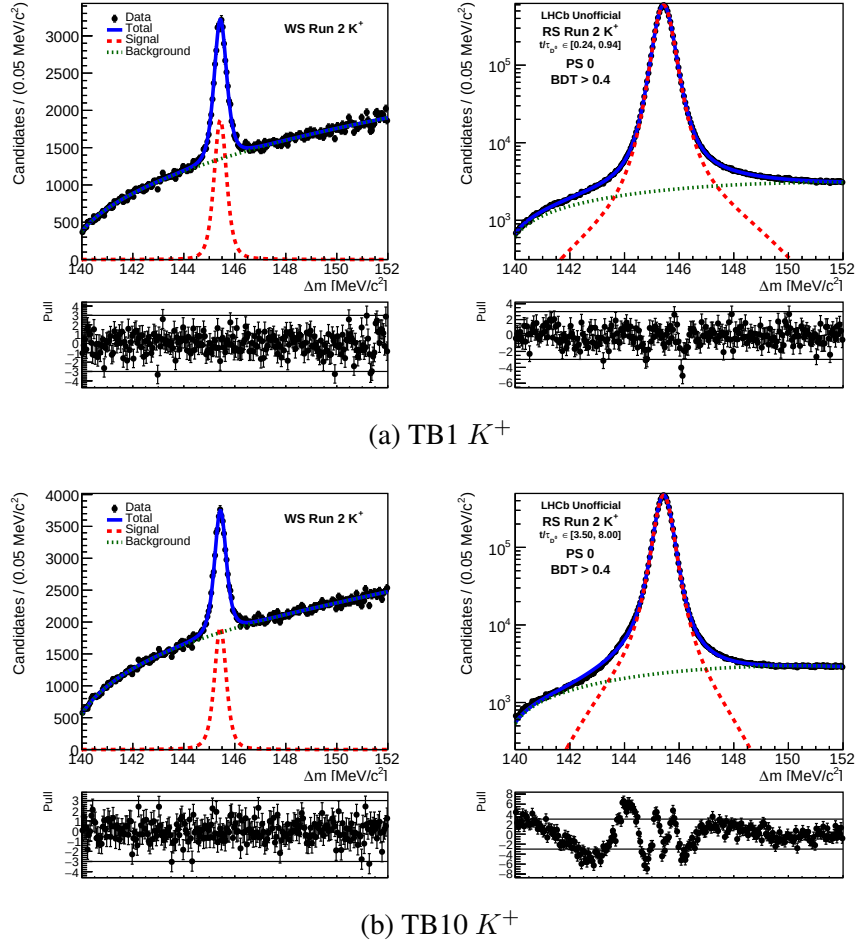


Figure 5.3: Simultaneous mass fit to K^+ phase-space integrated (left) WS Run 2 data and (right) RS data in (a) the first time bin and (b) the last time bin.

MagUp and *MagDown* polarities, and across D^0 and $\bar{D}^0$ flavours.

Tables 5.1 and 5.2 show the signal yields measured in the mass fits for all RS and WS Run 2 data. Figure 5.4 shows the ratio of these signal yields in bins of proper time. The increase in the WS/RS ratio as a function of decay time is indicative of mixing. Further corrections will be applied before the mixing observables are measured, notably the efficiency correction (see Sec. 5.3.3.3) and the double mis-ID removal (see Sec. 4.5.2).

Table 5.1: The measured RS signal yields in the mass fit, split by D^0 flavour.

Flavour		PS int.	PS 1	PS 2	PS 3	PS 4
D^0	TB int.	64788000 ± 9210	16651400 ± 4660	14403200 ± 4340	14748900 ± 4400	18988200 ± 5010
	TB 1	7962520 ± 3360	2072180 ± 1710	1800690 ± 1590	1796930 ± 1600	2289660 ± 1820
	TB 2	7201570 ± 3170	1863100 ± 1610	1623640 ± 1500	1628200 ± 1510	2086410 ± 1720
	TB 3	6615000 ± 3050	1705620 ± 1550	1480630 ± 1440	1502310 ± 1460	1927440 ± 1660
	TB 4	6428080 ± 3000	1650500 ± 1520	1431880 ± 1410	1463250 ± 1440	1882350 ± 1630
	TB 5	6044360 ± 2880	1548050 ± 1450	1343210 ± 1350	1377770 ± 1380	1774420 ± 1570
	TB 6	6252510 ± 2860	1601690 ± 1450	1383050 ± 1340	1426470 ± 1370	1840330 ± 1560
	TB 7	5716170 ± 2780	1460920 ± 1410	1263000 ± 1310	1308130 ± 1330	1686540 ± 1510
	TB 8	5501270 ± 2710	1405670 ± 1370	1213130 ± 1270	1257870 ± 1300	1624790 ± 1470
	TB 9	5148700 ± 2550	1314700 ± 1290	1130640 ± 1200	1177980 ± 1230	1525440 ± 1390
	TB 10	6014120 ± 2720	1538100 ± 1370	1316830 ± 1270	1376000 ± 1300	1782990 ± 1480
$\bar{D}^0$	TB int.	62802100 ± 9080	16140300 ± 4600	13970200 ± 4270	14295500 ± 4340	18397000 ± 4930
	TB 1	8216760 ± 3410	2139950 ± 1730	1858510 ± 1610	1855230 ± 1630	2361880 ± 1850
	TB 2	7429720 ± 3230	1923880 ± 1640	1671570 ± 1530	1680460 ± 1540	2153320 ± 1750
	TB 3	6810020 ± 3090	1756190 ± 1570	1523150 ± 1460	1544640 ± 1480	1985910 ± 1680
	TB 4	6631710 ± 3040	1700750 ± 1540	1478370 ± 1440	1508390 ± 1450	1944060 ± 1650
	TB 5	6242950 ± 2920	1600890 ± 1480	1385590 ± 1370	1424680 ± 1400	1833630 ± 1590
	TB 6	6446720 ± 2900	1649180 ± 1470	1426310 ± 1360	1472000 ± 1390	1899590 ± 1580
	TB 7	5897300 ± 2810	1507070 ± 1420	1301100 ± 1320	1349470 ± 1350	1741480 ± 1530
	TB 8	5681050 ± 2740	1451240 ± 1390	1250990 ± 1290	1298120 ± 1310	1680610 ± 1500
	TB 9	5311790 ± 2590	1357310 ± 1310	1165500 ± 1210	1215290 ± 1240	1574490 ± 1420
	TB 10	6206770 ± 2760	1586910 ± 1410	1358760 ± 1290	1421180 ± 1320	1840010 ± 1500

Table 5.2: The measured WS signal yields in the mass fit, split by D^0 flavour.

Flavour		PS int.	PS 1	PS 2	PS 3	PS 4
D^0	TB int.	221441 ± 825	61788 ± 424	62041 ± 408	56440 ± 404	41055 ± 413
	TB 1	25912 ± 264	7211 ± 136	7334 ± 201	6474 ± 129	4925 ± 131
	TB 2	23485 ± 262	6454 ± 135	6531 ± 130	5965 ± 128	4523 ± 131
	TB 3	21942 ± 258	6001 ± 133	6187 ± 128	5572 ± 126	4174 ± 129
	TB 4	21546 ± 258	6009 ± 133	6047 ± 128	5413 ± 126	4055 ± 130
	TB 5	20633 ± 255	5763 ± 131	5646 ± 126	5154 ± 125	4083 ± 129
	TB 6	22244 ± 265	6125 ± 136	6134 ± 130	5824 ± 130	4154 ± 133
	TB 7	20433 ± 256	5600 ± 131	5696 ± 127	5430 ± 126	3719 ± 129
	TB 8	20126 ± 256	5947 ± 133	5500 ± 125	5076 ± 125	3580 ± 128
	TB 9	20062 ± 252	5624 ± 130	5682 ± 125	5096 ± 124	3662 ± 126
	TB 10	25334 ± 283	7053 ± 146	7371 ± 141	6584 ± 139	4334 ± 141
$\bar{D}^0$	TB int.	220453 ± 816	60969 ± 420	62196 ± 405	56316 ± 400	41465 ± 409
	TB 1	25447 ± 262	6910 ± 134	7015 ± 154	6680 ± 130	4939 ± 130
	TB 2	23323 ± 259	6265 ± 133	6586 ± 130	5962 ± 127	4494 ± 129
	TB 3	22041 ± 255	6247 ± 132	5950 ± 126	5452 ± 124	4380 ± 129
	TB 4	21449 ± 256	5868 ± 131	5998 ± 127	5484 ± 125	4052 ± 129
	TB 5	20670 ± 253	5590 ± 129	5901 ± 125	5290 ± 124	3887 ± 127
	TB 6	21934 ± 262	5917 ± 134	6067 ± 129	5674 ± 129	4286 ± 132
	TB 7	20744 ± 255	5899 ± 131	5789 ± 126	5207 ± 125	3864 ± 128
	TB 8	20482 ± 254	5634 ± 130	5725 ± 125	5280 ± 124	3817 ± 127
	TB 9	19808 ± 248	5692 ± 129	5561 ± 122	5106 ± 122	3432 ± 123
	TB 10	25446 ± 281	7020 ± 144	7651 ± 140	6405 ± 137	4370 ± 139

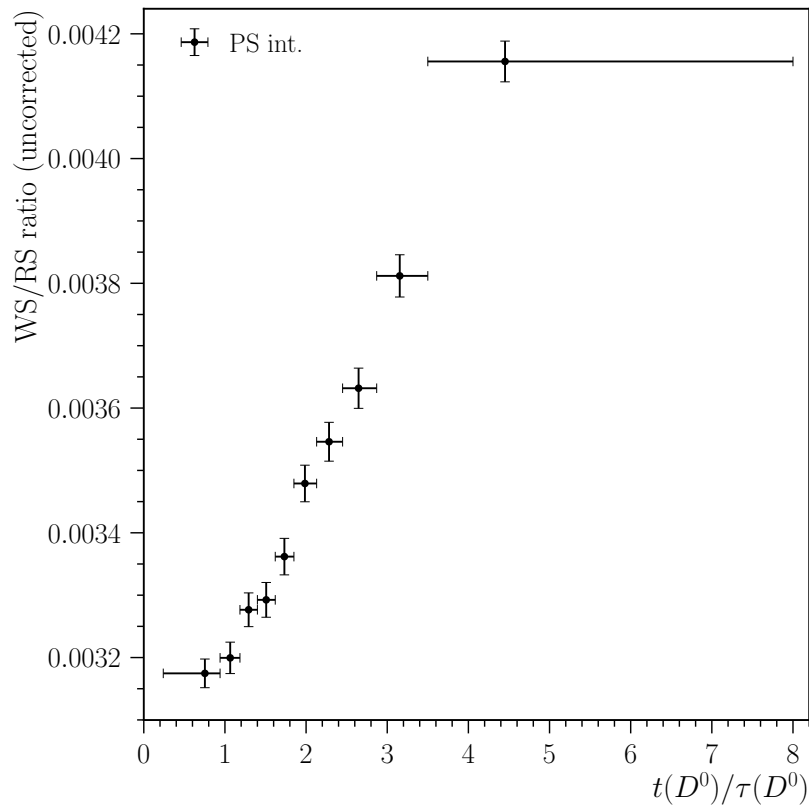


Figure 5.4: Ratio of WS/RS yields before further corrections (including the double mis-ID background and efficiency corrections), measured in all Run 2 data.

5.2.1 Fitter bias

To ensure the mass fit procedure is unbiased, simulated data are generated and fit with the same model (the baseline model) many times. Deviations in the measured WS and RS yields per time bin (N_{fit}^{WS} and N_{fit}^{RS}), and the ratio of the two from the generator yields (N_{gen}^{WS} and N_{gen}^{RS}) would be indicators of biases in the method. The procedure is as follows:

- The model described above is used to generate simulated data sets, according to the fitted parameter values in each time bin from the simultaneous mass fit to data;
- The yields of the WS and RS signal and background are Poisson fluctuated around N_{gen}^{WS} and N_{gen}^{RS} ;
- The generated data are fit with the same model to measure yields N_{fit}^{WS} and N_{fit}^{RS} ;
- This process is repeated 500 times for every time bin individually.

This study is performed with a model derived from a fit to all Run 2 WS data and a subsample of the Run 2 RS data. Around 3% of the fits do not converge or have good covariance quality; these are excluded from this study. Residual plots of all the fit parameters and signal yields are studied, where the residual pull of a parameter p is given by

$$\text{Pull}_p = \frac{p_{\text{fit}} - p_{\text{gen}}}{\sigma_{\text{fit}}} \quad (5.4)$$

where p_{gen} is the value used to generate the data set, p_{fit} (σ_{fit}) is the value (uncertainty) found in the fit to the generated data set. The fit procedure is unbiased if the pull distributions are Gaussian with a mean of zero and a standard deviation of unity. Figure 5.5 shows the distributions of Pull_p for $p =$

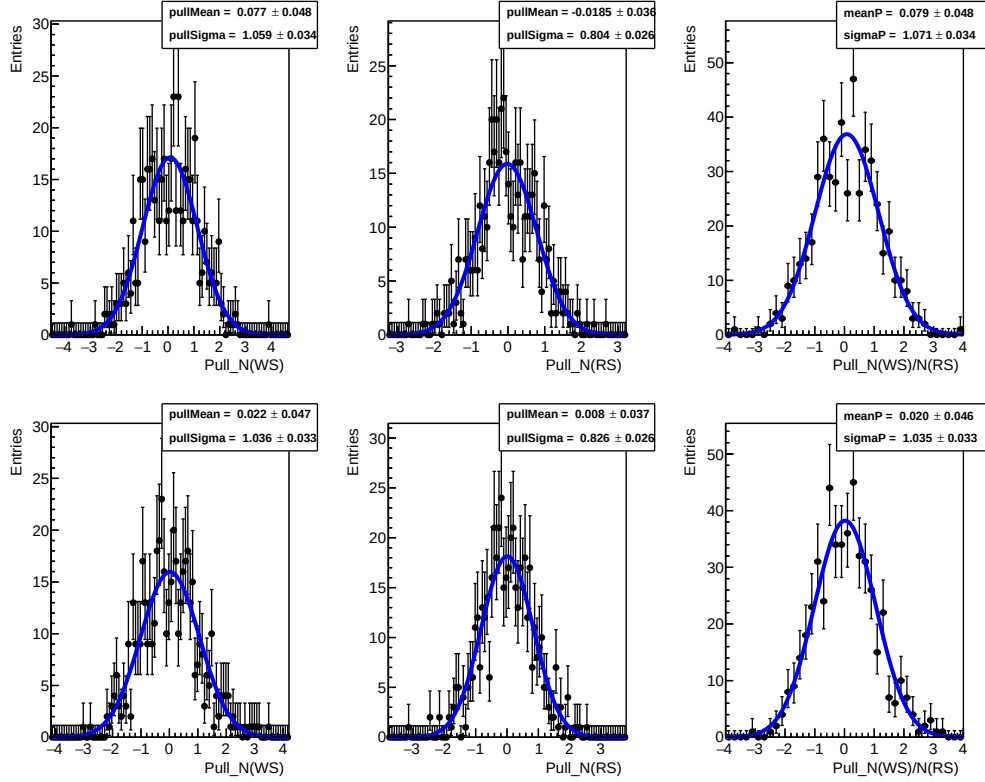


Figure 5.5: The pull distributions from pseudo-experiments of (left-right) N^{WS} , N^{RS} , N^{WS}/N^{RS} for (top) TB 1 and (bottom) TB 10.

N^{WS} , N^{RS} , N^{WS}/N^{RS} in time bins 1 and 10. In general, the pull of N^{WS} per time bin is unbiased whilst the pull of N^{RS} is slightly too narrow. However, as the pull of N^{WS}/N^{RS} is unbiased and has a width compatible with unity, no additional uncertainty is assigned.

5.3 Efficiency correction

The yields obtained in data are not directly proportional to the decay rates due to efficiency effects. Monte Carlo simulation (MC) is used to measure the efficiency effects with known mixing inputs. It is observed in MC samples that the reconstructed ratio of DCS / CF decays is not uniform in time, as it should be if

no efficiency effects are present (the mixing equation (Eqn. 2.29) simplifies to $R = r^2$ if $x = y = 0$). Studies are performed to understand the source of this bias, and to correct for it. Both Full LHCb MC and PGun MC samples are used, both of which are described in Sec. 3.3 with further details given below.

5.3.1 Full simulation

The ‘Full MC’ simulation sample is generated with full event generation, detector simulation, digitisation and reconstruction. 13 TeV pp -collisions are simulated in PYTHIA 8 [51] to generate Monte Carlo samples for prompt D^0 candidates. Three models are used to generate the kinematics of the D^0 decay: one uniformly distributed over the available phase space (PHSP) and two according to the decay models generated in the amplitude analyses of $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ (CF) and $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decays (DCS) [27]. ReDecay [61], a fast simulation technique, is employed to optimise processing time (as discussed previously in Sec. 3.3.3.1).

Table 5.3 details the yields of the Full MC samples after the triggers have been applied. The yield is approximately eight times larger than the WS signal yield. The same selection (detailed in Chapter 4) is applied to the Full MC as it is to data.

5.3.1.1 Weight to data

The MC and data after selection have reasonably good agreement, but this is further improved by a 5D reweighting of the sWeighted MC to a sample of RS sWeighted data (see Sec. 4.4 for details about the *sPlot* procedure). GBReweighter (hep_ml [67]) is trained to weight a sample of MC to data, with the results used to weight the entire sample. The variables chosen for the weighting are $P(D^0)$, $p_T(D^0)$, $\eta(D^0)$, $\phi(D^0)$, $t(D^0)$, and further details given in Sec. 5.3.2.1. A single weighting is derived from weighting CF MC to RS data, which is ap-

Table 5.3: $D^* \rightarrow D^0\pi$ Full MC samples.

Final state (model & event type)	Year	Yield after triggers	
		<i>MagUp</i>	<i>MagDown</i>
$D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ (PHSP)	2018	550795	554765
	2017	557058	546241
	2016	495271	490681
	2015	84561	86395
$D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ (CF)	2018	538872	536355
	2017	545699	545455
	2016	510911	518026
	2015	90701	88161
$D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ (DCS)	2018	536429	544653
	2017	537360	543977
	2016	516539	514961
	2015	90963	89567

plied to both DCS and CF MC samples. The distributions before and after the weighting are shown for the CF MC sample in Fig. 5.6.

5.3.1.2 Measurement of PV resolution

The primary vertex (PV) resolution, defined as the difference between the reconstructed PV location and the true PV location, in the three spatial coordinates (X,Y,Z) is studied in the Full MC samples (to be input to the smearing function in the Particle Gun generation). A window of interest around zero is chosen in each of the three coordinates. The PV resolution distributions are fit with one or two Gaussian cores and another Gaussian to model the background. The goal is not to make a perfect fit with perfect residuals, but rather to have a good enough representation of the distributions such that they can be implemented in fast MC generation. This is easiest to do with multiple Gaussians and so these shapes are adopted in the fit. The MC sample distributions and the fit shapes are shown in Fig. 5.7. The widths of the narrowest fitted Gaussians are $6.5 \mu\text{m}$ (X), $6.4 \mu\text{m}$ (Y) and $33 \mu\text{m}$ (Z). 2D distributions of the resolutions are shown as well,

showing that there are no significant correlations between the three dimensions.

5.3.2 Particle Gun simulation

The Particle Gun (PGun) MC (described previously in Sec. 3.3.3.2) is generated by sampling the D^* kinematics (p_T and η) from generator-level samples of $D^* \rightarrow D^0\pi$, $D^0 \rightarrow K\pi$ and $D^0 \rightarrow K^-\pi^+\pi^-\pi^+$ decays. The PV resolution in all three spatial dimension is included in generation via smearing, according to the fits to Full MC (see Fig. 5.7).

Samples of CF and DCS $D^* \rightarrow D^0\pi$ PGun simulation are generated, using trigger requirements to match 2017-18 and 2015-16 data-taking conditions. Yields of the samples (after HLT1 and HLT2 triggers have been applied) are given in Table. 5.4. Some selection and weighting to data is performed to increase the agreement between data and these samples, in particular to model the missing L0 trigger decision. The L0 trigger decision is not present as the Particle Gun generation method only propagates particles in the signal decay, thus triggering independently of the signal (TIS) is not possible. The L0 TOS trigger can be applied; however the trigger selects only high E_T hadrons and thus has a large impact on the D^0 kinematics. Therefore it is easier to include all generated PGun events and then weight the D^0 kinematics of the sample to the combined (L0 TOS + TIS) data sample. In all PGun samples a truth cut on the D^* ID is employed to remove a large proportion of the background (poorly reconstructed events) with minimum loss in signal.

5.3.2.1 Weight to data

The CF and DCS PGun samples are weighted in key variables to a sample of sWeighted RS data in a 5D reweighting, using GBReweigher (hep-ml [67]). The five variables chosen to train the reweighter are $P(D^0)$, $p_T(D^0)$, $\eta(D^0)$, $t(D^0)$, and $\phi(D^0)$. The differences between the raw PGun and data distributions are

Table 5.4: $D^* \rightarrow D^0 \pi$ Particle Gun MC samples. The 2018 sample is representative of triggers used in 2017-2018 data-taking ('TrigD'). The 2016 sample is split between the dominant trigger used in 2016 data-taking ('TrigD'), and a sample with triggers representative of all trigger selections used in 2015 and 2016 data-taking (referred to as 'all trigs').

Final state (model & event type)	Year	Yield after triggers	
		<i>MagUp</i>	<i>MagDown</i>
$D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ (CF)	2018	1868233	4286533
	2016 (TrigD)	550030	412633
	2016 (all trigs)	743671	659894
$D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ (DCS)	2018	1711140	4094673
	2016 (TrigD)	403931	403652
	2016 (all trigs)	722557	683390

larger than the differences between the Full MC and data due to the lack of L0 trigger in the PGun simulation. The weighting of the $p_T(D^0)$ distribution models this well. A sample of 100,000 CF PGun MC and 100,000 data events are used to train the reweighter. In order to have a large enough yield of PGun MC across phase space to perform the weighting, an upper cut on the $p_T(D^0)$ is applied ($p_T(D^0) < 18000 \text{ MeV}/c$) and a lower bound on the decay time is applied ($t(D^0)/\tau(D^0) > 0.24$). This is applied to all data and MC. A single weighting is performed (CF MC to RS data), which is applied to both CF and DCS MC samples. Figure 5.8 shows the distribution across phase space of the CF PGun sample, before and after the weighting, compared to the RS sWeighted data.

After weighting to data in the D^0 variables, the consistency between PGun and data is checked with the Cabibbo-Maksymowicz (CM) variables, which describe well the underlying physics of the decay. These variables are discussed in Sec. 5.3.3.2. No significant differences between CF PGun and RS data are observed as shown in Fig. 5.9.

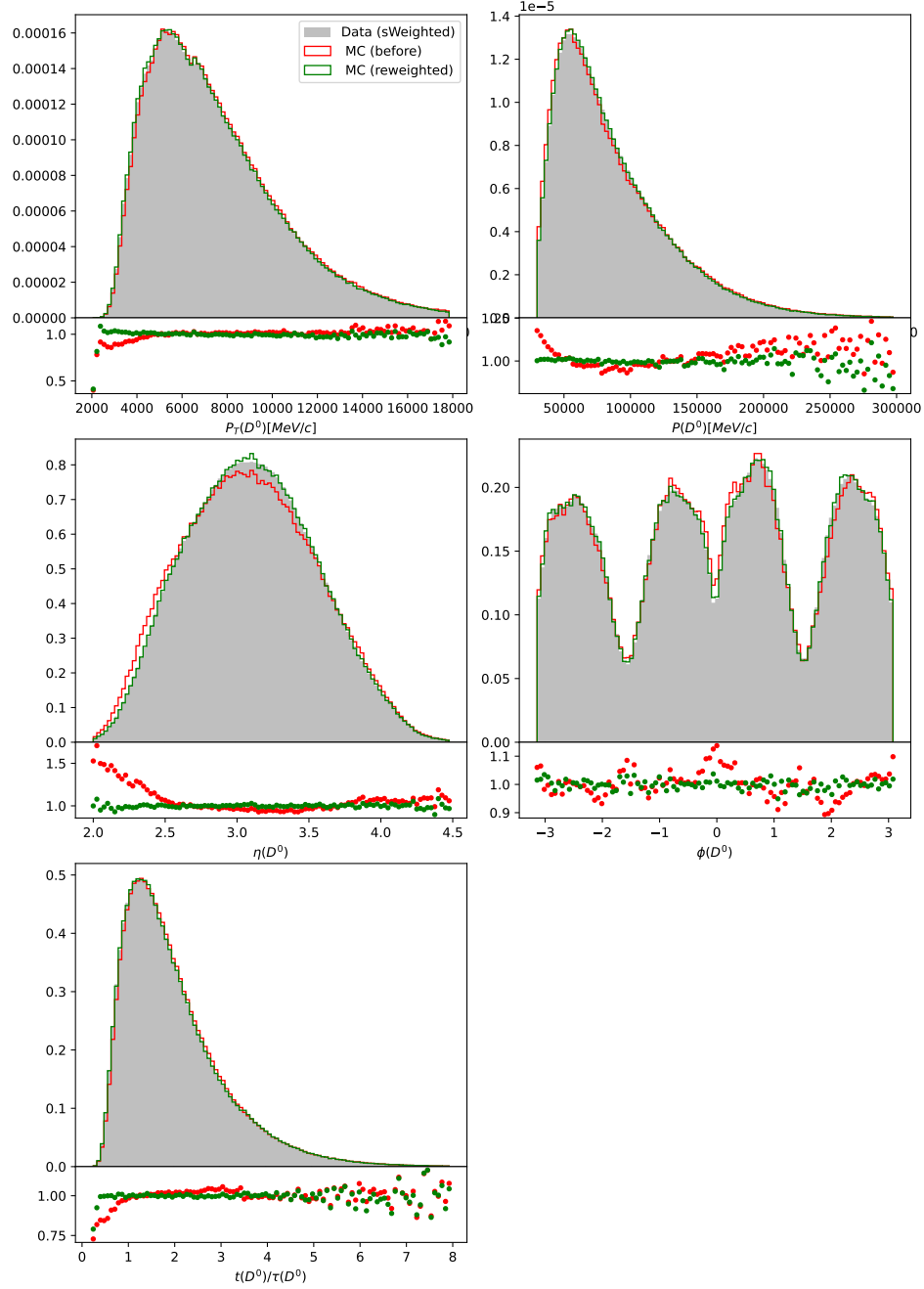
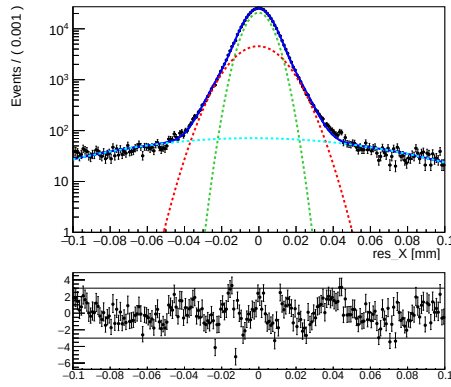
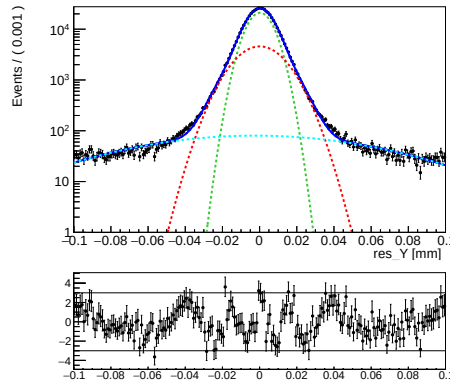


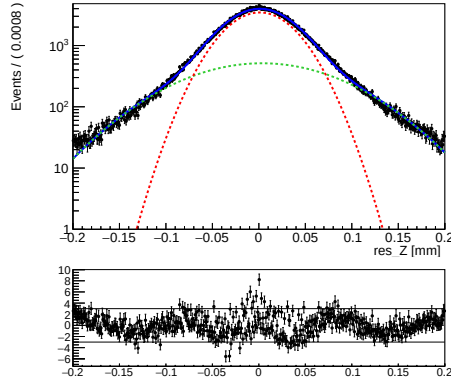
Figure 5.6: The CF Full MC sample before (red) and after (green) reweighting, compared to RS sWeighted data (grey), in the five variables used to characterise the samples.



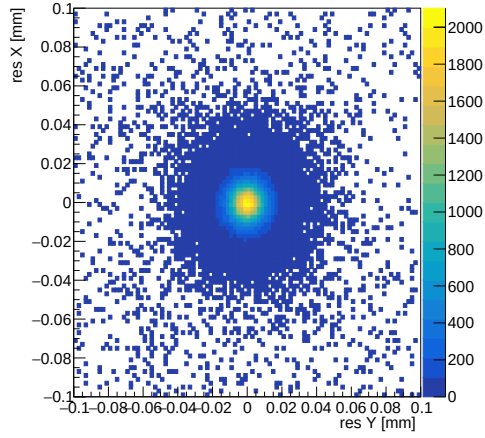
(a) Fit to PV resolution in X



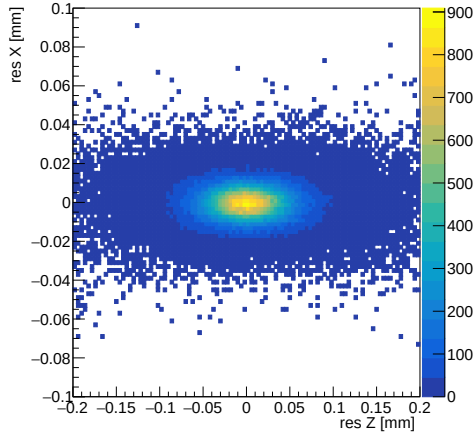
(b) Fit to PV resolution in Y



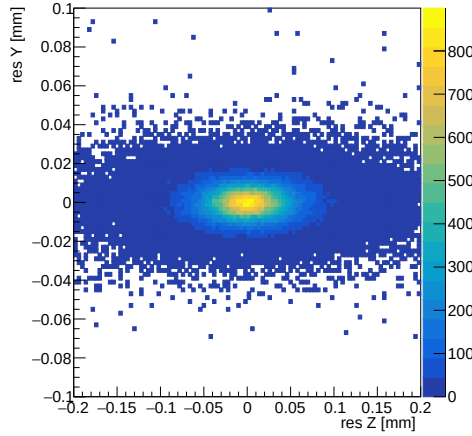
(c) Fit to PV resolution in Z



(d) 2D distribution of PV res X vs res Y



(e) 2D distribution of PV res X vs res Z



(f) 2D distribution of PV res Y vs res Z

Figure 5.7: Fits to the PV resolution and 2D plots showing their correlations in the 2018 CF Full MC sample (with BDT cut applied). The PV resolution fits use *MagUp* and *MagDown*, the 2D plots show only *MagUp*.

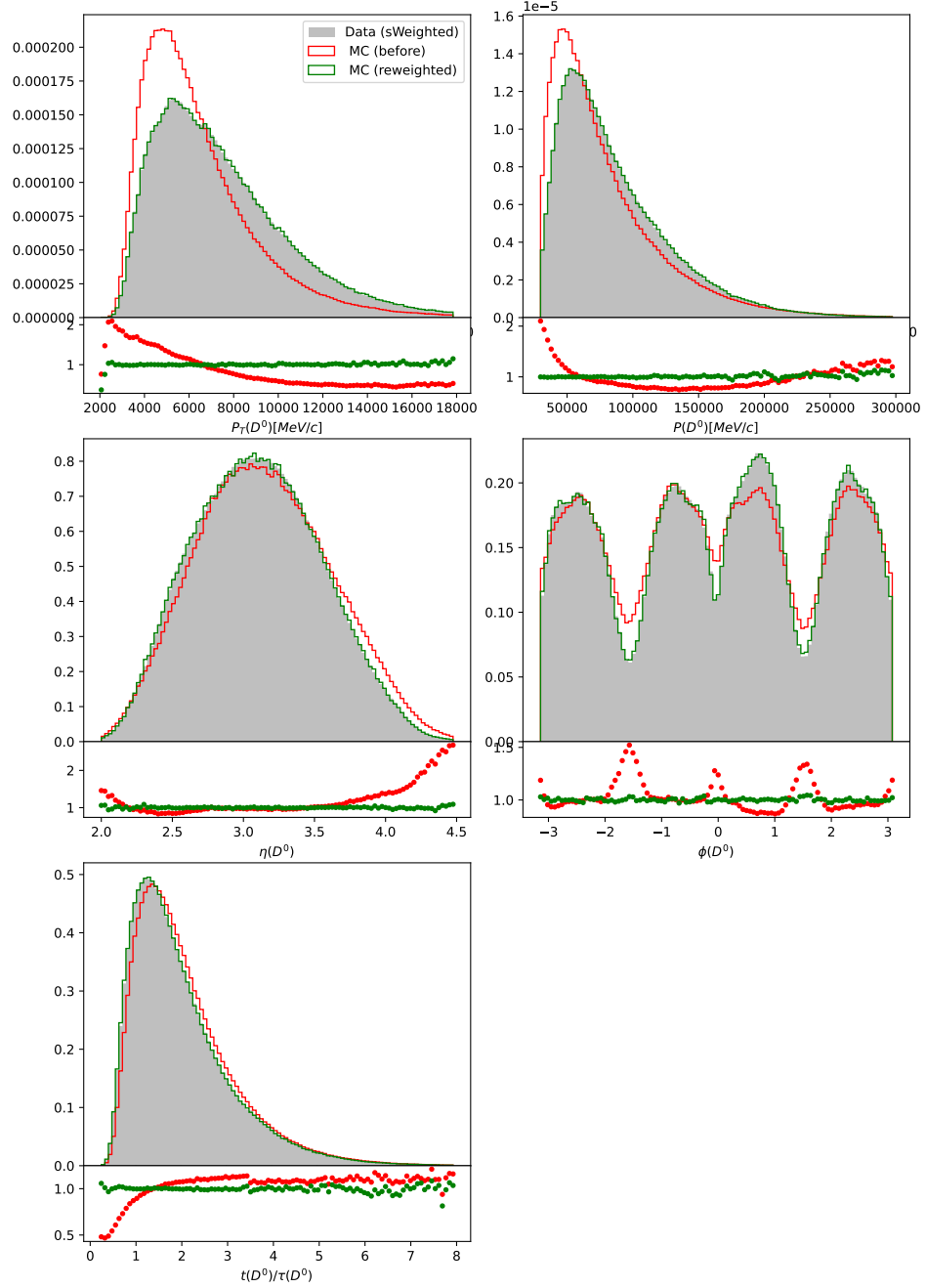


Figure 5.8: The CF Particle Gun MC sample before (red) and after (green) reweighting, compared to RS sWeighted data (grey), in the five variables used to characterise the samples.

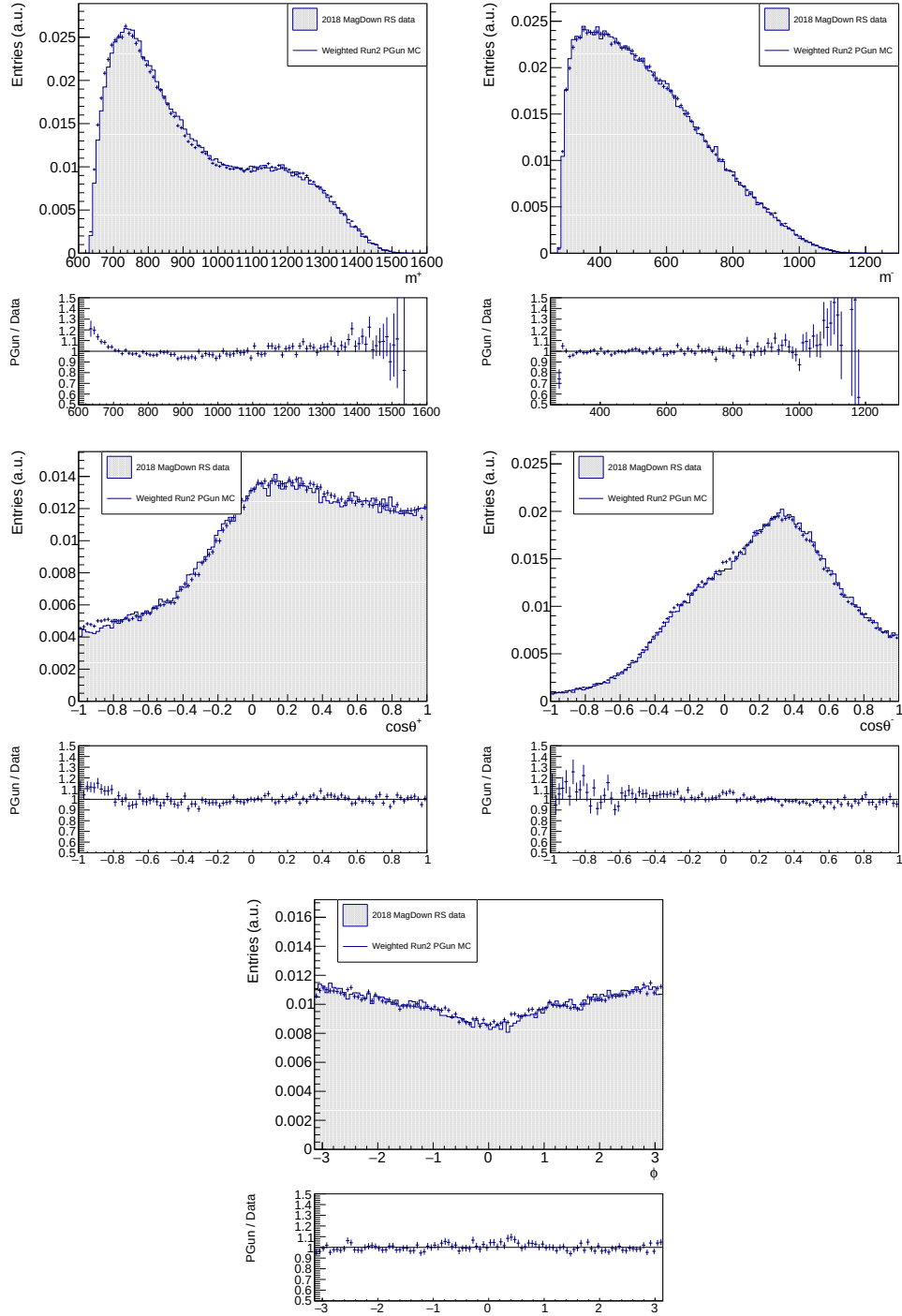


Figure 5.9: The reweighted CF Particle Gun distributions (blue) relative to RS sWeighted data (grey) in the Cabibbo-Maksymowicz variables which describe the phase-space distribution of events.

5.3.3 Efficiency effects

As presented in Sec. 4.1.1 (in particular Eqn. 4.8), the measured WS/RS ratio of yields must be corrected by the ratio of the efficiency of the WS and RS final states, $\frac{\epsilon^{RS}}{\epsilon^{WS}}$, defined by

$$\frac{\epsilon^{RS}}{\epsilon^{WS}} = \sqrt{\frac{\epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{RS} \times \epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{RS}}{\epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{WS} \times \epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{WS}}} \quad (5.5)$$

where each final-state efficiency is phase-space and time-dependent. The simplification of the efficiency factor to $\frac{\epsilon^{RS}}{\epsilon^{WS}}$ defines

$$\frac{\epsilon^{RS}}{\epsilon^{WS}} = \frac{\epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{RS}}{\epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{WS}} = \frac{\epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{RS}}{\epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{WS}} \quad (5.6)$$

i.e. the RS/WS efficiency of the final state is independent of the kaon charge. These assumptions are tested in Appendix B. In the following, we will always consider the ratio of RS/WS efficiencies, and the behaviour of $\frac{\epsilon^{RS}}{\epsilon^{WS}}$ can be calculated from the Run 2 MC samples.

LHCb MC with reconstruction and detection effects is compared to generator-level MC to make a measurement of the efficiency. In particular, it is known that detector acceptance effects can induce correlations between phase-space variables (e.g. p_T and IP) and the decay-time of the D^0 meson, which can create biases if not accounted for. Intuitively, correlations between D^0 p_T , D^0 decay time, and the IP and p_T of the D^0 decay products may be expected. For a given decay time, a D^0 meson with a larger (transverse) momentum will travel further in the detector before decaying. Its decay products will have a larger IP (or χ_{IP}^2) due to the large flight distance of the D^0 , and will also have greater (transverse) momenta. Conversely, a D^0 (at a given momenta) with a longer decay time will also travel further in the detector, and thus have decay products with a larger IP (or χ_{IP}^2). Thus, the acceptance efficiency is higher for those low-momenta D^0

mesons that possess a long decay time, or those high-momentum D^0 mesons that have a short decay time.

Knowledge of the efficiency is useful when generating toys for the study of systematic biases, in addition to correcting the data where the ratio of the efficiencies is used.

The efficiency can be written as the product of three components:

- Phase-space dependent (time-integrated) efficiency;
- Time dependent (phase-space integrated) efficiency;
- Correlations between phase-space and time dependent efficiency.

Hence, the ratio of the efficiency is

$$\begin{aligned} \frac{\epsilon^{RS}}{\epsilon^{WS}} &= \frac{\epsilon^{RS}(\Omega)}{\epsilon^{WS}(\Omega)} \times \frac{\epsilon^{RS}(t)}{\epsilon^{WS}(t)} \times \frac{1}{f_{\text{correl}}(t, \Omega)}, \\ &= \frac{1}{\tilde{\epsilon}(\text{WS/CF})} \times \frac{1}{f_{\text{correl}}(t, \Omega)} \end{aligned} \quad (5.7)$$

where $\epsilon(\Omega)$ is the phase-space dependent efficiency, $\epsilon(t)$ is the time dependent efficiency and $f_{\text{correl}}(t, \Omega)$ is the efficiency correlations function defined by the WS to RS efficiency correlations. At a point in phase space, $\epsilon^{RS}(\Omega) = \epsilon^{WS}(\Omega)$, however the distributions across phase space differ for WS and RS decays. When integrating over a region of phase space, Φ , a single time-independent value of the efficiency can be measured defined by

$$\tilde{\epsilon}(\text{WS/CF}) \equiv \frac{\int_{\Phi} N^{WS}(t, \Omega) dt d\Omega / \int_{\Phi} G^{WS}(t, \Omega) dt d\Omega}{\int_{\Phi} N^{RS}(t, \Omega) dt d\Omega / \int_{\Phi} G^{RS}(t, \Omega) dt d\Omega}, \quad (5.8)$$

where N are the signal MC yields measured and G are the expected (generator-level MC) yields. Knowledge of $\epsilon(t)$ is important as it varies significantly over the time range considered in this analysis, with low efficiency at low decay times, and thus it is an important effect to consider when simulating the decay-

time distributions of generator-level MC. However, $\epsilon^{RS}(t)$ is not expected to differ from $\epsilon^{WS}(t)$ (the decay-time distributions for WS and RS events are almost identical), and thus the effect cancels in the ratio. Checks are made to ensure no bias is introduced by this efficiency, see Sec. 6.2.2.3. The measurement of $\tilde{\epsilon}(\text{WS/CF})$ also, unavoidably, includes the time-dependent efficiency. The $f_{\text{correl}}(t, \Omega)$ is the most important efficiency effect to measure as this can bias the WS/RS ratio and must be corrected for.

5.3.3.1 Time-dependent efficiency

The time-dependent efficiency is measured by fitting an sWeighted RS data sample with an acceptance function equal to the efficiency multiplied by the exponential associated to the decay of the D^0 . The efficiency function is given by

$$\epsilon(t) = \theta(t - t_0) \frac{(t - t_0)^{n+m}}{(1 + (a(t - t_0))^n)(1 + (b(t - t_0))^m)} \exp(-\beta t) \quad (5.9)$$

where $t = t(D^0)/\tau(D^0)$ and we fit to find a, b, n, m, β . $t_0 = 0.24$ is a fixed low time threshold. The acceptance function is

$$\text{acc}(t) = \epsilon(t) \times \exp(-t). \quad (5.10)$$

A fit of the acceptance function to a RS data sample is shown in Fig. 5.10a and the efficiency function $\epsilon(t)$ is shown in Fig. 5.10b.

5.3.3.2 Phase-space dependent efficiency

The phase-space dependent efficiency, $\epsilon(\Omega)$, can be modelled in a number of ways. The parametrisation chosen is the Cabibbo-Maksymowicz (CM) variables (m^+ , m^- , $\cos \theta^+$, $\cos \theta^-$, ϕ) which are described below and shown in Fig. 5.11:

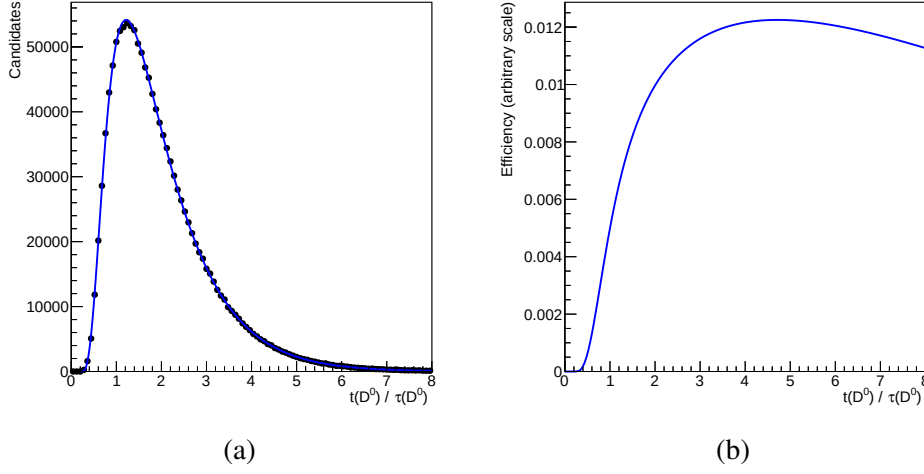


Figure 5.10: (a) Distribution of the decay time, with a fit to measure the acceptance function and (b) the measured efficiency function, measured in a 2018 *MagUp* RS data sample.

- m^+ is the invariant mass of the final-state K^+ and π^{+1} ,
- m^- is the invariant mass of the final-state π^- and π^- ,
- $\cos \theta^+$, where θ^+ is the angle between the K^+ momentum and the combined momenta of the final-state K^+ and π^+ in the D^0 rest frame,
- $\cos \theta^-$, where θ^- is the angle between one of the π^- momenta and the combined momenta of the final-state π^- and π^- in the D^0 rest frame,
- ϕ is the angle between the two planes formed by the K^+ / π^+ momenta and the π^- / π^- momenta.

These variables span the full five-dimensional (5D) phase space, with *CP*-even and *CP*-odd (ϕ) variables.

The efficiency across 5D phase space is measured by comparing generator-level MC to PGun MC (which is already weighted to RS data). In Fig. 5.12, which compares the generated and reconstructed phase-space projections in the

¹ m^+ always represents the angle between the kaon and the same-sign pion regardless of the charge of the kaon in the decay.

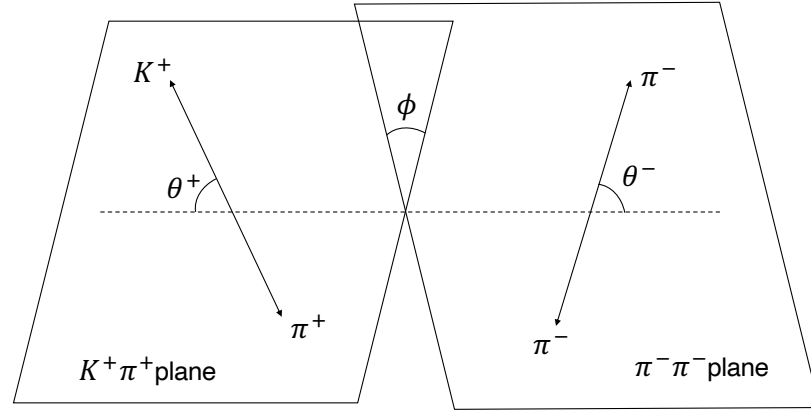


Figure 5.11: Cabibbo-Maksymowicz variables

WS sample, only small efficiency effects are observed. The largest effects are observed in $\cos \theta^+$ and $\cos \theta^-$, with variations of up to 20%. The efficiency effects are very similar across the CF and WS samples.

Monte Carlo simulation is used to calculate the integrated efficiency, $\tilde{\epsilon}(\text{WS}/\text{CF})$, which is a single time-independent value per phase-space bin associated to the time and phase-space integrated efficiency (as defined in Eqn. 5.8). The Full MC sample is used to measure $\tilde{\epsilon}(\text{WS}/\text{CF})$ as it has all the triggers in place, and can be split by year and polarity more easily than the PGun sample. The DCS MC sample is weighted to have the full WS amplitude (details are given in the following section). Events are counted over the whole time range, in the phase-space region of interest, in the MC and generator-level samples in order to derive the value of $\tilde{\epsilon}(\text{WS}/\text{CF})$. The values of $\tilde{\epsilon}(\text{WS}/\text{CF})$ are shown split by phase-space bin in Table 5.5.

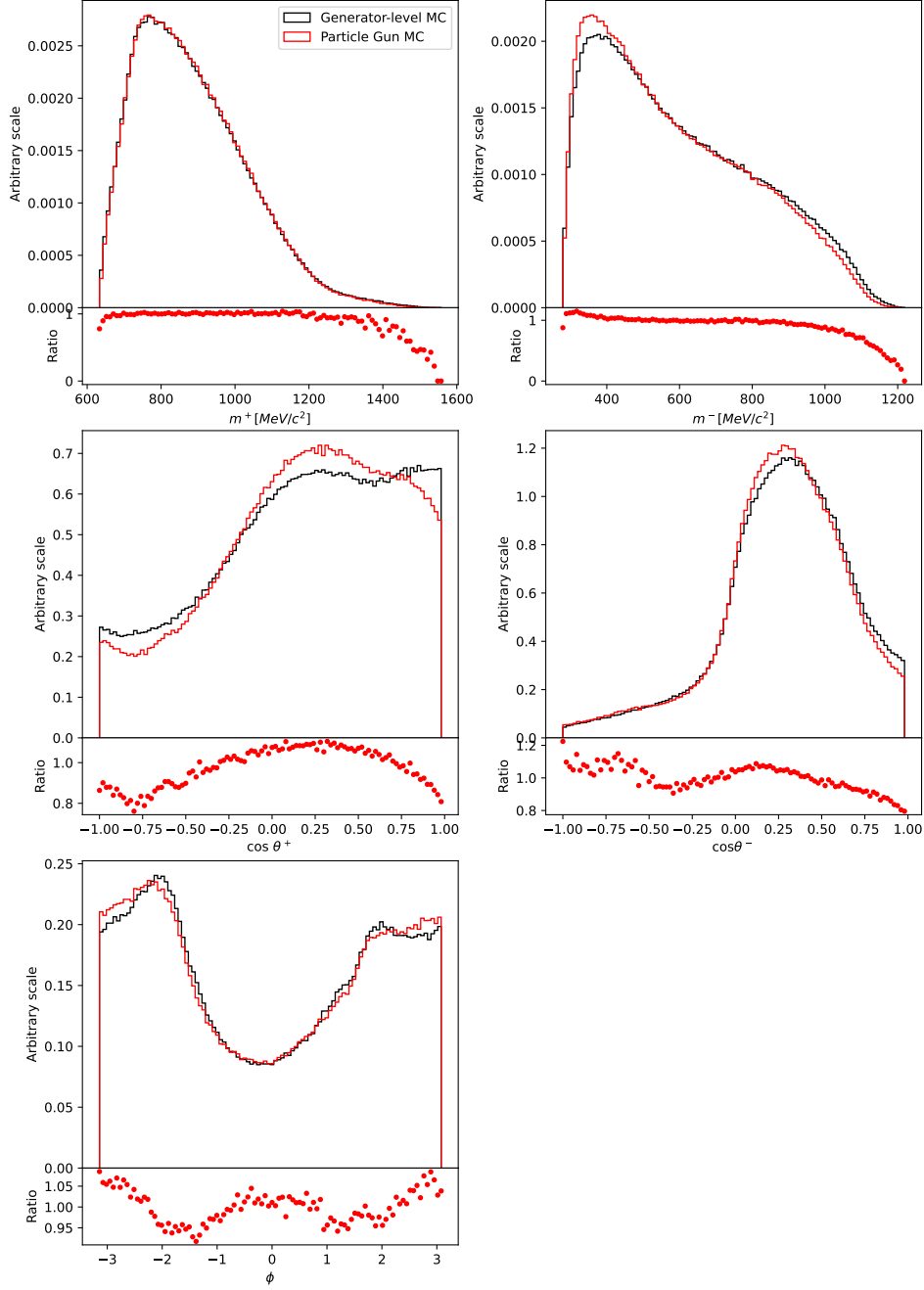


Figure 5.12: A comparison of the phase-space CM projections in WS (black) generator-level MC and (red) Particle Gun MC, giving an indication of the size of the phase-space efficiency effects.

5.3.3.3 Phase-space and time-dependent efficiency correlations

The Monte Carlo samples are generated with DCS and CF models, rather than WS and RS models. This means that they are generated with identical decay-time distributions, and there are no mixing effects present. The samples can be analysed in the same way as data, and a DCS/CF ‘mixing’ plot can be made, which should have constant ratios of yields as a function of time. However, when this is constructed from Full LHCb MC, an approximately linear trend with positive slope is observed (see Fig. 5.13), which is about 1/10th the size of the expected slope due to mixing.

Identification of cause of bias

The efficiency effect observed is due to the correlation of decay-time and phase-space acceptance effects that are caused by the following cuts on the final state tracks (from the D^0 decay products) in the HLT1 and HLT2 lines:

- $p_T(\pi), p_T(K) > 250 \text{ MeV}/c$,
- $\chi_{\text{IP}}^2(\pi), \chi_{\text{IP}}^2(K) > 3$.

This explanation is verified in the PGun sample by removing all the HLT1 and HLT2 requirements (there are no L0 requirements imposed) and then reintroducing the HLT2 cuts one-by-one. Three signal cuts are employed during the

Table 5.5: The ratio of time-integrated WS and CF efficiencies, measured from the Full MC samples, in different regions of phase space. The uncertainty arises from the limited size of the Full MC samples, generator-level MC samples and uncertainty on the weights.

Phase-space division	$\tilde{\epsilon}(\text{WS/CF})$
PS int.	0.991 ± 0.001
PS 1	0.977 ± 0.003
PS 2	0.985 ± 0.003
PS 3	0.998 ± 0.003
PS 4	0.994 ± 0.003

studies to reduce poorly reconstructed events in the PGun sample:

- $139 < \Delta m < 152 \text{ MeV}/c^2$,
- $|m(D^0) - m(D^0)_{\text{PDG}}| < 40 \text{ MeV}/c^2$,
- $\text{Dst_TRUEORIGINVERTEX_X} \neq 0$.

With only these cuts in place, and no other signal selection, it is observed that there are no biasing efficiency effects. This means that all the correlation effects arise from the HLT1 and HLT2 lines.

It is only the combination of the p_T and χ_{IP}^2 constraints on the D^0 daughters that reproduces the effects observed, as shown in Fig. 5.13. The difference in the correlation effects between this selection and the full selection in the PGun samples is attributed to the HLT1 requirements on the same variables. In particular, the `D0_Hlt1TrackMVADecision_TOS` line directly selects single tracks with a high p_T (here only in GeV/c) and large χ_{IP}^2 where the following condition must be satisfied:

$$\left\{ (p_T > 25) \wedge (\chi_{\text{IP}}^2 > 7.4) \right\} \vee \left\{ [1 < p_T < 25] \wedge \left[\ln \chi_{\text{IP}}^2 > \ln(7.4) + \frac{1}{(p_T - 1)^2} + \alpha_0 \left(1 - \frac{p_T}{25} \right) \right] \right\}. \quad (5.11)$$

The alternative HLT1 requirement (`D0_Hlt1TwoTrackMVADecision_TOS`) requires two tracks of high p_T that form a good-quality vertex that is significantly displaced from their associated primary vertex. The selection uses a bonsai boosted decision tree [69] based on a few variables: the χ^2 of the two-track vertex fit, the number of tracks with $\chi_{\text{IP}}^2 > 16$, the sum of the p_T of the two tracks, the flight-distance significance with respect to their associated primary vertex. In general, the requirements on the p_T and χ_{IP}^2 of the tracks are looser in this line than in the single track line.

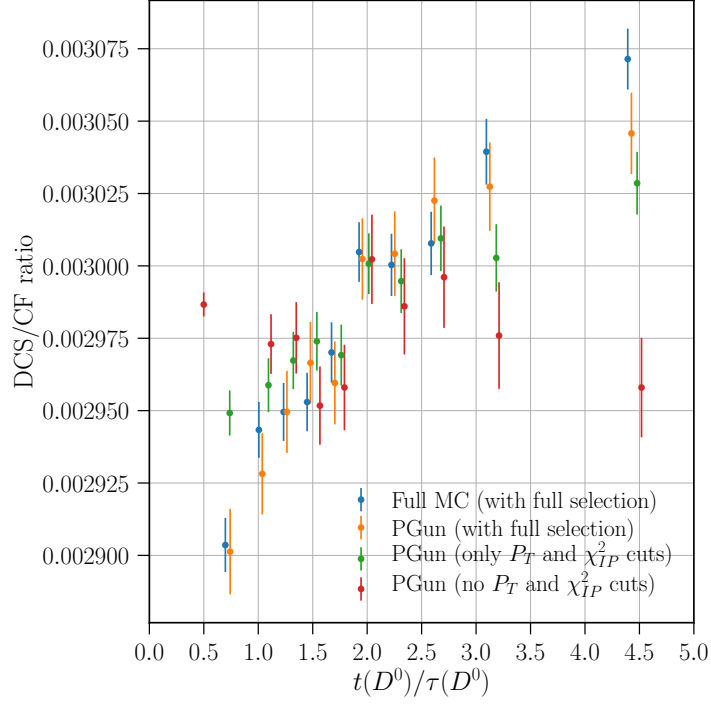


Figure 5.13: The effects of imposing the HLT2 requirements on the p_T and χ_{IP}^2 of the final state particles are observed in the difference between the no p_T and χ_{IP}^2 cuts (red) and only p_T and χ_{IP}^2 cuts (green). This can be compared to the correlations observed with the full selection on a Particle Gun sample (orange) and Full MC (blue). The difference between the partial and full selection can be attributed to the HLT1 requirements. The time coordinate for the points is slightly separated to increase clarity.

With the cause of the bias understood, the correction that must be made to the data to account for the bias is calculated. In order to evaluate the required correction, the full selection is applied to the PGun samples (which have been weighted to data), in order to encompass all effects present.

Measurement of efficiency effects due to DCS and CF differences

The DCS and CF PGun samples are normalised to have the same time-integrated weight per phase-space bin before the efficiency ratio is calculated. This results in an efficiency-ratio slope centred around unity, which gives the dominant contribution to $f_{\text{correl}}(t, \Omega)$ per time and phase-space bin. Figure 5.14 shows the efficiency correlations per time and phase-space bin for the normalisation de-

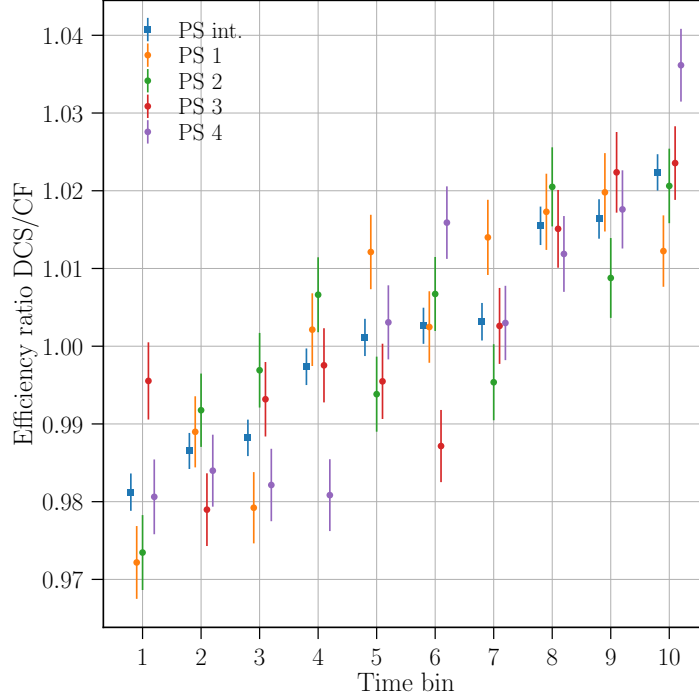


Figure 5.14: The DCS/CF efficiency correlations per time and phase-space bin. In each phase-space bin the DCS and CF samples are normalised to have the same total weight, resulting in a variation around unity.

scribed above, which shows consistent behaviour across the phase-space bins.

Measurement of efficiency effects due to WS and RS differences

The DCS/CF efficiency correction, $\epsilon(\text{DCS/CF})$, derived from MC described so far serves to correct the dominant efficiency effects in the WS/RS data ratio. However the WS decay is composed of the DCS amplitude, the mixing (CF) amplitude and the interference between the two amplitudes. Thus, after the DCS/CF efficiency correction has been applied there are residual efficiency effects remaining in the WS sample, mostly due to the interference term, that evolve with time. It is expected that the additional contributions to the efficiency from interference and mixing effects are smaller than the DCS/CF efficiency corrections, and thus can also be calculated from MC.

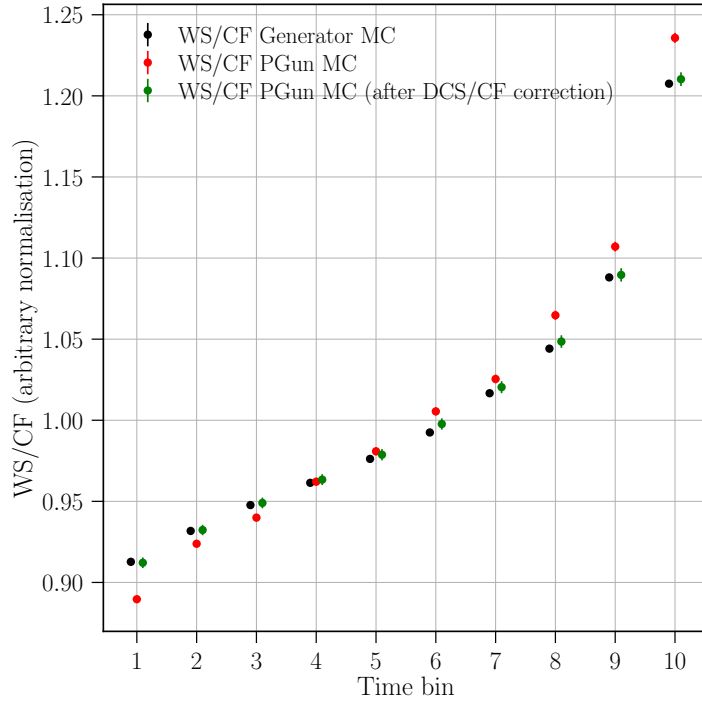
In order to generate MC samples that have the interference and mixing effects included, DCS PGun samples are reweighted to have the WS amplitude

(including DCS, interference and mixing effects) according to Eqn. 2.26. Some external inputs are required; the functions $g_{\pm}(t)$ require the mixing parameters x and y and in order to evaluate the amplitudes from the models the values of the hadronic parameters $r_D^{K3\pi}$ and $\delta_D^{K3\pi}$ are required. The values of the mixing parameters are taken from Ref. [25] (given in Sec. 2.2) and the hadronic parameters are taken from the BESIII+CLEO+LHCb combination [34] (given in Table 2.3).

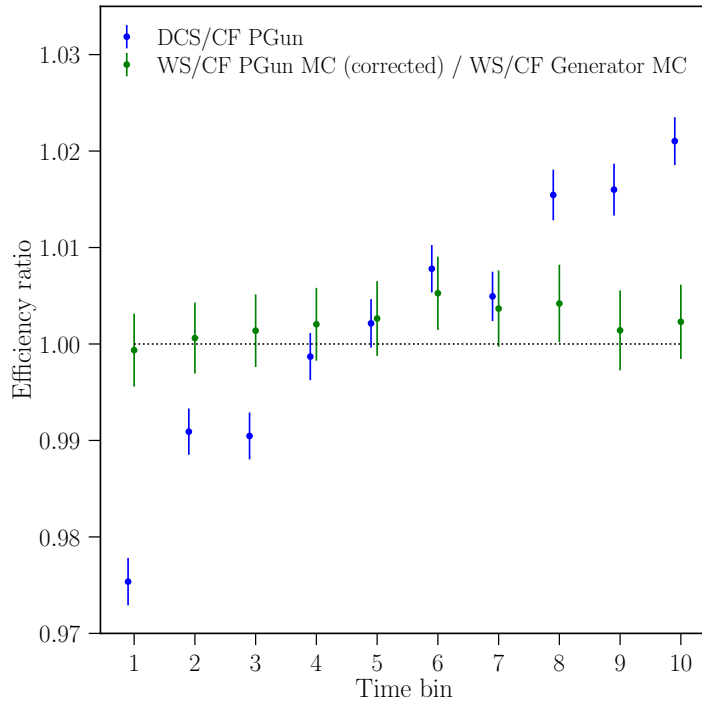
This reweighting produces a PGun sample with WS behaviour; an analogous sample of generator-level WS MC is generated from the DCS generator-level MC using exactly the same input parameters in the amplitude weighting. This ensures the true mixing effects are not diluted when the WS PGun is compared to the WS generator-level MC, and also that the dependence on the input mixing parameter values cancels (to first order). Then, the ratio of WS/CF PGun² is compared to WS/CF generator-level MC as shown in Fig. 5.15a. The ratio differs from unity with time through a combination of the DCS/CF efficiency ratio already discussed, and the additional interference/mixing effects. To isolate the interference/mixing effects, the previously calculated DCS/CF efficiency correction ($\epsilon(\text{DCS/CF})$) is applied to the WS/RS PGun ratio. The resulting difference between the ratio of (WS/CF PGun with correction)/(WS/CF gen-only) is the interference/mixing effects. Figure 5.15b shows that the efficiency effects of the interference and mixing part of the WS amplitude are smaller than that of the DCS part, as expected.

The value of $f_{\text{correl}}(t, \Omega)$ is derived per time and phase-space bin incorporating both the DCS/CF efficiency effects and the interference and mixing effects in a single measurement. As $\tilde{\epsilon}(\text{WS/CF})$ accounts for the integrated efficiency differences, the value of $f_{\text{correl}}(t, \Omega)$ is centred at unity. The values of $f_{\text{correl}}(t, \Omega)$ are shown in Fig. 5.16. The variation across phase-space bins is small, thus the

²As the RS decay is dominated by the CF amplitude, the efficiency effects present in the RS data are well-modelled by those of the CF MC.



(a)



(b)

Figure 5.15: (a) The WS/CF PGun (red) before and (green) after the DCS/CF correction, relative to the (black) WS/CF generator MC. (b) The (blue) DCS/CF correction relative to the (green) interference and mixing correction, which is derived by comparing the corrected ratio and generator ratio of (a).

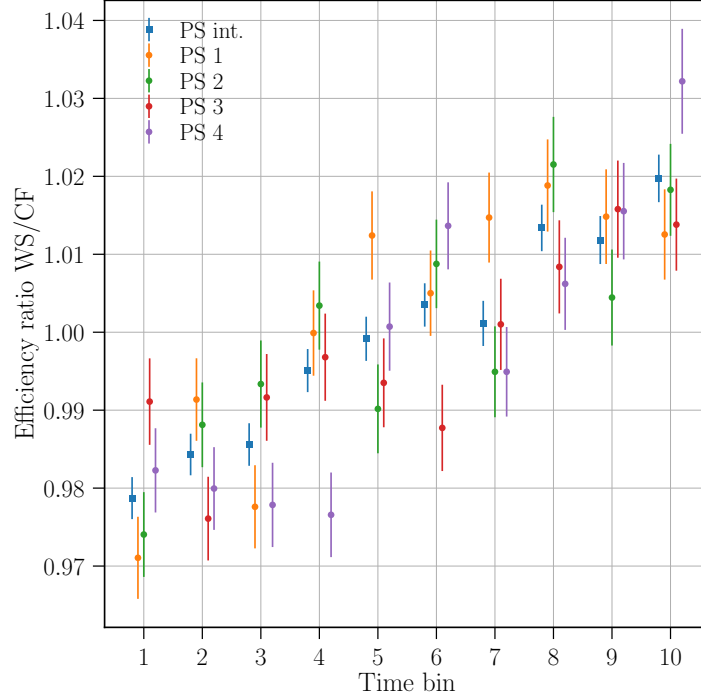


Figure 5.16: The values and uncertainty (due to the size of PGun samples) of $f_{\text{correl}}(t, \Omega)$, combining both the DCS/CF and interference+mixing/CF efficiency effects.

correction used is chosen to be the phase-space integrated values. A systematic uncertainty associated with this choice, and also associated with the sample size and the external knowledge of the parameters used to weight the DCS MC to WS MC, is assigned in Sec. 6.2.9.

5.3.3.4 Summary

The efficiency correlations function, $f_{\text{correl}}(t, \Omega)$, acts to correct the (normalised) biases that arise from the DCS and interference/mixing effects in one. Additionally, the differing phase-space distributions of WS and RS events require integrated efficiency corrections per phase-space bin. The measurement of $\frac{\epsilon^{RS}}{\epsilon^{WS}}$ allows for the WS/RS ratio in data to be corrected per time (and phase-space bin) before fitting (according to Eqn. 4.8), which will be performed in the next chapter.

6 | Determination of the mixing and hadronic parameters

In this chapter, a fit to measure the mixing parameters is presented with results and their statistical uncertainties given. Systematic uncertainties associated with the data selection and analysis method are assigned. Two consistency checks are performed, by splitting the data into different subsets. Finally, the measurements of the hadronic parameters per bin are interpreted in terms of the coherence factor κ and strong-phase difference δ per bin using external knowledge of the mixing parameters, and vice versa.

6.1 Mixing fit

The signal yields measured in the mass fit are used to construct the WS/RS ratio as a function of the mean decay time of RS events in each time bin. The WS mass fit yield comprises the WS signal yield with additional yields from secondary candidates and double mis-ID background. The RS mass fit yield comprises the RS signal yield and yield of secondary candidates. The WS yield is corrected to account for the double mis-ID background (with yields N^{DMI}). The WS/CF efficiency modifies the ratio in each time bin to remove the efficiency correlations and phase-space dependent efficiency effects per bin. The measured ratio in data, $\mathcal{R}_{ij}$, in bins of time j and phase space i , is

$$\mathcal{R}_{ij} = \frac{(N_{ij}^{\text{WS}} - N_{ij}^{\text{DMI}})}{N_{ij}^{\text{RS}}} \times \frac{1}{f_{\text{correl}}(t, \Omega)_{ij} \tilde{\epsilon}(\text{WS/CF})}, \quad (6.1)$$

where N^{WS} and N^{RS} are the yields measured in the mass fit.

Subtleties arise from the fact that the sample should be split by flavour and then the yields multiplied, according to Eqn. 4.8, to remove nuisance efficiency

effects. The ratio in data becomes

$$\mathcal{R}_{ij} = \sqrt{\frac{(N_{ij}^{WSD^0} - N_{ij}^{\text{DMI } D^0}) (N_{ij}^{WS\bar{D}^0} - N_{ij}^{\text{DMI } \bar{D}^0})}{N_{ij}^{RSD^0} \times N_{ij}^{RS\bar{D}^0}}} \frac{1}{f_{\text{correl}}(t, \Omega)_{ij} \tilde{\epsilon}(\text{WS/CF})}, \quad (6.2)$$

where the yields are measured in fits to the Δm distributions split by flavour, and the efficiency correlation function is CP -averaged by design.

The ratio in data is fit to the theoretical parametrisation, R_{ij} , given by the mixing equation,

$$R_{ij} = r_i^2 - r_i(\kappa y')_i \left\langle \frac{t_{ij}}{\tau_{D^0}} \right\rangle + \frac{x^2 + y^2}{4} \left\langle \left(\frac{t_{ij}}{\tau_{D^0}} \right)^2 \right\rangle, \quad (6.3)$$

where the normalised decay time (and decay-time squared) is estimated by the mean normalised decay time, $\left\langle \frac{t_{ij}}{\tau_{D^0}} \right\rangle$, (and normalised decay-time squared, $\left\langle \left(\frac{t_{ij}}{\tau_{D^0}} \right)^2 \right\rangle$) of flavour-integrated sWeighted RS data in each bin, the values of which are displayed in Table 6.1.

To account for secondary contamination within the signal yields, the theoretical parametrisation is altered to $\hat{R}_{ij}$ (according to Eqn. 4.10),

$$\begin{aligned} \hat{R}_{ij} = R_{ij} &+ f_{ij}^{\text{sec}}(-r_i)(\kappa y')_i \left(\left\langle \frac{t_{ij}^{\text{sec}}}{\tau_{D^0}} \right\rangle - \left\langle \frac{t_{ij}}{\tau_{D^0}} \right\rangle \right) \\ &+ f_{ij}^{\text{sec}} \frac{x^2 + y^2}{4} \left(\left\langle \left(\frac{t_{ij}^{\text{sec}}}{\tau_{D^0}} \right)^2 \right\rangle - \left\langle \left(\frac{t_{ij}}{\tau_{D^0}} \right)^2 \right\rangle \right). \end{aligned} \quad (6.4)$$

Table 6.1: The mean normalised decay time, $\left\langle \frac{t_{ij}}{\tau_{D^0}} \right\rangle$, and the mean normalised decay-time squared, $\left\langle \left(\frac{t_{ij}}{\tau_{D^0}} \right)^2 \right\rangle$, per time bin j and phase-space bin i , measured in Run 2 RS data.

Time unit	TB	PS int.	PS 1	PS 2	PS 3	PS 4
$\left\langle \frac{t_{ij}}{\tau_{D^0}} \right\rangle$	1	0.7530 ± 0.0001	0.7533 ± 0.0002	0.7525 ± 0.0003	0.7531 ± 0.0003	0.7529 ± 0.0002
	2	1.0648 ± 0.0001	1.0649 ± 0.0001	1.0644 ± 0.0001	1.0649 ± 0.0001	1.0651 ± 0.0001
	3	1.2922 ± 0.0001	1.2920 ± 0.0001	1.2923 ± 0.0001	1.2922 ± 0.0001	1.2922 ± 0.0001
	4	1.5085 ± 0.0001	1.5083 ± 0.0001	1.5083 ± 0.0001	1.5088 ± 0.0001	1.5085 ± 0.0001
	5	1.7326 ± 0.0001	1.7324 ± 0.0001	1.7327 ± 0.0002	1.7324 ± 0.0002	1.7329 ± 0.0001
	6	1.9856 ± 0.0001	1.9854 ± 0.0002	1.9857 ± 0.0002	1.9858 ± 0.0002	1.9856 ± 0.0002
	7	2.2829 ± 0.0001	2.2828 ± 0.0002	2.2829 ± 0.0002	2.2830 ± 0.0002	2.2828 ± 0.0002
	8	2.6469 ± 0.0001	2.6469 ± 0.0003	2.6469 ± 0.0003	2.6468 ± 0.0003	2.6470 ± 0.0003
	9	3.1542 ± 0.0002	3.1549 ± 0.0004	3.1548 ± 0.0005	3.1538 ± 0.0005	3.1536 ± 0.0004
	10	4.4517 ± 0.0009	4.4526 ± 0.0019	4.4479 ± 0.0020	4.4543 ± 0.0020	4.4516 ± 0.0017
$\left\langle \left(\frac{t_{ij}}{\tau_{D^0}} \right)^2 \right\rangle$	1	0.5846 ± 0.0002	0.5850 ± 0.0004	0.5839 ± 0.0004	0.5847 ± 0.0004	0.5846 ± 0.0003
	2	1.1388 ± 0.0002	1.1389 ± 0.0003	1.1379 ± 0.0003	1.1389 ± 0.0003	1.1395 ± 0.0003
	3	1.6735 ± 0.0002	1.6731 ± 0.0003	1.6738 ± 0.0004	1.6736 ± 0.0004	1.6736 ± 0.0003
	4	2.2795 ± 0.0002	2.2790 ± 0.0004	2.2790 ± 0.0004	2.2806 ± 0.0004	2.2796 ± 0.0004
	5	3.0064 ± 0.0003	3.0057 ± 0.0005	3.0066 ± 0.0005	3.0056 ± 0.0005	3.0074 ± 0.0005
	6	3.9491 ± 0.0003	3.9483 ± 0.0007	3.9494 ± 0.0007	3.9499 ± 0.0007	3.9490 ± 0.0006
	7	5.2201 ± 0.0005	5.2197 ± 0.0010	5.2202 ± 0.0010	5.2208 ± 0.0010	5.2198 ± 0.0009
	8	7.0207 ± 0.0007	7.0207 ± 0.0015	7.0205 ± 0.0016	7.0203 ± 0.0016	7.0211 ± 0.0014
	9	9.9817 ± 0.0014	9.9857 ± 0.0027	9.9854 ± 0.0029	9.9787 ± 0.0029	9.9777 ± 0.0026
	10	20.5840 ± 0.0096	20.5931 ± 0.0191	20.5498 ± 0.0206	20.6080 ± 0.0202	20.5828 ± 0.0177

Table 6.2: Measured values of the mixing parameters and their statistical uncertainties from the fits to data.

Phase-space division	$x^2 + y^2 (\times 10^{-5})$	$r (\times 10^{-2})$	$\kappa y' (\times 10^{-3})$
PS integrated	7.2 ± 2.8	5.489 ± 0.035	-2.68 ± 0.67
PS 1		5.652 ± 0.045	-4.15 ± 0.73
PS 2		5.966 ± 0.049	-5.70 ± 0.74
PS 3	5.9 ± 2.8	5.812 ± 0.046	-3.00 ± 0.72
PS 4		4.609 ± 0.051	0.51 ± 0.83

6.1.1 χ^2 minimisation

The fit to measure the mixing parameters minimises the χ^2 ,

$$\chi^2 = \sum_{i,j} \left(\frac{\mathcal{R}_{ij} - \hat{R}_{ij}}{\sigma(\mathcal{R}_{ij})} \right)^2 \quad (6.5)$$

where $\sigma(\mathcal{R}_{ij})$ is the uncertainty on $\frac{N^{WSD^0} N^{WSD^0}}{N^{RSD^0} N^{RSD^0}}$ (*i.e.* from the mass fit alone). A single fit is performed on the entire data set as a function of decay time to measure the phase-space integrated parameters $r_D^{K3\pi}$, $(\kappa y')_D^{K3\pi}$ and $x^2 + y^2$. A second fit is performed on the data set split by phase-space bin. This is a simultaneous fit across the four bins as a function of decay time, measuring the nine parameters r_i , $(\kappa y')_i$ and $x^2 + y^2$, where $i = 1, 2, 3, 4$ are the four phase-space bins.

6.1.2 Fits to data

The WS/RS ratio in data, with fit superimposed, is shown in Figs. 6.1 and 6.2. The measured parameters with their statistical uncertainty from the two fits to data are presented in Table 6.2, where the entire Run 2 sample is used. The correlation matrix of each fit is given in Table 6.3, along with the χ^2 per degree of freedom in each fit which is good.

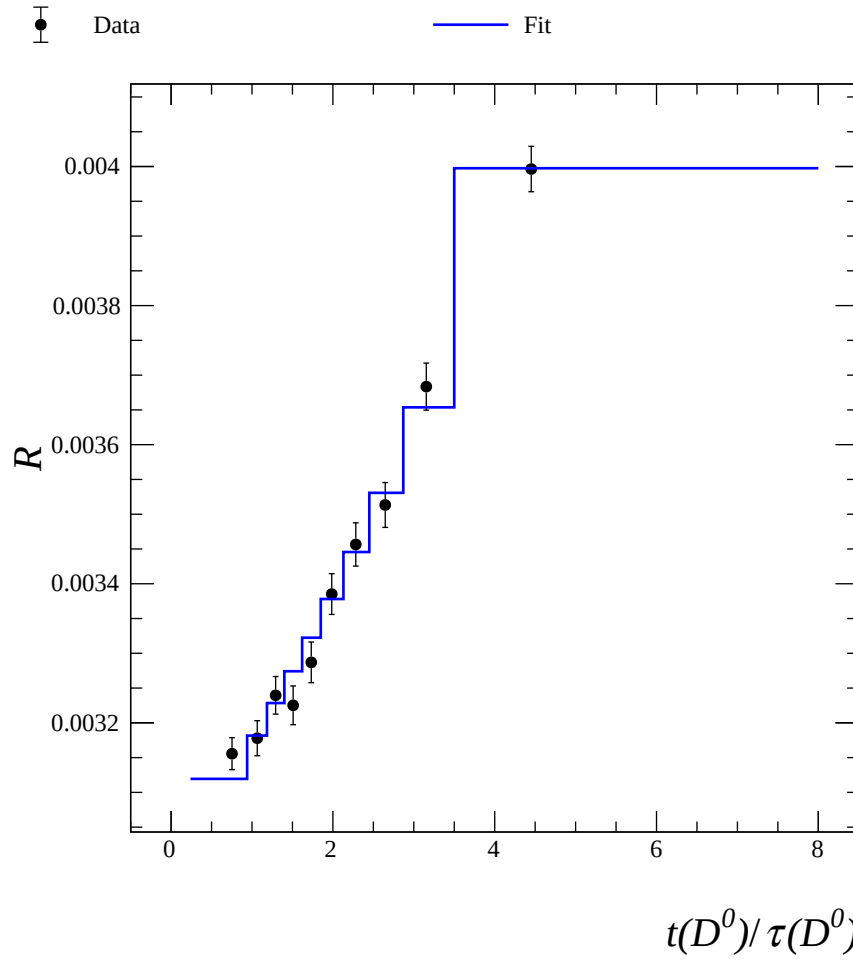


Figure 6.1: Phase-space integrated WS/RS ratio as a function of decay time, with fit result superimposed.

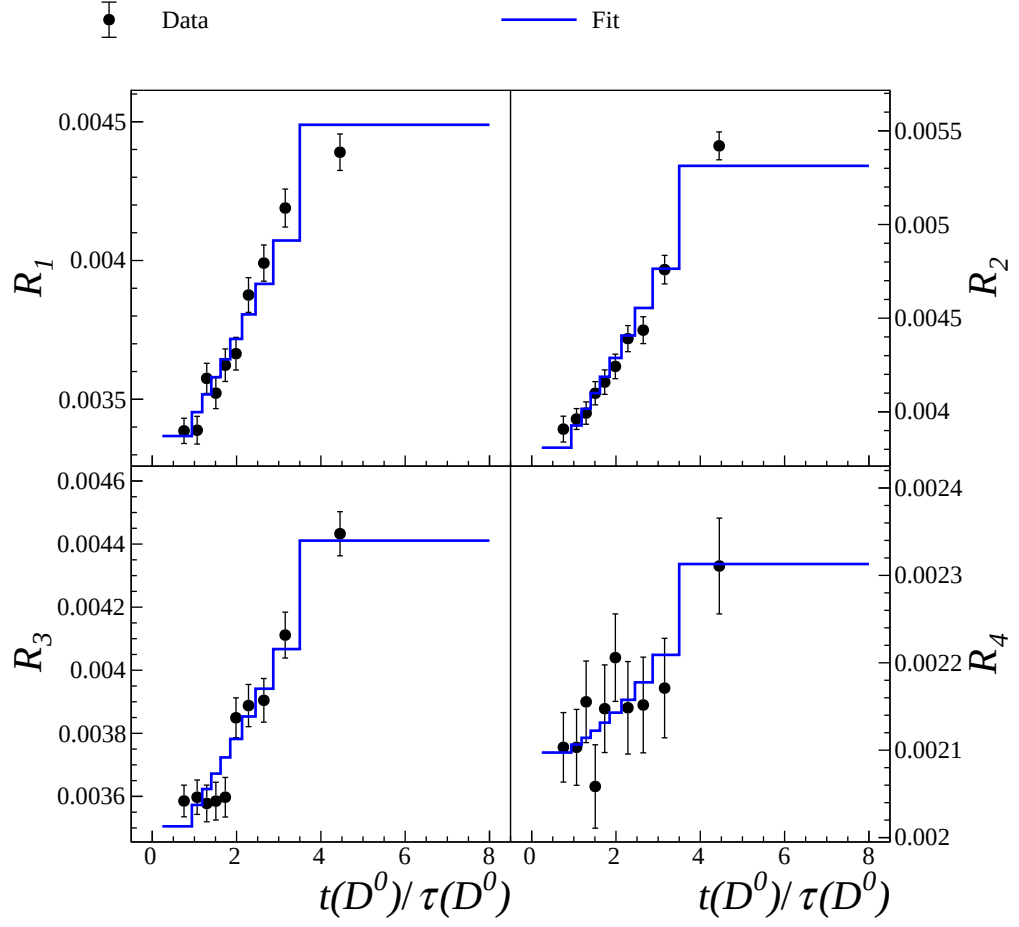


Figure 6.2: Phase-space binned WS/RS ratio as a function of decay time, with simultaneous fit result superimposed.

Table 6.3: The correlation coefficients of the (top) phase-space integrated fit and (bottom) phase-space binned fit.

Parameter	$x^2 + y^2$	$r_D^{K3\pi}$	$(\kappa y')_D^{K3\pi}$
$x^2 + y^2$	1		
$r_D^{K3\pi}$	0.865	1	
$(\kappa y')_D^{K3\pi}$	0.965	0.952	1
χ^2/ndof	7.9/7		

Parameter	$x^2 + y^2$	r_1	r_2	r_3	r_4	$(\kappa y')_1$	$(\kappa y')_2$	$(\kappa y')_3$
r_1	0.654	1						
r_2	0.610	0.399	1					
r_3	0.629	0.411	0.384	1				
r_4	0.715	0.468	0.436	0.450	1			
$(\kappa y')_1$	0.879	0.898	0.537	0.553	0.628	1		
$(\kappa y')_2$	0.845	0.553	0.902	0.532	0.604	0.743	1	
$(\kappa y')_3$	0.866	0.567	0.529	0.892	0.620	0.762	0.733	1
$(\kappa y')_4$	0.916	0.599	0.559	0.576	0.901	0.805	0.774	0.793
χ^2/ndof	42.2/31							

6.2 Systematic uncertainties

There are several source of systematic uncertainty in this measurement, which are broken down into two groups studied in different ways. The first category is associated to effects that cannot be separated from the data; *e.g.* the decay-time efficiency, decay-time resolution, phase-space migration and fitter bias. These are assessed using pseudo-experiments generated with known input values, which are then fitted and any significant offset in the result from the input value is considered a bias. These studies are outlined in Sec. 6.2.1 and Sec. 6.2.2. The second category is associated with the choices and corrections made to the data set during the analysis procedure, including the secondary-candidate correction, double mis-ID removal, mass-fit choices and the efficiency-correlations correction. These are studied by re-analysing the data set with one

correction varied every time.

6.2.1 Fitter bias

To check that there is no bias in the measured parameters being introduced by the fitter itself, 900 generator-level pseudo-data sets are generated according to the models from Ref. [33] for WS (including mixing) and RS (with no mixing) decays. The yield of each of the WS pseudo-experiments is $N_{\text{gen}}^{WS} = 600,000$ (which is approximately 1.4 times larger than the expected yield from data), and a single RS pseudo-experiment consists of $N_{\text{gen}}^{RS} = 186.4 \times 10^6$ events, calculated from the equation below and the generator values used.

The number of events used to generate each pseudo-experiment, when the decay-time distribution is described by an exponential decay, is defined by:

$$\begin{aligned}
 N_{\text{gen}}^{WS} &= c \int_{t_{\text{low}}}^{t_{\text{high}}} \Gamma(WS) dt \\
 &= c \int_{t_{\text{low}}}^{t_{\text{high}}} \exp(-t) \left(|A_{DCS}|^2 - \kappa_D^{K3\pi} y'_D |A_{DCS}| |A_{CF}| t + \frac{x^2 + y^2}{4} |A_{CF}|^2 t^2 \right) dt, \\
 N_{\text{gen}}^{RS} &= c \int_{t_{\text{low}}}^{t_{\text{high}}} \Gamma(RS) dt \\
 &= c \int_{t_{\text{low}}}^{t_{\text{high}}} \exp(-t) |A_{CF}|^2 dt \\
 &\equiv N_{\text{gen}}^{WS} \times \frac{c \int_{t_{\text{low}}}^{t_{\text{high}}} \Gamma(RS) dt}{c \int_{t_{\text{low}}}^{t_{\text{high}}} \Gamma(WS) dt},
 \end{aligned} \tag{6.6}$$

$$\tag{6.7}$$

where c is an arbitrary scaling factor and t is shorthand for $t(D^0)/\tau(D^0)$ (with $t_{\text{low}} = 0$ and $t_{\text{high}} = 8.0$).

If the fitter is unbiased, the pull distributions constructed in the same manner as in Sec. 5.2.1 should be unbiased in relation to the generator values used to generate the pseudo-experiments. The generator values are given in Table 6.4. The pull statistics are shown for phase-space integrated and binned fits in Table 6.5. From these we can see that the fitter is unbiased, as the means of the

Table 6.4: Generator values used in the generation of the pseudo-experiments. The phase-space integrated $r_D^{K3\pi}$ and $\delta_D^{K3\pi}$ are specified as inputs, as well as the mixing parameters x and y . The phase-space binned r_i, κ_i, δ_i ($i = 1, 2, 3, 4$) parameters are not direct inputs in generation but are intrinsic to the models.

Phase-space division	x	y	r	κ	δ ($^\circ$)
PS integrated			0.054900	0.4590	128
PS 1			0.054665	0.6732	56
PS 2	0.0037	0.0066	0.059435	0.8537	108
PS 3			0.058330	0.8161	149
PS 4			0.048165	0.6267	208

pull distributions are centred at zero and the widths are compatible with unity. No systematic uncertainty is assigned associated with the mixing-fit procedure.

These pseudo-experiments are not fully representative of the data for two reasons; the first is that no (time or phase-space) efficiency or resolution effects have been included in the pseudo-experiments, in particular the decay-time acceptance. This means that these data sets have much better precision on the fitted value of r in particular, due to the larger yields in the lowest decay time bin. The contributions from phase-space bin 5 are also not removed. The second is that the uncertainty on the yield per time bin is assumed to be $\sqrt{\sum_{i=1}^N w_i^2}$ for event i with weight w_i , which for unweighted data is equal to $\sqrt{N}$ for N events per time bin. This is in contrast to the uncertainty on the data yields per time bin which come directly from the mass fit (which are not equal to the square root of the yields measured in the mass fit.) In the following, the effects of adding these additional attributes are assessed.

6.2.2 Pseudo-experiments with detector effects

Pseudo-experiments are generated with a framework that accounts for the efficiency and resolution effects of the detection and selection. Many generator-level pseudo-experiments (with a WS yield of 600,000 which is slightly larger than the expected yield) are subject to the decay-time efficiency effects and

Table 6.5: The pull statistics ((top) mean and (bottom) width) of fits to generator-level pseudo-experiments.

Phase-space division	$x^2 + y^2$	Pull mean	
		r	$\kappa y'$
PS integrated	0.03 ± 0.03	0.02 ± 0.03	0.02 ± 0.03
PS 1		0.10 ± 0.03	0.03 ± 0.03
PS 2	0.03 ± 0.03	0.04 ± 0.02	0.04 ± 0.03
PS 3		-0.01 ± 0.03	0.03 ± 0.03
PS 4		-0.11 ± 0.03	0.02 ± 0.03

Phase-space division	$x^2 + y^2$	Pull width	
		r	$\kappa y'$
PS integrated	1.01 ± 0.02	0.91 ± 0.02	1.02 ± 0.02
PS 1		0.97 ± 0.02	1.01 ± 0.02
PS 2	1.01 ± 0.02	0.96 ± 0.02	1.02 ± 0.02
PS 3		0.97 ± 0.02	1.02 ± 0.02
PS 4		0.97 ± 0.02	1.02 ± 0.02

phase-space efficiency effects (presented in Sec. 5.3) and also to decay time and phase-space resolution effects, the details of which are given below.

6.2.2.1 Decay time resolution

The decay-time resolution is measured by fitting the distribution of the difference in the Monte Carlo truth level and reconstructed values of the D^0 decay time, where the resolution is equal to the width of the fitted Gaussian. The resolution is measured to be $(0.1295 \pm 0.0002)\tau_{D^0}$.

6.2.2.2 Phase-space resolution

The phase-space resolution can be measured by comparing the truth level and reconstructed invariant-mass values in simulated samples. The measured resolutions are shown in Table 6.6 for a sample of LHCb MC. However the resolutions measured in this form are inconvenient to implement, as the phase-space binning

Table 6.6: Phase-space resolution measured in a fully selected 2018 *MagUp* CF Prompt MC sample.

Quantity	Resolution (GeV ² /c ⁴)
$m^2(K^-\pi^+)$	0.004268 ± 0.000006
$m^2(K^-\pi^-)$	0.004177 ± 0.000006
$m^2(\pi^+\pi^-)$	0.003459 ± 0.000005
$m^2(K^-\pi^+\pi^-)$	0.006042 ± 0.000008
$m^2(\pi^+\pi^-\pi^+)$	0.004819 ± 0.000007
$m^2(K^-\pi^+\pi^+)$	0.005942 ± 0.000008

uses the kinematics of individual particles rather than invariant masses. Instead, a migration matrix is used to implement the effects of phase-space resolution.

The migration of events from the phase-space bin assigned using the truth-level kinematics (‘truth’ bin) to the bin assigned using the reconstructed kinematics (‘reconstructed’ bin) is studied. The MC sample used is 2017 + 2018 *MagUp* and *MagDown*, because the truth-level variables in the 2016 MC sample are not of good quality, and they are not available in the 2015 sample. It is not expected that the phase-space bin migration will change significantly between years (when the 2017 + 2018 sample are split by different years and polarities the individual migration matrices are consistent).

Table 6.7 shows the fraction of events from each truth-level bin (each row) that are assigned to each reconstructed bin (each column). More than 96% of events in bins 1–4 remain in the correct bin, and the migration into other bins is in the range 0–2%. This can be implemented in the pseudo-experiments by using a random number to decide what bin the event should be assigned, based on the generator (truth) bin. The difference in the migration for CF and DCS events is small, and thus the same migration matrix is used for all generated events (which is the average of the two).

Table 6.7: Phase-space bin migration measured in a fully selected 2017+2018 Prompt MC for the (top) CF sample and (bottom) DCS sample.

CF		Reconstructed bin				
		1	2	3	4	5
True bin	1	0.976	0.014	0.001	0.005	0.004
	2	0.016	0.961	0.018	0.002	0.004
	3	0.001	0.017	0.961	0.017	0.004
	4	0.004	0.000	0.013	0.979	0.003
	5	0.018	0.016	0.018	0.018	0.930

DCS		Reconstructed bin				
		1	2	3	4	5
True bin	1	0.974	0.017	0.001	0.004	0.003
	2	0.016	0.960	0.019	0.001	0.004
	3	0.001	0.020	0.960	0.016	0.004
	4	0.005	0.001	0.019	0.973	0.003
	5	0.016	0.023	0.018	0.014	0.929

6.2.2.3 Pseudo-experiment results

Pseudo-experiments are generated by applying smearing to the generated decay-time to model the decay-time resolution. The decay-time efficiency is applied using the accept-reject method. Phase-space bin migration is performed, after which candidates which fall into phase-space bin 5 (which collects potential $D^0 \rightarrow K_S^0 K^\pm \pi^\mp$ decays in the sample) are removed. The phase-space efficiency is not included, as in this study it is desired to measure only the bias of effects that are not corrected for; the difference in phase-space efficiency of WS and RS events is corrected for by the efficiency correlations function and integrated WS/CF efficiency ratio.

Table 6.8 shows the absolute value of the bias in the mean fitted value of 263 pseudo-experiments relative to the generator values in the model. As the bias on all observables is smaller than the statistical uncertainty, the absolute value of the bias is assigned as a systematic uncertainty.

Table 6.8: The value of the bias in the observables of generator-level pseudo-experiments with detector effects compared to the model values. The bias is negative if the mean value of the parameter from pseudo-experiments with detector effects is less than the generator value. The absolute value of the bias is assigned as a systematic uncertainty.

Phase-space division	Bias ($\times 10^{-5}$)		
	$x^2 + y^2$	r	$\kappa y'$
PS integrated	0.2	-2.3	5.6
PS 1		17.9	2.6
PS 2	0.2	-12.0	10.6
PS 3		-17.1	16.5
PS 4		29.0	7.7

6.2.3 Secondary-fraction and time-bias correction

Section 4.5.1 discussed the presence of secondary events within the data samples. Equation 4.10 shows how the bias on the measured decay time caused by this background can be corrected in the mixing fit, which is dependent on the measured secondary fraction per time bin f_j^{sec} and the mean normalised decay-time (squared) per time bin $\langle t_j^{\text{sec}(2)} \rangle$. As a reminder, the uncertainty associated with the fitted secondary yields in the $\log(\chi_{\text{IP}}^2(D^0))$ fits is small ($\sigma(f_j^{\text{sec}}) = 0.03 - 0.07\%$ for phase-space integrated and $\sigma(f_j^{\text{sec}}) = 0.06 - 0.17\%$ for phase-space binned fits). The uncertainty associated with the mean secondary decay-time per reconstructed decay time bin is also very small.

Repeated fits to data are used to measure the systematic uncertainty associated with this correction. Each data set is constructed by varying the values of the secondary yields and times within their associated uncertainties. The yields and times are varied independently per time bin and per phase-space bin. A total of 1000 data sets are fitted, and the distribution of fitted parameters values is fit with a Gaussian. The systematic uncertainty on each parameter is assigned as the width of the Gaussian. Table 6.9 shows the systematic uncertainties measured when the secondary yields and times are varied, which are all very small.

Table 6.9: Secondary systematic uncertainty considering only the variation in the measured uncertainty of $f^{\text{sec}}, t^{\text{sec}}, (t^{\text{sec}})^2$. These uncertainties are deemed to be over-optimistic and thus are not assigned as the systematic uncertainty.

Phase-space division	Uncertainty ($\times 10^{-5}$)		
	$x^2 + y^2$	$r_D^{K3\pi}$	$\kappa y'$
PS integrated	0.01	0.04	0.10
PS 1		0.15	0.28
PS 2	0.01	0.14	0.24
PS 3		0.13	0.25
PS 4		0.16	0.30

Table 6.10: The bias in the observables when setting $f^{\text{sec}}, t^{\text{sec}}, (t^{\text{sec}})^2 = 0$ for all TBs compared to nominal correction, showing the size of the effect that the secondary correction has on the observables. The systematic uncertainty is conservatively assigned as the absolute value of the bias.

Phase-space division	Bias ($\times 10^{-5}$)		
	$x^2 + y^2$	r	$\kappa y'$
PS integrated	0.2	2.0	8.9
PS 1		1.6	10.0
PS 2	0.2	5.9	17.3
PS 3		3.1	10.6
PS 4		-2.3	-2.0

It is possible that these results are an underestimation of the true systematic uncertainty of this correction as the uncertainties do not account for the choice of the fit method and any imperfections in the MC sample. For this reason, the systematic uncertainty is reassigned as the size of the bias caused by having no secondary correction applied (*e.g.* $f^{\text{sec}}, t^{\text{sec}}, (t^{\text{sec}})^2 = 0$ for all time bins) relative to the nominal correction. The result is shown in Table 6.10. This is a more conservative approach, and the assigned values remain a sub-dominant systematic.

6.2.4 Double mis-ID correction

In Sec. 4.5.2, the measurement of the double mis-ID yield in the data is described, and Eqn. 6.2 shows how the correction is applied. In this section, the

systematic uncertainty associated with the yield correction is assessed.

The systematic uncertainty associated with the model used to measure the mis-ID yields is first considered. The nominal model used to measure the yields comprises a Gaussian function for the signal and a RooBkg function for the background. Alternative parametrisations are used to see how the yields vary with different choices of model. An alternative signal function of an Extended Cruijff is considered, with some parameters fixed to ensure fit stability. In detail, α_L and α_R are fixed to values from the full mass fit (setting $\alpha = \alpha_L = \alpha_R = (\alpha_L(\text{mass fit}) + \alpha_R(\text{mass fit}))/2$), and a single (floating) width parameter $\sigma = \sigma_L = \sigma_R$ is used. For the background description, a RooRSP function¹ is considered. All combinations of signal and background shapes give four parametrisations in total. The yields are measured with each of these parametrisations, which are shown in Fig. 6.3a. The systematic uncertainty due to the different models is assigned as the RMS of the four corrections (per time bin),

$$\sigma_{syst} = \sqrt{\frac{\sum_{i=1}^4 (Y_i - \hat{Y}_i)^2}{4}}, \quad (6.8)$$

where Y_i is the yield per model i and $\hat{Y}_i$ is the mean value from all the models. The total uncertainty is then given by the addition in quadrature of σ_{stat} (from the nominal fit) and σ_{syst} . The increase in the uncertainty from the inclusion of σ_{syst} is shown in Fig. 6.3b.

From the double mis-ID yield and its total uncertainty, the systematic uncertainty associated with the subtraction of double mis-ID events from the measured WS data yield is calculated. A total of 1000 pseudo-experiments are constructed from a single data set, by subtracting the measured double mis-ID yield (split by flavour) many times (with the subtracted value varied according to a

¹This function is described in Sec. 6.2.8.

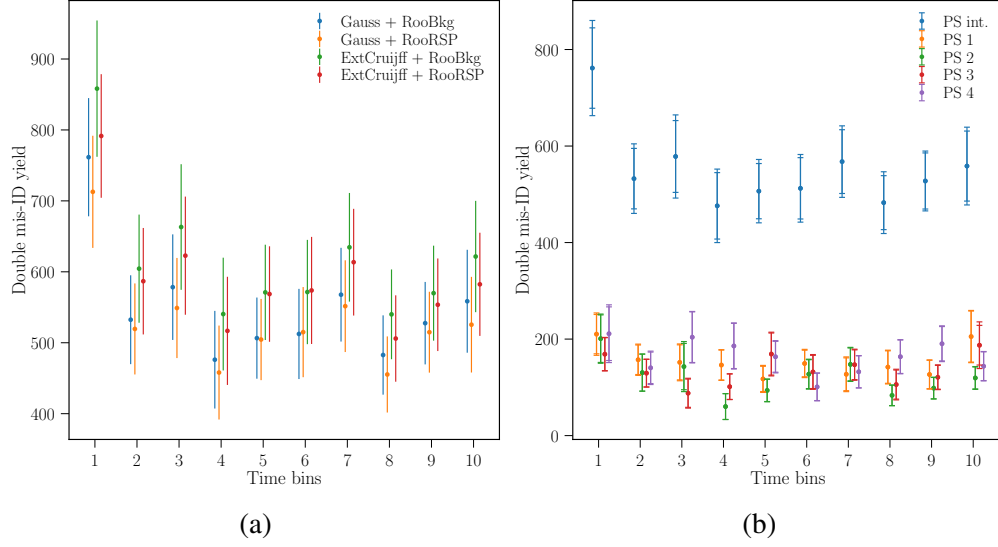


Figure 6.3: (a) The double mis-ID yields (phase-space integrated) using different models, showing statistical uncertainties only, for D^0 WS data. (b) The double mis-ID yields (phase-space integrated and binned), with statistical and total uncertainty, for D^0 WS data.

Gaussian with mean equal to the double mis-ID yield and width equal to the yield uncertainty (independent per time bin and per phase-space bin)). The double mis-ID uncertainty is small relative to the uncertainty on the WS signal yield from the mass fit so no alteration is made to the uncertainty on the WS signal yield when the correction is applied. Each pseudo-experiment is fit with the mixing equation (after other nominal corrections have been applied), and the fitted values of the observables are studied, with their distributions fitted with a Gaussian. The width of the Gaussian is taken as the systematic uncertainty per parameter. The systematic uncertainty on each observable is approximately a third of the statistical uncertainty.

6.2.5 Slow-pion ghosts

In Section 4.5.4, a cut on $\text{Prob}_{\text{ghost}}(\pi_s)$ is introduced to reduce bias caused by slow-pion ghosts that can enhance signal yields and incorrectly tag the D^0

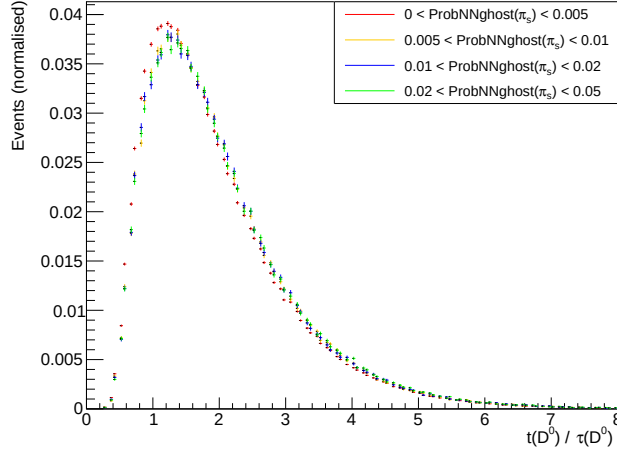


Figure 6.4: The decay-time distribution as a function of slow-pion ghost probability, in a sample of 2018 *MagUp* RS data.

flavour. Justification is provided for the choice of requirement at $\text{Prob}_{\text{ghost}}(\pi_s) < 0.05$; here a systematic uncertainty associated with any residual background surviving this requirement is assigned.

The increase in the cumulative WS/RS ratio from events with $\text{Prob}_{\text{ghost}}(\pi_s) < 0.001$ to all events ($\text{Prob}_{\text{ghost}}(\pi_s) < 0.05$) is taken as the bias (which is equal to a bias of (0.001 ± 0.005) on the WS/RS ratio). Pseudo-experiments are generated from the data, where the WS yield is varied by the bias for each time and phase-space bin (with 100% correlation as the decay-time distribution does not change with $\text{Prob}_{\text{ghost}}(\pi_s)$ as shown in Fig. 6.4). The magnitude of the bias is assumed to be uniform across phase space. The RMS of fitted parameter distributions is assigned as the systematic uncertainty. The systematic uncertainties on the observables are 40% (phase-space integrated) and 20% (phase-space binned) of the statistical uncertainties.

6.2.6 Charm backgrounds

The systematic uncertainty from mis-identified and partially reconstructed charm backgrounds is assigned. The contribution from $D^0 \rightarrow K_S^0 K \pi$ decays produces a bias on WS/RS, but this is small and so is not corrected. Partially reconstructed charm backgrounds should not bias the WS/RS ratio, but are conservatively treated as only biasing the WS yield. This conservative treatment accounts for the fact that the resonance structure and strong phase of the decays are unknown.

The $D^0 \rightarrow K_S^0 K \pi$ background is measured to contribute $(0.07 \pm 0.01)\%$ of the WS yield. As this is not corrected for, it leads to a small bias on the measured parameters, in particular on r . In Sec. 4.5.6, the contribution to the signal yield from $D^0 \rightarrow K \pi \eta'$ decays is measured to be $(0.022 \pm 0.001)\%$. Using the relative branching fraction, the contribution to the signal yield from $D^0 \rightarrow K \pi \eta$ decays is estimated to be $(0.009 \pm 0.001)\%$. The combined contribution to the WS yields is thus $(0.10 \pm 0.01)\%$.

The bias is measured by comparing the mixing observables from the mixing fit with and without the background accounted for. There is a positive bias on r of up to 8% of the statistical uncertainty, a very small negative bias on $\kappa y'$, and the bias on $x^2 + y^2$ is negligible. The statistical uncertainty is assigned as the absolute value of the bias (with biases less than 0.1×10^{-5} assigned as negligible).

6.2.7 Fixed parameters in mass fit

There are two fixed parameters in the mass fit. The first is the low-mass threshold of the background shape that is set to $140.0 \text{ MeV}/c^2$. This is imposed to improve the fit quality as the low-mass turn-on at $m(\pi)$ is hard to model. No uncertainty is assigned due to this choice.

The second is the β parameter of the Extended Cruijff function, which is

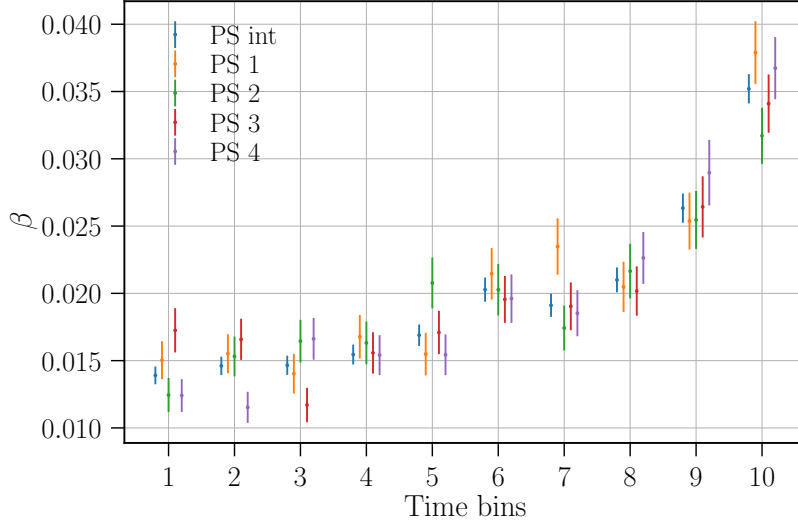


Figure 6.5: The fitted value of β in the signal PDF for phase-space integrated and binned mass fits per time bin.

taken from signal-only fits to (phase-space integrated) MC truth samples. There is a different value for each time bin, and each value is shared across phase-space integrated and phase-space binned data. To confirm that this choice is reasonable, phase-space binned MC truth samples are fitted and the value of β from the fits is verified to be consistent with that from the phase-space integrated fits. This is shown in Fig. 6.5.

A systematic uncertainty associated with the value of β is assigned by varying the value within Gaussian constraints. Many mass fits to data are performed, then the mixing fit is re-run. The RMS of the mixing observables is assigned as the systematic uncertainty, which is very small. The systematic uncertainties on the observables are less than 1% of the statistical uncertainties.

6.2.8 Mass-fit shape

Due to the large statistics of the RS signal, the signal yields are dominated by the RS peak and thus it is only necessary to find a single shape that well describes the signal peak. However the background parametrisation can affect the WS

signal yield and thus an alternative shape is sought. The ‘RooRSP’ shape is used in place of the RooBkg shape in the total PDF, and is described by

$$\text{RSP}(\Delta m; \Delta m_0, \alpha, \beta) = \exp(-\beta(\Delta m - \Delta m_0)) \times (\Delta m - \Delta m_0)^\alpha, \quad (6.9)$$

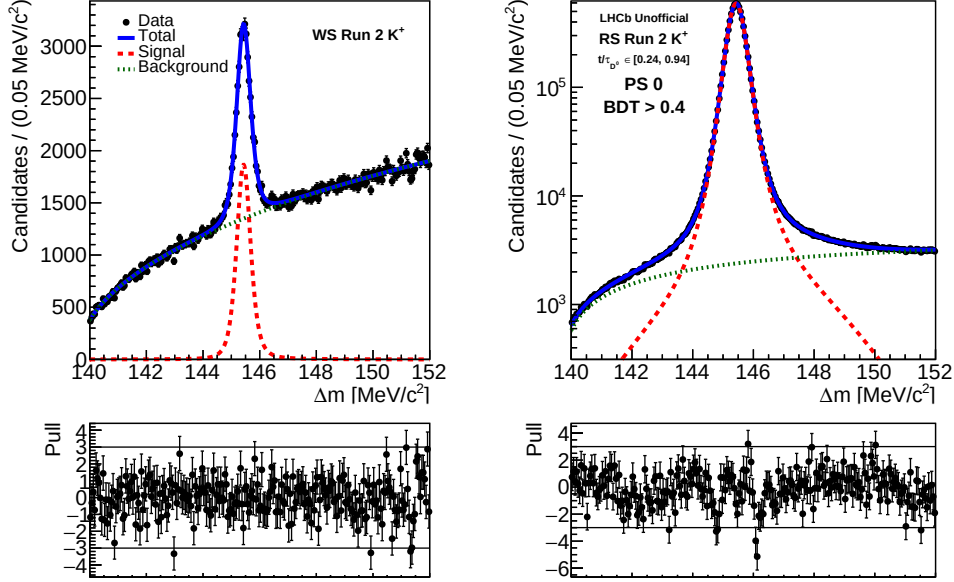
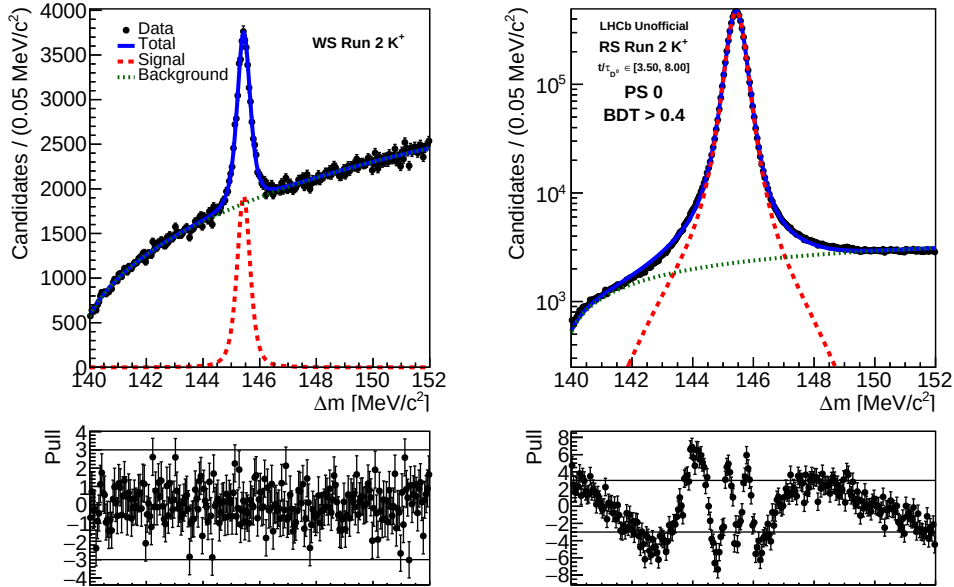
for $\Delta m > \Delta m_0$, and equals zero otherwise. Δm_0 is the low mass threshold and α and β float and take different values for the RS and WS fits.

Example fits to the Δm distribution of the lowest and highest time bins are shown in Fig. 6.6. The yields, and ratio, measured using the alternative fit are compared to the measurements from the baseline fit in Fig. 6.7, where both models are fitting the same (flavour-integrated) data. The RS yield has variation up to $2\sigma_{stat}$ and the WS yield has variation up to $0.2\sigma_{stat}$, leading to maximal variation on the WS/RS ratio of $0.22\sigma_{stat}$.

To assess the systematic uncertainty associated with the choice of mass fit model, the Δm distribution for each time (and phase-space) bin is generated many times using parameters of the alternative model previously measured in fits to data. Each pseudo-experiment is then fit with the baseline model. The signal yields obtained from each pseudo-experiment are used to construct the WS/RS ratio and the mixing fit is performed. The mean value of the observables after the mixing fit is compared to those of the nominal fit, with the magnitude of the difference set as the systematic uncertainty. The systematic uncertainties relative to the statistical uncertainties are small for $x^2 + y^2$, 5–20% for r and under 15% for $\kappa y'$.

6.2.9 Efficiency correction

Figure 5.14 shows the value of f_{correl} per time and phase-space bin. As there is no observed phase-space dependence, it is decided to use the phase-space integrated measurement as a correction for all phase-space bins. A systematic

(a) TB1 K^+ (phase-space integrated)(b) TB10 K^+ (phase-space integrated)Figure 6.6: A fit to the Δm distribution in the (a) lowest and (b) highest time bins using an alternative mass fit model.

uncertainty associated with this choice will be assigned.

A systematic uncertainty arises from the finite size of the Particle Gun sample used to derive the efficiency correlations correction, and from the limited knowledge of the mixing and hadronic parameters which are inputs when accounting for the interference and mixing effects.

Additionally, the systematic uncertainty on the measurement of $\epsilon(WS/CF)$ due to the finite size of the Full MC sample (and generator-level samples) is assigned.

Finite size of Particle Gun samples

To assess the uncertainty due to the finite size of the Particle Gun samples, 500 pseudo-experiments are generated from the same data sample, where the efficiency correlations correction is varied within its uncertainty, independently per time bin. Each pseudo-experiment is fit according to the mixing equation, and the distribution of each fitted parameter is fit with a Gaussian function. The systematic uncertainty assigned is the width of the Gaussian, and is shown in Tables 6.11 and 6.12. The uncertainties across r_i and $(\kappa y')_i$ in each phase-space bin are correlated.

Agreement of Particle Gun with data

The Particle Gun samples are weighted to look like data, and the agreement across phase space is confirmed, in Sec. 5.3.2.1. However, any small differences between the Particle Gun and data lead to uncertainties in the efficiency correction. To quantify a limit on this uncertainty, the weights applied to the Particle Gun sample are varied uniformly in a range of 90-110% around the nominal weight. The difference in the WS/CF ratio in Particle Gun per time bin when using the varied versus the nominal weights has a maximal difference of 9.5% of the uncertainty due to the sample size (and in most bins the difference is much smaller at 1–4%). Thus, it is concluded that the systematic uncertainty associated with the finite size of the samples is large enough to account for this

possible source of bias.

Knowledge of mixing and hadronic input parameters

To assess the uncertainty from the mixing and hadronic parameter inputs, the central values of the efficiency correlations correction are regenerated eight times, each time varying one of the parameters by $+1\sigma$ or -1σ . The uncertainty on the mixing parameters is taken from HFLAV [25] and the uncertainty on the hadronic parameters is taken from the BESIII, CLEO-c and LHCb combination in Ref. [34]. Each of the eight pseudo-experiments is fit according to the mixing equation, and compared to the value from the fit with the nominal input parameters. The difference in the fitted values is set as the systematic uncertainty for each pseudo-experiment, and the uncertainties are added in quadrature to give a combined uncertainty from all inputs. This is shown in Tables 6.11 and 6.12. The input parameter with the largest effect on the systematic uncertainty is δ , but overall the contribution is negligible compared to the uncertainty from the size of the PGun samples. The uncertainties across r_i and $(\kappa y')_i$ in each phase-space bin are correlated.

Choice to use phase-space integrated results

The choice of using the phase-space integrated values of f_{correl} could lead to biases relative to using the phase-space binned values. To investigate this possibility, the fit is repeated many times varying the phase-space binned correction. The mean values of the observables for each choice are compared and assigned as a bias. The biases are, in general, small apart from in PS bin 4 where the bias is up to 50% of the statistical uncertainty on r_4 . The absolute value of the bias is assigned as the systematic uncertainty. The uncertainties across r_i and $(Ry')_i$ in each phase-space bin are uncorrelated.

Finite size of Full MC (and generator-level) samples

Additionally, the Full MC and generator-level samples used to derived the phase-space integrated efficiency difference, $\tilde{\epsilon}(\text{WS}/\text{CF})$, have finite sizes. The asso-

Table 6.11: Systematic uncertainty associated with the efficiency-correlations correction. Uncertainties of less than 0.1×10^{-5} are negligible and set to zero.

Source	Uncertainty ($\times 10^{-5}$)		
	$x^2 + y^2$	$r_D^{K3\pi}$	$(\kappa y')_D^{K3\pi}$
Finite size of PGun sample	1.3	17.0	31.0
Knowledge of input parameters			
$r + 1\sigma$	0.0	0.2	-0.2
$r - 1\sigma$	0.1	0.2	-1.1
$\delta + 1\sigma$	0.0	-0.4	0.5
$\delta - 1\sigma$	0.0	0.4	0.4
$x + 1\sigma$	0.1	0.1	-1.7
$x - 1\sigma$	0.0	0.0	0.5
$y + 1\sigma$	0.1	0.3	-2.0
$y - 1\sigma$	0.0	0.2	0.0
Combined uncertainty from input parameters	0.2	0.7	3.0
Combined uncertainty from size and inputs	1.3	17.0	31.1

ciated uncertainties are given in Table 5.5. The central values of the correction are varied within these uncertainties to generate 1000 toys from data, which are fit. The distribution of the observables is fit with a Gaussian and the systematic uncertainty is assigned as the RMS, shown in Table 6.13. The systematic uncertainties are approximately 12% of the phase-space integrated statistical uncertainties and 20% of the phase-space binned statistical uncertainties.

The uncertainties on f_{correl} and on $\tilde{\epsilon}(\text{WS/CF})$ are combined in quadrature to yield a total uncertainty from the efficiency correction.

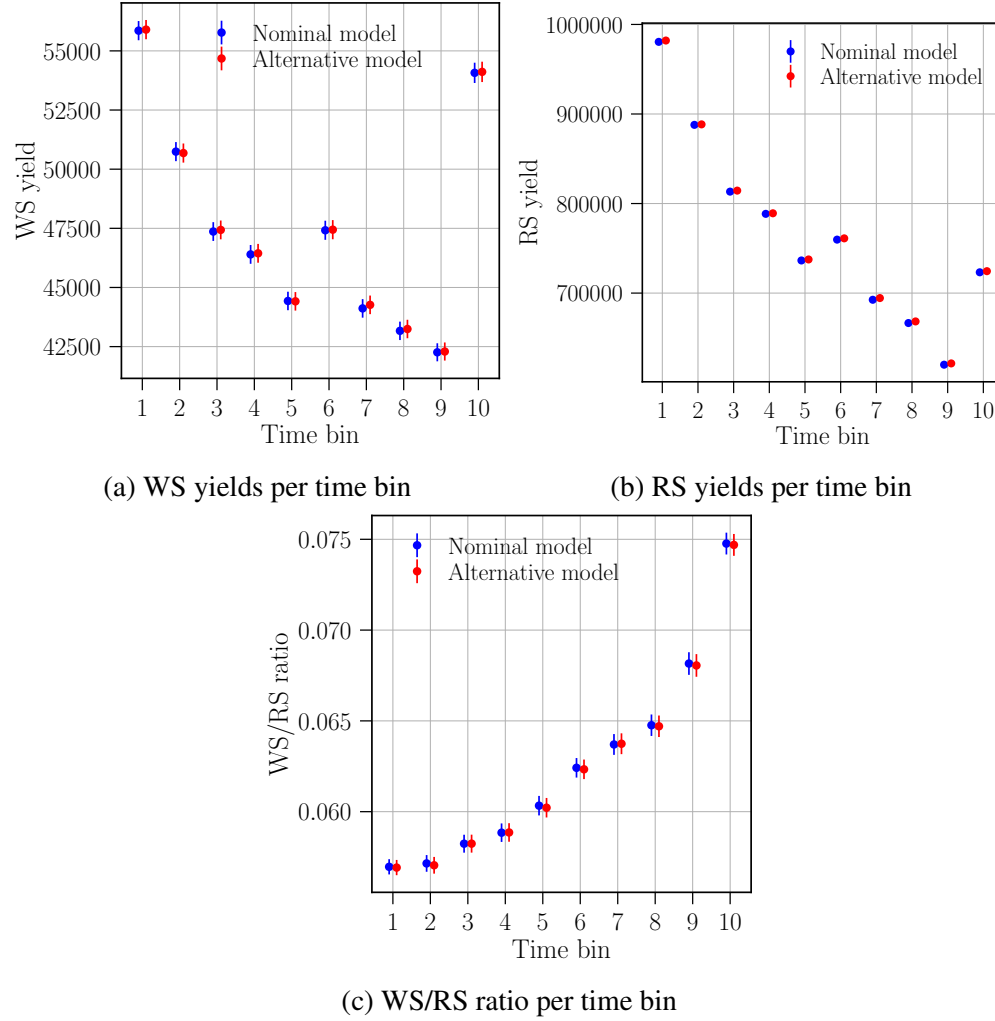


Figure 6.7: Yields and ratio as measured using the alternative mass fit model, when fitting the same (phase-space integrated) data set. The WS/RS ratio plotted is larger than the true ratio as only a subset of the Run 2 RS data is used.

Table 6.12: Systematic uncertainty associated with the efficiency-correlations correction (for the phase-space binned measurement). Uncertainties of less than 0.1×10^{-5} are set to zero.

Source	$x^2 + y^2$	r_1	r_2	Uncertainty ($\times 10^{-5}$)					
				r_3	r_4	$(\kappa y')_1$	$(\kappa y')_2$	$(\kappa y')_3$	$(\kappa y')_4$
Finite size of PGun sample	0.5	8.2	8.4	8.1	8.0	12.6	12.6	12.3	13.0
Variation of input parameters									
$r + 1\sigma$	0.0	0.2	0.3	0.2	0.2	0.1	0.1	0.1	0.0
$r - 1\sigma$	0.1	0.1	0.0	0.1	0.4	0.9	0.8	0.9	1.3
$\delta + 1\sigma$	0.0	-0.3	-0.2	-0.4	-0.6	-0.3	-0.1	-0.4	-1.0
$\delta - 1\sigma$	0.0	0.4	0.4	0.4	0.4	-0.5	-0.6	-0.5	-0.3
$x + 1\sigma$	0.1	0.0	-0.2	-0.1	0.5	1.5	1.2	1.4	2.2
$x - 1\sigma$	0.0	0.1	0.2	0.1	-0.1	-0.5	-0.5	-0.5	-0.7
$y + 1\sigma$	0.1	0.2	0.0	0.1	0.8	1.7	1.4	1.6	2.5
$y - 1\sigma$	0.0	0.3	0.4	0.3	0.1	0.0	0.1	0.0	-0.3
Combined uncertainty from inputs	0.2	0.7	0.7	0.7	1.3	2.6	2.2	2.4	3.8
Choice of using PS int. results (bias)	-0.3	1.7	11.5	11.8	-21.7	-7.5	7.1	-14.6	-22.8
Combined uncertainty	0.6	8.4	14.3	14.3	23.1	14.9	14.6	19.2	26.5

Table 6.13: The systematic uncertainty associated with the phase-space integrated efficiency correction ($\tilde{\epsilon}(\text{WS}/\text{CF})$).

Phase-space division	Uncertainty ($\times 10^{-5}$)		
	$x^2 + y^2$	r	$\kappa y'$
PS integrated	0.3	4.2	8.3
PS 1		9.1	13.0
PS 2	0.5	9.1	13.4
PS 3		8.3	11.8
PS 4		7.8	13.1

Table 6.14: The bias on the phase-space integrated observables due to removing candidates which fall into phase-space bin 5. The bias is positive if the value of the baseline result is larger than the value of the result from data including bin 5.

Phase-space division	Bias ($\times 10^{-5}$)		
	$x^2 + y^2$	$r_D^{K3\pi}$	$(\kappa y')_D^{K3\pi}$
PS integrated	0.2	-5.3	2.2

6.2.10 Phase-space bin 5 veto

The choice to remove candidates from phase-space bin 5 benefits the phase-space integrated measurement by excluding $D \rightarrow K_S^0 K \pi$ candidates that could bias the measurement and are assigned as a background. However, it also removes true $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays and because the proportion of WS decays that fall into bin 5 (approximately 5.3%) is slightly larger than the proportion of RS decays that fall into bin (4.9%), a bias in the measurement could occur.

The size of the bias was estimated using pseudo-experiments with and without bin 5 included. The pseudo-experiments include time efficiency, time resolution and phase-space migration effects. The results are presented in Table 6.14. As the bias is much smaller than the statistical uncertainty on all parameters the magnitude of the bias is assigned as a systematic uncertainty.

6.2.11 Summary of systematic uncertainties

Tables 6.15 and 6.16 show a summary of all systematic uncertainties, the statistical uncertainties, and the covariance matrix of systematic uncertainties for the phase-space integrated fit. Similar results are shown for the phase-space binned fit in Tables 6.17 and 6.18. The covariance matrices are symmetric and thus only one half is shown. The statistical uncertainty is larger than the systematic uncertainty for all parameters. The dominant systematic uncertainties on r are the efficiency correction and the detector effects, driven by the phase-space migration. The dominant systematic uncertainty on $x^2 + y^2$ and $\kappa y'$ is the efficiency correction, driven by the finite size of the Particle Gun MC samples.

A few of the systematic uncertainties are correlated between phase-space bins, notably the systematic uncertainty associated with the slow-pion ghosts, charm backgrounds, fixed parameters in the mass fit and part of the efficiency correction. These have a correlation matrix where $\sigma_{ij} = 1$ for all i, j . The other systematics are uncorrelated per phase-space bin and so $\sigma_{ij} = 1$ for $i = j$ and $\sigma_{ij} = 0$ otherwise.

Table 6.15: Summary of all systematic uncertainties for phase-space integrated fits.

Source	Uncertainty ($\times 10^{-5}$)		
	$x^2 + y^2$	$r_D^{K3\pi}$	$\kappa y'$
Detector effects	0.2	2.3	5.6
Secondary correction	0.2	2.0	8.9
Double mis-ID correction	0.8	10.8	19.7
Slow-pion ghosts	0.0	13.8	0.6
Charm backgrounds	0.0	2.8	0.1
Fixed parameters in mass fit	0.0	0.3	0.8
Mass-fit shape	0.1	5.5	0.3
Efficiency correction	1.3	17.5	32.2
Phase-space bin 5 veto	0.2	5.3	2.2
Total systematic	1.6	26.3	39.3
Statistical	2.8	34.6	66.8
Total	3.2	43.4	77.5

Table 6.16: Covariance matrix of systematic uncertainties for phase-space integrated fits.

$(\times 10^{-10})$	$x^2 + y^2$	$r_D^{K3\pi}$	$(\kappa y')_D^{K3\pi}$
$x^2 + y^2$	1.0		
$r_D^{K3\pi}$	0.0	620	
$(\kappa y')_D^{K3\pi}$	0.0	0.0	1478

Table 6.17: Summary of all systematic uncertainties for phase-space binned fits.

Source	$x^2 + y^2$	r_1	r_2	Uncertainty ($\times 10^{-5}$)					
				r_3	r_4	$(\kappa y')_1$	$(\kappa y')_2$	$(\kappa y')_3$	$(\kappa y')_4$
Detector effects	0.2	17.9	12.0	17.1	29.0	2.6	10.6	16.5	7.7
Secondary correction	0.2	1.6	5.9	3.1	2.3	10.0	17.3	10.6	2.0
Double mis-ID correction	0.7	12.8	13.4	12.6	16.5	19.8	19.2	19.0	23.9
Slow-pion ghosts	0.0	14.0	14.7	14.2	11.5	0.8	1.3	0.8	0.2
Charm backgrounds	0.0	2.9	3.1	3.0	2.4	0.1	0.3	0.1	0.0
Fixed parameters in mass fit	0.0	0.5	0.6	0.6	0.5	0.4	0.4	0.4	0.4
Mass-fit shape	0.1	4.2	10.1	9.8	2.6	1.3	9.7	6.1	6.9
Efficiency correction	0.8	12.4	16.9	16.5	24.4	19.8	19.8	22.6	29.6
Total systematic	1.1	29.4	31.2	32.3	43.1	29.9	35.6	35.9	39.4
Statistical	2.8	44.8	48.9	45.9	50.9	72.7	74.2	71.6	82.5
Total	3.0	53.6	58.0	56.1	66.7	78.6	82.3	80.1	91.4

Table 6.18: Covariance matrix of systematic uncertainties for phase-space binned fits.

$(\times 10^{-10})$	$x^2 + y^2$	r_1	r_2	r_3	r_4	$(\kappa y')_1$	$(\kappa y')_2$	$(\kappa y')_3$	$(\kappa y')_4$
$x^2 + y^2$	0.2								
r_1	4.1	804							
r_2	4.2	284	937						
r_3	4.0	274	286	985					
r_4	4.0	235	245	236	1799				
$(\kappa y')_1$	6.5	118	121	116	114	851			
$(\kappa y')_2$	6.4	124	128	123	120	166	1215		
$(\kappa y')_3$	6.2	114	117	113	111	162	161	1227	
$(\kappa y')_4$	6.8	114	117	112	112	175	173	169	1453

6.3 Consistency checks

In order to validate the robustness of the results, the measurement is re-performed with data split by data-taking condition (year and magnet polarity) and also for data split by trigger category. Care is taken to measure the three corrections to the data (secondary fraction, double mis-ID yields and efficiency correlations) per category. If the results from each category are not consistent, it indicates potential biases that are not accounted for in the method.

6.3.1 Split by data-taking condition

The data are split by year and magnet polarity. Mass fits are performed to construct the uncorrected WS/RS ratio, which is corrected by the efficiency-correlations function measured in Particle Gun samples representing 2015–16 and 2017–18 data-taking conditions. The double mis-ID yields and secondary fractions are measured for each data-taking condition and corrected for. The measured mixing parameters per year and polarity are shown in Fig. 6.8, with the Run 2 integrated results. The plots display the statistical uncertainties only, and the results are compatible.

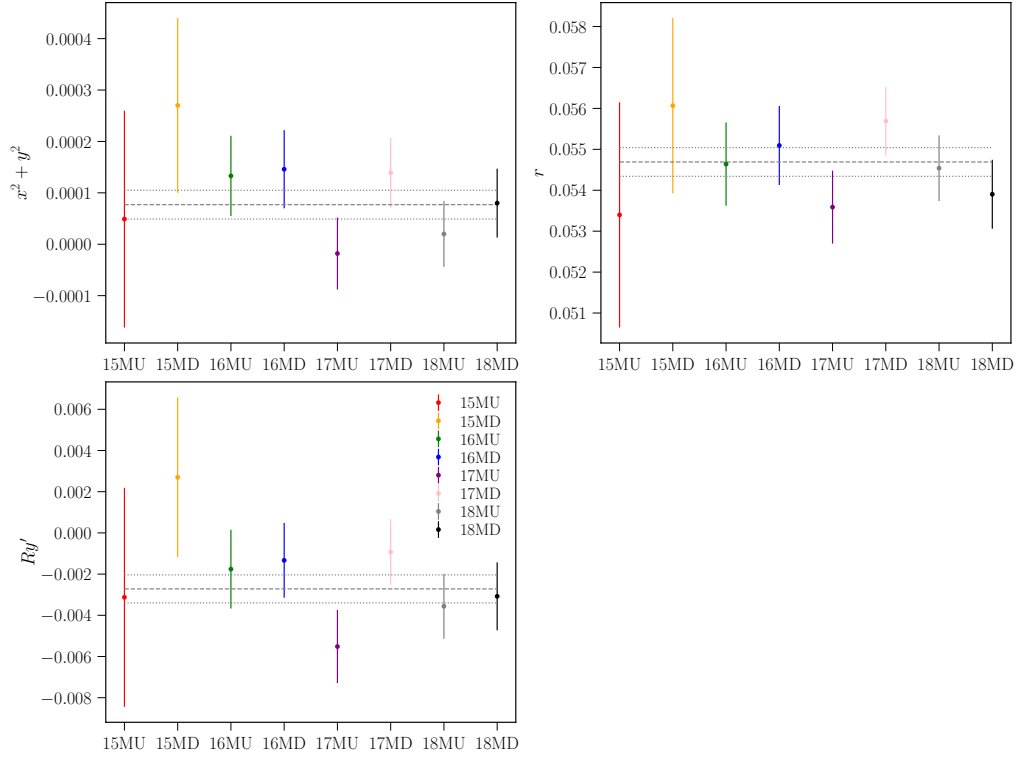


Figure 6.8: Consistency check of (phase-space integrated) mixing parameters, split by year and polarity. The uncertainties are statistical only.

6.3.2 Split by trigger

The data are split by exclusive trigger categories:

- ‘L0 TIS (and not L0 TOS)’ - a trigger at L0 independent of the D^0 decay;
- ‘L0 TOS’ - a trigger at L0 by a hadron from the D^0 decay;
- ‘HLT1 Track’ - at least one track from the D^0 passes the one track decision;
- ‘HLT1 TwoTrack (and not HLT1 Track)’ - at least one pair of tracks passes the two track decision.

Efficiency corrections and background yields are calculated per trigger to avoid making any assumptions about how these are distributed across trigger categories. As the L0 trigger is not accessible in the Particle Gun sample, Full MC

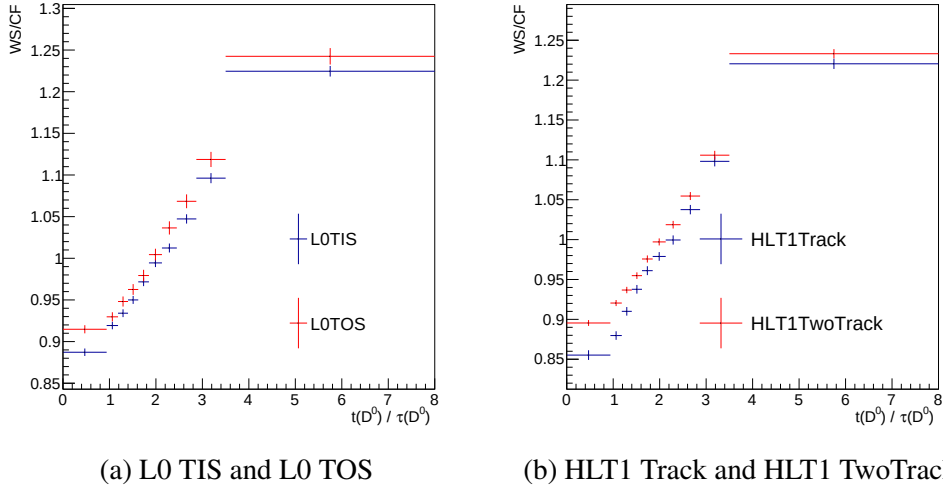


Figure 6.9: WS/CF efficiency correlations per trigger category, measured in Full MC.

is used to calculate the efficiency-correlations function. The WS/CF efficiency correlations per trigger are shown in Fig. 6.9, with indicative slopes. The HLT1 Track selection has the largest efficiency correlations, as expected.

The estimation of the secondary fraction (and secondary time distribution) is also limited as the secondary template is derived from Particle Gun MC as well. To overcome this, the $\chi^2_{\text{IP}}(D^0)$ fit relies heavily on the convoluted Gaussian function to fit the L0 integrated template to the data split by L0 trigger (in contrast, the HLT1 split can have its own templates generated). However, the size of the secondary fraction is not the dominant systematic uncertainty in the measurement and so this should not have a large effect on the cross-check performed here. The mixing parameters per trigger type are shown in Fig. 6.10, showing consistency between the different trigger selections.

6.4 Final results and discussion

The final results of this measurement are given in Table 6.19. The phase-space integrated results are in agreement with results obtained from LHCb Run 1

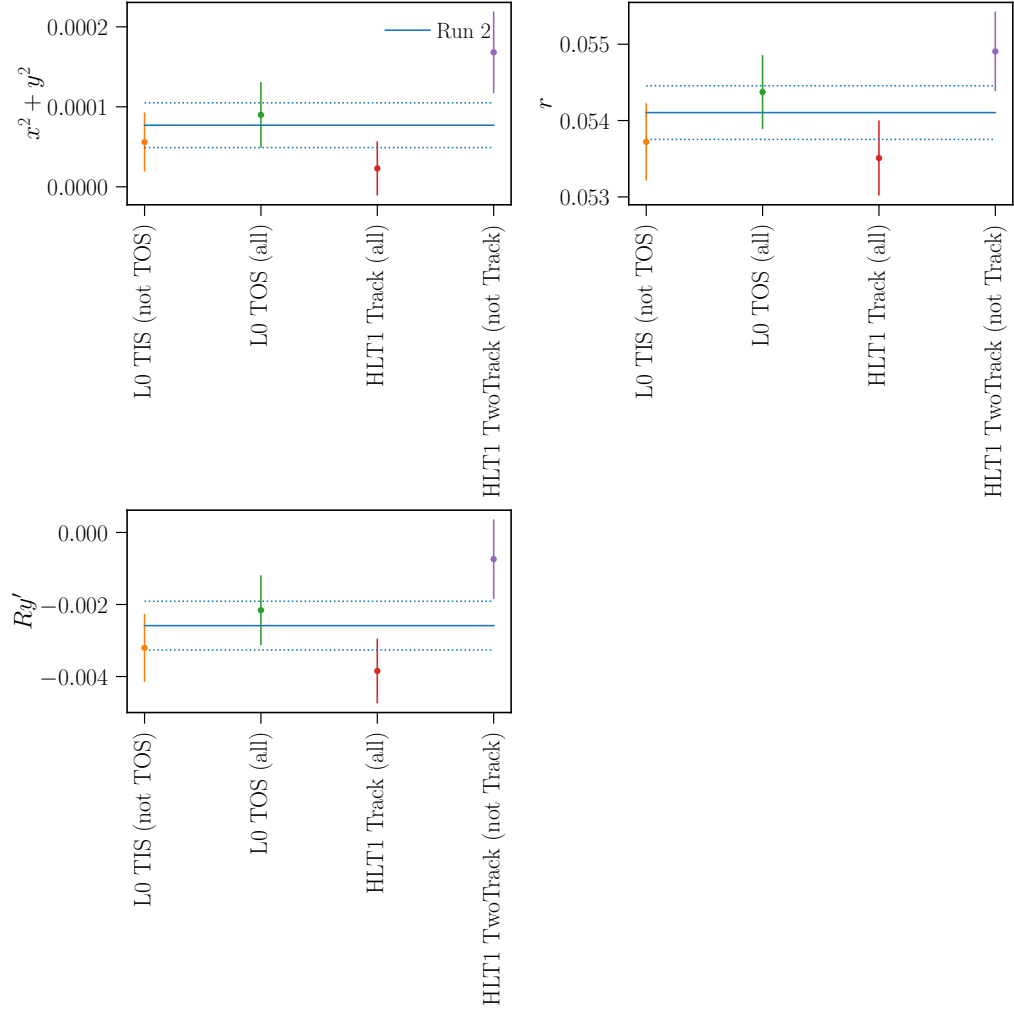


Figure 6.10: Consistency check of (phase-space integrated) mixing parameters, split by trigger type. The uncertainties are statistical only.

Table 6.19: Measured values of the effective mixing parameters and their uncertainties (statistical and systematic, respectively).

Phase-space division	$x^2 + y^2 (\times 10^{-5})$	$r (\times 10^{-2})$	$\kappa y' (\times 10^{-3})$
PS integrated	$7.2 \pm 2.8 \pm 1.6$	$5.489 \pm 0.035 \pm 0.026$	$-2.68 \pm 0.67 \pm 0.39$
PS 1		$5.652 \pm 0.045 \pm 0.029$	$-4.15 \pm 0.73 \pm 0.30$
PS 2	$5.9 \pm 2.8 \pm 1.1$	$5.966 \pm 0.049 \pm 0.031$	$-5.70 \pm 0.74 \pm 0.36$
PS 3		$5.812 \pm 0.046 \pm 0.032$	$-3.00 \pm 0.72 \pm 0.36$
PS 4		$4.609 \pm 0.051 \pm 0.043$	$0.51 \pm 0.83 \pm 0.39$

data [32], with a factor of 2.2–2.8 times increase in precision.

The remainder of the chapter details the interpretation of the measured observables into the hadronic parameters, κ and δ , and the mixing parameters, x and y , respectively.

6.4.1 Interpretation of the hadronic parameters

Using the measured value of $\kappa y' = \kappa(y \cos \delta - x \sin \delta)$, the values of κ and δ per phase-space region are constrained along a curve on the $\kappa - \delta$ plane as shown in Fig. 6.11. This is in contrast to the charm threshold measurements at BESIII and CLEO-c that measure a single best-fit value of κ and δ , which is also shown in Fig. 6.11. Despite the very different methods used to obtain these parameters, the agreement is good.

The results of this thesis are combined with the previous measurements to produce a new combination. These are reported in Table 6.20, with $\Delta\chi^2$ -scans shown in Fig. 6.12. A comparison can be made between this combination and the previous combination, detailed in Table 2.3. This measurement leads to an increase of precision on κ across all bins. The precision on δ also increases across the first three bins, however the precision decreases in bin 4 due to the shift towards zero of the central value of κ in this bin. The uncertainty on r (per phase-space region) is decreased by a factor of two relative to the previous

Table 6.20: The central values of the hadronic parameters after combination of this measurement with previous results from BESIII [34], CLEO-c [33], and LHCb Run 1 (mixing) [32] (PS integrated only).

Phase-space division	$r (\times 10^{-2})$	κ	$\delta (^\circ)$
PS int.	5.48 ± 0.03	$0.44^{+0.07}_{-0.06}$	162^{+18}_{-17}
PS 1	5.65 ± 0.04	0.68 ± 0.12	121^{+11}_{-9}
PS 2	5.96 ± 0.05	0.82 ± 0.09	147^{+11}_{-9}
PS 3	5.80 ± 0.04	0.65 ± 0.08	188^{+12}_{-13}
PS 4	4.60 ± 0.05	0.06 ± 0.06	344^{+129}_{-129}

Table 6.21: Values of the mixing parameters after interpretation and their uncertainties, from the phase-space binned data.

$x (\%)$	$y (\%)$
$0.48^{+0.14}_{-0.13}$	$0.48^{+0.11}_{-0.13}$

combination.

6.4.2 Interpretation of the mixing parameters

Additionally, the phase-space binned results are interpreted in terms of x and y , using constraints from this measurement and the previous measurements detailed above. This thesis measures $x^2 + y^2$ (constraining x and y along a circular arc in the $x - y$ plane) and $(\kappa y')_i = \kappa_i (y \cos \delta_i - x \sin \delta_i)$ which, with knowledge of κ_i and δ_i from previous measurements, allows for a central value to be selected. The results are reported in Table 6.21 with the contour plot shown in Fig. 6.13. Mixing ($x \neq 0, y \neq 0$) is clearly observed.

The current knowledge of the hadronic parameters (κ and δ) from external inputs limits the sensitivity achieved on x and y individually using this decay mode. An updated measurement of κ and δ from BESIII is expected in the near future, with an appreciable increase in sensitivity forecast from a significant increase in the sample size [70].

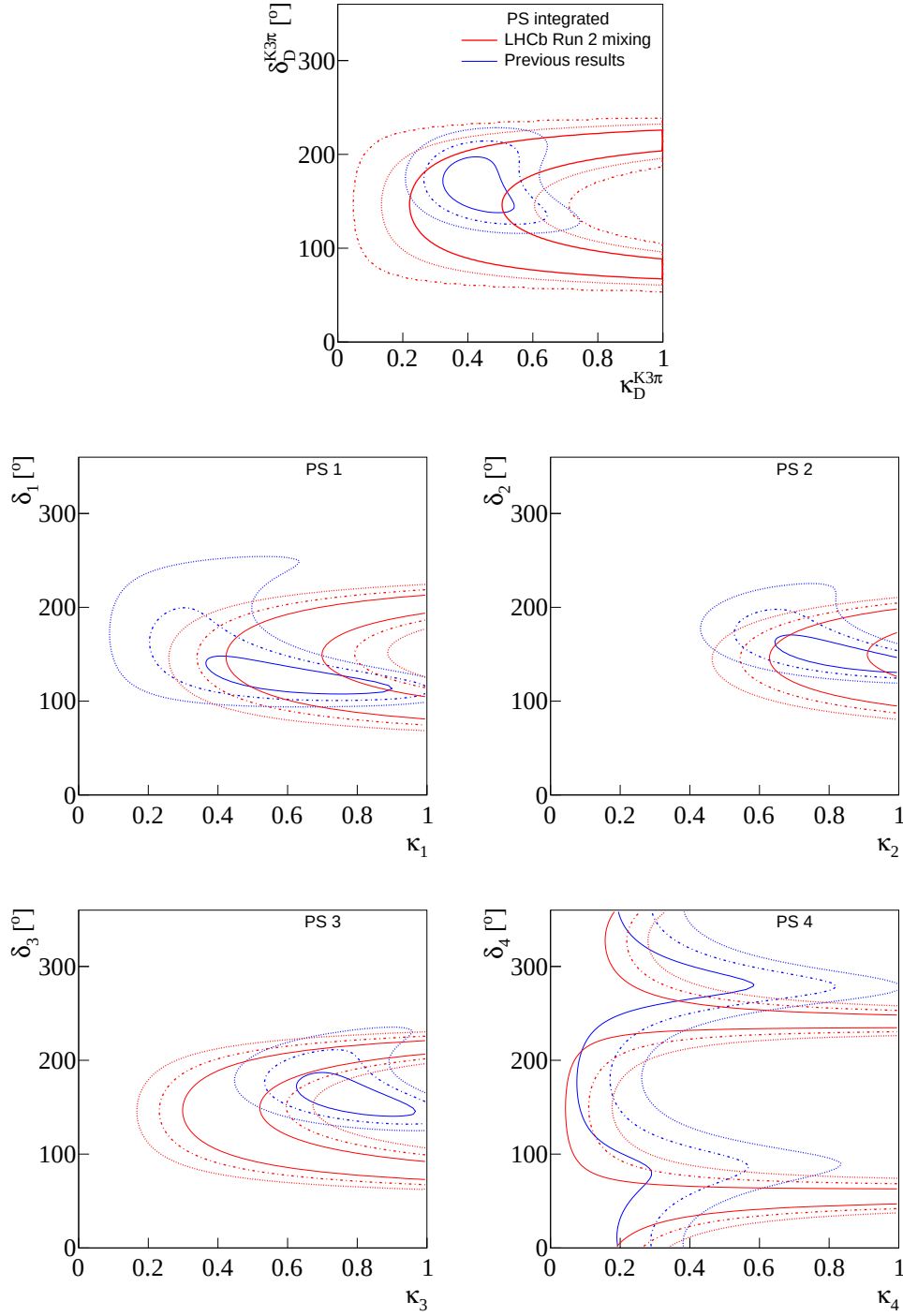


Figure 6.11: Interpretation of the phase-space integrated and binned measurements of the hadronic parameters (red) compared to the combination of previous results (blue) from BESIII [34], CLEO-c [33], and LHCb Run 1 (mixing) [32] (PS integrated only). The contours correspond to the intervals $\Delta\chi^2 = 2.30, 6.18, 11.83$. For Gaussian uncertainties, the contours correspond to 68.3%, 95.4% and 99.7% confidence levels.

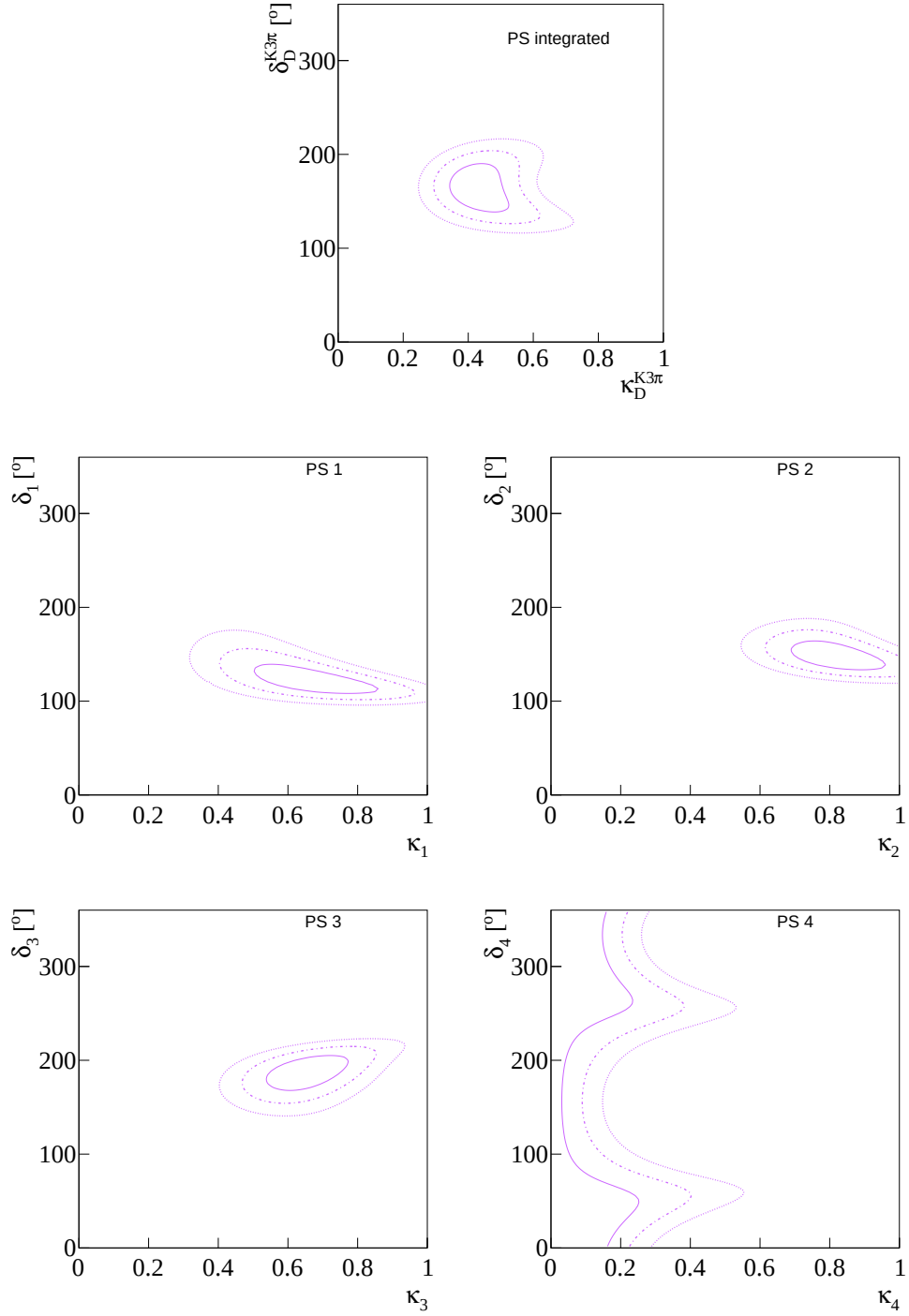


Figure 6.12: The combined interpretation of this measurement with previous results from BESIII [34], CLEO-c [33], and LHCb Run 1 (mixing) [32] (PS integrated only). The contours correspond to the intervals $\Delta\chi^2 = 2.30, 6.18, 11.83$. For Gaussian uncertainties, the contours correspond to 68.3%, 95.4% and 99.7% confidence levels.

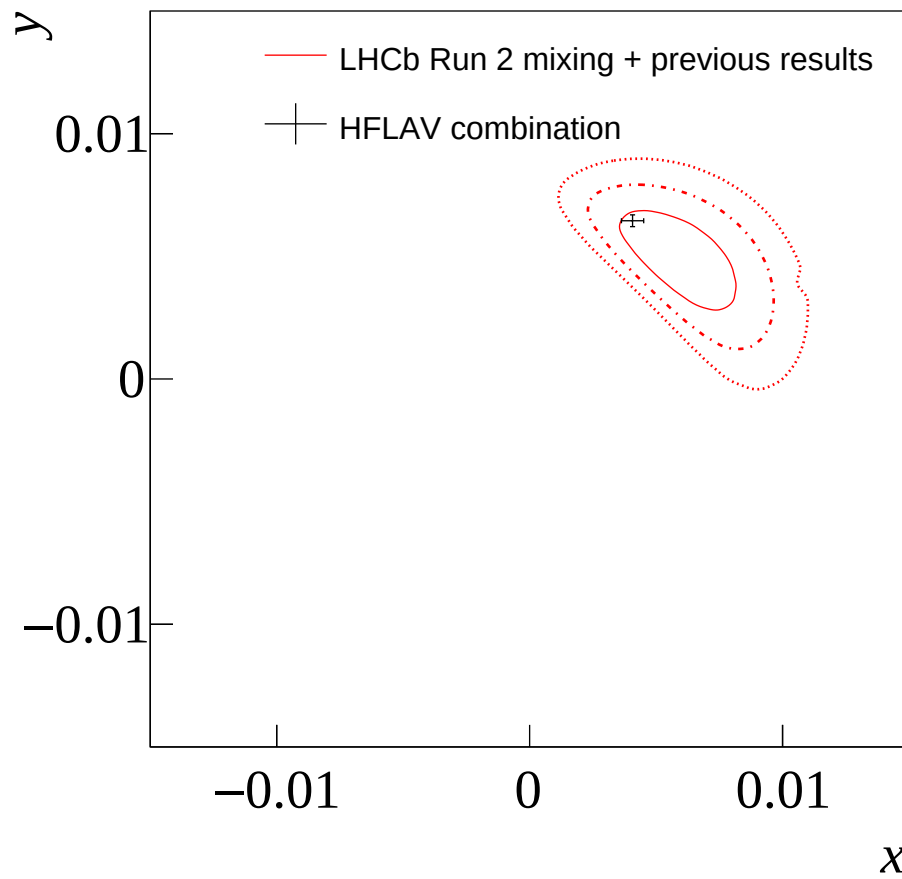


Figure 6.13: Interpretation of the results into the mixing parameters x and y , with constraints on κ and δ from previous measurements. The current HFLAV average [25] is also shown. The contours correspond to the intervals $\Delta\chi^2 = 2.30, 6.18, 11.83$. For Gaussian uncertainties, the contours correspond to 68.3%, 95.4% and 99.7% confidence levels.

7 | Performance of a TORCH prototype

As discussed in Sec. 3.2.2, the RICH detectors in LHCb provide PID of charged hadrons across a range of momenta, but crucially cannot separate kaons from protons at momenta below 10 GeV/c. In this low momentum region, any particle not producing Cherenkov radiation is assigned to be a kaon.

The TORCH detector (Time Of internally Reflected CHerenkov light) is proposed for installation in LHCb Upgrade II to perform particle identification of low-momentum (2-20 GeV/c) charged hadrons using time-of-flight information. Similar technology was used first by the BaBar DIRC [71] and Belle II iTOP [72, 73] collaborations, and similar detectors are being considered for detectors at future colliders, *e.g.* FCC-ee [74].

This chapter details the method and design of TORCH, and detail analysis of a prototype detector that has been built to test the sensitivity of current technology. The test-beam apparatus used is explained and the results are given, along with prospects for the future of TORCH. The data analysis detailed in Sec. 7.6 was performed by the author, after the data was collected and processed (described in Sec. 7.5) by members of the TORCH collaboration in October 2018. The results presented here have been published by the author in Ref. [1]. The author also participated in collecting data and performing initial studies during a test-beam campaign in November 2022; these results are not presented in this thesis.

7.1 Introduction to TORCH

The TORCH detector, located at a fixed distance from the LHCb interaction point, nominally 9.5 m, will measure the time-of-flight (ToF) of charged particles which can be used to discriminate between particle species. The charged particles traverse a 10 mm thick quartz plate, producing many prompt Cherenkov

photons. Total internal reflection traps a fraction of the photons within the plate, which propagate to focusing optics positioned at the periphery of the plate. Inside the focusing optics, the photons are reflected onto photo-sensitive detectors by a cylindrical mirrored surface. The geometry allows for the Cherenkov angle of emission to be reconstructed. The photon detectors measure the location and time-of-arrival of the photon, combining DIRC-style reconstruction with timing measurements to measure the ToF [75, 76].

7.1.1 Measurement of time-of-flight

In order to measure the ToF of the charged hadron, the time-of-propagation, t_p , of the photon in the quartz must be subtracted from the time difference of the photon arrival on the photodetectors and the time of the primary interaction that produced the charged particle. The propagation time is related to the path length of the photon in the quartz, L , by

$$t_p = \frac{Ln_g}{c} \quad (7.1)$$

where n_g is the group refractive index of the quartz. The variation of t_p with n_g means that photons of different wavelengths propagate at different speeds, a phenomenon referred to as chromatic dispersion. From knowledge of the particle entry position in the radiator¹ and the photon hit coordinate, the Cherenkov emission angle θ_c and hence the photon path length in the quartz L can be calculated by unfolding the multiple internal reflections of the photon [76].

The expression $\cos \theta_c = 1/(\beta n_p)$ relates θ_c to the phase refractive index n_p which in turn yields n_g via a known dispersion relation,

$$n_g \approx n_p - \lambda \frac{\partial n_p}{\partial \lambda}, \quad (7.2)$$

¹Obtained from external knowledge provided by the LHCb tracking system.

for photons with wavelength λ . The phase refractive index is modelled by the Sellmeier equation [77],

$$n_p^2 = 1 + \frac{a_1 \lambda^2}{\lambda^2 - b_1} + \frac{a_2 \lambda^2}{\lambda^2 - b_2} + \frac{a_3 \lambda^2}{\lambda^2 - b_3}, \quad (7.3)$$

where the coefficients $a_{1,2,3}$ and $b_{1,2,3}$ are particular to the material and known empirically. In practice, the values of n_p and n_g are stored in a look-up table for the typical wavelengths of photons that propagate in TORCH (175 – 625 nm).

The inferred group velocity and L are used to calculate the time-of-propagation, from which the ToF is calculated. The value of β is obtained from the known particle momentum (from the LHCb tracking system) for each particle mass hypothesis. A time-of-flight for each hypothesis is calculated, and the preferred mass is determined via maximisation of the log-likelihood difference between the detected photon hits and simulated photon hits.

7.1.2 Design for LHCb

The design of a single full-sized TORCH module is shown in Fig. 7.1a, with radiator dimensions $660 \times 2500 \times 10 \text{ mm}^3$ (width $\times$ length $\times$ thickness). A modular design has been employed to allow coverage across large areas with flexible geometries. Figure 7.1b shows the proposed design for use in LHCb Upgrade II [78]. Here the TORCH detector is composed of eighteen modules covering an area of $6 \text{ m} \times 5 \text{ m}$. Each module will be equipped with eleven Micro-Channel Plate Photomultiplier Tubes (MCP-PMTs).

When positioned at a distance of 9.5 m from the LHCb interaction point, the ToF difference between $10 \text{ GeV}/c$ pions and kaons is 35 ps as shown in Fig. 7.2. Thus a per-track time resolution of 10 – 15 ps for three standard deviations (σ) of separation between mass hypotheses is required. Given ~ 30 detected photons per track, the requirement on the single-photon time resolution is thus

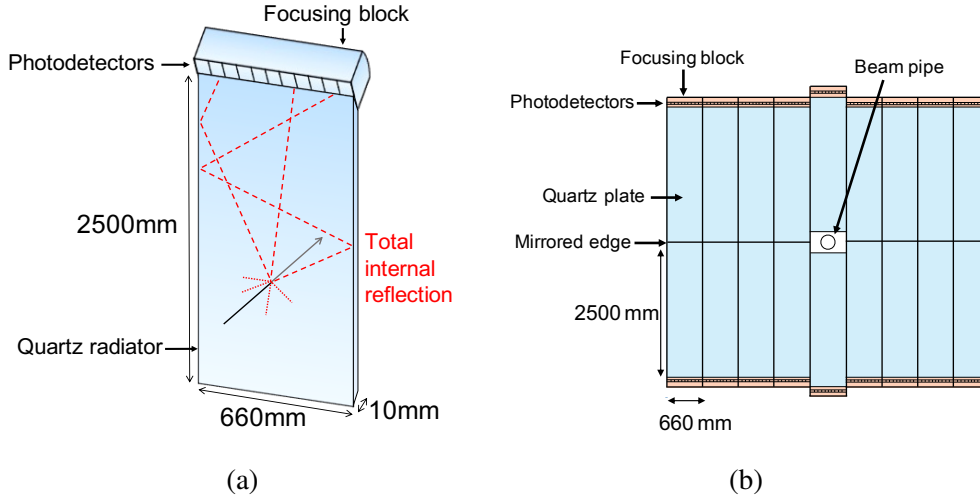


Figure 7.1: TORCH detector design for the LHCb Upgrade II. (a) A single TORCH module with the charged particle path (black line) and the Cherenkov photons (red lines), (b) the full TORCH detector.

~ 70 ps [79].

7.1.3 Previous developments

Two prototype detectors have been manufactured and tested alongside different generations of MCP-PMTs produced by Photech, UK [80]. A small-scale ($120 \times 350 \times 10$ mm³) TORCH demonstrator was previously tested with both a circular MCP-PMT and a single square MCP-PMT [76, 81]. A larger, half-length full-width, TORCH module instrumented with two square Photech MCP-PMTs and readout electronics was tested in October 2018. Further tests of this prototype were performed with up to 7 MCP-PMTs read out in November 2022. The work presented here is the analysis of data collected in October 2018.

7.2 Design of the prototype

The tests described in this thesis were undertaken with a TORCH prototype module, hereafter referred to as *Proto-TORCH*. The prototype is constructed

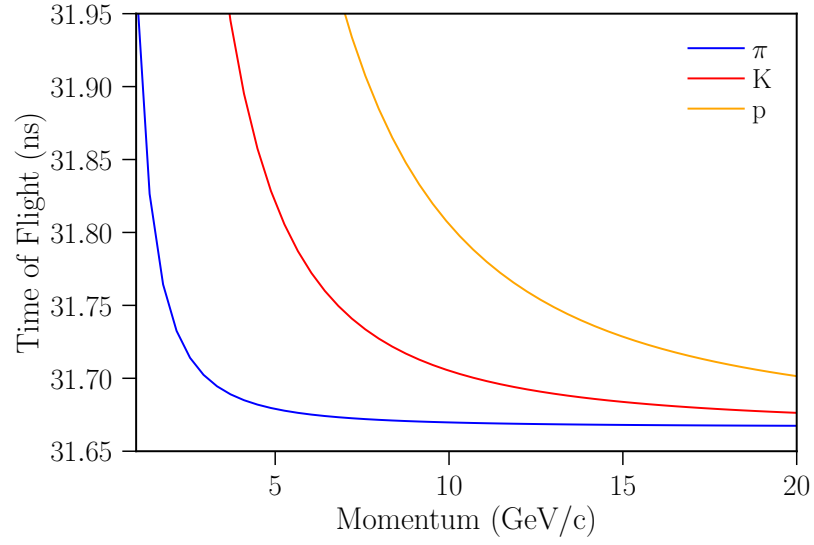
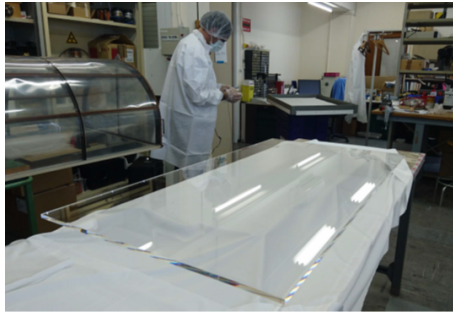


Figure 7.2: The time of flight over a 9.5m flight distance for pions, kaons and protons.



(a)



(b)

Figure 7.3: (a) The manufactured radiator plate. (b) The focusing block.

from a $660 \times 1250 \times 10 \text{ mm}^3$ (width $\times$ length $\times$ thickness) fused silica radiator plate. The plate, which has half the length of a full TORCH module, is glued to a full-sized focusing block with matching width at the top of the plate. In addition, specialised mechanics were manufactured for mounting the MCP-PMTs and the accompanying electronics. In this section the quartz radiator plate, the focusing block, and the support structure are reported, together with a description of the MCP-PMTs and readout electronics.

7.2.1 Quartz radiator plate and focusing block

Figure 7.3 shows photographs of the radiator plate and focusing block. The flat plate surfaces of the radiator plate are polished to a surface roughness of ~ 0.5 nm to minimize surface scattering. With this level of control, simulations show that 90 – 95% of photons that undergo ~ 200 internal reflections are retained [82]. All side faces of the plate are also polished, including the bottom surface to allow upward reflections of downward travelling photons.

An illustration of the focusing block is shown in Fig. 7.4a. The block has a cylindrical mirrored (aluminized) surface with a focal point 5.5 mm beyond the exit surface of the block, which is at an angle of 36° to the radiator face. The MCP-PMTs have their input windows spaced 0.5 mm distant from the focusing block with an air gap in between (to avoid gluing). Photons propagate at an angle θ_z relative to the radiator face and upon entering the block are reflected before reaching the MCP-PMT plane. The curvature of the mirror is such that photons emitted with the same angle θ_z from any point of the radiator are reflected onto the same vertical (y') coordinate on the MCP-PMT plane. The acceptance of photons on the detector plane is limited to photons that enter the focusing block at angles in the range $0.45 < \theta_z < 0.85$ rad. The reflectivity of the mirrored surface has been measured to be above 85% over the photon energy range of interest (and is further discussed in Sec. 7.3). The side faces are also critical reflective surfaces and as such are polished. The top and bottom faces are kept rough so light might be diffused outside the block. Whilst this prevents correlated reflections, associated scattering does result in some out-of-time background.

The focusing block is supported by a mounting plate located inside a groove in the block, shown in Fig. 7.4b. The groove can contribute towards multiple reflections within the focusing block and, to mitigate this, the groove surface is

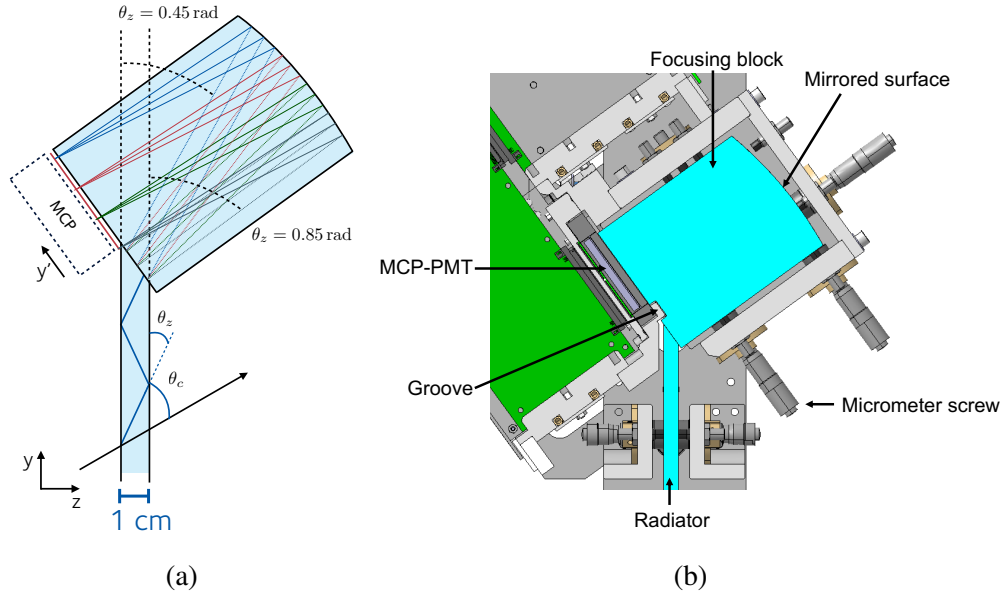


Figure 7.4: (a) A schematic representation of the focusing block, not to scale, which shows the translation of the photon's angle of exit from the quartz plate in the vertical plane into the vertical position on the MCP-PMT plane. The charged particle path is shown by the black line with the Cherenkov photons shown by the coloured lines. (b) A technical drawing showing the support of the radiator and focusing block.

is kept rough and painted black.

The focusing optics and radiator plate are bonded using Pactan 8030 silicone-based adhesive which has good transmission characteristics up to 6 eV in photon energy [83] (further discussed in Sec. 7.3). The advantage of this adhesive is that it does not set rigidly and is suited to disassembly if the need arises.

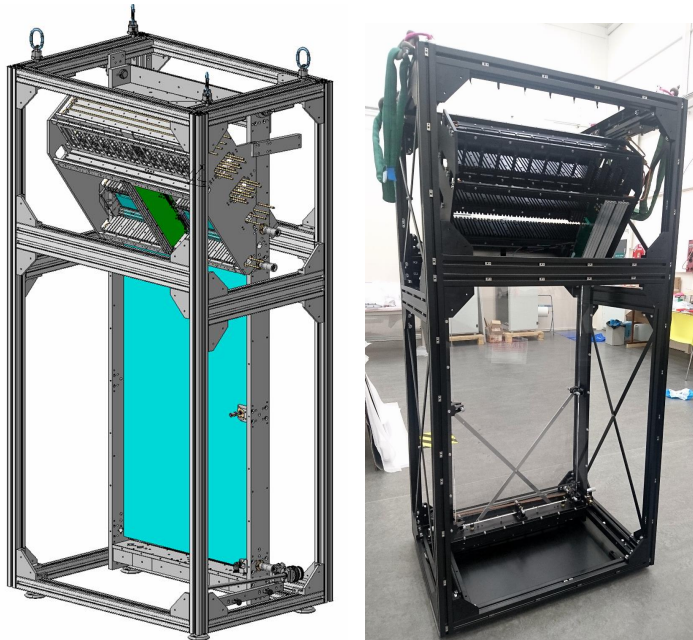
7.2.2 Mechanical frame

The optical components are mounted in a rigid anodised aluminium mechanical frame, as shown in Fig. 7.5a. The focusing block and the quartz plate assembly is finely located via micro-metric screws providing (almost) point contact to the side surfaces of the plate.

A mechanical structure that houses the MCP-PMTs and all the readout elec-



(a)



(b)

Figure 7.5: (a) The optical support frame for the quartz radiator, lying horizontally; the micrometer adjustments are visible at the bottom left, and the support structure for the MCP-PMTs and electronics, on the right. (b) The fully assembled prototype in its casing (left) the CAD model, and (right) after construction.

electronics is attached to the optical assembly described above. This structure implements so-called *comb plates*, which allow the accurate location of the electronics boards to directly couple to the photo-detectors. The readout boards are fitted with metallic plates, coupled with thermal paste, to adequately transfer heat to the comb plates. Copper pipes (visible in Fig. 7.5a) are embedded in

these combs, to provide liquid cooling and to evacuate the heat.

The complete assembly: optical components, aluminium frame, photodetectors and electronics, shown in Fig. 7.5b, is housed finally within a large and rigid light-tight crate. The assembly is mounted at a rotating point which is located at the top. This allows a tilt around the horizontal x -axis (perpendicular to the beam direction) and hence a variation of the incident particle angle through the radiator. A mechanism at the base ensures a smooth and precise rotation of the assembly up to 10° .

7.2.3 Micro-Channel Plate PMTs

During the beam tests, the Proto-TORCH module was instrumented with a pair of Photek-developed MCP-PMTs. The two MCP-PMTs were located adjacent to the upper corner of the radiator plate; their associated electronics readout boards are visible in Fig. 7.5.

The MCP-PMTs are a particular design of photomultiplier tube that are suited to fast-timing applications. They are compact and can be configured in different shapes and sizes. The working principle is as follows: photons enter the tube through the detector window and are converted into a photoelectron at the photocathode. The electron is accelerated across a high voltage through a pair of micro-channel plates in a chevron-like configuration. Electrons are amplified as they pass through a channel. The micro-channel plates in the TORCH MCP-PMTs have pores of diameter $15\text{ }\mu\text{m}$ and length 0.7 mm , with an open area ratio of 65%. The gain per plate is $\sim 10^3$, leading to a charge deposit on the anode of $\sim 10^6$. The spatial spread in the charge avalanche can be tuned, and the anode pixel size can be varied depending on the required resolution. A schematic of the MCP-PMT cross-section is shown in Fig. 7.6a. The design is liable to late and/or displaced pulses, caused by *backscattered* electrons that first hit the solid region of the micro-channel plate before entering a channel. These

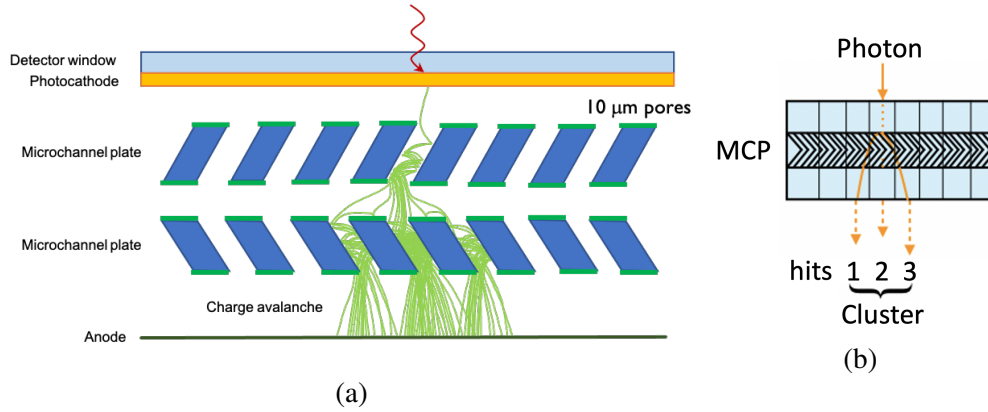


Figure 7.6: (a) The MCP-PMT cross-section. Photons arrive and are converted on the photocathode. Photoelectrons are accelerated towards the anode through microchannel plates arranged in a double-layer chevron-like structure. A charge avalanche develops which is detected over a number of pixels. (b) The pixel hits are combined into clusters corresponding to one photon. More details of the clustering algorithm are presented in Sec. 7.5.4.

often enter the channels after a spatial displacement, or much later in time with no spatial displacement.

In the Proto-TORCH MCP-PMTs, the electron avalanche within the MCP-PMT deposits charge on a resistive layer inside the PMT, which is capacitively coupled to the anode pads via an insulating layer. Both MCP-PMTs have an intrinsic pixelisation of 64×64 pads over a $53 \times 53 \text{ mm}^2$ active area, within a $60 \times 60 \text{ mm}^2$ physical footprint. The MCP-PMT anode pads are connected directly to an external printed circuit board (PCB) using anisotropic conductive film. The MCP-PMTs are read out with a granularity of 8×64 pixels, achieved by electronically grouping channels on the PCB in the horizontal direction, and seen schematically in Fig. 7.7a.

Since the avalanche of charge produced by a single photon in the MCP-PMT typically causes hits on multiple channels, demonstrated in Fig. 7.6b, the centroid of these hits allows the spatial (vertical direction) and timing precision of the photon arrival to be improved. This achieves an effective 8×128 granularity, which provides a sufficient angular precision on the reconstructed

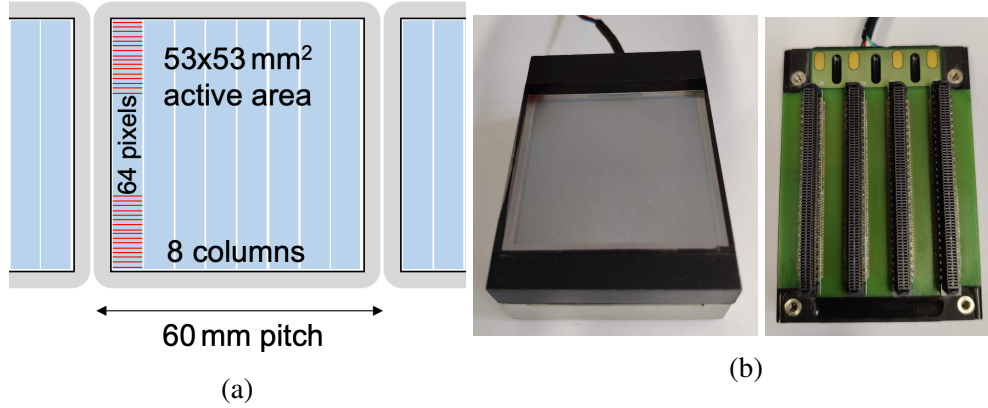


Figure 7.7: (a) The MCP-PMT pixel layout where the 64×64 pixels are grouped to form 8 columns and charge sharing is used to increase the effective granularity to 128 rows. (b) The MCP-PMT front surface and the connectors on the rear side.

Cherenkov angle to meet the TORCH single-photon timing requirements.

The front surface and back-plane connectors of the MCP-PMT are shown in Fig. 7.7b. The Proto-TORCH MCP-PMTs are designed to withstand an integrated charge of 5 C/cm^2 [84] by utilising an atomic layer deposition (ALD) coating on the micro-channel plates.

The two MCP-PMTs instrumented (hereafter labelled MCPs ‘A’ and ‘B’) have slightly different properties, presented in Table 7.1. The variation in Point-Spread Function (PSF) due to the thickness of the insulating layer leads to different numbers of photon hits per cluster on the two MCP-PMTs, as shown in Fig. 7.8. The gain was measured in dedicated laboratory experiments to be 1.25×10^6 (average) but with much larger variation in MCP A compared to MCP B.

In addition, the quantum efficiency (QE) of both MCP-PMTs was measured in the laboratory over the detector plane by illuminating small groups of pixels. The QE measured on MCP B was uniform, with the average measurement presented in Fig. 7.9. For MCP A, the response varied across the detector plane with significantly lower QE on the bottom half of the plane. The average QE

Table 7.1: The gain is estimated from measurements made in the laboratory across several pixels on each MCP-PMT. The collection efficiency, the probability of a converted photoelectron being multiplied through the MCP, is given by Photek.

MCP	A	B
Serial Number	A2170331	A3170517
Average gain (electrons)	1.25×10^6	1.25×10^6
Variation in gain (%)	47	17
Collection efficiency	0.65	0.65
Insulating layer thickness (mm)	0.3	0.5
PSF (FWHM) (mm)	1.30 ± 0.13	1.90 ± 0.06

measured at four points in the middle region of the MCP (the section illuminated by direct photon paths from Positions 1, 3, 4, 5 (see later section)) is shown in Fig. 7.9. Due to MCP B having a more favourable QE, the photon yield obtained on this MCP was larger than that obtained on MCP A.

7.2.4 Electronics and trigger

The signals from the MCP-PMTs are read out using customised electronics boards [85] that use chip-sets developed for the ALICE TOF detector [86].

The timing information of each hit on an MCP channel is read out in the following manner. Charge deposited on the MCP anode channel is amplified by a NINO ASIC chip [87], which outputs a square pulse. The leading edge of the pulse corresponds to the time that the leading edge of the charge deposit crosses the threshold; the length of the pulse indicates the Time-Over-Threshold (TOT). The NINO charge thresholds are estimated to be 20-30 fC over all channels [83]. Each NINO chip reads out 32 channels, so a single MCP-PMT requires 16 chips (grouped into four groups of four chips, each group is positioned on a NINO board).

A High Performance Time to Digital Convertor (HPTDC) ASIC [88] is used to time-stamp the two edges of the NINO output pulse. The HPTDC time dig-

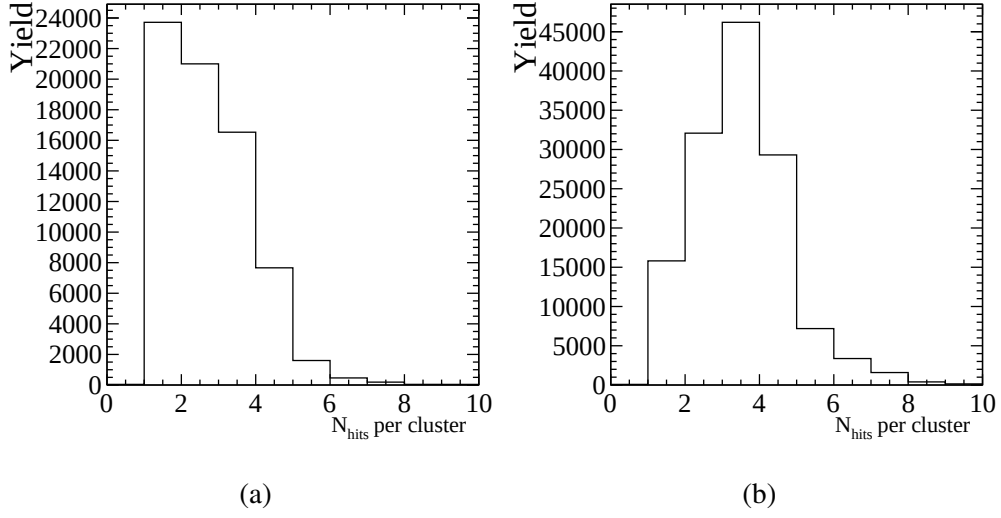


Figure 7.8: The number of hits per cluster (measured in data) on (a) MCP A and (b) MCP B.

itization was set throughout the beam tests close to 100 ps binning (256 bins in 25 ns), which provides a contribution to the final measured Proto-TORCH timing resolution of $100/\sqrt{12}$ ps. Eight HPTDC boards, each with two chips, are used to read out a single MCP-PMT.

A *Readout board* collects digitised data from four HPTDC boards and formats them into Ethernet packets to be sent to a DAQ PC. There are two Readout boards per MCP-PMT. The data include the leading edge time of the signal and the TOT. The TOT is correlated to the charge deposited in the pixel and is later used to correct for time-walk of the signal. The non-linearities in the HPTDC binning are also corrected offline, with details given in Sec. 7.5.

Figure 7.10 shows the data-flow of the electronics system and a photograph of the readout chain for a single NINO board, showing also the connector to an MCP-PMT.

The Readout board communicates with an AIDA Trigger Logic Unit (TLU)[89] using the EUDAQ2 framework [90], which provides a trigger and synchronisation with additional beamline counters. The TLU distributes clock, trigger and enable signals to the Readout boards in the system. Once data-taking starts, the

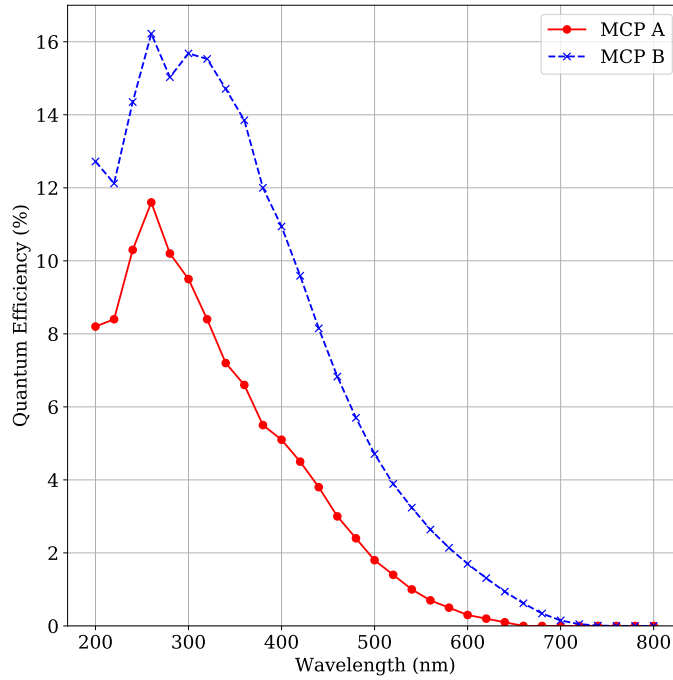


Figure 7.9: The measured quantum efficiencies as a function of wavelength of the MCP-PMTs instrumented on Proto-TORCH, as measured at CERN.

HPTDC measures the photon arrival time and TOT when a trigger signal is received. The Readout boards raise a busy signal to the TLU if a maximum trigger rate is reached.

7.3 Efficiencies and simulation

Monte Carlo simulation is used to provide expectations of the single-photon arrival-time distributions and of the photon yields. In addition, the external contribution towards the time resolution due to the beam profile used in the test beam (described in Sec. 7.4) is estimated using simulated events, where the unknown event-by-event beam-particle entry points result in small deviations in the path length of Cherenkov photons.

The simulation is generated using Geant4 [55, 56]. For a charged particle with specified mass, momentum and entry position, Cherenkov photons are

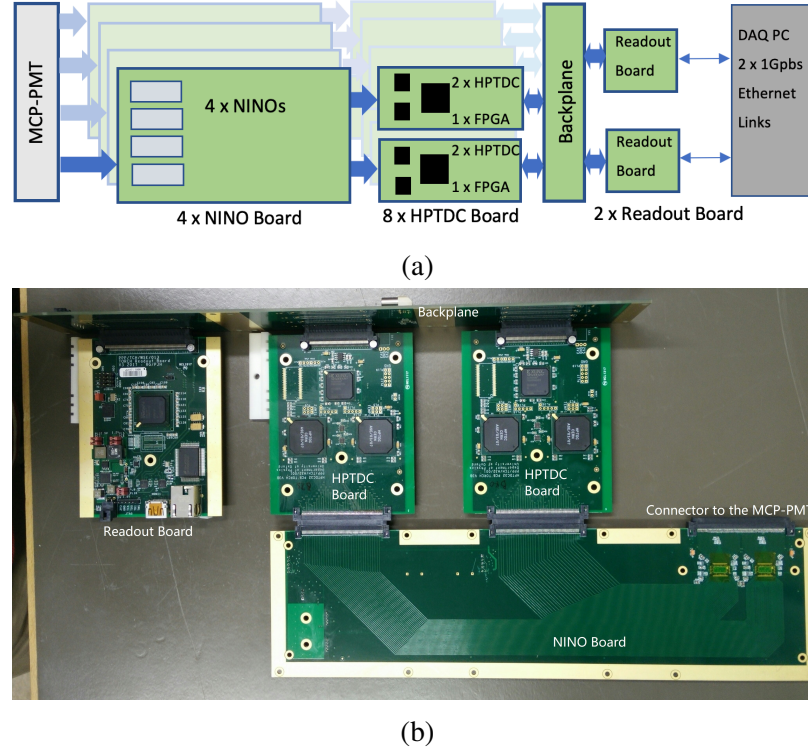


Figure 7.10: (a) A schematic and (b) a photograph of the TORCH electronics Readout boards.

simulated resulting in an output of their location and time-of-propagation.

The profile of the charged particle entry position is modelled as a 2D Gaussian with widths 2.73 mm in x and 2.01 mm in y , based on measurement in Ref. [81], to represent the beam profile during the beam test. The production of Cherenkov radiation is modelled according to the Frank-Tamm relation [91],

$$\frac{d^2 N_\gamma}{dE dx} = \frac{\alpha}{hc} Z^2 \left(1 - \frac{1}{\beta^2 n^2} \right), \quad (7.4)$$

where the number of photons, N_γ , of an energy E , is proportional to the properties of the charged particle with charge Z (in units of e^-) and momentum β , and the refractive index of the medium n . The distance travelled, x , is integrated over by the length of the medium. The photon emission spectrum is uniform over energy (so is inversely proportional to the wavelength squared). In

TORCH, with a 1 cm distance travelled and $n \approx 1.45$ (which varies with photon energy), $N_\gamma \approx 1000$ per charged hadron, for photon energies in the range 2-7 eV. However only a small proportion of these photons, approximately 30, will satisfy the internal reflection requirements and be detected at the detector plane.

In the simulation, the photons are propagated through a radiator and focusing block, with refractive indices implemented from the manufacturer's specifications. The surfaces of the radiator and focusing block are modelled as dielectric-dielectric boundaries, with properties to match the surface types as described in Sec. 7.2.1. The transmission of photons through the glue that is used to optically couple the radiator and focusing block is accounted for, which imposes a sharp cut-off at low wavelengths around 220 nm. The mirrored surface of the focusing block has a wavelength-dependent reflectivity which plateaus in the middle of the range. The air-gap between the focusing-block exit surface and the MCP-PMT window is also included.

Furthermore, the response of the MCP-PMT and NINO chip is modelled. The probability of photon conversion at the MCP photocathode is modelled according to the measured QE, which peaks in the wavelength range 250–300 nm. The behaviour of the photo-electron through the Micro-Channel Plates is determined by the collection efficiency and the probability of back-scattering, which can lead to spatial displacement and time delay of the charge avalanche. A back-scattering model has been determined based on data taken during laboratory tests of the MCP-PMTs. The gain amplification of an individual photo-electron is randomly sampled from a model developed from measurements of the gain. The charge avalanche is spread according to the PSF of each MCP-PMT, and a Gaussian integral of the charge-spread is performed to determine which pixels receive sufficient depositions to be above the NINO threshold, and thus have a hit recorded. The same clustering algorithm used in the data analysis (described

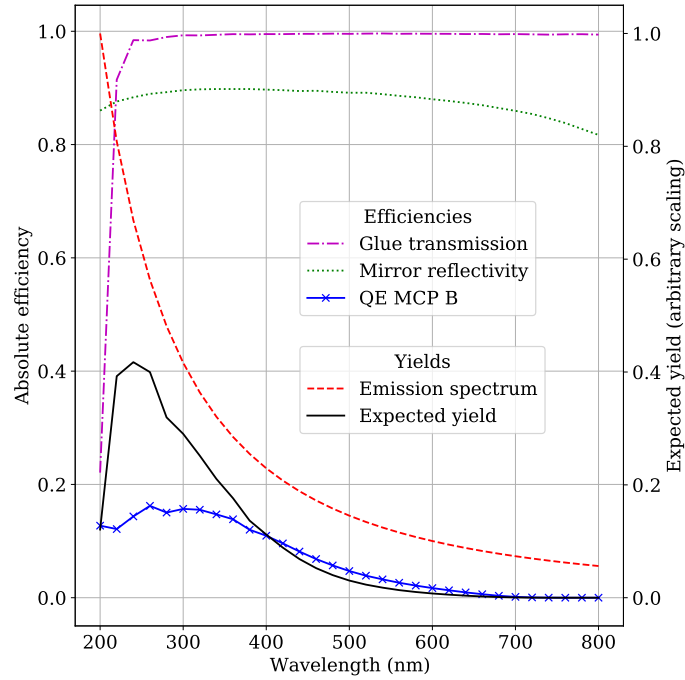


Figure 7.11: The absolute efficiencies of glue transmission, mirror reflectivity and photo-electron conversion (for MCP B) as a function of wavelength. The photon emission spectrum and expected photon yield due to the efficiency sources are also shown, both with arbitrary scaling.

in Sec. 7.5) is used to cluster the hits into reconstructed photons.

The dependence of the mirror reflectivity and glue transmission on the photon wavelength are presented in Fig. 7.11. Also shown is the QE of MCP B and the emission yield, along with the resulting expected yield. Discussion of whether the efficiencies adequately represent those in reality is presented later in Sec. 7.6.5.

7.4 Test-beam setup

The beam test was performed at the T9 area of the CERN PS using a nominal 8 GeV/c momentum beam composed of $(47.3 \pm 1.5)\%$ pions and $(52.7 \pm 1.5)\%$ protons [92]. The beam-line infrastructure was similar to that of previous test-

beam campaigns with the smaller module [81]. The beam-line components are illustrated and are visible in the photograph of the test-beam area in Fig. 7.12.

Two *timing stations* provided a time reference and were positioned 11 m apart, with Proto-TORCH positioned 1.65 m upstream of the downstream station. Each timing station comprises a $8 \times 8 \times 100 \text{ mm}^3$ borosilicate bar, coupled to a single-channel MCP-PMT to provide accurate timing, and a pair of crossed scintillators. The borosilicate bar was located relative to the beam such that Cherenkov light produced by traversing particles was oriented directly towards the MCP-PMT. The crossed scintillators, each connected to a PMT, had an overlap of $8 \times 8 \text{ mm}^2$ perpendicular to the beam. The trigger required a signal in both PMTs of the downstream timing station, which served to narrow the beam spatial definition and ensure the triggered particles pass through the borosilicate finger. The timing signals from both timing stations were injected into the Proto-TORCH readout electronics, eight channels per MCP-PMT, to allow straightforward synchronisation of the data, which also means that the timing signals were digitised using the 100 ps binning of the HPTDC chips. This scheme has the slight disadvantage that several pixels of the MCP-PMT were not read out, to be replaced by data from the time-reference stations.

A pair of CO₂-filled threshold Cherenkov counters were located upstream of the first timing station to discriminate pions from protons in the beam. This enables a method of particle identification independent of the ToF provided by the timing stations. In addition, a large scintillator coupled to a single-channel PMT was used in conjunction with signals from both Cherenkov counters to reduce random noise. An EUDET/AIDA pixel beam telescope [93] is used to measure the beam profile [81], and some of its scintillating layers were used in the trigger definition.

In the test beam, the TORCH light-tight crate was mounted on a translation table, allowing free movement in both directions perpendicular to the beam

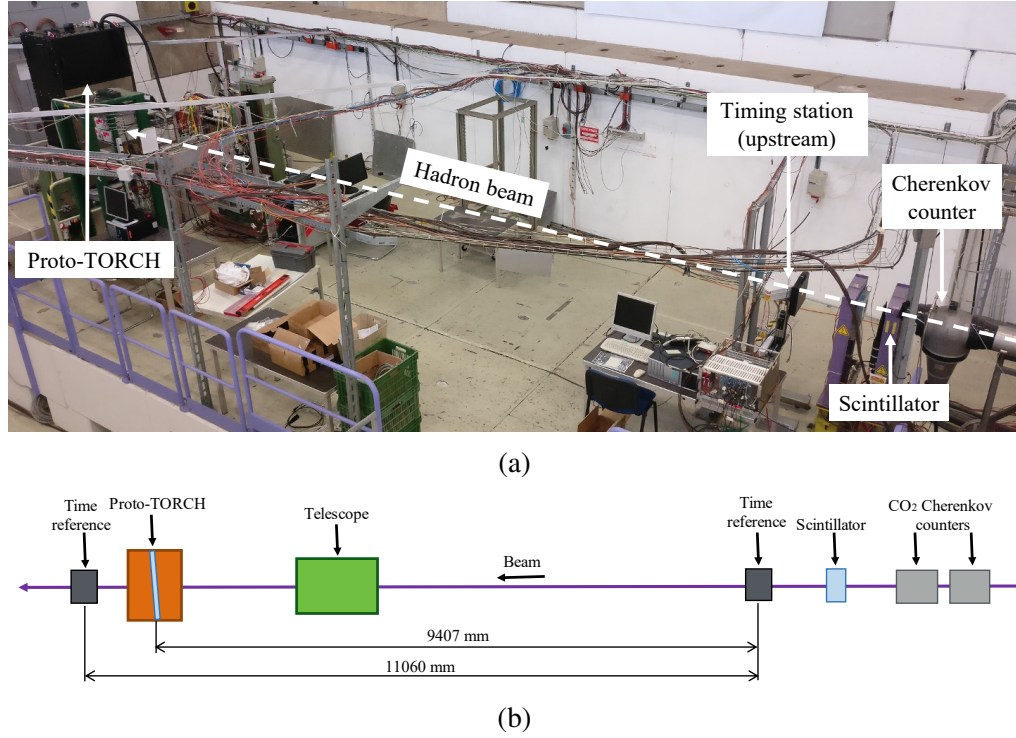


Figure 7.12: (a) Views of the T9 test-beam area along the beam direction. The beam trajectory has been overlaid and travels from right to left. One of the Cherenkov counters, the telescope and the downstream timing station are not visible in the photograph. (b) The layout of the components of the test beam, with the beam travelling from right to left.

direction. The radiator was tilted to 5° with respect to the vertical in order to improve the light collection efficiency from incident charged particles.

The performance of Proto-TORCH was studied at a total of seven beam-entry positions, as indicated in Fig. 7.13. The positions relative to the upper left corner of the plate (nearest to the MCP-PMTs) are shown in Table 7.2. These geometrical configurations were chosen such that the Cherenkov patterns are well-contained within the instrumented region of the MCP-PMT detector plane.

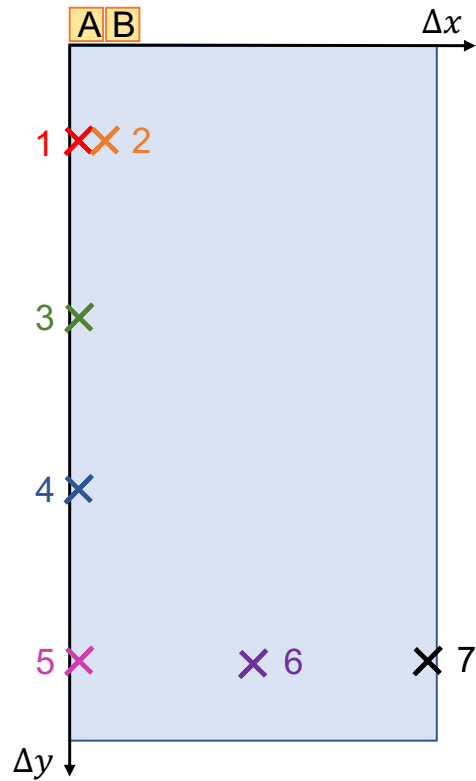


Figure 7.13: Proto-TORCH track positions, with coordinates given in Table 7.2.

Table 7.2: Positions of tracks in Proto-TORCH with the origin taken at the upper corner of the radiator closest to the MCP-PMTs.

Position	Δx (mm)	Δy (mm)
1	5	175
2	60	175
3	5	488.6
4	5	801.9
5	5	1115.4
6	330	1115.4
7	655	1115.4

7.5 Data processing

At the first stage of data analysis, various processing steps were undertaken to calibrate the data. All calibrations were data-driven and performed on a large, dedicated set of data, independent from the datasets used for TORCH analysis. These calibrations included corrections for effects introduced by the readout electronics: the time-walk of the NINO chip, time alignment of pixels, and the integral non-linearity of the HPTDC chips. All algorithms are described in Refs. [76, 81] and summarised below.

7.5.1 Pixel-to-pixel time alignment

Different routing-line lengths on the NINO and HPTDC boards give small pixel-to-pixel time offsets. These corrections were determined from the requirement that hits within a photon cluster have the same arrival time. In addition, arbitrary time offsets between different MCP-PMTs must be corrected for.

7.5.2 Time-walk calibration

The NINO chips introduce a timing slew (referred to as ‘time-walk’) dependent on the amount of charge deposited. Large charge deposits cross the NINO charge threshold earlier than smaller deposits, and thus the output pulse is initiated earlier leading to an earlier photon time-of-arrival. This is demonstrated in Fig. 7.14 where the larger blue pulse has an earlier time-of-arrival than the smaller red and black deposits. The time-walk effect must be corrected for in order to have accurate timing responses. In addition, larger charge deposits have a larger time-over-threshold (or NINO output pulse width).

Using the same procedure, the time-walk calibration was made for each pixel, and a correction subsequently made to the arrival time of each photon hit

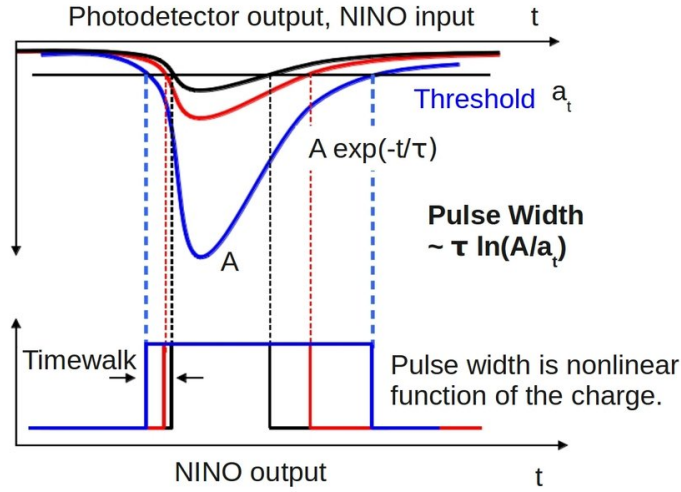


Figure 7.14: A schematic showing how the size of the charge deposit impacts the NINO output pulse. Taken from [94].

according to the width of its time-over-threshold.

7.5.3 HPTDC integral non-linearity calibration

The HPTDC ASICs which digitise the arrival times and time-over-threshold from the NINO chip employ time-binning which is not equally spaced in time (integral non-linearity). A calibration was adopted to correct for the channel-to-channel non-linearity.

7.5.4 Clustering algorithm

The MCP-PMT is designed such that a single incident photon will induce charge deposits over several neighbouring pixels, in order to improve the effective granularity in the vertical direction. In the analysis presented here, a simple average position is taken for the centroid of the pixel hits within a cluster. In detail, if n hits (located on pixels y_i) are reconstructed as a cluster, then the location of the

cluster, y_{cluster} , is assigned as

$$y_{\text{cluster}} = \frac{\sum_{i=1}^n y_i}{n}, \quad (7.5)$$

i.e. clusters with an even number of hits are reconstructed with locations at non-physical half-integer pixels² and clusters with an odd number of hits are reconstructed with locations on physical (integer) pixels. The same clustering algorithm is applied to combine the arrival time of each hit to give a cluster time.

In the analysis of future test-beam data, a more accurate cluster centroiding could be performed. The width of the NINO output pulse is proportional to the charge deposited, demonstrated in Fig. 7.14. This allows for a weighting of pixel hits proportional to their charge (via the recorded NINO pulse width), which would improve the hit position resolution compared to the current averaging. However the responses of every NINO channel must be calibrated as the width of the pulse scales non-linearly with the charge deposited (see Fig. 7.15) and this behaviour can be different across different channels; the calibration a lengthy procedure which needs dedicated hardware and is currently being performed.

In the final TORCH electronics, it is hoped that the MCP-PMT pixel sizes will be small enough so that the required spatial resolution is achieved via single hits, rather than requiring charge-sharing across pixels to increase the spatial resolution.

7.6 Test-beam analysis

Analysis of the test beam data is categorised into three parts. First, the single-photon time resolution is determined and compared to the target of 70 ps per photon. The contributions from the intrinsic MCP-PMT response, electronics

²Although this is not always the case as the reconstruction algorithm allows for missing pixel hits.

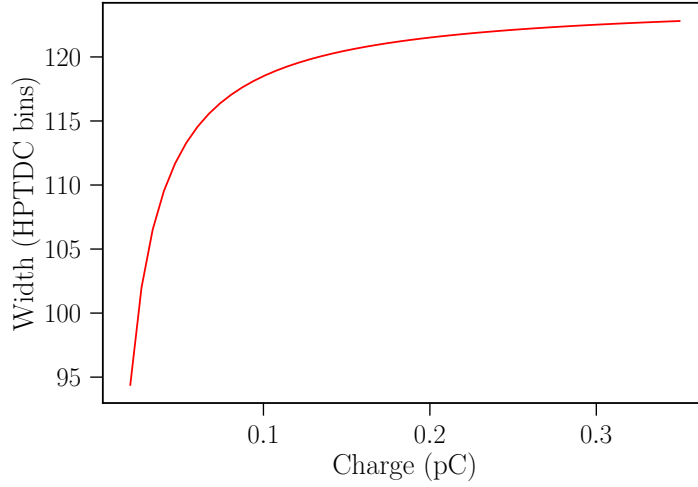


Figure 7.15: An indication of how the width of the NINO output pulse scales with the charge deposit.

readout and photon reconstruction are inferred. Studies of the resolution for paths of photons involving side reflections in the radiator are then performed. Second, the yields of photons per charged track are determined and compared to simulation. The relative yields of photons undergoing higher-order reflections in the radiator are also assessed. Finally, the per-track timing resolution is measured for tracks yielding multiple photon clusters, verifying the improvement in performance when clusters are combined. All of the above analyses employ data taken at Positions 1, 3, 4, and 5 in the radiator; the exception being for studies of variation of timing resolution and yield with photon path, where the Position 6 and 7 datasets are used. Additionally, a discussion of the photon-detection efficiency is presented, comparing the photon yields in data as a function of photon energy to expectations.

7.6.1 Photon hits on the MCP-PMTs

During the beam test, Proto-TORCH was instrumented with two MCP-PMTs corresponding to 1024 readout channels, 16 of which were allocated to time-reference channels. Figure 7.16a shows the spatial distribution of reconstructed

photon clusters on the MCP-PMT plane for data taken with the beam entering the plate at Position 6. The cones of Cherenkov radiation emitted by the charged hadrons are focused into hyperbolic patterns, which are folded in on themselves when the light is reflected from the sides of the module. Signals from different paths in the radiator can be detected in the same pixel but will arrive at different times. The different QEs of the two MCP-PMTs is responsible for the lower level of light seen in MCP A. The empty (dead) channels, represented by white entries in Fig. 7.16a, reduces from MCP A to MCP B as a result of improvements made in the wire-bonding procedure of the NINO chips when mounting on the boards during manufacture.

The time-projection plot (Fig. 7.16b) shows the reconstructed photon arrival time (relative to the reference time) versus the clusters' vertical pixel position, for data collected on a typical single MCP-PMT column (horizontal pixel). The red lines are predicted arrival patterns, simulated from knowledge of the hadron track position and angle. In this projection, the different *orders of reflection* (photon paths in the radiator) are well-separated. The widths of each order are then measured to determine the single-photon time resolution. The photon yields are determined by defining contained regions around each order in this projection, where the regions are tuned to minimise background counts and to avoid incorrect counting across neighbouring orders.

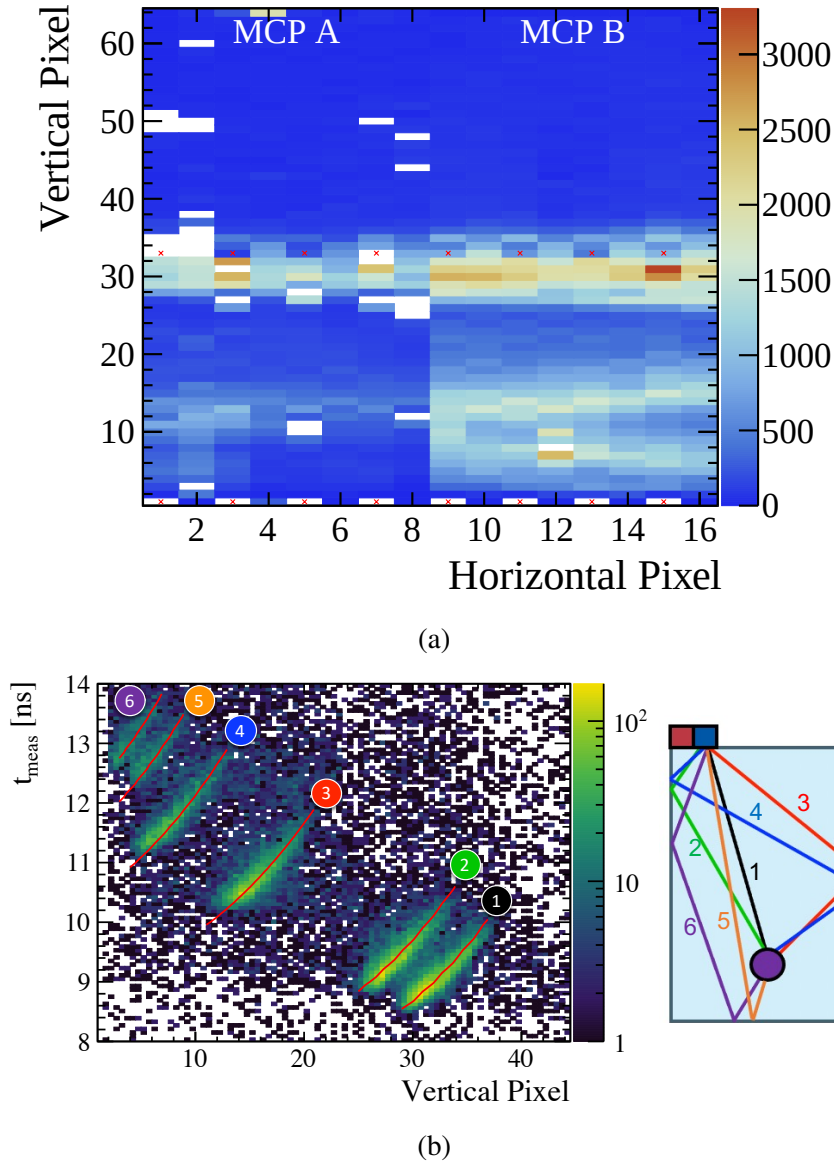


Figure 7.16: (a) The spatial distribution of reconstructed photon clusters on MCPs A and B. Pixels marked with a red cross are those where the time reference signals were injected. (b) The time projection of reconstructed photon clusters on MCP-PMT column 16, with the photon trajectories that fall within the MCP-PMT acceptance labelled, and the associated reflections on the right.

7.6.2 Single-photon time resolution

The single-photon time resolution is determined using simultaneous unbinned maximum-likelihood fits to the photon arrival times with respect to the time-reference signal (t_0). A separate fit model is created for each vertical pixel comprising:

- Peaking structures to fit the signal distributions, modelled by Crystal Ball functions [95] which fit asymmetric Gaussian distributions. This function models the tails caused by late arriving back-scattered photoelectrons from the MCP surface. Analytical reconstruction predictions are used to determine how many peaking structures are expected for each pixel;
- Peaking background shapes to account for structures observed in the data, modelled by Gaussian functions. The structures are bands across all pixels that appear at the same time, attributed to photons from secondary sources and/or scattered signal light;
- Polynomial background distributions to account for observed random background levels.

A simultaneous fit to the photon time distribution is performed across all vertical pixels of a column. In each vertical pixel, the yields, mean times and widths of the Crystal Ball probability distribution functions (PDFs) are free to float, however the widths for individual reflection orders are shared in the fit across all vertical pixels in the same MCP-PMT column. The parameters of the background PDFs are also shared across all vertical pixels in a column. The width of the fitted Crystal Ball function gives the measured resolution per order of reflection per column, σ_{meas} . An example fit to a single pixel is shown in Fig. 7.17 in which two peaking orders of reflection are observed, with one peaking background structure and the polynomial background.

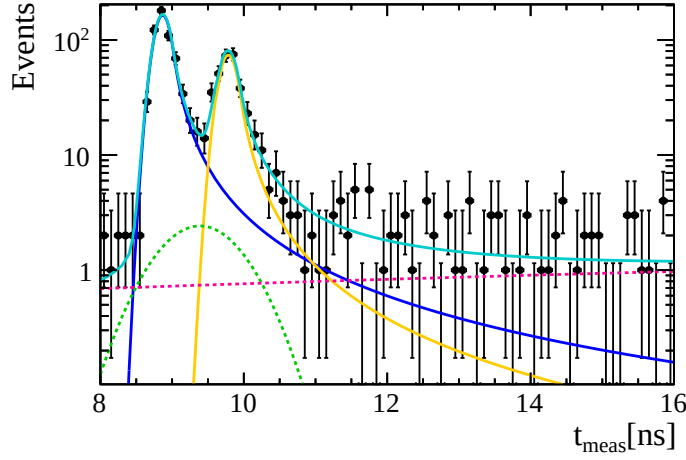


Figure 7.17: An example of a fit to the time of arrival of photons on a single vertical pixel (pixel 30 as shown in Fig. 7.16b), used to measure the single photon time resolution. On this pixel there are two orders of reflection, with times of arrival that peak at 8.9 ns and 9.8 ns. The total fit (light blue) comprises of two signal peaks (blue / yellow) and background contributions (dotted).

7.6.2.1 External contributions to the measured resolution

The measured resolution, σ_{meas} , receives contributions from the experimental setup that are external to TORCH, which need to be subtracted. Here the start-time resolution is the primary contribution. During nominal data taking, the rate of hits is maximised by requiring only the downstream timing-reference scintillator stations to have fired. From a study of the time difference between the two timing signals for different trigger configurations, the t_0 resolution, $\sigma_{\text{time ref}}$, is measured to be 53.2 ± 0.6 ps. The second contribution is due to the finite size of the beam; the uncertainty in the particle entry position in the quartz plate results in a variation of the photon path length and hence its arrival time. From simulation and a knowledge of the beam profile, the contribution to the time resolution, denoted by σ_{beam} , is between 15.1 and 30.4 ps [92], dependent on the height at which the beam enters the plate. The above contributions are subtracted in quadrature from σ_{meas} to obtain the TORCH single photon time

resolution:

$$\sigma_{\text{TORCH}}^2 = \sigma_{\text{meas}}^2 - \sigma_{\text{time ref}}^2 - \sigma_{\text{beam}}^2. \quad (7.6)$$

7.6.2.2 Direct photon paths

The time resolution is measured for several beam entry positions, indicated by Positions 1, 3, 4 and 5 in Fig. 7.13. Here the distance from the top of the plate to the beam is varied from 175 mm to 1115.4 mm in four steps. Only photons that travel directly to the photodetectors (or take paths with an immediate side reflection) are considered; here side reflection refers to the number of reflections off the narrow sides of the plate rather than off the large front and back surfaces. The TORCH time resolution for each MCP-PMT column and beam position is shown in Fig. 7.18, separately for the samples of pions and protons. It can be seen that the time resolution of the prototype approaches the design goal of 70 ps but there is some degradation for entry positions far from the MCP-PMTs, which is to be expected. Due to the finite size of the pixels, the uncertainty on the position and the Cherenkov angle introduces an uncertainty in the measurement of the photon energy, hence the time resolution. This uncertainty scales linearly with path length and propagation time. As a compensation for this degradation, in any future PID reconstruction, there will be additional information from the time of propagation measurement for the larger photon path lengths, where the difference in Cherenkov angle of any two particle species will result in a spatial and timing separation at the photodetector plane.

To quantify the contributions to the TORCH time resolution from different sources, a two-dimensional fit is performed as a function of the photon propagation time (directly related to the beam-entry position) and photon-cluster size, shown for protons in Fig. 7.19. The time resolution is parametrised by

$$\sigma_{\text{TORCH}}^2 = \sigma_{\text{prop}}(t_p)^2 + \sigma_{\text{MCP}}^2 + \sigma_{\text{RO}}(N_{\text{Hits}})^2, \quad (7.7)$$

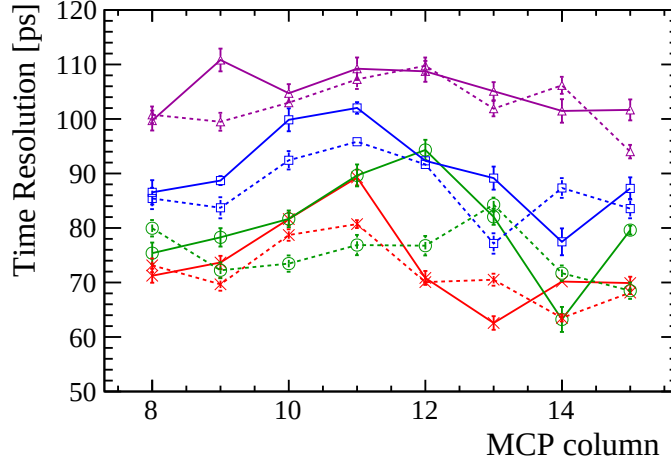


Figure 7.18: The single-photon time resolution determined in different columns of MCPB and for different beam positions (Positions 1 (×), 3 (○), 4 (□), 5 (△) as indicated in Fig. 7.13). Only data from direct photon paths are considered. The full (dotted) lines are the results from the pion (proton) samples.

where the term $\sigma_{\text{prop}}(t_p)$ characterises the contribution that grows with photon propagation time, t_p ; a constant term σ_{MCP} is associated to the resolution of the MCP-PMT; a term $\sigma_{\text{RO}}(N_{\text{Hits}})$ describes effects from the charge-spread and read-out process which is a function of the number of hits in a cluster, N_{Hits} . A linear dependence on t_p and a $1/\sqrt{N_{\text{Hits}}}$ dependence on N_{Hits} are assumed in σ_{prop} and σ_{RO} , respectively. The results of the fits are presented in Table 7.3, together with target values. The target values are driven by the requirements on the final detector (70 ps resolution per photon); this is split between a 50 ps MCP+Readout resolution (assuming 2.5 hits per photon cluster) and a 50 ps Time of Propagation resolution, assuming a photon propagation time over the maximum 2.5 m quartz length. Whilst the data do not entirely match expectations for the running of the σ_{prop} and σ_{RO} terms, the MCP resolution term is in agreement. The latter two resolutions are expected to improve with future electronics calibrations, and the sources of these contributions will be further studied in laboratory tests.

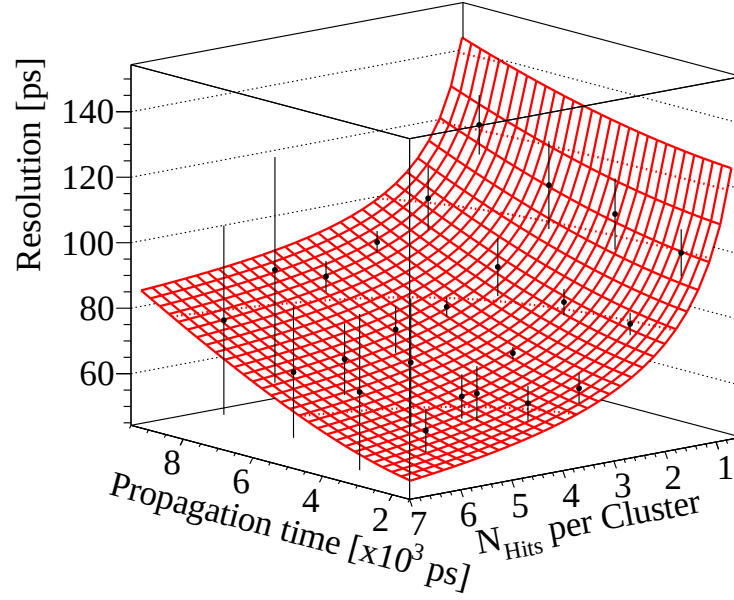


Figure 7.19: The TORCH time resolution of the proton sample, displayed as a function of the propagation time and number of hits per photon cluster. The overlaid surface is the result of the 2D fit described in the text.

Table 7.3: The results of a 2D fit to the single photon time resolution as a function of propagation time (t_p in ps) and the number of hits per photon cluster. The results of analysis of both the pion and proton samples are shown separately and are consistent with each other. The target performance is also shown.

Contribution	Fitted values (ps)		Target values (ps)
	Pion	Proton	
$\sigma_{\text{prop}}(t_p)$	$(8.3 \pm 0.7)t_p \times 10^{-3}$	$(7.6 \pm 0.5)t_p \times 10^{-3}$	$(3.75 \pm 0.8)t_p \times 10^{-3}$
σ_{MCP}	34.5 ± 8.6	31.0 ± 7.6	33
$\sigma_{\text{RO}}(N_{\text{Hits}})$	$(96.2 \pm 6.7)/\sqrt{N_{\text{Hits}}}$	$(95.0 \pm 6.0)/\sqrt{N_{\text{Hits}}}$	$60/\sqrt{N_{\text{Hits}}}$

7.6.2.3 Photon paths with multiple reflections

Data taken with the beam entering the plate at Position 6, a location which is centrally located towards the bottom of the plate, are used to study the time resolutions of photons that have undergone multiple side and/or bottom reflections in the plate, as shown in Fig. 7.16b. Photons collected on MCP B are studied, where there is good separation between different orders of reflection across all columns.

The simultaneous fit, as described above, is used to determine the resolutions of the six orders of reflection in each column. For each order, a weighted average of the measured resolutions across all columns is calculated and is shown in Fig. 7.20a. Figure 7.20b shows the resolution as a function of the total number of photon reflections in the quartz radiator, determined from simulation. The number of reflections from the front and back surfaces as well as those from the side and/or bottom faces are counted, although dominated by reflections from the front/back. The vertical error bars indicate the systematic uncertainties on the measurements associated with the column averaging, whilst the horizontal error bars indicate the range of the number of reflections.

The measured resolution for the direct photon path (path number 1) is approximately 100 ps which is similar to the measured resolution of direct photons from Position 5. Photon paths with side and bottom reflections have slightly degraded resolutions, to be expected from the increased path length and total numbers of reflections.

7.6.3 Photon yields

The number of photons collected in Proto-TORCH are counted and compared to yields obtained from simulated samples. To exclude photons that are likely to have originated from background sources, photons are only counted if they

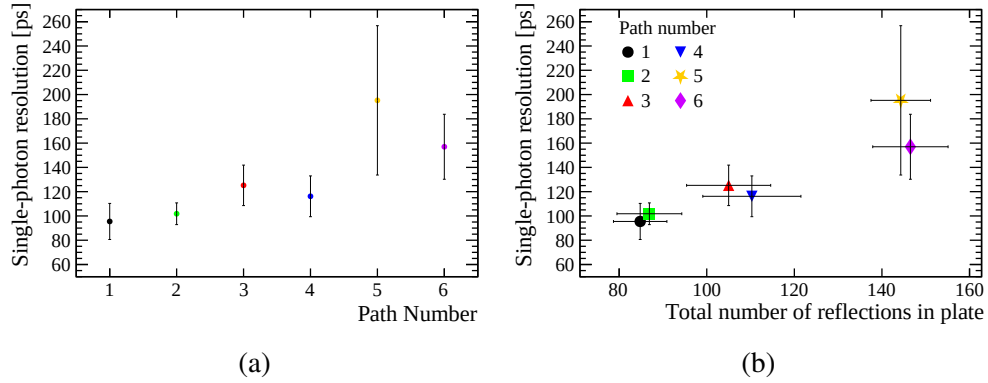


Figure 7.20: (a) Single-photon time resolution for different photon paths originating from pions entering the plate at Position 6. (b) Single-photon time resolution as a function of the total number of reflections made inside the quartz radiator (taken from simulation) for different photon paths. The colours indicate the paths as shown in Fig. 7.16b.

arrive within 0.5 ns of the peak time of arrival, and on pixels close to expectation (determined from simulation).

7.6.3.1 Direct photon yields

For beam positions close to the side edge of the quartz, only photons that travel directly or are reflected once off the nearest side edge are counted. The yields of the number of photons detected per incident hadron in data, N_{photons} , are compared to expectations from simulation in Fig. 7.21. Some discrepancies between data and simulation are observed; in particular there are more events in data where no photons are observed than in simulation. This is attributed to two effects: firstly the likely occurrence of a small number of false triggers, and secondly that small charge-shared pulses can have amplitudes below the NINO thresholds which are not accurately modelled in the simulation. The mean number of photons detected per hadron for each beam position is given in Table 7.4. Reasonable agreement is seen when events with $N_{\text{photons}} = 0$ are excluded, better than 80% at all beam positions. The photon yield reduces when the beam is positioned further from the MCP-PMTs because the angular acceptance of the

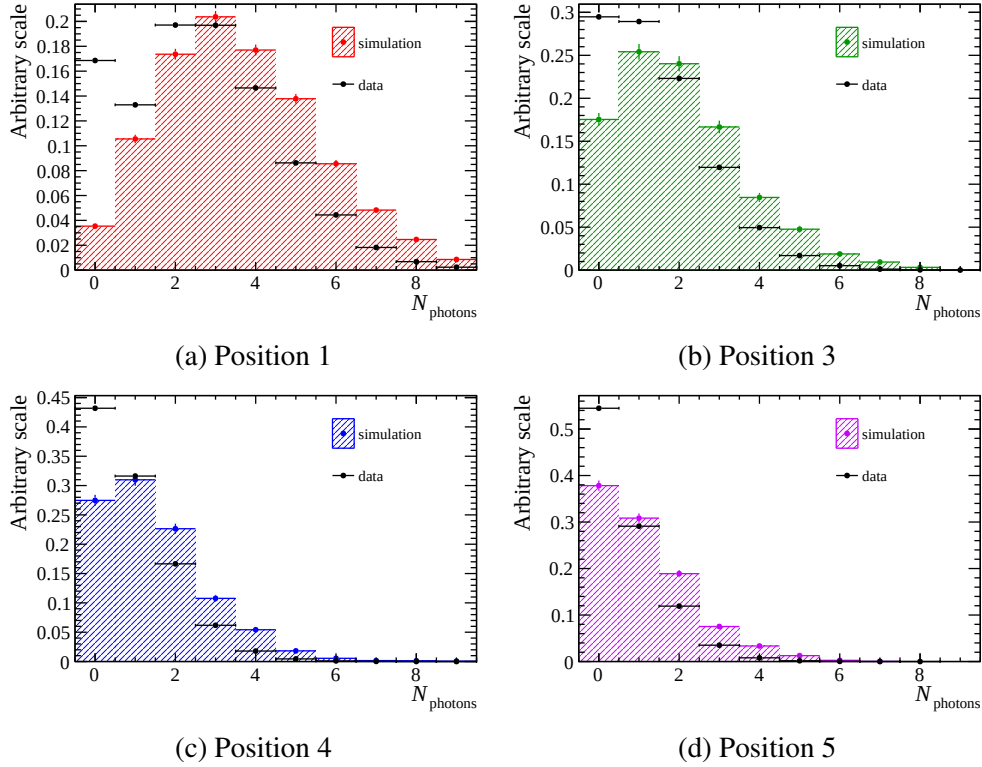


Figure 7.21: Photon counting yields for the four beam entry positions from data and simulation.

detection surface decreases. The mean photon yield per hadron is significantly reduced compared to future expectations of 30 photons per charged track for a fully instrumented module. This is primarily because only two (of the eleven) MCP-PMTs are instrumented, and only one of the possible photon paths is here considered. In addition, future MCP-PMTs are expected to have a much improved quantum efficiency which will result in larger yields.

7.6.3.2 Photon yields from multiple reflections

The yields of photons that have undergone multiple side reflections within the quartz plate are also studied, using data taken with the beam at Position 6. This central location towards the base of the plate allows for reflections from both side surfaces and the bottom surface of the radiator to be observed. As shown

Table 7.4: The mean number of photons detected originating from Positions 1, 3, 4 and 5 in data and simulation. The final column gives the ratio of the mean yields when the contributions from events with zero photons detected in data and simulation are excluded.

Position	Mean N_{photons}		Mean(Data)/Mean(Simulation)	
	Data	Simulation	All	Excluding $N_{\text{photons}}=0$
1	2.605 ± 0.007	3.586 ± 0.020	0.726 ± 0.004	0.843 ± 0.005
3	1.419 ± 0.005	2.016 ± 0.029	0.704 ± 0.010	0.824 ± 0.010
4	0.937 ± 0.004	1.454 ± 0.024	0.644 ± 0.011	0.823 ± 0.008
5	0.677 ± 0.002	1.127 ± 0.022	0.600 ± 0.012	0.820 ± 0.009

in Fig. 7.16b, some orders of reflection arrive on the same pixel close in time. To minimise double counting and mis-association, the boundaries around each order are adapted from the nominal ± 0.5 ns region for each column individually. The yields from different photon paths are presented in Table 7.5 and good agreement is generally seen in the photon yields when compared to simulation. There is some discrepancy in the photon yields from paths that reflect off the bottom surface of the plate.

Similar behaviour is observed for photons originating from Position 7 on the plate. To study these data, pairs of orders of reflections are grouped together as they arrive at indistinguishable times on the same pixels. This gives good separation for the direct photon path and those that reflect off the opposite side, and also for paths that reflect from the bottom surface. The yields obtained in data and simulation are presented in Table 7.6.

The deficiency in photon yields in data for photons reflecting from the bottom surface is not yet understood, but may be due to a slightly low-quality finish of this surface. In addition, the orders of reflection that include reflections from the bottom surface fall close to the edge of the MCP-PMT plane, where variations in the MCP-PMT performance are possible.

Table 7.5: The yields of different photon paths originating from Position 6 in data and simulation. The path number corresponds to the paths shown in Fig. 7.16b.

Reflections (path number)	Data		Simulation	
	Yield	%	Yield	%
Direct (1)	47533	27	23790	25
One side (left) (2)	46300	26	21422	23
One side (right) (3)	37383	21	16458	18
Two side (4)	28936	16	13957	15
One bottom (5)	9193	5	10624	11
One bottom + one side (6)	9100	5	7574	8

Table 7.6: The yields of different photon paths originating from Position 7 in data and simulation.

Reflections	Data		Simulation	
	Yield	%	Yield	%
Direct; one side (right)	60586	49.5	1621	38.4
One side (left); two sides	50804	41.5	1475	35.0
One side + one bottom	10998	9.0	1121	26.6

7.6.4 Per-track time resolution

The final stage of the analysis involves measuring the so-called *per-track* timing resolution, where the best estimate of the track time is determined from a combination of arrival times of all photon clusters in the event. The overall TORCH timing performance relies on there being a $1/\sqrt{N_{\text{photons}}}$ dependence, assuming that the measurement properties of individual photon hits are uncorrelated with others from the same track. In what follows, the time-stamps of the individual photons from a single incident track as identified in Sec. 7.6.2 are combined, and the corresponding time resolution determined.

The analysis combines information from photons that arrive on different MCP columns. In order to increase the yield of events that have several detected photons, hits on both MCPs A and B are used. The per-track time resolution is

measured as a function of track-entry position in the quartz radiator, and as a function of N_{photons} .

The time of arrival of each photon, t_{meas} , is compared to the expected time of arrival, t_{exp} , obtained from the peak position of the relevant Crystal Ball function in the single photon analysis. A variable Δt is introduced to provide the best estimate of the average time difference $\langle t_{\text{exp}} - t_{\text{meas}} \rangle$ of all photons that originate from the same hadron. The spread in Δt then indicates the best estimate of the per-track time resolution. The value of Δt for each event is found by constructing a χ^2 variable,

$$\chi^2 = \sum_i^{N_{\text{photons}}} \left(\frac{t_{\text{meas}}^i + \Delta t - t_{\text{exp}}^i}{\sigma_{\text{meas}}^i} \right)^2, \quad (7.8)$$

minimisation of which with respect to Δt allows the most probable value of Δt to be obtained, which is equivalent to the weighted mean.

The pion data (with direct photons paths only) are divided into separate samples by the numbers of photons detected per event (N_{photons}), and by track entry point. For each sample, the distribution of Δt is fit with a Crystal Ball function to characterise the signal. Examples of the Δt distribution and fitted PDFs for data taken at Position 1 are shown in Fig. 7.22. The width of the Gaussian component of the Crystal Ball is taken as the measured per-track time resolution, which is then corrected by subtracting contributions from the beam spread and time reference according to Eqn. (7.6).

The per-track time resolution as a function of the number of photons per event is shown in Fig. 7.23 for different beam entry positions. The measured resolution decreases as the number of photons increases, but more slowly than a $1/\sqrt{N_{\text{photons}}}$ dependence. A possible cause is correlations within the electronics between closely spaced multiple photon hits. A common clock which defines the 100 ps TDC binning services 64 adjacent channels of the HPTDC chips,

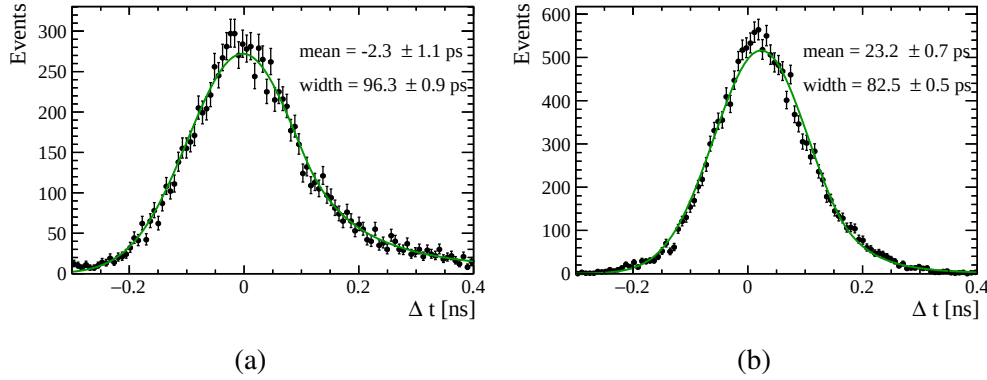


Figure 7.22: The distribution of Δt for events with beam entry at Position 1, with (a) one photon detected per event and (b) three. The means and widths specified refer to the means and sigmas respectively of the primary Gaussians in the Crystal-Ball fits. The width quoted is not yet corrected for external time resolution sources.

which may result in deviations from an uncorrelated $100/\sqrt{12}$ ps bin resolution. A significantly finer time bin designed into the next generation of TDCs will alleviate this problem.

7.6.5 Measurement of efficiency using data

In Sec. 7.3, the best understanding of the physics processes and efficiency effects associated with Proto-TORCH are presented. In particular, the quantum efficiency plays a large role in the yield of photons collected. Here, the photon yield in data is studied as a function of photon energy, in order to see if any regions of the photon spectrum are deficient. To minimise the effect of variation in photon detection efficiency across the MCP-PMT plane, only data from Position 1 on Tube B (which is collected on a small, central region of the detector plane) is used.

A key feature of the TORCH principle is that the photon energy is reconstructed explicitly from the timing and position measurement of each Cherenkov photon. The yield of photons in data as a function of reconstructed photon energy and wavelength is then plotted in Fig. 7.24, with the expected values

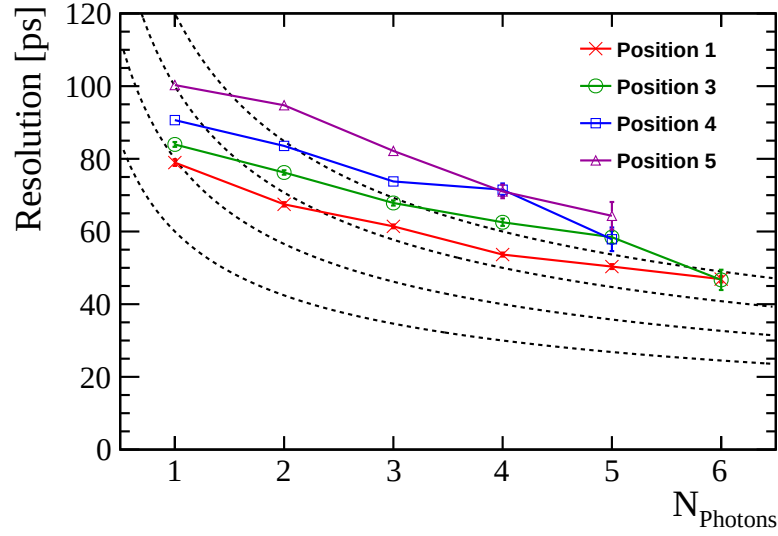


Figure 7.23: Proto-TORCH time resolution as a function of the number of collected photons. The dotted lines indicate the function $\sigma_t / \sqrt{N_{\text{photons}}}$, where $\sigma_t = 60, 80, 100, 120$ ps from the bottom upwards.

from the understanding of the photon emission and efficiencies overlaid. Here the efficiency is the emission spectrum, multiplied by the efficiency of the glue transmission, mirror reflectivity and QE of MCP B (as is shown in Fig. 7.11). This shows that there is a deficit in the photon yields collected at low energies (high wavelengths) compared to the expectation, which could explain the photon counting differences observed in Sec. 7.6.3.

It is possible that the simulation is neglecting a source of inefficiency at low photon energies, towards the red part of the spectrum. One explanation could be that the MCP-PMT efficiencies and NINO responses are dependent of photon/photoelectron energy, whereas the simulation models them as independent except for the QE. Low energy photons are also less likely to be reflected by the mirror. Imperfect modelling of this behaviour in simulation could lead to over-optimistic yields.

Another effect at play is that of the reconstruction of the photon energy. To study the effect of this reconstruction, the photon yield in simulation is studied, once using the ‘true’ energy of the simulated photon and once using the recon-

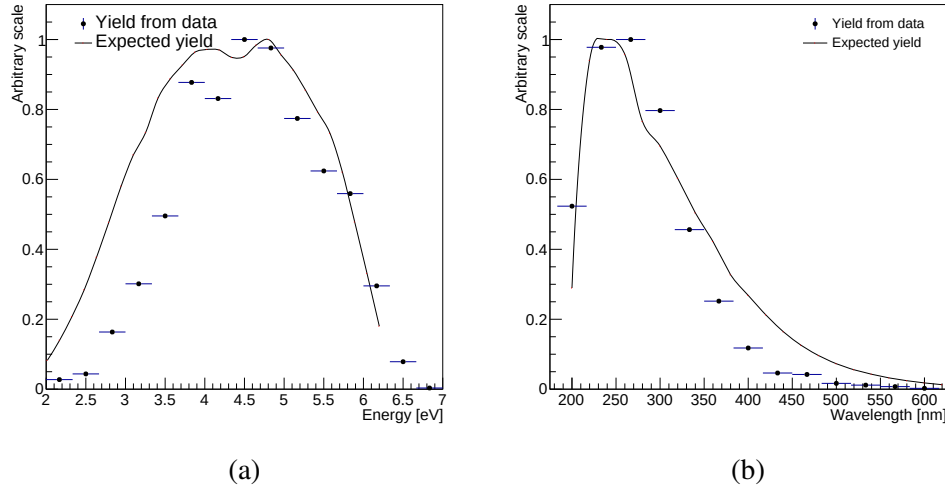


Figure 7.24: The photon yield in data as a function of (a) energy and (b) wavelength. The black line is the expected yield from the efficiency inputs to the simulation and the blue points are the yields from data.

structed energy of the photon. Figure 7.25a shows that the true photon yields and expectation match well as expected. However, in Fig. 7.25b it is observed the yield of photons at high reconstructed energies meets the expectation whilst the yield of photons at low reconstructed energies does not. This could explain part of the deficiency seen in data but is not the full effect. The energy of the photon is reconstructed via its y -pixel on the MCP-PMT plane (higher energies correspond to larger y -pixels), as the y -pixel is proportional to θ_z (the angle that the photon enters the focusing block). A small rotation in the MCP-PMT plane relative to the detector could lead to a shift in the reconstructed photon energies; however this explanation is excluded as the cause of the discrepancy using a simulated sample generated with an additional rotation in the $y - z$ plane of 0.1 mrad. Further studies of this effect will be performed in a future test-beam programme.

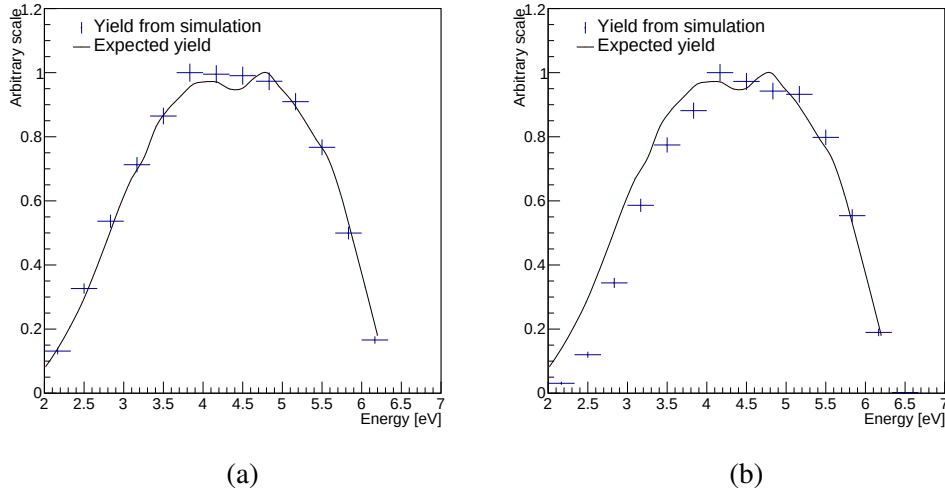


Figure 7.25: The photon yield from simulation as a function of energy, where (a) the true photon energy is used and (b) the reconstructed energy is used. The black line is the expected yield from the efficiency inputs to the simulation and the blue points are the yields from simulation.

7.7 Final results and discussion

Results have been reported from a half-height, full-width LHCb TORCH prototype module, using samples of 8 GeV/c pions and protons from the CERN T9 test beam. The prototype was instrumented with two custom-built MCP-PMTs (out of a possible eleven). Photon yields agree well with the expectations from simulation, indicating that most acceptance effects and sources of efficiency are well understood. Some deviation from expectation is observed for low-energy photons.

Single-photon time resolutions of the prototype have been measured for different beam entry positions in the quartz radiator plate. For entry positions close to the MCP-PMTs, the desired 70 ps time resolution is achieved. For positions further from the MCP-PMTs, and for photon paths that undergo reflections from the side and/or bottom of the radiator, a dependence on the path-length of the photons is expected, however the dependence seen in data is worse than

predicted by simulation. This performance should be improved by future calibrations of the MCP-PMT readout electronics. The measurement of the time resolution of the charged track as a function of number of photon hits shows that the desired $1/\sqrt{N_{\text{photon}}}$ dependence of the resolution is not quite yet achieved. This could be due to correlations of closely spaced hits of photons within the MCP-PMTs that would degrade the expected performance. This will be alleviated when more MCP-PMTs and thus more photons are collected per charged hadron.

In November 2022, another Proto-TORCH test beam was performed, collecting data with 7 MCP-PMTs. Large yields of data taken at a number of positions on the plate will allow further measurements of the photon yields and time resolutions. More data taken at positions in the centre of the plate will result in better understanding of the behaviour as a function of path length and number of side/bottom reflections. A range of beam energies were used in data-taking, which will allow for the assessment of the performance of the time-of-flight PID separation for pions and protons across the momentum range 3–15 GeV/ c . In addition, special runs which allow for independent kaon discrimination in the Cherenkov counters will allow for the PID separation between pions and kaons to be studied.

The prospects for TORCH in LHCb Upgrade II rely on the development of MCP-PMTs with higher granularity, and procuring many large radiator plates with a high-quality surface finish [63]. The planned location of TORCH within LHCb Upgrade II is between the last tracking system and in front of RICH2; in this location a compromise between the distance from the PV (which is desired to be maximal) and the total area that must be covered (which increases with distance, and increases the cost) is optimised. Work is underway to design a mechanical frame that could be installed within the LHCb cavern. Future developments in fast timing chips such as Pico-TDC chipsets [96] will help to

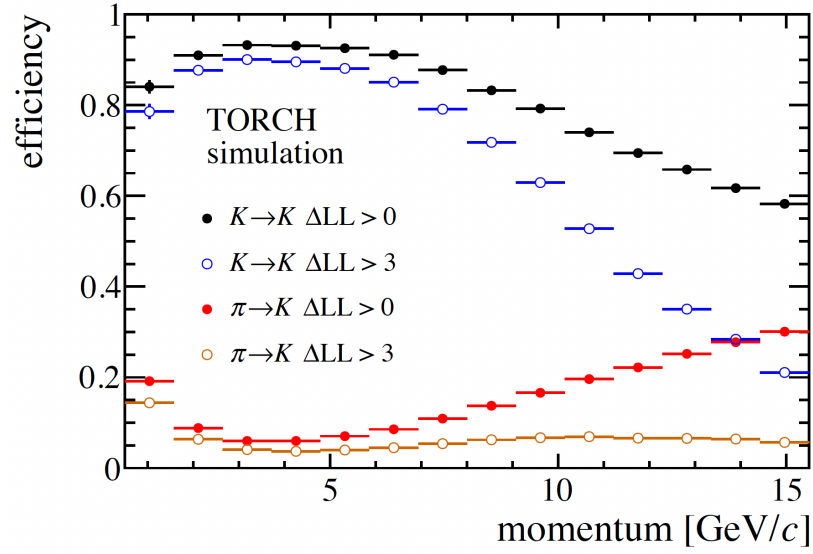


Figure 7.26: The predicted PID performance of TORCH in LHCb Upgrade II conditions, for two different ΔLL selections (the difference in log-likelihood between kaon and pion mass hypotheses). Taken from [63].

improve the time resolution. The current reconstruction algorithm is also being reconfigured to ensure that it can perform accurately in the predicted Upgrade II luminosity. The predicted PID performance for kaon/pions over the momentum range 1 – 15 GeV/ c is shown in Fig. 7.26 [63].

8 | Conclusions

8.1 Measurement of charm-mixing parameters

A study of promptly produced $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays has been performed using data collected at LHCb during 2015–2018 to measure the charm mixing and hadronic parameters. The favoured RS $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ decay provides a control channel for the measurement of the features of the WS $D^0 \rightarrow K^+ \pi^- \pi^+ \pi^-$ decay, with an amplitude consisting of the doubly suppressed tree-level decay, the neutral charm mixing followed by a favoured decay, and the interference between the two. To reduce the high levels of background in the WS sample, selection criteria are imposed, including use of multi-variate analysis techniques. Specific backgrounds which could bias the WS/RS ratio have been studied in detail, in particular background of secondary events and double mis-ID candidates, and systematics associated to the residual contamination were assigned.

The signal yields were isolated through a simultaneous mass fit to the WS and RS samples per flavour, time and phase-space bin. Differences in the detection efficiency of WS and RS events were studied as a function of time and phase-space, with trigger requirements causing biases of up to 10% of the true mixing effects.

The effective mixing parameters are measured integrated over phase-space and in four bins of phase-space. The phase-space integrated results are

$$\begin{aligned} r_D^{K3\pi} &= (5.489 \pm 0.043) \times 10^{-2}, \\ (\kappa y')_D^{K3\pi} &= (-2.68 \pm 0.78) \times 10^{-3}, \\ x^2 + y^2 &= (7.2 \pm 3.2) \times 10^{-5}. \end{aligned}$$

The measured parameters are interpreted as the following hadronic param-

eters (integrated over phase space) using external inputs of x and y :

$$\begin{aligned}\kappa_D^{K3\pi} &= 0.44_{-0.06}^{+0.07}, \\ \delta_D^{K3\pi} &= (162_{-17}^{+18})^\circ.\end{aligned}$$

This is a 60% improvement on the previous combination of results from LHCb, BESIII and CLEO-c data. This thesis presented the first measurement of the binned effective mixing parameters in $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays, and the results were interpreted in terms of κ_i and δ_i per phase-space bin i . These results are consistent with those from charm-threshold measurements performed using BESIII and CLEO-c data. The new combination has a factor of two improvement in precision relative to the previous combination from BESIII and CLEO-c data across κ and δ in three out of four bins.

The precision on $x^2 + y^2$ is increased by a factor of 2.25 relative to the previous measurement of $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays at LHCb [32]. The phase-space binned measurements of $x^2 + y^2$ and $(\kappa y')_i$ are interpreted in terms of the charm-mixing parameters x and y using external inputs of κ_i and δ_i , yielding $x = (0.48_{-0.13}^{+0.14})\%$ and $y = (0.48_{-0.13}^{+0.11})\%$. The results of this analysis will be published in a paper shortly.

8.1.1 Outlook

The phase-space binned measurements of κ_i and δ_i are important inputs to current and future measurements of the CKM angle γ using $B \rightarrow DK$ decays at LHCb, and also at Belle II. The results of this thesis, combined with those previously measured at BESIII and CLEO-c, will both improve the precision and give confidence in the measurement of γ . The new combination can be used to refine the current measurement of γ using the $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ final state [29].

The measurement of $x^2 + y^2$ presented here will contribute to global fits of

the charm-mixing parameters x and y , whose precision are dominated by measurements of $D^0 \rightarrow K_S^0 \pi^\pm \pi^\mp$ decays [24] and $D^0 \rightarrow K^\pm K^\mp, \pi^\pm \pi^\mp$ decays [97] at LHCb, respectively. Additionally, the mixing measurement can be extended to search for CP violation (CPV). This would be done by measuring two R ratios, split by D^0 flavour or kaon charge; any significant difference between the two would be an indication of CPV. Here, the precision on the CPV parameters is expected to be better than for x and y as many of the systematics are in common for D^0 and $\bar{D}^0$ decays. As the decays are tree-level decays, Standard Model (SM) CPV is expected to be very small so any CPV at the current sensitivity would be an indication of physics effects from beyond the SM.

During 2023–2025, LHCb Run 3 aims to collect an integrated luminosity of around 15 fb^{-1} . The larger data set, coupled with improvements in the trigger efficiency (which could have up to a factor of two improvement), gives the potential for over a factor of two increase in statistical precision on the charm-mixing measurements presented here. As well as pursuing $D^0 \rightarrow K^\pm \pi^\mp \pi^\pm \pi^\mp$ decays, this method could be extended to other multi-body charm decays of this sort, such as $D^0 \rightarrow K \pi \pi^0$. As the instantaneous luminosity increases, tight trigger requirements will be required in order to reduce the level of background recorded. This can introduce biases as a function of decay time and phase space, as detailed in this thesis. Finding ways to reduce these biases will be essential during future data collection.

8.2 Performance of a TORCH prototype

The performance of a TORCH prototype was analysed using data collected during a beam test at the CERN PS in October 2018. Three metrics were chosen to assess the performance: the number of photons detected per hadron, relative to the expected yield; the time resolution of a single photon; and the time reso-

lution of the hadron track, using information from all photons originating from the hadron.

In a single fully instrumented TORCH module, approximately 30 detected photons are required to meet the desired hadron-track resolution. This is equivalent to 5.5 photons in Proto-TORCH, which had only 2 out of the 11 photon detectors installed. Assessing the photon yields from paths that only travel directly to the photon detectors, 0.7–2.6 photons were collected per hadron track. The yield is 83% of the expected yield from simulation.

The time resolution of a single photon met the desired 70ps target for photons originating close to the photon detectors. However some decrease in performance is observed as photons travel longer paths in the quartz radiator. This shows that some performance gains are required which should be solved by future electronics chips and more advanced calibrations. The timing information from all photons detected per hadron is combined, and a time resolution per hadron track is deduced. At present, the time resolution does not quite meet the required dependence of $1/\sqrt{N_{\text{photons}}}$. The results presented in this thesis have been published in the journal Nuclear Instrumental Methods A [1].

8.2.1 Outlook

In a more recent testbeam, Proto-TORCH was instrumented with seven MCP-PMTs. This allows for future analysis with larger photon yields, and the results of which will be crucial to understanding the predicted performance of TORCH. Data were collected with a range of beam energies, with some data-taking including pions, kaons and protons, allowing for a measurement of the PID performance across a range of momenta.

For TORCH to be installed in LHCb Upgrade II, improvements in the MCP-PMTs and readout electronics must be made. Additionally, these components must be manufactured in bulk (the entire detector as currently planned would

require 200 MCP-PMTs) and high-quality radiators must be procured. The research and development of TORCH is ongoing.

A | Mass fits per time bin

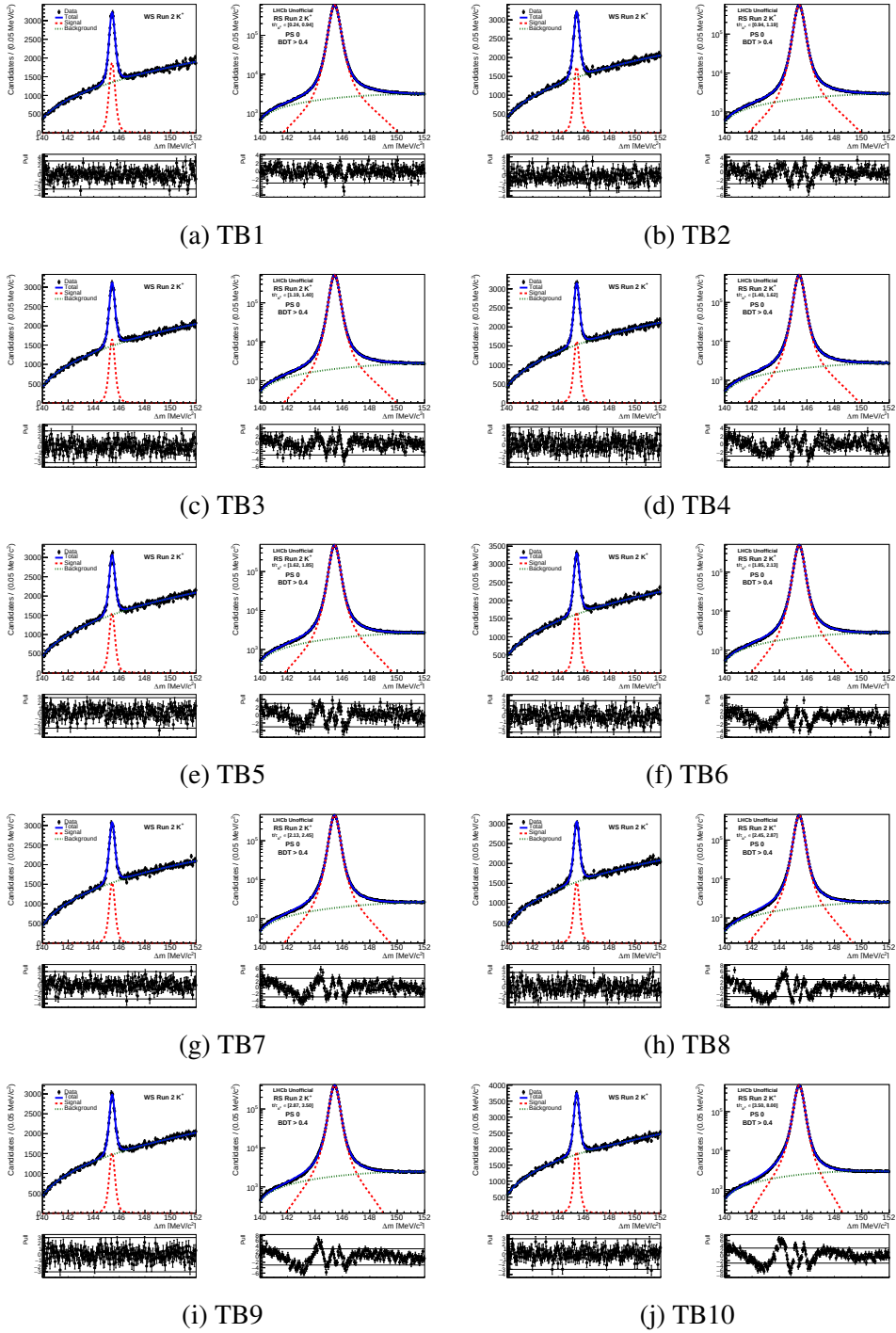


Figure A.1: The Δm distribution of Run 2 (left) WS (K^+ , D^0) and (right) RS (K^+ , $\bar{D}^0$) events per time bin. The mass fit PDFs are superimposed.

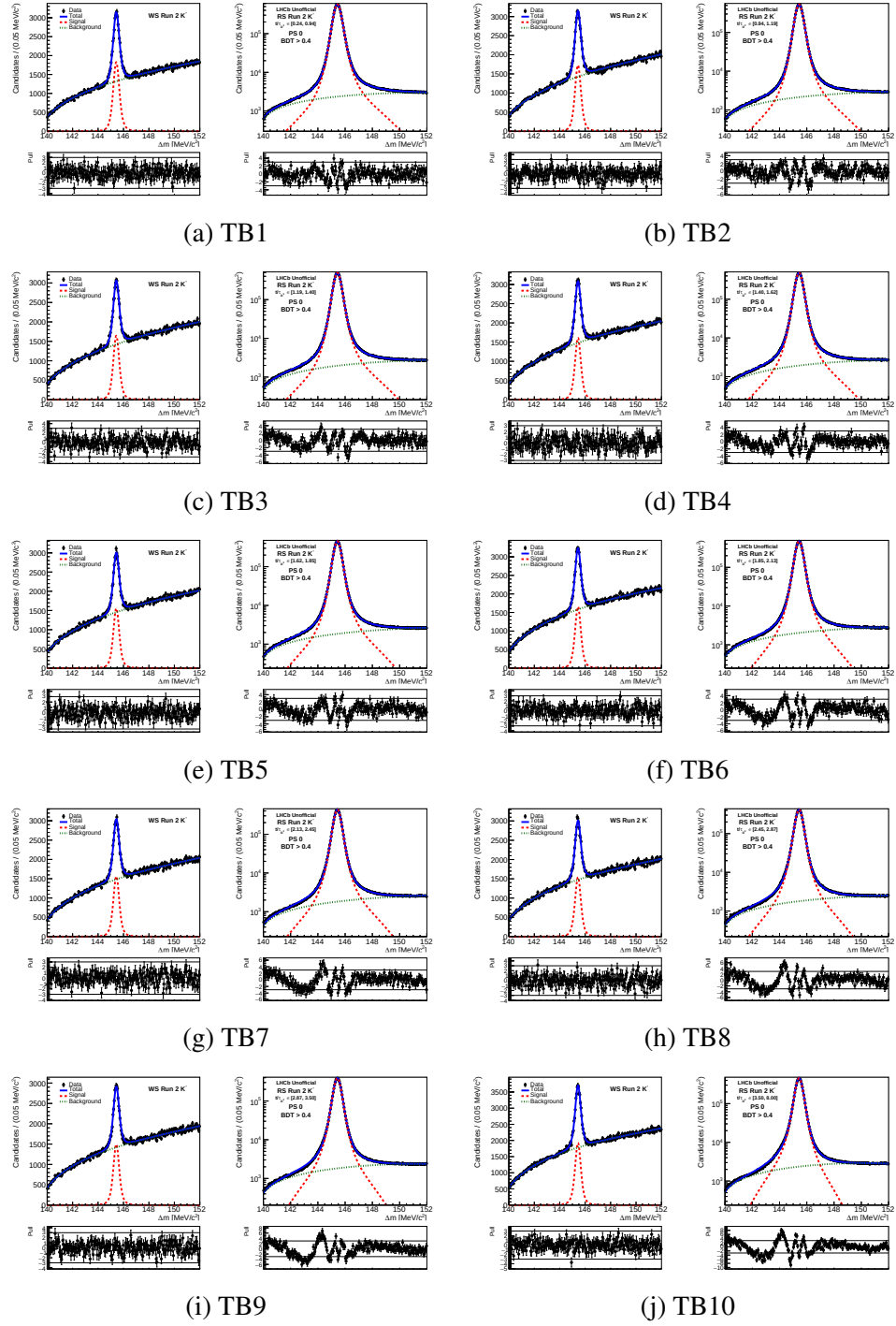


Figure A.2: The Δm distribution of Run 2 (left) WS (K^- , $\bar{D}^0$) and (right) RS (K^- , D^0) events per time bin. The mass fit PDFs are superimposed.

B | Efficiency assumptions

In this section, the simplification of the efficiency factor presented in Eqn. 4.8, namely

$$\sqrt{\frac{\epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{RS} \times \epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{RS}}{\epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{WS} \times \epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{WS}}} = \frac{\epsilon^{RS}}{\epsilon^{WS}}, \quad (\text{B.1})$$

is examined.

It is indeed possible that $\epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{RS} \neq \epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{RS}$, due to potential differences in the detection of oppositely charged kaons. If there is a global (phase-space and time integrated) offset of the form $\epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{RS} = c_{global} \epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{RS}$, then we expect c_{global} to be the same for WS decays too and thus c_{global} cancels in the ratio of RS to WS efficiencies. A phase-space dependent difference in the efficiencies is ruled out at current sensitivity by comparing the distribution of RS D^0 and $\bar{D}^0$ candidates across phase-space. The CM variables are used to parameterise the phase-space and distributions of D^0 and $\bar{D}^0$ candidates are shown in Fig. B.1. There is no difference between the samples and thus the phase-space dependent efficiencies are equal.

Further, the decay-time and phase-space dependent correlations are compared for samples split by kaon charge, probing the equality of

$$\frac{\epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{RS}}{\epsilon_{K^-\pi^+\pi^-\pi^+\pi_s}^{WS}} \quad \text{and} \quad \frac{\epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{RS}}{\epsilon_{K^+\pi^-\pi^+\pi^-\pi_s}^{WS}}$$

up to a constant factor. Figure B.2 shows the consistent behaviour between samples split by kaon charge, providing further justification that the simplification in Eqn. B.1 is reasonable.

An additional assumption is that the efficiency of the total final state is factorisable into a component associated to the slow pion charge and a component associated with the D^0 final state and slow pion (independent of its charge).

A sample of PGun (‘WS as RS’) is generated such that a RS D^* decay ($D^{*+} \rightarrow D^0 \pi^+$) has a D^0 which was forced to decay as a WS D^0 decay. This alters the number of same sign pions in the total final-state. The efficiency correlations of this sample relative to the CF sample are studied to see if it behaved like the WS sample or not. No significant differences are observed between the WS and the ‘WS as RS’ sample, demonstrating that the charge of the pion does not affect the efficiency. As an additional check, the kinematics of the slow pion are compared between D^0 and $\bar{D}^0$ candidates in CF Full MC samples. If the kinematics differ, this would show that efficiency of the final state and the slow-pion charge are not independent. However, the distributions of the slow pion p_x, p_y, p_z are consistent.

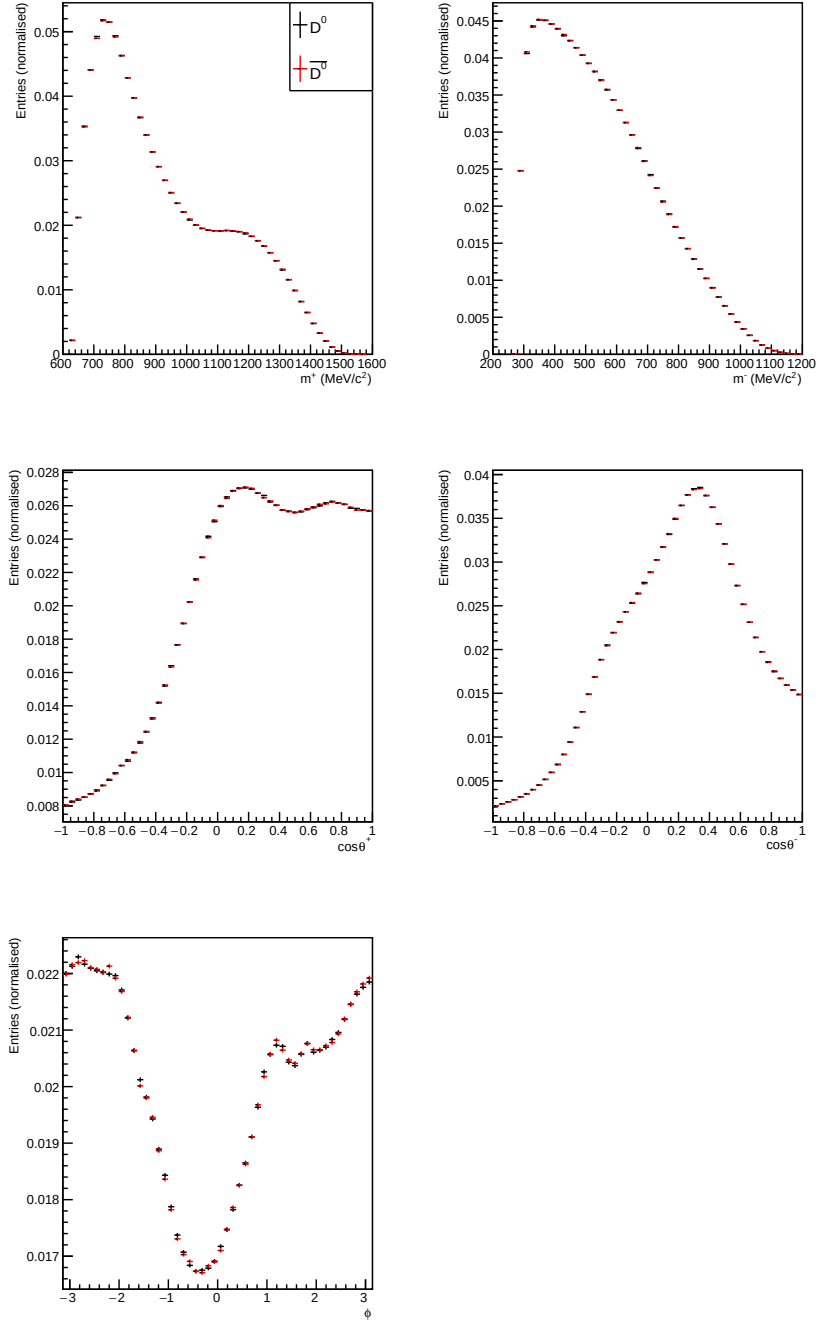


Figure B.1: The distributions of RS D^0 and $\bar{D}^0$ candidates (all 2018 *MagUp* data) across phase-space, parameterised by the CM variables (m^+ , m^- , $\cos \theta^+$, $\cos \theta^-$, ϕ). As ϕ is a CP -odd quantity, for the purposes of this comparison for $\bar{D}^0$ candidates $\phi = -\phi$ has been plotted.

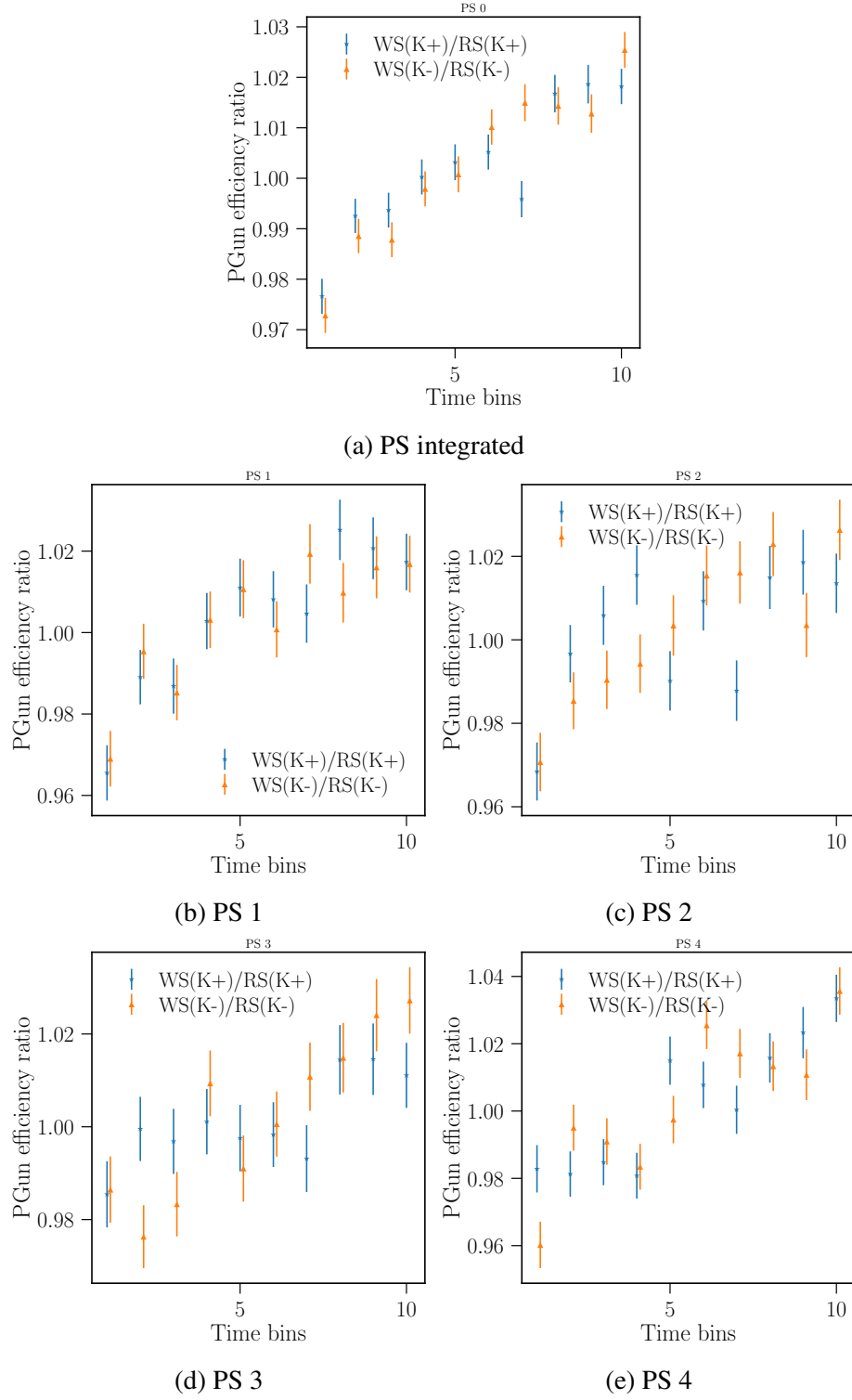


Figure B.2: The efficiency correlation function (DCS/CF) measured in the PGun sample, split by kaon charge for (top) phase-space integrated sample and (lower) phase-space binned samples.

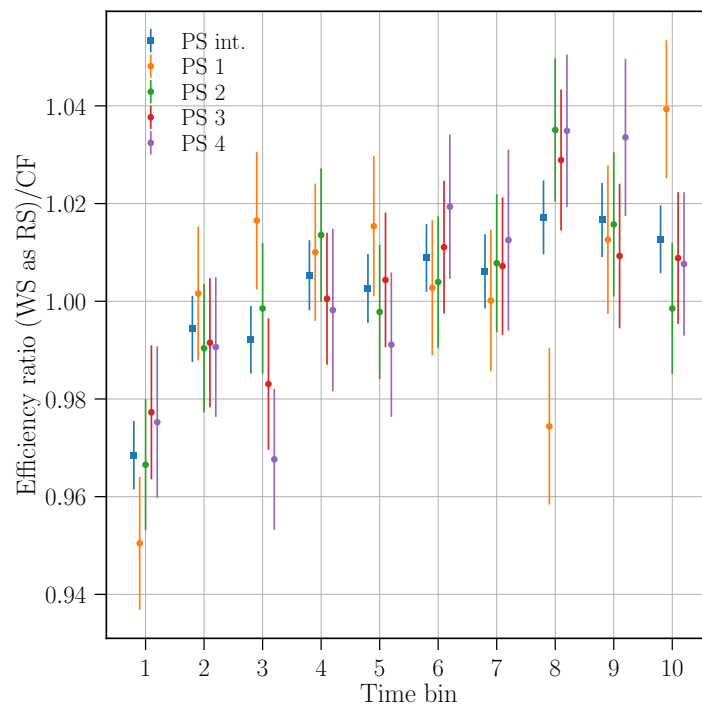


Figure B.3: The efficiency correlations of the ‘WS as RS’/CF PGun sample.

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